Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/21-1.1.2.4-e-x-^m-a+b-x^2-^p-c+d-x^2-^q

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1156]. This is test number [21].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1156)	0.00 (0)
Mathematica	100.00 (1156)	0.00 (0)
Maple	90.05 (1041)	9.95 (115)
Fricas	82.35 (952)	17.65 (204)
Giac	71.11 (822)	28.89 (334)
Mupad	63.15 (730)	36.85 (426)
Maxima	60.90 (704)	39.10 (452)
Sympy	54.58 (631)	45.42 (525)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

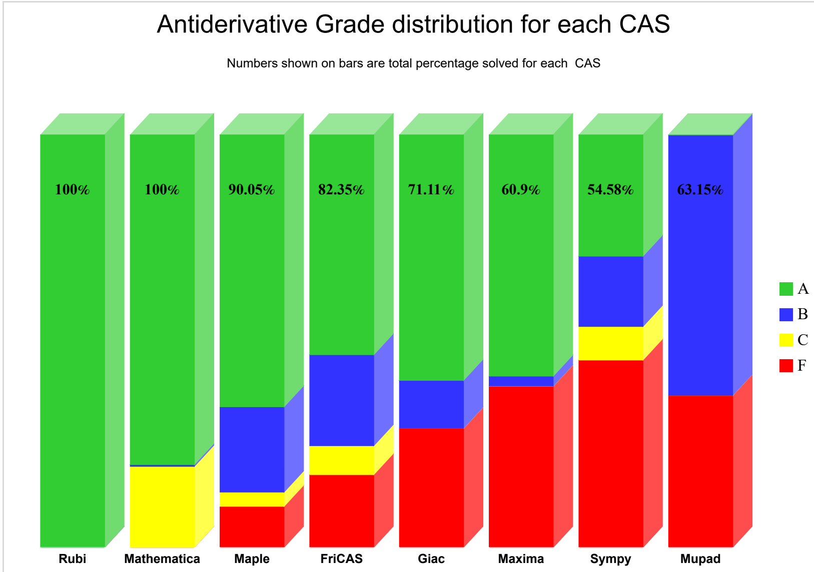
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

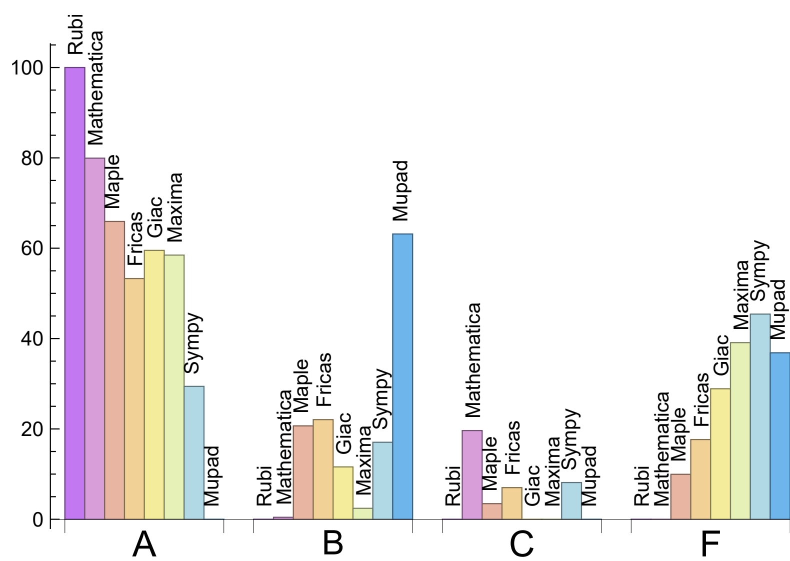
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	79.93	0.43	19.64	0.00
Maple	65.92	20.67	3.46	9.95
Giac	59.52	11.59	0.00	28.89
Maxima	58.48	2.42	0.00	39.10
Fricas	53.29	22.06	7.01	17.65
Sympy	29.41	17.04	8.13	45.42
Mupad	N/A	63.15	0.00	36.85

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	115	100.00 %	0.00 %	0.00 %
Fricas	204	44.12 %	44.12 %	11.76 %
Giac	334	94.91 %	0.00 %	5.09 %
Maxima	452	86.50 %	0.00 %	13.50 %
Sympy	525	64.19 %	33.52 %	2.29 %
Mupad	426	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

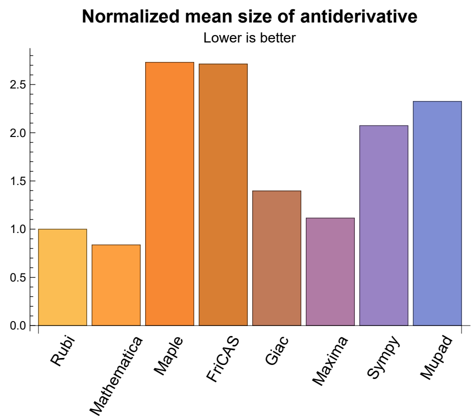
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

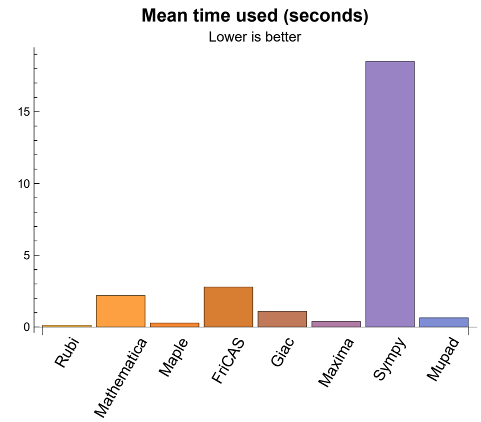
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	179.48	1.00	124.00	1.00
Mathematica	2.19	125.58	0.84	109.00	0.87
Maple	0.28	508.58	2.73	161.00	1.17
Maxima	0.38	167.67	1.11	119.50	1.03
Fricas	2.78	493.86	2.71	205.50	1.82
Sympy	18.49	246.97	2.07	134.00	1.31
Giac	1.09	214.01	1.40	135.00	1.14
Mupad	0.64	447.91	2.33	119.00	1.12

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {337, 338, 1014, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1041, 1048, 1052, 1067, 1068, 1087, 1088, 1142}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

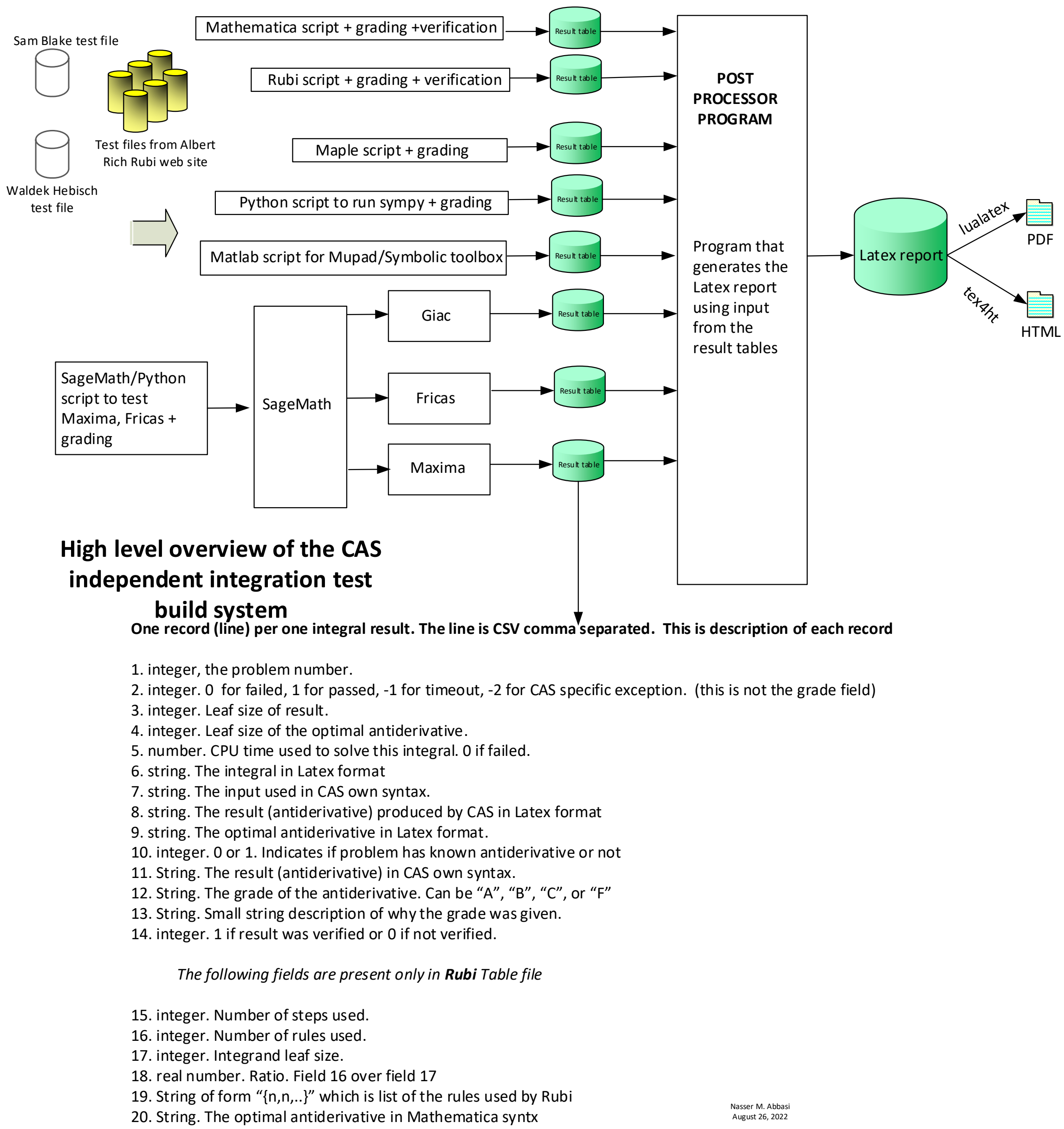
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55,
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B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 934, 935, 936, 937, 938, 939, 942, 944, 945, 946, 947, 948, 949, 954, 955, 956, 957, 958, 959, 964, 965, 966, 967, 969, 970, 971, 972, 973, 974, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 993, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1031, 1032, 1033, 1034, 1035, 1036, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1050, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1069, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1089, 1093, 1094, 1095, 1096, 1097, 1098, 1105, 1106, 1107, 1108, 1109, 1116, 1117, 1118, 1119, 1120, 1126, 1127, 1128, 1129, 1130, 1131, 1139, 1140, 1141, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

B grade: { 31, 47, 108, 1007, 1142 }

C grade: { 331, 332, 337, 338, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 943, 950, 951, 952, 953, 960, 961, 962, 963, 968, 975, 976, 977, 978, 979, 992, 994, 1004, 1005, 1006, 1014, 1015, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1040, 1041, 1048, 1049, 1051, 1052, 1067, 1068, 1070, 1071, 1072, 1087, 1088, 1090, 1091, 1092, 1099, 1100, 1101, 1102, 1103, 1104, 1110, 1111, 1112, 1113, 1114, 1115, 1121, 1122, 1123, 1124, 1125, 1132, 1133, 1134, 1135, 1136, 1137, 1138 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 327, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 534, 536, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 551, 553, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 820, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 881, 940, 941, 942, 943, 950, 951, 952, 953, 960, 961, 962, 963, 964, 965, 966, 967, 968, 975, 976, 977, 978, 979, 982, 984, 985, 986, 987, 988, 989, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1007, 1096, 1097, 1098, 1107, 1108, 1109, 1118, 1119, 1120, 1128, 1129, 1130, 1131 }

B grade: { 29, 31, 47, 108, 118, 319, 325, 326, 514, 516, 533, 535, 537, 549, 550, 552, 554, 591, 604, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 815, 816, 817, 818, 819, 821, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 944, 945, 946, 947, 948, 949, 954, 955, 956, 957, 958, 959, 969, 970, 971, 972, 973, 974, 980, 981, 983, 990, 991 }

C grade: { 1006, 1008, 1009, 1010, 1012, 1016, 1019, 1020, 1021, 1022, 1031, 1032, 1033, 1034, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1050, 1053, 1054, 1061, 1062, 1063, 1064, 1065, 1066, 1069, 1073, 1074, 1081, 1082, 1083, 1084, 1085, 1086, 1089 }

F grade: { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 1011, 1013, 1014, 1015, 1017, 1018, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1035, 1036, 1037, 1038, 1040, 1041, 1048, 1049, 1051, 1052, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1070, 1071, 1072, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 254, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 307, 309, 319, 320, 321, 325, 326, 327, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 965, 967, 1008, 1009, 1010, 1019, 1020, 1021, 1022, 1031, 1032, 1033, 1034, 1042, 1043, 1044, 1045, 1061, 1062, 1063, 1064, 1081, 1082, 1083, 1084, 1095, 1096, 1097, 1098, 1105, 1106, 1107, 1108, 1109, 1116, 1117, 1118, 1119, 1120, 1126, 1127, 1128, 1129, 1130, 1131 }

B grade: { 31, 47, 108, 251, 253, 255, 256, 259, 302, 304, 305, 306, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 533, 552, 554, 662, 1093, 1094 }

C grade: { }

F grade: { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 966, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1099, 1100, 1101, 1102, 1103, 1104, 1110, 1111, 1112, 1113, 1114, 1115, 1121, 1122, 1123, 1124, 1125, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 188, 189, 190, 192, 195, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 250, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 282, 286, 288, 289, 290, 291, 292, 293, 295, 297, 299, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 706, 707, 708, 709, 715, 716, 731, 732, 738, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 763, 765, 766, 934, 935, 936, 938, 939, 944, 945, 946, 947, 949, 954, 955, 956, 957, 958, 959, 964, 965, 966, 969, 970, 973, 974, 982, 984, 986, 995, 997, 999, 1001, 1002, 1003, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1042, 1043, 1044, 1045, 1046, 1047, 1057, 1058, 1061, 1062, 1063, 1081, 1082, 1083, 1084, 1085, 1086, 1096, 1097, 1098, 1107, 1108, 1109, 1118, 1119, 1120, 1128, 1129, 1130, 1131 }

B grade: { 31, 47, 95, 96, 108, 184, 187, 191, 193, 194, 196, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 278, 281, 283, 284, 285, 287, 294, 296, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 484, 488, 489, 490, 491, 493, 542, 553, 616, 625, 627, 703, 704, 705, 710, 711, 712, 713, 714, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 733, 734, 735, 736, 737, 739, 759, 760, 761, 762, 764, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 937, 948, 967, 971, 972, 980, 981, 983, 985, 987, 988, 989, 990, 991, 1016, 1031, 1032, 1033, 1034, 1035, 1036, 1039, 1050, 1053, 1054, 1055, 1056, 1059, 1060, 1064, 1065, 1066, 1069, 1073, 1075, 1076, 1077, 1078, 1079, 1080, 1089, 1105, 1106, 1126, 1127 }

C grade: { 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 1074 }

F grade: { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 481, 483, 485, 486, 487, 492, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 942, 943, 950, 951, 952, 953, 960, 961, 962, 963, 968, 975, 976, 977, 978, 979, 992, 993, 994, 996, 998, 1000, 1004, 1005, 1006, 1014, 1015, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1040, 1041, 1048, 1049, 1051, 1052, 1067, 1068, 1070, 1071, 1072, 1087, 1088, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 57, 59, 60, 61, 63, 65, 67, 69, 71, 72, 73, 75, 76, 77, 78, 79, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 106, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 133, 134, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 190, 191, 192, 194, 195, 196, 197, 198, 200, 201, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 261, 262, 263, 264, 265, 267, 268, 269, 272, 274, 276, 278, 281, 283, 285, 287, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 378, 380, 386, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 428, 430, 436, 445, 447, 449, 455, 457, 505, 509, 510, 511, 512, 513, 514, 518, 520, 527, 529, 530, 544, 546, 548, 555, 557, 558, 559, 560, 561, 562, 563, 564, 565, 569, 570, 571, 572, 573, 574, 575, 576, 577, 580, 582, 592, 601, 602, 603, 607, 608, 618, 620, 622, 629, 631, 633, 635, 640, 641, 642, 643, 644, 645, 653, 665, 677, 679, 681, 686, 688, 690, 695, 697, 699, 705, 706, 716, 718, 725, 727, 967, 1010, 1042, 1043, 1044, 1045, 1107, 1118 }

B grade: { 29, 31, 47, 56, 58, 62, 64, 66, 68, 70, 74, 80, 84, 86, 101, 104, 105, 108, 116, 130, 132, 135, 137, 162, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 193, 199, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 230, 231, 232, 233, 235, 243, 245, 251, 253, 255, 266, 270, 271, 273, 275, 277, 279, 280, 282, 284, 286, 288, 290, 292, 302, 304, 311, 313, 319, 320, 321, 325, 326, 327, 375, 377, 379, 381, 385, 387, 427, 429, 431, 435, 437, 441, 442, 443, 444, 446, 448, 454, 456, 458, 504, 506, 507, 508, 515, 516, 517, 519, 521, 522, 523, 524, 525, 526, 528, 531, 532, 533, 534, 535, 536, 538, 540, 541, 542, 543, 545, 547, 549, 550, 551, 552, 553, 556, 566, 567, 568, 578, 579, 581, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 604, 605, 606, 609, 610, 611, 612, 614, 615, 616, 617, 619, 621, 623, 624, 625, 627, 628, 630, 632, 634, 636, 637, 638, 639, 646, 647, 649, 651, 661, 663, 1096, 1119, 1128, 1129 }

C grade: { 107, 322, 323, 324, 328, 331, 332, 333, 334, 339, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1105, 1106, 1111, 1112, 1113, 1114, 1116, 1117, 1121, 1122, 1123, 1124, 1127, 1134, 1135, 1136 }

F grade: { 46, 48, 49, 50, 51, 52, 53, 54, 55, 228, 234, 236, 237, 238, 239, 240, 241, 242, 244, 246, 247, 248, 249, 250, 252, 254, 256, 257, 258, 259, 260, 289, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 305, 306, 307, 308, 309, 310, 312, 314, 315, 316, 317, 318, 329, 330, 335, 336, 337, 338, 340, 341, 342, 376, 382, 383, 384, 388, 389, 390, 425, 426, 432, 433, 434, 438, 439, 440, 450, 451, 452, 453, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 537, 539, 554, 613, 626, 648, 650, 652, 654, 655, 656, 657, 658, 659, 660, 662, 664, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 678, 680, 682, 683, 684, 685, 687, 689, 691, 692, 693, 694, 696, 698, 700, 701, 702, 703, 704, 707, 708, 709, 710, 711, 712, 713, 714, 715, 717, 719, 720, 721, 722, 723, 724, 726, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 815, 831, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1097, 1098, 1104, 1108, 1109, 1110, 1115, 1120, 1125, 1126, 1130, 1131, 1132, 1133, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 309, 310, 312, 313, 314, 316, 318, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 483, 484, 485, 486, 487, 488, 493, 494, 495, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 514, 516, 518, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 531, 533, 535, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 550, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 567, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 580, 582, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 647, 648, 649, 650, 651, 652, 653, 654, 655, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 671, 672, 673, 674, 675, 677, 679, 681, 683, 686, 688, 690, 692, 695, 697, 699, 701, 703, 704, 705, 706, 707, 710, 711, 714, 715, 716, 717, 718, 719, 720, 721, 723, 725, 727, 729, 732, 734, 736, 738, 741, 743, 745, 747, 750, 752, 754, 756, 759, 761, 763, 765, 768, 770, 772, 774, 775, 777, 779, 781, 783, 934, 935, 936, 944, 945, 946, 954, 955, 956, 965, 967, 969, 970, 971, 980, 990, 991, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1031, 1032, 1033, 1034, 1035, 1036, 1042, 1043, 1044, 1045, 1046, 1047, 1061, 1062, 1063, 1064, 1065, 1066, 1081, 1082, 1083, 1084, 1085, 1086 }

B grade: { 31, 47, 108, 129, 162, 177, 179, 249, 251, 257, 259, 281, 283, 296, 306, 308, 311, 315, 317, 319, 320, 321, 325, 326, 327, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 481, 489, 490, 491, 492, 496, 497, 498, 499, 500, 501, 513, 515, 517, 519, 530, 532, 534, 536, 549, 551, 553, 564, 566, 568, 579, 581, 583, 595, 597, 608, 609, 610, 611, 612, 623, 634, 644, 646, 656, 658, 668, 670, 682, 684, 693, 712, 722, 724, 726, 728, 730, 731, 733, 735, 737, 739, 740, 742, 744, 746, 748, 749, 751, 753, 757, 758, 760, 762, 764, 766, 767, 769, 771, 773, 776, 778, 780, 782, 784, 938, 939, 948, 949, 958, 959, 972, 973, 974, 981, 982, 983, 984, 985, 986, 987, 988, 989 }

C grade: { }

F grade: { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 676, 678, 680, 685, 687, 689, 691, 694, 696, 698, 700, 702, 708, 709, 713, 755, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 937, 940, 941, 942, 943, 947, 950, 951, 952, 953, 957, 960, 961, 962, 963, 964, 966, 968, 975, 976, 977, 978, 979, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1014, 1015, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1039, 1040, 1041, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 506, 508, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 523, 525, 527, 528, 529, 531, 533, 534, 535, 536, 537, 538, 540, 542, 544, 545, 546, 548, 550, 552, 553, 554, 555, 557, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 572, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 586, 588, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 610, 611, 612, 614, 616, 618, 620, 622, 624, 625, 627, 629, 631, 633, 635, 637, 639, 641, 642, 643, 645, 646, 647, 649, 651, 653, 655, 656, 657, 658, 659, 661, 663, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 677, 679, 681, 683, 686, 688, 690, 692, 695, 697, 699, 701, 703, 704, 705, 706, 707, 714, 716, 718, 720, 723, 725, 727, 729, 732, 734, 736, 738, 741, 743, 745, 747, 750, 752, 754, 756, 759, 761, 763, 765, 768, 770, 772, 774, 777, 779, 781, 783, 934, 935, 936, 937, 938, 939, 965, 967, 969, 970, 971, 972, 973, 974, 982, 984, 985, 986, 987, 988, 989, 990, 991, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1031, 1032, 1033, 1034, 1035, 1036, 1042, 1043, 1044, 1045, 1046, 1047, 1061, 1062, 1063, 1064, 1065, 1066, 1081, 1082, 1083, 1084, 1085, 1086, 1096, 1097, 1098, 1107, 1108, 1109, 1118, 1119, 1120, 1128, 1129, 1130, 1131 }

C grade: { }

F grade: { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 505, 507, 509, 522, 524, 526, 530, 532, 539, 541, 543, 547, 549, 551, 556, 558, 569, 571, 573, 585, 587, 589, 605, 606, 607, 608, 609, 613, 615, 617, 619, 621, 623, 626, 628, 630, 632, 634, 636, 638, 640, 644, 648, 650, 652, 654, 660, 662, 664, 676, 678, 680, 682, 684, 685, 687, 689, 691, 693, 694, 696, 698, 700, 702, 708, 709, 710, 711, 712, 713, 715, 717, 719, 721, 722, 724, 726, 728, 730, 731, 733, 735, 737, 739, 740, 742, 744, 746, 748, 749, 751, 753, 755, 757, 758, 760, 762, 764, 766, 767, 769, 771, 773, 775, 776, 778, 780, 782, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 966, 968, 975, 976, 977, 978, 979, 980, 981, 983, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1014, 1015, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1039, 1040, 1041, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.012	0.005	0.089	0.304	0.881	0.007	0.720	0.111

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.022	0.007	0.051	0.300	0.876	0.006	0.959	0.021

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.009	0.005	0.091	0.293	0.825	0.006	1.455	0.021

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	28	25	27	30	26
N.S.	1	1.00	1.00	0.97	0.97	0.86	0.93	1.03	0.90
time (sec)	N/A	0.015	0.007	0.035	0.292	0.724	0.031	1.234	0.035

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	20	23	24
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.77	0.88	0.92
time (sec)	N/A	0.011	0.007	0.015	0.277	0.650	0.030	0.715	0.044

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	28	30	26	42	25
N.S.	1	1.00	1.00	0.90	0.97	1.03	0.90	1.45	0.86
time (sec)	N/A	0.015	0.009	0.020	0.289	0.792	0.073	0.839	0.034

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	27	25	26	29	27	28	26
N.S.	1	1.00	1.04	0.96	1.00	1.12	1.04	1.08	1.00
time (sec)	N/A	0.011	0.010	0.018	0.290	1.180	0.087	1.037	0.018

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	26	30	31	29	39	29
N.S.	1	1.00	1.07	0.90	1.03	1.07	1.00	1.34	1.00
time (sec)	N/A	0.016	0.013	0.020	0.314	1.182	0.172	0.804	0.040

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	28	29	29	32	31	29
N.S.	1	1.00	1.06	0.90	0.94	0.94	1.03	1.00	0.94
time (sec)	N/A	0.011	0.008	0.018	0.292	1.049	0.175	1.198	0.020

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	28	29	29	32	31	30
N.S.	1	1.00	1.06	0.85	0.88	0.88	0.97	0.94	0.91
time (sec)	N/A	0.015	0.008	0.020	0.289	1.355	0.246	1.091	0.020

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.025	0.007	0.093	0.357	0.952	0.010	1.053	0.042

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	51	51	53	53	51
N.S.	1	1.00	1.21	1.24	1.21	1.21	1.26	1.26	1.21
time (sec)	N/A	0.047	0.009	0.075	0.394	1.497	0.011	0.822	0.024

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.015	0.007	0.103	0.400	3.140	0.010	0.834	0.023

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	52	49	49	53	48
N.S.	1	1.00	1.19	1.19	1.21	1.14	1.14	1.23	1.12
time (sec)	N/A	0.022	0.012	0.060	0.291	1.287	0.045	0.955	0.021

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	48	53	48	48	48
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00
time (sec)	N/A	0.018	0.012	0.060	0.279	0.834	0.042	1.321	0.026

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	48	52	54	48	70	48
N.S.	1	1.00	0.96	0.94	1.02	1.06	0.94	1.37	0.94
time (sec)	N/A	0.032	0.020	0.089	0.294	1.043	0.097	1.182	0.024

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	50	52	51	50	50
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04
time (sec)	N/A	0.020	0.015	0.065	0.283	0.745	0.106	1.028	0.037

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	46	54	55	51	72	51
N.S.	1	1.00	0.98	0.90	1.06	1.08	1.00	1.41	1.00
time (sec)	N/A	0.029	0.019	0.062	0.282	0.635	0.265	0.621	0.046

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	51	53	54	53	50
N.S.	1	1.00	1.00	0.94	1.06	1.10	1.12	1.10	1.04
time (sec)	N/A	0.019	0.015	0.063	0.283	0.626	0.305	0.857	0.025

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	46	55	55	56	66	51
N.S.	1	1.00	1.06	0.90	1.08	1.08	1.10	1.29	1.00
time (sec)	N/A	0.026	0.021	0.060	0.284	0.893	0.520	1.024	0.048

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	48	53	53	58	55	52
N.S.	1	1.00	1.06	0.91	1.00	1.00	1.09	1.04	0.98
time (sec)	N/A	0.019	0.013	0.058	0.280	0.725	0.565	1.210	0.019

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	55	48	53	53	58	55	53
N.S.	1	1.00	1.15	1.00	1.10	1.10	1.21	1.15	1.10
time (sec)	N/A	0.021	0.013	0.059	0.300	0.917	0.859	1.127	0.035

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	136	125	107
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.114	0.014	0.099	0.285	0.882	0.017	0.658	0.087

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	138	125	107
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.18	1.07	0.91
time (sec)	N/A	0.061	0.014	0.101	0.298	0.819	0.017	0.525	0.022

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	117	124	119	119	136	125	107
N.S.	1	1.00	0.96	1.02	0.98	0.98	1.11	1.02	0.88
time (sec)	N/A	0.194	0.012	0.100	0.297	0.760	0.019	0.473	0.022

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	136	125	107
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.047	0.012	0.098	0.293	0.591	0.018	0.654	0.022

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	124	119	119	133	124	107
N.S.	1	1.00	1.13	1.31	1.25	1.25	1.40	1.31	1.13
time (sec)	N/A	0.149	0.018	0.096	0.281	0.669	0.019	0.643	0.024

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	136	125	107
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.047	0.012	0.117	0.267	0.793	0.017	0.568	0.022

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	114	124	118	118	131	123	106
N.S.	1	1.00	1.70	1.85	1.76	1.76	1.96	1.84	1.58
time (sec)	N/A	0.100	0.012	0.079	0.281	0.723	0.019	0.647	0.023

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	119	134	124	106
N.S.	1	1.00	1.00	1.06	1.02	1.02	1.15	1.06	0.91
time (sec)	N/A	0.048	0.012	0.099	0.292	0.637	0.017	0.617	0.024

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	107	124	119	119	133	124	107
N.S.	1	1.00	2.55	2.95	2.83	2.83	3.17	2.95	2.55
time (sec)	N/A	0.046	0.017	0.081	0.285	0.804	0.019	0.603	0.022

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	121	115	115	129	121	103
N.S.	1	1.00	1.00	1.11	1.06	1.06	1.18	1.11	0.94
time (sec)	N/A	0.042	0.012	0.096	0.290	0.629	0.017	0.677	0.023

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	113	124	120	117	134	126	105
N.S.	1	1.00	1.28	1.41	1.36	1.33	1.52	1.43	1.19
time (sec)	N/A	0.044	0.022	0.063	0.284	0.717	0.087	0.634	0.025

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	121	116	121	126	120	104
N.S.	1	1.00	1.00	1.12	1.07	1.12	1.17	1.11	0.96
time (sec)	N/A	0.041	0.022	0.065	0.290	0.626	0.084	0.894	0.024

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	115	121	120	123	131	145	105
N.S.	1	1.00	1.02	1.07	1.06	1.09	1.16	1.28	0.93
time (sec)	N/A	0.075	0.033	0.065	0.298	0.702	0.142	0.720	0.045

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	110	118	118	121	128	122	106
N.S.	1	1.00	1.02	1.09	1.09	1.12	1.19	1.13	0.98
time (sec)	N/A	0.042	0.025	0.060	0.298	0.793	0.151	0.739	0.024

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	117	122	123	128	149	113
N.S.	1	1.00	1.00	1.04	1.09	1.10	1.14	1.33	1.01
time (sec)	N/A	0.074	0.028	0.070	0.295	0.705	0.333	0.701	0.027

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	111	113	120	121	129	123	111
N.S.	1	1.00	1.00	1.02	1.08	1.09	1.16	1.11	1.00
time (sec)	N/A	0.045	0.025	0.062	0.275	0.752	0.368	0.652	0.040

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	116	111	123	123	128	151	118
N.S.	1	1.00	1.02	0.97	1.08	1.08	1.12	1.32	1.04
time (sec)	N/A	0.072	0.029	0.066	0.298	0.957	0.720	0.647	0.044

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	111	108	120	121	131	124	116
N.S.	1	1.00	1.00	0.97	1.08	1.09	1.18	1.12	1.05
time (sec)	N/A	0.044	0.027	0.065	0.301	0.984	0.853	0.684	0.053

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	116	106	123	123	129	150	122
N.S.	1	1.00	1.04	0.95	1.10	1.10	1.15	1.34	1.09
time (sec)	N/A	0.068	0.038	0.086	0.292	0.799	1.545	0.639	0.033

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	115	102	119	121	129	123	119
N.S.	1	1.00	1.06	0.94	1.10	1.12	1.19	1.14	1.10
time (sec)	N/A	0.048	0.024	0.064	0.273	0.827	3.146	0.603	0.039

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	116	102	123	123	129	147	121
N.S.	1	1.00	1.03	0.90	1.09	1.09	1.14	1.30	1.07
time (sec)	N/A	0.060	0.041	0.067	0.297	0.887	5.033	0.640	0.059

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	122	101	119	121	131	125	119
N.S.	1	1.00	1.13	0.94	1.10	1.12	1.21	1.16	1.10
time (sec)	N/A	0.044	0.030	0.063	0.288	0.684	34.891	0.586	0.041

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	118	102	123	123	133	138	121
N.S.	1	1.00	1.30	1.12	1.35	1.35	1.46	1.52	1.33
time (sec)	N/A	0.038	0.045	0.068	0.291	0.750	26.604	0.626	0.068

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	121	121	0	127	120
N.S.	1	1.00	1.05	0.92	1.07	1.07	0.00	1.12	1.06
time (sec)	N/A	0.044	0.022	0.063	0.319	0.678	0.000	0.743	0.051

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	118	104	121	121	134	127	121
N.S.	1	1.00	2.46	2.17	2.52	2.52	2.79	2.65	2.52
time (sec)	N/A	0.021	0.023	0.069	0.322	0.772	134.476	1.100	0.035

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	121
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.042	0.024	0.066	0.301	0.620	0.000	1.494	0.035

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	121	104	121	121	0	127	120
N.S.	1	1.00	1.59	1.37	1.59	1.59	0.00	1.67	1.58
time (sec)	N/A	0.036	0.022	0.067	0.294	0.953	0.000	1.646	0.052

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	122
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.042	0.036	0.067	0.306	0.735	0.000	1.056	0.051

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	121
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.060	0.023	0.071	0.297	0.661	0.000	1.570	0.034

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	122
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.041	0.031	0.069	0.299	0.779	0.000	1.148	0.058

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	122
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.056	0.024	0.069	0.288	0.786	0.000	0.860	0.052

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	122
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.043	0.031	0.066	0.309	0.691	0.000	1.152	0.053

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	122
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.055	0.023	0.068	0.303	1.021	0.000	1.672	0.035

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	99	100	228	180	108	118
N.S.	1	1.00	1.00	1.01	1.02	2.33	1.84	1.10	1.20
time (sec)	N/A	0.040	0.051	0.086	0.512	1.753	0.209	0.950	0.027

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	74	74	75	70	77	76
N.S.	1	1.00	0.95	0.99	0.99	1.00	0.93	1.03	1.01
time (sec)	N/A	0.061	0.023	0.069	0.294	1.503	0.171	1.371	0.031

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	75	78	178	153	85	96
N.S.	1	1.00	1.00	0.97	1.01	2.31	1.99	1.10	1.25
time (sec)	N/A	0.033	0.037	0.082	0.588	1.167	0.236	0.833	0.054

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	47	50	50	51	46	52	52
N.S.	1	1.00	0.87	0.93	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.038	0.016	0.084	0.295	0.976	0.220	0.793	0.054

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	51	53	129	90	57	70
N.S.	1	1.00	0.98	0.88	0.91	2.22	1.55	0.98	1.21
time (sec)	N/A	0.024	0.034	0.082	0.506	0.934	0.185	0.720	0.056

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	31	30	27	32	31
N.S.	1	1.00	0.89	0.91	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.023	0.009	0.066	0.308	1.111	0.133	1.396	0.029

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	34	99	82	34	31
N.S.	1	1.00	1.03	0.87	0.87	2.54	2.10	0.87	0.79
time (sec)	N/A	0.011	0.020	0.080	0.539	1.635	0.168	1.602	0.028

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	35	32	26	36	32
N.S.	1	1.00	1.00	0.97	1.03	0.94	0.76	1.06	0.94
time (sec)	N/A	0.023	0.010	0.070	0.299	1.005	0.502	1.072	0.064

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	37	36	105	82	36	35
N.S.	1	1.00	0.98	0.86	0.84	2.44	1.91	0.84	0.81
time (sec)	N/A	0.014	0.020	0.076	0.506	0.907	0.181	1.353	0.033

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	46	48	47	41	71	46
N.S.	1	1.00	0.98	0.92	0.96	0.94	0.82	1.42	0.92
time (sec)	N/A	0.034	0.017	0.074	0.280	0.774	0.402	1.526	0.073

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	54	56	135	129	57	53
N.S.	1	1.00	1.02	0.92	0.95	2.29	2.19	0.97	0.90
time (sec)	N/A	0.024	0.040	0.077	0.511	0.899	0.206	1.045	0.055

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	70	64	70	73	61	100	70
N.S.	1	1.00	1.01	0.93	1.01	1.06	0.88	1.45	1.01
time (sec)	N/A	0.043	0.022	0.075	0.286	0.610	0.593	1.214	0.070

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	74	79	184	163	81	70
N.S.	1	1.00	0.98	0.92	0.99	2.30	2.04	1.01	0.88
time (sec)	N/A	0.033	0.041	0.092	0.509	0.670	0.306	1.258	0.052

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	96	86	96	98	88	126	92
N.S.	1	1.00	1.03	0.92	1.03	1.05	0.95	1.35	0.99
time (sec)	N/A	0.057	0.028	0.077	0.311	0.579	0.583	0.924	0.086

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	101	91	103	234	187	106	89
N.S.	1	1.00	1.02	0.92	1.04	2.36	1.89	1.07	0.90
time (sec)	N/A	0.043	0.051	0.097	0.503	0.635	0.292	1.060	0.072

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	113	125	131	172	131	159	181
N.S.	1	1.00	0.90	0.99	1.04	1.37	1.04	1.26	1.44
time (sec)	N/A	0.120	0.056	0.066	0.329	0.630	0.422	1.231	0.056

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	134	123	136	350	238	139	203
N.S.	1	1.00	1.02	0.94	1.04	2.67	1.82	1.06	1.55
time (sec)	N/A	0.101	0.078	0.089	0.516	0.607	0.529	1.112	0.030

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	103	107	148	104	135	121
N.S.	1	1.00	0.89	0.99	1.03	1.42	1.00	1.30	1.16
time (sec)	N/A	0.087	0.047	0.066	0.294	0.703	0.409	1.055	0.051

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	111	100	112	298	211	115	141
N.S.	1	1.00	1.01	0.91	1.02	2.71	1.92	1.05	1.28
time (sec)	N/A	0.078	0.063	0.085	0.508	0.723	0.475	1.110	0.027

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	76	82	121	78	106	86
N.S.	1	1.00	0.88	0.93	1.00	1.48	0.95	1.29	1.05
time (sec)	N/A	0.062	0.048	0.083	0.294	0.665	0.487	0.974	0.038

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	75	85	240	129	88	104
N.S.	1	1.00	1.02	0.86	0.98	2.76	1.48	1.01	1.20
time (sec)	N/A	0.048	0.054	0.084	0.530	0.710	0.360	2.019	0.038

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	59	60	81	56	91	62
N.S.	1	1.00	0.83	0.98	1.00	1.35	0.93	1.52	1.03
time (sec)	N/A	0.041	0.026	0.069	0.292	0.645	0.309	1.625	0.041

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	57	61	208	114	59	59
N.S.	1	1.00	1.01	0.85	0.91	3.10	1.70	0.88	0.88
time (sec)	N/A	0.034	0.051	0.085	0.512	0.668	0.282	1.556	0.065

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	38	40	44	36	65	37
N.S.	1	1.00	1.00	0.93	0.98	1.07	0.88	1.59	0.90
time (sec)	N/A	0.027	0.010	0.064	0.280	0.718	0.226	1.291	0.046

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	182	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.89	1.78	0.90	0.81
time (sec)	N/A	0.014	0.034	0.086	0.496	0.634	0.224	1.843	0.063

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	48	51	70	46	63	47
N.S.	1	1.00	0.90	0.94	1.00	1.37	0.90	1.24	0.92
time (sec)	N/A	0.031	0.022	0.068	0.326	0.829	0.332	0.919	0.086

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	62	63	210	114	62	63
N.S.	1	1.00	0.99	0.87	0.89	2.96	1.61	0.87	0.89
time (sec)	N/A	0.036	0.026	0.092	0.515	1.175	0.340	1.226	0.067

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	76	76	117	70	82	78
N.S.	1	1.00	0.84	1.00	1.00	1.54	0.92	1.08	1.03
time (sec)	N/A	0.053	0.036	0.073	0.299	1.281	0.562	1.399	0.075

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	78	93	250	184	85	83
N.S.	1	1.00	1.00	0.87	1.03	2.78	2.04	0.94	0.92
time (sec)	N/A	0.074	0.055	0.077	0.505	1.218	0.314	1.171	0.087

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	96	106	154	100	150	100
N.S.	1	1.00	0.88	0.99	1.09	1.59	1.03	1.55	1.03
time (sec)	N/A	0.070	0.076	0.076	0.328	1.004	0.687	2.623	0.074

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	112	99	119	308	218	112	104
N.S.	1	1.00	0.99	0.88	1.05	2.73	1.93	0.99	0.92
time (sec)	N/A	0.127	0.056	0.094	0.497	1.255	0.352	3.264	0.084

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	117	136	184	129	178	126
N.S.	1	1.00	0.89	0.94	1.10	1.48	1.04	1.44	1.02
time (sec)	N/A	0.091	0.072	0.082	0.275	1.339	0.705	1.263	0.093

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	136	151	165	231	170	183	225
N.S.	1	1.00	0.91	1.01	1.10	1.54	1.13	1.22	1.50
time (sec)	N/A	0.160	0.064	0.076	0.275	0.888	0.910	2.072	0.068

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	116	129	141	205	143	159	155
N.S.	1	1.00	0.91	1.01	1.10	1.60	1.12	1.24	1.21
time (sec)	N/A	0.114	0.055	0.068	0.293	0.700	0.978	1.205	0.040

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	102	116	179	119	132	118
N.S.	1	1.00	0.86	0.94	1.06	1.64	1.09	1.21	1.08
time (sec)	N/A	0.086	0.048	0.087	0.287	1.369	0.815	1.632	0.042

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	92	85	94	142	94	93	95
N.S.	1	1.00	1.05	0.97	1.07	1.61	1.07	1.06	1.08
time (sec)	N/A	0.064	0.029	0.070	0.286	0.884	1.038	1.187	0.069

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	61	72	89	70	61	70
N.S.	1	1.00	0.97	0.92	1.09	1.35	1.06	0.92	1.06
time (sec)	N/A	0.046	0.019	0.066	0.277	0.830	0.491	1.423	0.057

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	39	42	42	42	28	44
N.S.	1	1.00	0.94	1.22	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.014	0.010	0.072	0.280	0.926	0.276	1.439	0.039

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	63	77	119	75	76	71
N.S.	1	1.00	0.87	0.93	1.13	1.75	1.10	1.12	1.04
time (sec)	N/A	0.045	0.035	0.070	0.289	0.766	0.291	1.382	0.096

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	87	102	109	197	107	138	107
N.S.	1	1.00	0.86	1.01	1.08	1.95	1.06	1.37	1.06
time (sec)	N/A	0.072	0.044	0.076	0.308	0.638	0.605	1.486	0.060

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	108	123	137	229	136	133	131
N.S.	1	1.00	0.87	0.99	1.10	1.85	1.10	1.07	1.06
time (sec)	N/A	0.092	0.059	0.082	0.290	1.055	0.720	0.804	0.085

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	135	143	170	267	165	201	155
N.S.	1	1.00	0.91	0.96	1.14	1.79	1.11	1.35	1.04
time (sec)	N/A	0.115	0.090	0.080	0.281	0.712	0.707	1.387	0.100

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	158	144	171	468	280	162	246
N.S.	1	1.00	1.00	0.91	1.08	2.96	1.77	1.03	1.56
time (sec)	N/A	0.193	0.068	0.089	0.500	0.852	0.813	1.653	0.067

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	133	119	147	416	252	138	177
N.S.	1	1.00	0.96	0.86	1.07	3.01	1.83	1.00	1.28
time (sec)	N/A	0.146	0.077	0.101	0.495	0.709	0.810	1.714	0.038

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	95	120	358	214	111	138
N.S.	1	1.00	0.97	0.82	1.03	3.09	1.84	0.96	1.19
time (sec)	N/A	0.103	0.066	0.091	0.518	0.648	0.854	1.280	0.046

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	77	94	328	194	80	92
N.S.	1	1.00	0.97	0.82	1.00	3.49	2.06	0.85	0.98
time (sec)	N/A	0.059	0.055	0.086	0.510	0.836	0.594	0.952	0.079

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	83	76	92	301	155	78	82
N.S.	1	1.00	0.93	0.85	1.03	3.38	1.74	0.88	0.92
time (sec)	N/A	0.045	0.062	0.093	0.541	0.703	0.393	1.201	0.079

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	77	92	300	150	78	82
N.S.	1	1.00	0.91	0.84	1.00	3.26	1.63	0.85	0.89
time (sec)	N/A	0.024	0.047	0.101	0.537	0.581	0.386	1.471	0.079

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	82	96	324	194	82	113
N.S.	1	1.00	0.99	0.85	0.99	3.34	2.00	0.85	1.16
time (sec)	N/A	0.069	0.043	0.077	0.500	0.686	0.368	1.228	0.096

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	116	98	128	368	226	108	114
N.S.	1	1.00	0.99	0.84	1.09	3.15	1.93	0.92	0.97
time (sec)	N/A	0.115	0.067	0.088	0.506	0.575	0.478	0.930	0.102

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	139	119	154	426	260	135	135
N.S.	1	1.00	0.98	0.84	1.08	3.00	1.83	0.95	0.95
time (sec)	N/A	0.224	0.073	0.099	0.507	0.997	0.468	0.948	0.105

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	26	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	2.17	1.00	1.00
time (sec)	N/A	0.005	0.005	0.085	0.498	1.154	0.065	0.801	0.042

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	28	32	23	23	22	25	11
N.S.	1	1.00	2.55	2.91	2.09	2.09	2.00	2.27	1.00
time (sec)	N/A	0.005	0.008	0.082	0.277	1.062	0.084	1.418	0.062

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	16	10	10	7	11	10
N.S.	1	1.00	0.91	1.45	0.91	0.91	0.64	1.00	0.91
time (sec)	N/A	0.002	0.004	0.084	0.304	1.107	0.025	1.197	0.041

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	5	7	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.56	0.78	1.00
time (sec)	N/A	0.002	0.004	0.067	0.279	1.031	0.022	1.507	0.043

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	20	14	15	16
N.S.	1	1.00	1.00	0.84	0.79	1.05	0.74	0.79	0.84
time (sec)	N/A	0.003	0.006	0.067	0.492	0.889	0.041	1.134	0.042

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	15	15	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.79	0.79	0.89
time (sec)	N/A	0.003	0.006	0.070	0.490	0.632	0.032	1.259	0.039

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	19	10	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.36	0.71	1.00	1.00
time (sec)	N/A	0.003	0.005	0.070	0.493	1.183	0.028	1.198	0.013

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	13	14	14	8	14	12
N.S.	1	1.00	1.17	1.08	1.17	1.17	0.67	1.17	1.00
time (sec)	N/A	0.003	0.005	0.061	0.276	1.116	0.077	1.176	0.024

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	13	14	14	8	14	12
N.S.	1	1.00	1.17	1.08	1.17	1.17	0.67	1.17	1.00
time (sec)	N/A	0.003	0.004	0.069	0.289	1.295	0.076	1.526	0.001

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	49	98	75	36	31
N.S.	1	1.00	1.00	0.95	1.26	2.51	1.92	0.92	0.79
time (sec)	N/A	0.013	0.017	0.085	0.501	0.792	0.146	1.031	0.077

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	25	30	39	27	25	30
N.S.	1	1.00	1.00	0.71	0.86	1.11	0.77	0.71	0.86
time (sec)	N/A	0.005	0.008	0.085	0.503	0.705	0.045	1.058	0.043

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	30	16	16	17	11	20
N.S.	1	1.00	1.00	2.14	1.14	1.14	1.21	0.79	1.43
time (sec)	N/A	0.006	0.005	0.112	0.276	0.894	0.037	1.682	0.020

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.000	0.000	0.097	0.292	0.842	0.010	1.319	0.001

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.001	0.000	0.059	0.294	0.804	0.012	1.324	0.012

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.001	0.000	0.062	0.292	1.247	0.012	1.266	0.006

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.001	0.000	0.059	0.315	0.911	0.012	1.071	0.007

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00
time (sec)	N/A	0.001	0.000	0.062	0.287	1.265	0.009	1.377	0.003

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	7	4	3	5	4
N.S.	1	1.00	1.00	1.25	1.75	1.00	0.75	1.25	1.00
time (sec)	N/A	0.001	0.000	0.073	0.277	1.081	0.013	1.492	0.007

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	3	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.50	1.00	1.00
time (sec)	N/A	0.001	0.000	0.060	0.301	1.060	0.012	1.255	0.008

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.001	0.000	0.069	0.317	1.047	0.018	1.360	0.010

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	25	24	22	47	23
N.S.	1	1.00	1.00	0.90	0.86	0.83	0.76	1.62	0.79
time (sec)	N/A	0.014	0.004	0.064	0.282	0.965	0.053	1.685	0.022

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	28	86	58	28	25
N.S.	1	1.00	1.00	0.88	0.85	2.61	1.76	0.85	0.76
time (sec)	N/A	0.011	0.007	0.083	0.630	1.108	0.067	1.100	0.023

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	12	63	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.75	3.94	0.88
time (sec)	N/A	0.003	0.003	0.066	0.289	0.965	0.039	1.141	0.017

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	16	69	54	16	17
N.S.	1	1.00	1.00	0.68	0.64	2.76	2.16	0.64	0.68
time (sec)	N/A	0.006	0.004	0.080	0.505	0.930	0.094	1.301	0.025

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	25	21	17	26	19
N.S.	1	1.00	1.00	0.96	1.04	0.88	0.71	1.08	0.79
time (sec)	N/A	0.010	0.005	0.067	0.316	1.185	0.098	1.248	0.055

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	31	86	66	31	28
N.S.	1	1.00	1.00	0.89	0.86	2.39	1.83	0.86	0.78
time (sec)	N/A	0.010	0.010	0.071	0.570	1.044	0.073	0.955	0.052

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	34	36	36	32	47	34
N.S.	1	1.00	0.97	0.89	0.95	0.95	0.84	1.24	0.89
time (sec)	N/A	0.018	0.006	0.074	0.278	0.775	0.148	0.871	0.057

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	32	34	40	31	32	31
N.S.	1	1.00	0.80	0.91	0.97	1.14	0.89	0.91	0.89
time (sec)	N/A	0.021	0.008	0.064	0.283	1.118	0.085	0.787	0.051

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	38	38	128	80	37	35
N.S.	1	1.00	1.00	0.81	0.81	2.72	1.70	0.79	0.74
time (sec)	N/A	0.011	0.016	0.087	0.518	0.955	0.099	1.286	0.023

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	16	16	15	15	15
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.88	0.88	0.88
time (sec)	N/A	0.003	0.002	0.064	0.294	0.829	0.087	0.900	0.017

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	38	37	128	80	37	35
N.S.	1	1.00	1.00	0.81	0.79	2.72	1.70	0.79	0.74
time (sec)	N/A	0.010	0.018	0.084	0.550	0.791	0.132	0.968	0.022

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	44	40	54	36	51	37
N.S.	1	1.00	0.83	1.07	0.98	1.32	0.88	1.24	0.90
time (sec)	N/A	0.020	0.013	0.074	0.287	0.958	0.148	0.874	0.051

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	47	52	144	94	50	48
N.S.	1	1.00	0.93	0.78	0.87	2.40	1.57	0.83	0.80
time (sec)	N/A	0.015	0.028	0.083	0.506	0.925	0.185	1.109	0.037

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	57	57	80	53	56	55
N.S.	1	1.00	0.79	1.08	1.08	1.51	1.00	1.06	1.04
time (sec)	N/A	0.031	0.027	0.076	0.270	0.954	0.176	0.860	0.061

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.025	0.007	0.096	0.266	0.730	0.011	0.950	0.052

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	53	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.96	0.96	0.93
time (sec)	N/A	0.043	0.007	0.082	0.291	0.873	0.017	0.782	0.025

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.022	0.006	0.096	0.271	0.887	0.011	1.158	0.054

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	51	51	53	53	51
N.S.	1	1.00	1.21	1.24	1.21	1.21	1.26	1.26	1.21
time (sec)	N/A	0.042	0.010	0.085	0.305	0.810	0.013	0.754	0.025

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.016	0.007	0.059	0.307	0.668	0.012	1.289	0.030

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	52	49	49	53	48
N.S.	1	1.00	1.19	1.19	1.21	1.14	1.14	1.23	1.12
time (sec)	N/A	0.022	0.012	0.069	0.366	0.971	0.048	0.922	0.021

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	48	53	48	48	48
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00
time (sec)	N/A	0.018	0.013	0.061	0.272	1.201	0.046	0.945	0.025

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	48	52	54	48	70	48
N.S.	1	1.00	0.96	0.94	1.02	1.06	0.94	1.37	0.94
time (sec)	N/A	0.031	0.018	0.085	0.286	0.977	0.103	0.803	0.048

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	50	52	51	50	50
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04
time (sec)	N/A	0.020	0.015	0.068	0.265	1.055	0.113	1.570	0.026

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	90	85	85	100	94	78
N.S.	1	1.00	1.00	1.03	0.98	0.98	1.15	1.08	0.90
time (sec)	N/A	0.043	0.013	0.104	0.278	0.675	0.014	1.018	0.028

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	90	85	85	92	94	78
N.S.	1	1.00	0.93	1.03	0.98	0.98	1.06	1.08	0.90
time (sec)	N/A	0.075	0.019	0.088	0.295	1.053	0.021	0.869	0.016

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	90	85	85	100	94	78
N.S.	1	1.00	1.00	1.03	0.98	0.98	1.15	1.08	0.90
time (sec)	N/A	0.036	0.014	0.107	0.268	1.030	0.014	0.859	0.016

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	81	90	85	85	94	94	78
N.S.	1	1.00	1.14	1.27	1.20	1.20	1.32	1.32	1.10
time (sec)	N/A	0.073	0.017	0.086	0.272	0.965	0.018	1.470	0.016

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	82	82	97	91	75
N.S.	1	1.00	1.00	1.06	1.00	1.00	1.18	1.11	0.91
time (sec)	N/A	0.027	0.012	0.066	0.274	0.649	0.020	1.077	0.016

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	90	85	82	85	92	74
N.S.	1	1.00	1.00	1.12	1.06	1.02	1.06	1.15	0.92
time (sec)	N/A	0.052	0.018	0.126	0.283	0.854	0.071	1.277	0.018

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	91	83	87	92	90	76
N.S.	1	1.00	1.00	1.12	1.02	1.07	1.14	1.11	0.94
time (sec)	N/A	0.030	0.029	0.093	0.300	0.923	0.064	0.994	0.017

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	88	85	88	87	114	82
N.S.	1	1.00	0.99	1.05	1.01	1.05	1.04	1.36	0.98
time (sec)	N/A	0.054	0.031	0.072	0.347	0.749	0.155	1.399	0.020

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	81	84	87	92	88	82
N.S.	1	1.00	1.00	1.01	1.05	1.09	1.15	1.10	1.02
time (sec)	N/A	0.035	0.031	0.074	0.301	0.761	0.152	1.172	0.028

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	128	127	127	143	135	119
N.S.	1	1.00	1.00	1.01	1.00	1.00	1.13	1.06	0.94
time (sec)	N/A	0.062	0.020	0.102	0.325	0.775	0.018	1.013	0.053

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	119	128	127	127	138	135	119
N.S.	1	1.00	1.12	1.21	1.20	1.20	1.30	1.27	1.12
time (sec)	N/A	0.164	0.024	0.086	0.344	0.781	0.019	1.091	0.022

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	128	127	127	143	135	119
N.S.	1	1.00	1.00	1.01	1.00	1.00	1.13	1.06	0.94
time (sec)	N/A	0.049	0.015	0.120	0.329	0.648	0.028	0.749	0.021

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	119	128	127	127	136	134	118
N.S.	1	1.00	1.68	1.80	1.79	1.79	1.92	1.89	1.66
time (sec)	N/A	0.088	0.021	0.093	0.404	0.875	0.018	1.322	0.021

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	124	124	136	131	116
N.S.	1	1.00	1.00	1.02	1.02	1.02	1.11	1.07	0.95
time (sec)	N/A	0.038	0.016	0.066	0.328	0.724	0.017	0.896	0.023

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	123	132	128	125	133	134	116
N.S.	1	1.00	1.00	1.07	1.04	1.02	1.08	1.09	0.94
time (sec)	N/A	0.072	0.023	0.081	0.268	0.769	0.128	3.874	0.024

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	131	124	129	131	130	115
N.S.	1	1.00	1.00	1.09	1.03	1.08	1.09	1.08	0.96
time (sec)	N/A	0.040	0.029	0.070	0.297	0.750	0.105	1.734	0.047

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	120	130	128	131	133	160	121
N.S.	1	1.00	0.98	1.06	1.04	1.07	1.08	1.30	0.98
time (sec)	N/A	0.069	0.038	0.075	0.281	0.904	0.213	0.885	0.049

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	124	126	129	131	129	121
N.S.	1	1.00	1.00	1.03	1.05	1.08	1.09	1.08	1.01
time (sec)	N/A	0.040	0.032	0.076	0.292	0.975	0.209	0.651	0.022

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	143	139	302	246	153	169
N.S.	1	1.00	1.00	1.38	1.34	2.90	2.37	1.47	1.62
time (sec)	N/A	0.048	0.071	0.130	0.506	1.018	0.416	0.677	0.032

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	82	102	100	101	83	107	106
N.S.	1	1.00	1.04	1.29	1.27	1.28	1.05	1.35	1.34
time (sec)	N/A	0.060	0.031	0.082	0.282	1.406	0.313	0.724	0.035

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	102	104	228	194	113	128
N.S.	1	1.00	1.00	1.23	1.25	2.75	2.34	1.36	1.54
time (sec)	N/A	0.044	0.052	0.104	0.486	1.152	0.323	0.891	0.035

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	49	64	65	66	49	67	68
N.S.	1	1.00	0.80	1.05	1.07	1.08	0.80	1.10	1.11
time (sec)	N/A	0.033	0.018	0.097	0.281	1.288	0.186	0.690	0.064

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	68	179	172	72	90
N.S.	1	1.00	0.94	1.02	1.08	2.84	2.73	1.14	1.43
time (sec)	N/A	0.027	0.037	0.072	0.514	1.463	0.331	0.797	0.041

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	59	61	59	41	62	58
N.S.	1	1.00	0.98	1.16	1.20	1.16	0.80	1.22	1.14
time (sec)	N/A	0.033	0.017	0.082	0.284	1.080	0.771	0.695	0.089

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	65	63	164	165	63	80
N.S.	1	1.00	1.00	1.18	1.15	2.98	3.00	1.15	1.45
time (sec)	N/A	0.033	0.035	0.102	0.523	1.418	0.396	0.579	0.067

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	66	70	74	49	91	67
N.S.	1	1.00	1.03	1.14	1.21	1.28	0.84	1.57	1.16
time (sec)	N/A	0.041	0.022	0.089	0.308	1.051	0.840	0.626	0.097

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	68	71	192	172	71	90
N.S.	1	1.00	0.97	1.03	1.08	2.91	2.61	1.08	1.36
time (sec)	N/A	0.038	0.040	0.086	0.683	1.062	0.393	0.672	0.085

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	91	96	98	66	139	93
N.S.	1	1.00	0.96	1.21	1.28	1.31	0.88	1.85	1.24
time (sec)	N/A	0.047	0.033	0.085	0.311	1.200	0.831	0.613	0.087

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	107	236	207	112	129
N.S.	1	1.00	0.99	1.15	1.23	2.71	2.38	1.29	1.48
time (sec)	N/A	0.046	0.053	0.092	0.518	1.332	0.486	0.548	0.050

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	108	123	134	136	105	184	129
N.S.	1	1.00	1.10	1.26	1.37	1.39	1.07	1.88	1.32
time (sec)	N/A	0.056	0.048	0.086	0.290	1.061	0.988	0.513	0.078

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	138	141	149	400	286	156	200
N.S.	1	1.00	0.95	0.97	1.03	2.76	1.97	1.08	1.38
time (sec)	N/A	0.090	0.069	0.114	0.506	1.490	0.573	0.552	0.075

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	80	107	161	99	163	112
N.S.	1	1.00	0.97	0.89	1.19	1.79	1.10	1.81	1.24
time (sec)	N/A	0.072	0.047	0.091	0.281	1.408	0.515	0.507	0.067

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	105	101	109	342	246	114	146
N.S.	1	1.00	0.89	0.86	0.92	2.90	2.08	0.97	1.24
time (sec)	N/A	0.074	0.053	0.112	0.510	1.039	0.593	0.510	0.044

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	63	74	101	68	110	77
N.S.	1	1.00	0.90	1.02	1.19	1.63	1.10	1.77	1.24
time (sec)	N/A	0.042	0.035	0.076	0.284	1.154	0.531	0.529	0.046

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	89	92	96	302	236	95	124
N.S.	1	1.00	1.09	1.12	1.17	3.68	2.88	1.16	1.51
time (sec)	N/A	0.070	0.046	0.066	0.491	1.116	0.378	1.065	0.088

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	70	67	86	116	80	99	80
N.S.	1	1.00	1.04	1.00	1.28	1.73	1.19	1.48	1.19
time (sec)	N/A	0.044	0.032	0.079	0.299	1.045	0.823	1.208	0.068

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	104	91	97	100	305	238	102	128
N.S.	1	0.98	0.86	0.92	0.94	2.88	2.25	0.96	1.21
time (sec)	N/A	0.052	0.045	0.110	0.541	0.964	0.524	1.011	0.116

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	72	77	100	159	92	109	100
N.S.	1	1.00	0.89	0.95	1.23	1.96	1.14	1.35	1.23
time (sec)	N/A	0.059	0.066	0.090	0.274	1.245	0.822	1.654	0.052

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	107	118	356	248	111	147
N.S.	1	1.00	0.85	0.85	0.94	2.83	1.97	0.88	1.17
time (sec)	N/A	0.092	0.047	0.091	0.508	1.315	0.541	1.771	0.108

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	148	137	159	522	240	154	159
N.S.	1	1.00	0.91	0.84	0.98	3.20	1.47	0.94	0.98
time (sec)	N/A	0.110	0.067	0.114	0.531	0.961	1.203	1.435	0.086

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	114	113	120	178	122	107	123
N.S.	1	1.00	1.15	1.14	1.21	1.80	1.23	1.08	1.24
time (sec)	N/A	0.071	0.038	0.080	0.279	1.389	3.635	1.060	0.083

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	130	123	143	475	223	133	135
N.S.	1	1.00	1.02	0.97	1.13	3.74	1.76	1.05	1.06
time (sec)	N/A	0.083	0.075	0.102	0.547	1.176	1.225	1.183	0.098

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	75	76	87	108	87	76	83
N.S.	1	1.00	1.12	1.13	1.30	1.61	1.30	1.13	1.24
time (sec)	N/A	0.046	0.019	0.086	0.292	0.909	0.887	0.692	0.042

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	121	124	138	449	223	126	130
N.S.	1	1.00	1.04	1.07	1.19	3.87	1.92	1.09	1.12
time (sec)	N/A	0.051	0.072	0.065	0.533	0.966	0.547	1.270	0.110

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	103	98	109	163	107	110	106
N.S.	1	1.00	1.20	1.14	1.27	1.90	1.24	1.28	1.23
time (sec)	N/A	0.058	0.035	0.086	0.369	1.032	0.695	1.727	0.069

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	149	133	128	146	475	224	135	135
N.S.	1	0.98	0.88	0.84	0.96	3.12	1.47	0.89	0.89
time (sec)	N/A	0.073	0.064	0.111	0.553	1.467	0.733	1.451	0.117

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	99	114	142	256	139	177	132
N.S.	1	1.00	0.93	1.08	1.34	2.42	1.31	1.67	1.25
time (sec)	N/A	0.082	0.066	0.088	0.279	0.842	1.247	1.033	0.065

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	148	142	167	536	240	151	156
N.S.	1	1.00	0.92	0.88	1.04	3.33	1.49	0.94	0.97
time (sec)	N/A	0.133	0.053	0.117	0.506	1.088	0.869	1.390	0.111

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	74	74	75	70	77	76
N.S.	1	1.00	0.95	0.99	0.99	1.00	0.93	1.03	1.01
time (sec)	N/A	0.061	0.023	0.073	0.291	1.345	0.161	1.144	0.063

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	75	77	178	153	84	96
N.S.	1	1.00	1.00	0.97	1.00	2.31	1.99	1.09	1.25
time (sec)	N/A	0.033	0.039	0.089	0.533	1.079	0.258	1.038	0.064

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	47	50	50	51	46	52	52
N.S.	1	1.00	0.87	0.93	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.040	0.015	0.093	0.307	0.950	0.148	1.224	0.035

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	51	54	129	90	58	70
N.S.	1	1.00	0.98	0.88	0.93	2.22	1.55	1.00	1.21
time (sec)	N/A	0.023	0.032	0.083	0.501	1.284	0.186	1.133	0.039

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	31	29	27	32	31
N.S.	1	1.00	0.89	0.91	0.89	0.83	0.77	0.91	0.89
time (sec)	N/A	0.022	0.009	0.066	0.287	1.252	0.117	0.990	0.057

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	33	98	82	33	32
N.S.	1	1.00	1.03	0.87	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.011	0.020	0.060	0.535	1.192	0.173	1.476	0.030

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	35	33	26	36	32
N.S.	1	1.00	1.00	0.97	1.03	0.97	0.76	1.06	0.94
time (sec)	N/A	0.023	0.010	0.077	0.301	0.894	0.405	1.683	0.044

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	37	37	105	82	37	34
N.S.	1	1.00	0.98	0.86	0.86	2.44	1.91	0.86	0.79
time (sec)	N/A	0.015	0.021	0.091	0.492	1.266	0.176	1.368	0.061

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	46	48	48	41	72	45
N.S.	1	1.00	0.98	0.92	0.96	0.96	0.82	1.44	0.90
time (sec)	N/A	0.033	0.017	0.082	0.284	1.528	0.523	1.364	0.089

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	55	56	136	129	57	53
N.S.	1	1.00	1.02	0.93	0.95	2.31	2.19	0.97	0.90
time (sec)	N/A	0.025	0.039	0.077	0.497	0.969	0.205	1.510	0.069

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	116	139	137	138	122	148	146
N.S.	1	1.00	1.13	1.35	1.33	1.34	1.18	1.44	1.42
time (sec)	N/A	0.090	0.044	0.131	0.285	1.039	0.341	0.650	0.061

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	142	140	304	246	153	169
N.S.	1	1.00	1.00	1.35	1.33	2.90	2.34	1.46	1.61
time (sec)	N/A	0.049	0.072	0.131	0.499	1.035	0.357	0.948	0.055

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	102	101	102	83	107	106
N.S.	1	1.00	1.02	1.28	1.26	1.28	1.04	1.34	1.32
time (sec)	N/A	0.060	0.028	0.082	0.290	1.111	0.216	0.890	0.062

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	102	105	230	194	113	128
N.S.	1	1.00	1.00	1.21	1.25	2.74	2.31	1.35	1.52
time (sec)	N/A	0.043	0.054	0.089	0.499	1.047	0.256	0.869	0.065

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	49	63	66	67	49	67	68
N.S.	1	1.00	0.80	1.03	1.08	1.10	0.80	1.10	1.11
time (sec)	N/A	0.034	0.017	0.108	0.281	1.274	0.194	0.980	0.036

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	69	181	172	72	90
N.S.	1	1.00	0.94	1.02	1.10	2.87	2.73	1.14	1.43
time (sec)	N/A	0.027	0.037	0.000	0.499	1.516	0.224	0.944	0.040

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	59	61	59	41	62	58
N.S.	1	1.00	0.98	1.16	1.20	1.16	0.80	1.22	1.14
time (sec)	N/A	0.034	0.017	0.085	0.319	1.153	0.773	0.667	0.095

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	65	63	164	165	63	80
N.S.	1	1.00	1.00	1.18	1.15	2.98	3.00	1.15	1.45
time (sec)	N/A	0.034	0.039	0.083	0.501	1.098	0.313	0.615	0.041

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	66	69	73	49	90	67
N.S.	1	1.00	1.03	1.14	1.19	1.26	0.84	1.55	1.16
time (sec)	N/A	0.042	0.021	0.083	0.276	1.370	0.903	0.579	0.098

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	70	70	190	172	72	90
N.S.	1	1.00	1.03	1.09	1.09	2.97	2.69	1.12	1.41
time (sec)	N/A	0.040	0.048	0.087	0.504	1.455	0.421	0.535	0.079

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	128	234	219	220	201	238	236
N.S.	1	1.00	0.93	1.70	1.59	1.59	1.46	1.72	1.71
time (sec)	N/A	0.127	0.057	0.088	0.288	1.305	0.351	0.538	0.069

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	231	222	468	343	241	260
N.S.	1	1.00	1.00	1.65	1.59	3.34	2.45	1.72	1.86
time (sec)	N/A	0.065	0.036	0.106	0.527	1.718	0.392	0.620	0.060

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	125	177	168	169	144	180	178
N.S.	1	1.00	1.09	1.54	1.46	1.47	1.25	1.57	1.55
time (sec)	N/A	0.087	0.041	0.087	0.297	1.380	0.312	0.583	0.031

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	173	172	364	274	184	199
N.S.	1	1.00	0.99	1.45	1.45	3.06	2.30	1.55	1.67
time (sec)	N/A	0.058	0.030	0.102	0.520	0.996	0.359	0.575	0.056

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	119	119	120	94	124	123
N.S.	1	1.00	0.94	1.37	1.37	1.38	1.08	1.43	1.41
time (sec)	N/A	0.057	0.023	0.086	0.315	0.913	0.269	0.532	0.035

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	92	116	122	292	238	129	146
N.S.	1	1.00	0.94	1.18	1.24	2.98	2.43	1.32	1.49
time (sec)	N/A	0.039	0.049	0.064	0.549	1.046	0.371	0.548	0.067

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	65	86	98	101	65	99	97
N.S.	1	1.00	0.89	1.18	1.34	1.38	0.89	1.36	1.33
time (sec)	N/A	0.054	0.023	0.095	0.326	1.160	1.104	0.529	0.086

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	95	101	253	221	104	118
N.S.	1	1.00	0.99	1.23	1.31	3.29	2.87	1.35	1.53
time (sec)	N/A	0.046	0.026	0.088	0.536	1.181	0.431	0.585	0.039

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	94	97	105	63	120	95
N.S.	1	1.00	1.03	1.29	1.33	1.44	0.86	1.64	1.30
time (sec)	N/A	0.052	0.026	0.095	0.273	0.897	1.280	0.569	0.094

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	98	98	256	221	100	122
N.S.	1	1.00	1.00	1.32	1.32	3.46	2.99	1.35	1.65
time (sec)	N/A	0.048	0.030	0.092	0.516	1.046	0.663	0.621	0.085

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	68	72	0	70	68
N.S.	1	1.00	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.047	0.024	0.105	0.317	1.337	0.000	0.575	0.186

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	74	73	72	391	921	72	343
N.S.	1	1.00	0.95	0.94	0.92	5.01	11.81	0.92	4.40
time (sec)	N/A	0.057	0.066	0.135	0.492	1.093	153.144	0.546	0.348

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	50	49	42	144	51	51
N.S.	1	1.00	0.81	0.94	0.92	0.79	2.72	0.96	0.96
time (sec)	N/A	0.035	0.016	0.099	0.287	0.829	1.412	0.560	0.152

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	54	309	570	54	133
N.S.	1	1.00	0.87	0.79	0.77	4.41	8.14	0.77	1.90
time (sec)	N/A	0.023	0.034	0.130	0.515	1.227	1.829	0.594	0.194

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	42	41	31	138	51	148
N.S.	1	1.00	0.69	0.93	0.91	0.69	3.07	1.13	3.29
time (sec)	N/A	0.017	0.013	0.094	0.320	0.930	0.540	0.579	0.120

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	54	292	712	54	135
N.S.	1	1.00	0.87	0.79	0.77	4.17	10.17	0.77	1.93
time (sec)	N/A	0.016	0.032	0.070	0.482	1.068	2.809	0.677	0.185

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	59	61	54	0	73	58
N.S.	1	1.00	0.87	0.95	0.98	0.87	0.00	1.18	0.94
time (sec)	N/A	0.044	0.021	0.109	0.276	1.236	0.000	0.699	0.176

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	76	75	384	1093	75	338
N.S.	1	1.00	0.94	0.94	0.93	4.74	13.49	0.93	4.17
time (sec)	N/A	0.057	0.062	0.157	0.583	1.015	138.922	0.630	0.343

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	83	87	99	0	112	87
N.S.	1	1.00	1.01	0.95	1.00	1.14	0.00	1.29	1.00
time (sec)	N/A	0.066	0.030	0.118	0.296	1.298	0.000	1.344	0.208

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	101	96	96	560	0	98	367
N.S.	1	1.00	1.01	0.96	0.96	5.60	0.00	0.98	3.67
time (sec)	N/A	0.118	0.093	0.142	0.515	0.948	0.000	1.462	0.343

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	114	117	127	0	167	118
N.S.	1	1.00	1.00	0.96	0.98	1.07	0.00	1.40	0.99
time (sec)	N/A	0.088	0.041	0.133	0.279	1.893	0.000	1.043	0.239

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	135	127	131	669	0	139	397
N.S.	1	1.00	1.01	0.95	0.98	4.99	0.00	1.04	2.96
time (sec)	N/A	0.151	0.092	0.145	0.565	1.306	0.000	1.034	0.356

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	147	162	165	155	0	239	165
N.S.	1	1.00	0.95	1.05	1.06	1.00	0.00	1.54	1.06
time (sec)	N/A	0.119	0.048	0.130	0.292	2.238	0.000	1.237	0.275

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	91	86	130	162	0	152	169
N.S.	1	1.00	0.98	0.92	1.40	1.74	0.00	1.63	1.82
time (sec)	N/A	0.062	0.036	0.131	0.362	1.094	0.000	1.467	0.260

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	95	132	718	0	121	2500
N.S.	1	1.00	1.00	0.88	1.22	6.65	0.00	1.12	23.15
time (sec)	N/A	0.060	0.094	0.163	0.524	1.184	0.000	1.173	0.569

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	68	105	117	253	91	173
N.S.	1	1.00	1.00	0.92	1.42	1.58	3.42	1.23	2.34
time (sec)	N/A	0.048	0.028	0.116	0.319	1.602	1.340	1.565	0.156

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	90	85	119	705	0	110	3154
N.S.	1	1.00	0.87	0.82	1.14	6.78	0.00	1.06	30.33
time (sec)	N/A	0.045	0.096	0.161	0.551	1.553	0.000	1.640	0.446

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	73	99	103	248	85	160
N.S.	1	1.00	0.94	1.04	1.41	1.47	3.54	1.21	2.29
time (sec)	N/A	0.037	0.025	0.124	0.281	1.000	1.100	1.249	0.149

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	93	133	711	0	122	2500
N.S.	1	1.00	0.87	0.85	1.22	6.52	0.00	1.12	22.94
time (sec)	N/A	0.054	0.122	0.072	0.561	1.378	0.000	1.140	0.545

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	98	99	138	219	0	185	127
N.S.	1	1.00	0.98	0.99	1.38	2.19	0.00	1.85	1.27
time (sec)	N/A	0.071	0.068	0.138	0.284	1.987	0.000	1.338	0.398

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	123	109	178	1005	0	164	432
N.S.	1	1.00	0.85	0.76	1.24	6.98	0.00	1.14	3.00
time (sec)	N/A	0.133	0.188	0.192	0.536	1.426	0.000	1.100	0.464

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	117	120	188	302	0	257	171
N.S.	1	1.00	0.93	0.95	1.49	2.40	0.00	2.04	1.36
time (sec)	N/A	0.103	0.174	0.150	0.284	3.020	0.000	1.007	0.522

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	142	127	236	1281	0	165	469
N.S.	1	1.00	0.75	0.67	1.25	6.78	0.00	0.87	2.48
time (sec)	N/A	0.179	0.283	0.161	0.546	2.327	0.000	1.198	0.516

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	99	124	236	290	418	232	370
N.S.	1	1.00	0.85	1.07	2.03	2.50	3.60	2.00	3.19
time (sec)	N/A	0.079	0.076	0.125	0.305	1.132	3.070	1.351	0.249

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	154	154	264	1573	0	204	2500
N.S.	1	1.00	0.98	0.98	1.68	10.02	0.00	1.30	15.92
time (sec)	N/A	0.112	0.181	0.194	0.554	1.702	0.000	0.901	1.072

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	77	105	217	256	411	174	343
N.S.	1	1.00	0.77	1.05	2.17	2.56	4.11	1.74	3.43
time (sec)	N/A	0.066	0.087	0.128	0.312	1.005	2.697	0.779	0.207

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	151	151	266	1587	0	206	2500
N.S.	1	1.00	0.97	0.97	1.72	10.24	0.00	1.33	16.13
time (sec)	N/A	0.092	0.166	0.205	0.553	1.480	0.000	0.772	1.062

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	111	211	254	391	174	340
N.S.	1	1.00	1.00	1.13	2.15	2.59	3.99	1.78	3.47
time (sec)	N/A	0.053	0.040	0.137	0.330	1.201	2.372	0.912	0.155

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	158	158	277	1585	0	217	2500
N.S.	1	1.00	0.99	0.99	1.73	9.91	0.00	1.36	15.62
time (sec)	N/A	0.129	0.169	0.075	0.538	1.800	0.000	1.296	1.148

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	141	165	278	520	0	315	246
N.S.	1	1.00	0.95	1.11	1.87	3.49	0.00	2.11	1.65
time (sec)	N/A	0.111	0.205	0.165	0.334	6.311	0.000	1.229	0.783

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	172	170	352	1991	0	236	738
N.S.	1	1.00	0.82	0.81	1.67	9.44	0.00	1.12	3.50
time (sec)	N/A	0.204	0.288	0.237	0.513	2.904	0.000	0.775	0.704

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	171	187	364	640	0	357	314
N.S.	1	1.00	0.96	1.05	2.04	3.60	0.00	2.01	1.76
time (sec)	N/A	0.151	0.322	0.172	0.304	9.490	0.000	1.502	0.853

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	196	190	440	2397	0	256	785
N.S.	1	1.00	0.73	0.70	1.63	8.88	0.00	0.95	2.91
time (sec)	N/A	0.296	0.317	0.236	0.512	5.717	0.000	1.268	0.775

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81
time (sec)	N/A	0.009	0.004	0.076	0.299	0.861	0.031	0.882	0.084

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	76	84	240	129	88	104
N.S.	1	1.00	1.02	0.87	0.97	2.76	1.48	1.01	1.20
time (sec)	N/A	0.051	0.054	0.096	0.510	1.565	0.349	0.820	0.048

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	60	59	78	56	90	63
N.S.	1	1.00	0.83	1.00	0.98	1.30	0.93	1.50	1.05
time (sec)	N/A	0.044	0.026	0.076	0.301	0.923	0.305	0.896	0.044

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	59	60	202	114	58	59
N.S.	1	1.00	1.01	0.88	0.90	3.01	1.70	0.87	0.88
time (sec)	N/A	0.033	0.052	0.095	0.499	0.908	0.285	0.879	0.103

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	38	40	45	36	65	37
N.S.	1	1.00	1.00	0.93	0.98	1.10	0.88	1.59	0.90
time (sec)	N/A	0.027	0.010	0.069	0.316	1.153	0.181	0.739	0.081

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	181	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.87	1.78	0.90	0.81
time (sec)	N/A	0.013	0.034	0.000	0.523	1.089	0.203	0.756	0.097

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	49	51	71	46	63	47
N.S.	1	1.00	0.90	0.96	1.00	1.39	0.90	1.24	0.92
time (sec)	N/A	0.033	0.022	0.075	0.274	0.802	0.207	0.672	0.030

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	60	65	214	114	64	61
N.S.	1	1.00	0.99	0.85	0.92	3.01	1.61	0.90	0.86
time (sec)	N/A	0.036	0.025	0.081	0.483	1.092	0.252	1.108	0.114

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	75	78	122	70	84	74
N.S.	1	1.00	0.84	0.99	1.03	1.61	0.92	1.11	0.97
time (sec)	N/A	0.051	0.036	0.086	0.292	1.002	0.488	1.597	0.065

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	79	93	250	184	86	84
N.S.	1	1.00	1.00	0.88	1.03	2.78	2.04	0.96	0.93
time (sec)	N/A	0.074	0.053	0.085	0.498	0.911	0.301	1.465	0.111

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	138	141	149	400	286	156	200
N.S.	1	1.00	0.95	0.97	1.03	2.76	1.97	1.08	1.38
time (sec)	N/A	0.092	0.069	0.105	0.504	0.940	0.551	1.337	0.097

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	87	82	107	160	99	163	112
N.S.	1	1.00	0.99	0.93	1.22	1.82	1.12	1.85	1.27
time (sec)	N/A	0.075	0.047	0.097	0.298	1.086	0.506	1.319	0.042

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	105	102	109	342	246	114	148
N.S.	1	1.00	0.91	0.88	0.94	2.95	2.12	0.98	1.28
time (sec)	N/A	0.077	0.053	0.113	0.499	0.833	0.469	1.449	0.097

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	63	73	101	68	111	77
N.S.	1	1.00	0.92	1.03	1.20	1.66	1.11	1.82	1.26
time (sec)	N/A	0.042	0.036	0.079	0.302	1.063	0.408	0.974	0.099

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	88	94	95	297	236	94	124
N.S.	1	1.00	1.07	1.15	1.16	3.62	2.88	1.15	1.51
time (sec)	N/A	0.069	0.045	0.000	0.509	0.751	0.391	0.969	0.113

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	70	66	86	117	80	99	80
N.S.	1	1.00	1.04	0.99	1.28	1.75	1.19	1.48	1.19
time (sec)	N/A	0.046	0.031	0.089	0.291	1.077	0.689	0.952	0.143

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	100	91	95	101	308	238	103	128
N.S.	1	0.97	0.88	0.92	0.98	2.99	2.31	1.00	1.24
time (sec)	N/A	0.054	0.052	0.089	0.509	0.918	0.474	0.875	0.074

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	77	100	159	92	109	100
N.S.	1	1.00	0.90	0.96	1.25	1.99	1.15	1.36	1.25
time (sec)	N/A	0.057	0.072	0.087	0.290	0.613	0.760	0.762	0.104

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	125	107	107	118	356	248	112	146
N.S.	1	0.98	0.84	0.84	0.93	2.80	1.95	0.88	1.15
time (sec)	N/A	0.099	0.053	0.089	0.493	0.878	0.542	0.715	0.130

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	151	227	228	580	389	241	328
N.S.	1	1.00	0.89	1.34	1.35	3.43	2.30	1.43	1.94
time (sec)	N/A	0.107	0.062	0.109	0.564	1.063	0.760	0.622	0.105

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	106	137	174	254	163	249	194
N.S.	1	1.00	0.91	1.17	1.49	2.17	1.39	2.13	1.66
time (sec)	N/A	0.103	0.067	0.092	0.286	1.065	0.946	0.675	0.097

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	125	170	176	508	338	184	232
N.S.	1	1.00	0.85	1.16	1.20	3.46	2.30	1.25	1.58
time (sec)	N/A	0.130	0.050	0.111	0.533	1.047	0.677	0.543	0.036

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	127	89	124	181	112	183	130
N.S.	1	1.00	1.44	1.01	1.41	2.06	1.27	2.08	1.48
time (sec)	N/A	0.065	0.034	0.095	0.275	1.204	0.671	0.556	0.094

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	139	147	442	314	152	182
N.S.	1	1.00	1.00	1.31	1.39	4.17	2.96	1.43	1.72
time (sec)	N/A	0.065	0.046	0.071	0.503	0.974	0.581	0.504	0.111

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	111	94	122	178	110	150	122
N.S.	1	1.00	1.26	1.07	1.39	2.02	1.25	1.70	1.39
time (sec)	N/A	0.061	0.074	0.085	0.289	1.395	1.493	0.523	0.061

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	94	129	140	412	309	143	173
N.S.	1	1.00	0.72	0.98	1.07	3.15	2.36	1.09	1.32
time (sec)	N/A	0.091	0.046	0.116	0.518	0.967	0.884	0.620	0.118

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	100	141	209	128	157	135
N.S.	1	1.00	0.89	1.02	1.44	2.13	1.31	1.60	1.38
time (sec)	N/A	0.077	0.071	0.090	0.279	0.959	4.071	0.587	0.153

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	109	144	159	458	321	150	183
N.S.	1	1.00	0.74	0.98	1.08	3.12	2.18	1.02	1.24
time (sec)	N/A	0.094	0.047	0.092	0.506	0.807	1.146	0.492	0.138

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	94	133	726	0	122	2500
N.S.	1	1.00	0.87	0.86	1.22	6.66	0.00	1.12	22.94
time (sec)	N/A	0.060	0.110	0.173	0.523	0.910	0.000	0.478	0.550

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	69	105	117	253	92	172
N.S.	1	1.00	1.00	0.93	1.42	1.58	3.42	1.24	2.32
time (sec)	N/A	0.048	0.028	0.122	0.274	0.641	1.082	0.442	0.171

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	85	119	704	0	110	3153
N.S.	1	1.00	1.00	0.82	1.14	6.77	0.00	1.06	30.32
time (sec)	N/A	0.045	0.101	0.175	0.539	1.021	0.000	0.474	0.490

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	72	99	103	248	85	161
N.S.	1	1.00	0.94	1.03	1.41	1.47	3.54	1.21	2.30
time (sec)	N/A	0.038	0.021	0.121	0.285	0.795	1.042	0.551	0.163

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	109	95	132	699	0	121	2500
N.S.	1	1.00	1.01	0.88	1.22	6.47	0.00	1.12	23.15
time (sec)	N/A	0.051	0.105	0.072	0.489	0.958	0.000	0.452	0.538

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	97	100	137	218	0	183	127
N.S.	1	1.00	0.98	1.01	1.38	2.20	0.00	1.85	1.28
time (sec)	N/A	0.075	0.079	0.150	0.288	1.413	0.000	0.452	0.405

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	123	108	178	1003	0	164	2400
N.S.	1	1.00	0.85	0.75	1.24	6.97	0.00	1.14	16.67
time (sec)	N/A	0.134	0.138	0.174	0.526	1.973	0.000	0.737	0.557

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	119	120	189	303	0	257	171
N.S.	1	1.00	0.94	0.95	1.50	2.40	0.00	2.04	1.36
time (sec)	N/A	0.103	0.113	0.154	0.309	3.494	0.000	0.743	0.553

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	142	128	236	1281	0	165	2500
N.S.	1	1.00	0.75	0.68	1.25	6.78	0.00	0.87	13.23
time (sec)	N/A	0.185	0.201	0.204	0.528	2.371	0.000	1.526	0.640

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	155	153	258	356	0	281	217
N.S.	1	1.00	0.97	0.96	1.61	2.22	0.00	1.76	1.36
time (sec)	N/A	0.133	0.142	0.162	0.289	6.814	0.000	2.550	0.591

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	179	161	303	1489	0	207	2737
N.S.	1	1.00	0.72	0.64	1.21	5.96	0.00	0.83	10.95
time (sec)	N/A	0.280	0.225	0.155	0.534	4.559	0.000	0.998	0.659

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	202	201	339	410	0	354	278
N.S.	1	1.00	0.96	0.96	1.61	1.95	0.00	1.69	1.32
time (sec)	N/A	0.175	0.197	0.162	0.310	8.809	0.000	0.878	0.665

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	133	117	249	1407	0	198	2500
N.S.	1	1.00	0.82	0.72	1.54	8.69	0.00	1.22	15.43
time (sec)	N/A	0.114	0.136	0.218	0.527	1.402	0.000	0.635	1.034

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	86	113	228	296	507	178	522
N.S.	1	1.00	0.80	1.06	2.13	2.77	4.74	1.66	4.88
time (sec)	N/A	0.075	0.061	0.135	0.300	1.129	2.488	0.544	0.313

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	137	117	249	1387	0	196	2500
N.S.	1	1.00	0.93	0.80	1.69	9.44	0.00	1.33	17.01
time (sec)	N/A	0.093	0.131	0.234	0.559	2.138	0.000	0.546	0.944

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	77	106	215	253	410	163	378
N.S.	1	1.00	0.84	1.15	2.34	2.75	4.46	1.77	4.11
time (sec)	N/A	0.055	0.051	0.128	0.326	0.982	1.963	0.511	0.158

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	136	133	294	1681	0	232	2500
N.S.	1	1.00	0.81	0.80	1.76	10.07	0.00	1.39	14.97
time (sec)	N/A	0.136	0.231	0.001	0.531	2.038	0.000	0.596	1.158

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	133	136	295	540	0	321	193
N.S.	1	1.00	0.94	0.96	2.09	3.83	0.00	2.28	1.37
time (sec)	N/A	0.120	0.164	0.172	0.318	5.661	0.000	0.587	0.784

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	158	141	378	2113	0	321	2500
N.S.	1	1.00	0.72	0.65	1.73	9.69	0.00	1.47	11.47
time (sec)	N/A	0.207	0.217	0.216	0.546	2.972	0.000	1.026	0.971

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	157	157	381	667	0	333	313
N.S.	1	1.00	1.01	1.01	2.44	4.28	0.00	2.13	2.01
time (sec)	N/A	0.146	0.142	0.187	0.313	9.474	0.000	0.717	0.900

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	178	159	460	2457	0	275	2500
N.S.	1	1.00	0.66	0.59	1.70	9.07	0.00	1.01	9.23
time (sec)	N/A	0.301	0.285	0.213	0.550	7.992	0.000	0.810	1.189

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	166	177	443	2859	0	301	2500
N.S.	1	1.00	0.80	0.86	2.14	13.81	0.00	1.45	12.08
time (sec)	N/A	0.187	0.249	0.236	0.584	2.840	0.000	0.840	1.586

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	121	163	415	598	784	267	926
N.S.	1	1.00	0.85	1.15	2.92	4.21	5.52	1.88	6.52
time (sec)	N/A	0.107	0.082	0.159	0.313	1.601	28.871	0.898	0.396

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	171	182	473	2891	0	317	2500
N.S.	1	1.00	0.86	0.91	2.36	14.46	0.00	1.58	12.50
time (sec)	N/A	0.175	0.286	0.251	0.564	3.689	0.000	0.650	1.553

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	144	394	507	643	229	707
N.S.	1	1.00	0.85	1.14	3.13	4.02	5.10	1.82	5.61
time (sec)	N/A	0.080	0.095	0.148	0.322	1.031	110.917	0.648	0.328

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	197	198	529	3239	0	332	2500
N.S.	1	1.00	0.86	0.86	2.30	14.08	0.00	1.44	10.87
time (sec)	N/A	0.214	0.289	0.076	0.564	4.746	0.000	0.725	1.708

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	187	202	527	1058	0	470	472
N.S.	1	1.00	0.97	1.05	2.74	5.51	0.00	2.45	2.46
time (sec)	N/A	0.166	0.203	0.223	0.349	12.126	0.000	0.657	1.106

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	210	202	639	3753	0	430	2500
N.S.	1	1.00	0.71	0.68	2.15	12.64	0.00	1.45	8.42
time (sec)	N/A	0.346	0.317	0.232	0.540	7.409	0.000	0.884	1.554

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	208	223	651	1227	0	638	549
N.S.	1	1.00	0.97	1.04	3.03	5.71	0.00	2.97	2.55
time (sec)	N/A	0.205	0.224	0.230	0.331	25.420	0.000	0.792	1.315

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	230	222	738	4225	0	367	1161
N.S.	1	1.00	0.61	0.59	1.96	11.21	0.00	0.97	3.08
time (sec)	N/A	0.462	0.321	0.224	0.555	15.128	0.000	0.720	1.231

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	89	473	129	379	2069	593	289
N.S.	1	1.00	0.93	4.93	1.34	3.95	21.55	6.18	3.01
time (sec)	N/A	0.044	0.131	0.073	0.275	1.003	0.610	0.575	0.274

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	82	91	215	1044	332	177
N.S.	1	1.00	0.93	1.15	1.28	3.03	14.70	4.68	2.49
time (sec)	N/A	0.031	0.073	0.076	0.288	1.184	0.405	0.679	0.185

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	53	53	92	410	143	95
N.S.	1	1.00	0.93	1.18	1.18	2.04	9.11	3.18	2.11
time (sec)	N/A	0.015	0.050	0.020	0.325	0.976	0.253	0.657	0.156

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	190	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	2.88	0.00	-0.02
time (sec)	N/A	0.023	0.079	0.022	0.000	0.000	1.969	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	906	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	9.74	0.00	-0.01
time (sec)	N/A	0.030	0.102	0.027	0.000	0.000	15.327	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	3053	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	32.83	0.00	-0.01
time (sec)	N/A	0.030	0.111	0.042	0.000	0.000	52.137	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	141	975	215	773	4345	1192	443
N.S.	1	1.00	0.93	6.46	1.42	5.12	28.77	7.89	2.93
time (sec)	N/A	0.061	0.178	0.089	0.278	0.908	0.906	0.613	0.349

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	101	568	153	442	2363	703	302
N.S.	1	1.00	0.93	5.21	1.40	4.06	21.68	6.45	2.77
time (sec)	N/A	0.045	0.114	0.081	0.273	1.156	0.613	0.630	0.249

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	82	91	215	1044	332	177
N.S.	1	1.00	0.93	1.15	1.28	3.03	14.70	4.68	2.49
time (sec)	N/A	0.029	0.080	0.073	0.282	1.286	0.405	0.525	0.194

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	118	0	0	0	299	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	3.18	0.00	-0.01
time (sec)	N/A	0.045	0.131	0.036	0.000	0.000	3.001	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	98	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.187	0.036	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	166	124	0	0	0	0	0	-1
N.S.	1	0.97	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.400	0.054	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	0	0	0	411	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	3.09	0.00	-0.01
time (sec)	N/A	0.059	1.193	0.042	0.000	0.000	4.819	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	85	0	0	0	299	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	3.18	0.00	-0.01
time (sec)	N/A	0.041	0.365	0.033	0.000	0.000	3.129	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	190	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	2.88	0.00	-0.02
time (sec)	N/A	0.021	0.071	0.023	0.000	0.000	2.085	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	85	0	0	0	354	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	3.47	0.00	-0.01
time (sec)	N/A	0.033	0.087	0.047	0.000	0.000	3.554	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	127	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.169	0.060	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	170	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	0.514	0.024	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	2524	0	0	0	0	0	-1
N.S.	1	1.00	12.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.152	3.721	0.046	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	895	0	0	0	0	0	-1
N.S.	1	1.00	7.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	1.567	0.040	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	906	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	9.74	0.00	-0.01
time (sec)	N/A	0.028	0.105	0.032	0.000	0.000	14.983	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	127	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	173	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	0.407	0.023	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	215	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.448	0.755	0.024	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	28	27	32	46	29	31
N.S.	1	1.00	1.05	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.011	0.024	0.087	0.316	1.120	0.737	1.434	0.109

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.011	0.023	0.085	0.311	1.288	0.477	0.752	0.121

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.012	0.022	0.088	0.323	0.969	0.300	0.841	0.023

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	30	37	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.77	0.95	0.74	0.79
time (sec)	N/A	0.011	0.022	0.086	0.286	1.329	0.937	0.777	0.022

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	27	29	44	29	31
N.S.	1	1.00	0.95	0.76	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.011	0.024	0.077	0.284	0.839	0.143	1.198	0.104

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	44	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.011	0.028	0.043	0.295	0.926	0.240	1.360	0.103

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	42	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.14	0.78	0.84
time (sec)	N/A	0.011	0.029	0.048	0.296	0.981	0.280	1.893	0.022

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	29	29	42	31	31
N.S.	1	1.00	0.95	0.76	0.78	0.78	1.14	0.84	0.84
time (sec)	N/A	0.011	0.034	0.046	0.317	1.257	0.349	1.744	0.021

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.021	0.040	0.099	0.309	1.042	1.269	0.982	0.113

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.021	0.039	0.098	0.322	0.756	0.787	1.163	0.025

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.022	0.036	0.098	0.301	1.053	0.516	0.858	0.025

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	54	66	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.86	1.05	0.84	0.81
time (sec)	N/A	0.020	0.039	0.092	0.285	0.690	1.357	0.965	0.027

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	52	51	53	78	53	51
N.S.	1	1.00	0.97	0.85	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.021	0.036	0.096	0.286	0.669	0.304	1.200	0.025

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	78	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.020	0.049	0.076	0.275	0.870	0.394	0.859	0.026

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	76	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.25	0.87	0.84
time (sec)	N/A	0.023	0.047	0.078	0.281	0.924	0.459	0.820	0.027

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	51	53	53	76	55	55
N.S.	1	1.00	0.97	0.84	0.87	0.87	1.25	0.90	0.90
time (sec)	N/A	0.021	0.045	0.077	0.282	1.066	0.600	0.872	0.119

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.029	0.049	0.102	0.296	1.020	1.790	0.709	0.021

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.029	0.046	0.111	0.290	0.880	1.202	0.771	0.017

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.030	0.044	0.109	0.290	0.838	0.753	0.625	0.017

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	76	95	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.89	1.12	0.91	0.81
time (sec)	N/A	0.029	0.044	0.101	0.286	1.431	1.558	0.570	0.017

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	80	76	73	75	112	77	69
N.S.	1	1.00	0.96	0.92	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.029	0.045	0.096	0.281	1.700	0.479	0.639	0.017

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	78	73	75	110	77	69
N.S.	1	1.00	1.00	0.94	0.88	0.90	1.33	0.93	0.83
time (sec)	N/A	0.029	0.063	0.076	0.285	1.225	0.632	0.733	0.018

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	78	73	75	110	77	69
N.S.	1	1.00	0.94	0.94	0.88	0.90	1.33	0.93	0.83
time (sec)	N/A	0.029	0.061	0.083	0.295	1.382	0.701	0.644	0.018

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	75	75	75	107	79	72
N.S.	1	1.00	0.96	0.93	0.93	0.93	1.32	0.98	0.89
time (sec)	N/A	0.029	0.056	0.081	0.282	1.056	0.871	0.871	0.031

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	173	164	259	714	326	298	788
N.S.	1	1.00	0.63	0.59	0.94	2.59	1.18	1.08	2.86
time (sec)	N/A	0.179	0.276	0.089	0.501	1.422	38.642	1.000	0.197

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	152	142	214	899	348	264	92
N.S.	1	1.00	0.59	0.55	0.83	3.50	1.35	1.03	0.36
time (sec)	N/A	0.151	0.192	0.091	0.511	0.977	18.047	1.001	0.095

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	150	141	235	660	275	263	789
N.S.	1	1.00	0.59	0.55	0.92	2.59	1.08	1.03	3.09
time (sec)	N/A	0.143	0.185	0.086	0.524	1.239	5.159	1.068	0.208

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	135	124	194	834	303	251	71
N.S.	1	1.00	0.57	0.52	0.82	3.52	1.28	1.06	0.30
time (sec)	N/A	0.125	0.169	0.081	0.528	2.908	2.253	1.166	0.081

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	134	127	218	645	238	251	739
N.S.	1	1.00	0.57	0.54	0.93	2.74	1.01	1.07	3.14
time (sec)	N/A	0.126	0.161	0.084	0.503	1.144	1.872	1.336	0.123

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	135	127	194	843	150	251	71
N.S.	1	1.00	0.57	0.54	0.83	3.59	0.64	1.07	0.30
time (sec)	N/A	0.129	0.191	0.095	0.532	0.856	7.054	0.712	0.082

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	136	124	218	653	257	251	811
N.S.	1	1.00	0.57	0.52	0.92	2.76	1.08	1.06	3.42
time (sec)	N/A	0.127	0.191	0.088	0.509	2.509	9.189	0.843	0.211

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	152	140	213	883	258	268	90
N.S.	1	1.00	0.60	0.55	0.84	3.46	1.01	1.05	0.35
time (sec)	N/A	0.144	0.229	0.085	0.500	1.537	41.177	0.685	0.180

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	183	171	271	748	770	298	823
N.S.	1	1.00	0.59	0.55	0.87	2.41	2.48	0.96	2.65
time (sec)	N/A	0.166	0.583	0.115	0.559	1.003	160.169	0.584	0.202

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	162	153	223	925	0	283	106
N.S.	1	1.00	0.56	0.53	0.77	3.20	0.00	0.98	0.37
time (sec)	N/A	0.147	0.531	0.101	0.502	0.871	0.000	0.583	0.099

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	159	152	250	725	760	283	744
N.S.	1	1.00	0.56	0.54	0.88	2.55	2.68	1.00	2.62
time (sec)	N/A	0.146	0.550	0.102	0.519	0.636	30.807	0.557	0.198

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	152	146	217	912	162	273	91
N.S.	1	1.00	0.58	0.56	0.83	3.49	0.62	1.05	0.35
time (sec)	N/A	0.131	0.509	0.081	0.501	0.816	12.727	0.651	0.173

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	151	146	241	717	734	273	750
N.S.	1	1.00	0.58	0.56	0.92	2.75	2.81	1.05	2.87
time (sec)	N/A	0.125	0.461	0.080	0.566	0.862	25.598	0.591	0.233

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	162	153	222	920	573	278	104
N.S.	1	1.00	0.56	0.53	0.77	3.18	1.98	0.96	0.36
time (sec)	N/A	0.148	0.515	0.111	0.555	1.665	79.383	0.524	0.179

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	165	153	251	741	855	283	859
N.S.	1	1.00	0.57	0.53	0.87	2.56	2.96	0.98	2.97
time (sec)	N/A	0.150	0.557	0.105	0.527	2.584	118.420	0.543	0.249

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	186	170	250	974	0	303	121
N.S.	1	1.00	0.60	0.55	0.81	3.14	0.00	0.98	0.39
time (sec)	N/A	0.163	0.545	0.112	0.512	1.439	0.000	0.611	0.096

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	183	172	283	793	0	304	760
N.S.	1	1.00	0.58	0.54	0.90	2.51	0.00	0.96	2.41
time (sec)	N/A	0.170	0.622	0.122	0.513	0.961	0.000	0.679	0.221

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	171	166	251	990	0	293	122
N.S.	1	1.00	0.58	0.57	0.86	3.38	0.00	1.00	0.42
time (sec)	N/A	0.154	0.615	0.084	0.505	0.871	0.000	0.685	0.095

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	173	167	280	806	1445	298	799
N.S.	1	1.00	0.58	0.56	0.94	2.70	4.85	1.00	2.68
time (sec)	N/A	0.145	0.604	0.078	0.523	0.703	159.961	0.580	0.261

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	175	168	253	1005	299	298	124
N.S.	1	1.00	0.59	0.56	0.85	3.37	1.00	1.00	0.42
time (sec)	N/A	0.150	0.600	0.081	0.535	0.835	57.485	1.079	0.177

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	172	166	276	793	1406	293	780
N.S.	1	1.00	0.59	0.57	0.94	2.71	4.80	1.00	2.66
time (sec)	N/A	0.146	0.611	0.091	0.502	0.779	141.892	1.270	0.270

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	186	173	255	988	0	300	133
N.S.	1	1.00	0.58	0.54	0.79	3.07	0.00	0.93	0.41
time (sec)	N/A	0.166	0.635	0.129	0.555	0.733	0.000	1.779	0.102

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	186	173	285	809	0	304	888
N.S.	1	1.00	0.58	0.54	0.89	2.51	0.00	0.94	2.76
time (sec)	N/A	0.173	0.544	0.138	0.506	0.668	0.000	0.907	0.300

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	207	190	285	1043	0	326	152
N.S.	1	1.00	0.60	0.55	0.83	3.04	0.00	0.95	0.44
time (sec)	N/A	0.187	0.571	0.147	0.494	0.573	0.000	1.295	0.204

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.021	0.039	0.092	0.294	0.476	1.111	1.484	0.033

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.022	0.037	0.087	0.290	0.438	0.792	0.944	0.025

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.022	0.035	0.088	0.293	1.170	0.580	0.968	0.025

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	54	66	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.86	1.05	0.84	0.81
time (sec)	N/A	0.022	0.038	0.088	0.288	0.979	1.350	1.194	0.029

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	52	51	53	78	53	51
N.S.	1	1.00	0.97	0.85	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.022	0.035	0.086	0.287	1.099	0.284	1.166	0.025

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	78	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.021	0.048	0.075	0.289	1.535	0.464	1.217	0.027

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	76	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.25	0.87	0.84
time (sec)	N/A	0.022	0.046	0.080	0.295	1.552	0.539	0.916	0.026

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	51	53	53	76	55	55
N.S.	1	1.00	0.97	0.84	0.87	0.87	1.25	0.90	0.90
time (sec)	N/A	0.022	0.046	0.081	0.283	1.354	0.574	0.764	0.028

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	90	85	90	136	94	78
N.S.	1	1.00	0.96	0.93	0.88	0.93	1.40	0.97	0.80
time (sec)	N/A	0.035	0.061	0.106	0.319	1.003	1.672	1.052	0.028

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	90	85	90	136	94	78
N.S.	1	1.00	0.96	0.93	0.88	0.93	1.40	0.97	0.80
time (sec)	N/A	0.034	0.058	0.100	0.293	0.741	1.202	0.762	0.017

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	90	85	90	136	94	78
N.S.	1	1.00	0.96	0.93	0.88	0.93	1.40	0.97	0.80
time (sec)	N/A	0.034	0.058	0.103	0.306	0.693	0.761	0.947	0.017

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	90	85	88	110	94	78
N.S.	1	1.00	0.96	0.93	0.88	0.91	1.13	0.97	0.80
time (sec)	N/A	0.034	0.054	0.099	0.308	0.780	1.723	1.020	0.017

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	93	90	85	87	134	94	78
N.S.	1	1.00	0.98	0.95	0.89	0.92	1.41	0.99	0.82
time (sec)	N/A	0.035	0.059	0.105	0.301	0.703	0.549	1.117	0.017

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	92	95	85	87	134	94	78
N.S.	1	1.00	0.97	1.00	0.89	0.92	1.41	0.99	0.82
time (sec)	N/A	0.034	0.049	0.080	0.301	0.799	0.648	0.565	0.019

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	93	95	85	87	133	94	78
N.S.	1	1.00	0.98	1.00	0.89	0.92	1.40	0.99	0.82
time (sec)	N/A	0.033	0.045	0.082	0.287	0.731	0.730	0.752	0.018

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	93	89	87	87	133	96	86
N.S.	1	1.00	0.98	0.94	0.92	0.92	1.40	1.01	0.91
time (sec)	N/A	0.033	0.068	0.090	0.277	0.601	0.928	0.639	0.031

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	126	128	127	132	192	135	119
N.S.	1	1.00	0.91	0.92	0.91	0.95	1.38	0.97	0.86
time (sec)	N/A	0.045	0.088	0.115	0.289	0.532	2.334	0.512	0.123

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	126	128	127	132	192	135	119
N.S.	1	1.00	0.91	0.92	0.91	0.95	1.38	0.97	0.86
time (sec)	N/A	0.046	0.083	0.112	0.296	0.784	1.667	0.810	0.021

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	126	128	127	132	192	135	119
N.S.	1	1.00	0.91	0.92	0.91	0.95	1.38	0.97	0.86
time (sec)	N/A	0.044	0.077	0.109	0.271	0.499	1.187	0.578	0.022

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	126	128	127	130	155	135	119
N.S.	1	1.00	0.91	0.92	0.91	0.94	1.12	0.97	0.86
time (sec)	N/A	0.043	0.083	0.105	0.281	0.506	2.160	0.715	0.022

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	126	128	127	129	190	135	119
N.S.	1	1.00	0.92	0.93	0.93	0.94	1.39	0.99	0.87
time (sec)	N/A	0.044	0.072	0.120	0.290	0.473	0.791	0.802	0.022

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	126	136	127	129	189	135	119
N.S.	1	1.00	0.92	0.99	0.93	0.94	1.38	0.99	0.87
time (sec)	N/A	0.044	0.065	0.084	0.280	0.459	1.030	0.690	0.022

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	126	136	127	129	189	135	119
N.S.	1	1.00	0.92	0.99	0.93	0.94	1.38	0.99	0.87
time (sec)	N/A	0.043	0.093	0.082	0.272	0.486	1.122	1.321	0.023

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	126	132	129	129	185	137	125
N.S.	1	1.00	0.92	0.96	0.94	0.94	1.35	1.00	0.91
time (sec)	N/A	0.044	0.067	0.083	0.306	0.504	1.305	0.950	0.024

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	219	230	360	1334	561	436	1202
N.S.	1	1.00	0.70	0.74	1.16	4.29	1.80	1.40	3.86
time (sec)	N/A	0.220	0.236	0.128	0.516	0.627	93.270	1.068	0.232

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	187	192	263	1701	440	385	435
N.S.	1	1.00	0.64	0.66	0.91	5.87	1.52	1.33	1.50
time (sec)	N/A	0.183	0.207	0.107	0.499	0.652	226.342	0.785	0.178

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	187	194	324	1268	488	385	1175
N.S.	1	1.00	0.65	0.67	1.12	4.40	1.69	1.34	4.08
time (sec)	N/A	0.174	0.206	0.092	0.485	0.471	17.104	0.882	0.197

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	156	156	229	1629	87	361	390
N.S.	1	1.00	0.58	0.58	0.85	6.08	0.32	1.35	1.46
time (sec)	N/A	0.160	0.193	0.089	0.502	0.538	4.889	1.036	0.093

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	155	159	291	1245	423	360	1107
N.S.	1	1.00	0.58	0.60	1.09	4.68	1.59	1.35	4.16
time (sec)	N/A	0.153	0.180	0.095	0.503	0.519	4.694	1.238	0.204

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	154	153	223	1636	282	344	416
N.S.	1	1.00	0.59	0.59	0.86	6.29	1.08	1.32	1.60
time (sec)	N/A	0.186	0.191	0.099	0.525	0.531	12.516	0.881	0.189

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	155	155	286	1253	408	344	1201
N.S.	1	1.00	0.60	0.60	1.10	4.82	1.57	1.32	4.62
time (sec)	N/A	0.189	0.181	0.093	0.512	0.494	9.479	0.943	0.211

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	158	159	229	1648	299	353	417
N.S.	1	1.00	0.59	0.60	0.86	6.17	1.12	1.32	1.56
time (sec)	N/A	0.203	0.192	0.091	0.499	0.505	42.316	0.667	0.190

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	159	156	293	1252	449	354	1209
N.S.	1	1.00	0.59	0.58	1.09	4.65	1.67	1.32	4.49
time (sec)	N/A	0.193	0.194	0.098	0.505	0.481	68.541	0.737	0.238

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	192	186	263	1686	428	390	451
N.S.	1	1.00	0.67	0.65	0.91	5.85	1.49	1.35	1.57
time (sec)	N/A	0.215	0.227	0.100	0.499	0.538	184.599	0.628	0.203

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	255	240	377	1373	0	440	1367
N.S.	1	1.00	0.68	0.64	1.01	3.66	0.00	1.17	3.65
time (sec)	N/A	0.300	0.632	0.118	0.535	0.523	0.000	0.632	0.241

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	223	200	272	1733	0	413	160
N.S.	1	1.00	0.64	0.58	0.79	5.01	0.00	1.19	0.46
time (sec)	N/A	0.214	0.571	0.116	0.517	1.377	0.000	0.614	0.116

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	221	199	336	1334	1280	408	1238
N.S.	1	1.00	0.64	0.58	0.97	3.86	3.70	1.18	3.58
time (sec)	N/A	0.212	0.576	0.120	0.543	1.533	70.037	0.619	0.140

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	204	187	258	1723	173	388	137
N.S.	1	1.00	0.66	0.60	0.83	5.56	0.56	1.25	0.44
time (sec)	N/A	0.187	0.611	0.110	0.515	1.233	16.354	0.589	0.222

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	202	185	327	1341	1248	388	1267
N.S.	1	1.00	0.65	0.59	1.05	4.30	4.00	1.24	4.06
time (sec)	N/A	0.221	0.588	0.109	0.522	0.919	25.557	0.639	0.245

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	331	209	185	260	1739	976	389	138
N.S.	1	0.99	0.63	0.56	0.78	5.22	2.93	1.17	0.41
time (sec)	N/A	0.225	0.590	0.112	0.523	1.083	129.415	0.626	0.216

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	329	208	188	326	1340	1418	384	1340
N.S.	1	0.99	0.63	0.57	0.98	4.04	4.27	1.16	4.04
time (sec)	N/A	0.217	0.608	0.115	0.517	1.655	118.601	0.618	0.314

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	360	227	200	275	1737	0	401	152
N.S.	1	0.99	0.63	0.55	0.76	4.79	0.00	1.10	0.42
time (sec)	N/A	0.255	0.608	0.134	0.521	2.320	0.000	0.608	0.120

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	256	234	389	1427	0	451	1426
N.S.	1	1.00	0.58	0.53	0.88	3.24	0.00	1.02	3.24
time (sec)	N/A	0.257	0.735	0.154	0.524	2.121	0.000	0.597	0.254

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	235	220	306	1813	0	427	197
N.S.	1	1.00	0.59	0.55	0.76	4.52	0.00	1.06	0.49
time (sec)	N/A	0.231	0.767	0.148	0.498	1.196	0.000	0.615	0.219

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	232	216	374	1420	2302	426	1236
N.S.	1	1.00	0.58	0.54	0.93	3.53	5.73	1.06	3.07
time (sec)	N/A	0.221	0.822	0.128	0.510	1.184	160.525	0.562	0.247

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	224	213	297	1811	325	416	184
N.S.	1	1.00	0.62	0.59	0.82	4.98	0.89	1.14	0.51
time (sec)	N/A	0.193	0.686	0.091	0.517	1.202	58.560	0.554	0.220

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	225	213	366	1416	2258	416	1419
N.S.	1	1.00	0.62	0.59	1.01	3.89	6.20	1.14	3.90
time (sec)	N/A	0.207	0.652	0.092	0.527	0.873	144.213	0.594	0.327

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	396	238	216	307	1819	0	427	192
N.S.	1	0.99	0.60	0.54	0.77	4.56	0.00	1.07	0.48
time (sec)	N/A	0.262	0.781	0.144	0.499	0.770	0.000	0.619	0.216

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	398	241	220	377	1433	0	426	1508
N.S.	1	0.99	0.60	0.55	0.94	3.56	0.00	1.06	3.75
time (sec)	N/A	0.261	0.768	0.151	0.516	1.181	0.000	1.408	0.347

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	435	261	235	324	1832	0	444	208
N.S.	1	0.99	0.59	0.54	0.74	4.17	0.00	1.01	0.47
time (sec)	N/A	0.305	0.828	0.170	0.519	1.136	0.000	2.634	0.126

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	230	259	331	2528	743	531	634
N.S.	1	1.00	0.70	0.79	1.01	7.71	2.27	1.62	1.93
time (sec)	N/A	0.200	0.251	0.090	0.513	0.821	87.959	2.073	0.213

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	231	268	437	1898	736	531	1564
N.S.	1	1.00	0.71	0.82	1.34	5.82	2.26	1.63	4.80
time (sec)	N/A	0.193	0.249	0.092	0.547	0.633	37.783	1.214	0.216

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	188	208	282	2441	649	490	574
N.S.	1	1.00	0.61	0.68	0.92	7.98	2.12	1.60	1.88
time (sec)	N/A	0.175	0.225	0.095	0.539	0.646	18.282	1.610	0.086

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	188	214	390	1862	649	490	1460
N.S.	1	1.00	0.62	0.70	1.28	6.12	2.13	1.61	4.80
time (sec)	N/A	0.174	0.232	0.095	0.515	0.579	13.444	1.305	0.208

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	178	186	261	2442	435	462	580
N.S.	1	1.00	0.63	0.65	0.92	8.60	1.53	1.63	2.04
time (sec)	N/A	0.192	0.236	0.096	0.516	0.597	32.073	1.299	0.096

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	178	186	368	1866	604	461	1561
N.S.	1	1.00	0.63	0.65	1.30	6.57	2.13	1.62	5.50
time (sec)	N/A	0.179	0.240	0.092	0.536	0.552	21.604	2.188	0.123

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	177	188	259	2451	432	455	583
N.S.	1	1.00	0.63	0.66	0.92	8.66	1.53	1.61	2.06
time (sec)	N/A	0.191	0.228	0.097	0.503	0.586	59.728	1.414	0.200

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	177	188	368	1861	607	455	1564
N.S.	1	1.00	0.63	0.66	1.30	6.58	2.14	1.61	5.53
time (sec)	N/A	0.177	0.227	0.096	0.514	0.551	66.493	1.613	0.133

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	193	208	281	2459	469	483	591
N.S.	1	1.00	0.64	0.69	0.93	8.12	1.55	1.59	1.95
time (sec)	N/A	0.201	0.243	0.093	0.530	0.655	179.308	1.596	0.209

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	194	205	389	1866	0	483	1580
N.S.	1	1.00	0.64	0.67	1.28	6.12	0.00	1.58	5.18
time (sec)	N/A	0.183	0.247	0.094	0.494	0.757	0.000	1.107	0.287

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	238	249	330	2512	0	536	639
N.S.	1	1.00	0.73	0.77	1.02	7.73	0.00	1.65	1.97
time (sec)	N/A	0.212	0.258	0.096	0.523	0.876	0.000	2.344	0.215

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	301	327	499	2014	0	600	1850
N.S.	1	1.00	0.74	0.80	1.22	4.92	0.00	1.47	4.52
time (sec)	N/A	0.313	0.572	0.146	0.501	0.792	0.000	0.945	0.162

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	257	266	337	2542	0	552	681
N.S.	1	1.00	0.69	0.71	0.90	6.80	0.00	1.48	1.82
time (sec)	N/A	0.286	0.525	0.133	0.530	0.648	0.000	1.539	0.219

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	256	269	445	1961	1833	552	1691
N.S.	1	1.00	0.66	0.70	1.15	5.08	4.75	1.43	4.38
time (sec)	N/A	0.359	0.536	0.122	0.552	0.542	158.912	1.552	0.235

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	227	235	309	2531	173	516	616
N.S.	1	1.00	0.60	0.62	0.82	6.73	0.46	1.37	1.64
time (sec)	N/A	0.288	0.466	0.114	0.517	0.577	40.664	1.098	0.118

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	227	231	412	1944	1775	511	1636
N.S.	1	1.00	0.67	0.68	1.21	5.72	5.22	1.50	4.81
time (sec)	N/A	0.258	0.469	0.109	0.503	0.545	56.920	1.703	0.135

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	229	225	304	2547	1443	504	657
N.S.	1	1.00	0.62	0.61	0.83	6.92	3.92	1.37	1.79
time (sec)	N/A	0.294	0.483	0.146	0.505	0.562	202.380	1.815	0.225

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	229	225	415	1967	2008	501	1759
N.S.	1	1.00	0.62	0.61	1.13	5.36	5.47	1.37	4.79
time (sec)	N/A	0.286	0.493	0.125	0.501	0.506	116.771	0.952	0.248

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	236	232	315	2549	0	505	656
N.S.	1	1.00	0.63	0.62	0.84	6.78	0.00	1.34	1.74
time (sec)	N/A	0.292	0.562	0.125	0.552	0.576	0.000	1.339	0.136

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	236	236	424	1955	0	509	1746
N.S.	1	1.00	0.63	0.63	1.13	5.20	0.00	1.35	4.64
time (sec)	N/A	0.285	0.552	0.119	0.551	0.513	0.000	1.018	0.332

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	249	249	390	1422	0	476	2500
N.S.	1	1.00	0.52	0.52	0.82	2.97	0.00	1.00	5.23
time (sec)	N/A	0.381	0.505	0.110	0.542	1.810	0.000	1.771	1.080

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	247	245	384	1388	0	476	2500
N.S.	1	1.00	0.52	0.51	0.81	2.92	0.00	1.00	5.25
time (sec)	N/A	0.329	0.460	0.112	0.570	0.687	0.000	1.637	0.864

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	219	234	369	1385	0	457	2609
N.S.	1	1.00	0.47	0.51	0.80	2.99	0.00	0.99	5.63
time (sec)	N/A	0.244	0.386	0.095	0.501	0.551	0.000	1.116	0.677

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	219	226	367	1249	0	441	2500
N.S.	1	1.00	0.47	0.49	0.79	2.70	0.00	0.95	5.40
time (sec)	N/A	0.244	0.347	0.089	0.500	0.497	0.000	1.443	0.766

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	219	226	369	1285	0	481	2500
N.S.	1	1.00	0.47	0.49	0.80	2.78	0.00	1.04	5.40
time (sec)	N/A	0.251	0.406	0.091	0.518	0.510	0.000	1.582	0.638

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	219	234	371	1365	0	441	2500
N.S.	1	1.00	0.47	0.51	0.80	2.95	0.00	0.95	5.40
time (sec)	N/A	0.239	0.403	0.109	0.502	0.753	0.000	1.456	0.878

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	247	245	390	1421	0	492	2500
N.S.	1	1.00	0.52	0.51	0.82	2.99	0.00	1.03	5.25
time (sec)	N/A	0.367	0.525	0.113	0.521	0.892	0.000	2.367	0.897

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	249	249	396	1431	0	476	2500
N.S.	1	1.00	0.52	0.52	0.83	2.99	0.00	1.00	5.23
time (sec)	N/A	0.325	0.530	0.113	0.527	3.533	0.000	3.395	1.312

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	285	269	411	1484	0	487	2500
N.S.	1	1.00	0.57	0.54	0.83	2.98	0.00	0.98	5.02
time (sec)	N/A	0.456	0.607	0.127	0.563	10.086	0.000	1.376	1.400

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	301	286	494	3393	0	718	2500
N.S.	1	1.00	0.53	0.50	0.87	5.95	0.00	1.26	4.39
time (sec)	N/A	0.551	1.086	0.121	0.561	69.522	0.000	2.175	1.620

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	307	273	450	3524	0	681	2500
N.S.	1	1.00	0.57	0.51	0.84	6.57	0.00	1.27	4.66
time (sec)	N/A	0.408	0.894	0.091	0.525	39.888	0.000	1.593	1.343

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	304	271	468	3224	0	669	2500
N.S.	1	1.00	0.57	0.51	0.88	6.06	0.00	1.26	4.70
time (sec)	N/A	0.355	0.856	0.097	0.529	10.401	0.000	1.321	1.288

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	269	264	436	3393	0	683	2500
N.S.	1	1.00	0.51	0.50	0.83	6.43	0.00	1.29	4.73
time (sec)	N/A	0.390	0.795	0.093	0.507	12.185	0.000	2.101	1.268

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	268	263	461	3171	0	655	2500
N.S.	1	1.00	0.51	0.50	0.87	6.01	0.00	1.24	4.73
time (sec)	N/A	0.313	0.788	0.085	0.520	4.532	0.000	1.418	1.259

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	273	270	450	3457	0	701	2500
N.S.	1	1.00	0.51	0.50	0.84	6.45	0.00	1.31	4.66
time (sec)	N/A	0.408	1.006	0.092	0.540	12.243	0.000	1.601	1.234

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	273	273	489	3310	0	673	2500
N.S.	1	1.00	0.51	0.51	0.91	6.18	0.00	1.26	4.66
time (sec)	N/A	0.362	1.009	0.092	0.563	12.906	0.000	1.777	1.418

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	332	286	494	3630	0	725	2500
N.S.	1	1.00	0.58	0.50	0.87	6.37	0.00	1.27	4.39
time (sec)	N/A	0.516	1.166	0.128	0.538	24.132	0.000	1.785	2.336

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	334	286	539	3439	0	718	2500
N.S.	1	1.00	0.59	0.50	0.95	6.03	0.00	1.26	4.39
time (sec)	N/A	0.550	1.208	0.135	0.516	71.491	0.000	1.641	3.829

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	378	306	551	3728	0	715	2500
N.S.	1	1.00	0.61	0.50	0.89	6.03	0.00	1.16	4.05
time (sec)	N/A	0.651	1.276	0.138	0.523	100.565	0.000	1.805	3.003

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	328	330	653	5496	0	944	2500
N.S.	1	1.00	0.52	0.52	1.03	8.71	0.00	1.50	3.96
time (sec)	N/A	0.546	1.607	0.091	0.542	100.799	0.000	2.294	2.318

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	329	330	583	0	0	963	2500
N.S.	1	1.00	0.52	0.53	0.93	0.00	0.00	1.53	3.98
time (sec)	N/A	0.522	1.166	0.089	0.519	0.000	0.000	1.363	2.296

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	627	627	327	327	654	5450	0	946	2500
N.S.	1	1.00	0.52	0.52	1.04	8.69	0.00	1.51	3.99
time (sec)	N/A	0.484	1.180	0.092	0.513	88.785	0.000	1.811	2.346

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	633	361	336	594	0	0	968	2500
N.S.	1	1.00	0.57	0.53	0.94	0.00	0.00	1.53	3.95
time (sec)	N/A	0.553	1.378	0.086	0.521	0.000	0.000	1.048	2.372

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	633	362	336	675	5548	0	960	2500
N.S.	1	1.00	0.57	0.53	1.07	8.76	0.00	1.52	3.95
time (sec)	N/A	0.572	1.408	0.086	0.522	257.054	0.000	2.852	2.333

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	410	348	668	0	0	987	2500
N.S.	1	1.00	0.60	0.51	0.98	0.00	0.00	1.45	3.67
time (sec)	N/A	0.677	1.152	0.185	0.538	0.000	0.000	1.989	6.014

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	410	348	755	0	0	995	2500
N.S.	1	1.00	0.60	0.51	1.11	0.00	0.00	1.46	3.67
time (sec)	N/A	0.624	1.615	0.187	0.535	0.000	0.000	1.735	5.865

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	743	743	462	368	756	0	0	1000	2500
N.S.	1	1.00	0.62	0.50	1.02	0.00	0.00	1.35	3.36
time (sec)	N/A	0.826	1.175	0.227	0.609	0.000	0.000	2.921	7.028

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	344	302	617	5375	0	912	2500
N.S.	1	1.00	0.55	0.48	0.99	8.61	0.00	1.46	4.01
time (sec)	N/A	0.489	1.306	0.111	0.538	32.217	0.000	1.921	2.169

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	340	302	567	5814	0	952	2500
N.S.	1	1.00	0.56	0.50	0.93	9.55	0.00	1.56	4.11
time (sec)	N/A	0.520	1.606	0.118	0.511	63.140	0.000	1.107	2.114

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	338	300	620	5474	0	904	2500
N.S.	1	1.00	0.56	0.50	1.03	9.11	0.00	1.50	4.16
time (sec)	N/A	0.451	1.732	0.114	0.510	83.045	0.000	2.561	2.072

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	357	317	610	6028	0	973	2500
N.S.	1	1.00	0.57	0.51	0.98	9.66	0.00	1.56	4.01
time (sec)	N/A	0.574	1.423	0.115	0.544	84.733	0.000	1.638	2.454

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	361	312	678	0	0	977	2500
N.S.	1	1.00	0.57	0.50	1.08	0.00	0.00	1.56	3.98
time (sec)	N/A	0.581	1.672	0.111	0.525	0.000	0.000	1.100	2.333

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	421	323	694	6207	0	1035	2500
N.S.	1	1.00	0.62	0.48	1.03	9.18	0.00	1.53	3.70
time (sec)	N/A	0.712	1.196	0.191	0.561	270.749	0.000	1.297	5.959

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	425	323	761	0	0	1012	2500
N.S.	1	1.00	0.63	0.48	1.13	0.00	0.00	1.50	3.70
time (sec)	N/A	0.644	1.278	0.189	0.531	0.000	0.000	1.515	5.609

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	460	343	774	0	0	1015	2500
N.S.	1	1.00	0.63	0.47	1.06	0.00	0.00	1.39	3.42
time (sec)	N/A	0.857	1.332	0.204	0.580	0.000	0.000	2.163	6.625

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	383	364	855	0	0	1193	2500
N.S.	1	1.00	0.53	0.51	1.19	0.00	0.00	1.66	3.48
time (sec)	N/A	0.676	1.775	0.175	0.580	0.000	0.000	1.617	3.340

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	703	703	396	368	791	0	0	1238	2500
N.S.	1	1.00	0.56	0.52	1.13	0.00	0.00	1.76	3.56
time (sec)	N/A	0.677	2.593	0.179	0.514	0.000	0.000	1.969	3.633

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	703	703	392	364	889	0	0	1217	2500
N.S.	1	1.00	0.56	0.52	1.26	0.00	0.00	1.73	3.56
time (sec)	N/A	0.684	2.803	0.179	0.555	0.000	0.000	1.876	3.537

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	451	381	845	0	0	1233	2500
N.S.	1	1.00	0.61	0.52	1.14	0.00	0.00	1.67	3.38
time (sec)	N/A	0.757	2.045	0.181	0.535	0.000	0.000	2.353	4.253

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	450	375	951	0	0	1253	2500
N.S.	1	1.00	0.61	0.51	1.29	0.00	0.00	1.70	3.38
time (sec)	N/A	0.674	3.197	0.187	0.544	0.000	0.000	2.588	8.100

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	805	805	518	385	955	0	0	1333	2500
N.S.	1	1.00	0.64	0.48	1.19	0.00	0.00	1.66	3.11
time (sec)	N/A	0.933	2.111	0.271	0.587	0.000	0.000	2.900	11.671

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	805	805	521	385	1064	0	0	1278	2500
N.S.	1	1.00	0.65	0.48	1.32	0.00	0.00	1.59	3.11
time (sec)	N/A	0.897	6.115	0.349	0.544	0.000	0.000	1.797	8.728

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	881	881	598	405	1066	0	0	1289	2500
N.S.	1	1.00	0.68	0.46	1.21	0.00	0.00	1.46	2.84
time (sec)	N/A	1.138	1.967	0.408	0.541	0.000	0.000	2.609	10.135

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	75	144	132	99	212	104	96
N.S.	1	1.00	0.73	1.40	1.28	0.96	2.06	1.01	0.93
time (sec)	N/A	0.060	0.059	0.077	0.330	3.274	0.233	1.437	0.303

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	124	192	166	257	286	132	-1
N.S.	1	1.00	0.80	1.24	1.07	1.66	1.85	0.85	-0.01
time (sec)	N/A	0.051	0.173	0.092	0.332	3.109	25.566	1.171	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	96	90	75	162	73	76
N.S.	1	1.00	0.77	1.32	1.23	1.03	2.22	1.00	1.04
time (sec)	N/A	0.042	0.046	0.083	0.323	2.346	0.151	1.188	0.292

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	100	144	124	206	226	100	-1
N.S.	1	1.00	0.82	1.18	1.02	1.69	1.85	0.82	-0.01
time (sec)	N/A	0.040	0.117	0.083	0.289	2.192	7.346	2.956	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	52	50	50	110	44	53
N.S.	1	1.00	0.74	1.13	1.09	1.09	2.39	0.96	1.15
time (sec)	N/A	0.025	0.027	0.076	0.297	1.773	0.098	1.443	0.265

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	74	98	81	155	144	69	-1
N.S.	1	1.00	0.85	1.13	0.93	1.78	1.66	0.79	-0.01
time (sec)	N/A	0.020	0.094	0.079	0.277	2.620	3.290	1.884	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	45	123	76	60	47
N.S.	1	1.00	1.00	0.97	0.76	2.08	1.29	1.02	0.80
time (sec)	N/A	0.033	0.061	0.082	0.283	1.781	10.926	1.607	0.424

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	67	100	59	134	107	84	94
N.S.	1	1.00	0.80	1.19	0.70	1.60	1.27	1.00	1.12
time (sec)	N/A	0.024	0.115	0.083	0.276	2.265	2.048	1.487	0.561

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	106	83	141	107	68	68
N.S.	1	1.00	0.77	1.26	0.99	1.68	1.27	0.81	0.81
time (sec)	N/A	0.046	0.114	0.098	0.295	1.185	15.653	2.364	0.621

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	81	48	137	107	151	76
N.S.	1	1.00	1.06	1.23	0.73	2.08	1.62	2.29	1.15
time (sec)	N/A	0.019	0.099	0.081	0.274	2.126	1.511	1.379	0.679

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	78	154	130	170	144	120	93
N.S.	1	1.00	0.89	1.75	1.48	1.93	1.64	1.36	1.06
time (sec)	N/A	0.050	0.138	0.091	0.274	1.180	41.406	1.262	0.783

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	58	56	55	119	232	97
N.S.	1	1.00	0.75	1.09	1.06	1.04	2.25	4.38	1.83
time (sec)	N/A	0.016	0.099	0.085	0.280	1.207	1.306	1.838	0.433

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	102	202	174	221	226	140	134
N.S.	1	1.00	0.85	1.68	1.45	1.84	1.88	1.17	1.12
time (sec)	N/A	0.068	0.163	0.089	0.289	1.499	54.834	1.770	1.017

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	62	102	96	81	442	288	132
N.S.	1	1.00	0.74	1.21	1.14	0.96	5.26	3.43	1.57
time (sec)	N/A	0.023	0.119	0.090	0.318	1.507	1.618	0.879	0.554

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	126	250	216	269	286	194	173
N.S.	1	1.00	0.81	1.60	1.38	1.72	1.83	1.24	1.11
time (sec)	N/A	0.087	0.219	0.095	0.298	1.383	99.138	0.850	1.235

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	81	150	138	105	957	344	174
N.S.	1	1.00	0.69	1.28	1.18	0.90	8.18	2.94	1.49
time (sec)	N/A	0.039	0.152	0.093	0.326	1.500	2.106	1.310	0.675

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	146	298	258	317	347	230	209
N.S.	1	1.00	0.77	1.58	1.37	1.68	1.84	1.22	1.11
time (sec)	N/A	0.102	0.261	0.105	0.312	1.219	209.280	1.418	1.594

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	80	144	132	124	260	104	117
N.S.	1	1.00	0.78	1.40	1.28	1.20	2.52	1.01	1.14
time (sec)	N/A	0.053	0.061	0.080	0.280	1.262	0.404	1.516	0.351

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	142	224	204	299	345	159	-1
N.S.	1	1.00	0.76	1.19	1.09	1.59	1.84	0.85	-0.01
time (sec)	N/A	0.064	0.180	0.079	0.278	1.016	164.253	2.354	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	96	90	99	209	73	96
N.S.	1	1.00	0.77	1.32	1.23	1.36	2.86	1.00	1.32
time (sec)	N/A	0.041	0.044	0.079	0.274	1.537	0.293	1.367	0.306

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	122	176	162	260	287	133	-1
N.S.	1	1.00	0.79	1.14	1.05	1.68	1.85	0.86	-0.01
time (sec)	N/A	0.047	0.160	0.082	0.291	1.195	33.489	1.020	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	52	50	73	158	44	76
N.S.	1	1.00	0.74	1.13	1.09	1.59	3.43	0.96	1.65
time (sec)	N/A	0.025	0.030	0.077	0.289	1.375	0.186	2.268	0.294

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	99	130	116	207	253	102	-1
N.S.	1	1.00	0.84	1.10	0.98	1.75	2.14	0.86	-0.01
time (sec)	N/A	0.026	0.141	0.080	0.314	1.520	10.284	1.117	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	83	71	58	170	71	79	60
N.S.	1	1.00	1.09	0.93	0.76	2.24	0.93	1.04	0.79
time (sec)	N/A	0.037	0.080	0.076	0.351	1.371	24.401	1.506	0.452

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	84	132	91	182	216	114	80
N.S.	1	1.00	0.77	1.21	0.83	1.67	1.98	1.05	0.73
time (sec)	N/A	0.030	0.169	0.087	0.312	1.315	5.183	1.899	0.702

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	81	134	109	167	184	103	94
N.S.	1	1.00	0.74	1.22	0.99	1.52	1.67	0.94	0.85
time (sec)	N/A	0.059	0.123	0.095	0.275	1.577	19.806	1.505	0.748

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	84	180	115	166	202	207	-1
N.S.	1	1.00	0.71	1.51	0.97	1.39	1.70	1.74	-0.01
time (sec)	N/A	0.035	0.182	0.087	0.274	1.118	3.206	1.234	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	182	161	189	216	131	104
N.S.	1	1.00	0.70	1.58	1.40	1.64	1.88	1.14	0.90
time (sec)	N/A	0.060	0.175	0.104	0.307	1.282	50.514	1.473	0.973

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	92	121	88	184	184	236	-1
N.S.	1	1.00	1.07	1.41	1.02	2.14	2.14	2.74	-0.01
time (sec)	N/A	0.026	0.167	0.093	0.284	1.353	2.592	1.294	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	102	230	210	222	253	159	130
N.S.	1	1.00	0.85	1.92	1.75	1.85	2.11	1.32	1.08
time (sec)	N/A	0.067	0.180	0.096	0.305	1.448	73.369	2.418	1.267

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	58	56	78	518	344	128
N.S.	1	1.00	0.75	1.09	1.06	1.47	9.77	6.49	2.42
time (sec)	N/A	0.014	0.156	0.092	0.280	1.203	2.752	3.263	0.733

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	126	278	252	271	287	194	169
N.S.	1	1.00	0.81	1.78	1.62	1.74	1.84	1.24	1.08
time (sec)	N/A	0.086	0.232	0.097	0.271	1.683	135.607	1.364	1.721

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	62	102	96	105	1408	400	170
N.S.	1	1.00	0.74	1.21	1.14	1.25	16.76	4.76	2.02
time (sec)	N/A	0.025	0.195	0.094	0.278	1.231	3.422	0.929	0.953

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	142	326	294	317	0	212	205
N.S.	1	1.00	0.77	1.77	1.60	1.72	0.00	1.15	1.11
time (sec)	N/A	0.104	0.277	0.105	0.278	1.591	0.000	0.611	2.138

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	80	144	132	147	313	104	136
N.S.	1	1.00	0.78	1.40	1.28	1.43	3.04	1.01	1.32
time (sec)	N/A	0.052	0.067	0.078	0.279	1.192	0.722	0.626	0.383

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	164	256	242	355	0	195	-1
N.S.	1	1.00	0.74	1.16	1.10	1.61	0.00	0.88	-0.00
time (sec)	N/A	0.074	0.256	0.089	0.289	1.384	0.000	0.642	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	96	90	122	260	73	115
N.S.	1	1.00	0.77	1.32	1.23	1.67	3.56	1.00	1.58
time (sec)	N/A	0.041	0.048	0.080	0.325	1.192	0.526	0.655	0.365

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	143	208	200	308	348	165	-1
N.S.	1	1.00	0.76	1.11	1.06	1.64	1.85	0.88	-0.01
time (sec)	N/A	0.060	0.221	0.085	0.297	1.298	168.714	0.705	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	52	50	97	209	44	44
N.S.	1	1.00	0.74	1.13	1.09	2.11	4.54	0.96	0.96
time (sec)	N/A	0.025	0.031	0.082	0.326	1.254	0.372	0.704	0.332

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	123	162	151	257	316	134	-1
N.S.	1	1.00	0.83	1.09	1.01	1.72	2.12	0.90	-0.01
time (sec)	N/A	0.035	0.191	0.079	0.312	1.587	35.808	0.680	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	103	85	73	220	88	97	78
N.S.	1	1.00	1.08	0.89	0.77	2.32	0.93	1.02	0.82
time (sec)	N/A	0.048	0.094	0.079	0.368	1.278	33.899	0.945	0.483

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	110	164	124	236	306	146	80
N.S.	1	1.00	0.81	1.21	0.91	1.74	2.25	1.07	0.59
time (sec)	N/A	0.037	0.232	0.086	0.375	1.869	12.619	0.889	0.871

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	105	162	138	221	296	139	132
N.S.	1	1.00	0.78	1.20	1.02	1.64	2.19	1.03	0.98
time (sec)	N/A	0.069	0.180	0.099	0.295	1.103	23.276	1.019	0.849

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	109	212	151	220	299	238	-1
N.S.	1	1.00	0.75	1.45	1.03	1.51	2.05	1.63	-0.01
time (sec)	N/A	0.040	0.285	0.090	0.331	1.711	6.557	1.517	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	106	210	190	221	279	171	144
N.S.	1	1.00	0.74	1.47	1.33	1.55	1.95	1.20	1.01
time (sec)	N/A	0.069	0.199	0.105	0.293	1.898	54.514	1.291	1.157

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	108	260	198	220	292	321	-1
N.S.	1	1.00	0.71	1.71	1.30	1.45	1.92	2.11	-0.01
time (sec)	N/A	0.044	0.240	0.090	0.302	1.132	4.360	1.457	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	107	258	243	241	306	167	150
N.S.	1	1.00	0.72	1.73	1.63	1.62	2.05	1.12	1.01
time (sec)	N/A	0.073	0.209	0.099	0.353	1.286	79.275	1.306	1.494

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	112	161	128	234	592	320	-1
N.S.	1	1.00	1.04	1.49	1.19	2.17	5.48	2.96	-0.01
time (sec)	N/A	0.032	0.231	0.092	0.290	1.421	3.753	1.764	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	306	288	272	316	195	169
N.S.	1	1.00	0.81	2.01	1.89	1.79	2.08	1.28	1.11
time (sec)	N/A	0.082	0.217	0.099	0.291	1.819	158.077	1.376	2.046

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	58	56	102	1489	456	170
N.S.	1	1.00	0.75	1.09	1.06	1.92	28.09	8.60	3.21
time (sec)	N/A	0.014	0.224	0.099	0.318	1.481	4.348	1.200	1.278

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	143	354	330	319	0	230	205
N.S.	1	1.00	0.76	1.87	1.75	1.69	0.00	1.22	1.08
time (sec)	N/A	0.099	0.312	0.125	0.309	1.346	0.000	2.240	2.733

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	80	144	132	76	172	101	80
N.S.	1	1.00	0.80	1.44	1.32	0.76	1.72	1.01	0.80
time (sec)	N/A	0.054	0.054	0.085	0.292	1.337	0.350	1.405	0.321

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	100	154	128	211	235	107	-1
N.S.	1	1.00	0.82	1.26	1.05	1.73	1.93	0.88	-0.01
time (sec)	N/A	0.035	0.135	0.088	0.290	1.208	10.883	1.328	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	56	96	90	52	121	69	57
N.S.	1	1.00	0.79	1.35	1.27	0.73	1.70	0.97	0.80
time (sec)	N/A	0.038	0.039	0.081	0.299	1.513	0.304	1.279	0.303

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	76	106	86	162	150	75	-1
N.S.	1	1.00	0.85	1.19	0.97	1.82	1.69	0.84	-0.01
time (sec)	N/A	0.025	0.097	0.087	0.296	1.119	4.206	0.925	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	51	49	29	70	38	34
N.S.	1	1.00	0.77	1.19	1.14	0.67	1.63	0.88	0.79
time (sec)	N/A	0.025	0.025	0.080	0.316	1.325	0.233	1.267	0.281

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	63	47	110	126	48	86
N.S.	1	1.00	1.02	1.09	0.81	1.90	2.17	0.83	1.48
time (sec)	N/A	0.012	0.050	0.078	0.311	1.274	1.521	1.731	0.497

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	45	33	102	61	38	35
N.S.	1	1.00	1.00	1.05	0.77	2.37	1.42	0.88	0.81
time (sec)	N/A	0.023	0.038	0.079	0.334	2.901	4.745	1.275	0.519

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	50	41	33	109	99	58	40
N.S.	1	1.00	1.06	0.87	0.70	2.32	2.11	1.23	0.85
time (sec)	N/A	0.012	0.060	0.082	0.275	1.578	0.762	0.934	0.358

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	80	56	124	66	62	60
N.S.	1	1.00	1.00	1.38	0.97	2.14	1.14	1.07	1.03
time (sec)	N/A	0.030	0.071	0.095	0.285	1.447	11.299	1.226	0.605

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	58	56	34	70	120	35
N.S.	1	1.00	0.75	1.09	1.06	0.64	1.32	2.26	0.66
time (sec)	N/A	0.014	0.071	0.083	0.285	1.860	0.962	1.226	0.287

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	124	96	171	150	121	99
N.S.	1	1.00	0.87	1.38	1.07	1.90	1.67	1.34	1.10
time (sec)	N/A	0.048	0.135	0.092	0.296	1.965	25.875	1.410	0.705

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	62	102	96	58	355	176	58
N.S.	1	1.00	0.74	1.21	1.14	0.69	4.23	2.10	0.69
time (sec)	N/A	0.023	0.101	0.092	0.320	1.679	1.293	1.433	0.343

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	102	172	138	223	235	158	140
N.S.	1	1.00	0.83	1.40	1.12	1.81	1.91	1.28	1.14
time (sec)	N/A	0.064	0.200	0.090	0.285	1.702	46.844	1.199	0.806

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	86	150	138	82	819	232	105
N.S.	1	1.00	0.74	1.28	1.18	0.70	7.00	1.98	0.90
time (sec)	N/A	0.033	0.115	0.094	0.302	1.204	1.659	1.170	0.355

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	124	198	170	325	233	136	-1
N.S.	1	1.00	0.82	1.30	1.12	2.14	1.53	0.89	-0.01
time (sec)	N/A	0.048	0.202	0.099	0.287	1.973	20.201	1.037	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	77	142	132	88	172	113	89
N.S.	1	1.00	0.78	1.43	1.33	0.89	1.74	1.14	0.90
time (sec)	N/A	0.052	0.054	0.089	0.320	1.680	0.442	0.852	0.409

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	150	126	274	177	104	-1
N.S.	1	1.00	0.83	1.26	1.06	2.30	1.49	0.87	-0.01
time (sec)	N/A	0.035	0.166	0.095	0.314	1.552	7.340	1.259	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	94	89	63	117	77	59
N.S.	1	1.00	0.82	1.40	1.33	0.94	1.75	1.15	0.88
time (sec)	N/A	0.037	0.042	0.088	0.301	2.229	0.332	1.229	0.334

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	102	82	213	114	70	-1
N.S.	1	1.00	0.90	1.23	0.99	2.57	1.37	0.84	-0.01
time (sec)	N/A	0.039	0.130	0.095	0.309	1.426	3.898	1.362	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	51	49	40	66	36	30
N.S.	1	1.00	0.73	1.24	1.20	0.98	1.61	0.88	0.73
time (sec)	N/A	0.022	0.030	0.089	0.319	3.553	0.276	0.928	0.293

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	58	55	46	168	60	51	53
N.S.	1	1.00	1.07	1.02	0.85	3.11	1.11	0.94	0.98
time (sec)	N/A	0.013	0.086	0.094	0.296	1.585	2.427	1.333	0.367

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	61	48	167	48	52	50
N.S.	1	1.00	1.00	1.15	0.91	3.15	0.91	0.98	0.94
time (sec)	N/A	0.026	0.068	0.086	0.280	1.594	7.575	1.144	0.485

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	53	51	43	68	57	46
N.S.	1	1.00	0.77	1.13	1.09	0.91	1.45	1.21	0.98
time (sec)	N/A	0.013	0.064	0.093	0.319	2.120	2.800	0.824	0.272

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	114	86	232	262	99	90
N.S.	1	1.00	0.90	1.33	1.00	2.70	3.05	1.15	1.05
time (sec)	N/A	0.047	0.137	0.102	0.289	2.011	15.971	0.843	0.713

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	62	98	92	68	284	181	57
N.S.	1	1.00	0.76	1.20	1.12	0.83	3.46	2.21	0.70
time (sec)	N/A	0.021	0.098	0.095	0.290	1.732	3.764	1.094	0.331

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	100	162	130	287	180	137	134
N.S.	1	1.00	0.85	1.37	1.10	2.43	1.53	1.16	1.14
time (sec)	N/A	0.061	0.177	0.105	0.294	1.269	32.942	1.065	0.912

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	86	146	134	94	593	294	82
N.S.	1	1.00	0.75	1.27	1.17	0.82	5.16	2.56	0.71
time (sec)	N/A	0.031	0.127	0.097	0.294	1.557	4.865	0.897	0.437

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	126	210	174	341	236	180	178
N.S.	1	1.00	0.82	1.37	1.14	2.23	1.54	1.18	1.16
time (sec)	N/A	0.083	0.227	0.117	0.291	2.341	60.826	0.668	1.076

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	105	194	176	117	1030	407	148
N.S.	1	1.00	0.71	1.31	1.19	0.79	6.96	2.75	1.00
time (sec)	N/A	0.042	0.166	0.106	0.322	1.478	6.741	0.567	0.506

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	98	190	174	123	437	141	122
N.S.	1	1.00	0.77	1.48	1.36	0.96	3.41	1.10	0.95
time (sec)	N/A	0.069	0.069	0.099	0.301	1.770	0.579	0.512	0.484

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	119	194	210	392	804	148	-1
N.S.	1	1.00	0.80	1.30	1.41	2.63	5.40	0.99	-0.01
time (sec)	N/A	0.044	0.227	0.130	0.320	2.117	16.137	0.472	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	142	131	98	337	104	89
N.S.	1	1.00	0.75	1.46	1.35	1.01	3.47	1.07	0.92
time (sec)	N/A	0.054	0.054	0.104	0.276	1.277	0.563	0.939	0.402

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	97	146	160	333	675	112	-1
N.S.	1	1.00	0.85	1.28	1.40	2.92	5.92	0.98	-0.01
time (sec)	N/A	0.061	0.172	0.107	0.317	1.770	8.939	0.814	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	95	89	75	240	62	59
N.S.	1	1.00	0.82	1.40	1.31	1.10	3.53	0.91	0.87
time (sec)	N/A	0.040	0.042	0.095	0.305	1.860	0.395	1.344	0.344

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	75	117	103	245	352	69	-1
N.S.	1	1.00	0.97	1.52	1.34	3.18	4.57	0.90	-0.01
time (sec)	N/A	0.022	0.122	0.089	0.299	1.964	5.753	1.387	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	34	52	50	52	143	32	32
N.S.	1	1.00	0.77	1.18	1.14	1.18	3.25	0.73	0.73
time (sec)	N/A	0.024	0.030	0.092	0.273	1.404	0.391	1.416	0.279

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	90	68	54	144	40	33
N.S.	1	1.00	0.79	1.91	1.45	1.15	3.06	0.85	0.70
time (sec)	N/A	0.007	0.075	0.084	0.285	1.663	4.358	1.004	0.284

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	80	63	241	66	66	65
N.S.	1	1.00	0.96	1.11	0.88	3.35	0.92	0.92	0.90
time (sec)	N/A	0.037	0.090	0.092	0.390	1.638	14.694	1.371	0.540

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	62	92	85	77	265	101	68
N.S.	1	1.00	0.81	1.19	1.10	1.00	3.44	1.31	0.88
time (sec)	N/A	0.019	0.109	0.100	0.285	1.613	7.360	1.181	0.318

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	99	152	117	349	1608	101	126
N.S.	1	1.00	0.88	1.35	1.04	3.09	14.23	0.89	1.12
time (sec)	N/A	0.059	0.139	0.111	0.294	1.256	25.550	1.384	0.718

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	79	140	128	101	524	224	123
N.S.	1	1.00	0.73	1.30	1.19	0.94	4.85	2.07	1.14
time (sec)	N/A	0.033	0.120	0.101	0.296	1.244	10.722	1.235	0.368

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	119	200	164	407	1323	165	176
N.S.	1	1.00	0.82	1.37	1.12	2.79	9.06	1.13	1.21
time (sec)	N/A	0.077	0.199	0.121	0.277	1.821	49.686	1.459	0.920

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	105	188	172	129	944	336	231
N.S.	1	1.00	0.72	1.29	1.18	0.88	6.47	2.30	1.58
time (sec)	N/A	0.039	0.166	0.108	0.353	1.953	16.296	1.633	0.489

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	132	257	249	179	389	204	171
N.S.	1	1.00	0.84	1.64	1.59	1.14	2.48	1.30	1.09
time (sec)	N/A	0.090	0.084	0.085	0.346	1.355	0.370	1.412	0.373

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	99	185	181	140	308	150	137
N.S.	1	1.00	0.87	1.62	1.59	1.23	2.70	1.32	1.20
time (sec)	N/A	0.066	0.066	0.086	0.313	1.340	0.270	1.326	0.330

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	117	115	103	226	98	101
N.S.	1	1.00	0.87	1.52	1.49	1.34	2.94	1.27	1.31
time (sec)	N/A	0.041	0.046	0.092	0.311	1.604	0.163	1.227	0.312

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	97	88	207	90	101	135
N.S.	1	1.00	1.00	1.05	0.96	2.25	0.98	1.10	1.47
time (sec)	N/A	0.057	0.115	0.097	0.287	1.455	17.680	2.731	0.337

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	87	127	109	211	148	89	103
N.S.	1	1.00	0.80	1.17	1.00	1.94	1.36	0.82	0.94
time (sec)	N/A	0.060	0.164	0.112	0.284	1.462	25.210	1.177	0.506

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	140	104	200	173	225	219	153	137
N.S.	1	0.98	0.73	1.40	1.21	1.57	1.53	1.07	0.96
time (sec)	N/A	0.109	0.220	0.108	0.292	1.727	67.571	1.391	0.617

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	126	272	247	276	291	222	193
N.S.	1	1.00	0.85	1.83	1.66	1.85	1.95	1.49	1.30
time (sec)	N/A	0.103	0.222	0.102	0.276	1.505	80.623	0.942	0.858

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	188	156	257	237	341	411	174	-1
N.S.	1	0.98	0.82	1.35	1.24	1.79	2.15	0.91	-0.01
time (sec)	N/A	0.126	0.192	0.092	0.272	1.941	28.588	1.565	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	121	187	168	262	291	128	-1
N.S.	1	1.00	0.81	1.26	1.13	1.76	1.95	0.86	-0.01
time (sec)	N/A	0.063	0.155	0.090	0.297	1.512	8.414	1.404	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	130	106	165	120	215	219	126	-1
N.S.	1	0.98	0.80	1.24	0.90	1.62	1.65	0.95	-0.01
time (sec)	N/A	0.059	0.160	0.091	0.314	1.404	4.168	1.253	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	90	124	86	210	170	188	-1
N.S.	1	1.00	0.81	1.12	0.77	1.89	1.53	1.69	-0.01
time (sec)	N/A	0.046	0.186	0.091	0.331	1.296	2.509	1.445	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	124	94	221	199	403	-1
N.S.	1	1.00	1.01	1.20	0.91	2.15	1.93	3.91	-0.01
time (sec)	N/A	0.041	0.176	0.096	0.298	1.751	2.132	1.201	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	76	126	124	107	510	490	181
N.S.	1	1.01	0.77	1.27	1.25	1.08	5.15	4.95	1.83
time (sec)	N/A	0.049	0.159	0.100	0.304	1.765	2.120	1.281	0.777

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	144	108	194	190	147	1061	579	249
N.S.	1	1.01	0.76	1.36	1.33	1.03	7.42	4.05	1.74
time (sec)	N/A	0.090	0.194	0.106	0.286	2.010	2.704	1.337	1.053

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	190	141	266	258	185	1856	668	317
N.S.	1	1.01	0.75	1.41	1.37	0.98	9.82	3.53	1.68
time (sec)	N/A	0.111	0.243	0.127	0.311	1.832	3.380	1.315	1.381

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	278	224	377	367	494	0	263	-1
N.S.	1	0.99	0.80	1.34	1.31	1.76	0.00	0.94	-0.00
time (sec)	N/A	0.177	0.314	0.099	0.301	2.584	0.000	1.152	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	100	185	181	179	384	150	170
N.S.	1	1.00	0.88	1.62	1.59	1.57	3.37	1.32	1.49
time (sec)	N/A	0.062	0.075	0.094	0.288	1.520	0.429	1.208	0.391

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	232	192	305	299	419	505	219	-1
N.S.	1	0.99	0.82	1.30	1.27	1.78	2.15	0.93	-0.00
time (sec)	N/A	0.147	0.269	0.090	0.322	1.415	169.313	2.151	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	117	115	141	303	98	136
N.S.	1	1.00	0.87	1.52	1.49	1.83	3.94	1.27	1.77
time (sec)	N/A	0.042	0.053	0.089	0.292	1.088	0.304	1.468	0.367

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	158	235	227	344	440	175	-1
N.S.	1	1.00	0.81	1.20	1.16	1.76	2.24	0.89	-0.01
time (sec)	N/A	0.080	0.229	0.092	0.281	1.936	35.650	1.585	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	103	111	103	282	109	121	191
N.S.	1	1.00	0.93	1.00	0.93	2.54	0.98	1.09	1.72
time (sec)	N/A	0.067	0.143	0.099	0.303	1.268	41.808	1.080	0.342

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	172	139	213	178	293	367	173	-1
N.S.	1	0.98	0.79	1.22	1.02	1.67	2.10	0.99	-0.01
time (sec)	N/A	0.077	0.248	0.097	0.317	1.203	12.768	1.628	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	108	155	138	267	303	126	201
N.S.	1	1.00	0.79	1.14	1.01	1.96	2.23	0.93	1.48
time (sec)	N/A	0.076	0.189	0.124	0.296	1.315	31.321	1.619	0.548

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	181	119	239	177	266	352	262	-1
N.S.	1	0.98	0.65	1.30	0.96	1.45	1.91	1.42	-0.01
time (sec)	N/A	0.088	0.274	0.100	0.272	2.009	6.302	1.061	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	178	116	242	222	267	332	182	208
N.S.	1	0.98	0.64	1.34	1.23	1.48	1.83	1.01	1.15
time (sec)	N/A	0.137	0.228	0.124	0.274	1.745	77.684	1.075	0.761

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	112	204	147	266	304	407	-1
N.S.	1	1.00	0.76	1.39	1.00	1.81	2.07	2.77	-0.01
time (sec)	N/A	0.064	0.274	0.101	0.295	1.730	4.251	1.160	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	184	133	314	301	301	367	259	215
N.S.	1	0.98	0.71	1.68	1.61	1.61	1.96	1.39	1.15
time (sec)	N/A	0.149	0.233	0.117	0.302	1.116	104.805	1.011	1.091

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	99	185	181	216	468	150	207
N.S.	1	1.00	0.87	1.62	1.59	1.89	4.11	1.32	1.82
time (sec)	N/A	0.061	0.078	0.092	0.303	1.192	0.693	0.950	0.452

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	278	225	353	361	495	0	265	-1
N.S.	1	0.99	0.80	1.26	1.28	1.76	0.00	0.94	-0.00
time (sec)	N/A	0.170	0.347	0.094	0.283	2.015	0.000	1.165	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	117	115	178	384	98	98
N.S.	1	1.00	0.87	1.52	1.49	2.31	4.99	1.27	1.27
time (sec)	N/A	0.038	0.055	0.095	0.281	1.240	0.565	0.876	0.389

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	191	283	286	420	537	221	-1
N.S.	1	1.00	0.80	1.18	1.19	1.75	2.24	0.92	-0.00
time (sec)	N/A	0.096	0.302	0.095	0.285	1.984	179.905	0.671	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	115	125	120	360	128	141	249
N.S.	1	1.00	0.87	0.95	0.91	2.73	0.97	1.07	1.89
time (sec)	N/A	0.076	0.148	0.089	0.301	1.323	59.132	0.658	0.369

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	214	175	261	235	375	496	219	-1
N.S.	1	0.99	0.81	1.20	1.08	1.73	2.29	1.01	-0.00
time (sec)	N/A	0.095	0.341	0.097	0.334	1.233	41.459	0.582	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	133	183	170	349	518	165	274
N.S.	1	1.00	0.82	1.13	1.05	2.15	3.20	1.02	1.69
time (sec)	N/A	0.091	0.206	0.118	0.357	1.214	39.164	0.622	0.705

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	219	154	287	234	346	490	307	-1
N.S.	1	0.98	0.69	1.29	1.05	1.55	2.20	1.38	-0.00
time (sec)	N/A	0.118	0.313	0.106	0.308	1.207	15.153	0.694	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	219	153	284	271	319	473	242	262
N.S.	1	0.99	0.69	1.28	1.22	1.44	2.13	1.09	1.18
time (sec)	N/A	0.167	0.216	0.130	0.304	2.056	84.674	0.646	0.880

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	225	156	359	285	318	474	510	-1
N.S.	1	0.99	0.68	1.57	1.25	1.39	2.08	2.24	-0.00
time (sec)	N/A	0.111	0.390	0.103	0.307	1.317	8.275	0.572	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	219	152	356	353	347	468	286	301
N.S.	1	0.99	0.68	1.60	1.59	1.56	2.11	1.29	1.36
time (sec)	N/A	0.175	0.255	0.123	0.306	1.840	117.696	0.567	1.313

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	158	272	243	344	422	178	-1
N.S.	1	1.00	0.81	1.40	1.25	1.77	2.18	0.92	-0.01
time (sec)	N/A	0.105	0.234	0.101	0.300	1.152	49.874	0.544	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	185	181	103	240	137	105
N.S.	1	1.00	0.88	1.65	1.62	0.92	2.14	1.22	0.94
time (sec)	N/A	0.061	0.068	0.091	0.284	1.306	0.383	0.593	0.362

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	124	200	175	267	301	135	-1
N.S.	1	1.00	0.85	1.37	1.20	1.83	2.06	0.92	-0.01
time (sec)	N/A	0.096	0.173	0.088	0.295	2.408	12.533	0.766	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	116	114	68	158	84	68
N.S.	1	1.00	0.89	1.57	1.54	0.92	2.14	1.14	0.92
time (sec)	N/A	0.038	0.044	0.087	0.296	1.827	0.296	1.293	0.323

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	90	133	109	194	238	91	-1
N.S.	1	1.00	0.84	1.24	1.02	1.81	2.22	0.85	-0.01
time (sec)	N/A	0.038	0.095	0.090	0.335	1.775	4.378	1.060	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	63	86	75	157	76	82	77
N.S.	1	1.00	0.84	1.15	1.00	2.09	1.01	1.09	1.03
time (sec)	N/A	0.049	0.072	0.090	0.325	1.333	18.963	1.032	0.365

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	79	87	73	165	155	93	125
N.S.	1	1.00	0.96	1.06	0.89	2.01	1.89	1.13	1.52
time (sec)	N/A	0.032	0.121	0.096	0.352	1.270	2.168	1.021	0.734

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	99	77	175	99	81	65
N.S.	1	1.00	0.96	1.24	0.96	2.19	1.24	1.01	0.81
time (sec)	N/A	0.049	0.123	0.107	0.273	1.629	39.449	0.919	0.449

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	73	84	77	173	158	156	-1
N.S.	1	1.00	0.87	1.00	0.92	2.06	1.88	1.86	-0.01
time (sec)	N/A	0.033	0.127	0.095	0.276	1.609	1.426	0.931	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	92	159	123	204	178	140	129
N.S.	1	1.00	0.87	1.50	1.16	1.92	1.68	1.32	1.22
time (sec)	N/A	0.072	0.184	0.103	0.298	1.638	74.230	0.788	0.511

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	74	126	124	73	391	312	77
N.S.	1	1.01	0.75	1.27	1.25	0.74	3.95	3.15	0.78
time (sec)	N/A	0.048	0.123	0.093	0.333	1.354	1.938	1.033	0.385

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	128	227	190	279	301	241	207
N.S.	1	1.00	0.85	1.50	1.26	1.85	1.99	1.60	1.37
time (sec)	N/A	0.108	0.290	0.106	0.282	1.635	132.561	1.418	0.555

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	155	266	241	431	0	175	-1
N.S.	1	1.00	0.79	1.35	1.22	2.19	0.00	0.89	-0.01
time (sec)	N/A	0.102	0.260	0.114	0.285	2.118	0.000	1.181	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	182	180	115	236	149	107
N.S.	1	1.00	0.90	1.69	1.67	1.06	2.19	1.38	0.99
time (sec)	N/A	0.061	0.067	0.108	0.275	1.354	0.484	0.728	0.418

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	121	194	170	350	0	131	-1
N.S.	1	1.00	0.80	1.28	1.12	2.30	0.00	0.86	-0.01
time (sec)	N/A	0.081	0.202	0.113	0.303	1.489	0.000	0.556	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	65	115	115	79	155	92	75
N.S.	1	1.00	0.89	1.58	1.58	1.08	2.12	1.26	1.03
time (sec)	N/A	0.038	0.052	0.101	0.316	0.995	0.400	0.533	0.358

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	92	123	108	275	0	92	-1
N.S.	1	1.00	0.87	1.16	1.02	2.59	0.00	0.87	-0.01
time (sec)	N/A	0.039	0.156	0.112	0.286	1.501	0.000	0.557	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	100	90	232	70	82	76
N.S.	1	1.00	1.00	1.33	1.20	3.09	0.93	1.09	1.01
time (sec)	N/A	0.055	0.088	0.099	0.311	1.482	14.813	0.592	0.413

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	87	88	97	91	239	0	104	-1
N.S.	1	0.96	0.97	1.07	1.00	2.63	0.00	1.14	-0.01
time (sec)	N/A	0.046	0.136	0.111	0.304	1.287	0.000	0.573	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	106	97	135	112	292	0	140	119
N.S.	1	1.03	0.94	1.31	1.09	2.83	0.00	1.36	1.16
time (sec)	N/A	0.070	0.217	0.125	0.276	1.479	0.000	0.534	0.519

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	98	74	119	117	85	0	199	76
N.S.	1	1.01	0.76	1.23	1.21	0.88	0.00	2.05	0.78
time (sec)	N/A	0.048	0.124	0.101	0.279	1.150	0.000	0.560	0.393

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	125	212	177	364	0	163	179
N.S.	1	1.00	0.86	1.46	1.22	2.51	0.00	1.12	1.23
time (sec)	N/A	0.112	0.281	0.124	0.272	1.202	0.000	0.550	0.584

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	103	188	184	121	0	452	116
N.S.	1	1.00	0.73	1.33	1.30	0.86	0.00	3.21	0.82
time (sec)	N/A	0.077	0.164	0.113	0.270	1.379	0.000	0.903	0.477

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	191	158	284	247	447	0	267	246
N.S.	1	1.01	0.83	1.49	1.30	2.35	0.00	1.41	1.29
time (sec)	N/A	0.151	0.271	0.133	0.288	1.195	0.000	0.744	0.711

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	155	260	296	522	0	190	-1
N.S.	1	1.00	0.77	1.29	1.47	2.58	0.00	0.94	-0.00
time (sec)	N/A	0.101	0.278	0.169	0.300	1.941	0.000	0.657	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	183	181	124	454	140	107
N.S.	1	1.00	0.89	1.66	1.65	1.13	4.13	1.27	0.97
time (sec)	N/A	0.064	0.072	0.109	0.295	1.847	0.515	1.028	0.441

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	117	207	211	409	0	130	-1
N.S.	1	1.00	0.97	1.71	1.74	3.38	0.00	1.07	-0.01
time (sec)	N/A	0.077	0.209	0.127	0.306	2.746	0.000	1.260	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	116	114	91	303	79	76
N.S.	1	1.00	0.93	1.61	1.58	1.26	4.21	1.10	1.06
time (sec)	N/A	0.039	0.051	0.106	0.282	1.990	0.442	1.263	0.361

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	156	147	321	0	105	-1
N.S.	1	1.00	0.97	1.49	1.40	3.06	0.00	1.00	-0.01
time (sec)	N/A	0.036	0.174	0.095	0.275	2.487	0.000	1.219	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	87	120	108	316	87	102	90
N.S.	1	1.00	0.99	1.36	1.23	3.59	0.99	1.16	1.02
time (sec)	N/A	0.067	0.125	0.108	0.279	1.599	18.865	1.006	0.448

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	153	132	92	0	117	77
N.S.	1	1.00	0.84	1.70	1.47	1.02	0.00	1.30	0.86
time (sec)	N/A	0.033	0.134	0.109	0.355	2.023	0.000	0.786	0.351

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	134	120	173	146	426	0	128	147
N.S.	1	1.02	0.92	1.32	1.11	3.25	0.00	0.98	1.12
time (sec)	N/A	0.085	0.204	0.134	0.273	1.371	0.000	0.995	0.487

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	130	107	179	175	130	0	258	187
N.S.	1	0.99	0.82	1.37	1.34	0.99	0.00	1.97	1.43
time (sec)	N/A	0.084	0.153	0.119	0.332	1.676	0.000	0.876	0.419

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	159	269	231	537	0	210	216
N.S.	1	1.00	0.86	1.45	1.25	2.90	0.00	1.14	1.17
time (sec)	N/A	0.149	0.212	0.163	0.296	0.881	0.000	1.063	0.616

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	142	251	244	171	0	509	298
N.S.	1	1.00	0.78	1.37	1.33	0.93	0.00	2.78	1.63
time (sec)	N/A	0.110	0.223	0.132	0.285	2.052	0.000	1.359	0.544

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	54	67	147	0	56	51
N.S.	1	1.00	0.78	0.75	0.93	2.04	0.00	0.78	0.71
time (sec)	N/A	0.019	0.017	0.096	0.492	1.102	0.000	0.993	0.361

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	38	49	126	0	40	37
N.S.	1	1.00	0.85	0.73	0.94	2.42	0.00	0.77	0.71
time (sec)	N/A	0.012	0.013	0.094	0.517	2.827	0.000	0.782	0.324

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	24	23	94	0	22	23
N.S.	1	1.00	1.00	0.71	0.68	2.76	0.00	0.65	0.68
time (sec)	N/A	0.006	0.006	0.104	0.489	1.827	0.000	0.877	0.341

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	36	35	132	0	43	38
N.S.	1	1.00	0.92	0.72	0.70	2.64	0.00	0.86	0.76
time (sec)	N/A	0.011	0.012	0.089	0.483	1.254	0.000	0.817	0.354

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	57	52	157	0	54	53
N.S.	1	1.00	0.85	0.84	0.76	2.31	0.00	0.79	0.78
time (sec)	N/A	0.017	0.016	0.094	0.493	1.639	0.000	0.934	0.348

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	166	767	0	857	0	0	-1
N.S.	1	1.00	1.06	4.89	0.00	5.46	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.355	0.138	0.000	1.202	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	666	0	295	87	96	86
N.S.	1	1.00	0.97	7.57	0.00	3.35	0.99	1.09	0.98
time (sec)	N/A	0.061	0.135	0.107	0.000	1.557	3.550	0.804	0.347

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	128	700	0	690	0	0	-1
N.S.	1	1.00	1.14	6.25	0.00	6.16	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.189	0.117	0.000	1.191	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	647	0	255	61	64	53
N.S.	1	1.00	1.00	9.95	0.00	3.92	0.94	0.98	0.82
time (sec)	N/A	0.040	0.074	0.101	0.000	1.458	2.256	0.627	0.356

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	102	653	0	596	0	0	-1
N.S.	1	1.00	1.26	8.06	0.00	7.36	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.166	0.092	0.000	1.368	0.000	0.000	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	689	0	578	78	78	103
N.S.	1	1.00	0.98	8.61	0.00	7.22	0.98	0.98	1.29
time (sec)	N/A	0.055	0.078	0.107	0.000	1.878	3.878	0.845	0.403

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	89	724	0	273	0	117	-1
N.S.	1	1.00	1.27	10.34	0.00	3.90	0.00	1.67	-0.01
time (sec)	N/A	0.037	0.155	0.112	0.000	1.174	0.000	1.405	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	759	0	708	0	106	268
N.S.	1	1.00	0.95	6.72	0.00	6.27	0.00	0.94	2.37
time (sec)	N/A	0.086	0.279	0.120	0.000	2.390	0.000	1.384	0.539

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	114	750	0	325	0	215	-1
N.S.	1	1.00	1.09	7.14	0.00	3.10	0.00	2.05	-0.01
time (sec)	N/A	0.080	0.224	0.106	0.000	1.426	0.000	2.335	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	215	1389	0	1119	0	0	-1
N.S.	1	1.00	1.02	6.61	0.00	5.33	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.414	0.132	0.000	5.369	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	109	1256	0	397	104	151	179
N.S.	1	1.00	0.95	10.92	0.00	3.45	0.90	1.31	1.56
time (sec)	N/A	0.073	0.214	0.102	0.000	1.393	19.702	0.808	0.366

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	158	1306	0	894	0	0	-1
N.S.	1	1.00	1.00	8.27	0.00	5.66	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.335	0.122	0.000	2.285	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	1237	0	303	80	112	98
N.S.	1	1.00	0.90	13.59	0.00	3.33	0.88	1.23	1.08
time (sec)	N/A	0.052	0.144	0.100	0.000	1.451	11.394	0.583	0.349

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	129	1243	0	721	0	0	-1
N.S.	1	1.00	1.14	11.00	0.00	6.38	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.271	0.123	0.000	1.481	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	103	1293	0	682	92	110	711
N.S.	1	1.00	1.07	13.47	0.00	7.10	0.96	1.15	7.41
time (sec)	N/A	0.076	0.171	0.104	0.000	2.341	11.130	0.577	0.424

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	129	1330	0	718	0	0	-1
N.S.	1	1.00	1.26	13.04	0.00	7.04	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.229	0.118	0.000	1.485	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	108	1377	0	732	0	120	560
N.S.	1	1.00	0.95	12.08	0.00	6.42	0.00	1.05	4.91
time (sec)	N/A	0.098	0.248	0.129	0.000	2.238	0.000	0.749	0.638

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	111	1439	0	331	0	256	-1
N.S.	1	1.00	1.09	14.11	0.00	3.25	0.00	2.51	-0.01
time (sec)	N/A	0.084	0.277	0.115	0.000	1.930	0.000	1.656	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	266	2252	0	1443	0	0	-1
N.S.	1	1.00	0.91	7.74	0.00	4.96	0.00	0.00	-0.00
time (sec)	N/A	0.372	0.645	0.139	0.000	12.582	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	140	2087	0	527	144	228	251
N.S.	1	1.00	0.97	14.49	0.00	3.66	1.00	1.58	1.74
time (sec)	N/A	0.097	0.205	0.106	0.000	1.636	35.005	0.520	0.343

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	206	2153	0	1161	0	0	-1
N.S.	1	1.00	0.95	9.92	0.00	5.35	0.00	0.00	-0.00
time (sec)	N/A	0.262	0.411	0.131	0.000	4.849	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	116	2068	0	405	117	184	137
N.S.	1	1.00	0.97	17.38	0.00	3.40	0.98	1.55	1.15
time (sec)	N/A	0.069	0.127	0.097	0.000	1.357	23.581	0.551	0.343

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	160	2074	0	931	0	0	-1
N.S.	1	1.00	1.03	13.29	0.00	5.97	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.349	0.126	0.000	2.558	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	114	2138	0	837	119	163	2094
N.S.	1	1.00	0.92	17.24	0.00	6.75	0.96	1.31	16.89
time (sec)	N/A	0.131	0.163	0.098	0.000	2.990	22.287	0.611	0.519

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	148	2177	0	887	0	0	-1
N.S.	1	1.00	1.02	15.01	0.00	6.12	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.336	0.128	0.000	2.033	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	125	2236	0	891	0	158	1428
N.S.	1	1.00	0.87	15.53	0.00	6.19	0.00	1.10	9.92
time (sec)	N/A	0.154	0.239	0.135	0.000	3.242	0.000	0.646	0.735

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	145	2302	0	901	0	0	-1
N.S.	1	1.00	1.12	17.71	0.00	6.93	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.383	0.129	0.000	2.382	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	88	361	0	390	0	105	100
N.S.	1	1.00	0.88	3.61	0.00	3.90	0.00	1.05	1.00
time (sec)	N/A	0.077	0.176	0.096	0.000	1.096	0.000	0.812	0.385

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	318	0	306	0	64	57
N.S.	1	1.00	1.01	4.68	0.00	4.50	0.00	0.94	0.84
time (sec)	N/A	0.045	0.080	0.102	0.000	1.156	0.000	0.678	0.368

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	300	0	231	36	39	39
N.S.	1	1.00	0.98	6.12	0.00	4.71	0.73	0.80	0.80
time (sec)	N/A	0.030	0.040	0.090	0.000	1.247	2.567	0.694	0.367

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	331	0	603	63	70	651
N.S.	1	1.00	0.98	4.14	0.00	7.54	0.79	0.88	8.14
time (sec)	N/A	0.053	0.114	0.096	0.000	1.797	4.888	1.019	0.498

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	109	384	0	734	0	103	396
N.S.	1	1.00	0.95	3.34	0.00	6.38	0.00	0.90	3.44
time (sec)	N/A	0.083	0.236	0.120	0.000	1.347	0.000	0.981	0.632

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	130	385	0	717	0	0	-1
N.S.	1	1.00	1.14	3.38	0.00	6.29	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.309	0.126	0.000	1.430	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	102	337	0	616	0	0	-1
N.S.	1	1.00	1.24	4.11	0.00	7.51	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.164	0.100	0.000	1.237	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	70	306	0	241	0	70	-1
N.S.	1	1.00	1.43	6.24	0.00	4.92	0.00	1.43	-0.02
time (sec)	N/A	0.014	0.005	0.087	0.000	1.844	0.000	0.874	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	93	334	0	324	0	111	-1
N.S.	1	1.00	1.26	4.51	0.00	4.38	0.00	1.50	-0.01
time (sec)	N/A	0.038	0.161	0.102	0.000	1.089	0.000	0.893	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	117	378	0	414	0	195	-1
N.S.	1	1.00	1.06	3.44	0.00	3.76	0.00	1.77	-0.01
time (sec)	N/A	0.086	0.272	0.110	0.000	1.993	0.000	1.864	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	132	804	0	977	0	0	-1
N.S.	1	1.00	1.21	7.38	0.00	8.96	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.346	0.093	0.000	1.626	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	746	0	428	0	78	64
N.S.	1	1.00	0.99	9.69	0.00	5.56	0.00	1.01	0.83
time (sec)	N/A	0.052	0.134	0.094	0.000	1.476	0.000	1.339	0.426

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	93	758	0	334	0	103	-1
N.S.	1	1.00	1.26	10.24	0.00	4.51	0.00	1.39	-0.01
time (sec)	N/A	0.037	0.187	0.096	0.000	1.663	0.000	1.061	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	727	0	323	61	71	61
N.S.	1	1.00	1.00	10.10	0.00	4.49	0.85	0.99	0.85
time (sec)	N/A	0.040	0.092	0.096	0.000	1.655	6.687	0.990	0.428

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	99	733	0	442	0	107	-1
N.S.	1	1.00	1.25	9.28	0.00	5.59	0.00	1.35	-0.01
time (sec)	N/A	0.026	0.039	0.089	0.000	1.570	0.000	1.108	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	106	773	0	959	94	110	2296
N.S.	1	1.00	0.99	7.22	0.00	8.96	0.88	1.03	21.46
time (sec)	N/A	0.077	0.254	0.097	0.000	1.643	6.954	0.771	0.818

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	127	779	0	560	0	152	-1
N.S.	1	1.00	1.02	6.28	0.00	4.52	0.00	1.23	-0.01
time (sec)	N/A	0.078	0.417	0.125	0.000	1.110	0.000	2.477	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	142	847	0	1291	0	172	2500
N.S.	1	1.00	0.91	5.43	0.00	8.28	0.00	1.10	16.03
time (sec)	N/A	0.152	0.391	0.135	0.000	2.271	0.000	1.283	1.213

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	175	847	0	706	0	275	-1
N.S.	1	1.00	0.99	4.81	0.00	4.01	0.00	1.56	-0.01
time (sec)	N/A	0.145	0.508	0.142	0.000	1.240	0.000	1.600	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	115	1502	0	524	0	304	-1
N.S.	1	1.00	0.98	12.84	0.00	4.48	0.00	2.60	-0.01
time (sec)	N/A	0.075	0.373	0.098	0.000	1.855	0.000	1.423	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	100	1409	0	535	0	127	110
N.S.	1	1.00	0.97	13.68	0.00	5.19	0.00	1.23	1.07
time (sec)	N/A	0.066	0.227	0.094	0.000	1.450	0.000	1.066	0.538

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	122	1439	0	550	0	291	-1
N.S.	1	1.00	1.06	12.51	0.00	4.78	0.00	2.53	-0.01
time (sec)	N/A	0.059	0.379	0.091	0.000	1.377	0.000	0.961	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	90	1390	0	511	85	118	103
N.S.	1	1.00	0.92	14.18	0.00	5.21	0.87	1.20	1.05
time (sec)	N/A	0.053	0.159	0.098	0.000	1.374	9.675	0.755	0.517

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	132	1396	0	764	0	321	-1
N.S.	1	1.00	1.08	11.44	0.00	6.26	0.00	2.63	-0.01
time (sec)	N/A	0.063	0.378	0.099	0.000	1.517	0.000	0.759	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	138	1455	0	1711	133	176	2500
N.S.	1	1.00	0.95	10.03	0.00	11.80	0.92	1.21	17.24
time (sec)	N/A	0.128	0.416	0.102	0.000	2.392	9.861	0.545	1.437

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	180	1463	0	934	0	366	-1
N.S.	1	1.00	1.01	8.22	0.00	5.25	0.00	2.06	-0.01
time (sec)	N/A	0.149	0.533	0.155	0.000	1.914	0.000	1.121	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	194	1548	0	2219	0	211	2500
N.S.	1	1.00	0.92	7.34	0.00	10.52	0.00	1.00	11.85
time (sec)	N/A	0.229	0.705	0.178	0.000	3.061	0.000	0.498	1.999

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	233	1552	0	1128	0	490	-1
N.S.	1	1.00	0.95	6.33	0.00	4.60	0.00	2.00	-0.00
time (sec)	N/A	0.235	0.803	0.184	0.000	1.897	0.000	1.296	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	154	2002	0	1002	0	288	-1
N.S.	1	1.00	1.03	13.35	0.00	6.68	0.00	1.92	-0.01
time (sec)	N/A	0.115	0.721	0.135	0.000	1.880	0.000	0.688	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	1959	0	436	0	101	102
N.S.	1	1.00	0.72	14.40	0.00	3.21	0.00	0.74	0.75
time (sec)	N/A	0.076	0.254	0.122	0.000	1.279	0.000	0.613	0.510

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	138	1959	0	1069	0	251	-1
N.S.	1	1.00	1.15	16.32	0.00	8.91	0.00	2.09	-0.01
time (sec)	N/A	0.058	0.489	0.096	0.000	1.588	0.000	0.663	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	1320	0	356	0	79	70
N.S.	1	1.00	1.00	16.50	0.00	4.45	0.00	0.99	0.88
time (sec)	N/A	0.042	0.166	0.094	0.000	1.165	0.000	0.516	0.443

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	102	1965	0	369	0	218	-1
N.S.	1	1.00	1.24	23.96	0.00	4.50	0.00	2.66	-0.01
time (sec)	N/A	0.024	0.412	0.094	0.000	1.256	0.000	1.139	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	2001	0	1054	0	113	996
N.S.	1	1.00	0.93	16.82	0.00	8.86	0.00	0.95	8.37
time (sec)	N/A	0.077	0.502	0.101	0.000	1.249	0.000	0.511	0.638

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	123	2024	0	458	0	329	-1
N.S.	1	1.00	1.09	17.91	0.00	4.05	0.00	2.91	-0.01
time (sec)	N/A	0.074	0.430	0.129	0.000	1.401	0.000	1.232	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	132	2071	0	1043	0	183	1193
N.S.	1	1.00	0.83	13.03	0.00	6.56	0.00	1.15	7.50
time (sec)	N/A	0.145	0.558	0.150	0.000	1.694	0.000	0.503	0.951

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	154	2052	0	602	0	361	-1
N.S.	1	1.00	1.05	13.96	0.00	4.10	0.00	2.46	-0.01
time (sec)	N/A	0.130	0.681	0.128	0.000	1.572	0.000	1.307	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	197	3440	0	1249	0	394	-1
N.S.	1	1.00	1.00	17.46	0.00	6.34	0.00	2.00	-0.01
time (sec)	N/A	0.221	0.608	0.150	0.000	1.500	0.000	0.540	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	125	3381	0	413	0	173	183
N.S.	1	1.00	0.77	20.74	0.00	2.53	0.00	1.06	1.12
time (sec)	N/A	0.095	0.296	0.128	0.000	1.430	0.000	0.542	0.560

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	160	3381	0	996	0	336	-1
N.S.	1	1.00	1.07	22.69	0.00	6.68	0.00	2.26	-0.01
time (sec)	N/A	0.112	0.563	0.135	0.000	1.591	0.000	0.619	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	96	2152	0	333	0	122	117
N.S.	1	1.00	0.97	21.74	0.00	3.36	0.00	1.23	1.18
time (sec)	N/A	0.057	0.255	0.122	0.000	1.402	0.000	0.568	0.514

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	148	3387	0	903	0	315	-1
N.S.	1	1.00	1.13	25.85	0.00	6.89	0.00	2.40	-0.01
time (sec)	N/A	0.060	0.523	0.098	0.000	1.756	0.000	0.496	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	122	3437	0	883	0	154	488
N.S.	1	1.00	0.95	26.64	0.00	6.84	0.00	1.19	3.78
time (sec)	N/A	0.098	0.399	0.104	0.000	1.286	0.000	0.515	0.626

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	122	3462	0	351	0	412	-1
N.S.	1	1.00	0.95	27.05	0.00	2.74	0.00	3.22	-0.01
time (sec)	N/A	0.085	0.471	0.132	0.000	1.220	0.000	1.270	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	3521	0	1034	0	216	441
N.S.	1	1.00	0.91	20.71	0.00	6.08	0.00	1.27	2.59
time (sec)	N/A	0.172	0.669	0.146	0.000	1.386	0.000	0.534	0.957

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	151	3573	0	443	0	442	-1
N.S.	1	1.00	0.91	21.52	0.00	2.67	0.00	2.66	-0.01
time (sec)	N/A	0.160	0.479	0.128	0.000	1.366	0.000	1.346	0.000

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	269	5359	0	1697	0	521	-1
N.S.	1	1.00	1.04	20.77	0.00	6.58	0.00	2.02	-0.00
time (sec)	N/A	0.300	0.921	0.142	0.000	4.734	0.000	0.598	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	183	5284	0	573	0	264	276
N.S.	1	1.00	0.92	26.69	0.00	2.89	0.00	1.33	1.39
time (sec)	N/A	0.127	0.375	0.138	0.000	1.337	0.000	0.542	0.641

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	221	5284	0	1379	0	446	-1
N.S.	1	1.00	1.13	27.10	0.00	7.07	0.00	2.29	-0.01
time (sec)	N/A	0.163	0.813	0.140	0.000	3.611	0.000	0.526	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	128	3224	0	453	0	197	172
N.S.	1	1.00	1.02	25.59	0.00	3.60	0.00	1.56	1.37
time (sec)	N/A	0.074	0.301	0.134	0.000	1.960	0.000	0.523	0.573

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	191	5290	0	1228	0	407	-1
N.S.	1	1.00	1.10	30.40	0.00	7.06	0.00	2.34	-0.01
time (sec)	N/A	0.144	0.623	0.158	0.000	2.095	0.000	0.517	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	146	5354	0	1132	0	206	1321
N.S.	1	1.00	0.91	33.46	0.00	7.08	0.00	1.29	8.26
time (sec)	N/A	0.146	0.326	0.100	0.000	3.167	0.000	0.495	0.700

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	192	5381	0	1184	0	0	-1
N.S.	1	1.00	1.14	32.03	0.00	7.05	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.601	0.133	0.000	1.725	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	160	5452	0	1266	0	283	1152
N.S.	1	1.00	0.89	30.29	0.00	7.03	0.00	1.57	6.40
time (sec)	N/A	0.180	0.485	0.158	0.000	2.499	0.000	0.515	1.080

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	157	5508	0	483	0	496	-1
N.S.	1	1.00	0.89	31.30	0.00	2.74	0.00	2.82	-0.01
time (sec)	N/A	0.160	0.686	0.140	0.000	1.663	0.000	1.425	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	148	843	0	1053	0	284	-1
N.S.	1	1.00	1.12	6.39	0.00	7.98	0.00	2.15	-0.01
time (sec)	N/A	0.075	0.752	0.094	0.000	1.537	0.000	0.551	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	816	0	450	0	116	93
N.S.	1	1.00	1.01	8.24	0.00	4.55	0.00	1.17	0.94
time (sec)	N/A	0.058	0.233	0.093	0.000	1.829	0.000	0.523	0.501

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	109	816	0	418	0	231	-1
N.S.	1	1.00	1.22	9.17	0.00	4.70	0.00	2.60	-0.01
time (sec)	N/A	0.037	0.491	0.095	0.000	1.195	0.000	1.207	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	524	0	404	0	93	82
N.S.	1	1.00	0.99	6.02	0.00	4.64	0.00	1.07	0.94
time (sec)	N/A	0.046	0.172	0.093	0.000	1.257	0.000	0.720	0.435

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	122	822	0	459	0	225	-1
N.S.	1	1.00	1.22	8.22	0.00	4.59	0.00	2.25	-0.01
time (sec)	N/A	0.036	0.043	0.086	0.000	1.328	0.000	0.522	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	124	847	0	1037	0	138	3023
N.S.	1	1.00	0.95	6.52	0.00	7.98	0.00	1.06	23.25
time (sec)	N/A	0.091	0.369	0.099	0.000	1.845	0.000	0.559	1.071

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	156	838	0	600	0	396	-1
N.S.	1	1.00	1.06	5.70	0.00	4.08	0.00	2.69	-0.01
time (sec)	N/A	0.089	0.735	0.115	0.000	1.533	0.000	1.252	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	903	0	1407	0	257	2500
N.S.	1	1.00	0.88	4.88	0.00	7.61	0.00	1.39	13.51
time (sec)	N/A	0.163	0.750	0.139	0.000	2.233	0.000	0.547	1.579

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	199	885	0	758	0	375	-1
N.S.	1	1.00	0.97	4.30	0.00	3.68	0.00	1.82	-0.00
time (sec)	N/A	0.162	0.904	0.125	0.000	1.728	0.000	1.407	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	126	1943	0	552	0	298	-1
N.S.	1	1.00	0.97	14.95	0.00	4.25	0.00	2.29	-0.01
time (sec)	N/A	0.066	0.681	0.101	0.000	1.324	0.000	1.280	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	1922	0	732	0	181	142
N.S.	1	1.00	0.82	14.34	0.00	5.46	0.00	1.35	1.06
time (sec)	N/A	0.079	0.322	0.094	0.000	1.610	0.000	0.526	0.620

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	133	1922	0	744	0	299	-1
N.S.	1	1.00	1.08	15.63	0.00	6.05	0.00	2.43	-0.01
time (sec)	N/A	0.062	0.592	0.094	0.000	1.981	0.000	1.249	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	1203	0	537	0	153	130
N.S.	1	1.00	0.90	10.65	0.00	4.75	0.00	1.35	1.15
time (sec)	N/A	0.056	0.339	0.097	0.000	1.379	0.000	0.501	0.565

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	154	1928	0	854	0	318	-1
N.S.	1	1.00	1.08	13.58	0.00	6.01	0.00	2.24	-0.01
time (sec)	N/A	0.072	0.585	0.090	0.000	1.700	0.000	1.226	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	157	1968	0	1992	0	225	2500
N.S.	1	1.00	0.92	11.58	0.00	11.72	0.00	1.32	14.71
time (sec)	N/A	0.150	0.768	0.099	0.000	3.548	0.000	0.559	1.897

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	216	1962	0	1018	0	554	-1
N.S.	1	1.00	1.05	9.57	0.00	4.97	0.00	2.70	-0.00
time (sec)	N/A	0.171	0.953	0.152	0.000	1.878	0.000	1.381	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	223	2042	0	2554	0	367	2500
N.S.	1	1.00	0.93	8.47	0.00	10.60	0.00	1.52	10.37
time (sec)	N/A	0.243	0.961	0.173	0.000	6.461	0.000	0.527	2.747

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	265	2032	0	1252	0	486	-1
N.S.	1	1.00	0.96	7.34	0.00	4.52	0.00	1.75	-0.00
time (sec)	N/A	0.265	1.447	0.200	0.000	1.996	0.000	1.534	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	176	3522	0	1008	0	594	-1
N.S.	1	1.00	1.01	20.24	0.00	5.79	0.00	3.41	-0.01
time (sec)	N/A	0.141	1.145	0.102	0.000	2.324	0.000	1.343	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	3483	0	993	0	260	193
N.S.	1	1.00	0.91	20.49	0.00	5.84	0.00	1.53	1.14
time (sec)	N/A	0.113	0.472	0.099	0.000	1.436	0.000	0.683	0.794

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	183	3483	0	1292	0	595	-1
N.S.	1	1.00	1.12	21.37	0.00	7.93	0.00	3.65	-0.01
time (sec)	N/A	0.106	1.025	0.100	0.000	2.747	0.000	1.454	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	137	2101	0	895	0	226	171
N.S.	1	1.00	0.98	15.01	0.00	6.39	0.00	1.61	1.22
time (sec)	N/A	0.075	0.302	0.099	0.000	1.535	0.000	0.622	0.683

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	214	3489	0	1434	0	619	-1
N.S.	1	1.00	1.06	17.36	0.00	7.13	0.00	3.08	-0.00
time (sec)	N/A	0.149	1.164	0.097	0.000	2.826	0.000	1.305	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	215	3548	0	3403	0	298	2500
N.S.	1	1.00	0.96	15.77	0.00	15.12	0.00	1.32	11.11
time (sec)	N/A	0.243	0.749	0.160	0.000	9.203	0.000	1.122	2.936

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	287	3544	0	1662	0	938	-1
N.S.	1	1.00	1.03	12.70	0.00	5.96	0.00	3.36	-0.00
time (sec)	N/A	0.294	1.260	0.256	0.000	2.560	0.000	3.634	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	294	3641	0	4115	0	505	2500
N.S.	1	1.00	0.97	11.98	0.00	13.54	0.00	1.66	8.22
time (sec)	N/A	0.356	1.363	0.310	0.000	18.981	0.000	1.179	3.762

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	348	3635	0	1890	0	789	-1
N.S.	1	1.00	0.96	10.04	0.00	5.22	0.00	2.18	-0.00
time (sec)	N/A	0.405	1.741	0.349	0.000	4.008	0.000	4.539	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	110	276	0	94	97	0	-1
N.S.	1	1.00	0.52	1.30	0.00	0.44	0.46	0.00	-0.00
time (sec)	N/A	0.110	10.102	0.133	0.000	0.369	6.478	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	93	414	0	80	95	0	-1
N.S.	1	1.00	0.28	1.23	0.00	0.24	0.28	0.00	-0.00
time (sec)	N/A	0.182	10.075	0.123	0.000	0.280	1.781	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	93	246	0	68	97	0	-1
N.S.	1	1.00	0.53	1.40	0.00	0.39	0.55	0.00	-0.01
time (sec)	N/A	0.076	10.047	0.106	0.000	0.398	1.853	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	96	391	0	67	100	0	-1
N.S.	1	1.00	0.29	1.17	0.00	0.20	0.30	0.00	-0.00
time (sec)	N/A	0.184	9.542	0.121	0.000	0.300	1.881	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	82	234	0	59	100	0	-1
N.S.	1	1.00	0.48	1.36	0.00	0.34	0.58	0.00	-0.01
time (sec)	N/A	0.075	9.819	0.118	0.000	0.373	3.829	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	95	417	0	74	107	0	-1
N.S.	1	1.00	0.28	1.23	0.00	0.22	0.32	0.00	-0.00
time (sec)	N/A	0.185	10.044	0.118	0.000	0.343	13.117	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	79	242	0	67	97	0	-1
N.S.	1	1.00	0.52	1.59	0.00	0.44	0.64	0.00	-0.01
time (sec)	N/A	0.062	10.073	0.125	0.000	0.198	11.719	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	80	439	0	99	100	0	-1
N.S.	1	1.00	0.24	1.33	0.00	0.30	0.30	0.00	-0.00
time (sec)	N/A	0.164	10.076	0.121	0.000	0.235	32.695	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	80	270	0	92	100	0	-1
N.S.	1	1.00	0.43	1.44	0.00	0.49	0.53	0.00	-0.01
time (sec)	N/A	0.078	10.097	0.117	0.000	0.155	85.902	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	114	300	0	117	199	0	-1
N.S.	1	1.00	0.45	1.19	0.00	0.46	0.79	0.00	-0.00
time (sec)	N/A	0.117	10.126	0.102	0.000	0.305	13.916	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	97	438	0	105	197	0	-1
N.S.	1	1.00	0.26	1.16	0.00	0.28	0.52	0.00	-0.00
time (sec)	N/A	0.207	10.088	0.109	0.000	0.403	4.089	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	96	272	0	91	199	0	-1
N.S.	1	1.00	0.45	1.27	0.00	0.43	0.93	0.00	-0.00
time (sec)	N/A	0.091	10.067	0.112	0.000	0.268	4.613	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	84	421	0	90	202	0	-1
N.S.	1	1.00	0.23	1.15	0.00	0.25	0.55	0.00	-0.00
time (sec)	N/A	0.210	10.056	0.110	0.000	0.374	4.781	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	85	255	0	84	202	0	-1
N.S.	1	1.00	0.40	1.21	0.00	0.40	0.96	0.00	-0.00
time (sec)	N/A	0.096	10.056	0.116	0.000	0.261	6.955	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	84	422	0	79	212	0	-1
N.S.	1	1.00	0.23	1.16	0.00	0.22	0.58	0.00	-0.00
time (sec)	N/A	0.199	10.052	0.113	0.000	0.420	21.926	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	96	417	0	83	94	0	-1
N.S.	1	1.00	0.28	1.23	0.00	0.25	0.28	0.00	-0.00
time (sec)	N/A	0.178	10.102	0.109	0.000	0.390	14.890	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	96	250	0	69	94	0	-1
N.S.	1	1.00	0.55	1.44	0.00	0.40	0.54	0.00	-0.01
time (sec)	N/A	0.075	10.087	0.103	0.000	0.228	4.152	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	80	379	0	55	92	0	-1
N.S.	1	1.00	0.27	1.27	0.00	0.18	0.31	0.00	-0.00
time (sec)	N/A	0.155	10.067	0.100	0.000	0.286	1.728	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	79	214	0	45	94	0	-1
N.S.	1	1.00	0.57	1.54	0.00	0.32	0.68	0.00	-0.01
time (sec)	N/A	0.058	10.044	0.112	0.000	0.321	1.291	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	82	378	0	58	97	0	-1
N.S.	1	1.00	0.28	1.30	0.00	0.20	0.33	0.00	-0.00
time (sec)	N/A	0.160	10.031	0.104	0.000	0.248	1.900	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	81	223	0	55	97	0	-1
N.S.	1	1.00	0.59	1.62	0.00	0.40	0.70	0.00	-0.01
time (sec)	N/A	0.060	10.051	0.123	0.000	0.240	4.796	0.000	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	82	417	0	74	104	0	-1
N.S.	1	1.00	0.24	1.22	0.00	0.22	0.30	0.00	-0.00
time (sec)	N/A	0.181	10.047	0.117	0.000	0.209	17.326	0.000	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	111	252	0	125	94	0	-1
N.S.	1	1.00	0.53	1.19	0.00	0.59	0.45	0.00	-0.00
time (sec)	N/A	0.095	10.114	0.141	0.000	0.334	180.204	0.000	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	84	391	0	109	94	0	-1
N.S.	1	1.00	0.25	1.16	0.00	0.32	0.28	0.00	-0.00
time (sec)	N/A	0.182	10.106	0.130	0.000	0.304	52.690	0.000	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	85	225	0	98	94	0	-1
N.S.	1	1.00	0.49	1.29	0.00	0.56	0.54	0.00	-0.01
time (sec)	N/A	0.077	10.103	0.118	0.000	0.305	12.584	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	76	382	0	98	94	0	-1
N.S.	1	1.00	0.25	1.27	0.00	0.33	0.31	0.00	-0.00
time (sec)	N/A	0.163	10.091	0.100	0.000	0.281	4.606	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	75	213	0	84	94	0	-1
N.S.	1	1.00	0.52	1.48	0.00	0.58	0.65	0.00	-0.01
time (sec)	N/A	0.061	10.044	0.101	0.000	0.268	5.149	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	77	386	0	109	97	0	-1
N.S.	1	1.00	0.23	1.16	0.00	0.33	0.29	0.00	-0.00
time (sec)	N/A	0.176	10.031	0.127	0.000	0.396	9.399	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	91	232	0	109	97	0	-1
N.S.	1	1.00	0.52	1.32	0.00	0.62	0.55	0.00	-0.01
time (sec)	N/A	0.079	10.054	0.128	0.000	0.165	22.650	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	78	417	0	132	104	0	-1
N.S.	1	1.00	0.21	1.10	0.00	0.35	0.27	0.00	-0.00
time (sec)	N/A	0.212	10.044	0.136	0.000	0.258	63.729	0.000	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	116	439	0	156	0	0	-1
N.S.	1	1.00	0.56	2.11	0.00	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.093	10.135	0.164	0.000	0.332	0.000	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	97	767	0	160	94	0	-1
N.S.	1	1.00	0.28	2.20	0.00	0.46	0.27	0.00	-0.00
time (sec)	N/A	0.186	10.110	0.116	0.000	0.266	190.727	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	105	429	0	144	94	0	-1
N.S.	1	1.00	0.57	2.32	0.00	0.78	0.51	0.00	-0.01
time (sec)	N/A	0.081	10.136	0.117	0.000	0.263	60.391	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	84	764	0	153	94	0	-1
N.S.	1	1.00	0.24	2.22	0.00	0.44	0.27	0.00	-0.00
time (sec)	N/A	0.183	10.092	0.122	0.000	0.228	24.315	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	108	425	0	144	94	0	-1
N.S.	1	1.00	0.58	2.27	0.00	0.77	0.50	0.00	-0.01
time (sec)	N/A	0.078	10.069	0.115	0.000	0.185	42.553	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	86	771	0	167	97	0	-1
N.S.	1	1.00	0.23	2.05	0.00	0.44	0.26	0.00	-0.00
time (sec)	N/A	0.211	10.056	0.153	0.000	0.276	79.602	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	120	446	0	163	97	0	-1
N.S.	1	1.00	0.56	2.09	0.00	0.77	0.46	0.00	-0.00
time (sec)	N/A	0.100	10.078	0.144	0.000	0.405	148.757	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	225	448	0	160	150	0	-1
N.S.	1	1.00	0.78	1.56	0.00	0.56	0.52	0.00	-0.00
time (sec)	N/A	0.210	20.250	0.139	0.000	0.374	12.454	0.000	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	145	658	0	134	148	0	-1
N.S.	1	1.00	0.34	1.55	0.00	0.32	0.35	0.00	-0.00
time (sec)	N/A	0.299	20.120	0.123	0.000	0.261	3.092	0.000	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	189	401	0	119	150	0	-1
N.S.	1	1.00	0.77	1.64	0.00	0.49	0.61	0.00	-0.00
time (sec)	N/A	0.142	10.160	0.112	0.000	0.274	3.461	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	129	624	0	108	153	0	-1
N.S.	1	1.00	0.31	1.48	0.00	0.26	0.36	0.00	-0.00
time (sec)	N/A	0.274	20.102	0.115	0.000	0.271	3.683	0.000	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	171	383	0	103	153	0	-1
N.S.	1	1.00	0.73	1.64	0.00	0.44	0.65	0.00	-0.00
time (sec)	N/A	0.135	10.144	0.117	0.000	0.363	5.907	0.000	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	125	648	0	110	160	0	-1
N.S.	1	1.00	0.30	1.54	0.00	0.26	0.38	0.00	-0.00
time (sec)	N/A	0.275	20.099	0.124	0.000	0.387	18.354	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	210	160	385	0	102	144	0	-1
N.S.	1	0.99	0.75	1.81	0.00	0.48	0.68	0.00	-0.00
time (sec)	N/A	0.115	10.156	0.134	0.000	0.344	12.676	0.000	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	383	148	659	0	126	151	0	-1
N.S.	1	0.99	0.38	1.71	0.00	0.33	0.39	0.00	-0.00
time (sec)	N/A	0.222	20.123	0.131	0.000	0.288	35.297	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	213	187	403	0	118	151	0	-1
N.S.	1	0.98	0.86	1.86	0.00	0.54	0.70	0.00	-0.00
time (sec)	N/A	0.130	10.138	0.129	0.000	0.313	92.678	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	437	182	706	0	164	0	0	-1
N.S.	1	0.99	0.41	1.60	0.00	0.37	0.00	0.00	-0.00
time (sec)	N/A	0.270	20.159	0.132	0.000	0.310	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	210	743	0	210	306	0	-1
N.S.	1	1.00	0.40	1.40	0.00	0.40	0.58	0.00	-0.00
time (sec)	N/A	0.384	20.158	0.115	0.000	0.479	99.077	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	259	489	0	199	306	0	-1
N.S.	1	1.00	0.76	1.44	0.00	0.59	0.90	0.00	-0.00
time (sec)	N/A	0.222	10.206	0.113	0.000	0.301	32.515	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	179	699	0	173	304	0	-1
N.S.	1	1.00	0.37	1.45	0.00	0.36	0.63	0.00	-0.00
time (sec)	N/A	0.329	20.132	0.111	0.000	0.354	7.093	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	223	444	0	157	306	0	-1
N.S.	1	1.00	0.78	1.55	0.00	0.55	1.07	0.00	-0.00
time (sec)	N/A	0.178	10.189	0.112	0.000	0.301	9.835	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	161	669	0	146	309	0	-1
N.S.	1	1.00	0.34	1.41	0.00	0.31	0.65	0.00	-0.00
time (sec)	N/A	0.319	20.134	0.115	0.000	0.320	10.153	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	202	415	0	141	309	0	-1
N.S.	1	1.00	0.70	1.44	0.00	0.49	1.07	0.00	-0.00
time (sec)	N/A	0.168	10.177	0.126	0.000	0.284	13.522	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	141	668	0	130	320	0	-1
N.S.	1	1.00	0.30	1.43	0.00	0.28	0.68	0.00	-0.00
time (sec)	N/A	0.315	20.126	0.131	0.000	0.310	32.505	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	143	661	0	133	144	0	-1
N.S.	1	1.00	0.33	1.54	0.00	0.31	0.33	0.00	-0.00
time (sec)	N/A	0.279	20.116	0.109	0.000	0.293	31.377	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	190	405	0	122	144	0	-1
N.S.	1	1.00	0.79	1.69	0.00	0.51	0.60	0.00	-0.00
time (sec)	N/A	0.147	10.186	0.105	0.000	0.200	9.234	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	111	604	0	99	143	0	-1
N.S.	1	1.00	0.30	1.61	0.00	0.26	0.38	0.00	-0.00
time (sec)	N/A	0.246	20.107	0.111	0.000	0.257	2.692	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	148	350	0	83	144	0	-1
N.S.	1	1.00	0.77	1.81	0.00	0.43	0.75	0.00	-0.01
time (sec)	N/A	0.108	10.203	0.105	0.000	0.270	2.677	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	115	595	0	90	148	0	-1
N.S.	1	1.00	0.31	1.60	0.00	0.24	0.40	0.00	-0.00
time (sec)	N/A	0.231	20.112	0.115	0.000	0.397	2.983	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	165	352	0	83	148	0	-1
N.S.	1	1.00	0.90	1.91	0.00	0.45	0.80	0.00	-0.01
time (sec)	N/A	0.097	10.124	0.116	0.000	0.248	6.950	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	116	626	0	100	155	0	-1
N.S.	1	1.00	0.30	1.62	0.00	0.26	0.40	0.00	-0.00
time (sec)	N/A	0.236	20.106	0.135	0.000	0.288	26.835	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	159	370	0	94	155	0	-1
N.S.	1	1.00	0.82	1.92	0.00	0.49	0.80	0.00	-0.01
time (sec)	N/A	0.113	10.152	0.131	0.000	0.264	90.593	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	155	667	0	128	0	0	-1
N.S.	1	1.00	0.35	1.52	0.00	0.29	0.00	0.00	-0.00
time (sec)	N/A	0.291	10.154	0.136	0.000	0.481	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	196	411	0	119	0	0	-1
N.S.	1	1.00	0.81	1.70	0.00	0.49	0.00	0.00	-0.00
time (sec)	N/A	0.155	10.184	0.131	0.000	0.256	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	226	407	0	206	0	0	-1
N.S.	1	1.00	0.76	1.38	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.172	10.186	0.161	0.000	0.318	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	133	618	0	177	0	0	-1
N.S.	1	1.00	0.31	1.42	0.00	0.41	0.00	0.00	-0.00
time (sec)	N/A	0.271	20.132	0.151	0.000	0.287	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	191	363	0	163	0	0	-1
N.S.	1	1.00	0.78	1.48	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.138	10.162	0.148	0.000	0.255	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	119	597	0	156	0	0	-1
N.S.	1	1.00	0.31	1.55	0.00	0.41	0.00	0.00	-0.00
time (sec)	N/A	0.226	20.108	0.137	0.000	0.238	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	174	341	0	144	0	0	-1
N.S.	1	1.00	0.90	1.77	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.103	10.123	0.150	0.000	0.327	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	126	594	0	158	0	0	-1
N.S.	1	1.00	0.32	1.51	0.00	0.40	0.00	0.00	-0.00
time (sec)	N/A	0.243	10.105	0.145	0.000	0.264	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	181	353	0	156	0	0	-1
N.S.	1	1.00	0.87	1.71	0.00	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.122	10.162	0.151	0.000	0.189	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	141	638	0	186	0	0	-1
N.S.	1	1.00	0.32	1.47	0.00	0.43	0.00	0.00	-0.00
time (sec)	N/A	0.299	10.106	0.157	0.000	0.293	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	222	696	0	249	0	0	-1
N.S.	1	1.00	0.74	2.30	0.00	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.163	10.223	0.191	0.000	0.266	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	153	1191	0	242	0	0	-1
N.S.	1	1.00	0.35	2.69	0.00	0.55	0.00	0.00	-0.00
time (sec)	N/A	0.271	20.157	0.178	0.000	0.378	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	204	674	0	229	0	0	-1
N.S.	1	1.00	0.82	2.72	0.00	0.92	0.00	0.00	-0.00
time (sec)	N/A	0.132	10.199	0.166	0.000	0.340	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	147	1176	0	232	0	0	-1
N.S.	1	1.00	0.36	2.92	0.00	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.231	20.155	0.126	0.000	0.253	0.000	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	169	660	0	218	0	0	-1
N.S.	1	1.00	0.79	3.10	0.00	1.02	0.00	0.00	-0.00
time (sec)	N/A	0.114	10.215	0.132	0.000	0.212	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	161	1187	0	244	0	0	-1
N.S.	1	1.00	0.36	2.69	0.00	0.55	0.00	0.00	-0.00
time (sec)	N/A	0.298	10.128	0.174	0.000	0.262	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	211	686	0	238	0	0	-1
N.S.	1	1.00	0.82	2.66	0.00	0.92	0.00	0.00	-0.00
time (sec)	N/A	0.159	10.221	0.170	0.000	0.276	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	181	1231	0	273	0	0	-1
N.S.	1	1.00	0.37	2.52	0.00	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.330	10.157	0.176	0.000	0.369	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	187	1468	0	0	0	0	-1
N.S.	1	1.00	0.50	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	10.183	0.221	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	143	1480	0	0	0	0	-1
N.S.	1	1.00	0.35	3.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.552	10.133	0.174	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	143	1275	0	0	0	0	-1
N.S.	1	1.00	0.45	4.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	10.112	0.148	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	69	690	0	0	0	0	-1
N.S.	1	1.00	0.19	1.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	10.032	0.132	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	67	640	0	0	0	0	-1
N.S.	1	1.00	0.24	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.255	10.032	0.122	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	143	1263	0	0	0	0	-1
N.S.	1	1.00	0.36	3.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	10.099	0.126	0.000	0.000	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	146	1156	0	0	0	0	-1
N.S.	1	1.00	0.47	3.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	10.120	0.130	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	190	1542	0	0	0	0	-1
N.S.	1	1.00	0.42	3.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.692	10.169	0.127	0.000	0.000	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	183	2172	0	0	0	0	-1
N.S.	1	1.00	0.38	4.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.753	10.174	0.156	0.000	0.000	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	182	1909	0	0	0	0	-1
N.S.	1	1.00	0.49	5.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	10.171	0.144	0.000	0.000	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	155	1916	0	0	0	0	-1
N.S.	1	1.00	0.37	4.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	10.181	0.152	0.000	0.000	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	153	1710	0	0	0	0	-1
N.S.	1	1.00	0.47	5.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	10.129	0.142	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	151	1747	0	0	0	0	-1
N.S.	1	1.00	0.36	4.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	10.112	0.137	0.000	0.000	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	153	1729	0	0	0	0	-1
N.S.	1	1.00	0.46	5.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	10.129	0.120	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	187	2017	0	0	0	0	-1
N.S.	1	1.00	0.41	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	10.185	0.136	0.000	0.000	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	147	842	0	0	0	0	-1
N.S.	1	1.00	0.48	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.339	10.108	0.136	0.000	0.000	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	70	459	0	0	0	0	-1
N.S.	1	1.00	0.20	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	10.053	0.117	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	70	406	0	0	0	0	-1
N.S.	1	1.00	0.27	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	10.054	0.118	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	70	326	0	0	0	0	-1
N.S.	1	1.00	0.34	1.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.183	10.042	0.117	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	68	335	0	0	0	0	-1
N.S.	1	1.00	0.36	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	10.041	0.120	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	146	826	0	0	0	0	-1
N.S.	1	1.00	0.39	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.470	10.109	0.120	0.000	0.000	0.000	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	148	729	0	0	0	0	-1
N.S.	1	1.00	0.50	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	10.117	0.128	0.000	0.000	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	188	1098	0	0	0	0	-1
N.S.	1	1.00	0.42	2.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.636	10.171	0.128	0.000	0.000	0.000	0.000	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	148	1030	0	0	0	0	-1
N.S.	1	1.00	0.33	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.556	10.150	0.129	0.000	0.000	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	148	815	0	0	0	0	-1
N.S.	1	1.00	0.44	2.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	10.118	0.128	0.000	0.000	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	133	828	0	0	0	0	-1
N.S.	1	1.00	0.32	2.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	10.109	0.126	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	133	693	0	0	0	0	-1
N.S.	1	1.00	0.42	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	10.089	0.135	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	148	819	0	0	0	0	-1
N.S.	1	1.00	0.35	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.516	10.156	0.128	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	147	697	0	0	0	0	-1
N.S.	1	1.00	0.45	2.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	10.116	0.131	0.000	0.000	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	198	1047	0	0	0	0	-1
N.S.	1	1.00	0.40	2.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.695	10.187	0.129	0.000	0.000	0.000	0.000	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	197	885	0	0	0	0	-1
N.S.	1	1.00	0.50	2.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	10.184	0.141	0.000	0.000	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	184	2549	0	0	0	0	-1
N.S.	1	1.00	0.51	7.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.462	10.171	0.186	0.000	0.000	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	163	2530	0	0	0	0	-1
N.S.	1	1.00	0.39	6.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	10.135	0.125	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	163	2243	0	0	0	0	-1
N.S.	1	1.00	0.50	6.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	10.127	0.131	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	163	2522	0	0	0	0	-1
N.S.	1	1.00	0.39	6.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.496	10.110	0.128	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	161	2239	0	0	0	0	-1
N.S.	1	1.00	0.48	6.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.316	10.099	0.128	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	182	2556	0	0	0	0	-1
N.S.	1	1.00	0.41	5.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.617	10.137	0.127	0.000	0.000	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	181	2304	0	0	0	0	-1
N.S.	1	1.00	0.51	6.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.436	10.150	0.134	0.000	0.000	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	233	3778	0	0	0	0	-1
N.S.	1	1.00	0.54	8.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.661	10.210	0.174	0.000	0.000	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	196	3874	0	0	0	0	-1
N.S.	1	1.00	0.40	7.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.693	10.208	0.198	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	195	3454	0	0	0	0	-1
N.S.	1	1.00	0.51	9.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.498	10.201	0.162	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	189	3846	0	0	0	0	-1
N.S.	1	1.00	0.40	8.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.592	10.187	0.138	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	187	2519	0	0	0	0	-1
N.S.	1	1.00	0.51	6.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	10.150	0.123	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	197	3867	0	0	0	0	-1
N.S.	1	1.00	0.38	7.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.759	10.177	0.132	0.000	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	199	3472	0	0	0	0	-1
N.S.	1	1.00	0.48	8.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	10.177	0.134	0.000	0.000	0.000	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	184	2944	0	0	0	0	-1
N.S.	1	1.00	0.38	6.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.603	10.174	0.130	0.000	0.000	0.000	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	184	2508	0	0	0	0	-1
N.S.	1	1.00	0.49	6.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	10.168	0.121	0.000	0.000	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	168	2536	0	0	0	0	-1
N.S.	1	1.00	0.37	5.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	10.124	0.122	0.000	0.000	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	169	2246	0	0	0	0	-1
N.S.	1	1.00	0.47	6.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	10.125	0.123	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	181	2533	0	0	0	0	-1
N.S.	1	1.00	0.39	5.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	10.196	0.126	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	180	2254	0	0	0	0	-1
N.S.	1	1.00	0.49	6.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	10.156	0.125	0.000	0.000	0.000	0.000	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	535	235	2970	0	0	0	0	-1
N.S.	1	1.00	0.44	5.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.748	10.225	0.137	0.000	0.000	0.000	0.000	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	234	2610	0	0	0	0	-1
N.S.	1	1.00	0.55	6.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.561	10.234	0.132	0.000	0.000	0.000	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	189	2952	0	0	0	0	-1
N.S.	1	1.00	0.36	5.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.751	10.188	0.134	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	191	2518	0	0	0	0	-1
N.S.	1	1.00	0.45	6.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	10.182	0.130	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	185	2549	0	0	0	0	-1
N.S.	1	1.00	0.38	5.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.666	10.196	0.131	0.000	0.000	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	186	2265	0	0	0	0	-1
N.S.	1	1.00	0.48	5.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	10.172	0.140	0.000	0.000	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	230	2938	0	0	0	0	-1
N.S.	1	1.00	0.43	5.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.764	10.247	0.133	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	229	2542	0	0	0	0	-1
N.S.	1	1.00	0.54	5.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.526	10.201	0.142	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	319	3373	0	0	0	0	-1
N.S.	1	1.00	0.51	5.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.943	10.343	0.136	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	318	2859	0	0	0	0	-1
N.S.	1	1.00	0.62	5.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	10.367	0.134	0.000	0.000	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	256	5114	0	0	0	0	-1
N.S.	1	1.00	0.45	9.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.929	10.298	0.154	0.000	0.000	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	252	4391	0	0	0	0	-1
N.S.	1	1.00	0.56	9.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	10.291	0.138	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	278	5066	0	0	0	0	-1
N.S.	1	1.00	0.50	9.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.893	10.336	0.138	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	275	4391	0	0	0	0	-1
N.S.	1	1.00	0.62	9.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	10.363	0.143	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	327	5677	0	0	0	0	-1
N.S.	1	1.00	0.52	9.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.987	10.439	0.136	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	328	4764	0	0	0	0	-1
N.S.	1	1.00	0.64	9.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.674	10.391	0.134	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	735	735	407	6322	0	0	0	0	-1
N.S.	1	1.00	0.55	8.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.243	10.910	0.148	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	606	606	427	5236	0	0	0	0	-1
N.S.	1	1.00	0.70	8.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.986	11.008	0.152	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	187	455	0	442	0	226	993
N.S.	1	1.00	0.89	2.18	0.00	2.11	0.00	1.08	4.75
time (sec)	N/A	0.175	1.604	0.158	0.000	5.438	0.000	0.978	27.366

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	119	290	0	334	0	159	639
N.S.	1	1.00	0.87	2.12	0.00	2.44	0.00	1.16	4.66
time (sec)	N/A	0.092	0.953	0.121	0.000	6.058	0.000	1.754	14.077

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	170	0	259	0	106	280
N.S.	1	1.00	1.00	1.98	0.00	3.01	0.00	1.23	3.26
time (sec)	N/A	0.055	0.380	0.107	0.000	2.605	0.000	1.332	2.737

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	156	0	777	0	0	2500
N.S.	1	1.00	1.00	1.70	0.00	8.45	0.00	0.00	27.17
time (sec)	N/A	0.067	0.463	0.112	0.000	1.331	0.000	0.000	10.312

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	179	0	280	0	434	477
N.S.	1	1.00	1.00	2.01	0.00	3.15	0.00	4.88	5.36
time (sec)	N/A	0.050	0.642	0.145	0.000	4.329	0.000	2.389	3.388

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	125	306	0	358	0	1107	955
N.S.	1	1.00	0.87	2.14	0.00	2.50	0.00	7.74	6.68
time (sec)	N/A	0.089	1.202	0.125	0.000	4.403	0.000	2.098	11.529

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	246	526	0	0	0	0	-1
N.S.	1	1.00	0.72	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	1.603	0.141	0.000	0.000	0.000	0.000	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	199	335	0	0	0	0	-1
N.S.	1	1.00	0.77	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.108	1.056	0.126	0.000	0.000	0.000	0.000	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	111	168	0	0	0	0	-1
N.S.	1	1.00	0.48	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.105	1.176	0.132	0.000	0.000	0.000	0.000	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	228	418	0	0	0	0	-1
N.S.	1	1.00	0.74	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.177	1.794	0.129	0.000	0.000	0.000	0.000	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	231	658	0	574	0	305	-1
N.S.	1	1.00	0.84	2.38	0.00	2.08	0.00	1.11	-0.00
time (sec)	N/A	0.229	3.094	0.129	0.000	0.798	0.000	2.006	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	148	455	0	440	0	225	-1
N.S.	1	1.00	0.79	2.43	0.00	2.35	0.00	1.20	-0.01
time (sec)	N/A	0.122	1.705	0.118	0.000	0.714	0.000	0.867	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	104	288	0	334	0	149	-1
N.S.	1	1.00	0.83	2.30	0.00	2.67	0.00	1.19	-0.01
time (sec)	N/A	0.074	0.964	0.131	0.000	0.731	0.000	0.785	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	132	252	0	918	0	0	-1
N.S.	1	1.00	0.99	1.89	0.00	6.90	0.00	0.00	-0.01
time (sec)	N/A	0.099	1.230	0.117	0.000	2.246	0.000	0.000	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	135	263	0	958	0	498	-1
N.S.	1	1.00	0.99	1.93	0.00	7.04	0.00	3.66	-0.01
time (sec)	N/A	0.097	1.388	0.145	0.000	1.827	0.000	1.508	0.000

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	110	303	0	360	0	1101	-1
N.S.	1	1.00	0.84	2.31	0.00	2.75	0.00	8.40	-0.01
time (sec)	N/A	0.071	1.444	0.134	0.000	1.991	0.000	1.831	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	305	782	0	0	0	0	-1
N.S.	1	1.00	0.71	1.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	3.014	0.131	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	245	544	0	0	0	0	-1
N.S.	1	1.00	0.73	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	2.287	0.123	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	206	352	0	0	0	0	-1
N.S.	1	1.00	0.84	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.107	2.249	0.125	0.000	0.000	0.000	0.000	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	227	433	0	0	0	0	-1
N.S.	1	1.00	0.73	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	2.556	0.130	0.000	0.000	0.000	0.000	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	271	900	0	734	0	398	-1
N.S.	1	1.00	0.80	2.65	0.00	2.16	0.00	1.17	-0.00
time (sec)	N/A	0.286	3.908	0.146	0.000	1.397	0.000	1.058	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	214	658	0	574	0	304	-1
N.S.	1	1.00	0.90	2.78	0.00	2.42	0.00	1.28	-0.00
time (sec)	N/A	0.155	3.345	0.122	0.000	1.027	0.000	1.115	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	137	452	0	440	0	210	-1
N.S.	1	1.00	0.84	2.76	0.00	2.68	0.00	1.28	-0.01
time (sec)	N/A	0.100	1.682	0.134	0.000	1.335	0.000	1.038	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	162	390	0	1075	0	0	-1
N.S.	1	1.00	0.87	2.09	0.00	5.75	0.00	0.00	-0.01
time (sec)	N/A	0.151	2.124	0.125	0.000	2.627	0.000	0.000	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	166	374	0	1097	0	558	-1
N.S.	1	1.00	0.89	2.00	0.00	5.87	0.00	2.98	-0.01
time (sec)	N/A	0.158	2.234	0.172	0.000	1.947	0.000	1.450	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	206	408	0	1123	0	1175	-1
N.S.	1	1.00	1.07	2.12	0.00	5.85	0.00	6.12	-0.01
time (sec)	N/A	0.144	3.541	0.144	0.000	1.703	0.000	1.695	0.000

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	379	1047	0	0	0	0	-1
N.S.	1	1.00	0.69	1.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	4.319	0.131	0.000	0.000	0.000	0.000	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	306	782	0	0	0	0	-1
N.S.	1	1.00	0.70	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	3.063	0.130	0.000	0.000	0.000	0.000	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	254	568	0	0	0	0	-1
N.S.	1	1.00	0.77	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	3.005	0.123	0.000	0.000	0.000	0.000	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	261	583	0	0	0	0	-1
N.S.	1	1.00	0.78	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	3.171	0.135	0.000	0.000	0.000	0.000	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	135	0	35	0	0	-1
N.S.	1	1.00	0.93	1.36	0.00	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.513	0.184	0.000	0.129	0.000	0.000	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	66	81	46	72	0	40	414
N.S.	1	1.00	1.02	1.25	0.71	1.11	0.00	0.62	6.37
time (sec)	N/A	0.034	0.473	0.161	0.478	0.433	0.000	0.960	6.856

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	86	129	0	28	0	0	-1
N.S.	1	1.00	1.23	1.84	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.334	0.145	0.000	0.122	0.000	0.000	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	53	60	27	65	58	33	206
N.S.	1	1.00	1.36	1.54	0.69	1.67	1.49	0.85	5.28
time (sec)	N/A	0.021	0.120	0.135	0.479	0.422	2.117	0.808	1.491

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	127	306	0	0	0	0	-1
N.S.	1	1.00	0.53	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.098	0.823	0.148	0.000	0.000	0.000	0.000	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	120	291	0	336	0	160	550
N.S.	1	1.00	0.85	2.06	0.00	2.38	0.00	1.13	3.90
time (sec)	N/A	0.112	1.288	0.114	0.000	1.108	0.000	1.152	12.869

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	172	0	256	0	104	279
N.S.	1	1.00	1.00	1.95	0.00	2.91	0.00	1.18	3.17
time (sec)	N/A	0.068	0.869	0.117	0.000	1.108	0.000	0.983	2.898

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	89	0	194	0	54	49
N.S.	1	1.00	1.00	1.98	0.00	4.31	0.00	1.20	1.09
time (sec)	N/A	0.038	0.375	0.110	0.000	0.979	0.000	1.172	0.697

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	89	0	204	0	89	136
N.S.	1	1.00	1.00	1.93	0.00	4.43	0.00	1.93	2.96
time (sec)	N/A	0.032	0.555	0.110	0.000	1.092	0.000	1.197	1.966

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	181	0	278	0	413	481
N.S.	1	1.00	1.00	1.99	0.00	3.05	0.00	4.54	5.29
time (sec)	N/A	0.057	0.828	0.125	0.000	2.267	0.000	1.094	3.701

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	126	306	0	360	0	1015	962
N.S.	1	1.00	0.85	2.05	0.00	2.42	0.00	6.81	6.46
time (sec)	N/A	0.093	1.391	0.122	0.000	1.566	0.000	1.064	11.810

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	249	546	0	0	0	0	-1
N.S.	1	1.00	0.73	1.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	2.153	0.122	0.000	0.000	0.000	0.000	0.000

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	201	333	0	0	0	0	-1
N.S.	1	1.00	0.77	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.108	1.553	0.119	0.000	0.000	0.000	0.000	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	122	129	0	0	0	0	-1
N.S.	1	1.00	1.05	1.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.756	0.103	0.000	0.000	0.000	0.000	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	146	224	0	0	0	0	-1
N.S.	1	1.00	0.95	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	1.219	0.115	0.000	0.000	0.000	0.000	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	229	435	0	0	0	0	-1
N.S.	1	1.00	0.75	1.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.175	1.804	0.128	0.000	0.000	0.000	0.000	0.000

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	153	511	0	498	0	192	-1
N.S.	1	1.00	1.19	3.96	0.00	3.86	0.00	1.49	-0.01
time (sec)	N/A	0.108	1.895	0.165	0.000	1.002	0.000	0.896	0.000

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	292	0	367	0	135	-1
N.S.	1	1.00	1.00	3.52	0.00	4.42	0.00	1.63	-0.01
time (sec)	N/A	0.064	1.438	0.122	0.000	1.239	0.000	0.663	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	0	48	0	70	45
N.S.	1	1.00	1.00	0.88	0.00	1.41	0.00	2.06	1.32
time (sec)	N/A	0.018	0.075	0.109	0.000	1.091	0.000	0.609	0.692

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	114	609	0	706	0	333	-1
N.S.	1	1.00	0.83	4.45	0.00	5.15	0.00	2.43	-0.01
time (sec)	N/A	0.097	2.401	0.130	0.000	3.096	0.000	0.704	0.000

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	50	0	128	0	214	139
N.S.	1	1.00	0.61	0.56	0.00	1.44	0.00	2.40	1.56
time (sec)	N/A	0.049	2.026	0.116	0.000	3.156	0.000	0.957	0.791

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	47	0	126	0	129	137
N.S.	1	1.00	0.70	0.64	0.00	1.70	0.00	1.74	1.85
time (sec)	N/A	0.031	1.600	0.118	0.000	4.056	0.000	0.661	0.759

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	91	134	0	262	0	597	220
N.S.	1	1.00	0.59	0.87	0.00	1.70	0.00	3.88	1.43
time (sec)	N/A	0.122	2.597	0.133	0.000	3.501	0.000	0.659	0.880

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	91	140	0	269	0	472	227
N.S.	1	1.00	0.66	1.01	0.00	1.95	0.00	3.42	1.64
time (sec)	N/A	0.074	2.215	0.123	0.000	2.108	0.000	0.651	0.842

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	83	128	0	259	0	243	216
N.S.	1	1.00	0.73	1.13	0.00	2.29	0.00	2.15	1.91
time (sec)	N/A	0.047	1.814	0.117	0.000	5.020	0.000	0.586	0.823

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	151	241	0	451	0	1036	336
N.S.	1	1.00	0.70	1.11	0.00	2.08	0.00	4.77	1.55
time (sec)	N/A	0.180	3.673	0.170	0.000	3.354	0.000	0.653	1.019

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	92	0	201	0	57	48
N.S.	1	1.00	0.98	1.96	0.00	4.28	0.00	1.21	1.02
time (sec)	N/A	0.042	0.413	0.113	0.000	1.396	0.000	0.594	0.745

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	95	0	203	0	57	51
N.S.	1	1.00	1.00	1.98	0.00	4.23	0.00	1.19	1.06
time (sec)	N/A	0.041	0.413	0.117	0.000	0.820	0.000	0.600	0.678

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	72	70	0	0	0	0	-1
N.S.	1	1.00	0.65	0.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.606	0.109	0.000	0.000	0.000	0.000	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	59	59	0	0	0	0	-1
N.S.	1	1.00	0.68	0.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.524	0.131	0.000	0.000	0.000	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	70	76	0	0	0	0	-1
N.S.	1	1.00	0.80	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.528	0.121	0.000	0.000	0.000	0.000	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	25	0	23	0	0	-1
N.S.	1	1.00	0.77	0.81	0.00	0.74	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.274	0.129	0.000	0.207	0.000	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	37	23	0	0	0	0	-1
N.S.	1	1.00	1.19	0.74	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.267	0.127	0.000	0.000	0.000	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	29	0	23	0	0	-1
N.S.	1	1.00	0.80	0.83	0.00	0.66	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.285	0.121	0.000	0.314	0.000	0.000	0.000

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	38	33	0	0	0	0	-1
N.S.	1	1.00	1.09	0.94	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.256	0.133	0.000	0.000	0.000	0.000	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	29	0	23	0	0	-1
N.S.	1	1.00	0.80	0.83	0.00	0.66	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.274	0.127	0.000	0.271	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	27	0	0	0	0	-1
N.S.	1	1.00	0.80	0.77	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.255	0.129	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	35	0	21	0	0	-1
N.S.	1	1.00	0.88	0.83	0.00	0.50	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.278	0.124	0.000	0.149	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	35	0	21	0	0	-1
N.S.	1	1.00	0.88	0.81	0.00	0.49	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.257	0.128	0.000	0.130	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	35	0	23	0	0	-1
N.S.	1	1.00	0.85	0.74	0.00	0.49	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.278	0.126	0.000	0.232	0.000	0.000	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	34	30	0	0	0	0	-1
N.S.	1	1.00	0.42	0.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.249	0.114	0.000	0.000	0.000	0.000	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	38	26	0	0	0	0	-1
N.S.	1	1.00	0.46	0.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.268	0.113	0.000	0.000	0.000	0.000	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	50	36	0	0	0	0	-1
N.S.	1	1.00	0.57	0.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.266	0.114	0.000	0.000	0.000	0.000	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	34	0	23	0	0	-1
N.S.	1	1.00	2.18	2.00	0.00	1.35	0.00	0.00	-0.06
time (sec)	N/A	0.019	0.266	0.140	0.000	0.526	0.000	0.000	0.000

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	121	482	108	102	0	108	128
N.S.	1	1.00	1.11	4.42	0.99	0.94	0.00	0.99	1.17
time (sec)	N/A	0.070	0.147	7.511	0.478	0.886	0.000	0.616	0.501

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	105	648	97	122	0	97	117
N.S.	1	1.00	1.12	6.89	1.03	1.30	0.00	1.03	1.24
time (sec)	N/A	0.048	0.104	7.608	0.478	0.888	0.000	0.586	0.476

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	82	736	86	86	78	86	106
N.S.	1	1.00	1.04	9.32	1.09	1.09	0.99	1.09	1.34
time (sec)	N/A	0.038	0.084	3.575	0.488	0.747	41.524	0.545	0.586

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	163	0	0	177	0	149	256
N.S.	1	1.00	1.20	0.00	0.00	1.30	0.00	1.10	1.88
time (sec)	N/A	0.070	0.172	0.033	0.000	0.678	0.000	0.571	0.548

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	108	755	0	115	0	100	120
N.S.	1	1.00	1.11	7.78	0.00	1.19	0.00	1.03	1.24
time (sec)	N/A	0.056	0.146	13.127	0.000	0.693	0.000	0.590	0.493

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	214	0	0	217	0	177	397
N.S.	1	1.00	1.24	0.00	0.00	1.26	0.00	1.03	2.31
time (sec)	N/A	0.097	0.288	0.010	0.000	0.713	0.000	0.993	0.550

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	156	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	5.057	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	28	0	0	0	0	0	-1
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.117	4.437	0.030	0.000	0.000	0.000	0.000	0.000

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	938	0	1943	0	0	-1
N.S.	1	1.00	1.04	8.30	0.00	17.19	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.028	12.878	0.000	1.383	0.000	0.000	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	538	538	161	0	0	0	0	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	10.091	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	166	0	0	0	0	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	10.098	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	124	489	126	133	0	126	148
N.S.	1	1.00	0.93	3.68	0.95	1.00	0.00	0.95	1.11
time (sec)	N/A	0.062	0.271	6.605	0.471	0.566	0.000	1.367	0.523

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	119	656	115	153	0	115	137
N.S.	1	1.00	1.03	5.66	0.99	1.32	0.00	0.99	1.18
time (sec)	N/A	0.055	0.235	6.419	0.465	0.668	0.000	1.227	0.496

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	112	477	104	121	0	104	126
N.S.	1	1.00	1.11	4.72	1.03	1.20	0.00	1.03	1.25
time (sec)	N/A	0.045	0.196	6.464	0.479	0.507	0.000	1.262	0.505

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	112	484	104	125	0	104	126
N.S.	1	1.00	1.11	4.79	1.03	1.24	0.00	1.03	1.25
time (sec)	N/A	0.042	0.139	6.670	0.467	0.441	0.000	1.106	0.483

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	184	0	0	227	0	167	375
N.S.	1	1.00	1.16	0.00	0.00	1.44	0.00	1.06	2.37
time (sec)	N/A	0.078	0.301	0.013	0.000	0.444	0.000	0.923	0.550

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	194	0	0	238	0	190	409
N.S.	1	1.00	1.06	0.00	0.00	1.30	0.00	1.04	2.23
time (sec)	N/A	0.094	0.356	0.013	0.000	0.493	0.000	1.076	0.543

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	197	0	0	265	0	181	416
N.S.	1	1.00	0.95	0.00	0.00	1.27	0.00	0.87	2.00
time (sec)	N/A	0.107	0.378	0.011	0.000	0.491	0.000	0.814	0.560

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	157	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	5.416	0.013	0.000	0.000	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	156	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	4.997	0.009	0.000	0.000	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	157	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.170	10.101	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	168	0	0	0	0	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.209	10.116	0.013	0.000	0.000	0.000	0.000	0.000

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	173	0	0	0	0	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	10.109	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	110	211	151	253	0	160	82
N.S.	1	1.00	0.81	1.55	1.11	1.86	0.00	1.18	0.60
time (sec)	N/A	0.064	0.182	3.250	0.486	1.498	0.000	0.733	0.501

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	103	204	140	246	0	140	71
N.S.	1	1.00	0.85	1.69	1.16	2.03	0.00	1.16	0.59
time (sec)	N/A	0.045	0.132	2.732	0.478	1.391	0.000	0.825	0.092

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	96	199	129	241	0	129	60
N.S.	1	1.00	0.91	1.88	1.22	2.27	0.00	1.22	0.57
time (sec)	N/A	0.034	0.103	2.697	0.486	2.032	0.000	1.020	0.083

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	76	188	118	189	0	118	49
N.S.	1	1.00	0.84	2.07	1.30	2.08	0.00	1.30	0.54
time (sec)	N/A	0.010	0.088	1.431	0.498	1.646	0.000	1.145	0.081

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	117	0	0	270	0	216	91
N.S.	1	1.00	0.81	0.00	0.00	1.86	0.00	1.49	0.63
time (sec)	N/A	0.063	0.156	0.055	0.000	0.996	0.000	1.087	0.543

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	155	0	0	307	0	192	109
N.S.	1	1.00	0.95	0.00	0.00	1.88	0.00	1.18	0.67
time (sec)	N/A	0.094	0.269	0.020	0.000	1.027	0.000	0.840	0.568

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	184	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	6.447	0.015	0.000	0.000	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	37	0	0	0	0	0	-1
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	6.006	0.043	0.000	0.000	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	119	188	0	553	0	0	-1
N.S.	1	1.00	0.99	1.57	0.00	4.61	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.025	1.484	0.000	2.659	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	56	0	0	0	0	0	-1
N.S.	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	20.048	0.015	0.000	0.000	0.000	0.000	0.000

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	156	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	20.133	0.015	0.000	0.000	0.000	0.000	0.000

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	57	147	74	64	88	75	62
N.S.	1	1.00	0.73	1.88	0.95	0.82	1.13	0.96	0.79
time (sec)	N/A	0.038	0.051	1.399	0.469	0.574	12.601	0.706	0.068

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	141	63	57	75	64	51
N.S.	1	1.00	0.84	2.24	1.00	0.90	1.19	1.02	0.81
time (sec)	N/A	0.033	0.044	1.275	0.460	0.700	9.652	0.629	0.491

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	44	135	52	52	58	53	36
N.S.	1	1.00	0.92	2.81	1.08	1.08	1.21	1.10	0.75
time (sec)	N/A	0.023	0.029	1.363	0.465	0.575	7.031	0.753	0.474

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	126	41	41	42	42	25
N.S.	1	1.00	1.00	3.82	1.24	1.24	1.27	1.27	0.76
time (sec)	N/A	0.015	0.024	0.714	0.474	0.678	4.176	0.808	0.086

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	110	302	0	215	0	155	77
N.S.	1	1.00	0.64	1.75	0.00	1.24	0.00	0.90	0.45
time (sec)	N/A	0.098	0.129	4.179	0.000	0.588	0.000	0.736	0.104

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	127	315	0	252	0	169	82
N.S.	1	1.00	0.66	1.65	0.00	1.32	0.00	0.88	0.43
time (sec)	N/A	0.110	0.185	8.084	0.000	0.546	0.000	0.746	0.122

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	177	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.121	6.140	0.016	0.000	0.000	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	52	0	0	0	0	0	-1
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.071	5.749	0.047	0.000	0.000	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	138	0	104	0	0	-1
N.S.	1	1.00	0.92	2.26	0.00	1.70	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.013	0.944	0.000	8.401	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	64	0	0	0	0	0	-1
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.085	10.028	0.016	0.000	0.000	0.000	0.000	0.000

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	148	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.143	10.119	0.016	0.000	0.000	0.000	0.000	0.000

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	109	186	0	282	0	0	-1
N.S.	1	1.00	0.84	1.44	0.00	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.015	1.881	5.451	0.000	1.778	0.000	0.000	0.000

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	109	186	0	282	0	0	-1
N.S.	1	1.00	0.91	1.55	0.00	2.35	0.00	0.00	-0.01
time (sec)	N/A	0.016	1.926	5.438	0.000	1.152	0.000	0.000	0.000

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	112	0	0	393	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.021	2.105	0.035	0.000	0.908	0.000	0.000	0.000

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	114	0	0	403	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	3.39	0.00	0.00	-0.01
time (sec)	N/A	0.021	2.177	0.036	0.000	0.879	0.000	0.000	0.000

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	119	0	0	171	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.020	2.090	0.035	0.000	4.469	0.000	0.000	0.000

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	119	0	0	171	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.018	2.187	0.033	0.000	2.359	0.000	0.000	0.000

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	121	0	0	207	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	1.80	0.00	0.00	-0.01
time (sec)	N/A	0.024	2.089	0.032	0.000	1.336	0.000	0.000	0.000

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	125	0	0	211	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.027	2.109	0.031	0.000	1.076	0.000	0.000	0.000

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	110	212	151	198	0	160	82
N.S.	1	1.00	0.59	1.13	0.80	1.05	0.00	0.85	0.44
time (sec)	N/A	0.142	0.184	3.381	0.480	0.919	0.000	1.674	0.486

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	105	208	140	193	0	140	71
N.S.	1	1.00	0.61	1.20	0.81	1.12	0.00	0.81	0.41
time (sec)	N/A	0.124	0.136	2.922	0.473	1.087	0.000	1.963	0.094

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	97	202	129	186	0	129	60
N.S.	1	1.00	0.61	1.28	0.82	1.18	0.00	0.82	0.38
time (sec)	N/A	0.110	0.109	2.916	0.494	0.703	0.000	1.180	0.089

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	78	190	118	230	0	118	49
N.S.	1	1.00	0.55	1.33	0.83	1.61	0.00	0.83	0.34
time (sec)	N/A	0.083	0.096	1.139	0.484	0.712	0.000	1.219	0.494

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	121	562	0	315	0	210	91
N.S.	1	1.00	0.61	2.85	0.00	1.60	0.00	1.07	0.46
time (sec)	N/A	0.130	0.168	9.984	0.000	2.593	0.000	0.962	0.122

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	155	577	0	352	0	192	107
N.S.	1	1.00	0.72	2.68	0.00	1.64	0.00	0.89	0.50
time (sec)	N/A	0.159	0.255	25.009	0.000	1.644	0.000	0.842	0.557

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	190	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	7.027	0.025	0.000	0.000	0.000	0.000	0.000

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	184	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	6.828	0.012	0.000	0.000	0.000	0.000	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	109	186	0	282	0	0	-1
N.S.	1	1.00	0.91	1.55	0.00	2.35	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.035	0.001	0.000	0.956	0.000	0.000	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	63	0	0	0	0	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	6.426	0.039	0.000	0.000	0.000	0.000	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	37	0	0	0	0	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	20.060	0.017	0.000	0.000	0.000	0.000	0.000

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	37	0	0	0	0	0	-1
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	20.059	0.019	0.000	0.000	0.000	0.000	0.000

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	138	0	104	0	0	-1
N.S.	1	1.00	0.89	2.26	0.00	1.70	0.00	0.00	-0.02
time (sec)	N/A	0.011	1.773	1.414	0.000	0.621	0.000	0.000	0.000

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	139	0	115	0	0	-1
N.S.	1	1.00	0.89	2.28	0.00	1.89	0.00	0.00	-0.02
time (sec)	N/A	0.013	1.801	1.412	0.000	0.611	0.000	0.000	0.000

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	0	0	275	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.017	2.024	0.035	0.000	0.580	0.000	0.000	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	0	0	274	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	3.70	0.00	0.00	-0.01
time (sec)	N/A	0.018	1.977	0.033	0.000	0.710	0.000	0.000	0.000

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	0	0	145	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.019	1.985	0.031	0.000	0.587	0.000	0.000	0.000

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	0	0	145	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.016	2.061	0.027	0.000	0.620	0.000	0.000	0.000

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	83	0	0	207	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.022	2.038	0.032	0.000	0.645	0.000	0.000	0.000

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	86	0	0	211	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	2.15	0.00	0.00	-0.01
time (sec)	N/A	0.023	2.001	0.029	0.000	0.522	0.000	0.000	0.000

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	57	147	74	64	0	75	62
N.S.	1	1.00	0.73	1.88	0.95	0.82	0.00	0.96	0.79
time (sec)	N/A	0.037	0.056	1.651	0.481	0.498	0.000	1.513	0.053

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	142	63	59	0	64	51
N.S.	1	1.00	0.83	2.25	1.00	0.94	0.00	1.02	0.81
time (sec)	N/A	0.032	0.048	1.533	0.465	0.498	0.000	1.124	0.073

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	44	136	52	52	0	53	36
N.S.	1	1.00	0.92	2.83	1.08	1.08	0.00	1.10	0.75
time (sec)	N/A	0.023	0.034	1.420	0.490	0.580	0.000	1.037	0.056

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	125	41	41	0	42	25
N.S.	1	1.00	1.00	3.79	1.24	1.24	0.00	1.27	0.76
time (sec)	N/A	0.015	0.025	0.550	0.492	0.458	0.000	1.864	0.459

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	111	302	0	215	0	155	77
N.S.	1	1.00	0.64	1.75	0.00	1.24	0.00	0.90	0.45
time (sec)	N/A	0.094	0.116	3.683	0.000	0.456	0.000	1.065	0.509

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	127	314	0	252	0	169	81
N.S.	1	1.00	0.66	1.64	0.00	1.32	0.00	0.88	0.42
time (sec)	N/A	0.106	0.190	10.113	0.000	0.505	0.000	1.318	0.114

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	184	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	6.807	0.019	0.000	0.000	0.000	0.000	0.000

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	179	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	6.610	0.014	0.000	0.000	0.000	0.000	0.000

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	138	0	104	0	0	-1
N.S.	1	1.00	0.89	2.26	0.00	1.70	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.013	0.000	0.000	1.174	0.000	0.000	0.000

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	68	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	6.120	0.053	0.000	0.000	0.000	0.000	0.000

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	52	0	0	0	0	0	-1
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	10.059	0.012	0.000	0.000	0.000	0.000	0.000

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	52	0	0	0	0	0	-1
N.S.	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	10.053	0.014	0.000	0.000	0.000	0.000	0.000

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	131	0	275	0	94	0	-1
N.S.	1	1.00	0.76	0.00	1.59	0.00	0.54	0.00	-0.01
time (sec)	N/A	0.084	0.635	0.015	0.495	0.000	17.565	0.000	0.000

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	112	0	189	0	92	0	-1
N.S.	1	1.00	0.82	0.00	1.39	0.00	0.68	0.00	-0.01
time (sec)	N/A	0.059	0.387	0.008	0.490	0.000	2.066	0.000	0.000

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	100	0	93	0	85	0	-1
N.S.	1	1.00	0.88	0.00	0.82	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.051	0.310	0.011	0.477	0.000	3.886	0.000	0.000

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	39	58	37	121	0	49
N.S.	1	1.00	0.66	0.58	0.87	0.55	1.81	0.00	0.73
time (sec)	N/A	0.020	0.267	0.101	0.279	1.981	38.397	0.000	0.642

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	67	62	94	60	0	0	75
N.S.	1	1.00	0.64	0.60	0.90	0.58	0.00	0.00	0.72
time (sec)	N/A	0.031	0.372	0.106	0.268	1.214	0.000	0.000	0.692

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	91	86	128	84	0	0	100
N.S.	1	1.00	0.65	0.61	0.91	0.60	0.00	0.00	0.71
time (sec)	N/A	0.043	0.464	0.105	0.267	1.820	0.000	0.000	0.671

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	123	0	0	0	94	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.52	0.00	-0.01
time (sec)	N/A	0.093	10.107	0.015	0.000	0.000	57.559	0.000	0.000

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	97	0	0	0	94	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.68	0.00	-0.01
time (sec)	N/A	0.072	10.086	0.010	0.000	0.000	4.937	0.000	0.000

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	77	0	0	0	78	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.76	0.00	-0.01
time (sec)	N/A	0.062	10.051	0.011	0.000	0.000	2.197	0.000	0.000

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	84	0	0	0	82	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.77	0.00	-0.01
time (sec)	N/A	0.065	10.034	0.012	0.000	0.000	11.241	0.000	0.000

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	88	0	0	0	85	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.59	0.00	-0.01
time (sec)	N/A	0.078	10.060	0.011	0.000	0.000	127.394	0.000	0.000

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	88	0	0	0	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	10.056	0.015	0.000	0.000	0.000	0.000	0.000

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	148	0	229	891	94	0	-1
N.S.	1	1.00	0.87	0.00	1.34	5.21	0.55	0.00	-0.01
time (sec)	N/A	0.072	0.765	0.015	0.487	1.677	13.019	0.000	0.000

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	103	0	114	332	83	0	-1
N.S.	1	1.00	0.84	0.00	0.93	2.72	0.68	0.00	-0.01
time (sec)	N/A	0.049	0.441	0.023	0.498	1.485	6.923	0.000	0.000

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	39	60	51	117	0	70
N.S.	1	1.00	0.67	0.58	0.90	0.76	1.75	0.00	1.04
time (sec)	N/A	0.019	0.270	0.122	0.291	1.292	32.598	0.000	0.649

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	67	62	101	74	0	0	101
N.S.	1	1.00	0.64	0.60	0.97	0.71	0.00	0.00	0.97
time (sec)	N/A	0.031	0.440	0.112	0.278	1.552	0.000	0.000	0.657

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	91	86	138	99	0	0	125
N.S.	1	1.00	0.65	0.61	0.98	0.70	0.00	0.00	0.89
time (sec)	N/A	0.043	0.686	0.121	0.276	2.021	0.000	0.000	0.693

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	112	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	10.108	0.019	0.000	0.000	0.000	0.000	0.000

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	85	0	0	0	94	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.66	0.00	-0.01
time (sec)	N/A	0.049	10.086	0.011	0.000	0.000	49.065	0.000	0.000

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	77	0	0	0	94	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.95	0.00	-0.01
time (sec)	N/A	0.031	10.083	0.019	0.000	0.000	5.894	0.000	0.000

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	77	0	0	0	82	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.80	0.00	-0.01
time (sec)	N/A	0.035	10.038	0.015	0.000	0.000	13.462	0.000	0.000

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	78	0	0	0	85	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.59	0.00	-0.01
time (sec)	N/A	0.050	10.038	0.015	0.000	0.000	91.728	0.000	0.000

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	82	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	10.059	0.016	0.000	0.000	0.000	0.000	0.000

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	129	0	233	0	94	0	-1
N.S.	1	1.00	0.70	0.00	1.27	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.080	0.784	0.014	0.492	0.000	54.568	0.000	0.000

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	108	0	116	0	87	0	-1
N.S.	1	1.00	0.86	0.00	0.93	0.00	0.70	0.00	-0.01
time (sec)	N/A	0.054	0.498	0.017	0.486	0.000	11.008	0.000	0.000

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	40	60	50	119	0	69
N.S.	1	1.00	0.68	0.62	0.92	0.77	1.83	0.00	1.06
time (sec)	N/A	0.020	0.359	0.115	0.269	2.073	33.579	0.000	0.631

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	67	62	102	75	469	0	101
N.S.	1	1.00	0.64	0.60	0.98	0.72	4.51	0.00	0.97
time (sec)	N/A	0.033	0.403	0.114	0.273	0.860	166.519	0.000	0.667

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	91	86	139	99	0	0	125
N.S.	1	1.00	0.65	0.61	0.99	0.70	0.00	0.00	0.89
time (sec)	N/A	0.042	0.784	0.123	0.281	1.774	0.000	0.000	0.688

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	110	0	0	0	94	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.49	0.00	-0.01
time (sec)	N/A	0.093	10.123	0.015	0.000	0.000	173.341	0.000	0.000

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	85	0	0	0	94	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.62	0.00	-0.01
time (sec)	N/A	0.078	10.085	0.011	0.000	0.000	20.419	0.000	0.000

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	79	0	0	0	78	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.67	0.00	-0.01
time (sec)	N/A	0.063	10.054	0.022	0.000	0.000	16.636	0.000	0.000

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	91	0	0	0	97	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.67	0.00	-0.01
time (sec)	N/A	0.078	10.045	0.015	0.000	0.000	59.631	0.000	0.000

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	82	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	10.056	0.014	0.000	0.000	0.000	0.000	0.000

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	150	0	263	959	0	0	-1
N.S.	1	1.00	0.68	0.00	1.19	4.34	0.00	0.00	-0.00
time (sec)	N/A	0.092	1.688	0.014	0.495	1.170	0.000	0.000	0.000

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	124	0	129	428	116	0	-1
N.S.	1	1.00	0.83	0.00	0.87	2.87	0.78	0.00	-0.01
time (sec)	N/A	0.059	1.068	0.020	0.494	1.985	70.318	0.000	0.000

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	54	58	230	0	79
N.S.	1	1.00	0.56	0.49	0.68	0.73	2.91	0.00	1.00
time (sec)	N/A	0.023	0.428	0.103	0.271	1.039	71.808	0.000	0.645

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	67	62	91	86	435	0	115
N.S.	1	1.00	0.64	0.60	0.88	0.83	4.18	0.00	1.11
time (sec)	N/A	0.033	0.418	0.119	0.282	3.097	222.432	0.000	0.706

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	88	86	135	110	0	0	144
N.S.	1	1.00	0.62	0.61	0.96	0.78	0.00	0.00	1.02
time (sec)	N/A	0.046	0.607	0.125	0.312	1.300	0.000	0.000	0.689

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	115	110	172	134	0	0	156
N.S.	1	1.00	0.65	0.62	0.97	0.75	0.00	0.00	0.88
time (sec)	N/A	0.059	1.064	0.138	0.295	1.302	0.000	0.000	0.731

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	140	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.085	10.133	0.018	0.000	0.000	0.000	0.000	0.000

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	116	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	10.115	0.011	0.000	0.000	0.000	0.000	0.000

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	98	0	0	0	94	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.61	0.00	-0.01
time (sec)	N/A	0.050	10.102	0.018	0.000	0.000	169.858	0.000	0.000

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	86	0	0	0	94	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.82	0.00	-0.01
time (sec)	N/A	0.034	10.091	0.020	0.000	0.000	44.567	0.000	0.000

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	85	0	0	0	97	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.68	0.00	-0.01
time (sec)	N/A	0.049	10.044	0.015	0.000	0.000	128.493	0.000	0.000

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	86	0	0	0	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	10.042	0.014	0.000	0.000	0.000	0.000	0.000

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	87	0	0	0	0	0	-1
N.S.	1	1.00	0.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.084	10.041	0.019	0.000	0.000	0.000	0.000	0.000

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.122	0.055	0.000	0.000	0.000	0.000	0.000

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	86	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.082	0.030	0.000	0.000	0.000	0.000	0.000

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	86	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.062	0.026	0.000	0.000	0.000	0.000	0.000

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	-1
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	84	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.076	0.025	0.000	0.000	0.000	0.000	0.000

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	86	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.095	0.027	0.000	0.000	0.000	0.000	0.000

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	195	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.209	0.233	0.027	0.000	0.000	0.000	0.000	0.000

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	118	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.144	0.028	0.000	0.000	0.000	0.000	0.000

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.047	0.024	0.000	0.000	0.000	0.000	0.000

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	95	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.082	0.028	0.000	0.000	0.000	0.000	0.000

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.087	0.026	0.000	0.000	0.000	0.000	0.000

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.115	0.028	0.000	0.000	0.000	0.000	0.000

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.192	0.026	0.000	0.000	0.000	0.000	0.000

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.158	0.027	0.000	0.000	0.000	0.000	0.000

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.110	0.026	0.000	0.000	0.000	0.000	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.115	0.030	0.000	0.000	0.000	0.000	0.000

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.171	0.026	0.000	0.000	0.000	0.000	0.000

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.191	0.025	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [865] had the largest ratio of [30]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	18	0.056
2	A	3	2	1.00	16	0.125
3	A	2	1	1.00	15	0.067
4	A	3	2	1.00	18	0.111
5	A	2	1	1.00	18	0.056
6	A	3	2	1.00	18	0.111
7	A	2	1	1.00	18	0.056
8	A	3	2	1.00	18	0.111
9	A	2	1	1.00	18	0.056
10	A	3	2	1.00	18	0.111
11	A	2	1	1.00	20	0.050
12	A	3	2	1.00	18	0.111
13	A	2	1	1.00	17	0.059
14	A	4	3	1.00	20	0.150
15	A	2	1	1.00	20	0.050
16	A	3	2	1.00	20	0.100
17	A	2	1	1.00	20	0.050
18	A	3	2	1.00	20	0.100
19	A	2	1	1.00	20	0.050
20	A	3	2	1.00	20	0.100
21	A	2	1	1.00	20	0.050
22	A	3	3	1.00	20	0.150
23	A	3	2	1.00	20	0.100
24	A	2	1	1.00	20	0.050
25	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	1	1.00	20	0.050
27	A	3	2	1.00	20	0.100
28	A	2	1	1.00	20	0.050
29	A	3	2	1.00	20	0.100
30	A	2	1	1.00	20	0.050
31	A	3	2	1.00	18	0.111
32	A	2	1	1.00	17	0.059
33	A	4	3	1.00	20	0.150
34	A	2	1	1.00	20	0.050
35	A	3	2	1.00	20	0.100
36	A	2	1	1.00	20	0.050
37	A	3	2	1.00	20	0.100
38	A	2	1	1.00	20	0.050
39	A	3	2	1.00	20	0.100
40	A	2	1	1.00	20	0.050
41	A	3	2	1.00	20	0.100
42	A	2	1	1.00	20	0.050
43	A	3	2	1.00	20	0.100
44	A	2	1	1.00	20	0.050
45	A	4	3	1.00	20	0.150
46	A	2	1	1.00	20	0.050
47	A	3	3	1.00	20	0.150
48	A	2	1	1.00	20	0.050
49	A	4	4	1.00	20	0.200
50	A	2	1	1.00	20	0.050
51	A	3	2	1.00	20	0.100
52	A	2	1	1.00	20	0.050
53	A	3	2	1.00	20	0.100
54	A	2	1	1.00	20	0.050
55	A	3	2	1.00	20	0.100
56	A	4	3	1.00	20	0.150
57	A	3	2	1.00	20	0.100
58	A	4	3	1.00	20	0.150
59	A	3	2	1.00	20	0.100
60	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	18	0.111
62	A	2	2	1.00	17	0.118
63	A	3	2	1.00	20	0.100
64	A	2	2	1.00	20	0.100
65	A	3	2	1.00	20	0.100
66	A	3	3	1.00	20	0.150
67	A	3	2	1.00	20	0.100
68	A	4	3	1.00	20	0.150
69	A	3	2	1.00	20	0.100
70	A	5	3	1.00	20	0.150
71	A	3	2	1.00	20	0.100
72	A	4	3	1.00	20	0.150
73	A	3	2	1.00	20	0.100
74	A	4	3	1.00	20	0.150
75	A	3	2	1.00	20	0.100
76	A	4	3	1.00	20	0.150
77	A	3	2	1.00	20	0.100
78	A	3	3	1.00	20	0.150
79	A	3	2	1.00	18	0.111
80	A	2	2	1.00	17	0.118
81	A	3	2	1.00	20	0.100
82	A	3	3	1.00	20	0.150
83	A	3	2	1.00	20	0.100
84	A	4	3	1.00	20	0.150
85	A	3	2	1.00	20	0.100
86	A	4	3	1.00	20	0.150
87	A	3	2	1.00	20	0.100
88	A	3	2	1.00	20	0.100
89	A	3	2	1.00	20	0.100
90	A	3	2	1.00	20	0.100
91	A	3	2	1.00	20	0.100
92	A	3	2	1.00	20	0.100
93	A	2	2	1.00	18	0.111
94	A	3	2	1.00	20	0.100
95	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	2	1.00	20	0.100
97	A	3	2	1.00	20	0.100
98	A	5	4	1.00	20	0.200
99	A	5	4	1.00	20	0.200
100	A	5	4	1.00	20	0.200
101	A	4	4	1.00	20	0.200
102	A	3	3	1.00	20	0.150
103	A	3	3	1.00	17	0.176
104	A	4	3	1.00	20	0.150
105	A	5	4	1.00	20	0.200
106	A	5	4	1.00	20	0.200
107	A	2	2	1.00	15	0.133
108	A	2	2	1.00	17	0.118
109	A	1	1	1.00	13	0.077
110	A	1	1	1.00	15	0.067
111	A	2	2	1.00	15	0.133
112	A	2	2	1.00	13	0.154
113	A	2	2	1.00	13	0.154
114	A	1	1	1.00	19	0.053
115	A	1	1	1.00	18	0.056
116	A	2	2	1.00	18	0.111
117	A	3	3	1.00	13	0.231
118	A	2	2	1.00	18	0.111
119	A	2	2	1.00	17	0.118
120	A	2	2	1.00	23	0.087
121	A	2	2	1.00	23	0.087
122	A	2	2	1.00	21	0.095
123	A	2	2	1.00	20	0.100
124	A	2	2	1.00	23	0.087
125	A	2	2	1.00	23	0.087
126	A	2	2	1.00	23	0.087
127	A	4	3	1.00	23	0.130
128	A	3	3	1.00	23	0.130
129	A	2	2	1.00	21	0.095
130	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	5	5	1.00	23	0.217
132	A	3	3	1.00	23	0.130
133	A	4	3	1.00	23	0.130
134	A	4	3	1.00	23	0.130
135	A	3	3	1.00	23	0.130
136	A	2	2	1.00	21	0.095
137	A	3	3	1.00	20	0.150
138	A	4	3	1.00	23	0.130
139	A	4	4	1.00	23	0.174
140	A	4	3	1.00	23	0.130
141	A	2	1	1.00	20	0.050
142	A	3	2	1.00	20	0.100
143	A	2	1	1.00	20	0.050
144	A	3	2	1.00	18	0.111
145	A	2	1	1.00	17	0.059
146	A	4	3	1.00	20	0.150
147	A	2	1	1.00	20	0.050
148	A	3	2	1.00	20	0.100
149	A	2	1	1.00	20	0.050
150	A	2	1	1.00	22	0.045
151	A	3	2	1.00	22	0.091
152	A	2	1	1.00	22	0.045
153	A	3	2	1.00	20	0.100
154	A	2	1	1.00	19	0.053
155	A	3	2	1.00	22	0.091
156	A	2	1	1.00	22	0.045
157	A	3	2	1.00	22	0.091
158	A	2	1	1.00	22	0.045
159	A	2	1	1.00	22	0.045
160	A	3	2	1.00	22	0.091
161	A	2	1	1.00	22	0.045
162	A	3	2	1.00	20	0.100
163	A	2	1	1.00	19	0.053
164	A	3	2	1.00	22	0.091
165	A	2	1	1.00	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	2	1.00	22	0.091
167	A	2	1	1.00	22	0.045
168	A	3	2	1.00	22	0.091
169	A	3	2	1.00	22	0.091
170	A	3	2	1.00	22	0.091
171	A	3	2	1.00	20	0.100
172	A	3	2	1.00	19	0.105
173	A	3	2	1.00	22	0.091
174	A	3	2	1.00	22	0.091
175	A	3	2	1.00	22	0.091
176	A	3	2	1.00	22	0.091
177	A	3	2	1.00	22	0.091
178	A	3	2	1.00	22	0.091
179	A	3	2	1.00	22	0.091
180	A	5	4	1.00	22	0.182
181	A	3	2	1.00	22	0.091
182	A	4	4	1.00	22	0.182
183	A	3	2	1.00	20	0.100
184	A	4	3	1.00	19	0.158
185	A	3	2	1.00	22	0.091
186	A	3	3	0.98	22	0.136
187	A	3	2	1.00	22	0.091
188	A	4	4	1.00	22	0.182
189	A	5	4	1.00	22	0.182
190	A	3	2	1.00	22	0.091
191	A	4	4	1.00	22	0.182
192	A	3	2	1.00	20	0.100
193	A	3	3	1.00	19	0.158
194	A	3	2	1.00	22	0.091
195	A	4	4	0.98	22	0.182
196	A	3	2	1.00	22	0.091
197	A	5	4	1.00	22	0.182
198	A	3	2	1.00	20	0.100
199	A	4	3	1.00	20	0.150
200	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	3	3	1.00	20	0.150
202	A	3	2	1.00	18	0.111
203	A	2	2	1.00	17	0.118
204	A	3	2	1.00	20	0.100
205	A	2	2	1.00	20	0.100
206	A	3	2	1.00	20	0.100
207	A	3	3	1.00	20	0.150
208	A	3	2	1.00	22	0.091
209	A	3	2	1.00	22	0.091
210	A	3	2	1.00	22	0.091
211	A	3	2	1.00	22	0.091
212	A	3	2	1.00	20	0.100
213	A	3	2	1.00	19	0.105
214	A	3	2	1.00	22	0.091
215	A	3	2	1.00	22	0.091
216	A	3	2	1.00	22	0.091
217	A	3	2	1.00	22	0.091
218	A	3	2	1.00	22	0.091
219	A	3	2	1.00	22	0.091
220	A	3	2	1.00	22	0.091
221	A	3	2	1.00	22	0.091
222	A	3	2	1.00	20	0.100
223	A	3	2	1.00	19	0.105
224	A	3	2	1.00	22	0.091
225	A	3	2	1.00	22	0.091
226	A	3	2	1.00	22	0.091
227	A	3	2	1.00	22	0.091
228	A	3	2	1.00	22	0.091
229	A	4	3	1.00	22	0.136
230	A	3	2	1.00	22	0.091
231	A	3	2	1.00	22	0.091
232	A	4	3	1.00	20	0.150
233	A	3	2	1.00	19	0.105
234	A	3	2	1.00	22	0.091
235	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	2	1.00	22	0.091
237	A	5	4	1.00	22	0.182
238	A	3	2	1.00	22	0.091
239	A	6	4	1.00	22	0.182
240	A	3	2	1.00	22	0.091
241	A	3	2	1.00	22	0.091
242	A	4	3	1.00	22	0.136
243	A	3	2	1.00	22	0.091
244	A	4	3	1.00	22	0.136
245	A	3	2	1.00	20	0.100
246	A	4	3	1.00	19	0.158
247	A	3	2	1.00	22	0.091
248	A	5	4	1.00	22	0.182
249	A	3	2	1.00	22	0.091
250	A	6	4	1.00	22	0.182
251	A	3	2	1.00	22	0.091
252	A	5	4	1.00	22	0.182
253	A	3	2	1.00	22	0.091
254	A	5	4	1.00	22	0.182
255	A	3	2	1.00	20	0.100
256	A	5	4	1.00	19	0.210
257	A	3	2	1.00	22	0.091
258	A	6	5	1.00	22	0.227
259	A	3	2	1.00	22	0.091
260	A	7	5	1.00	22	0.227
261	A	4	3	1.00	16	0.188
262	A	4	3	1.00	20	0.150
263	A	3	2	1.00	20	0.100
264	A	3	3	1.00	20	0.150
265	A	3	2	1.00	18	0.111
266	A	2	2	1.00	17	0.118
267	A	3	2	1.00	20	0.100
268	A	3	3	1.00	20	0.150
269	A	3	2	1.00	20	0.100
270	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	5	4	1.00	22	0.182
272	A	3	2	1.00	22	0.091
273	A	4	4	1.00	22	0.182
274	A	3	2	1.00	20	0.100
275	A	4	3	1.00	19	0.158
276	A	3	2	1.00	22	0.091
277	A	3	3	0.97	22	0.136
278	A	3	2	1.00	22	0.091
279	A	4	4	0.98	22	0.182
280	A	4	3	1.00	22	0.136
281	A	3	2	1.00	22	0.091
282	A	5	4	1.00	22	0.182
283	A	3	2	1.00	20	0.100
284	A	4	3	1.00	19	0.158
285	A	3	2	1.00	22	0.091
286	A	4	3	1.00	22	0.136
287	A	3	2	1.00	22	0.091
288	A	4	3	1.00	22	0.136
289	A	4	3	1.00	22	0.136
290	A	3	2	1.00	22	0.091
291	A	4	3	1.00	22	0.136
292	A	3	2	1.00	20	0.100
293	A	4	3	1.00	19	0.158
294	A	3	2	1.00	22	0.091
295	A	5	4	1.00	22	0.182
296	A	3	2	1.00	22	0.091
297	A	6	4	1.00	22	0.182
298	A	3	2	1.00	22	0.091
299	A	7	4	1.00	22	0.182
300	A	3	2	1.00	22	0.091
301	A	5	4	1.00	22	0.182
302	A	3	2	1.00	22	0.091
303	A	5	4	1.00	22	0.182
304	A	3	2	1.00	20	0.100
305	A	5	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	3	2	1.00	22	0.091
307	A	6	5	1.00	22	0.227
308	A	3	2	1.00	22	0.091
309	A	7	5	1.00	22	0.227
310	A	6	4	1.00	22	0.182
311	A	3	2	1.00	22	0.091
312	A	6	4	1.00	22	0.182
313	A	3	2	1.00	20	0.100
314	A	6	4	1.00	19	0.210
315	A	3	2	1.00	22	0.091
316	A	7	5	1.00	22	0.227
317	A	3	2	1.00	22	0.091
318	A	8	5	1.00	22	0.227
319	A	2	1	1.00	20	0.050
320	A	2	1	1.00	20	0.050
321	A	2	1	1.00	18	0.056
322	A	2	2	1.00	20	0.100
323	A	2	2	1.00	20	0.100
324	A	2	2	1.00	20	0.100
325	A	2	1	1.00	22	0.045
326	A	2	1	1.00	22	0.045
327	A	2	1	1.00	20	0.050
328	A	3	2	1.00	22	0.091
329	A	3	3	1.00	22	0.136
330	A	3	3	0.97	22	0.136
331	A	3	2	1.00	22	0.091
332	A	3	2	1.00	22	0.091
333	A	2	2	1.00	20	0.100
334	A	3	2	1.00	22	0.091
335	A	5	3	1.00	22	0.136
336	A	6	4	1.00	22	0.182
337	A	4	3	1.00	22	0.136
338	A	3	3	1.00	22	0.136
339	A	2	2	1.00	20	0.100
340	A	5	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	6	4	1.00	22	0.182
342	A	7	4	1.00	22	0.182
343	A	2	1	1.00	20	0.050
344	A	2	1	1.00	20	0.050
345	A	2	1	1.00	20	0.050
346	A	2	1	1.00	20	0.050
347	A	2	1	1.00	20	0.050
348	A	2	1	1.00	20	0.050
349	A	2	1	1.00	20	0.050
350	A	2	1	1.00	20	0.050
351	A	2	1	1.00	22	0.045
352	A	2	1	1.00	22	0.045
353	A	2	1	1.00	22	0.045
354	A	2	1	1.00	22	0.045
355	A	2	1	1.00	22	0.045
356	A	2	1	1.00	22	0.045
357	A	2	1	1.00	22	0.045
358	A	2	1	1.00	22	0.045
359	A	2	1	1.00	22	0.045
360	A	2	1	1.00	22	0.045
361	A	2	1	1.00	22	0.045
362	A	2	1	1.00	22	0.045
363	A	2	1	1.00	22	0.045
364	A	2	1	1.00	22	0.045
365	A	2	1	1.00	22	0.045
366	A	2	1	1.00	22	0.045
367	A	13	9	1.00	22	0.409
368	A	12	9	1.00	22	0.409
369	A	12	9	1.00	22	0.409
370	A	11	8	1.00	22	0.364
371	A	11	8	1.00	22	0.364
372	A	11	8	1.00	22	0.364
373	A	11	8	1.00	22	0.364
374	A	12	9	1.00	22	0.409
375	A	13	9	1.00	22	0.409

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	12	9	1.00	22	0.409
377	A	12	9	1.00	22	0.409
378	A	11	8	1.00	22	0.364
379	A	11	8	1.00	22	0.364
380	A	12	9	1.00	22	0.409
381	A	12	9	1.00	22	0.409
382	A	13	9	1.00	22	0.409
383	A	13	10	1.00	22	0.454
384	A	12	9	1.00	22	0.409
385	A	12	9	1.00	22	0.409
386	A	12	9	1.00	22	0.409
387	A	12	9	1.00	22	0.409
388	A	13	10	1.00	22	0.454
389	A	13	10	1.00	22	0.454
390	A	14	10	1.00	22	0.454
391	A	2	1	1.00	22	0.045
392	A	2	1	1.00	22	0.045
393	A	2	1	1.00	22	0.045
394	A	2	1	1.00	22	0.045
395	A	2	1	1.00	22	0.045
396	A	2	1	1.00	22	0.045
397	A	2	1	1.00	22	0.045
398	A	2	1	1.00	22	0.045
399	A	2	1	1.00	24	0.042
400	A	2	1	1.00	24	0.042
401	A	2	1	1.00	24	0.042
402	A	2	1	1.00	24	0.042
403	A	2	1	1.00	24	0.042
404	A	2	1	1.00	24	0.042
405	A	2	1	1.00	24	0.042
406	A	2	1	1.00	24	0.042
407	A	2	1	1.00	24	0.042
408	A	2	1	1.00	24	0.042
409	A	2	1	1.00	24	0.042
410	A	2	1	1.00	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	2	1	1.00	24	0.042
412	A	2	1	1.00	24	0.042
413	A	2	1	1.00	24	0.042
414	A	2	1	1.00	24	0.042
415	A	14	9	1.00	24	0.375
416	A	13	9	1.00	24	0.375
417	A	13	9	1.00	24	0.375
418	A	12	8	1.00	24	0.333
419	A	12	8	1.00	24	0.333
420	A	12	9	1.00	24	0.375
421	A	12	9	1.00	24	0.375
422	A	12	9	1.00	24	0.375
423	A	12	9	1.00	24	0.375
424	A	13	10	1.00	24	0.417
425	A	14	10	1.00	24	0.417
426	A	13	10	1.00	24	0.417
427	A	13	10	1.00	24	0.417
428	A	12	9	1.00	24	0.375
429	A	12	9	1.00	24	0.375
430	A	12	9	0.99	24	0.375
431	A	12	9	0.99	24	0.375
432	A	13	10	0.99	24	0.417
433	A	14	10	1.00	24	0.417
434	A	13	10	1.00	24	0.417
435	A	13	10	1.00	24	0.417
436	A	12	9	1.00	24	0.375
437	A	12	9	1.00	24	0.375
438	A	13	10	0.99	24	0.417
439	A	13	10	0.99	24	0.417
440	A	14	11	0.99	24	0.458
441	A	13	9	1.00	24	0.375
442	A	13	9	1.00	24	0.375
443	A	12	8	1.00	24	0.333
444	A	12	8	1.00	24	0.333
445	A	12	8	1.00	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
446	A	12	8	1.00	24	0.333
447	A	12	8	1.00	24	0.333
448	A	12	8	1.00	24	0.333
449	A	12	8	1.00	24	0.333
450	A	12	8	1.00	24	0.333
451	A	12	8	1.00	24	0.333
452	A	13	9	1.00	24	0.375
453	A	13	9	1.00	24	0.375
454	A	14	10	1.00	24	0.417
455	A	13	9	1.00	24	0.375
456	A	13	9	1.00	24	0.375
457	A	13	9	1.00	24	0.375
458	A	13	9	1.00	24	0.375
459	A	13	9	1.00	24	0.375
460	A	13	9	1.00	24	0.375
461	A	22	9	1.00	24	0.375
462	A	21	9	1.00	24	0.375
463	A	20	8	1.00	24	0.333
464	A	20	8	1.00	24	0.333
465	A	20	8	1.00	24	0.333
466	A	20	8	1.00	24	0.333
467	A	22	9	1.00	24	0.375
468	A	21	9	1.00	24	0.375
469	A	23	10	1.00	24	0.417
470	A	22	10	1.00	24	0.417
471	A	22	9	1.00	24	0.375
472	A	21	9	1.00	24	0.375
473	A	22	9	1.00	24	0.375
474	A	21	9	1.00	24	0.375
475	A	22	9	1.00	24	0.375
476	A	21	9	1.00	24	0.375
477	A	23	10	1.00	24	0.417
478	A	22	10	1.00	24	0.417
479	A	24	10	1.00	24	0.417
480	A	22	10	1.00	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	23	10	1.00	24	0.417
482	A	22	10	1.00	24	0.417
483	A	23	10	1.00	24	0.417
484	A	22	10	1.00	24	0.417
485	A	24	11	1.00	24	0.458
486	A	23	11	1.00	24	0.458
487	A	25	11	1.00	24	0.458
488	A	22	10	1.00	24	0.417
489	A	23	10	1.00	24	0.417
490	A	22	10	1.00	24	0.417
491	A	23	10	1.00	24	0.417
492	A	22	10	1.00	24	0.417
493	A	24	11	1.00	24	0.458
494	A	23	11	1.00	24	0.458
495	A	25	11	1.00	24	0.458
496	A	23	10	1.00	24	0.417
497	A	24	10	1.00	24	0.417
498	A	23	10	1.00	24	0.417
499	A	24	10	1.00	24	0.417
500	A	23	10	1.00	24	0.417
501	A	25	11	1.00	24	0.458
502	A	24	11	1.00	24	0.458
503	A	26	11	1.00	24	0.458
504	A	3	2	1.00	22	0.091
505	A	6	5	1.00	22	0.227
506	A	3	2	1.00	22	0.091
507	A	5	5	1.00	22	0.227
508	A	3	2	1.00	20	0.100
509	A	4	4	1.00	19	0.210
510	A	5	5	1.00	22	0.227
511	A	4	4	1.00	22	0.182
512	A	5	5	1.00	22	0.227
513	A	4	4	1.00	22	0.182
514	A	5	5	1.00	22	0.227
515	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	6	6	1.00	22	0.273
517	A	3	3	1.00	22	0.136
518	A	7	6	1.00	22	0.273
519	A	4	3	1.00	22	0.136
520	A	8	6	1.00	22	0.273
521	A	3	2	1.00	22	0.091
522	A	7	5	1.00	22	0.227
523	A	3	2	1.00	22	0.091
524	A	6	5	1.00	22	0.227
525	A	3	2	1.00	20	0.100
526	A	5	4	1.00	19	0.210
527	A	6	5	1.00	22	0.227
528	A	5	4	1.00	22	0.182
529	A	6	5	1.00	22	0.227
530	A	5	5	1.00	22	0.227
531	A	6	6	1.00	22	0.273
532	A	5	4	1.00	22	0.182
533	A	6	5	1.00	22	0.227
534	A	2	2	1.00	22	0.091
535	A	7	6	1.00	22	0.273
536	A	3	3	1.00	22	0.136
537	A	8	6	1.00	22	0.273
538	A	3	2	1.00	22	0.091
539	A	8	5	1.00	22	0.227
540	A	3	2	1.00	22	0.091
541	A	7	5	1.00	22	0.227
542	A	3	2	1.00	20	0.100
543	A	6	4	1.00	19	0.210
544	A	7	5	1.00	22	0.227
545	A	6	4	1.00	22	0.182
546	A	7	5	1.00	22	0.227
547	A	6	5	1.00	22	0.227
548	A	7	6	1.00	22	0.273
549	A	6	5	1.00	22	0.227
550	A	7	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	6	4	1.00	22	0.182
552	A	7	5	1.00	22	0.227
553	A	2	2	1.00	22	0.091
554	A	8	6	1.00	22	0.273
555	A	3	2	1.00	22	0.091
556	A	5	4	1.00	22	0.182
557	A	3	2	1.00	22	0.091
558	A	4	4	1.00	22	0.182
559	A	3	2	1.00	20	0.100
560	A	3	3	1.00	19	0.158
561	A	4	4	1.00	22	0.182
562	A	3	3	1.00	22	0.136
563	A	4	4	1.00	22	0.182
564	A	2	2	1.00	22	0.091
565	A	5	5	1.00	22	0.227
566	A	3	3	1.00	22	0.136
567	A	6	5	1.00	22	0.227
568	A	4	3	1.00	22	0.136
569	A	6	5	1.00	22	0.227
570	A	3	2	1.00	22	0.091
571	A	5	5	1.00	22	0.227
572	A	3	2	1.00	22	0.091
573	A	4	4	1.00	22	0.182
574	A	3	2	1.00	20	0.100
575	A	3	3	1.00	19	0.158
576	A	4	4	1.00	22	0.182
577	A	2	2	1.00	22	0.091
578	A	5	5	1.00	22	0.227
579	A	3	3	1.00	22	0.136
580	A	6	6	1.00	22	0.273
581	A	4	3	1.00	22	0.136
582	A	7	6	1.00	22	0.273
583	A	5	3	1.00	22	0.136
584	A	3	2	1.00	22	0.091
585	A	6	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	3	2	1.00	22	0.091
587	A	5	5	1.00	22	0.227
588	A	3	2	1.00	22	0.091
589	A	4	4	1.00	22	0.182
590	A	3	2	1.00	20	0.100
591	A	2	2	1.00	19	0.105
592	A	5	5	1.00	22	0.227
593	A	3	3	1.00	22	0.136
594	A	6	5	1.00	22	0.227
595	A	4	4	1.00	22	0.182
596	A	7	6	1.00	22	0.273
597	A	5	4	1.00	22	0.182
598	A	3	2	1.00	24	0.083
599	A	3	2	1.00	24	0.083
600	A	3	2	1.00	22	0.091
601	A	6	5	1.00	24	0.208
602	A	6	6	1.00	24	0.250
603	A	6	6	0.98	24	0.250
604	A	6	6	1.00	24	0.250
605	A	6	6	0.98	24	0.250
606	A	5	5	1.00	21	0.238
607	A	5	5	0.98	24	0.208
608	A	5	5	1.00	24	0.208
609	A	5	5	1.00	24	0.208
610	A	3	3	1.01	24	0.125
611	A	4	4	1.01	24	0.167
612	A	5	4	1.01	24	0.167
613	A	8	6	0.99	24	0.250
614	A	3	2	1.00	24	0.083
615	A	7	6	0.99	24	0.250
616	A	3	2	1.00	22	0.091
617	A	6	5	1.00	21	0.238
618	A	7	5	1.00	24	0.208
619	A	6	5	0.98	24	0.208
620	A	7	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	6	5	0.98	24	0.208
622	A	7	6	0.98	24	0.250
623	A	6	6	1.00	24	0.250
624	A	7	7	0.98	24	0.292
625	A	3	2	1.00	24	0.083
626	A	8	6	0.99	24	0.250
627	A	3	2	1.00	22	0.091
628	A	7	5	1.00	21	0.238
629	A	8	5	1.00	24	0.208
630	A	7	5	0.99	24	0.208
631	A	8	6	1.00	24	0.250
632	A	7	5	0.98	24	0.208
633	A	8	6	0.99	24	0.250
634	A	7	6	0.99	24	0.250
635	A	8	7	0.99	24	0.292
636	A	6	5	1.00	24	0.208
637	A	3	2	1.00	24	0.083
638	A	5	5	1.00	24	0.208
639	A	3	2	1.00	22	0.091
640	A	4	4	1.00	21	0.190
641	A	5	4	1.00	24	0.167
642	A	4	4	1.00	24	0.167
643	A	5	5	1.00	24	0.208
644	A	4	4	1.00	24	0.167
645	A	5	5	1.00	24	0.208
646	A	3	3	1.01	24	0.125
647	A	6	6	1.00	24	0.250
648	A	6	5	1.00	24	0.208
649	A	3	2	1.00	24	0.083
650	A	5	5	1.00	24	0.208
651	A	3	2	1.00	22	0.091
652	A	4	4	1.00	21	0.190
653	A	5	4	1.00	24	0.167
654	A	4	4	0.96	24	0.167
655	A	5	5	1.03	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	3	3	1.01	24	0.125
657	A	6	6	1.00	24	0.250
658	A	4	4	1.00	24	0.167
659	A	7	7	1.01	24	0.292
660	A	6	6	1.00	24	0.250
661	A	3	2	1.00	24	0.083
662	A	5	5	1.00	24	0.208
663	A	3	2	1.00	22	0.091
664	A	4	4	1.00	21	0.190
665	A	5	4	1.00	24	0.167
666	A	3	3	1.00	24	0.125
667	A	6	6	1.02	24	0.250
668	A	4	4	0.99	24	0.167
669	A	7	6	1.00	24	0.250
670	A	5	5	1.00	24	0.208
671	A	4	3	1.00	22	0.136
672	A	3	3	1.00	22	0.136
673	A	2	2	1.00	20	0.100
674	A	3	3	1.00	22	0.136
675	A	4	3	1.00	22	0.136
676	A	7	7	1.00	24	0.292
677	A	5	5	1.00	24	0.208
678	A	6	6	1.00	24	0.250
679	A	4	4	1.00	22	0.182
680	A	5	5	1.00	21	0.238
681	A	6	4	1.00	24	0.167
682	A	4	4	1.00	24	0.167
683	A	7	5	1.00	24	0.208
684	A	5	5	1.00	24	0.208
685	A	8	7	1.00	24	0.292
686	A	6	5	1.00	24	0.208
687	A	7	7	1.00	24	0.292
688	A	5	4	1.00	22	0.182
689	A	6	6	1.00	21	0.286
690	A	7	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	6	6	1.00	24	0.250
692	A	7	5	1.00	24	0.208
693	A	5	5	1.00	24	0.208
694	A	9	8	1.00	24	0.333
695	A	7	5	1.00	24	0.208
696	A	8	8	1.00	24	0.333
697	A	6	4	1.00	22	0.182
698	A	7	7	1.00	21	0.333
699	A	8	6	1.00	24	0.250
700	A	7	7	1.00	24	0.292
701	A	8	6	1.00	24	0.250
702	A	7	7	1.00	24	0.292
703	A	5	4	1.00	24	0.167
704	A	4	4	1.00	24	0.167
705	A	3	3	1.00	22	0.136
706	A	6	4	1.00	24	0.167
707	A	7	5	1.00	24	0.208
708	A	6	6	1.00	24	0.250
709	A	5	5	1.00	24	0.208
710	A	2	2	1.00	21	0.095
711	A	4	4	1.00	24	0.167
712	A	5	5	1.00	24	0.208
713	A	6	6	1.00	24	0.250
714	A	4	4	1.00	24	0.167
715	A	4	4	1.00	24	0.167
716	A	4	4	1.00	22	0.182
717	A	3	3	1.00	21	0.143
718	A	7	5	1.00	24	0.208
719	A	5	5	1.00	24	0.208
720	A	8	6	1.00	24	0.250
721	A	6	5	1.00	24	0.208
722	A	5	5	1.00	24	0.208
723	A	5	5	1.00	24	0.208
724	A	5	5	1.00	24	0.208
725	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	5	5	1.00	21	0.238
727	A	8	6	1.00	24	0.250
728	A	6	6	1.00	24	0.250
729	A	9	6	1.00	24	0.250
730	A	7	6	1.00	24	0.250
731	A	7	7	1.00	24	0.292
732	A	5	5	1.00	24	0.208
733	A	6	6	1.00	24	0.250
734	A	4	4	1.00	22	0.182
735	A	3	3	1.00	21	0.143
736	A	7	5	1.00	24	0.208
737	A	5	5	1.00	24	0.208
738	A	8	6	1.00	24	0.250
739	A	6	5	1.00	24	0.208
740	A	8	8	1.00	24	0.333
741	A	6	5	1.00	24	0.208
742	A	7	7	1.00	24	0.292
743	A	5	5	1.00	22	0.227
744	A	6	6	1.00	21	0.286
745	A	7	5	1.00	24	0.208
746	A	5	5	1.00	24	0.208
747	A	8	6	1.00	24	0.250
748	A	6	5	1.00	24	0.208
749	A	9	8	1.00	24	0.333
750	A	7	5	1.00	24	0.208
751	A	8	7	1.00	24	0.292
752	A	6	5	1.00	22	0.227
753	A	7	7	1.00	21	0.333
754	A	8	6	1.00	24	0.250
755	A	7	7	1.00	24	0.292
756	A	8	6	1.00	24	0.250
757	A	6	6	1.00	24	0.250
758	A	6	6	1.00	24	0.250
759	A	4	4	1.00	24	0.167
760	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	4	4	1.00	22	0.182
762	A	3	3	1.00	21	0.143
763	A	7	5	1.00	24	0.208
764	A	5	5	1.00	24	0.208
765	A	8	6	1.00	24	0.250
766	A	6	5	1.00	24	0.208
767	A	5	5	1.00	24	0.208
768	A	5	5	1.00	24	0.208
769	A	5	5	1.00	24	0.208
770	A	5	5	1.00	22	0.227
771	A	5	5	1.00	21	0.238
772	A	8	6	1.00	24	0.250
773	A	6	6	1.00	24	0.250
774	A	9	7	1.00	24	0.292
775	A	7	6	1.00	24	0.250
776	A	6	5	1.00	24	0.208
777	A	6	5	1.00	24	0.208
778	A	6	5	1.00	24	0.208
779	A	6	5	1.00	22	0.227
780	A	6	5	1.00	21	0.238
781	A	9	6	1.00	24	0.250
782	A	7	6	1.00	24	0.250
783	A	10	7	1.00	24	0.292
784	A	8	6	1.00	24	0.250
785	A	5	5	1.00	26	0.192
786	A	6	6	1.00	26	0.231
787	A	4	4	1.00	26	0.154
788	A	6	6	1.00	26	0.231
789	A	4	4	1.00	26	0.154
790	A	6	6	1.00	26	0.231
791	A	4	4	1.00	24	0.167
792	A	7	7	1.00	24	0.292
793	A	5	5	1.00	24	0.208
794	A	6	5	1.00	26	0.192
795	A	7	6	1.00	26	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
796	A	5	4	1.00	26	0.154
797	A	7	6	1.00	26	0.231
798	A	5	4	1.00	26	0.154
799	A	7	7	1.00	26	0.269
800	A	6	6	1.00	26	0.231
801	A	4	4	1.00	26	0.154
802	A	5	5	1.00	26	0.192
803	A	3	3	1.00	26	0.115
804	A	5	5	1.00	26	0.192
805	A	3	3	1.00	26	0.115
806	A	6	6	1.00	26	0.231
807	A	5	5	1.00	26	0.192
808	A	6	6	1.00	26	0.231
809	A	4	4	1.00	26	0.154
810	A	5	5	1.00	26	0.192
811	A	3	3	1.00	26	0.115
812	A	6	6	1.00	26	0.231
813	A	4	4	1.00	26	0.154
814	A	7	7	1.00	26	0.269
815	A	5	4	1.00	26	0.154
816	A	6	6	1.00	26	0.231
817	A	4	4	1.00	26	0.154
818	A	6	6	1.00	26	0.231
819	A	4	4	1.00	26	0.154
820	A	7	6	1.00	26	0.231
821	A	5	4	1.00	26	0.154
822	A	6	6	1.00	28	0.214
823	A	7	7	1.00	28	0.250
824	A	5	5	1.00	28	0.179
825	A	7	7	1.00	28	0.250
826	A	5	5	1.00	28	0.179
827	A	7	7	1.00	28	0.250
828	A	5	5	0.99	26	0.192
829	A	7	7	0.99	26	0.269
830	A	5	5	0.98	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	8	8	0.99	26	0.308
832	A	9	8	1.00	28	0.286
833	A	7	6	1.00	28	0.214
834	A	8	7	1.00	28	0.250
835	A	6	5	1.00	28	0.179
836	A	8	7	1.00	28	0.250
837	A	6	5	1.00	28	0.179
838	A	8	7	1.00	28	0.250
839	A	7	7	1.00	28	0.250
840	A	5	5	1.00	28	0.179
841	A	6	6	1.00	28	0.214
842	A	4	4	1.00	28	0.143
843	A	6	6	1.00	28	0.214
844	A	4	4	1.00	28	0.143
845	A	6	6	1.00	28	0.214
846	A	4	4	1.00	28	0.143
847	A	7	7	1.00	28	0.250
848	A	5	5	1.00	28	0.179
849	A	6	5	1.00	28	0.179
850	A	7	7	1.00	28	0.250
851	A	5	5	1.00	28	0.179
852	A	6	6	1.00	28	0.214
853	A	4	4	1.00	28	0.143
854	A	6	6	1.00	28	0.214
855	A	4	4	1.00	28	0.143
856	A	7	7	1.00	28	0.250
857	A	6	6	1.00	28	0.214
858	A	7	7	1.00	28	0.250
859	A	5	5	1.00	28	0.179
860	A	6	6	1.00	28	0.214
861	A	4	4	1.00	28	0.143
862	A	7	7	1.00	28	0.250
863	A	5	5	1.00	28	0.179
864	A	8	7	1.00	28	0.250
865	A	11	9	1.00	30	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	15	12	1.00	30	0.400
867	A	10	8	1.00	30	0.267
868	A	13	11	1.00	30	0.367
869	A	9	7	1.00	30	0.233
870	A	15	12	1.00	30	0.400
871	A	10	8	1.00	30	0.267
872	A	16	13	1.00	30	0.433
873	A	16	13	1.00	30	0.433
874	A	11	9	1.00	30	0.300
875	A	15	12	1.00	30	0.400
876	A	10	8	1.00	30	0.267
877	A	15	12	1.00	30	0.400
878	A	10	8	1.00	30	0.267
879	A	16	13	1.00	30	0.433
880	A	10	8	1.00	30	0.267
881	A	13	11	1.00	30	0.367
882	A	9	7	1.00	30	0.233
883	A	6	4	1.00	30	0.133
884	A	6	4	1.00	30	0.133
885	A	15	12	1.00	30	0.400
886	A	10	8	1.00	30	0.267
887	A	16	13	1.00	30	0.433
888	A	15	12	1.00	30	0.400
889	A	10	8	1.00	30	0.267
890	A	15	12	1.00	30	0.400
891	A	10	8	1.00	30	0.267
892	A	15	12	1.00	30	0.400
893	A	10	8	1.00	30	0.267
894	A	16	13	1.00	30	0.433
895	A	11	9	1.00	30	0.300
896	A	11	9	1.00	30	0.300
897	A	15	12	1.00	30	0.400
898	A	10	8	1.00	30	0.267
899	A	15	12	1.00	30	0.400
900	A	10	8	1.00	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	16	13	1.00	30	0.433
902	A	11	9	1.00	30	0.300
903	A	12	10	1.00	30	0.333
904	A	16	13	1.00	30	0.433
905	A	11	9	1.00	30	0.300
906	A	15	12	1.00	30	0.400
907	A	10	8	1.00	30	0.267
908	A	16	13	1.00	30	0.433
909	A	11	9	1.00	30	0.300
910	A	15	12	1.00	30	0.400
911	A	10	8	1.00	30	0.267
912	A	15	12	1.00	30	0.400
913	A	10	8	1.00	30	0.267
914	A	15	12	1.00	30	0.400
915	A	10	8	1.00	30	0.267
916	A	16	13	1.00	30	0.433
917	A	11	9	1.00	30	0.300
918	A	16	13	1.00	30	0.433
919	A	11	9	1.00	30	0.300
920	A	16	13	1.00	30	0.433
921	A	11	9	1.00	30	0.300
922	A	16	13	1.00	30	0.433
923	A	11	9	1.00	30	0.300
924	A	17	14	1.00	30	0.467
925	A	12	10	1.00	30	0.333
926	A	17	13	1.00	30	0.433
927	A	12	9	1.00	30	0.300
928	A	17	13	1.00	30	0.433
929	A	12	9	1.00	30	0.300
930	A	17	13	1.00	30	0.433
931	A	12	9	1.00	30	0.300
932	A	18	14	1.00	30	0.467
933	A	13	10	1.00	30	0.333
934	A	7	7	1.00	26	0.269
935	A	6	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	5	5	1.00	24	0.208
937	A	8	8	1.00	26	0.308
938	A	4	4	1.00	26	0.154
939	A	5	5	1.00	26	0.192
940	A	6	6	1.00	26	0.231
941	A	5	5	1.00	26	0.192
942	A	6	6	1.00	26	0.231
943	A	6	6	1.00	26	0.231
944	A	8	7	1.00	26	0.269
945	A	7	6	1.00	26	0.231
946	A	6	5	1.00	24	0.208
947	A	8	8	1.00	26	0.308
948	A	8	8	1.00	26	0.308
949	A	5	4	1.00	26	0.154
950	A	7	6	1.00	26	0.231
951	A	6	6	1.00	26	0.231
952	A	5	5	1.00	26	0.192
953	A	6	6	1.00	26	0.231
954	A	9	7	1.00	26	0.269
955	A	8	6	1.00	26	0.231
956	A	7	5	1.00	24	0.208
957	A	9	9	1.00	26	0.346
958	A	9	9	1.00	26	0.346
959	A	9	9	1.00	26	0.346
960	A	8	7	1.00	26	0.269
961	A	7	7	1.00	26	0.269
962	A	6	6	1.00	26	0.231
963	A	6	6	1.00	26	0.231
964	A	5	5	1.00	26	0.192
965	A	6	6	1.00	26	0.231
966	A	4	4	1.00	26	0.154
967	A	5	5	1.00	24	0.208
968	A	5	5	1.00	26	0.192
969	A	6	6	1.00	26	0.231
970	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	4	4	1.00	24	0.167
972	A	3	3	1.00	26	0.115
973	A	4	4	1.00	26	0.154
974	A	6	6	1.00	26	0.231
975	A	6	6	1.00	26	0.231
976	A	5	5	1.00	26	0.192
977	A	2	2	1.00	26	0.077
978	A	4	4	1.00	26	0.154
979	A	6	6	1.00	26	0.231
980	A	6	6	1.00	26	0.231
981	A	5	5	1.00	26	0.192
982	A	2	2	1.00	24	0.083
983	A	6	6	1.00	26	0.231
984	A	3	3	1.00	26	0.115
985	A	3	3	1.00	24	0.125
986	A	4	4	1.00	26	0.154
987	A	4	4	1.00	26	0.154
988	A	4	3	1.00	24	0.125
989	A	5	5	1.00	26	0.192
990	A	4	4	1.00	25	0.160
991	A	4	4	1.00	26	0.154
992	A	2	2	1.00	26	0.077
993	A	5	5	1.00	26	0.192
994	A	2	2	1.00	24	0.083
995	A	3	3	1.00	26	0.115
996	A	3	3	1.00	26	0.115
997	A	3	3	1.00	26	0.115
998	A	3	3	1.00	26	0.115
999	A	3	3	1.00	26	0.115
1000	A	3	3	1.00	26	0.115
1001	A	3	3	1.00	24	0.125
1002	A	3	3	1.00	24	0.125
1003	A	3	3	1.00	26	0.115
1004	A	2	2	1.00	24	0.083
1005	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1006	A	2	2	1.00	26	0.077
1007	A	3	3	1.00	26	0.115
1008	A	7	6	1.00	22	0.273
1009	A	6	6	1.00	22	0.273
1010	A	5	5	1.00	20	0.250
1011	A	10	7	1.00	22	0.318
1012	A	7	7	1.00	22	0.318
1013	A	12	9	1.00	22	0.409
1014	A	7	7	1.00	22	0.318
1015	A	6	6	1.00	22	0.273
1016	A	1	1	1.00	19	0.053
1017	A	7	7	1.00	22	0.318
1018	A	8	8	1.00	22	0.364
1019	A	7	7	1.00	22	0.318
1020	A	7	7	1.00	22	0.318
1021	A	6	6	1.00	22	0.273
1022	A	6	6	1.00	20	0.300
1023	A	11	8	1.00	22	0.364
1024	A	12	9	1.00	22	0.409
1025	A	13	9	1.00	22	0.409
1026	A	7	7	1.00	22	0.318
1027	A	7	7	1.00	22	0.318
1028	A	7	7	1.00	19	0.368
1029	A	8	8	1.00	22	0.364
1030	A	9	8	1.00	22	0.364
1031	A	10	5	1.00	24	0.208
1032	A	7	5	1.00	24	0.208
1033	A	4	3	1.00	24	0.125
1034	A	1	1	1.00	22	0.045
1035	A	8	7	1.00	24	0.292
1036	A	14	8	1.00	24	0.333
1037	A	6	4	1.00	24	0.167
1038	A	4	3	1.00	24	0.125
1039	A	1	1	1.00	21	0.048
1040	A	5	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1041	A	8	4	1.00	24	0.167
1042	A	7	6	1.00	24	0.250
1043	A	7	6	1.00	24	0.250
1044	A	6	6	1.00	24	0.250
1045	A	5	5	1.00	22	0.227
1046	A	16	12	1.00	24	0.500
1047	A	17	13	1.00	24	0.542
1048	A	12	7	1.00	24	0.292
1049	A	7	6	1.00	24	0.250
1050	A	1	1	1.00	21	0.048
1051	A	8	7	1.00	24	0.292
1052	A	14	7	1.00	24	0.292
1053	A	1	1	1.00	24	0.042
1054	A	1	1	1.00	24	0.042
1055	A	1	1	1.00	24	0.042
1056	A	1	1	1.00	26	0.038
1057	A	1	1	1.00	26	0.038
1058	A	1	1	1.00	26	0.038
1059	A	1	1	1.00	26	0.038
1060	A	1	1	1.00	28	0.036
1061	A	20	12	1.00	24	0.500
1062	A	17	12	1.00	24	0.500
1063	A	14	10	1.00	24	0.417
1064	A	11	8	1.00	22	0.364
1065	A	18	13	1.00	24	0.542
1066	A	24	14	1.00	24	0.583
1067	A	11	5	1.00	24	0.208
1068	A	8	5	1.00	24	0.208
1069	A	1	1	1.00	24	0.042
1070	A	3	3	1.00	21	0.143
1071	A	7	5	1.00	24	0.208
1072	A	10	5	1.00	24	0.208
1073	A	1	1	1.00	24	0.042
1074	A	1	1	1.00	24	0.042
1075	A	1	1	1.00	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1076	A	1	1	1.00	26	0.038
1077	A	1	1	1.00	28	0.036
1078	A	1	1	1.00	28	0.036
1079	A	1	1	1.00	28	0.036
1080	A	1	1	1.00	30	0.033
1081	A	7	6	1.00	24	0.250
1082	A	7	6	1.00	24	0.250
1083	A	6	6	1.00	24	0.250
1084	A	5	5	1.00	22	0.227
1085	A	16	12	1.00	24	0.500
1086	A	17	13	1.00	24	0.542
1087	A	15	6	1.00	24	0.250
1088	A	11	6	1.00	24	0.250
1089	A	1	1	1.00	24	0.042
1090	A	4	4	1.00	21	0.190
1091	A	9	6	1.00	24	0.250
1092	A	13	6	1.00	24	0.250
1093	A	7	7	1.00	26	0.269
1094	A	6	6	1.00	26	0.231
1095	A	6	6	1.00	26	0.231
1096	A	2	2	1.00	26	0.077
1097	A	3	3	1.00	26	0.115
1098	A	4	3	1.00	26	0.115
1099	A	8	7	1.00	26	0.269
1100	A	7	7	1.00	26	0.269
1101	A	6	6	1.00	26	0.231
1102	A	6	6	1.00	26	0.231
1103	A	7	7	1.00	26	0.269
1104	A	8	7	1.00	26	0.269
1105	A	7	7	1.00	26	0.269
1106	A	6	6	1.00	26	0.231
1107	A	2	2	1.00	26	0.077
1108	A	3	3	1.00	26	0.115
1109	A	4	3	1.00	26	0.115
1110	A	6	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1111	A	5	5	1.00	26	0.192
1112	A	4	4	1.00	26	0.154
1113	A	4	4	1.00	26	0.154
1114	A	5	5	1.00	26	0.192
1115	A	6	5	1.00	26	0.192
1116	A	7	7	1.00	26	0.269
1117	A	6	6	1.00	26	0.231
1118	A	2	2	1.00	26	0.077
1119	A	3	3	1.00	26	0.115
1120	A	4	3	1.00	26	0.115
1121	A	8	7	1.00	26	0.269
1122	A	7	7	1.00	26	0.269
1123	A	6	6	1.00	26	0.231
1124	A	7	7	1.00	26	0.269
1125	A	8	8	1.00	26	0.308
1126	A	8	8	1.00	26	0.308
1127	A	7	7	1.00	26	0.269
1128	A	2	2	1.00	26	0.077
1129	A	3	3	1.00	26	0.115
1130	A	4	3	1.00	26	0.115
1131	A	5	3	1.00	26	0.115
1132	A	7	5	1.00	26	0.192
1133	A	6	5	1.00	26	0.192
1134	A	5	5	1.00	26	0.192
1135	A	4	4	1.00	26	0.154
1136	A	5	5	1.00	26	0.192
1137	A	6	6	1.00	26	0.231
1138	A	7	6	1.00	26	0.231
1139	A	3	2	1.00	24	0.083
1140	A	3	2	1.00	22	0.091
1141	A	3	2	1.00	22	0.091
1142	A	3	2	1.00	19	0.105
1143	A	3	2	1.00	22	0.091
1144	A	3	2	1.00	22	0.091
1145	A	5	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1146	A	4	4	1.00	22	0.182
1147	A	3	3	1.00	20	0.150
1148	A	3	3	1.00	22	0.136
1149	A	3	3	1.00	22	0.136
1150	A	3	3	1.00	22	0.136
1151	A	3	2	1.00	26	0.077
1152	A	3	2	1.00	26	0.077
1153	A	3	2	1.00	26	0.077
1154	A	3	2	1.00	26	0.077
1155	A	3	2	1.00	26	0.077
1156	A	3	2	1.00	26	0.077

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2(a+bx^2)(A+Bx^2)\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}aAx^3+\frac{1}{5}(Ab+aB)x^5+\frac{1}{7}bBx^7$$

```
[Out] 1/3*a*A*x^3+1/5*(A*b+B*a)*x^5+1/7*b*B*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.056$, Rules used = $\{459\}$

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*(A + B*x^2),x]
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)\left(A+Bx^2\right)\,dx&=\int\left(aAx^2+(Ab+aB)x^4+bBx^6\right)\,dx\\&=\frac{1}{3}aAx^3+\frac{1}{5}(Ab+aB)x^5+\frac{1}{7}bBx^7\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{3}aAx^3+\frac{1}{5}(Ab+aB)x^5+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*(A + B*x^2),x]
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

Maple [A]
time = 0.09, size = 28, normalized size = 0.85

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{(Ab+Ba)x^{5}}{5}+\frac{bB\,x^{7}}{7}$	28
norman	$\frac{bB\,x^{7}}{7}+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^{5}+\frac{aA\,x^{3}}{3}$	29
gosper	$\frac{1}{7}bB\,x^{7}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{3}aA\,x^{3}$	30
risch	$\frac{1}{7}bB\,x^{7}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{3}aA\,x^{3}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
[Out] 1/3*a*A*x^3+1/5*(A*b+B*a)*x^5+1/7*b*B*x^7
```

Maxima [A]
time = 0.30, size = 27, normalized size = 0.82

$$\frac{1}{7}\,Bbx^{7}+\frac{1}{5}\,(Ba+Ab)x^{5}+\frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
[Out] 1/7*B*b*x^7 + 1/5*(B*a + A*b)*x^5 + 1/3*A*a*x^3
```

Fricas [A]
time = 0.88, size = 27, normalized size = 0.82

$$\frac{1}{7}\,Bbx^{7}+\frac{1}{5}\,(Ba+Ab)x^{5}+\frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
[Out] 1/7*B*b*x^7 + 1/5*(B*a + A*b)*x^5 + 1/3*A*a*x^3
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{Aax^{3}}{3}+\frac{Bbx^{7}}{7}+x^{5}\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x**3/3 + B*b*x**7/7 + x**5*(A*b/5 + B*a/5)
```

Giac [**A**]
time = 0.72, size = 29, normalized size = 0.88

$$\frac{1}{7}\,Bbx^{7} + \frac{1}{5}\,Bax^{5} + \frac{1}{5}\,Abx^{5} + \frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*B*b*x^7 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/3*A*a*x^3
```

Mupad [**B**]
time = 0.11, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^7}{7} + \left(\frac{A\,b}{5} + \frac{B\,a}{5}\right)\,x^5 + \frac{A\,a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] x^5*((A*b)/5 + (B*a)/5) + (A*a*x^3)/3 + (B*b*x^7)/7
```

3.2 $\int x(a+bx^2)(A+Bx^2)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}aAx^2+\frac{1}{4}(Ab+aB)x^4+\frac{1}{6}bBx^6$$

```
[Out] 1/2*a*A*x^2+1/4*(A*b+B*a)*x^4+1/6*b*B*x^6
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {455, 45}

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\left(A+Bx^2\right)\,dx&=\frac{1}{2}\text{Subst}\left(\int(a+bx)(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(aA+(Ab+aB)x+bBx^2\right)\,dx,x,x^2\right)\\&=\frac{1}{2}aAx^2+\frac{1}{4}(Ab+aB)x^4+\frac{1}{6}bBx^6\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{2}aAx^2 + \frac{1}{4}(Ab + aB)x^4 + \frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(A + B*x^2),x]
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

Maple [A]
time = 0.05, size = 28, normalized size = 0.85

method	result	size
default	$\frac{aA\,x^{2}}{2}+\frac{(Ab+Ba)x^{4}}{4}+\frac{bB\,x^{6}}{6}$	28
norman	$\frac{bB\,x^{6}}{6}+\left(\frac{Ab}{4}+\frac{Ba}{4}\right)x^{4}+\frac{aA\,x^{2}}{2}$	29
gosper	$\frac{1}{6}bB\,x^{6}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ba+\frac{1}{2}aA\,x^{2}$	30
risch	$\frac{1}{6}bB\,x^{6}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ba+\frac{1}{2}aA\,x^{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
[Out] 1/2*a*A*x^2+1/4*(A*b+B*a)*x^4+1/6*b*B*x^6
```

Maxima [A]
time = 0.30, size = 27, normalized size = 0.82

$$\frac{1}{6}\,Bbx^6 + \frac{1}{4}\,(Ba + Ab)x^4 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
[Out] 1/6*B*b*x^6 + 1/4*(B*a + A*b)*x^4 + 1/2*A*a*x^2
```

Fricas [A]
time = 0.88, size = 27, normalized size = 0.82

$$\frac{1}{6}\,Bbx^6 + \frac{1}{4}\,(Ba + Ab)x^4 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/6*B*b*x^6 + 1/4*(B*a + A*b)*x^4 + 1/2*A*a*x^2
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{Aax^2}{2} + \frac{Bbx^6}{6} + x^4\left(\frac{Ab}{4} + \frac{Ba}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x**2/2 + B*b*x**6/6 + x**4*(A*b/4 + B*a/4)
```

Giac [A]
time = 0.96, size = 29, normalized size = 0.88

$$\frac{1}{6}Bbx^6 + \frac{1}{4}Bax^4 + \frac{1}{4}Abx^4 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/6*B*b*x^6 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + 1/2*A*a*x^2
```

Mupad [B]
time = 0.02, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^6}{6} + \left(\frac{A\,b}{4} + \frac{B\,a}{4}\right)\,x^4 + \frac{A\,a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] x^4*((A*b)/4 + (B*a)/4) + (A*a*x^2)/2 + (B*b*x^6)/6
```

3.3 $\int (a+bx^2)(A+Bx^2)\,dx$

Optimal. Leaf size=28

$$aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}bBx^5$$

[Out] a*A*x+1/3*(A*b+B*a)*x^3+1/5*b*B*x^5

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {380}

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)*(A + B*x^2),x]

[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx &= \int \left(aA+(Ab+aB)x^2+bBx^4\right)\,dx\\ &= aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}bBx^5\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)*(A + B*x^2),x]

[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5

Maple [A]
time = 0.09, size = 25, normalized size = 0.89

method	result	size
default	$aAx+\frac{(Ab+Ba)x^3}{3}+\frac{bB\,x^5}{5}$	25
norman	$\frac{bB\,x^5}{5}+\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^3+aAx$	26
gosper	$\frac{1}{5}bB\,x^5+\frac{1}{3}x^3Ab+\frac{1}{3}x^3Ba+aAx$	27
risch	$\frac{1}{5}bB\,x^5+\frac{1}{3}x^3Ab+\frac{1}{3}x^3Ba+aAx$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
[Out] a*A*x+1/3*(A*b+B*a)*x^3+1/5*b*B*x^5
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.86

$$\frac{1}{5}\,Bbx^5+\frac{1}{3}\,(Ba+Ab)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
[Out] 1/5*B*b*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

Fricas [A]
time = 0.83, size = 24, normalized size = 0.86

$$\frac{1}{5}\,Bbx^5+\frac{1}{3}\,(Ba+Ab)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
[Out] 1/5*B*b*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.93

$$Aax+\frac{Bbx^5}{5}+x^3\left(\frac{Ab}{3}+\frac{Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x + B*b*x**5/5 + x**3*(A*b/3 + B*a/3)
```

Giac [A]
time = 1.46, size = 26, normalized size = 0.93

$$\frac{1}{5}\,Bbx^5 + \frac{1}{3}\,Bax^3 + \frac{1}{3}\,Abx^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A),x, algorithm="giac")
[Out] 1/5*B*b*x^5 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*x
```

Mupad [B]
time = 0.02, size = 25, normalized size = 0.89

$$\frac{B\,b\,x^5}{5} + \left(\frac{A\,b}{3} + \frac{B\,a}{3}\right)\,x^3 + A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2),x)
[Out] x^3*((A*b)/3 + (B*a)/3) + A*a*x + (B*b*x^5)/5
```

3.4 $\int \frac{(a+bx^2)(A+Bx^2)}{x} dx$

Optimal. Leaf size=29

$$\frac{1}{2}(Ab+aB)x^2+\frac{1}{4}bBx^4+aA\log(x)$$

```
[Out] 1/2*(A*b+B*a)*x^2+1/4*b*B*x^4+a*A*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {457, 77}

$$\frac{1}{2}x^2(aB+Ab)+aA\log(x)+\frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x,x]
```

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(Ab+aB+\frac{aA}{x}+bBx\right)\,dx,x,x^2\right)\\ &= \frac{1}{2}(Ab+aB)x^2+\frac{1}{4}bBx^4+aA\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{2}(Ab+aB)x^2+\frac{1}{4}bBx^4+aA\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x,x]
```

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

Maple [A]
time = 0.04, size = 28, normalized size = 0.97

method	result	size
norman	$\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x^2+\frac{bB\,x^4}{4}+aA\ln\left(x\right)$	27
default	$\frac{bB\,x^4}{4}+\frac{Ab\,x^2}{2}+\frac{Ba\,x^2}{2}+aA\ln\left(x\right)$	28
risch	$\frac{bB\,x^4}{4}+\frac{Ab\,x^2}{2}+\frac{Ba\,x^2}{2}+\frac{b\,A^2}{4B}+\frac{Aa}{2}+\frac{B\,a^2}{4b}+aA\ln\left(x\right)$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*B*x^4+1/2*A*b*x^2+1/2*B*a*x^2+a*A*ln(x)
```

Maxima [A]
time = 0.29, size = 28, normalized size = 0.97

$$\frac{1}{4}\,Bbx^4+\frac{1}{2}\,(Ba+Ab)x^2+\frac{1}{2}\,Aa\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] 1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + 1/2*A*a*log(x^2)
```

Fricas [A]
time = 0.72, size = 25, normalized size = 0.86

$$\frac{1}{4}\,Bbx^4+\frac{1}{2}\,(Ba+Ab)x^2+Aa\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] 1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + A*a*log(x)
```

Sympy [A]
time = 0.03, size = 27, normalized size = 0.93

$$Aa\log(x) + \frac{Bbx^4}{4} + x^2\left(\frac{Ab}{2} + \frac{Ba}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x,x)
```

```
[Out] A*a*log(x) + B*b*x**4/4 + x**2*(A*b/2 + B*a/2)
```

Giac [A]
time = 1.23, size = 30, normalized size = 1.03

$$\frac{1}{4}Bbx^4 + \frac{1}{2}Bax^2 + \frac{1}{2}Abx^2 + \frac{1}{2}Aa\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] 1/4*B*b*x^4 + 1/2*B*a*x^2 + 1/2*A*b*x^2 + 1/2*A*a*log(x^2)
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.90

$$x^2\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right) + \frac{B\,b\,x^4}{4} + A\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x,x)
```

```
[Out] x^2*((A*b)/2 + (B*a)/2) + (B*b*x^4)/4 + A*a*log(x)
```

3.5 $\int \frac{(a+bx^2)(A+Bx^2)}{x^2} \, dx$

Optimal. Leaf size=26

$$-\frac{aA}{x}+(Ab+aB)x+\frac{1}{3}bBx^3$$

```
[Out] -a*A/x+(A*b+B*a)*x+1/3*b*B*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^2} \, dx &= \int \left(Ab\left(1+\frac{aB}{Ab}\right)+\frac{aA}{x^2}+bBx^2\right) \, dx\\ &= -\frac{aA}{x}+(Ab+aB)x+\frac{1}{3}bBx^3\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\frac{aA}{x}+(Ab+aB)x+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

Maple [A]
time = 0.02, size = 24, normalized size = 0.92

method	result	size
default	$\frac{bB\,x^3}{3}+Abx+Bax-\frac{aA}{x}$	24
risch	$\frac{bB\,x^3}{3}+Abx+Bax-\frac{aA}{x}$	24
norman	$\frac{\frac{bB\,x^4}{3}+(Ab+Ba)x^2-Aa}{x}$	28
gosper	$-\frac{-bB\,x^4-3Ab\,x^2-3Ba\,x^2+3Aa}{3x}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*B*x^3+A*b*x+B*a*x-a*A/x
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.92

$$\frac{1}{3}\,Bbx^3+(Ba+Ab)x-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*B*b*x^3 + (B*a + A*b)*x - A*a/x
```

Fricas [A]
time = 0.65, size = 28, normalized size = 1.08

$$\frac{Bbx^4+3\,(Ba+Ab)x^2-3\,Aa}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^4 + 3*(B*a + A*b)*x^2 - 3*A*a)/x
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.77

$$-\frac{Aa}{x}+\frac{Bbx^3}{3}+x(Ab+Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a/x + B*b*x**3/3 + x*(A*b + B*a)
```

Giac [A]
time = 0.72, size = 23, normalized size = 0.88

$$\frac{1}{3}\,Bbx^3 + Bax + Abx - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*B*b*x^3 + B*a*x + A*b*x - A*a/x
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.92

$$x\,(A\,b+B\,a)-\frac{A\,a}{x}+\frac{B\,b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^2,x)
```

```
[Out] x*(A*b + B*a) - (A*a)/x + (B*b*x^3)/3
```

3.6 $\int \frac{(a+bx^2)(A+Bx^2)}{x^3} dx$

Optimal. Leaf size=29

$$-\frac{aA}{2x^2}+\frac{1}{2}bBx^2+(Ab+aB)\log(x)$$

```
[Out] -1/2*a*A/x^2+1/2*b*B*x^2+(A*b+B*a)*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 77}

$$\log(x)(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(bB+\frac{aA}{x^2}+\frac{Ab+aB}{x}\right)dx, x, x^2\right)\\
&= -\frac{aA}{2x^2}+\frac{1}{2}bBx^2+(Ab+aB)\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$-\frac{aA}{2x^2}+\frac{1}{2}bBx^2+(Ab+aB)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^3,x]
[Out] -1/2*(a*A)/x^2 + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.90

method	result	size
default	$-\frac{aA}{2x^2}+\frac{bB\,x^2}{2}+(Ab+Ba)\ln(x)$	26
risch	$-\frac{aA}{2x^2}+\frac{bB\,x^2}{2}+A\ln(x)\,b+B\ln(x)\,a$	26
norman	$\frac{-\frac{Aa}{2}+\frac{bB\,x^4}{2}}{x^2}+(Ab+Ba)\ln(x)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a*A/x^2+1/2*b*B*x^2+(A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.29, size = 28, normalized size = 0.97

$$\frac{1}{2}\,Bbx^2+\frac{1}{2}\,(Ba+Ab)\log\left(x^2\right)-\frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="maxima")
[Out] 1/2*B*b*x^2 + 1/2*(B*a + A*b)*log(x^2) - 1/2*A*a/x^2
```

Fricas [A]
time = 0.79, size = 30, normalized size = 1.03

$$\frac{Bbx^4+2\,(Ba+Ab)x^2\log\left(x\right)-Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="fricas")
[Out] 1/2*(B*b*x^4 + 2*(B*a + A*b)*x^2*log(x) - A*a)/x^2
```

Sympy [A]
time = 0.07, size = 26, normalized size = 0.90

$$-\frac{Aa}{2x^2} + \frac{Bbx^2}{2} + (Ab + Ba)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a/(2*x**2) + B*b*x**2/2 + (A*b + B*a)*log(x)
```

Giac [A]
time = 0.84, size = 42, normalized size = 1.45

$$\frac{1}{2}\,Bbx^2 + \frac{1}{2}\,(Ba + Ab)\log\left(x^2\right) - \frac{Bax^2 + Abx^2 + Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*B*b*x^2 + 1/2*(B*a + A*b)*log(x^2) - 1/2*(B*a*x^2 + A*b*x^2 + A*a)/x^2
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.86

$$\ln(x)\ (A\,b + B\,a) - \frac{A\,a}{2\,x^2} + \frac{B\,b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^3,x)
```

```
[Out] log(x)*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^2)/2
```

3.7 $\int \frac{(a+bx^2)(A+Bx^2)}{x^4} dx$

Optimal. Leaf size=26

$$-\frac{aA}{3x^3}-\frac{Ab+aB}{x}+bBx$$

```
[Out] -1/3*a*A/x^3+(-A*b-B*a)/x+b*B*x
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$-\frac{aB+Ab}{x}-\frac{aA}{3x^3}+bBx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 - (A*b + a*B)/x + b*B*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^4} dx &= \int \left(bB+\frac{aA}{x^4}+\frac{Ab+aB}{x^2}\right) dx \\ &= -\frac{aA}{3x^3}-\frac{Ab+aB}{x}+bBx\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.04

$$-\frac{aA}{3x^3}+\frac{-Ab-aB}{x}+bBx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 + (-(A*b) - a*B)/x + b*B*x
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.96

method	result	size
default	$bBx - \frac{aA}{3x^3} - \frac{Ab+Ba}{x}$	25
risch	$bBx + \frac{(-Ab-Ba)x^2 - \frac{Aa}{3}}{x^3}$	28
norman	$\frac{bB\,x^4+(-Ab-Ba)x^2-\frac{Aa}{3}}{x^3}$	29
gosper	$-\frac{-3bB\,x^4+3Ab\,x^2+3Ba\,x^2+Aa}{3x^3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
[Out] b*B*x-1/3*a*A/x^3-(A*b+B*a)/x
```

Maxima [A]
time = 0.29, size = 26, normalized size = 1.00

$$Bbx - \frac{3\,(Ba + Ab)x^2 + Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="maxima")
[Out] B*b*x - 1/3*(3*(B*a + A*b)*x^2 + A*a)/x^3
```

Fricas [A]
time = 1.18, size = 29, normalized size = 1.12

$$\frac{3\,Bbx^4 - 3\,(Ba + Ab)x^2 - Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*B*b*x^4 - 3*(B*a + A*b)*x^2 - A*a)/x^3
```

Sympy [A]
time = 0.09, size = 27, normalized size = 1.04

$$Bbx + \frac{-Aa + x^2(-3Ab - 3Ba)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**4,x)
[Out] B*b*x + (-A*a + x**2*(-3*A*b - 3*B*a))/(3*x**3)
```

Giac [A]
time = 1.04, size = 28, normalized size = 1.08

$$Bbx - \frac{3\,Bax^2 + 3\,Abx^2 + Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="giac")
[Out] B*b*x - 1/3*(3*B*a*x^2 + 3*A*b*x^2 + A*a)/x^3
```

Mupad [B]
time = 0.02, size = 26, normalized size = 1.00

$$B\,b\,x - \frac{(A\,b + B\,a)\,x^2 + \frac{A\,a}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^4,x)
[Out] B*b*x - ((A*a)/3 + x^2*(A*b + B*a))/x^3
```

3.8 $\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=29

$$-\frac{aA}{4x^4}-\frac{Ab+aB}{2x^2}+bB\log(x)$$

```
[Out] -1/4*a*A/x^4+1/2*(-A*b-B*a)/x^2+b*B*ln(x)
```

Rubi [A]

time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 77}

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 - (A*b + a*B)/(2*x^2) + b*B*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{aA}{x^3}+\frac{Ab+aB}{x^2}+\frac{bB}{x}\right)dx,x,x^2\right)\\ &= -\frac{aA}{4x^4}-\frac{Ab+aB}{2x^2}+bB\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.07

$$-\frac{aA}{4x^4}+\frac{-Ab-aB}{2x^2}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^5,x]
[Out] -1/4*(a*A)/x^4 + (-(A*b) - a*B)/(2*x^2) + b*B*Log[x]
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.90

method	result	size
default	$-\frac{aA}{4x^4}-\frac{Ab+Ba}{2x^2}+bB\ln\left(x\right)$	26
norman	$\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^2-\frac{Aa}{4}}{x^4}+bB\ln\left(x\right)$	29
risch	$\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^2-\frac{Aa}{4}}{x^4}+bB\ln\left(x\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a*A/x^4-1/2*(A*b+B*a)/x^2+b*B*ln(x)
```

Maxima [A]
time = 0.31, size = 30, normalized size = 1.03

$$\frac{1}{2}\,Bb\log\left(x^2\right)-\frac{2\,(Ba+Ab)x^2+Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="maxima")
[Out] 1/2*B*b*log(x^2) - 1/4*(2*(B*a + A*b)*x^2 + A*a)/x^4
```

Fricas [A]
time = 1.18, size = 31, normalized size = 1.07

$$\frac{4\,Bbx^4\log\left(x\right)-2\,(Ba+Ab)x^2-Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*B*b*x^4*log(x) - 2*(B*a + A*b)*x^2 - A*a)/x^4
```

Sympy [A]
time = 0.17, size = 29, normalized size = 1.00

$$Bb\log{(x)} + \frac{-Aa + x^2(-2Ab - 2Ba)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**5,x)
[Out] B*b*log(x) + (-A*a + x**2*(-2*A*b - 2*B*a))/(4*x**4)
```

Giac [A]
time = 0.80, size = 39, normalized size = 1.34

$$\frac{1}{2}\,Bb\log\left(x^2\right) - \frac{3\,Bbx^4 + 2\,Bax^2 + 2\,Abx^2 + Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="giac")
[Out] 1/2*B*b*log(x^2) - 1/4*(3*B*b*x^4 + 2*B*a*x^2 + 2*A*b*x^2 + A*a)/x^4
```

Mupad [B]
time = 0.04, size = 29, normalized size = 1.00

$$B\,b\,\ln(x) - \frac{\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right)\,x^2 + \frac{A\,a}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^5,x)
[Out] B*b*log(x) - ((A*a)/4 + x^2*((A*b)/2 + (B*a)/2))/x^4
```

3.9 $\int \frac{(a+bx^2)(A+Bx^2)}{x^6} \, dx$

Optimal. Leaf size=31

$$-\frac{aA}{5x^5}-\frac{Ab+aB}{3x^3}-\frac{bB}{x}$$

```
[Out] -1/5*a*A/x^5+1/3*(-A*b-B*a)/x^3-b*B/x
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {459}

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(a*A)/x^5 - (A*b + a*B)/(3*x^3) - (b*B)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)(A+Bx^2)}{x^6} \, dx &= \int \left(\frac{aA}{x^6}+\frac{Ab+aB}{x^4}+\frac{bB}{x^2}\right) dx \\ &= -\frac{aA}{5x^5}-\frac{Ab+aB}{3x^3}-\frac{bB}{x} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.06

$$-\frac{aA}{5x^5}+\frac{-Ab-aB}{3x^3}-\frac{bB}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(a*A)/x^5 + (-(A*b) - a*B)/(3*x^3) - (b*B)/x
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.90

method	result	size
default	$-\frac{aA}{5x^5}-\frac{Ab+Ba}{3x^3}-\frac{bB}{x}$	28
norman	$\frac{-bB\,x^4+\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^2-\frac{Aa}{5}}{x^5}$	30
risch	$\frac{-bB\,x^4+\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^2-\frac{Aa}{5}}{x^5}$	30
gosper	$-\frac{15bB\,x^4+5Ab\,x^2+5Ba\,x^2+3Aa}{15x^5}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a*A/x^5-1/3*(A*b+B*a)/x^3-b*B/x
```

Maxima [A]
time = 0.29, size = 29, normalized size = 0.94

$$-\frac{15\,Bbx^4+5\,(Ba+Ab)x^2+3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

Fricas [A]
time = 1.05, size = 29, normalized size = 0.94

$$-\frac{15\,Bbx^4+5\,(Ba+Ab)x^2+3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

Sympy [A]
time = 0.17, size = 32, normalized size = 1.03

$$\frac{-3Aa-15Bbx^4+x^2(-5Ab-5Ba)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**6,x)
```

```
[Out] (-3*A*a - 15*B*b*x**4 + x**2*(-5*A*b - 5*B*a))/(15*x**5)
```

Giac [A]
time = 1.20, size = 31, normalized size = 1.00

$$-\frac{15\,Bbx^4+5\,Bax^2+5\,Abx^2+3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*B*a*x^2 + 5*A*b*x^2 + 3*A*a)/x^5
```

Mupad [B]
time = 0.02, size = 29, normalized size = 0.94

$$-\frac{B\,b\,x^4+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x^2+\frac{A\,a}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^6,x)
```

```
[Out] -((A*a)/5 + x^2*((A*b)/3 + (B*a)/3) + B*b*x^4)/x^5
```

3.10 $\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=33

$$-\frac{aA}{6x^6}-\frac{Ab+aB}{4x^4}-\frac{bB}{2x^2}$$

```
[Out] -1/6*a*A/x^6+1/4*(-A*b-B*a)/x^4-1/2*b*B/x^2
```

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {457, 77}

$$-\frac{aB+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^7,x]
```

```
[Out] -1/6*(a*A)/x^6 - (A*b + a*B)/(4*x^4) - (b*B)/(2*x^2)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^4}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{aA}{x^4}+\frac{Ab+aB}{x^3}+\frac{bB}{x^2}\right)\,dx,x,x^2\right)\\
&= -\frac{aA}{6x^6}-\frac{Ab+aB}{4x^4}-\frac{bB}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.06

$$-\frac{aA}{6x^6}+\frac{-Ab-aB}{4x^4}-\frac{bB}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^7,x]
[Out] -1/6*(a*A)/x^6 + (-(A*b) - a*B)/(4*x^4) - (b*B)/(2*x^2)
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.85

method	result	size
default	$-\frac{Ab+Ba}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$	28
norman	$\frac{-\frac{bB\,x^4}{2}+\left(-\frac{Ab}{4}-\frac{Ba}{4}\right)x^2-\frac{Aa}{6}}{x^6}$	30
risch	$\frac{-\frac{bB\,x^4}{2}+\left(-\frac{Ab}{4}-\frac{Ba}{4}\right)x^2-\frac{Aa}{6}}{x^6}$	30
gosper	$-\frac{6bB\,x^4+3Ab\,x^2+3Ba\,x^2+2Aa}{12x^6}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/4*(A*b+B*a)/x^4-1/6*a*A/x^6-1/2*b*B/x^2
```

Maxima [A]
time = 0.29, size = 29, normalized size = 0.88

$$-\frac{6\,Bbx^4+3\,(Ba+Ab)x^2+2\,Aa}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="maxima")
[Out] -1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

Fricas [A]
time = 1.35, size = 29, normalized size = 0.88

$$-\frac{6\,Bbx^4+3\,(Ba+Ab)x^2+2\,Aa}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

Sympy [A]
time = 0.25, size = 32, normalized size = 0.97

$$\frac{-2Aa - 6Bbx^4 + x^2(-3Ab - 3Ba)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**7,x)
[Out] (-2*A*a - 6*B*b*x**4 + x**2*(-3*A*b - 3*B*a))/(12*x**6)
```

Giac [A]
time = 1.09, size = 31, normalized size = 0.94

$$-\frac{6\,Bbx^4 + 3\,Bax^2 + 3\,Abx^2 + 2\,Aa}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="giac")
[Out] -1/12*(6*B*b*x^4 + 3*B*a*x^2 + 3*A*b*x^2 + 2*A*a)/x^6
```

Mupad [B]
time = 0.02, size = 30, normalized size = 0.91

$$-\frac{\frac{B\,b\,x^4}{2} + \left(\frac{A\,b}{4} + \frac{B\,a}{4}\right)\,x^2 + \frac{A\,a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^7,x)
[Out] -((A*a)/6 + x^2*((A*b)/4 + (B*a)/4) + (B*b*x^4)/2)/x^6
```

3.11 $\int x^2(a+bx^2)^2\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}a^2Ax^3+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{9}b^2Bx^9$$

```
[Out] 1/3*a^2*A*x^3+1/5*a*(2*A*b+B*a)*x^5+1/7*b*(A*b+2*B*a)*x^7+1/9*b^2*B*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*
x^9)/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx&=\int\left(a^2Ax^2+a(2Ab+aB)x^4+b(Ab+2aB)x^6+b^2Bx^8\right)\,dx\\&=\frac{1}{3}a^2Ax^3+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{9}b^2Bx^9\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{3}a^2Ax^3+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*
x^9)/9
```

Maple [A]
time = 0.09, size = 52, normalized size = 0.95

method	result	size
default	$\frac{b^2 B x^9}{9}+\frac{(b^2 A+2abB)x^7}{7}+\frac{(2abA+a^2 B)x^5}{5}+\frac{a^2 A x^3}{3}$	52
norman	$\frac{b^2 B x^9}{9}+\left(\frac{1}{7}b^2 A+\frac{2}{7}abB\right)x^7+\left(\frac{2}{5}abA+\frac{1}{5}a^2 B\right)x^5+\frac{a^2 A x^3}{3}$	52
gosper	$\frac{1}{9}b^2 B\,x^9+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+\frac{1}{3}a^2 A\,x^3$	54
risch	$\frac{1}{9}b^2 B\,x^9+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+\frac{1}{3}a^2 A\,x^3$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*B*x^9+1/7*(A*b^2+2*B*a*b)*x^7+1/5*(2*A*a*b+B*a^2)*x^5+1/3*a^2*A*x^3
```

Maxima [A]
time = 0.36, size = 51, normalized size = 0.93

$$\frac{1}{9}\,Bb^2x^9+\frac{1}{7}\,(2\,Bab+Ab^2)x^7+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ba^2+2\,Aab)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*B*b^2*x^9 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/3*A*a^2*x^3 + 1/5*(B*a^2 + 2*
A*a*b)*x^5
```

Fricas [A]
time = 0.95, size = 51, normalized size = 0.93

$$\frac{1}{9}\,Bb^2x^9+\frac{1}{7}\,(2\,Bab+Ab^2)x^7+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ba^2+2\,Aab)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9*B*b^2*x^9 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/3*A*a^2*x^3 + 1/5*(B*a^2 + 2*
A*a*b)*x^5
```

Sympy [A]
time = 0.01, size = 56, normalized size = 1.02

$$\frac{Aa^2x^3}{3}+\frac{Bb^2x^9}{9}+x^7\left(\frac{Ab^2}{7}+\frac{2Bab}{7}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ba^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**3/3 + B*b**2*x**9/9 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**5*(2*A*a*b
/5 + B*a**2/5)
```

Giac [A]
time = 1.05, size = 53, normalized size = 0.96

$$\frac{1}{9}\,Bb^2x^9+\frac{2}{7}\,Babx^7+\frac{1}{7}\,Ab^2x^7+\frac{1}{5}\,Ba^2x^5+\frac{2}{5}\,Aabx^5+\frac{1}{3}\,Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9*B*b^2*x^9 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/5*B*a^2*x^5 + 2/5*A*a*b*x
^5 + 1/3*A*a^2*x^3
```

Mupad [B]
time = 0.04, size = 51, normalized size = 0.93

$$x^5\,\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^7\,\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (A*a^2*x^3)
/3 + (B*b^2*x^9)/9
```

3.12 $\int x(a+bx^2)^2 (A+Bx^2)\, dx$

Optimal. Leaf size=42

$$\frac{(Ab-aB)(a+bx^2)^3}{6b^2}+\frac{B(a+bx^2)^4}{8b^2}$$

```
[Out] 1/6*(A*b-B*a)*(b*x^2+a)^3/b^2+1/8*B*(b*x^2+a)^4/b^2
```

Rubi [**A**]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {455, 45}

$$\frac{(a+bx^2)^3(Ab-aB)}{6b^2}+\frac{B(a+bx^2)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^3)/(6*b^2) + (B*(a + b*x^2)^4)/(8*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx^2)^2(A+Bx^2)\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^2(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^2}{b}+\frac{B(a+bx)^3}{b}\right)dx,x,x^2\right)\\ &= \frac{(Ab-aB)(a+bx^2)^3}{6b^2}+\frac{B(a+bx^2)^4}{8b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12a^2A+6a(2Ab+aB)x^2+4b(Ab+2aB)x^4+3b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (x^2*(12*a^2*A + 6*a*(2*A*b + a*B)*x^2 + 4*b*(A*b + 2*a*B)*x^4 + 3*b^2*B*x^
6))/24
```

Maple [A]
time = 0.08, size = 52, normalized size = 1.24

method	result	size
default	$\frac{b^2 B x^8}{8}+\frac{(b^2 A+2abB)x^6}{6}+\frac{(2abA+a^2 B)x^4}{4}+\frac{a^2 A x^2}{2}$	52
norman	$\frac{b^2 B x^8}{8}+\left(\frac{1}{6}b^2 A+\frac{1}{3}abB\right)x^6+\left(\frac{1}{2}abA+\frac{1}{4}a^2 B\right)x^4+\frac{a^2 A x^2}{2}$	52
gosper	$\frac{1}{8}b^2 B x^8+\frac{1}{6}x^6 b^2 A+\frac{1}{3}x^6 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+\frac{1}{2}a^2 A x^2$	54
risch	$\frac{1}{8}b^2 B x^8+\frac{1}{6}x^6 b^2 A+\frac{1}{3}x^6 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+\frac{1}{2}a^2 A x^2$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^2*B*x^8+1/6*(A*b^2+2*B*a*b)*x^6+1/4*(2*A*a*b+B*a^2)*x^4+1/2*a^2*A*x^2
```

Maxima [A]
time = 0.39, size = 51, normalized size = 1.21

$$\frac{1}{8}\,Bb^2x^8+\frac{1}{6}\left(2\,Bab+Ab^2\right)x^6+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\left(Ba^2+2\,Aab\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*x^8 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/2*A*a^2*x^2 + 1/4*(B*a^2 + 2*
A*a*b)*x^4
```

Fricas [A]
time = 1.50, size = 51, normalized size = 1.21

$$\frac{1}{8}\,Bb^2x^8+\frac{1}{6}\left(2\,Bab+Ab^2\right)x^6+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\left(Ba^2+2\,Aab\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/8*B*b^2*x^8 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/2*A*a^2*x^2 + 1/4*(B*a^2 + 2*
A*a*b)*x^4
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.26

$$\frac{Aa^2x^2}{2} + \frac{Bb^2x^8}{8} + x^6\left(\frac{Ab^2}{6} + \frac{Bab}{3}\right) + x^4\left(\frac{Aab}{2} + \frac{Ba^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**2/2 + B*b**2*x**8/8 + x**6*(A*b**2/6 + B*a*b/3) + x**4*(A*a*b/2 +
 B*a**2/4)
```

Giac [A]
time = 0.82, size = 53, normalized size = 1.26

$$\frac{1}{8}Bb^2x^8 + \frac{1}{3}Babx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{4}Ba^2x^4 + \frac{1}{2}Aabx^4 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/8*B*b^2*x^8 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/4*B*a^2*x^4 + 1/2*A*a*b*x
^4 + 1/2*A*a^2*x^2
```

Mupad [B]
time = 0.02, size = 51, normalized size = 1.21

$$x^4\left(\frac{B\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^6\left(\frac{A\,b^2}{6} + \frac{B\,a\,b}{3}\right) + \frac{A\,a^2\,x^2}{2} + \frac{B\,b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^4*((B*a^2)/4 + (A*a*b)/2) + x^6*((A*b^2)/6 + (B*a*b)/3) + (A*a^2*x^2)/2 +
 (B*b^2*x^8)/8
```

3.13 $\int (a+bx^2)^2 (A+Bx^2) \, dx$

Optimal. Leaf size=50

$$a^2Ax+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{7}b^2Bx^7$$

```
[Out] a^2*A*x+1/3*a*(2*A*b+B*a)*x^3+1/5*b*(A*b+2*B*a)*x^5+1/7*b^2*B*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(A + B*x^2),x]
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx &= \int \left(a^2A+a(2Ab+aB)x^2+b(Ab+2aB)x^4+b^2Bx^6\right)\,dx\\ &= a^2Ax+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{7}b^2Bx^7\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$a^2Ax+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(A + B*x^2),x]
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/7
```

Maple [A]
time = 0.10, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b^{2} B \,x^{7}}{7}+\frac{\left(b^{2} A+2 a b B\right) x^{5}}{5}+\frac{\left(2 a b A+a^{2} B\right) x^{3}}{3}+a^{2} A x$	49
norman	$\frac{b^{2} B \,x^{7}}{7}+\left(\frac{1}{5} b^{2} A+\frac{2}{5} a b B\right) x^{5}+\left(\frac{2}{3} a b A+\frac{1}{3} a^{2} B\right) x^{3}+a^{2} A x$	49
gosper	$\frac{1}{7} b^{2} B \,x^{7}+\frac{1}{5} x^{5} b^{2} A+\frac{2}{5} x^{5} a b B+\frac{2}{3} x^{3} a b A+\frac{1}{3} x^{3} a^{2} B+a^{2} A x$	51
risch	$\frac{1}{7} b^{2} B \,x^{7}+\frac{1}{5} x^{5} b^{2} A+\frac{2}{5} x^{5} a b B+\frac{2}{3} x^{3} a b A+\frac{1}{3} x^{3} a^{2} B+a^{2} A x$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^2*B*x^7+1/5*(A*b^2+2*B*a*b)*x^5+1/3*(2*A*a*b+B*a^2)*x^3+a^2*A*x
```

Maxima [A]
time = 0.40, size = 48, normalized size = 0.96

$$\frac{1}{7} \, B b^{2} x^{7} + \frac{1}{5} \, {\left(2 \, B a b + A b^{2}\right)} x^{5} + A a^{2} x + \frac{1}{3} \, {\left(B a^{2} + 2 \, A a b\right)} x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*B*b^2*x^7 + 1/5*(2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b)
*x^3
```

Fricas [A]
time = 3.14, size = 48, normalized size = 0.96

$$\frac{1}{7} \, B b^{2} x^{7} + \frac{1}{5} \, {\left(2 \, B a b + A b^{2}\right)} x^{5} + A a^{2} x + \frac{1}{3} \, {\left(B a^{2} + 2 \, A a b\right)} x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*B*b^2*x^7 + 1/5*(2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b)
*x^3
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.06

$$A a^{2} x + \frac{B b^{2} x^{7}}{7} + x^{5} \left(\frac{A b^{2}}{5} + \frac{2 B a b}{5}\right) + x^{3} \cdot \left(\frac{2 A a b}{3} + \frac{B a^{2}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x + B*b**2*x**7/7 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**3*(2*A*a*b/3 +
B*a**2/3)
```

Giac [A]
time = 0.83, size = 50, normalized size = 1.00

$$\frac{1}{7}Bb^2x^7 + \frac{2}{5}Babx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{3}Ba^2x^3 + \frac{2}{3}Aabx^3 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*B*b^2*x^7 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/3*B*a^2*x^3 + 2/3*A*a*b*x
^3 + A*a^2*x
```

Mupad [B]
time = 0.02, size = 48, normalized size = 0.96

$$x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^5\left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}\right)+\frac{B\,b^2\,x^7}{7}+A\,a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^5*((A*b^2)/5 + (2*B*a*b)/5) + (B*b^2*x^7)
/7 + A*a^2*x
```

3.14 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=43

$$aAbx^2+\frac{1}{4}Ab^2x^4+\frac{B(a+bx^2)^3}{6b}+a^2A\log(x)$$

```
[Out] a*A*b*x^2+1/4*A*b^2*x^4+1/6*B*(b*x^2+a)^3/b+a^2*A*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {457, 81, 45}

$$a^2A\log(x)+aAbx^2+\frac{B(a+bx^2)^3}{6b}+\frac{1}{4}Ab^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

```
[Out] a*A*b*x^2 + (A*b^2*x^4)/4 + (B*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x} dx, x, x^2\right)$$
$$= \frac{B(a+bx^2)^3}{6b} + \frac{1}{2}A\text{Subst}\left(\int \frac{(a+bx)^2}{x} dx, x, x^2\right)$$
$$= \frac{B(a+bx^2)^3}{6b} + \frac{1}{2}A\text{Subst}\left(\int \left(2ab + \frac{a^2}{x} + b^2x\right) dx, x, x^2\right)$$
$$= aAbx^2 + \frac{1}{4}Ab^2x^4 + \frac{B(a+bx^2)^3}{6b} + a^2A\log(x)$$

Mathematica [**A**]
time = 0.01, size = 51, normalized size = 1.19

$$\frac{1}{2}a(2Ab+aB)x^2 + \frac{1}{4}b(Ab+2aB)x^4 + \frac{1}{6}b^2Bx^6 + a^2A\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

```
[Out] (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^6)/6 + a^2*A*L
og[x]
```

Maple [**A**]
time = 0.06, size = 51, normalized size = 1.19

method	result	size
norman	$\left(\frac{1}{4}b^2A+\frac{1}{2}abB\right)x^4+\left(abA+\frac{1}{2}a^2B\right)x^2+\frac{b^2Bx^6}{6}+a^2A\ln(x)$	49
default	$\frac{b^2Bx^6}{6}+\frac{Ab^2x^4}{4}+\frac{Babx^4}{2}+aAbx^2+\frac{Ba^2x^2}{2}+a^2A\ln(x)$	51
risch	$\frac{b^2Bx^6}{6}+\frac{Ab^2x^4}{4}+\frac{Babx^4}{2}+aAbx^2+\frac{Ba^2x^2}{2}+a^2A\ln(x)$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2*B*x^6+1/4*A*b^2*x^4+1/2*B*a*b*x^4+a*A*b*x^2+1/2*B*a^2*x^2+a^2*A*ln(
x)
```

Maxima [**A**]
time = 0.29, size = 52, normalized size = 1.21

$$\frac{1}{6}Bb^2x^6 + \frac{1}{4}(2Bab + Ab^2)x^4 + \frac{1}{2}Aa^2\log(x^2) + \frac{1}{2}(Ba^2 + 2Aab)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + 1/2*A*a^2*log(x^2) + 1/2*(B*a^2
 + 2*A*a*b)*x^2
```

Fricas [A]
time = 1.29, size = 49, normalized size = 1.14

$$\frac{1}{6}Bb^2x^6 + \frac{1}{4}\left(2\,Bab + Ab^2\right)x^4 + Aa^2\log\left(x\right) + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + A*a^2*log(x) + 1/2*(B*a^2 + 2*A
*a*b)*x^2
```

Sympy [A]
time = 0.05, size = 49, normalized size = 1.14

$$Aa^2\log\left(x\right) + \frac{Bb^2x^6}{6} + x^4\left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*b**2*x**6/6 + x**4*(A*b**2/4 + B*a*b/2) + x**2*(A*a*b + B
*a**2/2)
```

Giac [A]
time = 0.96, size = 53, normalized size = 1.23

$$\frac{1}{6}Bb^2x^6 + \frac{1}{2}Babx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{2}Ba^2x^2 + Aabx^2 + \frac{1}{2}Aa^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] 1/6*B*b^2*x^6 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*B*a^2*x^2 + A*a*b*x^2 +
 1/2*A*a^2*log(x^2)
```

Mupad [B]
time = 0.02, size = 48, normalized size = 1.12

$$x^2\left(\frac{B\,a^2}{2} + A\,b\,a\right) + x^4\left(\frac{A\,b^2}{4} + \frac{B\,a\,b}{2}\right) + \frac{B\,b^2\,x^6}{6} + A\,a^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x,x)
```

```
[Out] x^2*((B*a^2)/2 + A*a*b) + x^4*((A*b^2)/4 + (B*a*b)/2) + (B*b^2*x^6)/6 + A*a
^2*log(x)
```

3.15 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^2} \, dx$

Optimal. Leaf size=48

$$-\frac{a^2 A}{x}+a(2 A b+a B) x+\frac{1}{3} b(A b+2 a B) x^3+\frac{1}{5} b^2 B x^5$$

```
[Out] -a^2*A/x+a*(2*A*b+B*a)*x+1/3*b*(A*b+2*B*a)*x^3+1/5*b^2*B*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^2 A}{x}+\frac{1}{3} b x^3(2 a B+A b)+a x(a B+2 A b)+\frac{1}{5} b^2 B x^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^2} \, dx &= \int \left(a(2Ab+aB)+\frac{a^2 A}{x^2}+b(Ab+2aB)x^2+b^2Bx^4\right) \, dx\\
&= -\frac{a^2 A}{x}+a(2Ab+aB)x+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{5}b^2Bx^5
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.00

$$-\frac{a^2 A}{x}+a(2 A b+a B) x+\frac{1}{3} b(A b+2 a B) x^3+\frac{1}{5} b^2 B x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^5)/5
```

Maple [A]
time = 0.06, size = 49, normalized size = 1.02

method	result	size
default	$\frac{b^{2} B \,x^{5}}{5}+\frac{A \,b^{2} x^{3}}{3}+\frac{2 B a b \,x^{3}}{3}+2 a b A x +a^{2} B x -\frac{a^{2} A}{x}$	49
risch	$\frac{b^{2} B \,x^{5}}{5}+\frac{A \,b^{2} x^{3}}{3}+\frac{2 B a b \,x^{3}}{3}+2 a b A x +a^{2} B x -\frac{a^{2} A}{x}$	49
norman	$\frac{\frac{b^{2} B \,x^{6}}{5}+\left(\frac{1}{3} b^{2} A +\frac{2}{3} a b B \right) x^{4}+\left(2 a b A +a^{2} B \right) x^{2}-a^{2} A}{x}$	52
gosper	$-\frac{-3 b^{2} B \,x^{6}-5 A \,b^{2} x^{4}-10 B a b \,x^{4}-30 a A b \,x^{2}-15 B \,a^{2} x^{2}+15 a^{2} A}{15 x}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*B*x^5+1/3*A*b^2*x^3+2/3*B*a*b*x^3+2*a*b*A*x+a^2*B*x-a^2*A/x
```

Maxima [A]
time = 0.28, size = 48, normalized size = 1.00

$$\frac{1}{5}\,Bb^{2}x^{5}+\frac{1}{3}\,\left(2\,Bab+Ab^{2}\right)x^{3}-\frac{Aa^{2}}{x}+\left(Ba^{2}+2\,Aab\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^2*x^5 + 1/3*(2*B*a*b + A*b^2)*x^3 - A*a^2/x + (B*a^2 + 2*A*a*b)*x
```

Fricas [A]
time = 0.83, size = 53, normalized size = 1.10

$$\frac{3\,Bb^{2}x^{6}+5\,(2\,Bab+Ab^{2})x^{4}-15\,Aa^{2}+15\,(Ba^{2}+2\,Aab)x^{2}}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^6 + 5*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 15*(B*a^2 + 2*A*a*
b)*x^2)/x
```

Sympy [A]
time = 0.04, size = 48, normalized size = 1.00

$$-\frac{Aa^{2}}{x}+\frac{Bb^{2}x^{5}}{5}+x^{3}\left(\frac{Ab^{2}}{3}+\frac{2Bab}{3}\right)+x\left(2Aab+Ba^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**5/5 + x**3*(A*b**2/3 + 2*B*a*b/3) + x*(2*A*a*b + B*a*
*2)
```

Giac [A]
time = 1.32, size = 48, normalized size = 1.00

$$\frac{1}{5}\,Bb^2x^5+\frac{2}{3}\,Babx^3+\frac{1}{3}\,Ab^2x^3+Ba^2x+2\,Aabx-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*B*b^2*x^5 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + B*a^2*x + 2*A*a*b*x - A*a^2
/x
```

Mupad [B]
time = 0.03, size = 48, normalized size = 1.00

$$x^3\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+x\,\left(B\,a^2+2\,A\,b\,a\right)-\frac{A\,a^2}{x}+\frac{B\,b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^2,x)
```

```
[Out] x^3*((A*b^2)/3 + (2*B*a*b)/3) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/x + (B*b^2*x^
5)/5
```

3.16 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{2x^2}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{4}b^2Bx^4+a(2Ab+aB)\log(x)$$

```
[Out] -1/2*a^2*A/x^2+1/2*b*(A*b+2*B*a)*x^2+1/4*b^2*B*x^4+a*(2*A*b+B*a)*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 77}

$$-\frac{a^2A}{2x^2}+\frac{1}{2}bx^2(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x^4)/4 + a*(2*A*b + a*B
)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(b(Ab+2aB)+\frac{a^2A}{x^2}+\frac{a(2Ab+aB)}{x}+b^2Bx\right)\,dx, x, x^2\right)$$
$$= -\frac{a^2A}{2x^2}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{4}b^2Bx^4+a(2Ab+aB)\log(x)$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2a^2A}{x^2}+2b(Ab+2aB)x^2+b^2Bx^4+4a(2Ab+aB)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
[Out] ((-2*a^2*A)/x^2 + 2*b*(A*b + 2*a*B)*x^2 + b^2*B*x^4 + 4*a*(2*A*b + a*B)*Log
[x])/4
```

Maple [A]
time = 0.09, size = 48, normalized size = 0.94

method	result	size
default	$\frac{b^2 B\,x^4}{4}+\frac{A\,b^2 x^2}{2}+Bab\,x^2-\frac{a^2 A}{2x^2}+a(2Ab+Ba)\ln(x)$	48
norman	$\frac{\left(\frac{1}{2}b^2A+abB\right)x^4-\frac{a^2A}{2}+\frac{b^2B\,x^6}{4}}{x^2}+\left(2abA+a^2B\right)\ln(x)$	51
risch	$\frac{b^2 B\,x^4}{4}+\frac{A\,b^2 x^2}{2}+Bab\,x^2+\frac{A^2b^2}{4B}+abA+a^2B-\frac{a^2 A}{2x^2}+2A\ln(x)\,ab+B\ln(x)\,a^2$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/4*b^2*B*x^4+1/2*A*b^2*x^2+B*a*b*x^2-1/2*a^2*A/x^2+a*(2*A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.29, size = 52, normalized size = 1.02

$$\frac{1}{4}\,Bb^2x^4+\frac{1}{2}\left(2\,Bab+Ab^2\right)x^2+\frac{1}{2}\left(Ba^2+2\,Aab\right)\log\left(x^2\right)-\frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^2*x^4 + 1/2*(2*B*a*b + A*b^2)*x^2 + 1/2*(B*a^2 + 2*A*a*b)*log(x^2)
- 1/2*A*a^2/x^2
```

Fricas [A]
time = 1.04, size = 54, normalized size = 1.06

$$\frac{Bb^2x^6 + 2\,(2\,Bab + Ab^2)x^4 + 4\,(Ba^2 + 2\,Aab)x^2\log\left(x\right) - 2\,Aa^2}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^2*x^6 + 2*(2*B*a*b + A*b^2)*x^4 + 4*(B*a^2 + 2*A*a*b)*x^2*log(x) -
 2*A*a^2)/x^2
```

Sympy [A]
time = 0.10, size = 48, normalized size = 0.94

$$-\frac{Aa^2}{2x^2} + \frac{Bb^2x^4}{4} + a(2Ab + Ba)\log\left(x\right) + x^2\left(\frac{Ab^2}{2} + Bab\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*x**4/4 + a*(2*A*b + B*a)*log(x) + x**2*(A*b**2/2
+ B*a*b)
```

Giac [A]
time = 1.18, size = 70, normalized size = 1.37

$$\frac{1}{4}\,Bb^2x^4 + Babx^2 + \frac{1}{2}\,Ab^2x^2 + \frac{1}{2}\,\left(Ba^2 + 2\,Aab\right)\log\left(x^2\right) - \frac{Ba^2x^2 + 2\,Aabx^2 + Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*B*b^2*x^4 + B*a*b*x^2 + 1/2*A*b^2*x^2 + 1/2*(B*a^2 + 2*A*a*b)*log(x^2)
- 1/2*(B*a^2*x^2 + 2*A*a*b*x^2 + A*a^2)/x^2
```

Mupad [B]
time = 0.02, size = 48, normalized size = 0.94

$$x^2\left(\frac{A\,b^2}{2} + B\,a\,b\right) + \ln\left(x\right)\,\left(B\,a^2 + 2\,A\,b\,a\right) - \frac{A\,a^2}{2\,x^2} + \frac{B\,b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^3,x)
```

```
[Out] x^2*((A*b^2)/2 + B*a*b) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*b
^2*x^4)/4
```

3.17 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^4} \, dx$

Optimal. Leaf size=48

$$-\frac{a^2A}{3x^3}-\frac{a(2Ab+aB)}{x}+b(Ab+2aB)x+\frac{1}{3}b^2Bx^3$$

```
[Out] -1/3*a^2*A/x^3-a*(2*A*b+B*a)/x+b*(A*b+2*B*a)*x+1/3*b^2*B*x^3
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^2A}{3x^3}+bx(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 - (a*(2*A*b + a*B))/x + b*(A*b + 2*a*B)*x + (b^2*B*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^2(A+Bx^2)}{x^4}\,dx &= \int \left(b(Ab+2aB)+\frac{a^2A}{x^4}+\frac{a(2Ab+aB)}{x^2}+b^2Bx^2\right)dx\\ &= -\frac{a^2A}{3x^3}-\frac{a(2Ab+aB)}{x}+b(Ab+2aB)x+\frac{1}{3}b^2Bx^3 \end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.04

$$-\frac{a^2A}{3x^3}+\frac{-2aAb-a^2B}{x}+b(Ab+2aB)x+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 + (-2*a*A*b - a^2*B)/x + b*(A*b + 2*a*B)*x + (b^2*B*x^3)/3
```

Maple [A]
time = 0.06, size = 46, normalized size = 0.96

method	result	size
default	$\frac{b^2 B x^3}{3}+b^2 A x+2 a b B x-\frac{a^2 A}{3 x^3}-\frac{a(2 A b+B a)}{x}$	46
risch	$\frac{b^2 B x^3}{3}+b^2 A x+2 a b B x+\frac{(-2 a b A-a^2 B) x^2-\frac{a^2 A}{3}}{x^3}$	50
norman	$\frac{\frac{b^2 B x^6}{3}+(b^2 A+2 a b B) x^4+(-2 a b A-a^2 B) x^2-\frac{a^2 A}{3}}{x^3}$	52
gosper	$-\frac{-b^2 B x^6-3 A b^2 x^4-6 B a b x^4+6 a A b x^2+3 B a^2 x^2+a^2 A}{3 x^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*B*x^3+b^2*A*x+2*a*b*B*x-1/3*a^2*A/x^3-a*(2*A*b+B*a)/x
```

Maxima [A]
time = 0.28, size = 50, normalized size = 1.04

$$\frac{1}{3} B b^2 x^3+\left(2 B a b+A b^2\right) x-\frac{A a^2+3 (B a^2+2 A a b) x^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*x - 1/3*(A*a^2 + 3*(B*a^2 + 2*A*a*b)*x^2)
/x^3
```

Fricas [A]
time = 0.75, size = 52, normalized size = 1.08

$$\frac{B b^2 x^6+3 (2 B a b+A b^2) x^4-A a^2-3 (B a^2+2 A a b) x^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^6 + 3*(2*B*a*b + A*b^2)*x^4 - A*a^2 - 3*(B*a^2 + 2*A*a*b)*x^2)
/x^3
```

Sympy [A]
time = 0.11, size = 51, normalized size = 1.06

$$\frac{B b^2 x^3}{3}+x\left(A b^2+2 B a b\right)+\frac{-A a^2+x^2(-6 A a b-3 B a^2)}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**4,x)
```

```
[Out] B*b**2*x**3/3 + x*(A*b**2 + 2*B*a*b) + (-A*a**2 + x**2*(-6*A*a*b - 3*B*a**2
))/(3*x**3)
```

Giac [A]
time = 1.03, size = 50, normalized size = 1.04

$$\frac{1}{3}\,Bb^2x^3+2\,Babx+Ab^2x-\frac{3\,Ba^2x^2+6\,Aabx^2+Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*B*b^2*x^3 + 2*B*a*b*x + A*b^2*x - 1/3*(3*B*a^2*x^2 + 6*A*a*b*x^2 + A*a^
2)/x^3
```

Mupad [B]
time = 0.04, size = 50, normalized size = 1.04

$$x\,\left(A\,b^2+2\,B\,a\,b\right)-\frac{x^2\,\left(B\,a^2+2\,A\,b\,a\right)+\frac{A\,a^2}{3}}{x^3}+\frac{B\,b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^4,x)
```

```
[Out] x*(A*b^2 + 2*B*a*b) - (x^2*(B*a^2 + 2*A*a*b) + (A*a^2)/3)/x^3 + (B*b^2*x^3)
/3
```

3.18 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5} \, dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{2x^2}+\frac{1}{2}b^2Bx^2+b(Ab+2aB)\log(x)$$

```
[Out] -1/4*a^2*A/x^4-1/2*a*(2*A*b+B*a)/x^2+1/2*b^2*B*x^2+b*(A*b+2*B*a)*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^2A}{4x^4}-\frac{a(aB+2Ab)}{2x^2}+b\log(x)(2aB+Ab)+\frac{1}{2}b^2Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```

```
[Out] -1/4*(a^2*A)/x^4 - (a*(2*A*b + a*B))/(2*x^2) + (b^2*B*x^2)/2 + b*(A*b + 2*a
*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^5} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^3} \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(b^2B + \frac{a^2A}{x^3} + \frac{a(2Ab+aB)}{x^2} + \frac{b(Ab+2aB)}{x}\right) \, dx, x, x^2\right)$$
$$= -\frac{a^2A}{4x^4} - \frac{a(2Ab+aB)}{2x^2} + \frac{1}{2}b^2Bx^2 + b(Ab+2aB)\log(x)$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.98

$$-\frac{4aAbx^2 - 2b^2Bx^6 + a^2(A+2Bx^2)}{4x^4} + b(Ab+2aB)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
[Out] -1/4*(4*a*A*b*x^2 - 2*b^2*B*x^6 + a^2*(A + 2*B*x^2))/x^4 + b*(A*b + 2*a*B)*
Log[x]
```

Maple [A]
time = 0.06, size = 46, normalized size = 0.90

method	result	size
default	$-\frac{a^2A}{4x^4} - \frac{a(2Ab+Ba)}{2x^2} + \frac{b^2Bx^2}{2} + b(Ab+2Ba)\ln(x)$	46
norman	$\frac{(-abA-\frac{1}{2}a^2B)x^2 - \frac{a^2A}{4} + \frac{b^2Bx^6}{2}}{x^4} + (b^2A+2abB)\ln(x)$	52
risch	$\frac{b^2Bx^2}{2} + \frac{(-abA-\frac{1}{2}a^2B)x^2 - \frac{a^2A}{4}}{x^4} + A\ln(x)b^2 + 2B\ln(x)ab$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a^2*A/x^4-1/2*a*(2*A*b+B*a)/x^2+1/2*b^2*B*x^2+b*(A*b+2*B*a)*ln(x)
```

Maxima [A]
time = 0.28, size = 54, normalized size = 1.06

$$\frac{1}{2}Bb^2x^2 + \frac{1}{2}(2Bab + Ab^2)\log(x^2) - \frac{Aa^2 + 2(Ba^2 + 2Aab)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*log(x^2) - 1/4*(A*a^2 + 2*(B*a^2 + 2*
A*a*b)*x^2)/x^4
```

Fricas [A]
time = 0.64, size = 55, normalized size = 1.08

$$\frac{2\,Bb^{2}x^{6}+4\,(2\,Bab+Ab^{2})x^{4}\log\left(x\right)-Aa^{2}-2\,(Ba^{2}+2\,Aab)x^{2}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="fricas")
[Out] 1/4*(2*B*b^2*x^6 + 4*(2*B*a*b + A*b^2)*x^4*log(x) - A*a^2 - 2*(B*a^2 + 2*A*
a*b)*x^2)/x^4
```

Sympy [A]
time = 0.27, size = 51, normalized size = 1.00

$$\frac{Bb^{2}x^{2}}{2}+b(Ab+2Ba)\log\left(x\right)+\frac{-Aa^{2}+x^{2}(-4Aab-2Ba^{2})}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**5,x)
[Out] B*b**2*x**2/2 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**2*(-4*A*a*b - 2*B*a*
*2))/(4*x**4)
```

Giac [A]
time = 0.62, size = 72, normalized size = 1.41

$$\frac{1}{2}\,Bb^{2}x^{2}+\frac{1}{2}\left(2\,Bab+Ab^{2}\right)\log\left(x^{2}\right)-\frac{6\,Babx^{4}+3\,Ab^{2}x^{4}+2\,Ba^{2}x^{2}+4\,Aabx^{2}+Aa^{2}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="giac")
[Out] 1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*log(x^2) - 1/4*(6*B*a*b*x^4 + 3*A*b^2
*x^4 + 2*B*a^2*x^2 + 4*A*a*b*x^2 + A*a^2)/x^4
```

Mupad [B]
time = 0.05, size = 51, normalized size = 1.00

$$\ln\left(x\right)\,\left(A\,b^{2}+2\,B\,a\,b\right)-\frac{x^{2}\,\left(\frac{B\,a^{2}}{2}+A\,b\,a\right)+\frac{A\,a^{2}}{4}}{x^{4}}+\frac{B\,b^{2}\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^5,x)
[Out] log(x)*(A*b^2 + 2*B*a*b) - (x^2*((B*a^2)/2 + A*a*b) + (A*a^2)/4)/x^4 + (B*b
^2*x^2)/2
```

3.19 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{3x^3}-\frac{b(Ab+2aB)}{x}+b^2Bx$$

```
[Out] -1/5*a^2*A/x^5-1/3*a*(2*A*b+B*a)/x^3-b*(A*b+2*B*a)/x+b^2*B*x
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{3x^3}-\frac{b(2aB+Ab)}{x}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/x + b^2*B*
x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^6}\,dx &= \int \left(b^2B+\frac{a^2A}{x^6}+\frac{a(2Ab+aB)}{x^4}+\frac{b(Ab+2aB)}{x^2}\right)dx\\ &= -\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{3x^3}-\frac{b(Ab+2aB)}{x}+b^2Bx\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.00

$$-\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{3x^3}-\frac{b(Ab+2aB)}{x}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/x + b^2*B*
x
```

Maple [A]
time = 0.06, size = 45, normalized size = 0.94

method	result	size
default	$-\frac{a^{2} A}{5 x^{5}}-\frac{a \left(2 A b +B a \right)}{3 x^{3}}-\frac{b \left(A b +2 B a \right)}{x}+b^{2} B x$	45
risch	$b^{2} B x +\frac{\left(-b^{2} A -2 a b B \right) x^{4}+\left(-\frac{2}{3} a b A -\frac{1}{3} a^{2} B \right) x^{2}-\frac{a^{2} A}{5}}{x^{5}}$	51
norman	$\frac{b^{2} B \,x^{6}+\left(-b^{2} A -2 a b B \right) x^{4}+\left(-\frac{2}{3} a b A -\frac{1}{3} a^{2} B \right) x^{2}-\frac{a^{2} A}{5}}{x^{5}}$	52
gosper	$-\frac{-15 b^{2} B \,x^{6}+15 A \,b^{2} x^{4}+30 B a b \,x^{4}+10 a A b \,x^{2}+5 B \,a^{2} x^{2}+3 a^{2} A}{15 x^{5}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^2*A/x^5-1/3*a*(2*A*b+B*a)/x^3-b*(A*b+2*B*a)/x+b^2*B*x
```

Maxima [A]
time = 0.28, size = 51, normalized size = 1.06

$$B b^{2} x - \frac{15 \, {\left(2 \, B a b + A b^{2}\right)} x^{4} + 3 \, A a^{2} + 5 \, {\left(B a^{2} + 2 \, A a b\right)} x^{2}}{15 \, x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] B*b^2*x - 1/15*(15*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 5*(B*a^2 + 2*A*a*b)*x^
2)/x^5
```

Fricas [A]
time = 0.63, size = 53, normalized size = 1.10

$$\frac{15 \, B b^{2} x^{6} - 15 \, {\left(2 \, B a b + A b^{2}\right)} x^{4} - 3 \, A a^{2} - 5 \, {\left(B a^{2} + 2 \, A a b\right)} x^{2}}{15 \, x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(15*B*b^2*x^6 - 15*(2*B*a*b + A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 + 2*A*a*
b)*x^2)/x^5
```

Sympy [A]
time = 0.31, size = 54, normalized size = 1.12

$$Bb^{2}x + \frac{-3Aa^{2} + x^{4}\left(-15Ab^{2} - 30Bab\right) + x^{2}\left(-10Aab - 5Ba^{2}\right)}{15x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**6,x)
```

```
[Out] B*b**2*x + (-3*A*a**2 + x**4*(-15*A*b**2 - 30*B*a*b) + x**2*(-10*A*a*b - 5*
B*a**2))/(15*x**5)
```

Giac [A]
time = 0.86, size = 53, normalized size = 1.10

$$Bb^2x - \frac{30\,Babx^4 + 15\,Ab^2x^4 + 5\,Ba^2x^2 + 10\,Aabx^2 + 3\,Aa^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] B*b^2*x - 1/15*(30*B*a*b*x^4 + 15*A*b^2*x^4 + 5*B*a^2*x^2 + 10*A*a*b*x^2 +
3*A*a^2)/x^5
```

Mupad [B]
time = 0.03, size = 50, normalized size = 1.04

$$B\,b^2\,x - \frac{x^2\,\left(\frac{B\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + x^4\,\left(A\,b^2 + 2\,B\,a\,b\right) + \frac{A\,a^2}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^6,x)
```

```
[Out] B*b^2*x - (x^2*((B*a^2)/3 + (2*A*a*b)/3) + x^4*(A*b^2 + 2*B*a*b) + (A*a^2)/
5)/x^5
```

3.20 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^7} dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{6x^6}-\frac{a(2Ab+aB)}{4x^4}-\frac{b(Ab+2aB)}{2x^2}+b^2B\log(x)$$

```
[Out] -1/6*a^2*A/x^6-1/4*a*(2*A*b+B*a)/x^4-1/2*b*(A*b+2*B*a)/x^2+b^2*B*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{2x^2}+b^2B\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

```
[Out] -1/6*(a^2*A)/x^6 - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^7} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^4} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2A}{x^4} + \frac{a(2Ab+aB)}{x^3} + \frac{b(Ab+2aB)}{x^2} + \frac{b^2B}{x}\right) dx, x, x^2\right)$$
$$= -\frac{a^2A}{6x^6} - \frac{a(2Ab+aB)}{4x^4} - \frac{b(Ab+2aB)}{2x^2} + b^2B\log(x)$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.06

$$-\frac{6Ab^2x^4 + 6abx^2(A+2Bx^2) + a^2(2A+3Bx^2)}{12x^6} + b^2B\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

```
[Out] -1/12*(6*A*b^2*x^4 + 6*a*b*x^2*(A + 2*B*x^2) + a^2*(2*A + 3*B*x^2))/x^6 + b
^2*B*Log[x]
```

Maple [A]
time = 0.06, size = 46, normalized size = 0.90

method	result	size
default	$-\frac{a^2A}{6x^6} - \frac{a(2Ab+Ba)}{4x^4} - \frac{b(Ab+2Ba)}{2x^2} + b^2B\ln(x)$	46
norman	$\frac{(-\frac{1}{2}b^2A-abB)x^4+(-\frac{1}{2}abA-\frac{1}{4}a^2B)x^2-\frac{a^2A}{6}}{x^6} + b^2B\ln(x)$	52
risch	$\frac{(-\frac{1}{2}b^2A-abB)x^4+(-\frac{1}{2}abA-\frac{1}{4}a^2B)x^2-\frac{a^2A}{6}}{x^6} + b^2B\ln(x)$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2*A/x^6-1/4*a*(2*A*b+B*a)/x^4-1/2*b*(A*b+2*B*a)/x^2+b^2*B*ln(x)
```

Maxima [A]
time = 0.28, size = 55, normalized size = 1.08

$$\frac{1}{2}Bb^2\log\left(x^2\right) - \frac{6\,(2\,Bab + Ab^2)x^4 + 2\,Aa^2 + 3\,(Ba^2 + 2\,Aab)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*log(x^2) - 1/12*(6*(2*B*a*b + A*b^2)*x^4 + 2*A*a^2 + 3*(B*a^2 + 2
*A*a*b)*x^2)/x^6
```

Fricas [A]
time = 0.89, size = 55, normalized size = 1.08

$$\frac{12\,Bb^2x^6\log\left(x\right)-6\,(2\,Bab+Ab^2)x^4-2\,Aa^2-3\,(Ba^2+2\,Aab)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*B*b^2*x^6*log(x) - 6*(2*B*a*b + A*b^2)*x^4 - 2*A*a^2 - 3*(B*a^2 +
2*A*a*b)*x^2)/x^6
```

Sympy [A]
time = 0.52, size = 56, normalized size = 1.10

$$Bb^2\log\left(x\right)+\frac{-2Aa^2+x^4(-6Ab^2-12Bab)+x^2(-6Aab-3Ba^2)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**7,x)
```

```
[Out] B*b**2*log(x) + (-2*A*a**2 + x**4*(-6*A*b**2 - 12*B*a*b) + x**2*(-6*A*a*b -
 3*B*a**2))/(12*x**6)
```

Giac [A]
time = 1.02, size = 66, normalized size = 1.29

$$\frac{1}{2}\,Bb^2\log\left(x^2\right)-\frac{11\,Bb^2x^6+12\,Babx^4+6\,Ab^2x^4+3\,Ba^2x^2+6\,Aabx^2+2\,Aa^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*log(x^2) - 1/12*(11*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x^4 + 3*B*
a^2*x^2 + 6*A*a*b*x^2 + 2*A*a^2)/x^6
```

Mupad [B]
time = 0.05, size = 51, normalized size = 1.00

$$B\,b^2\,\ln\left(x\right)-\frac{x^2\,\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^4\,\left(\frac{A\,b^2}{2}+B\,a\,b\right)+\frac{A\,a^2}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^7,x)
```

```
[Out] B*b^2*log(x) - (x^2*((B*a^2)/4 + (A*a*b)/2) + x^4*((A*b^2)/2 + B*a*b) + (A*
a^2)/6)/x^6
```

3.21 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^8} dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{3x^3}-\frac{b^2B}{x}$$

```
[Out] -1/7*a^2*A/x^7-1/5*a*(2*A*b+B*a)/x^5-1/3*b*(A*b+2*B*a)/x^3-b^2*B/x
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{3x^3}-\frac{b^2B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

```
[Out] -1/7*(a^2*A)/x^7 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(3*x^3) - (b^2*B)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^8}dx &= \int \left(\frac{a^2A}{x^8}+\frac{a(2Ab+aB)}{x^6}+\frac{b(Ab+2aB)}{x^4}+\frac{b^2B}{x^2}\right)dx\\ &= -\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{3x^3}-\frac{b^2B}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.06

$$-\frac{35b^2x^4(A+3Bx^2)+14abx^2(3A+5Bx^2)+3a^2(5A+7Bx^2)}{105x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

```
[Out] -1/105*(35*b^2*x^4*(A + 3*B*x^2) + 14*a*b*x^2*(3*A + 5*B*x^2) + 3*a^2*(5*A
+ 7*B*x^2))/x^7
```

Maple [A]
time = 0.06, size = 48, normalized size = 0.91

method	result	size
default	$-\frac{a^2 A}{7x^7}-\frac{a(2Ab+Ba)}{5x^5}-\frac{b(Ab+2Ba)}{3x^3}-\frac{b^2 B}{x}$	48
norman	$\frac{-b^2 B\,x^6+\left(-\frac{1}{3}b^2 A-\frac{2}{3}abB\right)x^4+\left(-\frac{2}{5}abA-\frac{1}{5}a^2 B\right)x^2-\frac{a^2 A}{7}}{x^7}$	53
risch	$\frac{-b^2 B\,x^6+\left(-\frac{1}{3}b^2 A-\frac{2}{3}abB\right)x^4+\left(-\frac{2}{5}abA-\frac{1}{5}a^2 B\right)x^2-\frac{a^2 A}{7}}{x^7}$	53
gosper	$-\frac{105b^2 B\,x^6+35A\,b^2 x^4+70Bab\,x^4+42aAb\,x^2+21B\,a^2 x^2+15a^2 A}{105x^7}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^2*A/x^7-1/5*a*(2*A*b+B*a)/x^5-1/3*b*(A*b+2*B*a)/x^3-b^2*B/x
```

Maxima [A]
time = 0.28, size = 53, normalized size = 1.00

$$-\frac{105\,Bb^2x^6+35\,(2\,Bab+Ab^2)x^4+15\,Aa^2+21\,(Ba^2+2\,Aab)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*(B*a^2 + 2
*A*a*b)*x^2)/x^7
```

Fricas [A]
time = 0.72, size = 53, normalized size = 1.00

$$-\frac{105\,Bb^2x^6+35\,(2\,Bab+Ab^2)x^4+15\,Aa^2+21\,(Ba^2+2\,Aab)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*(B*a^2 + 2
*A*a*b)*x^2)/x^7
```

Sympy [A]
time = 0.57, size = 58, normalized size = 1.09

$$\frac{-15Aa^2-105Bb^2x^6+x^4(-35Ab^2-70Bab)+x^2(-42Aab-21Ba^2)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**8,x)
```

```
[Out] (-15*A*a**2 - 105*B*b**2*x**6 + x**4*(-35*A*b**2 - 70*B*a*b) + x**2*(-42*A*
a*b - 21*B*a**2))/(105*x**7)
```

Giac [A]
time = 1.21, size = 55, normalized size = 1.04

$$-\frac{105\,Bb^2x^6 + 70\,Babx^4 + 35\,Ab^2x^4 + 21\,Ba^2x^2 + 42\,Aabx^2 + 15\,Aa^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 70*B*a*b*x^4 + 35*A*b^2*x^4 + 21*B*a^2*x^2 + 42*A*a
*b*x^2 + 15*A*a^2)/x^7
```

Mupad [B]
time = 0.02, size = 52, normalized size = 0.98

$$-\frac{x^2\,\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^4\,\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+\frac{A\,a^2}{7}+B\,b^2\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^8,x)
```

```
[Out] -(x^2*((B*a^2)/5 + (2*A*a*b)/5) + x^4*((A*b^2)/3 + (2*B*a*b)/3) + (A*a^2)/7
 + B*b^2*x^6)/x^7
```

3.22 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=48

$$-\frac{A(a+bx^2)^3}{8ax^8}+\frac{(Ab-4aB)\,(a+bx^2)^3}{24a^2x^6}$$

```
[Out] -1/8*A*(b*x^2+a)^3/a/x^8+1/24*(A*b-4*B*a)*(b*x^2+a)^3/a^2/x^6
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 79, 37}

$$\frac{(a+bx^2)^3\,(Ab-4aB)}{24a^2x^6}-\frac{A(a+bx^2)^3}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

```
[Out] -1/8*(A*(a + b*x^2)^3)/(a*x^8) + ((A*b - 4*a*B)*(a + b*x^2)^3)/(24*a^2*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx,x,x^2\right)\\ &= -\frac{A(a+bx^2)^3}{8ax^8}+\frac{(-Ab+4aB)\text{Subst}\left(\int \frac{(a+bx)^2}{x^4}\,dx,x,x^2\right)}{8a}\\ &= -\frac{A(a+bx^2)^3}{8ax^8}+\frac{(Ab-4aB)(a+bx^2)^3}{24a^2x^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.15

$$-\frac{6b^2x^4(A+2Bx^2)+4abx^2(2A+3Bx^2)+a^2(3A+4Bx^2)}{24x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

```
[Out] -1/24*(6*b^2*x^4*(A + 2*B*x^2) + 4*a*b*x^2*(2*A + 3*B*x^2) + a^2*(3*A + 4*B
*x^2))/x^8
```

Maple [A]
time = 0.06, size = 48, normalized size = 1.00

method	result	size
default	$-\frac{b(Ab+2Ba)}{4x^4}-\frac{a(2Ab+Ba)}{6x^6}-\frac{b^2B}{2x^2}-\frac{a^2A}{8x^8}$	48
norman	$\frac{-\frac{b^2Bx^6}{2}+(-\frac{1}{4}b^2A-\frac{1}{2}abB)x^4+(-\frac{1}{3}abA-\frac{1}{6}a^2B)x^2-\frac{a^2A}{8}}{x^8}$	53
risch	$\frac{-\frac{b^2Bx^6}{2}+(-\frac{1}{4}b^2A-\frac{1}{2}abB)x^4+(-\frac{1}{3}abA-\frac{1}{6}a^2B)x^2-\frac{a^2A}{8}}{x^8}$	53
gosper	$-\frac{12b^2Bx^6+6Ab^2x^4+12Babx^4+8aAbx^2+4Ba^2x^2+3a^2A}{24x^8}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b*(A*b+2*B*a)/x^4-1/6*a*(2*A*b+B*a)/x^6-1/2*b^2*B/x^2-1/8*a^2*A/x^8
```

Maxima [A]
time = 0.30, size = 53, normalized size = 1.10

$$-\frac{12\,Bb^2x^6+6\,(2\,Bab+Ab^2)x^4+3\,Aa^2+4\,(Ba^2+2\,Aab)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^2 + 2*A*a*
b)*x^2)/x^8
```

Fricas [A]
time = 0.92, size = 53, normalized size = 1.10

$$-\frac{12\,Bb^2x^6+6\,(2\,Bab+Ab^2)x^4+3\,Aa^2+4\,(Ba^2+2\,Aab)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^2 + 2*A*a*
b)*x^2)/x^8
```

Sympy [A]
time = 0.86, size = 58, normalized size = 1.21

$$\frac{-3Aa^2-12Bb^2x^6+x^4(-6Ab^2-12Bab)+x^2(-8Aab-4Ba^2)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**9,x)
```

```
[Out] (-3*A*a**2 - 12*B*b**2*x**6 + x**4*(-6*A*b**2 - 12*B*a*b) + x**2*(-8*A*a*b
- 4*B*a**2))/(24*x**8)
```

Giac [A]
time = 1.13, size = 55, normalized size = 1.15

$$-\frac{12\,Bb^2x^6+12\,Babx^4+6\,Ab^2x^4+4\,Ba^2x^2+8\,Aabx^2+3\,Aa^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x^4 + 4*B*a^2*x^2 + 8*A*a*b*x^
2 + 3*A*a^2)/x^8
```

Mupad [B]
time = 0.03, size = 53, normalized size = 1.10

$$-\frac{x^2\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+x^4\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}\right)+\frac{A\,a^2}{8}+\frac{B\,b^2\,x^6}{2}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^9,x)
```

```
[Out] -(x^2*((B*a^2)/6 + (A*a*b)/3) + x^4*((A*b^2)/4 + (B*a*b)/2) + (A*a^2)/8 + (
B*b^2*x^6)/2)/x^8
```

3.23 $\int x^9(a+bx^2)^5\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=117

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4(5Ab+aB)x^{12}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}+\frac{5}{18}ab^3(Ab+2aB)x^{18}+\frac{1}{20}b^4(Ab+5$$

```
[Out] 1/10*a^5*A*x^10+1/12*a^4*(5*A*b+B*a)*x^12+5/14*a^3*b*(2*A*b+B*a)*x^14+5/8*a
^2*b^2*(A*b+B*a)*x^16+5/18*a*b^3*(A*b+2*B*a)*x^18+1/20*b^4*(A*b+5*B*a)*x^20
+1/22*b^5*B*x^22
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b + a*B)*x^14
)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^18)/18 + (
b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^9 \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx = \frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^5(A+Bx)\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(a^5Ax^4 + a^4(5Ab+aB)x^5 + 5a^3b(2Ab+aB)x^6 + 10a^2b^2(Ab+$$
$$= \frac{1}{10}a^5Ax^{10} + \frac{1}{12}a^4(5Ab+aB)x^{12} + \frac{5}{14}a^3b(2Ab+aB)x^{14} + \frac{5}{8}a^2b^2(Ab+aB$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{10}a^5Ax^{10} + \frac{1}{12}a^4(5Ab+aB)x^{12} + \frac{5}{14}a^3b(2Ab+aB)x^{14} + \frac{5}{8}a^2b^2(Ab+aB)x^{16} + \frac{5}{18}ab^3(Ab+2aB)x^{18} + \frac{1}{20}b^4(Ab+5aB)x^{20} + \frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b + a*B)*x^14
)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^18)/18 + (
b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

Maple [A]
time = 0.10, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5Ax^{10}}{10} + \left(\frac{5}{12}a^4bA + \frac{1}{12}a^5B\right)x^{12} + \left(\frac{5}{7}a^3b^2A + \frac{5}{14}a^4bB\right)x^{14} + \left(\frac{5}{8}a^2b^3A + \frac{5}{8}a^3b^2B\right)x^{16} + \left(\frac{5}{18}a\,b^4A+\right.$
default	$\frac{b^5Bx^{22}}{22} + \frac{(b^5A+5a\,b^4B)x^{20}}{20} + \frac{(5a\,b^4A+10a^2b^3B)x^{18}}{18} + \frac{(10a^2b^3A+10a^3b^2B)x^{16}}{16} + \frac{(10a^3b^2A+5a^4bB)x^{14}}{14} + \frac{(5a^4bA+a^5B}{12}$
gosper	$\frac{1}{10}a^5Ax^{10} + \frac{5}{12}x^{12}a^4bA + \frac{1}{12}x^{12}a^5B + \frac{5}{7}x^{14}a^3b^2A + \frac{5}{14}x^{14}a^4bB + \frac{5}{8}x^{16}a^2b^3A + \frac{5}{8}x^{16}a^3b^2B + \frac{5}{18}x^{18}$
risch	$\frac{1}{10}a^5Ax^{10} + \frac{5}{12}x^{12}a^4bA + \frac{1}{12}x^{12}a^5B + \frac{5}{7}x^{14}a^3b^2A + \frac{5}{14}x^{14}a^4bB + \frac{5}{8}x^{16}a^2b^3A + \frac{5}{8}x^{16}a^3b^2B + \frac{5}{18}x^{18}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/22*b^5*B*x^22+1/20*(A*b^5+5*B*a*b^4)*x^20+1/18*(5*A*a*b^4+10*B*a^2*b^3)*x
^18+1/16*(10*A*a^2*b^3+10*B*a^3*b^2)*x^16+1/14*(10*A*a^3*b^2+5*B*a^4*b)*x^1
4+1/12*(5*A*a^4*b+B*a^5)*x^12+1/10*a^5*A*x^10
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.02

$$\frac{1}{22}Bb^5x^{22} + \frac{1}{20}\left(5\,Bab^4 + Ab^5\right)x^{20} + \frac{5}{18}\left(2\,Ba^2b^3 + Aab^4\right)x^{18} + \frac{5}{8}\left(Ba^3b^2 + Aa^2b^3\right)x^{16} + \frac{1}{10}Aa^5x^{10} + \frac{5}{14}\left(Ba^4b + 2\,Aa^3b^2\right)x^{14} + \frac{1}{12}\left(Ba^5 + 5\,Aa^4b\right)x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/22*B*b^5*x^22 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 1/10*A*a^5*x^10 + 5/14*(B*a^4
*b + 2*A*a^3*b^2)*x^14 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Fricas [A]
time = 0.88, size = 119, normalized size = 1.02

$$\frac{1}{22}\,Bb^{5}x^{22}+\frac{1}{20}\,(5\,Bab^{4}+Ab^{5})x^{20}+\frac{5}{18}\,(2\,Ba^{2}b^{3}+Aab^{4})x^{18}+\frac{5}{8}\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{16}+\frac{1}{10}\,Aa^{5}x^{10}+\frac{5}{14}\,(Ba^{4}b+2\,Aa^{3}b^{2})x^{14}+\frac{1}{12}\,(Ba^{5}+5\,Aa^{4}b)x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/22*B*b^5*x^22 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 1/10*A*a^5*x^10 + 5/14*(B*a^4
*b + 2*A*a^3*b^2)*x^14 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.16

$$\frac{Aa^{5}x^{10}}{10}+\frac{Bb^{5}x^{22}}{22}+x^{20}\left(\frac{Ab^{5}}{20}+\frac{Bab^{4}}{4}\right)+x^{18}\cdot\left(\frac{5Aab^{4}}{18}+\frac{5Ba^{2}b^{3}}{9}\right)+x^{16}\cdot\left(\frac{5Aa^{2}b^{3}}{8}+\frac{5Ba^{3}b^{2}}{8}\right)+x^{14}\cdot\left(\frac{5Aa^{3}b^{2}}{7}+\frac{5Ba^{4}b}{14}\right)+x^{12}\cdot\left(\frac{5Aa^{4}b}{12}+\frac{Ba^{5}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**22/22 + x**20*(A*b**5/20 + B*a*b**4/4) + x**18*
(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*b**2/
8) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**12*(5*A*a**4*b/12 + B*a**
5/12)
```

Giac [A]
time = 0.66, size = 125, normalized size = 1.07

$$\frac{1}{22}\,Bb^{5}x^{22}+\frac{1}{4}\,Bab^{4}x^{20}+\frac{1}{20}\,Ab^{5}x^{20}+\frac{5}{9}\,Ba^{2}b^{3}x^{18}+\frac{5}{18}\,Aab^{4}x^{18}+\frac{5}{8}\,Ba^{3}b^{2}x^{16}+\frac{5}{8}\,Aa^{2}b^{3}x^{16}+\frac{5}{14}\,Ba^{4}bx^{14}+\frac{5}{7}\,Aa^{3}b^{2}x^{14}+\frac{1}{12}\,Ba^{5}x^{12}+\frac{5}{12}\,Aa^{4}bx^{12}+\frac{1}{10}\,Aa^{5}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/22*B*b^5*x^22 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 5/9*B*a^2*b^3*x^18 +
 5/18*A*a*b^4*x^18 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/14*B*a^4*b
*x^14 + 5/7*A*a^3*b^2*x^14 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/10*A*a
^5*x^10
```

Mupad [B]
time = 0.09, size = 107, normalized size = 0.91

$$x^{12}\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{20}\left(\frac{A\,b^5}{20}+\frac{B\,a\,b^4}{4}\right)+\frac{A\,a^5\,x^{10}}{10}+\frac{B\,b^5\,x^{22}}{22}+\frac{5\,a^2\,b^2\,x^{16}\,(A\,b+B\,a)}{8}+\frac{5\,a^3\,b\,x^{14}\,(2\,A\,b+B\,a)}{14}+\frac{5\,a\,b^3\,x^{18}\,(A\,b+2\,B\,a)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^12*((B*a^5)/12 + (5*A*a^4*b)/12) + x^20*((A*b^5)/20 + (B*a*b^4)/4) + (A*a
^5*x^10)/10 + (B*b^5*x^22)/22 + (5*a^2*b^2*x^16*(A*b + B*a))/8 + (5*a^3*b*x
^14*(2*A*b + B*a))/14 + (5*a*b^3*x^18*(A*b + 2*B*a))/18
```

3.24 $\int x^8(a+bx^2)^5\left(A+Bx^2\right)dx$

Optimal. Leaf size=117

$$\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{2}{3}a^2b^2(Ab+aB)x^{15}+\frac{5}{17}ab^3(Ab+2aB)x^{17}+\frac{1}{19}b^4(Ab+5a$$

```
[Out] 1/9*a^5*A*x^9+1/11*a^4*(5*A*b+B*a)*x^11+5/13*a^3*b*(2*A*b+B*a)*x^13+2/3*a^2
*b^2*(A*b+B*a)*x^15+5/17*a*b^3*(A*b+2*B*a)*x^17+1/19*b^4*(A*b+5*B*a)*x^19+1
/21*b^5*B*x^21
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{21}b^5Bx^{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^2\right)^5\left(A+Bx^2\right)dx &= \int \left(a^5Ax^8+a^4(5Ab+aB)x^{10}+5a^3b(2Ab+aB)x^{12}+10a^2b^2(Ab+aB)x\right.\\ &= \frac{1}{9}a^5Ax^9+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{2}{3}a^2b^2(Ab+aB)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{2}{3}a^2b^2(Ab+aB)x^{15}+\frac{5}{17}ab^3(Ab+2aB)x^{17}+\frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{21}b^5Bx^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

Maple [A]
time = 0.10, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A x^9}{9} + \left(\frac{5}{11} a^4 b A + \frac{1}{11} a^5 B\right) x^{11} + \left(\frac{10}{13} a^3 b^2 A + \frac{5}{13} a^4 b B\right) x^{13} + \left(\frac{2}{3} a^2 b^3 A + \frac{2}{3} a^3 b^2 B\right) x^{15} + \left(\frac{5}{17} a\, b^4 A +\right.$
default	$\frac{b^5 B x^{21}}{21} + \frac{(b^5 A + 5 a\, b^4 B) x^{19}}{19} + \frac{(5 a\, b^4 A + 10 a^2 b^3 B) x^{17}}{17} + \frac{(10 a^2 b^3 A + 10 a^3 b^2 B) x^{15}}{15} + \frac{(10 a^3 b^2 A + 5 a^4 b B) x^{13}}{13} + \frac{(5 a^4 b A + a^5 B}{11}$
gosper	$\frac{1}{9} a^5 A x^9 + \frac{5}{11} x^{11} a^4 b A + \frac{1}{11} x^{11} a^5 B + \frac{10}{13} x^{13} a^3 b^2 A + \frac{5}{13} x^{13} a^4 b B + \frac{2}{3} x^{15} a^2 b^3 A + \frac{2}{3} x^{15} a^3 b^2 B + \frac{5}{17} x^{17} a$
risch	$\frac{1}{9} a^5 A x^9 + \frac{5}{11} x^{11} a^4 b A + \frac{1}{11} x^{11} a^5 B + \frac{10}{13} x^{13} a^3 b^2 A + \frac{5}{13} x^{13} a^4 b B + \frac{2}{3} x^{15} a^2 b^3 A + \frac{2}{3} x^{15} a^3 b^2 B + \frac{5}{17} x^{17} a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*b^5*B*x^21+1/19*(A*b^5+5*B*a*b^4)*x^19+1/17*(5*A*a*b^4+10*B*a^2*b^3)*x
^17+1/15*(10*A*a^2*b^3+10*B*a^3*b^2)*x^15+1/13*(10*A*a^3*b^2+5*B*a^4*b)*x^1
3+1/11*(5*A*a^4*b+B*a^5)*x^11+1/9*a^5*A*x^9
```

Maxima [A]
time = 0.30, size = 119, normalized size = 1.02

$$\frac{1}{21} B b^5 x^{21} + \frac{1}{19} (5 B a b^4 + A b^5) x^{19} + \frac{5}{17} (2 B a^2 b^3 + A a b^4) x^{17} + \frac{2}{3} (B a^3 b^2 + A a^2 b^3) x^{15} + \frac{1}{9} A a^5 x^9 + \frac{5}{13} (B a^4 b + 2 A a^3 b^2) x^{13} + \frac{1}{11} (B a^5 + 5 A a^4 b) x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/21*B*b^5*x^21 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 1/9*A*a^5*x^9 + 5/13*(B*a^4*b
 + 2*A*a^3*b^2)*x^13 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Fricas [A]
time = 0.82, size = 119, normalized size = 1.02

$$\frac{1}{21} B b^5 x^{21} + \frac{1}{19} (5 B a b^4 + A b^5) x^{19} + \frac{5}{17} (2 B a^2 b^3 + A a b^4) x^{17} + \frac{2}{3} (B a^3 b^2 + A a^2 b^3) x^{15} + \frac{1}{9} A a^5 x^9 + \frac{5}{13} (B a^4 b + 2 A a^3 b^2) x^{13} + \frac{1}{11} (B a^5 + 5 A a^4 b) x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/21*B*b^5*x^21 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 1/9*A*a^5*x^9 + 5/13*(B*a^4*b
 + 2*A*a^3*b^2)*x^13 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Sympy [A]
time = 0.02, size = 138, normalized size = 1.18

$$\frac{Aa^5x^9}{9}+\frac{Bb^5x^{21}}{21}+x^{19}\left(\frac{Ab^5}{19}+\frac{5Bab^4}{19}\right)+x^{17}\cdot\left(\frac{5Aab^4}{17}+\frac{10Ba^2b^3}{17}\right)+x^{15}\cdot\left(\frac{2Aa^2b^3}{3}+\frac{2Ba^3b^2}{3}\right)+x^{13}\cdot\left(\frac{10Aa^3b^2}{13}+\frac{5Ba^4b}{13}\right)+x^{11}\cdot\left(\frac{5Aa^4b}{11}+\frac{Ba^5}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**21/21 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**17
*(5*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b*
*2/3) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**11*(5*A*a**4*b/11 +
B*a**5/11)
```

Giac [A]
time = 0.52, size = 125, normalized size = 1.07

$$\frac{1}{21}Bb^5x^{21}+\frac{5}{19}Bab^4x^{19}+\frac{1}{19}Ab^5x^{19}+\frac{10}{17}Ba^2b^3x^{17}+\frac{5}{17}Aab^4x^{17}+\frac{2}{3}Ba^3b^2x^{15}+\frac{2}{3}Aa^2b^3x^{15}+\frac{5}{13}Ba^4bx^{13}+\frac{10}{13}Aa^3b^2x^{13}+\frac{1}{11}Ba^5x^{11}+\frac{5}{11}Aa^4bx^{11}+\frac{1}{9}Aa^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/21*B*b^5*x^21 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 10/17*B*a^2*b^3*x^1
7 + 5/17*A*a*b^4*x^17 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/13*B*a^
4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1/9
*A*a^5*x^9
```

Mupad [B]
time = 0.02, size = 107, normalized size = 0.91

$$x^{11}\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{19}\left(\frac{A\,b^5}{19}+\frac{5\,B\,a\,b^4}{19}\right)+\frac{A\,a^5\,x^9}{9}+\frac{B\,b^5\,x^{21}}{21}+\frac{2\,a^2\,b^2\,x^{15}\,(A\,b+B\,a)}{3}+\frac{5\,a^3\,b\,x^{13}\,(2\,A\,b+B\,a)}{13}+\frac{5\,a\,b^3\,x^{17}\,(A\,b+2\,B\,a)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^11*((B*a^5)/11 + (5*A*a^4*b)/11) + x^19*((A*b^5)/19 + (5*B*a*b^4)/19) + (
A*a^5*x^9)/9 + (B*b^5*x^21)/21 + (2*a^2*b^2*x^15*(A*b + B*a))/3 + (5*a^3*b*
x^13*(2*A*b + B*a))/13 + (5*a*b^3*x^17*(A*b + 2*B*a))/17
```

3.25 $\int x^7(a+bx^2)^5 (A+Bx^2)\, dx$

Optimal. Leaf size=122

$$-\frac{a^3(Ab-aB)(a+bx^2)^6}{12b^5}+\frac{a^2(3Ab-4aB)(a+bx^2)^7}{14b^5}-\frac{3a(Ab-2aB)(a+bx^2)^8}{16b^5}+\frac{(Ab-4aB)(a+bx^2)^9}{18b^5}+$$

```
[Out] -1/12*a^3*(A*b-B*a)*(b*x^2+a)^6/b^5+1/14*a^2*(3*A*b-4*B*a)*(b*x^2+a)^7/b^5-
3/16*a*(A*b-2*B*a)*(b*x^2+a)^8/b^5+1/18*(A*b-4*B*a)*(b*x^2+a)^9/b^5+1/20*B*
(b*x^2+a)^10/b^5
```

Rubi [A]
time = 0.19, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^3(a+bx^2)^6(Ab-aB)}{12b^5}+\frac{a^2(a+bx^2)^7(3Ab-4aB)}{14b^5}+\frac{(a+bx^2)^9(Ab-4aB)}{18b^5}-\frac{3a(a+bx^2)^8(Ab-2aB)}{16b^5}+\frac{B(a+bx^2)^{10}}{20b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] -1/12*(a^3*(A*b - a*B)*(a + b*x^2)^6)/b^5 + (a^2*(3*A*b - 4*a*B)*(a + b*x^2
)^7)/(14*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x^2)^8)/(16*b^5) + ((A*b - 4*a*B)
*(a + b*x^2)^9)/(18*b^5) + (B*(a + b*x^2)^10)/(20*b^5)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^7 \left(a+bx^2\right)^5 \left(A+Bx^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^5(A+Bx)\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3(-Ab+aB)(a+bx)^5}{b^4} - \frac{a^2(-3Ab+4aB)(a+bx)^6}{b^4} + \frac{3a}{}\right.\right.$$
$$= -\frac{a^3(Ab-aB)\left(a+bx^2\right)^6}{12b^5} + \frac{a^2(3Ab-4aB)\left(a+bx^2\right)^7}{14b^5} - \frac{3a(Ab-2aB)\,(}{16b^5}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 0.96

$$\frac{1}{8}a^5Ax^8 + \frac{1}{10}a^4(5Ab+aB)x^{10} + \frac{5}{12}a^3b(2Ab+aB)x^{12} + \frac{5}{7}a^2b^2(Ab+aB)x^{14} + \frac{5}{16}ab^3(Ab+2aB)x^{16} + \frac{1}{18}b^4(Ab+5aB)x^{18} + \frac{1}{20}b^5Bx^{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^12)/
12 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^
4*(A*b + 5*a*B)*x^18)/18 + (b^5*B*x^20)/20
```

Maple [A]
time = 0.10, size = 124, normalized size = 1.02

method	result
norman	$\frac{a^5Ax^8}{8} + \left(\frac{1}{2}a^4bA + \frac{1}{10}a^5B\right)x^{10} + \left(\frac{5}{6}a^3b^2A + \frac{5}{12}a^4bB\right)x^{12} + \left(\frac{5}{7}a^2b^3A + \frac{5}{7}a^3b^2B\right)x^{14} + \left(\frac{5}{16}a\,b^4A +\right.$
default	$\frac{b^5Bx^{20}}{20} + \frac{(b^5A+5a\,b^4B)x^{18}}{18} + \frac{(5a\,b^4A+10a^2b^3B)x^{16}}{16} + \frac{(10a^2b^3A+10a^3b^2B)x^{14}}{14} + \frac{(10a^3b^2A+5a^4bB)x^{12}}{12} + \frac{(5a^4bA+a^5}{10}$
gosper	$\frac{1}{8}a^5Ax^8 + \frac{1}{2}x^{10}a^4bA + \frac{1}{10}x^{10}a^5B + \frac{5}{6}x^{12}a^3b^2A + \frac{5}{12}x^{12}a^4bB + \frac{5}{7}x^{14}a^2b^3A + \frac{5}{7}x^{14}a^3b^2B + \frac{5}{16}x^{16}a$
risch	$\frac{1}{8}a^5Ax^8 + \frac{1}{2}x^{10}a^4bA + \frac{1}{10}x^{10}a^5B + \frac{5}{6}x^{12}a^3b^2A + \frac{5}{12}x^{12}a^4bB + \frac{5}{7}x^{14}a^2b^3A + \frac{5}{7}x^{14}a^3b^2B + \frac{5}{16}x^{16}a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*b^5*B*x^20+1/18*(A*b^5+5*B*a*b^4)*x^18+1/16*(5*A*a*b^4+10*B*a^2*b^3)*x
^16+1/14*(10*A*a^2*b^3+10*B*a^3*b^2)*x^14+1/12*(10*A*a^3*b^2+5*B*a^4*b)*x^1
2+1/10*(5*A*a^4*b+B*a^5)*x^10+1/8*a^5*A*x^8
```

Maxima [A]
time = 0.30, size = 119, normalized size = 0.98

$$\frac{1}{20}Bb^5x^{20} + \frac{1}{18}\left(5Bab^4 + Ab^5\right)x^{18} + \frac{5}{16}\left(2Ba^2b^3 + Aab^4\right)x^{16} + \frac{5}{7}\left(Ba^3b^2 + Aa^2b^3\right)x^{14} + \frac{1}{8}Aa^5x^8 + \frac{5}{12}\left(Ba^4b + 2Aa^3b^2\right)x^{12} + \frac{1}{10}\left(Ba^5 + 5Aa^4b\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/20*B*b^5*x^20 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 1/8*A*a^5*x^8 + 5/12*(B*a^4*b
 + 2*A*a^3*b^2)*x^12 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Fricas [A]
time = 0.76, size = 119, normalized size = 0.98

$$\frac{1}{20}Bb^5x^{20}+\frac{1}{18}\left(5\,Bab^4+Ab^5\right)x^{18}+\frac{5}{16}\left(2\,Ba^2b^3+Aab^4\right)x^{16}+\frac{5}{7}\left(Ba^3b^2+Aa^2b^3\right)x^{14}+\frac{1}{8}Aa^5x^8+\frac{5}{12}\left(Ba^4b+2\,Aa^3b^2\right)x^{12}+\frac{1}{10}\left(Ba^5+5\,Aa^4b\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/20*B*b^5*x^20 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 1/8*A*a^5*x^8 + 5/12*(B*a^4*b
 + 2*A*a^3*b^2)*x^12 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.11

$$\frac{Aa^5x^8}{8}+\frac{Bb^5x^{20}}{20}+x^{18}\left(\frac{Ab^5}{18}+\frac{5Bab^4}{18}\right)+x^{16}\cdot\left(\frac{5Aab^4}{16}+\frac{5Ba^2b^3}{8}\right)+x^{14}\cdot\left(\frac{5Aa^2b^3}{7}+\frac{5Ba^3b^2}{7}\right)+x^{12}\cdot\left(\frac{5Aa^3b^2}{6}+\frac{5Ba^4b}{12}\right)+x^{10}\left(\frac{Aa^4b}{2}+\frac{Ba^5}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**20/20 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**16
*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**2
/7) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**10*(A*a**4*b/2 + B*a**5/
10)
```

Giac [A]
time = 0.47, size = 125, normalized size = 1.02

$$\frac{1}{20}Bb^5x^{20}+\frac{5}{18}Bab^4x^{18}+\frac{1}{18}Ab^5x^{18}+\frac{5}{8}Ba^2b^3x^{16}+\frac{5}{16}Aab^4x^{16}+\frac{5}{7}Ba^3b^2x^{14}+\frac{5}{7}Aa^2b^3x^{14}+\frac{5}{12}Ba^4bx^{12}+\frac{5}{6}Aa^3b^2x^{12}+\frac{1}{10}Ba^5x^{10}+\frac{1}{2}Aa^4bx^{10}+\frac{1}{8}Aa^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/20*B*b^5*x^20 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 5/8*B*a^2*b^3*x^16
+ 5/16*A*a*b^4*x^16 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/12*B*a^4*
b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1/8*A*a^
5*x^8
```

Mupad [B]
time = 0.02, size = 107, normalized size = 0.88

$$x^{10}\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^{18}\left(\frac{A\,b^5}{18}+\frac{5\,B\,a\,b^4}{18}\right)+\frac{A\,a^5\,x^8}{8}+\frac{B\,b^5\,x^{20}}{20}+\frac{5\,a^2\,b^2\,x^{14}\,\left(A\,b+B\,a\right)}{7}+\frac{5\,a^3\,b\,x^{12}\,\left(2\,A\,b+B\,a\right)}{12}+\frac{5\,a\,b^3\,x^{16}\,\left(A\,b+2\,B\,a\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^10*((B*a^5)/10 + (A*a^4*b)/2) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*a
^5*x^8)/8 + (B*b^5*x^20)/20 + (5*a^2*b^2*x^14*(A*b + B*a))/7 + (5*a^3*b*x^1
2*(2*A*b + B*a))/12 + (5*a*b^3*x^16*(A*b + 2*B*a))/16
```

3.26 $\int x^6(a+bx^2)^5(A+Bx^2)\,dx$

Optimal. Leaf size=117

$$\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4(5Ab+aB)x^9+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{1}{3}ab^3(Ab+2aB)x^{15}+\frac{1}{17}b^4(Ab+5aB)x$$

```
[Out] 1/7*a^5*A*x^7+1/9*a^4*(5*A*b+B*a)*x^9+5/11*a^3*b*(2*A*b+B*a)*x^11+10/13*a^2
*b^2*(A*b+B*a)*x^13+1/3*a*b^3*(A*b+2*B*a)*x^15+1/17*b^4*(A*b+5*B*a)*x^17+1/
19*b^5*B*x^19
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a*B)*x^15)/3 + (b^4*(
A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx &= \int \left(a^5Ax^6+a^4(5Ab+aB)x^8+5a^3b(2Ab+aB)x^{10}+10a^2b^2(Ab+aB)x^{12}\right.\\ &= \frac{1}{7}a^5Ax^7+\frac{1}{9}a^4(5Ab+aB)x^9+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{10}{13}a^2b^2(Ab+aB)x^1\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4(5Ab+aB)x^9+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{1}{3}ab^3(Ab+2aB)x^{15}+\frac{1}{17}b^4(Ab+5aB)x^{17}+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a*B)*x^15)/3 + (b^4*(
A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

Maple [A]
time = 0.10, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A x^7}{7}+\left(\frac{5}{9}a^4bA+\frac{1}{9}a^5B\right)x^9+\left(\frac{10}{11}a^3b^2A+\frac{5}{11}a^4bB\right)x^{11}+\left(\frac{10}{13}a^2b^3A+\frac{10}{13}a^3b^2B\right)x^{13}+\left(\frac{1}{3}a\,b^4A+\right.$
default	$\frac{b^5 B x^{19}}{19}+\frac{(b^5A+5a\,b^4B)x^{17}}{17}+\frac{(5a\,b^4A+10a^2b^3B)x^{15}}{15}+\frac{(10a^2b^3A+10a^3b^2B)x^{13}}{13}+\frac{(10a^3b^2A+5a^4bB)x^{11}}{11}+\frac{(5a^4bA+a^5}{9}$
gosper	$\frac{1}{7}a^5A\,x^7+\frac{5}{9}x^9a^4bA+\frac{1}{9}x^9a^5B+\frac{10}{11}x^{11}a^3b^2A+\frac{5}{11}x^{11}a^4bB+\frac{10}{13}x^{13}a^2b^3A+\frac{10}{13}x^{13}a^3b^2B+\frac{1}{3}x^{15}a\,b$
risch	$\frac{1}{7}a^5A\,x^7+\frac{5}{9}x^9a^4bA+\frac{1}{9}x^9a^5B+\frac{10}{11}x^{11}a^3b^2A+\frac{5}{11}x^{11}a^4bB+\frac{10}{13}x^{13}a^2b^3A+\frac{10}{13}x^{13}a^3b^2B+\frac{1}{3}x^{15}a\,b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/19*b^5*B*x^19+1/17*(A*b^5+5*B*a*b^4)*x^17+1/15*(5*A*a*b^4+10*B*a^2*b^3)*x
^15+1/13*(10*A*a^2*b^3+10*B*a^3*b^2)*x^13+1/11*(10*A*a^3*b^2+5*B*a^4*b)*x^1
1+1/9*(5*A*a^4*b+B*a^5)*x^9+1/7*a^5*A*x^7
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.02

$$\frac{1}{19}Bb^5x^{19}+\frac{1}{17}\left(5\,Bab^4+Ab^5\right)x^{17}+\frac{1}{3}\left(2\,Ba^2b^3+Aab^4\right)x^{15}+\frac{10}{13}\left(Ba^3b^2+Aa^2b^3\right)x^{13}+\frac{1}{7}Aa^5x^7+\frac{5}{11}\left(Ba^4b+2\,Aa^3b^2\right)x^{11}+\frac{1}{9}\left(Ba^5+5\,Aa^4b\right)x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/19*B*b^5*x^19 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/7*A*a^5*x^7 + 5/11*(B*a^4*
b + 2*A*a^3*b^2)*x^11 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Fricas [A]
time = 0.59, size = 119, normalized size = 1.02

$$\frac{1}{19}Bb^5x^{19}+\frac{1}{17}\left(5\,Bab^4+Ab^5\right)x^{17}+\frac{1}{3}\left(2\,Ba^2b^3+Aab^4\right)x^{15}+\frac{10}{13}\left(Ba^3b^2+Aa^2b^3\right)x^{13}+\frac{1}{7}Aa^5x^7+\frac{5}{11}\left(Ba^4b+2\,Aa^3b^2\right)x^{11}+\frac{1}{9}\left(Ba^5+5\,Aa^4b\right)x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/19*B*b^5*x^19 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/7*A*a^5*x^7 + 5/11*(B*a^4*
b + 2*A*a^3*b^2)*x^11 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.16

$$\frac{Aa^5x^7}{7}+\frac{Bb^5x^{19}}{19}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{15}\left(\frac{Aab^4}{3}+\frac{2Ba^2b^3}{3}\right)+x^{13}\cdot\left(\frac{10Aa^2b^3}{13}+\frac{10Ba^3b^2}{13}\right)+x^{11}\cdot\left(\frac{10Aa^3b^2}{11}+\frac{5Ba^4b}{11}\right)+x^9\cdot\left(\frac{5Aa^4b}{9}+\frac{Ba^5}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**19/19 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**15
*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b**2
/13) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**9*(5*A*a**4*b/9 + B*a
**5/9)
```

Giac [A]
time = 0.65, size = 125, normalized size = 1.07

$$\frac{1}{19}Bb^5x^{19}+\frac{5}{17}Bab^4x^{17}+\frac{1}{17}Ab^5x^{17}+\frac{2}{3}Ba^2b^3x^{15}+\frac{1}{3}Aab^4x^{15}+\frac{10}{13}Ba^3b^2x^{13}+\frac{10}{13}Aa^2b^3x^{13}+\frac{5}{11}Ba^4bx^{11}+\frac{10}{11}Aa^3b^2x^{11}+\frac{1}{9}Ba^5x^9+\frac{5}{9}Aa^4bx^9+\frac{1}{7}Aa^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/19*B*b^5*x^19 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 2/3*B*a^2*b^3*x^15
+ 1/3*A*a*b^4*x^15 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 5/11*B*a
^4*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/7*A*
a^5*x^7
```

Mupad [B]
time = 0.02, size = 107, normalized size = 0.91

$$x^9\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^{17}\left(\frac{A\,b^5}{17}+\frac{5\,B\,a\,b^4}{17}\right)+\frac{A\,a^5\,x^7}{7}+\frac{B\,b^5\,x^{19}}{19}+\frac{10\,a^2\,b^2\,x^{13}\,\left(A\,b+B\,a\right)}{13}+\frac{5\,a^3\,b\,x^{11}\,\left(2\,A\,b+B\,a\right)}{11}+\frac{a\,b^3\,x^{15}\,\left(A\,b+2\,B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^9*((B*a^5)/9 + (5*A*a^4*b)/9) + x^17*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*a
^5*x^7)/7 + (B*b^5*x^19)/19 + (10*a^2*b^2*x^13*(A*b + B*a))/13 + (5*a^3*b*x
^11*(2*A*b + B*a))/11 + (a*b^3*x^15*(A*b + 2*B*a))/3
```

3.27 $\int x^5(a+bx^2)^5 (A+Bx^2)\,dx$

Optimal. Leaf size=95

$$\frac{a^2(Ab-aB)(a+bx^2)^6}{12b^4}-\frac{a(2Ab-3aB)(a+bx^2)^7}{14b^4}+\frac{(Ab-3aB)(a+bx^2)^8}{16b^4}+\frac{B(a+bx^2)^9}{18b^4}$$

```
[Out] 1/12*a^2*(A*b-B*a)*(b*x^2+a)^6/b^4-1/14*a*(2*A*b-3*B*a)*(b*x^2+a)^7/b^4+1/1
6*(A*b-3*B*a)*(b*x^2+a)^8/b^4+1/18*B*(b*x^2+a)^9/b^4
```

Rubi [A]
time = 0.15, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$\frac{a^2(a+bx^2)^6(Ab-aB)}{12b^4}+\frac{(a+bx^2)^8(Ab-3aB)}{16b^4}-\frac{a(a+bx^2)^7(2Ab-3aB)}{14b^4}+\frac{B(a+bx^2)^9}{18b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^6)/(12*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2)^7
)/(14*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^8)/(16*b^4) + (B*(a + b*x^2)^9)/(18
*b^4)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(a+bx^2\right)^5 \left(A+Bx^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^5(A+Bx)\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^5}{b^3} + \frac{a(-2Ab+3aB)(a+bx)^6}{b^3} + \frac{(Ab$$
$$= \frac{a^2(Ab-aB)\left(a+bx^2\right)^6}{12b^4} - \frac{a(2Ab-3aB)\left(a+bx^2\right)^7}{14b^4} + \frac{(Ab-3aB)\left(a+bx$$

Mathematica [A]
time = 0.02, size = 107, normalized size = 1.13

$$\frac{x^6(168a^5A + 126a^4(5Ab+aB)x^2 + 504a^3b(2Ab+aB)x^4 + 840a^2b^2(Ab+aB)x^6 + 360ab^3(Ab+2aB)x^8 + 63b^4(Ab+5aB)x^{10} + 56b^5Bx^{12})}{1008}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (x^6*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^2 + 504*a^3*b*(2*A*b + a*B)*x^4 +
 840*a^2*b^2*(A*b + a*B)*x^6 + 360*a*b^3*(A*b + 2*a*B)*x^8 + 63*b^4*(A*b +
5*a*B)*x^10 + 56*b^5*B*x^12))/1008
```

Maple [A]
time = 0.10, size = 124, normalized size = 1.31

method	result
norman	$\frac{a^5Ax^6}{6} + \left(\frac{5}{8}a^4bA + \frac{1}{8}a^5B\right)x^8 + \left(a^3b^2A + \frac{1}{2}a^4bB\right)x^{10} + \left(\frac{5}{6}a^2b^3A + \frac{5}{6}a^3b^2B\right)x^{12} + \left(\frac{5}{14}a\,b^4A + \frac{5}{7}a^2b$
default	$\frac{b^5Bx^{18}}{18} + \frac{(b^5A+5a\,b^4B)x^{16}}{16} + \frac{(5a\,b^4A+10a^2b^3B)x^{14}}{14} + \frac{(10a^2b^3A+10a^3b^2B)x^{12}}{12} + \frac{(10a^3b^2A+5a^4bB)x^{10}}{10} + \frac{(5a^4bA+a^5B$
gosper	$\frac{1}{6}a^5A\,x^6 + \frac{5}{8}x^8a^4bA + \frac{1}{8}x^8a^5B + x^{10}a^3b^2A + \frac{1}{2}x^{10}a^4bB + \frac{5}{6}x^{12}a^2b^3A + \frac{5}{6}x^{12}a^3b^2B + \frac{5}{14}x^{14}a\,b^4A +$
risch	$\frac{1}{6}a^5A\,x^6 + \frac{5}{8}x^8a^4bA + \frac{1}{8}x^8a^5B + x^{10}a^3b^2A + \frac{1}{2}x^{10}a^4bB + \frac{5}{6}x^{12}a^2b^3A + \frac{5}{6}x^{12}a^3b^2B + \frac{5}{14}x^{14}a\,b^4A +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*b^5*B*x^18+1/16*(A*b^5+5*B*a*b^4)*x^16+1/14*(5*A*a*b^4+10*B*a^2*b^3)*x
^14+1/12*(10*A*a^2*b^3+10*B*a^3*b^2)*x^12+1/10*(10*A*a^3*b^2+5*B*a^4*b)*x^1
0+1/8*(5*A*a^4*b+B*a^5)*x^8+1/6*a^5*A*x^6
```

Maxima [A]
time = 0.28, size = 119, normalized size = 1.25

$$\frac{1}{18}\,Bb^5x^{18} + \frac{1}{16}\,(5\,Bab^4 + Ab^5)x^{16} + \frac{5}{14}\,(2\,Ba^2b^3 + Aab^4)x^{14} + \frac{5}{6}\,(Ba^3b^2 + Aa^2b^3)x^{12} + \frac{1}{6}\,Aa^5x^6 + \frac{1}{2}\,(Ba^4b + 2\,Aa^3b^2)x^{10} + \frac{1}{8}\,(Ba^5 + 5\,Aa^4b)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/18*B*b^5*x^18 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 1/6*A*a^5*x^6 + 1/2*(B*a^4*b
+ 2*A*a^3*b^2)*x^10 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Fricas [A]
time = 0.67, size = 119, normalized size = 1.25

$$\frac{1}{18}\,Bb^{5}x^{18}+\frac{1}{16}\,(5\,Bab^{4}+Ab^{5})x^{16}+\frac{5}{14}\,(2\,Ba^{2}b^{3}+Aab^{4})x^{14}+\frac{5}{6}\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{12}+\frac{1}{6}\,Aa^{5}x^{6}+\frac{1}{2}\,(Ba^{4}b+2\,Aa^{3}b^{2})x^{10}+\frac{1}{8}\,(Ba^{5}+5\,Aa^{4}b)x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/18*B*b^5*x^18 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 1/6*A*a^5*x^6 + 1/2*(B*a^4*b
+ 2*A*a^3*b^2)*x^10 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Sympy [A]
time = 0.02, size = 133, normalized size = 1.40

$$\frac{Aa^{5}x^{6}}{6}+\frac{Bb^{5}x^{18}}{18}+x^{16}\left(\frac{Ab^{5}}{16}+\frac{5Bab^{4}}{16}\right)+x^{14}\cdot\left(\frac{5Aab^{4}}{14}+\frac{5Ba^{2}b^{3}}{7}\right)+x^{12}\cdot\left(\frac{5Aa^{2}b^{3}}{6}+\frac{5Ba^{3}b^{2}}{6}\right)+x^{10}\left(Aa^{3}b^{2}+\frac{Ba^{4}b}{2}\right)+x^{8}\cdot\left(\frac{5Aa^{4}b}{8}+\frac{Ba^{5}}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**6/6 + B*b**5*x**18/18 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**14
*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2
/6) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**8*(5*A*a**4*b/8 + B*a**5/8)
```

Giac [A]
time = 0.64, size = 124, normalized size = 1.31

$$\frac{1}{18}\,Bb^{5}x^{18}+\frac{5}{16}\,Bab^{4}x^{16}+\frac{1}{16}\,Ab^{5}x^{16}+\frac{5}{7}\,Ba^{2}b^{3}x^{14}+\frac{5}{14}\,Aab^{4}x^{14}+\frac{5}{6}\,Ba^{3}b^{2}x^{12}+\frac{5}{6}\,Aa^{2}b^{3}x^{12}+\frac{1}{2}\,Ba^{4}bx^{10}+Aa^{3}b^{2}x^{10}+\frac{1}{8}\,Ba^{5}x^{8}+\frac{5}{8}\,Aa^{4}bx^{8}+\frac{1}{6}\,Aa^{5}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/18*B*b^5*x^18 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 5/7*B*a^2*b^3*x^14
+ 5/14*A*a*b^4*x^14 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 1/2*B*a^4*b
*x^10 + A*a^3*b^2*x^10 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/6*A*a^5*x^6
```

Mupad [B]
time = 0.02, size = 107, normalized size = 1.13

$$x^{8}\left(\frac{B\,a^{5}}{8}+\frac{5\,A\,b\,a^{4}}{8}\right)+x^{16}\left(\frac{A\,b^{5}}{16}+\frac{5\,B\,a\,b^{4}}{16}\right)+\frac{A\,a^{5}\,x^{6}}{6}+\frac{B\,b^{5}\,x^{18}}{18}+\frac{5\,a^{2}\,b^{2}\,x^{12}\,(A\,b+B\,a)}{6}+\frac{a^{3}\,b\,x^{10}\,(2\,A\,b+B\,a)}{2}+\frac{5\,a\,b^{3}\,x^{14}\,(A\,b+2\,B\,a)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^8*((B*a^5)/8 + (5*A*a^4*b)/8) + x^16*((A*b^5)/16 + (5*B*a*b^4)/16) + (A*a
^5*x^6)/6 + (B*b^5*x^18)/18 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (a^3*b*x^10*
(2*A*b + B*a))/2 + (5*a*b^3*x^14*(A*b + 2*B*a))/14
```

3.28 $\int x^4(a+bx^2)^5\,(A+Bx^2)\;dx$

Optimal. Leaf size=117

$$\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{5}{9}a^3b(2Ab+aB)x^9+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{13}ab^3(Ab+2aB)x^{13}+\frac{1}{15}b^4(Ab+5aB)$$

```
[Out] 1/5*a^5*A*x^5+1/7*a^4*(5*A*b+B*a)*x^7+5/9*a^3*b*(2*A*b+B*a)*x^9+10/11*a^2*b
^2*(A*b+B*a)*x^11+5/13*a*b^3*(A*b+2*B*a)*x^13+1/15*b^4*(A*b+5*B*a)*x^15+1/1
7*b^5*B*x^17
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^9)/9 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*
(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx &= \int \left(a^5Ax^4+a^4(5Ab+aB)x^6+5a^3b(2Ab+aB)x^8+10a^2b^2(Ab+aB)x^{10}\right.\\ &= \frac{1}{5}a^5Ax^5+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{5}{9}a^3b(2Ab+aB)x^9+\frac{10}{11}a^2b^2(Ab+aB)x^{1}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{5}{9}a^3b(2Ab+aB)x^9+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{13}ab^3(Ab+2aB)x^{13}+\frac{1}{15}b^4(Ab+5aB)x^{15}+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(a + b*x^2)^5*(A + B*x^2),x]

[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^9)/9 + (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17

Maple [A]
time = 0.12, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A x^5}{5}+\left(\frac{5}{7}a^4bA+\frac{1}{7}a^5B\right)x^7+\left(\frac{10}{9}a^3b^2A+\frac{5}{9}a^4bB\right)x^9+\left(\frac{10}{11}a^2b^3A+\frac{10}{11}a^3b^2B\right)x^{11}+\left(\frac{5}{13}a\,b^4A+\frac{10}{13}\right.$
default	$\frac{b^5 B x^{17}}{17}+\frac{(b^5A+5a\,b^4B)x^{15}}{15}+\frac{(5a\,b^4A+10a^2b^3B)x^{13}}{13}+\frac{(10a^2b^3A+10a^3b^2B)x^{11}}{11}+\frac{(10a^3b^2A+5a^4bB)x^9}{9}+\frac{(5a^4bA+a^5B)}{7}$
gosper	$\frac{1}{5}a^5A\,x^5+\frac{5}{7}x^7a^4bA+\frac{1}{7}x^7a^5B+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{10}{11}x^{11}a^2b^3A+\frac{10}{11}x^{11}a^3b^2B+\frac{5}{13}x^{13}a\,b^4A$
risch	$\frac{1}{5}a^5A\,x^5+\frac{5}{7}x^7a^4bA+\frac{1}{7}x^7a^5B+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{10}{11}x^{11}a^2b^3A+\frac{10}{11}x^{11}a^3b^2B+\frac{5}{13}x^{13}a\,b^4A$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)

[Out] 1/17*b^5*B*x^17+1/15*(A*b^5+5*B*a*b^4)*x^15+1/13*(5*A*a*b^4+10*B*a^2*b^3)*x^13+1/11*(10*A*a^2*b^3+10*B*a^3*b^2)*x^11+1/9*(10*A*a^3*b^2+5*B*a^4*b)*x^9+1/7*(5*A*a^4*b+B*a^5)*x^7+1/5*a^5*A*x^5

Maxima [A]
time = 0.27, size = 119, normalized size = 1.02

$$\frac{1}{17}Bb^5x^{17}+\frac{1}{15}\left(5\,Bab^4+Ab^5\right)x^{15}+\frac{5}{13}\left(2\,Ba^2b^3+Aab^4\right)x^{13}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{1}{5}\,Aa^5x^5+\frac{5}{9}\left(Ba^4b+2\,Aa^3b^2\right)x^9+\frac{1}{7}\left(Ba^5+5\,Aa^4b\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")

[Out] 1/17*B*b^5*x^17 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/13*(2*B*a^2*b^3 + A*a*b^4)*x^13 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 1/5*A*a^5*x^5 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^9 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7

Fricas [A]
time = 0.79, size = 119, normalized size = 1.02

$$\frac{1}{17}Bb^5x^{17}+\frac{1}{15}\left(5\,Bab^4+Ab^5\right)x^{15}+\frac{5}{13}\left(2\,Ba^2b^3+Aab^4\right)x^{13}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{1}{5}\,Aa^5x^5+\frac{5}{9}\left(Ba^4b+2\,Aa^3b^2\right)x^9+\frac{1}{7}\left(Ba^5+5\,Aa^4b\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")

```
[Out] 1/17*B*b^5*x^17 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 1/5*A*a^5*x^5 + 5/9*(B*a^4*
b + 2*A*a^3*b^2)*x^9 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.16

$$\frac{Aa^5x^5}{5}+\frac{Bb^5x^{17}}{17}+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+x^{13}\cdot\left(\frac{5Aab^4}{13}+\frac{10Ba^2b^3}{13}\right)+x^{11}\cdot\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^9\cdot\left(\frac{10Aa^3b^2}{9}+\frac{5Ba^4b}{9}\right)+x^7\cdot\left(\frac{5Aa^4b}{7}+\frac{Ba^5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**17/17 + x**15*(A*b**5/15 + B*a*b**4/3) + x**13*(5
*A*a*b**4/13 + 10*B*a**2*b**3/13) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*b*
*2/11) + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**7*(5*A*a**4*b/7 + B*a*
*5/7)
```

Giac [A]
time = 0.57, size = 125, normalized size = 1.07

$$\frac{1}{17}Bb^5x^{17}+\frac{1}{3}Bab^4x^{15}+\frac{1}{15}Ab^5x^{15}+\frac{10}{13}Ba^2b^3x^{13}+\frac{5}{13}Aab^4x^{13}+\frac{10}{11}Ba^3b^2x^{11}+\frac{10}{11}Aa^2b^3x^{11}+\frac{5}{9}Ba^4bx^9+\frac{10}{9}Aa^3b^2x^9+\frac{1}{7}Ba^5x^7+\frac{5}{7}Aa^4bx^7+\frac{1}{5}Aa^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/17*B*b^5*x^17 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 10/13*B*a^2*b^3*x^13
 + 5/13*A*a*b^4*x^13 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/9*B*
a^4*b*x^9 + 10/9*A*a^3*b^2*x^9 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/5*A*a^
5*x^5
```

Mupad [B]
time = 0.02, size = 107, normalized size = 0.91

$$x^7\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^{15}\left(\frac{A\,b^5}{15}+\frac{B\,a\,b^4}{3}\right)+\frac{A\,a^5\,x^5}{5}+\frac{B\,b^5\,x^{17}}{17}+\frac{10\,a^2\,b^2\,x^{11}\,(A\,b+B\,a)}{11}+\frac{5\,a^3\,b\,x^9\,(2\,A\,b+B\,a)}{9}+\frac{5\,a\,b^3\,x^{13}\,(A\,b+2\,B\,a)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^15*((A*b^5)/15 + (B*a*b^4)/3) + (A*a^5*
x^5)/5 + (B*b^5*x^17)/17 + (10*a^2*b^2*x^11*(A*b + B*a))/11 + (5*a^3*b*x^9*
(2*A*b + B*a))/9 + (5*a*b^3*x^13*(A*b + 2*B*a))/13
```

3.29 $\int x^3(a+bx^2)^5\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=67

$$-\frac{a(Ab-aB)\left(a+bx^2\right)^6}{12b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^7}{14b^3}+\frac{B(a+bx^2)^8}{16b^3}$$

```
[Out] -1/12*a*(A*b-B*a)*(b*x^2+a)^6/b^3+1/14*(A*b-2*B*a)*(b*x^2+a)^7/b^3+1/16*B*(
b*x^2+a)^8/b^3
```

Rubi [A]
time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 77}

$$\frac{\left(a+bx^2\right)^7(Ab-2aB)}{14b^3}-\frac{a\left(a+bx^2\right)^6(Ab-aB)}{12b^3}+\frac{B(a+bx^2)^8}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] -1/12*(a*(A*b - a*B)*(a + b*x^2)^6)/b^3 + ((A*b - 2*a*B)*(a + b*x^2)^7)/(14
*b^3) + (B*(a + b*x^2)^8)/(16*b^3)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^5(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a(-Ab+aB)(a+bx)^5}{b^2}+\frac{(Ab-2aB)(a+bx)^6}{b^2}+\frac{B(a+bx\right.\right.\\ &= -\frac{a(Ab-aB)\left(a+bx^2\right)^6}{12b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^7}{14b^3}+\frac{B\left(a+bx^2\right)^8}{16b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 114, normalized size = 1.70

$$\frac{1}{4}a^5Ax^4+\frac{1}{6}a^4(5Ab+aB)x^6+\frac{5}{8}a^3b(2Ab+aB)x^8+a^2b^2(Ab+aB)x^{10}+\frac{5}{12}ab^3(Ab+2aB)x^{12}+\frac{1}{14}b^4(Ab+5aB)x^{14}+\frac{1}{16}b^5Bx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^6)/6 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 a^2*b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5
*a*B)*x^14)/14 + (b^5*B*x^16)/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(61) = 122$.
time = 0.08, size = 124, normalized size = 1.85

method	result
norman	$\frac{a^5Ax^4}{4}+\left(\frac{5}{6}a^4bA+\frac{1}{6}a^5B\right)x^6+\left(\frac{5}{4}a^3b^2A+\frac{5}{8}a^4bB\right)x^8+\left(a^2b^3A+a^3b^2B\right)x^{10}+\left(\frac{5}{12}a\,b^4A+\frac{5}{6}a^2b^3\right.$
gosper	$\frac{1}{4}a^5A\,x^4+\frac{5}{6}x^6a^4bA+\frac{1}{6}x^6a^5B+\frac{5}{4}x^8a^3b^2A+\frac{5}{8}x^8a^4bB+A\,a^2b^3x^{10}+B\,a^3b^2x^{10}+\frac{5}{12}x^{12}a\,b^4A+$
default	$\frac{b^5Bx^{16}}{16}+\frac{(b^5A+5a\,b^4B)x^{14}}{14}+\frac{(5a\,b^4A+10a^2b^3B)x^{12}}{12}+\frac{(10a^2b^3A+10a^3b^2B)x^{10}}{10}+\frac{(10a^3b^2A+5a^4bB)x^8}{8}+\frac{(5a^4bA+a^5B}{6}$
risch	$\frac{1}{4}a^5A\,x^4+\frac{5}{6}x^6a^4bA+\frac{1}{6}x^6a^5B+\frac{5}{4}x^8a^3b^2A+\frac{5}{8}x^8a^4bB+A\,a^2b^3x^{10}+B\,a^3b^2x^{10}+\frac{5}{12}x^{12}a\,b^4A+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*b^5*B*x^16+1/14*(A*b^5+5*B*a*b^4)*x^14+1/12*(5*A*a*b^4+10*B*a^2*b^3)*x
^12+1/10*(10*A*a^2*b^3+10*B*a^3*b^2)*x^10+1/8*(10*A*a^3*b^2+5*B*a^4*b)*x^8+
1/6*(5*A*a^4*b+B*a^5)*x^6+1/4*a^5*A*x^4
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.76

$$\frac{1}{16}Bb^5x^{16}+\frac{1}{14}\left(5\,Bab^4+Ab^5\right)x^{14}+\frac{5}{12}\left(2\,Ba^2b^3+Aab^4\right)x^{12}+\left(Ba^3b^2+Aa^2b^3\right)x^{10}+\frac{1}{4}Aa^5x^4+\frac{5}{8}\left(Ba^4b+2\,Aa^3b^2\right)x^8+\frac{1}{6}\left(Ba^5+5\,Aa^4b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/16*B*b^5*x^16 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 1/4*A*a^5*x^4 + 5/8*(B*a^4*b + 2*
A*a^3*b^2)*x^8 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Fricas [A]
time = 0.72, size = 118, normalized size = 1.76

$$\frac{1}{16}Bb^5x^{16}+\frac{1}{14}(5Bab^4+Ab^5)x^{14}+\frac{5}{12}(2Ba^2b^3+Aab^4)x^{12}+(Ba^3b^2+Aa^2b^3)x^{10}+\frac{1}{4}Aa^5x^4+\frac{5}{8}(Ba^4b+2Aa^3b^2)x^8+\frac{1}{6}(Ba^5+5Aa^4b)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/16*B*b^5*x^16 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 1/4*A*a^5*x^4 + 5/8*(B*a^4*b + 2*
A*a^3*b^2)*x^8 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(58) = 116$.
time = 0.02, size = 131, normalized size = 1.96

$$\frac{Aa^5x^4}{4}+\frac{Bb^5x^{16}}{16}+x^{14}\left(\frac{Ab^5}{14}+\frac{5Bab^4}{14}\right)+x^{12}\cdot\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right)+x^{10}(Aa^2b^3+Ba^3b^2)+x^8\cdot\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right)+x^6\cdot\left(\frac{5Aa^4b}{6}+\frac{Ba^5}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**4/4 + B*b**5*x**16/16 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**12
*(5*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x*
*8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

Giac [A]
time = 0.65, size = 123, normalized size = 1.84

$$\frac{1}{16}Bb^5x^{16}+\frac{5}{14}Bab^4x^{14}+\frac{1}{14}Ab^5x^{14}+\frac{5}{6}Ba^2b^3x^{12}+\frac{5}{12}Aab^4x^{12}+Ba^3b^2x^{10}+Aa^2b^3x^{10}+\frac{5}{8}Ba^4bx^8+\frac{5}{4}Aa^3b^2x^8+\frac{1}{6}Ba^5x^6+\frac{5}{6}Aa^4bx^6+\frac{1}{4}Aa^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/16*B*b^5*x^16 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 5/6*B*a^2*b^3*x^12
+ 5/12*A*a*b^4*x^12 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/8*B*a^4*b*x^8 + 5
/4*A*a^3*b^2*x^8 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/4*A*a^5*x^4
```

Mupad [B]
time = 0.02, size = 106, normalized size = 1.58

$$x^6\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+x^{14}\left(\frac{A\,b^5}{14}+\frac{5\,B\,a\,b^4}{14}\right)+\frac{A\,a^5\,x^4}{4}+\frac{B\,b^5\,x^{16}}{16}+a^2\,b^2\,x^{10}\,(A\,b+B\,a)+\frac{5\,a^3\,b\,x^8\,(2\,A\,b+B\,a)}{8}+\frac{5\,a\,b^3\,x^{12}\,(A\,b+2\,B\,a)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) + (A*a
^5*x^4)/4 + (B*b^5*x^16)/16 + a^2*b^2*x^10*(A*b + B*a) + (5*a^3*b*x^8*(2*A*
b + B*a))/8 + (5*a*b^3*x^12*(A*b + 2*B*a))/12
```

3.30 $\int x^2(a+bx^2)^5\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=117

$$\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{7}a^3b(2Ab+aB)x^7+\frac{10}{9}a^2b^2(Ab+aB)x^9+\frac{5}{11}ab^3(Ab+2aB)x^{11}+\frac{1}{13}b^4(Ab+5aB)x^{13}$$

```
[Out] 1/3*a^5*A*x^3+1/5*a^4*(5*A*b+B*a)*x^5+5/7*a^3*b*(2*A*b+B*a)*x^7+10/9*a^2*b^
2*(A*b+B*a)*x^9+5/11*a*b^3*(A*b+2*B*a)*x^11+1/13*b^4*(A*b+5*B*a)*x^13+1/15*
b^5*B*x^15
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 +
 (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A
*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx&=\int\left(a^5Ax^2+a^4(5Ab+aB)x^4+5a^3b(2Ab+aB)x^6+10a^2b^2(Ab+aB)x^8+\right.\\&=\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{7}a^3b(2Ab+aB)x^7+\frac{10}{9}a^2b^2(Ab+aB)x^9+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{7}a^3b(2Ab+aB)x^7+\frac{10}{9}a^2b^2(Ab+aB)x^9+\frac{5}{11}ab^3(Ab+2aB)x^{11}+\frac{1}{13}b^4(Ab+5aB)x^{13}+\frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 +
 (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A
*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

Maple [A]
time = 0.10, size = 124, normalized size = 1.06

method	result
norman	$\frac{a^5 A x^3}{3} + \left(a^4 bA + \frac{1}{5}a^5 B\right) x^5 + \left(\frac{10}{7}a^3 b^2 A + \frac{5}{7}a^4 bB\right) x^7 + \left(\frac{10}{9}a^2 b^3 A + \frac{10}{9}a^3 b^2 B\right) x^9 + \left(\frac{5}{11}a\, b^4 A + \frac{10}{11}\right.$
default	$\frac{b^5 B\, x^{15}}{15} + \frac{(b^5 A+5a\, b^4 B)x^{13}}{13} + \frac{(5a\, b^4 A+10a^2 b^3 B)x^{11}}{11} + \frac{(10a^2 b^3 A+10a^3 b^2 B)x^9}{9} + \frac{(10a^3 b^2 A+5a^4 bB)x^7}{7} + \frac{(5a^4 bA+a^5 B}{5}$
gosper	$\frac{1}{3}a^5 A\, x^3 + x^5 a^4 bA + \frac{1}{5}x^5 a^5 B + \frac{10}{7}x^7 a^3 b^2 A + \frac{5}{7}x^7 a^4 bB + \frac{10}{9}x^9 a^2 b^3 A + \frac{10}{9}x^9 a^3 b^2 B + \frac{5}{11}x^{11} a\, b^4 A +$
risch	$\frac{1}{3}a^5 A\, x^3 + x^5 a^4 bA + \frac{1}{5}x^5 a^5 B + \frac{10}{7}x^7 a^3 b^2 A + \frac{5}{7}x^7 a^4 bB + \frac{10}{9}x^9 a^2 b^3 A + \frac{10}{9}x^9 a^3 b^2 B + \frac{5}{11}x^{11} a\, b^4 A +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^5*B*x^15+1/13*(A*b^5+5*B*a*b^4)*x^13+1/11*(5*A*a*b^4+10*B*a^2*b^3)*x
^11+1/9*(10*A*a^2*b^3+10*B*a^3*b^2)*x^9+1/7*(10*A*a^3*b^2+5*B*a^4*b)*x^7+1/
5*(5*A*a^4*b+B*a^5)*x^5+1/3*a^5*A*x^3
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.02

$$\frac{1}{15}\,Bb^5x^{15} + \frac{1}{13}\,(5\,Bab^4 + Ab^5)x^{13} + \frac{5}{11}\,(2\,Ba^2b^3 + Aab^4)x^{11} + \frac{10}{9}\,(Ba^3b^2 + Aa^2b^3)x^9 + \frac{1}{3}\,Aa^5x^3 + \frac{5}{7}\,(Ba^4b + 2\,Aa^3b^2)x^7 + \frac{1}{5}\,(Ba^5 + 5\,Aa^4b)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/15*B*b^5*x^15 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 5/11*(2*B*a^2*b^3 + A*a*b
^4)*x^11 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1/3*A*a^5*x^3 + 5/7*(B*a^4*b
+ 2*A*a^3*b^2)*x^7 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Fricas [A]
time = 0.64, size = 119, normalized size = 1.02

$$\frac{1}{15}\,Bb^5x^{15} + \frac{1}{13}\,(5\,Bab^4 + Ab^5)x^{13} + \frac{5}{11}\,(2\,Ba^2b^3 + Aab^4)x^{11} + \frac{10}{9}\,(Ba^3b^2 + Aa^2b^3)x^9 + \frac{1}{3}\,Aa^5x^3 + \frac{5}{7}\,(Ba^4b + 2\,Aa^3b^2)x^7 + \frac{1}{5}\,(Ba^5 + 5\,Aa^4b)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/15*B*b^5*x^15 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 5/11*(2*B*a^2*b^3 + A*a*b
^4)*x^11 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1/3*A*a^5*x^3 + 5/7*(B*a^4*b
+ 2*A*a^3*b^2)*x^7 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Sympy [A]
time = 0.02, size = 134, normalized size = 1.15

$$\frac{Aa^5x^3}{3}+\frac{Bb^5x^{15}}{15}+x^{13}\left(\frac{Ab^5}{13}+\frac{5Bab^4}{13}\right)+x^{11}\cdot\left(\frac{5Aab^4}{11}+\frac{10Ba^2b^3}{11}\right)+x^9\cdot\left(\frac{10Aa^2b^3}{9}+\frac{10Ba^3b^2}{9}\right)+x^7\cdot\left(\frac{10Aa^3b^2}{7}+\frac{5Ba^4b}{7}\right)+x^5\left(Aa^4b+\frac{Ba^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**15/15 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x**11
*(5*A*a*b**4/11 + 10*B*a**2*b**3/11) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b
**2/9) + x**7*(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**5*(A*a**4*b + B*a**5/5
)
```

Giac [A]
time = 0.62, size = 124, normalized size = 1.06

$$\frac{1}{15}Bb^5x^{15}+\frac{5}{13}Bab^4x^{13}+\frac{1}{13}Ab^5x^{13}+\frac{10}{11}Ba^2b^3x^{11}+\frac{5}{11}Aab^4x^{11}+\frac{10}{9}Ba^3b^2x^9+\frac{10}{9}Aa^2b^3x^9+\frac{5}{7}Ba^4bx^7+\frac{10}{7}Aa^3b^2x^7+\frac{1}{5}Ba^5x^5+Aa^4bx^5+\frac{1}{3}Aa^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/15*B*b^5*x^15 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + 10/11*B*a^2*b^3*x^1
1 + 5/11*A*a*b^4*x^11 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/7*B*a^4
*b*x^7 + 10/7*A*a^3*b^2*x^7 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/3*A*a^5*x^3
```

Mupad [B]
time = 0.02, size = 106, normalized size = 0.91

$$x^5\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)+x^{13}\left(\frac{A\,b^5}{13}+\frac{5\,B\,a\,b^4}{13}\right)+\frac{A\,a^5\,x^3}{3}+\frac{B\,b^5\,x^{15}}{15}+\frac{10\,a^2\,b^2\,x^9\,\left(A\,b+B\,a\right)}{9}+\frac{5\,a^3\,b\,x^7\,\left(2\,A\,b+B\,a\right)}{7}+\frac{5\,a\,b^3\,x^{11}\,\left(A\,b+2\,B\,a\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^5*((B*a^5)/5 + A*a^4*b) + x^13*((A*b^5)/13 + (5*B*a*b^4)/13) + (A*a^5*x^3
)/3 + (B*b^5*x^15)/15 + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x^7*(2*A*
b + B*a))/7 + (5*a*b^3*x^11*(A*b + 2*B*a))/11
```

3.31 $\int x(a+bx^2)^5 (A+Bx^2)\, dx$

Optimal. Leaf size=42

$$\frac{(Ab-aB)(a+bx^2)^6}{12b^2}+\frac{B(a+bx^2)^7}{14b^2}$$

```
[Out] 1/12*(A*b-B*a)*(b*x^2+a)^6/b^2+1/14*B*(b*x^2+a)^7/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {455, 45}

$$\frac{(a+bx^2)^6(Ab-aB)}{12b^2}+\frac{B(a+bx^2)^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^6)/(12*b^2) + (B*(a + b*x^2)^7)/(14*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx^2)^5(A+Bx^2)\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^5(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^5}{b}+\frac{B(a+bx)^6}{b}\right)dx,x,x^2\right)\\ &= \frac{(Ab-aB)(a+bx^2)^6}{12b^2}+\frac{B(a+bx^2)^7}{14b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(42) = 84.
time = 0.02, size = 107, normalized size = 2.55

$$\frac{1}{84}x^2\left(42a^5A+21a^4(5Ab+aB)x^2+70a^3b(2Ab+aB)x^4+105a^2b^2(Ab+aB)x^6+42ab^3(Ab+2aB)x^8+7b^4(Ab+5aB)x^{10}+6b^5Bx^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (x^2*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^2 + 70*a^3*b*(2*A*b + a*B)*x^4 + 10
5*a^2*b^2*(A*b + a*B)*x^6 + 42*a*b^3*(A*b + 2*a*B)*x^8 + 7*b^4*(A*b + 5*a*B
)*x^10 + 6*b^5*B*x^12))/84
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(38) = 76.
time = 0.08, size = 124, normalized size = 2.95

method	result
norman	$\frac{a^5Ax^2}{2}+\left(\frac{5}{4}a^4bA+\frac{1}{4}a^5B\right)x^4+\left(\frac{5}{3}a^3b^2A+\frac{5}{6}a^4bB\right)x^6+\left(\frac{5}{4}a^2b^3A+\frac{5}{4}a^3b^2B\right)x^8+\left(\frac{1}{2}ab^4A+a^2b^3B\right.$
default	$\frac{b^5Bx^{14}}{14}+\frac{(b^5A+5ab^4B)x^{12}}{12}+\frac{(5ab^4A+10a^2b^3B)x^{10}}{10}+\frac{(10a^2b^3A+10a^3b^2B)x^8}{8}+\frac{(10a^3b^2A+5a^4bB)x^6}{6}+\frac{(5a^4bA+a^5B)x}{4}$
gosper	$\frac{1}{2}a^5Ax^2+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{5}{3}x^6a^3b^2A+\frac{5}{6}x^6a^4bB+\frac{5}{4}x^8a^2b^3A+\frac{5}{4}x^8a^3b^2B+\frac{1}{2}x^{10}ab^4A+x^1$
risch	$\frac{1}{2}a^5Ax^2+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{5}{3}x^6a^3b^2A+\frac{5}{6}x^6a^4bB+\frac{5}{4}x^8a^2b^3A+\frac{5}{4}x^8a^3b^2B+\frac{1}{2}x^{10}ab^4A+x^1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*b^5*B*x^14+1/12*(A*b^5+5*B*a*b^4)*x^12+1/10*(5*A*a*b^4+10*B*a^2*b^3)*x
^10+1/8*(10*A*a^2*b^3+10*B*a^3*b^2)*x^8+1/6*(10*A*a^3*b^2+5*B*a^4*b)*x^6+1/
4*(5*A*a^4*b+B*a^5)*x^4+1/2*a^5*A*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(38) = 76.
time = 0.29, size = 119, normalized size = 2.83

$$\frac{1}{14}Bb^5x^{14}+\frac{1}{12}\left(5Bab^4+Ab^5\right)x^{12}+\frac{1}{2}\left(2Ba^2b^3+Aab^4\right)x^{10}+\frac{5}{4}\left(Ba^3b^2+Aa^2b^3\right)x^8+\frac{1}{2}Aa^5x^2+\frac{5}{6}\left(Ba^4b+2Aa^3b^2\right)x^6+\frac{1}{4}\left(Ba^5+5Aa^4b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/14*B*b^5*x^14 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 1/2*(2*B*a^2*b^3 + A*a*b^
4)*x^10 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 1/2*A*a^5*x^2 + 5/6*(B*a^4*b +
2*A*a^3*b^2)*x^6 + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(38) = 76$.

time = 0.80, size = 119, normalized size = 2.83

$$\frac{1}{14}Bb^5x^{14} + \frac{1}{12}\left(5\,Bab^4 + Ab^5\right)x^{12} + \frac{1}{2}\left(2\,Ba^2b^3 + Aab^4\right)x^{10} + \frac{5}{4}\left(Ba^3b^2 + Aa^2b^3\right)x^8 + \frac{1}{2}Aa^5x^2 + \frac{5}{6}\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + \frac{1}{4}\left(Ba^5 + 5\,Aa^4b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/14*B*b^5*x^14 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 1/2*(2*B*a^2*b^3 + A*a*b^
4)*x^10 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 1/2*A*a^5*x^2 + 5/6*(B*a^4*b +
2*A*a^3*b^2)*x^6 + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(36) = 72$.

time = 0.02, size = 133, normalized size = 3.17

$$\frac{Aa^5x^2}{2} + \frac{Bb^5x^{14}}{14} + x^{12}\left(\frac{Ab^5}{12} + \frac{5Bab^4}{12}\right) + x^{10}\left(\frac{Aab^4}{2} + Ba^2b^3\right) + x^8\cdot\left(\frac{5Aa^2b^3}{4} + \frac{5Ba^3b^2}{4}\right) + x^6\cdot\left(\frac{5Aa^3b^2}{3} + \frac{5Ba^4b}{6}\right) + x^4\cdot\left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**2/2 + B*b**5*x**14/14 + x**12*(A*b**5/12 + 5*B*a*b**4/12) + x**10
*(A*a*b**4/2 + B*a**2*b**3) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4) + x*
*6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(38) = 76$.

time = 0.60, size = 124, normalized size = 2.95

$$\frac{1}{14}Bb^5x^{14} + \frac{5}{12}Bab^4x^{12} + \frac{1}{12}Ab^5x^{12} + Ba^2b^3x^{10} + \frac{1}{2}Aab^4x^{10} + \frac{5}{4}Ba^3b^2x^8 + \frac{5}{4}Aa^2b^3x^8 + \frac{5}{6}Ba^4bx^6 + \frac{5}{3}Aa^3b^2x^6 + \frac{1}{4}Ba^5x^4 + \frac{5}{4}Aa^4bx^4 + \frac{1}{2}Aa^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/14*B*b^5*x^14 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + B*a^2*b^3*x^10 + 1/
2*A*a*b^4*x^10 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + 5/6*B*a^4*b*x^6 +
5/3*A*a^3*b^2*x^6 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + 1/2*A*a^5*x^2
```

Mupad [B]

time = 0.02, size = 107, normalized size = 2.55

$$x^4\left(\frac{B\,a^5}{4} + \frac{5\,A\,b\,a^4}{4}\right) + x^{12}\left(\frac{A\,b^5}{12} + \frac{5\,B\,a\,b^4}{12}\right) + \frac{A\,a^5\,x^2}{2} + \frac{B\,b^5\,x^{14}}{14} + \frac{5\,a^2\,b^2\,x^8\,(A\,b + B\,a)}{4} + \frac{5\,a^3\,b\,x^6\,(2\,A\,b + B\,a)}{6} + \frac{a\,b^3\,x^{10}\,(A\,b + 2\,B\,a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^12*((A*b^5)/12 + (5*B*a*b^4)/12) + (A*a
^5*x^2)/2 + (B*b^5*x^14)/14 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + (5*a^3*b*x^6*
(2*A*b + B*a))/6 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

3.32 $\int \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=109

$$a^5Ax+\frac{1}{3}a^4(5Ab+aB)x^3+a^3b(2Ab+aB)x^5+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{13}b$$

```
[Out] a^5*A*x+1/3*a^4*(5*A*b+B*a)*x^3+a^3*b*(2*A*b+B*a)*x^5+10/7*a^2*b^2*(A*b+B*a
)*x^7+5/9*a*b^3*(A*b+2*B*a)*x^9+1/11*b^4*(A*b+5*B*a)*x^11+1/13*b^5*B*x^13
```

Rubi [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{380\}$

$$a^5Ax+\frac{1}{3}a^4x^3(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (10*a^2*b^2
*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^
11)/11 + (b^5*B*x^13)/13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx &= \int \left(a^5A+a^4(5Ab+aB)x^2+5a^3b(2Ab+aB)x^4+10a^2b^2(Ab+aB)x^6+5ab\right. \\ &= a^5Ax+\frac{1}{3}a^4(5Ab+aB)x^3+a^3b(2Ab+aB)x^5+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{5}{9}ab^3\end{aligned}$$

Mathematica [A]
time = 0.01, size = 109, normalized size = 1.00

$$a^5Ax+\frac{1}{3}a^4(5Ab+aB)x^3+a^3b(2Ab+aB)x^5+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (10*a^2*b^2
*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^
11)/11 + (b^5*B*x^13)/13
```

Maple [A]
time = 0.10, size = 121, normalized size = 1.11

method	result
norman	$\frac{b^5 B x^{13}}{13}+\left(\frac{1}{11}b^5 A+\frac{5}{11}a b^4 B\right)x^{11}+\left(\frac{5}{9}a b^4 A+\frac{10}{9}a^2 b^3 B\right)x^9+\left(\frac{10}{7}a^2 b^3 A+\frac{10}{7}a^3 b^2 B\right)x^7+(2a^3 b^2 A$
default	$\frac{b^5 B x^{13}}{13}+\frac{(b^5 A+5a b^4 B)x^{11}}{11}+\frac{(5a b^4 A+10a^2 b^3 B)x^9}{9}+\frac{(10a^2 b^3 A+10a^3 b^2 B)x^7}{7}+\frac{(10a^3 b^2 A+5a^4 b B)x^5}{5}+\frac{(5a^4 b A+a^5 B)}{3}$
gosper	$\frac{1}{13}b^5 B x^{13}+\frac{1}{11}x^{11}b^5 A+\frac{5}{11}x^{11}a b^4 B+\frac{5}{9}x^9 a b^4 A+\frac{10}{9}x^9 a^2 b^3 B+\frac{10}{7}x^7 a^2 b^3 A+\frac{10}{7}x^7 a^3 b^2 B+2A a^3$
risch	$\frac{1}{13}b^5 B x^{13}+\frac{1}{11}x^{11}b^5 A+\frac{5}{11}x^{11}a b^4 B+\frac{5}{9}x^9 a b^4 A+\frac{10}{9}x^9 a^2 b^3 B+\frac{10}{7}x^7 a^2 b^3 A+\frac{10}{7}x^7 a^3 b^2 B+2A a^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^5*B*x^13+1/11*(A*b^5+5*B*a*b^4)*x^11+1/9*(5*A*a*b^4+10*B*a^2*b^3)*x^
9+1/7*(10*A*a^2*b^3+10*B*a^3*b^2)*x^7+1/5*(10*A*a^3*b^2+5*B*a^4*b)*x^5+1/3*
(5*A*a^4*b+B*a^5)*x^3+a^5*A*x
```

Maxima [A]
time = 0.29, size = 115, normalized size = 1.06

$$\frac{1}{13}Bb^5x^{13}+\frac{1}{11}\left(5\,Bab^4+Ab^5\right)x^{11}+\frac{5}{9}\left(2\,Ba^2b^3+Aab^4\right)x^9+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7+Aa^5x+\left(Ba^4b+2\,Aa^3b^2\right)x^5+\frac{1}{3}\left(Ba^5+5\,Aa^4b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/13*B*b^5*x^13 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/9*(2*B*a^2*b^3 + A*a*b^
4)*x^9 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + A*a^5*x + (B*a^4*b + 2*A*a^3*b^
2)*x^5 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Fricas [A]
time = 0.63, size = 115, normalized size = 1.06

$$\frac{1}{13}Bb^5x^{13}+\frac{1}{11}\left(5\,Bab^4+Ab^5\right)x^{11}+\frac{5}{9}\left(2\,Ba^2b^3+Aab^4\right)x^9+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7+Aa^5x+\left(Ba^4b+2\,Aa^3b^2\right)x^5+\frac{1}{3}\left(Ba^5+5\,Aa^4b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/13*B*b^5*x^13 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/9*(2*B*a^2*b^3 + A*a*b^
4)*x^9 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + A*a^5*x + (B*a^4*b + 2*A*a^3*b^
2)*x^5 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Sympy [A]
time = 0.02, size = 129, normalized size = 1.18

$$Aa^5x + \frac{Bb^5x^{13}}{13} + x^{11}\left(\frac{Ab^5}{11} + \frac{5Bab^4}{11}\right) + x^9\cdot\left(\frac{5Aab^4}{9} + \frac{10Ba^2b^3}{9}\right) + x^7\cdot\left(\frac{10Aa^2b^3}{7} + \frac{10Ba^3b^2}{7}\right) + x^5\cdot\left(2Aa^3b^2 + Ba^4b\right) + x^3\cdot\left(\frac{5Aa^4b}{3} + \frac{Ba^5}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x + B*b**5*x**13/13 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**9*(5*A*
a*b**4/9 + 10*B*a**2*b**3/9) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7) +
 x**5*(2*A*a**3*b**2 + B*a**4*b) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

Giac [A]
time = 0.68, size = 121, normalized size = 1.11

$$\frac{1}{13}Bb^5x^{13} + \frac{5}{11}Bab^4x^{11} + \frac{1}{11}Ab^5x^{11} + \frac{10}{9}Ba^2b^3x^9 + \frac{5}{9}Aab^4x^9 + \frac{10}{7}Ba^3b^2x^7 + \frac{10}{7}Aa^2b^3x^7 + Ba^4bx^5 + 2\,Aa^3b^2x^5 + \frac{1}{3}Ba^5x^3 + \frac{5}{3}Aa^4bx^3 + Aa^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/13*B*b^5*x^13 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 10/9*B*a^2*b^3*x^9
+ 5/9*A*a*b^4*x^9 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + B*a^4*b*x^5 +
 2*A*a^3*b^2*x^5 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*x
```

Mupad [B]
time = 0.02, size = 103, normalized size = 0.94

$$x^3\left(\frac{B\,a^5}{3} + \frac{5\,A\,b\,a^4}{3}\right) + x^{11}\left(\frac{A\,b^5}{11} + \frac{5\,B\,a\,b^4}{11}\right) + \frac{B\,b^5\,x^{13}}{13} + A\,a^5\,x + \frac{10\,a^2\,b^2\,x^7\,(A\,b + B\,a)}{7} + a^3\,b\,x^5\,(2\,A\,b + B\,a) + \frac{5\,a\,b^3\,x^9\,(A\,b + 2\,B\,a)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^11*((A*b^5)/11 + (5*B*a*b^4)/11) + (B*b
^5*x^13)/13 + A*a^5*x + (10*a^2*b^2*x^7*(A*b + B*a))/7 + a^3*b*x^5*(2*A*b +
 B*a) + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

3.33 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=88

$$\frac{5}{2}a^4Abx^2+\frac{5}{2}a^3Ab^2x^4+\frac{5}{3}a^2Ab^3x^6+\frac{5}{8}aAb^4x^8+\frac{1}{10}Ab^5x^{10}+\frac{B(a+bx^2)^6}{12b}+a^5A\log(x)$$

```
[Out] 5/2*a^4*A*b*x^2+5/2*a^3*A*b^2*x^4+5/3*a^2*A*b^3*x^6+5/8*a*A*b^4*x^8+1/10*A*
b^5*x^10+1/12*B*(b*x^2+a)^6/b+a^5*A*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 81, 45}

$$a^5A\log(x)+\frac{5}{2}a^4Abx^2+\frac{5}{2}a^3Ab^2x^4+\frac{5}{3}a^2Ab^3x^6+\frac{5}{8}aAb^4x^8+\frac{B(a+bx^2)^6}{12b}+\frac{1}{10}Ab^5x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

```
[Out] (5*a^4*A*b*x^2)/2 + (5*a^3*A*b^2*x^4)/2 + (5*a^2*A*b^3*x^6)/3 + (5*a*A*b^4*
x^8)/8 + (A*b^5*x^10)/10 + (B*(a + b*x^2)^6)/(12*b) + a^5*A*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x}\,dx, x, x^2\right)$$
$$= \frac{B(a+bx^2)^6}{12b} + \frac{1}{2}A\text{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx, x, x^2\right)$$
$$= \frac{B(a+bx^2)^6}{12b} + \frac{1}{2}A\text{Subst}\left(\int \left(5a^4b + \frac{a^5}{x} + 10a^3b^2x + 10a^2b^3x^2 + 5ab^4x^3 + b^5\right.\right.$$
$$= \frac{5}{2}a^4Abx^2 + \frac{5}{2}a^3Ab^2x^4 + \frac{5}{3}a^2Ab^3x^6 + \frac{5}{8}aAb^4x^8 + \frac{1}{10}Ab^5x^{10} + \frac{B(a+bx^2)^6}{12b} + a$$

Mathematica [A]
time = 0.02, size = 113, normalized size = 1.28

$$\frac{1}{2}a^4(5Ab+aB)x^2+\frac{5}{4}a^3b(2Ab+aB)x^4+\frac{5}{3}a^2b^2(Ab+aB)x^6+\frac{5}{8}ab^3(Ab+2aB)x^8+\frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{12}b^5Bx^{12}+a^5A\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

```
[Out] (a^4*(5*A*b + a*B)*x^2)/2 + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (5*a^2*b^2*(A*b
 + a*B)*x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a*B)*x^10)/1
0 + (b^5*B*x^12)/12 + a^5*A*Log[x]
```

Maple [A]
time = 0.06, size = 124, normalized size = 1.41

method	result
norman	$\left(\frac{1}{10}b^5A+\frac{1}{2}a\,b^4B\right)x^{10}+\left(\frac{5}{8}a\,b^4A+\frac{5}{4}a^2b^3B\right)x^8+\left(\frac{5}{3}a^2b^3A+\frac{5}{3}a^3b^2B\right)x^6+\left(\frac{5}{2}a^3b^2A+\frac{5}{4}a^4bB\right)x^4$
default	$\frac{b^5Bx^{12}}{12}+\frac{A\,b^5x^{10}}{10}+\frac{Ba\,b^4x^{10}}{2}+\frac{5aA\,b^4x^8}{8}+\frac{5B\,a^2b^3x^8}{4}+\frac{5a^2A\,b^3x^6}{3}+\frac{5B\,a^3b^2x^6}{3}+\frac{5a^3A\,b^2x^4}{2}+\frac{5B\,a^4b\,x^4}{4}+\frac{5a^4Ab}{2}$
risch	$\frac{b^5Bx^{12}}{12}+\frac{A\,b^5x^{10}}{10}+\frac{Ba\,b^4x^{10}}{2}+\frac{5aA\,b^4x^8}{8}+\frac{5B\,a^2b^3x^8}{4}+\frac{5a^2A\,b^3x^6}{3}+\frac{5B\,a^3b^2x^6}{3}+\frac{5a^3A\,b^2x^4}{2}+\frac{5B\,a^4b\,x^4}{4}+\frac{5a^4Ab}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^5*B*x^12+1/10*A*b^5*x^10+1/2*B*a*b^4*x^10+5/8*a*A*b^4*x^8+5/4*B*a^2*
b^3*x^8+5/3*a^2*A*b^3*x^6+5/3*B*a^3*b^2*x^6+5/2*a^3*A*b^2*x^4+5/4*B*a^4*b*x
^4+5/2*a^4*A*b*x^2+1/2*B*a^5*x^2+a^5*A*ln(x)
```

Maxima [A]
time = 0.28, size = 120, normalized size = 1.36

$$\frac{1}{12}Bb^5x^{12}+\frac{1}{10}\left(5\,Bab^4+Ab^5\right)x^{10}+\frac{5}{8}\left(2\,Ba^2b^3+Aab^4\right)x^8+\frac{5}{3}\left(Ba^3b^2+Aa^2b^3\right)x^6+\frac{1}{2}Aa^5\log\left(x^2\right)+\frac{5}{4}\left(Ba^4b+2\,Aa^3b^2\right)x^4+\frac{1}{2}\left(Ba^5+5\,Aa^4b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] 1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1/2*A*a^5*log(x^2) + 5/4*(B*a^4*
b + 2*A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Fricas [A]
time = 0.72, size = 117, normalized size = 1.33

$$\frac{1}{12}Bb^5x^{12}+\frac{1}{10}\left(5\,Bab^4+Ab^5\right)x^{10}+\frac{5}{8}\left(2\,Ba^2b^3+Aab^4\right)x^8+\frac{5}{3}\left(Ba^3b^2+Aa^2b^3\right)x^6+Aa^5\log\left(x\right)+\frac{5}{4}\left(Ba^4b+2\,Aa^3b^2\right)x^4+\frac{1}{2}\left(Ba^5+5\,Aa^4b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] 1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + A*a^5*log(x) + 5/4*(B*a^4*b + 2*
A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Sympy [A]
time = 0.09, size = 134, normalized size = 1.52

$$Aa^5\log\left(x\right)+\frac{Bb^5x^{12}}{12}+x^{10}\left(\frac{Ab^5}{10}+\frac{Bab^4}{2}\right)+x^8\cdot\left(\frac{5Aab^4}{8}+\frac{5Ba^2b^3}{4}\right)+x^6\cdot\left(\frac{5Aa^2b^3}{3}+\frac{5Ba^3b^2}{3}\right)+x^4\cdot\left(\frac{5Aa^3b^2}{2}+\frac{5Ba^4b}{4}\right)+x^2\cdot\left(\frac{5Aa^4b}{2}+\frac{Ba^5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x,x)
```

```
[Out] A*a**5*log(x) + B*b**5*x**12/12 + x**10*(A*b**5/10 + B*a*b**4/2) + x**8*(5*
A*a*b**4/8 + 5*B*a**2*b**3/4) + x**6*(5*A*a**2*b**3/3 + 5*B*a**3*b**2/3) +
x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

Giac [A]
time = 0.63, size = 126, normalized size = 1.43

$$\frac{1}{12}Bb^5x^{12}+\frac{1}{2}Bab^4x^{10}+\frac{1}{10}Ab^5x^{10}+\frac{5}{4}Ba^2b^3x^8+\frac{5}{8}Aab^4x^8+\frac{5}{3}Ba^3b^2x^6+\frac{5}{3}Aa^2b^3x^6+\frac{5}{4}Ba^4bx^4+\frac{5}{2}Aa^3b^2x^4+\frac{1}{2}Ba^5x^2+\frac{5}{2}Aa^4bx^2+\frac{1}{2}Aa^5\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] 1/12*B*b^5*x^12 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 5/4*B*a^2*b^3*x^8 +
5/8*A*a*b^4*x^8 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/4*B*a^4*b*x^4 +
 5/2*A*a^3*b^2*x^4 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 + 1/2*A*a^5*log(x^2)
```

Mupad [B]
time = 0.03, size = 105, normalized size = 1.19

$$x^2\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)+x^{10}\left(\frac{A\,b^5}{10}+\frac{B\,a\,b^4}{2}\right)+\frac{B\,b^5\,x^{12}}{12}+A\,a^5\,\ln\left(x\right)+\frac{5\,a^2\,b^2\,x^6\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a^3\,b\,x^4\,\left(2\,A\,b+B\,a\right)}{4}+\frac{5\,a\,b^3\,x^8\,\left(A\,b+2\,B\,a\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x,x)
```

```
[Out] x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^10*((A*b^5)/10 + (B*a*b^4)/2) + (B*b^5*
x^12)/12 + A*a^5*log(x) + (5*a^2*b^2*x^6*(A*b + B*a))/3 + (5*a^3*b*x^4*(2*A
*b + B*a))/4 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

3.34 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^2} \, dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{x}+a^4(5Ab+aB)x+\frac{5}{3}a^3b(2Ab+aB)x^3+2a^2b^2(Ab+aB)x^5+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{9}b^4(Ab+5aB)x^9+\frac{1}{11}b^5B$$

```
[Out] -a^5*A/x+a^4*(5*A*b+B*a)*x+5/3*a^3*b*(2*A*b+B*a)*x^3+2*a^2*b^2*(A*b+B*a)*x^
5+5/7*a*b^3*(A*b+2*B*a)*x^7+1/9*b^4*(A*b+5*B*a)*x^9+1/11*b^5*B*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^5A}{x}+a^4x(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + 2*a^2*
b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*x^
9)/9 + (b^5*B*x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^2} \, dx = \int \left(a^4(5Ab+aB)+\frac{a^5A}{x^2}+5a^3b(2Ab+aB)x^2+10a^2b^2(Ab+aB)x^4+5ab^3\right.$$

$$= -\frac{a^5A}{x}+a^4(5Ab+aB)x+\frac{5}{3}a^3b(2Ab+aB)x^3+2a^2b^2(Ab+aB)x^5+\frac{5}{7}ab^3($$

Mathematica [A]
time = 0.02, size = 108, normalized size = 1.00

$$-\frac{a^5A}{x}+a^4(5Ab+aB)x+\frac{5}{3}a^3b(2Ab+aB)x^3+2a^2b^2(Ab+aB)x^5+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{9}b^4(Ab+5aB)x^9+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + 2*a^2*
b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*x^
9)/9 + (b^5*B*x^11)/11
```

Maple [A]
time = 0.06, size = 121, normalized size = 1.12

method	result
default	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^9}{9}+\frac{5 B a b^4 x^9}{9}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+2 A a^2 b^3 x^5+2 B a^3 b^2 x^5+\frac{10 A a^3 b^2 x^3}{3}+\frac{5 B a^4 b x^3}{3}+$
norman	$\frac{\frac{b^5 B x^{12}}{11}+\left(\frac{1}{9} b^5 A+\frac{5}{9} a b^4 B\right) x^{10}+\left(\frac{5}{7} a b^4 A+\frac{10}{7} a^2 b^3 B\right) x^8+\left(2 a^2 b^3 A+2 a^3 b^2 B\right) x^6+\left(\frac{10}{3} a^3 b^2 A+\frac{5}{3} a^4 b B\right) x^4+\left(5 a^4 b A+a^5 B\right) x^2-a^5 A}{x}$
risch	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^9}{9}+\frac{5 B a b^4 x^9}{9}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+2 A a^2 b^3 x^5+2 B a^3 b^2 x^5+\frac{10 A a^3 b^2 x^3}{3}+\frac{5 B a^4 b x^3}{3}+$
gosper	$-\frac{-63 b^5 B x^{12}-77 A b^5 x^{10}-385 B a b^4 x^{10}-495 a A b^4 x^8-990 B a^2 b^3 x^8-1386 a^2 A b^3 x^6-1386 B a^3 b^2 x^6-2310 a^3 A b^2 x^4-1155 B a^4 b x^4-}{693 x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^5*B*x^11+1/9*A*b^5*x^9+5/9*B*a*b^4*x^9+5/7*A*a*b^4*x^7+10/7*B*a^2*b^
3*x^7+2*A*a^2*b^3*x^5+2*B*a^3*b^2*x^5+10/3*A*a^3*b^2*x^3+5/3*B*a^4*b*x^3+5*
a^4*b*A*x+a^5*B*x-a^5*A/x
```

Maxima [A]
time = 0.29, size = 116, normalized size = 1.07

$$\frac{1}{11} B b^5 x^{11}+\frac{1}{9}\left(5 B a b^4+A b^5\right) x^9+\frac{5}{7}\left(2 B a^2 b^3+A a b^4\right) x^7+2\left(B a^3 b^2+A a^2 b^3\right) x^5-\frac{A a^5}{x}+\frac{5}{3}\left(B a^4 b+2 A a^3 b^2\right) x^3+\left(B a^5+5 A a^4 b\right) x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*B*b^5*x^11 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/7*(2*B*a^2*b^3 + A*a*b^4)
*x^7 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 - A*a^5/x + 5/3*(B*a^4*b + 2*A*a^3*b^2
)*x^3 + (B*a^5 + 5*A*a^4*b)*x
```

Fricas [A]
time = 0.63, size = 121, normalized size = 1.12

$$\frac{63 B b^5 x^{12}+77\left(5 B a b^4+A b^5\right) x^{10}+495\left(2 B a^2 b^3+A a b^4\right) x^8+1386\left(B a^3 b^2+A a^2 b^3\right) x^6-693 A a^5+1155\left(B a^4 b+2 A a^3 b^2\right) x^4+693\left(B a^5+5 A a^4 b\right) x^2}{693 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/693*(63*B*b^5*x^12 + 77*(5*B*a*b^4 + A*b^5)*x^10 + 495*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 693*A*a^5 + 1155*(B*a^4*b +
2*A*a^3*b^2)*x^4 + 693*(B*a^5 + 5*A*a^4*b)*x^2)/x
```

Sympy [A]
time = 0.08, size = 126, normalized size = 1.17

$$-\frac{Aa^5}{x} + \frac{Bb^5x^{11}}{11} + x^9\left(\frac{Ab^5}{9} + \frac{5Bab^4}{9}\right) + x^7\cdot\left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7}\right) + x^5\cdot\left(2Aa^2b^3 + 2Ba^3b^2\right) + x^3\cdot\left(\frac{10Aa^3b^2}{3} + \frac{5Ba^4b}{3}\right) + x\left(5Aa^4b + Ba^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**11/11 + x**9*(A*b**5/9 + 5*B*a*b**4/9) + x**7*(5*A*a*
b**4/7 + 10*B*a**2*b**3/7) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**3*(1
0*A*a**3*b**2/3 + 5*B*a**4*b/3) + x*(5*A*a**4*b + B*a**5)
```

Giac [A]
time = 0.89, size = 120, normalized size = 1.11

$$\frac{1}{11}Bb^5x^{11} + \frac{5}{9}Bab^4x^9 + \frac{1}{9}Ab^5x^9 + \frac{10}{7}Ba^2b^3x^7 + \frac{5}{7}Aab^4x^7 + 2Ba^3b^2x^5 + 2Aa^2b^3x^5 + \frac{5}{3}Ba^4bx^3 + \frac{10}{3}Aa^3b^2x^3 + Ba^5x + 5Aa^4bx - \frac{Aa^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/11*B*b^5*x^11 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 10/7*B*a^2*b^3*x^7 + 5/
7*A*a*b^4*x^7 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/3*B*a^4*b*x^3 + 10/3*
A*a^3*b^2*x^3 + B*a^5*x + 5*A*a^4*b*x - A*a^5/x
```

Mupad [B]
time = 0.02, size = 104, normalized size = 0.96

$$x\left(B\,a^5 + 5\,A\,b\,a^4\right) + x^9\left(\frac{A\,b^5}{9} + \frac{5\,B\,a\,b^4}{9}\right) - \frac{A\,a^5}{x} + \frac{B\,b^5\,x^{11}}{11} + 2\,a^2\,b^2\,x^5\,(A\,b + B\,a) + \frac{5\,a^3\,b\,x^3\,(2\,A\,b + B\,a)}{3} + \frac{5\,a\,b^3\,x^7\,(A\,b + 2\,B\,a)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^2,x)
```

```
[Out] x*(B*a^5 + 5*A*a^4*b) + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) - (A*a^5)/x + (B*b^
5*x^11)/11 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^3*(2*A*b + B*a))/3 + (5
*a*b^3*x^7*(A*b + 2*B*a))/7
```

3.35 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{2x^2}+\frac{5}{2}a^3b(2Ab+aB)x^2+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{10}b^5Bx^{10}+a^4(5Ab+aB$$

```
[Out] -1/2*a^5*A/x^2+5/2*a^3*b*(2*A*b+B*a)*x^2+5/2*a^2*b^2*(A*b+B*a)*x^4+5/6*a*b^
3*(A*b+2*B*a)*x^6+1/8*b^4*(A*b+5*B*a)*x^8+1/10*b^5*B*x^10+a^4*(5*A*b+B*a)*l
n(x)
```

Rubi [A]
time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{2x^2}+a^4\log(x)(aB+5Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{1}{8}b^4x^8(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

```
[Out] -1/2*(a^5*A)/x^2 + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*b + a*B)*x
^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*B*
x^10)/10 + a^4*(5*A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(5a^3b(2Ab+aB) + \frac{a^5A}{x^2} + \frac{a^4(5Ab+aB)}{x} + 10a^2b^2(Ab+aB)x\right.\right.$$
$$= -\frac{a^5A}{2x^2} + \frac{5}{2}a^3b(2Ab+aB)x^2 + \frac{5}{2}a^2b^2(Ab+aB)x^4 + \frac{5}{6}ab^3(Ab+2aB)x^6 + \frac{1}{8}$$

Mathematica [A]
time = 0.03, size = 115, normalized size = 1.02

$$-\frac{a^5A}{2x^2} + \frac{5}{2}a^3b(2Ab+aB)x^2 + \frac{5}{2}a^2b^2(Ab+aB)x^4 + \frac{5}{6}ab^3(Ab+2aB)x^6 + \frac{1}{8}b^4(Ab+5aB)x^8 + \frac{1}{10}b^5Bx^{10} + \left(5a^4Ab+a^5B\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

```
[Out] -1/2*(a^5*A)/x^2 + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*b + a*B)*x
^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*B*
x^10)/10 + (5*a^4*A*b + a^5*B)*Log[x]
```

Maple [A]
time = 0.06, size = 121, normalized size = 1.07

method	result
default	$\frac{b^5Bx^{10}}{10} + \frac{Ab^5x^8}{8} + \frac{5Bab^4x^8}{8} + \frac{5Aab^4x^6}{6} + \frac{5Ba^2b^3x^6}{3} + \frac{5Aa^2b^3x^4}{2} + \frac{5Ba^3b^2x^4}{2} + 5Aa^3b^2x^2 + \frac{5Ba^4bx^2}{2} - \frac{a^5}{2a}$
norman	$\frac{\left(\frac{1}{8}b^5A+\frac{5}{8}ab^4B\right)x^{10}+\left(\frac{5}{6}ab^4A+\frac{5}{3}a^2b^3B\right)x^8+\left(\frac{5}{2}a^2b^3A+\frac{5}{2}a^3b^2B\right)x^6+\left(5a^3b^2A+\frac{5}{2}a^4bB\right)x^4-\frac{a^5A}{2}+\frac{b^5Bx^{12}}{10}}{x^2} + (5a^4bA + a^5B)$
risch	$\frac{b^5Bx^{10}}{10} + \frac{Ab^5x^8}{8} + \frac{5Bab^4x^8}{8} + \frac{5Aab^4x^6}{6} + \frac{5Ba^2b^3x^6}{3} + \frac{5Aa^2b^3x^4}{2} + \frac{5Ba^3b^2x^4}{2} + 5Aa^3b^2x^2 + \frac{5Ba^4bx^2}{2} - \frac{a^5}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^5*B*x^10+1/8*A*b^5*x^8+5/8*B*a*b^4*x^8+5/6*A*a*b^4*x^6+5/3*B*a^2*b^3
*x^6+5/2*A*a^2*b^3*x^4+5/2*B*a^3*b^2*x^4+5*A*a^3*b^2*x^2+5/2*B*a^4*b*x^2-1/
2*a^5*A/x^2+a^4*(5*A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.30, size = 120, normalized size = 1.06

$$\frac{1}{10}Bb^5x^{10} + \frac{1}{8}\left(5Bab^4 + Ab^5\right)x^8 + \frac{5}{6}\left(2Ba^2b^3 + Aab^4\right)x^6 + \frac{5}{2}\left(Ba^3b^2 + Aa^2b^3\right)x^4 - \frac{Aa^5}{2x^2} + \frac{5}{2}\left(Ba^4b + 2Aa^3b^2\right)x^2 + \frac{1}{2}\left(Ba^5 + 5Aa^4b\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*B*b^5*x^10 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + 5/6*(2*B*a^2*b^3 + A*a*b^4)
*x^6 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 - 1/2*A*a^5/x^2 + 5/2*(B*a^4*b + 2*A
*a^3*b^2)*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*log(x^2)
```

Fricas [A]
time = 0.70, size = 123, normalized size = 1.09

$$\frac{12\,Bb^{5}x^{12}+15\,(5\,Bab^{4}+Ab^{5})x^{10}+100\,(2\,Ba^{2}b^{3}+Aab^{4})x^{8}+300\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{6}-60\,Aa^{5}+300\,(Ba^{4}b+2\,Aa^{3}b^{2})x^{4}+120\,(Ba^{5}+5\,Aa^{4}b)x^{2}\log{(x)}}{120\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(12*B*b^5*x^12 + 15*(5*B*a*b^4 + A*b^5)*x^10 + 100*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 60*A*a^5 + 300*(B*a^4*b + 2*A
*a^3*b^2)*x^4 + 120*(B*a^5 + 5*A*a^4*b)*x^2*log(x))/x^2
```

Sympy [A]
time = 0.14, size = 131, normalized size = 1.16

$$-\frac{Aa^{5}}{2x^{2}}+\frac{Bb^{5}x^{10}}{10}+a^{4}\cdot(5Ab+Ba)\log{(x)}+x^{8}\left(\frac{Ab^{5}}{8}+\frac{5Bab^{4}}{8}\right)+x^{6}\cdot\left(\frac{5Aab^{4}}{6}+\frac{5Ba^{2}b^{3}}{3}\right)+x^{4}\cdot\left(\frac{5Aa^{2}b^{3}}{2}+\frac{5Ba^{3}b^{2}}{2}\right)+x^{2}\cdot\left(5Aa^{3}b^{2}+\frac{5Ba^{4}b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**10/10 + a**4*(5*A*b + B*a)*log(x) + x**8*(A*b*
*5/8 + 5*B*a*b**4/8) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**4*(5*A*a*
*2*b**3/2 + 5*B*a**3*b**2/2) + x**2*(5*A*a**3*b**2 + 5*B*a**4*b/2)
```

Giac [A]
time = 0.72, size = 145, normalized size = 1.28

$$\frac{1}{10}\,Bb^{5}x^{10}+\frac{5}{8}\,Bab^{4}x^{8}+\frac{1}{8}\,Ab^{5}x^{8}+\frac{5}{3}\,Ba^{2}b^{3}x^{6}+\frac{5}{6}\,Aab^{4}x^{6}+\frac{5}{2}\,Ba^{3}b^{2}x^{4}+\frac{5}{2}\,Aa^{2}b^{3}x^{4}+\frac{5}{2}\,Ba^{4}bx^{2}+5\,Aa^{3}b^{2}x^{2}+\frac{1}{2}\,(Ba^{5}+5\,Aa^{4}b)\log{(x^{2})}-\frac{Ba^{5}x^{2}+5\,Aa^{4}bx^{2}+Aa^{5}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/10*B*b^5*x^10 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 5/3*B*a^2*b^3*x^6 + 5/6
*A*a*b^4*x^6 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5/2*B*a^4*b*x^2 + 5*
A*a^3*b^2*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*log(x^2) - 1/2*(B*a^5*x^2 + 5*A*a^4
*b*x^2 + A*a^5)/x^2
```

Mupad [B]
time = 0.05, size = 105, normalized size = 0.93

$$x^{8}\left(\frac{A\,b^{5}}{8}+\frac{5\,B\,a\,b^{4}}{8}\right)+\ln\left(x\right)\,\left(B\,a^{5}+5\,A\,b\,a^{4}\right)-\frac{A\,a^{5}}{2\,x^{2}}+\frac{B\,b^{5}\,x^{10}}{10}+\frac{5\,a^{2}\,b^{2}\,x^{4}\,\left(A\,b+B\,a\right)}{2}+\frac{5\,a^{3}\,b\,x^{2}\,\left(2\,A\,b+B\,a\right)}{2}+\frac{5\,a\,b^{3}\,x^{6}\,\left(A\,b+2\,B\,a\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^3,x)
```

```
[Out] x^8*((A*b^5)/8 + (5*B*a*b^4)/8) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/(2*x
^2) + (B*b^5*x^10)/10 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + (5*a^3*b*x^2*(2*A*b
 + B*a))/2 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

3.36 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^4} dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{3x^3}-\frac{a^4(5Ab+aB)}{x}+5a^3b(2Ab+aB)x+\frac{10}{3}a^2b^2(Ab+aB)x^3+ab^3(Ab+2aB)x^5+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{9}b^5Bx^9$$

```
[Out] -1/3*a^5*A/x^3-a^4*(5*A*b+B*a)/x+5*a^3*b*(2*A*b+B*a)*x+10/3*a^2*b^2*(A*b+B*
a)*x^3+a*b^3*(A*b+2*B*a)*x^5+1/7*b^4*(A*b+5*B*a)*x^7+1/9*b^5*B*x^9
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{3x^3}-\frac{a^4(aB+5Ab)}{x}+5a^3bx(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(a^5*A)/x^3 - (a^4*(5*A*b + a*B))/x + 5*a^3*b*(2*A*b + a*B)*x + (10*a^
2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x^7
)/7 + (b^5*B*x^9)/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^4}dx = \int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^2}+10a^2b^2(Ab+aB)x^2+5ab^3(A\right.$$
$$= -\frac{a^5A}{3x^3}-\frac{a^4(5Ab+aB)}{x}+5a^3b(2Ab+aB)x+\frac{10}{3}a^2b^2(Ab+aB)x^3+ab^3(Ab$$

Mathematica [A]
time = 0.02, size = 110, normalized size = 1.02

$$-\frac{a^5A}{3x^3}+\frac{-5a^4Ab-a^5B}{x}+5a^3b(2Ab+aB)x+\frac{10}{3}a^2b^2(Ab+aB)x^3+ab^3(Ab+2aB)x^5+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(a^5*A)/x^3 + (-5*a^4*A*b - a^5*B)/x + 5*a^3*b*(2*A*b + a*B)*x + (10*a
^2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x^
7)/7 + (b^5*B*x^9)/9
```

Maple [A]
time = 0.06, size = 118, normalized size = 1.09

method	result
default	$\frac{b^5 B x^9}{9} + \frac{A b^5 x^7}{7} + \frac{5Ba b^4 x^7}{7} + Aa b^4 x^5 + 2B a^2 b^3 x^5 + \frac{10A a^2 b^3 x^3}{3} + \frac{10B a^3 b^2 x^3}{3} + 10a^3 b^2 Ax + 5a^4 bBx -$
norman	$\frac{\frac{b^5 B x^{12}}{9} + \left(\frac{1}{7} b^5 A + \frac{5}{7} a b^4 B\right) x^{10} + \left(a b^4 A + 2a^2 b^3 B\right) x^8 + \left(\frac{10}{3} a^2 b^3 A + \frac{10}{3} a^3 b^2 B\right) x^6 + \left(10a^3 b^2 A + 5a^4 bB\right) x^4 + \left(-5a^4 bA - a^5 B\right) x^2 - \frac{a^5 A}{3}}{x^3}$
risch	$\frac{b^5 B x^9}{9} + \frac{A b^5 x^7}{7} + \frac{5Ba b^4 x^7}{7} + Aa b^4 x^5 + 2B a^2 b^3 x^5 + \frac{10A a^2 b^3 x^3}{3} + \frac{10B a^3 b^2 x^3}{3} + 10a^3 b^2 Ax + 5a^4 bBx +$
gosper	$-\frac{-7b^5 B x^{12} - 9A b^5 x^{10} - 45Ba b^4 x^{10} - 63aA b^4 x^8 - 126B a^2 b^3 x^8 - 210a^2 A b^3 x^6 - 210B a^3 b^2 x^6 - 630a^3 A b^2 x^4 - 315B a^4 b x^4 + 315a^4 A}{63x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^5*B*x^9+1/7*A*b^5*x^7+5/7*B*a*b^4*x^7+A*a*b^4*x^5+2*B*a^2*b^3*x^5+10/
3*A*a^2*b^3*x^3+10/3*B*a^3*b^2*x^3+10*a^3*b^2*A*x+5*a^4*b*B*x-1/3*a^5*A/x^3
-a^4*(5*A*b+B*a)/x
```

Maxima [A]
time = 0.30, size = 118, normalized size = 1.09

$$\frac{1}{9} Bb^5 x^9 + \frac{1}{7} \left(5 Bab^4 + Ab^5\right) x^7 + \left(2 Ba^2 b^3 + Aab^4\right) x^5 + \frac{10}{3} \left(Ba^3 b^2 + Aa^2 b^3\right) x^3 + 5 \left(Ba^4 b + 2 Aa^3 b^2\right) x - \frac{Aa^5 + 3 \left(Ba^5 + 5 Aa^4 b\right) x^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^5*x^9 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + (2*B*a^2*b^3 + A*a*b^4)*x^5 +
 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/3*(A*a^
5 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

Fricas [A]
time = 0.79, size = 121, normalized size = 1.12

$$\frac{7 Bb^5 x^{12} + 9 \left(5 Bab^4 + Ab^5\right) x^{10} + 63 \left(2 Ba^2 b^3 + Aab^4\right) x^8 + 210 \left(Ba^3 b^2 + Aa^2 b^3\right) x^6 - 21 Aa^5 + 315 \left(Ba^4 b + 2 Aa^3 b^2\right) x^4 - 63 \left(Ba^5 + 5 Aa^4 b\right) x^2}{63 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/63*(7*B*b^5*x^12 + 9*(5*B*a*b^4 + A*b^5)*x^10 + 63*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5 + 315*(B*a^4*b + 2*A*a^3
*b^2)*x^4 - 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

Sympy [A]
time = 0.15, size = 128, normalized size = 1.19

$$\frac{Bb^5x^9}{9} + x^7\left(\frac{Ab^5}{7} + \frac{5Bab^4}{7}\right) + x^5\left(Aab^4 + 2Ba^2b^3\right) + x^3 \cdot \left(\frac{10Aa^2b^3}{3} + \frac{10Ba^3b^2}{3}\right) + x\left(10Aa^3b^2 + 5Ba^4b\right) + \frac{-Aa^5 + x^2(-15Aa^4b - 3Ba^5)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**4,x)
```

```
[Out] B*b**5*x**9/9 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**5*(A*a*b**4 + 2*B*a**2*
b**3) + x**3*(10*A*a**2*b**3/3 + 10*B*a**3*b**2/3) + x*(10*A*a**3*b**2 + 5*
B*a**4*b) + (-A*a**5 + x**2*(-15*A*a**4*b - 3*B*a**5))/(3*x**3)
```

Giac [A]
time = 0.74, size = 122, normalized size = 1.13

$$\frac{1}{9}\,Bb^5x^9 + \frac{5}{7}\,Bab^4x^7 + \frac{1}{7}\,Ab^5x^7 + 2\,Ba^2b^3x^5 + Aab^4x^5 + \frac{10}{3}\,Ba^3b^2x^3 + \frac{10}{3}\,Aa^2b^3x^3 + 5\,Ba^4bx + 10\,Aa^3b^2x - \frac{3\,Ba^5x^2 + 15\,Aa^4bx^2 + Aa^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*B*b^5*x^9 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 2*B*a^2*b^3*x^5 + A*a*b^4
*x^5 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*B*a^4*b*x + 10*A*a^3*b^2
*x - 1/3*(3*B*a^5*x^2 + 15*A*a^4*b*x^2 + A*a^5)/x^3
```

Mupad [B]
time = 0.02, size = 106, normalized size = 0.98

$$x^7\left(\frac{A\,b^5}{7} + \frac{5\,B\,a\,b^4}{7}\right) - \frac{\frac{A\,a^5}{3} + x^2\,(B\,a^5 + 5\,A\,b\,a^4)}{x^3} + \frac{B\,b^5\,x^9}{9} + \frac{10\,a^2\,b^2\,x^3\,(A\,b + B\,a)}{3} + 5\,a^3\,b\,x\,(2\,A\,b + B\,a) + a\,b^3\,x^5\,(A\,b + 2\,B\,a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^4,x)
```

```
[Out] x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/3 + x^2*(B*a^5 + 5*A*a^4*b))/x^3
 + (B*b^5*x^9)/9 + (10*a^2*b^2*x^3*(A*b + B*a))/3 + 5*a^3*b*x*(2*A*b + B*a)
 + a*b^3*x^5*(A*b + 2*B*a)
```

3.37 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=112

$$-\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{2x^2}+5a^2b^2(Ab+aB)x^2+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{8}b^5Bx^8+5a^3b(2Ab+aB)\log(x)$$

```
[Out] -1/4*a^5*A/x^4-1/2*a^4*(5*A*b+B*a)/x^2+5*a^2*b^2*(A*b+B*a)*x^2+5/4*a*b^3*(A
*b+2*B*a)*x^4+1/6*b^4*(A*b+5*B*a)*x^6+1/8*b^5*B*x^8+5*a^3*b*(2*A*b+B*a)*ln(
x)
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)+5a^2b^2x^2(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

```
[Out] -1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b + a*B)*x^2
+ (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*x^8)/8
 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^5} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^3} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(10a^2b^2(Ab+aB) + \frac{a^5A}{x^3} + \frac{a^4(5Ab+aB)}{x^2} + \frac{5a^3b(2Ab+aB)}{x} + \right.\right.$$
$$= -\frac{a^5A}{4x^4} - \frac{a^4(5Ab+aB)}{2x^2} + 5a^2b^2(Ab+aB)x^2 + \frac{5}{4}ab^3(Ab+2aB)x^4 + \frac{1}{6}b^4(Ab$$

Mathematica [A]
time = 0.03, size = 112, normalized size = 1.00

$$-\frac{a^5A}{4x^4} - \frac{a^4(5Ab+aB)}{2x^2} + 5a^2b^2(Ab+aB)x^2 + \frac{5}{4}ab^3(Ab+2aB)x^4 + \frac{1}{6}b^4(Ab+5aB)x^6 + \frac{1}{8}b^5Bx^8 + 5a^3b(2Ab+aB)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

```
[Out] -1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b + a*B)*x^2
+ (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*x^8)/8
 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Maple [A]
time = 0.07, size = 117, normalized size = 1.04

method	result
default	$\frac{b^5Bx^8}{8} + \frac{Ab^5x^6}{6} + \frac{5Bab^4x^6}{6} + \frac{5Aab^4x^4}{4} + \frac{5Ba^2b^3x^4}{2} + 5Aa^2b^3x^2 + 5Ba^3b^2x^2 - \frac{a^5A}{4x^4} - \frac{a^4(5Ab+Ba)}{2x^2} + 5a^3b$
norman	$\frac{\left(\frac{1}{6}b^5A+\frac{5}{6}ab^4B\right)x^{10}+\left(\frac{5}{4}ab^4A+\frac{5}{2}a^2b^3B\right)x^8+\left(-\frac{5}{2}a^4bA-\frac{1}{2}a^5B\right)x^2+\left(5a^2b^3A+5a^3b^2B\right)x^6-\frac{a^5A}{4}+\frac{b^5Bx^{12}}{8}}{x^4} + (10a^3b^2A + 5a^4b$
risch	$\frac{b^5Bx^8}{8} + \frac{Ab^5x^6}{6} + \frac{5Bab^4x^6}{6} + \frac{5Aab^4x^4}{4} + \frac{5Ba^2b^3x^4}{2} + 5Aa^2b^3x^2 + 5Ba^3b^2x^2 + \frac{\left(-\frac{5}{2}a^4bA-\frac{1}{2}a^5B\right)x^2-\frac{a^5A}{4}}{x^4} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^5*B*x^8+1/6*A*b^5*x^6+5/6*B*a*b^4*x^6+5/4*A*a*b^4*x^4+5/2*B*a^2*b^3*x
^4+5*A*a^2*b^3*x^2+5*B*a^3*b^2*x^2-1/4*a^5*A/x^4-1/2*a^4*(5*A*b+B*a)/x^2+5*
a^3*b*(2*A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.29, size = 122, normalized size = 1.09

$$\frac{1}{8}Bb^5x^8 + \frac{1}{6}\left(5Bab^4 + Ab^5\right)x^6 + \frac{5}{4}\left(2Ba^2b^3 + Aab^4\right)x^4 + 5\left(Ba^3b^2 + Aa^2b^3\right)x^2 + \frac{5}{2}\left(Ba^4b + 2Aa^3b^2\right)\log\left(x^2\right) - \frac{Aa^5 + 2\left(Ba^5 + 5Aa^4b\right)x^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^5*x^8 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/4*(2*B*a^2*b^3 + A*a*b^4)*x
^4 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*log(x^2) -
 1/4*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^2)/x^4
```

Fricas [A]
time = 0.71, size = 123, normalized size = 1.10

$$\frac{3\,Bb^5x^{12}+4\,(5\,Bab^4+Ab^5)x^{10}+30\,(2\,Ba^2b^3+Aab^4)x^8+120\,(Ba^3b^2+Aa^2b^3)x^6-6\,Aa^5+120\,(Ba^4b+2\,Aa^3b^2)x^4\log(x)-12\,(Ba^5+5\,Aa^4b)x^2}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^5*x^12 + 4*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 6*A*a^5 + 120*(B*a^4*b + 2*A*a^3*
b^2)*x^4*log(x) - 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^4
```

Sympy [A]
time = 0.33, size = 128, normalized size = 1.14

$$\frac{Bb^5x^8}{8}+5a^3b(2Ab+Ba)\log(x)+x^6\left(\frac{Ab^5}{6}+\frac{5Bab^4}{6}\right)+x^4\cdot\left(\frac{5Aab^4}{4}+\frac{5Ba^2b^3}{2}\right)+x^2\cdot(5Aa^2b^3+5Ba^3b^2)+\frac{-Aa^5+x^2(-10Aa^4b-2Ba^5)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**5,x)
```

```
[Out] B*b**5*x**8/8 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b**4
/6) + x**4*(5*A*a*b**4/4 + 5*B*a**2*b**3/2) + x**2*(5*A*a**2*b**3 + 5*B*a**
3*b**2) + (-A*a**5 + x**2*(-10*A*a**4*b - 2*B*a**5))/(4*x**4)
```

Giac [A]
time = 0.70, size = 149, normalized size = 1.33

$$\frac{1}{8}\,Bb^5x^8+\frac{5}{6}\,Bab^4x^6+\frac{1}{6}\,Ab^5x^6+\frac{5}{2}\,Ba^2b^3x^4+\frac{5}{4}\,Aab^4x^4+5\,Ba^3b^2x^2+5\,Aa^2b^3x^2+\frac{5}{2}\,(Ba^4b+2\,Aa^3b^2)\log(x^2)-\frac{15\,Ba^4bx^4+30\,Aa^3b^2x^4+2\,Ba^5x^2+10\,Aa^4bx^2+Aa^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*B*b^5*x^8 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 5/2*B*a^2*b^3*x^4 + 5/4*A
*a*b^4*x^4 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 + 5/2*(B*a^4*b + 2*A*a^3*b^2
)*log(x^2) - 1/4*(15*B*a^4*b*x^4 + 30*A*a^3*b^2*x^4 + 2*B*a^5*x^2 + 10*A*a^
4*b*x^2 + A*a^5)/x^4
```

Mupad [B]
time = 0.03, size = 113, normalized size = 1.01

$$\ln(x)\,\left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right)-\frac{\frac{A\,a^5}{4}+x^2\,\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)}{x^4}+x^6\,\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+\frac{B\,b^5\,x^8}{8}+5\,a^2\,b^2\,x^2\,(A\,b+B\,a)+\frac{5\,a\,b^3\,x^4\,(A\,b+2\,B\,a)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^5,x)
```

```
[Out] log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - ((A*a^5)/4 + x^2*((B*a^5)/2 + (5*A*a^4*
b)/2))/x^4 + x^6*((A*b^5)/6 + (5*B*a*b^4)/6) + (B*b^5*x^8)/8 + 5*a^2*b^2*x^
2*(A*b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4
```

3.38 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=111

$$-\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{3x^3}-\frac{5a^3b(2Ab+aB)}{x}+10a^2b^2(Ab+aB)x+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{5}b^4(Ab+5aB)x^5+\frac{1}{7}b^5Bx$$

```
[Out] -1/5*a^5*A/x^5-1/3*a^4*(5*A*b+B*a)/x^3-5*a^3*b*(2*A*b+B*a)/x+10*a^2*b^2*(A*
b+B*a)*x+5/3*a*b^3*(A*b+2*B*a)*x^3+1/5*b^4*(A*b+5*B*a)*x^5+1/7*b^5*B*x^7
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{5x^5}-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{x}+10a^2b^2x(aB+Ab)+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/x
+ 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*
a*B)*x^5)/5 + (b^5*B*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5\,(A+Bx^2)}{x^6}\,dx &= \int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^4}+\frac{5a^3b(2Ab+aB)}{x^2}+5ab^3(Ab\right.\\ &= -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{3x^3}-\frac{5a^3b(2Ab+aB)}{x}+10a^2b^2(Ab+aB)x+\frac{5}{3}ab^3(Ab\end{aligned}$$

Mathematica [A]
time = 0.03, size = 111, normalized size = 1.00

$$-\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{3x^3}-\frac{5a^3b(2Ab+aB)}{x}+10a^2b^2(Ab+aB)x+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{5}b^4(Ab+5aB)x^5+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/x
+ 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*
a*B)*x^5)/5 + (b^5*B*x^7)/7
```

Maple [A]
time = 0.06, size = 113, normalized size = 1.02

method	result
default	$\frac{b^5 B x^7}{7} + \frac{A b^5 x^5}{5} + B a b^4 x^5 + \frac{5 A a b^4 x^3}{3} + \frac{10 B a^2 b^3 x^3}{3} + 10 a^2 b^3 A x + 10 a^3 b^2 B x - \frac{a^5 A}{5 x^5} - \frac{a^4 (5 A b + B a)}{3 x^3} - 5 a^3 b$
norman	$\frac{\frac{b^5 B x^{12}}{7} + (\frac{1}{5} b^5 A + a b^4 B) x^{10} + (\frac{5}{3} a b^4 A + \frac{10}{3} a^2 b^3 B) x^8 + (10 a^2 b^3 A + 10 a^3 b^2 B) x^6 + (-10 a^3 b^2 A - 5 a^4 b B) x^4 + (-\frac{5}{3} a^4 b A - \frac{1}{3} a^5 B) x^2 - \frac{a^5 A}{5}}{x^5}$
risch	$\frac{b^5 B x^7}{7} + \frac{A b^5 x^5}{5} + B a b^4 x^5 + \frac{5 A a b^4 x^3}{3} + \frac{10 B a^2 b^3 x^3}{3} + 10 a^2 b^3 A x + 10 a^3 b^2 B x + \frac{(-10 a^3 b^2 A - 5 a^4 b B) x^4 + (-\frac{5}{3} a}{x^5}$
gosper	$-\frac{-15 b^5 B x^{12} - 21 A b^5 x^{10} - 105 B a b^4 x^{10} - 175 a A b^4 x^8 - 350 B a^2 b^3 x^8 - 1050 a^2 A b^3 x^6 - 1050 B a^3 b^2 x^6 + 1050 a^3 A b^2 x^4 + 525 B a^4 b x^4 + 1}{105 x^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^5*B*x^7+1/5*A*b^5*x^5+B*a*b^4*x^5+5/3*A*a*b^4*x^3+10/3*B*a^2*b^3*x^3+
10*a^2*b^3*A*x+10*a^3*b^2*B*x-1/5*a^5*A/x^5-1/3*a^4*(5*A*b+B*a)/x^3-5*a^3*b
*(2*A*b+B*a)/x
```

Maxima [A]
time = 0.28, size = 120, normalized size = 1.08

$$\frac{1}{7} B b^5 x^7 + \frac{1}{5} \left(5 B a b^4 + A b^5\right) x^5 + \frac{5}{3} \left(2 B a^2 b^3 + A a b^4\right) x^3 + 10 \left(B a^3 b^2 + A a^2 b^3\right) x - \frac{3 A a^5 + 75 \left(B a^4 b + 2 A a^3 b^2\right) x^4 + 5 \left(B a^5 + 5 A a^4 b\right) x^2}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^5*x^7 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*x
^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/15*(3*A*a^5 + 75*(B*a^4*b + 2*A*a^3*b
^2)*x^4 + 5*(B*a^5 + 5*A*a^4*b)*x^2)/x^5
```

Fricas [A]
time = 0.75, size = 121, normalized size = 1.09

$$\frac{15 B b^5 x^{12} + 21 \left(5 B a b^4 + A b^5\right) x^{10} + 175 \left(2 B a^2 b^3 + A a b^4\right) x^8 + 1050 \left(B a^3 b^2 + A a^2 b^3\right) x^6 - 21 A a^5 - 525 \left(B a^4 b + 2 A a^3 b^2\right) x^4 - 35 \left(B a^5 + 5 A a^4 b\right) x^2}{105 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 175*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5 - 525*(B*a^4*b + 2*
A*a^3*b^2)*x^4 - 35*(B*a^5 + 5*A*a^4*b)*x^2)/x^5
```

Sympy [A]
time = 0.37, size = 129, normalized size = 1.16

$$\frac{Bb^5x^7}{7} + x^5\left(\frac{Ab^5}{5} + Bab^4\right) + x^3\cdot\left(\frac{5Aab^4}{3} + \frac{10Ba^2b^3}{3}\right) + x\left(10Aa^2b^3 + 10Ba^3b^2\right) + \frac{-3Aa^5 + x^4(-150Aa^3b^2 - 75Ba^4b) + x^2(-25Aa^4b - 5Ba^5)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**6,x)
```

```
[Out] B*b**5*x**7/7 + x**5*(A*b**5/5 + B*a*b**4) + x**3*(5*A*a*b**4/3 + 10*B*a**2
*b**3/3) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) + (-3*A*a**5 + x**4*(-150*A*
a**3*b**2 - 75*B*a**4*b) + x**2*(-25*A*a**4*b - 5*B*a**5))/(15*x**5)
```

Giac [A]
time = 0.65, size = 123, normalized size = 1.11

$$\frac{1}{7}Bb^5x^7 + Bab^4x^5 + \frac{1}{5}Ab^5x^5 + \frac{10}{3}Ba^2b^3x^3 + \frac{5}{3}Aab^4x^3 + 10\,Ba^3b^2x + 10\,Aa^2b^3x - \frac{75\,Ba^4bx^4 + 150\,Aa^3b^2x^4 + 5\,Ba^5x^2 + 25\,Aa^4bx^2 + 3\,Aa^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/7*B*b^5*x^7 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 10/3*B*a^2*b^3*x^3 + 5/3*A*a*
b^4*x^3 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/15*(75*B*a^4*b*x^4 + 150*A*a^
3*b^2*x^4 + 5*B*a^5*x^2 + 25*A*a^4*b*x^2 + 3*A*a^5)/x^5
```

Mupad [B]
time = 0.04, size = 111, normalized size = 1.00

$$x^5\left(\frac{A\,b^5}{5} + B\,a\,b^4\right) - \frac{\frac{A\,a^5}{5} + x^4\left(5\,B\,a^4\,b + 10\,A\,a^3\,b^2\right) + x^2\left(\frac{B\,a^5}{3} + \frac{5\,A\,b\,a^4}{3}\right)}{x^5} + \frac{B\,b^5\,x^7}{7} + 10\,a^2\,b^2\,x\,(A\,b + B\,a) + \frac{5\,a\,b^3\,x^3\,(A\,b + 2\,B\,a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^6,x)
```

```
[Out] x^5*((A*b^5)/5 + B*a*b^4) - ((A*a^5)/5 + x^4*(10*A*a^3*b^2 + 5*B*a^4*b) + x
^2*((B*a^5)/3 + (5*A*a^4*b)/3))/x^5 + (B*b^5*x^7)/7 + 10*a^2*b^2*x*(A*b + B
*a) + (5*a*b^3*x^3*(A*b + 2*B*a))/3
```

3.39 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=114

$$-\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{2x^2}+\frac{5}{2}ab^3(Ab+2aB)x^2+\frac{1}{4}b^4(Ab+5aB)x^4+\frac{1}{6}b^5Bx^6+10a^2b^2(Ab+aB)\,\mathrm{lo}$$

```
[Out] -1/6*a^5*A/x^6-1/4*a^4*(5*A*b+B*a)/x^4-5/2*a^3*b*(2*A*b+B*a)/x^2+5/2*a*b^3*
(A*b+2*B*a)*x^2+1/4*b^4*(A*b+5*B*a)*x^4+1/6*b^5*B*x^6+10*a^2*b^2*(A*b+B*a)*
ln(x)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{6x^6}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2\log(x)(aB+Ab)+\frac{1}{4}b^4x^4(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{6}b^5Bx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

```
[Out] -1/6*(a^5*A)/x^6 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4*(A*b + 5*a*B)*x^4)/4 + (b^5*B*
x^6)/6 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(5ab^3(Ab+2aB) + \frac{a^5A}{x^4} + \frac{a^4(5Ab+aB)}{x^3} + \frac{5a^3b(2Ab+aB)}{x^2} + \right.\right.$$
$$= -\frac{a^5A}{6x^6} - \frac{a^4(5Ab+aB)}{4x^4} - \frac{5a^3b(2Ab+aB)}{2x^2} + \frac{5}{2}ab^3(Ab+2aB)x^2 + \frac{1}{4}b^4(Ab$$

Mathematica [A]
time = 0.03, size = 116, normalized size = 1.02

$$\frac{1}{12}\left(-\frac{60a^3Ab^2}{x^2} + 60a^2b^3Bx^2 + 15ab^4x^2(2A+Bx^2) - \frac{15a^4b(A+2Bx^2)}{x^4} + b^5x^4(3A+2Bx^2) - \frac{a^5(2A+3Bx^2)}{x^6} + 120a^2b^2(Ab+aB)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

```
[Out] ((-60*a^3*A*b^2)/x^2 + 60*a^2*b^3*B*x^2 + 15*a*b^4*x^2*(2*A + B*x^2) - (15*
a^4*b*(A + 2*B*x^2))/x^4 + b^5*x^4*(3*A + 2*B*x^2) - (a^5*(2*A + 3*B*x^2))/
x^6 + 120*a^2*b^2*(A*b + a*B)*Log[x])/12
```

Maple [A]
time = 0.07, size = 111, normalized size = 0.97

method	result
default	$\frac{b^5Bx^6}{6} + \frac{Ab^5x^4}{4} + \frac{5Bab^4x^4}{4} + \frac{5Aab^4x^2}{2} + 5Ba^2b^3x^2 - \frac{a^4(5Ab+Ba)}{4x^4} - \frac{a^5A}{6x^6} - \frac{5a^3b(2Ab+Ba)}{2x^2} + 10a^2b^2(Ab+$
norman	$\frac{(\frac{1}{4}b^5A+\frac{5}{4}ab^4B)x^{10}+(\frac{5}{2}ab^4A+5a^2b^3B)x^8+(-5a^3b^2A-\frac{5}{2}a^4bB)x^4+(-\frac{5}{4}a^4bA-\frac{1}{4}a^5B)x^2-\frac{a^5A}{6}+\frac{b^5Bx^{12}}{6}}{x^6} + (10a^2b^3A+10$
risch	$\frac{b^5Bx^6}{6} + \frac{Ab^5x^4}{4} + \frac{5Bab^4x^4}{4} + \frac{5Aab^4x^2}{2} + 5Ba^2b^3x^2 + \frac{(-5a^3b^2A-\frac{5}{2}a^4bB)x^4+(-\frac{5}{4}a^4bA-\frac{1}{4}a^5B)x^2-\frac{a^5A}{6}}{x^6} + 10A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^5*B*x^6+1/4*A*b^5*x^4+5/4*B*a*b^4*x^4+5/2*A*a*b^4*x^2+5*B*a^2*b^3*x^2
-1/4*a^4*(5*A*b+B*a)/x^4-1/6*a^5*A/x^6-5/2*a^3*b*(2*A*b+B*a)/x^2+10*a^2*b^2
*(A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.30, size = 123, normalized size = 1.08

$$\frac{1}{6}Bb^5x^6 + \frac{1}{4}(5Bab^4 + Ab^5)x^4 + \frac{5}{2}(2Ba^2b^3 + Aab^4)x^2 + 5(Ba^3b^2 + Aa^2b^3)\log(x^2) - \frac{2Aa^5 + 30(Ba^4b + 2Aa^3b^2)x^4 + 3(Ba^5 + 5Aa^4b)x^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="maxima")

[Out] 1/6*B*b^5*x^6 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*x^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*log(x^2) - 1/12*(2*A*a^5 + 30*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^6

Fricas [A]
time = 0.96, size = 123, normalized size = 1.08

$$\frac{2\,Bb^5x^{12}+3\,(5\,Bab^4+Ab^5)x^{10}+30\,(2\,Ba^2b^3+Aab^4)x^8+120\,(Ba^3b^2+Aa^2b^3)x^6\log(x)-2\,Aa^5-30\,(Ba^4b+2\,Aa^3b^2)x^4-3\,(Ba^5+5\,Aa^4b)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="fricas")

[Out] 1/12*(2*B*b^5*x^12 + 3*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3 + A*a*b^4)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6*log(x) - 2*A*a^5 - 30*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^6

Sympy [A]
time = 0.72, size = 128, normalized size = 1.12

$$\frac{Bb^5x^6}{6}+10a^2b^2(Ab+Ba)\log(x)+x^4\left(\frac{Ab^5}{4}+\frac{5Bab^4}{4}\right)+x^2\cdot\left(\frac{5Aab^4}{2}+5Ba^2b^3\right)+\frac{-2Aa^5+x^4(-60Aa^3b^2-30Ba^4b)+x^2(-15Aa^4b-3Ba^5)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**7,x)

[Out] B*b**5*x**6/6 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**4*(A*b**5/4 + 5*B*a*b**4/4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*b**3) + (-2*A*a**5 + x**4*(-60*A*a**3*b**2 - 30*B*a**4*b) + x**2*(-15*A*a**4*b - 3*B*a**5))/(12*x**6)

Giac [A]
time = 0.65, size = 151, normalized size = 1.32

$$\frac{1}{6}Bb^5x^6+\frac{5}{4}Bab^4x^4+\frac{1}{4}Ab^5x^4+5\,Ba^2b^3x^2+\frac{5}{2}Aab^4x^2+5\,(Ba^3b^2+Aa^2b^3)\log(x^2)-\frac{110\,Ba^3b^2x^6+110\,Aa^2b^3x^6+30\,Ba^4bx^4+60\,Aa^3b^2x^4+3\,Ba^5x^2+15\,Aa^4bx^2+2\,Aa^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="giac")

[Out] 1/6*B*b^5*x^6 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 5*B*a^2*b^3*x^2 + 5/2*A*a*b^4*x^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*log(x^2) - 1/12*(110*B*a^3*b^2*x^6 + 110*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a^3*b^2*x^4 + 3*B*a^5*x^2 + 15*A*a^4*b*x^2 + 2*A*a^5)/x^6

Mupad [B]
time = 0.04, size = 118, normalized size = 1.04

$$x^4\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)-\frac{\frac{A\,a^5}{6}+x^4\,\left(\frac{5\,B\,a^4\,b}{2}+5\,A\,a^3\,b^2\right)+x^2\,\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)}{x^6}+\ln(x)\,\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right)+\frac{B\,b^5\,x^6}{6}+\frac{5\,a\,b^3\,x^2\,(A\,b+2\,B\,a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^7,x)
```

```
[Out] x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/6 + x^4*(5*A*a^3*b^2 + (5*B*a^4*
b)/2) + x^2*((B*a^5)/4 + (5*A*a^4*b)/4))/x^6 + log(x)*(10*A*a^2*b^3 + 10*B*
a^3*b^2) + (B*b^5*x^6)/6 + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

3.40 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=111

$$-\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{3x^3}-\frac{10a^2b^2(Ab+aB)}{x}+5ab^3(Ab+2aB)x+\frac{1}{3}b^4(Ab+5aB)x^3+\frac{1}{5}b^5Bx^5$$

```
[Out] -1/7*a^5*A/x^7-1/5*a^4*(5*A*b+B*a)/x^5-5/3*a^3*b*(2*A*b+B*a)/x^3-10*a^2*b^2
*(A*b+B*a)/x+5*a*b^3*(A*b+2*B*a)*x+1/3*b^4*(A*b+5*B*a)*x^3+1/5*b^5*B*x^5
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

```
[Out] -1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b +
5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^8}\,dx &= \int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^8}+\frac{a^4(5Ab+aB)}{x^6}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab\right.\\ &= -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{3x^3}-\frac{10a^2b^2(Ab+aB)}{x}+5ab^3(Ab+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 111, normalized size = 1.00

$$-\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{3x^3}-\frac{10a^2b^2(Ab+aB)}{x}+5ab^3(Ab+2aB)x+\frac{1}{3}b^4(Ab+5aB)x^3+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

```
[Out] -1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b +
5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

Maple [A]
time = 0.06, size = 108, normalized size = 0.97

method	result
default	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^3}{3}+\frac{5Ba b^4 x^3}{3}+5a b^4 Ax+10a^2b^3Bx-\frac{a^4(5Ab+Ba)}{5x^5}-\frac{a^5 A}{7x^7}-\frac{5a^3b(2Ab+Ba)}{3x^3}-\frac{10a^2b^2(Ab+Ba}{x}$
risch	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^3}{3}+\frac{5Ba b^4 x^3}{3}+5a b^4 Ax+10a^2b^3Bx+\frac{(-10a^2b^3A-10a^3b^2B)x^6+(-\frac{10}{3}a^3b^2A-\frac{5}{3}a^4bB)x^4+(-a^4bA-}{x^7}$
norman	$\frac{\frac{b^5 B x^{12}}{5}+(\frac{1}{3}b^5A+\frac{5}{3}a b^4B)x^{10}+(5a b^4A+10a^2b^3B)x^8+(-10a^2b^3A-10a^3b^2B)x^6+(-\frac{10}{3}a^3b^2A-\frac{5}{3}a^4bB)x^4+(-a^4bA-\frac{1}{5}a^5B)x^2-}{x^7}$
gosper	$-\frac{-21b^5 B x^{12}-35A b^5 x^{10}-175Ba b^4 x^{10}-525aA b^4 x^8-1050B a^2b^3x^8+1050a^2A b^3x^6+1050B a^3b^2x^6+350a^3A b^2x^4+175B a^4b x^4+}{105x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^5*B*x^5+1/3*A*b^5*x^3+5/3*B*a*b^4*x^3+5*a*b^4*A*x+10*a^2*b^3*B*x-1/5*
a^4*(5*A*b+B*a)/x^5-1/7*a^5*A/x^7-5/3*a^3*b*(2*A*b+B*a)/x^3-10*a^2*b^2*(A*b
+B*a)/x
```

Maxima [A]
time = 0.30, size = 120, normalized size = 1.08

$$\frac{1}{5}\,Bb^5x^5+\frac{1}{3}\,(5\,Bab^4+Ab^5)x^3+5\,(2\,Ba^2b^3+Aab^4)x-\frac{1050\,(Ba^3b^2+Aa^2b^3)x^6+15\,Aa^5+175\,(Ba^4b+2\,Aa^3b^2)x^4+21\,(Ba^5+5\,Aa^4b)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^5*x^5 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4)*x -
 1/105*(1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 15*A*a^5 + 175*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 21*(B*a^5 + 5*A*a^4*b)*x^2)/x^7
```

Fricas [A]
time = 0.98, size = 121, normalized size = 1.09

$$\frac{21\,Bb^5x^{12}+35\,(5\,Bab^4+Ab^5)x^{10}+525\,(2\,Ba^2b^3+Aab^4)x^8-1050\,(Ba^3b^2+Aa^2b^3)x^6-15\,Aa^5-175\,(Ba^4b+2\,Aa^3b^2)x^4-21\,(Ba^5+5\,Aa^4b)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(21*B*b^5*x^12 + 35*(5*B*a*b^4 + A*b^5)*x^10 + 525*(2*B*a^2*b^3 + A*a
*b^4)*x^8 - 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 15*A*a^5 - 175*(B*a^4*b + 2*
A*a^3*b^2)*x^4 - 21*(B*a^5 + 5*A*a^4*b)*x^2)/x^7
```

Sympy [A]
time = 0.85, size = 131, normalized size = 1.18

$$\frac{Bb^5x^5}{5}+x^3\left(\frac{Ab^5}{3}+\frac{5Bab^4}{3}\right)+x\left(5Aab^4+10Ba^2b^3\right)+\frac{-15Aa^5+x^6(-1050Aa^2b^3-1050Ba^3b^2)+x^4(-350Aa^3b^2-175Ba^4b)+x^2(-105Aa^4b-21Ba^5)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**8,x)
```

```
[Out] B*b**5*x**5/5 + x**3*(A*b**5/3 + 5*B*a*b**4/3) + x*(5*A*a*b**4 + 10*B*a**2*
b**3) + (-15*A*a**5 + x**6*(-1050*A*a**2*b**3 - 1050*B*a**3*b**2) + x**4*(-
350*A*a**3*b**2 - 175*B*a**4*b) + x**2*(-105*A*a**4*b - 21*B*a**5))/(105*x*
*7)
```

Giac [A]
time = 0.68, size = 124, normalized size = 1.12

$$\frac{1}{5}Bb^5x^5+\frac{5}{3}Bab^4x^3+\frac{1}{3}Ab^5x^3+10Ba^2b^3x+5Aab^4x-\frac{1050Ba^3b^2x^6+1050Aa^2b^3x^6+175Ba^4bx^4+350Aa^3b^2x^4+21Ba^5x^2+105Aa^4bx^2+15Aa^5}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/5*B*b^5*x^5 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 10*B*a^2*b^3*x + 5*A*a*b^
4*x - 1/105*(1050*B*a^3*b^2*x^6 + 1050*A*a^2*b^3*x^6 + 175*B*a^4*b*x^4 + 35
0*A*a^3*b^2*x^4 + 21*B*a^5*x^2 + 105*A*a^4*b*x^2 + 15*A*a^5)/x^7
```

Mupad [B]
time = 0.05, size = 116, normalized size = 1.05

$$x^3\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)-\frac{\frac{A\,a^5}{7}+x^4\left(\frac{5\,B\,a^4\,b}{3}+\frac{10\,A\,a^3\,b^2}{3}\right)+x^2\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)+x^6\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right)}{x^7}+\frac{B\,b^5\,x^5}{5}+5\,a\,b^3\,x\,\left(A\,b+2\,B\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^8,x)
```

```
[Out] x^3*((A*b^5)/3 + (5*B*a*b^4)/3) - ((A*a^5)/7 + x^4*((10*A*a^3*b^2)/3 + (5*B
*a^4*b)/3) + x^2*((B*a^5)/5 + A*a^4*b) + x^6*(10*A*a^2*b^3 + 10*B*a^3*b^2))
/x^7 + (B*b^5*x^5)/5 + 5*a*b^3*x*(A*b + 2*B*a)
```

3.41 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9} \, dx$

Optimal. Leaf size=112

$$-\frac{a^5 A}{8x^8}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{4x^4}-\frac{5a^2b^2(Ab+aB)}{x^2}+\frac{1}{2}b^4(Ab+5aB)x^2+\frac{1}{4}b^5Bx^4+5ab^3(Ab+2aB)\log$$

```
[Out] -1/8*a^5*A/x^8-1/6*a^4*(5*A*b+B*a)/x^6-5/4*a^3*b*(2*A*b+B*a)/x^4-5*a^2*b^2*
(A*b+B*a)/x^2+1/2*b^4*(A*b+5*B*a)*x^2+1/4*b^5*B*x^4+5*a*b^3*(A*b+2*B*a)*ln(
x)
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{8x^8}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{2}b^4x^2(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

```
[Out] -1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(4
*x^4) - (5*a^2*b^2*(A*b + a*B))/x^2 + (b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^
4)/4 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(b^4(Ab+5aB) + \frac{a^5A}{x^5} + \frac{a^4(5Ab+aB)}{x^4} + \frac{5a^3b(2Ab+aB)}{x^3} + 10a$$
$$= -\frac{a^5A}{8x^8} - \frac{a^4(5Ab+aB)}{6x^6} - \frac{5a^3b(2Ab+aB)}{4x^4} - \frac{5a^2b^2(Ab+aB)}{x^2} + \frac{1}{2}b^4(Ab+5a$$

Mathematica [A]
time = 0.04, size = 116, normalized size = 1.04

$$-\frac{120a^2Ab^3x^6 - 60ab^4Bx^{10} - 6b^5x^{10}(2A+Bx^2) + 60a^3b^2x^4(A+2Bx^2) + 10a^4bx^2(2A+3Bx^2) + a^5(3A+4Bx^2)}{24x^8} + 5ab^3(Ab+2aB)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

```
[Out] -1/24*(120*a^2*A*b^3*x^6 - 60*a*b^4*B*x^10 - 6*b^5*x^10*(2*A + B*x^2) + 60*
a^3*b^2*x^4*(A + 2*B*x^2) + 10*a^4*b*x^2*(2*A + 3*B*x^2) + a^5*(3*A + 4*B*x
^2))/x^8 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Maple [A]
time = 0.09, size = 106, normalized size = 0.95

method	result
default	$\frac{b^5Bx^4}{4} + \frac{Ab^5x^2}{2} + \frac{5Bab^4x^2}{2} - \frac{5a^3b(2Ab+Ba)}{4x^4} - \frac{a^4(5Ab+Ba)}{6x^6} - \frac{5a^2b^2(Ab+Ba)}{x^2} - \frac{a^5A}{8x^8} + 5ab^3(Ab+2Ba)\ln(x$
norman	$\frac{(\frac{1}{2}b^5A+\frac{5}{2}ab^4B)x^{10}+(-\frac{5}{2}a^3b^2A-\frac{5}{4}a^4bB)x^4+(-\frac{5}{6}a^4bA-\frac{1}{6}a^5B)x^2+(-5a^2b^3A-5a^3b^2B)x^6-\frac{a^5A}{8}+\frac{b^5Bx^{12}}{4}}{x^8} + (5ab^4A+10a$
risch	$\frac{b^5Bx^4}{4} + \frac{Ab^5x^2}{2} + \frac{5Bab^4x^2}{2} + \frac{b^5A^2}{4B} + \frac{5ab^4A}{2} + \frac{25a^2b^3B}{4} + \frac{(-5a^2b^3A-5a^3b^2B)x^6+(-\frac{5}{2}a^3b^2A-\frac{5}{4}a^4bB)x^4+(-\frac{5}{6}a^4bA-}{x^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^5*B*x^4+1/2*A*b^5*x^2+5/2*B*a*b^4*x^2-5/4*a^3*b*(2*A*b+B*a)/x^4-1/6*a
^4*(5*A*b+B*a)/x^6-5*a^2*b^2*(A*b+B*a)/x^2-1/8*a^5*A/x^8+5*a*b^3*(A*b+2*B*a
)*ln(x)
```

Maxima [A]
time = 0.29, size = 123, normalized size = 1.10

$$\frac{1}{4}Bb^5x^4 + \frac{1}{2}\left(5Bab^4 + Ab^5\right)x^2 + \frac{5}{2}\left(2Ba^2b^3 + Aab^4\right)\log\left(x^2\right) - \frac{120\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 3Aa^5 + 30\left(Ba^4b + 2Aa^3b^2\right)x^4 + 4\left(Ba^5 + 5Aa^4b\right)x^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^5*x^4 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*l
og(x^2) - 1/24*(120*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3*A*a^5 + 30*(B*a^4*b + 2
*A*a^3*b^2)*x^4 + 4*(B*a^5 + 5*A*a^4*b)*x^2)/x^8
```

Fricas [A]
time = 0.80, size = 123, normalized size = 1.10

$$\frac{6\,Bb^5x^{12} + 12\,(5\,Bab^4 + Ab^5)x^{10} + 120\,(2\,Ba^2b^3 + Aab^4)x^8\log(x) - 120\,(Ba^3b^2 + Aa^2b^3)x^6 - 3\,Aa^5 - 30\,(Ba^4b + 2\,Aa^3b^2)x^4 - 4\,(Ba^5 + 5\,Aa^4b)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(6*B*b^5*x^12 + 12*(5*B*a*b^4 + A*b^5)*x^10 + 120*(2*B*a^2*b^3 + A*a*b
^4)*x^8*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 3*A*a^5 - 30*(B*a^4*b +
2*A*a^3*b^2)*x^4 - 4*(B*a^5 + 5*A*a^4*b)*x^2)/x^8
```

Sympy [A]
time = 1.54, size = 129, normalized size = 1.15

$$\frac{Bb^5x^4}{4} + 5ab^3(Ab + 2Ba)\log(x) + x^2\left(\frac{Ab^5}{2} + \frac{5Bab^4}{2}\right) + \frac{-3Aa^5 + x^6(-120Aa^2b^3 - 120Ba^3b^2) + x^4(-60Aa^3b^2 - 30Ba^4b) + x^2(-20Aa^4b - 4Ba^5)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**9,x)
```

```
[Out] B*b**5*x**4/4 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**2*(A*b**5/2 + 5*B*a*b**4
/2) + (-3*A*a**5 + x**6*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**4*(-60*A*
a**3*b**2 - 30*B*a**4*b) + x**2*(-20*A*a**4*b - 4*B*a**5))/(24*x**8)
```

Giac [A]
time = 0.64, size = 150, normalized size = 1.34

$$\frac{1}{4}\,Bb^5x^4 + \frac{5}{2}\,Bab^4x^2 + \frac{1}{2}\,Ab^5x^2 + \frac{5}{2}\,(2\,Ba^2b^3 + Aab^4)\log(x^2) - \frac{250\,Ba^2b^3x^8 + 125\,Aab^4x^8 + 120\,Ba^3b^2x^6 + 120\,Aa^2b^3x^6 + 30\,Ba^4bx^4 + 60\,Aa^3b^2x^4 + 4\,Ba^5x^2 + 20\,Aa^4bx^2 + 3\,Aa^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*x^4 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 + 5/2*(2*B*a^2*b^3 + A*a*b^
4)*log(x^2) - 1/24*(250*B*a^2*b^3*x^8 + 125*A*a*b^4*x^8 + 120*B*a^3*b^2*x^6
 + 120*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a^3*b^2*x^4 + 4*B*a^5*x^2 + 20
*A*a^4*b*x^2 + 3*A*a^5)/x^8
```

Mupad [B]
time = 0.03, size = 122, normalized size = 1.09

$$\ln(x)\,\left(10\,B\,a^2\,b^3 + 5\,A\,a\,b^4\right) - \frac{\frac{A\,a^5}{8} + x^4\,\left(\frac{5\,B\,a^4\,b}{4} + \frac{5\,A\,a^3\,b^2}{2}\right) + x^2\,\left(\frac{B\,a^5}{6} + \frac{5\,A\,b\,a^4}{6}\right) + x^6\,\left(5\,B\,a^3\,b^2 + 5\,A\,a^2\,b^3\right)}{x^8} + x^2\,\left(\frac{A\,b^5}{2} + \frac{5\,B\,a\,b^4}{2}\right) + \frac{B\,b^5\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^9,x)
```

```
[Out] log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - ((A*a^5)/8 + x^4*((5*A*a^3*b^2)/2 + (5*
B*a^4*b)/4) + x^2*((B*a^5)/6 + (5*A*a^4*b)/6) + x^6*(5*A*a^2*b^3 + 5*B*a^3*
b^2))/x^8 + x^2*((A*b^5)/2 + (5*B*a*b^4)/2) + (B*b^5*x^4)/4
```

3.42 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{10}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{10a^2b^2(Ab+aB)}{3x^3}-\frac{5ab^3(Ab+2aB)}{x}+b^4(Ab+5aB)x+\frac{1}{3}b^5Bx^3$$

```
[Out] -1/9*a^5*A/x^9-1/7*a^4*(5*A*b+B*a)/x^7-a^3*b*(2*A*b+B*a)/x^5-10/3*a^2*b^2*(
A*b+B*a)/x^3-5*a*b^3*(A*b+2*B*a)/x+b^4*(A*b+5*B*a)*x+1/3*b^5*B*x^3
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{9x^9}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{10a^2b^2(aB+Ab)}{3x^3}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{3}b^5Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

```
[Out] -1/9*(a^5*A)/x^9 - (a^4*(5*A*b + a*B))/(7*x^7) - (a^3*b*(2*A*b + a*B))/x^5
- (10*a^2*b^2*(A*b + a*B))/(3*x^3) - (5*a*b^3*(A*b + 2*a*B))/x + b^4*(A*b +
 5*a*B)*x + (b^5*B*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{10}}\,dx = \int \left(b^4(Ab+5aB)+\frac{a^5A}{x^{10}}+\frac{a^4(5Ab+aB)}{x^8}+\frac{5a^3b(2Ab+aB)}{x^6}+\frac{10a^2b^2(Ab}{x^4}\right.$$
$$= -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{10a^2b^2(Ab+aB)}{3x^3}-\frac{5ab^3(Ab+}{x}$$

Mathematica [A]
time = 0.02, size = 115, normalized size = 1.06

$$-\frac{315ab^4x^8(A-Bx^2)-21b^5x^{10}(3A+Bx^2)+210a^2b^3x^6(A+3Bx^2)+42a^3b^2x^4(3A+5Bx^2)+9a^4bx^2(5A+7Bx^2)+a^5(7A+9Bx^2)}{63x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

```
[Out] -1/63*(315*a*b^4*x^8*(A - B*x^2) - 21*b^5*x^10*(3*A + B*x^2) + 210*a^2*b^3*
x^6*(A + 3*B*x^2) + 42*a^3*b^2*x^4*(3*A + 5*B*x^2) + 9*a^4*b*x^2*(5*A + 7*B
*x^2) + a^5*(7*A + 9*B*x^2))/x^9
```

Maple [A]
time = 0.06, size = 102, normalized size = 0.94

method	result
default	$\frac{b^5 B x^3}{3}+b^5 A x+5 a b^4 B x-\frac{a^3 b(2Ab+Ba)}{x^5}-\frac{a^5 A}{9x^9}-\frac{a^4(5Ab+Ba)}{7x^7}-\frac{10a^2b^2(Ab+Ba)}{3x^3}-\frac{5a b^3(Ab+2Ba)}{x}$
risch	$\frac{b^5 B x^3}{3}+b^5 A x+5 a b^4 B x+\frac{(-5a b^4 A-10a^2b^3B)x^8+(-\frac{10}{3}a^2b^3A-\frac{10}{3}a^3b^2B)x^6+(-2a^3b^2A-a^4bB)x^4+(-\frac{5}{7}a^4bA-\frac{1}{7}a^5B)x}{x^9}$
norman	$\frac{\frac{b^5 B x^{12}}{3}+(b^5A+5a b^4B)x^{10}+(-5a b^4A-10a^2b^3B)x^8+(-\frac{10}{3}a^2b^3A-\frac{10}{3}a^3b^2B)x^6+(-2a^3b^2A-a^4bB)x^4+(-\frac{5}{7}a^4bA-\frac{1}{7}a^5B)x^2-\frac{a^5A}{9}}{x^9}$
gosper	$-\frac{-21b^5Bx^{12}-63Ab^5x^{10}-315Bab^4x^{10}+315aAb^4x^8+630Ba^2b^3x^8+210a^2Ab^3x^6+210Ba^3b^2x^6+126a^3Ab^2x^4+63Ba^4bx^4+45a^4A}{63x^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^5*B*x^3+b^5*A*x+5*a*b^4*B*x-a^3*b*(2*A*b+B*a)/x^5-1/9*a^5*A/x^9-1/7*a
^4*(5*A*b+B*a)/x^7-10/3*a^2*b^2*(A*b+B*a)/x^3-5*a*b^3*(A*b+2*B*a)/x
```

Maxima [A]
time = 0.27, size = 119, normalized size = 1.10

$$\frac{1}{3}Bb^5x^3+(5Bab^4+Ab^5)x-\frac{315(2Ba^2b^3+Aab^4)x^8+210(Ba^3b^2+Aa^2b^3)x^6+7Aa^5+63(Ba^4b+2Aa^3b^2)x^4+9(Ba^5+5Aa^4b)x^2}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*x - 1/63*(315*(2*B*a^2*b^3 + A*a*b^4)*x
^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 7*A*a^5 + 63*(B*a^4*b + 2*A*a^3*b^2)
*x^4 + 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

Fricas [A]
time = 0.83, size = 121, normalized size = 1.12

$$\frac{21Bb^5x^{12}+63(5Bab^4+Ab^5)x^{10}-315(2Ba^2b^3+Aab^4)x^8-210(Ba^3b^2+Aa^2b^3)x^6-7Aa^5-63(Ba^4b+2Aa^3b^2)x^4-9(Ba^5+5Aa^4b)x^2}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(21*B*b^5*x^12 + 63*(5*B*a*b^4 + A*b^5)*x^10 - 315*(2*B*a^2*b^3 + A*a*
b^4)*x^8 - 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 7*A*a^5 - 63*(B*a^4*b + 2*A*a^
3*b^2)*x^4 - 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

Sympy [A]
time = 3.15, size = 129, normalized size = 1.19

$$\frac{Bb^5x^3}{3} + x\left(Ab^5 + 5Bab^4\right) + \frac{-7Aa^5 + x^8\left(-315Aab^4 - 630Ba^2b^3\right) + x^6\left(-210Aa^2b^3 - 210Ba^3b^2\right) + x^4\left(-126Aa^3b^2 - 63Ba^4b\right) + x^2\left(-45Aa^4b - 9Ba^5\right)}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**10,x)
```

```
[Out] B*b**5*x**3/3 + x*(A*b**5 + 5*B*a*b**4) + (-7*A*a**5 + x**8*(-315*A*a*b**4
- 630*B*a**2*b**3) + x**6*(-210*A*a**2*b**3 - 210*B*a**3*b**2) + x**4*(-126
*A*a**3*b**2 - 63*B*a**4*b) + x**2*(-45*A*a**4*b - 9*B*a**5))/(63*x**9)
```

Giac [A]
time = 0.60, size = 123, normalized size = 1.14

$$\frac{1}{3}Bb^5x^3 + 5Bab^4x + Ab^5x - \frac{630Ba^2b^3x^8 + 315Aab^4x^8 + 210Ba^3b^2x^6 + 210Aa^2b^3x^6 + 63Ba^4bx^4 + 126Aa^3b^2x^4 + 9Ba^5x^2 + 45Aa^4bx^2 + 7Aa^5}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="giac")
```

```
[Out] 1/3*B*b^5*x^3 + 5*B*a*b^4*x + A*b^5*x - 1/63*(630*B*a^2*b^3*x^8 + 315*A*a*b
^4*x^8 + 210*B*a^3*b^2*x^6 + 210*A*a^2*b^3*x^6 + 63*B*a^4*b*x^4 + 126*A*a^3
*b^2*x^4 + 9*B*a^5*x^2 + 45*A*a^4*b*x^2 + 7*A*a^5)/x^9
```

Mupad [B]
time = 0.04, size = 119, normalized size = 1.10

$$x\left(A b^5 + 5 B a b^4\right) - \frac{\frac{A a^5}{9} + x^4\left(B a^4 b + 2 A a^3 b^2\right) + x^8\left(10 B a^2 b^3 + 5 A a b^4\right) + x^2\left(\frac{B a^5}{7} + \frac{5 A b a^4}{7}\right) + x^6\left(\frac{10 B a^3 b^2}{3} + \frac{10 A a^2 b^3}{3}\right)}{x^9} + \frac{B b^5 x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^10,x)
```

```
[Out] x*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/9 + x^4*(2*A*a^3*b^2 + B*a^4*b) + x^8*(10*
B*a^2*b^3 + 5*A*a*b^4) + x^2*((B*a^5)/7 + (5*A*a^4*b)/7) + x^6*((10*A*a^2*b
^3)/3 + (10*B*a^3*b^2)/3))/x^9 + (B*b^5*x^3)/3
```

3.43 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{11}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{5a^3b(2Ab+aB)}{6x^6}-\frac{5a^2b^2(Ab+aB)}{2x^4}-\frac{5ab^3(Ab+2aB)}{2x^2}+\frac{1}{2}b^5Bx^2+b^4(Ab+5aB)\log(x)$$

```
[Out] -1/10*a^5*A/x^10-1/8*a^4*(5*A*b+B*a)/x^8-5/6*a^3*b*(2*A*b+B*a)/x^6-5/2*a^2*
b^2*(A*b+B*a)/x^4-5/2*a*b^3*(A*b+2*B*a)/x^2+1/2*b^5*B*x^2+b^4*(A*b+5*B*a)*l
n(x)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{5a^2b^2(aB+Ab)}{2x^4}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{2}b^5Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

```
[Out] -1/10*(a^5*A)/x^10 - (a^4*(5*A*b + a*B))/(8*x^8) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/(2*x^2)
 + (b^5*B*x^2)/2 + b^4*(A*b + 5*a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(b^5B + \frac{a^5A}{x^6} + \frac{a^4(5Ab+aB)}{x^5} + \frac{5a^3b(2Ab+aB)}{x^4} + \frac{10a^2b^2(Ab-}{x^3}\right.\right.$$
$$= -\frac{a^5A}{10x^{10}} - \frac{a^4(5Ab+aB)}{8x^8} - \frac{5a^3b(2Ab+aB)}{6x^6} - \frac{5a^2b^2(Ab+aB)}{2x^4} - \frac{5ab^3(Ab-}{2x}$$

Mathematica [A]
time = 0.04, size = 116, normalized size = 1.03

$$-\frac{300aAb^4x^8 - 60b^5Bx^{12} + 300a^2b^3x^6(A+2Bx^2) + 100a^3b^2x^4(2A+3Bx^2) + 25a^4bx^2(3A+4Bx^2) + 3a^5(4A+5Bx^2)}{120x^{10}} + b^4(Ab+5aB)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

```
[Out] -1/120*(300*a*A*b^4*x^8 - 60*b^5*B*x^12 + 300*a^2*b^3*x^6*(A + 2*B*x^2) + 1
00*a^3*b^2*x^4*(2*A + 3*B*x^2) + 25*a^4*b*x^2*(3*A + 4*B*x^2) + 3*a^5*(4*A
+ 5*B*x^2))/x^10 + b^4*(A*b + 5*a*B)*Log[x]
```

Maple [A]
time = 0.07, size = 102, normalized size = 0.90

method	result
default	$-\frac{a^5A}{10x^{10}} - \frac{a^4(5Ab+Ba)}{8x^8} - \frac{5a^3b(2Ab+Ba)}{6x^6} - \frac{5a^2b^2(Ab+Ba)}{2x^4} - \frac{5a\,b^3(Ab+2Ba)}{2x^2} + \frac{b^5B\,x^2}{2} + b^4(Ab+5Ba)\ln(x)$
norman	$\frac{(-\frac{5}{2}a\,b^4A-5a^2b^3B)x^8+(-\frac{5}{2}a^2b^3A-\frac{5}{2}a^3b^2B)x^6+(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB)x^4+(-\frac{5}{8}a^4bA-\frac{1}{8}a^5B)x^2-\frac{a^5A}{10}+\frac{b^5B\,x^{12}}{2}}{x^{10}} + (b^5A+5a$
risch	$\frac{b^5B\,x^2}{2} + \frac{(-\frac{5}{2}a\,b^4A-5a^2b^3B)x^8+(-\frac{5}{2}a^2b^3A-\frac{5}{2}a^3b^2B)x^6+(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB)x^4+(-\frac{5}{8}a^4bA-\frac{1}{8}a^5B)x^2-\frac{a^5A}{10}}{x^{10}} + A\ln(x)\,b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^5*A/x^10-1/8*a^4*(5*A*b+B*a)/x^8-5/6*a^3*b*(2*A*b+B*a)/x^6-5/2*a^2*
b^2*(A*b+B*a)/x^4-5/2*a*b^3*(A*b+2*B*a)/x^2+1/2*b^5*B*x^2+b^4*(A*b+5*B*a)*l
n(x)
```

Maxima [A]
time = 0.30, size = 123, normalized size = 1.09

$$\frac{1}{2}Bb^5x^2 + \frac{1}{2}(5Bab^4 + Ab^5)\log(x^2) - \frac{300(2Ba^2b^3 + Aab^4)x^8 + 300(Ba^3b^2 + Aa^2b^3)x^6 + 12Aa^5 + 100(Ba^4b + 2Aa^3b^2)x^4 + 15(Ba^5 + 5Aa^4b)x^2}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*log(x^2) - 1/120*(300*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 12*A*a^5 + 100*(B*a^4*b
+ 2*A*a^3*b^2)*x^4 + 15*(B*a^5 + 5*A*a^4*b)*x^2)/x^10
```

Fricas [A]
time = 0.89, size = 123, normalized size = 1.09

$$\frac{60\,Bb^5x^{12}+120\,(5\,Bab^4+Ab^5)x^{10}\log{(x)}-300\,(2\,Ba^2b^3+Aab^4)x^8-300\,(Ba^3b^2+Aa^2b^3)x^6-12\,Aa^5-100\,(Ba^4b+2\,Aa^3b^2)x^4-15\,(Ba^5+5\,Aa^4b)x^2}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(60*B*b^5*x^12 + 120*(5*B*a*b^4 + A*b^5)*x^10*log(x) - 300*(2*B*a^2*b
^3 + A*a*b^4)*x^8 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 12*A*a^5 - 100*(B*a^4
*b + 2*A*a^3*b^2)*x^4 - 15*(B*a^5 + 5*A*a^4*b)*x^2)/x^10
```

Sympy [A]
time = 5.03, size = 129, normalized size = 1.14

$$\frac{Bb^5x^2}{2}+b^4(Ab+5Ba)\log{(x)}+\frac{-12Aa^5+x^8(-300Aab^4-600Ba^2b^3)+x^6(-300Aa^2b^3-300Ba^3b^2)+x^4(-200Aa^3b^2-100Ba^4b)+x^2(-75Aa^4b-15Ba^5)}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**11,x)
```

```
[Out] B*b**5*x**2/2 + b**4*(A*b + 5*B*a)*log(x) + (-12*A*a**5 + x**8*(-300*A*a*b*
*4 - 600*B*a**2*b**3) + x**6*(-300*A*a**2*b**3 - 300*B*a**3*b**2) + x**4*(-
200*A*a**3*b**2 - 100*B*a**4*b) + x**2*(-75*A*a**4*b - 15*B*a**5))/(120*x**
10)
```

Giac [A]
time = 0.64, size = 147, normalized size = 1.30

$$\frac{1}{2}\,Bb^5x^2+\frac{1}{2}\,(5\,Bab^4+Ab^5)\log{(x^2)}-\frac{685\,Bab^4x^{10}+137\,Ab^5x^{10}+600\,Ba^2b^3x^8+300\,Aab^4x^8+300\,Ba^3b^2x^6+300\,Aa^2b^3x^6+100\,Ba^4bx^4+200\,Aa^3b^2x^4+15\,Ba^5x^2+75\,Aa^4bx^2+12\,Aa^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*log(x^2) - 1/120*(685*B*a*b^4*x^10
+ 137*A*b^5*x^10 + 600*B*a^2*b^3*x^8 + 300*A*a*b^4*x^8 + 300*B*a^3*b^2*x^6
+ 300*A*a^2*b^3*x^6 + 100*B*a^4*b*x^4 + 200*A*a^3*b^2*x^4 + 15*B*a^5*x^2 +
75*A*a^4*b*x^2 + 12*A*a^5)/x^10
```

Mupad [B]
time = 0.06, size = 121, normalized size = 1.07

$$\ln\left(x\right)\,\left(A\,b^5+5\,B\,a\,b^4\right)-\frac{\frac{A\,a^5}{10}+x^8\,\left(5\,B\,a^2\,b^3+\frac{5\,A\,a\,b^4}{2}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{6}+\frac{5\,A\,a^3\,b^2}{3}\right)+x^2\,\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)+x^6\,\left(\frac{5\,B\,a^3\,b^2}{2}+\frac{5\,A\,a^2\,b^3}{2}\right)}{x^{10}}+\frac{B\,b^5\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^11,x)
```

```
[Out] log(x)*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/10 + x^8*(5*B*a^2*b^3 + (5*A*a*b^4)/2
) + x^4*((5*A*a^3*b^2)/3 + (5*B*a^4*b)/6) + x^2*((B*a^5)/8 + (5*A*a^4*b)/8)
 + x^6*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2))/x^10 + (B*b^5*x^2)/2
```

3.44 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{3x^3}-\frac{b^4(Ab+5aB)}{x}+b^5Bx$$

```
[Out] -1/11*a^5*A/x^11-1/9*a^4*(5*A*b+B*a)/x^9-5/7*a^3*b*(2*A*b+B*a)/x^7-2*a^2*b^
2*(A*b+B*a)/x^5-5/3*a*b^3*(A*b+2*B*a)/x^3-b^4*(A*b+5*B*a)/x+b^5*B*x
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{3x^3}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

```
[Out] -1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(7*x^7) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(3*x^3) - (
b^4*(A*b + 5*a*B))/x + b^5*B*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx &= \int \left(b^5B+\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^{10}}+\frac{5a^3b(2Ab+aB)}{x^8}+\frac{10a^2b^2(Ab+aB)}{x^6}+\frac{5a\right.\\ &= -\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 122, normalized size = 1.13

$$-\frac{Ab^5}{x}+b^5Bx-\frac{5ab^4(A+3Bx^2)}{3x^3}-\frac{2a^2b^3(3A+5Bx^2)}{3x^5}-\frac{2a^3b^2(5A+7Bx^2)}{7x^7}-\frac{5a^4b(7A+9Bx^2)}{63x^9}-\frac{a^5(9A+11Bx^2)}{99x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

```
[Out] -((A*b^5)/x) + b^5*B*x - (5*a*b^4*(A + 3*B*x^2))/(3*x^3) - (2*a^2*b^3*(3*A
+ 5*B*x^2))/(3*x^5) - (2*a^3*b^2*(5*A + 7*B*x^2))/(7*x^7) - (5*a^4*b*(7*A +
 9*B*x^2))/(63*x^9) - (a^5*(9*A + 11*B*x^2))/(99*x^11)
```

Maple [A]
time = 0.06, size = 101, normalized size = 0.94

method	result
default	$-\frac{a^5 A}{11x^{11}}-\frac{a^4(5Ab+Ba)}{9x^9}-\frac{5a^3 b(2Ab+Ba)}{7x^7}-\frac{2a^2 b^2(Ab+Ba)}{x^5}-\frac{5a\,b^3(Ab+2Ba)}{3x^3}-\frac{b^4(Ab+5Ba)}{x}+b^5 Bx$
risch	$b^5 Bx+\frac{(-b^5 A-5a\,b^4 B)x^{10}+(-\frac{5}{3}a\,b^4 A-\frac{10}{3}a^2 b^3 B)x^8+(-2a^2 b^3 A-2a^3 b^2 B)x^6+(-\frac{10}{7}a^3 b^2 A-\frac{5}{7}a^4 bB)x^4+(-\frac{5}{9}a^4 bA-\frac{1}{9}a^5 B)x^2}{x^{11}}$
norman	$\frac{b^5 B\,x^{12}+(-b^5 A-5a\,b^4 B)x^{10}+(-\frac{5}{3}a\,b^4 A-\frac{10}{3}a^2 b^3 B)x^8+(-2a^2 b^3 A-2a^3 b^2 B)x^6+(-\frac{10}{7}a^3 b^2 A-\frac{5}{7}a^4 bB)x^4+(-\frac{5}{9}a^4 bA-\frac{1}{9}a^5 B)x^2-}{x^{11}}$
gosper	$-\frac{-693b^5 B\,x^{12}+693A\,b^5 x^{10}+3465Ba\,b^4 x^{10}+1155aA\,b^4 x^8+2310B\,a^2 b^3 x^8+1386a^2 A\,b^3 x^6+1386B\,a^3 b^2 x^6+990a^3 A\,b^2 x^4+495B\,a^4}{693x^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*a^5*A/x^11-1/9*a^4*(5*A*b+B*a)/x^9-5/7*a^3*b*(2*A*b+B*a)/x^7-2*a^2*b^
2*(A*b+B*a)/x^5-5/3*a*b^3*(A*b+2*B*a)/x^3-b^4*(A*b+5*B*a)/x+b^5*B*x
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.10

$$Bb^5x-\frac{693\,(5\,Bab^4+Ab^5)x^{10}+1155\,(2\,Ba^2b^3+Aab^4)x^8+1386\,(Ba^3b^2+Aa^2b^3)x^6+63\,Aa^5+495\,(Ba^4b+2\,Aa^3b^2)x^4+77\,(Ba^5+5\,Aa^4b)x^2}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="maxima")
```

```
[Out] B*b^5*x - 1/693*(693*(5*B*a*b^4 + A*b^5)*x^10 + 1155*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 63*A*a^5 + 495*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 77*(B*a^5 + 5*A*a^4*b)*x^2)/x^11
```

Fricas [A]
time = 0.68, size = 121, normalized size = 1.12

$$\frac{693\,Bb^5x^{12}-693\,(5\,Bab^4+Ab^5)x^{10}-1155\,(2\,Ba^2b^3+Aab^4)x^8-1386\,(Ba^3b^2+Aa^2b^3)x^6-63\,Aa^5-495\,(Ba^4b+2\,Aa^3b^2)x^4-77\,(Ba^5+5\,Aa^4b)x^2}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="fricas")
```

[Out] 1/693*(693*B*b^5*x^12 - 693*(5*B*a*b^4 + A*b^5)*x^10 - 1155*(2*B*a^2*b^3 + A*a*b^4)*x^8 - 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 63*A*a^5 - 495*(B*a^4*b + 2*A*a^3*b^2)*x^4 - 77*(B*a^5 + 5*A*a^4*b)*x^2)/x^11

Sympy [A]
time = 34.89, size = 131, normalized size = 1.21

$$Bb^5x + \frac{-63Aa^5 + x^{10}(-693Ab^5 - 3465Bab^4) + x^8(-1155Aab^4 - 2310Ba^2b^3) + x^6(-1386Aa^2b^3 - 1386Ba^3b^2) + x^4(-990Aa^3b^2 - 495Ba^4b) + x^2(-385Aa^4b - 77Ba^5)}{693x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**12,x)

[Out] B*b**5*x + (-63*A*a**5 + x**10*(-693*A*b**5 - 3465*B*a*b**4) + x**8*(-1155*A*a*b**4 - 2310*B*a**2*b**3) + x**6*(-1386*A*a**2*b**3 - 1386*B*a**3*b**2) + x**4*(-990*A*a**3*b**2 - 495*B*a**4*b) + x**2*(-385*A*a**4*b - 77*B*a**5))/(693*x**11)

Giac [A]
time = 0.59, size = 125, normalized size = 1.16

$$Bb^5x - \frac{3465\,Bab^4x^{10} + 693\,Ab^5x^{10} + 2310\,Ba^2b^3x^8 + 1155\,Aab^4x^8 + 1386\,Ba^3b^2x^6 + 1386\,Aa^2b^3x^6 + 495\,Ba^4bx^4 + 990\,Aa^3b^2x^4 + 77\,Ba^5x^2 + 385\,Aa^4bx^2 + 63\,Aa^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="giac")

[Out] B*b^5*x - 1/693*(3465*B*a*b^4*x^10 + 693*A*b^5*x^10 + 2310*B*a^2*b^3*x^8 + 1155*A*a*b^4*x^8 + 1386*B*a^3*b^2*x^6 + 1386*A*a^2*b^3*x^6 + 495*B*a^4*b*x^4 + 990*A*a^3*b^2*x^4 + 77*B*a^5*x^2 + 385*A*a^4*b*x^2 + 63*A*a^5)/x^11

Mupad [B]
time = 0.04, size = 119, normalized size = 1.10

$$B\,b^5\,x - \frac{\frac{A\,a^5}{11} + x^8\left(\frac{10\,B\,a^2\,b^3}{3} + \frac{5\,A\,a\,b^4}{3}\right) + x^4\left(\frac{5\,B\,a^4\,b}{7} + \frac{10\,A\,a^3\,b^2}{7}\right) + x^2\left(\frac{B\,a^5}{9} + \frac{5\,A\,b\,a^4}{9}\right) + x^{10}\left(A\,b^5 + 5\,B\,a\,b^4\right) + x^6\left(2\,B\,a^3\,b^2 + 2\,A\,a^2\,b^3\right)}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^12,x)

[Out] B*b^5*x - ((A*a^5)/11 + x^8*((10*B*a^2*b^3)/3 + (5*A*a*b^4)/3) + x^4*((10*A*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^2*((B*a^5)/9 + (5*A*a^4*b)/9) + x^10*(A*b^5 + 5*B*a*b^4) + x^6*(2*A*a^2*b^3 + 2*B*a^3*b^2))/x^11

3.45 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{13}} dx$

Optimal. Leaf size=91

$$-\frac{a^5B}{10x^{10}}-\frac{5a^4bB}{8x^8}-\frac{5a^3b^2B}{3x^6}-\frac{5a^2b^3B}{2x^4}-\frac{5ab^4B}{2x^2}-\frac{A(a+bx^2)^6}{12ax^{12}}+b^5B\log(x)$$

```
[Out] -1/10*a^5*B/x^10-5/8*a^4*b*B/x^8-5/3*a^3*b^2*B/x^6-5/2*a^2*b^3*B/x^4-5/2*a*
b^4*B/x^2-1/12*A*(b*x^2+a)^6/a/x^12+b^5*B*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 79, 45}

$$-\frac{a^5B}{10x^{10}}-\frac{5a^4bB}{8x^8}-\frac{5a^3b^2B}{3x^6}-\frac{5a^2b^3B}{2x^4}-\frac{A(a+bx^2)^6}{12ax^{12}}-\frac{5ab^4B}{2x^2}+b^5B\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

```
[Out] -1/10*(a^5*B)/x^10 - (5*a^4*b*B)/(8*x^8) - (5*a^3*b^2*B)/(3*x^6) - (5*a^2*b
^3*B)/(2*x^4) - (5*a*b^4*B)/(2*x^2) - (A*(a + b*x^2)^6)/(12*a*x^12) + b^5*B
*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{13}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx,x,x^2\right)\\ &= -\frac{A(a+bx^2)^6}{12ax^{12}}+\frac{1}{2}B\text{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx,x,x^2\right)\\ &= -\frac{A(a+bx^2)^6}{12ax^{12}}+\frac{1}{2}B\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)\right.\\ &= -\frac{a^5B}{10x^{10}}-\frac{5a^4bB}{8x^8}-\frac{5a^3b^2B}{3x^6}-\frac{5a^2b^3B}{2x^4}-\frac{5ab^4B}{2x^2}-\frac{A(a+bx^2)^6}{12ax^{12}}+b^5B\log(x)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 118, normalized size = 1.30

$$-\frac{60Ab^5x^{10}+150ab^4x^8(A+2Bx^2)+100a^2b^3x^6(2A+3Bx^2)+50a^3b^2x^4(3A+4Bx^2)+15a^4bx^2(4A+5Bx^2)+2a^5(5A+6Bx^2)}{120x^{12}}+b^5B\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

```
[Out] -1/120*(60*A*b^5*x^10 + 150*a*b^4*x^8*(A + 2*B*x^2) + 100*a^2*b^3*x^6*(2*A
+ 3*B*x^2) + 50*a^3*b^2*x^4*(3*A + 4*B*x^2) + 15*a^4*b*x^2*(4*A + 5*B*x^2)
+ 2*a^5*(5*A + 6*B*x^2))/x^12 + b^5*B*Log[x]
```

Maple [A]
time = 0.07, size = 102, normalized size = 1.12

method	result
default	$-\frac{5ab^3(Ab+2Ba)}{4x^4}-\frac{a^5A}{12x^{12}}-\frac{5a^2b^2(Ab+Ba)}{3x^6}-\frac{b^4(Ab+5Ba)}{2x^2}-\frac{5a^3b(2Ab+Ba)}{8x^8}-\frac{a^4(5Ab+Ba)}{10x^{10}}+b^5B\ln(x)$
norman	$\frac{(-\frac{1}{2}b^5A-\frac{5}{2}ab^4B)x^{10}+(-\frac{5}{4}ab^4A-\frac{5}{2}a^2b^3B)x^8+(-\frac{5}{3}a^2b^3A-\frac{5}{3}a^3b^2B)x^6+(-\frac{5}{4}a^3b^2A-\frac{5}{8}a^4bB)x^4+(-\frac{1}{2}a^4bA-\frac{1}{10}a^5B)x^2-\frac{a^5A}{12}}{x^{12}}+$
risch	$\frac{(-\frac{1}{2}b^5A-\frac{5}{2}ab^4B)x^{10}+(-\frac{5}{4}ab^4A-\frac{5}{2}a^2b^3B)x^8+(-\frac{5}{3}a^2b^3A-\frac{5}{3}a^3b^2B)x^6+(-\frac{5}{4}a^3b^2A-\frac{5}{8}a^4bB)x^4+(-\frac{1}{2}a^4bA-\frac{1}{10}a^5B)x^2-\frac{a^5A}{12}}{x^{12}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -5/4*a*b^3*(A*b+2*B*a)/x^4-1/12*a^5*A/x^12-5/3*a^2*b^2*(A*b+B*a)/x^6-1/2*b^
4*(A*b+5*B*a)/x^2-5/8*a^3*b*(2*A*b+B*a)/x^8-1/10*a^4*(5*A*b+B*a)/x^10+b^5*B
*ln(x)
```

Maxima [A]
time = 0.29, size = 123, normalized size = 1.35

$$\frac{1}{2}\,Bb^{5}\log\left(x^{2}\right)-\frac{60\,(5\,Bab^{4}+Ab^{5})x^{10}+150\,(2\,Ba^{2}b^{3}+Aab^{4})x^{8}+200\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{6}+10\,Aa^{5}+75\,(Ba^{4}b+2\,Aa^{3}b^{2})x^{4}+12\,(Ba^{5}+5\,Aa^{4}b)x^{2}}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*log(x^2) - 1/120*(60*(5*B*a*b^4 + A*b^5)*x^10 + 150*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 10*A*a^5 + 75*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^12
```

Fricas [A]
time = 0.75, size = 123, normalized size = 1.35

$$\frac{120\,Bb^{5}x^{12}\log\left(x\right)-60\,(5\,Bab^{4}+Ab^{5})x^{10}-150\,(2\,Ba^{2}b^{3}+Aab^{4})x^{8}-200\,(Ba^{3}b^{2}+Aa^{2}b^{3})x^{6}-10\,Aa^{5}-75\,(Ba^{4}b+2\,Aa^{3}b^{2})x^{4}-12\,(Ba^{5}+5\,Aa^{4}b)x^{2}}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="fricas")
```

```
[Out] 1/120*(120*B*b^5*x^12*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^10 - 150*(2*B*a^2*b
^3 + A*a*b^4)*x^8 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 10*A*a^5 - 75*(B*a^4*
b + 2*A*a^3*b^2)*x^4 - 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^12
```

Sympy [A]
time = 26.60, size = 133, normalized size = 1.46

$$Bb^{5}\log\left(x\right)+\frac{-10Aa^{5}+x^{10}(-60Ab^{5}-300Bab^{4})+x^{8}(-150Aab^{4}-300Ba^{2}b^{3})+x^{6}(-200Aa^{2}b^{3}-200Ba^{3}b^{2})+x^{4}(-150Aa^{3}b^{2}-75Ba^{4}b)+x^{2}(-60Aa^{4}b-12Ba^{5})}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**13,x)
```

```
[Out] B*b**5*log(x) + (-10*A*a**5 + x**10*(-60*A*b**5 - 300*B*a*b**4) + x**8*(-15
0*A*a*b**4 - 300*B*a**2*b**3) + x**6*(-200*A*a**2*b**3 - 200*B*a**3*b**2) +
 x**4*(-150*A*a**3*b**2 - 75*B*a**4*b) + x**2*(-60*A*a**4*b - 12*B*a**5))/(
120*x**12)
```

Giac [A]
time = 0.63, size = 138, normalized size = 1.52

$$\frac{1}{2}\,Bb^{5}\log\left(x^{2}\right)-\frac{147\,Bb^{5}x^{12}+300\,Bab^{4}x^{10}+60\,Ab^{5}x^{10}+300\,Ba^{2}b^{3}x^{8}+150\,Aab^{4}x^{8}+200\,Ba^{3}b^{2}x^{6}+200\,Aa^{2}b^{3}x^{6}+75\,Ba^{4}bx^{4}+150\,Aa^{3}b^{2}x^{4}+12\,Ba^{5}x^{2}+60\,Aa^{4}bx^{2}+10\,Aa^{5}}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*log(x^2) - 1/120*(147*B*b^5*x^12 + 300*B*a*b^4*x^10 + 60*A*b^5*x^
10 + 300*B*a^2*b^3*x^8 + 150*A*a*b^4*x^8 + 200*B*a^3*b^2*x^6 + 200*A*a^2*b^
3*x^6 + 75*B*a^4*b*x^4 + 150*A*a^3*b^2*x^4 + 12*B*a^5*x^2 + 60*A*a^4*b*x^2
+ 10*A*a^5)/x^12
```

Mupad [B]
time = 0.07, size = 121, normalized size = 1.33

$$B\,b^5\,\ln\left(x\right)-\frac{\frac{A\,a^5}{12}+x^8\,\left(\frac{5\,B\,a^2\,b^3}{2}+\frac{5\,A\,a\,b^4}{4}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{8}+\frac{5\,A\,a^3\,b^2}{4}\right)+x^2\,\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^{10}\,\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)+x^6\,\left(\frac{5\,B\,a^3\,b^2}{3}+\frac{5\,A\,a^2\,b^3}{3}\right)}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^13,x)
```

```
[Out] B*b^5*log(x) - ((A*a^5)/12 + x^8*((5*B*a^2*b^3)/2 + (5*A*a*b^4)/4) + x^4*((
5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^2*((B*a^5)/10 + (A*a^4*b)/2) + x^10*((A
*b^5)/2 + (5*B*a*b^4)/2) + x^6*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3))/x^12
```

3.46 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{ab^3(Ab+2aB)}{x^5}-\frac{b^4(Ab+5aB)}{3x^3}-\frac{b^5B}{x}$$

```
[Out] -1/13*a^5*A/x^13-1/11*a^4*(5*A*b+B*a)/x^11-5/9*a^3*b*(2*A*b+B*a)/x^9-10/7*a
^2*b^2*(A*b+B*a)/x^7-a*b^3*(A*b+2*B*a)/x^5-1/3*b^4*(A*b+5*B*a)/x^3-b^5*B/x
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{3x^3}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^5B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

```
[Out] -1/13*(a^5*A)/x^13 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (a*b^3*(A*b + 2*a*B))/x^5 -
(b^4*(A*b + 5*a*B))/(3*x^3) - (b^5*B)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx &= \int \left(\frac{a^5A}{x^{14}}+\frac{a^4(5Ab+aB)}{x^{12}}+\frac{5a^3b(2Ab+aB)}{x^{10}}+\frac{10a^2b^2(Ab+aB)}{x^8}+\frac{5ab^3(A\right.\\ &= -\frac{a^5A}{13x^{13}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{ab^3(Ab}{x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 119, normalized size = 1.05

$$-\frac{3003b^5x^{10}(A+3Bx^2)+3003ab^4x^8(3A+5Bx^2)+2574a^2b^3x^6(5A+7Bx^2)+1430a^3b^2x^4(7A+9Bx^2)+455a^4bx^2(9A+11Bx^2)+63a^5(11A+13Bx^2)}{9009x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

```
[Out] -1/9009*(3003*b^5*x^10*(A + 3*B*x^2) + 3003*a*b^4*x^8*(3*A + 5*B*x^2) + 257
4*a^2*b^3*x^6*(5*A + 7*B*x^2) + 1430*a^3*b^2*x^4*(7*A + 9*B*x^2) + 455*a^4*
b*x^2*(9*A + 11*B*x^2) + 63*a^5*(11*A + 13*B*x^2))/x^13
```

Maple [A]
time = 0.06, size = 104, normalized size = 0.92

method	result
default	$-\frac{a^5 A}{13x^{13}}-\frac{a^4(5Ab+Ba)}{11x^{11}}-\frac{5a^3b(2Ab+Ba)}{9x^9}-\frac{10a^2b^2(Ab+Ba)}{7x^7}-\frac{a\,b^3(Ab+2Ba)}{x^5}-\frac{b^4(Ab+5Ba)}{3x^3}-\frac{b^5B}{x}$
norman	$\frac{-b^5Bx^{12}+(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B)x^{10}+(-a\,b^4A-2a^2b^3B)x^8+(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B)x^6+(-\frac{10}{9}a^3b^2A-\frac{5}{9}a^4bB)x^4+(-\frac{5}{11}a^4bA-\frac{1}{11}a^5B)}{x^{13}}$
risch	$\frac{-b^5Bx^{12}+(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B)x^{10}+(-a\,b^4A-2a^2b^3B)x^8+(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B)x^6+(-\frac{10}{9}a^3b^2A-\frac{5}{9}a^4bB)x^4+(-\frac{5}{11}a^4bA-\frac{1}{11}a^5B)}{x^{13}}$
gosper	$-\frac{9009b^5Bx^{12}+3003Ab^5x^{10}+15015Bab^4x^{10}+9009aAb^4x^8+18018Ba^2b^3x^8+12870a^2Ab^3x^6+12870Ba^3b^2x^6+10010a^3Ab^2x^4+500}{9009x^{13}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*a^5*A/x^13-1/11*a^4*(5*A*b+B*a)/x^11-5/9*a^3*b*(2*A*b+B*a)/x^9-10/7*a
^2*b^2*(A*b+B*a)/x^7-a*b^3*(A*b+2*B*a)/x^5-1/3*b^4*(A*b+5*B*a)/x^3-b^5*B/x
```

Maxima [A]
time = 0.32, size = 121, normalized size = 1.07

$$-\frac{9009\,Bb^5x^{12}+3003\,(5\,Bab^4+Ab^5)x^{10}+9009\,(2\,Ba^2b^3+Aab^4)x^8+12870\,(Ba^3b^2+Aa^2b^3)x^6+693\,Aa^5+5005\,(Ba^4b+2\,Aa^3b^2)x^4+819\,(Ba^5+5\,Aa^4b)x^2}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(2*B*a^2*b^
3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 693*A*a^5 + 5005*(B*
a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a^4*b)*x^2)/x^13
```

Fricas [A]
time = 0.68, size = 121, normalized size = 1.07

$$-\frac{9009\,Bb^5x^{12}+3003\,(5\,Bab^4+Ab^5)x^{10}+9009\,(2\,Ba^2b^3+Aab^4)x^8+12870\,(Ba^3b^2+Aa^2b^3)x^6+693\,Aa^5+5005\,(Ba^4b+2\,Aa^3b^2)x^4+819\,(Ba^5+5\,Aa^4b)x^2}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(2*B*a^2*b^
3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 693*A*a^5 + 5005*(B*
a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a^4*b)*x^2)/x^13
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**14,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.74, size = 127, normalized size = 1.12

$$-\frac{9009\,Bb^5x^{12}+15015\,Bab^4x^{10}+3003\,Ab^5x^{10}+18018\,Ba^2b^3x^8+9009\,Aab^4x^8+12870\,Ba^3b^2x^6+12870\,Aa^2b^3x^6+5005\,Ba^4bx^4+10010\,Aa^3b^2x^4+819\,Ba^5x^2+4095\,Aa^4bx^2+693\,Aa^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 15015*B*a*b^4*x^10 + 3003*A*b^5*x^10 + 18018*B*a
^2*b^3*x^8 + 9009*A*a*b^4*x^8 + 12870*B*a^3*b^2*x^6 + 12870*A*a^2*b^3*x^6 +
 5005*B*a^4*b*x^4 + 10010*A*a^3*b^2*x^4 + 819*B*a^5*x^2 + 4095*A*a^4*b*x^2
+ 693*A*a^5)/x^13
```

Mupad [**B**]
time = 0.05, size = 120, normalized size = 1.06

$$-\frac{\frac{A\,a^5}{13}+x^8\,\left(2\,B\,a^2\,b^3+A\,a\,b^4\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{9}+\frac{10\,A\,a^3\,b^2}{9}\right)+x^2\,\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{10}\,\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)+x^6\,\left(\frac{10\,B\,a^3\,b^2}{7}+\frac{10\,A\,a^2\,b^3}{7}\right)+B\,b^5\,x^{12}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^14,x)
```

```
[Out] -((A*a^5)/13 + x^8*(2*B*a^2*b^3 + A*a*b^4) + x^4*((10*A*a^3*b^2)/9 + (5*B*a
^4*b)/9) + x^2*((B*a^5)/11 + (5*A*a^4*b)/11) + x^10*((A*b^5)/3 + (5*B*a*b^4
)/3) + x^6*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7) + B*b^5*x^12)/x^13
```

3.47 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{15}}\,dx$

Optimal. Leaf size=48

$$-\frac{A(a+bx^2)^6}{14ax^{14}}+\frac{(Ab-7aB)\,(a+bx^2)^6}{84a^2x^{12}}$$

[Out] -1/14*A*(b*x^2+a)^6/a/x^14+1/84*(A*b-7*B*a)*(b*x^2+a)^6/a^2/x^12

Rubi [A]

time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 79, 37}

$$\frac{(a+bx^2)^6\,(Ab-7aB)}{84a^2x^{12}}-\frac{A(a+bx^2)^6}{14ax^{14}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^15,x]

[Out] -1/14*(A*(a + b*x^2)^6)/(a*x^14) + ((A*b - 7*a*B)*(a + b*x^2)^6)/(84*a^2*x^12)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{15}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx, x, x^2\right)$$
$$= -\frac{A(a+bx^2)^6}{14ax^{14}} + \frac{(-Ab+7aB)\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^2\right)}{14a}$$
$$= -\frac{A(a+bx^2)^6}{14ax^{14}} + \frac{(Ab-7aB)(a+bx^2)^6}{84a^2x^{12}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 118 vs. $2(48) = 96$.
time = 0.02, size = 118, normalized size = 2.46

$$-\frac{21b^5x^{10}(A+2Bx^2)+35ab^4x^8(2A+3Bx^2)+35a^2b^3x^6(3A+4Bx^2)+21a^3b^2x^4(4A+5Bx^2)+7a^4bx^2(5A+6Bx^2)+a^5(6A+7Bx^2)}{84x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

```
[Out] -1/84*(21*b^5*x^10*(A + 2*B*x^2) + 35*a*b^4*x^8*(2*A + 3*B*x^2) + 35*a^2*b^
3*x^6*(3*A + 4*B*x^2) + 21*a^3*b^2*x^4*(4*A + 5*B*x^2) + 7*a^4*b*x^2*(5*A +
 6*B*x^2) + a^5*(6*A + 7*B*x^2))/x^14
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(44) = 88$.
time = 0.07, size = 104, normalized size = 2.17

method	result
default	$-\frac{a^5A}{14x^{14}}-\frac{b^4(Ab+5Ba)}{4x^4}-\frac{a^4(5Ab+Ba)}{12x^{12}}-\frac{5a\,b^3(Ab+2Ba)}{6x^6}-\frac{b^5B}{2x^2}-\frac{5a^2b^2(Ab+Ba)}{4x^8}-\frac{a^3b(2Ab+Ba)}{2x^{10}}$
norman	$\frac{-\frac{a^5A}{14}+\left(-\frac{5}{12}a^4bA-\frac{1}{12}a^5B\right)x^2+\left(-a^3b^2A-\frac{1}{2}a^4bB\right)x^4+\left(-\frac{5}{4}a^2b^3A-\frac{5}{4}a^3b^2B\right)x^6+\left(-\frac{5}{6}a\,b^4A-\frac{5}{3}a^2b^3B\right)x^8+\left(-\frac{1}{4}b^5A-\frac{5}{4}a\,b^4B\right)x^{10}-}{x^{14}}$
risch	$\frac{-\frac{a^5A}{14}+\left(-\frac{5}{12}a^4bA-\frac{1}{12}a^5B\right)x^2+\left(-a^3b^2A-\frac{1}{2}a^4bB\right)x^4+\left(-\frac{5}{4}a^2b^3A-\frac{5}{4}a^3b^2B\right)x^6+\left(-\frac{5}{6}a\,b^4A-\frac{5}{3}a^2b^3B\right)x^8+\left(-\frac{1}{4}b^5A-\frac{5}{4}a\,b^4B\right)x^{10}-}{x^{14}}$
gosper	$-\frac{42b^5B\,x^{12}+21A\,b^5x^{10}+105Ba\,b^4x^{10}+70aA\,b^4x^8+140B\,a^2b^3x^8+105a^2A\,b^3x^6+105B\,a^3b^2x^6+84a^3A\,b^2x^4+42B\,a^4b\,x^4+35a^4Ab}{84x^{14}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*a^5*A/x^14-1/4*b^4*(A*b+5*B*a)/x^4-1/12*a^4*(5*A*b+B*a)/x^12-5/6*a*b^
3*(A*b+2*B*a)/x^6-1/2*b^5*B/x^2-5/4*a^2*b^2*(A*b+B*a)/x^8-1/2*a^3*b*(2*A*b+
B*a)/x^10
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(45) = 90.
time = 0.32, size = 121, normalized size = 2.52

$$-\frac{42 Bb^5x^{12} + 21 (5 Bab^4 + Ab^5)x^{10} + 70 (2 Ba^2b^3 + Aab^4)x^8 + 105 (Ba^3b^2 + Aa^2b^3)x^6 + 6 Aa^5 + 42 (Ba^4b + 2 Aa^3b^2)x^4 + 7 (Ba^5 + 5 Aa^4b)x^2}{84 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="maxima")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*b^3 + A*a*
b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 + 42*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(45) = 90.
time = 0.77, size = 121, normalized size = 2.52

$$-\frac{42 Bb^5x^{12} + 21 (5 Bab^4 + Ab^5)x^{10} + 70 (2 Ba^2b^3 + Aab^4)x^8 + 105 (Ba^3b^2 + Aa^2b^3)x^6 + 6 Aa^5 + 42 (Ba^4b + 2 Aa^3b^2)x^4 + 7 (Ba^5 + 5 Aa^4b)x^2}{84 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*b^3 + A*a*
b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 + 42*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(41) = 82.
time = 134.48, size = 134, normalized size = 2.79

$$\frac{-6Aa^5 - 42Bb^5x^{12} + x^{10}(-21Ab^5 - 105Bab^4) + x^8(-70Aab^4 - 140Ba^2b^3) + x^6(-105Aa^2b^3 - 105Ba^3b^2) + x^4(-84Aa^3b^2 - 42Ba^4b) + x^2(-35Aa^4b - 7Ba^5)}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**15,x)
```

```
[Out] (-6*A*a**5 - 42*B*b**5*x**12 + x**10*(-21*A*b**5 - 105*B*a*b**4) + x**8*(-7
0*A*a*b**4 - 140*B*a**2*b**3) + x**6*(-105*A*a**2*b**3 - 105*B*a**3*b**2) +
 x**4*(-84*A*a**3*b**2 - 42*B*a**4*b) + x**2*(-35*A*a**4*b - 7*B*a**5))/(84
*x**14)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(45) = 90.
time = 1.10, size = 127, normalized size = 2.65

$$-\frac{42 Bb^5x^{12} + 105 Bab^4x^{10} + 21 Ab^5x^{10} + 140 Ba^2b^3x^8 + 70 Aab^4x^8 + 105 Ba^3b^2x^6 + 105 Aa^2b^3x^6 + 42 Ba^4bx^4 + 84 Aa^3b^2x^4 + 7 Ba^5x^2 + 35 Aa^4bx^2 + 6 Aa^5}{84 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 105*B*a*b^4*x^10 + 21*A*b^5*x^10 + 140*B*a^2*b^3*x^8
 + 70*A*a*b^4*x^8 + 105*B*a^3*b^2*x^6 + 105*A*a^2*b^3*x^6 + 42*B*a^4*b*x^4
+ 84*A*a^3*b^2*x^4 + 7*B*a^5*x^2 + 35*A*a^4*b*x^2 + 6*A*a^5)/x^14
```

Mupad [B]
time = 0.03, size = 121, normalized size = 2.52

$$-\frac{\frac{A\,a^5}{14}+x^4\,\left(\frac{B\,a^4\,b}{2}+A\,a^3\,b^2\right)+x^8\,\left(\frac{5\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{6}\right)+x^2\,\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{10}\,\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)+x^6\,\left(\frac{5\,B\,a^3\,b^2}{4}+\frac{5\,A\,a^2\,b^3}{4}\right)+\frac{B\,b^5\,x^{12}}{2}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^15,x)
```

```
[Out] -((A*a^5)/14 + x^4*(A*a^3*b^2 + (B*a^4*b)/2) + x^8*((5*B*a^2*b^3)/3 + (5*A*
a*b^4)/6) + x^2*((B*a^5)/12 + (5*A*a^4*b)/12) + x^10*((A*b^5)/4 + (5*B*a*b^
4)/4) + x^6*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4) + (B*b^5*x^12)/2)/x^14
```

3.48 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{16}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{10a^2b^2(Ab+aB)}{9x^9}-\frac{5ab^3(Ab+2aB)}{7x^7}-\frac{b^4(Ab+5aB)}{5x^5}-\frac{b^5B}{3x^3}$$

```
[Out] -1/15*a^5*A/x^15-1/13*a^4*(5*A*b+B*a)/x^13-5/11*a^3*b*(2*A*b+B*a)/x^11-10/9
*a^2*b^2*(A*b+B*a)/x^9-5/7*a*b^3*(A*b+2*B*a)/x^7-1/5*b^4*(A*b+5*B*a)/x^5-1/
3*b^5*B/x^3
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{15x^{15}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{10a^2b^2(aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

```
[Out] -1/15*(a^5*A)/x^15 - (a^4*(5*A*b + a*B))/(13*x^13) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (10*a^2*b^2*(A*b + a*B))/(9*x^9) - (5*a*b^3*(A*b + 2*a*B))/(7
*x^7) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(3*x^3)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{16}}\,dx = \int \left(\frac{a^5A}{x^{16}}+\frac{a^4(5Ab+aB)}{x^{14}}+\frac{5a^3b(2Ab+aB)}{x^{12}}+\frac{10a^2b^2(Ab+aB)}{x^{10}}+\frac{5ab^3(Ab$$
$$= -\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{10a^2b^2(Ab+aB)}{9x^9}-\frac{5ab^3(Ab-}{7x^7$$

Mathematica [A]
time = 0.02, size = 121, normalized size = 1.03

$$-\frac{3003b^5x^{10}(3A+5Bx^2)+6435ab^4x^8(5A+7Bx^2)+7150a^2b^3x^6(7A+9Bx^2)+4550a^3b^2x^4(9A+11Bx^2)+1575a^4bx^2(11A+13Bx^2)+231a^5(13A+15Bx^2)}{45045x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

```
[Out] -1/45045*(3003*b^5*x^10*(3*A + 5*B*x^2) + 6435*a*b^4*x^8*(5*A + 7*B*x^2) +
7150*a^2*b^3*x^6*(7*A + 9*B*x^2) + 4550*a^3*b^2*x^4*(9*A + 11*B*x^2) + 1575
*a^4*b*x^2*(11*A + 13*B*x^2) + 231*a^5*(13*A + 15*B*x^2))/x^15
```

Maple [A]
time = 0.07, size = 104, normalized size = 0.89

method	result
default	$-\frac{a^{5} A}{15 x^{15}}-\frac{a^{4} (5 A b+B a)}{13 x^{13}}-\frac{5 a^{3} b (2 A b+B a)}{11 x^{11}}-\frac{10 a^{2} b^{2} (A b+B a)}{9 x^{9}}-\frac{5 a\, b^{3} (A b+2 B a)}{7 x^{7}}-\frac{b^{4} (A b+5 B a)}{5 x^{5}}-\frac{b^{5} B}{3 x^{3}}$
norman	$\frac{-\frac{a^{5} A}{15}+(-\frac{5}{13} a^{4} b A-\frac{1}{13} a^{5} B) x^{2}+(-\frac{10}{11} a^{3} b^{2} A-\frac{5}{11} a^{4} b B) x^{4}+(-\frac{10}{9} a^{2} b^{3} A-\frac{10}{9} a^{3} b^{2} B) x^{6}+(-\frac{5}{7} a\, b^{4} A-\frac{10}{7} a^{2} b^{3} B) x^{8}+(-\frac{1}{5} b^{5} A-a\, b^{4} B)}{x^{15}}$
risch	$\frac{-\frac{a^{5} A}{15}+(-\frac{5}{13} a^{4} b A-\frac{1}{13} a^{5} B) x^{2}+(-\frac{10}{11} a^{3} b^{2} A-\frac{5}{11} a^{4} b B) x^{4}+(-\frac{10}{9} a^{2} b^{3} A-\frac{10}{9} a^{3} b^{2} B) x^{6}+(-\frac{5}{7} a\, b^{4} A-\frac{10}{7} a^{2} b^{3} B) x^{8}+(-\frac{1}{5} b^{5} A-a\, b^{4} B)}{x^{15}}$
gosper	$-\frac{15015 b^{5} B\, x^{12}+9009 A\, b^{5} x^{10}+45045 B a\, b^{4} x^{10}+32175 a A\, b^{4} x^{8}+64350 B\, a^{2} b^{3} x^{8}+50050 a^{2} A\, b^{3} x^{6}+50050 B\, a^{3} b^{2} x^{6}+40950 a^{3} A\, b^{2} x^{4}}{45045 x^{15}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^5*A/x^15-1/13*a^4*(5*A*b+B*a)/x^13-5/11*a^3*b*(2*A*b+B*a)/x^11-10/9
*a^2*b^2*(A*b+B*a)/x^9-5/7*a*b^3*(A*b+2*B*a)/x^7-1/5*b^4*(A*b+5*B*a)/x^5-1/
3*b^5*B/x^3
```

Maxima [A]
time = 0.30, size = 121, normalized size = 1.03

$$-\frac{15015 \, B b^{5} x^{12} + 9009 \, {\left(5 \, B a b^{4} + A b^{5}\right)} x^{10} + 32175 \, {\left(2 \, B a^{2} b^{3} + A a b^{4}\right)} x^{8} + 50050 \, {\left(B a^{3} b^{2} + A a^{2} b^{3}\right)} x^{6} + 3003 \, A a^{5} + 20475 \, {\left(B a^{4} b + 2 \, A a^{3} b^{2}\right)} x^{4} + 3465 \, {\left(B a^{5} + 5 \, A a^{4} b\right)} x^{2}}{45045 \, x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 32175*(2*B*a^2
*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3003*A*a^5 + 2047
5*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 + 5*A*a^4*b)*x^2)/x^15
```

Fricas [A]
time = 0.62, size = 121, normalized size = 1.03

$$-\frac{15015 \, B b^{5} x^{12} + 9009 \, {\left(5 \, B a b^{4} + A b^{5}\right)} x^{10} + 32175 \, {\left(2 \, B a^{2} b^{3} + A a b^{4}\right)} x^{8} + 50050 \, {\left(B a^{3} b^{2} + A a^{2} b^{3}\right)} x^{6} + 3003 \, A a^{5} + 20475 \, {\left(B a^{4} b + 2 \, A a^{3} b^{2}\right)} x^{4} + 3465 \, {\left(B a^{5} + 5 \, A a^{4} b\right)} x^{2}}{45045 \, x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 32175*(2*B*a^2
*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3003*A*a^5 + 2047
5*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 + 5*A*a^4*b)*x^2)/x^15
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**16,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.49, size = 127, normalized size = 1.09

$$-\frac{15015\,Bb^5x^{12} + 45045\,Bab^4x^{10} + 9009\,Ab^5x^{10} + 64350\,Ba^2b^3x^8 + 32175\,Aab^4x^8 + 50050\,Ba^3b^2x^6 + 50050\,Aa^2b^3x^6 + 20475\,Ba^4bx^4 + 40950\,Aa^3b^2x^4 + 3465\,Ba^5x^2 + 17325\,Aa^4bx^2 + 3003\,Aa^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 45045*B*a*b^4*x^10 + 9009*A*b^5*x^10 + 64350*B
*a^2*b^3*x^8 + 32175*A*a*b^4*x^8 + 50050*B*a^3*b^2*x^6 + 50050*A*a^2*b^3*x^
6 + 20475*B*a^4*b*x^4 + 40950*A*a^3*b^2*x^4 + 3465*B*a^5*x^2 + 17325*A*a^4*
b*x^2 + 3003*A*a^5)/x^15
```

Mupad [**B**]
time = 0.04, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^5}{15} + x^8\left(\frac{10\,B\,a^2\,b^3}{7} + \frac{5\,A\,a\,b^4}{7}\right) + x^4\left(\frac{5\,B\,a^4\,b}{11} + \frac{10\,A\,a^3\,b^2}{11}\right) + x^2\left(\frac{B\,a^5}{13} + \frac{5\,A\,b\,a^4}{13}\right) + x^{10}\left(\frac{A\,b^5}{5} + B\,a\,b^4\right) + x^6\left(\frac{10\,B\,a^3\,b^2}{9} + \frac{10\,A\,a^2\,b^3}{9}\right) + \frac{B\,b^5\,x^{12}}{3}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^16,x)
```

```
[Out] -((A*a^5)/15 + x^8*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^4*((10*A*a^3*b^2)
/11 + (5*B*a^4*b)/11) + x^2*((B*a^5)/13 + (5*A*a^4*b)/13) + x^10*((A*b^5)/5
 + B*a*b^4) + x^6*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9) + (B*b^5*x^12)/3)/x
^15
```

3.49 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{17}}\,dx$

Optimal. Leaf size=76

$$-\frac{A(a+bx^2)^6}{16ax^{16}}+\frac{(Ab-4aB)(a+bx^2)^6}{56a^2x^{14}}-\frac{b(Ab-4aB)(a+bx^2)^6}{336a^3x^{12}}$$

```
[Out] -1/16*A*(b*x^2+a)^6/a/x^16+1/56*(A*b-4*B*a)*(b*x^2+a)^6/a^2/x^14-1/336*b*(A
*b-4*B*a)*(b*x^2+a)^6/a^3/x^12
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {457, 79, 47, 37}

$$-\frac{b(a+bx^2)^6(Ab-4aB)}{336a^3x^{12}}+\frac{(a+bx^2)^6(Ab-4aB)}{56a^2x^{14}}-\frac{A(a+bx^2)^6}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

```
[Out] -1/16*(A*(a + b*x^2)^6)/(a*x^16) + ((A*b - 4*a*B)*(a + b*x^2)^6)/(56*a^2*x^
14) - (b*(A*b - 4*a*B)*(a + b*x^2)^6)/(336*a^3*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{17}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^9}\,dx, x, x^2\right)\\
&= -\frac{A(a+bx^2)^6}{16ax^{16}} + \frac{(-2Ab+8aB)\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^2\right)}{16a}\\
&= -\frac{A(a+bx^2)^6}{16ax^{16}} + \frac{(Ab-4aB)(a+bx^2)^6}{56a^2x^{14}} + \frac{(b(Ab-4aB))\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, \right.}{56a^2}\\
&= -\frac{A(a+bx^2)^6}{16ax^{16}} + \frac{(Ab-4aB)(a+bx^2)^6}{56a^2x^{14}} - \frac{b(Ab-4aB)(a+bx^2)^6}{336a^3x^{12}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 121, normalized size = 1.59

$$-\frac{28b^5x^{10}(2A+3Bx^2)+70ab^4x^8(3A+4Bx^2)+84a^2b^3x^6(4A+5Bx^2)+56a^3b^2x^4(5A+6Bx^2)+20a^4bx^2(6A+7Bx^2)+3a^5(7A+8Bx^2)}{336x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

```
[Out] -1/336*(28*b^5*x^10*(2*A + 3*B*x^2) + 70*a*b^4*x^8*(3*A + 4*B*x^2) + 84*a^2
*b^3*x^6*(4*A + 5*B*x^2) + 56*a^3*b^2*x^4*(5*A + 6*B*x^2) + 20*a^4*b*x^2*(6
*A + 7*B*x^2) + 3*a^5*(7*A + 8*B*x^2))/x^16
```

Maple [A]
time = 0.07, size = 104, normalized size = 1.37

method	result

default	$-\frac{a^4(5Ab+Ba)}{14x^{14}}-\frac{b^5B}{4x^4}-\frac{5a^3b(2Ab+Ba)}{12x^{12}}-\frac{b^4(Ab+5Ba)}{6x^6}-\frac{5a\,b^3(Ab+2Ba)}{8x^8}-\frac{a^2b^2(Ab+Ba)}{x^{10}}-\frac{a^5A}{16x^{16}}$
norman	$\frac{-\frac{a^5A}{16}+\left(-\frac{5}{14}a^4bA-\frac{1}{14}a^5B\right)x^2+\left(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB\right)x^4+\left(-a^2b^3A-a^3b^2B\right)x^6+\left(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B\right)x^8+\left(-\frac{1}{6}b^5A-\frac{5}{6}a\,b^4B\right)x^{10}-}{x^{16}}$
risch	$\frac{-\frac{a^5A}{16}+\left(-\frac{5}{14}a^4bA-\frac{1}{14}a^5B\right)x^2+\left(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB\right)x^4+\left(-a^2b^3A-a^3b^2B\right)x^6+\left(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B\right)x^8+\left(-\frac{1}{6}b^5A-\frac{5}{6}a\,b^4B\right)x^{10}-}{x^{16}}$
gosper	$-\frac{84b^5B\,x^{12}+56A\,b^5x^{10}+280Ba\,b^4x^{10}+210aA\,b^4x^8+420B\,a^2b^3x^8+336a^2A\,b^3x^6+336B\,a^3b^2x^6+280a^3A\,b^2x^4+140B\,a^4b\,x^4+120a}{336x^{16}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*a^4*(5*A*b+B*a)/x^14-1/4*b^5*B/x^4-5/12*a^3*b*(2*A*b+B*a)/x^12-1/6*b^
4*(A*b+5*B*a)/x^6-5/8*a*b^3*(A*b+2*B*a)/x^8-a^2*b^2*(A*b+B*a)/x^10-1/16*a^5
*A/x^16
```

Maxima [A]
time = 0.29, size = 121, normalized size = 1.59

$$-\frac{84\,Bb^5x^{12}+56\,(5\,Bab^4+Ab^5)x^{10}+210\,(2\,Ba^2b^3+Aab^4)x^8+336\,(Ba^3b^2+Aa^2b^3)x^6+21\,Aa^5+140\,(Ba^4b+2\,Aa^3b^2)x^4+24\,(Ba^5+5\,Aa^4b)x^2}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="maxima")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^2*b^3 + A*
a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5 + 140*(B*a^4*b + 2*
A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/x^16
```

Fricas [A]
time = 0.95, size = 121, normalized size = 1.59

$$-\frac{84\,Bb^5x^{12}+56\,(5\,Bab^4+Ab^5)x^{10}+210\,(2\,Ba^2b^3+Aab^4)x^8+336\,(Ba^3b^2+Aa^2b^3)x^6+21\,Aa^5+140\,(Ba^4b+2\,Aa^3b^2)x^4+24\,(Ba^5+5\,Aa^4b)x^2}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^2*b^3 + A*
a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5 + 140*(B*a^4*b + 2*
A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/x^16
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**17,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.65, size = 127, normalized size = 1.67

$$-\frac{84\,Bb^{5}x^{12}+280\,Bab^{4}x^{10}+56\,Ab^{5}x^{10}+420\,Ba^{2}b^{3}x^{8}+210\,Aab^{4}x^{8}+336\,Ba^{3}b^{2}x^{6}+336\,Aa^{2}b^{3}x^{6}+140\,Ba^{4}bx^{4}+280\,Aa^{3}b^{2}x^{4}+24\,Ba^{5}x^{2}+120\,Aa^{4}bx^{2}+21\,Aa^{5}}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 280*B*a*b^4*x^10 + 56*A*b^5*x^10 + 420*B*a^2*b^3*x^
8 + 210*A*a*b^4*x^8 + 336*B*a^3*b^2*x^6 + 336*A*a^2*b^3*x^6 + 140*B*a^4*b*x
^4 + 280*A*a^3*b^2*x^4 + 24*B*a^5*x^2 + 120*A*a^4*b*x^2 + 21*A*a^5)/x^16
```

Mupad [B]
time = 0.05, size = 120, normalized size = 1.58

$$-\frac{\frac{A\,a^5}{16}+x^8\,\left(\frac{5\,B\,a^2\,b^3}{4}+\frac{5\,A\,a\,b^4}{8}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{12}+\frac{5\,A\,a^3\,b^2}{6}\right)+x^2\,\left(\frac{B\,a^5}{14}+\frac{5\,A\,b\,a^4}{14}\right)+x^{10}\,\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+x^6\,\left(B\,a^3\,b^2+A\,a^2\,b^3\right)+\frac{B\,b^5\,x^{12}}{4}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^17,x)
```

```
[Out] -((A*a^5)/16 + x^8*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^4*((5*A*a^3*b^2)/6
 + (5*B*a^4*b)/12) + x^2*((B*a^5)/14 + (5*A*a^4*b)/14) + x^10*((A*b^5)/6 +
(5*B*a*b^4)/6) + x^6*(A*a^2*b^3 + B*a^3*b^2) + (B*b^5*x^12)/4)/x^16
```

3.50 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{18}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{15x^{15}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2aB)}{9x^9}-\frac{b^4(Ab+5aB)}{7x^7}-\frac{b^5B}{5x^5}$$

```
[Out] -1/17*a^5*A/x^17-1/15*a^4*(5*A*b+B*a)/x^15-5/13*a^3*b*(2*A*b+B*a)/x^13-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/9*a*b^3*(A*b+2*B*a)/x^9-1/7*b^4*(A*b+5*B*a)/x^7-
1/5*b^5*B/x^5
```

Rubi [A]

time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^5B}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

```
[Out] -1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(5*x^5)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{18}}\,dx = \int \left(\frac{a^5A}{x^{18}}+\frac{a^4(5Ab+aB)}{x^{16}}+\frac{5a^3b(2Ab+aB)}{x^{14}}+\frac{10a^2b^2(Ab+aB)}{x^{12}}+\frac{5ab^3(A\right.$$
$$= -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{15x^{15}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab}{9}$$

Mathematica [A]

time = 0.04, size = 117, normalized size = 1.00

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{15x^{15}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2aB)}{9x^9}-\frac{b^4(Ab+5aB)}{7x^7}-\frac{b^5B}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

```
[Out] -1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(5*x^5)
```

Maple [A]
time = 0.07, size = 104, normalized size = 0.89

method	result
default	$-\frac{a^5 A}{17x^{17}}-\frac{a^4(5Ab+Ba)}{15x^{15}}-\frac{5a^3b(2Ab+Ba)}{13x^{13}}-\frac{10a^2b^2(Ab+Ba)}{11x^{11}}-\frac{5a\,b^3(Ab+2Ba)}{9x^9}-\frac{b^4(Ab+5Ba)}{7x^7}-\frac{b^5B}{5x^5}$
norman	$\frac{-\frac{a^5A}{17}+\left(-\frac{1}{3}a^4bA-\frac{1}{15}a^5B\right)x^2+\left(-\frac{10}{13}a^3b^2A-\frac{5}{13}a^4bB\right)x^4+\left(-\frac{10}{11}a^2b^3A-\frac{10}{11}a^3b^2B\right)x^6+\left(-\frac{5}{9}a\,b^4A-\frac{10}{9}a^2b^3B\right)x^8+\left(-\frac{1}{7}b^5A-\frac{5}{7}a\,b^4B\right)x}{x^{17}}$
risch	$\frac{-\frac{a^5A}{17}+\left(-\frac{1}{3}a^4bA-\frac{1}{15}a^5B\right)x^2+\left(-\frac{10}{13}a^3b^2A-\frac{5}{13}a^4bB\right)x^4+\left(-\frac{10}{11}a^2b^3A-\frac{10}{11}a^3b^2B\right)x^6+\left(-\frac{5}{9}a\,b^4A-\frac{10}{9}a^2b^3B\right)x^8+\left(-\frac{1}{7}b^5A-\frac{5}{7}a\,b^4B\right)x}{x^{17}}$
gosper	$-\frac{153153b^5B\,x^{12}+109395A\,b^5x^{10}+546975Ba\,b^4x^{10}+425425aA\,b^4x^8+850850B\,a^2b^3x^8+696150a^2A\,b^3x^6+696150B\,a^3b^2x^6+589050a}{765765x^{17}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -1/17*a^5*A/x^17-1/15*a^4*(5*A*b+B*a)/x^15-5/13*a^3*b*(2*A*b+B*a)/x^13-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/9*a*b^3*(A*b+2*B*a)/x^9-1/7*b^4*(A*b+5*B*a)/x^7-
1/5*b^5*B/x^5
```

Maxima [A]
time = 0.31, size = 121, normalized size = 1.03

$$-\frac{153153\,Bb^5x^{12}+109395\,(5\,Bab^4+Ab^5)x^{10}+425425\,(2\,Ba^2b^3+Aab^4)x^8+696150\,(Ba^3b^2+Aa^2b^3)x^6+45045\,Aa^5+294525\,(Ba^4b+2\,Aa^3b^2)x^4+51051\,(Ba^5+5\,Aa^4b)x^2}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="maxima")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 + 425425*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 45045*A*a^5
 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051*(B*a^5 + 5*A*a^4*b)*x^2)/x^17
```

Fricas [A]
time = 0.74, size = 121, normalized size = 1.03

$$-\frac{153153\,Bb^5x^{12}+109395\,(5\,Bab^4+Ab^5)x^{10}+425425\,(2\,Ba^2b^3+Aab^4)x^8+696150\,(Ba^3b^2+Aa^2b^3)x^6+45045\,Aa^5+294525\,(Ba^4b+2\,Aa^3b^2)x^4+51051\,(Ba^5+5\,Aa^4b)x^2}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="fricas")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 + 425425*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 45045*A*a^5
 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051*(B*a^5 + 5*A*a^4*b)*x^2)/x^17
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**18,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.06, size = 127, normalized size = 1.09

$$-\frac{153153\,Bb^5x^{12}+546975\,Bab^4x^{10}+109395\,Ab^5x^{10}+850850\,Ba^2b^3x^8+425425\,Aab^4x^8+696150\,Ba^3b^2x^6+696150\,Aa^2b^3x^6+294525\,Ba^4bx^4+589050\,Aa^3b^2x^4+51051\,Ba^5x^2+255255\,Aa^4bx^2+45045\,Aa^5}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="giac")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 546975*B*a*b^4*x^10 + 109395*A*b^5*x^10 + 85
0850*B*a^2*b^3*x^8 + 425425*A*a*b^4*x^8 + 696150*B*a^3*b^2*x^6 + 696150*A*a
^2*b^3*x^6 + 294525*B*a^4*b*x^4 + 589050*A*a^3*b^2*x^4 + 51051*B*a^5*x^2 +
255255*A*a^4*b*x^2 + 45045*A*a^5)/x^17
```

Mupad [**B**]
time = 0.05, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^5}{17}+x^8\,\left(\frac{10\,B\,a^2\,b^3}{9}+\frac{5\,A\,a\,b^4}{9}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{13}+\frac{10\,A\,a^3\,b^2}{13}\right)+x^2\,\left(\frac{B\,a^5}{15}+\frac{A\,b\,a^4}{3}\right)+x^{10}\,\left(\frac{A\,b^5}{7}+\frac{5\,B\,a\,b^4}{7}\right)+x^6\,\left(\frac{10\,B\,a^3\,b^2}{11}+\frac{10\,A\,a^2\,b^3}{11}\right)+\frac{B\,b^5\,x^{12}}{5}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^18,x)
```

```
[Out] -((A*a^5)/17 + x^8*((10*B*a^2*b^3)/9 + (5*A*a*b^4)/9) + x^4*((10*A*a^3*b^2)
/13 + (5*B*a^4*b)/13) + x^2*((B*a^5)/15 + (A*a^4*b)/3) + x^10*((A*b^5)/7 +
(5*B*a*b^4)/7) + x^6*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^12)
/5)/x^17
```

3.51 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{19}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{18x^{18}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{14x^{14}}-\frac{5a^2b^2(Ab+aB)}{6x^{12}}-\frac{ab^3(Ab+2aB)}{2x^{10}}-\frac{b^4(Ab+5aB)}{8x^8}-\frac{b^5B}{6x^6}$$

```
[Out] -1/18*a^5*A/x^18-1/16*a^4*(5*A*b+B*a)/x^16-5/14*a^3*b*(2*A*b+B*a)/x^14-5/6*
a^2*b^2*(A*b+B*a)/x^12-1/2*a*b^3*(A*b+2*B*a)/x^10-1/8*b^4*(A*b+5*B*a)/x^8-1
/6*b^5*B/x^6
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{18x^{18}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{5a^2b^2(aB+Ab)}{6x^{12}}-\frac{b^4(5aB+Ab)}{8x^8}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

```
[Out] -1/18*(a^5*A)/x^18 - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (5*a^2*b^2*(A*b + a*B))/(6*x^12) - (a*b^3*(A*b + 2*a*B))/(2*x
^10) - (b^4*(A*b + 5*a*B))/(8*x^8) - (b^5*B)/(6*x^6)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{19}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^{10}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5A}{x^{10}} + \frac{a^4(5Ab+aB)}{x^9} + \frac{5a^3b(2Ab+aB)}{x^8} + \frac{10a^2b^2(Ab+aB)}{x^7}\right.\right.$$
$$= -\frac{a^5A}{18x^{18}} - \frac{a^4(5Ab+aB)}{16x^{16}} - \frac{5a^3b(2Ab+aB)}{14x^{14}} - \frac{5a^2b^2(Ab+aB)}{6x^{12}} - \frac{ab^3(Ab+}{2x^{10}}$$

Mathematica [A]
time = 0.02, size = 121, normalized size = 1.03

$$-\frac{42b^5x^{10}(3A+4Bx^2)+126ab^4x^8(4A+5Bx^2)+168a^2b^3x^6(5A+6Bx^2)+120a^3b^2x^4(6A+7Bx^2)+45a^4bx^2(7A+8Bx^2)+7a^5(8A+9Bx^2)}{1008x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

```
[Out] -1/1008*(42*b^5*x^10*(3*A + 4*B*x^2) + 126*a*b^4*x^8*(4*A + 5*B*x^2) + 168*
a^2*b^3*x^6*(5*A + 6*B*x^2) + 120*a^3*b^2*x^4*(6*A + 7*B*x^2) + 45*a^4*b*x^
2*(7*A + 8*B*x^2) + 7*a^5*(8*A + 9*B*x^2))/x^18
```

Maple [A]
time = 0.07, size = 104, normalized size = 0.89

method	result
default	$-\frac{a^5A}{18x^{18}}-\frac{a^4(5Ab+Ba)}{16x^{16}}-\frac{5a^3b(2Ab+Ba)}{14x^{14}}-\frac{5a^2b^2(Ab+Ba)}{6x^{12}}-\frac{a\,b^3(Ab+2Ba)}{2x^{10}}-\frac{b^4(Ab+5Ba)}{8x^8}-\frac{b^5B}{6x^6}$
norman	$\frac{-\frac{a^5A}{18}+(-\frac{5}{16}a^4bA-\frac{1}{16}a^5B)x^2+(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB)x^4+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^6+(-\frac{1}{2}a\,b^4A-a^2b^3B)x^8+(-\frac{1}{8}b^5A-\frac{5}{8}a\,b^4B)x^{10}}{x^{18}}$
risch	$\frac{-\frac{a^5A}{18}+(-\frac{5}{16}a^4bA-\frac{1}{16}a^5B)x^2+(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB)x^4+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^6+(-\frac{1}{2}a\,b^4A-a^2b^3B)x^8+(-\frac{1}{8}b^5A-\frac{5}{8}a\,b^4B)x^{10}}{x^{18}}$
gosper	$-\frac{168b^5B\,x^{12}+126A\,b^5x^{10}+630Ba\,b^4x^{10}+504aA\,b^4x^8+1008B\,a^2b^3x^8+840a^2A\,b^3x^6+840B\,a^3b^2x^6+720a^3A\,b^2x^4+360B\,a^4b\,x^4+3}{1008x^{18}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*a^5*A/x^18-1/16*a^4*(5*A*b+B*a)/x^16-5/14*a^3*b*(2*A*b+B*a)/x^14-5/6*
a^2*b^2*(A*b+B*a)/x^12-1/2*a*b^3*(A*b+2*B*a)/x^10-1/8*b^4*(A*b+5*B*a)/x^8-1
/6*b^5*B/x^6
```

Maxima [A]
time = 0.30, size = 121, normalized size = 1.03

$$-\frac{168\,Bb^5x^{12}+126\,(5\,Bab^4+Ab^5)x^{10}+504\,(2\,Ba^2b^3+Aab^4)x^8+840\,(Ba^3b^2+Aa^2b^3)x^6+56\,Aa^5+360\,(Ba^4b+2\,Aa^3b^2)x^4+63\,(Ba^5+5\,Aa^4b)x^2}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="maxima")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B*a^2*b^3 +
 A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*a^5 + 360*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^18
```

Fricas [**A**]
time = 0.66, size = 121, normalized size = 1.03

$$-\frac{168\,Bb^5x^{12} + 126\,(5\,Bab^4 + Ab^5)x^{10} + 504\,(2\,Ba^2b^3 + Aab^4)x^8 + 840\,(Ba^3b^2 + Aa^2b^3)x^6 + 56\,Aa^5 + 360\,(Ba^4b + 2\,Aa^3b^2)x^4 + 63\,(Ba^5 + 5\,Aa^4b)x^2}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B*a^2*b^3 +
 A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*a^5 + 360*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^18
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**19,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.57, size = 127, normalized size = 1.09

$$-\frac{168\,Bb^5x^{12} + 630\,Bab^4x^{10} + 126\,Ab^5x^{10} + 1008\,Ba^2b^3x^8 + 504\,Aab^4x^8 + 840\,Ba^3b^2x^6 + 840\,Aa^2b^3x^6 + 360\,Ba^4bx^4 + 720\,Aa^3b^2x^4 + 63\,Ba^5x^2 + 315\,Aa^4bx^2 + 56\,Aa^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 630*B*a*b^4*x^10 + 126*A*b^5*x^10 + 1008*B*a^2*b^
3*x^8 + 504*A*a*b^4*x^8 + 840*B*a^3*b^2*x^6 + 840*A*a^2*b^3*x^6 + 360*B*a^4
*b*x^4 + 720*A*a^3*b^2*x^4 + 63*B*a^5*x^2 + 315*A*a^4*b*x^2 + 56*A*a^5)/x^1
8
```

Mupad [**B**]
time = 0.03, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^5}{18} + x^8\,\left(B\,a^2\,b^3 + \frac{A\,a\,b^4}{2}\right) + x^4\,\left(\frac{5\,B\,a^4\,b}{14} + \frac{5\,A\,a^3\,b^2}{7}\right) + x^2\,\left(\frac{B\,a^5}{16} + \frac{5\,A\,b\,a^4}{16}\right) + x^{10}\,\left(\frac{A\,b^5}{8} + \frac{5\,B\,a\,b^4}{8}\right) + x^6\,\left(\frac{5\,B\,a^3\,b^2}{6} + \frac{5\,A\,a^2\,b^3}{6}\right) + \frac{B\,b^5\,x^{12}}{6}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^19,x)
```

```
[Out] -((A*a^5)/18 + x^8*(B*a^2*b^3 + (A*a*b^4)/2) + x^4*((5*A*a^3*b^2)/7 + (5*B*
a^4*b)/14) + x^2*((B*a^5)/16 + (5*A*a^4*b)/16) + x^10*((A*b^5)/8 + (5*B*a*b
^4)/8) + x^6*((5*A*a^2*b^3)/6 + (5*B*a^3*b^2)/6) + (B*b^5*x^12)/6)/x^18
```

3.52 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{20}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{a^3b(2Ab+aB)}{3x^{15}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{5ab^3(Ab+2aB)}{11x^{11}}-\frac{b^4(Ab+5aB)}{9x^9}-\frac{b^5B}{7x^7}$$

```
[Out] -1/19*a^5*A/x^19-1/17*a^4*(5*A*b+B*a)/x^17-1/3*a^3*b*(2*A*b+B*a)/x^15-10/13
*a^2*b^2*(A*b+B*a)/x^13-5/11*a*b^3*(A*b+2*B*a)/x^11-1/9*b^4*(A*b+5*B*a)/x^9
-1/7*b^5*B/x^7
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^5B}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

```
[Out] -1/19*(a^5*A)/x^19 - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*b + a*B))/
(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^3*(A*b + 2*a*B))/(11
*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B)/(7*x^7)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{20}}\,dx = \int \left(\frac{a^5A}{x^{20}}+\frac{a^4(5Ab+aB)}{x^{18}}+\frac{5a^3b(2Ab+aB)}{x^{16}}+\frac{10a^2b^2(Ab+aB)}{x^{14}}+\frac{5ab^3(Ab\right.$$

$$=-\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{a^3b(2Ab+aB)}{3x^{15}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{5ab^3(Ab+}{11x^{11}}$$

Mathematica [A]
time = 0.03, size = 117, normalized size = 1.00

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{a^3b(2Ab+aB)}{3x^{15}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{5ab^3(Ab+2aB)}{11x^{11}}-\frac{b^4(Ab+5aB)}{9x^9}-\frac{b^5B}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

```
[Out] -1/19*(a^5*A)/x^19 - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*b + a*B))/
(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^3*(A*b + 2*a*B))/(11
*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B)/(7*x^7)
```

Maple [A]
time = 0.07, size = 104, normalized size = 0.89

method	result
default	$-\frac{a^5 A}{19x^{19}}-\frac{a^4(5Ab+Ba)}{17x^{17}}-\frac{a^3b(2Ab+Ba)}{3x^{15}}-\frac{10a^2b^2(Ab+Ba)}{13x^{13}}-\frac{5ab^3(Ab+2Ba)}{11x^{11}}-\frac{b^4(Ab+5Ba)}{9x^9}-\frac{b^5B}{7x^7}$
norman	$\frac{-\frac{a^5A}{19}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^2+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^4+(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B)x^6+(-\frac{5}{11}ab^4A-\frac{10}{11}a^2b^3B)x^8+(-\frac{1}{9}b^5A-\frac{5}{9}ab^4B}{x^{19}}$
risch	$\frac{-\frac{a^5A}{19}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^2+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^4+(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B)x^6+(-\frac{5}{11}ab^4A-\frac{10}{11}a^2b^3B)x^8+(-\frac{1}{9}b^5A-\frac{5}{9}ab^4B}{x^{19}}$
gosper	$-\frac{415701b^5Bx^{12}+323323Ab^5x^{10}+1616615Bab^4x^{10}+1322685aAb^4x^8+2645370Ba^2b^3x^8+2238390a^2Ab^3x^6+2238390Ba^3b^2x^6+}{2909907x^{19}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/19*a^5*A/x^19-1/17*a^4*(5*A*b+B*a)/x^17-1/3*a^3*b*(2*A*b+B*a)/x^15-10/13
*a^2*b^2*(A*b+B*a)/x^13-5/11*a*b^3*(A*b+2*B*a)/x^11-1/9*b^4*(A*b+5*B*a)/x^9
-1/7*b^5*B/x^7
```

Maxima [A]
time = 0.30, size = 121, normalized size = 1.03

$$-\frac{415701\,Bb^5x^{12}+323323\,(5\,Bab^4+Ab^5)x^{10}+1322685\,(2\,Ba^2b^3+Aab^4)x^8+2238390\,(Ba^3b^2+Aa^2b^3)x^6+153153\,Aa^5+969969\,(Ba^4b+2\,Aa^3b^2)x^4+171171\,(Ba^5+5\,Aa^4b)x^2}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="maxima")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 + 1322685*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 153153*A
*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 171171*(B*a^5 + 5*A*a^4*b)*x^2)
/x^19
```

Fricas [A]
time = 0.78, size = 121, normalized size = 1.03

$$-\frac{415701\,Bb^5x^{12}+323323\,(5\,Bab^4+Ab^5)x^{10}+1322685\,(2\,Ba^2b^3+Aab^4)x^8+2238390\,(Ba^3b^2+Aa^2b^3)x^6+153153\,Aa^5+969969\,(Ba^4b+2\,Aa^3b^2)x^4+171171\,(Ba^5+5\,Aa^4b)x^2}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="fricas")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 + 1322685*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 153153*A
*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 171171*(B*a^5 + 5*A*a^4*b)*x^2)
/x^19
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**20,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.15, size = 127, normalized size = 1.09

$$-\frac{415701\,Bb^5x^{12}+1616615\,Bab^4x^{10}+323323\,Ab^5x^{10}+2645370\,Ba^2b^3x^8+1322685\,Aab^4x^8+2238390\,Ba^3b^2x^6+2238390\,Aa^2b^3x^6+969969\,Ba^4bx^4+1939938\,Aa^3b^2x^4+171171\,Ba^5x^2+855855\,Aa^4bx^2+153153\,Aa^5}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="giac")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 1616615*B*a*b^4*x^10 + 323323*A*b^5*x^10 +
2645370*B*a^2*b^3*x^8 + 1322685*A*a*b^4*x^8 + 2238390*B*a^3*b^2*x^6 + 22383
90*A*a^2*b^3*x^6 + 969969*B*a^4*b*x^4 + 1939938*A*a^3*b^2*x^4 + 171171*B*a^
5*x^2 + 855855*A*a^4*b*x^2 + 153153*A*a^5)/x^19
```

Mupad [**B**]
time = 0.06, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^5}{19}+x^4\,\left(\frac{B\,a^4\,b}{3}+\frac{2\,A\,a^3\,b^2}{3}\right)+x^8\,\left(\frac{10\,B\,a^2\,b^3}{11}+\frac{5\,A\,a\,b^4}{11}\right)+x^2\,\left(\frac{B\,a^5}{17}+\frac{5\,A\,b\,a^4}{17}\right)+x^{10}\,\left(\frac{A\,b^5}{9}+\frac{5\,B\,a\,b^4}{9}\right)+x^6\,\left(\frac{10\,B\,a^3\,b^2}{13}+\frac{10\,A\,a^2\,b^3}{13}\right)+\frac{B\,b^5\,x^{12}}{7}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^20,x)
```

```
[Out] -((A*a^5)/19 + x^4*((2*A*a^3*b^2)/3 + (B*a^4*b)/3) + x^8*((10*B*a^2*b^3)/11
 + (5*A*a*b^4)/11) + x^2*((B*a^5)/17 + (5*A*a^4*b)/17) + x^10*((A*b^5)/9 +
(5*B*a*b^4)/9) + x^6*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^12)
/7)/x^19
```

3.53 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{21}} \, dx$

Optimal. Leaf size=117

$$-\frac{a^5 A}{20x^{20}}-\frac{a^4(5Ab+aB)}{18x^{18}}-\frac{5a^3b(2Ab+aB)}{16x^{16}}-\frac{5a^2b^2(Ab+aB)}{7x^{14}}-\frac{5ab^3(Ab+2aB)}{12x^{12}}-\frac{b^4(Ab+5aB)}{10x^{10}}-\frac{b^5 B}{8x^8}$$

```
[Out] -1/20*a^5*A/x^20-1/18*a^4*(5*A*b+B*a)/x^18-5/16*a^3*b*(2*A*b+B*a)/x^16-5/7*
a^2*b^2*(A*b+B*a)/x^14-5/12*a*b^3*(A*b+2*B*a)/x^12-1/10*b^4*(A*b+5*B*a)/x^1
0-1/8*b^5*B/x^8
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5 A}{20x^{20}}-\frac{a^4(aB+5Ab)}{18x^{18}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{5a^2b^2(aB+Ab)}{7x^{14}}-\frac{b^4(5aB+Ab)}{10x^{10}}-\frac{5ab^3(2aB+Ab)}{12x^{12}}-\frac{b^5 B}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

```
[Out] -1/20*(a^5*A)/x^20 - (a^4*(5*A*b + a*B))/(18*x^18) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (5*a^2*b^2*(A*b + a*B))/(7*x^14) - (5*a*b^3*(A*b + 2*a*B))/(1
2*x^12) - (b^4*(A*b + 5*a*B))/(10*x^10) - (b^5*B)/(8*x^8)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{21}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^{11}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5A}{x^{11}} + \frac{a^4(5Ab+aB)}{x^{10}} + \frac{5a^3b(2Ab+aB)}{x^9} + \frac{10a^2b^2(Ab+aB)}{x^8} + \right.\right.$$
$$= -\frac{a^5A}{20x^{20}} - \frac{a^4(5Ab+aB)}{18x^{18}} - \frac{5a^3b(2Ab+aB)}{16x^{16}} - \frac{5a^2b^2(Ab+aB)}{7x^{14}} - \frac{5ab^3(Ab+}{12x^{12}}$$

Mathematica [A]
time = 0.02, size = 121, normalized size = 1.03

$$-\frac{126b^5x^{10}(4A+5Bx^2)+420ab^4x^8(5A+6Bx^2)+600a^2b^3x^6(6A+7Bx^2)+450a^3b^2x^4(7A+8Bx^2)+175a^4bx^2(8A+9Bx^2)+28a^5(9A+10Bx^2)}{5040x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

```
[Out] -1/5040*(126*b^5*x^10*(4*A + 5*B*x^2) + 420*a*b^4*x^8*(5*A + 6*B*x^2) + 600
*a^2*b^3*x^6*(6*A + 7*B*x^2) + 450*a^3*b^2*x^4*(7*A + 8*B*x^2) + 175*a^4*b*
x^2*(8*A + 9*B*x^2) + 28*a^5*(9*A + 10*B*x^2))/x^20
```

Maple [A]
time = 0.07, size = 104, normalized size = 0.89

method	result
default	$-\frac{a^5A}{20x^{20}} - \frac{a^4(5Ab+Ba)}{18x^{18}} - \frac{5a^3b(2Ab+Ba)}{16x^{16}} - \frac{5a^2b^2(Ab+Ba)}{7x^{14}} - \frac{5ab^3(Ab+2Ba)}{12x^{12}} - \frac{b^4(Ab+5Ba)}{10x^{10}} - \frac{b^5B}{8x^8}$
norman	$\frac{-\frac{a^5A}{20}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^2+(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB)x^4+(-\frac{5}{7}a^2b^3A-\frac{5}{7}a^3b^2B)x^6+(-\frac{5}{12}ab^4A-\frac{5}{6}a^2b^3B)x^8+(-\frac{1}{10}b^5A-\frac{1}{2}ab^4B)x^1}{x^{20}}$
risch	$\frac{-\frac{a^5A}{20}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^2+(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB)x^4+(-\frac{5}{7}a^2b^3A-\frac{5}{7}a^3b^2B)x^6+(-\frac{5}{12}ab^4A-\frac{5}{6}a^2b^3B)x^8+(-\frac{1}{10}b^5A-\frac{1}{2}ab^4B)x^1}{x^{20}}$
gosper	$-\frac{630b^5Bx^{12}+504Ab^5x^{10}+2520Bab^4x^{10}+2100aAb^4x^8+4200Ba^2b^3x^8+3600a^2Ab^3x^6+3600Ba^3b^2x^6+3150a^3Ab^2x^4+1575Ba^4b}{5040x^{20}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^21,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*a^5*A/x^20-1/18*a^4*(5*A*b+B*a)/x^18-5/16*a^3*b*(2*A*b+B*a)/x^16-5/7*
a^2*b^2*(A*b+B*a)/x^14-5/12*a*b^3*(A*b+2*B*a)/x^12-1/10*b^4*(A*b+5*B*a)/x^1
0-1/8*b^5*B/x^8
```

Maxima [A]
time = 0.29, size = 121, normalized size = 1.03

$$-\frac{630\,Bb^5x^{12}+504\,(5\,Bab^4+Ab^5)x^{10}+2100\,(2\,Ba^2b^3+Aab^4)x^8+3600\,(Ba^3b^2+Aa^2b^3)x^6+252\,Aa^5+1575\,(Ba^4b+2\,Aa^3b^2)x^4+280\,(Ba^5+5\,Aa^4b)x^2}{5040\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="maxima")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252*A*a^5 + 1575*(B*a^4
*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*b)*x^2)/x^20
```

Fricas [A]
time = 0.79, size = 121, normalized size = 1.03

$$-\frac{630\,Bb^5x^{12}+504\,(5\,Bab^4+Ab^5)x^{10}+2100\,(2\,Ba^2b^3+Aab^4)x^8+3600\,(Ba^3b^2+Aa^2b^3)x^6+252\,Aa^5+1575\,(Ba^4b+2\,Aa^3b^2)x^4+280\,(Ba^5+5\,Aa^4b)x^2}{5040\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="fricas")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252*A*a^5 + 1575*(B*a^4
*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*b)*x^2)/x^20
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**21,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.86, size = 127, normalized size = 1.09

$$-\frac{630\,Bb^5x^{12}+2520\,Bab^4x^{10}+504\,Ab^5x^{10}+4200\,Ba^2b^3x^8+2100\,Aab^4x^8+3600\,Ba^3b^2x^6+3600\,Aa^2b^3x^6+1575\,Ba^4bx^4+3150\,Aa^3b^2x^4+280\,Ba^5x^2+1400\,Aa^4bx^2+252\,Aa^5}{5040\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="giac")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 2520*B*a*b^4*x^10 + 504*A*b^5*x^10 + 4200*B*a^2*b
^3*x^8 + 2100*A*a*b^4*x^8 + 3600*B*a^3*b^2*x^6 + 3600*A*a^2*b^3*x^6 + 1575*
B*a^4*b*x^4 + 3150*A*a^3*b^2*x^4 + 280*B*a^5*x^2 + 1400*A*a^4*b*x^2 + 252*A
*a^5)/x^20
```

Mupad [B]
time = 0.05, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^5}{20}+x^8\,\left(\frac{5\,B\,a^2\,b^3}{6}+\frac{5\,A\,a\,b^4}{12}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{16}+\frac{5\,A\,a^3\,b^2}{8}\right)+x^2\,\left(\frac{B\,a^5}{18}+\frac{5\,A\,b\,a^4}{18}\right)+x^{10}\,\left(\frac{A\,b^5}{10}+\frac{B\,a\,b^4}{2}\right)+x^6\,\left(\frac{5\,B\,a^3\,b^2}{7}+\frac{5\,A\,a^2\,b^3}{7}\right)+\frac{B\,b^5\,x^{12}}{8}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^21,x)
```

```
[Out] -((A*a^5)/20 + x^8*((5*B*a^2*b^3)/6 + (5*A*a*b^4)/12) + x^4*((5*A*a^3*b^2)/
8 + (5*B*a^4*b)/16) + x^2*((B*a^5)/18 + (5*A*a^4*b)/18) + x^10*((A*b^5)/10
+ (B*a*b^4)/2) + x^6*((5*A*a^2*b^3)/7 + (5*B*a^3*b^2)/7) + (B*b^5*x^12)/8)/
x^20
```

3.54 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{22}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{21x^{21}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{17x^{17}}-\frac{2a^2b^2(Ab+aB)}{3x^{15}}-\frac{5ab^3(Ab+2aB)}{13x^{13}}-\frac{b^4(Ab+5aB)}{11x^{11}}-\frac{b^5B}{9x^9}$$

```
[Out] -1/21*a^5*A/x^21-1/19*a^4*(5*A*b+B*a)/x^19-5/17*a^3*b*(2*A*b+B*a)/x^17-2/3*
a^2*b^2*(A*b+B*a)/x^15-5/13*a*b^3*(A*b+2*B*a)/x^13-1/11*b^4*(A*b+5*B*a)/x^1
1-1/9*b^5*B/x^9
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^5A}{21x^{21}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^5B}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

```
[Out] -1/21*(a^5*A)/x^21 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b^3*(A*b + 2*a*B))/(1
3*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^5*B)/(9*x^9)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{22}}\,dx &= \int \left(\frac{a^5A}{x^{22}}+\frac{a^4(5Ab+aB)}{x^{20}}+\frac{5a^3b(2Ab+aB)}{x^{18}}+\frac{10a^2b^2(Ab+aB)}{x^{16}}+\frac{5ab^3(A\right.\\ &= -\frac{a^5A}{21x^{21}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{17x^{17}}-\frac{2a^2b^2(Ab+aB)}{3x^{15}}-\frac{5ab^3(Ab}{13x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 117, normalized size = 1.00

$$-\frac{a^5A}{21x^{21}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{17x^{17}}-\frac{2a^2b^2(Ab+aB)}{3x^{15}}-\frac{5ab^3(Ab+2aB)}{13x^{13}}-\frac{b^4(Ab+5aB)}{11x^{11}}-\frac{b^5B}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

```
[Out] -1/21*(a^5*A)/x^21 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b^3*(A*b + 2*a*B))/(1
3*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^5*B)/(9*x^9)
```

Maple [A]
time = 0.07, size = 104, normalized size = 0.89

method	result
default	$-\frac{a^5 A}{21x^{21}}-\frac{a^4(5Ab+Ba)}{19x^{19}}-\frac{5a^3b(2Ab+Ba)}{17x^{17}}-\frac{2a^2b^2(Ab+Ba)}{3x^{15}}-\frac{5a\,b^3(Ab+2Ba)}{13x^{13}}-\frac{b^4(Ab+5Ba)}{11x^{11}}-\frac{b^5B}{9x^9}$
norman	$\frac{-\frac{a^5A}{21}+(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B)x^2+(-\frac{10}{17}a^3b^2A-\frac{5}{17}a^4bB)x^4+(-\frac{2}{3}a^2b^3A-\frac{2}{3}a^3b^2B)x^6+(-\frac{5}{13}a\,b^4A-\frac{10}{13}a^2b^3B)x^8+(-\frac{1}{11}b^5A-\frac{5}{11}a\,b^4B}{x^{21}}$
risch	$\frac{-\frac{a^5A}{21}+(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B)x^2+(-\frac{10}{17}a^3b^2A-\frac{5}{17}a^4bB)x^4+(-\frac{2}{3}a^2b^3A-\frac{2}{3}a^3b^2B)x^6+(-\frac{5}{13}a\,b^4A-\frac{10}{13}a^2b^3B)x^8+(-\frac{1}{11}b^5A-\frac{5}{11}a\,b^4B}{x^{21}}$
gosper	$-\frac{323323b^5B\,x^{12}+264537A\,b^5x^{10}+1322685Ba\,b^4x^{10}+1119195aA\,b^4x^8+2238390B\,a^2b^3x^8+1939938a^2A\,b^3x^6+1939938B\,a^3b^2x^6+17}{2909907x^{21}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^22,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*a^5*A/x^21-1/19*a^4*(5*A*b+B*a)/x^19-5/17*a^3*b*(2*A*b+B*a)/x^17-2/3*
a^2*b^2*(A*b+B*a)/x^15-5/13*a*b^3*(A*b+2*B*a)/x^13-1/11*b^4*(A*b+5*B*a)/x^1
1-1/9*b^5*B/x^9
```

Maxima [A]
time = 0.31, size = 121, normalized size = 1.03

$$-\frac{323323\,Bb^5x^{12}+264537\,(5\,Bab^4+Ab^5)x^{10}+1119195\,(2\,Ba^2b^3+Aab^4)x^8+1939938\,(Ba^3b^2+Aa^2b^3)x^6+138567\,Aa^5+855855\,(Ba^4b+2\,Aa^3b^2)x^4+153153\,(Ba^5+5\,Aa^4b)x^2}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="maxima")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 + 1119195*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 138567*A
*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 153153*(B*a^5 + 5*A*a^4*b)*x^2)
/x^21
```

Fricas [A]
time = 0.69, size = 121, normalized size = 1.03

$$-\frac{323323\,Bb^5x^{12}+264537\,(5\,Bab^4+Ab^5)x^{10}+1119195\,(2\,Ba^2b^3+Aab^4)x^8+1939938\,(Ba^3b^2+Aa^2b^3)x^6+138567\,Aa^5+855855\,(Ba^4b+2\,Aa^3b^2)x^4+153153\,(Ba^5+5\,Aa^4b)x^2}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="fricas")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 + 1119195*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 138567*A
*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 153153*(B*a^5 + 5*A*a^4*b)*x^2)
/x^21
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**22,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.15, size = 127, normalized size = 1.09

$$-\frac{323323\,Bb^5x^{12}+1322685\,Bab^4x^{10}+264537\,Ab^5x^{10}+2238390\,Ba^2b^3x^8+1119195\,Aab^4x^8+1939938\,Ba^3b^2x^6+1939938\,Aa^2b^3x^6+855855\,Ba^4bx^4+1711710\,Aa^3b^2x^4+153153\,Ba^5x^2+765765\,Aa^4bx^2+138567\,Aa^5}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="giac")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 1322685*B*a*b^4*x^10 + 264537*A*b^5*x^10 +
2238390*B*a^2*b^3*x^8 + 1119195*A*a*b^4*x^8 + 1939938*B*a^3*b^2*x^6 + 19399
38*A*a^2*b^3*x^6 + 855855*B*a^4*b*x^4 + 1711710*A*a^3*b^2*x^4 + 153153*B*a^
5*x^2 + 765765*A*a^4*b*x^2 + 138567*A*a^5)/x^21
```

Mupad [**B**]
time = 0.05, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^5}{21}+x^8\,\left(\frac{10\,B\,a^2\,b^3}{13}+\frac{5\,A\,a\,b^4}{13}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{17}+\frac{10\,A\,a^3\,b^2}{17}\right)+x^2\,\left(\frac{B\,a^5}{19}+\frac{5\,A\,b\,a^4}{19}\right)+x^{10}\,\left(\frac{A\,b^5}{11}+\frac{5\,B\,a\,b^4}{11}\right)+x^6\,\left(\frac{2\,B\,a^3\,b^2}{3}+\frac{2\,A\,a^2\,b^3}{3}\right)+\frac{B\,b^5\,x^{12}}{9}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^22,x)
```

```
[Out] -((A*a^5)/21 + x^8*((10*B*a^2*b^3)/13 + (5*A*a*b^4)/13) + x^4*((10*A*a^3*b^
2)/17 + (5*B*a^4*b)/17) + x^2*((B*a^5)/19 + (5*A*a^4*b)/19) + x^10*((A*b^5)
/11 + (5*B*a*b^4)/11) + x^6*((2*A*a^2*b^3)/3 + (2*B*a^3*b^2)/3) + (B*b^5*x^
12)/9)/x^21
```

3.55 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{23}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{20x^{20}}-\frac{5a^3b(2Ab+aB)}{18x^{18}}-\frac{5a^2b^2(Ab+aB)}{8x^{16}}-\frac{5ab^3(Ab+2aB)}{14x^{14}}-\frac{b^4(Ab+5aB)}{12x^{12}}-\frac{b^5B}{10x^{10}}$$

```
[Out] -1/22*a^5*A/x^22-1/20*a^4*(5*A*b+B*a)/x^20-5/18*a^3*b*(2*A*b+B*a)/x^18-5/8*
a^2*b^2*(A*b+B*a)/x^16-5/14*a*b^3*(A*b+2*B*a)/x^14-1/12*b^4*(A*b+5*B*a)/x^1
2-1/10*b^5*B/x^10
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(aB+5Ab)}{20x^{20}}-\frac{5a^3b(aB+2Ab)}{18x^{18}}-\frac{5a^2b^2(aB+Ab)}{8x^{16}}-\frac{b^4(5aB+Ab)}{12x^{12}}-\frac{5ab^3(2aB+Ab)}{14x^{14}}-\frac{b^5B}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

```
[Out] -1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(20*x^20) - (5*a^3*b*(2*A*b + a*B)
)/(18*x^18) - (5*a^2*b^2*(A*b + a*B))/(8*x^16) - (5*a*b^3*(A*b + 2*a*B))/(1
4*x^14) - (b^4*(A*b + 5*a*B))/(12*x^12) - (b^5*B)/(10*x^10)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{23}} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^{12}} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5A}{x^{12}} + \frac{a^4(5Ab+aB)}{x^{11}} + \frac{5a^3b(2Ab+aB)}{x^{10}} + \frac{10a^2b^2(Ab+aB)}{x^9}\right.\right.$$
$$= -\frac{a^5A}{22x^{22}} - \frac{a^4(5Ab+aB)}{20x^{20}} - \frac{5a^3b(2Ab+aB)}{18x^{18}} - \frac{5a^2b^2(Ab+aB)}{8x^{16}} - \frac{5ab^3(Ab}{14x}$$

Mathematica [A]
time = 0.02, size = 121, normalized size = 1.03

$$-\frac{462b^5x^{10}(5A+6Bx^2)+1650ab^4x^8(6A+7Bx^2)+2475a^2b^3x^6(7A+8Bx^2)+1925a^3b^2x^4(8A+9Bx^2)+770a^4bx^2(9A+10Bx^2)+126a^5(10A+11Bx^2)}{27720x^{22}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

```
[Out] -1/27720*(462*b^5*x^10*(5*A + 6*B*x^2) + 1650*a*b^4*x^8*(6*A + 7*B*x^2) + 2
475*a^2*b^3*x^6*(7*A + 8*B*x^2) + 1925*a^3*b^2*x^4*(8*A + 9*B*x^2) + 770*a^
4*b*x^2*(9*A + 10*B*x^2) + 126*a^5*(10*A + 11*B*x^2))/x^22
```

Maple [A]
time = 0.07, size = 104, normalized size = 0.89

method	result
default	$-\frac{a^5A}{22x^{22}} - \frac{a^4(5Ab+Ba)}{20x^{20}} - \frac{5a^3b(2Ab+Ba)}{18x^{18}} - \frac{5a^2b^2(Ab+Ba)}{8x^{16}} - \frac{5ab^3(Ab+2Ba)}{14x^{14}} - \frac{b^4(Ab+5Ba)}{12x^{12}} - \frac{b^5B}{10x^{10}}$
norman	$\frac{-\frac{a^5A}{22}+(-\frac{1}{4}a^4bA-\frac{1}{20}a^5B)x^2+(-\frac{5}{9}a^3b^2A-\frac{5}{18}a^4bB)x^4+(-\frac{5}{8}a^2b^3A-\frac{5}{8}a^3b^2B)x^6+(-\frac{5}{14}ab^4A-\frac{5}{7}a^2b^3B)x^8+(-\frac{1}{12}b^5A-\frac{5}{12}ab^4B)}{x^{22}}$
risch	$\frac{-\frac{a^5A}{22}+(-\frac{1}{4}a^4bA-\frac{1}{20}a^5B)x^2+(-\frac{5}{9}a^3b^2A-\frac{5}{18}a^4bB)x^4+(-\frac{5}{8}a^2b^3A-\frac{5}{8}a^3b^2B)x^6+(-\frac{5}{14}ab^4A-\frac{5}{7}a^2b^3B)x^8+(-\frac{1}{12}b^5A-\frac{5}{12}ab^4B)}{x^{22}}$
gosper	$-\frac{2772b^5Bx^{12}+2310Ab^5x^{10}+11550Bab^4x^{10}+9900aAb^4x^8+19800Ba^2b^3x^8+17325a^2Ab^3x^6+17325Ba^3b^2x^6+15400a^3Ab^2x^4+7}{27720x^{22}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^23,x,method=_RETURNVERBOSE)
```

```
[Out] -1/22*a^5*A/x^22-1/20*a^4*(5*A*b+B*a)/x^20-5/18*a^3*b*(2*A*b+B*a)/x^18-5/8*
a^2*b^2*(A*b+B*a)/x^16-5/14*a*b^3*(A*b+2*B*a)/x^14-1/12*b^4*(A*b+5*B*a)/x^1
2-1/10*b^5*B/x^10
```

Maxima [A]
time = 0.30, size = 121, normalized size = 1.03

$$-\frac{2772\,Bb^5x^{12}+2310\,(5\,Bab^4+Ab^5)x^{10}+9900\,(2\,Ba^2b^3+Aab^4)x^8+17325\,(Ba^3b^2+Aa^2b^3)x^6+1260\,Aa^5+7700\,(Ba^4b+2\,Aa^3b^2)x^4+1386\,(Ba^5+5\,Aa^4b)x^2}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="maxima")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*(2*B*a^2*b
^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1260*A*a^5 + 7700*(
B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*A*a^4*b)*x^2)/x^22
```

Fricas [A]
time = 1.02, size = 121, normalized size = 1.03

$$-\frac{2772\,Bb^5x^{12}+2310\,(5\,Bab^4+Ab^5)x^{10}+9900\,(2\,Ba^2b^3+Aab^4)x^8+17325\,(Ba^3b^2+Aa^2b^3)x^6+1260\,Aa^5+7700\,(Ba^4b+2\,Aa^3b^2)x^4+1386\,(Ba^5+5\,Aa^4b)x^2}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="fricas")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*(2*B*a^2*b
^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1260*A*a^5 + 7700*(
B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*A*a^4*b)*x^2)/x^22
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**23,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.67, size = 127, normalized size = 1.09

$$-\frac{2772\,Bb^5x^{12}+11550\,Bab^4x^{10}+2310\,Ab^5x^{10}+19800\,Ba^2b^3x^8+9900\,Aab^4x^8+17325\,Ba^3b^2x^6+17325\,Aa^2b^3x^6+7700\,Ba^4bx^4+15400\,Aa^3b^2x^4+1386\,Ba^5x^2+6930\,Aa^4bx^2+1260\,Aa^5}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="giac")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 11550*B*a*b^4*x^10 + 2310*A*b^5*x^10 + 19800*B*
a^2*b^3*x^8 + 9900*A*a*b^4*x^8 + 17325*B*a^3*b^2*x^6 + 17325*A*a^2*b^3*x^6
+ 7700*B*a^4*b*x^4 + 15400*A*a^3*b^2*x^4 + 1386*B*a^5*x^2 + 6930*A*a^4*b*x^
2 + 1260*A*a^5)/x^22
```

Mupad [B]
time = 0.04, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^5}{22}+x^8\,\left(\frac{5\,B\,a^2\,b^3}{7}+\frac{5\,A\,a\,b^4}{14}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{18}+\frac{5\,A\,a^3\,b^2}{9}\right)+x^2\,\left(\frac{B\,a^5}{20}+\frac{A\,b\,a^4}{4}\right)+x^{10}\,\left(\frac{A\,b^5}{12}+\frac{5\,B\,a\,b^4}{12}\right)+x^6\,\left(\frac{5\,B\,a^3\,b^2}{8}+\frac{5\,A\,a^2\,b^3}{8}\right)+\frac{B\,b^5\,x^{12}}{10}}{x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^23,x)
```

```
[Out] -((A*a^5)/22 + x^8*((5*B*a^2*b^3)/7 + (5*A*a*b^4)/14) + x^4*((5*A*a^3*b^2)/
9 + (5*B*a^4*b)/18) + x^2*((B*a^5)/20 + (A*a^4*b)/4) + x^10*((A*b^5)/12 + (
5*B*a*b^4)/12) + x^6*((5*A*a^2*b^3)/8 + (5*B*a^3*b^2)/8) + (B*b^5*x^12)/10)
/x^22
```

3.56 $\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=98

$$\frac{a^2(Ab-aB)x}{b^4}-\frac{a(Ab-aB)x^3}{3b^3}+\frac{(Ab-aB)x^5}{5b^2}+\frac{Bx^7}{7b}-\frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

```
[Out] a^2*(A*b-B*a)*x/b^4-1/3*a*(A*b-B*a)*x^3/b^3+1/5*(A*b-B*a)*x^5/b^2+1/7*B*x^7
/b-a^(5/2)*(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {470, 308, 211}

$$-\frac{a^{5/2}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{a^2x(Ab-aB)}{b^4}-\frac{ax^3(Ab-aB)}{3b^3}+\frac{x^5(Ab-aB)}{5b^2}+\frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*x)/b^4 - (a*(A*b - a*B)*x^3)/(3*b^3) + ((A*b - a*B)*x^5)/(
5*b^2) + (B*x^7)/(7*b) - (a^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
b^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \frac{x^6}{a+bx^2}\,dx}{7b}$$
$$= \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \left(\frac{a^2}{b^3} - \frac{ax^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3(a+bx^2)}\right)dx}{7b}$$
$$= \frac{a^2(Ab-aB)x}{b^4} - \frac{a(Ab-aB)x^3}{3b^3} + \frac{(Ab-aB)x^5}{5b^2} + \frac{Bx^7}{7b} - \frac{(a^3(Ab-aB))\int \frac{1}{a+bx^2}\,dx}{b^4}$$
$$= \frac{a^2(Ab-aB)x}{b^4} - \frac{a(Ab-aB)x^3}{3b^3} + \frac{(Ab-aB)x^5}{5b^2} + \frac{Bx^7}{7b} - \frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

Mathematica [A]
time = 0.05, size = 98, normalized size = 1.00

$$-\frac{a^2(-Ab+aB)x}{b^4} + \frac{a(-Ab+aB)x^3}{3b^3} + \frac{(Ab-aB)x^5}{5b^2} + \frac{Bx^7}{7b} + \frac{a^{5/2}(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(A*b) + a*B)*x)/b^4) + (a*(-(A*b) + a*B)*x^3)/(3*b^3) + ((A*b - a*
B)*x^5)/(5*b^2) + (B*x^7)/(7*b) + (a^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/b^(9/2)
```

Maple [A]
time = 0.09, size = 99, normalized size = 1.01

method	result
default	$\frac{\frac{1}{7}B\,b^3x^7+\frac{1}{5}A\,b^3x^5-\frac{1}{5}Ba\,b^2x^5-\frac{1}{3}Aa\,b^2x^3+\frac{1}{3}B\,a^2b\,x^3+A\,a^2bx-B\,a^3x}{b^4} - \frac{a^3(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^4\sqrt{ab}}$
risch	$\frac{B\,x^7}{7b} + \frac{A\,x^5}{5b} - \frac{Ba\,x^5}{5b^2} - \frac{Aa\,x^3}{3b^2} + \frac{B\,a^2x^3}{3b^3} + \frac{A\,a^2x}{b^3} - \frac{B\,a^3x}{b^4} + \frac{\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-a\right)A}{2b^4} - \frac{\sqrt{-ab}\,a^3\ln\left(-\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/7*B*b^3*x^7+1/5*A*b^3*x^5-1/5*B*a*b^2*x^5-1/3*A*a*b^2*x^3+1/3*B*a^
2*b*x^3+A*a^2*b*x-B*a^3*x)-a^3*(A*b-B*a)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))
```

Maxima [A]
time = 0.51, size = 100, normalized size = 1.02

$$\frac{(Ba^4 - Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,Bb^3x^7 - 21\,(Bab^2 - Ab^3)x^5 + 35\,(Ba^2b - Aab^2)x^3 - 105\,(Ba^3 - Aa^2b)x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (B*a^4 - A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*B*b^3*x
^7 - 21*(B*a*b^2 - A*b^3)*x^5 + 35*(B*a^2*b - A*a*b^2)*x^3 - 105*(B*a^3 - A
*a^2*b)*x)/b^4
```

Fricas [A]
time = 1.75, size = 228, normalized size = 2.33

$$\left[\frac{30\,Bb^3x^7 - 42\,(Bab^2 - Ab^3)x^5 + 70\,(Ba^2b - Aab^2)x^3 - 105\,(Ba^3 - Aa^2b)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 210\,(Ba^3 - Aa^2b)x}{210\,b^4},\ \frac{15\,Bb^3x^7 - 21\,(Bab^2 - Ab^3)x^5 + 35\,(Ba^2b - Aab^2)x^3 + 105\,(Ba^3 - Aa^2b)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 105\,(Ba^3 - Aa^2b)x}{105\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*B*b^3*x^7 - 42*(B*a*b^2 - A*b^3)*x^5 + 70*(B*a^2*b - A*a*b^2)*x^
3 - 105*(B*a^3 - A*a^2*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*
x^2 + a)) - 210*(B*a^3 - A*a^2*b)*x)/b^4, 1/105*(15*B*b^3*x^7 - 21*(B*a*b^2
 - A*b^3)*x^5 + 35*(B*a^2*b - A*a*b^2)*x^3 + 105*(B*a^3 - A*a^2*b)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) - 105*(B*a^3 - A*a^2*b)*x)/b^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(87) = 174$.
time = 0.21, size = 180, normalized size = 1.84

$$\frac{Bx^7}{7b} + x^5\left(\frac{A}{5b} - \frac{Ba}{5b^2}\right) + x^3\left(-\frac{Aa}{3b^2} + \frac{Ba^2}{3b^3}\right) + x\left(\frac{Aa^2}{b^3} - \frac{Ba^3}{b^4}\right) - \frac{\sqrt{-\frac{a^5}{b^9}}\,(-Ab + Ba)\log\left(-\frac{b^4\sqrt{-\frac{a^5}{b^9}}\,(-Ab+Ba)}{-Aa^2b+Ba^3} + x\right)}{2} + \frac{\sqrt{-\frac{a^5}{b^9}}\,(-Ab + Ba)\log\left(\frac{b^4\sqrt{-\frac{a^5}{b^9}}\,(-Ab+Ba)}{-Aa^2b+Ba^3} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**7/(7*b) + x**5*(A/(5*b) - B*a/(5*b**2)) + x**3*(-A*a/(3*b**2) + B*a**2
/(3*b**3)) + x*(A*a**2/b**3 - B*a**3/b**4) - sqrt(-a**5/b**9)*(-A*b + B*a)*
log(-b**4*sqrt(-a**5/b**9)*(-A*b + B*a)/(-A*a**2*b + B*a**3) + x)/2 + sqrt(
-a**5/b**9)*(-A*b + B*a)*log(b**4*sqrt(-a**5/b**9)*(-A*b + B*a)/(-A*a**2*b
+ B*a**3) + x)/2
```

Giac [A]
time = 0.95, size = 108, normalized size = 1.10

$$\frac{(Ba^4 - Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,Bb^6x^7 - 21\,Bab^5x^5 + 21\,Ab^6x^5 + 35\,Ba^2b^4x^3 - 35\,Aab^5x^3 - 105\,Ba^3b^3x + 105\,Aa^2b^4x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a^4 - A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*B*b^6*x
^7 - 21*B*a*b^5*x^5 + 21*A*b^6*x^5 + 35*B*a^2*b^4*x^3 - 35*A*a*b^5*x^3 - 10
5*B*a^3*b^3*x + 105*A*a^2*b^4*x)/b^7
```

Mupad [B]
time = 0.03, size = 118, normalized size = 1.20

$$x^5\left(\frac{A}{5\,b} - \frac{B\,a}{5\,b^2}\right) + \frac{B\,x^7}{7\,b} + \frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,(A\,b - B\,a)}{B\,a^4 - A\,a^3\,b}\right)\,(A\,b - B\,a)}{b^{9/2}} - \frac{a\,x^3\,\left(\frac{A}{b} - \frac{B\,a}{b^2}\right)}{3\,b} + \frac{a^2\,x\,\left(\frac{A}{b} - \frac{B\,a}{b^2}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^5*(A/(5*b) - (B*a)/(5*b^2)) + (B*x^7)/(7*b) + (a^(5/2)*atan((a^(5/2)*b^(1
/2)*x*(A*b - B*a))/(B*a^4 - A*a^3*b))*(A*b - B*a))/b^(9/2) - (a*x^3*(A/b -
(B*a)/b^2))/(3*b) + (a^2*x*(A/b - (B*a)/b^2))/b^2
```

3.57 $\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=75

$$-\frac{a(Ab-aB)x^2}{2b^3}+\frac{(Ab-aB)x^4}{4b^2}+\frac{Bx^6}{6b}+\frac{a^2(Ab-aB)\log(a+bx^2)}{2b^4}$$

```
[Out] -1/2*a*(A*b-B*a)*x^2/b^3+1/4*(A*b-B*a)*x^4/b^2+1/6*B*x^6/b+1/2*a^2*(A*b-B*a
)*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{a^2(Ab-aB)\log(a+bx^2)}{2b^4}-\frac{ax^2(Ab-aB)}{2b^3}+\frac{x^4(Ab-aB)}{4b^2}+\frac{Bx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -1/2*(a*(A*b - a*B)*x^2)/b^3 + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^6)/(6*b) +
(a^2*(A*b - a*B)*Log[a + b*x^2])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^3} + \frac{(Ab-aB)x}{b^2} + \frac{Bx^2}{b} - \frac{a^2(-Ab+aB)}{b^3(a+bx)}\right) dx, x, x^2\right)$$
$$= -\frac{a(Ab-aB)x^2}{2b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^6}{6b} + \frac{a^2(Ab-aB)\log(a+bx^2)}{2b^4}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.95

$$\frac{bx^2(6a^2B-3ab(2A+Bx^2)+b^2x^2(3A+2Bx^2))+6a^2(Ab-aB)\log(a+bx^2)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*B - 3*a*b*(2*A + B*x^2) + b^2*x^2*(3*A + 2*B*x^2)) + 6*a^2*(A
*b - a*B)*Log[a + b*x^2])/(12*b^4)
```

Maple [A]
time = 0.07, size = 74, normalized size = 0.99

method	result	size
norman	$-\frac{a(Ab-Ba)x^2}{2b^3}+\frac{(Ab-Ba)x^4}{4b^2}+\frac{B\,x^6}{6b}+\frac{a^2(Ab-Ba)\ln(b\,x^2+a)}{2b^4}$	68
default	$-\frac{-\frac{1}{3}b^2B\,x^6-\frac{1}{2}A\,b^2x^4+\frac{1}{2}Bab\,x^4+aAb\,x^2-B\,a^2x^2}{2b^3}+\frac{a^2(Ab-Ba)\ln(b\,x^2+a)}{2b^4}$	74
risch	$\frac{B\,x^6}{6b}+\frac{A\,x^4}{4b}-\frac{Ba\,x^4}{4b^2}-\frac{aA\,x^2}{2b^2}+\frac{B\,a^2x^2}{2b^3}+\frac{a^2\ln(b\,x^2+a)A}{2b^3}-\frac{a^3\ln(b\,x^2+a)B}{2b^4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^3*(-1/3*b^2*B*x^6-1/2*A*b^2*x^4+1/2*B*a*b*x^4+a*A*b*x^2-B*a^2*x^2)+1
/2*a^2*(A*b-B*a)*ln(b*x^2+a)/b^4
```

Maxima [A]
time = 0.29, size = 74, normalized size = 0.99

$$\frac{2\,Bb^2x^6-3\,(Bab-Ab^2)x^4+6\,(Ba^2-Aab)x^2}{12\,b^3}-\frac{(Ba^3-Aa^2b)\log(bx^2+a)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*B*b^2*x^6 - 3*(B*a*b - A*b^2)*x^4 + 6*(B*a^2 - A*a*b)*x^2)/b^3 - 1/
2*(B*a^3 - A*a^2*b)*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 1.50, size = 75, normalized size = 1.00

$$\frac{2\,Bb^3x^6 - 3\,(Bab^2 - Ab^3)x^4 + 6\,(Ba^2b - Aab^2)x^2 - 6\,(Ba^3 - Aa^2b)\log\left(bx^2 + a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^3*x^6 - 3*(B*a*b^2 - A*b^3)*x^4 + 6*(B*a^2*b - A*a*b^2)*x^2 - 6
*(B*a^3 - A*a^2*b)*log(b*x^2 + a))/b^4
```

Sympy [A]
time = 0.17, size = 70, normalized size = 0.93

$$\frac{Bx^6}{6b} - \frac{a^2(-Ab + Ba)\log\left(a + bx^2\right)}{2b^4} + x^4\left(\frac{A}{4b} - \frac{Ba}{4b^2}\right) + x^2\left(-\frac{Aa}{2b^2} + \frac{Ba^2}{2b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**6/(6*b) - a**2*(-A*b + B*a)*log(a + b*x**2)/(2*b**4) + x**4*(A/(4*b) -
 B*a/(4*b**2)) + x**2*(-A*a/(2*b**2) + B*a**2/(2*b**3))
```

Giac [A]
time = 1.37, size = 77, normalized size = 1.03

$$\frac{2\,Bb^2x^6 - 3\,Babx^4 + 3\,Ab^2x^4 + 6\,Ba^2x^2 - 6\,Aabx^2}{12\,b^3} - \frac{(Ba^3 - Aa^2b)\log\left(|bx^2 + a|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*B*b^2*x^6 - 3*B*a*b*x^4 + 3*A*b^2*x^4 + 6*B*a^2*x^2 - 6*A*a*b*x^2)/
b^3 - 1/2*(B*a^3 - A*a^2*b)*log(abs(b*x^2 + a))/b^4
```

Mupad [B]
time = 0.03, size = 76, normalized size = 1.01

$$x^4\left(\frac{A}{4\,b} - \frac{B\,a}{4\,b^2}\right) + \frac{B\,x^6}{6\,b} - \frac{\ln\left(b\,x^2 + a\right)\,\left(B\,a^3 - A\,a^2\,b\right)}{2\,b^4} - \frac{a\,x^2\,\left(\frac{A}{b} - \frac{B\,a}{b^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^4*(A/(4*b) - (B*a)/(4*b^2)) + (B*x^6)/(6*b) - (log(a + b*x^2)*(B*a^3 - A*
a^2*b))/(2*b^4) - (a*x^2*(A/b - (B*a)/b^2))/(2*b)
```

3.58 $\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=77

$$-\frac{a(Ab-aB)x}{b^3}+\frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^5}{5b}+\frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] -a*(A*b-B*a)*x/b^3+1/3*(A*b-B*a)*x^3/b^2+1/5*B*x^5/b+a^(3/2)*(A*b-B*a)*arct
an(x*b^(1/2)/a^(1/2))/b^(7/2)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {470, 308, 211}

$$\frac{a^{3/2}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{ax(Ab-aB)}{b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5*b) + (a^(
3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^5}{5b} - \frac{(-5Ab+5aB)\int \frac{x^4}{a+bx^2}\,dx}{5b}$$

$$= \frac{Bx^5}{5b} - \frac{(-5Ab+5aB)\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right)dx}{5b}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^3}{3b^2} + \frac{Bx^5}{5b} + \frac{(a^2(Ab-aB))\int \frac{1}{a+bx^2}\,dx}{b^3}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^3}{3b^2} + \frac{Bx^5}{5b} + \frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 1.00

$$\frac{a(-Ab+aB)x}{b^3} + \frac{(Ab-aB)x^3}{3b^2} + \frac{Bx^5}{5b} - \frac{a^{3/2}(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (a*(-(A*b) + a*B)*x)/b^3 + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5*b) - (a^(
3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Maple [A]
time = 0.08, size = 75, normalized size = 0.97

method	result
default	$-\frac{-\frac{1}{5}b^2Bx^5-\frac{1}{3}Ab^2x^3+\frac{1}{3}Babx^3+abAx-a^2Bx}{b^3}+\frac{a^2(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{Bx^5}{5b}+\frac{Ax^3}{3b}-\frac{Bax^3}{3b^2}-\frac{aAx}{b^2}+\frac{a^2Bx}{b^3}+\frac{\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)A}{2b^3}-\frac{\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x+a\right)B}{2b^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/5*b^2*B*x^5-1/3*A*b^2*x^3+1/3*B*a*b*x^3+a*b*A*x-a^2*B*x)+a^2*(A*
b-B*a)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.59, size = 78, normalized size = 1.01

$$-\frac{(Ba^3 - Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,Bb^2x^5 - 5\,(Bab - Ab^2)x^3 + 15\,(Ba^2 - Aab)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*B*b^2*x^
5 - 5*(B*a*b - A*b^2)*x^3 + 15*(B*a^2 - A*a*b)*x)/b^3
```

Fricas [A]
time = 1.17, size = 178, normalized size = 2.31

$$\left[\frac{6\,Bb^2x^5 - 10\,(Bab - Ab^2)x^3 - 15\,(Ba^2 - Aab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 30\,(Ba^2 - Aab)x}{30\,b^3}, \frac{3\,Bb^2x^5 - 5\,(Bab - Ab^2)x^3 - 15\,(Ba^2 - Aab)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 15\,(Ba^2 - Aab)x}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*B*b^2*x^5 - 10*(B*a*b - A*b^2)*x^3 - 15*(B*a^2 - A*a*b)*sqrt(-a/b)
*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(B*a^2 - A*a*b)*x)/b^
3, 1/15*(3*B*b^2*x^5 - 5*(B*a*b - A*b^2)*x^3 - 15*(B*a^2 - A*a*b)*sqrt(a/b)
*arctan(b*x*sqrt(a/b)/a) + 15*(B*a^2 - A*a*b)*x)/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(68) = 136$.
time = 0.24, size = 153, normalized size = 1.99

$$\frac{Bx^5}{5b} + x^3\left(\frac{A}{3b} - \frac{Ba}{3b^2}\right) + x\left(-\frac{Aa}{b^2} + \frac{Ba^2}{b^3}\right) + \frac{\sqrt{-\frac{a^3}{b^7}}\,(-Ab + Ba)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}\,(-Ab+Ba)}{-Aab+Ba^2} + x\right)}{2} - \frac{\sqrt{-\frac{a^3}{b^7}}\,(-Ab + Ba)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}\,(-Ab+Ba)}{-Aab+Ba^2} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**5/(5*b) + x**3*(A/(3*b) - B*a/(3*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
+ sqrt(-a**3/b**7)*(-A*b + B*a)*log(-b**3*sqrt(-a**3/b**7)*(-A*b + B*a)/(-A
*a*b + B*a**2) + x)/2 - sqrt(-a**3/b**7)*(-A*b + B*a)*log(b**3*sqrt(-a**3/b
**7)*(-A*b + B*a)/(-A*a*b + B*a**2) + x)/2
```

Giac [A]
time = 0.83, size = 85, normalized size = 1.10

$$-\frac{\left(Ba^{3}-Aa^{2}b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{3\,Bb^{4}x^{5}-5\,Bab^{3}x^{3}+5\,Ab^{4}x^{3}+15\,Ba^{2}b^{2}x-15\,Aab^{3}x}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(B*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*B*b^4*x^
5 - 5*B*a*b^3*x^3 + 5*A*b^4*x^3 + 15*B*a^2*b^2*x - 15*A*a*b^3*x)/b^5
```

Mupad [B]
time = 0.05, size = 96, normalized size = 1.25

$$x^{3}\left(\frac{A}{3\,b}-\frac{B\,a}{3\,b^{2}}\right)+\frac{B\,x^{5}}{5\,b}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(A\,b-B\,a\right)}{B\,a^{3}-A\,a^{2}\,b}\right)\,\left(A\,b-B\,a\right)}{b^{7/2}}-\frac{a\,x\,\left(\frac{A}{b}-\frac{B\,a}{b^{2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^3*(A/(3*b) - (B*a)/(3*b^2)) + (B*x^5)/(5*b) - (a^(3/2)*atan((a^(3/2)*b^(1
/2)*x*(A*b - B*a))/(B*a^3 - A*a^2*b))*(A*b - B*a))/b^(7/2) - (a*x*(A/b - (B
*a)/b^2))/b
```

3.59 $\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=54

$$\frac{(Ab-aB)x^2}{2b^2}+\frac{Bx^4}{4b}-\frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] 1/2*(A*b-B*a)*x^2/b^2+1/4*B*x^4/b-1/2*a*(A*b-B*a)*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(Ab-aB)}{2b^2}+\frac{Bx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^4)/(4*b) - (a*(A*b - a*B)*Log[a + b*x^2])/
(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab-aB}{b^2}+\frac{Bx}{b}+\frac{a(-Ab+aB)}{b^2(a+bx)}\right)\,dx, x, x^2\right)$$
$$= \frac{(Ab-aB)x^2}{2b^2}+\frac{Bx^4}{4b}-\frac{a(Ab-aB)\log(a+bx^2)}{2b^3}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.87

$$\frac{bx^2(2Ab-2aB+bBx^2)+2a(-Ab+aB)\log(a+bx^2)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2),x]
[Out] (b*x^2*(2*A*b - 2*a*B + b*B*x^2) + 2*a*(-(A*b) + a*B)*Log[a + b*x^2])/(4*b^
3)
```

Maple [A]
time = 0.08, size = 50, normalized size = 0.93

method	result	size
norman	$\frac{(Ab-Ba)x^2}{2b^2}+\frac{Bx^4}{4b}-\frac{a(Ab-Ba)\ln(bx^2+a)}{2b^3}$	49
default	$\frac{\frac{1}{2}bBx^4+Abx^2-Bax^2}{2b^2}-\frac{a(Ab-Ba)\ln(bx^2+a)}{2b^3}$	50
risch	$\frac{Bx^4}{4b}+\frac{Ax^2}{2b}-\frac{Bax^2}{2b^2}+\frac{A^2}{4bB}-\frac{Aa}{2b^2}+\frac{Ba^2}{4b^3}-\frac{a\ln(bx^2+a)A}{2b^2}+\frac{a^2\ln(bx^2+a)B}{2b^3}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2/b^2*(1/2*b*B*x^4+A*b*x^2-B*a*x^2)-1/2*a*(A*b-B*a)*ln(b*x^2+a)/b^3
```

Maxima [A]
time = 0.30, size = 50, normalized size = 0.93

$$\frac{Bbx^4-2\,(Ba-Ab)x^2}{4\,b^2}+\frac{(Ba^2-Aab)\log(bx^2+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*x^4 - 2*(B*a - A*b)*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*log(b*x^2 + a)/
b^3
```

Fricas [A]
time = 0.98, size = 51, normalized size = 0.94

$$\frac{Bb^2x^4 - 2\,(Bab - Ab^2)x^2 + 2\,(Ba^2 - Aab)\log\left(bx^2 + a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^2*x^4 - 2*(B*a*b - A*b^2)*x^2 + 2*(B*a^2 - A*a*b)*log(b*x^2 + a))/
b^3
```

Sympy [A]
time = 0.22, size = 46, normalized size = 0.85

$$\frac{Bx^4}{4b} + \frac{a(-Ab + Ba)\log\left(a + bx^2\right)}{2b^3} + x^2\left(\frac{A}{2b} - \frac{Ba}{2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**4/(4*b) + a*(-A*b + B*a)*log(a + b*x**2)/(2*b**3) + x**2*(A/(2*b) - B*
a/(2*b**2))
```

Giac [A]
time = 0.79, size = 52, normalized size = 0.96

$$\frac{Bbx^4 - 2\,Bax^2 + 2\,Abx^2}{4\,b^2} + \frac{(Ba^2 - Aab)\log\left(|bx^2 + a|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(B*b*x^4 - 2*B*a*x^2 + 2*A*b*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*log(abs(b*x
^2 + a))/b^3
```

Mupad [B]
time = 0.05, size = 52, normalized size = 0.96

$$x^2\left(\frac{A}{2\,b} - \frac{B\,a}{2\,b^2}\right) + \frac{\ln\left(b\,x^2 + a\right)\,\left(B\,a^2 - A\,a\,b\right)}{2\,b^3} + \frac{B\,x^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^2*(A/(2*b) - (B*a)/(2*b^2)) + (log(a + b*x^2)*(B*a^2 - A*a*b))/(2*b^3) +
(B*x^4)/(4*b)
```

3.60 $\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=58

$$\frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}-\frac{\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] (A*b-B*a)*x/b^2+1/3*B*x^3/b-(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(
5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {470, 327, 211}

$$-\frac{\sqrt{a}\,(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) - (Sqrt[a]*(A*b - a*B)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx &= \frac{Bx^3}{3b}-\frac{(-3Ab+3aB)\int \frac{x^2}{a+bx^2}\,dx}{3b}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}-\frac{(a(Ab-aB))\int \frac{1}{a+bx^2}\,dx}{b^2}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}-\frac{\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.98

$$
\frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}+\frac{\sqrt{a}\,(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2),x]
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) + (Sqrt[a]*(-(A*b) + a*B)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2)
```

Maple [A]
time = 0.08, size = 51, normalized size = 0.88

method	result
default	$\frac{\frac{1}{3}bB\,x^3+Abx-Bax}{b^2}-\frac{a(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{B\,x^3}{3b}+\frac{Ax}{b}-\frac{Bax}{b^2}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)A}{2b^2}-\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)Ba}{2b^3}-\frac{\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-\right.}{2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/b^2*(1/3*b*B*x^3+A*b*x-B*a*x)-a*(A*b-B*a)/b^2/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))
```

Maxima [A]
time = 0.51, size = 53, normalized size = 0.91

$$
\frac{(Ba^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{Bbx^3-3\,(Ba-Ab)x}{3\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (B*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(B*b*x^3 - 3*(B
*a - A*b)*x)/b^2
```

Fricas [A]
time = 0.93, size = 129, normalized size = 2.22

$$\left[\frac{2\,Bbx^3-3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Ba-Ab)x}{6\,b^2},\ \frac{Bbx^3+3\,(Ba-Ab)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,(Ba-Ab)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*B*b*x^3 - 3*(B*a - A*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) - 6*(B*a - A*b)*x)/b^2, 1/3*(B*b*x^3 + 3*(B*a - A*b)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) - 3*(B*a - A*b)*x)/b^2]
```

Sympy [A]
time = 0.19, size = 90, normalized size = 1.55

$$\frac{Bx^3}{3b}+x\left(\frac{A}{b}-\frac{Ba}{b^2}\right)-\frac{\sqrt{-\frac{a}{b^5}}\,(-Ab+Ba)\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^5}}\,(-Ab+Ba)\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**3/(3*b) + x*(A/b - B*a/b**2) - sqrt(-a/b**5)*(-A*b + B*a)*log(-b**2*sq
rt(-a/b**5) + x)/2 + sqrt(-a/b**5)*(-A*b + B*a)*log(b**2*sqrt(-a/b**5) + x)
/2
```

Giac [A]
time = 0.72, size = 57, normalized size = 0.98

$$\frac{(Ba^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{Bb^2x^3-3\,Babx+3\,Ab^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(B*b^2*x^3 - 3*
B*a*b*x + 3*A*b^2*x)/b^3
```

Mupad [B]
time = 0.06, size = 70, normalized size = 1.21

$$x\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)+\frac{B\,x^3}{3\,b}+\frac{\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(A\,b-B\,a)}{B\,a^2-A\,a\,b}\right)\,(A\,b-B\,a)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x*(A/b - (B*a)/b^2) + (B*x^3)/(3*b) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(A*b
 - B*a))/(B*a^2 - A*a*b))*(A*b - B*a))/b^(5/2)
```

3.61 $\int \frac{x(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=35

$$\frac{Bx^2}{2b}+\frac{(Ab-aB)\log(a+bx^2)}{2b^2}$$

```
[Out] 1/2*B*x^2/b+1/2*(A*b-B*a)*ln(b*x^2+a)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {455, 45}

$$\frac{(Ab-aB)\log(a+bx^2)}{2b^2}+\frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (B*x^2)/(2*b) + ((A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{Bx^2}{2b}+\frac{(Ab-aB)\log(a+bx^2)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.89

$$\frac{bBx^2 + (Ab - aB)\log(a + bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2),x]
[Out] (b*B*x^2 + (A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.07, size = 32, normalized size = 0.91

method	result	size
default	$\frac{B\,x^2}{2b}+\frac{(Ab-Ba)\ln(b\,x^2+a)}{2b^2}$	32
norman	$\frac{B\,x^2}{2b}+\frac{(Ab-Ba)\ln(b\,x^2+a)}{2b^2}$	32
risch	$\frac{B\,x^2}{2b}+\frac{\ln(b\,x^2+a)A}{2b}-\frac{\ln(b\,x^2+a)Ba}{2b^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*B*x^2/b+1/2*(A*b-B*a)*ln(b*x^2+a)/b^2
```

Maxima [A]
time = 0.31, size = 31, normalized size = 0.89

$$\frac{Bx^2}{2\,b} - \frac{(Ba - Ab)\log(bx^2 + a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*B*x^2/b - 1/2*(B*a - A*b)*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 1.11, size = 30, normalized size = 0.86

$$\frac{Bbx^2 - (Ba - Ab)\log(bx^2 + a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*(B*b*x^2 - (B*a - A*b)*log(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.13, size = 27, normalized size = 0.77

$$\frac{Bx^2}{2b} - \frac{(-Ab + Ba)\log(a + bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a),x)
[Out] B*x**2/(2*b) - (-A*b + B*a)*log(a + b*x**2)/(2*b**2)
```

Giac [A]
time = 1.40, size = 32, normalized size = 0.91

$$\frac{Bx^2}{2\,b} - \frac{(Ba - Ab)\log(|bx^2 + a|)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*B*x^2/b - 1/2*(B*a - A*b)*log(abs(b*x^2 + a))/b^2
```

Mupad [B]
time = 0.03, size = 31, normalized size = 0.89

$$\frac{B\,x^2}{2\,b} + \frac{\ln(b\,x^2 + a)\,(A\,b - B\,a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2),x)
[Out] (B*x^2)/(2*b) + (log(a + b*x^2)*(A*b - B*a))/(2*b^2)
```

3.62 $\int \frac{A+Bx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{Bx}{b}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

```
[Out] B*x/b+(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {396, 211}

$$\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2),x]
```

```
[Out] (B*x)/b + ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{a+bx^2}\,dx &= \frac{Bx}{b}-\frac{(-Ab+aB)\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{Bx}{b}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.03

$$\frac{Bx}{b}-\frac{(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2),x]
[Out] (B*x)/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Maple [A]
time = 0.08, size = 34, normalized size = 0.87

method	result	size
default	$\frac{Bx}{b}+\frac{(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
risch	$\frac{Bx}{b}-\frac{\ln\left(bx+\sqrt{-ab}\right)A}{2\sqrt{-ab}}+\frac{\ln\left(bx+\sqrt{-ab}\right)Ba}{2b\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)A}{2\sqrt{-ab}}-\frac{\ln\left(-bx+\sqrt{-ab}\right)Ba}{2b\sqrt{-ab}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] B*x/b+(A*b-B*a)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 34, normalized size = 0.87

$$\frac{Bx}{b}-\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
[Out] B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 1.63, size = 99, normalized size = 2.54

$$\left[\frac{2\,Babx+(Ba-Ab)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab^2},\,\frac{Babx-(Ba-Ab)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b*x + (B*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (B*a*b*x - (B*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.
time = 0.17, size = 82, normalized size = 2.10

$$\frac{Bx}{b}+\frac{\sqrt{-\frac{1}{ab^3}}\,(-Ab+Ba)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(-Ab+Ba)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x/b + sqrt(-1/(a*b**3))*(-A*b + B*a)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 -
sqrt(-1/(a*b**3))*(-A*b + B*a)*log(a*b*sqrt(-1/(a*b**3)) + x)/2
```

Giac [A]
time = 1.60, size = 34, normalized size = 0.87

$$\frac{Bx}{b}-\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [B]
time = 0.03, size = 31, normalized size = 0.79

$$\frac{B\,x}{b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b-B\,a)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2),x)
```

```
[Out] (B*x)/b + (atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/(a^(1/2)*b^(3/2))
```

3.63 $\int \frac{A+Bx^2}{x(a+bx^2)}\,dx$

Optimal. Leaf size=34

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}$$

```
[Out] A*ln(x)/a-1/2*(A*b-B*a)*ln(b*x^2+a)/a/b
```

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^2])/(2*a*b)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aB}{a(a+bx)}\right)\,dx,x,x^2\right)\\ &= \frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{A\log(x)}{a}+\frac{(-Ab+aB)\log\left(a+bx^2\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^2])/(2*a*b)
```

Maple [A]
time = 0.07, size = 33, normalized size = 0.97

method	result	size
default	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln\left(b\,x^{2}+a\right)}{2ab}$	33
norman	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln\left(b\,x^{2}+a\right)}{2ab}$	33
risch	$\frac{A\ln(x)}{a}-\frac{\ln\left(b\,x^{2}+a\right)A}{2a}+\frac{\ln\left(b\,x^{2}+a\right)B}{2b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] A*ln(x)/a-1/2*(A*b-B*a)*ln(b*x^2+a)/a/b
```

Maxima [A]
time = 0.30, size = 35, normalized size = 1.03

$$\frac{A\log\left(x^2\right)}{2\,a}+\frac{(Ba-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*A*log(x^2)/a + 1/2*(B*a - A*b)*log(b*x^2 + a)/(a*b)
```

Fricas [A]
time = 1.01, size = 32, normalized size = 0.94

$$\frac{2\,Ab\log\left(x\right)+(Ba-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*b*log(x) + (B*a - A*b)*log(b*x^2 + a))/(a*b)
```

Sympy [A]
time = 0.50, size = 26, normalized size = 0.76

$$\frac{A\log\left(x\right)}{a}+\frac{\left(-Ab+Ba\right)\log\left(\frac{a}{b}+x^{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a),x)
[Out] A*log(x)/a + (-A*b + B*a)*log(a/b + x**2)/(2*a*b)
```

Giac [A]
time = 1.07, size = 36, normalized size = 1.06

$$\frac{A\log\left(x^{2}\right)}{2\,a}+\frac{\left(Ba-Ab\right)\log\left(\left|bx^{2}+a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*A*log(x^2)/a + 1/2*(B*a - A*b)*log(abs(b*x^2 + a))/(a*b)
```

Mupad [B]
time = 0.06, size = 32, normalized size = 0.94

$$\frac{A\,\ln\left(x\right)}{a}-\frac{\ln\left(b\,x^2+a\right)\,\left(A\,b-B\,a\right)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)),x)
[Out] (A*log(x))/a - (log(a + b*x^2)*(A*b - B*a))/(2*a*b)
```

3.64 $\int \frac{A+Bx^2}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=43

$$-\frac{A}{ax}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

```
[Out] -A/a/x-(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {464, 211}

$$-\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)),x]
[Out] -(A/(a*x)) - ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}\,dx &= -\frac{A}{ax}-\frac{(Ab-aB)\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{A}{ax}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.98

$$-\frac{A}{ax}+\frac{(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)),x]
[Out] -(A/(a*x)) + ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Maple [A]
time = 0.08, size = 37, normalized size = 0.86

method	result	si
default	$\frac{(-Ab+Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{A}{ax}$	37
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^3\_Z^2b+A^2b^2-2ABab+B^2a^2\right)}\_R\ln\left(\left(3\_R^2a^3b+2A^2b^2-4ABab+2B^2a^2\right)x+\left(A\,a^2b-B\,a^3\right)\_R\right)\right)}{2}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] (-A*b+B*a)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-A/a/x
```

Maxima [A]
time = 0.51, size = 36, normalized size = 0.84

$$\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="maxima")
[Out] (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - A/(a*x)
```

Fricas [A]
time = 0.91, size = 105, normalized size = 2.44

$$\left[\frac{(Ba-Ab)\sqrt{-ab}\,x\log\left(\frac{bx^2+2\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,Aab}{2\,a^2bx},\frac{(Ba-Ab)\sqrt{ab}\,x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-Aab}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*((B*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*A*a*b)/(a^2*b*x), ((B*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) - A*
a*b)/(a^2*b*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.
time = 0.18, size = 82, normalized size = 1.91

$$-\frac{A}{ax}-\frac{\sqrt{-\frac{1}{a^3b}}\,(-Ab+Ba)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\,(-Ab+Ba)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a),x)
```

```
[Out] -A/(a*x) - sqrt(-1/(a**3*b))*(-A*b + B*a)*log(-a**2*sqrt(-1/(a**3*b)) + x)/
2 + sqrt(-1/(a**3*b))*(-A*b + B*a)*log(a**2*sqrt(-1/(a**3*b)) + x)/2
```

Giac [A]
time = 1.35, size = 36, normalized size = 0.84

$$\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - A/(a*x)
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.81

$$-\frac{A}{a\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b-B\,a)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)),x)
```

```
[Out] - A/(a*x) - (atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/(a^(3/2)*b^(1/2))
```

3.65 $\int \frac{A+Bx^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=50

$$-\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] -1/2*A/a/x^2-(A*b-B*a)*ln(x)/a^2+1/2*(A*b-B*a)*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*A/(a*x^2) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^2])/(2
*a^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3(a+bx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^2} + \frac{-Ab+aB}{a^2x} - \frac{b(-Ab+aB)}{a^2(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\log(x)}{a^2} + \frac{(Ab-aB)\log(a+bx^2)}{2a^2}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.98

$$-\frac{A}{2ax^2} + \frac{(-Ab+aB)\log(x)}{a^2} + \frac{(Ab-aB)\log(a+bx^2)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)),x]
[Out] -1/2*A/(a*x^2) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^2])
/(2*a^2)
```

Maple [A]
time = 0.07, size = 46, normalized size = 0.92

method	result	size
default	$\frac{(Ab-Ba)\ln(b\,x^2+a)}{2a^2} - \frac{A}{2a\,x^2} + \frac{(-Ab+Ba)\ln(x)}{a^2}$	46
norman	$-\frac{A}{2a\,x^2} - \frac{(Ab-Ba)\ln(x)}{a^2} + \frac{(Ab-Ba)\ln(b\,x^2+a)}{2a^2}$	47
risch	$-\frac{A}{2a\,x^2} - \frac{\ln(x)Ab}{a^2} + \frac{\ln(x)B}{a} + \frac{\ln(-b\,x^2-a)Ab}{2a^2} - \frac{\ln(-b\,x^2-a)B}{2a}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(A*b-B*a)*ln(b*x^2+a)/a^2-1/2*A/a/x^2+1/a^2*(-A*b+B*a)*ln(x)
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.96

$$-\frac{(Ba-Ab)\log(bx^2+a)}{2\,a^2} + \frac{(Ba-Ab)\log(x^2)}{2\,a^2} - \frac{A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)*log(b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2 - 1/2*A/
(a*x^2)
```

Fricas [A]
time = 0.77, size = 47, normalized size = 0.94

$$-\frac{(Ba - Ab)x^2 \log(bx^2 + a) - 2(Ba - Ab)x^2 \log(x) + Aa}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*((B*a - A*b)*x^2*log(b*x^2 + a) - 2*(B*a - A*b)*x^2*log(x) + A*a)/(a^2
*x^2)
```

Sympy [A]
time = 0.40, size = 41, normalized size = 0.82

$$-\frac{A}{2ax^2} + \frac{(-Ab + Ba)\log(x)}{a^2} - \frac{(-Ab + Ba)\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a),x)
```

```
[Out] -A/(2*a*x**2) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**2)/(2*
a**2)
```

Giac [A]
time = 1.53, size = 71, normalized size = 1.42

$$\frac{(Ba - Ab)\log(x^2)}{2a^2} - \frac{(Bab - Ab^2)\log(|bx^2 + a|)}{2a^2b} - \frac{Bax^2 - Abx^2 + Aa}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*a - A*b)*log(x^2)/a^2 - 1/2*(B*a*b - A*b^2)*log(abs(b*x^2 + a))/(a^2
*b) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

Mupad [B]
time = 0.07, size = 46, normalized size = 0.92

$$\frac{\ln(b\,x^2 + a)\,(A\,b - B\,a)}{2\,a^2} - \frac{A}{2\,a\,x^2} - \frac{\ln(x)\,(A\,b - B\,a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(A*b - B*a))/(2*a^2) - A/(2*a*x^2) - (log(x)*(A*b - B*a))/a
^2
```

3.66 $\int \frac{A+Bx^2}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=59

$$-\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{\sqrt{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -1/3*A/a/x^3+(A*b-B*a)/a^2/x+(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^
(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {464, 331, 211}

$$\frac{\sqrt{b}\,(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*A/(a*x^3) + (A*b - a*B)/(a^2*x) + (Sqrt[b]*(A*b - a*B)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/a^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)}\,dx &= -\frac{A}{3ax^3}-\frac{(3Ab-3aB)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{3a}\\
&= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{(b(Ab-aB))\int \frac{1}{a+bx^2}\,dx}{a^2}\\
&= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{\sqrt{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 1.02

$$-\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}-\frac{\sqrt{b}\,(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)),x]
[Out] -1/3*A/(a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[b]*(-(A*b) + a*B)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.92

method	result
default	$\frac{b(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}-\frac{A}{3a\,x^3}-\frac{-Ab+Ba}{a^2x}$
risch	$\frac{\frac{(Ab-Ba)x^2}{a^2}-\frac{A}{3a}}{x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2+A^2b^3-2ABa\,b^2+B^2a^2b\right)}\_R\ln\left(\left(3\_R^2a^5+2A^2b^3-4ABa\,b^2+2B^2a^2b\right)x+\left(-A\,a^3b+B\,a^4\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] b*(A*b-B*a)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/3*A/a/x^3-1/a^2*(-A*b
+B*a)/x
```

Maxima [A]
time = 0.51, size = 56, normalized size = 0.95

$$-\frac{(Bab-Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}-\frac{3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3*(3*(B*a - A*b)
*x^2 + A*a)/(a^2*x^3)
```

Fricas [A]
time = 0.90, size = 135, normalized size = 2.29

$$\left[-\frac{3\,(Ba-Ab)x^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,(Ba-Ab)x^2+2\,Aa}{6\,a^2x^3},-\frac{3\,(Ba-Ab)x^3\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(B*a - A*b)*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) + 6*(B*a - A*b)*x^2 + 2*A*a)/(a^2*x^3), -1/3*(3*(B*a - A*b)*x^3*sq
rt(b/a)*arctan(x*sqrt(b/a)) + 3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.
time = 0.21, size = 129, normalized size = 2.19

$$\frac{\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2}+\frac{-Aa+x^2\cdot(3Ab-3Ba)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**5)*(-A*b + B*a)*log(-a**3*sqrt(-b/a**5)*(-A*b + B*a)/(-A*b**2 +
B*a*b) + x)/2 - sqrt(-b/a**5)*(-A*b + B*a)*log(a**3*sqrt(-b/a**5)*(-A*b + B
*a)/(-A*b**2 + B*a*b) + x)/2 + (-A*a + x**2*(3*A*b - 3*B*a))/(3*a**2*x**3)
```

Giac [A]
time = 1.05, size = 57, normalized size = 0.97

$$-\frac{\left(Bab - Ab^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}} - \frac{3\,Bax^{2} - 3\,Abx^{2} + Aa}{3\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3*(3*B*a*x^2 - 3
*A*b*x^2 + A*a)/(a^2*x^3)
```

Mupad [B]
time = 0.06, size = 53, normalized size = 0.90

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{a^{5/2}}-\frac{\frac{A}{3\,a}-\frac{x^2\,\left(A\,b-B\,a\right)}{a^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/a^(5/2) - (A/(3*a) - (x^2*(
A*b - B*a))/a^2)/x^3
```

3.67 $\int \frac{A+Bx^2}{x^5(a+bx^2)} dx$

Optimal. Leaf size=69

$$-\frac{A}{4ax^4}+\frac{Ab-aB}{2a^2x^2}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log(a+bx^2)}{2a^3}$$

```
[Out] -1/4*A/a/x^4+1/2*(A*b-B*a)/a^2/x^2+b*(A*b-B*a)*ln(x)/a^3-1/2*b*(A*b-B*a)*ln
(b*x^2+a)/a^3
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{b(Ab-aB)\log(a+bx^2)}{2a^3}+\frac{b\log(x)(Ab-aB)}{a^3}+\frac{Ab-aB}{2a^2x^2}-\frac{A}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

```
[Out] -1/4*A/(a*x^4) + (A*b - a*B)/(2*a^2*x^2) + (b*(A*b - a*B)*Log[x])/a^3 - (b*
(A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5(a+bx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^3} + \frac{-Ab+aB}{a^2x^2} - \frac{b(-Ab+aB)}{a^3x} + \frac{b^2(-Ab+aB)}{a^3(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{2a^2x^2} + \frac{b(Ab-aB)\log(x)}{a^3} - \frac{b(Ab-aB)\log(a+bx^2)}{2a^3}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.01

$$\frac{-a(aA-2Abx^2+2aBx^2)+4b(Ab-aB)x^4\log(x)+2b(-Ab+aB)x^4\log(a+bx^2)}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

```
[Out] (-(a*(a*A - 2*A*b*x^2 + 2*a*B*x^2)) + 4*b*(A*b - a*B)*x^4*Log[x] + 2*b*(-(A
*b) + a*B)*x^4*Log[a + b*x^2])/(4*a^3*x^4)
```

Maple [A]
time = 0.08, size = 64, normalized size = 0.93

method	result	size
default	$-\frac{b(Ab-Ba)\ln(b\,x^2+a)}{2a^3}-\frac{A}{4a\,x^4}-\frac{-Ab+Ba}{2a^2x^2}+\frac{b(Ab-Ba)\ln(x)}{a^3}$	64
norman	$\frac{-\frac{A}{4a}+\frac{(Ab-Ba)x^2}{2a^2}}{x^4}+\frac{b(Ab-Ba)\ln(x)}{a^3}-\frac{b(Ab-Ba)\ln(b\,x^2+a)}{2a^3}$	66
risch	$\frac{-\frac{A}{4a}+\frac{(Ab-Ba)x^2}{2a^2}}{x^4}+\frac{b^2\ln(x)A}{a^3}-\frac{b\ln(x)B}{a^2}-\frac{b^2\ln(b\,x^2+a)A}{2a^3}+\frac{b\ln(b\,x^2+a)B}{2a^2}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*(A*b-B*a)*ln(b*x^2+a)/a^3-1/4*A/a/x^4-1/2*(-A*b+B*a)/a^2/x^2+b*(A*b-
B*a)*ln(x)/a^3
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.01

$$\frac{(Bab-Ab^2)\log(bx^2+a)}{2\,a^3}-\frac{(Bab-Ab^2)\log(x^2)}{2\,a^3}-\frac{2\,(Ba-Ab)x^2+Aa}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*a*b - A*b^2)*log(b*x^2 + a)/a^3 - 1/2*(B*a*b - A*b^2)*log(x^2)/a^3 -
 1/4*(2*(B*a - A*b)*x^2 + A*a)/(a^2*x^4)
```

Fricas [A]
time = 0.61, size = 73, normalized size = 1.06

$$\frac{2\,(Bab - Ab^2)x^4\log\left(bx^2 + a\right) - 4\,(Bab - Ab^2)x^4\log\left(x\right) - Aa^2 - 2\,(Ba^2 - Aab)x^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*a*b - A*b^2)*x^4*log(b*x^2 + a) - 4*(B*a*b - A*b^2)*x^4*log(x) -
A*a^2 - 2*(B*a^2 - A*a*b)*x^2)/(a^3*x^4)
```

Sympy [A]
time = 0.59, size = 61, normalized size = 0.88

$$\frac{-Aa + x^2\cdot(2Ab - 2Ba)}{4a^2x^4} - \frac{b(-Ab + Ba)\log\left(x\right)}{a^3} + \frac{b(-Ab + Ba)\log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a),x)
```

```
[Out] (-A*a + x**2*(2*A*b - 2*B*a))/(4*a**2*x**4) - b*(-A*b + B*a)*log(x)/a**3 +
b*(-A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

Giac [A]
time = 1.21, size = 100, normalized size = 1.45

$$-\frac{(Bab - Ab^2)\log\left(x^2\right)}{2\,a^3} + \frac{(Bab^2 - Ab^3)\log\left(|bx^2 + a|\right)}{2\,a^3b} + \frac{3\,Babx^4 - 3\,Ab^2x^4 - 2\,Ba^2x^2 + 2\,Aabx^2 - Aa^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b - A*b^2)*log(x^2)/a^3 + 1/2*(B*a*b^2 - A*b^3)*log(abs(b*x^2 + a
))/(a^3*b) + 1/4*(3*B*a*b*x^4 - 3*A*b^2*x^4 - 2*B*a^2*x^2 + 2*A*a*b*x^2 - A
*a^2)/(a^3*x^4)
```

Mupad [B]
time = 0.07, size = 70, normalized size = 1.01

$$\frac{\ln\left(x\right)\,\left(A\,b^2 - B\,a\,b\right)}{a^3} - \frac{\ln\left(b\,x^2 + a\right)\,\left(A\,b^2 - B\,a\,b\right)}{2\,a^3} - \frac{\frac{A}{4\,a} - \frac{x^2\,\left(A\,b - B\,a\right)}{2\,a^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)),x)
```

```
[Out] (log(x)*(A*b^2 - B*a*b))/a^3 - (log(a + b*x^2)*(A*b^2 - B*a*b))/(2*a^3) - (
A/(4*a) - (x^2*(A*b - B*a))/(2*a^2))/x^4
```

3.68 $\int \frac{A+Bx^2}{x^6(a+bx^2)}\,dx$

Optimal. Leaf size=80

$$-\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{a^3x}-\frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -1/5*A/a/x^5+1/3*(A*b-B*a)/a^2/x^3-b*(A*b-B*a)/a^3/x-b^(3/2)*(A*b-B*a)*arct
an(x*b^(1/2)/a^(1/2))/a^(7/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {464, 331, 211}

$$-\frac{b^{3/2}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}-\frac{b(Ab-aB)}{a^3x}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/5*A/(a*x^5) + (A*b - a*B)/(3*a^2*x^3) - (b*(A*b - a*B))/(a^3*x) - (b^(3/
2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^6(a+bx^2)}\,dx &= -\frac{A}{5ax^5}-\frac{(5Ab-5aB)\int \frac{1}{x^4(a+bx^2)}\,dx}{5a}\\ &= -\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}+\frac{(b(Ab-aB))\int \frac{1}{x^2(a+bx^2)}\,dx}{a^2}\\ &= -\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{a^3x}-\frac{(b^2(Ab-aB))\int \frac{1}{a+bx^2}\,dx}{a^3}\\ &= -\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{a^3x}-\frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.98

$$-\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}+\frac{b(-Ab+aB)}{a^3x}+\frac{b^{3/2}(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)),x]
[Out] -1/5*A/(a*x^5) + (A*b - a*B)/(3*a^2*x^3) + (b*(-(A*b) + a*B))/(a^3*x) + (b^
(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

Maple [A]
time = 0.09, size = 74, normalized size = 0.92

method	result
default	$-\frac{b^2(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}-\frac{A}{5a\,x^5}-\frac{-Ab+Ba}{3a^2x^3}-\frac{b(Ab-Ba)}{a^3x}$
risch	$\frac{-\frac{b(Ab-Ba)x^4}{a^3}+\frac{(Ab-Ba)x^2}{3a^2}-\frac{A}{5a}}{x^5}+\frac{\sqrt{-ab}\,b^2\ln\left(-bx+\sqrt{-ab}\right)A}{2a^4}-\frac{\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)B}{2a^3}-\frac{\sqrt{-ab}\,b^2\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -b^2*(A*b-B*a)/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/5*A/a/x^5-1/3*(-A*
b+B*a)/a^2/x^3-b*(A*b-B*a)/a^3/x
```

Maxima [A]
time = 0.51, size = 79, normalized size = 0.99

$$\frac{(Bab^2 - Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{15\,(Bab - Ab^2)x^4 - 3\,Aa^2 - 5\,(Ba^2 - Aab)x^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(15*(B*a*b -
 A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 - A*a*b)*x^2)/(a^3*x^5)
```

Fricas [A]
time = 0.67, size = 184, normalized size = 2.30

$$\left[-\frac{15\,(Bab - Ab^2)x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) - 30\,(Bab - Ab^2)x^4 + 6\,Aa^2 + 10\,(Ba^2 - Aab)x^2}{30\,a^3x^5}, \frac{15\,(Bab - Ab^2)x^5\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + 15\,(Bab - Ab^2)x^4 - 3\,Aa^2 - 5\,(Ba^2 - Aab)x^2}{15\,a^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(B*a*b - A*b^2)*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)) - 30*(B*a*b - A*b^2)*x^4 + 6*A*a^2 + 10*(B*a^2 - A*a*b)*x^2)
/(a^3*x^5), 1/15*(15*(B*a*b - A*b^2)*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 15
*(B*a*b - A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 - A*a*b)*x^2)/(a^3*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(68) = 136$.
time = 0.31, size = 163, normalized size = 2.04

$$-\frac{\sqrt{-\frac{b^3}{a^7}}\,(-Ab + Ba)\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}\,(-Ab+Ba)}{-Ab^3+Bab^2} + x\right)}{2} + \frac{\sqrt{-\frac{b^3}{a^7}}\,(-Ab + Ba)\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}\,(-Ab+Ba)}{-Ab^3+Bab^2} + x\right)}{2} + \frac{-3Aa^2 + x^4(-15Ab^2 + 15Bab) + x^2\cdot(5Aab - 5Ba^2)}{15a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**7)*(-A*b + B*a)*log(-a**4*sqrt(-b**3/a**7)*(-A*b + B*a)/(-A*
b**3 + B*a*b**2) + x)/2 + sqrt(-b**3/a**7)*(-A*b + B*a)*log(a**4*sqrt(-b**3
/a**7)*(-A*b + B*a)/(-A*b**3 + B*a*b**2) + x)/2 + (-3*A*a**2 + x**4*(-15*A*
b**2 + 15*B*a*b) + x**2*(5*A*a*b - 5*B*a**2))/(15*a**3*x**5)
```

Giac [A]
time = 1.26, size = 81, normalized size = 1.01

$$\frac{(Bab^2 - Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{15\,Babx^4 - 15\,Ab^2x^4 - 5\,Ba^2x^2 + 5\,Aabx^2 - 3\,Aa^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(15*B*a*b*x^
4 - 15*A*b^2*x^4 - 5*B*a^2*x^2 + 5*A*a*b*x^2 - 3*A*a^2)/(a^3*x^5)
```

Mupad [B]
time = 0.05, size = 70, normalized size = 0.88

$$-\frac{\frac{A}{5\,a} - \frac{x^2\,(A\,b-B\,a)}{3\,a^2} + \frac{b\,x^4\,(A\,b-B\,a)}{a^3}}{x^5} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b-B\,a)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)),x)
```

```
[Out] - (A/(5*a) - (x^2*(A*b - B*a))/(3*a^2) + (b*x^4*(A*b - B*a))/a^3)/x^5 - (b^
(3/2)*atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/a^(7/2)
```

3.69 $\int \frac{A+Bx^2}{x^7(a+bx^2)}\,dx$

Optimal. Leaf size=93

$$-\frac{A}{6ax^6}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{2a^3x^2}-\frac{b^2(Ab-aB)\log(x)}{a^4}+\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}$$

```
[Out] -1/6*A/a/x^6+1/4*(A*b-B*a)/a^2/x^4-1/2*b*(A*b-B*a)/a^3/x^2-b^2*(A*b-B*a)*ln
(x)/a^4+1/2*b^2*(A*b-B*a)*ln(b*x^2+a)/a^4
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 78}

$$\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{b^2\log(x)(Ab-aB)}{a^4}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/6*A/(a*x^6) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(2*a^3*x^2) - (b
^2*(A*b - a*B)*Log[x])/a^4 + (b^2*(A*b - a*B)*Log[a + b*x^2])/(2*a^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^4}+\frac{-Ab+aB}{a^2x^3}-\frac{b(-Ab+aB)}{a^3x^2}+\frac{b^2(-Ab+aB)}{a^4x}-\frac{b^3(-Ab+aB)}{a^4(a+bx)}\right)\right.$$
$$= -\frac{A}{6ax^6}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{2a^3x^2}-\frac{b^2(Ab-aB)\log(x)}{a^4}+\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}$$

Mathematica [A]
time = 0.03, size = 96, normalized size = 1.03

$$-\frac{A}{6ax^6}+\frac{Ab-aB}{4a^2x^4}+\frac{b(-Ab+aB)}{2a^3x^2}+\frac{(-Ab^3+ab^2B)\log(x)}{a^4}+\frac{(Ab^3-ab^2B)\log\left(a+bx^2\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/6*A/(a*x^6) + (A*b - a*B)/(4*a^2*x^4) + (b*(-(A*b) + a*B))/(2*a^3*x^2) +
 ((-(A*b^3) + a*b^2*B)*Log[x])/a^4 + ((A*b^3 - a*b^2*B)*Log[a + b*x^2])/(2*
a^4)
```

Maple [A]
time = 0.08, size = 86, normalized size = 0.92

method	result	size
default	$\frac{b^2(Ab-Ba)\ln\left(b\,x^2+a\right)}{2a^4}-\frac{A}{6a\,x^6}-\frac{-Ab+Ba}{4a^2x^4}-\frac{b(Ab-Ba)}{2a^3x^2}-\frac{b^2(Ab-Ba)\ln(x)}{a^4}$	86
norman	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^2}{4a^2}-\frac{b(Ab-Ba)x^4}{2a^3}}{x^6}-\frac{b^2(Ab-Ba)\ln(x)}{a^4}+\frac{b^2(Ab-Ba)\ln\left(b\,x^2+a\right)}{2a^4}$	88
risch	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^2}{4a^2}-\frac{b(Ab-Ba)x^4}{2a^3}}{x^6}-\frac{b^3\ln(x)A}{a^4}+\frac{b^2\ln(x)B}{a^3}+\frac{b^3\ln\left(-b\,x^2-a\right)A}{2a^4}-\frac{b^2\ln\left(-b\,x^2-a\right)B}{2a^3}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2*(A*b-B*a)*ln(b*x^2+a)/a^4-1/6*A/a/x^6-1/4*(-A*b+B*a)/a^2/x^4-1/2*b*
(A*b-B*a)/a^3/x^2-b^2*(A*b-B*a)*ln(x)/a^4
```

Maxima [A]
time = 0.31, size = 96, normalized size = 1.03

$$-\frac{(Bab^2-Ab^3)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Bab^2-Ab^3)\log\left(x^2\right)}{2\,a^4}+\frac{6\,(Bab-Ab^2)x^4-2\,Aa^2-3\,(Ba^2-Aab)x^2}{12\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a*b^2 - A*b^3)*log(b*x^2 + a)/a^4 + 1/2*(B*a*b^2 - A*b^3)*log(x^2)/
a^4 + 1/12*(6*(B*a*b - A*b^2)*x^4 - 2*A*a^2 - 3*(B*a^2 - A*a*b)*x^2)/(a^3*x
^6)
```

Fricas [A]
time = 0.58, size = 98, normalized size = 1.05

$$-\frac{6\,(Bab^2-Ab^3)x^6\log\left(bx^2+a\right)-12\,(Bab^2-Ab^3)x^6\log\left(x\right)-6\,(Ba^2b-Aab^2)x^4+2\,Aa^3+3\,(Ba^3-Aa^2b)x^2}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*(B*a*b^2 - A*b^3)*x^6*log(b*x^2 + a) - 12*(B*a*b^2 - A*b^3)*x^6*lo
g(x) - 6*(B*a^2*b - A*a*b^2)*x^4 + 2*A*a^3 + 3*(B*a^3 - A*a^2*b)*x^2)/(a^4*
x^6)
```

Sympy [A]
time = 0.58, size = 88, normalized size = 0.95

$$\frac{-2Aa^2+x^4(-6Ab^2+6Bab)+x^2\cdot(3Aab-3Ba^2)}{12a^3x^6}+\frac{b^2(-Ab+Ba)\log\left(x\right)}{a^4}-\frac{b^2(-Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a),x)
```

```
[Out] (-2*A*a**2 + x**4*(-6*A*b**2 + 6*B*a*b) + x**2*(3*A*a*b - 3*B*a**2))/(12*a*
*3*x**6) + b**2*(-A*b + B*a)*log(x)/a**4 - b**2*(-A*b + B*a)*log(a/b + x**2
)/(2*a**4)
```

Giac [A]
time = 0.92, size = 126, normalized size = 1.35

$$\frac{(Bab^2-Ab^3)\log\left(x^2\right)}{2\,a^4}-\frac{(Bab^3-Ab^4)\log\left(|bx^2+a|\right)}{2\,a^4b}-\frac{11\,Bab^2x^6-11\,Ab^3x^6-6\,Ba^2bx^4+6\,Aab^2x^4+3\,Ba^3x^2-3\,Aa^2bx^2+2\,Aa^3}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*a*b^2 - A*b^3)*log(x^2)/a^4 - 1/2*(B*a*b^3 - A*b^4)*log(abs(b*x^2 +
a))/(a^4*b) - 1/12*(11*B*a*b^2*x^6 - 11*A*b^3*x^6 - 6*B*a^2*b*x^4 + 6*A*a*b
^2*x^4 + 3*B*a^3*x^2 - 3*A*a^2*b*x^2 + 2*A*a^3)/(a^4*x^6)
```

Mupad [B]
time = 0.09, size = 92, normalized size = 0.99

$$\frac{\ln\left(b\,x^2+a\right)\,\left(A\,b^3-B\,a\,b^2\right)}{2\,a^4}-\frac{\frac{A}{6\,a}-\frac{x^2\,\left(A\,b-B\,a\right)}{4\,a^2}+\frac{b\,x^4\,\left(A\,b-B\,a\right)}{2\,a^3}}{x^6}-\frac{\ln\left(x\right)\,\left(A\,b^3-B\,a\,b^2\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(A*b^3 - B*a*b^2))/(2*a^4) - (A/(6*a) - (x^2*(A*b - B*a))/(
4*a^2) + (b*x^4*(A*b - B*a))/(2*a^3))/x^6 - (log(x)*(A*b^3 - B*a*b^2))/a^4
```

3.70 $\int \frac{A+Bx^2}{x^8(a+bx^2)} dx$

Optimal. Leaf size=99

$$-\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{b^2(Ab-aB)}{a^4x}+\frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[Out] -1/7*A/a/x^7+1/5*(A*b-B*a)/a^2/x^5-1/3*b*(A*b-B*a)/a^3/x^3+b^2*(A*b-B*a)/a^
4/x+b^(5/2)*(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {464, 331, 211}

$$\frac{b^{5/2}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b^2(Ab-aB)}{a^4x}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}-\frac{A}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*A/(a*x^7) + (A*b - a*B)/(5*a^2*x^5) - (b*(A*b - a*B))/(3*a^3*x^3) + (b
^2*(A*b - a*B))/(a^4*x) + (b^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/a^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^8(a+bx^2)}\,dx &= -\frac{A}{7ax^7}-\frac{(7Ab-7aB)\int \frac{1}{x^6(a+bx^2)}\,dx}{7a}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}+\frac{(b(Ab-aB))\int \frac{1}{x^4(a+bx^2)}\,dx}{a^2}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}-\frac{b(Ab-aB)}{3a^3x^3}-\frac{(b^2(Ab-aB))\int \frac{1}{x^2(a+bx^2)}\,dx}{a^3}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{b^2(Ab-aB)}{a^4x}+\frac{(b^3(Ab-aB))\int \frac{1}{a+bx^2}\,dx}{a^4}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{b^2(Ab-aB)}{a^4x}+\frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 1.02

$$-\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}+\frac{b(-Ab+aB)}{3a^3x^3}-\frac{b^2(-Ab+aB)}{a^4x}-\frac{b^{5/2}(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^8*(a + b*x^2)),x]
[Out] -1/7*A/(a*x^7) + (A*b - a*B)/(5*a^2*x^5) + (b*(-(A*b) + a*B))/(3*a^3*x^3) -
 (b^2*(-(A*b) + a*B))/(a^4*x) - (b^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(9/2)
```

Maple [A]
time = 0.10, size = 91, normalized size = 0.92

method	result
default	$\frac{b^3(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}-\frac{A}{7a\,x^7}-\frac{-Ab+Ba}{5a^2x^5}-\frac{b(Ab-Ba)}{3a^3x^3}+\frac{b^2(Ab-Ba)}{a^4x}$
risch	$\frac{\frac{b^2(Ab-Ba)x^6}{a^4}-\frac{b(Ab-Ba)x^4}{3a^3}+\frac{(Ab-Ba)x^2}{5a^2}-\frac{A}{7a}}{x^7}+\frac{\sqrt{-ab}\,b^3\ln\left(-bx-\sqrt{-ab}\right)A}{2a^5}-\frac{\sqrt{-ab}\,b^2\ln\left(-bx-\sqrt{-ab}\right)B}{2a^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^8/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(A*b-B*a)/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/7*A/a/x^7-1/5*(-A*b
+B*a)/a^2/x^5-1/3*b*(A*b-B*a)/a^3/x^3+b^2*(A*b-B*a)/a^4/x
```

Maxima [A]
time = 0.50, size = 103, normalized size = 1.04

$$-\frac{(Bab^3 - Ab^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4} - \frac{105\,(Bab^2 - Ab^3)x^6 - 35\,(Ba^2b - Aab^2)x^4 + 15\,Aa^3 + 21\,(Ba^3 - Aa^2b)x^2}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/105*(105*(B*a*
b^2 - A*b^3)*x^6 - 35*(B*a^2*b - A*a*b^2)*x^4 + 15*A*a^3 + 21*(B*a^3 - A*a^
2*b)*x^2)/(a^4*x^7)
```

Fricas [A]
time = 0.64, size = 234, normalized size = 2.36

$$\left[-\frac{105\,(Bab^2 - Ab^3)x^7\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2 + 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) + 210\,(Bab^2 - Ab^3)x^6 - 70\,(Ba^2b - Aab^2)x^4 + 30\,Aa^3 + 42\,(Ba^3 - Aa^2b)x^2}{210\,a^4x^7}, -\frac{105\,(Bab^2 - Ab^3)x^7\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right) + 105\,(Bab^2 - Ab^3)x^6 - 35\,(Ba^2b - Aab^2)x^4 + 15\,Aa^3 + 21\,(Ba^3 - Aa^2b)x^2}{105\,a^4x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/210*(105*(B*a*b^2 - A*b^3)*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)) + 210*(B*a*b^2 - A*b^3)*x^6 - 70*(B*a^2*b - A*a*b^2)*x^4
 + 30*A*a^3 + 42*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7), -1/105*(105*(B*a*b^2 - A
*b^3)*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*(B*a*b^2 - A*b^3)*x^6 - 35*(B
*a^2*b - A*a*b^2)*x^4 + 15*A*a^3 + 21*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(87) = 174$.
time = 0.29, size = 187, normalized size = 1.89

$$\frac{\sqrt{-\frac{b^5}{a^9}}\,(-Ab + Ba)\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}\,(-Ab+Ba)}{-Ab^4+Bab^3} + x\right)}{2} - \frac{\sqrt{-\frac{b^5}{a^9}}\,(-Ab + Ba)\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}\,(-Ab+Ba)}{-Ab^4+Bab^3} + x\right)}{2} + \frac{-15Aa^3 + x^6\cdot(105Ab^3 - 105Bab^2) + x^4(-35Aab^2 + 35Ba^2b) + x^2\cdot(21Aa^2b - 21Ba^3)}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**9)*(-A*b + B*a)*log(-a**5*sqrt(-b**5/a**9)*(-A*b + B*a)/(-A*b
**4 + B*a*b**3) + x)/2 - sqrt(-b**5/a**9)*(-A*b + B*a)*log(a**5*sqrt(-b**5/
a**9)*(-A*b + B*a)/(-A*b**4 + B*a*b**3) + x)/2 + (-15*A*a**3 + x**6*(105*A*
b**3 - 105*B*a*b**2) + x**4*(-35*A*a*b**2 + 35*B*a**2*b) + x**2*(21*A*a**2*
b - 21*B*a**3))/(105*a**4*x**7)
```

Giac [A]
time = 1.06, size = 106, normalized size = 1.07

$$-\frac{(Bab^3 - Ab^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4} - \frac{105\,Bab^2x^6 - 105\,Ab^3x^6 - 35\,Ba^2bx^4 + 35\,Aab^2x^4 + 21\,Ba^3x^2 - 21\,Aa^2bx^2 + 15\,Aa^3}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/105*(105*B*a*b
^2*x^6 - 105*A*b^3*x^6 - 35*B*a^2*b*x^4 + 35*A*a*b^2*x^4 + 21*B*a^3*x^2 - 2
1*A*a^2*b*x^2 + 15*A*a^3)/(a^4*x^7)
```

Mupad [B]
time = 0.07, size = 89, normalized size = 0.90

$$\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{a^{9/2}}-\frac{\frac{A}{7\,a}-\frac{x^2\,\left(A\,b-B\,a\right)}{5\,a^2}-\frac{b^2\,x^6\,\left(A\,b-B\,a\right)}{a^4}+\frac{b\,x^4\,\left(A\,b-B\,a\right)}{3\,a^3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^8*(a + b*x^2)),x)
```

```
[Out] (b^(5/2)*atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/a^(9/2) - (A/(7*a) - (x^2*(
A*b - B*a))/(5*a^2) - (b^2*x^6*(A*b - B*a))/a^4 + (b*x^4*(A*b - B*a))/(3*a^
3))/x^7
```

3.71 $\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=126

$$\frac{a^2(3Ab-4aB)x^2}{2b^5}-\frac{a(2Ab-3aB)x^4}{4b^4}+\frac{(Ab-2aB)x^6}{6b^3}+\frac{Bx^8}{8b^2}-\frac{a^4(Ab-aB)}{2b^6(a+bx^2)}-\frac{a^3(4Ab-5aB)\log(a+bx^2)}{2b^6}$$

```
[Out] 1/2*a^2*(3*A*b-4*B*a)*x^2/b^5-1/4*a*(2*A*b-3*B*a)*x^4/b^4+1/6*(A*b-2*B*a)*x
^6/b^3+1/8*B*x^8/b^2-1/2*a^4*(A*b-B*a)/b^6/(b*x^2+a)-1/2*a^3*(4*A*b-5*B*a)*
ln(b*x^2+a)/b^6
```

Rubi [A]
time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$-\frac{a^4(Ab-aB)}{2b^6(a+bx^2)}-\frac{a^3(4Ab-5aB)\log(a+bx^2)}{2b^6}+\frac{a^2x^2(3Ab-4aB)}{2b^5}-\frac{ax^4(2Ab-3aB)}{4b^4}+\frac{x^6(Ab-2aB)}{6b^3}+\frac{Bx^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a^2*(3*A*b - 4*a*B)*x^2)/(2*b^5) - (a*(2*A*b - 3*a*B)*x^4)/(4*b^4) + ((A*b
 - 2*a*B)*x^6)/(6*b^3) + (B*x^8)/(8*b^2) - (a^4*(A*b - a*B))/(2*b^6*(a + b*
x^2)) - (a^3*(4*A*b - 5*a*B)*Log[a + b*x^2])/(2*b^6)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-3Ab+4aB)}{b^5} + \frac{a(-2Ab+3aB)x}{b^4} + \frac{(Ab-2aB)x^2}{b^3} + \frac{Bx^3}{b^2} - \frac{a^4(}{b}\right.\right.$$
$$= \frac{a^2(3Ab-4aB)x^2}{2b^5} - \frac{a(2Ab-3aB)x^4}{4b^4} + \frac{(Ab-2aB)x^6}{6b^3} + \frac{Bx^8}{8b^2} - \frac{a^4(Ab-aB)}{2b^6(a+bx^2)} - \frac{a^3}{}$$

Mathematica [A]
time = 0.06, size = 113, normalized size = 0.90

$$\frac{-12a^2b(-3Ab+4aB)x^2+6ab^2(-2Ab+3aB)x^4+4b^3(Ab-2aB)x^6+3b^4Bx^8+\frac{12a^4(-Ab+aB)}{a+bx^2}+12a^3(-4Ab+5aB)\log(a+bx^2)}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (-12*a^2*b*(-3*A*b + 4*a*B)*x^2 + 6*a*b^2*(-2*A*b + 3*a*B)*x^4 + 4*b^3*(A*b
 - 2*a*B)*x^6 + 3*b^4*B*x^8 + (12*a^4*(-(A*b) + a*B))/(a + b*x^2) + 12*a^3*
(-4*A*b + 5*a*B)*Log[a + b*x^2])/(24*b^6)
```

Maple [A]
time = 0.07, size = 125, normalized size = 0.99

method	result
norman	$\frac{-\frac{a(4Aa^3b-5Ba^4)}{2b^6}+\frac{Bx^{10}}{8b}+\frac{(4Ab-5Ba)x^8}{24b^2}-\frac{a(4Ab-5Ba)x^6}{12b^3}+\frac{a^2(4Ab-5Ba)x^4}{4b^4}}{bx^2+a} - \frac{a^3(4Ab-5Ba)\ln(bx^2+a)}{2b^6}$
default	$\frac{\frac{Bb^3x^8}{8}+\frac{(Ab^3-2Bab^2)x^6}{6}+\frac{(-2Aab^2+3Ba^2b)x^4}{4}+\frac{(3Aa^2b-4Ba^3)x^2}{2}}{b^5} - \frac{a^3\left(\frac{(4Ab-5Ba)\ln(bx^2+a)}{b}+\frac{a(Ab-Ba)}{b(bx^2+a)}\right)}{2b^5}$
risch	$\frac{Bx^8}{8b^2}+\frac{x^6A}{6b^2}-\frac{x^6Ba}{3b^3}-\frac{x^4Aa}{2b^3}+\frac{3x^4Ba^2}{4b^4}+\frac{3x^2Aa^2}{2b^4}-\frac{2x^2Ba^3}{b^5}-\frac{a^4A}{2b^5(bx^2+a)}+\frac{a^5B}{2b^6(bx^2+a)}-\frac{2a^3\ln(bx^2+a)A}{b^5}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/8*B*b^3*x^8+1/6*(A*b^3-2*B*a*b^2)*x^6+1/4*(-2*A*a*b^2+3*B*a^2*b)*x
^4+1/2*(3*A*a^2*b-4*B*a^3)*x^2)-1/2*a^3/b^5*((4*A*b-5*B*a)/b*ln(b*x^2+a)+a*
(A*b-B*a)/b/(b*x^2+a))
```

Maxima [A]
time = 0.33, size = 131, normalized size = 1.04

$$\frac{Ba^5-Aa^4b}{2(b^7x^2+ab^6)}+\frac{3Bb^3x^8-4(2Bab^2-Ab^3)x^6+6(3Ba^2b-2Aab^2)x^4-12(4Ba^3-3Aa^2b)x^2}{24b^5}+\frac{(5Ba^4-4Aa^3b)\log(bx^2+a)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^5 - A*a^4*b)/(b^7*x^2 + a*b^6) + 1/24*(3*B*b^3*x^8 - 4*(2*B*a*b^2
- A*b^3)*x^6 + 6*(3*B*a^2*b - 2*A*a*b^2)*x^4 - 12*(4*B*a^3 - 3*A*a^2*b)*x^2
)/b^5 + 1/2*(5*B*a^4 - 4*A*a^3*b)*log(b*x^2 + a)/b^6
```

Fricas [A]
time = 0.63, size = 172, normalized size = 1.37

$$\frac{3\,Bb^5x^{10} - (5\,Bab^4 - 4\,Ab^5)x^8 + 2\,(5\,Ba^2b^3 - 4\,Aab^4)x^6 + 12\,Ba^5 - 12\,Aa^4b - 6\,(5\,Ba^3b^2 - 4\,Aa^2b^3)x^4 - 12\,(4\,Ba^4b - 3\,Aa^3b^2)x^2 + 12\,(5\,Ba^5 - 4\,Aa^4b + (5\,Ba^4b - 4\,Aa^3b^2)x^2)\log\left(bx^2 + a\right)}{24\,(b^7x^2 + ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^5*x^10 - (5*B*a*b^4 - 4*A*b^5)*x^8 + 2*(5*B*a^2*b^3 - 4*A*a*b^4
)*x^6 + 12*B*a^5 - 12*A*a^4*b - 6*(5*B*a^3*b^2 - 4*A*a^2*b^3)*x^4 - 12*(4*B
*a^4*b - 3*A*a^3*b^2)*x^2 + 12*(5*B*a^5 - 4*A*a^4*b + (5*B*a^4*b - 4*A*a^3*
b^2)*x^2)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

Sympy [A]
time = 0.42, size = 131, normalized size = 1.04

$$\frac{Bx^8}{8b^2} + \frac{a^3(-4Ab + 5Ba)\log\left(a + bx^2\right)}{2b^6} + x^6\left(\frac{A}{6b^2} - \frac{Ba}{3b^3}\right) + x^4\left(-\frac{Aa}{2b^3} + \frac{3Ba^2}{4b^4}\right) + x^2 \cdot \left(\frac{3Aa^2}{2b^4} - \frac{2Ba^3}{b^5}\right) + \frac{-Aa^4b + Ba^5}{2ab^6 + 2b^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**8/(8*b**2) + a**3*(-4*A*b + 5*B*a)*log(a + b*x**2)/(2*b**6) + x**6*(A/
(6*b**2) - B*a/(3*b**3)) + x**4*(-A*a/(2*b**3) + 3*B*a**2/(4*b**4)) + x**2*
(3*A*a**2/(2*b**4) - 2*B*a**3/b**5) + (-A*a**4*b + B*a**5)/(2*a*b**6 + 2*b*
*7*x**2)
```

Giac [A]
time = 1.23, size = 159, normalized size = 1.26

$$\frac{(5\,Ba^4 - 4\,Aa^3b)\log\left(|bx^2 + a|\right)}{2\,b^6} - \frac{5\,Ba^4bx^2 - 4\,Aa^3b^2x^2 + 4\,Ba^5 - 3\,Aa^4b}{2\,(bx^2 + a)b^6} + \frac{3\,Bb^6x^8 - 8\,Bab^5x^6 + 4\,Ab^6x^6 + 18\,Ba^2b^4x^4 - 12\,Aab^5x^4 - 48\,Ba^3b^3x^2 + 36\,Aa^2b^4x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a^4 - 4*A*a^3*b)*log(abs(b*x^2 + a))/b^6 - 1/2*(5*B*a^4*b*x^2 - 4*
A*a^3*b^2*x^2 + 4*B*a^5 - 3*A*a^4*b)/((b*x^2 + a)*b^6) + 1/24*(3*B*b^6*x^8
- 8*B*a*b^5*x^6 + 4*A*b^6*x^6 + 18*B*a^2*b^4*x^4 - 12*A*a*b^5*x^4 - 48*B*a^
3*b^3*x^2 + 36*A*a^2*b^4*x^2)/b^8
```

Mupad [B]
time = 0.06, size = 181, normalized size = 1.44

$$x^2\left(\frac{a\left(\frac{2a\left(\frac{A}{b^2}-\frac{2Ba}{b^3}\right)}{b}+\frac{Ba^2}{b^4}\right)}{b}-\frac{a^2\left(\frac{A}{b^2}-\frac{2Ba}{b^3}\right)}{2b^2}\right)+x^6\left(\frac{A}{6b^2}-\frac{Ba}{3b^3}\right)-x^4\left(\frac{a\left(\frac{A}{b^2}-\frac{2Ba}{b^3}\right)}{2b}+\frac{Ba^2}{4b^4}\right)+\frac{Bx^8}{8b^2}+\frac{\ln\left(bx^2+a\right)\left(5Ba^4-4Aa^3b\right)}{2b^6}+\frac{Ba^5-Aa^4b}{2b\left(b^6x^2+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^2*((a*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (2
*B*a)/b^3))/(2*b^2)) + x^6*(A/(6*b^2) - (B*a)/(3*b^3)) - x^4*((a*(A/b^2 - (
2*B*a)/b^3))/(2*b) + (B*a^2)/(4*b^4)) + (B*x^8)/(8*b^2) + (log(a + b*x^2)*(
5*B*a^4 - 4*A*a^3*b))/(2*b^6) + (B*a^5 - A*a^4*b)/(2*b*(a*b^5 + b^6*x^2))
```

3.72 $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=131

$$\frac{a^2(3Ab-4aB)x}{b^5}-\frac{a(2Ab-3aB)x^3}{3b^4}+\frac{(Ab-2aB)x^5}{5b^3}+\frac{Bx^7}{7b^2}+\frac{a^3(Ab-aB)x}{2b^5(a+bx^2)}-\frac{a^{5/2}(7Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

```
[Out] a^2*(3*A*b-4*B*a)*x/b^5-1/3*a*(2*A*b-3*B*a)*x^3/b^4+1/5*(A*b-2*B*a)*x^5/b^3
+1/7*B*x^7/b^2+1/2*a^3*(A*b-B*a)*x/b^5/(b*x^2+a)-1/2*a^(5/2)*(7*A*b-9*B*a)*
arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {466, 1824, 211}

$$-\frac{a^{5/2}(7Ab-9aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{a^3x(Ab-aB)}{2b^5(a+bx^2)}+\frac{a^2x(3Ab-4aB)}{b^5}-\frac{ax^3(2Ab-3aB)}{3b^4}+\frac{x^5(Ab-2aB)}{5b^3}+\frac{Bx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a^2*(3*A*b - 4*a*B)*x)/b^5 - (a*(2*A*b - 3*a*B)*x^3)/(3*b^4) + ((A*b - 2*a
*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + (a^3*(A*b - a*B)*x)/(2*b^5*(a + b*x^2)
) - (a^(5/2)*(7*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{a^3(Ab-aB)x}{2b^5(a+bx^2)} - \frac{\int \frac{a^3(Ab-aB)-2a^2b(Ab-aB)x^2+2ab^2(Ab-aB)x^4-2b^3(Ab-aB)x^6-2b^4Bx^8}{a+bx^2}\,dx}{2b^5}$$

$$= \frac{a^3(Ab-aB)x}{2b^5(a+bx^2)} - \frac{\int \left(-2a^2(3Ab-4aB)+2ab(2Ab-3aB)x^2-2b^2(Ab-2aB)x^4-2\right.}{2b^5}$$

$$= \frac{a^2(3Ab-4aB)x}{b^5} - \frac{a(2Ab-3aB)x^3}{3b^4} + \frac{(Ab-2aB)x^5}{5b^3} + \frac{Bx^7}{7b^2} + \frac{a^3(Ab-aB)x}{2b^5(a+bx^2)} - \frac{(a}{}$$

$$= \frac{a^2(3Ab-4aB)x}{b^5} - \frac{a(2Ab-3aB)x^3}{3b^4} + \frac{(Ab-2aB)x^5}{5b^3} + \frac{Bx^7}{7b^2} + \frac{a^3(Ab-aB)x}{2b^5(a+bx^2)} - \frac{a^5}{}$$

Mathematica [A]
time = 0.08, size = 134, normalized size = 1.02

$$-\frac{a^2(-3Ab+4aB)x}{b^5} + \frac{a(-2Ab+3aB)x^3}{3b^4} + \frac{(Ab-2aB)x^5}{5b^3} + \frac{Bx^7}{7b^2} + \frac{(a^3Ab-a^4B)\,x}{2b^5(a+bx^2)} + \frac{a^{5/2}(-7Ab+9aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((a^2*(-3*A*b + 4*a*B)*x)/b^5) + (a*(-2*A*b + 3*a*B)*x^3)/(3*b^4) + ((A*b
- 2*a*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + ((a^3*A*b - a^4*B)*x)/(2*b^5*(a +
 b*x^2)) + (a^(5/2)*(-7*A*b + 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/
2))
```

Maple [A]
time = 0.09, size = 123, normalized size = 0.94

method	result
default	$\frac{\frac{1}{7}B\,b^3x^7+\frac{1}{5}A\,b^3x^5-\frac{2}{5}Ba\,b^2x^5-\frac{2}{3}Aa\,b^2x^3+B\,a^2b\,x^3+3A\,a^2bx-4B\,a^3x}{b^5} - \frac{a^3\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(7Ab-9Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^5}$
risch	$\frac{B\,x^7}{7b^2}+\frac{A\,x^5}{5b^2}-\frac{2Ba\,x^5}{5b^3}-\frac{2Aa\,x^3}{3b^3}+\frac{B\,a^2x^3}{b^4}+\frac{3A\,a^2x}{b^4}-\frac{4B\,a^3x}{b^5}+\frac{\left(\frac{1}{2}A\,a^3b-\frac{1}{2}B\,a^4\right)x}{b^5(b\,x^2+a)}+\frac{7\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\right.}{4b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/7*B*b^3*x^7+1/5*A*b^3*x^5-2/5*B*a*b^2*x^5-2/3*A*a*b^2*x^3+B*a^2*b*
x^3+3*A*a^2*b*x-4*B*a^3*x)-a^3/b^5*((-1/2*A*b+1/2*B*a)*x/(b*x^2+a)+1/2*(7*A
*b-9*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 136, normalized size = 1.04

$$-\frac{(Ba^4-Aa^3b)x}{2\,(b^6x^2+ab^5)}+\frac{(9\,Ba^4-7\,Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{15\,Bb^3x^7-21\,(2\,Bab^2-Ab^3)x^5+35\,(3\,Ba^2b-2\,Aab^2)x^3-105\,(4\,Ba^3-3\,Aa^2b)x}{105\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^4 - A*a^3*b)*x/(b^6*x^2 + a*b^5) + 1/2*(9*B*a^4 - 7*A*a^3*b)*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/105*(15*B*b^3*x^7 - 21*(2*B*a*b^2 - A
*b^3)*x^5 + 35*(3*B*a^2*b - 2*A*a*b^2)*x^3 - 105*(4*B*a^3 - 3*A*a^2*b)*x)/b
^5
```

Fricas [A]
time = 0.61, size = 350, normalized size = 2.67

$$\left[\frac{60\,Bb^4x^9-12\,(9\,Bab^3-7\,Ab^4)x^7+28\,(9\,Ba^2b^2-7\,Aab^3)x^5-140\,(9\,Ba^3b-7\,Aa^2b^2)x^3-105\,(9\,Ba^4-7\,Aa^3b+(9\,Ba^3b-7\,Aa^2b^2)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\,(9\,Ba^4-7\,Aa^3b)x}{420\,(b^6x^2+ab^5)},\ \frac{30\,Bb^4x^9-6\,(9\,Bab^3-7\,Ab^4)x^7+14\,(9\,Ba^2b^2-7\,Aab^3)x^5-70\,(9\,Ba^3b-7\,Aa^2b^2)x^3+105\,(9\,Ba^4-7\,Aa^3b+(9\,Ba^3b-7\,Aa^2b^2)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-105\,(9\,Ba^4-7\,Aa^3b)x}{210\,(b^6x^2+ab^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*B*b^4*x^9 - 12*(9*B*a*b^3 - 7*A*b^4)*x^7 + 28*(9*B*a^2*b^2 - 7*A
*a*b^3)*x^5 - 140*(9*B*a^3*b - 7*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 7*A*a^3*b
+ (9*B*a^3*b - 7*A*a^2*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) - 210*(9*B*a^4 - 7*A*a^3*b)*x)/(b^6*x^2 + a*b^5), 1/210*(3
0*B*b^4*x^9 - 6*(9*B*a*b^3 - 7*A*b^4)*x^7 + 14*(9*B*a^2*b^2 - 7*A*a*b^3)*x^
5 - 70*(9*B*a^3*b - 7*A*a^2*b^2)*x^3 + 105*(9*B*a^4 - 7*A*a^3*b + (9*B*a^3*
b - 7*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(9*B*a^4 - 7*
A*a^3*b)*x)/(b^6*x^2 + a*b^5)]
```

Sympy [A]
time = 0.53, size = 238, normalized size = 1.82

$$\frac{Bx^7}{7b^2}+x^5\left(\frac{A}{5b^2}-\frac{2Ba}{5b^3}\right)+x^3\left(-\frac{2Aa}{3b^3}+\frac{Ba^2}{b^4}\right)+x\left(\frac{3Aa^2}{b^4}-\frac{4Ba^3}{b^5}\right)+\frac{x(Aa^3b-Ba^4)}{2ab^5+2b^6x^2}-\frac{\sqrt{-\frac{a^5}{b^{11}}}\,(-7Ab+9Ba)\log\left(-\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}\,(-7Ab+9Ba)}{-7Aa^2b+9Ba^3}+x\right)}{4}+\frac{\sqrt{-\frac{a^5}{b^{11}}}\,(-7Ab+9Ba)\log\left(\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}\,(-7Ab+9Ba)}{-7Aa^2b+9Ba^3}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) + x**5*(A/(5*b**2) - 2*B*a/(5*b**3)) + x**3*(-2*A*a/(3*b**3
) + B*a**2/b**4) + x*(3*A*a**2/b**4 - 4*B*a**3/b**5) + x*(A*a**3*b - B*a**4
)/(2*a*b**5 + 2*b**6*x**2) - sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)*log(-b**5*s
qrt(-a**5/b**11)*(-7*A*b + 9*B*a)/(-7*A*a**2*b + 9*B*a**3) + x)/4 + sqrt(-a
**5/b**11)*(-7*A*b + 9*B*a)*log(b**5*sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)/(-7
*A*a**2*b + 9*B*a**3) + x)/4
```

Giac [A]
time = 1.11, size = 139, normalized size = 1.06

$$\frac{(9\,Ba^4 - 7\,Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5} - \frac{Ba^4x - Aa^3bx}{2\,(bx^2+a)b^5} + \frac{15\,Bb^{12}x^7 - 42\,Bab^{11}x^5 + 21\,Ab^{12}x^5 + 105\,Ba^2b^{10}x^3 - 70\,Aab^{11}x^3 - 420\,Ba^3b^9x + 315\,Aa^2b^{10}x}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*(B*a^
4*x - A*a^3*b*x)/((b*x^2 + a)*b^5) + 1/105*(15*B*b^12*x^7 - 42*B*a*b^11*x^5
 + 21*A*b^12*x^5 + 105*B*a^2*b^10*x^3 - 70*A*a*b^11*x^3 - 420*B*a^3*b^9*x +
 315*A*a^2*b^10*x)/b^14
```

Mupad [B]
time = 0.03, size = 203, normalized size = 1.55

$$x\left(\frac{2\,a\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b}-\frac{a^2\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b^2}\right)+x^5\,\left(\frac{A}{5\,b^2}-\frac{2\,B\,a}{5\,b^3}\right)-x^3\,\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{3\,b}+\frac{B\,a^2}{3\,b^4}\right)+\frac{B\,x^7}{7\,b^2}-\frac{x\,\left(\frac{B\,a^4}{2}-\frac{A\,a^3\,b}{2}\right)}{b^6\,x^2+a\,b^5}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,\left(7\,A\,b-9\,B\,a\right)}{9\,B\,a^4-7\,A\,a^3\,b}\right)\,\left(7\,A\,b-9\,B\,a\right)}{2\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x*((2*a*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (2
*B*a)/b^3))/b^2) + x^5*(A/(5*b^2) - (2*B*a)/(5*b^3)) - x^3*((2*a*(A/b^2 - (
2*B*a)/b^3))/(3*b) + (B*a^2)/(3*b^4)) + (B*x^7)/(7*b^2) - (x*((B*a^4)/2 - (
A*a^3*b)/2))/(a*b^5 + b^6*x^2) + (a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(7*A*b -
9*B*a))/(9*B*a^4 - 7*A*a^3*b))*(7*A*b - 9*B*a))/(2*b^(11/2))
```

3.73 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a(2Ab-3aB)x^2}{2b^4}+\frac{(Ab-2aB)x^4}{4b^3}+\frac{Bx^6}{6b^2}+\frac{a^3(Ab-aB)}{2b^5(a+bx^2)}+\frac{a^2(3Ab-4aB)\log(a+bx^2)}{2b^5}$$

```
[Out] -1/2*a*(2*A*b-3*B*a)*x^2/b^4+1/4*(A*b-2*B*a)*x^4/b^3+1/6*B*x^6/b^2+1/2*a^3*
(A*b-B*a)/b^5/(b*x^2+a)+1/2*a^2*(3*A*b-4*B*a)*ln(b*x^2+a)/b^5
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 78}

$$\frac{a^3(Ab-aB)}{2b^5(a+bx^2)}+\frac{a^2(3Ab-4aB)\log(a+bx^2)}{2b^5}-\frac{ax^2(2Ab-3aB)}{2b^4}+\frac{x^4(Ab-2aB)}{4b^3}+\frac{Bx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(a*(2*A*b - 3*a*B)*x^2)/b^4 + ((A*b - 2*a*B)*x^4)/(4*b^3) + (B*x^6)/(6
*b^2) + (a^3*(A*b - a*B))/(2*b^5*(a + b*x^2)) + (a^2*(3*A*b - 4*a*B)*Log[a
+ b*x^2])/(2*b^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-2Ab+3aB)}{b^4} + \frac{(Ab-2aB)x}{b^3} + \frac{Bx^2}{b^2} + \frac{a^3(-Ab+aB)}{b^4(a+bx)^2} - \frac{a^2(-3Ab}{b^4(a+}\right.\right.$$
$$= -\frac{a(2Ab-3aB)x^2}{2b^4} + \frac{(Ab-2aB)x^4}{4b^3} + \frac{Bx^6}{6b^2} + \frac{a^3(Ab-aB)}{2b^5(a+bx^2)} + \frac{a^2(3Ab-4aB)\log(a}{2b^5}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 0.89

$$\frac{6ab(-2Ab+3aB)x^2+3b^2(Ab-2aB)x^4+2b^3Bx^6+\frac{6a^3(Ab-aB)}{a+bx^2}+6a^2(3Ab-4aB)\log(a+bx^2)}{12b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (6*a*b*(-2*A*b + 3*a*B)*x^2 + 3*b^2*(A*b - 2*a*B)*x^4 + 2*b^3*B*x^6 + (6*a^
3*(A*b - a*B))/(a + b*x^2) + 6*a^2*(3*A*b - 4*a*B)*Log[a + b*x^2])/(12*b^5)
```

Maple [A]
time = 0.07, size = 103, normalized size = 0.99

method	result	siz
norman	$\frac{\frac{a(3Aa^2b-4Ba^3)}{2b^5}+\frac{Bx^8}{6b}+\frac{(3Ab-4Ba)x^6}{12b^2}-\frac{a(3Ab-4Ba)x^4}{4b^3}}{bx^2+a}+\frac{a^2(3Ab-4Ba)\ln(bx^2+a)}{2b^5}$	10
default	$-\frac{-\frac{b^2Bx^6}{6}+\frac{(-b^2A+2abB)x^4}{4}+\frac{(2abA-3a^2B)x^2}{2}}{b^4}+\frac{a^2\left(\frac{(3Ab-4Ba)\ln(bx^2+a)}{b}+\frac{a(Ab-Ba)}{b(bx^2+a)}\right)}{2b^4}$	10
risch	$\frac{Bx^6}{6b^2}+\frac{Ax^4}{4b^2}-\frac{Bax^4}{2b^3}-\frac{aAx^2}{b^3}+\frac{3Ba^2x^2}{2b^4}+\frac{a^3A}{2b^4(bx^2+a)}-\frac{a^4B}{2b^5(bx^2+a)}+\frac{3a^2\ln(bx^2+a)A}{2b^4}-\frac{2a^3\ln(bx^2+a)B}{b^5}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/6*b^2*B*x^6+1/4*(-A*b^2+2*B*a*b)*x^4+1/2*(2*A*a*b-3*B*a^2)*x^2)+
1/2*a^2/b^4*((3*A*b-4*B*a)/b*ln(b*x^2+a)+a*(A*b-B*a)/b/(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 107, normalized size = 1.03

$$-\frac{Ba^4-Aa^3b}{2(b^6x^2+ab^5)}+\frac{2Bb^2x^6-3(2Bab-Ab^2)x^4+6(3Ba^2-2Aab)x^2}{12b^4}-\frac{(4Ba^3-3Aa^2b)\log(bx^2+a)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^4 - A*a^3*b)/(b^6*x^2 + a*b^5) + 1/12*(2*B*b^2*x^6 - 3*(2*B*a*b -
 A*b^2)*x^4 + 6*(3*B*a^2 - 2*A*a*b)*x^2)/b^4 - 1/2*(4*B*a^3 - 3*A*a^2*b)*lo
g(b*x^2 + a)/b^5
```

Fricas [**A**]
time = 0.70, size = 148, normalized size = 1.42

$$\frac{2\,Bb^4x^8 - (4\,Bab^3 - 3\,Ab^4)x^6 - 6\,Ba^4 + 6\,Aa^3b + 3\,(4\,Ba^2b^2 - 3\,Aab^3)x^4 + 6\,(3\,Ba^3b - 2\,Aa^2b^2)x^2 - 6\,(4\,Ba^4 - 3\,Aa^3b + (4\,Ba^3b - 3\,Aa^2b^2)x^2)\log\left(bx^2 + a\right)}{12\,(b^6x^2 + ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^4*x^8 - (4*B*a*b^3 - 3*A*b^4)*x^6 - 6*B*a^4 + 6*A*a^3*b + 3*(4*
B*a^2*b^2 - 3*A*a*b^3)*x^4 + 6*(3*B*a^3*b - 2*A*a^2*b^2)*x^2 - 6*(4*B*a^4 -
 3*A*a^3*b + (4*B*a^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a))/(b^6*x^2 + a*b^
5)
```

Sympy [**A**]
time = 0.41, size = 104, normalized size = 1.00

$$\frac{Bx^6}{6b^2} - \frac{a^2(-3Ab + 4Ba)\log\left(a + bx^2\right)}{2b^5} + x^4\left(\frac{A}{4b^2} - \frac{Ba}{2b^3}\right) + x^2\left(-\frac{Aa}{b^3} + \frac{3Ba^2}{2b^4}\right) + \frac{Aa^3b - Ba^4}{2ab^5 + 2b^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) - a**2*(-3*A*b + 4*B*a)*log(a + b*x**2)/(2*b**5) + x**4*(A/
(4*b**2) - B*a/(2*b**3)) + x**2*(-A*a/b**3 + 3*B*a**2/(2*b**4)) + (A*a**3*b
 - B*a**4)/(2*a*b**5 + 2*b**6*x**2)
```

Giac [**A**]
time = 1.06, size = 135, normalized size = 1.30

$$-\frac{(4\,Ba^3 - 3\,Aa^2b)\log\left(|bx^2 + a|\right)}{2\,b^5} + \frac{2\,Bb^4x^6 - 6\,Bab^3x^4 + 3\,Ab^4x^4 + 18\,Ba^2b^2x^2 - 12\,Aab^3x^2}{12\,b^6} + \frac{4\,Ba^3bx^2 - 3\,Aa^2b^2x^2 + 3\,Ba^4 - 2\,Aa^3b}{2\,(bx^2 + a)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*B*a^3 - 3*A*a^2*b)*log(abs(b*x^2 + a))/b^5 + 1/12*(2*B*b^4*x^6 - 6*
B*a*b^3*x^4 + 3*A*b^4*x^4 + 18*B*a^2*b^2*x^2 - 12*A*a*b^3*x^2)/b^6 + 1/2*(4
*B*a^3*b*x^2 - 3*A*a^2*b^2*x^2 + 3*B*a^4 - 2*A*a^3*b)/((b*x^2 + a)*b^5)
```

Mupad [B]
time = 0.05, size = 121, normalized size = 1.16

$$x^4\left(\frac{A}{4\,b^2}-\frac{B\,a}{2\,b^3}\right)-x^2\left(\frac{a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{2\,b^4}\right)+\frac{B\,x^6}{6\,b^2}-\frac{\ln\left(b\,x^2+a\right)\,\left(4\,B\,a^3-3\,A\,a^2\,b\right)}{2\,b^5}-\frac{B\,a^4-A\,a^3\,b}{2\,b\,\left(b^5\,x^2+a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x^2*((a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2
)/(2*b^4)) + (B*x^6)/(6*b^2) - (log(a + b*x^2)*(4*B*a^3 - 3*A*a^2*b))/(2*b^
5) - (B*a^4 - A*a^3*b)/(2*b*(a*b^4 + b^5*x^2))
```

3.74 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{a(2Ab-3aB)x}{b^4}+\frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^5}{5b^2}-\frac{a^2(Ab-aB)x}{2b^4(a+bx^2)}+\frac{a^{3/2}(5Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

```
[Out] -a*(2*A*b-3*B*a)*x/b^4+1/3*(A*b-2*B*a)*x^3/b^3+1/5*B*x^5/b^2-1/2*a^2*(A*b-B
*a)*x/b^4/(b*x^2+a)+1/2*a^(3/2)*(5*A*b-7*B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(
9/2)
```

Rubi [A]
time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {466, 1824, 211}

$$\frac{a^{3/2}(5Ab-7aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{a^2x(Ab-aB)}{2b^4(a+bx^2)}-\frac{ax(2Ab-3aB)}{b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((a*(2*A*b - 3*a*B)*x)/b^4) + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^5)/(5*b^2
) - (a^2*(A*b - a*B)*x)/(2*b^4*(a + b*x^2)) + (a^(3/2)*(5*A*b - 7*a*B)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{a^2(Ab-aB)x}{2b^4(a+bx^2)} - \frac{\int \frac{-a^2(Ab-aB)+2ab(Ab-aB)x^2-2b^2(Ab-aB)x^4-2b^3Bx^6}{a+bx^2}\,dx}{2b^4}$$

$$= -\frac{a^2(Ab-aB)x}{2b^4(a+bx^2)} - \frac{\int \left(2a(2Ab-3aB)-2b(Ab-2aB)x^2-2b^2Bx^4+\frac{-5a^2Ab+7a^3B}{a+bx^2}\right)}{2b^4}$$

$$= -\frac{a(2Ab-3aB)x}{b^4} + \frac{(Ab-2aB)x^3}{3b^3} + \frac{Bx^5}{5b^2} - \frac{a^2(Ab-aB)x}{2b^4(a+bx^2)} + \frac{(a^2(5Ab-7aB))\int \frac{}{a}}{2b^4}$$

$$= -\frac{a(2Ab-3aB)x}{b^4} + \frac{(Ab-2aB)x^3}{3b^3} + \frac{Bx^5}{5b^2} - \frac{a^2(Ab-aB)x}{2b^4(a+bx^2)} + \frac{a^{3/2}(5Ab-7aB)\tan}{2b^{9/2}}$$

Mathematica [A]
time = 0.06, size = 111, normalized size = 1.01

$$\frac{a(-2Ab+3aB)x}{b^4} + \frac{(Ab-2aB)x^3}{3b^3} + \frac{Bx^5}{5b^2} - \frac{(a^2Ab-a^3B)\,x}{2b^4(a+bx^2)} - \frac{a^{3/2}(-5Ab+7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*A*b + 3*a*B)*x)/b^4 + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^5)/(5*b^2)
- ((a^2*A*b - a^3*B)*x)/(2*b^4*(a + b*x^2)) - (a^(3/2)*(-5*A*b + 7*a*B)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A]
time = 0.08, size = 100, normalized size = 0.91

method	result
default	$-\frac{-\frac{1}{5}b^2Bx^5-\frac{1}{3}Ab^2x^3+\frac{2}{3}Babx^3+2abAx-3a^2Bx}{b^4}+\frac{a^2\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x}{bx^2+a}+\frac{(5Ab-7Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$
risch	$\frac{Bx^5}{5b^2}+\frac{Ax^3}{3b^2}-\frac{2Bax^3}{3b^3}-\frac{2aAx}{b^3}+\frac{3a^2Bx}{b^4}+\frac{\left(-\frac{1}{2}Aa^2b+\frac{1}{2}Ba^3\right)x}{b^4(bx^2+a)}+\frac{5\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)A}{4b^4}-\frac{7\sqrt{-ab}\,a^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/5*b^2*B*x^5-1/3*A*b^2*x^3+2/3*B*a*b*x^3+2*a*b*A*x-3*a^2*B*x)+a^2
/b^4*((-1/2*A*b+1/2*B*a)*x/(b*x^2+a)+1/2*(5*A*b-7*B*a)/(a*b)^(1/2)*arctan(b
*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 112, normalized size = 1.02

$$\frac{(Ba^3 - Aa^2b)x}{2\,(b^5x^2 + ab^4)} - \frac{(7\,Ba^3 - 5\,Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4} + \frac{3\,Bb^2x^5 - 5\,(2\,Bab - Ab^2)x^3 + 15\,(3\,Ba^2 - 2\,Aab)x}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^3 - A*a^2*b)*x/(b^5*x^2 + a*b^4) - 1/2*(7*B*a^3 - 5*A*a^2*b)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/15*(3*B*b^2*x^5 - 5*(2*B*a*b - A*b^2)*
x^3 + 15*(3*B*a^2 - 2*A*a*b)*x)/b^4
```

Fricas [A]
time = 0.72, size = 298, normalized size = 2.71

$$\left[\frac{12\,Bb^3x^7 - 4\,(7\,Bab^2 - 5\,Ab^3)x^5 + 20\,(7\,Ba^2b - 5\,Aab^2)x^3 - 15\,(7\,Ba^3 - 5\,Aa^2b + (7\,Ba^2b - 5\,Aab^2)x^2)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2 + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 30\,(7\,Ba^3 - 5\,Aa^2b)x}{60\,(b^5x^2 + ab^4)}, \frac{6\,Bb^3x^7 - 2\,(7\,Bab^2 - 5\,Ab^3)x^5 + 10\,(7\,Ba^2b - 5\,Aab^2)x^3 - 15\,(7\,Ba^3 - 5\,Aa^2b + (7\,Ba^2b - 5\,Aab^2)x^2)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 15\,(7\,Ba^3 - 5\,Aa^2b)x}{30\,(b^5x^2 + ab^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*B*b^3*x^7 - 4*(7*B*a*b^2 - 5*A*b^3)*x^5 + 20*(7*B*a^2*b - 5*A*a*b
^2)*x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(-a/b)
*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(7*B*a^3 - 5*A*a^2*b)
*x)/(b^5*x^2 + a*b^4), 1/30*(6*B*b^3*x^7 - 2*(7*B*a*b^2 - 5*A*b^3)*x^5 + 10
*(7*B*a^2*b - 5*A*a*b^2)*x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a
*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*(7*B*a^3 - 5*A*a^2*b)*x)/
(b^5*x^2 + a*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(104) = 208$.
time = 0.48, size = 211, normalized size = 1.92

$$\frac{Bx^5}{5b^2} + x^3\left(\frac{A}{3b^2} - \frac{2Ba}{3b^3}\right) + x\left(-\frac{2Aa}{b^3} + \frac{3Ba^2}{b^4}\right) + \frac{x(-Aa^2b + Ba^3)}{2ab^4 + 2b^5x^2} + \frac{\sqrt{-\frac{a^3}{b^9}}\,(-5Ab + 7Ba)\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}\,(-5Ab+7Ba)}{-5Aab+7Ba^2} + x\right)}{4} - \frac{\sqrt{-\frac{a^3}{b^9}}\,(-5Ab + 7Ba)\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}\,(-5Ab+7Ba)}{-5Aab+7Ba^2} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) + x**3*(A/(3*b**2) - 2*B*a/(3*b**3)) + x*(-2*A*a/b**3 + 3*B
*a**2/b**4) + x*(-A*a**2*b + B*a**3)/(2*a*b**4 + 2*b**5*x**2) + sqrt(-a**3/
b**9)*(-5*A*b + 7*B*a)*log(-b**4*sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)/(-5*A*a*
b + 7*B*a**2) + x)/4 - sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)*log(b**4*sqrt(-a**
3/b**9)*(-5*A*b + 7*B*a)/(-5*A*a*b + 7*B*a**2) + x)/4
```

Giac [A]
time = 1.11, size = 115, normalized size = 1.05

$$-\frac{(7\,Ba^3-5\,Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{Ba^3x-Aa^2bx}{2\,(bx^2+a)b^4}+\frac{3\,Bb^8x^5-10\,Bab^7x^3+5\,Ab^8x^3+45\,Ba^2b^6x-30\,Aab^7x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*(B*a
^3*x - A*a^2*b*x)/((b*x^2 + a)*b^4) + 1/15*(3*B*b^8*x^5 - 10*B*a*b^7*x^3 +
5*A*b^8*x^3 + 45*B*a^2*b^6*x - 30*A*a*b^7*x)/b^10
```

Mupad [B]
time = 0.03, size = 141, normalized size = 1.28

$$x^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right)-x\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)+\frac{B\,x^5}{5\,b^2}+\frac{x\,\left(\frac{B\,a^3}{2}-\frac{A\,a^2\,b}{2}\right)}{b^5\,x^2+a\,b^4}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(5\,A\,b-7\,B\,a)}{7\,B\,a^3-5\,A\,a^2\,b}\right)\,(5\,A\,b-7\,B\,a)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a
^2)/b^4) + (B*x^5)/(5*b^2) + (x*((B*a^3)/2 - (A*a^2*b)/2))/(a*b^4 + b^5*x^2
) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(5*A*b - 7*B*a))/(7*B*a^3 - 5*A*a^2*b)
)*(5*A*b - 7*B*a))/(2*b^(9/2))
```

3.75 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=82

$$\frac{(Ab-2aB)x^2}{2b^3}+\frac{Bx^4}{4b^2}-\frac{a^2(Ab-aB)}{2b^4(a+bx^2)}-\frac{a(2Ab-3aB)\log(a+bx^2)}{2b^4}$$

```
[Out] 1/2*(A*b-2*B*a)*x^2/b^3+1/4*B*x^4/b^2-1/2*a^2*(A*b-B*a)/b^4/(b*x^2+a)-1/2*a
*(2*A*b-3*B*a)*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 78}

$$-\frac{a^2(Ab-aB)}{2b^4(a+bx^2)}-\frac{a(2Ab-3aB)\log(a+bx^2)}{2b^4}+\frac{x^2(Ab-2aB)}{2b^3}+\frac{Bx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x^2)/(2*b^3) + (B*x^4)/(4*b^2) - (a^2*(A*b - a*B))/(2*b^4*(a
 + b*x^2)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab-2aB}{b^3}+\frac{Bx}{b^2}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^2}+\frac{a(-2Ab+3aB)}{b^3(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{(Ab-2aB)x^2}{2b^3}+\frac{Bx^4}{4b^2}-\frac{a^2(Ab-aB)}{2b^4\,(a+bx^2)}-\frac{a(2Ab-3aB)\log\left(a+bx^2\right)}{2b^4}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.88

$$\frac{2b(Ab-2aB)x^2+b^2Bx^4+\frac{2a^2(-Ab+aB)}{a+bx^2}+2a(-2Ab+3aB)\log\left(a+bx^2\right)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*(A*b - 2*a*B)*x^2 + b^2*B*x^4 + (2*a^2*(-(A*b) + a*B))/(a + b*x^2) + 2
*a*(-2*A*b + 3*a*B)*Log[a + b*x^2])/(4*b^4)
```

Maple [A]
time = 0.08, size = 76, normalized size = 0.93

method	result	si
default	$\frac{\left(bB\,x^2+Ab-2Ba\right)^2}{4b^4B}-\frac{a\left(\frac{(2Ab-3Ba)\ln\left(b\,x^2+a\right)}{b}+\frac{a(Ab-Ba)}{b\left(b\,x^2+a\right)}\right)}{2b^3}$	7
norman	$\frac{\frac{B\,x^6}{4b}-\frac{a\left(2abA-3a^2B\right)}{2b^4}+\frac{(2Ab-3Ba)x^4}{4b^2}}{b\,x^2+a}-\frac{a(2Ab-3Ba)\ln\left(b\,x^2+a\right)}{2b^4}$	8
risch	$\frac{B\,x^4}{4b^2}+\frac{A\,x^2}{2b^2}-\frac{Ba\,x^2}{b^3}+\frac{A^2}{4b^2B}-\frac{Aa}{b^3}+\frac{B\,a^2}{b^4}-\frac{a^2A}{2b^3(b\,x^2+a)}+\frac{a^3B}{2b^4(b\,x^2+a)}-\frac{a\ln\left(b\,x^2+a\right)A}{b^3}+\frac{3a^2\ln\left(b\,x^2+a\right)B}{2b^4}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(B*b*x^2+A*b-2*B*a)^2/b^4/B-1/2*a/b^3*((2*A*b-3*B*a)/b*ln(b*x^2+a)+a*(A
*b-B*a)/b/(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 82, normalized size = 1.00

$$\frac{Ba^3-Aa^2b}{2\left(b^5x^2+ab^4\right)}+\frac{Bbx^4-2\left(2\,Ba-Ab\right)x^2}{4\,b^3}+\frac{\left(3\,Ba^2-2\,Aab\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^3 - A*a^2*b)/(b^5*x^2 + a*b^4) + 1/4*(B*b*x^4 - 2*(2*B*a - A*b)*x^
2)/b^3 + 1/2*(3*B*a^2 - 2*A*a*b)*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 0.67, size = 121, normalized size = 1.48

$$\frac{Bb^3x^6-(3\,Bab^2-2\,Ab^3)x^4+2\,Ba^3-2\,Aa^2b-2\,(2\,Ba^2b-Aab^2)x^2+2\,(3\,Ba^3-2\,Aa^2b+(3\,Ba^2b-2\,Aab^2)x^2)\log{(bx^2+a)}}{4\,(b^5x^2+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^3*x^6 - (3*B*a*b^2 - 2*A*b^3)*x^4 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*a
^2*b - A*a*b^2)*x^2 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^2)
*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A]
time = 0.49, size = 78, normalized size = 0.95

$$\frac{Bx^4}{4b^2}+\frac{a(-2Ab+3Ba)\log{(a+bx^2)}}{2b^4}+x^2\left(\frac{A}{2b^2}-\frac{Ba}{b^3}\right)+\frac{-Aa^2b+Ba^3}{2ab^4+2b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**4) + x**2*(A/(2*
b**2) - B*a/b**3) + (-A*a**2*b + B*a**3)/(2*a*b**4 + 2*b**5*x**2)
```

Giac [A]
time = 0.97, size = 106, normalized size = 1.29

$$\frac{(3\,Ba^2-2\,Aab)\log{(|bx^2+a|)}}{2\,b^4}+\frac{Bb^2x^4-4\,Babx^2+2\,Ab^2x^2}{4\,b^4}-\frac{3\,Ba^2bx^2-2\,Aab^2x^2+2\,Ba^3-Aa^2b}{2\,(bx^2+a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^2 + a))/b^4 + 1/4*(B*b^2*x^4 - 4*B*a*b*
x^2 + 2*A*b^2*x^2)/b^4 - 1/2*(3*B*a^2*b*x^2 - 2*A*a*b^2*x^2 + 2*B*a^3 - A*a
^2*b)/((b*x^2 + a)*b^4)
```

Mupad [B]
time = 0.04, size = 86, normalized size = 1.05

$$x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right)+\frac{\ln{(b\,x^2+a)}\,\left(3\,B\,a^2-2\,A\,a\,b\right)}{2\,b^4}+\frac{B\,x^4}{4\,b^2}+\frac{B\,a^3-A\,a^2\,b}{2\,b\,\left(b^4\,x^2+a\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^2*(A/(2*b^2) - (B*a)/b^3) + (log(a + b*x^2)*(3*B*a^2 - 2*A*a*b))/(2*b^4)
+ (B*x^4)/(4*b^2) + (B*a^3 - A*a^2*b)/(2*b*(a*b^3 + b^4*x^2))
```

3.76 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=87

$$\frac{(Ab-2aB)x}{b^3}+\frac{Bx^3}{3b^2}+\frac{a(Ab-aB)x}{2b^3(a+bx^2)}-\frac{\sqrt{a}\,(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

```
[Out] (A*b-2*B*a)*x/b^3+1/3*B*x^3/b^2+1/2*a*(A*b-B*a)*x/b^3/(b*x^2+a)-1/2*(3*A*b-
5*B*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {466, 1167, 211}

$$-\frac{\sqrt{a}\,(3Ab-5aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{ax(Ab-aB)}{2b^3(a+bx^2)}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + (a*(A*b - a*B)*x)/(2*b^3*(a + b*x
^2)) - (Sqrt[a]*(3*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx &= \frac{a(Ab-aB)x}{2b^3(a+bx^2)} - \frac{\int \frac{a(Ab-aB)-2b(Ab-aB)x^2-2b^2Bx^4}{a+bx^2}\,dx}{2b^3}\\
&= \frac{a(Ab-aB)x}{2b^3(a+bx^2)} - \frac{\int \left(-2(Ab-2aB)-2bBx^2+\frac{3aAb-5a^2B}{a+bx^2}\right)dx}{2b^3}\\
&= \frac{(Ab-2aB)x}{b^3} + \frac{Bx^3}{3b^2} + \frac{a(Ab-aB)x}{2b^3(a+bx^2)} - \frac{(a(3Ab-5aB))\int \frac{1}{a+bx^2}\,dx}{2b^3}\\
&= \frac{(Ab-2aB)x}{b^3} + \frac{Bx^3}{3b^2} + \frac{a(Ab-aB)x}{2b^3(a+bx^2)} - \frac{\sqrt{a}\,(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 1.02

$$\frac{(Ab-2aB)x}{b^3} + \frac{Bx^3}{3b^2} + \frac{(aAb-a^2B)\,x}{2b^3(a+bx^2)} + \frac{\sqrt{a}\,(-3Ab+5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + ((a*A*b - a^2*B)*x)/(2*b^3*(a + b
*x^2)) + (Sqrt[a]*(-3*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Maple [A]
time = 0.08, size = 75, normalized size = 0.86

method	result
default	$\frac{\frac{1}{3}bB\,x^3+Abx-2Bax}{b^3} - \frac{a\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(3Ab-5Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$
risch	$\frac{B\,x^3}{3b^2} + \frac{Ax}{b^2} - \frac{2Bax}{b^3} + \frac{\left(\frac{1}{2}abA-\frac{1}{2}a^2B\right)x}{b^3(b\,x^2+a)} + \frac{3\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)A}{4b^3} - \frac{5\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)Ba}{4b^4} - 3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*b*B*x^3+A*b*x-2*B*a*x)-a/b^3*((-1/2*A*b+1/2*B*a)*x/(b*x^2+a)+1/2
*(3*A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.53, size = 85, normalized size = 0.98

$$-\frac{(Ba^2 - Aab)x}{2\,(b^4x^2 + ab^3)} + \frac{(5\,Ba^2 - 3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3} + \frac{Bbx^3 - 3\,(2\,Ba - Ab)x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^2 - A*a*b)*x/(b^4*x^2 + a*b^3) + 1/2*(5*B*a^2 - 3*A*a*b)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/3*(B*b*x^3 - 3*(2*B*a - A*b)*x)/b^3
```

Fricas [A]
time = 0.71, size = 240, normalized size = 2.76

$$\left[\frac{4\,Bb^2x^5 - 4\,(5\,Bab - 3\,Ab^2)x^3 - 3\,(5\,Ba^2 - 3\,Aab + (5\,Bab - 3\,Ab^2)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 6\,(5\,Ba^2 - 3\,Aab)x}{12\,(b^4x^2 + ab^3)}, \frac{2\,Bb^2x^5 - 2\,(5\,Bab - 3\,Ab^2)x^3 + 3\,(5\,Ba^2 - 3\,Aab + (5\,Bab - 3\,Ab^2)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 3\,(5\,Ba^2 - 3\,Aab)x}{6\,(b^4x^2 + ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*B*b^2*x^5 - 4*(5*B*a*b - 3*A*b^2)*x^3 - 3*(5*B*a^2 - 3*A*a*b + (5*
B*a*b - 3*A*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2
+ a)) - 6*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a*b^3), 1/6*(2*B*b^2*x^5 - 2*(5
*B*a*b - 3*A*b^2)*x^3 + 3*(5*B*a^2 - 3*A*a*b + (5*B*a*b - 3*A*b^2)*x^2)*sqr
t(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a*b^3)
]
```

Sympy [A]
time = 0.36, size = 129, normalized size = 1.48

$$\frac{Bx^3}{3b^2} + x\left(\frac{A}{b^2} - \frac{2Ba}{b^3}\right) + \frac{x(Aab - Ba^2)}{2ab^3 + 2b^4x^2} - \frac{\sqrt{-\frac{a}{b^7}}\,(-3Ab + 5Ba)\log\left(-b^3\sqrt{-\frac{a}{b^7}} + x\right)}{4} + \frac{\sqrt{-\frac{a}{b^7}}\,(-3Ab + 5Ba)\log\left(b^3\sqrt{-\frac{a}{b^7}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(2*a*b**3 +
2*b**4*x**2) - sqrt(-a/b**7)*(-3*A*b + 5*B*a)*log(-b**3*sqrt(-a/b**7) + x)/
4 + sqrt(-a/b**7)*(-3*A*b + 5*B*a)*log(b**3*sqrt(-a/b**7) + x)/4
```

Giac [A]
time = 2.02, size = 88, normalized size = 1.01

$$\frac{(5\,Ba^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}-\frac{Ba^2x-Aabx}{2\,(bx^2+a)b^3}+\frac{Bb^4x^3-6\,Bab^3x+3\,Ab^4x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(B*a^2*
x - A*a*b*x)/((b*x^2 + a)*b^3) + 1/3*(B*b^4*x^3 - 6*B*a*b^3*x + 3*A*b^4*x)/
b^6
```

Mupad [B]
time = 0.04, size = 104, normalized size = 1.20

$$x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)-\frac{x\left(\frac{B\,a^2}{2}-\frac{A\,a\,b}{2}\right)}{b^4\,x^2+a\,b^3}+\frac{B\,x^3}{3\,b^2}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(3\,A\,b-5\,B\,a)}{5\,B\,a^2-3\,A\,a\,b}\right)\,(3\,A\,b-5\,B\,a)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/2 - (A*a*b)/2))/(a*b^3 + b^4*x^2) + (
B*x^3)/(3*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*A*b - 5*B*a))/(5*B*a^2
 - 3*A*a*b))*(3*A*b - 5*B*a))/(2*b^(7/2))
```

3.77 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=60

$$\frac{Bx^2}{2b^2}+\frac{a(Ab-aB)}{2b^3(a+bx^2)}+\frac{(Ab-2aB)\log(a+bx^2)}{2b^3}$$

```
[Out] 1/2*B*x^2/b^2+1/2*a*(A*b-B*a)/b^3/(b*x^2+a)+1/2*(A*b-2*B*a)*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{a(Ab-aB)}{2b^3(a+bx^2)}+\frac{(Ab-2aB)\log(a+bx^2)}{2b^3}+\frac{Bx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x^2)/(2*b^2) + (a*(A*b - a*B))/(2*b^3*(a + b*x^2)) + ((A*b - 2*a*B)*Log[
a + b*x^2])/(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{b^2} + \frac{a(-Ab+aB)}{b^2(a+bx)^2} + \frac{Ab-2aB}{b^2(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{Bx^2}{2b^2} + \frac{a(Ab-aB)}{2b^3(a+bx^2)} + \frac{(Ab-2aB)\log(a+bx^2)}{2b^3}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.83

$$\frac{bBx^2 + \frac{a(Ab-aB)}{a+bx^2} + (Ab-2aB)\log(a+bx^2)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (b*B*x^2 + (a*(A*b - a*B))/(a + b*x^2) + (A*b - 2*a*B)*Log[a + b*x^2])/(2*b
^3)
```

Maple [A]
time = 0.07, size = 59, normalized size = 0.98

method	result	size
norman	$\frac{\frac{B x^4}{2b}+\frac{a(Ab-2Ba)}{2b^3}}{b x^2+a}+\frac{(Ab-2Ba)\ln(b x^2+a)}{2b^3}$	57
default	$\frac{B x^2}{2b^2}+\frac{\frac{(Ab-2Ba)\ln(b x^2+a)}{b}+\frac{a(Ab-Ba)}{b(b x^2+a)}}{2b^2}$	59
risch	$\frac{B x^2}{2b^2}+\frac{aA}{2b^2(b x^2+a)}-\frac{a^2 B}{2b^3(b x^2+a)}+\frac{\ln(b x^2+a)A}{2b^2}-\frac{\ln(b x^2+a)Ba}{b^3}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/b^2+1/2/b^2*(1/b*(A*b-2*B*a)*ln(b*x^2+a)+a*(A*b-B*a)/b/(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 60, normalized size = 1.00

$$\frac{Bx^2}{2\,b^2} - \frac{Ba^2 - Aab}{2\,(b^4x^2 + ab^3)} - \frac{(2\,Ba - Ab)\log(bx^2 + a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b^2 - 1/2*(B*a^2 - A*a*b)/(b^4*x^2 + a*b^3) - 1/2*(2*B*a - A*b)*l
og(b*x^2 + a)/b^3
```

Fricas [A]
time = 0.64, size = 81, normalized size = 1.35

$$\frac{Bb^2x^4 + Babx^2 - Ba^2 + Aab - (2\,Ba^2 - Aab + (2\,Bab - Ab^2)x^2)\log{(bx^2 + a)}}{2\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*b^2*x^4 + B*a*b*x^2 - B*a^2 + A*a*b - (2*B*a^2 - A*a*b + (2*B*a*b -
A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

Sympy [A]
time = 0.31, size = 56, normalized size = 0.93

$$\frac{Bx^2}{2b^2} + \frac{Aab - Ba^2}{2ab^3 + 2b^4x^2} - \frac{(-Ab + 2Ba)\log{(a + bx^2)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**2/(2*b**2) + (A*a*b - B*a**2)/(2*a*b**3 + 2*b**4*x**2) - (-A*b + 2*B*a
)*log(a + b*x**2)/(2*b**3)
```

Giac [A]
time = 1.62, size = 91, normalized size = 1.52

$$\frac{\frac{(bx^2+a)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^2+a} - \frac{Aab^2}{bx^2+a}}{b^3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*ab
s(b)))/b^2 - (B*a^2*b/(b*x^2 + a) - A*a*b^2/(b*x^2 + a))/b^3)/b
```

Mupad [B]
time = 0.04, size = 62, normalized size = 1.03

$$\frac{B\,x^2}{2\,b^2} + \frac{\ln{(b\,x^2 + a)}\,\left(A\,b - 2\,B\,a\right)}{2\,b^3} - \frac{B\,a^2 - A\,a\,b}{2\,b\,\left(b^3\,x^2 + a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (B*x^2)/(2*b^2) + (log(a + b*x^2)*(A*b - 2*B*a))/(2*b^3) - (B*a^2 - A*a*b)/
(2*b*(a*b^2 + b^3*x^2))
```

3.78 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=67

$$\frac{Bx}{b^2}-\frac{(Ab-aB)x}{2b^2\left(a+bx^2\right)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}$$

```
[Out] B*x/b^2-1/2*(A*b-B*a)*x/b^2/(b*x^2+a)+1/2*(A*b-3*B*a)*arctan(x*b^(1/2)/a^(1
/2))/b^(5/2)/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {466, 396, 211}

$$\frac{(Ab-3aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}-\frac{x(Ab-aB)}{2b^2\left(a+bx^2\right)}+\frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) + ((A*b - 3*a*B)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
```

```
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx &= -\frac{(Ab-aB)x}{2b^2(a+bx^2)}-\frac{\int \frac{-Ab+aB-2bBx^2}{a+bx^2}\,dx}{2b^2}\\ &= \frac{Bx}{b^2}-\frac{(Ab-aB)x}{2b^2(a+bx^2)}+\frac{(Ab-3aB)\int \frac{1}{a+bx^2}\,dx}{2b^2}\\ &= \frac{Bx}{b^2}-\frac{(Ab-aB)x}{2b^2(a+bx^2)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 1.01

$$\frac{Bx}{b^2}-\frac{(Ab-aB)x}{2b^2(a+bx^2)}-\frac{(-Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) - ((-(A*b) + 3*a*B)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Maple [A]
time = 0.08, size = 57, normalized size = 0.85

method	result
default	$\frac{Bx}{b^2}+\frac{\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(Ab-3Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$
risch	$\frac{Bx}{b^2}+\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x}{b^2(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)A}{4b\sqrt{-ab}}+\frac{3\ln\left(bx+\sqrt{-ab}\right)Ba}{4b^2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)A}{4b\sqrt{-ab}}-\frac{3\ln\left(-bx+\sqrt{-ab}\right)Ba}{4b^2\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b^2+1/b^2*((-1/2*A*b+1/2*B*a)*x/(b*x^2+a)+1/2*(A*b-3*B*a)/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 61, normalized size = 0.91

$$\frac{(Ba-Ab)x}{2\,(b^3x^2+ab^2)}+\frac{Bx}{b^2}-\frac{(3\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)*x/(b^3*x^2 + a*b^2) + B*x/b^2 - 1/2*(3*B*a - A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.67, size = 208, normalized size = 3.10

$$\left[\frac{4\,Bab^2x^3+(3\,Ba^2-Aab+(3\,Bab-Ab^2)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,(3\,Ba^2b-Aab^2)x}{4\,(ab^4x^2+a^2b^3)},\ \frac{2\,Bab^2x^3-(3\,Ba^2-Aab+(3\,Bab-Ab^2)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(3\,Ba^2b-Aab^2)x}{2\,(ab^4x^2+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*B*a*b^2*x^3 + (3*B*a^2 - A*a*b + (3*B*a*b - A*b^2)*x^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*B*a^2*b - A*a*b^2)*x)/
(a*b^4*x^2 + a^2*b^3), 1/2*(2*B*a*b^2*x^3 - (3*B*a^2 - A*a*b + (3*B*a*b - A
*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*B*a^2*b - A*a*b^2)*x)/(a*b^
4*x^2 + a^2*b^3)]
```

Sympy [A]
time = 0.28, size = 114, normalized size = 1.70

$$\frac{Bx}{b^2}+\frac{x(-Ab+Ba)}{2ab^2+2b^3x^2}+\frac{\sqrt{-\frac{1}{ab^5}}\,(-Ab+3Ba)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^5}}\,(-Ab+3Ba)\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x/b**2 + x*(-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b**5))*(-A*
b + 3*B*a)*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(-1/(a*b**5))*(-A*b +
 3*B*a)*log(a*b**2*sqrt(-1/(a*b**5)) + x)/4
```

Giac [A]
time = 1.56, size = 59, normalized size = 0.88

$$\frac{Bx}{b^2}-\frac{(3\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{Bax-Abx}{2\,(bx^2+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] B*x/b^2 - 1/2*(3*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(B*
a*x - A*b*x)/((b*x^2 + a)*b^2)
```

Mupad [B]
time = 0.07, size = 59, normalized size = 0.88

$$\frac{B\,x}{b^2}-\frac{x\,\left(\frac{A\,b}{2}-\frac{B\,a}{2}\right)}{b^3\,x^2+a\,b^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-3\,B\,a\right)}{2\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (B*x)/b^2 - (x*((A*b)/2 - (B*a)/2))/(a*b^2 + b^3*x^2) + (atan((b^(1/2)*x)/a
^(1/2))*(A*b - 3*B*a))/(2*a^(1/2)*b^(5/2))
```

3.79 $\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=41

$$-\frac{Ab-aB}{2b^2(a+bx^2)}+\frac{B\log(a+bx^2)}{2b^2}$$

```
[Out] 1/2*(-A*b+B*a)/b^2/(b*x^2+a)+1/2*B*ln(b*x^2+a)/b^2
```

Rubi [A]

time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {455, 45}

$$\frac{B\log(a+bx^2)}{2b^2}-\frac{Ab-aB}{2b^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(A*b - a*B)/(b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)\,dx,x,x^2\right)\\
&= -\frac{Ab-aB}{2b^2(a+bx^2)}+\frac{B\log(a+bx^2)}{2b^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{-Ab+aB}{2b^2\left(a+bx^2\right)}+\frac{B\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (-(A*b) + a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.06, size = 38, normalized size = 0.93

method	result	size
default	$\frac{B\ln\left(b\,x^{2}+a\right)}{2b^{2}}-\frac{Ab-Ba}{2b^{2}\left(b\,x^{2}+a\right)}$	38
norman	$\frac{B\ln\left(b\,x^{2}+a\right)}{2b^{2}}-\frac{Ab-Ba}{2b^{2}\left(b\,x^{2}+a\right)}$	38
risch	$\frac{B\ln\left(b\,x^{2}+a\right)}{2b^{2}}-\frac{A}{2b\left(b\,x^{2}+a\right)}+\frac{Ba}{2b^{2}\left(b\,x^{2}+a\right)}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*ln(b*x^2+a)/b^2-1/2/b^2*(A*b-B*a)/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.98

$$\frac{Ba-Ab}{2\left(b^3x^2+ab^2\right)}+\frac{B\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)/(b^3*x^2 + a*b^2) + 1/2*B*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 0.72, size = 44, normalized size = 1.07

$$\frac{Ba-Ab+\left(Bbx^2+Ba\right)\log\left(bx^2+a\right)}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*a - A*b + (B*b*x^2 + B*a)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

Sympy [A]
time = 0.23, size = 36, normalized size = 0.88

$$\frac{B\log\left(a+bx^2\right)}{2b^2}+\frac{-Ab+Ba}{2ab^2+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*log(a + b*x**2)/(2*b**2) + (-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2)
```

Giac [A]
time = 1.29, size = 65, normalized size = 1.59

$$-\frac{B\left(\frac{\log\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{b}-\frac{a}{(bx^2+a)b}\right)}{2\,b}-\frac{A}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*B*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
 - 1/2*A/((b*x^2 + a)*b)
```

Mupad [B]
time = 0.05, size = 37, normalized size = 0.90

$$\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^2}-\frac{A\,b-B\,a}{2\,b^2\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (B*log(a + b*x^2))/(2*b^2) - (A*b - B*a)/(2*b^2*(a + b*x^2))
```

3.80 $\int \frac{A+Bx^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=63

$$\frac{(Ab-aB)x}{2ab\left(a+bx^2\right)}+\frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

```
[Out] 1/2*(A*b-B*a)*x/a/b/(b*x^2+a)+1/2*(A*b+B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/
2)/b^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {393, 211}

$$\frac{(aB+Ab)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aB)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{(Ab-aB)x}{2ab\,(a+bx^2)} + \frac{(Ab+aB)\int \frac{1}{a+bx^2}\,dx}{2ab}$$
$$= \frac{(Ab-aB)x}{2ab\,(a+bx^2)} + \frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.00

$$-\frac{(-Ab+aB)x}{2ab\,(a+bx^2)} + \frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((-(A*b) + a*B)*x)/(a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.90

method	result	size
default	$\frac{(Ab-Ba)x}{2ab(b\,x^2+a)} + \frac{(Ab+Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	57
risch	$\frac{(Ab-Ba)x}{2ab(b\,x^2+a)} - \frac{\ln\left(bx+\sqrt{-ab}\right)A}{4\sqrt{-ab}\,a} - \frac{\ln\left(bx+\sqrt{-ab}\right)B}{4\sqrt{-ab}\,b} + \frac{\ln\left(-bx+\sqrt{-ab}\right)A}{4\sqrt{-ab}\,a} + \frac{\ln\left(-bx+\sqrt{-ab}\right)B}{4\sqrt{-ab}\,b}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*b-B*a)*x/a/b/(b*x^2+a)+1/2*(A*b+B*a)/a/b/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))
```

Maxima [A]
time = 0.50, size = 57, normalized size = 0.90

$$-\frac{(Ba-Ab)x}{2\,(ab^2x^2+a^2b)} + \frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)*x/(a*b^2*x^2 + a^2*b) + 1/2*(B*a + A*b)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.63, size = 182, normalized size = 2.89

$$\left[-\frac{\left(Ba^2 + Aab + (Bab + Ab^2)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 2\left(Ba^2b - Aab^2\right)x}{4\left(a^2b^3x^2 + a^3b^2\right)}, \frac{\left(Ba^2 + Aab + (Bab + Ab^2)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - \left(Ba^2b - Aab^2\right)x}{2\left(a^2b^3x^2 + a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) + 2*(B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*b^
2), 1/2*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) - (B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.
time = 0.22, size = 112, normalized size = 1.78

$$\frac{x(Ab - Ba)}{2a^2b + 2ab^2x^2} - \frac{\sqrt{-\frac{1}{a^3b^3}}\,(Ab + Ba)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}\,(Ab + Ba)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] x*(A*b - B*a)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(A*b + B*a)
*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(A*b + B*a)
*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

Giac [A]
time = 1.84, size = 57, normalized size = 0.90

$$\frac{(Ba + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab} - \frac{Bax - Abx}{2\left(bx^2 + a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(B*a*x - A*b*x)
/((b*x^2 + a)*a*b)
```

Mupad [B]
time = 0.06, size = 51, normalized size = 0.81

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b+B\,a\right)}{2\,a^{3/2}\,b^{3/2}}+\frac{x\,\left(A\,b-B\,a\right)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^2,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(A*b + B*a))/(2*a^(3/2)*b^(3/2)) + (x*(A*b - B*a
))/(2*a*b*(a + b*x^2))
```

3.81 $\int \frac{A+Bx^2}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=51

$$\frac{Ab-aB}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] 1/2*(A*b-B*a)/a/b/(b*x^2+a)+A*ln(x)/a^2-1/2*A*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (A*b - a*B)/(2*a*b*(a + b*x^2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/(2*a^
2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^2x}+\frac{-Ab+aB}{a(a+bx)^2}-\frac{Ab}{a^2(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{Ab-aB}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.90

$$\frac{\frac{a(Ab-aB)}{b(a+bx^2)}+2A\log(x)-A\log\left(a+bx^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b - a*B))/(b*(a + b*x^2)) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2)
```

Maple [A]
time = 0.07, size = 48, normalized size = 0.94

method	result	size
default	$-\frac{A\ln\left(b\,x^2+a\right)-\frac{a(Ab-Ba)}{b\left(b\,x^2+a\right)}}{2a^2}+\frac{A\ln(x)}{a^2}$	48
norman	$-\frac{(Ab-Ba)x^2}{2a^2(b\,x^2+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^2+a\right)}{2a^2}$	48
risch	$\frac{A}{2a(b\,x^2+a)}-\frac{B}{2b(b\,x^2+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^2+a\right)}{2a^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*(A*ln(b*x^2+a)-a*(A*b-B*a)/b/(b*x^2+a))+A*ln(x)/a^2
```

Maxima [A]
time = 0.33, size = 51, normalized size = 1.00

$$-\frac{Ba-Ab}{2\left(ab^2x^2+a^2b\right)}-\frac{A\log\left(bx^2+a\right)}{2\,a^2}+\frac{A\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a)/a^2 + 1/2*A*log
(x^2)/a^2
```

Fricas [A]
time = 0.83, size = 70, normalized size = 1.37

$$-\frac{Ba^2 - Aab + (Ab^2x^2 + Aab)\log(bx^2 + a) - 2(Ab^2x^2 + Aab)\log(x)}{2(a^2b^2x^2 + a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(B*a^2 - A*a*b + (A*b^2*x^2 + A*a*b)*log(b*x^2 + a) - 2*(A*b^2*x^2 + A
*a*b)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

Sympy [A]
time = 0.33, size = 46, normalized size = 0.90

$$\frac{A\log(x)}{a^2} - \frac{A\log\left(\frac{a}{b} + x^2\right)}{2a^2} + \frac{Ab - Ba}{2a^2b + 2ab^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**2,x)
```

```
[Out] A*log(x)/a**2 - A*log(a/b + x**2)/(2*a**2) + (A*b - B*a)/(2*a**2*b + 2*a*b*
*2*x**2)
```

Giac [A]
time = 0.92, size = 63, normalized size = 1.24

$$\frac{A\log(x^2)}{2\,a^2} - \frac{A\log(|bx^2 + a|)}{2\,a^2} + \frac{Ab^2x^2 - Ba^2 + 2\,Aab}{2\,(bx^2 + a)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/a^2 - 1/2*A*log(abs(b*x^2 + a))/a^2 + 1/2*(A*b^2*x^2 - B*a^2
 + 2*A*a*b)/((b*x^2 + a)*a^2*b)
```

Mupad [B]
time = 0.09, size = 47, normalized size = 0.92

$$\frac{A\,\ln(x)}{a^2} - \frac{A\,\ln(b\,x^2 + a)}{2\,a^2} + \frac{A\,b - B\,a}{2\,a\,b\,(b\,x^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^2),x)
```

```
[Out] (A*log(x))/a^2 - (A*log(a + b*x^2))/(2*a^2) + (A*b - B*a)/(2*a*b*(a + b*x^2
))
```

3.82 $\int \frac{A+Bx^2}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=71

$$-\frac{A}{a^2x}-\frac{(Ab-aB)x}{2a^2(a+bx^2)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

```
[Out] -A/a^2/x-1/2*(A*b-B*a)*x/a^2/(b*x^2+a)-1/2*(3*A*b-B*a)*arctan(x*b^(1/2)/a^(
1/2))/a^(5/2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {467, 464, 211}

$$-\frac{(3Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{x(Ab-aB)}{2a^2(a+bx^2)}-\frac{A}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) - ((A*b - a*B)*x)/(2*a^2*(a + b*x^2)) - ((3*A*b - a*B)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^2}\,dx &= -\frac{(Ab-aB)x}{2a^2\left(a+bx^2\right)}-\frac{1}{2}\int \frac{-\frac{2A}{a}+\frac{(Ab-aB)x^2}{a^2}}{x^2\left(a+bx^2\right)}\,dx\\ &= -\frac{A}{a^2x}-\frac{(Ab-aB)x}{2a^2\left(a+bx^2\right)}-\frac{(3Ab-aB)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\ &= -\frac{A}{a^2x}-\frac{(Ab-aB)x}{2a^2\left(a+bx^2\right)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 0.99

$$-\frac{A}{a^2x}+\frac{(-Ab+aB)x}{2a^2\left(a+bx^2\right)}+\frac{(-3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + ((-(A*b) + a*B)*x)/(2*a^2*(a + b*x^2)) + ((-3*A*b + a*B)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Maple [A]
time = 0.09, size = 62, normalized size = 0.87

method	result
default	$-\frac{\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(3Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^2}-\frac{A}{a^2x}$
risch	$\frac{-\frac{(3Ab-Ba)x^2}{2a^2}-\frac{A}{a}}{x(b\,x^2+a)}-\frac{3\ln\left(-\sqrt{-ab}\,x-a\right)Ab}{4\sqrt{-ab}\,a^2}+\frac{\ln\left(-\sqrt{-ab}\,x-a\right)B}{4\sqrt{-ab}\,a}+\frac{3\ln\left(-\sqrt{-ab}\,x+a\right)Ab}{4\sqrt{-ab}\,a^2}-\frac{\ln\left(-\sqrt{-ab}\,x+a\right)}{4\sqrt{-ab}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/2*A*b-1/2*B*a)*x/(b*x^2+a)+1/2*(3*A*b-B*a)/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2)))-A/a^2/x
```

Maxima [A]
time = 0.51, size = 63, normalized size = 0.89

$$\frac{(Ba-3\,Ab)x^2-2\,Aa}{2\,(a^2bx^3+a^3x)}+\frac{(Ba-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*a - 3*A*b)*x^2 - 2*A*a)/(a^2*b*x^3 + a^3*x) + 1/2*(B*a - 3*A*b)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 1.18, size = 210, normalized size = 2.96

$$\left[-\frac{4\,Aa^2b-2\,(Ba^2b-3\,Aab^2)x^2-((Bab-3\,Ab^2)x^3+(Ba^2-3\,Aab)x)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,(a^3b^2x^3+a^4bx)},-\frac{2\,Aa^2b-(Ba^2b-3\,Aab^2)x^2-((Bab-3\,Ab^2)x^3+(Ba^2-3\,Aab)x)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(a^3b^2x^3+a^4bx)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*a^2*b - 2*(B*a^2*b - 3*A*a*b^2)*x^2 - ((B*a*b - 3*A*b^2)*x^3 + (
B*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^2*b - (B*a^2*b - 3*A*a*b^2)*x^2 - (
(B*a*b - 3*A*b^2)*x^3 + (B*a^2 - 3*A*a*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
))/(a^3*b^2*x^3 + a^4*b*x)]
```

Sympy [A]
time = 0.34, size = 114, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(-3Ab+Ba)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b}}\,(-3Ab+Ba)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{-2Aa+x^2(-3Ab+Ba)}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b))*(-3*A*b + B*a)*log(-a**3*sqrt(-1/(a**5*b)) + x)/4 + sqrt
(-1/(a**5*b))*(-3*A*b + B*a)*log(a**3*sqrt(-1/(a**5*b)) + x)/4 + (-2*A*a +
x**2*(-3*A*b + B*a))/(2*a**3*x + 2*a**2*b*x**3)
```

Giac [A]
time = 1.23, size = 62, normalized size = 0.87

$$\frac{(Ba-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}+\frac{Bax^2-3\,Abx^2-2\,Aa}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(B*a*x^2 - 3*
A*b*x^2 - 2*A*a)/((b*x^3 + a*x)*a^2)
```

Mupad [B]
time = 0.07, size = 63, normalized size = 0.89

$$-\frac{\frac{A}{a}+\frac{x^2\,\left(3\,A\,b-B\,a\right)}{2\,a^2}}{b\,x^3+a\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,A\,b-B\,a\right)}{2\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] - (A/a + (x^2*(3*A*b - B*a))/(2*a^2))/(a*x + b*x^3) - (atan((b^(1/2)*x)/a^(
1/2))*(3*A*b - B*a))/(2*a^(5/2)*b^(1/2))
```

3.83 $\int \frac{A+Bx^2}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=76

$$-\frac{A}{2a^2x^2}-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}$$

```
[Out] -1/2*A/a^2/x^2+1/2*(-A*b+B*a)/a^2/(b*x^2+a)-(2*A*b-B*a)*ln(x)/a^3+1/2*(2*A*
b-B*a)*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}-\frac{A}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*A/(a^2*x^2) - (A*b - a*B)/(2*a^2*(a + b*x^2)) - ((2*A*b - a*B)*Log[x])
/a^3 + ((2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^2x^2}+\frac{-2Ab+aB}{a^3x}-\frac{b(-Ab+aB)}{a^2(a+bx)^2}-\frac{b(-2Ab+aB)}{a^3(a+bx)}\right)\,dx,x,x^2\right)$$
$$= -\frac{A}{2a^2x^2}-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.84

$$\frac{-\frac{aA}{x^2}+\frac{a(-Ab+aB)}{a+bx^2}+2(-2Ab+aB)\log(x)+(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) + (a*(-(A*b) + a*B))/(a + b*x^2) + 2*(-2*A*b + a*B)*Log[x] +
(2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Maple [A]
time = 0.07, size = 76, normalized size = 1.00

method	result	size
default	$\frac{b\left(\frac{(2Ab-Ba)\ln\left(b\,x^2+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(b\,x^2+a\right)}\right)}{2a^3}-\frac{A}{2a^2x^2}+\frac{(-2Ab+Ba)\ln(x)}{a^3}$	76
norman	$\frac{-\frac{A}{2a}+\frac{b(2Ab-Ba)x^4}{2a^3}}{x^2(b\,x^2+a)}-\frac{(2Ab-Ba)\ln(x)}{a^3}+\frac{(2Ab-Ba)\ln(b\,x^2+a)}{2a^3}$	78
risch	$\frac{-\frac{(2Ab-Ba)x^2}{2a^2}-\frac{A}{2a}}{x^2(b\,x^2+a)}-\frac{2\ln(x)Ab}{a^3}+\frac{\ln(x)B}{a^2}+\frac{\ln(-b\,x^2-a)Ab}{a^3}-\frac{\ln(-b\,x^2-a)B}{2a^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^3*b*((2*A*b-B*a)/b*ln(b*x^2+a)-a*(A*b-B*a)/b/(b*x^2+a))-1/2*A/a^2/x^2
+(-2*A*b+B*a)/a^3*ln(x)
```

Maxima [A]
time = 0.30, size = 76, normalized size = 1.00

$$\frac{(Ba-2\,Ab)x^2-Aa}{2\left(a^2bx^4+a^3x^2\right)}-\frac{(Ba-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3}+\frac{(Ba-2\,Ab)\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a^3*x^2) - 1/2*(B*a - 2*A*b)*log
(b*x^2 + a)/a^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

Fricas [A]
time = 1.28, size = 117, normalized size = 1.54

$$-\frac{Aa^2 - (Ba^2 - 2\,Aab)x^2 + ((Bab - 2\,Ab^2)x^4 + (Ba^2 - 2\,Aab)x^2)\log(bx^2 + a) - 2\,((Bab - 2\,Ab^2)x^4 + (Ba^2 - 2\,Aab)x^2)\log(x)}{2\,(a^3bx^4 + a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(A*a^2 - (B*a^2 - 2*A*a*b)*x^2 + ((B*a*b - 2*A*b^2)*x^4 + (B*a^2 - 2*A
*a*b)*x^2)*log(b*x^2 + a) - 2*((B*a*b - 2*A*b^2)*x^4 + (B*a^2 - 2*A*a*b)*x^
2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A]
time = 0.56, size = 70, normalized size = 0.92

$$\frac{-Aa + x^2(-2Ab + Ba)}{2a^3x^2 + 2a^2bx^4} + \frac{(-2Ab + Ba)\log(x)}{a^3} - \frac{(-2Ab + Ba)\log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-A*a + x**2*(-2*A*b + B*a))/(2*a**3*x**2 + 2*a**2*b*x**4) + (-2*A*b + B*a)
*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

Giac [A]
time = 1.40, size = 82, normalized size = 1.08

$$\frac{(Ba - 2\,Ab)\log(x^2)}{2\,a^3} + \frac{Bax^2 - 2\,Abx^2 - Aa}{2\,(bx^4 + ax^2)a^2} - \frac{(Bab - 2\,Ab^2)\log(|bx^2 + a|)}{2\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 2*A*b)*log(x^2)/a^3 + 1/2*(B*a*x^2 - 2*A*b*x^2 - A*a)/((b*x^4 +
a*x^2)*a^2) - 1/2*(B*a*b - 2*A*b^2)*log(abs(b*x^2 + a))/(a^3*b)
```

Mupad [B]
time = 0.07, size = 78, normalized size = 1.03

$$\frac{\ln\left(b\,x^2 + a\right)\,\left(2\,A\,b - B\,a\right)}{2\,a^3} - \frac{\frac{A}{2\,a} + \frac{x^2\,\left(2\,A\,b - B\,a\right)}{2\,a^2}}{b\,x^4 + a\,x^2} - \frac{\ln\left(x\right)\,\left(2\,A\,b - B\,a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (log(a + b*x^2)*(2*A*b - B*a))/(2*a^3) - (A/(2*a) + (x^2*(2*A*b - B*a))/(2*
a^2))/(a*x^2 + b*x^4) - (log(x)*(2*A*b - B*a))/a^3
```

3.84 $\int \frac{A+Bx^2}{x^4(a+bx^2)^2} \, dx$

Optimal. Leaf size=90

$$-\frac{A}{3a^2x^3}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)}+\frac{\sqrt{b}\,(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] -1/3*A/a^2/x^3+(2*A*b-B*a)/a^3/x+1/2*b*(A*b-B*a)*x/a^3/(b*x^2+a)+1/2*(5*A*b
-3*B*a)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(7/2)
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {467, 1275, 211}

$$\frac{\sqrt{b}\,(5Ab-3aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{bx(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{a^3x}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*A/(a^2*x^3) + (2*A*b - a*B)/(a^3*x) + (b*(A*b - a*B)*x)/(2*a^3*(a + b*
x^2)) + (Sqrt[b]*(5*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^2}\,dx &= \frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)}-\frac{1}{2}b\int \frac{-\frac{2A}{ab}+\frac{2(Ab-aB)x^2}{a^2b}-\frac{(Ab-aB)x^4}{a^3}}{x^4\left(a+bx^2\right)}\,dx\\
&= \frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)}-\frac{1}{2}b\int \left(-\frac{2A}{a^2bx^4}-\frac{2(-2Ab+aB)}{a^3bx^2}+\frac{-5Ab+3aB}{a^3\left(a+bx^2\right)}\right)dx\\
&= -\frac{A}{3a^2x^3}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)}+\frac{(b(5Ab-3aB))\int \frac{1}{a+bx^2}\,dx}{2a^3}\\
&= -\frac{A}{3a^2x^3}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)}+\frac{\sqrt{b}\,(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 90, normalized size = 1.00

$$-\frac{A}{3a^2x^3}+\frac{2Ab-aB}{a^3x}-\frac{b(-Ab+aB)x}{2a^3\left(a+bx^2\right)}-\frac{\sqrt{b}\,(-5Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*A/(a^2*x^3) + (2*A*b - a*B)/(a^3*x) - (b*(-(A*b) + a*B)*x)/(2*a^3*(a +
 b*x^2)) - (Sqrt[b]*(-5*A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2
))
```

Maple [A]
time = 0.08, size = 78, normalized size = 0.87

method	result
default	$\frac{b\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(5Ab-3Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{A}{3a^2x^3}-\frac{-2Ab+Ba}{a^3x}$
risch	$\frac{\frac{b(5Ab-3Ba)x^4}{2a^3}+\frac{(5Ab-3Ba)x^2}{3a^2}-\frac{A}{3a}}{x^3(b\,x^2+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^7\_Z^2+25A^2b^3-30ABa\,b^2+9B^2a^2b\right)}\_R\ln\left(\left(3\_R^2a^7+50A^2b^3-60ABa\,b^2+18\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*b*((1/2*A*b-1/2*B*a)*x/(b*x^2+a)+1/2*(5*A*b-3*B*a)/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2)))-1/3*A/a^2/x^3-(-2*A*b+B*a)/a^3/x
```

Maxima [A]
time = 0.51, size = 93, normalized size = 1.03

$$-\frac{3\,(3\,Bab-5\,Ab^{2})x^{4}+2\,Aa^{2}+2\,(3\,Ba^{2}-5\,Aab)x^{2}}{6\,(a^{3}bx^{5}+a^{4}x^{3})}-\frac{(3\,Bab-5\,Ab^{2})\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*B*a*b - 5*A*b^2)*x^4 + 2*A*a^2 + 2*(3*B*a^2 - 5*A*a*b)*x^2)/(a^3
*b*x^5 + a^4*x^3) - 1/2*(3*B*a*b - 5*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^3)
```

Fricas [A]
time = 1.22, size = 250, normalized size = 2.78

$$\left[-\frac{6\,(3\,Bab-5\,Ab^{2})x^{4}+4\,Aa^{2}+4\,(3\,Ba^{2}-5\,Aab)x^{2}+3\,((3\,Bab-5\,Ab^{2})x^{5}+(3\,Ba^{2}-5\,Aab)x^{3})\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{12\,(a^{3}bx^{5}+a^{4}x^{3})},\ -\frac{3\,(3\,Bab-5\,Ab^{2})x^{4}+2\,Aa^{2}+2\,(3\,Ba^{2}-5\,Aab)x^{2}+3\,((3\,Bab-5\,Ab^{2})x^{5}+(3\,Ba^{2}-5\,Aab)x^{3})\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{6\,(a^{3}bx^{5}+a^{4}x^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(3*B*a*b - 5*A*b^2)*x^4 + 4*A*a^2 + 4*(3*B*a^2 - 5*A*a*b)*x^2 + 3
*((3*B*a*b - 5*A*b^2)*x^5 + (3*B*a^2 - 5*A*a*b)*x^3)*sqrt(-b/a)*log((b*x^2
+ 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*x^5 + a^4*x^3), -1/6*(3*(3*B*a
*b - 5*A*b^2)*x^4 + 2*A*a^2 + 2*(3*B*a^2 - 5*A*a*b)*x^2 + 3*((3*B*a*b - 5*A
*b^2)*x^5 + (3*B*a^2 - 5*A*a*b)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^3*b*
x^5 + a^4*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(82) = 164.
time = 0.31, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{b}{a^{7}}}\,(-5Ab+3Ba)\log\left(-\frac{a^{4}\sqrt{-\frac{b}{a^{7}}}\,(-5Ab+3Ba)}{-5Ab^{2}+3Bab}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^{7}}}\,(-5Ab+3Ba)\log\left(\frac{a^{4}\sqrt{-\frac{b}{a^{7}}}\,(-5Ab+3Ba)}{-5Ab^{2}+3Bab}+x\right)}{4}+\frac{-2Aa^{2}+x^{4}\cdot(15Ab^{2}-9Bab)+x^{2}\cdot(10Aab-6Ba^{2})}{6a^{4}x^{3}+6a^{3}bx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**7)*(-5*A*b + 3*B*a)*log(-a**4*sqrt(-b/a**7)*(-5*A*b + 3*B*a)/(-5
*A*b**2 + 3*B*a*b) + x)/4 - sqrt(-b/a**7)*(-5*A*b + 3*B*a)*log(a**4*sqrt(-b
/a**7)*(-5*A*b + 3*B*a)/(-5*A*b**2 + 3*B*a*b) + x)/4 + (-2*A*a**2 + x**4*(1
5*A*b**2 - 9*B*a*b) + x**2*(10*A*a*b - 6*B*a**2))/(6*a**4*x**3 + 6*a**3*b*x
**5)
```

Giac [A]
time = 1.17, size = 85, normalized size = 0.94

$$-\frac{(3\,Bab-5\,Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}-\frac{Babx-Ab^2x}{2\,(bx^2+a)a^3}-\frac{3\,Bax^2-6\,Abx^2+Aa}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*B*a*b - 5*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(B*a*b
*x - A*b^2*x)/((b*x^2 + a)*a^3) - 1/3*(3*B*a*x^2 - 6*A*b*x^2 + A*a)/(a^3*x^
3)
```

Mupad [B]
time = 0.09, size = 83, normalized size = 0.92

$$\frac{\frac{x^2\,(5\,A\,b-3\,B\,a)}{3\,a^2}-\frac{A}{3\,a}+\frac{b\,x^4\,(5\,A\,b-3\,B\,a)}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(5\,A\,b-3\,B\,a)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(5*A*b - 3*B*a))/(3*a^2) - A/(3*a) + (b*x^4*(5*A*b - 3*B*a))/(2*a^3))
/(a*x^3 + b*x^5) + (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(5*A*b - 3*B*a))/(2*a
^(7/2))
```

3.85 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{A}{4a^2x^4}+\frac{2Ab-aB}{2a^3x^2}+\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}$$

```
[Out] -1/4*A/a^2/x^4+1/2*(2*A*b-B*a)/a^3/x^2+1/2*b*(A*b-B*a)/a^3/(b*x^2+a)+b*(3*A
*b-2*B*a)*ln(x)/a^4-1/2*b*(3*A*b-2*B*a)*ln(b*x^2+a)/a^4
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}+\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{2a^3x^2}-\frac{A}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -1/4*A/(a^2*x^4) + (2*A*b - a*B)/(2*a^3*x^2) + (b*(A*b - a*B))/(2*a^3*(a +
b*x^2)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^2
])/(2*a^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^2x^3}+\frac{-2Ab+aB}{a^3x^2}-\frac{b(-3Ab+2aB)}{a^4x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^2}+\frac{b^2(-3Ab+2\right.\right.$$
$$= -\frac{A}{4a^2x^4}+\frac{2Ab-aB}{2a^3x^2}+\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a\right.}{2a^4}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 0.88

$$-\frac{\frac{a^2A}{x^4}+\frac{2a(-2Ab+aB)}{x^2}+\frac{2ab(-Ab+aB)}{a+bx^2}-4b(3Ab-2aB)\log(x)+2b(3Ab-2aB)\log\left(a+bx^2\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -1/4*((a^2*A)/x^4 + (2*a*(-2*A*b + a*B))/x^2 + (2*a*b*(-(A*b) + a*B))/(a +
b*x^2) - 4*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^2])/a
^4
```

Maple [A]
time = 0.08, size = 96, normalized size = 0.99

method	result	size
default	$-\frac{b^2\left(\frac{(3Ab-2Ba)\ln\left(b\,x^2+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(b\,x^2+a\right)}\right)}{2a^4}-\frac{A}{4a^2x^4}-\frac{-2Ab+Ba}{2a^3x^2}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}$	96
norman	$\frac{-\frac{A}{4a}+\frac{(3Ab-2Ba)x^2}{4a^2}-\frac{b\left(3b^2A-2abB\right)x^6}{2a^4}}{x^4(b\,x^2+a)}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}-\frac{b(3Ab-2Ba)\ln(b\,x^2+a)}{2a^4}$	99
risch	$\frac{\frac{b(3Ab-2Ba)x^4}{2a^3}+\frac{(3Ab-2Ba)x^2}{4a^2}-\frac{A}{4a}}{x^4(b\,x^2+a)}+\frac{3b^2\ln(x)A}{a^4}-\frac{2b\ln(x)B}{a^3}-\frac{3b^2\ln(b\,x^2+a)A}{2a^4}+\frac{b\ln(b\,x^2+a)B}{a^3}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^4*b^2*((3*A*b-2*B*a)/b*ln(b*x^2+a)-a*(A*b-B*a)/b/(b*x^2+a))-1/4*A/a^
2/x^4-1/2*(-2*A*b+B*a)/a^3/x^2+b*(3*A*b-2*B*a)*ln(x)/a^4
```

Maxima [A]
time = 0.33, size = 106, normalized size = 1.09

$$-\frac{2\left(2\,Bab-3\,Ab^2\right)x^4+Aa^2+\left(2\,Ba^2-3\,Aab\right)x^2}{4\left(a^3bx^6+a^4x^4\right)}+\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(bx^2+a\right)}{2\,a^4}-\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(2*B*a*b - 3*A*b^2)*x^4 + A*a^2 + (2*B*a^2 - 3*A*a*b)*x^2)/(a^3*b*x
^6 + a^4*x^4) + 1/2*(2*B*a*b - 3*A*b^2)*log(b*x^2 + a)/a^4 - 1/2*(2*B*a*b -
 3*A*b^2)*log(x^2)/a^4
```

Fricas [A]
time = 1.00, size = 154, normalized size = 1.59

$$-\frac{2\,(2\,Ba^2b-3\,Aab^2)x^4+Aa^3+(2\,Ba^3-3\,Aa^2b)x^2-2\,((2\,Bab^2-3\,Ab^3)x^6+(2\,Ba^2b-3\,Aab^2)x^4)\log\left(bx^2+a\right)+4\,((2\,Bab^2-3\,Ab^3)x^6+(2\,Ba^2b-3\,Aab^2)x^4)\log\left(x\right)}{4\,(a^4bx^6+a^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a^2*b - 3*A*a*b^2)*x^4 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^2 - 2
*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b - 3*A*a*b^2)*x^4)*log(b*x^2 + a) +
 4*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b - 3*A*a*b^2)*x^4)*log(x))/(a^4*b
*x^6 + a^5*x^4)
```

Sympy [A]
time = 0.69, size = 100, normalized size = 1.03

$$\frac{-Aa^2+x^4\cdot(6Ab^2-4Bab)+x^2\cdot(3Aab-2Ba^2)}{4a^4x^4+4a^3bx^6}-\frac{b(-3Ab+2Ba)\log\left(x\right)}{a^4}+\frac{b(-3Ab+2Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**2,x)
```

```
[Out] (-A*a**2 + x**4*(6*A*b**2 - 4*B*a*b) + x**2*(3*A*a*b - 2*B*a**2))/(4*a**4*x
**4 + 4*a**3*b*x**6) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a)*
log(a/b + x**2)/(2*a**4)
```

Giac [A]
time = 2.62, size = 150, normalized size = 1.55

$$-\frac{(2\,Bab-3\,Ab^2)\log\left(x^2\right)}{2\,a^4}+\frac{(2\,Bab^2-3\,Ab^3)\log\left(|bx^2+a|\right)}{2\,a^4b}-\frac{2\,Bab^2x^2-3\,Ab^3x^2+3\,Ba^2b-4\,Aab^2}{2\,(bx^2+a)a^4}+\frac{6\,Babx^4-9\,Ab^2x^4-2\,Ba^2x^2+4\,Aabx^2-Aa^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*a*b - 3*A*b^2)*log(x^2)/a^4 + 1/2*(2*B*a*b^2 - 3*A*b^3)*log(abs(b
*x^2 + a))/(a^4*b) - 1/2*(2*B*a*b^2*x^2 - 3*A*b^3*x^2 + 3*B*a^2*b - 4*A*a*b
^2)/((b*x^2 + a)*a^4) + 1/4*(6*B*a*b*x^4 - 9*A*b^2*x^4 - 2*B*a^2*x^2 + 4*A*
a*b*x^2 - A*a^2)/(a^4*x^4)
```

Mupad [B]
time = 0.07, size = 100, normalized size = 1.03

$$\frac{\frac{x^2\,\left(3\,A\,b-2\,B\,a\right)}{4\,a^2}-\frac{A}{4\,a}+\frac{b\,x^4\,\left(3\,A\,b-2\,B\,a\right)}{2\,a^3}}{b\,x^6+a\,x^4}-\frac{\ln\left(b\,x^2+a\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{2\,a^4}+\frac{\ln\left(x\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(3*A*b - 2*B*a))/(4*a^2) - A/(4*a) + (b*x^4*(3*A*b - 2*B*a))/(2*a^3))
/(a*x^4 + b*x^6) - (log(a + b*x^2)*(3*A*b^2 - 2*B*a*b))/(2*a^4) + (log(x)*(
3*A*b^2 - 2*B*a*b))/a^4
```

3.86 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{A}{5a^2x^5}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{a^4x}-\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)}-\frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}$$

```
[Out] -1/5*A/a^2/x^5+1/3*(2*A*b-B*a)/a^3/x^3-b*(3*A*b-2*B*a)/a^4/x-1/2*b^2*(A*b-B
*a)*x/a^4/(b*x^2+a)-1/2*b^(3/2)*(7*A*b-5*B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(
9/2)
```

Rubi [A]
time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {467, 1816, 211}

$$-\frac{b^{3/2}(7Ab-5aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{b^2x(Ab-aB)}{2a^4\left(a+bx^2\right)}-\frac{b(3Ab-2aB)}{a^4x}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*A/(a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) - (b*(3*A*b - 2*a*B))/(a^4*x)
- (b^2*(A*b - a*B)*x)/(2*a^4*(a + b*x^2)) - (b^(3/2)*(7*A*b - 5*a*B)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)} - \frac{1}{2}b^2\int \frac{-\frac{2A}{ab^2}+\frac{2(Ab-aB)x^2}{a^2b^2}-\frac{2(Ab-aB)x^4}{a^3b}+\frac{(Ab-aB)x^6}{a^4}}{x^6\left(a+bx^2\right)}\,dx$$

$$= -\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)} - \frac{1}{2}b^2\int \left(-\frac{2A}{a^2b^2x^6} - \frac{2(-2Ab+aB)}{a^3b^2x^4} + \frac{2(-3Ab+2aB)}{a^4bx^2} + \frac{7Ab-5a}{a^4\left(a+b\right.}\right.$$

$$= -\frac{A}{5a^2x^5} + \frac{2Ab-aB}{3a^3x^3} - \frac{b(3Ab-2aB)}{a^4x} - \frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)} - \frac{\left(b^2(7Ab-5aB)\right)\int \frac{1}{a+bx^2}\,dx}{2a^4}$$

$$= -\frac{A}{5a^2x^5} + \frac{2Ab-aB}{3a^3x^3} - \frac{b(3Ab-2aB)}{a^4x} - \frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)} - \frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{2a^{9/2}}$$

Mathematica [A]
time = 0.06, size = 112, normalized size = 0.99

$$-\frac{A}{5a^2x^5} + \frac{2Ab-aB}{3a^3x^3} + \frac{b(-3Ab+2aB)}{a^4x} + \frac{b^2(-Ab+aB)x}{2a^4\left(a+bx^2\right)} + \frac{b^{3/2}(-7Ab+5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
[Out] -1/5*A/(a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(-3*A*b + 2*a*B))/(a^4*x)
 + (b^2*(-(A*b) + a*B)*x)/(2*a^4*(a + b*x^2)) + (b^(3/2)*(-7*A*b + 5*a*B)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Maple [A]
time = 0.09, size = 99, normalized size = 0.88

method	result
default	$-\frac{b^2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(7Ab-5Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4} - \frac{A}{5a^2x^5} - \frac{-2Ab+Ba}{3a^3x^3} - \frac{b(3Ab-2Ba)}{a^4x}$
risch	$\frac{-\frac{b^2(7Ab-5Ba)x^6}{2a^4}-\frac{b(7Ab-5Ba)x^4}{3a^3}+\frac{(7Ab-5Ba)x^2}{15a^2}-\frac{A}{5a}}{x^5\left(b\,x^2+a\right)} + \frac{7\sqrt{-ab}\,b^2\ln\left(-bx+\sqrt{-ab}\right)A}{4a^5} - \frac{5\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{4a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^4*b^2*((1/2*A*b-1/2*B*a)*x/(b*x^2+a)+1/2*(7*A*b-5*B*a)/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2)))-1/5*A/a^2/x^5-1/3*(-2*A*b+B*a)/a^3/x^3-b*(3*A*b-2*B*a
)/a^4/x
```

Maxima [A]
time = 0.50, size = 119, normalized size = 1.05

$$\frac{15\,(5\,Bab^2-7\,Ab^3)x^6+10\,(5\,Ba^2b-7\,Aab^2)x^4-6\,Aa^3-2\,(5\,Ba^3-7\,Aa^2b)x^2}{30\,(a^4bx^7+a^5x^5)}+\frac{(5\,Bab^2-7\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(5*B*a*b^2 - 7*A*b^3)*x^6 + 10*(5*B*a^2*b - 7*A*a*b^2)*x^4 - 6*A*a
^3 - 2*(5*B*a^3 - 7*A*a^2*b)*x^2)/(a^4*b*x^7 + a^5*x^5) + 1/2*(5*B*a*b^2 -
7*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 1.26, size = 308, normalized size = 2.73

$$\left[\frac{30\,(5\,Bab^2-7\,Ab^3)x^6+20\,(5\,Ba^2b-7\,Aab^2)x^4-12\,Aa^3-4\,(5\,Ba^3-7\,Aa^2b)x^2-15\,((5\,Bab^2-7\,Ab^3)x^7+(5\,Ba^2b-7\,Aab^2)x^5)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{60\,(a^4bx^7+a^5x^5)},\ \frac{15\,(5\,Bab^2-7\,Ab^3)x^6+10\,(5\,Ba^2b-7\,Aab^2)x^4-6\,Aa^3-2\,(5\,Ba^3-7\,Aa^2b)x^2+15\,((5\,Bab^2-7\,Ab^3)x^7+(5\,Ba^2b-7\,Aab^2)x^5)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{30\,(a^4bx^7+a^5x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(30*(5*B*a*b^2 - 7*A*b^3)*x^6 + 20*(5*B*a^2*b - 7*A*a*b^2)*x^4 - 12*A
*a^3 - 4*(5*B*a^3 - 7*A*a^2*b)*x^2 - 15*((5*B*a*b^2 - 7*A*b^3)*x^7 + (5*B*a
^2*b - 7*A*a*b^2)*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)))/(a^4*b*x^7 + a^5*x^5), 1/30*(15*(5*B*a*b^2 - 7*A*b^3)*x^6 + 10*(5*B
*a^2*b - 7*A*a*b^2)*x^4 - 6*A*a^3 - 2*(5*B*a^3 - 7*A*a^2*b)*x^2 + 15*((5*B*
a*b^2 - 7*A*b^3)*x^7 + (5*B*a^2*b - 7*A*a*b^2)*x^5)*sqrt(b/a)*arctan(x*sqrt
(b/a)))/(a^4*b*x^7 + a^5*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(104) = 208$.
time = 0.35, size = 218, normalized size = 1.93

$$-\frac{\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)\log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)}{-7Ab^3+5Bab^2}+x\right)}{4}+\frac{\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)\log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)}{-7Ab^3+5Bab^2}+x\right)}{4}+\frac{-6Aa^3+x^6(-105Ab^3+75Bab^2)+x^4(-70Aab^2+50Ba^2b)+x^2\cdot(14Aa^2b-10Ba^3)}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)*log(-a**5*sqrt(-b**3/a**9)*(-7*A*b + 5*B
*a)/(-7*A*b**3 + 5*B*a*b**2) + x)/4 + sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)*log
(a**5*sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)/(-7*A*b**3 + 5*B*a*b**2) + x)/4 + (
-6*A*a**3 + x**6*(-105*A*b**3 + 75*B*a*b**2) + x**4*(-70*A*a*b**2 + 50*B*a*
*2*b) + x**2*(14*A*a**2*b - 10*B*a**3))/(30*a**5*x**5 + 30*a**4*b*x**7)
```

Giac [A]
time = 3.26, size = 112, normalized size = 0.99

$$\frac{(5\,Bab^2-7\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}+\frac{Bab^2x-Ab^3x}{2\,(bx^2+a)a^4}+\frac{30\,Babx^4-45\,Ab^2x^4-5\,Ba^2x^2+10\,Aabx^2-3\,Aa^2}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a*b^2 - 7*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/2*(B*a*
b^2*x - A*b^3*x)/((b*x^2 + a)*a^4) + 1/15*(30*B*a*b*x^4 - 45*A*b^2*x^4 - 5*
B*a^2*x^2 + 10*A*a*b*x^2 - 3*A*a^2)/(a^4*x^5)
```

Mupad [B]
time = 0.08, size = 104, normalized size = 0.92

$$-\frac{\frac{A}{5\,a}-\frac{x^2\,(7\,A\,b-5\,B\,a)}{15\,a^2}+\frac{b^2\,x^6\,(7\,A\,b-5\,B\,a)}{2\,a^4}+\frac{b\,x^4\,(7\,A\,b-5\,B\,a)}{3\,a^3}}{b\,x^7+a\,x^5}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(7\,A\,b-5\,B\,a)}{2\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^2),x)
```

```
[Out] - (A/(5*a) - (x^2*(7*A*b - 5*B*a))/(15*a^2) + (b^2*x^6*(7*A*b - 5*B*a))/(2*
a^4) + (b*x^4*(7*A*b - 5*B*a))/(3*a^3))/(a*x^5 + b*x^7) - (b^(3/2)*atan((b^
(1/2)*x)/a^(1/2))*(7*A*b - 5*B*a))/(2*a^(9/2))
```

3.87 $\int \frac{A+Bx^2}{x^7 (a+bx^2)^2} \, dx$

Optimal. Leaf size=124

$$-\frac{A}{6a^2x^6}+\frac{2Ab-aB}{4a^3x^4}-\frac{b(3Ab-2aB)}{2a^4x^2}-\frac{b^2(Ab-aB)}{2a^4(a+bx^2)}-\frac{b^2(4Ab-3aB)\log(x)}{a^5}+\frac{b^2(4Ab-3aB)\log(a+bx^2)}{2a^5}$$

```
[Out] -1/6*A/a^2/x^6+1/4*(2*A*b-B*a)/a^3/x^4-1/2*b*(3*A*b-2*B*a)/a^4/x^2-1/2*b^2*
(A*b-B*a)/a^4/(b*x^2+a)-b^2*(4*A*b-3*B*a)*ln(x)/a^5+1/2*b^2*(4*A*b-3*B*a)*l
n(b*x^2+a)/a^5
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{b^2(4Ab-3aB)\log(a+bx^2)}{2a^5}-\frac{b^2\log(x)(4Ab-3aB)}{a^5}-\frac{b^2(Ab-aB)}{2a^4(a+bx^2)}-\frac{b(3Ab-2aB)}{2a^4x^2}+\frac{2Ab-aB}{4a^3x^4}-\frac{A}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

```
[Out] -1/6*A/(a^2*x^6) + (2*A*b - a*B)/(4*a^3*x^4) - (b*(3*A*b - 2*a*B))/(2*a^4*x
^2) - (b^2*(A*b - a*B))/(2*a^4*(a + b*x^2)) - (b^2*(4*A*b - 3*a*B)*Log[x])/
a^5 + (b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(2*a^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\,(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^2x^4}+\frac{-2Ab+aB}{a^3x^3}-\frac{b(-3Ab+2aB)}{a^4x^2}+\frac{b^2(-4Ab+3aB)}{a^5x}-\frac{b^3(-Ab+}{a^4(a+b}\right.\right.$$
$$= -\frac{A}{6a^2x^6}+\frac{2Ab-aB}{4a^3x^4}-\frac{b(3Ab-2aB)}{2a^4x^2}-\frac{b^2(Ab-aB)}{2a^4\,(a+bx^2)}-\frac{b^2(4Ab-3aB)\log(x)}{a^5}+\frac{b^2(}{}$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 0.89

$$\frac{-\frac{2a^3A}{x^6}-\frac{3a^2(-2Ab+aB)}{x^4}+\frac{6ab(-3Ab+2aB)}{x^2}+\frac{6ab^2(-Ab+aB)}{a+bx^2}+12b^2(-4Ab+3aB)\log(x)+6b^2(4Ab-3aB)\log\left(a+bx^2\right)}{12a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

```
[Out] ((-2*a^3*A)/x^6 - (3*a^2*(-2*A*b + a*B))/x^4 + (6*a*b*(-3*A*b + 2*a*B))/x^2
 + (6*a*b^2*(-(A*b) + a*B))/(a + b*x^2) + 12*b^2*(-4*A*b + 3*a*B)*Log[x] +
6*b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(12*a^5)
```

Maple [A]
time = 0.08, size = 117, normalized size = 0.94

method	result
default	$\frac{b^3\left(\frac{(4Ab-3Ba)\ln\left(b\,x^2+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(b\,x^2+a\right)}\right)}{2a^5}-\frac{A}{6a^2x^6}-\frac{-2Ab+Ba}{4a^3x^4}-\frac{b(3Ab-2Ba)}{2a^4x^2}-\frac{b^2(4Ab-3Ba)\ln(x)}{a^5}$
norman	$\frac{-\frac{A}{6a}+\frac{(4Ab-3Ba)x^2}{12a^2}-\frac{b(4Ab-3Ba)x^4}{4a^3}+\frac{b\left(4A\,b^3-3Ba\,b^2\right)x^8}{2a^5}}{x^6(b\,x^2+a)}-\frac{b^2(4Ab-3Ba)\ln(x)}{a^5}+\frac{b^2(4Ab-3Ba)\ln(b\,x^2+a)}{2a^5}$
risch	$\frac{-\frac{b^2(4Ab-3Ba)x^6}{2a^4}-\frac{b(4Ab-3Ba)x^4}{4a^3}+\frac{(4Ab-3Ba)x^2}{12a^2}-\frac{A}{6a}}{x^6(b\,x^2+a)}-\frac{4b^3\ln(x)A}{a^5}+\frac{3b^2\ln(x)B}{a^4}+\frac{2b^3\ln(-b\,x^2-a)A}{a^5}-\frac{3b^2\ln(-b\,x^2-a)B}{2a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^5*b^3*((4*A*b-3*B*a)/b*ln(b*x^2+a)-a*(A*b-B*a)/b/(b*x^2+a))-1/6*A/a^2
/x^6-1/4*(-2*A*b+B*a)/a^3/x^4-1/2*b*(3*A*b-2*B*a)/a^4/x^2-b^2*(4*A*b-3*B*a)
*ln(x)/a^5
```

Maxima [A]
time = 0.28, size = 136, normalized size = 1.10

$$\frac{6\,(3\,Bab^2-4\,Ab^3)x^6+3\,(3\,Ba^2b-4\,Aab^2)x^4-2\,Aa^3-(3\,Ba^3-4\,Aa^2b)x^2}{12\,(a^4bx^8+a^5x^6)}-\frac{(3\,Bab^2-4\,Ab^3)\log\left(bx^2+a\right)}{2\,a^5}+\frac{(3\,Bab^2-4\,Ab^3)\log\left(x^2\right)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*(3*B*a*b^2 - 4*A*b^3)*x^6 + 3*(3*B*a^2*b - 4*A*a*b^2)*x^4 - 2*A*a^3
 - (3*B*a^3 - 4*A*a^2*b)*x^2)/(a^4*b*x^8 + a^5*x^6) - 1/2*(3*B*a*b^2 - 4*A*
b^3)*log(b*x^2 + a)/a^5 + 1/2*(3*B*a*b^2 - 4*A*b^3)*log(x^2)/a^5
```

Fricas [A]
time = 1.34, size = 184, normalized size = 1.48

$$\frac{6\,(3\,Ba^{2}b^{2}-4\,Aab^{3})x^{6}-2\,Aa^{4}+3\,(3\,Ba^{3}b-4\,Aa^{2}b^{2})x^{4}-(3\,Ba^{4}-4\,Aa^{3}b)x^{2}-6\,((3\,Bab^{3}-4\,Ab^{4})x^{8}+(3\,Ba^{2}b^{2}-4\,Aab^{3})x^{6})\log\left(bx^{2}+a\right)+12\,((3\,Bab^{3}-4\,Ab^{4})x^{8}+(3\,Ba^{2}b^{2}-4\,Aab^{3})x^{6})\log\left(x\right)}{12\,(a^{5}bx^{8}+a^{6}x^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(3*B*a^2*b^2 - 4*A*a*b^3)*x^6 - 2*A*a^4 + 3*(3*B*a^3*b - 4*A*a^2*b^
2)*x^4 - (3*B*a^4 - 4*A*a^3*b)*x^2 - 6*((3*B*a*b^3 - 4*A*b^4)*x^8 + (3*B*a^
2*b^2 - 4*A*a*b^3)*x^6)*log(b*x^2 + a) + 12*((3*B*a*b^3 - 4*A*b^4)*x^8 + (3
*B*a^2*b^2 - 4*A*a*b^3)*x^6)*log(x))/(a^5*b*x^8 + a^6*x^6)
```

Sympy [A]
time = 0.71, size = 129, normalized size = 1.04

$$\frac{-2Aa^{3}+x^{6}(-24Ab^{3}+18Bab^{2})+x^{4}(-12Aab^{2}+9Ba^{2}b)+x^{2}\cdot(4Aa^{2}b-3Ba^{3})}{12a^{5}x^{6}+12a^{4}bx^{8}}+\frac{b^{2}(-4Ab+3Ba)\log\left(x\right)}{a^{5}}-\frac{b^{2}(-4Ab+3Ba)\log\left(\frac{a}{b}+x^{2}\right)}{2a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**2,x)
```

```
[Out] (-2*A*a**3 + x**6*(-24*A*b**3 + 18*B*a*b**2) + x**4*(-12*A*a*b**2 + 9*B*a**
2*b) + x**2*(4*A*a**2*b - 3*B*a**3))/(12*a**5*x**6 + 12*a**4*b*x**8) + b**2
*(-4*A*b + 3*B*a)*log(x)/a**5 - b**2*(-4*A*b + 3*B*a)*log(a/b + x**2)/(2*a*
*5)
```

Giac [A]
time = 1.26, size = 178, normalized size = 1.44

$$\frac{(3\,Bab^{2}-4\,Ab^{3})\log\left(x^{2}\right)}{2\,a^{5}}-\frac{(3\,Bab^{3}-4\,Ab^{4})\log\left(|bx^{2}+a|\right)}{2\,a^{5}b}+\frac{3\,Bab^{3}x^{2}-4\,Ab^{4}x^{2}+4\,Ba^{2}b^{2}-5\,Aab^{3}}{2\,(bx^{2}+a)a^{5}}-\frac{33\,Bab^{2}x^{6}-44\,Ab^{3}x^{6}-12\,Ba^{2}bx^{4}+18\,Aab^{2}x^{4}+3\,Ba^{3}x^{2}-6\,Aa^{2}bx^{2}+2\,Aa^{3}}{12\,a^{5}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*a*b^2 - 4*A*b^3)*log(x^2)/a^5 - 1/2*(3*B*a*b^3 - 4*A*b^4)*log(abs(
b*x^2 + a))/(a^5*b) + 1/2*(3*B*a*b^3*x^2 - 4*A*b^4*x^2 + 4*B*a^2*b^2 - 5*A*
a*b^3)/((b*x^2 + a)*a^5) - 1/12*(33*B*a*b^2*x^6 - 44*A*b^3*x^6 - 12*B*a^2*b
*x^4 + 18*A*a*b^2*x^4 + 3*B*a^3*x^2 - 6*A*a^2*b*x^2 + 2*A*a^3)/(a^5*x^6)
```

Mupad [B]
time = 0.09, size = 126, normalized size = 1.02

$$\frac{\ln\left(b\,x^2+a\right)\,\left(4\,A\,b^3-3\,B\,a\,b^2\right)}{2\,a^5}-\frac{\frac{A}{6\,a}-\frac{x^2\,\left(4\,A\,b-3\,B\,a\right)}{12\,a^2}+\frac{b^2\,x^6\,\left(4\,A\,b-3\,B\,a\right)}{2\,a^4}+\frac{b\,x^4\,\left(4\,A\,b-3\,B\,a\right)}{4\,a^3}}{b\,x^8+a\,x^6}-\frac{\ln\left(x\right)\,\left(4\,A\,b^3-3\,B\,a\,b^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)^2),x)
```

```
[Out] (log(a + b*x^2)*(4*A*b^3 - 3*B*a*b^2))/(2*a^5) - (A/(6*a) - (x^2*(4*A*b - 3
*B*a))/(12*a^2) + (b^2*x^6*(4*A*b - 3*B*a))/(2*a^4) + (b*x^4*(4*A*b - 3*B*a
))/(4*a^3))/(a*x^6 + b*x^8) - (log(x)*(4*A*b^3 - 3*B*a*b^2))/a^5
```

3.88 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=150

$$\frac{a^2(3Ab-5aB)x^2}{b^6}-\frac{3a(Ab-2aB)x^4}{4b^5}+\frac{(Ab-3aB)x^6}{6b^4}+\frac{Bx^8}{8b^3}+\frac{a^5(Ab-aB)}{4b^7\,(a+bx^2)^2}-\frac{a^4(5Ab-6aB)}{2b^7\,(a+bx^2)}-\frac{5a^3(2Ab-3}{}$$

```
[Out] a^2*(3*A*b-5*B*a)*x^2/b^6-3/4*a*(A*b-2*B*a)*x^4/b^5+1/6*(A*b-3*B*a)*x^6/b^4
+1/8*B*x^8/b^3+1/4*a^5*(A*b-B*a)/b^7/(b*x^2+a)^2-1/2*a^4*(5*A*b-6*B*a)/b^7/
(b*x^2+a)-5/2*a^3*(2*A*b-3*B*a)*ln(b*x^2+a)/b^7
```

Rubi [A]
time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{a^5(Ab-aB)}{4b^7\,(a+bx^2)^2}-\frac{a^4(5Ab-6aB)}{2b^7\,(a+bx^2)}-\frac{5a^3(2Ab-3aB)\log(a+bx^2)}{2b^7}+\frac{a^2x^2(3Ab-5aB)}{b^6}-\frac{3ax^4(Ab-2aB)}{4b^5}+\frac{x^6(Ab-3aB)}{6b^4}+\frac{Bx^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a^2*(3*A*b - 5*a*B)*x^2)/b^6 - (3*a*(A*b - 2*a*B)*x^4)/(4*b^5) + ((A*b - 3
*a*B)*x^6)/(6*b^4) + (B*x^8)/(8*b^3) + (a^5*(A*b - a*B))/(4*b^7*(a + b*x^2)
^2) - (a^4*(5*A*b - 6*a*B))/(2*b^7*(a + b*x^2)) - (5*a^3*(2*A*b - 3*a*B)*Lo
g[a + b*x^2])/(2*b^7)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^5(A+Bx)}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{2a^2(-3Ab+5aB)}{b^6}+\frac{3a(-Ab+2aB)x}{b^5}+\frac{(Ab-3aB)x^2}{b^4}+\frac{Bx^3}{b^3}+\frac{a^5}{\cdots}\right.\right.$$
$$= \frac{a^2(3Ab-5aB)x^2}{b^6}-\frac{3a(Ab-2aB)x^4}{4b^5}+\frac{(Ab-3aB)x^6}{6b^4}+\frac{Bx^8}{8b^3}+\frac{a^5(Ab-aB)}{4b^7\left(a+bx^2\right)^2}-\frac{a^4}{2\cdots}$$

Mathematica [A]
time = 0.06, size = 136, normalized size = 0.91

$$\frac{-24a^2b(-3Ab+5aB)x^2+18ab^2(-Ab+2aB)x^4+4b^3(Ab-3aB)x^6+3b^4Bx^8+\frac{6a^5(Ab-aB)}{(a+bx^2)^2}+\frac{12a^4(-5Ab+6aB)}{a+bx^2}+60a^3(-2Ab+3aB)\log\left(a+bx^2\right)}{24b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-24*a^2*b*(-3*A*b + 5*a*B)*x^2 + 18*a*b^2*(-(A*b) + 2*a*B)*x^4 + 4*b^3*(A*
b - 3*a*B)*x^6 + 3*b^4*B*x^8 + (6*a^5*(A*b - a*B))/(a + b*x^2)^2 + (12*a^4*
(-5*A*b + 6*a*B))/(a + b*x^2) + 60*a^3*(-2*A*b + 3*a*B)*Log[a + b*x^2])/(24
*b^7)
```

Maple [A]
time = 0.08, size = 151, normalized size = 1.01

method	result
norman	$\frac{-\frac{a^2\left(30Aa^3b-45Ba^4\right)}{4b^7}+\frac{(2Ab-3Ba)x^{10}}{12b^2}+\frac{Bx^{12}}{8b}-\frac{5a(2Ab-3Ba)x^8}{24b^3}-\frac{a\left(10Aa^3b-15Ba^4\right)x^2}{b^6}+\frac{5a^2(2Ab-3Ba)x^6}{6b^4}}{\left(bx^2+a\right)^2}-\frac{5a^3(2Ab-3Ba)\ln\left(bx^2+\cdots\right)}{2b^7}$
default	$\frac{\frac{Bb^3x^8}{8}+\frac{\left(Ab^3-3Bab^2\right)x^6}{6}+\frac{\left(-3Aab^2+6Ba^2b\right)x^4}{4}+\frac{\left(6Aa^2b-10Ba^3\right)x^2}{2}}{b^6}-\frac{a^3\left(\frac{(10Ab-15Ba)\ln\left(bx^2+a\right)}{b}-\frac{a^2(Ab-Ba)}{2b\left(bx^2+a\right)^2}+\frac{a(5Ab-6Ba)}{b\left(bx^2+a\right)}\right)}{2b^6}$
risch	$\frac{Bx^8}{8b^3}+\frac{x^6A}{6b^3}-\frac{x^6Ba}{2b^4}-\frac{3x^4Aa}{4b^4}+\frac{3x^4Ba^2}{2b^5}+\frac{3Aa^2x^2}{b^5}-\frac{5Ba^3x^2}{b^6}+\frac{\left(-\frac{5}{2}a^4bA+3a^5B\right)x^2-\frac{a^5(9Ab-11Ba)}{4b}}{b^6\left(bx^2+a\right)^2}-\frac{5a^3\ln\left(bx^2+\cdots\right)}{b^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/8*B*b^3*x^8+1/6*(A*b^3-3*B*a*b^2)*x^6+1/4*(-3*A*a*b^2+6*B*a^2*b)*x
^4+1/2*(6*A*a^2*b-10*B*a^3)*x^2)-1/2*a^3/b^6*((10*A*b-15*B*a)/b*ln(b*x^2+a)
-1/2*a^2*(A*b-B*a)/b/(b*x^2+a)^2+a*(5*A*b-6*B*a)/b/(b*x^2+a))
```

Maxima [A]
time = 0.27, size = 165, normalized size = 1.10

$$\frac{11\,Ba^6-9\,Aa^5b+2\,(6\,Ba^5b-5\,Aa^4b^2)x^2}{4\,(b^9x^4+2\,ab^8x^2+a^2b^7)}+\frac{3\,Bb^3x^8-4\,(3\,Bab^2-Ab^3)x^6+18\,(2\,Ba^2b-Aab^2)x^4-24\,(5\,Ba^3-3\,Aa^2b)x^2}{24\,b^6}+\frac{5\,(3\,Ba^4-2\,Aa^3b)\log\left(bx^2+a\right)}{2\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(11*B*a^6 - 9*A*a^5*b + 2*(6*B*a^5*b - 5*A*a^4*b^2)*x^2)/(b^9*x^4 + 2*a
*b^8*x^2 + a^2*b^7) + 1/24*(3*B*b^3*x^8 - 4*(3*B*a*b^2 - A*b^3)*x^6 + 18*(2
*B*a^2*b - A*a*b^2)*x^4 - 24*(5*B*a^3 - 3*A*a^2*b)*x^2)/b^6 + 5/2*(3*B*a^4
- 2*A*a^3*b)*log(b*x^2 + a)/b^7
```

Fricas [A]
time = 0.89, size = 231, normalized size = 1.54

$$\frac{3\,Bb^{6}x^{12}-2\,(3\,Bab^{5}-2\,Ab^{6})x^{10}+5\,(3\,Ba^{2}b^{4}-2\,Aab^{5})x^{8}+66\,Ba^{6}-54\,Aa^{5}b-20\,(3\,Ba^{3}b^{3}-2\,Aa^{2}b^{4})x^{6}-6\,(34\,Ba^{4}b^{2}-21\,Aa^{3}b^{3})x^{4}-12\,(4\,Ba^{5}b-Aa^{4}b^{2})x^{2}+60\,(3\,Ba^{6}-2\,Aa^{5}b+(3\,Ba^{4}b^{2}-2\,Aa^{3}b^{3})x^{4}+2\,(3\,Ba^{5}b-2\,Aa^{4}b^{2})x^{2})\log\left(bx^{2}+a\right)}{24\,(b^{9}x^{4}+2\,ab^{8}x^{2}+a^{2}b^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^6*x^12 - 2*(3*B*a*b^5 - 2*A*b^6)*x^10 + 5*(3*B*a^2*b^4 - 2*A*a*
b^5)*x^8 + 66*B*a^6 - 54*A*a^5*b - 20*(3*B*a^3*b^3 - 2*A*a^2*b^4)*x^6 - 6*(
34*B*a^4*b^2 - 21*A*a^3*b^3)*x^4 - 12*(4*B*a^5*b - A*a^4*b^2)*x^2 + 60*(3*B
*a^6 - 2*A*a^5*b + (3*B*a^4*b^2 - 2*A*a^3*b^3)*x^4 + 2*(3*B*a^5*b - 2*A*a^4
*b^2)*x^2)*log(b*x^2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

Sympy [A]
time = 0.91, size = 170, normalized size = 1.13

$$\frac{Bx^{8}}{8b^{3}}+\frac{5a^{3}(-2Ab+3Ba)\log\left(a+bx^{2}\right)}{2b^{7}}+x^{6}\left(\frac{A}{6b^{3}}-\frac{Ba}{2b^{4}}\right)+x^{4}\left(-\frac{3Aa}{4b^{4}}+\frac{3Ba^{2}}{2b^{5}}\right)+x^{2}\cdot\left(\frac{3Aa^{2}}{b^{5}}-\frac{5Ba^{3}}{b^{6}}\right)+\frac{-9Aa^{5}b+11Ba^{6}+x^{2}(-10Aa^{4}b^{2}+12Ba^{5}b)}{4a^{2}b^{7}+8ab^{8}x^{2}+4b^{9}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**8/(8*b**3) + 5*a**3*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**7) + x**6*(
A/(6*b**3) - B*a/(2*b**4)) + x**4*(-3*A*a/(4*b**4) + 3*B*a**2/(2*b**5)) + x
**2*(3*A*a**2/b**5 - 5*B*a**3/b**6) + (-9*A*a**5*b + 11*B*a**6 + x**2*(-10*
A*a**4*b**2 + 12*B*a**5*b))/(4*a**2*b**7 + 8*a*b**8*x**2 + 4*b**9*x**4)
```

Giac [A]
time = 2.07, size = 183, normalized size = 1.22

$$\frac{5\,(3\,Ba^{4}-2\,Aa^{3}b)\log\left(|bx^{2}+a|\right)}{2\,b^{7}}-\frac{45\,Ba^{4}b^{2}x^{4}-30\,Aa^{3}b^{3}x^{4}+78\,Ba^{5}bx^{2}-50\,Aa^{4}b^{2}x^{2}+34\,Ba^{6}-21\,Aa^{5}b}{4\,(bx^{2}+a)^{2}b^{7}}+\frac{3\,Bb^{9}x^{8}-12\,Bab^{8}x^{6}+4\,Ab^{9}x^{6}+36\,Ba^{2}b^{7}x^{4}-18\,Aab^{8}x^{4}-120\,Ba^{3}b^{6}x^{2}+72\,Aa^{2}b^{7}x^{2}}{24\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/2*(3*B*a^4 - 2*A*a^3*b)*log(abs(b*x^2 + a))/b^7 - 1/4*(45*B*a^4*b^2*x^4 -
 30*A*a^3*b^3*x^4 + 78*B*a^5*b*x^2 - 50*A*a^4*b^2*x^2 + 34*B*a^6 - 21*A*a^5
*b)/((b*x^2 + a)^2*b^7) + 1/24*(3*B*b^9*x^8 - 12*B*a*b^8*x^6 + 4*A*b^9*x^6
```

```
+ 36*B*a^2*b^7*x^4 - 18*A*a*b^8*x^4 - 120*B*a^3*b^6*x^2 + 72*A*a^2*b^7*x^2)
/b^12
```

Mupad [B]
time = 0.07, size = 225, normalized size = 1.50

$$\frac{\frac{11\,B\,a^6-9\,A\,a^5\,b}{4\,b}+x^2\,\left(3\,B\,a^5-\frac{5\,A\,a^4\,b}{2}\right)}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}-x^2\,\left(\frac{B\,a^3}{2\,b^6}-\frac{3\,a\,\left(\frac{3\,a\,\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)}{2\,b}+\frac{3\,a^2\,\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{2\,b^2}\right)+x^6\,\left(\frac{A}{6\,b^3}-\frac{B\,a}{2\,b^4}\right)-x^4\,\left(\frac{3\,a\,\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{4\,b}+\frac{3\,B\,a^2}{4\,b^5}\right)+\frac{B\,x^8}{8\,b^3}+\frac{\ln\left(b\,x^2+a\right)\,\left(15\,B\,a^4-10\,A\,a^3\,b\right)}{2\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] ((11*B*a^6 - 9*A*a^5*b)/(4*b) + x^2*(3*B*a^5 - (5*A*a^4*b)/2))/(a^2*b^6 + b
^8*x^4 + 2*a*b^7*x^2) - x^2*((B*a^3)/(2*b^6) - (3*a*((3*a*(A/b^3 - (3*B*a)/
b^4))/b + (3*B*a^2)/b^5))/(2*b) + (3*a^2*(A/b^3 - (3*B*a)/b^4))/(2*b^2)) +
x^6*(A/(6*b^3) - (B*a)/(2*b^4)) - x^4*((3*a*(A/b^3 - (3*B*a)/b^4))/(4*b) +
(3*B*a^2)/(4*b^5)) + (B*x^8)/(8*b^3) + (log(a + b*x^2)*(15*B*a^4 - 10*A*a^3
*b))/(2*b^7)
```

3.89 $\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=128

$$-\frac{3a(Ab-2aB)x^2}{2b^5}+\frac{(Ab-3aB)x^4}{4b^4}+\frac{Bx^6}{6b^3}-\frac{a^4(Ab-aB)}{4b^6\,(a+bx^2)^2}+\frac{a^3(4Ab-5aB)}{2b^6\,(a+bx^2)}+\frac{a^2(3Ab-5aB)\log\left(a+bx^2\right)}{b^6}$$

```
[Out] -3/2*a*(A*b-2*B*a)*x^2/b^5+1/4*(A*b-3*B*a)*x^4/b^4+1/6*B*x^6/b^3-1/4*a^4*(A
*b-B*a)/b^6/(b*x^2+a)^2+1/2*a^3*(4*A*b-5*B*a)/b^6/(b*x^2+a)+a^2*(3*A*b-5*B*
a)*ln(b*x^2+a)/b^6
```

Rubi [A]
time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$-\frac{a^4(Ab-aB)}{4b^6\,(a+bx^2)^2}+\frac{a^3(4Ab-5aB)}{2b^6\,(a+bx^2)}+\frac{a^2(3Ab-5aB)\log\left(a+bx^2\right)}{b^6}-\frac{3ax^2(Ab-2aB)}{2b^5}+\frac{x^4(Ab-3aB)}{4b^4}+\frac{Bx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*(A*b - 2*a*B)*x^2)/(2*b^5) + ((A*b - 3*a*B)*x^4)/(4*b^4) + (B*x^6)/(6
*b^3) - (a^4*(A*b - a*B))/(4*b^6*(a + b*x^2)^2) + (a^3*(4*A*b - 5*a*B))/(2*
b^6*(a + b*x^2)) + (a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/b^6
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{3a(-Ab+2aB)}{b^5} + \frac{(Ab-3aB)x}{b^4} + \frac{Bx^2}{b^3} - \frac{a^4(-Ab+aB)}{b^5(a+bx)^3} + \frac{a^3(-4Ab+}{b^5(a+b}\right.\right.$$
$$= -\frac{3a(Ab-2aB)x^2}{2b^5} + \frac{(Ab-3aB)x^4}{4b^4} + \frac{Bx^6}{6b^3} - \frac{a^4(Ab-aB)}{4b^6(a+bx^2)^2} + \frac{a^3(4Ab-5aB)}{2b^6(a+bx^2)} + \frac{a^2(}{}$$

Mathematica [A]
time = 0.06, size = 116, normalized size = 0.91

$$\frac{18ab(-Ab+2aB)x^2+3b^2(Ab-3aB)x^4+2b^3Bx^6+\frac{3a^4(-Ab+aB)}{(a+bx^2)^2}+\frac{6a^3(4Ab-5aB)}{a+bx^2}+12a^2(3Ab-5aB)\log(a+bx^2)}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (18*a*b*(-(A*b) + 2*a*B)*x^2 + 3*b^2*(A*b - 3*a*B)*x^4 + 2*b^3*B*x^6 + (3*a
^4*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a^3*(4*A*b - 5*a*B))/(a + b*x^2) + 12
*a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/(12*b^6)
```

Maple [A]
time = 0.07, size = 129, normalized size = 1.01

method	result
norman	$\frac{\frac{a^2(9Aa^2b-15Ba^3)}{2b^6}+\frac{Bx^{10}}{6b}+\frac{(3Ab-5Ba)x^8}{12b^2}-\frac{a(3Ab-5Ba)x^6}{3b^3}+\frac{2a(3Aa^2b-5Ba^3)x^2}{b^5}}{(bx^2+a)^2}+\frac{a^2(3Ab-5Ba)\ln(bx^2+a)}{b^6}$
default	$-\frac{-\frac{b^2Bx^6}{6}+\frac{(-b^2A+3abB)x^4}{4}+\frac{(3abA-6a^2B)x^2}{2}}{b^5}+\frac{a^2\left(\frac{(6Ab-10Ba)\ln(bx^2+a)}{b}-\frac{a^2(Ab-Ba)}{2b(bx^2+a)^2}+\frac{a(4Ab-5Ba)}{b(bx^2+a)}\right)}{2b^5}$
risch	$\frac{Bx^6}{6b^3}+\frac{Ax^4}{4b^3}-\frac{3Bax^4}{4b^4}-\frac{3aAx^2}{2b^4}+\frac{3Ba^2x^2}{b^5}+\frac{(2Aa^3b-\frac{5}{2}Ba^4)x^2+\frac{a^4(7Ab-9Ba)}{4b}}{b^5(bx^2+a)^2}+\frac{3a^2\ln(bx^2+a)A}{b^5}-\frac{5a^3\ln(bx^2+a)B}{b^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/6*b^2*B*x^6+1/4*(-A*b^2+3*B*a*b)*x^4+1/2*(3*A*a*b-6*B*a^2)*x^2)+
1/2*a^2/b^5*((6*A*b-10*B*a)/b*ln(b*x^2+a)-1/2*a^2*(A*b-B*a)/b/(b*x^2+a)^2+a
*(4*A*b-5*B*a)/b/(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 141, normalized size = 1.10

$$-\frac{9\,Ba^5-7\,Aa^4b+2\,(5\,Ba^4b-4\,Aa^3b^2)x^2}{4\,(b^8x^4+2\,ab^7x^2+a^2b^6)}+\frac{2\,Bb^2x^6-3\,(3\,Bab-Ab^2)x^4+18\,(2\,Ba^2-Aab)x^2}{12\,b^5}-\frac{(5\,Ba^3-3\,Aa^2b)\log(bx^2+a)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(9*B*a^5 - 7*A*a^4*b + 2*(5*B*a^4*b - 4*A*a^3*b^2)*x^2)/(b^8*x^4 + 2*a
*b^7*x^2 + a^2*b^6) + 1/12*(2*B*b^2*x^6 - 3*(3*B*a*b - A*b^2)*x^4 + 18*(2*B
*a^2 - A*a*b)*x^2)/b^5 - (5*B*a^3 - 3*A*a^2*b)*log(b*x^2 + a)/b^6
```

Fricas [A]
time = 0.70, size = 205, normalized size = 1.60

$$\frac{2\,Bb^5x^{10} - (5\,Bab^4 - 3\,Ab^5)x^8 + 4\,(5\,Ba^2b^3 - 3\,Aab^4)x^6 - 27\,Ba^5 + 21\,Aa^4b + 3\,(21\,Ba^3b^2 - 11\,Aa^2b^3)x^4 + 6\,(Ba^4b + Aa^3b^2)x^2 - 12\,(5\,Ba^5 - 3\,Aa^4b + (5\,Ba^3b^2 - 3\,Aa^2b^3)x^4 + 2\,(5\,Ba^4b - 3\,Aa^3b^2)x^2)\log\left(bx^2 + a\right)}{12\,(b^8x^4 + 2\,ab^7x^2 + a^2b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^5*x^10 - (5*B*a*b^4 - 3*A*b^5)*x^8 + 4*(5*B*a^2*b^3 - 3*A*a*b^4
)*x^6 - 27*B*a^5 + 21*A*a^4*b + 3*(21*B*a^3*b^2 - 11*A*a^2*b^3)*x^4 + 6*(B*
a^4*b + A*a^3*b^2)*x^2 - 12*(5*B*a^5 - 3*A*a^4*b + (5*B*a^3*b^2 - 3*A*a^2*b
^3)*x^4 + 2*(5*B*a^4*b - 3*A*a^3*b^2)*x^2)*log(b*x^2 + a))/(b^8*x^4 + 2*a*b
^7*x^2 + a^2*b^6)
```

Sympy [A]
time = 0.98, size = 143, normalized size = 1.12

$$\frac{Bx^6}{6b^3} - \frac{a^2(-3Ab + 5Ba)\log\left(a + bx^2\right)}{b^6} + x^4\left(\frac{A}{4b^3} - \frac{3Ba}{4b^4}\right) + x^2\left(-\frac{3Aa}{2b^4} + \frac{3Ba^2}{b^5}\right) + \frac{7Aa^4b - 9Ba^5 + x^2\cdot\left(8Aa^3b^2 - 10Ba^4b\right)}{4a^2b^6 + 8ab^7x^2 + 4b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) - a**2*(-3*A*b + 5*B*a)*log(a + b*x**2)/b**6 + x**4*(A/(4*b
**3) - 3*B*a/(4*b**4)) + x**2*(-3*A*a/(2*b**4) + 3*B*a**2/b**5) + (7*A*a**4
*b - 9*B*a**5 + x**2*(8*A*a**3*b**2 - 10*B*a**4*b))/(4*a**2*b**6 + 8*a*b**7
*x**2 + 4*b**8*x**4)
```

Giac [A]
time = 1.20, size = 159, normalized size = 1.24

$$-\frac{(5\,Ba^3 - 3\,Aa^2b)\log\left(|bx^2 + a|\right)}{b^6} + \frac{30\,Ba^3b^2x^4 - 18\,Aa^2b^3x^4 + 50\,Ba^4bx^2 - 28\,Aa^3b^2x^2 + 21\,Ba^5 - 11\,Aa^4b}{4\,(bx^2 + a)^2b^6} + \frac{2\,Bb^6x^6 - 9\,Bab^5x^4 + 3\,Ab^6x^4 + 36\,Ba^2b^4x^2 - 18\,Aab^5x^2}{12\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -(5*B*a^3 - 3*A*a^2*b)*log(abs(b*x^2 + a))/b^6 + 1/4*(30*B*a^3*b^2*x^4 - 18
*A*a^2*b^3*x^4 + 50*B*a^4*b*x^2 - 28*A*a^3*b^2*x^2 + 21*B*a^5 - 11*A*a^4*b)
/((b*x^2 + a)^2*b^6) + 1/12*(2*B*b^6*x^6 - 9*B*a*b^5*x^4 + 3*A*b^6*x^4 + 36
*B*a^2*b^4*x^2 - 18*A*a*b^5*x^2)/b^9
```

Mupad [B]
time = 0.04, size = 155, normalized size = 1.21

$$x^4\left(\frac{A}{4\,b^3}-\frac{3\,B\,a}{4\,b^4}\right)-\frac{\frac{9\,B\,a^5-7\,A\,a^4\,b}{4\,b}+x^2\,\left(\frac{5\,B\,a^4}{2}-2\,A\,a^3\,b\right)}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}-x^2\,\left(\frac{3\,a\,\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{2\,b}+\frac{3\,B\,a^2}{2\,b^5}\right)+\frac{B\,x^6}{6\,b^3}-\frac{\ln\left(b\,x^2+a\right)\,\left(5\,B\,a^3-3\,A\,a^2\,b\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] x^4*(A/(4*b^3) - (3*B*a)/(4*b^4)) - ((9*B*a^5 - 7*A*a^4*b)/(4*b) + x^2*((5*
B*a^4)/2 - 2*A*a^3*b))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x^2*((3*a*(A/b^3
 - (3*B*a)/b^4))/(2*b) + (3*B*a^2)/(2*b^5)) + (B*x^6)/(6*b^3) - (log(a + b*
x^2)*(5*B*a^3 - 3*A*a^2*b))/b^6
```

3.90 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=109

$$\frac{(Ab-3aB)x^2}{2b^4}+\frac{Bx^4}{4b^3}+\frac{a^3(Ab-aB)}{4b^5(a+bx^2)^2}-\frac{a^2(3Ab-4aB)}{2b^5(a+bx^2)}-\frac{3a(Ab-2aB)\log(a+bx^2)}{2b^5}$$

```
[Out] 1/2*(A*b-3*B*a)*x^2/b^4+1/4*B*x^4/b^3+1/4*a^3*(A*b-B*a)/b^5/(b*x^2+a)^2-1/2
*a^2*(3*A*b-4*B*a)/b^5/(b*x^2+a)-3/2*a*(A*b-2*B*a)*ln(b*x^2+a)/b^5
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 78}

$$\frac{a^3(Ab-aB)}{4b^5(a+bx^2)^2}-\frac{a^2(3Ab-4aB)}{2b^5(a+bx^2)}-\frac{3a(Ab-2aB)\log(a+bx^2)}{2b^5}+\frac{x^2(Ab-3aB)}{2b^4}+\frac{Bx^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x^2)/(2*b^4) + (B*x^4)/(4*b^3) + (a^3*(A*b - a*B))/(4*b^5*(a
 + b*x^2)^2) - (a^2*(3*A*b - 4*a*B))/(2*b^5*(a + b*x^2)) - (3*a*(A*b - 2*a*
B)*Log[a + b*x^2])/(2*b^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{Ab-3aB}{b^4} + \frac{Bx}{b^3} + \frac{a^3(-Ab+aB)}{b^4(a+bx)^3} - \frac{a^2(-3Ab+4aB)}{b^4(a+bx)^2} + \frac{3a(-Ab+2a}{b^4(a+bx)}\right.\right.$$
$$= \frac{(Ab-3aB)x^2}{2b^4} + \frac{Bx^4}{4b^3} + \frac{a^3(Ab-aB)}{4b^5\left(a+bx^2\right)^2} - \frac{a^2(3Ab-4aB)}{2b^5\left(a+bx^2\right)} - \frac{3a(Ab-2aB)\log\left(a+bx\right.}{2b^5}$$

Mathematica [A]
time = 0.05, size = 94, normalized size = 0.86

$$\frac{2b(Ab-3aB)x^2 + b^2Bx^4 + \frac{a^3(Ab-aB)}{(a+bx^2)^2} + \frac{2a^2(-3Ab+4aB)}{a+bx^2} + 6a(-Ab+2aB)\log\left(a+bx^2\right)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (2*b*(A*b - 3*a*B)*x^2 + b^2*B*x^4 + (a^3*(A*b - a*B))/(a + b*x^2)^2 + (2*a
^2*(-3*A*b + 4*a*B))/(a + b*x^2) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^2])/(4*
b^5)
```

Maple [A]
time = 0.09, size = 102, normalized size = 0.94

method	result
norman	$\frac{-\frac{a^2\left(9abA-18a^2B\right)}{4b^5}+\frac{Bx^8}{4b}+\frac{(Ab-2Ba)x^6}{2b^2}-\frac{a\left(3abA-6a^2B\right)x^2}{b^4}}{\left(bx^2+a\right)^2} - \frac{3a(Ab-2Ba)\ln\left(bx^2+a\right)}{2b^5}$
default	$\frac{\left(bBx^2+Ab-3Ba\right)^2}{4b^5B} - \frac{a\left(\frac{(3Ab-6Ba)\ln\left(bx^2+a\right)}{b}-\frac{a^2(Ab-Ba)}{2b\left(bx^2+a\right)^2}+\frac{a(3Ab-4Ba)}{b\left(bx^2+a\right)}\right)}{2b^4}$
risch	$\frac{Bx^4}{4b^3} + \frac{Ax^2}{2b^3} - \frac{3Bax^2}{2b^4} + \frac{A^2}{4b^3B} - \frac{3Aa}{2b^4} + \frac{9Ba^2}{4b^5} + \frac{\left(-\frac{3}{2}Aa^2b+2Ba^3\right)x^2-\frac{a^3(5Ab-7Ba)}{4b}}{b^4\left(bx^2+a\right)^2} - \frac{3a\ln\left(bx^2+a\right)A}{2b^4} + \frac{3a^2\ln\left(bx^2+\right.}{b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(B*b*x^2+A*b-3*B*a)^2/b^5/B-1/2*a/b^4*((3*A*b-6*B*a)/b*ln(b*x^2+a)-1/2*
a^2*(A*b-B*a)/b/(b*x^2+a)^2+a*(3*A*b-4*B*a)/b/(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 116, normalized size = 1.06

$$\frac{7\,Ba^4 - 5\,Aa^3b + 2\,(4\,Ba^3b - 3\,Aa^2b^2)x^2}{4\,(b^7x^4 + 2\,ab^6x^2 + a^2b^5)} + \frac{Bbx^4 - 2\,(3\,Ba - Ab)x^2}{4\,b^4} + \frac{3\,(2\,Ba^2 - Aab)\log\left(bx^2 + a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^2)/(b^7*x^4 + 2*a*
b^6*x^2 + a^2*b^5) + 1/4*(B*b*x^4 - 2*(3*B*a - A*b)*x^2)/b^4 + 3/2*(2*B*a^2
 - A*a*b)*log(b*x^2 + a)/b^5
```

Fricas [A]
time = 1.37, size = 179, normalized size = 1.64

$$\frac{Bb^4x^8 - 2\,(2\,Bab^3 - Ab^4)x^6 + 7\,Ba^4 - 5\,Aa^3b - (11\,Ba^2b^2 - 4\,Aab^3)x^4 + 2\,(Ba^3b - 2\,Aa^2b^2)x^2 + 6\,(2\,Ba^4 - Aa^3b + (2\,Ba^2b^2 - Aab^3)x^4 + 2\,(2\,Ba^3b - Aa^2b^2)x^2)\log\left(bx^2 + a\right)}{4\,(b^7x^4 + 2\,ab^6x^2 + a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^4*x^8 - 2*(2*B*a*b^3 - A*b^4)*x^6 + 7*B*a^4 - 5*A*a^3*b - (11*B*a^
2*b^2 - 4*A*a*b^3)*x^4 + 2*(B*a^3*b - 2*A*a^2*b^2)*x^2 + 6*(2*B*a^4 - A*a^3
*b + (2*B*a^2*b^2 - A*a*b^3)*x^4 + 2*(2*B*a^3*b - A*a^2*b^2)*x^2)*log(b*x^2
 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
```

Sympy [A]
time = 0.81, size = 119, normalized size = 1.09

$$\frac{Bx^4}{4b^3} + \frac{3a(-Ab + 2Ba)\log\left(a + bx^2\right)}{2b^5} + x^2\left(\frac{A}{2b^3} - \frac{3Ba}{2b^4}\right) + \frac{-5Aa^3b + 7Ba^4 + x^2(-6Aa^2b^2 + 8Ba^3b)}{4a^2b^5 + 8ab^6x^2 + 4b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) + 3*a*(-A*b + 2*B*a)*log(a + b*x**2)/(2*b**5) + x**2*(A/(2*
b**3) - 3*B*a/(2*b**4)) + (-5*A*a**3*b + 7*B*a**4 + x**2*(-6*A*a**2*b**2 +
8*B*a**3*b))/(4*a**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4)
```

Giac [A]
time = 1.63, size = 132, normalized size = 1.21

$$\frac{3\,(2\,Ba^2 - Aab)\log\left(|bx^2 + a|\right)}{2\,b^5} + \frac{Bb^3x^4 - 6\,Bab^2x^2 + 2\,Ab^3x^2}{4\,b^6} - \frac{18\,Ba^2b^2x^4 - 9\,Aab^3x^4 + 28\,Ba^3bx^2 - 12\,Aa^2b^2x^2 + 11\,Ba^4 - 4\,Aa^3b}{4\,(bx^2 + a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*(2*B*a^2 - A*a*b)*log(abs(b*x^2 + a))/b^5 + 1/4*(B*b^3*x^4 - 6*B*a*b^2*
x^2 + 2*A*b^3*x^2)/b^6 - 1/4*(18*B*a^2*b^2*x^4 - 9*A*a*b^3*x^4 + 28*B*a^3*b
*x^2 - 12*A*a^2*b^2*x^2 + 11*B*a^4 - 4*A*a^3*b)/((b*x^2 + a)^2*b^5)
```

Mupad [B]
time = 0.04, size = 118, normalized size = 1.08

$$\frac{\frac{7\,B\,a^4-5\,A\,a^3\,b}{4\,b}+x^2\,\left(2\,B\,a^3-\frac{3\,A\,a^2\,b}{2}\right)}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+x^2\,\left(\frac{A}{2\,b^3}-\frac{3\,B\,a}{2\,b^4}\right)+\frac{\ln\left(b\,x^2+a\right)\,\left(6\,B\,a^2-3\,A\,a\,b\right)}{2\,b^5}+\frac{B\,x^4}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] ((7*B*a^4 - 5*A*a^3*b)/(4*b) + x^2*(2*B*a^3 - (3*A*a^2*b)/2))/(a^2*b^4 + b^
6*x^4 + 2*a*b^5*x^2) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4)) + (log(a + b*x^2)*
(6*B*a^2 - 3*A*a*b))/(2*b^5) + (B*x^4)/(4*b^3)
```

3.91 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=88

$$\frac{Bx^2}{2b^3}-\frac{a^2(Ab-aB)}{4b^4(a+bx^2)^2}+\frac{a(2Ab-3aB)}{2b^4(a+bx^2)}+\frac{(Ab-3aB)\log(a+bx^2)}{2b^4}$$

```
[Out] 1/2*B*x^2/b^3-1/4*a^2*(A*b-B*a)/b^4/(b*x^2+a)^2+1/2*a*(2*A*b-3*B*a)/b^4/(b*
x^2+a)+1/2*(A*b-3*B*a)*ln(b*x^2+a)/b^4
```

Rubi [A]

time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 78}

$$-\frac{a^2(Ab-aB)}{4b^4(a+bx^2)^2}+\frac{a(2Ab-3aB)}{2b^4(a+bx^2)}+\frac{(Ab-3aB)\log(a+bx^2)}{2b^4}+\frac{Bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x^2)/(2*b^3) - (a^2*(A*b - a*B))/(4*b^4*(a + b*x^2)^2) + (a*(2*A*b - 3*a
*B))/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{b^3} - \frac{a^2(-Ab+aB)}{b^3(a+bx)^3} + \frac{a(-2Ab+3aB)}{b^3(a+bx)^2} + \frac{Ab-3aB}{b^3(a+bx)}\right) dx, x, x^2\right)$$
$$= \frac{Bx^2}{2b^3} - \frac{a^2(Ab-aB)}{4b^4(a+bx^2)^2} + \frac{a(2Ab-3aB)}{2b^4(a+bx^2)} + \frac{(Ab-3aB)\log(a+bx^2)}{2b^4}$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 1.05

$$\frac{Bx^2}{2b^3} + \frac{-a^2Ab+a^3B}{4b^4(a+bx^2)^2} + \frac{2aAb-3a^2B}{2b^4(a+bx^2)} + \frac{(Ab-3aB)\log(a+bx^2)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x^2)/(2*b^3) + (-(a^2*A*b) + a^3*B)/(4*b^4*(a + b*x^2)^2) + (2*a*A*b - 3
*a^2*B)/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Maple [A]
time = 0.07, size = 85, normalized size = 0.97

method	result	size
norman	$\frac{\frac{a(Ab-3Ba)x^2}{b^3}+\frac{B x^6}{2b}+\frac{a^2(3Ab-9Ba)}{4b^4}}{(b x^2+a)^2}+\frac{(Ab-3Ba)\ln(b x^2+a)}{2b^4}$	76
default	$\frac{B x^2}{2b^3}+\frac{\frac{(Ab-3Ba)\ln(b x^2+a)}{b}-\frac{a^2(Ab-Ba)}{2b(b x^2+a)^2}+\frac{a(2Ab-3Ba)}{b(b x^2+a)}}{2b^3}$	85
risch	$\frac{B x^2}{2b^3}+\frac{(abA-\frac{3}{2}a^2B)x^2+\frac{(3Ab-5Ba)a^2}{4b}}{b^3(b x^2+a)^2}+\frac{\ln(b x^2+a)A}{2b^3}-\frac{3\ln(b x^2+a)Ba}{2b^4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/b^3+1/2/b^3*(1/b*(A*b-3*B*a)*ln(b*x^2+a)-1/2*a^2*(A*b-B*a)/b/(b*x
^2+a)^2+a*(2*A*b-3*B*a)/b/(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 94, normalized size = 1.07

$$-\frac{5\,Ba^3 - 3\,Aa^2b + 2\,(3\,Ba^2b - 2\,Aab^2)x^2}{4\,(b^6x^4 + 2\,ab^5x^2 + a^2b^4)} + \frac{Bx^2}{2\,b^3} - \frac{(3\,Ba - Ab)\log(bx^2 + a)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^2)/(b^6*x^4 + 2*a*b
^5*x^2 + a^2*b^4) + 1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 0.88, size = 142, normalized size = 1.61

$$\frac{2\,Bb^3x^6+4\,Bab^2x^4-5\,Ba^3+3\,Aa^2b-4\,(Ba^2b-Aab^2)x^2-2\,((3\,Bab^2-Ab^3)x^4+3\,Ba^3-Aa^2b+2\,(3\,Ba^2b-Aab^2)x^2)\log\left(bx^2+a\right)}{4\,(b^6x^4+2\,ab^5x^2+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b^3*x^6 + 4*B*a*b^2*x^4 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*b
^2)*x^2 - 2*((3*B*a*b^2 - A*b^3)*x^4 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A
*a*b^2)*x^2)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [A]
time = 1.04, size = 94, normalized size = 1.07

$$\frac{Bx^2}{2b^3}+\frac{3Aa^2b-5Ba^3+x^2\cdot\left(4Aab^2-6Ba^2b\right)}{4a^2b^4+8ab^5x^2+4b^6x^4}-\frac{\left(-Ab+3Ba\right)\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**2/(2*b**3) + (3*A*a**2*b - 5*B*a**3 + x**2*(4*A*a*b**2 - 6*B*a**2*b))/
(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - (-A*b + 3*B*a)*log(a + b*x**2
)/(2*b**4)
```

Giac [A]
time = 1.19, size = 93, normalized size = 1.06

$$\frac{Bx^2}{2\,b^3}-\frac{(3\,Ba-Ab)\log\left(|bx^2+a|\right)}{2\,b^4}+\frac{9\,Bab^2x^4-3\,Ab^3x^4+12\,Ba^2bx^2-2\,Aab^2x^2+4\,Ba^3}{4\,(bx^2+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*log(abs(b*x^2 + a))/b^4 + 1/4*(9*B*a*b^2*
x^4 - 3*A*b^3*x^4 + 12*B*a^2*b*x^2 - 2*A*a*b^2*x^2 + 4*B*a^3)/((b*x^2 + a)^
2*b^4)
```

Mupad [B]
time = 0.07, size = 95, normalized size = 1.08

$$\frac{B\,x^2}{2\,b^3}-\frac{x^2\,\left(\frac{3\,B\,a^2}{2}-A\,a\,b\right)+\frac{5\,B\,a^3-3\,A\,a^2\,b}{4\,b}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{\ln\left(b\,x^2+a\right)\,\left(A\,b-3\,B\,a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (B*x^2)/(2*b^3) - (x^2*((3*B*a^2)/2 - A*a*b) + (5*B*a^3 - 3*A*a^2*b)/(4*b))
/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + (log(a + b*x^2)*(A*b - 3*B*a))/(2*b^4)
```

3.92 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=66

$$\frac{a(Ab-aB)}{4b^3\left(a+bx^2\right)^2}-\frac{Ab-2aB}{2b^3\left(a+bx^2\right)}+\frac{B\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] 1/4*a*(A*b-B*a)/b^3/(b*x^2+a)^2+1/2*(-A*b+2*B*a)/b^3/(b*x^2+a)+1/2*B*ln(b*x
^2+a)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{Ab-2aB}{2b^3\left(a+bx^2\right)}+\frac{a(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{B\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a*(A*b - a*B))/(4*b^3*(a + b*x^2)^2) - (A*b - 2*a*B)/(2*b^3*(a + b*x^2)) +
 (B*Log[a + b*x^2])/(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^3} + \frac{Ab-2aB}{b^2(a+bx)^2} + \frac{B}{b^2(a+bx)}\right) dx, x, x^2\right)$$
$$= \frac{a(Ab-aB)}{4b^3(a+bx^2)^2} - \frac{Ab-2aB}{2b^3(a+bx^2)} + \frac{B\log(a+bx^2)}{2b^3}$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 0.97

$$\frac{3a^2B - 2Ab^2x^2 - ab(A-4Bx^2) + 2B(a+bx^2)^2\log(a+bx^2)}{4b^3(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (3*a^2*B - 2*A*b^2*x^2 - a*b*(A - 4*B*x^2) + 2*B*(a + b*x^2)^2*Log[a + b*x^
2]])/(4*b^3*(a + b*x^2)^2)
```

Maple [A]
time = 0.07, size = 61, normalized size = 0.92

method	result	size
norman	$\frac{-\frac{a(Ab-3Ba)}{4b^3}-\frac{(Ab-2Ba)x^2}{2b^2}}{(bx^2+a)^2}+\frac{B\ln(bx^2+a)}{2b^3}$	57
risch	$\frac{-\frac{a(Ab-3Ba)}{4b^3}-\frac{(Ab-2Ba)x^2}{2b^2}}{(bx^2+a)^2}+\frac{B\ln(bx^2+a)}{2b^3}$	57
default	$\frac{B\ln(bx^2+a)}{2b^3}+\frac{a(Ab-Ba)}{4b^3(bx^2+a)^2}-\frac{Ab-2Ba}{2b^3(bx^2+a)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*ln(b*x^2+a)/b^3+1/4*a*(A*b-B*a)/b^3/(b*x^2+a)^2-1/2*(A*b-2*B*a)/b^3/(
b*x^2+a)
```

Maxima [A]
time = 0.28, size = 72, normalized size = 1.09

$$\frac{3\,Ba^2 - Aab + 2\,(2\,Bab - Ab^2)x^2}{4\,(b^5x^4 + 2\,ab^4x^2 + a^2b^3)} + \frac{B\log(bx^2+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^
2*b^3) + 1/2*B*log(b*x^2 + a)/b^3
```

Fricas [A]
time = 0.83, size = 89, normalized size = 1.35

$$\frac{3\,Ba^2 - Aab + 2\,(2\,Bab - Ab^2)x^2 + 2\,(Bb^2x^4 + 2\,Babx^2 + Ba^2)\log\left(bx^2 + a\right)}{4\,(b^5x^4 + 2\,ab^4x^2 + a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2 + 2*(B*b^2*x^4 + 2*B*a*b*x^2
 + B*a^2)*log(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [A]
time = 0.49, size = 70, normalized size = 1.06

$$\frac{B\log\left(a + bx^2\right)}{2b^3} + \frac{-Aab + 3Ba^2 + x^2(-2Ab^2 + 4Bab)}{4a^2b^3 + 8ab^4x^2 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*log(a + b*x**2)/(2*b**3) + (-A*a*b + 3*B*a**2 + x**2*(-2*A*b**2 + 4*B*a*b
))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
```

Giac [A]
time = 1.42, size = 61, normalized size = 0.92

$$\frac{B\log\left(|bx^2 + a|\right)}{2\,b^3} + \frac{2\,(2\,Ba - Ab)x^2 + \frac{3\,Ba^2 - Aab}{b}}{4\,(bx^2 + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(b*x^2 + a))/b^3 + 1/4*(2*(2*B*a - A*b)*x^2 + (3*B*a^2 - A*a*b
)/b)/((b*x^2 + a)^2*b^2)
```

Mupad [B]
time = 0.06, size = 70, normalized size = 1.06

$$\frac{\frac{3\,B\,a^2 - A\,a\,b}{4\,b^3} - \frac{x^2\,(A\,b - 2\,B\,a)}{2\,b^2}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4} + \frac{B\,\ln\left(b\,x^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] ((3*B*a^2 - A*a*b)/(4*b^3) - (x^2*(A*b - 2*B*a))/(2*b^2))/(a^2 + b^2*x^4 +
2*a*b*x^2) + (B*log(a + b*x^2))/(2*b^3)
```

3.93 $\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=32

$$-\frac{\left(A+Bx^2\right)^2}{4(Ab-aB)\left(a+bx^2\right)^2}$$

[Out] -1/4*(B*x^2+A)^2/(A*b-B*a)/(b*x^2+a)^2

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {455, 37}

$$-\frac{\left(A+Bx^2\right)^2}{4\left(a+bx^2\right)^2(Ab-aB)}$$

Antiderivative was successfully verified.

[In] Int[(x*(A + B*x^2))/(a + b*x^2)^3,x]

[Out] -1/4*(A + B*x^2)^2/((A*b - a*B)*(a + b*x^2)^2)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx^2)}{(a+bx^2)^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^3} \, dx, x, x^2\right) \\
&= -\frac{\left(A+Bx^2\right)^2}{4(Ab-aB)\left(a+bx^2\right)^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.94

$$-\frac{Ab+B(a+2bx^2)}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(A*b + B*(a + 2*b*x^2))/(b^2*(a + b*x^2)^2)
```

Maple [A]
time = 0.07, size = 39, normalized size = 1.22

method	result	size
gosper	$-\frac{2bB\,x^{2}+Ab+Ba}{4b^{2}\left(b\,x^{2}+a\right)^{2}}$	29
norman	$\frac{-\frac{B\,x^{2}}{2b}-\frac{Ab+Ba}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$	33
risch	$\frac{-\frac{B\,x^{2}}{2b}-\frac{Ab+Ba}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$	33
default	$-\frac{Ab-Ba}{4b^{2}\left(b\,x^{2}+a\right)^{2}}-\frac{B}{2b^{2}\left(b\,x^{2}+a\right)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^2*(A*b-B*a)/(b*x^2+a)^2-1/2*B/b^2/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^2+Ba+Ab}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [A]
time = 0.93, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^2+Ba+Ab}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [A]
time = 0.28, size = 42, normalized size = 1.31

$$\frac{-Ab-Ba-2Bbx^2}{4a^2b^2+8ab^3x^2+4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] (-A*b - B*a - 2*B*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

Giac [A]
time = 1.44, size = 28, normalized size = 0.88

$$-\frac{2\,Bbx^2+Ba+Ab}{4\,(bx^2+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/((b*x^2 + a)^2*b^2)
```

Mupad [B]
time = 0.04, size = 44, normalized size = 1.38

$$-\frac{\frac{A\,b+B\,a}{4\,b^2}+\frac{B\,x^2}{2\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] -((A*b + B*a)/(4*b^2) + (B*x^2)/(2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.94 $\int \frac{A+Bx^2}{x(a+bx^2)^3} \, dx$

Optimal. Leaf size=68

$$\frac{Ab-aB}{4ab(a+bx^2)^2}+\frac{A}{2a^2(a+bx^2)}+\frac{A\log(x)}{a^3}-\frac{A\log(a+bx^2)}{2a^3}$$

```
[Out] 1/4*(A*b-B*a)/a/b/(b*x^2+a)^2+1/2*A/a^2/(b*x^2+a)+A*ln(x)/a^3-1/2*A*ln(b*x^
2+a)/a^3
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$-\frac{A\log(a+bx^2)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{A}{2a^2(a+bx^2)}+\frac{Ab-aB}{4ab(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*B)/(4*a*b*(a + b*x^2)^2) + A/(2*a^2*(a + b*x^2)) + (A*Log[x])/a^3
- (A*Log[a + b*x^2])/(2*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^3x}+\frac{-Ab+aB}{a(a+bx)^3}-\frac{Ab}{a^2(a+bx)^2}-\frac{Ab}{a^3(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{Ab-aB}{4ab\left(a+bx^2\right)^2}+\frac{A}{2a^2\left(a+bx^2\right)}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^2\right)}{2a^3}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.87

$$\frac{\frac{a\left(3aAb-a^2B+2Ab^2x^2\right)}{b\left(a+bx^2\right)^2}+4A\log(x)-2A\log\left(a+bx^2\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^2))/(b*(a + b*x^2)^2) + 4*A*Log[x] - 2*A*L
og[a + b*x^2])/(4*a^3)
```

Maple [A]
time = 0.07, size = 63, normalized size = 0.93

method	result	size
risch	$\frac{\frac{Ab\,x^2}{2a^2}+\frac{3Ab-Ba}{4ab}}{\left(b\,x^2+a\right)^2}+\frac{A\ln(x)}{a^3}-\frac{A\ln\left(b\,x^2+a\right)}{2a^3}$	61
default	$-\frac{A\ln\left(b\,x^2+a\right)-\frac{a^2(Ab-Ba)}{2b\left(b\,x^2+a\right)^2}-\frac{Aa}{b\,x^2+a}}{2a^3}+\frac{A\ln(x)}{a^3}$	63
norman	$\frac{-\frac{(2Ab-Ba)x^2}{2a^2}-\frac{b(3Ab-Ba)x^4}{4a^3}}{\left(b\,x^2+a\right)^2}+\frac{A\ln(x)}{a^3}-\frac{A\ln\left(b\,x^2+a\right)}{2a^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*(A*ln(b*x^2+a)-1/2*a^2*(A*b-B*a)/b/(b*x^2+a)^2-A*a/(b*x^2+a))+A*ln
(x)/a^3
```

Maxima [A]
time = 0.29, size = 77, normalized size = 1.13

$$\frac{2\,Ab^2x^2-Ba^2+3\,Aab}{4\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}-\frac{A\log\left(bx^2+a\right)}{2\,a^3}+\frac{A\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*A*b^2*x^2 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b) -
 1/2*A*log(b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3
```

Fricas [**A**]
time = 0.77, size = 119, normalized size = 1.75

$$\frac{2\,Aab^2x^2 - Ba^3 + 3\,Aa^2b - 2\,(Ab^3x^4 + 2\,Aab^2x^2 + Aa^2b)\log\left(bx^2 + a\right) + 4\,(Ab^3x^4 + 2\,Aab^2x^2 + Aa^2b)\log\left(x\right)}{4\,(a^3b^3x^4 + 2\,a^4b^2x^2 + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*A*a*b^2*x^2 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a
^2*b)*log(b*x^2 + a) + 4*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*log(x))/(a^3
*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [**A**]
time = 0.29, size = 75, normalized size = 1.10

$$\frac{A\log\left(x\right)}{a^3} - \frac{A\log\left(\frac{a}{b} + x^2\right)}{2a^3} + \frac{3Aab + 2Ab^2x^2 - Ba^2}{4a^4b + 8a^3b^2x^2 + 4a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**3,x)
```

```
[Out] A*log(x)/a**3 - A*log(a/b + x**2)/(2*a**3) + (3*A*a*b + 2*A*b**2*x**2 - B*a
**2)/(4*a**4*b + 8*a**3*b**2*x**2 + 4*a**2*b**3*x**4)
```

Giac [**A**]
time = 1.38, size = 76, normalized size = 1.12

$$\frac{A\log\left(x^2\right)}{2\,a^3} - \frac{A\log\left(\left|bx^2 + a\right|\right)}{2\,a^3} + \frac{3\,Ab^3x^4 + 8\,Aab^2x^2 - Ba^3 + 6\,Aa^2b}{4\,\left(bx^2 + a\right)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/a^3 - 1/2*A*log(abs(b*x^2 + a))/a^3 + 1/4*(3*A*b^3*x^4 + 8*A
*a*b^2*x^2 - B*a^3 + 6*A*a^2*b)/((b*x^2 + a)^2*a^3*b)
```

Mupad [**B**]
time = 0.10, size = 71, normalized size = 1.04

$$\frac{\frac{3\,A\,b-B\,a}{4\,a\,b} + \frac{A\,b\,x^2}{2\,a^2}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4} - \frac{A\,\ln\left(b\,x^2 + a\right)}{2\,a^3} + \frac{A\,\ln\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^3),x)
```

```
[Out] ((3*A*b - B*a)/(4*a*b) + (A*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (
A*log(a + b*x^2))/(2*a^3) + (A*log(x))/a^3
```

3.95 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{A}{2a^3x^2}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}$$

```
[Out] -1/2*A/a^3/x^2+1/4*(-A*b+B*a)/a^2/(b*x^2+a)^2+1/2*(-2*A*b+B*a)/a^3/(b*x^2+a
)-(3*A*b-B*a)*ln(x)/a^4+1/2*(3*A*b-B*a)*ln(b*x^2+a)/a^4
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{A}{2a^3x^2}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*A/(a^3*x^2) - (A*b - a*B)/(4*a^2*(a + b*x^2)^2) - (2*A*b - a*B)/(2*a^3
*(a + b*x^2)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2])
/(2*a^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^3x^2} + \frac{-3Ab+aB}{a^4x} - \frac{b(-Ab+aB)}{a^2(a+bx)^3} - \frac{b(-2Ab+aB)}{a^3(a+bx)^2} - \frac{b(-3Ab+aB)}{a^4(a+bx)}\right.\right.$$
$$= -\frac{A}{2a^3x^2} - \frac{Ab-aB}{4a^2\left(a+bx^2\right)^2} - \frac{2Ab-aB}{2a^3\left(a+bx^2\right)} - \frac{(3Ab-aB)\log(x)}{a^4} + \frac{(3Ab-aB)\log\left(a\right.}{2a^4}$$

Mathematica [A]
time = 0.04, size = 87, normalized size = 0.86

$$\frac{-\frac{2aA}{x^2} + \frac{a^2(-Ab+aB)}{(a+bx^2)^2} + \frac{2a(-2Ab+aB)}{a+bx^2} + 4(-3Ab+aB)\log(x) + 2(3Ab-aB)\log\left(a+bx^2\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a*A)/x^2 + (a^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (2*a*(-2*A*b + a*B))/(
a + b*x^2) + 4*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^2])/(4*a
^4)
```

Maple [A]
time = 0.08, size = 102, normalized size = 1.01

method	result	size
norman	$\frac{\frac{b(3Ab-Ba)x^4}{a^3} - \frac{A}{2a} + \frac{b^2(9Ab-3Ba)x^6}{4a^4}}{x^2\left(b\,x^2+a\right)^2} - \frac{(3Ab-Ba)\ln(x)}{a^4} + \frac{(3Ab-Ba)\ln\left(b\,x^2+a\right)}{2a^4}$	97
default	$\frac{b\left(\frac{(3Ab-Ba)\ln\left(b\,x^2+a\right)}{b} - \frac{a^2(Ab-Ba)}{2b\left(b\,x^2+a\right)^2} - \frac{a(2Ab-Ba)}{b\left(b\,x^2+a\right)}\right)}{2a^4} - \frac{A}{2a^3x^2} + \frac{(-3Ab+Ba)\ln(x)}{a^4}$	102
risch	$\frac{-\frac{b(3Ab-Ba)x^4}{2a^3} - \frac{3(3Ab-Ba)x^2}{4a^2} - \frac{A}{2a}}{x^2\left(b\,x^2+a\right)^2} - \frac{3\ln(x)Ab}{a^4} + \frac{\ln(x)B}{a^3} + \frac{3\ln\left(-b\,x^2-a\right)Ab}{2a^4} - \frac{\ln\left(-b\,x^2-a\right)B}{2a^3}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^4*b*((3*A*b-B*a)/b*ln(b*x^2+a)-1/2*a^2*(A*b-B*a)/b/(b*x^2+a)^2-a*(2*A
*b-B*a)/b/(b*x^2+a))-1/2*A/a^3/x^2+(-3*A*b+B*a)/a^4*ln(x)
```

Maxima [A]
time = 0.31, size = 109, normalized size = 1.08

$$\frac{2\left(Bab-3\,Ab^2\right)x^4 - 2\,Aa^2 + 3\left(Ba^2-3\,Aab\right)x^2}{4\left(a^3b^2x^6 + 2\,a^4bx^4 + a^5x^2\right)} - \frac{(Ba-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4} + \frac{(Ba-3\,Ab)\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(B*a*b - 3*A*b^2)*x^4 - 2*A*a^2 + 3*(B*a^2 - 3*A*a*b)*x^2)/(a^3*b^2*
x^6 + 2*a^4*b*x^4 + a^5*x^2) - 1/2*(B*a - 3*A*b)*log(b*x^2 + a)/a^4 + 1/2*(
B*a - 3*A*b)*log(x^2)/a^4
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(89) = 178$.
time = 0.64, size = 197, normalized size = 1.95

$$\frac{2\,(Ba^2b-3\,Aab^2)x^4-2\,Aa^3+3\,(Ba^3-3\,Aa^2b)x^2-2\,((Bab^2-3\,Ab^3)x^6+2\,(Ba^2b-3\,Aab^2)x^4+(Ba^3-3\,Aa^2b)x^2)\log\left(bx^2+a\right)+4\,((Bab^2-3\,Ab^3)x^6+2\,(Ba^2b-3\,Aab^2)x^4+(Ba^3-3\,Aa^2b)x^2)\log\left(x\right)}{4\,(a^4b^2x^6+2\,a^5bx^4+a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*a^2*b - 3*A*a*b^2)*x^4 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^2 - 2*
((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2)*x^4 + (B*a^3 - 3*A*a^2*b
)*x^2)*log(b*x^2 + a) + 4*((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2
)*x^4 + (B*a^3 - 3*A*a^2*b)*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x
^2)
```

Sympy [**A**]
time = 0.61, size = 107, normalized size = 1.06

$$\frac{-2Aa^2+x^4(-6Ab^2+2Bab)+x^2(-9Aab+3Ba^2)}{4a^5x^2+8a^4bx^4+4a^3b^2x^6}+\frac{(-3Ab+Ba)\log\left(x\right)}{a^4}-\frac{(-3Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] (-2*A*a**2 + x**4*(-6*A*b**2 + 2*B*a*b) + x**2*(-9*A*a*b + 3*B*a**2))/(4*a*
*5*x**2 + 8*a**4*b*x**4 + 4*a**3*b**2*x**6) + (-3*A*b + B*a)*log(x)/a**4 -
(-3*A*b + B*a)*log(a/b + x**2)/(2*a**4)
```

Giac [**A**]
time = 1.49, size = 138, normalized size = 1.37

$$\frac{(Ba-3\,Ab)\log\left(x^2\right)}{2\,a^4}-\frac{(Bab-3\,Ab^2)\log\left(|bx^2+a|\right)}{2\,a^4b}+\frac{3\,Bab^2x^4-9\,Ab^3x^4+8\,Ba^2bx^2-22\,Aab^2x^2+6\,Ba^3-14\,Aa^2b}{4\,(bx^2+a)^2a^4}-\frac{Bax^2-3\,Abx^2+Aa}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*b - 3*A*b^2)*log(abs(b*x^2 + a))/
(a^4*b) + 1/4*(3*B*a*b^2*x^4 - 9*A*b^3*x^4 + 8*B*a^2*b*x^2 - 22*A*a*b^2*x^2
```

```
 + 6*B*a^3 - 14*A*a^2*b)/((b*x^2 + a)^2*a^4) - 1/2*(B*a*x^2 - 3*A*b*x^2 + A
*a)/(a^4*x^2)
```

Mupad [B]
time = 0.06, size = 107, normalized size = 1.06

$$\frac{\ln\left(b\,x^2+a\right)\,\left(3\,A\,b-B\,a\right)}{2\,a^4}-\frac{\frac{A}{2\,a}+\frac{3\,x^2\,\left(3\,A\,b-B\,a\right)}{4\,a^2}+\frac{b\,x^4\,\left(3\,A\,b-B\,a\right)}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6}-\frac{\ln\left(x\right)\,\left(3\,A\,b-B\,a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] (log(a + b*x^2)*(3*A*b - B*a))/(2*a^4) - (A/(2*a) + (3*x^2*(3*A*b - B*a))/(
4*a^2) + (b*x^4*(3*A*b - B*a))/(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4) - (
log(x)*(3*A*b - B*a))/a^4
```

3.96 $\int \frac{A+Bx^2}{x^5 (a+bx^2)^3} \, dx$

Optimal. Leaf size=124

$$-\frac{A}{4a^3x^4}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}+\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{3b(2Ab-aB)\log\left(a+bx^2\right)}{2a^5}$$

```
[Out] -1/4*A/a^3/x^4+1/2*(3*A*b-B*a)/a^4/x^2+1/4*b*(A*b-B*a)/a^3/(b*x^2+a)^2+1/2*
b*(3*A*b-2*B*a)/a^4/(b*x^2+a)+3*b*(2*A*b-B*a)*ln(x)/a^5-3/2*b*(2*A*b-B*a)*l
n(b*x^2+a)/a^5
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$-\frac{3b(2Ab-aB)\log\left(a+bx^2\right)}{2a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}+\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{A}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

```
[Out] -1/4*A/(a^3*x^4) + (3*A*b - a*B)/(2*a^4*x^2) + (b*(A*b - a*B))/(4*a^3*(a +
b*x^2)^2) + (b*(3*A*b - 2*a*B))/(2*a^4*(a + b*x^2)) + (3*b*(2*A*b - a*B)*Lo
g[x])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x^2])/(2*a^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^3x^3} + \frac{-3Ab+aB}{a^4x^2} - \frac{3b(-2Ab+aB)}{a^5x} + \frac{b^2(-Ab+aB)}{a^3(a+bx)^3} + \frac{b^2(-3Ab+}{a^4(a+b}\right.\right.$$
$$= -\frac{A}{4a^3x^4} + \frac{3Ab-aB}{2a^4x^2} + \frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2} + \frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)} + \frac{3b(2Ab-aB)\log(x)}{a^5} - \frac{3}{}$$

Mathematica [A]
time = 0.06, size = 108, normalized size = 0.87

$$\frac{-\frac{a^2A}{x^4} - \frac{2a(-3Ab+aB)}{x^2} + \frac{a^2b(Ab-aB)}{(a+bx^2)^2} + \frac{2ab(3Ab-2aB)}{a+bx^2} + 12b(2Ab-aB)\log(x) + 6b(-2Ab+aB)\log\left(a+bx^2\right)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

```
[Out] (-((a^2*A)/x^4) - (2*a*(-3*A*b + a*B))/x^2 + (a^2*b*(A*b - a*B))/(a + b*x^2
)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^2) + 12*b*(2*A*b - a*B)*Log[x] + 6*b
*(-2*A*b + a*B)*Log[a + b*x^2])/(4*a^5)
```

Maple [A]
time = 0.08, size = 123, normalized size = 0.99

method	result	size
default	$-\frac{b^2\left(-\frac{a(3Ab-2Ba)}{b\left(b\,x^2+a\right)}+\frac{(6Ab-3Ba)\ln\left(b\,x^2+a\right)}{b}-\frac{a^2(Ab-Ba)}{2b\left(b\,x^2+a\right)^2}\right)}{2a^5}-\frac{A}{4a^3x^4}-\frac{-3Ab+Ba}{2a^4x^2}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}$	123
norman	$\frac{-\frac{A}{4a}+\frac{(2Ab-Ba)x^2}{2a^2}-\frac{b\left(6b^2A-3abB\right)x^6}{a^4}-\frac{b^2\left(18b^2A-9abB\right)x^8}{4a^5}}{x^4\left(b\,x^2+a\right)^2}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}-\frac{3b(2Ab-Ba)\ln\left(b\,x^2+a\right)}{2a^5}$	123
risch	$\frac{\frac{3b^2(2Ab-Ba)x^6}{2a^4}+\frac{9b(2Ab-Ba)x^4}{4a^3}+\frac{(2Ab-Ba)x^2}{2a^2}-\frac{A}{4a}}{x^4\left(b\,x^2+a\right)^2}+\frac{6b^2\ln(x)A}{a^5}-\frac{3b\ln(x)B}{a^4}-\frac{3b^2\ln\left(b\,x^2+a\right)A}{a^5}+\frac{3b\ln\left(b\,x^2+a\right)B}{2a^4}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^5*b^2*(-a*(3*A*b-2*B*a)/b/(b*x^2+a)+(6*A*b-3*B*a)/b*ln(b*x^2+a)-1/2*
a^2*(A*b-B*a)/b/(b*x^2+a)^2)-1/4*A/a^3/x^4-1/2*(-3*A*b+B*a)/a^4/x^2+3*b*(2*
A*b-B*a)*ln(x)/a^5
```

Maxima [A]
time = 0.29, size = 137, normalized size = 1.10

$$-\frac{6\,(Bab^2-2\,Ab^3)x^6+9\,(Ba^2b-2\,Aab^2)x^4+Aa^3+2\,(Ba^3-2\,Aa^2b)x^2}{4\,(a^4b^2x^8+2\,a^5bx^6+a^6x^4)}+\frac{3\,(Bab-2\,Ab^2)\log\left(bx^2+a\right)}{2\,a^5}-\frac{3\,(Bab-2\,Ab^2)\log\left(x^2\right)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*(B*a*b^2 - 2*A*b^3)*x^6 + 9*(B*a^2*b - 2*A*a*b^2)*x^4 + A*a^3 + 2*(
B*a^3 - 2*A*a^2*b)*x^2)/(a^4*b^2*x^8 + 2*a^5*b*x^6 + a^6*x^4) + 3/2*(B*a*b
- 2*A*b^2)*log(b*x^2 + a)/a^5 - 3/2*(B*a*b - 2*A*b^2)*log(x^2)/a^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.
time = 1.06, size = 229, normalized size = 1.85

$$-\frac{6\,(Ba^2b^2-2\,Aab^3)x^6+Aa^4+9\,(Ba^3b-2\,Aa^2b^2)x^4+2\,(Ba^4-2\,Aa^3b)x^2-6\,((Bab^3-2\,Ab^4)x^8+2\,(Ba^2b^2-2\,Aab^3)x^6+(Ba^3b-2\,Aa^2b^2)x^4)\log\left(bx^2+a\right)+12\,((Bab^3-2\,Ab^4)x^8+2\,(Ba^2b^2-2\,Aab^3)x^6+(Ba^3b-2\,Aa^2b^2)x^4)\log\left(x\right)}{4\,(a^5b^2x^8+2\,a^6bx^6+a^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + A*a^4 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^4
 + 2*(B*a^4 - 2*A*a^3*b)*x^2 - 6*((B*a*b^3 - 2*A*b^4)*x^8 + 2*(B*a^2*b^2 -
2*A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^2)*x^4)*log(b*x^2 + a) + 12*((B*a*b^3
 - 2*A*b^4)*x^8 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^2)*x
^4)*log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)
```

Sympy [A]
time = 0.72, size = 136, normalized size = 1.10

$$\frac{-Aa^3+x^6\cdot(12Ab^3-6Bab^2)+x^4\cdot(18Aab^2-9Ba^2b)+x^2\cdot(4Aa^2b-2Ba^3)}{4a^6x^4+8a^5bx^6+4a^4b^2x^8}-\frac{3b(-2Ab+Ba)\log\left(x\right)}{a^5}+\frac{3b(-2Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**3,x)
```

```
[Out] (-A*a**3 + x**6*(12*A*b**3 - 6*B*a*b**2) + x**4*(18*A*a*b**2 - 9*B*a**2*b)
+ x**2*(4*A*a**2*b - 2*B*a**3))/(4*a**6*x**4 + 8*a**5*b*x**6 + 4*a**4*b**2*
x**8) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + 3*b*(-2*A*b + B*a)*log(a/b + x**2)
/(2*a**5)
```

Giac [A]
time = 0.80, size = 133, normalized size = 1.07

$$-\frac{3\,(Bab-2\,Ab^2)\log\left(x^2\right)}{2\,a^5}+\frac{3\,(Bab^2-2\,Ab^3)\log\left(|bx^2+a|\right)}{2\,a^5b}-\frac{6\,Bab^2x^6-12\,Ab^3x^6+9\,Ba^2bx^4-18\,Aab^2x^4+2\,Ba^3x^2-4\,Aa^2bx^2+Aa^3}{4\,(bx^4+ax^2)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*(B*a*b - 2*A*b^2)*log(x^2)/a^5 + 3/2*(B*a*b^2 - 2*A*b^3)*log(abs(b*x^2
 + a))/(a^5*b) - 1/4*(6*B*a*b^2*x^6 - 12*A*b^3*x^6 + 9*B*a^2*b*x^4 - 18*A*a
*b^2*x^4 + 2*B*a^3*x^2 - 4*A*a^2*b*x^2 + A*a^3)/((b*x^4 + a*x^2)^2*a^4)
```

Mupad [B]
time = 0.08, size = 131, normalized size = 1.06

$$\frac{\frac{x^2\,\left(2\,A\,b-B\,a\right)}{2\,a^2}-\frac{A}{4\,a}+\frac{3\,b^2\,x^6\,\left(2\,A\,b-B\,a\right)}{2\,a^4}+\frac{9\,b\,x^4\,\left(2\,A\,b-B\,a\right)}{4\,a^3}}{a^2\,x^4+2\,a\,b\,x^6+b^2\,x^8}-\frac{\ln\left(b\,x^2+a\right)\,\left(6\,A\,b^2-3\,B\,a\,b\right)}{2\,a^5}+\frac{\ln\left(x\right)\,\left(6\,A\,b^2-3\,B\,a\,b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^3),x)
```

```
[Out] ((x^2*(2*A*b - B*a))/(2*a^2) - A/(4*a) + (3*b^2*x^6*(2*A*b - B*a))/(2*a^4)
+ (9*b*x^4*(2*A*b - B*a))/(4*a^3))/(a^2*x^4 + b^2*x^8 + 2*a*b*x^6) - (log(a
 + b*x^2)*(6*A*b^2 - 3*B*a*b))/(2*a^5) + (log(x)*(6*A*b^2 - 3*B*a*b))/a^5
```

3.97 $\int \frac{A+Bx^2}{x^7(a+bx^2)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{A}{6a^3x^6}+\frac{3Ab-aB}{4a^4x^4}-\frac{3b(2Ab-aB)}{2a^5x^2}-\frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)}-\frac{2b^2(5Ab-3aB)\log(x)}{a^6}+\frac{b^2(5Ab-3$$

```
[Out] -1/6*A/a^3/x^6+1/4*(3*A*b-B*a)/a^4/x^4-3/2*b*(2*A*b-B*a)/a^5/x^2-1/4*b^2*(A
*b-B*a)/a^4/(b*x^2+a)^2-1/2*b^2*(4*A*b-3*B*a)/a^5/(b*x^2+a)-2*b^2*(5*A*b-3*
B*a)*ln(x)/a^6+b^2*(5*A*b-3*B*a)*ln(b*x^2+a)/a^6
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{b^2(5Ab-3aB)\log\left(a+bx^2\right)}{a^6}-\frac{2b^2\log(x)(5Ab-3aB)}{a^6}-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)}-\frac{3b(2Ab-aB)}{2a^5x^2}-\frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}+\frac{3Ab-aB}{4a^4x^4}-\frac{A}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -1/6*A/(a^3*x^6) + (3*A*b - a*B)/(4*a^4*x^4) - (3*b*(2*A*b - a*B))/(2*a^5*x
^2) - (b^2*(A*b - a*B))/(4*a^4*(a + b*x^2)^2) - (b^2*(4*A*b - 3*a*B))/(2*a^
5*(a + b*x^2)) - (2*b^2*(5*A*b - 3*a*B)*Log[x])/a^6 + (b^2*(5*A*b - 3*a*B)*
Log[a + b*x^2])/a^6
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{a^3x^4}+\frac{-3Ab+aB}{a^4x^3}-\frac{3b(-2Ab+aB)}{a^5x^2}+\frac{2b^2(-5Ab+3aB)}{a^6x}-\frac{b^3(-Ab}{a^4(a-}\right.\right.$$
$$= -\frac{A}{6a^3x^6}+\frac{3Ab-aB}{4a^4x^4}-\frac{3b(2Ab-aB)}{2a^5x^2}-\frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)}-\frac{2b^2(5A}{}$$

Mathematica [A]
time = 0.09, size = 135, normalized size = 0.91

$$\frac{-\frac{2a^3A}{x^6}-\frac{3a^2(-3Ab+aB)}{x^4}+\frac{18ab(-2Ab+aB)}{x^2}+\frac{3a^2b^2(-Ab+aB)}{(a+bx^2)^2}+\frac{6ab^2(-4Ab+3aB)}{a+bx^2}+24b^2(-5Ab+3aB)\log(x)+12b^2(5Ab-3aB)\log\left(a+bx^2\right)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a^3*A)/x^6 - (3*a^2*(-3*A*b + a*B))/x^4 + (18*a*b*(-2*A*b + a*B))/x^2
+ (3*a^2*b^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a*b^2*(-4*A*b + 3*a*B))/(a
+ b*x^2) + 24*b^2*(-5*A*b + 3*a*B)*Log[x] + 12*b^2*(5*A*b - 3*a*B)*Log[a +
b*x^2])/(12*a^6)
```

Maple [A]
time = 0.08, size = 143, normalized size = 0.96

method	result
default	$\frac{b^3\left(\frac{(10Ab-6Ba)\ln\left(b\,x^2+a\right)}{b}-\frac{a^2(Ab-Ba)}{2b\left(b\,x^2+a\right)^2}-\frac{a(4Ab-3Ba)}{b\left(b\,x^2+a\right)}\right)}{2a^6}-\frac{A}{6a^3x^6}-\frac{-3Ab+Ba}{4a^4x^4}-\frac{3b(2Ab-Ba)}{2a^5x^2}-\frac{2b^2(5Ab-3Ba)\ln(x)}{a^6}$
norman	$\frac{-\frac{A}{6a}+\frac{(5Ab-3Ba)x^2}{12a^2}-\frac{b(5Ab-3Ba)x^4}{3a^3}+\frac{2b\left(5A\,b^3-3Ba\,b^2\right)x^8}{a^5}+\frac{b^2\left(15A\,b^3-9Ba\,b^2\right)x^{10}}{2a^6}}{x^6\left(b\,x^2+a\right)^2}+\frac{b^2(5Ab-3Ba)\ln\left(b\,x^2+a\right)}{a^6}-\frac{2b^2(5Ab-3Ba)}{a^6}$
risch	$\frac{-\frac{b^3(5Ab-3Ba)x^8}{a^5}-\frac{3b^2(5Ab-3Ba)x^6}{2a^4}-\frac{b(5Ab-3Ba)x^4}{3a^3}+\frac{(5Ab-3Ba)x^2}{12a^2}-\frac{A}{6a}}{x^6\left(b\,x^2+a\right)^2}-\frac{10b^3\ln(x)A}{a^6}+\frac{6b^2\ln(x)B}{a^5}+\frac{5b^3\ln\left(-b\,x^2-a\right)A}{a^6}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^6*b^3*((10*A*b-6*B*a)/b*ln(b*x^2+a)-1/2*a^2*(A*b-B*a)/b/(b*x^2+a)^2-a
*(4*A*b-3*B*a)/b/(b*x^2+a))-1/6*A/a^3/x^6-1/4*(-3*A*b+B*a)/a^4/x^4-3/2*b*(2
*A*b-B*a)/a^5/x^2-2*b^2*(5*A*b-3*B*a)*ln(x)/a^6
```

Maxima [A]
time = 0.28, size = 170, normalized size = 1.14

$$\frac{12\left(3\,Bab^3-5\,Ab^4\right)x^8+18\left(3\,Ba^2b^2-5\,Aab^3\right)x^6-2\,Aa^4+4\left(3\,Ba^3b-5\,Aa^2b^2\right)x^4-\left(3\,Ba^4-5\,Aa^3b\right)x^2}{12\left(a^5b^2x^{10}+2\,a^6bx^8+a^7x^6\right)}-\frac{\left(3\,Bab^2-5\,Ab^3\right)\log\left(bx^2+a\right)}{a^6}+\frac{\left(3\,Bab^2-5\,Ab^3\right)\log\left(x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(3*B*a*b^3 - 5*A*b^4)*x^8 + 18*(3*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 2*A
*a^4 + 4*(3*B*a^3*b - 5*A*a^2*b^2)*x^4 - (3*B*a^4 - 5*A*a^3*b)*x^2)/(a^5*b^
2*x^10 + 2*a^6*b*x^8 + a^7*x^6) - (3*B*a*b^2 - 5*A*b^3)*log(b*x^2 + a)/a^6
+ (3*B*a*b^2 - 5*A*b^3)*log(x^2)/a^6
```

Fricas [A]
time = 0.71, size = 267, normalized size = 1.79

$$\frac{12\,(3\,Ba^2b^3-5\,Aab^4)x^8+18\,(3\,Ba^3b^2-5\,Aa^2b^3)x^6-2\,Aa^5+4\,(3\,Ba^4b-5\,Aa^3b^2)x^4-(3\,Ba^5-5\,Aa^4b)x^2-12\,((3\,Bab^4-5\,Ab^5)x^{10}+2\,(3\,Ba^2b^3-5\,Aab^4)x^8+(3\,Ba^3b^2-5\,Aa^2b^3)x^6)\log\left(bx^2+a\right)+24\,((3\,Bab^4-5\,Ab^5)x^{10}+2\,(3\,Ba^2b^3-5\,Aab^4)x^8+(3\,Ba^3b^2-5\,Aa^2b^3)x^6)\log\left(x\right)}{12\,(a^6b^2x^{10}+2\,a^7bx^8+a^8x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + 18*(3*B*a^3*b^2 - 5*A*a^2*b^3)*x^6
 - 2*A*a^5 + 4*(3*B*a^4*b - 5*A*a^3*b^2)*x^4 - (3*B*a^5 - 5*A*a^4*b)*x^2 -
12*((3*B*a*b^4 - 5*A*b^5)*x^10 + 2*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + (3*B*a^3
*b^2 - 5*A*a^2*b^3)*x^6)*log(b*x^2 + a) + 24*((3*B*a*b^4 - 5*A*b^5)*x^10 +
2*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + (3*B*a^3*b^2 - 5*A*a^2*b^3)*x^6)*log(x))/
(a^6*b^2*x^10 + 2*a^7*b*x^8 + a^8*x^6)
```

Sympy [A]
time = 0.71, size = 165, normalized size = 1.11

$$\frac{-2Aa^4+x^8(-60Ab^4+36Bab^3)+x^6(-90Aab^3+54Ba^2b^2)+x^4(-20Aa^2b^2+12Ba^3b)+x^2\cdot(5Aa^3b-3Ba^4)}{12a^7x^6+24a^6bx^8+12a^5b^2x^{10}}+\frac{2b^2(-5Ab+3Ba)\log\left(x\right)}{a^6}-\frac{b^2(-5Ab+3Ba)\log\left(\frac{a}{b}+x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**3,x)
```

```
[Out] (-2*A*a**4 + x**8*(-60*A*b**4 + 36*B*a*b**3) + x**6*(-90*A*a*b**3 + 54*B*a*
*2*b**2) + x**4*(-20*A*a**2*b**2 + 12*B*a**3*b) + x**2*(5*A*a**3*b - 3*B*a*
*4))/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) + 2*b**2*(-5*A*b
+ 3*B*a)*log(x)/a**6 - b**2*(-5*A*b + 3*B*a)*log(a/b + x**2)/a**6
```

Giac [A]
time = 1.39, size = 201, normalized size = 1.35

$$\frac{(3\,Bab^2-5\,Ab^3)\log\left(x^2\right)}{a^6}-\frac{(3\,Bab^3-5\,Ab^4)\log\left(|bx^2+a|\right)}{a^6b}+\frac{18\,Bab^4x^4-30\,Ab^5x^4+42\,Ba^2b^3x^2-68\,Aab^4x^2+25\,Ba^3b^2-39\,Aa^2b^3}{4\,(bx^2+a)^2a^6}-\frac{66\,Bab^2x^6-110\,Ab^3x^6-18\,Ba^2bx^4+36\,Aab^2x^4+3\,Ba^3x^2-9\,Aa^2bx^2+2\,Aa^3}{12\,a^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (3*B*a*b^2 - 5*A*b^3)*log(x^2)/a^6 - (3*B*a*b^3 - 5*A*b^4)*log(abs(b*x^2 +
a))/(a^6*b) + 1/4*(18*B*a*b^4*x^4 - 30*A*b^5*x^4 + 42*B*a^2*b^3*x^2 - 68*A*
```

```
a*b^4*x^2 + 25*B*a^3*b^2 - 39*A*a^2*b^3)/((b*x^2 + a)^2*a^6) - 1/12*(66*B*a
*b^2*x^6 - 110*A*b^3*x^6 - 18*B*a^2*b*x^4 + 36*A*a*b^2*x^4 + 3*B*a^3*x^2 -
9*A*a^2*b*x^2 + 2*A*a^3)/(a^6*x^6)
```

Mupad [B]
time = 0.10, size = 155, normalized size = 1.04

$$\frac{\ln\left(b\,x^2+a\right)\,\left(5\,A\,b^3-3\,B\,a\,b^2\right)}{a^6}-\frac{\frac{A}{6\,a}-\frac{x^2\,\left(5\,A\,b-3\,B\,a\right)}{12\,a^2}+\frac{3\,b^2\,x^6\,\left(5\,A\,b-3\,B\,a\right)}{2\,a^4}+\frac{b^3\,x^8\,\left(5\,A\,b-3\,B\,a\right)}{a^5}+\frac{b\,x^4\,\left(5\,A\,b-3\,B\,a\right)}{3\,a^3}}{a^2\,x^6+2\,a\,b\,x^8+b^2\,x^{10}}-\frac{\ln\left(x\right)\,\left(10\,A\,b^3-6\,B\,a\,b^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)^3),x)
```

```
[Out] (log(a + b*x^2)*(5*A*b^3 - 3*B*a*b^2))/a^6 - (A/(6*a) - (x^2*(5*A*b - 3*B*a
))/(12*a^2) + (3*b^2*x^6*(5*A*b - 3*B*a))/(2*a^4) + (b^3*x^8*(5*A*b - 3*B*a
))/a^5 + (b*x^4*(5*A*b - 3*B*a))/(3*a^3))/(a^2*x^6 + b^2*x^10 + 2*a*b*x^8)
- (log(x)*(10*A*b^3 - 6*B*a*b^2))/a^6
```

3.98 $\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=158

$$\frac{2a^2(3Ab-5aB)x}{b^6}-\frac{a(Ab-2aB)x^3}{b^5}+\frac{(Ab-3aB)x^5}{5b^4}+\frac{Bx^7}{7b^3}-\frac{a^4(Ab-aB)x}{4b^6(a+bx^2)^2}+\frac{a^3(17Ab-21aB)x}{8b^6(a+bx^2)}-\frac{9a^{5/2}(7Ab$$

[Out] 2*a^2*(3*A*b-5*B*a)*x/b^6-a*(A*b-2*B*a)*x^3/b^5+1/5*(A*b-3*B*a)*x^5/b^4+1/7*B*x^7/b^3-1/4*a^4*(A*b-B*a)*x/b^6/(b*x^2+a)^2+1/8*a^3*(17*A*b-21*B*a)*x/b^6/(b*x^2+a)-9/8*a^(5/2)*(7*A*b-11*B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)

Rubi [A]
time = 0.19, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {466, 1828, 1824, 211}

$$-\frac{9a^{5/2}(7Ab-11aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{a^4x(Ab-aB)}{4b^6(a+bx^2)^2}+\frac{a^3x(17Ab-21aB)}{8b^6(a+bx^2)}+\frac{2a^2x(3Ab-5aB)}{b^6}-\frac{ax^3(Ab-2aB)}{b^5}+\frac{x^5(Ab-3aB)}{5b^4}+\frac{Bx^7}{7b^3}$$

Antiderivative was successfully verified.

[In] Int[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]

[Out] (2*a^2*(3*A*b - 5*a*B)*x)/b^6 - (a*(A*b - 2*a*B)*x^3)/b^5 + ((A*b - 3*a*B)*x^5)/(5*b^4) + (B*x^7)/(7*b^3) - (a^4*(A*b - a*B)*x)/(4*b^6*(a + b*x^2)^2) + (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2)) - (9*a^(5/2)*(7*A*b - 11*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{a^4(Ab-aB)x}{4b^6(a+bx^2)^2} - \frac{\int \frac{-a^4(Ab-aB)+4a^3b(Ab-aB)x^2-4a^2b^2(Ab-aB)x^4+4ab^3(Ab-aB)x^6-4b^4(Ab-aB)x}{(a+bx^2)^2}}{4b^6}$$

$$= -\frac{a^4(Ab-aB)x}{4b^6(a+bx^2)^2} + \frac{a^3(17Ab-21aB)x}{8b^6(a+bx^2)} + \frac{\int \frac{-a^4(15Ab-19aB)+8a^3b(3Ab-4aB)x^2-8a^2b^2(2Ab-3}{a+bx^2}}{8ab^6}$$

$$= -\frac{a^4(Ab-aB)x}{4b^6(a+bx^2)^2} + \frac{a^3(17Ab-21aB)x}{8b^6(a+bx^2)} + \frac{\int \Big(16a^3(3Ab-5aB) - 24a^2b(Ab-2aB)x}{}$$

$$= \frac{2a^2(3Ab-5aB)x}{b^6} - \frac{a(Ab-2aB)x^3}{b^5} + \frac{(Ab-3aB)x^5}{5b^4} + \frac{Bx^7}{7b^3} - \frac{a^4(Ab-aB)x}{4b^6(a+bx^2)^2} + \frac{a}{}$$

$$= \frac{2a^2(3Ab-5aB)x}{b^6} - \frac{a(Ab-2aB)x^3}{b^5} + \frac{(Ab-3aB)x^5}{5b^4} + \frac{Bx^7}{7b^3} - \frac{a^4(Ab-aB)x}{4b^6(a+bx^2)^2} + \frac{a}{}$$

Mathematica [A]
time = 0.07, size = 158, normalized size = 1.00

$$-\frac{2a^2(-3Ab+5aB)x}{b^6} + \frac{a(-Ab+2aB)x^3}{b^5} + \frac{(Ab-3aB)x^5}{5b^4} + \frac{Bx^7}{7b^3} + \frac{a^4(-Ab+aB)x}{4b^6(a+bx^2)^2} + \frac{a^3(17Ab-21aB)x}{8b^6(a+bx^2)} + \frac{9a^{5/2}(-7Ab+11aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
[Out] (-2*a^2*(-3*A*b + 5*a*B)*x)/b^6 + (a*(-(A*b) + 2*a*B)*x^3)/b^5 + ((A*b - 3*
a*B)*x^5)/(5*b^4) + (B*x^7)/(7*b^3) + (a^4*(-(A*b) + a*B)*x)/(4*b^6*(a + b*
x^2)^2) + (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2)) + (9*a^(5/2)*(-7*A*
b + 11*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Maple [A]
time = 0.09, size = 144, normalized size = 0.91

method	result
default	$\frac{\frac{1}{7} B\,b^{3} x^{7}+\frac{1}{5} A\,b^{3} x^{5}-\frac{3}{5} B a\,b^{2} x^{5}-A a\,b^{2} x^{3}+2 B\,a^{2} b\,x^{3}+6 A\,a^{2} b x-10 B\,a^{3} x}{b^{6}}-\frac{a^{3} \left(\frac{\left(-\frac{17}{8} b^{2} A+\frac{21}{8} a b B\right) x^{3}-\frac{a (15 A b-19 B a) x}{8}}{\left(b\,x^{2}+a\right)^{2}}+\frac{9 (7 A b-11 B a)}{8 \cdots}\right.}{b^{6}}$ …
risch	$\frac{B\,x^{7}}{7 b^{3}}+\frac{A\,x^{5}}{5 b^{3}}-\frac{3 B a\,x^{5}}{5 b^{4}}-\frac{A a\,x^{3}}{b^{4}}+\frac{2 B\,a^{2} x^{3}}{b^{5}}+\frac{6 A\,a^{2} x}{b^{5}}-\frac{10 B\,a^{3} x}{b^{6}}+\frac{\left(\frac{17}{8} a^{3} b^{2} A-\frac{21}{8} a^{4} b B\right) x^{3}+\frac{a^{4} (15 A b-19 B a) x}{8}}{b^{6} \left(b\,x^{2}+a\right)^{2}}+\frac{63 \sqrt{-\cdots}}{\cdots}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/7*B*b^3*x^7+1/5*A*b^3*x^5-3/5*B*a*b^2*x^5-A*a*b^2*x^3+2*B*a^2*b*x^
3+6*A*a^2*b*x-10*B*a^3*x)-a^3/b^6*(((-17/8*b^2*A+21/8*a*b*B)*x^3-1/8*a*(15*
A*b-19*B*a)*x)/(b*x^2+a)^2+9/8*(7*A*b-11*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2)))
```

Maxima [A]
time = 0.50, size = 171, normalized size = 1.08

$$-\frac{(21\,Ba^4b-17\,Aa^3b^2)x^3+(19\,Ba^5-15\,Aa^4b)x}{8\,(b^8x^4+2\,ab^7x^2+a^2b^6)}+\frac{9\,(11\,Ba^4-7\,Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}+\frac{5\,Bb^3x^7-7\,(3\,Bab^2-Ab^3)x^5+35\,(2\,Ba^2b-Aab^2)x^3-70\,(5\,Ba^3-3\,Aa^2b)x}{35\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((21*B*a^4*b - 17*A*a^3*b^2)*x^3 + (19*B*a^5 - 15*A*a^4*b)*x)/(b^8*x^4
 + 2*a*b^7*x^2 + a^2*b^6) + 9/8*(11*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^6) + 1/35*(5*B*b^3*x^7 - 7*(3*B*a*b^2 - A*b^3)*x^5 + 35*(2*B
*a^2*b - A*a*b^2)*x^3 - 70*(5*B*a^3 - 3*A*a^2*b)*x)/b^6
```

Fricas [A]
time = 0.85, size = 468, normalized size = 2.96

$$\left[\frac{80\,Bb^5x^{11}-16\,(11\,Bab^4-7\,Ab^5)x^9+48\,(11\,Ba^2b^3-7\,Aab^4)x^7-336\,(11\,Ba^3b^2-7\,Aa^2b^3)x^5-1050\,(11\,Ba^4b-7\,Aa^3b^2)x^3-315\,(11\,Ba^5-7\,Aa^4b+(11\,Ba^3b^2-7\,Aa^2b^3)x^4+2\,(11\,Ba^4b-7\,Aa^3b^2)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-630\,(11\,Ba^5-7\,Aa^4b)x}{560\,(b^8x^4+2\,ab^7x^2+a^2b^6)}, \frac{40\,Bb^5x^{11}-8\,(11\,Bab^4-7\,Ab^5)x^9+24\,(11\,Ba^2b^3-7\,Aab^4)x^7-168\,(11\,Ba^3b^2-7\,Aa^2b^3)x^5-525\,(11\,Ba^4b-7\,Aa^3b^2)x^3+315\,(11\,Ba^5-7\,Aa^4b+(11\,Ba^3b^2-7\,Aa^2b^3)x^4+2\,(11\,Ba^4b-7\,Aa^3b^2)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-315\,(11\,Ba^5-7\,Aa^4b)x}{280\,(b^8x^4+2\,ab^7x^2+a^2b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(80*B*b^5*x^11 - 16*(11*B*a*b^4 - 7*A*b^5)*x^9 + 48*(11*B*a^2*b^3 -
7*A*a*b^4)*x^7 - 336*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5 - 1050*(11*B*a^4*b -
7*A*a^3*b^2)*x^3 - 315*(11*B*a^5 - 7*A*a^4*b + (11*B*a^3*b^2 - 7*A*a^2*b^3)
```

```
*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt
(-a/b) - a)/(b*x^2 + a)) - 630*(11*B*a^5 - 7*A*a^4*b)*x)/(b^8*x^4 + 2*a*b^7
*x^2 + a^2*b^6), 1/280*(40*B*b^5*x^11 - 8*(11*B*a*b^4 - 7*A*b^5)*x^9 + 24*(
11*B*a^2*b^3 - 7*A*a*b^4)*x^7 - 168*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5 - 525*
(11*B*a^4*b - 7*A*a^3*b^2)*x^3 + 315*(11*B*a^5 - 7*A*a^4*b + (11*B*a^3*b^2
- 7*A*a^2*b^3)*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)*x^2)*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) - 315*(11*B*a^5 - 7*A*a^4*b)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*
b^6)]
```

Sympy [A]
time = 0.81, size = 280, normalized size = 1.77

$$\frac{Bx^7}{7b^3}+x^5\left(\frac{A}{5b^3}-\frac{3Ba}{5b^4}\right)+x^3\left(-\frac{Aa}{b^4}+\frac{2Ba^2}{b^5}\right)+x\left(\frac{6Aa^2}{b^5}-\frac{10Ba^3}{b^6}\right)-\frac{9\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)\log\left(-\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right)}{16}+\frac{9\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)\log\left(\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right)}{16}+\frac{x^3\cdot(17Aa^3b^2-21Ba^4b)+x(15Aa^4b-19Ba^5)}{8a^2b^6+16ab^7x^2+8b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**7/(7*b**3) + x**5*(A/(5*b**3) - 3*B*a/(5*b**4)) + x**3*(-A*a/b**4 + 2*
B*a**2/b**5) + x*(6*A*a**2/b**5 - 10*B*a**3/b**6) - 9*sqrt(-a**5/b**13)*(-7
*A*b + 11*B*a)*log(-9*b**6*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)/(-63*A*a**2*
b + 99*B*a**3) + x)/16 + 9*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)*log(9*b**6*s
qrt(-a**5/b**13)*(-7*A*b + 11*B*a)/(-63*A*a**2*b + 99*B*a**3) + x)/16 + (x*
*3*(17*A*a**3*b**2 - 21*B*a**4*b) + x*(15*A*a**4*b - 19*B*a**5))/(8*a**2*b*
*6 + 16*a*b**7*x**2 + 8*b**8*x**4)
```

Giac [A]
time = 1.65, size = 162, normalized size = 1.03

$$\frac{9\,(11\,Ba^4-7\,Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}-\frac{21\,Ba^4bx^3-17\,Aa^3b^2x^3+19\,Ba^5x-15\,Aa^4bx}{8\,(bx^2+a)^2b^6}+\frac{5\,Bb^{18}x^7-21\,Bab^{17}x^5+7\,Ab^{18}x^5+70\,Ba^2b^{16}x^3-35\,Aab^{17}x^3-350\,Ba^3b^{15}x+210\,Aa^2b^{16}x}{35\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 9/8*(11*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21*
B*a^4*b*x^3 - 17*A*a^3*b^2*x^3 + 19*B*a^5*x - 15*A*a^4*b*x)/((b*x^2 + a)^2*
b^6) + 1/35*(5*B*b^18*x^7 - 21*B*a*b^17*x^5 + 7*A*b^18*x^5 + 70*B*a^2*b^16*
x^3 - 35*A*a*b^17*x^3 - 350*B*a^3*b^15*x + 210*A*a^2*b^16*x)/b^21
```

Mupad [B]
time = 0.07, size = 246, normalized size = 1.56

$$x^5\left(\frac{A}{5\,b^3}-\frac{3\,B\,a}{5\,b^4}\right)-\frac{x\left(\frac{19\,B\,a^5}{8}-\frac{15\,A\,a^4\,b}{8}\right)-x^3\left(\frac{17\,A\,a^3\,b^2}{8}-\frac{21\,B\,a^4\,b}{8}\right)}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}-x^3\left(\frac{a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{B\,a^2}{b^5}\right)-x\left(\frac{B\,a^3}{b^6}-\frac{3\,a\left(\frac{3\,a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)}{b}+\frac{3\,a^2\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b^2}\right)+\frac{B\,x^7}{7\,b^3}+\frac{9\,a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,(7\,A\,b-11\,B\,a)}{11\,B\,a^4-7\,A\,a^3\,b}\right)\,(7\,A\,b-11\,B\,a)}{8\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] x^5*(A/(5*b^3) - (3*B*a)/(5*b^4)) - (x*((19*B*a^5)/8 - (15*A*a^4*b)/8) - x^
3*((17*A*a^3*b^2)/8 - (21*B*a^4*b)/8))/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) -
x^3*((a*(A/b^3 - (3*B*a)/b^4))/b + (B*a^2)/b^5) - x*((B*a^3)/b^6 - (3*a*((3
*a*(A/b^3 - (3*B*a)/b^4))/b + (3*B*a^2)/b^5))/b + (3*a^2*(A/b^3 - (3*B*a)/b
^4))/b^2) + (B*x^7)/(7*b^3) + (9*a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(7*A*b - 1
1*B*a))/(11*B*a^4 - 7*A*a^3*b))*(7*A*b - 11*B*a))/(8*b^(13/2))
```

3.99 $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{3a(Ab-2aB)x}{b^5}+\frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^5}{5b^3}+\frac{a^3(Ab-aB)x}{4b^5(a+bx^2)^2}-\frac{a^2(13Ab-17aB)x}{8b^5(a+bx^2)}+\frac{7a^{3/2}(5Ab-9aB)\tan^{-1}\left(\right.}{8b^{11/2}}$$

```
[Out] -3*a*(A*b-2*B*a)*x/b^5+1/3*(A*b-3*B*a)*x^3/b^4+1/5*B*x^5/b^3+1/4*a^3*(A*b-B
*a)*x/b^5/(b*x^2+a)^2-1/8*a^2*(13*A*b-17*B*a)*x/b^5/(b*x^2+a)+7/8*a^(3/2)*(
5*A*b-9*B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.15, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {466, 1828, 1824, 211}

$$\frac{7a^{3/2}(5Ab-9aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{a^3x(Ab-aB)}{4b^5(a+bx^2)^2}-\frac{a^2x(13Ab-17aB)}{8b^5(a+bx^2)}-\frac{3ax(Ab-2aB)}{b^5}+\frac{x^3(Ab-3aB)}{3b^4}+\frac{Bx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*(A*b - 2*a*B)*x)/b^5 + ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^5)/(5*b^3)
+ (a^3*(A*b - a*B)*x)/(4*b^5*(a + b*x^2)^2) - (a^2*(13*A*b - 17*a*B)*x)/(8*
b^5*(a + b*x^2)) + (7*a^(3/2)*(5*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{a^3(Ab-aB)x}{4b^5(a+bx^2)^2} - \frac{\int \frac{a^3(Ab-aB)-4a^2b(Ab-aB)x^2+4ab^2(Ab-aB)x^4-4b^3(Ab-aB)x^6-4b^4Bx^8}{(a+bx^2)^2}\,dx}{4b^5}$$

$$= \frac{a^3(Ab-aB)x}{4b^5(a+bx^2)^2} - \frac{a^2(13Ab-17aB)x}{8b^5(a+bx^2)} + \frac{\int \frac{a^3(11Ab-15aB)-8a^2b(2Ab-3aB)x^2+8ab^2(Ab-2aB)x^4+8a}{a+bx^2}}{8ab^5}$$

$$= \frac{a^3(Ab-aB)x}{4b^5(a+bx^2)^2} - \frac{a^2(13Ab-17aB)x}{8b^5(a+bx^2)} + \frac{\int \Big(-24a^2(Ab-2aB)+8ab(Ab-3aB)x^2+8}{8ab^5}$$

$$= -\frac{3a(Ab-2aB)x}{b^5} + \frac{(Ab-3aB)x^3}{3b^4} + \frac{Bx^5}{5b^3} + \frac{a^3(Ab-aB)x}{4b^5(a+bx^2)^2} - \frac{a^2(13Ab-17aB)x}{8b^5(a+bx^2)} +$$

$$= -\frac{3a(Ab-2aB)x}{b^5} + \frac{(Ab-3aB)x^3}{3b^4} + \frac{Bx^5}{5b^3} + \frac{a^3(Ab-aB)x}{4b^5(a+bx^2)^2} - \frac{a^2(13Ab-17aB)x}{8b^5(a+bx^2)} +$$

Mathematica [A]
time = 0.08, size = 133, normalized size = 0.96

$$\frac{x(945a^4B-525a^3b(A-3Bx^2)+8b^4x^6(5A+3Bx^2)-8ab^3x^4(35A+9Bx^2)+7a^2b^2x^2(-125A+72Bx^2))}{120b^5(a+bx^2)^2} - \frac{7a^{3/2}(-5Ab+9aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*B - 525*a^3*b*(A - 3*B*x^2) + 8*b^4*x^6*(5*A + 3*B*x^2) - 8*a*b
^3*x^4*(35*A + 9*B*x^2) + 7*a^2*b^2*x^2*(-125*A + 72*B*x^2)))/(120*b^5*(a +
 b*x^2)^2) - (7*a^(3/2)*(-5*A*b + 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^
(11/2))
```

Maple [A]
time = 0.10, size = 119, normalized size = 0.86

method	result
default	$-\frac{-\frac{1}{5}b^{2}B\,x^{5}-\frac{1}{3}A\,b^{2}x^{3}+Bab\,x^{3}+3abAx-6a^{2}Bx}{b^{5}}+\frac{a^{2}\left(\frac{\left(-\frac{13}{8}b^{2}A+\frac{17}{8}abB\right)x^{3}-\frac{a(11Ab-15Ba)x}{8}}{\left(b\,x^{2}+a\right)^{2}}+\frac{7(5Ab-9Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{5}}$
risch	$\frac{B\,x^{5}}{5b^{3}}+\frac{A\,x^{3}}{3b^{3}}-\frac{Ba\,x^{3}}{b^{4}}-\frac{3aAx}{b^{4}}+\frac{6a^{2}Bx}{b^{5}}+\frac{(-\frac{13}{8}a^{2}A\,b^{2}+\frac{17}{8}B\,a^{3}b)x^{3}-\frac{a^{3}(11Ab-15Ba)x}{8}}{b^{5}(b\,x^{2}+a)^{2}}+\frac{35\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x\right.}{16b^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/5*b^2*B*x^5-1/3*A*b^2*x^3+B*a*b*x^3+3*a*b*A*x-6*a^2*B*x)+a^2/b^5
*(((-13/8*b^2*A+17/8*a*b*B)*x^3-1/8*a*(11*A*b-15*B*a)*x)/(b*x^2+a)^2+7/8*(5
*A*b-9*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 147, normalized size = 1.07

$$\frac{(17\,Ba^{3}b-13\,Aa^{2}b^{2})x^{3}+(15\,Ba^{4}-11\,Aa^{3}b)x}{8\,(b^{7}x^{4}+2\,ab^{6}x^{2}+a^{2}b^{5})}-\frac{7\,(9\,Ba^{3}-5\,Aa^{2}b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{5}}+\frac{3\,Bb^{2}x^{5}-5\,(3\,Bab-Ab^{2})x^{3}+45\,(2\,Ba^{2}-Aab)x}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((17*B*a^3*b - 13*A*a^2*b^2)*x^3 + (15*B*a^4 - 11*A*a^3*b)*x)/(b^7*x^4
+ 2*a*b^6*x^2 + a^2*b^5) - 7/8*(9*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^5) + 1/15*(3*B*b^2*x^5 - 5*(3*B*a*b - A*b^2)*x^3 + 45*(2*B*a^2
 - A*a*b)*x)/b^5
```

Fricas [A]
time = 0.71, size = 416, normalized size = 3.01

$$\left[\frac{48\,Bb^{4}x^{9}-16\,(9\,Bab^{3}-5\,Ab^{4})x^{7}+112\,(9\,Ba^{2}b^{2}-5\,Aab^{3})x^{5}+350\,(9\,Ba^{3}b-5\,Aa^{2}b^{2})x^{3}-105\,(9\,Ba^{4}-5\,Aa^{3}b+(9\,Ba^{2}b^{2}-5\,Aab^{3})x^{4}+2\,(9\,Ba^{3}b-5\,Aa^{2}b^{2})x^{2})\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)+210\,(9\,Ba^{4}-5\,Aa^{3}b)x}{240\,(b^{7}x^{4}+2\,ab^{6}x^{2}+a^{2}b^{5})},\ \frac{24\,Bb^{4}x^{9}-8\,(9\,Bab^{3}-5\,Ab^{4})x^{7}+56\,(9\,Ba^{2}b^{2}-5\,Aab^{3})x^{5}+175\,(9\,Ba^{3}b-5\,Aa^{2}b^{2})x^{3}-105\,(9\,Ba^{4}-5\,Aa^{3}b+(9\,Ba^{2}b^{2}-5\,Aab^{3})x^{4}+2\,(9\,Ba^{3}b-5\,Aa^{2}b^{2})x^{2})\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\,(9\,Ba^{4}-5\,Aa^{3}b)x}{120\,(b^{7}x^{4}+2\,ab^{6}x^{2}+a^{2}b^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*B*b^4*x^9 - 16*(9*B*a*b^3 - 5*A*b^4)*x^7 + 112*(9*B*a^2*b^2 - 5*
A*a*b^3)*x^5 + 350*(9*B*a^3*b - 5*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 5*A*a^3*b
 + (9*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 2*(9*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(9*B*a^4 - 5*A*a^
```

3*b)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5), 1/120*(24*B*b^4*x^9 - 8*(9*B*a*b^3 - 5*A*b^4)*x^7 + 56*(9*B*a^2*b^2 - 5*A*a*b^3)*x^5 + 175*(9*B*a^3*b - 5*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 5*A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 2*(9*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(9*B*a^4 - 5*A*a^3*b)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)]

Sympy [A]
time = 0.81, size = 252, normalized size = 1.83

$$\frac{Bx^5}{5b^3}+x^3\left(\frac{A}{3b^3}-\frac{Ba}{b^4}\right)+x\left(-\frac{3Aa}{b^4}+\frac{6Ba^2}{b^5}\right)+\frac{7\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)\log\left(-\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)}{-35Aab+63Ba^2}+x\right)}{16}-\frac{7\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)\log\left(\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)}{-35Aab+63Ba^2}+x\right)}{16}+\frac{x^3(-13Aa^2b^2+17Ba^3b)+x(-11Aa^3b+15Ba^4)}{8a^2b^5+16ab^6x^2+8b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) + x**3*(A/(3*b**3) - B*a/b**4) + x*(-3*A*a/b**4 + 6*B*a**2/
b**5) + 7*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)*log(-7*b**5*sqrt(-a**3/b**11)*
(-5*A*b + 9*B*a)/(-35*A*a*b + 63*B*a**2) + x)/16 - 7*sqrt(-a**3/b**11)*(-5*
A*b + 9*B*a)*log(7*b**5*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)/(-35*A*a*b + 63*
B*a**2) + x)/16 + (x**3*(-13*A*a**2*b**2 + 17*B*a**3*b) + x*(-11*A*a**3*b +
 15*B*a**4))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x**4)
```

Giac [A]
time = 1.71, size = 138, normalized size = 1.00

$$-\frac{7\,(9\,Ba^3-5\,Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^5}+\frac{17\,Ba^3bx^3-13\,Aa^2b^2x^3+15\,Ba^4x-11\,Aa^3bx}{8\,(bx^2+a)^2b^5}+\frac{3\,Bb^{12}x^5-15\,Bab^{11}x^3+5\,Ab^{12}x^3+90\,Ba^2b^{10}x-45\,Aab^{11}x}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -7/8*(9*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/8*(17*
B*a^3*b*x^3 - 13*A*a^2*b^2*x^3 + 15*B*a^4*x - 11*A*a^3*b*x)/((b*x^2 + a)^2*
b^5) + 1/15*(3*B*b^12*x^5 - 15*B*a*b^11*x^3 + 5*A*b^12*x^3 + 90*B*a^2*b^10*
x - 45*A*a*b^11*x)/b^15
```

Mupad [B]
time = 0.04, size = 177, normalized size = 1.28

$$\frac{x\left(\frac{15\,B\,a^4}{8}-\frac{11\,A\,a^3\,b}{8}\right)-x^3\left(\frac{13\,A\,a^2\,b^2}{8}-\frac{17\,B\,a^3\,b}{8}\right)}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}-x\left(\frac{3\,a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)+x^3\left(\frac{A}{3\,b^3}-\frac{B\,a}{b^4}\right)+\frac{B\,x^5}{5\,b^3}-\frac{7\,a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(5\,A\,b-9\,B\,a)}{9\,B\,a^3-5\,A\,a^2\,b}\right)\,(5\,A\,b-9\,B\,a)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (x*((15*B*a^4)/8 - (11*A*a^3*b)/8) - x^3*((13*A*a^2*b^2)/8 - (17*B*a^3*b)/8
))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x*((3*a*(A/b^3 - (3*B*a)/b^4))/b + (
3*B*a^2)/b^5) + x^3*(A/(3*b^3) - (B*a)/b^4) + (B*x^5)/(5*b^3) - (7*a^(3/2)*
atan((a^(3/2)*b^(1/2)*x*(5*A*b - 9*B*a))/(9*B*a^3 - 5*A*a^2*b))*(5*A*b - 9*
B*a))/(8*b^(11/2))
```

3.100 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=116

$$\frac{(Ab-3aB)x}{b^4}+\frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2}+\frac{a(9Ab-13aB)x}{8b^4(a+bx^2)}-\frac{5\sqrt{a}\,(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}$$

```
[Out] (A*b-3*B*a)*x/b^4+1/3*B*x^3/b^3-1/4*a^2*(A*b-B*a)*x/b^4/(b*x^2+a)^2+1/8*a*(
9*A*b-13*B*a)*x/b^4/(b*x^2+a)-5/8*(3*A*b-7*B*a)*arctan(x*b^(1/2)/a^(1/2))*a
^(1/2)/b^(9/2)
```

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {466, 1828, 1167, 211}

$$-\frac{a^2x(Ab-aB)}{4b^4(a+bx^2)^2}-\frac{5\sqrt{a}\,(3Ab-7aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}+\frac{ax(9Ab-13aB)}{8b^4(a+bx^2)}+\frac{x(Ab-3aB)}{b^4}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x)/b^4 + (B*x^3)/(3*b^3) - (a^2*(A*b - a*B)*x)/(4*b^4*(a + b
*x^2)^2) + (a*(9*A*b - 13*a*B)*x)/(8*b^4*(a + b*x^2)) - (5*Sqrt[a]*(3*A*b -
 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2} - \frac{\int \frac{-a^2(Ab-aB)+4ab(Ab-aB)x^2-4b^2(Ab-aB)x^4-4b^3Bx^6}{(a+bx^2)^2}\,dx}{4b^4}$$

$$= -\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2} + \frac{a(9Ab-13aB)x}{8b^4(a+bx^2)} + \frac{\int \frac{-a^2(7Ab-11aB)+8ab(Ab-2aB)x^2+8ab^2Bx^4}{a+bx^2}\,dx}{8ab^4}$$

$$= -\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2} + \frac{a(9Ab-13aB)x}{8b^4(a+bx^2)} + \frac{\int \left(8a(Ab-3aB)+8abBx^2+\frac{5(-3a^2Ab+7a^3B)}{a+bx^2}\right)$$

$$= \frac{(Ab-3aB)x}{b^4} + \frac{Bx^3}{3b^3} - \frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2} + \frac{a(9Ab-13aB)x}{8b^4(a+bx^2)} - \frac{(5a(3Ab-7aB))\int \frac{1}{a+bx}$$

$$= \frac{(Ab-3aB)x}{b^4} + \frac{Bx^3}{3b^3} - \frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2} + \frac{a(9Ab-13aB)x}{8b^4(a+bx^2)} - \frac{5\sqrt{a}\,(3Ab-7aB)\tan^{-1}}{8b^{9/2}}$$

Mathematica [A]
time = 0.07, size = 113, normalized size = 0.97

$$\frac{-105a^3Bx+ab^2x^3(75A-56Bx^2)+5a^2bx(9A-35Bx^2)+8b^3x^5(3A+Bx^2)}{24b^4(a+bx^2)^2} + \frac{5\sqrt{a}\,(-3Ab+7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-105*a^3*B*x + a*b^2*x^3*(75*A - 56*B*x^2) + 5*a^2*b*x*(9*A - 35*B*x^2) +
8*b^3*x^5*(3*A + B*x^2))/(24*b^4*(a + b*x^2)^2) + (5*Sqrt[a]*(-3*A*b + 7*a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Maple [A]

time = 0.09, size = 95, normalized size = 0.82

method	result
default	$\frac{\frac{1}{3} bB\,x^{3}+Abx-3Bax}{b^{4}}-\frac{a\left(\frac{\left(-\frac{9}{8} b^{2} A+\frac{13}{8} abB\right) x^{3}-\frac{a(7Ab-11Ba)x}{8}}{\left(b\,x^{2}+a\right)^{2}}+\frac{5(3Ab-7Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{4}}$
risch	$\frac{B\,x^{3}}{3b^{3}}+\frac{Ax}{b^{3}}-\frac{3Bax}{b^{4}}+\frac{\left(\frac{9}{8}Aa\,b^{2}-\frac{13}{8}B\,a^{2}b\right)x^{3}+\frac{a^{2}(7Ab-11Ba)x}{8}}{b^{4}(b\,x^{2}+a)^{2}}+\frac{15\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)A}{16b^{4}}-\frac{35\sqrt{-ab}\,\ln\left(-\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/3*b*B*x^3+A*b*x-3*B*a*x)-a/b^4*(((-9/8*b^2*A+13/8*a*b*B)*x^3-1/8*a
*(7*A*b-11*B*a)*x)/(b*x^2+a)^2+5/8*(3*A*b-7*B*a)/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2)))
```

Maxima [A]

time = 0.52, size = 120, normalized size = 1.03

$$-\frac{(13\,Ba^{2}b-9\,Aab^{2})x^{3}+(11\,Ba^{3}-7\,Aa^{2}b)x}{8\,(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4})}+\frac{5\,(7\,Ba^{2}-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{4}}+\frac{Bbx^{3}-3\,(3\,Ba-Ab)x}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*B*a^2*b - 9*A*a*b^2)*x^3 + (11*B*a^3 - 7*A*a^2*b)*x)/(b^6*x^4 + 2
*a*b^5*x^2 + a^2*b^4) + 5/8*(7*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^4) + 1/3*(B*b*x^3 - 3*(3*B*a - A*b)*x)/b^4
```

Fricas [A]

time = 0.65, size = 358, normalized size = 3.09

$$\left[\frac{16\,Bb^{3}x^{7}-16\,(7\,Bab^{2}-3\,Ab^{3})x^{5}-50\,(7\,Ba^{2}b-3\,Aab^{2})x^{3}-15\,((7\,Bab^{2}-3\,Ab^{3})x^{4}+7\,Ba^{3}-3\,Aa^{2}b+2\,(7\,Ba^{2}b-3\,Aab^{2})x^{2})\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-30\,(7\,Ba^{3}-3\,Aa^{2}b)x}{48\,(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4})}, \frac{8\,Bb^{3}x^{7}-8\,(7\,Bab^{2}-3\,Ab^{3})x^{5}-25\,(7\,Ba^{2}b-3\,Aab^{2})x^{3}+15\,((7\,Bab^{2}-3\,Ab^{3})x^{4}+7\,Ba^{3}-3\,Aa^{2}b+2\,(7\,Ba^{2}b-3\,Aab^{2})x^{2})\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-15\,(7\,Ba^{3}-3\,Aa^{2}b)x}{24\,(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*B*b^3*x^7 - 16*(7*B*a*b^2 - 3*A*b^3)*x^5 - 50*(7*B*a^2*b - 3*A*a*
b^2)*x^3 - 15*((7*B*a*b^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*A*a^2*b + 2*(7*B*a^2
*b - 3*A*a*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
 a)) - 30*(7*B*a^3 - 3*A*a^2*b)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/24*
```

```
(8*B*b^3*x^7 - 8*(7*B*a*b^2 - 3*A*b^3)*x^5 - 25*(7*B*a^2*b - 3*A*a*b^2)*x^3
 + 15*((7*B*a*b^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*A*a^2*b + 2*(7*B*a^2*b - 3*A
*a*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 15*(7*B*a^3 - 3*A*a^2*b)*x
)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

Sympy [A]
time = 0.85, size = 214, normalized size = 1.84

$$\frac{Bx^3}{3b^3} + x\left(\frac{A}{b^3} - \frac{3Ba}{b^4}\right) - \frac{5\sqrt{-\frac{a}{b^9}}\,(-3Ab+7Ba)\log\left(-\frac{5b^4\sqrt{-\frac{a}{b^9}}\,(-3Ab+7Ba)}{-15Ab+35Ba}+x\right)}{16} + \frac{5\sqrt{-\frac{a}{b^9}}\,(-3Ab+7Ba)\log\left(\frac{5b^4\sqrt{-\frac{a}{b^9}}\,(-3Ab+7Ba)}{-15Ab+35Ba}+x\right)}{16} + \frac{x^3\cdot(9Aab^2-13Ba^2b)+x(7Aa^2b-11Ba^3)}{8a^2b^4+16ab^5x^2+8b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**3/(3*b**3) + x*(A/b**3 - 3*B*a/b**4) - 5*sqrt(-a/b**9)*(-3*A*b + 7*B*a
)*log(-5*b**4*sqrt(-a/b**9)*(-3*A*b + 7*B*a)/(-15*A*b + 35*B*a) + x)/16 + 5
*sqrt(-a/b**9)*(-3*A*b + 7*B*a)*log(5*b**4*sqrt(-a/b**9)*(-3*A*b + 7*B*a)/(
-15*A*b + 35*B*a) + x)/16 + (x**3*(9*A*a*b**2 - 13*B*a**2*b) + x*(7*A*a**2*
b - 11*B*a**3))/(8*a**2*b**4 + 16*a*b**5*x**2 + 8*b**6*x**4)
```

Giac [A]
time = 1.28, size = 111, normalized size = 0.96

$$\frac{5\,(7\,Ba^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^4} - \frac{13\,Ba^2bx^3-9\,Aab^2x^3+11\,Ba^3x-7\,Aa^2bx}{8\,(bx^2+a)^2b^4} + \frac{Bb^6x^3-9\,Bab^5x+3\,Ab^6x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/8*(7*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/8*(13*B*a
^2*b*x^3 - 9*A*a*b^2*x^3 + 11*B*a^3*x - 7*A*a^2*b*x)/((b*x^2 + a)^2*b^4) +
1/3*(B*b^6*x^3 - 9*B*a*b^5*x + 3*A*b^6*x)/b^9
```

Mupad [B]
time = 0.05, size = 138, normalized size = 1.19

$$\frac{x^3\left(\frac{9\,A\,a\,b^2}{8}-\frac{13\,B\,a^2\,b}{8}\right)-x\left(\frac{11\,B\,a^3}{8}-\frac{7\,A\,a^2\,b}{8}\right)}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4} + x\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right) + \frac{B\,x^3}{3\,b^3} + \frac{5\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(3\,A\,b-7\,B\,a)}{7\,B\,a^2-3\,A\,a\,b}\right)\,(3\,A\,b-7\,B\,a)}{8\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (x^3*((9*A*a*b^2)/8 - (13*B*a^2*b)/8) - x*((11*B*a^3)/8 - (7*A*a^2*b)/8))/(
a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^3)/(3*b^3
) + (5*a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*A*b - 7*B*a))/(7*B*a^2 - 3*A*a*b)
)*(3*A*b - 7*B*a))/(8*b^(9/2))
```

3.101 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=94

$$\frac{Bx}{b^3}+\frac{a(Ab-aB)x}{4b^3(a+bx^2)^2}-\frac{(5Ab-9aB)x}{8b^3(a+bx^2)}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}$$

```
[Out] B*x/b^3+1/4*a*(A*b-B*a)*x/b^3/(b*x^2+a)^2-1/8*(5*A*b-9*B*a)*x/b^3/(b*x^2+a)
+3/8*(A*b-5*B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {466, 1171, 396, 211}

$$\frac{3(Ab-5aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}-\frac{x(5Ab-9aB)}{8b^3(a+bx^2)}+\frac{ax(Ab-aB)}{4b^3(a+bx^2)^2}+\frac{Bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x)/b^3 + (a*(A*b - a*B)*x)/(4*b^3*(a + b*x^2)^2) - ((5*A*b - 9*a*B)*x)/(
8*b^3*(a + b*x^2)) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[
a]*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}\,dx &= \frac{a(Ab-aB)x}{4b^3(a+bx^2)^2}-\frac{\int \frac{a(Ab-aB)-4b(Ab-aB)x^2-4b^2Bx^4}{(a+bx^2)^2}\,dx}{4b^3}\\
&= \frac{a(Ab-aB)x}{4b^3(a+bx^2)^2}-\frac{(5Ab-9aB)x}{8b^3(a+bx^2)}+\frac{\int \frac{a(3Ab-7aB)+8abBx^2}{a+bx^2}\,dx}{8ab^3}\\
&= \frac{Bx}{b^3}+\frac{a(Ab-aB)x}{4b^3(a+bx^2)^2}-\frac{(5Ab-9aB)x}{8b^3(a+bx^2)}+\frac{(3(Ab-5aB))\int \frac{1}{a+bx^2}\,dx}{8b^3}\\
&= \frac{Bx}{b^3}+\frac{a(Ab-aB)x}{4b^3(a+bx^2)^2}-\frac{(5Ab-9aB)x}{8b^3(a+bx^2)}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 91, normalized size = 0.97

$$
\frac{x(15a^2B+b^2x^2(-5A+8Bx^2)+a(-3Ab+25bBx^2))}{8b^3(a+bx^2)^2}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^2*B + b^2*x^2*(-5*A + 8*B*x^2) + a*(-3*A*b + 25*b*B*x^2)))/(8*b^3*
(a + b*x^2)^2) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b
^(7/2))
```

Maple [A]
time = 0.09, size = 77, normalized size = 0.82

method	result
default	$\frac{Bx}{b^{3}}+\frac{\frac{\left(-\frac{5}{8}b^{2}A+\frac{9}{8}abB\right)x^{3}-\frac{a(3Ab-7Ba)x}{8}}{\left(b\,x^{2}+a\right)^{2}}+\frac{3(Ab-5Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^{3}}$
risch	$\frac{Bx}{b^{3}}+\frac{(-\frac{5}{8}b^{2}A+\frac{9}{8}abB)x^{3}-\frac{a(3Ab-7Ba)x}{8}}{b^{3}(b\,x^{2}+a)^{2}}-\frac{3\ln\left(bx+\sqrt{-ab}\right)A}{16b^{2}\sqrt{-ab}}+\frac{15\ln\left(bx+\sqrt{-ab}\right)Ba}{16b^{3}\sqrt{-ab}}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)A}{16b^{2}\sqrt{-ab}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b^3+1/b^3*(((-5/8*b^2*A+9/8*a*b*B)*x^3-1/8*a*(3*A*b-7*B*a)*x)/(b*x^2+a)
^2+3/8*(A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 94, normalized size = 1.00

$$\frac{(9\,Bab-5\,Ab^{2})x^{3}+(7\,Ba^{2}-3\,Aab)x}{8\,(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3})}+\frac{Bx}{b^{3}}-\frac{3\,(5\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*B*a*b - 5*A*b^2)*x^3 + (7*B*a^2 - 3*A*a*b)*x)/(b^5*x^4 + 2*a*b^4*x^
2 + a^2*b^3) + B*x/b^3 - 3/8*(5*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^3)
```

Fricas [A]
time = 0.84, size = 328, normalized size = 3.49

$$\left[\frac{16\,Bab^{3}x^{5}+10\,(5\,Ba^{2}b^{2}-Aab^{3})x^{3}+3\,((5\,Bab^{2}-Ab^{3})x^{4}+5\,Ba^{3}-Aa^{2}b+2\,(5\,Ba^{2}b-Aab^{2})x^{2})\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+6\,(5\,Ba^{3}b-Aa^{2}b^{2})x}{16\,(ab^{6}x^{4}+2\,a^{2}b^{5}x^{2}+a^{3}b^{4})},\ \frac{8\,Bab^{3}x^{5}+5\,(5\,Ba^{2}b^{2}-Aab^{3})x^{3}-3\,((5\,Bab^{2}-Ab^{3})x^{4}+5\,Ba^{3}-Aa^{2}b+2\,(5\,Ba^{2}b-Aab^{2})x^{2})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\,(5\,Ba^{3}b-Aa^{2}b^{2})x}{8\,(ab^{6}x^{4}+2\,a^{2}b^{5}x^{2}+a^{3}b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*B*a*b^3*x^5 + 10*(5*B*a^2*b^2 - A*a*b^3)*x^3 + 3*((5*B*a*b^2 - A*
b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5*B*a^2*b - A*a*b^2)*x^2)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*B*a^3*b - A*a^2*b^2)*x)/(a
*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4), 1/8*(8*B*a*b^3*x^5 + 5*(5*B*a^2*b^2 -
A*a*b^3)*x^3 - 3*((5*B*a*b^2 - A*b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5*B*a^2*
```

```
b - A*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*B*a^3*b - A*a^2*b^
2)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.
time = 0.59, size = 194, normalized size = 2.06

$$\frac{Bx}{b^3} + \frac{3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)\log\left(-\frac{3ab^3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16} - \frac{3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)\log\left(\frac{3ab^3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16} + \frac{x^3(-5Ab^2+9Bab)+x(-3Aab+7Ba^2)}{8a^2b^3+16ab^4x^2+8b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x/b**3 + 3*sqrt(-1/(a*b**7))*(-A*b + 5*B*a)*log(-3*a*b**3*sqrt(-1/(a*b**7
))*(-A*b + 5*B*a)/(-3*A*b + 15*B*a) + x)/16 - 3*sqrt(-1/(a*b**7))*(-A*b + 5
*B*a)*log(3*a*b**3*sqrt(-1/(a*b**7))*(-A*b + 5*B*a)/(-3*A*b + 15*B*a) + x)/
16 + (x**3*(-5*A*b**2 + 9*B*a*b) + x*(-3*A*a*b + 7*B*a**2))/(8*a**2*b**3 +
16*a*b**4*x**2 + 8*b**5*x**4)
```

Giac [A]
time = 0.95, size = 80, normalized size = 0.85

$$\frac{Bx}{b^3} - \frac{3\,(5\,Ba - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3} + \frac{9\,Babx^3 - 5\,Ab^2x^3 + 7\,Ba^2x - 3\,Aabx}{8\,(bx^2+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] B*x/b^3 - 3/8*(5*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/8*(9*
B*a*b*x^3 - 5*A*b^2*x^3 + 7*B*a^2*x - 3*A*a*b*x)/((b*x^2 + a)^2*b^3)
```

Mupad [B]
time = 0.08, size = 92, normalized size = 0.98

$$\frac{B\,x}{b^3} - \frac{x^3\,\left(\frac{5\,A\,b^2}{8} - \frac{9\,B\,a\,b}{8}\right) - x\,\left(\frac{7\,B\,a^2}{8} - \frac{3\,A\,a\,b}{8}\right)}{a^2\,b^3 + 2\,a\,b^4\,x^2 + b^5\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b - 5\,B\,a\right)}{8\,\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (B*x)/b^3 - (x^3*((5*A*b^2)/8 - (9*B*a*b)/8) - x*((7*B*a^2)/8 - (3*A*a*b)/8
))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + (3*atan((b^(1/2)*x)/a^(1/2))*(A*b -
5*B*a))/(8*a^(1/2)*b^(7/2))
```

3.102 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=89

$$-\frac{(Ab-aB)x}{4b^2\left(a+bx^2\right)^2}+\frac{(Ab-5aB)x}{8ab^2\left(a+bx^2\right)}+\frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}$$

```
[Out] -1/4*(A*b-B*a)*x/b^2/(b*x^2+a)^2+1/8*(A*b-5*B*a)*x/a/b^2/(b*x^2+a)+1/8*(A*b
+3*B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {466, 393, 211}

$$\frac{(3aB+Ab)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}+\frac{x(Ab-5aB)}{8ab^2\left(a+bx^2\right)}-\frac{x(Ab-aB)}{4b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*((A*b - a*B)*x)/(b^2*(a + b*x^2)^2) + ((A*b - 5*a*B)*x)/(8*a*b^2*(a +
b*x^2)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3}\,dx &= -\frac{(Ab-aB)x}{4b^2(a+bx^2)^2}-\frac{\int \frac{-Ab+aB-4bBx^2}{(a+bx^2)^2}\,dx}{4b^2}\\ &= -\frac{(Ab-aB)x}{4b^2(a+bx^2)^2}+\frac{(Ab-5aB)x}{8ab^2(a+bx^2)}+\frac{(Ab+3aB)\int \frac{1}{a+bx^2}\,dx}{8ab^2}\\ &= -\frac{(Ab-aB)x}{4b^2(a+bx^2)^2}+\frac{(Ab-5aB)x}{8ab^2(a+bx^2)}+\frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 83, normalized size = 0.93

$$\frac{\frac{\sqrt{b}\,x(-3a^2B+Ab^2x^2-ab(A+5Bx^2))}{a(a+bx^2)^2}+\frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[b]*x*(-3*a^2*B + A*b^2*x^2 - a*b*(A + 5*B*x^2)))/(a*(a + b*x^2)^2) +
 ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))/(8*b^(5/2))
```

Maple [A]
time = 0.09, size = 76, normalized size = 0.85

method	result
default	$\frac{\frac{(Ab-5Ba)x^3}{8ab}-\frac{(Ab+3Ba)x}{8b^2}}{(b\,x^2+a)^2}+\frac{(Ab+3Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^2a\sqrt{ab}}$
risch	$\frac{\frac{(Ab-5Ba)x^3}{8ab}-\frac{(Ab+3Ba)x}{8b^2}}{(b\,x^2+a)^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)A}{16\sqrt{-ab}\,ba}-\frac{3\ln\left(bx+\sqrt{-ab}\right)B}{16\sqrt{-ab}\,b^2}+\frac{\ln\left(-bx+\sqrt{-ab}\right)A}{16\sqrt{-ab}\,ba}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

[Out] (1/8*(A*b-5*B*a)/a/b*x^3-1/8*(A*b+3*B*a)/b^2*x)/(b*x^2+a)^2+1/8*(A*b+3*B*a)
/b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Maxima [A]
time = 0.54, size = 92, normalized size = 1.03

$$-\frac{(5\,Bab-Ab^2)x^3+(3\,Ba^2+Aab)x}{8\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)}+\frac{(3\,Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")

[Out] -1/8*((5*B*a*b - A*b^2)*x^3 + (3*B*a^2 + A*a*b)*x)/(a*b^4*x^4 + 2*a^2*b^3*x
^2 + a^3*b^2) + 1/8*(3*B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)

Fricas [A]
time = 0.70, size = 301, normalized size = 3.38

$$\left[-\frac{2\,(5\,Ba^2b^2-Aab^3)x^3+((3\,Bab^2+Ab^3)x^4+3\,Ba^3+Aa^2b+2\,(3\,Ba^2b+Aab^2)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,(3\,Ba^3b+Aa^2b^2)x}{16\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)},-\frac{(5\,Ba^2b^2-Aab^3)x^3-((3\,Bab^2+Ab^3)x^4+3\,Ba^3+Aa^2b+2\,(3\,Ba^2b+Aab^2)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(3\,Ba^3b+Aa^2b^2)x}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/16*(2*(5*B*a^2*b^2 - A*a*b^3)*x^3 + ((3*B*a*b^2 + A*b^3)*x^4 + 3*B*a^3
+ A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)) + 2*(3*B*a^3*b + A*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*
b^4*x^2 + a^4*b^3), -1/8*((5*B*a^2*b^2 - A*a*b^3)*x^3 - ((3*B*a*b^2 + A*b^3
)*x^4 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^2)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a) + (3*B*a^3*b + A*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a
^4*b^3)]

Sympy [A]
time = 0.39, size = 155, normalized size = 1.74

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Ab+3Ba)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Ab+3Ba)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{x^3(Ab^2-5Bab)+x(-Aab-3Ba^2)}{8a^3b^2+16a^2b^3x^2+8ab^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**3,x)

[Out] -sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x
)/16 + sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(a**2*b**2*sqrt(-1/(a**3*b**5)
) + x)/16 + (x**3*(A*b**2 - 5*B*a*b) + x*(-A*a*b - 3*B*a**2))/(8*a**3*b**2
+ 16*a**2*b**3*x**2 + 8*a*b**4*x**4)

Giac [A]
time = 1.20, size = 78, normalized size = 0.88

$$\frac{(3\,Ba + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2} - \frac{5\,Babx^3 - Ab^2x^3 + 3\,Ba^2x + Aabx}{8\,(bx^2 + a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*B*a*b*x^
3 - A*b^2*x^3 + 3*B*a^2*x + A*a*b*x)/((b*x^2 + a)^2*a*b^2)
```

Mupad [B]
time = 0.08, size = 82, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b + 3\,B\,a)}{8\,a^{3/2}\,b^{5/2}} - \frac{\frac{x\,(A\,b+3\,B\,a)}{8\,b^2} - \frac{x^3\,(A\,b-5\,B\,a)}{8\,a\,b}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(A*b + 3*B*a))/(8*a^(3/2)*b^(5/2)) - ((x*(A*b +
3*B*a))/(8*b^2) - (x^3*(A*b - 5*B*a))/(8*a*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.103 $\int \frac{A+Bx^2}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=92

$$\frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2}+\frac{(3Ab+aB)x}{8a^2b\left(a+bx^2\right)}+\frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

```
[Out] 1/4*(A*b-B*a)*x/a/b/(b*x^2+a)^2+1/8*(3*A*b+B*a)*x/a^2/b/(b*x^2+a)+1/8*(3*A*
b+B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {393, 205, 211}

$$\frac{(aB+3Ab)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(aB+3Ab)}{8a^2b\left(a+bx^2\right)}+\frac{x(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x)/(4*a*b*(a + b*x^2)^2) + ((3*A*b + a*B)*x)/(8*a^2*b*(a + b*x
^2)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
+ p, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2\right)^3}\,dx = \frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2} + \frac{(3Ab+aB)\int \frac{1}{\left(a+bx^2\right)^2}\,dx}{4ab}$$
$$= \frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2} + \frac{(3Ab+aB)x}{8a^2b\left(a+bx^2\right)} + \frac{(3Ab+aB)\int \frac{1}{a+bx^2}\,dx}{8a^2b}$$
$$= \frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2} + \frac{(3Ab+aB)x}{8a^2b\left(a+bx^2\right)} + \frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 0.91

$$\frac{x\left(-a^2B+3Ab^2x^2+ab\left(5A+Bx^2\right)\right)}{8a^2b\left(a+bx^2\right)^2} + \frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^3,x]
[Out] (x*(-(a^2*B) + 3*A*b^2*x^2 + a*b*(5*A + B*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Maple [A]
time = 0.10, size = 77, normalized size = 0.84

method	result
default	$\frac{\frac{(3Ab+Ba)x^3}{8a^2}+\frac{(5Ab-Ba)x}{8ab}}{\left(b\,x^2+a\right)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$
risch	$\frac{\frac{(3Ab+Ba)x^3}{8a^2}+\frac{(5Ab-Ba)x}{8ab}}{\left(b\,x^2+a\right)^2}-\frac{3\ln\left(bx+\sqrt{-ab}\right)A}{16\sqrt{-ab}\,a^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)B}{16\sqrt{-ab}\,ba}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)A}{16\sqrt{-ab}\,a^2}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] (1/8*(3*A*b+B*a)/a^2*x^3+1/8*(5*A*b-B*a)/a/b*x)/(b*x^2+a)^2+1/8*(3*A*b+B*a)
/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 92, normalized size = 1.00

$$\frac{(Bab+3\,Ab^2)x^3-(Ba^2-5\,Aab)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}+\frac{(Ba+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*a*b + 3*A*b^2)*x^3 - (B*a^2 - 5*A*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*
x^2 + a^4*b) + 1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Fricas [A]
time = 0.58, size = 300, normalized size = 3.26

$$\left[\frac{2\,(Ba^2b^2+3\,Aab^3)x^3-((Bab^2+3\,Ab^3)x^4+Ba^3+3\,Aa^2b+2\,(Ba^2b+3\,Aab^2)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,(Ba^3b-5\,Aa^2b^2)x}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)},\ \frac{(Ba^2b^2+3\,Aab^3)x^3+((Bab^2+3\,Ab^3)x^4+Ba^3+3\,Aa^2b+2\,(Ba^2b+3\,Aab^2)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-(Ba^3b-5\,Aa^2b^2)x}{8\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(B*a^2*b^2 + 3*A*a*b^3)*x^3 - ((B*a*b^2 + 3*A*b^3)*x^4 + B*a^3 + 3
*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*(B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((B*a^2*b^2 + 3*A*a*b^3)*x^3 + ((B*a*b^2 + 3*A*b^3)*
x^4 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) - (B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

Sympy [A]
time = 0.39, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ba)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ba)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{x^3\cdot(3Ab^2+Bab)+x(5Aab-Ba^2)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (x**3*(3*A*b**2 + B*a*b) + x*(5*A*a*b - B*a**2))/(8*a**4*b + 16*a**3*
b**2*x**2 + 8*a**2*b**3*x**4)
```

Giac [A]
time = 1.47, size = 78, normalized size = 0.85

$$\frac{(Ba+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}+\frac{Babx^3+3\,Ab^2x^3-Ba^2x+5\,Aabx}{8\,{(bx^2+a)}^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(B*a*b*x^3
+ 3*A*b^2*x^3 - B*a^2*x + 5*A*a*b*x)/((b*x^2 + a)^2*a^2*b)
```

Mupad [B]
time = 0.08, size = 82, normalized size = 0.89

$$\frac{\frac{x^3\,(3\,A\,b+B\,a)}{8\,a^2}+\frac{x\,(5\,A\,b-B\,a)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,A\,b+B\,a)}{8\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^3,x)
```

```
[Out] ((x^3*(3*A*b + B*a))/(8*a^2) + (x*(5*A*b - B*a))/(8*a*b))/(a^2 + b^2*x^4 +
2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(3*A*b + B*a))/(8*a^(5/2)*b^(3/2))
```

3.104 $\int \frac{A+Bx^2}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{A}{a^3x}-\frac{(Ab-aB)x}{4a^2(a+bx^2)^2}-\frac{(7Ab-3aB)x}{8a^3(a+bx^2)}-\frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}$$

```
[Out] -A/a^3/x-1/4*(A*b-B*a)*x/a^2/(b*x^2+a)^2-1/8*(7*A*b-3*B*a)*x/a^3/(b*x^2+a)-
3/8*(5*A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {467, 464, 211}

$$-\frac{3(5Ab-aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{x(7Ab-3aB)}{8a^3(a+bx^2)}-\frac{A}{a^3x}-\frac{x(Ab-aB)}{4a^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) - ((A*b - a*B)*x)/(4*a^2*(a + b*x^2)^2) - ((7*A*b - 3*a*B)*x)/
(8*a^3*(a + b*x^2)) - (3*(5*A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7
/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
```

```
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^3}\,dx &= -\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2}-\frac{1}{4}\int \frac{-\frac{4A}{a}+\frac{3(Ab-aB)x^2}{a^2}}{x^2\left(a+bx^2\right)^2}\,dx\\ &= -\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2}-\frac{(7Ab-3aB)x}{8a^3\left(a+bx^2\right)}+\frac{1}{8}\int \frac{\frac{8A}{a^2}-\frac{(7Ab-3aB)x^2}{a^3}}{x^2\left(a+bx^2\right)}\,dx\\ &= -\frac{A}{a^3x}-\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2}-\frac{(7Ab-3aB)x}{8a^3\left(a+bx^2\right)}-\frac{(3(5Ab-aB))\int \frac{1}{a+bx^2}\,dx}{8a^3}\\ &= -\frac{A}{a^3x}-\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2}-\frac{(7Ab-3aB)x}{8a^3\left(a+bx^2\right)}-\frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 96, normalized size = 0.99

$$-\frac{A}{a^3x}+\frac{(-Ab+aB)x}{4a^2\left(a+bx^2\right)^2}+\frac{(-7Ab+3aB)x}{8a^3\left(a+bx^2\right)}+\frac{3(-5Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + ((-(A*b) + a*B)*x)/(4*a^2*(a + b*x^2)^2) + ((-7*A*b + 3*a*B)
*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(7/2)*Sqrt[b])
```

Maple [A]
time = 0.08, size = 82, normalized size = 0.85

method	result
default	$-\frac{\frac{\left(\frac{7}{8}b^2A-\frac{3}{8}abB\right)x^3+\frac{a(9Ab-5Ba)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{3(5Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{a^3}-\frac{A}{a^3x}$

risch	$\frac{-\frac{3b(5Ab-Ba)x^4}{8a^3}-\frac{5(5Ab-Ba)x^2}{8a^2}-\frac{A}{a}}{x(b\,x^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(a^7b\_Z^2+25A^2b^2-10ABab+B^2a^2\right)}\_R\ln\left(\left(3\_R^2a^7b+50A^2b^2-20ABab+2\right.\right.\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((((7/8*b^2*A-3/8*a*b*B)*x^3+1/8*a*(9*A*b-5*B*a)*x)/(b*x^2+a)^2+3/8*(
5*A*b-B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-A/a^3/x
```

Maxima [A]
time = 0.50, size = 96, normalized size = 0.99

$$\frac{3\,(Bab-5\,Ab^2)x^4-8\,Aa^2+5\,(Ba^2-5\,Aab)x^2}{8\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}+\frac{3\,(Ba-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*a*b - 5*A*b^2)*x^4 - 8*A*a^2 + 5*(B*a^2 - 5*A*a*b)*x^2)/(a^3*b^2*
x^5 + 2*a^4*b*x^3 + a^5*x) + 3/8*(B*a - 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a^3)
```

Fricas [A]
time = 0.69, size = 324, normalized size = 3.34

$$\left[-\frac{16\,Aa^3b-6\,(Ba^2b^2-5\,Aab^3)x^4-10\,(Ba^3b-5\,Aa^2b^2)x^2-3\,((Bab^2-5\,Ab^3)x^5+2\,(Ba^2b-5\,Aab^2)x^3+(Ba^3-5\,Aa^2b)x)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\,(a^4b^3x^5+2\,a^5b^2x^3+a^6bx)},-\frac{8\,Aa^3b-3\,(Ba^2b^2-5\,Aab^3)x^4-5\,(Ba^3b-5\,Aa^2b^2)x^2-3\,((Bab^2-5\,Ab^3)x^5+2\,(Ba^2b-5\,Aab^2)x^3+(Ba^3-5\,Aa^2b)x)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\,(a^4b^3x^5+2\,a^5b^2x^3+a^6bx)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*A*a^3*b - 6*(B*a^2*b^2 - 5*A*a*b^3)*x^4 - 10*(B*a^3*b - 5*A*a^2*
b^2)*x^2 - 3*((B*a*b^2 - 5*A*b^3)*x^5 + 2*(B*a^2*b - 5*A*a*b^2)*x^3 + (B*a^
3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))
/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*A*a^3*b - 3*(B*a^2*b^2 -
5*A*a*b^3)*x^4 - 5*(B*a^3*b - 5*A*a^2*b^2)*x^2 - 3*((B*a*b^2 - 5*A*b^3)*x^5
 + 2*(B*a^2*b - 5*A*a*b^2)*x^3 + (B*a^3 - 5*A*a^2*b)*x)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(94) = 188.
time = 0.37, size = 194, normalized size = 2.00

$$-\frac{3\sqrt{-\frac{1}{a^7b}}\,(-5Ab+Ba)\log\left(-\frac{3a^4\sqrt{-\frac{1}{a^7b}}\,(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^7b}}\,(-5Ab+Ba)\log\left(\frac{3a^4\sqrt{-\frac{1}{a^7b}}\,(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16}+\frac{-8Aa^2+x^4(-15Ab^2+3Bab)+x^2(-25Aab+5Ba^2)}{8a^5x+16a^4bx^3+8a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**7*b))*(-5*A*b + B*a)*log(-3*a**4*sqrt(-1/(a**7*b))*(-5*A*b +
 B*a)/(-15*A*b + 3*B*a) + x)/16 + 3*sqrt(-1/(a**7*b))*(-5*A*b + B*a)*log(3*
a**4*sqrt(-1/(a**7*b))*(-5*A*b + B*a)/(-15*A*b + 3*B*a) + x)/16 + (-8*A*a**
2 + x**4*(-15*A*b**2 + 3*B*a*b) + x**2*(-25*A*a*b + 5*B*a**2))/(8*a**5*x +
16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

Giac [A]
time = 1.23, size = 82, normalized size = 0.85

$$\frac{3\,(Ba-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}-\frac{A}{a^3x}+\frac{3\,Babx^3-7\,Ab^2x^3+5\,Ba^2x-9\,Aabx}{8\,(bx^2+a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(B*a - 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - A/(a^3*x) + 1/8*(
3*B*a*b*x^3 - 7*A*b^2*x^3 + 5*B*a^2*x - 9*A*a*b*x)/((b*x^2 + a)^2*a^3)
```

Mupad [B]
time = 0.10, size = 113, normalized size = 1.16

$$-\frac{\frac{A}{a}+\frac{5\,x^2\,(5\,A\,b-B\,a)}{8\,a^2}+\frac{3\,b\,x^4\,(5\,A\,b-B\,a)}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{b}\,x\,(5\,A\,b-B\,a)}{\sqrt{a}\,(15\,A\,b-3\,B\,a)}\right)\,(5\,A\,b-B\,a)}{8\,a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] - (A/a + (5*x^2*(5*A*b - B*a))/(8*a^2) + (3*b*x^4*(5*A*b - B*a))/(8*a^3))/(
a^2*x + b^2*x^5 + 2*a*b*x^3) - (3*atan((3*b^(1/2)*x*(5*A*b - B*a))/(a^(1/2)
*(15*A*b - 3*B*a)))*(5*A*b - B*a))/(8*a^(7/2)*b^(1/2))
```

3.105 $\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=117

$$-\frac{A}{3a^3x^3}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}+\frac{5\sqrt{b}\,(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}$$

```
[Out] -1/3*A/a^3/x^3+(3*A*b-B*a)/a^4/x+1/4*b*(A*b-B*a)*x/a^3/(b*x^2+a)^2+1/8*b*(1
1*A*b-7*B*a)*x/a^4/(b*x^2+a)+5/8*(7*A*b-3*B*a)*arctan(x*b^(1/2)/a^(1/2))*b^
(1/2)/a^(9/2)
```

Rubi [A]
time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {467, 1273, 1275, 211}

$$\frac{5\sqrt{b}\,(7Ab-3aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{bx(11Ab-7aB)}{8a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{a^4x}+\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{A}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -1/3*A/(a^3*x^3) + (3*A*b - a*B)/(a^4*x) + (b*(A*b - a*B)*x)/(4*a^3*(a + b*
x^2)^2) + (b*(11*A*b - 7*a*B)*x)/(8*a^4*(a + b*x^2)) + (5*Sqrt[b]*(7*A*b -
3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx &= \frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}-\frac{1}{4}b\int \frac{-\frac{4A}{ab}+\frac{4(Ab-aB)x^2}{a^2b}-\frac{3(Ab-aB)x^4}{a^3}}{x^4\left(a+bx^2\right)^2}\,dx\\
&= \frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}-\frac{\int \frac{-8aAb+8b(2Ab-aB)x^2-\frac{b^2(11Ab-7aB)x^4}{a}}{x^4(a+bx^2)}\,dx}{8a^3b}\\
&= \frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}-\frac{\int \left(-\frac{8Ab}{x^4}-\frac{8b(-3Ab+aB)}{ax^2}+\frac{5b^2(-7Ab+3aB)}{a(a+bx^2)}\right)\,dx}{8a^3b}\\
&= -\frac{A}{3a^3x^3}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}+\frac{(5b(7Ab-3aB))\int \frac{1}{a+bx^2}}{8a^4}\\
&= -\frac{A}{3a^3x^3}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}+\frac{5\sqrt{b}\,(7Ab-3aB)\tan^{-1}}{8a^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 116, normalized size = 0.99

$$\frac{105Ab^3x^6+a^2bx^2(56A-75Bx^2)+5ab^2x^4(35A-9Bx^2)-8a^3(A+3Bx^2)}{24a^4x^3\left(a+bx^2\right)^2}+\frac{5\sqrt{b}\,(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (105*A*b^3*x^6 + a^2*b*x^2*(56*A - 75*B*x^2) + 5*a*b^2*x^4*(35*A - 9*B*x^2)
 - 8*a^3*(A + 3*B*x^2))/(24*a^4*x^3*(a + b*x^2)^2) + (5*Sqrt[b]*(7*A*b - 3*
a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Maple [A]
time = 0.09, size = 98, normalized size = 0.84

method	result
default	$\frac{b\left(\frac{\left(\frac{11}{8}b^2A-\frac{7}{8}abB\right)x^3+\frac{a(13Ab-9Ba)x}{8}}{\left(bx^2+a\right)^2}+\frac{5(7Ab-3Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}-\frac{A}{3a^3x^3}-\frac{-3Ab+Ba}{a^4x}$
risch	$\frac{\frac{5b^2(7Ab-3Ba)x^6}{8a^4}+\frac{25b(7Ab-3Ba)x^4}{24a^3}+\frac{(7Ab-3Ba)x^2}{3a^2}-\frac{A}{3a}}{x^3(bx^2+a)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^2+49A^2b^3-42ABab^2+9B^2a^2b\right)}\_R\ln\left(\left(3\_R^2a^9+\right.\right.\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*b*(((11/8*b^2*A-7/8*a*b*B)*x^3+1/8*a*(13*A*b-9*B*a)*x)/(b*x^2+a)^2+5/
8*(7*A*b-3*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3*A/a^3/x^3-(-3*A*b+
B*a)/a^4/x
```

Maxima [A]
time = 0.51, size = 128, normalized size = 1.09

$$-\frac{15\,(3\,Bab^2-7\,Ab^3)x^6+25\,(3\,Ba^2b-7\,Aab^2)x^4+8\,Aa^3+8\,(3\,Ba^3-7\,Aa^2b)x^2}{24\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}-\frac{5\,(3\,Bab-7\,Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*(3*B*a*b^2 - 7*A*b^3)*x^6 + 25*(3*B*a^2*b - 7*A*a*b^2)*x^4 + 8*A*
a^3 + 8*(3*B*a^3 - 7*A*a^2*b)*x^2)/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3) -
5/8*(3*B*a*b - 7*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 0.58, size = 368, normalized size = 3.15

$$\left[-\frac{30\,(3\,Bab^2-7\,Ab^3)x^6+50\,(3\,Ba^2b-7\,Aab^2)x^4+16\,Aa^3+16\,(3\,Ba^3-7\,Aa^2b)x^2+15\,((3\,Bab^2-7\,Ab^3)x^7+2\,(3\,Ba^2b-7\,Aab^2)x^5+(3\,Ba^3-7\,Aa^2b)x^3)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)},\ -\frac{15\,(3\,Bab^2-7\,Ab^3)x^6+25\,(3\,Ba^2b-7\,Aab^2)x^4+8\,Aa^3+8\,(3\,Ba^3-7\,Aa^2b)x^2+15\,((3\,Bab^2-7\,Ab^3)x^7+2\,(3\,Ba^2b-7\,Aab^2)x^5+(3\,Ba^3-7\,Aa^2b)x^3)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{24\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(30*(3*B*a*b^2 - 7*A*b^3)*x^6 + 50*(3*B*a^2*b - 7*A*a*b^2)*x^4 + 16*
A*a^3 + 16*(3*B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 - 7*A*b^3)*x^7 + 2*(3
*B*a^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^2*b)*x^3)*sqrt(-b/a)*log((b*x^
2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^
3), -1/24*(15*(3*B*a*b^2 - 7*A*b^3)*x^6 + 25*(3*B*a^2*b - 7*A*a*b^2)*x^4 +
8*A*a^3 + 8*(3*B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 - 7*A*b^3)*x^7 + 2*(
3*B*a^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^2*b)*x^3)*sqrt(b/a)*arctan(x*
sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(109) = 218$.
time = 0.48, size = 226, normalized size = 1.93

$$\frac{5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)\log\left(-\frac{5a^5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)}{-35Ab^2+15Bab}+x\right)}{16}-\frac{5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)\log\left(\frac{5a^5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)}{-35Ab^2+15Bab}+x\right)}{16}+\frac{-8Aa^3+x^6\cdot(105Ab^3-45Bab^2)+x^4\cdot(175Aab^2-75Ba^2b)+x^2\cdot(56Aa^2b-24Ba^3)}{24a^6x^3+48a^5bx^5+24a^4b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**3,x)
```

```
[Out] 5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)*log(-5*a**5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)
/(-35*A*b**2 + 15*B*a*b) + x)/16 - 5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)*log(5*a
**5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)/(-35*A*b**2 + 15*B*a*b) + x)/16 + (-8*A*
a**3 + x**6*(105*A*b**3 - 45*B*a*b**2) + x**4*(175*A*a*b**2 - 75*B*a**2*b)
+ x**2*(56*A*a**2*b - 24*B*a**3))/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a**4*
b**2*x**7)
```

Giac [A]
time = 0.93, size = 108, normalized size = 0.92

$$-\frac{5\,(3\,Bab-7\,Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}-\frac{7\,Bab^2x^3-11\,Ab^3x^3+9\,Ba^2bx-13\,Aab^2x}{8\,(bx^2+a)^2a^4}-\frac{3\,Bax^2-9\,Abx^2+Aa}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5/8*(3*B*a*b - 7*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/8*(7*B*a
*b^2*x^3 - 11*A*b^3*x^3 + 9*B*a^2*b*x - 13*A*a*b^2*x)/((b*x^2 + a)^2*a^4) -
 1/3*(3*B*a*x^2 - 9*A*b*x^2 + A*a)/(a^4*x^3)
```

Mupad [B]
time = 0.10, size = 114, normalized size = 0.97

$$\frac{\frac{x^2\,(7\,A\,b-3\,B\,a)}{3\,a^2}-\frac{A}{3\,a}+\frac{5\,b^2\,x^6\,(7\,A\,b-3\,B\,a)}{8\,a^4}+\frac{25\,b\,x^4\,(7\,A\,b-3\,B\,a)}{24\,a^3}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}+\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(7\,A\,b-3\,B\,a)}{8\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^3),x)
```

```
[Out] ((x^2*(7*A*b - 3*B*a))/(3*a^2) - A/(3*a) + (5*b^2*x^6*(7*A*b - 3*B*a))/(8*a
^4) + (25*b*x^4*(7*A*b - 3*B*a))/(24*a^3))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5)
+ (5*b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(7*A*b - 3*B*a))/(8*a^(9/2))
```

3.106 $\int \frac{A+Bx^2}{x^6(a+bx^2)^3}\,dx$

Optimal. Leaf size=142

$$-\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}-\frac{7b^{3/2}(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}$$

```
[Out] -1/5*A/a^3/x^5+1/3*(3*A*b-B*a)/a^4/x^3-3*b*(2*A*b-B*a)/a^5/x-1/4*b^2*(A*b-B
*a)*x/a^4/(b*x^2+a)^2-1/8*b^2*(15*A*b-11*B*a)*x/a^5/(b*x^2+a)-7/8*b^(3/2)*(
9*A*b-5*B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A]
time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {467, 1819, 1816, 211}

$$-\frac{7b^{3/2}(9Ab-5aB)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}-\frac{b^2x(15Ab-11aB)}{8a^5\left(a+bx^2\right)}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2x(Ab-aB)}{4a^4\left(a+bx^2\right)^2}+\frac{3Ab-aB}{3a^4x^3}-\frac{A}{5a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -1/5*A/(a^3*x^5) + (3*A*b - a*B)/(3*a^4*x^3) - (3*b*(2*A*b - a*B))/(a^5*x)
- (b^2*(A*b - a*B)*x)/(4*a^4*(a + b*x^2)^2) - (b^2*(15*A*b - 11*a*B)*x)/(8*
a^5*(a + b*x^2)) - (7*b^(3/2)*(9*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*a^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx = -\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2} - \frac{1}{4}b^2\int \frac{-\frac{4A}{ab^2}+\frac{4(Ab-aB)x^2}{a^2b^2}-\frac{4(Ab-aB)x^4}{a^3b}+\frac{3(Ab-aB)x^6}{a^4}}{x^6\left(a+bx^2\right)^2}\,dx$$

$$= -\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2} - \frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)} + \frac{b^2\int \frac{\frac{8A}{ab^2}-\frac{8(2Ab-aB)x^2}{a^2b^2}+\frac{8(3Ab-2aB)x^4}{a^3b}-\frac{(15Ab-11aB)x^6}{a^4}}{x^6(a+bx^2)}}{8a}$$

$$= -\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2} - \frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)} + \frac{b^2\int\left(\frac{8A}{a^2b^2x^6}+\frac{8(-3Ab+aB)}{a^3b^2x^4}-\frac{24(-2Ab+aB)}{a^4bx^2}+7(\right.}{8a}$$

$$= -\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}-\frac{(7b}{}$$

$$= -\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}-\frac{7b^{3/}}{}$$

Mathematica [A]
time = 0.07, size = 139, normalized size = 0.98

$$\frac{-945Ab^4x^8+525ab^3x^6(-3A+Bx^2)-8a^4(3A+5Bx^2)+8a^3bx^2(9A+35Bx^2)+7a^2b^2x^4(-72A+125Bx^2)}{120a^5x^5\left(a+bx^2\right)^2}+\frac{7b^{3/2}(-9Ab+5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] (-945*A*b^4*x^8 + 525*a*b^3*x^6*(-3*A + B*x^2) - 8*a^4*(3*A + 5*B*x^2) + 8*
a^3*b*x^2*(9*A + 35*B*x^2) + 7*a^2*b^2*x^4*(-72*A + 125*B*x^2))/(120*a^5*x^
```

```
5*(a + b*x^2)^2) + (7*b^(3/2)*(-9*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(8*a^(11/2))
```

Maple [A]
time = 0.10, size = 119, normalized size = 0.84

method	result
default	$-\frac{b^2\left(\frac{\left(\frac{15}{8}b^2A-\frac{11}{8}abB\right)x^3+\frac{a(17Ab-13Ba)x}{8}}{\left(bx^2+a\right)^2}+\frac{7(9Ab-5Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^5}-\frac{A}{5a^3x^5}-\frac{-3Ab+Ba}{3a^4x^3}-\frac{3b(2Ab-Ba)}{a^5x}$
risch	$\frac{-\frac{7b^3(9Ab-5Ba)x^8}{8a^5}-\frac{35b^2(9Ab-5Ba)x^6}{24a^4}-\frac{7b(9Ab-5Ba)x^4}{15a^3}+\frac{(9Ab-5Ba)x^2}{15a^2}-\frac{A}{5a}}{x^5\left(bx^2+a\right)^2}+\frac{63\sqrt{-ab}\,b^2\ln\left(-bx+\sqrt{-ab}\right)A}{16a^6}-\frac{35\sqrt{-ab}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^5*b^2*(((15/8*b^2*A-11/8*a*b*B)*x^3+1/8*a*(17*A*b-13*B*a)*x)/(b*x^2+a)
^2+7/8*(9*A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/5*A/a^3/x^5-1/3
*(-3*A*b+B*a)/a^4/x^3-3*b*(2*A*b-B*a)/a^5/x
```

Maxima [A]
time = 0.51, size = 154, normalized size = 1.08

$$\frac{105\,(5\,Bab^3-9\,Ab^4)x^8+175\,(5\,Ba^2b^2-9\,Aab^3)x^6-24\,Aa^4+56\,(5\,Ba^3b-9\,Aa^2b^2)x^4-8\,(5\,Ba^4-9\,Aa^3b)x^2}{120\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}+\frac{7\,(5\,Bab^2-9\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(105*(5*B*a*b^3 - 9*A*b^4)*x^8 + 175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 -
24*A*a^4 + 56*(5*B*a^3*b - 9*A*a^2*b^2)*x^4 - 8*(5*B*a^4 - 9*A*a^3*b)*x^2)/
(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5) + 7/8*(5*B*a*b^2 - 9*A*b^3)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Fricas [A]
time = 1.00, size = 426, normalized size = 3.00

$$\left[\frac{210\,(5\,Bab^3-9\,Ab^4)x^8+350\,(5\,Ba^2b^2-9\,Aab^3)x^6-48\,Aa^4+112\,(5\,Ba^3b-9\,Aa^2b^2)x^4-16\,(5\,Ba^4-9\,Aa^3b)x^2-105\,((5\,Bab^3-9\,Ab^4)x^9+2\,(5\,Ba^2b^2-9\,Aab^3)x^7+(5\,Ba^3b-9\,Aa^2b^2)x^5)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{240\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)},\ \frac{105\,(5\,Bab^3-9\,Ab^4)x^8+175\,(5\,Ba^2b^2-9\,Aab^3)x^6-24\,Aa^4+56\,(5\,Ba^3b-9\,Aa^2b^2)x^4-8\,(5\,Ba^4-9\,Aa^3b)x^2+105\,((5\,Bab^3-9\,Ab^4)x^9+2\,(5\,Ba^2b^2-9\,Aab^3)x^7+(5\,Ba^3b-9\,Aa^2b^2)x^5)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{120\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(210*(5*B*a*b^3 - 9*A*b^4)*x^8 + 350*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 -
 48*A*a^4 + 112*(5*B*a^3*b - 9*A*a^2*b^2)*x^4 - 16*(5*B*a^4 - 9*A*a^3*b)*x^
```

```
2 - 105*((5*B*a*b^3 - 9*A*b^4)*x^9 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^7 + (5*B
*a^3*b - 9*A*a^2*b^2)*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5), 1/120*(105*(5*B*a*b^3 -
9*A*b^4)*x^8 + 175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 - 24*A*a^4 + 56*(5*B*a^3*b
 - 9*A*a^2*b^2)*x^4 - 8*(5*B*a^4 - 9*A*a^3*b)*x^2 + 105*((5*B*a*b^3 - 9*A*b
^4)*x^9 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^7 + (5*B*a^3*b - 9*A*a^2*b^2)*x^5)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)]
```

Sympy [A]
time = 0.47, size = 260, normalized size = 1.83

$$-\frac{7\sqrt{-\frac{b^3}{a^{11}}}\,(-9Ab+5Ba)\log\left(-\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}\,(-9Ab+5Ba)}{-63Ab^3+35Bab^2}+x\right)}{16}+\frac{7\sqrt{-\frac{b^3}{a^{11}}}\,(-9Ab+5Ba)\log\left(\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}\,(-9Ab+5Ba)}{-63Ab^3+35Bab^2}+x\right)}{16}+\frac{-24Aa^4+x^8(-945Ab^4+525Bab^3)+x^6(-1575Aab^3+875Ba^2b^2)+x^4(-504Aa^2b^2+280Ba^3b)+x^2\cdot(72Aa^3b-40Ba^4)}{120a^7x^5+240a^6bx^7+120a^5b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**3,x)
```

```
[Out] -7*sqrt(-b**3/a**11)*(-9*A*b + 5*B*a)*log(-7*a**6*sqrt(-b**3/a**11)*(-9*A*b
 + 5*B*a)/(-63*A*b**3 + 35*B*a*b**2) + x)/16 + 7*sqrt(-b**3/a**11)*(-9*A*b
+ 5*B*a)*log(7*a**6*sqrt(-b**3/a**11)*(-9*A*b + 5*B*a)/(-63*A*b**3 + 35*B*a
*b**2) + x)/16 + (-24*A*a**4 + x**8*(-945*A*b**4 + 525*B*a*b**3) + x**6*(-1
575*A*a*b**3 + 875*B*a**2*b**2) + x**4*(-504*A*a**2*b**2 + 280*B*a**3*b) +
x**2*(72*A*a**3*b - 40*B*a**4))/(120*a**7*x**5 + 240*a**6*b*x**7 + 120*a**5
*b**2*x**9)
```

Giac [A]
time = 0.95, size = 135, normalized size = 0.95

$$\frac{7\,(5\,Bab^2-9\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}+\frac{11\,Bab^3x^3-15\,Ab^4x^3+13\,Ba^2b^2x-17\,Aab^3x}{8\,(bx^2+a)^2a^5}+\frac{45\,Babx^4-90\,Ab^2x^4-5\,Ba^2x^2+15\,Aabx^2-3\,Aa^2}{15\,a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 7/8*(5*B*a*b^2 - 9*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/8*(11*B
*a*b^3*x^3 - 15*A*b^4*x^3 + 13*B*a^2*b^2*x - 17*A*a*b^3*x)/((b*x^2 + a)^2*a
^5) + 1/15*(45*B*a*b*x^4 - 90*A*b^2*x^4 - 5*B*a^2*x^2 + 15*A*a*b*x^2 - 3*A*
a^2)/(a^5*x^5)
```

Mupad [B]
time = 0.10, size = 135, normalized size = 0.95

$$-\frac{\frac{A}{5\,a}-\frac{x^2\,(9\,A\,b-5\,B\,a)}{15\,a^2}+\frac{35\,b^2\,x^6\,(9\,A\,b-5\,B\,a)}{24\,a^4}+\frac{7\,b^3\,x^8\,(9\,A\,b-5\,B\,a)}{8\,a^5}+\frac{7\,b\,x^4\,(9\,A\,b-5\,B\,a)}{15\,a^3}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9}-\frac{7\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(9\,A\,b-5\,B\,a)}{8\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^3),x)
[Out] - (A/(5*a) - (x^2*(9*A*b - 5*B*a))/(15*a^2) + (35*b^2*x^6*(9*A*b - 5*B*a))/
(24*a^4) + (7*b^3*x^8*(9*A*b - 5*B*a))/(8*a^5) + (7*b*x^4*(9*A*b - 5*B*a))/
(15*a^3))/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (7*b^(3/2)*atan((b^(1/2)*x)/a^(
1/2))*(9*A*b - 5*B*a))/(8*a^(11/2))
```

3.107 $\int \frac{a+bx^2}{1+x^2} \, dx$

Optimal. Leaf size=12

$$bx + (a-b)\tan^{-1}(x)$$

```
[Out] b*x+(a-b)*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {396, 209}

$$(a-b)\text{ArcTan}(x) + bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2),x]
```

```
[Out] b*x + (a - b)*ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{1+x^2} \, dx &= bx - (-a+b) \int \frac{1}{1+x^2} \, dx \\ &= bx + (a-b)\tan^{-1}(x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$bx + (a-b)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(1 + x^2),x]
[Out] b*x + (a - b)*ArcTan[x]
```

Maple [A]
time = 0.08, size = 13, normalized size = 1.08

method	result	size
default	$bx + (a-b)\arctan(x)$	13
risch	$bx + a\arctan(x) - b\arctan(x)$	14
meijerg	$\frac{b(2x-2\arctan(x))}{2} + a\arctan(x)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] b*x+(a-b)*arctan(x)
```

Maxima [A]
time = 0.50, size = 12, normalized size = 1.00

$$bx + (a-b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^2+1),x, algorithm="maxima")
[Out] b*x + (a - b)*arctan(x)
```

Fricas [A]
time = 1.15, size = 12, normalized size = 1.00

$$bx + (a-b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^2+1),x, algorithm="fricas")
[Out] b*x + (a - b)*arctan(x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 26, normalized size = 2.17

$$bx - \frac{i(a-b)\log(x-i)}{2} + \frac{i(a-b)\log(x+i)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**2+1),x)
```

```
[Out] b*x - I*(a - b)*log(x - I)/2 + I*(a - b)*log(x + I)/2
```

Giac [A]
time = 0.80, size = 12, normalized size = 1.00

$$bx + (a - b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] b*x + (a - b)*arctan(x)
```

Mupad [B]
time = 0.04, size = 12, normalized size = 1.00

$$b\,x + \mathrm{atan}(x)\ (a - b)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + 1),x)
```

```
[Out] b*x + atan(x)*(a - b)
```

3.108 $\int \frac{a+bx^2}{1-x^2}\,dx$

Optimal. Leaf size=11

$$-bx+(a+b)\tanh^{-1}(x)$$

```
[Out] -b*x+(a+b)*arctanh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {396, 212}

$$(a+b)\tanh^{-1}(x)-bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 - x^2),x]
```

```
[Out] -(b*x) + (a + b)*ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{1-x^2}\,dx &= -bx-(-a-b)\int \frac{1}{1-x^2}\,dx\\ &= -bx+(a+b)\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.
time = 0.01, size = 28, normalized size = 2.55

$$\frac{1}{2}(-2bx-(a+b)\log(1-x)+(a+b)\log(1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(1 - x^2),x]
```

```
[Out] (-2*b*x - (a + b)*Log[1 - x] + (a + b)*Log[1 + x])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.
time = 0.08, size = 32, normalized size = 2.91

method	result	size
meijerg	$\frac{ib(2ix-2i\,\mathrm{arctanh}(x))}{2}+a\,\mathrm{arctanh}\,(x)$	20
norman	$-bx+\left(-\frac{a}{2}-\frac{b}{2}\right)\ln\,(x-1)+\left(\frac{a}{2}+\frac{b}{2}\right)\ln\,(x+1)$	30
default	$-bx+\frac{(-a-b)\ln(x-1)}{2}-\frac{(-a-b)\ln(x+1)}{2}$	32
risch	$-bx-\frac{\ln(x-1)a}{2}-\frac{\ln(x-1)b}{2}+\frac{\ln(x+1)a}{2}+\frac{\ln(x+1)b}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -b*x+1/2*(-a-b)*ln(x-1)-1/2*(-a-b)*ln(x+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 0.28, size = 23, normalized size = 2.09

$$-bx+\frac{1}{2}\,(a+b)\log\,(x+1)-\frac{1}{2}\,(a+b)\log\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 1.06, size = 23, normalized size = 2.09

$$-bx+\frac{1}{2}\,(a+b)\log\,(x+1)-\frac{1}{2}\,(a+b)\log\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] -b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.
time = 0.08, size = 22, normalized size = 2.00

$$-bx - \frac{(a+b)\log(x-1)}{2} + \frac{(a+b)\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(-x**2+1),x)
```

```
[Out] -b*x - (a + b)*log(x - 1)/2 + (a + b)*log(x + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 1.42, size = 25, normalized size = 2.27

$$-bx + \frac{1}{2}(a+b)\log(|x+1|) - \frac{1}{2}(a+b)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-x^2+1),x, algorithm="giac")
```

```
[Out] -b*x + 1/2*(a + b)*log(abs(x + 1)) - 1/2*(a + b)*log(abs(x - 1))
```

Mupad [B]
time = 0.06, size = 11, normalized size = 1.00

$$\mathrm{atanh}(x)\ (a+b) - b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x^2)/(x^2 - 1),x)
```

```
[Out] atanh(x)*(a + b) - b*x
```

3.109 $\int \frac{1+x^2}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

```
[Out] x/(-x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {391}

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(-1 + x^2)^2,x]
```

```
[Out] x/(1 - x^2)
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)^2} \, dx = \frac{x}{1-x^2}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 0.91

$$-\frac{x}{-1+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-1 + x^2)^2,x]
```

```
[Out] -(x/(-1 + x^2))
```

Maple [A]
time = 0.08, size = 16, normalized size = 1.45

method	result	size
gosper	$-\frac{x}{x^2-1}$	11
norman	$-\frac{x}{x^2-1}$	11
risch	$-\frac{x}{x^2-1}$	11
default	$-\frac{1}{2(x+1)}-\frac{1}{2(x-1)}$	16
meijerg	$\frac{i\left(-\frac{ix}{-x^2+1}+i\operatorname{arctanh}(x)\right)}{2}-\frac{i\left(\frac{2ix}{-2x^2+2}+i\operatorname{arctanh}(x)\right)}{2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x+1)-1/2/(x-1)
```

Maxima [A]
time = 0.30, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -x/(x^2 - 1)
```

Fricas [A]
time = 1.11, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -x/(x^2 - 1)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.64

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)**2,x)
```

```
[Out] -x/(x**2 - 1)
```

Giac [**A**]
time = 1.20, size = 11, normalized size = 1.00

$$-\frac{1}{x-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)^2,x, algorithm="giac")
[Out] -1/(x - 1/x)
```

Mupad [**B**]
time = 0.04, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 - 1)^2,x)
[Out] -x/(x^2 - 1)
```

3.110 $\int \frac{1-x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=9

$$\frac{x}{1+x^2}$$

[Out] x/(x^2+1)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{391\}$

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x^2)^2,x]

[Out] x/(1 + x^2)

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{1-x^2}{(1+x^2)^2} \, dx = \frac{x}{1+x^2}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{x}{1+x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x^2)^2,x]

[Out] x/(1 + x^2)

Maple [A]
time = 0.07, size = 10, normalized size = 1.11

method	result	size
gosper	$\frac{x}{x^2+1}$	10
default	$\frac{x}{x^2+1}$	10
norman	$\frac{x}{x^2+1}$	10
risch	$\frac{x}{x^2+1}$	10
meijerg	$\frac{2x}{2x^2+2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/(x^2+1)
```

Maxima [A]
time = 0.28, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1)
```

Fricas [A]
time = 1.03, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] x/(x^2 + 1)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.56

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+1)**2,x)
```

```
[Out] x/(x**2 + 1)
```

Giac [A]
time = 1.51, size = 7, normalized size = 0.78

$$\frac{1}{x+\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1/x)
```

Mupad [B]
time = 0.04, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^2 + 1)^2,x)
```

```
[Out] x/(x^2 + 1)
```

3.111 $\int \frac{3+2x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$\frac{x}{2(1+x^2)}+\frac{5}{2}\tan^{-1}(x)$$

```
[Out] 1/2*x/(x^2+1)+5/2*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {393, 209}

$$\frac{5\text{ArcTan}(x)}{2}+\frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x^2}{(1+x^2)^2}\,dx &= \frac{x}{2(1+x^2)}+\frac{5}{2}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{2(1+x^2)}+\frac{5}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{x}{2(1+x^2)}+\frac{5}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/(1 + x^2)^2,x]
[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

Maple [A]
time = 0.07, size = 16, normalized size = 0.84

method	result	size
default	$\frac{x}{2x^2+2}+\frac{5\arctan(x)}{2}$	16
risch	$\frac{x}{2x^2+2}+\frac{5\arctan(x)}{2}$	16
meijerg	$-\frac{x}{x^2+1}+\frac{5\arctan(x)}{2}+\frac{3x}{2x^2+2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x/(x^2+1)+5/2*arctan(x)
```

Maxima [A]
time = 0.49, size = 15, normalized size = 0.79

$$\frac{x}{2(x^2+1)}+\frac{5}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

Fricas [A]
time = 0.89, size = 20, normalized size = 1.05

$$\frac{5(x^2+1)\arctan(x)+x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(5*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

<u>Sympy</u> [A]
time = 0.04, size = 14, normalized size = 0.74

$$\frac{x}{2x^2+2}+\frac{5\,\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + 5*atan(x)/2
```

<u>Giac</u> [A]
time = 1.13, size = 15, normalized size = 0.79

$$\frac{x}{2\,(x^2+1)}+\frac{5}{2}\,\arctan\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

<u>Mupad</u> [B]
time = 0.04, size = 16, normalized size = 0.84

$$\frac{5\,\mathrm{atan}(x)}{2}+\frac{x}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 3)/(x^2 + 1)^2,x)
```

```
[Out] (5*atan(x))/2 + x/(2*(x^2 + 1))
```

3.112 $\int \frac{-2+x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$-\frac{3x}{2(1+x^2)} - \frac{1}{2}\tan^{-1}(x)$$

```
[Out] -3/2*x/(x^2+1)-1/2*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {393, 209}

$$-\frac{\text{ArcTan}(x)}{2} - \frac{3x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2)/(1 + x^2)^2,x]
```

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{-2+x^2}{(1+x^2)^2} \, dx &= -\frac{3x}{2(1+x^2)} - \frac{1}{2}\int \frac{1}{1+x^2} \, dx \\ &= -\frac{3x}{2(1+x^2)} - \frac{1}{2}\tan^{-1}(x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-\frac{3x}{2(1+x^2)}-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(1 + x^2)^2,x]
```

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

Maple [A]
time = 0.07, size = 16, normalized size = 0.84

method	result	size
default	$-\frac{3x}{2(x^2+1)}-\frac{\arctan(x)}{2}$	16
risch	$-\frac{3x}{2(x^2+1)}-\frac{\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2(x^2+1)}-\frac{\arctan(x)}{2}-\frac{2x}{2x^2+2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x/(x^2+1)-1/2*arctan(x)
```

Maxima [A]
time = 0.49, size = 15, normalized size = 0.79

$$-\frac{3x}{2(x^2+1)}-\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

Fricas [A]
time = 0.63, size = 21, normalized size = 1.11

$$-\frac{(x^2+1)\arctan(x)+3x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*arctan(x) + 3*x)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.79

$$-\frac{3x}{2x^2+2}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)/(x**2+1)**2,x)
[Out] -3*x/(2*x**2 + 2) - atan(x)/2
```

Giac [A]
time = 1.26, size = 15, normalized size = 0.79

$$-\frac{3x}{2(x^2+1)}-\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/(x^2+1)^2,x, algorithm="giac")
[Out] -3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

Mupad [B]
time = 0.04, size = 17, normalized size = 0.89

$$-\frac{\operatorname{atan}(x)}{2}-\frac{3x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 2)/(x^2 + 1)^2,x)
[Out] - atan(x)/2 - (3*x)/(2*(x^2 + 1))
```

3.113 $\int \frac{3+x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=14

$$\frac{x}{1+x^2} + 2\tan^{-1}(x)$$

[Out] x/(x^2+1)+2*arctan(x)

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {393, 209}

$$2\text{ArcTan}(x) + \frac{x}{x^2+1}$$

Antiderivative was successfully verified.

[In] Int[(3 + x^2)/(1 + x^2)^2,x]

[Out] x/(1 + x^2) + 2*ArcTan[x]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{3+x^2}{(1+x^2)^2} \, dx &= \frac{x}{1+x^2} + 2\int \frac{1}{1+x^2} \, dx \\ &= \frac{x}{1+x^2} + 2\tan^{-1}(x) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 14, normalized size = 1.00

$$\frac{x}{1+x^2} + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(1 + x^2) + 2*ArcTan[x]
```

Maple [A]
time = 0.07, size = 15, normalized size = 1.07

method	result	size
default	$\frac{x}{x^2+1}+2\arctan\left(x\right)$	15
risch	$\frac{x}{x^2+1}+2\arctan\left(x\right)$	15
meijerg	$-\frac{x}{2(x^2+1)}+2\arctan\left(x\right)+\frac{3x}{2x^2+2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/(x^2+1)+2*arctan(x)
```

Maxima [A]
time = 0.49, size = 14, normalized size = 1.00

$$\frac{x}{x^2+1}+2\ \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1) + 2*arctan(x)
```

Fricas [A]
time = 1.18, size = 19, normalized size = 1.36

$$\frac{2\left(x^2+1\right)\arctan\left(x\right)+x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] (2*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{x^2+1}+2\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**2+1)**2,x)
```

```
[Out] x/(x**2 + 1) + 2*atan(x)
```

Giac [A]
time = 1.20, size = 14, normalized size = 1.00

$$\frac{x}{x^2+1}+2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] x/(x^2 + 1) + 2*arctan(x)
```

Mupad [B]
time = 0.01, size = 14, normalized size = 1.00

$$2\,\mathrm{atan}(x)+\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3)/(x^2 + 1)^2,x)
```

```
[Out] 2*atan(x) + x/(x^2 + 1)
```

3.114 $\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx$

Optimal. Leaf size=12

$$\frac{x}{a-bx^2}$$

[Out] x/(-b*x^2+a)

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{391\}$

$$\frac{x}{a-bx^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/(-a + b*x^2)^2,x]

[Out] x/(a - b*x^2)

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx = \frac{x}{a-bx^2}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.17

$$-\frac{x}{-a+bx^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/(-a + b*x^2)^2,x]

[Out] -(x/(-a + b*x^2))

Maple [A]
time = 0.06, size = 13, normalized size = 1.08

method	result	size
gosper	$\frac{x}{-b\,x^2+a}$	13
default	$\frac{x}{-b\,x^2+a}$	13
norman	$\frac{x}{-b\,x^2+a}$	13
risch	$\frac{x}{-b\,x^2+a}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(b*x^2-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/(-b*x^2+a)
```

Maxima [A]
time = 0.28, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="maxima")
```

```
[Out] -x/(b*x^2 - a)
```

Fricas [A]
time = 1.12, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="fricas")
```

```
[Out] -x/(b*x^2 - a)
```

Sympy [A]
time = 0.08, size = 8, normalized size = 0.67

$$-\frac{x}{-a+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(b*x**2-a)**2,x)
```

```
[Out] -x/(-a + b*x**2)
```

Giac [A]
time = 1.18, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="giac")
[Out] -x/(b*x^2 - a)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 1.00

$$\frac{x}{a-b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(a - b*x^2)^2,x)
[Out] x/(a - b*x^2)
```

3.115 $\int \frac{a+bx^2}{(a-bx^2)^2} dx$

Optimal. Leaf size=12

$$\frac{x}{a-bx^2}$$

```
[Out] x/(-b*x^2+a)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {391}

$$\frac{x}{a-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(a - b*x^2)^2,x]
```

```
[Out] x/(a - b*x^2)
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{a+bx^2}{(a-bx^2)^2} dx = \frac{x}{a-bx^2}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.17

$$-\frac{x}{-a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(a - b*x^2)^2,x]
```

```
[Out] -(x/(-a + b*x^2))
```

Maple [A]
time = 0.07, size = 13, normalized size = 1.08

method	result	size
gosper	$\frac{x}{-b\,x^2+a}$	13
default	$\frac{x}{-b\,x^2+a}$	13
norman	$\frac{x}{-b\,x^2+a}$	13
risch	$\frac{x}{-b\,x^2+a}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/(-b*x^2+a)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -x/(b*x^2 - a)
```

Fricas [A]
time = 1.30, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -x/(b*x^2 - a)
```

Sympy [A]
time = 0.08, size = 8, normalized size = 0.67

$$-\frac{x}{-a+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-a + b*x**2)
```

Giac [A]
time = 1.53, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -x/(b*x^2 - a)
```

Mupad [B]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{x}{a - b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(a - b*x^2)^2,x)
```

```
[Out] x/(a - b*x^2)
```

3.116 $\int \frac{A+Bx^2}{a-bx^2}\,dx$

Optimal. Leaf size=39

$$-\frac{Bx}{b}+\frac{(Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

```
[Out] -B*x/b+(A*b+B*a)*arctanh(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {396, 214}

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - b*x^2),x]
```

```
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{a-bx^2}\,dx &= -\frac{Bx}{b}+\frac{(Ab+aB)\int \frac{1}{a-bx^2}\,dx}{b}\\ &= -\frac{Bx}{b}+\frac{(Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$-\frac{Bx}{b}+\frac{(Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - b*x^2),x]
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Maple [A]
time = 0.08, size = 37, normalized size = 0.95

method	result	size
default	$-\frac{Bx}{b}-\frac{(-Ab-Ba)\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37
risch	$-\frac{Bx}{b}-\frac{\ln\left(bx-\sqrt{ab}\right)A}{2\sqrt{ab}}-\frac{\ln\left(bx-\sqrt{ab}\right)Ba}{2b\sqrt{ab}}+\frac{\ln\left(-bx-\sqrt{ab}\right)A}{2\sqrt{ab}}+\frac{\ln\left(-bx-\sqrt{ab}\right)Ba}{2b\sqrt{ab}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(-b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -B*x/b-(-A*b-B*a)/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 49, normalized size = 1.26

$$-\frac{Bx}{b}-\frac{(Ba+Ab)\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="maxima")
[Out] -B*x/b - 1/2*(B*a + A*b)*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b
)*b)
```

Fricas [A]
time = 0.79, size = 98, normalized size = 2.51

$$\left[-\frac{2\,Babx-(Ba+Ab)\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{2\,ab^2},\,-\frac{Babx+(Ba+Ab)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*a*b*x - (B*a + A*b)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b
*x^2 - a)))/(a*b^2), -(B*a*b*x + (B*a + A*b)*sqrt(-a*b)*arctan(sqrt(-a*b)*x
/a))/(a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.
time = 0.15, size = 75, normalized size = 1.92

$$-\frac{Bx}{b} - \frac{\sqrt{\frac{1}{ab^3}}\,(Ab+Ba)\log\left(-ab\sqrt{\frac{1}{ab^3}}+x\right)}{2} + \frac{\sqrt{\frac{1}{ab^3}}\,(Ab+Ba)\log\left(ab\sqrt{\frac{1}{ab^3}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(-b*x**2+a),x)
```

```
[Out] -B*x/b - sqrt(1/(a*b**3))*(A*b + B*a)*log(-a*b*sqrt(1/(a*b**3)) + x)/2 + sq
rt(1/(a*b**3))*(A*b + B*a)*log(a*b*sqrt(1/(a*b**3)) + x)/2
```

Giac [A]
time = 1.03, size = 36, normalized size = 0.92

$$-\frac{Bx}{b} - \frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -B*x/b - (B*a + A*b)*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b)
```

Mupad [B]
time = 0.08, size = 31, normalized size = 0.79

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b+B\,a)}{\sqrt{a}\,b^{3/2}} - \frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a - b*x^2),x)
```

```
[Out] (atanh((b^(1/2)*x)/a^(1/2))*(A*b + B*a))/(a^(1/2)*b^(3/2)) - (B*x)/b
```

3.117 $\int \frac{1+x^2}{\left(16+x^2\right)^3} \, dx$

Optimal. Leaf size=35

$$-\frac{15x}{64\left(16+x^2\right)^2}+\frac{19x}{2048\left(16+x^2\right)}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

```
[Out] -15/64*x/(x^2+16)^2+19/2048*x/(x^2+16)+19/8192*arctan(1/4*x)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {393, 205, 209}

$$\frac{19\text{ArcTan}\left(\frac{x}{4}\right)}{8192}+\frac{19x}{2048\left(x^2+16\right)}-\frac{15x}{64\left(x^2+16\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(16 + x^2)^3,x]
```

```
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan[x/4])/819
2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{(16+x^2)^3}\,dx &= -\frac{15x}{64(16+x^2)^2}+\frac{19}{64}\int \frac{1}{(16+x^2)^2}\,dx\\ &= -\frac{15x}{64(16+x^2)^2}+\frac{19x}{2048(16+x^2)}+\frac{19\int \frac{1}{16+x^2}\,dx}{2048}\\ &= -\frac{15x}{64(16+x^2)^2}+\frac{19x}{2048(16+x^2)}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$-\frac{15x}{64(16+x^2)^2}+\frac{19x}{2048(16+x^2)}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(16 + x^2)^3,x]
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan[x/4])/819
2
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.71

method	result	size
default	$\frac{\frac{19}{2048}x^3-\frac{11}{128}x}{(x^2+16)^2}+\frac{19\arctan\left(\frac{x}{4}\right)}{8192}$	25
risch	$\frac{\frac{19}{2048}x^3-\frac{11}{128}x}{(x^2+16)^2}+\frac{19\arctan\left(\frac{x}{4}\right)}{8192}$	25
meijerg	$\frac{x\left(\frac{3x^2}{16}+5\right)}{32768\left(\frac{x^2}{16}+1\right)^2}+\frac{19\arctan\left(\frac{x}{4}\right)}{8192}-\frac{x\left(-\frac{3x^2}{16}+3\right)}{6144\left(\frac{x^2}{16}+1\right)^2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+16)^3,x,method=_RETURNVERBOSE)
[Out] (19/2048*x^3-11/128*x)/(x^2+16)^2+19/8192*arctan(1/4*x)
```

Maxima [A]
time = 0.50, size = 30, normalized size = 0.86

$$\frac{19\,x^3-176\,x}{2048\,(x^4+32\,x^2+256)}+\frac{19}{8192}\arctan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+16)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(19*x^3 - 176*x)/(x^4 + 32*x^2 + 256) + 19/8192*arctan(1/4*x)
```

Fricas [A]
time = 0.70, size = 39, normalized size = 1.11

$$\frac{76\,x^3+19\,(x^4+32\,x^2+256)\arctan\left(\frac{1}{4}\,x\right)-704\,x}{8192\,(x^4+32\,x^2+256)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+16)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(76*x^3 + 19*(x^4 + 32*x^2 + 256)*arctan(1/4*x) - 704*x)/(x^4 + 32*x
^2 + 256)
```

Sympy [A]
time = 0.04, size = 27, normalized size = 0.77

$$\frac{19x^3-176x}{2048x^4+65536x^2+524288}+\frac{19\,\mathrm{atan}\left(\frac{x}{4}\right)}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+16)**3,x)
```

```
[Out] (19*x**3 - 176*x)/(2048*x**4 + 65536*x**2 + 524288) + 19*atan(x/4)/8192
```

Giac [A]
time = 1.06, size = 25, normalized size = 0.71

$$\frac{19\,x^3-176\,x}{2048\,(x^2+16)^2}+\frac{19}{8192}\arctan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+16)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(19*x^3 - 176*x)/(x^2 + 16)^2 + 19/8192*arctan(1/4*x)
```

Mupad [B]
time = 0.04, size = 30, normalized size = 0.86

$$\frac{19\,\mathrm{atan}\left(\frac{x}{4}\right)}{8192}-\frac{\frac{11\,x}{128}-\frac{19\,x^3}{2048}}{x^4+32\,x^2+256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 + 16)^3,x)
```

```
[Out] (19*atan(x/4))/8192 - ((11*x)/128 - (19*x^3)/2048)/(32*x^2 + x^4 + 256)
```

3.118 $\int \frac{1+2x^2}{x^5(1+x^2)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{4x^4\left(1+x^2\right)^2}$$

```
[Out] -1/4/x^4/(x^2+1)^2
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {457, 75}

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]
```

```
[Out] -1/4*1/(x^4*(1 + x^2)^2)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{x^5\left(1+x^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1+2x}{x^3(1+x)^3}\,dx,x,x^2\right)\\ &= -\frac{1}{4x^4\left(1+x^2\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{4x^4 (1 + x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]
```

```
[Out] -1/4*1/(x^4*(1 + x^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.
time = 0.11, size = 30, normalized size = 2.14

method	result	size
gosper	$-\frac{1}{4x^4(x^2+1)^2}$	13
norman	$-\frac{1}{4x^4(x^2+1)^2}$	13
risch	$-\frac{1}{4x^4(x^2+1)^2}$	13
default	$-\frac{1}{4x^4}+\frac{1}{2x^2}-\frac{1}{4(x^2+1)^2}-\frac{1}{2(x^2+1)}$	30
meijerg	$-\frac{x^2(7x^2+8)}{4(x^2+1)^2}-\frac{3}{4}-\frac{1}{4x^4}+\frac{1}{2x^2}+\frac{x^2(5x^2+6)}{2(x^2+1)^2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/x^5/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4+1/2/x^2-1/4/(x^2+1)^2-1/2/(x^2+1)
```

Maxima [A]
time = 0.28, size = 16, normalized size = 1.14

$$-\frac{1}{4(x^8 + 2x^6 + x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Fricas [A]
time = 0.89, size = 16, normalized size = 1.14

$$-\frac{1}{4(x^8 + 2x^6 + x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Sympy [A]
time = 0.04, size = 17, normalized size = 1.21

$$-\frac{1}{4x^8+8x^6+4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/x**5/(x**2+1)**3,x)
```

```
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

Giac [A]
time = 1.68, size = 11, normalized size = 0.79

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + x^2)^2
```

Mupad [B]
time = 0.02, size = 20, normalized size = 1.43

$$-\frac{1}{4\,x^8+8\,x^6+4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(x^5*(x^2 + 1)^3),x)
```

```
[Out] -1/(4*x^4 + 8*x^6 + 4*x^8)
```

3.119 $\int \frac{(1-x^2)^2}{(-1+x^2)^2} \, dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {21, 8}

$$x$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)^2/(-1 + x^2)^2,x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(1-x^2)^2}{(-1+x^2)^2} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^2/(-1 + x^2)^2,x]
```

```
[Out] x
```

Maple [A]
time = 0.10, size = 2, normalized size = 2.00

method	result	size
default	x	2
risch	x	2
norman	$\frac{x^3-x}{x^2-1}$	16
meijerg	$-\frac{i\left(\frac{ix\left(-10x^2+15\right)}{-5x^2+5}-3i\operatorname{arctanh}(x)\right)}{2}-i\left(-\frac{ix}{-x^2+1}+i\operatorname{arctanh}\left(x\right)\right)-\frac{i\left(\frac{2ix}{-2x^2+2}+i\operatorname{arctanh}(x)\right)}{2}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^2/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x
```

Maxima [A]
time = 0.29, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 0.84, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A]
time = 0.01, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**2/(x**2-1)**2,x)
```

```
[Out] x
```

Giac [A]
time = 1.32, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

3.120 $\int \frac{x^3(ac+bcx^2)}{a+bx^2} dx$

Optimal. Leaf size=8

$$\frac{cx^4}{4}$$

[Out] 1/4*c*x^4

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {21, 30}

$$\frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^4)/4

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3(ac+bcx^2)}{a+bx^2} dx &= c\int x^3 dx \\ &= \frac{cx^4}{4} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^4)/4
```

Maple [A]
time = 0.06, size = 7, normalized size = 0.88

method	result	size
gosper	$\frac{x^4 c}{4}$	7
default	$\frac{x^4 c}{4}$	7
norman	$\frac{x^4 c}{4}$	7
risch	$\frac{x^4 c}{4}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*c
```

Maxima [A]
time = 0.29, size = 6, normalized size = 0.75

$$\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4
```

Fricas [A]
time = 0.80, size = 6, normalized size = 0.75

$$\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.62

$$\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**4/4
```

Giac [A]
time = 1.32, size = 6, normalized size = 0.75

$$\frac{1}{4}cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*c*x^4
```

Mupad [B]
time = 0.01, size = 6, normalized size = 0.75

$$\frac{c\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*c + b*c*x^2))/(a + b*x^2),x)
```

```
[Out] (c*x^4)/4
```

3.121 $\int \frac{x^2(ac+bcx^2)}{a+bx^2} dx$

Optimal. Leaf size=8

$$\frac{cx^3}{3}$$

[Out] 1/3*c*x^3

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 30}

$$\frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^3)/3

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(ac+bcx^2)}{a+bx^2}\,dx &= c\int x^2\,dx\\ &= \frac{cx^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^3)/3
```

Maple [A]
time = 0.06, size = 7, normalized size = 0.88

method	result	size
gosper	$\frac{c\,x^{3}}{3}$	7
default	$\frac{c\,x^{3}}{3}$	7
norman	$\frac{c\,x^{3}}{3}$	7
risch	$\frac{c\,x^{3}}{3}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*c*x^2+a*c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c*x^3
```

Maxima [A]
time = 0.29, size = 6, normalized size = 0.75

$$\frac{1}{3}\,cx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3
```

Fricas [A]
time = 1.25, size = 6, normalized size = 0.75

$$\frac{1}{3}\,cx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.62

$$\frac{cx^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**3/3
```

Giac [A]
time = 1.27, size = 6, normalized size = 0.75

$$\frac{1}{3}\,cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3
```

Mupad [B]
time = 0.01, size = 6, normalized size = 0.75

$$\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*c + b*c*x^2))/(a + b*x^2),x)
```

```
[Out] (c*x^3)/3
```

3.122 $\int \frac{x(ac+bcx^2)}{a+bx^2} dx$

Optimal. Leaf size=8

$$\frac{cx^2}{2}$$

[Out] 1/2*c*x^2

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {21, 30}

$$\frac{cx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2),x]

[Out] (c*x^2)/2

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(ac+bcx^2)}{a+bx^2}\,dx &= c\int x\,dx\\ &= \frac{cx^2}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^2)/2
```

Maple [A]
time = 0.06, size = 7, normalized size = 0.88

method	result	size
gosper	$\frac{c x^{2}}{2}$	7
default	$\frac{c x^{2}}{2}$	7
norman	$\frac{c x^{2}}{2}$	7
risch	$\frac{c x^{2}}{2}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*x^2
```

Maxima [A]
time = 0.31, size = 6, normalized size = 0.75

$$\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2
```

Fricas [A]
time = 0.91, size = 6, normalized size = 0.75

$$\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*c*x^2
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.62

$$\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**2/2
```

Giac [A]
time = 1.07, size = 6, normalized size = 0.75

$$\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*c*x^2
```

Mupad [B]
time = 0.01, size = 6, normalized size = 0.75

$$\frac{c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*c + b*c*x^2))/(a + b*x^2),x)
```

```
[Out] (c*x^2)/2
```

3.123 $\int \frac{ac+bcx^2}{a+bx^2} dx$

Optimal. Leaf size=3

$$cx$$

[Out] c*x

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 8}

$$cx$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(a + b*x^2),x]

[Out] c*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{ac+bcx^2}{a+bx^2} \, dx &= c \int 1 \, dx \\ &= cx \end{aligned}$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$cx$$

Antiderivative was successfully verified.

[In] Integrate[(a*c + b*c*x^2)/(a + b*x^2),x]

```
[Out] c*x
```

Maple [A]
time = 0.06, size = 4, normalized size = 1.33

method	result	size
default	cx	4
norman	cx	4
risch	cx	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*x
```

Maxima [A]
time = 0.29, size = 3, normalized size = 1.00

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] c*x
```

Fricas [A]
time = 1.26, size = 3, normalized size = 1.00

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] c*x
```

Sympy [A]
time = 0.01, size = 2, normalized size = 0.67

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x
```

Giac [A]
time = 1.38, size = 3, normalized size = 1.00

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c*x
```

Mupad [B]
time = 0.00, size = 3, normalized size = 1.00

$$c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(a + b*x^2),x)
```

```
[Out] c*x
```

3.124 $\int \frac{ac+bcx^2}{x(a+bx^2)} dx$

Optimal. Leaf size=4

$$c\log(x)$$

[Out] c*ln(x)

Rubi [A]

time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {21, 29}

$$c\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] c*Log[x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{ac+bcx^2}{x\,(a+bx^2)}\,dx &= c\int \frac{1}{x}\,dx \\ &= c\log(x) \end{aligned}$$

Mathematica [A]

time = 0.00, size = 4, normalized size = 1.00

$$c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] c*Log[x]
```

Maple [A]
time = 0.07, size = 5, normalized size = 1.25

method	result	size
default	$c \ln \left(x \right)$	5
norman	$c \ln \left(x \right)$	5
risch	$c \ln \left(x \right)$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*ln(x)
```

Maxima [A]
time = 0.28, size = 7, normalized size = 1.75

$$\frac{1}{2} \, c \log \left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2)
```

Fricas [A]
time = 1.08, size = 4, normalized size = 1.00

$$c \log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] c*log(x)
```

Sympy [A]
time = 0.01, size = 3, normalized size = 0.75

$$c \log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x/(b*x**2+a),x)
```

```
[Out] c*log(x)
```

Giac [**A**]
time = 1.49, size = 5, normalized size = 1.25

$$c\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))
```

Mupad [**B**]
time = 0.01, size = 4, normalized size = 1.00

$$c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x*(a + b*x^2)),x)
```

```
[Out] c*log(x)
```

3.125 $\int \frac{ac+bcx^2}{x^2(a+bx^2)} \, dx$

Optimal. Leaf size=6

$$-\frac{c}{x}$$

[Out] -c/x

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {21, 30}

$$-\frac{c}{x}$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]

[Out] -(c/x)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x^2(a+bx^2)} \, dx &= c\int \frac{1}{x^2} \, dx \\ &= -\frac{c}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/x)
```

Maple [A]
time = 0.06, size = 7, normalized size = 1.17

method	result	size
gosper	$-\frac{c}{x}$	7
default	$-\frac{c}{x}$	7
norman	$-\frac{c}{x}$	7
risch	$-\frac{c}{x}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -c/x
```

Maxima [A]
time = 0.30, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -c/x
```

Fricas [A]
time = 1.06, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -c/x
```

Sympy [A]
time = 0.01, size = 3, normalized size = 0.50

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**2/(b*x**2+a),x)
```

```
[Out] -c/x
```

Giac [A]
time = 1.25, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c/x
```

Mupad [B]
time = 0.01, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^2*(a + b*x^2)),x)
```

```
[Out] -c/x
```

3.126 $\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=8

$$-\frac{c}{2x^2}$$

[Out] -1/2*c/x^2

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {21, 30}

$$-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]

[Out] -1/2*c/x^2

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx &= c\int \frac{1}{x^3}\,dx\\ &= -\frac{c}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*c/x^2
```

Maple [A]
time = 0.07, size = 7, normalized size = 0.88

method	result	size
gosper	$-\frac{c}{2x^2}$	7
default	$-\frac{c}{2x^2}$	7
norman	$-\frac{c}{2x^2}$	7
risch	$-\frac{c}{2x^2}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c/x^2
```

Maxima [A]
time = 0.32, size = 6, normalized size = 0.75

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*c/x^2
```

Fricas [A]
time = 1.05, size = 6, normalized size = 0.75

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*c/x^2
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.88

$$-\frac{c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a),x)
```

```
[Out] -c/(2*x**2)
```

Giac [A]
time = 1.36, size = 6, normalized size = 0.75

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*c/x^2
```

Mupad [B]
time = 0.01, size = 6, normalized size = 0.75

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^3*(a + b*x^2)),x)
```

```
[Out] -c/(2*x^2)
```

3.127 $\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=29

$$\frac{cx^2}{2b} - \frac{ac\log(a+bx^2)}{2b^2}$$

[Out] 1/2*c*x^2/b-1/2*a*c*ln(b*x^2+a)/b^2

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {21, 272, 45}

$$\frac{cx^2}{2b} - \frac{ac\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]

[Out] (c*x^2)/(2*b) - (a*c*Log[a + b*x^2])/(2*b^2)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2}\,dx = c\int \frac{x^3}{a+bx^2}\,dx$$
$$= \frac{1}{2}c\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}c\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx, x, x^2\right)$$
$$= \frac{cx^2}{2b}-\frac{ac\log(a+bx^2)}{2b^2}$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 1.00

$$c\left(\frac{x^2}{2b}-\frac{a\log(a+bx^2)}{2b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
[Out] c*(x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2))
```

Maple [A]
time = 0.06, size = 26, normalized size = 0.90

method	result	size
default	$c\left(\frac{x^2}{2b}-\frac{a\ln(b\,x^2+a)}{2b^2}\right)$	26
risch	$\frac{c\,x^2}{2b}-\frac{ac\ln(b\,x^2+a)}{2b^2}$	26
norman	$\frac{\frac{x^4c}{2}-\frac{a^2c}{2b^2}}{b\,x^2+a}-\frac{ac\ln(b\,x^2+a)}{2b^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] c*(1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.86

$$\frac{cx^2}{2\,b}-\frac{ac\log(bx^2+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2/b - 1/2*a*c*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 0.97, size = 24, normalized size = 0.83

$$\frac{bcx^2 - ac\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2 - a*c*log(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.05, size = 22, normalized size = 0.76

$$c\left(-\frac{a\log\left(a + bx^2\right)}{2b^2} + \frac{x^2}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(-a*log(a + b*x**2)/(2*b**2) + x**2/(2*b))
```

Giac [A]
time = 1.69, size = 47, normalized size = 1.62

$$\frac{\frac{ac\log\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{b} + \frac{(bx^2+a)c}{b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b + (b*x^2 + a)*c/b)/b
```

Mupad [B]
time = 0.02, size = 23, normalized size = 0.79

$$-\frac{c\left(a\,\ln\left(b\,x^2 + a\right) - b\,x^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x)
```

```
[Out] -(c*(a*log(a + b*x^2) - b*x^2))/(2*b^2)
```

3.128 $\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=33

$$\frac{cx}{b} - \frac{\sqrt{a}\, c \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] c*x/b-c*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {21, 327, 211}

$$\frac{cx}{b} - \frac{\sqrt{a}\, c\text{ArcTan}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*x)/b - (Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx = c\int \frac{x^2}{a+bx^2}\,dx$$
$$= \frac{cx}{b} - \frac{(ac)\int \frac{1}{a+bx^2}\,dx}{b}$$
$$= \frac{cx}{b} - \frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$c\left(\frac{x}{b} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
[Out] c*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2))
```

Maple [A]
time = 0.08, size = 29, normalized size = 0.88

method	result	size
default	$c\left(\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}\right)$	29
risch	$\frac{cx}{b}+\frac{\sqrt{-ab}\,c\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^2}-\frac{\sqrt{-ab}\,c\ln\left(\sqrt{-ab}\,x-a\right)}{2b^2}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] c*(x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.63, size = 28, normalized size = 0.85

$$-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} + \frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -a*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + c*x/b
```

Fricas [A]
time = 1.11, size = 86, normalized size = 2.61

$$\left[\frac{c\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,cx}{2\,b}, -\frac{c\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-cx}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(c*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*c*x)
/b, -(c*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - c*x)/b]
```

Sympy [A]
time = 0.07, size = 58, normalized size = 1.76

$$c\left(\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/
b**3) + x)/2 + x/b)
```

Giac [A]
time = 1.10, size = 28, normalized size = 0.85

$$-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -a*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + c*x/b
```

Mupad [B]
time = 0.02, size = 25, normalized size = 0.76

$$\frac{c\,x}{b}-\frac{\sqrt{a}\,\,c\,\mathrm{atan}\left(\frac{\sqrt{b}\,\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (c*x)/b - (a^(1/2)*c*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

3.129 $\int \frac{x(ac+bcx^2)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=16

$$\frac{c\log(a+bx^2)}{2b}$$

[Out] 1/2*c*ln(b*x^2+a)/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {21, 266}

$$\frac{c\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]

[Out] (c*Log[a + b*x^2])/(2*b)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{x(ac+bcx^2)}{(a+bx^2)^2} \, dx &= c\int \frac{x}{a+bx^2} \, dx \\ &= \frac{c\log(a+bx^2)}{2b} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{c\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*Log[a + b*x^2])/(2*b)
```

Maple [A]
time = 0.07, size = 15, normalized size = 0.94

method	result	size
default	$\frac{c\ln\left(b\,x^{2}+a\right)}{2b}$	15
norman	$\frac{c\ln\left(b\,x^{2}+a\right)}{2b}$	15
risch	$\frac{c\ln\left(b\,x^{2}+a\right)}{2b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*ln(b*x^2+a)/b
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.88

$$\frac{c\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(b*x^2 + a)/b
```

Fricas [A]
time = 0.96, size = 14, normalized size = 0.88

$$\frac{c\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*c*log(b*x^2 + a)/b
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.75

$$\frac{c\log\left(a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*log(a + b*x**2)/(2*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(14) = 28$.
time = 1.14, size = 63, normalized size = 3.94

$$-\frac{1}{2}c\left(\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b} - \frac{a}{(bx^2+a)b}\right) - \frac{ac}{2(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*c*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b)) -
 1/2*a*c/((b*x^2 + a)*b)
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.88

$$\frac{c\ln\left(b\,x^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*c + b*c*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (c*log(a + b*x^2))/(2*b)
```

3.130 $\int \frac{ac+bcx^2}{(a+bx^2)^2} dx$

Optimal. Leaf size=25

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] c*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 211}

$$\frac{c\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(a + b*x^2)^2,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx &= c\int \frac{1}{a+bx^2}\,dx \\ &= \frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(a + b*x^2)^2,x]
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.08, size = 17, normalized size = 0.68

method	result	size
default	$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	17
risch	$-\frac{c\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{c\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 16, normalized size = 0.64

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.93, size = 69, normalized size = 2.76

$$\left[-\frac{\sqrt{-ab}\,c\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,c\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqr
t(a*b)*c*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.
time = 0.09, size = 54, normalized size = 2.16

$$c\left(-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt
(-1/(a*b)) + x)/2)
```

Giac [A]
time = 1.30, size = 16, normalized size = 0.64

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.68

$$\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(a + b*x^2)^2,x)
```

```
[Out] (c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.131 $\int \frac{ac+bcx^2}{x(a+bx^2)^2} dx$

Optimal. Leaf size=24

$$\frac{c\log(x)}{a} - \frac{c\log(a+bx^2)}{2a}$$

[Out] c*ln(x)/a-1/2*c*ln(b*x^2+a)/a

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {21, 272, 36, 29, 31}

$$\frac{c\log(x)}{a} - \frac{c\log(a+bx^2)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]

[Out] (c*Log[x])/a - (c*Log[a + b*x^2])/(2*a)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}\,dx &= c\int \frac{1}{x\left(a+bx^2\right)}\,dx\\ &= \frac{1}{2}c\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^2\right)\\ &= \frac{c\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}-\frac{(bc)\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^2\right)}{2a}\\ &= \frac{c\log(x)}{a}-\frac{c\log\left(a+bx^2\right)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$c\left(\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]
[Out] c*(Log[x]/a - Log[a + b*x^2]/(2*a))
```

Maple [A]
time = 0.07, size = 23, normalized size = 0.96

method	result	size
default	$c\left(\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}\right)$	23
norman	$\frac{c\ln(x)}{a}-\frac{c\ln(b\,x^2+a)}{2a}$	23
risch	$\frac{c\ln(x)}{a}-\frac{c\ln(b\,x^2+a)}{2a}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] c*(ln(x)/a-1/2*ln(b*x^2+a)/a)
```

Maxima [A]
time = 0.32, size = 25, normalized size = 1.04

$$-\frac{c\log\left(bx^2+a\right)}{2\,a}+\frac{c\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*c*log(b*x^2 + a)/a + 1/2*c*log(x^2)/a
```

Fricas [A]
time = 1.19, size = 21, normalized size = 0.88

$$-\frac{c\log\left(bx^2+a\right)-2\,c\log\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(c*log(b*x^2 + a) - 2*c*log(x))/a
```

Sympy [A]
time = 0.10, size = 17, normalized size = 0.71

$$c\left(\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x/(b*x**2+a)**2,x)
```

```
[Out] c*(log(x)/a - log(a/b + x**2)/(2*a))
```

Giac [A]
time = 1.25, size = 26, normalized size = 1.08

$$\frac{c\log\left(x^2\right)}{2\,a}-\frac{c\log\left(|bx^2+a|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a - 1/2*c*log(abs(b*x^2 + a))/a
```

Mupad [B]
time = 0.06, size = 19, normalized size = 0.79

$$-\frac{c\left(\ln\left(b\,x^2+a\right)-2\,\ln\left(x\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x*(a + b*x^2)^2),x)
```

```
[Out] -(c*(log(a + b*x^2) - 2*log(x)))/(2*a)
```

3.132 $\int \frac{ac+bcx^2}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=36

$$-\frac{c}{ax}-\frac{\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -c/a/x-c*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {21, 331, 211}

$$-\frac{\sqrt{b}\,c\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a*x)) - (Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)^2}\,dx = c\int \frac{1}{x^2(a+bx^2)}\,dx$$
$$= -\frac{c}{ax} - \frac{(bc)\int \frac{1}{a+bx^2}\,dx}{a}$$
$$= -\frac{c}{ax} - \frac{\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.00

$$c\left(-\frac{1}{ax} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
[Out] c*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))
```

Maple [A]
time = 0.07, size = 32, normalized size = 0.89

method	result	size
default	$c\left(-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}\right)$	32
risch	$-\frac{c}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^3\_Z^2+b\,c^2\right)}\_R\ln\left(\left(3\_R^2a^3+2b\,c^2\right)x+a^2c\_R\right)\right)}{2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] c*(-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x)
```

Maxima [A]
time = 0.57, size = 31, normalized size = 0.86

$$-\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} - \frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

Fricas [A]
time = 1.04, size = 86, normalized size = 2.39

$$\left[\frac{cx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,c}{2\,ax}, -\frac{cx\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+c}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(c*x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*c)
/(a*x), -(c*x*sqrt(b/a)*arctan(x*sqrt(b/a)) + c)/(a*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.
time = 0.07, size = 66, normalized size = 1.83

$$c\left(\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] c*(sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*
sqrt(-b/a**3)/b + x)/2 - 1/(a*x))
```

Giac [A]
time = 0.95, size = 31, normalized size = 0.86

$$-\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

Mupad [B]
time = 0.05, size = 28, normalized size = 0.78

$$-\frac{c}{a\,x}-\frac{\sqrt{b}\;c\,\mathrm{atan}\left(\frac{\sqrt{b}\;x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] - c/(a*x) - (b^(1/2)*c*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.133 $\int \frac{ac+bcx^2}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{c}{2ax^2}-\frac{bc\log(x)}{a^2}+\frac{bc\log(a+bx^2)}{2a^2}$$

[Out] -1/2*c/a/x^2-b*c*ln(x)/a^2+1/2*b*c*ln(b*x^2+a)/a^2

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 272, 46}

$$\frac{bc\log(a+bx^2)}{2a^2}-\frac{bc\log(x)}{a^2}-\frac{c}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]

[Out] -1/2*c/(a*x^2) - (b*c*Log[x])/a^2 + (b*c*Log[a + b*x^2])/(2*a^2)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x^3 (a+bx^2)^2}\,dx &= c\int \frac{1}{x^3 (a+bx^2)}\,dx\\ &= \frac{1}{2}c\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}c\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)\,dx,x,x^2\right)\\ &= -\frac{c}{2ax^2}-\frac{bc\log(x)}{a^2}+\frac{bc\log(a+bx^2)}{2a^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.97

$$c\left(-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx^2)}{2a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]
[Out] c*(-1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2))
```

Maple [A]
time = 0.07, size = 34, normalized size = 0.89

method	result	size
default	$c\left(\frac{b\ln(b\,x^2+a)}{2a^2}-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}\right)$	34
risch	$-\frac{c}{2a\,x^2}-\frac{bc\ln(x)}{a^2}+\frac{bc\ln(-b\,x^2-a)}{2a^2}$	38
norman	$\frac{-\frac{c}{2}+\frac{b^2c\,x^4}{2a^2}}{x^2(b\,x^2+a)}-\frac{bc\ln(x)}{a^2}+\frac{bc\ln(b\,x^2+a)}{2a^2}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] c*(1/2*b/a^2*ln(b*x^2+a)-1/2/a/x^2-b*ln(x)/a^2)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.95

$$\frac{bc\log(bx^2+a)}{2\,a^2}-\frac{bc\log(x^2)}{2\,a^2}-\frac{c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*log(b*x^2 + a)/a^2 - 1/2*b*c*log(x^2)/a^2 - 1/2*c/(a*x^2)
```

Fricas [A]
time = 0.77, size = 36, normalized size = 0.95

$$\frac{bcx^2\log\left(bx^2+a\right)-2\,bcx^2\log\left(x\right)-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2*log(b*x^2 + a) - 2*b*c*x^2*log(x) - a*c)/(a^2*x^2)
```

Sympy [A]
time = 0.15, size = 32, normalized size = 0.84

$$c\left(-\frac{1}{2ax^2}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**2,x)
```

```
[Out] c*(-1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2))
```

Giac [A]
time = 0.87, size = 47, normalized size = 1.24

$$-\frac{bc\log\left(x^2\right)}{2\,a^2}+\frac{bc\log\left(|bx^2+a|\right)}{2\,a^2}+\frac{bcx^2-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*c*log(x^2)/a^2 + 1/2*b*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b*c*x^2 - a*
c)/(a^2*x^2)
```

Mupad [B]
time = 0.06, size = 34, normalized size = 0.89

$$\frac{b\,c\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{c}{2\,a\,x^2}-\frac{b\,c\,\ln\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (b*c*log(a + b*x^2))/(2*a^2) - c/(2*a*x^2) - (b*c*log(x))/a^2
```

3.134 $\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=35

$$\frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

[Out] 1/2*a*c/b^2/(b*x^2+a)+1/2*c*ln(b*x^2+a)/b^2

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {21, 272, 45}

$$\frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]

[Out] (a*c)/(2*b^2*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*b^2)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx = c\int \frac{x^3}{(a+bx^2)^2}\,dx$$
$$= \frac{1}{2}c\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}c\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)\,dx, x, x^2\right)$$
$$= \frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

Mathematica [**A**]
time = 0.01, size = 28, normalized size = 0.80

$$\frac{c\left(\frac{a}{a+bx^2}+\log(a+bx^2)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (c*(a/(a + b*x^2) + Log[a + b*x^2]))/(2*b^2)
```

Maple [**A**]
time = 0.06, size = 32, normalized size = 0.91

method	result	size
default	$c\left(\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}\right)$	32
risch	$\frac{ac}{2b^2(b\,x^2+a)}+\frac{c\ln(b\,x^2+a)}{2b^2}$	32
norman	$\frac{\frac{a^2c}{2b^2}+\frac{ca\,x^2}{2b}}{(b\,x^2+a)^2}+\frac{c\ln(b\,x^2+a)}{2b^2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2)
```

Maxima [**A**]
time = 0.28, size = 34, normalized size = 0.97

$$\frac{ac}{2\,(b^3x^2+ab^2)}+\frac{c\log(bx^2+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c/(b^3*x^2 + a*b^2) + 1/2*c*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 1.12, size = 40, normalized size = 1.14

$$\frac{ac + (bcx^2 + ac)\log(bx^2 + a)}{2(b^3x^2 + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*c + (b*c*x^2 + a*c)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

Sympy [A]
time = 0.09, size = 31, normalized size = 0.89

$$c\left(\frac{a}{2ab^2 + 2b^3x^2} + \frac{\log(a + bx^2)}{2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2))
```

Giac [A]
time = 0.79, size = 32, normalized size = 0.91

$$\frac{c\log(|bx^2 + a|)}{2b^2} + \frac{ac}{2(bx^2 + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*log(abs(b*x^2 + a))/b^2 + 1/2*a*c/((b*x^2 + a)*b^2)
```

Mupad [B]
time = 0.05, size = 31, normalized size = 0.89

$$\frac{c\ln(b\,x^2 + a)}{2\,b^2} + \frac{a\,c}{2\,b^2\,\left(b\,x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (c*log(a + b*x^2))/(2*b^2) + (a*c)/(2*b^2*(a + b*x^2))
```

3.135 $\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=47

$$-\frac{cx}{2b\,(a+bx^2)}+\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

```
[Out] -1/2*c*x/b/(b*x^2+a)+1/2*c*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 294, 211}

$$\frac{c\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{cx}{2b\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -1/2*(c*x)/(b*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(
3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx = c\int \frac{x^2}{(a+bx^2)^2}\,dx$$
$$= -\frac{cx}{2b(a+bx^2)} + \frac{c\int \frac{1}{a+bx^2}\,dx}{2b}$$
$$= -\frac{cx}{2b(a+bx^2)} + \frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.00

$$c\left(-\frac{x}{2b(a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
[Out] c*(-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
)
```

Maple [A]
time = 0.09, size = 38, normalized size = 0.81

method	result	size
default	$c\left(-\frac{x}{2b(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)$	38
risch	$-\frac{cx}{2b(b\,x^2+a)}-\frac{c\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}+\frac{c\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] c*(-1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 38, normalized size = 0.81

$$-\frac{cx}{2(b^2x^2+ab)} + \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*c*x/(b^2*x^2 + a*b) + 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 0.96, size = 128, normalized size = 2.72

$$\left[-\frac{2\,abcx+(bcx^2+ac)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,(ab^3x^2+a^2b^2)},-\frac{abcx-(bcx^2+ac)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(ab^3x^2+a^2b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*c*x + (b*c*x^2 + a*c)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*c*x - (b*c*x^2 + a*c)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.
time = 0.10, size = 80, normalized size = 1.70

$$c\left(-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(-x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3))
+ x)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4)
```

Giac [A]
time = 1.29, size = 37, normalized size = 0.79

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b}-\frac{cx}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*c*x/((b*x^2 + a)*b)
```

Mupad [B]
time = 0.02, size = 35, normalized size = 0.74

$$\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}-\frac{c\,x}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x)
[Out] (c*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2)) - (c*x)/(2*b*(a + b*x^2))
```

3.136 $\int \frac{x(ac+bcx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=17

$$-\frac{c}{2b(a+bx^2)}$$

```
[Out] -1/2*c/b/(b*x^2+a)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {21, 267}

$$-\frac{c}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -1/2*c/(b*(a + b*x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(ac+bcx^2)}{(a+bx^2)^3} dx &= c\int \frac{x}{(a+bx^2)^2} dx\\ &= -\frac{c}{2b(a+bx^2)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{c}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -1/2*c/(b*(a + b*x^2))
```

Maple [A]
time = 0.06, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{c}{2b\left(b\,x^{2}+a\right)}$	16
default	$-\frac{c}{2b\left(b\,x^{2}+a\right)}$	16
risch	$-\frac{c}{2b\left(b\,x^{2}+a\right)}$	16
norman	$\frac{-\frac{ac}{2b}-\frac{c\,x^{2}}{2}}{\left(b\,x^{2}+a\right)^{2}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c/b/(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.94

$$-\frac{c}{2\,(b^2x^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*c/(b^2*x^2 + a*b)
```

Fricas [A]
time = 0.83, size = 16, normalized size = 0.94

$$-\frac{c}{2\,(b^2x^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*c/(b^2*x^2 + a*b)
```

Sympy [A]
time = 0.09, size = 15, normalized size = 0.88

$$-\frac{c}{2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] -c/(2*a*b + 2*b**2*x**2)
```

Giac [A]
time = 0.90, size = 15, normalized size = 0.88

$$-\frac{c}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*c/((b*x^2 + a)*b)
```

Mupad [B]
time = 0.02, size = 15, normalized size = 0.88

$$-\frac{c}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*c + b*c*x^2))/(a + b*x^2)^3,x)
```

```
[Out] -c/(2*b*(a + b*x^2))
```

3.137 $\int \frac{ac+bcx^2}{(a+bx^2)^3} dx$

Optimal. Leaf size=47

$$\frac{cx}{2a(a+bx^2)}+\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

```
[Out] 1/2*c*x/a/(b*x^2+a)+1/2*c*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {21, 205, 211}

$$\frac{c\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{cx}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (c*x)/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx = c\int \frac{1}{(a+bx^2)^2}\,dx$$
$$= \frac{cx}{2a(a+bx^2)} + \frac{c\int \frac{1}{a+bx^2}\,dx}{2a}$$
$$= \frac{cx}{2a(a+bx^2)} + \frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.00

$$c\left(\frac{x}{2a(a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
[Out] c*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]))
```

Maple [A]
time = 0.08, size = 38, normalized size = 0.81

method	result	size
default	$c\left(\frac{x}{2a(b\,x^2+a)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)$	38
risch	$\frac{cx}{2a(b\,x^2+a)} - \frac{\ln\left(bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\,a} + \frac{\ln\left(-bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\,a}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] c*(1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.55, size = 37, normalized size = 0.79

$$\frac{cx}{2(abx^2+a^2)} + \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c*x/(a*b*x^2 + a^2) + 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Fricas [A]
time = 0.79, size = 128, normalized size = 2.72

$$\left[\frac{2\,abcx - (bcx^2 + ac)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{4\,(a^2b^2x^2 + a^3b)}, \frac{abcx + (bcx^2 + ac)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(a^2b^2x^2 + a^3b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*c*x - (b*c*x^2 + a*c)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x -
a)/(b*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*c*x + (b*c*x^2 + a*c)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.
time = 0.13, size = 80, normalized size = 1.70

$$c\left(\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b))
+ x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4)
```

Giac [A]
time = 0.97, size = 37, normalized size = 0.79

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a} + \frac{cx}{2\,(bx^2 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*c*x/((b*x^2 + a)*a)
```

Mupad [B]
time = 0.02, size = 35, normalized size = 0.74

$$\frac{c\,x}{2\,a\,\left(b\,x^2+a\right)}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(a + b*x^2)^3,x)
[Out] (c*x)/(2*a*(a + b*x^2)) + (c*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2))
```

3.138 $\int \frac{ac+bcx^2}{x(a+bx^2)^3} \, dx$

Optimal. Leaf size=41

$$\frac{c}{2a(a+bx^2)}+\frac{c\log(x)}{a^2}-\frac{c\log(a+bx^2)}{2a^2}$$

```
[Out] 1/2*c/a/(b*x^2+a)+c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {21, 272, 46}

$$-\frac{c\log(a+bx^2)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{c}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] c/(2*a*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ac+bcx^2}{x(a+bx^2)^3}\,dx &= c\int \frac{1}{x(a+bx^2)^2}\,dx \\
&= \frac{1}{2}c\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}c\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)\,dx, x, x^2\right) \\
&= \frac{c}{2a(a+bx^2)} + \frac{c\log(x)}{a^2} - \frac{c\log(a+bx^2)}{2a^2}
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 34, normalized size = 0.83

$$
\frac{c\left(\frac{a}{a+bx^2} + 2\log(x) - \log(a+bx^2)\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] (c*(a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2]))/(2*a^2)
```

Maple [**A**]
time = 0.07, size = 44, normalized size = 1.07

method	result	size
risch	$\frac{c}{2a(b\,x^2+a)} + \frac{c\ln(x)}{a^2} - \frac{c\ln(b\,x^2+a)}{2a^2}$	38
default	$c\left(-\frac{b\left(\frac{\ln(b\,x^2+a)}{b} - \frac{a}{b(b\,x^2+a)}\right)}{2a^2} + \frac{\ln(x)}{a^2}\right)$	44
norman	$\frac{\frac{c}{4} - \frac{b^2c\,x^4}{4a^2}}{(b\,x^2+a)^2} + \frac{c\ln(x)}{a^2} - \frac{c\ln(b\,x^2+a)}{2a^2}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/2*b/a^2*(ln(b*x^2+a)/b-a/b/(b*x^2+a))+ln(x)/a^2)
```

Maxima [**A**]
time = 0.29, size = 40, normalized size = 0.98

$$
\frac{c}{2(abx^2+a^2)} - \frac{c\log(bx^2+a)}{2a^2} + \frac{c\log(x^2)}{2a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/2*c/(a*b*x^2 + a^2) - 1/2*c*log(b*x^2 + a)/a^2 + 1/2*c*log(x^2)/a^2
```

Fricas [A]
time = 0.96, size = 54, normalized size = 1.32

$$\frac{ac - (bcx^2 + ac)\log(bx^2 + a) + 2(bcx^2 + ac)\log(x)}{2(a^2bx^2 + a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/2*(a*c - (b*c*x^2 + a*c)*log(b*x^2 + a) + 2*(b*c*x^2 + a*c)*log(x))/(a^2*
b*x^2 + a^3)
```

Sympy [A]
time = 0.15, size = 36, normalized size = 0.88

$$c\left(\frac{1}{2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x/(b*x**2+a)**3,x)
[Out] c*(1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2))
```

Giac [A]
time = 0.87, size = 51, normalized size = 1.24

$$\frac{c\log(x^2)}{2a^2} - \frac{c\log(|bx^2 + a|)}{2a^2} + \frac{bcx^2 + 2ac}{2(bx^2 + a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/2*c*log(x^2)/a^2 - 1/2*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b*c*x^2 + 2*a*c)/
((b*x^2 + a)*a^2)
```

Mupad [B]
time = 0.05, size = 37, normalized size = 0.90

$$\frac{c}{2a\,(b\,x^2 + a)} - \frac{c\,\ln(b\,x^2 + a)}{2\,a^2} + \frac{c\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x*(a + b*x^2)^3),x)
[Out] c/(2*a*(a + b*x^2)) - (c*log(a + b*x^2))/(2*a^2) + (c*log(x))/a^2
```

3.139 $\int \frac{ac+bcx^2}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=60

$$-\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] -3/2*c/a^2/x+1/2*c/a/x/(b*x^2+a)-3/2*c*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^
(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {21, 296, 331, 211}

$$-\frac{3\sqrt{b}\,c\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] (-3*c)/(2*a^2*x) + c/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^3}\,dx &= c\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx\\ &= \frac{c}{2ax\left(a+bx^2\right)}+\frac{(3c)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a}\\ &= -\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}-\frac{(3bc)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\ &= -\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 56, normalized size = 0.93

$$c\left(-\frac{1}{a^2x}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] c*(-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(5/2)))
```

Maple [**A**]
time = 0.08, size = 47, normalized size = 0.78

method	result	size

default	$c\left(-\frac{b\left(\frac{x}{2b\,x^{2}+2a}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{1}{a^{2}x}\right)$	47
risch	$\frac{-\frac{3bc\,x^{2}}{2a^{2}}-\frac{c}{a}}{x(b\,x^{2}+a)}+\frac{3\sqrt{-ab}\,c\ln\left(-bx+\sqrt{-ab}\right)}{4a^{3}}-\frac{3\sqrt{-ab}\,c\ln\left(-bx-\sqrt{-ab}\right)}{4a^{3}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/a^2/x
)
```

Maxima [A]
time = 0.51, size = 52, normalized size = 0.87

$$-\frac{3\,bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}-\frac{3\,bcx^{2}+2\,ac}{2\,(a^{2}bx^{3}+a^{3}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 + 2*a*c)/(a
^2*b*x^3 + a^3*x)
```

Fricas [A]
time = 0.92, size = 144, normalized size = 2.40

$$\left[-\frac{6\,bcx^{2}-3\,(bcx^{3}+acx)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+4\,ac}{4\,(a^{2}bx^{3}+a^{3}x)},-\frac{3\,bcx^{2}+3\,(bcx^{3}+acx)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,ac}{2\,(a^{2}bx^{3}+a^{3}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*c*x^2 - 3*(b*c*x^3 + a*c*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-
b/a) - a)/(b*x^2 + a)) + 4*a*c)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*c*x^2 + 3*(b
*c*x^3 + a*c*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a*c)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A]
time = 0.19, size = 94, normalized size = 1.57

$$c\left(\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] c*(3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a
**3*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3))
```

Giac [A]
time = 1.11, size = 50, normalized size = 0.83

$$-\frac{3\,bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bcx^2+2\,ac}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 + 2*a*c)/((
b*x^3 + a*x)*a^2)
```

Mupad [B]
time = 0.04, size = 48, normalized size = 0.80

$$-\frac{\frac{c}{a}+\frac{3\,b\,c\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,\,c\,\mathrm{atan}\left(\frac{\sqrt{b}\,\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] - (c/a + (3*b*c*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*c*atan((b^(1/2)*x)
/a^(1/2)))/(2*a^(5/2))
```

3.140 $\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=53

$$-\frac{c}{2a^2x^2}-\frac{bc}{2a^2\left(a+bx^2\right)}-\frac{2bc\log(x)}{a^3}+\frac{bc\log\left(a+bx^2\right)}{a^3}$$

```
[Out] -1/2*c/a^2/x^2-1/2*b*c/a^2/(b*x^2+a)-2*b*c*ln(x)/a^3+b*c*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 272, 46}

$$\frac{bc\log\left(a+bx^2\right)}{a^3}-\frac{2bc\log(x)}{a^3}-\frac{bc}{2a^2\left(a+bx^2\right)}-\frac{c}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*c/(a^2*x^2) - (b*c)/(2*a^2*(a + b*x^2)) - (2*b*c*Log[x])/a^3 + (b*c*Lo
g[a + b*x^2])/a^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^3}\,dx &= c\int \frac{1}{x^3\left(a+bx^2\right)^2}\,dx \\
&= \frac{1}{2}c\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}c\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)\,dx, x, x^2\right) \\
&= -\frac{c}{2a^2x^2}-\frac{bc}{2a^2\left(a+bx^2\right)}-\frac{2bc\log(x)}{a^3}+\frac{bc\log\left(a+bx^2\right)}{a^3}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.79

$$
-\frac{c\left(a\left(\frac{1}{x^2}+\frac{b}{a+bx^2}\right)+4b\log(x)-2b\log\left(a+bx^2\right)\right)}{2a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*(c*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2]))/a^3
```

Maple [A]
time = 0.08, size = 57, normalized size = 1.08

method	result	size
default	$c\left(\frac{b^2\left(\frac{2\ln\left(b\,x^2+a\right)}{b}-\frac{a}{b\left(b\,x^2+a\right)}\right)}{2a^3}-\frac{1}{2a^2x^2}-\frac{2b\ln(x)}{a^3}\right)$	57
risch	$\frac{-\frac{bc\,x^2}{a^2}-\frac{c}{2a}}{x^2(b\,x^2+a)}-\frac{2bc\ln(x)}{a^3}+\frac{bc\ln(-b\,x^2-a)}{a^3}$	58
norman	$\frac{-\frac{c}{2}+\frac{2b^2c\,x^4}{a^2}+\frac{3b^3c\,x^6}{2a^3}}{x^2(b\,x^2+a)^2}+\frac{bc\ln(b\,x^2+a)}{a^3}-\frac{2bc\ln(x)}{a^3}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/2*b^2/a^3*(2*ln(b*x^2+a)/b-a/b/(b*x^2+a))-1/2/a^2/x^2-2*b*ln(x)/a^3)
```

Maxima [A]
time = 0.27, size = 57, normalized size = 1.08

$$
-\frac{2\,bcx^2+ac}{2\left(a^2bx^4+a^3x^2\right)}+\frac{bc\log\left(bx^2+a\right)}{a^3}-\frac{bc\log\left(x^2\right)}{a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*c*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + b*c*log(b*x^2 + a)/a^3 - b*c
*log(x^2)/a^3
```

Fricas [A]
time = 0.95, size = 80, normalized size = 1.51

$$-\frac{2\,abcx^2+a^2c-2\left(b^2cx^4+abcx^2\right)\log\left(bx^2+a\right)+4\left(b^2cx^4+abcx^2\right)\log\left(x\right)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*c*x^2 + a^2*c - 2*(b^2*c*x^4 + a*b*c*x^2)*log(b*x^2 + a) + 4*(b
^2*c*x^4 + a*b*c*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A]
time = 0.18, size = 53, normalized size = 1.00

$$c\left(\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log\left(x\right)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] c*((-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(
a/b + x**2)/a**3)
```

Giac [A]
time = 0.86, size = 56, normalized size = 1.06

$$-\frac{bc\log\left(x^2\right)}{a^3}+\frac{bc\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bcx^2+ac}{2\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -b*c*log(x^2)/a^3 + b*c*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*c*x^2 + a*c)/((b
*x^4 + a*x^2)*a^2)
```

Mupad [B]
time = 0.06, size = 55, normalized size = 1.04

$$\frac{b\,c\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{c}{2\,a}+\frac{b\,c\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\,c\,\ln\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] (b*c*log(a + b*x^2))/a^3 - (c/(2*a) + (b*c*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b
*c*log(x))/a^3
```

3.141 $\int x^4(a+bx^2)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{5}a^2cx^5+\frac{1}{7}a(2bc+ad)x^7+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{11}b^2dx^{11}$$

```
[Out] 1/5*a^2*c*x^5+1/7*a*(a*d+2*b*c)*x^7+1/9*b*(2*a*d+b*c)*x^9+1/11*b^2*d*x^11
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*
x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\int\left(a^2cx^4+a(2bc+ad)x^6+b(bc+2ad)x^8+b^2dx^{10}\right)\,dx\\&=\frac{1}{5}a^2cx^5+\frac{1}{7}a(2bc+ad)x^7+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{11}b^2dx^{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{5}a^2cx^5+\frac{1}{7}a(2bc+ad)x^7+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{11}b^2dx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*
x^11)/11
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.95

method	result	size
default	$\frac{b^2 d\,x^{11}}{11}+\frac{(2abd+b^2c)x^9}{9}+\frac{(a^2d+2abc)x^7}{7}+\frac{a^2c\,x^5}{5}$	52
norman	$\frac{b^2 d\,x^{11}}{11}+\left(\frac{2}{9}abd+\frac{1}{9}b^2c\right)x^9+\left(\frac{1}{7}a^2d+\frac{2}{7}abc\right)x^7+\frac{a^2c\,x^5}{5}$	52
gosper	$\frac{1}{11}b^2d\,x^{11}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{7}x^7a^2d+\frac{2}{7}x^7abc+\frac{1}{5}a^2c\,x^5$	54
risch	$\frac{1}{11}b^2d\,x^{11}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{7}x^7a^2d+\frac{2}{7}x^7abc+\frac{1}{5}a^2c\,x^5$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*d*x^11+1/9*(2*a*b*d+b^2*c)*x^9+1/7*(a^2*d+2*a*b*c)*x^7+1/5*a^2*c*x
^5
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.93

$$\frac{1}{11}\,b^2dx^{11}+\frac{1}{9}\left(b^2c+2\,abd\right)x^9+\frac{1}{5}\,a^2cx^5+\frac{1}{7}\left(2\,abc+a^2d\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d*x^11 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*a^2*c*x^5 + 1/7*(2*a*b*c
+ a^2*d)*x^7
```

Fricas [A]
time = 0.73, size = 51, normalized size = 0.93

$$\frac{1}{11}\,b^2dx^{11}+\frac{1}{9}\left(b^2c+2\,abd\right)x^9+\frac{1}{5}\,a^2cx^5+\frac{1}{7}\left(2\,abc+a^2d\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/11*b^2*d*x^11 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*a^2*c*x^5 + 1/7*(2*a*b*c
+ a^2*d)*x^7
```

Sympy [A]
time = 0.01, size = 56, normalized size = 1.02

$$\frac{a^2cx^5}{5}+\frac{b^2dx^{11}}{11}+x^9\cdot\left(\frac{2abd}{9}+\frac{b^2c}{9}\right)+x^7\left(\frac{a^2d}{7}+\frac{2abc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**5/5 + b**2*d*x**11/11 + x**9*(2*a*b*d/9 + b**2*c/9) + x**7*(a**2*
d/7 + 2*a*b*c/7)
```

Giac [A]
time = 0.95, size = 53, normalized size = 0.96

$$\frac{1}{11}\,b^{2}dx^{11}+\frac{1}{9}\,b^{2}cx^{9}+\frac{2}{9}\,abdx^{9}+\frac{2}{7}\,abcx^{7}+\frac{1}{7}\,a^{2}dx^{7}+\frac{1}{5}\,a^{2}cx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/11*b^2*d*x^11 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/7*a*b*c*x^7 + 1/7*a^2*d
*x^7 + 1/5*a^2*c*x^5
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.93

$$x^{7}\left(\frac{d\,a^{2}}{7}+\frac{2\,b\,c\,a}{7}\right)+x^{9}\left(\frac{c\,b^{2}}{9}+\frac{2\,a\,d\,b}{9}\right)+\frac{a^{2}\,c\,x^{5}}{5}+\frac{b^{2}\,d\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^7*((a^2*d)/7 + (2*a*b*c)/7) + x^9*((b^2*c)/9 + (2*a*b*d)/9) + (a^2*c*x^5)
/5 + (b^2*d*x^11)/11
```

3.142 $\int x^3(a+bx^2)^2(c+dx^2)\,dx$

Optimal. Leaf size=55

$$\frac{1}{4}a^2cx^4+\frac{1}{6}a(2bc+ad)x^6+\frac{1}{8}b(bc+2ad)x^8+\frac{1}{10}b^2dx^{10}$$

```
[Out] 1/4*a^2*c*x^4+1/6*a*(a*d+2*b*c)*x^6+1/8*b*(2*a*d+b*c)*x^8+1/10*b^2*d*x^10
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {457, 77}

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8 + (b^2*d*
x^10)/10
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^2cx+a(2bc+ad)x^2+b(bc+2ad)x^3+b^2dx^4\right)\,dx,x,x^2\right)\\&=\frac{1}{4}a^2cx^4+\frac{1}{6}a(2bc+ad)x^6+\frac{1}{8}b(bc+2ad)x^8+\frac{1}{10}b^2dx^{10}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{4}a^2cx^4 + \frac{1}{6}a(2bc + ad)x^6 + \frac{1}{8}b(bc + 2ad)x^8 + \frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8 + (b^2*d*
x^10)/10
```

Maple [A]
time = 0.08, size = 52, normalized size = 0.95

method	result	size
default	$\frac{b^2 d\,x^{10}}{10}+\frac{(2abd+b^2c)x^8}{8}+\frac{(a^2d+2abc)x^6}{6}+\frac{a^2c\,x^4}{4}$	52
norman	$\frac{b^2 d\,x^{10}}{10}+\left(\frac{1}{4}abd+\frac{1}{8}b^2c\right)x^8+\left(\frac{1}{6}a^2d+\frac{1}{3}abc\right)x^6+\frac{a^2c\,x^4}{4}$	52
gosper	$\frac{1}{10}b^2d\,x^{10}+\frac{1}{4}x^8abd+\frac{1}{8}x^8b^2c+\frac{1}{6}x^6a^2d+\frac{1}{3}x^6abc+\frac{1}{4}a^2c\,x^4$	54
risch	$\frac{1}{10}b^2d\,x^{10}+\frac{1}{4}x^8abd+\frac{1}{8}x^8b^2c+\frac{1}{6}x^6a^2d+\frac{1}{3}x^6abc+\frac{1}{4}a^2c\,x^4$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*d*x^10+1/8*(2*a*b*d+b^2*c)*x^8+1/6*(a^2*d+2*a*b*c)*x^6+1/4*a^2*c*x
^4
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.93

$$\frac{1}{10}\,b^2dx^{10} + \frac{1}{8}\left(b^2c + 2\,abd\right)x^8 + \frac{1}{4}\,a^2cx^4 + \frac{1}{6}\left(2\,abc + a^2d\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/8*(b^2*c + 2*a*b*d)*x^8 + 1/4*a^2*c*x^4 + 1/6*(2*a*b*c
+ a^2*d)*x^6
```

Fricas [A]
time = 0.87, size = 51, normalized size = 0.93

$$\frac{1}{10}\,b^2dx^{10} + \frac{1}{8}\left(b^2c + 2\,abd\right)x^8 + \frac{1}{4}\,a^2cx^4 + \frac{1}{6}\left(2\,abc + a^2d\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d*x^10 + 1/8*(b^2*c + 2*a*b*d)*x^8 + 1/4*a^2*c*x^4 + 1/6*(2*a*b*c
+ a^2*d)*x^6
```

Sympy [A]
time = 0.02, size = 53, normalized size = 0.96

$$\frac{a^2cx^4}{4} + \frac{b^2dx^{10}}{10} + x^8\left(\frac{abd}{4} + \frac{b^2c}{8}\right) + x^6\left(\frac{a^2d}{6} + \frac{abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**4/4 + b**2*d*x**10/10 + x**8*(a*b*d/4 + b**2*c/8) + x**6*(a**2*d/
6 + a*b*c/3)
```

Giac [A]
time = 0.78, size = 53, normalized size = 0.96

$$\frac{1}{10}\,b^2dx^{10} + \frac{1}{8}\,b^2cx^8 + \frac{1}{4}\,abdx^8 + \frac{1}{3}\,abcx^6 + \frac{1}{6}\,a^2dx^6 + \frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/8*b^2*c*x^8 + 1/4*a*b*d*x^8 + 1/3*a*b*c*x^6 + 1/6*a^2*d
*x^6 + 1/4*a^2*c*x^4
```

Mupad [B]
time = 0.02, size = 51, normalized size = 0.93

$$x^6\left(\frac{d\,a^2}{6} + \frac{b\,c\,a}{3}\right) + x^8\left(\frac{c\,b^2}{8} + \frac{a\,d\,b}{4}\right) + \frac{a^2\,c\,x^4}{4} + \frac{b^2\,d\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^6*((a^2*d)/6 + (a*b*c)/3) + x^8*((b^2*c)/8 + (a*b*d)/4) + (a^2*c*x^4)/4 +
 (b^2*d*x^10)/10
```

3.143 $\int x^2(a+bx^2)^2(c+dx^2)\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}a^2cx^3+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{9}b^2dx^9$$

```
[Out] 1/3*a^2*c*x^3+1/5*a*(a*d+2*b*c)*x^5+1/7*b*(2*a*d+b*c)*x^7+1/9*b^2*d*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$\frac{1}{3}a^2cx^3+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{9}b^2dx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*
x^9)/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\int\left(a^2cx^2+a(2bc+ad)x^4+b(bc+2ad)x^6+b^2dx^8\right)\,dx\\&=\frac{1}{3}a^2cx^3+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{9}b^2dx^9\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{3}a^2cx^3+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{9}b^2dx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*
x^9)/9
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.95

method	result	size
default	$\frac{b^{2} d \,x^{9}}{9}+\frac{\left(2 a b d +b^{2} c \right) x^{7}}{7}+\frac{\left(a^{2} d +2 a b c \right) x^{5}}{5}+\frac{a^{2} c \,x^{3}}{3}$	52
norman	$\frac{b^{2} d \,x^{9}}{9}+\left(\frac{2}{7} a b d +\frac{1}{7} b^{2} c \right) x^{7}+\left(\frac{1}{5} a^{2} d +\frac{2}{5} a b c \right) x^{5}+\frac{a^{2} c \,x^{3}}{3}$	52
gosper	$\frac{1}{9} b^{2} d \,x^{9}+\frac{2}{7} x^{7} a b d +\frac{1}{7} x^{7} b^{2} c +\frac{1}{5} x^{5} a^{2} d +\frac{2}{5} x^{5} a b c +\frac{1}{3} a^{2} c \,x^{3}$	54
risch	$\frac{1}{9} b^{2} d \,x^{9}+\frac{2}{7} x^{7} a b d +\frac{1}{7} x^{7} b^{2} c +\frac{1}{5} x^{5} a^{2} d +\frac{2}{5} x^{5} a b c +\frac{1}{3} a^{2} c \,x^{3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*d*x^9+1/7*(2*a*b*d+b^2*c)*x^7+1/5*(a^2*d+2*a*b*c)*x^5+1/3*a^2*c*x^3
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.93

$$\frac{1}{9}\,b^{2}dx^{9}+\frac{1}{7}\,{\left(b^{2}c+2\,abd\right)}x^{7}+\frac{1}{3}\,a^{2}cx^{3}+\frac{1}{5}\,{\left(2\,abc+a^{2}d\right)}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d*x^9 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/3*a^2*c*x^3 + 1/5*(2*a*b*c +
a^2*d)*x^5
```

Fricas [A]
time = 0.89, size = 51, normalized size = 0.93

$$\frac{1}{9}\,b^{2}dx^{9}+\frac{1}{7}\,{\left(b^{2}c+2\,abd\right)}x^{7}+\frac{1}{3}\,a^{2}cx^{3}+\frac{1}{5}\,{\left(2\,abc+a^{2}d\right)}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/9*b^2*d*x^9 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/3*a^2*c*x^3 + 1/5*(2*a*b*c +
a^2*d)*x^5
```

Sympy [A]
time = 0.01, size = 56, normalized size = 1.02

$$\frac{a^{2}cx^{3}}{3}+\frac{b^{2}dx^{9}}{9}+x^{7}\cdot\left(\frac{2abd}{7}+\frac{b^{2}c}{7}\right)+x^{5}\left(\frac{a^{2}d}{5}+\frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**3/3 + b**2*d*x**9/9 + x**7*(2*a*b*d/7 + b**2*c/7) + x**5*(a**2*d/
5 + 2*a*b*c/5)
```

Giac [A]
time = 1.16, size = 53, normalized size = 0.96

$$\frac{1}{9}\,b^2dx^9+\frac{1}{7}\,b^2cx^7+\frac{2}{7}\,abdx^7+\frac{2}{5}\,abcx^5+\frac{1}{5}\,a^2dx^5+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/9*b^2*d*x^9 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 2/5*a*b*c*x^5 + 1/5*a^2*d*x
^5 + 1/3*a^2*c*x^3
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.93

$$x^5\left(\frac{d\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,d\,b}{7}\right)+\frac{a^2\,c\,x^3}{3}+\frac{b^2\,d\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^5*((a^2*d)/5 + (2*a*b*c)/5) + x^7*((b^2*c)/7 + (2*a*b*d)/7) + (a^2*c*x^3)
/3 + (b^2*d*x^9)/9
```

3.144 $\int x(a+bx^2)^2 (c+dx^2)\, dx$

Optimal. Leaf size=42

$$\frac{(bc-ad)(a+bx^2)^3}{6b^2}+\frac{d(a+bx^2)^4}{8b^2}$$

```
[Out] 1/6*(-a*d+b*c)*(b*x^2+a)^3/b^2+1/8*d*(b*x^2+a)^4/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {455, 45}

$$\frac{(a+bx^2)^3(bc-ad)}{6b^2}+\frac{d(a+bx^2)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)*(a + b*x^2)^3)/(6*b^2) + (d*(a + b*x^2)^4)/(8*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x(a+bx^2)^2(c+dx^2)\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^2(c+dx)\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(bc-ad)(a+bx)^2}{b}+\frac{d(a+bx)^3}{b}\right)dx, x, x^2\right)\\
&= \frac{(bc-ad)(a+bx^2)^3}{6b^2}+\frac{d(a+bx^2)^4}{8b^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12a^2c+6a(2bc+ad)x^2+4b(bc+2ad)x^4+3b^2dx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (x^2*(12*a^2*c + 6*a*(2*b*c + a*d)*x^2 + 4*b*(b*c + 2*a*d)*x^4 + 3*b^2*d*x^
6))/24
```

Maple [A]
time = 0.08, size = 52, normalized size = 1.24

method	result	size
default	$\frac{b^2 d\,x^8}{8}+\frac{(2abd+b^2c)x^6}{6}+\frac{(a^2d+2abc)x^4}{4}+\frac{a^2c\,x^2}{2}$	52
norman	$\frac{b^2 d\,x^8}{8}+\left(\frac{1}{3}abd+\frac{1}{6}b^2c\right)x^6+\left(\frac{1}{4}a^2d+\frac{1}{2}abc\right)x^4+\frac{a^2c\,x^2}{2}$	52
gosper	$\frac{1}{8}b^2d\,x^8+\frac{1}{3}x^6abd+\frac{1}{6}x^6b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+\frac{1}{2}a^2c\,x^2$	54
risch	$\frac{1}{8}b^2d\,x^8+\frac{1}{3}x^6abd+\frac{1}{6}x^6b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+\frac{1}{2}a^2c\,x^2$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^2*d*x^8+1/6*(2*a*b*d+b^2*c)*x^6+1/4*(a^2*d+2*a*b*c)*x^4+1/2*a^2*c*x^2
```

Maxima [A]
time = 0.30, size = 51, normalized size = 1.21

$$\frac{1}{8}\,b^2dx^8+\frac{1}{6}\,\left(b^2c+2\,abd\right)x^6+\frac{1}{2}\,a^2cx^2+\frac{1}{4}\,\left(2\,abc+a^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d*x^8 + 1/6*(b^2*c + 2*a*b*d)*x^6 + 1/2*a^2*c*x^2 + 1/4*(2*a*b*c +
a^2*d)*x^4
```

Fricas [A]
time = 0.81, size = 51, normalized size = 1.21

$$\frac{1}{8}\,b^2dx^8+\frac{1}{6}\,\left(b^2c+2\,abd\right)x^6+\frac{1}{2}\,a^2cx^2+\frac{1}{4}\,\left(2\,abc+a^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/8*b^2*d*x^8 + 1/6*(b^2*c + 2*a*b*d)*x^6 + 1/2*a^2*c*x^2 + 1/4*(2*a*b*c +
a^2*d)*x^4
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.26

$$\frac{a^2cx^2}{2} + \frac{b^2dx^8}{8} + x^6\left(\frac{abd}{3} + \frac{b^2c}{6}\right) + x^4\left(\frac{a^2d}{4} + \frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**2/2 + b**2*d*x**8/8 + x**6*(a*b*d/3 + b**2*c/6) + x**4*(a**2*d/4
+ a*b*c/2)
```

Giac [A]
time = 0.75, size = 53, normalized size = 1.26

$$\frac{1}{8}\,b^2dx^8 + \frac{1}{6}\,b^2cx^6 + \frac{1}{3}\,abdx^6 + \frac{1}{2}\,abcx^4 + \frac{1}{4}\,a^2dx^4 + \frac{1}{2}\,a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/8*b^2*d*x^8 + 1/6*b^2*c*x^6 + 1/3*a*b*d*x^6 + 1/2*a*b*c*x^4 + 1/4*a^2*d*x
^4 + 1/2*a^2*c*x^2
```

Mupad [B]
time = 0.02, size = 51, normalized size = 1.21

$$x^4\left(\frac{d\,a^2}{4} + \frac{b\,c\,a}{2}\right) + x^6\left(\frac{c\,b^2}{6} + \frac{a\,d\,b}{3}\right) + \frac{a^2\,c\,x^2}{2} + \frac{b^2\,d\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^4*((a^2*d)/4 + (a*b*c)/2) + x^6*((b^2*c)/6 + (a*b*d)/3) + (a^2*c*x^2)/2 +
 (b^2*d*x^8)/8
```

3.145 $\int (a+bx^2)^2 (c+dx^2)\, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7$$

```
[Out] a^2*c*x+1/3*a*(a*d+2*b*c)*x^3+1/5*b*(2*a*d+b*c)*x^5+1/7*b^2*d*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2),x]
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^2 (c+dx^2)\, dx &= \int \left(a^2c+a(2bc+ad)x^2+b(bc+2ad)x^4+b^2dx^6\right) dx\\ &= a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2),x]
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Maple [A]
time = 0.06, size = 49, normalized size = 0.98

method	result	size
default	$\frac{b^{2} d \,x^{7}}{7}+\frac{\left(2 a b d +b^{2} c \right) x^{5}}{5}+\frac{\left(a^{2} d +2 a b c \right) x^{3}}{3}+a^{2} c x$	49
norman	$\frac{b^{2} d \,x^{7}}{7}+\left(\frac{2}{5} a b d +\frac{1}{5} b^{2} c \right) x^{5}+\left(\frac{1}{3} a^{2} d +\frac{2}{3} a b c \right) x^{3}+a^{2} c x$	49
gosper	$\frac{1}{7} b^{2} d \,x^{7}+\frac{2}{5} x^{5} a b d +\frac{1}{5} x^{5} b^{2} c +\frac{1}{3} x^{3} a^{2} d +\frac{2}{3} x^{3} a b c +a^{2} c x$	51
risch	$\frac{1}{7} b^{2} d \,x^{7}+\frac{2}{5} x^{5} a b d +\frac{1}{5} x^{5} b^{2} c +\frac{1}{3} x^{3} a^{2} d +\frac{2}{3} x^{3} a b c +a^{2} c x$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2*c*x
```

Maxima [A]
time = 0.31, size = 48, normalized size = 0.96

$$\frac{1}{7}\,b^{2}dx^{7}+\frac{1}{5}\,\left(b^{2}c+2\,abd\right)x^{5}+a^{2}cx+\frac{1}{3}\,\left(2\,abc+a^{2}d\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

Fricas [A]
time = 0.67, size = 48, normalized size = 0.96

$$\frac{1}{7}\,b^{2}dx^{7}+\frac{1}{5}\,\left(b^{2}c+2\,abd\right)x^{5}+a^{2}cx+\frac{1}{3}\,\left(2\,abc+a^{2}d\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.06

$$a^{2}cx+\frac{b^{2}dx^{7}}{7}+x^{5}\cdot\left(\frac{2abd}{5}+\frac{b^{2}c}{5}\right)+x^{3}\left(\frac{a^{2}d}{3}+\frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a**2*d/3 + 2
*a*b*c/3)
```

Giac [A]
time = 1.29, size = 50, normalized size = 1.00

$$\frac{1}{7}\,b^2dx^7+\frac{1}{5}\,b^2cx^5+\frac{2}{5}\,abdx^5+\frac{2}{3}\,abcx^3+\frac{1}{3}\,a^2dx^3+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1/3*a^2*d*x
^3 + a^2*c*x
```

Mupad [B]
time = 0.03, size = 48, normalized size = 0.96

$$x^3\left(\frac{d\,a^2}{3}+\frac{2\,b\,c\,a}{3}\right)+x^5\left(\frac{c\,b^2}{5}+\frac{2\,a\,d\,b}{5}\right)+\frac{b^2\,d\,x^7}{7}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^3*((a^2*d)/3 + (2*a*b*c)/3) + x^5*((b^2*c)/5 + (2*a*b*d)/5) + (b^2*d*x^7)
/7 + a^2*c*x
```

3.146 $\int \frac{(a+bx^2)^2(c+dx^2)}{x} \, dx$

Optimal. Leaf size=43

$$abcx^2 + \frac{1}{4}b^2cx^4 + \frac{d(a+bx^2)^3}{6b} + a^2c\log(x)$$

```
[Out] a*b*c*x^2+1/4*b^2*c*x^4+1/6*d*(b*x^2+a)^3/b+a^2*c*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 81, 45}

$$a^2c\log(x) + abcx^2 + \frac{d(a+bx^2)^3}{6b} + \frac{1}{4}b^2cx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

```
[Out] a*b*c*x^2 + (b^2*c*x^4)/4 + (d*(a + b*x^2)^3)/(6*b) + a^2*c*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\,(c+dx^2)}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)}{x}\,dx, x, x^2\right)$$
$$= \frac{d(a+bx^2)^3}{6b} + \frac{1}{2}c\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, x^2\right)$$
$$= \frac{d(a+bx^2)^3}{6b} + \frac{1}{2}c\text{Subst}\left(\int \left(2ab + \frac{a^2}{x} + b^2x\right)\,dx, x, x^2\right)$$
$$= abcx^2 + \frac{1}{4}b^2cx^4 + \frac{d(a+bx^2)^3}{6b} + a^2c\log(x)$$

Mathematica [**A**]
time = 0.01, size = 51, normalized size = 1.19

$$\frac{1}{2}a(2bc+ad)x^2 + \frac{1}{4}b(bc+2ad)x^4 + \frac{1}{6}b^2dx^6 + a^2c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

```
[Out] (a*(2*b*c + a*d)*x^2)/2 + (b*(b*c + 2*a*d)*x^4)/4 + (b^2*d*x^6)/6 + a^2*c*L
og[x]
```

Maple [**A**]
time = 0.07, size = 51, normalized size = 1.19

method	result	size
norman	$\left(\frac{1}{2}a^2d+abc\right)x^2+\left(\frac{1}{2}abd+\frac{1}{4}b^2c\right)x^4+\frac{b^2d\,x^6}{6}+a^2c\ln\left(x\right)$	49
default	$\frac{b^2d\,x^6}{6}+\frac{abd\,x^4}{2}+\frac{b^2c\,x^4}{4}+\frac{a^2d\,x^2}{2}+abc\,x^2+a^2c\ln\left(x\right)$	51
risch	$\frac{b^2d\,x^6}{6}+\frac{abd\,x^4}{2}+\frac{b^2c\,x^4}{4}+\frac{a^2d\,x^2}{2}+abc\,x^2+a^2c\ln\left(x\right)$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2*d*x^6+1/2*a*b*d*x^4+1/4*b^2*c*x^4+1/2*a^2*d*x^2+a*b*c*x^2+a^2*c*ln(
x)
```

Maxima [**A**]
time = 0.37, size = 52, normalized size = 1.21

$$\frac{1}{6}\,b^2dx^6 + \frac{1}{4}\,(b^2c + 2\,abd)x^4 + \frac{1}{2}\,a^2c\log\left(x^2\right) + \frac{1}{2}\,(2\,abc + a^2d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + 1/2*a^2*c*log(x^2) + 1/2*(2*a*b
*c + a^2*d)*x^2
```

Fricas [A]
time = 0.97, size = 49, normalized size = 1.14

$$\frac{1}{6}\,b^2dx^6+\frac{1}{4}\left(b^2c+2\,abd\right)x^4+a^2c\log\left(x\right)+\frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + a^2*c*log(x) + 1/2*(2*a*b*c + a
^2*d)*x^2
```

Sympy [A]
time = 0.05, size = 49, normalized size = 1.14

$$a^2c\log\left(x\right)+\frac{b^2dx^6}{6}+x^4\left(\frac{abd}{2}+\frac{b^2c}{4}\right)+x^2\left(\frac{a^2d}{2}+abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x,x)
```

```
[Out] a**2*c*log(x) + b**2*d*x**6/6 + x**4*(a*b*d/2 + b**2*c/4) + x**2*(a**2*d/2
+ a*b*c)
```

Giac [A]
time = 0.92, size = 53, normalized size = 1.23

$$\frac{1}{6}\,b^2dx^6+\frac{1}{4}\,b^2cx^4+\frac{1}{2}\,abdx^4+abcx^2+\frac{1}{2}\,a^2dx^2+\frac{1}{2}\,a^2c\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*b^2*c*x^4 + 1/2*a*b*d*x^4 + a*b*c*x^2 + 1/2*a^2*d*x^2 +
 1/2*a^2*c*log(x^2)
```

Mupad [B]
time = 0.02, size = 48, normalized size = 1.12

$$x^2\,\left(\frac{d\,a^2}{2}+b\,c\,a\right)+x^4\,\left(\frac{c\,b^2}{4}+\frac{a\,d\,b}{2}\right)+\frac{b^2\,d\,x^6}{6}+a^2\,c\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x,x)
```

```
[Out] x^2*((a^2*d)/2 + a*b*c) + x^4*((b^2*c)/4 + (a*b*d)/2) + (b^2*d*x^6)/6 + a^2
*c*log(x)
```

3.147 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^2} dx$

Optimal. Leaf size=48

$$-\frac{a^2c}{x} + a(2bc + ad)x + \frac{1}{3}b(bc + 2ad)x^3 + \frac{1}{5}b^2dx^5$$

```
[Out] -a^2*c/x+a*(a*d+2*b*c)*x+1/3*b*(2*a*d+b*c)*x^3+1/5*b^2*d*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$-\frac{a^2c}{x} + \frac{1}{3}bx^3(2ad + bc) + ax(ad + 2bc) + \frac{1}{5}b^2dx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2*d*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^2}\,dx &= \int \left(a(2bc+ad) + \frac{a^2c}{x^2} + b(bc+2ad)x^2 + b^2dx^4\right) dx\\ &= -\frac{a^2c}{x} + a(2bc+ad)x + \frac{1}{3}b(bc+2ad)x^3 + \frac{1}{5}b^2dx^5\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.00

$$-\frac{a^2c}{x} + a(2bc + ad)x + \frac{1}{3}b(bc + 2ad)x^3 + \frac{1}{5}b^2dx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2*d*x^5)/5
```

Maple [A]
time = 0.06, size = 49, normalized size = 1.02

method	result	size
default	$\frac{b^2 d x^5}{5}+\frac{2abd x^3}{3}+\frac{b^2 c x^3}{3}+a^2 dx+2abcx-\frac{a^2 c}{x}$	49
risch	$\frac{b^2 d x^5}{5}+\frac{2abd x^3}{3}+\frac{b^2 c x^3}{3}+a^2 dx+2abcx-\frac{a^2 c}{x}$	49
norman	$\frac{\frac{b^2 d x^6}{5}+\left(\frac{2}{3}abd+\frac{1}{3}b^2 c\right)x^4+\left(a^2 d+2abc\right)x^2-a^2 c}{x}$	52
gosper	$-\frac{-3b^2 d x^6-10abd x^4-5b^2 c x^4-15a^2 d x^2-30abc x^2+15a^2 c}{15x}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/5*b^2*d*x^5+2/3*a*b*d*x^3+1/3*b^2*c*x^3+a^2*d*x+2*a*b*c*x-a^2*c/x
```

Maxima [A]
time = 0.27, size = 48, normalized size = 1.00

$$\frac{1}{5}\,b^2dx^5+\frac{1}{3}\,\left(b^2c+2\,abd\right)x^3-\frac{a^2c}{x}+\left(2\,abc+a^2d\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="maxima")
[Out] 1/5*b^2*d*x^5 + 1/3*(b^2*c + 2*a*b*d)*x^3 - a^2*c/x + (2*a*b*c + a^2*d)*x
```

Fricas [A]
time = 1.20, size = 53, normalized size = 1.10

$$\frac{3\,b^2dx^6+5\,(b^2c+2\,abd)x^4-15\,a^2c+15\,(2\,abc+a^2d)x^2}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="fricas")
[Out] 1/15*(3*b^2*d*x^6 + 5*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 15*(2*a*b*c + a^2*
d)*x^2)/x
```

Sympy [A]
time = 0.05, size = 48, normalized size = 1.00

$$-\frac{a^2c}{x}+\frac{b^2dx^5}{5}+x^3\cdot\left(\frac{2abd}{3}+\frac{b^2c}{3}\right)+x\left(a^2d+2abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**2,x)
```

```
[Out] -a**2*c/x + b**2*d*x**5/5 + x**3*(2*a*b*d/3 + b**2*c/3) + x*(a**2*d + 2*a*b
*c)
```

Giac [A]
time = 0.94, size = 48, normalized size = 1.00

$$\frac{1}{5}\,b^{2}dx^{5}+\frac{1}{3}\,b^{2}cx^{3}+\frac{2}{3}\,abdx^{3}+2\,abcx+a^{2}dx-\frac{a^{2}c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d*x^5 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + 2*a*b*c*x + a^2*d*x - a^2*c
/x
```

Mupad [B]
time = 0.03, size = 48, normalized size = 1.00

$$x\,\left(d\,a^2+2\,b\,c\,a\right)+x^3\,\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right)-\frac{a^2\,c}{x}+\frac{b^2\,d\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^2,x)
```

```
[Out] x*(a^2*d + 2*a*b*c) + x^3*((b^2*c)/3 + (2*a*b*d)/3) - (a^2*c)/x + (b^2*d*x^
5)/5
```

3.148 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2c}{2x^2}+\frac{1}{2}b(bc+2ad)x^2+\frac{1}{4}b^2dx^4+a(2bc+ad)\log(x)$$

```
[Out] -1/2*a^2*c/x^2+1/2*b*(2*a*d+b*c)*x^2+1/4*b^2*d*x^4+a*(a*d+2*b*c)*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$-\frac{a^2c}{2x^2}+\frac{1}{2}bx^2(2ad+bc)+a\log(x)(ad+2bc)+\frac{1}{4}b^2dx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

```
[Out] -1/2*(a^2*c)/x^2 + (b*(b*c + 2*a*d)*x^2)/2 + (b^2*d*x^4)/4 + a*(2*b*c + a*d
)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^3} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)}{x^2} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(b(bc+2ad)+\frac{a^2c}{x^2}+\frac{a(2bc+ad)}{x}+b^2dx\right) dx, x, x^2\right)$$
$$= -\frac{a^2c}{2x^2}+\frac{1}{2}b(bc+2ad)x^2+\frac{1}{4}b^2dx^4+a(2bc+ad)\log(x)$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2a^2c}{x^2}+2b(bc+2ad)x^2+b^2dx^4+4a(2bc+ad)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

```
[Out] ((-2*a^2*c)/x^2 + 2*b*(b*c + 2*a*d)*x^2 + b^2*d*x^4 + 4*a*(2*b*c + a*d)*Log
[x])/4
```

Maple [A]
time = 0.08, size = 48, normalized size = 0.94

method	result	size
default	$\frac{b^2 d\,x^4}{4}+abd\,x^2+\frac{b^2 c\,x^2}{2}-\frac{a^2 c}{2x^2}+a(ad+2bc)\ln(x)$	48
norman	$\frac{(abd+\frac{1}{2}b^2c)x^4-\frac{a^2c}{2}+\frac{b^2 d\,x^6}{4}}{x^2}+(a^2d+2abc)\ln(x)$	51
risch	$\frac{b^2 d\,x^4}{4}+abd\,x^2+\frac{b^2 c\,x^2}{2}+a^2d+abc+\frac{b^2c^2}{4d}-\frac{a^2c}{2x^2}+\ln(x)\,a^2d+2\ln(x)\,abc$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2*d*x^4+a*b*d*x^2+1/2*b^2*c*x^2-1/2*a^2*c/x^2+a*(a*d+2*b*c)*ln(x)
```

Maxima [A]
time = 0.29, size = 52, normalized size = 1.02

$$\frac{1}{4}\,b^2dx^4+\frac{1}{2}\,(b^2c+2\,abd)x^2+\frac{1}{2}\,(2\,abc+a^2d)\log\left(x^2\right)-\frac{a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d*x^4 + 1/2*(b^2*c + 2*a*b*d)*x^2 + 1/2*(2*a*b*c + a^2*d)*log(x^2)
- 1/2*a^2*c/x^2
```

Fricas [A]
time = 0.98, size = 54, normalized size = 1.06

$$\frac{b^2dx^6 + 2\,(b^2c + 2\,abd)x^4 + 4\,(2\,abc + a^2d)x^2\log{(x)} - 2\,a^2c}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^6 + 2*(b^2*c + 2*a*b*d)*x^4 + 4*(2*a*b*c + a^2*d)*x^2*log(x) -
 2*a^2*c)/x^2
```

Sympy [A]
time = 0.10, size = 48, normalized size = 0.94

$$-\frac{a^2c}{2x^2} + a(ad + 2bc)\log{(x)} + \frac{b^2dx^4}{4} + x^2\left(abd + \frac{b^2c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**3,x)
```

```
[Out] -a**2*c/(2*x**2) + a*(a*d + 2*b*c)*log(x) + b**2*d*x**4/4 + x**2*(a*b*d + b
**2*c/2)
```

Giac [A]
time = 0.80, size = 70, normalized size = 1.37

$$\frac{1}{4}\,b^2dx^4 + \frac{1}{2}\,b^2cx^2 + abdx^2 + \frac{1}{2}\,(2\,abc + a^2d)\log{(x^2)} - \frac{2\,abcx^2 + a^2dx^2 + a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^2*d*x^4 + 1/2*b^2*c*x^2 + a*b*d*x^2 + 1/2*(2*a*b*c + a^2*d)*log(x^2)
- 1/2*(2*a*b*c*x^2 + a^2*d*x^2 + a^2*c)/x^2
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.94

$$x^2\left(\frac{c\,b^2}{2} + a\,d\,b\right) + \ln{(x)}\,\left(d\,a^2 + 2\,b\,c\,a\right) - \frac{a^2\,c}{2\,x^2} + \frac{b^2\,d\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^3,x)
```

```
[Out] x^2*((b^2*c)/2 + a*b*d) + log(x)*(a^2*d + 2*a*b*c) - (a^2*c)/(2*x^2) + (b^2
*d*x^4)/4
```

3.149 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^4}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2c}{3x^3}-\frac{a(2bc+ad)}{x}+b(bc+2ad)x+\frac{1}{3}b^2dx^3$$

```
[Out] -1/3*a^2*c/x^3-a*(a*d+2*b*c)/x+b*(2*a*d+b*c)*x+1/3*b^2*d*x^3
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$-\frac{a^2c}{3x^3}+bx(2ad+bc)-\frac{a(ad+2bc)}{x}+\frac{1}{3}b^2dx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

```
[Out] -1/3*(a^2*c)/x^3 - (a*(2*b*c + a*d))/x + b*(b*c + 2*a*d)*x + (b^2*d*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^4}\,dx &= \int \left(b(bc+2ad)+\frac{a^2c}{x^4}+\frac{a(2bc+ad)}{x^2}+b^2dx^2\right)dx\\ &= -\frac{a^2c}{3x^3}-\frac{a(2bc+ad)}{x}+b(bc+2ad)x+\frac{1}{3}b^2dx^3\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.04

$$-\frac{a^2c}{3x^3}+\frac{-2abc-a^2d}{x}+b(bc+2ad)x+\frac{1}{3}b^2dx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
[Out] -1/3*(a^2*c)/x^3 + (-2*a*b*c - a^2*d)/x + b*(b*c + 2*a*d)*x + (b^2*d*x^3)/3
```

Maple [A]
time = 0.07, size = 46, normalized size = 0.96

method	result	size
default	$\frac{b^2 d\,x^3}{3}+2abdx+b^2cx-\frac{a^2c}{3x^3}-\frac{a(ad+2bc)}{x}$	46
risch	$\frac{b^2 d\,x^3}{3}+2abdx+b^2cx+\frac{(-a^2d-2abc)x^2-\frac{a^2c}{3}}{x^3}$	50
norman	$\frac{\frac{b^2 d\,x^6}{3}+(2abd+b^2c)x^4+(-a^2d-2abc)x^2-\frac{a^2c}{3}}{x^3}$	52
gosper	$-\frac{-b^2 d\,x^6-6abd\,x^4-3b^2c\,x^4+3a^2d\,x^2+6abc\,x^2+a^2c}{3x^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/3*b^2*d*x^3+2*a*b*d*x+b^2*c*x-1/3*a^2*c/x^3-a*(a*d+2*b*c)/x
```

Maxima [A]
time = 0.27, size = 50, normalized size = 1.04

$$\frac{1}{3}\,b^2dx^3+\left(b^2c+2\,abd\right)x-\frac{a^2c+3\,(2\,abc+a^2d)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="maxima")
[Out] 1/3*b^2*d*x^3 + (b^2*c + 2*a*b*d)*x - 1/3*(a^2*c + 3*(2*a*b*c + a^2*d)*x^2)
/x^3
```

Fricas [A]
time = 1.05, size = 52, normalized size = 1.08

$$\frac{b^2dx^6+3\,(b^2c+2\,abd)x^4-a^2c-3\,(2\,abc+a^2d)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="fricas")
[Out] 1/3*(b^2*d*x^6 + 3*(b^2*c + 2*a*b*d)*x^4 - a^2*c - 3*(2*a*b*c + a^2*d)*x^2)
/x^3
```

Sympy [A]
time = 0.11, size = 51, normalized size = 1.06

$$\frac{b^2dx^3}{3}+x\left(2abd+b^2c\right)+\frac{-a^2c+x^2(-3a^2d-6abc)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**4,x)
```

```
[Out] b**2*d*x**3/3 + x*(2*a*b*d + b**2*c) + (-a**2*c + x**2*(-3*a**2*d - 6*a*b*c
))/(3*x**3)
```

Giac [A]
time = 1.57, size = 50, normalized size = 1.04

$$\frac{1}{3}b^2dx^3 + b^2cx + 2\,abdx - \frac{6\,abcx^2 + 3\,a^2dx^2 + a^2c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^2*d*x^3 + b^2*c*x + 2*a*b*d*x - 1/3*(6*a*b*c*x^2 + 3*a^2*d*x^2 + a^2*
c)/x^3
```

Mupad [B]
time = 0.03, size = 50, normalized size = 1.04

$$x\left(c\,b^2 + 2\,a\,d\,b\right) - \frac{\frac{a^2\,c}{3} + x^2\left(d\,a^2 + 2\,b\,c\,a\right)}{x^3} + \frac{b^2\,d\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^4,x)
```

```
[Out] x*(b^2*c + 2*a*b*d) - ((a^2*c)/3 + x^2*(a^2*d + 2*a*b*c))/x^3 + (b^2*d*x^3)
/3
```

3.150 $\int x^4(a+bx^2)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=87

$$\frac{1}{5}a^2c^2x^5+\frac{2}{7}ac(bc+ad)x^7+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{11}bd(bc+ad)x^{11}+\frac{1}{13}b^2d^2x^{13}$$

```
[Out] 1/5*a^2*c^2*x^5+2/7*a*c*(a*d+b*c)*x^7+1/9*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^9+2
/11*b*d*(a*d+b*c)*x^11+1/13*b^2*d^2*x^13
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^13)/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx&=\int\left(a^2c^2x^4+2ac(bc+ad)x^6+\left(b^2c^2+4abcd+a^2d^2\right)x^8+2bd(bc+ad)x^{10}\right.\\&=\frac{1}{5}a^2c^2x^5+\frac{2}{7}ac(bc+ad)x^7+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{11}bd(bc+ad)x^{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 87, normalized size = 1.00

$$\frac{1}{5}a^2c^2x^5+\frac{2}{7}ac(bc+ad)x^7+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{11}bd(bc+ad)x^{11}+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^13)/13
```

Maple [A]
time = 0.10, size = 90, normalized size = 1.03

method	result
norman	$\frac{b^2d^2x^{13}}{13}+\left(\frac{2}{11}ab\,d^2+\frac{2}{11}b^2cd\right)x^{11}+\left(\frac{1}{9}a^2d^2+\frac{4}{9}abcd+\frac{1}{9}b^2c^2\right)x^9+\left(\frac{2}{7}a^2cd+\frac{2}{7}ab\,c^2\right)x^7+\frac{a^2c^2x^5}{5}$
default	$\frac{b^2d^2x^{13}}{13}+\frac{(2ab\,d^2+2b^2cd)x^{11}}{11}+\frac{(a^2d^2+4abcd+b^2c^2)x^9}{9}+\frac{(2a^2cd+2ab\,c^2)x^7}{7}+\frac{a^2c^2x^5}{5}$
gosper	$\frac{1}{13}b^2d^2x^{13}+\frac{2}{11}x^{11}ab\,d^2+\frac{2}{11}x^{11}b^2cd+\frac{1}{9}x^9a^2d^2+\frac{4}{9}x^9abcd+\frac{1}{9}x^9b^2c^2+\frac{2}{7}x^7a^2cd+\frac{2}{7}x^7ab\,c^2+\frac{1}{5}a^2$
risch	$\frac{1}{13}b^2d^2x^{13}+\frac{2}{11}x^{11}ab\,d^2+\frac{2}{11}x^{11}b^2cd+\frac{1}{9}x^9a^2d^2+\frac{4}{9}x^9abcd+\frac{1}{9}x^9b^2c^2+\frac{2}{7}x^7a^2cd+\frac{2}{7}x^7ab\,c^2+\frac{1}{5}a^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^2*d^2*x^13+1/11*(2*a*b*d^2+2*b^2*c*d)*x^11+1/9*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^9+1/7*(2*a^2*c*d+2*a*b*c^2)*x^7+1/5*a^2*c^2*x^5
```

Maxima [A]
time = 0.28, size = 85, normalized size = 0.98

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{2}{11}\,(b^2cd+abd^2)x^{11}+\frac{1}{9}\,(b^2c^2+4\,abcd+a^2d^2)x^9+\frac{1}{5}\,a^2c^2x^5+\frac{2}{7}\,(abc^2+a^2cd)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 2/11*(b^2*c*d + a*b*d^2)*x^11 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 1/5*a^2*c^2*x^5 + 2/7*(a*b*c^2 + a^2*c*d)*x^7
```

Fricas [A]
time = 0.68, size = 85, normalized size = 0.98

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{2}{11}\,(b^2cd+abd^2)x^{11}+\frac{1}{9}\,(b^2c^2+4\,abcd+a^2d^2)x^9+\frac{1}{5}\,a^2c^2x^5+\frac{2}{7}\,(abc^2+a^2cd)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*b^2*d^2*x^13 + 2/11*(b^2*c*d + a*b*d^2)*x^11 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 1/5*a^2*c^2*x^5 + 2/7*(a*b*c^2 + a^2*c*d)*x^7
```

Sympy [A]
time = 0.01, size = 100, normalized size = 1.15

$$\frac{a^2c^2x^5}{5}+\frac{b^2d^2x^{13}}{13}+x^{11}\cdot\left(\frac{2abd^2}{11}+\frac{2b^2cd}{11}\right)+x^9\left(\frac{a^2d^2}{9}+\frac{4abcd}{9}+\frac{b^2c^2}{9}\right)+x^7\cdot\left(\frac{2a^2cd}{7}+\frac{2abc^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**5/5 + b**2*d**2*x**13/13 + x**11*(2*a*b*d**2/11 + 2*b**2*c*d/1
1) + x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**7*(2*a**2*c*d/7 +
2*a*b*c**2/7)
```

Giac [A]
time = 1.02, size = 94, normalized size = 1.08

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{2}{11}\,b^2cdx^{11}+\frac{2}{11}\,abd^2x^{11}+\frac{1}{9}\,b^2c^2x^9+\frac{4}{9}\,abcdx^9+\frac{1}{9}\,a^2d^2x^9+\frac{2}{7}\,abc^2x^7+\frac{2}{7}\,a^2cdx^7+\frac{1}{5}\,a^2c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 2/11*b^2*c*d*x^11 + 2/11*a*b*d^2*x^11 + 1/9*b^2*c^2*x^9
 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/7*a*b*c^2*x^7 + 2/7*a^2*c*d*x^7 +
1/5*a^2*c^2*x^5
```

Mupad [B]
time = 0.03, size = 78, normalized size = 0.90

$$x^9\left(\frac{a^2\,d^2}{9}+\frac{4\,a\,b\,c\,d}{9}+\frac{b^2\,c^2}{9}\right)+\frac{a^2\,c^2\,x^5}{5}+\frac{b^2\,d^2\,x^{13}}{13}+\frac{2\,a\,c\,x^7\,\left(a\,d+b\,c\right)}{7}+\frac{2\,b\,d\,x^{11}\,\left(a\,d+b\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^9*((a^2*d^2)/9 + (b^2*c^2)/9 + (4*a*b*c*d)/9) + (a^2*c^2*x^5)/5 + (b^2*d^
2*x^13)/13 + (2*a*c*x^7*(a*d + b*c))/7 + (2*b*d*x^11*(a*d + b*c))/11
```

3.151 $\int x^3(a+bx^2)^2\,(c+dx^2)^2\,dx$

Optimal. Leaf size=87

$$\frac{1}{4}a^2c^2x^4+\frac{1}{3}ac(bc+ad)x^6+\frac{1}{8}\left(b^2c^2+4abcd+a^2d^2\right)x^8+\frac{1}{5}bd(bc+ad)x^{10}+\frac{1}{12}b^2d^2x^{12}$$

```
[Out] 1/4*a^2*c^2*x^4+1/3*a*c*(a*d+b*c)*x^6+1/8*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^8+1
/5*b*d*(a*d+b*c)*x^10+1/12*b^2*d^2*x^12
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {457, 78}

$$\frac{1}{8}x^8\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{4}a^2c^2x^4+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{3}acx^6(ad+bc)+\frac{1}{12}b^2d^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^4)/4 + (a*c*(b*c + a*d)*x^6)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x^8)/8 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^12)/12
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)^2 \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(a^2c^2x + 2ac(bc+ad)x^2 + \left(b^2c^2 + 4abcd + a^2d^2\right)x^3 + 2bd(bc+\right.\right.$$
$$= \frac{1}{4}a^2c^2x^4 + \frac{1}{3}ac(bc+ad)x^6 + \frac{1}{8}\left(b^2c^2 + 4abcd + a^2d^2\right)x^8 + \frac{1}{5}bd(bc+ad)x^{10}$$

Mathematica [A]
time = 0.02, size = 81, normalized size = 0.93

$$\frac{1}{120}x^4\left(30a^2c^2 + 40ac(bc+ad)x^2 + 15\left(b^2c^2 + 4abcd + a^2d^2\right)x^4 + 24bd(bc+ad)x^6 + 10b^2d^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (x^4*(30*a^2*c^2 + 40*a*c*(b*c + a*d)*x^2 + 15*(b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^4 + 24*b*d*(b*c + a*d)*x^6 + 10*b^2*d^2*x^8))/120
```

Maple [A]
time = 0.09, size = 90, normalized size = 1.03

method	result
norman	$\frac{b^2d^2x^{12}}{12} + \left(\frac{1}{5}ab\,d^2 + \frac{1}{5}b^2cd\right)x^{10} + \left(\frac{1}{8}a^2d^2 + \frac{1}{2}abcd + \frac{1}{8}b^2c^2\right)x^8 + \left(\frac{1}{3}a^2cd + \frac{1}{3}ab\,c^2\right)x^6 + \frac{a^2c^2x^4}{4}$
default	$\frac{b^2d^2x^{12}}{12} + \frac{(2ab\,d^2+2b^2cd)x^{10}}{10} + \frac{(a^2d^2+4abcd+b^2c^2)x^8}{8} + \frac{(2a^2cd+2ab\,c^2)x^6}{6} + \frac{a^2c^2x^4}{4}$
gosper	$\frac{1}{12}b^2d^2x^{12} + \frac{1}{5}x^{10}ab\,d^2 + \frac{1}{5}x^{10}b^2cd + \frac{1}{8}x^8a^2d^2 + \frac{1}{2}x^8abcd + \frac{1}{8}x^8b^2c^2 + \frac{1}{3}x^6a^2cd + \frac{1}{3}x^6ab\,c^2 + \frac{1}{4}a^2c^2x$
risch	$\frac{1}{12}b^2d^2x^{12} + \frac{1}{5}x^{10}ab\,d^2 + \frac{1}{5}x^{10}b^2cd + \frac{1}{8}x^8a^2d^2 + \frac{1}{2}x^8abcd + \frac{1}{8}x^8b^2c^2 + \frac{1}{3}x^6a^2cd + \frac{1}{3}x^6ab\,c^2 + \frac{1}{4}a^2c^2x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^2*d^2*x^12+1/10*(2*a*b*d^2+2*b^2*c*d)*x^10+1/8*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^8+1/6*(2*a^2*c*d+2*a*b*c^2)*x^6+1/4*a^2*c^2*x^4
```

Maxima [A]
time = 0.30, size = 85, normalized size = 0.98

$$\frac{1}{12}\,b^2d^2x^{12} + \frac{1}{5}\,\left(b^2cd + abd^2\right)x^{10} + \frac{1}{8}\,\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^8 + \frac{1}{4}\,a^2c^2x^4 + \frac{1}{3}\,\left(abc^2 + a^2cd\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*d^2*x^12 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/8*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^8 + 1/4*a^2*c^2*x^4 + 1/3*(a*b*c^2 + a^2*c*d)*x^6
```

Fricas [A]
time = 1.05, size = 85, normalized size = 0.98

$$\frac{1}{12}\,b^2d^2x^{12}+\frac{1}{5}\,(b^2cd+abd^2)x^{10}+\frac{1}{8}\,(b^2c^2+4\,abcd+a^2d^2)x^8+\frac{1}{4}\,a^2c^2x^4+\frac{1}{3}\,(abc^2+a^2cd)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*b^2*d^2*x^12 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/8*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^8 + 1/4*a^2*c^2*x^4 + 1/3*(a*b*c^2 + a^2*c*d)*x^6
```

Sympy [A]
time = 0.02, size = 92, normalized size = 1.06

$$\frac{a^2c^2x^4}{4}+\frac{b^2d^2x^{12}}{12}+x^{10}\left(\frac{abd^2}{5}+\frac{b^2cd}{5}\right)+x^8\left(\frac{a^2d^2}{8}+\frac{abcd}{2}+\frac{b^2c^2}{8}\right)+x^6\left(\frac{a^2cd}{3}+\frac{abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**4/4 + b**2*d**2*x**12/12 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x
**8*(a**2*d**2/8 + a*b*c*d/2 + b**2*c**2/8) + x**6*(a**2*c*d/3 + a*b*c**2/3
)
```

Giac [A]
time = 0.87, size = 94, normalized size = 1.08

$$\frac{1}{12}\,b^2d^2x^{12}+\frac{1}{5}\,b^2cdx^{10}+\frac{1}{5}\,abd^2x^{10}+\frac{1}{8}\,b^2c^2x^8+\frac{1}{2}\,abcdx^8+\frac{1}{8}\,a^2d^2x^8+\frac{1}{3}\,abc^2x^6+\frac{1}{3}\,a^2cdx^6+\frac{1}{4}\,a^2c^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*d^2*x^12 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/8*b^2*c^2*x^8 +
 1/2*a*b*c*d*x^8 + 1/8*a^2*d^2*x^8 + 1/3*a*b*c^2*x^6 + 1/3*a^2*c*d*x^6 + 1/
4*a^2*c^2*x^4
```

Mupad [B]
time = 0.02, size = 78, normalized size = 0.90

$$x^8\left(\frac{a^2\,d^2}{8}+\frac{a\,b\,c\,d}{2}+\frac{b^2\,c^2}{8}\right)+\frac{a^2\,c^2\,x^4}{4}+\frac{b^2\,d^2\,x^{12}}{12}+\frac{a\,c\,x^6\,\left(a\,d+b\,c\right)}{3}+\frac{b\,d\,x^{10}\,\left(a\,d+b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^8*((a^2*d^2)/8 + (b^2*c^2)/8 + (a*b*c*d)/2) + (a^2*c^2*x^4)/4 + (b^2*d^2*
x^12)/12 + (a*c*x^6*(a*d + b*c))/3 + (b*d*x^10*(a*d + b*c))/5
```

3.152 $\int x^2(a+bx^2)^2(c+dx^2)^2\,dx$

Optimal. Leaf size=87

$$\frac{1}{3}a^2c^2x^3+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{2}{9}bd(bc+ad)x^9+\frac{1}{11}b^2d^2x^{11}$$

```
[Out] 1/3*a^2*c^2*x^3+2/5*a*c*(a*d+b*c)*x^5+1/7*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^7+2
/9*b*d*(a*d+b*c)*x^9+1/11*b^2*d^2*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx&=\int\left(a^2c^2x^2+2ac(bc+ad)x^4+\left(b^2c^2+4abcd+a^2d^2\right)x^6+2bd(bc+ad)x^8+\right.\\&=\frac{1}{3}a^2c^2x^3+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{2}{9}bd(bc+ad)x^9+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 87, normalized size = 1.00

$$\frac{1}{3}a^2c^2x^3+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{2}{9}bd(bc+ad)x^9+\frac{1}{11}b^2d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)/11
```

Maple [A]
time = 0.11, size = 90, normalized size = 1.03

method	result
norman	$\frac{b^2d^2x^{11}}{11}+\left(\frac{2}{9}ab\,d^2+\frac{2}{9}b^2cd\right)x^9+\left(\frac{1}{7}a^2d^2+\frac{4}{7}abcd+\frac{1}{7}b^2c^2\right)x^7+\left(\frac{2}{5}a^2cd+\frac{2}{5}ab\,c^2\right)x^5+\frac{a^2c^2x^3}{3}$
default	$\frac{b^2d^2x^{11}}{11}+\frac{(2ab\,d^2+2b^2cd)x^9}{9}+\frac{(a^2d^2+4abcd+b^2c^2)x^7}{7}+\frac{(2a^2cd+2ab\,c^2)x^5}{5}+\frac{a^2c^2x^3}{3}$
gosper	$\frac{1}{11}b^2d^2x^{11}+\frac{2}{9}x^9ab\,d^2+\frac{2}{9}x^9b^2cd+\frac{1}{7}x^7a^2d^2+\frac{4}{7}x^7abcd+\frac{1}{7}x^7b^2c^2+\frac{2}{5}x^5a^2cd+\frac{2}{5}x^5ab\,c^2+\frac{1}{3}a^2c^2x$
risch	$\frac{1}{11}b^2d^2x^{11}+\frac{2}{9}x^9ab\,d^2+\frac{2}{9}x^9b^2cd+\frac{1}{7}x^7a^2d^2+\frac{4}{7}x^7abcd+\frac{1}{7}x^7b^2c^2+\frac{2}{5}x^5a^2cd+\frac{2}{5}x^5ab\,c^2+\frac{1}{3}a^2c^2x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*d^2*x^11+1/9*(2*a*b*d^2+2*b^2*c*d)*x^9+1/7*(a^2*d^2+4*a*b*c*d+b^2*
c^2)*x^7+1/5*(2*a^2*c*d+2*a*b*c^2)*x^5+1/3*a^2*c^2*x^3
```

Maxima [A]
time = 0.27, size = 85, normalized size = 0.98

$$\frac{1}{11}\,b^2d^2x^{11}+\frac{2}{9}\,(b^2cd+abd^2)x^9+\frac{1}{7}\,(b^2c^2+4\,abcd+a^2d^2)x^7+\frac{1}{3}\,a^2c^2x^3+\frac{2}{5}\,(abc^2+a^2cd)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^2*x^11 + 2/9*(b^2*c*d + a*b*d^2)*x^9 + 1/7*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^7 + 1/3*a^2*c^2*x^3 + 2/5*(a*b*c^2 + a^2*c*d)*x^5
```

Fricas [A]
time = 1.03, size = 85, normalized size = 0.98

$$\frac{1}{11}\,b^2d^2x^{11}+\frac{2}{9}\,(b^2cd+abd^2)x^9+\frac{1}{7}\,(b^2c^2+4\,abcd+a^2d^2)x^7+\frac{1}{3}\,a^2c^2x^3+\frac{2}{5}\,(abc^2+a^2cd)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/11*b^2*d^2*x^11 + 2/9*(b^2*c*d + a*b*d^2)*x^9 + 1/7*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^7 + 1/3*a^2*c^2*x^3 + 2/5*(a*b*c^2 + a^2*c*d)*x^5
```

Sympy [A]
time = 0.01, size = 100, normalized size = 1.15

$$\frac{a^2c^2x^3}{3}+\frac{b^2d^2x^{11}}{11}+x^9\cdot\left(\frac{2abd^2}{9}+\frac{2b^2cd}{9}\right)+x^7\left(\frac{a^2d^2}{7}+\frac{4abcd}{7}+\frac{b^2c^2}{7}\right)+x^5\cdot\left(\frac{2a^2cd}{5}+\frac{2abc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 + b**2*d**2*x**11/11 + x**9*(2*a*b*d**2/9 + 2*b**2*c*d/9)
+ x**7*(a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**5*(2*a**2*c*d/5 + 2*a
*b*c**2/5)
```

Giac [A]
time = 0.86, size = 94, normalized size = 1.08

$$\frac{1}{11}\,b^{2}d^{2}x^{11}+\frac{2}{9}\,b^{2}cdx^{9}+\frac{2}{9}\,abd^{2}x^{9}+\frac{1}{7}\,b^{2}c^{2}x^{7}+\frac{4}{7}\,abcdx^{7}+\frac{1}{7}\,a^{2}d^{2}x^{7}+\frac{2}{5}\,abc^{2}x^{5}+\frac{2}{5}\,a^{2}cdx^{5}+\frac{1}{3}\,a^{2}c^{2}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^2*x^11 + 2/9*b^2*c*d*x^9 + 2/9*a*b*d^2*x^9 + 1/7*b^2*c^2*x^7 + 4
/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5 + 1/3*
a^2*c^2*x^3
```

Mupad [B]
time = 0.02, size = 78, normalized size = 0.90

$$x^7\,\left(\frac{a^2\,d^2}{7}+\frac{4\,a\,b\,c\,d}{7}+\frac{b^2\,c^2}{7}\right)+\frac{a^2\,c^2\,x^3}{3}+\frac{b^2\,d^2\,x^{11}}{11}+\frac{2\,a\,c\,x^5\,\left(a\,d+b\,c\right)}{5}+\frac{2\,b\,d\,x^9\,\left(a\,d+b\,c\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^7*((a^2*d^2)/7 + (b^2*c^2)/7 + (4*a*b*c*d)/7) + (a^2*c^2*x^3)/3 + (b^2*d^
2*x^11)/11 + (2*a*c*x^5*(a*d + b*c))/5 + (2*b*d*x^9*(a*d + b*c))/9
```

3.153 $\int x(a+bx^2)^2 (c+dx^2)^2 \, dx$

Optimal. Leaf size=71

$$\frac{(bc-ad)^2 (a+bx^2)^3}{6b^3}+\frac{d(bc-ad) (a+bx^2)^4}{4b^3}+\frac{d^2(a+bx^2)^5}{10b^3}$$

```
[Out] 1/6*(-a*d+b*c)^2*(b*x^2+a)^3/b^3+1/4*d*(-a*d+b*c)*(b*x^2+a)^4/b^3+1/10*d^2*
(b*x^2+a)^5/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\frac{d(a+bx^2)^4 (bc-ad)}{4b^3}+\frac{(a+bx^2)^3 (bc-ad)^2}{6b^3}+\frac{d^2(a+bx^2)^5}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x^2)^3)/(6*b^3) + (d*(b*c - a*d)*(a + b*x^2)^4)/(4*b^
3) + (d^2*(a + b*x^2)^5)/(10*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx^2)^2 (c+dx^2)^2 \, dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^2(c+dx)^2 \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(bc-ad)^2(a+bx)^2}{b^2}+\frac{2d(bc-ad)(a+bx)^3}{b^2}+\frac{d^2(a+bx)^4}{b^2}\right)\right.\\ &= \frac{(bc-ad)^2 (a+bx^2)^3}{6b^3}+\frac{d(bc-ad) (a+bx^2)^4}{4b^3}+\frac{d^2(a+bx^2)^5}{10b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 81, normalized size = 1.14

$$\frac{1}{60}x^2\left(30a^2c^2+30ac(bc+ad)x^2+10\left(b^2c^2+4abcd+a^2d^2\right)x^4+15bd(bc+ad)x^6+6b^2d^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (x^2*(30*a^2*c^2 + 30*a*c*(b*c + a*d)*x^2 + 10*(b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^4 + 15*b*d*(b*c + a*d)*x^6 + 6*b^2*d^2*x^8))/60
```

Maple [A]
time = 0.09, size = 90, normalized size = 1.27

method	result
norman	$\frac{b^2d^2x^{10}}{10}+\left(\frac{1}{4}ab\,d^2+\frac{1}{4}b^2cd\right)x^8+\left(\frac{1}{6}a^2d^2+\frac{2}{3}abcd+\frac{1}{6}b^2c^2\right)x^6+\left(\frac{1}{2}a^2cd+\frac{1}{2}ab\,c^2\right)x^4+\frac{a^2c^2x^2}{2}$
default	$\frac{b^2d^2x^{10}}{10}+\frac{(2ab\,d^2+2b^2cd)x^8}{8}+\frac{(a^2d^2+4abcd+b^2c^2)x^6}{6}+\frac{(2a^2cd+2ab\,c^2)x^4}{4}+\frac{a^2c^2x^2}{2}$
gosper	$\frac{1}{10}b^2d^2x^{10}+\frac{1}{4}x^8ab\,d^2+\frac{1}{4}x^8b^2cd+\frac{1}{6}x^6a^2d^2+\frac{2}{3}x^6abcd+\frac{1}{6}x^6b^2c^2+\frac{1}{2}x^4a^2cd+\frac{1}{2}x^4ab\,c^2+\frac{1}{2}a^2c^2x^2$
risch	$\frac{1}{10}b^2d^2x^{10}+\frac{1}{4}x^8ab\,d^2+\frac{1}{4}x^8b^2cd+\frac{1}{6}x^6a^2d^2+\frac{2}{3}x^6abcd+\frac{1}{6}x^6b^2c^2+\frac{1}{2}x^4a^2cd+\frac{1}{2}x^4ab\,c^2+\frac{1}{2}a^2c^2x^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*d^2*x^10+1/8*(2*a*b*d^2+2*b^2*c*d)*x^8+1/6*(a^2*d^2+4*a*b*c*d+b^2*
c^2)*x^6+1/4*(2*a^2*c*d+2*a*b*c^2)*x^4+1/2*a^2*c^2*x^2
```

Maxima [A]
time = 0.27, size = 85, normalized size = 1.20

$$\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\,\left(b^2cd+abd^2\right)x^8+\frac{1}{6}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^6+\frac{1}{2}\,a^2c^2x^2+\frac{1}{2}\,\left(abc^2+a^2cd\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^2*x^10 + 1/4*(b^2*c*d + a*b*d^2)*x^8 + 1/6*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^6 + 1/2*a^2*c^2*x^2 + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Fricas [A]
time = 0.97, size = 85, normalized size = 1.20

$$\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\,\left(b^2cd+abd^2\right)x^8+\frac{1}{6}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^6+\frac{1}{2}\,a^2c^2x^2+\frac{1}{2}\,\left(abc^2+a^2cd\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d^2*x^10 + 1/4*(b^2*c*d + a*b*d^2)*x^8 + 1/6*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^6 + 1/2*a^2*c^2*x^2 + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Sympy [A]
time = 0.02, size = 94, normalized size = 1.32

$$\frac{a^2c^2x^2}{2}+\frac{b^2d^2x^{10}}{10}+x^8\left(\frac{abd^2}{4}+\frac{b^2cd}{4}\right)+x^6\left(\frac{a^2d^2}{6}+\frac{2abcd}{3}+\frac{b^2c^2}{6}\right)+x^4\left(\frac{a^2cd}{2}+\frac{abc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**2/2 + b**2*d**2*x**10/10 + x**8*(a*b*d**2/4 + b**2*c*d/4) + x*
*6*(a**2*d**2/6 + 2*a*b*c*d/3 + b**2*c**2/6) + x**4*(a**2*c*d/2 + a*b*c**2/
2)
```

Giac [A]
time = 1.47, size = 94, normalized size = 1.32

$$\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\,b^2cdx^8+\frac{1}{4}\,abd^2x^8+\frac{1}{6}\,b^2c^2x^6+\frac{2}{3}\,abcdx^6+\frac{1}{6}\,a^2d^2x^6+\frac{1}{2}\,abc^2x^4+\frac{1}{2}\,a^2cdx^4+\frac{1}{2}\,a^2c^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^2*x^10 + 1/4*b^2*c*d*x^8 + 1/4*a*b*d^2*x^8 + 1/6*b^2*c^2*x^6 + 2
/3*a*b*c*d*x^6 + 1/6*a^2*d^2*x^6 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 + 1/2*
a^2*c^2*x^2
```

Mupad [B]
time = 0.02, size = 78, normalized size = 1.10

$$x^6\left(\frac{a^2\,d^2}{6}+\frac{2\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{6}\right)+\frac{a^2\,c^2\,x^2}{2}+\frac{b^2\,d^2\,x^{10}}{10}+\frac{a\,c\,x^4\,\left(a\,d+b\,c\right)}{2}+\frac{b\,d\,x^8\,\left(a\,d+b\,c\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^6*((a^2*d^2)/6 + (b^2*c^2)/6 + (2*a*b*c*d)/3) + (a^2*c^2*x^2)/2 + (b^2*d^
2*x^10)/10 + (a*c*x^4*(a*d + b*c))/2 + (b*d*x^8*(a*d + b*c))/4
```

3.154 $\int (a+bx^2)^2 (c+dx^2)^2 \, dx$

Optimal. Leaf size=82

$$a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9$$

```
[Out] a^2*c^2*x+2/3*a*c*(a*d+b*c)*x^3+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^5+2/7*b*d
*(a*d+b*c)*x^7+1/9*b^2*d^2*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{380\}$

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\int \left(a^2c^2+2ac(bc+ad)x^2+\left(b^2c^2+4abcd+a^2d^2\right)x^4+2bd(bc+ad)x^6+b^2d^2\right.$$
$$=a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.00

$$a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Maple [A]
time = 0.07, size = 87, normalized size = 1.06

method	result	s
norman	$\frac{b^2d^2x^9}{9}+\left(\frac{2}{7}ab\,d^2+\frac{2}{7}b^2cd\right)x^7+\left(\frac{1}{5}a^2d^2+\frac{4}{5}abcd+\frac{1}{5}b^2c^2\right)x^5+\left(\frac{2}{3}a^2cd+\frac{2}{3}ab\,c^2\right)x^3+a^2c^2x$	8
default	$\frac{b^2d^2x^9}{9}+\frac{(2ab\,d^2+2b^2cd)x^7}{7}+\frac{(a^2d^2+4abcd+b^2c^2)x^5}{5}+\frac{(2a^2cd+2ab\,c^2)x^3}{3}+a^2c^2x$	8
gosper	$\frac{1}{9}b^2d^2x^9+\frac{2}{7}x^7ab\,d^2+\frac{2}{7}x^7b^2cd+\frac{1}{5}x^5a^2d^2+\frac{4}{5}x^5abcd+\frac{1}{5}x^5b^2c^2+\frac{2}{3}x^3a^2cd+\frac{2}{3}x^3ab\,c^2+a^2c^2x$	9
risch	$\frac{1}{9}b^2d^2x^9+\frac{2}{7}x^7ab\,d^2+\frac{2}{7}x^7b^2cd+\frac{1}{5}x^5a^2d^2+\frac{4}{5}x^5abcd+\frac{1}{5}x^5b^2c^2+\frac{2}{3}x^3a^2cd+\frac{2}{3}x^3ab\,c^2+a^2c^2x$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*d^2*x^9+1/7*(2*a*b*d^2+2*b^2*c*d)*x^7+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^5+1/3*(2*a^2*c*d+2*a*b*c^2)*x^3+a^2*c^2*x
```

Maxima [A]
time = 0.27, size = 82, normalized size = 1.00

$$\frac{1}{9}\,b^2d^2x^9+\frac{2}{7}\,\left(b^2cd+abd^2\right)x^7+\frac{1}{5}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+a^2c^2x+\frac{2}{3}\,\left(abc^2+a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Fricas [A]
time = 0.65, size = 82, normalized size = 1.00

$$\frac{1}{9}\,b^2d^2x^9+\frac{2}{7}\,\left(b^2cd+abd^2\right)x^7+\frac{1}{5}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+a^2c^2x+\frac{2}{3}\,\left(abc^2+a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Sympy [A]
time = 0.02, size = 97, normalized size = 1.18

$$a^2c^2x+\frac{b^2d^2x^9}{9}+x^7\cdot\left(\frac{2abd^2}{7}+\frac{2b^2cd}{7}\right)+x^5\left(\frac{a^2d^2}{5}+\frac{4abcd}{5}+\frac{b^2c^2}{5}\right)+x^3\cdot\left(\frac{2a^2cd}{3}+\frac{2abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/7) + x**5*
(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**2*c*d/3 + 2*a*b*c**2
/3)
```

Giac [A]
time = 1.08, size = 91, normalized size = 1.11

$$\frac{1}{9}\,b^2d^2x^9+\frac{2}{7}\,b^2cdx^7+\frac{2}{7}\,abd^2x^7+\frac{1}{5}\,b^2c^2x^5+\frac{4}{5}\,abcdx^5+\frac{1}{5}\,a^2d^2x^5+\frac{2}{3}\,abc^2x^3+\frac{2}{3}\,a^2cdx^3+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2*x^5 + 4/5
*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*a^2*c*d*x^3 + a^2*c^
2*x
```

Mupad [B]
time = 0.02, size = 75, normalized size = 0.91

$$x^5\left(\frac{a^2\,d^2}{5}+\frac{4\,a\,b\,c\,d}{5}+\frac{b^2\,c^2}{5}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^9}{9}+\frac{2\,a\,c\,x^3\,(a\,d+b\,c)}{3}+\frac{2\,b\,d\,x^7\,(a\,d+b\,c)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^5*((a^2*d^2)/5 + (b^2*c^2)/5 + (4*a*b*c*d)/5) + a^2*c^2*x + (b^2*d^2*x^9)
/9 + (2*a*c*x^3*(a*d + b*c))/3 + (2*b*d*x^7*(a*d + b*c))/7
```

3.155 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x} dx$

Optimal. Leaf size=80

$$ac(bc+ad)x^2+\frac{1}{4}\left(b^2c^2+4abcd+a^2d^2\right)x^4+\frac{1}{3}bd(bc+ad)x^6+\frac{1}{8}b^2d^2x^8+a^2c^2\log(x)$$

```
[Out] a*c*(a*d+b*c)*x^2+1/4*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^4+1/3*b*d*(a*d+b*c)*x^6
+1/8*b^2*d^2*x^8+a^2*c^2*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d*(b*c +
 a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^2}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(2ac(bc+ad)+\frac{a^2c^2}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+2bd(bc+ad)x\right.\right.\\ &= ac(bc+ad)x^2+\frac{1}{4}\left(b^2c^2+4abcd+a^2d^2\right)x^4+\frac{1}{3}bd(bc+ad)x^6+\frac{1}{8}b^2d^2x^8+a^2\end{aligned}$$

Mathematica [A]
time = 0.02, size = 80, normalized size = 1.00

$$ac(bc+ad)x^2+\frac{1}{4}\left(b^2c^2+4abcd+a^2d^2\right)x^4+\frac{1}{3}bd(bc+ad)x^6+\frac{1}{8}b^2d^2x^8+a^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d*(b*c +
 a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

Maple [A]
time = 0.13, size = 90, normalized size = 1.12

method	result
norman	$\left(\frac{1}{3}ab\,d^2+\frac{1}{3}b^2cd\right)x^6+\left(\frac{1}{4}a^2d^2+abcd+\frac{1}{4}b^2c^2\right)x^4+\left(a^2cd+ab\,c^2\right)x^2+\frac{b^2d^2x^8}{8}+a^2c^2\ln\left(x\right)$
default	$\frac{b^2d^2x^8}{8}+\frac{ab\,d^2x^6}{3}+\frac{b^2cd\,x^6}{3}+\frac{a^2d^2x^4}{4}+abcd\,x^4+\frac{b^2c^2x^4}{4}+a^2cd\,x^2+ab\,c^2x^2+a^2c^2\ln\left(x\right)$
risch	$-\frac{b^2c^4}{24d^2}-\frac{d^2a^4}{24b^2}+\frac{ba\,c^3}{3d}+\frac{d\,a^3c}{3b}+\frac{3a^2c^2}{4}+\frac{ab\,d^2x^6}{3}+\frac{b^2cd\,x^6}{3}+\frac{a^2d^2x^4}{4}+\frac{b^2c^2x^4}{4}+\frac{b^2d^2x^8}{8}+abcd\,x^4+a^2cd\,x^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^2*d^2*x^8+1/3*a*b*d^2*x^6+1/3*b^2*c*d*x^6+1/4*a^2*d^2*x^4+a*b*c*d*x^4
+1/4*b^2*c^2*x^4+a^2*c*d*x^2+a*b*c^2*x^2+a^2*c^2*ln(x)
```

Maxima [A]
time = 0.28, size = 85, normalized size = 1.06

$$\frac{1}{8}\,b^2d^2x^8+\frac{1}{3}\,\left(b^2cd+abd^2\right)x^6+\frac{1}{4}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+\frac{1}{2}\,a^2c^2\log\left(x^2\right)+\left(abc^2+a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^4 + 1/2*a^2*c^2*log(x^2) + (a*b*c^2 + a^2*c*d)*x^2
```

Fricas [A]
time = 0.85, size = 82, normalized size = 1.02

$$\frac{1}{8}\,b^2d^2x^8+\frac{1}{3}\,\left(b^2cd+abd^2\right)x^6+\frac{1}{4}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+a^2c^2\log\left(x\right)+\left(abc^2+a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^4 + a^2*c^2*log(x) + (a*b*c^2 + a^2*c*d)*x^2
```

Sympy [A]
time = 0.07, size = 85, normalized size = 1.06

$$a^2c^2\log(x) + \frac{b^2d^2x^8}{8} + x^6\left(\frac{abd^2}{3} + \frac{b^2cd}{3}\right) + x^4\left(\frac{a^2d^2}{4} + abcd + \frac{b^2c^2}{4}\right) + x^2\left(a^2cd + abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x,x)
```

```
[Out] a**2*c**2*log(x) + b**2*d**2*x**8/8 + x**6*(a*b*d**2/3 + b**2*c*d/3) + x**4
*(a**2*d**2/4 + a*b*c*d + b**2*c**2/4) + x**2*(a**2*c*d + a*b*c**2)
```

Giac [A]
time = 1.28, size = 92, normalized size = 1.15

$$\frac{1}{8}b^2d^2x^8 + \frac{1}{3}b^2cdx^6 + \frac{1}{3}abd^2x^6 + \frac{1}{4}b^2c^2x^4 + abcdx^4 + \frac{1}{4}a^2d^2x^4 + abc^2x^2 + a^2cdx^2 + \frac{1}{2}a^2c^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*b^2*c*d*x^6 + 1/3*a*b*d^2*x^6 + 1/4*b^2*c^2*x^4 + a*b
*c*d*x^4 + 1/4*a^2*d^2*x^4 + a*b*c^2*x^2 + a^2*c*d*x^2 + 1/2*a^2*c^2*log(x^
2)
```

Mupad [B]
time = 0.02, size = 74, normalized size = 0.92

$$x^4\left(\frac{a^2\,d^2}{4} + a\,b\,c\,d + \frac{b^2\,c^2}{4}\right) + \frac{b^2\,d^2\,x^8}{8} + a^2\,c^2\,\ln(x) + a\,c\,x^2\,(a\,d + b\,c) + \frac{b\,d\,x^6\,(a\,d + b\,c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x,x)
```

```
[Out] x^4*((a^2*d^2)/4 + (b^2*c^2)/4 + a*b*c*d) + (b^2*d^2*x^8)/8 + a^2*c^2*log(x
) + a*c*x^2*(a*d + b*c) + (b*d*x^6*(a*d + b*c))/3
```

3.156 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^2} dx$

Optimal. Leaf size=81

$$-\frac{a^2c^2}{x}+2ac(bc+ad)x+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3+\frac{2}{5}bd(bc+ad)x^5+\frac{1}{7}b^2d^2x^7$$

```
[Out] -a^2*c^2/x+2*a*c*(a*d+b*c)*x+1/3*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^3+2/5*b*d*(a
*d+b*c)*x^5+1/7*b^2*d^2*x^7
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3
)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^2}\,dx &= \int \left(2ac(bc+ad)+\frac{a^2c^2}{x^2}+\left(b^2c^2+4abcd+a^2d^2\right)x^2+2bd(bc+ad)x^4+b^2d^2x\right.\\ &= -\frac{a^2c^2}{x}+2ac(bc+ad)x+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3+\frac{2}{5}bd(bc+ad)x^5+\frac{1}{7}b^2d\end{aligned}$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 1.00

$$-\frac{a^2c^2}{x}+2ac(bc+ad)x+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3+\frac{2}{5}bd(bc+ad)x^5+\frac{1}{7}b^2d^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3
)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

Maple [A]
time = 0.09, size = 91, normalized size = 1.12

method	result	size
norman	$\frac{\frac{b^2 d^2 x^8}{7}+\left(\frac{2}{5} ab\, d^2+\frac{2}{5} b^2 cd\right) x^6+\left(\frac{1}{3} a^2 d^2+\frac{4}{3} abcd+\frac{1}{3} b^2 c^2\right) x^4+\left(2a^2 cd+2ab\, c^2\right) x^2-a^2 c^2}{x}$	90
default	$\frac{b^2 d^2 x^7}{7}+\frac{2ab\, d^2 x^5}{5}+\frac{2b^2 cd\, x^5}{5}+\frac{a^2 d^2 x^3}{3}+\frac{4abcd\, x^3}{3}+\frac{b^2 c^2 x^3}{3}+2a^2 cdx+2ab\, c^2 x-\frac{a^2 c^2}{x}$	91
risch	$\frac{b^2 d^2 x^7}{7}+\frac{2ab\, d^2 x^5}{5}+\frac{2b^2 cd\, x^5}{5}+\frac{a^2 d^2 x^3}{3}+\frac{4abcd\, x^3}{3}+\frac{b^2 c^2 x^3}{3}+2a^2 cdx+2ab\, c^2 x-\frac{a^2 c^2}{x}$	91
gosper	$-\frac{-15b^2 d^2 x^8-42ab\, d^2 x^6-42b^2 cd\, x^6-35a^2 d^2 x^4-140abcd\, x^4-35b^2 c^2 x^4-210a^2 cd\, x^2-210ab\, c^2 x^2+105a^2 c^2}{105x}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^2*d^2*x^7+2/5*a*b*d^2*x^5+2/5*b^2*c*d*x^5+1/3*a^2*d^2*x^3+4/3*a*b*c*d
*x^3+1/3*b^2*c^2*x^3+2*a^2*c*d*x+2*a*b*c^2*x-a^2*c^2/x
```

Maxima [A]
time = 0.30, size = 83, normalized size = 1.02

$$\frac{1}{7}\, b^2 d^2 x^7+\frac{2}{5}\left(b^2 cd+abd^2\right) x^5+\frac{1}{3}\left(b^2 c^2+4\, abcd+a^2 d^2\right) x^3-\frac{a^2 c^2}{x}+2\left(abc^2+a^2 cd\right) x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^2*x^7 + 2/5*(b^2*c*d + a*b*d^2)*x^5 + 1/3*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^3 - a^2*c^2/x + 2*(a*b*c^2 + a^2*c*d)*x
```

Fricas [A]
time = 0.92, size = 87, normalized size = 1.07

$$\frac{15\, b^2 d^2 x^8+42\,(b^2 cd+abd^2) x^6+35\,(b^2 c^2+4\, abcd+a^2 d^2) x^4-105\, a^2 c^2+210\,(abc^2+a^2 cd) x^2}{105\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^2*x^8 + 42*(b^2*c*d + a*b*d^2)*x^6 + 35*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^4 - 105*a^2*c^2 + 210*(a*b*c^2 + a^2*c*d)*x^2)/x
```

Sympy [A]
time = 0.06, size = 92, normalized size = 1.14

$$-\frac{a^2c^2}{x}+\frac{b^2d^2x^7}{7}+x^5\cdot\left(\frac{2abd^2}{5}+\frac{2b^2cd}{5}\right)+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{b^2c^2}{3}\right)+x\left(2a^2cd+2abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**2,x)
```

```
[Out] -a**2*c**2/x + b**2*d**2*x**7/7 + x**5*(2*a*b*d**2/5 + 2*b**2*c*d/5) + x**3
*(a**2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x*(2*a**2*c*d + 2*a*b*c**2)
```

Giac [A]
time = 0.99, size = 90, normalized size = 1.11

$$\frac{1}{7}\,b^2d^2x^7+\frac{2}{5}\,b^2cdx^5+\frac{2}{5}\,abd^2x^5+\frac{1}{3}\,b^2c^2x^3+\frac{4}{3}\,abcdx^3+\frac{1}{3}\,a^2d^2x^3+2\,abc^2x+2\,a^2cdx-\frac{a^2c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^2*x^7 + 2/5*b^2*c*d*x^5 + 2/5*a*b*d^2*x^5 + 1/3*b^2*c^2*x^3 + 4/3
*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + 2*a*b*c^2*x + 2*a^2*c*d*x - a^2*c^2/x
```

Mupad [B]
time = 0.02, size = 76, normalized size = 0.94

$$x^3\left(\frac{a^2\,d^2}{3}+\frac{4\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{3}\right)-\frac{a^2\,c^2}{x}+\frac{b^2\,d^2\,x^7}{7}+2\,a\,c\,x\,\left(a\,d+b\,c\right)+\frac{2\,b\,d\,x^5\,\left(a\,d+b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x)
```

```
[Out] x^3*((a^2*d^2)/3 + (b^2*c^2)/3 + (4*a*b*c*d)/3) - (a^2*c^2)/x + (b^2*d^2*x^
7)/7 + 2*a*c*x*(a*d + b*c) + (2*b*d*x^5*(a*d + b*c))/5
```

3.157 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^3}\,dx$

Optimal. Leaf size=84

$$-\frac{a^2c^2}{2x^2}+\frac{1}{2}\left(b^2c^2+4abcd+a^2d^2\right)x^2+\frac{1}{2}bd(bc+ad)x^4+\frac{1}{6}b^2d^2x^6+2ac(bc+ad)\log(x)$$

```
[Out] -1/2*a^2*c^2/x^2+1/2*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^2+1/2*b*d*(a*d+b*c)*x^4+
1/6*b^2*d^2*x^6+2*a*c*(a*d+b*c)*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 90}

$$\frac{1}{2}x^2\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{2x^2}+\frac{1}{2}bdx^4(ad+bc)+2ac\log(x)(ad+bc)+\frac{1}{6}b^2d^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*c^2)/x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)/2 + (b*d*(b*c +
a*d)*x^4)/2 + (b^2*d^2*x^6)/6 + 2*a*c*(b*c + a*d)*Log[x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^2}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(4bc+ad)}{b^2c^2}\right)+\frac{a^2c^2}{x^2}+\frac{2ac(bc+ad)}{x}+2bd(bc+ad\right.\right.\\ &= -\frac{a^2c^2}{2x^2}+\frac{1}{2}\left(b^2c^2+4abcd+a^2d^2\right)x^2+\frac{1}{2}bd(bc+ad)x^4+\frac{1}{6}b^2d^2x^6+2ac(bc+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 0.99

$$\frac{1}{6}\left(3abdx^2(4c+dx^2)+\frac{3a^2(-c^2+d^2x^4)}{x^2}+b^2x^2(3c^2+3cdx^2+d^2x^4)+12ac(bc+ad)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

```
[Out] (3*a*b*d*x^2*(4*c + d*x^2) + (3*a^2*(-c^2 + d^2*x^4))/x^2 + b^2*x^2*(3*c^2
+ 3*c*d*x^2 + d^2*x^4) + 12*a*c*(b*c + a*d)*Log[x])/6
```

Maple [A]
time = 0.07, size = 88, normalized size = 1.05

method	result	size
default	$\frac{b^2d^2x^6}{6}+\frac{ab\,d^2x^4}{2}+\frac{b^2cd\,x^4}{2}+\frac{a^2d^2x^2}{2}+2abcd\,x^2+\frac{b^2c^2x^2}{2}-\frac{a^2c^2}{2x^2}+2ac(ad+bc)\ln(x)$	88
norman	$\frac{(\frac{1}{2}ab\,d^2+\frac{1}{2}b^2cd)x^6+(\frac{1}{2}a^2d^2+2abcd+\frac{1}{2}b^2c^2)x^4-\frac{a^2c^2}{2}+\frac{b^2d^2x^8}{6}}{x^2}+(2a^2cd+2ab\,c^2)\ln(x)$	90
risch	$\frac{b^2d^2x^6}{6}+\frac{ab\,d^2x^4}{2}+\frac{b^2cd\,x^4}{2}+\frac{a^2d^2x^2}{2}+2abcd\,x^2+\frac{b^2c^2x^2}{2}-\frac{a^2c^2}{2x^2}+2\ln(x)\,a^2cd+2\ln(x)\,ab\,c^2$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2*d^2*x^6+1/2*a*b*d^2*x^4+1/2*b^2*c*d*x^4+1/2*a^2*d^2*x^2+2*a*b*c*d*x
^2+1/2*b^2*c^2*x^2-1/2*a^2*c^2/x^2+2*a*c*(a*d+b*c)*ln(x)
```

Maxima [A]
time = 0.35, size = 85, normalized size = 1.01

$$\frac{1}{6}\,b^2d^2x^6+\frac{1}{2}\,(b^2cd+abd^2)x^4+\frac{1}{2}\,(b^2c^2+4\,abcd+a^2d^2)x^2-\frac{a^2c^2}{2\,x^2}+(abc^2+a^2cd)\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^2*x^6 + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/2*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^2 - 1/2*a^2*c^2/x^2 + (a*b*c^2 + a^2*c*d)*log(x^2)
```

Fricas [A]
time = 0.75, size = 88, normalized size = 1.05

$$\frac{b^2d^2x^8+3\,(b^2cd+abd^2)x^6+3\,(b^2c^2+4\,abcd+a^2d^2)x^4-3\,a^2c^2+12\,(abc^2+a^2cd)x^2\log(x)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^2*d^2*x^8 + 3*(b^2*c*d + a*b*d^2)*x^6 + 3*(b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*x^4 - 3*a^2*c^2 + 12*(a*b*c^2 + a^2*c*d)*x^2*log(x))/x^2
```

Sympy [A]
time = 0.16, size = 87, normalized size = 1.04

$$-\frac{a^2c^2}{2x^2} + 2ac(ad+bc)\log{(x)} + \frac{b^2d^2x^6}{6} + x^4\left(\frac{abd^2}{2} + \frac{b^2cd}{2}\right) + x^2\left(\frac{a^2d^2}{2} + 2abcd + \frac{b^2c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**3,x)
```

```
[Out] -a**2*c**2/(2*x**2) + 2*a*c*(a*d + b*c)*log(x) + b**2*d**2*x**6/6 + x**4*(a
*b*d**2/2 + b**2*c*d/2) + x**2*(a**2*d**2/2 + 2*a*b*c*d + b**2*c**2/2)
```

Giac [A]
time = 1.40, size = 114, normalized size = 1.36

$$\frac{1}{6}\,b^2d^2x^6 + \frac{1}{2}\,b^2cdx^4 + \frac{1}{2}\,abd^2x^4 + \frac{1}{2}\,b^2c^2x^2 + 2\,abcdx^2 + \frac{1}{2}\,a^2d^2x^2 + \left(abc^2 + a^2cd\right)\log\left(x^2\right) - \frac{2\,abc^2x^2 + 2\,a^2cdx^2 + a^2c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d^2*x^6 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/2*b^2*c^2*x^2 + 2*a
*b*c*d*x^2 + 1/2*a^2*d^2*x^2 + (a*b*c^2 + a^2*c*d)*log(x^2) - 1/2*(2*a*b*c^
2*x^2 + 2*a^2*c*d*x^2 + a^2*c^2)/x^2
```

Mupad [B]
time = 0.02, size = 82, normalized size = 0.98

$$x^2\left(\frac{a^2\,d^2}{2} + 2\,a\,b\,c\,d + \frac{b^2\,c^2}{2}\right) + \ln\left(x\right)\,\left(2\,d\,a^2\,c + 2\,b\,a\,c^2\right) - \frac{a^2\,c^2}{2\,x^2} + \frac{b^2\,d^2\,x^6}{6} + \frac{b\,d\,x^4\,\left(a\,d + b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x)
```

```
[Out] x^2*((a^2*d^2)/2 + (b^2*c^2)/2 + 2*a*b*c*d) + log(x)*(2*a*b*c^2 + 2*a^2*c*d
) - (a^2*c^2)/(2*x^2) + (b^2*d^2*x^6)/6 + (b*d*x^4*(a*d + b*c))/2
```

3.158 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^4}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2c^2}{3x^3}-\frac{2ac(bc+ad)}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+\frac{2}{3}bd(bc+ad)x^3+\frac{1}{5}b^2d^2x^5$$

```
[Out] -1/3*a^2*c^2/x^3-2*a*c*(a*d+b*c)/x+(a^2*d^2+4*a*b*c*d+b^2*c^2)*x+2/3*b*d*(a
*d+b*c)*x^3+1/5*b^2*d^2*x^5
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

```
[Out] -1/3*(a^2*c^2)/x^3 - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^4}\,dx &= \int \left(b^2c^2\left(1+\frac{ad(4bc+ad)}{b^2c^2}\right)+\frac{a^2c^2}{x^4}+\frac{2ac(bc+ad)}{x^2}+2bd(bc+ad)x^2+b^2d^2\right.\\ &= -\frac{a^2c^2}{3x^3}-\frac{2ac(bc+ad)}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+\frac{2}{3}bd(bc+ad)x^3+\frac{1}{5}b^2d^2x^5\end{aligned}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 1.00

$$-\frac{a^2c^2}{3x^3}-\frac{2ac(bc+ad)}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+\frac{2}{3}bd(bc+ad)x^3+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

```
[Out] -1/3*(a^2*c^2)/x^3 - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

Maple [A]
time = 0.07, size = 81, normalized size = 1.01

method	result	size
default	$\frac{b^2 x^5 d^2}{5}+\frac{2ab\,d^2 x^3}{3}+\frac{2b^2 cd\,x^3}{3}+a^2 d^2 x+4abcdx+b^2 c^2 x-\frac{a^2 c^2}{3x^3}-\frac{2ac(ad+bc)}{x}$	81
norman	$\frac{\frac{b^2 d^2 x^8}{5}+\left(\frac{2}{3}ab\,d^2+\frac{2}{3}b^2 cd\right)x^6+\left(a^2 d^2+4abcd+b^2 c^2\right)x^4+\left(-2a^2 cd-2ab\,c^2\right)x^2-\frac{a^2 c^2}{3}}{x^3}$	88
risch	$\frac{b^2 x^5 d^2}{5}+\frac{2ab\,d^2 x^3}{3}+\frac{2b^2 cd\,x^3}{3}+a^2 d^2 x+4abcdx+b^2 c^2 x+\frac{\left(-2a^2 cd-2ab\,c^2\right)x^2-\frac{a^2 c^2}{3}}{x^3}$	88
gosper	$-\frac{-3b^2 d^2 x^8-10ab\,d^2 x^6-10b^2 cd\,x^6-15a^2 d^2 x^4-60abcd\,x^4-15b^2 c^2 x^4+30a^2 cd\,x^2+30ab\,c^2 x^2+5a^2 c^2}{15x^3}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*x^5*d^2+2/3*a*b*d^2*x^3+2/3*b^2*c*d*x^3+a^2*d^2*x+4*a*b*c*d*x+b^2*c
^2*x-1/3*a^2*c^2/x^3-2*a*c*(a*d+b*c)/x
```

Maxima [A]
time = 0.30, size = 84, normalized size = 1.05

$$\frac{1}{5}\,b^2 d^2 x^5+\frac{2}{3}\,\left(b^2 cd+abd^2\right)x^3+\left(b^2 c^2+4\,abcd+a^2 d^2\right)x-\frac{a^2 c^2+6\,\left(abc^2+a^2 cd\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + 2/3*(b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*x - 1/3*(a^2*c^2 + 6*(a*b*c^2 + a^2*c*d)*x^2)/x^3
```

Fricas [A]
time = 0.76, size = 87, normalized size = 1.09

$$\frac{3\,b^2 d^2 x^8+10\,\left(b^2 cd+abd^2\right)x^6+15\,\left(b^2 c^2+4\,abcd+a^2 d^2\right)x^4-5\,a^2 c^2-30\,\left(abc^2+a^2 cd\right)x^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^2*x^8 + 10*(b^2*c*d + a*b*d^2)*x^6 + 15*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^4 - 5*a^2*c^2 - 30*(a*b*c^2 + a^2*c*d)*x^2)/x^3
```

Sympy [A]
time = 0.15, size = 92, normalized size = 1.15

$$\frac{b^2d^2x^5}{5}+x^3\cdot\left(\frac{2abd^2}{3}+\frac{2b^2cd}{3}\right)+x\left(a^2d^2+4abcd+b^2c^2\right)+\frac{-a^2c^2+x^2(-6a^2cd-6abc^2)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**4,x)
```

```
[Out] b**2*d**2*x**5/5 + x**3*(2*a*b*d**2/3 + 2*b**2*c*d/3) + x*(a**2*d**2 + 4*a*
b*c*d + b**2*c**2) + (-a**2*c**2 + x**2*(-6*a**2*c*d - 6*a*b*c**2))/(3*x**3
)
```

Giac [A]
time = 1.17, size = 88, normalized size = 1.10

$$\frac{1}{5}\,b^2d^2x^5+\frac{2}{3}\,b^2cdx^3+\frac{2}{3}\,abd^2x^3+b^2c^2x+4\,abcdx+a^2d^2x-\frac{6\,abc^2x^2+6\,a^2cdx^2+a^2c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d^2*x^5 + 2/3*b^2*c*d*x^3 + 2/3*a*b*d^2*x^3 + b^2*c^2*x + 4*a*b*c*d
*x + a^2*d^2*x - 1/3*(6*a*b*c^2*x^2 + 6*a^2*c*d*x^2 + a^2*c^2)/x^3
```

Mupad [B]
time = 0.03, size = 82, normalized size = 1.02

$$x\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)-\frac{x^2\,\left(2\,d\,a^2\,c+2\,b\,a\,c^2\right)+\frac{a^2\,c^2}{3}}{x^3}+\frac{b^2\,d^2\,x^5}{5}+\frac{2\,b\,d\,x^3\,\left(a\,d+b\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x)
```

```
[Out] x*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) - (x^2*(2*a*b*c^2 + 2*a^2*c*d) + (a^2*c^2
)/3)/x^3 + (b^2*d^2*x^5)/5 + (2*b*d*x^3*(a*d + b*c))/3
```

3.159 $\int x^4(a+bx^2)^2(c+dx^2)^3\,dx$

Optimal. Leaf size=127

$$\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2(2bc+3ad)x^7+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9+\frac{1}{11}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{11}+\frac{1}{13}bd^2(3bc+2ac$$

```
[Out] 1/5*a^2*c^3*x^5+1/7*a*c^2*(3*a*d+2*b*c)*x^7+1/9*c*(3*a^2*d^2+6*a*b*c*d+b^2*
c^2)*x^9+1/11*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^11+1/13*b*d^2*(2*a*d+3*b*c)
*x^13+1/15*b^2*d^3*x^15
```

Rubi [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{15}b^2d^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d^2
*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx&=\int\left(a^2c^3x^4+ac^2(2bc+3ad)x^6+c\left(b^2c^2+6abcd+3a^2d^2\right)x^8+d\left(3b^2c^2+6\right.\right.\\&=\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2(2bc+3ad)x^7+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9+\frac{1}{11}d\left(3b^2c^2\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 127, normalized size = 1.00

$$\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2(2bc+3ad)x^7+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9+\frac{1}{11}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{11}+\frac{1}{13}bd^2(3bc+2ad)x^{13}+\frac{1}{15}b^2d^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d^2
*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

Maple [A]
time = 0.10, size = 128, normalized size = 1.01

method	result
norman	$\frac{a^2c^3x^5}{5}+\left(\frac{3}{7}a^2c^2d+\frac{2}{7}ab\,c^3\right)x^7+\left(\frac{1}{3}a^2c\,d^2+\frac{2}{3}ab\,c^2d+\frac{1}{9}b^2c^3\right)x^9+\left(\frac{1}{11}a^2d^3+\frac{6}{11}abc\,d^2+\frac{3}{11}b^2c^2d\right)x^1$
default	$\frac{b^2d^3x^{15}}{15}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{13}}{13}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^{11}}{11}+\frac{(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^9}{9}+\frac{(3a^2c^2d+2ab\,c^3)x^7}{7}+\frac{a^2c^3x^5}{5}$
gosper	$\frac{1}{5}a^2c^3x^5+\frac{3}{7}x^7a^2c^2d+\frac{2}{7}x^7ab\,c^3+\frac{1}{3}x^9a^2c\,d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{11}x^{11}a^2d^3+\frac{6}{11}x^{11}abc\,d^2+\frac{3}{11}$
risch	$\frac{1}{5}a^2c^3x^5+\frac{3}{7}x^7a^2c^2d+\frac{2}{7}x^7ab\,c^3+\frac{1}{3}x^9a^2c\,d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{11}x^{11}a^2d^3+\frac{6}{11}x^{11}abc\,d^2+\frac{3}{11}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^2*d^3*x^15+1/13*(2*a*b*d^3+3*b^2*c*d^2)*x^13+1/11*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^11+1/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^9+1/7*(3*a^2*c
^2*d+2*a*b*c^3)*x^7+1/5*a^2*c^3*x^5
```

Maxima [A]
time = 0.32, size = 127, normalized size = 1.00

$$\frac{1}{15}b^2d^3x^{15}+\frac{1}{13}\left(3b^2cd^2+2abd^3\right)x^{13}+\frac{1}{11}\left(3b^2c^2d+6abcd^2+a^2d^3\right)x^{11}+\frac{1}{5}a^2c^3x^5+\frac{1}{9}\left(b^2c^3+6abc^2d+3a^2cd^2\right)x^9+\frac{1}{7}\left(2abc^3+3a^2c^2d\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*d^3*x^15 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/11*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^11 + 1/5*a^2*c^3*x^5 + 1/9*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^9 + 1/7*(2*a*b*c^3 + 3*a^2*c^2*d)*x^7
```

Fricas [A]
time = 0.77, size = 127, normalized size = 1.00

$$\frac{1}{15}b^2d^3x^{15}+\frac{1}{13}\left(3b^2cd^2+2abd^3\right)x^{13}+\frac{1}{11}\left(3b^2c^2d+6abcd^2+a^2d^3\right)x^{11}+\frac{1}{5}a^2c^3x^5+\frac{1}{9}\left(b^2c^3+6abc^2d+3a^2cd^2\right)x^9+\frac{1}{7}\left(2abc^3+3a^2c^2d\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/15*b^2*d^3*x^15 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/11*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^11 + 1/5*a^2*c^3*x^5 + 1/9*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^9 + 1/7*(2*a*b*c^3 + 3*a^2*c^2*d)*x^7
```

Sympy [A]
time = 0.02, size = 143, normalized size = 1.13

$$\frac{a^2c^3x^5}{5}+\frac{b^2d^3x^{15}}{15}+x^{13}\cdot\left(\frac{2abd^3}{13}+\frac{3b^2cd^2}{13}\right)+x^{11}\left(\frac{a^2d^3}{11}+\frac{6abcd^2}{11}+\frac{3b^2c^2d}{11}\right)+x^9\left(\frac{a^2cd^2}{3}+\frac{2abc^2d}{3}+\frac{b^2c^3}{9}\right)+x^7\cdot\left(\frac{3a^2c^2d}{7}+\frac{2abc^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**5/5 + b**2*d**3*x**15/15 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**
2/13) + x**11*(a**2*d**3/11 + 6*a*b*c*d**2/11 + 3*b**2*c**2*d/11) + x**9*(a
**2*c*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**7*(3*a**2*c**2*d/7 + 2*a*
b*c**3/7)
```

Giac [A]
time = 1.01, size = 135, normalized size = 1.06

$$\frac{1}{15}b^2d^3x^{15}+\frac{3}{13}b^2cd^2x^{13}+\frac{2}{13}abd^3x^{13}+\frac{3}{11}b^2c^2dx^{11}+\frac{6}{11}abcd^2x^{11}+\frac{1}{11}a^2d^3x^{11}+\frac{1}{9}b^2c^3x^9+\frac{2}{3}abc^2dx^9+\frac{1}{3}a^2cd^2x^9+\frac{2}{7}abc^3x^7+\frac{3}{7}a^2c^2dx^7+\frac{1}{5}a^2c^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/15*b^2*d^3*x^15 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/11*b^2*c^2*
d*x^11 + 6/11*a*b*c*d^2*x^11 + 1/11*a^2*d^3*x^11 + 1/9*b^2*c^3*x^9 + 2/3*a*
b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/7*a*b*c^3*x^7 + 3/7*a^2*c^2*d*x^7 + 1/5
*a^2*c^3*x^5
```

Mupad [B]
time = 0.05, size = 119, normalized size = 0.94

$$x^9\left(\frac{a^2\,c\,d^2}{3}+\frac{2\,a\,b\,c^2\,d}{3}+\frac{b^2\,c^3}{9}\right)+x^{11}\left(\frac{a^2\,d^3}{11}+\frac{6\,a\,b\,c\,d^2}{11}+\frac{3\,b^2\,c^2\,d}{11}\right)+\frac{a^2\,c^3\,x^5}{5}+\frac{b^2\,d^3\,x^{15}}{15}+\frac{a\,c^2\,x^7\,\left(3\,a\,d+2\,b\,c\right)}{7}+\frac{b\,d^2\,x^{13}\,\left(2\,a\,d+3\,b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^9*((b^2*c^3)/9 + (a^2*c*d^2)/3 + (2*a*b*c^2*d)/3) + x^11*((a^2*d^3)/11 +
(3*b^2*c^2*d)/11 + (6*a*b*c*d^2)/11) + (a^2*c^3*x^5)/5 + (b^2*d^3*x^15)/15
+ (a*c^2*x^7*(3*a*d + 2*b*c))/7 + (b*d^2*x^13*(2*a*d + 3*b*c))/13
```

3.160 $\int x^3(a+bx^2)^2(c+dx^2)^3\,dx$

Optimal. Leaf size=106

$$-\frac{c(bc-ad)^2(c+dx^2)^4}{8d^4}+\frac{(bc-ad)(3bc-ad)(c+dx^2)^5}{10d^4}-\frac{b(3bc-2ad)(c+dx^2)^6}{12d^4}+\frac{b^2(c+dx^2)^7}{14d^4}$$

```
[Out] -1/8*c*(-a*d+b*c)^2*(d*x^2+c)^4/d^4+1/10*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2+c)^
5/d^4-1/12*b*(-2*a*d+3*b*c)*(d*x^2+c)^6/d^4+1/14*b^2*(d*x^2+c)^7/d^4
```

Rubi [A]
time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{b(c+dx^2)^6(3bc-2ad)}{12d^4}+\frac{(c+dx^2)^5(bc-ad)(3bc-ad)}{10d^4}-\frac{c(c+dx^2)^4(bc-ad)^2}{8d^4}+\frac{b^2(c+dx^2)^7}{14d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] -1/8*(c*(b*c - a*d)^2*(c + d*x^2)^4)/d^4 + ((b*c - a*d)*(3*b*c - a*d)*(c +
d*x^2)^5)/(10*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^6)/(12*d^4) + (b^2*(c +
 d*x^2)^7)/(14*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)^3 \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2(c+dx)^3}{d^3} + \frac{(bc-ad)(3bc-ad)(c+dx)^4}{d^3} - \frac{b(\right.\right.$$
$$= -\frac{c(bc-ad)^2 \left(c+dx^2\right)^4}{8d^4} + \frac{(bc-ad)(3bc-ad) \left(c+dx^2\right)^5}{10d^4} - \frac{b(3bc-2ad)}{12d}$$

Mathematica [A]
time = 0.02, size = 119, normalized size = 1.12

$$\frac{1}{840}x^4\left(210a^2c^3+140ac^2(2bc+3ad)x^2+105c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+84d\left(3b^2c^2+6abcd+a^2d^2\right)x^6+70bd^2(3bc+2ad)x^8+60b^2d^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (x^4*(210*a^2*c^3 + 140*a*c^2*(2*b*c + 3*a*d)*x^2 + 105*c*(b^2*c^2 + 6*a*b*
c*d + 3*a^2*d^2)*x^4 + 84*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 70*b*d^
2*(3*b*c + 2*a*d)*x^8 + 60*b^2*d^3*x^10))/840
```

Maple [A]
time = 0.09, size = 128, normalized size = 1.21

method	result
norman	$\frac{a^2c^3x^4}{4}+\left(\frac{1}{2}a^2c^2d+\frac{1}{3}ab\,c^3\right)x^6+\left(\frac{3}{8}a^2c\,d^2+\frac{3}{4}ab\,c^2d+\frac{1}{8}b^2c^3\right)x^8+\left(\frac{1}{10}a^2d^3+\frac{3}{5}abc\,d^2+\frac{3}{10}b^2c^2d\right)x$
default	$\frac{b^2d^3x^{14}}{14}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{12}}{12}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^{10}}{10}+\frac{(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^8}{8}+\frac{(3a^2c^2d+2ab\,c^3)x^6}{6}+\frac{a^2c^3}{4}$
gosper	$\frac{1}{4}a^2c^3x^4+\frac{1}{2}x^6a^2c^2d+\frac{1}{3}x^6ab\,c^3+\frac{3}{8}x^8a^2c\,d^2+\frac{3}{4}x^8ab\,c^2d+\frac{1}{8}x^8b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+$
risch	$\frac{1}{4}a^2c^3x^4+\frac{1}{2}x^6a^2c^2d+\frac{1}{3}x^6ab\,c^3+\frac{3}{8}x^8a^2c\,d^2+\frac{3}{4}x^8ab\,c^2d+\frac{1}{8}x^8b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*b^2*d^3*x^14+1/12*(2*a*b*d^3+3*b^2*c*d^2)*x^12+1/10*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^10+1/8*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^8+1/6*(3*a^2*c
^2*d+2*a*b*c^3)*x^6+1/4*a^2*c^3*x^4
```

Maxima [A]
time = 0.34, size = 127, normalized size = 1.20

$$\frac{1}{14}\,b^2d^3x^{14}+\frac{1}{12}\left(3\,b^2cd^2+2\,abd^3\right)x^{12}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{4}\,a^2c^3x^4+\frac{1}{8}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^8+\frac{1}{6}\left(2\,abc^3+3\,a^2c^2d\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^2*d^3*x^14 + 1/12*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/4*a^2*c^3*x^4 + 1/8*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^8 + 1/6*(2*a*b*c^3 + 3*a^2*c^2*d)*x^6
```

Fricas [A]
time = 0.78, size = 127, normalized size = 1.20

$$\frac{1}{14}b^2d^3x^{14}+\frac{1}{12}\left(3\,b^2cd^2+2\,abd^3\right)x^{12}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{4}a^2c^3x^4+\frac{1}{8}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^8+\frac{1}{6}\left(2\,abc^3+3\,a^2c^2d\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/14*b^2*d^3*x^14 + 1/12*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/4*a^2*c^3*x^4 + 1/8*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^8 + 1/6*(2*a*b*c^3 + 3*a^2*c^2*d)*x^6
```

Sympy [A]
time = 0.02, size = 138, normalized size = 1.30

$$\frac{a^2c^3x^4}{4}+\frac{b^2d^3x^{14}}{14}+x^{12}\left(\frac{abd^3}{6}+\frac{b^2cd^2}{4}\right)+x^{10}\left(\frac{a^2d^3}{10}+\frac{3abcd^2}{5}+\frac{3b^2c^2d}{10}\right)+x^8\cdot\left(\frac{3a^2cd^2}{8}+\frac{3abc^2d}{4}+\frac{b^2c^3}{8}\right)+x^6\left(\frac{a^2c^2d}{2}+\frac{abc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**4/4 + b**2*d**3*x**14/14 + x**12*(a*b*d**3/6 + b**2*c*d**2/4)
+ x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**8*(3*a**2*c
*d**2/8 + 3*a*b*c**2*d/4 + b**2*c**3/8) + x**6*(a**2*c**2*d/2 + a*b*c**3/3)
```

Giac [A]
time = 1.09, size = 135, normalized size = 1.27

$$\frac{1}{14}b^2d^3x^{14}+\frac{1}{4}b^2cd^2x^{12}+\frac{1}{6}abd^3x^{12}+\frac{3}{10}b^2c^2dx^{10}+\frac{3}{5}abcd^2x^{10}+\frac{1}{10}a^2d^3x^{10}+\frac{1}{8}b^2c^3x^8+\frac{3}{4}abc^2dx^8+\frac{3}{8}a^2cd^2x^8+\frac{1}{3}abc^3x^6+\frac{1}{2}a^2c^2dx^6+\frac{1}{4}a^2c^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^2*d^3*x^14 + 1/4*b^2*c*d^2*x^12 + 1/6*a*b*d^3*x^12 + 3/10*b^2*c^2*d*
x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/8*b^2*c^3*x^8 + 3/4*a*b*c
^2*d*x^8 + 3/8*a^2*c*d^2*x^8 + 1/3*a*b*c^3*x^6 + 1/2*a^2*c^2*d*x^6 + 1/4*a^
2*c^3*x^4
```

Mupad [B]
time = 0.02, size = 119, normalized size = 1.12

$$x^8\left(\frac{3\,a^2\,c\,d^2}{8}+\frac{3\,a\,b\,c^2\,d}{4}+\frac{b^2\,c^3}{8}\right)+x^{10}\left(\frac{a^2\,d^3}{10}+\frac{3\,a\,b\,c\,d^2}{5}+\frac{3\,b^2\,c^2\,d}{10}\right)+\frac{a^2\,c^3\,x^4}{4}+\frac{b^2\,d^3\,x^{14}}{14}+\frac{a\,c^2\,x^6\,\left(3\,a\,d+2\,b\,c\right)}{6}+\frac{b\,d^2\,x^{12}\,\left(2\,a\,d+3\,b\,c\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^8*((b^2*c^3)/8 + (3*a^2*c*d^2)/8 + (3*a*b*c^2*d)/4) + x^10*((a^2*d^3)/10
+ (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + (a^2*c^3*x^4)/4 + (b^2*d^3*x^14)/14
 + (a*c^2*x^6*(3*a*d + 2*b*c))/6 + (b*d^2*x^12*(2*a*d + 3*b*c))/12
```

3.161 $\int x^2(a+bx^2)^2(c+dx^2)^3\,dx$

Optimal. Leaf size=127

$$\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{9}d\left(3b^2c^2+6abcd+a^2d^2\right)x^9+\frac{1}{11}bd^2(3bc+2ad)x^1$$

```
[Out] 1/3*a^2*c^3*x^3+1/5*a*c^2*(3*a*d+2*b*c)*x^5+1/7*c*(3*a^2*d^2+6*a*b*c*d+b^2*
c^2)*x^7+1/9*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^9+1/11*b*d^2*(2*a*d+3*b*c)*x
^11+1/13*b^2*d^3*x^13
```

Rubi [A]
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{13}b^2d^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^9)/9 + (b*d^2*(
3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^2+ac^2(2bc+3ad)x^4+c\left(b^2c^2+6abcd+3a^2d^2\right)x^6+d\left(3b^2c^2+6a\right.\right.\\ &= \frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{9}d\left(3b^2c^2+\right.\end{aligned}$$

Mathematica [A]
time = 0.01, size = 127, normalized size = 1.00

$$\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{9}d\left(3b^2c^2+6abcd+a^2d^2\right)x^9+\frac{1}{11}bd^2(3bc+2ad)x^{11}+\frac{1}{13}b^2d^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^9)/9 + (b*d^2*(
3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

Maple [A]
time = 0.12, size = 128, normalized size = 1.01

method	result
norman	$\frac{b^2 d^3 x^{13}}{13}+\left(\frac{2}{11} ab\,d^3+\frac{3}{11} b^2 c\,d^2\right) x^{11}+\left(\frac{1}{9} a^2 d^3+\frac{2}{3} abc\,d^2+\frac{1}{3} b^2 c^2 d\right) x^9+\left(\frac{3}{7} a^2 c\,d^2+\frac{6}{7} ab\,c^2 d+\frac{1}{7} b^2 c^3\right)$
default	$\frac{b^2 d^3 x^{13}}{13}+\frac{(2ab\,d^3+3b^2 c\,d^2) x^{11}}{11}+\frac{(a^2 d^3+6abc\,d^2+3b^2 c^2 d) x^9}{9}+\frac{(3a^2 c\,d^2+6ab\,c^2 d+b^2 c^3) x^7}{7}+\frac{(3a^2 c^2 d+2ab\,c^3) x^5}{5}+\frac{a^2 c^3 x}{3}$
gosper	$\frac{1}{13} b^2 d^3 x^{13}+\frac{2}{11} x^{11} ab\,d^3+\frac{3}{11} x^{11} b^2 c\,d^2+\frac{1}{9} x^9 a^2 d^3+\frac{2}{3} x^9 abc\,d^2+\frac{1}{3} x^9 b^2 c^2 d+\frac{3}{7} x^7 a^2 c\,d^2+\frac{6}{7} x^7 ab\,c^2 d$
risch	$\frac{1}{13} b^2 d^3 x^{13}+\frac{2}{11} x^{11} ab\,d^3+\frac{3}{11} x^{11} b^2 c\,d^2+\frac{1}{9} x^9 a^2 d^3+\frac{2}{3} x^9 abc\,d^2+\frac{1}{3} x^9 b^2 c^2 d+\frac{3}{7} x^7 a^2 c\,d^2+\frac{6}{7} x^7 ab\,c^2 d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^2*d^3*x^13+1/11*(2*a*b*d^3+3*b^2*c*d^2)*x^11+1/9*(a^2*d^3+6*a*b*c*d^
2+3*b^2*c^2*d)*x^9+1/7*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^7+1/5*(3*a^2*c^2
*d+2*a*b*c^3)*x^5+1/3*a^2*c^3*x^3
```

Maxima [A]
time = 0.33, size = 127, normalized size = 1.00

$$\frac{1}{13}\,b^2 d^3 x^{13}+\frac{1}{11}\,(3\,b^2 cd^2+2\,abd^3)x^{11}+\frac{1}{9}\,(3\,b^2 c^2 d+6\,abcd^2+a^2 d^3)x^9+\frac{1}{3}\,a^2 c^3 x^3+\frac{1}{7}\,(b^2 c^3+6\,abc^2 d+3\,a^2 cd^2)x^7+\frac{1}{5}\,(2\,abc^3+3\,a^2 c^2 d)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^3*x^13 + 1/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 1/9*(3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*x^9 + 1/3*a^2*c^3*x^3 + 1/7*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^7 + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Fricas [A]
time = 0.65, size = 127, normalized size = 1.00

$$\frac{1}{13}\,b^2 d^3 x^{13}+\frac{1}{11}\,(3\,b^2 cd^2+2\,abd^3)x^{11}+\frac{1}{9}\,(3\,b^2 c^2 d+6\,abcd^2+a^2 d^3)x^9+\frac{1}{3}\,a^2 c^3 x^3+\frac{1}{7}\,(b^2 c^3+6\,abc^2 d+3\,a^2 cd^2)x^7+\frac{1}{5}\,(2\,abc^3+3\,a^2 c^2 d)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b^2*d^3*x^13 + 1/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 1/9*(3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*x^9 + 1/3*a^2*c^3*x^3 + 1/7*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^7 + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Sympy [A]
time = 0.03, size = 143, normalized size = 1.13

$$\frac{a^2c^3x^3}{3}+\frac{b^2d^3x^{13}}{13}+x^{11}\cdot\left(\frac{2abd^3}{11}+\frac{3b^2cd^2}{11}\right)+x^9\left(\frac{a^2d^3}{9}+\frac{2abcd^2}{3}+\frac{b^2c^2d}{3}\right)+x^7\cdot\left(\frac{3a^2cd^2}{7}+\frac{6abc^2d}{7}+\frac{b^2c^3}{7}\right)+x^5\cdot\left(\frac{3a^2c^2d}{5}+\frac{2abc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**3/3 + b**2*d**3*x**13/13 + x**11*(2*a*b*d**3/11 + 3*b**2*c*d**
2/11) + x**9*(a**2*d**3/9 + 2*a*b*c*d**2/3 + b**2*c**2*d/3) + x**7*(3*a**2*
c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**5*(3*a**2*c**2*d/5 + 2*a*b*c*
*3/5)
```

Giac [A]
time = 0.75, size = 135, normalized size = 1.06

$$\frac{1}{13}b^2d^3x^{13}+\frac{3}{11}b^2cd^2x^{11}+\frac{2}{11}abd^3x^{11}+\frac{1}{3}b^2c^2dx^9+\frac{2}{3}abcd^2x^9+\frac{1}{9}a^2d^3x^9+\frac{1}{7}b^2c^3x^7+\frac{6}{7}abc^2dx^7+\frac{3}{7}a^2cd^2x^7+\frac{2}{5}abc^3x^5+\frac{3}{5}a^2c^2dx^5+\frac{1}{3}a^2c^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^3*x^13 + 3/11*b^2*c*d^2*x^11 + 2/11*a*b*d^3*x^11 + 1/3*b^2*c^2*d
*x^9 + 2/3*a*b*c*d^2*x^9 + 1/9*a^2*d^3*x^9 + 1/7*b^2*c^3*x^7 + 6/7*a*b*c^2*
d*x^7 + 3/7*a^2*c*d^2*x^7 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 + 1/3*a^2*c
^3*x^3
```

Mupad [B]
time = 0.02, size = 119, normalized size = 0.94

$$x^7\left(\frac{3\,a^2\,c\,d^2}{7}+\frac{6\,a\,b\,c^2\,d}{7}+\frac{b^2\,c^3}{7}\right)+x^9\left(\frac{a^2\,d^3}{9}+\frac{2\,a\,b\,c\,d^2}{3}+\frac{b^2\,c^2\,d}{3}\right)+\frac{a^2\,c^3\,x^3}{3}+\frac{b^2\,d^3\,x^{13}}{13}+\frac{a\,c^2\,x^5\,\left(3\,a\,d+2\,b\,c\right)}{5}+\frac{b\,d^2\,x^{11}\,\left(2\,a\,d+3\,b\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^9*((a^2*d^3)/9 +
(b^2*c^2*d)/3 + (2*a*b*c*d^2)/3) + (a^2*c^3*x^3)/3 + (b^2*d^3*x^13)/13 + (a
*c^2*x^5*(3*a*d + 2*b*c))/5 + (b*d^2*x^11*(2*a*d + 3*b*c))/11
```

3.162 $\int x(a+bx^2)^2(c+dx^2)^3\,dx$

Optimal. Leaf size=71

$$\frac{(bc-ad)^2(c+dx^2)^4}{8d^3}-\frac{b(bc-ad)(c+dx^2)^5}{5d^3}+\frac{b^2(c+dx^2)^6}{12d^3}$$

```
[Out] 1/8*(-a*d+b*c)^2*(d*x^2+c)^4/d^3-1/5*b*(-a*d+b*c)*(d*x^2+c)^5/d^3+1/12*b^2*
(d*x^2+c)^6/d^3
```

Rubi [A]
time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$-\frac{b(c+dx^2)^5(bc-ad)}{5d^3}+\frac{(c+dx^2)^4(bc-ad)^2}{8d^3}+\frac{b^2(c+dx^2)^6}{12d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^4)/(8*d^3) - (b*(b*c - a*d)*(c + d*x^2)^5)/(5*d^
3) + (b^2*(c + d*x^2)^6)/(12*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx^2)^2(c+dx^2)^3\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^2(c+dx)^3\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2(c+dx)^3}{d^2}-\frac{2b(bc-ad)(c+dx)^4}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right.\right.\\ &= \frac{(bc-ad)^2(c+dx^2)^4}{8d^3}-\frac{b(bc-ad)(c+dx^2)^5}{5d^3}+\frac{b^2(c+dx^2)^6}{12d^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 119, normalized size = 1.68

$$\frac{1}{120}x^2\left(60a^2c^3+30ac^2(2bc+3ad)x^2+20c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+15d\left(3b^2c^2+6abcd+a^2d^2\right)x^6+12bd^2(3bc+2ad)x^8+10b^2d^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (x^2*(60*a^2*c^3 + 30*a*c^2*(2*b*c + 3*a*d)*x^2 + 20*c*(b^2*c^2 + 6*a*b*c*d
 + 3*a^2*d^2)*x^4 + 15*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 12*b*d^2*(
3*b*c + 2*a*d)*x^8 + 10*b^2*d^3*x^10))/120
```

Maple [A]
time = 0.09, size = 128, normalized size = 1.80

method	result
norman	$\frac{b^2d^3x^{12}}{12}+\left(\frac{1}{5}ab\,d^3+\frac{3}{10}b^2c\,d^2\right)x^{10}+\left(\frac{1}{8}a^2d^3+\frac{3}{4}abc\,d^2+\frac{3}{8}b^2c^2d\right)x^8+\left(\frac{1}{2}a^2c\,d^2+ab\,c^2d+\frac{1}{6}b^2c^3\right)x^6$
default	$\frac{b^2d^3x^{12}}{12}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{10}}{10}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^8}{8}+\frac{(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^6}{6}+\frac{(3a^2c^2d+2ab\,c^3)x^4}{4}+\frac{a^2c^3x^2}{2}$
gosper	$\frac{1}{12}b^2d^3x^{12}+\frac{1}{5}x^{10}ab\,d^3+\frac{3}{10}x^{10}b^2c\,d^2+\frac{1}{8}x^8a^2d^3+\frac{3}{4}x^8abc\,d^2+\frac{3}{8}x^8b^2c^2d+\frac{1}{2}x^6a^2c\,d^2+x^6ab\,c^2d+$
risch	$\frac{1}{12}b^2d^3x^{12}+\frac{1}{5}x^{10}ab\,d^3+\frac{3}{10}x^{10}b^2c\,d^2+\frac{1}{8}x^8a^2d^3+\frac{3}{4}x^8abc\,d^2+\frac{3}{8}x^8b^2c^2d+\frac{1}{2}x^6a^2c\,d^2+x^6ab\,c^2d+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^2*d^3*x^12+1/10*(2*a*b*d^3+3*b^2*c*d^2)*x^10+1/8*(a^2*d^3+6*a*b*c*d^
2+3*b^2*c^2*d)*x^8+1/6*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^6+1/4*(3*a^2*c^2
*d+2*a*b*c^3)*x^4+1/2*a^2*c^3*x^2
```

Maxima [A]
time = 0.40, size = 127, normalized size = 1.79

$$\frac{1}{12}\,b^2d^3x^{12}+\frac{1}{10}\,\left(3\,b^2cd^2+2\,abd^3\right)x^{10}+\frac{1}{8}\,\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^8+\frac{1}{2}\,a^2c^3x^2+\frac{1}{6}\,\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^6+\frac{1}{4}\,\left(2\,abc^3+3\,a^2c^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*d^3*x^12 + 1/10*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 1/8*(3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*x^8 + 1/2*a^2*c^3*x^2 + 1/6*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^6 + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Fricas [A]
time = 0.87, size = 127, normalized size = 1.79

$$\frac{1}{12}\,b^2d^3x^{12}+\frac{1}{10}\,\left(3\,b^2cd^2+2\,abd^3\right)x^{10}+\frac{1}{8}\,\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^8+\frac{1}{2}\,a^2c^3x^2+\frac{1}{6}\,\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^6+\frac{1}{4}\,\left(2\,abc^3+3\,a^2c^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*b^2*d^3*x^12 + 1/10*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 1/8*(3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*x^8 + 1/2*a^2*c^3*x^2 + 1/6*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^6 + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(60) = 120$.
time = 0.02, size = 136, normalized size = 1.92

$$\frac{a^2c^3x^2}{2} + \frac{b^2d^3x^{12}}{12} + x^{10}\left(\frac{abd^3}{5} + \frac{3b^2cd^2}{10}\right) + x^8\left(\frac{a^2d^3}{8} + \frac{3abcd^2}{4} + \frac{3b^2c^2d}{8}\right) + x^6\left(\frac{a^2cd^2}{2} + abc^2d + \frac{b^2c^3}{6}\right) + x^4\cdot\left(\frac{3a^2c^2d}{4} + \frac{abc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**2/2 + b**2*d**3*x**12/12 + x**10*(a*b*d**3/5 + 3*b**2*c*d**2/1
0) + x**8*(a**2*d**3/8 + 3*a*b*c*d**2/4 + 3*b**2*c**2*d/8) + x**6*(a**2*c*d
**2/2 + a*b*c**2*d + b**2*c**3/6) + x**4*(3*a**2*c**2*d/4 + a*b*c**3/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(65) = 130$.
time = 1.32, size = 134, normalized size = 1.89

$$\frac{1}{12}b^2d^3x^{12} + \frac{3}{10}b^2cd^2x^{10} + \frac{1}{5}abd^3x^{10} + \frac{3}{8}b^2c^2dx^8 + \frac{3}{4}abcd^2x^8 + \frac{1}{8}a^2d^3x^8 + \frac{1}{6}b^2c^3x^6 + abc^2dx^6 + \frac{1}{2}a^2cd^2x^6 + \frac{1}{2}abc^3x^4 + \frac{3}{4}a^2c^2dx^4 + \frac{1}{2}a^2c^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^2*d^3*x^12 + 3/10*b^2*c*d^2*x^10 + 1/5*a*b*d^3*x^10 + 3/8*b^2*c^2*d*
x^8 + 3/4*a*b*c*d^2*x^8 + 1/8*a^2*d^3*x^8 + 1/6*b^2*c^3*x^6 + a*b*c^2*d*x^6
 + 1/2*a^2*c*d^2*x^6 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + 1/2*a^2*c^3*x^
2
```

Mupad [B]
time = 0.02, size = 118, normalized size = 1.66

$$x^6\left(\frac{a^2\,c\,d^2}{2} + a\,b\,c^2\,d + \frac{b^2\,c^3}{6}\right) + x^8\left(\frac{a^2\,d^3}{8} + \frac{3\,a\,b\,c\,d^2}{4} + \frac{3\,b^2\,c^2\,d}{8}\right) + \frac{a^2\,c^3\,x^2}{2} + \frac{b^2\,d^3\,x^{12}}{12} + \frac{a\,c^2\,x^4\,(3\,a\,d + 2\,b\,c)}{4} + \frac{b\,d^2\,x^{10}\,(2\,a\,d + 3\,b\,c)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^6*((b^2*c^3)/6 + (a^2*c*d^2)/2 + a*b*c^2*d) + x^8*((a^2*d^3)/8 + (3*b^2*c
^2*d)/8 + (3*a*b*c*d^2)/4) + (a^2*c^3*x^2)/2 + (b^2*d^3*x^12)/12 + (a*c^2*x
^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^10*(2*a*d + 3*b*c))/10
```

3.163 $\int (a+bx^2)^2 (c+dx^2)^3 \, dx$

Optimal. Leaf size=122

$$a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7+\frac{1}{9}bd^2(3bc+2ad)x^9+\frac{1}{1}$$

```
[Out] a^2*c^3*x+1/3*a*c^2*(3*a*d+2*b*c)*x^3+1/5*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x
^5+1/7*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^7+1/9*b*d^2*(2*a*d+3*b*c)*x^9+1/11
*b^2*d^3*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx &= \int \left(a^2c^3+ac^2(2bc+3ad)x^2+c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+d\left(3b^2c^2+6abcd\right.\right.\\ &= a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6abc\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 122, normalized size = 1.00

$$a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7+\frac{1}{9}bd^2(3bc+2ad)x^9+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Maple [A]
time = 0.07, size = 125, normalized size = 1.02

method	result
norman	$\frac{b^2d^3x^{11}}{11}+\left(\frac{2}{9}ab\,d^3+\frac{1}{3}b^2c\,d^2\right)x^9+\left(\frac{1}{7}a^2d^3+\frac{6}{7}abc\,d^2+\frac{3}{7}b^2c^2d\right)x^7+\left(\frac{3}{5}a^2c\,d^2+\frac{6}{5}ab\,c^2d+\frac{1}{5}b^2c^3\right)x^5$
default	$\frac{b^2d^3x^{11}}{11}+\frac{(2ab\,d^3+3b^2c\,d^2)x^9}{9}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^7}{7}+\frac{(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^5}{5}+\frac{(3a^2c^2d+2ab\,c^3)x^3}{3}+a^2c^3x$
gosper	$\frac{1}{11}b^2d^3x^{11}+\frac{2}{9}x^9ab\,d^3+\frac{1}{3}x^9b^2c\,d^2+\frac{1}{7}x^7a^2d^3+\frac{6}{7}x^7abc\,d^2+\frac{3}{7}x^7b^2c^2d+\frac{3}{5}x^5a^2c\,d^2+\frac{6}{5}x^5ab\,c^2d+$
risch	$\frac{1}{11}b^2d^3x^{11}+\frac{2}{9}x^9ab\,d^3+\frac{1}{3}x^9b^2c\,d^2+\frac{1}{7}x^7a^2d^3+\frac{6}{7}x^7abc\,d^2+\frac{3}{7}x^7b^2c^2d+\frac{3}{5}x^5a^2c\,d^2+\frac{6}{5}x^5ab\,c^2d+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*d^3*x^11+1/9*(2*a*b*d^3+3*b^2*c*d^2)*x^9+1/7*(a^2*d^3+6*a*b*c*d^2+
3*b^2*c^2*d)*x^7+1/5*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^5+1/3*(3*a^2*c^2*d
+2*a*b*c^3)*x^3+a^2*c^3*x
```

Maxima [A]
time = 0.33, size = 124, normalized size = 1.02

$$\frac{1}{11}\,b^2d^3x^{11}+\frac{1}{9}\,(3\,b^2cd^2+2\,abd^3)x^9+\frac{1}{7}\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^7+a^2c^3x+\frac{1}{5}\,(b^2c^3+6\,abc^2d+3\,a^2cd^2)x^5+\frac{1}{3}\,(2\,abc^3+3\,a^2c^2d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

Fricas [A]
time = 0.72, size = 124, normalized size = 1.02

$$\frac{1}{11}\,b^2d^3x^{11}+\frac{1}{9}\,(3\,b^2cd^2+2\,abd^3)x^9+\frac{1}{7}\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^7+a^2c^3x+\frac{1}{5}\,(b^2c^3+6\,abc^2d+3\,a^2cd^2)x^5+\frac{1}{3}\,(2\,abc^3+3\,a^2c^2d)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

Sympy [A]
time = 0.02, size = 136, normalized size = 1.11

$$a^2c^3x + \frac{b^2d^3x^{11}}{11} + x^9 \cdot \left(\frac{2abd^3}{9} + \frac{b^2cd^2}{3}\right) + x^7\left(\frac{a^2d^3}{7} + \frac{6abcd^2}{7} + \frac{3b^2c^2d}{7}\right) + x^5 \cdot \left(\frac{3a^2cd^2}{5} + \frac{6abc^2d}{5} + \frac{b^2c^3}{5}\right) + x^3\left(a^2c^2d + \frac{2abc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d**2/3) + x*
*7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x**5*(3*a**2*c*d**2/5
 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2*c**2*d + 2*a*b*c**3/3)
```

Giac [A]
time = 0.90, size = 131, normalized size = 1.07

$$\frac{1}{11}b^2d^3x^{11} + \frac{1}{3}b^2cd^2x^9 + \frac{2}{9}abd^3x^9 + \frac{3}{7}b^2c^2dx^7 + \frac{6}{7}abcd^2x^7 + \frac{1}{7}a^2d^3x^7 + \frac{1}{5}b^2c^3x^5 + \frac{6}{5}abc^2dx^5 + \frac{3}{5}a^2cd^2x^5 + \frac{2}{3}abc^3x^3 + a^2c^2dx^3 + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2*c^2*d*x^7
 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^5 + 6/5*a*b*c^2*d*x^
5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2*c^2*d*x^3 + a^2*c^3*x
```

Mupad [B]
time = 0.02, size = 116, normalized size = 0.95

$$x^5\left(\frac{3\,a^2\,c\,d^2}{5} + \frac{6\,a\,b\,c^2\,d}{5} + \frac{b^2\,c^3}{5}\right) + x^7\left(\frac{a^2\,d^3}{7} + \frac{6\,a\,b\,c\,d^2}{7} + \frac{3\,b^2\,c^2\,d}{7}\right) + a^2\,c^3\,x + \frac{b^2\,d^3\,x^{11}}{11} + \frac{a\,c^2\,x^3\,(3\,a\,d + 2\,b\,c)}{3} + \frac{b\,d^2\,x^9\,(2\,a\,d + 3\,b\,c)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^5*((b^2*c^3)/5 + (3*a^2*c*d^2)/5 + (6*a*b*c^2*d)/5) + x^7*((a^2*d^3)/7 +
(3*b^2*c^2*d)/7 + (6*a*b*c*d^2)/7) + a^2*c^3*x + (b^2*d^3*x^11)/11 + (a*c^2
*x^3*(3*a*d + 2*b*c))/3 + (b*d^2*x^9*(2*a*d + 3*b*c))/9
```

3.164 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x}\,dx$

Optimal. Leaf size=123

$$\frac{1}{2}ac^2(2bc+3ad)x^2+\frac{1}{4}c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+\frac{1}{6}d\left(3b^2c^2+6abcd+a^2d^2\right)x^6+\frac{1}{8}bd^2(3bc+2ad)x^8+\frac{1}{10}b^2d^3$$

```
[Out] 1/2*a*c^2*(3*a*d+2*b*c)*x^2+1/4*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^4+1/6*d*(
a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^6+1/8*b*d^2*(2*a*d+3*b*c)*x^8+1/10*b^2*d^3*x
^10+a^2*c^3*ln(x)
```

Rubi [A]

time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)+\frac{1}{10}b^2d^3x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4
 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2*(3*b*c + 2*a*d)*x^8
)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^3}{x} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(ac^2(2bc+3ad) + \frac{a^2c^3}{x} + c\left(b^2c^2+6abcd+3a^2d^2\right)x + d\left(3b^2c^2+\right.\right.\right.$$
$$= \frac{1}{2}ac^2(2bc+3ad)x^2 + \frac{1}{4}c\left(b^2c^2+6abcd+3a^2d^2\right)x^4 + \frac{1}{6}d\left(3b^2c^2+6abcd+a^2d^2\right)$$

Mathematica [A]
time = 0.02, size = 123, normalized size = 1.00

$$\frac{1}{2}ac^2(2bc+3ad)x^2 + \frac{1}{4}c\left(b^2c^2+6abcd+3a^2d^2\right)x^4 + \frac{1}{6}d\left(3b^2c^2+6abcd+a^2d^2\right)x^6 + \frac{1}{8}bd^2(3bc+2ad)x^8 + \frac{1}{10}b^2d^3x^{10} + a^2c^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4
 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2*(3*b*c + 2*a*d)*x^8
)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

Maple [A]
time = 0.08, size = 132, normalized size = 1.07

method	result
norman	$\left(\frac{1}{4}ab\,d^3+\frac{3}{8}b^2c\,d^2\right)x^8+\left(\frac{3}{2}a^2c^2d+ab\,c^3\right)x^2+\left(\frac{1}{6}a^2d^3+abc\,d^2+\frac{1}{2}b^2c^2d\right)x^6+\left(\frac{3}{4}a^2c\,d^2+\frac{3}{2}ab\,c^2d+\right.$
default	$\frac{b^2d^3x^{10}}{10}+\frac{ab\,d^3x^8}{4}+\frac{3b^2c\,d^2x^8}{8}+\frac{a^2d^3x^6}{6}+abc\,d^2x^6+\frac{b^2c^2d\,x^6}{2}+\frac{3a^2c\,d^2x^4}{4}+\frac{3ab\,c^2d\,x^4}{2}+\frac{b^2c^3x^4}{4}+\frac{3a^2c^2d\,x^2}{2}+$
risch	$\frac{b^2d^3x^{10}}{10}+\frac{ab\,d^3x^8}{4}+\frac{3b^2c\,d^2x^8}{8}+\frac{a^2d^3x^6}{6}+abc\,d^2x^6+\frac{b^2c^2d\,x^6}{2}+\frac{3a^2c\,d^2x^4}{4}+\frac{3ab\,c^2d\,x^4}{2}+\frac{b^2c^3x^4}{4}+\frac{3a^2c^2d\,x^2}{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*d^3*x^10+1/4*a*b*d^3*x^8+3/8*b^2*c*d^2*x^8+1/6*a^2*d^3*x^6+a*b*c*d
^2*x^6+1/2*b^2*c^2*d*x^6+3/4*a^2*c*d^2*x^4+3/2*a*b*c^2*d*x^4+1/4*b^2*c^3*x^
4+3/2*a^2*c^2*d*x^2+a*b*c^3*x^2+a^2*c^3*ln(x)
```

Maxima [A]
time = 0.27, size = 128, normalized size = 1.04

$$\frac{1}{10}b^2d^3x^{10} + \frac{1}{8}\left(3\,b^2cd^2+2\,abd^3\right)x^8 + \frac{1}{6}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6 + \frac{1}{2}a^2c^3\log\left(x^2\right) + \frac{1}{4}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4 + \frac{1}{2}\left(2\,abc^3+3\,a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^6 + 1/2*a^2*c^3*log(x^2) + 1/4*(b^2*c^3 + 6*a*b*c^
2*d + 3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Fricas [A]
time = 0.77, size = 125, normalized size = 1.02

$$\frac{1}{10}\,b^2d^3x^{10}+\frac{1}{8}\,\left(3\,b^2cd^2+2\,abd^3\right)x^8+\frac{1}{6}\,\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6+a^2c^3\log\left(x\right)+\frac{1}{4}\,\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+\frac{1}{2}\,\left(2\,abc^3+3\,a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3*log(x) + 1/4*(b^2*c^3 + 6*a*b*c^2*d +
3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Sympy [A]
time = 0.13, size = 133, normalized size = 1.08

$$a^2c^3\log\left(x\right)+\frac{b^2d^3x^{10}}{10}+x^8\left(\frac{abd^3}{4}+\frac{3b^2cd^2}{8}\right)+x^6\left(\frac{a^2d^3}{6}+abcd^2+\frac{b^2c^2d}{2}\right)+x^4\cdot\left(\frac{3a^2cd^2}{4}+\frac{3abc^2d}{2}+\frac{b^2c^3}{4}\right)+x^2\cdot\left(\frac{3a^2c^2d}{2}+abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x,x)
```

```
[Out] a**2*c**3*log(x) + b**2*d**3*x**10/10 + x**8*(a*b*d**3/4 + 3*b**2*c*d**2/8)
 + x**6*(a**2*d**3/6 + a*b*c*d**2 + b**2*c**2*d/2) + x**4*(3*a**2*c*d**2/4
+ 3*a*b*c**2*d/2 + b**2*c**3/4) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

Giac [A]
time = 3.87, size = 134, normalized size = 1.09

$$\frac{1}{10}\,b^2d^3x^{10}+\frac{3}{8}\,b^2cd^2x^8+\frac{1}{4}\,abd^3x^8+\frac{1}{2}\,b^2c^2dx^6+abcd^2x^6+\frac{1}{6}\,a^2d^3x^6+\frac{1}{4}\,b^2c^3x^4+\frac{3}{2}\,abc^2dx^4+\frac{3}{4}\,a^2cd^2x^4+abc^3x^2+\frac{3}{2}\,a^2c^2dx^2+\frac{1}{2}\,a^2c^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^3*x^10 + 3/8*b^2*c*d^2*x^8 + 1/4*a*b*d^3*x^8 + 1/2*b^2*c^2*d*x^6
 + a*b*c*d^2*x^6 + 1/6*a^2*d^3*x^6 + 1/4*b^2*c^3*x^4 + 3/2*a*b*c^2*d*x^4 +
3/4*a^2*c*d^2*x^4 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + 1/2*a^2*c^3*log(x^2)
```

Mupad [B]
time = 0.02, size = 116, normalized size = 0.94

$$x^4\left(\frac{3\,a^2\,c\,d^2}{4}+\frac{3\,a\,b\,c^2\,d}{2}+\frac{b^2\,c^3}{4}\right)+x^6\left(\frac{a^2\,d^3}{6}+a\,b\,c\,d^2+\frac{b^2\,c^2\,d}{2}\right)+\frac{b^2\,d^3\,x^{10}}{10}+a^2\,c^3\,\ln\left(x\right)+\frac{a\,c^2\,x^2\,\left(3\,a\,d+2\,b\,c\right)}{2}+\frac{b\,d^2\,x^8\,\left(2\,a\,d+3\,b\,c\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x,x)
```

```
[Out] x^4*((b^2*c^3)/4 + (3*a^2*c*d^2)/4 + (3*a*b*c^2*d)/2) + x^6*((a^2*d^3)/6 +
(b^2*c^2*d)/2 + a*b*c*d^2) + (b^2*d^3*x^10)/10 + a^2*c^3*log(x) + (a*c^2*x^
2*(3*a*d + 2*b*c))/2 + (b*d^2*x^8*(2*a*d + 3*b*c))/8
```

3.165 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2} dx$

Optimal. Leaf size=120

$$-\frac{a^2c^3}{x}+ac^2(2bc+3ad)x+\frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3+\frac{1}{5}d\left(3b^2c^2+6abcd+a^2d^2\right)x^5+\frac{1}{7}bd^2(3bc+2ad)x^7+\frac{1}{9}$$

```
[Out] -a^2*c^3/x+a*c^2*(3*a*d+2*b*c)*x+1/3*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^3+1/
5*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^5+1/7*b*d^2*(2*a*d+3*b*c)*x^7+1/9*b^2*d
^3*x^9
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*
d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)/5 + (b*d^2*(3*b*c +
 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2} dx &= \int \left(ac^2(2bc+3ad)+\frac{a^2c^3}{x^2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^2+d\left(3b^2c^2+6abcd+\right.\right.\\ &= -\frac{a^2c^3}{x}+ac^2(2bc+3ad)x+\frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3+\frac{1}{5}d\left(3b^2c^2+6abcd\right.\end{aligned}$$

Mathematica [A]
time = 0.03, size = 120, normalized size = 1.00

$$-\frac{a^2c^3}{x}+ac^2(2bc+3ad)x+\frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3+\frac{1}{5}d\left(3b^2c^2+6abcd+a^2d^2\right)x^5+\frac{1}{7}bd^2(3bc+2ad)x^7+\frac{1}{9}b^2d^3x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*
d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)/5 + (b*d^2*(3*b*c +
 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

Maple [A]
time = 0.07, size = 131, normalized size = 1.09

method	result
norman	$\frac{\frac{b^2 d^3 x^{10}}{9}+\left(\frac{2}{7}ab\,d^3+\frac{3}{7}b^2 c\,d^2\right)x^8+\left(\frac{1}{5}a^2 d^3+\frac{6}{5}abc\,d^2+\frac{3}{5}b^2 c^2 d\right)x^6+\left(a^2 c\,d^2+2ab\,c^2 d+\frac{1}{3}b^2 c^3\right)x^4+\left(3a^2 c^2 d+2ab\,c^3\right)x^2-a^2 c^3}{x}$
default	$\frac{b^2 d^3 x^9}{9}+\frac{2ab\,d^3 x^7}{7}+\frac{3b^2 c\,d^2 x^7}{7}+\frac{a^2 d^3 x^5}{5}+\frac{6abc\,d^2 x^5}{5}+\frac{3b^2 c^2 d\,x^5}{5}+a^2 c\,d^2 x^3+2ab\,c^2 d\,x^3+\frac{b^2 c^3 x^3}{3}+3a^2 c^2 dx$
risch	$\frac{b^2 d^3 x^9}{9}+\frac{2ab\,d^3 x^7}{7}+\frac{3b^2 c\,d^2 x^7}{7}+\frac{a^2 d^3 x^5}{5}+\frac{6abc\,d^2 x^5}{5}+\frac{3b^2 c^2 d\,x^5}{5}+a^2 c\,d^2 x^3+2ab\,c^2 d\,x^3+\frac{b^2 c^3 x^3}{3}+3a^2 c^2 dx$
gosper	$-\frac{-35b^2 d^3 x^{10}-90ab\,d^3 x^8-135b^2 c\,d^2 x^8-63a^2 d^3 x^6-378abc\,d^2 x^6-189b^2 c^2 d\,x^6-315a^2 c\,d^2 x^4-630ab\,c^2 d\,x^4-105b^2 c^3 x^4-945a^2 c^2 d\,x^2}{315x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*d^3*x^9+2/7*a*b*d^3*x^7+3/7*b^2*c*d^2*x^7+1/5*a^2*d^3*x^5+6/5*a*b*c
*d^2*x^5+3/5*b^2*c^2*d*x^5+a^2*c*d^2*x^3+2*a*b*c^2*d*x^3+1/3*b^2*c^3*x^3+3*
a^2*c^2*d*x+2*a*b*c^3*x-a^2*c^3/x
```

Maxima [A]
time = 0.30, size = 124, normalized size = 1.03

$$\frac{1}{9}\,b^2d^3x^9+\frac{1}{7}\left(3\,b^2cd^2+2\,abd^3\right)x^7+\frac{1}{5}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^5-\frac{a^2c^3}{x}+\frac{1}{3}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^3+\left(2\,abc^3+3\,a^2c^2d\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^3*x^9 + 1/7*(3*b^2*c*d^2 + 2*a*b*d^3)*x^7 + 1/5*(3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*x^5 - a^2*c^3/x + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + (2*a*b*c^3 + 3*a^2*c^2*d)*x
```

Fricas [A]
time = 0.75, size = 129, normalized size = 1.08

$$\frac{35\,b^2d^3x^{10}+45\left(3\,b^2cd^2+2\,abd^3\right)x^8+63\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-315\,a^2c^3+105\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+315\left(2\,abc^3+3\,a^2c^2d\right)x^2}{315\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^3*x^10 + 45*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 63*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 315*a^2*c^3 + 105*(b^2*c^3 + 6*a*b*c^2*d +
3*a^2*c*d^2)*x^4 + 315*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x
```

Sympy [A]
time = 0.11, size = 131, normalized size = 1.09

$$-\frac{a^2c^3}{x}+\frac{b^2d^3x^9}{9}+x^7\cdot\left(\frac{2abd^3}{7}+\frac{3b^2cd^2}{7}\right)+x^5\left(\frac{a^2d^3}{5}+\frac{6abcd^2}{5}+\frac{3b^2c^2d}{5}\right)+x^3\left(a^2cd^2+2abc^2d+\frac{b^2c^3}{3}\right)+x\left(3a^2c^2d+2abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**2,x)
```

```
[Out] -a**2*c**3/x + b**2*d**3*x**9/9 + x**7*(2*a*b*d**3/7 + 3*b**2*c*d**2/7) + x
**5*(a**2*d**3/5 + 6*a*b*c*d**2/5 + 3*b**2*c**2*d/5) + x**3*(a**2*c*d**2 +
2*a*b*c**2*d + b**2*c**3/3) + x*(3*a**2*c**2*d + 2*a*b*c**3)
```

Giac [A]
time = 1.73, size = 130, normalized size = 1.08

$$\frac{1}{9}b^2d^3x^9+\frac{3}{7}b^2cd^2x^7+\frac{2}{7}abd^3x^7+\frac{3}{5}b^2c^2dx^5+\frac{6}{5}abcd^2x^5+\frac{1}{5}a^2d^3x^5+\frac{1}{3}b^2c^3x^3+2\,abc^2dx^3+a^2cd^2x^3+2\,abc^3x+3\,a^2c^2dx-\frac{a^2c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^3*x^9 + 3/7*b^2*c*d^2*x^7 + 2/7*a*b*d^3*x^7 + 3/5*b^2*c^2*d*x^5 +
 6/5*a*b*c*d^2*x^5 + 1/5*a^2*d^3*x^5 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3 +
a^2*c*d^2*x^3 + 2*a*b*c^3*x + 3*a^2*c^2*d*x - a^2*c^3/x
```

Mupad [B]
time = 0.05, size = 115, normalized size = 0.96

$$x^3\left(a^2\,c\,d^2+2\,a\,b\,c^2\,d+\frac{b^2\,c^3}{3}\right)+x^5\left(\frac{a^2\,d^3}{5}+\frac{6\,a\,b\,c\,d^2}{5}+\frac{3\,b^2\,c^2\,d}{5}\right)-\frac{a^2\,c^3}{x}+\frac{b^2\,d^3\,x^9}{9}+\frac{b\,d^2\,x^7\,\left(2\,a\,d+3\,b\,c\right)}{7}+a\,c^2\,x\,\left(3\,a\,d+2\,b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x)
```

```
[Out] x^3*((b^2*c^3)/3 + a^2*c*d^2 + 2*a*b*c^2*d) + x^5*((a^2*d^3)/5 + (3*b^2*c^2
*d)/5 + (6*a*b*c*d^2)/5) - (a^2*c^3)/x + (b^2*d^3*x^9)/9 + (b*d^2*x^7*(2*a*
d + 3*b*c))/7 + a*c^2*x*(3*a*d + 2*b*c)
```

3.166 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^3}\,dx$

Optimal. Leaf size=123

$$-\frac{a^2c^3}{2x^2}+\frac{1}{2}c\left(b^2c^2+6abcd+3a^2d^2\right)x^2+\frac{1}{4}d\left(3b^2c^2+6abcd+a^2d^2\right)x^4+\frac{1}{6}bd^2(3bc+2ad)x^6+\frac{1}{8}b^2d^3x^8+ac^2(2bc+3$$

```
[Out] -1/2*a^2*c^3/x^2+1/2*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^2+1/4*d*(a^2*d^2+6*a
*b*c*d+3*b^2*c^2)*x^4+1/6*b*d^2*(2*a*d+3*b*c)*x^6+1/8*b^2*d^3*x^8+a*c^2*(3*
a*d+2*b*c)*ln(x)
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}cx^2\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{2x^2}+ac^2\log(x)(3ad+2bc)+\frac{1}{6}bd^2x^6(2ad+3bc)+\frac{1}{8}b^2d^3x^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

```
[Out] -1/2*(a^2*c^3)/x^2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^2)/2 + (d*(3*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c + 2*a*d)*x^6)/6 + (b^2*
d^3*x^8)/8 + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^3} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^3}{x^2} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(c(b^2c^2+6abcd+3a^2d^2) + \frac{a^2c^3}{x^2} + \frac{ac^2(2bc+3ad)}{x} + d(3b^2c^2 + \right.\right.$$
$$= -\frac{a^2c^3}{2x^2} + \frac{1}{2}c(b^2c^2+6abcd+3a^2d^2)x^2 + \frac{1}{4}d(3b^2c^2+6abcd+a^2d^2)x^4 + \frac{1}{6}bd^2$$

Mathematica [A]
time = 0.04, size = 120, normalized size = 0.98

$$\frac{4abdx^4(18c^2+9cdx^2+2d^2x^4)+3b^2x^4(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6)+6a^2(-2c^3+6cd^2x^4+d^3x^6)}{24x^2} + ac^2(2bc+3ad)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

```
[Out] (4*a*b*d*x^4*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + 3*b^2*x^4*(4*c^3 + 6*c^2*d*
x^2 + 4*c*d^2*x^4 + d^3*x^6) + 6*a^2*(-2*c^3 + 6*c*d^2*x^4 + d^3*x^6))/(24*
x^2) + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

Maple [A]
time = 0.08, size = 130, normalized size = 1.06

method	result
norman	$\frac{\left(\frac{1}{3}ab\,d^3+\frac{1}{2}b^2c\,d^2\right)x^8+\left(\frac{1}{4}a^2d^3+\frac{3}{2}abc\,d^2+\frac{3}{4}b^2c^2d\right)x^6+\left(\frac{3}{2}a^2c\,d^2+3ab\,c^2d+\frac{1}{2}b^2c^3\right)x^4-\frac{a^2c^3}{2}+\frac{b^2d^3x^{10}}{8}}{x^2}+\left(3a^2c^2d+2ab\,c^3\right)\ln$
default	$\frac{b^2d^3x^8}{8}+\frac{ab\,d^3x^6}{3}+\frac{b^2c\,d^2x^6}{2}+\frac{a^2d^3x^4}{4}+\frac{3abc\,d^2x^4}{2}+\frac{3b^2c^2d\,x^4}{4}+\frac{3a^2c\,d^2x^2}{2}+3ab\,c^2d\,x^2+\frac{b^2c^3x^2}{2}-\frac{a^2c^3}{2x^2}+a$
risch	$\frac{b^2d^3x^8}{8}+\frac{ab\,d^3x^6}{3}+\frac{b^2c\,d^2x^6}{2}+\frac{a^2d^3x^4}{4}+\frac{3abc\,d^2x^4}{2}+\frac{3b^2c^2d\,x^4}{4}+\frac{3a^2c\,d^2x^2}{2}+3ab\,c^2d\,x^2+\frac{b^2c^3x^2}{2}-\frac{a^2c^3}{2x^2}+3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^2*d^3*x^8+1/3*a*b*d^3*x^6+1/2*b^2*c*d^2*x^6+1/4*a^2*d^3*x^4+3/2*a*b*c
*d^2*x^4+3/4*b^2*c^2*d*x^4+3/2*a^2*c*d^2*x^2+3*a*b*c^2*d*x^2+1/2*b^2*c^3*x^
2-1/2*a^2*c^3/x^2+a*c^2*(3*a*d+2*b*c)*ln(x)
```

Maxima [A]
time = 0.28, size = 128, normalized size = 1.04

$$\frac{1}{8}b^2d^3x^8 + \frac{1}{6}(3b^2cd^2+2abd^3)x^6 + \frac{1}{4}(3b^2c^2d+6abcd^2+a^2d^3)x^4 - \frac{a^2c^3}{2x^2} + \frac{1}{2}(b^2c^3+6abc^2d+3a^2cd^2)x^2 + \frac{1}{2}(2abc^3+3a^2c^2d)\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d^3*x^8 + 1/6*(3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 1/4*(3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*x^4 - 1/2*a^2*c^3/x^2 + 1/2*(b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*log(x^2)
```

Fricas [A]
time = 0.90, size = 131, normalized size = 1.07

$$\frac{3\,b^2d^3x^{10}+4\,(3\,b^2cd^2+2\,abd^3)x^8+6\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^6-12\,a^2c^3+12\,(b^2c^3+6\,abc^2d+3\,a^2cd^2)x^4+24\,(2\,abc^3+3\,a^2c^2d)x^2\log\left(x\right)}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*d^3*x^10 + 4*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 6*(3*b^2*c^2*d + 6
*a*b*c*d^2 + a^2*d^3)*x^6 - 12*a^2*c^3 + 12*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*
c*d^2)*x^4 + 24*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2*log(x))/x^2
```

Sympy [A]
time = 0.21, size = 133, normalized size = 1.08

$$-\frac{a^2c^3}{2x^2}+ac^2\cdot(3ad+2bc)\log\left(x\right)+\frac{b^2d^3x^8}{8}+x^6\left(\frac{abd^3}{3}+\frac{b^2cd^2}{2}\right)+x^4\left(\frac{a^2d^3}{4}+\frac{3abcd^2}{2}+\frac{3b^2c^2d}{4}\right)+x^2\cdot\left(\frac{3a^2cd^2}{2}+3abc^2d+\frac{b^2c^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**3,x)
```

```
[Out] -a**2*c**3/(2*x**2) + a*c**2*(3*a*d + 2*b*c)*log(x) + b**2*d**3*x**8/8 + x*
*6*(a*b*d**3/3 + b**2*c*d**2/2) + x**4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b*
*2*c**2*d/4) + x**2*(3*a**2*c*d**2/2 + 3*a*b*c**2*d + b**2*c**3/2)
```

Giac [A]
time = 0.89, size = 160, normalized size = 1.30

$$\frac{1}{8}\,b^2d^3x^8+\frac{1}{2}\,b^2cd^2x^6+\frac{1}{3}\,abd^3x^6+\frac{3}{4}\,b^2c^2dx^4+\frac{3}{2}\,abcd^2x^4+\frac{1}{4}\,a^2d^3x^4+\frac{1}{2}\,b^2c^3x^2+3\,abc^2dx^2+\frac{3}{2}\,a^2cd^2x^2+\frac{1}{2}\,(2\,abc^3+3\,a^2c^2d)\log\left(x^2\right)-\frac{2\,abc^3x^2+3\,a^2c^2dx^2+a^2c^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^2*d^3*x^8 + 1/2*b^2*c*d^2*x^6 + 1/3*a*b*d^3*x^6 + 3/4*b^2*c^2*d*x^4 +
 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/2*b^2*c^3*x^2 + 3*a*b*c^2*d*x^2 +
3/2*a^2*c*d^2*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*log(x^2) - 1/2*(2*a*b*c^3
*x^2 + 3*a^2*c^2*d*x^2 + a^2*c^3)/x^2
```

Mupad [B]
time = 0.05, size = 121, normalized size = 0.98

$$x^2\left(\frac{3\,a^2\,c\,d^2}{2}+3\,a\,b\,c^2\,d+\frac{b^2\,c^3}{2}\right)+x^4\left(\frac{a^2\,d^3}{4}+\frac{3\,a\,b\,c\,d^2}{2}+\frac{3\,b^2\,c^2\,d}{4}\right)+\ln\left(x\right)\,\left(3\,d\,a^2\,c^2+2\,b\,a\,c^3\right)-\frac{a^2\,c^3}{2\,x^2}+\frac{b^2\,d^3\,x^8}{8}+\frac{b\,d^2\,x^6\,\left(2\,a\,d+3\,b\,c\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x)
```

```
[Out] x^2*((b^2*c^3)/2 + (3*a^2*c*d^2)/2 + 3*a*b*c^2*d) + x^4*((a^2*d^3)/4 + (3*b
^2*c^2*d)/4 + (3*a*b*c*d^2)/2) + log(x)*(3*a^2*c^2*d + 2*a*b*c^3) - (a^2*c^
3)/(2*x^2) + (b^2*d^3*x^8)/8 + (b*d^2*x^6*(2*a*d + 3*b*c))/6
```

3.167 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4}\,dx$

Optimal. Leaf size=120

$$-\frac{a^2c^3}{3x^3}-\frac{ac^2(2bc+3ad)}{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x+\frac{1}{3}d\left(3b^2c^2+6abcd+a^2d^2\right)x^3+\frac{1}{5}bd^2(3bc+2ad)x^5+\frac{1}{7}b^2d^3$$

```
[Out] -1/3*a^2*c^3/x^3-a*c^2*(3*a*d+2*b*c)/x+c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x+1/
3*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^3+1/5*b*d^2*(2*a*d+3*b*c)*x^5+1/7*b^2*d
^3*x^7
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

```
[Out] -1/3*(a^2*c^3)/x^3 - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a*b*c*d + 3
*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*d^2*(3*b*c +
 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4}\,dx &= \int \left(c\left(b^2c^2+6abcd+3a^2d^2\right)+\frac{a^2c^3}{x^4}+\frac{ac^2(2bc+3ad)}{x^2}+d\left(3b^2c^2+6abcd+a^2\right.\right.\\ &= -\frac{a^2c^3}{3x^3}-\frac{ac^2(2bc+3ad)}{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x+\frac{1}{3}d\left(3b^2c^2+6abcd+a\right.\end{aligned}$$

Mathematica [A]
time = 0.03, size = 120, normalized size = 1.00

$$-\frac{a^2c^3}{3x^3}-\frac{ac^2(2bc+3ad)}{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x+\frac{1}{3}d\left(3b^2c^2+6abcd+a^2d^2\right)x^3+\frac{1}{5}bd^2(3bc+2ad)x^5+\frac{1}{7}b^2d^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

```
[Out] -1/3*(a^2*c^3)/x^3 - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a*b*c*d + 3
*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*d^2*(3*b*c +
 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

Maple [A]
time = 0.08, size = 124, normalized size = 1.03

method	result
default	$\frac{b^2 d^3 x^7}{7}+\frac{2ab\,d^3 x^5}{5}+\frac{3b^2 c\,d^2 x^5}{5}+\frac{a^2 d^3 x^3}{3}+2abc\,d^2 x^3+b^2 c^2 d\,x^3+3a^2 c\,d^2 x+6ab\,c^2 dx+b^2 c^3 x-\frac{a^2 c^3}{3x^3}$ -
norman	$\frac{\frac{b^2 d^3 x^{10}}{7}+\left(\frac{2}{5}ab\,d^3+\frac{3}{5}b^2 c\,d^2\right)x^8+\left(\frac{1}{3}a^2 d^3+2abc\,d^2+b^2 c^2 d\right)x^6+\left(3a^2 c\,d^2+6ab\,c^2 d+b^2 c^3\right)x^4+\left(-3a^2 c^2 d-2ab\,c^3\right)x^2-\frac{a^2 c^3}{3}}{x^3}$
risch	$\frac{b^2 d^3 x^7}{7}+\frac{2ab\,d^3 x^5}{5}+\frac{3b^2 c\,d^2 x^5}{5}+\frac{a^2 d^3 x^3}{3}+2abc\,d^2 x^3+b^2 c^2 d\,x^3+3a^2 c\,d^2 x+6ab\,c^2 dx+b^2 c^3 x+\frac{(-3a^2}{}$
gosper	$-\frac{-15b^2 d^3 x^{10}-42ab\,d^3 x^8-63b^2 c\,d^2 x^8-35a^2 d^3 x^6-210abc\,d^2 x^6-105b^2 c^2 d\,x^6-315a^2 c\,d^2 x^4-630ab\,c^2 d\,x^4-105b^2 c^3 x^4+315a^2 c^2 d\,x}{105x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^2*d^3*x^7+2/5*a*b*d^3*x^5+3/5*b^2*c*d^2*x^5+1/3*a^2*d^3*x^3+2*a*b*c*d
^2*x^3+b^2*c^2*d*x^3+3*a^2*c*d^2*x+6*a*b*c^2*d*x+b^2*c^3*x-1/3*a^2*c^3/x^3-
a*c^2*(3*a*d+2*b*c)/x
```

Maxima [A]
time = 0.29, size = 126, normalized size = 1.05

$$\frac{1}{7}\,b^2 d^3 x^7+\frac{1}{5}\left(3\,b^2 c d^2+2\,a b d^3\right)x^5+\frac{1}{3}\left(3\,b^2 c^2 d+6\,a b c d^2+a^2 d^3\right)x^3+\left(b^2 c^3+6\,a b c^2 d+3\,a^2 c d^2\right)x-\frac{a^2 c^3+3\,(2\,a b c^3+3\,a^2 c^2 d)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^3*x^7 + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/3*(3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x - 1/3*(a
^2*c^3 + 3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^3
```

Fricas [A]
time = 0.97, size = 129, normalized size = 1.08

$$\frac{15\,b^2 d^3 x^{10}+21\,(3\,b^2 c d^2+2\,a b d^3)x^8+35\,(3\,b^2 c^2 d+6\,a b c d^2+a^2 d^3)x^6-35\,a^2 c^3+105\,(b^2 c^3+6\,a b c^2 d+3\,a^2 c d^2)x^4-105\,(2\,a b c^3+3\,a^2 c^2 d)x^2}{105\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^10 + 21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 35*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 35*a^2*c^3 + 105*(b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^4 - 105*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^3
```

Sympy [A]
time = 0.21, size = 131, normalized size = 1.09

$$\frac{b^2d^3x^7}{7} + x^5 \cdot \left(\frac{2abd^3}{5} + \frac{3b^2cd^2}{5}\right) + x^3\left(\frac{a^2d^3}{3} + 2abcd^2 + b^2c^2d\right) + x\left(3a^2cd^2 + 6abc^2d + b^2c^3\right) + \frac{-a^2c^3 + x^2(-9a^2c^2d - 6abc^3)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**4,x)
```

```
[Out] b**2*d**3*x**7/7 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x**3*(a**2*d**3/
3 + 2*a*b*c*d**2 + b**2*c**2*d) + x*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c*
*3) + (-a**2*c**3 + x**2*(-9*a**2*c**2*d - 6*a*b*c**3))/(3*x**3)
```

Giac [A]
time = 0.65, size = 129, normalized size = 1.08

$$\frac{1}{7}b^2d^3x^7 + \frac{3}{5}b^2cd^2x^5 + \frac{2}{5}abd^3x^5 + b^2c^2dx^3 + 2\,abcd^2x^3 + \frac{1}{3}a^2d^3x^3 + b^2c^3x + 6\,abc^2dx + 3\,a^2cd^2x - \frac{6\,abc^3x^2 + 9\,a^2c^2dx^2 + a^2c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^3*x^7 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + b^2*c^2*d*x^3 + 2*a
*b*c*d^2*x^3 + 1/3*a^2*d^3*x^3 + b^2*c^3*x + 6*a*b*c^2*d*x + 3*a^2*c*d^2*x
- 1/3*(6*a*b*c^3*x^2 + 9*a^2*c^2*d*x^2 + a^2*c^3)/x^3
```

Mupad [B]
time = 0.02, size = 121, normalized size = 1.01

$$x^3\left(\frac{a^2\,d^3}{3} + 2\,a\,b\,c\,d^2 + b^2\,c^2\,d\right) - \frac{x^2\,\left(3\,d\,a^2\,c^2 + 2\,b\,a\,c^3\right) + \frac{a^2\,c^3}{3}}{x^3} + x\,\left(3\,a^2\,c\,d^2 + 6\,a\,b\,c^2\,d + b^2\,c^3\right) + \frac{b^2\,d^3\,x^7}{7} + \frac{b\,d^2\,x^5\,\left(2\,a\,d + 3\,b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x)
```

```
[Out] x^3*((a^2*d^3)/3 + b^2*c^2*d + 2*a*b*c*d^2) - (x^2*(3*a^2*c^2*d + 2*a*b*c^3
) + (a^2*c^3)/3)/x^3 + x*(b^2*c^3 + 3*a^2*c*d^2 + 6*a*b*c^2*d) + (b^2*d^3*x
^7)/7 + (b*d^2*x^5*(2*a*d + 3*b*c))/5
```

3.168 $\int \frac{x^4(a+bx^2)^2}{c+dx^2} dx$

Optimal. Leaf size=104

$$-\frac{c(bc-ad)^2x}{d^4}+\frac{(bc-ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{9/2}}$$

```
[Out] -c*(-a*d+b*c)^2*x/d^4+1/3*(-a*d+b*c)^2*x^3/d^3-1/5*b*(-2*a*d+b*c)*x^5/d^2+1
/7*b^2*x^7/d+c^(3/2)*(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))/d^(9/2)
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {472, 211}

$$\frac{c^{3/2}(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{cx(bc-ad)^2}{d^4}+\frac{x^3(bc-ad)^2}{3d^3}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{b^2x^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((b*c - a*d)^2*x^3)/(3*d^3) - (b*(b*c - 2*a*d)
*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/d^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^2}{c+dx^2}\,dx = \int \left(-\frac{c(bc-ad)^2}{d^4}+\frac{(bc-ad)^2x^2}{d^3}-\frac{b(bc-2ad)x^4}{d^2}+\frac{b^2x^6}{d}+\frac{b^2c^4-2abc^3d+a^2c^2d^2}{d^4\,(c+dx^2)}\right)$$
$$= -\frac{c(bc-ad)^2x}{d^4}+\frac{(bc-ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{(c^2(bc-ad)^2)\int\frac{1}{c+dx^2}\,dx}{d^4}$$
$$= -\frac{c(bc-ad)^2x}{d^4}+\frac{(bc-ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{9/2}}$$

Mathematica [A]
time = 0.07, size = 104, normalized size = 1.00

$$-\frac{c(bc-ad)^2x}{d^4}+\frac{(-bc+ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((-(b*c) + a*d)^2*x^3)/(3*d^3) - (b*(b*c - 2*a
*d)*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/d^(9/2)
```

Maple [A]
time = 0.13, size = 143, normalized size = 1.38

method	result
default	$-\frac{-\frac{b^2d^3x^7}{7}+\frac{\left(-(ad-bc)b\,d^2-ab\,d^3\right)x^5}{5}+\frac{\left(-(ad-bc)a\,d^2+bd\left(acd-b\,c^2\right)\right)x^3}{3}+(ad-bc)\left(acd-b\,c^2\right)x}{d^4}+\frac{c^2\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{c}\ldots}\right)}{d^4\sqrt{cd}}$
risch	$\frac{b^2x^7}{7d}+\frac{2ab\,x^5}{5d}-\frac{b^2c\,x^5}{5d^2}+\frac{a^2x^3}{3d}-\frac{2abc\,x^3}{3d^2}+\frac{b^2c^2x^3}{3d^3}-\frac{a^2cx}{d^2}+\frac{2ab\,c^2x}{d^3}-\frac{b^2c^3x}{d^4}+\frac{\sqrt{-cd}\;c\ln\left(-\sqrt{-cd}\;x+c\right)a^2}{2d^3}-\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^4*(-1/7*b^2*d^3*x^7+1/5*(-(a*d-b*c)*b*d^2-a*b*d^3)*x^5+1/3*(-(a*d-b*c)
*a*d^2+b*d*(a*c*d-b*c^2))*x^3+(a*d-b*c)*(a*c*d-b*c^2)*x)+c^2*(a^2*d^2-2*a*b
*c*d+b^2*c^2)/d^4/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.51, size = 139, normalized size = 1.34

$$\frac{(b^2c^4-2\,abc^3d+a^2c^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^4}+\frac{15\,b^2d^3x^7-21\,(b^2cd^2-2\,abd^3)x^5+35\,(b^2c^2d-2\,abcd^2+a^2d^3)x^3-105\,(b^2c^3-2\,abc^2d+a^2cd^2)x}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4)
 + 1/105*(15*b^2*d^3*x^7 - 21*(b^2*c*d^2 - 2*a*b*d^3)*x^5 + 35*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*x^3 - 105*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*x)/d^
4
```

Fricas [A]
time = 1.02, size = 302, normalized size = 2.90

$$\left[\frac{30\,b^2d^3x^7 - 42\,(b^2cd^2 - 2\,abd^3)x^5 + 70\,(b^2c^2d - 2\,abcd^2 + a^2d^3)x^3 + 105\,(b^2c^3 - 2\,abc^2d + a^2cd^2)\sqrt{-\frac{c}{d}}\,\log\left(\frac{dx^2 + 2\,dx\sqrt{-\frac{c}{d}} - c}{dx^2 + c}\right) - 210\,(b^2c^3 - 2\,abc^2d + a^2cd^2)x}{210\,d^4}, \frac{15\,b^2d^3x^7 - 21\,(b^2cd^2 - 2\,abd^3)x^5 + 35\,(b^2c^2d - 2\,abcd^2 + a^2d^3)x^3 + 105\,(b^2c^3 - 2\,abc^2d + a^2cd^2)\sqrt{\frac{c}{d}}\,\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right) - 105\,(b^2c^3 - 2\,abc^2d + a^2cd^2)x}{105\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^2*d^3*x^7 - 42*(b^2*c*d^2 - 2*a*b*d^3)*x^5 + 70*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(-c
/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 210*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)*x)/d^4, 1/105*(15*b^2*d^3*x^7 - 21*(b^2*c*d^2 - 2*a*b*d^
3)*x^5 + 35*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) - 105*(b^2*c^3 - 2*a*b
*c^2*d + a^2*c*d^2)*x)/d^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(92) = 184$.
time = 0.42, size = 246, normalized size = 2.37

$$\frac{b^2x^7}{7d} + x^5\cdot\left(\frac{2ab}{5d} - \frac{b^2c}{5d^2}\right) + x^3\left(\frac{a^2}{3d} - \frac{2abc}{3d^2} + \frac{b^2c^2}{3d^3}\right) + x\left(-\frac{a^2c}{d^2} + \frac{2abc^2}{d^3} - \frac{b^2c^3}{d^4}\right) - \frac{\sqrt{-\frac{c^3}{d^9}}\,(ad - bc)^2\log\left(-\frac{d^4\sqrt{-\frac{c^3}{d^9}}\,(ad-bc)^2}{a^2cd^2 - 2abc^2d + b^2c^3} + x\right)}{2} + \frac{\sqrt{-\frac{c^3}{d^9}}\,(ad - bc)^2\log\left(\frac{d^4\sqrt{-\frac{c^3}{d^9}}\,(ad-bc)^2}{a^2cd^2 - 2abc^2d + b^2c^3} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**7/(7*d) + x**5*(2*a*b/(5*d) - b**2*c/(5*d**2)) + x**3*(a**2/(3*d) -
 2*a*b*c/(3*d**2) + b**2*c**2/(3*d**3)) + x*(-a**2*c/d**2 + 2*a*b*c**2/d**3
 - b**2*c**3/d**4) - sqrt(-c**3/d**9)*(a*d - b*c)**2*log(-d**4*sqrt(-c**3/d
**9)*(a*d - b*c)**2/(a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3) + x)/2 + sqrt(
-c**3/d**9)*(a*d - b*c)**2*log(d**4*sqrt(-c**3/d**9)*(a*d - b*c)**2/(a**2*c
*d**2 - 2*a*b*c**2*d + b**2*c**3) + x)/2
```

Giac [A]
time = 0.68, size = 153, normalized size = 1.47

$$\frac{(b^2c^4 - 2\,abc^3d + a^2c^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^4} + \frac{15\,b^2d^6x^7 - 21\,b^2cd^5x^5 + 42\,abd^6x^5 + 35\,b^2c^2d^4x^3 - 70\,abcd^5x^3 + 35\,a^2d^6x^3 - 105\,b^2c^3d^3x + 210\,abc^2d^4x - 105\,a^2cd^5x}{105\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4)
 + 1/105*(15*b^2*d^6*x^7 - 21*b^2*c*d^5*x^5 + 42*a*b*d^6*x^5 + 35*b^2*c^2*d
^4*x^3 - 70*a*b*c*d^5*x^3 + 35*a^2*d^6*x^3 - 105*b^2*c^3*d^3*x + 210*a*b*c^
2*d^4*x - 105*a^2*c*d^5*x)/d^7
```

Mupad [B]
time = 0.03, size = 169, normalized size = 1.62

$$x^3\left(\frac{a^2}{3\,d}+\frac{c\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{3\,d}\right)-x^5\,\left(\frac{b^2\,c}{5\,d^2}-\frac{2\,a\,b}{5\,d}\right)+\frac{b^2\,x^7}{7\,d}+\frac{c^{3/2}\,\mathrm{atan}\left(\frac{c^{3/2}\,\sqrt{d}\,x\,{\left(a\,d-b\,c\right)}^2}{a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4}\right)\,{\left(a\,d-b\,c\right)}^2}{d^{9/2}}-\frac{c\,x\,\left(\frac{a^2}{d}+\frac{c\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x^3*(a^2/(3*d) + (c*((b^2*c)/d^2 - (2*a*b)/d))/(3*d)) - x^5*((b^2*c)/(5*d^2
) - (2*a*b)/(5*d)) + (b^2*x^7)/(7*d) + (c^(3/2)*atan((c^(3/2)*d^(1/2)*x*(a*
d - b*c)^2)/(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))*(a*d - b*c)^2)/d^(9/2) -
 (c*x*(a^2/d + (c*((b^2*c)/d^2 - (2*a*b)/d))/d))/d
```

3.169 $\int \frac{x^3(a+bx^2)^2}{c+dx^2} dx$

Optimal. Leaf size=79

$$\frac{(bc-ad)^2x^2}{2d^3} - \frac{b(bc-2ad)x^4}{4d^2} + \frac{b^2x^6}{6d} - \frac{c(bc-ad)^2\log(c+dx^2)}{2d^4}$$

```
[Out] 1/2*(-a*d+b*c)^2*x^2/d^3-1/4*b*(-2*a*d+b*c)*x^4/d^2+1/6*b^2*x^6/d-1/2*c*(-a
*d+b*c)^2*ln(d*x^2+c)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{c(bc-ad)^2\log(c+dx^2)}{2d^4} + \frac{x^2(bc-ad)^2}{2d^3} - \frac{bx^4(bc-2ad)}{4d^2} + \frac{b^2x^6}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x^2)/(2*d^3) - (b*(b*c - 2*a*d)*x^4)/(4*d^2) + (b^2*x^6)/(6*
d) - (c*(b*c - a*d)^2*Log[c + d*x^2])/(2*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{c+dx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^3} - \frac{b(bc-2ad)x}{d^2} + \frac{b^2x^2}{d} - \frac{c(bc-ad)^2}{d^3(c+dx)}\right)dx, x, x^2\right)$$
$$= \frac{(bc-ad)^2x^2}{2d^3} - \frac{b(bc-2ad)x^4}{4d^2} + \frac{b^2x^6}{6d} - \frac{c(bc-ad)^2\log(c+dx^2)}{2d^4}$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 1.04

$$\frac{dx^2(6a^2d^2+6abd(-2c+dx^2)+b^2(6c^2-3cdx^2+2d^2x^4))-6c(bc-ad)^2\log(c+dx^2)}{12d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (d*x^2*(6*a^2*d^2 + 6*a*b*d*(-2*c + d*x^2) + b^2*(6*c^2 - 3*c*d*x^2 + 2*d^2
*x^4)) - 6*c*(b*c - a*d)^2*Log[c + d*x^2])/(12*d^4)
```

Maple [A]
time = 0.08, size = 102, normalized size = 1.29

method	result	size
norman	$\frac{b^2x^6}{6d}+\frac{(a^2d^2-2abcd+b^2c^2)x^2}{2d^3}+\frac{b(2ad-bc)x^4}{4d^2}-\frac{c(a^2d^2-2abcd+b^2c^2)\ln(dx^2+c)}{2d^4}$	95
default	$\frac{\frac{1}{3}b^2d^2x^6+ab\,d^2x^4-\frac{1}{2}b^2cd\,x^4+a^2d^2x^2-2abcd\,x^2+b^2c^2x^2}{2d^3}-\frac{c(a^2d^2-2abcd+b^2c^2)\ln(dx^2+c)}{2d^4}$	102
risch	$\frac{b^2x^6}{6d}+\frac{ab\,x^4}{2d}-\frac{b^2c\,x^4}{4d^2}+\frac{a^2x^2}{2d}-\frac{abc\,x^2}{d^2}+\frac{b^2c^2x^2}{2d^3}-\frac{c\ln(dx^2+c)a^2}{2d^2}+\frac{c^2\ln(dx^2+c)ab}{d^3}-\frac{c^3\ln(dx^2+c)b^2}{2d^4}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d^3*(1/3*b^2*d^2*x^6+a*b*d^2*x^4-1/2*b^2*c*d*x^4+a^2*d^2*x^2-2*a*b*c*d*
x^2+b^2*c^2*x^2)-1/2*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4*ln(d*x^2+c)
```

Maxima [A]
time = 0.28, size = 100, normalized size = 1.27

$$\frac{2\,b^2d^2x^6-3\,(b^2cd-2\,abd^2)x^4+6\,(b^2c^2-2\,abcd+a^2d^2)x^2}{12\,d^3}-\frac{(b^2c^3-2\,abc^2d+a^2cd^2)\log(dx^2+c)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 - 3*(b^2*c*d - 2*a*b*d^2)*x^4 + 6*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x^2 + c
)/d^4
```

Fricas [A]
time = 1.41, size = 101, normalized size = 1.28

$$\frac{2\,b^2d^3x^6-3\,(b^2cd^2-2\,abd^3)x^4+6\,(b^2c^2d-2\,abcd^2+a^2d^3)x^2-6\,(b^2c^3-2\,abc^2d+a^2cd^2)\log\left(dx^2+c\right)}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^4 + 6*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*x^2 - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x^2 + c)
)/d^4
```

Sympy [A]
time = 0.31, size = 83, normalized size = 1.05

$$\frac{b^2x^6}{6d}-\frac{c(ad-bc)^2\log\left(c+dx^2\right)}{2d^4}+x^4\left(\frac{ab}{2d}-\frac{b^2c}{4d^2}\right)+x^2\left(\frac{a^2}{2d}-\frac{abc}{d^2}+\frac{b^2c^2}{2d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**6/(6*d) - c*(a*d - b*c)**2*log(c + d*x**2)/(2*d**4) + x**4*(a*b/(2*
d) - b**2*c/(4*d**2)) + x**2*(a**2/(2*d) - a*b*c/d**2 + b**2*c**2/(2*d**3))
```

Giac [A]
time = 0.72, size = 107, normalized size = 1.35

$$\frac{2\,b^2d^2x^6-3\,b^2cdx^4+6\,abd^2x^4+6\,b^2c^2x^2-12\,abcdx^2+6\,a^2d^2x^2}{12\,d^3}-\frac{(b^2c^3-2\,abc^2d+a^2cd^2)\log\left(|dx^2+c|\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 - 3*b^2*c*d*x^4 + 6*a*b*d^2*x^4 + 6*b^2*c^2*x^2 - 12*a*
b*c*d*x^2 + 6*a^2*d^2*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*lo
g(abs(d*x^2 + c))/d^4
```

Mupad [B]
time = 0.03, size = 106, normalized size = 1.34

$$x^2\left(\frac{a^2}{2\,d}+\frac{c\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{2\,d}\right)-x^4\left(\frac{b^2\,c}{4\,d^2}-\frac{a\,b}{2\,d}\right)+\frac{b^2\,x^6}{6\,d}-\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x^2*(a^2/(2*d) + (c*((b^2*c)/d^2 - (2*a*b)/d))/(2*d)) - x^4*((b^2*c)/(4*d^2
) - (a*b)/(2*d)) + (b^2*x^6)/(6*d) - (log(c + d*x^2)*(b^2*c^3 + a^2*c*d^2 -
 2*a*b*c^2*d))/(2*d^4)
```

3.170 $\int \frac{x^2(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=83

$$\frac{(bc-ad)^2x}{d^3}-\frac{b(bc-2ad)x^3}{3d^2}+\frac{b^2x^5}{5d}-\frac{\sqrt{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{7/2}}$$

```
[Out] (-a*d+b*c)^2*x/d^3-1/3*b*(-2*a*d+b*c)*x^3/d^2+1/5*b^2*x^5/d-(-a*d+b*c)^2*ar
ctan(x*d^(1/2)/c^(1/2))*c^(1/2)/d^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {472, 211}

$$-\frac{\sqrt{c}\,(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{x(bc-ad)^2}{d^3}-\frac{bx^3(bc-2ad)}{3d^2}+\frac{b^2x^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)/(5*d) - (
Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{x^2(a+bx^2)^2}{c+dx^2}\,dx = \int \left(\frac{(bc-ad)^2}{d^3} - \frac{b(bc-2ad)x^2}{d^2} + \frac{b^2x^4}{d} + \frac{-b^2c^3+2abc^2d-a^2cd^2}{d^3\,(c+dx^2)}\right)dx$$

$$= \frac{(bc-ad)^2x}{d^3} - \frac{b(bc-2ad)x^3}{3d^2} + \frac{b^2x^5}{5d} - \frac{(c(bc-ad)^2)\int \frac{1}{c+dx^2}\,dx}{d^3}$$

$$= \frac{(bc-ad)^2x}{d^3} - \frac{b(bc-2ad)x^3}{3d^2} + \frac{b^2x^5}{5d} - \frac{\sqrt{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{7/2}}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 1.00

$$\frac{(-bc+ad)^2x}{d^3} - \frac{b(bc-2ad)x^3}{3d^2} + \frac{b^2x^5}{5d} - \frac{\sqrt{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((-(b*c) + a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)/(5*d)
- (Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7/2)
```

Maple [A]
time = 0.10, size = 102, normalized size = 1.23

method	result
default	$\frac{\frac{1}{5}b^2x^5d^2+\frac{2}{3}ab\,d^2x^3-\frac{1}{3}b^2cd\,x^3+a^2d^2x-2abcdx+b^2c^2x}{d^3} - \frac{c\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^3\sqrt{cd}}$
risch	$\frac{b^2x^5}{5d} + \frac{2ab\,x^3}{3d} - \frac{b^2c\,x^3}{3d^2} + \frac{a^2x}{d} - \frac{2abcx}{d^2} + \frac{b^2c^2x}{d^3} + \frac{\sqrt{-cd}\,\ln\left(-\sqrt{-cd}\,x-c\right)a^2}{2d^2} - \frac{\sqrt{-cd}\,\ln\left(-\sqrt{-cd}\,x-c\right)ab}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(1/5*b^2*x^5*d^2+2/3*a*b*d^2*x^3-1/3*b^2*c*d*x^3+a^2*d^2*x-2*a*b*c*d*
x+b^2*c^2*x)-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(c*d)^(1/2)*arctan(d*x/(c*d)
^(1/2))
```

Maxima [A]
time = 0.49, size = 104, normalized size = 1.25

$$-\frac{(b^2c^3-2\,abc^2d+a^2cd^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3} + \frac{3\,b^2d^2x^5-5\,(b^2cd-2\,abd^2)x^3+15\,(b^2c^2-2\,abcd+a^2d^2)x}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
+ 1/15*(3*b^2*d^2*x^5 - 5*(b^2*c*d - 2*a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x)/d^3
```

Fricas [A]
time = 1.15, size = 228, normalized size = 2.75

$$\left[\frac{6\,b^2d^2x^5 - 10\,(b^2cd - 2\,abd^2)x^3 + 15\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2 - 2\,dx\sqrt{-\frac{c}{d}} - c}{dx^2 + c}\right) + 30\,(b^2c^2 - 2\,abcd + a^2d^2)x}{30\,d^3}, \frac{3\,b^2d^2x^5 - 5\,(b^2cd - 2\,abd^2)x^3 - 15\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right) + 15\,(b^2c^2 - 2\,abcd + a^2d^2)x}{15\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d^2*x^5 - 10*(b^2*c*d - 2*a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) +
30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/d^3, 1/15*(3*b^2*d^2*x^5 - 5*(b^2*c*d
 - 2*a*b*d^2)*x^3 - 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c/d)*arctan(d*x
*sqrt(c/d)/c) + 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/d^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(73) = 146$.
time = 0.32, size = 194, normalized size = 2.34

$$\frac{b^2x^5}{5d} + x^3\cdot\left(\frac{2ab}{3d} - \frac{b^2c}{3d^2}\right) + x\left(\frac{a^2}{d} - \frac{2abc}{d^2} + \frac{b^2c^2}{d^3}\right) + \frac{\sqrt{-\frac{c}{d^7}}\,(ad - bc)^2\log\left(-\frac{d^3\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2} + x\right)}{2} - \frac{\sqrt{-\frac{c}{d^7}}\,(ad - bc)^2\log\left(\frac{d^3\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**5/(5*d) + x**3*(2*a*b/(3*d) - b**2*c/(3*d**2)) + x*(a**2/d - 2*a*b*
c/d**2 + b**2*c**2/d**3) + sqrt(-c/d**7)*(a*d - b*c)**2*log(-d**3*sqrt(-c/d
**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-c/d*
*7)*(a*d - b*c)**2*log(d**3*sqrt(-c/d**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b
*c*d + b**2*c**2) + x)/2
```

Giac [A]
time = 0.89, size = 113, normalized size = 1.36

$$-\frac{(b^2c^3 - 2\,abc^2d + a^2cd^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3} + \frac{3\,b^2d^4x^5 - 5\,b^2cd^3x^3 + 10\,abd^4x^3 + 15\,b^2c^2d^2x - 30\,abcd^3x + 15\,a^2d^4x}{15\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
+ 1/15*(3*b^2*d^4*x^5 - 5*b^2*c*d^3*x^3 + 10*a*b*d^4*x^3 + 15*b^2*c^2*d^2*x
 - 30*a*b*c*d^3*x + 15*a^2*d^4*x)/d^5
```

Mupad [B]
time = 0.03, size = 128, normalized size = 1.54

$$x\left(\frac{a^2}{d}+\frac{c\left(\frac{b^2c}{d^2}-\frac{2ab}{d}\right)}{d}\right)-x^3\left(\frac{b^2c}{3d^2}-\frac{2ab}{3d}\right)+\frac{b^2x^5}{5d}-\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,x\,(a\,d-b\,c)^2}{a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3}\right)\,(a\,d-b\,c)^2}{d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x*(a^2/d + (c*((b^2*c)/d^2 - (2*a*b)/d))/d) - x^3*((b^2*c)/(3*d^2) - (2*a*b
)/(3*d)) + (b^2*x^5)/(5*d) - (c^(1/2)*atan((c^(1/2)*d^(1/2)*x*(a*d - b*c)^2
)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d))*(a*d - b*c)^2)/d^(7/2)
```

3.171 $\int \frac{x\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=61

$$-\frac{b(bc-ad)x^2}{2d^2}+\frac{\left(a+bx^2\right)^2}{4d}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2d^3}$$

```
[Out] -1/2*b*(-a*d+b*c)*x^2/d^2+1/4*(b*x^2+a)^2/d+1/2*(-a*d+b*c)^2*ln(d*x^2+c)/d^
3
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2d^3}-\frac{bx^2(bc-ad)}{2d^2}+\frac{\left(a+bx^2\right)^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -1/2*(b*(b*c - a*d)*x^2)/d^2 + (a + b*x^2)^2/(4*d) + ((b*c - a*d)^2*Log[c +
 d*x^2])/(2*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx,x,x^2\right)\\ &= -\frac{b(bc-ad)x^2}{2d^2}+\frac{\left(a+bx^2\right)^2}{4d}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2d^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.80

$$\frac{bdx^2(-2bc+4ad+bdx^2)+2(bc-ad)^2\log(c+dx^2)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2),x]
[Out] (b*d*x^2*(-2*b*c + 4*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[c + d*x^2])/(4*d^
3)
```

Maple [A]
time = 0.10, size = 64, normalized size = 1.05

method	result	size
default	$\frac{b\left(\frac{1}{2}bd\,x^{4}+2ad\,x^{2}-c\,x^{2}b\right)}{2d^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(d\,x^{2}+c\right)}{2d^{3}}$	64
norman	$\frac{b^{2}x^{4}}{4d}+\frac{b(2ad-bc)x^{2}}{2d^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(d\,x^{2}+c\right)}{2d^{3}}$	65
risch	$\frac{b^{2}x^{4}}{4d}+\frac{ab\,x^{2}}{d}-\frac{b^{2}c\,x^{2}}{2d^{2}}+\frac{a^{2}}{d}-\frac{abc}{d^{2}}+\frac{b^{2}c^{2}}{4d^{3}}+\frac{\ln\left(d\,x^{2}+c\right)a^{2}}{2d}-\frac{\ln\left(d\,x^{2}+c\right)abc}{d^{2}}+\frac{\ln\left(d\,x^{2}+c\right)b^{2}c^{2}}{2d^{3}}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/2*b/d^2*(1/2*b*d*x^4+2*a*d*x^2-c*x^2*b)+1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d
^3*ln(d*x^2+c)
```

Maxima [A]
time = 0.28, size = 65, normalized size = 1.07

$$\frac{b^2dx^4-2\,(b^2c-2\,abd)x^2}{4\,d^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log(dx^2+c)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
[Out] 1/4*(b^2*d*x^4 - 2*(b^2*c - 2*a*b*d)*x^2)/d^2 + 1/2*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(d*x^2 + c)/d^3
```

Fricas [A]
time = 1.29, size = 66, normalized size = 1.08

$$\frac{b^2d^2x^4-2\,(b^2cd-2\,abd^2)x^2+2\,(b^2c^2-2\,abcd+a^2d^2)\log(dx^2+c)}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^4 - 2*(b^2*c*d - 2*a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(d*x^2 + c))/d^3
```

Sympy [A]
time = 0.19, size = 49, normalized size = 0.80

$$\frac{b^2x^4}{4d}+x^2\left(\frac{ab}{d}-\frac{b^2c}{2d^2}\right)+\frac{(ad-bc)^2\log\left(c+dx^2\right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**4/(4*d) + x**2*(a*b/d - b**2*c/(2*d**2)) + (a*d - b*c)**2*log(c + d
*x**2)/(2*d**3)
```

Giac [A]
time = 0.69, size = 67, normalized size = 1.10

$$\frac{b^2dx^4-2\,b^2cx^2+4\,abdx^2}{4\,d^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(|dx^2+c|\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d*x^4 - 2*b^2*c*x^2 + 4*a*b*d*x^2)/d^2 + 1/2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(d*x^2 + c))/d^3
```

Mupad [B]
time = 0.06, size = 68, normalized size = 1.11

$$\frac{b^2\,x^4}{4\,d}-x^2\,\left(\frac{b^2\,c}{2\,d^2}-\frac{a\,b}{d}\right)+\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] (b^2*x^4)/(4*d) - x^2*((b^2*c)/(2*d^2) - (a*b)/d) + (log(c + d*x^2)*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d))/(2*d^3)
```

3.172 $\int \frac{(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=63

$$-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+1/3*b^2*x^3/d+(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))/
d^(5/2)/c^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {398, 211}

$$\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2} + \frac{b^2x^2}{d} + \frac{b^2c^2-2abcd+a^2d^2}{d^2(c+dx^2)}\right) dx$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^3}{3d} + \frac{(bc-ad)^2 \int \frac{1}{c+dx^2}\,dx}{d^2}$$
$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^3}{3d} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}$$

<u>**Mathematica**</u> [A]
time = 0.04, size = 59, normalized size = 0.94

$$\frac{bx(-3bc+6ad+bdx^2)}{3d^2} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] (b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

<u>**Maple**</u> [A]
time = 0.07, size = 64, normalized size = 1.02

method	result
default	$\frac{b\left(\frac{1}{3}bd\,x^3+2adx-bcx\right)}{d^2} + \frac{(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^2\sqrt{cd}}$
risch	$\frac{b^2x^3}{3d} + \frac{2bax}{d} - \frac{b^2cx}{d^2} - \frac{\ln\left(dx+\sqrt{-cd}\right)a^2}{2\sqrt{-cd}} + \frac{\ln\left(dx+\sqrt{-cd}\right)abc}{d\sqrt{-cd}} - \frac{\ln\left(dx+\sqrt{-cd}\right)b^2c^2}{2d^2\sqrt{-cd}} + \frac{\ln\left(-dx+\sqrt{-cd}\right)}{2\sqrt{-cd}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^2*(1/3*b*d*x^3+2*a*d*x-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2/(c*d)^(1/
2)*arctan(d*x/(c*d)^(1/2))
```

<u>**Maxima**</u> [A]
time = 0.51, size = 68, normalized size = 1.08

$$\frac{(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2} + \frac{b^2dx^3 - 3\,(b^2c - 2\,abd)x}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x)/d^2
```

Fricas [A]
time = 1.46, size = 179, normalized size = 2.84

$$\left[\frac{2\,b^{2}cd^{2}x^{3} - 3\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\sqrt{-cd}\,\log\left(\frac{dx^{2} - 2\,\sqrt{-cd}\,x - c}{dx^{2} + c}\right) - 6\,(b^{2}c^{2}d - 2\,abcd^{2})x}{6\,cd^{3}}, \frac{b^{2}cd^{2}x^{3} + 3\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right) - 3\,(b^{2}c^{2}d - 2\,abcd^{2})x}{3\,cd^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^2*c*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*log((d
*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c
*d^3), 1/3*(b^2*c*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) - 3*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c*d^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.
time = 0.33, size = 172, normalized size = 2.73

$$\frac{b^{2}x^{3}}{3d} + x\left(\frac{2ab}{d} - \frac{b^{2}c}{d^{2}}\right) - \frac{\sqrt{-\frac{1}{cd^{5}}}\,(ad - bc)^{2}\log\left(-\frac{cd^{2}\sqrt{-\frac{1}{cd^{5}}}\,(ad-bc)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^{5}}}\,(ad - bc)^{2}\log\left(\frac{cd^{2}\sqrt{-\frac{1}{cd^{5}}}\,(ad-bc)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) + x*(2*a*b/d - b**2*c/d**2) - sqrt(-1/(c*d**5))*(a*d - b*c)
**2*log(-c*d**2*sqrt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b
**2*c**2) + x)/2 + sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(c*d
**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

Giac [A]
time = 0.80, size = 72, normalized size = 1.14

$$\frac{(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^{2}} + \frac{b^{2}d^{2}x^{3} - 3\,b^{2}cdx + 6\,abd^{2}x}{3\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

Mupad [B]
time = 0.04, size = 90, normalized size = 1.43

$$\frac{b^2\,x^3}{3\,d}-x\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2),x)
```

```
[Out] (b^2*x^3)/(3*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((d^(1/2)*x*(a*d - b*c
)^2)/(c^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(1/2)*d^(
5/2))
```

3.173 $\int \frac{(a+bx^2)^2}{x(c+dx^2)} dx$

Optimal. Leaf size=51

$$\frac{b^2x^2}{2d}+\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log(c+dx^2)}{2cd^2}$$

```
[Out] 1/2*b^2*x^2/d+a^2*ln(x)/c-1/2*(-a*d+b*c)^2*ln(d*x^2+c)/c/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log(c+dx^2)}{2cd^2}+\frac{b^2x^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

```
[Out] (b^2*x^2)/(2*d) + (a^2*Log[x])/c - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c*d^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x(c+dx^2)}dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)}dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2}{d}+\frac{a^2}{cx}-\frac{(bc-ad)^2}{cd(c+dx)}\right)dx, x, x^2\right)\\ &= \frac{b^2x^2}{2d}+\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log(c+dx^2)}{2cd^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.98

$$\frac{b^2cdx^2+2a^2d^2\log(x)-(bc-ad)^2\log(c+dx^2)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)),x]
[Out] (b^2*c*d*x^2 + 2*a^2*d^2*Log[x] - (b*c - a*d)^2*Log[c + d*x^2])/(2*c*d^2)
```

Maple [A]
time = 0.08, size = 59, normalized size = 1.16

method	result	size
default	$\frac{b^2x^2}{2d}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(d\,x^2+c)}{2c\,d^2}+\frac{a^2\ln(x)}{c}$	59
norman	$\frac{b^2x^2}{2d}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(d\,x^2+c)}{2c\,d^2}+\frac{a^2\ln(x)}{c}$	59
risch	$\frac{b^2x^2}{2d}-\frac{\ln(d\,x^2+c)a^2}{2c}+\frac{\ln(d\,x^2+c)ab}{d}-\frac{c\ln(d\,x^2+c)b^2}{2d^2}+\frac{a^2\ln(x)}{c}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/2*b^2*x^2/d-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^2*ln(d*x^2+c)+a^2*ln(x)/c
```

Maxima [A]
time = 0.28, size = 61, normalized size = 1.20

$$\frac{b^2x^2}{2\,d}+\frac{a^2\log(x^2)}{2\,c}-\frac{(b^2c^2-2\,abcd+a^2d^2)\log(dx^2+c)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="maxima")
[Out] 1/2*b^2*x^2/d + 1/2*a^2*log(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(d*x^2 + c)/(c*d^2)
```

Fricas [A]
time = 1.08, size = 59, normalized size = 1.16

$$\frac{b^2cdx^2+2\,a^2d^2\log(x)-(b^2c^2-2\,abcd+a^2d^2)\log(dx^2+c)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c*d*x^2 + 2*a^2*d^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(d
*x^2 + c))/(c*d^2)
```

Sympy [A]
time = 0.77, size = 41, normalized size = 0.80

$$\frac{a^2\log{(x)}}{c}+\frac{b^2x^2}{2d}-\frac{(ad-bc)^2\log{\left(\frac{c}{d}+x^2\right)}}{2cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c),x)
```

```
[Out] a**2*log(x)/c + b**2*x**2/(2*d) - (a*d - b*c)**2*log(c/d + x**2)/(2*c*d**2)
```

Giac [A]
time = 0.69, size = 62, normalized size = 1.22

$$\frac{b^2x^2}{2\,d}+\frac{a^2\log{(x^2)}}{2\,c}-\frac{(b^2c^2-2\,abcd+a^2d^2)\log{(|dx^2+c|)}}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2/d + 1/2*a^2*log(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(d*x^2 + c))/(c*d^2)
```

Mupad [B]
time = 0.09, size = 58, normalized size = 1.14

$$\frac{b^2\,x^2}{2\,d}+\frac{a^2\,\ln{(x)}}{c}-\frac{\ln{(d\,x^2+c)}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)),x)
```

```
[Out] (b^2*x^2)/(2*d) + (a^2*log(x))/c - (log(c + d*x^2)*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))/(2*c*d^2)
```

3.174 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)}\,dx$

Optimal. Leaf size=55

$$-\frac{a^2}{cx}+\frac{b^2x}{d}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}$$

```
[Out] -a^2/c/x+b^2*x/d-(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {472, 211}

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
3/2)*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)}\,dx = \int \left(\frac{b^2}{d}+\frac{a^2}{cx^2}-\frac{(bc-ad)^2}{cd\,(c+dx^2)}\right)dx$$
$$= -\frac{a^2}{cx}+\frac{b^2x}{d}-\frac{(bc-ad)^2\int \frac{1}{c+dx^2}\,dx}{cd}$$
$$= -\frac{a^2}{cx}+\frac{b^2x}{d}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 1.00

$$-\frac{a^2}{cx}+\frac{b^2x}{d}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
3/2)*d^(3/2))
```

Maple [A]
time = 0.10, size = 65, normalized size = 1.18

method	result
default	$\frac{b^2x}{d}+\frac{(-a^2d^2+2abcd-b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{cd\sqrt{cd}}-\frac{a^2}{cx}$
risch	$\frac{b^2x}{d}-\frac{a^2}{cx}-\frac{d\ln\left(-\sqrt{-cd}\,x-c\right)a^2}{2\sqrt{-cd}\,c}+\frac{\ln\left(-\sqrt{-cd}\,x-c\right)ab}{\sqrt{-cd}}-\frac{c\ln\left(-\sqrt{-cd}\,x-c\right)b^2}{2d\sqrt{-cd}}+\frac{d\ln\left(-\sqrt{-cd}\,x+c\right)a^2}{2\sqrt{-cd}\,c}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/d+1/c/d*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/
2))-a^2/c/x
```

Maxima [A]
time = 0.52, size = 63, normalized size = 1.15

$$\frac{b^2x}{d}-\frac{a^2}{cx}-\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b^2*x/d - a^2/(c*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*c*d)
```

Fricas [A]
time = 1.42, size = 164, normalized size = 2.98

$$\left[\frac{2\,b^{2}c^{2}dx^{2}-2\,a^{2}cd^{2}-(b^{2}c^{2}-2\,abcd+a^{2}d^{2})\sqrt{-cd}\,x\log\left(\frac{dx^{2}+2\,\sqrt{-cd}\,x-c}{dx^{2}+c}\right)}{2\,c^{2}d^{2}x},\frac{b^{2}c^{2}dx^{2}-a^{2}cd^{2}-(b^{2}c^{2}-2\,abcd+a^{2}d^{2})\sqrt{cd}\,x\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{c^{2}d^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b^2*c^2*d*x^2 - 2*a^2*c*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(
-c*d)*x*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^2*d^2*x), (b^2*c^
2*d*x^2 - a^2*c*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*x*arctan(sq
rt(c*d)*x/c))/(c^2*d^2*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(44) = 88$.
time = 0.40, size = 165, normalized size = 3.00

$$-\frac{a^{2}}{cx}+\frac{b^{2}x}{d}+\frac{\sqrt{-\frac{1}{c^{3}d^{3}}}\,(ad-bc)^{2}\log\left(-\frac{c^{2}d\sqrt{-\frac{1}{c^{3}d^{3}}}\,(ad-bc)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{c^{3}d^{3}}}\,(ad-bc)^{2}\log\left(\frac{c^{2}d\sqrt{-\frac{1}{c^{3}d^{3}}}\,(ad-bc)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c),x)
```

```
[Out] -a**2/(c*x) + b**2*x/d + sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(-c**2*d*sq
rt(-1/(c**3*d**3))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/
2 - sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(c**2*d*sqrt(-1/(c**3*d**3))*(a*
d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

Giac [A]
time = 0.58, size = 63, normalized size = 1.15

$$\frac{b^{2}x}{d}-\frac{a^{2}}{cx}-\frac{(b^{2}c^{2}-2\,abcd+a^{2}d^{2})\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b^2*x/d - a^2/(c*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*c*d)
```

Mupad [B]
time = 0.07, size = 80, normalized size = 1.45

$$\frac{b^2\,x}{d}-\frac{a^2}{c\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{c^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)),x)
```

```
[Out] (b^2*x)/d - a^2/(c*x) - (atan((d^(1/2)*x*(a*d - b*c)^2)/(c^(1/2)*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(3/2)*d^(3/2))
```

3.175 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)} \, dx$

Optimal. Leaf size=58

$$-\frac{a^2}{2cx^2}+\frac{a(2bc-ad)\log(x)}{c^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}$$

```
[Out] -1/2*a^2/c/x^2+a*(-a*d+2*b*c)*ln(x)/c^2+1/2*(-a*d+b*c)^2*ln(d*x^2+c)/c^2/d
```

Rubi [A]

time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{a^2}{2cx^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}+\frac{a\log(x)(2bc-ad)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

```
[Out] -1/2*a^2/(c*x^2) + (a*(2*b*c - a*d)*Log[x])/c^2 + ((b*c - a*d)^2*Log[c + d*
x^2])/(2*c^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^3\,(c+dx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{cx^2}-\frac{a(-2bc+ad)}{c^2x}+\frac{(bc-ad)^2}{c^2(c+dx)}\right)\,dx,x,x^2\right)\\ &= -\frac{a^2}{2cx^2}+\frac{a(2bc-ad)\log(x)}{c^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 1.03

$$\frac{-a^2cd-2ad(-2bc+ad)x^2\log(x)+(bc-ad)^2x^2\log\left(c+dx^2\right)}{2c^2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

```
[Out] (-(a^2*c*d) - 2*a*d*(-2*b*c + a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2*Log[c + d
*x^2])/(2*c^2*d*x^2)
```

Maple [A]
time = 0.09, size = 66, normalized size = 1.14

method	result	size
default	$\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(dx^2+c\right)}{2c^2d}-\frac{a^2}{2cx^2}-\frac{a(ad-2bc)\ln(x)}{c^2}$	66
norman	$\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(dx^2+c\right)}{2c^2d}-\frac{a^2}{2cx^2}-\frac{a(ad-2bc)\ln(x)}{c^2}$	66
risch	$-\frac{a^2}{2cx^2}-\frac{a^2\ln(x)d}{c^2}+\frac{2a\ln(x)b}{c}+\frac{d\ln(-dx^2-c)a^2}{2c^2}-\frac{\ln(-dx^2-c)ab}{c}+\frac{\ln(-dx^2-c)b^2}{2d}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^2/d*ln(d*x^2+c)-1/2*a^2/c/x^2-a*(a*d-2*b*
c)/c^2*ln(x)
```

Maxima [A]
time = 0.31, size = 70, normalized size = 1.21

$$\frac{(2\,abc-a^2d)\log\left(x^2\right)}{2\,c^2}-\frac{a^2}{2\,cx^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(dx^2+c\right)}{2\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*c - a^2*d)*log(x^2)/c^2 - 1/2*a^2/(c*x^2) + 1/2*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*log(d*x^2 + c)/(c^2*d)
```

Fricas [A]
time = 1.05, size = 74, normalized size = 1.28

$$-\frac{a^2cd-(b^2c^2-2\,abcd+a^2d^2)x^2\log\left(dx^2+c\right)-2\,(2\,abcd-a^2d^2)x^2\log\left(x\right)}{2\,c^2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*d - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(d*x^2 + c) - 2*(2*a
*b*c*d - a^2*d^2)*x^2*log(x))/(c^2*d*x^2)
```

Sympy [A]
time = 0.84, size = 49, normalized size = 0.84

$$-\frac{a^2}{2cx^2} - \frac{a(ad-2bc)\log(x)}{c^2} + \frac{(ad-bc)^2\log\left(\frac{c}{d}+x^2\right)}{2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c),x)
```

```
[Out] -a**2/(2*c*x**2) - a*(a*d - 2*b*c)*log(x)/c**2 + (a*d - b*c)**2*log(c/d + x
**2)/(2*c**2*d)
```

Giac [A]
time = 0.63, size = 91, normalized size = 1.57

$$\frac{(2\,abc-a^2d)\log(x^2)}{2\,c^2} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log(|dx^2+c|)}{2\,c^2d} - \frac{2\,abcx^2-a^2dx^2+a^2c}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*c - a^2*d)*log(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(d*x^2 + c))/(c^2*d) - 1/2*(2*a*b*c*x^2 - a^2*d*x^2 + a^2*c)/(c^2*x^2)
```

Mupad [B]
time = 0.10, size = 67, normalized size = 1.16

$$\frac{\ln(d\,x^2+c)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^2\,d} - \frac{a^2}{2\,c\,x^2} - \frac{\ln(x)\,\left(a^2\,d-2\,a\,b\,c\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)),x)
```

```
[Out] (log(c + d*x^2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c^2*d) - a^2/(2*c*x^2)
- (log(x)*(a^2*d - 2*a*b*c))/c^2
```

3.176 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{3cx^3}-\frac{a(2bc-ad)}{c^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}$$

```
[Out] -1/3*a^2/c/x^3-a*(-a*d+2*b*c)/c^2/x+(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))/
c^(5/2)/d^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {472, 211}

$$-\frac{a^2}{3cx^3}+\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}-\frac{a(2bc-ad)}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

```
[Out] -1/3*a^2/(c*x^3) - (a*(2*b*c - a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}\,dx = \int \left(\frac{a^2}{cx^4} - \frac{a(-2bc+ad)}{c^2x^2} + \frac{(bc-ad)^2}{c^2(c+dx^2)}\right)dx$$
$$= -\frac{a^2}{3cx^3} - \frac{a(2bc-ad)}{c^2x} + \frac{(bc-ad)^2\int \frac{1}{c+dx^2}\,dx}{c^2}$$
$$= -\frac{a^2}{3cx^3} - \frac{a(2bc-ad)}{c^2x} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.97

$$-\frac{a^2}{3cx^3} + \frac{a(-2bc+ad)}{c^2x} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
[Out] -1/3*a^2/(c*x^3) + (a*(-2*b*c + a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt
[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

Maple [A]
time = 0.09, size = 68, normalized size = 1.03

method	result
default	$\frac{(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^2\sqrt{cd}} - \frac{a^2}{3c\,x^3} + \frac{a(ad-2bc)}{c^2x}$
risch	$\frac{\frac{a(ad-2bc)x^2}{c^2}-\frac{a^2}{3c}}{x^3} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^5d\_Z^2+a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)} \_R\ln\left(\left(3\_R^2c^5d+2a^4d^4-8a^3bc\,d^3+12a\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] (a^2*d^2-2*a*b*c*d+b^2*c^2)/c^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))-1/3*a^2
/c/x^3+a*(a*d-2*b*c)/c^2/x
```

Maxima [A]
time = 0.68, size = 71, normalized size = 1.08

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^2} - \frac{a^2c+3\,(2\,abc-a^2d)x^2}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2) - 1/3
*(a^2*c + 3*(2*a*b*c - a^2*d)*x^2)/(c^2*x^3)
```

Fricas [A]
time = 1.06, size = 192, normalized size = 2.91

$$\left[-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-cd}\,x^3\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\,a^2c^2d+6\,(2\,abc^2d-a^2cd^2)x^2}{6\,c^3dx^3},\ \frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}\,x^3\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-a^2c^2d-3\,(2\,abc^2d-a^2cd^2)x^2}{3\,c^3dx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*x^3*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*a^2*c^2*d + 6*(2*a*b*c^2*d - a^2*c*d^2)*x^2)
/(c^3*d*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*x^3*arctan(s
qrt(c*d)*x/c) - a^2*c^2*d - 3*(2*a*b*c^2*d - a^2*c*d^2)*x^2)/(c^3*d*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.
time = 0.39, size = 172, normalized size = 2.61

$$-\frac{\sqrt{-\frac{1}{c^5d}}\,(ad-bc)^2\log\left(-\frac{c^3\sqrt{-\frac{1}{c^5d}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{c^5d}}\,(ad-bc)^2\log\left(\frac{c^3\sqrt{-\frac{1}{c^5d}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{-a^2c+x^2\cdot(3a^2d-6abc)}{3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c),x)
```

```
[Out] -sqrt(-1/(c**5*d))*(a*d - b*c)**2*log(-c**3*sqrt(-1/(c**5*d))*(a*d - b*c)**
2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(c**5*d))*(a*d - b*c
)**2*log(c**3*sqrt(-1/(c**5*d))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**
2*c**2) + x)/2 + (-a**2*c + x**2*(3*a**2*d - 6*a*b*c))/(3*c**2*x**3)
```

Giac [A]
time = 0.67, size = 71, normalized size = 1.08

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^2}-\frac{6\,abcx^2-3\,a^2dx^2+a^2c}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2) - 1/3
*(6*a*b*c*x^2 - 3*a^2*d*x^2 + a^2*c)/(c^2*x^3)
```

Mupad [B]
time = 0.09, size = 90, normalized size = 1.36

$$\frac{a^2\,d}{c^2\,x}-\frac{a^2}{3\,c\,x^3}+\frac{a^2\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}}+\frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{d}}-\frac{2\,a\,b}{c\,x}-\frac{2\,a\,b\,\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)),x)
```

```
[Out] (a^2*d)/(c^2*x) - a^2/(3*c*x^3) + (a^2*d^(3/2)*atan((d^(1/2)*x)/c^(1/2)))/c
^(5/2) + (b^2*atan((d^(1/2)*x)/c^(1/2)))/(c^(1/2)*d^(1/2)) - (2*a*b)/(c*x)
- (2*a*b*d^(1/2)*atan((d^(1/2)*x)/c^(1/2)))/c^(3/2)
```

3.177 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)} \, dx$

Optimal. Leaf size=75

$$-\frac{a^2}{4cx^4}-\frac{a(2bc-ad)}{2c^2x^2}+\frac{(bc-ad)^2\log(x)}{c^3}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^3}$$

```
[Out] -1/4*a^2/c/x^4-1/2*a*(-a*d+2*b*c)/c^2/x^2+(-a*d+b*c)^2*ln(x)/c^3-1/2*(-a*d+
b*c)^2*ln(d*x^2+c)/c^3
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{a^2}{4cx^4}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^3}+\frac{\log(x)(bc-ad)^2}{c^3}-\frac{a(2bc-ad)}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

```
[Out] -1/4*a^2/(c*x^4) - (a*(2*b*c - a*d))/(2*c^2*x^2) + ((b*c - a*d)^2*Log[x])/c
^3 - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^3(c+dx)} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{cx^3}-\frac{a(-2bc+ad)}{c^2x^2}+\frac{(bc-ad)^2}{c^3x}-\frac{d(bc-ad)^2}{c^3(c+dx)}\right) \, dx, x, x^2\right)\\
&= -\frac{a^2}{4cx^4}-\frac{a(2bc-ad)}{2c^2x^2}+\frac{(bc-ad)^2\log(x)}{c^3}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 0.96

$$-\frac{ac(ac+4bcx^2-2adx^2)-4(bc-ad)^2x^4\log(x)+2(bc-ad)^2x^4\log\left(c+dx^2\right)}{4c^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

```
[Out] -1/4*(a*c*(a*c + 4*b*c*x^2 - 2*a*d*x^2) - 4*(b*c - a*d)^2*x^4*Log[x] + 2*(b
*c - a*d)^2*x^4*Log[c + d*x^2])/(c^3*x^4)
```

Maple [A]
time = 0.08, size = 91, normalized size = 1.21

method	result	size
default	$-\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(d\,x^2+c\right)}{2c^3}-\frac{a^2}{4x^4c}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x\right)}{c^3}+\frac{a(ad-2bc)}{2c^2x^2}$	91
norman	$\frac{-\frac{a^2}{4c}+\frac{a(ad-2bc)x^2}{2c^2}}{x^4}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x\right)}{c^3}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(d\,x^2+c\right)}{2c^3}$	93
risch	$\frac{-\frac{a^2}{4c}+\frac{a(ad-2bc)x^2}{2c^2}}{x^4}+\frac{\ln\left(x\right)a^2d^2}{c^3}-\frac{2\ln\left(x\right)abd}{c^2}+\frac{\ln\left(x\right)b^2}{c}-\frac{\ln\left(d\,x^2+c\right)a^2d^2}{2c^3}+\frac{\ln\left(d\,x^2+c\right)abd}{c^2}-\frac{\ln\left(d\,x^2+c\right)b^2}{2c}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3*ln(d*x^2+c)-1/4*a^2/x^4/c+(a^2*d^2-2*a
*b*c*d+b^2*c^2)/c^3*ln(x)+1/2*a*(a*d-2*b*c)/c^2/x^2
```

Maxima [A]
time = 0.31, size = 96, normalized size = 1.28

$$-\frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(dx^2+c\right)}{2\,c^3}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2\right)}{2\,c^3}-\frac{a^2c+2\,(2\,abc-a^2d)x^2}{4\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(d*x^2 + c)/c^3 + 1/2*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*log(x^2)/c^3 - 1/4*(a^2*c + 2*(2*a*b*c - a^2*d)*x^2)/(c^
2*x^4)
```

Fricas [A]
time = 1.20, size = 98, normalized size = 1.31

$$-\frac{2\,(b^2c^2-2\,abcd+a^2d^2)x^4\log\left(dx^2+c\right)-4\,(b^2c^2-2\,abcd+a^2d^2)x^4\log\left(x\right)+a^2c^2+2\,(2\,abc^2-a^2cd)x^2}{4\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4*log(d*x^2 + c) - 4*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x^4*log(x) + a^2*c^2 + 2*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^
3*x^4)
```

Sympy [**A**]
time = 0.83, size = 66, normalized size = 0.88

$$\frac{-a^2c + x^2 \cdot (2a^2d - 4abc)}{4c^2x^4} + \frac{(ad - bc)^2 \log{(x)}}{c^3} - \frac{(ad - bc)^2 \log{\left(\frac{c}{d} + x^2\right)}}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c),x)
```

```
[Out] (-a**2*c + x**2*(2*a**2*d - 4*a*b*c))/(4*c**2*x**4) + (a*d - b*c)**2*log(x)
/c**3 - (a*d - b*c)**2*log(c/d + x**2)/(2*c**3)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(69) = 138.
time = 0.61, size = 139, normalized size = 1.85

$$\frac{(b^2c^2 - 2\,abcd + a^2d^2)\log{(x^2)}}{2\,c^3} - \frac{(b^2c^2d - 2\,abcd^2 + a^2d^3)\log{(|dx^2 + c|)}}{2\,c^3d} - \frac{3\,b^2c^2x^4 - 6\,abcdx^4 + 3\,a^2d^2x^4 + 4\,abc^2x^2 - 2\,a^2cdx^2 + a^2c^2}{4\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2)/c^3 - 1/2*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*log(abs(d*x^2 + c))/(c^3*d) - 1/4*(3*b^2*c^2*x^4 - 6*a*b*c*
d*x^4 + 3*a^2*d^2*x^4 + 4*a*b*c^2*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(c^3*x^4)
```

Mupad [**B**]
time = 0.09, size = 93, normalized size = 1.24

$$\frac{\ln{(x)}\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{c^3} - \frac{\frac{a^2}{4\,c} - \frac{a\,x^2\,(a\,d - 2\,b\,c)}{2\,c^2}}{x^4} - \frac{\ln{\left(d\,x^2 + c\right)}\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^5*(c + d*x^2)),x)
```

```
[Out] (log(x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/c^3 - (a^2/(4*c) - (a*x^2*(a*d - 2
*b*c))/(2*c^2))/x^4 - (log(c + d*x^2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c
^3)
```

3.178 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx$

Optimal. Leaf size=87

$$-\frac{a^2}{5cx^5}-\frac{a(2bc-ad)}{3c^2x^3}-\frac{(bc-ad)^2}{c^3x}-\frac{\sqrt{d}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}$$

```
[Out] -1/5*a^2/c/x^5-1/3*a*(-a*d+2*b*c)/c^2/x^3-(-a*d+b*c)^2/c^3/x-(-a*d+b*c)^2*a
rctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(7/2)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {472, 211}

$$-\frac{a^2}{5cx^5}-\frac{\sqrt{d}\,(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}-\frac{(bc-ad)^2}{c^3x}-\frac{a(2bc-ad)}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

```
[Out] -1/5*a^2/(c*x^5) - (a*(2*b*c - a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c^3*x) -
(Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx = \int \left(\frac{a^2}{cx^6} - \frac{a(-2bc+ad)}{c^2x^4} + \frac{(bc-ad)^2}{c^3x^2} - \frac{d(bc-ad)^2}{c^3(c+dx^2)}\right)dx$$
$$= -\frac{a^2}{5cx^5} - \frac{a(2bc-ad)}{3c^2x^3} - \frac{(bc-ad)^2}{c^3x} - \frac{(d(bc-ad)^2)\int \frac{1}{c+dx^2}\,dx}{c^3}$$
$$= -\frac{a^2}{5cx^5} - \frac{a(2bc-ad)}{3c^2x^3} - \frac{(bc-ad)^2}{c^3x} - \frac{\sqrt{d}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}$$

Mathematica [A]
time = 0.05, size = 86, normalized size = 0.99

$$-\frac{a^2}{5cx^5} + \frac{a(-2bc+ad)}{3c^2x^3} - \frac{(bc-ad)^2}{c^3x} - \frac{\sqrt{d}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

```
[Out] -1/5*a^2/(c*x^5) + (a*(-2*b*c + a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c^3*x) -
 (Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/2)
```

Maple [A]
time = 0.09, size = 100, normalized size = 1.15

method	result
default	$-\frac{d(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^3\sqrt{cd}} - \frac{a^2}{5cx^5} - \frac{a^2d^2-2abcd+b^2c^2}{c^3x} + \frac{a(ad-2bc)}{3c^2x^3}$
risch	$\frac{-\frac{(a^2d^2-2abcd+b^2c^2)x^4}{c^3} + \frac{a(ad-2bc)x^2}{3c^2} - \frac{a^2}{5c}}{x^5} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^7\_Z^2+a^4d^5-4a^3bc\,d^4+6a^2b^2c^2d^3-4a\,b^3c^3d^2+b^4c^4d\right)} \_R\ln\left(\left(3\_R^2c^7\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))-1/5*
a^2/c/x^5-(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3/x+1/3*a*(a*d-2*b*c)/c^2/x^3
```

Maxima [A]
time = 0.52, size = 107, normalized size = 1.23

$$-\frac{(b^2c^2d - 2\,abcd^2 + a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^3} - \frac{15\,(b^2c^2 - 2\,abcd + a^2d^2)x^4 + 3\,a^2c^2 + 5\,(2\,abc^2 - a^2cd)x^2}{15\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
- 1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 + 3*a^2*c^2 + 5*(2*a*b*c^2 -
 a^2*c*d)*x^2)/(c^3*x^5)
```

Fricas [A]
time = 1.33, size = 236, normalized size = 2.71

$$\left[\frac{15\,(b^2c^2-2\,abcd+a^2d^2)x^5\sqrt{-\frac{d}{c}}\,\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-30\,(b^2c^2-2\,abcd+a^2d^2)x^4-6\,a^2c^2-10\,(2\,abc^2-a^2cd)x^2}{30\,c^3x^5}, -\frac{15\,(b^2c^2-2\,abcd+a^2d^2)x^5\sqrt{\frac{d}{c}}\,\arctan\left(x\sqrt{\frac{d}{c}}\right)+15\,(b^2c^2-2\,abcd+a^2d^2)x^4+3\,a^2c^2+5\,(2\,abc^2-a^2cd)x^2}{15\,c^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x
*sqrt(-d/c) - c)/(d*x^2 + c)) - 30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 - 6*
a^2*c^2 - 10*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^5), -1/15*(15*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*x^5*sqrt(d/c)*arctan(x*sqrt(d/c)) + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x^4 + 3*a^2*c^2 + 5*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(75) = 150$.
time = 0.49, size = 207, normalized size = 2.38

$$\frac{\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2\log\left(-\frac{c^4\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2}{a^2d^3-2abcd^2+b^2c^2d}+x\right)}{2}-\frac{\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2\log\left(\frac{c^4\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2}{a^2d^3-2abcd^2+b^2c^2d}+x\right)}{2}+\frac{-3a^2c^2+x^4(-15a^2d^2+30abcd-15b^2c^2)+x^2\cdot(5a^2cd-10abc^2)}{15c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c),x)
```

```
[Out] sqrt(-d/c**7)*(a*d - b*c)**2*log(-c**4*sqrt(-d/c**7)*(a*d - b*c)**2/(a**2*d
**3 - 2*a*b*c*d**2 + b**2*c**2*d) + x)/2 - sqrt(-d/c**7)*(a*d - b*c)**2*log
(c**4*sqrt(-d/c**7)*(a*d - b*c)**2/(a**2*d**3 - 2*a*b*c*d**2 + b**2*c**2*d)
 + x)/2 + (-3*a**2*c**2 + x**4*(-15*a**2*d**2 + 30*a*b*c*d - 15*b**2*c**2)
+ x**2*(5*a**2*c*d - 10*a*b*c**2))/(15*c**3*x**5)
```

Giac [A]
time = 0.55, size = 112, normalized size = 1.29

$$-\frac{(b^2c^2d-2\,abcd^2+a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^3}-\frac{15\,b^2c^2x^4-30\,abcdx^4+15\,a^2d^2x^4+10\,abc^2x^2-5\,a^2cdx^2+3\,a^2c^2}{15\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
- 1/15*(15*b^2*c^2*x^4 - 30*a*b*c*d*x^4 + 15*a^2*d^2*x^4 + 10*a*b*c^2*x^2 -
 5*a^2*c*d*x^2 + 3*a^2*c^2)/(c^3*x^5)
```

Mupad [B]
time = 0.05, size = 129, normalized size = 1.48

$$\frac{a^2\,d}{3\,c^2\,x^3}-\frac{b^2}{c\,x}-\frac{a^2}{5\,c\,x^5}-\frac{a^2\,d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}-\frac{b^2\,\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a^2\,d^2}{c^3\,x}-\frac{2\,a\,b}{3\,c\,x^3}+\frac{2\,a\,b\,d}{c^2\,x}+\frac{2\,a\,b\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^6*(c + d*x^2)),x)
```

```
[Out] (a^2*d)/(3*c^2*x^3) - b^2/(c*x) - a^2/(5*c*x^5) - (a^2*d^(5/2)*atan((d^(1/2
)*x)/c^(1/2)))/c^(7/2) - (b^2*d^(1/2)*atan((d^(1/2)*x)/c^(1/2)))/c^(3/2) -
(a^2*d^2)/(c^3*x) - (2*a*b)/(3*c*x^3) + (2*a*b*d)/(c^2*x) + (2*a*b*d^(3/2)*
atan((d^(1/2)*x)/c^(1/2)))/c^(5/2)
```

3.179 $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)} dx$

Optimal. Leaf size=98

$$-\frac{a^2}{6cx^6}-\frac{a(2bc-ad)}{4c^2x^4}-\frac{(bc-ad)^2}{2c^3x^2}-\frac{d(bc-ad)^2\log(x)}{c^4}+\frac{d(bc-ad)^2\log(c+dx^2)}{2c^4}$$

```
[Out] -1/6*a^2/c/x^6-1/4*a*(-a*d+2*b*c)/c^2/x^4-1/2*(-a*d+b*c)^2/c^3/x^2-d*(-a*d+b*c)^2*ln(x)/c^4+1/2*d*(-a*d+b*c)^2*ln(d*x^2+c)/c^4
```

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{a^2}{6cx^6}+\frac{d(bc-ad)^2\log(c+dx^2)}{2c^4}-\frac{d\log(x)(bc-ad)^2}{c^4}-\frac{(bc-ad)^2}{2c^3x^2}-\frac{a(2bc-ad)}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

```
[Out] -1/6*a^2/(c*x^6) - (a*(2*b*c - a*d))/(4*c^2*x^4) - (b*c - a*d)^2/(2*c^3*x^2) - (d*(b*c - a*d)^2*Log[x])/c^4 + (d*(b*c - a*d)^2*Log[c + d*x^2])/(2*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^7(c+dx^2)}dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^4(c+dx)}dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{cx^4}-\frac{a(-2bc+ad)}{c^2x^3}+\frac{(bc-ad)^2}{c^3x^2}-\frac{d(bc-ad)^2}{c^4x}+\frac{d^2(bc-ad)^2}{c^4(c+dx)}\right)dx, x\right.\\
&= -\frac{a^2}{6cx^6}-\frac{a(2bc-ad)}{4c^2x^4}-\frac{(bc-ad)^2}{2c^3x^2}-\frac{d(bc-ad)^2\log(x)}{c^4}+\frac{d(bc-ad)^2\log(c+dx^2)}{2c^4}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 108, normalized size = 1.10

$$-\frac{c(6b^2c^2x^4+6abcx^2(c-2dx^2)+a^2(2c^2-3cdx^2+6d^2x^4))+12d(bc-ad)^2x^6\log(x)-6d(bc-ad)^2x^6\log(c+dx^2)}{12c^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

```
[Out] -1/12*(c*(6*b^2*c^2*x^4 + 6*a*b*c*x^2*(c - 2*d*x^2) + a^2*(2*c^2 - 3*c*d*x^
2 + 6*d^2*x^4)) + 12*d*(b*c - a*d)^2*x^6*Log[x] - 6*d*(b*c - a*d)^2*x^6*Log
[c + d*x^2])/(c^4*x^6)
```

Maple [A]
time = 0.09, size = 123, normalized size = 1.26

method	result
default	$\frac{d(a^2d^2-2abcd+b^2c^2)\ln(dx^2+c)}{2c^4}-\frac{a^2}{6cx^6}-\frac{a^2d^2-2abcd+b^2c^2}{2c^3x^2}+\frac{a(ad-2bc)}{4c^2x^4}-\frac{d(a^2d^2-2abcd+b^2c^2)\ln(x)}{c^4}$
norman	$\frac{-\frac{a^2}{6c}-\frac{(a^2d^2-2abcd+b^2c^2)x^4}{2c^3}+\frac{a(ad-2bc)x^2}{4c^2}}{x^6}-\frac{d(a^2d^2-2abcd+b^2c^2)\ln(x)}{c^4}+\frac{d(a^2d^2-2abcd+b^2c^2)\ln(dx^2+c)}{2c^4}$
risch	$\frac{-\frac{a^2}{6c}-\frac{(a^2d^2-2abcd+b^2c^2)x^4}{2c^3}+\frac{a(ad-2bc)x^2}{4c^2}}{x^6}-\frac{d^3\ln(x)a^2}{c^4}+\frac{2d^2\ln(x)ab}{c^3}-\frac{d\ln(x)b^2}{c^2}+\frac{d^3\ln(-dx^2-c)a^2}{2c^4}-\frac{d^2\ln(-dx^2-c)ab}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^4*ln(d*x^2+c)-1/6*a^2/c/x^6-1/2*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/c^3/x^2+1/4*a*(a*d-2*b*c)/c^2/x^4-d*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/c^4*ln(x)
```

Maxima [A]
time = 0.29, size = 134, normalized size = 1.37

$$\frac{(b^2c^2d-2\,abcd^2+a^2d^3)\log(dx^2+c)}{2\,c^4}-\frac{(b^2c^2d-2\,abcd^2+a^2d^3)\log(x^2)}{2\,c^4}-\frac{6\,(b^2c^2-2\,abcd+a^2d^2)x^4+2\,a^2c^2+3\,(2\,abc^2-a^2cd)x^2}{12\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(d*x^2 + c)/c^4 - 1/2*(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*log(x^2)/c^4 - 1/12*(6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*x^4 + 2*a^2*c^2 + 3*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^6)
```

Fricas [A]
time = 1.06, size = 136, normalized size = 1.39

$$\frac{6\,(b^2c^2d-2\,abcd^2+a^2d^3)x^6\log(dx^2+c)-12\,(b^2c^2d-2\,abcd^2+a^2d^3)x^6\log(x)-2\,a^2c^3-6\,(b^2c^3-2\,abc^2d+a^2cd^2)x^4-3\,(2\,abc^3-a^2c^2d)x^2}{12\,c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(d*x^2 + c) - 12*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(x) - 2*a^2*c^3 - 6*(b^2*c^3 - 2*a*b*c^
2*d + a^2*c*d^2)*x^4 - 3*(2*a*b*c^3 - a^2*c^2*d)*x^2)/(c^4*x^6)
```

Sympy [A]
time = 0.99, size = 105, normalized size = 1.07

$$\frac{-2a^2c^2 + x^4(-6a^2d^2 + 12abcd - 6b^2c^2) + x^2 \cdot (3a^2cd - 6abc^2)}{12c^3x^6} - \frac{d(ad - bc)^2 \log{(x)}}{c^4} + \frac{d(ad - bc)^2 \log{\left(\frac{c}{d} + x^2\right)}}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c),x)
```

```
[Out] (-2*a**2*c**2 + x**4*(-6*a**2*d**2 + 12*a*b*c*d - 6*b**2*c**2) + x**2*(3*a*
*2*c*d - 6*a*b*c**2))/(12*c**3*x**6) - d*(a*d - b*c)**2*log(x)/c**4 + d*(a*
d - b*c)**2*log(c/d + x**2)/(2*c**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.
time = 0.51, size = 184, normalized size = 1.88

$$-\frac{(b^2c^2d - 2\,abcd^2 + a^2d^3)\log{(x^2)}}{2\,c^4} + \frac{(b^2c^2d^2 - 2\,abcd^3 + a^2d^4)\log{(|dx^2 + c|)}}{2\,c^4d} + \frac{11\,b^2c^2dx^6 - 22\,abcd^2x^6 + 11\,a^2d^3x^6 - 6\,b^2c^3x^4 + 12\,abc^2dx^4 - 6\,a^2cd^2x^4 - 6\,abc^3x^2 + 3\,a^2c^2dx^2 - 2\,a^2c^3}{12\,c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(x^2)/c^4 + 1/2*(b^2*c^2*d^2 -
2*a*b*c*d^3 + a^2*d^4)*log(abs(d*x^2 + c))/(c^4*d) + 1/12*(11*b^2*c^2*d*x^6
 - 22*a*b*c*d^2*x^6 + 11*a^2*d^3*x^6 - 6*b^2*c^3*x^4 + 12*a*b*c^2*d*x^4 - 6
*a^2*c*d^2*x^4 - 6*a*b*c^3*x^2 + 3*a^2*c^2*d*x^2 - 2*a^2*c^3)/(c^4*x^6)
```

Mupad [B]
time = 0.08, size = 129, normalized size = 1.32

$$\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{2\,c^4}-\frac{\frac{a^2}{6\,c}+\frac{x^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^3}-\frac{a\,x^2\,\left(a\,d-2\,b\,c\right)}{4\,c^2}}{x^6}-\frac{\ln\left(x\right)\,\left(a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^7*(c + d*x^2)),x)
```

```
[Out] (log(c + d*x^2)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/(2*c^4) - (a^2/(6*c) +
 (x^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c^3) - (a*x^2*(a*d - 2*b*c))/(4*c
^2))/x^6 - (log(x)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/c^4
```

3.180 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=145

$$\frac{(7bc-3ad)(bc-ad)x}{2d^4}-\frac{(7bc-3ad)(bc-ad)x^3}{6cd^3}+\frac{b^2x^5}{5d^2}+\frac{(bc-ad)^2x^5}{2cd^2(c+dx^2)}-\frac{\sqrt{c}\,(7bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{2d^{9/2}}$$

```
[Out] 1/2*(-3*a*d+7*b*c)*(-a*d+b*c)*x/d^4-1/6*(-3*a*d+7*b*c)*(-a*d+b*c)*x^3/c/d^3
+1/5*b^2*x^5/d^2+1/2*(-a*d+b*c)^2*x^5/c/d^2/(d*x^2+c)-1/2*(-3*a*d+7*b*c)*(-
a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))*c^(1/2)/d^(9/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {474, 470, 308, 211}

$$-\frac{\sqrt{c}\,(7bc-3ad)(bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{9/2}}+\frac{x(7bc-3ad)(bc-ad)}{2d^4}-\frac{x^3(7bc-3ad)(bc-ad)}{6cd^3}+\frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x^5}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((7*b*c - 3*a*d)*(b*c - a*d)*x)/(2*d^4) - ((7*b*c - 3*a*d)*(b*c - a*d)*x^3)
/(6*c*d^3) + (b^2*x^5)/(5*d^2) + ((b*c - a*d)^2*x^5)/(2*c*d^2*(c + d*x^2))
- (Sqrt[c]*(7*b*c - 3*a*d)*(b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9
/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(bc-ad)^2x^5}{2cd^2(c+dx^2)} - \frac{\int \frac{x^4(-2a^2d^2+5(bc-ad)^2-2b^2cdx^2)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2(c+dx^2)} - \frac{((7bc-3ad)(bc-ad))\int \frac{x^4}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2(c+dx^2)} - \frac{((7bc-3ad)(bc-ad))\int \left(-\frac{c}{d^2}+\frac{x^2}{d}+\frac{c^2}{d^2(c+dx^2)}\right)dx}{2cd^2}$$

$$= \frac{(7bc-3ad)(bc-ad)x}{2d^4} - \frac{(7bc-3ad)(bc-ad)x^3}{6cd^3} + \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2(c+dx^2)} - \frac{(c(7bc-$$

$$= \frac{(7bc-3ad)(bc-ad)x}{2d^4} - \frac{(7bc-3ad)(bc-ad)x^3}{6cd^3} + \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2(c+dx^2)} - \frac{\sqrt{c}\,(7bc}{}$$

Mathematica [A]
time = 0.07, size = 138, normalized size = 0.95

$$\frac{(3b^2c^2-4abcd+a^2d^2)\,x}{d^4} - \frac{2b(bc-ad)x^3}{3d^3} + \frac{b^2x^5}{5d^2} + \frac{c(bc-ad)^2x}{2d^4(c+dx^2)} - \frac{\sqrt{c}\,(7b^2c^2-10abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x)/d^4 - (2*b*(b*c - a*d)*x^3)/(3*d^3) +
 (b^2*x^5)/(5*d^2) + (c*(b*c - a*d)^2*x)/(2*d^4*(c + d*x^2)) - (Sqrt[c]*(7*
b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9/2))
```

Maple [A]
time = 0.11, size = 141, normalized size = 0.97

method	result
default	$\frac{\frac{1}{5}b^2x^5d^2+\frac{2}{3}ab\,d^2x^3-\frac{2}{3}b^2cd\,x^3+a^2d^2x-4abcdx+3b^2c^2x}{d^4}-\frac{c\left(\frac{\left(-\frac{1}{2}a^2d^2+abcd-\frac{1}{2}b^2c^2\right)x}{d\,x^2+c}+\frac{\left(3a^2d^2-10abcd+7b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{d^4}$
risch	$\frac{b^2x^5}{5d^2}+\frac{2ab\,x^3}{3d^2}-\frac{2b^2c\,x^3}{3d^3}+\frac{a^2x}{d^2}-\frac{4abcx}{d^3}+\frac{3b^2c^2x}{d^4}+\frac{\left(\frac{1}{2}a^2c\,d^2-ab\,c^2d+\frac{1}{2}b^2c^3\right)x}{d^4(d\,x^2+c)}+\frac{3\sqrt{-cd}\,\ln\left(-\sqrt{-cd}\,x-c\right)a^2}{4d^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(1/5*b^2*x^5*d^2+2/3*a*b*d^2*x^3-2/3*b^2*c*d*x^3+a^2*d^2*x-4*a*b*c*d*
x+3*b^2*c^2*x)-c/d^4*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2*c^2)*x/(d*x^2+c)+1/2*(3
*a^2*d^2-10*a*b*c*d+7*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.51, size = 149, normalized size = 1.03

$$\frac{(b^2c^3-2\,abc^2d+a^2cd^2)x}{2\,(d^5x^2+cd^4)}-\frac{(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,d^4}+\frac{3\,b^2d^2x^5-10\,(b^2cd-abd^2)x^3+15\,(3\,b^2c^2-4\,abcd+a^2d^2)x}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*x/(d^5*x^2 + c*d^4) - 1/2*(7*b^2*c^
3 - 10*a*b*c^2*d + 3*a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4) + 1/1
5*(3*b^2*d^2*x^5 - 10*(b^2*c*d - a*b*d^2)*x^3 + 15*(3*b^2*c^2 - 4*a*b*c*d +
 a^2*d^2)*x)/d^4
```

Fricas [A]
time = 1.49, size = 400, normalized size = 2.76

$$\left[\frac{12\,b^2d^3x^7-4\,(7\,b^2cd^2-10\,abd^3)x^5+20\,(7\,b^2c^2d-10\,abcd^2+3\,a^2d^3)x^3+15\,(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2+(7\,b^2c^2d-10\,abcd^2+3\,a^2d^3)x^2)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)+30\,(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2)x}{60\,(d^5x^2+cd^4)},\ \frac{6\,b^2d^3x^7-2\,(7\,b^2cd^2-10\,abd^3)x^5+10\,(7\,b^2c^2d-10\,abcd^2+3\,a^2d^3)x^3-15\,(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2+(7\,b^2c^2d-10\,abcd^2+3\,a^2d^3)x^2)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)+15\,(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2)x}{30\,(d^5x^2+cd^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^2*d^3*x^7 - 4*(7*b^2*c*d^2 - 10*a*b*d^3)*x^5 + 20*(7*b^2*c^2*d
- 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 + 15*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*d^
2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^2)*sqrt(-c/d)*log((d*x^2 - 2
*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 30*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*
d^2)*x)/(d^5*x^2 + c*d^4), 1/30*(6*b^2*d^3*x^7 - 2*(7*b^2*c*d^2 - 10*a*b*d^
```

```
3)*x^5 + 10*(7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 - 15*(7*b^2*c^3 -
10*a*b*c^2*d + 3*a^2*c*d^2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^2)*
sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + 15*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*
d^2)*x)/(d^5*x^2 + c*d^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(133) = 266$.
time = 0.57, size = 286, normalized size = 1.97

$$\frac{b^2x^5}{5d^2} + x^3 \cdot \left(\frac{2ab}{3d^2} - \frac{2b^2c}{3d^3}\right) + x\left(\frac{a^2}{d^2} - \frac{4abc}{d^3} + \frac{3b^2c^2}{d^4}\right) + \frac{x(a^2cd^2 - 2abc^2d + b^2c^3)}{2cd^4 + 2d^5x^2} + \frac{\sqrt{-\frac{c}{d^9}}\,(ad-bc)\,(3ad-7bc)\log\left(-\frac{d^4\sqrt{-\frac{c}{d^9}}\,(ad-bc)(3ad-7bc)}{3a^2d^2-10abcd+7b^2c^2} + x\right)}{4} - \frac{\sqrt{-\frac{c}{d^9}}\,(ad-bc)\,(3ad-7bc)\log\left(\frac{d^4\sqrt{-\frac{c}{d^9}}\,(ad-bc)(3ad-7bc)}{3a^2d^2-10abcd+7b^2c^2} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**5/(5*d**2) + x**3*(2*a*b/(3*d**2) - 2*b**2*c/(3*d**3)) + x*(a**2/d*
*2 - 4*a*b*c/d**3 + 3*b**2*c**2/d**4) + x*(a**2*c*d**2 - 2*a*b*c**2*d + b**
2*c**3)/(2*c*d**4 + 2*d**5*x**2) + sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c
)*log(-d**4*sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)/(3*a**2*d**2 - 10*a*b
*c*d + 7*b**2*c**2) + x)/4 - sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)*log(
d**4*sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)/(3*a**2*d**2 - 10*a*b*c*d +
7*b**2*c**2) + x)/4
```

Giac [A]
time = 0.55, size = 156, normalized size = 1.08

$$-\frac{(7\,b^2c^3 - 10\,abc^2d + 3\,a^2cd^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,d^4} + \frac{b^2c^3x - 2\,abc^2dx + a^2cd^2x}{2\,(dx^2+c)d^4} + \frac{3\,b^2d^8x^5 - 10\,b^2cd^7x^3 + 10\,abd^8x^3 + 45\,b^2c^2d^6x - 60\,abcd^7x + 15\,a^2d^8x}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c
*d)*d^4) + 1/2*(b^2*c^3*x - 2*a*b*c^2*d*x + a^2*c*d^2*x)/((d*x^2 + c)*d^4)
+ 1/15*(3*b^2*d^8*x^5 - 10*b^2*c*d^7*x^3 + 10*a*b*d^8*x^3 + 45*b^2*c^2*d^6*
x - 60*a*b*c*d^7*x + 15*a^2*d^8*x)/d^10
```

Mupad [B]
time = 0.07, size = 200, normalized size = 1.38

$$x\left(\frac{a^2}{d^2} + \frac{2\,c\left(\frac{2\,b^2\,c}{d^3} - \frac{2\,a\,b}{d^2}\right)}{d} - \frac{b^2\,c^2}{d^4}\right) - x^3\left(\frac{2\,b^2\,c}{3\,d^3} - \frac{2\,a\,b}{3\,d^2}\right) + \frac{b^2\,x^5}{5\,d^2} + \frac{x\left(\frac{a^2\,c\,d^2}{2} - a\,b\,c^2\,d + \frac{b^2\,c^3}{2}\right)}{d^5\,x^2 + c\,d^4} - \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,x\,(a\,d-b\,c)\,(3\,a\,d-7\,b\,c)}{3\,a^2\,c\,d^2-10\,a\,b\,c^2\,d+7\,b^2\,c^3}\right)\,(a\,d-b\,c)\,(3\,a\,d-7\,b\,c)}{2\,d^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] x*(a^2/d^2 + (2*c*((2*b^2*c)/d^3 - (2*a*b)/d^2))/d - (b^2*c^2)/d^4) - x^3*(
(2*b^2*c)/(3*d^3) - (2*a*b)/(3*d^2)) + (b^2*x^5)/(5*d^2) + (x*((b^2*c^3)/2
+ (a^2*c*d^2)/2 - a*b*c^2*d))/(c*d^4 + d^5*x^2) - (c^(1/2)*atan((c^(1/2)*d^
(1/2)*x*(a*d - b*c)*(3*a*d - 7*b*c))/(7*b^2*c^3 + 3*a^2*c*d^2 - 10*a*b*c^2*
d))*(a*d - b*c)*(3*a*d - 7*b*c))/(2*d^(9/2))
```

3.181 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{b(bc-ad)x^2}{d^3}+\frac{b^2x^4}{4d^2}+\frac{c(bc-ad)^2}{2d^4(c+dx^2)}+\frac{(bc-ad)(3bc-ad)\log(c+dx^2)}{2d^4}$$

```
[Out] -b*(-a*d+b*c)*x^2/d^3+1/4*b^2*x^4/d^2+1/2*c*(-a*d+b*c)^2/d^4/(d*x^2+c)+1/2*
(-a*d+b*c)*(-a*d+3*b*c)*ln(d*x^2+c)/d^4
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{c(bc-ad)^2}{2d^4(c+dx^2)}+\frac{(bc-ad)(3bc-ad)\log(c+dx^2)}{2d^4}-\frac{bx^2(bc-ad)}{d^3}+\frac{b^2x^4}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*(b*c - a*d)*x^2)/d^3) + (b^2*x^4)/(4*d^2) + (c*(b*c - a*d)^2)/(2*d^4*(
c + d*x^2)) + ((b*c - a*d)*(3*b*c - a*d)*Log[c + d*x^2])/(2*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{2b(bc-ad)}{d^3} + \frac{b^2x}{d^2} - \frac{c(bc-ad)^2}{d^3(c+dx)^2} + \frac{(bc-ad)(3bc-ad)}{d^3(c+dx)}\right) dx, x, x^2\right)$$
$$= -\frac{b(bc-ad)x^2}{d^3} + \frac{b^2x^4}{4d^2} + \frac{c(bc-ad)^2}{2d^4(c+dx^2)} + \frac{(bc-ad)(3bc-ad)\log(c+dx^2)}{2d^4}$$

Mathematica [A]
time = 0.05, size = 87, normalized size = 0.97

$$\frac{4bd(-bc+ad)x^2 + b^2d^2x^4 + \frac{2c(bc-ad)^2}{c+dx^2} + 2(3b^2c^2 - 4abcd + a^2d^2)\log(c+dx^2)}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (4*b*d*(-(b*c) + a*d)*x^2 + b^2*d^2*x^4 + (2*c*(b*c - a*d)^2)/(c + d*x^2) +
 2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(4*d^4)
```

Maple [A]
time = 0.09, size = 80, normalized size = 0.89

method	result
default	$\frac{(bd\,x^2+2ad-2bc)^2}{4d^4} + \frac{(ad-bc)\left(\frac{c(ad-bc)}{d(d\,x^2+c)} + \frac{(ad-3bc)\ln(d\,x^2+c)}{d}\right)}{2d^3}$
norman	$\frac{\frac{b^2x^6}{4d} + \frac{b(4ad-3bc)x^4}{4d^2} - \frac{(a^2c\,d^2-4ab\,c^2d+3b^2c^3)x^2}{2d^3c}}{d\,x^2+c} + \frac{(a^2d^2-4abcd+3b^2c^2)\ln(d\,x^2+c)}{2d^4}$
risch	$\frac{b^2x^4}{4d^2} + \frac{ab\,x^2}{d^2} - \frac{b^2c\,x^2}{d^3} + \frac{a^2}{d^2} - \frac{2abc}{d^3} + \frac{b^2c^2}{d^4} + \frac{c\,a^2}{2d^2(d\,x^2+c)} - \frac{c^2ab}{d^3(d\,x^2+c)} + \frac{c^3b^2}{2d^4(d\,x^2+c)} + \frac{\ln(d\,x^2+c)a^2}{2d^2} - \frac{2\ln(d\,x^2+}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*d*x^2+2*a*d-2*b*c)^2/d^4+1/2/d^3*(a*d-b*c)*(c*(a*d-b*c)/d/(d*x^2+c)+
(a*d-3*b*c)/d*ln(d*x^2+c))
```

Maxima [A]
time = 0.28, size = 107, normalized size = 1.19

$$\frac{b^2c^3 - 2\,abc^2d + a^2cd^2}{2\,(d^5x^2 + cd^4)} + \frac{b^2dx^4 - 4\,(b^2c - abd)x^2}{4\,d^3} + \frac{(3\,b^2c^2 - 4\,abcd + a^2d^2)\log(dx^2 + c)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(d^5*x^2 + c*d^4) + 1/4*(b^2*d*x^4
- 4*(b^2*c - a*b*d)*x^2)/d^3 + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*log(d*
x^2 + c)/d^4
```

Fricas [A]
time = 1.41, size = 161, normalized size = 1.79

$$\frac{b^2d^3x^6+2\,b^2c^3-4\,abc^2d+2\,a^2cd^2-(3\,b^2cd^2-4\,abd^3)x^4-4\,(b^2c^2d-abcd^2)x^2+2\,(3\,b^2c^3-4\,abc^2d+a^2cd^2+(3\,b^2c^2d-4\,abcd^2+a^2d^3)x^2)\log\left(dx^2+c\right)}{4\,(d^5x^2+cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^3*x^6 + 2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d^2 - (3*b^2*c*d^2 - 4
*a*b*d^3)*x^4 - 4*(b^2*c^2*d - a*b*c*d^2)*x^2 + 2*(3*b^2*c^3 - 4*a*b*c^2*d
+ a^2*c*d^2 + (3*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2)*log(d*x^2 + c))/(d
^5*x^2 + c*d^4)
```

Sympy [A]
time = 0.52, size = 99, normalized size = 1.10

$$\frac{b^2x^4}{4d^2}+x^2\left(\frac{ab}{d^2}-\frac{b^2c}{d^3}\right)+\frac{a^2cd^2-2abc^2d+b^2c^3}{2cd^4+2d^5x^2}+\frac{(ad-3bc)\,(ad-bc)\log\left(c+dx^2\right)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**4/(4*d**2) + x**2*(a*b/d**2 - b**2*c/d**3) + (a**2*c*d**2 - 2*a*b*c
**2*d + b**2*c**3)/(2*c*d**4 + 2*d**5*x**2) + (a*d - 3*b*c)*(a*d - b*c)*log
(c + d*x**2)/(2*d**4)
```

Giac [A]
time = 0.51, size = 163, normalized size = 1.81

$$\frac{\frac{(dx^2+c)^2\left(b^2-\frac{2\,(3\,b^2cd-2\,abd^2)}{(dx^2+c)d}\right)}{d^3}-\frac{2\,(3\,b^2c^2-4\,abcd+a^2d^2)\log\left(\frac{|dx^2+c|}{(dx^2+c)^2|d|}\right)}{d^3}+\frac{2\left(\frac{b^2c^3d^2}{dx^2+c}-\frac{2\,abc^2d^3}{dx^2+c}+\frac{a^2cd^4}{dx^2+c}\right)}{d^5}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((d*x^2 + c)^2*(b^2 - 2*(3*b^2*c*d - 2*a*b*d^2)/((d*x^2 + c)*d))/d^3 -
2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*log(abs(d*x^2 + c)/((d*x^2 + c)^2*abs(d
```

```
)))/d^3 + 2*(b^2*c^3*d^2/(d*x^2 + c) - 2*a*b*c^2*d^3/(d*x^2 + c) + a^2*c*d^
4/(d*x^2 + c))/d^5)/d
```

Mupad [B]
time = 0.07, size = 112, normalized size = 1.24

$$\frac{a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3}{2\,d\,\left(d^4\,x^2+c\,d^3\right)}-x^2\,\left(\frac{b^2\,c}{d^3}-\frac{a\,b}{d^2}\right)+\frac{b^2\,x^4}{4\,d^2}+\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)/(2*d*(c*d^3 + d^4*x^2)) - x^2*((b^2*c)/
d^3 - (a*b)/d^2) + (b^2*x^4)/(4*d^2) + (log(c + d*x^2)*(a^2*d^2 + 3*b^2*c^2
 - 4*a*b*c*d))/(2*d^4)
```

3.182 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=118

$$-\frac{(bc-ad)(5bc-ad)x}{2cd^3}+\frac{b^2x^3}{3d^2}+\frac{(bc-ad)^2x^3}{2cd^2\,(c+dx^2)}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,d^{7/2}}$$

```
[Out] -1/2*(-a*d+b*c)*(-a*d+5*b*c)*x/c/d^3+1/3*b^2*x^3/d^2+1/2*(-a*d+b*c)^2*x^3/c
/d^2/(d*x^2+c)+1/2*(-a*d+b*c)*(-a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/d^(7/2
)/c^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {474, 470, 327, 211}

$$\frac{(bc-ad)(5bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,d^{7/2}}-\frac{x(bc-ad)(5bc-ad)}{2cd^3}+\frac{x^3(bc-ad)^2}{2cd^2\,(c+dx^2)}+\frac{b^2x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*(5*b*c - a*d)*x)/(c*d^3) + (b^2*x^3)/(3*d^2) + ((b*c - a*
d)^2*x^3)/(2*c*d^2*(c + d*x^2)) + ((b*c - a*d)*(5*b*c - a*d)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(2*Sqrt[c]*d^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx &= \frac{(bc-ad)^2x^3}{2cd^2(c+dx^2)}-\frac{\int \frac{x^2(3b^2c^2-6abcd+a^2d^2-2b^2cdx^2)}{c+dx^2}\,dx}{2cd^2}\\ &= \frac{b^2x^3}{3d^2}+\frac{(bc-ad)^2x^3}{2cd^2(c+dx^2)}-\frac{((bc-ad)(5bc-ad))\int \frac{x^2}{c+dx^2}\,dx}{2cd^2}\\ &= -\frac{(bc-ad)(5bc-ad)x}{2cd^3}+\frac{b^2x^3}{3d^2}+\frac{(bc-ad)^2x^3}{2cd^2(c+dx^2)}+\frac{((bc-ad)(5bc-ad))\int \frac{1}{c+dx^2}\,dx}{2d^3}\\ &= -\frac{(bc-ad)(5bc-ad)x}{2cd^3}+\frac{b^2x^3}{3d^2}+\frac{(bc-ad)^2x^3}{2cd^2(c+dx^2)}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,d^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 105, normalized size = 0.89

$$-\frac{2b(bc-ad)x}{d^3}+\frac{b^2x^3}{3d^2}-\frac{(bc-ad)^2x}{2d^3(c+dx^2)}+\frac{(5b^2c^2-6abcd+a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-2*b*(b*c - a*d)*x)/d^3 + (b^2*x^3)/(3*d^2) - ((b*c - a*d)^2*x)/(2*d^3*(c
+ d*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])
/(2*Sqrt[c]*d^(7/2))
```

Maple [A]
time = 0.11, size = 101, normalized size = 0.86

method	result
default	$\frac{b\left(\frac{1}{3}bd\,x^{3}+2adx-2bcx\right)}{d^{3}}+\frac{\frac{\left(-\frac{1}{2}a^{2}d^{2}+abcd-\frac{1}{2}b^{2}c^{2}\right)x}{d\,x^{2}+c}+\frac{\left(a^{2}d^{2}-6abcd+5b^{2}c^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}}{d^{3}}$
risch	$\frac{b^{2}x^{3}}{3d^{2}}+\frac{2bax}{d^{2}}-\frac{2b^{2}cx}{d^{3}}+\frac{\left(-\frac{1}{2}a^{2}d^{2}+abcd-\frac{1}{2}b^{2}c^{2}\right)x}{d^{3}\left(d\,x^{2}+c\right)}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^{2}}{4d\sqrt{-cd}}+\frac{3\ln\left(dx+\sqrt{-cd}\right)abc}{2d^{2}\sqrt{-cd}}-\frac{5\ln\left(dx+\sqrt{-c\right.}}{4d^{3}\sqrt{-c}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] b/d^3*(1/3*b*d*x^3+2*a*d*x-2*b*c*x)+1/d^3*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2*c^2)*x/(d*x^2+c)+1/2*(a^2*d^2-6*a*b*c*d+5*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))

Maxima [A]
time = 0.51, size = 109, normalized size = 0.92

$$-\frac{(b^2c^2-2\,abcd+a^2d^2)x}{2\,(d^4x^2+cd^3)}+\frac{(5\,b^2c^2-6\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,d^3}+\frac{b^2dx^3-6\,(b^2c-abd)x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")

[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(d^4*x^2 + c*d^3) + 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3) + 1/3*(b^2*d*x^3 - 6*(b^2*c - a*b*d)*x)/d^3

Fricas [A]
time = 1.04, size = 342, normalized size = 2.90

$$\left[\frac{4\,b^2cd^3x^5-4\,(5\,b^2c^2d^2-6\,abcd^3)x^3-3\,(5\,b^2c^3-6\,abc^2d+a^2cd^2+(5\,b^2c^2d-6\,abcd^2+a^2d^3)x^2)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)-6\,(5\,b^2c^3d-6\,abc^2d^2+a^2cd^3)x}{12\,(cd^5x^2+c^2d^4)},\ \frac{2\,b^2cd^3x^5-2\,(5\,b^2c^2d^2-6\,abcd^3)x^3+3\,(5\,b^2c^3-6\,abc^2d+a^2cd^2+(5\,b^2c^2d-6\,abcd^2+a^2d^3)x^2)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-3\,(5\,b^2c^3d-6\,abc^2d^2+a^2cd^3)x}{6\,(cd^5x^2+c^2d^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")

[Out] [1/12*(4*b^2*c*d^3*x^5 - 4*(5*b^2*c^2*d^2 - 6*a*b*c*d^3)*x^3 - 3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + (5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(5*b^2*c^3*d - 6*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c*d^5*x^2 + c^2*d^4), 1/6*(2*b^2*c*d^3*x^5 - 2*(5*b^2*c^2*d^2 - 6*a*b*c*d^3)*x^3 + 3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + (5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(5*b^2*c^3*d - 6*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c*d^5*x^2 + c^2*d^4)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(104) = 208$.
time = 0.59, size = 246, normalized size = 2.08

$$\frac{b^2x^3}{3d^2} + x\left(\frac{2ab}{d^2} - \frac{2b^2c}{d^3}\right) + \frac{x(-a^2d^2 + 2abcd - b^2c^2)}{2cd^3 + 2d^4x^2} - \frac{\sqrt{-\frac{1}{cd^7}}\,(ad - 5bc)\,(ad - bc)\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2} + x\right)}{4} + \frac{\sqrt{-\frac{1}{cd^7}}\,(ad - 5bc)\,(ad - bc)\log\left(\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**3/(3*d**2) + x*(2*a*b/d**2 - 2*b**2*c/d**3) + x*(-a**2*d**2 + 2*a*b
*c*d - b**2*c**2)/(2*c*d**3 + 2*d**4*x**2) - sqrt(-1/(c*d**7))*(a*d - 5*b*c
)*(a*d - b*c)*log(-c*d**3*sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)/(a**2
*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(
a*d - b*c)*log(c*d**3*sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)/(a**2*d**
2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4
```

Giac [A]
time = 0.51, size = 114, normalized size = 0.97

$$\frac{(5\,b^2c^2 - 6\,abcd + a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,d^3} - \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(dx^2+c)d^3} + \frac{b^2d^4x^3 - 6\,b^2cd^3x + 6\,abd^4x}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
 - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*d^3) + 1/3*(b^2*d
^4*x^3 - 6*b^2*c*d^3*x + 6*a*b*d^4*x)/d^6
```

Mupad [B]
time = 0.04, size = 146, normalized size = 1.24

$$\frac{b^2\,x^3}{3\,d^2} - \frac{x\left(\frac{a^2\,d^2}{2} - a\,b\,c\,d + \frac{b^2\,c^2}{2}\right)}{d^4\,x^2 + c\,d^3} - x\left(\frac{2\,b^2\,c}{d^3} - \frac{2\,a\,b}{d^2}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(a\,d-5\,b\,c)}{\sqrt{c}\,(a^2\,d^2-6\,a\,b\,c\,d+5\,b^2\,c^2)}\right)(a\,d - b\,c)\,(a\,d - 5\,b\,c)}{2\sqrt{c}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (b^2*x^3)/(3*d^2) - (x*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(c*d^3 + d^4*
x^2) - x*((2*b^2*c)/d^3 - (2*a*b)/d^2) + (atan((d^(1/2)*x*(a*d - b*c)*(a*d
- 5*b*c))/(c^(1/2)*(a^2*d^2 + 5*b^2*c^2 - 6*a*b*c*d)))*(a*d - b*c)*(a*d - 5
*b*c))/(2*c^(1/2)*d^(7/2))
```

3.183 $\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=62

$$\frac{b^2x^2}{2d^2}-\frac{(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{b(bc-ad)\log\left(c+dx^2\right)}{d^3}$$

```
[Out] 1/2*b^2*x^2/d^2-1/2*(-a*d+b*c)^2/d^3/(d*x^2+c)-b*(-a*d+b*c)*ln(d*x^2+c)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$-\frac{(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{b(bc-ad)\log\left(c+dx^2\right)}{d^3}+\frac{b^2x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x^2)/(2*d^2) - (b*c - a*d)^2/(2*d^3*(c + d*x^2)) - (b*(b*c - a*d)*Log[
c + d*x^2])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2}{d^2}+\frac{(-bc+ad)^2}{d^2(c+dx)^2}-\frac{2b(bc-ad)}{d^2(c+dx)}\right)\,dx,x,x^2\right)\\ &= \frac{b^2x^2}{2d^2}-\frac{(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{b(bc-ad)\log\left(c+dx^2\right)}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.90

$$\frac{b^2dx^2 - \frac{(bc-ad)^2}{c+dx^2} + 2b(-bc+ad)\log\left(c+dx^2\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*d*x^2 - (b*c - a*d)^2/(c + d*x^2) + 2*b*(-(b*c) + a*d)*Log[c + d*x^2])
/(2*d^3)
```

Maple [A]
time = 0.08, size = 63, normalized size = 1.02

method	result	size
default	$\frac{b^{2} x^{2}}{2 d^{2}}+\frac{\left(a d -b c \right) \left(-\frac{a d -b c}{d \left(d \,x^{2}+c \right)}+\frac{2 b \ln \left(d \,x^{2}+c \right)}{d}\right)}{2 d^{2}}$	63
norman	$\frac{-\frac{a^{2} d^{2}-2 a b c d +2 b^{2} c^{2}}{2 d^{3}}+\frac{b^{2} x^{4}}{2 d}}{d \,x^{2}+c}+\frac{b \left(a d -b c \right) \ln \left(d \,x^{2}+c \right)}{d^{3}}$	72
risch	$\frac{b^{2} x^{2}}{2 d^{2}}-\frac{a^{2}}{2 d \left(d \,x^{2}+c \right)}+\frac{a b c}{d^{2} \left(d \,x^{2}+c \right)}-\frac{b^{2} c^{2}}{2 d^{3} \left(d \,x^{2}+c \right)}+\frac{b \ln \left(d \,x^{2}+c \right) a}{d^{2}}-\frac{b^{2} \ln \left(d \,x^{2}+c \right) c}{d^{3}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2*x^2/d^2+1/2/d^2*(a*d-b*c)*(-(a*d-b*c)/d/(d*x^2+c)+2*b/d*ln(d*x^2+c)
)
```

Maxima [A]
time = 0.28, size = 74, normalized size = 1.19

$$\frac{b^2x^2}{2\,d^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{2\,(d^4x^2 + cd^3)} - \frac{(b^2c - abd)\log\left(dx^2 + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x^2 + c*d^3) - (
b^2*c - a*b*d)*log(d*x^2 + c)/d^3
```

Fricas [A]
time = 1.15, size = 101, normalized size = 1.63

$$\frac{b^2d^2x^4 + b^2cdx^2 - b^2c^2 + 2\,abcd - a^2d^2 - 2\,(b^2c^2 - abcd + (b^2cd - abd^2)x^2)\log\left(dx^2 + c\right)}{2\,(d^4x^2 + cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^4 + b^2*c*d*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2
 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)*log(d*x^2 + c))/(d^4*x^2 + c*d^3)
```

Sympy [**A**]
time = 0.53, size = 68, normalized size = 1.10

$$\frac{b^2x^2}{2d^2}+\frac{b(ad-bc)\log\left(c+dx^2\right)}{d^3}+\frac{-a^2d^2+2abcd-b^2c^2}{2cd^3+2d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**2/(2*d**2) + b*(a*d - b*c)*log(c + d*x**2)/d**3 + (-a**2*d**2 + 2*a
*b*c*d - b**2*c**2)/(2*c*d**3 + 2*d**4*x**2)
```

Giac [**A**]
time = 0.53, size = 110, normalized size = 1.77

$$\frac{(dx^2+c)b^2}{2\,d^3}+\frac{(b^2c-abd)\log\left(\frac{|dx^2+c|}{(dx^2+c)^2|d|}\right)}{d^3}-\frac{\frac{b^2c^2d}{dx^2+c}-\frac{2\,abcd^2}{dx^2+c}+\frac{a^2d^3}{dx^2+c}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + c)*b^2/d^3 + (b^2*c - a*b*d)*log(abs(d*x^2 + c)/((d*x^2 + c)^2
*abs(d)))/d^3 - 1/2*(b^2*c^2*d/(d*x^2 + c) - 2*a*b*c*d^2/(d*x^2 + c) + a^2*
d^3/(d*x^2 + c))/d^4
```

Mupad [**B**]
time = 0.05, size = 77, normalized size = 1.24

$$\frac{b^2\,x^2}{2\,d^2}-\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{2\,d\,\left(d^3\,x^2+c\,d^2\right)}-\frac{\ln\left(d\,x^2+c\right)\,\left(b^2\,c-a\,b\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (b^2*x^2)/(2*d^2) - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*d*(c*d^2 + d^3*x^2))
 - (log(c + d*x^2)*(b^2*c - a*b*d))/d^3
```

3.184 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2} \, dx$

Optimal. Leaf size=82

$$\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2(c+dx^2)}-\frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

```
[Out] b^2*x/d^2+1/2*(-a*d+b*c)^2*x/c/d^2/(d*x^2+c)-1/2*(-a*d+b*c)*(a*d+3*b*c)*arc
tan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {398, 393, 211}

$$-\frac{(bc-ad)(ad+3bc)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*c
 + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx &= \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{d^2(c+dx^2)^2}\right)dx\\
&= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{(c+dx^2)^2}\,dx}{d^2}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2(c+dx^2)}-\frac{((bc-ad)(3bc+ad))\int \frac{1}{c+dx^2}\,dx}{2cd^2}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2(c+dx^2)}-\frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 1.09

$$
\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2(c+dx^2)}-\frac{(3b^2c^2-2abcd-a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c^2 - 2*a*b
*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Maple [A]
time = 0.07, size = 92, normalized size = 1.12

method	result
default	$\frac{b^2x}{d^2}+\frac{\frac{(a^2d^2-2abcd+b^2c^2)x}{2c(dx^2+c)}+\frac{(a^2d^2+2abcd-3b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}}{d^2}$
risch	$\frac{b^2x}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{2c\,d^2(d\,x^2+c)}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^2}{4\sqrt{-cd}\,c}-\frac{\ln\left(dx+\sqrt{-cd}\right)ab}{2d\sqrt{-cd}}+\frac{3c\ln\left(dx+\sqrt{-cd}\right)b^2}{4d^2\sqrt{-cd}}+\frac{\ln\left(-dx+\sqrt{-\right.}}{4\sqrt{-cd}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/d^2+1/d^2*(1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/(d*x^2+c)+1/2*(a^2*d^2
+2*a*b*c*d-3*b^2*c^2)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.49, size = 96, normalized size = 1.17

$$\frac{(b^2c^2 - 2\,abcd + a^2d^2)x}{2\,(cd^3x^2 + c^2d^2)} + \frac{b^2x}{d^2} - \frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^2 + c^2*d^2) + b^2*x/d^2 - 1
/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d^2
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(70) = 140.
time = 1.12, size = 302, normalized size = 3.68

$$\left[\frac{4\,b^2c^2d^2x^3 + (3\,b^2c^3 - 2\,abc^2d - a^2cd^2 + (3\,b^2c^2d - 2\,abcd^2 - a^2d^3)x^2)\sqrt{-cd}\,\log\left(\frac{dx^2 - 2\,\sqrt{-cd}\,x - c}{dx^2 + c}\right) + 2\,(3\,b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x}{4\,(c^2d^4x^2 + c^3d^3)}, \frac{2\,b^2c^2d^2x^3 - (3\,b^2c^3 - 2\,abc^2d - a^2cd^2 + (3\,b^2c^2d - 2\,abcd^2 - a^2d^3)x^2)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right) + (3\,b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x}{2\,(c^2d^4x^2 + c^3d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^2*c^2*d^2*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2
*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c
)/(d*x^2 + c)) + 2*(3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^
2 + c^3*d^3), 1/2*(2*b^2*c^2*d^2*x^3 - (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/
c) + (3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^2 + c^3*d^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(73) = 146.
time = 0.38, size = 236, normalized size = 2.88

$$\frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{2c^2d^2 + 2cd^3x^2} - \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad - bc)\,(ad + 3bc)\log\left(-\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2} + x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad - bc)\,(ad + 3bc)\log\left(\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d**3
*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(-c**2*d**2*sqrt
(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*
c**2) + x)/4 + sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*d**2
```

```
*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*
b**2*c**2) + x)/4
```

Giac [A]
time = 1.07, size = 95, normalized size = 1.16

$$\frac{b^2x}{d^2} - \frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(dx^2+c)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sq
rt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c*d
^2)
```

Mupad [B]
time = 0.09, size = 124, normalized size = 1.51

$$\frac{b^2\,x}{d^2} + \frac{x\,(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2)}{2\,c\,(d^3\,x^2 + c\,d^2)} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{\sqrt{c}\,(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2)}\right)\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{2\,c^{3/2}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^2 + d^3*x^2)) +
 (atan((d^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(c^(1/2)*(a^2*d^2 - 3*b^2*c^2
+ 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*c^(3/2)*d^(5/2))
```

3.185 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^2} dx$

Optimal. Leaf size=67

$$\frac{(bc-ad)^2}{2cd^2(c+dx^2)} + \frac{a^2\log(x)}{c^2} - \frac{1}{2}\left(\frac{a^2}{c^2} - \frac{b^2}{d^2}\right)\log\left(c+dx^2\right)$$

```
[Out] 1/2*(-a*d+b*c)^2/c/d^2/(d*x^2+c)+a^2*ln(x)/c^2-1/2*(a^2/c^2-b^2/d^2)*ln(d*x
^2+c)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{1}{2}\left(\frac{a^2}{c^2} - \frac{b^2}{d^2}\right)\log\left(c+dx^2\right) + \frac{a^2\log(x)}{c^2} + \frac{(bc-ad)^2}{2cd^2(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

```
[Out] (b*c - a*d)^2/(2*c*d^2*(c + d*x^2)) + (a^2*Log[x])/c^2 - ((a^2/c^2 - b^2/d^
2)*Log[c + d*x^2])/2
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{c^2x} - \frac{(bc-ad)^2}{cd(c+dx)^2} + \frac{b^2c^2-a^2d^2}{c^2d(c+dx)}\right)dx, x, x^2\right)$$
$$= \frac{(bc-ad)^2}{2cd^2(c+dx^2)} + \frac{a^2\log(x)}{c^2} - \frac{1}{2}\left(\frac{a^2}{c^2} - \frac{b^2}{d^2}\right)\log(c+dx^2)$$

Mathematica [**A**]
time = 0.03, size = 70, normalized size = 1.04

$$\frac{2a^2\log(x) + \frac{(bc-ad)(c(bc-ad)+(bc+ad)(c+dx^2)\log(c+dx^2))}{d^2(c+dx^2)}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

```
[Out] (2*a^2*Log[x] + ((b*c - a*d)*(c*(b*c - a*d) + (b*c + a*d)*(c + d*x^2)*Log[c
 + d*x^2]))/(d^2*(c + d*x^2)))/(2*c^2)
```

Maple [**A**]
time = 0.08, size = 67, normalized size = 1.00

method	result	size
default	$-\frac{(ad-bc)\left(-\frac{c(ad-bc)}{d^2(dx^2+c)}+\frac{(ad+bc)\ln(dx^2+c)}{d^2}\right)}{2c^2}+\frac{a^2\ln(x)}{c^2}$	67
norman	$\frac{a^2d^2-2abcd+b^2c^2}{2cd^2(dx^2+c)}+\frac{a^2\ln(x)}{c^2}-\frac{(a^2d^2-b^2c^2)\ln(dx^2+c)}{2c^2d^2}$	81
risch	$\frac{a^2}{2c(dx^2+c)}-\frac{ab}{d(dx^2+c)}+\frac{cb^2}{2d^2(dx^2+c)}+\frac{a^2\ln(x)}{c^2}-\frac{\ln(dx^2+c)a^2}{2c^2}+\frac{\ln(dx^2+c)b^2}{2d^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2*(a*d-b*c)*(-c*(a*d-b*c)/d^2/(d*x^2+c)+(a*d+b*c)/d^2*ln(d*x^2+c))+a
^2*ln(x)/c^2
```

Maxima [**A**]
time = 0.30, size = 86, normalized size = 1.28

$$\frac{a^2\log(x^2)}{2c^2} + \frac{b^2c^2 - 2abcd + a^2d^2}{2(cd^3x^2 + c^2d^2)} + \frac{(b^2c^2 - a^2d^2)\log(dx^2+c)}{2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(c*d^3*x^2 + c^2
*d^2) + 1/2*(b^2*c^2 - a^2*d^2)*log(d*x^2 + c)/(c^2*d^2)
```

Fricas [A]
time = 1.05, size = 116, normalized size = 1.73

$$\frac{b^2c^3 - 2\,abc^2d + a^2cd^2 + (b^2c^3 - a^2cd^2 + (b^2c^2d - a^2d^3)x^2)\log\left(dx^2 + c\right) + 2\left(a^2d^3x^2 + a^2cd^2\right)\log\left(x\right)}{2\left(c^2d^3x^2 + c^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^3 - a^2*c*d^2 + (b^2*c^2*d
- a^2*d^3)*x^2)*log(d*x^2 + c) + 2*(a^2*d^3*x^2 + a^2*c*d^2)*log(x))/(c^2*d
^3*x^2 + c^3*d^2)
```

Sympy [A]
time = 0.82, size = 80, normalized size = 1.19

$$\frac{a^2\log\left(x\right)}{c^2} + \frac{a^2d^2 - 2abcd + b^2c^2}{2c^2d^2 + 2cd^3x^2} - \frac{(ad - bc)\,(ad + bc)\log\left(\frac{c}{d} + x^2\right)}{2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**2,x)
```

```
[Out] a**2*log(x)/c**2 + (a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d
**3*x**2) - (a*d - b*c)*(a*d + b*c)*log(c/d + x**2)/(2*c**2*d**2)
```

Giac [A]
time = 1.21, size = 99, normalized size = 1.48

$$\frac{a^2\log\left(x^2\right)}{2\,c^2} + \frac{(b^2c^2 - a^2d^2)\log\left(|dx^2 + c|\right)}{2\,c^2d^2} - \frac{b^2c^2x^2 - a^2d^2x^2 + 2\,abc^2 - 2\,a^2cd}{2\,(dx^2 + c)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(x^2)/c^2 + 1/2*(b^2*c^2 - a^2*d^2)*log(abs(d*x^2 + c))/(c^2*d^2
) - 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d)/((d*x^2 + c)*c^
2*d)
```

Mupad [B]
time = 0.07, size = 80, normalized size = 1.19

$$\frac{a^2\,\ln\left(x\right)}{c^2} + \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{2\,c\,d^2\,\left(d\,x^2 + c\right)} - \frac{\ln\left(d\,x^2 + c\right)\,\left(a^2\,d^2 - b^2\,c^2\right)}{2\,c^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^2),x)
```

```
[Out] (a^2*log(x))/c^2 + (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*c*d^2*(c + d*x^2)) -
(log(c + d*x^2)*(a^2*d^2 - b^2*c^2))/(2*c^2*d^2)
```

3.186 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2} \, dx$

Optimal. Leaf size=106

$$-\frac{a^2}{cx(c+dx^2)}-\frac{(b^2c^2-2abcd+3a^2d^2)\,x}{2c^2d(c+dx^2)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

```
[Out] -a^2/c/x/(d*x^2+c)-1/2*(3*a^2*d^2-2*a*b*c*d+b^2*c^2)*x/c^2/d/(d*x^2+c)+1/2*
(-a*d+b*c)*(3*a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/d^(3/2)
```

Rubi [A]

time = 0.05, antiderivative size = 104, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {473, 393, 211}

$$\frac{x\left(-\frac{3a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2c(c+dx^2)}-\frac{a^2}{cx(c+dx^2)}+\frac{(bc-ad)(3ad+bc)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2))) + ((2*a*b - (b^2*c)/d - (3*a^2*d)/c)*x)/(2*c*(c +
d*x^2)) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2
)*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
```

```
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2}\,dx &= -\frac{a^2}{cx(c+dx^2)} + \frac{\int \frac{a(2bc-3ad)+b^2cx^2}{(c+dx^2)^2}\,dx}{c} \\
&= -\frac{a^2}{cx(c+dx^2)} - \frac{(b^2c^2-2abcd+3a^2d^2)\,x}{2c^2d(c+dx^2)} + \frac{((bc-ad)(bc+3ad))\int \frac{1}{c+dx^2}\,dx}{2c^2d} \\
&= -\frac{a^2}{cx(c+dx^2)} - \frac{(b^2c^2-2abcd+3a^2d^2)\,x}{2c^2d(c+dx^2)} + \frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 91, normalized size = 0.86

$$-\frac{a^2}{c^2x} - \frac{(bc-ad)^2x}{2c^2d(c+dx^2)} + \frac{(b^2c^2+2abcd-3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

```
[Out] -(a^2/(c^2*x)) - ((b*c - a*d)^2*x)/(2*c^2*d*(c + d*x^2)) + ((b^2*c^2 + 2*a*
b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*d^(3/2))
```

Maple [A]
time = 0.11, size = 97, normalized size = 0.92

method	result
default	$-\frac{\frac{(a^2d^2-2abcd+b^2c^2)x}{2d(dx^2+c)}+\frac{(3a^2d^2-2abcd-b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2d\sqrt{cd}}}{c^2}-\frac{a^2}{c^2x}$
risch	$\frac{-\frac{(3a^2d^2-2abcd+b^2c^2)x^2}{2c^2d}-\frac{a^2}{c}}{(dx^2+c)x}-\frac{3d\ln\left(-\sqrt{-cd}\,x-c\right)a^2}{4\sqrt{-cd}\,c^2}+\frac{\ln\left(-\sqrt{-cd}\,x-c\right)ab}{2\sqrt{-cd}\,c}+\frac{\ln\left(-\sqrt{-cd}\,x-c\right)b^2}{4\sqrt{-cd}\,d}+\frac{3d\ln(-}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

[Out] -1/c^2*(1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*x/(d*x^2+c)+1/2*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))-a^2/c^2/x

Maxima [A]
time = 0.54, size = 100, normalized size = 0.94

$$-\frac{2\,a^2cd+(b^2c^2-2\,abcd+3\,a^2d^2)x^2}{2\,(c^2d^2x^3+c^3dx)}+\frac{(b^2c^2+2\,abcd-3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="maxima")

[Out] -1/2*(2*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*x^2)/(c^2*d^2*x^3 + c^3*d*x) + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d)

Fricas [A]
time = 0.96, size = 305, normalized size = 2.88

$$\left[-\frac{4\,a^2c^2d^2+2\,(b^2c^3d-2\,abc^2d^2+3\,a^2cd^3)x^2-((b^2c^2d+2\,abcd^2-3\,a^2d^3)x^3+(b^2c^3+2\,abc^2d-3\,a^2cd^2)x)\sqrt{-cd}\,\log\left(\frac{dx^2+2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)}{4\,(c^3d^3x^3+c^4d^2x)},-\frac{2\,a^2c^2d^2+(b^2c^3d-2\,abc^2d^2+3\,a^2cd^3)x^2-((b^2c^2d+2\,abcd^2-3\,a^2d^3)x^3+(b^2c^3+2\,abc^2d-3\,a^2cd^2)x)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{2\,(c^3d^3x^3+c^4d^2x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="fricas")

[Out] [-1/4*(4*a^2*c^2*d^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2 - ((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-c*d)*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^3*d^3*x^3 + c^4*d^2*x), -1/2*(2*a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2 - ((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^3*d^3*x^3 + c^4*d^2*x)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(94) = 188$.
time = 0.52, size = 238, normalized size = 2.25

$$\frac{\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)\,(3ad+bc)\log\left(-\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)\,(3ad+bc)\log\left(\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}+\frac{-2a^2cd+x^2(-3a^2d^2+2abcd-b^2c^2)}{2c^3dx+2c^2d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**2,x)

[Out] sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4

```
 - sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)*log(c**3*d*sqrt(-1/(c**5*
d**3))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)
/4 + (-2*a**2*c*d + x**2*(-3*a**2*d**2 + 2*a*b*c*d - b**2*c**2))/(2*c**3*d*
x + 2*c**2*d**2*x**3)
```

Giac [A]
time = 1.01, size = 102, normalized size = 0.96

$$\frac{(b^2c^2 + 2\,abcd - 3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,c^2d} - \frac{b^2c^2x^2 - 2\,abcdx^2 + 3\,a^2d^2x^2 + 2\,a^2cd}{2\,(dx^3 + cx)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*
d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 3*a^2*d^2*x^2 + 2*a^2*c*d)/((d*x^3
+ c*x)*c^2*d)
```

Mupad [B]
time = 0.12, size = 128, normalized size = 1.21

$$\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{\sqrt{c}\,\left(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{2\,c^{5/2}\,d^{3/2}} - \frac{\frac{a^2}{c}+\frac{x^2\,(3\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,c^2\,d}}{d\,x^3+c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^2),x)
```

```
[Out] (atan((d^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(c^(1/2)*(b^2*c^2 - 3*a^2*d^2 +
 2*a*b*c*d)))*(a*d - b*c)*(3*a*d + b*c))/(2*c^(5/2)*d^(3/2)) - (a^2/c + (x^
2*(3*a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c^2*d))/(c*x + d*x^3)
```

3.187 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2} dx$

Optimal. Leaf size=81

$$-\frac{a^2}{2c^2x^2}-\frac{(bc-ad)^2}{2c^2d\,(c+dx^2)}+\frac{2a(bc-ad)\log(x)}{c^3}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}$$

```
[Out] -1/2*a^2/c^2/x^2-1/2*(-a*d+b*c)^2/c^2/d/(d*x^2+c)+2*a*(-a*d+b*c)*ln(x)/c^3-
a*(-a*d+b*c)*ln(d*x^2+c)/c^3
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 90}

$$-\frac{a^2}{2c^2x^2}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}+\frac{2a\log(x)(bc-ad)}{c^3}-\frac{(bc-ad)^2}{2c^2d\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

```
[Out] -1/2*a^2/(c^2*x^2) - (b*c - a*d)^2/(2*c^2*d*(c + d*x^2)) + (2*a*(b*c - a*d)
*Log[x])/c^3 - (a*(b*c - a*d)*Log[c + d*x^2])/c^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{c^2x^2}-\frac{2a(-bc+ad)}{c^3x}+\frac{(bc-ad)^2}{c^2(c+dx)^2}+\frac{2ad(-bc+ad)}{c^3(c+dx)}\right)dx,x,x^2\right)$$
$$= -\frac{a^2}{2c^2x^2}-\frac{(bc-ad)^2}{2c^2d(c+dx^2)}+\frac{2a(bc-ad)\log(x)}{c^3}-\frac{a(bc-ad)\log(c+dx^2)}{c^3}$$

Mathematica [A]
time = 0.07, size = 72, normalized size = 0.89

$$-\frac{\frac{a^2c}{x^2}+\frac{c(bc-ad)^2}{d(c+dx^2)}+4a(-bc+ad)\log(x)-2a(-bc+ad)\log(c+dx^2)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

```
[Out] -1/2*((a^2*c)/x^2 + (c*(b*c - a*d)^2)/(d*(c + d*x^2)) + 4*a*(-(b*c) + a*d)*
Log[x] - 2*a*(-(b*c) + a*d)*Log[c + d*x^2])/c^3
```

Maple [A]
time = 0.09, size = 77, normalized size = 0.95

method	result	size
default	$\frac{(ad-bc)\left(-\frac{c(ad-bc)}{d(dx^2+c)}+2a\ln(dx^2+c)\right)}{2c^3}-\frac{a^2}{2c^2x^2}-\frac{2a(ad-bc)\ln(x)}{c^3}$	77
norman	$\frac{-\frac{a^2}{2c}+\frac{(2a^2d^2-2abcd+b^2c^2)x^4}{2c^3}}{x^2(dx^2+c)}+\frac{a(ad-bc)\ln(dx^2+c)}{c^3}-\frac{2a(ad-bc)\ln(x)}{c^3}$	91
risch	$\frac{-\frac{(2a^2d^2-2abcd+b^2c^2)x^2}{2c^2d}-\frac{a^2}{2c}}{x^2(dx^2+c)}-\frac{2a^2\ln(x)d}{c^3}+\frac{2a\ln(x)b}{c^2}+\frac{a^2\ln(-dx^2-c)d}{c^3}-\frac{a\ln(-dx^2-c)b}{c^2}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^3*(a*d-b*c)*(-c*(a*d-b*c)/d/(d*x^2+c)+2*a*ln(d*x^2+c))-1/2*a^2/c^2/x^
2-2*a*(a*d-b*c)/c^3*ln(x)
```

Maxima [A]
time = 0.27, size = 100, normalized size = 1.23

$$-\frac{a^2cd+(b^2c^2-2\,abcd+2\,a^2d^2)x^2}{2\,(c^2d^2x^4+c^3dx^2)}-\frac{(abc-a^2d)\log(dx^2+c)}{c^3}+\frac{(abc-a^2d)\log(x^2)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*x^2)/(c^2*d^2*x^4 + c^3*d
*x^2) - (a*b*c - a^2*d)*log(d*x^2 + c)/c^3 + (a*b*c - a^2*d)*log(x^2)/c^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(77) = 154$.
time = 1.25, size = 159, normalized size = 1.96

$$-\frac{a^{2} c^{2} d + (b^{2} c^{3} - 2\,abc^{2} d + 2\,a^{2} cd^{2})x^{2} + 2\,((abcd^{2} - a^{2} d^{3})x^{4} + (abc^{2} d - a^{2} cd^{2})x^{2})\log\left(dx^{2} + c\right) - 4\,((abcd^{2} - a^{2} d^{3})x^{4} + (abc^{2} d - a^{2} cd^{2})x^{2})\log\left(x\right)}{2\,(c^{3} d^{2} x^{4} + c^{4} dx^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c^2*d + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x^2 + 2*((a*b*c*d^2
 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x^2)*log(d*x^2 + c) - 4*((a*b*c*d
^2 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x^2)*log(x))/(c^3*d^2*x^4 + c^4
*d*x^2)
```

Sympy [A]
time = 0.82, size = 92, normalized size = 1.14

$$-\frac{2a(ad - bc)\log\left(x\right)}{c^{3}} + \frac{a(ad - bc)\log\left(\frac{c}{d} + x^{2}\right)}{c^{3}} + \frac{-a^{2}cd + x^{2}(-2a^{2}d^{2} + 2abcd - b^{2}c^{2})}{2c^{3}dx^{2} + 2c^{2}d^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**2,x)
```

```
[Out] -2*a*(a*d - b*c)*log(x)/c**3 + a*(a*d - b*c)*log(c/d + x**2)/c**3 + (-a**2*
c*d + x**2*(-2*a**2*d**2 + 2*a*b*c*d - b**2*c**2))/(2*c**3*d*x**2 + 2*c**2*
d**2*x**4)
```

Giac [A]
time = 1.65, size = 109, normalized size = 1.35

$$\frac{(abc - a^{2}d)\log\left(x^{2}\right)}{c^{3}} - \frac{(abcd - a^{2}d^{2})\log\left(|dx^{2} + c|\right)}{c^{3}d} - \frac{b^{2}c^{2}x^{2} - 2\,abcdx^{2} + 2\,a^{2}d^{2}x^{2} + a^{2}cd}{2\,(dx^{4} + cx^{2})c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (a*b*c - a^2*d)*log(x^2)/c^3 - (a*b*c*d - a^2*d^2)*log(abs(d*x^2 + c))/(c^3
*d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 2*a^2*d^2*x^2 + a^2*c*d)/((d*x^4 +
 c*x^2)*c^2*d)
```

Mupad [B]
time = 0.05, size = 100, normalized size = 1.23

$$\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d-a\,b\,c\right)}{c^3}-\frac{\frac{a^2}{2\,c}+\frac{x^2\,\left(2\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^2\,d}}{d\,x^4+c\,x^2}-\frac{\ln\left(x\right)\,\left(2\,a^2\,d-2\,a\,b\,c\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^2),x)
```

```
[Out] (log(c + d*x^2)*(a^2*d - a*b*c))/c^3 - (a^2/(2*c) + (x^2*(2*a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))/(2*c^2*d))/(c*x^2 + d*x^4) - (log(x)*(2*a^2*d - 2*a*b*c))/
c^3
```

3.188 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx$

Optimal. Leaf size=126

$$-\frac{a(6bc-5ad)}{3c^3x}-\frac{a^2}{3cx^3\,(c+dx^2)}+\frac{(3b^2c^2-6abcd+5a^2d^2)\,x}{6c^3\,(c+dx^2)}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}$$

```
[Out] -1/3*a*(-5*a*d+6*b*c)/c^3/x-1/3*a^2/c/x^3/(d*x^2+c)+1/6*(5*a^2*d^2-6*a*b*c*
d+3*b^2*c^2)*x/c^3/(d*x^2+c)+1/2*(-5*a*d+b*c)*(-a*d+b*c)*arctan(x*d^(1/2)/c
^(1/2))/c^(7/2)/d^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {473, 467, 464, 211}

$$\frac{x(5a^2d^2-6abcd+3b^2c^2)}{6c^3\,(c+dx^2)}-\frac{a^2}{3cx^3\,(c+dx^2)}+\frac{(bc-5ad)(bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}-\frac{a(6bc-5ad)}{3c^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

```
[Out] -1/3*(a*(6*b*c - 5*a*d))/(c^3*x) - a^2/(3*c*x^3*(c + d*x^2)) + ((3*b^2*c^2
- 6*a*b*c*d + 5*a^2*d^2)*x)/(6*c^3*(c + d*x^2)) + ((b*c - 5*a*d)*(b*c - a*d
)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
```

```
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx = -\frac{a^2}{3cx^3(c+dx^2)}+\frac{\int \frac{a(6bc-5ad)+3b^2cx^2}{x^2(c+dx^2)^2}\,dx}{3c}$$

$$= -\frac{a^2}{3cx^3(c+dx^2)}+\frac{(3b^2c^2-6abcd+5a^2d^2)\,x}{6c^3(c+dx^2)}-\frac{\int \frac{-\frac{2a(6bc-5ad)}{c}-\left(3b^2-\frac{6abd}{c}+\frac{5a^2d^2}{c^2}\right)x^2}{x^2(c+dx^2)}\,dx}{6c}$$

$$= -\frac{a(6bc-5ad)}{3c^3x}-\frac{a^2}{3cx^3(c+dx^2)}+\frac{(3b^2c^2-6abcd+5a^2d^2)\,x}{6c^3(c+dx^2)}+\frac{((bc-5ad)(bc-ad}{2c^3}$$

$$= -\frac{a(6bc-5ad)}{3c^3x}-\frac{a^2}{3cx^3(c+dx^2)}+\frac{(3b^2c^2-6abcd+5a^2d^2)\,x}{6c^3(c+dx^2)}+\frac{(bc-5ad)(bc-ad)}{2c^{7/2}\sqrt{}}$$

Mathematica [A]
time = 0.05, size = 107, normalized size = 0.85

$$-\frac{a^2}{3c^2x^3}+\frac{2a(-bc+ad)}{c^3x}+\frac{(bc-ad)^2x}{2c^3(c+dx^2)}+\frac{(b^2c^2-6abcd+5a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

```
[Out] -1/3*a^2/(c^2*x^3) + (2*a*(-(b*c) + a*d))/(c^3*x) + ((b*c - a*d)^2*x)/(2*c^
3*(c + d*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(2*c^(7/2)*Sqrt[d])
```

Maple [A]
time = 0.09, size = 107, normalized size = 0.85

method	result
default	$\frac{\frac{\left(\frac{1}{2}a^2d^2-abcd+\frac{1}{2}b^2c^2\right)x}{d\,x^2+c}+\frac{\left(5a^2d^2-6abcd+b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}}{c^3}-\frac{a^2}{3c^2x^3}+\frac{2a(ad-bc)}{c^3x}$
risch	$\frac{\frac{\left(5a^2d^2-6abcd+b^2c^2\right)x^4}{2c^3}+\frac{a(5ad-6bc)x^2}{3c^2}-\frac{a^2}{3c}}{x^3(d\,x^2+c)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^7d\_Z^2+25a^4d^4-60a^3bc\,d^3+46a^2b^2c^2d^2-12a\,b^3c^3d+b^4c^4\right)}\_R\ln\left(\left(3\_R\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*((1/2*a^2*d^2-a*b*c*d+1/2*b^2*c^2)*x/(d*x^2+c)+1/2*(5*a^2*d^2-6*a*b*c
*d+b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))-1/3*a^2/c^2/x^3+2*a*(a*d-b
*c)/c^3/x
```

Maxima [A]
time = 0.51, size = 118, normalized size = 0.94

$$\frac{3\,(b^2c^2-6\,abcd+5\,a^2d^2)x^4-2\,a^2c^2-2\,(6\,abc^2-5\,a^2cd)x^2}{6\,(c^3dx^5+c^4x^3)}+\frac{(b^2c^2-6\,abcd+5\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^4 - 2*a^2*c^2 - 2*(6*a*b*c^2 - 5
*a^2*c*d)*x^2)/(c^3*d*x^5 + c^4*x^3) + 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2
)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
```

Fricas [A]
time = 1.32, size = 356, normalized size = 2.83

$$\left[-\frac{4\,a^2c^3d-6\,(b^2c^3d-6\,abc^2d^2+5\,a^2cd^3)x^4+4\,(6\,abc^3d-5\,a^2c^2d^2)x^2+3\,((b^2c^2d-6\,abcd^2+5\,a^2d^3)x^5+(b^2c^3-6\,abc^2d+5\,a^2cd^2)x^3)\sqrt{-cd}\,\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)}{12\,(c^4d^2x^5+c^5dx^3)},-\frac{2\,a^2c^3d-3\,(b^2c^3d-6\,abc^2d^2+5\,a^2cd^3)x^4+2\,(6\,abc^3d-5\,a^2c^2d^2)x^2-3\,((b^2c^2d-6\,abcd^2+5\,a^2d^3)x^5+(b^2c^3-6\,abc^2d+5\,a^2cd^2)x^3)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{6\,(c^4d^2x^5+c^5dx^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*c^3*d - 6*(b^2*c^3*d - 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^4 + 4*(
6*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2 + 3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)
*x^5 + (b^2*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*sqrt(-c*d)*log((d*x^2 - 2
*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^4*d^2*x^5 + c^5*d*x^3), -1/6*(2*a^2*c^3
```

```
*d - 3*(b^2*c^3*d - 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^4 + 2*(6*a*b*c^3*d - 5*a
^2*c^2*d^2)*x^2 - 3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)*x^5 + (b^2*c^3 -
 6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^4*d^2*
x^5 + c^5*d*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(114) = 228$.

time = 0.54, size = 248, normalized size = 1.97

$$-\frac{\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)\log\left(-\frac{c^4\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)\log\left(\frac{c^4\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{-2a^2c^2+x^4\cdot(15a^2d^2-18abcd+3b^2c^2)+x^2\cdot(10a^2cd-12abc^2)}{6c^4x^3+6c^3dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**2,x)
```

```
[Out] -sqrt(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(-c**4*sqrt(-1/(c**7*d))*(a
*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + sqrt
(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(c**4*sqrt(-1/(c**7*d))*(a*d - b
*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + (-2*a**2*c
**2 + x**4*(15*a**2*d**2 - 18*a*b*c*d + 3*b**2*c**2) + x**2*(10*a**2*c*d -
12*a*b*c**2))/(6*c**4*x**3 + 6*c**3*d*x**5)
```

Giac [A]

time = 1.77, size = 111, normalized size = 0.88

$$\frac{(b^2c^2-6\,abcd+5\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,c^3}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(dx^2+c)c^3}-\frac{6\,abcx^2-6\,a^2dx^2+a^2c}{3\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
 + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c^3) - 1/3*(6*a*b
*c*x^2 - 6*a^2*d*x^2 + a^2*c)/(c^3*x^3)
```

Mupad [B]

time = 0.11, size = 147, normalized size = 1.17

$$\frac{\frac{x^4\,\left(5\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^3}-\frac{a^2}{3\,c}+\frac{a\,x^2\,\left(5\,a\,d-6\,b\,c\right)}{3\,c^2}}{d\,x^5+c\,x^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,\left(a\,d-b\,c\right)\,\left(5\,a\,d-b\,c\right)}{\sqrt{c}\,\left(5\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,\left(a\,d-b\,c\right)\,\left(5\,a\,d-b\,c\right)}{2\,c^{7/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^2),x)
```

```
[Out] ((x^4*(5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(2*c^3) - a^2/(3*c) + (a*x^2*(5*a*
d - 6*b*c))/(3*c^2))/(c*x^3 + d*x^5) + (atan((d^(1/2)*x*(a*d - b*c)*(5*a*d
- b*c))/(c^(1/2)*(5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)))*(a*d - b*c)*(5*a*d - b
*c))/(2*c^(7/2)*d^(1/2))
```

3.189 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{(13b^2c^2-10abcd+a^2d^2)\,x}{4cd^4}+\frac{b^2x^3}{3d^3}+\frac{(bc-ad)^2x^5}{4cd^2\,(c+dx^2)^2}-\frac{(bc-ad)(9bc-ad)x}{8d^4\,(c+dx^2)}+\frac{(35b^2c^2-30abcd+3a^2d^2)\tan}{8\sqrt{c}\,d^{9/2}}$$

[Out] -1/4*(a^2*d^2-10*a*b*c*d+13*b^2*c^2)*x/c/d^4+1/3*b^2*x^3/d^3+1/4*(-a*d+b*c)^2*x^5/c/d^2/(d*x^2+c)^2-1/8*(-a*d+b*c)*(-a*d+9*b*c)*x/d^4/(d*x^2+c)+1/8*(3*a^2*d^2-30*a*b*c*d+35*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/d^(9/2)/c^(1/2)

Rubi [A]
time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {474, 466, 1167, 211}

$$\frac{(3a^2d^2-30abcd+35b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\,d^{9/2}}-\frac{x(a^2d^2-10abcd+13b^2c^2)}{4cd^4}-\frac{x(bc-ad)(9bc-ad)}{8d^4\,(c+dx^2)}+\frac{x^5(bc-ad)^2}{4cd^2\,(c+dx^2)^2}+\frac{b^2x^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -1/4*((13*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*x)/(c*d^4) + (b^2*x^3)/(3*d^3) +
((b*c - a*d)^2*x^5)/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9*b*c - a*d)*x)
/(8*d^4*(c + d*x^2)) + ((35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(8*Sqrt[c]*d^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(bc-ad)^2x^5}{4cd^2(c+dx^2)^2} - \frac{\int \frac{x^4(-4a^2d^2+5(bc-ad)^2-4b^2cdx^2)}{(c+dx^2)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^5}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4(c+dx^2)} + \frac{\int \frac{cd(bc-ad)(9bc-ad)-2d^2(bc-ad)(9bc-ad)x^2+8b^2cd^3x^{\cdots}}{c+dx^2}}{8cd^5}$$

$$= \frac{(bc-ad)^2x^5}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4(c+dx^2)} + \frac{\int \left(-2d(13b^2c^2-10abcd+a^2d^2)+8b^2cd^2\cdots\right.}{8cd^5}$$

$$= -\frac{(13b^2c^2-10abcd+a^2d^2)x}{4cd^4} + \frac{b^2x^3}{3d^3} + \frac{(bc-ad)^2x^5}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4(c+dx^2)} + \frac{(35\cdots}{\cdots}$$

$$= -\frac{(13b^2c^2-10abcd+a^2d^2)x}{4cd^4} + \frac{b^2x^3}{3d^3} + \frac{(bc-ad)^2x^5}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4(c+dx^2)} + \frac{(35\cdots}{\cdots}$$

Mathematica [A]
time = 0.07, size = 148, normalized size = 0.91

$$-\frac{b(3bc-2ad)x}{d^4} + \frac{b^2x^3}{3d^3} + \frac{c(bc-ad)^2x}{4d^4(c+dx^2)^2} - \frac{(13b^2c^2-18abcd+5a^2d^2)x}{8d^4(c+dx^2)} + \frac{(35b^2c^2-30abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\,d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*(3*b*c - 2*a*d)*x)/d^4) + (b^2*x^3)/(3*d^3) + (c*(b*c - a*d)^2*x)/(4*d
^4*(c + d*x^2)^2) - ((13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*x)/(8*d^4*(c + d
```

```
*x^2)) + ((35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(8*Sqrt[c]*d^(9/2))
```

Maple [A]
time = 0.11, size = 137, normalized size = 0.84

method	result
default	$\frac{b\left(\frac{1}{3}bd\,x^{3}+2adx-3bcx\right)}{d^{4}}+\frac{\frac{\left(-\frac{5}{8}a^{2}d^{3}+\frac{9}{4}abc\,d^{2}-\frac{13}{8}b^{2}c^{2}d\right)x^{3}-\frac{c\left(3a^{2}d^{2}-14abcd+11b^{2}c^{2}\right)x}{8}}{\left(d\,x^{2}+c\right)^{2}}+\frac{\left(3a^{2}d^{2}-30abcd+35b^{2}c^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}}}{d^{4}}$
risch	$\frac{b^{2}x^{3}}{3d^{3}}+\frac{2bax}{d^{3}}-\frac{3b^{2}cx}{d^{4}}+\frac{\left(-\frac{5}{8}a^{2}d^{3}+\frac{9}{4}abc\,d^{2}-\frac{13}{8}b^{2}c^{2}d\right)x^{3}-\frac{c\left(3a^{2}d^{2}-14abcd+11b^{2}c^{2}\right)x}{8}}{d^{4}\left(d\,x^{2}+c\right)^{2}}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^{2}}{16d^{2}\sqrt{-cd}}+\frac{15\ln\left(dx+\right.}{8d^{3}\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b/d^4*(1/3*b*d*x^3+2*a*d*x-3*b*c*x)+1/d^4*(((-5/8*a^2*d^3+9/4*a*b*c*d^2-13/
8*b^2*c^2*d)*x^3-1/8*c*(3*a^2*d^2-14*a*b*c*d+11*b^2*c^2)*x)/(d*x^2+c)^2+1/8
*(3*a^2*d^2-30*a*b*c*d+35*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.53, size = 159, normalized size = 0.98

$$-\frac{\left(13\,b^{2}c^{2}d-18\,abcd^{2}+5\,a^{2}d^{3}\right)x^{3}+\left(11\,b^{2}c^{3}-14\,abc^{2}d+3\,a^{2}cd^{2}\right)x}{8\left(d^{6}x^{4}+2\,cd^{5}x^{2}+c^{2}d^{4}\right)}+\frac{\left(35\,b^{2}c^{2}-30\,abcd+3\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,d^{4}}+\frac{b^{2}dx^{3}-3\left(3\,b^{2}c-2\,abd\right)x}{3\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*b^2*c^2*d - 18*a*b*c*d^2 + 5*a^2*d^3)*x^3 + (11*b^2*c^3 - 14*a*b*
c^2*d + 3*a^2*c*d^2)*x)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) + 1/8*(35*b^2*c^2
 - 30*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4) + 1/3*(b^2
*d*x^3 - 3*(3*b^2*c - 2*a*b*d)*x)/d^4
```

Fricas [A]
time = 0.96, size = 522, normalized size = 3.20

$$\left[\frac{16\,b^{2}cd^{4}x^{7}-16\left(7\,b^{2}c^{2}d^{3}-6\,abcd^{4}\right)x^{5}-10\left(35\,b^{2}c^{3}d^{2}-30\,abc^{2}d^{3}+3\,a^{2}cd^{4}\right)x^{3}-3\left(35\,b^{2}c^{4}-30\,abc^{3}d+3\,a^{2}c^{2}d^{2}+\left(35\,b^{2}c^{2}d^{2}-30\,abcd^{3}+3\,a^{2}d^{4}\right)x^{4}+2\left(35\,b^{2}c^{3}d-30\,abc^{2}d^{2}+3\,a^{2}cd^{3}\right)x^{2}\right)\sqrt{-cd}\,\log\left(\frac{dx^{2}-2\sqrt{-cd}\,x-c}{dx^{2}+c}\right)-6\left(35\,b^{2}c^{4}d-30\,abc^{3}d^{2}+3\,a^{2}c^{2}d^{3}\right)x}{48\left(cd^{7}x^{4}+2\,c^{2}d^{6}x^{2}+c^{3}d^{5}\right)},\ \frac{8\,b^{2}cd^{4}x^{7}-8\left(7\,b^{2}c^{2}d^{3}-6\,abcd^{4}\right)x^{5}-5\left(35\,b^{2}c^{3}d^{2}-30\,abc^{2}d^{3}+3\,a^{2}cd^{4}\right)x^{3}+3\left(35\,b^{2}c^{4}-30\,abc^{3}d+3\,a^{2}c^{2}d^{2}+\left(35\,b^{2}c^{2}d^{2}-30\,abcd^{3}+3\,a^{2}d^{4}\right)x^{4}+2\left(35\,b^{2}c^{3}d-30\,abc^{2}d^{2}+3\,a^{2}cd^{3}\right)x^{2}\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-3\left(35\,b^{2}c^{4}d-30\,abc^{3}d^{2}+3\,a^{2}c^{2}d^{3}\right)x}{24\left(cd^{7}x^{4}+2\,c^{2}d^{6}x^{2}+c^{3}d^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*b^2*c*d^4*x^7 - 16*(7*b^2*c^2*d^3 - 6*a*b*c*d^4)*x^5 - 10*(35*b^2
*c^3*d^2 - 30*a*b*c^2*d^3 + 3*a^2*c*d^4)*x^3 - 3*(35*b^2*c^4 - 30*a*b*c^3*d
```

```
 + 3*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(35*
b^2*c^3*d - 30*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sq
rt(-c*d)*x - c)/(d*x^2 + c)) - 6*(35*b^2*c^4*d - 30*a*b*c^3*d^2 + 3*a^2*c^2
*d^3)*x)/(c*d^7*x^4 + 2*c^2*d^6*x^2 + c^3*d^5), 1/24*(8*b^2*c*d^4*x^7 - 8*(
7*b^2*c^2*d^3 - 6*a*b*c*d^4)*x^5 - 5*(35*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 3*a
^2*c*d^4)*x^3 + 3*(35*b^2*c^4 - 30*a*b*c^3*d + 3*a^2*c^2*d^2 + (35*b^2*c^2*
d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 30*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(35*b^2*c^4*d - 30*a*b*
c^3*d^2 + 3*a^2*c^2*d^3)*x)/(c*d^7*x^4 + 2*c^2*d^6*x^2 + c^3*d^5)]
```

Sympy [**A**]
time = 1.20, size = 240, normalized size = 1.47

$$\frac{b^2x^3}{3d^3}+x\left(\frac{2ab}{d^3}-\frac{3b^2c}{d^4}\right)-\frac{\sqrt{-\frac{1}{cd^9}}\cdot(3a^2d^2-30abcd+35b^2c^2)\log\left(-cd^4\sqrt{-\frac{1}{cd^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{cd^9}}\cdot(3a^2d^2-30abcd+35b^2c^2)\log\left(cd^4\sqrt{-\frac{1}{cd^9}}+x\right)}{16}+\frac{x^3(-5a^2d^3+18abcd^2-13b^2c^2d)+x(-3a^2cd^2+14abc^2d-11b^2c^3)}{8c^2d^4+16cd^5x^2+8d^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x**3/(3*d**3) + x*(2*a*b/d**3 - 3*b**2*c/d**4) - sqrt(-1/(c*d**9))*(3*
a**2*d**2 - 30*a*b*c*d + 35*b**2*c**2)*log(-c*d**4*sqrt(-1/(c*d**9)) + x)/1
6 + sqrt(-1/(c*d**9))*(3*a**2*d**2 - 30*a*b*c*d + 35*b**2*c**2)*log(c*d**4*
sqrt(-1/(c*d**9)) + x)/16 + (x**3*(-5*a**2*d**3 + 18*a*b*c*d**2 - 13*b**2*c
**2*d) + x*(-3*a**2*c*d**2 + 14*a*b*c**2*d - 11*b**2*c**3))/(8*c**2*d**4 +
16*c*d**5*x**2 + 8*d**6*x**4)
```

Giac [**A**]
time = 1.43, size = 154, normalized size = 0.94

$$\frac{(35\,b^2c^2-30\,abcd+3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,d^4}-\frac{13\,b^2c^2dx^3-18\,abcd^2x^3+5\,a^2d^3x^3+11\,b^2c^3x-14\,abc^2dx+3\,a^2cd^2x}{8\,(dx^2+c)^2d^4}+\frac{b^2d^6x^3-9\,b^2cd^5x+6\,abd^6x}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*
d^4) - 1/8*(13*b^2*c^2*d*x^3 - 18*a*b*c*d^2*x^3 + 5*a^2*d^3*x^3 + 11*b^2*c^
3*x - 14*a*b*c^2*d*x + 3*a^2*c*d^2*x)/((d*x^2 + c)^2*d^4) + 1/3*(b^2*d^6*x^
3 - 9*b^2*c*d^5*x + 6*a*b*d^6*x)/d^9
```

Mupad [**B**]
time = 0.09, size = 159, normalized size = 0.98

$$\frac{b^2\,x^3}{3\,d^3}-\frac{\left(\frac{5\,a^2\,d^3}{8}-\frac{9\,a\,b\,c\,d^2}{4}+\frac{13\,b^2\,c^2\,d}{8}\right)\,x^3+\left(\frac{3\,a^2\,c\,d^2}{8}-\frac{7\,a\,b\,c^2\,d}{4}+\frac{11\,b^2\,c^3}{8}\right)\,x}{c^2\,d^4+2\,c\,d^5\,x^2+d^6\,x^4}-x\,\left(\frac{3\,b^2\,c}{d^4}-\frac{2\,a\,b}{d^3}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(3\,a^2\,d^2-30\,a\,b\,c\,d+35\,b^2\,c^2\right)}{8\,\sqrt{c}\,d^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (b^2*x^3)/(3*d^3) - (x^3*((5*a^2*d^3)/8 + (13*b^2*c^2*d)/8 - (9*a*b*c*d^2)/
4) + x*((11*b^2*c^3)/8 + (3*a^2*c*d^2)/8 - (7*a*b*c^2*d)/4))/(c^2*d^4 + d^6
*x^4 + 2*c*d^5*x^2) - x*((3*b^2*c)/d^4 - (2*a*b)/d^3) + (atan((d^(1/2)*x)/c
^(1/2))*(3*a^2*d^2 + 35*b^2*c^2 - 30*a*b*c*d))/(8*c^(1/2)*d^(9/2))
```

3.190 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=99

$$\frac{b^2x^2}{2d^3}+\frac{c(bc-ad)^2}{4d^4(c+dx^2)^2}-\frac{(bc-ad)(3bc-ad)}{2d^4(c+dx^2)}-\frac{b(3bc-2ad)\log(c+dx^2)}{2d^4}$$

```
[Out] 1/2*b^2*x^2/d^3+1/4*c*(-a*d+b*c)^2/d^4/(d*x^2+c)^2-1/2*(-a*d+b*c)*(-a*d+3*b
*c)/d^4/(d*x^2+c)-1/2*b*(-2*a*d+3*b*c)*ln(d*x^2+c)/d^4
```

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\frac{c(bc-ad)^2}{4d^4(c+dx^2)^2}-\frac{(3bc-ad)(bc-ad)}{2d^4(c+dx^2)}-\frac{b(3bc-2ad)\log(c+dx^2)}{2d^4}+\frac{b^2x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (b^2*x^2)/(2*d^3) + (c*(b*c - a*d)^2)/(4*d^4*(c + d*x^2)^2) - ((b*c - a*d)*
(3*b*c - a*d))/(2*d^4*(c + d*x^2)) - (b*(3*b*c - 2*a*d)*Log[c + d*x^2])/(2*
d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2}{d^3} - \frac{c(bc-ad)^2}{d^3(c+dx)^3} + \frac{(bc-ad)(3bc-ad)}{d^3(c+dx)^2} - \frac{b(3bc-2ad)}{d^3(c+dx)}\right) dx, x, x^2\right)$$
$$= \frac{b^2x^2}{2d^3} + \frac{c(bc-ad)^2}{4d^4(c+dx^2)^2} - \frac{(bc-ad)(3bc-ad)}{2d^4(c+dx^2)} - \frac{b(3bc-2ad)\log(c+dx^2)}{2d^4}$$

Mathematica [A]
time = 0.04, size = 114, normalized size = 1.15

$$\frac{-a^2d^2(c+2dx^2)+2abcd(3c+4dx^2)+b^2(-5c^3-4c^2dx^2+4cd^2x^4+2d^3x^6)-2b(3bc-2ad)(c+dx^2)^2\log(c+dx^2)}{4d^4(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-(a^2*d^2*(c + 2*d*x^2)) + 2*a*b*c*d*(3*c + 4*d*x^2) + b^2*(-5*c^3 - 4*c^2
*d*x^2 + 4*c*d^2*x^4 + 2*d^3*x^6) - 2*b*(3*b*c - 2*a*d)*(c + d*x^2)^2*Log[c
 + d*x^2])/(4*d^4*(c + d*x^2)^2)
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.14

method	result	size
norman	$\frac{\frac{b^2x^6}{2d}-\frac{c(a^2d^2-6abcd+9b^2c^2)}{4d^4}-\frac{(a^2d^2-4abcd+6b^2c^2)x^2}{2d^3}}{(dx^2+c)^2}+\frac{b(2ad-3bc)\ln(dx^2+c)}{2d^4}$	105
default	$\frac{b^2x^2}{2d^3}+\frac{\frac{c(a^2d^2-2abcd+b^2c^2)}{2d(dx^2+c)^2}-\frac{a^2d^2-4abcd+3b^2c^2}{d(dx^2+c)}+\frac{b(2ad-3bc)\ln(dx^2+c)}{d}}{2d^3}$	113
risch	$\frac{b^2x^2}{2d^3}+\frac{(-\frac{1}{2}a^2d^2+2abcd-\frac{3}{2}b^2c^2)x^2-\frac{c(a^2d^2-6abcd+5b^2c^2)}{4d}}{d^3(dx^2+c)^2}+\frac{b\ln(dx^2+c)a}{d^3}-\frac{3b^2\ln(dx^2+c)c}{2d^4}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2*x^2/d^3+1/2/d^3*(1/2*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c)^2-(a
^2*d^2-4*a*b*c*d+3*b^2*c^2)/d/(d*x^2+c)+b/d*(2*a*d-3*b*c)*ln(d*x^2+c))
```

Maxima [A]
time = 0.28, size = 120, normalized size = 1.21

$$\frac{b^2x^2}{2\,d^3} - \frac{5\,b^2c^3 - 6\,abc^2d + a^2cd^2 + 2\,(3\,b^2c^2d - 4\,abcd^2 + a^2d^3)x^2}{4\,(d^6x^4 + 2\,cd^5x^2 + c^2d^4)} - \frac{(3\,b^2c - 2\,abd)\log(dx^2 + c)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d^3 - 1/4*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3*b^2*c^2*d
 - 4*a*b*c*d^2 + a^2*d^3)*x^2)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) - 1/2*(3*b
^2*c - 2*a*b*d)*log(d*x^2 + c)/d^4
```

Fricas [A]
time = 1.39, size = 178, normalized size = 1.80

$$\frac{2\,b^{2}d^{3}x^{6}+4\,b^{2}cd^{2}x^{4}-5\,b^{2}c^{3}+6\,abc^{2}d-a^{2}cd^{2}-2\,(2\,b^{2}c^{2}d-4\,abcd^{2}+a^{2}d^{3})x^{2}-2\,(3\,b^{2}c^{3}-2\,abc^{2}d+(3\,b^{2}cd^{2}-2\,abd^{3})x^{4}+2\,(3\,b^{2}c^{2}d-2\,abcd^{2})x^{2})\log\left(dx^{2}+c\right)}{4\,(d^{6}x^{4}+2\,cd^{5}x^{2}+c^{2}d^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^3*x^6 + 4*b^2*c*d^2*x^4 - 5*b^2*c^3 + 6*a*b*c^2*d - a^2*c*d^2
- 2*(2*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2 - 2*(3*b^2*c^3 - 2*a*b*c^2*d
+ (3*b^2*c*d^2 - 2*a*b*d^3)*x^4 + 2*(3*b^2*c^2*d - 2*a*b*c*d^2)*x^2)*log(d*
x^2 + c))/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

Sympy [A]
time = 3.63, size = 122, normalized size = 1.23

$$\frac{b^{2}x^{2}}{2d^{3}}+\frac{b(2ad-3bc)\log\left(c+dx^{2}\right)}{2d^{4}}+\frac{-a^{2}cd^{2}+6abc^{2}d-5b^{2}c^{3}+x^{2}(-2a^{2}d^{3}+8abcd^{2}-6b^{2}c^{2}d)}{4c^{2}d^{4}+8cd^{5}x^{2}+4d^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x**2/(2*d**3) + b*(2*a*d - 3*b*c)*log(c + d*x**2)/(2*d**4) + (-a**2*c*
d**2 + 6*a*b*c**2*d - 5*b**2*c**3 + x**2*(-2*a**2*d**3 + 8*a*b*c*d**2 - 6*b
**2*c**2*d))/(4*c**2*d**4 + 8*c*d**5*x**2 + 4*d**6*x**4)
```

Giac [A]
time = 1.06, size = 107, normalized size = 1.08

$$\frac{b^{2}x^{2}}{2\,d^{3}}-\frac{(3\,b^{2}c-2\,abd)\log\left(|dx^{2}+c|\right)}{2\,d^{4}}-\frac{5\,b^{2}c^{3}-6\,abc^{2}d+a^{2}cd^{2}+2\,(3\,b^{2}c^{2}d-4\,abcd^{2}+a^{2}d^{3})x^{2}}{4\,(dx^{2}+c)^{2}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2/d^3 - 1/2*(3*b^2*c - 2*a*b*d)*log(abs(d*x^2 + c))/d^4 - 1/4*(5*
b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)
*x^2)/((d*x^2 + c)^2*d^4)
```

Mupad [B]
time = 0.08, size = 123, normalized size = 1.24

$$\frac{b^2\,x^2}{2\,d^3}-\frac{\ln\left(d\,x^2+c\right)\,\left(3\,b^2\,c-2\,a\,b\,d\right)}{2\,d^4}-\frac{x^2\,\left(\frac{a^2\,d^2}{2}-2\,a\,b\,c\,d+\frac{3\,b^2\,c^2}{2}\right)+\frac{a^2\,c\,d^2-6\,a\,b\,c^2\,d+5\,b^2\,c^3}{4\,d}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (b^2*x^2)/(2*d^3) - (log(c + d*x^2)*(3*b^2*c - 2*a*b*d))/(2*d^4) - (x^2*((a
^2*d^2)/2 + (3*b^2*c^2)/2 - 2*a*b*c*d) + (5*b^2*c^3 + a^2*c*d^2 - 6*a*b*c^2
*d)/(4*d))/(c^2*d^3 + d^5*x^4 + 2*c*d^4*x^2)
```

3.191 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=127

$$\frac{b^2x}{d^3}+\frac{(bc-ad)^2x^3}{4cd^2\left(c+dx^2\right)^2}+\frac{(bc-ad)(7bc+ad)x}{8cd^3\left(c+dx^2\right)}-\frac{\left(15b^2c^2-6abcd-a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}$$

```
[Out] b^2*x/d^3+1/4*(-a*d+b*c)^2*x^3/c/d^2/(d*x^2+c)^2+1/8*(-a*d+b*c)*(a*d+7*b*c)
*x/c/d^3/(d*x^2+c)-1/8*(-a^2*d^2-6*a*b*c*d+15*b^2*c^2)*arctan(x*d^(1/2)/c^(
1/2))/c^(3/2)/d^(7/2)
```

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {474, 466, 396, 211}

$$-\frac{\left(-a^2d^2-6abcd+15b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}+\frac{x(bc-ad)(ad+7bc)}{8cd^3\left(c+dx^2\right)}+\frac{x^3(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}+\frac{b^2x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (b^2*x)/d^3 + ((b*c - a*d)^2*x^3)/(4*c*d^2*(c + d*x^2)^2) + ((b*c - a*d)*(7
*b*c + a*d)*x)/(8*c*d^3*(c + d*x^2)) - ((15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
```

```
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx &= \frac{(bc-ad)^2x^3}{4cd^2(c+dx^2)^2} - \frac{\int \frac{x^2(-4a^2d^2+3(bc-ad)^2-4b^2cdx^2)}{(c+dx^2)^2}\,dx}{4cd^2} \\
&= \frac{(bc-ad)^2x^3}{4cd^2(c+dx^2)^2} + \frac{(bc-ad)(7bc+ad)x}{8cd^3(c+dx^2)} + \frac{\int \frac{-d(bc-ad)(7bc+ad)+8b^2cd^2x^2}{c+dx^2}\,dx}{8cd^4} \\
&= \frac{b^2x}{d^3} + \frac{(bc-ad)^2x^3}{4cd^2(c+dx^2)^2} + \frac{(bc-ad)(7bc+ad)x}{8cd^3(c+dx^2)} - \frac{(15b^2c^2-6abcd-a^2d^2)\int \frac{1}{c+dx^2}\,dx}{8cd^3} \\
&= \frac{b^2x}{d^3} + \frac{(bc-ad)^2x^3}{4cd^2(c+dx^2)^2} + \frac{(bc-ad)(7bc+ad)x}{8cd^3(c+dx^2)} - \frac{(15b^2c^2-6abcd-a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 130, normalized size = 1.02

$$\frac{x(a^2d^2(-c+dx^2)-2abcd(3c+5dx^2)+b^2c(15c^2+25cdx^2+8d^2x^4))}{8cd^3(c+dx^2)^2} - \frac{(15b^2c^2-6abcd-a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(a^2*d^2*(-c + d*x^2) - 2*a*b*c*d*(3*c + 5*d*x^2) + b^2*c*(15*c^2 + 25*c
*d*x^2 + 8*d^2*x^4)))/(8*c*d^3*(c + d*x^2)^2) - ((15*b^2*c^2 - 6*a*b*c*d -
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/2))
```

Maple [A]
time = 0.10, size = 123, normalized size = 0.97

method	result
default	$\frac{b^2x}{d^3}+\frac{\frac{\frac{d\left(a^2d^2-10abcd+9b^2c^2\right)x^3}{8c}+\left(-\frac{1}{8}a^2d^2-\frac{3}{4}abcd+\frac{7}{8}b^2c^2\right)x}{\left(d\,x^2+c\right)^2}+\frac{\left(a^2d^2+6abcd-15b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c\sqrt{cd}}}{d^3}$
risch	$\frac{b^2x}{d^3}+\frac{\frac{d\left(a^2d^2-10abcd+9b^2c^2\right)x^3}{8c}+(-\frac{1}{8}a^2d^2-\frac{3}{4}abcd+\frac{7}{8}b^2c^2)x}{d^3(d\,x^2+c)^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^2}{16d\sqrt{-cd}\,c}-\frac{3\ln\left(dx+\sqrt{-cd}\right)ab}{8d^2\sqrt{-cd}}+\frac{15c\ln\left(dx\right.}{16d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/d^3+1/d^3*((1/8*d*(a^2*d^2-10*a*b*c*d+9*b^2*c^2)/c*x^3+(-1/8*a^2*d^2-
3/4*a*b*c*d+7/8*b^2*c^2)*x)/(d*x^2+c)^2+1/8*(a^2*d^2+6*a*b*c*d-15*b^2*c^2)/
c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.55, size = 143, normalized size = 1.13

$$\frac{(9\,b^2c^2d-10\,abcd^2+a^2d^3)x^3+(7\,b^2c^3-6\,abc^2d-a^2cd^2)x}{8\,(cd^5x^4+2\,c^2d^4x^2+c^3d^3)}+\frac{b^2x}{d^3}-\frac{(15\,b^2c^2-6\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*b^2*c^2*d - 10*a*b*c*d^2 + a^2*d^3)*x^3 + (7*b^2*c^3 - 6*a*b*c^2*d
- a^2*c*d^2)*x)/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3) + b^2*x/d^3 - 1/8*(15
*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(113) = 226.
time = 1.18, size = 475, normalized size = 3.74

$$\left[\frac{16\,b^2c^2d^3x^5+2\,(25\,b^2c^3d^2-10\,abc^2d^3+a^2cd^4)x^3+(15\,b^2c^4-6\,abc^3d-a^2c^2d^2+(15\,b^2c^2d^2-6\,abcd^3-a^2d^4)x^4+2\,(15\,b^2c^3d-6\,abc^2d^2-a^2cd^3)x^2)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\,(15\,b^2c^4d-6\,abc^3d^2-a^2c^2d^3)x}{16\,(c^2d^6x^4+2\,c^3d^5x^2+c^4d^4)},\ \frac{8\,b^2c^2d^3x^5+(25\,b^2c^3d^2-10\,abc^2d^3+a^2cd^4)x^3-(15\,b^2c^4-6\,abc^3d-a^2c^2d^2+(15\,b^2c^2d^2-6\,abcd^3-a^2d^4)x^4+2\,(15\,b^2c^3d-6\,abc^2d^2-a^2cd^3)x^2)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+(15\,b^2c^4d-6\,abc^3d^2-a^2c^2d^3)x}{8\,(c^2d^6x^4+2\,c^3d^5x^2+c^4d^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^2*c^2*d^3*x^5 + 2*(25*b^2*c^3*d^2 - 10*a*b*c^2*d^3 + a^2*c*d^4)
*x^3 + (15*b^2*c^4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 6*a*b*c*
d^3 - a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt
(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(15*b^2*c^4*d - 6*
a*b*c^3*d^2 - a^2*c^2*d^3)*x)/(c^2*d^6*x^4 + 2*c^3*d^5*x^2 + c^4*d^4), 1/8*
(8*b^2*c^2*d^3*x^5 + (25*b^2*c^3*d^2 - 10*a*b*c^2*d^3 + a^2*c*d^4)*x^3 - (1
```

```
5*b^2*c^4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 6*a*b*c*d^3 - a^2
*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) + (15*b^2*c^4*d - 6*a*b*c^3*d^2 - a^2*c^2*d^3)*x)/(c^2*d
^6*x^4 + 2*c^3*d^5*x^2 + c^4*d^4)]
```

Sympy [A]
time = 1.23, size = 223, normalized size = 1.76

$$\frac{b^2x}{d^3} - \frac{\sqrt{-\frac{1}{c^3d^7}}\,(a^2d^2 + 6abcd - 15b^2c^2)\log\left(-c^2d^3\sqrt{-\frac{1}{c^3d^7}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{c^3d^7}}\,(a^2d^2 + 6abcd - 15b^2c^2)\log\left(c^2d^3\sqrt{-\frac{1}{c^3d^7}} + x\right)}{16} + \frac{x^3(a^2d^3 - 10abcd^2 + 9b^2c^2d) + x(-a^2cd^2 - 6abc^2d + 7b^2c^3)}{8c^3d^3 + 16c^2d^4x^2 + 8cd^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x/d**3 - sqrt(-1/(c**3*d**7))*(a**2*d**2 + 6*a*b*c*d - 15*b**2*c**2)*l
og(-c**2*d**3*sqrt(-1/(c**3*d**7)) + x)/16 + sqrt(-1/(c**3*d**7))*(a**2*d**
2 + 6*a*b*c*d - 15*b**2*c**2)*log(c**2*d**3*sqrt(-1/(c**3*d**7)) + x)/16 +
(x**3*(a**2*d**3 - 10*a*b*c*d**2 + 9*b**2*c**2*d) + x*(-a**2*c*d**2 - 6*a*b
*c**2*d + 7*b**2*c**3))/(8*c**3*d**3 + 16*c**2*d**4*x**2 + 8*c*d**5*x**4)
```

Giac [A]
time = 1.18, size = 133, normalized size = 1.05

$$\frac{b^2x}{d^3} - \frac{(15\,b^2c^2 - 6\,abcd - a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,cd^3} + \frac{9\,b^2c^2dx^3 - 10\,abcd^2x^3 + a^2d^3x^3 + 7\,b^2c^3x - 6\,abc^2dx - a^2cd^2x}{8\,(dx^2 + c)^2cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^2*x/d^3 - 1/8*(15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(s
qrt(c*d)*c*d^3) + 1/8*(9*b^2*c^2*d*x^3 - 10*a*b*c*d^2*x^3 + a^2*d^3*x^3 + 7
*b^2*c^3*x - 6*a*b*c^2*d*x - a^2*c*d^2*x)/((d*x^2 + c)^2*c*d^3)
```

Mupad [B]
time = 0.10, size = 135, normalized size = 1.06

$$\frac{b^2\,x}{d^3} - \frac{x\,\left(\frac{a^2\,d^2}{8} + \frac{3\,a\,b\,c\,d}{4} - \frac{7\,b^2\,c^2}{8}\right) - \frac{x^3\,\left(a^2\,d^3 - 10\,a\,b\,c\,d^2 + 9\,b^2\,c^2\,d\right)}{8\,c}}{c^2\,d^3 + 2\,c\,d^4\,x^2 + d^5\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(a^2\,d^2 + 6\,a\,b\,c\,d - 15\,b^2\,c^2\right)}{8\,c^{3/2}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (b^2*x)/d^3 - (x*((a^2*d^2)/8 - (7*b^2*c^2)/8 + (3*a*b*c*d)/4) - (x^3*(a^2*
d^3 + 9*b^2*c^2*d - 10*a*b*c*d^2))/(8*c))/(c^2*d^3 + d^5*x^4 + 2*c*d^4*x^2)
 + (atan((d^(1/2)*x)/c^(1/2))*(a^2*d^2 - 15*b^2*c^2 + 6*a*b*c*d))/(8*c^(3/2
)*d^(7/2))
```

3.192 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{(bc-ad)^2}{4d^3(c+dx^2)^2}+\frac{b(bc-ad)}{d^3(c+dx^2)}+\frac{b^2\log(c+dx^2)}{2d^3}$$

[Out] -1/4*(-a*d+b*c)^2/d^3/(d*x^2+c)^2+b*(-a*d+b*c)/d^3/(d*x^2+c)+1/2*b^2*ln(d*x^2+c)/d^3

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 45}

$$\frac{b(bc-ad)}{d^3(c+dx^2)}-\frac{(bc-ad)^2}{4d^3(c+dx^2)^2}+\frac{b^2\log(c+dx^2)}{2d^3}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]

[Out] -1/4*(b*c - a*d)^2/(d^3*(c + d*x^2)^2) + (b*(b*c - a*d))/(d^3*(c + d*x^2)) + (b^2*Log[c + d*x^2])/(2*d^3)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^3} - \frac{2b(bc-ad)}{d^2(c+dx)^2} + \frac{b^2}{d^2(c+dx)}\right) dx, x, x^2\right)$$
$$= -\frac{(bc-ad)^2}{4d^3(c+dx^2)^2} + \frac{b(bc-ad)}{d^3(c+dx^2)} + \frac{b^2\log(c+dx^2)}{2d^3}$$

Mathematica [A]
time = 0.02, size = 75, normalized size = 1.12

$$\frac{-a^2d^2 - 2abd(c+2dx^2) + b^2c(3c+4dx^2) + 2b^2(c+dx^2)^2\log(c+dx^2)}{4d^3(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-(a^2*d^2) - 2*a*b*d*(c + 2*d*x^2) + b^2*c*(3*c + 4*d*x^2) + 2*b^2*(c + d*
x^2)^2*Log[c + d*x^2])/(4*d^3*(c + d*x^2)^2)
```

Maple [A]
time = 0.09, size = 76, normalized size = 1.13

method	result	size
risch	$\frac{-\frac{b(ad-bc)x^2}{d^2} - \frac{a^2d^2+2abcd-3b^2c^2}{4d^3}}{(dx^2+c)^2} + \frac{b^2\ln(dx^2+c)}{2d^3}$	73
norman	$\frac{-\frac{a^2d^2+2abcd-3b^2c^2}{4d^3} - \frac{(abd-b^2c)x^2}{d^2}}{(dx^2+c)^2} + \frac{b^2\ln(dx^2+c)}{2d^3}$	75
default	$-\frac{a^2d^2-2abcd+b^2c^2}{4d^3(dx^2+c)^2} - \frac{b(ad-bc)}{d^3(dx^2+c)} + \frac{b^2\ln(dx^2+c)}{2d^3}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(d*x^2+c)^2-b*(a*d-b*c)/d^3/(d*x^2+c)+
1/2*b^2*ln(d*x^2+c)/d^3
```

Maxima [A]
time = 0.29, size = 87, normalized size = 1.30

$$\frac{3\,b^2c^2 - 2\,abcd - a^2d^2 + 4\,(b^2cd - abd^2)x^2}{4\,(d^5x^4 + 2\,cd^4x^2 + c^2d^3)} + \frac{b^2\log(dx^2+c)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2)/(d^5*x^4
+ 2*c*d^4*x^2 + c^2*d^3) + 1/2*b^2*log(d*x^2 + c)/d^3
```

Fricas [A]
time = 0.91, size = 108, normalized size = 1.61

$$\frac{3\,b^2c^2-2\,abcd-a^2d^2+4\,(b^2cd-abd^2)x^2+2\,(b^2d^2x^4+2\,b^2cdx^2+b^2c^2)\log{(dx^2+c)}}{4\,(d^5x^4+2\,cd^4x^2+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d
^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(d^5*x^4 + 2*c*d^4*x^2 +
c^2*d^3)
```

Sympy [A]
time = 0.89, size = 87, normalized size = 1.30

$$\frac{b^2\log{(c+dx^2)}}{2d^3}+\frac{-a^2d^2-2abcd+3b^2c^2+x^2(-4abd^2+4b^2cd)}{4c^2d^3+8cd^4x^2+4d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*log(c + d*x**2)/(2*d**3) + (-a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2 + x**
2*(-4*a*b*d**2 + 4*b**2*c*d))/(4*c**2*d**3 + 8*c*d**4*x**2 + 4*d**5*x**4)
```

Giac [A]
time = 0.69, size = 76, normalized size = 1.13

$$\frac{b^2\log{(|dx^2+c|)}}{2\,d^3}+\frac{4\,(b^2c-abd)x^2+\frac{3\,b^2c^2-2\,abcd-a^2d^2}{d}}{4\,(dx^2+c)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(abs(d*x^2 + c))/d^3 + 1/4*(4*(b^2*c - a*b*d)*x^2 + (3*b^2*c^2 -
 2*a*b*c*d - a^2*d^2)/d)/((d*x^2 + c)^2*d^2)
```

Mupad [B]
time = 0.04, size = 83, normalized size = 1.24

$$\frac{b^2\,\ln{(d\,x^2+c)}}{2\,d^3}-\frac{\frac{a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2}{4\,d^3}+\frac{b\,x^2\,(a\,d-b\,c)}{d^2}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (b^2*log(c + d*x^2))/(2*d^3) - ((a^2*d^2 - 3*b^2*c^2 + 2*a*b*c*d)/(4*d^3) +
 (b*x^2*(a*d - b*c))/d^2)/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

3.193 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx$

Optimal. Leaf size=116

$$-\frac{(bc-ad)x(a+bx^2)}{4cd\,(c+dx^2)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\,(c+dx^2)}+\frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^2+a)/c/d/(d*x^2+c)^2+3/8*(a^2/c^2-b^2/d^2)*x/(d*x^2+
c)+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/d^
(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {424, 393, 211}

$$\frac{(3a^2d^2+2abcd+3b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}+\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\,(c+dx^2)}-\frac{x(a+bx^2)\,(bc-ad)}{4cd\,(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -1/4*((b*c - a*d)*x*(a + b*x^2))/(c*d*(c + d*x^2)^2) + (3*(a^2/c^2 - b^2/d^
2)*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx &= -\frac{(bc-ad)x(a+bx^2)}{4cd\,(c+dx^2)^2}+\frac{\int \frac{a(bc+3ad)+b(3bc+ad)x^2}{(c+dx^2)^2}\,dx}{4cd}\\ &= -\frac{(bc-ad)x(a+bx^2)}{4cd\,(c+dx^2)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\,(c+dx^2)}+\frac{(3b^2c^2+2abcd+3a^2d^2)\int\frac{1}{c+dx^2}\,dx}{8c^2d^2}\\ &= -\frac{(bc-ad)x(a+bx^2)}{4cd\,(c+dx^2)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\,(c+dx^2)}+\frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 121, normalized size = 1.04

$$\frac{x(-2abcd(c-dx^2)+a^2d^2(5c+3dx^2)-b^2c^2(3c+5dx^2))}{8c^2d^2\,(c+dx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] (x*(-2*a*b*c*d*(c - d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*(3*c + 5*d*x
^2)))/(8*c^2*d^2*(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Maple [A]
time = 0.06, size = 124, normalized size = 1.07

method	result
default	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8d^2c}}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2d^2\sqrt{cd}}$
risch	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8d^2c}}{\left(dx^2+c\right)^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2}{16\sqrt{-cd}\,c^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)ab}{8\sqrt{-cd}\,dc}-\frac{3\ln\left(dx+\sqrt{-cd}\right)b^2}{16\sqrt{-cd}\,d^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a*b*c*d-3*b
^2*c^2)/d^2/c*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/c^2/d^2/(c
*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.53, size = 138, normalized size = 1.19

$$-\frac{(5\,b^2c^2d-2\,abcd^2-3\,a^2d^3)x^3+(3\,b^2c^3+2\,abc^2d-5\,a^2cd^2)x}{8\,(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2)}+\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2*c^3 + 2*a*b*c^2*
d - 5*a^2*c*d^2)*x)/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1/8*(3*b^2*c^
2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(102) = 204$.
time = 0.97, size = 449, normalized size = 3.87

$$\left[-\frac{2\,(5\,b^2c^3d^2-2\,abc^2d^3-3\,a^2cd^4)x^3+(3\,b^2c^4+2\,abc^3d+3\,a^2c^2d^2+(3\,b^2c^2d^2+2\,abcd^3+3\,a^2d^4)x^4+2\,(3\,b^2c^3d+2\,abc^2d^2+3\,a^2cd^3)x^2)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\,(3\,b^2c^4d+2\,abc^3d^2-5\,a^2c^2d^3)x}{16\,(c^3d^5x^4+2\,c^4d^4x^2+c^5d^3)},-\frac{(5\,b^2c^3d^2-2\,abc^2d^3-3\,a^2cd^4)x^3-(3\,b^2c^4+2\,abc^3d+3\,a^2c^2d^2+(3\,b^2c^2d^2+2\,abcd^3+3\,a^2d^4)x^4+2\,(3\,b^2c^3d+2\,abc^2d^2+3\,a^2cd^3)x^2)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+(3\,b^2c^4d+2\,abc^3d^2-5\,a^2c^2d^3)x}{8\,(c^3d^5x^4+2\,c^4d^4x^2+c^5d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 + (3*b^2*c^4 +
2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2
 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(3*b^2*c^4*d + 2*a*b*c^3*d^2 - 5*a^
2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^3), -1/8*((5*b^2*c^3*d^2
 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 - (3*b^2*c^4 + 2*a*b*c^3*d + 3*a^2*c^2*
d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 2*a*
b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (3*b^2*c^4*
d + 2*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^
3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.
time = 0.55, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(-c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{x^3\cdot(3a^2d^3+2abcd^2-5b^2c^2d)+x(5a^2cd^2-2abc^2d-3b^2c^3)}{8c^4d^2+16c^3d^3x^2+8c^2d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-c**3*d**
2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + (x**3*(3*a*
*2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2*d -
 3*b**2*c**3))/(8*c**4*d**2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

Giac [A]
time = 1.27, size = 126, normalized size = 1.09

$$\frac{(3\,b^2c^2 + 2\,abcd + 3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^2d^2} - \frac{5\,b^2c^2dx^3 - 2\,abcd^2x^3 - 3\,a^2d^3x^3 + 3\,b^2c^3x + 2\,abc^2dx - 5\,a^2cd^2x}{8\,(dx^2+c)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^
2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2*d^3*x^3 + 3*b^2*c^3
*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 + c)^2*c^2*d^2)
```

Mupad [B]
time = 0.11, size = 130, normalized size = 1.12

$$\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c^{5/2}\,d^{5/2}} - \frac{\frac{x\,\left(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c\,d^2} - \frac{x^3\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2\right)}{8\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^3,x)
```

```
[Out] (atan((d^(1/2)*x)/c^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*c^(5/2)*
d^(5/2)) - ((x*(3*b^2*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*c*d^2) - (x^3*(3*a^2
*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

3.194 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx$

Optimal. Leaf size=86

$$\frac{(bc-ad)^2}{4cd^2(c+dx^2)^2}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2(c+dx^2)}+\frac{a^2\log(x)}{c^3}-\frac{a^2\log(c+dx^2)}{2c^3}$$

```
[Out] 1/4*(-a*d+b*c)^2/c/d^2/(d*x^2+c)^2+1/2*(a^2/c^2-b^2/d^2)/(d*x^2+c)+a^2*ln(x
)/c^3-1/2*a^2*ln(d*x^2+c)/c^3
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 90}

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2(c+dx^2)}-\frac{a^2\log(c+dx^2)}{2c^3}+\frac{a^2\log(x)}{c^3}+\frac{(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

```
[Out] (b*c - a*d)^2/(4*c*d^2*(c + d*x^2)^2) + (a^2/c^2 - b^2/d^2)/(2*(c + d*x^2))
 + (a^2*Log[x])/c^3 - (a^2*Log[c + d*x^2])/(2*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{a^2}{c^3x} - \frac{(bc-ad)^2}{cd(c+dx)^3} + \frac{b^2c^2-a^2d^2}{c^2d(c+dx)^2} - \frac{a^2d}{c^3(c+dx)}\right)dx, x, x^2\right)$$
$$= \frac{(bc-ad)^2}{4cd^2(c+dx^2)^2} + \frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2(c+dx^2)} + \frac{a^2\log(x)}{c^3} - \frac{a^2\log(c+dx^2)}{2c^3}$$

Mathematica [A]
time = 0.04, size = 103, normalized size = 1.20

$$\frac{b^2c^2-2abcd+a^2d^2}{4cd^2(c+dx^2)^2} + \frac{-b^2c^2+a^2d^2}{2c^2d^2(c+dx^2)} + \frac{a^2\log(x)}{c^3} - \frac{a^2\log(c+dx^2)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(4*c*d^2*(c + d*x^2)^2) + (-(b^2*c^2) + a^2
*d^2)/(2*c^2*d^2*(c + d*x^2)) + (a^2*Log[x])/c^3 - (a^2*Log[c + d*x^2])/(2*
c^3)
```

Maple [A]
time = 0.09, size = 98, normalized size = 1.14

method	result	size
norman	$\frac{-\frac{(a^2d-abc)x^2}{c^2}-\frac{(3a^2d^2-2abcd-b^2c^2)x^4}{4c^3}}{(dx^2+c)^2}+\frac{a^2\ln(x)}{c^3}-\frac{a^2\ln(dx^2+c)}{2c^3}$	88
risch	$\frac{\frac{(a^2d^2-b^2c^2)x^2}{2c^2d}+\frac{3a^2d^2-2abcd-b^2c^2}{4d^2c}}{(dx^2+c)^2}+\frac{a^2\ln(x)}{c^3}-\frac{a^2\ln(dx^2+c)}{2c^3}$	96
default	$-\frac{-\frac{c^2(a^2d^2-2abcd+b^2c^2)}{2d^2(dx^2+c)^2}-\frac{c(a^2d^2-b^2c^2)}{d^2(dx^2+c)}+a^2\ln(dx^2+c)}{2c^3}+\frac{a^2\ln(x)}{c^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^3*(-1/2*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2/(d*x^2+c)^2-c*(a^2*d^2-b
^2*c^2)/d^2/(d*x^2+c)+a^2*ln(d*x^2+c))+a^2*ln(x)/c^3
```

Maxima [A]
time = 0.37, size = 109, normalized size = 1.27

$$-\frac{b^2c^3+2\,abc^2d-3\,a^2cd^2+2\,(b^2c^2d-a^2d^3)x^2}{4\,(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2)} - \frac{a^2\log(dx^2+c)}{2\,c^3} + \frac{a^2\log(x^2)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^2 + 2*(b^2*c^2*d - a^2*d^3)*x^2)/(c
^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) - 1/2*a^2*log(d*x^2 + c)/c^3 + 1/2*a^
2*log(x^2)/c^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(80) = 160$.
time = 1.03, size = 163, normalized size = 1.90

$$-\frac{b^2c^4 + 2\,abc^3d - 3\,a^2c^2d^2 + 2\,(b^2c^3d - a^2cd^3)x^2 + 2\,(a^2d^4x^4 + 2\,a^2cd^3x^2 + a^2c^2d^2)\log\left(dx^2 + c\right) - 4\,(a^2d^4x^4 + 2\,a^2cd^3x^2 + a^2c^2d^2)\log\left(x\right)}{4\,(c^3d^4x^4 + 2\,c^4d^3x^2 + c^5d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^4 + 2*a*b*c^3*d - 3*a^2*c^2*d^2 + 2*(b^2*c^3*d - a^2*c*d^3)*x^2
 + 2*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(d*x^2 + c) - 4*(a^2*
d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(x))/(c^3*d^4*x^4 + 2*c^4*d^3*x
^2 + c^5*d^2)
```

Sympy [A]
time = 0.70, size = 107, normalized size = 1.24

$$\frac{a^2\log\left(x\right)}{c^3} - \frac{a^2\log\left(\frac{c}{d} + x^2\right)}{2c^3} + \frac{3a^2cd^2 - 2abc^2d - b^2c^3 + x^2\cdot\left(2a^2d^3 - 2b^2c^2d\right)}{4c^4d^2 + 8c^3d^3x^2 + 4c^2d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**3,x)
```

```
[Out] a**2*log(x)/c**3 - a**2*log(c/d + x**2)/(2*c**3) + (3*a**2*c*d**2 - 2*a*b*c
**2*d - b**2*c**3 + x**2*(2*a**2*d**3 - 2*b**2*c**2*d))/(4*c**4*d**2 + 8*c*
*3*d**3*x**2 + 4*c**2*d**4*x**4)
```

Giac [A]
time = 1.73, size = 110, normalized size = 1.28

$$\frac{a^2\log\left(x^2\right)}{2\,c^3} - \frac{a^2\log\left(|dx^2 + c|\right)}{2\,c^3} + \frac{3\,a^2d^4x^4 - 2\,b^2c^3dx^2 + 8\,a^2cd^3x^2 - b^2c^4 - 2\,abc^3d + 6\,a^2c^2d^2}{4\,\left(dx^2 + c\right)^2c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(x^2)/c^3 - 1/2*a^2*log(abs(d*x^2 + c))/c^3 + 1/4*(3*a^2*d^4*x^4
 - 2*b^2*c^3*d*x^2 + 8*a^2*c*d^3*x^2 - b^2*c^4 - 2*a*b*c^3*d + 6*a^2*c^2*d^
2)/((d*x^2 + c)^2*c^3*d^2)
```

Mupad [B]
time = 0.07, size = 106, normalized size = 1.23

$$\frac{a^2\,\ln\left(x\right)}{c^3}-\frac{a^2\,\ln\left(d\,x^2+c\right)}{2\,c^3}-\frac{\frac{-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2}{4\,c\,d^2}-\frac{x^2\,\left(a^2\,d^2-b^2\,c^2\right)}{2\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^3),x)
```

```
[Out] (a^2*log(x))/c^3 - (a^2*log(c + d*x^2))/(2*c^3) - ((b^2*c^2 - 3*a^2*d^2 + 2
*a*b*c*d)/(4*c*d^2) - (x^2*(a^2*d^2 - b^2*c^2))/(2*c^2*d))/(c^2 + d^2*x^4 +
 2*c*d*x^2)
```

3.195 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{a^2}{cx\,(c+dx^2)^2}-\frac{(b^2c^2-2abcd+5a^2d^2)\,x}{4c^2d\,(c+dx^2)^2}+\frac{(b^2c^2+3ad(2bc-5ad))\,x}{8c^3d\,(c+dx^2)}+\frac{(b^2c^2+3ad(2bc-5ad))\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}$$

```
[Out] -a^2/c/x/(d*x^2+c)^2-1/4*(5*a^2*d^2-2*a*b*c*d+b^2*c^2)*x/c^2/d/(d*x^2+c)^2+
1/8*(b^2*c^2+3*a*d*(-5*a*d+2*b*c))*x/c^3/d/(d*x^2+c)+1/8*(b^2*c^2+3*a*d*(-5
*a*d+2*b*c))*arctan(x*d^(1/2)/c^(1/2))/c^(7/2)/d^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 149, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {473, 393, 205, 211}

$$\frac{x\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4c\,(c+dx^2)^2}-\frac{a^2}{cx\,(c+dx^2)^2}+\frac{(3ad(2bc-5ad)+b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}+\frac{x\left(\frac{3a(2bc-5ad)}{c^2}+\frac{b^2}{d}\right)}{8c\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2)^2)) + ((2*a*b - (b^2*c)/d - (5*a^2*d)/c)*x)/(4*c*(c
+ d*x^2)^2) + ((b^2/d + (3*a*(2*b*c - 5*a*d))/c^2)*x)/(8*c*(c + d*x^2)) + (
(b^2*c^2 + 3*a*d*(2*b*c - 5*a*d))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*d
^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx &= -\frac{a^2}{cx(c+dx^2)^2}+\frac{\int \frac{a(2bc-5ad)+b^2cx^2}{(c+dx^2)^3}\,dx}{c}\\
&= -\frac{a^2}{cx(c+dx^2)^2}-\frac{(b^2c^2-2abcd+5a^2d^2)\,x}{4c^2d(c+dx^2)^2}+\frac{1}{4}\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)\int \frac{1}{(c+dx^2)^2}\,d\\
&= -\frac{a^2}{cx(c+dx^2)^2}-\frac{(b^2c^2-2abcd+5a^2d^2)\,x}{4c^2d(c+dx^2)^2}+\frac{\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)x}{8c(c+dx^2)}+\frac{\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right.}{8c}\\
&= -\frac{a^2}{cx(c+dx^2)^2}-\frac{(b^2c^2-2abcd+5a^2d^2)\,x}{4c^2d(c+dx^2)^2}+\frac{\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)x}{8c(c+dx^2)}+\frac{(b^2c^2+6abcd-1}{8c}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 133, normalized size = 0.88

$$-\frac{a^2}{c^3x}-\frac{(bc-ad)^2x}{4c^2d(c+dx^2)^2}+\frac{(b^2c^2+6abcd-7a^2d^2)\,x}{8c^3d(c+dx^2)}+\frac{(b^2c^2+6abcd-15a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

```
[Out] -(a^2/(c^3*x)) - ((b*c - a*d)^2*x)/(4*c^2*d*(c + d*x^2)^2) + ((b^2*c^2 + 6*
a*b*c*d - 7*a^2*d^2)*x)/(8*c^3*d*(c + d*x^2)) + ((b^2*c^2 + 6*a*b*c*d - 15*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*d^(3/2))
```

Maple [A]
time = 0.11, size = 128, normalized size = 0.84

method	result
default	$-\frac{\frac{\left(\frac{7}{8}a^2d^2-\frac{3}{4}abcd-\frac{1}{8}b^2c^2\right)x^3+\frac{c\left(9a^2d^2-10abcd+b^2c^2\right)x}{8d}}{\left(dx^2+c\right)^2}+\frac{\left(15a^2d^2-6abcd-b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8d\sqrt{cd}}}{c^3}-\frac{a^2}{c^3x}$
risch	$\frac{-\frac{\left(15a^2d^2-6abcd-b^2c^2\right)x^4}{8c^3}-\frac{\left(25a^2d^2-10abcd+b^2c^2\right)x^2}{8c^2d}-\frac{a^2}{c}}{x(dx^2+c)^2}-\frac{15d\ln\left(-\sqrt{-cd}\,x-c\right)a^2}{16\sqrt{-cd}\,c^3}+\frac{3\ln\left(-\sqrt{-cd}\,x-c\right)ab}{8\sqrt{-cd}\,c^2}+\frac{\ln\left(-\sqrt{\right.}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*((((7/8*a^2*d^2-3/4*a*b*c*d-1/8*b^2*c^2)*x^3+1/8*c*(9*a^2*d^2-10*a*b*
c*d+b^2*c^2)/d*x)/(d*x^2+c)^2+1/8*(15*a^2*d^2-6*a*b*c*d-b^2*c^2)/d/(c*d)^(1
/2)*arctan(d*x/(c*d)^(1/2)))-a^2/c^3/x
```

Maxima [A]
time = 0.55, size = 146, normalized size = 0.96

$$-\frac{8\,a^2c^2d-(b^2c^2d+6\,abcd^2-15\,a^2d^3)x^4+(b^2c^3-10\,abc^2d+25\,a^2cd^2)x^2}{8\,(c^3d^3x^5+2\,c^4d^2x^3+c^5dx)}+\frac{(b^2c^2+6\,abcd-15\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^2*c^2*d - (b^2*c^2*d + 6*a*b*c*d^2 - 15*a^2*d^3)*x^4 + (b^2*c^3 -
 10*a*b*c^2*d + 25*a^2*c*d^2)*x^2)/(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)
+ 1/8*(b^2*c^2 + 6*a*b*c*d - 15*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c
^3*d)
```

Fricas [A]
time = 1.47, size = 475, normalized size = 3.12

$$\left[-\frac{16\,a^2c^3d^2-2\,(b^2c^3d^2+6\,abc^2d^3-15\,a^2cd^4)x^4+2\,(b^2c^4d-10\,abc^3d^2+25\,a^2c^2d^3)x^2-((b^2c^2d^2+6\,abcd^3-15\,a^2d^4)x^5+2\,(b^2c^3d+6\,abc^2d^2-15\,a^2cd^3)x^3+(b^2c^4+6\,abc^3d-15\,a^2c^2d^2)x)\sqrt{-cd}\,\log\left(\frac{dx^2+2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)}{16\,(c^4d^4x^5+2\,c^5d^3x^3+c^6d^2x)},\ -\frac{8\,a^2c^3d^2-(b^2c^3d^2+6\,abc^2d^3-15\,a^2cd^4)x^4+(b^2c^4d-10\,abc^3d^2+25\,a^2c^2d^3)x^2-((b^2c^2d^2+6\,abcd^3-15\,a^2d^4)x^5+2\,(b^2c^3d+6\,abc^2d^2-15\,a^2cd^3)x^3+(b^2c^4+6\,abc^3d-15\,a^2c^2d^2)x)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{8\,(c^4d^4x^5+2\,c^5d^3x^3+c^6d^2x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^2*c^3*d^2 - 2*(b^2*c^3*d^2 + 6*a*b*c^2*d^3 - 15*a^2*c*d^4)*x^4
 + 2*(b^2*c^4*d - 10*a*b*c^3*d^2 + 25*a^2*c^2*d^3)*x^2 - ((b^2*c^2*d^2 + 6*
a*b*c*d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 6*a*b*c^2*d^2 - 15*a^2*c*d^3)*
x^3 + (b^2*c^4 + 6*a*b*c^3*d - 15*a^2*c^2*d^2)*x)*sqrt(-c*d)*log((d*x^2 + 2
*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^4*d^4*x^5 + 2*c^5*d^3*x^3 + c^6*d^2*x),
 -1/8*(8*a^2*c^3*d^2 - (b^2*c^3*d^2 + 6*a*b*c^2*d^3 - 15*a^2*c*d^4)*x^4 + (
```

```
b^2*c^4*d - 10*a*b*c^3*d^2 + 25*a^2*c^2*d^3)*x^2 - ((b^2*c^2*d^2 + 6*a*b*c*
d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 6*a*b*c^2*d^2 - 15*a^2*c*d^3)*x^3 +
(b^2*c^4 + 6*a*b*c^3*d - 15*a^2*c^2*d^2)*x)*sqrt(c*d)*arctan(sqrt(c*d)*x/c)
)/(c^4*d^4*x^5 + 2*c^5*d^3*x^3 + c^6*d^2*x)]
```

Sympy [A]
time = 0.73, size = 224, normalized size = 1.47

$$\frac{\sqrt{-\frac{1}{c^7d^3}}\cdot(15a^2d^2-6abcd-b^2c^2)\log\left(-c^4d\sqrt{-\frac{1}{c^7d^3}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{c^7d^3}}\cdot(15a^2d^2-6abcd-b^2c^2)\log\left(c^4d\sqrt{-\frac{1}{c^7d^3}}+x\right)}{16}+\frac{-8a^2c^2d+x^4(-15a^2d^3+6abcd^2+b^2c^2d)+x^2(-25a^2cd^2+10abc^2d-b^2c^3)}{8c^5dx+16c^4d^2x^3+8c^3d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**3,x)
```

```
[Out] sqrt(-1/(c**7*d**3))*(15*a**2*d**2 - 6*a*b*c*d - b**2*c**2)*log(-c**4*d*sqr
t(-1/(c**7*d**3)) + x)/16 - sqrt(-1/(c**7*d**3))*(15*a**2*d**2 - 6*a*b*c*d
- b**2*c**2)*log(c**4*d*sqrt(-1/(c**7*d**3)) + x)/16 + (-8*a**2*c**2*d + x*
*4*(-15*a**2*d**3 + 6*a*b*c*d**2 + b**2*c**2*d) + x**2*(-25*a**2*c*d**2 + 1
0*a*b*c**2*d - b**2*c**3))/(8*c**5*d*x + 16*c**4*d**2*x**3 + 8*c**3*d**3*x*
*5)
```

Giac [A]
time = 1.45, size = 135, normalized size = 0.89

$$-\frac{a^2}{c^3x}+\frac{(b^2c^2+6\,abcd-15\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,c^3d}+\frac{b^2c^2dx^3+6\,abcd^2x^3-7\,a^2d^3x^3-b^2c^3x+10\,abc^2dx-9\,a^2cd^2x}{8\,(dx^2+c)^2c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -a^2/(c^3*x) + 1/8*(b^2*c^2 + 6*a*b*c*d - 15*a^2*d^2)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*c^3*d) + 1/8*(b^2*c^2*d*x^3 + 6*a*b*c*d^2*x^3 - 7*a^2*d^3*x^3 -
 b^2*c^3*x + 10*a*b*c^2*d*x - 9*a^2*c*d^2*x)/((d*x^2 + c)^2*c^3*d)
```

Mupad [B]
time = 0.12, size = 135, normalized size = 0.89

$$\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(-15\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{8\,c^{7/2}\,d^{3/2}}-\frac{\frac{a^2}{c}-\frac{x^4\,\left(-15\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{8\,c^3}+\frac{x^2\,\left(25\,a^2\,d^2-10\,a\,b\,c\,d+b^2\,c^2\right)}{8\,c^2\,d}}{c^2\,x+2\,c\,d\,x^3+d^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^3),x)
```

```
[Out] (atan((d^(1/2)*x)/c^(1/2))*(b^2*c^2 - 15*a^2*d^2 + 6*a*b*c*d))/(8*c^(7/2)*d
^(3/2)) - (a^2/c - (x^4*(b^2*c^2 - 15*a^2*d^2 + 6*a*b*c*d))/(8*c^3) + (x^2*
(25*a^2*d^2 + b^2*c^2 - 10*a*b*c*d))/(8*c^2*d))/(c^2*x + d^2*x^5 + 2*c*d*x^
3)
```

3.196 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx$

Optimal. Leaf size=106

$$-\frac{a^2}{2c^3x^2}-\frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}+\frac{a(bc-ad)}{c^3\left(c+dx^2\right)}+\frac{a(2bc-3ad)\log(x)}{c^4}-\frac{a(2bc-3ad)\log\left(c+dx^2\right)}{2c^4}$$

```
[Out] -1/2*a^2/c^3/x^2-1/4*(-a*d+b*c)^2/c^2/d/(d*x^2+c)^2+a*(-a*d+b*c)/c^3/(d*x^2
+c)+a*(-3*a*d+2*b*c)*ln(x)/c^4-1/2*a*(-3*a*d+2*b*c)*ln(d*x^2+c)/c^4
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{a^2}{2c^3x^2}-\frac{a(2bc-3ad)\log\left(c+dx^2\right)}{2c^4}+\frac{a\log(x)(2bc-3ad)}{c^4}+\frac{a(bc-ad)}{c^3\left(c+dx^2\right)}-\frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

```
[Out] -1/2*a^2/(c^3*x^2) - (b*c - a*d)^2/(4*c^2*d*(c + d*x^2)^2) + (a*(b*c - a*d)
)/(c^3*(c + d*x^2)) + (a*(2*b*c - 3*a*d)*Log[x])/c^4 - (a*(2*b*c - 3*a*d)*L
og[c + d*x^2])/(2*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^3}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{c^3x^2} - \frac{a(-2bc+3ad)}{c^4x} + \frac{(bc-ad)^2}{c^2(c+dx)^3} + \frac{2ad(-bc+ad)}{c^3(c+dx)^2} + \frac{ad(-2bc+3a}{c^4(c+dx)}\right.\right.$$
$$= -\frac{a^2}{2c^3x^2} - \frac{(bc-ad)^2}{4c^2d(c+dx^2)^2} + \frac{a(bc-ad)}{c^3(c+dx^2)} + \frac{a(2bc-3ad)\log(x)}{c^4} - \frac{a(2bc-3ad)\log(c}{2c^4}$$

Mathematica [A]
time = 0.07, size = 99, normalized size = 0.93

$$\frac{-\frac{2a^2c}{x^2} - \frac{c^2(bc-ad)^2}{d(c+dx^2)^2} + \frac{4ac(bc-ad)}{c+dx^2} + 4a(2bc-3ad)\log(x) + 2a(-2bc+3ad)\log(c+dx^2)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

```
[Out] ((-2*a^2*c)/x^2 - (c^2*(b*c - a*d)^2)/(d*(c + d*x^2)^2) + (4*a*c*(b*c - a*d
))/(c + d*x^2) + 4*a*(2*b*c - 3*a*d)*Log[x] + 2*a*(-2*b*c + 3*a*d)*Log[c +
d*x^2])/(4*c^4)
```

Maple [A]
time = 0.09, size = 114, normalized size = 1.08

method	result	size
default	$\frac{-\frac{c^2(a^2d^2-2abcd+b^2c^2)}{2d(dx^2+c)^2}-\frac{2ac(ad-bc)}{dx^2+c}+a(3ad-2bc)\ln(dx^2+c)}{2c^4}-\frac{a^2}{2c^3x^2}-\frac{a(3ad-2bc)\ln(x)}{c^4}$	114
norman	$\frac{-\frac{a^2}{2c}+\frac{(6a^2d^2-4abcd+b^2c^2)x^4}{2c^3}+\frac{d(9a^2d^2-6abcd+b^2c^2)x^6}{4c^4}}{x^2(dx^2+c)^2}-\frac{a(3ad-2bc)\ln(x)}{c^4}+\frac{a(3ad-2bc)\ln(dx^2+c)}{2c^4}$	125
risch	$\frac{-\frac{da(3ad-2bc)x^4}{2c^3}-\frac{(9a^2d^2-6abcd+b^2c^2)x^2}{4c^2d}-\frac{a^2}{2c}}{x^2(dx^2+c)^2}-\frac{3a^2\ln(x)d}{c^4}+\frac{2a\ln(x)b}{c^3}+\frac{3a^2\ln(-dx^2-c)d}{2c^4}-\frac{a\ln(-dx^2-c)b}{c^3}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^4*(-1/2*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c)^2-2*a*c*(a*d-b*c)
/(d*x^2+c)+a*(3*a*d-2*b*c)*ln(d*x^2+c))-1/2*a^2/c^3/x^2-a*(3*a*d-2*b*c)/c^4
*ln(x)
```

Maxima [A]
time = 0.28, size = 142, normalized size = 1.34

$$-\frac{2a^2c^2d - 2(2abcd^2 - 3a^2d^3)x^4 + (b^2c^3 - 6abc^2d + 9a^2cd^2)x^2}{4(c^3d^3x^6 + 2c^4d^2x^4 + c^5dx^2)} - \frac{(2abc - 3a^2d)\log(dx^2+c)}{2c^4} + \frac{(2abc - 3a^2d)\log(x^2)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*c^2*d - 2*(2*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (b^2*c^3 - 6*a*b*c^2*
d + 9*a^2*c*d^2)*x^2)/(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2) - 1/2*(2*a*
b*c - 3*a^2*d)*log(d*x^2 + c)/c^4 + 1/2*(2*a*b*c - 3*a^2*d)*log(x^2)/c^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(100) = 200$.
time = 0.84, size = 256, normalized size = 2.42

$$-\frac{2\,a^{2}c^{3}d-2\,(2\,abc^{2}d^{2}-3\,a^{2}cd^{3})x^{4}+(b^{2}c^{4}-6\,abc^{3}d+9\,a^{2}c^{2}d^{2})x^{2}+2\,((2\,abcd^{3}-3\,a^{2}d^{4})x^{6}+2\,(2\,abc^{2}d^{2}-3\,a^{2}cd^{3})x^{4}+(2\,abc^{3}d-3\,a^{2}c^{2}d^{2})x^{2})\log\left(dx^{2}+c\right)-4\,((2\,abcd^{3}-3\,a^{2}d^{4})x^{6}+2\,(2\,abc^{2}d^{2}-3\,a^{2}cd^{3})x^{4}+(2\,abc^{3}d-3\,a^{2}c^{2}d^{2})x^{2})\log\left(x\right)}{4\,(c^{4}d^{3}x^{6}+2\,c^{5}d^{2}x^{4}+c^{6}dx^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*c^3*d - 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4 + (b^2*c^4 - 6*a*b*
c^3*d + 9*a^2*c^2*d^2)*x^2 + 2*((2*a*b*c*d^3 - 3*a^2*d^4)*x^6 + 2*(2*a*b*c^
2*d^2 - 3*a^2*c*d^3)*x^4 + (2*a*b*c^3*d - 3*a^2*c^2*d^2)*x^2)*log(d*x^2 + c
) - 4*((2*a*b*c*d^3 - 3*a^2*d^4)*x^6 + 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4
+ (2*a*b*c^3*d - 3*a^2*c^2*d^2)*x^2)*log(x))/(c^4*d^3*x^6 + 2*c^5*d^2*x^4 +
 c^6*d*x^2)
```

Sympy [A]
time = 1.25, size = 139, normalized size = 1.31

$$-\frac{a(3ad-2bc)\log{(x)}}{c^4}+\frac{a(3ad-2bc)\log{\left(\frac{c}{d}+x^2\right)}}{2c^4}+\frac{-2a^2c^2d+x^4(-6a^2d^3+4abcd^2)+x^2(-9a^2cd^2+6abc^2d-b^2c^3)}{4c^5dx^2+8c^4d^2x^4+4c^3d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**3,x)
```

```
[Out] -a*(3*a*d - 2*b*c)*log(x)/c**4 + a*(3*a*d - 2*b*c)*log(c/d + x**2)/(2*c**4)
 + (-2*a**2*c**2*d + x**4*(-6*a**2*d**3 + 4*a*b*c*d**2) + x**2*(-9*a**2*c*d
**2 + 6*a*b*c**2*d - b**2*c**3))/(4*c**5*d*x**2 + 8*c**4*d**2*x**4 + 4*c**3
*d**3*x**6)
```

Giac [A]
time = 1.03, size = 177, normalized size = 1.67

$$\frac{(2\,abc-3\,a^{2}d)\log\left(x^{2}\right)}{2\,c^{4}}-\frac{(2\,abcd-3\,a^{2}d^{2})\log\left(|dx^{2}+c|\right)}{2\,c^{4}d}-\frac{2\,abcx^{2}-3\,a^{2}dx^{2}+a^{2}c}{2\,c^{4}x^{2}}+\frac{6\,abcd^{3}x^{4}-9\,a^{2}d^{4}x^{4}+16\,abc^{2}d^{2}x^{2}-22\,a^{2}cd^{3}x^{2}-b^{2}c^{4}+12\,abc^{3}d-14\,a^{2}c^{2}d^{2}}{4\,(dx^{2}+c)^{2}c^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*c - 3*a^2*d)*log(x^2)/c^4 - 1/2*(2*a*b*c*d - 3*a^2*d^2)*log(abs(
d*x^2 + c))/(c^4*d) - 1/2*(2*a*b*c*x^2 - 3*a^2*d*x^2 + a^2*c)/(c^4*x^2) + 1
```

```
/4*(6*a*b*c*d^3*x^4 - 9*a^2*d^4*x^4 + 16*a*b*c^2*d^2*x^2 - 22*a^2*c*d^3*x^2
 - b^2*c^4 + 12*a*b*c^3*d - 14*a^2*c^2*d^2)/((d*x^2 + c)^2*c^4*d)
```

Mupad [B]
time = 0.07, size = 132, normalized size = 1.25

$$\frac{\ln\left(d\,x^2+c\right)\,\left(3\,a^2\,d-2\,a\,b\,c\right)}{2\,c^4}-\frac{\frac{a^2}{2\,c}+\frac{x^2\,\left(9\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{4\,c^2\,d}+\frac{a\,d\,x^4\,\left(3\,a\,d-2\,b\,c\right)}{2\,c^3}}{c^2\,x^2+2\,c\,d\,x^4+d^2\,x^6}-\frac{\ln\left(x\right)\,\left(3\,a^2\,d-2\,a\,b\,c\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^3),x)
```

```
[Out] (log(c + d*x^2)*(3*a^2*d - 2*a*b*c))/(2*c^4) - (a^2/(2*c) + (x^2*(9*a^2*d^2
 + b^2*c^2 - 6*a*b*c*d))/(4*c^2*d) + (a*d*x^4*(3*a*d - 2*b*c))/(2*c^3))/(c^
2*x^2 + d^2*x^6 + 2*c*d*x^4) - (log(x)*(3*a^2*d - 2*a*b*c))/c^4
```

3.197 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx$

Optimal. Leaf size=161

$$-\frac{a(6bc-7ad)}{3c^4x}-\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2-6abcd+7a^2d^2\right)x}{12c^3\left(c+dx^2\right)^2}+\frac{(3bc-7ad)^2x}{24c^4\left(c+dx^2\right)}+\frac{\left(3b^2c^2-30abcd+35a^2d^2\right)\text{ta}}{8c^{9/2}\sqrt{d}}$$

```
[Out] -1/3*a*(-7*a*d+6*b*c)/c^4/x-1/3*a^2/c/x^3/(d*x^2+c)^2+1/12*(7*a^2*d^2-6*a*b
*c*d+3*b^2*c^2)*x/c^3/(d*x^2+c)^2+1/24*(-7*a*d+3*b*c)^2*x/c^4/(d*x^2+c)+1/8
*(35*a^2*d^2-30*a*b*c*d+3*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(9/2)/d^(1/2
)
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {473, 467, 464, 211}

$$\frac{\left(35a^2d^2-30abcd+3b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}+\frac{x\left(7a^2d^2-6abcd+3b^2c^2\right)}{12c^3\left(c+dx^2\right)^2}-\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{x(3bc-7ad)^2}{24c^4\left(c+dx^2\right)}-\frac{a(6bc-7ad)}{3c^4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

```
[Out] -1/3*(a*(6*b*c - 7*a*d))/(c^4*x) - a^2/(3*c*x^3*(c + d*x^2)^2) + ((3*b^2*c^
2 - 6*a*b*c*d + 7*a^2*d^2)*x)/(12*c^3*(c + d*x^2)^2) + ((3*b*c - 7*a*d)^2*x
)/(24*c^4*(c + d*x^2)) + ((3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(8*c^(9/2)*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx = -\frac{a^2}{3cx^3(c+dx^2)^2} + \frac{\int \frac{a(6bc-7ad)+3b^2cx^2}{x^2(c+dx^2)^3}\,dx}{3c}$$

$$= -\frac{a^2}{3cx^3(c+dx^2)^2} + \frac{(3b^2c^2-6abcd+7a^2d^2)\,x}{12c^3(c+dx^2)^2} - \frac{\int \frac{-\frac{4a(6bc-7ad)}{c}-3\left(3b^2-\frac{6abd}{c}+\frac{7a^2d^2}{c^2}\right)x^2}{x^2(c+dx^2)^2}\,dx}{12c}$$

$$= -\frac{a^2}{3cx^3(c+dx^2)^2} + \frac{(3b^2c^2-6abcd+7a^2d^2)\,x}{12c^3(c+dx^2)^2} + \frac{(3bc-7ad)^2x}{24c^4(c+dx^2)} + \frac{\int \frac{\frac{8a(6bc-7ad)}{c^2}+\frac{(3bc-7ad)}{c^3}}{x^2(c+dx^2)}}{24c}$$

$$= -\frac{a(6bc-7ad)}{3c^4x} - \frac{a^2}{3cx^3(c+dx^2)^2} + \frac{(3b^2c^2-6abcd+7a^2d^2)\,x}{12c^3(c+dx^2)^2} + \frac{(3bc-7ad)^2x}{24c^4(c+dx^2)} + \frac{(3b}{}$$

$$= -\frac{a(6bc-7ad)}{3c^4x} - \frac{a^2}{3cx^3(c+dx^2)^2} + \frac{(3b^2c^2-6abcd+7a^2d^2)\,x}{12c^3(c+dx^2)^2} + \frac{(3bc-7ad)^2x}{24c^4(c+dx^2)} + \frac{(3b}{}$$

Mathematica [A]
time = 0.05, size = 148, normalized size = 0.92

$$-\frac{a^2}{3c^3x^3} + \frac{a(-2bc+3ad)}{c^4x} + \frac{(bc-ad)^2x}{4c^3(c+dx^2)^2} + \frac{(3b^2c^2-14abcd+11a^2d^2)\,x}{8c^4(c+dx^2)} + \frac{(3b^2c^2-30abcd+35a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]

[Out] -1/3*a^2/(c^3*x^3) + (a*(-2*b*c + 3*a*d))/(c^4*x) + ((b*c - a*d)^2*x)/(4*c^3*(c + d*x^2)^2) + ((3*b^2*c^2 - 14*a*b*c*d + 11*a^2*d^2)*x)/(8*c^4*(c + d*x^2)) + ((3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*Sqrt[d])

Maple [A]
time = 0.12, size = 142, normalized size = 0.88

method	result
default	$\frac{\frac{\left(\frac{11}{8}a^2d^3-\frac{7}{4}abc\,d^2+\frac{3}{8}b^2c^2d\right)x^3+\frac{c\left(13a^2d^2-18abcd+5b^2c^2\right)x}{8}}{\left(d\,x^2+c\right)^2}+\frac{\left(35a^2d^2-30abcd+3b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}}}{c^4}-\frac{a^2}{3c^3x^3}+\frac{a(3ad-2bc)}{c^4x}$
risch	$\frac{\frac{d\left(35a^2d^2-30abcd+3b^2c^2\right)x^6}{8c^4}+\frac{5\left(35a^2d^2-30abcd+3b^2c^2\right)x^4}{24c^3}+\frac{a(7ad-6bc)x^2}{3c^2}-\frac{a^2}{3c}}{x^3\left(d\,x^2+c\right)^2}-\frac{35\ln\left(-\sqrt{-cd}\,x+c\right)a^2d^2}{16\sqrt{-cd}\,c^4}+\frac{15\ln\left(-\sqrt{-cd}\right.}{8\sqrt{-cd}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/c^4*(((11/8*a^2*d^3-7/4*a*b*c*d^2+3/8*b^2*c^2*d)*x^3+1/8*c*(13*a^2*d^2-18*a*b*c*d+5*b^2*c^2)*x)/(d*x^2+c)^2+1/8*(35*a^2*d^2-30*a*b*c*d+3*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))-1/3*a^2/c^3/x^3+a*(3*a*d-2*b*c)/c^4/x

Maxima [A]
time = 0.51, size = 167, normalized size = 1.04

$$\frac{3\,(3\,b^2c^2d-30\,abcd^2+35\,a^2d^3)x^6-8\,a^2c^3+5\,(3\,b^2c^3-30\,abc^2d+35\,a^2cd^2)x^4-8\,(6\,abc^3-7\,a^2c^2d)x^2}{24\,(c^4d^2x^7+2\,c^5dx^5+c^6x^3)}+\frac{(3\,b^2c^2-30\,abcd+35\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="maxima")

[Out] 1/24*(3*(3*b^2*c^2*d - 30*a*b*c*d^2 + 35*a^2*d^3)*x^6 - 8*a^2*c^3 + 5*(3*b^2*c^3 - 30*a*b*c^2*d + 35*a^2*c*d^2)*x^4 - 8*(6*a*b*c^3 - 7*a^2*c^2*d)*x^2)/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3) + 1/8*(3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^4)

Fricas [A]
time = 1.09, size = 536, normalized size = 3.33

$$\left[-\frac{16\,a^2c^4d-6\,(3\,b^2c^3d^2-30\,abc^2d^3+35\,a^2cd^4)x^6-10\,(3\,b^2c^4d-30\,abc^3d^2+35\,a^2c^2d^3)x^4+16\,(6\,abc^4d-7\,a^2c^3d^2)x^2+3\,((3\,b^2c^2d^2-30\,abcd^3+35\,a^2d^4)x^7+2\,(3\,b^2c^3d-30\,abc^2d^2+35\,a^2cd^3)x^5+(3\,b^2c^4-30\,abc^3d+35\,a^2c^2d^2)x^3)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)}{48\,(c^5d^3x^7+2\,c^6d^2x^5+c^7dx^3)},\ -\frac{8\,a^2c^4d-3\,(3\,b^2c^3d^2-30\,abc^2d^3+35\,a^2cd^4)x^6-5\,(3\,b^2c^4d-30\,abc^3d^2+35\,a^2c^2d^3)x^4+8\,(6\,abc^4d-7\,a^2c^3d^2)x^2-3\,((3\,b^2c^2d^2-30\,abcd^3+35\,a^2d^4)x^7+2\,(3\,b^2c^3d-30\,abc^2d^2+35\,a^2cd^3)x^5+(3\,b^2c^4-30\,abc^3d+35\,a^2c^2d^2)x^3)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{24\,(c^5d^3x^7+2\,c^6d^2x^5+c^7dx^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="fricas")

```
[Out] [-1/48*(16*a^2*c^4*d - 6*(3*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 35*a^2*c*d^4)*x^
6 - 10*(3*b^2*c^4*d - 30*a*b*c^3*d^2 + 35*a^2*c^2*d^3)*x^4 + 16*(6*a*b*c^4*
d - 7*a^2*c^3*d^2)*x^2 + 3*((3*b^2*c^2*d^2 - 30*a*b*c*d^3 + 35*a^2*d^4)*x^7
 + 2*(3*b^2*c^3*d - 30*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 30*a*
b*c^3*d + 35*a^2*c^2*d^2)*x^3)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/
(d*x^2 + c)))/(c^5*d^3*x^7 + 2*c^6*d^2*x^5 + c^7*d*x^3), -1/24*(8*a^2*c^4*d
 - 3*(3*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 35*a^2*c*d^4)*x^6 - 5*(3*b^2*c^4*d -
 30*a*b*c^3*d^2 + 35*a^2*c^2*d^3)*x^4 + 8*(6*a*b*c^4*d - 7*a^2*c^3*d^2)*x^2
 - 3*((3*b^2*c^2*d^2 - 30*a*b*c*d^3 + 35*a^2*d^4)*x^7 + 2*(3*b^2*c^3*d - 30
*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 30*a*b*c^3*d + 35*a^2*c^2*d
^2)*x^3)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^5*d^3*x^7 + 2*c^6*d^2*x^5 + c^
7*d*x^3)]
```

Sympy [A]
time = 0.87, size = 240, normalized size = 1.49

$$- \frac{\sqrt{- \frac{1}{c^{9} d}} \left(35 a^{2} d^{2} - 30 a b c d + 3 b^{2} c^{2}\right) \log{\left(- c^{5} \sqrt{- \frac{1}{c^{9} d}} + x \right)}}{16} + \frac{\sqrt{- \frac{1}{c^{9} d}} \left(35 a^{2} d^{2} - 30 a b c d + 3 b^{2} c^{2}\right) \log{\left(c^{5} \sqrt{- \frac{1}{c^{9} d}} + x \right)}}{16} + \frac{- 8 a^{2} c^{3} + x^{6} \cdot \left(105 a^{2} d^{3} - 90 a b c d^{2} + 9 b^{2} c^{2} d\right) + x^{4} \cdot \left(175 a^{2} c d^{2} - 150 a b c^{2} d + 15 b^{2} c^{3}\right) + x^{2} \cdot \left(56 a^{2} c^{2} d - 48 a b c^{3}\right)}{24 c^{6} x^{3} + 48 c^{5} d x^{5} + 24 c^{4} d^{2} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**9*d))*(35*a**2*d**2 - 30*a*b*c*d + 3*b**2*c**2)*log(-c**5*sqrt
(-1/(c**9*d)) + x)/16 + sqrt(-1/(c**9*d))*(35*a**2*d**2 - 30*a*b*c*d + 3*b*
*2*c**2)*log(c**5*sqrt(-1/(c**9*d)) + x)/16 + (-8*a**2*c**3 + x**6*(105*a**
2*d**3 - 90*a*b*c*d**2 + 9*b**2*c**2*d) + x**4*(175*a**2*c*d**2 - 150*a*b*c
**2*d + 15*b**2*c**3) + x**2*(56*a**2*c**2*d - 48*a*b*c**3))/(24*c**6*x**3
+ 48*c**5*d*x**5 + 24*c**4*d**2*x**7)
```

Giac [A]
time = 1.39, size = 151, normalized size = 0.94

$$\frac{\left(3 \, b^{2} c^{2} - 30 \, a b c d + 35 \, a^{2} d^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 \, \sqrt{c d} c^{4}} + \frac{3 \, b^{2} c^{2} d x^{3} - 14 \, a b c d^{2} x^{3} + 11 \, a^{2} d^{3} x^{3} + 5 \, b^{2} c^{3} x - 18 \, a b c^{2} d x + 13 \, a^{2} c d^{2} x}{8 \, {\left(d x^{2} + c\right)}^{2} c^{4}} - \frac{6 \, a b c x^{2} - 9 \, a^{2} d x^{2} + a^{2} c}{3 \, c^{4} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*
c^4) + 1/8*(3*b^2*c^2*d*x^3 - 14*a*b*c*d^2*x^3 + 11*a^2*d^3*x^3 + 5*b^2*c^3
*x - 18*a*b*c^2*d*x + 13*a^2*c*d^2*x)/((d*x^2 + c)^2*c^4) - 1/3*(6*a*b*c*x^
2 - 9*a^2*d*x^2 + a^2*c)/(c^4*x^3)
```

Mupad [B]
time = 0.11, size = 156, normalized size = 0.97

$$\frac{\frac{5\,x^4\,\left(35\,a^2\,d^2-30\,a\,b\,c\,d+3\,b^2\,c^2\right)}{24\,c^3}-\frac{a^2}{3\,c}+\frac{a\,x^2\,\left(7\,a\,d-6\,b\,c\right)}{3\,c^2}+\frac{d\,x^6\,\left(35\,a^2\,d^2-30\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c^4}}{c^2\,x^3+2\,c\,d\,x^5+d^2\,x^7}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(35\,a^2\,d^2-30\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c^{9/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^3),x)
```

```
[Out] ((5*x^4*(35*a^2*d^2 + 3*b^2*c^2 - 30*a*b*c*d))/(24*c^3) - a^2/(3*c) + (a*x^
2*(7*a*d - 6*b*c))/(3*c^2) + (d*x^6*(35*a^2*d^2 + 3*b^2*c^2 - 30*a*b*c*d))/
(8*c^4))/(c^2*x^3 + d^2*x^7 + 2*c*d*x^5) + (atan((d^(1/2)*x)/c^(1/2))*(35*a
^2*d^2 + 3*b^2*c^2 - 30*a*b*c*d))/(8*c^(9/2)*d^(1/2))
```

3.198 $\int \frac{x^5(c+dx^2)}{a+bx^2} dx$

Optimal. Leaf size=75

$$-\frac{a(bc-ad)x^2}{2b^3}+\frac{(bc-ad)x^4}{4b^2}+\frac{dx^6}{6b}+\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}$$

```
[Out] -1/2*a*(-a*d+b*c)*x^2/b^3+1/4*(-a*d+b*c)*x^4/b^2+1/6*d*x^6/b+1/2*a^2*(-a*d+
b*c)*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2(bc-ad)}{2b^3}+\frac{x^4(bc-ad)}{4b^2}+\frac{dx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] -1/2*(a*(b*c - a*d)*x^2)/b^3 + ((b*c - a*d)*x^4)/(4*b^2) + (d*x^6)/(6*b) +
(a^2*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(c+dx^2)}{a+bx^2} dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(c+dx)}{a+bx} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-bc+ad)}{b^3} + \frac{(bc-ad)x}{b^2} + \frac{dx^2}{b} - \frac{a^2(-bc+ad)}{b^3(a+bx)}\right) dx, x, x^2\right)$$
$$= -\frac{a(bc-ad)x^2}{2b^3} + \frac{(bc-ad)x^4}{4b^2} + \frac{dx^6}{6b} + \frac{a^2(bc-ad)\log(a+bx^2)}{2b^4}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.95

$$\frac{bx^2(6a^2d - 3ab(2c+dx^2) + b^2x^2(3c+2dx^2)) + 6a^2(bc-ad)\log(a+bx^2)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2))/(a + b*x^2),x]
[Out] (b*x^2*(6*a^2*d - 3*a*b*(2*c + d*x^2) + b^2*x^2*(3*c + 2*d*x^2)) + 6*a^2*(b
*c - a*d)*Log[a + b*x^2])/(12*b^4)
```

Maple [A]
time = 0.07, size = 74, normalized size = 0.99

method	result	size
norman	$-\frac{(ad-bc)x^4}{4b^2} + \frac{d x^6}{6b} + \frac{a(ad-bc)x^2}{2b^3} - \frac{a^2(ad-bc)\ln(b x^2+a)}{2b^4}$	68
default	$\frac{\frac{1}{3}b^2 d x^6 - \frac{1}{2}abd x^4 + \frac{1}{2}b^2 c x^4 + a^2 d x^2 - abc x^2}{2b^3} - \frac{a^2(ad-bc)\ln(b x^2+a)}{2b^4}$	74
risch	$\frac{d x^6}{6b} - \frac{ad x^4}{4b^2} + \frac{c x^4}{4b} + \frac{a^2 d x^2}{2b^3} - \frac{ac x^2}{2b^2} - \frac{a^3\ln(b x^2+a) d}{2b^4} + \frac{a^2\ln(b x^2+a) c}{2b^3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2/b^3*(1/3*b^2*d*x^6-1/2*a*b*d*x^4+1/2*b^2*c*x^4+a^2*d*x^2-a*b*c*x^2)-1/2
*a^2*(a*d-b*c)/b^4*ln(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 74, normalized size = 0.99

$$\frac{2\,b^2dx^6 + 3\,(b^2c - abd)x^4 - 6\,(abc - a^2d)x^2}{12\,b^3} + \frac{(a^2bc - a^3d)\log(bx^2+a)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d*x^6 + 3*(b^2*c - a*b*d)*x^4 - 6*(a*b*c - a^2*d)*x^2)/b^3 + 1/
2*(a^2*b*c - a^3*d)*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 1.35, size = 75, normalized size = 1.00

$$\frac{2\,b^3dx^6+3\,(b^3c-ab^2d)x^4-6\,(ab^2c-a^2bd)x^2+6\,(a^2bc-a^3d)\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d*x^6 + 3*(b^3*c - a*b^2*d)*x^4 - 6*(a*b^2*c - a^2*b*d)*x^2 + 6
*(a^2*b*c - a^3*d)*log(b*x^2 + a))/b^4
```

Sympy [A]
time = 0.16, size = 70, normalized size = 0.93

$$-\frac{a^2(ad-bc)\log\left(a+bx^2\right)}{2b^4}+x^4\left(-\frac{ad}{4b^2}+\frac{c}{4b}\right)+x^2\left(\frac{a^2d}{2b^3}-\frac{ac}{2b^2}\right)+\frac{dx^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)*log(a + b*x**2)/(2*b**4) + x**4*(-a*d/(4*b**2) + c/(4*b))
 + x**2*(a**2*d/(2*b**3) - a*c/(2*b**2)) + d*x**6/(6*b)
```

Giac [A]
time = 1.14, size = 77, normalized size = 1.03

$$\frac{2\,b^2dx^6+3\,b^2cx^4-3\,abdx^4-6\,abcx^2+6\,a^2dx^2}{12\,b^3}+\frac{(a^2bc-a^3d)\log\left(|bx^2+a|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d*x^6 + 3*b^2*c*x^4 - 3*a*b*d*x^4 - 6*a*b*c*x^2 + 6*a^2*d*x^2)/
b^3 + 1/2*(a^2*b*c - a^3*d)*log(abs(b*x^2 + a))/b^4
```

Mupad [B]
time = 0.06, size = 76, normalized size = 1.01

$$x^4\left(\frac{c}{4\,b}-\frac{a\,d}{4\,b^2}\right)+\frac{d\,x^6}{6\,b}-\frac{\ln\left(b\,x^2+a\right)\,\left(a^3\,d-a^2\,b\,c\right)}{2\,b^4}-\frac{a\,x^2\,\left(\frac{c}{b}-\frac{a\,d}{b^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^2))/(a + b*x^2),x)
```

```
[Out] x^4*(c/(4*b) - (a*d)/(4*b^2)) + (d*x^6)/(6*b) - (log(a + b*x^2)*(a^3*d - a^
2*b*c))/(2*b^4) - (a*x^2*(c/b - (a*d)/b^2))/(2*b)
```

3.199 $\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=77

$$-\frac{a(bc-ad)x}{b^3}+\frac{(bc-ad)x^3}{3b^2}+\frac{dx^5}{5b}+\frac{a^{3/2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] -a*(-a*d+b*c)*x/b^3+1/3*(-a*d+b*c)*x^3/b^2+1/5*d*x^5/b+a^(3/2)*(-a*d+b*c)*a
rctan(x*b^(1/2)/a^(1/2))/b^(7/2)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {470, 308, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)}{b^{7/2}}-\frac{ax(bc-ad)}{b^3}+\frac{x^3(bc-ad)}{3b^2}+\frac{dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)*x)/b^3) + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5*b) + (a^(
3/2)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx = \frac{dx^5}{5b} - \frac{(-5bc+5ad)\int \frac{x^4}{a+bx^2}\,dx}{5b}$$
$$= \frac{dx^5}{5b} - \frac{(-5bc+5ad)\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right)dx}{5b}$$
$$= -\frac{a(bc-ad)x}{b^3} + \frac{(bc-ad)x^3}{3b^2} + \frac{dx^5}{5b} + \frac{(a^2(bc-ad))\int \frac{1}{a+bx^2}\,dx}{b^3}$$
$$= -\frac{a(bc-ad)x}{b^3} + \frac{(bc-ad)x^3}{3b^2} + \frac{dx^5}{5b} + \frac{a^{3/2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 1.00

$$\frac{a(-bc+ad)x}{b^3} + \frac{(bc-ad)x^3}{3b^2} + \frac{dx^5}{5b} - \frac{a^{3/2}(-bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (a*(-(b*c) + a*d)*x)/b^3 + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5*b) - (a^(
3/2)*(-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Maple [A]
time = 0.09, size = 75, normalized size = 0.97

method	result
default	$\frac{\frac{1}{5}b^2 d\,x^5-\frac{1}{3}abd\,x^3+\frac{1}{3}b^2 c\,x^3+a^2 dx-abcx}{b^3}-\frac{a^2(ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{d\,x^5}{5b}-\frac{ad\,x^3}{3b^2}+\frac{c\,x^3}{3b}+\frac{a^2dx}{b^3}-\frac{acx}{b^2}+\frac{\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-a\right)d}{2b^4}-\frac{\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x-a\right)c}{2b^3}-\frac{\sqrt{-ab}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*b^2*d*x^5-1/3*a*b*d*x^3+1/3*b^2*c*x^3+a^2*d*x-a*b*c*x)-a^2*(a*d-
b*c)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.53, size = 77, normalized size = 1.00

$$\frac{(a^2bc - a^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^2dx^5 + 5\,(b^2c - abd)x^3 - 15\,(abc - a^2d)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b*c - a^3*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^2*d*x^5
 + 5*(b^2*c - a*b*d)*x^3 - 15*(a*b*c - a^2*d)*x)/b^3
```

Fricas [A]
time = 1.08, size = 178, normalized size = 2.31

$$\left[\frac{6\,b^2dx^5 + 10\,(b^2c - abd)x^3 - 15\,(abc - a^2d)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 30\,(abc - a^2d)x}{30\,b^3}, \frac{3\,b^2dx^5 + 5\,(b^2c - abd)x^3 + 15\,(abc - a^2d)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 15\,(abc - a^2d)x}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d*x^5 + 10*(b^2*c - a*b*d)*x^3 - 15*(a*b*c - a^2*d)*sqrt(-a/b)
*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 30*(a*b*c - a^2*d)*x)/b^
3, 1/15*(3*b^2*d*x^5 + 5*(b^2*c - a*b*d)*x^3 + 15*(a*b*c - a^2*d)*sqrt(a/b)
*arctan(b*x*sqrt(a/b)/a) - 15*(a*b*c - a^2*d)*x)/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(68) = 136$.
time = 0.26, size = 153, normalized size = 1.99

$$x^3\left(-\frac{ad}{3b^2} + \frac{c}{3b}\right) + x\left(\frac{a^2d}{b^3} - \frac{ac}{b^2}\right) + \frac{\sqrt{-\frac{a^3}{b^7}}\,(ad - bc)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)}{a^2d-abc} + x\right)}{2} - \frac{\sqrt{-\frac{a^3}{b^7}}\,(ad - bc)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)}{a^2d-abc} + x\right)}{2} + \frac{dx^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**3*(-a*d/(3*b**2) + c/(3*b)) + x*(a**2*d/b**3 - a*c/b**2) + sqrt(-a**3/b*
*7)*(a*d - b*c)*log(-b**3*sqrt(-a**3/b**7)*(a*d - b*c)/(a**2*d - a*b*c) + x
)/2 - sqrt(-a**3/b**7)*(a*d - b*c)*log(b**3*sqrt(-a**3/b**7)*(a*d - b*c)/(a
**2*d - a*b*c) + x)/2 + d*x**5/(5*b)
```

Giac [A]
time = 1.04, size = 84, normalized size = 1.09

$$\frac{(a^2bc - a^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^3} + \frac{3\, b^4dx^5 + 5\, b^4cx^3 - 5\, ab^3dx^3 - 15\, ab^3cx + 15\, a^2b^2dx}{15\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b*c - a^3*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*d*x^5
 + 5*b^4*c*x^3 - 5*a*b^3*d*x^3 - 15*a*b^3*c*x + 15*a^2*b^2*d*x)/b^5
```

Mupad [B]
time = 0.06, size = 96, normalized size = 1.25

$$x^3\left(\frac{c}{3\,b} - \frac{a\,d}{3\,b^2}\right) + \frac{d\,x^5}{5\,b} - \frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(a\,d-b\,c)}{a^3\,d-a^2\,b\,c}\right)\,(a\,d-b\,c)}{b^{7/2}} - \frac{a\,x\,\left(\frac{c}{b}-\frac{a\,d}{b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2))/(a + b*x^2),x)
```

```
[Out] x^3*(c/(3*b) - (a*d)/(3*b^2)) + (d*x^5)/(5*b) - (a^(3/2)*atan((a^(3/2)*b^(1
/2)*x*(a*d - b*c))/(a^3*d - a^2*b*c))*(a*d - b*c))/b^(7/2) - (a*x*(c/b - (a
*d)/b^2))/b
```

3.200 $\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=54

$$\frac{(bc-ad)x^2}{2b^2}+\frac{dx^4}{4b}-\frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] 1/2*(-a*d+b*c)*x^2/b^2+1/4*d*x^4/b-1/2*a*(-a*d+b*c)*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$-\frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(bc-ad)}{2b^2}+\frac{dx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x^2)/(2*b^2) + (d*x^4)/(4*b) - (a*(b*c - a*d)*Log[a + b*x^2])/
(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{bc-ad}{b^2}+\frac{dx}{b}+\frac{a(-bc+ad)}{b^2(a+bx)}\right)\,dx, x, x^2\right)$$
$$= \frac{(bc-ad)x^2}{2b^2}+\frac{dx^4}{4b}-\frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.87

$$\frac{bx^2(2bc-2ad+bdx^2)+2a(-bc+ad)\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(2*b*c - 2*a*d + b*d*x^2) + 2*a*(-(b*c) + a*d)*Log[a + b*x^2])/(4*b^
3)
```

Maple [A]
time = 0.09, size = 50, normalized size = 0.93

method	result	size
norman	$-\frac{(ad-bc)x^2}{2b^2}+\frac{d\,x^4}{4b}+\frac{a(ad-bc)\ln\left(b\,x^2+a\right)}{2b^3}$	49
default	$-\frac{-\frac{1}{2}bd\,x^4+ad\,x^2-c\,x^2b}{2b^2}+\frac{a(ad-bc)\ln\left(b\,x^2+a\right)}{2b^3}$	50
risch	$\frac{d\,x^4}{4b}-\frac{da\,x^2}{2b^2}+\frac{c\,x^2}{2b}+\frac{d\,a^2}{4b^3}-\frac{ac}{2b^2}+\frac{c^2}{4db}+\frac{a^2\ln\left(b\,x^2+a\right)d}{2b^3}-\frac{ac\ln\left(b\,x^2+a\right)}{2b^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^2*(-1/2*b*d*x^4+a*d*x^2-c*x^2*b)+1/2*a*(a*d-b*c)/b^3*ln(b*x^2+a)
```

Maxima [A]
time = 0.31, size = 50, normalized size = 0.93

$$\frac{bdx^4+2\,(bc-ad)x^2}{4\,b^2}-\frac{(abc-a^2d)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d*x^4 + 2*(b*c - a*d)*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*log(b*x^2 + a)/
b^3
```

Fricas [A]
time = 0.95, size = 51, normalized size = 0.94

$$\frac{b^2dx^4 + 2\,(b^2c - abd)x^2 - 2\,(abc - a^2d)\log\left(bx^2 + a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^4 + 2*(b^2*c - a*b*d)*x^2 - 2*(a*b*c - a^2*d)*log(b*x^2 + a))/
b^3
```

Sympy [A]
time = 0.15, size = 46, normalized size = 0.85

$$\frac{a(ad - bc)\log\left(a + bx^2\right)}{2b^3} + x^2\left(-\frac{ad}{2b^2} + \frac{c}{2b}\right) + \frac{dx^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)*log(a + b*x**2)/(2*b**3) + x**2*(-a*d/(2*b**2) + c/(2*b)) + d
*x**4/(4*b)
```

Giac [A]
time = 1.22, size = 52, normalized size = 0.96

$$\frac{bdx^4 + 2\,bcx^2 - 2\,adx^2}{4\,b^2} - \frac{(abc - a^2d)\log\left(|bx^2 + a|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d*x^4 + 2*b*c*x^2 - 2*a*d*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*log(abs(b*x
^2 + a))/b^3
```

Mupad [B]
time = 0.04, size = 52, normalized size = 0.96

$$x^2\left(\frac{c}{2\,b} - \frac{a\,d}{2\,b^2}\right) + \frac{\ln\left(b\,x^2 + a\right)\,\left(a^2\,d - a\,b\,c\right)}{2\,b^3} + \frac{d\,x^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2))/(a + b*x^2),x)
```

```
[Out] x^2*(c/(2*b) - (a*d)/(2*b^2)) + (log(a + b*x^2)*(a^2*d - a*b*c))/(2*b^3) +
(d*x^4)/(4*b)
```

3.201 $\int \frac{x^2(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=58

$$\frac{(bc-ad)x}{b^2}+\frac{dx^3}{3b}-\frac{\sqrt{a}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] (-a*d+b*c)*x/b^2+1/3*d*x^3/b-(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b
^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {470, 327, 211}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)}{b^{5/2}}+\frac{x(bc-ad)}{b^2}+\frac{dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) - (Sqrt[a]*(b*c - a*d)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(c+dx^2)}{a+bx^2}\,dx &= \frac{dx^3}{3b} - \frac{(-3bc+3ad)\int \frac{x^2}{a+bx^2}\,dx}{3b} \\
&= \frac{(bc-ad)x}{b^2} + \frac{dx^3}{3b} - \frac{(a(bc-ad))\int \frac{1}{a+bx^2}\,dx}{b^2} \\
&= \frac{(bc-ad)x}{b^2} + \frac{dx^3}{3b} - \frac{\sqrt{a}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.98

$$
\frac{(bc-ad)x}{b^2} + \frac{dx^3}{3b} + \frac{\sqrt{a}\,(-bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2))/(a + b*x^2),x]
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) + (Sqrt[a]*(-(b*c) + a*d)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2)
```

Maple [A]
time = 0.08, size = 51, normalized size = 0.88

method	result
default	$-\frac{-\frac{1}{3}bd\,x^3+adx-bcx}{b^2}+\frac{a(ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{d\,x^3}{3b}-\frac{adx}{b^2}+\frac{cx}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x+a\right)ad}{2b^3}-\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x+a\right)c}{2b^2}-\frac{\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x+a\right)}{2b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/3*b*d*x^3+a*d*x-b*c*x)+a*(a*d-b*c)/b^2/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 54, normalized size = 0.93

$$
-\frac{(abc-a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{bdx^3+3\,(bc-ad)x}{3\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*d*x^3 + 3*(
b*c - a*d)*x)/b^2
```

Fricas [A]
time = 1.28, size = 129, normalized size = 2.22

$$\left[\frac{2\,bdx^3 - 3\,(bc - ad)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2 + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 6\,(bc - ad)x}{6\,b^2}, \frac{bdx^3 - 3\,(bc - ad)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 3\,(bc - ad)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*d*x^3 - 3*(b*c - a*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) + 6*(b*c - a*d)*x)/b^2, 1/3*(b*d*x^3 - 3*(b*c - a*d)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*c - a*d)*x)/b^2]
```

Sympy [A]
time = 0.19, size = 90, normalized size = 1.55

$$x\left(-\frac{ad}{b^2} + \frac{c}{b}\right) - \frac{\sqrt{-\frac{a}{b^5}}\,(ad - bc)\log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{a}{b^5}}\,(ad - bc)\log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{dx^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x*(-a*d/b**2 + c/b) - sqrt(-a/b**5)*(a*d - b*c)*log(-b**2*sqrt(-a/b**5) + x
)/2 + sqrt(-a/b**5)*(a*d - b*c)*log(b**2*sqrt(-a/b**5) + x)/2 + d*x**3/(3*b
)
```

Giac [A]
time = 1.13, size = 58, normalized size = 1.00

$$-\frac{(abc - a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{b^2dx^3 + 3\,b^2cx - 3\,abdx}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*d*x^3 + 3
*b^2*c*x - 3*a*b*d*x)/b^3
```

Mupad [B]
time = 0.04, size = 70, normalized size = 1.21

$$x\left(\frac{c}{b}-\frac{a\,d}{b^2}\right)+\frac{d\,x^3}{3\,b}+\frac{\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(a\,d-b\,c)}{a^2\,d-a\,b\,c}\right)\,(a\,d-b\,c)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2))/(a + b*x^2),x)
```

```
[Out] x*(c/b - (a*d)/b^2) + (d*x^3)/(3*b) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d
 - b*c))/(a^2*d - a*b*c))*(a*d - b*c))/b^(5/2)
```

3.202 $\int \frac{x(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=35

$$\frac{dx^2}{2b}+\frac{(bc-ad)\log(a+bx^2)}{2b^2}$$

[Out] 1/2*d*x^2/b+1/2*(-a*d+b*c)*ln(b*x^2+a)/b^2

Rubi [A]

time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {455, 45}

$$\frac{(bc-ad)\log(a+bx^2)}{2b^2}+\frac{dx^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x^2))/(a + b*x^2),x]

[Out] (d*x^2)/(2*b) + ((b*c - a*d)*Log[a + b*x^2])/(2*b^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(c+dx^2)}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)\,dx,x,x^2\right)\\ &= \frac{dx^2}{2b}+\frac{(bc-ad)\log(a+bx^2)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.89

$$\frac{bdx^2+(bc-ad)\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2))/(a + b*x^2),x]
[Out] (b*d*x^2 + (b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.07, size = 32, normalized size = 0.91

method	result	size
default	$\frac{d\,x^2}{2b}+\frac{(-ad+bc)\ln(b\,x^2+a)}{2b^2}$	32
norman	$\frac{d\,x^2}{2b}-\frac{(ad-bc)\ln(b\,x^2+a)}{2b^2}$	32
risch	$\frac{d\,x^2}{2b}-\frac{\ln(b\,x^2+a)ad}{2b^2}+\frac{c\ln(b\,x^2+a)}{2b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*d*x^2/b+1/2*(-a*d+b*c)*ln(b*x^2+a)/b^2
```

Maxima [A]
time = 0.29, size = 31, normalized size = 0.89

$$\frac{dx^2}{2\,b}+\frac{(bc-ad)\log(bx^2+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*d*x^2/b + 1/2*(b*c - a*d)*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 1.25, size = 29, normalized size = 0.83

$$\frac{bdx^2+(bc-ad)\log(bx^2+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*(b*d*x^2 + (b*c - a*d)*log(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.12, size = 27, normalized size = 0.77

$$\frac{dx^2}{2b} - \frac{(ad - bc)\log(a + bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)/(b*x**2+a),x)
[Out] d*x**2/(2*b) - (a*d - b*c)*log(a + b*x**2)/(2*b**2)
```

Giac [A]
time = 0.99, size = 32, normalized size = 0.91

$$\frac{dx^2}{2\,b} + \frac{(bc - ad)\log(|bx^2 + a|)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*d*x^2/b + 1/2*(b*c - a*d)*log(abs(b*x^2 + a))/b^2
```

Mupad [B]
time = 0.06, size = 31, normalized size = 0.89

$$\frac{d\,x^2}{2\,b} - \frac{\ln(b\,x^2 + a)\,(a\,d - b\,c)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2))/(a + b*x^2),x)
[Out] (d*x^2)/(2*b) - (log(a + b*x^2)*(a*d - b*c))/(2*b^2)
```

3.203 $\int \frac{c+dx^2}{a+bx^2} \, dx$

Optimal. Leaf size=39

$$\frac{dx}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

```
[Out] d*x/b+(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {396, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)}{\sqrt{a}\,b^{3/2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{a+bx^2}\,dx &= \frac{dx}{b}-\frac{(-bc+ad)\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{dx}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.03

$$\frac{dx}{b}-\frac{(-bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2),x]
[Out] (d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Maple [A]
time = 0.06, size = 34, normalized size = 0.87

method	result	size
default	$\frac{dx}{b}+\frac{(-ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
risch	$\frac{dx}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)ad}{2b\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)c}{2\sqrt{-ab}}+\frac{\ln\left(-bx-\sqrt{-ab}\right)ad}{2b\sqrt{-ab}}-\frac{\ln\left(-bx-\sqrt{-ab}\right)c}{2\sqrt{-ab}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] d*x/b+(-a*d+b*c)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.53, size = 33, normalized size = 0.85

$$\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 1.19, size = 98, normalized size = 2.51

$$\left[\frac{2\,abdx+\sqrt{-ab}\,(bc-ad)\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab^2},\ \frac{abdx+\sqrt{ab}\,(bc-ad)\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*d*x + sqrt(-a*b)*(b*c - a*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*d*x + sqrt(a*b)*(b*c - a*d)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.
time = 0.17, size = 82, normalized size = 2.10

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + d*x/b
```

Giac [A]
time = 1.48, size = 33, normalized size = 0.85

$$\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [B]
time = 0.03, size = 32, normalized size = 0.82

$$\frac{d\,x}{b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d-b\,c)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2),x)
```

```
[Out] (d*x)/b - (atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(1/2)*b^(3/2))
```

3.204 $\int \frac{c+dx^2}{x(a+bx^2)} dx$

Optimal. Leaf size=34

$$\frac{c\log(x)}{a} - \frac{(bc-ad)\log(a+bx^2)}{2ab}$$

```
[Out] c*ln(x)/a-1/2*(-a*d+b*c)*ln(b*x^2+a)/a/b
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{c\log(x)}{a} - \frac{(bc-ad)\log(a+bx^2)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] (c*Log[x])/a - ((b*c - a*d)*Log[a + b*x^2])/(2*a*b)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{x(a+bx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{x(a+bx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c}{ax} + \frac{-bc+ad}{a(a+bx)}\right)\,dx, x, x^2\right)\\
&= \frac{c\log(x)}{a} - \frac{(bc-ad)\log(a+bx^2)}{2ab}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{c\log(x)}{a}+\frac{(-bc+ad)\log(a+bx^2)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x*(a + b*x^2)),x]
[Out] (c*Log[x])/a + ((-(b*c) + a*d)*Log[a + b*x^2])/(2*a*b)
```

Maple [A]
time = 0.08, size = 33, normalized size = 0.97

method	result	size
default	$\frac{(ad-bc)\ln(b\,x^2+a)}{2ab}+\frac{c\ln(x)}{a}$	33
norman	$\frac{(ad-bc)\ln(b\,x^2+a)}{2ab}+\frac{c\ln(x)}{a}$	33
risch	$\frac{c\ln(x)}{a}+\frac{\ln(-b\,x^2-a)d}{2b}-\frac{\ln(-b\,x^2-a)c}{2a}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*d-b*c)/a/b*ln(b*x^2+a)+c*ln(x)/a
```

Maxima [A]
time = 0.30, size = 35, normalized size = 1.03

$$\frac{c\log(x^2)}{2\,a}-\frac{(bc-ad)\log(bx^2+a)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*c*log(x^2)/a - 1/2*(b*c - a*d)*log(b*x^2 + a)/(a*b)
```

Fricas [A]
time = 0.89, size = 33, normalized size = 0.97

$$\frac{2\,bc\log(x)-(bc-ad)\log(bx^2+a)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*(2*b*c*log(x) - (b*c - a*d)*log(b*x^2 + a))/(a*b)
```

Sympy [A]
time = 0.40, size = 26, normalized size = 0.76

$$\frac{c\log{(x)}}{a}+\frac{(ad-bc)\log{\left(\frac{a}{b}+x^2\right)}}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x/(b*x**2+a),x)
[Out] c*log(x)/a + (a*d - b*c)*log(a/b + x**2)/(2*a*b)
```

Giac [A]
time = 1.68, size = 36, normalized size = 1.06

$$\frac{c\log{(x^2)}}{2\,a}-\frac{(bc-ad)\log{(|bx^2+a|)}}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*c*log(x^2)/a - 1/2*(b*c - a*d)*log(abs(b*x^2 + a))/(a*b)
```

Mupad [B]
time = 0.04, size = 32, normalized size = 0.94

$$\frac{c\,\ln{(x)}}{a}+\frac{\ln{(b\,x^2+a)}\,\,(a\,d-b\,c)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x*(a + b*x^2)),x)
[Out] (c*log(x))/a + (log(a + b*x^2)*(a*d - b*c))/(2*a*b)
```

3.205 $\int \frac{c+dx^2}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=43

$$-\frac{c}{ax}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

```
[Out] -c/a/x-(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {464, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) - ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^2\left(a+bx^2\right)}\,dx&=-\frac{c}{ax}-\frac{(bc-ad)\int \frac{1}{a+bx^2}\,dx}{a}\\&=-\frac{c}{ax}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.98

$$-\frac{c}{ax}+\frac{(-bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^2*(a + b*x^2)),x]
[Out] -(c/(a*x)) + ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Maple [A]
time = 0.09, size = 37, normalized size = 0.86

method	result	size
default	$\frac{(ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{c}{ax}$	37
risch	$-\frac{c}{ax}-\frac{\ln\left(-\sqrt{-ab}\,x+a\right)d}{2\sqrt{-ab}}+\frac{\ln\left(-\sqrt{-ab}\,x+a\right)bc}{2\sqrt{-ab}\,a}+\frac{\ln\left(-\sqrt{-ab}\,x-a\right)d}{2\sqrt{-ab}}-\frac{\ln\left(-\sqrt{-ab}\,x-a\right)bc}{2\sqrt{-ab}\,a}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] (a*d-b*c)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c/a/x
```

Maxima [A]
time = 0.49, size = 37, normalized size = 0.86

$$-\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
[Out] -(b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

Fricas [A]
time = 1.27, size = 105, normalized size = 2.44

$$\left[\frac{\sqrt{-ab}\,(bc-ad)x\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,abc}{2\,a^2bx},\,-\frac{\sqrt{ab}\,(bc-ad)x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+abc}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a*b)*(b*c - a*d)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*a*b*c)/(a^2*b*x), -(sqrt(a*b)*(b*c - a*d)*x*arctan(sqrt(a*b)*x/a) + a
*b*c)/(a^2*b*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.
time = 0.18, size = 82, normalized size = 1.91

$$-\frac{\sqrt{-\frac{1}{a^3b}}\,(ad-bc)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\,(ad-bc)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**3*b))*(a*d - b*c)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2 + sqrt(-1
/(a**3*b))*(a*d - b*c)*log(a**2*sqrt(-1/(a**3*b)) + x)/2 - c/(a*x)
```

Giac [A]
time = 1.37, size = 37, normalized size = 0.86

$$-\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

Mupad [B]
time = 0.06, size = 34, normalized size = 0.79

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d-b\,c)}{a^{3/2}\,\sqrt{b}}-\frac{c}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^2*(a + b*x^2)),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(3/2)*b^(1/2)) - c/(a*x)
```

3.206 $\int \frac{c+dx^2}{x^3(a+bx^2)} dx$

Optimal. Leaf size=50

$$-\frac{c}{2ax^2}-\frac{(bc-ad)\log(x)}{a^2}+\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] -1/2*c/a/x^2-(-a*d+b*c)*ln(x)/a^2+1/2*(-a*d+b*c)*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(bc-ad)}{a^2}-\frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*c/(a*x^2) - ((b*c - a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*x^2])/(2
*a^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c}{ax^2}+\frac{-bc+ad}{a^2x}-\frac{b(-bc+ad)}{a^2(a+bx)}\right)\,dx,x,x^2\right)$$
$$= -\frac{c}{2ax^2}-\frac{(bc-ad)\log(x)}{a^2}+\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.98

$$-\frac{c}{2ax^2}+\frac{(-bc+ad)\log(x)}{a^2}+\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^3*(a + b*x^2)),x]
[Out] -1/2*c/(a*x^2) + ((-(b*c) + a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*x^2])
/(2*a^2)
```

Maple [A]
time = 0.08, size = 46, normalized size = 0.92

method	result	size
default	$-\frac{(ad-bc)\ln\left(b\,x^2+a\right)}{2a^2}-\frac{c}{2a\,x^2}+\frac{(ad-bc)\ln(x)}{a^2}$	46
norman	$-\frac{(ad-bc)\ln\left(b\,x^2+a\right)}{2a^2}-\frac{c}{2a\,x^2}+\frac{(ad-bc)\ln(x)}{a^2}$	46
risch	$-\frac{\ln\left(b\,x^2+a\right)d}{2a}+\frac{bc\ln\left(b\,x^2+a\right)}{2a^2}-\frac{c}{2a\,x^2}+\frac{\ln(x)d}{a}-\frac{bc\ln(x)}{a^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/2*(a*d-b*c)/a^2*ln(b*x^2+a)-1/2*c/a/x^2+(a*d-b*c)/a^2*ln(x)
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.96

$$\frac{(bc-ad)\log\left(bx^2+a\right)}{2\,a^2}-\frac{(bc-ad)\log\left(x^2\right)}{2\,a^2}-\frac{c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

[Out] 1/2*(b*c - a*d)*log(b*x^2 + a)/a^2 - 1/2*(b*c - a*d)*log(x^2)/a^2 - 1/2*c/(a*x^2)

Fricas [A]
time = 1.53, size = 48, normalized size = 0.96

$$\frac{(bc-ad)x^2\log\left(bx^2+a\right)-2\,(bc-ad)x^2\log\left(x\right)-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="fricas")

[Out] 1/2*((b*c - a*d)*x^2*log(b*x^2 + a) - 2*(b*c - a*d)*x^2*log(x) - a*c)/(a^2*x^2)

Sympy [A]
time = 0.52, size = 41, normalized size = 0.82

$$-\frac{c}{2ax^2}+\frac{(ad-bc)\log\left(x\right)}{a^2}-\frac{(ad-bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)/x**3/(b*x**2+a),x)

[Out] -c/(2*a*x**2) + (a*d - b*c)*log(x)/a**2 - (a*d - b*c)*log(a/b + x**2)/(2*a**2)

Giac [A]
time = 1.36, size = 72, normalized size = 1.44

$$-\frac{(bc-ad)\log\left(x^2\right)}{2\,a^2}+\frac{(b^2c-abd)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bcx^2-adx^2-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="giac")

[Out] -1/2*(b*c - a*d)*log(x^2)/a^2 + 1/2*(b^2*c - a*b*d)*log(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*c*x^2 - a*d*x^2 - a*c)/(a^2*x^2)

Mupad [B]
time = 0.09, size = 45, normalized size = 0.90

$$\frac{\ln\left(x\right)\,\left(a\,d-b\,c\right)}{a^2}-\frac{\ln\left(b\,x^2+a\right)\,\left(a\,d-b\,c\right)}{2\,a^2}-\frac{c}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^2)/(x^3*(a + b*x^2)),x)

[Out] (log(x)*(a*d - b*c))/a^2 - (log(a + b*x^2)*(a*d - b*c))/(2*a^2) - c/(2*a*x^2)

3.207 $\int \frac{c+dx^2}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=59

$$-\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{\sqrt{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

[Out] -1/3*c/a/x^3+(-a*d+b*c)/a^2/x+(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {464, 331, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)}{a^{5/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^2)/(x^4*(a + b*x^2)),x]

[Out] -1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) + (Sqrt[b]*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^4\left(a+bx^2\right)}\,dx &= -\frac{c}{3ax^3}-\frac{(3bc-3ad)\int \frac{1}{x^2(a+bx^2)}\,dx}{3a}\\ &= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{(b(bc-ad))\int \frac{1}{a+bx^2}\,dx}{a^2}\\ &= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{\sqrt{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 1.02

$$-\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}-\frac{\sqrt{b}\,(-bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) - (Sqrt[b]*(-(b*c) + a*d)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A]
time = 0.08, size = 55, normalized size = 0.93

method	result
default	$-\frac{b(ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}-\frac{c}{3a\,x^3}-\frac{ad-bc}{a^2x}$
risch	$\frac{-\frac{(ad-bc)x^2}{a^2}-\frac{c}{3a}}{x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2+a^2b\,d^2-2a\,b^2cd+b^3c^2\right)}\_R\ln\left(\left(3\_R^2a^5+2a^2b\,d^2-4a\,b^2cd+2b^3c^2\right)x+\left(a^4d-a^3bc\right)\_R\right)\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b*(a*d-b*c)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/3*c/a/x^3-(a*d-b*c)/
a^2/x
```

Maxima [A]
time = 0.50, size = 56, normalized size = 0.95

$$\frac{(b^2c - abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} + \frac{3\,(bc - ad)x^2 - ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^2*c - a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*(b*c - a*d)*
x^2 - a*c)/(a^2*x^3)
```

Fricas [A]
time = 0.97, size = 136, normalized size = 2.31

$$\left[-\frac{3\,(bc - ad)x^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) - 6\,(bc - ad)x^2 + 2\,ac}{6\,a^2x^3}, \frac{3\,(bc - ad)x^3\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right) + 3\,(bc - ad)x^2 - ac}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b*c - a*d)*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) - 6*(b*c - a*d)*x^2 + 2*a*c)/(a^2*x^3), 1/3*(3*(b*c - a*d)*x^3*sqr
t(b/a)*arctan(x*sqrt(b/a)) + 3*(b*c - a*d)*x^2 - a*c)/(a^2*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.
time = 0.21, size = 129, normalized size = 2.19

$$\frac{\sqrt{-\frac{b}{a^5}}\,(ad - bc)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}\,(ad-bc)}{abd-b^2c} + x\right)}{2} - \frac{\sqrt{-\frac{b}{a^5}}\,(ad - bc)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}\,(ad-bc)}{abd-b^2c} + x\right)}{2} + \frac{-ac + x^2(-3ad + 3bc)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**5)*(a*d - b*c)*log(-a**3*sqrt(-b/a**5)*(a*d - b*c)/(a*b*d - b**2
*c) + x)/2 - sqrt(-b/a**5)*(a*d - b*c)*log(a**3*sqrt(-b/a**5)*(a*d - b*c)/(
a*b*d - b**2*c) + x)/2 + (-a*c + x**2*(-3*a*d + 3*b*c))/(3*a**2*x**3)
```

Giac [A]
time = 1.51, size = 57, normalized size = 0.97

$$\frac{(b^2c - abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} + \frac{3\,bcx^2 - 3\,adx^2 - ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*c*x^2 - 3*
a*d*x^2 - a*c)/(a^2*x^3)
```

Mupad [B]
time = 0.07, size = 53, normalized size = 0.90

$$-\frac{\frac{c}{3\,a} + \frac{x^2\,(a\,d-b\,c)}{a^2}}{x^3} - \frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d - b\,c)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^4*(a + b*x^2)),x)
```

```
[Out] - (c/(3*a) + (x^2*(a*d - b*c))/a^2)/x^3 - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)
)*(a*d - b*c))/a^(5/2)
```

3.208 $\int \frac{x^5 (c+dx^2)^2}{a+bx^2} \, dx$

Optimal. Leaf size=103

$$-\frac{a(bc-ad)^2x^2}{2b^4}+\frac{(bc-ad)^2x^4}{4b^3}+\frac{d(2bc-ad)x^6}{6b^2}+\frac{d^2x^8}{8b}+\frac{a^2(bc-ad)^2\log(a+bx^2)}{2b^5}$$

```
[Out] -1/2*a*(-a*d+b*c)^2*x^2/b^4+1/4*(-a*d+b*c)^2*x^4/b^3+1/6*d*(-a*d+2*b*c)*x^6
/b^2+1/8*d^2*x^8/b+1/2*a^2*(-a*d+b*c)^2*ln(b*x^2+a)/b^5
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{a^2(bc-ad)^2\log(a+bx^2)}{2b^5}-\frac{ax^2(bc-ad)^2}{2b^4}+\frac{x^4(bc-ad)^2}{4b^3}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{d^2x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -1/2*(a*(b*c - a*d)^2*x^2)/b^4 + ((b*c - a*d)^2*x^4)/(4*b^3) + (d*(2*b*c -
a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + (a^2*(b*c - a*d)^2*Log[a + b*x^2])/(2
*b^5)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(c+dx)^2}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(-bc+ad)^2}{b^4} + \frac{(bc-ad)^2x}{b^3} + \frac{d(2bc-ad)x^2}{b^2} + \frac{d^2x^3}{b} + \frac{a^2(-bc+ad)^2}{b^4(a+bx)}\right.\right.$$
$$= -\frac{a(bc-ad)^2x^2}{2b^4} + \frac{(bc-ad)^2x^4}{4b^3} + \frac{d(2bc-ad)x^6}{6b^2} + \frac{d^2x^8}{8b} + \frac{a^2(bc-ad)^2\log(a+bx^2)}{2b^5}$$

Mathematica [A]
time = 0.04, size = 116, normalized size = 1.13

$$-\frac{a(-bc+ad)^2x^2}{2b^4} + \frac{(bc-ad)^2x^4}{4b^3} + \frac{d(2bc-ad)x^6}{6b^2} + \frac{d^2x^8}{8b} + \frac{(a^2b^2c^2-2a^3bcd+a^4d^2)\log(a+bx^2)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -1/2*(a*(-(b*c) + a*d)^2*x^2)/b^4 + ((b*c - a*d)^2*x^4)/(4*b^3) + (d*(2*b*c
 - a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + ((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
d^2)*Log[a + b*x^2])/(2*b^5)
```

Maple [A]
time = 0.13, size = 139, normalized size = 1.35

method	result
norman	$\frac{(a^2d^2-2abcd+b^2c^2)x^4}{4b^3}+\frac{d^2x^8}{8b}-\frac{a(a^2d^2-2abcd+b^2c^2)x^2}{2b^4}-\frac{d(ad-2bc)x^6}{6b^2}+\frac{a^2(a^2d^2-2abcd+b^2c^2)\ln(bx^2+a)}{2b^5}$
default	$-\frac{-\frac{d^2x^8b^3}{4}+\frac{((ad-bc)b^2d-b^3dc)x^6}{3}+\frac{((ad-bc)b^2c-bd(a^2d-abc))x^4}{2}+(ad-bc)(a^2d-abc)x^2}{2b^4}+\frac{a^2(a^2d^2-2abcd+b^2c^2)\ln(bx^2+a)}{2b^5}$
risch	$-\frac{d^2ax^6}{6b^2}+\frac{dcx^6}{3b}+\frac{d^2a^2x^4}{4b^3}+\frac{c^2x^4}{4b}-\frac{d^2a^3x^2}{2b^4}+\frac{d^2x^8}{8b}-\frac{c^4}{24d^2b}+\frac{7d^2a^4}{24b^5}-\frac{ac^3}{6db^2}-\frac{5da^3c}{6b^4}+\frac{3a^2c^2}{4b^3}-\frac{dacx^4}{2b^2}+\frac{da^2}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^4*(-1/4*d^2*x^8*b^3+1/3*((a*d-b*c)*b^2*d-b^3*d*c)*x^6+1/2*((a*d-b*c)
*b^2*c-b*d*(a^2*d-a*b*c))*x^4+(a*d-b*c)*(a^2*d-a*b*c)*x^2)+1/2*a^2*(a^2*d^2
-2*a*b*c*d+b^2*c^2)/b^5*ln(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 137, normalized size = 1.33

$$\frac{3\,b^3d^2x^8+4\,(2\,b^3cd-ab^2d^2)x^6+6\,(b^3c^2-2\,ab^2cd+a^2bd^2)x^4-12\,(ab^2c^2-2\,a^2bcd+a^3d^2)x^2}{24\,b^4} + \frac{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\log(bx^2+a)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^3*d^2*x^8 + 4*(2*b^3*c*d - a*b^2*d^2)*x^6 + 6*(b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*x^4 - 12*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)/b^4 + 1/
2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b*x^2 + a)/b^5
```

Fricas [A]
time = 1.04, size = 138, normalized size = 1.34

$$\frac{3\,b^4d^2x^8+4\,(2\,b^4cd-ab^3d^2)x^6+6\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4-12\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2+12\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)\log\left(bx^2+a\right)}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*d^2*x^8 + 4*(2*b^4*c*d - a*b^3*d^2)*x^6 + 6*(b^4*c^2 - 2*a*b^3*
c*d + a^2*b^2*d^2)*x^4 - 12*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2 + 1
2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b*x^2 + a))/b^5
```

Sympy [A]
time = 0.34, size = 122, normalized size = 1.18

$$\frac{a^2(ad-bc)^2\log\left(a+bx^2\right)}{2b^5}+x^6\left(-\frac{ad^2}{6b^2}+\frac{cd}{3b}\right)+x^4\left(\frac{a^2d^2}{4b^3}-\frac{acd}{2b^2}+\frac{c^2}{4b}\right)+x^2\left(-\frac{a^3d^2}{2b^4}+\frac{a^2cd}{b^3}-\frac{ac^2}{2b^2}\right)+\frac{d^2x^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] a**2*(a*d - b*c)**2*log(a + b*x**2)/(2*b**5) + x**6*(-a*d**2/(6*b**2) + c*d
/(3*b)) + x**4*(a**2*d**2/(4*b**3) - a*c*d/(2*b**2) + c**2/(4*b)) + x**2*(-
a**3*d**2/(2*b**4) + a**2*c*d/b**3 - a*c**2/(2*b**2)) + d**2*x**8/(8*b)
```

Giac [A]
time = 0.65, size = 148, normalized size = 1.44

$$\frac{3\,b^3d^2x^8+8\,b^3cdx^6-4\,ab^2d^2x^6+6\,b^3c^2x^4-12\,ab^2cdx^4+6\,a^2bd^2x^4-12\,ab^2c^2x^2+24\,a^2bcdx^2-12\,a^3d^2x^2}{24\,b^4}+\frac{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\log\left(|bx^2+a|\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*d^2*x^8 + 8*b^3*c*d*x^6 - 4*a*b^2*d^2*x^6 + 6*b^3*c^2*x^4 - 12*
a*b^2*c*d*x^4 + 6*a^2*b*d^2*x^4 - 12*a*b^2*c^2*x^2 + 24*a^2*b*c*d*x^2 - 12*
a^3*d^2*x^2)/b^4 + 1/2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(abs(b*x^2
+ a))/b^5
```

Mupad [B]
time = 0.06, size = 146, normalized size = 1.42

$$x^4\left(\frac{c^2}{4\,b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{4\,b}\right)-x^6\,\left(\frac{a\,d^2}{6\,b^2}-\frac{c\,d}{3\,b}\right)+\frac{\ln\left(b\,x^2+a\right)\,\left(a^4\,d^2-2\,a^3\,b\,c\,d+a^2\,b^2\,c^2\right)}{2\,b^5}+\frac{d^2\,x^8}{8\,b}-\frac{a\,x^2\,\left(\frac{c^2}{b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] x^4*(c^2/(4*b) + (a*((a*d^2)/b^2 - (2*c*d)/b))/(4*b)) - x^6*((a*d^2)/(6*b^2
) - (c*d)/(3*b)) + (log(a + b*x^2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))/(
2*b^5) + (d^2*x^8)/(8*b) - (a*x^2*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))/b)
)/(2*b)
```

3.209 $\int \frac{x^4(c+dx^2)^2}{a+bx^2} dx$

Optimal. Leaf size=105

$$-\frac{a(bc-ad)^2x}{b^4}+\frac{(bc-ad)^2x^3}{3b^3}+\frac{d(2bc-ad)x^5}{5b^2}+\frac{d^2x^7}{7b}+\frac{a^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

```
[Out] -a*(-a*d+b*c)^2*x/b^4+1/3*(-a*d+b*c)^2*x^3/b^3+1/5*d*(-a*d+2*b*c)*x^5/b^2+1
/7*d^2*x^7/b+a^(3/2)*(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {472, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^2}{b^{9/2}}-\frac{ax(bc-ad)^2}{b^4}+\frac{x^3(bc-ad)^2}{3b^3}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{d^2x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*b*c - a*d)
*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/b^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{x^4(c+dx^2)^2}{a+bx^2}\,dx = \int \left(-\frac{a(bc-ad)^2}{b^4} + \frac{(bc-ad)^2x^2}{b^3} + \frac{d(2bc-ad)x^4}{b^2} + \frac{d^2x^6}{b} + \frac{a^2b^2c^2-2a^3bcd+a^4d^2}{b^4(a+bx^2)}\right.$$

$$= -\frac{a(bc-ad)^2x}{b^4} + \frac{(bc-ad)^2x^3}{3b^3} + \frac{d(2bc-ad)x^5}{5b^2} + \frac{d^2x^7}{7b} + \frac{(a^2(bc-ad)^2)\int \frac{1}{a+bx^2}\,dx}{b^4}$$

$$= -\frac{a(bc-ad)^2x}{b^4} + \frac{(bc-ad)^2x^3}{3b^3} + \frac{d(2bc-ad)x^5}{5b^2} + \frac{d^2x^7}{7b} + \frac{a^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.}{b^{9/2}}$$

Mathematica [A]
time = 0.07, size = 105, normalized size = 1.00

$$-\frac{a(-bc+ad)^2x}{b^4} + \frac{(bc-ad)^2x^3}{3b^3} + \frac{d(2bc-ad)x^5}{5b^2} + \frac{d^2x^7}{7b} + \frac{a^{3/2}(-bc+ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -((a*(-(b*c) + a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*b*c - a
*d)*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(-(b*c) + a*d)^2*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A]
time = 0.13, size = 142, normalized size = 1.35

method	result
default	$-\frac{-\frac{d^2x^7b^3}{7}+\frac{\left((ad-bc)b^2d-b^3dc\right)x^5}{5}+\frac{\left((ad-bc)b^2c-bd\left(a^2d-abc\right)\right)x^3}{3}+(ad-bc)\left(a^2d-abc\right)x}{b^4}+\frac{a^2\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^4\sqrt{ab}}$
risch	$\frac{d^2x^7}{7b}-\frac{x^5a\,d^2}{5b^2}+\frac{2x^5dc}{5b}-\frac{2x^3acd}{3b^2}+\frac{x^3c^2}{3b}+\frac{x^3a^2d^2}{3b^3}-\frac{a^3d^2x}{b^4}+\frac{2a^2cdx}{b^3}-\frac{a\,c^2x}{b^2}+\frac{\sqrt{-ab}\,a^3\ln\left(-\sqrt{-ab}\,x+a\right)d^2}{2b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/7*d^2*x^7*b^3+1/5*((a*d-b*c)*b^2*d-b^3*d*c)*x^5+1/3*((a*d-b*c)*b
^2*c-b*d*(a^2*d-a*b*c))*x^3+(a*d-b*c)*(a^2*d-a*b*c)*x)+a^2*(a^2*d^2-2*a*b*c
*d+b^2*c^2)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 140, normalized size = 1.33

$$\frac{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,b^3d^2x^7+21\,(2\,b^3cd-ab^2d^2)x^5+35\,(b^3c^2-2\,ab^2cd+a^2bd^2)x^3-105\,(ab^2c^2-2\,a^2bcd+a^3d^2)x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/105*(15*b^3*d^2*x^7 + 21*(2*b^3*c*d - a*b^2*d^2)*x^5 + 35*(b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*x^3 - 105*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x)/b^
4
```

Fricas [A]
time = 1.04, size = 304, normalized size = 2.90

$$\left[\frac{30\,b^3d^2x^7+42\,(2\,b^3cd-ab^2d^2)x^5+70\,(b^3c^2-2\,ab^2cd+a^2bd^2)x^3+105\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\,(ab^2c^2-2\,a^2bcd+a^3d^2)x}{210\,b^4}, \frac{15\,b^3d^2x^7+21\,(2\,b^3cd-ab^2d^2)x^5+35\,(b^3c^2-2\,ab^2cd+a^2bd^2)x^3+105\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-105\,(ab^2c^2-2\,a^2bcd+a^3d^2)x}{105\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*d^2*x^7 + 42*(2*b^3*c*d - a*b^2*d^2)*x^5 + 70*(b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*(a*b^2*c^2 - 2*a^
2*b*c*d + a^3*d^2)*x)/b^4, 1/105*(15*b^3*d^2*x^7 + 21*(2*b^3*c*d - a*b^2*d^
2)*x^5 + 35*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 - 2*a^
2*b*c*d + a^3*d^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(a*b^2*c^2 - 2*a
^2*b*c*d + a^3*d^2)*x)/b^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(92) = 184$.
time = 0.36, size = 246, normalized size = 2.34

$$x^5\left(-\frac{ad^2}{5b^2}+\frac{2cd}{5b}\right)+x^3\left(\frac{a^2d^2}{3b^3}-\frac{2acd}{3b^2}+\frac{c^2}{3b}\right)+x\left(-\frac{a^3d^2}{b^4}+\frac{2a^2cd}{b^3}-\frac{ac^2}{b^2}\right)-\frac{\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2}+\frac{d^2x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x**5*(-a*d**2/(5*b**2) + 2*c*d/(5*b)) + x**3*(a**2*d**2/(3*b**3) - 2*a*c*d/
(3*b**2) + c**2/(3*b)) + x*(-a**3*d**2/b**4 + 2*a**2*c*d/b**3 - a*c**2/b**2
) - sqrt(-a**3/b**9)*(a*d - b*c)**2*log(-b**4*sqrt(-a**3/b**9)*(a*d - b*c)*
*2/(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2) + x)/2 + sqrt(-a**3/b**9)*(a*d
- b*c)**2*log(b**4*sqrt(-a**3/b**9)*(a*d - b*c)**2/(a**3*d**2 - 2*a**2*b*c*
d + a*b**2*c**2) + x)/2 + d**2*x**7/(7*b)
```

Giac [A]
time = 0.95, size = 153, normalized size = 1.46

$$\frac{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^6d^2x^7+42\,b^6cdx^5-21\,ab^5d^2x^5+35\,b^6c^2x^3-70\,ab^5cdx^3+35\,a^2b^4d^2x^3-105\,ab^5c^2x+210\,a^2b^4cdx-105\,a^3b^3d^2x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/105*(15*b^6*d^2*x^7 + 42*b^6*c*d*x^5 - 21*a*b^5*d^2*x^5 + 35*b^6*c^2*x
^3 - 70*a*b^5*c*d*x^3 + 35*a^2*b^4*d^2*x^3 - 105*a*b^5*c^2*x + 210*a^2*b^4*
c*d*x - 105*a^3*b^3*d^2*x)/b^7
```

Mupad [B]
time = 0.06, size = 169, normalized size = 1.61

$$x^3\left(\frac{c^2}{3\,b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{3\,b}\right)-x^5\,\left(\frac{a\,d^2}{5\,b^2}-\frac{2\,c\,d}{5\,b}\right)+\frac{d^2\,x^7}{7\,b}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2}{a^4\,d^2-2\,a^3\,b\,c\,d+a^2\,b^2\,c^2}\right)\,{\left(a\,d-b\,c\right)}^2}{b^{9/2}}-\frac{a\,x\,\left(\frac{c^2}{b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] x^3*(c^2/(3*b) + (a*((a*d^2)/b^2 - (2*c*d)/b))/(3*b)) - x^5*((a*d^2)/(5*b^2
) - (2*c*d)/(5*b)) + (d^2*x^7)/(7*b) + (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(a*
d - b*c)^2)/(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))*(a*d - b*c)^2)/b^(9/2) -
 (a*x*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))/b))/b
```

3.210 $\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=80

$$\frac{(bc-ad)^2x^2}{2b^3}+\frac{d(2bc-ad)x^4}{4b^2}+\frac{d^2x^6}{6b}-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}$$

```
[Out] 1/2*(-a*d+b*c)^2*x^2/b^3+1/4*d*(-a*d+2*b*c)*x^4/b^2+1/6*d^2*x^6/b-1/2*a*(-a
*d+b*c)^2*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(bc-ad)^2}{2b^3}+\frac{dx^4(2bc-ad)}{4b^2}+\frac{d^2x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x^2)/(2*b^3) + (d*(2*b*c - a*d)*x^4)/(4*b^2) + (d^2*x^6)/(6*
b) - (a*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^2}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(bc-ad)^2}{b^3}+\frac{d(2bc-ad)x}{b^2}+\frac{d^2x^2}{b}-\frac{a(-bc+ad)^2}{b^3(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{(bc-ad)^2x^2}{2b^3}+\frac{d(2bc-ad)x^4}{4b^2}+\frac{d^2x^6}{6b}-\frac{a(bc-ad)^2\log(a+bx^2)}{2b^4}$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 1.02

$$\frac{bx^2(6a^2d^2-3abd(4c+dx^2)+2b^2(3c^2+3cdx^2+d^2x^4))-6a(bc-ad)^2\log(a+bx^2)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*d^2 - 3*a*b*d*(4*c + d*x^2) + 2*b^2*(3*c^2 + 3*c*d*x^2 + d^2*
x^4)) - 6*a*(b*c - a*d)^2*Log[a + b*x^2])/(12*b^4)
```

Maple [A]
time = 0.08, size = 102, normalized size = 1.28

method	result	size
norman	$\frac{(a^2d^2-2abcd+b^2c^2)x^2}{2b^3}+\frac{d^2x^6}{6b}-\frac{d(ad-2bc)x^4}{4b^2}-\frac{a(a^2d^2-2abcd+b^2c^2)\ln(bx^2+a)}{2b^4}$	94
default	$\frac{\frac{1}{3}b^2d^2x^6-\frac{1}{2}ab\,d^2x^4+b^2cd\,x^4+a^2d^2x^2-2abcd\,x^2+b^2c^2x^2}{2b^3}-\frac{a(a^2d^2-2abcd+b^2c^2)\ln(bx^2+a)}{2b^4}$	102
risch	$\frac{d^2x^6}{6b}-\frac{a\,d^2x^4}{4b^2}+\frac{cd\,x^4}{2b}+\frac{a^2d^2x^2}{2b^3}-\frac{acd\,x^2}{b^2}+\frac{c^2x^2}{2b}-\frac{a^3\ln(bx^2+a)d^2}{2b^4}+\frac{a^2\ln(bx^2+a)cd}{b^3}-\frac{a\ln(bx^2+a)c^2}{2b^2}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^3*(1/3*b^2*d^2*x^6-1/2*a*b*d^2*x^4+b^2*c*d*x^4+a^2*d^2*x^2-2*a*b*c*d*
x^2+b^2*c^2*x^2)-1/2*a*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4*ln(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 101, normalized size = 1.26

$$\frac{2\,b^2d^2x^6+3\,(2\,b^2cd-abd^2)x^4+6\,(b^2c^2-2\,abcd+a^2d^2)x^2}{12\,b^3}-\frac{(ab^2c^2-2\,a^2bcd+a^3d^2)\log(bx^2+a)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 + 3*(2*b^2*c*d - a*b*d^2)*x^4 + 6*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*log(b*x^2 + a
)/b^4
```

Fricas [A]

time = 1.11, size = 102, normalized size = 1.28

$$\frac{2\,b^{3}d^{2}x^{6}+3\,(2\,b^{3}cd-ab^{2}d^{2})x^{4}+6\,(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2})x^{2}-6\,(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2})\log\left(bx^{2}+a\right)}{12\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^6 + 3*(2*b^3*c*d - a*b^2*d^2)*x^4 + 6*(b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*x^2 - 6*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*log(b*x^2 + a)
)/b^4
```

Sympy [A]

time = 0.22, size = 83, normalized size = 1.04

$$-\frac{a(ad-bc)^{2}\log\left(a+bx^{2}\right)}{2b^{4}}+x^{4}\left(-\frac{ad^{2}}{4b^{2}}+\frac{cd}{2b}\right)+x^{2}\left(\frac{a^{2}d^{2}}{2b^{3}}-\frac{acd}{b^{2}}+\frac{c^{2}}{2b}\right)+\frac{d^{2}x^{6}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -a*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4) + x**4*(-a*d**2/(4*b**2) + c*d/(
2*b)) + x**2*(a**2*d**2/(2*b**3) - a*c*d/b**2 + c**2/(2*b)) + d**2*x**6/(6*
b)
```

Giac [A]

time = 0.89, size = 107, normalized size = 1.34

$$\frac{2\,b^{2}d^{2}x^{6}+6\,b^{2}cdx^{4}-3\,abd^{2}x^{4}+6\,b^{2}c^{2}x^{2}-12\,abcdx^{2}+6\,a^{2}d^{2}x^{2}}{12\,b^{3}}-\frac{(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2})\log\left(|bx^{2}+a|\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 + 6*b^2*c*d*x^4 - 3*a*b*d^2*x^4 + 6*b^2*c^2*x^2 - 12*a*
b*c*d*x^2 + 6*a^2*d^2*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*lo
g(abs(b*x^2 + a))/b^4
```

Mupad [B]

time = 0.06, size = 106, normalized size = 1.32

$$x^{2}\left(\frac{c^{2}}{2\,b}+\frac{a\left(\frac{a\,d^{2}}{b^{2}}-\frac{2\,c\,d}{b}\right)}{2\,b}\right)-x^{4}\left(\frac{a\,d^{2}}{4\,b^{2}}-\frac{c\,d}{2\,b}\right)-\frac{\ln\left(b\,x^{2}+a\right)\,\left(a^{3}\,d^{2}-2\,a^{2}\,b\,c\,d+a\,b^{2}\,c^{2}\right)}{2\,b^{4}}+\frac{d^{2}\,x^{6}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] x^2*(c^2/(2*b) + (a*((a*d^2)/b^2 - (2*c*d)/b))/(2*b)) - x^4*((a*d^2)/(4*b^2
) - (c*d)/(2*b)) - (log(a + b*x^2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))/(2*
b^4) + (d^2*x^6)/(6*b)
```

3.211 $\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=84

$$\frac{(bc-ad)^2x}{b^3}+\frac{d(2bc-ad)x^3}{3b^2}+\frac{d^2x^5}{5b}-\frac{\sqrt{a}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] (-a*d+b*c)^2*x/b^3+1/3*d*(-a*d+2*b*c)*x^3/b^2+1/5*d^2*x^5/b-(-a*d+b*c)^2*ar
ctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {472, 211}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^2}{b^{7/2}}+\frac{x(bc-ad)^2}{b^3}+\frac{dx^3(2bc-ad)}{3b^2}+\frac{d^2x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)/(5*b) - (
Sqrt[a]*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx = \int \left(\frac{(bc-ad)^2}{b^3} + \frac{d(2bc-ad)x^2}{b^2} + \frac{d^2x^4}{b} + \frac{-ab^2c^2+2a^2bcd-a^3d^2}{b^3\,(a+bx^2)}\right)dx$$
$$= \frac{(bc-ad)^2x}{b^3} + \frac{d(2bc-ad)x^3}{3b^2} + \frac{d^2x^5}{5b} - \frac{(a(bc-ad)^2)\int \frac{1}{a+bx^2}\,dx}{b^3}$$
$$= \frac{(bc-ad)^2x}{b^3} + \frac{d(2bc-ad)x^3}{3b^2} + \frac{d^2x^5}{5b} - \frac{\sqrt{a}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 1.00

$$\frac{(bc-ad)^2x}{b^3} + \frac{d(2bc-ad)x^3}{3b^2} + \frac{d^2x^5}{5b} - \frac{\sqrt{a}\,(-bc+ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)/(5*b) - (
Sqrt[a]*(-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Maple [A]
time = 0.09, size = 102, normalized size = 1.21

method	result
default	$\frac{\frac{1}{5}b^2x^5d^2-\frac{1}{3}ab\,d^2x^3+\frac{2}{3}b^2cd\,x^3+a^2d^2x-2abcdx+b^2c^2x}{b^3} - \frac{a\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{d^2x^5}{5b} - \frac{a\,d^2x^3}{3b^2} + \frac{2cd\,x^3}{3b} + \frac{a^2d^2x}{b^3} - \frac{2acdx}{b^2} + \frac{c^2x}{b} + \frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)a^2d^2}{2b^4} - \frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right.}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*b^2*x^5*d^2-1/3*a*b*d^2*x^3+2/3*b^2*c*d*x^3+a^2*d^2*x-2*a*b*c*d*
x+b^2*c^2*x)-a*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))
```

Maxima [A]
time = 0.50, size = 105, normalized size = 1.25

$$-\frac{(ab^2c^2-2\,a^2bcd+a^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^2d^2x^5+5\,(2\,b^2cd-abd^2)x^3+15\,(b^2c^2-2\,abcd+a^2d^2)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
+ 1/15*(3*b^2*d^2*x^5 + 5*(2*b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x)/b^3
```

Fricas [**A**]
time = 1.05, size = 230, normalized size = 2.74

$$\left[\frac{6\,b^{2}d^{2}x^{5} + 10\,(2\,b^{2}cd - abd^{2})x^{3} + 15\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2} - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^{2} + a}\right) + 30\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})x}{30\,b^{3}}, \frac{3\,b^{2}d^{2}x^{5} + 5\,(2\,b^{2}cd - abd^{2})x^{3} - 15\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 15\,(b^{2}c^{2} - 2\,abcd + a^{2}d^{2})x}{15\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d^2*x^5 + 10*(2*b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) +
30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^3, 1/15*(3*b^2*d^2*x^5 + 5*(2*b^2*c
*d - a*b*d^2)*x^3 - 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) + 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^3]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(73) = 146$.
time = 0.26, size = 194, normalized size = 2.31

$$x^{3}\left(-\frac{ad^{2}}{3b^{2}} + \frac{2cd}{3b}\right) + x\left(\frac{a^{2}d^{2}}{b^{3}} - \frac{2acd}{b^{2}} + \frac{c^{2}}{b}\right) + \frac{\sqrt{-\frac{a}{b^{7}}}\,(ad - bc)^{2}\log\left(-\frac{b^{3}\sqrt{-\frac{a}{b^{7}}}\,(ad-bc)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}} + x\right)}{2} - \frac{\sqrt{-\frac{a}{b^{7}}}\,(ad - bc)^{2}\log\left(\frac{b^{3}\sqrt{-\frac{a}{b^{7}}}\,(ad-bc)^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}} + x\right)}{2} + \frac{d^{2}x^{5}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x**3*(-a*d**2/(3*b**2) + 2*c*d/(3*b)) + x*(a**2*d**2/b**3 - 2*a*c*d/b**2 +
c**2/b) + sqrt(-a/b**7)*(a*d - b*c)**2*log(-b**3*sqrt(-a/b**7)*(a*d - b*c)*
*2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-a/b**7)*(a*d - b*c)**
2*log(b**3*sqrt(-a/b**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2)
 + x)/2 + d**2*x**5/(5*b)
```

Giac [**A**]
time = 0.87, size = 113, normalized size = 1.35

$$-\frac{(ab^{2}c^{2} - 2\,a^{2}bcd + a^{3}d^{2})\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}} + \frac{3\,b^{4}d^{2}x^{5} + 10\,b^{4}cdx^{3} - 5\,ab^{3}d^{2}x^{3} + 15\,b^{4}c^{2}x - 30\,ab^{3}cdx + 15\,a^{2}b^{2}d^{2}x}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
+ 1/15*(3*b^4*d^2*x^5 + 10*b^4*c*d*x^3 - 5*a*b^3*d^2*x^3 + 15*b^4*c^2*x - 3
0*a*b^3*c*d*x + 15*a^2*b^2*d^2*x)/b^5
```

Mupad [B]
time = 0.06, size = 128, normalized size = 1.52

$$x\left(\frac{c^2}{b}+\frac{a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)-x^3\left(\frac{a\,d^2}{3\,b^2}-\frac{2\,c\,d}{3\,b}\right)+\frac{d^2\,x^5}{5\,b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2}{a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2}\right)\,{\left(a\,d-b\,c\right)}^2}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] x*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))/b) - x^3*((a*d^2)/(3*b^2) - (2*c*d
)/(3*b)) + (d^2*x^5)/(5*b) - (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d - b*c)^2
)/(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))*(a*d - b*c)^2)/b^(7/2)
```

3.212 $\int \frac{x(c+dx^2)^2}{a+bx^2} dx$

Optimal. Leaf size=61

$$\frac{d(bc-ad)x^2}{2b^2}+\frac{(c+dx^2)^2}{4b}+\frac{(bc-ad)^2\log(a+bx^2)}{2b^3}$$

```
[Out] 1/2*d*(-a*d+b*c)*x^2/b^2+1/4*(d*x^2+c)^2/b+1/2*(-a*d+b*c)^2*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 45}

$$\frac{(bc-ad)^2\log(a+bx^2)}{2b^3}+\frac{dx^2(bc-ad)}{2b^2}+\frac{(c+dx^2)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (d*(b*c - a*d)*x^2)/(2*b^2) + (c + d*x^2)^2/(4*b) + ((b*c - a*d)^2*Log[a +
b*x^2])/(2*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx, x, x^2\right)\\
&= \frac{d(bc-ad)x^2}{2b^2}+\frac{(c+dx^2)^2}{4b}+\frac{(bc-ad)^2\log(a+bx^2)}{2b^3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.80

$$\frac{bdx^2(4bc-2ad+bdx^2)+2(bc-ad)^2\log(a+bx^2)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (b*d*x^2*(4*b*c - 2*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[a + b*x^2])/(4*b^3
)
```

Maple [A]
time = 0.11, size = 63, normalized size = 1.03

method	result	size
default	$-\frac{d\left(-\frac{1}{2}bd\,x^4+ad\,x^2-2c\,x^2b\right)}{2b^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2b^3}$	63
norman	$\frac{d^2x^4}{4b}-\frac{d(ad-2bc)x^2}{2b^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2b^3}$	64
risch	$\frac{d^2x^4}{4b}-\frac{a\,d^2x^2}{2b^2}+\frac{cd\,x^2}{b}+\frac{a^2d^2}{4b^3}-\frac{acd}{b^2}+\frac{c^2}{b}+\frac{\ln(b\,x^2+a)a^2d^2}{2b^3}-\frac{\ln(b\,x^2+a)acd}{b^2}+\frac{\ln(b\,x^2+a)c^2}{2b}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d/b^2*(-1/2*b*d*x^4+a*d*x^2-2*c*x^2*b)+1/2/b^3*(a^2*d^2-2*a*b*c*d+b^2*
c^2)*ln(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 66, normalized size = 1.08

$$\frac{bd^2x^4+2\,(2\,bcd-ad^2)x^2}{4\,b^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log(bx^2+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d^2*x^4 + 2*(2*b*c*d - a*d^2)*x^2)/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(b*x^2 + a)/b^3
```

Fricas [A]
time = 1.27, size = 67, normalized size = 1.10

$$\frac{b^2d^2x^4+2\,(2\,b^2cd-abd^2)x^2+2\,(b^2c^2-2\,abcd+a^2d^2)\log(bx^2+a)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^4 + 2*(2*b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(b*x^2 + a))/b^3
```

Sympy [A]
time = 0.19, size = 49, normalized size = 0.80

$$x^2\left(-\frac{ad^2}{2b^2}+\frac{cd}{b}\right)+\frac{d^2x^4}{4b}+\frac{(ad-bc)^2\log(a+bx^2)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x**2*(-a*d**2/(2*b**2) + c*d/b) + d**2*x**4/(4*b) + (a*d - b*c)**2*log(a +
b*x**2)/(2*b**3)
```

Giac [A]
time = 0.98, size = 67, normalized size = 1.10

$$\frac{bd^2x^4+4\,bcdx^2-2\,ad^2x^2}{4\,b^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log(|bx^2+a|)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d^2*x^4 + 4*b*c*d*x^2 - 2*a*d^2*x^2)/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(b*x^2 + a))/b^3
```

Mupad [B]
time = 0.04, size = 68, normalized size = 1.11

$$\frac{d^2\,x^4}{4\,b}-x^2\left(\frac{a\,d^2}{2\,b^2}-\frac{c\,d}{b}\right)+\frac{\ln(b\,x^2+a)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] (d^2*x^4)/(4*b) - x^2*((a*d^2)/(2*b^2) - (c*d)/b) + (log(a + b*x^2)*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d))/(2*b^3)
```

3.213 $\int \frac{(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=63

$$\frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2+1/3*d^2*x^3/b+(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/b
^(5/2)/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {398, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^2}{\sqrt{a}\,b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \int \left(\frac{d(2bc-ad)}{b^2} + \frac{d^2x^2}{b} + \frac{b^2c^2-2abcd+a^2d^2}{b^2(a+bx^2)}\right)dx$$
$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^3}{3b} + \frac{(bc-ad)^2 \int \frac{1}{a+bx^2}\,dx}{b^2}$$
$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^3}{3b} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.94

$$\frac{dx(6bc-3ad+bdx^2)}{3b^2} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2),x]
[Out] (d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Maple [A]
time = 0.00, size = 64, normalized size = 1.02

method	result
default	$-\frac{d\left(-\frac{1}{3}bd\,x^3+adx-2bcx\right)}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{d^2x^3}{3b}-\frac{d^2ax}{b^2}+\frac{2dcx}{b}-\frac{\ln\left(bx+\sqrt{-ab}\right)a^2d^2}{2b^2\sqrt{-ab}}+\frac{\ln\left(bx+\sqrt{-ab}\right)acd}{b\sqrt{-ab}}-\frac{\ln\left(bx+\sqrt{-ab}\right)c^2}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2b^2\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -d/b^2*(-1/3*b*d*x^3+a*d*x-2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 69, normalized size = 1.10

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{bd^2x^3+3\,(2\,bcd-ad^2)x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x)/b^2
```

Fricas [A]
time = 1.52, size = 181, normalized size = 2.87

$$\left[\frac{2\,ab^2d^2x^3 - 3\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 6\,(2\,ab^2cd - a^2bd^2)x}{6\,ab^3},\ \frac{ab^2d^2x^3 + 3\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + 3\,(2\,ab^2cd - a^2bd^2)x}{3\,ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*b^2*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a
*b^3), 1/3*(a*b^2*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 3*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.
time = 0.22, size = 172, normalized size = 2.73

$$x\left(-\frac{ad^2}{b^2} + \frac{2cd}{b}\right) - \frac{\sqrt{-\frac{1}{ab^5}}\,(ad - bc)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2}{a^2d^2 - 2abcd + b^2c^2} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ab^5}}\,(ad - bc)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2}{a^2d^2 - 2abcd + b^2c^2} + x\right)}{2} + \frac{d^2x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) - sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*s
qrt(-1/(a*b**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
+ sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b*c)
**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x**3/(3*b)
```

Giac [A]
time = 0.94, size = 72, normalized size = 1.14

$$\frac{(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{b^2d^2x^3 + 6\,b^2cdx - 3\,abd^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

Mupad [B]
time = 0.04, size = 90, normalized size = 1.43

$$\frac{d^2\,x^3}{3\,b}-x\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2),x)
```

```
[Out] (d^2*x^3)/(3*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((b^(1/2)*x*(a*d - b*c
)^2)/(a^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(1/2)*b^(
5/2))
```

3.214 $\int \frac{(c+dx^2)^2}{x(a+bx^2)} dx$

Optimal. Leaf size=51

$$\frac{d^2x^2}{2b} + \frac{c^2 \log(x)}{a} - \frac{(bc-ad)^2 \log(a+bx^2)}{2ab^2}$$

[Out] 1/2*d^2*x^2/b+c^2*ln(x)/a-1/2*(-a*d+b*c)^2*ln(b*x^2+a)/a/b^2

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 84}

$$-\frac{(bc-ad)^2 \log(a+bx^2)}{2ab^2} + \frac{c^2 \log(x)}{a} + \frac{d^2x^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^2)^2/(x*(a + b*x^2)),x]

[Out] (d^2*x^2)/(2*b) + (c^2*Log[x])/a - ((b*c - a*d)^2*Log[a + b*x^2])/(2*a*b^2)

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^2)^2}{x(a+bx^2)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{x(a+bx)} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{b} + \frac{c^2}{ax} - \frac{(-bc+ad)^2}{ab(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{d^2x^2}{2b} + \frac{c^2 \log(x)}{a} - \frac{(bc-ad)^2 \log(a+bx^2)}{2ab^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.98

$$\frac{abd^2x^2+2b^2c^2\log(x)-(bc-ad)^2\log(a+bx^2)}{2ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

```
[Out] (a*b*d^2*x^2 + 2*b^2*c^2*Log[x] - (b*c - a*d)^2*Log[a + b*x^2])/(2*a*b^2)
```

Maple [A]
time = 0.08, size = 59, normalized size = 1.16

method	result	size
default	$\frac{d^2x^2}{2b}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2a\,b^2}+\frac{c^2\ln(x)}{a}$	59
norman	$\frac{d^2x^2}{2b}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2a\,b^2}+\frac{c^2\ln(x)}{a}$	59
risch	$\frac{d^2x^2}{2b}-\frac{a\ln(b\,x^2+a)d^2}{2b^2}+\frac{\ln(b\,x^2+a)cd}{b}-\frac{\ln(b\,x^2+a)c^2}{2a}+\frac{c^2\ln(x)}{a}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d^2*x^2/b-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a/b^2*ln(b*x^2+a)+c^2*ln(x)/a
```

Maxima [A]
time = 0.32, size = 61, normalized size = 1.20

$$\frac{d^2x^2}{2\,b}+\frac{c^2\log(x^2)}{2\,a}-\frac{(b^2c^2-2\,abcd+a^2d^2)\log(bx^2+a)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^2/b + 1/2*c^2*log(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(b*x^2 + a)/(a*b^2)
```

Fricas [A]
time = 1.15, size = 59, normalized size = 1.16

$$\frac{abd^2x^2+2\,b^2c^2\log(x)-(b^2c^2-2\,abcd+a^2d^2)\log(bx^2+a)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*d^2*x^2 + 2*b^2*c^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(b
*x^2 + a))/(a*b^2)
```

Sympy [A]
time = 0.77, size = 41, normalized size = 0.80

$$\frac{d^2x^2}{2b}+\frac{c^2\log(x)}{a}-\frac{(ad-bc)^2\log\left(\frac{a}{b}+x^2\right)}{2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x/(b*x**2+a),x)
```

```
[Out] d**2*x**2/(2*b) + c**2*log(x)/a - (a*d - b*c)**2*log(a/b + x**2)/(2*a*b**2)
```

Giac [A]
time = 0.67, size = 62, normalized size = 1.22

$$\frac{d^2x^2}{2\,b}+\frac{c^2\log(x^2)}{2\,a}-\frac{(b^2c^2-2\,abcd+a^2d^2)\log(|bx^2+a|)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*x^2/b + 1/2*c^2*log(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(b*x^2 + a))/(a*b^2)
```

Mupad [B]
time = 0.10, size = 58, normalized size = 1.14

$$\frac{d^2\,x^2}{2\,b}+\frac{c^2\,\ln(x)}{a}-\frac{\ln(b\,x^2+a)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x*(a + b*x^2)),x)
```

```
[Out] (d^2*x^2)/(2*b) + (c^2*log(x))/a - (log(a + b*x^2)*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))/(2*a*b^2)
```

3.215 $\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=55

$$-\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}$$

```
[Out] -c^2/a/x+d^2*x/b-(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {472, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^2}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(
3/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx = \int \left(\frac{d^2}{b}+\frac{c^2}{ax^2}-\frac{(-bc+ad)^2}{ab(a+bx^2)}\right)dx$$
$$= -\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(bc-ad)^2\int\frac{1}{a+bx^2}\,dx}{ab}$$
$$= -\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 1.00

$$-\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(-bc+ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(3/2)*b^(3/2))
```

Maple [A]
time = 0.08, size = 65, normalized size = 1.18

method	result
default	$\frac{d^2x}{b}+\frac{(-a^2d^2+2abcd-b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{ba\sqrt{ab}}-\frac{c^2}{ax}$
risch	$\frac{d^2x}{b}-\frac{c^2}{ax}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4+a^3\_Z^2b\right)}\_R\ln\left(\left(2a^4d^4-8a^3bc\,d^3+12a^2b^2c^2d^2-8a\,b^3c^3d+2b^4\right.\right.}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*x/b+1/b/a*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/
2))-c^2/a/x
```

Maxima [A]
time = 0.50, size = 63, normalized size = 1.15

$$\frac{d^2x}{b}-\frac{c^2}{ax}-\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="maxima")

[Out] d^2*x/b - c^2/(a*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)

Fricas [A]
time = 1.10, size = 164, normalized size = 2.98

$$\left[\frac{2\,a^2bd^2x^2 - 2\,ab^2c^2 - (b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-ab}\,x\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,a^2b^2x}, \frac{a^2bd^2x^2 - ab^2c^2 - (b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}\,x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{a^2b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="fricas")

[Out] [1/2*(2*a^2*b*d^2*x^2 - 2*a*b^2*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^2*x), (a^2*b*d^2*x^2 - a*b^2*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a))/(a^2*b^2*x)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(44) = 88$.
time = 0.31, size = 165, normalized size = 3.00

$$\frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad-bc)^2\log\left(-\frac{a^2b\sqrt{-\frac{1}{a^3b^3}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} - \frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad-bc)^2\log\left(\frac{a^2b\sqrt{-\frac{1}{a^3b^3}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{d^2x}{b} - \frac{c^2}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)**2/x**2/(b*x**2+a),x)

[Out] sqrt(-1/(a**3*b**3))*(a*d - b*c)**2*log(-a**2*b*sqrt(-1/(a**3*b**3))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-1/(a**3*b**3))*(a*d - b*c)**2*log(a**2*b*sqrt(-1/(a**3*b**3))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x/b - c**2/(a*x)

Giac [A]
time = 0.61, size = 63, normalized size = 1.15

$$\frac{d^2x}{b} - \frac{c^2}{ax} - \frac{(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d^2*x/b - c^2/(a*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a*b)
```

Mupad [B]
time = 0.04, size = 80, normalized size = 1.45

$$\frac{d^2\,x}{b}-\frac{c^2}{a\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^2*(a + b*x^2)),x)
```

```
[Out] (d^2*x)/b - c^2/(a*x) - (atan((b^(1/2)*x*(a*d - b*c)^2)/(a^(1/2)*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(3/2)*b^(3/2))
```

3.216 $\int \frac{(c+dx^2)^2}{x^3(a+bx^2)} \, dx$

Optimal. Leaf size=58

$$-\frac{c^2}{2ax^2}-\frac{c(bc-2ad)\log(x)}{a^2}+\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2a^2b}$$

```
[Out] -1/2*c^2/a/x^2-c*(-2*a*d+b*c)*ln(x)/a^2+1/2*(-a*d+b*c)^2*ln(b*x^2+a)/a^2/b
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2a^2b}-\frac{c\log(x)(bc-2ad)}{a^2}-\frac{c^2}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*c^2/(a*x^2) - (c*(b*c - 2*a*d)*Log[x])/a^2 + ((b*c - a*d)^2*Log[a + b*
x^2])/(2*a^2*b)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c^2}{ax^2}+\frac{c(-bc+2ad)}{a^2x}+\frac{(-bc+ad)^2}{a^2(a+bx)}\right)\,dx,x,x^2\right)\\
&= -\frac{c^2}{2ax^2}-\frac{c(bc-2ad)\log(x)}{a^2}+\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2a^2b}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 1.03

$$\frac{-abc^2 - 2bc(bc - 2ad)x^2 \log(x) + (bc - ad)^2 x^2 \log(a + bx^2)}{2a^2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

```
[Out] (-(a*b*c^2) - 2*b*c*(b*c - 2*a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2*Log[a + b*
x^2])/(2*a^2*b*x^2)
```

Maple [A]
time = 0.08, size = 66, normalized size = 1.14

method	result	size
default	$\frac{(a^2d^2-2abcd+b^2c^2)\ln(bx^2+a)}{2a^2b} - \frac{c^2}{2ax^2} + \frac{c(2ad-bc)\ln(x)}{a^2}$	66
norman	$\frac{(a^2d^2-2abcd+b^2c^2)\ln(bx^2+a)}{2a^2b} - \frac{c^2}{2ax^2} + \frac{c(2ad-bc)\ln(x)}{a^2}$	66
risch	$-\frac{c^2}{2ax^2} + \frac{2c\ln(x)d}{a} - \frac{c^2\ln(x)b}{a^2} + \frac{\ln(-bx^2-a)d^2}{2b} - \frac{\ln(-bx^2-a)cd}{a} + \frac{b\ln(-bx^2-a)c^2}{2a^2}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^2/b*ln(b*x^2+a)-1/2*c^2/a/x^2+c*(2*a*d-b*
c)/a^2*ln(x)
```

Maxima [A]
time = 0.28, size = 69, normalized size = 1.19

$$-\frac{(bc^2 - 2\,acd)\log(x^2)}{2\,a^2} - \frac{c^2}{2\,ax^2} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log(bx^2 + a)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c^2 - 2*a*c*d)*log(x^2)/a^2 - 1/2*c^2/(a*x^2) + 1/2*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*log(b*x^2 + a)/(a^2*b)
```

Fricas [A]
time = 1.37, size = 73, normalized size = 1.26

$$-\frac{abc^2 - (b^2c^2 - 2\,abcd + a^2d^2)x^2\log(bx^2 + a) + 2\,(b^2c^2 - 2\,abcd)x^2\log(x)}{2\,a^2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(b*x^2 + a) + 2*(b^2
*c^2 - 2*a*b*c*d)*x^2*log(x))/(a^2*b*x^2)
```

Sympy [A]
time = 0.90, size = 49, normalized size = 0.84

$$-\frac{c^2}{2ax^2}+\frac{c(2ad-bc)\log{(x)}}{a^2}+\frac{(ad-bc)^2\log{\left(\frac{a}{b}+x^2\right)}}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**3/(b*x**2+a),x)
```

```
[Out] -c**2/(2*a*x**2) + c*(2*a*d - b*c)*log(x)/a**2 + (a*d - b*c)**2*log(a/b + x
**2)/(2*a**2*b)
```

Giac [A]
time = 0.58, size = 90, normalized size = 1.55

$$-\frac{(bc^2-2\,acd)\log{(x^2)}}{2\,a^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log{(|bx^2+a|)}}{2\,a^2b}+\frac{bc^2x^2-2\,acdx^2-ac^2}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2 - 2*a*c*d)*log(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*c^2*x^2 - 2*a*c*d*x^2 - a*c^2)/(a^2*x^2
)
```

Mupad [B]
time = 0.10, size = 67, normalized size = 1.16

$$\frac{\ln{(b\,x^2+a)}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,a^2\,b}-\frac{c^2}{2\,a\,x^2}-\frac{\ln{(x)}\,\left(b\,c^2-2\,a\,c\,d\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^3*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a^2*b) - c^2/(2*a*x^2)
- (log(x)*(b*c^2 - 2*a*c*d))/a^2
```

3.217 $\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=64

$$-\frac{c^2}{3ax^3}+\frac{c(bc-2ad)}{a^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}$$

```
[Out] -1/3*c^2/a/x^3+c*(-2*a*d+b*c)/a^2/x+(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/
a^(5/2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {472, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^2}{a^{5/2}\sqrt{b}}+\frac{c(bc-2ad)}{a^2x}-\frac{c^2}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c^2/(a*x^3) + (c*(b*c - 2*a*d))/(a^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx = \int \left(\frac{c^2}{ax^4} + \frac{c(-bc+2ad)}{a^2x^2} + \frac{(-bc+ad)^2}{a^2(a+bx^2)}\right) dx$$
$$= -\frac{c^2}{3ax^3} + \frac{c(bc-2ad)}{a^2x} + \frac{(bc-ad)^2 \int \frac{1}{a+bx^2}\,dx}{a^2}$$
$$= -\frac{c^2}{3ax^3} + \frac{c(bc-2ad)}{a^2x} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 1.03

$$-\frac{c^2}{3ax^3} - \frac{c(-bc+2ad)}{a^2x} + \frac{(-bc+ad)^2 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
[Out] -1/3*c^2/(a*x^3) - (c*(-(b*c) + 2*a*d))/(a^2*x) + ((-(b*c) + a*d)^2*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Maple [A]
time = 0.09, size = 70, normalized size = 1.09

method	result
default	$\frac{(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2\sqrt{ab}} - \frac{c^2}{3a\,x^3} - \frac{c(2ad-bc)}{a^2x}$
risch	$\frac{-\frac{c(2ad-bc)x^2}{a^2}-\frac{c^2}{3a}}{x^3} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2b+a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)} \_R\ln\left(\left(3\_R^2a^5b+2a^4d^4-8a^3bc\,d^3+12 \right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] (a^2*d^2-2*a*b*c*d+b^2*c^2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/3*c^2
/a/x^3-c*(2*a*d-b*c)/a^2/x
```

Maxima [A]
time = 0.50, size = 70, normalized size = 1.09

$$\frac{(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} - \frac{ac^2 - 3\,(bc^2 - 2\,acd)x^2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3
*(a*c^2 - 3*(b*c^2 - 2*a*c*d)*x^2)/(a^2*x^3)
```

Fricas [A]
time = 1.45, size = 190, normalized size = 2.97

$$\left[-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-ab}\,x^3\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,a^2bc^2-6\,(ab^2c^2-2\,a^2bcd)x^2}{6\,a^3bx^3},\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}\,x^3\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-a^2bc^2+3\,(ab^2c^2-2\,a^2bcd)x^2}{3\,a^3bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*x^3*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) + 2*a^2*b*c^2 - 6*(a*b^2*c^2 - 2*a^2*b*c*d)*x^2)
/(a^3*b*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*x^3*arctan(s
qrt(a*b)*x/a) - a^2*b*c^2 + 3*(a*b^2*c^2 - 2*a^2*b*c*d)*x^2)/(a^3*b*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.
time = 0.42, size = 172, normalized size = 2.69

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-bc)^2\log\left(-\frac{a^3\sqrt{-\frac{1}{a^5b}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-bc)^2\log\left(\frac{a^3\sqrt{-\frac{1}{a^5b}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{-ac^2+x^2(-6acd+3bc^2)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**5*b))*(a*d - b*c)**2*log(-a**3*sqrt(-1/(a**5*b))*(a*d - b*c)**
2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(a**5*b))*(a*d - b*c
)**2*log(a**3*sqrt(-1/(a**5*b))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**
2*c**2) + x)/2 + (-a*c**2 + x**2*(-6*a*c*d + 3*b*c**2))/(3*a**2*x**3)
```

Giac [A]
time = 0.53, size = 72, normalized size = 1.12

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bc^2x^2-6\,acdx^2-ac^2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3
*(3*b*c^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^2*x^3)
```

Mupad [B]
time = 0.08, size = 90, normalized size = 1.41

$$\frac{b\,c^2}{a^2\,x}-\frac{c^2}{3\,a\,x^3}+\frac{b^{3/2}\,c^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2\,c\,d}{a\,x}-\frac{2\,\sqrt{b}\,c\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^4*(a + b*x^2)),x)
```

```
[Out] (b*c^2)/(a^2*x) - c^2/(3*a*x^3) + (b^(3/2)*c^2*atan((b^(1/2)*x)/a^(1/2)))/a
^(5/2) + (d^2*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (2*c*d)/(a*x)
- (2*b^(1/2)*c*d*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.218 $\int \frac{x^5\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=138

$$-\frac{a(bc-ad)^3x^2}{2b^5}+\frac{(bc-ad)^3x^4}{4b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^6}{6b^3}+\frac{d^2(3bc-ad)x^8}{8b^2}+\frac{d^3x^{10}}{10b}+\frac{a^2(bc-ad)^3\log\left(a+b\right.}{2b^6}$$

```
[Out] -1/2*a*(-a*d+b*c)^3*x^2/b^5+1/4*(-a*d+b*c)^3*x^4/b^4+1/6*d*(a^2*d^2-3*a*b*c
*d+3*b^2*c^2)*x^6/b^3+1/8*d^2*(-a*d+3*b*c)*x^8/b^2+1/10*d^3*x^10/b+1/2*a^2*
(-a*d+b*c)^3*ln(b*x^2+a)/b^6
```

Rubi [A]
time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{a^2(bc-ad)^3\log\left(a+bx^2\right)}{2b^6}+\frac{dx^6\left(a^2d^2-3abcd+3b^2c^2\right)}{6b^3}-\frac{ax^2(bc-ad)^3}{2b^5}+\frac{x^4(bc-ad)^3}{4b^4}+\frac{d^2x^8(3bc-ad)}{8b^2}+\frac{d^3x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] -1/2*(a*(b*c - a*d)^3*x^2)/b^5 + ((b*c - a*d)^3*x^4)/(4*b^4) + (d*(3*b^2*c^
2 - 3*a*b*c*d + a^2*d^2)*x^6)/(6*b^3) + (d^2*(3*b*c - a*d)*x^8)/(8*b^2) + (
d^3*x^10)/(10*b) + (a^2*(b*c - a*d)^3*Log[a + b*x^2])/(2*b^6)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(c+dx)^3}{a+bx}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-bc+ad)^3}{b^5} + \frac{(bc-ad)^3x}{b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^2}{b^3} + \frac{d^2(3bc-\cdots}{b^2}\right.\right.$$

$$= -\frac{a(bc-ad)^3x^2}{2b^5} + \frac{(bc-ad)^3x^4}{4b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^6}{6b^3} + \frac{d^2(3bc-ad)x^8}{8b^2} + \frac{d^3\cdots}{1\cdots}$$

Mathematica [A]
time = 0.06, size = 128, normalized size = 0.93

$$\frac{60ab(-bc+ad)^3x^2 + 30b^2(bc-ad)^3x^4 + 20b^3d(3b^2c^2-3abcd+a^2d^2)\,x^6 + 15b^4d^2(3bc-ad)x^8 + 12b^5d^3x^{10} + 60a^2(bc-ad)^3\log\left(a+bx^2\right)}{120b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (60*a*b*(-(b*c) + a*d)^3*x^2 + 30*b^2*(b*c - a*d)^3*x^4 + 20*b^3*d*(3*b^2*c
^2 - 3*a*b*c*d + a^2*d^2)*x^6 + 15*b^4*d^2*(3*b*c - a*d)*x^8 + 12*b^5*d^3*x
^10 + 60*a^2*(b*c - a*d)^3*Log[a + b*x^2])/(120*b^6)
```

Maple [A]
time = 0.09, size = 234, normalized size = 1.70

method	result
norman	$-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^4}{4b^4}+\frac{d^3x^{10}}{10b}+\frac{a\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^2}{2b^5}-\frac{d^2(ad-3bc)x^8}{8b^2}+\frac{d\left(a^2d^2-3abcd+3\cdots\right.}{6b^3}$
default	$\frac{\frac{1}{5}d^3x^{10}b^4-\frac{1}{4}a\,b^3d^3x^8+\frac{3}{4}b^4c\,d^2x^8+\frac{1}{3}a^2b^2d^3x^6-a\,b^3c\,d^2x^6+b^4c^2d\,x^6-\frac{1}{2}a^3b\,d^3x^4+\frac{3}{2}a^2b^2c\,d^2x^4-\frac{3}{2}a\,b^3c^2d\,x^4+\frac{1}{2}b^4c^3x^4+a^4d^3x^2-3a\cdots}{2b^5}$
risch	$\frac{d^3x^{10}}{10b}-\frac{a\,d^3x^8}{8b^2}+\frac{3c\,d^2x^8}{8b}+\frac{a^2d^3x^6}{6b^3}-\frac{ac\,d^2x^6}{2b^2}+\frac{c^2d\,x^6}{2b}-\frac{a^3d^3x^4}{4b^4}+\frac{3a^2c\,d^2x^4}{4b^3}-\frac{3a\,c^2d\,x^4}{4b^2}+\frac{c^3x^4}{4b}+\frac{a^4d^3x^2}{2b^5}-3\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^5*(1/5*d^3*x^10*b^4-1/4*a*b^3*d^3*x^8+3/4*b^4*c*d^2*x^8+1/3*a^2*b^2*d
^3*x^6-a*b^3*c*d^2*x^6+b^4*c^2*d*x^6-1/2*a^3*b*d^3*x^4+3/2*a^2*b^2*c*d^2*x^
4-3/2*a*b^3*c^2*d*x^4+1/2*b^4*c^3*x^4+a^4*d^3*x^2-3*a^3*b*c*d^2*x^2+3*a^2*b
^2*c^2*d*x^2-a*b^3*c^3*x^2)-1/2*a^2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)/b^6*ln(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 219, normalized size = 1.59

$$\frac{12\,b^4d^3x^{10} + 15\,(3\,b^4cd^2 - ab^3d^3)x^8 + 20\,(3\,b^4c^2d - 3\,ab^3cd^2 + a^2b^2d^3)x^6 + 30\,(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3)x^4 - 60\,(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)x^2}{120\,b^5} + \frac{(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3)\log\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/120*(12*b^4*d^3*x^10 + 15*(3*b^4*c*d^2 - a*b^3*d^3)*x^8 + 20*(3*b^4*c^2*d
 - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^6 + 30*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b
^2*c*d^2 - a^3*b*d^3)*x^4 - 60*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
 - a^4*d^3)*x^2)/b^5 + 1/2*(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*log(b*x^2 + a)/b^6
```

Fricas [A]
time = 1.30, size = 220, normalized size = 1.59

$$\frac{12\,b^5d^3x^{10}+15\,(3\,b^5cd^2-ab^4d^3)x^8+20\,(3\,b^5c^2d-3\,ab^4cd^2+a^2b^3d^3)x^6+30\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)x^4-60\,(ab^4c^3-3\,a^2b^3c^2d+3\,a^3b^2cd^2-a^4bd^3)x^2+60\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\log\left(bx^2+a\right)}{120\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/120*(12*b^5*d^3*x^10 + 15*(3*b^5*c*d^2 - a*b^4*d^3)*x^8 + 20*(3*b^5*c^2*d
 - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x^6 + 30*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b
^3*c*d^2 - a^3*b^2*d^3)*x^4 - 60*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c
*d^2 - a^4*b*d^3)*x^2 + 60*(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*log(b*x^2 + a))/b^6
```

Sympy [A]
time = 0.35, size = 201, normalized size = 1.46

$$-\frac{a^2(ad-bc)^3\log\left(a+bx^2\right)}{2b^6}+x^8\left(-\frac{ad^3}{8b^2}+\frac{3cd^2}{8b}\right)+x^6\left(\frac{a^2d^3}{6b^3}-\frac{acd^2}{2b^2}+\frac{c^2d}{2b}\right)+x^4\left(-\frac{a^3d^3}{4b^4}+\frac{3a^2cd^2}{4b^3}-\frac{3ac^2d}{4b^2}+\frac{c^3}{4b}\right)+x^2\left(\frac{a^4d^3}{2b^5}-\frac{3a^3cd^2}{2b^4}+\frac{3a^2c^2d}{2b^3}-\frac{ac^3}{2b^2}\right)+\frac{d^3x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)**3*log(a + b*x**2)/(2*b**6) + x**8*(-a*d**3/(8*b**2) + 3*
c*d**2/(8*b)) + x**6*(a**2*d**3/(6*b**3) - a*c*d**2/(2*b**2) + c**2*d/(2*b)
) + x**4*(-a**3*d**3/(4*b**4) + 3*a**2*c*d**2/(4*b**3) - 3*a*c**2*d/(4*b**2
) + c**3/(4*b)) + x**2*(a**4*d**3/(2*b**5) - 3*a**3*c*d**2/(2*b**4) + 3*a**
2*c**2*d/(2*b**3) - a*c**3/(2*b**2)) + d**3*x**10/(10*b)
```

Giac [A]
time = 0.54, size = 238, normalized size = 1.72

$$\frac{12\,b^4d^3x^{10}+45\,b^4cd^2x^8-15\,ab^3d^3x^8+60\,b^4c^2dx^6-60\,ab^3cd^2x^6+20\,a^2b^2d^3x^6+30\,b^4c^3x^4-90\,ab^3c^2dx^4+90\,a^2b^2cd^2x^4-30\,a^3bd^3x^4-60\,ab^3c^3x^2+180\,a^2b^2c^2dx^2-180\,a^3bcd^2x^2+60\,a^4d^3x^2}{120\,b^5}+\frac{(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\log\left(|bx^2+a|\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/120*(12*b^4*d^3*x^10 + 45*b^4*c*d^2*x^8 - 15*a*b^3*d^3*x^8 + 60*b^4*c^2*d
*x^6 - 60*a*b^3*c*d^2*x^6 + 20*a^2*b^2*d^3*x^6 + 30*b^4*c^3*x^4 - 90*a*b^3*
c^2*d*x^4 + 90*a^2*b^2*c*d^2*x^4 - 30*a^3*b*d^3*x^4 - 60*a*b^3*c^3*x^2 + 18
0*a^2*b^2*c^2*d*x^2 - 180*a^3*b*c*d^2*x^2 + 60*a^4*d^3*x^2)/b^5 + 1/2*(a^2*
b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*log(abs(b*x^2 + a))/b^
6
```

Mupad [B]
time = 0.07, size = 236, normalized size = 1.71

$$x^4\left(\frac{c^3}{4\,b}-\frac{a\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{4\,b}\right)-x^8\left(\frac{a\,d^3}{8\,b^2}-\frac{3\,c\,d^2}{8\,b}\right)+x^6\left(\frac{c^2\,d}{2\,b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{6\,b}\right)-\frac{\ln\left(b\,x^2+a\right)\,\left(a^5\,d^3-3\,a^4\,b\,c\,d^2+3\,a^3\,b^2\,c^2\,d-a^2\,b^3\,c^3\right)}{2\,b^6}+\frac{d^3\,x^{10}}{10\,b}-\frac{a\,x^2\left(\frac{c^3}{b}-\frac{a\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^4*(c^3/(4*b) - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/(4*b
)) - x^8*((a*d^3)/(8*b^2) - (3*c*d^2)/(8*b)) + x^6*((c^2*d)/(2*b) + (a*((a*
d^3)/b^2 - (3*c*d^2)/b))/(6*b)) - (log(a + b*x^2)*(a^5*d^3 - a^2*b^3*c^3 +
3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))/(2*b^6) + (d^3*x^10)/(10*b) - (a*x^2*(c^3
/b - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/b))/(2*b)
```

3.219 $\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=140

$$-\frac{a(bc-ad)^3x}{b^5}+\frac{(bc-ad)^3x^3}{3b^4}+\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^5}{5b^3}+\frac{d^2(3bc-ad)x^7}{7b^2}+\frac{d^3x^9}{9b}+\frac{a^{3/2}(bc-ad)^3\tan^{-1}\left(\right.}{b^{11/2}}$$

```
[Out] -a*(-a*d+b*c)^3*x/b^5+1/3*(-a*d+b*c)^3*x^3/b^4+1/5*d*(a^2*d^2-3*a*b*c*d+3*b
^2*c^2)*x^5/b^3+1/7*d^2*(-a*d+3*b*c)*x^7/b^2+1/9*d^3*x^9/b+a^(3/2)*(-a*d+b*
c)^3*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {472, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^3}{b^{11/2}}+\frac{dx^5(a^2d^2-3abcd+3b^2c^2)}{5b^3}-\frac{ax(bc-ad)^3}{b^5}+\frac{x^3(bc-ad)^3}{3b^4}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^3*x)/b^5) + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*b^2*c^2 -
3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*x^7)/(7*b^2) + (d^3*
x^9)/(9*b) + (a^(3/2)*(b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx = \int \left(-\frac{a(bc-ad)^3}{b^5} + \frac{(bc-ad)^3x^2}{b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^4}{b^3} + \frac{d^2(3bc-ad)x^6}{b^2} + \right.$$

$$= -\frac{a(bc-ad)^3x}{b^5} + \frac{(bc-ad)^3x^3}{3b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^5}{5b^3} + \frac{d^2(3bc-ad)x^7}{7b^2} + \frac{d^3x}{9b}$$

$$= -\frac{a(bc-ad)^3x}{b^5} + \frac{(bc-ad)^3x^3}{3b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^5}{5b^3} + \frac{d^2(3bc-ad)x^7}{7b^2} + \frac{d^3x}{9b}$$

Mathematica [A]
time = 0.04, size = 140, normalized size = 1.00

$$\frac{a(-bc+ad)^3x}{b^5} + \frac{(bc-ad)^3x^3}{3b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^5}{5b^3} + \frac{d^2(3bc-ad)x^7}{7b^2} + \frac{d^3x^9}{9b} - \frac{a^{3/2}(-bc+ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (a*(-(b*c) + a*d)^3*x)/b^5 + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*b^2*c^2 -
3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*x^7)/(7*b^2) + (d^3*
x^9)/(9*b) - (a^(3/2)*(-(b*c) + a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2
)
```

Maple [A]
time = 0.11, size = 231, normalized size = 1.65

method	result
default	$\frac{\frac{1}{9}d^3x^9b^4-\frac{1}{7}a\,b^3d^3x^7+\frac{3}{7}b^4c\,d^2x^7+\frac{1}{5}a^2b^2d^3x^5-\frac{3}{5}a\,b^3c\,d^2x^5+\frac{3}{5}b^4c^2d\,x^5-\frac{1}{3}a^3b\,d^3x^3+a^2b^2c\,d^2x^3-a\,b^3c^2d\,x^3+\frac{1}{3}b^4c^3x^3+a^4d^3x-3a^3}{b^5}$
risch	$\frac{d^3x^9}{9b}-\frac{a\,d^3x^7}{7b^2}+\frac{3c\,d^2x^7}{7b}+\frac{a^2d^3x^5}{5b^3}-\frac{3ac\,d^2x^5}{5b^2}+\frac{3c^2d\,x^5}{5b}-\frac{a^3d^3x^3}{3b^4}+\frac{a^2c\,d^2x^3}{b^3}-\frac{a\,c^2d\,x^3}{b^2}+\frac{c^3x^3}{3b}+\frac{a^4d^3x}{b^5}-\frac{3a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/9*d^3*x^9*b^4-1/7*a*b^3*d^3*x^7+3/7*b^4*c*d^2*x^7+1/5*a^2*b^2*d^3*
x^5-3/5*a*b^3*c*d^2*x^5+3/5*b^4*c^2*d*x^5-1/3*a^3*b*d^3*x^3+a^2*b^2*c*d^2*x
^3-a*b^3*c^2*d*x^3+1/3*b^4*c^3*x^3+a^4*d^3*x-3*a^3*b*c*d^2*x+3*a^2*b^2*c^2*
d*x-a*b^3*c^3*x)-a^2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^5/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.53, size = 222, normalized size = 1.59

$$\frac{(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5}+\frac{35\,b^4d^3x^9+45\,(3\,b^4cd^2-ab^3d^3)x^7+63\,(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3)x^5+105\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)x^3-315\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)x}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^4*d^3*x^9 + 45*(3*b^4*c*d^2 - a*b^3*d^3)
*x^7 + 63*(3*b^4*c^2*d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 105*(b^4*c^3 -
3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^3 - 315*(a*b^3*c^3 - 3*a^2*b
^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5
```

Fricas [A]
time = 1.72, size = 468, normalized size = 3.34

$$\left[\frac{70\,b^4d^3x^9+90\,(3\,b^4cd^2-ab^3d^3)x^7+126\,(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3)x^5+210\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)x^3-315\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-630\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)x}{630\,b^5},\ \frac{35\,b^4d^3x^9+45\,(3\,b^4cd^2-ab^3d^3)x^7+63\,(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3)x^5+105\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)x^3+315\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-315\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)x}{315\,b^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*d^3*x^9 + 90*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 126*(3*b^4*c^2*
d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 210*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*x^3 - 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*
d^2 - a^4*d^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) -
 630*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5, 1/315*
(35*b^4*d^3*x^9 + 45*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 63*(3*b^4*c^2*d - 3*a*
b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 105*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d
^2 - a^3*b*d^3)*x^3 + 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^
4*d^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(128) = 256$.
time = 0.39, size = 343, normalized size = 2.45

$$x^7\left(-\frac{ad^3}{7b^2}+\frac{3cd^2}{7b}\right)+x^5\left(\frac{a^2d^3}{5b^3}-\frac{3acd^2}{5b^2}+\frac{3c^2d}{5b}\right)+x^3\left(-\frac{a^3d^3}{3b^4}+\frac{a^2cd^2}{b^3}-\frac{ac^2d}{b^2}+\frac{c^3}{3b}\right)+x\left(\frac{a^4d^3}{b^5}-\frac{3a^3cd^2}{b^4}+\frac{3a^2c^2d}{b^3}-\frac{ac^3}{b^2}\right)+\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2}+\frac{d^3x^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**7*(-a*d**3/(7*b**2) + 3*c*d**2/(7*b)) + x**5*(a**2*d**3/(5*b**3) - 3*a*c
*d**2/(5*b**2) + 3*c**2*d/(5*b)) + x**3*(-a**3*d**3/(3*b**4) + a**2*c*d**2/
```

```
b**3 - a*c**2*d/b**2 + c**3/(3*b)) + x*(a**4*d**3/b**5 - 3*a**3*c*d**2/b**4
 + 3*a**2*c**2*d/b**3 - a*c**3/b**2) + sqrt(-a**3/b**11)*(a*d - b*c)**3*log
(-b**5*sqrt(-a**3/b**11)*(a*d - b*c)**3/(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a*
*2*b**2*c**2*d - a*b**3*c**3) + x)/2 - sqrt(-a**3/b**11)*(a*d - b*c)**3*log
(b**5*sqrt(-a**3/b**11)*(a*d - b*c)**3/(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**
2*b**2*c**2*d - a*b**3*c**3) + x)/2 + d**3*x**9/(9*b)
```

Giac [A]
time = 0.62, size = 241, normalized size = 1.72

$$\frac{\left(a^{2} b^{3} c^{3} - 3 \, a^{3} b^{2} c^{2} d + 3 \, a^{4} b c d^{2} - a^{5} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{5}} + \frac{35 \, b^{8} d^{3} x^{9} + 135 \, b^{8} c d^{2} x^{7} - 45 \, a b^{7} d^{3} x^{7} + 189 \, b^{8} c^{2} d x^{5} - 189 \, a b^{7} c d^{2} x^{5} + 63 \, a^{2} b^{6} d^{3} x^{5} + 105 \, b^{8} c^{3} x^{3} - 315 \, a b^{7} c^{2} d x^{3} + 315 \, a^{2} b^{6} c d^{2} x^{3} - 105 \, a^{3} b^{5} d^{3} x^{3} - 315 \, a b^{7} c^{3} x + 945 \, a^{2} b^{6} c^{2} d x - 945 \, a^{3} b^{5} c d^{2} x + 315 \, a^{4} b^{4} d^{3} x}{315 \, b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*d^3*x^9 + 135*b^8*c*d^2*x^7 - 45*a*b^7
*d^3*x^7 + 189*b^8*c^2*d*x^5 - 189*a*b^7*c*d^2*x^5 + 63*a^2*b^6*d^3*x^5 + 1
05*b^8*c^3*x^3 - 315*a*b^7*c^2*d*x^3 + 315*a^2*b^6*c*d^2*x^3 - 105*a^3*b^5*
d^3*x^3 - 315*a*b^7*c^3*x + 945*a^2*b^6*c^2*d*x - 945*a^3*b^5*c*d^2*x + 315
*a^4*b^4*d^3*x)/b^9
```

Mupad [B]
time = 0.06, size = 260, normalized size = 1.86

$$x^3\,\left(\frac{c^3}{3\,b}-\frac{a\,\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{3\,b}\right)-x^7\,\left(\frac{a\,d^3}{7\,b^2}-\frac{3\,c\,d^2}{7\,b}\right)+x^5\,\left(\frac{3\,c^2\,d}{5\,b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{5\,b}\right)+\frac{d^3\,x^9}{9\,b}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3}{a^5\,d^3-3\,a^4\,b\,c\,d^2+3\,a^3\,b^2\,c^2\,d-a^2\,b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^3}{b^{11/2}}-\frac{a\,x\,\left(\frac{c^3}{b}-\frac{a\,\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^3*(c^3/(3*b) - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/(3*b
)) - x^7*((a*d^3)/(7*b^2) - (3*c*d^2)/(7*b)) + x^5*((3*c^2*d)/(5*b) + (a*((
a*d^3)/b^2 - (3*c*d^2)/b))/(5*b)) + (d^3*x^9)/(9*b) - (a^(3/2)*atan((a^(3/2
)*b^(1/2)*x*(a*d - b*c)^3)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4
*b*c*d^2))*(a*d - b*c)^3)/b^(11/2) - (a*x*(c^3/b - (a*((3*c^2*d)/b + (a*((a
*d^3)/b^2 - (3*c*d^2)/b))/b))/b))/b
```

3.220 $\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=115

$$\frac{(bc-ad)^3x^2}{2b^4}+\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^4}{4b^3}+\frac{d^2(3bc-ad)x^6}{6b^2}+\frac{d^3x^8}{8b}-\frac{a(bc-ad)^3\log\left(a+bx^2\right)}{2b^5}$$

```
[Out] 1/2*(-a*d+b*c)^3*x^2/b^4+1/4*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^4/b^3+1/6*d^
2*(-a*d+3*b*c)*x^6/b^2+1/8*d^3*x^8/b-1/2*a*(-a*d+b*c)^3*ln(b*x^2+a)/b^5
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\frac{dx^4(a^2d^2-3abcd+3b^2c^2)}{4b^3}-\frac{a(bc-ad)^3\log\left(a+bx^2\right)}{2b^5}+\frac{x^2(bc-ad)^3}{2b^4}+\frac{d^2x^6(3bc-ad)}{6b^2}+\frac{d^3x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x^2)/(2*b^4) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^4)/(4*
b^3) + (d^2*(3*b*c - a*d)*x^6)/(6*b^2) + (d^3*x^8)/(8*b) - (a*(b*c - a*d)^3
*Log[a + b*x^2])/(2*b^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^3}{a+bx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(bc-ad)^3}{b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^2}{b^2} + \frac{d^3x^3}{b} + \frac{a(\cdot}{b}\right.\right.$$
$$= \frac{(bc-ad)^3x^2}{2b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^4}{4b^3} + \frac{d^2(3bc-ad)x^6}{6b^2} + \frac{d^3x^8}{8b} - \frac{a(bc-ad)^3\,\mathrm{lo}}{2b}$$

Mathematica [A]
time = 0.04, size = 125, normalized size = 1.09

$$\frac{bx^2(-12a^3d^3+6a^2bd^2(6c+dx^2)-2ab^2d(18c^2+9cdx^2+2d^2x^4)+3b^3(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6))+12a(-bc+ad)^3\log(a+bx^2)}{24b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (b*x^2*(-12*a^3*d^3 + 6*a^2*b*d^2*(6*c + d*x^2) - 2*a*b^2*d*(18*c^2 + 9*c*d
*x^2 + 2*d^2*x^4) + 3*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c*d^2*x^4 + d^3*x^6)) +
12*a*(-(b*c) + a*d)^3*Log[a + b*x^2])/(24*b^5)
```

Maple [A]
time = 0.09, size = 177, normalized size = 1.54

method	result
norman	$-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x^2}{2b^4}+\frac{d^3x^8}{8b}-\frac{d^2(ad-3bc)x^6}{6b^2}+\frac{d(a^2d^2-3abcd+3b^2c^2)x^4}{4b^3}+\frac{a(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b}{2b^5}$
default	$-\frac{-\frac{1}{4}d^3x^8b^3+\frac{1}{3}a\,b^2d^3x^6-b^3c\,d^2x^6-\frac{1}{2}a^2b\,d^3x^4+\frac{3}{2}a\,b^2c\,d^2x^4-\frac{3}{2}b^3c^2d\,x^4+a^3d^3x^2-3a^2bc\,d^2x^2+3a\,b^2c^2d\,x^2-b^3c^3x^2}{2b^4}+\frac{a(a^3d^3-3}{}$
risch	$\frac{d^3x^8}{8b}-\frac{a\,d^3x^6}{6b^2}+\frac{c\,d^2x^6}{2b}+\frac{a^2d^3x^4}{4b^3}-\frac{3ac\,d^2x^4}{4b^2}+\frac{3c^2d\,x^4}{4b}-\frac{a^3d^3x^2}{2b^4}+\frac{3a^2c\,d^2x^2}{2b^3}-\frac{3a\,c^2d\,x^2}{2b^2}+\frac{c^3x^2}{2b}+\frac{a^4\ln(b\,x^2+a)}{2b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^4*(-1/4*d^3*x^8*b^3+1/3*a*b^2*d^3*x^6-b^3*c*d^2*x^6-1/2*a^2*b*d^3*x^
4+3/2*a*b^2*c*d^2*x^4-3/2*b^3*c^2*d*x^4+a^3*d^3*x^2-3*a^2*b*c*d^2*x^2+3*a*b
^2*c^2*d*x^2-b^3*c^3*x^2)+1/2*a*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)/b^5*ln(b*x^2+a)
```

Maxima [A]
time = 0.30, size = 168, normalized size = 1.46

$$\frac{3\,b^3d^3x^8+4\,(3\,b^3cd^2-ab^2d^3)x^6+6\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x^4+12\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^2}{24\,b^4}-\frac{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\log(bx^2+a)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^3*d^3*x^8 + 4*(3*b^3*c*d^2 - a*b^2*d^3)*x^6 + 6*(3*b^3*c^2*d - 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^4 + 12*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*x^2)/b^4 - 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^
4*d^3)*log(b*x^2 + a)/b^5
```

Fricas [A]
time = 1.38, size = 169, normalized size = 1.47

$$\frac{3\,b^4d^3x^8+4\,(3\,b^4cd^2-ab^3d^3)x^6+6\,(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3)x^4+12\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)x^2-12\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\log\left(bx^2+a\right)}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*d^3*x^8 + 4*(3*b^4*c*d^2 - a*b^3*d^3)*x^6 + 6*(3*b^4*c^2*d - 3*
a*b^3*c*d^2 + a^2*b^2*d^3)*x^4 + 12*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3)*x^2 - 12*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^
4*d^3)*log(b*x^2 + a))/b^5
```

Sympy [A]
time = 0.31, size = 144, normalized size = 1.25

$$\frac{a(ad-bc)^3\log\left(a+bx^2\right)}{2b^5}+x^6\left(-\frac{ad^3}{6b^2}+\frac{cd^2}{2b}\right)+x^4\left(\frac{a^2d^3}{4b^3}-\frac{3acd^2}{4b^2}+\frac{3c^2d}{4b}\right)+x^2\left(-\frac{a^3d^3}{2b^4}+\frac{3a^2cd^2}{2b^3}-\frac{3ac^2d}{2b^2}+\frac{c^3}{2b}\right)+\frac{d^3x^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)**3*log(a + b*x**2)/(2*b**5) + x**6*(-a*d**3/(6*b**2) + c*d**2
/(2*b)) + x**4*(a**2*d**3/(4*b**3) - 3*a*c*d**2/(4*b**2) + 3*c**2*d/(4*b))
+ x**2*(-a**3*d**3/(2*b**4) + 3*a**2*c*d**2/(2*b**3) - 3*a*c**2*d/(2*b**2)
+ c**3/(2*b)) + d**3*x**8/(8*b)
```

Giac [A]
time = 0.58, size = 180, normalized size = 1.57

$$\frac{3\,b^3d^3x^8+12\,b^3cd^2x^6-4\,ab^2d^3x^6+18\,b^3c^2dx^4-18\,ab^2cd^2x^4+6\,a^2bd^3x^4+12\,b^3c^3x^2-36\,ab^2c^2dx^2+36\,a^2bcd^2x^2-12\,a^3d^3x^2}{24\,b^4}-\frac{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\log\left(|bx^2+a|\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*d^3*x^8 + 12*b^3*c*d^2*x^6 - 4*a*b^2*d^3*x^6 + 18*b^3*c^2*d*x^4
 - 18*a*b^2*c*d^2*x^4 + 6*a^2*b*d^3*x^4 + 12*b^3*c^3*x^2 - 36*a*b^2*c^2*d*x
^2 + 36*a^2*b*c*d^2*x^2 - 12*a^3*d^3*x^2)/b^4 - 1/2*(a*b^3*c^3 - 3*a^2*b^2*
c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*log(abs(b*x^2 + a))/b^5
```

Mupad [B]

time = 0.03, size = 178, normalized size = 1.55

$$x^2\left(\frac{c^3}{2\,b}-\frac{a\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{2\,b}\right)-x^6\left(\frac{a\,d^3}{6\,b^2}-\frac{c\,d^2}{2\,b}\right)+x^4\left(\frac{3\,c^2\,d}{4\,b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{4\,b}\right)+\frac{\ln\left(b\,x^2+a\right)\,\left(a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3\right)}{2\,b^5}+\frac{d^3\,x^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^2*(c^3/(2*b) - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/(2*b
)) - x^6*((a*d^3)/(6*b^2) - (c*d^2)/(2*b)) + x^4*((3*c^2*d)/(4*b) + (a*((a*
d^3)/b^2 - (3*c*d^2)/b))/(4*b)) + (log(a + b*x^2)*(a^4*d^3 - a*b^3*c^3 + 3*
a^2*b^2*c^2*d - 3*a^3*b*c*d^2))/(2*b^5) + (d^3*x^8)/(8*b)
```

3.221 $\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=119

$$\frac{(bc-ad)^3x}{b^4}+\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^3}{3b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^7}{7b}-\frac{\sqrt{a}\,(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

```
[Out] (-a*d+b*c)^3*x/b^4+1/3*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^3/b^3+1/5*d^2*(-a*
d+3*b*c)*x^5/b^2+1/7*d^3*x^7/b-(-a*d+b*c)^3*arctan(x*b^(1/2)/a^(1/2))*a^(1/
2)/b^(9/2)
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {472, 211}

$$\frac{dx^3(a^2d^2-3abcd+3b^2c^2)}{3b^3}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^3}{b^{9/2}}+\frac{x(bc-ad)^3}{b^4}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) +
 (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) - (Sqrt[a]*(b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx = \int \left(\frac{(bc-ad)^3}{b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^2}{b^3} + \frac{d^2(3bc-ad)x^4}{b^2} + \frac{d^3x^6}{b} + \frac{-ab^3c^3+3\right.$$
$$= \frac{(bc-ad)^3x}{b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^3}{3b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^7}{7b} - \frac{(a(bc-ad)^3)}{b^4}$$
$$= \frac{(bc-ad)^3x}{b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^3}{3b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^7}{7b} - \frac{\sqrt{a}\,(bc-ad)^3}{b}$$

Mathematica [A]
time = 0.03, size = 118, normalized size = 0.99

$$\frac{(bc-ad)^3x}{b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^3}{3b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^7}{7b} + \frac{\sqrt{a}\,(-bc+ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) +
 (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) + (Sqrt[a]*(-(b*c) + a*d
)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A]
time = 0.10, size = 173, normalized size = 1.45

method	result
default	$-\frac{-\frac{1}{7}d^3x^7b^3+\frac{1}{5}ab^2d^3x^5-\frac{3}{5}b^3cd^2x^5-\frac{1}{3}a^2bd^3x^3+ab^2cd^2x^3-b^3c^2dx^3+a^3d^3x-3a^2bcd^2x+3ab^2c^2dx-b^3c^3x}{b^4} + \frac{a(a^3d^3-3a^2bcd^2}{}$
risch	$\frac{d^3x^7}{7b} - \frac{a\,d^3x^5}{5b^2} + \frac{3c\,d^2x^5}{5b} + \frac{a^2d^3x^3}{3b^3} - \frac{ac\,d^2x^3}{b^2} + \frac{c^2d\,x^3}{b} - \frac{a^3d^3x}{b^4} + \frac{3a^2c\,d^2x}{b^3} - \frac{3ac^2dx}{b^2} + \frac{c^3x}{b} + \frac{\sqrt{-ab}\,\ln\left(-\sqrt{\right.}}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/7*d^3*x^7*b^3+1/5*a*b^2*d^3*x^5-3/5*b^3*c*d^2*x^5-1/3*a^2*b*d^3*
x^3+a*b^2*c*d^2*x^3-b^3*c^2*d*x^3+a^3*d^3*x-3*a^2*b*c*d^2*x+3*a*b^2*c^2*d*x
-b^3*c^3*x)+a*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^4/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.52, size = 172, normalized size = 1.45

$$-\frac{(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,b^3d^3x^7+21\,(3\,b^3cd^2-ab^2d^3)x^5+35\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x^3+105\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*d^3*x^7 + 21*(3*b^3*c*d^2 - a*b^2*d^3)*
x^5 + 35*(3*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 + 105*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4
```

Fricas [A]
time = 1.00, size = 364, normalized size = 3.06

$$\left[\frac{30\,b^3d^3x^7+42\,(3\,b^3cd^2-ab^2d^3)x^5+70\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x^3-105\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+210\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{210\,b^4},\ \frac{15\,b^3d^3x^7+21\,(3\,b^3cd^2-ab^2d^3)x^5+35\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x^3-105\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{105\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*d^3*x^7 + 42*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 70*(3*b^3*c^2*d
 - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 - 105*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a))
 + 210*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4, 1/105*(1
5*b^3*d^3*x^7 + 21*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 35*(3*b^3*c^2*d - 3*a*b^
2*c*d^2 + a^2*b*d^3)*x^3 - 105*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(109) = 218$.
time = 0.36, size = 274, normalized size = 2.30

$$x^5\left(-\frac{ad^3}{5b^2}+\frac{3cd^2}{5b}\right)+x^3\left(\frac{a^2d^3}{3b^3}-\frac{acd^2}{b^2}+\frac{c^2d}{b}\right)+x\left(-\frac{a^3d^3}{b^4}+\frac{3a^2cd^2}{b^3}-\frac{3ac^2d}{b^2}+\frac{c^3}{b}\right)-\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3\log\left(-\frac{b^4\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3\log\left(\frac{b^4\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x**3*(a**2*d**3/(3*b**3) - a*c*d
**2/b**2 + c**2*d/b) + x*(-a**3*d**3/b**4 + 3*a**2*c*d**2/b**3 - 3*a*c**2*d
/b**2 + c**3/b) - sqrt(-a/b**9)*(a*d - b*c)**3*log(-b**4*sqrt(-a/b**9)*(a*d
 - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)
/2 + sqrt(-a/b**9)*(a*d - b*c)**3*log(b**4*sqrt(-a/b**9)*(a*d - b*c)**3/(a*
*3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x**7
/(7*b)
```

Giac [A]
time = 0.57, size = 184, normalized size = 1.55

$$-\frac{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^6d^3x^7+63\,b^6cd^2x^5-21\,ab^5d^3x^5+105\,b^6c^2dx^3-105\,ab^5cd^2x^3+35\,a^2b^4d^3x^3+105\,b^6c^3x-315\,ab^5c^2dx+315\,a^2b^4cd^2x-105\,a^3b^3d^3x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^3*x^7 + 63*b^6*c*d^2*x^5 - 21*a*b^5*d
^3*x^5 + 105*b^6*c^2*d*x^3 - 105*a*b^5*c*d^2*x^3 + 35*a^2*b^4*d^3*x^3 + 105
*b^6*c^3*x - 315*a*b^5*c^2*d*x + 315*a^2*b^4*c*d^2*x - 105*a^3*b^3*d^3*x)/b
^7
```

Mupad [B]
time = 0.06, size = 199, normalized size = 1.67

$$x^3\left(\frac{c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{3\,b}\right)-x^5\,\left(\frac{a\,d^3}{5\,b^2}-\frac{3\,c\,d^2}{5\,b}\right)+x\,\left(\frac{c^3}{b}-\frac{a\,\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{b}\right)+\frac{d^3\,x^7}{7\,b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3}{a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^3}{b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^3*((c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/(3*b)) - x^5*((a*d^3)/(5*b
^2) - (3*c*d^2)/(5*b)) + x*(c^3/b - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*
c*d^2)/b))/b))/b) + (d^3*x^7)/(7*b) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d
 - b*c)^3)/(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))*(a*d -
b*c)^3)/b^(9/2)
```

3.222 $\int \frac{x(c+dx^2)^3}{a+bx^2} dx$

Optimal. Leaf size=87

$$\frac{d(bc-ad)^2x^2}{2b^3}+\frac{(bc-ad)\left(c+dx^2\right)^2}{4b^2}+\frac{\left(c+dx^2\right)^3}{6b}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}$$

```
[Out] 1/2*d*(-a*d+b*c)^2*x^2/b^3+1/4*(-a*d+b*c)*(d*x^2+c)^2/b^2+1/6*(d*x^2+c)^3/b
+1/2*(-a*d+b*c)^3*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}+\frac{dx^2(bc-ad)^2}{2b^3}+\frac{\left(c+dx^2\right)^2(bc-ad)}{4b^2}+\frac{\left(c+dx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (d*(b*c - a*d)^2*x^2)/(2*b^3) + ((b*c - a*d)*(c + d*x^2)^2)/(4*b^2) + (c +
d*x^2)^3/(6*b) + ((b*c - a*d)^3*Log[a + b*x^2])/(2*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^3}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx,x,x^2\right)\\
&= \frac{d(bc-ad)^2x^2}{2b^3}+\frac{(bc-ad)\left(c+dx^2\right)^2}{4b^2}+\frac{\left(c+dx^2\right)^3}{6b}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 82, normalized size = 0.94

$$\frac{bdx^2(6a^2d^2 - 3abd(6c + dx^2) + b^2(18c^2 + 9cdx^2 + 2d^2x^4)) + 6(bc - ad)^3 \log (a + bx^2)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (b*d*x^2*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x^2) + b^2*(18*c^2 + 9*c*d*x^2 + 2*d
^2*x^4)) + 6*(b*c - a*d)^3*Log[a + b*x^2])/(12*b^4)
```

Maple [A]
time = 0.09, size = 119, normalized size = 1.37

method	result
norman	$\frac{d^3x^6}{6b} + \frac{d(a^2d^2-3abcd+3b^2c^2)x^2}{2b^3} - \frac{d^2(ad-3bc)x^4}{4b^2} - \frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\ln(b\,x^2+a)}{2b^4}$
default	$\frac{d(\frac{1}{3}b^2d^2x^6-\frac{1}{2}ab\,d^2x^4+\frac{3}{2}b^2cd\,x^4+a^2d^2x^2-3abcd\,x^2+3b^2c^2x^2)}{2b^3} + \frac{(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3)\ln(b\,x^2+a)}{2b^4}$
risch	$\frac{d^3x^6}{6b} - \frac{d^3a\,x^4}{4b^2} + \frac{3d^2c\,x^4}{4b} + \frac{d^3a^2x^2}{2b^3} - \frac{3d^2ac\,x^2}{2b^2} + \frac{3d\,c^2x^2}{2b} - \frac{\ln(b\,x^2+a)a^3d^3}{2b^4} + \frac{3\ln(b\,x^2+a)a^2c\,d^2}{2b^3} - \frac{3\ln(b\,x^2+a)a\,c^2}{2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d/b^3*(1/3*b^2*d^2*x^6-1/2*a*b*d^2*x^4+3/2*b^2*c*d*x^4+a^2*d^2*x^2-3*a*
b*c*d*x^2+3*b^2*c^2*x^2)+1/2*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)
/b^4*ln(b*x^2+a)
```

Maxima [A]
time = 0.32, size = 119, normalized size = 1.37

$$\frac{2\,b^2d^3x^6 + 3\,(3\,b^2cd^2 - abd^3)x^4 + 6\,(3\,b^2c^2d - 3\,abcd^2 + a^2d^3)x^2}{12\,b^3} + \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log(bx^2 + a)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 6*(3*b^2*c^2*d - 3*a*
b*c*d^2 + a^2*d^3)*x^2)/b^3 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 0.91, size = 120, normalized size = 1.38

$$\frac{2\,b^3d^3x^6 + 3\,(3\,b^3cd^2 - ab^2d^3)x^4 + 6\,(3\,b^3c^2d - 3\,ab^2cd^2 + a^2bd^3)x^2 + 6\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log(bx^2 + a)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^3*x^6 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^4 + 6*(3*b^3*c^2*d - 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(b*x^2 + a))/b^4
```

Sympy [A]
time = 0.27, size = 94, normalized size = 1.08

$$x^4\left(-\frac{ad^3}{4b^2}+\frac{3cd^2}{4b}\right)+x^2\left(\frac{a^2d^3}{2b^3}-\frac{3acd^2}{2b^2}+\frac{3c^2d}{2b}\right)+\frac{d^3x^6}{6b}-\frac{(ad-bc)^3\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**4*(-a*d**3/(4*b**2) + 3*c*d**2/(4*b)) + x**2*(a**2*d**3/(2*b**3) - 3*a*c
*d**2/(2*b**2) + 3*c**2*d/(2*b)) + d**3*x**6/(6*b) - (a*d - b*c)**3*log(a +
 b*x**2)/(2*b**4)
```

Giac [A]
time = 0.53, size = 124, normalized size = 1.43

$$\frac{2\,b^2d^3x^6+9\,b^2cd^2x^4-3\,abd^3x^4+18\,b^2c^2dx^2-18\,abcd^2x^2+6\,a^2d^3x^2}{12\,b^3}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(|bx^2+a|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 + 9*b^2*c*d^2*x^4 - 3*a*b*d^3*x^4 + 18*b^2*c^2*d*x^2 -
18*a*b*c*d^2*x^2 + 6*a^2*d^3*x^2)/b^3 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/b^4
```

Mupad [B]
time = 0.03, size = 123, normalized size = 1.41

$$x^2\left(\frac{3\,c^2\,d}{2\,b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{2\,b}\right)-x^4\,\left(\frac{a\,d^3}{4\,b^2}-\frac{3\,c\,d^2}{4\,b}\right)-\frac{\ln\left(b\,x^2+a\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,b^4}+\frac{d^3\,x^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^2*((3*c^2*d)/(2*b) + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/(2*b)) - x^4*((a*d^3
)/(4*b^2) - (3*c*d^2)/(4*b)) - (log(a + b*x^2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2))/(2*b^4) + (d^3*x^6)/(6*b)
```

3.223 $\int \frac{(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=98

$$\frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/3*d^2*(-a*d+3*b*c)*x^3/b^2+1/5*d^3*
x^5/b+(-a*d+b*c)^3*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {398, 211}

$$\frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^3}{\sqrt{a}\,b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b^
2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)}{b^3} + \frac{d^2(3bc-ad)x^2}{b^2} + \frac{d^3x^4}{b} + \frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d}{b^3\,(a+bx^2)}\right.$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^3}{3b^2} + \frac{d^3x^5}{5b} + \frac{(bc-ad)^3\int \frac{1}{a+bx^2}\,dx}{b^3}$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^3}{3b^2} + \frac{d^3x^5}{5b} + \frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

Mathematica [A]
time = 0.05, size = 92, normalized size = 0.94

$$\frac{dx(15a^2d^2-5abd(9c+dx^2)+3b^2(15c^2+5cdx^2+d^2x^4))}{15b^3} + \frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*x^2 + d^2*
x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7
/2))
```

Maple [A]
time = 0.06, size = 116, normalized size = 1.18

method	result
default	$\frac{d\left(\frac{1}{5}b^2x^5d^2-\frac{1}{3}ab\,d^2x^3+b^2cd\,x^3+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{d^3x^5}{5b}-\frac{d^3a\,x^3}{3b^2}+\frac{d^2c\,x^3}{b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)a^3d^3}{2b^3\sqrt{-ab}}+\frac{3\ln\left(bx-\sqrt{-ab}\right)a^2c\,d^2}{2b^2\sqrt{-ab}}-\frac{3\ln\left(bx-\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b^3*(1/5*b^2*x^5*d^2-1/3*a*b*d^2*x^3+b^2*c*d*x^3+a^2*d^2*x-3*a*b*c*d*x+3*
b^2*c^2*x)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.55, size = 122, normalized size = 1.24

$$\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{3\,b^{2}d^{3}x^{5}+5\,\left(3\,b^{2}cd^{2}-abd^{3}\right)x^{3}+15\,\left(3\,b^{2}c^{2}d-3\,abcd^{2}+a^{2}d^{3}\right)x}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15*(
3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

Fricas [A]
time = 1.05, size = 292, normalized size = 2.98

$$\left[\frac{6\,ab^{3}d^{3}x^{5}+10\,\left(3\,ab^{3}cd^{2}-a^{2}b^{2}d^{3}\right)x^{3}+15\,\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+30\,\left(3\,ab^{3}c^{2}d-3\,a^{2}b^{2}cd^{2}+a^{3}bd^{3}\right)x}{30\,ab^{4}},\ \frac{3\,ab^{3}d^{3}x^{5}+5\,\left(3\,ab^{3}cd^{2}-a^{2}b^{2}d^{3}\right)x^{3}+15\,\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+15\,\left(3\,ab^{3}c^{2}d-3\,a^{2}b^{2}cd^{2}+a^{3}bd^{3}\right)x}{15\,ab^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*d^3*x^5 + 10*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 30*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3
)*x)/(a*b^4), 1/15*(3*a*b^3*d^3*x^5 + 5*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 +
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) + 15*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x)/(a*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(92) = 184$.
time = 0.37, size = 238, normalized size = 2.43

$$x^{3}\left(-\frac{ad^{3}}{3b^{2}}+\frac{cd^{2}}{b}\right)+x\left(\frac{a^{2}d^{3}}{b^{3}}-\frac{3acd^{2}}{b^{2}}+\frac{3c^{2}d}{b}\right)+\frac{\sqrt{-\frac{1}{ab^{7}}}\,(ad-bc)^{3}\log\left(-\frac{ab^{3}\sqrt{-\frac{1}{ab^{7}}}\,(ad-bc)^{3}}{a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^{7}}}\,(ad-bc)^{3}\log\left(\frac{ab^{3}\sqrt{-\frac{1}{ab^{7}}}\,(ad-bc)^{3}}{a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}}+x\right)}{2}+\frac{d^{3}x^{5}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**3*(-a*d**3/(3*b**2) + c*d**2/b) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b**2 +
3*c**2*d/b) + sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**7)
)*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3
) + x)/2 - sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt(-1/(a*b**7))*(a
*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) +
x)/2 + d**3*x**5/(5*b)
```

Giac [A]
time = 0.55, size = 129, normalized size = 1.32

$$\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^4d^3x^5 + 15\,b^4cd^2x^3 - 5\,ab^3d^3x^3 + 45\,b^4c^2dx - 45\,ab^3cd^2x + 15\,a^2b^2d^3x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3 - 5*a*b^3*d^3*x^3 +
 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b^2*d^3*x)/b^5
```

Mupad [B]
time = 0.07, size = 146, normalized size = 1.49

$$x\left(\frac{3\,c^2\,d}{b} + \frac{a\left(\frac{a\,d^3}{b^2} - \frac{3\,c\,d^2}{b}\right)}{b}\right) - x^3\left(\frac{a\,d^3}{3\,b^2} - \frac{c\,d^2}{b}\right) + \frac{d^3\,x^5}{5\,b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3}{\sqrt{a}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^3}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^3*((a*d^3)/(3*b^2)
- (c*d^2)/b) + (d^3*x^5)/(5*b) - (atan((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(1/2
)*b^(7/2))
```

3.224 $\int \frac{(c+dx^2)^3}{x(a+bx^2)}\,dx$

Optimal. Leaf size=73

$$\frac{d^2(3bc-ad)x^2}{2b^2}+\frac{d^3x^4}{4b}+\frac{c^3\log(x)}{a}-\frac{(bc-ad)^3\log(a+bx^2)}{2ab^3}$$

```
[Out] 1/2*d^2*(-a*d+3*b*c)*x^2/b^2+1/4*d^3*x^4/b+c^3*ln(x)/a-1/2*(-a*d+b*c)^3*ln(
b*x^2+a)/a/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$-\frac{(bc-ad)^3\log(a+bx^2)}{2ab^3}+\frac{d^2x^2(3bc-ad)}{2b^2}+\frac{c^3\log(x)}{a}+\frac{d^3x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

```
[Out] (d^2*(3*b*c - a*d)*x^2)/(2*b^2) + (d^3*x^4)/(4*b) + (c^3*Log[x])/a - ((b*c
- a*d)^3*Log[a + b*x^2])/(2*a*b^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x(a+bx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^3}{x(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2(3bc-ad)}{b^2}+\frac{c^3}{ax}+\frac{d^3x}{b}+\frac{(-bc+ad)^3}{ab^2(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{d^2(3bc-ad)x^2}{2b^2}+\frac{d^3x^4}{4b}+\frac{c^3\log(x)}{a}-\frac{(bc-ad)^3\log(a+bx^2)}{2ab^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.89

$$\frac{abd^2x^2(6bc-2ad+bdx^2)+4b^3c^3\log(x)-2(bc-ad)^3\log\left(a+bx^2\right)}{4ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

```
[Out] (a*b*d^2*x^2*(6*b*c - 2*a*d + b*d*x^2) + 4*b^3*c^3*Log[x] - 2*(b*c - a*d)^3
*Log[a + b*x^2])/(4*a*b^3)
```

Maple [A]
time = 0.10, size = 86, normalized size = 1.18

method	result
default	$\frac{d\left(-b d\,x^{2}+a d-3 b c\right)^{2}}{4 b^{3}}+\frac{\left(a^{3} d^{3}-3 a^{2} b c\,d^{2}+3 a\,b^{2} c^{2} d-b^{3} c^{3}\right) \ln\left(b\,x^{2}+a\right)}{2 a\,b^{3}}+\frac{c^{3} \ln\left(x\right)}{a}$
norman	$\frac{d^{3} x^{4}}{4 b}-\frac{d^{2}\left(a d-3 b c\right) x^{2}}{2 b^{2}}+\frac{c^{3} \ln\left(x\right)}{a}+\frac{\left(a^{3} d^{3}-3 a^{2} b c\,d^{2}+3 a\,b^{2} c^{2} d-b^{3} c^{3}\right) \ln\left(b\,x^{2}+a\right)}{2 a\,b^{3}}$
risch	$\frac{d^{3} x^{4}}{4 b}-\frac{d^{3} a\,x^{2}}{2 b^{2}}+\frac{3 d^{2} c\,x^{2}}{2 b}+\frac{d^{3} a^{2}}{4 b^{3}}-\frac{3 d^{2} a c}{2 b^{2}}+\frac{9 d\,c^{2}}{4 b}+\frac{c^{3} \ln\left(x\right)}{a}+\frac{a^{2} \ln\left(-b\,x^{2}-a\right) d^{3}}{2 b^{3}}-\frac{3 a \ln\left(-b\,x^{2}-a\right) c\,d^{2}}{2 b^{2}}+\frac{3 \ln\left(-b\,x^{2}-\right.}{2 b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*(-b*d*x^2+a*d-3*b*c)^2/b^3+1/2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b
^3*c^3)/a/b^3*ln(b*x^2+a)+c^3*ln(x)/a
```

Maxima [A]
time = 0.33, size = 98, normalized size = 1.34

$$\frac{c^3\log\left(x^2\right)}{2\,a}+\frac{bd^3x^4+2\,(3\,bcd^2-ad^3)x^2}{4\,b^2}-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(bx^2+a\right)}{2\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(x^2)/a + 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - a*d^3)*x^2)/b^2 - 1/2*
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a)/(a*b^3)
```

Fricas [A]
time = 1.16, size = 101, normalized size = 1.38

$$\frac{ab^2d^3x^4+4\,b^3c^3\log\left(x\right)+2\,(3\,ab^2cd^2-a^2bd^3)x^2-2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(bx^2+a\right)}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(a*b^2*d^3*x^4 + 4*b^3*c^3*log(x) + 2*(3*a*b^2*c*d^2 - a^2*b*d^3)*x^2 -
 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a))/(a*b
^3)
```

Sympy [A]
time = 1.10, size = 65, normalized size = 0.89

$$x^2\left(-\frac{ad^3}{2b^2}+\frac{3cd^2}{2b}\right)+\frac{d^3x^4}{4b}+\frac{c^3\log{(x)}}{a}+\frac{(ad-bc)^3\log{\left(\frac{a}{b}+x^2\right)}}{2ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x/(b*x**2+a),x)
```

```
[Out] x**2*(-a*d**3/(2*b**2) + 3*c*d**2/(2*b)) + d**3*x**4/(4*b) + c**3*log(x)/a
+ (a*d - b*c)**3*log(a/b + x**2)/(2*a*b**3)
```

Giac [A]
time = 0.53, size = 99, normalized size = 1.36

$$\frac{c^3\log{(x^2)}}{2\,a}+\frac{bd^3x^4+6\,bcd^2x^2-2\,ad^3x^2}{4\,b^2}-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log{(|bx^2+a|)}}{2\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*c^3*log(x^2)/a + 1/4*(b*d^3*x^4 + 6*b*c*d^2*x^2 - 2*a*d^3*x^2)/b^2 - 1/
2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/(
a*b^3)
```

Mupad [B]
time = 0.09, size = 97, normalized size = 1.33

$$\frac{d^3\,x^4}{4\,b}-x^2\left(\frac{a\,d^3}{2\,b^2}-\frac{3\,c\,d^2}{2\,b}\right)+\frac{c^3\,\ln{(x)}}{a}+\frac{\ln{(b\,x^2+a)}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x*(a + b*x^2)),x)
```

```
[Out] (d^3*x^4)/(4*b) - x^2*((a*d^3)/(2*b^2) - (3*c*d^2)/(2*b)) + (c^3*log(x))/a
+ (log(a + b*x^2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a
*b^3)
```

3.225 $\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=77

$$-\frac{c^3}{ax}+\frac{d^2(3bc-ad)x}{b^2}+\frac{d^3x^3}{3b}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

```
[Out] -c^3/a/x+d^2*(-a*d+3*b*c)*x/b^2+1/3*d^3*x^3/b-(-a*d+b*c)^3*arctan(x*b^(1/2)
/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {472, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^3}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) - ((b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx = \int \left(\frac{d^2(3bc-ad)}{b^2} + \frac{c^3}{ax^2} + \frac{d^3x^2}{b} + \frac{(-bc+ad)^3}{ab^2(a+bx^2)}\right) dx$$
$$= -\frac{c^3}{ax} + \frac{d^2(3bc-ad)x}{b^2} + \frac{d^3x^3}{3b} - \frac{(bc-ad)^3 \int \frac{1}{a+bx^2}\,dx}{ab^2}$$
$$= -\frac{c^3}{ax} + \frac{d^2(3bc-ad)x}{b^2} + \frac{d^3x^3}{3b} - \frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

Mathematica [A]
time = 0.03, size = 76, normalized size = 0.99

$$-\frac{c^3}{ax} + \frac{d^2(3bc-ad)x}{b^2} + \frac{d^3x^3}{3b} + \frac{(-bc+ad)^3 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) + ((-(b*c) + a*d
)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Maple [A]
time = 0.09, size = 95, normalized size = 1.23

method	result
default	$-\frac{d^2\left(-\frac{1}{3}bd\,x^3+adx-3bcx\right)}{b^2}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\,b^2\sqrt{ab}}-\frac{c^3}{ax}$
risch	$\frac{d^3x^3}{3b}-\frac{d^3ax}{b^2}+\frac{3d^2cx}{b}-\frac{c^3}{ax}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6+a^3\_Z^2b\right)}\_R\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -d^2/b^2*(-1/3*b*d*x^3+a*d*x-3*b*c*x)+1/a/b^2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^
2*c^2*d-b^3*c^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c^3/a/x
```

Maxima [A]
time = 0.54, size = 101, normalized size = 1.31

$$-\frac{c^3}{ax} + \frac{bd^3x^3 + 3\,(3\,bcd^2 - ad^3)x}{3\,b^2} - \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -c^3/(a*x) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - a*d^3)*x)/b^2 - (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^
2)
```

Fricas [A]
time = 1.18, size = 253, normalized size = 3.29

$$\left[\frac{2\,a^{2}b^{2}d^{3}x^{4}-6\,ab^{3}c^{3}+3\,(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\sqrt{-ab}\,x\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+6\,(3\,a^{2}b^{2}cd^{2}-a^{3}bd^{3})x^{2}}{6\,a^{2}b^{3}x},\ \frac{a^{2}b^{2}d^{3}x^{4}-3\,ab^{3}c^{3}-3\,(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\sqrt{ab}\,x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\,(3\,a^{2}b^{2}cd^{2}-a^{3}bd^{3})x^{2}}{3\,a^{2}b^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b^2*d^3*x^4 - 6*a*b^3*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*sqrt(-a*b)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)) + 6*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^2)/(a^2*b^3*x), 1/3*(a^2*b^2*d^3*x^4
 - 3*a*b^3*c^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b)*x*arctan(sqrt(a*b)*x/a) + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^2)/(a^2*b
^3*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(65) = 130$.
time = 0.43, size = 221, normalized size = 2.87

$$x\left(-\frac{ad^{3}}{b^{2}}+\frac{3cd^{2}}{b}\right)-\frac{\sqrt{-\frac{1}{a^{3}b^{5}}}\,(ad-bc)^{3}\log\left(-\frac{a^{2}b^{2}\sqrt{-\frac{1}{a^{3}b^{5}}}\,(ad-bc)^{3}}{a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^{3}b^{5}}}\,(ad-bc)^{3}\log\left(\frac{a^{2}b^{2}\sqrt{-\frac{1}{a^{3}b^{5}}}\,(ad-bc)^{3}}{a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}}+x\right)}{2}+\frac{d^{3}x^{3}}{3b}-\frac{c^{3}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**2/(b*x**2+a),x)
```

```
[Out] x*(-a*d**3/b**2 + 3*c*d**2/b) - sqrt(-1/(a**3*b**5))*(a*d - b*c)**3*log(-a*
*2*b**2*sqrt(-1/(a**3*b**5))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 +
3*a*b**2*c**2*d - b**3*c**3) + x)/2 + sqrt(-1/(a**3*b**5))*(a*d - b*c)**3*l
og(a**2*b**2*sqrt(-1/(a**3*b**5))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d*
*2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x**3/(3*b) - c**3/(a*x)
```

Giac [A]
time = 0.58, size = 104, normalized size = 1.35

$$-\frac{c^{3}}{ax}-\frac{(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^{2}}+\frac{b^{2}d^{3}x^{3}+9\,b^{2}cd^{2}x-3\,abd^{3}x}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c^3/(a*x) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/3*(b^2*d^3*x^3 + 9*b^2*c*d^2*x - 3*a*b*d^
3*x)/b^3
```

Mupad [B]
time = 0.04, size = 118, normalized size = 1.53

$$\frac{d^3\,x^3}{3\,b}-\frac{c^3}{a\,x}-x\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3}{\sqrt{a}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^3}{a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^2*(a + b*x^2)),x)
```

```
[Out] (d^3*x^3)/(3*b) - c^3/(a*x) - x*((a*d^3)/b^2 - (3*c*d^2)/b) + (atan((b^(1/2
)*x*(a*d - b*c)^3)/(a^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2)))*(a*d - b*c)^3)/(a^(3/2)*b^(5/2))
```

3.226 $\int \frac{(c+dx^2)^3}{x^3(a+bx^2)} \, dx$

Optimal. Leaf size=73

$$-\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}-\frac{c^2(bc-3ad)\log(x)}{a^2}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2}$$

```
[Out] -1/2*c^3/a/x^2+1/2*d^3*x^2/b-c^2*(-3*a*d+b*c)*ln(x)/a^2+1/2*(-a*d+b*c)^3*ln
(b*x^2+a)/a^2/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2}-\frac{c^2\log(x)(bc-3ad)}{a^2}-\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*c^3/(a*x^2) + (d^3*x^2)/(2*b) - (c^2*(b*c - 3*a*d)*Log[x])/a^2 + ((b*c
 - a*d)^3*Log[a + b*x^2])/(2*a^2*b^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^3}{b}+\frac{c^3}{ax^2}+\frac{c^2(-bc+3ad)}{a^2x}-\frac{(-bc+ad)^3}{a^2b(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}-\frac{c^2(bc-3ad)\log(x)}{a^2}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 75, normalized size = 1.03

$$\frac{ab(-bc^3 + ad^3x^4) - 2b^2c^2(bc - 3ad)x^2 \log(x) + (bc - ad)^3x^2 \log(a + bx^2)}{2a^2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

```
[Out] (a*b*(-(b*c^3) + a*d^3*x^4) - 2*b^2*c^2*(b*c - 3*a*d)*x^2*Log[x] + (b*c - a
*d)^3*x^2*Log[a + b*x^2])/(2*a^2*b^2*x^2)
```

Maple [A]
time = 0.10, size = 94, normalized size = 1.29

method	result	siz
default	$\frac{d^3x^2}{2b}-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln\left(b\,x^2+a\right)}{2a^2b^2}-\frac{c^3}{2a\,x^2}+\frac{c^2(3ad-bc)\ln(x)}{a^2}$	94
norman	$\frac{-\frac{c^3}{2a}+\frac{d^3x^4}{2b}}{x^2}+\frac{c^2(3ad-bc)\ln(x)}{a^2}-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln\left(b\,x^2+a\right)}{2a^2b^2}$	96
risch	$\frac{d^3x^2}{2b}-\frac{a\ln\left(b\,x^2+a\right)d^3}{2b^2}+\frac{3\ln\left(b\,x^2+a\right)c\,d^2}{2b}-\frac{3\ln\left(b\,x^2+a\right)c^2d}{2a}+\frac{b\ln\left(b\,x^2+a\right)c^3}{2a^2}-\frac{c^3}{2a\,x^2}+\frac{3c^2\ln(x)d}{a}-\frac{c^3\ln(x)b}{a^2}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d^3*x^2/b-1/2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^2/b^2*ln(
b*x^2+a)-1/2*c^3/a/x^2+c^2*(3*a*d-b*c)/a^2*ln(x)
```

Maxima [A]
time = 0.27, size = 97, normalized size = 1.33

$$\frac{d^3x^2}{2\,b} - \frac{c^3}{2\,ax^2} - \frac{(bc^3 - 3\,ac^2d)\log(x^2)}{2\,a^2} + \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log(bx^2 + a)}{2\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^2/b - 1/2*c^3/(a*x^2) - 1/2*(b*c^3 - 3*a*c^2*d)*log(x^2)/a^2 + 1/
2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a)/(a^2*b
^2)
```

Fricas [A]
time = 0.90, size = 105, normalized size = 1.44

$$\frac{a^2bd^3x^4 - ab^2c^3 + (b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^2\log(bx^2 + a) - 2\,(b^3c^3 - 3\,ab^2c^2d)x^2\log(x)}{2\,a^2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*d^3*x^4 - a*b^2*c^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*x^2*log(b*x^2 + a) - 2*(b^3*c^3 - 3*a*b^2*c^2*d)*x^2*log(x))/(a^2
*b^2*x^2)
```

Sympy [A]
time = 1.28, size = 63, normalized size = 0.86

$$\frac{d^3x^2}{2b} - \frac{c^3}{2ax^2} + \frac{c^2 \cdot (3ad - bc)\log(x)}{a^2} - \frac{(ad - bc)^3 \log\left(\frac{a}{b} + x^2\right)}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**3/(b*x**2+a),x)
```

```
[Out] d**3*x**2/(2*b) - c**3/(2*a*x**2) + c**2*(3*a*d - b*c)*log(x)/a**2 - (a*d -
 b*c)**3*log(a/b + x**2)/(2*a**2*b**2)
```

Giac [A]
time = 0.57, size = 120, normalized size = 1.64

$$\frac{d^3x^2}{2\,b} - \frac{(bc^3 - 3\,ac^2d)\log(x^2)}{2\,a^2} + \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log(|bx^2 + a|)}{2\,a^2b^2} + \frac{bc^3x^2 - 3\,ac^2dx^2 - ac^3}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^3*x^2/b - 1/2*(b*c^3 - 3*a*c^2*d)*log(x^2)/a^2 + 1/2*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/(a^2*b^2) + 1/2*(b*
c^3*x^2 - 3*a*c^2*d*x^2 - a*c^3)/(a^2*x^2)
```

Mupad [B]
time = 0.09, size = 95, normalized size = 1.30

$$\frac{d^3\,x^2}{2\,b} - \frac{c^3}{2\,a\,x^2} - \frac{\ln(x)\,\left(b\,c^3 - 3\,a\,c^2\,d\right)}{a^2} - \frac{\ln\left(b\,x^2 + a\right)\,\left(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3\right)}{2\,a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^3*(a + b*x^2)),x)
```

```
[Out] (d^3*x^2)/(2*b) - c^3/(2*a*x^2) - (log(x)*(b*c^3 - 3*a*c^2*d))/a^2 - (log(a
 + b*x^2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2*b^2)
```

3.227 $\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=74

$$-\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

```
[Out] -1/3*c^3/a/x^3+c^2*(-3*a*d+b*c)/a^2/x+d^3*x/b+(-a*d+b*c)^3*arctan(x*b^(1/2)
/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A]

time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {472, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^3}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c^3/(a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx = \int \left(\frac{d^3}{b}+\frac{c^3}{ax^4}+\frac{c^2(-bc+3ad)}{a^2x^2}-\frac{(-bc+ad)^3}{a^2b(a+bx^2)}\right)dx$$
$$= -\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^2}\,dx}{a^2b}$$
$$= -\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 1.00

$$-\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c^3/(a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Maple [A]
time = 0.09, size = 98, normalized size = 1.32

method	result
default	$\frac{d^3x}{b}+\frac{(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2b\sqrt{ab}}-\frac{c^3}{3a\,x^3}-\frac{c^2(3ad-bc)}{a^2x}$
risch	$\frac{d^3x}{b}+\frac{-\frac{b\,c^2(3ad-bc)x^2}{a^2}-\frac{b\,c^3}{3a}}{b\,x^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6+a^5\_Z^2b\right)}\_R\ln((}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^3*x/b+1/a^2/b*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2))-1/3*c^3/a/x^3-c^2*(3*a*d-b*c)/a^2/x
```

Maxima [A]
time = 0.52, size = 98, normalized size = 1.32

$$\frac{d^3x}{b}+\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b}-\frac{ac^3-3\,(bc^3-3\,ac^2d)x^2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] d^3*x/b + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2*b) - 1/3*(a*c^3 - 3*(b*c^3 - 3*a*c^2*d)*x^2)/(a^2*x
^3)
```

Fricas [A]
time = 1.05, size = 256, normalized size = 3.46

$$\left[\frac{6\,a^{3}bd^{3}x^{4} - 2\,a^{2}b^{2}c^{3} + 3\,(b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3})\sqrt{-ab}\,x^{3}\log\left(\frac{bx^{2} + 2\,\sqrt{-ab}\,x - a}{bx^{2} + a}\right) + 6\,(ab^{3}c^{3} - 3\,a^{2}b^{2}c^{2}d)x^{2}}{6\,a^{3}b^{2}x^{3}}, \frac{3\,a^{3}bd^{3}x^{4} - a^{2}b^{2}c^{3} + 3\,(b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3})\sqrt{ab}\,x^{3}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + 3\,(ab^{3}c^{3} - 3\,a^{2}b^{2}c^{2}d)x^{2}}{3\,a^{3}b^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^3*b*d^3*x^4 - 2*a^2*b^2*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*sqrt(-a*b)*x^3*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) + 6*(a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)/(a^3*b^2*x^3), 1/3*(3*a^3*b*d^3
*x^4 - a^2*b^2*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)/
(a^3*b^2*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(65) = 130$.
time = 0.66, size = 221, normalized size = 2.99

$$\frac{\sqrt{-\frac{1}{a^{5}b^{3}}}\,(ad-bc)^{3}\log\left(-\frac{a^{3}b\sqrt{-\frac{1}{a^{5}b^{3}}}\,(ad-bc)^{3}}{a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{a^{5}b^{3}}}\,(ad-bc)^{3}\log\left(\frac{a^{3}b\sqrt{-\frac{1}{a^{5}b^{3}}}\,(ad-bc)^{3}}{a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}} + x\right)}{2} + \frac{d^{3}x}{b} + \frac{-ac^{3} + x^{2}(-9ac^{2}d + 3bc^{3})}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(-a**3*b*sqrt(-1/(a**5*b**3))*(a*d -
 b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2
 - sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(a**3*b*sqrt(-1/(a**5*b**3))*(a*d
 - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)
/2 + d**3*x/b + (-a*c**3 + x**2*(-9*a*c**2*d + 3*b*c**3))/(3*a**2*x**3)
```

Giac [A]
time = 0.62, size = 100, normalized size = 1.35

$$\frac{d^{3}x}{b} + \frac{(b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3})\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}b} + \frac{3\,bc^{3}x^{2} - 9\,ac^{2}dx^{2} - ac^{3}}{3\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d^3*x/b + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2*b) + 1/3*(3*b*c^3*x^2 - 9*a*c^2*d*x^2 - a*c^3)/(a^2
*x^3)
```

Mupad [B]
time = 0.08, size = 122, normalized size = 1.65

$$\frac{d^3\,x}{b}-\frac{\frac{b\,c^3}{3\,a}+\frac{b\,c^2\,x^2\,\left(3\,a\,d-b\,c\right)}{a^2}}{b\,x^3}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3}{\sqrt{a}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^3}{a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^4*(a + b*x^2)),x)
```

```
[Out] (d^3*x)/b - ((b*c^3)/(3*a) + (b*c^2*x^2*(3*a*d - b*c))/a^2)/(b*x^3) - (atan
((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(5/2)*b^(3/2))
```

3.228 $\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{x^2}{2bd}+\frac{a^2\log(a+bx^2)}{2b^2(bc-ad)}-\frac{c^2\log(c+dx^2)}{2d^2(bc-ad)}$$

```
[Out] 1/2*x^2/b/d+1/2*a^2*ln(b*x^2+a)/b^2/(-a*d+b*c)-1/2*c^2*ln(d*x^2+c)/d^2/(-a*
d+b*c)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {457, 84}

$$\frac{a^2\log(a+bx^2)}{2b^2(bc-ad)}-\frac{c^2\log(c+dx^2)}{2d^2(bc-ad)}+\frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] x^2/(2*b*d) + (a^2*Log[a + b*x^2])/(2*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^2
])/(2*d^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx,x,x^2\right)\\ &= \frac{x^2}{2bd}+\frac{a^2\log(a+bx^2)}{2b^2(bc-ad)}-\frac{c^2\log(c+dx^2)}{2d^2(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log(a+bx^2)-b(d(-bc+ad)x^2+bc^2\log(c+dx^2))}{2b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^2] - b*(d*(-(b*c) + a*d)*x^2 + b*c^2*Log[c + d*x^2]))/
(2*b^2*d^2*(b*c - a*d))
```

Maple [A]
time = 0.10, size = 65, normalized size = 0.93

method	result	size
default	$\frac{x^2}{2bd}-\frac{a^2\ln(b\,x^2+a)}{2b^2(ad-bc)}+\frac{c^2\ln(d\,x^2+c)}{2d^2(ad-bc)}$	65
norman	$\frac{x^2}{2bd}-\frac{a^2\ln(b\,x^2+a)}{2b^2(ad-bc)}+\frac{c^2\ln(d\,x^2+c)}{2d^2(ad-bc)}$	65
risch	$\frac{x^2}{2bd}+\frac{c^2\ln(d\,x^2+c)}{2d^2(ad-bc)}-\frac{a^2\ln(-b\,x^2-a)}{2b^2(ad-bc)}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b/d-1/2*a^2/b^2/(a*d-b*c)*ln(b*x^2+a)+1/2*c^2/d^2/(a*d-b*c)*ln(d*x^
2+c)
```

Maxima [A]
time = 0.32, size = 68, normalized size = 0.97

$$\frac{a^2\log(bx^2+a)}{2(b^3c-ab^2d)}-\frac{c^2\log(dx^2+c)}{2(bcd^2-ad^3)}+\frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(b*x^2 + a)/(b^3*c - a*b^2*d) - 1/2*c^2*log(d*x^2 + c)/(b*c*d^2
- a*d^3) + 1/2*x^2/(b*d)
```

Fricas [A]
time = 1.34, size = 72, normalized size = 1.03

$$\frac{a^2d^2\log(bx^2+a)-b^2c^2\log(dx^2+c)+(b^2cd-abd^2)x^2}{2(b^3cd^2-ab^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*log(b*x^2 + a) - b^2*c^2*log(d*x^2 + c) + (b^2*c*d - a*b*d^2)*
x^2)/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.57, size = 70, normalized size = 1.00

$$\frac{a^2 \log \left(|bx^2 + a|\right)}{2\,(b^3c - ab^2d)} - \frac{c^2 \log \left(|dx^2 + c|\right)}{2\,(bcd^2 - ad^3)} + \frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(b*x^2 + a))/(b^3*c - a*b^2*d) - 1/2*c^2*log(abs(d*x^2 + c))
/(b*c*d^2 - a*d^3) + 1/2*x^2/(b*d)
```

Mupad [**B**]
time = 0.19, size = 68, normalized size = 0.97

$$\frac{a^2\,\ln\left(b\,x^2+a\right)}{2\,b^3\,c-2\,a\,b^2\,d}+\frac{c^2\,\ln\left(d\,x^2+c\right)}{2\,a\,d^3-2\,b\,c\,d^2}+\frac{x^2}{2\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (a^2*log(a + b*x^2))/(2*b^3*c - 2*a*b^2*d) + (c^2*log(c + d*x^2))/(2*a*d^3
- 2*b*c*d^2) + x^2/(2*b*d)
```

3.229 $\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=78

$$\frac{x}{bd}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}$$

```
[Out] x/b/d+a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/(-a*d+b*c)-c^(3/2)*arctan(x
*d^(1/2)/c^(1/2))/d^(3/2)/(-a*d+b*c)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {490, 536, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)}-\frac{c^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] x/(b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*(b*c - a*d)) - (c^
(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(d^(3/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx = \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx}{bd}$$
$$= \frac{x}{bd} + \frac{a^2 \int \frac{1}{a+bx^2}\,dx}{b(bc-ad)} - \frac{c^2 \int \frac{1}{c+dx^2}\,dx}{d(bc-ad)}$$
$$= \frac{x}{bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)} - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}$$

Mathematica [A]
time = 0.07, size = 74, normalized size = 0.95

$$\frac{-\frac{ax}{b} + \frac{cx}{d} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}} - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{3/2}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)),x]
[Out] (-((a*x)/b) + (c*x)/d + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) - (c^
(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(3/2))/(b*c - a*d)
```

Maple [A]
time = 0.14, size = 73, normalized size = 0.94

method	result
default	$\frac{x}{bd} - \frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b(ad-bc)\sqrt{ab}} + \frac{c^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d(ad-bc)\sqrt{cd}}$
risch	$\frac{x}{bd} + \frac{\sqrt{-cd}\, c \ln\left(\left(-(-cd)^{\frac{3}{2}} a b^3 c^2 d - (-cd)^{\frac{3}{2}} b^4 c^3 - a^4 \sqrt{-cd}\, d^5 - b^4 c^4 \sqrt{-cd}\, d\right)x + a^4 c d^5 - a b^3 c^4 d^2\right)}{2d^2(ad-bc)} - \frac{\sqrt{-cd}\, c \ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] x/b/d-1/b*a^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/d*c^2/(a*d-b*
c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.49, size = 72, normalized size = 0.92

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c - abd)\sqrt{ab}} - \frac{c^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bcd - ad^2)\sqrt{cd}} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - c^2*arctan(d*x/sqrt
(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + x/(b*d)
```

Fricas [A]
time = 1.09, size = 391, normalized size = 5.01

$$\left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2(bc-ad)x}{2(b^2cd-abd^2)}, \frac{2ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)+2(bc-ad)x}{2(b^2cd-abd^2)}, -\frac{2bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)+ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2(bc-ad)x}{2(b^2cd-abd^2)}, \frac{ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)+(bc-ad)x}{b^2cd-abd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + b*c
*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(b*c - a*d)
*x)/(b^2*c*d - a*b*d^2), 1/2*(2*a*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - b*c
*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 2*(b*c - a*d)
*x)/(b^2*c*d - a*b*d^2), -1/2*(2*b*c*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + a*
d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b*c - a*d
)*x)/(b^2*c*d - a*b*d^2), (a*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - b*c*sqrt
(c/d)*arctan(d*x*sqrt(c/d)/c) + (b*c - a*d)*x)/(b^2*c*d - a*b*d^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(65) = 130$.
time = 153.14, size = 921, normalized size = 11.81

$$-\frac{\sqrt{-\frac{a^3}{b^3}}\log\left(x+\frac{-\frac{a^4d^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}-\frac{a^3b^3d^6\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^4cd^5\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ab^5c^2d^4\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^6c^3d^3\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^4c^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{2(ad-bc)}+\frac{\sqrt{-\frac{a^3}{b^3}}\log\left(x+\frac{\frac{a^4d^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}+\frac{a^3b^3d^6\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^4cd^5\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ab^5c^2d^4\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^6c^3d^3\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^4c^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{c^3}{d^3}}\log\left(x+\frac{-\frac{a^4d^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}-\frac{a^3b^3d^6\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^4cd^5\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ab^5c^2d^4\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^6c^3d^3\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^4c^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{2(ad-bc)}+\frac{\sqrt{-\frac{c^3}{d^3}}\log\left(x+\frac{\frac{a^4d^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}+\frac{a^3b^3d^6\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^4cd^5\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ab^5c^2d^4\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^6c^3d^3\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^4c^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{2(ad-bc)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-a**3/b**3)*log(x + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) - a**3*b
**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-a**3/b**3)
**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d - b*c)**
3 - b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**4*c**4*sqrt(-a**
```

```
3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(2*(a*d -
 b*c)) + sqrt(-a**3/b**3)*log(x + (a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) +
 a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5*(-a**
3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d -
 b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + b**4*c**4*sq
rt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(2
*(a*d - b*c)) - sqrt(-c**3/d**3)*log(x + (-a**4*d**4*sqrt(-c**3/d**3)/(a*d
- b*c) - a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d*
*5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-c**3/d**3)**(3/2
)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - b**4
*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c
**3))/(2*(a*d - b*c)) + sqrt(-c**3/d**3)*log(x + (a**4*d**4*sqrt(-c**3/d**3
)/(a*d - b*c) + a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a**2*b*
*4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-c**3/d**3
)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)**3
 + b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a
*b**2*c**3))/(2*(a*d - b*c)) + x/(b*d)
```

Giac [A]
time = 0.55, size = 72, normalized size = 0.92

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c - abd)\sqrt{ab}} - \frac{c^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bcd - ad^2)\sqrt{cd}} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - c^2*arctan(d*x/sqrt
(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + x/(b*d)
```

Mupad [B]
time = 0.35, size = 343, normalized size = 4.40

$$\frac{\ln\left(a^5 b^4 d^3 - a^2 b^7 c^3 + d^3 x\,(-a^3 b^3)^{3/2} + b^6 c^3 x\sqrt{-a^3 b^3}\right)\sqrt{-a^3 b^3}}{2 b^4 c - 2 a b^3 d} - \frac{\ln\left(a^2 b^7 c^3 - a^5 b^4 d^3 + d^3 x\,(-a^3 b^3)^{3/2} + b^6 c^3 x\sqrt{-a^3 b^3}\right)\sqrt{-a^3 b^3}}{2\,(b^4 c - a b^3 d)} + \frac{x}{b d} - \frac{\ln\left(a^3 c^2 d^7 - b^3 c^5 d^4 + b^3 x\,(-c^3 d^3)^{3/2} + a^3 d^6 x\sqrt{-c^3 d^3}\right)\sqrt{-c^3 d^3}}{2\,(a d^4 - b c d^3)} + \frac{\ln\left(b^3 c^5 d^4 - a^3 c^2 d^7 + b^3 x\,(-c^3 d^3)^{3/2} + a^3 d^6 x\sqrt{-c^3 d^3}\right)\sqrt{-c^3 d^3}}{2 a d^4 - 2 b c d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a^5*b^4*d^3 - a^2*b^7*c^3 + d^3*x*(-a^3*b^3)^(3/2) + b^6*c^3*x*(-a^3*b
^3)^(1/2))*(-a^3*b^3)^(1/2))/(2*b^4*c - 2*a*b^3*d) - (log(a^2*b^7*c^3 - a^5
*b^4*d^3 + d^3*x*(-a^3*b^3)^(3/2) + b^6*c^3*x*(-a^3*b^3)^(1/2))*(-a^3*b^3)^
(1/2))/(2*(b^4*c - a*b^3*d)) + x/(b*d) - (log(a^3*c^2*d^7 - b^3*c^5*d^4 + b
^3*x*(-c^3*d^3)^(3/2) + a^3*d^6*x*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(2*(a
*d^4 - b*c*d^3)) + (log(b^3*c^5*d^4 - a^3*c^2*d^7 + b^3*x*(-c^3*d^3)^(3/2)
+ a^3*d^6*x*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(2*a*d^4 - 2*b*c*d^3)
```

3.230 $\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=53

$$-\frac{a\log(a+bx^2)}{2b(bc-ad)}+\frac{c\log(c+dx^2)}{2d(bc-ad)}$$

```
[Out] -1/2*a*ln(b*x^2+a)/b/(-a*d+b*c)+1/2*c*ln(d*x^2+c)/d/(-a*d+b*c)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{c\log(c+dx^2)}{2d(bc-ad)}-\frac{a\log(a+bx^2)}{2b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/2*(a*Log[a + b*x^2])/(b*(b*c - a*d)) + (c*Log[c + d*x^2])/(2*d*(b*c - a*
d))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)}\right)dx, x, x^2\right)\\
&= -\frac{a\log(a+bx^2)}{2b(bc-ad)} + \frac{c\log(c+dx^2)}{2d(bc-ad)}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.81

$$
-\frac{ad\log(a+bx^2) - bc\log(c+dx^2)}{2b^2cd - 2abd^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)),x]
[Out] -((a*d*Log[a + b*x^2] - b*c*Log[c + d*x^2])/(2*b^2*c*d - 2*a*b*d^2))
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.94

method	result	size
default	$\frac{a\ln(bx^2+a)}{2(ad-bc)b} - \frac{c\ln(dx^2+c)}{2(ad-bc)d}$	50
norman	$\frac{a\ln(bx^2+a)}{2(ad-bc)b} - \frac{c\ln(dx^2+c)}{2(ad-bc)d}$	50
risch	$\frac{a\ln(bx^2+a)}{2(ad-bc)b} - \frac{c\ln(-dx^2-c)}{2d(ad-bc)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/2*a/(a*d-b*c)/b*ln(b*x^2+a)-1/2*c/(a*d-b*c)/d*ln(d*x^2+c)
```

Maxima [A]
time = 0.29, size = 49, normalized size = 0.92

$$
-\frac{a\log(bx^2+a)}{2(b^2c-abd)} + \frac{c\log(dx^2+c)}{2(bcd-ad^2)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(b*x^2 + a)/(b^2*c - a*b*d) + 1/2*c*log(d*x^2 + c)/(b*c*d - a*d^2
)
```

Fricas [A]
time = 0.83, size = 42, normalized size = 0.79

$$-\frac{ad\log\left(bx^2+a\right)-bc\log\left(dx^2+c\right)}{2\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d*log(b*x^2 + a) - b*c*log(d*x^2 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(39) = 78$.
time = 1.41, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^2+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{2b\left(ad-bc\right)}-\frac{c\log\left(x^2+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{2d\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] a*log(x**2 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(2*b*(a*d - b*c)) - c*log(x**2 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(2*d*(a*d - b*c))
```

Giac [A]
time = 0.56, size = 51, normalized size = 0.96

$$-\frac{a\log\left(\left|bx^2+a\right|\right)}{2\left(b^2c-abd\right)}+\frac{c\log\left(\left|dx^2+c\right|\right)}{2\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*a*log(abs(b*x^2 + a))/(b^2*c - a*b*d) + 1/2*c*log(abs(d*x^2 + c))/(b*c
*d - a*d^2)
```

Mupad [B]
time = 0.15, size = 51, normalized size = 0.96

$$-\frac{a\,\ln\left(b\,x^2+a\right)}{2\,b^2\,c-2\,a\,b\,d}-\frac{c\,\ln\left(d\,x^2+c\right)}{2\,a\,d^2-2\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - (a*log(a + b*x^2))/(2*b^2*c - 2*a*b*d) - (c*log(c + d*x^2))/(2*a*d^2 - 2*
b*c*d)
```

3.231 $\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)}+\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)}$$

```
[Out] -arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)/b^(1/2)+arctan(x*d^(1/2)/c^(1
/2))*c^(1/2)/(-a*d+b*c)/d^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {492, 211}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) + (Sqrt[c]*A
rcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx = -\frac{a\int \frac{1}{a+bx^2}\,dx}{bc-ad} + \frac{c\int \frac{1}{c+dx^2}\,dx}{bc-ad}$$
$$= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)} + \frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.87

$$\frac{-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/Sqrt[d])/(b*c - a*d)
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.79

method	result
default	$\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}} - \frac{c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$
risch	$\frac{\sqrt{-ab}\,\ln\left(\left(-(-ab)^{\frac{3}{2}}a\,d^2-(-ab)^{\frac{3}{2}}bcd-a^2\sqrt{-ab}\,d^2b-b^3c^2\sqrt{-ab}\right)x-a^2b^2cd+a\,b^3c^2\right)}{2b(ad-bc)} - \frac{\sqrt{-ab}\,\ln\left(\left((-ab)^{\frac{3}{2}}a\,d^2+(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c/(a*d-b*c)/(c*d)^(1/2)*arc
tan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.52, size = 54, normalized size = 0.77

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)} + \frac{c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + c*arctan(d*x/sqrt(c*d))/
((b*c - a*d)*sqrt(c*d))
```

Fricas [A]
time = 1.23, size = 309, normalized size = 4.41

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2(bc-ad)}, -\frac{2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2(bc-ad)}, \frac{2\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{2(bc-ad)}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)}{bc-ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + sqrt(-c
/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(a/b)*arctan(b*x*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d)
 - c)/(d*x^2 + c)))/(b*c - a*d), 1/2*(2*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) -
 sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b*c - a*d), -
(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - sqrt(c/d)*arctan(d*x*sqrt(c/d)/c))/(b*
c - a*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(60) = 120$.
time = 1.83, size = 570, normalized size = 8.14

$$\frac{\sqrt{-\frac{a}{b}}\log\left(-\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}+\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x\right)}{2(ad-bc)}+\frac{\sqrt{-\frac{c}{d}}\log\left(-\frac{2a^2bd^3\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{c}{d}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{c}{d}}}{ad-bc}+x\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{c}{d}}\log\left(\frac{2a^2bd^3\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{c}{d}}}{ad-bc}+\frac{2b^3c^2d\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{bc\sqrt{-\frac{c}{d}}}{ad-bc}+x\right)}{2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-a/b)*log(-2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 + 4*a*b**2*c*d**
2*(-a/b)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-a/b)/(a*d - b*c) - 2*b**3*c**2*d
*(-a/b)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-a/b)/(a*d - b*c) + x)/(2*(a*d - b
*c)) - sqrt(-a/b)*log(2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 - 4*a*b**2
*c*d**2*(-a/b)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-a/b)/(a*d - b*c) + 2*b**3*
c**2*d*(-a/b)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-a/b)/(a*d - b*c) + x)/(2*(a
*d - b*c)) + sqrt(-c/d)*log(-2*a**2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)**3 + 4
*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-c/d)/(a*d - b*c) -
2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-c/d)/(a*d - b*c) + x
)/(2*(a*d - b*c)) - sqrt(-c/d)*log(2*a**2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)*
*3 - 4*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-c/d)/(a*d - b
```

```
*c) + 2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-c/d)/(a*d - b*
c) + x)/(2*(a*d - b*c))
```

Giac [A]
time = 0.59, size = 54, normalized size = 0.77

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}+\frac{c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + c*arctan(d*x/sqrt(c*d))/
((b*c - a*d)*sqrt(c*d))
```

Mupad [B]
time = 0.19, size = 133, normalized size = 1.90

$$\frac{\ln\left(a+x\sqrt{-a\,b}\right)\sqrt{-a\,b}}{2\,b^2\,c-2\,a\,b\,d}-\frac{\ln\left(a-x\sqrt{-a\,b}\right)\sqrt{-a\,b}}{2\,\left(b^2\,c-a\,b\,d\right)}-\frac{\ln\left(c-x\sqrt{-c\,d}\right)\sqrt{-c\,d}}{2\,\left(a\,d^2-b\,c\,d\right)}+\frac{\ln\left(c+x\sqrt{-c\,d}\right)\sqrt{-c\,d}}{2\,a\,d^2-2\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a + x*(-a*b)^(1/2))*(-a*b)^(1/2))/(2*b^2*c - 2*a*b*d) - (log(a - x*(-a
*b)^(1/2))*(-a*b)^(1/2))/(2*(b^2*c - a*b*d)) - (log(c - x*(-c*d)^(1/2))*(-c
*d)^(1/2))/(2*(a*d^2 - b*c*d)) + (log(c + x*(-c*d)^(1/2))*(-c*d)^(1/2))/(2*
a*d^2 - 2*b*c*d)
```

3.232 $\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=45

$$\frac{\log(a+bx^2)}{2(bc-ad)}-\frac{\log(c+dx^2)}{2(bc-ad)}$$

[Out] 1/2*ln(b*x^2+a)/(-a*d+b*c)-1/2*ln(d*x^2+c)/(-a*d+b*c)

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 36, 31}

$$\frac{\log(a+bx^2)}{2(bc-ad)}-\frac{\log(c+dx^2)}{2(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x/((a + b*x^2)*(c + d*x^2)),x]

[Out] Log[a + b*x^2]/(2*(b*c - a*d)) - Log[c + d*x^2]/(2*(b*c - a*d))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx, x, x^2\right)$$
$$= \frac{b\mathrm{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^2\right)}{2(bc-ad)} - \frac{d\mathrm{Subst}\left(\int \frac{1}{c+dx}\,dx, x, x^2\right)}{2(bc-ad)}$$
$$= \frac{\log(a+bx^2)}{2(bc-ad)} - \frac{\log(c+dx^2)}{2(bc-ad)}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^2)-\log(c+dx^2)}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)),x]
[Out] (Log[a + b*x^2] - Log[c + d*x^2])/(2*b*c - 2*a*d)
```

Maple [A]
time = 0.09, size = 42, normalized size = 0.93

method	result	size
default	$-\frac{\ln(b\,x^2+a)}{2(ad-bc)}+\frac{\ln(d\,x^2+c)}{2ad-2bc}$	42
norman	$-\frac{\ln(b\,x^2+a)}{2(ad-bc)}+\frac{\ln(d\,x^2+c)}{2ad-2bc}$	42
risch	$\frac{\ln(d\,x^2+c)}{2ad-2bc}-\frac{\ln(-b\,x^2-a)}{2(ad-bc)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] -1/2/(a*d-b*c)*ln(b*x^2+a)+1/2/(a*d-b*c)*ln(d*x^2+c)
```

Maxima [A]
time = 0.32, size = 41, normalized size = 0.91

$$\frac{\log(bx^2+a)}{2(bc-ad)} - \frac{\log(dx^2+c)}{2(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a)/(b*c - a*d) - 1/2*log(d*x^2 + c)/(b*c - a*d)
```

Fricas [A]
time = 0.93, size = 31, normalized size = 0.69

$$\frac{\log\left(bx^2+a\right)-\log\left(dx^2+c\right)}{2\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x^2 + a) - log(d*x^2 + c))/(b*c - a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(36) = 72$.
time = 0.54, size = 138, normalized size = 3.07

$$\frac{\log\left(x^2+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{2\left(ad-bc\right)}-\frac{\log\left(x^2+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{2\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] log(x**2 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(2*(a*d - b*c)) - log(x**2 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(2*(a*d - b*c))
```

Giac [A]
time = 0.58, size = 51, normalized size = 1.13

$$\frac{b\log\left(|bx^2+a|\right)}{2\left(b^2c-abd\right)}-\frac{d\log\left(|dx^2+c|\right)}{2\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b*log(abs(b*x^2 + a))/(b^2*c - a*b*d) - 1/2*d*log(abs(d*x^2 + c))/(b*c*
d - a*d^2)
```

Mupad [B]
time = 0.12, size = 148, normalized size = 3.29

$$\frac{2\,\mathrm{atanh}\left(\frac{8\,b^2\,d^2\,x^2}{\left(2\,a\,d-2\,b\,c\right)\,\left(\frac{32\,a\,b^2\,c\,d^2}{4\,a^2\,d^2-8\,a\,b\,c\,d+4\,b^2\,c^2}+\frac{16\,a\,b^2\,d^3\,x^2}{4\,a^2\,d^2-8\,a\,b\,c\,d+4\,b^2\,c^2}+\frac{16\,b^3\,c\,d^2\,x^2}{4\,a^2\,d^2-8\,a\,b\,c\,d+4\,b^2\,c^2}\right)}\right)}{2\,a\,d-2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (2*atanh((8*b^2*d^2*x^2)/((2*a*d - 2*b*c)*((32*a*b^2*c*d^2)/(4*a^2*d^2 + 4*
b^2*c^2 - 8*a*b*c*d) + (16*a*b^2*d^3*x^2)/(4*a^2*d^2 + 4*b^2*c^2 - 8*a*b*c*
d) + (16*b^3*c*d^2*x^2)/(4*a^2*d^2 + 4*b^2*c^2 - 8*a*b*c*d)))))/(2*a*d - 2*
b*c)
```

3.233 $\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)/a^(1/2)-arctan(x*d^(1/2)/c^(1/
2))*d^(1/2)/(-a*d+b*c)/c^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {400, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx &= \frac{b\int \frac{1}{a+bx^2}\,dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^2}\,dx}{bc-ad}\\ &= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

Maple [A]
time = 0.07, size = 55, normalized size = 0.79

method	result
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}}+\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$
risch	$\frac{\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2a(ad-bc)}-\frac{\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2a(ad-bc)}+\frac{\sqrt{-cd}\,\ln\left(dx+\sqrt{-cd}\right)}{2c(ad-bc)}-\frac{\sqrt{-cd}\,\ln\left(dx-\sqrt{-c\right.}}{2c(ad-bc)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)/(c*d)^(1/2)*ar
ctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.48, size = 54, normalized size = 0.77

$$\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/(
(b*c - a*d)*sqrt(c*d))
```

Fricas [A]
time = 1.07, size = 292, normalized size = 4.17

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2(bc-ad)},\ -\frac{2\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2(bc-ad)},\ \frac{2\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2(bc-ad)},\ \frac{\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)}{bc-ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(d/c)*arctan(x*sqrt(d/c)) + sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(b*c - a*d), 1/2*(2*sqrt(b/a)*arctan(x*sqrt(b/a)) - sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqrt(b/a
)*arctan(x*sqrt(b/a)) - sqrt(d/c)*arctan(x*sqrt(d/c)))/(b*c - a*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(60) = 120$.
time $= 2.81$, size $= 712$, normalized size $= 10.17$

$$\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}+\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2
*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d -
b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(a*d
 - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqr
t(-b/a)*log(x + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**
2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)
**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a*d - b
*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) + sqrt(-d
/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2*d**2*(
-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3
 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)
**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-d/c)
*log(x + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-d/c
)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 + a
**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3
+ b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
```

Giac [A]
time $= 0.68$, size $= 54$, normalized size $= 0.77$

$$\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/(
(b*c - a*d)*sqrt(c*d))
```

Mupad [B]
time = 0.19, size = 135, normalized size = 1.93

$$\frac{\ln\left(b\,x-\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,a^2\,d-2\,a\,b\,c}-\frac{\ln\left(d\,x+\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,\left(b\,c^2-a\,c\,d\right)}-\frac{\ln\left(b\,x+\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,\left(a^2\,d-a\,b\,c\right)}+\frac{\ln\left(d\,x-\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,b\,c^2-2\,a\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(b*x - (-a*b)^(1/2))*(-a*b)^(1/2))/(2*a^2*d - 2*a*b*c) - (log(d*x + (-c
*d)^(1/2))*(-c*d)^(1/2))/(2*(b*c^2 - a*c*d)) - (log(b*x + (-a*b)^(1/2))*(-a
*b)^(1/2))/(2*(a^2*d - a*b*c)) + (log(d*x - (-c*d)^(1/2))*(-c*d)^(1/2))/(2*
b*c^2 - 2*a*c*d)
```

3.234 $\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=62

$$\frac{\log(x)}{ac}-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}$$

```
[Out] ln(x)/a/c-1/2*b*ln(b*x^2+a)/a/(-a*d+b*c)+1/2*d*ln(d*x^2+c)/c/(-a*d+b*c)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^2])/(2*a*(b*c - a*d)) + (d*Log[c + d*x^2])/(2
*c*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx,x,x^2\right)\\ &= \frac{\log(x)}{ac}-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.87

$$\frac{2bc\log(x) - 2ad\log(x) - bc\log\left(a + bx^2\right) + ad\log\left(c + dx^2\right)}{2abc^2 - 2a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*b*c*Log[x] - 2*a*d*Log[x] - b*c*Log[a + b*x^2] + a*d*Log[c + d*x^2])/(2*
a*b*c^2 - 2*a^2*c*d)
```

Maple [A]
time = 0.11, size = 59, normalized size = 0.95

method	result	size
default	$\frac{b\ln(bx^2+a)}{2a(ad-bc)} - \frac{d\ln(dx^2+c)}{2c(ad-bc)} + \frac{\ln(x)}{ac}$	59
norman	$\frac{b\ln(bx^2+a)}{2a(ad-bc)} - \frac{d\ln(dx^2+c)}{2c(ad-bc)} + \frac{\ln(x)}{ac}$	59
risch	$\frac{b\ln(bx^2+a)}{2a(ad-bc)} - \frac{d\ln(dx^2+c)}{2c(ad-bc)} + \frac{\ln(x)}{ac}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/a/(a*d-b*c)*ln(b*x^2+a)-1/2*d/c/(a*d-b*c)*ln(d*x^2+c)+ln(x)/a/c
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.98

$$-\frac{b\log\left(bx^2 + a\right)}{2\left(abc - a^2d\right)} + \frac{d\log\left(dx^2 + c\right)}{2\left(bc^2 - acd\right)} + \frac{\log\left(x^2\right)}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*b*log(b*x^2 + a)/(a*b*c - a^2*d) + 1/2*d*log(d*x^2 + c)/(b*c^2 - a*c*d
) + 1/2*log(x^2)/(a*c)
```

Fricas [A]
time = 1.24, size = 54, normalized size = 0.87

$$-\frac{bc\log\left(bx^2 + a\right) - ad\log\left(dx^2 + c\right) - 2\left(bc - ad\right)\log\left(x\right)}{2\left(abc^2 - a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*log(b*x^2 + a) - a*d*log(d*x^2 + c) - 2*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.70, size = 73, normalized size = 1.18

$$-\frac{b^2 \log\left(|bx^2 + a|\right)}{2\left(ab^2c - a^2bd\right)} + \frac{d^2 \log\left(|dx^2 + c|\right)}{2\left(bc^2d - acd^2\right)} + \frac{\log\left(x^2\right)}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*b^2*log(abs(b*x^2 + a))/(a*b^2*c - a^2*b*d) + 1/2*d^2*log(abs(d*x^2 +
c))/(b*c^2*d - a*c*d^2) + 1/2*log(x^2)/(a*c)
```

Mupad [**B**]
time = 0.18, size = 58, normalized size = 0.94

$$\frac{b\,\ln\left(b\,x^2 + a\right)}{2\,a^2\,d - 2\,a\,b\,c} + \frac{d\,\ln\left(d\,x^2 + c\right)}{2\,b\,c^2 - 2\,a\,c\,d} + \frac{\ln\left(x\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (b*log(a + b*x^2))/(2*a^2*d - 2*a*b*c) + (d*log(c + d*x^2))/(2*b*c^2 - 2*a*
c*d) + log(x)/(a*c)
```

3.235 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)} dx$

Optimal. Leaf size=81

$$-\frac{1}{acx}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}$$

```
[Out] -1/a/c/x-b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)+d^(3/2)*arcta
n(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {491, 536, 211}

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}-\frac{1}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -(1/(a*c*x)) - (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d))
+ (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{1}{acx}+\frac{\int \frac{-bc-ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx}{ac}\\ &= -\frac{1}{acx}-\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^2}\,dx}{c(bc-ad)}\\ &= -\frac{1}{acx}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 0.94

$$\frac{-\frac{b}{a}+\frac{d}{c}-\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{d^{3/2}x\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}}{bcx-adx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
[Out] (-(b/a) + d/c - (b^(3/2)*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + (d^(3/2)*
x*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c*x - a*d*x)
```

Maple [A]
time = 0.16, size = 76, normalized size = 0.94

method	result
default	$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a(ad-bc)\sqrt{ab}}-\frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c(ad-bc)\sqrt{cd}}-\frac{1}{acx}$
risch	$-\frac{1}{acx}+\frac{\sqrt{-cd}\,d\ln\left(c\,d^2x+(-cd)^{\frac{3}{2}}\right)}{2c^2(ad-bc)}-\frac{\sqrt{-cd}\,d\ln\left(c\,d^2x-(-cd)^{\frac{3}{2}}\right)}{2c^2(ad-bc)}+\frac{\sqrt{-ab}\,b\ln\left(-a\,b^2x+(-ab)^{\frac{3}{2}}\right)}{2a^2(ad-bc)}-\frac{\sqrt{-ab}\,b\ln(}{2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/a*b^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/c*d^2/(a*d-b*c)/(c*
d)^(1/2)*arctan(d*x/(c*d)^(1/2))-1/a/c/x
```

Maxima [A]
time = 0.58, size = 75, normalized size = 0.93

$$-\frac{b^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(abc-a^{2}d)\sqrt{ab}}+\frac{d^{2} \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^{2}-acd)\sqrt{cd}}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -b^2*arctan(b*x/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + d^2*arctan(d*x/sqr
t(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/(a*c*x)
```

Fricas [A]
time = 1.02, size = 384, normalized size = 4.74

$$\left[-\frac{bcx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+adx\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2}-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^{2}+c}\right)+2\,bc-2\,ad}{2\,(abc^{2}-a^{2}cd)x},\ \frac{2\,adx\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-bcx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)-2\,bc+2\,ad}{2\,(abc^{2}-a^{2}cd)x},\ -\frac{2\,bcx\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+adx\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2}-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^{2}+c}\right)+2\,bc-2\,ad}{2\,(abc^{2}-a^{2}cd)x},\ -\frac{bcx\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-adx\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+bc-ad}{(abc^{2}-a^{2}cd)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*c*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + a
*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*b*c - 2
*a*d)/((a*b*c^2 - a^2*c*d)*x), 1/2*(2*a*d*x*sqrt(d/c)*arctan(x*sqrt(d/c)) -
 b*c*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*b*c +
 2*a*d)/((a*b*c^2 - a^2*c*d)*x), -1/2*(2*b*c*x*sqrt(b/a)*arctan(x*sqrt(b/a)
) + a*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*b*
c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x), -(b*c*x*sqrt(b/a)*arctan(x*sqrt(b/a)) -
 a*d*x*sqrt(d/c)*arctan(x*sqrt(d/c)) + b*c - a*d)/((a*b*c^2 - a^2*c*d)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(66) = 132$.
time = 138.92, size = 1093, normalized size = 13.49

$$-\frac{\sqrt{-\frac{b^{3}}{a^{3}}}\log\left(x+\frac{-\frac{a^{7}c^{3}d^{4}\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{2a^{6}bc^{4}d^{3}\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{2a^{5}b^{2}c^{5}d^{2}\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{5}d^{5}\sqrt{-\frac{b^{3}}{a^{3}}}}{ad-bc}+\frac{2a^{4}b^{3}c^{6}d\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{3}b^{4}c^{7}\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{5}c^{5}\sqrt{-\frac{b^{3}}{a^{3}}}}{ad-bc}}{a^{2}b^{2}d^{4}+ab^{3}cd^{3}+b^{4}c^{2}d^{2}}\right)}{2\,(ad-bc)}+\frac{\sqrt{-\frac{b^{3}}{a^{3}}}\log\left(x+\frac{\frac{a^{7}c^{3}d^{4}\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{2a^{6}bc^{4}d^{3}\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{2a^{5}b^{2}c^{5}d^{2}\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{5}d^{5}\sqrt{-\frac{b^{3}}{a^{3}}}}{ad-bc}-\frac{2a^{4}b^{3}c^{6}d\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{3}b^{4}c^{7}\left(-\frac{b^{3}}{a^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{b^{5}c^{5}\sqrt{-\frac{b^{3}}{a^{3}}}}{ad-bc}}{a^{2}b^{2}d^{4}+ab^{3}cd^{3}+b^{4}c^{2}d^{2}}\right)}{2\,(ad-bc)}-\frac{\sqrt{-\frac{d^{3}}{c^{3}}}\log\left(x+\frac{-\frac{a^{7}c^{3}d^{4}\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{2a^{6}bc^{4}d^{3}\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{2a^{5}b^{2}c^{5}d^{2}\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{5}d^{5}\sqrt{-\frac{d^{3}}{c^{3}}}}{ad-bc}+\frac{2a^{4}b^{3}c^{6}d\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{3}b^{4}c^{7}\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{5}c^{5}\sqrt{-\frac{d^{3}}{c^{3}}}}{ad-bc}}{a^{2}b^{2}d^{4}+ab^{3}cd^{3}+b^{4}c^{2}d^{2}}\right)}{2\,(ad-bc)}+\frac{\sqrt{-\frac{d^{3}}{c^{3}}}\log\left(x+\frac{\frac{a^{7}c^{3}d^{4}\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{2a^{6}bc^{4}d^{3}\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{2a^{5}b^{2}c^{5}d^{2}\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{5}d^{5}\sqrt{-\frac{d^{3}}{c^{3}}}}{ad-bc}-\frac{2a^{4}b^{3}c^{6}d\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{3}b^{4}c^{7}\left(-\frac{d^{3}}{c^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{b^{5}c^{5}\sqrt{-\frac{d^{3}}{c^{3}}}}{ad-bc}}{a^{2}b^{2}d^{4}+ab^{3}cd^{3}+b^{4}c^{2}d^{2}}\right)}{2\,(ad-bc)}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-b**3/a**3)*log(x + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*c)*
*3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2*c*
*5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-b**3/a**3)/(a*
d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**3*b**
4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b**3/a**3)/(a*d
```

```
 - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c))
 + sqrt(-b**3/a**3)*log(x + (a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*c)
**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + 2*a**5*b**2*c
**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-b**3/a**3)/(a
*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**3*b*
*4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-b**3/a**3)/(a*
d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c)
) - sqrt(-d**3/c**3)*log(x + (-a**7*c**3*d**4*(-d**3/c**3)**(3/2)/(a*d - b*
c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2
*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-d**3/c**3)/
(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**3*
b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-d**3/c**3)/(
a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*
c)) + sqrt(-d**3/c**3)*log(x + (a**7*c**3*d**4*(-d**3/c**3)**(3/2)/(a*d - b
*c)**3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + 2*a**5*b**
2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-d**3/c**3)
/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**3
*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-d**3/c**3)/
(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b
*c)) - 1/(a*c*x)
```

Giac [A]
time = 0.63, size = 75, normalized size = 0.93

$$-\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(abc - a^2d)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^2 - acd)\sqrt{cd}} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -b^2*arctan(b*x/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + d^2*arctan(d*x/sqr
t(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/(a*c*x)
```

Mupad [B]
time = 0.34, size = 338, normalized size = 4.17

$$\frac{\ln\left(a^3\,c^5\,d^4-b^3\,c^8\,d+a^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,b\,c^4-2\,a\,c^3\,d}-\frac{\ln\left(b^3\,c^8\,d-a^3\,c^5\,d^4+a^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,\left(b\,c^4-a\,c^3\,d\right)}-\frac{1}{a\,c\,x}-\frac{\ln\left(a^8\,b\,d^3-a^5\,b^4\,c^3+c^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+a^6\,d^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,\left(a^4\,d-a^3\,b\,c\right)}+\frac{\ln\left(a^5\,b^4\,c^3-a^8\,b\,d^3+c^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+a^6\,d^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,a^4\,d-2\,a^3\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a^3*c^5*d^4 - b^3*c^8*d + a^3*x*(-c^3*d^3)^(3/2) + b^3*c^6*x*(-c^3*d^3
)^(1/2))*(-c^3*d^3)^(1/2))/(2*b*c^4 - 2*a*c^3*d) - (log(b^3*c^8*d - a^3*c^5
*d^4 + a^3*x*(-c^3*d^3)^(3/2) + b^3*c^6*x*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2
```

```
))/(2*(b*c^4 - a*c^3*d)) - 1/(a*c*x) - (log(a^8*b*d^3 - a^5*b^4*c^3 + c^3*x
*(-a^3*b^3)^(3/2) + a^6*d^3*x*(-a^3*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(2*(a^4*d
 - a^3*b*c)) + (log(a^5*b^4*c^3 - a^8*b*d^3 + c^3*x*(-a^3*b^3)^(3/2) + a^6*
d^3*x*(-a^3*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(2*a^4*d - 2*a^3*b*c)
```

3.236 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{2acx^2}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}$$

```
[Out] -1/2/a/c/x^2-(a*d+b*c)*ln(x)/a^2/c^2+1/2*b^2*ln(b*x^2+a)/a^2/(-a*d+b*c)-1/2
*d^2*ln(d*x^2+c)/c^2/(-a*d+b*c)
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 84}

$$\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/2*1/(a*c*x^2) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^2])/(2
*a^2*(b*c - a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)dx, x, x^2\right)\\
&= -\frac{1}{2acx^2}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 1.01

$$-\frac{1}{2acx^2}+\frac{(-bc-ad)\log(x)}{a^2c^2}-\frac{b^2\log\left(a+bx^2\right)}{2a^2(-bc+ad)}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/2*1/(a*c*x^2) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^2])
/(2*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*d))
```

Maple [A]
time = 0.12, size = 83, normalized size = 0.95

method	result	size
norman	$-\frac{1}{2c\,x^2a}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^2(ad-bc)}+\frac{d^2\ln\left(d\,x^2+c\right)}{2c^2(ad-bc)}-\frac{(ad+bc)\ln(x)}{a^2c^2}$	82
default	$-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^2(ad-bc)}+\frac{d^2\ln\left(d\,x^2+c\right)}{2c^2(ad-bc)}-\frac{1}{2c\,x^2a}+\frac{(-ad-bc)\ln(x)}{a^2c^2}$	83
risch	$-\frac{1}{2c\,x^2a}-\frac{\ln(x)d}{a\,c^2}-\frac{\ln(x)b}{a^2c}+\frac{d^2\ln\left(-d\,x^2-c\right)}{2c^2(ad-bc)}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^2(ad-bc)}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2/a^2/(a*d-b*c)*ln(b*x^2+a)+1/2*d^2/c^2/(a*d-b*c)*ln(d*x^2+c)-1/2/c/
x^2/a+1/a^2/c^2*(-a*d-b*c)*ln(x)
```

Maxima [A]
time = 0.30, size = 87, normalized size = 1.00

$$\frac{b^2\log\left(bx^2+a\right)}{2\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^2+c\right)}{2\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^2\right)}{2\,a^2c^2}-\frac{1}{2\,acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(b*x^2 + a)/(a^2*b*c - a^3*d) - 1/2*d^2*log(d*x^2 + c)/(b*c^3 -
a*c^2*d) - 1/2*(b*c + a*d)*log(x^2)/(a^2*c^2) - 1/2/(a*c*x^2)
```

Fricas [A]
time = 1.30, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^2\log\left(bx^2+a\right)-a^2d^2x^2\log\left(dx^2+c\right)-abc^2+a^2cd-2\left(b^2c^2-a^2d^2\right)x^2\log\left(x\right)}{2\left(a^2bc^3-a^3c^2d\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*log(b*x^2 + a) - a^2*d^2*x^2*log(d*x^2 + c) - a*b*c^2 + a^
2*c*d - 2*(b^2*c^2 - a^2*d^2)*x^2*log(x))/((a^2*b*c^3 - a^3*c^2*d)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.34, size = 112, normalized size = 1.29

$$\frac{b^3\log\left(|bx^2+a|\right)}{2\left(a^2b^2c-a^3bd\right)}-\frac{d^3\log\left(|dx^2+c|\right)}{2\left(bc^3d-ac^2d^2\right)}-\frac{(bc+ad)\log\left(x^2\right)}{2\,a^2c^2}+\frac{bcx^2+adx^2-ac}{2\,a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(abs(b*x^2 + a))/(a^2*b^2*c - a^3*b*d) - 1/2*d^3*log(abs(d*x^2 +
 c))/(b*c^3*d - a*c^2*d^2) - 1/2*(b*c + a*d)*log(x^2)/(a^2*c^2) + 1/2*(b*c*
x^2 + a*d*x^2 - a*c)/(a^2*c^2*x^2)
```

Mupad [**B**]
time = 0.21, size = 87, normalized size = 1.00

$$-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{d^2\,\ln\left(d\,x^2+c\right)}{2\,\left(b\,c^3-a\,c^2\,d\right)}-\frac{1}{2\,a\,c\,x^2}-\frac{\ln\left(x\right)\,\left(a\,d+b\,c\right)}{a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - (b^2*log(a + b*x^2))/(2*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^2))/(2*(b*c
^3 - a*c^2*d)) - 1/(2*a*c*x^2) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

3.237 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)} \, dx$

Optimal. Leaf size=100

$$-\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}$$

```
[Out] -1/3/a/c/x^3+(a*d+b*c)/a^2/c^2/x+b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/
(-a*d+b*c)-d^(5/2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)
```

Rubi [A]
time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {491, 597, 536, 211}

$$\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/3*1/(a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5
/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{1}{3acx^3}+\frac{\int \frac{-3(bc+ad)-3bdx^2}{x^2(a+bx^2)(c+dx^2)}\,dx}{3ac}\\
&= -\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{-3\left(b^2c^2+abcd+a^2d^2\right)-3bd(bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx}{3a^2c^2}\\
&= -\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{1}{a+bx^2}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{1}{c+dx^2}\,dx}{c^2(bc-ad)}\\
&= -\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 101, normalized size = 1.01

$$-\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/3*1/(a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(a^(5/2)*(-(b*c) + a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c
^(5/2)*(b*c - a*d))
```

Maple [A]
time = 0.14, size = 96, normalized size = 0.96

method	result
default	$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2(ad-bc)\sqrt{ab}}+\frac{d^3 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^2(ad-bc)\sqrt{cd}}-\frac{1}{3ac\,x^3}-\frac{-ad-bc}{a^2c^2x}$
risch	$\frac{\frac{(ad+bc)x^2}{a^2c^2}-\frac{1}{3ac}}{x^3}+\frac{\sqrt{-cd}\,d^2\ln\left(\left(a^4c\,d^5+b\,c^2d^4a^3+b^2c^3d^3a^2+a\,b^3c^4d^2+b^4c^5d\right)x-(-cd)^{\frac{3}{2}}a^4d^3-(-cd)^{\frac{3}{2}}a^3bc\,d^2-(-cd)^{\frac{3}{2}}a^2b^2c\right.}{2c^3(ad-bc)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*b^3/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/c^2*d^3/(a*d-b*c
)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))-1/3/a/c/x^3-1/a^2/c^2*(-a*d-b*c)/x
```

Maxima [A]
time = 0.52, size = 96, normalized size = 0.96

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2bc-a^3d)\sqrt{ab}}-\frac{d^3 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^3-ac^2d)\sqrt{cd}}+\frac{3\,(bc+ad)x^2-ac}{3\,a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - d^3*arctan(d*x/sq
rt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/3*(3*(b*c + a*d)*x^2 - a*c)/(a^2
*c^2*x^3)
```

Fricas [A]
time = 0.95, size = 560, normalized size = 5.60

$$\left[-\frac{3\,b^2c^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+3\,a^2d^2x^3\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,abc^2-2\,a^2cd-6\,(b^2c^2-a^2d^2)x^2}{6\,(a^2bc^3-a^3c^2d)x^3},\ -\frac{6\,a^2d^2x^3\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+3\,b^2c^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+2\,abc^2-2\,a^2cd-6\,(b^2c^2-a^2d^2)x^2}{6\,(a^2bc^3-a^3c^2d)x^3},\ \frac{6\,b^2c^2x^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-3\,a^2d^2x^3\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\,abc^2+2\,a^2cd+6\,(b^2c^2-a^2d^2)x^2}{6\,(a^2bc^3-a^3c^2d)x^3},\ \frac{3\,b^2c^2x^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-3\,a^2d^2x^3\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-abc^2+a^2cd+3\,(b^2c^2-a^2d^2)x^2}{3\,(a^2bc^3-a^3c^2d)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*b^2*c^2*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
 a)) + 3*a^2*d^2*x^3*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 +
 c)) + 2*a*b*c^2 - 2*a^2*c*d - 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3
*c^2*d)*x^3), -1/6*(6*a^2*d^2*x^3*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*b^2*c^2
*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*a*b*c^2
 - 2*a^2*c*d - 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^3), 1/
6*(6*b^2*c^2*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*a^2*d^2*x^3*sqrt(-d/c)*l
```

```
og((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*a*b*c^2 + 2*a^2*c*d + 6*
(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^3), 1/3*(3*b^2*c^2*x^3*
sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*a^2*d^2*x^3*sqrt(d/c)*arctan(x*sqrt(d/c))
 - a*b*c^2 + a^2*c*d + 3*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*
x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.46, size = 98, normalized size = 0.98

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2bc - a^3d)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^3 - ac^2d)\sqrt{cd}} + \frac{3\,bcx^2 + 3\,adx^2 - ac}{3\,a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - d^3*arctan(d*x/sq
rt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/3*(3*b*c*x^2 + 3*a*d*x^2 - a*c)/
(a^2*c^2*x^3)
```

Mupad [B]
time = 0.34, size = 367, normalized size = 3.67

$$\frac{\ln\left(a^{13}\,b^2\,d^5-a^8\,b^7\,c^5+c^5\,x\,{\left(-a^5\,b^5\right)}^{3/2}+a^{10}\,d^5\,x\,\sqrt{-a^5\,b^5}\right)\,\sqrt{-a^5\,b^5}}{2\,a^6\,d-2\,a^5\,b\,c}-\frac{\ln\left(a^8\,b^7\,c^5-a^{13}\,b^2\,d^5+c^5\,x\,{\left(-a^5\,b^5\right)}^{3/2}+a^{10}\,d^5\,x\,\sqrt{-a^5\,b^5}\right)\,\sqrt{-a^5\,b^5}}{2\,\left(a^6\,d-a^5\,b\,c\right)}-\frac{\frac{1}{3\,a\,c}-\frac{x^2\,\left(a\,d+b\,c\right)}{a^2\,c^2}}{x^3}-\frac{\ln\left(a^5\,c^8\,d^7-b^5\,c^{13}\,d^2+a^5\,x\,{\left(-c^5\,d^5\right)}^{3/2}+b^5\,c^{10}\,x\,\sqrt{-c^5\,d^5}\right)\,\sqrt{-c^5\,d^5}}{2\,\left(b\,c^6-a\,c^5\,d\right)}+\frac{\ln\left(b^5\,c^{13}\,d^2-a^5\,c^8\,d^7+a^5\,x\,{\left(-c^5\,d^5\right)}^{3/2}+b^5\,c^{10}\,x\,\sqrt{-c^5\,d^5}\right)\,\sqrt{-c^5\,d^5}}{2\,b\,c^6-2\,a\,c^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a^13*b^2*d^5 - a^8*b^7*c^5 + c^5*x*(-a^5*b^5)^(3/2) + a^10*d^5*x*(-a^5
*b^5)^(1/2))*(-a^5*b^5)^(1/2))/(2*a^6*d - 2*a^5*b*c) - (log(a^8*b^7*c^5 - a
^13*b^2*d^5 + c^5*x*(-a^5*b^5)^(3/2) + a^10*d^5*x*(-a^5*b^5)^(1/2))*(-a^5*b
^5)^(1/2))/(2*(a^6*d - a^5*b*c)) - (1/(3*a*c) - (x^2*(a*d + b*c))/(a^2*c^2)
)/x^3 - (log(a^5*c^8*d^7 - b^5*c^13*d^2 + a^5*x*(-c^5*d^5)^(3/2) + b^5*c^10
*x*(-c^5*d^5)^(1/2))*(-c^5*d^5)^(1/2))/(2*(b*c^6 - a*c^5*d)) + (log(b^5*c^1
3*d^2 - a^5*c^8*d^7 + a^5*x*(-c^5*d^5)^(3/2) + b^5*c^10*x*(-c^5*d^5)^(1/2))
*(-c^5*d^5)^(1/2))/(2*b*c^6 - 2*a*c^5*d)
```

3.238 $\int \frac{1}{x^5(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{4acx^4}+\frac{bc+ad}{2a^2c^2x^2}+\frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3}-\frac{b^3\log(a+bx^2)}{2a^3(bc-ad)}+\frac{d^3\log(c+dx^2)}{2c^3(bc-ad)}$$

```
[Out] -1/4/a/c/x^4+1/2*(a*d+b*c)/a^2/c^2/x^2+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^3/
c^3-1/2*b^3*ln(b*x^2+a)/a^3/(-a*d+b*c)+1/2*d^3*ln(d*x^2+c)/c^3/(-a*d+b*c)
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$-\frac{b^3\log(a+bx^2)}{2a^3(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{\log(x)(a^2d^2+abcd+b^2c^2)}{a^3c^3}+\frac{d^3\log(c+dx^2)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/4*1/(a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^2])/(2*a^3*(b*c - a*d)) + (d^3*Lo
g[c + d*x^2])/(2*c^3*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx^2)(c+dx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)(c+dx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{acx^3}+\frac{-bc-ad}{a^2c^2x^2}+\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^4}{a^3(-bc+ad)(a+b\ldots}\right.\right.\\ &= -\frac{1}{4acx^4}+\frac{bc+ad}{2a^2c^2x^2}+\frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3}-\frac{b^3\log(a+bx^2)}{2a^3(bc-ad)}+\frac{d^3\log\ldots}{2c^3(\ldots}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 119, normalized size = 1.00

$$-\frac{1}{4acx^4}+\frac{bc+ad}{2a^2c^2x^2}+\frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3}+\frac{b^3\log(a+bx^2)}{2a^3(-bc+ad)}+\frac{d^3\log(c+dx^2)}{2c^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/4*1/(a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x])/(a^3*c^3) + (b^3*Log[a + b*x^2])/(2*a^3*(-(b*c) + a*d)) + (d^3
*Log[c + d*x^2])/(2*c^3*(b*c - a*d))
```

Maple [A]
time = 0.13, size = 114, normalized size = 0.96

method	result	size
default	$\frac{b^3\ln(bx^2+a)}{2a^3(ad-bc)}-\frac{d^3\ln(dx^2+c)}{2c^3(ad-bc)}-\frac{1}{4acx^4}-\frac{-ad-bc}{2a^2c^2x^2}+\frac{(a^2d^2+abcd+b^2c^2)\ln(x)}{a^3c^3}$	114
norman	$\frac{-\frac{1}{4ac}+\frac{(ad+bc)x^2}{2a^2c^2}}{x^4}+\frac{(a^2d^2+abcd+b^2c^2)\ln(x)}{a^3c^3}+\frac{b^3\ln(bx^2+a)}{2a^3(ad-bc)}-\frac{d^3\ln(dx^2+c)}{2c^3(ad-bc)}$	114
risch	$\frac{-\frac{1}{4ac}+\frac{(ad+bc)x^2}{2a^2c^2}}{x^4}+\frac{\ln(x)d^2}{ac^3}+\frac{\ln(x)bd}{a^2c^2}+\frac{\ln(x)b^2}{a^3c}-\frac{d^3\ln(-dx^2-c)}{2c^3(ad-bc)}+\frac{b^3\ln(bx^2+a)}{2a^3(ad-bc)}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^3/a^3/(a*d-b*c)*ln(b*x^2+a)-1/2*d^3/c^3/(a*d-b*c)*ln(d*x^2+c)-1/4/a/c
/x^4-1/2*(-a*d-b*c)/a^2/c^2/x^2+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^3/c^3
```

Maxima [A]
time = 0.28, size = 117, normalized size = 0.98

$$-\frac{b^3\log(bx^2+a)}{2(a^3bc-a^4d)}+\frac{d^3\log(dx^2+c)}{2(bc^4-ac^3d)}+\frac{(b^2c^2+abcd+a^2d^2)\log(x^2)}{2a^3c^3}+\frac{2(bc+ad)x^2-ac}{4a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(b*x^2 + a)/(a^3*b*c - a^4*d) + 1/2*d^3*log(d*x^2 + c)/(b*c^4 -
 a*c^3*d) + 1/2*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^2)/(a^3*c^3) + 1/4*(2*(
b*c + a*d)*x^2 - a*c)/(a^2*c^2*x^4)
```

Fricas [A]
time = 1.89, size = 127, normalized size = 1.07

$$-\frac{2b^3c^3x^4\log(bx^2+a)-2a^3d^3x^4\log(dx^2+c)+a^2bc^3-a^3c^2d-4(b^3c^3-a^3d^3)x^4\log(x)-2(ab^2c^3-a^3cd^2)x^2}{4(a^3bc^4-a^4c^3d)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^3*c^3*x^4*log(b*x^2 + a) - 2*a^3*d^3*x^4*log(d*x^2 + c) + a^2*b*c
^3 - a^3*c^2*d - 4*(b^3*c^3 - a^3*d^3)*x^4*log(x) - 2*(a*b^2*c^3 - a^3*c*d^
2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.04, size = 167, normalized size = 1.40

$$-\frac{b^4\log\left(|bx^2+a|\right)}{2\left(a^3b^2c-a^4bd\right)}+\frac{d^4\log\left(|dx^2+c|\right)}{2\left(bc^4d-ac^3d^2\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^2\right)}{2\,a^3c^3}-\frac{3\,b^2c^2x^4+3\,abcdx^4+3\,a^2d^2x^4-2\,abc^2x^2-2\,a^2cdx^2+a^2c^2}{4\,a^3c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*b^4*log(abs(b*x^2 + a))/(a^3*b^2*c - a^4*b*d) + 1/2*d^4*log(abs(d*x^2
+ c))/(b*c^4*d - a*c^3*d^2) + 1/2*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^2)/(a
^3*c^3) - 1/4*(3*b^2*c^2*x^4 + 3*a*b*c*d*x^4 + 3*a^2*d^2*x^4 - 2*a*b*c^2*x^
2 - 2*a^2*c*d*x^2 + a^2*c^2)/(a^3*c^3*x^4)
```

Mupad [**B**]
time = 0.24, size = 118, normalized size = 0.99

$$\frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,a^4\,d-2\,a^3\,b\,c}-\frac{\frac{1}{4\,a\,c}-\frac{x^2\,\left(a\,d+b\,c\right)}{2\,a^2\,c^2}}{x^4}+\frac{d^3\,\ln\left(d\,x^2+c\right)}{2\,b\,c^4-2\,a\,c^3\,d}+\frac{\ln\left(x\right)\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (b^3*log(a + b*x^2))/(2*a^4*d - 2*a^3*b*c) - (1/(4*a*c) - (x^2*(a*d + b*c))
/(2*a^2*c^2))/x^4 + (d^3*log(c + d*x^2))/(2*b*c^4 - 2*a*c^3*d) + (log(x)*(a
^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3)
```

3.239 $\int \frac{1}{x^6(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=134

$$-\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}$$

```
[Out] -1/5/a/c/x^5+1/3*(a*d+b*c)/a^2/c^2/x^3+(-a^2*d^2-a*b*c*d-b^2*c^2)/a^3/c^3/x
-b^(7/2)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/(-a*d+b*c)+d^(7/2)*arctan(x*d^(1
/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)
```

Rubi [A]

time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {491, 597, 536, 211}

$$-\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}+\frac{d^{7/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/5*1/(a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) - (b^2*c^2 + a*b*c*d + a^2*d
^2)/(a^3*c^3*x) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(b*c - a*d
)) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^2}{x^4(a+bx^2)(c+dx^2)}\,dx}{5ac}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{\int \frac{-15\left(b^2c^2+abcd+a^2d^2\right)-15bd(bc+ad)x^2}{x^2(a+bx^2)(c+dx^2)}\,dx}{15a^2c^2}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \frac{-15(bc+ad)\left(b^2c^2+a^2d^2\right)-15bd\left(b^2c^2+ab\right.}{(a+bx^2)(c+dx^2)}}{15a^3c^3}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^4\int \frac{1}{a+bx^2}\,dx}{a^3(bc-ad)}+\frac{d^4\int \frac{1}{c+dx^2}\,dx}{c^3(bc-ad)}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{d^{7/2}\tan^{-1}}{c^{7/2}(bc}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 135, normalized size = 1.01

$$
-\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}+\frac{-b^2c^2-abcd-a^2d^2}{a^3c^3x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(-bc+ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/5*1/(a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) + (-(b^2*c^2) - a*b*c*d - a^
2*d^2)/(a^3*c^3*x) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(-(b*c)
 + a*d)) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d))
```

Maple [A]
time = 0.14, size = 127, normalized size = 0.95

method	result
default	$\frac{b^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{3}(ad-bc)\sqrt{ab}}-\frac{d^{4}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^{3}(ad-bc)\sqrt{cd}}-\frac{1}{5ac\,x^{5}}-\frac{-ad-bc}{3a^{2}c^{2}x^{3}}-\frac{a^{2}d^{2}+abcd+b^{2}c^{2}}{a^{3}c^{3}x}$
risch	$\frac{-\frac{\left(a^{2}d^{2}+abcd+b^{2}c^{2}\right)x^{4}}{a^{3}c^{3}}+\frac{(ad+bc)x^{2}}{3a^{2}c^{2}}-\frac{1}{5ac}}{x^{5}}+\frac{\sqrt{-ab}\,b^{3}\ln\left(\left(-d^{6}a^{7}b-a^{6}d^{5}cb^{2}-d^{4}a^{5}c^{2}b^{3}-a^{4}d^{3}c^{3}b^{4}-d^{2}a^{3}c^{4}b^{5}-a^{2}dc^{5}b^{6}-ac^{6}b^{7}\right)x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*b^4/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/c^3*d^4/(a*d-b*c)
/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))-1/5/a/c/x^5-1/3*(-a*d-b*c)/a^2/c^2/x^3
-(a^2*d^2+a*b*c*d+b^2*c^2)/a^3/c^3/x
```

Maxima [A]
time = 0.57, size = 131, normalized size = 0.98

$$-\frac{b^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^{3}bc-a^{4}d)\sqrt{ab}}+\frac{d^{4}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^{4}-ac^{3}d)\sqrt{cd}}-\frac{15\,(b^{2}c^{2}+abcd+a^{2}d^{2})x^{4}+3\,a^{2}c^{2}-5\,(abc^{2}+a^{2}cd)x^{2}}{15\,a^{3}c^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a^3*b*c - a^4*d)*sqrt(a*b)) + d^4*arctan(d*x/s
qrt(c*d))/((b*c^4 - a*c^3*d)*sqrt(c*d)) - 1/15*(15*(b^2*c^2 + a*b*c*d + a^2
*d^2)*x^4 + 3*a^2*c^2 - 5*(a*b*c^2 + a^2*c*d)*x^2)/(a^3*c^3*x^5)
```

Fricas [A]
time = 1.31, size = 669, normalized size = 4.99

$$\left[-\frac{15\,b^{3}c^{3}x^{5}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+15\,a^{3}d^{3}x^{5}\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2}-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^{2}+c}\right)+6\,a^{2}bc^{3}-6\,a^{3}c^{2}d+30\,(b^{3}c^{3}-a^{3}d^{3})x^{4}-10\,(ab^{2}c^{3}-a^{3}cd^{2})x^{2}}{30\,(a^{3}bc^{4}-a^{4}c^{3}d)x^{5}},\ \frac{30\,a^{3}d^{3}x^{5}\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-15\,b^{3}c^{3}x^{5}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)-6\,a^{2}bc^{3}+6\,a^{3}c^{2}d-30\,(b^{3}c^{3}-a^{3}d^{3})x^{4}+10\,(ab^{2}c^{3}-a^{3}cd^{2})x^{2}}{30\,(a^{3}bc^{4}-a^{4}c^{3}d)x^{5}},\ -\frac{30\,b^{3}c^{3}x^{5}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+15\,a^{3}d^{3}x^{5}\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2}-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^{2}+c}\right)+6\,a^{2}bc^{3}-6\,a^{3}c^{2}d+30\,(b^{3}c^{3}-a^{3}d^{3})x^{4}-10\,(ab^{2}c^{3}-a^{3}cd^{2})x^{2}}{30\,(a^{3}bc^{4}-a^{4}c^{3}d)x^{5}},\ -\frac{15\,b^{3}c^{3}x^{5}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-15\,a^{3}d^{3}x^{5}\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+3\,a^{2}bc^{3}-3\,a^{3}c^{2}d+15\,(b^{3}c^{3}-a^{3}d^{3})x^{4}-5\,(ab^{2}c^{3}-a^{3}cd^{2})x^{2}}{15\,(a^{3}bc^{4}-a^{4}c^{3}d)x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + 15*a^3*d^3*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^
2 + c)) + 6*a^2*b*c^3 - 6*a^3*c^2*d + 30*(b^3*c^3 - a^3*d^3)*x^4 - 10*(a*b^
2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^5), 1/30*(30*a^3*d^3*x^5
```

```
*sqrt(d/c)*arctan(x*sqrt(d/c)) - 15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2 + 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6*a^2*b*c^3 + 6*a^3*c^2*d - 30*(b^3*c^3 -
 a^3*d^3)*x^4 + 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^
5), -1/30*(30*b^3*c^3*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 15*a^3*d^3*x^5*sq
rt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 6*a^2*b*c^3 - 6*
a^3*c^2*d + 30*(b^3*c^3 - a^3*d^3)*x^4 - 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((
a^3*b*c^4 - a^4*c^3*d)*x^5), -1/15*(15*b^3*c^3*x^5*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 15*a^3*d^3*x^5*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*a^2*b*c^3 - 3*a^3*
c^2*d + 15*(b^3*c^3 - a^3*d^3)*x^4 - 5*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b
*c^4 - a^4*c^3*d)*x^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.03, size = 139, normalized size = 1.04

$$-\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^3bc - a^4d)\sqrt{ab}} + \frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^4 - ac^3d)\sqrt{cd}} - \frac{15\,b^2c^2x^4 + 15\,abcdx^4 + 15\,a^2d^2x^4 - 5\,abc^2x^2 - 5\,a^2cdx^2 + 3\,a^2c^2}{15\,a^3c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a^3*b*c - a^4*d)*sqrt(a*b)) + d^4*arctan(d*x/s
qrt(c*d))/((b*c^4 - a*c^3*d)*sqrt(c*d)) - 1/15*(15*b^2*c^2*x^4 + 15*a*b*c*d
*x^4 + 15*a^2*d^2*x^4 - 5*a*b*c^2*x^2 - 5*a^2*c*d*x^2 + 3*a^2*c^2)/(a^3*c^3
*x^5)
```

Mupad [**B**]
time = 0.36, size = 397, normalized size = 2.96

$$\frac{\ln\left(a^{11}\,b^{10}\,c^7-a^{18}\,b^3\,d^7+c^7\,x\,{\left(-a^7\,b^7\right)}^{3/2}+a^{14}\,d^7\,x\,\sqrt{-a^7\,b^7}\right)\,\sqrt{-a^7\,b^7}}{2\,a^8\,d-2\,a^7\,b\,c}-\frac{\ln\left(a^{18}\,b^3\,d^7-a^{11}\,b^{10}\,c^7+c^7\,x\,{\left(-a^7\,b^7\right)}^{3/2}+a^{14}\,d^7\,x\,\sqrt{-a^7\,b^7}\right)\,\sqrt{-a^7\,b^7}}{2\,\left(a^8\,d-a^7\,b\,c\right)}-\frac{\frac{1}{5\,a\,c}-\frac{x^2\,\left(a\,d+b\,c\right)}{3\,a^2\,c^2}+\frac{x^4\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{a^3\,c^3}}{x^5}-\frac{\ln\left(b^7\,c^{18}\,d^3-a^7\,c^{11}\,d^{10}+a^7\,x\,{\left(-c^7\,d^7\right)}^{3/2}+b^7\,c^{14}\,x\,\sqrt{-c^7\,d^7}\right)\,\sqrt{-c^7\,d^7}}{2\,\left(b\,c^8-a\,c^7\,d\right)}+\frac{\ln\left(a^7\,c^{11}\,d^{10}-b^7\,c^{18}\,d^3+a^7\,x\,{\left(-c^7\,d^7\right)}^{3/2}+b^7\,c^{14}\,x\,\sqrt{-c^7\,d^7}\right)\,\sqrt{-c^7\,d^7}}{2\,b\,c^8-2\,a\,c^7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a^11*b^10*c^7 - a^18*b^3*d^7 + c^7*x*(-a^7*b^7)^(3/2) + a^14*d^7*x*(-a
^7*b^7)^(1/2))*(-a^7*b^7)^(1/2))/(2*a^8*d - 2*a^7*b*c) - (log(a^18*b^3*d^7
```

```
- a^11*b^10*c^7 + c^7*x*(-a^7*b^7)^(3/2) + a^14*d^7*x*(-a^7*b^7)^(1/2))*(-a
^7*b^7)^(1/2))/(2*(a^8*d - a^7*b*c)) - (1/(5*a*c) - (x^2*(a*d + b*c))/(3*a^
2*c^2) + (x^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3))/x^5 - (log(b^7*c^18
*d^3 - a^7*c^11*d^10 + a^7*x*(-c^7*d^7)^(3/2) + b^7*c^14*x*(-c^7*d^7)^(1/2)
)*(-c^7*d^7)^(1/2))/(2*(b*c^8 - a*c^7*d)) + (log(a^7*c^11*d^10 - b^7*c^18*d
^3 + a^7*x*(-c^7*d^7)^(3/2) + b^7*c^14*x*(-c^7*d^7)^(1/2))*(-c^7*d^7)^(1/2)
)/(2*b*c^8 - 2*a*c^7*d)
```

3.240 $\int \frac{1}{x^7(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=155

$$-\frac{1}{6acx^6}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{2a^3c^3x^2}-\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\log(x)}{a^4c^4}+\frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad)}-\frac{d^4\log\left(c+dx^2\right)}{2c^4(bc-ad)}$$

```
[Out] -1/6/a/c/x^6+1/4*(a*d+b*c)/a^2/c^2/x^4+1/2*(-a^2*d^2-a*b*c*d-b^2*c^2)/a^3/c
^3/x^2-(a*d+b*c)*(a^2*d^2+b^2*c^2)*ln(x)/a^4/c^4+1/2*b^4*ln(b*x^2+a)/a^4/(-
a*d+b*c)-1/2*d^4*ln(d*x^2+c)/c^4/(-a*d+b*c)
```

Rubi [A]
time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{\log(x)(ad+bc)\left(a^2d^2+b^2c^2\right)}{a^4c^4}-\frac{a^2d^2+abcd+b^2c^2}{2a^3c^3x^2}-\frac{d^4\log\left(c+dx^2\right)}{2c^4(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/6*1/(a*c*x^6) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d
^2)/(2*a^3*c^3*x^2) - ((b*c + a*d)*(b^2*c^2 + a^2*d^2)*Log[x])/(a^4*c^4) +
(b^4*Log[a + b*x^2])/(2*a^4*(b*c - a*d)) - (d^4*Log[c + d*x^2])/(2*c^4*(b*c
 - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7 (a+bx^2)(c+dx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)(c+dx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{acx^4} + \frac{-bc-ad}{a^2c^2x^3} + \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x^2} - \frac{(bc+ad)(b^2c^2+a^2d^2)}{a^4c^4x}\right.\right.$$
$$= -\frac{1}{6acx^6} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{2a^3c^3x^2} - \frac{(bc+ad)(b^2c^2+a^2d^2)\log(x)}{a^4c^4} + $$

Mathematica [A]
time = 0.05, size = 147, normalized size = 0.95

$$\frac{12(b^4c^4-a^4d^4)x^6\log(x) - 6b^4c^4x^6\log(a+bx^2) + a(2a^2bc^4 - 3ab^2c^4x^2 + 6b^3c^4x^4 + a^3cd(-2c^2+3cdx^2-6d^2x^4) + 6a^3d^4x^6\log(c+dx^2))}{12a^4c^4(-bc+ad)x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (12*(b^4*c^4 - a^4*d^4)*x^6*Log[x] - 6*b^4*c^4*x^6*Log[a + b*x^2] + a*(2*a^
2*b*c^4 - 3*a*b^2*c^4*x^2 + 6*b^3*c^4*x^4 + a^3*c*d*(-2*c^2 + 3*c*d*x^2 - 6
*d^2*x^4) + 6*a^3*d^4*x^6*Log[c + d*x^2]))/(12*a^4*c^4*(-(b*c) + a*d)*x^6)
```

Maple [A]
time = 0.13, size = 162, normalized size = 1.05

method	result
norman	$\frac{-\frac{1}{6ac}+\frac{(ad+bc)x^2}{4a^2c^2}-\frac{(a^2d^2+abcd+b^2c^2)x^4}{2a^3c^3}}{x^6} - \frac{b^4\ln(bx^2+a)}{2a^4(ad-bc)} + \frac{d^4\ln(dx^2+c)}{2c^4(ad-bc)} - \frac{(a^3d^3+a^2bcd^2+ab^2c^2d+b^3c^3)\ln(x)}{a^4c^4}$
default	$-\frac{b^4\ln(bx^2+a)}{2a^4(ad-bc)} + \frac{d^4\ln(dx^2+c)}{2c^4(ad-bc)} - \frac{1}{6acx^6} - \frac{-ad-bc}{4a^2c^2x^4} - \frac{a^2d^2+abcd+b^2c^2}{2a^3c^3x^2} + \frac{(-a^3d^3-a^2bcd^2-ab^2c^2d-b^3c^3)\ln(x)}{a^4c^4}$
risch	$\frac{-\frac{1}{6ac}+\frac{(ad+bc)x^2}{4a^2c^2}-\frac{(a^2d^2+abcd+b^2c^2)x^4}{2a^3c^3}}{x^6} - \frac{\ln(x)d^3}{ac^4} - \frac{\ln(x)bd^2}{a^2c^3} - \frac{\ln(x)b^2d}{a^3c^2} - \frac{\ln(x)b^3}{a^4c} + \frac{d^4\ln(-dx^2-c)}{2c^4(ad-bc)} - \frac{b^4\ln(bx^2+a)}{2a^4(ad-bc)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^4/a^4/(a*d-b*c)*ln(b*x^2+a)+1/2*d^4/c^4/(a*d-b*c)*ln(d*x^2+c)-1/6/a/
c/x^6-1/4*(-a*d-b*c)/a^2/c^2/x^4-1/2*(a^2*d^2+a*b*c*d+b^2*c^2)/a^3/c^3/x^2+
1/a^4/c^4*(-a^3*d^3-a^2*b*c*d^2-a*b^2*c^2*d-b^3*c^3)*ln(x)
```

Maxima [A]
time = 0.29, size = 165, normalized size = 1.06

$$\frac{b^4\log(bx^2+a)}{2(a^4bc-a^5d)} - \frac{d^4\log(dx^2+c)}{2(bc^5-ac^4d)} - \frac{(b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3)\log(x^2)}{2a^4c^4} - \frac{6(b^2c^2+abcd+a^2d^2)x^4+2a^2c^2-3(abc^2+a^2cd)x^2}{12a^3c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(b*x^2 + a)/(a^4*b*c - a^5*d) - 1/2*d^4*log(d*x^2 + c)/(b*c^5 -
a*c^4*d) - 1/2*(b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^
4*c^4) - 1/12*(6*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 - 3*(a*b*c^2
 + a^2*c*d)*x^2)/(a^3*c^3*x^6)
```

Fricas [**A**]
time = 2.24, size = 155, normalized size = 1.00

$$\frac{6\,b^4c^4x^6\log\left(bx^2+a\right)-6\,a^4d^4x^6\log\left(dx^2+c\right)-2\,a^3bc^4+2\,a^4c^3d-12\,(b^4c^4-a^4d^4)x^6\log\left(x\right)-6\,(ab^3c^4-a^4cd^3)x^4+3\,(a^2b^2c^4-a^4c^2d^2)x^2}{12\,(a^4bc^5-a^5c^4d)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^4*c^4*x^6*log(b*x^2 + a) - 6*a^4*d^4*x^6*log(d*x^2 + c) - 2*a^3*b
*c^4 + 2*a^4*c^3*d - 12*(b^4*c^4 - a^4*d^4)*x^6*log(x) - 6*(a*b^3*c^4 - a^4
*c*d^3)*x^4 + 3*(a^2*b^2*c^4 - a^4*c^2*d^2)*x^2)/((a^4*b*c^5 - a^5*c^4*d)*x
^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.24, size = 239, normalized size = 1.54

$$\frac{b^5\log\left(|bx^2+a|\right)}{2\,(a^4b^2c-a^5bd)}-\frac{d^5\log\left(|dx^2+c|\right)}{2\,(bc^5d-ac^4d^2)}-\frac{(b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3)\log\left(x^2\right)}{2\,a^4c^4}+\frac{11\,b^3c^3x^6+11\,ab^2c^2dx^6+11\,a^2bcd^2x^6+11\,a^3d^3x^6-6\,ab^2c^3x^4-6\,a^2bc^2dx^4-6\,a^3cd^2x^4+3\,a^2bc^3x^2+3\,a^3c^2dx^2-2\,a^3c^3}{12\,a^4c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(abs(b*x^2 + a))/(a^4*b^2*c - a^5*b*d) - 1/2*d^5*log(abs(d*x^2 +
 c))/(b*c^5*d - a*c^4*d^2) - 1/2*(b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3
*d^3)*log(x^2)/(a^4*c^4) + 1/12*(11*b^3*c^3*x^6 + 11*a*b^2*c^2*d*x^6 + 11*a
^2*b*c*d^2*x^6 + 11*a^3*d^3*x^6 - 6*a*b^2*c^3*x^4 - 6*a^2*b*c^2*d*x^4 - 6*a
^3*c*d^2*x^4 + 3*a^2*b*c^3*x^2 + 3*a^3*c^2*d*x^2 - 2*a^3*c^3)/(a^4*c^4*x^6)
```

Mupad [B]
time = 0.27, size = 165, normalized size = 1.06

$$-\frac{\frac{1}{6\,a\,c}-\frac{x^2\,\left(a\,d+b\,c\right)}{4\,a^2\,c^2}+\frac{x^4\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{2\,a^3\,c^3}}{x^6}-\frac{b^4\,\ln\left(b\,x^2+a\right)}{2\,\left(a^5\,d-a^4\,b\,c\right)}-\frac{d^4\,\ln\left(d\,x^2+c\right)}{2\,\left(b\,c^5-a\,c^4\,d\right)}-\frac{\ln\left(x\right)\,\left(a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3\right)}{a^4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - (1/(6*a*c) - (x^2*(a*d + b*c))/(4*a^2*c^2) + (x^4*(a^2*d^2 + b^2*c^2 + a*
b*c*d))/(2*a^3*c^3))/x^6 - (b^4*log(a + b*x^2))/(2*(a^5*d - a^4*b*c)) - (d^
4*log(c + d*x^2))/(2*(b*c^5 - a*c^4*d)) - (log(x)*(a^3*d^3 + b^3*c^3 + a*b^
2*c^2*d + a^2*b*c*d^2))/(a^4*c^4)
```

3.241 $\int \frac{x^5}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2}{2b^2(bc-ad)\left(a+bx^2\right)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}$$

```
[Out] -1/2*a^2/b^2/(-a*d+b*c)/(b*x^2+a)-1/2*a*(-a*d+2*b*c)*ln(b*x^2+a)/b^2/(-a*d+
b*c)^2+1/2*c^2*ln(d*x^2+c)/d/(-a*d+b*c)^2
```

Rubi [A]

time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{a^2}{2b^2\left(a+bx^2\right)(bc-ad)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/2*a^2/(b^2*(b*c - a*d)*(a + b*x^2)) - (a*(2*b*c - a*d)*Log[a + b*x^2])/(
2*b^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^2])/(2*d*(b*c - a*d)^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^2}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)}+\frac{c^2}{(bc-ad)^2(c+dx)}\right.\right.\\ &= -\frac{a^2}{2b^2(bc-ad)\left(a+bx^2\right)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 91, normalized size = 0.98

$$\frac{a^2d(-bc+ad)+ad(-2bc+ad)\left(a+bx^2\right)\log\left(a+bx^2\right)+b^2c^2\left(a+bx^2\right)\log\left(c+dx^2\right)}{2b^2d(bc-ad)^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (a^2*d*(-(b*c) + a*d) + a*d*(-2*b*c + a*d)*(a + b*x^2)*Log[a + b*x^2] + b^2
*c^2*(a + b*x^2)*Log[c + d*x^2])/(2*b^2*d*(b*c - a*d)^2*(a + b*x^2))
```

Maple [A]
time = 0.13, size = 86, normalized size = 0.92

method	result	size
default	$-\frac{a\left(\frac{(-ad+2bc)\ln\left(b\,x^2+a\right)}{b^2}-\frac{a(ad-bc)}{b^2\left(b\,x^2+a\right)}\right)}{2(ad-bc)^2}+\frac{c^2\ln\left(d\,x^2+c\right)}{2(ad-bc)^2d}$	86
norman	$\frac{a^2}{2b^2(ad-bc)(b\,x^2+a)}+\frac{c^2\ln\left(d\,x^2+c\right)}{2d(a^2d^2-2abcd+b^2c^2)}+\frac{a(ad-2bc)\ln\left(b\,x^2+a\right)}{2(a^2d^2-2abcd+b^2c^2)b^2}$	113
risch	$\frac{a^2}{2b^2(ad-bc)(b\,x^2+a)}+\frac{c^2\ln\left(-d\,x^2-c\right)}{2d(a^2d^2-2abcd+b^2c^2)}+\frac{a^2\ln\left(b\,x^2+a\right)d}{2(a^2d^2-2abcd+b^2c^2)b^2}-\frac{a\ln\left(b\,x^2+a\right)c}{(a^2d^2-2abcd+b^2c^2)b}$	149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(a*d-b*c)^2*((-a*d+2*b*c)/b^2*ln(b*x^2+a)-a*(a*d-b*c)/b^2/(b*x^2+a))
+1/2*c^2/(a*d-b*c)^2/d*ln(d*x^2+c)
```

Maxima [A]
time = 0.36, size = 130, normalized size = 1.40

$$\frac{c^2\log\left(dx^2+c\right)}{2\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}-\frac{a^2}{2\left(ab^3c-a^2b^2d+(b^4c-ab^3d)x^2\right)}-\frac{\left(2\,abc-a^2d\right)\log\left(bx^2+a\right)}{2\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(d*x^2 + c)/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/2*a^2/(a*b^3
*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^2) - 1/2*(2*a*b*c - a^2*d)*log(b*x^2 +
 a)/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
```

Fricas [A]
time = 1.09, size = 162, normalized size = 1.74

$$-\frac{a^2bcd-a^3d^2+\left(2\,a^2bcd-a^3d^2+(2\,ab^2cd-a^2bd^2)x^2\right)\log\left(bx^2+a\right)-\left(b^3c^2x^2+ab^2c^2\right)\log\left(dx^2+c\right)}{2\left(ab^4c^2d-2\,a^2b^3cd^2+a^3b^2d^3+(b^5c^2d-2\,ab^4cd^2+a^2b^3d^3)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")

[Out] -1/2*(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d^2)*x^2)*log(b*x^2 + a) - (b^3*c^2*x^2 + a*b^2*c^2)*log(d*x^2 + c))/(a*b^4*c^2*d - 2*a^2*b^3*c*d^2 + a^3*b^2*d^3 + (b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*x^2)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**2+a)**2/(d*x**2+c),x)

[Out] Timed out

Giac [**A**]
time = 1.47, size = 152, normalized size = 1.63

$$\frac{c^2\log\left(|dx^2+c|\right)}{2\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}-\frac{(2\,abc-a^2d)\log\left(|bx^2+a|\right)}{2\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)}+\frac{2\,abcx^2-a^2dx^2+a^2c}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")

[Out] 1/2*c^2*log(abs(d*x^2 + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/2*(2*a*b*c - a^2*d)*log(abs(b*x^2 + a))/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2) + 1/2*(2*a*b*c*x^2 - a^2*d*x^2 + a^2*c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(b*x^2 + a))

Mupad [**B**]
time = 0.26, size = 169, normalized size = 1.82

$$\frac{a^2}{2\,\left(d\,a^2\,b^2+d\,a\,b^3\,x^2-c\,a\,b^3-c\,b^4\,x^2\right)}+\frac{c^2\,\ln\left(d\,x^2+c\right)}{2\,a^2\,d^3-4\,a\,b\,c\,d^2+2\,b^2\,c^2\,d}+\frac{a^2\,d\,\ln\left(b\,x^2+a\right)}{2\,a^2\,b^2\,d^2-4\,a\,b^3\,c\,d+2\,b^4\,c^2}-\frac{2\,a\,b\,c\,\ln\left(b\,x^2+a\right)}{2\,a^2\,b^2\,d^2-4\,a\,b^3\,c\,d+2\,b^4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((a + b*x^2)^2*(c + d*x^2)),x)

[Out] a^2/(2*(a^2*b^2*d - b^4*c*x^2 - a*b^3*c + a*b^3*d*x^2)) + (c^2*log(c + d*x^2))/(2*a^2*d^3 + 2*b^2*c^2*d - 4*a*b*c*d^2) + (a^2*d*log(a + b*x^2))/(2*b^4*c^2 + 2*a^2*b^2*d^2 - 4*a*b^3*c*d) - (2*a*b*c*log(a + b*x^2))/(2*b^4*c^2 + 2*a^2*b^2*d^2 - 4*a*b^3*c*d)

3.242 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^2} dx$

Optimal. Leaf size=108

$$-\frac{cx}{2d(bc-ad)(c+dx^2)}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)^2}+\frac{\sqrt{c}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}$$

```
[Out] -1/2*c*x/d/(-a*d+b*c)/(d*x^2+c)+a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*c
)^2/b^(1/2)+1/2*(-3*a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))*c^(1/2)/d^(3/2)/(-a*
d+b*c)^2
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {481, 536, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)^2}+\frac{\sqrt{c}\,(bc-3ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}-\frac{cx}{2d\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(c*x)/(d*(b*c - a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(Sqrt[b]*(b*c - a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx &= -\frac{cx}{2d(bc-ad)(c+dx^2)}+\frac{\int \frac{ac+(bc-2ad)x^2}{(a+bx^2)(c+dx^2)}\,dx}{2d(bc-ad)}\\ &= -\frac{cx}{2d(bc-ad)(c+dx^2)}+\frac{a^2\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2}+\frac{(c(bc-3ad))\int \frac{1}{c+dx^2}\,dx}{2d(bc-ad)^2}\\ &= -\frac{cx}{2d(bc-ad)(c+dx^2)}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)^2}+\frac{\sqrt{c}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 108, normalized size = 1.00

$$\frac{cx}{2d(-bc+ad)(c+dx^2)}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(-bc+ad)^2}+\frac{\sqrt{c}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (c*x)/(2*d*(-(b*c) + a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(Sqrt[b]*(-(b*c) + a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

Maple [A]
time = 0.16, size = 95, normalized size = 0.88

method	result
default	$\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^2\sqrt{ab}}-\frac{c\left(-\frac{(ad-bc)x}{2d(dx^2+c)}+\frac{(3ad-bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2d\sqrt{cd}}\right)}{(ad-bc)^2}$
risch	$\frac{cx}{2d(ad-bc)(dx^2+c)}+\frac{3\sqrt{-cd}\,\ln\left(\left(-9(-cd)^{\frac{3}{2}}a^3bd^3-3(-cd)^{\frac{3}{2}}a^2b^2cd^2+5(-cd)^{\frac{3}{2}}ab^3c^2d-(-cd)^{\frac{3}{2}}b^4c^3-4a^4\sqrt{-cd}\,d^5-9\sqrt{\ldots}\right.\right.}{4d(ad-\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c/(a*d-b*c)^2*(-1/2*(a*
d-b*c)/d*x/(d*x^2+c)+1/2*(3*a*d-b*c)/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.52, size = 132, normalized size = 1.22

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}} - \frac{cx}{2\,(bc^2d - acd^2 + (bcd^2 - ad^3)x^2)} + \frac{(bc^2 - 3\,acd)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2d - 2\,abcd^2 + a^2d^3)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*c*x/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2) + 1/2*(b*c^2 - 3*a*c*d)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(c*d))
```

Fricas [A]
time = 1.18, size = 718, normalized size = 6.65

$$\left[\frac{2\,(ad^2x^2 + acd)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - (bc^2 - 3\,acd + (bcd - 3\,ad^2)x^2)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2 - 2\,dx\sqrt{-\frac{c}{d}} - c}{dx^2 + c}\right) - 2\,(bc^2 - acd)x}{4\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^2)}, \frac{4\,(ad^2x^2 + acd)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - (bc^2 - 3\,acd + (bcd - 3\,ad^2)x^2)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2 - 2\,dx\sqrt{-\frac{c}{d}} - c}{dx^2 + c}\right) - 2\,(bc^2 - acd)x}{4\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^2)}, \frac{(bc^2 - 3\,acd + (bcd - 3\,ad^2)x^2)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right) + (ad^2x^2 + acd)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - (bc^2 - acd)x}{2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^2)}, \frac{2\,(ad^2x^2 + acd)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + (bc^2 - 3\,acd + (bcd - 3\,ad^2)x^2)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right) - (bc^2 - acd)x}{2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*d^2*x^2 + a*c*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)) - (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(-c/d)*log((d*x
^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(b*c^2 - a*c*d)*x)/(b^2*c^3*d -
 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/
4*(4*(a*d^2*x^2 + a*c*d)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - (b*c^2 - 3*a*c
*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(
d*x^2 + c)) - 2*(b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 +
 (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/2*((b*c^2 - 3*a*c*d + (b*c*d
 - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + (a*d^2*x^2 + a*c*d)*sq
rt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - (b*c^2 - a*c*d)*
x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*x^2), 1/2*(2*(a*d^2*x^2 + a*c*d)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
 (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c
) - (b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.17, size = 121, normalized size = 1.12

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}} + \frac{(bc^2 - 3\,acd) \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2d - 2\,abcd^2 + a^2d^3)\sqrt{cd}} - \frac{cx}{2\,(bcd - ad^2)(dx^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1/2
*(b*c^2 - 3*a*c*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)*sqrt(c*d)) - 1/2*c*x/((b*c*d - a*d^2)*(d*x^2 + c))
```

Mupad [**B**]
time = 0.57, size = 2500, normalized size = 23.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (c*x)/(2*d*(c + d*x^2)*(a*d - b*c)) - (atan(((((x*(b^5*c^4 + 4*a^4*b*d^4 +
9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))
 - ((-c*d^3)^(1/2)*(3*a*d - b*c)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^
2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 - b^3*c^3*
d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (x*(-c*d^3)^(1/2)*(3*a*d - b*c)*(16*
a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2
*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4
)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*
c*d^4)))*(-c*d^3)^(1/2)*(3*a*d - b*c)*1i)/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b
*c*d^4)) + (((x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d)
)/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) + ((-c*d^3)^(1/2)*(3*a*d - b*c)*(
(2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4
 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^
```

3) + (x*(-c*d^3)^(1/2)*(3*a*d - b*c)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a*d - b*c)*1i)/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)))/(((a^2*b^3*c^3)/2 - (5*a^3*b^2*c^2*d)/2 + 3*a^4*b*c*d^2)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (((x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) - ((-c*d^3)^(1/2)*(3*a*d - b*c)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (x*(-c*d^3)^(1/2)*(3*a*d - b*c)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a*d - b*c))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)) + (((x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) + ((-c*d^3)^(1/2)*(3*a*d - b*c)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x*(-c*d^3)^(1/2)*(3*a*d - b*c)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a*d - b*c))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4))))*(-c*d^3)^(1/2)*(3*a*d - b*c)*1i)/(2*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)) - (atan(-(((((-a^3*b)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) - (x*(-a^3*b)^(1/2)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) - (x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(4*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)))*(-a^3*b)^(1/2)*1i)/(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d) - ((((-a^3*b)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) + (x*(-a^3*b)^(1/2)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) + (x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(4*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)))*(-a^3*b)^(1/2)*1i)/(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d))/(((a^2*b^3*c^3)/2 - (5*a^3*b^2*c^2*d)/2 + 3*a^4*b*c*d^2)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + ((((-a^3*b)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) - (x*(-a^3*b)^(1/2)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4

```
- 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(b^3*c^2
+ a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(b^3*c
^2 + a^2*b*d^2 - 2*a*b^2*c*d)) - (x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*
d^2 - 6*a*b^4*c^3*d))/(4*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)))*(-a^3*b)^(1/
2))/(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d) + ((((-a^3*b)^(1/2)*((2*a*b^6*c^5*d
^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c
^2*d^5)/(2*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) + (x*(-
a^3*b)^(1/2)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b
^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*...
```

3.243 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{c}{2d(bc-ad)(c+dx^2)}-\frac{a\log(a+bx^2)}{2(bc-ad)^2}+\frac{a\log(c+dx^2)}{2(bc-ad)^2}$$

```
[Out] -1/2*c/d/(-a*d+b*c)/(d*x^2+c)-1/2*a*ln(b*x^2+a)/(-a*d+b*c)^2+1/2*a*ln(d*x^2
+c)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{c}{2d(c+dx^2)(bc-ad)}-\frac{a\log(a+bx^2)}{2(bc-ad)^2}+\frac{a\log(c+dx^2)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*c/(d*(b*c - a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a*d)^2) +
 (a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{ab}{(bc-ad)^2(a+bx)} + \frac{c}{(bc-ad)(c+dx)^2} + \frac{ad}{(-bc+ad)^2(c+d\ldots}\right.\right.$$
$$= -\frac{c}{2d(bc-ad)(c+dx^2)} - \frac{a\log(a+bx^2)}{2(bc-ad)^2} + \frac{a\log(c+dx^2)}{2(bc-ad)^2}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 1.00

$$\frac{c}{2d(-bc+ad)(c+dx^2)} - \frac{a\log(a+bx^2)}{2(bc-ad)^2} + \frac{a\log(c+dx^2)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] c/(2*d*(-(b*c) + a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a*d)^2) +
 (a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Maple [A]
time = 0.12, size = 68, normalized size = 0.92

method	result	size
default	$-\frac{a\ln(b\,x^2+a)}{2(ad-bc)^2}+\frac{\frac{c(ad-bc)}{d(d\,x^2+c)}+a\ln(d\,x^2+c)}{2(ad-bc)^2}$	68
norman	$-\frac{x^2}{2(ad-bc)(d\,x^2+c)}-\frac{a\ln(b\,x^2+a)}{2(a^2d^2-2abcd+b^2c^2)}+\frac{a\ln(d\,x^2+c)}{2a^2d^2-4abcd+2b^2c^2}$	94
risch	$\frac{c}{2d(ad-bc)(d\,x^2+c)}-\frac{a\ln(-b\,x^2-a)}{2(a^2d^2-2abcd+b^2c^2)}+\frac{a\ln(d\,x^2+c)}{2a^2d^2-4abcd+2b^2c^2}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(a*d-b*c)^2*ln(b*x^2+a)+1/2/(a*d-b*c)^2*(c*(a*d-b*c)/d/(d*x^2+c)+a*l
n(d*x^2+c))
```

Maxima [A]
time = 0.32, size = 105, normalized size = 1.42

$$-\frac{a\log(bx^2+a)}{2(b^2c^2-2\,abcd+a^2d^2)} + \frac{a\log(dx^2+c)}{2(b^2c^2-2\,abcd+a^2d^2)} - \frac{c}{2(bc^2d-acd^2+(bcd^2-ad^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a*log(d*x^2 + c
)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a
*d^3)*x^2)
```

Fricas [A]
time = 1.60, size = 117, normalized size = 1.58

$$-\frac{bc^2 - acd + (ad^2x^2 + acd)\log(bx^2 + a) - (ad^2x^2 + acd)\log(dx^2 + c)}{2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c^2 - a*c*d + (a*d^2*x^2 + a*c*d)*log(b*x^2 + a) - (a*d^2*x^2 + a*c
*d)*log(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(58) = 116$.
time = 1.34, size = 253, normalized size = 3.42

$$\frac{a\log\left(x^2 + \frac{-\frac{a^4d^3}{(ad-bc)^2} + \frac{3a^3bcd^2}{(ad-bc)^2} - \frac{3a^2b^2c^2d}{(ad-bc)^2} + a^2d + \frac{ab^3c^3}{(ad-bc)^2} + abc}{2abd}\right)}{2\,(ad-bc)^2} - \frac{a\log\left(x^2 + \frac{\frac{a^4d^3}{(ad-bc)^2} - \frac{3a^3bcd^2}{(ad-bc)^2} + \frac{3a^2b^2c^2d}{(ad-bc)^2} + a^2d - \frac{ab^3c^3}{(ad-bc)^2} + abc}{2abd}\right)}{2\,(ad-bc)^2} + \frac{c}{2acd^2 - 2bc^2d + x^2 \cdot (2ad^3 - 2bcd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] a*log(x**2 + (-a**4*d**3/(a*d - b*c)**2 + 3*a**3*b*c*d**2/(a*d - b*c)**2 -
3*a**2*b**2*c**2*d/(a*d - b*c)**2 + a**2*d + a*b**3*c**3/(a*d - b*c)**2 + a
*b*c)/(2*a*b*d))/(2*(a*d - b*c)**2) - a*log(x**2 + (a**4*d**3/(a*d - b*c)**
2 - 3*a**3*b*c*d**2/(a*d - b*c)**2 + 3*a**2*b**2*c**2*d/(a*d - b*c)**2 + a*
*2*d - a*b**3*c**3/(a*d - b*c)**2 + a*b*c)/(2*a*b*d))/(2*(a*d - b*c)**2) +
c/(2*a*c*d**2 - 2*b*c**2*d + x**2*(2*a*d**3 - 2*b*c*d**2))
```

Giac [A]
time = 1.57, size = 91, normalized size = 1.23

$$-\frac{\frac{ad^2\log\left(\left|b - \frac{bc}{dx^2+c} + \frac{ad}{dx^2+c}\right|\right)}{b^2c^2d - 2\,abcd^2 + a^2d^3} + \frac{cd}{(bcd - ad^2)(dx^2+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*log(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3) + c*d/((b*c*d - a*d^2)*(d*x^2 + c)))/d
```

Mupad [B]
time = 0.16, size = 173, normalized size = 2.34

$$-\frac{b\,c^2-c\,\left(a\,d-a\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}\right)+a\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}}{2\,a^2\,c\,d^3+2\,a^2\,d^4\,x^2-4\,a\,b\,c^2\,d^2-4\,a\,b\,c\,d^3\,x^2+2\,b^2\,c^3\,d+2\,b^2\,c^2\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] -(b*c^2 - c*(a*d - a*d*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*
c*x^2))*2i) + a*d^2*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b
*c*x^2))*2i)/(2*a^2*c*d^3 + 2*b^2*c^3*d + 2*a^2*d^4*x^2 + 2*b^2*c^2*d^2*x^2
 - 4*a*b*c^2*d^2 - 4*a*b*c*d^3*x^2)
```

3.244 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=104

$$\frac{x}{2(bc-ad)(c+dx^2)}-\frac{\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^2}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}\,(bc-ad)^2}$$

```
[Out] 1/2*x/(-a*d+b*c)/(d*x^2+c)-arctan(x*b^(1/2)/a^(1/2))*a^(1/2)*b^(1/2)/(-a*d+
b*c)^2+1/2*(a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^2/c^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {482, 536, 211}

$$-\frac{\sqrt{a}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^2}+\frac{(ad+bc)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}\,(bc-ad)^2}+\frac{x}{2\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] x/(2*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(b*c - a*d)^2 + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*Sqr
t[d]*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx &= \frac{x}{2(bc-ad)(c+dx^2)} - \frac{\int \frac{a-bx^2}{(a+bx^2)(c+dx^2)}\,dx}{2(bc-ad)} \\
&= \frac{x}{2(bc-ad)(c+dx^2)} - \frac{(ab)\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2} + \frac{(bc+ad)\int \frac{1}{c+dx^2}\,dx}{2(bc-ad)^2} \\
&= \frac{x}{2(bc-ad)(c+dx^2)} - \frac{\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^2} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}\,(bc-ad)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 0.87

$$
\frac{\frac{(bc-ad)x}{c+dx^2} - 2\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{d}}}{2(bc-ad)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (((b*c - a*d)*x)/(c + d*x^2) - 2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
] + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(2*(b*c -
a*d)^2)
```

Maple [A]
time = 0.16, size = 85, normalized size = 0.82

method	result
default	$-\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^2\sqrt{ab}}+\frac{\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{d\,x^2+c}+\frac{(ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}}{(ad-bc)^2}$
risch	$-\frac{x}{2(ad-bc)(d\,x^2+c)}+\frac{\sqrt{-ab}\;\ln\left(\left(-4(-ab)^{\frac{3}{2}}a\,d^2-4(-ab)^{\frac{3}{2}}bcd-5a^2\sqrt{-ab}\;d^2b-2\sqrt{-ab}\;a\,b^2cd-b^3c^2\sqrt{-ab}\right)x-a^3b\right.}{2(ad-bc)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*b/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/(a*d-b*c)^2*((-1/2*a
*d+1/2*b*c)*x/(d*x^2+c)+1/2*(a*d+b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.55, size = 119, normalized size = 1.14

$$-\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}}+\frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}}+\frac{x}{2\,(bc^2-acd+(bcd-ad^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -a*b*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1/
2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c
*d)) + 1/2*x/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

Fricas [A]
time = 1.55, size = 705, normalized size = 6.78

$$\left[\frac{2\,(cd^2x^2+c^2d)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-(bc^2+acd+(bcd+ad^2)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\,(bc^2d-acd^2)x}{4\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2)},\ \frac{(bc^2+acd+(bcd+ad^2)x^2)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+(cd^2x^2+c^2d)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+(bc^2d-acd^2)x}{2\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2)},\ -\frac{4\,(cd^2x^2+c^2d)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(bc^2+acd+(bcd+ad^2)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)-2\,(bc^2d-acd^2)x}{4\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2)},\ -\frac{2\,(cd^2x^2+c^2d)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-(bc^2+acd+(bcd+ad^2)x^2)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-(bc^2d-acd^2)x}{2\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(c*d^2*x^2 + c^2*d)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)) - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2
*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a
*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2),
1/2*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c)
+ (c*d^2*x^2 + c^2*d)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
a)) + (b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^
2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2), -1/4*(4*(c*d^2*x^2 + c^2*d)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) + (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(
-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b*c^2*d - a*c*d^2)
*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3
 + a^2*c*d^4)*x^2), -1/2*(2*(c*d^2*x^2 + c^2*d)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a) - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
) - (b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*
c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.64, size = 110, normalized size = 1.06

$$-\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}}+\frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}}+\frac{x}{2\,(dx^2+c)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -a*b*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1/
2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c
*d)) + 1/2*x/((d*x^2 + c)*(b*c - a*d))
```

Mupad [B]
time = 0.45, size = 3154, normalized size = 30.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (atan(-(((-a*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*
a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 + 16
*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 3
2*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d)) - (x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(4*(
a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - ((-a
*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^
5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)) + (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2
 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^
2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)) + (x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(4*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(((a^2*b^3*d^2)/2
 + (a*b^4*c*d)/2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((-
a*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d
^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^
```

2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + ((-a*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(-a*b)^(1/2)*1i)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - x/(2*(c + d*x^2)*(a*d - b*c)) + (atan((((-c*d)^(1/2)*((x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - ((-c*d)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - (x*(-c*d)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c)*1i)/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)) + ((-c*d)^(1/2)*((x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + ((-c*d)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (x*(-c*d)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c)*1i)/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))/(((a^2*b^3*d^2)/2 + (a*b^4*c*d)/2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - ((-c*d)^(1/2)*((x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - ((-c*d)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - (x*(-c*d)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)) + ((-c*d)^(1/2)*((x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + ((-c*d)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (x*(-c*d)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2))))

```
*(-c*d)^(1/2)*(a*d + b*c)*1i)/(2*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2))
```

3.245 $\int \frac{x}{(a+bx^2)(c+dx^2)^2} \, dx$

Optimal. Leaf size=70

$$\frac{1}{2(bc-ad)(c+dx^2)} + \frac{b\log(a+bx^2)}{2(bc-ad)^2} - \frac{b\log(c+dx^2)}{2(bc-ad)^2}$$

```
[Out] 1/2/(-a*d+b*c)/(d*x^2+c)+1/2*b*ln(b*x^2+a)/(-a*d+b*c)^2-1/2*b*ln(d*x^2+c)/(
-a*d+b*c)^2
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 46}

$$\frac{1}{2(c+dx^2)(bc-ad)} + \frac{b\log(a+bx^2)}{2(bc-ad)^2} - \frac{b\log(c+dx^2)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x^2)) + (b*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (b*L
og[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^2)(c+dx^2)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2}{(bc-ad)^2(a+bx)} - \frac{d}{(bc-ad)(c+dx)^2} - \frac{bd}{(bc-ad)^2(c+dx)}\right) \right. \\
&= \frac{1}{2(bc-ad)(c+dx^2)} + \frac{b\log(a+bx^2)}{2(bc-ad)^2} - \frac{b\log(c+dx^2)}{2(bc-ad)^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.94

$$\frac{bc-ad+b(c+dx^2)\log(a+bx^2)-b(c+dx^2)\log(c+dx^2)}{2(bc-ad)^2(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^2),x]
[Out] (b*c - a*d + b*(c + d*x^2)*Log[a + b*x^2] - b*(c + d*x^2)*Log[c + d*x^2])/(
2*(b*c - a*d)^2*(c + d*x^2))
```

Maple [A]
time = 0.12, size = 73, normalized size = 1.04

method	result	size
default	$\frac{b\ln(bx^2+a)}{2(ad-bc)^2}+\frac{d\left(-\frac{ad-bc}{d(dx^2+c)}-\frac{b\ln(dx^2+c)}{d}\right)}{2(ad-bc)^2}$	73
risch	$-\frac{1}{2(ad-bc)(dx^2+c)}+\frac{b\ln(bx^2+a)}{2a^2d^2-4abcd+2b^2c^2}-\frac{b\ln(-dx^2-c)}{2(a^2d^2-2abcd+b^2c^2)}$	94
norman	$\frac{dx^2}{2c(ad-bc)(dx^2+c)}+\frac{b\ln(bx^2+a)}{2a^2d^2-4abcd+2b^2c^2}-\frac{b\ln(dx^2+c)}{2(a^2d^2-2abcd+b^2c^2)}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*b/(a*d-b*c)^2*ln(b*x^2+a)+1/2*d/(a*d-b*c)^2*(-(a*d-b*c)/d/(d*x^2+c)-b/d
*ln(d*x^2+c))
```

Maxima [A]
time = 0.28, size = 99, normalized size = 1.41

$$\frac{b\log(bx^2+a)}{2(b^2c^2-2abcd+a^2d^2)}-\frac{b\log(dx^2+c)}{2(b^2c^2-2abcd+a^2d^2)}+\frac{1}{2(bc^2-acd+(bcd-ad^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*b*log(d*x^2 + c)
/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2
)
```

Fricas [A]
time = 1.00, size = 103, normalized size = 1.47

$$\frac{bc-ad+(bdx^2+bc)\log(bx^2+a)-(bdx^2+bc)\log(dx^2+c)}{2(b^2c^3-2abc^2d+a^2cd^2+(b^2c^2d-2abcd^2+a^2d^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c - a*d + (b*d*x^2 + b*c)*log(b*x^2 + a) - (b*d*x^2 + b*c)*log(d*x^2
 + c))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(56) = 112$.
time = 1.10, size = 248, normalized size = 3.54

$$-\frac{b\log\left(x^2+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{2\left(ad-bc\right)^2}+\frac{b\log\left(x^2+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{2\left(ad-bc\right)^2}-\frac{1}{2acd-2bc^2+x^2\cdot(2ad^2-2bcd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] -b*log(x**2 + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)
**2 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b
**2*c)/(2*b**2*d))/(2*(a*d - b*c)**2) + b*log(x**2 + (a**3*b*d**3/(a*d - b*
c)**2 - 3*a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2
+ a*b*d - b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(2*(a*d - b*c)**2)
 - 1/(2*a*c*d - 2*b*c**2 + x**2*(2*a*d**2 - 2*b*c*d))
```

Giac [A]
time = 1.25, size = 85, normalized size = 1.21

$$\frac{bd\log\left(\left|b-\frac{bc}{dx^2+c}+\frac{ad}{dx^2+c}\right|\right)}{2\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}+\frac{d}{2\left(bcd-ad^2\right)\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*d*log(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3) + 1/2*d/((b*c*d - a*d^2)*(d*x^2 + c))
```

Mupad [B]
time = 0.15, size = 160, normalized size = 2.29

$$\frac{-a\,d+c\,\left(b+b\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}\right)+b\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}}{2\,a^2\,c\,d^2+2\,a^2\,d^3\,x^2-4\,a\,b\,c^2\,d-4\,a\,b\,c\,d^2\,x^2+2\,b^2\,c^3+2\,b^2\,c^2\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (c*(b + b*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i) -
 a*d + b*d*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*
2i)/(2*b^2*c^3 + 2*a^2*c*d^2 + 2*a^2*d^3*x^2 + 2*b^2*c^2*d*x^2 - 4*a*b*c^2*
d - 4*a*b*c*d^2*x^2)
```

3.246 $\int \frac{1}{(a+bx^2)(c+dx^2)^2} \, dx$

Optimal. Leaf size=109

$$-\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}$$

```
[Out] -1/2*d*x/c/(-a*d+b*c)/(d*x^2+c)+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*c
)^2/a^(1/2)-1/2*(-a*d+3*b*c)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(3/2)/(-a*
d+b*c)^2
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {425, 536, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x)/(c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx &= -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{\int \frac{2bc-ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx}{2c(bc-ad)}\\ &= -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2}-\frac{(d(3bc-ad))\int \frac{1}{c+dx^2}\,dx}{2c(bc-ad)^2}\\ &= -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 95, normalized size = 0.87

$$\frac{\frac{d(-bc+ad)x}{c(c+dx^2)}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}\,(-3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2)
)/(2*(b*c - a*d)^2)
```

Maple [A]
time = 0.07, size = 93, normalized size = 0.85

method	result
default	$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^2\sqrt{ab}}+\frac{d\left(\frac{(ad-bc)x}{2c\left(d\,x^2+c\right)}+\frac{(ad-3bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}\right)}{(ad-bc)^2}$
risch	$\frac{dx}{2c(ad-bc)(d\,x^2+c)}+\frac{\sqrt{-ab}\,b\ln\left(\left(-4(-ab)^{\frac{3}{2}}ab\,c^2d-4(-ab)^{\frac{3}{2}}b^2c^3-\sqrt{-ab}\,a^4d^3+6\sqrt{-ab}\,a^3bc\,d^2-13\sqrt{-ab}\,a^2b^2c^2d\right)\right.}{2a(ad-bc)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^2*(1/2*(a*d
-b*c)/c*x/(d*x^2+c)+1/2*(a*d-3*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.56, size = 133, normalized size = 1.22

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}} - \frac{dx}{2\,(bc^3 - ac^2d + (bc^2d - acd^2)x^2)} - \frac{(3\,bcd - ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3 - 2\,abc^2d + a^2cd^2)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) - 1/2*(3*b*c*d - a*d^2)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
```

Fricas [A]
time = 1.38, size = 711, normalized size = 6.52

$$\left[\frac{2\,(bcdx^2 + bc^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 + 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) - (3\,bc^2 - acd + (3\,bcd - ad^2)x^2)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2 + 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^2 + c}\right) - 2\,(bcd - ad^2)x}{4\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}, -\frac{(3\,bc^2 - acd + (3\,bcd - ad^2)x^2)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right) - (bcdx^2 + bc^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 + 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) + (bcd - ad^2)x}{2\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}, \frac{4\,(bcdx^2 + bc^2)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) - (3\,bc^2 - acd + (3\,bcd - ad^2)x^2)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2 + 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^2 + c}\right) - 2\,(bcd - ad^2)x}{4\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}, \frac{2\,(bcdx^2 + bc^2)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) - (3\,bc^2 - acd + (3\,bcd - ad^2)x^2)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right) - (bcd - ad^2)x}{2\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(
b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d*x
^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), -1
/2*((3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
 - (b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3
*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*
arctan(x*sqrt(b/a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*
log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2
*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*
x^2), 1/2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - (3*b*c^2 -
 a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b*c*d - a*
d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 +
 a^2*c*d^3)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.14, size = 122, normalized size = 1.12

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}} - \frac{(3\,bcd - ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3 - 2\,abc^2d + a^2cd^2)\sqrt{cd}} - \frac{dx}{2\,(bc^2 - acd)(dx^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x^2 + c))
```

Mupad [B]
time = 0.55, size = 2500, normalized size = 22.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (d*x)/(2*c*(c + d*x^2)*(a*d - b*c)) - (atan((((-c^3*d)^(1/2)*(a*d - 3*b*c)*
((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7
+ 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 -
a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3
*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*
c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a
*d - 3*b*c))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a
^2*c^3*d^2 - 2*a*b*c^4*d)) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5
 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)) + (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4
*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*
a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^
7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4
*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*
(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a
```

*b*c^4*d)))/(((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5 - a^3*c^2*d^3 + 3*a ^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3* d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^ 3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5* c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7 *c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48* a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^ 3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/ (4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a *b*c^4*d)) - ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^ 3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)) + (((4*b^7*c^ 6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^ 4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6 *c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 1 6*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2 *c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*(b^2*c^5 + a^2*c ^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))*(-c^3* d)^(1/2)*(a*d - 3*b*c)*1i)/(2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) - (ata n((((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (x*(-a*b^3)^(1/2)*(16*b ^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 4 8*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b* c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^ 4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a*b^2*c^2 - 2 *a^2*b*c*d) - ((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5* b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^ (1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4* c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d ^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2* (a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6 *a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a* b^2*c^2 - 2*a^2*b*c*d))/(((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d ^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3 *c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (x*(-a*b^3)^(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3) ^(1/2))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5* c^2*d^3 - 6*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))))/(a^3* d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - ((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5

```
 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-a*b^3)^(1/2)*((((4*b
^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a
^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^
3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^(1/2)*(16...
```

3.247 $\int \frac{1}{x(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{d}{2c(bc-ad)(c+dx^2)}+\frac{\log(x)}{ac^2}-\frac{b^2\log(a+bx^2)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log(c+dx^2)}{2c^2(bc-ad)^2}$$

```
[Out] -1/2*d/c/(-a*d+b*c)/(d*x^2+c)+ln(x)/a/c^2-1/2*b^2*ln(b*x^2+a)/a/(-a*d+b*c)^
2+1/2*d*(-a*d+2*b*c)*ln(d*x^2+c)/c^2/(-a*d+b*c)^2
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 84}

$$-\frac{b^2\log(a+bx^2)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log(c+dx^2)}{2c^2(bc-ad)^2}-\frac{d}{2c(c+dx^2)(bc-ad)}+\frac{\log(x)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*d/(c*(b*c - a*d)*(c + d*x^2)) + Log[x]/(a*c^2) - (b^2*Log[a + b*x^2])/
(2*a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2
)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx^2)(c+dx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ac^2x}-\frac{b^3}{a(-bc+ad)^2(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)^2}+\frac{d^2}{c^2(bc-\right.\right.\\ &= -\frac{d}{2c(bc-ad)(c+dx^2)}+\frac{\log(x)}{ac^2}-\frac{b^2\log(a+bx^2)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log(c+d}{2c^2(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 98, normalized size = 0.98

$$\frac{1}{2}\left(-\frac{d}{c(bc-ad)(c+dx^2)}+\frac{2\log(x)}{ac^2}-\frac{b^2\log(a+bx^2)}{a(bc-ad)^2}+\frac{d(2bc-ad)\log(c+dx^2)}{c^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-(d/(c*(b*c - a*d)*(c + d*x^2))) + (2*Log[x])/(a*c^2) - (b^2*Log[a + b*x^2
])/(a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^2)
)/2
```

Maple [A]
time = 0.14, size = 99, normalized size = 0.99

method	result	size
default	$-\frac{b^2\ln(b\,x^2+a)}{2a(ad-bc)^2}-\frac{d^2\left(-\frac{c(ad-bc)}{d(d\,x^2+c)}+\frac{(ad-2bc)\ln(d\,x^2+c)}{d}\right)}{2c^2(ad-bc)^2}+\frac{\ln(x)}{c^2a}$	99
norman	$-\frac{d^2x^2}{2c^2(ad-bc)(d\,x^2+c)}+\frac{\ln(x)}{c^2a}-\frac{b^2\ln(b\,x^2+a)}{2a(a^2d^2-2abcd+b^2c^2)}-\frac{d(ad-2bc)\ln(d\,x^2+c)}{2c^2(a^2d^2-2abcd+b^2c^2)}$	125
risch	$\frac{d}{2c(ad-bc)(d\,x^2+c)}+\frac{\ln(x)}{c^2a}-\frac{b^2\ln(b\,x^2+a)}{2a(a^2d^2-2abcd+b^2c^2)}-\frac{d^2\ln(-d\,x^2-c)a}{2c^2(a^2d^2-2abcd+b^2c^2)}+\frac{d\ln(-d\,x^2-c)b}{c(a^2d^2-2abcd+b^2c^2)}$	158

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2/a/(a*d-b*c)^2*ln(b*x^2+a)-1/2*d^2/c^2/(a*d-b*c)^2*(-c*(a*d-b*c)/d/
(d*x^2+c)+(a*d-2*b*c)/d*ln(d*x^2+c))+ln(x)/c^2/a
```

Maxima [A]
time = 0.28, size = 138, normalized size = 1.38

$$-\frac{b^2\log(bx^2+a)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)}+\frac{(2\,bcd-ad^2)\log(dx^2+c)}{2\,(b^2c^4-2\,abc^3d+a^2c^2d^2)}-\frac{d}{2\,(bc^3-ac^2d+(bc^2d-acd^2)x^2)}+\frac{\log(x^2)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) + 1/2*(2*b*c*d
- a*d^2)*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/2*d/(b*c^
3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) + 1/2*log(x^2)/(a*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(94) = 188.
time = 1.99, size = 219, normalized size = 2.19

$$-\frac{abc^2d-a^2cd^2+(b^2c^2dx^2+b^2c^3)\log(bx^2+a)-(2\,abc^2d-a^2cd^2+(2\,abcd^2-a^2d^3)x^2)\log(dx^2+c)-2\,(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x^2)\log(x)}{2\,(ab^2c^5-2\,a^2bc^4d+a^3c^3d^2+(ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b*c^2*d - a^2*c*d^2 + (b^2*c^2*d*x^2 + b^2*c^3)*log(b*x^2 + a) - (2
*a*b*c^2*d - a^2*c*d^2 + (2*a*b*c*d^2 - a^2*d^3)*x^2)*log(d*x^2 + c) - 2*(b
^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)
*log(x))/(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2 + (a*b^2*c^4*d - 2*a^2*b*
c^3*d^2 + a^3*c^2*d^3)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.34, size = 185, normalized size = 1.85

$$-\frac{b^3\log\left(|bx^2+a|\right)}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)}+\frac{\left(2\,bcd^2-ad^3\right)\log\left(|dx^2+c|\right)}{2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)}-\frac{2\,bcd^2x^2-ad^3x^2+3\,bc^2d-2\,acd^2}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\left(dx^2+c\right)}+\frac{\log\left(x^2\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(abs(b*x^2 + a))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2) + 1/2*
(2*b*c*d^2 - a*d^3)*log(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3) - 1/2*(2*b*c*d^2*x^2 - a*d^3*x^2 + 3*b*c^2*d - 2*a*c*d^2)/((b^2*c^4
- 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)) + 1/2*log(x^2)/(a*c^2)
```

Mupad [**B**]
time = 0.40, size = 127, normalized size = 1.27

$$\frac{\ln\left(x\right)}{a\,c^2}-\frac{\ln\left(d\,x^2+c\right)\,\left(a\,d^2-2\,b\,c\,d\right)}{2\,a^2\,c^2\,d^2-4\,a\,b\,c^3\,d+2\,b^2\,c^4}-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,a^3\,d^2-4\,a^2\,b\,c\,d+2\,a\,b^2\,c^2}+\frac{d}{2\,c\,\left(d\,x^2+c\right)\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] log(x)/(a*c^2) - (log(c + d*x^2)*(a*d^2 - 2*b*c*d))/(2*b^2*c^4 + 2*a^2*c^2*
d^2 - 4*a*b*c^3*d) - (b^2*log(a + b*x^2))/(2*a^3*d^2 + 2*a*b^2*c^2 - 4*a^2*
b*c*d) + d/(2*c*(c + d*x^2)*(a*d - b*c))
```

3.248 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2} dx$

Optimal. Leaf size=144

$$-\frac{2bc-3ad}{2ac^2(bc-ad)x}-\frac{d}{2c(bc-ad)x\,(c+dx^2)}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}$$

```
[Out] 1/2*(3*a*d-2*b*c)/a/c^2/(-a*d+b*c)/x-1/2*d/c/(-a*d+b*c)/x/(d*x^2+c)-b^(5/2)
*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)^2+1/2*d^(3/2)*(-3*a*d+5*b*c)*
arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 597, 536, 211}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5bc-3ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}-\frac{2bc-3ad}{2ac^2x(bc-ad)}-\frac{d}{2cx\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(2*b*c - 3*a*d)/(a*c^2*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x*(c + d*x^
2)) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^2) + (d^(3
/2)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2}\,dx &= -\frac{d}{2c(bc-ad)x(c+dx^2)}+\frac{\int \frac{2bc-3ad-3bdx^2}{x^2(a+bx^2)(c+dx^2)}\,dx}{2c(bc-ad)}\\ &= -\frac{2bc-3ad}{2ac^2(bc-ad)x}-\frac{d}{2c(bc-ad)x(c+dx^2)}-\frac{\int \frac{2b^2c^2+2abcd-3a^2d^2+bd(2bc-3ad)x}{(a+bx^2)(c+dx^2)}}{2ac^2(bc-ad)}\\ &= -\frac{2bc-3ad}{2ac^2(bc-ad)x}-\frac{d}{2c(bc-ad)x(c+dx^2)}-\frac{b^3\int \frac{1}{a+bx^2}\,dx}{a(bc-ad)^2}+\frac{(d^2(5bc-3ac}{2c^2(bc}\\ &= -\frac{2bc-3ad}{2ac^2(bc-ad)x}-\frac{d}{2c(bc-ad)x(c+dx^2)}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5b}{}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 123, normalized size = 0.85

$$-\frac{1}{ac^2x}+\frac{d^2x}{2c^2(bc-ad)(c+dx^2)}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(-bc+ad)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(1/(a*c^2*x)) + (d^2*x)/(2*c^2*(b*c - a*d)*(c + d*x^2)) - (b^(5/2)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*(5*b*c - 3*a*d)
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*d)^2)
```

Maple [A]
time = 0.19, size = 109, normalized size = 0.76

method	result
default	$-\frac{b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{a (a d-b c)^{2} \sqrt{a b}}-\frac{d^{2}\left(\frac{\left(\frac{a d}{2}-\frac{b c}{2}\right) x}{d x^{2}+c}+\frac{(3 a d-5 b c) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \sqrt{c d}}\right)}{c^{2} (a d-b c)^{2}}-\frac{1}{c^{2} a x}$
risch	$\frac{-\frac{d(3ad-2bc)x^{2}}{2c^{2}a(ad-bc)}-\frac{1}{ac}}{x(d x^{2}+c)}+\frac{3\sqrt{-cd}\, d^{2} \ln\left(\left(9a^{4}c d^{6}-21a^{3}b c^{2}d^{5}+4a^{2}b^{2}c^{3}d^{4}+4a b^{3}c^{4}d^{3}+4b^{4}c^{5}d^{2}\right)x+9(-cd)^{\frac{3}{2}}a^{4}d^{4}-21(-cd)^{\frac{3}{2}}a^{3}bcd^{3}-\right.}{4c^{3}(ad-bc)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*b^3/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-d^2/c^2/(a*d-b*c)^
2*((1/2*a*d-1/2*b*c)*x/(d*x^2+c)+1/2*(3*a*d-5*b*c)/(c*d)^(1/2)*arctan(d*x/(
c*d)^(1/2)))-1/c^2/a/x
```

Maxima [A]
time = 0.54, size = 178, normalized size = 1.24

$$-\frac{b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(a b^{2} c^{2}-2 \, a^{2} b c d+a^{3} d^{2}\right) \sqrt{a b}}+\frac{\left(5 \, b c d^{2}-3 \, a d^{3}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \, \left(b^{2} c^{4}-2 \, a b c^{3} d+a^{2} c^{2} d^{2}\right) \sqrt{c d}}-\frac{2 \, b c^{2}-2 \, a c d+\left(2 \, b c d-3 \, a d^{2}\right) x^{2}}{2 \, \left(\left(a b c^{3} d-a^{2} c^{2} d^{2}\right) x^{3}+\left(a b c^{4}-a^{2} c^{3} d\right) x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
+ 1/2*(5*b*c*d^2 - 3*a*d^3)*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2)*sqrt(c*d)) - 1/2*(2*b*c^2 - 2*a*c*d + (2*b*c*d - 3*a*d^2)*x^2
)/((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)
```

Fricas [A]
time = 1.43, size = 1005, normalized size = 6.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*b^2*c^3 - 8*a*b*c^2*d + 4*a^2*c*d^2 + 2*(2*b^2*c^2*d - 5*a*b*c*d^2
 + 3*a^2*d^3)*x^2 - 2*(b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-b/a)*log((b*x^2 - 2
*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (5*a*b
*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x
^2 + c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 -
```

```
 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/2*(2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d
^2 + (2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 - ((5*a*b*c*d^2 - 3*a^2*d^
3)*x^3 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b^
2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 -
 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/4*(4*b^2*c^3 - 8*a*b*c^2*d + 4*a^2*c*d
^2 + 2*(2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 4*(b^2*c^2*d*x^3 + b^2
*c^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (5
*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/
(d*x^2 + c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c
^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/2*(2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2
*c*d^2 + (2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 2*(b^2*c^2*d*x^3 + b
^2*c^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) - ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 +
(5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a*b^2*c^4*d
 - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^
3*d^2)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.10, size = 164, normalized size = 1.14

$$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ab^2c^2 - 2\,a^2bcd + a^3d^2)\sqrt{ab}} + \frac{(5\,bcd^2 - 3\,ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2)\sqrt{cd}} - \frac{2\,bcdx^2 - 3\,ad^2x^2 + 2\,bc^2 - 2\,acd}{2\,(abc^3 - a^2c^2d)(dx^3 + cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
+ 1/2*(5*b*c*d^2 - 3*a*d^3)*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2)*sqrt(c*d)) - 1/2*(2*b*c*d*x^2 - 3*a*d^2*x^2 + 2*b*c^2 - 2*a*c
*d)/((a*b*c^3 - a^2*c^2*d)*(d*x^3 + c*x))
```

Mupad [B]
time = 0.46, size = 432, normalized size = 3.00

$$-\frac{\frac{1}{a\,c}+\frac{x^2\,\left(3\,a\,d^2-2\,b\,c\,d\right)}{2\,a\,c^2\,\left(a\,d-b\,c\right)}}{d\,x^3+c\,x}+\frac{\mathrm{atan}\left(\frac{b\,c^5\,x\,{\left(-a^3\,b^5\right)}^{3/2}\,4{}\mathrm{i}+a^8\,b\,d^5\,x\,\sqrt{-a^3\,b^5}\,9{}\mathrm{i}+a^6\,b^3\,c^2\,d^3\,x\,\sqrt{-a^3\,b^5}\,25{}\mathrm{i}-a^7\,b^2\,c\,d^4\,x\,\sqrt{-a^3\,b^5}\,30{}\mathrm{i}}{-9\,a^{10}\,b^3\,d^5+30\,a^9\,b^4\,c\,d^4-25\,a^8\,b^5\,c^2\,d^3+4\,a^5\,b^8\,c^5}\right)\,\sqrt{-a^3\,b^5}\,1{}\mathrm{i}}{a^5\,d^2-2\,a^4\,b\,c\,d+a^3\,b^2\,c^2}+\frac{\mathrm{atan}\left(\frac{a^5\,d^3\,x\,{\left(-c^5\,d^3\right)}^{3/2}\,9{}\mathrm{i}+b^5\,c^{10}\,d\,x\,\sqrt{-c^5\,d^3}\,4{}\mathrm{i}-a^4\,b\,c\,d^2\,x\,{\left(-c^5\,d^3\right)}^{3/2}\,30{}\mathrm{i}+a^3\,b^2\,c^2\,d\,x\,{\left(-c^5\,d^3\right)}^{3/2}\,25{}\mathrm{i}}{9\,a^5\,c^8\,d^7-30\,a^4\,b\,c^9\,d^6+25\,a^3\,b^2\,c^{10}\,d^5-4\,b^5\,c^{13}\,d^2}\right)\,\left(3\,a\,d-5\,b\,c\right)\,\sqrt{-c^5\,d^3}\,1{}\mathrm{i}}{2\,\left(a^2\,c^5\,d^2-2\,a\,b\,c^6\,d+b^2\,c^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (atan((b*c^5*x*(-a^3*b^5)^(3/2)*4i + a^8*b*d^5*x*(-a^3*b^5)^(1/2)*9i + a^6*
b^3*c^2*d^3*x*(-a^3*b^5)^(1/2)*25i - a^7*b^2*c*d^4*x*(-a^3*b^5)^(1/2)*30i)/
(4*a^5*b^8*c^5 - 9*a^10*b^3*d^5 + 30*a^9*b^4*c*d^4 - 25*a^8*b^5*c^2*d^3))*(
-a^3*b^5)^(1/2)*1i)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - (1/(a*c) + (x^2
*(3*a*d^2 - 2*b*c*d))/(2*a*c^2*(a*d - b*c)))/(c*x + d*x^3) + (atan((a^5*d^3
*x*(-c^5*d^3)^(3/2)*9i + b^5*c^10*d*x*(-c^5*d^3)^(1/2)*4i - a^4*b*c*d^2*x*(
-c^5*d^3)^(3/2)*30i + a^3*b^2*c^2*d*x*(-c^5*d^3)^(3/2)*25i)/(9*a^5*c^8*d^7
- 4*b^5*c^13*d^2 - 30*a^4*b*c^9*d^6 + 25*a^3*b^2*c^10*d^5))*(3*a*d - 5*b*c)
*(-c^5*d^3)^(1/2)*1i)/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d))
```

3.249 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{2ac^2x^2}+\frac{d^2}{2c^2(bc-ad)(c+dx^2)}-\frac{(bc+2ad)\log(x)}{a^2c^3}+\frac{b^3\log(a+bx^2)}{2a^2(bc-ad)^2}-\frac{d^2(3bc-2ad)\log(c+dx^2)}{2c^3(bc-ad)^2}$$

```
[Out] -1/2/a/c^2/x^2+1/2*d^2/c^2/(-a*d+b*c)/(d*x^2+c)-(2*a*d+b*c)*ln(x)/a^2/c^3+1
/2*b^3*ln(b*x^2+a)/a^2/(-a*d+b*c)^2-1/2*d^2*(-2*a*d+3*b*c)*ln(d*x^2+c)/c^3/
(-a*d+b*c)^2
```

Rubi [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{b^3\log(a+bx^2)}{2a^2(bc-ad)^2}-\frac{\log(x)(2ad+bc)}{a^2c^3}-\frac{d^2(3bc-2ad)\log(c+dx^2)}{2c^3(bc-ad)^2}+\frac{d^2}{2c^2(c+dx^2)(bc-ad)}-\frac{1}{2ac^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*1/(a*c^2*x^2) + d^2/(2*c^2*(b*c - a*d)*(c + d*x^2)) - ((b*c + 2*a*d)*L
og[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*(3*b*c
 - 2*a*d)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ac^2x^2} + \frac{-bc-2ad}{a^2c^3x} + \frac{b^4}{a^2(-bc+ad)^2(a+bx)} - \frac{d^3}{c^2(bc-ad)(c\right.\right.$$
$$= -\frac{1}{2ac^2x^2} + \frac{d^2}{2c^2(bc-ad)(c+dx^2)} - \frac{(bc+2ad)\log(x)}{a^2c^3} + \frac{b^3\log(a+bx^2)}{2a^2(bc-ad)^2} -$$

Mathematica [A]
time = 0.17, size = 117, normalized size = 0.93

$$\frac{1}{2}\left(-\frac{2(bc+2ad)\log(x)}{a^2c^3} + \frac{b^3\log(a+bx^2)}{a^2(bc-ad)^2} + \frac{-\frac{c}{ax^2} + \frac{cd^2}{(bc-ad)(c+dx^2)} + \frac{d^2(-3bc+2ad)\log(c+dx^2)}{(bc-ad)^2}}{c^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((-2*(b*c + 2*a*d)*Log[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(a^2*(b*c - a*d
)^2) + (-(c/(a*x^2)) + (c*d^2)/((b*c - a*d)*(c + d*x^2)) + (d^2*(-3*b*c + 2
*a*d)*Log[c + d*x^2])/(b*c - a*d)^2)/c^3)/2
```

Maple [A]
time = 0.15, size = 120, normalized size = 0.95

method	result	si
default	$\frac{b^3\ln(bx^2+a)}{2a^2(ad-bc)^2} + \frac{d^3\left(-\frac{c(ad-bc)}{d(dx^2+c)} + \frac{(2ad-3bc)\ln(dx^2+c)}{d}\right)}{2c^3(ad-bc)^2} - \frac{1}{2a\,c^2x^2} + \frac{(-2ad-bc)\ln(x)}{a^2c^3}$	12
norman	$\frac{-\frac{1}{2ac} + \frac{(-2a\,d^3+bc\,d^2)x^2}{2c^2da(ad-bc)}}{x^2(dx^2+c)} + \frac{b^3\ln(bx^2+a)}{2a^2(a^2d^2-2abcd+b^2c^2)} - \frac{(2ad+bc)\ln(x)}{a^2c^3} + \frac{d^2(2ad-3bc)\ln(dx^2+c)}{2c^3(a^2d^2-2abcd+b^2c^2)}$	16
risch	$\frac{-\frac{d(2ad-bc)x^2}{2c^2a(ad-bc)} - \frac{1}{2ac}}{x^2(dx^2+c)} - \frac{2\ln(x)d}{a\,c^3} - \frac{\ln(x)b}{a^2c^2} + \frac{b^3\ln(bx^2+a)}{2a^2(a^2d^2-2abcd+b^2c^2)} + \frac{d^3\ln(-dx^2-c)a}{c^3(a^2d^2-2abcd+b^2c^2)} - \frac{3d^2\ln(-dx^2-c)b}{2c^2(a^2d^2-2abcd+b^2c^2)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^3/a^2/(a*d-b*c)^2*ln(b*x^2+a)+1/2*d^3/c^3/(a*d-b*c)^2*(-c*(a*d-b*c)/d
/(d*x^2+c)+(2*a*d-3*b*c)/d*ln(d*x^2+c))-1/2/a/c^2/x^2+(-2*a*d-b*c)/a^2/c^3*
ln(x)
```

Maxima [A]
time = 0.28, size = 188, normalized size = 1.49

$$\frac{b^3\log\left(bx^2+a\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}-\frac{\left(3\,bcd^2-2\,ad^3\right)\log\left(dx^2+c\right)}{2\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)}-\frac{bc^2-acd+(bcd-2\,ad^2)x^2}{2\left((abc^3d-a^2c^2d^2)x^4+(abc^4-a^2c^3d)x^2\right)}-\frac{(bc+2\,ad)\log\left(x^2\right)}{2\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) - 1/2*(3*b*c*d
^2 - 2*a*d^3)*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1/2*(b
*c^2 - a*c*d + (b*c*d - 2*a*d^2)*x^2)/((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b
*c^4 - a^2*c^3*d)*x^2) - 1/2*(b*c + 2*a*d)*log(x^2)/(a^2*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(118) = 236.
time = 3.02, size = 302, normalized size = 2.40

$$-\frac{ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+(ab^2c^3d-3\,a^2bc^2d^2+2\,a^3cd^3)x^2-(b^3c^3dx^4+b^3c^4x^2)\log\left(bx^2+a\right)+((3\,a^2bcd^3-2\,a^3d^4)x^4+(3\,a^2bc^2d^2-2\,a^3cd^3)x^2)\log\left(dx^2+c\right)+2\left((b^3c^3d-3\,a^2bcd^3+2\,a^3d^4)x^4+(b^3c^4-3\,a^2bc^2d^2+2\,a^3cd^3)x^2\right)\log\left(x\right)}{2\left((a^2b^2c^5d-2\,a^3bc^4d^2+a^4c^3d^3)x^4+(a^2b^2c^6-2\,a^3bc^5d+a^4c^4d^2)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 3*a^2*b*c^2*
d^2 + 2*a^3*c*d^3)*x^2 - (b^3*c^3*d*x^4 + b^3*c^4*x^2)*log(b*x^2 + a) + ((3
*a^2*b*c*d^3 - 2*a^3*d^4)*x^4 + (3*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*log(d*
x^2 + c) + 2*((b^3*c^3*d - 3*a^2*b*c*d^3 + 2*a^3*d^4)*x^4 + (b^3*c^4 - 3*a^
2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*log(x))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 +
 a^4*c^3*d^3)*x^4 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(118) = 236.
time = 1.01, size = 257, normalized size = 2.04

$$\frac{b^4\log\left(|bx^2+a|\right)}{2\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)}-\frac{\left(3\,bcd^3-2\,ad^4\right)\log\left(|dx^2+c|\right)}{2\left(b^2c^5d-2\,abc^4d^2+a^2c^3d^3\right)}+\frac{b^3c^2dx^4+b^3c^3x^2-2\,ab^2c^2dx^2+6\,a^2bcd^2x^2-4\,a^3d^3x^2-2\,ab^2c^3+4\,a^2bc^2d-2\,a^3cd^2}{4\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(dx^4+cx^2\right)}-\frac{(bc+2\,ad)\log\left(x^2\right)}{2\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2) - 1/2
*(3*b*c*d^3 - 2*a*d^4)*log(abs(d*x^2 + c))/(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2
*c^3*d^3) + 1/4*(b^3*c^2*d*x^4 + b^3*c^3*x^2 - 2*a*b^2*c^2*d*x^2 + 6*a^2*b*
c*d^2*x^2 - 4*a^3*d^3*x^2 - 2*a*b^2*c^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(d*x^4 + c*x^2)) - 1/2*(b*c + 2*a*
d)*log(x^2)/(a^2*c^3)
```

Mupad [B]
time = 0.52, size = 171, normalized size = 1.36

$$\frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,\left(a^4\,d^2-2\,a^3\,b\,c\,d+a^2\,b^2\,c^2\right)}-\frac{\frac{1}{2\,a\,c}+\frac{x^2\,\left(2\,a\,d^2-b\,c\,d\right)}{2\,a\,c^2\,\left(a\,d-b\,c\right)}}{d\,x^4+c\,x^2}+\frac{\ln\left(d\,x^2+c\right)\,\left(2\,a\,d^3-3\,b\,c\,d^2\right)}{2\,a^2\,c^3\,d^2-4\,a\,b\,c^4\,d+2\,b^2\,c^5}-\frac{\ln\left(x\right)\,\left(2\,a\,d+b\,c\right)}{a^2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (b^3*log(a + b*x^2))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (1/(2*a*c)
 + (x^2*(2*a*d^2 - b*c*d))/(2*a*c^2*(a*d - b*c)))/(c*x^2 + d*x^4) + (log(c
+ d*x^2)*(2*a*d^3 - 3*b*c*d^2))/(2*b^2*c^5 + 2*a^2*c^3*d^2 - 4*a*b*c^4*d) -
 (log(x)*(2*a*d + b*c))/(a^2*c^3)
```

3.250 $\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{2bc-5ad}{6ac^2(bc-ad)x^3}+\frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x}-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2}-\frac{d^{5/2}(7bc-5ad)}{2c^{7/2}(bc}$$

```
[Out] 1/6*(5*a*d-2*b*c)/a/c^2/(-a*d+b*c)/x^3+1/2*(-5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)
/a^2/c^3/(-a*d+b*c)/x-1/2*d/c/(-a*d+b*c)/x^3/(d*x^2+c)+b^(7/2)*arctan(x*b^(
1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^2-1/2*d^(5/2)*(-5*a*d+7*b*c)*arctan(x*d^(1
/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.18, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 597, 536, 211}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2}+\frac{-5a^2d^2+2abcd+2b^2c^2}{2a^2c^3x(bc-ad)}-\frac{d^{5/2}(7bc-5ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{2bc-5ad}{6ac^2x^3(bc-ad)}-\frac{d}{2cx^3\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/6*(2*b*c - 5*a*d)/(a*c^2*(b*c - a*d)*x^3) + (2*b^2*c^2 + 2*a*b*c*d - 5*a
^2*d^2)/(2*a^2*c^3*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x^3*(c + d*x^2)) + (
b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)^2) - (d^(5/2)*(7*
b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^2}\,dx = -\frac{d}{2c(bc-ad)x^3(c+dx^2)} + \frac{\int \frac{2bc-5ad-5bdx^2}{x^4(a+bx^2)(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= -\frac{2bc-5ad}{6ac^2(bc-ad)x^3} - \frac{d}{2c(bc-ad)x^3(c+dx^2)} - \frac{\int \frac{3(2b^2c^2+2abcd-5a^2d^2)+3bd(2bc-5\cdots}{x^2(a+bx^2)(c+dx^2)}}{6ac^2(bc-ad)}$$

$$= -\frac{2bc-5ad}{6ac^2(bc-ad)x^3} + \frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x} - \frac{d}{2c(bc-ad)x^3(c+dx^2)} + \frac{\int 3\cdots}{\cdots}$$

$$= -\frac{2bc-5ad}{6ac^2(bc-ad)x^3} + \frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x} - \frac{d}{2c(bc-ad)x^3(c+dx^2)} + \frac{b^4\cdots}{a^2(\cdots}$$

$$= -\frac{2bc-5ad}{6ac^2(bc-ad)x^3} + \frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x} - \frac{d}{2c(bc-ad)x^3(c+dx^2)} + \frac{b^{7/2}\cdots}{a\cdots}$$

Mathematica [A]
time = 0.28, size = 142, normalized size = 0.75

$$-\frac{1}{3ac^2x^3} + \frac{bc+2ad}{a^2c^3x} - \frac{d^3x}{2c^3(bc-ad)(c+dx^2)} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^2} - \frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/3*1/(a*c^2*x^3) + (b*c + 2*a*d)/(a^2*c^3*x) - (d^3*x)/(2*c^3*(b*c - a*d)
*(c + d*x^2)) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*
```

```
d)^2) - (d^(5/2)*(7*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*(b
*c - a*d)^2)
```

Maple [A]
time = 0.16, size = 127, normalized size = 0.67

method	result
default	$\frac{b^{4} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{a^{2} (a d-b c)^{2} \sqrt{a b}}+\frac{d^{3} \left(\frac{\left(\frac{a d}{2}-\frac{b c}{2}\right) x}{d \,x^{2}+c}+\frac{(5 a d-7 b c) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \sqrt{c d}}\right)}{c^{3} (a d-b c)^{2}}-\frac{1}{3 c^{2} a \,x^{3}}-\frac{-2 a d-b c}{a^{2} c^{3} x}$
risch	$\frac{\frac{d \left(5 a^{2} d^{2}-2 a b c d-2 b^{2} c^{2}\right) x^{4}}{2 a^{2} c^{3} (a d-b c)}+\frac{(5 a d+3 b c) x^{2}}{3 a^{2} c^{2}}-\frac{1}{3 a c}}{x^{3} (d \,x^{2}+c)}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{9} d^{4}-4 b \,d^{3} a^{8} c+6 b^{2} d^{2} a^{7} c^{2}-4 b^{3} d \,a^{6} c^{3}+b^{4} a^{5} c^{4}\right) \_Z^{2}+b^{7}\right)} \_R \ln\left(\left(\left(1\right.\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*b^4/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d^3/c^3/(a*d-b*c)
^2*((1/2*a*d-1/2*b*c)*x/(d*x^2+c)+1/2*(5*a*d-7*b*c)/(c*d)^(1/2)*arctan(d*x/
(c*d)^(1/2)))-1/3/c^2/a/x^3-(-2*a*d-b*c)/a^2/c^3/x
```

Maxima [A]
time = 0.55, size = 236, normalized size = 1.25

$$\frac{b^{4} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^{2}b^{2}c^{2} - 2\,a^{3}bcd + a^{4}d^{2})\sqrt{ab}} - \frac{(7\,bcd^{3} - 5\,ad^{4}) \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^{2}c^{5} - 2\,abc^{4}d + a^{2}c^{3}d^{2})\sqrt{cd}} - \frac{2\,abc^{3} - 2\,a^{2}c^{2}d - 3\,(2\,b^{2}c^{2}d + 2\,abcd^{2} - 5\,a^{2}d^{3})x^{4} - 2\,(3\,b^{2}c^{3} + 2\,abc^{2}d - 5\,a^{2}cd^{2})x^{2}}{6\,((a^{2}bc^{4}d - a^{3}c^{3}d^{2})x^{5} + (a^{2}bc^{5} - a^{3}c^{4}d)x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b))
 - 1/2*(7*b*c*d^3 - 5*a*d^4)*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d
+ a^2*c^3*d^2)*sqrt(c*d)) - 1/6*(2*a*b*c^3 - 2*a^2*c^2*d - 3*(2*b^2*c^2*d +
 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 - 2*(3*b^2*c^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x
^2)/((a^2*b*c^4*d - a^3*c^3*d^2)*x^5 + (a^2*b*c^5 - a^3*c^4*d)*x^3)
```

Fricas [A]
time = 2.33, size = 1281, normalized size = 6.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3*c^3*d - 7*a^
2*b*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 +
 5*a^3*c*d^3)*x^2 - 6*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-b/a)*log((b*x^2 +
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 +
(7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/
c) - c)/(d*x^2 + c)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5
+ (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^4 - 4*a
^2*b*c^3*d + 2*a^3*c^2*d^2 - 3*(2*b^3*c^3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^
4 - 2*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 + 3*((7
*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(d
/c)*arctan(x*sqrt(d/c)) - 3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-b/a)*log((b
*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^
2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^3), -1
/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3*c^3*d - 7*a^2*b
*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 + 5*
a^3*c*d^3)*x^2 - 12*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(b/a)*arctan(x*sqrt(b
/a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)
*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^2
*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d
+ a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^4 - 4*a^2*b*c^3*d + 2*a^3*c^2*d^2 - 3*
(2*b^3*c^3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^4 - 2*(3*b^3*c^4 - a*b^2*c^3*d
- 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 - 6*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt
(b/a)*arctan(x*sqrt(b/a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c
^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^2*c^5*d -
 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^
4*d^2)*x^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.20, size = 165, normalized size = 0.87

$$\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{ab}}-\frac{d^3x}{2\,(bc^4-ac^3d)(dx^2+c)}-\frac{(7\,bcd^3-5\,ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^5-2\,abc^4d+a^2c^3d^2)\sqrt{cd}}+\frac{3\,bcx^2+6\,adx^2-ac}{3\,a^2c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b))
 - 1/2*d^3*x/((b*c^4 - a*c^3*d)*(d*x^2 + c)) - 1/2*(7*b*c*d^3 - 5*a*d^4)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d)) + 1/3
*(3*b*c*x^2 + 6*a*d*x^2 - a*c)/(a^2*c^3*x^3)
```

Mupad [B]
time = 0.52, size = 469, normalized size = 2.48

$$-\frac{\frac{1}{3\,a\,c}-\frac{x^2\,\left(5\,a\,d+3\,b\,c\right)}{3\,a^2\,c^2}+\frac{x^4\,\left(-5\,a^2\,d^3+2\,a\,b\,c\,d^2+2\,b^2\,c^2\,d\right)}{2\,a^2\,c^3\,\left(a\,d-b\,c\right)}}{d\,x^5+c\,x^3}-\frac{\mathrm{atan}\left(\frac{b\,c^7\,x\,{\left(-a^5\,b^7\right)}^{3/2}\,4\mathrm{i}+a^{12}\,b\,d^7\,x\,\sqrt{-a^5\,b^7}\,25\mathrm{i}+a^{10}\,b^3\,c^2\,d^5\,x\,\sqrt{-a^5\,b^7}\,49\mathrm{i}-a^{11}\,b^2\,c\,d^6\,x\,\sqrt{-a^5\,b^7}\,70\mathrm{i}}{-25\,a^{15}\,b^4\,d^7+70\,a^{14}\,b^5\,c\,d^6-49\,a^{13}\,b^6\,c^2\,d^5+4\,a^8\,b^{11}\,c^7}\right)\,\sqrt{-a^5\,b^7}\,1\mathrm{i}}{a^7\,d^2-2\,a^6\,b\,c\,d+a^5\,b^2\,c^2}-\frac{\mathrm{atan}\left(\frac{a^7\,d^3\,x\,{\left(-c^7\,d^5\right)}^{3/2}\,25\mathrm{i}+b^7\,c^{14}\,d\,x\,\sqrt{-c^7\,d^5}\,4\mathrm{i}-a^6\,b\,c\,d^2\,x\,{\left(-c^7\,d^5\right)}^{3/2}\,70\mathrm{i}+a^5\,b^2\,c^2\,d\,x\,{\left(-c^7\,d^5\right)}^{3/2}\,49\mathrm{i}}{25\,a^7\,c^{11}\,d^{10}-70\,a^6\,b\,c^{12}\,d^9+49\,a^5\,b^2\,c^{13}\,d^8-4\,b^7\,c^{18}\,d^3}\right)\,\left(5\,a\,d-7\,b\,c\right)\,\sqrt{-c^7\,d^5}\,1\mathrm{i}}{2\,\left(a^2\,c^7\,d^2-2\,a\,b\,c^8\,d+b^2\,c^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] - (1/(3*a*c) - (x^2*(5*a*d + 3*b*c))/(3*a^2*c^2) + (x^4*(2*b^2*c^2*d - 5*a^
2*d^3 + 2*a*b*c*d^2))/(2*a^2*c^3*(a*d - b*c)))/(c*x^3 + d*x^5) - (atan((b*c
^7*x*(-a^5*b^7)^(3/2)*4i + a^12*b*d^7*x*(-a^5*b^7)^(1/2)*25i + a^10*b^3*c^2
*d^5*x*(-a^5*b^7)^(1/2)*49i - a^11*b^2*c*d^6*x*(-a^5*b^7)^(1/2)*70i)/(4*a^8
*b^11*c^7 - 25*a^15*b^4*d^7 + 70*a^14*b^5*c*d^6 - 49*a^13*b^6*c^2*d^5))*(-a
^5*b^7)^(1/2)*1i)/(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d) - (atan((a^7*d^3*x*
(-c^7*d^5)^(3/2)*25i + b^7*c^14*d*x*(-c^7*d^5)^(1/2)*4i - a^6*b*c*d^2*x*(-c
^7*d^5)^(3/2)*70i + a^5*b^2*c^2*d*x*(-c^7*d^5)^(3/2)*49i)/(25*a^7*c^11*d^10
 - 4*b^7*c^18*d^3 - 70*a^6*b*c^12*d^9 + 49*a^5*b^2*c^13*d^8))*(5*a*d - 7*b*
c)*(-c^7*d^5)^(1/2)*1i)/(2*(b^2*c^9 + a^2*c^7*d^2 - 2*a*b*c^8*d))
```

3.251 $\int \frac{x^5}{(a+bx^2)^3(c+dx^2)} dx$

Optimal. Leaf size=116

$$-\frac{a^2}{4b^2(bc-ad)(a+bx^2)^2}+\frac{a(2bc-ad)}{2b^2(bc-ad)^2(a+bx^2)}+\frac{c^2\log(a+bx^2)}{2(bc-ad)^3}-\frac{c^2\log(c+dx^2)}{2(bc-ad)^3}$$

```
[Out] -1/4*a^2/b^2/(-a*d+b*c)/(b*x^2+a)^2+1/2*a*(-a*d+2*b*c)/b^2/(-a*d+b*c)^2/(b*
x^2+a)+1/2*c^2*ln(b*x^2+a)/(-a*d+b*c)^3-1/2*c^2*ln(d*x^2+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 90}

$$-\frac{a^2}{4b^2(a+bx^2)^2(bc-ad)}+\frac{a(2bc-ad)}{2b^2(a+bx^2)(bc-ad)^2}+\frac{c^2\log(a+bx^2)}{2(bc-ad)^3}-\frac{c^2\log(c+dx^2)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] -1/4*a^2/(b^2*(b*c - a*d)*(a + b*x^2)^2) + (a*(2*b*c - a*d))/(2*b^2*(b*c -
a*d)^2*(a + b*x^2)) + (c^2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (c^2*Log[c +
 d*x^2])/(2*(b*c - a*d)^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^3(c+dx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^3} + \frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)^2} + \frac{bc^2}{(bc-ad)^3(a+bx}\right.\right.$$
$$= -\frac{a^2}{4b^2(bc-ad)(a+bx^2)^2} + \frac{a(2bc-ad)}{2b^2(bc-ad)^2(a+bx^2)} + \frac{c^2\log(a+bx^2)}{2(bc-ad)^3} - \frac{c^2 \log}{2(}$$

Mathematica [A]
time = 0.08, size = 99, normalized size = 0.85

$$\frac{-\frac{a^2(bc-ad)^2}{b^2(a+bx^2)^2} + \frac{2a(-2bc+ad)(-bc+ad)}{b^2(a+bx^2)} + 2c^2\log(a+bx^2) - 2c^2\log(c+dx^2)}{4(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (-((a^2*(b*c - a*d)^2)/(b^2*(a + b*x^2)^2)) + (2*a*(-2*b*c + a*d)*(-(b*c) +
 a*d))/(b^2*(a + b*x^2)) + 2*c^2*Log[a + b*x^2] - 2*c^2*Log[c + d*x^2])/(4*
(b*c - a*d)^3)
```

Maple [A]
time = 0.12, size = 124, normalized size = 1.07

method	result	si
default	$-\frac{c^2\ln(bx^2+a) - \frac{a^2(a^2d^2-2abcd+b^2c^2)}{2b^2(bx^2+a)^2} + \frac{a(a^2d^2-3abcd+2b^2c^2)}{b^2(bx^2+a)}}{2(ad-bc)^3} + \frac{c^2\ln(dx^2+c)}{2(ad-bc)^3}$	12
norman	$\frac{\frac{(-ad+3bc)a^2}{4b^2(a^2d^2-2abcd+b^2c^2)} + \frac{a(-ad+2bc)x^2}{2b(a^2d^2-2abcd+b^2c^2)}}{(bx^2+a)^2} - \frac{c^2\ln(bx^2+a)}{2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)} + \frac{c^2\ln(dx^2+c)}{2a^3d^3-6a^2bcd^2+6ab^2c^2d-2b^3c^3}$	19
risch	$\frac{-\frac{a(ad-2bc)x^2}{2b(a^2d^2-2abcd+b^2c^2)} - \frac{a^2(ad-3bc)}{4b^2(a^2d^2-2abcd+b^2c^2)}}{(bx^2+a)^2} + \frac{c^2\ln(dx^2+c)}{2a^3d^3-6a^2bcd^2+6ab^2c^2d-2b^3c^3} - \frac{c^2\ln(-bx^2-a)}{2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*d-b*c)^3*(c^2*ln(b*x^2+a)-1/2*a^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/(
b*x^2+a)^2+a*(a^2*d^2-3*a*b*c*d+2*b^2*c^2)/b^2/(b*x^2+a))+1/2*c^2/(a*d-b*c)
^3*ln(d*x^2+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(108) = 216$.

time = 0.30, size = 236, normalized size = 2.03

$$\frac{c^2 \log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}-\frac{c^2 \log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{3\,a^2bc-a^3d+2\left(2\,ab^2c-a^2bd\right)x^2}{4\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2+\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)x^4+2\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
- 1/2*c^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
) + 1/4*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^2)/(a^2*b^4*c^2 - 2*
a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 2*(
a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(108) = 216$.

time = 1.13, size = 290, normalized size = 2.50

$$\frac{3\,a^2b^2c^2-4\,a^3bcd+a^4d^2+2\left(2\,ab^3c^2-3\,a^2b^2cd+a^3bd^2\right)x^2+2\left(b^4c^2x^4+2\,ab^3c^2x^2+a^2b^2c^2\right)\log\left(bx^2+a\right)-2\left(b^4c^2x^4+2\,ab^3c^2x^2+a^2b^2c^2\right)\log\left(dx^2+c\right)}{4\left(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3+\left(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3\right)x^4+2\left(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*d
 + a^3*b*d^2)*x^2 + 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + a^2*b^2*c^2)*log(b*x
^2 + a) - 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + a^2*b^2*c^2)*log(d*x^2 + c))/(
a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3 + (b^7*c^3 -
3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 2*(a*b^6*c^3 - 3*a^2*b
^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(97) = 194$.

time = 3.07, size = 418, normalized size = 3.60

$$\frac{c^2 \log\left(x^2+\frac{-\frac{a^4c^2d^4}{(ad-bc)^3}+\frac{4a^3bc^3d^3}{(ad-bc)^3}-\frac{6a^2b^2c^4d^2}{(ad-bc)^3}+\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d-\frac{b^4c^6}{(ad-bc)^3}+bc^3}{2bc^2d}\right)}{2\left(ad-bc\right)^3}-\frac{c^2 \log\left(x^2+\frac{\frac{a^4c^2d^4}{(ad-bc)^3}-\frac{4a^3bc^3d^3}{(ad-bc)^3}+\frac{6a^2b^2c^4d^2}{(ad-bc)^3}-\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d+\frac{b^4c^6}{(ad-bc)^3}+bc^3}{2bc^2d}\right)}{2\left(ad-bc\right)^3}+\frac{-a^3d+3a^2bc+x^2(-2a^2bd+4ab^2c)}{4a^4b^2d^2-8a^3b^3cd+4a^2b^4c^2+x^4\cdot\left(4a^2b^4d^2-8ab^5cd+4b^6c^2\right)+x^2\cdot\left(8a^3b^3d^2-16a^2b^4cd+8ab^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] c**2*log(x**2 + (-a**4*c**2*d**4/(a*d - b*c)**3 + 4*a**3*b*c**3*d**3/(a*d -
 b*c)**3 - 6*a**2*b**2*c**4*d**2/(a*d - b*c)**3 + 4*a*b**3*c**5*d/(a*d - b*
c)**3 + a*c**2*d - b**4*c**6/(a*d - b*c)**3 + b*c**3)/(2*b*c**2*d))/(2*(a*d
 - b*c)**3) - c**2*log(x**2 + (a**4*c**2*d**4/(a*d - b*c)**3 - 4*a**3*b*c**
3*d**3/(a*d - b*c)**3 + 6*a**2*b**2*c**4*d**2/(a*d - b*c)**3 - 4*a*b**3*c**
```

```
5*d/(a*d - b*c)**3 + a*c**2*d + b**4*c**6/(a*d - b*c)**3 + b*c**3)/(2*b*c**
2*d))/(2*(a*d - b*c)**3) + (-a**3*d + 3*a**2*b*c + x**2*(-2*a**2*b*d + 4*a*
b**2*c))/(4*a**4*b**2*d**2 - 8*a**3*b**3*c*d + 4*a**2*b**4*c**2 + x**4*(4*a
**2*b**4*d**2 - 8*a*b**5*c*d + 4*b**6*c**2) + x**2*(8*a**3*b**3*d**2 - 16*a
**2*b**4*c*d + 8*a*b**5*c**2))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(108) = 216$.
time = 1.35, size = 232, normalized size = 2.00

$$\frac{bc^2\log\left(|bx^2+a|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}-\frac{c^2d\log\left(|dx^2+c|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)}-\frac{3\,b^4c^2x^4+2\,ab^3c^2x^2+6\,a^2b^2cdx^2-2\,a^3bd^2x^2+4\,a^3bcd-a^4d^2}{4\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b*c^2*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3) - 1/2*c^2*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 - a^3*d^4) - 1/4*(3*b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + 6*a^2*b^2*
c*d*x^2 - 2*a^3*b*d^2*x^2 + 4*a^3*b*c*d - a^4*d^2)/((b^5*c^3 - 3*a*b^4*c^2*
d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(b*x^2 + a)^2)
```

Mupad [**B**]
time = 0.25, size = 370, normalized size = 3.19

$$\frac{b^3\left(4\,a\,c^2\,x^2+a\,c^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}\right)+b\left(2\,a^3\,d^2\,x^2-4\,a^3\,c\,d\right)+a^4\,d^2+b^2\left(3\,a^2\,c^2-6\,a^2\,c\,d\,x^2+a^2\,c^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}\right)+b^4\,c^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}}{-4\,a^5\,b^2\,d^3+12\,a^4\,b^3\,c\,d^2-8\,a^4\,b^3\,d^3\,x^2-12\,a^3\,b^4\,c^2\,d+24\,a^3\,b^4\,c\,d^2\,x^2-4\,a^3\,b^4\,d^3\,x^4+4\,a^2\,b^5\,c^3-24\,a^2\,b^5\,c^2\,d\,x^2+12\,a^2\,b^5\,c\,d^2\,x^4+8\,a\,b^6\,c^3\,x^2-12\,a\,b^6\,c^2\,d\,x^4+4\,b^7\,c^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^3*(c + d*x^2)),x)
```

```
[Out] (b^3*(4*a*c^2*x^2 + a*c^2*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x
^2 + b*c*x^2))*8i) + b*(2*a^3*d^2*x^2 - 4*a^3*c*d) + a^4*d^2 + b^2*(3*a^2*c
^2 + a^2*c^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i
 - 6*a^2*c*d*x^2) + b^4*c^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d
*x^2 + b*c*x^2))*4i)/(4*a^2*b^5*c^3 - 4*a^5*b^2*d^3 + 4*b^7*c^3*x^4 - 12*a^
3*b^4*c^2*d + 12*a^4*b^3*c*d^2 + 8*a*b^6*c^3*x^2 - 8*a^4*b^3*d^3*x^2 - 4*a^
3*b^4*d^3*x^4 - 12*a*b^6*c^2*d*x^4 - 24*a^2*b^5*c^2*d*x^2 + 24*a^3*b^4*c*d^
2*x^2 + 12*a^2*b^5*c*d^2*x^4)
```

3.252 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=157

$$-\frac{cx}{4d(bc-ad)(c+dx^2)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2(c+dx^2)}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{(b^2c^2-6abcd-3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{3/2}(bc-ad)^3}$$

```
[Out] -1/4*c*x/d/(-a*d+b*c)/(d*x^2+c)^2+1/8*(-5*a*d+b*c)*x/d/(-a*d+b*c)^2/(d*x^2+
c)+a^(3/2)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)^3+1/8*(-3*a^2*d^2-6
*a*b*c*d+b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/d^(3/2)/(-a*d+b*c)^3/c^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {481, 541, 536, 211}

$$\frac{a^{3/2}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{(-3a^2d^2-6abcd+b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{3/2}(bc-ad)^3}+\frac{x(bc-5ad)}{8d(c+dx^2)(bc-ad)^2}-\frac{cx}{4d(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(c*x)/(d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(8*d*(b*c - a*
d)^2*(c + d*x^2)) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a*
d)^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*S
qrt[c]*d^(3/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}\,dx &= -\frac{cx}{4d(bc-ad)(c+dx^2)^2}+\frac{\int \frac{ac+(bc-4ad)x^2}{(a+bx^2)(c+dx^2)^2}\,dx}{4d(bc-ad)}\\
&= -\frac{cx}{4d(bc-ad)(c+dx^2)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2(c+dx^2)}+\frac{\int \frac{ac(bc+3ad)+bc(bc-5ad)x^2}{(a+bx^2)(c+dx^2)}\,dx}{8cd(bc-ad)^2}\\
&= -\frac{cx}{4d(bc-ad)(c+dx^2)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2(c+dx^2)}+\frac{(a^2b)\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3}+\frac{(b^2c^2}{}\\
&= -\frac{cx}{4d(bc-ad)(c+dx^2)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2(c+dx^2)}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 154, normalized size = 0.98

$$\frac{1}{8}\left(\frac{2cx}{d(-bc+ad)(c+dx^2)^2}+\frac{(bc-5ad)x}{d(bc-ad)^2(c+dx^2)}+\frac{8a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{(b^2c^2-6abcd-3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((2*c*x)/(d*(-(b*c) + a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(d*(b*c - a*d
)^2*(c + d*x^2)) + (8*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a
*d)^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sq
rt[c]*d^(3/2)*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.19, size = 154, normalized size = 0.98

method	result
default	$-\frac{a^{2} b \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(a d-b c\right)^{3} \sqrt{a b}}+\frac{\frac{\left(-\frac{5}{8} a^{2} d^{2}+\frac{3}{4} a b c d-\frac{1}{8} b^{2} c^{2}\right) x^{3}-\frac{c\left(3 a^{2} d^{2}-2 a b c d-b^{2} c^{2}\right) x}{8 d}}{\left(d x^{2}+c\right)^{2}}+\frac{\left(3 a^{2} d^{2}+6 a b c d-b^{2} c^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 d \sqrt{c d}}}{\left(a d-b c\right)^{3}}$
risch	$\frac{-\frac{\left(5 a d-b c\right) x^{3}}{8\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}-\frac{c\left(3 a d+b c\right) x}{8 d\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}}{\left(d x^{2}+c\right)^{2}}+\frac{\sqrt{-a b}\, a \ln\left(\left(-64\left(-a b\right)^{\frac{3}{2}} a^{3} d^{4}-64\left(-a b\right)^{\frac{3}{2}} a^{2} b c d^{3}-73 a^{4} \sqrt{-a b}\, d^{4} b-36 \sqrt{-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*b/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/(a*d-b*c)^3*(((-5/
8*a^2*d^2+3/4*a*b*c*d-1/8*b^2*c^2)*x^3-1/8*c*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/
d*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2+6*a*b*c*d-b^2*c^2)/d/(c*d)^(1/2)*arctan(d*x
/(c*d)^(1/2)))
```

Maxima [A]
time = 0.55, size = 264, normalized size = 1.68

$$\frac{a^{2} b \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) \sqrt{a b}}+\frac{\left(b^{2} c^{2}-6 a b c d-3 a^{2} d^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8\left(b^{3} c^{3} d-3 a b^{2} c^{2} d^{2}+3 a^{2} b c d^{3}-a^{3} d^{4}\right) \sqrt{c d}}+\frac{\left(b c d-5 a d^{2}\right) x^{3}-\left(b c^{2}+3 a c d\right) x}{8\left(b^{2} c^{4} d-2 a b c^{3} d^{2}+a^{2} c^{2} d^{3}+\left(b^{2} c^{2} d^{3}-2 a b c d^{4}+a^{2} d^{5}\right) x^{4}+2\left(b^{2} c^{3} d^{2}-2 a b c^{2} d^{3}+a^{2} c d^{4}\right) x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] a^2*b*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(a*b)) + 1/8*(b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*
d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(c*d)) + 1
/8*((b*c*d - 5*a*d^2)*x^3 - (b*c^2 + 3*a*c*d)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2
 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2
 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(135) = 270$.
time = 1.70, size = 1573, normalized size = 10.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 - 8*(a*c*d^4*x^4 +
 2*a*c^2*d^3*x^2 + a*c^3*d^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)) - (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b
*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*
sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b^2*c^4*d + 2
*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c
^4*d^4 - a^3*c^3*d^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a
^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^
2*d^6)*x^2), 1/8*((b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + (b^2*c^
4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x
^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt
(c*d)*x/c) - 4*(a*c*d^4*x^4 + 2*a*c^2*d^3*x^2 + a*c^3*d^2)*sqrt(-a*b)*log((
b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - (b^2*c^4*d + 2*a*b*c^3*d^2 - 3*a
^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d
^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 2*
(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^2), 1/16*
(2*(b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + 16*(a*c*d^4*x^4 + 2*a*
c^2*d^3*x^2 + a*c^3*d^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (b^2*c^4 - 6*a*b
*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b
^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) - 2*(b^2*c^4*d + 2*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*
x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5 + (b^3*c^
4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3
 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^2), 1/8*((b^2*c^3*d^2
 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + 8*(a*c*d^4*x^4 + 2*a*c^2*d^3*x^2 + a*
c^3*d^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c
^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b
*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - (b^2*c^4*d +
 2*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b
*c^4*d^4 - a^3*c^3*d^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 -
 a^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*
c^2*d^6)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.90, size = 204, normalized size = 1.30

$$\frac{a^2b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{ab}}+\frac{\left(b^2c^2-6\,abcd-3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)\sqrt{cd}}+\frac{bcdx^3-5\,ad^2x^3-bc^2x-3\,acdx}{8\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\left(dx^2+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] a^2*b*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(a*b)) + 1/8*(b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*
d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(c*d)) + 1
/8*(b*c*d*x^3 - 5*a*d^2*x^3 - b*c^2*x - 3*a*c*d*x)/((b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*(d*x^2 + c)^2)
```

Mupad [B]
time = 1.07, size = 2500, normalized size = 15.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] (atan((((-c*d^3)^(1/2)*((x*(b^7*c^4 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3 + 3
0*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)) - (((96*a^8*b^2*d^9 + 32*a*b^9*c
^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 + 800*
a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^6*d^7
 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 +
15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) - (x*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c
^2 + 6*a*b*c*d)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d^3 - 1280*a*b^8*c^6*d^4 -
1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*a^3*b^6*c^4*d^6 - 1280*a^4
*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(512*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b
^2*c^3*d^4 - 3*a^2*b*c^2*d^5)*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*
b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)))
*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*1i)/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b
^2*c^3*d^4 - 3*a^2*b*c^2*d^5)) + ((-c*d^3)^(1/2)*((x*(b^7*c^4 + 73*a^4*b^3*
d^4 + 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5
 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)) + (((9
6*a^8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 -
 96*a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6
*b^4*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d
^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) + (x*(-c*d^3
)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d
^3 - 1280*a*b^8*c^6*d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*
```

a^3*b^6*c^4*d^6 - 1280*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(512*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)))*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*1i)/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)))/((a^2*b^6*c^3 + 15*a^5*b^3*d^3 - 11*a^3*b^5*c^2*d + 27*a^4*b^4*c*d^2)/(32*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) - ((-c*d^3)^(1/2)*((x*(b^7*c^4 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)) - (((96*a^8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) - (x*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d^3 - 1280*a*b^8*c^6*d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*a^3*b^6*c^4*d^6 - 1280*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(512*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)))*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)) + ((-c*d^3)^(1/2)*((x*(b^7*c^4 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)) + (((96*a^8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) + (x*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d^3 - 1280*a*b^8*c^6*d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*a^3*b^6*c^4*d^6 - 1280*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(512*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)))*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5))))*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*1i)/(8*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)) - (atan((((-a^3*b)^(1/2)*(((-a^3*b)^(1/2)*((96*a^8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) - (x*(-a^3*b)^(1/2)*(256*a^7*b^2*d^10 + 256*b...

3.253 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{c}{4d(bc-ad)\left(c+dx^2\right)^2}-\frac{a}{2(bc-ad)^2\left(c+dx^2\right)}-\frac{ab\log\left(a+bx^2\right)}{2(bc-ad)^3}+\frac{ab\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

```
[Out] -1/4*c/d/(-a*d+b*c)/(d*x^2+c)^2-1/2*a/(-a*d+b*c)^2/(d*x^2+c)-1/2*a*b*ln(b*x
^2+a)/(-a*d+b*c)^3+1/2*a*b*ln(d*x^2+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{a}{2\left(c+dx^2\right)(bc-ad)^2}-\frac{c}{4d\left(c+dx^2\right)^2(bc-ad)}-\frac{ab\log\left(a+bx^2\right)}{2(bc-ad)^3}+\frac{ab\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*c/(d*(b*c - a*d)*(c + d*x^2)^2) - a/(2*(b*c - a*d)^2*(c + d*x^2)) - (a
*b*Log[a + b*x^2])/(2*(b*c - a*d)^3) + (a*b*Log[c + d*x^2])/(2*(b*c - a*d)^
3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{ab^2}{(bc-ad)^3(a+bx)} + \frac{c}{(bc-ad)(c+dx)^3} + \frac{ad}{(-bc+ad)^2(c+d\ldots}\right.\right. \\
&= -\frac{c}{4d(bc-ad)(c+dx^2)^2} - \frac{a}{2(bc-ad)^2(c+dx^2)} - \frac{ab\log(a+bx^2)}{2(bc-ad)^3} + \frac{ab\log(\ldots}{2(bc\ldots}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 77, normalized size = 0.77

$$
\frac{\frac{(-bc+ad)(bc^2+ad(c+2dx^2))}{d(c+dx^2)^2} - 2ab\log(a+bx^2) + 2ab\log(c+dx^2)}{4(bc-ad)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (((-(b*c) + a*d)*(b*c^2 + a*d*(c + 2*d*x^2)))/(d*(c + d*x^2)^2) - 2*a*b*Log
[a + b*x^2] + 2*a*b*Log[c + d*x^2])/(4*(b*c - a*d)^3)
```

Maple [A]
time = 0.13, size = 105, normalized size = 1.05

method	result	siz…
default	$\frac{ab\ln(bx^2+a)}{2(ad-bc)^3} + \frac{-\frac{(ad-bc)a}{dx^2+c} + \frac{c(a^2d^2-2abcd+b^2c^2)}{2d(dx^2+c)^2} - ab\ln(dx^2+c)}{2(ad-bc)^3}$	10…
risch	$\frac{-\frac{adx^2}{2(a^2d^2-2abcd+b^2c^2)} - \frac{c(ad+bc)}{4d(a^2d^2-2abcd+b^2c^2)}}{(dx^2+c)^2} - \frac{ab\ln(-dx^2-c)}{2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)} + \frac{ab\ln(bx^2+a)}{2a^3d^3-6a^2bcd^2+6ab^2c^2d-2b^3c^3}$	18…
norman	$\frac{-\frac{adx^2}{2(a^2d^2-2abcd+b^2c^2)} + \frac{(-ad^2-bcd)c}{4d^2(a^2d^2-2abcd+b^2c^2)}}{(dx^2+c)^2} + \frac{ab\ln(bx^2+a)}{2a^3d^3-6a^2bcd^2+6ab^2c^2d-2b^3c^3} - \frac{ab\ln(dx^2+c)}{2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}$	18…

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*b/(a*d-b*c)^3*ln(b*x^2+a)+1/2/(a*d-b*c)^3*(-(a*d-b*c)*a/(d*x^2+c)+1/2
*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c)^2-a*b*ln(d*x^2+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(92) = 184.
time = 0.31, size = 217, normalized size = 2.17

$$
-\frac{ab\log(bx^2+a)}{2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)} + \frac{ab\log(dx^2+c)}{2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)} - \frac{2ad^2x^2+bc^2+acd}{4(b^2c^4d-2abc^3d^2+a^2c^2d^3+(b^2c^2d^3-2abcd^4+a^2d^5)x^4+2(b^2c^3d^2-2abc^2d^3+a^2cd^4)x^2)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*b*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
 + 1/2*a*b*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3) - 1/4*(2*a*d^2*x^2 + b*c^2 + a*c*d)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2
*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c
^2*d^3 + a^2*c*d^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(92) = 184$.
time = 1.00, size = 256, normalized size = 2.56

$$-\frac{b^2c^3 - a^2cd^2 + 2\left(abcd^2 - a^2d^3\right)x^2 + 2\left(abd^3x^4 + 2\,abcd^2x^2 + abc^2d\right)\log\left(bx^2 + a\right) - 2\left(abd^3x^4 + 2\,abcd^2x^2 + abc^2d\right)\log\left(dx^2 + c\right)}{4\left(b^3c^5d - 3\,ab^2c^4d^2 + 3\,a^2bc^3d^3 - a^3c^2d^4 + \left(b^3c^3d^3 - 3\,ab^2c^2d^4 + 3\,a^2bcd^5 - a^3d^6\right)x^4 + 2\left(b^3c^4d^2 - 3\,ab^2c^3d^3 + 3\,a^2bc^2d^4 - a^3cd^5\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^3 - a^2*c*d^2 + 2*(a*b*c*d^2 - a^2*d^3)*x^2 + 2*(a*b*d^3*x^4 +
2*a*b*c*d^2*x^2 + a*b*c^2*d)*log(b*x^2 + a) - 2*(a*b*d^3*x^4 + 2*a*b*c*d^2*
x^2 + a*b*c^2*d)*log(d*x^2 + c))/(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3
*d^3 - a^3*c^2*d^4 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d
^6)*x^4 + 2*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*x
^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(82) = 164$.
time = 2.70, size = 411, normalized size = 4.11

$$-\frac{ab\log\left(x^2 + \frac{-\frac{a^5bd^4}{(ad-bc)^3} + \frac{4a^4b^2cd^3}{(ad-bc)^3} - \frac{6a^3b^3c^2d^2}{(ad-bc)^3} + \frac{4a^2b^4c^3d}{(ad-bc)^3} + a^2bd - \frac{ab^5c^4}{(ad-bc)^3} + ab^2c}{2ab^2d}\right)}{2\left(ad - bc\right)^3} + \frac{ab\log\left(x^2 + \frac{\frac{a^5bd^4}{(ad-bc)^3} - \frac{4a^4b^2cd^3}{(ad-bc)^3} + \frac{6a^3b^3c^2d^2}{(ad-bc)^3} - \frac{4a^2b^4c^3d}{(ad-bc)^3} + a^2bd + \frac{ab^5c^4}{(ad-bc)^3} + ab^2c}{2ab^2d}\right)}{2\left(ad - bc\right)^3} + \frac{-acd - 2ad^2x^2 - bc^2}{4a^2c^2d^3 - 8abc^3d^2 + 4b^2c^4d + x^4\cdot\left(4a^2d^5 - 8abcd^4 + 4b^2c^2d^3\right) + x^2\cdot\left(8a^2cd^4 - 16abc^2d^3 + 8b^2c^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -a*b*log(x**2 + (-a**5*b*d**4/(a*d - b*c)**3 + 4*a**4*b**2*c*d**3/(a*d - b*
c)**3 - 6*a**3*b**3*c**2*d**2/(a*d - b*c)**3 + 4*a**2*b**4*c**3*d/(a*d - b*
c)**3 + a**2*b*d - a*b**5*c**4/(a*d - b*c)**3 + a*b**2*c)/(2*a*b**2*d))/(2*
(a*d - b*c)**3) + a*b*log(x**2 + (a**5*b*d**4/(a*d - b*c)**3 - 4*a**4*b**2*
c*d**3/(a*d - b*c)**3 + 6*a**3*b**3*c**2*d**2/(a*d - b*c)**3 - 4*a**2*b**4*
c**3*d/(a*d - b*c)**3 + a**2*b*d + a*b**5*c**4/(a*d - b*c)**3 + a*b**2*c)/(
2*a*b**2*d))/(2*(a*d - b*c)**3) + (-a*c*d - 2*a*d**2*x**2 - b*c**2)/(4*a**2
*c**2*d**3 - 8*a*b*c**3*d**2 + 4*b**2*c**4*d + x**4*(4*a**2*d**5 - 8*a*b*c*
d**4 + 4*b**2*c**2*d**3) + x**2*(8*a**2*c*d**4 - 16*a*b*c**2*d**3 + 8*b**2*
c**3*d**2))
```

Giac [A]
time = 0.78, size = 174, normalized size = 1.74

$$-\frac{ab^2\log\left(|bx^2+a|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}+\frac{abd\log\left(|dx^2+c|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)}-\frac{b^2c^3-a^2cd^2+2\left(abcd^2-a^2d^3\right)x^2}{4\left(dx^2+c\right)^2(bc-ad)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*a*b^2*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3) + 1/2*a*b*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4) - 1/4*(b^2*c^3 - a^2*c*d^2 + 2*(a*b*c*d^2 - a^2*d^
3)*x^2)/((d*x^2 + c)^2*(b*c - a*d)^3*d)
```

Mupad [B]
time = 0.21, size = 343, normalized size = 3.43

$$-\frac{b^2\,c^3-a^2\,c\,d^2-2\,a^2\,d^3\,x^2+2\,a\,b\,c\,d^2\,x^2+a\,b\,c^2\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+a\,b\,d^3\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+a\,b\,c\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}}{-4\,a^3\,c^2\,d^4-8\,a^3\,c\,d^5\,x^2-4\,a^3\,d^6\,x^4+12\,a^2\,b\,c^3\,d^3+24\,a^2\,b\,c^2\,d^4\,x^2+12\,a^2\,b\,c\,d^5\,x^4-12\,a\,b^2\,c^4\,d^2-24\,a\,b^2\,c^3\,d^3\,x^2-12\,a\,b^2\,c^2\,d^4\,x^4+4\,b^3\,c^5\,d+8\,b^3\,c^4\,d^2\,x^2+4\,b^3\,c^3\,d^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] -(b^2*c^3 - a^2*c*d^2 - 2*a^2*d^3*x^2 + 2*a*b*c*d^2*x^2 + a*b*c^2*d*atan((a
*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a*b*d^3*x^4*atan(
(a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a*b*c*d^2*x^2*a
tan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i)/(4*b^3*c^5*d
 - 4*a^3*c^2*d^4 - 4*a^3*d^6*x^4 - 12*a*b^2*c^4*d^2 + 12*a^2*b*c^3*d^3 - 8*
a^3*c*d^5*x^2 + 8*b^3*c^4*d^2*x^2 + 4*b^3*c^3*d^3*x^4 + 12*a^2*b*c*d^5*x^4
- 24*a*b^2*c^3*d^3*x^2 + 24*a^2*b*c^2*d^4*x^2 - 12*a*b^2*c^2*d^4*x^4)
```

3.254 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=155

$$\frac{x}{4(bc-ad)(c+dx^2)^2}+\frac{(3bc+ad)x}{8c(bc-ad)^2(c+dx^2)}-\frac{\sqrt{a}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{(3b^2c^2+6abcd-a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}\,(bc-ad)^3}$$

```
[Out] 1/4*x/(-a*d+b*c)/(d*x^2+c)^2+1/8*(a*d+3*b*c)*x/c/(-a*d+b*c)^2/(d*x^2+c)-b^(
3/2)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)^3+1/8*(-a^2*d^2+6*a*b*c*d
+3*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)^3/d^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {482, 541, 536, 211}

$$\frac{(-a^2d^2+6abcd+3b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}\,(bc-ad)^3}-\frac{\sqrt{a}\,b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(ad+3bc)}{8c\,(c+dx^2)\,(bc-ad)^2}+\frac{x}{4\,(c+dx^2)^2\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] x/(4*(b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(8*c*(b*c - a*d)^2*(c +
 d*x^2)) - (Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a*d)^3 + ((
3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*Sq
rt[d]*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx &= \frac{x}{4(bc-ad)(c+dx^2)^2}-\frac{\int \frac{a-3bx^2}{(a+bx^2)(c+dx^2)^2}\,dx}{4(bc-ad)}\\ &= \frac{x}{4(bc-ad)(c+dx^2)^2}+\frac{(3bc+ad)x}{8c(bc-ad)^2(c+dx^2)}-\frac{\int \frac{a(5bc-ad)-b(3bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx}{8c(bc-ad)^2}\\ &= \frac{x}{4(bc-ad)(c+dx^2)^2}+\frac{(3bc+ad)x}{8c(bc-ad)^2(c+dx^2)}-\frac{(ab^2)\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3}+\frac{(3b^2c^2+\cdots}{\cdots}\\ &= \frac{x}{4(bc-ad)(c+dx^2)^2}+\frac{(3bc+ad)x}{8c(bc-ad)^2(c+dx^2)}-\frac{\sqrt{a}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^3}+\cdots\end{aligned}$$

Mathematica [A]
time = 0.17, size = 151, normalized size = 0.97

$$\frac{1}{8}\left(\frac{2x}{(bc-ad)(c+dx^2)^2}+\frac{(3bc+ad)x}{c(bc-ad)^2(c+dx^2)}+\frac{8\sqrt{a}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(-bc+ad)^3}+\frac{(3b^2c^2+6abcd-a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}\,(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((2*x)/((b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(c*(b*c - a*d)^2*(c
+ d*x^2)) + (8*Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(-(b*c) + a*d)^
3 + ((3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2
)*Sqrt[d]*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.20, size = 151, normalized size = 0.97

method	result
default	$\frac{a\,b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(a d -b c \right)^{3} \sqrt{a b}}+\frac{\frac{\frac{d \left(a^{2} d^{2}+2 a b c d -3 b^{2} c^{2}\right) x^{3}}{8 c}+\left(\frac{3}{4} a b c d -\frac{5}{8} b^{2} c^{2}-\frac{1}{8} a^{2} d^{2}\right) x}{\left(d \,x^{2}+c \right)^{2}}+\frac{\left(a^{2} d^{2}-6 a b c d -3 b^{2} c^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 c \sqrt{c d}}}{\left(a d -b c \right)^{3}}$
risch	$\frac{\frac{d \left(a d +3 b c \right) x^{3}}{8 c \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}-\frac{\left(a d -5 b c \right) x}{8 \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}}{\left(d \,x^{2}+c \right)^{2}}+\frac{\sqrt{-a b}\, b \ln\left(\left(-64 \left(-a b \right)^{\frac{3}{2}} a b \,c^{2} d^{2}-64 \left(-a b \right)^{\frac{3}{2}} b^{2} c^{3} d -\sqrt{-a b}\, a^{4} d^{4}+12 \sqrt{-a b}\,\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/(a*d-b*c)^3*((1/8*d
*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/c*x^3+(3/4*a*b*c*d-5/8*b^2*c^2-1/8*a^2*d^2)*
x)/(d*x^2+c)^2+1/8*(a^2*d^2-6*a*b*c*d-3*b^2*c^2)/c/(c*d)^(1/2)*arctan(d*x/(
c*d)^(1/2)))
```

Maxima [A]
time = 0.55, size = 266, normalized size = 1.72

$$-\frac{a b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{{\left(b^{3} c^{3} - 3 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} - a^{3} d^{3}\right)} \sqrt{a b}} + \frac{{\left(3 \, b^{2} c^{2} + 6 \, a b c d - a^{2} d^{2}\right)} \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 \, {\left(b^{3} c^{4} - 3 \, a b^{2} c^{3} d + 3 \, a^{2} b c^{2} d^{2} - a^{3} c d^{3}\right)} \sqrt{c d}} + \frac{{\left(3 \, b c d + a d^{2}\right)} x^{3} + {\left(5 \, b c^{2} - a c d\right)} x}{8 \, {\left(b^{2} c^{5} - 2 \, a b c^{4} d + a^{2} c^{3} d^{2} + {\left(b^{2} c^{3} d^{2} - 2 \, a b c^{2} d^{3} + a^{2} c d^{4}\right)} x^{4} + 2 \, {\left(b^{2} c^{4} d - 2 \, a b c^{3} d^{2} + a^{2} c^{2} d^{3}\right)} x^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -a*b^2*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a*b)) + 1/8*(3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c
*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c*d)) +
1/8*((3*b*c*d + a*d^2)*x^3 + (5*b*c^2 - a*c*d)*x)/(b^2*c^5 - 2*a*b*c^4*d +
a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d
- 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(133) = 266.
time = 1.48, size = 1587, normalized size = 10.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 8*(b*c^2*d^3*x^4
 + 2*b*c^3*d^2*x^2 + b*c^4*d)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)) - (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a
*b*c*d^3 - a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*
sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(5*b^2*c^4*d -
 6*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5
*d^3 - a^3*c^4*d^4 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3
*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^
3*d^5)*x^2), 1/8*((3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 + (3*b^2*
c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x
^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt
(c*d)*x/c) - 4*(b*c^2*d^3*x^4 + 2*b*c^3*d^2*x^2 + b*c^4*d)*sqrt(-a*b)*log((
b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + (5*b^2*c^4*d - 6*a*b*c^3*d^2 + a
^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4
 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^4 + 2*
(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2), 1/16*
(2*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 16*(b*c^2*d^3*x^4 + 2*
b*c^3*d^2*x^2 + b*c^4*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (3*b^2*c^4 + 6*a
*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x^4 + 2*(3
*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*(5*b^2*c^4*d - 6*a*b*c^3*d^2 + a^2*c^2*d^3)*
x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4 + (b^3*c^5*
d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2
 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2), 1/8*((3*b^2*c^3*d
^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 8*(b*c^2*d^3*x^4 + 2*b*c^3*d^2*x^2 +
b*c^4*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c
^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a
*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (5*b^2*c^4*d
 - 6*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c
^5*d^3 - a^3*c^4*d^4 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a
^3*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*
c^3*d^5)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.77, size = 206, normalized size = 1.33

$$-\frac{ab^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}}+\frac{(3\,b^2c^2+6\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{cd}}+\frac{3\,bcdx^3+ad^2x^3+5\,bc^2x-acdx}{8\,(b^2c^3-2\,abc^2d+a^2cd^2)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -a*b^2*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a*b)) + 1/8*(3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c
*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c*d)) +
1/8*(3*b*c*d*x^3 + a*d^2*x^3 + 5*b*c^2*x - a*c*d*x)/((b^2*c^3 - 2*a*b*c^2*d
 + a^2*c*d^2)*(d*x^2 + c)^2)
```

Mupad [B]
time = 1.06, size = 2500, normalized size = 16.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] (atan((((-a*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9 - 992*a^2*b
^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*a^5*b^5*c^4
*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8 + a^6*c^2*d
^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2
*c^4*d^4 - 6*a*b^5*c^7*d)) - (x*(-a*b^3)^(1/2)*(256*b^9*c^9*d^2 - 1280*a*b^
8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a^4*b^5*c^5*
d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2*c^2*d^9))/(
64*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(b^4*c^6 + a^4*c^2*d
^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-a*b^3)^(1/2))
/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(9*b^7*c^4*d
+ a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^4 + 94*a^2*b^5*c^2*d^3))/
(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*
c^5*d)))*1i)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - ((-a
*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9 - 992*a^2*b^8*c^7*d^3
+ 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*a^5*b^5*c^4*d^6 - 1440
*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*
b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 -
6*a*b^5*c^7*d)) + (x*(-a*b^3)^(1/2)*(256*b^9*c^9*d^2 - 1280*a*b^8*c^8*d^3 +
 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a^4*b^5*c^5*d^6 + 2304*
a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2*c^2*d^9))/(64*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*
b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^
3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(9*b^7*c^4*d + a^4*b^3*d
```

^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^4 + 94*a^2*b^5*c^2*d^3))/(32*(b^4*c^
6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))*1i
)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/((3*a^3*b^5*c*d^
3 - a^4*b^4*d^4 + 21*a^2*b^6*c^2*d^2 + 9*a*b^7*c^3*d)/(32*(b^6*c^8 + a^6*c^
2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*
b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + ((-a*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*
a^8*b^2*c*d^9 - 992*a^2*b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c
^5*d^5 + 3040*a^5*b^5*c^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)
/(64*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3
*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (x*(-a*b^3)^(1/2)*(25
6*b^9*c^9*d^2 - 1280*a*b^8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^
6*d^5 - 1280*a^4*b^5*c^5*d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8
+ 256*a^7*b^2*c^2*d^9))/(64*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3
*c^5*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
d^2)) - (x(9*b^7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^
4 + 94*a^2*b^5*c^2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a
^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + ((-a*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9
- 992*a^2*b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*
a^5*b^5*c^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8
+ a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 +
15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (x*(-a*b^3)^(1/2)*(256*b^9*c^9*d^2
- 1280*a*b^8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a
^4*b^5*c^5*d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2*
c^2*d^9))/(64*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(b^4*c^6
+ a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-a*
b^3)^(1/2))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(9
*b^7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^4 + 94*a^2*b^5
*c^2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2
- 4*a*b^3*c^5*d))))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
)))*(-a*b^3)^(1/2)*1i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
- ((x*(a*d - 5*b*c))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (x^3*(a*d^2 + 3*
b*c*d))/(8*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 + 2*c*d*x^2)
- (atan((((-c^3*d)^(1/2)*((x*(9*b^7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2
- 12*a^3*b^4*c*d^4 + 94*a^2*b^5*c^2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^
3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)) - (((160*a*b^9*c^8*d^2 -
32*a^8*b^2*c*d^9 - 992*a^2*b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^
6*c^5*d^5 + 3040*a^5*b^5*c^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d
^8)/(64*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*
a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (x*(-c^3*d)^(1/2)*
(3*b^2*c^2 - a^2*d^2 + 6*a*b*c*d)*(256*b^9*c^9*d^2 - 1280*a*b^8*c^8*d^3 + 2
304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a^4*b^5*c^5*d^6 + 2304*a^
5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*...

3.255 $\int \frac{x}{(a+bx^2)(c+dx^2)^3} \, dx$

Optimal. Leaf size=98

$$\frac{1}{4(bc-ad)(c+dx^2)^2}+\frac{b}{2(bc-ad)^2(c+dx^2)}+\frac{b^2\log(a+bx^2)}{2(bc-ad)^3}-\frac{b^2\log(c+dx^2)}{2(bc-ad)^3}$$

```
[Out] 1/4/(-a*d+b*c)/(d*x^2+c)^2+1/2*b/(-a*d+b*c)^2/(d*x^2+c)+1/2*b^2*ln(b*x^2+a)
/(-a*d+b*c)^3-1/2*b^2*ln(d*x^2+c)/(-a*d+b*c)^3
```

Rubi [A]

time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 46}

$$\frac{b^2\log(a+bx^2)}{2(bc-ad)^3}-\frac{b^2\log(c+dx^2)}{2(bc-ad)^3}+\frac{b}{2(c+dx^2)(bc-ad)^2}+\frac{1}{4(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] 1/(4*(b*c - a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x^2)) + (b^2*Lo
g[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^2)(c+dx^2)^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^3} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\right.\right. \\
&= \frac{1}{4(bc-ad)(c+dx^2)^2}+\frac{b}{2(bc-ad)^2(c+dx^2)}+\frac{b^2\log(a+bx^2)}{2(bc-ad)^3}-\frac{b^2\log(c+d}{2(bc-ad}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 1.00

$$-\frac{1}{4(-bc+ad)\left(c+dx^2\right)^2}+\frac{b}{2(bc-ad)^2\left(c+dx^2\right)}+\frac{b^2\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{b^2\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*1/((-(b*c) + a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x^2)) + (
b^2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^2])/(2*(b*c - a*d)
^3)
```

Maple [A]
time = 0.14, size = 111, normalized size = 1.13

method	result	size
default	$-\frac{b^2\ln\left(b\,x^2+a\right)}{2(ad-bc)^3}+\frac{d\left(\frac{b(ad-bc)}{d\left(d\,x^2+c\right)}-\frac{a^2d^2-2abcd+b^2c^2}{2d\left(d\,x^2+c\right)^2}+\frac{b^2\ln\left(d\,x^2+c\right)}{d}\right)}{2(ad-bc)^3}$	111
risch	$\frac{\frac{bd\,x^2}{2a^2d^2-4abcd+2b^2c^2}-\frac{ad-3bc}{4\left(a^2d^2-2abcd+b^2c^2\right)}}{\left(d\,x^2+c\right)^2}+\frac{b^2\ln\left(d\,x^2+c\right)}{2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3}-\frac{b^2\ln\left(-b\,x^2-a\right)}{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	181
norman	$\frac{\frac{-a\,d^3+3bc\,d^2}{4d^2\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{bd\,x^2}{2a^2d^2-4abcd+2b^2c^2}}{\left(d\,x^2+c\right)^2}-\frac{b^2\ln\left(b\,x^2+a\right)}{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{b^2\ln\left(d\,x^2+c\right)}{2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3}$	187

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2/(a*d-b*c)^3*ln(b*x^2+a)+1/2*d/(a*d-b*c)^3*(b*(a*d-b*c)/d/(d*x^2+c)
-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c)^2+b^2/d*ln(d*x^2+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(90) = 180.
time = 0.33, size = 211, normalized size = 2.15

$$\frac{b^2\log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}-\frac{b^2\log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{2\,bdx^2+3\,bc-ad}{4\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^4+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
- 1/2*b^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
) + 1/4*(2*b*d*x^2 + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^
2*c*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(90) = 180$.
time = 1.20, size = 254, normalized size = 2.59

$$\frac{3\,b^2c^2 - 4\,abcd + a^2d^2 + 2\,(b^2cd - abd^2)x^2 + 2\,(b^2d^2x^4 + 2\,b^2cdx^2 + b^2c^2)\log\left(bx^2 + a\right) - 2\,(b^2d^2x^4 + 2\,b^2cdx^2 + b^2c^2)\log\left(dx^2 + c\right)}{4\,(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3 + (b^3c^3d^2 - 3\,ab^2c^2d^3 + 3\,a^2bcd^4 - a^3d^5)x^4 + 2\,(b^3c^4d - 3\,ab^2c^3d^2 + 3\,a^2bc^2d^3 - a^3cd^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d
^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(b*x^2 + a) - 2*(b^2*d^2*x^4 + 2*b^2*c
*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^
2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)
*x^4 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(80) = 160$.
time = 2.37, size = 391, normalized size = 3.99

$$\frac{b^2\log\left(x^2 + \frac{-\frac{a^4b^2d^4}{(ad-bc)^3} + \frac{4a^3b^3cd^3}{(ad-bc)^3} - \frac{6a^2b^4c^2d^2}{(ad-bc)^3} + \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d - \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{2\,(ad-bc)^3} - \frac{b^2\log\left(x^2 + \frac{\frac{a^4b^2d^4}{(ad-bc)^3} - \frac{4a^3b^3cd^3}{(ad-bc)^3} + \frac{6a^2b^4c^2d^2}{(ad-bc)^3} - \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d + \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{2\,(ad-bc)^3} + \frac{-ad + 3bc + 2bdx^2}{4a^2c^2d^2 - 8abc^3d + 4b^2c^4 + x^4\cdot(4a^2d^4 - 8abcd^3 + 4b^2c^2d^2) + x^2\cdot(8a^2cd^3 - 16abc^2d^2 + 8b^2c^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] b**2*log(x**2 + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d -
 b*c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*
c)**3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(2*(a*d -
 b*c)**3) - b**2*log(x**2 + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*
d**3/(a*d - b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*
d/(a*d - b*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d)
)/(2*(a*d - b*c)**3) + (-a*d + 3*b*c + 2*b*d*x**2)/(4*a**2*c**2*d**2 - 8*a*
b*c**3*d + 4*b**2*c**4 + x**4*(4*a**2*d**4 - 8*a*b*c*d**3 + 4*b**2*c**2*d**
2) + x**2*(8*a**2*c*d**3 - 16*a*b*c**2*d**2 + 8*b**2*c**3*d))
```

Giac [A]
time = 0.91, size = 174, normalized size = 1.78

$$\frac{b^3\log\left(|bx^2 + a|\right)}{2\,(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3)} - \frac{b^2d\log\left(|dx^2 + c|\right)}{2\,(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4)} + \frac{3\,b^2c^2 - 4\,abcd + a^2d^2 + 2\,(b^2cd - abd^2)x^2}{4\,(dx^2 + c)^2(bc - ad)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^
3*b*d^3) - 1/2*b^2*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
```

```
^2*b*c*d^3 - a^3*d^4) + 1/4*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d -
 a*b*d^2)*x^2)/((d*x^2 + c)^2*(b*c - a*d)^3)
```

Mupad [B]
time = 0.16, size = 340, normalized size = 3.47

$$\frac{a^2 d^2 + 3 b^2 c^2 + b^2 c^2 \operatorname{atan}\left(\frac{a d x^2 1i - b c x^2 1i}{2 a c + a d x^2 + b c x^2}\right) 4i + b^2 d^2 x^4 \operatorname{atan}\left(\frac{a d x^2 1i - b c x^2 1i}{2 a c + a d x^2 + b c x^2}\right) 4i - 2 a b d^2 x^2 + 2 b^2 c d x^2 - 4 a b c d + b^2 c d x^2 \operatorname{atan}\left(\frac{a d x^2 1i - b c x^2 1i}{2 a c + a d x^2 + b c x^2}\right) 8i}{-4 a^3 c^2 d^3 - 8 a^3 c d^4 x^2 - 4 a^3 d^5 x^4 + 12 a^2 b c^3 d^2 + 24 a^2 b c^2 d^3 x^2 + 12 a^2 b c d^4 x^4 - 12 a b^2 c^4 d - 24 a b^2 c^3 d^2 x^2 - 12 a b^2 c^2 d^3 x^4 + 4 b^3 c^5 + 8 b^3 c^4 d x^2 + 4 b^3 c^3 d^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] (a^2*d^2 + 3*b^2*c^2 + b^2*c^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*
x^2 + b*c*x^2))*4i + b^2*d^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*
d*x^2 + b*c*x^2))*4i - 2*a*b*d^2*x^2 + 2*b^2*c*d*x^2 - 4*a*b*c*d + b^2*c*d*
x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i)/(4*b^3*
c^5 - 4*a^3*c^2*d^3 - 4*a^3*d^5*x^4 + 12*a^2*b*c^3*d^2 - 8*a^3*c*d^4*x^2 +
8*b^3*c^4*d*x^2 + 4*b^3*c^3*d^2*x^4 - 12*a*b^2*c^4*d + 12*a^2*b*c*d^4*x^4 -
 24*a*b^2*c^3*d^2*x^2 + 24*a^2*b*c^2*d^3*x^2 - 12*a*b^2*c^2*d^3*x^4)
```

3.256 $\int \frac{1}{(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=160

$$-\frac{dx}{4c(bc-ad)(c+dx^2)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-\frac{\sqrt{d}\,(15b^2c^2-10abcd+3a^2d^2)\tan^{-}}{8c^{5/2}(bc-ad)^3}$$

```
[Out] -1/4*d*x/c/(-a*d+b*c)/(d*x^2+c)^2-1/8*d*(-3*a*d+7*b*c)*x/c^2/(-a*d+b*c)^2/(
d*x^2+c)+b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*c)^3/a^(1/2)-1/8*(3*a^2*
d^2-10*a*b*c*d+15*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(5/2)/(-a*d+
b*c)^3
```

Rubi [A]
time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {425, 541, 536, 211}

$$-\frac{\sqrt{d}\,(3a^2d^2-10abcd+15b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\,(c+dx^2)\,(bc-ad)^2}-\frac{dx}{4c\,(c+dx^2)^2\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8*c^2*(b*
c - a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b
*c - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{dx}{4c(bc-ad)(c+dx^2)^2}+\frac{\int \frac{4bc-3ad-3bdx^2}{(a+bx^2)(c+dx^2)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)}+\frac{\int \frac{8b^2c^2-7abcd+3a^2d^2-bd(7bc-}{(a+bx^2)(c+dx^2)}}{8c^2(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)}+\frac{b^3\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3}-\frac{(d(15b^2c}{}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^2)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-$$

Mathematica [A]
time = 0.17, size = 158, normalized size = 0.99

$$\frac{1}{8}\left(-\frac{2dx}{c(bc-ad)(c+dx^2)^2}+\frac{d(-7bc+3ad)x}{c^2(bc-ad)^2(c+dx^2)}-\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(-bc+ad)^3}-\frac{\sqrt{d}\,(15b^2c^2-10abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-2*d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (d*(-7*b*c + 3*a*d)*x)/(c^2*(b*c
- a*d)^2*(c + d*x^2)) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-
(b*c) + a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.08, size = 158, normalized size = 0.99

method	result	si
default	$-\frac{b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{(a d-b c)^{3} \sqrt{a b}}+\frac{d\left(\frac{\frac{d\left(3 a^{2} d^{2}-10 a b c d+7 b^{2} c^{2}\right) x^{3}}{8 c^{2}}+\frac{\left(5 a^{2} d^{2}-14 a b c d+9 b^{2} c^{2}\right) x}{8 c}}{\left(d x^{2}+c\right)^{2}}+\frac{\left(3 a^{2} d^{2}-10 a b c d+15 b^{2} c^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 c^{2} \sqrt{c d}}\right)}{(a d-b c)^{3}}$	1
risch	Expression too large to display	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^3*((1/8*d*
(3*a^2*d^2-10*a*b*c*d+7*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-14*a*b*c*d+9*b^2*c^
2)/c*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2-10*a*b*c*d+15*b^2*c^2)/c^2/(c*d)^(1/2)*a
rctan(d*x/(c*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(138) = 276$.
time = 0.54, size = 277, normalized size = 1.73

$$\frac{b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) \sqrt{a b}}-\frac{\left(15 b^{2} c^{2} d-10 a b c d^{2}+3 a^{2} d^{3}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8\left(b^{3} c^{5}-3 a b^{2} c^{4} d+3 a^{2} b c^{3} d^{2}-a^{3} c^{2} d^{3}\right) \sqrt{c d}}-\frac{\left(7 b c d^{2}-3 a d^{3}\right) x^{3}+\left(9 b c^{2} d-5 a c d^{2}\right) x}{8\left(b^{2} c^{6}-2 a b c^{5} d+a^{2} c^{4} d^{2}+\left(b^{2} c^{4} d^{2}-2 a b c^{3} d^{3}+a^{2} c^{2} d^{4}\right) x^{4}+2\left(b^{2} c^{5} d-2 a b c^{4} d^{2}+a^{2} c^{3} d^{3}\right) x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*((7*b*c*d^2 - 3*a*d^3)*x^3 + (9*b*c^2*d - 5*a*c*d^2)*x)/(b^2*c^6
 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*
x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(138) = 276$.
time = 1.80, size = 1585, normalized size = 9.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2*c^2*d^2*x
^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*
d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c))
+ 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*
d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*
b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4
*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*
x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 +
b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (9*b^
2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2
*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4
 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^
3*c^3*d^4)*x^2), -1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 -
16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/
a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(9*b^2*c
^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*
c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 -
a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c
^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 8*(b^2
*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (
15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3
+ 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x
)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 -
 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*
b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 1.30, size = 217, normalized size = 1.36

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{ab}} - \frac{(15\,b^2c^2d - 10\,abcd^2 + 3\,a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3)\sqrt{cd}} - \frac{7\,bcd^2x^3 - 3\,ad^3x^3 + 9\,bc^2dx - 5\,acd^2x}{8\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2)(dx^2 + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((b^2*
c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

Mupad [B]
time = 1.15, size = 2500, normalized size = 15.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((x^3*(3*a*d^3 - 7*b*c*d^2))/(8*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(
5*a*d^2 - 9*b*c*d))/(8*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 +
 2*c*d*x^2) - (atan((((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 -
 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8
+ a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-
a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^
4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 30
40*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^
10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^
3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*
d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1
280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7
*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*
(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*
d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(
a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) + ((-a*b^5)^(1/2)*(
(x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6
+ 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^
2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 176
0*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^
6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d
^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 1
5*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d
```

)) + (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))/((9*a^3*b^5*d^6 - 105*b^8*c^3*d^3 + 115*a*b^7*c^2*d^4 - 51*a^2*b^6*c*d^5)/(32*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + ((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) - ((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))*(-a*b^5)^(1/2)*1i)/(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2) - (atan((((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 -...

3.257 $\int \frac{1}{x(a+bx^2)(c+dx^2)^3} dx$

Optimal. Leaf size=149

$$-\frac{d}{4c(bc-ad)(c+dx^2)^2}-\frac{d(2bc-ad)}{2c^2(bc-ad)^2(c+dx^2)}+\frac{\log(x)}{ac^3}-\frac{b^3\log(a+bx^2)}{2a(bc-ad)^3}+\frac{d(3b^2c^2-3abcd+a^2d^2)\log(c+dx^2)}{2c^3(bc-ad)^3}$$

```
[Out] -1/4*d/c/(-a*d+b*c)/(d*x^2+c)^2-1/2*d*(-a*d+2*b*c)/c^2/(-a*d+b*c)^2/(d*x^2+
c)+ln(x)/a/c^3-1/2*b^3*ln(b*x^2+a)/a/(-a*d+b*c)^3+1/2*d*(a^2*d^2-3*a*b*c*d+
3*b^2*c^2)*ln(d*x^2+c)/c^3/(-a*d+b*c)^3
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 84}

$$\frac{d(a^2d^2-3abcd+3b^2c^2)\log(c+dx^2)}{2c^3(bc-ad)^3}-\frac{b^3\log(a+bx^2)}{2a(bc-ad)^3}-\frac{d(2bc-ad)}{2c^2(c+dx^2)(bc-ad)^2}-\frac{d}{4c(c+dx^2)^2(bc-ad)}+\frac{\log(x)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*d/(c*(b*c - a*d)*(c + d*x^2)^2) - (d*(2*b*c - a*d))/(2*c^2*(b*c - a*d)
^2*(c + d*x^2)) + Log[x]/(a*c^3) - (b^3*Log[a + b*x^2])/(2*a*(b*c - a*d)^3)
 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^
3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(a+bx^2)\,(c+dx^2)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ac^3x}+\frac{b^4}{a(-bc+ad)^3(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)^3}+\frac{d^2}{c^2(bc-}\right.\right.\\ &= -\frac{d}{4c(bc-ad)\,(c+dx^2)^2}-\frac{d(2bc-ad)}{2c^2(bc-ad)^2\,(c+dx^2)}+\frac{\log(x)}{ac^3}-\frac{b^3\log(a+bx}{2a(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 141, normalized size = 0.95

$$\frac{\log(x)}{ac^3}+\frac{\frac{2b^3\log(a+bx^2)}{a}+\frac{d\left(\frac{c\left(a^2d^2\left(3c+2dx^2\right)-2abcd\left(4c+3dx^2\right)+b^2c^2\left(5c+4dx^2\right)\right)}{\left(c+dx^2\right)^2}-2\left(3b^2c^2-3abcd+a^2d^2\right)\log\left(c+dx^2\right)\right)}{c^3}}{4(-bc+ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] Log[x]/(a*c^3) + ((2*b^3*Log[a + b*x^2])/a + (d*((c*(a^2*d^2*(3*c + 2*d*x^2
) - 2*a*b*c*d*(4*c + 3*d*x^2) + b^2*c^2*(5*c + 4*d*x^2)))/(c + d*x^2)^2 - 2
*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c + d*x^2]))/c^3)/(4*(-(b*c) + a*d)^
3)
```

Maple [A]
time = 0.16, size = 165, normalized size = 1.11

method	result
default	$\frac{b^3\ln\left(b\,x^2+a\right)}{2a(ad-bc)^3}-\frac{d^2\left(-\frac{c\left(a^2d^2-3abcd+2b^2c^2\right)}{d\left(d\,x^2+c\right)}-\frac{c^2\left(a^2d^2-2abcd+b^2c^2\right)}{2d\left(d\,x^2+c\right)^2}+\frac{\left(a^2d^2-3abcd+3b^2c^2\right)\ln\left(d\,x^2+c\right)}{d}\right)}{2c^3(ad-bc)^3}+\frac{\ln(x)}{c^3a}$
norman	$\frac{\frac{\left(-3a\,d^2+5bcd\right)d^2x^4}{4c^3\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(-2a\,d^2+3bcd\right)d\,x^2}{2c^2\left(a^2d^2-2abcd+b^2c^2\right)}}{(d\,x^2+c)^2}+\frac{\ln(x)}{c^3a}+\frac{b^3\ln\left(b\,x^2+a\right)}{2a(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}-\frac{d(a^2d^2-3abcd+3b^2c^2)\ln(d}{2c^3(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d}$
risch	$\frac{\frac{d^2(ad-2bc)x^2}{2c^2\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d(3ad-5bc)}{4c\left(a^2d^2-2abcd+b^2c^2\right)}}{(d\,x^2+c)^2}+\frac{\ln(x)}{c^3a}-\frac{d^3\ln\left(-d\,x^2-c\right)a^2}{2c^3(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{3d^2\ln\left(-d\,x^2-c\right)ab}{2c^2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^3/a/(a*d-b*c)^3*ln(b*x^2+a)-1/2*d^2/c^3/(a*d-b*c)^3*(-c*(a^2*d^2-3*a*
b*c*d+2*b^2*c^2)/d/(d*x^2+c)-1/2*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c
)^2+(a^2*d^2-3*a*b*c*d+3*b^2*c^2)/d*ln(d*x^2+c))+ln(x)/c^3/a
```

Maxima [A]
time = 0.33, size = 278, normalized size = 1.87

$$-\frac{b^3\log\left(bx^2+a\right)}{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)}+\frac{\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)\log\left(dx^2+c\right)}{2\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3\right)}-\frac{5\,bc^2d-3\,acd^2+2\left(2\,bcd^2-ad^3\right)x^2}{4\left(b^2c^6-2\,abc^5d+a^2c^4d^2+\left(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4\right)x^4+2\left(b^2c^5d-2\,abc^4d^2+a^2c^3d^3\right)x^2\right)}+\frac{\log\left(x^2\right)}{2\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(b*x^2 + a)/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*
d^3) + 1/2*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x^2 + c)/(b^3*c^6 -
3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3) - 1/4*(5*b*c^2*d - 3*a*c*d^2
 + 2*(2*b*c*d^2 - a*d^3)*x^2)/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c
^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 +
a^2*c^3*d^3)*x^2) + 1/2*log(x^2)/(a*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(141) = 282.
time = 6.31, size = 520, normalized size = 3.49

$$-\frac{5\,ab^2c^4d-8\,a^2bc^3d^2+3\,a^3c^2d^3+2\left(2\,ab^2c^3d^2-3\,a^2bc^2d^3+a^3cd^4\right)x^2+2\left(b^3c^3d^2x^4+2\,b^3c^4dx^2+b^3c^5\right)\log\left(bx^2+a\right)-2\left(3\,ab^2c^4d-3\,a^2bc^3d^2+a^3c^2d^3+\left(3\,ab^2c^2d^3-3\,a^2bcd^4+a^3d^5\right)x^4+2\left(3\,ab^2c^3d^2-3\,a^2bc^2d^3+a^3cd^4\right)x^2\right)\log\left(dx^2+c\right)-4\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3+\left(b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bcd^4-a^3d^5\right)x^4+2\left(b^3c^4d-3\,ab^2c^3d^2+3\,a^2bc^2d^3-a^3cd^4\right)x^2\right)\log\left(x\right)}{4\left(ab^3c^8-3\,a^2b^2c^7d+3\,a^3bc^6d^2-a^4c^5d^3+\left(ab^3c^6d^2-3\,a^2b^2c^5d^3+3\,a^3bc^4d^4-a^4c^3d^5\right)x^4+2\left(ab^3c^7d-3\,a^2b^2c^6d^2+3\,a^3bc^5d^3-a^4c^4d^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(5*a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 3*a^3*c^2*d^3 + 2*(2*a*b^2*c^3*d^2
- 3*a^2*b*c^2*d^3 + a^3*c*d^4)*x^2 + 2*(b^3*c^3*d^2*x^4 + 2*b^3*c^4*d*x^2 +
 b^3*c^5)*log(b*x^2 + a) - 2*(3*a*b^2*c^4*d - 3*a^2*b*c^3*d^2 + a^3*c^2*d^3
 + (3*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 + a^3*d^5)*x^4 + 2*(3*a*b^2*c^3*d^2 - 3
*a^2*b*c^2*d^3 + a^3*c*d^4)*x^2)*log(d*x^2 + c) - 4*(b^3*c^5 - 3*a*b^2*c^4*
d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*
b*c*d^4 - a^3*d^5)*x^4 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 -
 a^3*c*d^4)*x^2)*log(x))/(a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*c^6*d^2 - a
^4*c^5*d^3 + (a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3
*d^5)*x^4 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*
d^4)*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(141) = 282.
time = 1.23, size = 315, normalized size = 2.11

$$-\frac{b^4\log\left(|bx^2+a|\right)}{2\left(ab^4c^3-3\,a^2b^3c^2d+3\,a^3b^2cd^2-a^4bd^3\right)}+\frac{\left(3\,b^2c^2d^2-3\,abcd^3+a^2d^4\right)\log\left(|dx^2+c|\right)}{2\left(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4\right)}-\frac{9\,b^2c^2d^3x^4-9\,abcd^4x^4+3\,a^2d^5x^4+22\,b^2c^3d^2x^2-24\,abc^2d^3x^2+8\,a^2cd^4x^2+14\,b^2c^4d-17\,abc^3d^2+6\,a^2c^2d^3}{4\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3\right)\left(dx^2+c\right)^2}+\frac{\log\left(x^2\right)}{2\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^4*log(abs(b*x^2 + a))/(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2
 - a^4*b*d^3) + 1/2*(3*b^2*c^2*d^2 - 3*a*b*c*d^3 + a^2*d^4)*log(abs(d*x^2 +
 c))/(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4) - 1/4*(9
*b^2*c^2*d^3*x^4 - 9*a*b*c*d^4*x^4 + 3*a^2*d^5*x^4 + 22*b^2*c^3*d^2*x^2 - 2
4*a*b*c^2*d^3*x^2 + 8*a^2*c*d^4*x^2 + 14*b^2*c^4*d - 17*a*b*c^3*d^2 + 6*a^2
*c^2*d^3)/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2
 + c)^2) + 1/2*log(x^2)/(a*c^3)
```

Mupad [B]
time = 0.78, size = 246, normalized size = 1.65

$$\frac{\frac{3\,a\,d^2-5\,b\,c\,d}{4\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{d^2\,x^2\,\left(a\,d-2\,b\,c\right)}{2\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{c^2+2\,c\,d\,x^2+d^2\,x^4}+\frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,a^4\,d^3-6\,a^3\,b\,c\,d^2+6\,a^2\,b^2\,c^2\,d-2\,a\,b^3\,c^3}+\frac{\ln\left(x\right)}{a\,c^3}+\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^3-3\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{-2\,a^3\,c^3\,d^3+6\,a^2\,b\,c^4\,d^2-6\,a\,b^2\,c^5\,d+2\,b^3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((3*a*d^2 - 5*b*c*d)/(4*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d^2*x^2*(a*d
- 2*b*c))/(2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 + 2*c*d*x
^2) + (b^3*log(a + b*x^2))/(2*a^4*d^3 - 2*a*b^3*c^3 + 6*a^2*b^2*c^2*d - 6*a
^3*b*c*d^2) + log(x)/(a*c^3) + (log(c + d*x^2)*(a^2*d^3 + 3*b^2*c^2*d - 3*a
*b*c*d^2))/(2*b^3*c^6 - 2*a^3*c^3*d^3 + 6*a^2*b*c^4*d^2 - 6*a*b^2*c^5*d)
```

3.258 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=211

$$-\frac{8b^2c^2-27abcd+15a^2d^2}{8ac^3(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\,(c+dx^2)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\,(c+dx^2)}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(3$$

```
[Out] 1/8*(-15*a^2*d^2+27*a*b*c*d-8*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x-1/4*d/c/(-a*d+b
*c)/x/(d*x^2+c)^2-1/8*d*(-5*a*d+9*b*c)/c^2/(-a*d+b*c)^2/x/(d*x^2+c)-b^(7/2)
*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)^3+1/8*d^(3/2)*(15*a^2*d^2-42*
a*b*c*d+35*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)^3
```

Rubi [A]

time = 0.20, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {483, 593, 597, 536, 211}

$$-\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(15a^2d^2-42abcd+35b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^3}-\frac{15a^2d^2-27abcd+8b^2c^2}{8ac^3x(bc-ad)^2}-\frac{d(9bc-5ad)}{8c^2x\,(c+dx^2)\,(bc-ad)^2}-\frac{d}{4cx\,(c+dx^2)^2\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/8*(8*b^2*c^2 - 27*a*b*c*d + 15*a^2*d^2)/(a*c^3*(b*c - a*d)^2*x) - d/(4*c
*(b*c - a*d)*x*(c + d*x^2)^2) - (d*(9*b*c - 5*a*d))/(8*c^2*(b*c - a*d)^2*x*
(c + d*x^2)) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^3
) + (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d + 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(8*c^(7/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = -\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-5ad-5bdx^2}{x^2(a+bx^2)(c+dx^2)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-27abcd+15a^2d^2}{x^2(a+bx^2)(}}{8c^2(bc-}$$

$$= -\frac{\frac{8b^2c}{a}-27bd+\frac{15ad^2}{c}}{8c^2(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}$$

$$= -\frac{\frac{8b^2c}{a}-27bd+\frac{15ad^2}{c}}{8c^2(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}$$

$$= -\frac{\frac{8b^2c}{a}-27bd+\frac{15ad^2}{c}}{8c^2(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}$$

Mathematica [A]

time = 0.29, size = 172, normalized size = 0.82

$$\frac{1}{8}\left(-\frac{8}{ac^3x}+\frac{2d^2x}{c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{d^2(11bc-7ad)x}{c^3(bc-ad)^2\left(c+dx^2\right)}+\frac{8b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(-bc+ad)^3}+\frac{d^{3/2}\left(35b^2c^2-42abcd+15a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a*c^3*x) + (2*d^2*x)/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(11*b*c -
7*a*d)*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (8*b^(7/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(a^(3/2)*(-(b*c) + a*d)^3) + (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d + 1
5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.24, size = 170, normalized size = 0.81

method	result
default	$\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a(ad-bc)^3\sqrt{ab}}-\frac{d^2\left(\frac{\left(\frac{7}{8}a^2d^3-\frac{9}{4}abc\,d^2+\frac{11}{8}b^2c^2d\right)x^3+\frac{c\left(9a^2d^2-22abcd+13b^2c^2\right)x}{8}}{\left(d\,x^2+c\right)^2}+\frac{\left(15a^2d^2-42abcd+35b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}}\right)}{c^3(ad-bc)^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*b^4/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-d^2/c^3/(a*d-b*c)^3
*(((7/8*a^2*d^3-9/4*a*b*c*d^2+11/8*b^2*c^2*d)*x^3+1/8*c*(9*a^2*d^2-22*a*b*c
*d+13*b^2*c^2)*x)/(d*x^2+c)^2+1/8*(15*a^2*d^2-42*a*b*c*d+35*b^2*c^2)/(c*d)^
(1/2)*arctan(d*x/(c*d)^(1/2)))-1/a/c^3/x
```

Maxima [A]
time = 0.51, size = 352, normalized size = 1.67

$$-\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\sqrt{ab}}+\frac{\left(35\,b^2c^2d^2-42\,abcd^3+15\,a^2d^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3\right)\sqrt{cd}}-\frac{8\,b^2c^4-16\,abc^3d+8\,a^2c^2d^2+\left(8\,b^2c^2d^2-27\,abcd^3+15\,a^2d^4\right)x^4+\left(16\,b^2c^3d-45\,abc^2d^2+25\,a^2cd^3\right)x^2}{8\left(\left(ab^2c^5d^2-2\,a^2bc^4d^3+a^3c^3d^4\right)x^5+2\left(ab^2c^6d-2\,a^2bc^5d^2+a^3c^4d^3\right)x^3+\left(ab^2c^7-2\,a^2bc^6d+a^3c^5d^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
a^4*d^3)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 42*a*b*c*d^3 + 15*a^2*d^4)*arct
an(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
)*sqrt(c*d)) - 1/8*(8*b^2*c^4 - 16*a*b*c^3*d + 8*a^2*c^2*d^2 + (8*b^2*c^2*d
```

```
^2 - 27*a*b*c*d^3 + 15*a^2*d^4)*x^4 + (16*b^2*c^3*d - 45*a*b*c^2*d^2 + 25*a
^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^5 + 2*(a*
b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^3 + (a*b^2*c^7 - 2*a^2*b*c^6*d
 + a^3*c^5*d^2)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(187) = 374$.
time = 2.90, size = 1991, normalized size = 9.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*b^3*c^5 - 48*a*b^2*c^4*d + 48*a^2*b*c^3*d^2 - 16*a^3*c^2*d^3 + 2
*(8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4 - 15*a^3*d^5)*x^4 + 2*(
16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3 - 25*a^3*c*d^4)*x^2 + 8*
(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-b/a)*log((b*x^2 + 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*
a^3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 +
 (35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(-d/c)*log((d*
x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d
^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^
2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a
^3*b*c^6*d^2 - a^4*c^5*d^3)*x), -1/8*(8*b^3*c^5 - 24*a*b^2*c^4*d + 24*a^2*b
*c^3*d^2 - 8*a^3*c^2*d^3 + (8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d
^4 - 15*a^3*d^5)*x^4 + (16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3
- 25*a^3*c*d^4)*x^2 - ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^3*d^5)*x^5
 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (35*a*b^2*c
^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ 4*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-b/a)*log((b*x^2 +
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 +
3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3
*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*
c^6*d^2 - a^4*c^5*d^3)*x), -1/16*(16*b^3*c^5 - 48*a*b^2*c^4*d + 48*a^2*b*c^
3*d^2 - 16*a^3*c^2*d^3 + 2*(8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d
^4 - 15*a^3*d^5)*x^4 + 2*(16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^
3 - 25*a^3*c*d^4)*x^2 + 16*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*
sqrt(b/a)*arctan(x*sqrt(b/a)) + ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^
3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (
35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(-d/c)*log((d*x^
2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3
 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2
+ 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3
*b*c^6*d^2 - a^4*c^5*d^3)*x), -1/8*(8*b^3*c^5 - 24*a*b^2*c^4*d + 24*a^2*b*c
```

```
^3*d^2 - 8*a^3*c^2*d^3 + (8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4
 - 15*a^3*d^5)*x^4 + (16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3 -
25*a^3*c*d^4)*x^2 + 8*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(
b/a)*arctan(x*sqrt(b/a)) - ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^3*d^5
)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (35*a*
b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d
/c)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*
x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x
^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*c^6*d^2 - a^4*c^5*d^3)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.78, size = 236, normalized size = 1.12

$$-\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{ab}}+\frac{(35\,b^2c^2d^2-42\,abcd^3+15\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)\sqrt{cd}}+\frac{11\,bcd^3x^3-7\,ad^4x^3+13\,bc^2d^2x-9\,acd^3x}{8\,(b^2c^5-2\,abc^4d+a^2c^3d^2)(dx^2+c)^2}-\frac{1}{ac^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
a^4*d^3)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 42*a*b*c*d^3 + 15*a^2*d^4)*arct
an(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
)*sqrt(c*d)) + 1/8*(11*b*c*d^3*x^3 - 7*a*d^4*x^3 + 13*b*c^2*d^2*x - 9*a*c*d
^3*x)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 1/(a*c^3*x)
```

Mupad [B]
time = 0.70, size = 738, normalized size = 3.50

$$-\frac{\frac{1}{a\,c}+\frac{x^4\,\left(15\,a^2\,d^4-27\,a\,b\,c\,d^3+8\,b^2\,c^2\,d^2\right)}{8\,a\,c^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{x^2\,\left(25\,a^2\,d^3-45\,a\,b\,c\,d^2+16\,b^2\,c^2\,d\right)}{8\,a\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{c^2\,x+2\,c\,d\,x^3+d^2\,x^5}-\frac{\mathrm{atan}\left(\frac{b\,c^7\,x\,{\left(-a^3\,b^7\right)}^{3/2}\,64{}\mathrm{i}+a^{10}\,b\,d^7\,x\,\sqrt{-a^3\,b^7}\,225{}\mathrm{i}+a^6\,b^5\,c^4\,d^3\,x\,\sqrt{-a^3\,b^7}\,1225{}\mathrm{i}-a^7\,b^4\,c^3\,d^4\,x\,\sqrt{-a^3\,b^7}\,2940{}\mathrm{i}+a^8\,b^3\,c^2\,d^5\,x\,\sqrt{-a^3\,b^7}\,2814{}\mathrm{i}-a^9\,b^2\,c\,d^6\,x\,\sqrt{-a^3\,b^7}\,1260{}\mathrm{i}}{a^3\,b^7\,\left(2940\,a^6\,c^3\,d^4-1225\,a^5\,b\,c^4\,d^3+64\,a^2\,b^4\,c^7\right)-225\,a^{12}\,b^4\,d^7+1260\,a^{11}\,b^5\,c\,d^6-2814\,a^{10}\,b^6\,c^2\,d^5}\right)\,\sqrt{-a^3\,b^7}\,1{}\mathrm{i}}{a^6\,d^3-3\,a^5\,b\,c\,d^2+3\,a^4\,b^2\,c^2\,d-a^3\,b^3\,c^3}-\frac{\mathrm{atan}\left(\frac{a^7\,d^5\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,225{}\mathrm{i}+b^7\,c^{14}\,d\,x\,\sqrt{-c^7\,d^3}\,64{}\mathrm{i}-a^4\,b^3\,c^3\,d^2\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,2940{}\mathrm{i}+a^5\,b^2\,c^2\,d^3\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,2814{}\mathrm{i}-a^6\,b\,c\,d^4\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,1260{}\mathrm{i}+a^3\,b^4\,c^4\,d\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,1225{}\mathrm{i}}{225\,a^7\,c^{11}\,d^9-1260\,a^6\,b\,c^{12}\,d^8+2814\,a^5\,b^2\,c^{13}\,d^7-2940\,a^4\,b^3\,c^{14}\,d^6+1225\,a^3\,b^4\,c^{15}\,d^5-64\,b^7\,c^{18}\,d^2}\right)\,\sqrt{-c^7\,d^3}\,\left(15\,a^2\,d^2-42\,a\,b\,c\,d+35\,b^2\,c^2\right)\,1{}\mathrm{i}}{8\,\left(-a^3\,c^7\,d^3+3\,a^2\,b\,c^8\,d^2-3\,a\,b^2\,c^9\,d+b^3\,c^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] - (1/(a*c) + (x^4*(15*a^2*d^4 + 8*b^2*c^2*d^2 - 27*a*b*c*d^3))/(8*a*c^3*(a^
2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(25*a^2*d^3 + 16*b^2*c^2*d - 45*a*b*c*
d^2))/(8*a*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2*x + d^2*x^5 + 2*c*d*x
```

```
^3) - (atan((b*c^7*x*(-a^3*b^7)^(3/2)*64i + a^10*b*d^7*x*(-a^3*b^7)^(1/2)*2
25i + a^6*b^5*c^4*d^3*x*(-a^3*b^7)^(1/2)*1225i - a^7*b^4*c^3*d^4*x*(-a^3*b^
7)^(1/2)*2940i + a^8*b^3*c^2*d^5*x*(-a^3*b^7)^(1/2)*2814i - a^9*b^2*c*d^6*x
*(-a^3*b^7)^(1/2)*1260i)/(a^3*b^7*(64*a^2*b^4*c^7 + 2940*a^6*c^3*d^4 - 1225
*a^5*b*c^4*d^3) - 225*a^12*b^4*d^7 + 1260*a^11*b^5*c*d^6 - 2814*a^10*b^6*c^
2*d^5))*(-a^3*b^7)^(1/2)*1i)/(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a
^5*b*c*d^2) - (atan((a^7*d^5*x*(-c^7*d^3)^(3/2)*225i + b^7*c^14*d*x*(-c^7*d
^3)^(1/2)*64i - a^4*b^3*c^3*d^2*x*(-c^7*d^3)^(3/2)*2940i + a^5*b^2*c^2*d^3*
x*(-c^7*d^3)^(3/2)*2814i - a^6*b*c*d^4*x*(-c^7*d^3)^(3/2)*1260i + a^3*b^4*c
^4*d*x*(-c^7*d^3)^(3/2)*1225i)/(225*a^7*c^11*d^9 - 64*b^7*c^18*d^2 - 1260*a
^6*b*c^12*d^8 + 1225*a^3*b^4*c^15*d^5 - 2940*a^4*b^3*c^14*d^6 + 2814*a^5*b^
2*c^13*d^7))*(-c^7*d^3)^(1/2)*(15*a^2*d^2 + 35*b^2*c^2 - 42*a*b*c*d)*1i)/(8
*(b^3*c^10 - a^3*c^7*d^3 + 3*a^2*b*c^8*d^2 - 3*a*b^2*c^9*d))
```

3.259 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=178

$$-\frac{1}{2ac^3x^2}+\frac{d^2}{4c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{d^2(3bc-2ad)}{2c^3(bc-ad)^2\left(c+dx^2\right)}-\frac{(bc+3ad)\log(x)}{a^2c^4}+\frac{b^4\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}-\frac{d^2(6b^2c^2-$$

```
[Out] -1/2/a/c^3/x^2+1/4*d^2/c^2/(-a*d+b*c)/(d*x^2+c)^2+1/2*d^2*(-2*a*d+3*b*c)/c^
3/(-a*d+b*c)^2/(d*x^2+c)-(3*a*d+b*c)*ln(x)/a^2/c^4+1/2*b^4*ln(b*x^2+a)/a^2/
(-a*d+b*c)^3-1/2*d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*ln(d*x^2+c)/c^4/(-a*d+
b*c)^3
```

Rubi [A]
time = 0.15, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{b^4\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}-\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^3}-\frac{\log(x)(3ad+bc)}{a^2c^4}+\frac{d^2(3bc-2ad)}{2c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{d^2}{4c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{1}{2ac^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/2*1/(a*c^3*x^2) + d^2/(4*c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(3*b*c -
2*a*d))/(2*c^3*(b*c - a*d)^2*(c + d*x^2)) - ((b*c + 3*a*d)*Log[x])/(a^2*c^4
) + (b^4*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3) - (d^2*(6*b^2*c^2 - 8*a*b*c*
d + 3*a^2*d^2)*Log[c + d*x^2])/(2*c^4*(b*c - a*d)^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ac^3x^2} + \frac{-bc-3ad}{a^2c^4x} - \frac{b^5}{a^2(-bc+ad)^3(a+bx)} - \frac{d^3}{c^2(bc-ad)(\cdots}\right.\right.$$
$$= -\frac{1}{2ac^3x^2} + \frac{d^2}{4c^2(bc-ad)(c+dx^2)^2} + \frac{d^2(3bc-2ad)}{2c^3(bc-ad)^2(c+dx^2)} - \frac{(bc+3ad)}{a^2c^4}\cdots$$

Mathematica [A]
time = 0.32, size = 171, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2}{ac^3x^2} + \frac{d^2}{c^2(bc-ad)(c+dx^2)^2} + \frac{2d^2(3bc-2ad)}{c^3(bc-ad)^2(c+dx^2)} - \frac{4(bc+3ad)\log(x)}{a^2c^4} - \frac{2b^4\log(a+bx^2)}{a^2(-bc+ad)^3} - \frac{2d^2(6b^2c^2-8abcd+3a^2d^2)\log(c+dx^2)}{c^4(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-2/(a*c^3*x^2) + d^2/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (2*d^2*(3*b*c - 2*a
*d))/(c^3*(b*c - a*d)^2*(c + d*x^2)) - (4*(b*c + 3*a*d)*Log[x])/(a^2*c^4) -
 (2*b^4*Log[a + b*x^2])/(a^2*(-(b*c) + a*d)^3) - (2*d^2*(6*b^2*c^2 - 8*a*b*
c*d + 3*a^2*d^2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^3))/4
```

Maple [A]
time = 0.17, size = 187, normalized size = 1.05

method	result
default	$-\frac{b^4\ln(bx^2+a)}{2a^2(ad-bc)^3} + \frac{d^3\left(-\frac{c(2a^2d^2-5abcd+3b^2c^2)}{d(dx^2+c)} - \frac{c^2(a^2d^2-2abcd+b^2c^2)}{2d(dx^2+c)^2} + \frac{(3a^2d^2-8abcd+6b^2c^2)\ln(dx^2+c)}{d}\right)}{2c^4(ad-bc)^3} - \frac{1}{2a\,c^3x^2} + \frac{(-3a\cdots}{}$
norman	$\frac{-\frac{1}{2ac} + \frac{(6a^2d^3-10abc\,d^2+3b^2c^2d)d\,x^4}{2a\,c^3(a^2d^2-2abcd+b^2c^2)} + \frac{(9a^2d^3-15abc\,d^2+4b^2c^2d)d^2x^6}{4c^4a(a^2d^2-2abcd+b^2c^2)}}{x^2(dx^2+c)^2} - \frac{b^4\ln(bx^2+a)}{2a^2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)} - \frac{(3ad+bc)\ln(x)}{a^2c^4} + \cdots$
risch	$\frac{-\frac{d^2(3a^2d^2-5abcd+b^2c^2)x^4}{2c^3a(a^2d^2-2abcd+b^2c^2)} - \frac{d(9a^2d^2-15abcd+4b^2c^2)x^2}{4c^2a(a^2d^2-2abcd+b^2c^2)} - \frac{1}{2ac}}{x^2(dx^2+c)^2} - \frac{3\ln(x)d}{a\,c^4} - \frac{\ln(x)b}{a^2c^3} + \frac{3d^4\ln(-dx^2-c)a^2}{2c^4(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)} - \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^4/a^2/(a*d-b*c)^3*ln(b*x^2+a)+1/2*d^3/c^4/(a*d-b*c)^3*(-c*(2*a^2*d^2
-5*a*b*c*d+3*b^2*c^2)/d/(d*x^2+c)-1/2*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*
x^2+c)^2+(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)/d*ln(d*x^2+c))-1/2/a/c^3/x^2+(-3*a
*d-b*c)/a^2/c^4*ln(x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(168) = 336$.
time = 0.30, size = 364, normalized size = 2.04

$$\frac{b^4 \log(bx^2+a)}{2(a^2b^3c^3 - 3a^3b^2c^2d + 3a^4bcd^2 - a^5d^3)} - \frac{(6b^2c^2d^2 - 8abcd^3 + 3a^2d^4)\log(dx^2+c)}{2(b^3c^7 - 3ab^2c^6d + 3a^2bc^5d^2 - a^3c^4d^3)} - \frac{2b^2c^4 - 4abc^3d + 2a^2c^2d^2 + 2(b^2c^2d^2 - 5abcd^3 + 3a^2d^4)x^4 + (4b^2c^3d - 15abc^2d^2 + 9a^2cd^3)x^2}{4((ab^2c^5d^2 - 2a^2bc^4d^3 + a^3c^3d^4)x^6 + 2(ab^2c^6d - 2a^2bc^5d^2 + a^3c^4d^3)x^4 + (ab^2c^7 - 2a^2bc^6d + a^3c^5d^2)x^2)} - \frac{(bc+3ad)\log(x^2)}{2a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5
*d^3) - 1/2*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*log(d*x^2 + c)/(b^3*c
^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3) - 1/4*(2*b^2*c^4 - 4*a*
b*c^3*d + 2*a^2*c^2*d^2 + 2*(b^2*c^2*d^2 - 5*a*b*c*d^3 + 3*a^2*d^4)*x^4 + (
4*b^2*c^3*d - 15*a*b*c^2*d^2 + 9*a^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*
c^4*d^3 + a^3*c^3*d^4)*x^6 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3
)*x^4 + (a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2)*x^2) - 1/2*(b*c + 3*a*d)*
log(x^2)/(a^2*c^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(168) = 336$.
time = 9.49, size = 640, normalized size = 3.60

$$-\frac{2ab^3c^6 - 6a^2b^2c^5d + 6a^3bc^4d^2 - 2a^4c^3d^3 + 2(ab^3c^4d^2 - 6a^2b^2c^3d^3 + 8a^3bc^2d^4 - 3a^4cd^5)x^4 + (4ab^3c^5d - 19a^2b^2c^4d^2 + 24a^3bc^3d^3 - 9a^4c^2d^4)x^2 - 2(b^4c^4d^2x^6 + 2b^4c^5dx^4 + b^4c^6x^2)\log(bx^2+a) + 2((6a^2b^2c^2d^4 - 8a^3bcd^5 + 3a^4d^6)x^6 + 2(6a^2b^2c^3d^3 - 8a^3bc^2d^4 + 3a^4cd^5)x^4 + (6a^2b^2c^4d^2 - 8a^3bc^3d^3 + 3a^4c^2d^4)x^2)\log(dx^2+c) + 4((b^4c^4d^2 - 6a^2b^2c^2d^4 + 8a^3bcd^5 - 3a^4d^6)x^6 + 2(b^4c^5d - 6a^2b^2c^3d^3 + 8a^3bc^2d^4 - 3a^4cd^5)x^4 + (b^4c^6 - 6a^2b^2c^4d^2 + 8a^3bc^3d^3 - 3a^4c^2d^4)x^2)\log(x)}{4((a^2b^3c^7d^2 - 3a^3b^2c^6d^3 + 3a^4bc^5d^4 - a^5c^4d^5)x^6 + 2(a^2b^3c^8d - 3a^3b^2c^7d^2 + 3a^4bc^6d^3 - a^5c^5d^4)x^4 + (a^2b^3c^9 - 3a^3b^2c^8d + 3a^4bc^7d^2 - a^5c^6d^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b^3*c^6 - 6*a^2*b^2*c^5*d + 6*a^3*b*c^4*d^2 - 2*a^4*c^3*d^3 + 2*(
a*b^3*c^4*d^2 - 6*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 - 3*a^4*c*d^5)*x^4 + (4
*a*b^3*c^5*d - 19*a^2*b^2*c^4*d^2 + 24*a^3*b*c^3*d^3 - 9*a^4*c^2*d^4)*x^2 -
 2*(b^4*c^4*d^2*x^6 + 2*b^4*c^5*d*x^4 + b^4*c^6*x^2)*log(b*x^2 + a) + 2*((6
*a^2*b^2*c^2*d^4 - 8*a^3*b*c*d^5 + 3*a^4*d^6)*x^6 + 2*(6*a^2*b^2*c^3*d^3 -
8*a^3*b*c^2*d^4 + 3*a^4*c*d^5)*x^4 + (6*a^2*b^2*c^4*d^2 - 8*a^3*b*c^3*d^3 +
 3*a^4*c^2*d^4)*x^2)*log(d*x^2 + c) + 4*((b^4*c^4*d^2 - 6*a^2*b^2*c^2*d^4 +
 8*a^3*b*c*d^5 - 3*a^4*d^6)*x^6 + 2*(b^4*c^5*d - 6*a^2*b^2*c^3*d^3 + 8*a^3*
b*c^2*d^4 - 3*a^4*c*d^5)*x^4 + (b^4*c^6 - 6*a^2*b^2*c^4*d^2 + 8*a^3*b*c^3*d
^3 - 3*a^4*c^2*d^4)*x^2)*log(x))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*
a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^6 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3
*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^4 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*
b*c^7*d^2 - a^5*c^6*d^3)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(168) = 336$.
time = 1.50, size = 357, normalized size = 2.01

$$\frac{b^5\log\left(|bx^2+a|\right)}{2\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)}-\frac{\left(6\,b^2c^2d^3-8\,abcd^4+3\,a^2d^5\right)\log\left(|dx^2+c|\right)}{2\left(b^3c^7d-3\,ab^2c^6d^2+3\,a^2bc^5d^3-a^3c^4d^4\right)}+\frac{18\,b^2c^2d^4x^4-24\,abcd^5x^4+9\,a^2d^6x^4+42\,b^2c^3d^3x^2-58\,abc^2d^4x^2+22\,a^2cd^5x^2+25\,b^2c^4d^2-36\,abc^3d^3+14\,a^2c^2d^4}{4\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3\right)(dx^2+c)^2}-\frac{(bc+3\,ad)\log\left(x^2\right)}{2\,a^2c^4}+\frac{bcx^2+3\,adx^2-ac}{2\,a^2c^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(abs(b*x^2 + a))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^
2 - a^5*b*d^3) - 1/2*(6*b^2*c^2*d^3 - 8*a*b*c*d^4 + 3*a^2*d^5)*log(abs(d*x^
2 + c))/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4) + 1/4
*(18*b^2*c^2*d^4*x^4 - 24*a*b*c*d^5*x^4 + 9*a^2*d^6*x^4 + 42*b^2*c^3*d^3*x^
2 - 58*a*b*c^2*d^4*x^2 + 22*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 36*a*b*c^3*d^3
 + 14*a^2*c^2*d^4)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^
3)*(d*x^2 + c)^2) - 1/2*(b*c + 3*a*d)*log(x^2)/(a^2*c^4) + 1/2*(b*c*x^2 + 3
*a*d*x^2 - a*c)/(a^2*c^4*x^2)
```

Mupad [B]
time = 0.85, size = 314, normalized size = 1.76

$$-\frac{\frac{1}{2\,a\,c}+\frac{x^4\,\left(3\,a^2\,d^4-5\,a\,b\,c\,d^3+b^2\,c^2\,d^2\right)}{2\,a\,c^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{x^2\,\left(9\,a^2\,d^3-15\,a\,b\,c\,d^2+4\,b^2\,c^2\,d\right)}{4\,a\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{c^2\,x^2+2\,c\,d\,x^4+d^2\,x^6}-\frac{\ln\left(d\,x^2+c\right)\,\left(3\,a^2\,d^4-8\,a\,b\,c\,d^3+6\,b^2\,c^2\,d^2\right)}{-2\,a^3\,c^4\,d^3+6\,a^2\,b\,c^5\,d^2-6\,a\,b^2\,c^6\,d+2\,b^3\,c^7}-\frac{b^4\,\ln\left(b\,x^2+a\right)}{2\,\left(a^5\,d^3-3\,a^4\,b\,c\,d^2+3\,a^3\,b^2\,c^2\,d-a^2\,b^3\,c^3\right)}-\frac{\ln\left(x\right)\,\left(3\,a\,d+b\,c\right)}{a^2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] - (1/(2*a*c) + (x^4*(3*a^2*d^4 + b^2*c^2*d^2 - 5*a*b*c*d^3))/(2*a*c^3*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(9*a^2*d^3 + 4*b^2*c^2*d - 15*a*b*c*d^2)
)/(4*a*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2*x^2 + d^2*x^6 + 2*c*d*x^4
) - (log(c + d*x^2)*(3*a^2*d^4 + 6*b^2*c^2*d^2 - 8*a*b*c*d^3))/(2*b^3*c^7 -
 2*a^3*c^4*d^3 + 6*a^2*b*c^5*d^2 - 6*a*b^2*c^6*d) - (b^4*log(a + b*x^2))/(2
*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (log(x)*(3*a*
d + b*c))/(a^2*c^4)
```

3.260 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=270

$$-\frac{8b^2c^2-55abcd+35a^2d^2}{24ac^3(bc-ad)^2x^3}+\frac{8b^3c^3+8ab^2c^2d-55a^2bcd^2+35a^3d^3}{8a^2c^4(bc-ad)^2x}-\frac{d}{4c(bc-ad)x^3\left(c+dx^2\right)^2}-\frac{d(11bc-}{8c^2(bc-ad)^2x}$$

```
[Out] 1/24*(-35*a^2*d^2+55*a*b*c*d-8*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x^3+1/8*(35*a^3*
d^3-55*a^2*b*c*d^2+8*a*b^2*c^2*d+8*b^3*c^3)/a^2/c^4/(-a*d+b*c)^2/x-1/4*d/c/
(-a*d+b*c)/x^3/(d*x^2+c)^2-1/8*d*(-7*a*d+11*b*c)/c^2/(-a*d+b*c)^2/x^3/(d*x^
2+c)+b^(9/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^3-1/8*d^(5/2)*(35
*a^2*d^2-90*a*b*c*d+63*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(9/2)/(-a*d+b*c
)^3
```

Rubi [A]

time = 0.30, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {483, 593, 597, 536, 211}

$$\frac{b^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^3}-\frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}-\frac{35a^2d^2-55abcd+8b^2c^2}{24ac^3x^3(bc-ad)^2}+\frac{35a^3d^3-55a^2bcd^2+8ab^2c^2d+8b^3c^3}{8a^2c^4x(bc-ad)^2}-\frac{d(11bc-7ad)}{8c^2x^3\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^3\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/24*(8*b^2*c^2 - 55*a*b*c*d + 35*a^2*d^2)/(a*c^3*(b*c - a*d)^2*x^3) + (8*
b^3*c^3 + 8*a*b^2*c^2*d - 55*a^2*b*c*d^2 + 35*a^3*d^3)/(8*a^2*c^4*(b*c - a*
d)^2*x) - d/(4*c*(b*c - a*d)*x^3*(c + d*x^2)^2) - (d*(11*b*c - 7*a*d))/(8*c
^2*(b*c - a*d)^2*x^3*(c + d*x^2)) + (b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*Ar
cTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^3} \, dx = -\frac{d}{4c(bc-ad)x^3 (c+dx^2)^2} + \frac{\int \frac{4bc-7ad-7bdx^2}{x^4(a+bx^2)(c+dx^2)^2} \, dx}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x^3 (c+dx^2)^2} - \frac{d(11bc-7ad)}{8c^2(bc-ad)^2x^3 (c+dx^2)} + \frac{\int \frac{8b^2c^2-55abcd+35a^2d \cdots}{x^4(a+bx^2) \cdots}}{8c^2(bc \cdots}$$

$$= -\frac{\frac{8b^2c}{a} - 55bd + \frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3} - \frac{d}{4c(bc-ad)x^3 (c+dx^2)^2} - \frac{d(11bc-7ad)}{8c^2(bc-ad)^2x^3 (c+dx^2 \cdots}$$

$$= -\frac{\frac{8b^2c}{a} - 55bd + \frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3} + \frac{8b^3c^3 + 8ab^2c^2d - 55a^2bcd^2 + 35a^3d^3}{8a^2c^4(bc-ad)^2x} - \frac{\cdots}{4c(bc-ad) \cdots}$$

$$= -\frac{\frac{8b^2c}{a} - 55bd + \frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3} + \frac{8b^3c^3 + 8ab^2c^2d - 55a^2bcd^2 + 35a^3d^3}{8a^2c^4(bc-ad)^2x} - \frac{\cdots}{4c(bc-ad) \cdots}$$

$$= -\frac{\frac{8b^2c}{a} - 55bd + \frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3} + \frac{8b^3c^3 + 8ab^2c^2d - 55a^2bcd^2 + 35a^3d^3}{8a^2c^4(bc-ad)^2x} - \frac{\cdots}{4c(bc-ad) \cdots}$$

Mathematica [A]
time = 0.32, size = 196, normalized size = 0.73

$$-\frac{1}{3ac^3x^3} + \frac{bc+3ad}{a^2c^4x} - \frac{d^3x}{4c^3(bc-ad)(c+dx^2)^2} - \frac{d^3(15bc-11ad)x}{8c^4(bc-ad)^2(c+dx^2)} - \frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^3} - \frac{d^{5/2}(63b^2c^2-90abcd+35a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/3*1/(a*c^3*x^3) + (b*c + 3*a*d)/(a^2*c^4*x) - (d^3*x)/(4*c^3*(b*c - a*d)
*(c + d*x^2)^2) - (d^3*(15*b*c - 11*a*d)*x)/(8*c^4*(b*c - a*d)^2*(c + d*x^2
)) - (b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)^3) - (d^
(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(
8*c^(9/2)*(b*c - a*d)^3)
```

Maple [A]
time = 0.24, size = 190, normalized size = 0.70

method	result
default	$-\frac{b^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2(ad-bc)^3\sqrt{ab}} + \frac{d^3\left(\frac{\left(\frac{11}{8}a^2d^3-\frac{13}{4}abc\,d^2+\frac{15}{8}b^2c^2d\right)x^3+\frac{c\left(13a^2d^2-30abcd+17b^2c^2\right)x}{8}}{\left(d\,x^2+c\right)^2}+\frac{\left(35a^2d^2-90abcd+63b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right.}{8\sqrt{cd}}\cdots\right.}{c^4(ad-bc)^3}\cdots$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*b^5/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d^3/c^4/(a*d-b*c
)^3*(((11/8*a^2*d^3-13/4*a*b*c*d^2+15/8*b^2*c^2*d)*x^3+1/8*c*(13*a^2*d^2-30
*a*b*c*d+17*b^2*c^2)*x)/(d*x^2+c)^2+1/8*(35*a^2*d^2-90*a*b*c*d+63*b^2*c^2)/
(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))-1/3/a/c^3/x^3-(-3*a*d-b*c)/a^2/c^4/x
```

Maxima [A]
time = 0.51, size = 440, normalized size = 1.63

$$\frac{b^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}}-\frac{(63\,b^2c^2d^3-90\,abcd^4+35\,a^2d^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3)\sqrt{cd}}-\frac{8\,ab^2c^5-16\,a^2bc^4d+8\,a^3c^3d^2-3\,(8\,b^3c^3d^2+8\,ab^2c^2d^3-55\,a^2bcd^4+35\,a^3d^5)x^6-(48\,b^3c^4d+40\,ab^2c^3d^2-275\,a^2bc^2d^3+175\,a^3cd^4)x^4-8\,(3\,b^3c^5+ab^2c^4d-11\,a^2bc^3d^2+7\,a^3c^2d^3)x^2}{24\,((a^2b^2c^6d^2-2\,a^3bc^5d^3+a^4c^4d^4)x^7+2\,(a^2b^2c^7d-2\,a^3bc^6d^2+a^4c^5d^3)x^5+(a^2b^2c^8-2\,a^3bc^7d+a^4c^6d^2)x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^5*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*sqrt(a*b)) - 1/8*(63*b^2*c^2*d^3 - 90*a*b*c*d^4 + 35*a^2*d^5)*arc
tan(d*x/sqrt(c*d))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^
3)*sqrt(c*d)) - 1/24*(8*a*b^2*c^5 - 16*a^2*b*c^4*d + 8*a^3*c^3*d^2 - 3*(8*b
^3*c^3*d^2 + 8*a*b^2*c^2*d^3 - 55*a^2*b*c*d^4 + 35*a^3*d^5)*x^6 - (48*b^3*c
^4*d + 40*a*b^2*c^3*d^2 - 275*a^2*b*c^2*d^3 + 175*a^3*c*d^4)*x^4 - 8*(3*b^3
*c^5 + a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 7*a^3*c^2*d^3)*x^2)/((a^2*b^2*c^6*d
^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^7 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^
2 + a^4*c^5*d^3)*x^5 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(244) = 488$.
time = 5.72, size = 2397, normalized size = 8.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a*b^3*c^6 - 48*a^2*b^2*c^5*d + 48*a^3*b*c^4*d^2 - 16*a^4*c^3*d^3
 - 6*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5 - 35*a^4*d^6)*x^6
 - 2*(48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^3 + 450*a^3*b*c^2*
d^4 - 175*a^4*c*d^5)*x^4 - 16*(3*b^4*c^6 - 2*a*b^3*c^5*d - 12*a^2*b^2*c^4*d
^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 + 24*(b^4*c^4*d^2*x^7 + 2*b^4*c^
5*d*x^5 + b^4*c^6*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + 3*((63*a^2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*
a^2*b^2*c^3*d^3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^
```

```
2 - 90*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*s
qrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*
b*c^5*d^4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4
*b*c^6*d^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^
7*d^2 - a^5*c^6*d^3)*x^3), -1/24*(8*a*b^3*c^6 - 24*a^2*b^2*c^5*d + 24*a^3*b
*c^4*d^2 - 8*a^4*c^3*d^3 - 3*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b
*c*d^5 - 35*a^4*d^6)*x^6 - (48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^
3*d^3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 8*(3*b^4*c^6 - 2*a*b^3*c^5
*d - 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 + 3*((63*a^
2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3*d^3 -
90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*b*c^3*d
^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 12*(b^4*c^4*d^2*x
^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a
) - a)/(b*x^2 + a)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^
4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d
^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 -
a^5*c^6*d^3)*x^3), -1/48*(16*a*b^3*c^6 - 48*a^2*b^2*c^5*d + 48*a^3*b*c^4*d^
2 - 16*a^4*c^3*d^3 - 6*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5
 - 35*a^4*d^6)*x^6 - 2*(48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^
3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 16*(3*b^4*c^6 - 2*a*b^3*c^5*d
- 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 - 48*(b^4*c^4*
d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*
((63*a^2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3
*d^3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*
b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) -
 c)/(d*x^2 + c)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 -
 a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3
- a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^5
*c^6*d^3)*x^3), -1/24*(8*a*b^3*c^6 - 24*a^2*b^2*c^5*d + 24*a^3*b*c^4*d^2 -
8*a^4*c^3*d^3 - 3*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5 - 35
*a^4*d^6)*x^6 - (48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^3 + 450
*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 8*(3*b^4*c^6 - 2*a*b^3*c^5*d - 12*a^2
*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 - 24*(b^4*c^4*d^2*x^7
+ 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*((63*a^2
*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3*d^3 - 9
0*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*b*c^3*d^
3 + 35*a^4*c^2*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^3*c^7*d^2 -
 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d
- 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3
*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^5*c^6*d^3)*x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.27, size = 256, normalized size = 0.95

$$\frac{b^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}}-\frac{(63\,b^2c^2d^3-90\,abcd^4+35\,a^2d^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3)\sqrt{cd}}-\frac{15\,bcd^4x^3-11\,ad^5x^3+17\,bc^2d^3x-13\,acd^4x}{8\,(b^2c^6-2\,abc^5d+a^2c^4d^2)(dx^2+c)^2}+\frac{3\,bcx^2+9\,adx^2-ac}{3\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^5*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*sqrt(a*b)) - 1/8*(63*b^2*c^2*d^3 - 90*a*b*c*d^4 + 35*a^2*d^5)*arc
tan(d*x/sqrt(c*d))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^
3)*sqrt(c*d)) - 1/8*(15*b*c*d^4*x^3 - 11*a*d^5*x^3 + 17*b*c^2*d^3*x - 13*a*
c*d^4*x)/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*(d*x^2 + c)^2) + 1/3*(3*b*c
*x^2 + 9*a*d*x^2 - a*c)/(a^2*c^4*x^3)
```

Mupad [B]
time = 0.78, size = 785, normalized size = 2.91

$$\frac{\frac{x^2(7ad+3bc)}{3a^2c^2}-\frac{1}{3ac}+\frac{x^4(175a^3d^4-275a^2bcd^3+40ab^2c^2d^2+48b^3c^3d)}{24a^2c^3(a^2d^2-2abcd+b^2c^2)}+\frac{x^6(35a^3d^5-55a^2bcd^4+8ab^2c^2d^3+8b^3c^3d^2)}{8a^2c^4(a^2d^2-2abcd+b^2c^2)}}{c^2x^3+2cdx^5+d^2x^7}+\frac{\mathrm{atan}\left(\frac{bc^9x(-a^5b^9)^{3/2}64\mathrm{i}+a^{14}bd^9x\sqrt{-a^5b^9}1225\mathrm{i}+a^{10}b^5c^4d^5x\sqrt{-a^5b^9}3969\mathrm{i}-a^{11}b^4c^3d^6x\sqrt{-a^5b^9}11340\mathrm{i}+a^{12}b^3c^2d^7x\sqrt{-a^5b^9}12510\mathrm{i}-a^{13}b^2cd^8x\sqrt{-a^5b^9}6300\mathrm{i}}{-1225a^{17}b^5d^9+6300a^{16}b^6cd^8-12510a^{15}b^7c^2d^7+11340a^{14}b^8c^3d^6-3969a^{13}b^9c^4d^5+64a^8b^{14}c^9}\right)\sqrt{-a^5b^9}\,1\mathrm{i}}{a^8d^3-3a^7bcd^2+3a^6b^2c^2d-a^5b^3c^3}+\frac{\mathrm{atan}\left(\frac{a^9d^5x(-c^9d^5)^{3/2}1225\mathrm{i}+b^9c^{18}dx\sqrt{-c^9d^5}64\mathrm{i}-a^6b^3c^3d^2x(-c^9d^5)^{3/2}11340\mathrm{i}+a^7b^2c^2d^3x(-c^9d^5)^{3/2}12510\mathrm{i}-a^8bcd^4x(-c^9d^5)^{3/2}6300\mathrm{i}+a^5b^4c^4dx(-c^9d^5)^{3/2}3969\mathrm{i}}{1225a^9c^{14}d^{12}-6300a^8bc^{15}d^{11}+12510a^7b^2c^{16}d^{10}-11340a^6b^3c^{17}d^9+3969a^5b^4c^{18}d^8-64b^9c^{23}d^3}\right)\sqrt{-c^9d^5}(35a^2d^2-90abcd+63b^2c^2)\,1\mathrm{i}}{8(-a^3c^9d^3+3a^2bc^{10}d^2-3ab^2c^{11}d+b^3c^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((x^2*(7*a*d + 3*b*c))/(3*a^2*c^2) - 1/(3*a*c) + (x^4*(175*a^3*d^4 + 48*b^3
*c^3*d + 40*a*b^2*c^2*d^2 - 275*a^2*b*c*d^3))/(24*a^2*c^3*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)) + (x^6*(35*a^3*d^5 + 8*b^3*c^3*d^2 + 8*a*b^2*c^2*d^3 - 55*a
^2*b*c*d^4))/(8*a^2*c^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2*x^3 + d^2*x^
7 + 2*c*d*x^5) + (atan((b*c^9*x*(-a^5*b^9)^(3/2)*64i + a^14*b*d^9*x*(-a^5*b
^9)^(1/2)*1225i + a^10*b^5*c^4*d^5*x*(-a^5*b^9)^(1/2)*3969i - a^11*b^4*c^3*
d^6*x*(-a^5*b^9)^(1/2)*11340i + a^12*b^3*c^2*d^7*x*(-a^5*b^9)^(1/2)*12510i
- a^13*b^2*c*d^8*x*(-a^5*b^9)^(1/2)*6300i)/(64*a^8*b^14*c^9 - 1225*a^17*b^5
*d^9 + 6300*a^16*b^6*c*d^8 - 3969*a^13*b^9*c^4*d^5 + 11340*a^14*b^8*c^3*d^6
 - 12510*a^15*b^7*c^2*d^7))*(-a^5*b^9)^(1/2)*1i)/(a^8*d^3 - a^5*b^3*c^3 + 3
*a^6*b^2*c^2*d - 3*a^7*b*c*d^2) + (atan((a^9*d^5*x*(-c^9*d^5)^(3/2)*1225i +
 b^9*c^18*d*x*(-c^9*d^5)^(1/2)*64i - a^6*b^3*c^3*d^2*x*(-c^9*d^5)^(3/2)*113
40i + a^7*b^2*c^2*d^3*x*(-c^9*d^5)^(3/2)*12510i - a^8*b*c*d^4*x*(-c^9*d^5)^
(3/2)*6300i + a^5*b^4*c^4*d*x*(-c^9*d^5)^(3/2)*3969i)/(1225*a^9*c^14*d^12 -
 64*b^9*c^23*d^3 - 6300*a^8*b*c^15*d^11 + 3969*a^5*b^4*c^18*d^8 - 11340*a^6
*b^3*c^17*d^9 + 12510*a^7*b^2*c^16*d^10))*(-c^9*d^5)^(1/2)*(35*a^2*d^2 + 63
*b^2*c^2 - 90*a*b*c*d)*1i)/(8*(b^3*c^12 - a^3*c^9*d^3 + 3*a^2*b*c^10*d^2 -
3*a*b^2*c^11*d))
```

3.261 $\int \frac{x}{(1+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)$$

```
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {455, 36, 31}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x^2)(4+x^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx,x,x^2\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\ &= \frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x^2)*(4 + x^2)),x]
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.86

method	result	size
default	$\frac{\ln(x^2+1)}{6}-\frac{\ln(x^2+4)}{6}$	18
norman	$\frac{\ln(x^2+1)}{6}-\frac{\ln(x^2+4)}{6}$	18
risch	$\frac{\ln(x^2+1)}{6}-\frac{\ln(x^2+4)}{6}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)/(x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Maxima [A]
time = 0.30, size = 17, normalized size = 0.81

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="maxima")
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Fricas [A]
time = 0.86, size = 17, normalized size = 0.81

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="fricas")
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{\log\left(x^{2}+1\right)}{6}-\frac{\log\left(x^{2}+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

Giac [A]
time = 0.88, size = 17, normalized size = 0.81

$$-\frac{1}{6}\,\log\left(x^{2}+4\right)+\frac{1}{6}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Mupad [B]
time = 0.08, size = 17, normalized size = 0.81

$$\frac{\mathrm{atanh}\left(\frac{3\,x^{2}}{5\,x^{2}+8}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x^2 + 4)),x)
```

```
[Out] atanh((3*x^2)/(5*x^2 + 8))/3
```

3.262 $\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=87

$$\frac{(bc-2ad)x}{b^3}+\frac{dx^3}{3b^2}+\frac{a(bc-ad)x}{2b^3(a+bx^2)}-\frac{\sqrt{a}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

```
[Out] (-2*a*d+b*c)*x/b^3+1/3*d*x^3/b^2+1/2*a*(-a*d+b*c)*x/b^3/(b*x^2+a)-1/2*(-5*a
*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {466, 1167, 211}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3bc-5ad)}{2b^{7/2}}+\frac{ax(bc-ad)}{2b^3(a+bx^2)}+\frac{x(bc-2ad)}{b^3}+\frac{dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + (a*(b*c - a*d)*x)/(2*b^3*(a + b*x
^2)) - (Sqrt[a]*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx &= \frac{a(bc-ad)x}{2b^3(a+bx^2)} - \frac{\int \frac{a(bc-ad)-2b(bc-ad)x^2-2b^2dx^4}{a+bx^2}\,dx}{2b^3} \\
&= \frac{a(bc-ad)x}{2b^3(a+bx^2)} - \frac{\int \left(-2(bc-2ad)-2bdx^2+\frac{3abc-5a^2d}{a+bx^2}\right)dx}{2b^3} \\
&= \frac{(bc-2ad)x}{b^3} + \frac{dx^3}{3b^2} + \frac{a(bc-ad)x}{2b^3(a+bx^2)} - \frac{(a(3bc-5ad))\int \frac{1}{a+bx^2}\,dx}{2b^3} \\
&= \frac{(bc-2ad)x}{b^3} + \frac{dx^3}{3b^2} + \frac{a(bc-ad)x}{2b^3(a+bx^2)} - \frac{\sqrt{a}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 1.02

$$\frac{(bc-2ad)x}{b^3} + \frac{dx^3}{3b^2} + \frac{(abc-a^2d)\,x}{2b^3(a+bx^2)} + \frac{\sqrt{a}\,(-3bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + ((a*b*c - a^2*d)*x)/(2*b^3*(a + b
*x^2)) + (Sqrt[a]*(-3*b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.87

method	result
default	$-\frac{-\frac{1}{3}bd\,x^3+2adx-bcx}{b^3}+\frac{a\left(\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(5ad-3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$
risch	$\frac{d\,x^3}{3b^2}-\frac{2adx}{b^3}+\frac{cx}{b^2}+\frac{\left(-\frac{1}{2}a^2d+\frac{1}{2}abc\right)x}{b^3(b\,x^2+a)}+\frac{5\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x+a\right)ad}{4b^4}-\frac{3\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x+a\right)c}{4b^3}-5\sqrt{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/3*b*d*x^3+2*a*d*x-b*c*x)+a/b^3*((-1/2*a*d+1/2*b*c)*x/(b*x^2+a)+1
/2*(5*a*d-3*b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 84, normalized size = 0.97

$$\frac{(abc-a^2d)x}{2\,(b^4x^2+ab^3)}-\frac{(3\,abc-5\,a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{bdx^3+3\,(bc-2\,ad)x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b*c - a^2*d)*x/(b^4*x^2 + a*b^3) - 1/2*(3*a*b*c - 5*a^2*d)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/3*(b*d*x^3 + 3*(b*c - 2*a*d)*x)/b^3
```

Fricas [A]
time = 1.56, size = 240, normalized size = 2.76

$$\left[\frac{4\,b^2dx^5+4\,(3\,b^2c-5\,abd)x^3-3\,(3\,abc-5\,a^2d+(3\,b^2c-5\,abd)x^2)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,(3\,abc-5\,a^2d)x}{12\,(b^4x^2+ab^3)},\ \frac{2\,b^2dx^5+2\,(3\,b^2c-5\,abd)x^3-3\,(3\,abc-5\,a^2d+(3\,b^2c-5\,abd)x^2)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,(3\,abc-5\,a^2d)x}{6\,(b^4x^2+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*d*x^5 + 4*(3*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c - 5*a^2*d + (3*
b^2*c - 5*a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2
+ a)) + 6*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a*b^3), 1/6*(2*b^2*d*x^5 + 2*(3
*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c - 5*a^2*d + (3*b^2*c - 5*a*b*d)*x^2)*sqr
t(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a*b^3)
]
```

Sympy [A]
time = 0.35, size = 129, normalized size = 1.48

$$x\left(-\frac{2ad}{b^3}+\frac{c}{b^2}\right)+\frac{x(-a^2d+abc)}{2ab^3+2b^4x^2}-\frac{\sqrt{-\frac{a}{b^7}}\cdot(5ad-3bc)\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}+\frac{\sqrt{-\frac{a}{b^7}}\cdot(5ad-3bc)\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}+\frac{dx^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*d/b**3 + c/b**2) + x*(-a**2*d + a*b*c)/(2*a*b**3 + 2*b**4*x**2) - s
qrt(-a/b**7)*(5*a*d - 3*b*c)*log(-b**3*sqrt(-a/b**7) + x)/4 + sqrt(-a/b**7)
*(5*a*d - 3*b*c)*log(b**3*sqrt(-a/b**7) + x)/4 + d*x**3/(3*b**2)
```

Giac [A]
time = 0.82, size = 88, normalized size = 1.01

$$-\frac{(3\,abc-5\,a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{abcx-a^2dx}{2\,(bx^2+a)b^3}+\frac{b^4dx^3+3\,b^4cx-6\,ab^3dx}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b*c - 5*a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(a*b*c
*x - a^2*d*x)/((b*x^2 + a)*b^3) + 1/3*(b^4*d*x^3 + 3*b^4*c*x - 6*a*b^3*d*x)
/b^6
```

Mupad [B]
time = 0.05, size = 104, normalized size = 1.20

$$x\,\left(\frac{c}{b^2}-\frac{2\,a\,d}{b^3}\right)+\frac{d\,x^3}{3\,b^2}-\frac{x\,\left(\frac{a^2\,d}{2}-\frac{a\,b\,c}{2}\right)}{b^4\,x^2+a\,b^3}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(5\,a\,d-3\,b\,c\right)}{5\,a^2\,d-3\,a\,b\,c}\right)\,\left(5\,a\,d-3\,b\,c\right)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x*(c/b^2 - (2*a*d)/b^3) + (d*x^3)/(3*b^2) - (x*((a^2*d)/2 - (a*b*c)/2))/(a*
b^3 + b^4*x^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(5*a*d - 3*b*c))/(5*a^2*d
 - 3*a*b*c))*(5*a*d - 3*b*c))/(2*b^(7/2))
```

3.263 $\int \frac{x^3(c+dx^2)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=60

$$\frac{dx^2}{2b^2} + \frac{a(bc-ad)}{2b^3(a+bx^2)} + \frac{(bc-2ad)\log(a+bx^2)}{2b^3}$$

```
[Out] 1/2*d*x^2/b^2+1/2*a*(-a*d+b*c)/b^3/(b*x^2+a)+1/2*(-2*a*d+b*c)*ln(b*x^2+a)/b
^3
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{a(bc-ad)}{2b^3(a+bx^2)} + \frac{(bc-2ad)\log(a+bx^2)}{2b^3} + \frac{dx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x^2)/(2*b^2) + (a*(b*c - a*d))/(2*b^3*(a + b*x^2)) + ((b*c - 2*a*d)*Log[
a + b*x^2])/(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{b^2} + \frac{a(-bc+ad)}{b^2(a+bx)^2} + \frac{bc-2ad}{b^2(a+bx)}\right)\,dx, x, x^2\right)$$
$$= \frac{dx^2}{2b^2} + \frac{a(bc-ad)}{2b^3(a+bx^2)} + \frac{(bc-2ad)\log(a+bx^2)}{2b^3}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.83

$$\frac{bdx^2 + \frac{a(bc-ad)}{a+bx^2} + (bc-2ad)\log(a+bx^2)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
[Out] (b*d*x^2 + (a*(b*c - a*d))/(a + b*x^2) + (b*c - 2*a*d)*Log[a + b*x^2])/(2*b
^3)
```

Maple [A]
time = 0.08, size = 60, normalized size = 1.00

method	result	size
norman	$\frac{-\frac{a(2ad-bc)}{2b^3}+\frac{d\,x^4}{2b}}{b\,x^2+a} - \frac{(2ad-bc)\ln(b\,x^2+a)}{2b^3}$	59
default	$\frac{d\,x^2}{2b^2} - \frac{\frac{(2ad-bc)\ln(b\,x^2+a)}{b}+\frac{a(ad-bc)}{b(b\,x^2+a)}}{2b^2}$	60
risch	$\frac{d\,x^2}{2b^2} - \frac{a^2d}{2b^3(b\,x^2+a)} + \frac{ac}{2b^2(b\,x^2+a)} - \frac{\ln(b\,x^2+a)ad}{b^3} + \frac{c\ln(b\,x^2+a)}{2b^2}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*d*x^2/b^2-1/2/b^2*(1/b*(2*a*d-b*c)*ln(b*x^2+a)+a*(a*d-b*c)/b/(b*x^2+a))
```

Maxima [A]
time = 0.30, size = 59, normalized size = 0.98

$$\frac{dx^2}{2\,b^2} + \frac{abc-a^2d}{2\,(b^4x^2+ab^3)} + \frac{(bc-2\,ad)\log(bx^2+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2/b^2 + 1/2*(a*b*c - a^2*d)/(b^4*x^2 + a*b^3) + 1/2*(b*c - 2*a*d)*l
og(b*x^2 + a)/b^3
```

Fricas [A]
time = 0.92, size = 78, normalized size = 1.30

$$\frac{b^2dx^4 + abdx^2 + abc - a^2d + (abc - 2\,a^2d + (b^2c - 2\,abd)x^2)\log\left(bx^2 + a\right)}{2\left(b^4x^2 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d*x^4 + a*b*d*x^2 + a*b*c - a^2*d + (a*b*c - 2*a^2*d + (b^2*c - 2*
a*b*d)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

Sympy [A]
time = 0.31, size = 56, normalized size = 0.93

$$\frac{-a^2d + abc}{2ab^3 + 2b^4x^2} + \frac{dx^2}{2b^2} - \frac{(2ad - bc)\log\left(a + bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] (-a**2*d + a*b*c)/(2*a*b**3 + 2*b**4*x**2) + d*x**2/(2*b**2) - (2*a*d - b*c
)*log(a + b*x**2)/(2*b**3)
```

Giac [A]
time = 0.90, size = 90, normalized size = 1.50

$$\frac{\frac{(bx^2+a)d}{b^2} - \frac{(bc-2\,ad)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^2} + \frac{\frac{ab^2c}{bx^2+a} - \frac{a^2bd}{bx^2+a}}{b^3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)*d/b^2 - (b*c - 2*a*d)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*ab
s(b)))/b^2 + (a*b^2*c/(b*x^2 + a) - a^2*b*d/(b*x^2 + a))/b^3)/b
```

Mupad [B]
time = 0.04, size = 63, normalized size = 1.05

$$\frac{d\,x^2}{2\,b^2} - \frac{\ln\left(b\,x^2 + a\right)\,\left(2\,a\,d - b\,c\right)}{2\,b^3} - \frac{a^2\,d - a\,b\,c}{2\,b\,\left(b^3\,x^2 + a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (d*x^2)/(2*b^2) - (log(a + b*x^2)*(2*a*d - b*c))/(2*b^3) - (a^2*d - a*b*c)/
(2*b*(a*b^2 + b^3*x^2))
```

3.264 $\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=67

$$\frac{dx}{b^2}-\frac{(bc-ad)x}{2b^2\left(a+bx^2\right)}+\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}$$

```
[Out] d*x/b^2-1/2*(-a*d+b*c)*x/b^2/(b*x^2+a)+1/2*(-3*a*d+b*c)*arctan(x*b^(1/2)/a^
(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {466, 396, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-3ad)}{2\sqrt{a}\,b^{5/2}}-\frac{x(bc-ad)}{2b^2\left(a+bx^2\right)}+\frac{dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) + ((b*c - 3*a*d)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
```

```
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx &= -\frac{(bc-ad)x}{2b^2(a+bx^2)} - \frac{\int \frac{-bc+ad-2bdx^2}{a+bx^2}\,dx}{2b^2}\\ &= \frac{dx}{b^2} - \frac{(bc-ad)x}{2b^2(a+bx^2)} + \frac{(bc-3ad)\int \frac{1}{a+bx^2}\,dx}{2b^2}\\ &= \frac{dx}{b^2} - \frac{(bc-ad)x}{2b^2(a+bx^2)} + \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 1.01

$$\frac{dx}{b^2} - \frac{(bc-ad)x}{2b^2(a+bx^2)} - \frac{(-bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) - ((-(b*c) + 3*a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Maple [A]
time = 0.10, size = 59, normalized size = 0.88

method	result
default	$\frac{dx}{b^2} - \frac{\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{b\,x^2+a} + \frac{(3ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$
risch	$\frac{dx}{b^2} + \frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b^2(b\,x^2+a)} - \frac{3\ln\left(bx-\sqrt{-ab}\right)ad}{4b^2\sqrt{-ab}} + \frac{\ln\left(bx-\sqrt{-ab}\right)c}{4b\sqrt{-ab}} + \frac{3\ln\left(-bx-\sqrt{-ab}\right)ad}{4b^2\sqrt{-ab}} - \frac{\ln\left(-bx-\sqrt{-ab}\right)c}{4b\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*x/b^2-1/b^2*((-1/2*a*d+1/2*b*c)*x/(b*x^2+a)+1/2*(3*a*d-b*c)/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 60, normalized size = 0.90

$$-\frac{(bc-ad)x}{2\,(b^3x^2+ab^2)}+\frac{dx}{b^2}+\frac{(bc-3\,ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c - a*d)*x/(b^3*x^2 + a*b^2) + d*x/b^2 + 1/2*(b*c - 3*a*d)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.91, size = 202, normalized size = 3.01

$$\left[\frac{4\,ab^2dx^3+(abc-3\,a^2d+(b^2c-3\,abd)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,(ab^2c-3\,a^2bd)x}{4\,(ab^4x^2+a^2b^3)},\frac{2\,ab^2dx^3+(abc-3\,a^2d+(b^2c-3\,abd)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-(ab^2c-3\,a^2bd)x}{2\,(ab^4x^2+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b^2*d*x^3 + (a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-a*b)*
log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - 3*a^2*b*d)*x)/
(a*b^4*x^2 + a^2*b^3), 1/2*(2*a*b^2*d*x^3 + (a*b*c - 3*a^2*d + (b^2*c - 3*a
*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (a*b^2*c - 3*a^2*b*d)*x)/(a*b^
4*x^2 + a^2*b^3)]
```

Sympy [A]
time = 0.29, size = 114, normalized size = 1.70

$$\frac{x(ad-bc)}{2ab^2+2b^3x^2}+\frac{\sqrt{-\frac{1}{ab^5}}\cdot(3ad-bc)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^5}}\cdot(3ad-bc)\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}+\frac{dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b**5))*(3*a*d - b*c)*lo
g(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(-1/(a*b**5))*(3*a*d - b*c)*log(a*
b**2*sqrt(-1/(a*b**5)) + x)/4 + d*x/b**2
```

Giac [A]
time = 0.88, size = 58, normalized size = 0.87

$$\frac{dx}{b^2} + \frac{(bc - 3\,ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2} - \frac{bcx - adx}{2\,(bx^2 + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d*x/b^2 + 1/2*(b*c - 3*a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(b*
c*x - a*d*x)/((b*x^2 + a)*b^2)
```

Mupad [B]
time = 0.10, size = 59, normalized size = 0.88

$$\frac{x\,\left(\frac{a\,d}{2} - \frac{b\,c}{2}\right)}{b^3\,x^2 + a\,b^2} + \frac{d\,x}{b^2} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,a\,d - b\,c)}{2\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (x*((a*d)/2 - (b*c)/2))/(a*b^2 + b^3*x^2) + (d*x)/b^2 - (atan((b^(1/2)*x)/a
^(1/2))*(3*a*d - b*c))/(2*a^(1/2)*b^(5/2))
```

3.265 $\int \frac{x(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=41

$$\frac{-bc+ad}{2b^2\left(a+bx^2\right)}+\frac{d\log\left(a+bx^2\right)}{2b^2}$$

[Out] 1/2*(a*d-b*c)/b^2/(b*x^2+a)+1/2*d*ln(b*x^2+a)/b^2

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {455, 45}

$$\frac{d\log\left(a+bx^2\right)}{2b^2}-\frac{bc-ad}{2b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

[In] Int[(x*(c + d*x^2))/(a + b*x^2)^2,x]

[Out] -1/2*(b*c - a*d)/(b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(c+dx^2)}{(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{bc-ad}{b(a+bx)^2}+\frac{d}{b(a+bx)}\right)\,dx,x,x^2\right)\\ &= -\frac{bc-ad}{2b^2\left(a+bx^2\right)}+\frac{d\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{-bc+ad}{2b^2(a+bx^2)}+\frac{d\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2))/(a + b*x^2)^2,x]
[Out] (-(b*c) + a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.07, size = 38, normalized size = 0.93

method	result	size
default	$\frac{d\ln(b\,x^2+a)}{2b^2}-\frac{-ad+bc}{2b^2(b\,x^2+a)}$	38
norman	$\frac{ad-bc}{2b^2(b\,x^2+a)}+\frac{d\ln(b\,x^2+a)}{2b^2}$	38
risch	$\frac{ad}{2b^2(b\,x^2+a)}-\frac{c}{2b(b\,x^2+a)}+\frac{d\ln(b\,x^2+a)}{2b^2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*d*ln(b*x^2+a)/b^2-1/2*(-a*d+b*c)/b^2/(b*x^2+a)
```

Maxima [A]
time = 0.32, size = 40, normalized size = 0.98

$$-\frac{bc-ad}{2(b^3x^2+ab^2)}+\frac{d\log(bx^2+a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*(b*c - a*d)/(b^3*x^2 + a*b^2) + 1/2*d*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 1.15, size = 45, normalized size = 1.10

$$-\frac{bc-ad-(bdx^2+ad)\log(bx^2+a)}{2(b^3x^2+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c - a*d - (b*d*x^2 + a*d)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

Sympy [A]
time = 0.18, size = 36, normalized size = 0.88

$$\frac{ad - bc}{2ab^2 + 2b^3x^2} + \frac{d\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] (a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + d*log(a + b*x**2)/(2*b**2)
```

Giac [A]
time = 0.74, size = 65, normalized size = 1.59

$$-\frac{d\left(\frac{\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b} - \frac{a}{(bx^2+a)b}\right)}{2\,b} - \frac{c}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
 - 1/2*c/((b*x^2 + a)*b)
```

Mupad [B]
time = 0.08, size = 37, normalized size = 0.90

$$\frac{d\,\ln\left(b\,x^2+a\right)}{2\,b^2} + \frac{a\,d - b\,c}{2\,b^2\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (d*log(a + b*x^2))/(2*b^2) + (a*d - b*c)/(2*b^2*(a + b*x^2))
```

3.266 $\int \frac{c+dx^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)x}{2ab\,(a+bx^2)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

```
[Out] 1/2*(-a*d+b*c)*x/a/b/(b*x^2+a)+1/2*(a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3
/2)/b^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(ad+bc)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\int \frac{1}{a+bx^2}\,dx}{2ab}$$
$$= \frac{(bc-ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.00

$$-\frac{(-bc+ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((-(b*c) + a*d)*x)/(a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A]
time = 0.00, size = 57, normalized size = 0.90

method	result	size
default	$-\frac{(ad-bc)x}{2ab(b\,x^2+a)}+\frac{(ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	57
risch	$-\frac{(ad-bc)x}{2ab(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\;b}-\frac{\ln\left(bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\;a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\;b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\;a}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2*(a*d+b*c)/a/b/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))
```

Maxima [A]
time = 0.52, size = 57, normalized size = 0.90

$$\frac{(bc-ad)x}{2\,(ab^2x^2+a^2b)} + \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)*x/(a*b^2*x^2 + a^2*b) + 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 1.09, size = 181, normalized size = 2.87

$$\left[-\frac{\left(abc+a^{2}d+\left(b^{2}c+abd\right)x^{2}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\sqrt{-ab}\,x-a}{bx^{2}+a}\right)-2\left(ab^{2}c-a^{2}bd\right)x}{4\left(a^{2}b^{3}x^{2}+a^{3}b^{2}\right)},\ \frac{\left(abc+a^{2}d+\left(b^{2}c+abd\right)x^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(ab^{2}c-a^{2}bd\right)x}{2\left(a^{2}b^{3}x^{2}+a^{3}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^
2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) + (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.
time = 0.20, size = 112, normalized size = 1.78

$$\frac{x(-ad+bc)}{2a^{2}b+2ab^{2}x^{2}}-\frac{\sqrt{-\frac{1}{a^{3}b^{3}}}\,(ad+bc)\log\left(-a^{2}b\sqrt{-\frac{1}{a^{3}b^{3}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}b^{3}}}\,(ad+bc)\log\left(a^{2}b\sqrt{-\frac{1}{a^{3}b^{3}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

Giac [A]
time = 0.76, size = 57, normalized size = 0.90

$$\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}+\frac{bcx-adx}{2\,(bx^{2}+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*x - a*d*x)
/((b*x^2 + a)*a*b)
```

Mupad [B]
time = 0.10, size = 51, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d+b\,c)}{2\,a^{3/2}\,b^{3/2}}-\frac{x\,(a\,d-b\,c)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^2,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(a*d + b*c))/(2*a^(3/2)*b^(3/2)) - (x*(a*d - b*c
))/(2*a*b*(a + b*x^2))
```

3.267 $\int \frac{c+dx^2}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=51

$$\frac{bc-ad}{2ab\,(a+bx^2)}+\frac{c\log(x)}{a^2}-\frac{c\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] 1/2*(-a*d+b*c)/a/b/(b*x^2+a)+c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{bc-ad}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (b*c - a*d)/(2*a*b*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^
2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^2}{x(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{x(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c}{a^2x} + \frac{-bc+ad}{a(a+bx)^2} - \frac{bc}{a^2(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{bc-ad}{2ab(a+bx^2)} + \frac{c\log(x)}{a^2} - \frac{c\log(a+bx^2)}{2a^2}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.90

$$\frac{\frac{a(bc-ad)}{b(a+bx^2)} + 2c\log(x) - c\log(a+bx^2)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x*(a + b*x^2)^2),x]
[Out] ((a*(b*c - a*d))/(b*(a + b*x^2)) + 2*c*Log[x] - c*Log[a + b*x^2])/(2*a^2)
```

Maple [A]
time = 0.08, size = 49, normalized size = 0.96

method	result	size
norman	$\frac{(ad-bc)x^2}{2a^2(bx^2+a)} + \frac{c\ln(x)}{a^2} - \frac{c\ln(bx^2+a)}{2a^2}$	48
default	$\frac{-c\ln(bx^2+a) - \frac{a(ad-bc)}{b(bx^2+a)}}{2a^2} + \frac{c\ln(x)}{a^2}$	49
risch	$-\frac{d}{2b(bx^2+a)} + \frac{c}{2a(bx^2+a)} + \frac{c\ln(x)}{a^2} - \frac{c\ln(bx^2+a)}{2a^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2/a^2*(-c*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a))+c*ln(x)/a^2
```

Maxima [A]
time = 0.27, size = 51, normalized size = 1.00

$$\frac{bc-ad}{2(ab^2x^2+a^2b)} - \frac{c\log(bx^2+a)}{2a^2} + \frac{c\log(x^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)/(a*b^2*x^2 + a^2*b) - 1/2*c*log(b*x^2 + a)/a^2 + 1/2*c*log(
x^2)/a^2
```

Fricas [A]
time = 0.80, size = 71, normalized size = 1.39

$$\frac{abc - a^2d - (b^2cx^2 + abc)\log(bx^2 + a) + 2(b^2cx^2 + abc)\log(x)}{2(a^2b^2x^2 + a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*c - a^2*d - (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) + 2*(b^2*c*x^2 + a*
b*c)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

Sympy [A]
time = 0.21, size = 46, normalized size = 0.90

$$\frac{-ad + bc}{2a^2b + 2ab^2x^2} + \frac{c\log(x)}{a^2} - \frac{c\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x/(b*x**2+a)**2,x)
```

```
[Out] (-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) + c*log(x)/a**2 - c*log(a/b + x**2)
/(2*a**2)
```

Giac [A]
time = 0.67, size = 63, normalized size = 1.24

$$\frac{c\log(x^2)}{2\,a^2} - \frac{c\log(|bx^2 + a|)}{2\,a^2} + \frac{b^2cx^2 + 2\,abc - a^2d}{2\,(bx^2 + a)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a^2 - 1/2*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b^2*c*x^2 + 2*a*b
*c - a^2*d)/((b*x^2 + a)*a^2*b)
```

Mupad [B]
time = 0.03, size = 47, normalized size = 0.92

$$\frac{c\,\ln(x)}{a^2} - \frac{c\,\ln(b\,x^2 + a)}{2\,a^2} - \frac{a\,d - b\,c}{2\,a\,b\,(b\,x^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x*(a + b*x^2)^2),x)
```

```
[Out] (c*log(x))/a^2 - (c*log(a + b*x^2))/(2*a^2) - (a*d - b*c)/(2*a*b*(a + b*x^2
))
```

3.268 $\int \frac{c+dx^2}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{c}{a^2x}-\frac{(bc-ad)x}{2a^2(a+bx^2)}-\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

```
[Out] -c/a^2/x-1/2*(-a*d+b*c)*x/a^2/(b*x^2+a)-1/2*(-a*d+3*b*c)*arctan(x*b^(1/2)/a
^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {467, 464, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3bc-ad)}{2a^{5/2}\sqrt{b}}-\frac{x(bc-ad)}{2a^2(a+bx^2)}-\frac{c}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) - ((b*c - a*d)*x)/(2*a^2*(a + b*x^2)) - ((3*b*c - a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^2\left(a+bx^2\right)^2}\,dx &= -\frac{(bc-ad)x}{2a^2\left(a+bx^2\right)}-\frac{1}{2}\int \frac{-\frac{2c}{a}+\frac{(bc-ad)x^2}{a^2}}{x^2\left(a+bx^2\right)}\,dx\\ &= -\frac{c}{a^2x}-\frac{(bc-ad)x}{2a^2\left(a+bx^2\right)}-\frac{(3bc-ad)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\ &= -\frac{c}{a^2x}-\frac{(bc-ad)x}{2a^2\left(a+bx^2\right)}-\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 0.99

$$-\frac{c}{a^2x}+\frac{(-bc+ad)x}{2a^2\left(a+bx^2\right)}+\frac{(-3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + ((-(b*c) + a*d)*x)/(2*a^2*(a + b*x^2)) + ((-3*b*c + a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Maple [A]
time = 0.08, size = 60, normalized size = 0.85

method	result
default	$\frac{\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(ad-3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^2}-\frac{c}{a^2x}$
risch	$\frac{\frac{(ad-3bc)x^2}{2a^2}-\frac{c}{a}}{x(b\,x^2+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2b+a^2d^2-6abcd+9b^2c^2\right)}\_R\ln\left(\left(3\_R^2a^5b+2a^2d^2-12abcd+18b^2c^2\right)x+\left(-a^4d+3a^3bc\right)\_R\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(a*d-3*b*c)/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2)))-c/a^2/x
```

Maxima [A]
time = 0.48, size = 65, normalized size = 0.92

$$-\frac{(3\,bc-ad)x^2+2\,ac}{2\,(a^2bx^3+a^3x)}-\frac{(3\,bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b*c - a*d)*x^2 + 2*a*c)/(a^2*b*x^3 + a^3*x) - 1/2*(3*b*c - a*d)*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 1.09, size = 214, normalized size = 3.01

$$\left[-\frac{4\,a^2bc+2\,(3\,ab^2c-a^2bd)x^2-((3\,b^2c-abd)x^3+(3\,abc-a^2d)x)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,(a^3b^2x^3+a^4bx)},-\frac{2\,a^2bc+(3\,ab^2c-a^2bd)x^2+((3\,b^2c-abd)x^3+(3\,abc-a^2d)x)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(a^3b^2x^3+a^4bx)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b*c + 2*(3*a*b^2*c - a^2*b*d)*x^2 - ((3*b^2*c - a*b*d)*x^3 + (
3*a*b*c - a^2*d)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*a^2*b*c + (3*a*b^2*c - a^2*b*d)*x^2 + (
(3*b^2*c - a*b*d)*x^3 + (3*a*b*c - a^2*d)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
))/(a^3*b^2*x^3 + a^4*b*x)]
```

Sympy [A]
time = 0.25, size = 114, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-3bc)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-3bc)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{-2ac+x^2(ad-3bc)}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b))*(a*d - 3*b*c)*log(-a**3*sqrt(-1/(a**5*b)) + x)/4 + sqrt(
-1/(a**5*b))*(a*d - 3*b*c)*log(a**3*sqrt(-1/(a**5*b)) + x)/4 + (-2*a*c + x*
*2*(a*d - 3*b*c))/(2*a**3*x + 2*a**2*b*x**3)
```

Giac [A]
time = 1.11, size = 64, normalized size = 0.90

$$-\frac{(3\,bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bcx^2-adx^2+2\,ac}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 -
 a*d*x^2 + 2*a*c)/((b*x^3 + a*x)*a^2)
```

Mupad [B]
time = 0.11, size = 61, normalized size = 0.86

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(a\,d-3\,b\,c\right)}{2\,a^{5/2}\,\sqrt{b}}-\frac{\frac{c}{a}-\frac{x^2\,\left(a\,d-3\,b\,c\right)}{2\,a^2}}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(a*d - 3*b*c))/(2*a^(5/2)*b^(1/2)) - (c/a - (x^2
*(a*d - 3*b*c))/(2*a^2))/(a*x + b*x^3)
```

3.269 $\int \frac{c+dx^2}{x^3(a+bx^2)^2} dx$

Optimal. Leaf size=76

$$-\frac{c}{2a^2x^2}-\frac{bc-ad}{2a^2(a+bx^2)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{(2bc-ad)\log(a+bx^2)}{2a^3}$$

```
[Out] -1/2*c/a^2/x^2+1/2*(a*d-b*c)/a^2/(b*x^2+a)-(-a*d+2*b*c)*ln(x)/a^3+1/2*(-a*d
+2*b*c)*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\frac{(2bc-ad)\log(a+bx^2)}{2a^3}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{bc-ad}{2a^2(a+bx^2)}-\frac{c}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*c/(a^2*x^2) - (b*c - a*d)/(2*a^2*(a + b*x^2)) - ((2*b*c - a*d)*Log[x])
/a^3 + ((2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c}{a^2x^2}+\frac{-2bc+ad}{a^3x}-\frac{b(-bc+ad)}{a^2(a+bx)^2}-\frac{b(-2bc+ad)}{a^3(a+bx)}\right)dx,x,x^2\right)$$
$$= -\frac{c}{2a^2x^2}-\frac{bc-ad}{2a^2\left(a+bx^2\right)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{(2bc-ad)\log\left(a+bx^2\right)}{2a^3}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.84

$$\frac{-\frac{ac}{x^2}+\frac{a(-bc+ad)}{a+bx^2}+2(-2bc+ad)\log(x)+(2bc-ad)\log\left(a+bx^2\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*c)/x^2) + (a*(-(b*c) + a*d))/(a + b*x^2) + 2*(-2*b*c + a*d)*Log[x] +
(2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Maple [A]
time = 0.09, size = 75, normalized size = 0.99

method	result	size
default	$-\frac{b\left(\frac{(ad-2bc)\ln\left(b\,x^2+a\right)}{b}-\frac{a(ad-bc)}{b\left(b\,x^2+a\right)}\right)}{2a^3}-\frac{c}{2a^2x^2}+\frac{(ad-2bc)\ln(x)}{a^3}$	75
norman	$\frac{-\frac{c}{2a}+\frac{b(-ad+2bc)x^4}{2a^3}}{x^2\left(b\,x^2+a\right)}+\frac{(ad-2bc)\ln(x)}{a^3}-\frac{(ad-2bc)\ln\left(b\,x^2+a\right)}{2a^3}$	75
risch	$\frac{\frac{(ad-2bc)x^2}{2a^2}-\frac{c}{2a}}{x^2\left(b\,x^2+a\right)}+\frac{\ln(x)d}{a^2}-\frac{2bc\ln(x)}{a^3}-\frac{\ln\left(b\,x^2+a\right)d}{2a^2}+\frac{bc\ln\left(b\,x^2+a\right)}{a^3}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*b*((a*d-2*b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a))-1/2*c/a^2/x^
2+(a*d-2*b*c)/a^3*ln(x)
```

Maxima [A]
time = 0.29, size = 78, normalized size = 1.03

$$-\frac{(2\,bc-ad)x^2+ac}{2\left(a^2bx^4+a^3x^2\right)}+\frac{(2\,bc-ad)\log\left(bx^2+a\right)}{2\,a^3}-\frac{(2\,bc-ad)\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*b*c - a*d)*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + 1/2*(2*b*c - a*d)*lo
g(b*x^2 + a)/a^3 - 1/2*(2*b*c - a*d)*log(x^2)/a^3
```

Fricas [**A**]
time = 1.00, size = 122, normalized size = 1.61

$$-\frac{a^2c+(2\,abc-a^2d)x^2-((2\,b^2c-abd)x^4+(2\,abc-a^2d)x^2)\log\left(bx^2+a\right)+2\,((2\,b^2c-abd)x^4+(2\,abc-a^2d)x^2)\log\left(x\right)}{2\,(a^3bx^4+a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c + (2*a*b*c - a^2*d)*x^2 - ((2*b^2*c - a*b*d)*x^4 + (2*a*b*c - a
^2*d)*x^2)*log(b*x^2 + a) + 2*((2*b^2*c - a*b*d)*x^4 + (2*a*b*c - a^2*d)*x^
2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [**A**]
time = 0.49, size = 70, normalized size = 0.92

$$\frac{-ac+x^2(ad-2bc)}{2a^3x^2+2a^2bx^4}+\frac{(ad-2bc)\log\left(x\right)}{a^3}-\frac{(ad-2bc)\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*c + x**2*(a*d - 2*b*c))/(2*a**3*x**2 + 2*a**2*b*x**4) + (a*d - 2*b*c)*l
og(x)/a**3 - (a*d - 2*b*c)*log(a/b + x**2)/(2*a**3)
```

Giac [**A**]
time = 1.60, size = 84, normalized size = 1.11

$$-\frac{(2\,bc-ad)\log\left(x^2\right)}{2\,a^3}-\frac{2\,bcx^2-adx^2+ac}{2\,(bx^4+ax^2)a^2}+\frac{(2\,b^2c-abd)\log\left(|bx^2+a|\right)}{2\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c - a*d)*log(x^2)/a^3 - 1/2*(2*b*c*x^2 - a*d*x^2 + a*c)/((b*x^4 +
 a*x^2)*a^2) + 1/2*(2*b^2*c - a*b*d)*log(abs(b*x^2 + a))/(a^3*b)
```

Mupad [**B**]
time = 0.07, size = 74, normalized size = 0.97

$$\frac{\ln\left(x\right)\,\left(a\,d-2\,b\,c\right)}{a^3}-\frac{\ln\left(b\,x^2+a\right)\,\left(a\,d-2\,b\,c\right)}{2\,a^3}-\frac{\frac{c}{2\,a}-\frac{x^2\,\left(a\,d-2\,b\,c\right)}{2\,a^2}}{b\,x^4+a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (log(x)*(a*d - 2*b*c))/a^3 - (log(a + b*x^2)*(a*d - 2*b*c))/(2*a^3) - (c/(2
*a) - (x^2*(a*d - 2*b*c))/(2*a^2))/(a*x^2 + b*x^4)
```

3.270 $\int \frac{c+dx^2}{x^4(a+bx^2)^2} dx$

Optimal. Leaf size=90

$$-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{b(bc-ad)x}{2a^3(a+bx^2)}+\frac{\sqrt{b}\,(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] -1/3*c/a^2/x^3+(-a*d+2*b*c)/a^3/x+1/2*b*(-a*d+b*c)*x/a^3/(b*x^2+a)+1/2*(-3*
a*d+5*b*c)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(7/2)
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {467, 1275, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(5bc-3ad)}{2a^{7/2}}+\frac{bx(bc-ad)}{2a^3(a+bx^2)}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (b*(b*c - a*d)*x)/(2*a^3*(a + b*
x^2)) + (Sqrt[b]*(5*b*c - 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^4\left(a+bx^2\right)^2}\,dx &= \frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}-\frac{1}{2}b\int \frac{-\frac{2c}{ab}+\frac{2(bc-ad)x^2}{a^2b}-\frac{(bc-ad)x^4}{a^3}}{x^4\left(a+bx^2\right)}\,dx\\ &= \frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}-\frac{1}{2}b\int \left(-\frac{2c}{a^2bx^4}-\frac{2(-2bc+ad)}{a^3bx^2}+\frac{-5bc+3ad}{a^3\left(a+bx^2\right)}\right)\,dx\\ &= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}+\frac{(b(5bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a^3}\\ &= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}+\frac{\sqrt{b}\,(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 90, normalized size = 1.00

$$-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}-\frac{b(-bc+ad)x}{2a^3\left(a+bx^2\right)}-\frac{\sqrt{b}\,(-5bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
[Out] -1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) - (b*(-(b*c) + a*d)*x)/(2*a^3*(a +
 b*x^2)) - (Sqrt[b]*(-5*b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2
))
```

Maple [A]
time = 0.08, size = 79, normalized size = 0.88

method	result
default	$-\frac{b\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(3ad-5bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{c}{3a^2x^3}-\frac{ad-2bc}{a^3x}$
risch	$\frac{-\frac{b(3ad-5bc)x^4}{2a^3}-\frac{(3ad-5bc)x^2}{3a^2}-\frac{c}{3a}}{x^3(b\,x^2+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^7\_Z^2+9a^2b\,d^2-30a\,b^2cd+25b^3c^2\right)}\_R\ln\left(\left(3\_R^2a^7+18a^2b\,d^2-60a\,b^2cd+50b\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*b*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(3*a*d-5*b*c)/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2)))-1/3*c/a^2/x^3-(a*d-2*b*c)/a^3/x
```

Maxima [A]
time = 0.50, size = 93, normalized size = 1.03

$$\frac{3\,(5\,b^2c-3\,abd)x^4-2\,a^2c+2\,(5\,abc-3\,a^2d)x^2}{6\,(a^3bx^5+a^4x^3)}+\frac{(5\,b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^2*c - 3*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 3*a^2*d)*x^2)/(a^3*
b*x^5 + a^4*x^3) + 1/2*(5*b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^3)
```

Fricas [A]
time = 0.91, size = 250, normalized size = 2.78

$$\left[\frac{6\,(5\,b^2c-3\,abd)x^4-4\,a^2c+4\,(5\,abc-3\,a^2d)x^2-3\,((5\,b^2c-3\,abd)x^5+(5\,abc-3\,a^2d)x^3)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{12\,(a^3bx^5+a^4x^3)},\ \frac{3\,(5\,b^2c-3\,abd)x^4-2\,a^2c+2\,(5\,abc-3\,a^2d)x^2+3\,((5\,b^2c-3\,abd)x^5+(5\,abc-3\,a^2d)x^3)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)}{6\,(a^3bx^5+a^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(6*(5*b^2*c - 3*a*b*d)*x^4 - 4*a^2*c + 4*(5*a*b*c - 3*a^2*d)*x^2 - 3*
((5*b^2*c - 3*a*b*d)*x^5 + (5*a*b*c - 3*a^2*d)*x^3)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*x^5 + a^4*x^3), 1/6*(3*(5*b^2*c
 - 3*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 3*a^2*d)*x^2 + 3*((5*b^2*c - 3*a*b
*d)*x^5 + (5*a*b*c - 3*a^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^3*b*x^
5 + a^4*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(82) = 164$.
time = 0.30, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{b}{a^7}}\cdot(3ad-5bc)\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}\cdot(3ad-5bc)}{3abd-5b^2c}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^7}}\cdot(3ad-5bc)\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}\cdot(3ad-5bc)}{3abd-5b^2c}+x\right)}{4}+\frac{-2a^2c+x^4(-9abd+15b^2c)+x^2(-6a^2d+10abc)}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**7)*(3*a*d - 5*b*c)*log(-a**4*sqrt(-b/a**7)*(3*a*d - 5*b*c)/(3*a*
b*d - 5*b**2*c) + x)/4 - sqrt(-b/a**7)*(3*a*d - 5*b*c)*log(a**4*sqrt(-b/a**
7)*(3*a*d - 5*b*c)/(3*a*b*d - 5*b**2*c) + x)/4 + (-2*a**2*c + x**4*(-9*a*b*
d + 15*b**2*c) + x**2*(-6*a**2*d + 10*a*b*c))/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Giac [A]
time = 1.46, size = 86, normalized size = 0.96

$$\frac{(5\,b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}+\frac{b^2cx-abdx}{2\,(bx^2+a)a^3}+\frac{6\,bcx^2-3\,adx^2-ac}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*(b^2*c*
x - a*b*d*x)/((b*x^2 + a)*a^3) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x^3
)
```

Mupad [B]
time = 0.11, size = 84, normalized size = 0.93

$$-\frac{\frac{c}{3\,a}+\frac{x^2\,(3\,a\,d-5\,b\,c)}{3\,a^2}+\frac{b\,x^4\,(3\,a\,d-5\,b\,c)}{2\,a^3}}{b\,x^5+a\,x^3}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,a\,d-5\,b\,c)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] - (c/(3*a) + (x^2*(3*a*d - 5*b*c))/(3*a^2) + (b*x^4*(3*a*d - 5*b*c))/(2*a^3
))/(a*x^3 + b*x^5) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(3*a*d - 5*b*c))/(2
*a^(7/2))
```

3.271 $\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=145

$$\frac{(3bc-7ad)(bc-ad)x}{2b^4}-\frac{(3bc-7ad)(bc-ad)x^3}{6ab^3}+\frac{d^2x^5}{5b^2}+\frac{(bc-ad)^2x^5}{2ab^2(a+bx^2)}-\frac{\sqrt{a}\,(3bc-7ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

```
[Out] 1/2*(-7*a*d+3*b*c)*(-a*d+b*c)*x/b^4-1/6*(-7*a*d+3*b*c)*(-a*d+b*c)*x^3/a/b^3
+1/5*d^2*x^5/b^2+1/2*(-a*d+b*c)^2*x^5/a/b^2/(b*x^2+a)-1/2*(-7*a*d+3*b*c)*(-
a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(9/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {474, 470, 308, 211}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3bc-7ad)(bc-ad)}{2b^{9/2}}+\frac{x(3bc-7ad)(bc-ad)}{2b^4}-\frac{x^3(3bc-7ad)(bc-ad)}{6ab^3}+\frac{x^5(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] ((3*b*c - 7*a*d)*(b*c - a*d)*x)/(2*b^4) - ((3*b*c - 7*a*d)*(b*c - a*d)*x^3)
/(6*a*b^3) + (d^2*x^5)/(5*b^2) + ((b*c - a*d)^2*x^5)/(2*a*b^2*(a + b*x^2))
- (Sqrt[a]*(3*b*c - 7*a*d)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9
/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(bc-ad)^2x^5}{2ab^2(a+bx^2)} - \frac{\int \frac{x^4(-2b^2c^2+5(bc-ad)^2-2abd^2x^2)}{a+bx^2}\,dx}{2ab^2}$$
$$= \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2(a+bx^2)} - \frac{((3bc-7ad)(bc-ad))\int \frac{x^4}{a+bx^2}\,dx}{2ab^2}$$
$$= \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2(a+bx^2)} - \frac{((3bc-7ad)(bc-ad))\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right)dx}{2ab^2}$$
$$= \frac{(3bc-7ad)(bc-ad)x}{2b^4} - \frac{(3bc-7ad)(bc-ad)x^3}{6ab^3} + \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2(a+bx^2)} - \frac{(a(3bc-$$
$$= \frac{(3bc-7ad)(bc-ad)x}{2b^4} - \frac{(3bc-7ad)(bc-ad)x^3}{6ab^3} + \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2(a+bx^2)} - \frac{\sqrt{a}\,(3b$$

Mathematica [A]
time = 0.07, size = 138, normalized size = 0.95

$$\frac{(b^2c^2-4abcd+3a^2d^2)\,x}{b^4} + \frac{2d(bc-ad)x^3}{3b^3} + \frac{d^2x^5}{5b^2} + \frac{a(bc-ad)^2x}{2b^4(a+bx^2)} - \frac{\sqrt{a}\,(3b^2c^2-10abcd+7a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d*(b*c - a*d)*x^3)/(3*b^3) +
 (d^2*x^5)/(5*b^2) + (a*(b*c - a*d)^2*x)/(2*b^4*(a + b*x^2)) - (Sqrt[a]*(3*
b^2*c^2 - 10*a*b*c*d + 7*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A]
time = 0.10, size = 141, normalized size = 0.97

method	result
default	$\frac{\frac{1}{5}b^2x^5d^2-\frac{2}{3}ab\,d^2x^3+\frac{2}{3}b^2cd\,x^3+3a^2d^2x-4abcdx+b^2c^2x}{b^4}-\frac{a\left(\frac{\left(-\frac{1}{2}a^2d^2+abcd-\frac{1}{2}b^2c^2\right)x}{b\,x^2+a}+\frac{\left(7a^2d^2-10abcd+3b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$
risch	$\frac{d^2x^5}{5b^2}-\frac{2a\,d^2x^3}{3b^3}+\frac{2cd\,x^3}{3b^2}+\frac{3a^2d^2x}{b^4}-\frac{4acdx}{b^3}+\frac{c^2x}{b^2}+\frac{\left(\frac{1}{2}d^2a^3-bcd\,a^2+\frac{1}{2}b^2c^2a\right)x}{b^4\left(b\,x^2+a\right)}+\frac{7\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)a^2d^2}{4b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/5*b^2*x^5*d^2-2/3*a*b*d^2*x^3+2/3*b^2*c*d*x^3+3*a^2*d^2*x-4*a*b*c*
d*x+b^2*c^2*x)-a/b^4*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2*c^2)*x/(b*x^2+a)+1/2*(7
*a^2*d^2-10*a*b*c*d+3*b^2*c^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 149, normalized size = 1.03

$$\frac{(ab^2c^2-2\,a^2bcd+a^3d^2)x}{2\,(b^5x^2+ab^4)}-\frac{(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{3\,b^2d^2x^5+10\,(b^2cd-abd^2)x^3+15\,(b^2c^2-4\,abcd+3\,a^2d^2)x}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x/(b^5*x^2 + a*b^4) - 1/2*(3*a*b^2*
c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/1
5*(3*b^2*d^2*x^5 + 10*(b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 4*a*b*c*d + 3
*a^2*d^2)*x)/b^4
```

Fricas [A]
time = 0.94, size = 400, normalized size = 2.76

$$\left[\frac{12\,b^3d^2x^7+4\,(10\,b^3cd-7\,ab^2d^2)x^5+20\,(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2)x^3+15\,(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2+(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\,(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2)x}{60\,(b^5x^2+ab^4)},\ \frac{6\,b^3d^2x^7+2\,(10\,b^3cd-7\,ab^2d^2)x^5+10\,(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2)x^3-15\,(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2+(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+15\,(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2)x}{30\,(b^5x^2+ab^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*d^2*x^7 + 4*(10*b^3*c*d - 7*a*b^2*d^2)*x^5 + 20*(3*b^3*c^2 -
10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 + 15*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^
2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2
*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*
d^2)*x)/(b^5*x^2 + a*b^4), 1/30*(6*b^3*d^2*x^7 + 2*(10*b^3*c*d - 7*a*b^2*d^
```

```
2)*x^5 + 10*(3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 - 15*(3*a*b^2*c^2
- 10*a^2*b*c*d + 7*a^3*d^2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*
sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*
d^2)*x)/(b^5*x^2 + a*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(133) = 266$.
time = 0.55, size = 286, normalized size = 1.97

$$x^3\left(-\frac{2ad^2}{3b^3}+\frac{2cd}{3b^2}\right)+x\left(\frac{3a^2d^2}{b^4}-\frac{4acd}{b^3}+\frac{c^2}{b^2}\right)+\frac{x(a^3d^2-2a^2bcd+ab^2c^2)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)\,(7ad-3bc)\log\left(-\frac{b^4\sqrt{-\frac{a}{b^9}}\,(ad-bc)(7ad-3bc)}{7a^2d^2-10abcd+3b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)\,(7ad-3bc)\log\left(\frac{b^4\sqrt{-\frac{a}{b^9}}\,(ad-bc)(7ad-3bc)}{7a^2d^2-10abcd+3b^2c^2}+x\right)}{4}+\frac{d^2x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*d**2/(3*b**3) + 2*c*d/(3*b**2)) + x*(3*a**2*d**2/b**4 - 4*a*c*d/
b**3 + c**2/b**2) + x*(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(2*a*b**4 +
2*b**5*x**2) + sqrt(-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(-b**4*sqrt(-a/
b**9)*(a*d - b*c)*(7*a*d - 3*b*c)/(7*a**2*d**2 - 10*a*b*c*d + 3*b**2*c**2)
+ x)/4 - sqrt(-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(b**4*sqrt(-a/b**9)*(
a*d - b*c)*(7*a*d - 3*b*c)/(7*a**2*d**2 - 10*a*b*c*d + 3*b**2*c**2) + x)/4
+ d**2*x**5/(5*b**2)
```

Giac [A]
time = 1.34, size = 156, normalized size = 1.08

$$-\frac{(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{ab^2c^2x-2\,a^2bcdx+a^3d^2x}{2\,(bx^2+a)b^4}+\frac{3\,b^8d^2x^5+10\,b^8cdx^3-10\,ab^7d^2x^3+15\,b^8c^2x-60\,ab^7cdx+45\,a^2b^6d^2x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^4) + 1/2*(a*b^2*c^2*x - 2*a^2*b*c*d*x + a^3*d^2*x)/((b*x^2 + a)*b^4)
+ 1/15*(3*b^8*d^2*x^5 + 10*b^8*c*d*x^3 - 10*a*b^7*d^2*x^3 + 15*b^8*c^2*x -
60*a*b^7*c*d*x + 45*a^2*b^6*d^2*x)/b^10
```

Mupad [B]
time = 0.10, size = 200, normalized size = 1.38

$$x\,\left(\frac{c^2}{b^2}+\frac{2\,a\,\left(\frac{2\,a\,d^2}{b^3}-\frac{2\,c\,d}{b^2}\right)}{b}-\frac{a^2\,d^2}{b^4}\right)-x^3\,\left(\frac{2\,a\,d^2}{3\,b^3}-\frac{2\,c\,d}{3\,b^2}\right)+\frac{d^2\,x^5}{5\,b^2}+\frac{x\,\left(\frac{a^3\,d^2}{2}-a^2\,b\,c\,d+\frac{a\,b^2\,c^2}{2}\right)}{b^5\,x^2+a\,b^4}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(a\,d-b\,c)\,(7\,a\,d-3\,b\,c)}{7\,a^3\,d^2-10\,a^2\,b\,c\,d+3\,a\,b^2\,c^2}\right)\,(a\,d-b\,c)\,(7\,a\,d-3\,b\,c)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

```
[Out] x*(c^2/b^2 + (2*a*((2*a*d^2)/b^3 - (2*c*d)/b^2))/b - (a^2*d^2)/b^4) - x^3*(
(2*a*d^2)/(3*b^3) - (2*c*d)/(3*b^2)) + (d^2*x^5)/(5*b^2) + (x*((a^3*d^2)/2
+ (a*b^2*c^2)/2 - a^2*b*c*d))/(a*b^4 + b^5*x^2) - (a^(1/2)*atan((a^(1/2)*b^
(1/2)*x*(a*d - b*c)*(7*a*d - 3*b*c))/(7*a^3*d^2 + 3*a*b^2*c^2 - 10*a^2*b*c*
d))*(a*d - b*c)*(7*a*d - 3*b*c))/(2*b^(9/2))
```

3.272 $\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=88

$$\frac{d(bc-ad)x^2}{b^3}+\frac{d^2x^4}{4b^2}+\frac{a(bc-ad)^2}{2b^4(a+bx^2)}+\frac{(bc-3ad)(bc-ad)\log(a+bx^2)}{2b^4}$$

```
[Out] d*(-a*d+b*c)*x^2/b^3+1/4*d^2*x^4/b^2+1/2*a*(-a*d+b*c)^2/b^4/(b*x^2+a)+1/2*(
-3*a*d+b*c)*(-a*d+b*c)*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{a(bc-ad)^2}{2b^4(a+bx^2)}+\frac{(bc-3ad)(bc-ad)\log(a+bx^2)}{2b^4}+\frac{dx^2(bc-ad)}{b^3}+\frac{d^2x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d*(b*c - a*d)*x^2)/b^3 + (d^2*x^4)/(4*b^2) + (a*(b*c - a*d)^2)/(2*b^4*(a +
 b*x^2)) + ((b*c - 3*a*d)*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^2}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{2d(bc-ad)}{b^3}+\frac{d^2x}{b^2}-\frac{a(-bc+ad)^2}{b^3(a+bx)^2}+\frac{(bc-3ad)(bc-ad)}{b^3(a+bx)}\right)\,dx, x, x^2\right)$$
$$= \frac{d(bc-ad)x^2}{b^3}+\frac{d^2x^4}{4b^2}+\frac{a(bc-ad)^2}{2b^4\,(a+bx^2)}+\frac{(bc-3ad)(bc-ad)\log\,(a+bx^2)}{2b^4}$$

Mathematica [A]
time = 0.05, size = 87, normalized size = 0.99

$$\frac{4bd(bc-ad)x^2+b^2d^2x^4+\frac{2a(bc-ad)^2}{a+bx^2}+2(b^2c^2-4abcd+3a^2d^2)\log\,(a+bx^2)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (4*b*d*(b*c - a*d)*x^2 + b^2*d^2*x^4 + (2*a*(b*c - a*d)^2)/(a + b*x^2) + 2*
(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*Log[a + b*x^2])/(4*b^4)
```

Maple [A]
time = 0.10, size = 82, normalized size = 0.93

method	result
default	$\frac{\left(-bd\,x^2+2ad-2bc\right)^2}{4b^4}+\frac{(ad-bc)\left(\frac{(3ad-bc)\ln\left(b\,x^2+a\right)}{b}+\frac{a(ad-bc)}{b\left(b\,x^2+a\right)}\right)}{2b^3}$
norman	$\frac{\frac{d^2x^6}{4b}-\frac{d(3ad-4bc)x^4}{4b^2}-\frac{\left(3d^2a^3-4bcd\,a^2+b^2c^2a\right)x^2}{2a\,b^3}}{b\,x^2+a}+\frac{\left(3a^2d^2-4abcd+b^2c^2\right)\ln\left(b\,x^2+a\right)}{2b^4}$
risch	$\frac{d^2x^4}{4b^2}-\frac{a\,d^2x^2}{b^3}+\frac{cd\,x^2}{b^2}+\frac{a^2d^2}{b^4}-\frac{2acd}{b^3}+\frac{c^2}{b^2}+\frac{a^3d^2}{2b^4(b\,x^2+a)}-\frac{a^2cd}{b^3(b\,x^2+a)}+\frac{a\,c^2}{2b^2(b\,x^2+a)}+\frac{3\ln(b\,x^2+a)a^2d^2}{2b^4}-\frac{2\ln(b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-b*d*x^2+2*a*d-2*b*c)^2/b^4+1/2/b^3*(a*d-b*c)*((3*a*d-b*c)/b*ln(b*x^2+
a)+a*(a*d-b*c)/b/(b*x^2+a))
```

Maxima [A]
time = 0.30, size = 107, normalized size = 1.22

$$\frac{ab^2c^2-2\,a^2bcd+a^3d^2}{2\,(b^5x^2+ab^4)}+\frac{bd^2x^4+4\,(bcd-ad^2)x^2}{4\,b^3}+\frac{(b^2c^2-4\,abcd+3\,a^2d^2)\log\,(bx^2+a)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/(b^5*x^2 + a*b^4) + 1/4*(b*d^2*x^4
+ 4*(b*c*d - a*d^2)*x^2)/b^3 + 1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*log(b*
x^2 + a)/b^4
```

Fricas [**A**]
time = 1.09, size = 160, normalized size = 1.82

$$\frac{b^3d^2x^6 + 2\,ab^2c^2 - 4\,a^2bcd + 2\,a^3d^2 + (4\,b^3cd - 3\,ab^2d^2)x^4 + 4\,(ab^2cd - a^2bd^2)x^2 + 2\,(ab^2c^2 - 4\,a^2bcd + 3\,a^3d^2 + (b^3c^2 - 4\,ab^2cd + 3\,a^2bd^2)x^2)\log\left(bx^2 + a\right)}{4\,(b^5x^2 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*d^2*x^6 + 2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 + (4*b^3*c*d - 3*a
*b^2*d^2)*x^4 + 4*(a*b^2*c*d - a^2*b*d^2)*x^2 + 2*(a*b^2*c^2 - 4*a^2*b*c*d
+ 3*a^3*d^2 + (b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*x^2)*log(b*x^2 + a))/(b
^5*x^2 + a*b^4)
```

Sympy [**A**]
time = 0.51, size = 99, normalized size = 1.12

$$x^{2}\left(-\frac{ad^{2}}{b^{3}} + \frac{cd}{b^{2}}\right) + \frac{a^{3}d^{2} - 2a^{2}bcd + ab^{2}c^{2}}{2ab^{4} + 2b^{5}x^{2}} + \frac{d^{2}x^{4}}{4b^{2}} + \frac{(ad - bc)\,(3ad - bc)\log\left(a + bx^{2}\right)}{2b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x**2*(-a*d**2/b**3 + c*d/b**2) + (a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(
2*a*b**4 + 2*b**5*x**2) + d**2*x**4/(4*b**2) + (a*d - b*c)*(3*a*d - b*c)*lo
g(a + b*x**2)/(2*b**4)
```

Giac [**A**]
time = 1.32, size = 163, normalized size = 1.85

$$\frac{\frac{\left(bx^2+a\right)^2\left(d^2+\frac{2\,\left(2\,b^2cd-3\,abd^2\right)}{\left(bx^2+a\right)b}\right)}{b^3} - \frac{2\,\left(b^2c^2-4\,abcd+3\,a^2d^2\right)\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^3} + \frac{2\,\left(\frac{ab^4c^2}{bx^2+a}-\frac{2\,a^2b^3cd}{bx^2+a}+\frac{a^3b^2d^2}{bx^2+a}\right)}{b^5}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 + a)^2*(d^2 + 2*(2*b^2*c*d - 3*a*b*d^2)/((b*x^2 + a)*b))/b^3 -
2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b
```

```
)))/b^3 + 2*(a*b^4*c^2/(b*x^2 + a) - 2*a^2*b^3*c*d/(b*x^2 + a) + a^3*b^2*d^
2/(b*x^2 + a))/b^5)/b
```

Mupad [B]
time = 0.04, size = 112, normalized size = 1.27

$$\frac{a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2}{2\,b\,\left(b^4\,x^2+a\,b^3\right)}-x^2\,\left(\frac{a\,d^2}{b^3}-\frac{c\,d}{b^2}\right)+\frac{d^2\,x^4}{4\,b^2}+\frac{\ln\left(b\,x^2+a\right)\,\left(3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

```
[Out] (a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)/(2*b*(a*b^3 + b^4*x^2)) - x^2*((a*d^2)/
b^3 - (c*d)/b^2) + (d^2*x^4)/(4*b^2) + (log(a + b*x^2)*(3*a^2*d^2 + b^2*c^2
 - 4*a*b*c*d))/(2*b^4)
```

3.273 $\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=116

$$-\frac{(bc-5ad)(bc-ad)x}{2ab^3}+\frac{d^2x^3}{3b^2}+\frac{(bc-ad)^2x^3}{2ab^2(a+bx^2)}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{7/2}}$$

```
[Out] -1/2*(-5*a*d+b*c)*(-a*d+b*c)*x/a/b^3+1/3*d^2*x^3/b^2+1/2*(-a*d+b*c)^2*x^3/a
/b^2/(b*x^2+a)+1/2*(-5*a*d+b*c)*(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2
)/a^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {474, 470, 327, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-5ad)(bc-ad)}{2\sqrt{a}\,b^{7/2}}-\frac{x(bc-5ad)(bc-ad)}{2ab^3}+\frac{x^3(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((b*c - 5*a*d)*(b*c - a*d)*x)/(a*b^3) + (d^2*x^3)/(3*b^2) + ((b*c - a*
d)^2*x^3)/(2*a*b^2*(a + b*x^2)) + ((b*c - 5*a*d)*(b*c - a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2}\,dx &= \frac{(bc-ad)^2x^3}{2ab^2(a+bx^2)} - \frac{\int \frac{x^2(b^2c^2-6abcd+3a^2d^2-2abd^2x^2)}{a+bx^2}\,dx}{2ab^2}\\
&= \frac{d^2x^3}{3b^2} + \frac{(bc-ad)^2x^3}{2ab^2(a+bx^2)} - \frac{((bc-5ad)(bc-ad))\int \frac{x^2}{a+bx^2}\,dx}{2ab^2}\\
&= -\frac{(bc-5ad)(bc-ad)x}{2ab^3} + \frac{d^2x^3}{3b^2} + \frac{(bc-ad)^2x^3}{2ab^2(a+bx^2)} + \frac{((bc-5ad)(bc-ad))\int \frac{1}{a+bx^2}\,dx}{2b^3}\\
&= -\frac{(bc-5ad)(bc-ad)x}{2ab^3} + \frac{d^2x^3}{3b^2} + \frac{(bc-ad)^2x^3}{2ab^2(a+bx^2)} + \frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 105, normalized size = 0.91

$$
\frac{2d(bc-ad)x}{b^3} + \frac{d^2x^3}{3b^2} - \frac{(bc-ad)^2x}{2b^3(a+bx^2)} + \frac{(b^2c^2-6abcd+5a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (2*d*(b*c - a*d)*x)/b^3 + (d^2*x^3)/(3*b^2) - ((b*c - a*d)^2*x)/(2*b^3*(a +
 b*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*Sqrt[a]*b^(7/2))
```

Maple [A]
time = 0.11, size = 102, normalized size = 0.88

method	result
default	$-\frac{d\left(-\frac{1}{3} b d\, x^{3}+2 a d x -2 b c x \right)}{b^{3}}+\frac{\frac{\left(-\frac{1}{2} a^{2} d^{2}+a b c d -\frac{1}{2} b^{2} c^{2}\right) x}{b\, x^{2}+a}+\frac{\left(5 a^{2} d^{2}-6 a b c d +b^{2} c^{2}\right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}}{b^{3}}$
risch	$\frac{d^{2} x^{3}}{3 b^{2}}-\frac{2 d^{2} a x}{b^{3}}+\frac{2 d c x}{b^{2}}+\frac{\left(-\frac{1}{2} a^{2} d^{2}+a b c d -\frac{1}{2} b^{2} c^{2}\right) x}{b^{3} \left(b\, x^{2}+a \right)}-\frac{5 \ln \left(b x +\sqrt{-a b}\right) a^{2} d^{2}}{4 b^{3} \sqrt{-a b}}+\frac{3 \ln \left(b x +\sqrt{-a b}\right) a c d}{2 b^{2} \sqrt{-a b}}-\frac{\ln \left(b x +\sqrt{-\right.}}{4 b \sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d/b^3*(-1/3*b*d*x^3+2*a*d*x-2*b*c*x)+1/b^3*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2*
c^2)*x/(b*x^2+a)+1/2*(5*a^2*d^2-6*a*b*c*d+b^2*c^2)/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 109, normalized size = 0.94

$$-\frac{{\left(b^{2} c^{2} - 2 \, a b c d + a^{2} d^{2}\right)} x}{2 \, {\left(b^{4} x^{2} + a b^{3}\right)}} + \frac{{\left(b^{2} c^{2} - 6 \, a b c d + 5 \, a^{2} d^{2}\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} b^{3}} + \frac{b d^{2} x^{3} + 6 \, {\left(b c d - a d^{2}\right)} x}{3 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(b^4*x^2 + a*b^3) + 1/2*(b^2*c^2 - 6
*a*b*c*d + 5*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/3*(b*d^2*x^
3 + 6*(b*c*d - a*d^2)*x)/b^3
```

Fricas [A]
time = 0.83, size = 342, normalized size = 2.95

$$\left[\frac{4 \, a b^{3} d^{2} x^{5} + 4 \, {\left(6 \, a b^{3} c d - 5 \, a^{2} b^{2} d^{2}\right)} x^{3} - 3 \, {\left(a b^{2} c^{2} - 6 \, a^{2} b c d + 5 \, a^{3} d^{2} + {\left(b^{3} c^{2} - 6 \, a b^{2} c d + 5 \, a^{2} b d^{2}\right)} x^{2}\right)} \sqrt{-a b} \log\left(\frac{b x^{2} - 2 \, \sqrt{-a b} x - a}{b x^{2} + a}\right) - 6 \, {\left(a b^{3} c^{2} - 6 \, a^{2} b^{2} c d + 5 \, a^{3} b d^{2}\right)} x}{12 \, {\left(a b^{5} x^{2} + a^{2} b^{4}\right)}}, \frac{2 \, a b^{3} d^{2} x^{5} + 2 \, {\left(6 \, a b^{3} c d - 5 \, a^{2} b^{2} d^{2}\right)} x^{3} + 3 \, {\left(a b^{2} c^{2} - 6 \, a^{2} b c d + 5 \, a^{3} d^{2} + {\left(b^{3} c^{2} - 6 \, a b^{2} c d + 5 \, a^{2} b d^{2}\right)} x^{2}\right)} \sqrt{a b} \arctan\left(\frac{\sqrt{a b} x}{a}\right) - 3 \, {\left(a b^{3} c^{2} - 6 \, a^{2} b^{2} c d + 5 \, a^{3} b d^{2}\right)} x}{6 \, {\left(a b^{5} x^{2} + a^{2} b^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a*b^3*d^2*x^5 + 4*(6*a*b^3*c*d - 5*a^2*b^2*d^2)*x^3 - 3*(a*b^2*c^2
 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqr
t(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(a*b^3*c^2 - 6*a^
2*b^2*c*d + 5*a^3*b*d^2)*x)/(a*b^5*x^2 + a^2*b^4), 1/6*(2*a*b^3*d^2*x^5 + 2
*(6*a*b^3*c*d - 5*a^2*b^2*d^2)*x^3 + 3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2
 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) - 3*(a*b^3*c^2 - 6*a^2*b^2*c*d + 5*a^3*b*d^2)*x)/(a*b^5*x^2 + a^2*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(104) = 208.
time = 0.47, size = 246, normalized size = 2.12

$$x\left(-\frac{2ad^2}{b^3}+\frac{2cd}{b^2}\right)+\frac{x(-a^2d^2+2abcd-b^2c^2)}{2ab^3+2b^4x^2}-\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)\,(5ad-bc)\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}\,(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)\,(5ad-bc)\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}\,(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{d^2x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*d**2/b**3 + 2*c*d/b**2) + x*(-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(2
*a*b**3 + 2*b**4*x**2) - sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)*log(-a
*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d
+ b**2*c**2) + x)/4 + sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)*log(a*b**
3*sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b*
*2*c**2) + x)/4 + d**2*x**3/(3*b**2)
```

Giac [A]
time = 1.45, size = 114, normalized size = 0.98

$$\frac{(b^2c^2-6\,abcd+5\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}-\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(bx^2+a)b^3}+\frac{b^4d^2x^3+6\,b^4cdx-6\,ab^3d^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*b^3) + 1/3*(b^4*d
^2*x^3 + 6*b^4*c*d*x - 6*a*b^3*d^2*x)/b^6
```

Mupad [B]
time = 0.10, size = 148, normalized size = 1.28

$$\frac{d^2\,x^3}{3\,b^2}-\frac{x\,\left(\frac{a^2\,d^2}{2}-a\,b\,c\,d+\frac{b^2\,c^2}{2}\right)}{b^4\,x^2+a\,b^3}-x\,\left(\frac{2\,a\,d^2}{b^3}-\frac{2\,c\,d}{b^2}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,\left(a\,d-b\,c\right)\,\left(5\,a\,d-b\,c\right)}{\sqrt{a}\,\left(5\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,\left(a\,d-b\,c\right)\,\left(5\,a\,d-b\,c\right)}{2\,\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

```
[Out] (d^2*x^3)/(3*b^2) - (x*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(a*b^3 + b^4*
x^2) - x*((2*a*d^2)/b^3 - (2*c*d)/b^2) + (atan((b^(1/2)*x*(a*d - b*c)*(5*a*
d - b*c))/(a^(1/2)*(5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)))*(a*d - b*c)*(5*a*d -
 b*c))/(2*a^(1/2)*b^(7/2))
```

3.274 $\int \frac{x\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=61

$$\frac{d^2x^2}{2b^2}-\frac{(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{d(bc-ad)\log\left(a+bx^2\right)}{b^3}$$

```
[Out] 1/2*d^2*x^2/b^2-1/2*(-a*d+b*c)^2/b^3/(b*x^2+a)+d*(-a*d+b*c)*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$-\frac{(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{d(bc-ad)\log\left(a+bx^2\right)}{b^3}+\frac{d^2x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x^2)/(2*b^2) - (b*c - a*d)^2/(2*b^3*(a + b*x^2)) + (d*(b*c - a*d)*Log[
a + b*x^2])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)^2}+\frac{2d(bc-ad)}{b^2(a+bx)}\right)\,dx,x,x^2\right)\\ &= \frac{d^2x^2}{2b^2}-\frac{(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{d(bc-ad)\log\left(a+bx^2\right)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.92

$$\frac{bd^2x^2 - \frac{(bc-ad)^2}{a+bx^2} + 2d(bc-ad)\log(a+bx^2)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (b*d^2*x^2 - (b*c - a*d)^2/(a + b*x^2) + 2*d*(b*c - a*d)*Log[a + b*x^2])/(2
*b^3)
```

Maple [A]
time = 0.08, size = 63, normalized size = 1.03

method	result	size
default	$\frac{d^2x^2}{2b^2}-\frac{(ad-bc)\left(\frac{2d\ln\left(b\,x^2+a\right)}{b}-\frac{-ad+bc}{b\left(b\,x^2+a\right)}\right)}{2b^2}$	63
norman	$\frac{-\frac{2a^2d^2-2abcd+b^2c^2}{2b^3}+\frac{d^2x^4}{2b}}{b\,x^2+a}-\frac{d(ad-bc)\ln\left(b\,x^2+a\right)}{b^3}$	73
risch	$\frac{d^2x^2}{2b^2}-\frac{a^2d^2}{2b^3(b\,x^2+a)}+\frac{acd}{b^2(b\,x^2+a)}-\frac{c^2}{2b(b\,x^2+a)}-\frac{d^2\ln\left(b\,x^2+a\right)a}{b^3}+\frac{d\ln\left(b\,x^2+a\right)c}{b^2}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d^2*x^2/b^2-1/2/b^2*(a*d-b*c)*(2*d/b*ln(b*x^2+a)-(-a*d+b*c)/b/(b*x^2+a)
)
```

Maxima [A]
time = 0.30, size = 73, normalized size = 1.20

$$\frac{d^2x^2}{2\,b^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{2\,(b^4x^2 + ab^3)} + \frac{(bcd - ad^2)\log(bx^2 + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^2/b^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x^2 + a*b^3) + (
b*c*d - a*d^2)*log(b*x^2 + a)/b^3
```

Fricas [A]
time = 1.06, size = 101, normalized size = 1.66

$$\frac{b^2d^2x^4 + abd^2x^2 - b^2c^2 + 2\,abcd - a^2d^2 + 2\,(abcd - a^2d^2 + (b^2cd - abd^2)x^2)\log(bx^2 + a)}{2\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^4 + a*b*d^2*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d
 - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

Sympy [A]
time = 0.41, size = 68, normalized size = 1.11

$$\frac{-a^2d^2+2abcd-b^2c^2}{2ab^3+2b^4x^2}+\frac{d^2x^2}{2b^2}-\frac{d(ad-bc)\log\left(a+bx^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] (-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(2*a*b**3 + 2*b**4*x**2) + d**2*x**2/(
2*b**2) - d*(a*d - b*c)*log(a + b*x**2)/b**3
```

Giac [A]
time = 0.97, size = 111, normalized size = 1.82

$$\frac{(bx^2+a)d^2}{2\,b^3}-\frac{(bcd-ad^2)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^3}-\frac{\frac{b^3c^2}{bx^2+a}-\frac{2\,ab^2cd}{bx^2+a}+\frac{a^2bd^2}{bx^2+a}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + a)*d^2/b^3 - (b*c*d - a*d^2)*log(abs(b*x^2 + a)/((b*x^2 + a)^2
*abs(b)))/b^3 - 1/2*(b^3*c^2/(b*x^2 + a) - 2*a*b^2*c*d/(b*x^2 + a) + a^2*b*
d^2/(b*x^2 + a))/b^4
```

Mupad [B]
time = 0.10, size = 77, normalized size = 1.26

$$\frac{d^2\,x^2}{2\,b^2}-\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{2\,b\,\left(b^3\,x^2+a\,b^2\right)}-\frac{\ln\left(b\,x^2+a\right)\,\left(a\,d^2-b\,c\,d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

```
[Out] (d^2*x^2)/(2*b^2) - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*b*(a*b^2 + b^3*x^2))
 - (log(a + b*x^2)*(a*d^2 - b*c*d))/b^3
```

3.275 $\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=82

$$\frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

```
[Out] d^2*x/b^2+1/2*(-a*d+b*c)^2*x/a/b^2/(b*x^2+a)+1/2*(-a*d+b*c)*(3*a*d+b*c)*arc
tan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {398, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)(3ad+bc)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c +
 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx &= \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{b^2(a+bx^2)^2}\right)dx \\
&= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{(a+bx^2)^2}\,dx}{b^2} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2(a+bx^2)}+\frac{((bc-ad)(bc+3ad))\int \frac{1}{a+bx^2}\,dx}{2ab^2} \\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2(a+bx^2)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 1.07

$$
\frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2(a+bx^2)}+\frac{(b^2c^2+2abcd-3a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2 + 2*a*b*c
*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Maple [A]
time = 0.00, size = 94, normalized size = 1.15

method	result
default	$\frac{d^2x}{b^2}-\frac{-\frac{(a^2d^2-2abcd+b^2c^2)x}{2a(bx^2+a)}+\frac{(3a^2d^2-2abcd-b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}$
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{2ab^2(bx^2+a)}-\frac{3a\ln\left(bx-\sqrt{-ab}\right)d^2}{4b^2\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)cd}{2b\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)c^2}{4\sqrt{-ab}\,a}+\frac{3a\ln\left(-bx-\sqrt{}\right.}{4b^2\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*x/b^2-1/b^2*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/(b*x^2+a)+1/2*(3*a^2*
d^2-2*a*b*c*d-b^2*c^2)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 95, normalized size = 1.16

$$\frac{(b^2c^2 - 2\,abcd + a^2d^2)x}{2\,(ab^3x^2 + a^2b^2)} + \frac{d^2x}{b^2} + \frac{(b^2c^2 + 2\,abcd - 3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^2 + a^2*b^2) + d^2*x/b^2 + 1
/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2
)
```

Fricas [A]
time = 0.75, size = 297, normalized size = 3.62

$$\left[\frac{4\,a^2b^2d^2x^3 + (ab^2c^2 + 2\,a^2bcd - 3\,a^3d^2 + (b^3c^2 + 2\,ab^2cd - 3\,a^2bd^2)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2 + 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 2\,(ab^3c^2 - 2\,a^2b^2cd + 3\,a^3bd^2)x}{4\,(a^2b^4x^2 + a^3b^3)}, \frac{2\,a^2b^2d^2x^3 + (ab^2c^2 + 2\,a^2bcd - 3\,a^3d^2 + (b^3c^2 + 2\,ab^2cd - 3\,a^2bd^2)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + (ab^3c^2 - 2\,a^2b^2cd + 3\,a^3bd^2)x}{2\,(a^2b^4x^2 + a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 +
 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a
)/(b*x^2 + a)) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^
2 + a^3*b^3), 1/2*(2*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2
 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + (a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(73) = 146$.
time = 0.39, size = 236, normalized size = 2.88

$$\frac{x(a^2d^2 - 2abcd + b^2c^2)}{2a^2b^2 + 2ab^3x^2} + \frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad - bc)\,(3ad + bc)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)(3ad+bc)}{3a^2d^2 - 2abcd - b^2c^2} + x\right)}{4} - \frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad - bc)\,(3ad + bc)\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)(3ad+bc)}{3a^2d^2 - 2abcd - b^2c^2} + x\right)}{4} + \frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqrt(
-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5
))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 -
 sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(a**3
*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x
)/4 + d**2*x/b**2
```

Giac [A]
time = 0.97, size = 94, normalized size = 1.15

$$\frac{d^2x}{b^2} + \frac{(b^2c^2 + 2\,abcd - 3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(bx^2 + a)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a*b
^2)
```

Mupad [B]
time = 0.11, size = 124, normalized size = 1.51

$$\frac{d^2\,x}{b^2} + \frac{x\,(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2)}{2\,a\,\left(b^3\,x^2 + a\,b^2\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{\sqrt{a}\,\left(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,(a\,d - b\,c)\,\left(3\,a\,d + b\,c\right)}{2\,a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a*(a*b^2 + b^3*x^2)) +
 (atan((b^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(a^(1/2)*(b^2*c^2 - 3*a^2*d^2
+ 2*a*b*c*d)))*(a*d - b*c)*(3*a*d + b*c))/(2*a^(3/2)*b^(5/2))
```

3.276 $\int \frac{(c+dx^2)^2}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=67

$$\frac{(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{c^2\log(x)}{a^2}-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)$$

```
[Out] 1/2*(-a*d+b*c)^2/a/b^2/(b*x^2+a)+c^2*ln(x)/a^2-1/2*(c^2/a^2-d^2/b^2)*ln(b*x
^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)+\frac{c^2\log(x)}{a^2}+\frac{(bc-ad)^2}{2ab^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

```
[Out] (b*c - a*d)^2/(2*a*b^2*(a + b*x^2)) + (c^2*Log[x])/a^2 - ((c^2/a^2 - d^2/b^
2)*Log[a + b*x^2])/2
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{x(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c^2}{a^2x} - \frac{(-bc+ad)^2}{ab(a+bx)^2} + \frac{-b^2c^2+a^2d^2}{a^2b(a+bx)}\right)\,dx, x, x^2\right)$$
$$= \frac{(bc-ad)^2}{2ab^2(a+bx^2)} + \frac{c^2\log(x)}{a^2} - \frac{1}{2}\left(\frac{c^2}{a^2} - \frac{d^2}{b^2}\right)\log\left(a+bx^2\right)$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 1.04

$$\frac{2c^2\log(x) + \frac{(-bc+ad)(a(-bc+ad)+(bc+ad)(a+bx^2)\log(a+bx^2))}{b^2(a+bx^2)}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

```
[Out] (2*c^2*Log[x] + ((-(b*c) + a*d)*(a*(-(b*c) + a*d) + (b*c + a*d)*(a + b*x^2)
*Log[a + b*x^2]))/(b^2*(a + b*x^2)))/(2*a^2)
```

Maple [A]
time = 0.09, size = 66, normalized size = 0.99

method	result	size
default	$\frac{(ad-bc)\left(\frac{(ad+bc)\ln\left(b\,x^2+a\right)}{b^2}+\frac{a(ad-bc)}{b^2\left(b\,x^2+a\right)}\right)}{2a^2}+\frac{c^2\ln(x)}{a^2}$	66
norman	$\frac{a^2d^2-2abcd+b^2c^2}{2a\,b^2(b\,x^2+a)}+\frac{c^2\ln(x)}{a^2}+\frac{(a^2d^2-b^2c^2)\ln(b\,x^2+a)}{2a^2b^2}$	81
risch	$\frac{a\,d^2}{2b^2(b\,x^2+a)}-\frac{cd}{b(b\,x^2+a)}+\frac{c^2}{2a(b\,x^2+a)}+\frac{c^2\ln(x)}{a^2}+\frac{\ln(-b\,x^2-a)d^2}{2b^2}-\frac{\ln(-b\,x^2-a)c^2}{2a^2}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2*(a*d-b*c)*((a*d+b*c)/b^2*ln(b*x^2+a)+a*(a*d-b*c)/b^2/(b*x^2+a))+c^2
*ln(x)/a^2
```

Maxima [A]
time = 0.29, size = 86, normalized size = 1.28

$$\frac{c^2\log(x^2)}{2\,a^2} + \frac{b^2c^2 - 2\,abcd + a^2d^2}{2\,(ab^3x^2 + a^2b^2)} - \frac{(b^2c^2 - a^2d^2)\log(bx^2 + a)}{2\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(a*b^3*x^2 + a^2
*b^2) - 1/2*(b^2*c^2 - a^2*d^2)*log(b*x^2 + a)/(a^2*b^2)
```

Fricas [A]
time = 1.08, size = 117, normalized size = 1.75

$$\frac{ab^2c^2 - 2\,a^2bcd + a^3d^2 - (ab^2c^2 - a^3d^2 + (b^3c^2 - a^2bd^2)x^2)\log\left(bx^2 + a\right) + 2\,(b^3c^2x^2 + ab^2c^2)\log\left(x\right)}{2\,(a^2b^3x^2 + a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 - (a*b^2*c^2 - a^3*d^2 + (b^3*c^2 -
a^2*b*d^2)*x^2)*log(b*x^2 + a) + 2*(b^3*c^2*x^2 + a*b^2*c^2)*log(x))/(a^2*b
^3*x^2 + a^3*b^2)
```

Sympy [A]
time = 0.69, size = 80, normalized size = 1.19

$$\frac{a^2d^2 - 2abcd + b^2c^2}{2a^2b^2 + 2ab^3x^2} + \frac{c^2\log\left(x\right)}{a^2} + \frac{(ad - bc)\,(ad + bc)\log\left(\frac{a}{b} + x^2\right)}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x/(b*x**2+a)**2,x)
```

```
[Out] (a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + c**2*lo
g(x)/a**2 + (a*d - b*c)*(a*d + b*c)*log(a/b + x**2)/(2*a**2*b**2)
```

Giac [A]
time = 0.95, size = 99, normalized size = 1.48

$$\frac{c^2\log\left(x^2\right)}{2\,a^2} - \frac{(b^2c^2 - a^2d^2)\log\left(|bx^2 + a|\right)}{2\,a^2b^2} + \frac{b^2c^2x^2 - a^2d^2x^2 + 2\,abc^2 - 2\,a^2cd}{2\,(bx^2 + a)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2)/a^2 - 1/2*(b^2*c^2 - a^2*d^2)*log(abs(b*x^2 + a))/(a^2*b^2
) + 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d)/((b*x^2 + a)*a^
2*b)
```

Mupad [B]
time = 0.14, size = 80, normalized size = 1.19

$$\frac{c^2\,\ln\left(x\right)}{a^2} + \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{2\,a\,b^2\,\left(b\,x^2 + a\right)} + \frac{\ln\left(b\,x^2 + a\right)\,\left(a^2\,d^2 - b^2\,c^2\right)}{2\,a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x*(a + b*x^2)^2),x)
```

```
[Out] (c^2*log(x))/a^2 + (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*a*b^2*(a + b*x^2)) +
(log(a + b*x^2)*(a^2*d^2 - b^2*c^2))/(2*a^2*b^2)
```

3.277 $\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=103

$$-\frac{c^2}{ax\left(a+bx^2\right)}-\frac{\left(\frac{3bc^2}{a}-2cd+\frac{ad^2}{b}\right)x}{2a\left(a+bx^2\right)}-\frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}$$

[Out] -c^2/a/x/(b*x^2+a)-1/2*(3*b*c^2/a-2*c*d+a*d^2/b)*x/a/(b*x^2+a)-1/2*(-a*d+b*c)*(a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {473, 393, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)(ad+3bc)}{2a^{5/2}b^{3/2}}-\frac{x\left(\frac{c(3bc-2ad)}{a^2}+\frac{d^2}{b}\right)}{2\left(a+bx^2\right)}-\frac{c^2}{ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]

[Out] -(c^2/(a*x*(a + b*x^2))) - ((d^2/b + (c*(3*b*c - 2*a*d))/a^2)*x)/(2*(a + b*x^2)) - ((b*c - a*d)*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(3/2))

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
```

```
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2}\,dx &= -\frac{c^2}{ax(a+bx^2)}+\frac{\int \frac{-c(3bc-2ad)+ad^2x^2}{(a+bx^2)^2}\,dx}{a}\\ &= -\frac{c^2}{ax(a+bx^2)}-\frac{\left(\frac{3bc^2}{a}-2cd+\frac{ad^2}{b}\right)x}{2a(a+bx^2)}-\frac{((bc-ad)(3bc+ad))\int \frac{1}{a+bx^2}\,dx}{2a^2b}\\ &= -\frac{c^2}{ax(a+bx^2)}-\frac{\left(\frac{3bc^2}{a}-2cd+\frac{ad^2}{b}\right)x}{2a(a+bx^2)}-\frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 91, normalized size = 0.88

$$-\frac{c^2}{a^2x}-\frac{(-bc+ad)^2x}{2a^2b(a+bx^2)}+\frac{(-3b^2c^2+2abcd+a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
[Out] -(c^2/(a^2*x)) - ((-(b*c) + a*d)^2*x)/(2*a^2*b*(a + b*x^2)) + ((-3*b^2*c^2
+ 2*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(3/2))
```

Maple [A]
time = 0.09, size = 95, normalized size = 0.92

method	result
default	$\frac{-\frac{(a^2d^2-2abcd+b^2c^2)x}{2b(bx^2+a)}+\frac{(a^2d^2+2abcd-3b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}}{a^2}-\frac{c^2}{a^2x}$
risch	$\frac{-\frac{(a^2d^2-2abcd+3b^2c^2)x^2}{2a^2b}-\frac{c^2}{a}}{x(bx^2+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(a^5\_Z^2b^3+a^4d^4+4a^3bcd^3-2a^2b^2c^2d^2-12ab^3c^3d+9b^4c^4)}\_R\ln\left(\left(3\_R^2a^5b^3+2a^4d^4\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b*x/(b*x^2+a)+1/2*(a^2*d^2+2*a*b*c*
d-3*b^2*c^2)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-c^2/a^2/x
```

Maxima [A]
time = 0.51, size = 101, normalized size = 0.98

$$-\frac{2\,abc^2+(3\,b^2c^2-2\,abcd+a^2d^2)x^2}{2\,(a^2b^2x^3+a^3bx)}-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*c^2 + (3*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(a^2*b^2*x^3 + a^3
*b*x) - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a^2*b)
```

Fricas [A]
time = 0.92, size = 308, normalized size = 2.99

$$\left[-\frac{4\,a^2b^2c^2+2\,(3\,ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2-((3\,b^3c^2-2\,ab^2cd-a^2bd^2)x^3+(3\,ab^2c^2-2\,a^2bcd-a^3d^2)x)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,(a^3b^3x^3+a^4b^2x)},-\frac{2\,a^2b^2c^2+(3\,ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2+((3\,b^3c^2-2\,ab^2cd-a^2bd^2)x^3+(3\,ab^2c^2-2\,a^2bcd-a^3d^2)x)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(a^3b^3x^3+a^4b^2x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b^2*c^2 + 2*(3*a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2 - ((
3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - 2*a^2*b*c*d - a^3
*d^2)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3
*x^3 + a^4*b^2*x), -1/2*(2*a^2*b^2*c^2 + (3*a*b^3*c^2 - 2*a^2*b^2*c*d + a^3
*b*d^2)*x^2 + ((3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - 2
*a^2*b*c*d - a^3*d^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^3 + a^
4*b^2*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(88) = 176.
time = 0.47, size = 238, normalized size = 2.31

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}+\frac{-2abc^2+x^2(-a^2d^2+2abcd-3b^2c^2)}{2a^3bx+2a^2b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*b
**3))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x)/
```

```
4 + sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)*log(a**3*b*sqrt(-1/(a**5
*b**3))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x
)/4 + (-2*a*b*c**2 + x**2*(-a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2))/(2*a**3*b
*x + 2*a**2*b**2*x**3)
```

Giac [A]
time = 0.87, size = 103, normalized size = 1.00

$$-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b}-\frac{3\,b^2c^2x^2-2\,abcdx^2+a^2d^2x^2+2\,abc^2}{2\,(bx^3+ax)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2
*b) - 1/2*(3*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2 + 2*a*b*c^2)/((b*x^3
 + a*x)*a^2*b)
```

Mupad [B]
time = 0.07, size = 128, normalized size = 1.24

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{\sqrt{a}\,(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2)}\right)\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{2\,a^{5/2}\,b^{3/2}}-\frac{\frac{c^2}{a}+\frac{x^2\,(a^2\,d^2-2\,a\,b\,c\,d+3\,b^2\,c^2)}{2\,a^2\,b}}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^2*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(a^(1/2)*(a^2*d^2 - 3*b^2*c^2 +
 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*a^(5/2)*b^(3/2)) - (c^2/a + (x^
2*(a^2*d^2 + 3*b^2*c^2 - 2*a*b*c*d))/(2*a^2*b))/(a*x + b*x^3)
```

3.278 $\int \frac{\left(c+dx^2\right)^2}{x^3 \left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=80

$$-\frac{c^2}{2a^2x^2}-\frac{(bc-ad)^2}{2a^2b\left(a+bx^2\right)}-\frac{2c(bc-ad)\log(x)}{a^3}+\frac{c(bc-ad)\log\left(a+bx^2\right)}{a^3}$$

```
[Out] -1/2*c^2/a^2/x^2-1/2*(-a*d+b*c)^2/a^2/b/(b*x^2+a)-2*c*(-a*d+b*c)*ln(x)/a^3+
c*(-a*d+b*c)*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{c(bc-ad)\log\left(a+bx^2\right)}{a^3}-\frac{2c\log(x)(bc-ad)}{a^3}-\frac{(bc-ad)^2}{2a^2b\left(a+bx^2\right)}-\frac{c^2}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*c^2/(a^2*x^2) - (b*c - a*d)^2/(2*a^2*b*(a + b*x^2)) - (2*c*(b*c - a*d)
*Log[x])/a^3 + (c*(b*c - a*d)*Log[a + b*x^2])/a^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c^2}{a^2x^2} + \frac{2c(-bc+ad)}{a^3x} + \frac{(-bc+ad)^2}{a^2(a+bx)^2} - \frac{2bc(-bc+ad)}{a^3(a+bx)}\right) dx, x, x^2\right)$$
$$= -\frac{c^2}{2a^2x^2} - \frac{(bc-ad)^2}{2a^2b(a+bx^2)} - \frac{2c(bc-ad)\log(x)}{a^3} + \frac{c(bc-ad)\log(a+bx^2)}{a^3}$$

Mathematica [A]
time = 0.07, size = 72, normalized size = 0.90

$$-\frac{\frac{ac^2}{x^2} + \frac{a(bc-ad)^2}{b(a+bx^2)} + 4c(bc-ad)\log(x) - 2c(bc-ad)\log(a+bx^2)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*((a*c^2)/x^2 + (a*(b*c - a*d)^2)/(b*(a + b*x^2)) + 4*c*(b*c - a*d)*Log
[x] - 2*c*(b*c - a*d)*Log[a + b*x^2])/a^3
```

Maple [A]
time = 0.09, size = 77, normalized size = 0.96

method	result	size
default	$\frac{(ad-bc)\left(-2c\ln(b\,x^2+a)-\frac{a(ad-bc)}{b\left(b\,x^2+a\right)}\right)}{2a^3} - \frac{c^2}{2a^2x^2} + \frac{2c(ad-bc)\ln(x)}{a^3}$	77
norman	$\frac{-\frac{c^2}{2a}+\frac{\left(a^2d^2-2abcd+2b^2c^2\right)x^4}{2a^3}}{x^2(b\,x^2+a)} + \frac{2c(ad-bc)\ln(x)}{a^3} - \frac{c(ad-bc)\ln(b\,x^2+a)}{a^3}$	92
risch	$\frac{-\frac{\left(a^2d^2-2abcd+2b^2c^2\right)x^2}{2a^2b}-\frac{c^2}{2a}}{x^2(b\,x^2+a)} + \frac{2c\ln(x)d}{a^2} - \frac{2c^2\ln(x)b}{a^3} - \frac{c\ln(b\,x^2+a)d}{a^2} + \frac{c^2\ln(b\,x^2+a)b}{a^3}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^3*(a*d-b*c)*(-2*c*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a))-1/2*c^2/a^2/x^
2+2*c*(a*d-b*c)/a^3*ln(x)
```

Maxima [A]
time = 0.29, size = 100, normalized size = 1.25

$$-\frac{abc^2 + (2\,b^2c^2 - 2\,abcd + a^2d^2)x^2}{2\,(a^2b^2x^4 + a^3bx^2)} + \frac{(bc^2 - acd)\log(bx^2+a)}{a^3} - \frac{(bc^2 - acd)\log(x^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b*c^2 + (2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(a^2*b^2*x^4 + a^3*b
*x^2) + (b*c^2 - a*c*d)*log(b*x^2 + a)/a^3 - (b*c^2 - a*c*d)*log(x^2)/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(76) = 152$.
time = 0.61, size = 159, normalized size = 1.99

$$-\frac{a^2bc^2 + (2\,ab^2c^2 - 2\,a^2bcd + a^3d^2)x^2 - 2\left((b^3c^2 - ab^2cd)x^4 + (ab^2c^2 - a^2bcd)x^2\right)\log\left(bx^2 + a\right) + 4\left((b^3c^2 - ab^2cd)x^4 + (ab^2c^2 - a^2bcd)x^2\right)\log\left(x\right)}{2\left(a^3b^2x^4 + a^4bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2 - 2*((b^3*c^2 -
 a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x^2)*log(b*x^2 + a) + 4*((b^3*c^2
 - a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x^2)*log(x))/(a^3*b^2*x^4 + a^4
*b*x^2)
```

Sympy [A]
time = 0.76, size = 92, normalized size = 1.15

$$\frac{-abc^2 + x^2(-a^2d^2 + 2abcd - 2b^2c^2)}{2a^3bx^2 + 2a^2b^2x^4} + \frac{2c(ad - bc)\log\left(x\right)}{a^3} - \frac{c(ad - bc)\log\left(\frac{a}{b} + x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*b*c**2 + x**2*(-a**2*d**2 + 2*a*b*c*d - 2*b**2*c**2))/(2*a**3*b*x**2 +
2*a**2*b**2*x**4) + 2*c*(a*d - b*c)*log(x)/a**3 - c*(a*d - b*c)*log(a/b + x
**2)/a**3
```

Giac [A]
time = 0.76, size = 109, normalized size = 1.36

$$-\frac{(bc^2 - acd)\log\left(x^2\right)}{a^3} + \frac{(b^2c^2 - abcd)\log\left(|bx^2 + a|\right)}{a^3b} - \frac{2\,b^2c^2x^2 - 2\,abcdx^2 + a^2d^2x^2 + abc^2}{2\,(bx^4 + ax^2)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(b*c^2 - a*c*d)*log(x^2)/a^3 + (b^2*c^2 - a*b*c*d)*log(abs(b*x^2 + a))/(a^
3*b) - 1/2*(2*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2 + a*b*c^2)/((b*x^4
+ a*x^2)*a^2*b)
```

Mupad [B]
time = 0.10, size = 100, normalized size = 1.25

$$\frac{\ln(bx^2+a)\ (bc^2-acd)}{a^3}-\frac{\frac{c^2}{2a}+\frac{x^2\,(a^2d^2-2abcd+2b^2c^2)}{2a^2b}}{bx^4+ax^2}-\frac{\ln(x)\ (2bc^2-2acd)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (log(a + b*x^2)*(b*c^2 - a*c*d))/a^3 - (c^2/(2*a) + (x^2*(a^2*d^2 + 2*b^2*c
^2 - 2*a*b*c*d))/(2*a^2*b))/(a*x^2 + b*x^4) - (log(x)*(2*b*c^2 - 2*a*c*d))/
a^3
```

3.279 $\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=127

$$\frac{c(5bc-6ad)}{3a^3x}-\frac{c^2}{3ax^3(a+bx^2)}+\frac{(5b^2c^2-6abcd+3a^2d^2)\,x}{6a^3(a+bx^2)}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}$$

```
[Out] 1/3*c*(-6*a*d+5*b*c)/a^3/x-1/3*c^2/a/x^3/(b*x^2+a)+1/6*(3*a^2*d^2-6*a*b*c*d
+5*b^2*c^2)*x/a^3/(b*x^2+a)+1/2*(-a*d+b*c)*(-a*d+5*b*c)*arctan(x*b^(1/2)/a^
(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {473, 467, 464, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)(5bc-ad)}{2a^{7/2}\sqrt{b}}+\frac{c(5bc-6ad)}{3a^3x}+\frac{x\left(\frac{bc(5bc-6ad)}{a^2}+3d^2\right)}{6a(a+bx^2)}-\frac{c^2}{3ax^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (c*(5*b*c - 6*a*d))/(3*a^3*x) - c^2/(3*a*x^3*(a + b*x^2)) + ((3*d^2 + (b*c*
(5*b*c - 6*a*d))/a^2)*x)/(6*a*(a + b*x^2)) + ((b*c - a*d)*(5*b*c - a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
```

```
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2}\,dx = -\frac{c^2}{3ax^3(a+bx^2)} + \frac{\int \frac{-c(5bc-6ad)+3ad^2x^2}{x^2(a+bx^2)^2}\,dx}{3a}$$

$$= -\frac{c^2}{3ax^3(a+bx^2)} + \frac{\left(3d^2+\frac{bc(5bc-6ad)}{a^2}\right)x}{6a(a+bx^2)} - \frac{\int \frac{\frac{2c(5bc-6ad)}{a}-\left(\frac{5b^2c^2}{a^2}-\frac{6bcd}{a}+3d^2\right)x^2}{x^2(a+bx^2)}\,dx}{6a}$$

$$= \frac{c(5bc-6ad)}{3a^3x} - \frac{c^2}{3ax^3(a+bx^2)} + \frac{\left(3d^2+\frac{bc(5bc-6ad)}{a^2}\right)x}{6a(a+bx^2)} + \frac{((bc-ad)(5bc-ad))\int \frac{1}{a+b\cdots}}{2a^3}$$

$$= \frac{c(5bc-6ad)}{3a^3x} - \frac{c^2}{3ax^3(a+bx^2)} + \frac{\left(3d^2+\frac{bc(5bc-6ad)}{a^2}\right)x}{6a(a+bx^2)} + \frac{(bc-ad)(5bc-ad)\tan^{-1}(\cdots}{2a^{7/2}\sqrt{b}}$$

Mathematica [A]
time = 0.05, size = 107, normalized size = 0.84

$$-\frac{c^2}{3a^2x^3} - \frac{2c(-bc+ad)}{a^3x} + \frac{(-bc+ad)^2x}{2a^3(a+bx^2)} + \frac{(5b^2c^2-6abcd+a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c^2/(a^2*x^3) - (2*c*(-(b*c) + a*d))/(a^3*x) + ((-(b*c) + a*d)^2*x)/(2
*a^3*(a + b*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(7/2)*Sqrt[b])
```

Maple [A]
time = 0.09, size = 107, normalized size = 0.84

method	result
default	$\frac{\frac{\left(\frac{1}{2}a^2d^2-abcd+\frac{1}{2}b^2c^2\right)x}{b\,x^2+a}+\frac{\left(a^2d^2-6abcd+5b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^3}-\frac{c^2}{3a^2x^3}-\frac{2c(ad-bc)}{a^3x}$
risch	$\frac{\frac{\left(a^2d^2-6abcd+5b^2c^2\right)x^4}{2a^3}-\frac{c(6ad-5bc)x^2}{3a^2}-\frac{c^2}{3a}}{x^3(b\,x^2+a)}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^7b\_Z^2+a^4d^4-12a^3bc\,d^3+46a^2b^2c^2d^2-60a\,b^3c^3d+25b^4c^4\right)}\_R\ln\left(\left(3\_R\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*((1/2*a^2*d^2-a*b*c*d+1/2*b^2*c^2)*x/(b*x^2+a)+1/2*(a^2*d^2-6*a*b*c*d
+5*b^2*c^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3*c^2/a^2/x^3-2*c*(a*d-b
*c)/a^3/x
```

Maxima [A]
time = 0.49, size = 118, normalized size = 0.93

$$\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)x^4-2\,a^2c^2+2\,(5\,abc^2-6\,a^2cd)x^2}{6\,(a^3bx^5+a^4x^3)}+\frac{(5\,b^2c^2-6\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^2 - 6
*a^2*c*d)*x^2)/(a^3*b*x^5 + a^4*x^3) + 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2
)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.88, size = 356, normalized size = 2.80

$$\left[-\frac{4\,a^3bc^2-6\,(5\,ab^3c^2-6\,a^2b^2cd+a^3bd^2)x^4-4\,(5\,a^2b^2c^2-6\,a^3bcd)x^2+3\,((5\,b^3c^2-6\,ab^2cd+a^2bd^2)x^5+(5\,ab^2c^2-6\,a^2bcd+a^3d^2)x^3)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{12\,(a^4b^2x^5+a^5bx^3)},\,-\frac{2\,a^3bc^2-3\,(5\,ab^3c^2-6\,a^2b^2cd+a^3bd^2)x^4-2\,(5\,a^2b^2c^2-6\,a^3bcd)x^2-3\,((5\,b^3c^2-6\,ab^2cd+a^2bd^2)x^5+(5\,ab^2c^2-6\,a^2bcd+a^3d^2)x^3)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{6\,(a^4b^2x^5+a^5bx^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b*c^2 - 6*(5*a*b^3*c^2 - 6*a^2*b^2*c*d + a^3*b*d^2)*x^4 - 4*(
5*a^2*b^2*c^2 - 6*a^3*b*c*d)*x^2 + 3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)
*x^5 + (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*sqrt(-a*b)*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^2*x^5 + a^5*b*x^3), -1/6*(2*a^3*b*c
```

```
^2 - 3*(5*a*b^3*c^2 - 6*a^2*b^2*c*d + a^3*b*d^2)*x^4 - 2*(5*a^2*b^2*c^2 - 6
*a^3*b*c*d)*x^2 - 3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^5 + (5*a*b^2*c
^2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^2*
x^5 + a^5*b*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(114) = 228$.
time = 0.54, size = 248, normalized size = 1.95

$$-\frac{\sqrt{-\frac{1}{a^7b}}\left(ad-5bc\right)\left(ad-bc\right)\log\left(-\frac{a^4\sqrt{-\frac{1}{a^7b}}\left(ad-5bc\right)\left(ad-bc\right)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b}}\left(ad-5bc\right)\left(ad-bc\right)\log\left(\frac{a^4\sqrt{-\frac{1}{a^7b}}\left(ad-5bc\right)\left(ad-bc\right)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}+\frac{-2a^2c^2+x^4\cdot\left(3a^2d^2-18abcd+15b^2c^2\right)+x^2\left(-12a^2cd+10abc^2\right)}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(-a**4*sqrt(-1/(a**7*b))*(a
*d - 5*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + sqrt
(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(a**4*sqrt(-1/(a**7*b))*(a*d - 5
*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + (-2*a**2*c
**2 + x**4*(3*a**2*d**2 - 18*a*b*c*d + 15*b**2*c**2) + x**2*(-12*a**2*c*d +
 10*a*b*c**2))/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Giac [A]
time = 0.72, size = 112, normalized size = 0.88

$$\frac{\left(5\,b^2c^2-6\,abcd+a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(bx^2+a)a^3}+\frac{6\,bc^2x^2-6\,acdx^2-ac^2}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
 + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a^3) + 1/3*(6*b*c
^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^3*x^3)
```

Mupad [B]
time = 0.13, size = 146, normalized size = 1.15

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,\left(a\,d-b\,c\right)\,\left(a\,d-5\,b\,c\right)}{\sqrt{a}\,\left(a^2\,d^2-6\,a\,b\,c\,d+5\,b^2\,c^2\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-5\,b\,c\right)}{2\,a^{7/2}\,\sqrt{b}}-\frac{\frac{c^2}{3\,a}-\frac{x^4\,\left(a^2\,d^2-6\,a\,b\,c\,d+5\,b^2\,c^2\right)}{2\,a^3}+\frac{c\,x^2\,\left(6\,a\,d-5\,b\,c\right)}{3\,a^2}}{b\,x^5+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^4*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x*(a*d - b*c)*(a*d - 5*b*c))/(a^(1/2)*(a^2*d^2 + 5*b^2*c^2 -
 6*a*b*c*d)))*(a*d - b*c)*(a*d - 5*b*c))/(2*a^(7/2)*b^(1/2)) - (c^2/(3*a) -
 (x^4*(a^2*d^2 + 5*b^2*c^2 - 6*a*b*c*d))/(2*a^3) + (c*x^2*(6*a*d - 5*b*c))/
(3*a^2))/(a*x^3 + b*x^5)
```

3.280 $\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=169

$$\frac{3(bc-3ad)(bc-ad)^2x}{2b^5}+\frac{d(5b^2c^2-7abcd+3a^2d^2)\,x^3}{2b^4}+\frac{3d^2(7bc-3ad)x^5}{10b^3}+\frac{9d^3x^7}{14b^2}-\frac{x^3(c+dx^2)^3}{2b\,(a+bx^2)}-\frac{3\sqrt{a}\,(bc-$$

```
[Out] 3/2*(-3*a*d+b*c)*(-a*d+b*c)^2*x/b^5+1/2*d*(3*a^2*d^2-7*a*b*c*d+5*b^2*c^2)*x
^3/b^4+3/10*d^2*(-3*a*d+7*b*c)*x^5/b^3+9/14*d^3*x^7/b^2-1/2*x^3*(d*x^2+c)^3
/b/(b*x^2+a)-3/2*(-3*a*d+b*c)*(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))*a^(1/2
)/b^(11/2)
```

Rubi [A]
time = 0.11, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {478, 584, 211}

$$\frac{dx^3(3a^2d^2-7abcd+5b^2c^2)}{2b^4}-\frac{3\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-3ad)(bc-ad)^2}{2b^{11/2}}+\frac{3x(bc-3ad)(bc-ad)^2}{2b^5}+\frac{3d^2x^5(7bc-3ad)}{10b^3}-\frac{x^3(c+dx^2)^3}{2b\,(a+bx^2)}+\frac{9d^3x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*(b*c - 3*a*d)*(b*c - a*d)^2*x)/(2*b^5) + (d*(5*b^2*c^2 - 7*a*b*c*d + 3*a
^2*d^2)*x^3)/(2*b^4) + (3*d^2*(7*b*c - 3*a*d)*x^5)/(10*b^3) + (9*d^3*x^7)/(
14*b^2) - (x^3*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - (3*Sqrt[a]*(b*c - 3*a*d)*
(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx = -\frac{x^3(c+dx^2)^3}{2b\,(a+bx^2)} + \frac{\int \frac{x^2(c+dx^2)^2(3c+9dx^2)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^3(c+dx^2)^3}{2b\,(a+bx^2)} + \frac{\int \left(\frac{3(bc-3ad)(bc-ad)^2}{b^4} + \frac{3d(5b^2c^2-7abcd+3a^2d^2)x^2}{b^3} + \frac{3d^2(7bc-3ad)x^4}{b^2} + \frac{9d^3x^6}{b} + \right.}{2b}$$

$$= \frac{3(bc-3ad)(bc-ad)^2x}{2b^5} + \frac{d(5b^2c^2-7abcd+3a^2d^2)\,x^3}{2b^4} + \frac{3d^2(7bc-3ad)x^5}{10b^3} + \frac{9d^3x^7}{14b^2}$$

$$= \frac{3(bc-3ad)(bc-ad)^2x}{2b^5} + \frac{d(5b^2c^2-7abcd+3a^2d^2)\,x^3}{2b^4} + \frac{3d^2(7bc-3ad)x^5}{10b^3} + \frac{9d^3x^7}{14b^2}$$

Mathematica [A]
time = 0.06, size = 151, normalized size = 0.89

$$\frac{(bc-4ad)(bc-ad)^2x}{b^5} + \frac{d(bc-ad)^2x^3}{b^4} + \frac{d^2(3bc-2ad)x^5}{5b^3} + \frac{d^3x^7}{7b^2} + \frac{a(bc-ad)^3x}{2b^5\,(a+bx^2)} + \frac{3\sqrt{a}\,(bc-ad)^2(-bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 4*a*d)*(b*c - a*d)^2*x)/b^5 + (d*(b*c - a*d)^2*x^3)/b^4 + (d^2*(3*b
*c - 2*a*d)*x^5)/(5*b^3) + (d^3*x^7)/(7*b^2) + (a*(b*c - a*d)^3*x)/(2*b^5*(
a + b*x^2)) + (3*Sqrt[a]*(b*c - a*d)^2*(-(b*c) + 3*a*d)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*b^(11/2))
```

Maple [A]
time = 0.11, size = 227, normalized size = 1.34

method	result
default	$-\frac{-\frac{1}{7}d^3x^7b^3+\frac{2}{5}a\,b^2d^3x^5-\frac{3}{5}b^3c\,d^2x^5-a^2b\,d^3x^3+2a\,b^2c\,d^2x^3-b^3c^2d\,x^3+4a^3d^3x-9a^2bc\,d^2x+6a\,b^2c^2dx-b^3c^3x}{b^5}+\frac{a\left(\frac{\left(-\frac{1}{2}a^3d^3+\frac{3}{2}a\right.}{}\right.}{}$

risch	$\frac{d^3x^7}{7b^2} - \frac{2a\,d^3x^5}{5b^3} + \frac{3c\,d^2x^5}{5b^2} + \frac{a^2d^3x^3}{b^4} - \frac{2ac\,d^2x^3}{b^3} + \frac{c^2d\,x^3}{b^2} - \frac{4a^3d^3x}{b^5} + \frac{9a^2c\,d^2x}{b^4} - \frac{6a\,c^2dx}{b^3} + \frac{c^3x}{b^2} + \frac{\left(-\frac{1}{2}a^4d^3+\frac{3}{2}a^3bc\right.}{b^5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/b^5*(-1/7*d^3*x^7*b^3+2/5*a*b^2*d^3*x^5-3/5*b^3*c*d^2*x^5-a^2*b*d^3*x^3+2*a*b^2*c*d^2*x^3-b^3*c^2*d*x^3+4*a^3*d^3*x-9*a^2*b*c*d^2*x+6*a*b^2*c^2*d*x-b^3*c^3*x)+a/b^5*((-1/2*a^3*d^3+3/2*a^2*b*c*d^2-3/2*a*b^2*c^2*d+1/2*b^3*c^3)*x/(b*x^2+a)+3/2*(3*a^3*d^3-7*a^2*b*c*d^2+5*a*b^2*c^2*d-b^3*c^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))

Maxima [A]
time = 0.56, size = 228, normalized size = 1.35

$$\frac{(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)x}{2\,(b^6x^2 + ab^5)} - \frac{3\,(ab^3c^3 - 5\,a^2b^2c^2d + 7\,a^3bcd^2 - 3\,a^4d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5} + \frac{5\,b^3d^3x^7 + 7\,(3\,b^3cd^2 - 2\,ab^2d^3)x^5 + 35\,(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^3 + 35\,(b^3c^3 - 6\,ab^2c^2d + 9\,a^2bcd^2 - 4\,a^3d^3)x}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x/(b^6*x^2 + a*b^5) - 3/2*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/35*(5*b^3*d^3*x^7 + 7*(3*b^3*c*d^2 - 2*a*b^2*d^3)*x^5 + 35*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3 + 35*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5

Fricas [A]
time = 1.06, size = 580, normalized size = 3.43

$$\left[\frac{20\,b^4d^3x^9 + 12\,(7\,b^4cd^2 - 3\,ab^3d^3)x^7 + 28\,(5\,b^4c^2d - 7\,ab^3cd^2 + 3\,a^2b^2d^3)x^5 + 140\,(b^4c^3 - 5\,ab^3c^2d + 7\,a^2b^2cd^2 - 3\,a^3bd^3)x^3 - 105\,(ab^3c^3 - 5\,a^2b^2c^2d + 7\,a^3bcd^2 - 3\,a^4d^3 + (b^4c^3 - 5\,ab^3c^2d + 7\,a^2b^2cd^2 - 3\,a^3bd^3)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 210\,(ab^3c^3 - 5\,a^2b^2c^2d + 7\,a^3bcd^2 - 3\,a^4d^3)x}{140\,(b^6x^2 + ab^5)}, \frac{10\,b^4d^3x^9 + 6\,(7\,b^4cd^2 - 3\,ab^3d^3)x^7 + 14\,(5\,b^4c^2d - 7\,ab^3cd^2 + 3\,a^2b^2d^3)x^5 + 70\,(b^4c^3 - 5\,ab^3c^2d + 7\,a^2b^2cd^2 - 3\,a^3bd^3)x^3 - 105\,(ab^3c^3 - 5\,a^2b^2c^2d + 7\,a^3bcd^2 - 3\,a^4d^3 + (b^4c^3 - 5\,ab^3c^2d + 7\,a^2b^2cd^2 - 3\,a^3bd^3)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 105\,(ab^3c^3 - 5\,a^2b^2c^2d + 7\,a^3bcd^2 - 3\,a^4d^3)x}{70\,(b^6x^2 + ab^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/140*(20*b^4*d^3*x^9 + 12*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 28*(5*b^4*c^2*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 140*(b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*x)/(b^6*x^2 + a*b^5), 1/70*(10*b^4*d^3*x^9 + 6*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 14*(5*b^4*c^2*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 70*(b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3

```
*b*d^3)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(a*b^3*c^3 - 5*a^2*b^2
*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*x)/(b^6*x^2 + a*b^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(167) = 334$.
time = 0.76, size = 389, normalized size = 2.30

$$x^5\left(-\frac{2ad^3}{5b^3}+\frac{3cd^2}{5b^2}\right)+x^3\left(\frac{a^2d^3}{b^4}-\frac{2acd^2}{b^3}+\frac{c^2d}{b^2}\right)+x\left(-\frac{4a^3d^3}{b^5}+\frac{9a^2cd^2}{b^4}-\frac{6ac^2d}{b^3}+\frac{c^3}{b^2}\right)+\frac{x(-a^4d^3+3a^3bcd^2-3a^2b^2c^2d+ab^3c^3)}{2ab^5+2b^6x^2}-\frac{3\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2\cdot(3ad-bc)\log\left(-\frac{3b^5\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2\cdot(3ad-bc)}{9a^3d^3-21a^2bcd^2+15ab^2c^2d-3b^3c^3}+x\right)}{4}+\frac{3\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2\cdot(3ad-bc)\log\left(\frac{3b^5\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2\cdot(3ad-bc)}{9a^3d^3-21a^2bcd^2+15ab^2c^2d-3b^3c^3}+x\right)}{4}+\frac{d^3x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x**5*(-2*a*d**3/(5*b**3) + 3*c*d**2/(5*b**2)) + x**3*(a**2*d**3/b**4 - 2*a*
c*d**2/b**3 + c**2*d/b**2) + x*(-4*a**3*d**3/b**5 + 9*a**2*c*d**2/b**4 - 6*
a*c**2*d/b**3 + c**3/b**2) + x*(-a**4*d**3 + 3*a**3*b*c*d**2 - 3*a**2*b**2*
c**2*d + a*b**3*c**3)/(2*a*b**5 + 2*b**6*x**2) - 3*sqrt(-a/b**11)*(a*d - b*
c)**2*(3*a*d - b*c)*log(-3*b**5*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)
/(9*a**3*d**3 - 21*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 3*b**3*c**3) + x)/4 +
 3*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)*log(3*b**5*sqrt(-a/b**11)*(a
*d - b*c)**2*(3*a*d - b*c)/(9*a**3*d**3 - 21*a**2*b*c*d**2 + 15*a*b**2*c**2
*d - 3*b**3*c**3) + x)/4 + d**3*x**7/(7*b**2)
```

Giac [A]
time = 0.62, size = 241, normalized size = 1.43

$$-\frac{3\,(ab^3c^3-5\,a^2b^2c^2d+7\,a^3bcd^2-3\,a^4d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{ab^3c^3x-3\,a^2b^2c^2dx+3\,a^3bcd^2x-a^4d^3x}{2\,(bx^2+a)b^5}+\frac{5\,b^{12}d^3x^7+21\,b^{12}cd^2x^5-14\,ab^{11}d^3x^5+35\,b^{12}c^2dx^3-70\,ab^{11}cd^2x^3+35\,a^2b^{10}d^3x^3+35\,b^{12}c^3x-210\,ab^{11}c^2dx+315\,a^2b^{10}cd^2x-140\,a^3b^9d^3x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^5) + 1/2*(a*b^3*c^3*x - 3*a^2*b^2*c^2*d*x + 3*a^3*b*
c*d^2*x - a^4*d^3*x)/((b*x^2 + a)*b^5) + 1/35*(5*b^12*d^3*x^7 + 21*b^12*c*d
^2*x^5 - 14*a*b^11*d^3*x^5 + 35*b^12*c^2*d*x^3 - 70*a*b^11*c*d^2*x^3 + 35*a
^2*b^10*d^3*x^3 + 35*b^12*c^3*x - 210*a*b^11*c^2*d*x + 315*a^2*b^10*c*d^2*x
 - 140*a^3*b^9*d^3*x)/b^14
```

Mupad [B]
time = 0.11, size = 328, normalized size = 1.94

$$x\left(\frac{c^3}{b^2}-\frac{2\,a\left(\frac{3\,c^2\,d}{b^2}+\frac{2\,a\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b}-\frac{a^2\,d^3}{b^4}\right)}{b}+\frac{a^2\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b^2}\right)-x^5\left(\frac{2\,a\,d^3}{5\,b^3}-\frac{3\,c\,d^2}{5\,b^2}\right)+x^3\left(\frac{c^2\,d}{b^2}+\frac{2\,a\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{3\,b}-\frac{a^2\,d^3}{3\,b^4}\right)-\frac{x\left(\frac{a^4\,d^3}{2}-\frac{3\,a^3\,b\,c\,d^2}{2}+\frac{3\,a^2\,b^2\,c^2\,d}{2}-\frac{a\,b^3\,c^3}{2}\right)}{b^6\,x^2+a\,b^5}+\frac{d^3\,x^7}{7\,b^2}+\frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d-b\,c\right)}{3\,a^4\,d^3-7\,a^3\,b\,c\,d^2+5\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d-b\,c\right)}{2\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x*(c^3/b^2 - (2*a*((3*c^2*d)/b^2 + (2*a*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b
- (a^2*d^3)/b^4))/b + (a^2*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b^2) - x^5*((2*
a*d^3)/(5*b^3) - (3*c*d^2)/(5*b^2)) + x^3*((c^2*d)/b^2 + (2*a*((2*a*d^3)/b^
3 - (3*c*d^2)/b^2))/(3*b) - (a^2*d^3)/(3*b^4)) - (x*((a^4*d^3)/2 - (a*b^3*c
^3)/2 + (3*a^2*b^2*c^2*d)/2 - (3*a^3*b*c*d^2)/2))/(a*b^5 + b^6*x^2) + (d^3*
x^7)/(7*b^2) + (3*a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d - b*c)^2*(3*a*d - b*
c))/(3*a^4*d^3 - a*b^3*c^3 + 5*a^2*b^2*c^2*d - 7*a^3*b*c*d^2))*(a*d - b*c)^
2*(3*a*d - b*c))/(2*b^(11/2))
```

3.281 $\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2} dx$

Optimal. Leaf size=117

$$\frac{3d(bc-ad)^2x^2}{2b^4}+\frac{d^2(3bc-2ad)x^4}{4b^3}+\frac{d^3x^6}{6b^2}+\frac{a(bc-ad)^3}{2b^5(a+bx^2)}+\frac{(bc-4ad)(bc-ad)^2\log(a+bx^2)}{2b^5}$$

```
[Out] 3/2*d*(-a*d+b*c)^2*x^2/b^4+1/4*d^2*(-2*a*d+3*b*c)*x^4/b^3+1/6*d^3*x^6/b^2+1
/2*a*(-a*d+b*c)^3/b^5/(b*x^2+a)+1/2*(-4*a*d+b*c)*(-a*d+b*c)^2*ln(b*x^2+a)/b
^5
```

Rubi [A]
time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{a(bc-ad)^3}{2b^5(a+bx^2)}+\frac{(bc-4ad)(bc-ad)^2\log(a+bx^2)}{2b^5}+\frac{3dx^2(bc-ad)^2}{2b^4}+\frac{d^2x^4(3bc-2ad)}{4b^3}+\frac{d^3x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*d*(b*c - a*d)^2*x^2)/(2*b^4) + (d^2*(3*b*c - 2*a*d)*x^4)/(4*b^3) + (d^3*
x^6)/(6*b^2) + (a*(b*c - a*d)^3)/(2*b^5*(a + b*x^2)) + ((b*c - 4*a*d)*(b*c
- a*d)^2*Log[a + b*x^2])/(2*b^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^3}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{3d(bc-ad)^2}{b^4} + \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^2}{b^2} + \frac{a(-bc+ad)^3}{b^4(a+bx)^2} + \frac{(bc-4ad)(bc}{b^4(a+bx}\right.\right.$$
$$= \frac{3d(bc-ad)^2x^2}{2b^4} + \frac{d^2(3bc-2ad)x^4}{4b^3} + \frac{d^3x^6}{6b^2} + \frac{a(bc-ad)^3}{2b^5\left(a+bx^2\right)} + \frac{(bc-4ad)(bc-ad)^2\log(}{2b^5}$$

Mathematica [A]
time = 0.07, size = 106, normalized size = 0.91

$$\frac{18bd(bc-ad)^2x^2 + 3b^2d^2(3bc-2ad)x^4 + 2b^3d^3x^6 - \frac{6a(-bc+ad)^3}{a+bx^2} + 6(bc-4ad)(bc-ad)^2\log\left(a+bx^2\right)}{12b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (18*b*d*(b*c - a*d)^2*x^2 + 3*b^2*d^2*(3*b*c - 2*a*d)*x^4 + 2*b^3*d^3*x^6 -
 (6*a*(-(b*c) + a*d)^3)/(a + b*x^2) + 6*(b*c - 4*a*d)*(b*c - a*d)^2*Log[a +
 b*x^2])/(12*b^5)
```

Maple [A]
time = 0.09, size = 137, normalized size = 1.17

method	result
default	$\frac{d\left(\frac{b^2d^2x^6}{6}+\frac{\left(-2ab\,d^2+3b^2cd\right)x^4}{4}+\frac{\left(3a^2d^2-6abcd+3b^2c^2\right)x^2}{2}\right)}{b^4}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{(4ad-bc)\ln\left(b\,x^2+a\right)}{b}+\frac{a(ad-bc)}{b\left(b\,x^2+a\right)}\right)}{2b^4}$
norman	$\frac{\frac{d^3x^8}{6b}+\frac{d\left(4a^2d^2-9abcd+6b^2c^2\right)x^4}{4b^3}-\frac{d^2(4ad-9bc)x^6}{12b^2}+\frac{\left(4a^4d^3-9a^3bc\,d^2+6a^2b^2c^2d-a\,b^3c^3\right)x^2}{2b^4a}}{b\,x^2+a}-\frac{\left(4a^3d^3-9a^2bc\,d^2+6a\,b^2c^2d-b^3c^3\right)\ln\left(b\,x^2\right.}{2b^5}$
risch	$\frac{d^3x^6}{6b^2}-\frac{d^3a\,x^4}{2b^3}+\frac{3d^2c\,x^4}{4b^2}+\frac{3d^3a^2x^2}{2b^4}-\frac{3d^2ac\,x^2}{b^3}+\frac{3d\,c^2x^2}{2b^2}-\frac{a^4d^3}{2b^5(b\,x^2+a)}+\frac{3a^3c\,d^2}{2b^4(b\,x^2+a)}-\frac{3a^2c^2d}{2b^3(b\,x^2+a)}+\frac{a\,c^3}{2b^2(b\,x^2+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d/b^4*(1/6*b^2*d^2*x^6+1/4*(-2*a*b*d^2+3*b^2*c*d)*x^4+1/2*(3*a^2*d^2-6*a*b*
c*d+3*b^2*c^2)*x^2)-1/2/b^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)*((4*a*d-b*c)/b*ln(b
*x^2+a)+a*(a*d-b*c)/b/(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 174, normalized size = 1.49

$$\frac{ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3}{2\left(b^6x^2 + ab^5\right)} + \frac{2\,b^2d^3x^6 + 3\left(3\,b^2cd^2 - 2\,abd^3\right)x^4 + 18\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^2}{12\,b^4} + \frac{\left(b^3c^3 - 6\,ab^2c^2d + 9\,a^2bcd^2 - 4\,a^3d^3\right)\log\left(bx^2 + a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)/(b^6*x^2 + a*b^
5) + 1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - 2*a*b*d^3)*x^4 + 18*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*x^2)/b^4 + 1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*
c*d^2 - 4*a^3*d^3)*log(b*x^2 + a)/b^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(107) = 214$.
time = 1.06, size = 254, normalized size = 2.17

$$\frac{2\,b^4d^3x^8+6\,ab^3c^3-18\,a^2b^2c^2d+18\,a^3bcd^2-6\,a^4d^3+(9\,b^4cd^2-4\,ab^3d^3)x^6+3\,(6\,b^4c^2d-9\,ab^3cd^2+4\,a^2b^2d^3)x^4+18\,(ab^3c^2d-2\,a^2b^2cd^2+a^3bd^3)x^2+6\,(ab^3c^3-6\,a^2b^2c^2d+9\,a^3bcd^2-4\,a^4d^3+(b^4c^3-6\,ab^3c^2d+9\,a^2b^2cd^2-4\,a^3bd^3)x^2)\log\left(bx^2+a\right)}{12\,(b^6x^2+ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^4*d^3*x^8 + 6*a*b^3*c^3 - 18*a^2*b^2*c^2*d + 18*a^3*b*c*d^2 - 6*a
^4*d^3 + (9*b^4*c*d^2 - 4*a*b^3*d^3)*x^6 + 3*(6*b^4*c^2*d - 9*a*b^3*c*d^2 +
 4*a^2*b^2*d^3)*x^4 + 18*(a*b^3*c^2*d - 2*a^2*b^2*c*d^2 + a^3*b*d^3)*x^2 +
6*(a*b^3*c^3 - 6*a^2*b^2*c^2*d + 9*a^3*b*c*d^2 - 4*a^4*d^3 + (b^4*c^3 - 6*a
*b^3*c^2*d + 9*a^2*b^2*c*d^2 - 4*a^3*b*d^3)*x^2)*log(b*x^2 + a))/(b^6*x^2 +
 a*b^5)
```

Sympy [A]
time = 0.95, size = 163, normalized size = 1.39

$$x^4\left(-\frac{ad^3}{2b^3}+\frac{3cd^2}{4b^2}\right)+x^2\cdot\left(\frac{3a^2d^3}{2b^4}-\frac{3acd^2}{b^3}+\frac{3c^2d}{2b^2}\right)+\frac{-a^4d^3+3a^3bcd^2-3a^2b^2c^2d+ab^3c^3}{2ab^5+2b^6x^2}+\frac{d^3x^6}{6b^2}-\frac{(ad-bc)^2\cdot(4ad-bc)\log\left(a+bx^2\right)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x**4*(-a*d**3/(2*b**3) + 3*c*d**2/(4*b**2)) + x**2*(3*a**2*d**3/(2*b**4) -
3*a*c*d**2/b**3 + 3*c**2*d/(2*b**2)) + (-a**4*d**3 + 3*a**3*b*c*d**2 - 3*a*
*2*b**2*c**2*d + a*b**3*c**3)/(2*a*b**5 + 2*b**6*x**2) + d**3*x**6/(6*b**2)
 - (a*d - b*c)**2*(4*a*d - b*c)*log(a + b*x**2)/(2*b**5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(107) = 214$.
time = 0.67, size = 249, normalized size = 2.13

$$\frac{\frac{\left(2\,d^3+\frac{3\,\left(3\,b^2cd^2-4\,abd^3\right)}{\left(bx^2+a\right)b}+\frac{18\,\left(b^4c^2d-3\,ab^3cd^2+2\,a^2b^2d^3\right)}{\left(bx^2+a\right)^2b^2}\right)\left(bx^2+a\right)^3}{b^4}-\frac{6\,\left(b^3c^3-6\,ab^2c^2d+9\,a^2bcd^2-4\,a^3d^3\right)\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^4}+\frac{6\,\left(\frac{ab^6c^3}{bx^2+a}-\frac{3\,a^2b^5c^2d}{bx^2+a}+\frac{3\,a^3b^4cd^2}{bx^2+a}-\frac{a^4b^3d^3}{bx^2+a}\right)}{b^7}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*((2*d^3 + 3*(3*b^2*c*d^2 - 4*a*b*d^3)/((b*x^2 + a)*b) + 18*(b^4*c^2*d
- 3*a*b^3*c*d^2 + 2*a^2*b^2*d^3)/((b*x^2 + a)^2*b^2))*(b*x^2 + a)^3/b^4 - 6
*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*log(abs(b*x^2 + a)/(
(b*x^2 + a)^2*abs(b)))/b^4 + 6*(a*b^6*c^3/(b*x^2 + a) - 3*a^2*b^5*c^2*d/(b*
x^2 + a) + 3*a^3*b^4*c*d^2/(b*x^2 + a) - a^4*b^3*d^3/(b*x^2 + a))/b^7)/b
```

Mupad [B]
time = 0.10, size = 194, normalized size = 1.66

$$x^2\left(\frac{3\,c^2\,d}{2\,b^2}+\frac{a\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b}-\frac{a^2\,d^3}{2\,b^4}\right)-x^4\,\left(\frac{a\,d^3}{2\,b^3}-\frac{3\,c\,d^2}{4\,b^2}\right)-\frac{\ln\left(b\,x^2+a\right)\,\left(4\,a^3\,d^3-9\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,b^5}-\frac{a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}{2\,b\,\left(b^5\,x^2+a\,b^4\right)}+\frac{d^3\,x^6}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x^2*((3*c^2*d)/(2*b^2) + (a*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b - (a^2*d^3)/
(2*b^4)) - x^4*((a*d^3)/(2*b^3) - (3*c*d^2)/(4*b^2)) - (log(a + b*x^2)*(4*a
^3*d^3 - b^3*c^3 + 6*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(2*b^5) - (a^4*d^3 - a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)/(2*b*(a*b^4 + b^5*x^2)) + (d^3*x^
6)/(6*b^2)
```

3.282 $\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=147

$$\frac{d(81b^2c^2-190abcd+105a^2d^2)\,x}{30b^4}+\frac{d(33bc-35ad)x(c+dx^2)}{30b^3}+\frac{7dx(c+dx^2)^2}{10b^2}-\frac{x(c+dx^2)^3}{2b\,(a+bx^2)}+\frac{(bc-7ad)(bc}{}$$

```
[Out] 1/30*d*(105*a^2*d^2-190*a*b*c*d+81*b^2*c^2)*x/b^4+1/30*d*(-35*a*d+33*b*c)*x
*(d*x^2+c)/b^3+7/10*d*x*(d*x^2+c)^2/b^2-1/2*x*(d*x^2+c)^3/b/(b*x^2+a)+1/2*(
-7*a*d+b*c)*(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)/a^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {478, 542, 396, 211}

$$\frac{dx(105a^2d^2-190abcd+81b^2c^2)}{30b^4}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-7ad)(bc-ad)^2}{2\sqrt{a}\,b^{9/2}}+\frac{dx(c+dx^2)(33bc-35ad)}{30b^3}-\frac{x(c+dx^2)^3}{2b\,(a+bx^2)}+\frac{7dx(c+dx^2)^2}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d*(81*b^2*c^2 - 190*a*b*c*d + 105*a^2*d^2)*x)/(30*b^4) + (d*(33*b*c - 35*a
*d)*x*(c + d*x^2))/(30*b^3) + (7*d*x*(c + d*x^2)^2)/(10*b^2) - (x*(c + d*x^
2)^3)/(2*b*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
```

```
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx = -\frac{x(c+dx^2)^3}{2b(a+bx^2)} + \frac{\int \frac{(c+dx^2)^2(c+7dx^2)}{a+bx^2}\,dx}{2b}$$

$$= \frac{7dx(c+dx^2)^2}{10b^2} - \frac{x(c+dx^2)^3}{2b(a+bx^2)} + \frac{\int \frac{(c+dx^2)(c(5bc-7ad)+d(33bc-35ad)x^2)}{a+bx^2}\,dx}{10b^2}$$

$$= \frac{d(33bc-35ad)x(c+dx^2)}{30b^3} + \frac{7dx(c+dx^2)^2}{10b^2} - \frac{x(c+dx^2)^3}{2b(a+bx^2)} + \frac{\int \frac{c(15b^2c^2-54abcd+35a^2d^2)+d(\cdots}{a+\cdots}}{30\cdots}$$

$$= \frac{d(81b^2c^2-190abcd+105a^2d^2)x}{30b^4} + \frac{d(33bc-35ad)x(c+dx^2)}{30b^3} + \frac{7dx(c+dx^2)^2}{10b^2} - \frac{x(c\cdots}{2b(\cdots}$$

$$= \frac{d(81b^2c^2-190abcd+105a^2d^2)x}{30b^4} + \frac{d(33bc-35ad)x(c+dx^2)}{30b^3} + \frac{7dx(c+dx^2)^2}{10b^2} - \frac{x(c\cdots}{2b(\cdots}$$

Mathematica [A]
time = 0.05, size = 125, normalized size = 0.85

$$\frac{3d(bc-ad)^2x}{b^4} + \frac{d^2(3bc-2ad)x^3}{3b^3} + \frac{d^3x^5}{5b^2} - \frac{(bc-ad)^3x}{2b^4(a+bx^2)} + \frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*d*(b*c - a*d)^2*x)/b^4 + (d^2*(3*b*c - 2*a*d)*x^3)/(3*b^3) + (d^3*x^5)/(
5*b^2) - ((b*c - a*d)^3*x)/(2*b^4*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c - a*d)
^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Maple [A]
time = 0.11, size = 170, normalized size = 1.16

method	result
default	$\frac{d\left(\frac{1}{5}b^2x^5d^2-\frac{2}{3}ab\,d^2x^3+b^2cd\,x^3+3a^2d^2x-6abcdx+3b^2c^2x\right)}{b^4}-\frac{\frac{\left(-\frac{1}{2}a^3d^3+\frac{3}{2}a^2bc\,d^2-\frac{3}{2}a\,b^2c^2d+\frac{1}{2}b^3c^3\right)x}{b\,x^2+a}+\frac{\left(7a^3d^3-15a^2bc\,d^2+9a\,b^2c^2d-\right.}{2\sqrt{a}}}{b^4}$
risch	$\frac{d^3x^5}{5b^2}-\frac{2d^3a\,x^3}{3b^3}+\frac{d^2c\,x^3}{b^2}+\frac{3d^3a^2x}{b^4}-\frac{6d^2acx}{b^3}+\frac{3d\,c^2x}{b^2}+\frac{\left(\frac{1}{2}a^3d^3-\frac{3}{2}a^2bc\,d^2+\frac{3}{2}a\,b^2c^2d-\frac{1}{2}b^3c^3\right)x}{b^4(b\,x^2+a)}-\frac{7\ln\left(bx-\sqrt{-ab}\right)}{4b^4\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d/b^4*(1/5*b^2*x^5*d^2-2/3*a*b*d^2*x^3+b^2*c*d*x^3+3*a^2*d^2*x-6*a*b*c*d*x+
3*b^2*c^2*x)-1/b^4*((-1/2*a^3*d^3+3/2*a^2*b*c*d^2-3/2*a*b^2*c^2*d+1/2*b^3*c
^3)*x/(b*x^2+a)+1/2*(7*a^3*d^3-15*a^2*b*c*d^2+9*a*b^2*c^2*d-b^3*c^3)/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.53, size = 176, normalized size = 1.20

$$-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{2\,(b^5x^2+ab^4)}+\frac{(b^3c^3-9\,ab^2c^2d+15\,a^2bcd^2-7\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{3\,b^2d^3x^5+5\,(3\,b^2cd^2-2\,abd^3)x^3+45\,(b^2c^2d-2\,abcd^2+a^2d^3)x}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(b^5*x^2 + a*b^4
) + 1/2*(b^3*c^3 - 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^4) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - 2*a*b*d^
3)*x^3 + 45*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)/b^4
```

Fricas [A]
time = 1.05, size = 508, normalized size = 3.46

$$\left[\frac{12\,ab^4d^3x^7+4\,(15\,ab^4cd^2-7\,a^2b^3d^3)x^5+20\,(9\,ab^4c^2d-15\,a^2b^3cd^2+7\,a^3b^2d^3)x^3+15\,(ab^3c^3-9\,a^2b^2c^2d+15\,a^3bcd^2-7\,a^4d^3+(b^4c^3-9\,ab^3c^2d+15\,a^2b^2cd^2-7\,a^3bd^3)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-30\,(ab^4c^3-9\,a^2b^3c^2d+15\,a^3b^2cd^2-7\,a^4bd^3)x}{60\,(ab^6x^2+a^2b^5)},\frac{6\,ab^4d^3x^7+2\,(15\,ab^4cd^2-7\,a^2b^3d^3)x^5+10\,(9\,ab^4c^2d-15\,a^2b^3cd^2+7\,a^3b^2d^3)x^3+15\,(ab^3c^3-9\,a^2b^2c^2d+15\,a^3bcd^2-7\,a^4d^3+(b^4c^3-9\,ab^3c^2d+15\,a^2b^2cd^2-7\,a^3bd^3)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-15\,(ab^4c^3-9\,a^2b^3c^2d+15\,a^3b^2cd^2-7\,a^4bd^3)x}{30\,(ab^6x^2+a^2b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a*b^4*d^3*x^7 + 4*(15*a*b^4*c*d^2 - 7*a^2*b^3*d^3)*x^5 + 20*(9*a*
b^4*c^2*d - 15*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*x^3 + 15*(a*b^3*c^3 - 9*a^2*b
^2*c^2*d + 15*a^3*b*c*d^2 - 7*a^4*d^3 + (b^4*c^3 - 9*a*b^3*c^2*d + 15*a^2*b
^2*c*d^2 - 7*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b
```

```
*x^2 + a)) - 30*(a*b^4*c^3 - 9*a^2*b^3*c^2*d + 15*a^3*b^2*c*d^2 - 7*a^4*b*d
^3)*x)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*d^3*x^7 + 2*(15*a*b^4*c*d^2 - 7
*a^2*b^3*d^3)*x^5 + 10*(9*a*b^4*c^2*d - 15*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*x
^3 + 15*(a*b^3*c^3 - 9*a^2*b^2*c^2*d + 15*a^3*b*c*d^2 - 7*a^4*d^3 + (b^4*c^
3 - 9*a*b^3*c^2*d + 15*a^2*b^2*c*d^2 - 7*a^3*b*d^3)*x^2)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a) - 15*(a*b^4*c^3 - 9*a^2*b^3*c^2*d + 15*a^3*b^2*c*d^2 - 7*a^4*
b*d^3)*x)/(a*b^6*x^2 + a^2*b^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(141) = 282$.
time = 0.68, size = 338, normalized size = 2.30

$$x^3\left(-\frac{2ad^3}{3b^3}+\frac{cd^2}{b^2}\right)+x\left(\frac{3a^2d^3}{b^4}-\frac{6acd^2}{b^3}+\frac{3c^2d}{b^2}\right)+\frac{x(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^2\cdot(7ad-bc)\log\left(-\frac{ab^4\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^2\cdot(7ad-bc)}{7a^3d^3-15a^2bcd^2+9ab^2c^2d-b^3c^3}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^2\cdot(7ad-bc)\log\left(\frac{ab^4\sqrt{-\frac{1}{ab^9}}\,(ad-bc)^2\cdot(7ad-bc)}{7a^3d^3-15a^2bcd^2+9ab^2c^2d-b^3c^3}+x\right)}{4}+\frac{d^3x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*d**3/(3*b**3) + c*d**2/b**2) + x*(3*a**2*d**3/b**4 - 6*a*c*d**2/
b**3 + 3*c**2*d/b**2) + x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d -
b**3*c**3)/(2*a*b**4 + 2*b**5*x**2) + sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a
*d - b*c)*log(-a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)/(7*a**
3*d**3 - 15*a**2*b*c*d**2 + 9*a*b**2*c**2*d - b**3*c**3) + x)/4 - sqrt(-1/(
a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)*log(a*b**4*sqrt(-1/(a*b**9))*(a*d - b
*c)**2*(7*a*d - b*c)/(7*a**3*d**3 - 15*a**2*b*c*d**2 + 9*a*b**2*c**2*d - b*
*3*c**3) + x)/4 + d**3*x**5/(5*b**2)
```

Giac [A]
time = 0.54, size = 184, normalized size = 1.25

$$\frac{(b^3c^3-9\,ab^2c^2d+15\,a^2bcd^2-7\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}-\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)b^4}+\frac{3\,b^8d^3x^5+15\,b^8cd^2x^3-10\,ab^7d^3x^3+45\,b^8c^2dx-90\,ab^7cd^2x+45\,a^2b^6d^3x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 - 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^4) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
- a^3*d^3*x)/((b*x^2 + a)*b^4) + 1/15*(3*b^8*d^3*x^5 + 15*b^8*c*d^2*x^3 - 1
0*a*b^7*d^3*x^3 + 45*b^8*c^2*d*x - 90*a*b^7*c*d^2*x + 45*a^2*b^6*d^3*x)/b^1
0
```

Mupad [B]
time = 0.04, size = 232, normalized size = 1.58

$$x\left(\frac{3\,c^2\,d}{b^2}+\frac{2\,a\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b}-\frac{a^2\,d^3}{b^4}\right)-x^3\left(\frac{2\,a\,d^3}{3\,b^3}-\frac{c\,d^2}{b^2}\right)+\frac{x\,\left(\frac{a^3\,d^3}{2}-\frac{3\,a^2\,b\,c\,d^2}{2}+\frac{3\,a\,b^2\,c^2\,d}{2}-\frac{b^3\,c^3}{2}\right)}{b^5\,x^2+a\,b^4}+\frac{d^3\,x^5}{5\,b^2}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d-b\,c\right)}{\sqrt{a}\,\left(7\,a^3\,d^3-15\,a^2\,b\,c\,d^2+9\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d-b\,c\right)}{2\,\sqrt{a}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x*((3*c^2*d)/b^2 + (2*a*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b - (a^2*d^3)/b^4)
 - x^3*((2*a*d^3)/(3*b^3) - (c*d^2)/b^2) + (x*((a^3*d^3)/2 - (b^3*c^3)/2 +
(3*a*b^2*c^2*d)/2 - (3*a^2*b*c*d^2)/2))/(a*b^4 + b^5*x^2) + (d^3*x^5)/(5*b^
2) - (atan((b^(1/2)*x*(a*d - b*c)^2*(7*a*d - b*c))/(a^(1/2)*(7*a^3*d^3 - b^
3*c^3 + 9*a*b^2*c^2*d - 15*a^2*b*c*d^2)))*(a*d - b*c)^2*(7*a*d - b*c))/(2*a
^(1/2)*b^(9/2))
```

3.283 $\int \frac{x(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=88

$$\frac{d^2(3bc-2ad)x^2}{2b^3}+\frac{d^3x^4}{4b^2}-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}$$

```
[Out] 1/2*d^2*(-2*a*d+3*b*c)*x^2/b^3+1/4*d^3*x^4/b^2-1/2*(-a*d+b*c)^3/b^4/(b*x^2+
a)+3/2*d*(-a*d+b*c)^2*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 45}

$$-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}+\frac{d^2x^2(3bc-2ad)}{2b^3}+\frac{d^3x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) - (b*c - a*d)^3/(2*b^
4*(a + b*x^2)) + (3*d*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(c+dx^2)^3}{(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^3}{(a+bx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x}{b^2}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)}\right)dx, x, x^2\right) \\
&= \frac{d^2(3bc-2ad)x^2}{2b^3}+\frac{d^3x^4}{4b^2}-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 127, normalized size = 1.44

$$
\frac{d^2(3bc-2ad)x^2}{2b^3}+\frac{d^3x^4}{4b^2}+\frac{-b^3c^3+3ab^2c^2d-3a^2bcd^2+a^3d^3}{2b^4(a+bx^2)}+\frac{3(b^2c^2d-2abcd^2+a^2d^3)\log(a+bx^2)}{2b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) + (-(b^3*c^3) + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)/(2*b^4*(a + b*x^2)) + (3*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*Log[a + b*x^2])/(2*b^4)
```

Maple [A]
time = 0.10, size = 89, normalized size = 1.01

method	result
default	$\frac{d(-bd\,x^2+2ad-3bc)^2}{4b^4}+\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{3d\ln(b\,x^2+a)}{b}-\frac{-ad+bc}{b(b\,x^2+a)}\right)}{2b^3}$
norman	$\frac{\frac{3a^3d^3-6a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{2b^4}+\frac{d^3x^6}{4b}-\frac{3d^2(ad-2bc)x^4}{4b^2}}{b\,x^2+a}+\frac{3d(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2b^4}$
risch	$\frac{d^3x^4}{4b^2}-\frac{d^3a\,x^2}{b^3}+\frac{3d^2c\,x^2}{2b^2}+\frac{d^3a^2}{b^4}-\frac{3d^2ac}{b^3}+\frac{9d\,c^2}{4b^2}+\frac{a^3d^3}{2b^4(b\,x^2+a)}-\frac{3a^2c\,d^2}{2b^3(b\,x^2+a)}+\frac{3a\,c^2d}{2b^2(b\,x^2+a)}-\frac{c^3}{2b(b\,x^2+a)}+3d^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*(-b*d*x^2+2*a*d-3*b*c)^2/b^4+1/2/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(3*d
/b*ln(b*x^2+a)-(-a*d+b*c)/b/(b*x^2+a))
```

Maxima [A]
time = 0.28, size = 124, normalized size = 1.41

$$
-\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{2\,(b^5x^2+ab^4)}+\frac{bd^3x^4+2\,(3\,bcd^2-2\,ad^3)x^2}{4\,b^3}+\frac{3\,(b^2c^2d-2\,abcd^2+a^2d^3)\log(bx^2+a)}{2\,b^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x^2 + a*b^4)
+ 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - 2*a*d^3)*x^2)/b^3 + 3/2*(b^2*c^2*d - 2*a*
b*c*d^2 + a^2*d^3)*log(b*x^2 + a)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(80) = 160$.
time = 1.20, size = 181, normalized size = 2.06

$$\frac{b^3d^3x^6 - 2\,b^3c^3 + 6\,ab^2c^2d - 6\,a^2bcd^2 + 2\,a^3d^3 + 3\,(2\,b^3cd^2 - ab^2d^3)x^4 + 2\,(3\,ab^2cd^2 - 2\,a^2bd^3)x^2 + 6\,(ab^2c^2d - 2\,a^2bcd^2 + a^3d^3 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^2)\log\left(bx^2 + a\right)}{4\,(b^5x^2 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*d^3*x^6 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
3*(2*b^3*c*d^2 - a*b^2*d^3)*x^4 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x^2 + 6*(
a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*
d^3)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A]
time = 0.67, size = 112, normalized size = 1.27

$$x^2\left(-\frac{ad^3}{b^3} + \frac{3cd^2}{2b^2}\right) + \frac{a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3}{2ab^4 + 2b^5x^2} + \frac{d^3x^4}{4b^2} + \frac{3d(ad - bc)^2\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x**2*(-a*d**3/b**3 + 3*c*d**2/(2*b**2)) + (a**3*d**3 - 3*a**2*b*c*d**2 + 3*
a*b**2*c**2*d - b**3*c**3)/(2*a*b**4 + 2*b**5*x**2) + d**3*x**4/(4*b**2) +
3*d*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(80) = 160$.
time = 0.56, size = 183, normalized size = 2.08

$$\frac{\left(d^3 + \frac{6\,(b^2cd^2 - abd^3)}{(bx^2+a)b}\right)(bx^2 + a)^2}{4\,b^4} - \frac{3\,(b^2c^2d - 2\,abcd^2 + a^2d^3)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{2\,b^4} - \frac{\frac{b^5c^3}{bx^2+a} - \frac{3\,ab^4c^2d}{bx^2+a} + \frac{3\,a^2b^3cd^2}{bx^2+a} - \frac{a^3b^2d^3}{bx^2+a}}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x^2 + a)*b))*(b*x^2 + a)^2/b^4 - 3/2
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(
```

```
b)))/b^4 - 1/2*(b^5*c^3/(b*x^2 + a) - 3*a*b^4*c^2*d/(b*x^2 + a) + 3*a^2*b^3
*c*d^2/(b*x^2 + a) - a^3*b^2*d^3/(b*x^2 + a))/b^6
```

Mupad [B]
time = 0.09, size = 130, normalized size = 1.48

$$\frac{\ln\left(b\,x^2+a\right)\,\left(3\,a^2\,d^3-6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{2\,b^4}-x^2\,\left(\frac{a\,d^3}{b^3}-\frac{3\,c\,d^2}{2\,b^2}\right)+\frac{d^3\,x^4}{4\,b^2}+\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{2\,b\,\left(b^4\,x^2+a\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] (log(a + b*x^2)*(3*a^2*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2))/(2*b^4) - x^2*((a*
d^3)/b^3 - (3*c*d^2)/(2*b^2)) + (d^3*x^4)/(4*b^2) + (a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2)/(2*b*(a*b^3 + b^4*x^2))
```

3.284 $\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=106

$$\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\,(a+bx^2)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/3*d^3*x^3/b^2+1/2*(-a*d+b*c)^3*x/a/b^3/(b*x^2+a)
+1/2*(-a*d+b*c)^2*(5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(7/2)
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {398, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(5ad+bc)(bc-ad)^2}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\,(a+bx^2)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx &= \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^2}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{b^3\,(a+bx^2)^2}\right)dx \\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{(a+bx^2)^2}\,dx}{b^3} \\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\,(a+bx^2)}+\frac{((bc-ad)^2(bc+5ad))\int \frac{1}{a+bx^2}\,dx}{2ab^3} \\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\,(a+bx^2)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 106, normalized size = 1.00

$$
\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\,(a+bx^2)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Maple [A]
time = 0.07, size = 139, normalized size = 1.31

method	result
default	$-\frac{d^2\left(-\frac{1}{3}bd\,x^3+2adx-3bcx\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(5a^3d^3-9a^2bc\,d^2+3a\,b^2c^2d+b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^3}$
risch	$\frac{d^3x^3}{3b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{2a\,b^3(b\,x^2+a)}-\frac{5a^2\ln\left(bx+\sqrt{-ab}\right)d^3}{4b^3\sqrt{-ab}}+\frac{9a\ln\left(bx+\sqrt{-ab}\right)c\,d^2}{4b^2\sqrt{-ab}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^3*(-1/3*b*d*x^3+2*a*d*x-3*b*c*x)+1/b^3*(-1/2*(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/a*x/(b*x^2+a)+1/2*(5*a^3*d^3-9*a^2*b*c*d^2+3*a*b^2*c
^2*d+b^3*c^3)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 147, normalized size = 1.39

$$\frac{\left(b^{3} c^{3}-3\, a b^{2} c^{2} d+3\, a^{2} b c d^{2}-a^{3} d^{3}\right) x}{2\, \left(a b^{4} x^{2}+a^{2} b^{3}\right)}+\frac{b d^{3} x^{3}+3\, \left(3\, b c d^{2}-2\, a d^{3}\right) x}{3\, b^{3}}+\frac{\left(b^{3} c^{3}+3\, a b^{2} c^{2} d-9\, a^{2} b c d^{2}+5\, a^{3} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2\, \sqrt{a b}\, a b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^2 + a^2*
b^3) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/2*(b^3*c^3 + 3*a
*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*
b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(92) = 184.
time = 0.97, size = 442, normalized size = 4.17

$$\left[\frac{4\, a^{2} b^{3} d^{3} x^{5}+4\, \left(9\, a^{2} b^{3} c d^{2}-5\, a^{3} b^{2} d^{3}\right) x^{3}-3\, \left(a b^{3} c^{3}+3\, a^{2} b^{2} c^{2} d-9\, a^{3} b c d^{2}+5\, a^{4} d^{3}+\left(b^{4} c^{3}+3\, a b^{3} c^{2} d-9\, a^{2} b^{2} c d^{2}+5\, a^{3} b d^{3}\right) x^{2}\right) \sqrt{-a b} \log\left(\frac{b x^{2}-2\, \sqrt{-a b}\, x-a}{b x^{2}+a}\right)+6\, \left(a b^{4} c^{3}-3\, a^{2} b^{3} c^{2} d+9\, a^{3} b^{2} c d^{2}-5\, a^{4} b d^{3}\right) x}{12\, \left(a^{2} b^{5} x^{2}+a^{3} b^{4}\right)}, \frac{2\, a^{2} b^{3} d^{3} x^{5}+2\, \left(9\, a^{2} b^{3} c d^{2}-5\, a^{3} b^{2} d^{3}\right) x^{3}+3\, \left(a b^{3} c^{3}+3\, a^{2} b^{2} c^{2} d-9\, a^{3} b c d^{2}+5\, a^{4} d^{3}+\left(b^{4} c^{3}+3\, a b^{3} c^{2} d-9\, a^{2} b^{2} c d^{2}+5\, a^{3} b d^{3}\right) x^{2}\right) \sqrt{a b} \arctan\left(\frac{\sqrt{a b}\, x}{a}\right)+3\, \left(a b^{4} c^{3}-3\, a^{2} b^{3} c^{2} d+9\, a^{3} b^{2} c d^{2}-5\, a^{4} b d^{3}\right) x}{6\, \left(a^{2} b^{5} x^{2}+a^{3} b^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*d^3*x^5 + 4*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 - 3*(a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c
^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c*d^2
 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2*a^2*b^3*d^3*x^5 + 2*(9*a
^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 + 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*
b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*
d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c^3 - 3*a^2*b^3*c^2*d
+ 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(95) = 190.
time = 0.58, size = 314, normalized size = 2.96

$$x \left(- \frac{2 a d^{3}}{b^{3}} + \frac{3 c d^{2}}{b^{2}}\right) + \frac{x \left(- a^{3} d^{3} + 3 a^{2} b c d^{2} - 3 a b^{2} c^{2} d + b^{3} c^{3}\right)}{2 a^{2} b^{3} + 2 a b^{4} x^{2}} - \frac{\sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \cdot \left(5 a d + b c\right) \log{\left(- \frac{a^{2} b^{3} \sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \cdot \left(5 a d + b c\right)}{5 a^{3} d^{3} - 9 a^{2} b c d^{2} + 3 a b^{2} c^{2} d + b^{3} c^{3}} + x \right)}}{4} + \frac{\sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \cdot \left(5 a d + b c\right) \log{\left(\frac{a^{2} b^{3} \sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \cdot \left(5 a d + b c\right)}{5 a^{3} d^{3} - 9 a^{2} b c d^{2} + 3 a b^{2} c^{2} d + b^{3} c^{3}} + x \right)}}{4} + \frac{d^{3} x^{3}}{3 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(2*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7
))*(a*d - b*c)**2*(5*a*d + b*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d -
b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b*
*3*c**3) + x)/4 + sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)*log(a**
2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a
**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3) + x)/4 + d**3*x**3/(3*b**2)
```

Giac [A]
time = 0.50, size = 152, normalized size = 1.43

$$\frac{\left(b^{3} c^{3}+3 \, a b^{2} c^{2} d-9 \, a^{2} b c d^{2}+5 \, a^{3} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} a b^{3}}+\frac{b^{3} c^{3} x-3 \, a b^{2} c^{2} d x+3 \, a^{2} b c d^{2} x-a^{3} d^{3} x}{2 \, \left(b x^{2}+a\right) a b^{3}}+\frac{b^{4} d^{3} x^{3}+9 \, b^{4} c d^{2} x-6 \, a b^{3} d^{3} x}{3 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*d^3*x^3 + 9*b^4*c*d^2*x - 6*a*
b^3*d^3*x)/b^6
```

Mupad [B]
time = 0.11, size = 182, normalized size = 1.72

$$\frac{d^3\,x^3}{3\,b^2}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)-\frac{x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a\,\left(b^4\,x^2+a\,b^3\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(5\,a\,d+b\,c\right)}{\sqrt{a}\,\left(5\,a^3\,d^3-9\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(5\,a\,d+b\,c\right)}{2\,a^{3/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^2,x)
```

```
[Out] (d^3*x^3)/(3*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/
2)*x*(a*d - b*c)^2*(5*a*d + b*c))/(a^(1/2)*(5*a^3*d^3 + b^3*c^3 + 3*a*b^2*c
^2*d - 9*a^2*b*c*d^2)))*(a*d - b*c)^2*(5*a*d + b*c))/(2*a^(3/2)*b^(7/2))
```

3.285 $\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=88

$$\frac{d^3x^2}{2b^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{c^3\log(x)}{a^2}-\frac{(bc-ad)^2(bc+2ad)\log\left(a+bx^2\right)}{2a^2b^3}$$

```
[Out] 1/2*d^3*x^2/b^2+1/2*(-a*d+b*c)^3/a/b^3/(b*x^2+a)+c^3*ln(x)/a^2-1/2*(-a*d+b*
c)^2*(2*a*d+b*c)*ln(b*x^2+a)/a^2/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 90}

$$-\frac{(bc-ad)^2(2ad+bc)\log\left(a+bx^2\right)}{2a^2b^3}+\frac{c^3\log(x)}{a^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

```
[Out] (d^3*x^2)/(2*b^2) + (b*c - a*d)^3/(2*a*b^3*(a + b*x^2)) + (c^3*Log[x])/a^2
- ((b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^2])/(2*a^2*b^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^3}{x(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^3}{b^2}+\frac{c^3}{a^2x}+\frac{(-bc+ad)^3}{ab^2(a+bx)^2}-\frac{(-bc+ad)^2(bc+2ad)}{a^2b^2(a+bx)}\right)\,dx,x,x^2\right)\\
&= \frac{d^3x^2}{2b^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{c^3\log(x)}{a^2}-\frac{(bc-ad)^2(bc+2ad)\log\left(a+bx^2\right)}{2a^2b^3}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 111, normalized size = 1.26

$$
\frac{2c^3\log(x)+\frac{\frac{a\left(b^3c^3-a^3d^3+a^2bd^2\left(3c+dx^2\right)+ab^2\left(-3c^2d+d^3x^4\right)\right)}{a+bx^2}-(bc-ad)^2(bc+2ad)\log\left(a+bx^2\right)}{b^3}}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

```
[Out] (2*c^3*Log[x] + ((a*(b^3*c^3 - a^3*d^3 + a^2*b*d^2*(3*c + d*x^2) + a*b^2*(-
3*c^2*d + d^3*x^4)))/(a + b*x^2) - (b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^
2])/b^3)/(2*a^2)
```

Maple [A]
time = 0.08, size = 94, normalized size = 1.07

method	result
default	$\frac{d^3x^2}{2b^2}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{(2ad+bc)\ln\left(b\,x^2+a\right)}{b}+\frac{a(ad-bc)}{b\left(b\,x^2+a\right)}\right)}{2a^2b^2}+\frac{c^3\ln(x)}{a^2}$
norman	$\frac{\frac{d^3x^4}{2b}-\frac{2a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{2a\,b^3}}{b\,x^2+a}+\frac{c^3\ln(x)}{a^2}-\frac{\left(2a^3d^3-3a^2bc\,d^2+b^3c^3\right)\ln\left(b\,x^2+a\right)}{2a^2b^3}$
risch	$\frac{d^3x^2}{2b^2}-\frac{a^2d^3}{2b^3(b\,x^2+a)}+\frac{3ac\,d^2}{2b^2(b\,x^2+a)}-\frac{3c^2d}{2b(b\,x^2+a)}+\frac{c^3}{2a(b\,x^2+a)}+\frac{c^3\ln(x)}{a^2}-\frac{a\ln(b\,x^2+a)d^3}{b^3}+\frac{3\ln(b\,x^2+a)c\,d^2}{2b^2}-\ln(b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d^3*x^2/b^2-1/2/a^2/b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*((2*a*d+b*c)/b*ln(b
*x^2+a)+a*(a*d-b*c)/b/(b*x^2+a))+c^3*ln(x)/a^2
```

Maxima [A]
time = 0.29, size = 122, normalized size = 1.39

$$
\frac{d^3x^2}{2\,b^2}+\frac{c^3\log\left(x^2\right)}{2\,a^2}+\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{2\,\left(ab^4x^2+a^2b^3\right)}-\frac{\left(b^3c^3-3\,a^2bcd^2+2\,a^3d^3\right)\log\left(bx^2+a\right)}{2\,a^2b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^2/b^2 + 1/2*c^3*log(x^2)/a^2 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)/(a*b^4*x^2 + a^2*b^3) - 1/2*(b^3*c^3 - 3*a^2*b*c*d^2
+ 2*a^3*d^3)*log(b*x^2 + a)/(a^2*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(82) = 164$.
time = 1.40, size = 178, normalized size = 2.02

$$\frac{a^2b^2d^3x^4 + a^3bd^3x^2 + ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3 - (ab^3c^3 - 3\,a^3bcd^2 + 2\,a^4d^3 + (b^4c^3 - 3\,a^2b^2cd^2 + 2\,a^3bd^3)x^2)\log\left(bx^2 + a\right) + 2\,(b^4c^3x^2 + ab^3c^3)\log\left(x\right)}{2\,(a^2b^4x^2 + a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b^2*d^3*x^4 + a^3*b*d^3*x^2 + a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*
b*c*d^2 - a^4*d^3 - (a*b^3*c^3 - 3*a^3*b*c*d^2 + 2*a^4*d^3 + (b^4*c^3 - 3*a
^2*b^2*c*d^2 + 2*a^3*b*d^3)*x^2)*log(b*x^2 + a) + 2*(b^4*c^3*x^2 + a*b^3*c^
3)*log(x))/(a^2*b^4*x^2 + a^3*b^3)
```

Sympy [A]
time = 1.49, size = 110, normalized size = 1.25

$$\frac{-a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3}{2a^2b^3 + 2ab^4x^2} + \frac{d^3x^2}{2b^2} + \frac{c^3\log\left(x\right)}{a^2} - \frac{\left(ad - bc\right)^2 \cdot \left(2ad + bc\right)\log\left(\frac{a}{b} + x^2\right)}{2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x/(b*x**2+a)**2,x)
```

```
[Out] (-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + b**3*c**3)/(2*a**2*b**3 +
 2*a*b**4*x**2) + d**3*x**2/(2*b**2) + c**3*log(x)/a**2 - (a*d - b*c)**2*(2
*a*d + b*c)*log(a/b + x**2)/(2*a**2*b**3)
```

Giac [A]
time = 0.52, size = 150, normalized size = 1.70

$$\frac{d^3x^2}{2\,b^2} + \frac{c^3\log\left(x^2\right)}{2\,a^2} - \frac{(b^3c^3 - 3\,a^2bcd^2 + 2\,a^3d^3)\log\left(|bx^2 + a|\right)}{2\,a^2b^3} + \frac{b^4c^3x^2 - 3\,a^2b^2cd^2x^2 + 2\,a^3bd^3x^2 + 2\,ab^3c^3 - 3\,a^2b^2c^2d + a^4d^3}{2\,(bx^2 + a)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^3*x^2/b^2 + 1/2*c^3*log(x^2)/a^2 - 1/2*(b^3*c^3 - 3*a^2*b*c*d^2 + 2*a
^3*d^3)*log(abs(b*x^2 + a))/(a^2*b^3) + 1/2*(b^4*c^3*x^2 - 3*a^2*b^2*c*d^2*
x^2 + 2*a^3*b*d^3*x^2 + 2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + a^4*d^3)/((b*x^2 +
a)*a^2*b^3)
```

Mupad [B]
time = 0.06, size = 122, normalized size = 1.39

$$\frac{d^3\,x^2}{2\,b^2}+\frac{c^3\,\ln\left(x\right)}{a^2}-\frac{\ln\left(b\,x^2+a\right)\,\left(2\,a^3\,d^3-3\,a^2\,b\,c\,d^2+b^3\,c^3\right)}{2\,a^2\,b^3}-\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{2\,a\,b\,\left(b^3\,x^2+a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x*(a + b*x^2)^2),x)
```

```
[Out] (d^3*x^2)/(2*b^2) + (c^3*log(x))/a^2 - (log(a + b*x^2)*(2*a^3*d^3 + b^3*c^3
 - 3*a^2*b*c*d^2))/(2*a^2*b^3) - (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2)/(2*a*b*(a*b^2 + b^3*x^2))
```

3.286 $\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2(bc-3ad)x}{2ab^2}+\frac{(bc-ad)\,(c+dx^2)^2}{2abx\,(a+bx^2)}-\frac{3(bc-ad)^2(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

```
[Out] -1/2*c^2*(-a*d+3*b*c)/a^2/b/x-1/2*d^2*(-3*a*d+b*c)*x/a/b^2+1/2*(-a*d+b*c)*(
d*x^2+c)^2/a/b/x/(b*x^2+a)-3/2*(-a*d+b*c)^2*(a*d+b*c)*arctan(x*b^(1/2)/a^(1
/2))/a^(5/2)/b^(5/2)
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {479, 584, 211}

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^2(ad+bc)}{2a^{5/2}b^{5/2}}-\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2x(bc-3ad)}{2ab^2}+\frac{(c+dx^2)^2\,(bc-ad)}{2abx\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -1/2*(c^2*(3*b*c - a*d))/(a^2*b*x) - (d^2*(b*c - 3*a*d)*x)/(2*a*b^2) + ((b*
c - a*d)*(c + d*x^2)^2)/(2*a*b*x*(a + b*x^2)) - (3*(b*c - a*d)^2*(b*c + a*d
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx &= \frac{(bc-ad)(c+dx^2)^2}{2abx(a+bx^2)} - \frac{\int \frac{(c+dx^2)(-c(3bc-ad)+d(bc-3ad)x^2)}{x^2(a+bx^2)}\,dx}{2ab}\\ &= \frac{(bc-ad)(c+dx^2)^2}{2abx(a+bx^2)} - \frac{\int \left(\frac{d^2(bc-3ad)}{b}+\frac{c^2(-3bc+ad)}{ax^2}+\frac{3(-bc+ad)^2(bc+ad)}{ab(a+bx^2)}\right)dx}{2ab}\\ &= -\frac{c^2(3bc-ad)}{2a^2bx} - \frac{d^2(bc-3ad)x}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx(a+bx^2)} - \frac{(3(bc-ad)^2(bc+ad))\int \frac{}{a\cdot}}{2a^2b^2}\\ &= -\frac{c^2(3bc-ad)}{2a^2bx} - \frac{d^2(bc-3ad)x}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx(a+bx^2)} - \frac{3(bc-ad)^2(bc+ad)\tan^{-1}}{2a^{5/2}b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 94, normalized size = 0.72

$$-\frac{c^3}{a^2x}+\frac{d^3x}{b^2}+\frac{(-bc+ad)^3x}{2a^2b^2(a+bx^2)}-\frac{3(-bc+ad)^2(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
[Out] -(c^3/(a^2*x)) + (d^3*x)/b^2 + ((-(b*c) + a*d)^3*x)/(2*a^2*b^2*(a + b*x^2))
 - (3*(-(b*c) + a*d)^2*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*
b^(5/2))
```

Maple [A]
time = 0.12, size = 129, normalized size = 0.98

method	result
default	$\frac{d^3x}{b^2}-\frac{\frac{\left(-\frac{1}{2}a^3d^3+\frac{3}{2}a^2bc\,d^2-\frac{3}{2}a\,b^2c^2d+\frac{1}{2}b^3c^3\right)x}{b\,x^2+a}+\frac{3\left(a^3d^3-a^2bc\,d^2-a\,b^2c^2d+b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^2b^2}-\frac{c^3}{a^2x}$

risch	$\frac{d^3x}{b^2}+\frac{\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-3b^3c^3\right)x^2}{2a^2}-\frac{b^2c^3}{a}}{b^2x(b\,x^2+a)}-\frac{3a\ln\left(-\sqrt{-ab}\;x-a\right)d^3}{4b^2\sqrt{-ab}}+\frac{3\ln\left(-\sqrt{-ab}\;x-a\right)c\,d^2}{4b\sqrt{-ab}}+\frac{3\ln\left(-\sqrt{-ab}\right.}{4\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^3/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] d^3/b^2*x-1/a^2/b^2*((-1/2*a^3*d^3+3/2*a^2*b*c*d^2-3/2*a*b^2*c^2*d+1/2*b^3*c^3)*x/(b*x^2+a)+3/2*(a^3*d^3-a^2*b*c*d^2-a*b^2*c^2*d+b^3*c^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-c^3/a^2/x

Maxima [A]
time = 0.52, size = 140, normalized size = 1.07

$$\frac{d^3x}{b^2}-\frac{2\,ab^2c^3+(3\,b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^2}{2\,(a^2b^3x^3+a^3b^2x)}-\frac{3\,(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="maxima")

[Out] d^3*x/b^2 - 1/2*(2*a*b^2*c^3 + (3*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2)/(a^2*b^3*x^3 + a^3*b^2*x) - 3/2*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)

Fricas [A]
time = 0.97, size = 412, normalized size = 3.15

$$\left[\frac{4\,a^3b^2d^3x^4-4\,a^2b^3c^3-6\,(ab^4c^3-a^2b^3c^2d+a^3b^2cd^2-a^4bd^3)x^2-3\,((b^4c^3-ab^3c^2d-a^2b^2cd^2+a^3bd^3)x^3+(ab^3c^3-a^2b^2c^2d-a^3bcd^2+a^4d^3)x)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,(a^3b^4x^3+a^4b^3x)},\ \frac{2\,a^3b^2d^3x^4-2\,a^2b^3c^3-3\,(ab^4c^3-a^2b^3c^2d+a^3b^2cd^2-a^4bd^3)x^2-3\,((b^4c^3-ab^3c^2d-a^2b^2cd^2+a^3bd^3)x^3+(ab^3c^3-a^2b^2c^2d-a^3bcd^2+a^4d^3)x)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(a^3b^4x^3+a^4b^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/4*(4*a^3*b^2*d^3*x^4 - 4*a^2*b^3*c^3 - 6*(a*b^4*c^3 - a^2*b^3*c^2*d + a^3*b^2*c*d^2 - a^4*b*d^3)*x^2 - 3*((b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 + a^3*b*d^3)*x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^4*d^3)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a^4*b^3*x), 1/2*(2*a^3*b^2*d^3*x^4 - 2*a^2*b^3*c^3 - 3*(a*b^4*c^3 - a^2*b^3*c^2*d + a^3*b^2*c*d^2 - a^4*b*d^3)*x^2 - 3*((b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 + a^3*b*d^3)*x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^4*d^3)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^4*x^3 + a^4*b^3*x)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(112) = 224$.
time = 0.88, size = 309, normalized size = 2.36

$$\frac{3\sqrt{-\frac{1}{a^5b^5}}\,(ad-bc)^2\,(ad+bc)\log\left(-\frac{3a^3b^2\sqrt{-\frac{1}{a^5b^5}}\,(ad-bc)^2(ad+bc)}{3a^3d^3-3a^2bcd^2-3ab^2c^2d+3b^3c^3}+x\right)}{4}-\frac{3\sqrt{-\frac{1}{a^5b^5}}\,(ad-bc)^2\,(ad+bc)\log\left(\frac{3a^3b^2\sqrt{-\frac{1}{a^5b^5}}\,(ad-bc)^2(ad+bc)}{3a^3d^3-3a^2bcd^2-3ab^2c^2d+3b^3c^3}+x\right)}{4}+\frac{-2ab^2c^3+x^2(a^3d^3-3a^2bcd^2+3ab^2c^2d-3b^3c^3)}{2a^3b^2x+2a^2b^3x^3}+\frac{d^3x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**2/(b*x**2+a)**2,x)
```

```
[Out] 3*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)*log(-3*a**3*b**2*sqrt(-1/
(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)/(3*a**3*d**3 - 3*a**2*b*c*d**2 - 3*
a*b**2*c**2*d + 3*b**3*c**3) + x)/4 - 3*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2
*(a*d + b*c)*log(3*a**3*b**2*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c
)/(3*a**3*d**3 - 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + 3*b**3*c**3) + x)/4 +
(-2*a*b**2*c**3 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - 3*b
**3*c**3))/(2*a**3*b**2*x + 2*a**2*b**3*x**3) + d**3*x/b**2
```

Giac [A]
time = 0.62, size = 143, normalized size = 1.09

$$\frac{d^3x}{b^2} - \frac{3\,(b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b^2} - \frac{3\,b^3c^3x^2 - 3\,ab^2c^2dx^2 + 3\,a^2bcd^2x^2 - a^3d^3x^2 + 2\,ab^2c^3}{2\,(bx^3 + ax)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^3*x/b^2 - 3/2*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c^3*x^2 - 3*a*b^2*c^2*d*x^2 + 3
*a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 2*a*b^2*c^3)/((b*x^3 + a*x)*a^2*b^2)
```

Mupad [B]
time = 0.12, size = 173, normalized size = 1.32

$$\frac{\frac{x^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-3\,b^3\,c^3\right)}{2\,a^2}-\frac{b^2\,c^3}{a}}{b^3\,x^3+a\,b^2\,x}+\frac{d^3\,x}{b^2}-\frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{b}\,x\,\left(a\,d+b\,c\right)\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,\left(3\,a^3\,d^3-3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+3\,b^3\,c^3\right)}\right)\,\left(a\,d+b\,c\right)\,{\left(a\,d-b\,c\right)}^2}{2\,a^{5/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^2*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(a^3*d^3 - 3*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2) - (b^2
*c^3)/a)/(b^3*x^3 + a*b^2*x) + (d^3*x)/b^2 - (3*atan((3*b^(1/2)*x*(a*d + b*
c)*(a*d - b*c)^2)/(a^(1/2)*(3*a^3*d^3 + 3*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)))*(a*d + b*c)*(a*d - b*c)^2)/(2*a^(5/2)*b^(5/2))
```

3.287 $\int \frac{(c+dx^2)^3}{x^3 (a+bx^2)^2} \, dx$

Optimal. Leaf size=98

$$-\frac{c^3}{2a^2x^2}-\frac{(bc-ad)^3}{2a^2b^2\left(a+bx^2\right)}-\frac{c^2(2bc-3ad)\log(x)}{a^3}+\frac{(bc-ad)^2(2bc+ad)\log\left(a+bx^2\right)}{2a^3b^2}$$

```
[Out] -1/2*c^3/a^2/x^2-1/2*(-a*d+b*c)^3/a^2/b^2/(b*x^2+a)-c^2*(-3*a*d+2*b*c)*ln(x
)/a^3+1/2*(-a*d+b*c)^2*(a*d+2*b*c)*ln(b*x^2+a)/a^3/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{(bc-ad)^2(ad+2bc)\log\left(a+bx^2\right)}{2a^3b^2}-\frac{c^2\log(x)(2bc-3ad)}{a^3}-\frac{(bc-ad)^3}{2a^2b^2\left(a+bx^2\right)}-\frac{c^3}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*c^3/(a^2*x^2) - (b*c - a*d)^3/(2*a^2*b^2*(a + b*x^2)) - (c^2*(2*b*c -
3*a*d)*Log[x])/a^3 + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*x^2])/(2*a^3*b^
2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c^3}{a^2x^2} + \frac{c^2(-2bc+3ad)}{a^3x} - \frac{(-bc+ad)^3}{a^2b(a+bx)^2} + \frac{(-bc+ad)^2(2bc+ad)}{a^3b(a+bx)}\right) dx,$$
$$= -\frac{c^3}{2a^2x^2} - \frac{(bc-ad)^3}{2a^2b^2(a+bx^2)} - \frac{c^2(2bc-3ad)\log(x)}{a^3} + \frac{(bc-ad)^2(2bc+ad)\log(a+bx}{2a^3b^2}$$

Mathematica [A]
time = 0.07, size = 87, normalized size = 0.89

$$\frac{-\frac{ac^3}{x^2} + \frac{a(-bc+ad)^3}{b^2(a+bx^2)} + 2c^2(-2bc+3ad)\log(x) + \frac{(bc-ad)^2(2bc+ad)\log(a+bx^2)}{b^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*c^3)/x^2) + (a*(-(b*c) + a*d)^3)/(b^2*(a + b*x^2)) + 2*c^2*(-2*b*c + 
3*a*d)*Log[x] + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*x^2])/b^2)/(2*a^3)
```

Maple [A]
time = 0.09, size = 100, normalized size = 1.02

method	result
default	$\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{(ad+2bc)\ln(bx^2+a)}{b^2}+\frac{a(ad-bc)}{b^2(bx^2+a)}\right)}{2a^3} - \frac{c^3}{2a^2x^2} + \frac{c^2(3ad-2bc)\ln(x)}{a^3}$
norman	$\frac{-\frac{c^3}{2a}+\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-2b^3c^3)x^2}{2a^2b^2}}{x^2(bx^2+a)} + \frac{c^2(3ad-2bc)\ln(x)}{a^3} + \frac{(a^3d^3-3ab^2c^2d+2b^3c^3)\ln(bx^2+a)}{2a^3b^2}$
risch	$\frac{-\frac{c^3}{2a}+\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-2b^3c^3)x^2}{2a^2b^2}}{x^2(bx^2+a)} + \frac{3c^2\ln(x)d}{a^2} - \frac{2c^3\ln(x)b}{a^3} + \frac{\ln(-bx^2-a)d^3}{2b^2} - \frac{3\ln(-bx^2-a)c^2d}{2a^2} + \frac{b\ln(-bx^2-a)}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*((a*d+2*b*c)/b^2*ln(b*x^2+a)+a*(a*d-b*c
)/b^2/(b*x^2+a))-1/2*c^3/a^2/x^2+c^2*(3*a*d-2*b*c)/a^3*ln(x)
```

Maxima [A]
time = 0.28, size = 141, normalized size = 1.44

$$-\frac{ab^2c^3 + (2\,b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^2}{2\,(a^2b^3x^4 + a^3b^2x^2)} - \frac{(2\,bc^3 - 3\,ac^2d)\log(x^2)}{2\,a^3} + \frac{(2\,b^3c^3 - 3\,ab^2c^2d + a^3d^3)\log(bx^2 + a)}{2\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b^2*c^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2
)/(a^2*b^3*x^4 + a^3*b^2*x^2) - 1/2*(2*b*c^3 - 3*a*c^2*d)*log(x^2)/a^3 + 1/
2*(2*b^3*c^3 - 3*a*b^2*c^2*d + a^3*d^3)*log(b*x^2 + a)/(a^3*b^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(92) = 184$.
time = 0.96, size = 209, normalized size = 2.13

$$-\frac{a^{2} b^{2} c^{3} + (2\, a b^{3} c^{3} - 3\, a^{2} b^{2} c^{2} d + 3\, a^{3} b c d^{2} - a^{4} d^{3}) x^{2} - ((2\, b^{4} c^{3} - 3\, a b^{3} c^{2} d + a^{3} b d^{3}) x^{4} + (2\, a b^{3} c^{3} - 3\, a^{2} b^{2} c^{2} d + a^{4} d^{3}) x^{2}) \log\left(b x^{2} + a\right) + 2\, ((2\, b^{4} c^{3} - 3\, a b^{3} c^{2} d) x^{4} + (2\, a b^{3} c^{3} - 3\, a^{2} b^{2} c^{2} d) x^{2}) \log\left(x\right)}{2\, (a^{3} b^{3} x^{4} + a^{4} b^{2} x^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^2*c^3 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^
3)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d + a^3*b*d^3)*x^4 + (2*a*b^3*c^3 - 3*a^
2*b^2*c^2*d + a^4*d^3)*x^2)*log(b*x^2 + a) + 2*((2*b^4*c^3 - 3*a*b^3*c^2*d)
*x^4 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)*log(x))/(a^3*b^3*x^4 + a^4*b^2*
x^2)
```

Sympy [**A**]
time = 4.07, size = 128, normalized size = 1.31

$$\frac{-ab^{2}c^{3} + x^{2}(a^{3}d^{3} - 3a^{2}bcd^{2} + 3ab^{2}c^{2}d - 2b^{3}c^{3})}{2a^{3}b^{2}x^{2} + 2a^{2}b^{3}x^{4}} + \frac{c^{2} \cdot (3ad - 2bc)\log\left(x\right)}{a^{3}} + \frac{(ad - bc)^{2}\,(ad + 2bc)\log\left(\frac{a}{b} + x^{2}\right)}{2a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*b**2*c**3 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - 2*b**
3*c**3))/(2*a**3*b**2*x**2 + 2*a**2*b**3*x**4) + c**2*(3*a*d - 2*b*c)*log(x
)/a**3 + (a*d - b*c)**2*(a*d + 2*b*c)*log(a/b + x**2)/(2*a**3*b**2)
```

Giac [**A**]
time = 0.59, size = 157, normalized size = 1.60

$$-\frac{(2\, b c^{3} - 3\, a c^{2} d) \log\left(x^{2}\right)}{2\, a^{3}} + \frac{(2\, b^{3} c^{3} - 3\, a b^{2} c^{2} d + a^{3} d^{3}) \log\left(|b x^{2} + a|\right)}{2\, a^{3} b^{2}} - \frac{a^{2} b d^{3} x^{4} + 4\, b^{3} c^{3} x^{2} - 6\, a b^{2} c^{2} d x^{2} + 6\, a^{2} b c d^{2} x^{2} - a^{3} d^{3} x^{2} + 2\, a b^{2} c^{3}}{4\, (b x^{4} + a x^{2}) a^{2} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c^3 - 3*a*c^2*d)*log(x^2)/a^3 + 1/2*(2*b^3*c^3 - 3*a*b^2*c^2*d +
a^3*d^3)*log(abs(b*x^2 + a))/(a^3*b^2) - 1/4*(a^2*b*d^3*x^4 + 4*b^3*c^3*x^2
```

```
 - 6*a*b^2*c^2*d*x^2 + 6*a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 2*a*b^2*c^3)/((b*x
^4 + a*x^2)*a^2*b^2)
```

Mupad [B]
time = 0.15, size = 135, normalized size = 1.38

$$\frac{\ln\left(b\,x^2+a\right)\,\left(a^3\,d^3-3\,a\,b^2\,c^2\,d+2\,b^3\,c^3\right)}{2\,a^3\,b^2}-\frac{\ln\left(x\right)\,\left(2\,b\,c^3-3\,a\,c^2\,d\right)}{a^3}-\frac{\frac{c^3}{2\,a}-\frac{x^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-2\,b^3\,c^3\right)}{2\,a^2\,b^2}}{b\,x^4+a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (log(a + b*x^2)*(a^3*d^3 + 2*b^3*c^3 - 3*a*b^2*c^2*d))/(2*a^3*b^2) - (log(x
)*(2*b*c^3 - 3*a*c^2*d))/a^3 - (c^3/(2*a) - (x^2*(a^3*d^3 - 2*b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2*b^2))/(a*x^2 + b*x^4)
```

3.288 $\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2} \, dx$

Optimal. Leaf size=147

$$-\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{c(5b^2c^2-9abcd+2a^2d^2)}{2a^3bx}+\frac{(bc-ad)\,(c+dx^2)^2}{2abx^3\,(a+bx^2)}+\frac{(bc-ad)^2(5bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

```
[Out] -1/6*c^2*(-3*a*d+5*b*c)/a^2/b/x^3+1/2*c*(2*a^2*d^2-9*a*b*c*d+5*b^2*c^2)/a^3
/b/x+1/2*(-a*d+b*c)*(d*x^2+c)^2/a/b/x^3/(b*x^2+a)+1/2*(-a*d+b*c)^2*(a*d+5*b
*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {479, 584, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-ad)^2(ad+5bc)}{2a^{7/2}b^{3/2}}-\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{c(2a^2d^2-9abcd+5b^2c^2)}{2a^3bx}+\frac{(c+dx^2)^2\,(bc-ad)}{2abx^3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/6*(c^2*(5*b*c - 3*a*d))/(a^2*b*x^3) + (c*(5*b^2*c^2 - 9*a*b*c*d + 2*a^2*
d^2))/(2*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^3*(a + b*x^2)) + (
(b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2)
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx = \frac{(bc-ad)(c+dx^2)^2}{2abx^3(a+bx^2)} - \frac{\int \frac{(c+dx^2)(-c(5bc-3ad)-d(bc+ad)x^2)}{x^4(a+bx^2)}\,dx}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^3(a+bx^2)} - \frac{\int \left(\frac{c^2(-5bc+3ad)}{ax^4} + \frac{c(5b^2c^2-9abcd+2a^2d^2)}{a^2x^2} - \frac{(-bc+ad)^2(5bc+ad)}{a^2(a+bx^2)}\right)dx}{2ab}$$

$$= -\frac{c^2(5bc-3ad)}{6a^2bx^3} + \frac{c(5b^2c^2-9abcd+2a^2d^2)}{2a^3bx} + \frac{(bc-ad)(c+dx^2)^2}{2abx^3(a+bx^2)} + \frac{((bc-ad)^2(5b$$

$$= -\frac{c^2(5bc-3ad)}{6a^2bx^3} + \frac{c(5b^2c^2-9abcd+2a^2d^2)}{2a^3bx} + \frac{(bc-ad)(c+dx^2)^2}{2abx^3(a+bx^2)} + \frac{(bc-ad)^2(5bc}{2}$$

Mathematica [A]
time = 0.05, size = 109, normalized size = 0.74

$$-\frac{c^3}{3a^2x^3} - \frac{c^2(-2bc+3ad)}{a^3x} - \frac{(-bc+ad)^3x}{2a^3b(a+bx^2)} + \frac{(-bc+ad)^2(5bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
[Out] -1/3*c^3/(a^2*x^3) - (c^2*(-2*b*c + 3*a*d))/(a^3*x) - ((-(b*c) + a*d)^3*x)/
(2*a^3*b*(a + b*x^2)) + ((-(b*c) + a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Maple [A]
time = 0.09, size = 144, normalized size = 0.98

method	result
default	$\frac{-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{2b(b\,x^2+a)}+\frac{(a^3d^3+3a^2bc\,d^2-9a\,b^2c^2d+5b^3c^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}}{a^3} - \frac{c^3}{3a^2x^3} - \frac{c^2(3ad-2bc)}{a^3x}$

risch	$\frac{-\frac{\left(a^{3} d^{3}-3 a^{2} b c\, d^{2}+9 a\, b^{2} c^{2} d-5 b^{3} c^{3}\right) x^{4}}{2 a^{3} b}-\frac{c^{2} (9 a d-5 b c) x^{2}}{3 a^{2}}-\frac{c^{3}}{3 a}}{x^{3} \left(b\, x^{2}+a\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{7} \_Z^{2} b^{3}+a^{6} d^{6}+6 a^{5} b c\, d^{5}-9 a^{4} b^{2} c^{2} d^{4}-44 a^{3} b^{3} c^{3} d^{3}+111 a^{2} b^{4}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b*x/(b*x^2+a)+1/2
*(a^3*d^3+3*a^2*b*c*d^2-9*a*b^2*c^2*d+5*b^3*c^3)/b/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))-1/3*c^3/a^2/x^3-c^2*(3*a*d-2*b*c)/a^3/x
```

Maxima [A]
time = 0.51, size = 159, normalized size = 1.08

$$-\frac{2\,a^{2} b c^{3} - 3\,{\left(5\,b^{3} c^{3} - 9\,a b^{2} c^{2} d + 3\,a^{2} b c d^{2} - a^{3} d^{3}\right)} x^{4} - 2\,{\left(5\,a b^{2} c^{3} - 9\,a^{2} b c^{2} d\right)} x^{2}}{6\,{\left(a^{3} b^{2} x^{5} + a^{4} b x^{3}\right)}} + \frac{{\left(5\,b^{3} c^{3} - 9\,a b^{2} c^{2} d + 3\,a^{2} b c d^{2} + a^{3} d^{3}\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2\,\sqrt{a b} a^{3} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*b*c^3 - 3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*x^4 - 2*(5*a*b^2*c^3 - 9*a^2*b*c^2*d)*x^2)/(a^3*b^2*x^5 + a^4*b*x^3) + 1/2
*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^3*b)
```

Fricas [A]
time = 0.81, size = 458, normalized size = 3.12

$$\left[-\frac{4\,a^{3} b^{2} c^{3} - 6\,{\left(5\,a b^{4} c^{3} - 9\,a^{2} b^{3} c^{2} d + 3\,a^{3} b^{2} c d^{2} - a^{4} b d^{3}\right)} x^{4} - 4\,{\left(5\,a^{2} b^{3} c^{3} - 9\,a^{3} b^{2} c^{2} d\right)} x^{2} + 3\,{\left({\left(5\,b^{4} c^{3} - 9\,a b^{3} c^{2} d + 3\,a^{2} b^{2} c d^{2} + a^{3} b d^{3}\right)} x^{5} + {\left(5\,a b^{3} c^{3} - 9\,a^{2} b^{2} c^{2} d + 3\,a^{3} b c d^{2} + a^{4} d^{3}\right)} x^{3}\right)} \sqrt{-a b} \log\left(\frac{b x^{2} - 2\,\sqrt{-a b} x - a}{b x^{2} + a}\right)}{12\,{\left(a^{4} b^{3} x^{5} + a^{5} b^{2} x^{3}\right)}}, -\frac{2\,a^{3} b^{2} c^{3} - 3\,{\left(5\,a b^{4} c^{3} - 9\,a^{2} b^{3} c^{2} d + 3\,a^{3} b^{2} c d^{2} - a^{4} b d^{3}\right)} x^{4} - 2\,{\left(5\,a^{2} b^{3} c^{3} - 9\,a^{3} b^{2} c^{2} d\right)} x^{2} - 3\,{\left({\left(5\,b^{4} c^{3} - 9\,a b^{3} c^{2} d + 3\,a^{2} b^{2} c d^{2} + a^{3} b d^{3}\right)} x^{5} + {\left(5\,a b^{3} c^{3} - 9\,a^{2} b^{2} c^{2} d + 3\,a^{3} b c d^{2} + a^{4} d^{3}\right)} x^{3}\right)} \sqrt{a b} \arctan\left(\frac{\sqrt{a b} x}{a}\right)}{6\,{\left(a^{4} b^{3} x^{5} + a^{5} b^{2} x^{3}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b^2*c^3 - 6*(5*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2
- a^4*b*d^3)*x^4 - 4*(5*a^2*b^3*c^3 - 9*a^3*b^2*c^2*d)*x^2 + 3*((5*b^4*c^3
- 9*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x^5 + (5*a*b^3*c^3 - 9*a^2*b
^2*c^2*d + 3*a^3*b*c*d^2 + a^4*d^3)*x^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + a^5*b^2*x^3), -1/6*(2*a^3*b^2*c^3 -
3*(5*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x^4 - 2*(5*
a^2*b^3*c^3 - 9*a^3*b^2*c^2*d)*x^2 - 3*((5*b^4*c^3 - 9*a*b^3*c^2*d + 3*a^2*
b^2*c*d^2 + a^3*b*d^3)*x^5 + (5*a*b^3*c^3 - 9*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
 + a^4*d^3)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a^5*b^2*x^
3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(133) = 266.

time = 1.15, size = 321, normalized size = 2.18

$$-\frac{\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2\,(ad+5bc)\log\left(-\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2\,(ad+5bc)\log\left(\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}+\frac{-2a^2bc^3+x^4(-3a^3d^3+9a^2bcd^2-27ab^2c^2d+15b^3c^3)+x^2(-18a^2bc^2d+10ab^2c^3)}{6a^4bx^3+6a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-a**4*b*sqrt(-1/(a**
7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b*
*2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d
+ 5*b*c)*log(a**4*b*sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3
*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4 + (-2*a**2*
b*c**3 + x**4*(-3*a**3*d**3 + 9*a**2*b*c*d**2 - 27*a*b**2*c**2*d + 15*b**3*
c**3) + x**2*(-18*a**2*b*c**2*d + 10*a*b**2*c**3))/(6*a**4*b*x**3 + 6*a**3*
b**2*x**5)
```

Giac [A]

time = 0.49, size = 150, normalized size = 1.02

$$\frac{(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3b}+\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)a^3b}+\frac{6\,bc^3x^2-9\,ac^2dx^2-ac^3}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^3*b) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a^3*b) + 1/3*(6*b*c^3*x^2 - 9*a*c^2*d*x^2 - a*c^
3)/(a^3*x^3)
```

Mupad [B]

time = 0.14, size = 183, normalized size = 1.24

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{\sqrt{a}\,(a^3\,d^3+3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}\right)\,(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{2\,a^{7/2}\,b^{3/2}}-\frac{\frac{c^3}{3\,a}+\frac{x^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+9\,a\,b^2\,c^2\,d-5\,b^3\,c^3)}{2\,a^3\,b}+\frac{c^2\,x^2\,(9\,a\,d-5\,b\,c)}{3\,a^2}}{b\,x^5+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^4*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x*(a*d - b*c)^2*(a*d + 5*b*c))/(a^(1/2)*(a^3*d^3 + 5*b^3*c^3
 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2)))*(a*d - b*c)^2*(a*d + 5*b*c))/(2*a^(7/2)
*b^(3/2)) - (c^3/(3*a) + (x^4*(a^3*d^3 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a^2*
b*c*d^2))/(2*a^3*b) + (c^2*x^2*(9*a*d - 5*b*c))/(3*a^2))/(a*x^3 + b*x^5)
```

3.289 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=109

$$\frac{ax}{2b(bc-ad)(a+bx^2)}-\frac{\sqrt{a}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)^2}$$

```
[Out] 1/2*a*x/b/(-a*d+b*c)/(b*x^2+a)-1/2*(-a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))*a
^(1/2)/b^(3/2)/(-a*d+b*c)^2+c^(3/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^2/
d^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {481, 536, 211}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3bc-ad)}{2b^{3/2}(bc-ad)^2}+\frac{c^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)^2}+\frac{ax}{2b\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (a*x)/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d)^2) + (c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[d]*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx &= \frac{ax}{2b(bc-ad)(a+bx^2)} - \frac{\int \frac{ac+(-2bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx}{2b(bc-ad)}\\ &= \frac{ax}{2b(bc-ad)(a+bx^2)} + \frac{c^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2} - \frac{(a(3bc-ad))\int \frac{1}{a+bx^2}\,dx}{2b(bc-ad)^2}\\ &= \frac{ax}{2b(bc-ad)(a+bx^2)} - \frac{\sqrt{a}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2} + \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 95, normalized size = 0.87

$$\frac{\frac{a(bc-ad)x}{b(a+bx^2)} + \frac{\sqrt{a}(-3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}} + \frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
[Out] ((a*(b*c - a*d)*x)/(b*(a + b*x^2)) + (Sqrt[a]*(-3*b*c + a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(3/2) + (2*c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[d])/(
2*(b*c - a*d)^2)
```

Maple [A]
time = 0.17, size = 94, normalized size = 0.86

method	result
default	$-\frac{a\left(\frac{(ad-bc)x}{2b\left(b\,x^2+a\right)}-\frac{(ad-3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(ad-bc)^2}+\frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^2\sqrt{cd}}$

risch	$-\frac{ax}{2(ad-bc)b(b\,x^2+a)}+\frac{\sqrt{-cd}\;c\ln\left(\left(-4(-cd)^{\frac{3}{2}}a\,b^3c^2d-4(-cd)^{\frac{3}{2}}b^4c^3-a^4\sqrt{-cd}\;d^5+6\sqrt{-cd}\;a^3bc\,d^4-9\sqrt{-cd}\;a^2b^2c^2d\right.\right.}{2d(ad-bc)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -a/(a*d-b*c)^2*(1/2*(a*d-b*c)/b*x/(b*x^2+a)-1/2*(a*d-3*b*c)/b/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2)))+c^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.52, size = 133, normalized size = 1.22

$$\frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}}+\frac{ax}{2\,(ab^2c-a^2bd+(b^3c-ab^2d)x^2)}-\frac{(3\,abc-a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] c^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*a*x/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2) - 1/2*(3*a*b*c - a^2*d)*ar
ctan(b*x/sqrt(a*b))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(a*b))
```

Fricas [A]
time = 0.91, size = 726, normalized size = 6.66

$$\left[-\frac{(3\,abc-a^2d+(3\,b^2c-abd)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,(b^2cx^2+abc)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2\,(abc-a^2d)x}{4\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)},\ -\frac{(3\,abc-a^2d+(3\,b^2c-abd)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-(b^2cx^2+abc)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-(abc-a^2d)x}{2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)},\ \frac{4\,(b^2cx^2+abc)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-(3\,abc-a^2d+(3\,b^2c-abd)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,(abc-a^2d)x}{4\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)},\ -\frac{(3\,abc-a^2d+(3\,b^2c-abd)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-2\,(b^2cx^2+abc)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-(abc-a^2d)x}{2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*
b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b^2*c*x^2 + a*b*c)*sqrt(-c/d)*log((d*
x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(a*b*c - a^2*d)*x)/(a*b^3*c^2
- 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), -
1/2*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/
b)/a) - (b^2*c*x^2 + a*b*c)*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(
d*x^2 + c)) - (a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (
b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), 1/4*(4*(b^2*c*x^2 + a*b*c)*sqrt(
c/d)*arctan(d*x*sqrt(c/d)/c) - (3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sq
rt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*(a*b*c - a^2*d
)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*x^2), -1/2*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(a/b)*ar
ctan(b*x*sqrt(a/b)/a) - 2*(b^2*c*x^2 + a*b*c)*sqrt(c/d)*arctan(d*x*sqrt(c/d
```

```
)/c) - (a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2
 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 122, normalized size = 1.12

$$\frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}}-\frac{(3\,abc-a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{ab}}+\frac{ax}{2\,(b^2c-abd)(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] c^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) - 1/2
*(3*a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^
2)*sqrt(a*b)) + 1/2*a*x/((b^2*c - a*b*d)*(b*x^2 + a))
```

Mupad [**B**]
time = 0.55, size = 2500, normalized size = 22.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (atan((((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^
4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(b^4*c^3 - a^3*b*d^3 + 3
*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)) - (x*(-c^3*d)^(1/2)*(16*a^5*b^3*d^7 + 16*b
^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*
a^3*b^5*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2
*d - 2*a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) - (x*(a^4*d^5
+ 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(4*(b^3*c^2 + a^2*b*d^2
 - 2*a*b^2*c*d)))*(-c^3*d)^(1/2)*1i)/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2) -
((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 +
 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^
2*c*d^2 - 3*a*b^3*c^2*d)) + (x*(-c^3*d)^(1/2)*(16*a^5*b^3*d^7 + 16*b^8*c^5*
```

d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*a^3*b^5*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) + (x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(4*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(-c^3*d)^(1/2)*1i)/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/(((a^3*c^2*d^3)/2 - (5*a^2*b*c^3*d^2)/2 + 3*a*b^2*c^4*d)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) + ((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)) - (x*(-c^3*d)^(1/2)*(16*a^5*b^3*d^7 + 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*a^3*b^5*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) - (x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(4*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(-c^3*d)^(1/2))/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2) + ((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)) + (x*(-c^3*d)^(1/2)*(16*a^5*b^3*d^7 + 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*a^3*b^5*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) + (x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(4*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(-c^3*d)^(1/2))/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)))*(-c^3*d)^(1/2)*1i)/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2) - (atan(((((x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) - ((-a*b^3)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) - (x*(-a*b^3)^(1/2)*(a*d - 3*b*c)*(16*a^5*b^3*d^7 + 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*a^3*b^5*c^2*d^5))/(8*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(a*d - 3*b*c))/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)))*(-a*b^3)^(1/2)*(a*d - 3*b*c)*1i)/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)) + (((x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) + ((-a*b^3)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) + (x*(-a*b^3)^(1/2)*(a*d - 3*b*c)*(16*a^5*b^3*d^7 + 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*a^3*b^5*c^2*d^5))/(8*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(a*d - 3*b*c))/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)))*(-a*b^3)^(1/2)*(a*d - 3*b*c)*1i)/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)))/(((a^3*c^2*d^3)/2 - (5*a^2*b*c^3*d^2)/2 + 3*a*b^2*c^4*d)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) - (((x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) - ((-a*b^3)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^

```
2*c*d^2 - 3*a*b^3*c^2*d) - (x*(-a*b^3)^(1/2)*(a*d - 3*b*c)*(16*a^5*b^3*d^7
+ 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4
 + 32*a^3*b^5*c^2*d^5))/(8*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)*(b^3*c^2 +
 a^2*b*d^2 - 2*a*b^2*c*d)))*(a*d - 3*b*c))/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*
b^4*c*d)))*(-a*b^3)^(1/2)*(a*d - 3*b*c))/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^
4*c*d)) + (((x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))
/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) + ((-a*b^3)^(1/2)*((2*a*b^6*c^5*d^
2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^
2*d^5)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) + (x*(-a*b^3
)^(1/2)*(a*d - 3*b*c)*(16*a^5*b^3*d^7 + 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 -
 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*a^3...
```

3.290 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)} dx$

Optimal. Leaf size=74

$$\frac{a}{2b(bc-ad)(a+bx^2)}+\frac{c\log(a+bx^2)}{2(bc-ad)^2}-\frac{c\log(c+dx^2)}{2(bc-ad)^2}$$

```
[Out] 1/2*a/b/(-a*d+b*c)/(b*x^2+a)+1/2*c*ln(b*x^2+a)/(-a*d+b*c)^2-1/2*c*ln(d*x^2+
c)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{a}{2b(a+bx^2)(bc-ad)}+\frac{c\log(a+bx^2)}{2(bc-ad)^2}-\frac{c\log(c+dx^2)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (c
*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^2} + \frac{bc}{(bc-ad)^2(a+bx)} - \frac{cd}{(bc-ad)^2(c+dx)}\right.\right.$$

$$= \frac{a}{2b(bc-ad)\left(a+bx^2\right)} + \frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2} - \frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 1.00

$$\frac{a}{2b(bc-ad)\left(a+bx^2\right)} + \frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2} - \frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (c
*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Maple [A]
time = 0.12, size = 69, normalized size = 0.93

method	result	size
default	$\frac{c\ln\left(b\,x^2+a\right)-\frac{a(ad-bc)}{b\left(b\,x^2+a\right)}}{2(ad-bc)^2}-\frac{c\ln\left(d\,x^2+c\right)}{2(ad-bc)^2}$	69
norman	$\frac{x^2}{2(ad-bc)(b\,x^2+a)}+\frac{c\ln\left(b\,x^2+a\right)}{2a^2d^2-4abcd+2b^2c^2}-\frac{c\ln\left(d\,x^2+c\right)}{2(a^2d^2-2abcd+b^2c^2)}$	94
risch	$-\frac{a}{2(ad-bc)b(b\,x^2+a)}+\frac{c\ln\left(b\,x^2+a\right)}{2a^2d^2-4abcd+2b^2c^2}-\frac{c\ln\left(-d\,x^2-c\right)}{2(a^2d^2-2abcd+b^2c^2)}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*d-b*c)^2*(c*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a))-1/2*c/(a*d-b*c)^2*l
n(d*x^2+c)
```

Maxima [A]
time = 0.27, size = 105, normalized size = 1.42

$$\frac{c\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)} - \frac{c\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)} + \frac{a}{2\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c*log(d*x^2 + c)
/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a/(a*b^2*c - a^2*b*d + (b^3*c - a*b^
2*d)*x^2)
```

Fricas [A]
time = 0.64, size = 117, normalized size = 1.58

$$\frac{abc - a^2d + (b^2cx^2 + abc)\log\left(bx^2 + a\right) - (b^2cx^2 + abc)\log\left(dx^2 + c\right)}{2\left(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*c - a^2*d + (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) - (b^2*c*x^2 + a*b*
c)*log(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*
b^3*c*d + a^2*b^2*d^2)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(58) = 116$.
time = 1.08, size = 253, normalized size = 3.42

$$-\frac{a}{2a^2bd - 2ab^2c + x^2\cdot(2ab^2d - 2b^3c)} - \frac{c\log\left(x^2 + \frac{-\frac{a^3cd^3}{(ad-bc)^2} + \frac{3a^2bc^2d^2}{(ad-bc)^2} - \frac{3ab^2c^3d}{(ad-bc)^2} + acd + \frac{b^3c^4}{(ad-bc)^2} + bc^2}{2bcd}\right)}{2\left(ad - bc\right)^2} + \frac{c\log\left(x^2 + \frac{\frac{a^3cd^3}{(ad-bc)^2} - \frac{3a^2bc^2d^2}{(ad-bc)^2} + \frac{3ab^2c^3d}{(ad-bc)^2} + acd - \frac{b^3c^4}{(ad-bc)^2} + bc^2}{2bcd}\right)}{2\left(ad - bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -a/(2*a**2*b*d - 2*a*b**2*c + x**2*(2*a*b**2*d - 2*b**3*c)) - c*log(x**2 +
(-a**3*c*d**3/(a*d - b*c)**2 + 3*a**2*b*c**2*d**2/(a*d - b*c)**2 - 3*a*b**2
*c**3*d/(a*d - b*c)**2 + a*c*d + b**3*c**4/(a*d - b*c)**2 + b*c**2)/(2*b*c*
d))/(2*(a*d - b*c)**2) + c*log(x**2 + (a**3*c*d**3/(a*d - b*c)**2 - 3*a**2*
b*c**2*d**2/(a*d - b*c)**2 + 3*a*b**2*c**3*d/(a*d - b*c)**2 + a*c*d - b**3*
c**4/(a*d - b*c)**2 + b*c**2)/(2*b*c*d))/(2*(a*d - b*c)**2)
```

Giac [A]
time = 0.44, size = 92, normalized size = 1.24

$$-\frac{\frac{b^2c\log\left(\left|\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right|\right)}{b^3c^2 - 2\,ab^2cd + a^2bd^2} - \frac{ab}{(b^2c - abd)(bx^2+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c^2 - 2*a*
b^2*c*d + a^2*b*d^2) - a*b/((b^2*c - a*b*d)*(b*x^2 + a)))/b
```

Mupad [B]

time = 0.17, size = 172, normalized size = 2.32

$$\frac{a\left(b\,c+b\,c\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}\right)-a^2\,d+b^2\,c\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}}{2\,a^3\,b\,d^2-4\,a^2\,b^2\,c\,d+2\,a^2\,b^2\,d^2\,x^2+2\,a\,b^3\,c^2-4\,a\,b^3\,c\,d\,x^2+2\,b^4\,c^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (a*(b*c + b*c*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2
i) - a^2*d + b^2*c*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*
c*x^2))*2i)/(2*a*b^3*c^2 + 2*a^3*b*d^2 + 2*b^4*c^2*x^2 + 2*a^2*b^2*d^2*x^2
- 4*a^2*b^2*c*d - 4*a*b^3*c*d*x^2)
```

3.291 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=104

$$-\frac{x}{2(bc-ad)\left(a+bx^2\right)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,(bc-ad)^2}-\frac{\sqrt{c}\,\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

```
[Out] -1/2*x/(-a*d+b*c)/(b*x^2+a)+1/2*(a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b
*c)^2/a^(1/2)/b^(1/2)-arctan(x*d^(1/2)/c^(1/2))*c^(1/2)*d^(1/2)/(-a*d+b*c)^
2
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {482, 536, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(ad+bc)}{2\sqrt{a}\,\sqrt{b}\,(bc-ad)^2}-\frac{\sqrt{c}\,\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{(bc-ad)^2}-\frac{x}{2\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/2*x/((b*c - a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*Sqrt[a]*Sqrt[b]*(b*c - a*d)^2) - (Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(b*c - a*d)^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx &= -\frac{x}{2(bc-ad)(a+bx^2)}+\frac{\int \frac{c-dx^2}{(a+bx^2)(c+dx^2)}\,dx}{2(bc-ad)}\\ &= -\frac{x}{2(bc-ad)(a+bx^2)}-\frac{(cd)\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2}+\frac{(bc+ad)\int \frac{1}{a+bx^2}\,dx}{2(bc-ad)^2}\\ &= -\frac{x}{2(bc-ad)(a+bx^2)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,(bc-ad)^2}-\frac{\sqrt{c}\,\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 104, normalized size = 1.00

$$\frac{x}{2(-bc+ad)(a+bx^2)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,(-bc+ad)^2}-\frac{\sqrt{c}\,\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
[Out] x/(2*(-(b*c) + a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*Sqrt[a]*Sqrt[b]*(-(b*c) + a*d)^2) - (Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(b*c - a*d)^2
```

Maple [A]
time = 0.18, size = 85, normalized size = 0.82

method	result
default	$\frac{\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(ad-bc)^2}-\frac{dc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^2\sqrt{cd}}$
risch	$\frac{x}{2(ad-bc)(b\,x^2+a)}-\frac{\ln\left(a\,b^2x-(-ab)^{\frac{3}{2}}\right)ad}{4\sqrt{-ab}\,(ad-bc)^2}-\frac{\ln\left(a\,b^2x-(-ab)^{\frac{3}{2}}\right)bc}{4\sqrt{-ab}\,(ad-bc)^2}+\frac{\ln\left(-a\,b^2x-(-ab)^{\frac{3}{2}}\right)ad}{4\sqrt{-ab}\,(ad-bc)^2}+\frac{\ln\left(-a\,b^2x-(-ab)^{\frac{3}{2}}\right)bc}{4\sqrt{-ab}\,(ad-bc)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d-b*c)^2*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(a*d+b*c)/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2)))-d/(a*d-b*c)^2*c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.54, size = 119, normalized size = 1.14

$$-\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}}+\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}}-\frac{x}{2\,(abc-a^2d+(b^2c-abd)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -c*d*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a
*b)) - 1/2*x/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)
```

Fricas [A]
time = 1.02, size = 704, normalized size = 6.77

$$\left[-\frac{(abc+a^2d+(b^2c+abd)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,(ab^2x^2+a^2b)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\,(ab^2c-a^2bd)x}{4\,(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)x^2)}, \frac{(abc+a^2d+(b^2c+abd)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(ab^2x^2+a^2b)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)-(ab^2c-a^2bd)x}{2\,(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)x^2)}, -\frac{4\,(ab^2x^2+a^2b)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+(abc+a^2d+(b^2c+abd)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,(ab^2c-a^2bd)x}{4\,(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)x^2)}, \frac{(abc+a^2d+(b^2c+abd)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-2\,(ab^2x^2+a^2b)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-(ab^2c-a^2bd)x}{2\,(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*x^2 + a^2*b)*sqrt(-c*d)*log((d*x^2 -
2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 -
2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2),
 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a)
 + (a*b^2*x^2 + a^2*b)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 +
 c)) - (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a
*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2), -1/4*(4*(a*b^2*x^2 + a^2*b)*s
qrt(c*d)*arctan(sqrt(c*d)*x/c) + (a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt
(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(a*b^2*c - a^2*b*d
)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d
+ a^3*b^2*d^2)*x^2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*a
rctan(sqrt(a*b)*x/a) - 2*(a*b^2*x^2 + a^2*b)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
) - (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 110, normalized size = 1.06

$$-\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}}+\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}}-\frac{x}{2\,(bx^2+a)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -c*d*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a
*b)) - 1/2*x/((b*x^2 + a)*(b*c - a*d))
```

Mupad [**B**]
time = 0.49, size = 3153, normalized size = 30.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] x/(2*(a + b*x^2)*(a*d - b*c)) + (atan((((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^6
*c^5*d^2 - 8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b
^3*c^2*d^5)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-
c*d)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3
*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (x*(a^2*b*d^5 + 5*b^3
*c^2*d^3 + 2*a*b^2*c*d^4))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d) - ((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^6*c^5*d^2 - 8
*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/
(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(-c*d)^(1/2)*(
16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*
a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2
)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2
```

```
*a*b^2*c*d^4))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^2 + b^2*c^2
- 2*a*b*c*d))/(((b^2*c^2*d^3)/2 + (a*b*c*d^4)/2)/(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2) + ((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^6*c^5*d^2 -
8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)
/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-c*d)^(1/2)*
(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32
*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^
2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (x*(a^2*b*d^5 + 5*b^3*c^2*d^3 +
2*a*b^2*c*d^4))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(a^2*d^2 + b^2*c^2 -
2*a*b*c*d) + ((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3
 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(2*(a^3*d^3 -
b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(-c*d)^(1/2)*(16*a^5*b^2*d^7
 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^
4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)) + (x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))
/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(-
c*d)^(1/2)*1i)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - (atan((((-a*b)^(1/2)*((x*(
a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)) - ((-a*b)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 +
12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2) - (x*(-a*b)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7*c^5
*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^
4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a*b^3*c^2 + a^3*b*d^2 - 2*a
^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))*(a*
d + b*c)*1i)/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)) + ((-a*b)^(1/2)*((
x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))/(2*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d)) + ((-a*b)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6
 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2) + (x*(-a*b)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7*
c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3
*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a*b^3*c^2 + a^3*b*d^2 -
2*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))*
(a*d + b*c)*1i)/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))/(((b^2*c^2*d^3
)/2 + (a*b*c*d^4)/2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) -
((-a*b)^(1/2)*((x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))/(2*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)) - ((-a*b)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 +
 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - (x*(-a*b)^(1/2)*(a*d + b*c)*(16*a^5*b
^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*
c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a*b^3*c^
2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2
*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)) +
((-a*b)^(1/2)*((x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))/(2*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)) + ((-a*b)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 +
 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(a^3*d^3 - b^3*c
```

```
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (x*(-a*b)^(1/2)*(a*d + b*c)*(16*a^5*b
^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*
c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a*b^3*c^
2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2
*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d))))*
(-a*b)^(1/2)*(a*d + b*c)*1i)/(2*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d))
```

3.292 $\int \frac{x}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{2(bc-ad)\left(a+bx^2\right)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

```
[Out] -1/2/(-a*d+b*c)/(b*x^2+a)-1/2*d*ln(b*x^2+a)/(-a*d+b*c)^2+1/2*d*ln(d*x^2+c)/
(-a*d+b*c)^2
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$,
Rules used = {455, 46}

$$-\frac{1}{2\left(a+bx^2\right)(bc-ad)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/2*1/((b*c - a*d)*(a + b*x^2)) - (d*Log[a + b*x^2])/(2*(b*c - a*d)^2) + (
d*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)\right.\\&=-\frac{1}{2(bc-ad)\left(a+bx^2\right)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.94

$$\frac{-bc+ad-d(a+bx^2)\log(a+bx^2)+d(a+bx^2)\log(c+dx^2)}{2(bc-ad)^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x^2)*Log[a + b*x^2] + d*(a + b*x^2)*Log[c + d*x^2]
)/(2*(b*c - a*d)^2*(a + b*x^2))
```

Maple [A]
time = 0.12, size = 72, normalized size = 1.03

method	result	size
default	$-\frac{b\left(\frac{d\ln(b\,x^2+a)}{b}-\frac{ad-bc}{b(b\,x^2+a)}\right)}{2(ad-bc)^2}+\frac{d\ln(d\,x^2+c)}{2(ad-bc)^2}$	72
risch	$\frac{1}{2(ad-bc)(b\,x^2+a)}-\frac{d\ln(-b\,x^2-a)}{2(a^2d^2-2abcd+b^2c^2)}+\frac{d\ln(d\,x^2+c)}{2a^2d^2-4abcd+2b^2c^2}$	94
norman	$-\frac{b\,x^2}{2a(ad-bc)(b\,x^2+a)}-\frac{d\ln(b\,x^2+a)}{2(a^2d^2-2abcd+b^2c^2)}+\frac{d\ln(d\,x^2+c)}{2a^2d^2-4abcd+2b^2c^2}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/(a*d-b*c)^2*(d/b*ln(b*x^2+a)-(a*d-b*c)/b/(b*x^2+a))+1/2*d/(a*d-b*c)^
2*ln(d*x^2+c)
```

Maxima [A]
time = 0.28, size = 99, normalized size = 1.41

$$-\frac{d\log(bx^2+a)}{2(b^2c^2-2\,abcd+a^2d^2)}+\frac{d\log(dx^2+c)}{2(b^2c^2-2\,abcd+a^2d^2)}-\frac{1}{2(abc-a^2d+(b^2c-abd)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*d*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*d*log(d*x^2 + c
)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^
2)
```

Fricas [A]
time = 0.80, size = 103, normalized size = 1.47

$$-\frac{bc-ad+(bdx^2+ad)\log(bx^2+a)-(bdx^2+ad)\log(dx^2+c)}{2(ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c - a*d + (b*d*x^2 + a*d)*log(b*x^2 + a) - (b*d*x^2 + a*d)*log(d*x^
2 + c))/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b
*d^2)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(56) = 112$.
time = 1.04, size = 248, normalized size = 3.54

$$\frac{d\log\left(x^2+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2\,(ad-bc)^2}-\frac{d\log\left(x^2+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2\,(ad-bc)^2}+\frac{1}{2a^2d-2abc+x^2\cdot(2abd-2b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d*log(x**2 + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 -
3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b
*c*d)/(2*b*d**2))/(2*(a*d - b*c)**2) - d*log(x**2 + (a**3*d**4/(a*d - b*c)*
*2 - 3*a**2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a
*d**2 - b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(2*(a*d - b*c)**2)
+ 1/(2*a**2*d - 2*a*b*c + x**2*(2*a*b*d - 2*b**2*c))
```

Giac [A]
time = 0.55, size = 85, normalized size = 1.21

$$\frac{bd\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)}-\frac{b}{2\,(b^2c-abd)(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2) - 1/2*b/((b^2*c - a*b*d)*(b*x^2 + a))
```

Mupad [B]
time = 0.16, size = 161, normalized size = 2.30

$$-\frac{b\,c-a\,\left(d-d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}\right)+b\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}}{2\,a^3\,d^2-4\,a^2\,b\,c\,d+2\,a^2\,b\,d^2\,x^2+2\,a\,b^2\,c^2-4\,a\,b^2\,c\,d\,x^2+2\,b^3\,c^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] -(b*c - a*(d - d*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2)
)*2i) + b*d*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))
*2i)/(2*a^3*d^2 + 2*a*b^2*c^2 + 2*b^3*c^2*x^2 + 2*a^2*b*d^2*x^2 - 4*a^2*b*c
*d - 4*a*b^2*c*d*x^2)
```

3.293 $\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=108

$$\frac{bx}{2a(bc-ad)\left(a+bx^2\right)}+\frac{\sqrt{b}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}$$

```
[Out] 1/2*b*x/a/(-a*d+b*c)/(b*x^2+a)+1/2*(-3*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*b
^(1/2)/a^(3/2)/(-a*d+b*c)^2+d^(3/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^2/
c^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {425, 536, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-3ad)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx &= \frac{bx}{2a(bc-ad)(a+bx^2)} - \frac{\int \frac{-bc+2ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx}{2a(bc-ad)} \\
&= \frac{bx}{2a(bc-ad)(a+bx^2)} + \frac{d^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2} + \frac{(b(bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^2} \\
&= \frac{bx}{2a(bc-ad)(a+bx^2)} + \frac{\sqrt{b}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 109, normalized size = 1.01

$$
-\frac{bx}{2a(-bc+ad)(a+bx^2)} - \frac{\sqrt{b}\,(-bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(-bc+ad)^2} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
[Out] -1/2*(b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*d)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Maple [A]
time = 0.07, size = 95, normalized size = 0.88

method	result
default	$-\frac{b\left(\frac{(ad-bc)x}{2a(bx^2+a)}+\frac{(3ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^2}+\frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^2\sqrt{cd}}$
risch	$-\frac{bx}{2a(ad-bc)(bx^2+a)}+\frac{3\sqrt{-ab}\,\ln\left(\left(-9(-ab)^{\frac{3}{2}}a^3d^3-3(-ab)^{\frac{3}{2}}a^2bcd^2+5(-ab)^{\frac{3}{2}}ab^2c^2d-(-ab)^{\frac{3}{2}}b^3c^3-13\sqrt{-ab}\,a^4bd^3+6\right.\right.}{4a(ad-bc)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a*d-b*c)^2*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(3*a*d-b*c)/a/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Maxima [A]
time = 0.49, size = 132, normalized size = 1.22

$$\frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}}+\frac{bx}{2\,(a^2bc-a^3d+(ab^2c-a^2bd)x^2)}+\frac{(b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*(b^2*c - 3*a*b*d)*ar
ctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
```

Fricas [A]
time = 0.96, size = 699, normalized size = 6.47

$$\left[-\frac{(abc-3\,a^2d+(b^2c-3\,abd)x^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,(abdx^2+a^2d)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\,(b^2c-abd)x}{4\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}, \frac{4\,(abdx^2+a^2d)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-(abc-3\,a^2d+(b^2c-3\,abd)x^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+2\,(b^2c-abd)x}{4\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}, \frac{(abc-3\,a^2d+(b^2c-3\,abd)x^2)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+(abdx^2+a^2d)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+(b^2c-abd)x}{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}, \frac{(abc-3\,a^2d+(b^2c-3\,abd)x^2)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,(abdx^2+a^2d)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+(b^2c-abd)x}{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*
x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^
2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1
/4*(4*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (a*b*c - 3*a^2*d
+ (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b
^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c -
3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (a*b*d*x^2 + a^2*d)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*d)*x)/(a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*
x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c - a*
b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 121, normalized size = 1.12

$$\frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{cd}} + \frac{(b^2c - 3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2 - 2\,a^2bcd + a^3d^2)\sqrt{ab}} + \frac{bx}{2\,(abc - a^2d)(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^
2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x^2 + a))
```

Mupad [B]
time = 0.54, size = 2500, normalized size = 23.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (atan((((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*
a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7
- 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*
d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*
b*c*d^2) - (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d
^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*
b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c
^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2
 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)
^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^
2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4
*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-
a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*
c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d
)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))
)*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))/(((3*a*b^3*d^5)/2 - (b^4*c
```

```
*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - ((-a^3
*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))
/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7 - 2*a*b^7*c^5
*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*
b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - (x
*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b
^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))
/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*
c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*
d))))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)^(1/2)*(3*a*d -
b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*
b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c
*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d
^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-a^3*b)^(1/2)*(3*
a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3
*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*
b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/
2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))))/(4*(a^5*d^2 +
 a^3*b^2*c^2 - 2*a^4*b*c*d))))*(-a^3*b)^(1/2)*(3*a*d - b*c)*1i)/(2*(a^5*d^2
 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - (atan((((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 -
 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d
^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^
4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2
*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5
))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^
2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2
*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*
b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d) - ((-c*d^3)^(1/2)*((((4*a^
6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^
3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c
^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d
^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*
b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2
 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) +
 (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d))/(((3*a*b^3*d^5
)/2 - (b^4*c*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^
6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d
^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(1
6*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3
+ 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3
 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c
*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))))/(b^2*c^3 + a^2*c*d^2 - 2
```

```
*a*b*c^2*d) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*
b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(
2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^
(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a...
```

3.294 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)} dx$

Optimal. Leaf size=99

$$\frac{b}{2a(bc-ad)(a+bx^2)} + \frac{\log(x)}{a^2c} - \frac{b(bc-2ad)\log(a+bx^2)}{2a^2(bc-ad)^2} - \frac{d^2\log(c+dx^2)}{2c(bc-ad)^2}$$

```
[Out] 1/2*b/a/(-a*d+b*c)/(b*x^2+a)+ln(x)/a^2/c-1/2*b*(-2*a*d+b*c)*ln(b*x^2+a)/a^2
/(-a*d+b*c)^2-1/2*d^2*ln(d*x^2+c)/c/(-a*d+b*c)^2
```

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 84}

$$-\frac{b(bc-2ad)\log(a+bx^2)}{2a^2(bc-ad)^2} + \frac{\log(x)}{a^2c} - \frac{d^2\log(c+dx^2)}{2c(bc-ad)^2} + \frac{b}{2a(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] b/(2*a*(b*c - a*d)*(a + b*x^2)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a +
 b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^2])/(2*c*(b*c - a*d)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx^2)^2(c+dx^2)}dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)}dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2cx} + \frac{b^2}{a(-bc+ad)(a+bx)^2} + \frac{b^2(-bc+2ad)}{a^2(-bc+ad)^2(a+bx)} - \frac{}{c(b}\right.\right.\\ &= \frac{b}{2a(bc-ad)(a+bx^2)} + \frac{\log(x)}{a^2c} - \frac{b(bc-2ad)\log(a+bx^2)}{2a^2(bc-ad)^2} - \frac{d^2\log(c+dx}{2c(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 97, normalized size = 0.98

$$\frac{2\log(x)-\frac{bc(bc-2ad)(a+bx^2)\log(a+bx^2)+a(bc(-bc+ad)+ad^2(a+bx^2)\log(c+dx^2))}{(bc-ad)^2(a+bx^2)}}{2a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (2*Log[x] - (b*c*(b*c - 2*a*d)*(a + b*x^2)*Log[a + b*x^2] + a*(b*c*(-(b*c)
+ a*d) + a*d^2*(a + b*x^2)*Log[c + d*x^2]))/((b*c - a*d)^2*(a + b*x^2)))/(2
*a^2*c)
```

Maple [A]
time = 0.15, size = 100, normalized size = 1.01

method	result	size
default	$\frac{b^2\left(\frac{(2ad-bc)\ln\left(b\,x^2+a\right)}{b}-\frac{a(ad-bc)}{b\left(b\,x^2+a\right)}\right)}{2a^2(ad-bc)^2}-\frac{d^2\ln\left(d\,x^2+c\right)}{2c(ad-bc)^2}+\frac{\ln(x)}{a^2c}$	100
norman	$\frac{b^2x^2}{2a^2(ad-bc)(b\,x^2+a)}+\frac{\ln(x)}{a^2c}-\frac{d^2\ln\left(d\,x^2+c\right)}{2c(a^2d^2-2abcd+b^2c^2)}+\frac{b(2ad-bc)\ln\left(b\,x^2+a\right)}{2a^2(a^2d^2-2abcd+b^2c^2)}$	126
risch	$-\frac{b}{2a(ad-bc)(b\,x^2+a)}+\frac{\ln(x)}{a^2c}+\frac{b\ln\left(b\,x^2+a\right)d}{a(a^2d^2-2abcd+b^2c^2)}-\frac{b^2\ln\left(b\,x^2+a\right)c}{2a^2(a^2d^2-2abcd+b^2c^2)}-\frac{d^2\ln\left(-d\,x^2-c\right)}{2c(a^2d^2-2abcd+b^2c^2)}$	155

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^2/(a*d-b*c)^2*(1/b*(2*a*d-b*c)*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a
))-1/2*d^2/c/(a*d-b*c)^2*ln(d*x^2+c)+ln(x)/a^2/c
```

Maxima [A]
time = 0.29, size = 137, normalized size = 1.38

$$-\frac{d^2\log\left(dx^2+c\right)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}-\frac{\left(b^2c-2\,abd\right)\log\left(bx^2+a\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}+\frac{b}{2\left(a^2bc-a^3d+\left(ab^2c-a^2bd\right)x^2\right)}+\frac{\log\left(x^2\right)}{2\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*d^2*log(d*x^2 + c)/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - 1/2*(b^2*c -
2*a*b*d)*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/2*b/(a^2*
b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*log(x^2)/(a^2*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(93) = 186.
time = 1.41, size = 218, normalized size = 2.20

$$\frac{ab^2c^2-a^2bcd-\left(ab^2c^2-2\,a^2bcd+\left(b^3c^2-2\,ab^2cd\right)x^2\right)\log\left(bx^2+a\right)-\left(a^2bd^2x^2+a^3d^2\right)\log\left(dx^2+c\right)+2\left(ab^2c^2-2\,a^2bcd+a^3d^2+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^2\right)\log\left(x\right)}{2\left(a^3b^2c^3-2\,a^4bc^2d+a^5cd^2+\left(a^2b^3c^3-2\,a^3b^2c^2d+a^4bcd^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")

[Out] 1/2*(a*b^2*c^2 - a^2*b*c*d - (a*b^2*c^2 - 2*a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d)*x^2)*log(b*x^2 + a) - (a^2*b*d^2*x^2 + a^3*d^2)*log(d*x^2 + c) + 2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*log(x))/(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2 + (a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^2)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c),x)

[Out] Timed out

Giac [**A**]
time = 0.45, size = 183, normalized size = 1.85

$$-\frac{d^3\log\left(|dx^2+c|\right)}{2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)}-\frac{\left(b^3c-2\,ab^2d\right)\log\left(|bx^2+a|\right)}{2\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)}+\frac{b^3cx^2-2\,ab^2dx^2+2\,ab^2c-3\,a^2bd}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\left(bx^2+a\right)}+\frac{\log\left(x^2\right)}{2\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")

[Out] -1/2*d^3*log(abs(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3) - 1/2*(b^3*c - 2*a*b^2*d)*log(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2) + 1/2*(b^3*c*x^2 - 2*a*b^2*d*x^2 + 2*a*b^2*c - 3*a^2*b*d)/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 + a)) + 1/2*log(x^2)/(a^2*c)

Mupad [**B**]
time = 0.40, size = 127, normalized size = 1.28

$$\frac{\ln\left(x\right)}{a^2\,c}-\frac{d^2\,\ln\left(d\,x^2+c\right)}{2\,a^2\,c\,d^2-4\,a\,b\,c^2\,d+2\,b^2\,c^3}-\frac{\ln\left(b\,x^2+a\right)\,\left(b^2\,c-2\,a\,b\,d\right)}{2\,a^4\,d^2-4\,a^3\,b\,c\,d+2\,a^2\,b^2\,c^2}-\frac{b}{2\,a\,\left(b\,x^2+a\right)\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)),x)

[Out] log(x)/(a^2*c) - (d^2*log(c + d*x^2))/(2*b^2*c^3 + 2*a^2*c*d^2 - 4*a*b*c^2*d) - (log(a + b*x^2)*(b^2*c - 2*a*b*d))/(2*a^4*d^2 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d) - b/(2*a*(a + b*x^2)*(a*d - b*c))

3.295 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=144

$$-\frac{3bc-2ad}{2a^2c(bc-ad)x}+\frac{b}{2a(bc-ad)x\left(a+bx^2\right)}-\frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}$$

```
[Out] 1/2*(2*a*d-3*b*c)/a^2/c/(-a*d+b*c)/x+1/2*b/a/(-a*d+b*c)/x/(b*x^2+a)-1/2*b^(
3/2)*(-5*a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^2-d^(5/2)*
arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {483, 597, 536, 211}

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3bc-5ad)}{2a^{5/2}(bc-ad)^2}-\frac{3bc-2ad}{2a^2cx(bc-ad)}-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}+\frac{b}{2ax\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/2*(3*b*c - 2*a*d)/(a^2*c*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x*(a + b*x^
2)) - (b^(3/2)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c
 - a*d)^2) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{\int \frac{-3bc+2ad-3bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}$$

$$= -\frac{3bc-2ad}{2a^2c(bc-ad)x} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} + \frac{\int \frac{-3b^2c^2+2abcd+2a^2d^2-bd(3bc-2ad)}{\left(a+bx^2\right)\left(c+dx^2\right)}}{2a^2c(bc-ad)}$$

$$= -\frac{3bc-2ad}{2a^2c(bc-ad)x} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{d^3\int \frac{1}{c+dx^2}\,dx}{c(bc-ad)^2} - \frac{\left(b^2(3bc-5a\right.}{2a^2(bc}$$

$$= -\frac{3bc-2ad}{2a^2c(bc-ad)x} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.}{2a^{5/2}(bc-ad)^2}$$

Mathematica [A]
time = 0.14, size = 123, normalized size = 0.85

$$-\frac{1}{a^2cx} + \frac{b^2x}{2a^2(-bc+ad)\left(a+bx^2\right)} + \frac{b^{3/2}(-3bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(-bc+ad)^2} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(1/(a^2*c*x)) + (b^2*x)/(2*a^2*(-(b*c) + a*d)*(a + b*x^2)) + (b^(3/2)*(-3*
b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(-(b*c) + a*d)^2) - (d
^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2)
```

Maple [A]
time = 0.17, size = 108, normalized size = 0.75

method	result
default	$\frac{b^{2}\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{(5ad-3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}(ad-bc)^{2}}-\frac{d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c(ad-bc)^{2}\sqrt{cd}}-\frac{1}{a^{2}cx}$
risch	$\frac{-\frac{b(2ad-3bc)x^{2}}{2a^{2}c(ad-bc)}-\frac{1}{ac}}{x(b\,x^{2}+a)}+\frac{5\sqrt{-ab}\,b\ln\left(\left(-4a^{5}b^{2}d^{4}-4a^{4}b^{3}c\,d^{3}-4a^{3}b^{4}c^{2}d^{2}+21a^{2}b^{5}c^{3}d-9a\,b^{6}c^{4}\right)x+10(-ab)^{\frac{3}{2}}a^{4}d^{4}+4(-ab)^{\frac{3}{2}}a^{3}bc\,d^{3}-\right.}{4a^{2}(ad-bc)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)

[Out] b^2/a^2/(a*d-b*c)^2*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(5*a*d-3*b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/c*d^3/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))-1/a^2/c/x

Maxima [A]
time = 0.53, size = 178, normalized size = 1.24

$$-\frac{d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)\sqrt{cd}}-\frac{\left(3\,b^{3}c-5\,ab^{2}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)\sqrt{ab}}-\frac{2\,abc-2\,a^{2}d+\left(3\,b^{2}c-2\,abd\right)x^{2}}{2\left(\left(a^{2}b^{2}c^{2}-a^{3}bcd\right)x^{3}+\left(a^{3}bc^{2}-a^{4}cd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")

[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d)) - 1/2*(3*b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b)) - 1/2*(2*a*b*c - 2*a^2*d + (3*b^2*c - 2*a*b*d)*x^2)/((a^2*b^2*c^2 - a^3*b*c*d)*x^3 + (a^3*b*c^2 - a^4*c*d)*x)

Fricas [A]
time = 1.97, size = 1003, normalized size = 6.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")

[Out] [-1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4*a^3*d^2 + 2*(3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x

```
^2 + c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3
 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4*a^3*d
^2 + 2*(3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + 4*(a^2*b*d^2*x^3 + a^3
*d^2*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) + ((3*b^3*c^2 - 5*a*b^2*c*d)*x^3 + (3
*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2
*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/2*(2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a
^3*d^2 + (3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^
2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
 (a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/
(d*x^2 + c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2
*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/2*(2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a
^3*d^2 + (3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^
2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) +
 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^3*c^3
 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*
c*d^2)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.74, size = 164, normalized size = 1.14

$$-\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}}-\frac{(3\,b^3c-5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{ab}}-\frac{3\,b^2cx^2-2\,abdx^2+2\,abc-2\,a^2d}{2\,(a^2bc^2-a^3cd)(bx^3+ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
- 1/2*(3*b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c
*d + a^4*d^2)*sqrt(a*b)) - 1/2*(3*b^2*c*x^2 - 2*a*b*d*x^2 + 2*a*b*c - 2*a^2
*d)/((a^2*b*c^2 - a^3*c*d)*(b*x^3 + a*x))
```

Mupad [**B**]
time = 0.56, size = 2400, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x)

[Out] (atan((a^5*d*x*(-c^3*d^5)^(3/2)*4i + b^5*c^8*d*x*(-c^3*d^5)^(1/2)*9i + a^2*b^3*c^6*d^3*x*(-c^3*d^5)^(1/2)*25i - a*b^4*c^7*d^2*x*(-c^3*d^5)^(1/2)*30i)/(4*a^5*c^5*d^8 - 9*b^5*c^10*d^3 + 30*a*b^4*c^9*d^4 - 25*a^2*b^3*c^8*d^5))*(-c^3*d^5)^(1/2)*1i)/(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d) - (1/(a*c) - (x^2*(3*b^2*c - 2*a*b*d))/(2*a^2*c*(a*d - b*c)))/(a*x + b*x^3) - (atan((((x*(144*a^6*b^10*c^10*d^3 - 912*a^7*b^9*c^9*d^4 + 2272*a^8*b^8*c^8*d^5 - 2784*a^9*b^7*c^7*d^6 + 1744*a^10*b^6*c^6*d^7 - 592*a^11*b^5*c^5*d^8 + 192*a^12*b^4*c^4*d^9 - 64*a^13*b^3*c^3*d^10) + ((5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(1280*a^9*b^9*c^11*d^3 - 192*a^8*b^10*c^12*d^2 - 3520*a^10*b^8*c^10*d^4 + 4992*a^11*b^7*c^9*d^5 - 3520*a^12*b^6*c^8*d^6 + 512*a^13*b^5*c^7*d^7 + 960*a^14*b^4*c^6*d^8 - 640*a^15*b^3*c^5*d^9 + 128*a^16*b^2*c^4*d^10 + (x*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(256*a^10*b^10*c^13*d^2 - 1536*a^11*b^9*c^12*d^3 + 3584*a^12*b^8*c^11*d^4 - 3584*a^13*b^7*c^10*d^5 + 3584*a^15*b^5*c^8*d^7 - 3584*a^16*b^4*c^7*d^8 + 1536*a^17*b^3*c^6*d^9 - 256*a^18*b^2*c^5*d^10))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d))))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)))*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*1i)/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)) + ((x*(144*a^6*b^10*c^10*d^3 - 912*a^7*b^9*c^9*d^4 + 2272*a^8*b^8*c^8*d^5 - 2784*a^9*b^7*c^7*d^6 + 1744*a^10*b^6*c^6*d^7 - 592*a^11*b^5*c^5*d^8 + 192*a^12*b^4*c^4*d^9 - 64*a^13*b^3*c^3*d^10) + ((5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(192*a^8*b^10*c^12*d^2 - 1280*a^9*b^9*c^11*d^3 + 3520*a^10*b^8*c^10*d^4 - 4992*a^11*b^7*c^9*d^5 + 3520*a^12*b^6*c^8*d^6 - 512*a^13*b^5*c^7*d^7 - 960*a^14*b^4*c^6*d^8 + 640*a^15*b^3*c^5*d^9 - 128*a^16*b^2*c^4*d^10 + (x*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(256*a^10*b^10*c^13*d^2 - 1536*a^11*b^9*c^12*d^3 + 3584*a^12*b^8*c^11*d^4 - 3584*a^13*b^7*c^10*d^5 + 3584*a^15*b^5*c^8*d^7 - 3584*a^16*b^4*c^7*d^8 + 1536*a^17*b^3*c^6*d^9 - 256*a^18*b^2*c^5*d^10))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d))))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)))*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*1i)/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)))/(((x*(144*a^6*b^10*c^10*d^3 - 912*a^7*b^9*c^9*d^4 + 2272*a^8*b^8*c^8*d^5 - 2784*a^9*b^7*c^7*d^6 + 1744*a^10*b^6*c^6*d^7 - 592*a^11*b^5*c^5*d^8 + 192*a^12*b^4*c^4*d^9 - 64*a^13*b^3*c^3*d^10) + ((5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(192*a^8*b^10*c^12*d^2 - 1280*a^9*b^9*c^11*d^3 + 3520*a^10*b^8*c^10*d^4 - 4992*a^11*b^7*c^9*d^5 + 3520*a^12*b^6*c^8*d^6 - 512*a^13*b^5*c^7*d^7 - 960*a^14*b^4*c^6*d^8 + 640*a^15*b^3*c^5*d^9 - 128*a^16*b^2*c^4*d^10 + (x*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(256*a^10*b^10*c^13*d^2 - 1536*a^11*b^9*c^12*d^3 + 3584*a^12*b^8*c^11*d^4 - 3584*a^13*b^7*c^10*d^5 + 3584*a^15*b^5*c^8*d^7 - 3584*a^16*b^4*c^7*d^8 + 1536*a^17*b^3*c^6*d^9 - 256*a^18*b^2*c^5*d^10))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d))))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)))*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)) - ((x*(144*a^6*b^10*c^10*d^3 - 912*a^7*b^9*c^9*d^4 + 2272*a^8*b^8*c^8*d^5 - 2784*a^9*b^7*c^7*d^6 + 1744*a^10*b^6*c^6*d^7 - 592*a^11*b^5*c^5*d^8 + 192*a^12*b^4*c^4*d^9 - 64*a^13*b^3*c^3*d^10

```
) + ((5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(1280*a^9*b^9*c^11*d^3 - 192*a^8*b^10
*c^12*d^2 - 3520*a^10*b^8*c^10*d^4 + 4992*a^11*b^7*c^9*d^5 - 3520*a^12*b^6*
c^8*d^6 + 512*a^13*b^5*c^7*d^7 + 960*a^14*b^4*c^6*d^8 - 640*a^15*b^3*c^5*d^
9 + 128*a^16*b^2*c^4*d^10 + (x*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(256*a^10*b
^10*c^13*d^2 - 1536*a^11*b^9*c^12*d^3 + 3584*a^12*b^8*c^11*d^4 - 3584*a^13*
b^7*c^10*d^5 + 3584*a^15*b^5*c^8*d^7 - 3584*a^16*b^4*c^7*d^8 + 1536*a^17*b^
3*c^6*d^9 - 256*a^18*b^2*c^5*d^10))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d
))))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)))*(5*a*d - 3*b*c)*(-a^5*b^3)^
(1/2))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)) + 144*a^6*b^8*c^7*d^5 - 62
4*a^7*b^7*c^6*d^6 + 976*a^8*b^6*c^5*d^7 - 656*a^9*b^5*c^4*d^8 + 160*a^10*b^
4*c^3*d^9))*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*1i)/(2*(a^7*d^2 + a^5*b^2*c^2
- 2*a^6*b*c*d))
```

3.296 $\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)} \, dx$

Optimal. Leaf size=126

$$-\frac{1}{2a^2cx^2}-\frac{b^2}{2a^2(bc-ad)\left(a+bx^2\right)}-\frac{(2bc+ad)\log(x)}{a^3c^2}+\frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}$$

```
[Out] -1/2/a^2/c/x^2-1/2*b^2/a^2/(-a*d+b*c)/(b*x^2+a)-(a*d+2*b*c)*ln(x)/a^3/c^2+1
/2*b^2*(-3*a*d+2*b*c)*ln(b*x^2+a)/a^3/(-a*d+b*c)^2+1/2*d^3*ln(d*x^2+c)/c^2/
(-a*d+b*c)^2
```

Rubi [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^2}-\frac{\log(x)(ad+2bc)}{a^3c^2}-\frac{b^2}{2a^2\left(a+bx^2\right)(bc-ad)}-\frac{1}{2a^2cx^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/2*1/(a^2*c*x^2) - b^2/(2*a^2*(b*c - a*d)*(a + b*x^2)) - ((2*b*c + a*d)*L
og[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(2*a^3*(b*c - a*d)^
2) + (d^3*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2cx^2} + \frac{-2bc-ad}{a^3c^2x} - \frac{b^3}{a^2(-bc+ad)(a+bx)^2} - \frac{b^3(-2bc}{a^3(-bc+ad}\right.\right.$$
$$= -\frac{1}{2a^2cx^2} - \frac{b^2}{2a^2(bc-ad)\left(a+bx^2\right)} - \frac{(2bc+ad)\log(x)}{a^3c^2} + \frac{b^2(2bc-3ad)\log}{2a^3(bc-a}$$

Mathematica [A]
time = 0.11, size = 119, normalized size = 0.94

$$\frac{1}{2}\left(-\frac{1}{a^2cx^2} + \frac{b^2}{a^2(-bc+ad)\left(a+bx^2\right)} - \frac{2(2bc+ad)\log(x)}{a^3c^2} + \frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^2} + \frac{d^3\log\left(c+dx^2\right)}{c^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(1/(a^2*c*x^2)) + b^2/(a^2*(-(b*c) + a*d)*(a + b*x^2)) - (2*(2*b*c + a*d)
*Log[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^
2) + (d^3*Log[c + d*x^2])/(c^2*(b*c - a*d)^2))/2
```

Maple [A]
time = 0.15, size = 120, normalized size = 0.95

method	result	s
default	$-\frac{b^3\left(\frac{(3ad-2bc)\ln\left(bx^2+a\right)}{b}-\frac{a(ad-bc)}{b\left(bx^2+a\right)}\right)}{2a^3(ad-bc)^2}+\frac{d^3\ln\left(dx^2+c\right)}{2c^2(ad-bc)^2}-\frac{1}{2a^2cx^2}+\frac{(-ad-2bc)\ln(x)}{a^3c^2}$	1
norman	$\frac{-\frac{1}{2ac}+\frac{\left(abd-2b^2c\right)bx^4}{2ca^3(ad-bc)}}{x^2\left(bx^2+a\right)}+\frac{d^3\ln\left(dx^2+c\right)}{2c^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{(ad+2bc)\ln(x)}{a^3c^2}-\frac{b^2(3ad-2bc)\ln\left(bx^2+a\right)}{2a^3\left(a^2d^2-2abcd+b^2c^2\right)}$	1
risch	$\frac{-\frac{b(ad-2bc)x^2}{2a^2c(ad-bc)}-\frac{1}{2ac}}{x^2\left(bx^2+a\right)}-\frac{\ln(x)d}{a^2c^2}-\frac{2\ln(x)b}{a^3c}+\frac{d^3\ln\left(-dx^2-c\right)}{2c^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{3b^2\ln\left(bx^2+a\right)d}{2a^2\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{b^3\ln\left(bx^2+a\right)c}{a^3\left(a^2d^2-2abcd+b^2c^2\right)}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3/a^3/(a*d-b*c)^2*((3*a*d-2*b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+
a))+1/2*d^3/c^2/(a*d-b*c)^2*ln(d*x^2+c)-1/2/a^2/c/x^2+1/a^3/c^2*(-a*d-2*b*c
)*ln(x)
```

Maxima [A]
time = 0.31, size = 189, normalized size = 1.50

$$\frac{d^3\log\left(dx^2+c\right)}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)} + \frac{\left(2\,b^3c-3\,ab^2d\right)\log\left(bx^2+a\right)}{2\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)} - \frac{abc-a^2d+\left(2\,b^2c-abd\right)x^2}{2\left(\left(a^2b^2c^2-a^3bcd\right)x^4+\left(a^3bc^2-a^4cd\right)x^2\right)} - \frac{(2\,bc+ad)\log\left(x^2\right)}{2\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + 1/2*(2*b^3*c
 - 3*a*b^2*d)*log(b*x^2 + a)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2) - 1/2*(a
*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)/((a^2*b^2*c^2 - a^3*b*c*d)*x^4 + (a^3
*b*c^2 - a^4*c*d)*x^2) - 1/2*(2*b*c + a*d)*log(x^2)/(a^3*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(118) = 236$.
time = 3.49, size = 303, normalized size = 2.40

$$-\frac{a^{2} b^{2} c^{3} - 2 \, a^{3} b c^{2} d + a^{4} c d^{2} + (2 \, a b^{3} c^{3} - 3 \, a^{2} b^{2} c^{2} d + a^{3} b c d^{2}) x^{2} - ((2 \, b^{4} c^{3} - 3 \, a b^{3} c^{2} d) x^{4} + (2 \, a b^{3} c^{3} - 3 \, a^{2} b^{2} c^{2} d) x^{2}) \log\left(b x^{2} + a\right) - (a^{3} b d^{3} x^{4} + a^{4} d^{3} x^{2}) \log\left(d x^{2} + c\right) + 2 \, ((2 \, b^{4} c^{3} - 3 \, a b^{3} c^{2} d + a^{3} b d^{3}) x^{4} + (2 \, a b^{3} c^{3} - 3 \, a^{2} b^{2} c^{2} d + a^{4} d^{3}) x^{2}) \log\left(x\right)}{2 \, ((a^{3} b^{3} c^{4} - 2 \, a^{4} b^{2} c^{3} d + a^{5} b c^{2} d^{2}) x^{4} + (a^{4} b^{2} c^{4} - 2 \, a^{5} b c^{3} d + a^{6} c^{2} d^{2}) x^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2 + (2*a*b^3*c^3 - 3*a^2*b^2*c^
2*d + a^3*b*c*d^2)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d)*x^4 + (2*a*b^3*c^3 -
3*a^2*b^2*c^2*d)*x^2)*log(b*x^2 + a) - (a^3*b*d^3*x^4 + a^4*d^3*x^2)*log(d*
x^2 + c) + 2*((2*b^4*c^3 - 3*a*b^3*c^2*d + a^3*b*d^3)*x^4 + (2*a*b^3*c^3 -
3*a^2*b^2*c^2*d + a^4*d^3)*x^2)*log(x))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a
^5*b*c^2*d^2)*x^4 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(118) = 236$.
time = 0.74, size = 257, normalized size = 2.04

$$\frac{d^{4} \log\left(|d x^{2} + c|\right)}{2 \, (b^{2} c^{4} d - 2 \, a b c^{3} d^{2} + a^{2} c^{2} d^{3})} + \frac{(2 \, b^{4} c - 3 \, a b^{3} d) \log\left(|b x^{2} + a|\right)}{2 \, (a^{3} b^{3} c^{2} - 2 \, a^{4} b^{2} c d + a^{5} b d^{2})} + \frac{a^{2} b d^{3} x^{4} - 4 \, b^{3} c^{3} x^{2} + 6 \, a b^{2} c^{2} d x^{2} - 2 \, a^{2} b c d^{2} x^{2} + a^{3} d^{3} x^{2} - 2 \, a b^{2} c^{3} + 4 \, a^{2} b c^{2} d - 2 \, a^{3} c d^{2}}{4 \, (a^{2} b^{2} c^{4} - 2 \, a^{3} b c^{3} d + a^{4} c^{2} d^{2})(b x^{4} + a x^{2})} - \frac{(2 \, b c + a d) \log\left(x^{2}\right)}{2 \, a^{3} c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3) + 1/2
*(2*b^4*c - 3*a*b^3*d)*log(abs(b*x^2 + a))/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a
^5*b*d^2) + 1/4*(a^2*b*d^3*x^4 - 4*b^3*c^3*x^2 + 6*a*b^2*c^2*d*x^2 - 2*a^2*
```

```
b*c*d^2*x^2 + a^3*d^3*x^2 - 2*a*b^2*c^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*x^4 + a*x^2)) - 1/2*(2*b*c + a*
d)*log(x^2)/(a^3*c^2)
```

Mupad [B]
time = 0.55, size = 171, normalized size = 1.36

$$\frac{\ln\left(b\,x^2+a\right)\,\left(2\,b^3\,c-3\,a\,b^2\,d\right)}{2\,a^5\,d^2-4\,a^4\,b\,c\,d+2\,a^3\,b^2\,c^2}-\frac{\frac{1}{2\,a\,c}-\frac{x^2\,\left(2\,b^2\,c-a\,b\,d\right)}{2\,a^2\,c\,\left(a\,d-b\,c\right)}}{b\,x^4+a\,x^2}+\frac{d^3\,\ln\left(d\,x^2+c\right)}{2\,\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}-\frac{\ln\left(x\right)\,\left(a\,d+2\,b\,c\right)}{a^3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(2*b^3*c - 3*a*b^2*d))/(2*a^5*d^2 + 2*a^3*b^2*c^2 - 4*a^4*b
*c*d) - (1/(2*a*c) - (x^2*(2*b^2*c - a*b*d))/(2*a^2*c*(a*d - b*c)))/(a*x^2
+ b*x^4) + (d^3*log(c + d*x^2))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)) -
 (log(x)*(a*d + 2*b*c))/(a^3*c^2)
```

3.297 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=189

$$-\frac{5bc-2ad}{6a^2c(bc-ad)x^3}+\frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x}+\frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)}+\frac{b^{5/2}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^2}+\frac{d^{7/2}\,\mathrm{t}\cdots}{c^{5/\cdots}}$$

```
[Out] 1/6*(2*a*d-5*b*c)/a^2/c/(-a*d+b*c)/x^3+1/2*(-2*a^2*d^2-2*a*b*c*d+5*b^2*c^2)
/a^3/c^2/(-a*d+b*c)/x+1/2*b/a/(-a*d+b*c)/x^3/(b*x^2+a)+1/2*b^(5/2)*(-7*a*d+
5*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/(-a*d+b*c)^2+d^(7/2)*arctan(x*d^(1
/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.19, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 597, 536, 211}

$$\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(5bc-7ad)}{2a^{7/2}(bc-ad)^2}-\frac{5bc-2ad}{6a^2cx^3(bc-ad)}+\frac{-2a^2d^2-2abcd+5b^2c^2}{2a^3c^2x(bc-ad)}+\frac{d^{7/2}\mathrm{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}+\frac{b}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/6*(5*b*c - 2*a*d)/(a^2*c*(b*c - a*d)*x^3) + (5*b^2*c^2 - 2*a*b*c*d - 2*a
^2*d^2)/(2*a^3*c^2*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (
b^(5/2)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*(b*c - a*d)
^2) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)}\,dx = \frac{b}{2a(bc-ad)x^3(a+bx^2)} - \frac{\int \frac{-5bc+2ad-5bdx^2}{x^4(a+bx^2)(c+dx^2)}\,dx}{2a(bc-ad)}$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{b}{2a(bc-ad)x^3(a+bx^2)} + \frac{\int \frac{-3(5b^2c^2-2abcd-2a^2d^2)-3bd(5b}{x^2(a+bx^2)(c+dx^2)}}{6a^2c(bc-ad)}$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3(a+bx^2)} - \int$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3(a+bx^2)} + \frac{d}{c}$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3(a+bx^2)} + \frac{b}{}$$

Mathematica [A]
time = 0.20, size = 142, normalized size = 0.75

$$-\frac{1}{3a^2cx^3} + \frac{2bc+ad}{a^3c^2x} - \frac{b^3x}{2a^3(-bc+ad)(a+bx^2)} - \frac{b^{5/2}(-5bc+7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}(-bc+ad)^2} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/3*1/(a^2*c*x^3) + (2*b*c + a*d)/(a^3*c^2*x) - (b^3*x)/(2*a^3*(-(b*c) + a
*d)*(a + b*x^2)) - (b^(5/2)*(-5*b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
```

```
2*a^(7/2)*(-(b*c) + a*d)^2) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2
)*(b*c - a*d)^2)
```

Maple [A]
time = 0.20, size = 128, normalized size = 0.68

method	result	size
default	$-\frac{b^{3}\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{(7ad-5bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}(ad-bc)^{2}}+\frac{d^{4}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^{2}(ad-bc)^{2}\sqrt{cd}}-\frac{1}{3a^{2}c\,x^{3}}-\frac{-ad-2bc}{a^{3}c^{2}x}$	128
risch	Expression too large to display	1579

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/a^3/(a*d-b*c)^2*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(7*a*d-5*b*c)/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2)))+1/c^2*d^4/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*
x/(c*d)^(1/2))-1/3/a^2/c/x^3-1/a^3/c^2*(-a*d-2*b*c)/x
```

Maxima [A]
time = 0.53, size = 236, normalized size = 1.25

$$\frac{d^{4}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}\right)\sqrt{cd}}+\frac{\left(5\,b^{4}c-7\,ab^{3}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^{3}b^{2}c^{2}-2\,a^{4}bcd+a^{5}d^{2}\right)\sqrt{ab}}-\frac{2\,a^{2}bc^{2}-2\,a^{3}cd-3\left(5\,b^{3}c^{2}-2\,ab^{2}cd-2\,a^{2}bd^{2}\right)x^{4}-2\left(5\,ab^{2}c^{2}-2\,a^{2}bcd-3\,a^{3}d^{2}\right)x^{2}}{6\left(\left(a^{3}b^{2}c^{3}-a^{4}bc^{2}d\right)x^{5}+\left(a^{4}bc^{3}-a^{5}c^{2}d\right)x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] d^4*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(c*d))
 + 1/2*(5*b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^3*b^2*c^2 - 2*a^4*b*
c*d + a^5*d^2)*sqrt(a*b)) - 1/6*(2*a^2*b*c^2 - 2*a^3*c*d - 3*(5*b^3*c^2 - 2
*a*b^2*c*d - 2*a^2*b*d^2)*x^4 - 2*(5*a*b^2*c^2 - 2*a^2*b*c*d - 3*a^3*d^2)*x
^2)/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^5 + (a^4*b*c^3 - a^5*c^2*d)*x^3)
```

Fricas [A]
time = 2.37, size = 1281, normalized size = 6.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*b^2*c^3 - 8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^3 - 7*a*b^
3*c^2*d + 2*a^3*b*d^3)*x^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2
```

```
 + 3*a^4*d^3)*x^2 + 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a
^2*b^2*c^2*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a
)) - 6*(a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/
c) - c)/(d*x^2 + c)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5
+ (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^3), -1/12*(4*a^2*b^2*c^3 -
8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x
^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2 - 12*(
a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*((5*b^4*c^3
- 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(-b/a)*log(
(b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*
d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^3),
-1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c^2*d + 2*a^4*c*d^2 - 3*(5*b^4*c^3 - 7*a*b^3*
c^2*d + 2*a^3*b*d^3)*x^4 - 2*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2 +
 3*a^4*d^3)*x^2 - 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2
*b^2*c^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*(a^3*b*d^3*x^5 + a^4*d^3
*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^3
*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d
+ a^6*c^2*d^2)*x^3), -1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c^2*d + 2*a^4*c*d^2 - 3*
(5*b^4*c^3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x^4 - 2*(5*a*b^3*c^3 - 7*a^2*b^2*
c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2 - 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 +
 (5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 6*(a^
3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^3*b^3*c^4 - 2
*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^
2*d^2)*x^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c),x)
[Out] Timed out
```

Giac [**A**]
time = 1.53, size = 165, normalized size = 0.87

$$\frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^4 - 2\,abc^3d + a^2c^2d^2)\sqrt{cd}} + \frac{b^3x}{2\,(a^3bc - a^4d)(bx^2 + a)} + \frac{(5\,b^4c - 7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^2c^2 - 2\,a^4bcd + a^5d^2)\sqrt{ab}} + \frac{6\,bcx^2 + 3\,adx^2 - ac}{3\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
[Out] d^4*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(c*d))
 + 1/2*b^3*x/((a^3*b*c - a^4*d)*(b*x^2 + a)) + 1/2*(5*b^4*c - 7*a*b^3*d)*ar
```

```
ctan(b*x/sqrt(a*b))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(a*b)) + 1/3
*(6*b*c*x^2 + 3*a*d*x^2 - a*c)/(a^3*c^2*x^3)
```

Mupad [B]
time = 0.64, size = 2500, normalized size = 13.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] ((x^2*(3*a*d + 5*b*c))/(3*a^2*c^2) - 1/(3*a*c) + (x^4*(2*a^2*b*d^2 - 5*b^3*
c^2 + 2*a*b^2*c*d))/(2*a^3*c^2*(a*d - b*c)))/(a*x^3 + b*x^5) - (atan(((((x*
(400*a^9*b^12*c^15*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^5
- 6112*a^12*b^9*c^12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d^8 +
 64*a^15*b^6*c^9*d^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 - 64*a
^18*b^3*c^6*d^12))/2 + ((-c^5*d^7)^(1/2)*((x*(-c^5*d^7)^(1/2)*(256*a^15*b^1
0*c^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a^18*b^
7*c^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536*a^22*b^
3*c^11*d^9 - 256*a^23*b^2*c^10*d^10))/(4*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6
*d)) - 160*a^12*b^11*c^17*d^2 + 1024*a^13*b^10*c^16*d^3 - 2720*a^14*b^9*c^1
5*d^4 + 3840*a^15*b^8*c^14*d^5 - 3104*a^16*b^7*c^13*d^6 + 1600*a^17*b^6*c^1
2*d^7 - 864*a^18*b^5*c^11*d^8 + 640*a^19*b^4*c^10*d^9 - 320*a^20*b^3*c^9*d^
10 + 64*a^21*b^2*c^8*d^11))/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)))*(-c^
5*d^7)^(1/2)*1i)/(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d) + (((x*(400*a^9*b^12
*c^15*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^5 - 6112*a^12*b
^9*c^12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d^8 + 64*a^15*b^6*
c^9*d^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 - 64*a^18*b^3*c^6*d
^12))/2 + ((-c^5*d^7)^(1/2)*((x*(-c^5*d^7)^(1/2)*(256*a^15*b^10*c^18*d^2 -
1536*a^16*b^9*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c^15*d^5 +
3584*a^20*b^5*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c^11*d^9 -
256*a^23*b^2*c^10*d^10))/(4*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)) + 160*a^
12*b^11*c^17*d^2 - 1024*a^13*b^10*c^16*d^3 + 2720*a^14*b^9*c^15*d^4 - 3840*
a^15*b^8*c^14*d^5 + 3104*a^16*b^7*c^13*d^6 - 1600*a^17*b^6*c^12*d^7 + 864*a
^18*b^5*c^11*d^8 - 640*a^19*b^4*c^10*d^9 + 320*a^20*b^3*c^9*d^10 - 64*a^21*
b^2*c^8*d^11))/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)))*(-c^5*d^7)^(1/2)*
1i)/(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d))/((((x*(400*a^9*b^12*c^15*d^3 - 2
320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^5 - 6112*a^12*b^9*c^12*d^6 +
 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d^8 + 64*a^15*b^6*c^9*d^9 - 192
*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 - 64*a^18*b^3*c^6*d^12))/2 + ((-
c^5*d^7)^(1/2)*((x*(-c^5*d^7)^(1/2)*(256*a^15*b^10*c^18*d^2 - 1536*a^16*b^9
*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c^15*d^5 + 3584*a^20*b^5
*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c^11*d^9 - 256*a^23*b^2*
c^10*d^10))/(4*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)) - 160*a^12*b^11*c^17*
d^2 + 1024*a^13*b^10*c^16*d^3 - 2720*a^14*b^9*c^15*d^4 + 3840*a^15*b^8*c^14
```

```
*d^5 - 3104*a^16*b^7*c^13*d^6 + 1600*a^17*b^6*c^12*d^7 - 864*a^18*b^5*c^11*
d^8 + 640*a^19*b^4*c^10*d^9 - 320*a^20*b^3*c^9*d^10 + 64*a^21*b^2*c^8*d^11)
)/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)))*(-c^5*d^7)^(1/2))/(b^2*c^7 + a
^2*c^5*d^2 - 2*a*b*c^6*d) - (((x*(400*a^9*b^12*c^15*d^3 - 2320*a^10*b^11*c^
14*d^4 + 5344*a^11*b^10*c^13*d^5 - 6112*a^12*b^9*c^12*d^6 + 3472*a^13*b^8*c
^11*d^7 - 784*a^14*b^7*c^10*d^8 + 64*a^15*b^6*c^9*d^9 - 192*a^16*b^5*c^8*d^
10 + 192*a^17*b^4*c^7*d^11 - 64*a^18*b^3*c^6*d^12))/2 + ((-c^5*d^7)^(1/2)*(
(x*(-c^5*d^7)^(1/2)*(256*a^15*b^10*c^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 3584
*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 3584
*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c^11*d^9 - 256*a^23*b^2*c^10*d^10))/(4*(
b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)) + 160*a^12*b^11*c^17*d^2 - 1024*a^13*
b^10*c^16*d^3 + 2720*a^14*b^9*c^15*d^4 - 3840*a^15*b^8*c^14*d^5 + 3104*a^16
*b^7*c^13*d^6 - 1600*a^17*b^6*c^12*d^7 + 864*a^18*b^5*c^11*d^8 - 640*a^19*b
^4*c^10*d^9 + 320*a^20*b^3*c^9*d^10 - 64*a^21*b^2*c^8*d^11))/(2*(b^2*c^7 +
a^2*c^5*d^2 - 2*a*b*c^6*d)))*(-c^5*d^7)^(1/2))/(b^2*c^7 + a^2*c^5*d^2 - 2*a
*b*c^6*d) - 400*a^9*b^10*c^11*d^6 + 1520*a^10*b^9*c^10*d^7 - 1904*a^11*b^8*
c^9*d^8 + 624*a^12*b^7*c^8*d^9 + 384*a^13*b^6*c^7*d^10 - 224*a^14*b^5*c^6*d
^11))*(-c^5*d^7)^(1/2)*1i)/(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d) + (atan(((
(7*a*d - 5*b*c)*(x*(400*a^9*b^12*c^15*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*
a^11*b^10*c^13*d^5 - 6112*a^12*b^9*c^12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*
a^14*b^7*c^10*d^8 + 64*a^15*b^6*c^9*d^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*
b^4*c^7*d^11 - 64*a^18*b^3*c^6*d^12) + ((7*a*d - 5*b*c)*(-a^7*b^5)^(1/2)*(2
048*a^13*b^10*c^16*d^3 - 320*a^12*b^11*c^17*d^2 - 5440*a^14*b^9*c^15*d^4 +
7680*a^15*b^8*c^14*d^5 - 6208*a^16*b^7*c^13*d^6 + 3200*a^17*b^6*c^12*d^7 -
1728*a^18*b^5*c^11*d^8 + 1280*a^19*b^4*c^10*d^9 - 640*a^20*b^3*c^9*d^10 + 1
28*a^21*b^2*c^8*d^11 + (x*(7*a*d - 5*b*c)*(-a^7*b^5)^(1/2)*(256*a^15*b^10*c
^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c
^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c
^11*d^9 - 256*a^23*b^2*c^10*d^10))/(4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d)
)))/(4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d)))*(-a^7*b^5)^(1/2)*1i)/(4*(a^9
*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d)) + ((7*a*d - 5*b*c)*(x*(400*a^9*b^12*c^15
*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^5 - 6112*a^12*b^9*c^
12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*...
```

3.298 $\int \frac{1}{x^5(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=160

$$-\frac{1}{4a^2cx^4}+\frac{2bc+ad}{2a^3c^2x^2}+\frac{b^3}{2a^3(bc-ad)\,(a+bx^2)}+\frac{(3b^2c^2+2abcd+a^2d^2)\log(x)}{a^4c^3}-\frac{b^3(3bc-4ad)\log\left(a+bx^2\right)}{2a^4(bc-ad)^2}-\frac{d}{2}$$

```
[Out] -1/4/a^2/c/x^4+1/2*(a*d+2*b*c)/a^3/c^2/x^2+1/2*b^3/a^3/(-a*d+b*c)/(b*x^2+a)
+(a^2*d^2+2*a*b*c*d+3*b^2*c^2)*ln(x)/a^4/c^3-1/2*b^3*(-4*a*d+3*b*c)*ln(b*x^
2+a)/a^4/(-a*d+b*c)^2-1/2*d^4*ln(d*x^2+c)/c^3/(-a*d+b*c)^2
```

Rubi [A]
time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{b^3(3bc-4ad)\log\left(a+bx^2\right)}{2a^4(bc-ad)^2}+\frac{b^3}{2a^3\left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{2a^3c^2x^2}-\frac{1}{4a^2cx^4}+\frac{\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}-\frac{d^4\log\left(c+dx^2\right)}{2c^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/4*1/(a^2*c*x^4) + (2*b*c + a*d)/(2*a^3*c^2*x^2) + b^3/(2*a^3*(b*c - a*d)
*(a + b*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x])/(a^4*c^3) - (b^3
*(3*b*c - 4*a*d)*Log[a + b*x^2])/(2*a^4*(b*c - a*d)^2) - (d^4*Log[c + d*x^2
])/(2*c^3*(b*c - a*d)^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^2)^2 (c+dx^2)} dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)^2(c+dx)} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2cx^3} + \frac{-2bc-ad}{a^3c^2x^2} + \frac{3b^2c^2+2abcd+a^2d^2}{a^4c^3x} + \frac{b^4}{a^3(-bc+ad)(\cdots}\right.\right.$$
$$= -\frac{1}{4a^2cx^4} + \frac{2bc+ad}{2a^3c^2x^2} + \frac{b^3}{2a^3(bc-ad)(a+bx^2)} + \frac{(3b^2c^2+2abcd+a^2d^2)\log\cdots}{a^4c^3}$$

Mathematica [A]
time = 0.14, size = 155, normalized size = 0.97

$$\frac{1}{4}\left(-\frac{1}{a^2cx^4} + \frac{4bc+2ad}{a^3c^2x^2} - \frac{2b^3}{a^3(-bc+ad)(a+bx^2)} + \frac{4(3b^2c^2+2abcd+a^2d^2)\log(x)}{a^4c^3} + \frac{2b^3(-3bc+4ad)\log(a+bx^2)}{a^4(bc-ad)^2} - \frac{2d^4\log(c+dx^2)}{c^3(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(1/(a^2*c*x^4)) + (4*b*c + 2*a*d)/(a^3*c^2*x^2) - (2*b^3)/(a^3*(-(b*c) +
a*d)*(a + b*x^2)) + (4*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x])/(a^4*c^3)
+ (2*b^3*(-3*b*c + 4*a*d)*Log[a + b*x^2])/(a^4*(b*c - a*d)^2) - (2*d^4*Log[
c + d*x^2])/(c^3*(b*c - a*d)^2))/4
```

Maple [A]
time = 0.16, size = 153, normalized size = 0.96

method	result
default	$\frac{b^4\left(\frac{(4ad-3bc)\ln\left(b\,x^2+a\right)}{b}-\frac{a(ad-bc)}{b\left(b\,x^2+a\right)}\right)}{2a^4(ad-bc)^2}-\frac{d^4\ln\left(d\,x^2+c\right)}{2c^3(ad-bc)^2}-\frac{1}{4a^2c\,x^4}-\frac{-ad-2bc}{2a^3c^2x^2}+\frac{\left(a^2d^2+2abcd+3b^2c^2\right)\ln(x)}{a^4c^3}$
norman	$\frac{-\frac{1}{4ac}+\frac{(2ad+3bc)x^2}{4a^2c^2}+\frac{\left(-a^2b\,d^2-a\,b^2cd+3b^3c^2\right)b\,x^6}{2c^2a^4(ad-bc)}}{x^4(b\,x^2+a)}+\frac{\left(a^2d^2+2abcd+3b^2c^2\right)\ln(x)}{a^4c^3}-\frac{d^4\ln\left(d\,x^2+c\right)}{2c^3\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{b^3(4ad-3bc)\ln\left(b\cdots\right.}{2a^4\left(a^2d^2-2abcd\cdots\right.}$
risch	$\frac{\frac{b\left(a^2d^2+abcd-3b^2c^2\right)x^4}{2a^3c^2(ad-bc)}+\frac{(2ad+3bc)x^2}{4a^2c^2}-\frac{1}{4ac}}{x^4(b\,x^2+a)}+\frac{\ln(x)d^2}{a^2c^3}+\frac{2\ln(x)bd}{a^3c^2}+\frac{3\ln(x)b^2}{a^4c}-\frac{d^4\ln\left(-d\,x^2-c\right)}{2c^3\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{2b^3\ln\left(b\,x^2+a\cdots\right.}{a^3\left(a^2d^2-2abcd+\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^4/a^4/(a*d-b*c)^2*((4*a*d-3*b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a
))-1/2*d^4/c^3/(a*d-b*c)^2*ln(d*x^2+c)-1/4/a^2/c/x^4-1/2*(-a*d-2*b*c)/a^3/c
^2/x^2+(a^2*d^2+2*a*b*c*d+3*b^2*c^2)*ln(x)/a^4/c^3
```

Maxima [A]
time = 0.29, size = 258, normalized size = 1.61

$$-\frac{d^4\log(dx^2+c)}{2(b^2c^5-2abc^4d+a^2c^3d^2)} - \frac{(3b^4c-4ab^3d)\log(bx^2+a)}{2(a^4b^2c^2-2a^5bcd+a^6d^2)} - \frac{a^2bc^2-a^3cd-2(3b^3c^2-ab^2cd-a^2bd^2)x^4-(3ab^2c^2-a^2bcd-2a^3d^2)x^2}{4((a^3b^2c^3-a^4bc^2d)x^6+(a^4bc^3-a^5c^2d)x^4)} + \frac{(3b^2c^2+2abcd+a^2d^2)\log(x^2)}{2a^4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*d^4*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1/2*(3*b^4*
c - 4*a*b^3*d)*log(b*x^2 + a)/(a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2) - 1/4*(
a^2*b*c^2 - a^3*c*d - 2*(3*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^4 - (3*a*b^2*
c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b
*c^3 - a^5*c^2*d)*x^4) + 1/2*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(x^2)/(a^
4*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(150) = 300$.
time = 6.81, size = 356, normalized size = 2.22

$$-\frac{a^3b^2c^4 - 2\,a^4bc^3d + a^5c^2d^2 - 2\,(3\,ab^4c^4 - 4\,a^2b^3c^3d + a^4bcd^3)x^4 - (3\,a^2b^3c^4 - 4\,a^3b^2c^3d - a^4bc^2d^2 + 2\,a^5cd^3)x^2 + 2\,((3\,b^5c^4 - 4\,ab^4c^3d)x^6 + (3\,ab^4c^4 - 4\,a^2b^3c^3d)x^4)\log\left(bx^2 + a\right) + 2\,(a^4bd^4x^6 + a^5d^4x^4)\log\left(dx^2 + c\right) - 4\,((3\,b^5c^4 - 4\,ab^4c^3d + a^4bd^4)x^6 + (3\,ab^4c^4 - 4\,a^2b^3c^3d + a^5d^4)x^4)\log\left(x\right)}{4\,((a^4b^3c^5 - 2\,a^5b^2c^4d + a^6bc^3d^2)x^6 + (a^5b^2c^5 - 2\,a^6bc^4d + a^7c^3d^2)x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 - 2*(3*a*b^4*c^4 - 4*a^2*b^
3*c^3*d + a^4*b*c*d^3)*x^4 - (3*a^2*b^3*c^4 - 4*a^3*b^2*c^3*d - a^4*b*c^2*d
^2 + 2*a^5*c*d^3)*x^2 + 2*((3*b^5*c^4 - 4*a*b^4*c^3*d)*x^6 + (3*a*b^4*c^4 -
 4*a^2*b^3*c^3*d)*x^4)*log(b*x^2 + a) + 2*(a^4*b*d^4*x^6 + a^5*d^4*x^4)*log
(d*x^2 + c) - 4*((3*b^5*c^4 - 4*a*b^4*c^3*d + a^4*b*d^4)*x^6 + (3*a*b^4*c^4
 - 4*a^2*b^3*c^3*d + a^5*d^4)*x^4)*log(x))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d
+ a^6*b*c^3*d^2)*x^6 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.55, size = 281, normalized size = 1.76

$$-\frac{d^5\log\left(|dx^2 + c|\right)}{2\,(b^2c^5d - 2\,abc^4d^2 + a^2c^3d^3)} - \frac{(3\,b^5c - 4\,ab^4d)\log\left(|bx^2 + a|\right)}{2\,(a^4b^3c^2 - 2\,a^5b^2cd + a^6bd^2)} + \frac{3\,b^5cx^2 - 4\,ab^4dx^2 + 4\,ab^4c - 5\,a^2b^3d}{2\,(a^4b^2c^2 - 2\,a^5bcd + a^6d^2)(bx^2 + a)} + \frac{(3\,b^2c^2 + 2\,abcd + a^2d^2)\log\left(x^2\right)}{2\,a^4c^3} - \frac{9\,b^2c^2x^4 + 6\,abcdx^4 + 3\,a^2d^2x^4 - 4\,abc^2x^2 - 2\,a^2cdx^2 + a^2c^2}{4\,a^4c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*d^5*log(abs(d*x^2 + c))/(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3) - 1/
2*(3*b^5*c - 4*a*b^4*d)*log(abs(b*x^2 + a))/(a^4*b^3*c^2 - 2*a^5*b^2*c*d +
a^6*b*d^2) + 1/2*(3*b^5*c*x^2 - 4*a*b^4*d*x^2 + 4*a*b^4*c - 5*a^2*b^3*d)/((
a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*(b*x^2 + a)) + 1/2*(3*b^2*c^2 + 2*a*b*
c*d + a^2*d^2)*log(x^2)/(a^4*c^3) - 1/4*(9*b^2*c^2*x^4 + 6*a*b*c*d*x^4 + 3*
a^2*d^2*x^4 - 4*a*b*c^2*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(a^4*c^3*x^4)
```

Mupad [B]
time = 0.59, size = 217, normalized size = 1.36

$$\frac{\frac{x^2\,\left(2\,a\,d+3\,b\,c\right)}{4\,a^2\,c^2}-\frac{1}{4\,a\,c}+\frac{x^4\,\left(a^2\,b\,d^2+a\,b^2\,c\,d-3\,b^3\,c^2\right)}{2\,a^3\,c^2\,\left(a\,d-b\,c\right)}}{b\,x^6+a\,x^4}-\frac{\ln\left(b\,x^2+a\right)\,\left(3\,b^4\,c-4\,a\,b^3\,d\right)}{2\,a^6\,d^2-4\,a^5\,b\,c\,d+2\,a^4\,b^2\,c^2}-\frac{d^4\,\ln\left(d\,x^2+c\right)}{2\,\left(a^2\,c^3\,d^2-2\,a\,b\,c^4\,d+b^2\,c^5\right)}+\frac{\ln\left(x\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{a^4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] ((x^2*(2*a*d + 3*b*c))/(4*a^2*c^2) - 1/(4*a*c) + (x^4*(a^2*b*d^2 - 3*b^3*c^
2 + a*b^2*c*d))/(2*a^3*c^2*(a*d - b*c)))/(a*x^4 + b*x^6) - (log(a + b*x^2)*
(3*b^4*c - 4*a*b^3*d))/(2*a^6*d^2 + 2*a^4*b^2*c^2 - 4*a^5*b*c*d) - (d^4*log
(c + d*x^2))/(2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) + (log(x)*(a^2*d^2 +
 3*b^2*c^2 + 2*a*b*c*d))/(a^4*c^3)
```

3.299 $\int \frac{1}{x^6(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=250

$$-\frac{7bc-2ad}{10a^2c(bc-ad)x^5}+\frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3}-\frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2-2a^3d^3}{2a^4c^3(bc-ad)x}+\frac{b}{2a(bc-ad)x^5(a+bx^2)}-$$

```
[Out] 1/10*(2*a*d-7*b*c)/a^2/c/(-a*d+b*c)/x^5+1/6*(-2*a^2*d^2-2*a*b*c*d+7*b^2*c^2
)/a^3/c^2/(-a*d+b*c)/x^3+1/2*(2*a^3*d^3+2*a^2*b*c*d^2+2*a*b^2*c^2*d-7*b^3*c
^3)/a^4/c^3/(-a*d+b*c)/x+1/2*b/a/(-a*d+b*c)/x^5/(b*x^2+a)-1/2*b^(7/2)*(-9*a
*d+7*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/(-a*d+b*c)^2-d^(9/2)*arctan(x*d
^(1/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.28, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 597, 536, 211}

$$-\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(7bc-9ad)}{2a^{9/2}(bc-ad)^2}-\frac{7bc-2ad}{10a^2cx^5(bc-ad)}+\frac{-2a^2d^2-2abcd+7b^2c^2}{6a^3c^2x^3(bc-ad)}-\frac{-2a^3d^3-2a^2bcd^2-2ab^2c^2d+7b^3c^3}{2a^4c^3x(bc-ad)}-\frac{d^{9/2}\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^2}+\frac{b}{2ax^5(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/10*(7*b*c - 2*a*d)/(a^2*c*(b*c - a*d)*x^5) + (7*b^2*c^2 - 2*a*b*c*d - 2*
a^2*d^2)/(6*a^3*c^2*(b*c - a*d)*x^3) - (7*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*b
*c*d^2 - 2*a^3*d^3)/(2*a^4*c^3*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^5*(a +
 b*x^2)) - (b^(7/2)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)
*(b*c - a*d)^2) - (d^(9/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d
)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{b}{2a(bc-ad)x^5\left(a+bx^2\right)} - \frac{\int \frac{-7bc+2ad-7bdx^2}{x^6(a+bx^2)(c+dx^2)}\,dx}{2a(bc-ad)}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{b}{2a(bc-ad)x^5\left(a+bx^2\right)} + \frac{\int \frac{-5\left(7b^2c^2-2abcd-2a^2d^2\right)-5bd(}{x^4(a+bx^2)(c+dx^2)}}{10a^2c(bc-ad}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} + \frac{b}{2a(bc-ad)x^5\left(a+bx^2\right)} -$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2}{2a^4c^3(bc-ad)x}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2}{2a^4c^3(bc-ad)x}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2}{2a^4c^3(bc-ad)x}$$

Mathematica [A]
time = 0.22, size = 179, normalized size = 0.72

$$-\frac{1}{5a^2cx^5} + \frac{2bc+ad}{3a^3c^2x^3} + \frac{-3b^2c^2-2abcd-a^2d^2}{a^4c^3x} + \frac{b^4x}{2a^4(-bc+ad)\left(a+bx^2\right)} + \frac{b^{7/2}(-7bc+9ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}(-bc+ad)^2} - \frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/5*1/(a^2*c*x^5) + (2*b*c + a*d)/(3*a^3*c^2*x^3) + (-3*b^2*c^2 - 2*a*b*c*
d - a^2*d^2)/(a^4*c^3*x) + (b^4*x)/(2*a^4*(-(b*c) + a*d)*(a + b*x^2)) + (b^
(7/2)*(-7*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*(-(b*c) + a*
d)^2) - (d^(9/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^2)
```

Maple [A]

time = 0.16, size = 161, normalized size = 0.64

method	result
default	$\frac{b^4\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(9ad-7bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4(ad-bc)^2}-\frac{d^5\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^3(ad-bc)^2\sqrt{cd}}-\frac{1}{5a^2c\,x^5}-\frac{-ad-2bc}{3a^3c^2x^3}-\frac{a^2d^2+2abcd+3b^2c^2}{a^4c^3x}$
risch	$\frac{-\frac{b\left(2a^3d^3+2a^2bc\,d^2+2a\,b^2c^2d-7b^3c^3\right)x^6}{2a^4c^3(ad-bc)}-\frac{\left(3a^2d^2+5abcd+7b^2c^2\right)x^4}{3a^3c^3}+\frac{(5ad+7bc)x^2}{15a^2c^2}-\frac{1}{5ac}}{x^5(b\,x^2+a)}+\Big(\_R=\mathrm{RootOf}\Big(\Big(d^4a^{13}-4c\,d^3a^{12}b+6c^2d^2a^{11}b^2-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] b^4/a^4/(a*d-b*c)^2*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(9*a*d-7*b*c)/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2)))-1/c^3*d^5/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x
/(c*d)^(1/2))-1/5/a^2/c/x^5-1/3*(-a*d-2*b*c)/a^3/c^2/x^3-(a^2*d^2+2*a*b*c*d
+3*b^2*c^2)/a^4/c^3/x
```

Maxima [A]

time = 0.53, size = 303, normalized size = 1.21

$$-\frac{d^5\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^5-2\,abc^4d+a^2c^3d^2)\sqrt{cd}}-\frac{(7\,b^5c-9\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^4b^2c^2-2\,a^5bcd+a^6d^2)\sqrt{ab}}-\frac{6\,a^3bc^3-6\,a^4c^2d+15\,(7\,b^4c^3-2\,ab^3c^2d-2\,a^2b^2cd^2-2\,a^3bd^3)x^6+10\,(7\,ab^3c^3-2\,a^2b^2c^2d-2\,a^3bcd^2-3\,a^4d^3)x^4-2\,(7\,a^2b^2c^3-2\,a^3bc^2d-5\,a^4cd^2)x^2}{30\,((a^4b^2c^4-a^5bc^3d)x^7+(a^5bc^4-a^6c^3d)x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -d^5*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d)
) - 1/2*(7*b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^4*b^2*c^2 - 2*a^5*b
*c*d + a^6*d^2)*sqrt(a*b)) - 1/30*(6*a^3*b*c^3 - 6*a^4*c^2*d + 15*(7*b^4*c^
3 - 2*a*b^3*c^2*d - 2*a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^6 + 10*(7*a*b^3*c^3 -
2*a^2*b^2*c^2*d - 2*a^3*b*c*d^2 - 3*a^4*d^3)*x^4 - 2*(7*a^2*b^2*c^3 - 2*a^3
*b*c^2*d - 5*a^4*c*d^2)*x^2)/((a^4*b^2*c^4 - a^5*b*c^3*d)*x^7 + (a^5*b*c^4
- a^6*c^3*d)*x^5)
```

Fricas [A]

time = 4.56, size = 1489, normalized size = 5.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/60*(12*a^3*b^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*b^5*c^4 -
9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*
b*c*d^3 + 3*a^5*d^4)*x^4 - 4*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2
*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^
4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)) - 30*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(-d/c)*log((d*x^2 - 2*c*x*
sqrt(-d/c) - c)/(d*x^2 + c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d
^2)*x^7 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5), -1/60*(12*a^3*b
^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*b^5*c^4 - 9*a*b^4*c^3*d +
2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*
d^4)*x^4 - 4*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d
^3)*x^2 + 60*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(d/c)*arctan(x*sqrt(d/c)) +
15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^4*b^3*c^5 -
 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*
c^3*d^2)*x^5), -1/30*(6*a^3*b^2*c^4 - 12*a^4*b*c^3*d + 6*a^5*c^2*d^2 + 15*(
7*b^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 10*(7*a*b^4*c^4 - 9*a^2*b^3*
c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 2*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d -
 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 +
 (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 15*(a
^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(
d*x^2 + c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^
2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5), -1/30*(6*a^3*b^2*c^4 - 12*a^4*b*
c^3*d + 6*a^5*c^2*d^2 + 15*(7*b^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 +
10*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 2*(7*a^2
*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^
5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 30*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(d/c)*arctan(x*
sqrt(d/c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2
*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 1.00, size = 207, normalized size = 0.83

$$-\frac{d^5\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^5-2\,abc^4d+a^2c^3d^2)\sqrt{cd}}-\frac{b^4x}{2\,(a^4bc-a^5d)(bx^2+a)}-\frac{(7\,b^5c-9\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^4b^2c^2-2\,a^5bcd+a^6d^2)\sqrt{ab}}-\frac{45\,b^2c^2x^4+30\,abcdx^4+15\,a^2d^2x^4-10\,abc^2x^2-5\,a^2cdx^2+3\,a^2c^2}{15\,a^4c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -d^5*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d)
) - 1/2*b^4*x/((a^4*b*c - a^5*d)*(b*x^2 + a)) - 1/2*(7*b^5*c - 9*a*b^4*d)*a
rctan(b*x/sqrt(a*b))/((a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*sqrt(a*b)) - 1/
15*(45*b^2*c^2*x^4 + 30*a*b*c*d*x^4 + 15*a^2*d^2*x^4 - 10*a*b*c^2*x^2 - 5*a
^2*c*d*x^2 + 3*a^2*c^2)/(a^4*c^3*x^5)
```

Mupad [B]
time = 0.66, size = 2737, normalized size = 10.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (atan((a^9*d*x*(-c^7*d^9)^(3/2)*4i + b^9*c^16*d*x*(-c^7*d^9)^(1/2)*49i + a^
2*b^7*c^14*d^3*x*(-c^7*d^9)^(1/2)*81i - a*b^8*c^15*d^2*x*(-c^7*d^9)^(1/2)*1
26i)/(4*a^9*c^11*d^14 - 49*b^9*c^20*d^5 + 126*a*b^8*c^19*d^6 - 81*a^2*b^7*c
^18*d^7))*(-c^7*d^9)^(1/2)*1i)/(b^2*c^9 + a^2*c^7*d^2 - 2*a*b*c^8*d) - (1/(
5*a*c) - (x^2*(5*a*d + 7*b*c))/(15*a^2*c^2) + (x^4*(3*a^2*d^2 + 7*b^2*c^2 +
 5*a*b*c*d))/(3*a^3*c^3) + (x^6*(2*a^3*b*d^3 - 7*b^4*c^3 + 2*a^2*b^2*c*d^2
+ 2*a*b^3*c^2*d))/(2*a^4*c^3*(a*d - b*c)))/(a*x^5 + b*x^7) - (atan((((x*(78
4*a^12*b^14*c^20*d^3 - 4368*a^13*b^13*c^19*d^4 + 9696*a^14*b^12*c^18*d^5 -
10720*a^15*b^11*c^17*d^6 + 5904*a^16*b^10*c^16*d^7 - 1296*a^17*b^9*c^15*d^8
 + 64*a^20*b^6*c^12*d^11 - 192*a^21*b^5*c^11*d^12 + 192*a^22*b^4*c^10*d^13
- 64*a^23*b^3*c^9*d^14) + ((9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*(2816*a^17*b^11
*c^21*d^3 - 448*a^16*b^12*c^22*d^2 - 7360*a^18*b^10*c^20*d^4 + 10240*a^19*b
^9*c^19*d^5 - 8000*a^20*b^8*c^18*d^6 + 3200*a^21*b^7*c^17*d^7 + 64*a^22*b^6
*c^16*d^8 - 1280*a^23*b^5*c^15*d^9 + 1280*a^24*b^4*c^14*d^10 - 640*a^25*b^3
*c^13*d^11 + 128*a^26*b^2*c^12*d^12 + (x*(9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*(
256*a^20*b^10*c^23*d^2 - 1536*a^21*b^9*c^22*d^3 + 3584*a^22*b^8*c^21*d^4 -
3584*a^23*b^7*c^20*d^5 + 3584*a^25*b^5*c^18*d^7 - 3584*a^26*b^4*c^17*d^8 +
1536*a^27*b^3*c^16*d^9 - 256*a^28*b^2*c^15*d^10))/(4*(a^11*d^2 + a^9*b^2*c^
2 - 2*a^10*b*c*d))))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d)))*(9*a*d -
7*b*c)*(-a^9*b^7)^(1/2)*1i)/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d)) + (
(x*(784*a^12*b^14*c^20*d^3 - 4368*a^13*b^13*c^19*d^4 + 9696*a^14*b^12*c^18*
d^5 - 10720*a^15*b^11*c^17*d^6 + 5904*a^16*b^10*c^16*d^7 - 1296*a^17*b^9*c^
15*d^8 + 64*a^20*b^6*c^12*d^11 - 192*a^21*b^5*c^11*d^12 + 192*a^22*b^4*c^10
```

*d^13 - 64*a^23*b^3*c^9*d^14) + ((9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*(448*a^16
*b^12*c^22*d^2 - 2816*a^17*b^11*c^21*d^3 + 7360*a^18*b^10*c^20*d^4 - 10240*
a^19*b^9*c^19*d^5 + 8000*a^20*b^8*c^18*d^6 - 3200*a^21*b^7*c^17*d^7 - 64*a^
22*b^6*c^16*d^8 + 1280*a^23*b^5*c^15*d^9 - 1280*a^24*b^4*c^14*d^10 + 640*a^
25*b^3*c^13*d^11 - 128*a^26*b^2*c^12*d^12 + (x*(9*a*d - 7*b*c)*(-a^9*b^7)^(
1/2)*(256*a^20*b^10*c^23*d^2 - 1536*a^21*b^9*c^22*d^3 + 3584*a^22*b^8*c^21*
d^4 - 3584*a^23*b^7*c^20*d^5 + 3584*a^25*b^5*c^18*d^7 - 3584*a^26*b^4*c^17*
d^8 + 1536*a^27*b^3*c^16*d^9 - 256*a^28*b^2*c^15*d^10))/(4*(a^11*d^2 + a^9*
b^2*c^2 - 2*a^10*b*c*d))))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d)))*(9*
a*d - 7*b*c)*(-a^9*b^7)^(1/2)*1i)/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d
)))/(((x*(784*a^12*b^14*c^20*d^3 - 4368*a^13*b^13*c^19*d^4 + 9696*a^14*b^12
*c^18*d^5 - 10720*a^15*b^11*c^17*d^6 + 5904*a^16*b^10*c^16*d^7 - 1296*a^17*
b^9*c^15*d^8 + 64*a^20*b^6*c^12*d^11 - 192*a^21*b^5*c^11*d^12 + 192*a^22*b^
4*c^10*d^13 - 64*a^23*b^3*c^9*d^14) + ((9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*(44
8*a^16*b^12*c^22*d^2 - 2816*a^17*b^11*c^21*d^3 + 7360*a^18*b^10*c^20*d^4 -
10240*a^19*b^9*c^19*d^5 + 8000*a^20*b^8*c^18*d^6 - 3200*a^21*b^7*c^17*d^7 -
64*a^22*b^6*c^16*d^8 + 1280*a^23*b^5*c^15*d^9 - 1280*a^24*b^4*c^14*d^10 +
640*a^25*b^3*c^13*d^11 - 128*a^26*b^2*c^12*d^12 + (x*(9*a*d - 7*b*c)*(-a^9*
b^7)^(1/2)*(256*a^20*b^10*c^23*d^2 - 1536*a^21*b^9*c^22*d^3 + 3584*a^22*b^8
*c^21*d^4 - 3584*a^23*b^7*c^20*d^5 + 3584*a^25*b^5*c^18*d^7 - 3584*a^26*b^4
*c^17*d^8 + 1536*a^27*b^3*c^16*d^9 - 256*a^28*b^2*c^15*d^10))/(4*(a^11*d^2
+ a^9*b^2*c^2 - 2*a^10*b*c*d))))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d)
))*(9*a*d - 7*b*c)*(-a^9*b^7)^(1/2))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*
c*d)) - ((x*(784*a^12*b^14*c^20*d^3 - 4368*a^13*b^13*c^19*d^4 + 9696*a^14*b
^12*c^18*d^5 - 10720*a^15*b^11*c^17*d^6 + 5904*a^16*b^10*c^16*d^7 - 1296*a^
17*b^9*c^15*d^8 + 64*a^20*b^6*c^12*d^11 - 192*a^21*b^5*c^11*d^12 + 192*a^22
*b^4*c^10*d^13 - 64*a^23*b^3*c^9*d^14) + ((9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*
(2816*a^17*b^11*c^21*d^3 - 448*a^16*b^12*c^22*d^2 - 7360*a^18*b^10*c^20*d^4
+ 10240*a^19*b^9*c^19*d^5 - 8000*a^20*b^8*c^18*d^6 + 3200*a^21*b^7*c^17*d^
7 + 64*a^22*b^6*c^16*d^8 - 1280*a^23*b^5*c^15*d^9 + 1280*a^24*b^4*c^14*d^10
- 640*a^25*b^3*c^13*d^11 + 128*a^26*b^2*c^12*d^12 + (x*(9*a*d - 7*b*c)*(-a
^9*b^7)^(1/2)*(256*a^20*b^10*c^23*d^2 - 1536*a^21*b^9*c^22*d^3 + 3584*a^22*
b^8*c^21*d^4 - 3584*a^23*b^7*c^20*d^5 + 3584*a^25*b^5*c^18*d^7 - 3584*a^26*
b^4*c^17*d^8 + 1536*a^27*b^3*c^16*d^9 - 256*a^28*b^2*c^15*d^10))/(4*(a^11*d
^2 + a^9*b^2*c^2 - 2*a^10*b*c*d))))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c
d)))(9*a*d - 7*b*c)*(-a^9*b^7)^(1/2))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10
*b*c*d)) + 784*a^12*b^12*c^15*d^7 - 2800*a^13*b^11*c^14*d^8 + 3312*a^14*b^1
0*c^13*d^9 - 1296*a^15*b^9*c^12*d^10 + 224*a^16*b^8*c^11*d^11 - 512*a^17*b^
7*c^10*d^12 + 288*a^18*b^6*c^9*d^13))*(9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*1i)/
(2*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d))

3.300 $\int \frac{1}{x^7\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=210

$$-\frac{1}{6a^2cx^6}+\frac{2bc+ad}{4a^3c^2x^4}-\frac{3b^2c^2+2abcd+a^2d^2}{2a^4c^3x^2}-\frac{b^4}{2a^4(bc-ad)\left(a+bx^2\right)}-\frac{\left(4b^3c^3+3ab^2c^2d+2a^2bcd^2+a^3d^3\right)\log}{a^5c^4}$$

```
[Out] -1/6/a^2/c/x^6+1/4*(a*d+2*b*c)/a^3/c^2/x^4+1/2*(-a^2*d^2-2*a*b*c*d-3*b^2*c^
2)/a^4/c^3/x^2-1/2*b^4/a^4/(-a*d+b*c)/(b*x^2+a)-(a^3*d^3+2*a^2*b*c*d^2+3*a*
b^2*c^2*d+4*b^3*c^3)*ln(x)/a^5/c^4+1/2*b^4*(-5*a*d+4*b*c)*ln(b*x^2+a)/a^5/(
-a*d+b*c)^2+1/2*d^5*ln(d*x^2+c)/c^4/(-a*d+b*c)^2
```

Rubi [A]
time = 0.17, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{b^4(4bc-5ad)\log\left(a+bx^2\right)}{2a^5(bc-ad)^2}-\frac{b^4}{2a^4\left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{4a^3c^2x^4}-\frac{1}{6a^2cx^6}-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}-\frac{\log(x)\left(a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3\right)}{a^5c^4}+\frac{d^5\log\left(c+dx^2\right)}{2c^4(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/6*1/(a^2*c*x^6) + (2*b*c + a*d)/(4*a^3*c^2*x^4) - (3*b^2*c^2 + 2*a*b*c*d
 + a^2*d^2)/(2*a^4*c^3*x^2) - b^4/(2*a^4*(b*c - a*d)*(a + b*x^2)) - ((4*b^3
*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Log[x])/(a^5*c^4) + (b^4*(4
*b*c - 5*a*d)*Log[a + b*x^2])/(2*a^5*(b*c - a*d)^2) + (d^5*Log[c + d*x^2])/
(2*c^4*(b*c - a*d)^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7 (a+bx^2)^2 (c+dx^2)} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)^2(c+dx)} \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2cx^4} + \frac{-2bc-ad}{a^3c^2x^3} + \frac{3b^2c^2+2abcd+a^2d^2}{a^4c^3x^2} + \frac{-4b^3c^3-3ab^2\right.\right.$$
$$= -\frac{1}{6a^2cx^6} + \frac{2bc+ad}{4a^3c^2x^4} - \frac{3b^2c^2+2abcd+a^2d^2}{2a^4c^3x^2} - \frac{b^4}{2a^4(bc-ad)(a+bx^2)} - \frac{(\cdot}{}$$

Mathematica [A]
time = 0.20, size = 202, normalized size = 0.96

$$\frac{1}{12}\left(-\frac{2}{a^2cx^6} + \frac{6bc+3ad}{a^3c^2x^4} - \frac{6(3b^2c^2+2abcd+a^2d^2)}{a^4c^3x^2} + \frac{6b^4}{a^4(-bc+ad)(a+bx^2)} - \frac{12(4b^3c^3+3ab^2c^2d+2a^2bcd^2+a^3d^3)\log(x)}{a^5c^4} + \frac{6b^4(4bc-5ad)\log(a+bx^2)}{a^5(bc-ad)^2} + \frac{6d^5\log(c+dx^2)}{c^4(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-2/(a^2*c*x^6) + (6*b*c + 3*a*d)/(a^3*c^2*x^4) - (6*(3*b^2*c^2 + 2*a*b*c*d
 + a^2*d^2))/(a^4*c^3*x^2) + (6*b^4)/(a^4*(-(b*c) + a*d)*(a + b*x^2)) - (12
*(4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Log[x])/(a^5*c^4) +
(6*b^4*(4*b*c - 5*a*d)*Log[a + b*x^2])/(a^5*(b*c - a*d)^2) + (6*d^5*Log[c +
 d*x^2])/(c^4*(b*c - a*d)^2))/12
```

Maple [A]
time = 0.16, size = 201, normalized size = 0.96

method	result
default	$-\frac{b^5\left(\frac{(5ad-4bc)\ln(bx^2+a)}{b}-\frac{a(ad-bc)}{b(bx^2+a)}\right)}{2a^5(ad-bc)^2}+\frac{d^5\ln(dx^2+c)}{2c^4(ad-bc)^2}-\frac{1}{6a^2cx^6}-\frac{-ad-2bc}{4a^3c^2x^4}-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}+\frac{(-a^3d^3-2a^2bcd^2}{}$
norman	$\frac{-\frac{1}{6ac}+\frac{(3ad+4bc)x^2}{12a^2c^2}-\frac{(2a^2d^2+3abcd+4b^2c^2)x^4}{4a^3c^3}+\frac{(a^3bd^3+a^2b^2cd^2+ab^3c^2d-4b^4c^3)bx^8}{2c^3(ad-bc)a^5}}{x^6(bx^2+a)}+\frac{d^5\ln(dx^2+c)}{2c^4(a^2d^2-2abcd+b^2c^2)}-\frac{(a^3d^3+2a^2bcd}{}$
risch	$\frac{-\frac{b(a^3d^3+a^2bcd^2+ab^2c^2d-4b^3c^3)x^6}{2a^4c^3(ad-bc)}-\frac{(2a^2d^2+3abcd+4b^2c^2)x^4}{4a^3c^3}+\frac{(3ad+4bc)x^2}{12a^2c^2}-\frac{1}{6ac}}{x^6(bx^2+a)}-\frac{\ln(x)d^3}{a^2c^4}-\frac{2\ln(x)bd^2}{a^3c^3}-\frac{3\ln(x)b^2d}{a^4c^2}-\frac{4\ln}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^5/a^5/(a*d-b*c)^2*((5*a*d-4*b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+
a))+1/2*d^5/c^4/(a*d-b*c)^2*ln(d*x^2+c)-1/6/a^2/c/x^6-1/4*(-a*d-2*b*c)/a^3/
c^2/x^4-1/2*(a^2*d^2+2*a*b*c*d+3*b^2*c^2)/a^4/c^3/x^2+1/a^5/c^4*(-a^3*d^3-2
*a^2*b*c*d^2-3*a*b^2*c^2*d-4*b^3*c^3)*ln(x)
```

Maxima [A]
time = 0.31, size = 339, normalized size = 1.61

$$\frac{d^5\log\left(dx^2+c\right)}{2\left(b^2c^6-2\,abc^5d+a^2c^4d^2\right)}+\frac{\left(4\,b^5c-5\,ab^4d\right)\log\left(bx^2+a\right)}{2\left(a^5b^2c^2-2\,a^6bcd+a^7d^2\right)}-\frac{2\,a^3bc^3-2\,a^4c^2d+6\left(4\,b^4c^3-ab^3c^2d-a^2b^2cd^2-a^3bd^3\right)x^6+3\left(4\,ab^3c^3-a^2b^2c^2d-a^3bcd^2-2\,a^4d^3\right)x^4-\left(4\,a^2b^2c^3-a^3bc^2d-3\,a^4cd^2\right)x^2}{12\left(\left(a^4b^2c^4-a^5bc^3d\right)x^8+\left(a^5bc^4-a^6c^3d\right)x^6\right)}-\frac{\left(4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3\right)\log\left(x^2\right)}{2\,a^5c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*d^5*log(d*x^2 + c)/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2) + 1/2*(4*b^5*c
 - 5*a*b^4*d)*log(b*x^2 + a)/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2) - 1/12*(
2*a^3*b*c^3 - 2*a^4*c^2*d + 6*(4*b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 - a^
3*b*d^3)*x^6 + 3*(4*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 - 2*a^4*d^3)*x^
4 - (4*a^2*b^2*c^3 - a^3*b*c^2*d - 3*a^4*c*d^2)*x^2)/((a^4*b^2*c^4 - a^5*b*
c^3*d)*x^8 + (a^5*b*c^4 - a^6*c^3*d)*x^6) - 1/2*(4*b^3*c^3 + 3*a*b^2*c^2*d
+ 2*a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^5*c^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(198) = 396$.
time = 8.81, size = 410, normalized size = 1.95

$$-\frac{2\,a^4b^2c^5-4\,a^5bc^4d+2\,a^6c^3d^2+6\left(4\,ab^5c^5-5\,a^2b^4c^4d+a^5bcd^4\right)x^6+3\left(4\,a^2b^4c^5-5\,a^3b^3c^4d-a^5bc^2d^3+2\,a^6cd^4\right)x^4-\left(4\,a^3b^3c^5-5\,a^4b^2c^4d-2\,a^5bc^3d^2+3\,a^6c^2d^3\right)x^2-6\left(\left(4\,b^6c^5-5\,ab^5c^4d\right)x^8+\left(4\,ab^5c^5-5\,a^2b^4c^4d\right)x^6\right)\log\left(bx^2+a\right)-6\left(a^5bd^5x^8+a^6d^5x^6\right)\log\left(dx^2+c\right)+12\left(\left(4\,b^6c^5-5\,ab^5c^4d+a^5bd^5\right)x^8+\left(4\,ab^5c^5-5\,a^2b^4c^4d+a^6d^5\right)x^6\right)\log\left(x\right)}{12\left(\left(a^5b^3c^6-2\,a^6b^2c^5d+a^7bc^4d^2\right)x^8+\left(a^6b^2c^6-2\,a^7bc^5d+a^8c^4d^2\right)x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/12*(2*a^4*b^2*c^5 - 4*a^5*b*c^4*d + 2*a^6*c^3*d^2 + 6*(4*a*b^5*c^5 - 5*a
^2*b^4*c^4*d + a^5*b*c*d^4)*x^6 + 3*(4*a^2*b^4*c^5 - 5*a^3*b^3*c^4*d - a^5*
b*c^2*d^3 + 2*a^6*c*d^4)*x^4 - (4*a^3*b^3*c^5 - 5*a^4*b^2*c^4*d - 2*a^5*b*c
^3*d^2 + 3*a^6*c^2*d^3)*x^2 - 6*((4*b^6*c^5 - 5*a*b^5*c^4*d)*x^8 + (4*a*b^5
*c^5 - 5*a^2*b^4*c^4*d)*x^6)*log(b*x^2 + a) - 6*(a^5*b*d^5*x^8 + a^6*d^5*x^
6)*log(d*x^2 + c) + 12*((4*b^6*c^5 - 5*a*b^5*c^4*d + a^5*b*d^5)*x^8 + (4*a*
b^5*c^5 - 5*a^2*b^4*c^4*d + a^6*d^5)*x^6)*log(x))/((a^5*b^3*c^6 - 2*a^6*b^2
*c^5*d + a^7*b*c^4*d^2)*x^8 + (a^6*b^2*c^6 - 2*a^7*b*c^5*d + a^8*c^4*d^2)*x
^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.88, size = 354, normalized size = 1.69

$$\frac{d^6\log\left(|dx^2+c|\right)}{2\left(b^2c^6d-2\,abc^5d^2+a^2c^4d^3\right)}+\frac{\left(4\,b^6c-5\,ab^5d\right)\log\left(|bx^2+a|\right)}{2\left(a^5b^3c^2-2\,a^6b^2cd+a^7bd^2\right)}-\frac{4\,b^6cx^2-5\,ab^5dx^2+5\,ab^5c-6\,a^2b^4d}{2\left(a^5b^2c^2-2\,a^6bcd+a^7d^2\right)\left(bx^2+a\right)}-\frac{\left(4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3\right)\log\left(x^2\right)}{2\,a^5c^4}+\frac{44\,b^3c^3x^6+33\,ab^2c^2dx^6+22\,a^2bcd^2x^6+11\,a^3d^3x^6-18\,ab^2c^3x^4-12\,a^2bc^2dx^4-6\,a^3cd^2x^4+6\,a^2bc^3x^2+3\,a^3c^2dx^2-2\,a^3c^3}{12\,a^5c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*d^6*log(abs(d*x^2 + c))/(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3) + 1/2
*(4*b^6*c - 5*a*b^5*d)*log(abs(b*x^2 + a))/(a^5*b^3*c^2 - 2*a^6*b^2*c*d + a
^7*b*d^2) - 1/2*(4*b^6*c*x^2 - 5*a*b^5*d*x^2 + 5*a*b^5*c - 6*a^2*b^4*d)/((a
^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*(b*x^2 + a)) - 1/2*(4*b^3*c^3 + 3*a*b^2
*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^5*c^4) + 1/12*(44*b^3*c^3*x^6
 + 33*a*b^2*c^2*d*x^6 + 22*a^2*b*c*d^2*x^6 + 11*a^3*d^3*x^6 - 18*a*b^2*c^3*
x^4 - 12*a^2*b*c^2*d*x^4 - 6*a^3*c*d^2*x^4 + 6*a^2*b*c^3*x^2 + 3*a^3*c^2*d*
x^2 - 2*a^3*c^3)/(a^5*c^4*x^6)
```

Mupad [B]
time = 0.66, size = 278, normalized size = 1.32

$$\frac{\ln\left(b\,x^2+a\right)\,\left(4\,b^5\,c-5\,a\,b^4\,d\right)}{2\,a^7\,d^2-4\,a^6\,b\,c\,d+2\,a^5\,b^2\,c^2}-\frac{\frac{1}{6\,a\,c}-\frac{x^2\,\left(3\,a\,d+4\,b\,c\right)}{12\,a^2\,c^2}+\frac{x^4\,\left(2\,a^2\,d^2+3\,a\,b\,c\,d+4\,b^2\,c^2\right)}{4\,a^3\,c^3}+\frac{x^6\,\left(a^3\,b\,d^3+a^2\,b^2\,c\,d^2+a\,b^3\,c^2\,d-4\,b^4\,c^3\right)}{2\,a^4\,c^3\,\left(a\,d-b\,c\right)}}{b\,x^8+a\,x^6}+\frac{d^5\,\ln\left(d\,x^2+c\right)}{2\,\left(a^2\,c^4\,d^2-2\,a\,b\,c^5\,d+b^2\,c^6\right)}-\frac{\ln\left(x\right)\,\left(a^3\,d^3+2\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+4\,b^3\,c^3\right)}{a^5\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(4*b^5*c - 5*a*b^4*d))/(2*a^7*d^2 + 2*a^5*b^2*c^2 - 4*a^6*b
*c*d) - (1/(6*a*c) - (x^2*(3*a*d + 4*b*c))/(12*a^2*c^2) + (x^4*(2*a^2*d^2 +
 4*b^2*c^2 + 3*a*b*c*d))/(4*a^3*c^3) + (x^6*(a^3*b*d^3 - 4*b^4*c^3 + a^2*b^
2*c*d^2 + a*b^3*c^2*d))/(2*a^4*c^3*(a*d - b*c)))/(a*x^6 + b*x^8) + (d^5*log
(c + d*x^2))/(2*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d)) - (log(x)*(a^3*d^3 +
 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2))/(a^5*c^4)
```

3.301 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=162

$$\frac{(bc+ad)x}{2b(bc-ad)^2\left(c+dx^2\right)}+\frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{\sqrt{a}\,(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc-ad)^3}+\frac{\sqrt{c}\,(bc+3ad)\tan^{-1}\cdots}{2\sqrt{d}\,(bc-\cdots}$$

```
[Out] 1/2*(a*d+b*c)*x/b/(-a*d+b*c)^2/(d*x^2+c)+1/2*a*x/b/(-a*d+b*c)/(b*x^2+a)/(d*
x^2+c)-1/2*(a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)^3/b^(1/
2)+1/2*(3*a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))*c^(1/2)/(-a*d+b*c)^3/d^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {481, 541, 536, 211}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(ad+3bc)}{2\sqrt{b}\,(bc-ad)^3}+\frac{\sqrt{c}\,(3ad+bc)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{d}\,(bc-ad)^3}+\frac{x(ad+bc)}{2b\left(c+dx^2\right)(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*c + a*d)*x)/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*x)/(2*b*(b*c - a*d)*(a
 + b*x^2)*(c + d*x^2)) - (Sqrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*Sqrt[b]*(b*c - a*d)^3) + (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(2*Sqrt[d]*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\int \frac{ac+(-2bc-ad)x^2}{(a+bx^2)(c+dx^2)^2}\,dx}{2b(bc-ad)}$$

$$= \frac{(bc+ad)x}{2b(bc-ad)^2(c+dx^2)} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\int \frac{4abc^2-2bc(bc+ad)x^2}{(a+bx^2)(c+dx^2)}}{4bc(bc-ad)^2}$$

$$= \frac{(bc+ad)x}{2b(bc-ad)^2(c+dx^2)} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)} - \frac{(a(3bc+ad))\int \frac{}{a}}{2(bc-ad)^3}$$

$$= \frac{(bc+ad)x}{2b(bc-ad)^2(c+dx^2)} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\sqrt{a}(3bc+ad)\,\mathrm{ta}}{2\sqrt{b}(bc-}$$

Mathematica [A]
time = 0.14, size = 133, normalized size = 0.82

$$\frac{1}{2}\left(\frac{ax}{(bc-ad)^2(a+bx^2)} + \frac{cx}{(bc-ad)^2(c+dx^2)} + \frac{\sqrt{a}(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}(-bc+ad)^3} + \frac{\sqrt{c}(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((a*x)/((b*c - a*d)^2*(a + b*x^2)) + (c*x)/((b*c - a*d)^2*(c + d*x^2)) + (S
qrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(-(b*c) + a*d)^3
) + (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d
)^3))/2
```

Maple [A]
time = 0.22, size = 117, normalized size = 0.72

method	result	size
default	$\frac{a\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{(ad+3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(ad-bc)^{3}}-\frac{c\left(\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{d\,x^{2}+c}+\frac{(3ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{(ad-bc)^{3}}$	117
risch	Expression too large to display	1816

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(a*d+3*b*c)/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2)))-c/(a*d-b*c)^3*((-1/2*a*d+1/2*b*c)*x/(d*x^2+c)+1/2*(3
*a*d+b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.53, size = 249, normalized size = 1.54

$$-\frac{(3\,abc+a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}}+\frac{(bc^2+3\,acd)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}}+\frac{(bc+ad)x^3+2\,acx}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a*b*c + a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) + 1/2*(b*c^2 + 3*a*c*d)*arctan(d*x/sqrt(c
*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) + 1/2*
((b*c + a*d)*x^3 + 2*a*c*x)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c
^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^
2 + a^3*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(138) = 276$.
time = 1.40, size = 1407, normalized size = 8.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 +
a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*
b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3
```

```
*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2
*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3
*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2
 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*
d^3 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - 2*((3*b^2*c*d + a*b*d
^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^
2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*
x*sqrt(-c/d) - c)/(d*x^2 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^
2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 +
3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3
 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 + 2*((b^2*c*d + 3*a*b*d^2)
*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(c/
d)*arctan(d*x*sqrt(c/d)/c) - ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c
*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*s
qrt(-a/b) - a)/(b*x^2 + a)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^2*b
^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a
^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 -
a^4*d^4)*x^2), 1/2*((b^2*c^2 - a^2*d^2)*x^3 - ((3*b^2*c*d + a*b*d^2)*x^4 +
3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(a/b)*arct
an(b*x*sqrt(a/b)/a) + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b
^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + 2*
(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4
*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 +
(b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 198, normalized size = 1.22

$$-\frac{(3\,abc+a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}}+\frac{(bc^2+3\,acd)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}}+\frac{bcx^3+adx^3+2\,acx}{2\,(bdx^4+bcx^2+adx^2+ac)(b^2c^2-2\,abcd+a^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b*c + a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) + 1/2*(b*c^2 + 3*a*c*d)*arctan(d*x/sqrt(c
*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) + 1/2*
(b*c*x^3 + a*d*x^3 + 2*a*c*x)/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2))
```

Mupad [B]
time = 1.03, size = 2500, normalized size = 15.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((x^3*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (a*c*x)/(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d))/(a*c + x^2*(a*d + b*c) + b*d*x^4) - (atan((((-c*d)^(
1/2)*((x*(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^
2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
 4*a^3*b*c*d^3)) - ((-c*d)^(1/2)*((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a
^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d
^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b
^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-c*d
)^(1/2)*(3*a*d + b*c)*(16*a^7*b^2*d^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 -
 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c^5*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c
^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
3*a^2*b*c*d^3)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a
^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*
a^2*b*c*d^3)))*(3*a*d + b*c)*1i)/(4*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2
- 3*a^2*b*c*d^3)) + ((-c*d)^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*c^3*
d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + ((-c*d)^(1/2)*((4*a*b^8*c^7
*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a^4*b
^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^6 +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5) + (x*(-c*d)^(1/2)*(3*a*d + b*c)*(16*a^7*b^2*d^9 + 16*b^9
*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c^5*d^4 - 80*a
^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8*(a^3*d^4 - b
^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^3*d^4 - b^3
*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)))*(3*a*d + b*c)*1i)/(4*(a^3*d^4 -
 b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)))/((((13*a^2*b^3*c^3*d^2)/4 +
(13*a^3*b^2*c^2*d^3)/4 + (3*a*b^4*c^4*d)/4 + (3*a^4*b*c*d^4)/4)/(a^6*d^6 +
b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*
a*b^5*c^5*d - 6*a^5*b*c*d^5) - ((-c*d)^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d + 6
*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4*c
```

```
^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - ((-c*d)^(1/2)*((
4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4
 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6 +
 b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-c*d)^(1/2)*(3*a*d + b*c)*(16*a^7*b^2*d
^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c^5
*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8*(
a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)*(a^4*d^4 + b^4*c^4 +
 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^
3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)))*(3*a*d + b*c))/(4*(a
^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) + ((-c*d)^(1/2)*((x*
(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2
*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3)) + ((-c*d)^(1/2)*((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^
6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a
^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^
3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (x*(-c*d)^(1/2)*(
3*a*d + b*c)*(16*a^7*b^2*d^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*a^6*b
^3*c*d^8 + 144*a^2*b^7*c^5*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d^6 +
144*a^5*b^4*c^2*d^7))/(8*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c
*d^3)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^
3)))*(3*a*d + b*c))/(4*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d
^3)))*(3*a*d + b*c))/(4*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*
d^3))))*(-c*d)^(1/2)*(3*a*d + b*c)*1i)/(2*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^
2*d^2 - 3*a^2*b*c*d^3)) - (atan((((-a*b)^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d +
 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4
*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - ((-a*b)^(1/2)*
((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d
^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6
 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-a*b)^(1/2)*(a*d + 3*b*c)*(16*a^7*b^2
*d^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c
^5*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8
*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*(a^4*d^4 + b^4*c^4
 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(a*d + 3*b*c))/(4*(
b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3...
```

3.302 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=107

$$\frac{a}{2(bc-ad)^2(a+bx^2)}+\frac{c}{2(bc-ad)^2(c+dx^2)}+\frac{(bc+ad)\log(a+bx^2)}{2(bc-ad)^3}-\frac{(bc+ad)\log(c+dx^2)}{2(bc-ad)^3}$$

```
[Out] 1/2*a/(-a*d+b*c)^2/(b*x^2+a)+1/2*c/(-a*d+b*c)^2/(d*x^2+c)+1/2*(a*d+b*c)*ln(
b*x^2+a)/(-a*d+b*c)^3-1/2*(a*d+b*c)*ln(d*x^2+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{a}{2(a+bx^2)(bc-ad)^2}+\frac{c}{2(c+dx^2)(bc-ad)^2}+\frac{(ad+bc)\log(a+bx^2)}{2(bc-ad)^3}-\frac{(ad+bc)\log(c+dx^2)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] a/(2*(b*c - a*d)^2*(a + b*x^2)) + c/(2*(b*c - a*d)^2*(c + d*x^2)) + ((b*c +
 a*d)*Log[a + b*x^2])/(2*(b*c - a*d)^3) - ((b*c + a*d)*Log[c + d*x^2])/(2*(
b*c - a*d)^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{ab}{(bc-ad)^2(a+bx)^2} + \frac{b(bc+ad)}{(bc-ad)^3(a+bx)} - \frac{cd}{(bc-ad)^2(c+d\right.\right.$$
$$= \frac{a}{2(bc-ad)^2(a+bx^2)} + \frac{c}{2(bc-ad)^2(c+dx^2)} + \frac{(bc+ad)\log(a+bx^2)}{2(bc-ad)^3} - \frac{(b}{}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.80

$$\frac{\frac{a(bc-ad)}{a+bx^2} + \frac{c(bc-ad)}{c+dx^2} + (bc+ad)\log(a+bx^2) - (bc+ad)\log(c+dx^2)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d))/(c + d*x^2) + (b*c + a*d)*Lo
g[a + b*x^2] - (b*c + a*d)*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Maple [A]
time = 0.14, size = 113, normalized size = 1.06

method	result
default	$-\frac{b\left(\frac{(ad+bc)\ln(bx^2+a)}{b} - \frac{a(ad-bc)}{b(bx^2+a)}\right)}{2(ad-bc)^3} + \frac{d\left(\frac{c(ad-bc)}{d(dx^2+c)} + \frac{(ad+bc)\ln(dx^2+c)}{d}\right)}{2(ad-bc)^3}$
norman	$\frac{\frac{ac}{a^2d^2-2abcd+b^2c^2} + \frac{(ad+bc)x^2}{2a^2d^2-4abcd+2b^2c^2}}{(bx^2+a)(dx^2+c)} - \frac{(ad+bc)\ln(bx^2+a)}{2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)} + \frac{(ad+bc)\ln(dx^2+c)}{2a^3d^3-6a^2bcd^2+6ab^2c^2d-2b^3c^3}$
risch	$\frac{\frac{ac}{a^2d^2-2abcd+b^2c^2} + \frac{(ad+bc)x^2}{2a^2d^2-4abcd+2b^2c^2}}{(bx^2+a)(dx^2+c)} + \frac{\ln(dx^2+c)ad}{2a^3d^3-6a^2bcd^2+6ab^2c^2d-2b^3c^3} + \frac{\ln(dx^2+c)bc}{2a^3d^3-6a^2bcd^2+6ab^2c^2d-2b^3c^3} - \frac{\ln}{2(a^3d^3-3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/(a*d-b*c)^3*((a*d+b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a))+1/2*d/
(a*d-b*c)^3*(c*(a*d-b*c)/d/(d*x^2+c)+(a*d+b*c)/d*ln(d*x^2+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(99) = 198.
time = 0.30, size = 228, normalized size = 2.13

$$\frac{(bc+ad)\log(bx^2+a)}{2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)} - \frac{(bc+ad)\log(dx^2+c)}{2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)} + \frac{(bc+ad)x^2+2ac}{2(ab^2c^3-2a^2bc^2d+a^3cd^2+(b^3c^2d-2ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c + a*d)*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3) - 1/2*(b*c + a*d)*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3) + 1/2*((b*c + a*d)*x^2 + 2*a*c)/(a*b^2*c^3 - 2*a^2*b*c^2*
d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*
b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(99) = 198$.
time = 1.13, size = 296, normalized size = 2.77

$$\frac{2\,abc^2 - 2\,a^2cd + (b^2c^2 - a^2d^2)x^2 + ((b^2cd + abd^2)x^4 + abc^2 + a^2cd + (b^2c^2 + 2\,abcd + a^2d^2)x^2)\log\left(bx^2 + a\right) - ((b^2cd + abd^2)x^4 + abc^2 + a^2cd + (b^2c^2 + 2\,abcd + a^2d^2)x^2)\log\left(dx^2 + c\right)}{2\,(ab^3c^4 - 3\,a^2b^2c^3d + 3\,a^3bc^2d^2 - a^4cd^3 + (b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4)x^4 + (b^4c^4 - 2\,ab^3c^3d + 2\,a^3bcd^3 - a^4d^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2 - 2*a^2*c*d + (b^2*c^2 - a^2*d^2)*x^2 + ((b^2*c*d + a*b*d^2)
*x^4 + a*b*c^2 + a^2*c*d + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*x^2)*log(b*x^2 +
 a) - ((b^2*c*d + a*b*d^2)*x^4 + a*b*c^2 + a^2*c*d + (b^2*c^2 + 2*a*b*c*d +
 a^2*d^2)*x^2)*log(d*x^2 + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d
^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4
)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(90) = 180$.
time = 2.49, size = 507, normalized size = 4.74

$$\frac{2ac + x^2(ad + bc)}{2a^3cd^2 - 4a^2bc^2d + 2ab^2c^3 + x^4 \cdot (2a^2bd^3 - 4ab^2cd^2 + 2b^3c^2d) + x^2 \cdot (2a^3d^3 - 2a^2bcd^2 - 2ab^2c^2d + 2b^3c^3)} + \frac{(ad + bc)\log\left(x^2 + \frac{-\frac{a^4d^4(ad+bc)}{(ad-bc)^3} + \frac{4a^3bcd^3(ad+bc)}{(ad-bc)^3} - \frac{6a^2b^2c^2d^2(ad+bc)}{(ad-bc)^3} + a^2d^2 + \frac{4ab^3c^3d(ad+bc)}{(ad-bc)^3} + 2abcd - \frac{b^4c^4(ad+bc)}{(ad-bc)^3} + b^2c^2}{2abd^2+2b^2cd}\right)}{2\,(ad - bc)^3} - \frac{(ad + bc)\log\left(x^2 + \frac{\frac{a^4d^4(ad+bc)}{(ad-bc)^3} - \frac{4a^3bcd^3(ad+bc)}{(ad-bc)^3} + \frac{6a^2b^2c^2d^2(ad+bc)}{(ad-bc)^3} + a^2d^2 - \frac{4ab^3c^3d(ad+bc)}{(ad-bc)^3} + 2abcd + \frac{b^4c^4(ad+bc)}{(ad-bc)^3} + b^2c^2}{2abd^2+2b^2cd}\right)}{2\,(ad - bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] (2*a*c + x**2*(a*d + b*c))/(2*a**3*c*d**2 - 4*a**2*b*c**2*d + 2*a*b**2*c**3
 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2 + 2*b**3*c**2*d) + x**2*(2*a**3*d*
*3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d + 2*b**3*c**3)) + (a*d + b*c)*log(x*
*2 + (-a**4*d**4*(a*d + b*c)/(a*d - b*c)**3 + 4*a**3*b*c*d**3*(a*d + b*c)/(
a*d - b*c)**3 - 6*a**2*b**2*c**2*d**2*(a*d + b*c)/(a*d - b*c)**3 + a**2*d**
2 + 4*a*b**3*c**3*d*(a*d + b*c)/(a*d - b*c)**3 + 2*a*b*c*d - b**4*c**4*(a*d
 + b*c)/(a*d - b*c)**3 + b**2*c**2)/(2*a*b*d**2 + 2*b**2*c*d))/(2*(a*d - b*
c)**3) - (a*d + b*c)*log(x**2 + (a**4*d**4*(a*d + b*c)/(a*d - b*c)**3 - 4*a
**3*b*c*d**3*(a*d + b*c)/(a*d - b*c)**3 + 6*a**2*b**2*c**2*d**2*(a*d + b*c)
/(a*d - b*c)**3 + a**2*d**2 - 4*a*b**3*c**3*d*(a*d + b*c)/(a*d - b*c)**3 +
2*a*b*c*d + b**4*c**4*(a*d + b*c)/(a*d - b*c)**3 + b**2*c**2)/(2*a*b*d**2 +
 2*b**2*c*d))/(2*(a*d - b*c)**3)
```

Giac [A]
time = 0.54, size = 178, normalized size = 1.66

$$\frac{\frac{ab^3}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)(bx^2+a)} - \frac{(b^3c+ab^2d)\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} - \frac{b^2cd}{(bc-ad)^3\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(b*x^2 + a)) - (b^3*c + a
*b^2*d)*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - b^2*c*d/((b*c - a*d)^3*(b*c/(b*x^2 +
 a) - a*d/(b*x^2 + a) + d)))/b
```

Mupad [B]
time = 0.31, size = 522, normalized size = 4.88

$$\frac{b^2\,c^2\,x^2-a^2\,d^2\,x^2+2\,a\,b\,c^2-2\,a^2\,c\,d+a^2\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+b^2\,c^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+a\,b\,c^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+a^2\,c\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+a\,b\,d^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+b^2\,c\,d\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+a\,b\,c\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}}{-2\,a^4\,c\,d^3-2\,a^4\,d^4\,x^2+6\,a^3\,b\,c^2\,d^2+4\,a^3\,b\,c\,d^3\,x^2-2\,a^3\,b\,d^4\,x^4-6\,a^2\,b^2\,c^3\,d+6\,a^2\,b^2\,c\,d^3\,x^4+2\,a\,b^3\,c^4-4\,a\,b^3\,c^3\,d\,x^2-6\,a\,b^3\,c^2\,d^2\,x^4+2\,b^4\,c^4\,x^2+2\,b^4\,c^3\,d\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] (b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d + a^2*d^2*x^2*atan((a*d*
x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + b^2*c^2*x^2*atan((a*
d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a*b*c^2*atan((a*d*
x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a^2*c*d*atan((a*d*x^
2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a*b*d^2*x^4*atan((a*d*
x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + b^2*c*d*x^4*atan((a*
d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a*b*c*d*x^2*atan((
a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i)/(2*a*b^3*c^4 - 2*
a^4*c*d^3 - 2*a^4*d^4*x^2 + 2*b^4*c^4*x^2 - 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d
^2 - 2*a^3*b*d^4*x^4 + 2*b^4*c^3*d*x^4 - 4*a*b^3*c^3*d*x^2 + 4*a^3*b*c*d^3*
x^2 - 6*a*b^3*c^2*d^2*x^4 + 6*a^2*b^2*c*d^3*x^4)
```

3.303 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{dx}{(bc-ad)^2(c+dx^2)}-\frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)}+\frac{\sqrt{b}\,(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)^3}-\frac{\sqrt{d}\,(3bc+ad)\tan^{-1}}{2\sqrt{c}\,(bc-ad}$$

```
[Out] -d*x/(-a*d+b*c)^2/(d*x^2+c)-1/2*x/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)+1/2*(3*a*d
+b*c)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)^3/a^(1/2)-1/2*(a*d+3*b*c
)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/(-a*d+b*c)^3/c^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {482, 541, 536, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3ad+bc)}{2\sqrt{a}\,(bc-ad)^3}-\frac{\sqrt{d}\,(ad+3bc)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)^3}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{dx}{\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -((d*x)/((b*c - a*d)^2*(c + d*x^2))) - x/(2*(b*c - a*d)*(a + b*x^2)*(c + d*
x^2)) + (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]
*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = -\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\int \frac{c-3dx^2}{(a+bx^2)(c+dx^2)^2}\,dx}{2(bc-ad)}$$

$$= -\frac{dx}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\int \frac{2c(bc+ad)-4bcdx^2}{(a+bx^2)(c+dx^2)}\,dx}{4c(bc-ad)^2}$$

$$= -\frac{dx}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(d(3bc+ad))\int \frac{1}{c+dx^2}\ldots}{2(bc-ad)^3}$$

$$= -\frac{dx}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt{b}\,(bc+3ad)\tan^{-1}\ldots}{2\sqrt{a}\,(bc-\ldots}$$

Mathematica [A]
time = 0.13, size = 137, normalized size = 0.93

$$\frac{1}{2}\left(-\frac{bx}{(bc-ad)^2\left(a+bx^2\right)} - \frac{dx}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{b}\,(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(-bc+ad)^3} - \frac{\sqrt{d}\,(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-((b*x)/((b*c - a*d)^2*(a + b*x^2))) - (d*x)/((b*c - a*d)^2*(c + d*x^2)) -
 (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) + a*d
)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c -
a*d)^3))/2
```

Maple [A]
time = 0.23, size = 117, normalized size = 0.80

method	result	size
default	$-\frac{b\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{(3ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(ad-bc)^{3}}+\frac{d\left(\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{d\,x^{2}+c}+\frac{(ad+3bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{(ad-bc)^{3}}$	117
risch	Expression too large to display	1521

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(3*a*d+b*c)/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2)))+d/(a*d-b*c)^3*((-1/2*a*d+1/2*b*c)*x/(d*x^2+c)+1/2*(
a*d+3*b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [A]
time = 0.56, size = 249, normalized size = 1.69

$$\frac{(b^2c+3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}}-\frac{(3\,bcd+ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}}-\frac{2\,bdx^3+(bc+ad)x}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c + 3*a*b*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) - 1/2*(3*b*c*d + a*d^2)*arctan(d*x/sqrt(c*
d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) - 1/2*(
2*b*d*x^3 + (b*c + a*d)*x)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^
2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2
 + a^3*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(125) = 250.
time = 2.14, size = 1387, normalized size = 9.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3
*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2
*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2
```

```
+ a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 +
2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 -
3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^
2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c
*d^3 - a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + 2*((3*b^2*c*d + a*b
*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sq
rt(d/c)*arctan(x*sqrt(d/c)) + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*
c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*
sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*
b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*
a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 -
 a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 - 2*((b^2*c*d + 3*a*b*d^2)*
x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(b/a
)*arctan(x*sqrt(b/a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d +
(3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^
3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^
2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d
^4)*x^2), -1/2*(2*(b^2*c*d - a*b*d^2)*x^3 - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*
b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(b/a)*arctan
(x*sqrt(b/a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c^2 - a
^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^
4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2
*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.55, size = 196, normalized size = 1.33

$$\frac{(b^2c + 3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{ab}} - \frac{(3\,bcd + ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{cd}} - \frac{2\,bdx^3 + bcx + adx}{2\,(bdx^4 + bcx^2 + adx^2 + ac)(b^2c^2 - 2\,abcd + a^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c + 3*a*b*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) - 1/2*(3*b*c*d + a*d^2)*arctan(d*x/sqrt(c*
d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) - 1/2*(
2*b*d*x^3 + b*c*x + a*d*x)/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2*c^2 -
2*a*b*c*d + a^2*d^2))
```

Mupad [B]
time = 0.94, size = 2500, normalized size = 17.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) - ((
-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*d^3 - 10*a^6*b^3
*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*b^5*c^3*d^6 + 18*
a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-a*b)^(1/2)*
(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40*a^6*b^
3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*d^6 + 72
*a^5*b^4*c^2*d^7))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^
2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))
)*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)
))*(3*a*d + b*c)*1i)/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*
d^2)) + ((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) + (
(-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*d^3 - 10*a^6*b^
3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*b^5*c^3*d^6 + 18
*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (x*(-a*b)^(1/2)
*(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40*a^6*b
^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*d^6 + 7
2*a^5*b^4*c^2*d^7))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d
^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)
))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2
)))*(3*a*d + b*c)*1i)/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c
*d^2)))/(((3*a^2*b^3*d^5)/2 + (3*b^5*c^2*d^3)/2 + 5*a*b^4*c*d^4)/(a^6*d^6 +
 b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
*a*b^5*c^5*d - 6*a^5*b*c*d^5) - ((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^
2*d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^
3*d - 4*a^3*b*c*d^3) - ((-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a
*b^8*c^6*d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 -
 10*a^4*b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c
```

^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-a*b)^(1/2)*(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*d^6 + 72*a^5*b^4*c^2*d^7))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) + ((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) + ((-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (x*(-a*b)^(1/2)*(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*d^6 + 72*a^5*b^4*c^2*d^7))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))*(-a*b)^(1/2)*(3*a*d + b*c)*1i)/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) - ((x*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^3)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*c + x^2*(a*d + b*c) + b*d*x^4) + (atan((((-c*d)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) - ((-c*d)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-c*d)^(1/2)*(a*d + 3*b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*d^6 + 72*a^5*b^4*c^2*d^7))/(4*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(a*d + 3*b*c))/(4*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(a*d + 3*b*c)*1i)/(4*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) + ((-c*d)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^...

3.304 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^2} \, dx$

Optimal. Leaf size=92

$$-\frac{b}{2(bc-ad)^2(a+bx^2)}-\frac{d}{2(bc-ad)^2(c+dx^2)}-\frac{bd\log(a+bx^2)}{(bc-ad)^3}+\frac{bd\log(c+dx^2)}{(bc-ad)^3}$$

```
[Out] -1/2*b/(-a*d+b*c)^2/(b*x^2+a)-1/2*d/(-a*d+b*c)^2/(d*x^2+c)-b*d*ln(b*x^2+a)/
(-a*d+b*c)^3+b*d*ln(d*x^2+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$,
Rules used = {455, 46}

$$-\frac{b}{2(a+bx^2)(bc-ad)^2}-\frac{d}{2(c+dx^2)(bc-ad)^2}-\frac{bd\log(a+bx^2)}{(bc-ad)^3}+\frac{bd\log(c+dx^2)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/2*b/((b*c - a*d)^2*(a + b*x^2)) - d/(2*(b*c - a*d)^2*(c + d*x^2)) - (b*d
*Log[a + b*x^2])/(b*c - a*d)^3 + (b*d*Log[c + d*x^2])/(b*c - a*d)^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^2)^2(c+dx^2)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^2} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2}{(bc-ad)^2(a+bx)^2}-\frac{2b^2d}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^2}\right.\right.\\ &= -\frac{b}{2(bc-ad)^2(a+bx^2)}-\frac{d}{2(bc-ad)^2(c+dx^2)}-\frac{bd\log(a+bx^2)}{(bc-ad)^3}+\frac{bd\log(c}{(bc-}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 0.84

$$\frac{\frac{b(-bc+ad)}{a+bx^2}+\frac{d(-bc+ad)}{c+dx^2}-2bd\log\left(a+bx^2\right)+2bd\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x^2) + (d*(-(b*c) + a*d))/(c + d*x^2) - 2*b*d*Lo
g[a + b*x^2] + 2*b*d*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Maple [A]
time = 0.13, size = 106, normalized size = 1.15

method	result	size
default	$\frac{b^2\left(\frac{2d\ln\left(b\,x^2+a\right)}{b}-\frac{ad-bc}{b\left(b\,x^2+a\right)}\right)}{2(ad-bc)^3}+\frac{d^2\left(-\frac{ad-bc}{d\left(d\,x^2+c\right)}-\frac{2b\ln\left(d\,x^2+c\right)}{d}\right)}{2(ad-bc)^3}$	106
risch	$\frac{-\frac{bd\,x^2}{a^2d^2-2abcd+b^2c^2}-\frac{ad+bc}{2\left(a^2d^2-2abcd+b^2c^2\right)}}{(b\,x^2+a)(d\,x^2+c)}-\frac{bd\ln\left(-d\,x^2-c\right)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}+\frac{bd\ln\left(b\,x^2+a\right)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$	186
norman	$\frac{-\frac{bd\,x^2}{a^2d^2-2abcd+b^2c^2}+\frac{-ab\,d^2-b^2cd}{2db\left(a^2d^2-2abcd+b^2c^2\right)}}{(b\,x^2+a)(d\,x^2+c)}+\frac{bd\ln\left(b\,x^2+a\right)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{bd\ln\left(d\,x^2+c\right)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$	197

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/(a*d-b*c)^3*(2*d/b*ln(b*x^2+a)-(a*d-b*c)/b/(b*x^2+a))+1/2*d^2/(a*d-
b*c)^3*(-(a*d-b*c)/d/(d*x^2+c)-2*b/d*ln(d*x^2+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(88) = 176.
time = 0.33, size = 215, normalized size = 2.34

$$-\frac{bd\log\left(bx^2+a\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{bd\log\left(dx^2+c\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{2\,bdx^2+bc+ad}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -b*d*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + b
*d*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/2
*(2*b*d*x^2 + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*
d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
 a^3*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(88) = 176$.

time = 0.98, size = 253, normalized size = 2.75

$$-\frac{b^2c^2 - a^2d^2 + 2\,(b^2cd - abd^2)x^2 + 2\,(b^2d^2x^4 + abcd + (b^2cd + abd^2)x^2)\log\left(bx^2 + a\right) - 2\,(b^2d^2x^4 + abcd + (b^2cd + abd^2)x^2)\log\left(dx^2 + c\right)}{2\,(ab^3c^4 - 3\,a^2b^2c^3d + 3\,a^3bc^2d^2 - a^4cd^3 + (b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4)x^4 + (b^4c^4 - 2\,ab^3c^3d + 2\,a^3bcd^3 - a^4d^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d^2*x^4 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x^2)*log(b*x^2 + a) - 2*(b^2*d^2*x^4 + a*b*c*d +
(b^2*c*d + a*b*d^2)*x^2)*log(d*x^2 + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a
^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 -
 a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(76) = 152$.

time = 1.96, size = 410, normalized size = 4.46

$$-\frac{bd\log\left(x^2 + \frac{-\frac{a^4bd^5}{(ad-bc)^3} + \frac{4a^3b^2cd^4}{(ad-bc)^3} - \frac{6a^2b^3c^2d^3}{(ad-bc)^3} + \frac{4ab^4c^3d^2}{(ad-bc)^3} + abd^2 - \frac{b^5c^4d}{(ad-bc)^3} + b^2cd}{2b^2d^2}\right)}{(ad-bc)^3} + \frac{bd\log\left(x^2 + \frac{\frac{a^4bd^5}{(ad-bc)^3} - \frac{4a^3b^2cd^4}{(ad-bc)^3} + \frac{6a^2b^3c^2d^3}{(ad-bc)^3} - \frac{4ab^4c^3d^2}{(ad-bc)^3} + abd^2 + \frac{b^5c^4d}{(ad-bc)^3} + b^2cd}{2b^2d^2}\right)}{(ad-bc)^3} + \frac{-ad - bc - 2bdx^2}{2a^3cd^2 - 4a^2bc^2d + 2ab^2c^3 + x^4\cdot(2a^2bd^3 - 4ab^2cd^2 + 2b^3c^2d) + x^2\cdot(2a^3d^3 - 2a^2bcd^2 - 2ab^2c^2d + 2b^3c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] -b*d*log(x**2 + (-a**4*b*d**5/(a*d - b*c)**3 + 4*a**3*b**2*c*d**4/(a*d - b*
c)**3 - 6*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 4*a*b**4*c**3*d**2/(a*d - b*
c)**3 + a*b*d**2 - b**5*c**4*d/(a*d - b*c)**3 + b**2*c*d)/(2*b**2*d**2))/(a
*d - b*c)**3 + b*d*log(x**2 + (a**4*b*d**5/(a*d - b*c)**3 - 4*a**3*b**2*c*d
**4/(a*d - b*c)**3 + 6*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 4*a*b**4*c**3*d
**2/(a*d - b*c)**3 + a*b*d**2 + b**5*c**4*d/(a*d - b*c)**3 + b**2*c*d)/(2*b
**2*d**2))/(a*d - b*c)**3 + (-a*d - b*c - 2*b*d*x**2)/(2*a**3*c*d**2 - 4*a*
*2*b*c**2*d + 2*a*b**2*c**3 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2 + 2*b**
3*c**2*d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d + 2*b**3*
c**3))
```

Giac [A]

time = 0.51, size = 163, normalized size = 1.77

$$\frac{b^2d\log\left(\left|\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right|\right)}{b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3} - \frac{b^3}{2\,(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)(bx^2 + a)} + \frac{bd^2}{2\,(bc - ad)^3\left(\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^4*c^3 - 3*a*b^3*c^
2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 1/2*b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*
```

```
b^2*d^2)*(b*x^2 + a)) + 1/2*b*d^2/((b*c - a*d)^3*(b*c/(b*x^2 + a) - a*d/(b*
x^2 + a) + d))
```

Mupad [B]
time = 0.16, size = 378, normalized size = 4.11

$$-\frac{b^2\,c^2-a^2\,d^2+b^2\,d^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}-2\,a\,b\,d^2\,x^2+2\,b^2\,c\,d\,x^2+a\,b\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+b^2\,c\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+a\,b\,c\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}}{-2\,a^4\,c\,d^3-2\,a^4\,d^4\,x^2+6\,a^3\,b\,c^2\,d^2+4\,a^3\,b\,c\,d^3\,x^2-2\,a^3\,b\,d^4\,x^4-6\,a^2\,b^2\,c^3\,d+6\,a^2\,b^2\,c\,d^3\,x^4+2\,a\,b^3\,c^4-4\,a\,b^3\,c^3\,d\,x^2-6\,a\,b^3\,c^2\,d^2\,x^4+2\,b^4\,c^4\,x^2+2\,b^4\,c^3\,d\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] -(b^2*c^2 - a^2*d^2 + b^2*d^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a
*d*x^2 + b*c*x^2))*4i - 2*a*b*d^2*x^2 + 2*b^2*c*d*x^2 + a*b*d^2*x^2*atan((a
*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + b^2*c*d*x^2*atan(
(a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a*b*c*d*atan((a
*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i)/(2*a*b^3*c^4 - 2*a
^4*c*d^3 - 2*a^4*d^4*x^2 + 2*b^4*c^4*x^2 - 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^
2 - 2*a^3*b*d^4*x^4 + 2*b^4*c^3*d*x^4 - 4*a*b^3*c^3*d*x^2 + 4*a^3*b*c*d^3*x
^2 - 6*a*b^3*c^2*d^2*x^4 + 6*a^2*b^2*c*d^3*x^4)
```

3.305 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=167

$$\frac{d(bc+ad)x}{2ac(bc-ad)^2\,(c+dx^2)}+\frac{bx}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)}+\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}$$

```
[Out] 1/2*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*b*x/a/(-a*d+b*c)/(b*x^2+a)
/(d*x^2+c)+1/2*b^(3/2)*(-5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d
+b*c)^3+1/2*d^(3/2)*(-a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*
c)^3
```

Rubi [A]

time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-5ad)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{dx(ad+bc)}{2ac\,(c+dx^2)\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^2} \, dx = \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\int \frac{-bc+2ad-3bdx^2}{(a+bx^2)(c+dx^2)^2} \, dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\int \frac{-2(b^2c^2-4abcd+a\ldots}{(a+bx^2\ldots}}{4ac(b\ldots}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{(b^2(bc-5ad)) \int \ldots}{2a(bc-a\ldots}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{b^{3/2}(bc-5ad)\,\mathrm{t}\ldots}{2a^{3/2}(bc\ldots}$$

Mathematica [A]
time = 0.23, size = 136, normalized size = 0.81

$$\frac{1}{2}\left(\frac{b^{3/2}(-bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(-bc+ad)^3} + \frac{(bc-ad)x\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) +
a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^2 + c*d*x^2)) + (d^(
3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c - a*d)^3)/2
```

Maple [A]
time = 0.00, size = 133, normalized size = 0.80

method	result	size
default	$\frac{b^2\left(\frac{(ad-bc)x}{2a(b\,x^2+a)}+\frac{(5ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^3}+\frac{d^2\left(\frac{(ad-bc)x}{2c(d\,x^2+c)}+\frac{(ad-5bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}\right)}{(ad-bc)^3}$	133
risch	Expression too large to display	2124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(5*a*d-b*c)/a/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/c*x/(d*x^2+c)+1/2*(
a*d-5*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(143) = 286$.
time = 0.53, size = 294, normalized size = 1.76

$$\frac{(b^3c-5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{ab}}+\frac{(5\,bcd^2-ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{cd}}+\frac{(b^2cd+abd^2)x^3+(b^2c^2+a^2d^2)x}{2\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^4+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*((b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 -
 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d
^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(143) = 286$.
time = 2.04, size = 1681, normalized size = 10.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2
*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sq
```

```
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (5*a^2*b*c^2*d -
 a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d
^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c
)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*
a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^
3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d
 + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 +
2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c
^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (a*b^2
*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2
*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^
3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*
a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2
*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d
^3)*x^3 + 2*(a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 +
(b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)
) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2
*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*
x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*
c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c
^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/2*((b^3*c^2*d -
 a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*
x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5
*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ (b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b
^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2
 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*
a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.60, size = 232, normalized size = 1.39

$$\frac{(b^3c-5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{ab}}+\frac{(5\,bcd^2-ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{cd}}+\frac{b^2cdx^3+abd^2x^3+b^2c^2x+a^2d^2x}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2)(bdx^4+bcx^2+adx^2+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*(b^2*c*d*x^3 + a*b*d^2*x^3 + b^2*c^2*x + a^2*d^2*x)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
```

Mupad [B]
time = 1.16, size = 2500, normalized size = 14.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^3
*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d + b*
c) + b*d*x^4) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 -
 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*
a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2
 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7
*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 -
 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b
*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) -
(x*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3
+ 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b
^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^
3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3
*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3
)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*
a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 1
0*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^
3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 +
 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c
^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 2
0*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c
^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x
*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 +
144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4
*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*
c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b
^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^
```

```
(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*
d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d -
 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^7)/4 + (5*b^7*c^3*d^4)/4 - (21*a*b^6*c^2*d
^5)/4 - (21*a^2*b^5*c*d^6)/4)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d
- 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^
4*d^4) - (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*
d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d
- 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c
*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220
*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^
2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*
a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) - (x*(5*a*d - b*
c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*
c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 8
0*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*
b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4
*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a
^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a
^3*b^3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*d^6 +
 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a
^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c*d^10
 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220*a^5*
b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^2*d^9
)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*a^4*b
^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x*(5*a*d - b*c)*(-
a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*c^7*d
^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 80*a^8
*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c
^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a^5*
b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a^6*d^
3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a^3*b^
3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(
5*a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(2*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2
*d - 3*a^5*b*c*d^2)) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^
3*d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d
^4 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*
c^9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*
a^4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6...
```

3.306 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=141

$$\frac{b^2}{2a(bc-ad)^2(a+bx^2)}+\frac{d^2}{2c(bc-ad)^2(c+dx^2)}+\frac{\log(x)}{a^2c^2}-\frac{b^2(bc-3ad)\log(a+bx^2)}{2a^2(bc-ad)^3}-\frac{d^2(3bc-ad)\log(c+dx^2)}{2c^2(bc-ad)^3}$$

```
[Out] 1/2*b^2/a/(-a*d+b*c)^2/(b*x^2+a)+1/2*d^2/c/(-a*d+b*c)^2/(d*x^2+c)+ln(x)/a^2
/c^2-1/2*b^2*(-3*a*d+b*c)*ln(b*x^2+a)/a^2/(-a*d+b*c)^3-1/2*d^2*(-a*d+3*b*c)
*ln(d*x^2+c)/c^2/(-a*d+b*c)^3
```

Rubi [A]
time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{b^2(bc-3ad)\log(a+bx^2)}{2a^2(bc-ad)^3}+\frac{\log(x)}{a^2c^2}+\frac{b^2}{2a(a+bx^2)(bc-ad)^2}-\frac{d^2(3bc-ad)\log(c+dx^2)}{2c^2(bc-ad)^3}+\frac{d^2}{2c(c+dx^2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] b^2/(2*a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(2*c*(b*c - a*d)^2*(c + d*x^2)) +
 Log[x]/(a^2*c^2) - (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3
) - (d^2*(3*b*c - a*d)*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^2}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2c^2x} - \frac{b^3}{a(-bc+ad)^2(a+bx)^2} - \frac{b^3(-bc+3ad)}{a^2(-bc+ad)^3(a+bx)} - \cdots\right.\right.$$

$$= \frac{b^2}{2a(bc-ad)^2\left(a+bx^2\right)} + \frac{d^2}{2c(bc-ad)^2\left(c+dx^2\right)} + \frac{\log(x)}{a^2c^2} - \frac{b^2(bc-3ad)\log\cdots}{2a^2(bc-\cdots}$$

Mathematica [A]
time = 0.16, size = 133, normalized size = 0.94

$$\frac{1}{2}\left(\frac{b^2}{a(bc-ad)^2\left(a+bx^2\right)} + \frac{d^2}{c(bc-ad)^2\left(c+dx^2\right)} + \frac{2\log(x)}{a^2c^2} + \frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{a^2(-bc+ad)^3} + \frac{d^2(-3bc+ad)\log\left(c+dx^2\right)}{c^2(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (b^2/(a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*x^2)) + (2
*Log[x])/(a^2*c^2) + (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/(a^2*(-(b*c) + a*d)
^3) + (d^2*(-3*b*c + a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^3))/2
```

Maple [A]
time = 0.17, size = 136, normalized size = 0.96

method	result
default	$-\frac{b^3\left(\frac{(3ad-bc)\ln\left(b\,x^2+a\right)}{b}-\frac{a(ad-bc)}{b\left(b\,x^2+a\right)}\right)}{2a^2(ad-bc)^3}-\frac{d^3\left(-\frac{c(ad-bc)}{d\left(d\,x^2+c\right)}+\frac{(ad-3bc)\ln\left(d\,x^2+c\right)}{d}\right)}{2c^2(ad-bc)^3}+\frac{\ln(x)}{a^2c^2}$
norman	$\frac{\frac{\left(-a^3d^3-b^3c^3\right)x^2}{2c^2a^2\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(-a^2d^2-b^2c^2\right)bd\,x^4}{2c^2a^2\left(a^2d^2-2abcd+b^2c^2\right)}}{(b\,x^2+a)(d\,x^2+c)}+\frac{\ln(x)}{a^2c^2}-\frac{b^2(3ad-bc)\ln(b\,x^2+a)}{2a^2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}-\frac{d^2(ad-3bc)\ln(d\cdots}{2c^2(a^3d^3-3a^2bc\,d^2+3a\cdots}$
risch	$\frac{\frac{bd(ad+bc)x^2}{2ac\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{a^2d^2+b^2c^2}{2ac\left(a^2d^2-2abcd+b^2c^2\right)}}{(b\,x^2+a)(d\,x^2+c)}+\frac{\ln(x)}{a^2c^2}-\frac{d^3\ln(-d\,x^2-c)a}{2c^2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{3d^2\ln(-d\,x^2-c)b}{2c(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3/a^2/(a*d-b*c)^3*((3*a*d-b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a)
)-1/2*d^3/c^2/(a*d-b*c)^3*(-c*(a*d-b*c)/d/(d*x^2+c)+(a*d-3*b*c)/d*ln(d*x^2+
c))+ln(x)/a^2/c^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(133) = 266.
time = 0.32, size = 295, normalized size = 2.09

$$-\frac{\left(b^3c-3\,ab^2d\right)\log\left(bx^2+a\right)}{2\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)}-\frac{\left(3\,bcd^2-ad^3\right)\log\left(dx^2+c\right)}{2\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)}+\frac{b^2c^2+a^2d^2+\left(b^2cd+abd^2\right)x^2}{2\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+\left(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3\right)x^4+\left(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3\right)x^2\right)}+\frac{\log\left(x^2\right)}{2\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")

[Out] -1/2*(b^3*c - 3*a*b^2*d)*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 1/2*(3*b*c*d^2 - a*d^3)*log(d*x^2 + c)/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/2*(b^2*c^2 + a^2*d^2 + (b^2*c*d + a*b*d^2)*x^2)/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2) + 1/2*log(x^2)/(a^2*c^2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(133) = 266$.
time = 5.66, size = 540, normalized size = 3.83

$$\frac{ab^3c^4 - a^2b^2c^3d + a^3bc^2d^2 - a^4cd^3 + (ab^3c^3d - a^3bcd^3)x^2 - (ab^3c^4 - 3\,a^2b^2c^3d + (b^4c^3d - 3\,ab^3c^2d^2)x^4 + (b^4c^4 - 2\,ab^3c^3d - 3\,a^2b^2c^2d^2)x^2)\log\left(bx^2 + a\right) - (3\,a^3bc^2d^2 - a^4cd^3 + (3\,a^2b^2cd^3 - a^3bd^4)x^4 + (3\,a^2b^2c^2d^2 + 2\,a^3bcd^3 - a^4d^4)x^2)\log\left(dx^2 + c\right) + 2\,(ab^3c^4 - 3\,a^2b^2c^3d + 3\,a^3bc^2d^2 - a^4cd^3 + (b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4)x^4 + (b^4c^4 - 2\,ab^3c^3d + 2\,a^3bcd^3 - a^4d^4)x^2)\log\left(x\right)}{2\,(a^3b^3c^6 - 3\,a^4b^2c^5d + 3\,a^5bc^4d^2 - a^6c^3d^3 + (a^2b^4c^5d - 3\,a^3b^3c^4d^2 + 3\,a^4b^2c^3d^3 - a^5bc^2d^4)x^4 + (a^2b^4c^6 - 2\,a^3b^3c^5d + 2\,a^5bc^3d^3 - a^6c^2d^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")

[Out] 1/2*(a*b^3*c^4 - a^2*b^2*c^3*d + a^3*b*c^2*d^2 - a^4*c*d^3 + (a*b^3*c^3*d - a^3*b*c*d^3)*x^2 - (a*b^3*c^4 - 3*a^2*b^2*c^3*d + (b^4*c^3*d - 3*a*b^3*c^2*d^2)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2)*x^2)*log(b*x^2 + a) - (3*a^3*b*c^2*d^2 - a^4*c*d^3 + (3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (3*a^2*b^2*c^2*d^2 + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)*log(d*x^2 + c) + 2*(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)*log(x))/(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3 + (a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^4 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^2)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**2,x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(133) = 266$.
time = 0.59, size = 321, normalized size = 2.28

$$-\frac{(b^4c - 3\,ab^3d)\log\left(|bx^2 + a|\right)}{2\,(a^2b^4c^3 - 3\,a^3b^3c^2d + 3\,a^4b^2cd^2 - a^5bd^3)} - \frac{(3\,bcd^3 - ad^4)\log\left(|dx^2 + c|\right)}{2\,(b^3c^5d - 3\,ab^2c^4d^2 + 3\,a^2bc^3d^3 - a^3c^2d^4)} + \frac{b^3c^2dx^4 - 2\,ab^2cd^2x^4 + a^2bd^3x^4 + b^3c^3x^2 + ab^2c^2dx^2 + a^2bcd^2x^2 + a^3d^3x^2 + 3\,ab^2c^3 - 2\,a^2bc^2d + 3\,a^3cd^2}{4\,(a^2b^2c^4 - 2\,a^3bc^3d + a^4c^2d^2)(bdx^4 + bcx^2 + adx^2 + ac)} + \frac{\log\left(x^2\right)}{2\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^4*c - 3*a*b^3*d)*log(abs(b*x^2 + a))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d
 + 3*a^4*b^2*c*d^2 - a^5*b*d^3) - 1/2*(3*b*c*d^3 - a*d^4)*log(abs(d*x^2 + c
))/(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4) + 1/4*(b^3
*c^2*d*x^4 - 2*a*b^2*c*d^2*x^4 + a^2*b*d^3*x^4 + b^3*c^3*x^2 + a*b^2*c^2*d*
x^2 + a^2*b*c*d^2*x^2 + a^3*d^3*x^2 + 3*a*b^2*c^3 - 2*a^2*b*c^2*d + 3*a^3*c
*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*d*x^4 + b*c*x^2 + a*d
*x^2 + a*c)) + 1/2*log(x^2)/(a^2*c^2)
```

Mupad [B]
time = 0.78, size = 193, normalized size = 1.37

$$\frac{\frac{a^2\,d^2+b^2\,c^2}{2\,a\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b\,d\,x^2\,\left(a\,d+b\,c\right)}{2\,a\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{b\,d\,x^4+\left(a\,d+b\,c\right)\,x^2+a\,c}+\frac{\ln\left(x\right)}{a^2\,c^2}-\frac{b^2\,\ln\left(b\,x^2+a\right)\,\left(3\,a\,d-b\,c\right)}{2\,a^2\,{\left(a\,d-b\,c\right)}^3}-\frac{d^2\,\ln\left(d\,x^2+c\right)\,\left(a\,d-3\,b\,c\right)}{2\,c^2\,{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2)/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^2*(a*
d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d + b*c) +
 b*d*x^4) + log(x)/(a^2*c^2) - (b^2*log(a + b*x^2)*(3*a*d - b*c))/(2*a^2*(a
*d - b*c)^3) - (d^2*log(c + d*x^2)*(a*d - 3*b*c))/(2*c^2*(a*d - b*c)^3)
```

3.307 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x}+\frac{d(bc+ad)}{2ac(bc-ad)^2x\,(c+dx^2)}+\frac{b}{2a(bc-ad)x\,(a+bx^2)\,(c+dx^2)}-\frac{b^{5/2}(3bc-7ad)\tan^{-}}{2a^{5/2}(bc-ac}$$

```
[Out] 1/2*(-3*a^2*d^2+4*a*b*c*d-3*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/x+1/2*d*(a*d+b*c)
/a/c/(-a*d+b*c)^2/x/(d*x^2+c)+1/2*b/a/(-a*d+b*c)/x/(b*x^2+a)/(d*x^2+c)-1/2*
b^(5/2)*(-7*a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^3-1/2*d
^(5/2)*(-3*a*d+7*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.21, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {483, 593, 597, 536, 211}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3bc-7ad)}{2a^{5/2}(bc-ad)^3}-\frac{3a^2d^2-4abcd+3b^2c^2}{2a^2c^2x(bc-ad)^2}-\frac{d^{5/2}(7bc-3ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^3}+\frac{b}{2ax\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{d(ad+bc)}{2acx\,(c+dx^2)\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(3*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)/(a^2*c^2*(b*c - a*d)^2*x) + (d*(b*
c + a*d))/(2*a*c*(b*c - a*d)^2*x*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x*(a + b
*x^2)*(c + d*x^2)) - (b^(5/2)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*(7*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(2*c^(5/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2} \, dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-3bc+2ad-5bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^2} \, dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-2(3b^2c^2-$$

$$= -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+\right.}$$

$$= -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+\right.}$$

$$= -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+\right.}$$

Mathematica [A]

time = 0.22, size = 158, normalized size = 0.72

$$\frac{1}{2}\left(-\frac{2}{a^2c^2x}-\frac{b^3x}{a^2(bc-ad)^2\left(a+bx^2\right)}-\frac{d^3x}{c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^3}+\frac{d^{5/2}(-7bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-2/(a^2*c^2*x) - (b^3*x)/(a^2*(b*c - a*d)^2*(a + b*x^2)) - (d^3*x)/(c^2*(b
*c - a*d)^2*(c + d*x^2)) + (b^(5/2)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(5/2)*(-(b*c) + a*d)^3) + (d^(5/2)*(-7*b*c + 3*a*d)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/2
```

Maple [A]
time = 0.22, size = 141, normalized size = 0.65

method	result	size
default	$-\frac{b^3\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(7ad-3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2(ad-bc)^3}-\frac{d^3\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{d\,x^2+c}+\frac{(3ad-7bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{c^2(ad-bc)^3}-\frac{1}{a^2c^2x}$	141
risch	Expression too large to display	2040

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/a^2/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(7*a*d-3*b*c)/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2)))-d^3/c^2/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(d
*x^2+c)+1/2*(3*a*d-7*b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))-1/a^2/c^2/x
```

Maxima [A]
time = 0.55, size = 378, normalized size = 1.73

$$-\frac{(3\,b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}}-\frac{(7\,bcd^3-3\,ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}}-\frac{2\,ab^2c^3-4\,a^2bc^2d+2\,a^3cd^2+(3\,b^3c^2d-4\,ab^2cd^2+3\,a^2bd^3)x^4+(3\,b^3c^3-2\,ab^2c^2d-2\,a^2bcd^2+3\,a^3d^3)x^2}{2\,((a^2b^3c^4d-2\,a^3b^2c^3d^2+a^4bc^2d^3)x^5+(a^2b^3c^5-a^3b^2c^4d-a^4bc^3d^2+a^5c^2d^3)x^3+(a^3b^2c^5-2\,a^4bc^4d+a^5c^3d^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*
c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - 3*a*d^4)*arc
tan(d*x/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^
3)*sqrt(c*d)) - 1/2*(2*a*b^2*c^3 - 4*a^2*b*c^2*d + 2*a^3*c*d^2 + (3*b^3*c^2
*d - 4*a*b^2*c*d^2 + 3*a^2*b*d^3)*x^4 + (3*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*
```

```
b*c*d^2 + 3*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d
^3)*x^5 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3 +
 (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(192) = 384$.
time = 2.97, size = 2113, normalized size = 9.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2*
(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(3*
b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 - ((3*b^4*c^3*d -
7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3
+ (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/
a) - a)/(b*x^2 + a)) - ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^
2*d^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)
*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^4*c^5*
d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c
^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 -
3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*x), -1/4*(4*a*b^3*c^4 - 12
*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2*(3*b^4*c^3*d - 7*a*b^3*
c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(3*b^4*c^4 - 5*a*b^3*c^3*d
 + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + 2*((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5
+ (7*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 -
3*a^4*c*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) - ((3*b^4*c^3*d - 7*a*b^3*c^2
*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3 + (3*a*b^3*
c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*
c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d
^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*x
), -1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 +
2*(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(
3*b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + 2*((3*b^4*c^3*
d - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*
x^3 + (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) - ((
7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 3
*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)*sqrt(-d/c)*log((d*x^2 -
2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3
*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*
a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b
*c^4*d^2 - a^6*c^3*d^3)*x), -1/2*(2*a*b^3*c^4 - 6*a^2*b^2*c^3*d + 6*a^3*b*c
```

```
^2*d^2 - 2*a^4*c*d^3 + (3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3
*a^3*b*d^4)*x^4 + (3*b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x
^2 + ((3*b^4*c^3*d - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*
a^2*b^2*c^2*d^2)*x^3 + (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(b/a)*arctan(
x*sqrt(b/a)) + ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d^2 +
4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)*sqrt(d/
c)*arctan(x*sqrt(d/c)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3
*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^
3 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a
^6*c^3*d^3)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.03, size = 321, normalized size = 1.47

$$-\frac{(3\,b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}}-\frac{(7\,bcd^3-3\,ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}}-\frac{3\,b^3c^2dx^4-4\,ab^2cd^2x^4+3\,a^2bd^3x^4+3\,b^3c^3x^2-2\,ab^2c^2dx^2-2\,a^2bcd^2x^2+3\,a^3d^3x^2+2\,ab^2c^3-4\,a^2bc^2d+2\,a^3cd^2}{2\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2)(bdx^5+bcx^3+adx^3+acx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*
c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - 3*a*d^4)*arc
tan(d*x/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^
3)*sqrt(c*d)) - 1/2*(3*b^3*c^2*d*x^4 - 4*a*b^2*c*d^2*x^4 + 3*a^2*b*d^3*x^4
+ 3*b^3*c^3*x^2 - 2*a*b^2*c^2*d*x^2 - 2*a^2*b*c*d^2*x^2 + 3*a^3*d^3*x^2 + 2
*a*b^2*c^3 - 4*a^2*b*c^2*d + 2*a^3*c*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a
^4*c^2*d^2)*(b*d*x^5 + b*c*x^3 + a*d*x^3 + a*c*x))
```

Mupad [**B**]
time = 0.97, size = 2500, normalized size = 11.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] - (1/(a*c) + (x^2*(3*a^3*d^3 + 3*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*b*c*d^2))/
(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^4*(3*a^2*d^2 + 3*b^2*c
^2 - 4*a*b*c*d))/(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^3*(a*d + b
*c) + a*c*x + b*d*x^5) - (atan((a^7*d^3*x*(-c^5*d^5)^(3/2)*9i + b^7*c^12*d*
x*(-c^5*d^5)^(1/2)*9i + a^2*b^5*c^10*d^3*x*(-c^5*d^5)^(1/2)*49i - a^6*b*c*d
^2*x*(-c^5*d^5)^(3/2)*42i + a^5*b^2*c^2*d*x*(-c^5*d^5)^(3/2)*49i - a*b^6*c^
11*d^2*x*(-c^5*d^5)^(1/2)*42i)/(9*a^7*c^8*d^10 - 9*b^7*c^15*d^3 + 42*a*b^6*
c^14*d^4 - 42*a^6*b*c^9*d^9 - 49*a^2*b^5*c^13*d^5 + 49*a^5*b^2*c^10*d^8))*(
3*a*d - 7*b*c)*(-c^5*d^5)^(1/2)*1i)/(2*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6
*d^2 - 3*a*b^2*c^7*d)) - (atan((((7*a*d - 3*b*c)*(x*(144*a^6*b^15*c^18*d^3
- 1536*a^7*b^14*c^17*d^4 + 6976*a^8*b^13*c^16*d^5 - 17664*a^9*b^12*c^15*d^6
 + 28144*a^10*b^11*c^14*d^7 - 32000*a^11*b^10*c^13*d^8 + 31872*a^12*b^9*c^1
2*d^9 - 32000*a^13*b^8*c^11*d^10 + 28144*a^14*b^7*c^10*d^11 - 17664*a^15*b^
6*c^9*d^12 + 6976*a^16*b^5*c^8*d^13 - 1536*a^17*b^4*c^7*d^14 + 144*a^18*b^3
*c^6*d^15) - ((7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*(192*a^8*b^15*c^21*d^2 - 217
6*a^9*b^14*c^20*d^3 + 10944*a^10*b^13*c^19*d^4 - 31808*a^11*b^12*c^18*d^5 +
 57600*a^12*b^11*c^17*d^6 - 62784*a^13*b^10*c^16*d^7 + 28032*a^14*b^9*c^15*
d^8 + 28032*a^15*b^8*c^14*d^9 - 62784*a^16*b^7*c^13*d^10 + 57600*a^17*b^6*c
^12*d^11 - 31808*a^18*b^5*c^11*d^12 + 10944*a^19*b^4*c^10*d^13 - 2176*a^20*
b^3*c^9*d^14 + 192*a^21*b^2*c^8*d^15 - (x*(7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*
(256*a^10*b^15*c^23*d^2 - 2816*a^11*b^14*c^22*d^3 + 13824*a^12*b^13*c^21*d^
4 - 39424*a^13*b^12*c^20*d^5 + 70400*a^14*b^11*c^19*d^6 - 76032*a^15*b^10*c
^18*d^7 + 33792*a^16*b^9*c^17*d^8 + 33792*a^17*b^8*c^16*d^9 - 76032*a^18*b^
7*c^15*d^10 + 70400*a^19*b^6*c^14*d^11 - 39424*a^20*b^5*c^13*d^12 + 13824*a
^21*b^4*c^12*d^13 - 2816*a^22*b^3*c^11*d^14 + 256*a^23*b^2*c^10*d^15))/(4*(
a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2))))/(4*(a^8*d^3 - a
^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b^5)^(1/2)*1i)/(4*(a^
8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)) + ((7*a*d - 3*b*c)*
(x*(144*a^6*b^15*c^18*d^3 - 1536*a^7*b^14*c^17*d^4 + 6976*a^8*b^13*c^16*d^5
 - 17664*a^9*b^12*c^15*d^6 + 28144*a^10*b^11*c^14*d^7 - 32000*a^11*b^10*c^1
3*d^8 + 31872*a^12*b^9*c^12*d^9 - 32000*a^13*b^8*c^11*d^10 + 28144*a^14*b^7
*c^10*d^11 - 17664*a^15*b^6*c^9*d^12 + 6976*a^16*b^5*c^8*d^13 - 1536*a^17*b
^4*c^7*d^14 + 144*a^18*b^3*c^6*d^15) + ((7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*(1
92*a^8*b^15*c^21*d^2 - 2176*a^9*b^14*c^20*d^3 + 10944*a^10*b^13*c^19*d^4 -
31808*a^11*b^12*c^18*d^5 + 57600*a^12*b^11*c^17*d^6 - 62784*a^13*b^10*c^16*
d^7 + 28032*a^14*b^9*c^15*d^8 + 28032*a^15*b^8*c^14*d^9 - 62784*a^16*b^7*c^
13*d^10 + 57600*a^17*b^6*c^12*d^11 - 31808*a^18*b^5*c^11*d^12 + 10944*a^19*
b^4*c^10*d^13 - 2176*a^20*b^3*c^9*d^14 + 192*a^21*b^2*c^8*d^15 + (x*(7*a*d
- 3*b*c)*(-a^5*b^5)^(1/2)*(256*a^10*b^15*c^23*d^2 - 2816*a^11*b^14*c^22*d^3
 + 13824*a^12*b^13*c^21*d^4 - 39424*a^13*b^12*c^20*d^5 + 70400*a^14*b^11*c^
19*d^6 - 76032*a^15*b^10*c^18*d^7 + 33792*a^16*b^9*c^17*d^8 + 33792*a^17*b^
8*c^16*d^9 - 76032*a^18*b^7*c^15*d^10 + 70400*a^19*b^6*c^14*d^11 - 39424*a^
20*b^5*c^13*d^12 + 13824*a^21*b^4*c^12*d^13 - 2816*a^22*b^3*c^11*d^14 + 256
*a^23*b^2*c^10*d^15))/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b
*c*d^2))))/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(
```

```
-a^5*b^5)^(1/2)*1i)/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c
*d^2)))/(((7*a*d - 3*b*c)*(x*(144*a^6*b^15*c^18*d^3 - 1536*a^7*b^14*c^17*d^
4 + 6976*a^8*b^13*c^16*d^5 - 17664*a^9*b^12*c^15*d^6 + 28144*a^10*b^11*c^14
*d^7 - 32000*a^11*b^10*c^13*d^8 + 31872*a^12*b^9*c^12*d^9 - 32000*a^13*b^8*
c^11*d^10 + 28144*a^14*b^7*c^10*d^11 - 17664*a^15*b^6*c^9*d^12 + 6976*a^16*
b^5*c^8*d^13 - 1536*a^17*b^4*c^7*d^14 + 144*a^18*b^3*c^6*d^15) + ((7*a*d -
3*b*c)*(-a^5*b^5)^(1/2)*(192*a^8*b^15*c^21*d^2 - 2176*a^9*b^14*c^20*d^3 + 1
0944*a^10*b^13*c^19*d^4 - 31808*a^11*b^12*c^18*d^5 + 57600*a^12*b^11*c^17*d
^6 - 62784*a^13*b^10*c^16*d^7 + 28032*a^14*b^9*c^15*d^8 + 28032*a^15*b^8*c^
14*d^9 - 62784*a^16*b^7*c^13*d^10 + 57600*a^17*b^6*c^12*d^11 - 31808*a^18*b
^5*c^11*d^12 + 10944*a^19*b^4*c^10*d^13 - 2176*a^20*b^3*c^9*d^14 + 192*a^21
*b^2*c^8*d^15 + (x*(7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*(256*a^10*b^15*c^23*d^2
 - 2816*a^11*b^14*c^22*d^3 + 13824*a^12*b^13*c^21*d^4 - 39424*a^13*b^12*c^2
0*d^5 + 70400*a^14*b^11*c^19*d^6 - 76032*a^15*b^10*c^18*d^7 + 33792*a^16*b^
9*c^17*d^8 + 33792*a^17*b^8*c^16*d^9 - 76032*a^18*b^7*c^15*d^10 + 70400*a^1
9*b^6*c^14*d^11 - 39424*a^20*b^5*c^13*d^12 + 13824*a^21*b^4*c^12*d^13 - 281
6*a^22*b^3*c^11*d^14 + 256*a^23*b^2*c^10*d^15))/(4*(a^8*d^3 - a^5*b^3*c^3 +
 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2))))/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*
c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b^5)^(1/2))/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^
6*b^2*c^2*d - 3*a^7*b*c*d^2)) - ((7*a*d - 3*b*c)*(x*(144*a^6*b^15*c^18*d^3
- 1536*a^7*b^14*c^17*d^4 + 6976*a^8*b^13*c^16*d^5 - 17664*a^9*b^12*c^15*d^6
 + 28144*a^10*b^11*c^14*d^7 - 32000*a^11*b^10*c...
```

3.308 $\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{1}{2a^2c^2x^2}-\frac{b^3}{2a^2(bc-ad)^2\left(a+bx^2\right)}-\frac{d^3}{2c^2(bc-ad)^2\left(c+dx^2\right)}-\frac{2(bc+ad)\log(x)}{a^3c^3}+\frac{b^3(bc-2ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^3}$$

```
[Out] -1/2/a^2/c^2/x^2-1/2*b^3/a^2/(-a*d+b*c)^2/(b*x^2+a)-1/2*d^3/c^2/(-a*d+b*c)^
2/(d*x^2+c)-2*(a*d+b*c)*ln(x)/a^3/c^3+b^3*(-2*a*d+b*c)*ln(b*x^2+a)/a^3/(-a*
d+b*c)^3+d^3*(-a*d+2*b*c)*ln(d*x^2+c)/c^3/(-a*d+b*c)^3
```

Rubi [A]

time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$\frac{b^3(bc-2ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^3}-\frac{2\log(x)(ad+bc)}{a^3c^3}-\frac{b^3}{2a^2\left(a+bx^2\right)(bc-ad)^2}-\frac{1}{2a^2c^2x^2}+\frac{d^3(2bc-ad)\log\left(c+dx^2\right)}{c^3(bc-ad)^3}-\frac{d^3}{2c^2\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/2*1/(a^2*c^2*x^2) - b^3/(2*a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(2*c^2*(
b*c - a*d)^2*(c + d*x^2)) - (2*(b*c + a*d)*Log[x])/(a^3*c^3) + (b^3*(b*c -
2*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^3) + (d^3*(2*b*c - a*d)*Log[c + d*x
^2])/(c^3*(b*c - a*d)^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2c^2x^2} - \frac{2(bc+ad)}{a^3c^3x} + \frac{b^4}{a^2(-bc+ad)^2(a+bx)^2} + \frac{2b^4(-bc}{a^3(-bc+a}\right.\right.$$
$$= -\frac{1}{2a^2c^2x^2} - \frac{b^3}{2a^2(bc-ad)^2(a+bx^2)} - \frac{d^3}{2c^2(bc-ad)^2(c+dx^2)} - \frac{2(bc+ad}{a^3c}$$

Mathematica [A]
time = 0.14, size = 157, normalized size = 1.01

$$\frac{1}{2}\left(-\frac{1}{a^2c^2x^2} - \frac{b^3}{a^2(bc-ad)^2(a+bx^2)} - \frac{d^3}{c^2(bc-ad)^2(c+dx^2)} - \frac{4(bc+ad)\log(x)}{a^3c^3} + \frac{2b^3(-bc+2ad)\log(a+bx^2)}{a^3(-bc+ad)^3} + \frac{2d^3(2bc-ad)\log(c+dx^2)}{c^3(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-(1/(a^2*c^2*x^2)) - b^3/(a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(c^2*(b*c -
 a*d)^2*(c + d*x^2)) - (4*(b*c + a*d)*Log[x])/(a^3*c^3) + (2*b^3*(-(b*c) +
2*a*d)*Log[a + b*x^2])/(a^3*(-(b*c) + a*d)^3) + (2*d^3*(2*b*c - a*d)*Log[c
+ d*x^2])/(c^3*(b*c - a*d)^3))/2
```

Maple [A]
time = 0.19, size = 157, normalized size = 1.01

method	result
default	$\frac{b^4\left(\frac{(4ad-2bc)\ln\left(bx^2+a\right)}{b}-\frac{a(ad-bc)}{b\left(bx^2+a\right)}\right)}{2a^3(ad-bc)^3}+\frac{d^4\left(-\frac{c(ad-bc)}{d\left(dx^2+c\right)}+\frac{(2ad-4bc)\ln\left(dx^2+c\right)}{d}\right)}{2c^3(ad-bc)^3}-\frac{1}{2a^2c^2x^2}+\frac{(-2ad-2bc)\ln(x)}{a^3c^3}$
norman	$\frac{-\frac{1}{2ac}+\frac{\left(2a^4d^4-a^3bc\,d^3-a\,b^3c^3d+2b^4c^4\right)x^4}{2c^3a^3\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(2a^3d^3-a^2bc\,d^2-a\,b^2c^2d+2b^3c^3\right)bd\,x^6}{2c^3a^3\left(a^2d^2-2abcd+b^2c^2\right)}}{x^2(b\,x^2+a)(d\,x^2+c)}+\frac{b^3(2ad-bc)\ln\left(b\,x^2+a\right)}{a^3(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{d^3(}{c^3(a^3d^3-}$
risch	$\frac{-\frac{bd\left(a^2d^2-abcd+b^2c^2\right)x^4}{a^2c^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(2a^3d^3-a^2bc\,d^2-a\,b^2c^2d+2b^3c^3\right)x^2}{2a^2c^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{1}{2ac}}{x^2(b\,x^2+a)(d\,x^2+c)}-\frac{2\ln(x)d}{a^2c^3}-\frac{2\ln(x)b}{a^3c^2}+\frac{d^4\ln\left(-d\,x^2-c\right)a}{c^3(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^4/a^3/(a*d-b*c)^3*((4*a*d-2*b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a
))+1/2*d^4/c^3/(a*d-b*c)^3*(-c*(a*d-b*c)/d/(d*x^2+c)+(2*a*d-4*b*c)/d*ln(d*x
^2+c))-1/2/a^2/c^2/x^2+(-2*a*d-2*b*c)/a^3/c^3*ln(x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(150) = 300$.
time = 0.31, size = 381, normalized size = 2.44

$$\frac{(b^4c - 2\,ab^3d)\log\left(bx^2 + a\right)}{a^3b^3c^3 - 3\,a^4b^2c^2d + 3\,a^5bcd^2 - a^6d^3} + \frac{(2\,bcd^3 - ad^4)\log\left(dx^2 + c\right)}{b^3c^6 - 3\,ab^2c^5d + 3\,a^2bc^4d^2 - a^3c^3d^3} - \frac{ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + 2\,(b^3c^2d - ab^2cd^2 + a^2bd^3)x^4 + (2\,b^3c^3 - ab^2c^2d - a^2bcd^2 + 2\,a^3d^3)x^2}{2\,((a^2b^3c^4d - 2\,a^3b^2c^3d^2 + a^4bc^2d^3)x^6 + (a^2b^3c^5 - a^3b^2c^4d - a^4bc^3d^2 + a^5c^2d^3)x^4 + (a^3b^2c^5 - 2\,a^4bc^4d + a^5c^3d^2)x^2)} - \frac{(bc + ad)\log\left(x^2\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b^4*c - 2*a*b^3*d)*log(b*x^2 + a)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b
*c*d^2 - a^6*d^3) + (2*b*c*d^3 - a*d^4)*log(d*x^2 + c)/(b^3*c^6 - 3*a*b^2*c
^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3) - 1/2*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^
3*c*d^2 + 2*(b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (2*b^3*c^3 - a*b^2*
c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 +
 a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^
2*d^3)*x^4 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*x^2) - (b*c + a*d)
*log(x^2)/(a^3*c^3)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(150) = 300$.
time = 9.47, size = 667, normalized size = 4.28

$$-\frac{a^2b^3c^5 - 3\,a^3b^2c^4d + 3\,a^4bc^3d^2 - a^5c^2d^3 + 2\,(ab^4c^4d - 2\,a^2b^3c^3d^2 + 2\,a^3b^2c^2d^3 - a^4bcd^4)x^4 + (2\,ab^4c^5 - 3\,a^2b^3c^4d + 3\,a^4bc^2d^3 - 2\,a^5cd^4)x^2 - 2\,((b^5c^4d - 2\,ab^4c^3d^2)x^6 + (b^5c^5 - ab^4c^4d - 2\,a^2b^3c^3d^2)x^4 + (ab^4c^5 - 2\,a^2b^3c^4d)x^2)\log\left(bx^2 + a\right) - 2\,((2\,a^3b^2cd^4 - a^4bd^5)x^6 + (2\,a^3b^2c^2d^3 + a^4bcd^4 - a^5d^5)x^4 + (2\,a^4bc^2d^3 - a^5cd^4)x^2)\log\left(dx^2 + c\right) + 4\,((b^5c^4d - 2\,ab^4c^3d^2 + 2\,a^3b^2cd^4 - a^4bd^5)x^6 + (b^5c^5 - ab^4c^4d - 2\,a^2b^3c^3d^2 + 2\,a^3b^2c^2d^3 + a^4bcd^4 - a^5d^5)x^4 + (ab^4c^5 - 2\,a^2b^3c^4d + 2\,a^4bc^2d^3 - a^5cd^4)x^2)\log\left(x\right)}{2\,((a^3b^4c^6d - 3\,a^4b^3c^5d^2 + 3\,a^5b^2c^4d^3 - a^6bc^3d^4)x^6 + (a^3b^4c^7 - 2\,a^4b^3c^6d + 2\,a^6bc^4d^3 - a^7c^3d^4)x^4 + (a^4b^3c^7 - 3\,a^5b^2c^6d + 3\,a^6bc^5d^2 - a^7c^4d^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + 2*(a*
b^4*c^4*d - 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (2*a
*b^4*c^5 - 3*a^2*b^3*c^4*d + 3*a^4*b*c^2*d^3 - 2*a^5*c*d^4)*x^2 - 2*((b^5*c
^4*d - 2*a*b^4*c^3*d^2)*x^6 + (b^5*c^5 - a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2)*x
^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d)*x^2)*log(b*x^2 + a) - 2*((2*a^3*b^2*c*d^
4 - a^4*b*d^5)*x^6 + (2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x^4 + (2*a
^4*b*c^2*d^3 - a^5*c*d^4)*x^2)*log(d*x^2 + c) + 4*((b^5*c^4*d - 2*a*b^4*c^3
*d^2 + 2*a^3*b^2*c*d^4 - a^4*b*d^5)*x^6 + (b^5*c^5 - a*b^4*c^4*d - 2*a^2*b^
3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x^4 + (a*b^4*c^5 - 2
*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)*log(x))/((a^3*b^4*c^6*d
- 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^6 + (a^3*b^4*c^7
 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^4 + (a^4*b^3*c^7 - 3*
a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(150) = 300.
time = 0.72, size = 333, normalized size = 2.13

$$\frac{(b^5c-2\,ab^4d)\log\left(|bx^2+a|\right)}{a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3}+\frac{(2\,bcd^4-ad^5)\log\left(|dx^2+c|\right)}{b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4}-\frac{2\,b^3c^2dx^4-2\,ab^2cd^2x^4+2\,a^2bd^3x^4+2\,b^3c^3x^2-ab^2c^2dx^2-a^2bcd^2x^2+2\,a^3d^3x^2+ab^2c^3-2\,a^2bc^2d+a^3cd^2}{2\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2)(bdx^6+bcx^4+adx^4+acx^2)}-\frac{(bc+ad)\log\left(x^2\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (b^5*c - 2*a*b^4*d)*log(abs(b*x^2 + a))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*
a^5*b^2*c*d^2 - a^6*b*d^3) + (2*b*c*d^4 - a*d^5)*log(abs(d*x^2 + c))/(b^3*c
^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4) - 1/2*(2*b^3*c^2*d*
x^4 - 2*a*b^2*c*d^2*x^4 + 2*a^2*b*d^3*x^4 + 2*b^3*c^3*x^2 - a*b^2*c^2*d*x^2
 - a^2*b*c*d^2*x^2 + 2*a^3*d^3*x^2 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)
/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*d*x^6 + b*c*x^4 + a*d*x^4
+ a*c*x^2)) - (b*c + a*d)*log(x^2)/(a^3*c^3)
```

Mupad [B]
time = 0.90, size = 313, normalized size = 2.01

$$-\frac{\frac{1}{2\,a\,c}+\frac{x^4\,\left(a^2\,b\,d^3-a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}{a^2\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{x^2\,\left(a\,d+b\,c\right)\,\left(2\,a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2\right)}{2\,a^2\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^4+a\,c\,x^2}-\frac{\ln\left(b\,x^2+a\right)\,\left(b^4\,c-2\,a\,b^3\,d\right)}{a^6\,d^3-3\,a^5\,b\,c\,d^2+3\,a^4\,b^2\,c^2\,d-a^3\,b^3\,c^3}-\frac{\ln\left(d\,x^2+c\right)\,\left(a\,d^4-2\,b\,c\,d^3\right)}{-a^3\,c^3\,d^3+3\,a^2\,b\,c^4\,d^2-3\,a\,b^2\,c^5\,d+b^3\,c^6}-\frac{\ln\left(x\right)\,\left(2\,a\,d+2\,b\,c\right)}{a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] - (1/(2*a*c) + (x^4*(a^2*b*d^3 + b^3*c^2*d - a*b^2*c*d^2))/(a^2*c^2*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(a*d + b*c)*(2*a^2*d^2 + 2*b^2*c^2 - 3*a*b
*c*d))/(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^4*(a*d + b*c) + a*c*
x^2 + b*d*x^6) - (log(a + b*x^2)*(b^4*c - 2*a*b^3*d))/(a^6*d^3 - a^3*b^3*c^
3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2) - (log(c + d*x^2)*(a*d^4 - 2*b*c*d^3))
/(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d) - (log(x)*(2*a*d
 + 2*b*c))/(a^3*c^3)
```

3.309 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^2} \, dx$

Optimal. Leaf size=271

$$-\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3}+\frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x}+\frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+dx^2\right)}+\frac{b}{2a(bc-ad)x^3\left(a\right.}$$

```
[Out] 1/6*(-5*a^2*d^2+4*a*b*c*d-5*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/x^3+1/2*(a*d+b*c)
*(5*a^2*d^2-9*a*b*c*d+5*b^2*c^2)/a^3/c^3/(-a*d+b*c)^2/x+1/2*d*(a*d+b*c)/a/c
/(-a*d+b*c)^2/x^3/(d*x^2+c)+1/2*b/a/(-a*d+b*c)/x^3/(b*x^2+a)/(d*x^2+c)+1/2*
b^(7/2)*(-9*a*d+5*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/(-a*d+b*c)^3+1/2*d
^(7/2)*(-5*a*d+9*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.30, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {483, 593, 597, 536, 211}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(5bc-9ad)}{2a^{7/2}(bc-ad)^3}-\frac{5a^2d^2-4abcd+5b^2c^2}{6a^2c^2x^3(bc-ad)^2}+\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{2a^3c^3x(bc-ad)^2}+\frac{d^{7/2}(9bc-5ad)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^3}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2acx^3\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/6*(5*b^2*c^2 - 4*a*b*c*d + 5*a^2*d^2)/(a^2*c^2*(b*c - a*d)^2*x^3) + ((b*
c + a*d)*(5*b^2*c^2 - 9*a*b*c*d + 5*a^2*d^2))/(2*a^3*c^3*(b*c - a*d)^2*x) +
 (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)) + b/(2*a*(b*c - a*d)
*x^3*(a + b*x^2)*(c + d*x^2)) + (b^(7/2)*(5*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(7/2)*(b*c - a*d)^3) + (d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-5bc+2ad-7bdx^2}{x^4(a+bx^2)(c+dx^2)^2}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-2(5b^2c}{}}{}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^3\left(a\right.}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x} + \frac{d(}{2ac(bc-}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x} + \frac{d(}{2ac(bc-}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x} + \frac{d(}{2ac(bc-}$$

Mathematica [A]
time = 0.28, size = 178, normalized size = 0.66

$$\frac{1}{6}\left(-\frac{2}{a^2c^2x^3}+\frac{12(bc+ad)}{a^3c^3x}+\frac{3b^4x}{a^3(bc-ad)^2\left(a+bx^2\right)}+\frac{3d^4x}{c^3(bc-ad)^2\left(c+dx^2\right)}+\frac{3b^{7/2}(-5bc+9ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(-bc+ad)^3}+\frac{3d^{7/2}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-2/(a^2*c^2*x^3) + (12*(b*c + a*d))/(a^3*c^3*x) + (3*b^4*x)/(a^3*(b*c - a*
d)^2*(a + b*x^2)) + (3*d^4*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (3*b^(7/2)*
(-5*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(-(b*c) + a*d)^3) +
(3*d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d
)^3))/6
```

Maple [A]
time = 0.21, size = 159, normalized size = 0.59

method	result	si
default	$\frac{b^4\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(9ad-5bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3(ad-bc)^3}+\frac{d^4\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{d\,x^2+c}+\frac{(5ad-9bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{c^3(ad-bc)^3}-\frac{1}{3a^2c^2x^3}-\frac{-2ad-2bc}{a^3c^3x}$	15

risch	Expression too large to display	243

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^4/a^3/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(9*a*d-5*b*c)/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2)))+d^4/c^3/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(d*
x^2+c)+1/2*(5*a*d-9*b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))-1/3/a^2/c^2/x
^3-(-2*a*d-2*b*c)/a^3/c^3/x
```

Maxima [A]
time = 0.55, size = 460, normalized size = 1.70

$$\frac{(5\,b^5c-9\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3)\sqrt{ab}}+\frac{(9\,bcd^4-5\,ad^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)\sqrt{cd}}-\frac{2\,a^2b^2c^4-4\,a^3bc^3d+2\,a^4c^2d^2-3\,(5\,b^4c^3d-4\,ab^3c^2d^2-4\,a^2b^2cd^3+5\,a^3bd^4)x^6-(15\,b^4c^4-2\,ab^3c^3d-20\,a^2b^2c^2d^2-2\,a^3bcd^3+15\,a^4d^4)x^4-10\,(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)x^2}{6\,((a^3b^3c^5d-2\,a^4b^2c^4d^2+a^5bc^3d^3)x^7+(a^3b^3c^6-a^4b^2c^5d-a^5bc^4d^2+a^6c^3d^3)x^5+(a^4b^2c^6-2\,a^5bc^5d+a^6c^4d^2)x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(5*b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^3*b^3*c^3 - 3*a^4*b^2*c
^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(a*b)) + 1/2*(9*b*c*d^4 - 5*a*d^5)*arct
an(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
)*sqrt(c*d)) - 1/6*(2*a^2*b^2*c^4 - 4*a^3*b*c^3*d + 2*a^4*c^2*d^2 - 3*(5*b^
4*c^3*d - 4*a*b^3*c^2*d^2 - 4*a^2*b^2*c*d^3 + 5*a^3*b*d^4)*x^6 - (15*b^4*c^
4 - 2*a*b^3*c^3*d - 20*a^2*b^2*c^2*d^2 - 2*a^3*b*c*d^3 + 15*a^4*d^4)*x^4 -
10*(a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)/((a^3*b^3*c
^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^7 + (a^3*b^3*c^6 - a^4*b^2*c^5*
d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^5 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c
^4*d^2)*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(243) = 486$.
time = 7.99, size = 2457, normalized size = 9.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d + 12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3
 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 -
2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 +
17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*
b*c^2*d^3 - a^5*c*d^4)*x^2 - 3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^
5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c
^4*d)*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*(
```

```
(9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^4 -
5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(-d/c)*log((d*x^2
 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2
+ 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d +
 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^
6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d +
12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*a^3
*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^3
*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a*b^
4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 6*((9*a^3*b^2*
c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^4 - 5*a^5*d^5)*
x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) -
3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2
*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d
^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*
d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3
*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d
 + 12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*
a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*
b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a
*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 6*((5*b^5*c
^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^
2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a))
 - 3*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*
d^4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(-d/c)*log(
(d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^
5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c
^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d
+ 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/6*(2*a^2*b^3*c^5 - 6*a^3*b^2*c^4*
d + 6*a^4*b*c^3*d^2 - 2*a^5*c^2*d^3 - 3*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*
a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - (15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^
3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 10*(a*b
^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 3*((5*b^5*c^4
*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^2)
*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
 3*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^
4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(d/c)*arctan(
x*sqrt(d/c)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6
*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^
3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3
)*x^3)]
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.81, size = 275, normalized size = 1.01

$$\frac{(5\,b^5c-9\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3)\sqrt{ab}}+\frac{(9\,bcd^4-5\,ad^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)\sqrt{cd}}+\frac{b^4c^3dx^3+a^3bd^4x^3+b^4c^4x+a^4d^4x}{2\,(a^3b^2c^5-2\,a^4bc^4d+a^5c^3d^2)(bdx^4+bcx^2+adx^2+ac)}+\frac{6\,bcx^2+6\,adx^2-ac}{3\,a^3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^3*b^3*c^3 - 3*a^4*b^2*c
^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(a*b)) + 1/2*(9*b*c*d^4 - 5*a*d^5)*arct
an(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
)*sqrt(c*d)) + 1/2*(b^4*c^3*d*x^3 + a^3*b*d^4*x^3 + b^4*c^4*x + a^4*d^4*x)/
((a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 +
 a*c)) + 1/3*(6*b*c*x^2 + 6*a*d*x^2 - a*c)/(a^3*c^3*x^3)
```

Mupad [B]
time = 1.19, size = 2500, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] (atan((a^9*d^3*x*(-c^7*d^7)^(3/2)*25i + b^9*c^16*d*x*(-c^7*d^7)^(1/2)*25i +
 a^2*b^7*c^14*d^3*x*(-c^7*d^7)^(1/2)*81i - a^8*b*c*d^2*x*(-c^7*d^7)^(3/2)*9
0i + a^7*b^2*c^2*d*x*(-c^7*d^7)^(3/2)*81i - a*b^8*c^15*d^2*x*(-c^7*d^7)^(1/
2)*90i)/(25*a^9*c^11*d^13 - 25*b^9*c^20*d^4 + 90*a*b^8*c^19*d^5 - 90*a^8*b*
c^12*d^12 - 81*a^2*b^7*c^18*d^6 + 81*a^7*b^2*c^13*d^11))*(5*a*d - 9*b*c)*(-
c^7*d^7)^(1/2)*1i)/(2*(b^3*c^10 - a^3*c^7*d^3 + 3*a^2*b*c^8*d^2 - 3*a*b^2*c
^9*d)) - (1/(3*a*c) - (5*x^2*(a*d + b*c))/(3*a^2*c^2) + (x^4*(20*a^2*b^2*c^
2*d^2 - 15*b^4*c^4 - 15*a^4*d^4 + 2*a*b^3*c^3*d + 2*a^3*b*c*d^3))/(6*a^3*c^
3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (b*d*x^6*(5*a^3*d^3 + 5*b^3*c^3 - 4*a*
b^2*c^2*d - 4*a^2*b*c*d^2))/(2*a^3*c^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x
^5*(a*d + b*c) + a*c*x^3 + b*d*x^7) + (atan((((x*(400*a^9*b^17*c^23*d^3 - 3
840*a^10*b^16*c^22*d^4 + 15936*a^11*b^15*c^21*d^5 - 37376*a^12*b^14*c^20*d^
6 + 54240*a^13*b^13*c^19*d^7 - 49920*a^14*b^12*c^18*d^8 + 29776*a^15*b^11*c
```

^17*d^9 - 18432*a^16*b^10*c^16*d^10 + 29776*a^17*b^9*c^15*d^11 - 49920*a^18
*b^8*c^14*d^12 + 54240*a^19*b^7*c^13*d^13 - 37376*a^20*b^6*c^12*d^14 + 1593
6*a^21*b^5*c^11*d^15 - 3840*a^22*b^4*c^10*d^16 + 400*a^23*b^3*c^9*d^17) - (
(9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*(320*a^12*b^16*c^26*d^2 - 3456*a^13*b^15*c
^25*d^3 + 16704*a^14*b^14*c^24*d^4 - 47616*a^15*b^13*c^23*d^5 + 89280*a^16*
b^12*c^22*d^6 - 118400*a^17*b^11*c^21*d^7 + 123072*a^18*b^10*c^20*d^8 - 119
808*a^19*b^9*c^19*d^9 + 123072*a^20*b^8*c^18*d^10 - 118400*a^21*b^7*c^17*d^
11 + 89280*a^22*b^6*c^16*d^12 - 47616*a^23*b^5*c^15*d^13 + 16704*a^24*b^4*c
^14*d^14 - 3456*a^25*b^3*c^13*d^15 + 320*a^26*b^2*c^12*d^16 - (x*(9*a*d - 5
*b*c)*(-a^7*b^7)^(1/2)*(256*a^15*b^15*c^28*d^2 - 2816*a^16*b^14*c^27*d^3 +
13824*a^17*b^13*c^26*d^4 - 39424*a^18*b^12*c^25*d^5 + 70400*a^19*b^11*c^24*
d^6 - 76032*a^20*b^10*c^23*d^7 + 33792*a^21*b^9*c^22*d^8 + 33792*a^22*b^8*c
^21*d^9 - 76032*a^23*b^7*c^20*d^10 + 70400*a^24*b^6*c^19*d^11 - 39424*a^25*
b^5*c^18*d^12 + 13824*a^26*b^4*c^17*d^13 - 2816*a^27*b^3*c^16*d^14 + 256*a^
28*b^2*c^15*d^15))/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c
d^2))))/(4(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2)))*(9
*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*1i)/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c
^2*d - 3*a^9*b*c*d^2)) + ((x*(400*a^9*b^17*c^23*d^3 - 3840*a^10*b^16*c^22*d
^4 + 15936*a^11*b^15*c^21*d^5 - 37376*a^12*b^14*c^20*d^6 + 54240*a^13*b^13*
c^19*d^7 - 49920*a^14*b^12*c^18*d^8 + 29776*a^15*b^11*c^17*d^9 - 18432*a^16
*b^10*c^16*d^10 + 29776*a^17*b^9*c^15*d^11 - 49920*a^18*b^8*c^14*d^12 + 542
40*a^19*b^7*c^13*d^13 - 37376*a^20*b^6*c^12*d^14 + 15936*a^21*b^5*c^11*d^15
 - 3840*a^22*b^4*c^10*d^16 + 400*a^23*b^3*c^9*d^17) + ((9*a*d - 5*b*c)*(-a^
7*b^7)^(1/2)*(320*a^12*b^16*c^26*d^2 - 3456*a^13*b^15*c^25*d^3 + 16704*a^14
*b^14*c^24*d^4 - 47616*a^15*b^13*c^23*d^5 + 89280*a^16*b^12*c^22*d^6 - 1184
00*a^17*b^11*c^21*d^7 + 123072*a^18*b^10*c^20*d^8 - 119808*a^19*b^9*c^19*d^
9 + 123072*a^20*b^8*c^18*d^10 - 118400*a^21*b^7*c^17*d^11 + 89280*a^22*b^6*
c^16*d^12 - 47616*a^23*b^5*c^15*d^13 + 16704*a^24*b^4*c^14*d^14 - 3456*a^25
*b^3*c^13*d^15 + 320*a^26*b^2*c^12*d^16 + (x*(9*a*d - 5*b*c)*(-a^7*b^7)^(1/
2)*(256*a^15*b^15*c^28*d^2 - 2816*a^16*b^14*c^27*d^3 + 13824*a^17*b^13*c^26
*d^4 - 39424*a^18*b^12*c^25*d^5 + 70400*a^19*b^11*c^24*d^6 - 76032*a^20*b^1
0*c^23*d^7 + 33792*a^21*b^9*c^22*d^8 + 33792*a^22*b^8*c^21*d^9 - 76032*a^23
*b^7*c^20*d^10 + 70400*a^24*b^6*c^19*d^11 - 39424*a^25*b^5*c^18*d^12 + 1382
4*a^26*b^4*c^17*d^13 - 2816*a^27*b^3*c^16*d^14 + 256*a^28*b^2*c^15*d^15))/(
4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2))))/(4*(a^10*d^
3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2)))*(9*a*d - 5*b*c)*(-a^7*
b^7)^(1/2)*1i)/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2
)))/(((x*(400*a^9*b^17*c^23*d^3 - 3840*a^10*b^16*c^22*d^4 + 15936*a^11*b^15
*c^21*d^5 - 37376*a^12*b^14*c^20*d^6 + 54240*a^13*b^13*c^19*d^7 - 49920*a^1
4*b^12*c^18*d^8 + 29776*a^15*b^11*c^17*d^9 - 18432*a^16*b^10*c^16*d^10 + 29
776*a^17*b^9*c^15*d^11 - 49920*a^18*b^8*c^14*d^12 + 54240*a^19*b^7*c^13*d^1
3 - 37376*a^20*b^6*c^12*d^14 + 15936*a^21*b^5*c^11*d^15 - 3840*a^22*b^4*c^1
0*d^16 + 400*a^23*b^3*c^9*d^17) + ((9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*(320*a^
12*b^16*c^26*d^2 - 3456*a^13*b^15*c^25*d^3 + 16704*a^14*b^14*c^24*d^4 - 476
16*a^15*b^13*c^23*d^5 + 89280*a^16*b^12*c^22*d^6 - 118400*a^17*b^11*c^21*d^

7 + 123072*a^18*b^10*c^20*d^8 - 119808*a^19*b^9*c^19*d^9 + 123072*a^20*b^8*c^18*d^10 - 118400*a^21*b^7*c^17*d^11 + 89280*a^22*b^6*c^16*d^12 - 47616*a^23*b^5*c^15*d^13 + 16704*a^24*b^4*c^14*d^14 - 3456*a^25*b^3*c^13*d^15 + 320*a^26*b^2*c^12*d^16 + (x*(9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*(256*a^15*b^15*c^28*d^2 - 2816*a^16*b^14*c^27*d^3 + 13824*a^17*b^13*c^26*d^4 - 39424*a^18*b^12*c^25*d^5 + 70400*a^19*b^11*c^24*d^6 - 76032*a^20*b^10*c^23*d^7 + 33792*a^21*b^9*c^22*d^8 + 33792*a^22*b^8*c^21*d^9 - 76032*a^23*b^7*c^20*d^10 + 70400*a^24*b^6*c^19*d^11 - 39424*a^25*b^5*c^18*d^12 + 13824*a^26*b^4*c^17*d^13 - 2816*a^27*b^3*c^16*d^14 + 256*a^28*b^2*c^15*...

3.310 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=207

$$\frac{(bc+2ad)x}{4b(bc-ad)^2(c+dx^2)^2}+\frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^2}+\frac{3(bc+3ad)x}{8(bc-ad)^3(c+dx^2)}-\frac{3\sqrt{a}\sqrt{b}(bc+ad)\tan^{-1}}{2(bc-ad)^4}$$

```
[Out] 1/4*(2*a*d+b*c)*x/b/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*a*x/b/(-a*d+b*c)/(b*x^2+a)
/(d*x^2+c)^2+3/8*(3*a*d+b*c)*x/(-a*d+b*c)^3/(d*x^2+c)-3/2*(a*d+b*c)*arctan(
x*b^(1/2)/a^(1/2))*a^(1/2)*b^(1/2)/(-a*d+b*c)^4+3/8*(a^2*d^2+6*a*b*c*d+b^2*
c^2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^4/c^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {481, 541, 536, 211}

$$\frac{3(a^2d^2+6abcd+b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\sqrt{d}(bc-ad)^4}-\frac{3\sqrt{a}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(ad+bc)}{2(bc-ad)^4}+\frac{3x(3ad+bc)}{8(c+dx^2)(bc-ad)^3}+\frac{x(2ad+bc)}{4b(c+dx^2)^2(bc-ad)^2}+\frac{ax}{2b(a+bx^2)(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((b*c + 2*a*d)*x)/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*x)/(2*b*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)^2) + (3*(b*c + 3*a*d)*x)/(8*(b*c - a*d)^3*(c + d*
x^2)) - (3*Sqrt[a]*Sqrt[b]*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*(b*c
 - a*d)^4) + (3*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(8*Sqrt[c]*Sqrt[d]*(b*c - a*d)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\int \frac{ac+(-2bc-3ad)x^2}{(a+bx^2)(c+dx^2)^3}\,dx}{2b(bc-ad)}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2(c+dx^2)^2} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\int \frac{6abc^2-6bc(bc+2ad)}{(a+bx^2)(c+dx^2)^2}}{8bc(bc-ad)}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2(c+dx^2)^2} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3(c+}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2(c+dx^2)^2} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3(c+}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2(c+dx^2)^2} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3(c+}$$

Mathematica [A]
time = 0.25, size = 166, normalized size = 0.80

$$\frac{\frac{4ab(bc-ad)x}{a+bx^2} + \frac{2c(bc-ad)^2x}{(c+dx^2)^2} + \frac{(bc-ad)(3bc+5ad)x}{c+dx^2} - 12\sqrt{a}\sqrt{b}(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) + \frac{3(b^2c^2+6abcd+a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}}{8(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((4*a*b*(b*c - a*d)*x)/(a + b*x^2) + (2*c*(b*c - a*d)^2*x)/(c + d*x^2)^2 +
((b*c - a*d)*(3*b*c + 5*a*d)*x)/(c + d*x^2) - 12*Sqrt[a]*Sqrt[b]*(b*c + a*d
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + (3*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(8*(b*c - a*d)^4)
```

Maple [A]
time = 0.24, size = 177, normalized size = 0.86

method	result
default	$-\frac{ab\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{3(ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(ad-bc)^{4}}+\frac{\frac{\left(-\frac{5}{8}a^{2}d^{3}+\frac{1}{4}abc\,d^{2}+\frac{3}{8}b^{2}c^{2}d\right)x^{3}-\frac{c\left(3a^{2}d^{2}+2abcd-5b^{2}c^{2}\right)x}{8}}{\left(d\,x^{2}+c\right)^{2}}+\frac{3\left(a^{2}d^{2}+6abcd+b^{2}c^{2}\right)\text{a}}{8\sqrt{cd}}}{(ad-bc)^{4}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -a*b/(a*d-b*c)^4*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+3/2*(a*d+b*c)/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2)))+1/(a*d-b*c)^4*(((-5/8*a^2*d^3+1/4*a*b*c*d^2+3/8*b^2
*c^2*d)*x^3-1/8*c*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)*x)/(d*x^2+c)^2+3/8*(a^2*d
^2+6*a*b*c*d+b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(181) = 362.
time = 0.58, size = 443, normalized size = 2.14

$$-\frac{3\,(ab^2c+a^2bd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{ab}}+\frac{3\,(b^2c^2+6\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{cd}}+\frac{3\,(b^2cd+3\,abd^2)x^5+(5\,b^2c^2+14\,abcd+5\,a^2d^2)x^3+3\,(3\,abc^2+a^2cd)x}{8\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^6+(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5)x^4+(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -3/2*(a*b^2*c + a^2*b*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) + 3/8*(b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(c*d)) + 1/8*(3*(b^2*c*d + 3*a*b
*d^2)*x^5 + (5*b^2*c^2 + 14*a*b*c*d + 5*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + a^2*c
*d)*x)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*
c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d
 - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*
c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. 2(181) = 362.
time = 2.84, size = 2859, normalized size = 13.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(b^3*c^3*d^2 + 2*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + 2*(5*b^3*c^4
*d + 9*a*b^2*c^3*d^2 - 9*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3 + 12*(a*b*c^4*d +
 a^2*c^3*d^2 + (b^2*c^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d
^3 + a^2*c*d^4)*x^4 + (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt
(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 3*(a*b^2*c^4 + 6*a^2
*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^6 + (2
*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8
*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 -
 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 6*(3*a*b^2*c^4*d - 2*a^2*b*c^3*d^2 - a^
3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^3 - 4*a^4*
b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2*b^3*c^3*d^
5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a*b^4*c^5*d^3
 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a^5*c*d^7)*x^4
 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b^2*c^4*d^4 - 7
*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/8*(3*(b^3*c^3*d^2 + 2*a*b^2*c^2*d^3
 - 3*a^2*b*c*d^4)*x^5 + (5*b^3*c^4*d + 9*a*b^2*c^3*d^2 - 9*a^2*b*c^2*d^3 -
5*a^3*c*d^4)*x^3 + 3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^
2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^
2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*
a^3*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + 6*(a*b*c^4*d + a^2*c^3*d^
2 + (b^2*c^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d^3 + a^2*c*
d^4)*x^4 + (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 3*(3*a*b^2*c^4*d - 2*a^2*b*c^3*
d^2 - a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^3
- 4*a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2*b^
3*c^3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a*b^4
*c^5*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a^5*c*
d^7)*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b^2*c^4
*d^4 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/16*(6*(b^3*c^3*d^2 + 2*a*b^
2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + 2*(5*b^3*c^4*d + 9*a*b^2*c^3*d^2 - 9*a^2*b
*c^2*d^3 - 5*a^3*c*d^4)*x^3 - 24*(a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3 +
a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + (b^2*c^4
*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) -
3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 +
 a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4
)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*sqr
t(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 6*(3*a*b^2*c^4*d -
2*a^2*b*c^3*d^2 - a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*
b^2*c^5*d^3 - 4*a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^
4 + 6*a^2*b^3*c^3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d
^2 - 7*a*b^4*c^5*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*
```

```
d^6 + a^5*c*d^7)*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8
*a^3*b^2*c^4*d^4 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/8*(3*(b^3*c^3*d
^2 + 2*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + (5*b^3*c^4*d + 9*a*b^2*c^3*d^2
- 9*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3 - 12*(a*b*c^4*d + a^2*c^3*d^2 + (b^2*c
^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d^3 + a^2*c*d^4)*x^4 +
 (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b
)*x/a) + 3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^
2*c*d^3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3
+ a^3*d^4)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)
*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + 3*(3*a*b^2*c^4*d - 2*a^2*b*c^3*d^2
- a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^3 - 4*
a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2*b^3*c^
3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a*b^4*c^5
*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a^5*c*d^7)
*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b^2*c^4*d^4
 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.84, size = 301, normalized size = 1.45

$$\frac{abx}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(bx^2+a)}-\frac{3\,(ab^2c+a^2bd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{ab}}+\frac{3\,(b^2c^2+6\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{cd}}+\frac{3\,bcdx^3+5\,ad^2x^3+5\,bc^2x+3\,acdx}{8\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*a*b*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a))
 - 3/2*(a*b^2*c + a^2*b*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) + 3/8*(b^2*c^2 +
6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(c*d)) + 1/8*(3*b*c*d*x^3 + 5*
a*d^2*x^3 + 5*b*c^2*x + 3*a*c*d*x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*(d*x^2 + c)^2)
```

Mupad [B]
time = 1.59, size = 2500, normalized size = 12.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^3),x)

[Out] (atan(((((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))) - (3*(-a*b)^(1/2)*(((3*a^10*b^2*d^11)/2 + (9*a*b^11*c^9*d^2)/2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^8*d^3)/2 + 114*a^3*b^9*c^7*d^4 - 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6 - 147*a^6*b^6*c^4*d^7 + 42*a^7*b^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) - (3*x*(-a*b)^(1/2)*(a*d + b*c)*(256*a^9*b^2*d^11 + 256*b^11*c^9*d^2 - 1792*a*b^10*c^8*d^3 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d^4 - 7168*a^3*b^8*c^6*d^5 + 3584*a^4*b^7*c^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7168*a^6*b^5*c^3*d^8 + 5120*a^7*b^4*c^2*d^9))/(128*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)))*(a*d + b*c))/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(-a*b)^(1/2)*(a*d + b*c)*3i)/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)) + (3*(-a*b)^(1/2)*(((3*a^10*b^2*d^11)/2 + (9*a*b^11*c^9*d^2)/2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^8*d^3)/2 + 114*a^3*b^9*c^7*d^4 - 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6 - 147*a^6*b^6*c^4*d^7 + 42*a^7*b^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (3*x*(-a*b)^(1/2)*(a*d + b*c)*(256*a^9*b^2*d^11 + 256*b^11*c^9*d^2 - 1792*a*b^10*c^8*d^3 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d^4 - 7168*a^3*b^8*c^6*d^5 + 3584*a^4*b^7*c^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7168*a^6*b^5*c^3*d^8 + 5120*a^7*b^4*c^2*d^9))/(128*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)))*(a*d + b*c))/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(-a*b)^(1/2)*(a*d + b*c)*3i)/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))/(((81*a^5*b^3*d^5)/64 + (297*a^4*b^4*c*d^4)/32 + (135*a^2*b^6*c^3*d^2)/32 + (189*a^3*b^5*c^2*d^3)/16 + (27*a*b^7*c^4*d)/64)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) - (3*((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -

```
 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)) - (3*(-a*b)^(1/2)*(((3*a^10*b^2*d^11)/2 +
(9*a*b^11*c^9*d^2)/2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^8*d^3)/2 + 11
4*a^3*b^9*c^7*d^4 - 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6 - 147*a^6*b^6
*c^4*d^7 + 42*a^7*b^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^9*c^9 - 36*
a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^
4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c
*d^8) - (3*x*(-a*b)^(1/2)*(a*d + b*c)*(256*a^9*b^2*d^11 + 256*b^11*c^9*d^2
- 1792*a*b^10*c^8*d^3 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d^4 - 7168*a
^3*b^8*c^6*d^5 + 3584*a^4*b^7*c^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7168*a^6*b^5
*c^3*d^8 + 5120*a^7*b^4*c^2*d^9))/(128*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d
^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)
))*(a*d + b*c))/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
 4*a^3*b*c*d^3)))*(-a*b)^(1/2)*(a*d + b*c))/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b
^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (3*((x*(9*b^7*c^4*d + 153*a^
4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(
32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)) + (3*(-a*b)^(1/2)*(((3*a^10*b^2
*d^11)/2 + (9*a*b^11*c^9*d^2)/2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^8*
d^3)/2 + 114*a^3*b^9*c^7*d^4 - 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6 -
147*a^6*b^6*c^4*d^7 + 42*a^7*b^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^
9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126
*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d
- 9*a^8*b*c*d^8) + (3*x*(-a*b)^(1/2)*(a*d + b*c)*(256*a^9*b^2*d^11 + 256*b^
11*c^9*d^2 - 1792*a*b^10*c^8*d^3 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d
^4 - 7168*a^3*b^8*c^6*d^5 + 3584*a^4*b^7*c^5*d^...
```

3.311 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=142

$$\frac{ab}{2(bc-ad)^3(a+bx^2)}+\frac{c}{4(bc-ad)^2(c+dx^2)^2}+\frac{bc+ad}{2(bc-ad)^3(c+dx^2)}+\frac{b(bc+2ad)\log(a+bx^2)}{2(bc-ad)^4}-\frac{b(bc+2ad}{2(bc}$$

```
[Out] 1/2*a*b/(-a*d+b*c)^3/(b*x^2+a)+1/4*c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*(a*d+b*c)
/(-a*d+b*c)^3/(d*x^2+c)+1/2*b*(2*a*d+b*c)*ln(b*x^2+a)/(-a*d+b*c)^4-1/2*b*(2
*a*d+b*c)*ln(d*x^2+c)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{ab}{2(a+bx^2)(bc-ad)^3}+\frac{ad+bc}{2(c+dx^2)(bc-ad)^3}+\frac{c}{4(c+dx^2)^2(bc-ad)^2}+\frac{b(2ad+bc)\log(a+bx^2)}{2(bc-ad)^4}-\frac{b(2ad+bc)\log(c+dx^2)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (a*b)/(2*(b*c - a*d)^3*(a + b*x^2)) + c/(4*(b*c - a*d)^2*(c + d*x^2)^2) + (
b*c + a*d)/(2*(b*c - a*d)^3*(c + d*x^2)) + (b*(b*c + 2*a*d)*Log[a + b*x^2])
/(2*(b*c - a*d)^4) - (b*(b*c + 2*a*d)*Log[c + d*x^2])/(2*(b*c - a*d)^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{ab^2}{(bc-ad)^3(a+bx)^2} + \frac{b^2(bc+2ad)}{(bc-ad)^4(a+bx)} - \frac{cd}{(bc-ad)^2(c+d\ldots}\right.\right.$$
$$= \frac{ab}{2(bc-ad)^3(a+bx^2)} + \frac{c}{4(bc-ad)^2(c+dx^2)^2} + \frac{bc+ad}{2(bc-ad)^3(c+dx^2)} + \frac{b(\ldots}{\ldots}$$

Mathematica [A]
time = 0.08, size = 121, normalized size = 0.85

$$\frac{\frac{2ab(bc-ad)}{a+bx^2} + \frac{c(bc-ad)^2}{(c+dx^2)^2} + \frac{2(bc-ad)(bc+ad)}{c+dx^2} + 2b(bc+2ad)\log(a+bx^2) - 2b(bc+2ad)\log(c+dx^2)}{4(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((2*a*b*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d)^2)/(c + d*x^2)^2 + (2*(b*
c - a*d)*(b*c + a*d))/(c + d*x^2) + 2*b*(b*c + 2*a*d)*Log[a + b*x^2] - 2*b*
(b*c + 2*a*d)*Log[c + d*x^2])/(4*(b*c - a*d)^4)
```

Maple [A]
time = 0.16, size = 163, normalized size = 1.15

method	result
default	$\frac{b^2\left(\frac{(2ad+bc)\ln(bx^2+a)}{b}-\frac{a(ad-bc)}{b(bx^2+a)}\right)}{2(ad-bc)^4}+\frac{d\left(-\frac{a^2d^2-b^2c^2}{d(dx^2+c)}-\frac{b(2ad+bc)\ln(dx^2+c)}{d}+\frac{c(a^2d^2-2abcd+b^2c^2)}{2d(dx^2+c)^2}\right)}{2(ad-bc)^4}$
norman	$\frac{\frac{(-2ab^2d^3-b^3cd^2)x^4}{2db(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}+\frac{ac(-abd^3-5b^2cd^2)}{4bd^2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}+\frac{(2ad+bc)(-abd^3-3b^2cd^2)x^2}{4bd^2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}}{(bx^2+a)(dx^2+c)^2}+\frac{b(2\ldots}{2a^4d^4-8a^3bcd^3\ldots}$
risch	$\frac{-\frac{bd(2ad+bc)x^4}{2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}-\frac{(ad+3bc)(2ad+bc)x^2}{4(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}-\frac{ac(ad+5bc)}{4(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}}{(bx^2+a)(dx^2+c)^2}-\frac{b\ln(-dx^2\ldots}{a^4d^4-4a^3bcd^3+6a^2b^2c\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/(a*d-b*c)^4*((2*a*d+b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a))+1/2
*d/(a*d-b*c)^4*(-(a^2*d^2-b^2*c^2)/d/(d*x^2+c)-b*(2*a*d+b*c)/d*ln(d*x^2+c)+
1/2*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(132) = 264$.

time = 0.31, size = 415, normalized size = 2.92

$$\frac{(b^2c + 2abd)\log(bx^2 + a)}{2(b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4)} - \frac{(b^2c + 2abd)\log(dx^2 + c)}{2(b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4)} + \frac{2(b^2cd + 2abd^2)x^4 + 5abc^2 + a^2cd + (3b^2c^2 + 7abcd + 2a^2d^2)x^2}{4(ab^3c^5 - 3a^2b^2c^4d + 3a^3bc^3d^2 - a^4c^2d^3 + (b^4c^3d^2 - 3ab^3c^2d^3 + 3a^2b^2cd^4 - a^3bd^5)x^6 + (2b^4c^4d - 5ab^3c^3d^2 + 3a^2b^2c^2d^3 + a^3bcd^4 - a^4d^5)x^4 + (b^4c^5 - ab^3c^4d - 3a^2b^2c^3d^2 + 5a^3bc^2d^3 - 2a^4cd^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c + 2*a*b*d)*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c
^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(b^2*c + 2*a*b*d)*log(d*x^2 + c)/(b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/4
*(2*(b^2*c*d + 2*a*b*d^2)*x^4 + 5*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 7*a*b*c*
d + 2*a^2*d^2)*x^2)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^
2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 +
 (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5
)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^
4*c*d^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(132) = 264$.

time = 1.60, size = 598, normalized size = 4.21

$$\frac{5ab^2c^3 - 4a^2bc^2d - a^3cd^2 + 2(b^3c^2d + ab^2cd^2 - 2a^2bd^3)x^4 + (3b^3c^3 + 4ab^2c^2d - 5a^2bcd^2 - 2a^3d^3)x^2 + 2((b^3cd^2 + 2ab^2d^3)x^6 + ab^2c^3 + 2a^2bc^2d + (2b^3c^2d + 5ab^2cd^2 + 2a^2bd^3)x^4 + (b^3c^3 + 4ab^2c^2d + 4a^2bcd^2)x^2)\log(bx^2 + a) - 2((b^3cd^2 + 2ab^2d^3)x^6 + ab^2c^3 + 2a^2bc^2d + (2b^3c^2d + 5ab^2cd^2 + 2a^2bd^3)x^4 + (b^3c^3 + 4ab^2c^2d + 4a^2bcd^2)x^2)\log(dx^2 + c)}{4(ab^4c^6 - 4a^2b^3c^5d + 6a^3b^2c^4d^2 - 4a^4bc^3d^3 + a^5c^2d^4 + (b^5c^4d^2 - 4ab^4c^3d^3 + 6a^2b^3c^2d^4 - 4a^3b^2cd^5 + a^4bd^6)x^6 + (2b^5c^5d - 7ab^4c^4d^2 + 8a^2b^3c^3d^3 - 2a^3b^2c^2d^4 - 2a^4bcd^5 + a^5d^6)x^4 + (b^5c^6 - 2ab^4c^5d - 2a^2b^3c^4d^2 + 8a^3b^2c^3d^3 - 7a^4bc^2d^4 + 2a^5cd^5)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a*b^2*c^3 - 4*a^2*b*c^2*d - a^3*c*d^2 + 2*(b^3*c^2*d + a*b^2*c*d^2 -
 2*a^2*b*d^3)*x^4 + (3*b^3*c^3 + 4*a*b^2*c^2*d - 5*a^2*b*c*d^2 - 2*a^3*d^3)
*x^2 + 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a*b^2*c^3 + 2*a^2*b*c^2*d + (2*b^
3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^4 + (b^3*c^3 + 4*a*b^2*c^2*d + 4*a
^2*b*c*d^2)*x^2)*log(b*x^2 + a) - 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a*b^2*
c^3 + 2*a^2*b*c^2*d + (2*b^3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^4 + (b^
3*c^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)*x^2)*log(d*x^2 + c))/(a*b^4*c^6 - 4*
a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^
4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*
x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^
4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4
*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(122) = 244$.

time = 28.87, size = 784, normalized size = 5.52

$$-\frac{b(2ad + bc)\log\left(x^2 + \frac{-\frac{a^5bd^5\cdot(2ad+bc)}{(ad-bc)^4} + \frac{5a^4b^2cd^4\cdot(2ad+bc)}{(ad-bc)^4} - \frac{10a^3b^3c^2d^3\cdot(2ad+bc)}{(ad-bc)^4} + \frac{10a^2b^4c^3d^2\cdot(2ad+bc)}{(ad-bc)^4} + 2a^2bd^2 - \frac{5ab^5c^4d(2ad+bc)}{(ad-bc)^4} + 3ab^2cd + \frac{b^6c^5\cdot(2ad+bc)}{(ad-bc)^4} + b^3c^2}{4ab^2d^2 + 2b^3cd}\right)}{2(ad - bc)^4} + \frac{b(2ad + bc)\log\left(x^2 + \frac{\frac{a^5bd^5\cdot(2ad+bc)}{(ad-bc)^4} - \frac{5a^4b^2cd^4\cdot(2ad+bc)}{(ad-bc)^4} + \frac{10a^3b^3c^2d^3\cdot(2ad+bc)}{(ad-bc)^4} - \frac{10a^2b^4c^3d^2\cdot(2ad+bc)}{(ad-bc)^4} + 2a^2bd^2 + \frac{5ab^5c^4d(2ad+bc)}{(ad-bc)^4} + 3ab^2cd - \frac{b^6c^5\cdot(2ad+bc)}{(ad-bc)^4} + b^3c^2}{4ab^2d^2 + 2b^3cd}\right)}{2(ad - bc)^4} + \frac{-a^2cd - 5abc^2 + x^4(-4abd^2 - 2b^2cd) + x^2(-2a^2d^2 - 7abcd - 3b^2c^2)}{4a^4c^2d^3 - 12a^3bc^3d^2 + 12a^2b^2c^4d - 4ab^3c^5 + x^6\cdot(4a^3bd^5 - 12a^2b^2cd^4 + 12ab^3c^2d^3 - 4b^4c^3d^2) + x^4\cdot(4a^4d^5 - 4a^3bcd^4 - 12a^2b^2c^2d^3 + 20ab^3c^3d^2 - 8b^4c^4d) + x^2\cdot(8a^4cd^4 - 20a^3bc^2d^3 + 12a^2b^2c^3d^2 + 4ab^3c^4d - 4b^4c^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -b*(2*a*d + b*c)*log(x**2 + (-a**5*b*d**5*(2*a*d + b*c)/(a*d - b*c)**4 + 5*
a**4*b**2*c*d**4*(2*a*d + b*c)/(a*d - b*c)**4 - 10*a**3*b**3*c**2*d**3*(2*a
*d + b*c)/(a*d - b*c)**4 + 10*a**2*b**4*c**3*d**2*(2*a*d + b*c)/(a*d - b*c)
**4 + 2*a**2*b*d**2 - 5*a*b**5*c**4*d*(2*a*d + b*c)/(a*d - b*c)**4 + 3*a*b*
*2*c*d + b**6*c**5*(2*a*d + b*c)/(a*d - b*c)**4 + b**3*c**2)/(4*a*b**2*d**2
 + 2*b**3*c*d))/(2*(a*d - b*c)**4) + b*(2*a*d + b*c)*log(x**2 + (a**5*b*d**
5*(2*a*d + b*c)/(a*d - b*c)**4 - 5*a**4*b**2*c*d**4*(2*a*d + b*c)/(a*d - b*
c)**4 + 10*a**3*b**3*c**2*d**3*(2*a*d + b*c)/(a*d - b*c)**4 - 10*a**2*b**4*
c**3*d**2*(2*a*d + b*c)/(a*d - b*c)**4 + 2*a**2*b*d**2 + 5*a*b**5*c**4*d*(2
*a*d + b*c)/(a*d - b*c)**4 + 3*a*b**2*c*d - b**6*c**5*(2*a*d + b*c)/(a*d -
b*c)**4 + b**3*c**2)/(4*a*b**2*d**2 + 2*b**3*c*d))/(2*(a*d - b*c)**4) + (-a
**2*c*d - 5*a*b*c**2 + x**4*(-4*a*b*d**2 - 2*b**2*c*d) + x**2*(-2*a**2*d**2
 - 7*a*b*c*d - 3*b**2*c**2))/(4*a**4*c**2*d**3 - 12*a**3*b*c**3*d**2 + 12*a
**2*b**2*c**4*d - 4*a*b**3*c**5 + x**6*(4*a**3*b*d**5 - 12*a**2*b**2*c*d**4
 + 12*a*b**3*c**2*d**3 - 4*b**4*c**3*d**2) + x**4*(4*a**4*d**5 - 4*a**3*b*c
*d**4 - 12*a**2*b**2*c**2*d**3 + 20*a*b**3*c**3*d**2 - 8*b**4*c**4*d) + x**
2*(8*a**4*c*d**4 - 20*a**3*b*c**2*d**3 + 12*a**2*b**2*c**3*d**2 + 4*a*b**3*
c**4*d - 4*b**4*c**5))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(132) = 264.
time = 0.90, size = 267, normalized size = 1.88

$$\frac{\frac{2\,ab^5}{(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)(bx^2+a)}-\frac{2\,(b^4c+2\,ab^3d)\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{3\,b^3cd^2+2\,ab^2d^3+\frac{2\,(2\,b^5c^2d-ab^4cd^2-a^2b^3d^3)}{(bx^2+a)b}}{(bc-ad)^4\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a*b^5/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*
x^2 + a)) - 2*(b^4*c + 2*a*b^3*d)*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a)
 + d))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4
*b*d^4) - (3*b^3*c*d^2 + 2*a*b^2*d^3 + 2*(2*b^5*c^2*d - a*b^4*c*d^2 - a^2*b
^3*d^3)/((b*x^2 + a)*b))/((b*c - a*d)^4*(b*c/(b*x^2 + a) - a*d/(b*x^2 + a)
+ d)^2))/b
```

Mupad [B]
time = 0.40, size = 926, normalized size = 6.52

$$\frac{5\,a\,b^2\,c^3-a^3\,c\,d^2-2\,a^3\,d^3\,x^2+3\,b^3\,c^3\,x^2-4\,a^2\,b\,d^3\,x^4+2\,b^3\,c^2\,d\,x^4+a\,b^2\,c^3\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}-4\,a^2\,b\,c^2\,d+b^3\,c^3\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+a^2\,b\,d^3\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}+a\,b^2\,d^3\,x^6\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}+b^3\,c^2\,d\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}+b^3\,c\,d^2\,x^6\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+4\,a\,b^2\,c^2\,d\,x^2-5\,a^2\,b\,c\,d^2\,x^2+2\,a\,b^2\,c\,d^2\,x^4+a^2\,b\,c^2\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}+a\,b^2\,c^2\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,16\mathrm{i}+a^2\,b\,c\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,16\mathrm{i}+a\,b^2\,c\,d^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,20\mathrm{i}}{4\,a^5\,c^2\,d^4+8\,a^5\,c\,d^5\,x^2+4\,a^5\,d^6\,x^4-16\,a^4\,b\,c^3\,d^3-28\,a^4\,b\,c^2\,d^4\,x^2-8\,a^4\,b\,c\,d^5\,x^4+4\,a^4\,b\,d^6\,x^6+24\,a^3\,b^2\,c^4\,d^2+32\,a^3\,b^2\,c^3\,d^3\,x^2-8\,a^3\,b^2\,c^2\,d^4\,x^4-16\,a^3\,b^2\,c\,d^5\,x^6-16\,a^2\,b^3\,c^5\,d-8\,a^2\,b^3\,c^4\,d^2\,x^2+32\,a^2\,b^3\,c^3\,d^3\,x^4+24\,a^2\,b^3\,c^2\,d^4\,x^6+4\,a\,b^4\,c^6-8\,a\,b^4\,c^5\,d\,x^2-28\,a\,b^4\,c^4\,d^2\,x^4-16\,a\,b^4\,c^3\,d^3\,x^6+4\,b^5\,c^6\,x^2+8\,b^5\,c^5\,d\,x^4+4\,b^5\,c^4\,d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^3),x)

[Out] (5*a*b^2*c^3 - a^3*c*d^2 - 2*a^3*d^3*x^2 + 3*b^3*c^3*x^2 - 4*a^2*b*d^3*x^4
+ 2*b^3*c^2*d*x^4 + a*b^2*c^3*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x
^2 + b*c*x^2))*4i - 4*a^2*b*c^2*d + b^3*c^3*x^2*atan((a*d*x^2*1i - b*c*x^2*
1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a^2*b*d^3*x^4*atan((a*d*x^2*1i - b*c*
x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + a*b^2*d^3*x^6*atan((a*d*x^2*1i -
b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + b^3*c^2*d*x^4*atan((a*d*x^2*1
i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + b^3*c*d^2*x^6*atan((a*d*x
^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + 4*a*b^2*c^2*d*x^2 - 5
*a^2*b*c*d^2*x^2 + 2*a*b^2*c*d^2*x^4 + a^2*b*c^2*d*atan((a*d*x^2*1i - b*c*x
^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + a*b^2*c^2*d*x^2*atan((a*d*x^2*1i -
 b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*16i + a^2*b*c*d^2*x^2*atan((a*d*x
^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*16i + a*b^2*c*d^2*x^4*atan
((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*20i)/(4*a*b^4*c^6 +
 4*a^5*c^2*d^4 + 4*b^5*c^6*x^2 + 4*a^5*d^6*x^4 - 16*a^2*b^3*c^5*d - 16*a^4*
b*c^3*d^3 + 4*a^4*b*d^6*x^6 + 8*a^5*c*d^5*x^2 + 8*b^5*c^5*d*x^4 + 24*a^3*b^
2*c^4*d^2 + 4*b^5*c^4*d^2*x^6 - 8*a^2*b^3*c^4*d^2*x^2 + 32*a^3*b^2*c^3*d^3*
x^2 + 32*a^2*b^3*c^3*d^3*x^4 - 8*a^3*b^2*c^2*d^4*x^4 + 24*a^2*b^3*c^2*d^4*x
^6 - 8*a*b^4*c^5*d*x^2 - 8*a^4*b*c*d^5*x^4 - 28*a^4*b*c^2*d^4*x^2 - 28*a*b^
4*c^4*d^2*x^4 - 16*a*b^4*c^3*d^3*x^6 - 16*a^3*b^2*c*d^5*x^6)
```

3.312 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=200

$$-\frac{3dx}{4(bc-ad)^2\left(c+dx^2\right)^2}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}-\frac{d(11bc+ad)x}{8c(bc-ad)^3\left(c+dx^2\right)}+\frac{b^{3/2}(bc+5ad)\tan^{-1}\left(\right.}{2\sqrt{a}\,(bc-ad)^4}$$

```
[Out] -3/4*d*x/(-a*d+b*c)^2/(d*x^2+c)^2-1/2*x/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^2-1/
8*d*(a*d+11*b*c)*x/c/(-a*d+b*c)^3/(d*x^2+c)+1/2*b^(3/2)*(5*a*d+b*c)*arctan(
x*b^(1/2)/a^(1/2))/(-a*d+b*c)^4/a^(1/2)-1/8*(-a^2*d^2+10*a*b*c*d+15*b^2*c^2
)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(3/2)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.17, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {482, 541, 536, 211}

$$-\frac{\sqrt{d}\left(-a^2d^2+10abcd+15b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}(bc-ad)^4}+\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(5ad+bc)}{2\sqrt{a}\,(bc-ad)^4}-\frac{dx(ad+11bc)}{8c\left(c+dx^2\right)(bc-ad)^3}-\frac{3dx}{4\left(c+dx^2\right)^2(bc-ad)^2}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*x)/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x/(2*(b*c - a*d)*(a + b*x^2)*(c
+ d*x^2)^2) - (d*(11*b*c + a*d)*x)/(8*c*(b*c - a*d)^3*(c + d*x^2)) + (b^(3/
2)*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)^4) - (
Sqrt[d]*(15*b^2*c^2 + 10*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8
*c^(3/2)*(b*c - a*d)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^2}+\frac{\int \frac{c-5dx^2}{(a+bx^2)(c+dx^2)^3}\,dx}{2(bc-ad)}$$

$$= -\frac{3dx}{4(bc-ad)^2(c+dx^2)^2}-\frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^2}+\frac{\int \frac{2c(2bc+ad)-18bcdx}{(a+bx^2)(c+dx^2)^2}}{8c(bc-ad)^2}$$

$$= -\frac{3dx}{4(bc-ad)^2(c+dx^2)^2}-\frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^2}-\frac{d(11bc+ad)}{8c(bc-ad)^3(c+}$$

$$= -\frac{3dx}{4(bc-ad)^2(c+dx^2)^2}-\frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^2}-\frac{d(11bc+ad)}{8c(bc-ad)^3(c+}$$

$$= -\frac{3dx}{4(bc-ad)^2(c+dx^2)^2}-\frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^2}-\frac{d(11bc+ad)}{8c(bc-ad)^3(c+}$$

Mathematica [A]
time = 0.29, size = 171, normalized size = 0.86

$$\frac{-\frac{4b^2(bc-ad)x}{a+bx^2}-\frac{2d(bc-ad)^2x}{(c+dx^2)^2}+\frac{d(-bc+ad)(7bc+ad)x}{c(c+dx^2)}+\frac{4b^{3/2}(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}\,(-15b^2c^2-10abcd+a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}}{8(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^2*(b*c - a*d)*x)/(a + b*x^2) - (2*d*(b*c - a*d)^2*x)/(c + d*x^2)^2 +
 (d*(-(b*c) + a*d)*(7*b*c + a*d)*x)/(c*(c + d*x^2)) + (4*b^(3/2)*(b*c + 5*a
```

```
*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (Sqrt[d]*(-15*b^2*c^2 - 10*a*b*c
*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(8*(b*c - a*d)^4)
```

Maple [A]
time = 0.25, size = 182, normalized size = 0.91

method	result
default	$\frac{b^2\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(5ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(ad-bc)^4}+\frac{d\left(\frac{\frac{d\left(a^2d^2+6abcd-7b^2c^2\right)x^3}{8c}+\left(\frac{5}{4}abcd-\frac{9}{8}b^2c^2-\frac{1}{8}a^2d^2\right)x}{\left(d\,x^2+c\right)^2}+\frac{\left(a^2d^2-10abcd-15b^2c^2\right)\text{arct}}{8c\sqrt{cd}}\right.}{(ad-bc)^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a*d-b*c)^4*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(5*a*d+b*c)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))+d/(a*d-b*c)^4*((1/8*d*(a^2*d^2+6*a*b*c*d-7*b^2*c^2
)/c*x^3+(5/4*a*b*c*d-9/8*b^2*c^2-1/8*a^2*d^2)*x)/(d*x^2+c)^2+1/8*(a^2*d^2-1
0*a*b*c*d-15*b^2*c^2)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(174) = 348$.
time = 0.56, size = 473, normalized size = 2.36

$$\frac{\left(b^3c+5\,ab^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{ab}}-\frac{\left(15\,b^2c^2d+10\,abcd^2-a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4\right)\sqrt{cd}}-\frac{\left(11\,b^2cd^2+abd^3\right)x^5+\left(17\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^3+\left(4\,b^2c^3+9\,abc^2d-a^2cd^2\right)x}{8\left(ab^3c^6-3\,a^2b^2c^5d+3\,a^3bc^4d^2-a^4c^3d^3+\left(b^4c^4d^2-3\,ab^3c^3d^3+3\,a^2b^2c^2d^4-a^3bcd^5\right)x^6+\left(2\,b^4c^5d-5\,ab^3c^4d^2+3\,a^2b^2c^3d^3+a^3bc^2d^4-a^4cd^5\right)x^4+\left(b^4c^6-ab^3c^5d-3\,a^2b^2c^4d^2+5\,a^3bc^3d^3-2\,a^4c^2d^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c + 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d
+ 10*a*b*c*d^2 - a^2*d^3)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b^3*c^4*d +
 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) - 1/8*((11*b^2
*c*d^2 + a*b*d^3)*x^5 + (17*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (4*b^2
*c^3 + 9*a*b*c^2*d - a^2*c*d^2)*x)/(a*b^3*c^6 - 3*a^2*b^2*c^5*d + 3*a^3*b*c
^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c^3*d^3 + 3*a^2*b^2*c^2*d^4 -
 a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3*c^4*d^2 + 3*a^2*b^2*c^3*d^3 + a^
3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2*c^4*d^2 +
 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(174) = 348$.
time = 3.69, size = 2891, normalized size = 14.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")

[Out] [-1/16*(2*(11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + 2*(17*b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 - 4*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (15*a*b^2*c^4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2), -1/8*((11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + (17*b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 + (15*a*b^2*c^4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2), -1/16*(2*(11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + 2*(17*b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 - 8*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (15*a*b^2*c^4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5

```
*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)
*x^4 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7
*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2), -1/8*((11*b^3*c^2*d^2 - 10*a*b^2*c*d^
3 - a^2*b*d^4)*x^5 + (17*b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3
*d^4)*x^3 - 4*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^
6 + (2*b^3*c^3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b
^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (15*a*b^2
*c^4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^
2*b*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*
x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*s
qrt(d/c)*arctan(x*sqrt(d/c)) + (4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^
2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*
b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^
4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 +
 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 +
 (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b
*c^3*d^4 + 2*a^5*c^2*d^5)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
[Out] Timed out
```

Giac [**A**]
time = 0.65, size = 317, normalized size = 1.58

$$-\frac{b^2x}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(bx^2+a)}+\frac{(b^3c+5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{ab}}-\frac{(15\,b^2c^2d+10\,abcd^2-a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)\sqrt{cd}}-\frac{7\,bcd^2x^3+ad^3x^3+9\,bc^2dx-acd^2x}{8\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
[Out] -1/2*b^2*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a)
) + 1/2*(b^3*c + 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) - 1/8*(15*b^2*c^
2*d + 10*a*b*c*d^2 - a^2*d^3)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b^3*c^4
*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) - 1/8*(7*b
*c*d^2*x^3 + a*d^3*x^3 + 9*b*c^2*d*x - a*c*d^2*x)/((b^3*c^4 - 3*a*b^2*c^3*d
 + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2 + c)^2)
```

Mupad [**B**]
time = 1.55, size = 2500, normalized size = 12.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^3),x)

[Out] ((x^5*(11*b^2*c*d^2 + a*b*d^3))/(8*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(4*b^2*c^2 - a^2*d^2 + 9*a*b*c*d))/(8*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(a^2*d^2 + 17*b^2*c^2 + 6*a*b*c*d))/(8*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x^6) + (atan((((-a*b^3)^(1/2)*(5*a*d + b*c)*((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6*c^3*d^4 - 20*a^3*b^4*c*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3)/2 - (a^10*b^2*c*d^12)/2 + (39*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a^4*b^8*c^7*d^6 + 231*a^5*b^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^4*d^9 - 48*a^8*b^4*c^3*d^10 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*d^8 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d) - (x*(-a*b^3)^(1/2)*(5*a*d + b*c)*(256*b^11*c^11*d^2 - 1792*a*b^10*c^10*d^3 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4*b^7*c^7*d^6 + 3584*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c^4*d^9 - 1792*a^8*b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(128*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)))*(-a*b^3)^(1/2)*(5*a*d + b*c))/(4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)))*1i)/(4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)) + ((-a*b^3)^(1/2)*(5*a*d + b*c)*((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6*c^3*d^4 - 20*a^3*b^4*c*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3)/2 - (a^10*b^2*c*d^12)/2 + (39*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a^4*b^8*c^7*d^6 + 231*a^5*b^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^4*d^9 - 48*a^8*b^4*c^3*d^10 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*d^8 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d) + (x*(-a*b^3)^(1/2)*(5*a*d + b*c)*(256*b^11*c^11*d^2 - 1792*a*b^10*c^10*d^3 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4*b^7*c^7*d^6 + 3584*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c^4*d^9 - 1792*a^8*b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(128*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)))*(-a*b^3)^(1/2)*(5*a*d + b*c))/(4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)))*1i)/(4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)))/(((165*b^

```
8*c^4*d^3)/64 - (5*a^4*b^4*d^7)/64 + (475*a*b^7*c^3*d^4)/32 - (3*a^3*b^5*c*
d^6)/32 + (39*a^2*b^6*c^2*d^5)/4)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*d^8
 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a^5*
b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d) - (
(-a*b^3)^(1/2)*(5*a*d + b*c)*((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6
*c^3*d^4 - 20*a^3*b^4*c*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*
d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^
2*c^4*d^4 - 6*a*b^5*c^7*d)) - (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3)/2 -
 (a^10*b^2*c*d^12)/2 + (39*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a
^4*b^8*c^7*d^6 + 231*a^5*b^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^
4*d^9 - 48*a^8*b^4*c^3*d^10 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*
d^9 + 9*a^8*b*c^3*d^8 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b
^5*c^7*d^4 - 126*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7
- 9*a*b^8*c^10*d) - (x*(-a*b^3)^(1/2)*(5*a*d + b*c)*(256*b^11*c^11*d^2 - 17
92*a*b^10*c^10*d^3 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4
*b^7*c^7*d^6 + 3584*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c
^4*d^9 - 1792*a^8*b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(128*(a^5*d^4 + a*b
^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*(b^6*c^8 + a^
6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*
a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)))*(-a*b^3)^(1/2)*(5*a*d + b*c))/(4*(a^5*d^
4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))))/(4*
(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)
) + ((-a*b^3)^(1/2)*(5*a*d + b*c)*((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*
a*b^6*c^3*d^4 - 20*a^3*b^4*c*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6
*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a
^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (((2*b^12*c^...
```

3.313 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=126

$$-\frac{b^2}{2(bc-ad)^3\left(a+bx^2\right)}-\frac{d}{4(bc-ad)^2\left(c+dx^2\right)^2}-\frac{bd}{(bc-ad)^3\left(c+dx^2\right)}-\frac{3b^2d\log\left(a+bx^2\right)}{2(bc-ad)^4}+\frac{3b^2d\log\left(c+dx^2\right)}{2(bc-ad)^4}$$

```
[Out] -1/2*b^2/(-a*d+b*c)^3/(b*x^2+a)-1/4*d/(-a*d+b*c)^2/(d*x^2+c)^2-b*d/(-a*d+b*
c)^3/(d*x^2+c)-3/2*b^2*d*ln(b*x^2+a)/(-a*d+b*c)^4+3/2*b^2*d*ln(d*x^2+c)/(-a
*d+b*c)^4
```

Rubi [A]
time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 46}

$$-\frac{b^2}{2\left(a+bx^2\right)(bc-ad)^3}-\frac{3b^2d\log\left(a+bx^2\right)}{2(bc-ad)^4}+\frac{3b^2d\log\left(c+dx^2\right)}{2(bc-ad)^4}-\frac{bd}{\left(c+dx^2\right)(bc-ad)^3}-\frac{d}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/2*b^2/((b*c - a*d)^3*(a + b*x^2)) - d/(4*(b*c - a*d)^2*(c + d*x^2)^2) -
(b*d)/((b*c - a*d)^3*(c + d*x^2)) - (3*b^2*d*Log[a + b*x^2])/(2*(b*c - a*d)
^4) + (3*b^2*d*Log[c + d*x^2])/(2*(b*c - a*d)^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^2\,(c+dx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3}{(bc-ad)^3(a+bx)^2} - \frac{3b^3d}{(bc-ad)^4(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)} \cdots\right.\right.$$

$$= -\frac{b^2}{2(bc-ad)^3\,(a+bx^2)} - \frac{d}{4(bc-ad)^2\,(c+dx^2)^2} - \frac{bd}{(bc-ad)^3\,(c+dx^2)} - \cdots$$

Mathematica [A]
time = 0.09, size = 107, normalized size = 0.85

$$-\frac{\frac{2b^2(bc-ad)}{a+bx^2} + \frac{d(bc-ad)^2}{(c+dx^2)^2} + \frac{4bd(bc-ad)}{c+dx^2} + 6b^2d\log(a+bx^2) - 6b^2d\log(c+dx^2)}{4(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/4*((2*b^2*(b*c - a*d))/(a + b*x^2) + (d*(b*c - a*d)^2)/(c + d*x^2)^2 + (
4*b*d*(b*c - a*d))/(c + d*x^2) + 6*b^2*d*Log[a + b*x^2] - 6*b^2*d*Log[c + d
*x^2])/(b*c - a*d)^4
```

Maple [A]
time = 0.15, size = 144, normalized size = 1.14

method	result
default	$-\frac{b^3\left(\frac{3d\ln\left(bx^2+a\right)}{b}-\frac{ad-bc}{b\left(bx^2+a\right)}\right)}{2(ad-bc)^4}+\frac{d^2\left(\frac{2b(ad-bc)}{d\left(dx^2+c\right)}+\frac{3b^2\ln\left(dx^2+c\right)}{d}-\frac{a^2d^2-2abcd+b^2c^2}{2d\left(dx^2+c\right)^2}\right)}{2(ad-bc)^4}$
risch	$\frac{\frac{3b^2d^2x^4}{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{3(ad+3bc)bd\,x^2}{4\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}-\frac{a^2d^2-5abcd-2b^2c^2}{4\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(bx^2+a)(dx^2+c)^2} - \frac{3b^2d\ln(-b \cdots}{2(a^4d^4-4a^3bc\,d^3+6a^2b^2 \cdots}$
norman	$\frac{\frac{-a^2b\,d^4+5a\,b^2c\,d^3+2b^3c^2d^2}{4b\,d^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{3b^2d^2x^4}{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{\left(3a\,b^2d^4+9b^3c\,d^3\right)x^2}{4b\,d^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(bx^2+a)(dx^2+c)^2} - \frac{3b^2 \cdots}{2(a^4d^4-4a^3bc\,d^3+ \cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3/(a*d-b*c)^4*(3*d/b*ln(b*x^2+a)-(a*d-b*c)/b/(b*x^2+a))+1/2*d^2/(a*d
-b*c)^4*(2*b*(a*d-b*c)/d/(d*x^2+c)+3*b^2/d*ln(d*x^2+c)-1/2*(a^2*d^2-2*a*b*c
*d+b^2*c^2)/d/(d*x^2+c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(118) = 236$.
time = 0.32, size = 394, normalized size = 3.13

$$-\frac{3\,b^2d\log\left(bx^2+a\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}+\frac{3\,b^2d\log\left(dx^2+c\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}-\frac{6\,b^2d^2x^4+2\,b^2c^2+5\,abcd-a^2d^2+3\left(3\,b^2cd+abd^2\right)x^2}{4\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^6+\left(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^4+\left(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -3/2*b^2*d*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4) + 3/2*b^2*d*log(d*x^2 + c)/(b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/4*(6*b^2*d^2*x^4 + 2*b^2
*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x^2)/(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(118) = 236$.
time = 1.03, size = 507, normalized size = 4.02

$$-\frac{2\,b^3c^3+3\,ab^2c^2d-6\,a^2bcd^2+a^3d^3+6\left(b^3cd^2-ab^2d^3\right)x^4+3\left(3\,b^3c^2d-2\,ab^2cd^2-a^2bd^3\right)x^2+6\left(b^3d^3x^6+ab^2c^2d+\left(2\,b^3cd^2+ab^2d^3\right)x^4+\left(b^3c^2d+2\,ab^2cd^2\right)x^2\right)\log\left(bx^2+a\right)-6\left(b^3d^3x^6+ab^2c^2d+\left(2\,b^3cd^2+ab^2d^3\right)x^4+\left(b^3c^2d+2\,ab^2cd^2\right)x^2\right)\log\left(dx^2+c\right)}{4\left(ab^4c^6-4\,a^2b^3c^5d+6\,a^3b^2c^4d^2-4\,a^4bc^3d^3+a^5c^2d^4+\left(b^5c^4d^2-4\,ab^4c^3d^3+6\,a^2b^3c^2d^4-4\,a^3b^2cd^5+a^4bd^6\right)x^6+\left(2\,b^5c^5d-7\,ab^4c^4d^2+8\,a^2b^3c^3d^3-2\,a^3b^2c^2d^4-2\,a^4bcd^5+a^5d^6\right)x^4+\left(b^5c^6-2\,ab^4c^5d-2\,a^2b^3c^4d^2+8\,a^3b^2c^3d^3-7\,a^4bc^2d^4+2\,a^5cd^5\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
a*b^2*d^3)*x^4 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x^2 + 6*(b^3*d
^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^4 + (b^3*c^2*d + 2*a*b^2
*c*d^2)*x^2)*log(b*x^2 + a) - 6*(b^3*d^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d^2 +
 a*b^2*d^3)*x^4 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x^2)*log(d*x^2 + c))/(a*b^4*c
^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 +
(b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*
b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2
*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*
b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(109) = 218$.
time = 110.92, size = 643, normalized size = 5.10

$$\frac{3b^2d\log\left(x^2+\frac{-\frac{3a^5b^2d^6}{(ad-bc)^4}+\frac{15a^4b^3cd^5}{(ad-bc)^4}-\frac{30a^3b^4c^2d^4}{(ad-bc)^4}+\frac{30a^2b^5c^3d^3}{(ad-bc)^4}-\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2+\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{2\left(ad-bc\right)^4}-\frac{3b^2d\log\left(x^2+\frac{\frac{3a^5b^2d^6}{(ad-bc)^4}-\frac{15a^4b^3cd^5}{(ad-bc)^4}+\frac{30a^3b^4c^2d^4}{(ad-bc)^4}-\frac{30a^2b^5c^3d^3}{(ad-bc)^4}+\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2-\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{2\left(ad-bc\right)^4}+\frac{-a^2d^2+5abcd+2b^2c^2+6b^2d^2x^4+x^2\cdot\left(3abd^2+9b^2cd\right)}{4a^4c^2d^3-12a^3bc^3d^2+12a^2b^2c^4d-4ab^3c^5+x^6\cdot\left(4a^3bd^5-12a^2b^2cd^4+12ab^3c^2d^3-4b^4c^3d^2\right)+x^4\cdot\left(4a^4d^5-4a^3bcd^4-12a^2b^2c^2d^3+20ab^3c^3d^2-8b^4c^4d\right)+x^2\cdot\left(8a^4cd^4-20a^3bc^2d^3+12a^2b^2c^3d^2+4ab^3c^4d-4b^4c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] 3*b**2*d*log(x**2 + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5
/(a*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3
*d**3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 +
 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(2*(a*d - b*c)**
4) - 3*b**2*d*log(x**2 + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*
d**5/(a*d - b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*
c**3*d**3/(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d*
*2 - 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(2*(a*d - b*
c)**4) + (-a**2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**4 + x**2*(3
*a*b*d**2 + 9*b**2*c*d))/(4*a**4*c**2*d**3 - 12*a**3*b*c**3*d**2 + 12*a**2*
b**2*c**4*d - 4*a*b**3*c**5 + x**6*(4*a**3*b*d**5 - 12*a**2*b**2*c*d**4 + 1
2*a*b**3*c**2*d**3 - 4*b**4*c**3*d**2) + x**4*(4*a**4*d**5 - 4*a**3*b*c*d**
4 - 12*a**2*b**2*c**2*d**3 + 20*a*b**3*c**3*d**2 - 8*b**4*c**4*d) + x**2*(8
*a**4*c*d**4 - 20*a**3*b*c**2*d**3 + 12*a**2*b**2*c**3*d**2 + 4*a*b**3*c**4
*d - 4*b**4*c**5))
```

Giac [A]
time = 0.65, size = 229, normalized size = 1.82

$$\frac{3\,b^3 d \log\left(\left|\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right|\right)}{2\left(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4\right)} - \frac{b^5}{2\left(b^6c^3 - 3\,ab^5c^2d + 3\,a^2b^4cd^2 - a^3b^3d^3\right)(bx^2+a)} + \frac{5\,b^2d^3 + \frac{6\,(b^4cd^2 - ab^3d^3)}{(bx^2+a)b}}{4\,(bc-ad)^4\left(\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 3/2*b^3*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/2*b^5/((b^6*
c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*x^2 + a)) + 1/4*(5*
b^2*d^3 + 6*(b^4*c*d^2 - a*b^3*d^3)/((b*x^2 + a)*b))/((b*c - a*d)^4*(b*c/(b
*x^2 + a) - a*d/(b*x^2 + a) + d)^2)
```

Mupad [B]
time = 0.33, size = 707, normalized size = 5.61

$$-\frac{a^3d^3 + 2b^3c^3 - 3a^2bd^3x^2 - 6ab^2d^3x^4 + 9b^3c^2dx^2 + 6b^3cd^2x^4 + 3ab^2c^2d - 6a^2bcd^2 + b^3d^3x^6\,\mathrm{atan}\left(\frac{adx^2\,1\mathrm{i} - bcx^2\,1\mathrm{i}}{2ac+adx^2+bcx^2}\right)12\mathrm{i} + ab^2d^3x^4\,\mathrm{atan}\left(\frac{adx^2\,1\mathrm{i} - bcx^2\,1\mathrm{i}}{2ac+adx^2+bcx^2}\right)12\mathrm{i} + b^3c^2dx^2\,\mathrm{atan}\left(\frac{adx^2\,1\mathrm{i} - bcx^2\,1\mathrm{i}}{2ac+adx^2+bcx^2}\right)12\mathrm{i} + b^3cd^2x^4\,\mathrm{atan}\left(\frac{adx^2\,1\mathrm{i} - bcx^2\,1\mathrm{i}}{2ac+adx^2+bcx^2}\right)24\mathrm{i} - 6ab^2cd^2x^2 + ab^2c^2d\,\mathrm{atan}\left(\frac{adx^2\,1\mathrm{i} - bcx^2\,1\mathrm{i}}{2ac+adx^2+bcx^2}\right)12\mathrm{i} + ab^2cd^2x^2\,\mathrm{atan}\left(\frac{adx^2\,1\mathrm{i} - bcx^2\,1\mathrm{i}}{2ac+adx^2+bcx^2}\right)24\mathrm{i}}{4a^5c^2d^4 + 8a^5cd^5x^2 + 4a^5d^6x^4 - 16a^4bc^3d^3 - 28a^4bc^2d^4x^2 - 8a^4bcd^5x^4 + 4a^4bd^6x^6 + 24a^3b^2c^4d^2 + 32a^3b^2c^3d^3x^2 - 8a^3b^2c^2d^4x^4 - 16a^3b^2cd^5x^6 - 16a^2b^3c^5d - 8a^2b^3c^4d^2x^2 + 32a^2b^3c^3d^3x^4 + 24a^2b^3c^2d^4x^6 + 4ab^4c^6 - 8ab^4c^5dx^2 - 28ab^4c^4d^2x^4 - 16ab^4c^3d^3x^6 + 4b^5c^6x^2 + 8b^5c^5dx^4 + 4b^5c^4d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] -(a^3*d^3 + 2*b^3*c^3 - 3*a^2*b*d^3*x^2 - 6*a*b^2*d^3*x^4 + 9*b^3*c^2*d*x^2
 + 6*b^3*c*d^2*x^4 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + b^3*d^3*x^6*atan((a*d*
x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*12i + a*b^2*d^3*x^4*atan(
(a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*12i + b^3*c^2*d*x^2*
atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*12i + b^3*c*d^2
```

```
*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*24i - 6*a*
b^2*c*d^2*x^2 + a*b^2*c^2*d*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2
 + b*c*x^2))*12i + a*b^2*c*d^2*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c +
a*d*x^2 + b*c*x^2))*24i)/(4*a*b^4*c^6 + 4*a^5*c^2*d^4 + 4*b^5*c^6*x^2 + 4*a
^5*d^6*x^4 - 16*a^2*b^3*c^5*d - 16*a^4*b*c^3*d^3 + 4*a^4*b*d^6*x^6 + 8*a^5*
c*d^5*x^2 + 8*b^5*c^5*d*x^4 + 24*a^3*b^2*c^4*d^2 + 4*b^5*c^4*d^2*x^6 - 8*a^
2*b^3*c^4*d^2*x^2 + 32*a^3*b^2*c^3*d^3*x^2 + 32*a^2*b^3*c^3*d^3*x^4 - 8*a^3
*b^2*c^2*d^4*x^4 + 24*a^2*b^3*c^2*d^4*x^6 - 8*a*b^4*c^5*d*x^2 - 8*a^4*b*c*d
^5*x^4 - 28*a^4*b*c^2*d^4*x^2 - 28*a*b^4*c^4*d^2*x^4 - 16*a*b^4*c^3*d^3*x^6
 - 16*a^3*b^2*c*d^5*x^6)
```

3.314 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=230

$$\frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}+\frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3\left(c+dx^2\right)}+\frac{b^{5/2}(bc-7ad)\tan^{-1}}{2a^{3/2}(bc-a}$$

```
[Out] 1/4*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x/a/(-a*d+b*c)/(b*x^
2+a)/(d*x^2+c)^2+1/8*d*(-a*d+4*b*c)*(3*a*d+b*c)*x/a/c^2/(-a*d+b*c)^3/(d*x^2
+c)+1/2*b^(5/2)*(-7*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)^4
+1/8*d^(3/2)*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^
(5/2)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.21, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {425, 541, 536, 211}

$$\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-7ad)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-bc+2ad-5bdx^2}{(a+bx^2)(c+dx^2)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-2(2b^2c^2-8abcd- \cdots}{(a+b \cdots}}{8ac \cdots}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc \cdots}{8ac^2(bc-ad)^3 \cdots}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc \cdots}{8ac^2(bc-ad)^3 \cdots}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc \cdots}{8ac^2(bc-ad)^3 \cdots}$$

Mathematica [A]
time = 0.29, size = 197, normalized size = 0.86

$$\frac{1}{8}\left(-\frac{4b^3x}{a(-bc+ad)^3\left(a+bx^2\right)}+\frac{2d^2x}{c(bc-ad)^2\left(c+dx^2\right)^2}+\frac{d^2(11bc-3ad)x}{c^2(bc-ad)^3\left(c+dx^2\right)}+\frac{4b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(35b^2c^2-14abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c - a*d)^2*(
c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*d)^3*(c + d*x^2)) +
(4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^
4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(5/2)*(b*c - a*d)^4))/8
```

Maple [A]
time = 0.08, size = 198, normalized size = 0.86

method	result
default	$-\frac{b^{3}\left(\frac{(ad-bc)x}{2a\left(b\,x^{2}+a\right)}+\frac{(7ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^{4}}+\frac{d^{2}\left(\frac{\frac{d\left(3a^{2}d^{2}-14abcd+11b^{2}c^{2}\right)x^{3}}{8c^{2}}+\frac{\left(5a^{2}d^{2}-18abcd+13b^{2}c^{2}\right)x}{8c}}{\left(d\,x^{2}+c\right)^{2}}+\frac{\left(3a^{2}d^{2}-14abcd+35 \right.}{8c^{2}}\right.}{(ad-bc)^{4}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a*d-b*c)^4*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(7*a*d-b*c)/a/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^4*((1/8*d*(3*a^2*d^2-14*a*b*c*d+11*
b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-18*a*b*c*d+13*b^2*c^2)/c*x)/(d*x^2+c)^2+1/8
*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)/c^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(204) = 408.
time = 0.56, size = 529, normalized size = 2.30

$$\frac{(b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4)\sqrt{ab}}+\frac{(35\,b^2c^2d^2-14\,abcd^3+3\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4)\sqrt{cd}}+\frac{(4\,b^3c^2d^2+11\,ab^2cd^3-3\,a^2bd^4)x^5+(8\,b^3c^3d+13\,ab^2c^2d^2+6\,a^2bcd^3-3\,a^3d^4)x^3+(4\,b^3c^4+13\,a^2bc^2d^2-5\,a^3cd^3)x}{8\,(a^2b^3c^7-3\,a^3b^2c^6d+3\,a^4bc^5d^2-a^5c^4d^3+(ab^4c^5d^2-3\,a^2b^3c^4d^3+3\,a^3b^2c^3d^4-a^4bc^2d^5)x^6+(2\,ab^4c^6d-5\,a^2b^3c^5d^2+3\,a^3b^2c^4d^3+a^4bc^3d^4-a^5c^2d^5)x^4+(ab^4c^7-a^2b^3c^6d-3\,a^3b^2c^5d^2+5\,a^4bc^4d^3-2\,a^5c^3d^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^3*c^3*d
 + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35*b^2*c^
2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 - 4*a*b^3
*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d)) + 1/
8*((4*b^3*c^2*d^2 + 11*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^5 + (8*b^3*c^3*d + 13*a
*b^2*c^2*d^2 + 6*a^2*b*c*d^3 - 3*a^3*d^4)*x^3 + (4*b^3*c^4 + 13*a^2*b*c^2*d
^2 - 5*a^3*c*d^3)*x)/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^2 - a^5
*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*b*c
^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a^4*
b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d
^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(204) = 408$.
time = 4.75, size = 3239, normalized size = 14.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4
 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3
*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 +
(b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2
+ 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 +
 (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (
35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 -
 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a
^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^
4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b
^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^
5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c
^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^
2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2
*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*c^4*d - 14*a^3*b*c^
3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x
^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4
*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b
^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^
2*c*d^4 + 3*a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c
^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 8*(a*b^3*c^5 - 7*a^2*b^2*c^4*d +
(b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a
```

```
^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4
*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*
b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^
3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c
*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^
3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*
c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^
2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(b
/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*
d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*b^3*c
^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4
*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^
3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^
6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3
+ 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7
*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^
3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2
 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.72, size = 332, normalized size = 1.44

$$\frac{b^3x}{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\left(bx^2+a\right)}+\frac{\left(b^4c-7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4\right)\sqrt{ab}}+\frac{\left(35\,b^2c^2d^2-14\,abcd^3+3\,a^2d^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4\right)\sqrt{cd}}+\frac{11\,bcd^3x^3-3\,ad^4x^3+13\,bc^2d^2x-5\,acd^3x}{8\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)\left(dx^2+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b*x^2 +
 a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35
*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 -
4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d
)) + 1/8*(11*b*c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x)/((b^
3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

Mupad [B]
time = 1.71, size = 2500, normalized size = 10.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^
4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7)
)/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*
a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - (((2*a*b^13*c
^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*
c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7
*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 +
(35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^
4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*
b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d
^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^
11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^
7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 +
256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a
^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9
*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2
*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^
4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d))
)*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 +
 a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)) + (((x
*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2
009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*
b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8
*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) + (((2*a*b^13*c^13*d^2 - 2
8*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2
 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*
a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^
```

3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) + (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))/(((63*a^5*b^5*d^9)/64 + (35*b^10*c^5*d^4)/16 - (651*a*b^9*c^4*d^5)/64 - (267*a^4*b^6*c*d^8)/32 - (1275*a^2*b^8*c^3*d^6)/32 + (451*a^3*b^7*c^2*d^7)/16)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - (((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b...

3.315 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=192

$$\frac{b^3}{2a(bc-ad)^3(a+bx^2)}+\frac{d^2}{4c(bc-ad)^2(c+dx^2)^2}+\frac{d^2(3bc-ad)}{2c^2(bc-ad)^3(c+dx^2)}+\frac{\log(x)}{a^2c^3}-\frac{b^3(bc-4ad)\log(a+bx^2)}{2a^2(bc-ad)^4}$$

```
[Out] 1/2*b^3/a/(-a*d+b*c)^3/(b*x^2+a)+1/4*d^2/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*d^2
*(-a*d+3*b*c)/c^2/(-a*d+b*c)^3/(d*x^2+c)+ln(x)/a^2/c^3-1/2*b^3*(-4*a*d+b*c)
*ln(b*x^2+a)/a^2/(-a*d+b*c)^4-1/2*d^2*(a^2*d^2-4*a*b*c*d+6*b^2*c^2)*ln(d*x^
2+c)/c^3/(-a*d+b*c)^4
```

Rubi [A]
time = 0.17, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 90}

$$-\frac{b^3(bc-4ad)\log(a+bx^2)}{2a^2(bc-ad)^4}-\frac{d^2(a^2d^2-4abcd+6b^2c^2)\log(c+dx^2)}{2c^3(bc-ad)^4}+\frac{\log(x)}{a^2c^3}+\frac{b^3}{2a(a+bx^2)(bc-ad)^3}+\frac{d^2(3bc-ad)}{2c^2(c+dx^2)(bc-ad)^3}+\frac{d^2}{4c(c+dx^2)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] b^3/(2*a*(b*c - a*d)^3*(a + b*x^2)) + d^2/(4*c*(b*c - a*d)^2*(c + d*x^2)^2)
 + (d^2*(3*b*c - a*d))/(2*c^2*(b*c - a*d)^3*(c + d*x^2)) + Log[x]/(a^2*c^3)
 - (b^3*(b*c - 4*a*d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^4) - (d^2*(6*b^2*c
^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2c^3x} + \frac{b^4}{a(-bc+ad)^3(a+bx)^2} + \frac{b^4(-bc+4ad)}{a^2(-bc+ad)^4(a+bx)} - \right.\right.$$
$$= \frac{b^3}{2a(bc-ad)^3\left(a+bx^2\right)} + \frac{d^2}{4c(bc-ad)^2\left(c+dx^2\right)^2} + \frac{d^2(3bc-ad)}{2c^2(bc-ad)^3\left(c+dx^2\right)}$$

Mathematica [A]
time = 0.20, size = 187, normalized size = 0.97

$$\frac{1}{4}\left(-\frac{2b^3}{a(-bc+ad)^3\left(a+bx^2\right)} + \frac{d^2}{c(bc-ad)^2\left(c+dx^2\right)^2} + \frac{2d^2(3bc-ad)}{c^2(bc-ad)^3\left(c+dx^2\right)} + \frac{4\log(x)}{a^2c^3} + \frac{2b^3(-bc+4ad)\log\left(a+bx^2\right)}{a^2(bc-ad)^4} - \frac{2d^2\left(6b^2c^2-4abcd+a^2d^2\right)\log\left(c+dx^2\right)}{c^3(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-2*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*x^
2)^2) + (2*d^2*(3*b*c - a*d))/(c^2*(b*c - a*d)^3*(c + d*x^2)) + (4*Log[x])/
(a^2*c^3) + (2*b^3*(-(b*c) + 4*a*d)*Log[a + b*x^2])/(a^2*(b*c - a*d)^4) - (
2*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(c^3*(b*c - a*d)^4)
)/4
```

Maple [A]
time = 0.22, size = 202, normalized size = 1.05

method	result
default	$\frac{b^4\left(\frac{(4ad-bc)\ln\left(bx^2+a\right)}{b}-\frac{a(ad-bc)}{b\left(bx^2+a\right)}\right)}{2a^2(ad-bc)^4}-\frac{d^3\left(-\frac{c\left(a^2d^2-4abcd+3b^2c^2\right)}{d\left(dx^2+c\right)}+\frac{\left(a^2d^2-4abcd+6b^2c^2\right)\ln\left(dx^2+c\right)}{d}-\frac{c^2\left(a^2d^2-2abcd+b^2c^2\right)}{2d\left(dx^2+c\right)^2}\right)}{2c^3(ad-bc)^4}+$
norman	$\frac{\frac{\left(-2a^4d^4+4a^3bc\,d^3+b^4c^4\right)x^2}{2c^2a^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{d\left(-3a^4d^4+3a^3bc\,d^3+8a^2b^2c^2d^2+4b^4c^4\right)x^4}{4c^3a^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{d^2b\left(-3a^3d^3+7a^2bc\,d^2+2b^3c^3\right)x^6}{4c^3a^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\ln(x)}{a^2c^3}+$
risch	$\frac{\frac{b\,d^2\left(a^2d^2-3abcd-b^2c^2\right)x^4}{2c^2a\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{d\left(2a^3d^3-3a^2bc\,d^2-7a\,b^2c^2d-4b^3c^3\right)x^2}{4a\,c^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{3a^3d^3-7a^2bc\,d^2-2b^3c^3}{4ac\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\ln(x)}{a^2c^3}+\frac{}{a(a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^4/a^2/(a*d-b*c)^4*((4*a*d-b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+a))
-1/2*d^3/c^3/(a*d-b*c)^4*(-c*(a^2*d^2-4*a*b*c*d+3*b^2*c^2)/d/(d*x^2+c)+(a^2
*d^2-4*a*b*c*d+6*b^2*c^2)/d*ln(d*x^2+c)-1/2*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)
/d/(d*x^2+c)^2)+ln(x)/a^2/c^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(182) = 364$.
time = 0.35, size = 527, normalized size = 2.74

$$-\frac{\left(b^{4} c - 4 \, a b^{3} d\right) \log\left(b x^{2} + a\right)}{2 \, \left(a^{2} b^{4} c^{4} - 4 \, a^{3} b^{3} c^{3} d + 6 \, a^{4} b^{2} c^{2} d^{2} - 4 \, a^{5} b c d^{3} + a^{6} d^{4}\right)} - \frac{\left(6 \, b^{2} c^{2} d^{2} - 4 \, a b c d^{3} + a^{2} d^{4}\right) \log\left(d x^{2} + c\right)}{2 \, \left(b^{4} c^{7} - 4 \, a b^{3} c^{6} d + 6 \, a^{2} b^{2} c^{5} d^{2} - 4 \, a^{3} b c^{4} d^{3} + a^{4} c^{3} d^{4}\right)} + \frac{2 \, b^{3} c^{4} + 7 \, a^{2} b c^{2} d^{2} - 3 \, a^{3} c d^{3} + 2 \, \left(b^{3} c^{2} d^{2} + 3 \, a b^{2} c d^{3} - a^{2} b d^{4}\right) x^{4} + \left(4 \, b^{3} c^{3} d + 7 \, a b^{2} c^{2} d^{2} + 3 \, a^{2} b c d^{3} - 2 \, a^{3} d^{4}\right) x^{2}}{4 \, \left(a^{2} b^{3} c^{7} - 3 \, a^{3} b^{2} c^{6} d + 3 \, a^{4} b c^{5} d^{2} - a^{5} c^{4} d^{3} + \left(a b^{4} c^{5} d^{2} - 3 \, a^{2} b^{3} c^{4} d^{3} + 3 \, a^{3} b^{2} c^{3} d^{4} - a^{4} b c^{2} d^{5}\right) x^{6} + \left(2 \, a b^{4} c^{6} d - 5 \, a^{2} b^{3} c^{5} d^{2} + 3 \, a^{3} b^{2} c^{4} d^{3} + a^{4} b c^{3} d^{4} - a^{5} c^{2} d^{5}\right) x^{4} + \left(a b^{4} c^{7} - a^{2} b^{3} c^{6} d - 3 \, a^{3} b^{2} c^{5} d^{2} + 5 \, a^{4} b c^{4} d^{3} - 2 \, a^{5} c^{3} d^{4}\right) x^{2}\right)} + \frac{\log\left(x^{2}\right)}{2 \, a^{2} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^4*c - 4*a*b^3*d)*log(b*x^2 + a)/(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*
a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4) - 1/2*(6*b^2*c^2*d^2 - 4*a*b*c*d
^3 + a^2*d^4)*log(d*x^2 + c)/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 -
 4*a^3*b*c^4*d^3 + a^4*c^3*d^4) + 1/4*(2*b^3*c^4 + 7*a^2*b*c^2*d^2 - 3*a^3*
c*d^3 + 2*(b^3*c^2*d^2 + 3*a*b^2*c*d^3 - a^2*b*d^4)*x^4 + (4*b^3*c^3*d + 7*
a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - 2*a^3*d^4)*x^2)/(a^2*b^3*c^7 - 3*a^3*b^2*c^
6*d + 3*a^4*b*c^5*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 +
3*a^3*b^2*c^3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2
 + 3*a^3*b^2*c^4*d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*
b^3*c^6*d - 3*a^3*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2) + 1/2
*log(x^2)/(a^2*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(182) = 364$.
time = 12.13, size = 1058, normalized size = 5.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b^4*c^6 - 2*a^2*b^3*c^5*d + 7*a^3*b^2*c^4*d^2 - 10*a^4*b*c^3*d^3 +
 3*a^5*c^2*d^4 + 2*(a*b^4*c^4*d^2 + 2*a^2*b^3*c^3*d^3 - 4*a^3*b^2*c^2*d^4 +
 a^4*b*c*d^5)*x^4 + (4*a*b^4*c^5*d + 3*a^2*b^3*c^4*d^2 - 4*a^3*b^2*c^3*d^3
- 5*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2 - 2*(a*b^4*c^6 - 4*a^2*b^3*c^5*d + (b^
5*c^4*d^2 - 4*a*b^4*c^3*d^3)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 - 4*a^2*b
^3*c^3*d^3)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 8*a^2*b^3*c^4*d^2)*x^2)*log(b*
x^2 + a) - 2*(6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (6*a^2*b^
3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (12*a^2*b^3*c^3*d^3 - 2*a^3*
b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (6*a^2*b^3*c^4*d^2 + 8*a^3*b^2
*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)*log(d*x^2 + c) + 4*(a*b^4*c^
6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (
b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b
*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*
c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b
^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)*log(x)
```

```
)/(a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^
7*c^5*d^4 + (a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^
5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^6 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 +
 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^4
 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 -
 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(182) = 364.
time = 0.66, size = 470, normalized size = 2.45

$$-\frac{(b^5c-4\,ab^4d)\log\left(|bx^2+a|\right)}{2\,(a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4)}-\frac{(6\,b^2c^2d^3-4\,abcd^4+a^2d^5)\log\left(|dx^2+c|\right)}{2\,(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5)}+\frac{b^5cx^2-4\,ab^4dx^2+2\,ab^4c-5\,a^2b^3d}{2\,(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4)(bx^2+a)}+\frac{18\,b^2c^2d^4x^4-12\,abcd^5x^4+3\,a^2d^6x^4+42\,b^2c^3d^3x^2-32\,abc^2d^4x^2+8\,a^2cd^5x^2+25\,b^2c^4d^2-22\,abc^3d^3+6\,a^2c^2d^4}{4\,(b^4c^7-4\,ab^3c^6d+6\,a^2b^2c^5d^2-4\,a^3bc^4d^3+a^4c^3d^4)(dx^2+c)^2}+\frac{\log\left(x^2\right)}{2\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5*c - 4*a*b^4*d)*log(abs(b*x^2 + a))/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d
 + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4) - 1/2*(6*b^2*c^2*d^3 -
4*a*b*c*d^4 + a^2*d^5)*log(abs(d*x^2 + c))/(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6
*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5) + 1/2*(b^5*c*x^2 - 4*a*b^
4*d*x^2 + 2*a*b^4*c - 5*a^2*b^3*d)/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*
b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*(b*x^2 + a)) + 1/4*(18*b^2*c^2*d^4*x
^4 - 12*a*b*c*d^5*x^4 + 3*a^2*d^6*x^4 + 42*b^2*c^3*d^3*x^2 - 32*a*b*c^2*d^4
*x^2 + 8*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 22*a*b*c^3*d^3 + 6*a^2*c^2*d^4)/(
(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^
4)*(d*x^2 + c)^2) + 1/2*log(x^2)/(a^2*c^3)
```

Mupad [**B**]
time = 1.11, size = 472, normalized size = 2.46

$$\frac{\ln\left(x\right)}{a^2\,c^3}-\frac{\ln\left(b\,x^2+a\right)\,\left(b^4\,c-4\,a\,b^3\,d\right)}{2\,a^6\,d^4-8\,a^5\,b\,c\,d^3+12\,a^4\,b^2\,c^2\,d^2-8\,a^3\,b^3\,c^3\,d+2\,a^2\,b^4\,c^4}-\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^4-4\,a\,b\,c\,d^3+6\,b^2\,c^2\,d^2\right)}{2\,a^4\,c^3\,d^4-8\,a^3\,b\,c^4\,d^3+12\,a^2\,b^2\,c^5\,d^2-8\,a\,b^3\,c^6\,d+2\,b^4\,c^7}-\frac{\frac{-3\,a^3\,d^3+7\,a^2\,b\,c\,d^2+2\,b^3\,c^3}{4\,a\,c\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{x^2\,\left(-2\,a^3\,d^4+3\,a^2\,b\,c\,d^3+7\,a\,b^2\,c^2\,d^2+4\,b^3\,c^3\,d\right)}{4\,a\,c^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{b\,d^2\,x^4\,\left(-a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{2\,a\,c^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}}{a\,c^2+x^2\,\left(b\,c^2+2\,a\,d\,c\right)+x^4\,\left(a\,d^2+2\,b\,c\,d\right)+b\,d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] log(x)/(a^2*c^3) - (log(a + b*x^2)*(b^4*c - 4*a*b^3*d))/(2*a^6*d^4 + 2*a^2*
b^4*c^4 - 8*a^3*b^3*c^3*d + 12*a^4*b^2*c^2*d^2 - 8*a^5*b*c*d^3) - (log(c +
```

```
d*x^2)*(a^2*d^4 + 6*b^2*c^2*d^2 - 4*a*b*c*d^3))/(2*b^4*c^7 + 2*a^4*c^3*d^4
- 8*a^3*b*c^4*d^3 + 12*a^2*b^2*c^5*d^2 - 8*a*b^3*c^6*d) - ((2*b^3*c^3 - 3*a
^3*d^3 + 7*a^2*b*c*d^2)/(4*a*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)) + (x^2*(4*b^3*c^3*d - 2*a^3*d^4 + 7*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3)
)/(4*a*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d^2*x^
4*(b^2*c^2 - a^2*d^2 + 3*a*b*c*d))/(2*a*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 + 2*b*c*
d) + b*d^2*x^6)
```

3.316 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=297

$$-\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x}+\frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2}+\frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2}+\frac{d(4}{8ac}$$

```
[Out] -3/8*(-a*d+2*b*c)*(5*a^2*d^2-3*a*b*c*d+2*b^2*c^2)/a^2/c^3/(-a*d+b*c)^3/x+1/
4*d*(a*d+2*b*c)/a/c/(-a*d+b*c)^2/x/(d*x^2+c)^2+1/2*b/a/(-a*d+b*c)/x/(b*x^2+
a)/(d*x^2+c)^2+1/8*d*(-5*a^2*d^2+13*a*b*c*d+4*b^2*c^2)/a/c^2/(-a*d+b*c)^3/x
/(d*x^2+c)-3/2*b^(7/2)*(-3*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d
+b*c)^4-3/8*d^(5/2)*(5*a^2*d^2-18*a*b*c*d+21*b^2*c^2)*arctan(x*d^(1/2)/c^(1
/2))/c^(7/2)/(-a*d+b*c)^4
```

Rubi [A]

time = 0.35, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {483, 593, 597, 536, 211}

$$-\frac{3b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bc-3ad)}{2a^{5/2}(bc-ad)^4}-\frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^4}+\frac{d\left(-5a^2d^2+13abcd+4b^2c^2\right)}{8ac^2x\left(c+dx^2\right)(bc-ad)^3}-\frac{3(2bc-ad)\left(5a^2d^2-3abcd+2b^2c^2\right)}{8a^2c^3x(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d(ad+2bc)}{4acx\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*(2*b*c - a*d)*(2*b^2*c^2 - 3*a*b*c*d + 5*a^2*d^2))/(8*a^2*c^3*(b*c - a*
d)^3*x) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x*(c + d*x^2)^2) + b/(2*a*
(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b^2*c^2 + 13*a*b*c*d - 5*a
^2*d^2))/(8*a*c^2*(b*c - a*d)^3*x*(c + d*x^2)) - (3*b^(7/2)*(b*c - 3*a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*
c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*(b*c
- a*d)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-3bc+2ad-7bdx^2}{x^2(a+bx^2)(c+dx^2)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2} - \int \frac{-2(6b^2c \cdots}{\cdots}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4b^2c^2 \cdots}{8ac^2(bc \cdots}$$

$$= -\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{\cdots}{2a(b \cdots}$$

$$= -\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{\cdots}{2a(b \cdots}$$

$$= -\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{\cdots}{2a(b \cdots}$$

Mathematica [A]
time = 0.32, size = 210, normalized size = 0.71

$$\frac{1}{8}\left(-\frac{8}{a^2c^3x}+\frac{4b^4x}{a^2(-bc+ad)^3\left(a+bx^2\right)}-\frac{2d^3x}{c^2(bc-ad)^2\left(c+dx^2\right)^2}+\frac{d^3(-15bc+7ad)x}{c^3(bc-ad)^3\left(c+dx^2\right)}+\frac{12b^{7/2}(-bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4}-\frac{3d^{5/2}\left(21b^2c^2-18abcd+5a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a^2*c^3*x) + (4*b^4*x)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - (2*d^3*x)/
(c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (d^3*(-15*b*c + 7*a*d)*x)/(c^3*(b*c - a
*d)^3*(c + d*x^2)) + (12*b^(7/2)*(-(b*c) + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*c^2 - 18*a*b*c*d + 5*a^2*d
^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^4))/8
```

Maple [A]
time = 0.23, size = 202, normalized size = 0.68

method	result
default	$\frac{b^4\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{3(3ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2(ad-bc)^4}-\frac{d^3\left(\frac{\left(\frac{7}{8}a^2d^3-\frac{11}{4}abc\,d^2+\frac{15}{8}b^2c^2d\right)x^3+\frac{c\left(9a^2d^2-26abcd+17b^2c^2\right)x}{8}}{\left(d\,x^2+c\right)^2}+\frac{3\left(5a^2d^2-18abcd+\cdots\right.}{\cdots}\right.}{c^3(ad-bc)^4}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^4/a^2/(a*d-b*c)^4*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+3/2*(3*a*d-b*c)/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2)))-d^3/c^3/(a*d-b*c)^4*(((7/8*a^2*d^3-11/4*a*b*c*
d^2+15/8*b^2*c^2*d)*x^3+1/8*c*(9*a^2*d^2-26*a*b*c*d+17*b^2*c^2)*x)/(d*x^2+c
)^2+3/8*(5*a^2*d^2-18*a*b*c*d+21*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2
)))-1/a^2/c^3/x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(269) = 538$.
time = 0.54, size = 639, normalized size = 2.15

$$-\frac{3\,(b^5c - 3\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^4c^4 - 4\,a^3b^3c^3d + 6\,a^4b^2c^2d^2 - 4\,a^5bcd^3 + a^6d^4)\sqrt{ab}} - \frac{3\,(21\,b^2c^2d^3 - 18\,abcd^4 + 5\,a^2d^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^7 - 4\,ab^3c^6d + 6\,a^2b^2c^5d^2 - 4\,a^3bc^4d^3 + a^4c^3d^4)\sqrt{cd}} - \frac{8\,ab^3c^5 - 24\,a^2b^2c^4d + 24\,a^3bc^3d^2 - 8\,a^4c^2d^3 + 3\,(4\,b^4c^3d^2 - 8\,ab^3c^2d^3 + 13\,a^2b^2cd^4 - 5\,a^3bd^5)x^6 + (24\,b^4c^4d - 40\,ab^3c^3d^2 + 41\,a^2b^2c^2d^3 + 14\,a^3bcd^4 - 15\,a^4d^5)x^4 + (12\,b^4c^5 - 8\,ab^3c^4d - 24\,a^2b^2c^3d^2 + 57\,a^3bc^2d^3 - 25\,a^4cd^4)x^2}{8\,((a^2b^4c^6d^2 - 3\,a^3b^3c^5d^3 + 3\,a^4b^2c^4d^4 - a^5bc^3d^5)x^7 + (2\,a^2b^4c^7d - 5\,a^3b^3c^6d^2 + 3\,a^4b^2c^5d^3 + a^5bc^4d^4 - a^6c^3d^5)x^5 + (a^2b^4c^8 - a^3b^3c^7d - 3\,a^4b^2c^6d^2 + 5\,a^5bc^5d^3 - 2\,a^6c^4d^4)x^3 + (a^3b^3c^8 - 3\,a^4b^2c^7d + 3\,a^5bc^6d^2 - a^6c^5d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -3/2*(b^5*c - 3*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^4 - 4*a^3*b^3*c^
3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(a*b)) - 3/8*(21*b^2
*c^2*d^3 - 18*a*b*c*d^4 + 5*a^2*d^5)*arctan(d*x/sqrt(c*d))/((b^4*c^7 - 4*a*
b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)*sqrt(c*d)) -
 1/8*(8*a*b^3*c^5 - 24*a^2*b^2*c^4*d + 24*a^3*b*c^3*d^2 - 8*a^4*c^2*d^3 + 3
*(4*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 + 13*a^2*b^2*c*d^4 - 5*a^3*b*d^5)*x^6 + (
24*b^4*c^4*d - 40*a*b^3*c^3*d^2 + 41*a^2*b^2*c^2*d^3 + 14*a^3*b*c*d^4 - 15*
a^4*d^5)*x^4 + (12*b^4*c^5 - 8*a*b^3*c^4*d - 24*a^2*b^2*c^3*d^2 + 57*a^3*b*
c^2*d^3 - 25*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*
b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^7 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3
*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^5 + (a^2*b^4*c^8 - a^3*b^
3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^3 + (a^3*b
^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(269) = 538$.
time = 7.41, size = 3753, normalized size = 12.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*b^4*c^6 - 64*a^2*b^3*c^5*d + 96*a^3*b^2*c^4*d^2 - 64*a^4*b*c^3
*d^3 + 16*a^5*c^2*d^4 + 6*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^
```

2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + 2*(24*b^5*c^5*d - 64*a*b^4*c^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^4 + 2*(12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 12*((b^5*c^4*d^2 - 3*a*b^4*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^4)*x), -1/8*(8*a*b^4*c^6 - 32*a^2*b^3*c^5*d + 48*a^3*b^2*c^4*d^2 - 32*a^4*b*c^3*d^3 + 8*a^5*c^2*d^4 + 3*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + (24*b^5*c^5*d - 64*a*b^4*c^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^4 + (12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 6*((b^5*c^4*d^2 - 3*a*b^4*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^4)*x), -1/16*(16*a*b^4*c^6 - 64*a^2*b^3*c^5*d + 96*a^3*b^2*c^4*d^2 - 64*a^4*b*c^3*d^3 + 16*a^5*c^2*d^4 + 6*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + 2*(24*b^5*c^5*d - 64*a*b^4*c^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^4 + 2*(12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 24*((b^5*c^4*d^2 - 3*a*b^4*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*

```
c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3
*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*
a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(-d/c)*log((d*x^
2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d
^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 + (2*a^2*b^
5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6
*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*
c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b
^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^
4)*x), -1/8*(8*a*b^4*c^6 - 32*a^2*b^3*c^5*d + 48*a^3*b^2*c^4*d^2 - 32*a^4*b
*c^3*d^3 + 8*a^5*c^2*d^4 + 3*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3
*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + (24*b^5*c^5*d - 64*a*b^4*c
^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*
d^6)*x^4 + (12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c
^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 12*((b^5*c^4*d^2 - 3*a*b^4*
c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b
^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*
d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c
*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*
b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.88, size = 430, normalized size = 1.45

$$-\frac{3\,(b^5c-3\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4)\sqrt{ab}}-\frac{3\,(21\,b^2c^2d^3-18\,abcd^4+5\,a^2d^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^7-4\,ab^3c^6d+6\,a^2b^2c^5d^2-4\,a^3bc^4d^3+a^4c^3d^4)\sqrt{cd}}-\frac{3\,b^4c^3x^2-6\,ab^3c^2dx^2+6\,a^2b^2cd^2x^2-2\,a^3bd^3x^2+2\,ab^3c^3-6\,a^2b^2c^2d+6\,a^3bcd^2-2\,a^4d^3}{2\,(a^2b^3c^6-3\,a^3b^2c^5d+3\,a^4bc^4d^2-a^5c^3d^3)(bx^3+ax)}-\frac{15\,bcd^4x^3-7\,ad^5x^3+17\,bc^2d^3x-9\,acd^4x}{8\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -3/2*(b^5*c - 3*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^4 - 4*a^3*b^3*c^
3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(a*b)) - 3/8*(21*b^2
*c^2*d^3 - 18*a*b*c*d^4 + 5*a^2*d^5)*arctan(d*x/sqrt(c*d))/((b^4*c^7 - 4*a*
b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)*sqrt(c*d)) -
 1/2*(3*b^4*c^3*x^2 - 6*a*b^3*c^2*d*x^2 + 6*a^2*b^2*c*d^2*x^2 - 2*a^3*b*d^3
*x^2 + 2*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 2*a^4*d^3)/((a^2*b^3
*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*(b*x^3 + a*x)) - 1/
```

```
8*(15*b*c*d^4*x^3 - 7*a*d^5*x^3 + 17*b*c^2*d^3*x - 9*a*c*d^4*x)/((b^3*c^6 -
 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^2)
```

Mupad [B]
time = 1.55, size = 2500, normalized size = 8.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x)
[Out] (atan((a^9*d^5*x*(-c^7*d^5)^(3/2)*25i + b^9*c^16*d*x*(-c^7*d^5)^(1/2)*16i -
 a^6*b^3*c^3*d^2*x*(-c^7*d^5)^(3/2)*756i + a^7*b^2*c^2*d^3*x*(-c^7*d^5)^(3/
2)*534i + a^2*b^7*c^14*d^3*x*(-c^7*d^5)^(1/2)*144i - a^8*b*c*d^4*x*(-c^7*d^
5)^(3/2)*180i + a^5*b^4*c^4*d*x*(-c^7*d^5)^(3/2)*441i - a*b^8*c^15*d^2*x*(-
c^7*d^5)^(1/2)*96i)/(25*a^9*c^11*d^12 - 16*b^9*c^20*d^3 + 96*a*b^8*c^19*d^4
 - 180*a^8*b*c^12*d^11 - 144*a^2*b^7*c^18*d^5 + 441*a^5*b^4*c^15*d^8 - 756*
a^6*b^3*c^14*d^9 + 534*a^7*b^2*c^13*d^10))*(-c^7*d^5)^(1/2)*(5*a^2*d^2 + 21
*b^2*c^2 - 18*a*b*c*d)*3i)/(8*(b^4*c^11 + a^4*c^7*d^4 - 4*a^3*b*c^8*d^3 + 6
*a^2*b^2*c^9*d^2 - 4*a*b^3*c^10*d)) - (atan((((x*(147456*a^6*b^20*c^26*d^3
- 2211840*a^7*b^19*c^25*d^4 + 14598144*a^8*b^18*c^24*d^5 - 56180736*a^9*b^1
7*c^23*d^6 + 144737280*a^10*b^16*c^22*d^7 - 285078528*a^11*b^15*c^21*d^8 +
505018368*a^12*b^14*c^20*d^9 - 885012480*a^13*b^13*c^19*d^10 + 1434332160*a
^14*b^12*c^18*d^11 - 1921047552*a^15*b^11*c^17*d^12 + 1999835136*a^16*b^10*
c^16*d^13 - 1581355008*a^17*b^9*c^15*d^14 + 938843136*a^18*b^8*c^14*d^15 -
412314624*a^19*b^7*c^13*d^16 + 130332672*a^20*b^6*c^12*d^17 - 28145664*a^21
*b^5*c^11*d^18 + 3732480*a^22*b^4*c^10*d^19 - 230400*a^23*b^3*c^9*d^20) + (
3*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(3145728*a^9*b^19*c^29*d^3 - 196608*a^8*b^
20*c^30*d^2 - 23003136*a^10*b^18*c^28*d^4 + 101203968*a^11*b^17*c^27*d^5 -
294961152*a^12*b^16*c^26*d^6 + 582500352*a^13*b^15*c^25*d^7 - 729071616*a^1
4*b^14*c^24*d^8 + 339296256*a^15*b^13*c^23*d^9 + 766132224*a^16*b^12*c^22*d
^10 - 2185936896*a^17*b^11*c^21*d^11 + 3127787520*a^18*b^10*c^20*d^12 - 308
4337152*a^19*b^9*c^19*d^13 + 2249834496*a^20*b^8*c^18*d^14 - 1236221952*a^2
1*b^7*c^17*d^15 + 508674048*a^22*b^6*c^16*d^16 - 152715264*a^23*b^5*c^15*d^
17 + 31703040*a^24*b^4*c^14*d^18 - 4079616*a^25*b^3*c^13*d^19 + 245760*a^26
*b^2*c^12*d^20 + (3*x*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(262144*a^10*b^20*c^33
*d^2 - 4194304*a^11*b^19*c^32*d^3 + 31195136*a^12*b^18*c^31*d^4 - 142606336
*a^13*b^17*c^30*d^5 + 445644800*a^14*b^16*c^29*d^6 - 998244352*a^15*b^15*c^
28*d^7 + 1622147072*a^16*b^14*c^27*d^8 - 1853882368*a^17*b^13*c^26*d^9 + 12
74544128*a^18*b^12*c^25*d^10 - 1274544128*a^20*b^10*c^23*d^12 + 1853882368*
a^21*b^9*c^22*d^13 - 1622147072*a^22*b^8*c^21*d^14 + 998244352*a^23*b^7*c^2
0*d^15 - 445644800*a^24*b^6*c^19*d^16 + 142606336*a^25*b^5*c^18*d^17 - 3119
5136*a^26*b^4*c^17*d^18 + 4194304*a^27*b^3*c^16*d^19 - 262144*a^28*b^2*c^15
*d^20))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4
*a^8*b*c*d^3))))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^
```

2*d^2 - 4*a^8*b*c*d^3)))*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*3i)/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)) + ((x*(147456*a^6*b^20*c^26*d^3 - 2211840*a^7*b^19*c^25*d^4 + 14598144*a^8*b^18*c^24*d^5 - 56180736*a^9*b^17*c^23*d^6 + 144737280*a^10*b^16*c^22*d^7 - 285078528*a^11*b^15*c^21*d^8 + 505018368*a^12*b^14*c^20*d^9 - 885012480*a^13*b^13*c^19*d^10 + 1434332160*a^14*b^12*c^18*d^11 - 1921047552*a^15*b^11*c^17*d^12 + 1999835136*a^16*b^10*c^16*d^13 - 1581355008*a^17*b^9*c^15*d^14 + 938843136*a^18*b^8*c^14*d^15 - 412314624*a^19*b^7*c^13*d^16 + 130332672*a^20*b^6*c^12*d^17 - 28145664*a^21*b^5*c^11*d^18 + 3732480*a^22*b^4*c^10*d^19 - 230400*a^23*b^3*c^9*d^20) + (3*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(196608*a^8*b^20*c^30*d^2 - 3145728*a^9*b^19*c^29*d^3 + 23003136*a^10*b^18*c^28*d^4 - 101203968*a^11*b^17*c^27*d^5 + 294961152*a^12*b^16*c^26*d^6 - 582500352*a^13*b^15*c^25*d^7 + 729071616*a^14*b^14*c^24*d^8 - 339296256*a^15*b^13*c^23*d^9 - 766132224*a^16*b^12*c^22*d^10 + 2185936896*a^17*b^11*c^21*d^11 - 3127787520*a^18*b^10*c^20*d^12 + 3084337152*a^19*b^9*c^19*d^13 - 2249834496*a^20*b^8*c^18*d^14 + 1236221952*a^21*b^7*c^17*d^15 - 508674048*a^22*b^6*c^16*d^16 + 152715264*a^23*b^5*c^15*d^17 - 31703040*a^24*b^4*c^14*d^18 + 4079616*a^25*b^3*c^13*d^19 - 245760*a^26*b^2*c^12*d^20 + (3*x*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(262144*a^10*b^20*c^33*d^2 - 4194304*a^11*b^19*c^32*d^3 + 31195136*a^12*b^18*c^31*d^4 - 142606336*a^13*b^17*c^30*d^5 + 445644800*a^14*b^16*c^29*d^6 - 998244352*a^15*b^15*c^28*d^7 + 1622147072*a^16*b^14*c^27*d^8 - 1853882368*a^17*b^13*c^26*d^9 + 1274544128*a^18*b^12*c^25*d^10 - 1274544128*a^20*b^10*c^23*d^12 + 1853882368*a^21*b^9*c^22*d^13 - 1622147072*a^22*b^8*c^21*d^14 + 998244352*a^23*b^7*c^20*d^15 - 445644800*a^24*b^6*c^19*d^16 + 142606336*a^25*b^5*c^18*d^17 - 31195136*a^26*b^4*c^17*d^18 + 4194304*a^27*b^3*c^16*d^19 - 262144*a^28*b^2*c^15*d^20))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3))))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*3i)/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))/(1161216*a^6*b^18*c^21*d^5 - 13768704*a^7*b^17*c^20*d^6 + 74221056*a^8*b^16*c^19*d^7 - 244574208*a^9*b^15*c^18*d^8 + 551397888*a^10*b^14*c^17*d^9 - 893251584*a^11*b^13*c^16*d^10 + 1058724864*a^12*b^12*c^15*d^11 - 918245376*a^13*b^11*c^14*d^12 + 575106048*...

3.317 $\int \frac{1}{x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3} \, dx$

Optimal. Leaf size=215

$$-\frac{1}{2a^2c^3x^2}-\frac{b^4}{2a^2(bc-ad)^3\,(a+bx^2)}-\frac{d^3}{4c^2(bc-ad)^2\,(c+dx^2)^2}-\frac{d^3(2bc-ad)}{c^3(bc-ad)^3\,(c+dx^2)}-\frac{(2bc+3ad)\log(x)}{a^3c^4}-$$

```
[Out] -1/2/a^2/c^3/x^2-1/2*b^4/a^2/(-a*d+b*c)^3/(b*x^2+a)-1/4*d^3/c^2/(-a*d+b*c)^
2/(d*x^2+c)^2-d^3*(-a*d+2*b*c)/c^3/(-a*d+b*c)^3/(d*x^2+c)-(3*a*d+2*b*c)*ln(
x)/a^3/c^4+1/2*b^4*(-5*a*d+2*b*c)*ln(b*x^2+a)/a^3/(-a*d+b*c)^4+1/2*d^3*(3*a
^2*d^2-10*a*b*c*d+10*b^2*c^2)*ln(d*x^2+c)/c^4/(-a*d+b*c)^4
```

Rubi [A]
time = 0.20, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$,
Rules used = {457, 90}

$$\frac{b^4(2bc-5ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^4}-\frac{\log(x)(3ad+2bc)}{a^3c^4}-\frac{b^4}{2a^2\left(a+bx^2\right)(bc-ad)^3}+\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^4}-\frac{1}{2a^2c^3x^2}-\frac{d^3(2bc-ad)}{c^3\left(c+dx^2\right)(bc-ad)^3}-\frac{d^3}{4c^2\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/2*1/(a^2*c^3*x^2) - b^4/(2*a^2*(b*c - a*d)^3*(a + b*x^2)) - d^3/(4*c^2*(
b*c - a*d)^2*(c + d*x^2)^2) - (d^3*(2*b*c - a*d))/(c^3*(b*c - a*d)^3*(c + d
*x^2)) - ((2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (b^4*(2*b*c - 5*a*d)*Log[a +
b*x^2])/(2*a^3*(b*c - a*d)^4) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*
Log[c + d*x^2])/(2*c^4*(b*c - a*d)^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2c^3x^2} + \frac{-2bc-3ad}{a^3c^4x} - \frac{b^5}{a^2(-bc+ad)^3(a+bx)^2} - \frac{b^5(-2b\cdots}{a^3(-bc+\cdots}\right.\right.$$

$$= -\frac{1}{2a^2c^3x^2} - \frac{b^4}{2a^2(bc-ad)^3\,(a+bx^2)} - \frac{d^3}{4c^2(bc-ad)^2\,(c+dx^2)^2} - \frac{d^3(2\cdots}{c^3(bc-\cdots}$$

Mathematica [A]
time = 0.22, size = 208, normalized size = 0.97

$$\frac{1}{4}\left(-\frac{2}{a^2c^3x^2} + \frac{2b^4}{a^2(-bc+ad)^3\,(a+bx^2)} - \frac{d^3}{c^2(bc-ad)^2\,(c+dx^2)^2} + \frac{4d^3(-2bc+ad)}{c^3(bc-ad)^3\,(c+dx^2)} - \frac{4(2bc+3ad)\log(x)}{a^3c^4} + \frac{2b^4(2bc-5ad)\log(a+bx^2)}{a^3(bc-ad)^4} + \frac{2d^3(10b^2c^2-10abcd+3a^2d^2)\log(c+dx^2)}{c^4(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-2/(a^2*c^3*x^2) + (2*b^4)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - d^3/(c^2*(
b*c - a*d)^2*(c + d*x^2)^2) + (4*d^3*(-2*b*c + a*d))/(c^3*(b*c - a*d)^3*(c
+ d*x^2)) - (4*(2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (2*b^4*(2*b*c - 5*a*d)*L
og[a + b*x^2])/(a^3*(b*c - a*d)^4) + (2*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^
2*d^2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^4))/4
```

Maple [A]
time = 0.23, size = 223, normalized size = 1.04

method	result
default	$-\frac{b^5\left(\frac{(5ad-2bc)\ln(b\,x^2+a)}{b}-\frac{a(ad-bc)}{b(b\,x^2+a)}\right)}{2a^3(ad-bc)^4}+\frac{d^4\left(-\frac{2c(a^2d^2-3abcd+2b^2c^2)}{d(d\,x^2+c)}+\frac{(3a^2d^2-10abcd+10b^2c^2)\ln(d\,x^2+c)}{d}-\frac{c^2(a^2d^2-2abcd+b^2c^2)}{2d(d\,x^2+c)^2}\cdots\right.}{2c^4(ad-bc)^4}$
norman	$\frac{-\frac{1}{2ac}+\frac{(6a^5d^5-12a^4bc\,d^4+4a^3b^2c^2d^3+a\,b^4c^4d-2b^5c^5)x^4}{2c^3a^3(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{d(9a^5d^5-7a^4bc\,d^4-18a^3b^2c^2d^3+8a^2b^3c^3d^2+4a\,b^4c^4d-8b^5c^5)x^6}{4c^4a^3(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{d^2b(9a^4d^4-19a\cdots}{4c^4a^3(\cdots}}{x^2(b\,x^2+a)(d\,x^2+c)^2}$
risch	$\frac{-\frac{b\,d^2(3a^3d^3-7a^2bc\,d^2+3a\,b^2c^2d-2b^3c^3)x^6}{2a^2c^3(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}-\frac{d(6a^4d^4-5a^3bc\,d^3-15a^2b^2c^2d^2+10a\,b^3c^3d-8b^4c^4)x^4}{4a^2c^3(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}-\frac{(9a^4d^4-19a^3bc\,d^3+6a^2b^2c^2d^2+2a\,b^3c^3d\cdots}{4a^2c^2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b\cdots}}{x^2(b\,x^2+a)(d\,x^2+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^5/a^3/(a*d-b*c)^4*((5*a*d-2*b*c)/b*ln(b*x^2+a)-a*(a*d-b*c)/b/(b*x^2+
a))+1/2*d^4/c^4/(a*d-b*c)^4*(-2*c*(a^2*d^2-3*a*b*c*d+2*b^2*c^2)/d/(d*x^2+c)
+(3*a^2*d^2-10*a*b*c*d+10*b^2*c^2)/d*ln(d*x^2+c)-1/2*c^2*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/d/(d*x^2+c)^2)-1/2/a^2/c^3/x^2+(-3*a*d-2*b*c)/a^3/c^4*ln(x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(205) = 410$.

time = 0.33, size = 651, normalized size = 3.03

$$\frac{(2\,b^5c-5\,ab^4d)\log\left(bx^2+a\right)}{2\,(a^3b^4c^4-4\,a^4b^3c^3d+6\,a^5b^2c^2d^2-4\,a^6bcd^3+a^7d^4)}+\frac{(10\,b^2c^2d^3-10\,abcd^4+3\,a^2d^5)\log\left(dx^2+c\right)}{2\,(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4)}-\frac{2\,ab^3c^5-6\,a^2b^2c^4d+6\,a^3bc^3d^2-2\,a^4c^2d^3+2\,(2\,b^4c^3d^2-3\,ab^3c^2d^3+7\,a^2b^2cd^4-3\,a^3bd^5)x^6+(8\,b^4c^4d-10\,ab^3c^3d^2+15\,a^2b^2c^2d^3+5\,a^3bcd^4-6\,a^4d^5)x^4+(4\,b^4c^5-2\,ab^3c^4d-6\,a^2b^2c^3d^2+19\,a^3bc^2d^3-9\,a^4cd^4)x^2}{4\,((a^2b^4c^6d^2-3\,a^3b^3c^5d^3+3\,a^4b^2c^4d^4-a^5bc^3d^5)x^8+(2\,a^2b^4c^7d-5\,a^3b^3c^6d^2+3\,a^4b^2c^5d^3+a^5bc^4d^4-a^6c^3d^5)x^6+(a^2b^4c^8-a^3b^3c^7d-3\,a^4b^2c^6d^2+5\,a^5bc^5d^3-2\,a^6c^4d^4)x^4+(a^3b^3c^8-3\,a^4b^2c^7d+3\,a^5bc^6d^2-a^6c^5d^3)x^2)}-\frac{(2\,bc+3\,ad)\log\left(x^2\right)}{2\,a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^5*c - 5*a*b^4*d)*log(b*x^2 + a)/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6
*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4) + 1/2*(10*b^2*c^2*d^3 - 10*a*b*
c*d^4 + 3*a^2*d^5)*log(d*x^2 + c)/(b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*
d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4) - 1/4*(2*a*b^3*c^5 - 6*a^2*b^2*c^4*d +
 6*a^3*b*c^3*d^2 - 2*a^4*c^2*d^3 + 2*(2*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 7*a
^2*b^2*c*d^4 - 3*a^3*b*d^5)*x^6 + (8*b^4*c^4*d - 10*a*b^3*c^3*d^2 + 15*a^2*
b^2*c^2*d^3 + 5*a^3*b*c*d^4 - 6*a^4*d^5)*x^4 + (4*b^4*c^5 - 2*a*b^3*c^4*d -
 6*a^2*b^2*c^3*d^2 + 19*a^3*b*c^2*d^3 - 9*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2
 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^8 + (2*a^2*b^4*
c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5
)*x^6 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3
 - 2*a^6*c^4*d^4)*x^4 + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a
^6*c^5*d^3)*x^2) - 1/2*(2*b*c + 3*a*d)*log(x^2)/(a^3*c^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. $2(205) = 410$.

time = 25.42, size = 1227, normalized size = 5.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*b^4*c^7 - 8*a^3*b^3*c^6*d + 12*a^4*b^2*c^5*d^2 - 8*a^5*b*c^4*d^
3 + 2*a^6*c^3*d^4 + 2*(2*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3 + 10*a^3*b^3*c^3
*d^4 - 10*a^4*b^2*c^2*d^5 + 3*a^5*b*c*d^6)*x^6 + (8*a*b^5*c^6*d - 18*a^2*b^
4*c^5*d^2 + 25*a^3*b^3*c^4*d^3 - 10*a^4*b^2*c^3*d^4 - 11*a^5*b*c^2*d^5 + 6*
a^6*c*d^6)*x^4 + (4*a*b^5*c^7 - 6*a^2*b^4*c^6*d - 4*a^3*b^3*c^5*d^2 + 25*a^
4*b^2*c^4*d^3 - 28*a^5*b*c^3*d^4 + 9*a^6*c^2*d^5)*x^2 - 2*((2*b^6*c^5*d^2 -
 5*a*b^5*c^4*d^3)*x^8 + (4*b^6*c^6*d - 8*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3)
*x^6 + (2*b^6*c^7 - a*b^5*c^6*d - 10*a^2*b^4*c^5*d^2)*x^4 + (2*a*b^5*c^7 -
5*a^2*b^4*c^6*d)*x^2)*log(b*x^2 + a) - 2*((10*a^3*b^3*c^2*d^5 - 10*a^4*b^2*
c*d^6 + 3*a^5*b*d^7)*x^8 + (20*a^3*b^3*c^3*d^4 - 10*a^4*b^2*c^2*d^5 - 4*a^5
*b*c*d^6 + 3*a^6*d^7)*x^6 + (10*a^3*b^3*c^4*d^3 + 10*a^4*b^2*c^3*d^4 - 17*a
^5*b*c^2*d^5 + 6*a^6*c*d^6)*x^4 + (10*a^4*b^2*c^4*d^3 - 10*a^5*b*c^3*d^4 +
3*a^6*c^2*d^5)*x^2)*log(d*x^2 + c) + 4*((2*b^6*c^5*d^2 - 5*a*b^5*c^4*d^3 +
```

```
10*a^3*b^3*c^2*d^5 - 10*a^4*b^2*c*d^6 + 3*a^5*b*d^7)*x^8 + (4*b^6*c^6*d - 8
*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4 - 10*a^4*b^2*c^2*d^
5 - 4*a^5*b*c*d^6 + 3*a^6*d^7)*x^6 + (2*b^6*c^7 - a*b^5*c^6*d - 10*a^2*b^4*
c^5*d^2 + 10*a^3*b^3*c^4*d^3 + 10*a^4*b^2*c^3*d^4 - 17*a^5*b*c^2*d^5 + 6*a^
6*c*d^6)*x^4 + (2*a*b^5*c^7 - 5*a^2*b^4*c^6*d + 10*a^4*b^2*c^4*d^3 - 10*a^5
*b*c^3*d^4 + 3*a^6*c^2*d^5)*x^2)*log(x))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*
d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^8 + (2*a^3*b
^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^
7*b*c^5*d^5 + a^8*c^4*d^6)*x^6 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^
3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^4 + (a^4
*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6
*d^4)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(205) = 410.
time = 0.79, size = 638, normalized size = 2.97

$$\frac{(2\,b^6c-5\,ab^5d)\log\left(|bx^2+a|\right)}{2\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4)}+\frac{(10\,b^2c^2d^4-10\,abcd^5+3\,a^2d^6)\log\left(|dx^2+c|\right)}{2\,(b^4c^8d-4\,ab^3c^7d^2+6\,a^2b^2c^6d^3-4\,a^3bc^5d^4+a^4c^4d^5)}+\frac{10\,a^2b^3c^2d^3x^4-10\,a^3b^2cd^4x^4+3\,a^4bd^5x^4-4\,b^5c^5x^2+10\,ab^4c^4dx^2-12\,a^2b^3c^3d^2x^2+18\,a^3b^2c^2d^3x^2-12\,a^4bcd^4x^2+3\,a^5d^5x^2-2\,ab^4c^5+8\,a^2b^3c^4d-12\,a^3b^2c^3d^2+8\,a^4bc^2d^3-2\,a^5cd^4}{4\,(a^2b^4c^8-4\,a^3b^3c^7d+6\,a^4b^2c^6d^2-4\,a^5bc^5d^3+a^6c^4d^4)(bx^4+ax^2)}-\frac{30\,b^2c^2d^5x^4-30\,abcd^6x^4+9\,a^2d^7x^4+68\,b^2c^3d^4x^2-72\,abc^2d^5x^2+22\,a^2cd^6x^2+39\,b^2c^4d^3-44\,abc^3d^4+14\,a^2c^2d^5}{4\,(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4)(dx^2+c)^2}-\frac{(2\,bc+3\,ad)\log\left(x^2\right)}{2\,a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^6*c - 5*a*b^5*d)*log(abs(b*x^2 + a))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*
d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4) + 1/2*(10*b^2*c^2*d^4
- 10*a*b*c*d^5 + 3*a^2*d^6)*log(abs(d*x^2 + c))/(b^4*c^8*d - 4*a*b^3*c^7*d^
2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5) + 1/4*(10*a^2*b^3*c^
2*d^3*x^4 - 10*a^3*b^2*c*d^4*x^4 + 3*a^4*b*d^5*x^4 - 4*b^5*c^5*x^2 + 10*a*b
^4*c^4*d*x^2 - 12*a^2*b^3*c^3*d^2*x^2 + 18*a^3*b^2*c^2*d^3*x^2 - 12*a^4*b*c
*d^4*x^2 + 3*a^5*d^5*x^2 - 2*a*b^4*c^5 + 8*a^2*b^3*c^4*d - 12*a^3*b^2*c^3*d
^2 + 8*a^4*b*c^2*d^3 - 2*a^5*c*d^4)/((a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4
*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4)*(b*x^4 + a*x^2)) - 1/4*(30*b^
2*c^2*d^5*x^4 - 30*a*b*c*d^6*x^4 + 9*a^2*d^7*x^4 + 68*b^2*c^3*d^4*x^2 - 72*
a*b*c^2*d^5*x^2 + 22*a^2*c*d^6*x^2 + 39*b^2*c^4*d^3 - 44*a*b*c^3*d^4 + 14*a
^2*c^2*d^5)/((b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3
 + a^4*c^4*d^4)*(d*x^2 + c)^2) - 1/2*(2*b*c + 3*a*d)*log(x^2)/(a^3*c^4)
```

Mupad [B]
time = 1.32, size = 549, normalized size = 2.55

$$\frac{\ln\left(b\,x^2+a\right)\,\left(2\,b^5\,c-5\,a\,b^4\,d\right)}{2\,a^7\,d^4-8\,a^6\,b\,c\,d^3+12\,a^5\,b^2\,c^2\,d^2-8\,a^4\,b^3\,c^3\,d+2\,a^3\,b^4\,c^4}-\frac{\frac{1}{2\,a\,c}-\frac{x^4\,\left(-6\,a^4\,d^5+5\,a^3\,b\,c\,d^4+15\,a^2\,b^2\,c^2\,d^3-10\,a\,b^3\,c^3\,d^2+8\,b^4\,c^4\,d\right)}{4\,a^2\,c^3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{x^2\,\left(9\,a^4\,d^4-19\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2+2\,a\,b^3\,c^3\,d-4\,b^4\,c^4\right)}{4\,a^2\,c^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{b\,d^2\,x^6\,\left(3\,a^3\,d^3-7\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-2\,b^3\,c^3\right)}{2\,a^2\,c^3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}}{x^4\,\left(b\,c^2+2\,a\,d\,c\right)+x^6\,\left(a\,d^2+2\,b\,c\,d\right)+a\,c^2\,x^2+b\,d^2\,x^8}+\frac{\ln\left(d\,x^2+c\right)\,\left(3\,a^2\,d^5-10\,a\,b\,c\,d^4+10\,b^2\,c^2\,d^3\right)}{2\,a^4\,c^4\,d^4-8\,a^3\,b\,c^5\,d^3+12\,a^2\,b^2\,c^6\,d^2-8\,a\,b^3\,c^7\,d+2\,b^4\,c^8}-\frac{\ln\left(x\right)\,\left(3\,a\,d+2\,b\,c\right)}{a^3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (log(a + b*x^2)*(2*b^5*c - 5*a*b^4*d))/(2*a^7*d^4 + 2*a^3*b^4*c^4 - 8*a^4*b
^3*c^3*d + 12*a^5*b^2*c^2*d^2 - 8*a^6*b*c*d^3) - (1/(2*a*c) - (x^4*(8*b^4*c
^4*d - 6*a^4*d^5 - 10*a*b^3*c^3*d^2 + 15*a^2*b^2*c^2*d^3 + 5*a^3*b*c*d^4))/
(4*a^2*c^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x^2*(9*a
^4*d^4 - 4*b^4*c^4 + 6*a^2*b^2*c^2*d^2 + 2*a*b^3*c^3*d - 19*a^3*b*c*d^3))/(
4*a^2*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d^2*x^6
*(3*a^3*d^3 - 2*b^3*c^3 + 3*a*b^2*c^2*d - 7*a^2*b*c*d^2))/(2*a^2*c^3*(a^3*d
^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/(x^4*(b*c^2 + 2*a*c*d) + x^
6*(a*d^2 + 2*b*c*d) + a*c^2*x^2 + b*d^2*x^8) + (log(c + d*x^2)*(3*a^2*d^5 +
 10*b^2*c^2*d^3 - 10*a*b*c*d^4))/(2*b^4*c^8 + 2*a^4*c^4*d^4 - 8*a^3*b*c^5*d
^3 + 12*a^2*b^2*c^6*d^2 - 8*a*b^3*c^7*d) - (log(x)*(3*a*d + 2*b*c))/(a^3*c^
4)
```

3.318 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=377

$$-\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3}+\frac{20b^4c^4-24ab^3c^3d-24a^2b^2c^2d^2+75a^3bcd^3-35a^4d^4}{8a^3c^4(bc-ad)^3x}+\frac{d(2}{4ac(bc-a}$$

```
[Out] 1/24*(35*a^3*d^3-75*a^2*b*c*d^2+24*a*b^2*c^2*d-20*b^3*c^3)/a^2/c^3/(-a*d+b*
c)^3/x^3+1/8*(-35*a^4*d^4+75*a^3*b*c*d^3-24*a^2*b^2*c^2*d^2-24*a*b^3*c^3*d+
20*b^4*c^4)/a^3/c^4/(-a*d+b*c)^3/x+1/4*d*(a*d+2*b*c)/a/c/(-a*d+b*c)^2/x^3/(
d*x^2+c)^2+1/2*b/a/(-a*d+b*c)/x^3/(b*x^2+a)/(d*x^2+c)^2+1/8*d*(-7*a^2*d^2+1
5*a*b*c*d+4*b^2*c^2)/a/c^2/(-a*d+b*c)^3/x^3/(d*x^2+c)+1/2*b^(9/2)*(-11*a*d+
5*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/(-a*d+b*c)^4+1/8*d^(7/2)*(35*a^2*d
^2-110*a*b*c*d+99*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(9/2)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.46, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {483, 593, 597, 536, 211}

$$\frac{b^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bc-11ad)}{2a^{7/2}(bc-ad)^4}+\frac{d^{7/2}(35a^2d^2-110abcd+99b^2c^2)\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^4}+\frac{d(-7a^2d^2+15abcd+4b^2c^2)}{8ac^2x^3(c+dx^2)(bc-ad)^3}-\frac{-35a^3d^3+75a^2bcd^2-24ab^2c^2d+20b^3c^3}{24a^2c^3x^3(bc-ad)^3}+\frac{-35a^4d^4+75a^3bcd^3-24a^2b^2c^2d^2-24ab^3c^3d+20b^4c^4}{8a^3c^4x(bc-ad)^3}+\frac{b}{2ax^3(a+bx^2)(c+dx^2)^2(bc-ad)}+\frac{d(ad+2bc)}{4acx^3(c+dx^2)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/24*(20*b^3*c^3 - 24*a*b^2*c^2*d + 75*a^2*b*c*d^2 - 35*a^3*d^3)/(a^2*c^3*
(b*c - a*d)^3*x^3) + (20*b^4*c^4 - 24*a*b^3*c^3*d - 24*a^2*b^2*c^2*d^2 + 75
*a^3*b*c*d^3 - 35*a^4*d^4)/(8*a^3*c^4*(b*c - a*d)^3*x) + (d*(2*b*c + a*d))/
(4*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2
)*(c + d*x^2)^2) + (d*(4*b^2*c^2 + 15*a*b*c*d - 7*a^2*d^2))/(8*a*c^2*(b*c -
 a*d)^3*x^3*(c + d*x^2)) + (b^(9/2)*(5*b*c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*a^(7/2)*(b*c - a*d)^4) + (d^(7/2)*(99*b^2*c^2 - 110*a*b*c*d + 35*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3} \, dx = \frac{b}{2a(bc-ad)x^3 \left(a+bx^2\right) \left(c+dx^2\right)^2} - \frac{\int \frac{-5bc+2ad-9bdx^2}{x^4 (a+bx^2)(c+dx^2)^3} \, dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2 x^3 \left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^3 \left(a+bx^2\right) \left(c+dx^2\right)^2} - \frac{\int \frac{-2(10b}{}}{}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2 x^3 \left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^3 \left(a+bx^2\right) \left(c+dx^2\right)^2} + \frac{d(4b^2c^2}{8ac^2(bc}$$

$$= -\frac{20b^3c^3 - 24ab^2c^2d + 75a^2bcd^2 - 35a^3d^3}{24a^2c^3(bc-ad)^3 x^3} + \frac{d(2bc+ad)}{4ac(bc-ad)^2 x^3 \left(c+dx^2\right)^2} + \frac{}{2}$$

$$= -\frac{20b^3c^3 - 24ab^2c^2d + 75a^2bcd^2 - 35a^3d^3}{24a^2c^3(bc-ad)^3 x^3} + \frac{20b^4c^4 - 24ab^3c^3d - 24a^2b^2c^2d^2}{8a^3c^4(bc-ad}$$

$$= -\frac{20b^3c^3 - 24ab^2c^2d + 75a^2bcd^2 - 35a^3d^3}{24a^2c^3(bc-ad)^3 x^3} + \frac{20b^4c^4 - 24ab^3c^3d - 24a^2b^2c^2d^2}{8a^3c^4(bc-ad}$$

$$= -\frac{20b^3c^3 - 24ab^2c^2d + 75a^2bcd^2 - 35a^3d^3}{24a^2c^3(bc-ad)^3 x^3} + \frac{20b^4c^4 - 24ab^3c^3d - 24a^2b^2c^2d^2}{8a^3c^4(bc-ad}$$

Mathematica [A]
time = 0.32, size = 230, normalized size = 0.61

$$\frac{1}{24} \left(-\frac{8}{a^2c^3x^3} + \frac{48bc+72ad}{a^3c^4x} - \frac{12b^5x}{a^3(-bc+ad)^3 \left(a+bx^2\right)} + \frac{6d^4x}{c^3(bc-ad)^2 \left(c+dx^2\right)^2} + \frac{3d^4(19bc-11ad)x}{c^4(bc-ad)^3 \left(c+dx^2\right)} + \frac{12b^{9/2}(5bc-11ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)^4} + \frac{3d^{7/2}\left(99b^2c^2-110abcd+35a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{9/2}(bc-ad)^4} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a^2*c^3*x^3) + (48*b*c + 72*a*d)/(a^3*c^4*x) - (12*b^5*x)/(a^3*(-(b*c)
 + a*d)^3*(a + b*x^2)) + (6*d^4*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)^2) + (3*d
^4*(19*b*c - 11*a*d)*x)/(c^4*(b*c - a*d)^3*(c + d*x^2)) + (12*b^(9/2)*(5*b*
c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(b*c - a*d)^4) + (3*d^(7/
2)*(99*b^2*c^2 - 110*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^
(9/2)*(b*c - a*d)^4))/24
```

Maple [A]
time = 0.22, size = 222, normalized size = 0.59

method	result

default	$-\frac{b^5\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(11ad-5bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3(ad-bc)^4}+\frac{d^4\left(\frac{\left(\frac{11}{8}a^2d^3-\frac{15}{4}abc\,d^2+\frac{19}{8}b^2c^2d\right)x^3+\frac{c\left(13a^2d^2-34abcd+21b^2c^2\right)x}{8}}{\left(d\,x^2+c\right)^2}+\frac{\left(35a^2d^2-1\right.}{}\right.}{c^4(ad-bc)^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^5/a^3/(a*d-b*c)^4*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(11*a*d-5*b*c)/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^4/c^4/(a*d-b*c)^4*(((11/8*a^2*d^3-15/4*a
*b*c*d^2+19/8*b^2*c^2*d)*x^3+1/8*c*(13*a^2*d^2-34*a*b*c*d+21*b^2*c^2)*x)/(d
*x^2+c)^2+1/8*(35*a^2*d^2-110*a*b*c*d+99*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c
*d)^(1/2)))-1/3/a^2/c^3/x^3-(-3*a*d-2*b*c)/a^3/c^4/x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(347) = 694$.
time = 0.55, size = 738, normalized size = 1.96

$$\frac{(5\,b^6c-11\,ab^5d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^4c^4-4\,a^4b^3c^3d+6\,a^5b^2c^2d^2-4\,a^6bcd^3+a^7d^4)\sqrt{ab}}+\frac{(99\,b^2c^2d^4-110\,abcd^5+35\,a^2d^6)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4)\sqrt{cd}}-\frac{8\,a^2b^3c^6-24\,a^3b^2c^5d+24\,a^4bc^4d^2-8\,a^5c^3d^3-3\,(20\,b^5c^4d^2-24\,ab^4c^3d^3-24\,a^2b^3c^2d^4+75\,a^3b^2cd^5-35\,a^4bd^6)x^8-(120\,b^5c^5d-104\,ab^4c^4d^2-192\,a^2b^3c^3d^3+303\,a^3b^2c^2d^4+50\,a^4bcd^5-105\,a^5d^6)x^6-(60\,b^5c^6+8\,ab^4c^5d-176\,a^2b^3c^4d^2+319\,a^4bc^2d^4-175\,a^5cd^5)x^4-8\,(5\,ab^4c^6-8\,a^2b^3c^5d-6\,a^3b^2c^4d^2+16\,a^4bc^3d^3-7\,a^5c^2d^4)x^2}{24\,((a^3b^4c^7d^2-3\,a^4b^3c^6d^3+3\,a^5b^2c^5d^4-a^6bc^4d^5)x^9+(2\,a^3b^4c^8d-5\,a^4b^3c^7d^2+3\,a^5b^2c^6d^3+a^6bc^5d^4-a^7c^4d^5)x^7+(a^3b^4c^9-a^4b^3c^8d-3\,a^5b^2c^7d^2+5\,a^6bc^6d^3-2\,a^7c^5d^4)x^5+(a^4b^3c^9-3\,a^5b^2c^8d+3\,a^6bc^7d^2-a^7c^6d^3)x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(5*b^6*c - 11*a*b^5*d)*arctan(b*x/sqrt(a*b))/((a^3*b^4*c^4 - 4*a^4*b^3*
c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*sqrt(a*b)) + 1/8*(99*b
^2*c^2*d^4 - 110*a*b*c*d^5 + 35*a^2*d^6)*arctan(d*x/sqrt(c*d))/((b^4*c^8 -
4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(c*d
)) - 1/24*(8*a^2*b^3*c^6 - 24*a^3*b^2*c^5*d + 24*a^4*b*c^4*d^2 - 8*a^5*c^3*
d^3 - 3*(20*b^5*c^4*d^2 - 24*a*b^4*c^3*d^3 - 24*a^2*b^3*c^2*d^4 + 75*a^3*b^
2*c*d^5 - 35*a^4*b*d^6)*x^8 - (120*b^5*c^5*d - 104*a*b^4*c^4*d^2 - 192*a^2*
b^3*c^3*d^3 + 303*a^3*b^2*c^2*d^4 + 50*a^4*b*c*d^5 - 105*a^5*d^6)*x^6 - (60
*b^5*c^6 + 8*a*b^4*c^5*d - 176*a^2*b^3*c^4*d^2 + 319*a^4*b*c^2*d^4 - 175*a^
5*c*d^5)*x^4 - 8*(5*a*b^4*c^6 - 8*a^2*b^3*c^5*d - 6*a^3*b^2*c^4*d^2 + 16*a^
4*b*c^3*d^3 - 7*a^5*c^2*d^4)*x^2)/((a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3
*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^9 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^
2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x^7 + (a^3*b^4*c^9 - a
^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^5 + (
a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(347) = 694$.
time = 15.13, size = 4225, normalized size = 11.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")

[Out] [-1/48*(16*a^2*b^4*c^7 - 64*a^3*b^3*c^6*d + 96*a^4*b^2*c^5*d^2 - 64*a^5*b*c^4*d^3 + 16*a^6*c^3*d^4 - 6*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - 2*(120*b^6*c^6*d - 224*a*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 - 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - 2*(60*b^6*c^7 - 52*a*b^5*c^6*d - 184*a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2*d^5 + 175*a^6*c*d^6)*x^4 - 16*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 + 12*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3), -1/24*(8*a^2*b^4*c^7 - 32*a^3*b^3*c^6*d + 48*a^4*b^2*c^5*d^2 - 32*a^5*b*c^4*d^3 + 8*a^6*c^3*d^4 - 3*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - (120*b^6*c^6*d - 224*a*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 - 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - (60*b^6*c^7 - 52*a*b^5*c^6*d - 184*a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2*d^5 + 175*a^6*c*d^6)*x^4 - 8*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 6*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a

```
^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a^4*b^4*c^10 - 4*a
^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3), -1/
48*(16*a^2*b^4*c^7 - 64*a^3*b^3*c^6*d + 96*a^4*b^2*c^5*d^2 - 64*a^5*b*c^4*d
^3 + 16*a^6*c^3*d^4 - 6*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3*c^2
*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - 2*(120*b^6*c^6*d - 224*a*b^5
*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 -
 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - 2*(60*b^6*c^7 - 52*a*b^5*c^6*d - 184*
a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2
*d^5 + 175*a^6*c*d^6)*x^4 - 16*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*b^3*
c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 - 24*(
(5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d^2 -
 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x
^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^9 + (198*a^3*b
^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a
^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5
 + (99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^5*c^8*d^2 - 4*a^
4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9
+ (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*
d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d
- 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*
x^5 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3
 + a^8*c^6*d^4)*x^3), -1/24*(8*a^2*b^4*c^7 - 32*a^3*b^3*c^6*d + 48*a^4*b^2*
c^5*d^2 - 32*a^5*b*c^4*d^3 + 8*a^6*c^3*d^4 - 3*(20*b^6*c^5*d^2 - 44*a*b^5*c
^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.72, size = 367, normalized size = 0.97

$$\frac{b^5 x}{2\,(a^3b^3c^3 - 3\,a^4b^2c^2d + 3\,a^5bcd^2 - a^6d^3)(bx^2 + a)} + \frac{(5\,b^6c - 11\,ab^5d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^4c^4 - 4\,a^4b^3c^3d + 6\,a^5b^2c^2d^2 - 4\,a^6bcd^3 + a^7d^4)\sqrt{ab}} + \frac{(99\,b^2c^2d^4 - 110\,abcd^5 + 35\,a^2d^6)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^8 - 4\,ab^3c^7d + 6\,a^2b^2c^6d^2 - 4\,a^3bc^5d^3 + a^4c^4d^4)\sqrt{cd}} + \frac{19\,bcd^5x^3 - 11\,ad^6x^3 + 21\,bc^2d^4x - 13\,acd^5x}{8\,(b^3c^7 - 3\,ab^2c^6d + 3\,a^2bc^5d^2 - a^3c^4d^3)(dx^2 + c)^2} + \frac{6\,bcx^2 + 9\,adx^2 - ac}{3\,a^3c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*(b*x^2
 + a)) + 1/2*(5*b^6*c - 11*a*b^5*d)*arctan(b*x/sqrt(a*b))/((a^3*b^4*c^4 - 4
*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*sqrt(a*b)) +
1/8*(99*b^2*c^2*d^4 - 110*a*b*c*d^5 + 35*a^2*d^6)*arctan(d*x/sqrt(c*d))/((b
^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)
*sqrt(c*d)) + 1/8*(19*b*c*d^5*x^3 - 11*a*d^6*x^3 + 21*b*c^2*d^4*x - 13*a*c*
d^5*x)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(d*x^2 +
c)^2) + 1/3*(6*b*c*x^2 + 9*a*d*x^2 - a*c)/(a^3*c^4*x^3)
```

Mupad [B]
time = 1.23, size = 1161, normalized size = 3.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] ((x^2*(7*a*d + 5*b*c))/(3*a^2*c^2) - 1/(3*a*c) + (x^8*(35*a^4*b*d^6 - 20*b^
5*c^4*d^2 + 24*a*b^4*c^3*d^3 - 75*a^3*b^2*c*d^5 + 24*a^2*b^3*c^2*d^4))/(8*a
^3*c^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x^4*(60*b^5*
c^5 - 175*a^5*d^5 - 176*a^2*b^3*c^3*d^2 + 8*a*b^4*c^4*d + 319*a^4*b*c*d^4))
/(24*a^3*c^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^6*(105*a^5
*d^5 - 120*b^5*c^5 + 192*a^2*b^3*c^3*d^2 - 303*a^3*b^2*c^2*d^3 + 104*a*b^4*
c^4*d - 50*a^4*b*c*d^4))/(24*a^3*c^4*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b
*c*d)))/(x^5*(b*c^2 + 2*a*c*d) + x^7*(a*d^2 + 2*b*c*d) + a*c^2*x^3 + b*d^2*
x^9) + (atan((b^3*c^11*x*(-a^7*b^9)^(3/2)*400i + a^18*b*d^11*x*(-a^7*b^9)^(
1/2)*1225i + a^14*b^5*c^4*d^7*x*(-a^7*b^9)^(1/2)*9801i - a^15*b^4*c^3*d^8*x
*(-a^7*b^9)^(1/2)*21780i + a^16*b^3*c^2*d^9*x*(-a^7*b^9)^(1/2)*19030i - a*b
^2*c^10*d*x*(-a^7*b^9)^(3/2)*1760i + a^2*b*c^9*d^2*x*(-a^7*b^9)^(3/2)*1936i
 - a^17*b^2*c*d^10*x*(-a^7*b^9)^(1/2)*7700i)/(400*a^11*b^16*c^11 - 1225*a^2
2*b^5*d^11 - 1760*a^12*b^15*c^10*d + 7700*a^21*b^6*c*d^10 + 1936*a^13*b^14*
c^9*d^2 - 9801*a^18*b^9*c^4*d^7 + 21780*a^19*b^8*c^3*d^8 - 19030*a^20*b^7*c
^2*d^9))*(11*a*d - 5*b*c)*(-a^7*b^9)^(1/2)*1i)/(2*(a^11*d^4 + a^7*b^4*c^4 -
 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3)) - (atan((a^11*d^5*x
*(-c^9*d^7)^(3/2)*1225i + b^11*c^20*d*x*(-c^9*d^7)^(1/2)*400i - a^8*b^3*c^3
*d^2*x*(-c^9*d^7)^(3/2)*21780i + a^9*b^2*c^2*d^3*x*(-c^9*d^7)^(3/2)*19030i
+ a^2*b^9*c^18*d^3*x*(-c^9*d^7)^(1/2)*1936i - a^10*b*c*d^4*x*(-c^9*d^7)^(3/
2)*7700i + a^7*b^4*c^4*d*x*(-c^9*d^7)^(3/2)*9801i - a*b^10*c^19*d^2*x*(-c^9
*d^7)^(1/2)*1760i)/(1225*a^11*c^14*d^15 - 400*b^11*c^25*d^4 + 1760*a*b^10*c
^24*d^5 - 7700*a^10*b*c^15*d^14 - 1936*a^2*b^9*c^23*d^6 + 9801*a^7*b^4*c^18
*d^11 - 21780*a^8*b^3*c^17*d^12 + 19030*a^9*b^2*c^16*d^13))*(-c^9*d^7)^(1/2
)*(35*a^2*d^2 + 99*b^2*c^2 - 110*a*b*c*d)*1i)/(8*(b^4*c^13 + a^4*c^9*d^4 -
4*a^3*b*c^10*d^3 + 6*a^2*b^2*c^11*d^2 - 4*a*b^3*c^12*d))
```

3.319 $\int x^m(a+bx^2)^3(A+Bx^2)\,dx$

Optimal. Leaf size=96

$$\frac{a^3Ax^{1+m}}{1+m}+\frac{a^2(3Ab+aB)x^{3+m}}{3+m}+\frac{3ab(Ab+aB)x^{5+m}}{5+m}+\frac{b^2(Ab+3aB)x^{7+m}}{7+m}+\frac{b^3Bx^{9+m}}{9+m}$$

```
[Out] a^3*A*x^(1+m)/(1+m)+a^2*(3*A*b+B*a)*x^(3+m)/(3+m)+3*a*b*(A*b+B*a)*x^(5+m)/(
5+m)+b^2*(A*b+3*B*a)*x^(7+m)/(7+m)+b^3*B*x^(9+m)/(9+m)
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$\frac{a^3Ax^{m+1}}{m+1}+\frac{a^2x^{m+3}(aB+3Ab)}{m+3}+\frac{b^2x^{m+7}(3aB+Ab)}{m+7}+\frac{3abx^{m+5}(aB+Ab)}{m+5}+\frac{b^3Bx^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (a^3*A*x^(1 + m))/(1 + m) + (a^2*(3*A*b + a*B)*x^(3 + m))/(3 + m) + (3*a*b*
(A*b + a*B)*x^(5 + m))/(5 + m) + (b^2*(A*b + 3*a*B)*x^(7 + m))/(7 + m) + (b
^3*B*x^(9 + m))/(9 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx&=\int\left(a^3Ax^m+a^2(3Ab+aB)x^{2+m}+3ab(Ab+aB)x^{4+m}+b^2(Ab+3aB)x\right.\\&=\frac{a^3Ax^{1+m}}{1+m}+\frac{a^2(3Ab+aB)x^{3+m}}{3+m}+\frac{3ab(Ab+aB)x^{5+m}}{5+m}+\frac{b^2(Ab+3aB)x}{7+m}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 89, normalized size = 0.93

$$x^{1+m}\left(\frac{a^3A}{1+m}+\frac{a^2(3Ab+aB)x^2}{3+m}+\frac{3ab(Ab+aB)x^4}{5+m}+\frac{b^2(Ab+3aB)x^6}{7+m}+\frac{b^3Bx^8}{9+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] x^(1 + m)*((a^3*A)/(1 + m) + (a^2*(3*A*b + a*B)*x^2)/(3 + m) + (3*a*b*(A*b
+ a*B)*x^4)/(5 + m) + (b^2*(A*b + 3*a*B)*x^6)/(7 + m) + (b^3*B*x^8)/(9 + m)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(96) = 192$.
time = 0.07, size = 473, normalized size = 4.93

method	result
risch	$\frac{x\left(B\,b^3m^4x^8+16B\,b^3m^3x^8+A\,b^3m^4x^6+3Ba\,b^2m^4x^6+86B\,b^3m^2x^8+18A\,b^3m^3x^6+54Ba\,b^2m^3x^6+176m\,x^8B\,b^3+3Aa\,b^2m^4x^4+104A\right.}{}$
gosper	$\frac{x^{1+m}\left(B\,b^3m^4x^8+16B\,b^3m^3x^8+A\,b^3m^4x^6+3Ba\,b^2m^4x^6+86B\,b^3m^2x^8+18A\,b^3m^3x^6+54Ba\,b^2m^3x^6+176m\,x^8B\,b^3+3Aa\,b^2m^4x^4+1\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^3*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] x*(B*b^3*m^4*x^8+16*B*b^3*m^3*x^8+A*b^3*m^4*x^6+3*B*a*b^2*m^4*x^6+86*B*b^3*
m^2*x^8+18*A*b^3*m^3*x^6+54*B*a*b^2*m^3*x^6+176*B*b^3*m*x^8+3*A*a*b^2*m^4*x
^4+104*A*b^3*m^2*x^6+3*B*a^2*b*m^4*x^4+312*B*a*b^2*m^2*x^6+105*B*b^3*x^8+60
*A*a*b^2*m^3*x^4+222*A*b^3*m*x^6+60*B*a^2*b*m^3*x^4+666*B*a*b^2*m*x^6+3*A*a
^2*b*m^4*x^2+390*A*a*b^2*m^2*x^4+135*A*b^3*x^6+B*a^3*m^4*x^2+390*B*a^2*b*m^
2*x^4+405*B*a*b^2*x^6+66*A*a^2*b*m^3*x^2+900*A*a*b^2*m*x^4+22*B*a^3*m^3*x^2
+900*B*a^2*b*m*x^4+A*a^3*m^4+492*A*a^2*b*m^2*x^2+567*A*a*b^2*x^4+164*B*a^3*
m^2*x^2+567*B*a^2*b*x^4+24*A*a^3*m^3+1374*A*a^2*b*m*x^2+458*B*a^3*m*x^2+206
*A*a^3*m^2+945*A*a^2*b*x^2+315*B*a^3*x^2+744*A*a^3*m+945*A*a^3)*x^m/(9+m)/(
7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [A]
time = 0.28, size = 129, normalized size = 1.34

$$\frac{Bb^3x^{m+9}}{m+9}+\frac{3\,Bab^2x^{m+7}}{m+7}+\frac{Ab^3x^{m+7}}{m+7}+\frac{3\,Ba^2bx^{m+5}}{m+5}+\frac{3\,Aab^2x^{m+5}}{m+5}+\frac{Ba^3x^{m+3}}{m+3}+\frac{3\,Aa^2bx^{m+3}}{m+3}+\frac{Aa^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] B*b^3*x^(m + 9)/(m + 9) + 3*B*a*b^2*x^(m + 7)/(m + 7) + A*b^3*x^(m + 7)/(m
+ 7) + 3*B*a^2*b*x^(m + 5)/(m + 5) + 3*A*a*b^2*x^(m + 5)/(m + 5) + B*a^3*x^
(m + 3)/(m + 3) + 3*A*a^2*b*x^(m + 3)/(m + 3) + A*a^3*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(96) = 192$.
time = 1.00, size = 379, normalized size = 3.95

$$\frac{\left(\left(Bb^3m^4+16\,Bb^3m^3+86\,Bb^3m^2+176\,Bb^3m+105\,Bb^3\right)x^9+\left(\left(3\,Bab^2+Ab^3\right)m^4+405\,Bab^2+135\,Ab^3+18\left(3\,Bab^2+Ab^3\right)m^3+104\left(3\,Bab^2+Ab^3\right)m^2+222\left(3\,Bab^2+Ab^3\right)m\right)x^7+3\left(\left(Ba^2b+Aab^2\right)m^4+189\,Ba^2b+189\,Aab^2+20\left(Ba^2b+Aab^2\right)m^3+130\left(Ba^2b+Aab^2\right)m^2+300\left(Ba^2b+Aab^2\right)m\right)x^5+\left(\left(Ba^3+3\,Aa^2b\right)m^4+315\,Ba^3+945\,Aa^2b+22\left(Ba^3+3\,Aa^2b\right)m^3+164\left(Ba^3+3\,Aa^2b\right)m^2+458\left(Ba^3+3\,Aa^2b\right)m\right)x^3+\left(Aa^3m^4+24\,Aa^3m^3+206\,Aa^3m^2+744\,Aa^3m+945\,Aa^3\right)x\right)x^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] ((B*b^3*m^4 + 16*B*b^3*m^3 + 86*B*b^3*m^2 + 176*B*b^3*m + 105*B*b^3)*x^9 +
((3*B*a*b^2 + A*b^3)*m^4 + 405*B*a*b^2 + 135*A*b^3 + 18*(3*B*a*b^2 + A*b^3)
*m^3 + 104*(3*B*a*b^2 + A*b^3)*m^2 + 222*(3*B*a*b^2 + A*b^3)*m)*x^7 + 3*((B
*a^2*b + A*a*b^2)*m^4 + 189*B*a^2*b + 189*A*a*b^2 + 20*(B*a^2*b + A*a*b^2)*
m^3 + 130*(B*a^2*b + A*a*b^2)*m^2 + 300*(B*a^2*b + A*a*b^2)*m)*x^5 + ((B*a^
3 + 3*A*a^2*b)*m^4 + 315*B*a^3 + 945*A*a^2*b + 22*(B*a^3 + 3*A*a^2*b)*m^3 +
 164*(B*a^3 + 3*A*a^2*b)*m^2 + 458*(B*a^3 + 3*A*a^2*b)*m)*x^3 + (A*a^3*m^4
+ 24*A*a^3*m^3 + 206*A*a^3*m^2 + 744*A*a^3*m + 945*A*a^3)*x)*x^m/(m^5 + 25*
m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2069 vs. $2(87) = 174$.
time = 0.61, size = 2069, normalized size = 21.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a**3/(8*x**8) - A*a**2*b/(2*x**6) - 3*A*a*b**2/(4*x**4) - A*b
**3/(2*x**2) - B*a**3/(6*x**6) - 3*B*a**2*b/(4*x**4) - 3*B*a*b**2/(2*x**2)
+ B*b**3*log(x), Eq(m, -9)), (-A*a**3/(6*x**6) - 3*A*a**2*b/(4*x**4) - 3*A*
a*b**2/(2*x**2) + A*b**3*log(x) - B*a**3/(4*x**4) - 3*B*a**2*b/(2*x**2) + 3
*B*a*b**2*log(x) + B*b**3*x**2/2, Eq(m, -7)), (-A*a**3/(4*x**4) - 3*A*a**2*
b/(2*x**2) + 3*A*a*b**2*log(x) + A*b**3*x**2/2 - B*a**3/(2*x**2) + 3*B*a**2
*b*log(x) + 3*B*a*b**2*x**2/2 + B*b**3*x**4/4, Eq(m, -5)), (-A*a**3/(2*x**2
) + 3*A*a**2*b*log(x) + 3*A*a*b**2*x**2/2 + A*b**3*x**4/4 + B*a**3*log(x) +
 3*B*a**2*b*x**2/2 + 3*B*a*b**2*x**4/4 + B*b**3*x**6/6, Eq(m, -3)), (A*a**3
*log(x) + 3*A*a**2*b*x**2/2 + 3*A*a*b**2*x**4/4 + A*b**3*x**6/6 + B*a**3*x*
*2/2 + 3*B*a**2*b*x**4/4 + B*a*b**2*x**6/2 + B*b**3*x**8/8, Eq(m, -1)), (A*
a**3*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 24
*A*a**3*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
 206*A*a**3*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 744*A*a**3*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 945*A*a**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 3*A*a**2*b*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 66*A*a**2*b*m**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 492*A*a**2*b*m**2*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 1374*A*a**2*b*m*x**3*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 945*A*a**2*b*x**3*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 3*A*a*b**2*m**4*x**5*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 60*A*a*b**2*m**3*x**5*x**m/(
```

```
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 390*A*a*b**2*m**2*x*
*5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 900*A*a*b**
2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 567*A
*a*b**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + A
*b**3*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 18*A*b**3*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 104*A*b**3*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 222*A*b**3*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 135*A*b**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + B*a**3*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950
*m**2 + 1689*m + 945) + 22*B*a**3*m**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 164*B*a**3*m**2*x**3*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 458*B*a**3*m*x**3*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 315*B*a**3*x**3*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 3*B*a**2*b*m**4*x**5*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 60*B*a**2*b*m**3*x**5*x**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 390*B*a**2*b*m**2
*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 900*B*a*
*2*b*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 56
7*B*a**2*b*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 3*B*a*b**2*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + 54*B*a*b**2*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 312*B*a*b**2*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 666*B*a*b**2*m*x**7*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 405*B*a*b**2*x**7*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + B*b**3*m**4*x**9*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 16*B*b**3*m**3*x**9*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*B*b**3*m**2*x**9*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*B*b**3*m*x**9*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*B*b**3*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(96) = 192.

time = 0.58, size = 593, normalized size = 6.18

$$\frac{Bb^3m^4x^9x^m + 16\,Bb^3m^3x^9x^m + 3\,Bab^2m^4x^7x^m + Ab^3m^4x^7x^m + 86\,Bb^3m^2x^9x^m + 54\,Bab^2m^3x^7x^m + 18\,Ab^3m^3x^7x^m + 176\,Bb^3mx^9x^m + 3\,Ba^2bm^4x^5x^m + 3\,Aab^2m^4x^5x^m + 312\,Bab^2m^2x^7x^m + 104\,Ab^3m^2x^7x^m + 105\,Bb^3x^9x^m + 60\,Ba^2bm^3x^5x^m + 60\,Aab^2m^3x^5x^m + 666\,Bab^2mx^7x^m + 222\,Ab^3mx^7x^m + Ba^3m^4x^3x^m + 3\,Aa^2bm^4x^3x^m + 390\,Ba^2bm^2x^5x^m + 390\,Aab^2m^2x^5x^m + 405\,Bab^2x^7x^m + 135\,Ab^3x^7x^m + 22\,Ba^3m^3x^3x^m + 66\,Aa^2bm^3x^3x^m + 900\,Ba^2bmx^5x^m + 900\,Aab^2mx^5x^m + Aa^3m^4xx^m + 164\,Ba^3m^2x^3x^m + 492\,Aa^2bm^2x^3x^m + 567\,Ba^2bx^5x^m + 567\,Aab^2x^5x^m + 24\,Aa^3m^3xx^m + 458\,Ba^3mx^3x^m + 1374\,Aa^2bmx^3x^m + 206\,Aa^3m^2xx^m + 315\,Ba^3x^3x^m + 945\,Aa^2bx^3x^m + 744\,Aa^3mxx^m + 945\,Aa^3xx^m}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] (B*b^3*m^4*x^9*x^m + 16*B*b^3*m^3*x^9*x^m + 3*B*a*b^2*m^4*x^7*x^m + A*b^3*m
^4*x^7*x^m + 86*B*b^3*m^2*x^9*x^m + 54*B*a*b^2*m^3*x^7*x^m + 18*A*b^3*m^3*x
^7*x^m + 176*B*b^3*m*x^9*x^m + 3*B*a^2*b*m^4*x^5*x^m + 3*A*a*b^2*m^4*x^5*x^
m + 312*B*a*b^2*m^2*x^7*x^m + 104*A*b^3*m^2*x^7*x^m + 105*B*b^3*x^9*x^m + 6
```

```
0*B*a^2*b*m^3*x^5*x^m + 60*A*a*b^2*m^3*x^5*x^m + 666*B*a*b^2*m*x^7*x^m + 22
2*A*b^3*m*x^7*x^m + B*a^3*m^4*x^3*x^m + 3*A*a^2*b*m^4*x^3*x^m + 390*B*a^2*b
*m^2*x^5*x^m + 390*A*a*b^2*m^2*x^5*x^m + 405*B*a*b^2*x^7*x^m + 135*A*b^3*x^
7*x^m + 22*B*a^3*m^3*x^3*x^m + 66*A*a^2*b*m^3*x^3*x^m + 900*B*a^2*b*m*x^5*x
^m + 900*A*a*b^2*m*x^5*x^m + A*a^3*m^4*x*x^m + 164*B*a^3*m^2*x^3*x^m + 492*
A*a^2*b*m^2*x^3*x^m + 567*B*a^2*b*x^5*x^m + 567*A*a*b^2*x^5*x^m + 24*A*a^3*
m^3*x*x^m + 458*B*a^3*m*x^3*x^m + 1374*A*a^2*b*m*x^3*x^m + 206*A*a^3*m^2*x*
x^m + 315*B*a^3*x^3*x^m + 945*A*a^2*b*x^3*x^m + 744*A*a^3*m*x*x^m + 945*A*a
^3*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Mupad [B]
time = 0.27, size = 289, normalized size = 3.01

$$\frac{A\,a^3\,x\,x^m\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{B\,b^3\,x^m\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{a^2\,x^m\,x^3\,\left(3\,A\,b+B\,a\right)\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{b^2\,x^m\,x^7\,\left(A\,b+3\,B\,a\right)\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{3\,a\,b\,x^m\,x^5\,\left(A\,b+B\,a\right)\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] (A*a^3*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 23
0*m^3 + 25*m^4 + m^5 + 945) + (B*b^3*x^m*x^9*(176*m + 86*m^2 + 16*m^3 + m^4
 + 105))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (a^2*x^m*x^3*(
3*A*b + B*a)*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 23
0*m^3 + 25*m^4 + m^5 + 945) + (b^2*x^m*x^7*(A*b + 3*B*a)*(222*m + 104*m^2 +
 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (
3*a*b*x^m*x^5*(A*b + B*a)*(300*m + 130*m^2 + 20*m^3 + m^4 + 189))/(1689*m +
 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945)
```

3.320 $\int x^m(a+bx^2)^2(A+Bx^2)\,dx$

Optimal. Leaf size=71

$$\frac{a^2Ax^{1+m}}{1+m}+\frac{a(2Ab+aB)x^{3+m}}{3+m}+\frac{b(Ab+2aB)x^{5+m}}{5+m}+\frac{b^2Bx^{7+m}}{7+m}$$

```
[Out] a^2*A*x^(1+m)/(1+m)+a*(2*A*b+B*a)*x^(3+m)/(3+m)+b*(A*b+2*B*a)*x^(5+m)/(5+m)
+b^2*B*x^(7+m)/(7+m)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+3}(aB+2Ab)}{m+3}+\frac{bx^{m+5}(2aB+Ab)}{m+5}+\frac{b^2Bx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(3 + m))/(3 + m) + (b*(A*b +
 2*a*B)*x^(5 + m))/(5 + m) + (b^2*B*x^(7 + m))/(7 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx&=\int\left(a^2Ax^m+a(2Ab+aB)x^{2+m}+b(Ab+2aB)x^{4+m}+b^2Bx^{6+m}\right)\,dx\\&=\frac{a^2Ax^{1+m}}{1+m}+\frac{a(2Ab+aB)x^{3+m}}{3+m}+\frac{b(Ab+2aB)x^{5+m}}{5+m}+\frac{b^2Bx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.93

$$x^{1+m}\left(\frac{a^2A}{1+m}+\frac{a(2Ab+aB)x^2}{3+m}+\frac{b(Ab+2aB)x^4}{5+m}+\frac{b^2Bx^6}{7+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^2)/(3 + m) + (b*(A*b + 2*a*
B)*x^4)/(5 + m) + (b^2*B*x^6)/(7 + m))
```

Maple [A]
time = 0.08, size = 82, normalized size = 1.15

method	result
norman	$\frac{a(2Ab+Ba)x^{3}\mathrm{e}^{m\ln(x)}}{3+m}+\frac{a^{2}Ax\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{b(Ab+2Ba)x^{5}\mathrm{e}^{m\ln(x)}}{5+m}+\frac{b^{2}B\,x^{7}\mathrm{e}^{m\ln(x)}}{7+m}$
risch	$x\left(B\,b^{2}m^{3}x^{6}+9B\,b^{2}m^{2}x^{6}+A\,b^{2}m^{3}x^{4}+2Bab\,m^{3}x^{4}+23m\,x^{6}b^{2}B+11A\,b^{2}m^{2}x^{4}+22Bab\,m^{2}x^{4}+15b^{2}B\,x^{6}+2Aab\,m^{3}x^{2}+31A\,b^{2}x^{4}m+\right.$
gosper	$x^{1+m}\left(B\,b^{2}m^{3}x^{6}+9B\,b^{2}m^{2}x^{6}+A\,b^{2}m^{3}x^{4}+2Bab\,m^{3}x^{4}+23m\,x^{6}b^{2}B+11A\,b^{2}m^{2}x^{4}+22Bab\,m^{2}x^{4}+15b^{2}B\,x^{6}+2Aab\,m^{3}x^{2}+31A\,b^{2}x^{4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] a*(2*A*b+B*a)/(3+m)*x^3*exp(m*ln(x))+a^2*A/(1+m)*x*exp(m*ln(x))+b*(A*b+2*B*
a)/(5+m)*x^5*exp(m*ln(x))+b^2*B/(7+m)*x^7*exp(m*ln(x))
```

Maxima [A]
time = 0.29, size = 91, normalized size = 1.28

$$\frac{Bb^{2}x^{m+7}}{m+7}+\frac{2\,Babx^{m+5}}{m+5}+\frac{Ab^{2}x^{m+5}}{m+5}+\frac{Ba^{2}x^{m+3}}{m+3}+\frac{2\,Aabx^{m+3}}{m+3}+\frac{Aa^{2}x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] B*b^2*x^(m + 7)/(m + 7) + 2*B*a*b*x^(m + 5)/(m + 5) + A*b^2*x^(m + 5)/(m +
5) + B*a^2*x^(m + 3)/(m + 3) + 2*A*a*b*x^(m + 3)/(m + 3) + A*a^2*x^(m + 1)/
(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(71) = 142$.
time = 1.18, size = 215, normalized size = 3.03

$$\frac{((Bb^{2}m^{3}+9\,Bb^{2}m^{2}+23\,Bb^{2}m+15\,Bb^{2})x^{7}+((2\,Bab+Ab^{2})m^{3}+42\,Bab+21\,Ab^{2}+11\,(2\,Bab+Ab^{2})m^{2}+31\,(2\,Bab+Ab^{2})m)x^{5}+((Ba^{2}+2\,Aab)m^{3}+35\,Ba^{2}+70\,Aab+13\,(Ba^{2}+2\,Aab)m^{2}+47\,(Ba^{2}+2\,Aab)m)x^{3}+(Aa^{2}m^{3}+15\,Aa^{2}m^{2}+71\,Aa^{2}m+105\,Aa^{2})x)x^{m}}{m^{4}+16\,m^{3}+86\,m^{2}+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] ((B*b^2*m^3 + 9*B*b^2*m^2 + 23*B*b^2*m + 15*B*b^2)*x^7 + ((2*B*a*b + A*b^2)
*m^3 + 42*B*a*b + 21*A*b^2 + 11*(2*B*a*b + A*b^2)*m^2 + 31*(2*B*a*b + A*b^2
```

)*m)*x^5 + ((B*a^2 + 2*A*a*b)*m^3 + 35*B*a^2 + 70*A*a*b + 13*(B*a^2 + 2*A*a*b)*m^2 + 47*(B*a^2 + 2*A*a*b)*m)*x^3 + (A*a^2*m^3 + 15*A*a^2*m^2 + 71*A*a^2*m + 105*A*a^2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(63) = 126$.
time = 0.40, size = 1044, normalized size = 14.70

$$\begin{cases} -\frac{Aa^2}{6x^6} - \frac{Aab}{2x^4} - \frac{Ab^2}{2x^2} - \frac{Ba^2}{4x^4} - \frac{Bab}{x^2} + Bb^2\log(x) & \text{for}\ m = -7 \\ -\frac{Aa^2}{4x^4} - \frac{Aab}{x^2} + Ab^2\log(x) - \frac{Ba^2}{2x^2} + 2Bab\log(x) + \frac{Bb^2x^2}{2} & \text{for}\ m = -5 \\ -\frac{Aa^2}{2x^2} + 2Aab\log(x) + \frac{Ab^2x^2}{2} + Ba^2\log(x) + Babx^2 + \frac{Bb^2x^4}{4} & \text{for}\ m = -3 \\ Aa^2\log(x) + Aabx^2 + \frac{Ab^2x^4}{4} + \frac{Ba^2x^2}{2} + \frac{Babx^4}{2} + \frac{Bb^2x^6}{6} & \text{for}\ m = -1 \\ \frac{Aa^2m^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{15Aa^2m^2xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{71Aa^2mxx^m}{m^4+16m^3+86m^2+176m+105} + \frac{105Aa^2xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{2Aabm^3x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{26Aabm^2x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{94Aabmx^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{70Aabx^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{Ab^2m^3x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{11Ab^2m^2x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{31Ab^2mx^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{21Ab^2x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{Ba^2m^3x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{13Ba^2m^2x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{47Ba^2mx^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{35Ba^2x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{2Babm^3x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{22Babm^2x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{62Babmx^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{42Babx^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{Bb^2m^3x^7x^m}{m^4+16m^3+86m^2+176m+105} + \frac{9Bb^2m^2x^7x^m}{m^4+16m^3+86m^2+176m+105} + \frac{23Bb^2mx^7x^m}{m^4+16m^3+86m^2+176m+105} + \frac{15Bb^2x^7x^m}{m^4+16m^3+86m^2+176m+105} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*b**2/(2*x**2) - B*a**2/(4*
x**4) - B*a*b/x**2 + B*b**2*log(x), Eq(m, -7)), (-A*a**2/(4*x**4) - A*a*b/x
**2 + A*b**2*log(x) - B*a**2/(2*x**2) + 2*B*a*b*log(x) + B*b**2*x**2/2, Eq(
m, -5)), (-A*a**2/(2*x**2) + 2*A*a*b*log(x) + A*b**2*x**2/2 + B*a**2*log(x)
 + B*a*b*x**2 + B*b**2*x**4/4, Eq(m, -3)), (A*a**2*log(x) + A*a*b*x**2 + A*
b**2*x**4/4 + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b**2*x**6/6, Eq(m, -1)), (A*
a**2*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*A*a**2*m**2*
x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*A*a**2*m*x*x**m/(m**4
+ 16*m**3 + 86*m**2 + 176*m + 105) + 105*A*a**2*x*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 2*A*a*b*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 1
76*m + 105) + 26*A*a*b*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 1
05) + 94*A*a*b*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 70*A*
a*b*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + A*b**2*m**3*x**5*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*A*b**2*m**2*x**5*x**m/(m*
*4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*A*b**2*m*x**5*x**m/(m**4 + 16*m*
*3 + 86*m**2 + 176*m + 105) + 21*A*b**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2
 + 176*m + 105) + B*a**2*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m +
 105) + 13*B*a**2*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 47*B*a**2*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*B*a**2
*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*B*a*b*m**3*x**5*x**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 22*B*a*b*m**2*x**5*x**m/(m**4
+ 16*m**3 + 86*m**2 + 176*m + 105) + 62*B*a*b*m*x**5*x**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + 42*B*a*b*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + B*b**2*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + 9*B*b**2*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*B*
b**2*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*B*b**2*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(71) = 142$.
time = 0.68, size = 332, normalized size = 4.68

$$\frac{Bb^2m^3x^7x^m + 9\,Bb^2m^2x^7x^m + 2\,Babm^3x^5x^m + Ab^2m^3x^5x^m + 23\,Bb^2mx^7x^m + 22\,Babm^2x^5x^m + 11\,Ab^2m^2x^5x^m + 15\,Bb^2x^7x^m + Ba^2m^3x^3x^m + 2\,Aabm^3x^3x^m + 62\,Babmx^5x^m + 31\,Ab^2mx^5x^m + 13\,Ba^2m^2x^3x^m + 26\,Aabm^2x^3x^m + 42\,Babx^5x^m + 21\,Ab^2x^5x^m + Aa^2m^3xx^m + 47\,Ba^2mx^3x^m + 94\,Aabmx^3x^m + 15\,Aa^2m^2xx^m + 35\,Ba^2x^3x^m + 70\,Aabx^3x^m + 71\,Aa^2mxx^m + 105\,Aa^2xx^m}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] (B*b^2*m^3*x^7*x^m + 9*B*b^2*m^2*x^7*x^m + 2*B*a*b*m^3*x^5*x^m + A*b^2*m^3*
x^5*x^m + 23*B*b^2*m*x^7*x^m + 22*B*a*b*m^2*x^5*x^m + 11*A*b^2*m^2*x^5*x^m
+ 15*B*b^2*x^7*x^m + B*a^2*m^3*x^3*x^m + 2*A*a*b*m^3*x^3*x^m + 62*B*a*b*m*x
^5*x^m + 31*A*b^2*m*x^5*x^m + 13*B*a^2*m^2*x^3*x^m + 26*A*a*b*m^2*x^3*x^m +
 42*B*a*b*x^5*x^m + 21*A*b^2*x^5*x^m + A*a^2*m^3*x*x^m + 47*B*a^2*m*x^3*x^m
 + 94*A*a*b*m*x^3*x^m + 15*A*a^2*m^2*x*x^m + 35*B*a^2*x^3*x^m + 70*A*a*b*x^
3*x^m + 71*A*a^2*m*x*x^m + 105*A*a^2*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)
```

Mupad [B]
time = 0.18, size = 177, normalized size = 2.49

$$x^m\left(\frac{B\,b^2\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{A\,a^2\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{a\,x^3\,\left(2\,A\,b+B\,a\right)\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b\,x^5\,\left(A\,b+2\,B\,a\right)\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^m*((B*b^2*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 +
 105) + (A*a^2*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 + m^
4 + 105) + (a*x^3*(2*A*b + B*a)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2
 + 16*m^3 + m^4 + 105) + (b*x^5*(A*b + 2*B*a)*(31*m + 11*m^2 + m^3 + 21))/(
176*m + 86*m^2 + 16*m^3 + m^4 + 105))
```

3.321 $\int x^m(a+bx^2)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=45

$$\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{3+m}}{3+m}+\frac{bBx^{5+m}}{5+m}$$

```
[Out] a*A*x^(1+m)/(1+m)+(A*b+B*a)*x^(3+m)/(3+m)+b*B*x^(5+m)/(5+m)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$\frac{x^{m+3}(aB+Ab)}{m+3}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+5}}{m+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(3 + m))/(3 + m) + (b*B*x^(5 + m))
/(5 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)\left(A+Bx^2\right)\,dx&=\int\left(aAx^m+(Ab+aB)x^{2+m}+bBx^{4+m}\right)\,dx\\&=\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{3+m}}{3+m}+\frac{bBx^{5+m}}{5+m}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 0.93

$$x^{1+m}\left(\frac{aA}{1+m}+\frac{(Ab+aB)x^2}{3+m}+\frac{bBx^4}{5+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^2)/(3 + m) + (b*B*x^4)/(5 + m))
```

Maple [A]
time = 0.02, size = 53, normalized size = 1.18

method	result	size
norman	$\frac{(Ab+Ba)x^3 e^{m\ln(x)}}{3+m}+\frac{Aax\,e^{m\ln(x)}}{1+m}+\frac{Bb\,x^5 e^{m\ln(x)}}{5+m}$	53
risch	$\frac{x\left(Bb\,m^2x^4+4Bbm\,x^4+Ab\,m^2x^2+Ba\,m^2x^2+3bB\,x^4+6Abm\,x^2+6Bam\,x^2+Aa\,m^2+5Ab\,x^2+5Ba\,x^2+8Aam+15Aa\right)x^m}{(5+m)(3+m)(1+m)}$	109
gosper	$\frac{x^{1+m}\left(Bb\,m^2x^4+4Bbm\,x^4+Ab\,m^2x^2+Ba\,m^2x^2+3bB\,x^4+6Abm\,x^2+6Bam\,x^2+Aa\,m^2+5Ab\,x^2+5Ba\,x^2+8Aam+15Aa\right)}{(5+m)(3+m)(1+m)}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] (A*b+B*a)/(3+m)*x^3*exp(m*ln(x))+A*a/(1+m)*x*exp(m*ln(x))+B*b/(5+m)*x^5*exp
(m*ln(x))
```

Maxima [A]
time = 0.33, size = 53, normalized size = 1.18

$$\frac{Bbx^{m+5}}{m+5}+\frac{Bax^{m+3}}{m+3}+\frac{Abx^{m+3}}{m+3}+\frac{Aax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] B*b*x^(m + 5)/(m + 5) + B*a*x^(m + 3)/(m + 3) + A*b*x^(m + 3)/(m + 3) + A*a
*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.
time = 0.98, size = 92, normalized size = 2.04

$$\frac{((Bbm^2+4\,Bbm+3\,Bb)x^5+((Ba+Ab)m^2+5\,Ba+5\,Ab+6\,(Ba+Ab)m)x^3+(Aam^2+8\,Aam+15\,Aa)x)x^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] ((B*b*m^2 + 4*B*b*m + 3*B*b)*x^5 + ((B*a + A*b)*m^2 + 5*B*a + 5*A*b + 6*(B*
a + A*b)*m)*x^3 + (A*a*m^2 + 8*A*a*m + 15*A*a)*x)*x^m/(m^3 + 9*m^2 + 23*m +
 15)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(37) = 74$.

time = 0.25, size = 410, normalized size = 9.11

$$\begin{cases} -\frac{Aa}{4x^4} - \frac{Ab}{2x^2} - \frac{Ba}{2x^2} + Bb\log(x) & \text{for } m = -5 \\ -\frac{Aa}{2x^2} + Ab\log(x) + Ba\log(x) + \frac{Bbx^2}{2} & \text{for } m = -3 \\ Aa\log(x) + \frac{Abx^2}{2} + \frac{Bax^2}{2} + \frac{Bbx^4}{4} & \text{for } m = -1 \\ \frac{Aam^2xx^m}{m^3+9m^2+23m+15} + \frac{8Aamxx^m}{m^3+9m^2+23m+15} + \frac{15Aaxx^m}{m^3+9m^2+23m+15} + \frac{Abm^2x^3x^m}{m^3+9m^2+23m+15} + \frac{6Abmx^3x^m}{m^3+9m^2+23m+15} + \frac{5Abx^3x^m}{m^3+9m^2+23m+15} + \frac{Bam^2x^3x^m}{m^3+9m^2+23m+15} + \frac{6Bamx^3x^m}{m^3+9m^2+23m+15} + \frac{5Bax^3x^m}{m^3+9m^2+23m+15} + \frac{Bbm^2x^5x^m}{m^3+9m^2+23m+15} + \frac{4Bbmx^5x^m}{m^3+9m^2+23m+15} + \frac{3Bbx^5x^m}{m^3+9m^2+23m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a/(4*x**4) - A*b/(2*x**2) - B*a/(2*x**2) + B*b*log(x), Eq(m,
-5)), (-A*a/(2*x**2) + A*b*log(x) + B*a*log(x) + B*b*x**2/2, Eq(m, -3)), (A
*a*log(x) + A*b*x**2/2 + B*a*x**2/2 + B*b*x**4/4, Eq(m, -1)), (A*a*m**2*x*x
**m/(m**3 + 9*m**2 + 23*m + 15) + 8*A*a*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15
) + 15*A*a*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + A*b*m**2*x**3*x**m/(m**3 +
9*m**2 + 23*m + 15) + 6*A*b*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*A*b
*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + B*a*m**2*x**3*x**m/(m**3 + 9*m**2
+ 23*m + 15) + 6*B*a*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*B*a*x**3*x
**m/(m**3 + 9*m**2 + 23*m + 15) + B*b*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m
+ 15) + 4*B*b*m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*B*b*x**5*x**m/(m*
*3 + 9*m**2 + 23*m + 15), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(45) = 90$.

time = 0.66, size = 143, normalized size = 3.18

$$\frac{Bbm^2x^5x^m + 4\,Bbmx^5x^m + Bam^2x^3x^m + Abm^2x^3x^m + 3\,Bbx^5x^m + 6\,Bamx^3x^m + 6\,Abmx^3x^m + Aam^2xx^m + 5\,Bax^3x^m + 5\,Abx^3x^m + 8\,Aamxx^m + 15\,Aaxx^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] (B*b*m^2*x^5*x^m + 4*B*b*m*x^5*x^m + B*a*m^2*x^3*x^m + A*b*m^2*x^3*x^m + 3*
B*b*x^5*x^m + 6*B*a*m*x^3*x^m + 6*A*b*m*x^3*x^m + A*a*m^2*x*x^m + 5*B*a*x^3
*x^m + 5*A*b*x^3*x^m + 8*A*a*m*x*x^m + 15*A*a*x*x^m)/(m^3 + 9*m^2 + 23*m +
15)
```

Mupad [B]

time = 0.16, size = 95, normalized size = 2.11

$$x^m\,\left(\frac{x^3\,\left(A\,b+B\,a\right)\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}+\frac{B\,b\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{A\,a\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] x^m*((x^3*(A*b + B*a)*(6*m + m^2 + 5))/(23*m + 9*m^2 + m^3 + 15) + (B*b*x^5
*(4*m + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (A*a*x*(8*m + m^2 + 15))/(23*
m + 9*m^2 + m^3 + 15))
```

3.322 $\int \frac{x^m(A+Bx^2)}{a+bx^2} dx$

Optimal. Leaf size=66

$$\frac{Bx^{1+m}}{b(1+m)} + \frac{(Ab-aB)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab(1+m)}$$

```
[Out] B*x^(1+m)/b/(1+m)+(A*b-B*a)*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b
*x^2/a)/a/b/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {470, 371}

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)} + \frac{Bx^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^{1+m}}{b(1+m)} - \frac{(-Ab(1+m)+aB(1+m))\int \frac{x^m}{a+bx^2}\,dx}{b(1+m)}$$
$$= \frac{Bx^{1+m}}{b(1+m)} + \frac{(Ab-aB)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab(1+m)}$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.83

$$\frac{x^{1+m}\left(aB+(Ab-aB)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{ab(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(a + b*x^2),x]
[Out] (x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)]))/(a*b*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m(B\,x^2+A)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(b*x^2+a),x)
[Out] int(x^m*(B*x^2+A)/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.97, size = 190, normalized size = 2.88

$$\frac{Amxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Axx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Bmx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3Bx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] A*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(4*a*gamma(m/2 + 3/2)) + A*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m
/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*m*x**3*x**m*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5
/2)) + 3*B*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma
(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(B\,x^2+A\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] int((x^m*(A + B*x^2))/(a + b*x^2), x)
```

3.323 $\int \frac{x^m (A+Bx^2)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=93

$$\frac{(Ab-aB)x^{1+m}}{2ab\,(a+bx^2)}+\frac{(aB(1+m)+A(b-bm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2b(1+m)}$$

```
[Out] 1/2*(A*b-B*a)*x^(1+m)/a/b/(b*x^2+a)+1/2*(a*B*(1+m)+A*(-b*m+b))*x^(1+m)*hype
rgeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/b/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {468, 371}

$$\frac{x^{m+1}(aB(m+1)+A(b-bm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*B*(1 + m) + A*(b - b*m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*
b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ab-aB)x^{1+m}}{2ab\,(a+bx^2)} + \frac{(aB(1+m)+A(b-bm))\int \frac{x^m}{a+bx^2}\,dx}{2ab}$$
$$= \frac{(Ab-aB)x^{1+m}}{2ab\,(a+bx^2)} + \frac{(aB(1+m)+A(b-bm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2b(1+m)}$$

Mathematica [A]
time = 0.10, size = 80, normalized size = 0.86

$$\frac{x^{1+m}\left(aB\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+(Ab-aB)\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{a^2b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(a + b*x^2)^2,x]
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] +
(A*b - a*B)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a^2
*b*(1 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m(B\,x^2+A)}{(b\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(b*x^2+a)^2,x)
[Out] int(x^m*(B*x^2+A)/(b*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.33, size = 906, normalized size = 9.74

$$A\left(-\frac{am^2xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2amxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{axx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2axx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}-\frac{bm^2x^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{bx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}\right)+B\left(-\frac{am^2x^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{4amx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{2amx^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{3ax^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{6ax^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{bm^2x^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{4bmx^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{3bx^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] A*(-a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/
2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*m
*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2
 + 3/2)) + a*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a
*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2
 + 3/2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2
)) + b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))) + B*(-a
*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2
+ 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4*a*m*x
**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/
(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 2*a*m*x**3*x**
m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2
)) - 3*a*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m
/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 6*a*
x**3*x**m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m
/2 + 5/2)) - b*m**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3
/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5
/2)) - 4*b*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gam
ma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) -
3*b*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 +
3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(B\,x^2+A\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^2))/(a + b*x^2)^2,x)
[Out] int((x^m*(A + B*x^2))/(a + b*x^2)^2, x)
```

3.324 $\int \frac{x^m (A+Bx^2)}{(a+bx^2)^3} dx$

Optimal. Leaf size=93

$$\frac{(Ab-aB)x^{1+m}}{4ab\,(a+bx^2)^2}+\frac{(Ab(3-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{4a^3b(1+m)}$$

```
[Out] 1/4*(A*b-B*a)*x^(1+m)/a/b/(b*x^2+a)^2+1/4*(A*b*(3-m)+a*B*(1+m))*x^(1+m)*hyp
ergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/b/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {468, 371}

$$\frac{x^{m+1}(aB(m+1)+Ab(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(4*a*b*(a + b*x^2)^2) + ((A*b*(3 - m) + a*B*(1 + m)
)*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(4*a^
3*b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(Ab-aB)x^{1+m}}{4ab\,(a+bx^2)^2} + \frac{(-Ab(-3+m)+aB(1+m))\int \frac{x^m}{(a+bx^2)^2}\,dx}{4ab}$$
$$= \frac{(Ab-aB)x^{1+m}}{4ab\,(a+bx^2)^2} + \frac{(Ab(3-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{4a^3b(1+m)}$$

Mathematica [A]
time = 0.11, size = 80, normalized size = 0.86

$$\frac{x^{1+m}\left(aB\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+(Ab-aB)\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{a^3b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] +
(A*b - a*B)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a^3
*b*(1 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m(B\,x^2+A)}{(b\,x^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] int(x^m*(B*x^2+A)/(b*x^2+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 52.14, size = 3053, normalized size = 32.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] A*(a**2*m**3*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32
*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*m**2*x*x**m*lerchphi(b*x**2*exp_
polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 6
4*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a*
*2*m**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*
gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - a**2*m*x*x**m*lerc
hphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamm
a(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/
2 + 3/2)) + 8*a**2*m*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64
*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*a**
2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*
x**4*gamma(m/2 + 3/2)) + 10*a**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2
 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3
/2)) + 2*a*b*m**3*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3
/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 6*a*b*m**2*x**3*x**m*lerchphi(b
*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2
+ 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/
2)) - 2*a*b*m**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*
a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a*b*
m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/
2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b*
*2*x**4*gamma(m/2 + 3/2)) + 4*a*b*m*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gam
ma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m
/2 + 3/2)) + 6*a*b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/
2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 +
3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 6*a*b*x**3*x**m*gamma(m/2 + 1/
2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b*
*2*x**4*gamma(m/2 + 3/2)) + b**2*m**3*x**5*x**m*lerchphi(b*x**2*exp_polar(I
*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*
```

```
b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*b**2*m**2
*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**
2*x**4*gamma(m/2 + 3/2)) - b**2*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)
/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x*
*2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*b**2*x**5*x**
m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**
5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*ga
mma(m/2 + 3/2))) + B*(a**2*m**3*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2
*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) + 3*a**2*m**2*x**3*
x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*
a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4
*gamma(m/2 + 5/2)) - 2*a**2*m**2*x**3*x**m*gamma(m/2 + 3/2)/(32*a**5*gamma(
m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2
+ 5/2)) - a**2*m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)
*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/
2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) - 3*a**2*x**3*x**m*lerchphi(b*x**2
*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2
) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) +
 18*a**2*x**3*x**m*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x
**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) + 2*a*b*m**3*x**
5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(3
2*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x*
*4*gamma(m/2 + 5/2)) + 6*a*b*m**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)
/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x*
*2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) - 2*a*b*m**2*x**5
*x**m*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma(m/2
 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) - 2*a*b*m*x**5*x**m*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2
 + 5/2) + 64*a**4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5
/2)) - 4*a*b*m*x**5*x**m*gamma(m/2 + 3/2)/(32*a**5*gamma(m/2 + 5/2) + 64*a*
*4*b*x**2*gamma(m/2 + 5/2) + 32*a**3*b**2*x**4*gamma(m/2 + 5/2)) - 6*a*b*x*
*5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(
32*a**5*gamma(m/2 + 5/2) + 64*a**4*b*x**2*gamma...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,(B\,x^2+A)}{{(b\,x^2+a)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^2))/(a + b*x^2)^3,x)
[Out] int((x^m*(A + B*x^2))/(a + b*x^2)^3, x)
```

3.325 $\int x^m(a+bx^2)^2(c+dx^2)^3\,dx$

Optimal. Leaf size=151

$$\frac{a^2c^3x^{1+m}}{1+m}+\frac{ac^2(2bc+3ad)x^{3+m}}{3+m}+\frac{c(b^2c^2+6abcd+3a^2d^2)\,x^{5+m}}{5+m}+\frac{d(3b^2c^2+6abcd+a^2d^2)\,x^{7+m}}{7+m}+\frac{bd^2(3bc+}{9+}$$

```
[Out] a^2*c^3*x^(1+m)/(1+m)+a*c^2*(3*a*d+2*b*c)*x^(3+m)/(3+m)+c*(3*a^2*d^2+6*a*b*
c*d+b^2*c^2)*x^(5+m)/(5+m)+d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(7+m)/(7+m)+b*
d^2*(2*a*d+3*b*c)*x^(9+m)/(9+m)+b^2*d^3*x^(11+m)/(11+m)
```

Rubi [A]
time = 0.06, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\frac{cx^{m+5}(3a^2d^2+6abcd+b^2c^2)}{m+5}+\frac{dx^{m+7}(a^2d^2+6abcd+3b^2c^2)}{m+7}+\frac{a^2c^3x^{m+1}}{m+1}+\frac{ac^2x^{m+3}(3ad+2bc)}{m+3}+\frac{bd^2x^{m+9}(2ad+3bc)}{m+9}+\frac{b^2d^3x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^(1 + m))/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^(3 + m))/(3 + m) + (
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5 + m))/(5 + m) + (d*(3*b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^(7 + m))/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^(9 + m))/(
9 + m) + (b^2*d^3*x^(11 + m))/(11 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^m\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=\int\left(a^2c^3x^m+ac^2(2bc+3ad)x^{2+m}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{4+m}+d(3b\right.$$
$$=\frac{a^2c^3x^{1+m}}{1+m}+\frac{ac^2(2bc+3ad)x^{3+m}}{3+m}+\frac{c\left(b^2c^2+6abcd+3a^2d^2\right)x^{5+m}}{5+m}+\frac{d(3b}{}$$

Mathematica [A]
time = 0.18, size = 141, normalized size = 0.93

$$x^m\left(\frac{a^2c^3x}{1+m}+\frac{ac^2(2bc+3ad)x^3}{3+m}+\frac{c(b^2c^2+6abcd+3a^2d^2)\,x^5}{5+m}+\frac{d(3b^2c^2+6abcd+a^2d^2)\,x^7}{7+m}+\frac{bd^2(3bc+2ad)x^9}{9+m}+\frac{b^2d^3x^{11}}{11+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] x^m*((a^2*c^3*x)/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^3)/(3 + m) + (c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^5)/(5 + m) + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^7)/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^9)/(9 + m) + (b^2*d^3*x^11)/(11
 + m))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(151) = 302$.
time = 0.09, size = 975, normalized size = 6.46

method	result
risch	$x\left(b^{2}d^{3}m^{5}x^{10}+25b^{2}d^{3}m^{4}x^{10}+2ab\,d^{3}m^{5}x^{8}+3b^{2}c\,d^{2}m^{5}x^{8}+230b^{2}d^{3}m^{3}x^{10}+54ab\,d^{3}m^{4}x^{8}+81b^{2}c\,d^{2}m^{4}x^{8}+950b^{2}d^{3}m^{2}x^{10}+a^{2}d^{3}m^{5}x^{6}\right.$
gosper	$x^{1+m}\left(b^{2}d^{3}m^{5}x^{10}+25b^{2}d^{3}m^{4}x^{10}+2ab\,d^{3}m^{5}x^{8}+3b^{2}c\,d^{2}m^{5}x^{8}+230b^{2}d^{3}m^{3}x^{10}+54ab\,d^{3}m^{4}x^{8}+81b^{2}c\,d^{2}m^{4}x^{8}+950b^{2}d^{3}m^{2}x^{10}+a^{2}d^{3}m\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*d^3*m^5*x^10+25*b^2*d^3*m^4*x^10+2*a*b*d^3*m^5*x^8+3*b^2*c*d^2*m^5*x
^8+230*b^2*d^3*m^3*x^10+54*a*b*d^3*m^4*x^8+81*b^2*c*d^2*m^4*x^8+950*b^2*d^3
*m^2*x^10+a^2*d^3*m^5*x^6+6*a*b*c*d^2*m^5*x^6+524*a*b*d^3*m^3*x^8+3*b^2*c^2
*d*m^5*x^6+786*b^2*c*d^2*m^3*x^8+1689*b^2*d^3*m*x^10+29*a^2*d^3*m^4*x^6+174
*a*b*c*d^2*m^4*x^6+2244*a*b*d^3*m^2*x^8+87*b^2*c^2*d*m^4*x^6+3366*b^2*c*d^2
*m^2*x^8+945*b^2*d^3*x^10+3*a^2*c*d^2*m^5*x^4+302*a^2*d^3*m^3*x^6+6*a*b*c^2
*d*m^5*x^4+1812*a*b*c*d^2*m^3*x^6+4082*a*b*d^3*m*x^8+b^2*c^3*m^5*x^4+906*b^
2*c^2*d*m^3*x^6+6123*b^2*c*d^2*m*x^8+93*a^2*c*d^2*m^4*x^4+1366*a^2*d^3*m^2*
x^6+186*a*b*c^2*d*m^4*x^4+8196*a*b*c*d^2*m^2*x^6+2310*a*b*d^3*x^8+31*b^2*c^
3*m^4*x^4+4098*b^2*c^2*d*m^2*x^6+3465*b^2*c*d^2*x^8+3*a^2*c^2*d*m^5*x^2+105
0*a^2*c*d^2*m^3*x^4+2577*a^2*d^3*m*x^6+2*a*b*c^3*m^5*x^2+2100*a*b*c^2*d*m^3
*x^4+15462*a*b*c*d^2*m*x^6+350*b^2*c^3*m^3*x^4+7731*b^2*c^2*d*m*x^6+99*a^2*
c^2*d*m^4*x^2+5190*a^2*c*d^2*m^2*x^4+1485*a^2*d^3*x^6+66*a*b*c^3*m^4*x^2+10
380*a*b*c^2*d*m^2*x^4+8910*a*b*c*d^2*x^6+1730*b^2*c^3*m^2*x^4+4455*b^2*c^2*
d*x^6+a^2*c^3*m^5+1218*a^2*c^2*d*m^3*x^2+10467*a^2*c*d^2*m*x^4+812*a*b*c^3*
m^3*x^2+20934*a*b*c^2*d*m*x^4+3489*b^2*c^3*m*x^4+35*a^2*c^3*m^4+6786*a^2*c^
2*d*m^2*x^2+6237*a^2*c*d^2*x^4+4524*a*b*c^3*m^2*x^2+12474*a*b*c^2*d*x^4+207
9*b^2*c^3*x^4+470*a^2*c^3*m^3+16059*a^2*c^2*d*m*x^2+10706*a*b*c^3*m*x^2+301
0*a^2*c^3*m^2+10395*a^2*c^2*d*x^2+6930*a*b*c^3*x^2+9129*a^2*c^3*m+10395*a^2
*c^3)*x^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [**A**]
time = 0.28, size = 215, normalized size = 1.42

$$\frac{b^{2}d^{3}x^{m+11}}{m+11}+\frac{3\,b^{2}cd^{2}x^{m+9}}{m+9}+\frac{2\,abd^{3}x^{m+9}}{m+9}+\frac{3\,b^{2}c^{2}dx^{m+7}}{m+7}+\frac{6\,abcd^{2}x^{m+7}}{m+7}+\frac{a^{2}d^{3}x^{m+7}}{m+7}+\frac{b^{2}c^{3}x^{m+5}}{m+5}+\frac{6\,abc^{2}dx^{m+5}}{m+5}+\frac{3\,a^{2}cd^{2}x^{m+5}}{m+5}+\frac{2\,abc^{3}x^{m+3}}{m+3}+\frac{3\,a^{2}c^{2}dx^{m+3}}{m+3}+\frac{a^{2}c^{3}x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^2*d^3*x^(m + 11)/(m + 11) + 3*b^2*c*d^2*x^(m + 9)/(m + 9) + 2*a*b*d^3*x^(
m + 9)/(m + 9) + 3*b^2*c^2*d*x^(m + 7)/(m + 7) + 6*a*b*c*d^2*x^(m + 7)/(m +
 7) + a^2*d^3*x^(m + 7)/(m + 7) + b^2*c^3*x^(m + 5)/(m + 5) + 6*a*b*c^2*d*x
^(m + 5)/(m + 5) + 3*a^2*c*d^2*x^(m + 5)/(m + 5) + 2*a*b*c^3*x^(m + 3)/(m +
 3) + 3*a^2*c^2*d*x^(m + 3)/(m + 3) + a^2*c^3*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(151) = 302$.
time = 0.91, size = 773, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] ((b^2*d^3*m^5 + 25*b^2*d^3*m^4 + 230*b^2*d^3*m^3 + 950*b^2*d^3*m^2 + 1689*b
^2*d^3*m + 945*b^2*d^3)*x^11 + ((3*b^2*c*d^2 + 2*a*b*d^3)*m^5 + 3465*b^2*c*
d^2 + 2310*a*b*d^3 + 27*(3*b^2*c*d^2 + 2*a*b*d^3)*m^4 + 262*(3*b^2*c*d^2 +
2*a*b*d^3)*m^3 + 1122*(3*b^2*c*d^2 + 2*a*b*d^3)*m^2 + 2041*(3*b^2*c*d^2 + 2
*a*b*d^3)*m)*x^9 + ((3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^5 + 4455*b^2*c^
2*d + 8910*a*b*c*d^2 + 1485*a^2*d^3 + 29*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d
^3)*m^4 + 302*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^3 + 1366*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*m^2 + 2577*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*
m)*x^7 + ((b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^5 + 2079*b^2*c^3 + 12474*
a*b*c^2*d + 6237*a^2*c*d^2 + 31*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^4 +
 350*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^3 + 1730*(b^2*c^3 + 6*a*b*c^2*
d + 3*a^2*c*d^2)*m^2 + 3489*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m)*x^5 +
((2*a*b*c^3 + 3*a^2*c^2*d)*m^5 + 6930*a*b*c^3 + 10395*a^2*c^2*d + 33*(2*a*b
*c^3 + 3*a^2*c^2*d)*m^4 + 406*(2*a*b*c^3 + 3*a^2*c^2*d)*m^3 + 2262*(2*a*b*c
^3 + 3*a^2*c^2*d)*m^2 + 5353*(2*a*b*c^3 + 3*a^2*c^2*d)*m)*x^3 + (a^2*c^3*m^
5 + 35*a^2*c^3*m^4 + 470*a^2*c^3*m^3 + 3010*a^2*c^3*m^2 + 9129*a^2*c^3*m +
10395*a^2*c^3)*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 1952
4*m + 10395)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4345 vs. $2(144) = 288$.
time = 0.91, size = 4345, normalized size = 28.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((-a**2*c**3/(10*x**10) - 3*a**2*c**2*d/(8*x**8) - a**2*c*d**2/(2*
x**6) - a**2*d**3/(4*x**4) - a*b*c**3/(4*x**8) - a*b*c**2*d/x**6 - 3*a*b*c*
d**2/(2*x**4) - a*b*d**3/x**2 - b**2*c**3/(6*x**6) - 3*b**2*c**2*d/(4*x**4)
 - 3*b**2*c*d**2/(2*x**2) + b**2*d**3*log(x), Eq(m, -11)), (-a**2*c**3/(8*x
**8) - a**2*c**2*d/(2*x**6) - 3*a**2*c*d**2/(4*x**4) - a**2*d**3/(2*x**2) -
 a*b*c**3/(3*x**6) - 3*a*b*c**2*d/(2*x**4) - 3*a*b*c*d**2/x**2 + 2*a*b*d**3
*log(x) - b**2*c**3/(4*x**4) - 3*b**2*c**2*d/(2*x**2) + 3*b**2*c*d**2*log(x
) + b**2*d**3*x**2/2, Eq(m, -9)), (-a**2*c**3/(6*x**6) - 3*a**2*c**2*d/(4*x
**4) - 3*a**2*c*d**2/(2*x**2) + a**2*d**3*log(x) - a*b*c**3/(2*x**4) - 3*a*
b*c**2*d/x**2 + 6*a*b*c*d**2*log(x) + a*b*d**3*x**2 - b**2*c**3/(2*x**2) +
3*b**2*c**2*d*log(x) + 3*b**2*c*d**2*x**2/2 + b**2*d**3*x**4/4, Eq(m, -7)),
 (-a**2*c**3/(4*x**4) - 3*a**2*c**2*d/(2*x**2) + 3*a**2*c*d**2*log(x) + a**
2*d**3*x**2/2 - a*b*c**3/x**2 + 6*a*b*c**2*d*log(x) + 3*a*b*c*d**2*x**2 + a
*b*d**3*x**4/2 + b**2*c**3*log(x) + 3*b**2*c**2*d*x**2/2 + 3*b**2*c*d**2*x*
*4/4 + b**2*d**3*x**6/6, Eq(m, -5)), (-a**2*c**3/(2*x**2) + 3*a**2*c**2*d*l
og(x) + 3*a**2*c*d**2*x**2/2 + a**2*d**3*x**4/4 + 2*a*b*c**3*log(x) + 3*a*b
*c**2*d*x**2 + 3*a*b*c*d**2*x**4/2 + a*b*d**3*x**6/3 + b**2*c**3*x**2/2 + 3
*b**2*c**2*d*x**4/4 + b**2*c*d**2*x**6/2 + b**2*d**3*x**8/8, Eq(m, -3)), (a
**2*c**3*log(x) + 3*a**2*c**2*d*x**2/2 + 3*a**2*c*d**2*x**4/4 + a**2*d**3*x
**6/6 + a*b*c**3*x**2 + 3*a*b*c**2*d*x**4/2 + a*b*c*d**2*x**6 + a*b*d**3*x*
*8/4 + b**2*c**3*x**4/4 + b**2*c**2*d*x**6/2 + 3*b**2*c*d**2*x**8/8 + b**2*
d**3*x**10/10, Eq(m, -1)), (a**2*c**3*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35*a**2*c**3*m**4*x*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*
a**2*c**3*m**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 3010*a**2*c**3*m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129*a**2*c**3*m*x*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a*
*2*c**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 3*a**2*c**2*d*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 99*a**2*c**2*d*m**4*x**3*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1218*a*
*2*c**2*d*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**
2 + 19524*m + 10395) + 6786*a**2*c**2*d*m**2*x**3*x**m/(m**6 + 36*m**5 + 50
5*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 16059*a**2*c**2*d*m*x*
*3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 10395*a**2*c**2*d*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 3*a**2*c*d**2*m**5*x**5*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 93*a**2*c*d**2*m
**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 1050*a**2*c*d**2*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 5190*a**2*c*d**2*m**2*x**5*x**m/(
m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 104
67*a**2*c*d**2*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 6237*a**2*c*d**2*x**5*x**m/(m**6 + 36*m**5 + 505*m
```

```
**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + a**2*d**3*m**5*x**7*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 29
*a**2*d**3*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 302*a**2*d**3*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1366*a**2*d**3*m**2*x**7
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 2577*a**2*d**3*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 1485*a**2*d**3*x**7*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*b*c**3*m**5*x**3*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 66*a*b*c**3*m**4*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 812*a*b*c**3*m**3*x**3*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4524*a*b*c**3*m**2*x**3
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 10706*a*b*c**3*m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 6930*a*b*c**3*x**3*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6*a*b*c**2*d*m**5*x**5*x
**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
+ 186*a*b*c**2*d*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 2100*a*b*c**2*d*m**3*x**5*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10380*a*b*c**2*d
*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 34...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(151) = 302$.

time = 0.61, size = 1192, normalized size = 7.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] (b^2*d^3*m^5*x^11*x^m + 25*b^2*d^3*m^4*x^11*x^m + 3*b^2*c*d^2*m^5*x^9*x^m +
 2*a*b*d^3*m^5*x^9*x^m + 230*b^2*d^3*m^3*x^11*x^m + 81*b^2*c*d^2*m^4*x^9*x^
m + 54*a*b*d^3*m^4*x^9*x^m + 950*b^2*d^3*m^2*x^11*x^m + 3*b^2*c^2*d*m^5*x^7
*x^m + 6*a*b*c*d^2*m^5*x^7*x^m + a^2*d^3*m^5*x^7*x^m + 786*b^2*c*d^2*m^3*x^
9*x^m + 524*a*b*d^3*m^3*x^9*x^m + 1689*b^2*d^3*m*x^11*x^m + 87*b^2*c^2*d*m^
4*x^7*x^m + 174*a*b*c*d^2*m^4*x^7*x^m + 29*a^2*d^3*m^4*x^7*x^m + 3366*b^2*c
*d^2*m^2*x^9*x^m + 2244*a*b*d^3*m^2*x^9*x^m + 945*b^2*d^3*x^11*x^m + b^2*c^
3*m^5*x^5*x^m + 6*a*b*c^2*d*m^5*x^5*x^m + 3*a^2*c*d^2*m^5*x^5*x^m + 906*b^2
*c^2*d*m^3*x^7*x^m + 1812*a*b*c*d^2*m^3*x^7*x^m + 302*a^2*d^3*m^3*x^7*x^m +
 6123*b^2*c*d^2*m*x^9*x^m + 4082*a*b*d^3*m*x^9*x^m + 31*b^2*c^3*m^4*x^5*x^m
 + 186*a*b*c^2*d*m^4*x^5*x^m + 93*a^2*c*d^2*m^4*x^5*x^m + 4098*b^2*c^2*d*m^
2*x^7*x^m + 8196*a*b*c*d^2*m^2*x^7*x^m + 1366*a^2*d^3*m^2*x^7*x^m + 3465*b^
2*c*d^2*x^9*x^m + 2310*a*b*d^3*x^9*x^m + 2*a*b*c^3*m^5*x^3*x^m + 3*a^2*c^2*
```

```
d*m^5*x^3*x^m + 350*b^2*c^3*m^3*x^5*x^m + 2100*a*b*c^2*d*m^3*x^5*x^m + 1050
*a^2*c*d^2*m^3*x^5*x^m + 7731*b^2*c^2*d*m*x^7*x^m + 15462*a*b*c*d^2*m*x^7*x
^m + 2577*a^2*d^3*m*x^7*x^m + 66*a*b*c^3*m^4*x^3*x^m + 99*a^2*c^2*d*m^4*x^3
*x^m + 1730*b^2*c^3*m^2*x^5*x^m + 10380*a*b*c^2*d*m^2*x^5*x^m + 5190*a^2*c*
d^2*m^2*x^5*x^m + 4455*b^2*c^2*d*x^7*x^m + 8910*a*b*c*d^2*x^7*x^m + 1485*a^
2*d^3*x^7*x^m + a^2*c^3*m^5*x*x^m + 812*a*b*c^3*m^3*x^3*x^m + 1218*a^2*c^2*
d*m^3*x^3*x^m + 3489*b^2*c^3*m*x^5*x^m + 20934*a*b*c^2*d*m*x^5*x^m + 10467*
a^2*c*d^2*m*x^5*x^m + 35*a^2*c^3*m^4*x*x^m + 4524*a*b*c^3*m^2*x^3*x^m + 678
6*a^2*c^2*d*m^2*x^3*x^m + 2079*b^2*c^3*x^5*x^m + 12474*a*b*c^2*d*x^5*x^m +
6237*a^2*c*d^2*x^5*x^m + 470*a^2*c^3*m^3*x*x^m + 10706*a*b*c^3*m*x^3*x^m +
16059*a^2*c^2*d*m*x^3*x^m + 3010*a^2*c^3*m^2*x*x^m + 6930*a*b*c^3*x^3*x^m +
 10395*a^2*c^2*d*x^3*x^m + 9129*a^2*c^3*m*x*x^m + 10395*a^2*c^3*x*x^m)/(m^6
 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Mupad [B]
time = 0.35, size = 443, normalized size = 2.93

$$\frac{a^2\,c^3\,x\,x^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{c\,x^m\,x^5\,\left(3\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+2079\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{d\,x^m\,x^7\,\left(a^2\,d^2+6\,a\,b\,c\,d+3\,b^2\,c^2\right)\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+1485\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{b^2\,d^3\,x^m\,x^{11}\,\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{a\,c^2\,x^m\,x^3\,\left(3\,a\,d+2\,b\,c\right)\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{b\,d^2\,x^m\,x^9\,\left(2\,a\,d+3\,b\,c\right)\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] (a^2*c^3*x*x^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395))/(19524
*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (c*x^m*x^5*(3
*a^2*d^2 + b^2*c^2 + 6*a*b*c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4 + m^5
 + 2079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
 + (d*x^m*x^7*(a^2*d^2 + 3*b^2*c^2 + 6*a*b*c*d)*(2577*m + 1366*m^2 + 302*m^
3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^
5 + m^6 + 10395) + (b^2*d^3*x^m*x^11*(1689*m + 950*m^2 + 230*m^3 + 25*m^4 +
 m^5 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 103
95) + (a*c^2*x^m*x^3*(3*a*d + 2*b*c)*(5353*m + 2262*m^2 + 406*m^3 + 33*m^4
+ m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 1
0395) + (b*d^2*x^m*x^9*(2*a*d + 3*b*c)*(2041*m + 1122*m^2 + 262*m^3 + 27*m^
4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 +
 10395)
```

3.326 $\int x^m(a+bx^2)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=109

$$\frac{a^2c^2x^{1+m}}{1+m}+\frac{2ac(bc+ad)x^{3+m}}{3+m}+\frac{\left(b^2c^2+4abcd+a^2d^2\right)x^{5+m}}{5+m}+\frac{2bd(bc+ad)x^{7+m}}{7+m}+\frac{b^2d^2x^{9+m}}{9+m}$$

```
[Out] a^2*c^2*x^(1+m)/(1+m)+2*a*c*(a*d+b*c)*x^(3+m)/(3+m)+(a^2*d^2+4*a*b*c*d+b^2*
c^2)*x^(5+m)/(5+m)+2*b*d*(a*d+b*c)*x^(7+m)/(7+m)+b^2*d^2*x^(9+m)/(9+m)
```

Rubi [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\frac{x^{m+5}(a^2d^2+4abcd+b^2c^2)}{m+5}+\frac{a^2c^2x^{m+1}}{m+1}+\frac{2acx^{m+3}(ad+bc)}{m+3}+\frac{2bdx^{m+7}(ad+bc)}{m+7}+\frac{b^2d^2x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^(1 + m))/(1 + m) + (2*a*c*(b*c + a*d)*x^(3 + m))/(3 + m) + ((b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(5 + m))/(5 + m) + (2*b*d*(b*c + a*d)*x^(7 +
m))/(7 + m) + (b^2*d^2*x^(9 + m))/(9 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx&=\int\left(a^2c^2x^m+2ac(bc+ad)x^{2+m}+\left(b^2c^2+4abcd+a^2d^2\right)x^{4+m}+2bd(bc+\right.\\&=\frac{a^2c^2x^{1+m}}{1+m}+\frac{2ac(bc+ad)x^{3+m}}{3+m}+\frac{\left(b^2c^2+4abcd+a^2d^2\right)x^{5+m}}{5+m}+\frac{2bd(bc+}{7+}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 101, normalized size = 0.93

$$x^m\left(\frac{a^2c^2x}{1+m}+\frac{2ac(bc+ad)x^3}{3+m}+\frac{\left(b^2c^2+4abcd+a^2d^2\right)x^5}{5+m}+\frac{2bd(bc+ad)x^7}{7+m}+\frac{b^2d^2x^9}{9+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] x^m*((a^2*c^2*x)/(1 + m) + (2*a*c*(b*c + a*d)*x^3)/(3 + m) + ((b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^5)/(5 + m) + (2*b*d*(b*c + a*d)*x^7)/(7 + m) + (b^2*d^
2*x^9)/(9 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(109) = 218$.
time = 0.08, size = 568, normalized size = 5.21

method	result
risch	$\frac{x\left(b^2d^2m^4x^8+16b^2d^2m^3x^8+2ab\,d^2m^4x^6+2b^2cd\,m^4x^6+86b^2d^2m^2x^8+36ab\,d^2m^3x^6+36b^2cd\,m^3x^6+176m\,x^8b^2d^2+a^2d^2m^4x^4+4abcd\right.}{}$
gosper	$\frac{x^{1+m}\left(b^2d^2m^4x^8+16b^2d^2m^3x^8+2ab\,d^2m^4x^6+2b^2cd\,m^4x^6+86b^2d^2m^2x^8+36ab\,d^2m^3x^6+36b^2cd\,m^3x^6+176m\,x^8b^2d^2+a^2d^2m^4x^4+4a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*d^2*m^4*x^8+16*b^2*d^2*m^3*x^8+2*a*b*d^2*m^4*x^6+2*b^2*c*d*m^4*x^6+8
6*b^2*d^2*m^2*x^8+36*a*b*d^2*m^3*x^6+36*b^2*c*d*m^3*x^6+176*b^2*d^2*m*x^8+a
^2*d^2*m^4*x^4+4*a*b*c*d*m^4*x^4+208*a*b*d^2*m^2*x^6+b^2*c^2*m^4*x^4+208*b^
2*c*d*m^2*x^6+105*b^2*d^2*x^8+20*a^2*d^2*m^3*x^4+80*a*b*c*d*m^3*x^4+444*a*b
*d^2*m*x^6+20*b^2*c^2*m^3*x^4+444*b^2*c*d*m*x^6+2*a^2*c*d*m^4*x^2+130*a^2*d
^2*m^2*x^4+2*a*b*c^2*m^4*x^2+520*a*b*c*d*m^2*x^4+270*a*b*d^2*x^6+130*b^2*c^
2*m^2*x^4+270*b^2*c*d*x^6+44*a^2*c*d*m^3*x^2+300*a^2*d^2*m*x^4+44*a*b*c^2*m
^3*x^2+1200*a*b*c*d*m*x^4+300*b^2*c^2*m*x^4+a^2*c^2*m^4+328*a^2*c*d*m^2*x^2
+189*a^2*d^2*x^4+328*a*b*c^2*m^2*x^2+756*a*b*c*d*x^4+189*b^2*c^2*x^4+24*a^2
*c^2*m^3+916*a^2*c*d*m*x^2+916*a*b*c^2*m*x^2+206*a^2*c^2*m^2+630*a^2*c*d*x^
2+630*a*b*c^2*x^2+744*a^2*c^2*m+945*a^2*c^2)*x^m/(9+m)/(7+m)/(5+m)/(3+m)/(1
+m)
```

Maxima [A]
time = 0.27, size = 153, normalized size = 1.40

$$\frac{b^2d^2x^{m+9}}{m+9}+\frac{2\,b^2cdx^{m+7}}{m+7}+\frac{2\,abd^2x^{m+7}}{m+7}+\frac{b^2c^2x^{m+5}}{m+5}+\frac{4\,abcdx^{m+5}}{m+5}+\frac{a^2d^2x^{m+5}}{m+5}+\frac{2\,abc^2x^{m+3}}{m+3}+\frac{2\,a^2cdx^{m+3}}{m+3}+\frac{a^2c^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*d^2*x^(m + 9)/(m + 9) + 2*b^2*c*d*x^(m + 7)/(m + 7) + 2*a*b*d^2*x^(m +
7)/(m + 7) + b^2*c^2*x^(m + 5)/(m + 5) + 4*a*b*c*d*x^(m + 5)/(m + 5) + a^2*
d^2*x^(m + 5)/(m + 5) + 2*a*b*c^2*x^(m + 3)/(m + 3) + 2*a^2*c*d*x^(m + 3)/(
m + 3) + a^2*c^2*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(109) = 218$.
time = 1.16, size = 442, normalized size = 4.06

$$\frac{\left((b^2d^2m^4+16\,b^2d^2m^3+86\,b^2d^2m^2+176\,b^2d^2m+105\,b^2d^2)x^9+2\,((b^2cd+abd^2)m^4+135\,b^2cd+135\,abd^2+18\,(b^2cd+abd^2)m^3+104\,(b^2cd+abd^2)m^2+222\,(b^2cd+abd^2)m)x^7+((b^2c^2+4\,abcd+a^2d^2)m^4+189\,b^2c^2+756\,abcd+189\,a^2d^2+20\,(b^2c^2+4\,abcd+a^2d^2)m^3+130\,(b^2c^2+4\,abcd+a^2d^2)m^2+300\,(b^2c^2+4\,abcd+a^2d^2)m)x^5+2\,((abc^2+a^2cd)m^4+315\,abc^2+315\,a^2cd+22\,(abc^2+a^2cd)m^3+164\,(abc^2+a^2cd)m^2+458\,(abc^2+a^2cd)m)x^3+(a^2c^2m^4+24\,a^2c^2m^3+206\,a^2c^2m^2+744\,a^2c^2m+945\,a^2c^2)x\right)x^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*d^2*m^4 + 16*b^2*d^2*m^3 + 86*b^2*d^2*m^2 + 176*b^2*d^2*m + 105*b^2*d
^2)*x^9 + 2*((b^2*c*d + a*b*d^2)*m^4 + 135*b^2*c*d + 135*a*b*d^2 + 18*(b^2*
c*d + a*b*d^2)*m^3 + 104*(b^2*c*d + a*b*d^2)*m^2 + 222*(b^2*c*d + a*b*d^2)*
m)*x^7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m^4 + 189*b^2*c^2 + 756*a*b*c*d +
 189*a^2*d^2 + 20*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m^3 + 130*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*m^2 + 300*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m)*x^5 + 2*((a*b
*c^2 + a^2*c*d)*m^4 + 315*a*b*c^2 + 315*a^2*c*d + 22*(a*b*c^2 + a^2*c*d)*m^
3 + 164*(a*b*c^2 + a^2*c*d)*m^2 + 458*(a*b*c^2 + a^2*c*d)*m)*x^3 + (a^2*c^2
*m^4 + 24*a^2*c^2*m^3 + 206*a^2*c^2*m^2 + 744*a^2*c^2*m + 945*a^2*c^2)*x)*x
^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2363 vs. $2(100) = 200$.
time = 0.61, size = 2363, normalized size = 21.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((-a**2*c**2/(8*x**8) - a**2*c*d/(3*x**6) - a**2*d**2/(4*x**4) - a
*b*c**2/(3*x**6) - a*b*c*d/x**4 - a*b*d**2/x**2 - b**2*c**2/(4*x**4) - b**2
*c*d/x**2 + b**2*d**2*log(x), Eq(m, -9)), (-a**2*c**2/(6*x**6) - a**2*c*d/(
2*x**4) - a**2*d**2/(2*x**2) - a*b*c**2/(2*x**4) - 2*a*b*c*d/x**2 + 2*a*b*d
**2*log(x) - b**2*c**2/(2*x**2) + 2*b**2*c*d*log(x) + b**2*d**2*x**2/2, Eq(
m, -7)), (-a**2*c**2/(4*x**4) - a**2*c*d/x**2 + a**2*d**2*log(x) - a*b*c**2
/x**2 + 4*a*b*c*d*log(x) + a*b*d**2*x**2 + b**2*c**2*log(x) + b**2*c*d*x**2
 + b**2*d**2*x**4/4, Eq(m, -5)), (-a**2*c**2/(2*x**2) + 2*a**2*c*d*log(x) +
 a**2*d**2*x**2/2 + 2*a*b*c**2*log(x) + 2*a*b*c*d*x**2 + a*b*d**2*x**4/2 +
b**2*c**2*x**2/2 + b**2*c*d*x**4/2 + b**2*d**2*x**6/6, Eq(m, -3)), (a**2*c*
*2*log(x) + a**2*c*d*x**2 + a**2*d**2*x**4/4 + a*b*c**2*x**2 + a*b*c*d*x**4
 + a*b*d**2*x**6/3 + b**2*c**2*x**4/4 + b**2*c*d*x**6/3 + b**2*d**2*x**8/8,
 Eq(m, -1)), (a**2*c**2*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 24*a**2*c**2*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*
m**2 + 1689*m + 945) + 206*a**2*c**2*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 744*a**2*c**2*m*x*x**m/(m**5 + 25*m**4 + 230*
```

```
m**3 + 950*m**2 + 1689*m + 945) + 945*a**2*c**2*x*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 2*a**2*c*d*m**4*x**3*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 44*a**2*c*d*m**3*x**3*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 328*a**2*c*d*m**2*x**3*x
**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a**2*c*d*m*
x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a**2*
c*d*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + a**2*
d**2*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
 20*a**2*d**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 130*a**2*d**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 300*a**2*d**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 +
950*m**2 + 1689*m + 945) + 189*a**2*d**2*x**5*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 2*a*b*c**2*m**4*x**3*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 44*a*b*c**2*m**3*x**3*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 328*a*b*c**2*m**2*x**3*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a*b*c**2*m*x**3
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a*b*c**2*
x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a*b*c*d
*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 80*
a*b*c*d*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 520*a*b*c*d*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 1200*a*b*c*d*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
+ 1689*m + 945) + 756*a*b*c*d*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 2*a*b*d**2*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 36*a*b*d**2*m**3*x**7*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 208*a*b*d**2*m**2*x**7*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 444*a*b*d**2*m*x**7*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 270*a*b*d**2*x**7*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**2*c**2*m**4*x**5*
x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 20*b**2*c**2*m
**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 130*b
**2*c**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 300*b**2*c**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 189*b**2*c**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 2*b**2*c*d*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950
*m**2 + 1689*m + 945) + 36*b**2*c*d*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 208*b**2*c*d*m**2*x**7*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 444*b**2*c*d*m*x**7*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 270*b**2*c*d*x**7*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**2*d**2*m**4*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**2*d**2*m**3*
x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**2*d
**2*m**2*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
176*b**2*d**2*m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
945) + 105*b**2*d**2*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
```

```
*m + 945), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(109) = 218.
time = 0.63, size = 703, normalized size = 6.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*m^4*x^9*x^m + 16*b^2*d^2*m^3*x^9*x^m + 2*b^2*c*d*m^4*x^7*x^m + 2*a
*b*d^2*m^4*x^7*x^m + 86*b^2*d^2*m^2*x^9*x^m + 36*b^2*c*d*m^3*x^7*x^m + 36*a
*b*d^2*m^3*x^7*x^m + 176*b^2*d^2*m*x^9*x^m + b^2*c^2*m^4*x^5*x^m + 4*a*b*c*
d*m^4*x^5*x^m + a^2*d^2*m^4*x^5*x^m + 208*b^2*c*d*m^2*x^7*x^m + 208*a*b*d^2
*m^2*x^7*x^m + 105*b^2*d^2*x^9*x^m + 20*b^2*c^2*m^3*x^5*x^m + 80*a*b*c*d*m^
3*x^5*x^m + 20*a^2*d^2*m^3*x^5*x^m + 444*b^2*c*d*m*x^7*x^m + 444*a*b*d^2*m*
x^7*x^m + 2*a*b*c^2*m^4*x^3*x^m + 2*a^2*c*d*m^4*x^3*x^m + 130*b^2*c^2*m^2*x
^5*x^m + 520*a*b*c*d*m^2*x^5*x^m + 130*a^2*d^2*m^2*x^5*x^m + 270*b^2*c*d*x^
7*x^m + 270*a*b*d^2*x^7*x^m + 44*a*b*c^2*m^3*x^3*x^m + 44*a^2*c*d*m^3*x^3*x
^m + 300*b^2*c^2*m*x^5*x^m + 1200*a*b*c*d*m*x^5*x^m + 300*a^2*d^2*m*x^5*x^m
 + a^2*c^2*m^4*x*x^m + 328*a*b*c^2*m^2*x^3*x^m + 328*a^2*c*d*m^2*x^3*x^m +
189*b^2*c^2*x^5*x^m + 756*a*b*c*d*x^5*x^m + 189*a^2*d^2*x^5*x^m + 24*a^2*c^
2*m^3*x*x^m + 916*a*b*c^2*m*x^3*x^m + 916*a^2*c*d*m*x^3*x^m + 206*a^2*c^2*m
^2*x*x^m + 630*a*b*c^2*x^3*x^m + 630*a^2*c*d*x^3*x^m + 744*a^2*c^2*m*x*x^m
+ 945*a^2*c^2*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Mupad [B]
time = 0.25, size = 302, normalized size = 2.77

$$\frac{x^m\,x^5\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{b^2\,d^2\,x^m\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{a^2\,c^2\,x\,x^m\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{2\,a\,c\,x^m\,x^3\,\left(a\,d+b\,c\right)\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{2\,b\,d\,x^m\,x^7\,\left(a\,d+b\,c\right)\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] (x^m*x^5*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d)*(300*m + 130*m^2 + 20*m^3 + m^4 +
189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (b^2*d^2*x^m*x^9*
(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4
 + m^5 + 945) + (a^2*c^2*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(168
9*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (2*a*c*x^m*x^3*(a*d + b*c)*
(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 230*m^3 + 25*m^
4 + m^5 + 945) + (2*b*d*x^m*x^7*(a*d + b*c)*(222*m + 104*m^2 + 18*m^3 + m^4
 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945)
```

3.327 $\int x^m(a+bx^2)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=71

$$\frac{a^2cx^{1+m}}{1+m}+\frac{a(2bc+ad)x^{3+m}}{3+m}+\frac{b(bc+2ad)x^{5+m}}{5+m}+\frac{b^2dx^{7+m}}{7+m}$$

```
[Out] a^2*c*x^(1+m)/(1+m)+a*(a*d+2*b*c)*x^(3+m)/(3+m)+b*(2*a*d+b*c)*x^(5+m)/(5+m)
+b^2*d*x^(7+m)/(7+m)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{a^2cx^{m+1}}{m+1}+\frac{ax^{m+3}(ad+2bc)}{m+3}+\frac{bx^{m+5}(2ad+bc)}{m+5}+\frac{b^2dx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^(1 + m))/(1 + m) + (a*(2*b*c + a*d)*x^(3 + m))/(3 + m) + (b*(b*c +
 2*a*d)*x^(5 + m))/(5 + m) + (b^2*d*x^(7 + m))/(7 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx &= \int \left(a^2cx^m+a(2bc+ad)x^{2+m}+b(bc+2ad)x^{4+m}+b^2dx^{6+m}\right)\,dx\\ &= \frac{a^2cx^{1+m}}{1+m}+\frac{a(2bc+ad)x^{3+m}}{3+m}+\frac{b(bc+2ad)x^{5+m}}{5+m}+\frac{b^2dx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.93

$$x^{1+m}\left(\frac{a^2c}{1+m}+\frac{a(2bc+ad)x^2}{3+m}+\frac{b(bc+2ad)x^4}{5+m}+\frac{b^2dx^6}{7+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] x^(1 + m)*((a^2*c)/(1 + m) + (a*(2*b*c + a*d)*x^2)/(3 + m) + (b*(b*c + 2*a*
d)*x^4)/(5 + m) + (b^2*d*x^6)/(7 + m))
```

Maple [A]
time = 0.07, size = 82, normalized size = 1.15

method	result
norman	$\frac{a(ad+2bc)x^3 e^{m\ln(x)}}{3+m}+\frac{a^2 cx\, e^{m\ln(x)}}{1+m}+\frac{b(2ad+bc)x^5 e^{m\ln(x)}}{5+m}+\frac{b^2 d\, x^7 e^{m\ln(x)}}{7+m}$
risch	$\frac{x\left(b^2 d\, m^3 x^6+9b^2 d\, m^2 x^6+2abd\, m^3 x^4+b^2 c\, m^3 x^4+23m\, x^6 b^2 d+22abd\, m^2 x^4+11b^2 c\, m^2 x^4+15b^2 d\, x^6+a^2 d\, m^3 x^2+2abc\, m^3 x^2+62abd\, x\right.}{(7+}$
gosper	$\frac{x^{1+m}\left(b^2 d\, m^3 x^6+9b^2 d\, m^2 x^6+2abd\, m^3 x^4+b^2 c\, m^3 x^4+23m\, x^6 b^2 d+22abd\, m^2 x^4+11b^2 c\, m^2 x^4+15b^2 d\, x^6+a^2 d\, m^3 x^2+2abc\, m^3 x^2+62a\right.}{(7-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*(a*d+2*b*c)/(3+m)*x^3*exp(m*ln(x))+a^2*c/(1+m)*x*exp(m*ln(x))+b*(2*a*d+b*
c)/(5+m)*x^5*exp(m*ln(x))+b^2*d/(7+m)*x^7*exp(m*ln(x))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 1.28

$$\frac{b^2 dx^{m+7}}{m+7}+\frac{b^2 cx^{m+5}}{m+5}+\frac{2\,abdx^{m+5}}{m+5}+\frac{2\,abcx^{m+3}}{m+3}+\frac{a^2 dx^{m+3}}{m+3}+\frac{a^2 cx^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b^2*d*x^(m + 7)/(m + 7) + b^2*c*x^(m + 5)/(m + 5) + 2*a*b*d*x^(m + 5)/(m +
5) + 2*a*b*c*x^(m + 3)/(m + 3) + a^2*d*x^(m + 3)/(m + 3) + a^2*c*x^(m + 1)/
(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(71) = 142.
time = 1.29, size = 215, normalized size = 3.03

$$\frac{\left(\left(b^2 dm^3+9\,b^2 dm^2+23\,b^2 dm+15\,b^2 d\right)x^7+\left(\left(b^2 c+2\,abd\right)m^3+21\,b^2 c+42\,abd+11\,\left(b^2 c+2\,abd\right)m^2+31\,\left(b^2 c+2\,abd\right)m\right)x^5+\left(\left(2\,abc+a^2 d\right)m^3+70\,abc+35\,a^2 d+13\,\left(2\,abc+a^2 d\right)m^2+47\,\left(2\,abc+a^2 d\right)m\right)x^3+\left(a^2 cm^3+15\,a^2 cm^2+71\,a^2 cm+105\,a^2 c\right)x\right)x^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] ((b^2*d*m^3 + 9*b^2*d*m^2 + 23*b^2*d*m + 15*b^2*d)*x^7 + ((b^2*c + 2*a*b*d)
*m^3 + 21*b^2*c + 42*a*b*d + 11*(b^2*c + 2*a*b*d)*m^2 + 31*(b^2*c + 2*a*b*d
```

```
)*m)*x^5 + ((2*a*b*c + a^2*d)*m^3 + 70*a*b*c + 35*a^2*d + 13*(2*a*b*c + a^2
*d)*m^2 + 47*(2*a*b*c + a^2*d)*m)*x^3 + (a^2*c*m^3 + 15*a^2*c*m^2 + 71*a^2*
c*m + 105*a^2*c)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(63) = 126$.
time = 0.41, size = 1044, normalized size = 14.70

$$\begin{cases} -\frac{a^2c}{6x^6} - \frac{a^2d}{4x^4} - \frac{abc}{2x^4} - \frac{abd}{x^2} - \frac{b^2c}{2x^2} + b^2d\log(x) & \text{for } m = -7 \\ -\frac{a^2c}{4x^4} - \frac{a^2d}{2x^2} - \frac{abc}{x^2} + 2abd\log(x) + b^2c\log(x) + \frac{b^2dx^2}{2} & \text{for } m = -5 \\ -\frac{a^2c}{2x^2} + a^2d\log(x) + 2abc\log(x) + abdx^2 + \frac{b^2cx^2}{2} + \frac{b^2dx^4}{4} & \text{for } m = -3 \\ a^2c\log(x) + \frac{a^2dx^2}{2} + abcx^2 + \frac{abdx^4}{2} + \frac{b^2cx^4}{4} + \frac{b^2dx^6}{6} & \text{for } m = -1 \\ \frac{a^2cm^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{15a^2cm^2xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{71a^2cmxx^m}{m^4+16m^3+86m^2+176m+105} + \frac{105a^2cxx^m}{m^4+16m^3+86m^2+176m+105} + \frac{a^2dm^3x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{13a^2dm^2x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{47a^2dmx^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{35a^2dx^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{2abcm^3x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{26abcm^2x^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{94abcmx^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{70abcx^3x^m}{m^4+16m^3+86m^2+176m+105} + \frac{2abdm^3x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{22abdm^2x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{62abdmx^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{42abdx^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{b^2cm^3x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{11b^2cm^2x^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{31b^2cmx^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{21b^2cx^5x^m}{m^4+16m^3+86m^2+176m+105} + \frac{b^2dm^3x^7x^m}{m^4+16m^3+86m^2+176m+105} + \frac{9b^2dm^2x^7x^m}{m^4+16m^3+86m^2+176m+105} + \frac{23b^2dmx^7x^m}{m^4+16m^3+86m^2+176m+105} + \frac{15b^2dx^7x^m}{m^4+16m^3+86m^2+176m+105} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] Piecewise((-a**2*c/(6*x**6) - a**2*d/(4*x**4) - a*b*c/(2*x**4) - a*b*d/x**2
 - b**2*c/(2*x**2) + b**2*d*log(x), Eq(m, -7)), (-a**2*c/(4*x**4) - a**2*d/
(2*x**2) - a*b*c/x**2 + 2*a*b*d*log(x) + b**2*c*log(x) + b**2*d*x**2/2, Eq(
m, -5)), (-a**2*c/(2*x**2) + a**2*d*log(x) + 2*a*b*c*log(x) + a*b*d*x**2 +
b**2*c*x**2/2 + b**2*d*x**4/4, Eq(m, -3)), (a**2*c*log(x) + a**2*d*x**2/2 +
 a*b*c*x**2 + a*b*d*x**4/2 + b**2*c*x**4/4 + b**2*d*x**6/6, Eq(m, -1)), (a*
*2*c*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*a**2*c*m**2*
x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*a**2*c*m*x*x**m/(m**4
+ 16*m**3 + 86*m**2 + 176*m + 105) + 105*a**2*c*x*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + a**2*d*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + 13*a**2*d*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 1
05) + 47*a**2*d*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a
**2*d*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*a*b*c*m**3*x**
3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 26*a*b*c*m**2*x**3*x**m/(
m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 94*a*b*c*m*x**3*x**m/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105) + 70*a*b*c*x**3*x**m/(m**4 + 16*m**3 + 86*m**2
 + 176*m + 105) + 2*a*b*d*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
+ 105) + 22*a*b*d*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 62*a*b*d*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 42*a*b*d*x
**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**2*c*m**3*x**5*x**m/(
m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*b**2*c*m**2*x**5*x**m/(m**4 +
16*m**3 + 86*m**2 + 176*m + 105) + 31*b**2*c*m*x**5*x**m/(m**4 + 16*m**3 +
86*m**2 + 176*m + 105) + 21*b**2*c*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + b**2*d*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + 9*b**2*d*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*b*
*2*d*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*b**2*d*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(71) = 142$.
time = 0.53, size = 332, normalized size = 4.68

$$\frac{b^2dm^3x^7x^m + 9\,b^2dm^2x^7x^m + b^2cm^3x^5x^m + 2\,abdm^3x^5x^m + 23\,b^2dmx^7x^m + 11\,b^2cm^2x^5x^m + 22\,abdm^2x^5x^m + 15\,b^2dx^7x^m + 2\,abcm^3x^3x^m + a^2dm^3x^3x^m + 31\,b^2cmx^5x^m + 62\,abdmx^5x^m + 26\,abcm^2x^3x^m + 13\,a^2dm^2x^3x^m + 21\,b^2cx^5x^m + 42\,abdx^5x^m + a^2cm^3xx^m + 94\,abcmx^3x^m + 47\,a^2dmx^3x^m + 15\,a^2cm^2xx^m + 70\,abcx^3x^m + 35\,a^2dx^3x^m + 71\,a^2cmxx^m + 105\,a^2cxx^m}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*d*m^3*x^7*x^m + 9*b^2*d*m^2*x^7*x^m + b^2*c*m^3*x^5*x^m + 2*a*b*d*m^3*
x^5*x^m + 23*b^2*d*m*x^7*x^m + 11*b^2*c*m^2*x^5*x^m + 22*a*b*d*m^2*x^5*x^m
+ 15*b^2*d*x^7*x^m + 2*a*b*c*m^3*x^3*x^m + a^2*d*m^3*x^3*x^m + 31*b^2*c*m*x
^5*x^m + 62*a*b*d*m*x^5*x^m + 26*a*b*c*m^2*x^3*x^m + 13*a^2*d*m^2*x^3*x^m +
 21*b^2*c*x^5*x^m + 42*a*b*d*x^5*x^m + a^2*c*m^3*x*x^m + 94*a*b*c*m*x^3*x^m
 + 47*a^2*d*m*x^3*x^m + 15*a^2*c*m^2*x*x^m + 70*a*b*c*x^3*x^m + 35*a^2*d*x^
3*x^m + 71*a^2*c*m*x*x^m + 105*a^2*c*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)
```

Mupad [B]
time = 0.19, size = 177, normalized size = 2.49

$$x^m\left(\frac{a\,x^3\,\left(a\,d+2\,b\,c\right)\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b\,x^5\,\left(2\,a\,d+b\,c\right)\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b^2\,d\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{a^2\,c\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^m*((a*x^3*(a*d + 2*b*c)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*
m^3 + m^4 + 105) + (b*x^5*(2*a*d + b*c)*(31*m + 11*m^2 + m^3 + 21))/(176*m
+ 86*m^2 + 16*m^3 + m^4 + 105) + (b^2*d*x^7*(23*m + 9*m^2 + m^3 + 15))/(176
*m + 86*m^2 + 16*m^3 + m^4 + 105) + (a^2*c*x*(71*m + 15*m^2 + m^3 + 105))/(
176*m + 86*m^2 + 16*m^3 + m^4 + 105))
```

3.328 $\int \frac{x^m (a+bx^2)^2}{c+dx^2} dx$

Optimal. Leaf size=94

$$-\frac{b(bc-2ad)x^{1+m}}{d^2(1+m)}+\frac{b^2x^{3+m}}{d(3+m)}+\frac{(bc-ad)^2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)}{cd^2(1+m)}$$

```
[Out] -b*(-2*a*d+b*c)*x^(1+m)/d^2/(1+m)+b^2*x^(3+m)/d/(3+m)+(-a*d+b*c)^2*x^(1+m)*
hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/d^2/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {472, 371}

$$\frac{x^{m+1}(bc-ad)^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{cd^2(m+1)}-\frac{bx^{m+1}(bc-2ad)}{d^2(m+1)}+\frac{b^2x^{m+3}}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x^(1 + m))/(d^2*(1 + m))) + (b^2*x^(3 + m))/(d*(3 + m))
+ ((b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*
x^2)/c)])/(c*d^2*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)x^m}{d^2}+\frac{b^2x^{2+m}}{d}+\frac{(b^2c^2-2abcd+a^2d^2)\,x^m}{d^2\,(c+dx^2)}\right)dx$$
$$= -\frac{b(bc-2ad)x^{1+m}}{d^2(1+m)}+\frac{b^2x^{3+m}}{d(3+m)}+\frac{(bc-ad)^2\int \frac{x^m}{c+dx^2}\,dx}{d^2}$$
$$= -\frac{b(bc-2ad)x^{1+m}}{d^2(1+m)}+\frac{b^2x^{3+m}}{d(3+m)}+\frac{(bc-ad)^2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)}{cd^2(1+m)}$$

Mathematica [A]
time = 0.13, size = 118, normalized size = 1.26

$$\frac{x^{1+m}\left(\frac{a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)}{1+m}+bx^2\left(\frac{2a\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{dx^2}{c}\right)}{3+m}+\frac{bx^2\,{}_2F_1\left(1,\frac{5+m}{2};\frac{7+m}{2};-\frac{dx^2}{c}\right)}{5+m}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (x^(1 + m)*((a^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/
(1 + m) + b*x^2*((2*a*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((d*x^2)/
c)])/(3 + m) + (b*x^2*Hypergeometric2F1[1, (5 + m)/2, (7 + m)/2, -((d*x^2)/
c)])/(5 + m))))/c
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(b\,x^2+a\right)^2}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] int(x^m*(b*x^2+a)^2/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d*x^2 + c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.00, size = 299, normalized size = 3.18

$$\frac{a^2mxx^m\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{a^2xx^m\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{abmx^3x^m\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3abx^3x^m\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{b^2mx^5x^m\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{5b^2x^5x^m\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] a**2*m*x*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*c*gamma(m/2 + 3/2)) + a**2*x*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + a*b*m*x**3*x**m*l
erchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*c*gamma
(m/2 + 5/2)) + 3*a*b*x**3*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(2*c*gamma(m/2 + 5/2)) + b**2*m*x**5*x**m*lerchphi(d*
x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2
)) + 5*b**2*x**5*x**m*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamm
a(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,{\left(b\,x^2+a\right)}^2}{d\,x^2+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] int((x^m*(a + b*x^2)^2)/(c + d*x^2), x)
```

3.329 $\int \frac{x^m \left(a+bx^2\right)^2}{\left(c+dx^2\right)^2} \, dx$

Optimal. Leaf size=120

$$\frac{b^2x^{1+m}}{d^2(1+m)}+\frac{(bc-ad)^2x^{1+m}}{2cd^2\left(c+dx^2\right)}-\frac{(bc-ad)(ad(1-m)+bc(3+m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)}{2c^2d^2(1+m)}$$

```
[Out] b^2*x^(1+m)/d^2/(1+m)+1/2*(-a*d+b*c)^2*x^(1+m)/c/d^2/(d*x^2+c)-1/2*(-a*d+b*
c)*(a*d*(1-m)+b*c*(3+m))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^
2/c)/c^2/d^2/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {474, 470, 371}

$$-\frac{x^{m+1}(bc-ad)(ad(1-m)+bc(m+3))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2d^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x^{m+1}}{d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x^(1 + m))/(d^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*c*d^2*(c + d*x
^2)) - ((b*c - a*d)*(a*d*(1 - m) + b*c*(3 + m))*x^(1 + m)*Hypergeometric2F1
[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*d^2*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
```

```
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(bc-ad)^2x^{1+m}}{2cd^2(c+dx^2)} - \frac{\int \frac{x^m(-2a^2d^2+(bc-ad)^2(1+m)-2b^2cdx^2)}{c+dx^2}\,dx}{2cd^2}$$
$$= \frac{b^2x^{1+m}}{d^2(1+m)} + \frac{(bc-ad)^2x^{1+m}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(ad(1-m)+bc(3+m)))\int \frac{x^m}{c+dx^2}\,dx}{2cd^2}$$
$$= \frac{b^2x^{1+m}}{d^2(1+m)} + \frac{(bc-ad)^2x^{1+m}}{2cd^2(c+dx^2)} - \frac{(bc-ad)(ad(1-m)+bc(3+m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};3\right.}{2c^2d^2(1+m)}$$

Mathematica [A]
time = 0.19, size = 98, normalized size = 0.82

$$\frac{x^{1+m}\left(b^2c^2-2bc(bc-ad)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)+(bc-ad)^2\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)\right)}{c^2d^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(b^2*c^2 - 2*b*c*(b*c - a*d)*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -((d*x^2)/c)] + (b*c - a*d)^2*Hypergeometric2F1[2, (1 + m)/2, (3 +
m)/2, -((d*x^2)/c)]))/(c^2*d^2*(1 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m(b\,x^2+a)^2}{(d\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] int(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Integral(x**m*(a + b*x**2)**2/(c + d*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] int((x^m*(a + b*x^2)^2)/(c + d*x^2)^2, x)
```

3.330 $\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=171

$$\frac{(bc-ad)^2x^{1+m}}{4cd^2\left(c+dx^2\right)^2}-\frac{(bc-ad)(ad(3-m)+bc(5+m))x^{1+m}}{8c^2d^2\left(c+dx^2\right)}+\frac{(2abcd(1-m^2)+a^2d^2(3-4m+m^2)+b^2c^2(3+}{8c^3d^2(1+}$$

```
[Out] 1/4*(-a*d+b*c)^2*x^(1+m)/c/d^2/(d*x^2+c)^2-1/8*(-a*d+b*c)*(a*d*(3-m)+b*c*(5
+m))*x^(1+m)/c^2/d^2/(d*x^2+c)+1/8*(2*a*b*c*d*(-m^2+1)+a^2*d^2*(m^2-4*m+3)+
b^2*c^2*(m^2+4*m+3))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)
/c^3/d^2/(1+m)
```

Rubi [A]
time = 0.11, antiderivative size = 166, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {474, 468, 371}

$$\frac{x^{m+1}\left(\frac{(1-m)\left(4a^2d^2-(m+1)(bc-ad)^2\right)}{c^2(m+1)}+4b^2\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8cd^2}-\frac{x^{m+1}(bc-ad)(ad(3-m)+bc(m+5))}{8c^2d^2\left(c+dx^2\right)}+\frac{x^{m+1}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^2*x^(1 + m))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(a*d*(3 -
m) + b*c*(5 + m))*x^(1 + m))/(8*c^2*d^2*(c + d*x^2)) + ((4*b^2 + ((1 - m)*(
4*a^2*d^2 - (b*c - a*d)^2*(1 + m)))/(c^2*(1 + m)))*x^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(8*c*d^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(bc-ad)^2x^{1+m}}{4cd^2(c+dx^2)^2} - \frac{\int \frac{x^m(-4a^2d^2+(bc-ad)^2(1+m)-4b^2cdx^2)}{(c+dx^2)^2}\,dx}{4cd^2}$$
$$= \frac{(bc-ad)^2x^{1+m}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(ad(3-m)+bc(5+m))x^{1+m}}{8c^2d^2(c+dx^2)} + -\frac{(-4b^2c^2d(1+m)-a}{}$$
$$= \frac{(bc-ad)^2x^{1+m}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(ad(3-m)+bc(5+m))x^{1+m}}{8c^2d^2(c+dx^2)} + \frac{(4b^2c^2(1+m)+(1-r}{}$$

Mathematica [A]
time = 0.40, size = 124, normalized size = 0.73

$$\frac{x^{1+m}\left(b^2c^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)-(bc-ad)\left(2bc\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)+(-bc+ad)\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)\right)\right)}{c^3d^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(b^2*c^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)
] - (b*c - a*d)*(2*b*c*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)
/c)] + (-(b*c) + a*d)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((d*x^2)/
c)])))/(c^3*d^2*(1 + m))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(b\,x^2+a\right)^2}{\left(d\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] int(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x
^2 + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**3,x)
[Out] Integral(x**m*(a + b*x**2)**2/(c + d*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x)
[Out] int((x^m*(a + b*x^2)^2)/(c + d*x^2)^3, x)
```

3.331 $\int \frac{x^m (c+dx^2)^3}{a+bx^2} \, dx$

Optimal. Leaf size=133

$$\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^{1+m}}{b^3(1+m)}+\frac{d^2(3bc-ad)x^{3+m}}{b^2(3+m)}+\frac{d^3x^{5+m}}{b(5+m)}+\frac{(bc-ad)^3x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab^3(1+m)}$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^(1+m)/b^3/(1+m)+d^2*(-a*d+3*b*c)*x^(3+m)/
b^2/(3+m)+d^3*x^(5+m)/b/(5+m)+(-a*d+b*c)^3*x^(1+m)*hypergeom([1, 1/2+1/2*m]
,[3/2+1/2*m],-b*x^2/a)/a/b^3/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {472, 371}

$$\frac{dx^{m+1}(a^2d^2-3abcd+3b^2c^2)}{b^3(m+1)}+\frac{x^{m+1}(bc-ad)^3\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^3(m+1)}+\frac{d^2x^{m+3}(3bc-ad)}{b^2(m+3)}+\frac{d^3x^{m+5}}{b(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(1 + m))/(b^3*(1 + m)) + (d^2*(3*b*c
 - a*d)*x^(3 + m))/(b^2*(3 + m)) + (d^3*x^(5 + m))/(b*(5 + m)) + ((b*c - a*
d)^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
*b^3*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

Rubi steps

$$\int \frac{x^m(c+dx^2)^3}{a+bx^2}\,dx = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^m}{b^3} + \frac{d^2(3bc-ad)x^{2+m}}{b^2} + \frac{d^3x^{4+m}}{b} + \frac{(b^3c^3-3ab^2c^2d+}{b^3\,(a}\right.$$
$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^{1+m}}{b^3(1+m)} + \frac{d^2(3bc-ad)x^{3+m}}{b^2(3+m)} + \frac{d^3x^{5+m}}{b(5+m)} + \frac{(bc-ad)^3\int \frac{x^m}{a+bx^2}}{b^3}$$
$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^{1+m}}{b^3(1+m)} + \frac{d^2(3bc-ad)x^{3+m}}{b^2(3+m)} + \frac{d^3x^{5+m}}{b(5+m)} + \frac{(bc-ad)^3x^{1+m}\,{}_2F}{ab^3(}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 1.19, size = 114, normalized size = 0.86

$$\frac{x^{1+m}\left(c^3\Phi\left(-\frac{bx^2}{a},1,\frac{1+m}{2}\right)+dx^2\left(3c^2\Phi\left(-\frac{bx^2}{a},1,\frac{3+m}{2}\right)+dx^2\left(3c\Phi\left(-\frac{bx^2}{a},1,\frac{5+m}{2}\right)+dx^2\Phi\left(-\frac{bx^2}{a},1,\frac{7+m}{2}\right)\right)\right)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*(c^3*HurwitzLerchPhi[-((b*x^2)/a), 1, (1 + m)/2] + d*x^2*(3*c^2*
HurwitzLerchPhi[-((b*x^2)/a), 1, (3 + m)/2] + d*x^2*(3*c*HurwitzLerchPhi[-(
(b*x^2)/a), 1, (5 + m)/2] + d*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/
2]))))/(2*a)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m(d\,x^2+c)^3}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] int(x^m*(d*x^2+c)^3/(b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*x^m/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.82, size = 411, normalized size = 3.09

$$\frac{c^3mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{c^3xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{3c^2dmx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{9c^2dx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3cd^2mx^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{15cd^2x^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{d^3mx^7x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{7}{2}\right)\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}+\frac{7d^3x^7x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{7}{2}\right)\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] c**3*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + c**3*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + 3*c**2*d*m*x**3*x
**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*
gamma(m/2 + 5/2)) + 9*c**2*d*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1
, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*c*d**2*m*x**5*x**m
*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gam
ma(m/2 + 7/2)) + 15*c*d**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + d**3*m*x**7*x**m*lerch
phi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(4*a*gamma(m/2
 + 9/2)) + 7*d**3*x**7*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7/2
)*gamma(m/2 + 7/2)/(4*a*gamma(m/2 + 9/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,{\left(d\,x^2+c\right)}^3}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + d*x^2)^3)/(a + b*x^2),x)
[Out] int((x^m*(c + d*x^2)^3)/(a + b*x^2), x)
```

3.332 $\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=94

$$\frac{d(2bc-ad)x^{1+m}}{b^2(1+m)}+\frac{d^2x^{3+m}}{b(3+m)}+\frac{(bc-ad)^2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab^2(1+m)}$$

```
[Out] d*(-a*d+2*b*c)*x^(1+m)/b^2/(1+m)+d^2*x^(3+m)/b/(3+m)+(-a*d+b*c)^2*x^(1+m)*h
ypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b^2/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {472, 371}

$$\frac{x^{m+1}(bc-ad)^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab^2(m+1)}+\frac{dx^{m+1}(2bc-ad)}{b^2(m+1)}+\frac{d^2x^{m+3}}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x^(1 + m))/(b^2*(1 + m)) + (d^2*x^(3 + m))/(b*(3 + m)) + (
(b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)])/(a*b^2*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx = \int \left(\frac{d(2bc-ad)x^m}{b^2}+\frac{d^2x^{2+m}}{b}+\frac{(b^2c^2-2abcd+a^2d^2)\,x^m}{b^2\,(a+bx^2)}\right)dx$$
$$= \frac{d(2bc-ad)x^{1+m}}{b^2(1+m)}+\frac{d^2x^{3+m}}{b(3+m)}+\frac{(bc-ad)^2\int \frac{x^m}{a+bx^2}\,dx}{b^2}$$
$$= \frac{d(2bc-ad)x^{1+m}}{b^2(1+m)}+\frac{d^2x^{3+m}}{b(3+m)}+\frac{(bc-ad)^2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab^2(1+m)}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.37, size = 85, normalized size = 0.90

$$\frac{x^{1+m}\left(c^2\Phi\left(-\frac{bx^2}{a},1,\frac{1+m}{2}\right)+dx^2\left(2c\Phi\left(-\frac{bx^2}{a},1,\frac{3+m}{2}\right)+dx^2\Phi\left(-\frac{bx^2}{a},1,\frac{5+m}{2}\right)\right)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c + d*x^2)^2)/(a + b*x^2),x]
[Out] (x^(1 + m)*(c^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (1 + m)/2] + d*x^2*(2*c*Hu
rwitzLerchPhi[-((b*x^2)/a), 1, (3 + m)/2] + d*x^2*HurwitzLerchPhi[-((b*x^2)
/a), 1, (5 + m)/2])))/(2*a)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m(d\,x^2+c)^2}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)^2/(b*x^2+a),x)
[Out] int(x^m*(d*x^2+c)^2/(b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*x^m/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.13, size = 299, normalized size = 3.18

$$\frac{c^2mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{c^2xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{cdmx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3cdx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{d^2mx^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{5d^2x^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] c**2*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + c**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c*d*m*x**3*x**m*l
erchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*a*gamma
(m/2 + 5/2)) + 3*c*d*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(2*a*gamma(m/2 + 5/2)) + d**2*m*x**5*x**m*lerchphi(b*
x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2
)) + 5*d**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamm
a(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,{\left(d\,x^2+c\right)}^2}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] int((x^m*(c + d*x^2)^2)/(a + b*x^2), x)
```

3.333 $\int \frac{x^m(c+dx^2)}{a+bx^2} dx$

Optimal. Leaf size=66

$$\frac{dx^{1+m}}{b(1+m)}+\frac{(bc-ad)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab(1+m)}$$

```
[Out] d*x^(1+m)/b/(1+m)+(-a*d+b*c)*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-
b*x^2/a)/a/b/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {470, 371}

$$\frac{x^{m+1}(bc-ad)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{dx^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (d*x^(1 + m))/(b*(1 + m)) + ((b*c - a*d)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx = \frac{dx^{1+m}}{b(1+m)} - \frac{(-bc(1+m)+ad(1+m))\int \frac{x^m}{a+bx^2}\,dx}{b(1+m)}$$
$$= \frac{dx^{1+m}}{b(1+m)} + \frac{(bc-ad)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ab(1+m)}$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 0.83

$$\frac{x^{1+m}\left(ad+(bc-ad)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{ab(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c + d*x^2))/(a + b*x^2),x]
[Out] (x^(1 + m)*(a*d + (b*c - a*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)]))/(a*b*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m(d\,x^2+c)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)/(b*x^2+a),x)
[Out] int(x^m*(d*x^2+c)/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")

[Out] integral((d*x^2 + c)*x^m/(b*x^2 + a), x)

Sympy [C] Result contains complex when optimal does not.
time = 2.09, size = 190, normalized size = 2.88

$$\frac{cmxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{cxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{dmx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3dx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(d*x**2+c)/(b*x**2+a),x)

[Out] c*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(4*a*gamma(m/2 + 3/2)) + c*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m
/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + d*m*x**3*x**m*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5
/2)) + 3*d*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma
(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")

[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(d\,x^2+c\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^m*(c + d*x^2))/(a + b*x^2),x)

[Out] int((x^m*(c + d*x^2))/(a + b*x^2), x)

3.334 $\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=102

$$\frac{bx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(bc-ad)(1+m)}-\frac{dx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)}{c(bc-ad)(1+m)}$$

```
[Out] b*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(-a*d+b*c)/(1+m)
-d*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/(-a*d+b*c)/(1+m
)
```

Rubi [A]
time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {493, 371}

$$\frac{bx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(
b*c - a*d)*(1 + m)) - (d*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/
2, -((d*x^2)/c)])/(c*(b*c - a*d)*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx = \frac{b\int \frac{x^m}{a+bx^2}\,dx}{bc-ad} - \frac{d\int \frac{x^m}{c+dx^2}\,dx}{bc-ad}$$
$$= \frac{bx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(bc-ad)(1+m)} - \frac{dx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)}{c(bc-ad)(1+m)}$$

Mathematica [**A**]
time = 0.09, size = 85, normalized size = 0.83

$$\frac{x^{1+m}\left(-bc\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+ad\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)\right)}{ac(-bc+ad)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(b*c*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])
 + a*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)]))/(a*c*(-(b
*c) + a*d)*(1 + m))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^2+a)\,(d\,x^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)/(d*x^2+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.55, size = 354, normalized size = 3.47

$$\frac{amx^m\Phi\left(\frac{ae^{i\pi}}{bx^2},1,\frac{3}{2}-\frac{m}{2}\right)\Gamma^2\left(\frac{3}{2}-\frac{m}{2}\right)}{x^3\cdot\left(4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)\right)}-\frac{3ax^m\Phi\left(\frac{ae^{i\pi}}{bx^2},1,\frac{3}{2}-\frac{m}{2}\right)\Gamma^2\left(\frac{3}{2}-\frac{m}{2}\right)}{x^3\cdot\left(4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)\right)}+\frac{bmx^m\Phi\left(\frac{ce^{i\pi}}{dx^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)}{x\left(4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)\right)}-\frac{bx^m\Phi\left(\frac{ce^{i\pi}}{dx^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)}{x\left(4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] a*m*x**m*lerchphi(a*exp_polar(I*pi)/(b*x**2), 1, 3/2 - m/2)*gamma(3/2 - m/2
)**2/(x**3*(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2 - m/2) - 4*b**2*c*gamma(3/2
- m/2)*gamma(5/2 - m/2))) - 3*a*x**m*lerchphi(a*exp_polar(I*pi)/(b*x**2), 1
, 3/2 - m/2)*gamma(3/2 - m/2)**2/(x**3*(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2
- m/2) - 4*b**2*c*gamma(3/2 - m/2)*gamma(5/2 - m/2))) + b*m*x**m*lerchphi(c
*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)*gamma(5/2 - m/2)/
(x*(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2 - m/2) - 4*b**2*c*gamma(3/2 - m/2)*g
amma(5/2 - m/2))) - b*x**m*lerchphi(c*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/
2)*gamma(1/2 - m/2)*gamma(5/2 - m/2)/(x*(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2
 - m/2) - 4*b**2*c*gamma(3/2 - m/2)*gamma(5/2 - m/2)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\left(b\,x^2+a\right)\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] int(x^m/((a + b*x^2)*(c + d*x^2)), x)
```

3.335 $\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)} dx$

Optimal. Leaf size=156

$$\frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)}+\frac{b(bc(1-m)-ad(3-m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^2(1+m)}+\frac{d^2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\right.}{c(bc-ad)^2(1+m)}$$

```
[Out] 1/2*b*x^(1+m)/a/(-a*d+b*c)/(b*x^2+a)+1/2*b*(b*c*(1-m)-a*d*(3-m))*x^(1+m)*hy
pergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^2/(1+m)+d^2*x^(
1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/(-a*d+b*c)^2/(1+m)
```

Rubi [A]
time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {483, 598, 371}

$$\frac{bx^{m+1}(bc(1-m)-ad(3-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) + (b*(b*c*(1 - m) - a*d*(3 - m)
)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^
2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)^2*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)} - \frac{\int \frac{x^m(2ad-bc(1-m)-bd(1-m)x^2)}{(a+bx^2)(c+dx^2)}\,dx}{2a(bc-ad)}$$

$$= \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)} - \frac{\int \left(\frac{b(-bc(1-m)+ad(3-m))x^m}{(bc-ad)(a+bx^2)} + \frac{2ad^2x^m}{(-bc+ad)(c+dx^2)}\right)dx}{2a(bc-ad)}$$

$$= \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)} + \frac{d^2\int \frac{x^m}{c+dx^2}\,dx}{(bc-ad)^2} + \frac{(b(bc(1-m)-ad(3-m)))\int \frac{x^m}{a+bx^2}\,dx}{2a(bc-ad)^2}$$

$$= \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)} - \frac{b(ad(3-m)-b(c-cm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^2(1+m)}$$

Mathematica [A]
time = 0.17, size = 127, normalized size = 0.81

$$\frac{x^{1+m}\left(-abcd\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+a^2d^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)+bc(bc-ad)\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{a^2c(bc-ad)^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(a*b*c*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/
a)]) + a^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)] + b
*c*(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(
a^2*c*(b*c - a*d)^2*(1 + m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^2+a)^2(d\,x^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
[Out] integral(x^m/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)
*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{{\left(b\,x^2+a\right)}^2\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*x^2)^2*(c + d*x^2)),x)
[Out] int(x^m/((a + b*x^2)^2*(c + d*x^2)), x)
```

3.336 $\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx$

Optimal. Leaf size=234

$$\frac{bx^{1+m}}{4a(bc-ad)\left(a+bx^2\right)^2}+\frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2\left(a+bx^2\right)}+\frac{b(a^2d^2(15-8m+m^2)-2abcd(5-6m+m^2)+b}{8a^3(bc-ad}$$

```
[Out] 1/4*b*x^(1+m)/a/(-a*d+b*c)/(b*x^2+a)^2+1/8*b*(b*c*(3-m)-a*d*(7-m))*x^(1+m)/
a^2/(-a*d+b*c)^2/(b*x^2+a)+1/8*b*(a^2*d^2*(m^2-8*m+15)-2*a*b*c*d*(m^2-6*m+5
)+b^2*c^2*(m^2-4*m+3))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/
a)/a^3/(-a*d+b*c)^3/(1+m)-d^3*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],
-d*x^2/c)/c/(-a*d+b*c)^3/(1+m)
```

Rubi [A]

time = 0.26, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 593, 598, 371}

$$\frac{bx^{m+1}(bc(3-m)-ad(7-m))}{8a^2\left(a+bx^2\right)(bc-ad)^2}+\frac{bx^{m+1}\left(a^2d^2(m^2-8m+15)-2abcd(m^2-6m+5)+b^2c^2(m^2-4m+3)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{8a^3(m+1)(bc-ad)^3}-\frac{d^3x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^3}+\frac{bx^{m+1}}{4a\left(a+bx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(b*c*(3 - m) - a*d*(7 -
m))*x^(1 + m))/(8*a^2*(b*c - a*d)^2*(a + b*x^2)) + (b*(a^2*d^2*(15 - 8*m +
m^2) - 2*a*b*c*d*(5 - 6*m + m^2) + b^2*c^2*(3 - 4*m + m^2))*x^(1 + m)*Hyper
geometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(8*a^3*(b*c - a*d)^3*(
1 + m)) - (d^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^
2)/c)])/(c*(b*c - a*d)^3*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} - \frac{\int \frac{x^m(4ad-bc(3-m)-bd(3-m)x^2)}{(a+bx^2)^2(c+dx^2)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} + \frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2(a+bx^2)} + \frac{\int \frac{x^m(8a^2d^2-abcd(7-}{}}{}$$

$$= \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} + \frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2(a+bx^2)} + \frac{\int \Big(\frac{b(a^2d^2(15-8m+m}{}}{}$$

$$= \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} + \frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2(a+bx^2)} - \frac{d^3\int \frac{x^m}{c+dx^2}\,dx}{(bc-ad)^3} + \frac{(}{}$$

$$= \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} + \frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2(a+bx^2)} + \frac{b(a^2d^2(15-8m+}{}$$

Mathematica [A]

time = 0.51, size = 170, normalized size = 0.73

$$\frac{x^{1+m}\left(-a^2bcd^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+a^3d^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)-bc(-bc+ad)\left(ad\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+(-bc+ad)\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)\right)}{a^3c(-bc+ad)^3(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(a^2*b*c*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)]) + a^3*d^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)]
 - b*c*(-(b*c) + a*d)*(a*d*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)] + (-(b*c) + a*d)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)])))/(a^3*c*(-(b*c) + a*d)^3*(1 + m))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(b\,x^{2}+a\right)^{3}\left(d\,x^{2}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)^3/(d*x^2+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*d*x^8 + (b^3*c + 3*a*b^2*d)*x^6 + 3*(a*b^2*c + a^2*b*d)*x
^4 + a^3*c + (3*a^2*b*c + a^3*d)*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
[Out] integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m}{\left(b\,x^2+a\right)^3\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*x^2)^3*(c + d*x^2)),x)
[Out] int(x^m/((a + b*x^2)^3*(c + d*x^2)), x)
```

3.337 $\int \frac{x^m (c+dx^2)^3}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=201

$$-\frac{d(2b^2c^2(1+m)-3abcd(3+m)+a^2d^2(5+m))\,x^{1+m}}{2ab^3(1+m)}-\frac{d^2(bc(3+m)-ad(5+m))x^{3+m}}{2ab^2(3+m)}+\frac{(bc-ad)x^{1+m}(c}{2ab\,(a+b}$$

```
[Out] -1/2*d*(2*b^2*c^2*(1+m)-3*a*b*c*d*(3+m)+a^2*d^2*(5+m))*x^(1+m)/a/b^3/(1+m)-
1/2*d^2*(b*c*(3+m)-a*d*(5+m))*x^(3+m)/a/b^2/(3+m)+1/2*(-a*d+b*c)*x^(1+m)*(d
*x^2+c)^2/a/b/(b*x^2+a)+1/2*(-a*d+b*c)^2*(a*d*(5+m)+b*(-c*m+c))*x^(1+m)*hyp
ergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/b^3/(1+m)
```

Rubi [A]
time = 0.15, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {479, 584, 371}

$$\frac{x^{m+1}(bc-ad)^2(ad(m+5)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^3(m+1)}-\frac{dx^{m+1}(a^2d^2(m+5)-3abcd(m+3)+2b^2c^2(m+1))}{2ab^3(m+1)}-\frac{d^2x^{m+3}(bc(m+3)-ad(m+5))}{2ab^2(m+3)}+\frac{x^{m+1}(c+dx^2)^2\,(bc-ad)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(d*(2*b^2*c^2*(1 + m) - 3*a*b*c*d*(3 + m) + a^2*d^2*(5 + m))*x^(1 + m)
)/(a*b^3*(1 + m)) - (d^2*(b*c*(3 + m) - a*d*(5 + m))*x^(3 + m))/(2*a*b^2*(3
 + m)) + ((b*c - a*d)*x^(1 + m)*(c + d*x^2)^2)/(2*a*b*(a + b*x^2)) + ((b*c
- a*d)^2*(a*d*(5 + m) + b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)
/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*b^3*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
```

```
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m(c+dx^2)^3}{(a+bx^2)^2}\,dx &= \frac{(bc-ad)x^{1+m}(c+dx^2)^2}{2ab\,(a+bx^2)} - \frac{\int \frac{x^m(c+dx^2)(-c(bc(1-m)+ad(1+m))+d(bc(3+m)-ad(5+m))x^2)}{a+bx^2}\,dx}{2ab}\\ &= \frac{(bc-ad)x^{1+m}(c+dx^2)^2}{2ab\,(a+bx^2)} - \frac{\int \left(\frac{d(2b^2c^2(1+m)-3abcd(3+m)+a^2d^2(5+m))x^m}{b^2} + \frac{d^2(bc(3+m)-ad(5+}{b}\right.}{}\\ &= -\frac{d(2b^2c^2(1+m)-3abcd(3+m)+a^2d^2(5+m))\,x^{1+m}}{2ab^3(1+m)} - \frac{d^2(bc(3+m)-ad(5+m}{2ab^2(3+m)}\\ &= -\frac{d(2b^2c^2(1+m)-3abcd(3+m)+a^2d^2(5+m))\,x^{1+m}}{2ab^3(1+m)} - \frac{d^2(bc(3+m)-ad(5+m}{2ab^2(3+m)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.72, size = 2524, normalized size = 12.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] -1/192*(x^(1 + m)*(a*(945 + 744*m + 206*m^2 + 24*m^3 + m^4)*(c^3*(-47 + 52*
m + 6*m^2 + 4*m^3 + m^4) + 3*c^2*d*(1 + m)^4*x^2 + 3*c*d^2*(1 + m)^4*x^4 +
d^3*(1 + m)^4*x^6)*HurwitzLerchPhi[-((b*x^2)/a), 1, (1 + m)/2] - 3*a*(945 +
 744*m + 206*m^2 + 24*m^3 + m^4)*(c^3*(3 + m)^4 + 3*c^2*d*(65 + 92*m + 54*m
^2 + 12*m^3 + m^4)*x^2 + 3*c*d^2*(3 + m)^4*x^4 + d^3*(3 + m)^4*x^6)*Hurwitz
LerchPhi[-((b*x^2)/a), 1, (3 + m)/2] + 1771875*a*c^3*HurwitzLerchPhi[-((b*x
^2)/a), 1, (5 + m)/2] + 2812500*a*c^3*m*HurwitzLerchPhi[-((b*x^2)/a), 1, (5
 + m)/2] + 1927500*a*c^3*m^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] +
745500*a*c^3*m^3*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 178050*a*c^3
*m^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 26892*a*c^3*m^5*HurwitzL
```

erchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2508*a*c^3*m^6*HurwitzLerchPhi[-((b*x
^2)/a), 1, (5 + m)/2] + 132*a*c^3*m^7*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 +
 m)/2] + 3*a*c^3*m^8*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 5315625*
a*c^2*d*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 8437500*a*c^2*d*m
*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 5782500*a*c^2*d*m^2*x^2*
HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2236500*a*c^2*d*m^3*x^2*Hurwi
tzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 534150*a*c^2*d*m^4*x^2*HurwitzLerc
hPhi[-((b*x^2)/a), 1, (5 + m)/2] + 80676*a*c^2*d*m^5*x^2*HurwitzLerchPhi[-(
(b*x^2)/a), 1, (5 + m)/2] + 7524*a*c^2*d*m^6*x^2*HurwitzLerchPhi[-((b*x^2)/
a), 1, (5 + m)/2] + 396*a*c^2*d*m^7*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5
 + m)/2] + 9*a*c^2*d*m^8*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] +
5723865*a*c*d^2*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 8894988*a
*c*d^2*m*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 5978628*a*c*d^2*
m^2*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2276532*a*c*d^2*m^3*x
^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 538038*a*c*d^2*m^4*x^4*Hur
witzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 80820*a*c*d^2*m^5*x^4*HurwitzLer
chPhi[-((b*x^2)/a), 1, (5 + m)/2] + 7524*a*c*d^2*m^6*x^4*HurwitzLerchPhi[-(
(b*x^2)/a), 1, (5 + m)/2] + 396*a*c*d^2*m^7*x^4*HurwitzLerchPhi[-((b*x^2)/a
), 1, (5 + m)/2] + 9*a*c*d^2*m^8*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 +
m)/2] + 1771875*a*d^3*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 281
2500*a*d^3*m*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 1927500*a*d^
3*m^2*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 745500*a*d^3*m^3*x^
6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 178050*a*d^3*m^4*x^6*Hurwit
zLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 26892*a*d^3*m^5*x^6*HurwitzLerchPhi
[-((b*x^2)/a), 1, (5 + m)/2] + 2508*a*d^3*m^6*x^6*HurwitzLerchPhi[-((b*x^2)
/a), 1, (5 + m)/2] + 132*a*d^3*m^7*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5
+ m)/2] + 3*a*d^3*m^8*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] - 226
8945*a*c^3*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3082884*a*c^3*m*Hu
rwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 1793204*a*c^3*m^2*HurwitzLerchP
hi[-((b*x^2)/a), 1, (7 + m)/2] - 585452*a*c^3*m^3*HurwitzLerchPhi[-((b*x^2)
/a), 1, (7 + m)/2] - 117670*a*c^3*m^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 +
 m)/2] - 14940*a*c^3*m^5*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 1172
*a*c^3*m^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 52*a*c^3*m^7*Hurwi
tzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - a*c^3*m^8*HurwitzLerchPhi[-((b*x^2
)/a), 1, (7 + m)/2] - 6806835*a*c^2*d*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1,
(7 + m)/2] - 9248652*a*c^2*d*m*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)
/2] - 5379612*a*c^2*d*m^2*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] -
 1756356*a*c^2*d*m^3*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3530
10*a*c^2*d*m^4*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 44820*a*c^
2*d*m^5*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3516*a*c^2*d*m^6*
x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 156*a*c^2*d*m^7*x^2*Hurwi
tzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3*a*c^2*d*m^8*x^2*HurwitzLerchPhi[
-((b*x^2)/a), 1, (7 + m)/2] - 6806835*a*c*d^2*x^4*HurwitzLerchPhi[-((b*x^2)
/a), 1, (7 + m)/2] - 9248652*a*c*d^2*m*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1,
 (7 + m)/2] - 5379612*a*c*d^2*m^2*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 +

```
 m)/2] - 1756356*a*c*d^2*m^3*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2
] - 353010*a*c*d^2*m^4*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 44
820*a*c*d^2*m^5*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3516*a*c*
d^2*m^6*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 156*a*c*d^2*m^7*x
^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 3*a*c*d^2*m^8*x^4*HurwitzL
erchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 2042145*a*d^3*x^6*HurwitzLerchPhi[-((
b*x^2)/a), 1, (7 + m)/2] - 2858964*a*d^3*m*x^6*HurwitzLerchPhi[-((b*x^2)/a)
, 1, (7 + m)/2] - 1708052*a*d^3*m^2*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7
 + m)/2] - 569804*a*d^3*m^3*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2]
 - 116278*a*d^3*m^4*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 14892
*a*d^3*m^5*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2] - 1172*a*d^3*m^6
*x^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 + m)/2...
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(d\,x^{2}+c \right)^{3}}{\left(b\,x^{2}+a \right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] int(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*x^m/(b^2*x^4 + 2*a*b*x
^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (c+dx^2)^3}{(a+bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)**3/(b*x**2+a)**2,x)
[Out] Integral(x**m*(c + d*x**2)**3/(a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m\,{\left(d\,x^2+c\right)}^3}{{\left(b\,x^2+a\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x)
[Out] int((x^m*(c + d*x^2)^3)/(a + b*x^2)^2, x)
```

3.338 $\int \frac{x^m(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=120

$$\frac{d^2x^{1+m}}{b^2(1+m)}+\frac{(bc-ad)^2x^{1+m}}{2ab^2(a+bx^2)}+\frac{(bc-ad)(ad(3+m)+b(c-cm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2b^2(1+m)}$$

```
[Out] d^2*x^(1+m)/b^2/(1+m)+1/2*(-a*d+b*c)^2*x^(1+m)/a/b^2/(b*x^2+a)+1/2*(-a*d+b*
c)*(a*d*(3+m)+b*(-c*m+c))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x
^2/a)/a^2/b^2/(1+m)
```

Rubi [A]
time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {474, 470, 371}

$$\frac{x^{m+1}(bc-ad)(ad(m+3)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b^2(m+1)}+\frac{x^{m+1}(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x^{m+1}}{b^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x^(1 + m))/(b^2*(1 + m)) + ((b*c - a*d)^2*x^(1 + m))/(2*a*b^2*(a + b*x
^2)) + ((b*c - a*d)*(a*d*(3 + m) + b*(c - c*m))*x^(1 + m)*Hypergeometric2F1
[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*b^2*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
```

```
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^m (c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(bc-ad)^2 x^{1+m}}{2ab^2(a+bx^2)} - \frac{\int \frac{x^m(-2b^2c^2+(bc-ad)^2(1+m)-2abd^2x^2)}{a+bx^2}\,dx}{2ab^2}$$
$$= \frac{d^2x^{1+m}}{b^2(1+m)} + \frac{(bc-ad)^2x^{1+m}}{2ab^2(a+bx^2)} - -\frac{(-2a^2bd^2(1+m) - b(1+m)\left(-2b^2c^2+(bc-ad)^2(\right.}{2ab^3(1+m)}$$
$$= \frac{d^2x^{1+m}}{b^2(1+m)} + \frac{(bc-ad)^2x^{1+m}}{2ab^2(a+bx^2)} + \frac{(bc-ad)(bc(1-m)+ad(3+m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3-}{}\right.}{2a^2b^2(1+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 1.57, size = 895, normalized size = 7.46

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a*(105 + 71*m + 15*m^2 + m^3)*(c^2*(9 - 5*m + 3*m^2 + m^3) + 2*
c*d*(1 + m)^3*x^2 + d^2*(1 + m)^3*x^4)*HurwitzLerchPhi[-((b*x^2)/a), 1, (1
+ m)/2] - 2*a*(105 + 71*m + 15*m^2 + m^3)*(c^2*(3 + m)^3 + 2*c*d*(31 + 31*m
 + 9*m^2 + m^3)*x^2 + d^2*(3 + m)^3*x^4)*HurwitzLerchPhi[-((b*x^2)/a), 1, (
3 + m)/2] + 13125*a*c^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 16750
*a*c^2*m*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 8775*a*c^2*m^2*Hurwi
tzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2420*a*c^2*m^3*HurwitzLerchPhi[-((
b*x^2)/a), 1, (5 + m)/2] + 371*a*c^2*m^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (
5 + m)/2] + 30*a*c^2*m^5*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + a*c^
2*m^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 26250*a*c*d*x^2*Hurwitz
LerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 33500*a*c*d*m*x^2*HurwitzLerchPhi[-(
(b*x^2)/a), 1, (5 + m)/2] + 17550*a*c*d*m^2*x^2*HurwitzLerchPhi[-((b*x^2)/a
), 1, (5 + m)/2] + 4840*a*c*d*m^3*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 +
 m)/2] + 742*a*c*d*m^4*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 60
*a*c*d*m^5*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2*a*c*d*m^6*x^
2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 10605*a*d^2*x^4*HurwitzLerc
hPhi[-((b*x^2)/a), 1, (5 + m)/2] + 14206*a*d^2*m*x^4*HurwitzLerchPhi[-((b*x
```

```
^2)/a), 1, (5 + m)/2] + 7847*a*d^2*m^2*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1,
 (5 + m)/2] + 2276*a*d^2*m^3*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2
] + 363*a*d^2*m^4*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 30*a*d^
2*m^5*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + a*d^2*m^6*x^4*Hurwi
tzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] - 128*b*c^2*x^2*HypergeometricPFQ[{2
, 2, 2, 3/2 + m/2}, {1, 1, 9/2 + m/2}, -((b*x^2)/a)] - 256*b*c*d*x^4*Hyperg
eometricPFQ[{2, 2, 2, 3/2 + m/2}, {1, 1, 9/2 + m/2}, -((b*x^2)/a)] - 128*b*
d^2*x^6*HypergeometricPFQ[{2, 2, 2, 3/2 + m/2}, {1, 1, 9/2 + m/2}, -((b*x^2
)/a)]))/(32*a^3*(3 + m)*(5 + m)*(7 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(d\,x^2+c\right)^2}{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x)
[Out] int(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**m*(c + d*x**2)**2/(a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,{\left(d\,x^2+c\right)}^2}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

```
[Out] int((x^m*(c + d*x^2)^2)/(a + b*x^2)^2, x)
```

3.339 $\int \frac{x^m(c+dx^2)}{(a+bx^2)^2} dx$

Optimal. Leaf size=93

$$\frac{(bc-ad)x^{1+m}}{2ab\left(a+bx^2\right)}+\frac{(ad(1+m)+b(c-cm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2b(1+m)}$$

```
[Out] 1/2*(-a*d+b*c)*x^(1+m)/a/b/(b*x^2+a)+1/2*(a*d*(1+m)+b*(-c*m+c))*x^(1+m)*hyp
ergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/b/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {468, 371}

$$\frac{x^{m+1}(ad(m+1)+b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*d*(1 + m) + b*(c - c*m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*
b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x^{1+m}}{2ab\,(a+bx^2)} + \frac{(ad(1+m)+b(c-cm))\int \frac{x^m}{a+bx^2}\,dx}{2ab}$$
$$= \frac{(bc-ad)x^{1+m}}{2ab\,(a+bx^2)} + \frac{(ad(1+m)+b(c-cm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2b(1+m)}$$

Mathematica [A]
time = 0.10, size = 80, normalized size = 0.86

$$\frac{x^{1+m}\left(ad\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+(bc-ad)\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{a^2b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] +
(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a^2
*b*(1 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m(d\,x^2+c)}{(b\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] int(x^m*(d*x^2+c)/(b*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.98, size = 906, normalized size = 9.74

$$c\left(-\frac{am^2xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2amxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{axx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2axx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}-\frac{bm^2x^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{bx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}\right)+d\left(-\frac{am^2x^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{4amx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{2amx^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{3ax^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{6ax^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{bm^2x^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{4bmx^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{3bx^5x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] c*(-a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/
2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*m
*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2
 + 3/2)) + a*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a
*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2
 + 3/2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2
)) + b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))) + d*(-a
*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2
+ 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4*a*m*x
**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/
(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 2*a*m*x**3*x**
m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2
)) - 3*a*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m
/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 6*a*
x**3*x**m*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m
/2 + 5/2)) - b*m**2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3
/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5
/2)) - 4*b*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gam
ma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) -
3*b*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 +
3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c + d*x^2))/(a + b*x^2)^2,x)
[Out] int((x^m*(c + d*x^2))/(a + b*x^2)^2, x)
```

3.340 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=156

$$\frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{b(bc(1-m)-ad(3-m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^2(1+m)}+\frac{d^2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\right.}{c(bc-ad)^2(1+m)}$$

```
[Out] 1/2*b*x^(1+m)/a/(-a*d+b*c)/(b*x^2+a)+1/2*b*(b*c*(1-m)-a*d*(3-m))*x^(1+m)*hy
pergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^2/(1+m)+d^2*x^(
1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/(-a*d+b*c)^2/(1+m)
```

Rubi [A]

time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {483, 598, 371}

$$\frac{bx^{m+1}(bc(1-m)-ad(3-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^2}+\frac{d^2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) + (b*(b*c*(1 - m) - a*d*(3 - m)
)*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^
2*(b*c - a*d)^2*(1 + m)) + (d^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)^2*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{(a+bx^2)^2\,(c+dx^2)}\,dx &= \frac{bx^{1+m}}{2a(bc-ad)\,(a+bx^2)} - \frac{\int \frac{x^m(2ad-bc(1-m)-bd(1-m)x^2)}{(a+bx^2)(c+dx^2)}\,dx}{2a(bc-ad)}\\
&= \frac{bx^{1+m}}{2a(bc-ad)\,(a+bx^2)} - \frac{\int \left(\frac{b(-bc(1-m)+ad(3-m))x^m}{(bc-ad)(a+bx^2)} + \frac{2ad^2x^m}{(-bc+ad)(c+dx^2)}\right)dx}{2a(bc-ad)}\\
&= \frac{bx^{1+m}}{2a(bc-ad)\,(a+bx^2)} + \frac{d^2\int \frac{x^m}{c+dx^2}\,dx}{(bc-ad)^2} + \frac{(b(bc(1-m)-ad(3-m)))\int \frac{x^m}{a+bx^2}\,dx}{2a(bc-ad)^2}\\
&= \frac{bx^{1+m}}{2a(bc-ad)\,(a+bx^2)} - \frac{b(ad(3-m)-b(c-cm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^2(1+m)}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 127, normalized size = 0.81

$$\frac{x^{1+m}\left(-abcd\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+a^2d^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)+bc(bc-ad)\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{a^2c(bc-ad)^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (x^(1 + m)*(-(a*b*c*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/
a)]) + a^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)] + b
*c*(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(
a^2*c*(b*c - a*d)^2*(1 + m))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^2+a)^2\,(d\,x^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
[Out] integral(x^m/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d)
*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*x^2)^2*(c + d*x^2)),x)
[Out] int(x^m/((a + b*x^2)^2*(c + d*x^2)), x)
```

3.341 $\int \frac{x^m}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^2} \, dx$

Optimal. Leaf size=230

$$\frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2\left(c+dx^2\right)}+\frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{b^2(ad(5-m)-b(c-cm))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\right.}{2a^2(bc-ad)^3(1+m)}$$

```
[Out] 1/2*d*(a*d+b*c)*x^(1+m)/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*b*x^(1+m)/a/(-a*d+b*
c)/(b*x^2+a)/(d*x^2+c)-1/2*b^2*(a*d*(5-m)-b*(-c*m+c))*x^(1+m)*hypergeom([1,
 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^3/(1+m)-1/2*d^2*(a*d*(1-m)
-b*c*(5-m))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^2/(-a*
d+b*c)^3/(1+m)
```

Rubi [A]
time = 0.27, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 593, 598, 371}

$$-\frac{b^2x^{m+1}(ad(5-m)-b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^3}-\frac{d^2x^{m+1}(ad(1-m)-bc(5-m))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{2c^2(m+1)(bc-ad)^3}+\frac{dx^{m+1}(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}+\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x^(1 + m))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x^(1 + m))
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^2*(a*d*(5 - m) - b*(c - c*m
))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a
^2*(b*c - a*d)^3*(1 + m)) - (d^2*(a*d*(1 - m) - b*c*(5 - m))*x^(1 + m)*Hype
rgeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*(b*c - a*d)^3*
(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{x^m\left(2ad-bc(1-m)-bd(3-m)x^2\right)}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{x^m\left(2\left(4abcd-b^2c^2 \cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \left(\frac{2b^2c(-bc(1-m) \cdots}{(bc-ad)(\cdots}\right.}{\cdots}$$

$$= \frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(d^2(ad(1-m) \cdots\right.}{2c \cdots}$$

$$= \frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{1+m}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{b^2(bc(1-m) - \cdots}{\cdots}$$

Mathematica [A]

time = 0.41, size = 173, normalized size = 0.75

$$\frac{x^{1+m}\left(2ab^2c^2d\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)-2a^2bcd^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)-(bc-ad)\left(b^2c^2\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+a^2d^2\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)\right)\right)}{a^2c^2(-bc+ad)^3(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (x^(1 + m)*(2*a*b^2*c^2*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)] - 2*a^2*b*c*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2
)/c)] - (b*c - a*d)*(b^2*c^2*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((
b*x^2)/a)] + a^2*d^2*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c
)])))/(a^2*c^2*(-(b*c) + a*d)^3*(1 + m))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(b\,x^{2}+a\right)^{2}\left(d\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d^2*x^8 + 2*(b^2*c*d + a*b*d^2)*x^6 + (b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^4 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m}{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] int(x^m/((a + b*x^2)^2*(c + d*x^2)^2), x)
```

3.342 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=325

$$\frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2\,(c+dx^2)^2}+\frac{bx^{1+m}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2}+\frac{d(4b^2c^2-a^2d^2(3-m)+abcd(11-m))\,x^{1+m}}{8ac^2(bc-ad)^3\,(c+dx^2)}-$$

```
[Out] 1/4*d*(a*d+2*b*c)*x^(1+m)/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x^(1+m)/a/(-a*
d+b*c)/(b*x^2+a)/(d*x^2+c)^2+1/8*d*(4*b^2*c^2-a^2*d^2*(3-m)+a*b*c*d*(11-m))
*x^(1+m)/a/c^2/(-a*d+b*c)^3/(d*x^2+c)-1/2*b^3*(a*d*(7-m)-b*(-c*m+c))*x^(1+m
)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^4/(1+m)+1/8
*d^2*(b^2*c^2*(m^2-12*m+35)-2*a*b*c*d*(m^2-8*m+7)+a^2*d^2*(m^2-4*m+3))*x^(1
+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^3/(-a*d+b*c)^4/(1+m)
```

Rubi [A]

time = 0.45, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 593, 598, 371}

$$-\frac{b^3x^{m+1}(ad(7-m)-b(c-cm))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{2a^2(m+1)(bc-ad)^4}+\frac{dx^{m+1}(-a^2d^2(3-m)+abcd(11-m)+4b^2c^2)}{8ac^2\,(c+dx^2)\,(bc-ad)^3}+\frac{d^2x^{m+1}(a^2d^2(m^2-4m+3)-2abcd(m^2-8m+7)+b^2c^2(m^2-12m+35))\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{dx^2}{c}\right)}{8c^3(m+1)(bc-ad)^4}+\frac{dx^{m+1}(ad+2bc)}{4ac\,(c+dx^2)^2\,(bc-ad)^2}+\frac{bx^{m+1}}{2a\,(a+bx^2)\,(c+dx^2)^2\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x^(1 + m))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x^(1 +
 m))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b^2*c^2 - a^2*d^2*
(3 - m) + a*b*c*d*(11 - m))*x^(1 + m))/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2))
- (b^3*(a*d*(7 - m) - b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2
, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*(b*c - a*d)^4*(1 + m)) + (d^2*(b^2*c^2*(
35 - 12*m + m^2) - 2*a*b*c*d*(7 - 8*m + m^2) + a^2*d^2*(3 - 4*m + m^2))*x^(
1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(8*c^3*(b*
c - a*d)^4*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\int \frac{x^m(2ad-bc(1-m)-bd(5-m)x^2)}{(a+bx^2)(c+dx^2)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\int \frac{x^m(2(8abcd-2b}{}}{}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4b^2c^2-a^2d}{8ac}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4b^2c^2-a^2d}{8ac}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4b^2c^2-a^2d}{8ac}$$

$$= \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4b^2c^2-a^2d}{8ac}$$

Mathematica [A]

time = 0.75, size = 215, normalized size = 0.66

$$\frac{x^{1+m}\left(-3ab^3c^3d\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+3a^2b^2c^2d^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)+(bc-ad)\left(b^3c^3\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+a^2d^2\left(2bc\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)+(bc-ad)\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{dx^2}{c}\right)\right)\right)\right)}{a^2c^3(bc-ad)^4(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (x^(1 + m)*(-3*a*b^3*c^3*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)] + 3*a^2*b^2*c^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((
d*x^2)/c)] + (b*c - a*d)*(b^3*c^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2
, -((b*x^2)/a)] + a^2*d^2*(2*b*c*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
 -((d*x^2)/c)] + (b*c - a*d)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((
d*x^2)/c)]))))/(a^2*c^3*(b*c - a*d)^4*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^2+a\right)^2\left(d\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] int(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*d^3*x^10 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + (3*b^2*c^2*d +
 6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^
2)*x^4 + (2*a*b*c^3 + 3*a^2*c^2*d)*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m}{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] int(x^m/((a + b*x^2)^2*(c + d*x^2)^3), x)
```

3.343 $\int x^{7/2}(a+bx^2)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{9}aAx^{9/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{17}bBx^{17/2}$$

```
[Out] 2/9*a*A*x^(9/2)+2/13*(A*b+B*a)*x^(13/2)+2/17*b*B*x^(17/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{17}bBx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx&=\int\left(aAx^{7/2}+(Ab+aB)x^{11/2}+bBx^{15/2}\right)\,dx\\&=\frac{2}{9}aAx^{9/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{17}bBx^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.05

$$\frac{2\left(221aAx^{9/2}+153Abx^{13/2}+153aBx^{13/2}+117bBx^{17/2}\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*(221*a*A*x^(9/2) + 153*A*b*x^(13/2) + 153*a*B*x^(13/2) + 117*b*B*x^(17/2
)))/1989
```

Maple [A]
time = 0.09, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
default	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
gosper	$\frac{2x^{\frac{9}{2}}\left(117bB\,x^{4}+153Ab\,x^{2}+153Ba\,x^{2}+221Aa\right)}{1989}$	32
trager	$\frac{2x^{\frac{9}{2}}\left(117bB\,x^{4}+153Ab\,x^{2}+153Ba\,x^{2}+221Aa\right)}{1989}$	32
risch	$\frac{2x^{\frac{9}{2}}\left(117bB\,x^{4}+153Ab\,x^{2}+153Ba\,x^{2}+221Aa\right)}{1989}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
[Out] 2/9*a*A*x^(9/2)+2/13*(A*b+B*a)*x^(13/2)+2/17*b*B*x^(17/2)
```

Maxima [A]
time = 0.32, size = 27, normalized size = 0.69

$$\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{13}\,(Ba+Ab)x^{\frac{13}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
[Out] 2/17*B*b*x^(17/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/9*A*a*x^(9/2)
```

Fricas [A]
time = 1.12, size = 32, normalized size = 0.82

$$\frac{2}{1989}\,\left(117\,Bbx^{8}+153\,(Ba+Ab)x^{6}+221\,Aax^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
[Out] 2/1989*(117*B*b*x^8 + 153*(B*a + A*b)*x^6 + 221*A*a*x^4)*sqrt(x)
```

Sympy [A]
time = 0.74, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{9}{2}}}{9}+\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Bax^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(9/2)/9 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(17/2
)/17
```

Giac [A]
time = 1.43, size = 29, normalized size = 0.74

$$\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{13}\,Bax^{\frac{13}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/9*A*a*x^(9/2)
```

Mupad [B]
time = 0.11, size = 31, normalized size = 0.79

$$\frac{2\,x^{9/2}\,\left(221\,A\,a+153\,A\,b\,x^2+153\,B\,a\,x^2+117\,B\,b\,x^4\right)}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] (2*x^(9/2)*(221*A*a + 153*A*b*x^2 + 153*B*a*x^2 + 117*B*b*x^4))/1989
```

3.344 $\int x^{5/2}(a+bx^2)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{7}aAx^{7/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{15}bBx^{15/2}$$

```
[Out] 2/7*a*A*x^(7/2)+2/11*(A*b+B*a)*x^(11/2)+2/15*b*B*x^(15/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{15}bBx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx &= \int \left(aAx^{5/2}+(Ab+aB)x^{9/2}+bBx^{13/2}\right)\,dx\\ &= \frac{2}{7}aAx^{7/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{15}bBx^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2x^{7/2}\left(165aA+105Abx^2+105aBx^2+77bBx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

[Out] (2*x^(7/2)*(165*a*A + 105*A*b*x^2 + 105*a*B*x^2 + 77*b*B*x^4))/1155

Maple [A]
time = 0.08, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
default	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
gosper	$\frac{2x^{\frac{7}{2}}\left(77bB\,x^{4}+105Ab\,x^{2}+105Ba\,x^{2}+165Aa\right)}{1155}$	32
trager	$\frac{2x^{\frac{7}{2}}\left(77bB\,x^{4}+105Ab\,x^{2}+105Ba\,x^{2}+165Aa\right)}{1155}$	32
risch	$\frac{2x^{\frac{7}{2}}\left(77bB\,x^{4}+105Ab\,x^{2}+105Ba\,x^{2}+165Aa\right)}{1155}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)

[Out] 2/7*a*A*x^(7/2)+2/11*(A*b+B*a)*x^(11/2)+2/15*b*B*x^(15/2)

Maxima [A]
time = 0.31, size = 27, normalized size = 0.69

$$\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")

[Out] 2/15*B*b*x^(15/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/7*A*a*x^(7/2)

Fricas [A]
time = 1.29, size = 32, normalized size = 0.82

$$\frac{2}{1155}\left(77\,Bbx^{7}+105\,(Ba+Ab)x^{5}+165\,Aax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")

[Out] 2/1155*(77*B*b*x^7 + 105*(B*a + A*b)*x^5 + 165*A*a*x^3)*sqrt(x)

Sympy [A]
time = 0.48, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{7}{2}}}{7}+\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Bax^{\frac{11}{2}}}{11}+\frac{2Bbx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(7/2)/7 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(15/2
)/15
```

Giac [A]
time = 0.75, size = 29, normalized size = 0.74

$$\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{11}\,Bax^{\frac{11}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/7*A*a*x^(7/2)
```

Mupad [B]
time = 0.12, size = 31, normalized size = 0.79

$$\frac{2\,x^{7/2}\,\left(165\,A\,a+105\,A\,b\,x^2+105\,B\,a\,x^2+77\,B\,b\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] (2*x^(7/2)*(165*A*a + 105*A*b*x^2 + 105*B*a*x^2 + 77*B*b*x^4))/1155
```

3.345 $\int x^{3/2}(a+bx^2)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{5}aAx^{5/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{13}bBx^{13/2}$$

```
[Out] 2/5*a*A*x^(5/2)+2/9*(A*b+B*a)*x^(9/2)+2/13*b*B*x^(13/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{13}bBx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx&=\int\left(aAx^{3/2}+(Ab+aB)x^{7/2}+bBx^{11/2}\right)\,dx\\&=\frac{2}{5}aAx^{5/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{13}bBx^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{585}x^{5/2}\left(117aA+65Abx^2+65aBx^2+45bBx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(5/2)*(117*a*A + 65*A*b*x^2 + 65*a*B*x^2 + 45*b*B*x^4))/585
```

Maple [A]
time = 0.09, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{13}{2}}}{13}$	28
default	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{13}{2}}}{13}$	28
gosper	$\frac{2x^{\frac{5}{2}}\left(45bB\,x^{4}+65Ab\,x^{2}+65Ba\,x^{2}+117Aa\right)}{585}$	32
trager	$\frac{2x^{\frac{5}{2}}\left(45bB\,x^{4}+65Ab\,x^{2}+65Ba\,x^{2}+117Aa\right)}{585}$	32
risch	$\frac{2x^{\frac{5}{2}}\left(45bB\,x^{4}+65Ab\,x^{2}+65Ba\,x^{2}+117Aa\right)}{585}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
[Out] 2/5*a*A*x^(5/2)+2/9*(A*b+B*a)*x^(9/2)+2/13*b*B*x^(13/2)
```

Maxima [A]
time = 0.32, size = 27, normalized size = 0.69

$$\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
[Out] 2/13*B*b*x^(13/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/5*A*a*x^(5/2)
```

Fricas [A]
time = 0.97, size = 32, normalized size = 0.82

$$\frac{2}{585}\left(45\,Bbx^{6}+65\,(Ba+Ab)x^{4}+117\,Aax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
[Out] 2/585*(45*B*b*x^6 + 65*(B*a + A*b)*x^4 + 117*A*a*x^2)*sqrt(x)
```

Sympy [A]
time = 0.30, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Bax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(5/2)/5 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(13/2)/13
```

Giac [A]
time = 0.84, size = 29, normalized size = 0.74

$$\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{9}\,Bax^{\frac{9}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/5*A*a*x^(5/2)
```

Mupad [B]
time = 0.02, size = 31, normalized size = 0.79

$$\frac{2\,x^{5/2}\,\left(117\,A\,a+65\,A\,b\,x^2+65\,B\,a\,x^2+45\,B\,b\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] (2*x^(5/2)*(117*A*a + 65*A*b*x^2 + 65*B*a*x^2 + 45*B*b*x^4))/585
```

3.346 $\int \sqrt{x} \left(a+bx^2\right) \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=39

$$\frac{2}{3}aAx^{3/2}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{11}bBx^{11/2}$$

```
[Out] 2/3*a*A*x^(3/2)+2/7*(A*b+B*a)*x^(7/2)+2/11*b*B*x^(11/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{11}bBx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(11/2))/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x} \left(a+bx^2\right) \left(A+Bx^2\right) \, dx &= \int \left(aA\sqrt{x}+(Ab+aB)x^{5/2}+bBx^{9/2}\right) \, dx\\ &= \frac{2}{3}aAx^{3/2}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{11}bBx^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{231}x^{3/2}\left(77aA+33Abx^2+33aBx^2+21bBx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(77*a*A + 33*A*b*x^2 + 33*a*B*x^2 + 21*b*B*x^4))/231
```

Maple [A]
time = 0.09, size = 28, normalized size = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{11}{2}}}{11}$	28
default	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{11}{2}}}{11}$	28
gosper	$\frac{2x^{\frac{3}{2}}\left(21bB\,x^{4}+33Ab\,x^{2}+33Ba\,x^{2}+77Aa\right)}{231}$	32
trager	$\frac{2x^{\frac{3}{2}}\left(21bB\,x^{4}+33Ab\,x^{2}+33Ba\,x^{2}+77Aa\right)}{231}$	32
risch	$\frac{2x^{\frac{3}{2}}\left(21bB\,x^{4}+33Ab\,x^{2}+33Ba\,x^{2}+77Aa\right)}{231}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*a*A*x^(3/2)+2/7*(A*b+B*a)*x^(7/2)+2/11*b*B*x^(11/2)
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.69

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
[Out] 2/11*B*b*x^(11/2) + 2/7*(B*a + A*b)*x^(7/2) + 2/3*A*a*x^(3/2)
```

Fricas [A]
time = 1.33, size = 30, normalized size = 0.77

$$\frac{2}{231}\left(21\,Bbx^{5}+33\,(Ba+Ab)x^{3}+77\,Aax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
[Out] 2/231*(21*B*b*x^5 + 33*(B*a + A*b)*x^3 + 77*A*a*x)*sqrt(x)
```

Sympy [A]
time = 0.94, size = 37, normalized size = 0.95

$$\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{11}{2}}}{11}+\frac{2x^{\frac{7}{2}}(Ab+Ba)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)*x**(1/2),x)
[Out] 2*A*a*x**(3/2)/3 + 2*B*b*x**(11/2)/11 + 2*x**(7/2)*(A*b + B*a)/7
```

Giac [A]
time = 0.78, size = 29, normalized size = 0.74

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{7}\,Bax^{\frac{7}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="giac")
[Out] 2/11*B*b*x^(11/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2/3*A*a*x^(3/2)
```

Mupad [B]
time = 0.02, size = 31, normalized size = 0.79

$$\frac{2\,x^{3/2}\,\left(77\,A\,a+33\,A\,b\,x^2+33\,B\,a\,x^2+21\,B\,b\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(a + b*x^2),x)
[Out] (2*x^(3/2)*(77*A*a + 33*A*b*x^2 + 33*B*a*x^2 + 21*B*b*x^4))/231
```

3.347 $\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}} \, dx$

Optimal. Leaf size=37

$$2aA\sqrt{x} + \frac{2}{5}(Ab + aB)x^{5/2} + \frac{2}{9}bBx^{9/2}$$

```
[Out] 2/5*(A*b+B*a)*x^(5/2)+2/9*b*B*x^(9/2)+2*a*A*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\frac{2}{5}x^{5/2}(aB + Ab) + 2aA\sqrt{x} + \frac{2}{9}bBx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(9/2))/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}} \, dx &= \int \left(\frac{aA}{\sqrt{x}} + (Ab + aB)x^{3/2} + bBx^{7/2}\right) dx \\ &= 2aA\sqrt{x} + \frac{2}{5}(Ab + aB)x^{5/2} + \frac{2}{9}bBx^{9/2} \end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.95

$$\frac{2}{45}\sqrt{x}\left(45aA + 9Abx^2 + 9aBx^2 + 5bBx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]
[Out] (2*Sqrt[x]*(45*a*A + 9*A*b*x^2 + 9*a*B*x^2 + 5*b*B*x^4))/45
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2(Ab+Ba)x^{\frac{5}{2}}}{5}+\frac{2bB\,x^{\frac{9}{2}}}{9}+2aA\sqrt{x}$	28
default	$\frac{2(Ab+Ba)x^{\frac{5}{2}}}{5}+\frac{2bB\,x^{\frac{9}{2}}}{9}+2aA\sqrt{x}$	28
trager	$\left(\frac{2}{9}bB\,x^{4}+\frac{2}{5}Ab\,x^{2}+\frac{2}{5}Ba\,x^{2}+2Aa\right)\sqrt{x}$	31
gosper	$\frac{2\sqrt{x}\,\left(5bB\,x^{4}+9Ab\,x^{2}+9Ba\,x^{2}+45Aa\right)}{45}$	32
risch	$\frac{2\sqrt{x}\,\left(5bB\,x^{4}+9Ab\,x^{2}+9Ba\,x^{2}+45Aa\right)}{45}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(A*b+B*a)*x^(5/2)+2/9*b*B*x^(9/2)+2*a*A*x^(1/2)
```

Maxima [A]
time = 0.28, size = 27, normalized size = 0.73

$$\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}}+2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
[Out] 2/9*B*b*x^(9/2) + 2/5*(B*a + A*b)*x^(5/2) + 2*A*a*sqrt(x)
```

Fricas [A]
time = 0.84, size = 29, normalized size = 0.78

$$\frac{2}{45}\left(5\,Bbx^{4}+9\,(Ba+Ab)x^{2}+45\,Aa\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*B*b*x^4 + 9*(B*a + A*b)*x^2 + 45*A*a)*sqrt(x)
```

Sympy [A]
time = 0.14, size = 44, normalized size = 1.19

$$2Aa\sqrt{x}+\frac{2Abx^{\frac{5}{2}}}{5}+\frac{2Bax^{\frac{5}{2}}}{5}+\frac{2Bbx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a*sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(9/2)/9
```

Giac [A]
time = 1.20, size = 29, normalized size = 0.78

$$\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{5}\,Bax^{\frac{5}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}+2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) + 2*A*a*sqrt(x)
```

Mupad [B]
time = 0.10, size = 31, normalized size = 0.84

$$\frac{2\,\sqrt{x}\,\left(45\,A\,a+9\,A\,b\,x^2+9\,B\,a\,x^2+5\,B\,b\,x^4\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(45*A*a + 9*A*b*x^2 + 9*B*a*x^2 + 5*B*b*x^4))/45
```

3.348 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}} \, dx$

Optimal. Leaf size=37

$$-\frac{2aA}{\sqrt{x}}+\frac{2}{3}(Ab+aB)x^{3/2}+\frac{2}{7}bBx^{7/2}$$

```
[Out] 2/3*(A*b+B*a)*x^(3/2)+2/7*b*B*x^(7/2)-2*a*A/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{7}bBx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(7/2))/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}} \, dx &= \int \left(\frac{aA}{x^{3/2}}+(Ab+aB)\sqrt{x}+bBx^{5/2}\right) dx\\ &= -\frac{2aA}{\sqrt{x}}+\frac{2}{3}(Ab+aB)x^{3/2}+\frac{2}{7}bBx^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.95

$$\frac{2(-21aA+7Abx^2+7aBx^2+3bBx^4)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
[Out] (2*(-21*a*A + 7*A*b*x^2 + 7*a*B*x^2 + 3*b*B*x^4))/(21*Sqrt[x])
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{7}{2}}}{7}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{\sqrt{x}}$	30
default	$\frac{2bB\,x^{\frac{7}{2}}}{7}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{\sqrt{x}}$	30
gosper	$-\frac{2(-3bB\,x^{4}-7Ab\,x^{2}-7Ba\,x^{2}+21Aa)}{21\sqrt{x}}$	32
trager	$-\frac{2(-3bB\,x^{4}-7Ab\,x^{2}-7Ba\,x^{2}+21Aa)}{21\sqrt{x}}$	32
risch	$-\frac{2(-3bB\,x^{4}-7Ab\,x^{2}-7Ba\,x^{2}+21Aa)}{21\sqrt{x}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/7*b*B*x^(7/2)+2/3*A*b*x^(3/2)+2/3*B*a*x^(3/2)-2*a*A/x^(1/2)
```

Maxima [A]
time = 0.30, size = 27, normalized size = 0.73

$$\frac{2}{7}\,Bbx^{\frac{7}{2}}+\frac{2}{3}\,(Ba+Ab)x^{\frac{3}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
[Out] 2/7*B*b*x^(7/2) + 2/3*(B*a + A*b)*x^(3/2) - 2*A*a/sqrt(x)
```

Fricas [A]
time = 0.93, size = 29, normalized size = 0.78

$$\frac{2\,(3\,Bbx^4+7\,(Ba+Ab)x^2-21\,Aa)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
[Out] 2/21*(3*B*b*x^4 + 7*(B*a + A*b)*x^2 - 21*A*a)/sqrt(x)
```

Sympy [A]
time = 0.24, size = 44, normalized size = 1.19

$$-\frac{2Aa}{\sqrt{x}} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(3/2),x)
[Out] -2*A*a/sqrt(x) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(7/2)/7
```

Giac [A]
time = 1.36, size = 29, normalized size = 0.78

$$\frac{2}{7} Bbx^{\frac{7}{2}} + \frac{2}{3} Bax^{\frac{3}{2}} + \frac{2}{3} Abx^{\frac{3}{2}} - \frac{2 Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="giac")
[Out] 2/7*B*b*x^(7/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2*A*a/sqrt(x)
```

Mupad [B]
time = 0.10, size = 31, normalized size = 0.84

$$\frac{14\, A\, b\, x^2 - 42\, A\, a + 14\, B\, a\, x^2 + 6\, B\, b\, x^4}{21\, \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^(3/2),x)
[Out] (14*A*b*x^2 - 42*A*a + 14*B*a*x^2 + 6*B*b*x^4)/(21*x^(1/2))
```

3.349 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2aA}{3x^{3/2}}+2(Ab+aB)\sqrt{x}+\frac{2}{5}bBx^{5/2}$$

```
[Out] -2/3*a*A/x^(3/2)+2/5*b*B*x^(5/2)+2*(A*b+B*a)*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$2\sqrt{x}\,(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{5}bBx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a*A)/(3*x^(3/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(5/2))/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx &= \int \left(\frac{aA}{x^{5/2}}+\frac{Ab+aB}{\sqrt{x}}+bBx^{3/2}\right)dx\\ &= -\frac{2aA}{3x^{3/2}}+2(Ab+aB)\sqrt{x}+\frac{2}{5}bBx^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.95

$$-\frac{2(5aA-15Abx^2-15aBx^2-3bBx^4)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
[Out] (-2*(5*a*A - 15*A*b*x^2 - 15*a*B*x^2 - 3*b*B*x^4))/(15*x^(3/2))
```

Maple [A]
time = 0.05, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
default	$\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
gosper	$-\frac{2\left(-3bB\,x^{4}-15Ab\,x^{2}-15Ba\,x^{2}+5Aa\right)}{15x^{\frac{3}{2}}}$	32
trager	$-\frac{2\left(-3bB\,x^{4}-15Ab\,x^{2}-15Ba\,x^{2}+5Aa\right)}{15x^{\frac{3}{2}}}$	32
risch	$-\frac{2\left(-3bB\,x^{4}-15Ab\,x^{2}-15Ba\,x^{2}+5Aa\right)}{15x^{\frac{3}{2}}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/5*b*B*x^(5/2)+2*A*b*x^(1/2)+2*B*a*x^(1/2)-2/3*a*A/x^(3/2)
```

Maxima [A]
time = 0.30, size = 27, normalized size = 0.73

$$\frac{2}{5}\,Bbx^{\frac{5}{2}}+2\,(Ba+Ab)\sqrt{x}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
[Out] 2/5*B*b*x^(5/2) + 2*(B*a + A*b)*sqrt(x) - 2/3*A*a/x^(3/2)
```

Fricas [A]
time = 0.98, size = 29, normalized size = 0.78

$$\frac{2\,(3\,Bbx^4+15\,(Ba+Ab)x^2-5\,Aa)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*B*b*x^4 + 15*(B*a + A*b)*x^2 - 5*A*a)/x^(3/2)
```

Sympy [A]
time = 0.28, size = 42, normalized size = 1.14

$$-\frac{2Aa}{3x^{\frac{3}{2}}} + 2Ab\sqrt{x} + 2Ba\sqrt{x} + \frac{2Bbx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(5/2)/5
```

Giac [A]
time = 1.89, size = 29, normalized size = 0.78

$$\frac{2}{5}\,Bbx^{\frac{5}{2}} + 2\,Ba\sqrt{x} + 2\,Ab\sqrt{x} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*B*b*x^(5/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/3*A*a/x^(3/2)
```

Mupad [B]
time = 0.02, size = 31, normalized size = 0.84

$$\frac{30\,A\,b\,x^2 - 10\,A\,a + 30\,B\,a\,x^2 + 6\,B\,b\,x^4}{15\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^(5/2),x)
```

```
[Out] (30*A*b*x^2 - 10*A*a + 30*B*a*x^2 + 6*B*b*x^4)/(15*x^(3/2))
```

3.350 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2aA}{5x^{5/2}}-\frac{2(Ab+aB)}{\sqrt{x}}+\frac{2}{3}bBx^{3/2}$$

```
[Out] -2/5*a*A/x^(5/2)+2/3*b*B*x^(3/2)-2*(A*b+B*a)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {459}

$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{5x^{5/2}}+\frac{2}{3}bBx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/Sqrt[x] + (2*b*B*x^(3/2))/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx&=\int \left(\frac{aA}{x^{7/2}}+\frac{Ab+aB}{x^{3/2}}+bB\sqrt{x}\right)dx\\&=-\frac{2aA}{5x^{5/2}}-\frac{2(Ab+aB)}{\sqrt{x}}+\frac{2}{3}bBx^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.95

$$-\frac{2(3aA+15Abx^2+15aBx^2-5bBx^4)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

[Out] (-2*(3*a*A + 15*A*b*x^2 + 15*a*B*x^2 - 5*b*B*x^4))/(15*x^(5/2))

Maple [A]
time = 0.05, size = 28, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{2aA}{5x^{\frac{5}{2}}}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2(Ab+Ba)}{\sqrt{x}}$	28
default	$-\frac{2aA}{5x^{\frac{5}{2}}}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2(Ab+Ba)}{\sqrt{x}}$	28
gosper	$-\frac{2(-5bB\,x^{4}+15Ab\,x^{2}+15Ba\,x^{2}+3Aa)}{15x^{\frac{5}{2}}}$	32
trager	$-\frac{2(-5bB\,x^{4}+15Ab\,x^{2}+15Ba\,x^{2}+3Aa)}{15x^{\frac{5}{2}}}$	32
risch	$-\frac{2(-5bB\,x^{4}+15Ab\,x^{2}+15Ba\,x^{2}+3Aa)}{15x^{\frac{5}{2}}}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)*(B*x^2+A)/x^(7/2),x,method=_RETURNVERBOSE)

[Out] -2/5*a*A/x^(5/2)+2/3*b*B*x^(3/2)-2*(A*b+B*a)/x^(1/2)

Maxima [A]
time = 0.32, size = 29, normalized size = 0.78

$$\frac{2}{3}\,Bbx^{\frac{3}{2}}-\frac{2\,(5\,(Ba+Ab)x^2+Aa)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="maxima")

[Out] 2/3*B*b*x^(3/2) - 2/5*(5*(B*a + A*b)*x^2 + A*a)/x^(5/2)

Fricas [A]
time = 1.26, size = 29, normalized size = 0.78

$$\frac{2\,(5\,Bbx^4-15\,(Ba+Ab)x^2-3\,Aa)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="fricas")

[Out] 2/15*(5*B*b*x^4 - 15*(B*a + A*b)*x^2 - 3*A*a)/x^(5/2)

Sympy [A]
time = 0.35, size = 42, normalized size = 1.14

$$-\frac{2Aa}{5x^{\frac{5}{2}}}-\frac{2Ab}{\sqrt{x}}-\frac{2Ba}{\sqrt{x}}+\frac{2Bbx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a/(5*x**(5/2)) - 2*A*b/sqrt(x) - 2*B*a/sqrt(x) + 2*B*b*x**(3/2)/3
```

Giac [A]
time = 1.74, size = 31, normalized size = 0.84

$$\frac{2}{3}\,Bbx^{\frac{3}{2}} - \frac{2\,(5\,Bax^2 + 5\,Abx^2 + Aa)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*B*b*x^(3/2) - 2/5*(5*B*a*x^2 + 5*A*b*x^2 + A*a)/x^(5/2)
```

Mupad [B]
time = 0.02, size = 31, normalized size = 0.84

$$-\frac{6\,A\,a + 30\,A\,b\,x^2 + 30\,B\,a\,x^2 - 10\,B\,b\,x^4}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^(7/2),x)
```

```
[Out] -(6*A*a + 30*A*b*x^2 + 30*B*a*x^2 - 10*B*b*x^4)/(15*x^(5/2))
```

3.351 $\int x^{7/2}(a+bx^2)^2 (A+Bx^2)\,dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{21}b^2Bx^{21/2}$$

```
[Out] 2/9*a^2*A*x^(9/2)+2/13*a*(2*A*b+B*a)*x^(13/2)+2/17*b*(A*b+2*B*a)*x^(17/2)+2
/21*b^2*B*x^(21/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}(a+bx^2)^2 (A+Bx^2)\,dx &= \int \left(a^2Ax^{7/2}+a(2Ab+aB)x^{11/2}+b(Ab+2aB)x^{15/2}+b^2Bx^{19/2}\right)dx\\ &= \frac{2}{9}a^2Ax^{9/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{21}b^2Bx^{21/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.95

$$\frac{2x^{9/2}(119a^2(13A+9Bx^2)+126abx^2(17A+13Bx^2)+39b^2x^4(21A+17Bx^2))}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(9/2)*(119*a^2*(13*A + 9*B*x^2) + 126*a*b*x^2*(17*A + 13*B*x^2) + 39*b
^2*x^4*(21*A + 17*B*x^2)))/13923
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{21}{2}}}{21}+\frac{2(b^2 A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2 B)x^{\frac{13}{2}}}{13}+\frac{2a^2 A x^{\frac{9}{2}}}{9}$	52
default	$\frac{2b^2 B x^{\frac{21}{2}}}{21}+\frac{2(b^2 A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2 B)x^{\frac{13}{2}}}{13}+\frac{2a^2 A x^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}(663b^2 B x^6+819A b^2 x^4+1638Bab x^4+2142aAb x^2+1071B a^2 x^2+1547a^2 A)}{13923}$	56
trager	$\frac{2x^{\frac{9}{2}}(663b^2 B x^6+819A b^2 x^4+1638Bab x^4+2142aAb x^2+1071B a^2 x^2+1547a^2 A)}{13923}$	56
risch	$\frac{2x^{\frac{9}{2}}(663b^2 B x^6+819A b^2 x^4+1638Bab x^4+2142aAb x^2+1071B a^2 x^2+1547a^2 A)}{13923}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*b^2*B*x^(21/2)+2/17*(A*b^2+2*B*a*b)*x^(17/2)+2/13*(2*A*a*b+B*a^2)*x^(1
3/2)+2/9*a^2*A*x^(9/2)
```

Maxima [A]
time = 0.31, size = 51, normalized size = 0.81

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{2}{17}\,\left(2\,Bab+Ab^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}+\frac{2}{13}\,\left(Ba^2+2\,Aab\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/9*A*a^2*x^(9/2) +
 2/13*(B*a^2 + 2*A*a*b)*x^(13/2)
```

Fricas [A]
time = 1.04, size = 56, normalized size = 0.89

$$\frac{2}{13923}\,\left(663\,Bb^2x^{10}+819\,\left(2\,Bab+Ab^2\right)x^8+1547\,Aa^2x^4+1071\,\left(Ba^2+2\,Aab\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*B*b^2*x^10 + 819*(2*B*a*b + A*b^2)*x^8 + 1547*A*a^2*x^4 + 1071
*(B*a^2 + 2*A*a*b)*x^6)*sqrt(x)
```

Sympy [A]
time = 1.27, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(17/2)/17 + 2*B*a*
*2*x**(13/2)/13 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(21/2)/21
```

Giac [A]
time = 0.98, size = 53, normalized size = 0.84

$$\frac{2}{21}Bb^2x^{\frac{21}{2}}+\frac{4}{17}Babx^{\frac{17}{2}}+\frac{2}{17}Ab^2x^{\frac{17}{2}}+\frac{2}{13}Ba^2x^{\frac{13}{2}}+\frac{4}{13}Aabx^{\frac{13}{2}}+\frac{2}{9}Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/13*B*a^
2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/9*A*a^2*x^(9/2)
```

Mupad [B]
time = 0.11, size = 51, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,a^2}{13}+\frac{4\,A\,b\,a}{13}\right)+x^{17/2}\left(\frac{2\,A\,b^2}{17}+\frac{4\,B\,a\,b}{17}\right)+\frac{2\,A\,a^2\,x^{9/2}}{9}+\frac{2\,B\,b^2\,x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^(13/2)*((2*B*a^2)/13 + (4*A*a*b)/13) + x^(17/2)*((2*A*b^2)/17 + (4*B*a*b)
/17) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(21/2))/21
```

3.352 $\int x^{5/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{15}b(Ab+2aB)x^{15/2}+\frac{2}{19}b^2Bx^{19/2}$$

```
[Out] 2/7*a^2*A*x^(7/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/15*b*(A*b+2*B*a)*x^(15/2)+2
/19*b^2*B*x^(19/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx&=\int\left(a^2Ax^{5/2}+a(2Ab+aB)x^{9/2}+b(Ab+2aB)x^{13/2}+b^2Bx^{17/2}\right)\,dx\\&=\frac{2}{7}a^2Ax^{7/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{15}b(Ab+2aB)x^{15/2}+\frac{2}{19}b^2Bx^{19/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.95

$$\frac{2x^{7/2}\left(285a^2\left(11A+7Bx^2\right)+266abx^2\left(15A+11Bx^2\right)+77b^2x^4\left(19A+15Bx^2\right)\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(7/2)*(285*a^2*(11*A + 7*B*x^2) + 266*a*b*x^2*(15*A + 11*B*x^2) + 77*b
^2*x^4*(19*A + 15*B*x^2)))/21945
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{19}{2}}}{19}+\frac{2(b^2A+2abB)x^{\frac{15}{2}}}{15}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}+\frac{2a^2Ax^{\frac{7}{2}}}{7}$	52
default	$\frac{2b^2Bx^{\frac{19}{2}}}{19}+\frac{2(b^2A+2abB)x^{\frac{15}{2}}}{15}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}+\frac{2a^2Ax^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}(1155b^2Bx^6+1463Ab^2x^4+2926Babx^4+3990aAbx^2+1995Ba^2x^2+3135a^2A)}{21945}$	56
trager	$\frac{2x^{\frac{7}{2}}(1155b^2Bx^6+1463Ab^2x^4+2926Babx^4+3990aAbx^2+1995Ba^2x^2+3135a^2A)}{21945}$	56
risch	$\frac{2x^{\frac{7}{2}}(1155b^2Bx^6+1463Ab^2x^4+2926Babx^4+3990aAbx^2+1995Ba^2x^2+3135a^2A)}{21945}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*b^2*B*x^(19/2)+2/15*(A*b^2+2*B*a*b)*x^(15/2)+2/11*(2*A*a*b+B*a^2)*x^(1
1/2)+2/7*a^2*A*x^(7/2)
```

Maxima [A]
time = 0.32, size = 51, normalized size = 0.81

$$\frac{2}{19}Bb^2x^{\frac{19}{2}}+\frac{2}{15}\left(2Bab+Ab^2\right)x^{\frac{15}{2}}+\frac{2}{7}Aa^2x^{\frac{7}{2}}+\frac{2}{11}\left(Ba^2+2Aab\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/7*A*a^2*x^(7/2) +
 2/11*(B*a^2 + 2*A*a*b)*x^(11/2)
```

Fricas [A]
time = 0.76, size = 56, normalized size = 0.89

$$\frac{2}{21945}\left(1155Bb^2x^9+1463\left(2Bab+Ab^2\right)x^7+3135Aa^2x^3+1995\left(Ba^2+2Aab\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*b^2*x^9 + 1463*(2*B*a*b + A*b^2)*x^7 + 3135*A*a^2*x^3 + 199
5*(B*a^2 + 2*A*a*b)*x^5)*sqrt(x)
```

Sympy [A]
time = 0.79, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(15/2)/15 + 2*B*a*
*2*x**(11/2)/11 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(19/2)/19
```

Giac [A]
time = 1.16, size = 53, normalized size = 0.84

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{4}{15}\,Babx^{\frac{15}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}+\frac{2}{11}\,Ba^2x^{\frac{11}{2}}+\frac{4}{11}\,Aabx^{\frac{11}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/11*B*a^
2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/7*A*a^2*x^(7/2)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{15/2}\left(\frac{2\,A\,b^2}{15}+\frac{4\,B\,a\,b}{15}\right)+\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{2\,B\,b^2\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(15/2)*((2*A*b^2)/15 + (4*B*a*b)
/15) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(19/2))/19
```

3.353 $\int x^{3/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{17}b^2Bx^{17/2}$$

```
[Out] 2/5*a^2*A*x^(5/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/13*b*(A*b+2*B*a)*x^(13/2)+2/1
7*b^2*B*x^(17/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(13/2))/13 + (2*b^2*B*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx&=\int\left(a^2Ax^{3/2}+a(2Ab+aB)x^{7/2}+b(Ab+2aB)x^{11/2}+b^2Bx^{15/2}\right)\,dx\\&=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{17}b^2Bx^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.95

$$\frac{2x^{5/2}\left(221a^2\left(9A+5Bx^2\right)+170abx^2\left(13A+9Bx^2\right)+45b^2x^4\left(17A+13Bx^2\right)\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(5/2)*(221*a^2*(9*A + 5*B*x^2) + 170*a*b*x^2*(13*A + 9*B*x^2) + 45*b^2
*x^4*(17*A + 13*B*x^2)))/9945
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{17}{2}}}{17}+\frac{2(b^2A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2B)x^{\frac{9}{2}}}{9}+\frac{2a^2Ax^{\frac{5}{2}}}{5}$	52
default	$\frac{2b^2Bx^{\frac{17}{2}}}{17}+\frac{2(b^2A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2B)x^{\frac{9}{2}}}{9}+\frac{2a^2Ax^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(585b^2Bx^6+765Ab^2x^4+1530Babx^4+2210aAbx^2+1105Ba^2x^2+1989a^2A)}{9945}$	56
trager	$\frac{2x^{\frac{5}{2}}(585b^2Bx^6+765Ab^2x^4+1530Babx^4+2210aAbx^2+1105Ba^2x^2+1989a^2A)}{9945}$	56
risch	$\frac{2x^{\frac{5}{2}}(585b^2Bx^6+765Ab^2x^4+1530Babx^4+2210aAbx^2+1105Ba^2x^2+1989a^2A)}{9945}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*b^2*B*x^(17/2)+2/13*(A*b^2+2*B*a*b)*x^(13/2)+2/9*(2*A*a*b+B*a^2)*x^(9/
2)+2/5*a^2*A*x^(5/2)
```

Maxima [A]
time = 0.30, size = 51, normalized size = 0.81

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{2}{13}\,\big(2\,Bab+Ab^2\big)x^{\frac{13}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}+\frac{2}{9}\,\big(Ba^2+2\,Aab\big)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/5*A*a^2*x^(5/2) +
 2/9*(B*a^2 + 2*A*a*b)*x^(9/2)
```

Fricas [A]
time = 1.05, size = 56, normalized size = 0.89

$$\frac{2}{9945}\,\big(585\,Bb^2x^8+765\,\big(2\,Bab+Ab^2\big)x^6+1989\,Aa^2x^2+1105\,\big(Ba^2+2\,Aab\big)x^4\big)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*B*b^2*x^8 + 765*(2*B*a*b + A*b^2)*x^6 + 1989*A*a^2*x^2 + 1105*(
B*a^2 + 2*A*a*b)*x^4)*sqrt(x)
```

Sympy [A]
time = 0.52, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{5}{2}}}{5} + \frac{4Aabx^{\frac{9}{2}}}{9} + \frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{2Ba^2x^{\frac{9}{2}}}{9} + \frac{4Babx^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2
*x**(9/2)/9 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(17/2)/17
```

Giac [A]
time = 0.86, size = 53, normalized size = 0.84

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}} + \frac{4}{13}\,Babx^{\frac{13}{2}} + \frac{2}{13}\,Ab^2x^{\frac{13}{2}} + \frac{2}{9}\,Ba^2x^{\frac{9}{2}} + \frac{4}{9}\,Aabx^{\frac{9}{2}} + \frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/9*B*a^2
*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/5*A*a^2*x^(5/2)
```

Mupad [B]
time = 0.02, size = 51, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,B\,a^2}{9}+\frac{4\,A\,b\,a}{9}\right)+x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b)/13
) + (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(17/2))/17
```

3.354 $\int \sqrt{x} \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{15}b^2Bx^{15/2}$$

```
[Out] 2/3*a^2*A*x^(3/2)+2/7*a*(2*A*b+B*a)*x^(7/2)+2/11*b*(A*b+2*B*a)*x^(11/2)+2/1
5*b^2*B*x^(15/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^
(11/2))/11 + (2*b^2*B*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx&=\int \left(a^2A\sqrt{x}+a(2Ab+aB)x^{5/2}+b(Ab+2aB)x^{9/2}+b^2Bx^{13/2}\right)\,dx\\&=\frac{2}{3}a^2Ax^{3/2}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{15}b^2Bx^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.94

$$\frac{2x^{3/2}(385a^2A+330aAbx^2+165a^2Bx^2+105Ab^2x^4+210abBx^4+77b^2Bx^6)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(385*a^2*A + 330*a*A*b*x^2 + 165*a^2*B*x^2 + 105*A*b^2*x^4 + 210
*a*b*B*x^4 + 77*b^2*B*x^6))/1155
```

Maple [A]
time = 0.09, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{15}{2}}}{15}+\frac{2(b^2A+2abB)x^{\frac{11}{2}}}{11}+\frac{2(2abA+a^2B)x^{\frac{7}{2}}}{7}+\frac{2a^2A x^{\frac{3}{2}}}{3}$	52
default	$\frac{2b^2 B x^{\frac{15}{2}}}{15}+\frac{2(b^2A+2abB)x^{\frac{11}{2}}}{11}+\frac{2(2abA+a^2B)x^{\frac{7}{2}}}{7}+\frac{2a^2A x^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}(77b^2B x^6+105A b^2x^4+210Bab x^4+330aAb x^2+165B a^2x^2+385a^2A)}{1155}$	56
trager	$\frac{2x^{\frac{3}{2}}(77b^2B x^6+105A b^2x^4+210Bab x^4+330aAb x^2+165B a^2x^2+385a^2A)}{1155}$	56
risch	$\frac{2x^{\frac{3}{2}}(77b^2B x^6+105A b^2x^4+210Bab x^4+330aAb x^2+165B a^2x^2+385a^2A)}{1155}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*b^2*B*x^(15/2)+2/11*(A*b^2+2*B*a*b)*x^(11/2)+2/7*(2*A*a*b+B*a^2)*x^(7/
2)+2/3*a^2*A*x^(3/2)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.81

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{2}{11}\,\left(2\,Bab+Ab^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}+\frac{2}{7}\,\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/3*A*a^2*x^(3/2) +
 2/7*(B*a^2 + 2*A*a*b)*x^(7/2)
```

Fricas [A]
time = 0.69, size = 54, normalized size = 0.86

$$\frac{2}{1155}\,\left(77\,Bb^2x^7+105\,\left(2\,Bab+Ab^2\right)x^5+385\,Aa^2x+165\,\left(Ba^2+2\,Aab\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b^2*x^7 + 105*(2*B*a*b + A*b^2)*x^5 + 385*A*a^2*x + 165*(B*a^2
 + 2*A*a*b)*x^3)*sqrt(x)
```

Sympy [A]
time = 1.36, size = 66, normalized size = 1.05

$$\frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{2Bb^2x^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}\left(Ab^2+2Bab\right)}{11}+\frac{2x^{\frac{7}{2}}\cdot\left(2Aab+Ba^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 2*B*b**2*x**(15/2)/15 + 2*x**(11/2)*(A*b**2 + 2*B*a*b
)/11 + 2*x**(7/2)*(2*A*a*b + B*a**2)/7
```

Giac [A]
time = 0.96, size = 53, normalized size = 0.84

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{4}{11}\,Babx^{\frac{11}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{7}\,Ba^2x^{\frac{7}{2}}+\frac{4}{7}\,Aabx^{\frac{7}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/7*B*a^2
*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2/3*A*a^2*x^(3/2)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.81

$$x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right)+x^{11/2}\left(\frac{2\,A\,b^2}{11}+\frac{4\,B\,a\,b}{11}\right)+\frac{2\,A\,a^2\,x^{3/2}}{3}+\frac{2\,B\,b^2\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/11
) + (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(15/2))/15
```

3.355 $\int \frac{(a+bx^2)^2(A+Bx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$2a^2A\sqrt{x}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{13}b^2Bx^{13/2}$$

```
[Out] 2/5*a*(2*A*b+B*a)*x^(5/2)+2/9*b*(A*b+2*B*a)*x^(9/2)+2/13*b^2*B*x^(13/2)+2*a
^2*A*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$2a^2A\sqrt{x}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(9/2
))/9 + (2*b^2*B*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{\sqrt{x}}\,dx &= \int \left(\frac{a^2A}{\sqrt{x}}+a(2Ab+aB)x^{3/2}+b(Ab+2aB)x^{7/2}+b^2Bx^{11/2}\right)dx\\ &= 2a^2A\sqrt{x}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{13}b^2Bx^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.97

$$\frac{2}{585}\sqrt{x}\left(117a^2(5A+Bx^2)+26abx^2(9A+5Bx^2)+5b^2x^4(13A+9Bx^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(117*a^2*(5*A + B*x^2) + 26*a*b*x^2*(9*A + 5*B*x^2) + 5*b^2*x^4*
(13*A + 9*B*x^2)))/585
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.85

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{13}{2}}}{13}+\frac{2(b^2 A+2abB)x^{\frac{9}{2}}}{9}+\frac{2(2abA+a^2 B)x^{\frac{5}{2}}}{5}+2a^2 A\sqrt{x}$	52
default	$\frac{2b^2 B x^{\frac{13}{2}}}{13}+\frac{2(b^2 A+2abB)x^{\frac{9}{2}}}{9}+\frac{2(2abA+a^2 B)x^{\frac{5}{2}}}{5}+2a^2 A\sqrt{x}$	52
trager	$\left(\frac{2}{13}b^2 B x^6+\frac{2}{9}A b^2 x^4+\frac{4}{9}Bab x^4+\frac{4}{5}aAb x^2+\frac{2}{5}B a^2 x^2+2a^2 A\right)\sqrt{x}$	55
gosper	$\frac{2\sqrt{x}\,(45b^2 B x^6+65A b^2 x^4+130Bab x^4+234aAb x^2+117B a^2 x^2+585a^2 A)}{585}$	56
risch	$\frac{2\sqrt{x}\,(45b^2 B x^6+65A b^2 x^4+130Bab x^4+234aAb x^2+117B a^2 x^2+585a^2 A)}{585}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*b^2*B*x^(13/2)+2/9*(A*b^2+2*B*a*b)*x^(9/2)+2/5*(2*A*a*b+B*a^2)*x^(5/2)
+2*a^2*A*x^(1/2)
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.84

$$\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{2}{9}\left(2\,Bab+Ab^2\right)x^{\frac{9}{2}}+2\,Aa^2\sqrt{x}+\frac{2}{5}\left(Ba^2+2\,Aab\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2*A*a^2*sqrt(x) + 2/5
*(B*a^2 + 2*A*a*b)*x^(5/2)
```

Fricas [A]
time = 0.67, size = 53, normalized size = 0.87

$$\frac{2}{585}\left(45\,Bb^2x^6+65\left(2\,Bab+Ab^2\right)x^4+585\,Aa^2+117\left(Ba^2+2\,Aab\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b^2*x^6 + 65*(2*B*a*b + A*b^2)*x^4 + 585*A*a^2 + 117*(B*a^2 + 2
*A*a*b)*x^2)*sqrt(x)
```

Sympy [A]
time = 0.30, size = 78, normalized size = 1.28

$$2Aa^2\sqrt{x} + \frac{4Aabx^{\frac{5}{2}}}{5} + \frac{2Ab^2x^{\frac{9}{2}}}{9} + \frac{2Ba^2x^{\frac{5}{2}}}{5} + \frac{4Babx^{\frac{9}{2}}}{9} + \frac{2Bb^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**2*sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(9/2)/9 + 2*B*a**2*x**(
5/2)/5 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(13/2)/13
```

Giac [A]
time = 1.20, size = 53, normalized size = 0.87

$$\frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{4}{9}\,Babx^{\frac{9}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}} + \frac{2}{5}\,Ba^2x^{\frac{5}{2}} + \frac{4}{5}\,Aabx^{\frac{5}{2}} + 2\,Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/5*B*a^2*x^(
5/2) + 4/5*A*a*b*x^(5/2) + 2*A*a^2*sqrt(x)
```

Mupad [B]
time = 0.02, size = 51, normalized size = 0.84

$$x^{5/2}\,\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right)+x^{9/2}\,\left(\frac{2\,A\,b^2}{9}+\frac{4\,B\,a\,b}{9}\right)+2\,A\,a^2\,\sqrt{x}+\frac{2\,B\,b^2\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^(1/2),x)
```

```
[Out] x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9) +
 2*A*a^2*x^(1/2) + (2*B*b^2*x^(13/2))/13
```

3.356 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{11}b^2Bx^{11/2}$$

```
[Out] 2/3*a*(2*A*b+B*a)*x^(3/2)+2/7*b*(A*b+2*B*a)*x^(7/2)+2/11*b^2*B*x^(11/2)-2*a
^2*A/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(11/2))/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{3/2}}\,dx &= \int \left(\frac{a^2A}{x^{3/2}}+a(2Ab+aB)\sqrt{x}+b(Ab+2aB)x^{5/2}+b^2Bx^{9/2}\right)dx\\ &= -\frac{2a^2A}{\sqrt{x}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{11}b^2Bx^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.97

$$-\frac{2(231a^2A-154aAbx^2-77a^2Bx^2-33Ab^2x^4-66abBx^4-21b^2Bx^6)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*(231*a^2*A - 154*a*A*b*x^2 - 77*a^2*B*x^2 - 33*A*b^2*x^4 - 66*a*b*B*x^4
 - 21*b^2*B*x^6))/(231*Sqrt[x])
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2b^{2} B \,x^{\frac{11}{2}}}{11}+\frac{2 A \,b^{2} x^{\frac{7}{2}}}{7}+\frac{4 B a b \,x^{\frac{7}{2}}}{7}+\frac{4 A a b \,x^{\frac{3}{2}}}{3}+\frac{2 B \,a^{2} x^{\frac{3}{2}}}{3}-\frac{2 a^{2} A}{\sqrt{x}}$	54
default	$\frac{2b^{2} B \,x^{\frac{11}{2}}}{11}+\frac{2 A \,b^{2} x^{\frac{7}{2}}}{7}+\frac{4 B a b \,x^{\frac{7}{2}}}{7}+\frac{4 A a b \,x^{\frac{3}{2}}}{3}+\frac{2 B \,a^{2} x^{\frac{3}{2}}}{3}-\frac{2 a^{2} A}{\sqrt{x}}$	54
gosper	$-\frac{2\left(-21 b^{2} B \,x^{6}-33 A \,b^{2} x^{4}-66 B a b \,x^{4}-154 a A b \,x^{2}-77 B \,a^{2} x^{2}+231 a^{2} A \right)}{231 \sqrt{x}}$	56
trager	$-\frac{2\left(-21 b^{2} B \,x^{6}-33 A \,b^{2} x^{4}-66 B a b \,x^{4}-154 a A b \,x^{2}-77 B \,a^{2} x^{2}+231 a^{2} A \right)}{231 \sqrt{x}}$	56
risch	$-\frac{2\left(-21 b^{2} B \,x^{6}-33 A \,b^{2} x^{4}-66 B a b \,x^{4}-154 a A b \,x^{2}-77 B \,a^{2} x^{2}+231 a^{2} A \right)}{231 \sqrt{x}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*b^2*B*x^(11/2)+2/7*A*b^2*x^(7/2)+4/7*B*a*b*x^(7/2)+4/3*A*a*b*x^(3/2)+2
/3*B*a^2*x^(3/2)-2*a^2*A/x^(1/2)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.84

$$\frac{2}{11}\,Bb^{2}x^{\frac{11}{2}}+\frac{2}{7}\left(2\,Bab+Ab^{2}\right)x^{\frac{7}{2}}-\frac{2\,Aa^{2}}{\sqrt{x}}+\frac{2}{3}\left(Ba^{2}+2\,Aab\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b^2*x^(11/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) - 2*A*a^2/sqrt(x) + 2/3
*(B*a^2 + 2*A*a*b)*x^(3/2)
```

Fricas [A]
time = 0.87, size = 53, normalized size = 0.87

$$\frac{2\,\left(21\,Bb^{2}x^{6}+33\,\left(2\,Bab+Ab^{2}\right)x^{4}-231\,Aa^{2}+77\,\left(Ba^{2}+2\,Aab\right)x^{2}\right)}{231\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*B*b^2*x^6 + 33*(2*B*a*b + A*b^2)*x^4 - 231*A*a^2 + 77*(B*a^2 + 2*
A*a*b)*x^2)/sqrt(x)
```

Sympy [A]
time = 0.39, size = 78, normalized size = 1.28

$$-\frac{2Aa^2}{\sqrt{x}} + \frac{4Aabx^{\frac{3}{2}}}{3} + \frac{2Ab^2x^{\frac{7}{2}}}{7} + \frac{2Ba^2x^{\frac{3}{2}}}{3} + \frac{4Babx^{\frac{7}{2}}}{7} + \frac{2Bb^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**2/sqrt(x) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*x**
(3/2)/3 + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(11/2)/11
```

Giac [A]
time = 0.86, size = 53, normalized size = 0.87

$$\frac{2}{11}Bb^2x^{\frac{11}{2}} + \frac{4}{7}Babx^{\frac{7}{2}} + \frac{2}{7}Ab^2x^{\frac{7}{2}} + \frac{2}{3}Ba^2x^{\frac{3}{2}} + \frac{4}{3}Aabx^{\frac{3}{2}} - \frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b^2*x^(11/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2/3*B*a^2*x^(
3/2) + 4/3*A*a*b*x^(3/2) - 2*A*a^2/sqrt(x)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.84

$$x^{3/2}\left(\frac{2\,B\,a^2}{3} + \frac{4\,A\,b\,a}{3}\right) + x^{7/2}\left(\frac{2\,A\,b^2}{7} + \frac{4\,B\,a\,b}{7}\right) - \frac{2\,A\,a^2}{\sqrt{x}} + \frac{2\,B\,b^2\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^(3/2),x)
```

```
[Out] x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7) -
 (2*A*a^2)/x^(1/2) + (2*B*b^2*x^(11/2))/11
```

3.357 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{3x^{3/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{5}b(Ab+2aB)x^{5/2}+\frac{2}{9}b^2Bx^{9/2}$$

```
[Out] -2/3*a^2*A/x^(3/2)+2/5*b*(A*b+2*B*a)*x^(5/2)+2/9*b^2*B*x^(9/2)+2*a*(2*A*b+B
*a)*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2aB+Ab)+2a\sqrt{x}\,(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
5/2))/5 + (2*b^2*B*x^(9/2))/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{5/2}}\,dx &= \int \left(\frac{a^2A}{x^{5/2}}+\frac{a(2Ab+aB)}{\sqrt{x}}+b(Ab+2aB)x^{3/2}+b^2Bx^{7/2}\right)dx\\ &= -\frac{2a^2A}{3x^{3/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{5}b(Ab+2aB)x^{5/2}+\frac{2}{9}b^2Bx^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.97

$$-\frac{2(15a^2A-90aAbx^2-45a^2Bx^2-9Ab^2x^4-18abBx^4-5b^2Bx^6)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*(15*a^2*A - 90*a*A*b*x^2 - 45*a^2*B*x^2 - 9*A*b^2*x^4 - 18*a*b*B*x^4 -
5*b^2*B*x^6))/(45*x^(3/2))
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2b^2 B\,x^{\frac{9}{2}}}{9}+\frac{2A\,b^2 x^{\frac{5}{2}}}{5}+\frac{4Bab\,x^{\frac{5}{2}}}{5}+4abA\sqrt{x}+2a^2 B\sqrt{x}-\frac{2a^2 A}{3x^{\frac{3}{2}}}$	54
default	$\frac{2b^2 B\,x^{\frac{9}{2}}}{9}+\frac{2A\,b^2 x^{\frac{5}{2}}}{5}+\frac{4Bab\,x^{\frac{5}{2}}}{5}+4abA\sqrt{x}+2a^2 B\sqrt{x}-\frac{2a^2 A}{3x^{\frac{3}{2}}}$	54
gosper	$-\frac{2\left(-5b^2 B\,x^6-9A\,b^2 x^4-18Bab\,x^4-90aAb\,x^2-45B\,a^2 x^2+15a^2 A\right)}{45x^{\frac{3}{2}}}$	56
trager	$-\frac{2\left(-5b^2 B\,x^6-9A\,b^2 x^4-18Bab\,x^4-90aAb\,x^2-45B\,a^2 x^2+15a^2 A\right)}{45x^{\frac{3}{2}}}$	56
risch	$-\frac{2\left(-5b^2 B\,x^6-9A\,b^2 x^4-18Bab\,x^4-90aAb\,x^2-45B\,a^2 x^2+15a^2 A\right)}{45x^{\frac{3}{2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*b^2*B*x^(9/2)+2/5*A*b^2*x^(5/2)+4/5*B*a*b*x^(5/2)+4*a*b*A*x^(1/2)+2*a^2
*B*x^(1/2)-2/3*a^2*A/x^(3/2)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.84

$$\frac{2}{9}\,Bb^2x^{\frac{9}{2}}+\frac{2}{5}\left(2\,Bab+Ab^2\right)x^{\frac{5}{2}}-\frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}+2\left(Ba^2+2\,Aab\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*b^2*x^(9/2) + 2/5*(2*B*a*b + A*b^2)*x^(5/2) - 2/3*A*a^2/x^(3/2) + 2*(
B*a^2 + 2*A*a*b)*sqrt(x)
```

Fricas [A]
time = 0.92, size = 53, normalized size = 0.87

$$\frac{2\left(5\,Bb^2x^6+9\left(2\,Bab+Ab^2\right)x^4-15\,Aa^2+45\left(Ba^2+2\,Aab\right)x^2\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*B*b^2*x^6 + 9*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 45*(B*a^2 + 2*A*a*
b)*x^2)/x^(3/2)
```

Sympy [A]
time = 0.46, size = 76, normalized size = 1.25

$$-\frac{2Aa^2}{3x^{\frac{3}{2}}}+4Aab\sqrt{x}+\frac{2Ab^2x^{\frac{5}{2}}}{5}+2Ba^2\sqrt{x}+\frac{4Babx^{\frac{5}{2}}}{5}+\frac{2Bb^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(5/2)/5 + 2*B*a**2*s
qrt(x) + 4*B*a*b*x**(5/2)/5 + 2*B*b**2*x**(9/2)/9
```

Giac [A]
time = 0.82, size = 53, normalized size = 0.87

$$\frac{2}{9}Bb^2x^{\frac{9}{2}}+\frac{4}{5}Babx^{\frac{5}{2}}+\frac{2}{5}Ab^2x^{\frac{5}{2}}+2Ba^2\sqrt{x}+4Aab\sqrt{x}-\frac{2Aa^2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b^2*x^(9/2) + 4/5*B*a*b*x^(5/2) + 2/5*A*b^2*x^(5/2) + 2*B*a^2*sqrt(x)
 + 4*A*a*b*sqrt(x) - 2/3*A*a^2/x^(3/2)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.84

$$\sqrt{x}\,\left(2\,B\,a^2+4\,A\,b\,a\right)+x^{5/2}\,\left(\frac{2\,A\,b^2}{5}+\frac{4\,B\,a\,b}{5}\right)-\frac{2\,A\,a^2}{3\,x^{3/2}}+\frac{2\,B\,b^2\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^(5/2),x)
```

```
[Out] x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(5/2)*((2*A*b^2)/5 + (4*B*a*b)/5) - (2*A*a^
2)/(3*x^(3/2)) + (2*B*b^2*x^(9/2))/9
```

3.358 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{5x^{5/2}}-\frac{2a(2Ab+aB)}{\sqrt{x}}+\frac{2}{3}b(Ab+2aB)x^{3/2}+\frac{2}{7}b^2Bx^{7/2}$$

```
[Out] -2/5*a^2*A/x^(5/2)+2/3*b*(A*b+2*B*a)*x^(3/2)+2/7*b^2*B*x^(7/2)-2*a*(2*A*b+B
*a)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2aB+Ab)-\frac{2a(aB+2Ab)}{\sqrt{x}}+\frac{2}{7}b^2Bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + (2*b*(A*b + 2*a*B)*x
^(3/2))/3 + (2*b^2*B*x^(7/2))/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx &= \int \left(\frac{a^2A}{x^{7/2}}+\frac{a(2Ab+aB)}{x^{3/2}}+b(Ab+2aB)\sqrt{x}+b^2Bx^{5/2}\right)dx\\ &= -\frac{2a^2A}{5x^{5/2}}-\frac{2a(2Ab+aB)}{\sqrt{x}}+\frac{2}{3}b(Ab+2aB)x^{3/2}+\frac{2}{7}b^2Bx^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.97

$$-\frac{2(21a^2A+210aAbx^2+105a^2Bx^2-35Ab^2x^4-70abBx^4-15b^2Bx^6)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*(21*a^2*A + 210*a*A*b*x^2 + 105*a^2*B*x^2 - 35*A*b^2*x^4 - 70*a*b*B*x^4
 - 15*b^2*B*x^6))/(105*x^(5/2))
```

Maple [A]
time = 0.08, size = 51, normalized size = 0.84

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{3}{2}}}{3}-\frac{2a^2A}{5x^{\frac{5}{2}}}-\frac{2a(2Ab+Ba)}{\sqrt{x}}$	51
default	$\frac{2b^2Bx^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{3}{2}}}{3}-\frac{2a^2A}{5x^{\frac{5}{2}}}-\frac{2a(2Ab+Ba)}{\sqrt{x}}$	51
gosper	$-\frac{2(-15b^2Bx^6-35Ab^2x^4-70Babx^4+210aAbx^2+105Ba^2x^2+21a^2A)}{105x^{\frac{5}{2}}}$	56
trager	$-\frac{2(-15b^2Bx^6-35Ab^2x^4-70Babx^4+210aAbx^2+105Ba^2x^2+21a^2A)}{105x^{\frac{5}{2}}}$	56
risch	$-\frac{2(-15b^2Bx^6-35Ab^2x^4-70Babx^4+210aAbx^2+105Ba^2x^2+21a^2A)}{105x^{\frac{5}{2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*b^2*B*x^(7/2)+2/3*A*b^2*x^(3/2)+4/3*B*a*b*x^(3/2)-2/5*a^2*A/x^(5/2)-2*a
*(2*A*b+B*a)/x^(1/2)
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.87

$$\frac{2}{7}\,Bb^2x^{\frac{7}{2}}+\frac{2}{3}\,\left(2\,Bab+Ab^2\right)x^{\frac{3}{2}}-\frac{2\,\left(Aa^2+5\,\left(Ba^2+2\,Aab\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*B*b^2*x^(7/2) + 2/3*(2*B*a*b + A*b^2)*x^(3/2) - 2/5*(A*a^2 + 5*(B*a^2 +
 2*A*a*b)*x^2)/x^(5/2)
```

Fricas [A]
time = 1.07, size = 53, normalized size = 0.87

$$\frac{2\,(15\,Bb^2x^6+35\,(2\,Bab+Ab^2)x^4-21\,Aa^2-105\,(Ba^2+2\,Aab)x^2)}{105\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 - 21*A*a^2 - 105*(B*a^2 + 2*
A*a*b)*x^2)/x^(5/2)
```

Sympy [A]
time = 0.60, size = 76, normalized size = 1.25

$$-\frac{2Aa^2}{5x^{\frac{5}{2}}}-\frac{4Aab}{\sqrt{x}}+\frac{2Ab^2x^{\frac{3}{2}}}{3}-\frac{2Ba^2}{\sqrt{x}}+\frac{4Babx^{\frac{3}{2}}}{3}+\frac{2Bb^2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) - 4*A*a*b/sqrt(x) + 2*A*b**2*x**(3/2)/3 - 2*B*a**2/s
qrt(x) + 4*B*a*b*x**(3/2)/3 + 2*B*b**2*x**(7/2)/7
```

Giac [A]
time = 0.87, size = 55, normalized size = 0.90

$$\frac{2}{7}\,Bb^2x^{\frac{7}{2}}+\frac{4}{3}\,Babx^{\frac{3}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}-\frac{2\,(5\,Ba^2x^2+10\,Aabx^2+Aa^2)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*B*b^2*x^(7/2) + 4/3*B*a*b*x^(3/2) + 2/3*A*b^2*x^(3/2) - 2/5*(5*B*a^2*x^
2 + 10*A*a*b*x^2 + A*a^2)/x^(5/2)
```

Mupad [B]
time = 0.12, size = 55, normalized size = 0.90

$$-\frac{210\,B\,a^2\,x^2+42\,A\,a^2-140\,B\,a\,b\,x^4+420\,A\,a\,b\,x^2-30\,B\,b^2\,x^6-70\,A\,b^2\,x^4}{105\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^(7/2),x)
```

```
[Out] -(42*A*a^2 + 210*B*a^2*x^2 - 70*A*b^2*x^4 - 30*B*b^2*x^6 + 420*A*a*b*x^2 -
140*B*a*b*x^4)/(105*x^(5/2))
```

3.359 $\int x^{7/2}(a+bx^2)^3 (A+Bx^2) \, dx$

Optimal. Leaf size=85

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{21}b^2(Ab+3aB)x^{21/2}+\frac{2}{25}b^3Bx^{25/2}$$

```
[Out] 2/9*a^3*A*x^(9/2)+2/13*a^2*(3*A*b+B*a)*x^(13/2)+6/17*a*b*(A*b+B*a)*x^(17/2)
+2/21*b^2*(A*b+3*B*a)*x^(21/2)+2/25*b^3*B*x^(25/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}(a+bx^2)^3 (A+Bx^2) \, dx &= \int (a^3Ax^{7/2}+a^2(3Ab+aB)x^{11/2}+3ab(Ab+aB)x^{15/2}+b^2(Ab+3aB)x\\ &= \frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{21}b^2(Ab+3\end{aligned}$$

Mathematica [A]
time = 0.05, size = 81, normalized size = 0.95

$$\frac{2x^{9/2}(2975a^3(13A+9Bx^2)+4725a^2bx^2(17A+13Bx^2)+2925ab^2x^4(21A+17Bx^2)+663b^3x^6(25A+21Bx^2))}{348075}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*x^(9/2)*(2975*a^3*(13*A + 9*B*x^2) + 4725*a^2*b*x^2*(17*A + 13*B*x^2) +
2925*a*b^2*x^4*(21*A + 17*B*x^2) + 663*b^3*x^6*(25*A + 21*B*x^2)))/348075
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{25}{2}}}{25}+\frac{2(A b^3+3B a b^2) x^{\frac{21}{2}}}{21}+\frac{2(3A a b^2+3B a^2 b) x^{\frac{17}{2}}}{17}+\frac{2(3A a^2 b+B a^3) x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{9}{2}}}{9}$
default	$\frac{2b^3 B x^{\frac{25}{2}}}{25}+\frac{2(A b^3+3B a b^2) x^{\frac{21}{2}}}{21}+\frac{2(3A a b^2+3B a^2 b) x^{\frac{17}{2}}}{17}+\frac{2(3A a^2 b+B a^3) x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{9}{2}}}{9}$
gosper	$\frac{2x^{\frac{9}{2}}(13923B b^3 x^8+16575x^6 A b^3+49725x^6 B a b^2+61425A a b^2 x^4+61425x^4 B a^2 b+80325x^2 A a^2 b+26775B a^3 x^2+38675\ldots}{348075}$
trager	$\frac{2x^{\frac{9}{2}}(13923B b^3 x^8+16575x^6 A b^3+49725x^6 B a b^2+61425A a b^2 x^4+61425x^4 B a^2 b+80325x^2 A a^2 b+26775B a^3 x^2+38675\ldots}{348075}$
risch	$\frac{2x^{\frac{9}{2}}(13923B b^3 x^8+16575x^6 A b^3+49725x^6 B a b^2+61425A a b^2 x^4+61425x^4 B a^2 b+80325x^2 A a^2 b+26775B a^3 x^2+38675\ldots}{348075}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*b^3*B*x^(25/2)+2/21*(A*b^3+3*B*a*b^2)*x^(21/2)+2/17*(3*A*a*b^2+3*B*a^2
*b)*x^(17/2)+2/13*(3*A*a^2*b+B*a^3)*x^(13/2)+2/9*a^3*A*x^(9/2)
```

Maxima [A]
time = 0.30, size = 73, normalized size = 0.86

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{2}{21}\,\left(3\,Bab^2+Ab^3\right)x^{\frac{21}{2}}+\frac{6}{17}\,\left(Ba^2b+Aab^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}+\frac{2}{13}\,\left(Ba^3+3\,Aa^2b\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 6/17*(B*a^2*b + A
*a*b^2)*x^(17/2) + 2/9*A*a^3*x^(9/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

Fricas [A]
time = 1.02, size = 78, normalized size = 0.92

$$\frac{2}{348075}\left(13923\,Bb^3x^{12}+16575\,\left(3\,Bab^2+Ab^3\right)x^{10}+61425\,\left(Ba^2b+Aab^2\right)x^8+38675\,Aa^3x^4+26775\,\left(Ba^3+3\,Aa^2b\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*B*b^3*x^12 + 16575*(3*B*a*b^2 + A*b^3)*x^10 + 61425*(B*a^2*
b + A*a*b^2)*x^8 + 38675*A*a^3*x^4 + 26775*(B*a^3 + 3*A*a^2*b)*x^6)*sqrt(x)
```

Sympy [A]
time = 1.79, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{6Aa^2bx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{13}{2}}}{13}+\frac{6Ba^2bx^{\frac{17}{2}}}{17}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(17/2)/17 + 2
*A*b**3*x**(21/2)/21 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(17/2)/17 + 2*
B*a*b**2*x**(21/2)/7 + 2*B*b**3*x**(25/2)/25
```

Giac [A]
time = 0.71, size = 77, normalized size = 0.91

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{2}{7}\,Bab^2x^{\frac{21}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}+\frac{6}{17}\,Ba^2bx^{\frac{17}{2}}+\frac{6}{17}\,Aab^2x^{\frac{17}{2}}+\frac{2}{13}\,Ba^3x^{\frac{13}{2}}+\frac{6}{13}\,Aa^2bx^{\frac{13}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 6/17*B*a
^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2*b*
x^(13/2) + 2/9*A*a^3*x^(9/2)
```

Mupad [B]
time = 0.02, size = 69, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,a^3}{13}+\frac{6\,A\,b\,a^2}{13}\right)+x^{21/2}\left(\frac{2\,A\,b^3}{21}+\frac{2\,B\,a\,b^2}{7}\right)+\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{2\,B\,b^3\,x^{25/2}}{25}+\frac{6\,a\,b\,x^{17/2}\,\left(A\,b+B\,a\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] x^(13/2)*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^(21/2)*((2*A*b^3)/21 + (2*B*a*
b^2)/7) + (2*A*a^3*x^(9/2))/9 + (2*B*b^3*x^(25/2))/25 + (6*a*b*x^(17/2)*(A*
b + B*a))/17
```

3.360 $\int x^{5/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{19}b^2(Ab+3aB)x^{19/2}+\frac{2}{23}b^3Bx^{23/2}$$

```
[Out] 2/7*a^3*A*x^(7/2)+2/11*a^2*(3*A*b+B*a)*x^(11/2)+2/5*a*b*(A*b+B*a)*x^(15/2)+
2/19*b^2*(A*b+3*B*a)*x^(19/2)+2/23*b^3*B*x^(23/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*(A*b + a*B
)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx &= \int \left(a^3Ax^{5/2}+a^2(3Ab+aB)x^{9/2}+3ab(Ab+aB)x^{13/2}+b^2(Ab+3aB)x\right.\\ &= \frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{19}b^2(Ab+3\end{aligned}$$

Mathematica [A]
time = 0.05, size = 81, normalized size = 0.95

$$\frac{2x^{7/2}(2185a^3(11A+7Bx^2)+3059a^2bx^2(15A+11Bx^2)+1771ab^2x^4(19A+15Bx^2)+385b^3x^6(23A+19Bx^2))}{168245}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*x^(7/2)*(2185*a^3*(11*A + 7*B*x^2) + 3059*a^2*b*x^2*(15*A + 11*B*x^2) +
1771*a*b^2*x^4*(19*A + 15*B*x^2) + 385*b^3*x^6*(23*A + 19*B*x^2)))/168245
```

Maple [A]
time = 0.11, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{23}{2}}}{23}+\frac{2(A b^3+3Ba b^2)x^{\frac{19}{2}}}{19}+\frac{2(3Aa b^2+3B a^2 b)x^{\frac{15}{2}}}{15}+\frac{2(3A a^2 b+B a^3)x^{\frac{11}{2}}}{11}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
default	$\frac{2b^3 B x^{\frac{23}{2}}}{23}+\frac{2(A b^3+3Ba b^2)x^{\frac{19}{2}}}{19}+\frac{2(3Aa b^2+3B a^2 b)x^{\frac{15}{2}}}{15}+\frac{2(3A a^2 b+B a^3)x^{\frac{11}{2}}}{11}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}}(7315B b^3 x^8+8855x^6 A b^3+26565x^6 Ba b^2+33649Aa b^2 x^4+33649x^4 B a^2 b+45885x^2 A a^2 b+15295B a^3 x^2+24035A a^3}{168245}$
trager	$\frac{2x^{\frac{7}{2}}(7315B b^3 x^8+8855x^6 A b^3+26565x^6 Ba b^2+33649Aa b^2 x^4+33649x^4 B a^2 b+45885x^2 A a^2 b+15295B a^3 x^2+24035A a^3}{168245}$
risch	$\frac{2x^{\frac{7}{2}}(7315B b^3 x^8+8855x^6 A b^3+26565x^6 Ba b^2+33649Aa b^2 x^4+33649x^4 B a^2 b+45885x^2 A a^2 b+15295B a^3 x^2+24035A a^3}{168245}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*b^3*B*x^(23/2)+2/19*(A*b^3+3*B*a*b^2)*x^(19/2)+2/15*(3*A*a*b^2+3*B*a^2
*b)*x^(15/2)+2/11*(3*A*a^2*b+B*a^3)*x^(11/2)+2/7*a^3*A*x^(7/2)
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.86

$$\frac{2}{23}Bb^3x^{\frac{23}{2}}+\frac{2}{19}\left(3Bab^2+Ab^3\right)x^{\frac{19}{2}}+\frac{2}{5}\left(Ba^2b+Aab^2\right)x^{\frac{15}{2}}+\frac{2}{7}Aa^3x^{\frac{7}{2}}+\frac{2}{11}\left(Ba^3+3Aa^2b\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 2/5*(B*a^2*b + A*
a*b^2)*x^(15/2) + 2/7*A*a^3*x^(7/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

Fricas [A]
time = 0.88, size = 78, normalized size = 0.92

$$\frac{2}{168245}\left(7315\,Bb^3x^{11}+8855\left(3\,Bab^2+Ab^3\right)x^9+33649\left(Ba^2b+Aab^2\right)x^7+24035\,Aa^3x^3+15295\left(Ba^3+3\,Aa^2b\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*B*b^3*x^11 + 8855*(3*B*a*b^2 + A*b^3)*x^9 + 33649*(B*a^2*b +
 A*a*b^2)*x^7 + 24035*A*a^3*x^3 + 15295*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(x)
```

Sympy [A]
time = 1.20, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{2Aab^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{19}{2}}}{19}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{2Ba^2bx^{\frac{15}{2}}}{5}+\frac{6Bab^2x^{\frac{19}{2}}}{19}+\frac{2Bb^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(11/2)/11 + 2*A*a*b**2*x**(15/2)/5 + 2*
A*b**3*x**(19/2)/19 + 2*B*a**3*x**(11/2)/11 + 2*B*a**2*b*x**(15/2)/5 + 6*B*
a*b**2*x**(19/2)/19 + 2*B*b**3*x**(23/2)/23
```

Giac [A]
time = 0.77, size = 77, normalized size = 0.91

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{6}{19}\,Bab^2x^{\frac{19}{2}}+\frac{2}{19}\,Ab^3x^{\frac{19}{2}}+\frac{2}{5}\,Ba^2bx^{\frac{15}{2}}+\frac{2}{5}\,Aab^2x^{\frac{15}{2}}+\frac{2}{11}\,Ba^3x^{\frac{11}{2}}+\frac{6}{11}\,Aa^2bx^{\frac{11}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 2/5*B*a
^2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b*x
^(11/2) + 2/7*A*a^3*x^(7/2)
```

Mupad [B]
time = 0.02, size = 69, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right)+x^{19/2}\left(\frac{2\,A\,b^3}{19}+\frac{6\,B\,a\,b^2}{19}\right)+\frac{2\,A\,a^3\,x^{7/2}}{7}+\frac{2\,B\,b^3\,x^{23/2}}{23}+\frac{2\,a\,b\,x^{15/2}\,(A\,b+B\,a)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(19/2)*((2*A*b^3)/19 + (6*B*a*
b^2)/19) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(23/2))/23 + (2*a*b*x^(15/2)*(A
*b + B*a))/5
```

3.361 $\int x^{3/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{17}b^2(Ab+3aB)x^{17/2}+\frac{2}{21}b^3Bx^{21/2}$$

```
[Out] 2/5*a^3*A*x^(5/2)+2/9*a^2*(3*A*b+B*a)*x^(9/2)+6/13*a*b*(A*b+B*a)*x^(13/2)+2
/17*b^2*(A*b+3*B*a)*x^(17/2)+2/21*b^3*B*x^(21/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A*b + a*B)*
x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx&=\int\left(a^3Ax^{3/2}+a^2(3Ab+aB)x^{7/2}+3ab(Ab+aB)x^{11/2}+b^2(Ab+3aB)x^1\right.\\&=\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{17}b^2(Ab+3aI\end{aligned}$$

Mathematica [A]
time = 0.04, size = 81, normalized size = 0.95

$$\frac{2x^{5/2}\left(1547a^3(9A+5Bx^2)+1785a^2bx^2(13A+9Bx^2)+945ab^2x^4(17A+13Bx^2)+195b^3x^6(21A+17Bx^2)\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*x^(5/2)*(1547*a^3*(9*A + 5*B*x^2) + 1785*a^2*b*x^2*(13*A + 9*B*x^2) + 94
5*a*b^2*x^4*(17*A + 13*B*x^2) + 195*b^3*x^6*(21*A + 17*B*x^2)))/69615
```

Maple [A]
time = 0.11, size = 76, normalized size = 0.89

method	result
derivativedivides	$\frac{2b^3Bx^{\frac{21}{2}}}{21}+\frac{2(Ab^3+3Bab^2)x^{\frac{17}{2}}}{17}+\frac{2(3Aab^2+3Ba^2b)x^{\frac{13}{2}}}{13}+\frac{2(3Aa^2b+Ba^3)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{5}{2}}}{5}$
default	$\frac{2b^3Bx^{\frac{21}{2}}}{21}+\frac{2(Ab^3+3Bab^2)x^{\frac{17}{2}}}{17}+\frac{2(3Aab^2+3Ba^2b)x^{\frac{13}{2}}}{13}+\frac{2(3Aa^2b+Ba^3)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{5}{2}}}{5}$
gosper	$\frac{2x^{\frac{5}{2}}(3315Bb^3x^8+4095x^6Ab^3+12285x^6Bab^2+16065Aab^2x^4+16065x^4Ba^2b+23205x^2Aa^2b+7735Ba^3x^2+13923Aa^3}{69615}$
trager	$\frac{2x^{\frac{5}{2}}(3315Bb^3x^8+4095x^6Ab^3+12285x^6Bab^2+16065Aab^2x^4+16065x^4Ba^2b+23205x^2Aa^2b+7735Ba^3x^2+13923Aa^3}{69615}$
risch	$\frac{2x^{\frac{5}{2}}(3315Bb^3x^8+4095x^6Ab^3+12285x^6Bab^2+16065Aab^2x^4+16065x^4Ba^2b+23205x^2Aa^2b+7735Ba^3x^2+13923Aa^3}{69615}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*b^3*B*x^(21/2)+2/17*(A*b^3+3*B*a*b^2)*x^(17/2)+2/13*(3*A*a*b^2+3*B*a^2
*b)*x^(13/2)+2/9*(3*A*a^2*b+B*a^3)*x^(9/2)+2/5*a^3*A*x^(5/2)
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.86

$$\frac{2}{21}Bb^3x^{\frac{21}{2}}+\frac{2}{17}\left(3Bab^2+Ab^3\right)x^{\frac{17}{2}}+\frac{6}{13}\left(Ba^2b+Aab^2\right)x^{\frac{13}{2}}+\frac{2}{5}Aa^3x^{\frac{5}{2}}+\frac{2}{9}\left(Ba^3+3Aa^2b\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/13*(B*a^2*b + A
*a*b^2)*x^(13/2) + 2/5*A*a^3*x^(5/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

Fricas [A]
time = 0.84, size = 78, normalized size = 0.92

$$\frac{2}{69615}\left(3315Bb^3x^{10}+4095\left(3Bab^2+Ab^3\right)x^8+16065\left(Ba^2b+Aab^2\right)x^6+13923Aa^3x^2+7735\left(Ba^3+3Aa^2b\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*B*b^3*x^10 + 4095*(3*B*a*b^2 + A*b^3)*x^8 + 16065*(B*a^2*b +
A*a*b^2)*x^6 + 13923*A*a^3*x^2 + 7735*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

Sympy [A]
time = 0.75, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{6Aab^2x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{6Ba^2bx^{\frac{13}{2}}}{13}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 2*A*a**2*b*x**(9/2)/3 + 6*A*a*b**2*x**(13/2)/13 + 2*A
*b**3*x**(17/2)/17 + 2*B*a**3*x**(9/2)/9 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*
b**2*x**(17/2)/17 + 2*B*b**3*x**(21/2)/21
```

Giac [A]
time = 0.62, size = 77, normalized size = 0.91

$$\frac{2}{21}Bb^3x^{\frac{21}{2}}+\frac{6}{17}Bab^2x^{\frac{17}{2}}+\frac{2}{17}Ab^3x^{\frac{17}{2}}+\frac{6}{13}Ba^2bx^{\frac{13}{2}}+\frac{6}{13}Aab^2x^{\frac{13}{2}}+\frac{2}{9}Ba^3x^{\frac{9}{2}}+\frac{2}{3}Aa^2bx^{\frac{9}{2}}+\frac{2}{5}Aa^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/13*B*
a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^
(9/2) + 2/5*A*a^3*x^(5/2)
```

Mupad [B]
time = 0.02, size = 69, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{17/2}\left(\frac{2\,A\,b^3}{17}+\frac{6\,B\,a\,b^2}{17}\right)+\frac{2\,A\,a^3\,x^{5/2}}{5}+\frac{2\,B\,b^3\,x^{21/2}}{21}+\frac{6\,a\,b\,x^{13/2}\,\left(A\,b+B\,a\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(17/2)*((2*A*b^3)/17 + (6*B*a*b^2
)/17) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(21/2))/21 + (6*a*b*x^(13/2)*(A*b
+ B*a))/13
```

3.362 $\int \sqrt{x} \left(a+bx^2\right)^3 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=85

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}+\frac{2}{19}b^3Bx^{19/2}$$

```
[Out] 2/3*a^3*A*x^(3/2)+2/7*a^2*(3*A*b+B*a)*x^(7/2)+6/11*a*b*(A*b+B*a)*x^(11/2)+2
/15*b^2*(A*b+3*B*a)*x^(15/2)+2/19*b^3*B*x^(19/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*
x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x} \left(a+bx^2\right)^3 \left(A+Bx^2\right) \, dx &= \int \left(a^3A\sqrt{x}+a^2(3Ab+aB)x^{5/2}+3ab(Ab+aB)x^{9/2}+b^2(Ab+3aB)x\right.\\ &= \frac{2}{3}a^3Ax^{3/2}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{15}b^2(Ab+3a\end{aligned}$$

Mathematica [A]
time = 0.04, size = 81, normalized size = 0.95

$$\frac{2x^{3/2}\left(1045a^3\left(7A+3Bx^2\right)+855a^2bx^2\left(11A+7Bx^2\right)+399ab^2x^4\left(15A+11Bx^2\right)+77b^3x^6\left(19A+15Bx^2\right)\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(1045*a^3*(7*A + 3*B*x^2) + 855*a^2*b*x^2*(11*A + 7*B*x^2) + 399
*a*b^2*x^4*(15*A + 11*B*x^2) + 77*b^3*x^6*(19*A + 15*B*x^2)))/21945
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.89

method	result	si
derivativedivides	$\frac{2b^3 B x^{\frac{19}{2}}}{19}+\frac{2(A b^3+3Ba b^2)x^{\frac{15}{2}}}{15}+\frac{2(3Aa b^2+3B a^2 b)x^{\frac{11}{2}}}{11}+\frac{2(3A a^2 b+B a^3)x^{\frac{7}{2}}}{7}+\frac{2a^3 A x^{\frac{3}{2}}}{3}$	76
default	$\frac{2b^3 B x^{\frac{19}{2}}}{19}+\frac{2(A b^3+3Ba b^2)x^{\frac{15}{2}}}{15}+\frac{2(3Aa b^2+3B a^2 b)x^{\frac{11}{2}}}{11}+\frac{2(3A a^2 b+B a^3)x^{\frac{7}{2}}}{7}+\frac{2a^3 A x^{\frac{3}{2}}}{3}$	76
gosper	$\frac{2x^{\frac{3}{2}}(1155B b^3x^8+1463x^6A b^3+4389x^6Ba b^2+5985Aa b^2x^4+5985x^4B a^2b+9405x^2A a^2b+3135B a^3x^2+7315A a^3)}{21945}$	80
trager	$\frac{2x^{\frac{3}{2}}(1155B b^3x^8+1463x^6A b^3+4389x^6Ba b^2+5985Aa b^2x^4+5985x^4B a^2b+9405x^2A a^2b+3135B a^3x^2+7315A a^3)}{21945}$	80
risch	$\frac{2x^{\frac{3}{2}}(1155B b^3x^8+1463x^6A b^3+4389x^6Ba b^2+5985Aa b^2x^4+5985x^4B a^2b+9405x^2A a^2b+3135B a^3x^2+7315A a^3)}{21945}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*b^3*B*x^(19/2)+2/15*(A*b^3+3*B*a*b^2)*x^(15/2)+2/11*(3*A*a*b^2+3*B*a^2
*b)*x^(11/2)+2/7*(3*A*a^2*b+B*a^3)*x^(7/2)+2/3*a^3*A*x^(3/2)
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.86

$$\frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{2}{15}\left(3\,Bab^2+Ab^3\right)x^{\frac{15}{2}}+\frac{6}{11}\left(Ba^2b+Aab^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,Aa^3x^{\frac{3}{2}}+\frac{2}{7}\left(Ba^3+3\,Aa^2b\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 6/11*(B*a^2*b + A
*a*b^2)*x^(11/2) + 2/3*A*a^3*x^(3/2) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

Fricas [A]
time = 1.43, size = 76, normalized size = 0.89

$$\frac{2}{21945}\left(1155\,Bb^3x^9+1463\left(3\,Bab^2+Ab^3\right)x^7+5985\left(Ba^2b+Aab^2\right)x^5+7315\,Aa^3x+3135\left(Ba^3+3\,Aa^2b\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*b^3*x^9 + 1463*(3*B*a*b^2 + A*b^3)*x^7 + 5985*(B*a^2*b + A*
a*b^2)*x^5 + 7315*A*a^3*x + 3135*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

Sympy [A]
time = 1.56, size = 95, normalized size = 1.12

$$\frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Bb^3x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{15}{2}}(Ab^3+3Bab^2)}{15}+\frac{2x^{\frac{11}{2}}\cdot(3Aab^2+3Ba^2b)}{11}+\frac{2x^{\frac{7}{2}}\cdot(3Aa^2b+Ba^3)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*B*b**3*x**(19/2)/19 + 2*x**(15/2)*(A*b**3 + 3*B*a*b
**2)/15 + 2*x**(11/2)*(3*A*a*b**2 + 3*B*a**2*b)/11 + 2*x**(7/2)*(3*A*a**2*b
 + B*a**3)/7
```

Giac [A]
time = 0.57, size = 77, normalized size = 0.91

$$\frac{2}{19}Bb^3x^{\frac{19}{2}}+\frac{2}{5}Bab^2x^{\frac{15}{2}}+\frac{2}{15}Ab^3x^{\frac{15}{2}}+\frac{6}{11}Ba^2bx^{\frac{11}{2}}+\frac{6}{11}Aab^2x^{\frac{11}{2}}+\frac{2}{7}Ba^3x^{\frac{7}{2}}+\frac{6}{7}Aa^2bx^{\frac{7}{2}}+\frac{2}{3}Aa^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 6/11*B*a
^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^(
7/2) + 2/3*A*a^3*x^(3/2)
```

Mupad [B]
time = 0.02, size = 69, normalized size = 0.81

$$x^{7/2}\left(\frac{2\,B\,a^3}{7}+\frac{6\,A\,b\,a^2}{7}\right)+x^{15/2}\left(\frac{2\,A\,b^3}{15}+\frac{2\,B\,a\,b^2}{5}\right)+\frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{19/2}}{19}+\frac{6\,a\,b\,x^{11/2}\,(A\,b+B\,a)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^2
)/5) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(19/2))/19 + (6*a*b*x^(11/2)*(A*b +
 B*a))/11
```

3.363 $\int \frac{(a+bx^2)^3(A+Bx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=83

$$2a^3A\sqrt{x}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}+\frac{2}{17}b^3Bx^{17/2}$$

```
[Out] 2/5*a^2*(3*A*b+B*a)*x^(5/2)+2/3*a*b*(A*b+B*a)*x^(9/2)+2/13*b^2*(A*b+3*B*a)*
x^(13/2)+2/17*b^3*B*x^(17/2)+2*a^3*A*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$2a^3A\sqrt{x}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{17}b^3Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (2*a*b*(A*b + a*B)*x^(9
/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx^2)}{\sqrt{x}}\,dx &= \int \left(\frac{a^3A}{\sqrt{x}}+a^2(3Ab+aB)x^{3/2}+3ab(Ab+aB)x^{7/2}+b^2(Ab+3aB)x^{11/2}+b^3\right.\\ &= 2a^3A\sqrt{x}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.96

$$\frac{2\sqrt{x}\left(663a^3(5A+Bx^2)+221a^2bx^2(9A+5Bx^2)+85ab^2x^4(13A+9Bx^2)+15b^3x^6(17A+13Bx^2)\right)}{3315}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(663*a^3*(5*A + B*x^2) + 221*a^2*b*x^2*(9*A + 5*B*x^2) + 85*a*b^
2*x^4*(13*A + 9*B*x^2) + 15*b^3*x^6*(17*A + 13*B*x^2)))/3315
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.92

method	result
derivativedivides	$\frac{2b^3Bx^{\frac{17}{2}}}{17}+\frac{2(Ab^3+3Bab^2)x^{\frac{13}{2}}}{13}+\frac{2(3Aab^2+3Ba^2b)x^{\frac{9}{2}}}{9}+\frac{2(3Aa^2b+Ba^3)x^{\frac{5}{2}}}{5}+2a^3A\sqrt{x}$
default	$\frac{2b^3Bx^{\frac{17}{2}}}{17}+\frac{2(Ab^3+3Bab^2)x^{\frac{13}{2}}}{13}+\frac{2(3Aab^2+3Ba^2b)x^{\frac{9}{2}}}{9}+\frac{2(3Aa^2b+Ba^3)x^{\frac{5}{2}}}{5}+2a^3A\sqrt{x}$
trager	$\left(\frac{2}{17}B\,b^3x^8+\frac{2}{13}x^6A\,b^3+\frac{6}{13}x^6Ba\,b^2+\frac{2}{3}Aa\,b^2x^4+\frac{2}{3}x^4B\,a^2b+\frac{6}{5}x^2A\,a^2b+\frac{2}{5}B\,a^3x^2+2A\right.$
gosper	$\frac{2\sqrt{x}\,(195Bb^3x^8+255x^6Ab^3+765x^6Bab^2+1105Aab^2x^4+1105x^4Ba^2b+1989x^2Aa^2b+663Ba^3x^2+3315Aa^3)}{3315}$
risch	$\frac{2\sqrt{x}\,(195Bb^3x^8+255x^6Ab^3+765x^6Bab^2+1105Aab^2x^4+1105x^4Ba^2b+1989x^2Aa^2b+663Ba^3x^2+3315Aa^3)}{3315}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*b^3*B*x^(17/2)+2/13*(A*b^3+3*B*a*b^2)*x^(13/2)+2/9*(3*A*a*b^2+3*B*a^2*
b)*x^(9/2)+2/5*(3*A*a^2*b+B*a^3)*x^(5/2)+2*a^3*A*x^(1/2)
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.88

$$\frac{2}{17}\,Bb^3x^{\frac{17}{2}}+\frac{2}{13}\left(3\,Bab^2+Ab^3\right)x^{\frac{13}{2}}+\frac{2}{3}\left(Ba^2b+Aab^2\right)x^{\frac{9}{2}}+2\,Aa^3\sqrt{x}+\frac{2}{5}\left(Ba^3+3\,Aa^2b\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^3*x^(17/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 2/3*(B*a^2*b + A*
a*b^2)*x^(9/2) + 2*A*a^3*sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

Fricas [A]
time = 1.70, size = 75, normalized size = 0.90

$$\frac{2}{3315}\left(195\,Bb^3x^8+255\left(3\,Bab^2+Ab^3\right)x^6+1105\left(Ba^2b+Aab^2\right)x^4+3315\,Aa^3+663\left(Ba^3+3\,Aa^2b\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*B*b^3*x^8 + 255*(3*B*a*b^2 + A*b^3)*x^6 + 1105*(B*a^2*b + A*a*b
^2)*x^4 + 3315*A*a^3 + 663*(B*a^3 + 3*A*a^2*b)*x^2)*sqrt(x)
```

Sympy [A]
time = 0.48, size = 112, normalized size = 1.35

$$2Aa^3\sqrt{x} + \frac{6Aa^2bx^{\frac{5}{2}}}{5} + \frac{2Aab^2x^{\frac{9}{2}}}{3} + \frac{2Ab^3x^{\frac{13}{2}}}{13} + \frac{2Ba^3x^{\frac{5}{2}}}{5} + \frac{2Ba^2bx^{\frac{9}{2}}}{3} + \frac{6Bab^2x^{\frac{13}{2}}}{13} + \frac{2Bb^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 2*A*a*b**2*x**(9/2)/3 + 2*A*b**3
*x**(13/2)/13 + 2*B*a**3*x**(5/2)/5 + 2*B*a**2*b*x**(9/2)/3 + 6*B*a*b**2*x*
*(13/2)/13 + 2*B*b**3*x**(17/2)/17
```

Giac [A]
time = 0.64, size = 77, normalized size = 0.93

$$\frac{2}{17}Bb^3x^{\frac{17}{2}} + \frac{6}{13}Bab^2x^{\frac{13}{2}} + \frac{2}{13}Ab^3x^{\frac{13}{2}} + \frac{2}{3}Ba^2bx^{\frac{9}{2}} + \frac{2}{3}Aab^2x^{\frac{9}{2}} + \frac{2}{5}Ba^3x^{\frac{5}{2}} + \frac{6}{5}Aa^2bx^{\frac{5}{2}} + 2\,Aa^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b^3*x^(17/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 2/3*B*a
^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^(5/2
) + 2*A*a^3*sqrt(x)
```

Mupad [B]
time = 0.02, size = 69, normalized size = 0.83

$$x^{5/2}\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right)+x^{13/2}\left(\frac{2\,A\,b^3}{13}+\frac{6\,B\,a\,b^2}{13}\right)+2\,A\,a^3\,\sqrt{x}+\frac{2\,B\,b^3\,x^{17/2}}{17}+\frac{2\,a\,b\,x^{9/2}\,\left(A\,b+B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^3)/x^(1/2),x)
```

```
[Out] x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^2
)/13) + 2*A*a^3*x^(1/2) + (2*B*b^3*x^(17/2))/17 + (2*a*b*x^(9/2)*(A*b + B*a
))/3
```

3.364 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{6}{7}ab(Ab+aB)x^{7/2}+\frac{2}{11}b^2(Ab+3aB)x^{11/2}+\frac{2}{15}b^3Bx^{15/2}$$

```
[Out] 2/3*a^2*(3*A*b+B*a)*x^(3/2)+6/7*a*b*(A*b+B*a)*x^(7/2)+2/11*b^2*(A*b+3*B*a)*
x^(11/2)+2/15*b^3*B*x^(15/2)-2*a^3*A/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{11}b^2x^{11/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{15}b^3Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(11/2))/11 + (2*b^3*B*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx &= \int \left(\frac{a^3A}{x^{3/2}}+a^2(3Ab+aB)\sqrt{x}+3ab(Ab+aB)x^{5/2}+b^2(Ab+3aB)x^{9/2}+b^3\right.\\ &= -\frac{2a^3A}{\sqrt{x}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{6}{7}ab(Ab+aB)x^{7/2}+\frac{2}{11}b^2(Ab+3aB)x^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 83, normalized size = 1.00

$$-\frac{2(1155a^3A-1155a^2Abx^2-385a^3Bx^2-495aAb^2x^4-495a^2bBx^4-105Ab^3x^6-315ab^2Bx^6-77b^3Bx^8)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*(1155*a^3*A - 1155*a^2*A*b*x^2 - 385*a^3*B*x^2 - 495*a*A*b^2*x^4 - 495*
a^2*b*B*x^4 - 105*A*b^3*x^6 - 315*a*b^2*B*x^6 - 77*b^3*B*x^8))/(1155*Sqrt[x
])
```

Maple [A]
time = 0.08, size = 78, normalized size = 0.94

method	result	size
derivativedivides	$\frac{2b^3Bx^{\frac{15}{2}}}{15}+\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{6Bab^2x^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{7}{2}}}{7}+\frac{6Ba^2bx^{\frac{7}{2}}}{7}+2Aa^2bx^{\frac{3}{2}}+\frac{2Ba^3x^{\frac{3}{2}}}{3}-\frac{2a^3A}{\sqrt{x}}$	78
default	$\frac{2b^3Bx^{\frac{15}{2}}}{15}+\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{6Bab^2x^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{7}{2}}}{7}+\frac{6Ba^2bx^{\frac{7}{2}}}{7}+2Aa^2bx^{\frac{3}{2}}+\frac{2Ba^3x^{\frac{3}{2}}}{3}-\frac{2a^3A}{\sqrt{x}}$	78
gosper	$-\frac{2(-77Bb^3x^8-105x^6Ab^3-315x^6Bab^2-495Aab^2x^4-495x^4Ba^2b-1155x^2Aa^2b-385Ba^3x^2+1155Aa^3)}{1155\sqrt{x}}$	80
trager	$-\frac{2(-77Bb^3x^8-105x^6Ab^3-315x^6Bab^2-495Aab^2x^4-495x^4Ba^2b-1155x^2Aa^2b-385Ba^3x^2+1155Aa^3)}{1155\sqrt{x}}$	80
risch	$-\frac{2(-77Bb^3x^8-105x^6Ab^3-315x^6Bab^2-495Aab^2x^4-495x^4Ba^2b-1155x^2Aa^2b-385Ba^3x^2+1155Aa^3)}{1155\sqrt{x}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*b^3*B*x^(15/2)+2/11*A*b^3*x^(11/2)+6/11*B*a*b^2*x^(11/2)+6/7*A*a*b^2*x
^(7/2)+6/7*B*a^2*b*x^(7/2)+2*A*a^2*b*x^(3/2)+2/3*B*a^3*x^(3/2)-2*a^3*A/x^(1
/2)
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.88

$$\frac{2}{15}Bb^3x^{\frac{15}{2}}+\frac{2}{11}\left(3Bab^2+Ab^3\right)x^{\frac{11}{2}}+\frac{6}{7}\left(Ba^2b+Aab^2\right)x^{\frac{7}{2}}-\frac{2Aa^3}{\sqrt{x}}+\frac{2}{3}\left(Ba^3+3Aa^2b\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^3*x^(15/2) + 2/11*(3*B*a*b^2 + A*b^3)*x^(11/2) + 6/7*(B*a^2*b + A*
a*b^2)*x^(7/2) - 2*A*a^3/sqrt(x) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

Fricas [A]
time = 1.22, size = 75, normalized size = 0.90

$$\frac{2\left(77Bb^3x^8+105\left(3Bab^2+Ab^3\right)x^6+495\left(Ba^2b+Aab^2\right)x^4-1155Aa^3+385\left(Ba^3+3Aa^2b\right)x^2\right)}{1155\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b^3*x^8 + 105*(3*B*a*b^2 + A*b^3)*x^6 + 495*(B*a^2*b + A*a*b^2
)*x^4 - 1155*A*a^3 + 385*(B*a^3 + 3*A*a^2*b)*x^2)/sqrt(x)
```

Sympy [A]
time = 0.63, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{\sqrt{x}}+2Aa^2bx^{\frac{3}{2}}+\frac{6Aab^2x^{\frac{7}{2}}}{7}+\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{2Ba^3x^{\frac{3}{2}}}{3}+\frac{6Ba^2bx^{\frac{7}{2}}}{7}+\frac{6Bab^2x^{\frac{11}{2}}}{11}+\frac{2Bb^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 2*A*a**2*b*x**(3/2) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b**3*
x**(11/2)/11 + 2*B*a**3*x**(3/2)/3 + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x**
(11/2)/11 + 2*B*b**3*x**(15/2)/15
```

Giac [A]
time = 0.73, size = 77, normalized size = 0.93

$$\frac{2}{15}Bb^3x^{\frac{15}{2}}+\frac{6}{11}Bab^2x^{\frac{11}{2}}+\frac{2}{11}Ab^3x^{\frac{11}{2}}+\frac{6}{7}Ba^2bx^{\frac{7}{2}}+\frac{6}{7}Aab^2x^{\frac{7}{2}}+\frac{2}{3}Ba^3x^{\frac{3}{2}}+2\,Aa^2bx^{\frac{3}{2}}-\frac{2\,Aa^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^3*x^(15/2) + 6/11*B*a*b^2*x^(11/2) + 2/11*A*b^3*x^(11/2) + 6/7*B*a
^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2)
- 2*A*a^3/sqrt(x)
```

Mupad [B]
time = 0.02, size = 69, normalized size = 0.83

$$x^{3/2}\left(\frac{2\,B\,a^3}{3}+2\,A\,b\,a^2\right)+x^{11/2}\left(\frac{2\,A\,b^3}{11}+\frac{6\,B\,a\,b^2}{11}\right)-\frac{2\,A\,a^3}{\sqrt{x}}+\frac{2\,B\,b^3\,x^{15/2}}{15}+\frac{6\,a\,b\,x^{7/2}\,(A\,b+B\,a)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^3)/x^(3/2),x)
```

```
[Out] x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(11/2)*((2*A*b^3)/11 + (6*B*a*b^2)/11
) - (2*A*a^3)/x^(1/2) + (2*B*b^3*x^(15/2))/15 + (6*a*b*x^(7/2)*(A*b + B*a))
/7
```

3.365 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{3x^{3/2}}+2a^2(3Ab+aB)\sqrt{x}+\frac{6}{5}ab(Ab+aB)x^{5/2}+\frac{2}{9}b^2(Ab+3aB)x^{9/2}+\frac{2}{13}b^3Bx^{13/2}$$

```
[Out] -2/3*a^3*A/x^(3/2)+6/5*a*b*(A*b+B*a)*x^(5/2)+2/9*b^2*(A*b+3*B*a)*x^(9/2)+2/
13*b^3*B*x^(13/2)+2*a^2*(3*A*b+B*a)*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{459\}$

$$-\frac{2a^3A}{3x^{3/2}}+2a^2\sqrt{x}\,(aB+3Ab)+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(5/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(9/2))/9 + (2*b^3*B*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{5/2}}\,dx = \int \left(\frac{a^3A}{x^{5/2}}+\frac{a^2(3Ab+aB)}{\sqrt{x}}+3ab(Ab+aB)x^{3/2}+b^2(Ab+3aB)x^{7/2}+b^3Bx^{11}\right.$$

$$= -\frac{2a^3A}{3x^{3/2}}+2a^2(3Ab+aB)\sqrt{x}+\frac{6}{5}ab(Ab+aB)x^{5/2}+\frac{2}{9}b^2(Ab+3aB)x^{9/2}+$$

Mathematica [A]
time = 0.06, size = 78, normalized size = 0.94

$$\frac{-390a^3(A-3Bx^2)+702a^2bx^2(5A+Bx^2)+78ab^2x^4(9A+5Bx^2)+10b^3x^6(13A+9Bx^2)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-390*a^3*(A - 3*B*x^2) + 702*a^2*b*x^2*(5*A + B*x^2) + 78*a*b^2*x^4*(9*A +
 5*B*x^2) + 10*b^3*x^6*(13*A + 9*B*x^2))/(585*x^(3/2))
```

Maple [A]
time = 0.08, size = 78, normalized size = 0.94

method	result	s
derivativedivides	$\frac{2b^3Bx^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{9}{2}}}{9}+\frac{2Bab^2x^{\frac{9}{2}}}{3}+\frac{6Aab^2x^{\frac{5}{2}}}{5}+\frac{6Ba^2bx^{\frac{5}{2}}}{5}+6Aa^2b\sqrt{x}+2Ba^3\sqrt{x}-\frac{2a^3A}{3x^{\frac{3}{2}}}$	7
default	$\frac{2b^3Bx^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{9}{2}}}{9}+\frac{2Bab^2x^{\frac{9}{2}}}{3}+\frac{6Aab^2x^{\frac{5}{2}}}{5}+\frac{6Ba^2bx^{\frac{5}{2}}}{5}+6Aa^2b\sqrt{x}+2Ba^3\sqrt{x}-\frac{2a^3A}{3x^{\frac{3}{2}}}$	7
gosper	$-\frac{2\left(-45Bb^3x^8-65x^6Ab^3-195x^6Bab^2-351Aab^2x^4-351x^4Ba^2b-1755x^2Aa^2b-585Ba^3x^2+195Aa^3\right)}{585x^{\frac{3}{2}}}$	8
trager	$-\frac{2\left(-45Bb^3x^8-65x^6Ab^3-195x^6Bab^2-351Aab^2x^4-351x^4Ba^2b-1755x^2Aa^2b-585Ba^3x^2+195Aa^3\right)}{585x^{\frac{3}{2}}}$	8
risch	$-\frac{2\left(-45Bb^3x^8-65x^6Ab^3-195x^6Bab^2-351Aab^2x^4-351x^4Ba^2b-1755x^2Aa^2b-585Ba^3x^2+195Aa^3\right)}{585x^{\frac{3}{2}}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*b^3*B*x^(13/2)+2/9*A*b^3*x^(9/2)+2/3*B*a*b^2*x^(9/2)+6/5*A*a*b^2*x^(5/
2)+6/5*B*a^2*b*x^(5/2)+6*A*a^2*b*x^(1/2)+2*B*a^3*x^(1/2)-2/3*a^3*A/x^(3/2)
```

Maxima [A]
time = 0.30, size = 73, normalized size = 0.88

$$\frac{2}{13}Bb^3x^{\frac{13}{2}}+\frac{2}{9}\left(3Bab^2+Ab^3\right)x^{\frac{9}{2}}+\frac{6}{5}\left(Ba^2b+Aab^2\right)x^{\frac{5}{2}}-\frac{2Aa^3}{3x^{\frac{3}{2}}}+2\left(Ba^3+3Aa^2b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^3*x^(13/2) + 2/9*(3*B*a*b^2 + A*b^3)*x^(9/2) + 6/5*(B*a^2*b + A*a*
b^2)*x^(5/2) - 2/3*A*a^3/x^(3/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

Fricas [A]
time = 1.38, size = 75, normalized size = 0.90

$$\frac{2\left(45Bb^3x^8+65\left(3Bab^2+Ab^3\right)x^6+351\left(Ba^2b+Aab^2\right)x^4-195Aa^3+585\left(Ba^3+3Aa^2b\right)x^2\right)}{585x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b^3*x^8 + 65*(3*B*a*b^2 + A*b^3)*x^6 + 351*(B*a^2*b + A*a*b^2)*
x^4 - 195*A*a^3 + 585*(B*a^3 + 3*A*a^2*b)*x^2)/x^(3/2)
```

Sympy [A]
time = 0.70, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{3x^{\frac{3}{2}}}+6Aa^2b\sqrt{x}+\frac{6Aab^2x^{\frac{5}{2}}}{5}+\frac{2Ab^3x^{\frac{9}{2}}}{9}+2Ba^3\sqrt{x}+\frac{6Ba^2bx^{\frac{5}{2}}}{5}+\frac{2Bab^2x^{\frac{9}{2}}}{3}+\frac{2Bb^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(5/2)/5 + 2*A*b
**3*x**(9/2)/9 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(5/2)/5 + 2*B*a*b**2*x**(
9/2)/3 + 2*B*b**3*x**(13/2)/13
```

Giac [A]
time = 0.64, size = 77, normalized size = 0.93

$$\frac{2}{13}Bb^3x^{\frac{13}{2}}+\frac{2}{3}Bab^2x^{\frac{9}{2}}+\frac{2}{9}Ab^3x^{\frac{9}{2}}+\frac{6}{5}Ba^2bx^{\frac{5}{2}}+\frac{6}{5}Aab^2x^{\frac{5}{2}}+2Ba^3\sqrt{x}+6Aa^2b\sqrt{x}-\frac{2Aa^3}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^3*x^(13/2) + 2/3*B*a*b^2*x^(9/2) + 2/9*A*b^3*x^(9/2) + 6/5*B*a^2*b
*x^(5/2) + 6/5*A*a*b^2*x^(5/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) - 2/3*
A*a^3/x^(3/2)
```

Mupad [B]
time = 0.02, size = 69, normalized size = 0.83

$$\sqrt{x}\,\left(2\,B\,a^3+6\,A\,b\,a^2\right)+x^{9/2}\,\left(\frac{2\,A\,b^3}{9}+\frac{2\,B\,a\,b^2}{3}\right)-\frac{2\,A\,a^3}{3\,x^{3/2}}+\frac{2\,B\,b^3\,x^{13/2}}{13}+\frac{6\,a\,b\,x^{5/2}\,\left(A\,b+B\,a\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^3)/x^(5/2),x)
```

```
[Out] x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(9/2)*((2*A*b^3)/9 + (2*B*a*b^2)/3) - (2*
A*a^3)/(3*x^(3/2)) + (2*B*b^3*x^(13/2))/13 + (6*a*b*x^(5/2)*(A*b + B*a))/5
```

3.366 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(3Ab+aB)}{\sqrt{x}}+2ab(Ab+aB)x^{3/2}+\frac{2}{7}b^2(Ab+3aB)x^{7/2}+\frac{2}{11}b^3Bx^{11/2}$$

```
[Out] -2/5*a^3*A/x^(5/2)+2*a*b*(A*b+B*a)*x^(3/2)+2/7*b^2*(A*b+3*B*a)*x^(7/2)+2/11
*b^3*B*x^(11/2)-2*a^2*(3*A*b+B*a)/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(aB+3Ab)}{\sqrt{x}}+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/Sqrt[x] + 2*a*b*(A*b + a*B)*
x^(3/2) + (2*b^2*(A*b + 3*a*B)*x^(7/2))/7 + (2*b^3*B*x^(11/2))/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{7/2}}\,dx &= \int \left(\frac{a^3A}{x^{7/2}}+\frac{a^2(3Ab+aB)}{x^{3/2}}+3ab(Ab+aB)\sqrt{x}+b^2(Ab+3aB)x^{5/2}+b^3Bx\right.\\ &= -\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(3Ab+aB)}{\sqrt{x}}+2ab(Ab+aB)x^{3/2}+\frac{2}{7}b^2(Ab+3aB)x^{7/2}+\frac{2}{11}b^3\end{aligned}$$

Mathematica [A]
time = 0.06, size = 78, normalized size = 0.96

$$\frac{2(385a^2bx^2(-3A+Bx^2)+55ab^2x^4(7A+3Bx^2)-77a^3(A+5Bx^2)+5b^3x^6(11A+7Bx^2))}{385x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (2*(385*a^2*b*x^2*(-3*A + B*x^2) + 55*a*b^2*x^4*(7*A + 3*B*x^2) - 77*a^3*(A
 + 5*B*x^2) + 5*b^3*x^6*(11*A + 7*B*x^2)))/(385*x^(5/2))
```

Maple [A]
time = 0.08, size = 75, normalized size = 0.93

method	result	size
derivativedivides	$\frac{2b^3 B x^{\frac{11}{2}}}{11}+\frac{2A b^3 x^{\frac{7}{2}}}{7}+\frac{6Ba b^2 x^{\frac{7}{2}}}{7}+2Aa b^2 x^{\frac{3}{2}}+2B a^2 b x^{\frac{3}{2}}-\frac{2a^3 A}{5x^{\frac{5}{2}}}-\frac{2a^2(3Ab+Ba)}{\sqrt{x}}$	75
default	$\frac{2b^3 B x^{\frac{11}{2}}}{11}+\frac{2A b^3 x^{\frac{7}{2}}}{7}+\frac{6Ba b^2 x^{\frac{7}{2}}}{7}+2Aa b^2 x^{\frac{3}{2}}+2B a^2 b x^{\frac{3}{2}}-\frac{2a^3 A}{5x^{\frac{5}{2}}}-\frac{2a^2(3Ab+Ba)}{\sqrt{x}}$	75
gosper	$-\frac{2(-35B b^3 x^8-55x^6 A b^3-165x^6 Ba b^2-385Aa b^2 x^4-385x^4 B a^2 b+1155x^2 A a^2 b+385B a^3 x^2+77A a^3)}{385x^{\frac{5}{2}}}$	80
trager	$-\frac{2(-35B b^3 x^8-55x^6 A b^3-165x^6 Ba b^2-385Aa b^2 x^4-385x^4 B a^2 b+1155x^2 A a^2 b+385B a^3 x^2+77A a^3)}{385x^{\frac{5}{2}}}$	80
risch	$-\frac{2(-35B b^3 x^8-55x^6 A b^3-165x^6 Ba b^2-385Aa b^2 x^4-385x^4 B a^2 b+1155x^2 A a^2 b+385B a^3 x^2+77A a^3)}{385x^{\frac{5}{2}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*b^3*B*x^(11/2)+2/7*A*b^3*x^(7/2)+6/7*B*a*b^2*x^(7/2)+2*A*a*b^2*x^(3/2)
+2*B*a^2*b*x^(3/2)-2/5*a^3*A/x^(5/2)-2*a^2*(3*A*b+B*a)/x^(1/2)
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.93

$$\frac{2}{11}\,Bb^3x^{\frac{11}{2}}+\frac{2}{7}\left(3\,Bab^2+Ab^3\right)x^{\frac{7}{2}}+2\left(Ba^2b+Aab^2\right)x^{\frac{3}{2}}-\frac{2\left(Aa^3+5\left(Ba^3+3\,Aa^2b\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b^3*x^(11/2) + 2/7*(3*B*a*b^2 + A*b^3)*x^(7/2) + 2*(B*a^2*b + A*a*b^
2)*x^(3/2) - 2/5*(A*a^3 + 5*(B*a^3 + 3*A*a^2*b)*x^2)/x^(5/2)
```

Fricas [A]
time = 1.06, size = 75, normalized size = 0.93

$$\frac{2\left(35\,Bb^3x^8+55\left(3\,Bab^2+Ab^3\right)x^6+385\left(Ba^2b+Aab^2\right)x^4-77\,Aa^3-385\left(Ba^3+3\,Aa^2b\right)x^2\right)}{385\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/385*(35*B*b^3*x^8 + 55*(3*B*a*b^2 + A*b^3)*x^6 + 385*(B*a^2*b + A*a*b^2)*
x^4 - 77*A*a^3 - 385*(B*a^3 + 3*A*a^2*b)*x^2)/x^(5/2)
```

Sympy [A]
time = 0.87, size = 107, normalized size = 1.32

$$-\frac{2Aa^3}{5x^{\frac{5}{2}}}-\frac{6Aa^2b}{\sqrt{x}}+2Aab^2x^{\frac{3}{2}}+\frac{2Ab^3x^{\frac{7}{2}}}{7}-\frac{2Ba^3}{\sqrt{x}}+2Ba^2bx^{\frac{3}{2}}+\frac{6Bab^2x^{\frac{7}{2}}}{7}+\frac{2Bb^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) - 6*A*a**2*b/sqrt(x) + 2*A*a*b**2*x**(3/2) + 2*A*b**
3*x**(7/2)/7 - 2*B*a**3/sqrt(x) + 2*B*a**2*b*x**(3/2) + 6*B*a*b**2*x**(7/2)
/7 + 2*B*b**3*x**(11/2)/11
```

Giac [A]
time = 0.87, size = 79, normalized size = 0.98

$$\frac{2}{11}\,Bb^3x^{\frac{11}{2}}+\frac{6}{7}\,Bab^2x^{\frac{7}{2}}+\frac{2}{7}\,Ab^3x^{\frac{7}{2}}+2\,Ba^2bx^{\frac{3}{2}}+2\,Aab^2x^{\frac{3}{2}}-\frac{2\,(5\,Ba^3x^2+15\,Aa^2bx^2+Aa^3)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b^3*x^(11/2) + 6/7*B*a*b^2*x^(7/2) + 2/7*A*b^3*x^(7/2) + 2*B*a^2*b*x
^(3/2) + 2*A*a*b^2*x^(3/2) - 2/5*(5*B*a^3*x^2 + 15*A*a^2*b*x^2 + A*a^3)/x^(
5/2)
```

Mupad [B]
time = 0.03, size = 72, normalized size = 0.89

$$x^{7/2}\left(\frac{2\,A\,b^3}{7}+\frac{6\,B\,a\,b^2}{7}\right)-\frac{\frac{2\,A\,a^3}{5}+x^2\,(2\,B\,a^3+6\,A\,b\,a^2)}{x^{5/2}}+\frac{2\,B\,b^3\,x^{11/2}}{11}+2\,a\,b\,x^{3/2}\,(A\,b+B\,a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^3)/x^(7/2),x)
```

```
[Out] x^(7/2)*((2*A*b^3)/7 + (6*B*a*b^2)/7) - ((2*A*a^3)/5 + x^2*(2*B*a^3 + 6*A*a
^2*b))/x^(5/2) + (2*B*b^3*x^(11/2))/11 + 2*a*b*x^(3/2)*(A*b + B*a)
```

3.367 $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=276

$$-\frac{2a(Ab-aB)\sqrt{x}}{b^3}+\frac{2(Ab-aB)x^{5/2}}{5b^2}+\frac{2Bx^{9/2}}{9b}-\frac{a^{5/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{13/4}}+\frac{a^{5/4}(Ab-aB)\,\mathrm{t}}{}$$

```
[Out] 2/5*(A*b-B*a)*x^(5/2)/b^2+2/9*B*x^(9/2)/b-1/2*a^(5/4)*(A*b-B*a)*arctan(1-b^
(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(13/4)*2^(1/2)+1/2*a^(5/4)*(A*b-B*a)*arcta
n(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(13/4)*2^(1/2)-1/4*a^(5/4)*(A*b-B*a)
*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/4
*a^(5/4)*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^
(13/4)*2^(1/2)-2*a*(A*b-B*a)*x^(1/2)/b^3
```

Rubi [A]

time = 0.18, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = $\{470, 327, 335, 217, 1179, 642, 1176, 631, 210\}$

$$-\frac{a^{5/4}(Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{13/4}}+\frac{a^{5/4}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{13/4}}-\frac{a^{5/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{a^{5/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{13/4}}-\frac{2a\sqrt{x}\,(Ab-aB)}{b^3}+\frac{2x^{5/2}(Ab-aB)}{5b^2}+\frac{2Bx^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (-2*a*(A*b - a*B)*Sqrt[x])/b^3 + (2*(A*b - a*B)*x^(5/2))/(5*b^2) + (2*B*x^(
9/2))/(9*b) - (a^(5/4)*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*b^(13/4)) + (a^(5/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(13/4)) - (a^(5/4)*(A*b - a*B)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(13/4)) + (a^(
5/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{2Bx^{9/2}}{9b} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{x^{7/2}}{a+bx^2}\,dx}{9b}$$
$$= \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(a(Ab-aB))\int \frac{x^{3/2}}{a+bx^2}\,dx}{b^2}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{(a^2(Ab-aB))\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{b^3}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{(2a^2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,d\right.}{b^3}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(a^{3/2}(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt{a}-}{a+}\right.}{b^3}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(a^{3/2}(Ab-aB)\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}}\right.}{2b^{7/2}}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} - \frac{a^{5/4}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\right.}{2\sqrt{2}\,b^{13/4}}$$
$$= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} - \frac{a^{5/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,b^{13/4}}$$

Mathematica [A]
time = 0.28, size = 173, normalized size = 0.63

$$\frac{2\sqrt{x}\left(-45aAb+45a^2B+9Ab^2x^2-9abBx^2+5b^2Bx^4\right)}{45b^3}+\frac{a^{5/4}(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,b^{13/4}}-\frac{a^{5/4}(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x]*(-45*a*A*b + 45*a^2*B + 9*A*b^2*x^2 - 9*a*b*B*x^2 + 5*b^2*B*x^4)
)/(45*b^3) + (a^(5/4)*(-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(13/4)) - (a^(5/4)*(-(A*b) + a*B)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(
13/4))
```

Maple [A]
time = 0.09, size = 164, normalized size = 0.59

method	result
derivativedivides	$-\frac{2\left(-\frac{b^{2} B x^{\frac{9}{2}}}{9}-\frac{A b^{2} x^{\frac{5}{2}}}{5}+\frac{B a b x^{\frac{5}{2}}}{5}+a b A \sqrt{x}-a^{2} B \sqrt{x}\right)}{b^{3}}+\frac{a(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{}}\right.\right.}{}$
default	$-\frac{2\left(-\frac{b^{2} B x^{\frac{9}{2}}}{9}-\frac{A b^{2} x^{\frac{5}{2}}}{5}+\frac{B a b x^{\frac{5}{2}}}{5}+a b A \sqrt{x}-a^{2} B \sqrt{x}\right)}{b^{3}}+\frac{a(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{}}\right.\right.}{}$
risch	$-\frac{2\left(-5 b^{2} B x^{4}-9 A b^{2} x^{2}+9 B a b x^{2}+45 a b A-45 a^{2} B\right)\sqrt{x}}{45 b^{3}}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, A \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 b^{2}}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^3*(-1/9*b^2*B*x^(9/2)-1/5*A*b^2*x^(5/2)+1/5*B*a*b*x^(5/2)+a*b*A*x^(1/2
)-a^2*B*x^(1/2))+1/4*a*(A*b-B*a)/b^3*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+
2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1))
```

Maxima [A]
time = 0.50, size = 259, normalized size = 0.94

$$-\frac{\left(\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(Ba-Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(Ba-Ab)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a^{2}}{4b^{3}}+\frac{2\left(5Bb^{2}x^{\frac{9}{2}}-9\left(Bab-Ab^{2}\right)x^{\frac{5}{2}}+45\left(Ba^{2}-Aab\right)\sqrt{x}\right)}{45b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
2*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(
2)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(
a^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
```

```
 + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a^2/b^3 + 2/45*(5*B*b^2*x^(9/2)
- 9*(B*a*b - A*b^2)*x^(5/2) + 45*(B*a^2 - A*a*b)*sqrt(x))/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(199) = 398$.
time = 1.42, size = 714, normalized size = 2.59

$$\frac{180\, b^{3} \left(-\frac{B^{4} a^{9} - 4 A B^{3} a^{8} b + 6 A^{2} B^{2} a^{7} b^{2} - 4 A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{b^{6} \sqrt{-\frac{B^{4} a^{9} - 4 A B^{3} a^{8} b + 6 A^{2} B^{2} a^{7} b^{2} - 4 A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}} + \left(B^{2} a^{4} - 2 A B a^{3} b + A^{2} a^{2} b^{2}\right) x}\, b^{10} \left(-\frac{B^{4} a^{9} - 4 A B^{3} a^{8} b + 6 A^{2} B^{2} a^{7} b^{2} - 4 A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}\right)^{\frac{3}{4}} + \left(B a^{2} b^{10} - A a b^{11}\right) \sqrt{x} \left(-\frac{B^{4} a^{9} - 4 A B^{3} a^{8} b + 6 A^{2} B^{2} a^{7} b^{2} - 4 A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}\right)^{\frac{3}{4}}}{B^{4} a^{9} - 4 A B^{3} a^{8} b + 6 A^{2} B^{2} a^{7} b^{2} - 4 A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}\right) + 45\, b^{3} \left(-\frac{B^{4} a^{9} - 4 A B^{3} a^{8} b + 6 A^{2} B^{2} a^{7} b^{2} - 4 A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}\right)^{\frac{1}{4}} \log\left(b^{3} \left(-\frac{B^{4} a^{9} - 4 A B^{3} a^{8} b + 6 A^{2} B^{2} a^{7} b^{2} - 4 A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}\right)^{\frac{1}{4}} - \left(B a^{2} - A a b\right) \sqrt{x}\right) - 45\, b^{3} \left(-\frac{B^{4} a^{9} - 4 A B^{3} a^{8} b + 6 A^{2} B^{2} a^{7} b^{2} - 4 A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}\right)^{\frac{1}{4}} \log\left(-b^{3} \left(-\frac{B^{4} a^{9} - 4 A B^{3} a^{8} b + 6 A^{2} B^{2} a^{7} b^{2} - 4 A^{3} B a^{6} b^{3} + A^{4} a^{5} b^{4}}{b^{13}}\right)^{\frac{1}{4}} - \left(B a^{2} - A a b\right) \sqrt{x}\right) + 4 \left(5 B b^{2} x^{4} + 45 B a^{2} - 45 A a b - 9 \left(B a b - A b^{2}\right) x^{2}\right) \sqrt{x}}{90\, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*
b^3 + A^4*a^5*b^4)/b^13)^(1/4)*arctan((sqrt(b^6*sqrt(-(B^4*a^9 - 4*A*B^3*a^
8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13) + (B^2*a^4 -
 2*A*B*a^3*b + A^2*a^2*b^2)*x)*b^10*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*
a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(3/4) + (B*a^2*b^10 - A*a*b^
11)*sqrt(x)*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^
3 + A^4*a^5*b^4)/b^13)^(3/4))/(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2
- 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)) + 45*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A
^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(b^3*(-(B^4*
a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^
13)^(1/4) - (B*a^2 - A*a*b)*sqrt(x)) - 45*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b +
6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(-b^3*(-(
B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4
)/b^13)^(1/4) - (B*a^2 - A*a*b)*sqrt(x)) + 4*(5*B*b^2*x^4 + 45*B*a^2 - 45*A
*a*b - 9*(B*a*b - A*b^2)*x^2)*sqrt(x))/b^3
```

Sympy [A]
time = 38.64, size = 326, normalized size = 1.18

$$\begin{cases} \tilde{\infty} \left(\frac{2 A x^{\frac{5}{2}}}{5} + \frac{2 B x^{\frac{9}{2}}}{9}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{\frac{2 A x^{\frac{9}{2}}}{9} + \frac{2 B x^{\frac{13}{2}}}{13}}{a} & \text{for}\: b = 0 \\ \frac{\frac{2 A x^{\frac{5}{2}}}{5} + \frac{2 B x^{\frac{9}{2}}}{9}}{b} & \text{for}\: a = 0 \\ - \frac{2 A a \sqrt{x}}{b^{2}} - \frac{A a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{2}} + \frac{A a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{2}} + \frac{A a \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{2}} + \frac{2 A x^{\frac{5}{2}}}{5 b} + \frac{2 B a^{2} \sqrt{x}}{b^{3}} + \frac{B a^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{3}} - \frac{B a^{2} \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{3}} - \frac{B a^{2} \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{3}} - \frac{2 B a x^{\frac{5}{2}}}{5 b^{2}} + \frac{2 B x^{\frac{9}{2}}}{9 b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(2*A*x**(5/2)/5 + 2*B*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(9/2)/9 + 2*B*x**(13/2)/13)/a, Eq(b, 0)), ((2*A*x**(5/2)/5 + 2*B*x**(
9/2)/9)/b, Eq(a, 0)), (-2*A*a*sqrt(x)/b**2 - A*a*(-a/b)**(1/4)*log(sqrt(x)
- (-a/b)**(1/4))/(2*b**2) + A*a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/
(2*b**2) + A*a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 + 2*A*x**(5/2
)/(5*b) + 2*B*a**2*sqrt(x)/b**3 + B*a**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)
**(1/4))/(2*b**3) - B*a**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b*
```

```
*3) - B*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**3 - 2*B*a*x**(5/2
)/(5*b**2) + 2*B*x**(9/2)/(9*b), True))
```

Giac [A]

time = 1.00, size = 298, normalized size = 1.08

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{4}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{4}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{4}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba^{2}-\left(ab^{3}\right)^{\frac{1}{4}}Aab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{4}}+\frac{2\left(5Bb^{8}x^{\frac{9}{2}}-9Bab^{7}x^{\frac{5}{2}}+9Ab^{8}x^{\frac{5}{2}}+45Ba^{2}b^{6}\sqrt{x}-45Aab^{7}\sqrt{x}\right)}{45b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*((a*b^3)^
(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)
^(1/4)*A*a*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 + 1/4*sq
rt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*log(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/b^4 + 2/45*(5*B*b^8*x^(9/2) - 9*B*a*b^7*x^(5/2) +
9*A*b^8*x^(5/2) + 45*B*a^2*b^6*sqrt(x) - 45*A*a*b^7*sqrt(x))/b^9
```

Mupad [B]

time = 0.20, size = 788, normalized size = 2.86

$$x^{5/2}\left(\frac{2A}{5b}-\frac{2Ba}{5b^{2}}\right)+\frac{2Bx^{9/2}}{9b}-\frac{(-a)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}a^{4}b^{2}-2ABa^{5}b+B^{2}a^{6}\right)}{b^{3}}-\frac{(-a)^{5/4}(Ab-Ba)\left(32Ba^{4}-32Aa^{3}b\right)1\mathrm{i}}{2b^{13/4}}\right)(Ab-Ba)}{2b^{13/4}}+\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}a^{4}b^{2}-2ABa^{5}b+B^{2}a^{6}\right)}{b^{3}}+\frac{(-a)^{5/4}(Ab-Ba)\left(32Ba^{4}-32Aa^{3}b\right)1\mathrm{i}}{2b^{13/4}}\right)(Ab-Ba)}{2b^{13/4}}}{\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}a^{4}b^{2}-2ABa^{5}b+B^{2}a^{6}\right)}{b^{3}}-\frac{(-a)^{5/4}(Ab-Ba)\left(32Ba^{4}-32Aa^{3}b\right)1\mathrm{i}}{2b^{13/4}}\right)(Ab-Ba)\,1\mathrm{i}}{2b^{13/4}}-\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}a^{4}b^{2}-2ABa^{5}b+B^{2}a^{6}\right)}{b^{3}}+\frac{(-a)^{5/4}(Ab-Ba)\left(32Ba^{4}-32Aa^{3}b\right)1\mathrm{i}}{2b^{13/4}}\right)(Ab-Ba)\,1\mathrm{i}}{2b^{13/4}}}\right)(Ab-Ba)}{b^{13/4}}-\frac{a\sqrt{x}\left(\frac{2A}{b}-\frac{2Ba}{b^{2}}\right)}{b}-\frac{(-a)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}a^{4}b^{2}-2ABa^{5}b+B^{2}a^{6}\right)}{b^{3}}-\frac{(-a)^{5/4}(Ab-Ba)\left(32Ba^{4}-32Aa^{3}b\right)}{2b^{13/4}}\right)(Ab-Ba)\,1\mathrm{i}}{2b^{13/4}}+\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}a^{4}b^{2}-2ABa^{5}b+B^{2}a^{6}\right)}{b^{3}}+\frac{(-a)^{5/4}(Ab-Ba)\left(32Ba^{4}-32Aa^{3}b\right)}{2b^{13/4}}\right)(Ab-Ba)\,1\mathrm{i}}{2b^{13/4}}}{\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}a^{4}b^{2}-2ABa^{5}b+B^{2}a^{6}\right)}{b^{3}}-\frac{(-a)^{5/4}(Ab-Ba)\left(32Ba^{4}-32Aa^{3}b\right)}{2b^{13/4}}\right)(Ab-Ba)}{2b^{13/4}}-\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^{2}a^{4}b^{2}-2ABa^{5}b+B^{2}a^{6}\right)}{b^{3}}+\frac{(-a)^{5/4}(Ab-Ba)\left(32Ba^{4}-32Aa^{3}b\right)}{2b^{13/4}}\right)(Ab-Ba)}{2b^{13/4}}}\right)(Ab-Ba)\,1\mathrm{i}}{b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^(5/2)*((2*A)/(5*b) - (2*B*a)/(5*b^2)) + (2*B*x^(9/2))/(9*b) - ((-a)^(5/4)
*atan((((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3
- ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b))/(2*b^(13/4)))*(A*b - B*a
)*1i)/(2*b^(13/4)) + ((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*
B*a^5*b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b))/(2*b^(13/4
)))*(A*b - B*a)*1i)/(2*b^(13/4)))/(((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*
a^4*b^2 - 2*A*B*a^5*b))/b^3 - ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*
b))/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4)) - ((-a)^(5/4)*((16*x^(1/2)*(B^2
*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4
- 32*A*a^3*b))/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4))))*(A*b - B*a)*1i)/b^
(13/4) - ((-a)^(5/4)*atan((((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2
- 2*A*B*a^5*b))/b^3 - ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b)*1i)/(
2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4)) + ((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6
+ A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*
A*a^3*b)*1i)/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4)))/(((-a)^(5/4)*((16*x^(
1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 - ((-a)^(5/4)*(A*b - B*a)*(
32*B*a^4 - 32*A*a^3*b)*1i)/(2*b^(13/4)))*(A*b - B*a)*1i)/(2*b^(13/4)) - ((-
a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 + ((-a)^(5
/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b)*1i)/(2*b^(13/4)))*(A*b - B*a)*1i)/(
2*b^(13/4))))*(A*b - B*a))/b^(13/4) - (a*x^(1/2)*((2*A)/b - (2*B*a)/b^2))/b
```

3.368 $\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=257

$$\frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{7/2}}{7b}+\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{11/4}}-\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{11/4}}$$

```
[Out] 2/3*(A*b-B*a)*x^(3/2)/b^2+2/7*B*x^(7/2)/b+1/2*a^(3/4)*(A*b-B*a)*arctan(1-b^
(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(11/4)*2^(1/2)-1/2*a^(3/4)*(A*b-B*a)*arcta
n(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(11/4)*2^(1/2)-1/4*a^(3/4)*(A*b-B*a)
*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)+1/4
*a^(3/4)*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^
(11/4)*2^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {470, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{a^{3/4}(Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{11/4}}-\frac{a^{3/4}(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{11/4}}-\frac{a^{3/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{2x^{3/2}(Ab-aB)}{3b^2}+\frac{2Bx^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(7/2))/(7*b) + (a^(3/4)*(A*b - a*B
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(11/4)) - (a^(3
/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(
11/4)) - (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4)) + (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{2Bx^{7/2}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{x^{5/2}}{a+bx^2}\,dx}{7b}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab-aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{b^2}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(2a(Ab-aB))\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} + \frac{(a(Ab-aB))\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}} - \frac{(a(A}{}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab-aB))\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^3}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} - \frac{a^{3/4}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{11/4}}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{7/2}}{7b} + \frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{11/4}} - \frac{a^{3/4}(Ab-}{}$$

Mathematica [A]
time = 0.19, size = 152, normalized size = 0.59

$$\frac{2x^{3/2}(7Ab-7aB+3bBx^2)}{21b^2} - \frac{a^{3/4}(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\,b^{11/4}} - \frac{a^{3/4}(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2)*(7*A*b - 7*a*B + 3*b*B*x^2))/(21*b^2) - (a^(3/4)*(-(A*b) + a*B)*
ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b
^(11/4)) - (a^(3/4)*(-(A*b) + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
)/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(11/4))
```

Maple [A]
time = 0.09, size = 142, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+\frac{2(Ab-Ba)x^{\frac{3}{2}}}{3}}{b^{2}}-\frac{a(Ab-Ba)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\text{arcta}\right.}{4b^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+\frac{2(Ab-Ba)x^{\frac{3}{2}}}{3}}{b^{2}}-\frac{a(Ab-Ba)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\text{arcta}\right.}{4b^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{2}+7Ab-7Ba\right)}{21b^{2}}-\frac{a\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{a\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/7*b*B*x^(7/2)+1/3*(A*b-B*a)*x^(3/2))-1/4*a*(A*b-B*a)/b^3/(a/b)^(1/
4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.51, size = 214, normalized size = 0.83

$$\frac{(Ba^{2}-Aab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,b^{2}}+\frac{2\left(3\,Bbx^{\frac{7}{2}}-7\,(Ba-Ab)x^{\frac{3}{2}}\right)}{21\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*a^2 - A*a*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(
x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sq
rt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*
b^(3/4)))/b^2 + 2/21*(3*B*b*x^(7/2) - 7*(B*a - A*b)*x^(3/2))/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 899 vs. 2(182) = 364.
time = 0.98, size = 899, normalized size = 3.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b
^3 + A^4*a^3*b^4)/b^11)^(1/4)*arctan((sqrt((B^6*a^10 - 6*A*B^5*a^9*b + 15*A
^2*B^4*a^8*b^2 - 20*A^3*B^3*a^7*b^3 + 15*A^4*B^2*a^6*b^4 - 6*A^5*B*a^5*b^5
+ A^6*a^4*b^6)*x - (B^4*a^7*b^5 - 4*A*B^3*a^6*b^6 + 6*A^2*B^2*a^5*b^7 - 4*A
^3*B*a^4*b^8 + A^4*a^3*b^9)*sqrt(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*
b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11))*b^3*(-(B^4*a^7 - 4*A*B^3*a^6*b
+ 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4) + (B^3*a^5
*b^3 - 3*A*B^2*a^4*b^4 + 3*A^2*B*a^3*b^5 - A^3*a^2*b^6)*sqrt(x)*(-(B^4*a^7
- 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^
(1/4))/(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4
*a^3*b^4)) - 21*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*
B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4)*log(b^8*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6
*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(3/4) - (B^3*a^5 -
3*A*B^2*a^4*b + 3*A^2*B*a^3*b^2 - A^3*a^2*b^3)*sqrt(x)) + 21*b^2*(-(B^4*a^7
 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)
^(1/4)*log(-b^8*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^
4*b^3 + A^4*a^3*b^4)/b^11)^(3/4) - (B^3*a^5 - 3*A*B^2*a^4*b + 3*A^2*B*a^3*b
^2 - A^3*a^2*b^3)*sqrt(x)) + 4*(3*B*b*x^3 - 7*(B*a - A*b)*x)*sqrt(x))/b^2
```

Sympy [A]
time = 18.05, size = 348, normalized size = 1.35

$$\begin{cases} \tilde{\infty} \left(\frac{2Ax^{\frac{3}{2}}}{3} + \frac{2Bx^{\frac{7}{2}}}{7}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7} + \frac{2Bx^{\frac{11}{2}}}{11}}{a} & \text{for}\: b = 0 \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3} + \frac{2Bx^{\frac{7}{2}}}{7}}{b} & \text{for}\: a = 0 \\ - \frac{2Aa \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{2} \sqrt[4]{- \frac{a}{b}}} + \frac{2Ax^{\frac{3}{2}}}{3b} + \frac{A \left(- \frac{a}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2b} - \frac{A \left(- \frac{a}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2b} - \frac{A \left(- \frac{a}{b}\right)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b} + \frac{2Ba^{2} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{3} \sqrt[4]{- \frac{a}{b}}} - \frac{2Bax^{\frac{3}{2}}}{3b^{2}} - \frac{Ba \left(- \frac{a}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2b^{2}} + \frac{Ba \left(- \frac{a}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2b^{2}} + \frac{Ba \left(- \frac{a}{b}\right)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{2}} + \frac{2Bx^{\frac{7}{2}}}{7b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(7/2)/7 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(
7/2)/7)/b, Eq(a, 0)), (-2*A*a*atan(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b)**(1/
4)) + 2*A*x**(3/2)/(3*b) + A*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*
b) - A*(-a/b)**(3/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - A*(-a/b)**(3/4)*a
tan(sqrt(x)/(-a/b)**(1/4))/b + 2*B*a**2*atan(sqrt(x)/(-a/b)**(1/4))/(b**3*(
-a/b)**(1/4)) - 2*B*a*x**(3/2)/(3*b**2) - B*a*(-a/b)**(3/4)*log(sqrt(x) - (
-a/b)**(1/4))/(2*b**2) + B*a*(-a/b)**(3/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*
b**2) + B*a*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 + 2*B*x**(7/2)/(
7*b), True))
```

Giac [A]
time = 1.00, size = 264, normalized size = 1.03

$$\frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{3}{4}} Ba - \left(ab^{3}\right)^{\frac{3}{4}} Ab\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 b^{5}} + \frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{3}{4}} Ba - \left(ab^{3}\right)^{\frac{3}{4}} Ab\right) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 b^{5}} - \frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{3}{4}} Ba - \left(ab^{3}\right)^{\frac{3}{4}} Ab\right) \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 b^{5}} + \frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{3}{4}} Ba - \left(ab^{3}\right)^{\frac{3}{4}} Ab\right) \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 b^{5}} + \frac{2 \left(3 B b^{6} x^{\frac{7}{2}} - 7 B a b^{5} x^{\frac{3}{2}} + 7 A b^{6} x^{\frac{3}{2}}\right)}{21 b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 + 1/2*sqrt(2)*((a*b^3)^(3/4)
*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/b^5 - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((a*b^3
)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/b^5 + 2/21*(3*B*b^6*x^(7/2) - 7*B*a*b^5*x^(3/2) + 7*A*b^6*x^(3/2))/
b^7
```

Mupad [B]
time = 0.10, size = 92, normalized size = 0.36

$$x^{3/2}\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)+\frac{2\,B\,x^{7/2}}{7\,b}+\frac{{\left(-a\right)}^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b-B\,a\right)}{b^{11/4}}+\frac{{\left(-a\right)}^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b-B\,a\right)\,1{}\mathrm{i}}{b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(7/2))/(7*b) + ((-a)^(3/4)
*atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/b^(11/4) + ((-a)^(3/4)*ata
n((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*(A*b - B*a)*1i)/b^(11/4)
```

3.369 $\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=255

$$\frac{2(Ab-aB)\sqrt{x}}{b^2}+\frac{2Bx^{5/2}}{5b}+\frac{\sqrt[4]{a}\,(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}-\frac{\sqrt[4]{a}\,(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{}}{\sqrt[4]{a}}\right.}{\sqrt{2}\,b^{9/4}}$$

```
[Out] 2/5*B*x^(5/2)/b+1/2*a^(1/4)*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1
/4))/b^(9/4)*2^(1/2)-1/2*a^(1/4)*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)
/a^(1/4))/b^(9/4)*2^(1/2)+1/4*a^(1/4)*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4
)*b^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)-1/4*a^(1/4)*(A*b-B*a)*ln(a^(1/2)
+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+2*(A*b-B*a)*x^(
1/2)/b^2
```

Rubi [A]

time = 0.14, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {470, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{a}\,(Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}-\frac{\sqrt[4]{a}\,(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{9/4}}+\frac{\sqrt[4]{a}\,(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{\sqrt[4]{a}\,(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{2\sqrt{x}\,(Ab-aB)}{b^2}+\frac{2Bx^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(5/2))/(5*b) + (a^(1/4)*(A*b - a*B)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) - (a^(1/4)*(
A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4))
 + (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(2*Sqrt[2]*b^(9/4)) - (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{2Bx^{5/2}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^2}\,dx}{5b}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{(a(Ab-aB))\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{(2a(Ab-aB))\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{\left(\sqrt{a}\,(Ab-aB)\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2} -$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{\left(\sqrt{a}\,(Ab-aB)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{\right.}{2b^{5/2}}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} + \frac{\sqrt[4]{a}\,(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} + \frac{\sqrt[4]{a}\,(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}} - \frac{\sqrt[4]{a}\,(Ab-}{}$$

Mathematica [A]
time = 0.19, size = 150, normalized size = 0.59

$$\frac{2\sqrt{x}\,(5Ab-5aB+bBx^2)}{5b^2} - \frac{\sqrt[4]{a}\,(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\,b^{9/4}} + \frac{\sqrt[4]{a}\,(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x]*(5*A*b - 5*a*B + b*B*x^2))/(5*b^2) - (a^(1/4)*(-(A*b) + a*B)*Arc
Tan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(9
/4)) + (a^(1/4)*(-(A*b) + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(S
qrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(9/4))
```

Maple [A]

time = 0.09, size = 141, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^2}-\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4b^2}$
default	$\frac{\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^2}-\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4b^2}$
risch	$\frac{2\left(bB\,x^2+5Ab-5Ba\right)\sqrt{x}}{5b^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/5*b*B*x^(5/2)+A*b*x^(1/2)-B*a*x^(1/2))-1/4*(A*b-B*a)/b^2*(a/b)^(1/
4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]

time = 0.52, size = 235, normalized size = 0.92

$$\frac{\left(\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a}{4\,b^2}+\frac{2\left(Bbx^{\frac{5}{2}}-5\,(Ba-Ab)\sqrt{x}\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^2 + 2/5*(B*b*x^(5/2) - 5*(B*a
 - A*b)*sqrt(x))/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(182) = 364$.

time = 1.24, size = 660, normalized size = 2.59

$$-\frac{20\,b^2\left(-\frac{B^4a^5-4AB^3a^4b+6A^2B^2a^3b^2-4A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4\sqrt{-\frac{B^4a^5-4AB^3a^4b+6A^2B^2a^3b^2-4A^3Ba^2b^3+A^4ab^4}{b^9}}+(B^2a^2-2ABab+A^2b^2)x}\;b^7\left(-\frac{B^4a^5-4AB^3a^4b+6A^2B^2a^3b^2-4A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{3}{4}}+(Bab^7-Ab^8)\sqrt{x}\left(-\frac{B^4a^5-4AB^3a^4b+6A^2B^2a^3b^2-4A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{3}{4}}}{B^4a^5-4AB^3a^4b+6A^2B^2a^3b^2-4A^3Ba^2b^3+A^4ab^4}\right)+5\,b^2\left(-\frac{B^4a^5-4AB^3a^4b+6A^2B^2a^3b^2-4A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{B^4a^5-4AB^3a^4b+6A^2B^2a^3b^2-4A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)-5\,b^2\left(-\frac{B^4a^5-4AB^3a^4b+6A^2B^2a^3b^2-4A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}\log\left(-b^2\left(-\frac{B^4a^5-4AB^3a^4b+6A^2B^2a^3b^2-4A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)-4\,(Bbx^2-5\,Ba+5\,Ab)\sqrt{x}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*
b^3 + A^4*a*b^4)/b^9)^(1/4)*arctan((sqrt(b^4*sqrt(-(B^4*a^5 - 4*A*B^3*a^4*b
 + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9) + (B^2*a^2 - 2*A*B
*a*b + A^2*b^2)*x)*b^7*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A
^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(3/4) + (B*a*b^7 - A*b^8)*sqrt(x)*(-(B^4*a^5
 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(3
/4))/(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a
*b^4)) + 5*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2
*b^3 + A^4*a*b^4)/b^9)^(1/4)*log(b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2
*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) -
 5*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A
^4*a*b^4)/b^9)^(1/4)*log(-b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^
2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) - 4*(B*b
*x^2 - 5*B*a + 5*A*b)*sqrt(x))/b^2
```

Sympy [A]
time = 5.16, size = 275, normalized size = 1.08

$$\begin{cases} \tilde{\infty} \left(2 A \sqrt{x} + \frac{2 B x^{\frac{5}{2}}}{5}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{\frac{2 A x^{\frac{5}{2}}}{5} + \frac{2 B x^{\frac{9}{2}}}{9}}{a} & \text{for}\: b = 0 \\ \frac{2 A \sqrt{x} + \frac{2 B x^{\frac{5}{2}}}{5}}{b} & \text{for}\: a = 0 \\ \frac{2 A \sqrt{x}}{b} + \frac{A \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 b} - \frac{A \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 b} - \frac{A \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b} - \frac{2 B a \sqrt{x}}{b^{2}} - \frac{B a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{2}} + \frac{B a \sqrt[4]{- \frac{a}{b}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{2}} + \frac{B a \sqrt[4]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{2}} + \frac{2 B x^{\frac{5}{2}}}{5 b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*A*
x**(5/2)/5 + 2*B*x**(9/2)/9)/a, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/
b, Eq(a, 0)), (2*A*sqrt(x)/b + A*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))
/(2*b) - A*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - A*(-a/b)**(1/
4)*atan(sqrt(x)/(-a/b)**(1/4))/b - 2*B*a*sqrt(x)/b**2 - B*a*(-a/b)**(1/4)*l
og(sqrt(x) - (-a/b)**(1/4))/(2*b**2) + B*a*(-a/b)**(1/4)*log(sqrt(x) + (-a/
b)**(1/4))/(2*b**2) + B*a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 +
2*B*x**(5/2)/(5*b), True))
```

Giac [A]
time = 1.07, size = 263, normalized size = 1.03

$$\frac{\sqrt{2} \left(\left(a b^{3}\right)^{\frac{1}{4}} B a - \left(a b^{3}\right)^{\frac{1}{4}} A b\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 b^{3}} + \frac{\sqrt{2} \left(\left(a b^{3}\right)^{\frac{1}{4}} B a - \left(a b^{3}\right)^{\frac{1}{4}} A b\right) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 b^{3}} + \frac{\sqrt{2} \left(\left(a b^{3}\right)^{\frac{1}{4}} B a - \left(a b^{3}\right)^{\frac{1}{4}} A b\right) \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 b^{3}} - \frac{\sqrt{2} \left(\left(a b^{3}\right)^{\frac{1}{4}} B a - \left(a b^{3}\right)^{\frac{1}{4}} A b\right) \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 b^{3}} + \frac{2 \left(B b^{4} x^{\frac{5}{2}} - 5 B a b^{3} \sqrt{x} + 5 A b^{4} \sqrt{x}\right)}{5 b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/2*sqrt(2)*((a*b^3)^(1/4)
*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*((a*b^3
)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/b^3 + 2/5*(B*b^4*x^(5/2) - 5*B*a*b^3*sqrt(x) + 5*A*b^4*sqrt(x))/b^5
```

Mupad [B]
time = 0.21, size = 789, normalized size = 3.09

$$\sqrt{x}\,\left(\frac{2\,A}{b}-\frac{2\,B\,a}{b^2}\right)+\frac{2\,B\,x^{5/2}}{5\,b}-\frac{{\left(-a\right)}^{1/4}\,\mathrm{atan}\left(\frac{\frac{{\left(-a\right)}^{1/4}\,\left(A\,b-B\,a\right)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}-\frac{{\left(-a\right)}^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)}{2\,b^{9/4}}\right)\,1{}\mathrm{i}}{2\,b^{9/4}}+\frac{{\left(-a\right)}^{1/4}\,\left(A\,b-B\,a\right)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}+\frac{{\left(-a\right)}^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)}{2\,b^{9/4}}\right)\,1{}\mathrm{i}}{2\,b^{9/4}}}{\frac{{\left(-a\right)}^{1/4}\,\left(A\,b-B\,a\right)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}-\frac{{\left(-a\right)}^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)}{2\,b^{9/4}}\right)}{2\,b^{9/4}}-\frac{{\left(-a\right)}^{1/4}\,\left(A\,b-B\,a\right)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}+\frac{{\left(-a\right)}^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)}{2\,b^{9/4}}\right)}{2\,b^{9/4}}}\right)\,\left(A\,b-B\,a\right)\,1{}\mathrm{i}}{b^{9/4}}-\frac{{\left(-a\right)}^{1/4}\,\mathrm{atan}\left(\frac{\frac{{\left(-a\right)}^{1/4}\,\left(A\,b-B\,a\right)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}-\frac{{\left(-a\right)}^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)\,1{}\mathrm{i}}{2\,b^{9/4}}\right)}{2\,b^{9/4}}+\frac{{\left(-a\right)}^{1/4}\,\left(A\,b-B\,a\right)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}+\frac{{\left(-a\right)}^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)\,1{}\mathrm{i}}{2\,b^{9/4}}\right)}{2\,b^{9/4}}}{\frac{{\left(-a\right)}^{1/4}\,\left(A\,b-B\,a\right)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}-\frac{{\left(-a\right)}^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)\,1{}\mathrm{i}}{2\,b^{9/4}}\right)\,1{}\mathrm{i}}{2\,b^{9/4}}-\frac{{\left(-a\right)}^{1/4}\,\left(A\,b-B\,a\right)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}+\frac{{\left(-a\right)}^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)\,1{}\mathrm{i}}{2\,b^{9/4}}\right)\,1{}\mathrm{i}}{2\,b^{9/4}}}\right)\,\left(A\,b-B\,a\right)}{b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(5/2))/(5*b) - ((-a)^(1/4)*atan(((
(-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/
b - ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a))/(2*b^(9/4)))*1i)/(
2*b^(9/4)) + ((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 -
2*A*B*a^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a))/(2*b
^(9/4)))*1i)/(2*b^(9/4)))/(((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 +
A^2*a^2*b^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*
b - B*a))/(2*b^(9/4))))/(2*b^(9/4)) - ((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*
(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B
*a^3*b)*(A*b - B*a))/(2*b^(9/4))))/(2*b^(9/4))))*(A*b - B*a)*1i)/b^(9/4) -
((-a)^(1/4)*atan((((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b
^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a)*
1i)/(2*b^(9/4))))/(2*b^(9/4)) + ((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a
^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b
)*(A*b - B*a)*1i)/(2*b^(9/4))))/(2*b^(9/4)))/(((-a)^(1/4)*(A*b - B*a)*((16*
x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(32*A*a^2*b^
2 - 32*B*a^3*b)*(A*b - B*a)*1i)/(2*b^(9/4)))*1i)/(2*b^(9/4)) - ((-a)^(1/4)*
(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^(
1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a)*1i)/(2*b^(9/4)))*1i)/(2*b^(9/4
))))*(A*b - B*a))/b^(9/4)
```

3.370 $\int \frac{\sqrt{x}\,(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=237

$$\frac{2Bx^{3/2}}{3b}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}+\frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}+\frac{(Ab-aB)\log\left(\sqrt{a}\right.}{2}$$

```
[Out] 2/3*B*x^(3/2)/b-1/2*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
1/4)/b^(7/4)*2^(1/2)+1/2*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4)
)/a^(1/4)/b^(7/4)*2^(1/2)+1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4
)*2^(1/2)*x^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)-1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/
2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {470, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}+\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}+\frac{(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}-\frac{(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}+\frac{2Bx^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(7/4)) -
 ((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*a^(1/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\,(A+Bx^2)}{a+bx^2}\,dx = \frac{2Bx^{3/2}}{3b} - \frac{\left(2\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{3b}$$

$$= \frac{2Bx^{3/2}}{3b} - \frac{\left(4\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{3b}$$

$$= \frac{2Bx^{3/2}}{3b} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{a}+}{a+}\right.}{b^{3/2}}$$

$$= \frac{2Bx^{3/2}}{3b} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} + \frac{(Ab-aB)\text{Subst}\left(\right.}{}$$

$$= \frac{2Bx^{3/2}}{3b} + \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{7/4}} - \frac{(Ab-aB)\log\left(\sqrt{a}\right.}{2\sqrt{}}$$

$$= \frac{2Bx^{3/2}}{3b} - \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{7/4}} + \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}\,b^{7/4}}$$

Mathematica [A]
time = 0.17, size = 135, normalized size = 0.57

$$\frac{2Bx^{3/2}}{3b} + \frac{(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{7/4}} + \frac{(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) + ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((-(A*b) + a*B)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(
1/4)*b^(7/4))
```

Maple [A]
time = 0.08, size = 124, normalized size = 0.52

method	result

derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3b}+\frac{(Ab-Ba)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\right.\right.}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{2Bx^{\frac{3}{2}}}{3b}+\frac{(Ab-Ba)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\right.\right.}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$\frac{2Bx^{\frac{3}{2}}}{3b}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,A\arctan\left(\right.}{2b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B*x^(3/2)/b+1/4*(A*b-B*a)/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1
))
```

Maxima [A]
time = 0.53, size = 194, normalized size = 0.82

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b}-\frac{(Ba-Ab)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/b - 1/4*(B*a - A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^
(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqr
t(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) -
sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)
*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(1/4)*b^(3/4)))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(166) = 332$.
time = 2.91, size = 834, normalized size = 3.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*B*x^(3/2) - 12*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*
A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*arctan((sqrt((B^6*a^6 - 6*A*B^5*a^5*b
 + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a
*b^5 + A^6*b^6)*x - (B^4*a^5*b^3 - 4*A*B^3*a^4*b^4 + 6*A^2*B^2*a^3*b^5 - 4*
A^3*B*a^2*b^6 + A^4*a*b^7)*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b
^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7)))*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*
A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4) + (B^3*a^3*b^2 -
3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5)*sqrt(x)*(-(B^4*a^4 - 4*A*B^3*a^3
*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4))/(B^4*a^4
- 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)) + 3*b*(-(B^
4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7
))^(1/4)*log(a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B
*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2
 - A^3*b^3)*sqrt(x)) - 3*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*log(-a*b^5*(-(B^4*a^4 - 4*A*B^3*a^
3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^
3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)))/b
```

Sympy [A]
time = 2.25, size = 303, normalized size = 1.28

$$\begin{cases} \tilde{\infty} \left(- \frac{2 A}{\sqrt{x}} + \frac{2 B x^{\frac{3}{2}}}{3}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{- \frac{2 A}{\sqrt{x}} + \frac{2 B x^{\frac{3}{2}}}{3}}{b} & \text{for}\: a = 0 \\ \frac{\frac{2 A x^{\frac{3}{2}}}{3} + \frac{2 B x^{\frac{7}{2}}}{7}}{a} & \text{for}\: b = 0 \\ \frac{2 A \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b \sqrt[4]{- \frac{a}{b}}} - \frac{A \left(- \frac{a}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 a} + \frac{A \left(- \frac{a}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 a} + \frac{A \left(- \frac{a}{b}\right)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{a} - \frac{2 B a \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{2} \sqrt[4]{- \frac{a}{b}}} + \frac{2 B x^{\frac{3}{2}}}{3 b} + \frac{B \left(- \frac{a}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 b} - \frac{B \left(- \frac{a}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 b} - \frac{B \left(- \frac{a}{b}\right)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)), ((-2*
A/sqrt(x) + 2*B*x**(3/2)/3)/b, Eq(a, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(7/2)/7
)/a, Eq(b, 0)), (2*A*atan(sqrt(x)/(-a/b)**(1/4))/(b*(-a/b)**(1/4)) - A*(-a/
b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) + A*(-a/b)**(3/4)*log(sqrt(x)
+ (-a/b)**(1/4))/(2*a) + A*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))/a - 2*
B*a*atan(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b)**(1/4)) + 2*B*x**(3/2)/(3*b) +
 B*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b) - B*(-a/b)**(3/4)*log(s
qrt(x) + (-a/b)**(1/4))/(2*b) - B*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))
/b, True))
```

Giac [A]
time = 1.17, size = 251, normalized size = 1.06

$$\frac{2 \, B x^{\frac{3}{2}}}{3 \, b} - \frac{\sqrt{2} {\left(\left(a b^{3}\right)^{\frac{3}{4}} B a - \left(a b^{3}\right)^{\frac{3}{4}} A b\right)} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \, \sqrt{x}\right)}}{2 \, \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 \, a b^{4}} - \frac{\sqrt{2} {\left(\left(a b^{3}\right)^{\frac{3}{4}} B a - \left(a b^{3}\right)^{\frac{3}{4}} A b\right)} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \, \sqrt{x}\right)}}{2 \, \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 \, a b^{4}} + \frac{\sqrt{2} {\left(\left(a b^{3}\right)^{\frac{3}{4}} B a - \left(a b^{3}\right)^{\frac{3}{4}} A b\right)} \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 \, a b^{4}} - \frac{\sqrt{2} {\left(\left(a b^{3}\right)^{\frac{3}{4}} B a - \left(a b^{3}\right)^{\frac{3}{4}} A b\right)} \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 \, a b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b - 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 1/2*
sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(3/4
)*B*a - (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a*b^4) - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4)
```

Mupad [B]
time = 0.08, size = 71, normalized size = 0.30

$$\frac{2\,B\,x^{3/2}}{3\,b}+\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b-B\,a\right)}{{\left(-a\right)}^{1/4}\,b^{7/4}}-\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b-B\,a\right)}{{\left(-a\right)}^{1/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] (2*B*x^(3/2))/(3*b) + (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a
)^(1/4)*b^(7/4)) - (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a)^
(1/4)*b^(7/4))
```

3.371 $\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)}\,dx$

Optimal. Leaf size=235

$$\frac{2B\sqrt{x}}{b}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{(Ab-aB)\log\left(\sqrt{a}\right.}{2}$$

```
[Out] -1/2*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2^
(1/2)+1/2*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(5/
4)*2^(1/2)-1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/
2))/a^(3/4)/b^(5/4)*2^(1/2)+1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1
/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+2*B*x^(1/2)/b
```

Rubi [A]
time = 0.13, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {470, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(Sqrt[2]*a^(3/4)*b^(5/4)) + ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(5/4)) - ((A*b - a*B)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((A
*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(3/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{2B\sqrt{x}}{b} - \frac{\left(2\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{b}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{\left(4\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\,b} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{a}+\cdots}{a+b\cdots}\right.}{\sqrt{a}\,b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\,b^{3/2}} + \frac{(Ab-aB)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(Ab-aB)\log\left(\sqrt{a}\cdots\right.}{2\sqrt{\cdots}}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}}\right.}{\sqrt{2}\,a^{3/4}b^{5/4}}$$

Mathematica [A]
time = 0.16, size = 134, normalized size = 0.57

$$\frac{2B\sqrt{x}}{b} + \frac{(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*B*Sqrt[x])/b + ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(3/4)*b^(5/4)) - ((-(A*b) + a*B)*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(3/4)
*b^(5/4))
```

Maple [A]
time = 0.08, size = 127, normalized size = 0.54

method	result

derivativedivides	$\frac{2B\sqrt{x}}{b}+\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{}{}\right.}\right.\right.}{4ba}$
default	$\frac{2B\sqrt{x}}{b}+\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{}{}\right.}\right.\right.}{4ba}$
risch	$\frac{2B\sqrt{x}}{b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B*x^(1/2)/b+1/4*(A*b-B*a)/b*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 218, normalized size = 0.93

$$\frac{2\,B\sqrt{x}}{b}-\frac{\frac{2\,\sqrt{2}\,(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*B*sqrt(x)/b - 1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1
/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + 2*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))) + sqrt(2)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(166) = 332.
time = 1.14, size = 645, normalized size = 2.74

$$\frac{4\,b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2b^2\sqrt{-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}}+(B^2a^2-2\,ABab+A^2b^2)x}\;a^2b^4\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{3}{4}}+(Ba^3b^4-Aa^2b^5)\sqrt{x}\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{3}{4}}}{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}\right)+b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)-b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}\log\left(-ab\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)+4\,B\sqrt{x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A
^4*b^4)/(a^3*b^5))^(1/4)*arctan((sqrt(a^2*b^2*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*
b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5)) + (B^2*a^2 - 2*
A*B*a*b + A^2*b^2)*x)*a^2*b^4*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(3/4) + (B*a^3*b^4 - A*a^2*b^5)*sqr
t(x)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a^3*b^5))^(3/4))/(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)) + b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*
A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4)*log(a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b
+ 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*
b)*sqrt(x)) - b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*
b^3 + A^4*b^4)/(a^3*b^5))^(1/4)*log(-a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2
*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*sqrt
(x)) + 4*B*sqrt(x))/b
```

Sympy [A]
time = 1.87, size = 238, normalized size = 1.01

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) & \text{for}\ a=0\wedge b=0 \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b} & \text{for}\ a=0 \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{a} & \text{for}\ b=0 \\ -\frac{A\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a}+\frac{A\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a}+\frac{A\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a}+\frac{2B\sqrt{x}}{b}+\frac{B\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{B\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{B\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(3*x**(3/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/
5)/a, Eq(b, 0)), (-A*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) + A*(
-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a) + A*(-a/b)**(1/4)*atan(sqrt
(x)/(-a/b)**(1/4))/a + 2*B*sqrt(x)/b + B*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)
**(1/4))/(2*b) - B*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - B*(-a
/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b, True))
```

Giac [A]
time = 1.34, size = 251, normalized size = 1.07

$$\frac{2B\sqrt{x}}{b}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^2}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^2}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab^2}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b - 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) - 1/2*sq
rt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*
```

```
(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*((a*b^3)^(1/4)*
B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
a*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
```

Mupad [B]
time = 0.12, size = 739, normalized size = 3.14

$$\frac{2B\sqrt{x}}{b}-\frac{\operatorname{atan}\left(\frac{\frac{(Ab-Ba)\left(\sqrt{x}\,(16A^2b^3-32ABab^2+16B^2a^2b)-\frac{(32Ba^2b^2-32Aab^3)(Ab-Ba)}{2(-a)^{3/4}b^{5/4}}\right)1i}{2(-a)^{3/4}b^{5/4}}+\frac{(Ab-Ba)\left(\sqrt{x}\,(16A^2b^3-32ABab^2+16B^2a^2b)+\frac{(32Ba^2b^2-32Aab^3)(Ab-Ba)}{2(-a)^{3/4}b^{5/4}}\right)1i}{2(-a)^{3/4}b^{5/4}}}{\frac{(Ab-Ba)\left(\sqrt{x}\,(16A^2b^3-32ABab^2+16B^2a^2b)-\frac{(32Ba^2b^2-32Aab^3)(Ab-Ba)}{2(-a)^{3/4}b^{5/4}}\right)}{2(-a)^{3/4}b^{5/4}}-\frac{(Ab-Ba)\left(\sqrt{x}\,(16A^2b^3-32ABab^2+16B^2a^2b)+\frac{(32Ba^2b^2-32Aab^3)(Ab-Ba)}{2(-a)^{3/4}b^{5/4}}\right)}{2(-a)^{3/4}b^{5/4}}}\right)(Ab-Ba)\,1i}{(-a)^{3/4}b^{5/4}}-\frac{\operatorname{atan}\left(\frac{\frac{(Ab-Ba)\left(\sqrt{x}\,(16A^2b^3-32ABab^2+16B^2a^2b)-\frac{(32Ba^2b^2-32Aab^3)(Ab-Ba)1i}{2(-a)^{3/4}b^{5/4}}\right)}{2(-a)^{3/4}b^{5/4}}+\frac{(Ab-Ba)\left(\sqrt{x}\,(16A^2b^3-32ABab^2+16B^2a^2b)+\frac{(32Ba^2b^2-32Aab^3)(Ab-Ba)1i}{2(-a)^{3/4}b^{5/4}}\right)}{2(-a)^{3/4}b^{5/4}}}{\frac{(Ab-Ba)\left(\sqrt{x}\,(16A^2b^3-32ABab^2+16B^2a^2b)-\frac{(32Ba^2b^2-32Aab^3)(Ab-Ba)1i}{2(-a)^{3/4}b^{5/4}}\right)1i}{2(-a)^{3/4}b^{5/4}}-\frac{(Ab-Ba)\left(\sqrt{x}\,(16A^2b^3-32ABab^2+16B^2a^2b)+\frac{(32Ba^2b^2-32Aab^3)(Ab-Ba)1i}{2(-a)^{3/4}b^{5/4}}\right)1i}{2(-a)^{3/4}b^{5/4}}}\right)(Ab-Ba)}{(-a)^{3/4}b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(a + b*x^2)),x)
```

```
[Out] (2*B*x^(1/2))/b - (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b -
 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a))/(2*(-a)^(3/4)*b^
(5/4)))*1i)/(2*(-a)^(3/4)*b^(5/4)) + ((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 16
*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a))/(2*(
-a)^(3/4)*b^(5/4)))*1i)/(2*(-a)^(3/4)*b^(5/4)))/(((A*b - B*a)*(x^(1/2)*(16*
A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b
- B*a))/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4)) - ((A*b - B*a)*(x^(
1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B*a^2*b^2 - 32*A*a*b
^3)*(A*b - B*a))/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4))))*(A*b - B
*a)*1i)/((-a)^(3/4)*b^(5/4)) - (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 1
6*B^2*a^2*b - 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a)*1i)/
(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4)) + ((A*b - B*a)*(x^(1/2)*(16
*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b
 - B*a)*1i)/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4)))/(((A*b - B*a)*
(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A
*a*b^3)*(A*b - B*a)*1i)/(2*(-a)^(3/4)*b^(5/4)))*1i)/(2*(-a)^(3/4)*b^(5/4))
- ((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B
*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a)*1i)/(2*(-a)^(3/4)*b^(5/4)))*1i)/(2*(-a)^
(3/4)*b^(5/4))))*(A*b - B*a))/((-a)^(3/4)*b^(5/4))
```

3.372 $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx$

Optimal. Leaf size=235

$$-\frac{2A}{a\sqrt{x}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{(Ab-aB)\log\left(\sqrt{a}\right.}{2}$$

```
[Out] 1/2*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(3/4)*2^(
1/2)-1/2*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(3/4
)*2^(1/2)-1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2
))/a^(5/4)/b^(3/4)*2^(1/2)+1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/
4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(3/4)*2^(1/2)-2*A/a/x^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {464, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{(Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{2A}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(3/4)) +
((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(5/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx = -\frac{2A}{a\sqrt{x}} - \frac{\left(2\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{\left(4\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a\sqrt{b}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{}}{a+b}\right.}{a\sqrt{b}}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2ab} - \frac{(Ab-aB)\text{Subst}\left(\right.}{}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{(Ab-aB)\log\left(\sqrt{a}\right.}{2\sqrt{}}$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}} - \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{}}{\sqrt[4]{a}}\right.}{\sqrt{2}\,a^{5/4}b^{3/4}}$$

Mathematica [A]
time = 0.19, size = 135, normalized size = 0.57

$$-\frac{2A}{a\sqrt{x}} - \frac{(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}} - \frac{(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) - ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((-(A*b) + a*B)*ArcT
anh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(5
/4)*b^(3/4))
```

Maple [A]
time = 0.10, size = 127, normalized size = 0.54

method	result

derivativedivides	$-\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}-$
default	$-\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}-$
risch	$-\frac{2A}{a\sqrt{x}}-\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}}\right.}{4a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(A*b-B*a)/a/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*A/a/x^(1/2
)
```

Maxima [A]
time = 0.53, size = 194, normalized size = 0.83

$$\frac{(Ba-Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4a}-\frac{2A}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*a - A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3
/4)))/a - 2*A/(a*sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(166) = 332$.
time = 0.86, size = 843, normalized size = 3.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
 A^4*b^4)/(a^5*b^3))^(1/4)*arctan((sqrt((B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6)*x - (B^4*a^7*b - 4*A*B^3*a^6*b^2 + 6*A^2*B^2*a^5*b^3 - 4*A^3*B*a^4*b^4
 + A^4*a^3*b^5)*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^5*b^3)))*a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a
^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4) + (B^3*a^4*b - 3*A*B^2*a
^3*b^2 + 3*A^2*B*a^2*b^3 - A^3*a*b^4)*sqrt(x)*(-(B^4*a^4 - 4*A*B^3*a^3*b +
6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4))/(B^4*a^4 - 4
*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)) - a*x*(-(B^4*a
^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3)
)^(1/4)*log(a^4*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*
b^2 - A^3*b^3)*sqrt(x)) + a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-(B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4)
- (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)) - 4*A*sqrt(x
))/(a*x)
```

Sympy [A]
time = 7.05, size = 150, normalized size = 0.64

$$A\left(\begin{cases} \frac{\infty}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for}\: b = 0 \\ -\frac{\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} + \frac{\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}} - \frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}\right) + 2B\operatorname{RootSum}\left(256t^4ab^3 + 1, \left(t \mapsto t\log\left(64t^3ab^2 + \sqrt{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(b*x**2+a),x)
```

```
[Out] A*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a,
0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) - (-a/b)**(1/4))/(2*a*(-a/b)
**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) - atan(sqrt(x)/
(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*sqrt(x)), True)) + 2*B*RootSum(256*
_t**4*a*b**3 + 1, Lambda(_t, _t*log(64*_t**3*a*b**2 + sqrt(x))))
```

Giac [A]
time = 0.71, size = 251, normalized size = 1.07

$$-\frac{2A}{a\sqrt{x}} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^2b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*A/(a*sqrt(x)) + 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arct
an(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1
/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*b^3)
^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(a^2*b^3) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

Mupad [B]
time = 0.08, size = 71, normalized size = 0.30

$$\frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{(-a)^{5/4}\,b^{3/4}}-\frac{2\,A}{a\,\sqrt{x}}-\frac{\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{(-a)^{5/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(a + b*x^2)),x)
```

```
[Out] (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a)^(5/4)*b^(3/4)) - (2*
A)/(a*x^(1/2)) - (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a)^(5
/4)*b^(3/4))
```

3.373 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)} \, dx$

Optimal. Leaf size=237

$$-\frac{2A}{3ax^{3/2}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{(Ab-aB)\log\left(\sqrt{\right.}}{}$$

```
[Out] -2/3*A/a/x^(3/2)+1/2*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^
(7/4)/b^(1/4)*2^(1/2)-1/2*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4
))/a^(7/4)/b^(1/4)*2^(1/2)+1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/
4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(1/4)*2^(1/2)-1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1
/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {464, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{(Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) - ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) + ((A*b - a*B)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(1/4))
- ((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx = -\frac{2A}{3ax^{3/2}} - \frac{\left(2\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{3a}$$

$$= -\frac{2A}{3ax^{3/2}} - \frac{\left(4\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{3a}$$

$$= -\frac{2A}{3ax^{3/2}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{a}+\cdots}{a+\cdots}\right.}{a^{3/2}}$$

$$= -\frac{2A}{3ax^{3/2}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}\sqrt{b}} - \frac{(Ab-aB)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{2A}{3ax^{3/2}} + \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}} - \frac{(Ab-aB)\log\left(\sqrt{a}\cdots\right.}{2\sqrt{\cdots}}$$

$$= -\frac{2A}{3ax^{3/2}} + \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}} - \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}}\right.}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}}$$

Mathematica [A]
time = 0.19, size = 136, normalized size = 0.57

$$-\frac{2A}{3ax^{3/2}} - \frac{(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) - ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(7/4)*b^(1/4)) + ((-(A*b) + a*B)*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^
(7/4)*b^(1/4))
```

Maple [A]
time = 0.09, size = 124, normalized size = 0.52

method	result

derivativedivides	$\frac{(-Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right.}{4a^2}$
default	$\frac{(-Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right.}{4a^2}$
risch	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)b}{2a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)b}{2a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-A*b+B*a)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/3*A/a/x^(3
/2)
```

Maxima [A]
time = 0.51, size = 218, normalized size = 0.92

$$\frac{\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4a}-\frac{2A}{3ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a - 2/3*A/(a*x^(3/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(166) = 332.
time = 2.51, size = 653, normalized size = 2.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4)/(a^7*b))^(1/4)*arctan((sqrt(a^4*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*
b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b)) + (B^2*a^2 - 2*A*
B*a*b + A^2*b^2)*x)*a^5*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(3/4) + (B*a^6*b - A*a^5*b^2)*sqrt(x)*(-(
B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7
*b))^(3/4))/(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
A^4*b^4)) + 3*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4)*log(a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^
2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(
x)) - 3*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^
3 + A^4*b^4)/(a^7*b))^(1/4)*log(-a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2
*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(x)) +
 4*A*sqrt(x))/(a*x^2)
```

Sympy [A]
time = 9.19, size = 257, normalized size = 1.08

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{3x^{\frac{3}{2}}}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{3x^{\frac{3}{2}}}}{b} & \text{for}\: a = 0 \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}} + 2B\sqrt{x}}{a} & \text{for}\: b = 0 \\ -\frac{2A}{3ax^{\frac{3}{2}}} + \frac{Ab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a^2} - \frac{Ab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a^2} - \frac{Ab\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2} - \frac{B\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{B\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{B\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*sqrt(x))/a, Eq(b, 0)), (-2*A/(3*a*x**(3/2)) + A*b*(-a/b)**(1/4)*log(
sqrt(x) - (-a/b)**(1/4))/(2*a**2) - A*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)*
*(1/4))/(2*a**2) - A*b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a**2 - B*(
-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) + B*(-a/b)**(1/4)*log(sqrt(
x) + (-a/b)**(1/4))/(2*a) + B*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a,
True))
```

Giac [A]
time = 0.84, size = 251, normalized size = 1.06

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^2b} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba - \left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^2b} - \frac{2A}{3ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/2*sqrt(2)*((a*b^3)^(
1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*
sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1
/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 1/4*sqr
t(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/
4) + x + sqrt(a/b))/(a^2*b) - 2/3*A/(a*x^(3/2))
```

Mupad [B]

time = 0.21, size = 811, normalized size = 3.42

$$-\frac{2\,A}{3\,a\,x^{3/2}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(A\,b-B\,a\right)\,\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{\left(A\,b-B\,a\right)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}\right)\,1{}\mathrm{i}}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}+\frac{\left(A\,b-B\,a\right)\,\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)+\frac{\left(A\,b-B\,a\right)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}\right)\,1{}\mathrm{i}}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}}{\frac{\left(A\,b-B\,a\right)\,\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{\left(A\,b-B\,a\right)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}\right)}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}-\frac{\left(A\,b-B\,a\right)\,\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)+\frac{\left(A\,b-B\,a\right)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}\right)}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}}\right)\,\left(A\,b-B\,a\right)\,1{}\mathrm{i}}{{\left(-a\right)}^{7/4}\,b^{1/4}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(A\,b-B\,a\right)\,\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{\left(A\,b-B\,a\right)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)\,1{}\mathrm{i}}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}\right)}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}+\frac{\left(A\,b-B\,a\right)\,\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)+\frac{\left(A\,b-B\,a\right)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)\,1{}\mathrm{i}}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}\right)}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}}{\frac{\left(A\,b-B\,a\right)\,\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{\left(A\,b-B\,a\right)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)\,1{}\mathrm{i}}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}\right)\,1{}\mathrm{i}}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}-\frac{\left(A\,b-B\,a\right)\,\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)+\frac{\left(A\,b-B\,a\right)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)\,1{}\mathrm{i}}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}\right)\,1{}\mathrm{i}}{2\,{\left(-a\right)}^{7/4}\,b^{1/4}}}\right)\,\left(A\,b-B\,a\right)}{{\left(-a\right)}^{7/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(a + b*x^2)),x)
```

```
[Out] - (2*A)/(3*a*x^(3/2)) - (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B
^2*a^5*b^3 - 32*A*B*a^4*b^4) - ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3))/
(2*(-a)^(7/4)*b^(1/4)))*1i)/(2*(-a)^(7/4)*b^(1/4)) + ((A*b - B*a)*(x^(1/2)*
(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A*a^5
*b^4 - 32*B*a^6*b^3))/(2*(-a)^(7/4)*b^(1/4)))*1i)/(2*(-a)^(7/4)*b^(1/4)))/(
((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4) -
((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3))/(2*(-a)^(7/4)*b^(1/4))))/(2*(-a
)^(7/4)*b^(1/4)) - ((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 -
 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3))/(2*(-a)^(7/4
)*b^(1/4))))/(2*(-a)^(7/4)*b^(1/4))))*(A*b - B*a)*1i)/((-a)^(7/4)*b^(1/4))
- (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^
4*b^4) - ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)*1i)/(2*(-a)^(7/4)*b^(1/
4))))/(2*(-a)^(7/4)*b^(1/4)) + ((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B
^2*a^5*b^3 - 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)*1
i)/(2*(-a)^(7/4)*b^(1/4))))/(2*(-a)^(7/4)*b^(1/4)))/(((A*b - B*a)*(x^(1/2)*
(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4) - ((A*b - B*a)*(32*A*a^5
*b^4 - 32*B*a^6*b^3)*1i)/(2*(-a)^(7/4)*b^(1/4)))*1i)/(2*(-a)^(7/4)*b^(1/4))
 - ((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4)
 + ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)*1i)/(2*(-a)^(7/4)*b^(1/4)))*1
i)/(2*(-a)^(7/4)*b^(1/4))))*(A*b - B*a))/((-a)^(7/4)*b^(1/4))
```

3.374 $\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=255

$$-\frac{2A}{5ax^{5/2}}+\frac{2(Ab-aB)}{a^2\sqrt{x}}-\frac{\sqrt[4]{b}\,(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+\frac{\sqrt[4]{b}\,(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}$$

```
[Out] -2/5*A/a/x^(5/2)-1/2*b^(1/4)*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(
1/4))/a^(9/4)*2^(1/2)+1/2*b^(1/4)*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2
)/a^(1/4))/a^(9/4)*2^(1/2)+1/4*b^(1/4)*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/
4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)*2^(1/2)-1/4*b^(1/4)*(A*b-B*a)*ln(a^(1/2
)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)*2^(1/2)+2*(A*b-B*a)/a^
2/x^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {464, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{b}\,(Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+\frac{\sqrt[4]{b}\,(Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{9/4}}+\frac{\sqrt[4]{b}\,(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{\sqrt[4]{b}\,(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}+\frac{2(Ab-aB)}{a^2\sqrt{x}}-\frac{2A}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(5*a*x^(5/2)) + (2*(A*b - a*B))/(a^2*Sqrt[x]) - (b^(1/4)*(A*b - a*B)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(1/4
)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/
4)) + (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)) - (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx = -\frac{2A}{5ax^{5/2}} - \frac{\left(2\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{5a}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{(b(Ab-aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a^2}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{(2b(Ab-aB))\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} - \frac{\left(\sqrt{b}\,(Ab-aB)\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{\left(\sqrt{\right.}{}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{(Ab-aB)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^2} + \frac{(A}{}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{\sqrt[4]{b}\,(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}} - \frac{\sqrt[4]{\ }}{}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} - \frac{\sqrt[4]{b}\,(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}} + \frac{\sqrt[4]{b}\,(Ab-aB}{}$$

Mathematica [A]
time = 0.23, size = 152, normalized size = 0.60

$$-\frac{2(aA-5Abx^2+5aBx^2)}{5a^2x^{5/2}} + \frac{\sqrt[4]{b}\,(-Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\,a^{9/4}} + \frac{\sqrt[4]{b}\,(-Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*A - 5*A*b*x^2 + 5*a*B*x^2))/(5*a^2*x^(5/2)) + (b^(1/4)*(-(A*b) + a*B
)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]
```

```
*a^(9/4)) + (b^(1/4)*(-(A*b) + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(9/4))
```

Maple [A]
time = 0.08, size = 140, normalized size = 0.55

method	result
derivativedivides	$\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-$
default	$\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-$
risch	$-\frac{2\left(-5Ab\,x^{2}+5Ba\,x^{2}+Aa\right)}{5a^{2}x^{\frac{5}{2}}}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)b}{2a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)b}{2a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\sqrt{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(A*b-B*a)/a^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/5*A/a/x^(5/
2)-2/a^2*(-A*b+B*a)/x^(1/2)
```

Maxima [A]
time = 0.50, size = 213, normalized size = 0.84

$$-\frac{(Bab-Ab^{2})\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4a^{2}}-\frac{2\left(5(Ba-Ab)x^{2}+Aa\right)}{5a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(B*a*b - A*b^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt
(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + s
qrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)
*b^(3/4)))/a^2 - 2/5*(5*(B*a - A*b)*x^2 + A*a)/(a^2*x^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(182) = 364$.
time = 1.54, size = 883, normalized size = 3.46

$$-\frac{20\,a^{2}x^{3}\left(-\frac{B^{4}a^{4}b-4\,AB^{3}a^{3}b^{2}+6\,A^{2}B^{2}a^{2}b^{3}-4\,A^{3}Bab^{4}+A^{4}b^{5}}{a^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^{6}a^{6}b^{2}-6\,AB^{5}a^{5}b^{3}+15\,A^{2}B^{4}a^{4}b^{4}-20\,A^{3}B^{3}a^{3}b^{5}+15\,A^{4}B^{2}a^{2}b^{6}-6\,A^{5}Bab^{7}+A^{6}b^{8}\right)x-\left(B^{4}a^{9}b-4\,AB^{3}a^{8}b^{2}+6\,A^{2}B^{2}a^{7}b^{3}-4\,A^{3}Ba^{6}b^{4}+A^{4}a^{5}b^{5}\right)\sqrt{-\frac{B^{4}a^{4}b-4\,AB^{3}a^{3}b^{2}+6\,A^{2}B^{2}a^{2}b^{3}-4\,A^{3}Bab^{4}+A^{4}b^{5}}{a^{9}}}}\,a^{2}\left(-\frac{B^{4}a^{4}b-4\,AB^{3}a^{3}b^{2}+6\,A^{2}B^{2}a^{2}b^{3}-4\,A^{3}Bab^{4}+A^{4}b^{5}}{a^{9}}\right)^{\frac{1}{4}}+\left(B^{3}a^{5}b-3\,AB^{2}a^{4}b^{2}+3\,A^{2}Ba^{3}b^{3}-A^{3}a^{2}b^{4}\right)\sqrt{x}\left(-\frac{B^{4}a^{4}b-4\,AB^{3}a^{3}b^{2}+6\,A^{2}B^{2}a^{2}b^{3}-4\,A^{3}Bab^{4}+A^{4}b^{5}}{a^{9}}\right)^{\frac{1}{4}}}{B^{4}a^{4}b-4\,AB^{3}a^{3}b^{2}+6\,A^{2}B^{2}a^{2}b^{3}-4\,A^{3}Bab^{4}+A^{4}b^{5}}\right)-5\,a^{2}x^{3}\left(-\frac{B^{4}a^{4}b-4\,AB^{3}a^{3}b^{2}+6\,A^{2}B^{2}a^{2}b^{3}-4\,A^{3}Bab^{4}+A^{4}b^{5}}{a^{9}}\right)^{\frac{1}{4}}\log\left(a^{7}\left(-\frac{B^{4}a^{4}b-4\,AB^{3}a^{3}b^{2}+6\,A^{2}B^{2}a^{2}b^{3}-4\,A^{3}Bab^{4}+A^{4}b^{5}}{a^{9}}\right)^{\frac{3}{4}}-\left(B^{3}a^{3}b-3\,AB^{2}a^{2}b^{2}+3\,A^{2}Bab^{3}-A^{3}b^{4}\right)\sqrt{x}\right)+5\,a^{2}x^{3}\left(-\frac{B^{4}a^{4}b-4\,AB^{3}a^{3}b^{2}+6\,A^{2}B^{2}a^{2}b^{3}-4\,A^{3}Bab^{4}+A^{4}b^{5}}{a^{9}}\right)^{\frac{1}{4}}\log\left(-a^{7}\left(-\frac{B^{4}a^{4}b-4\,AB^{3}a^{3}b^{2}+6\,A^{2}B^{2}a^{2}b^{3}-4\,A^{3}Bab^{4}+A^{4}b^{5}}{a^{9}}\right)^{\frac{3}{4}}-\left(B^{3}a^{3}b-3\,AB^{2}a^{2}b^{2}+3\,A^{2}Bab^{3}-A^{3}b^{4}\right)\sqrt{x}\right)+4\,\left(5\,(Ba-Ab)x^{2}+Aa\right)\sqrt{x}}{10\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^
3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*arctan((sqrt((B^6*a^6*b^2 - 6*A*B^5*a^5*b^3
 + 15*A^2*B^4*a^4*b^4 - 20*A^3*B^3*a^3*b^5 + 15*A^4*B^2*a^2*b^6 - 6*A^5*B*a
*b^7 + A^6*b^8)*x - (B^4*a^9*b - 4*A*B^3*a^8*b^2 + 6*A^2*B^2*a^7*b^3 - 4*A^
3*B*a^6*b^4 + A^4*a^5*b^5)*sqrt(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a
^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9))*a^2*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2
 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4) + (B^3*a^5*b - 3*
A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3 - A^3*a^2*b^4)*sqrt(x)*(-(B^4*a^4*b - 4*A*B
^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4))/(B^4*
a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)) - 5
*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4
 + A^4*b^5)/a^9)^(1/4)*log(a^7*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a
^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2
 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(x)) + 5*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3
*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(-a^7*(-(
B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/
a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(x
)) + 4*(5*(B*a - A*b)*x^2 + A*a)*sqrt(x))/(a^2*x^3)
```

Sympy [A]
time = 41.18, size = 258, normalized size = 1.01

$$A\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a=0 \wedge b=0\\ -\frac{2}{9bx^{\frac{9}{2}}} & \text{for}\: a=0\\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for}\: b=0\\ -\frac{2}{5ax^{\frac{5}{2}}}+\frac{b\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a^{2}\sqrt[4]{-\frac{a}{b}}}-\frac{b\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a^{2}\sqrt[4]{-\frac{a}{b}}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^{2}\sqrt[4]{-\frac{a}{b}}}+\frac{2b}{a^{2}\sqrt{x}} & \text{otherwise}\end{cases}\right)+B\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a=0 \wedge b=0\\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: a=0\\ -\frac{2}{a\sqrt{x}} & \text{for}\: b=0\\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}+\frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}-\frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}}-\frac{2}{a\sqrt{x}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a),x)
```

```
[Out] A*Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(a,
0)), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + b*log(sqrt(x) - (-
a/b)**(1/4))/(2*a**2*(-a/b)**(1/4)) - b*log(sqrt(x) + (-a/b)**(1/4))/(2*a**
2*(-a/b)**(1/4)) + b*atan(sqrt(x)/(-a/b)**(1/4))/(a**2*(-a/b)**(1/4)) + 2*b
/(a**2*sqrt(x)), True)) + B*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)),
(-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) -
```

```
(-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2*a*(-a/
b)**(1/4)) - atan(sqrt(x)/(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*sqrt(x)),
 True))
```

Giac [A]
time = 0.69, size = 268, normalized size = 1.05

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{3}b^{2}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{3}b^{2}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{3}b^{2}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{3}b^{2}}-\frac{2\left(5Bax^{2}-5Abx^{2}+Aa\right)}{5a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) - 1/2*sqrt(2)*((a*b^3
)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^
3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) -
1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 2/5*(5*B*a*x^2 - 5*A*b*x^2 + A*a)/(a
^2*x^(5/2))
```

Mupad [B]
time = 0.18, size = 90, normalized size = 0.35

$$\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,\left(A\,b-B\,a\right)}{a^{9/4}}-\frac{\frac{2\,A}{5\,a}-\frac{2\,x^2\,\left(A\,b-B\,a\right)}{a^2}}{x^{5/2}}-\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,\left(A\,b-B\,a\right)}{a^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4))*(A*b - B*a))/a^(9/4) - ((2*A
)/(5*a) - (2*x^2*(A*b - B*a))/a^2)/x^(5/2) - ((-b)^(1/4)*atanh(((-b)^(1/4)*
x^(1/2))/a^(1/4))*(A*b - B*a))/a^(9/4)
```

3.375 $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=310

$$\frac{(5Ab-9aB)\sqrt{x}}{2b^3}-\frac{(5Ab-9aB)x^{5/2}}{10ab^2}+\frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)}+\frac{\sqrt[4]{a}\,(5Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{13/4}}-\frac{\sqrt[4]{a}\,(5}{}$$

```
[Out] -1/10*(5*A*b-9*B*a)*x^(5/2)/a/b^2+1/2*(A*b-B*a)*x^(9/2)/a/b/(b*x^2+a)+1/8*a
^(1/4)*(5*A*b-9*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(13/4)*2^(
1/2)-1/8*a^(1/4)*(5*A*b-9*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^
(13/4)*2^(1/2)+1/16*a^(1/4)*(5*A*b-9*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)-1/16*a^(1/4)*(5*A*b-9*B*a)*ln(a^(1/2)
+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/2*(5*A*b-9*B
*a)*x^(1/2)/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$,
Rules used = {468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{a}\,(5Ab-9aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{13/4}}-\frac{\sqrt[4]{a}\,(5Ab-9aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,b^{13/4}}+\frac{\sqrt[4]{a}\,(5Ab-9aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{\sqrt[4]{a}\,(5Ab-9aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{\sqrt{x}\,(5Ab-9aB)}{2b^3}-\frac{x^{5/2}(5Ab-9aB)}{10ab^2}+\frac{x^{9/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((5*A*b - 9*a*B)*Sqrt[x])/(2*b^3) - ((5*A*b - 9*a*B)*x^(5/2))/(10*a*b^2) +
((A*b - a*B)*x^(9/2))/(2*a*b*(a + b*x^2)) + (a^(1/4)*(5*A*b - 9*a*B)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5
*A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(
13/4)) + (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)} + \frac{\left(-\frac{5Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)} + \frac{(5Ab-9aB)\int \frac{x^{3/2}}{a+bx^2}\,dx}{4b^2}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)} - \frac{(a(5Ab-9aB))\int \frac{1}{\sqrt{x}\,(a\cdots}}{4b^3}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)} - \frac{(a(5Ab-9aB))\text{Subst}\left(\int \cdots\right.}{2b^3}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)} - \frac{\left(\sqrt{a}\,(5Ab-9aB)\right)\text{Subs}\cdots}{\cdots}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)} - \frac{\left(\sqrt{a}\,(5Ab-9aB)\right)\text{Subs}\cdots}{\cdots}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)} + \frac{\sqrt[4]{a}\,(5Ab-9aB)\log\left(\sqrt{\cdots}\right.}{8\cdots}$$

$$= \frac{(5Ab-9aB)\sqrt{x}}{2b^3} - \frac{(5Ab-9aB)x^{5/2}}{10ab^2} + \frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)} + \frac{\sqrt[4]{a}\,(5Ab-9aB)\tan^{-1}\left(\cdots\right.}{4\sqrt{2}\,b^{13\cdots}}$$

Mathematica [A]
time = 0.58, size = 183, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{b}\,\sqrt{x}\,\left(-45a^2B+ab(25A-36Bx^2)+4b^2x^2(5A+Bx^2)\right)}{a+bx^2} - 5\sqrt{2}\,\sqrt[4]{a}\,(-5Ab+9aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right) + 5\sqrt{2}\,\sqrt[4]{a}\,(-5Ab+9aB)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{40b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((4*b^(1/4)*Sqrt[x]*(-45*a^2*B + a*b*(25*A - 36*B*x^2) + 4*b^2*x^2*(5*A + B
*x^2)))/(a + b*x^2) - 5*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*ArcTan[(Sqrt[a] -
Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*a^(1/4)*(-5*A*b +
 9*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(
40*b^(13/4))
```

Maple [A]
time = 0.12, size = 171, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}-4Ba\sqrt{x}}{b^{3}}-\frac{2a\left(\frac{\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^{2}+a}+\frac{(5Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)\cdots}{32a\cdots}\right)}{b^{3}}$
default	$\frac{\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}-4Ba\sqrt{x}}{b^{3}}-\frac{2a\left(\frac{\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^{2}+a}+\frac{(5Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)\cdots}{32a\cdots}\right)}{b^{3}}$
risch	$\frac{2\left(bB\,x^{2}+5Ab-10Ba\right)\sqrt{x}}{5b^{3}}+\frac{a\sqrt{x}\,A}{2b^{2}\left(b\,x^{2}+a\right)}-\frac{a^{2}\sqrt{x}\,B}{2b^{3}\left(b\,x^{2}+a\right)}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*b*B*x^(5/2)+A*b*x^(1/2)-2*B*a*x^(1/2))-2*a/b^3*((-1/4*A*b+1/4*B*
a)*x^(1/2)/(b*x^2+a)+1/32*(5*A*b-9*B*a)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)))
```

Maxima [A]
time = 0.56, size = 271, normalized size = 0.87

$$-\frac{\left(Ba^{2}-Aab\right)\sqrt{x}}{2\left(b^{4}x^{2}+ab^{3}\right)}+\frac{\left(\frac{2\sqrt{2}\,(9\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(9\,Ba-5\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(9\,Ba-5\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(9\,Ba-5\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a}{16\,b^{3}}+\frac{2\left(Bbx^{\frac{5}{2}}-5\,(2\,Ba-Ab)\sqrt{x}\right)}{5\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^2 - A*a*b)*sqrt(x)/(b^4*x^2 + a*b^3) + 1/16*(2*sqrt(2)*(9*B*a - 5
*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(9*B*a - 5*A
*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(9*B*a - 5*A*b)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4
)) - sqrt(2)*(9*B*a - 5*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^3 + 2/5*(B*b*x^(5/2) - 5*(2*B*a - A*b)
*sqrt(x))/b^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(226) = 452$.

time = 1.00, size = 748, normalized size = 2.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(b^4*x^2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2
*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*arctan((sqrt
(b^6*sqrt(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500
*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13) + (81*B^2*a^2 - 90*A*B*a*b + 25*A^2*b
^2)*x)*b^10*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4
500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(3/4) + (9*B*a*b^10 - 5*A*b^11)*sq
rt(x)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^
3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(3/4))/(6561*B^4*a^5 - 14580*A*B^3*a^4*b
 + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)) + 5*(b^4*x^
2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 45
00*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(b^3*(-(6561*B^4*a^5 - 145
80*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4
)/b^13)^(1/4) - (9*B*a - 5*A*b)*sqrt(x)) - 5*(b^4*x^2 + a*b^3)*(-(6561*B^4*
a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*
A^4*a*b^4)/b^13)^(1/4)*log(-b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150
*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4) - (9*B*a
 - 5*A*b)*sqrt(x)) - 4*(4*B*b^2*x^4 - 45*B*a^2 + 25*A*a*b - 4*(9*B*a*b - 5*
A*b^2)*x^2)*sqrt(x))/(b^4*x^2 + a*b^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(292) = 584$.

time = 160.17, size = 770, normalized size = 2.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*A*
x**(9/2)/9 + 2*B*x**(13/2)/13)/a**2, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2
)/5)/b**2, Eq(a, 0)), (100*A*a*b*sqrt(x)/(40*a*b**3 + 40*b**4*x**2) + 25*A*
a*b*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a*b**3 + 40*b**4*x**2) -
 25*A*a*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a*b**3 + 40*b**4*x
**2) - 50*A*a*b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(40*a*b**3 + 40*b
**4*x**2) + 80*A*b**2*x**(5/2)/(40*a*b**3 + 40*b**4*x**2) + 25*A*b**2*x**2*
(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a*b**3 + 40*b**4*x**2) - 25*
A*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a*b**3 + 40*b**4
*x**2) - 50*A*b**2*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(40*a*b**
3 + 40*b**4*x**2) - 180*B*a**2*sqrt(x)/(40*a*b**3 + 40*b**4*x**2) - 45*B*a*
*2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a*b**3 + 40*b**4*x**2) +
45*B*a**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a*b**3 + 40*b**4*x
**2) + 90*B*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(40*a*b**3 + 40*
b**4*x**2) - 144*B*a*b*x**(5/2)/(40*a*b**3 + 40*b**4*x**2) - 45*B*a*b*x**2*
(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a*b**3 + 40*b**4*x**2) + 45*
B*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a*b**3 + 40*b**4*
x**2) + 90*B*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(40*a*b**3
+ 40*b**4*x**2) + 16*B*b**2*x**(9/2)/(40*a*b**3 + 40*b**4*x**2), True))
```

Giac [A]
time = 0.58, size = 298, normalized size = 0.96

$$\frac{\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-5\,(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-5\,(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-5\,(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^4}-\frac{\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-5\,(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^4}-\frac{Ba^2\sqrt{x}-Aab\sqrt{x}}{2\,(bx^2+a)b^3}+\frac{2\left(Bb^8x^{\frac{5}{2}}-10\,Bab^7\sqrt{x}+5\,Ab^8\sqrt{x}\right)}{5\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/8*sqrt(2)*(9*(a*b^3)
^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
- 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/16*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^
3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/16*s
qrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/b^4 - 1/2*(B*a^2*sqrt(x) - A*a*b*sqrt(x))/((b*x^2
 + a)*b^3) + 2/5*(B*b^8*x^(5/2) - 10*B*a*b^7*sqrt(x) + 5*A*b^8*sqrt(x))/b^1
0
```

Mupad [B]
time = 0.20, size = 823, normalized size = 2.65

$$\sqrt{x}\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)+\frac{2\,B\,x^{5/2}}{5\,b^2}-\frac{\sqrt{x}\left(\frac{B\,a^2}{2}-\frac{A\,a\,b}{2}\right)}{b^4\,x^2+a\,b^3}+\frac{{(-a)}^{1/4}\,\mathrm{atan}\left(\frac{\frac{{(-a)}^{1/4}\left(\frac{\sqrt{x}\,\left(25\,A^2\,a^2\,b^2-90\,A\,B\,a^3\,b+81\,B^2\,a^4\right)}{b^3}-\frac{{(-a)}^{1/4}\,\left(5\,A\,b-9\,B\,a\right)\,\left(72\,B\,a^3-40\,A\,a^2\,b\right)}{8\,b^{13/4}}\right)\,\left(5\,A\,b-9\,B\,a\right)\,1{}\mathrm{i}}{8\,b^{13/4}}+\frac{{(-a)}^{1/4}\left(\frac{\sqrt{x}\,\left(25\,A^2\,a^2\,b^2-90\,A\,B\,a^3\,b+81\,B^2\,a^4\right)}{b^3}+\frac{{(-a)}^{1/4}\,\left(5\,A\,b-9\,B\,a\right)\,\left(72\,B\,a^3-40\,A\,a^2\,b\right)}{8\,b^{13/4}}\right)\,\left(5\,A\,b-9\,B\,a\right)\,1{}\mathrm{i}}{8\,b^{13/4}}}{\frac{{(-a)}^{1/4}\left(\frac{\sqrt{x}\,\left(25\,A^2\,a^2\,b^2-90\,A\,B\,a^3\,b+81\,B^2\,a^4\right)}{b^3}-\frac{{(-a)}^{1/4}\,\left(5\,A\,b-9\,B\,a\right)\,\left(72\,B\,a^3-40\,A\,a^2\,b\right)}{8\,b^{13/4}}\right)\,\left(5\,A\,b-9\,B\,a\right)}{8\,b^{13/4}}-\frac{{(-a)}^{1/4}\left(\frac{\sqrt{x}\,\left(25\,A^2\,a^2\,b^2-90\,A\,B\,a^3\,b+81\,B^2\,a^4\right)}{b^3}+\frac{{(-a)}^{1/4}\,\left(5\,A\,b-9\,B\,a\right)\,\left(72\,B\,a^3-40\,A\,a^2\,b\right)}{8\,b^{13/4}}\right)\,\left(5\,A\,b-9\,B\,a\right)}{8\,b^{13/4}}}\right)\,\left(5\,A\,b-9\,B\,a\right)\,1{}\mathrm{i}}{4\,b^{13/4}}+\frac{{(-a)}^{1/4}\,\mathrm{atan}\left(\frac{\frac{{(-a)}^{1/4}\left(\frac{\sqrt{x}\,\left(25\,A^2\,a^2\,b^2-90\,A\,B\,a^3\,b+81\,B^2\,a^4\right)}{b^3}-\frac{{(-a)}^{1/4}\,\left(5\,A\,b-9\,B\,a\right)\,\left(72\,B\,a^3-40\,A\,a^2\,b\right)\,1{}\mathrm{i}}{8\,b^{13/4}}\right)\,\left(5\,A\,b-9\,B\,a\right)}{8\,b^{13/4}}+\frac{{(-a)}^{1/4}\left(\frac{\sqrt{x}\,\left(25\,A^2\,a^2\,b^2-90\,A\,B\,a^3\,b+81\,B^2\,a^4\right)}{b^3}+\frac{{(-a)}^{1/4}\,\left(5\,A\,b-9\,B\,a\right)\,\left(72\,B\,a^3-40\,A\,a^2\,b\right)\,1{}\mathrm{i}}{8\,b^{13/4}}\right)\,\left(5\,A\,b-9\,B\,a\right)}{8\,b^{13/4}}}{\frac{{(-a)}^{1/4}\left(\frac{\sqrt{x}\,\left(25\,A^2\,a^2\,b^2-90\,A\,B\,a^3\,b+81\,B^2\,a^4\right)}{b^3}-\frac{{(-a)}^{1/4}\,\left(5\,A\,b-9\,B\,a\right)\,\left(72\,B\,a^3-40\,A\,a^2\,b\right)\,1{}\mathrm{i}}{8\,b^{13/4}}\right)\,\left(5\,A\,b-9\,B\,a\right)\,1{}\mathrm{i}}{8\,b^{13/4}}-\frac{{(-a)}^{1/4}\left(\frac{\sqrt{x}\,\left(25\,A^2\,a^2\,b^2-90\,A\,B\,a^3\,b+81\,B^2\,a^4\right)}{b^3}+\frac{{(-a)}^{1/4}\,\left(5\,A\,b-9\,B\,a\right)\,\left(72\,B\,a^3-40\,A\,a^2\,b\right)\,1{}\mathrm{i}}{8\,b^{13/4}}\right)\,\left(5\,A\,b-9\,B\,a\right)\,1{}\mathrm{i}}{8\,b^{13/4}}}\right)\,\left(5\,A\,b-9\,B\,a\right)}{4\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^(1/2)*((2*A)/b^2 - (4*B*a)/b^3) + (2*B*x^(5/2))/(5*b^2) - (x^(1/2)*((B*a^
2)/2 - (A*a*b)/2))/(a*b^3 + b^4*x^2) + ((-a)^(1/4)*atan((((-a)^(1/4)*((x^(1
/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b
- 9*B*a)*(72*B*a^3 - 40*A*a^2*b))/(8*b^(13/4)))*(5*A*b - 9*B*a)*1i)/(8*b^(1
3/4)) + ((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))
/b^3 + ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b))/(8*b^(13/4)))*(
5*A*b - 9*B*a)*1i)/(8*b^(13/4)))/(((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^
2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40
*A*a^2*b))/(8*b^(13/4)))*(5*A*b - 9*B*a))/(8*b^(13/4)) - ((-a)^(1/4)*((x^(1
/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 + ((-a)^(1/4)*(5*A*b
- 9*B*a)*(72*B*a^3 - 40*A*a^2*b))/(8*b^(13/4)))*(5*A*b - 9*B*a))/(8*b^(13/4
))))*(5*A*b - 9*B*a)*1i)/(4*b^(13/4)) + ((-a)^(1/4)*atan((((-a)^(1/4)*((x^(
1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b
 - 9*B*a)*(72*B*a^3 - 40*A*a^2*b)*1i)/(8*b^(13/4)))*(5*A*b - 9*B*a))/(8*b^(
13/4)) + ((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b)
)/b^3 + ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b)*1i)/(8*b^(13/4)
))*(5*A*b - 9*B*a))/(8*b^(13/4)))/(((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A
^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 4
0*A*a^2*b)*1i)/(8*b^(13/4)))*(5*A*b - 9*B*a)*1i)/(8*b^(13/4)) - ((-a)^(1/4)
*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 + ((-a)^(1/4)*
(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b)*1i)/(8*b^(13/4)))*(5*A*b - 9*B*a)*1
i)/(8*b^(13/4))))*(5*A*b - 9*B*a))/(4*b^(13/4))
```

3.376 $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{(3Ab-7aB)x^{3/2}}{6ab^2}+\frac{(Ab-aB)x^{7/2}}{2ab\,(a+bx^2)}-\frac{(3Ab-7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}}+\frac{(3Ab-7aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}}$$

```
[Out] -1/6*(3*A*b-7*B*a)*x^(3/2)/a/b^2+1/2*(A*b-B*a)*x^(7/2)/a/b/(b*x^2+a)-1/8*(3
*A*b-7*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b^(11/4)*2^(1
/2)+1/8*(3*A*b-7*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b^(
11/4)*2^(1/2)+1/16*(3*A*b-7*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/
2)*x^(1/2))/a^(1/4)/b^(11/4)*2^(1/2)-1/16*(3*A*b-7*B*a)*ln(a^(1/2)+x*b^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(3Ab-7aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}}+\frac{(3Ab-7aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}}+\frac{(3Ab-7aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}}-\frac{(3Ab-7aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}}-\frac{x^{3/2}(3Ab-7aB)}{6ab^2}+\frac{x^{7/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -1/6*((3*A*b - 7*a*B)*x^(3/2))/(a*b^2) + ((A*b - a*B)*x^(7/2))/(2*a*b*(a +
b*x^2)) - ((3*A*b - 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
4*Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(1
1/4)) - ((3*A*b - 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ab-aB)x^{7/2}}{2ab(a+bx^2)} + \frac{\left(-\frac{3Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab(a+bx^2)} + \frac{(3Ab-7aB)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4b^2}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab(a+bx^2)} + \frac{(3Ab-7aB)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab(a+bx^2)} - \frac{(3Ab-7aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{\right.}}{4b^{5/2}}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab(a+bx^2)} + \frac{(3Ab-7aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\right.}{8b^3}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab(a+bx^2)} + \frac{(3Ab-7aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{8\sqrt{2}\sqrt[4]{a}\,b^{11/4}}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab(a+bx^2)} - \frac{(3Ab-7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{11/4}} +$$

Mathematica [A]
time = 0.53, size = 162, normalized size = 0.56

$$\frac{\frac{4b^{3/4}x^{3/2}(-3Ab+7aB+4bBx^2)}{a+bx^2} + \frac{3\sqrt{2}(-3Ab+7aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}} + \frac{3\sqrt{2}(-3Ab+7aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{a}}}{24b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((4*b^(3/4)*x^(3/2)*(-3*A*b + 7*a*B + 4*b*B*x^2))/(a + b*x^2) + (3*Sqrt[2]*
(-3*A*b + 7*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])])/a^(1/4) + (3*Sqrt[2]*(-3*A*b + 7*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4))/(24*b^(11/4))
```

Maple [A]
time = 0.10, size = 153, normalized size = 0.53

method	result
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(-\frac{7Ba}{4}+\frac{3Ab}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b^2}$
default	$\frac{2Bx^{\frac{3}{2}}}{3b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(-\frac{7Ba}{4}+\frac{3Ab}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b^2}$
risch	$\frac{2Bx^{\frac{3}{2}}}{3b^2}-\frac{x^{\frac{3}{2}}A}{2b(bx^2+a)}+\frac{x^{\frac{3}{2}}Ba}{2b^2(bx^2+a)}-\frac{7\sqrt{2}\,Ba\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{16b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{7\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right)}\right.}{8b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B*x^(3/2)/b^2+2/b^2*((-1/4*A*b+1/4*B*a)*x^(3/2)/(b*x^2+a)+1/8*(-7/4*B*a
+3/4*A*b)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.50, size = 223, normalized size = 0.77

$$\frac{(Ba-Ab)x^{\frac{3}{2}}}{2\,(b^3x^2+ab^2)}+\frac{2\,Bx^{\frac{3}{2}}}{3\,b^2}-\frac{(7\,Ba-3\,Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)*x^(3/2)/(b^3*x^2 + a*b^2) + 2/3*B*x^(3/2)/b^2 - 1/16*(7*B*a
 - 3*A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
```

```
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1
/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(
-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/
b^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(209) = 418$.
time = 0.87, size = 925, normalized size = 3.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(b^3*x^2 + a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B
^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*arctan((sqrt((11
7649*B^6*a^6 - 302526*A*B^5*a^5*b + 324135*A^2*B^4*a^4*b^2 - 185220*A^3*B^3
*a^3*b^3 + 59535*A^4*B^2*a^2*b^4 - 10206*A^5*B*a*b^5 + 729*A^6*b^6)*x - (24
01*B^4*a^5*b^5 - 4116*A*B^3*a^4*b^6 + 2646*A^2*B^2*a^3*b^7 - 756*A^3*B*a^2*
b^8 + 81*A^4*a*b^9)*sqrt(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a
^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11)))*b^3*(-(2401*B^4*a^4 - 411
6*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^1
1))^(1/4) + (343*B^3*a^3*b^3 - 441*A*B^2*a^2*b^4 + 189*A^2*B*a*b^5 - 27*A^3
*b^6)*sqrt(x)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 7
56*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4))/(2401*B^4*a^4 - 4116*A*B^3*a^
3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)) - 3*(b^3*x^2 +
a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*
B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(a*b^8*(-(2401*B^4*a^4 - 4116*A*B^
3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(3
/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 - 27*A^3*b^3)*sqrt(x
)) + 3*(b^3*x^2 + a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*
a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(-a*b^8*(-(2401*
B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^
4*b^4)/(a*b^11))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 -
 27*A^3*b^3)*sqrt(x)) - 4*(4*B*b*x^3 + (7*B*a - 3*A*b)*x)*sqrt(x))/(b^3*x^2
 + a*b^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 283, normalized size = 0.98

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b^{2}}+\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{2\,(bx^{2}+a)b^{2}}-\frac{\sqrt{2}\left(7\,(ab^{3})^{\frac{3}{4}}\,Ba-3\,(ab^{3})^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{a}{b})^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,(\frac{a}{b})^{\frac{1}{4}}}\right)}{8\,ab^{5}}-\frac{\sqrt{2}\left(7\,(ab^{3})^{\frac{3}{4}}\,Ba-3\,(ab^{3})^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{a}{b})^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,(\frac{a}{b})^{\frac{1}{4}}}\right)}{8\,ab^{5}}+\frac{\sqrt{2}\left(7\,(ab^{3})^{\frac{3}{4}}\,Ba-3\,(ab^{3})^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,(\frac{a}{b})^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^{5}}-\frac{\sqrt{2}\left(7\,(ab^{3})^{\frac{3}{4}}\,Ba-3\,(ab^{3})^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,(\frac{a}{b})^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b^2 + 1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*b^2) - 1/8
*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) - 1/8*sqrt(2)*(7*(a*b^3
)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) + 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*
(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5)
 - 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sq
rt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5)
```

Mupad [B]
time = 0.10, size = 106, normalized size = 0.37

$$\frac{2\,B\,x^{3/2}}{3\,b^2}-\frac{x^{3/2}\,\left(\frac{A\,b}{2}-\frac{B\,a}{2}\right)}{b^3\,x^2+a\,b^2}+\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(3\,A\,b-7\,B\,a\right)}{4\,{\left(-a\right)}^{1/4}\,b^{11/4}}+\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}}\right)\,\left(3\,A\,b-7\,B\,a\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{1/4}\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (2*B*x^(3/2))/(3*b^2) - (x^(3/2)*((A*b)/2 - (B*a)/2))/(a*b^2 + b^3*x^2) + (
atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(3*A*b - 7*B*a))/(4*(-a)^(1/4)*b^(11/4))
 + (atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*(3*A*b - 7*B*a)*1i)/(4*(-a)^(1/4)
*b^(11/4))
```

3.377 $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=284

$$-\frac{(Ab-5aB)\sqrt{x}}{2ab^2}+\frac{(Ab-aB)x^{5/2}}{2ab\,(a+bx^2)}-\frac{(Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{4\sqrt{2}\,a^{3/4}b^{9/4}}$$

```
[Out] 1/2*(A*b-B*a)*x^(5/2)/a/b/(b*x^2+a)-1/8*(A*b-5*B*a)*arctan(1-b^(1/4)*2^(1/2
)*x^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)+1/8*(A*b-5*B*a)*arctan(1+b^(1/4)
*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)-1/16*(A*b-5*B*a)*ln(a^(1/
2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)+1/16*
(A*b-5*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b
^(9/4)*2^(1/2)-1/2*(A*b-5*B*a)*x^(1/2)/a/b^2
```

Rubi [A]

time = 0.15, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(Ab-5aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{(Ab-5aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{\sqrt{x}\,(Ab-5aB)}{2ab^2}+\frac{x^{5/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((A*b - 5*a*B)*Sqrt[x])/(a*b^2) + ((A*b - a*B)*x^(5/2))/(2*a*b*(a + b*
x^2)) - ((A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sq
rt[2]*a^(3/4)*b^(9/4)) + ((A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(9/4)) - ((A*b - 5*a*B)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(9/4)) + ((A
*b - 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*
Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ab-aB)x^{5/2}}{2ab\,(a+bx^2)} + \frac{\left(-\frac{Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\,(a+bx^2)} + \frac{(Ab-5aB)\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{4b^2}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\,(a+bx^2)} + \frac{(Ab-5aB)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\,(a+bx^2)} + \frac{(Ab-5aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{4\sqrt{a}\,b^2}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\,(a+bx^2)} + \frac{(Ab-5aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx\right.}{8\sqrt{a}\,b^{5/2}}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\,(a+bx^2)} - \frac{(Ab-5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{8\sqrt{2}\,a^{3/4}b^{9/4}}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\,(a+bx^2)} - \frac{(Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}} + \cdots$$

Mathematica [A]
time = 0.55, size = 159, normalized size = 0.56

$$\frac{\frac{4\sqrt[4]{b}\sqrt{x}\,(-Ab+5aB+4bBx^2)}{a+bx^2}+\frac{\sqrt{2}\,(-Ab+5aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}}+\frac{\sqrt{2}\,(Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}}}{8b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((4*b^(1/4)*Sqrt[x]*(-(A*b) + 5*a*B + 4*b*B*x^2))/(a + b*x^2) + (Sqrt[2]*(-
(A*b) + 5*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])])/a^(3/4) + (Sqrt[2]*(A*b - 5*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4))/(8*b^(9/4))
```

Maple [A]
time = 0.10, size = 152, normalized size = 0.54

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(Ab-5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2}{16a}}{b^2}$
default	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(Ab-5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2}{16a}}{b^2}$
risch	$\frac{2B\sqrt{x}}{b^2}-\frac{\sqrt{x}\,A}{2b(b\,x^2+a)}+\frac{\sqrt{x}\,Ba}{2b^2(b\,x^2+a)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ba}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right)}\right)}{8ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/b^2*x^(1/2)+2/b^2*((-1/4*A*b+1/4*B*a)*x^(1/2)/(b*x^2+a)+1/32*(A*b-5*B*a
)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-
(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.52, size = 250, normalized size = 0.88

$$\frac{(Ba-Ab)\sqrt{x}}{2\,(b^3x^2+ab^2)}+\frac{2\,B\sqrt{x}}{b^2}-\frac{\frac{2\,\sqrt{2}\,(5\,Ba-Ab)\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(5\,Ba-Ab)\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(5\,Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(5\,Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)*sqrt(x)/(b^3*x^2 + a*b^2) + 2*B*sqrt(x)/b^2 - 1/16*(2*sqrt(
2)*(5*B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(
5*B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(
x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(5*B*a
 - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)
*b^(1/4)) - sqrt(2)*(5*B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(209) = 418$.

time = 0.64, size = 725, normalized size = 2.55

$$\frac{4\,(b^3x^2+ab^2)\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2b^4\sqrt{-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}}+(25\,B^2a^2-10\,ABab+A^2b^2)x}\,a^2b^7\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{3}{4}}+(5\,Ba^3b^7-Aa^2b^8)\sqrt{x}\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{3}{4}}}{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}\right)+(b^3x^2+ab^2)\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}-(5\,Ba-Ab)\sqrt{x}\right)-(b^3x^2+ab^2)\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}-(5\,Ba-Ab)\sqrt{x}\right)+4\,(4\,Bbx^2+5\,Ba-Ab)\sqrt{x}}{8\,(b^3x^2+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^3*x^2 + a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2
*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*arctan((sqrt(a^2*b^4*sqrt
(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A
^4*b^4)/(a^3*b^9)) + (25*B^2*a^2 - 10*A*B*a*b + A^2*b^2)*x)*a^2*b^7*(-(625*
B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)
/(a^3*b^9))^(3/4) + (5*B*a^3*b^7 - A*a^2*b^8)*sqrt(x)*(-(625*B^4*a^4 - 500*
A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(3
/4))/(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3
+ A^4*b^4)) + (b^3*x^2 + a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*
B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*log(a*b^2*(-(625*B
^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/
(a^3*b^9))^(1/4) - (5*B*a - A*b)*sqrt(x)) - (b^3*x^2 + a*b^2)*(-(625*B^4*a^
4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*
b^9))^(1/4)*log(-a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b
^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4) - (5*B*a - A*b)*sqrt(x)) +
4*(4*B*b*x^2 + 5*B*a - A*b)*sqrt(x))/(b^3*x^2 + a*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(270) = 540$.

time = 30.81, size = 760, normalized size = 2.68

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) & \text{for}\ a=0 \wedge b=0 \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{a^2} & \text{for}\ b=0 \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b^2} & \text{for}\ a=0 \\ -\frac{4Aab\sqrt{x}}{8a^2b^2+8ab^3x^2}-\frac{Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}+\frac{Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}+\frac{2Aab\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b^2+8ab^3x^2}-\frac{Ab^2x^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}+\frac{Ab^2x^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}+\frac{2Ab^2x^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b^2+8ab^3x^2}+\frac{20Ba^2\sqrt{x}}{8a^2b^2+8ab^3x^2}+\frac{5Ba^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}-\frac{5Ba^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}-\frac{10Ba^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b^2+8ab^3x^2}+\frac{16Babx^{\frac{5}{2}}}{8a^2b^2+8ab^3x^2}+\frac{5Babx^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}-\frac{5Babx^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}-\frac{10Babx^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b^2+8ab^3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(5/2)/5 + 2*B*x**(9/2)/9)/a**2, Eq(b, 0)), ((-2*A/(3*x**(3/2)) + 2*B*
sqrt(x))/b**2, Eq(a, 0)), (-4*A*a*b*sqrt(x)/(8*a**2*b**2 + 8*a*b**3*x**2) -
 A*a*b*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x
**2) + A*a*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*
b**3*x**2) + 2*A*a*b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b**2
 + 8*a*b**3*x**2) - A*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/
(8*a**2*b**2 + 8*a*b**3*x**2) + A*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a
/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) + 2*A*b**2*x**2*(-a/b)**(1/4)*ata
```

```
n(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) + 20*B*a**2*sqrt(x)/
(8*a**2*b**2 + 8*a*b**3*x**2) + 5*B*a**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)
**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) - 5*B*a**2*(-a/b)**(1/4)*log(sqrt(x)
 + (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) - 10*B*a**2*(-a/b)**(1/4)*a
tan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) + 16*B*a*b*x**(5/2
)/(8*a**2*b**2 + 8*a*b**3*x**2) + 5*B*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) -
(-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) - 5*B*a*b*x**2*(-a/b)**(1/4)*l
og(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) - 10*B*a*b*x**2*(
-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2), Tru
e))
```

Giac [A]
time = 0.56, size = 283, normalized size = 1.00

$$\frac{2\,B\sqrt{x}}{b^2}-\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^3}+\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^3}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\,(bx^2+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^2 - 1/8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/
8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/16*sqrt(2)*(5*(a*b^
3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(a*b^3) + 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*lo
g(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/2*(B*a*sqrt(x)
- A*b*sqrt(x))/((b*x^2 + a)*b^2)
```

Mupad [B]
time = 0.20, size = 744, normalized size = 2.62

$$\frac{2\,B\,\sqrt{x}}{b^2}-\frac{\sqrt{x}\,\left(\frac{A\,b}{2}-\frac{B\,a}{2}\right)}{b^3\,x^2+a\,b^2}+\frac{\mathrm{atan}\left(\frac{\frac{\left(A\,b-5\,B\,a\right)\,\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{\left(A\,b-5\,B\,a\right)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}+\frac{\left(A\,b-5\,B\,a\right)\,\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{\left(A\,b-5\,B\,a\right)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}}{\frac{\left(A\,b-5\,B\,a\right)\,\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{\left(A\,b-5\,B\,a\right)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}\right)}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}-\frac{\left(A\,b-5\,B\,a\right)\,\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{\left(A\,b-5\,B\,a\right)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}\right)}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}}\right)\,\left(A\,b-5\,B\,a\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(A\,b-5\,B\,a\right)\,\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{\left(A\,b-5\,B\,a\right)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}\right)}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}+\frac{\left(A\,b-5\,B\,a\right)\,\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{\left(A\,b-5\,B\,a\right)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}\right)}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}}{\frac{\left(A\,b-5\,B\,a\right)\,\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{\left(A\,b-5\,B\,a\right)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}-\frac{\left(A\,b-5\,B\,a\right)\,\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{\left(A\,b-5\,B\,a\right)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{3/4}\,b^{9/4}}}\right)\,\left(A\,b-5\,B\,a\right)}{4\,{\left(-a\right)}^{3/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (2*B*x^(1/2))/b^2 - (x^(1/2)*((A*b)/2 - (B*a)/2))/(a*b^2 + b^3*x^2) + (atan
((((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/b - ((A*b -
 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*b^(9/4)))*1i)/(8*(-a)^(3/4)
*b^(9/4)) + ((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/b
 + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*b^(9/4)))*1i)/(8*
(-a)^(3/4)*b^(9/4)))/(((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A
*B*a*b))/b - ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*b^(9/4)
)))/(8*(-a)^(3/4)*b^(9/4)) - ((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2
 - 10*A*B*a*b))/b + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*
```

```
b^(9/4))))/(8*(-a)^(3/4)*b^(9/4))))*(A*b - 5*B*a)*1i)/(4*(-a)^(3/4)*b^(9/4)
) + (atan((((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/b
- ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*1i)/(8*(-a)^(3/4)*b^(9/4))))/(8*(
-a)^(3/4)*b^(9/4)) + ((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*
B*a*b))/b + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*1i)/(8*(-a)^(3/4)*b^(9/
4))))/(8*(-a)^(3/4)*b^(9/4)))/(((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a
^2 - 10*A*B*a*b))/b - ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*1i)/(8*(-a)^(
3/4)*b^(9/4)))*1i)/(8*(-a)^(3/4)*b^(9/4)) - ((A*b - 5*B*a)*((x^(1/2)*(A^2*b
^2 + 25*B^2*a^2 - 10*A*B*a*b))/b + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*
1i)/(8*(-a)^(3/4)*b^(9/4)))*1i)/(8*(-a)^(3/4)*b^(9/4))))*(A*b - 5*B*a))/(4*
(-a)^(3/4)*b^(9/4))
```

3.378 $\int \frac{\sqrt{x}\,(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=261

$$\frac{(Ab-aB)x^{3/2}}{2ab\,(a+bx^2)}-\frac{(Ab+3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{(Ab+3aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{(Ab+3a}{}$$

```
[Out] 1/2*(A*b-B*a)*x^(3/2)/a/b/(b*x^2+a)-1/8*(A*b+3*B*a)*arctan(1-b^(1/4)*2^(1/2
)*x^(1/2)/a^(1/4))/a^(5/4)/b^(7/4)*2^(1/2)+1/8*(A*b+3*B*a)*arctan(1+b^(1/4)
*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(7/4)*2^(1/2)+1/16*(A*b+3*B*a)*ln(a^(1/
2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)-1/16*
(A*b+3*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b
^(7/4)*2^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {468, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(3aB+Ab)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{(3aB+Ab)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{(3aB+Ab)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{x^{3/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(2*a*b*(a + b*x^2)) - ((A*b + 3*a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/4)) + ((A*b + 3*a*B)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/4))
 + ((A*b + 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(5/4)*b^(7/4)) - ((A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\,(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ab-aB)x^{3/2}}{2ab\,(a+bx^2)} + \frac{\left(\frac{Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{2ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\,(a+bx^2)} + \frac{\left(\frac{Ab}{2}+\frac{3aB}{2}\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\,(a+bx^2)} - \frac{(Ab+3aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4ab^{3/2}} + \frac{(Ab+3aB)\text{Subst}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\,(a+bx^2)} + \frac{(Ab+3aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8ab^2} + \frac{(Ab+3a$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\,(a+bx^2)} + \frac{(Ab+3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{7/4}} - \frac{(Ab+3aB)$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\,(a+bx^2)} - \frac{(Ab+3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}} + \frac{(Ab+3aB)\tan^{-1}\left(1+\right.}{4\sqrt{2}\,a^{5/4}b^7$$

Mathematica [A]
time = 0.51, size = 152, normalized size = 0.58

$$\frac{\frac{4\sqrt[4]{a}\,b^{3/4}(Ab-aB)x^{3/2}}{a+bx^2} - \sqrt{2}\,(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - \sqrt{2}\,(Ab+3aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{8a^{5/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((4*a^(1/4)*b^(3/4)*(A*b - a*B)*x^(3/2))/(a + b*x^2) - Sqrt[2]*(A*b + 3*a*B
)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - Sqrt[2]
*(A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]
*x)])/(8*a^(5/4)*b^(7/4))
```

Maple [A]
time = 0.08, size = 146, normalized size = 0.56

method	result

derivativedivides	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{2ab(b\,x^2+a)}+\frac{(Ab+3Ba)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right.}\right.\right.}{16a\,b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{2ab(b\,x^2+a)}+\frac{(Ab+3Ba)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right.}\right.\right.}{16a\,b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*b-B*a)*x^(3/2)/a/b/(b*x^2+a)+1/16*(A*b+3*B*a)/a/b^2/(a/b)^(1/4)*2^(1
/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*
2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 217, normalized size = 0.83

$$-\frac{(Ba-Ab)x^{\frac{3}{2}}}{2\,(ab^2x^2+a^2b)}+\frac{(3\,Ba+Ab)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)*x^(3/2)/(a*b^2*x^2 + a^2*b) + 1/16*(3*B*a + A*b)*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^
(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(185) = 370$.
time = 0.82, size = 912, normalized size = 3.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(B*a - A*b)*x^(3/2) + 4*(a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B
^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*
arctan((sqrt((729*B^6*a^6 + 1458*A*B^5*a^5*b + 1215*A^2*B^4*a^4*b^2 + 540*A
^3*B^3*a^3*b^3 + 135*A^4*B^2*a^2*b^4 + 18*A^5*B*a*b^5 + A^6*b^6)*x - (81*B^
4*a^7*b^3 + 108*A*B^3*a^6*b^4 + 54*A^2*B^2*a^5*b^5 + 12*A^3*B*a^4*b^6 + A^4
*a^3*b^7)*sqrt(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3
*B*a*b^3 + A^4*b^4)/(a^5*b^7)))*a*b^2*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4) - (27*B^3*a^4*
b^2 + 27*A*B^2*a^3*b^3 + 9*A^2*B*a^2*b^4 + A^3*a*b^5)*sqrt(x)*(-(81*B^4*a^4
 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^
7))^(1/4))/(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*
b^3 + A^4*b^4)) - (a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(a^4*b^5*(-
(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b
^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*a*b^2 + A^3*b
^3)*sqrt(x)) + (a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2
*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-(8
1*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4
)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*a*b^2 + A^3*b^3
)*sqrt(x)))/(a*b^2*x^2 + a^2*b)
```

Sympy [A]
time = 12.73, size = 162, normalized size = 0.62

$$\frac{2Ax^{\frac{3}{2}}}{4a^2+4abx^2}+2A\,\mathrm{RootSum}\left(65536t^4a^5b^3+1,\left(t\mapsto t\log\left(4096t^3a^4b^2+\sqrt{x}\right)\right)\right)-\frac{2Bax^{\frac{3}{2}}}{4a^2b+4ab^2x^2}-\frac{2Ba\,\mathrm{RootSum}\left(65536t^4a^5b^3+1,\left(t\mapsto t\log\left(4096t^3a^4b^2+\sqrt{x}\right)\right)\right)}{b}+\frac{2B\,\mathrm{RootSum}\left(256t^4ab^3+1,\left(t\mapsto t\log\left(64t^3ab^2+\sqrt{x}\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] 2*A*x**(3/2)/(4*a**2 + 4*a*b*x**2) + 2*A*RootSum(65536*_t**4*a**5*b**3 + 1,
 Lambda(_t, _t*log(4096*_t**3*a**4*b**2 + sqrt(x)))) - 2*B*a*x**(3/2)/(4*a*
*2*b + 4*a*b**2*x**2) - 2*B*a*RootSum(65536*_t**4*a**5*b**3 + 1, Lambda(_t,
 _t*log(4096*_t**3*a**4*b**2 + sqrt(x))))/b + 2*B*RootSum(256*_t**4*a*b**3
+ 1, Lambda(_t, _t*log(64*_t**3*a*b**2 + sqrt(x))))/b
```

Giac [A]
time = 0.65, size = 273, normalized size = 1.05

$$-\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{2\,(bx^2+a)ab}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^4}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^4}-\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^4}+\frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*a*b) + 1/8*sqrt(2)*(3*(a*b^3)
^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2
```

```
*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) + 1/8*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^
3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(
1/4))/(a^2*b^4) - 1/16*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*lo
g(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 1/16*sqrt(2)*(3*
(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^4)
```

Mupad [B]
time = 0.17, size = 91, normalized size = 0.35

$$\frac{\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,\left(A\,b+3\,B\,a\right)}{4\,{(-a)}^{5/4}\,b^{7/4}}-\frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,\left(A\,b+3\,B\,a\right)}{4\,{(-a)}^{5/4}\,b^{7/4}}+\frac{x^{3/2}\,\left(A\,b-B\,a\right)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 3*B*a))/(4*(-a)^(5/4)*b^(7/4))
- (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 3*B*a))/(4*(-a)^(5/4)*b^(7/4))
 + (x^(3/2)*(A*b - B*a))/(2*a*b*(a + b*x^2))
```

3.379 $\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx$

Optimal. Leaf size=261

$$\frac{(Ab-aB)\sqrt{x}}{2ab\,(a+bx^2)}-\frac{(3Ab+aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(3Ab+aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(3Ab+a$$

```
[Out] -1/8*(3*A*b+B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(5/4)*
2^(1/2)+1/8*(3*A*b+B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b
^(5/4)*2^(1/2)-1/16*(3*A*b+B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2
)*x^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*(3*A*b+B*a)*ln(a^(1/2)+x*b^(1/2)+a^
(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/2*(A*b-B*a)*x^(1/2
)/a/b/(b*x^2+a)
```

Rubi [A]
time = 0.12, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {468, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(aB+3Ab)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(aB+3Ab)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\sqrt{x}\,(Ab-aB)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(2*a*b*(a + b*x^2)) - ((3*A*b + a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*A*b + a*B)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/4))
 - ((3*A*b + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{2ab\,(a+bx^2)} + \frac{\left(\frac{3Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{2ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\,(a+bx^2)} + \frac{\left(\frac{3Ab}{2}+\frac{aB}{2}\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\,(a+bx^2)} + \frac{(3Ab+aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{3/2}b} + \frac{(3Ab+aB)\text{Subs}}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\,(a+bx^2)} + \frac{(3Ab+aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^{3/2}b^{3/2}} + \frac{(3Ab+a}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\,(a+bx^2)} - \frac{(3Ab+aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{(3Ab+aB)}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\,(a+bx^2)} - \frac{(3Ab+aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{(3Ab+aB)\tan^{-1}\left(1+\right.}{4\sqrt{2}\,a^{7/4}b^{5}}$$

Mathematica [A]
time = 0.46, size = 151, normalized size = 0.58

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\,(Ab-aB)\sqrt{x}}{a+bx^2} - \sqrt{2}\,(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + \sqrt{2}\,(3Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{8a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(A*b - a*B)*Sqrt[x])/(a + b*x^2) - Sqrt[2]*(3*A*b + a*B
)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]
*(3*A*b + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]
*x)])/(8*a^(7/4)*b^(5/4))
```

Maple [A]
time = 0.08, size = 146, normalized size = 0.56

method	result

derivativedivides	$\frac{(Ab-Ba)\sqrt{x}}{2ab(b\,x^2+a)}+\frac{(3Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{16a^2b}$
default	$\frac{(Ab-Ba)\sqrt{x}}{2ab(b\,x^2+a)}+\frac{(3Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{16a^2b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(A*b-B*a)*x^(1/2)/a/b/(b*x^2+a)+1/16*(3*A*b+B*a)/a^2/b*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))

Maxima [A]
time = 0.57, size = 241, normalized size = 0.92

$$-\frac{(Ba-Ab)\sqrt{x}}{2\,(ab^2x^2+a^2b)}+\frac{\frac{2\sqrt{2}\,(Ba+3\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(Ba+3\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba+3\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba+3\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")

[Out] -1/2*(B*a - A*b)*sqrt(x)/(a*b^2*x^2 + a^2*b) + 1/16*(2*sqrt(2)*(B*a + 3*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(B*a + 3*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a + 3*A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a + 3*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(185) = 370.
time = 0.86, size = 717, normalized size = 2.75

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")

[Out] 1/8*(4*(a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*arctan((sqrt(a^4*b^2*sqrt

```
(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4
*b^4)/(a^7*b^5)) + (B^2*a^2 + 6*A*B*a*b + 9*A^2*b^2)*x)*a^5*b^4*(-(B^4*a^4
+ 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*
b^5))^(3/4) - (B*a^6*b^4 + 3*A*a^5*b^5)*sqrt(x)*(-(B^4*a^4 + 12*A*B^3*a^3*b
 + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(3/4))/(B^
4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)
) + (a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 +
108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(B^4*a^4 + 12*A*
B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(
1/4) + (B*a + 3*A*b)*sqrt(x)) - (a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a
^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*
log(-a^2*b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b
^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4) + (B*a + 3*A*b)*sqrt(x)) - 4*(B*a - A*b)*
sqrt(x))/(a*b^2*x^2 + a^2*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(250) = 500$.
time = 25.60, size = 734, normalized size = 2.81

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{3x^{\frac{3}{2}}}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{3x^{\frac{3}{2}}}}{b^2} & \text{for } a = 0 \\ \frac{2A\sqrt{x} + \frac{2Bx^{\frac{5}{2}}}{5}}{a^2} & \text{for } b = 0 \\ \frac{4Aab\sqrt{x}}{8a^3b+8a^2b^2x^2} - \frac{3Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2} + \frac{3Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2} + \frac{6Aab\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^3b+8a^2b^2x^2} - \frac{3Ab^2x^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2} + \frac{3Ab^2x^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2} + \frac{6Ab^2x^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^3b+8a^2b^2x^2} - \frac{4Ba^2\sqrt{x}}{8a^3b+8a^2b^2x^2} - \frac{Ba^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2} + \frac{Ba^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2} + \frac{2Ba^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^3b+8a^2b^2x^2} - \frac{Babx^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2} + \frac{Babx^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2} + \frac{2Babx^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^3b+8a^2b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/b**2, Eq(a, 0)), ((2*A*sqrt(x) +
2*B*x**(5/2)/5)/a**2, Eq(b, 0)), (4*A*a*b*sqrt(x)/(8*a**3*b + 8*a**2*b**2*x
**2) - 3*A*a*b*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*b + 8*a**
2*b**2*x**2) + 3*A*a*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3*b
 + 8*a**2*b**2*x**2) + 6*A*a*b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8
*a**3*b + 8*a**2*b**2*x**2) - 3*A*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a
/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) + 3*A*b**2*x**2*(-a/b)**(1/4)*log
(sqrt(x) + (-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) + 6*A*b**2*x**2*(-a
/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) - 4*B*
a**2*sqrt(x)/(8*a**3*b + 8*a**2*b**2*x**2) - B*a**2*(-a/b)**(1/4)*log(sqrt(
x) - (-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) + B*a**2*(-a/b)**(1/4)*lo
g(sqrt(x) + (-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) + 2*B*a**2*(-a/b)*
*(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) - B*a*b*x*
*2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2)
 + B*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3*b + 8*a**2
*b**2*x**2) + 2*B*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**
3*b + 8*a**2*b**2*x**2), True))
```

Giac [A]

time = 0.59, size = 273, normalized size = 1.05

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}b^{2}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}b^{2}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{2}b^{2}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{2}b^{2}}-\frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\left(bx^{2}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 1/8*sqrt(2)*((a*b^
3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*((a*b^3)^(1/4)*B*a + 3
*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b
^2) - 1/16*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) - 1/2*(B*a*sqrt(x) - A*b*sqrt
(x))/((b*x^2 + a)*a*b)
```

Mupad [B]

time = 0.23, size = 750, normalized size = 2.87

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(3\,A\,b+B\,a\right)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{\left(3\,A\,b+B\,a\right)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}+\frac{\left(3\,A\,b+B\,a\right)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{\left(3\,A\,b+B\,a\right)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}}{\frac{\left(3\,A\,b+B\,a\right)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{\left(3\,A\,b+B\,a\right)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}-\frac{\left(3\,A\,b+B\,a\right)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{\left(3\,A\,b+B\,a\right)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}}\right)\,\left(3\,A\,b+B\,a\right)}{4\,{\left(-a\right)}^{7/4}\,b^{5/4}}+\frac{\sqrt{x}\,\left(A\,b-B\,a\right)}{2\,a\,b\,\left(b\,x^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\frac{\left(3\,A\,b+B\,a\right)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{\left(3\,A\,b+B\,a\right)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}+\frac{\left(3\,A\,b+B\,a\right)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{\left(3\,A\,b+B\,a\right)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)\,1{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}}{\frac{\left(3\,A\,b+B\,a\right)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{\left(3\,A\,b+B\,a\right)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}-\frac{\left(3\,A\,b+B\,a\right)\,\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{\left(3\,A\,b+B\,a\right)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,b^{5/4}}}\right)\,\left(3\,A\,b+B\,a\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{7/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(a + b*x^2)^2),x)
```

```
[Out] (atan((((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2
- ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(8*(-a)^(7/4)*b^(5/4)))*1i)/(8*(-a
)^(7/4)*b^(5/4)) + ((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*
a*b^2))/a^2 + ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(8*(-a)^(7/4)*b^(5/4))
)*1i)/(8*(-a)^(7/4)*b^(5/4)))/(((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^
2*b + 6*A*B*a*b^2))/a^2 - ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(8*(-a)^(7
/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4)) - ((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3
 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 + ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(
8*(-a)^(7/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4))))*(3*A*b + B*a)*1i)/(4*(-a)^
(7/4)*b^(5/4)) + (atan((((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6
*A*B*a*b^2))/a^2 - ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2)*1i)/(8*(-a)^(7/4)*
b^(5/4))))/(8*(-a)^(7/4)*b^(5/4)) + ((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B
^2*a^2*b + 6*A*B*a*b^2))/a^2 + ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2)*1i)/(8
*(-a)^(7/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4)))/(((3*A*b + B*a)*((x^(1/2)*(9
*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 - ((3*A*b + B*a)*(24*A*b^3 + 8*B*a
*b^2)*1i)/(8*(-a)^(7/4)*b^(5/4)))*1i)/(8*(-a)^(7/4)*b^(5/4)) - ((3*A*b + B*
a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 + ((3*A*b + B*a)*(2
4*A*b^3 + 8*B*a*b^2)*1i)/(8*(-a)^(7/4)*b^(5/4)))*1i)/(8*(-a)^(7/4)*b^(5/4))
))*(3*A*b + B*a))/(4*(-a)^(7/4)*b^(5/4)) + (x^(1/2)*(A*b - B*a))/(2*a*b*(a
+ b*x^2))
```

3.380 $\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,\left(a+bx^2\right)}+\frac{(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}}-\frac{(5Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}}$$

```
[Out] 1/8*(5*A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(3/4)*2
^(1/2)-1/8*(5*A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^
(3/4)*2^(1/2)-1/16*(5*A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)
*x^(1/2))/a^(9/4)/b^(3/4)*2^(1/2)+1/16*(5*A*b-B*a)*ln(a^(1/2)+x*b^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(3/4)*2^(1/2)+1/2*(-5*A*b+B*a)/a^2/
b/x^(1/2)+1/2*(A*b-B*a)/a/b/(b*x^2+a)/x^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{(5Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}}-\frac{(5Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}}-\frac{(5Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{3/4}}-\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/2*(5*A*b - a*B)/(a^2*b*Sqrt[x]) + (A*b - a*B)/(2*a*b*Sqrt[x]*(a + b*x^2)
) + ((5*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(3/4)) + ((5*A*b
 - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt
[2]*a^(9/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)} + \frac{\left(\frac{5Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{2ab}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)} - \frac{(5Ab-aB)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^2}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)} - \frac{(5Ab-aB)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^2}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)} + \frac{(5Ab-aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^2\sqrt{b}} -$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)} - \frac{(5Ab-aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{\right.}{8a^2b}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)} - \frac{(5Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\right.}{8\sqrt{2}\,a^{9/4}b^{3/4}}$$

$$= -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)} + \frac{(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}} - \frac{(5Ab-}{}$$

Mathematica [A]
time = 0.52, size = 162, normalized size = 0.56

$$\frac{\frac{4\sqrt[4]{a}\left(-4aA-5Abx^2+aBx^2\right)}{\sqrt{x}\,(a+bx^2)} + \frac{\sqrt{2}\,(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}} + \frac{\sqrt{2}\,(5Ab-aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{3/4}}}{8a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] ((4*a^(1/4)*(-4*a*A - 5*A*b*x^2 + a*B*x^2))/(Sqrt[x]*(a + b*x^2)) + (Sqrt[2
]*(5*A*b - a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x])])/b^(3/4) + (Sqrt[2]*(5*A*b - a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4))/(8*a^(9/4))
```

Maple [A]
time = 0.11, size = 153, normalized size = 0.53

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^{2}+a}+\frac{\left(\frac{5Ab}{4}-\frac{Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{a}{b}\right)\cdots}\right.\cdots\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right)}{a^{2}}$
default	$-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^{2}+a}+\frac{\left(\frac{5Ab}{4}-\frac{Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{a}{b}\right)\cdots}\right.\cdots\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right)}{a^{2}}$
risch	$-\frac{2A}{a^{2}\sqrt{x}}-\frac{x^{\frac{3}{2}}Ab}{2a^{2}(b\,x^{2}+a)}+\frac{x^{\frac{3}{2}}B}{2a(b\,x^{2}+a)}-\frac{5\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{16a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{5\sqrt{2}\,A\arctan\left(\sqrt{\cdots}\right.}{8a^{2}\left(\frac{a}{b}\right)\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2*((1/4*A*b-1/4*B*a)*x^(3/2)/(b*x^2+a)+1/8*(5/4*A*b-1/4*B*a)/b/(a/b)^(
1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2*A/a^2/x^(1/2)
```

Maxima [A]
time = 0.55, size = 222, normalized size = 0.77

$$\frac{(Ba-5\,Ab)x^{2}-4\,Aa}{2\left(a^{2}bx^{\frac{5}{2}}+a^{3}\sqrt{x}\right)}+\frac{(Ba-5\,Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

[Out] 1/2*((B*a - 5*A*b)*x^2 - 4*A*a)/(a^2*b*x^(5/2) + a^3*sqrt(x)) + 1/16*(B*a - 5*A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(204) = 408$.
time = 1.67, size = 920, normalized size = 3.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/8*(4*(a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*arctan((sqrt((B^6*a^6 - 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 - 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 - 18750*A^5*B*a*b^5 + 15625*A^6*b^6)*x - (B^4*a^9*b - 20*A*B^3*a^8*b^2 + 150*A^2*B^2*a^7*b^3 - 500*A^3*B*a^6*b^4 + 625*A^4*a^5*b^5)*sqrt(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3)))*a^2*b*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4) + (B^3*a^5*b - 15*A*B^2*a^4*b^2 + 75*A^2*B*a^3*b^3 - 125*A^3*a^2*b^4)*sqrt(x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4))/(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)) - (a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*log(a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3)*sqrt(x)) + (a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*log(-a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3)*sqrt(x)) + 4*((B*a - 5*A*b)*x^2 - 4*A*a)*sqrt(x))/(a^2*b*x^3 + a^3*x)

Sympy [A]
time = 79.38, size = 573, normalized size = 1.98

$$A\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{9 b^{2} x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{a^{2} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{5 a \sqrt{x} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} + \frac{5 a \sqrt{x} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{10 a \sqrt{x} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{16 a \sqrt[4]{- \frac{a}{b}}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{5 b x^{\frac{5}{2}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} + \frac{5 b x^{\frac{5}{2}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{10 b x^{\frac{5}{2}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} - \frac{20 b x^{2} \sqrt[4]{- \frac{a}{b}}}{8 a^{3} \sqrt{x} \sqrt[4]{- \frac{a}{b}} + 8 a^{2} b x^{\frac{5}{2}} \sqrt[4]{- \frac{a}{b}}} & \text{otherwise} \end{cases}\right) + \frac{2 B x^{\frac{3}{2}}}{4 a^{2} + 4 a b x^{2}} + 2 B \operatorname{RootSum}{\left(65536 t^{4} a^{5} b^{3} + 1, \left(t \mapsto t \log{\left(4096 t^{3} a^{4} b^{2} + \sqrt{x} \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] A*Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b**2*x**(9/2)), Eq(
a, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-5*a*sqrt(x)*log(sqrt(x) - (-a/b)**
(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) + 5
*a*sqrt(x)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a
**2*b*x**(5/2)*(-a/b)**(1/4)) - 10*a*sqrt(x)*atan(sqrt(x)/(-a/b)**(1/4))/(8
*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 16*a*(-a/b
)**(1/4)/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) -
 5*b*x**(5/2)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) +
8*a**2*b*x**(5/2)*(-a/b)**(1/4)) + 5*b*x**(5/2)*log(sqrt(x) + (-a/b)**(1/4)
)/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 10*b*x
**(5/2)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*
b*x**(5/2)*(-a/b)**(1/4)) - 20*b*x**2*(-a/b)**(1/4)/(8*a**3*sqrt(x)*(-a/b)*
*(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)), True)) + 2*B*x**(3/2)/(4*a**2 +
4*a*b*x**2) + 2*B*RootSum(65536*_t**4*a**5*b**3 + 1, Lambda(_t, _t*log(4096
*_t**3*a**4*b**2 + sqrt(x))))
```

Giac [A]
time = 0.52, size = 278, normalized size = 0.96

$$\frac{Bax^2-5\,Abx^2-4\,Aa}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^2}+\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-5\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3}+\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-5\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3}-\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-5\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^3}+\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-5\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a*x^2 - 5*A*b*x^2 - 4*A*a)/((b*x^(5/2) + a*sqrt(x))*a^2) + 1/8*sqrt(
2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) + 1/8*sqrt(2)*((a*b^3)^(3/4)*
B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqr
t(x))/(a/b)^(1/4))/(a^3*b^3) - 1/16*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^
(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 1/1
6*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

Mupad [B]
time = 0.18, size = 104, normalized size = 0.36

$$\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)\,\left(5\,A\,b-B\,a\right)}{4\,{(-a)}^{9/4}\,b^{3/4}}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)\,\left(5\,A\,b-B\,a\right)}{4\,{(-a)}^{9/4}\,b^{3/4}}-\frac{\frac{2\,A}{a}+\frac{x^2\,\left(5\,A\,b-B\,a\right)}{2\,a^2}}{a\,\sqrt{x}+b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x)
```

```
[Out] (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b - B*a))/(4*(-a)^(9/4)*b^(3/4))
- (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b - B*a))/(4*(-a)^(9/4)*b^(3/4))
 - ((2*A)/a + (x^2*(5*A*b - B*a))/(2*a^2))/(a*x^(1/2) + b*x^(5/2))
```

3.381 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}+\frac{(7Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\right.}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}}$$

```
[Out] 1/6*(-7*A*b+3*B*a)/a^2/b/x^(3/2)+1/2*(A*b-B*a)/a/b/x^(3/2)/(b*x^2+a)+1/8*(7
*A*b-3*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1
/2)-1/8*(7*A*b-3*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^
(1/4)*2^(1/2)+1/16*(7*A*b-3*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/
2)*x^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)-1/16*(7*A*b-3*B*a)*ln(a^(1/2)+x*b^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {468, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{(7Ab-3aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{(7Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/6*(7*A*b - 3*a*B)/(a^2*b*x^(3/2)) + (A*b - a*B)/(2*a*b*x^(3/2)*(a + b*x^
2)) + ((7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sq
rt[2]*a^(11/4)*b^(1/4)) - ((7*A*b - 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(1/4)) + ((7*A*b - 3*a*B)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(1/4)
) - ((7*A*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(8*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx = \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} + \frac{\left(\frac{7Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}(a+bx^2)}\,dx}{2ab}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} - \frac{(7Ab-3aB)\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{4a^2}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} - \frac{(7Ab-3aB)\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^2}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} - \frac{(7Ab-3aB)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{5/2}}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} - \frac{(7Ab-3aB)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x\right.}{8a^{5/2}\sqrt{b}}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} + \frac{(7Ab-3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{\right.}}{8\sqrt{2}\,a^{11/4}\sqrt[4]{b}}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} + \frac{(7Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}} - \frac{(7Ab}{}$$

Mathematica [A]
time = 0.56, size = 165, normalized size = 0.57

$$\frac{\frac{4a^{3/4}\left(-4aA-7Abx^2+3aBx^2\right)}{x^{3/2}(a+bx^2)} + \frac{3\sqrt{2}\,(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}\,(-7Ab+3aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{b}}}{24a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

[Out] ((4*a^(3/4)*(-4*a*A - 7*A*b*x^2 + 3*a*B*x^2))/(x^(3/2)*(a + b*x^2)) + (3*Sqrt[2]*(7*A*b - 3*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (3*Sqrt[2]*(-7*A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4))/(24*a^(11/4))

Maple [A]
time = 0.10, size = 153, normalized size = 0.53

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)\sqrt{x}}{b x^2+a}+\frac{(7Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{32a}}{a^2}$
default	$-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)\sqrt{x}}{b x^2+a}+\frac{(7Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{32a}}{a^2}$
risch	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{\sqrt{x}\,Ab}{2a^2(b x^2+a)}+\frac{\sqrt{x}\,B}{2a(b x^2+a)}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)b}{8a^3}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\right.}{8a^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -2/a^2*((1/4*A*b-1/4*B*a)*x^(1/2)/(b*x^2+a)+1/32*(7*A*b-3*B*a)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/3*A/a^2/x^(3/2)

Maxima [A]
time = 0.53, size = 251, normalized size = 0.87

$$\frac{(3\,Ba-7\,Ab)x^2-4\,Aa}{6\left(a^2bx^{\frac{7}{2}}+a^3x^{\frac{3}{2}}\right)}+\frac{\frac{2\sqrt{2}\,(3\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(3\,Ba-7\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(3\,Ba-7\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,Ba-7\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/6*((3*B*a - 7*A*b)*x^2 - 4*A*a)/(a^2*b*x^(7/2) + a^3*x^(3/2)) + 1/16*(2*sqrt(2)*(3*B*a - 7*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqr

```
t(2)*(3*B*a - 7*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(3*B*a - 7*A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a)
)/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*B*a - 7*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(209) = 418$.
time = 2.58, size = 741, normalized size = 2.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(a^2*b*x^4 + a^3*x^2)*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*
B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)*arctan((sqrt
(a^6*sqrt(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*
B*a*b^3 + 2401*A^4*b^4)/(a^11*b)) + (9*B^2*a^2 - 42*A*B*a*b + 49*A^2*b^2)*x
)*a^8*b*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B
*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(3/4) + (3*B*a^9*b - 7*A*a^8*b^2)*sqrt(x)*
(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 +
 2401*A^4*b^4)/(a^11*b))^(3/4))/(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^
2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)) + 3*(a^2*b*x^4 + a^3*x^2)*(-(
81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 24
01*A^4*b^4)/(a^11*b))^(1/4)*log(a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*
A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a
 - 7*A*b)*sqrt(x)) - 3*(a^2*b*x^4 + a^3*x^2)*(-(81*B^4*a^4 - 756*A*B^3*a^3*
b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)
*log(-a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3
*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a - 7*A*b)*sqrt(x)) - 4*((3
*B*a - 7*A*b)*x^2 - 4*A*a)*sqrt(x))/(a^2*b*x^4 + a^3*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(277) = 554$.
time = 118.42, size = 855, normalized size = 2.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0
)), ((-2*A/(3*x**(3/2)) + 2*B*sqrt(x))/a**2, Eq(b, 0)), ((-2*A/(11*x**(11/2
)) - 2*B/(7*x**(7/2)))/b**2, Eq(a, 0)), (-16*A*a**2/(24*a**4*x**(3/2) + 24*
```

```
a**3*b*x**(7/2)) + 21*A*a*b*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1
/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 21*A*a*b*x**(3/2)*(-a/b)**(1
/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) -
42*A*a*b*x**(3/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*x**(3/
2) + 24*a**3*b*x**(7/2)) - 28*A*a*b*x**2/(24*a**4*x**(3/2) + 24*a**3*b*x**(
7/2)) + 21*A*b**2*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24*a
**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 21*A*b**2*x**(7/2)*(-a/b)**(1/4)*log(s
qrt(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 42*A*b**2
*x**(7/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*
a**3*b*x**(7/2)) - 9*B*a**2*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1
/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) + 9*B*a**2*x**(3/2)*(-a/b)**(1
/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) +
18*B*a**2*x**(3/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*x**(3
/2) + 24*a**3*b*x**(7/2)) + 12*B*a**2*x**2/(24*a**4*x**(3/2) + 24*a**3*b*x*
*(7/2)) - 9*B*a*b*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24*a
**4*x**(3/2) + 24*a**3*b*x**(7/2)) + 9*B*a*b*x**(7/2)*(-a/b)**(1/4)*log(sqr
t(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) + 18*B*a*b*x*
*(7/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**
3*b*x**(7/2)), True))
```

Giac [A]
time = 0.54, size = 283, normalized size = 0.98

$$\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{3}b}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{3}b}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{3}b}-\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{3}b}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\left(bx^{2}+a\right)a^{2}}-\frac{2A}{3a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 1/8*sqrt(2)*(3*(a*
b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B*a -
 7*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3
*b) - 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/2*(B*a*sqrt(x) - A*b*sqrt
(x))/((b*x^2 + a)*a^2) - 2/3*A/(a^2*x^(3/2))
```

Mupad [B]
time = 0.25, size = 859, normalized size = 2.97

$$-\frac{\frac{2A}{3a}+\frac{x^{2}\left(7Ab-3Ba\right)}{6a^{2}}}{ax^{3/2}+bx^{7/2}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(7Ab-3Ba\right)\left(\sqrt{x}\left(1568A^{2}a^{6}b^{5}-1344ABa^{7}b^{4}+288B^{2}a^{8}b^{3}\right)-\frac{\left(7Ab-3Ba\right)\left(1792Aa^{9}b^{4}-768Ba^{10}b^{3}\right)}{8\left(-a\right)^{11/4}b^{1/4}}\right)1\mathrm{i}}{8\left(-a\right)^{11/4}b^{1/4}}+\frac{\left(7Ab-3Ba\right)\left(\sqrt{x}\left(1568A^{2}a^{6}b^{5}-1344ABa^{7}b^{4}+288B^{2}a^{8}b^{3}\right)+\frac{\left(7Ab-3Ba\right)\left(1792Aa^{9}b^{4}-768Ba^{10}b^{3}\right)}{8\left(-a\right)^{11/4}b^{1/4}}\right)1\mathrm{i}}{8\left(-a\right)^{11/4}b^{1/4}}}{\frac{\left(7Ab-3Ba\right)\left(\sqrt{x}\left(1568A^{2}a^{6}b^{5}-1344ABa^{7}b^{4}+288B^{2}a^{8}b^{3}\right)-\frac{\left(7Ab-3Ba\right)\left(1792Aa^{9}b^{4}-768Ba^{10}b^{3}\right)}{8\left(-a\right)^{11/4}b^{1/4}}\right)}{8\left(-a\right)^{11/4}b^{1/4}}-\frac{\left(7Ab-3Ba\right)\left(\sqrt{x}\left(1568A^{2}a^{6}b^{5}-1344ABa^{7}b^{4}+288B^{2}a^{8}b^{3}\right)+\frac{\left(7Ab-3Ba\right)\left(1792Aa^{9}b^{4}-768Ba^{10}b^{3}\right)}{8\left(-a\right)^{11/4}b^{1/4}}\right)}{8\left(-a\right)^{11/4}b^{1/4}}}\right)\left(7Ab-3Ba\right)1\mathrm{i}}{4\left(-a\right)^{11/4}b^{1/4}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(7Ab-3Ba\right)\left(\sqrt{x}\left(1568A^{2}a^{6}b^{5}-1344ABa^{7}b^{4}+288B^{2}a^{8}b^{3}\right)-\frac{\left(7Ab-3Ba\right)\left(1792Aa^{9}b^{4}-768Ba^{10}b^{3}\right)1\mathrm{i}}{8\left(-a\right)^{11/4}b^{1/4}}\right)}{8\left(-a\right)^{11/4}b^{1/4}}+\frac{\left(7Ab-3Ba\right)\left(\sqrt{x}\left(1568A^{2}a^{6}b^{5}-1344ABa^{7}b^{4}+288B^{2}a^{8}b^{3}\right)+\frac{\left(7Ab-3Ba\right)\left(1792Aa^{9}b^{4}-768Ba^{10}b^{3}\right)1\mathrm{i}}{8\left(-a\right)^{11/4}b^{1/4}}\right)}{8\left(-a\right)^{11/4}b^{1/4}}}{\frac{\left(7Ab-3Ba\right)\left(\sqrt{x}\left(1568A^{2}a^{6}b^{5}-1344ABa^{7}b^{4}+288B^{2}a^{8}b^{3}\right)-\frac{\left(7Ab-3Ba\right)\left(1792Aa^{9}b^{4}-768Ba^{10}b^{3}\right)1\mathrm{i}}{8\left(-a\right)^{11/4}b^{1/4}}\right)1\mathrm{i}}{8\left(-a\right)^{11/4}b^{1/4}}-\frac{\left(7Ab-3Ba\right)\left(\sqrt{x}\left(1568A^{2}a^{6}b^{5}-1344ABa^{7}b^{4}+288B^{2}a^{8}b^{3}\right)+\frac{\left(7Ab-3Ba\right)\left(1792Aa^{9}b^{4}-768Ba^{10}b^{3}\right)1\mathrm{i}}{8\left(-a\right)^{11/4}b^{1/4}}\right)1\mathrm{i}}{8\left(-a\right)^{11/4}b^{1/4}}}\right)\left(7Ab-3Ba\right)}{4\left(-a\right)^{11/4}b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x)
```

```
[Out] - ((2*A)/(3*a) + (x^2*(7*A*b - 3*B*a))/(6*a^2))/(a*x^(3/2) + b*x^(7/2)) - (
atan((((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*
A*B*a^7*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3))/(8*(-a)^
(11/4)*b^(1/4)))*1i)/(8*(-a)^(11/4)*b^(1/4)) + ((7*A*b - 3*B*a)*(x^(1/2)*(1
568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*(1
792*A*a^9*b^4 - 768*B*a^10*b^3))/(8*(-a)^(11/4)*b^(1/4)))*1i)/(8*(-a)^(11/4
)*b^(1/4)))/(((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3
- 1344*A*B*a^7*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3))/(
8*(-a)^(11/4)*b^(1/4))))/(8*(-a)^(11/4)*b^(1/4)) - ((7*A*b - 3*B*a)*(x^(1/2
)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*a
)*(1792*A*a^9*b^4 - 768*B*a^10*b^3))/(8*(-a)^(11/4)*b^(1/4))))/(8*(-a)^(11/
4)*b^(1/4))))*(7*A*b - 3*B*a)*1i)/(4*(-a)^(11/4)*b^(1/4)) - (atan((((7*A*b
- 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) -
 ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3)*1i)/(8*(-a)^(11/4)*b^(1
/4))))/(8*(-a)^(11/4)*b^(1/4)) + ((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^
5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*(1792*A*a^9*b^4
- 768*B*a^10*b^3)*1i)/(8*(-a)^(11/4)*b^(1/4))))/(8*(-a)^(11/4)*b^(1/4)))/((
(7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7
*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3)*1i)/(8*(-a)^(11/
4)*b^(1/4)))*1i)/(8*(-a)^(11/4)*b^(1/4)) - ((7*A*b - 3*B*a)*(x^(1/2)*(1568*
A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*(1792*
A*a^9*b^4 - 768*B*a^10*b^3)*1i)/(8*(-a)^(11/4)*b^(1/4)))*1i)/(8*(-a)^(11/4)
*b^(1/4))))*(7*A*b - 3*B*a))/(4*(-a)^(11/4)*b^(1/4))
```

3.382 $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}-\frac{\sqrt[4]{b}\left(9Ab-5aB\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}+\frac{\sqrt[4]{b}\left(9Ab-5\right.}{}$$

```
[Out] 1/10*(-9*A*b+5*B*a)/a^2/b/x^(5/2)+1/2*(A*b-B*a)/a/b/x^(5/2)/(b*x^2+a)-1/8*b
^(1/4)*(9*A*b-5*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(
1/2)+1/8*b^(1/4)*(9*A*b-5*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^
(13/4)*2^(1/2)+1/16*b^(1/4)*(9*A*b-5*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)-1/16*b^(1/4)*(9*A*b-5*B*a)*ln(a^(1/2)
+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)+1/2*(9*A*b-5*B
*a)/a^3/x^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{b}\,(9Ab-5aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}+\frac{\sqrt[4]{b}\,(9Ab-5aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{13/4}}+\frac{\sqrt[4]{b}\,(9Ab-5aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{\sqrt[4]{b}\,(9Ab-5aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}-\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/10*(9*A*b - 5*a*B)/(a^2*b*x^(5/2)) + (9*A*b - 5*a*B)/(2*a^3*Sqrt[x]) + (
A*b - a*B)/(2*a*b*x^(5/2)*(a + b*x^2)) - (b^(1/4)*(9*A*b - 5*a*B)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)) + (b^(1/4)*(9*A*
b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/
4)) + (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)) - (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx = \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{\left(\frac{9Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{2ab}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} - \frac{(9Ab-5aB)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{(b(9Ab-5aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{(b(9Ab-5aB))\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,d\right.}{2a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\left(\sqrt{b}\,(9Ab-5aB)\right)\text{Subst}\left(\int \right.}{4a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{(9Ab-5aB)\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}-} \right.}{8a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{\sqrt[4]{b}\,(9Ab-5aB)\log\left(\sqrt{a}-\right.}{8\sqrt{2}\,a^{13/}}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\sqrt[4]{b}\,(9Ab-5aB)\tan^{-1}\left(1-\right.}{4\sqrt{2}\,a^{13/4}}$$

Mathematica [A]

time = 0.55, size = 186, normalized size = 0.60

$$\frac{-\frac{4\sqrt[4]{a}\left(-45Ab^2x^4+4a^2\left(A+5Bx^2\right)+a\left(-36Abx^2+25bBx^4\right)\right)}{x^{5/2}\left(a+bx^2\right)}+5\sqrt{2}\,\sqrt[4]{b}\,(-9Ab+5aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+5\sqrt{2}\,\sqrt[4]{b}\,(-9Ab+5aB)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{40a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-4*a^(1/4)*(-45*A*b^2*x^4 + 4*a^2*(A + 5*B*x^2) + a*(-36*A*b*x^2 + 25*b*B
*x^4)))/(x^(5/2)*(a + b*x^2)) + 5*Sqrt[2]*b^(1/4)*(-9*A*b + 5*a*B)*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^(1/4)
*(-9*A*b + 5*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)])/(40*a^(13/4))
```

Maple [A]

time = 0.11, size = 170, normalized size = 0.55

method	result
derivativedivides	$\frac{2b\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^{2}+a}+\frac{\left(\frac{9Ab}{4}-\frac{5Ba}{4}\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.}{a^{3}}$
default	$\frac{2b\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^{2}+a}+\frac{\left(\frac{9Ab}{4}-\frac{5Ba}{4}\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.}{a^{3}}$
risch	$-\frac{2\left(-10Ab\,x^{2}+5Ba\,x^{2}+Aa\right)}{5a^{3}x^{\frac{5}{2}}}+\frac{b^{2}x^{\frac{3}{2}}A}{2a^{3}\left(b\,x^{2}+a\right)}-\frac{b\,x^{\frac{3}{2}}B}{2a^{2}\left(b\,x^{2}+a\right)}+\frac{9b\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{16a^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{9b\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^3*b*((1/4*A*b-1/4*B*a)*x^(3/2)/(b*x^2+a)+1/8*(9/4*A*b-5/4*B*a)/b/(a/b)^
(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/5*A/a^2/x^(5/2)-2*(-2*A*b+B*a)/a^3
/x^(1/2)
```

Maxima [A]

time = 0.51, size = 250, normalized size = 0.81

$$-\frac{5\,(5\,Bab-9\,Ab^{2})x^{4}+4\,Aa^{2}+4\,(5\,Ba^{2}-9\,Aab)x^{2}}{10\left(a^{3}bx^{\frac{9}{2}}+a^{4}x^{\frac{5}{2}}\right)}-\frac{(5\,Bab-9\,Ab^{2})\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/10*(5*(5*B*a*b - 9*A*b^2)*x^4 + 4*A*a^2 + 4*(5*B*a^2 - 9*A*a*b)*x^2)/(a^
3*b*x^(9/2) + a^4*x^(5/2)) - 1/16*(5*B*a*b - 9*A*b^2)*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x
) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(226) = 452$.
time = 1.44, size = 974, normalized size = 3.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 121
50*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*arctan((
sqrt((15625*B^6*a^6*b^2 - 168750*A*B^5*a^5*b^3 + 759375*A^2*B^4*a^4*b^4 - 1
822500*A^3*B^3*a^3*b^5 + 2460375*A^4*B^2*a^2*b^6 - 1771470*A^5*B*a*b^7 + 53
1441*A^6*b^8)*x - (625*B^4*a^11*b - 4500*A*B^3*a^10*b^2 + 12150*A^2*B^2*a^9
*b^3 - 14580*A^3*B*a^8*b^4 + 6561*A^4*a^7*b^5)*sqrt(-(625*B^4*a^4*b - 4500*
A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a
^13))*a^3*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 1
4580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4) + (125*B^3*a^6*b - 675*A*B^2*a
^5*b^2 + 1215*A^2*B*a^4*b^3 - 729*A^3*a^3*b^4)*sqrt(x)*(-(625*B^4*a^4*b - 4
500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^
5)/a^13)^(1/4))/(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3
 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)) - 5*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*
a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 65
61*A^4*b^5)/a^13)^(1/4)*log(a^10*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12
150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(3/4) - (125*
B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)) +
5*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B
^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(-a^10*(-(625
*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4
 + 6561*A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^
2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)) + 4*(5*(5*B*a*b - 9*A*b^2)*x^4 + 4*A*a^2
+ 4*(5*B*a^2 - 9*A*a*b)*x^2)*sqrt(x))/(a^3*b*x^5 + a^4*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 303, normalized size = 0.98

$$-\frac{Babx^{\frac{3}{2}} - Ab^2x^{\frac{3}{2}}}{2\,(bx^2+a)a^3} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,Ba - 9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^2} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,Ba - 9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^2} + \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,Ba - 9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^2} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,Ba - 9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^2} - \frac{2\,(5\,Bax^2 - 10\,Abx^2 + Aa)}{5\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b*x^(3/2) - A*b^2*x^(3/2))/((b*x^2 + a)*a^3) - 1/8*sqrt(2)*(5*(a*
b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 1/8*sqrt(2)*(5*(a*b^3)^(3/4)*B*a -
 9*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^4*b^2) + 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/
4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*s
qrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 2/5*(5*B*a*x^2 - 10*A*b*x^2 + A*a)/(a
^3*x^(5/2))
```

Mupad [B]
time = 0.10, size = 121, normalized size = 0.39

$$\frac{\frac{2\,x^2\,(9\,A\,b-5\,B\,a)}{5\,a^2} - \frac{2\,A}{5\,a} + \frac{b\,x^4\,(9\,A\,b-5\,B\,a)}{2\,a^3}}{a\,x^{5/2}+b\,x^{9/2}} + \frac{{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(9\,A\,b-5\,B\,a)}{4\,a^{13/4}} - \frac{{(-b)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(9\,A\,b-5\,B\,a)}{4\,a^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x)
```

```
[Out] ((2*x^2*(9*A*b - 5*B*a))/(5*a^2) - (2*A)/(5*a) + (b*x^4*(9*A*b - 5*B*a))/(2
*a^3))/(a*x^(5/2) + b*x^(9/2)) + ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1
/4))*(9*A*b - 5*B*a))/(4*a^(13/4)) - ((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))
/a^(1/4))*(9*A*b - 5*B*a))/(4*a^(13/4))
```

3.383 $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=316

$$-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}-\frac{5(Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{5(Ab-9$$

```
[Out] 1/4*(A*b-B*a)*x^(9/2)/a/b/(b*x^2+a)^2+1/16*(A*b-9*B*a)*x^(5/2)/a/b^2/(b*x^2
+a)-5/64*(A*b-9*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(1
3/4)*2^(1/2)+5/64*(A*b-9*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
3/4)/b^(13/4)*2^(1/2)-5/128*(A*b-9*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4
)*2^(1/2)*x^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)+5/128*(A*b-9*B*a)*ln(a^(1/2)+x*
b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)-5/16*(A*b
-9*B*a)*x^(1/2)/a/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {468, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{5(Ab-9aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{5(Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{5\sqrt{x}(Ab-9aB)}{16ab^3}+\frac{x^{5/2}(Ab-9aB)}{16ab^2\left(a+bx^2\right)}+\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-5*(A*b - 9*a*B)*Sqrt[x])/(16*a*b^3) + ((A*b - a*B)*x^(9/2))/(4*a*b*(a + b
*x^2)^2) + ((A*b - 9*a*B)*x^(5/2))/(16*a*b^2*(a + b*x^2)) - (5*(A*b - 9*a*B
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(13/
4)) + (5*(A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*S
qrt[2]*a^(3/4)*b^(13/4)) - (5*(A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(13/4)) + (5*(A*b - 9*a*
B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*
a^(3/4)*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(Ab-aB)x^{9/2}}{4ab\,(a+bx^2)^2} + \frac{\left(-\frac{Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)x^{9/2}}{4ab\,(a+bx^2)^2} + \frac{(Ab-9aB)x^{5/2}}{16ab^2\,(a+bx^2)} - \frac{(5(Ab-9aB))\int \frac{x^{3/2}}{a+bx^2}\,dx}{32ab^2}$$

$$= -\frac{5(Ab-9aB)\sqrt{x}}{16ab^3} + \frac{(Ab-aB)x^{9/2}}{4ab\,(a+bx^2)^2} + \frac{(Ab-9aB)x^{5/2}}{16ab^2\,(a+bx^2)} + \frac{(5(Ab-9aB))\int \frac{1}{\sqrt{x}\,(a+}}{32b^3}$$

$$= -\frac{5(Ab-9aB)\sqrt{x}}{16ab^3} + \frac{(Ab-aB)x^{9/2}}{4ab\,(a+bx^2)^2} + \frac{(Ab-9aB)x^{5/2}}{16ab^2\,(a+bx^2)} + \frac{(5(Ab-9aB))\text{Subst}\left(\int\right.}{16b^3}$$

$$= -\frac{5(Ab-9aB)\sqrt{x}}{16ab^3} + \frac{(Ab-aB)x^{9/2}}{4ab\,(a+bx^2)^2} + \frac{(Ab-9aB)x^{5/2}}{16ab^2\,(a+bx^2)} + \frac{(5(Ab-9aB))\text{Subst}\left(\int\right.}{32\sqrt{}}$$

$$= -\frac{5(Ab-9aB)\sqrt{x}}{16ab^3} + \frac{(Ab-aB)x^{9/2}}{4ab\,(a+bx^2)^2} + \frac{(Ab-9aB)x^{5/2}}{16ab^2\,(a+bx^2)} + \frac{(5(Ab-9aB))\text{Subst}\left(\int\right.}{6}$$

$$= -\frac{5(Ab-9aB)\sqrt{x}}{16ab^3} + \frac{(Ab-aB)x^{9/2}}{4ab\,(a+bx^2)^2} + \frac{(Ab-9aB)x^{5/2}}{16ab^2\,(a+bx^2)} - \frac{5(Ab-9aB)\log\left(\sqrt{a}\right. -}{64\sqrt{2}}$$

$$= -\frac{5(Ab-9aB)\sqrt{x}}{16ab^3} + \frac{(Ab-aB)x^{9/2}}{4ab\,(a+bx^2)^2} + \frac{(Ab-9aB)x^{5/2}}{16ab^2\,(a+bx^2)} - \frac{5(Ab-9aB)\tan^{-1}\left(1-\right.}{32\sqrt{2}\,a^{3/4}b^{1}}$$

Mathematica [A]

time = 0.62, size = 183, normalized size = 0.58

$$\frac{\frac{4\sqrt[4]{b}\,\sqrt{x}\,(-5aAb+45a^2B-9Ab^2x^2+81abBx^2+32b^2Bx^4)}{(a+bx^2)^2} + \frac{5\sqrt{2}\,(-Ab+9aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{3/4}} + \frac{5\sqrt{2}\,(Ab-9aB)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}}}{64b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((4*b^(1/4)*Sqrt[x]*(-5*a*A*b + 45*a^2*B - 9*A*b^2*x^2 + 81*a*b*B*x^2 + 32*
b^2*B*x^4))/(a + b*x^2)^2 + (5*Sqrt[2]*(-(A*b) + 9*a*B)*ArcTan[(Sqrt[a] - S
qrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (5*Sqrt[2]*(A*b - 9
*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(
3/4))/(64*b^(13/4))
```

Maple [A]

time = 0.12, size = 172, normalized size = 0.54

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b^3}+\frac{\frac{2\left(\left(-\frac{9}{32}b^2A+\frac{17}{32}abB\right)x^{\frac{5}{2}}-\frac{a(5Ab-13Ba)\sqrt{x}}{32}\right)}{(bx^2+a)^2}+\frac{5(Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{12}}{b^3}$
default	$\frac{2B\sqrt{x}}{b^3}+\frac{\frac{2\left(\left(-\frac{9}{32}b^2A+\frac{17}{32}abB\right)x^{\frac{5}{2}}-\frac{a(5Ab-13Ba)\sqrt{x}}{32}\right)}{(bx^2+a)^2}+\frac{5(Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{12}}{b^3}$
risch	$\frac{2B\sqrt{x}}{b^3}-\frac{9x^{\frac{5}{2}}A}{16b(bx^2+a)^2}+\frac{17x^{\frac{5}{2}}aB}{16b^2(bx^2+a)^2}-\frac{5A\sqrt{x}\,a}{16b^2(bx^2+a)^2}+\frac{13B\sqrt{x}\,a^2}{16b^3(bx^2+a)^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right.}\right.}{64b^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/b^3*x^(1/2)+2/b^3*(((-9/32*b^2*A+17/32*a*b*B)*x^(5/2)-1/32*a*(5*A*b-13*
B*a)*x^(1/2))/(b*x^2+a)^2+5/256*(A*b-9*B*a)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)
^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.51, size = 283, normalized size = 0.90

$$\frac{(17\,Bab-9\,Ab^2)x^{\frac{5}{2}}+(13\,Ba^2-5\,Aab)\sqrt{x}}{16\,(b^5x^4+2\,ab^4x^2+a^2b^3)}+\frac{2\,B\sqrt{x}}{b^3}-\frac{5\left(\frac{2\sqrt{2}\,(9\,Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(9\,Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(9\,Ba-Ab)\log\left(\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}-\frac{\sqrt{2}\,(9\,Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}\right)}{128\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((17*B*a*b - 9*A*b^2)*x^(5/2) + (13*B*a^2 - 5*A*a*b)*sqrt(x))/(b^5*x^4
 + 2*a*b^4*x^2 + a^2*b^3) + 2*B*sqrt(x)/b^3 - 5/128*(2*sqrt(2)*(9*B*a - A*b
)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(9*B*a - A*b)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(9*B*a - A*b)*log(sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqr
t(2)*(9*B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/(a^(3/4)*b^(1/4)))/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(238) = 476$.
time = 0.96, size = 793, normalized size = 2.51

$$\frac{20\,(b^5x^4+2\,ab^4x^2+a^2b^3)\left(-\frac{6561\,B^4a^4-2916\,AB^3a^3b+486\,A^2B^2a^2b^2-36\,A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2b^6\sqrt{-\frac{6561\,B^4a^4-2916\,AB^3a^3b+486\,A^2B^2a^2b^2-36\,A^3Bab^3+A^4b^4}{a^3b^{13}}}+(81\,B^2a^2-18\,ABab+A^2b^2)x}\,a^2b^{10}\left(-\frac{6561\,B^4a^4-2916\,AB^3a^3b+486\,A^2B^2a^2b^2-36\,A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{3}{4}}+(9\,Ba^3b^{10}-Aa^2b^{11})\sqrt{x}\left(-\frac{6561\,B^4a^4-2916\,AB^3a^3b+486\,A^2B^2a^2b^2-36\,A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{3}{4}}}{6561\,B^4a^4-2916\,AB^3a^3b+486\,A^2B^2a^2b^2-36\,A^3Bab^3+A^4b^4}\right)+5\,(b^5x^4+2\,ab^4x^2+a^2b^3)\left(-\frac{6561\,B^4a^4-2916\,AB^3a^3b+486\,A^2B^2a^2b^2-36\,A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(5\,ab^3\left(-\frac{6561\,B^4a^4-2916\,AB^3a^3b+486\,A^2B^2a^2b^2-36\,A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}-5\,(9\,Ba-Ab)\sqrt{x}\right)-5\,(b^5x^4+2\,ab^4x^2+a^2b^3)\left(-\frac{6561\,B^4a^4-2916\,AB^3a^3b+486\,A^2B^2a^2b^2-36\,A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(-5\,ab^3\left(-\frac{6561\,B^4a^4-2916\,AB^3a^3b+486\,A^2B^2a^2b^2-36\,A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}-5\,(9\,Ba-Ab)\sqrt{x}\right)+4\,(32\,Bb^2x^4+45\,Ba^2-5\,Aab+9\,(9\,Bab-Ab^2)x^2)\sqrt{x}}{64\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3
*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*arct
an((sqrt(a^2*b^6*sqrt(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b
^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13)) + (81*B^2*a^2 - 18*A*B*a*b + A^2
*b^2)*x)*a^2*b^10*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2
 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(3/4) + (9*B*a^3*b^10 - A*a^2*b^11)
*sqrt(x)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*
B*a*b^3 + A^4*b^4)/(a^3*b^13))^(3/4))/(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 48
6*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)) + 5*(b^5*x^4 + 2*a*b^4*x^2 +
 a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3
*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*log(5*a*b^3*(-(6561*B^4*a^4 - 2916*A*
B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/
4) - 5*(9*B*a - A*b)*sqrt(x)) - 5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561
*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^
4)/(a^3*b^13))^(1/4)*log(-5*a*b^3*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*
A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4) - 5*(9*B*a -
A*b)*sqrt(x)) + 4*(32*B*b^2*x^4 + 45*B*a^2 - 5*A*a*b + 9*(9*B*a*b - A*b^2)*
x^2)*sqrt(x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.68, size = 304, normalized size = 0.96

$$\frac{2\,B\sqrt{x}}{b^3}-\frac{5\,\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^4}-\frac{5\,\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^4}-\frac{5\,\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^4}+\frac{5\,\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^4}+\frac{17\,Babx^{\frac{5}{2}}-9\,Ab^2x^{\frac{5}{2}}+13\,Ba^2\sqrt{x}-5\,Aab\sqrt{x}}{16\,(bx^2+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^3 - 5/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 5
/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 5/128*sqrt(2)*(9*(a
*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
sqrt(a/b))/(a*b^4) + 5/128*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b
)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) + 1/16*(17*B*a*
b*x^(5/2) - 9*A*b^2*x^(5/2) + 13*B*a^2*sqrt(x) - 5*A*a*b*sqrt(x))/((b*x^2 +
 a)^2*b^3)
```

Mupad [B]
time = 0.22, size = 760, normalized size = 2.41

$$\frac{\sqrt{x}\,\left(\frac{13\,B\,a^2}{16}-\frac{5\,A\,a\,b}{16}\right)-x^{5/2}\,\left(\frac{9\,A\,b^2}{16}-\frac{17\,B\,a\,b}{16}\right)}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{2\,B\,\sqrt{x}}{b^3}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{5\,\left(45\,B\,a^2-5\,A\,a\,b\right)\,(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)5{}\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}+\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{5\,\left(45\,B\,a^2-5\,A\,a\,b\right)\,(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)5{}\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}}{\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{5\,\left(45\,B\,a^2-5\,A\,a\,b\right)\,(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}-\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{5\,\left(45\,B\,a^2-5\,A\,a\,b\right)\,(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}}\right)\,(A\,b-9\,B\,a)\,5{}\mathrm{i}}{32\,(-a)^{3/4}\,b^{13/4}}-\frac{5\,\mathrm{atan}\left(\frac{\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)\,(A\,b-9\,B\,a)\,5{}\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}+\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)\,(A\,b-9\,B\,a)\,5{}\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}}{\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)\,(A\,b-9\,B\,a)\,5{}\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)5{}\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}-\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)\,(A\,b-9\,B\,a)\,5{}\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)5{}\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}}\right)\,(A\,b-9\,B\,a)}{32\,(-a)^{3/4}\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (x^(1/2)*((13*B*a^2)/16 - (5*A*a*b)/16) - x^(5/2)*((9*A*b^2)/16 - (17*B*a*b
)/16))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + (2*B*x^(1/2))/b^3 - (atan((((A*b
 - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) - (5*(
45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a))/(64*(-a)^(3/4)*b^(13/4)))*5i)/(64*(-a)^(
3/4)*b^(13/4)) + ((A*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B
*a*b))/(64*b^3) + (5*(45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a))/(64*(-a)^(3/4)*b^(
13/4)))*5i)/(64*(-a)^(3/4)*b^(13/4)))/((5*(A*b - 9*B*a)*((25*x^(1/2)*(A^2*b
^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) - (5*(45*B*a^2 - 5*A*a*b)*(A*b - 9*
B*a))/(64*(-a)^(3/4)*b^(13/4))))/(64*(-a)^(3/4)*b^(13/4)) - (5*(A*b - 9*B*a
)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) + (5*(45*B*a^2
 - 5*A*a*b)*(A*b - 9*B*a))/(64*(-a)^(3/4)*b^(13/4))))/(64*(-a)^(3/4)*b^(13/
4))))*(A*b - 9*B*a)*5i)/(32*(-a)^(3/4)*b^(13/4)) - (5*atan(((5*(A*b - 9*B*a
)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) - ((45*B*a^2 -
 5*A*a*b)*(A*b - 9*B*a)*5i)/(64*(-a)^(3/4)*b^(13/4))))/(64*(-a)^(3/4)*b^(13
/4)) + (5*(A*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(
64*b^3) + ((45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a)*5i)/(64*(-a)^(3/4)*b^(13/4)))
)/(64*(-a)^(3/4)*b^(13/4)))/(((A*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*
a^2 - 18*A*B*a*b))/(64*b^3) - ((45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a)*5i)/(64*(
-a)^(3/4)*b^(13/4)))*5i)/(64*(-a)^(3/4)*b^(13/4)) - ((A*b - 9*B*a)*((25*x^(
1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) + ((45*B*a^2 - 5*A*a*b)*
(A*b - 9*B*a)*5i)/(64*(-a)^(3/4)*b^(13/4)))*5i)/(64*(-a)^(3/4)*b^(13/4))))*
(A*b - 9*B*a))/(32*(-a)^(3/4)*b^(13/4))
```

3.384 $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=293

$$\frac{(Ab-aB)x^{7/2}}{4ab\,(a+bx^2)^2}-\frac{(Ab+7aB)x^{3/2}}{16ab^2\,(a+bx^2)}-\frac{3(Ab+7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{3(Ab+7aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}\right.}{32\sqrt{2}\,a^{5/4}b^{11/4}}$$

```
[Out] 1/4*(A*b-B*a)*x^(7/2)/a/b/(b*x^2+a)^2-1/16*(A*b+7*B*a)*x^(3/2)/a/b^2/(b*x^2
+a)-3/64*(A*b+7*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(1
1/4)*2^(1/2)+3/64*(A*b+7*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
5/4)/b^(11/4)*2^(1/2)+3/128*(A*b+7*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4
)*2^(1/2)*x^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)-3/128*(A*b+7*B*a)*ln(a^(1/2)+x*
b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3(7aB+Ab)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{3(7aB+Ab)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{3(7aB+Ab)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{x^{3/2}(7aB+Ab)}{16ab^2\,(a+bx^2)}+\frac{x^{7/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(7/2))/(4*a*b*(a + b*x^2)^2) - ((A*b + 7*a*B)*x^(3/2))/(16*a
*b^2*(a + b*x^2)) - (3*(A*b + 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(5/4)*b^(11/4)) - (3*(A*b + 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(Ab-aB)x^{7/2}}{4ab\,(a+bx^2)^2} + \frac{\left(\frac{Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\,(a+bx^2)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\,(a+bx^2)} + \frac{(3(Ab+7aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\,(a+bx^2)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\,(a+bx^2)} + \frac{(3(Ab+7aB))\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\,(a+bx^2)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\,(a+bx^2)} - \frac{(3(Ab+7aB))\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{32ab^{5/2}}$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\,(a+bx^2)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\,(a+bx^2)} + \frac{(3(Ab+7aB))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,\right.}{64ab^3}$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\,(a+bx^2)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\,(a+bx^2)} + \frac{3(Ab+7aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{\right.}}{64\sqrt{2}\,a^{5/4}b^{11/4}}$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\,(a+bx^2)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\,(a+bx^2)} - \frac{3(Ab+7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{11/4}} + \frac{3(A}{}$$

Mathematica [A]
time = 0.61, size = 171, normalized size = 0.58

$$\frac{-\frac{4\sqrt[4]{a}\,b^{3/4}x^{3/2}(7a^2B-3Ab^2x^2+ab(A+11Bx^2))}{(a+bx^2)^2} - 3\sqrt{2}\,(Ab+7aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 3\sqrt{2}\,(Ab+7aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{64a^{5/4}b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((-4*a^(1/4)*b^(3/4)*x^(3/2)*(7*a^2*B - 3*A*b^2*x^2 + a*b*(A + 11*B*x^2)))/
(a + b*x^2)^2 - 3*Sqrt[2]*(A*b + 7*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*(A*b + 7*a*B)*ArcTanh[(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(5/4)*b^(11/4))
```

Maple [A]
time = 0.08, size = 166, normalized size = 0.57

method	result
derivativedivides	$\frac{\frac{(3Ab-11Ba)x^{\frac{7}{2}}}{16ab}-\frac{(Ab+7Ba)x^{\frac{3}{2}}}{16b^2}}{(bx^2+a)^2}+\frac{3(Ab+7Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\right.}{128b^3a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(3Ab-11Ba)x^{\frac{7}{2}}}{16ab}-\frac{(Ab+7Ba)x^{\frac{3}{2}}}{16b^2}}{(bx^2+a)^2}+\frac{3(Ab+7Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\right.}{128b^3a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(3*A*b-11*B*a)/a/b*x^(7/2)-1/32*(A*b+7*B*a)/b^2*x^(3/2))/(b*x^2+a)^
2+3/128*(A*b+7*B*a)/b^3/a/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^
(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 251, normalized size = 0.86

$$-\frac{(11\,Bab-3\,Ab^2)x^{\frac{7}{2}}+(7\,Ba^2+Aab)x^{\frac{3}{2}}}{16\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)}+\frac{3\,(7\,Ba+Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((11*B*a*b - 3*A*b^2)*x^(7/2) + (7*B*a^2 + A*a*b)*x^(3/2))/(a*b^4*x^4
 + 2*a^2*b^3*x^2 + a^3*b^2) + 3/128*(7*B*a + A*b)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(
1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt
(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(213) = 426$.

time = 0.87, size = 990, normalized size = 3.38

$$-\frac{12\,\left(a b^{4} x^{4}+2 a^{2} b^{3} x^{2}+a^{3} b^{2}\right)\left(-\frac{2401 B^{4} a^{4}+1372 A B^{3} a^{3} b+294 A^{2} B^{2} a^{2} b^{2}+28 A^{3} B a b^{3}+A^{4} b^{4}}{a^{5} b^{11}}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{\left(117649 B^{6} a^{6}+100842 A B^{5} a^{5} b+36015 A^{2} B^{4} a^{4} b^{2}+6860 A^{3} B^{3} a^{3} b^{3}+735 A^{4} B^{2} a^{2} b^{4}+42 A^{5} B a b^{5}+A^{6} b^{6}\right) x-\left(2401 B^{4} a^{7} b^{5}+1372 A B^{3} a^{6} b^{6}+294 A^{2} B^{2} a^{5} b^{7}+28 A^{3} B a^{4} b^{8}+A^{4} a^{3} b^{9}\right) \sqrt{-\frac{2401 B^{4} a^{4}+1372 A B^{3} a^{3} b+294 A^{2} B^{2} a^{2} b^{2}+28 A^{3} B a b^{3}+A^{4} b^{4}}{a^{5} b^{11}}}}\, a b^{3}\left(-\frac{2401 B^{4} a^{4}+1372 A B^{3} a^{3} b+294 A^{2} B^{2} a^{2} b^{2}+28 A^{3} B a b^{3}+A^{4} b^{4}}{a^{5} b^{11}}\right)^{\frac{1}{4}}-\left(343 B^{3} a^{4} b^{3}+147 A B^{2} a^{3} b^{4}+21 A^{2} B a^{2} b^{5}+A^{3} a b^{6}\right) \sqrt{x}\left(-\frac{2401 B^{4} a^{4}+1372 A B^{3} a^{3} b+294 A^{2} B^{2} a^{2} b^{2}+28 A^{3} B a b^{3}+A^{4} b^{4}}{a^{5} b^{11}}\right)^{\frac{1}{4}}}{2401 B^{4} a^{4}+1372 A B^{3} a^{3} b+294 A^{2} B^{2} a^{2} b^{2}+28 A^{3} B a b^{3}+A^{4} b^{4}}\right)-3\,\left(a b^{4} x^{4}+2 a^{2} b^{3} x^{2}+a^{3} b^{2}\right)\left(-\frac{2401 B^{4} a^{4}+1372 A B^{3} a^{3} b+294 A^{2} B^{2} a^{2} b^{2}+28 A^{3} B a b^{3}+A^{4} b^{4}}{a^{5} b^{11}}\right)^{\frac{1}{4}} \log\left(27 a^{4} b^{8}\left(-\frac{2401 B^{4} a^{4}+1372 A B^{3} a^{3} b+294 A^{2} B^{2} a^{2} b^{2}+28 A^{3} B a b^{3}+A^{4} b^{4}}{a^{5} b^{11}}\right)^{\frac{3}{4}}+27\left(343 B^{3} a^{3}+147 A B^{2} a^{2} b+21 A^{2} B a b^{2}+A^{3} b^{3}\right) \sqrt{x}\right)+3\,\left(a b^{4} x^{4}+2 a^{2} b^{3} x^{2}+a^{3} b^{2}\right)\left(-\frac{2401 B^{4} a^{4}+1372 A B^{3} a^{3} b+294 A^{2} B^{2} a^{2} b^{2}+28 A^{3} B a b^{3}+A^{4} b^{4}}{a^{5} b^{11}}\right)^{\frac{1}{4}} \log\left(-27 a^{4} b^{8}\left(-\frac{2401 B^{4} a^{4}+1372 A B^{3} a^{3} b+294 A^{2} B^{2} a^{2} b^{2}+28 A^{3} B a b^{3}+A^{4} b^{4}}{a^{5} b^{11}}\right)^{\frac{3}{4}}+27\left(343 B^{3} a^{3}+147 A B^{2} a^{2} b+21 A^{2} B a b^{2}+A^{3} b^{3}\right) \sqrt{x}\right)+4\left(\left(11 B a b-3 A b^{2}\right) x^{3}+\left(7 B a^{2}+A a b\right) x\right) \sqrt{x}}{64\,\left(a b^{4} x^{4}+2 a^{2} b^{3} x^{2}+a^{3} b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^
3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)
*arctan((sqrt((117649*B^6*a^6 + 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2
+ 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 + 42*A^5*B*a*b^5 + A^6*b^6)*x
- (2401*B^4*a^7*b^5 + 1372*A*B^3*a^6*b^6 + 294*A^2*B^2*a^5*b^7 + 28*A^3*B*a
^4*b^8 + A^4*a^3*b^9)*sqrt(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*
a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11)))*a*b^3*(-(2401*B^4*a^4 + 13
72*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11)
)^(1/4) - (343*B^3*a^4*b^3 + 147*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 + A^3*a*b
^6)*sqrt(x)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A
^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4))/(2401*B^4*a^4 + 1372*A*B^3*a^3*b +
 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)) - 3*(a*b^4*x^4 + 2*a^2*b^
3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 +
 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)*log(27*a^4*b^8*(-(2401*B^4*a^4
 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*
b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A*B^2*a^2*b + 21*A^2*B*a*b^2 + A^3*b^3
)*sqrt(x)) + 3*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372
*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^
(1/4)*log(-27*a^4*b^8*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*
b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A
*B^2*a^2*b + 21*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) + 4*((11*B*a*b - 3*A*b^2)*x
^3 + (7*B*a^2 + A*a*b)*x)*sqrt(x))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.68, size = 293, normalized size = 1.00

$$-\frac{11\,B a b x^{\frac{7}{2}}-3\,A b^{2} x^{\frac{7}{2}}+7\,B a^{2} x^{\frac{3}{2}}+A a b x^{\frac{3}{2}}}{16\,\left(b x^{2}+a\right)^{2} a b^{2}}+\frac{3\,\sqrt{2}\left(7\,\left(a b^{3}\right)^{\frac{3}{4}} B a+\left(a b^{3}\right)^{\frac{3}{4}} A b\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2} b^{5}}+\frac{3\,\sqrt{2}\left(7\,\left(a b^{3}\right)^{\frac{3}{4}} B a+\left(a b^{3}\right)^{\frac{3}{4}} A b\right) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2} b^{5}}-\frac{3\,\sqrt{2}\left(7\,\left(a b^{3}\right)^{\frac{3}{4}} B a+\left(a b^{3}\right)^{\frac{3}{4}} A b\right) \log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{2} b^{5}}+\frac{3\,\sqrt{2}\left(7\,\left(a b^{3}\right)^{\frac{3}{4}} B a+\left(a b^{3}\right)^{\frac{3}{4}} A b\right) \log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{2} b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*B*a*b*x^(7/2) - 3*A*b^2*x^(7/2) + 7*B*a^2*x^(3/2) + A*a*b*x^(3/2)
)/((b*x^2 + a)^2*a*b^2) + 3/64*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)
*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^
2*b^5) + 3/64*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) - 3/128*s
qrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(a^2*b^5) + 3/128*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*
b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^5)
```

Mupad [B]
time = 0.10, size = 122, normalized size = 0.42

$$\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b+7\,B\,a\right)}{32\,{\left(-a\right)}^{5/4}\,b^{11/4}}-\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b+7\,B\,a\right)}{32\,{\left(-a\right)}^{5/4}\,b^{11/4}}-\frac{\frac{x^{3/2}\,\left(A\,b+7\,B\,a\right)}{16\,b^2}-\frac{x^{7/2}\,\left(3\,A\,b-11\,B\,a\right)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 7*B*a))/(32*(-a)^(5/4)*b^(11/
4)) - (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 7*B*a))/(32*(-a)^(5/4)*b
^(11/4)) - ((x^(3/2)*(A*b + 7*B*a))/(16*b^2) - (x^(7/2)*(3*A*b - 11*B*a))/(
16*a*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.385 $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=298

$$\frac{(Ab-aB)x^{5/2}}{4ab\,(a+bx^2)^2}-\frac{(3Ab+5aB)\sqrt{x}}{16ab^2\,(a+bx^2)}-\frac{(3Ab+5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(3Ab+5aB)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,a^{7/4}b^{9/4}}$$

```
[Out] 1/4*(A*b-B*a)*x^(5/2)/a/b/(b*x^2+a)^2-1/64*(3*A*b+5*B*a)*arctan(1-b^(1/4)*2
^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)+1/64*(3*A*b+5*B*a)*arctan(1
+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)-1/128*(3*A*b+5*B*
a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(9/4)*2^
(1/2)+1/128*(3*A*b+5*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1
/2))/a^(7/4)/b^(9/4)*2^(1/2)-1/16*(3*A*b+5*B*a)*x^(1/2)/a/b^2/(b*x^2+a)
```

Rubi [A]

time = 0.14, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(5aB+3Ab)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(5aB+3Ab)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{\sqrt{x}\,(5aB+3Ab)}{16ab^2\,(a+bx^2)}+\frac{x^{5/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(4*a*b*(a + b*x^2)^2) - ((3*A*b + 5*a*B)*Sqrt[x])/(16
*a*b^2*(a + b*x^2)) - ((3*A*b + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) + ((3*A*b + 5*a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) - ((3*A*b + 5*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(7/4)*b^(9/4)) + ((3*A*b + 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(Ab-aB)x^{5/2}}{4ab\,(a+bx^2)^2} + \frac{\left(\frac{3Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\,(a+bx^2)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\,(a+bx^2)} + \frac{(3Ab+5aB)\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{32ab^2}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\,(a+bx^2)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\,(a+bx^2)} + \frac{(3Ab+5aB)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16ab^2}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\,(a+bx^2)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\,(a+bx^2)} + \frac{(3Ab+5aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^{3/2}b^2}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\,(a+bx^2)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\,(a+bx^2)} + \frac{(3Ab+5aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,\right.}{64a^{3/2}b^{5/2}}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\,(a+bx^2)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\,(a+bx^2)} - \frac{(3Ab+5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{64\sqrt{2}\,a^{7/4}b^{9/4}}$$

$$= \frac{(Ab-aB)x^{5/2}}{4ab\,(a+bx^2)^2} - \frac{(3Ab+5aB)\sqrt{x}}{16ab^2\,(a+bx^2)} - \frac{(3Ab+5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{9/4}} + \frac{(3}{}$$

Mathematica [A]
time = 0.60, size = 173, normalized size = 0.58

$$\frac{-\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}\,(5a^2B-Ab^2x^2+3ab(A+3Bx^2))}{(a+bx^2)^2}-\sqrt{2}\,(3Ab+5aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}\,(3Ab+5aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{64a^{7/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((-4*a^(3/4)*b^(1/4)*Sqrt[x]*(5*a^2*B - A*b^2*x^2 + 3*a*b*(A + 3*B*x^2)))/(
a + b*x^2)^2 - Sqrt[2]*(3*A*b + 5*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*(3*A*b + 5*a*B)*ArcTanh[(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(7/4)*b^(9/4))
```

Maple [A]
time = 0.08, size = 167, normalized size = 0.56

method	result
derivativedivides	$\frac{\frac{(Ab-9Ba)x^{\frac{5}{2}}}{16ab}-\frac{(3Ab+5Ba)\sqrt{x}}{16b^2}}{(b\,x^2+a)^2}+\frac{(3Ab+5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128b^2a^2}$
default	$\frac{\frac{(Ab-9Ba)x^{\frac{5}{2}}}{16ab}-\frac{(3Ab+5Ba)\sqrt{x}}{16b^2}}{(b\,x^2+a)^2}+\frac{(3Ab+5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128b^2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(A*b-9*B*a)/a/b*x^(5/2)-1/32*(3*A*b+5*B*a)/b^2*x^(1/2))/(b*x^2+a)^2
+1/128*(3*A*b+5*B*a)/b^2/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.52, size = 280, normalized size = 0.94

$$-\frac{(9\,Bab-Ab^2)x^{\frac{5}{2}}+(5\,Ba^2+3\,Aab)\sqrt{x}}{16\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)}+\frac{\frac{2\,\sqrt{2}\,(5\,Ba+3\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(5\,Ba+3\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(5\,Ba+3\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(5\,Ba+3\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{128\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((9*B*a*b - A*b^2)*x^(5/2) + (5*B*a^2 + 3*A*a*b)*sqrt(x))/(a*b^4*x^4
+ 2*a^2*b^3*x^2 + a^3*b^2) + 1/128*(2*sqrt(2)*(5*B*a + 3*A*b)*arctan(1/2*sq
rt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(5*B*a + 3*A*b)*arctan(-1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(5*B*a + 3*A*b)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*B*a
 + 3*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3
/4)*b^(1/4)))/(a*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(218) = 436$.

time = 0.70, size = 806, normalized size = 2.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a
^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4
)*arctan((sqrt(a^4*b^4*sqrt(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2
*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9)) + (25*B^2*a^2 + 30*A*B*
a*b + 9*A^2*b^2)*x)*a^5*b^7*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^
2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(3/4) - (5*B*a^6*b^7 +
 3*A*a^5*b^8)*sqrt(x)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*
b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(3/4))/(625*B^4*a^4 + 1500*A
*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)) + (a*b^4
*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^
2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4)*log(a^2*b^2*
(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3
+ 81*A^4*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x)) - (a*b^4*x^4 + 2*
a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2
*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(625*B
^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4
*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x)) - 4*(5*B*a^2 + 3*A*a*b +
(9*B*a*b - A*b^2)*x^2)*sqrt(x))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1445 vs. $2(287) = 574$.
time = 159.96, size = 1445, normalized size = 4.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((2*A*x**(5/2)/5 + 2*B*x**(9/2)/9)/a**3, Eq(b, 0)), ((-2*A/(7*x**(7/2)) -
 2*B/(3*x**(3/2)))/b**3, Eq(a, 0)), (-12*A*a**2*b*sqrt(x)/(64*a**4*b**2 + 1
28*a**3*b**3*x**2 + 64*a**2*b**4*x**4) - 3*A*a**2*b*(-a/b)**(1/4)*log(sqrt(
x) - (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4)
 + 3*A*a**2*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b**2 + 12
8*a**3*b**3*x**2 + 64*a**2*b**4*x**4) + 6*A*a**2*b*(-a/b)**(1/4)*atan(sqrt(
x)/(-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) +
 4*A*a*b**2*x**(5/2)/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4
) - 6*A*a*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**4*b**
```

```
2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) + 6*A*a*b**2*x**2*(-a/b)**(1/4)
*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*
b**4*x**4) + 12*A*a*b**2*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64
*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) - 3*A*b**3*x**4*(-a/b)
**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 6
4*a**2*b**4*x**4) + 3*A*b**3*x**4*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4)
)/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) + 6*A*b**3*x**4*(
-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2
 + 64*a**2*b**4*x**4) - 20*B*a**3*sqrt(x)/(64*a**4*b**2 + 128*a**3*b**3*x**
2 + 64*a**2*b**4*x**4) - 5*B*a**3*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4)
)/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) + 5*B*a**3*(-a/b)
**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 6
4*a**2*b**4*x**4) + 10*B*a**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64
*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) - 36*B*a**2*b*x**(5/2)
/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) - 10*B*a**2*b*x**2
*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x
**2 + 64*a**2*b**4*x**4) + 10*B*a**2*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a
/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) + 20*B*
a**2*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b**2 + 128*a
**3*b**3*x**2 + 64*a**2*b**4*x**4) - 5*B*a*b**2*x**4*(-a/b)**(1/4)*log(sqrt
(x) - (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4
) + 5*B*a*b**2*x**4*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b**
2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) + 10*B*a*b**2*x**4*(-a/b)**(1/4
)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*
b**4*x**4), True))
```

Giac [A]
time = 0.58, size = 298, normalized size = 1.00

$$\frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2}b^{3}}+\frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2}b^{3}}+\frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{2}b^{3}}-\frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{2}b^{3}}-\frac{9\,Babx^{\frac{5}{2}}-Ab^{2}x^{\frac{5}{2}}+5\,Ba^{2}\sqrt{x}+3\,Aab\sqrt{x}}{16\left(bx^{2}+a\right)^{2}ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/64*sqrt(2)*(5
*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/128*sqrt(2)*(5*(a*b^3)^(1/4
)*B*a + 3*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b
))/(a^2*b^3) - 1/128*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*lo
g(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) - 1/16*(9*B*a*b*x
^(5/2) - A*b^2*x^(5/2) + 5*B*a^2*sqrt(x) + 3*A*a*b*sqrt(x))/((b*x^2 + a)^2*
a*b^2)
```

Mupad [B]

time = 0.26, size = 799, normalized size = 2.68

$$-\frac{\frac{\sqrt{x}\,\left(3Ab+5Ba\right)}{16b^2}-\frac{x^{5/2}\,\left(Ab-9Ba\right)}{16ab}}{a^2+2abx^2+b^2x^4}+\frac{\mathrm{atan}\left(\frac{\frac{\left(3Ab+5Ba\right)\left(\frac{\sqrt{x}\,\left(9A^2b^2+30ABab+25B^2a^2\right)}{64a^2b}-\frac{\left(3Ab^2+5Bab\right)\left(3Ab+5Ba\right)}{64\,(-a)^{7/4}\,b^{9/4}}\right)1i}{64\,(-a)^{7/4}\,b^{9/4}}+\frac{\left(3Ab+5Ba\right)\left(\frac{\sqrt{x}\,\left(9A^2b^2+30ABab+25B^2a^2\right)}{64a^2b}+\frac{\left(3Ab^2+5Bab\right)\left(3Ab+5Ba\right)}{64\,(-a)^{7/4}\,b^{9/4}}\right)1i}{64\,(-a)^{7/4}\,b^{9/4}}}{\frac{\left(3Ab+5Ba\right)\left(\frac{\sqrt{x}\,\left(9A^2b^2+30ABab+25B^2a^2\right)}{64a^2b}-\frac{\left(3Ab^2+5Bab\right)\left(3Ab+5Ba\right)}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}-\frac{\left(3Ab+5Ba\right)\left(\frac{\sqrt{x}\,\left(9A^2b^2+30ABab+25B^2a^2\right)}{64a^2b}+\frac{\left(3Ab^2+5Bab\right)\left(3Ab+5Ba\right)}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}}\right)\left(3Ab+5Ba\right)1i}{32\,(-a)^{7/4}\,b^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(3Ab+5Ba\right)\left(\frac{\sqrt{x}\,\left(9A^2b^2+30ABab+25B^2a^2\right)}{64a^2b}-\frac{\left(3Ab^2+5Bab\right)\left(3Ab+5Ba\right)1i}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}+\frac{\left(3Ab+5Ba\right)\left(\frac{\sqrt{x}\,\left(9A^2b^2+30ABab+25B^2a^2\right)}{64a^2b}+\frac{\left(3Ab^2+5Bab\right)\left(3Ab+5Ba\right)1i}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}}{\frac{\left(3Ab+5Ba\right)\left(\frac{\sqrt{x}\,\left(9A^2b^2+30ABab+25B^2a^2\right)}{64a^2b}-\frac{\left(3Ab^2+5Bab\right)\left(3Ab+5Ba\right)1i}{64\,(-a)^{7/4}\,b^{9/4}}\right)1i}{64\,(-a)^{7/4}\,b^{9/4}}-\frac{\left(3Ab+5Ba\right)\left(\frac{\sqrt{x}\,\left(9A^2b^2+30ABab+25B^2a^2\right)}{64a^2b}+\frac{\left(3Ab^2+5Bab\right)\left(3Ab+5Ba\right)1i}{64\,(-a)^{7/4}\,b^{9/4}}\right)1i}{64\,(-a)^{7/4}\,b^{9/4}}}\right)\left(3Ab+5Ba\right)}{32\,(-a)^{7/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (atan((((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(6
4*a^2*b) - ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a))/(64*(-a)^(7/4)*b^(9/4)))*1
i)/(64*(-a)^(7/4)*b^(9/4)) + ((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2
*a^2 + 30*A*B*a*b))/(64*a^2*b) + ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a))/(64*
(-a)^(7/4)*b^(9/4)))*1i)/(64*(-a)^(7/4)*b^(9/4)))/(((3*A*b + 5*B*a)*((x^(1/
2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) - ((3*A*b^2 + 5*B*a*b)
*(3*A*b + 5*B*a))/(64*(-a)^(7/4)*b^(9/4))))/(64*(-a)^(7/4)*b^(9/4)) - ((3*A
*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) + (
(3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a))/(64*(-a)^(7/4)*b^(9/4))))/(64*(-a)^(7/
4)*b^(9/4))))*(3*A*b + 5*B*a)*1i)/(32*(-a)^(7/4)*b^(9/4)) - ((x^(1/2)*(3*A*
b + 5*B*a))/(16*b^2) - (x^(5/2)*(A*b - 9*B*a))/(16*a*b))/(a^2 + b^2*x^4 + 2
*a*b*x^2) + (atan((((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*
A*B*a*b))/(64*a^2*b) - ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a)*1i)/(64*(-a)^(7
/4)*b^(9/4))))/(64*(-a)^(7/4)*b^(9/4)) + ((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*
b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) + ((3*A*b^2 + 5*B*a*b)*(3*A*b +
5*B*a)*1i)/(64*(-a)^(7/4)*b^(9/4))))/(64*(-a)^(7/4)*b^(9/4)))/(((3*A*b + 5*
B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) - ((3*A*b^
2 + 5*B*a*b)*(3*A*b + 5*B*a)*1i)/(64*(-a)^(7/4)*b^(9/4)))*1i)/(64*(-a)^(7/4
)*b^(9/4)) - ((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*
b))/(64*a^2*b) + ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a)*1i)/(64*(-a)^(7/4)*b^
(9/4)))*1i)/(64*(-a)^(7/4)*b^(9/4))))*(3*A*b + 5*B*a))/(32*(-a)^(7/4)*b^(9/
4))
```

3.386 $\int \frac{\sqrt{x}\,(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=298

$$\frac{(Ab-aB)x^{3/2}}{4ab\,(a+bx^2)^2}+\frac{(5Ab+3aB)x^{3/2}}{16a^2b\,(a+bx^2)}-\frac{(5Ab+3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{(5Ab+3aB)\tan^{-1}\left(1+\sqrt{2}\right.}{32\sqrt{2}\,a^{9/4}b^{7/4}}$$

```
[Out] 1/4*(A*b-B*a)*x^(3/2)/a/b/(b*x^2+a)^2+1/16*(5*A*b+3*B*a)*x^(3/2)/a^2/b/(b*x
^2+a)-1/64*(5*A*b+3*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/
b^(7/4)*2^(1/2)+1/64*(5*A*b+3*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4)
)/a^(9/4)/b^(7/4)*2^(1/2)+1/128*(5*A*b+3*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)-1/128*(5*A*b+3*B*a)*ln(a^(
1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(3aB+5Ab)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{(3aB+5Ab)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{(3aB+5Ab)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{x^{3/2}(3aB+5Ab)}{16a^2b\,(a+bx^2)}+\frac{x^{3/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(4*a*b*(a + b*x^2)^2) + ((5*A*b + 3*a*B)*x^(3/2))/(16
*a^2*b*(a + b*x^2)) - ((5*A*b + 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(9/4)*b^(7/4)) - ((5*A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\,(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(Ab-aB)x^{3/2}}{4ab\,(a+bx^2)^2} + \frac{\left(\frac{5Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\,(a+bx^2)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\,(a+bx^2)} + \frac{(5Ab+3aB)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^2b}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\,(a+bx^2)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\,(a+bx^2)} + \frac{(5Ab+3aB)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2b}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\,(a+bx^2)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\,(a+bx^2)} - \frac{(5Ab+3aB)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{32a^2b^{3/2}}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\,(a+bx^2)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\,(a+bx^2)} + \frac{(5Ab+3aB)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx\right.}{64a^2b^2}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\,(a+bx^2)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\,(a+bx^2)} + \frac{(5Ab+3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{64\sqrt{2}\,a^{9/4}b^{7/4}}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\,(a+bx^2)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\,(a+bx^2)} - \frac{(5Ab+3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{7/4}} + \cdots$$

Mathematica [A]
time = 0.60, size = 175, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{a}\,b^{3/4}x^{3/2}(9aAb-a^2B+5Ab^2x^2+3abBx^2)}{(a+bx^2)^2} - \sqrt{2}\,(5Ab+3aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - \sqrt{2}\,(5Ab+3aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{64a^{9/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((4*a^(1/4)*b^(3/4)*x^(3/2)*(9*a*A*b - a^2*B + 5*A*b^2*x^2 + 3*a*b*B*x^2))/
(a + b*x^2)^2 - Sqrt[2]*(5*A*b + 3*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])] - Sqrt[2]*(5*A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(9/4)*b^(7/4))
```

Maple [A]
time = 0.08, size = 168, normalized size = 0.56

method	result
derivativedivides	$\frac{\frac{(5Ab+3Ba)x^{\frac{7}{2}}}{16a^2}+\frac{(9Ab-Ba)x^{\frac{3}{2}}}{16ab}}{(b\,x^2+a)^2}+\frac{(5Ab+3Ba)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\right.}{128a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(5Ab+3Ba)x^{\frac{7}{2}}}{16a^2}+\frac{(9Ab-Ba)x^{\frac{3}{2}}}{16ab}}{(b\,x^2+a)^2}+\frac{(5Ab+3Ba)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\right.}{128a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(5*A*b+3*B*a)/a^2*x^(7/2)+1/32*(9*A*b-B*a)/a/b*x^(3/2))/(b*x^2+a)^2
+1/128*(5*A*b+3*B*a)/a^2/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.54, size = 253, normalized size = 0.85

$$\frac{(3\,Bab+5\,Ab^2)x^{\frac{7}{2}}-(Ba^2-9\,Aab)x^{\frac{3}{2}}}{16\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}+\frac{(3\,Ba+5\,Ab)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*B*a*b + 5*A*b^2)*x^(7/2) - (B*a^2 - 9*A*a*b)*x^(3/2))/(a^2*b^3*x^4
 + 2*a^3*b^2*x^2 + a^4*b) + 1/128*(3*B*a + 5*A*b)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(
1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt
(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(218) = 436$.

time = 0.84, size = 1005, normalized size = 3.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^
3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/
4)*arctan((sqrt((729*B^6*a^6 + 7290*A*B^5*a^5*b + 30375*A^2*B^4*a^4*b^2 + 6
7500*A^3*B^3*a^3*b^3 + 84375*A^4*B^2*a^2*b^4 + 56250*A^5*B*a*b^5 + 15625*A^
6*b^6)*x - (81*B^4*a^9*b^3 + 540*A*B^3*a^8*b^4 + 1350*A^2*B^2*a^7*b^5 + 150
0*A^3*B*a^6*b^6 + 625*A^4*a^5*b^7)*sqrt(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 13
50*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7)))*a^2*b^2*(-
(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 6
25*A^4*b^4)/(a^9*b^7))^(1/4) - (27*B^3*a^5*b^2 + 135*A*B^2*a^4*b^3 + 225*A^
2*B*a^3*b^4 + 125*A^3*a^2*b^5)*sqrt(x)*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 13
50*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/4))/(81*
B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A
^4*b^4)) - (a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*
a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(
1/4)*log(a^7*b^5*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1
500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(3/4) + (27*B^3*a^3 + 135*A*B^2*a
^2*b + 225*A^2*B*a*b^2 + 125*A^3*b^3)*sqrt(x)) + (a^2*b^3*x^4 + 2*a^3*b^2*x
^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A
^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/4)*log(-a^7*b^5*(-(81*B^4*a^4 + 540
*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*
b^7))^(3/4) + (27*B^3*a^3 + 135*A*B^2*a^2*b + 225*A^2*B*a*b^2 + 125*A^3*b^3
)*sqrt(x)) - 4*((3*B*a*b + 5*A*b^2)*x^3 - (B*a^2 - 9*A*a*b)*x)*sqrt(x))/(a^
2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [A]
time = 57.48, size = 299, normalized size = 1.00

$$\frac{18Aax^{\frac{3}{2}}}{32a^4+64a^3bx^2+32a^2b^2x^4}+\frac{10Abx^{\frac{7}{2}}}{32a^4+64a^3bx^2+32a^2b^2x^4}+2A\operatorname{RootSum}\left(268435456t^4a^9b^3+625,\left(t\mapsto t\log\left(\frac{2097152t^3a^7b^2}{125}+\sqrt{x}\right)\right)\right)-\frac{18Ba^2x^{\frac{3}{2}}}{32a^4b+64a^3b^2x^2+32a^2b^3x^4}-\frac{10Bax^{\frac{7}{2}}}{32a^4+64a^3bx^2+32a^2b^2x^4}-\frac{2Ba\operatorname{RootSum}\left(268435456t^4a^9b^3+625,\left(t\mapsto t\log\left(\frac{2097152t^3a^7b^2}{125}+\sqrt{x}\right)\right)\right)}{b}+\frac{2Bx^{\frac{3}{2}}}{4a^2b+4ab^2x^2}+\frac{2B\operatorname{RootSum}\left(65536t^4a^5b^3+1,\left(t\mapsto t\log\left(4096t^3a^4b^2+\sqrt{x}\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] 18*A*a*x**(3/2)/(32*a**4 + 64*a**3*b*x**2 + 32*a**2*b**2*x**4) + 10*A*b*x**
(7/2)/(32*a**4 + 64*a**3*b*x**2 + 32*a**2*b**2*x**4) + 2*A*RootSum(26843545
6*_t**4*a**9*b**3 + 625, Lambda(_t, _t*log(2097152*_t**3*a**7*b**2/125 + sq
rt(x)))) - 18*B*a**2*x**(3/2)/(32*a**4*b + 64*a**3*b**2*x**2 + 32*a**2*b**3
*x**4) - 10*B*a*x**(7/2)/(32*a**4 + 64*a**3*b*x**2 + 32*a**2*b**2*x**4) - 2
*B*a*RootSum(268435456*_t**4*a**9*b**3 + 625, Lambda(_t, _t*log(2097152*_t*
```

```
*3*a**7*b**2/125 + sqrt(x))))/b + 2*B*x**(3/2)/(4*a**2*b + 4*a*b**2*x**2) +
 2*B*RootSum(65536*_t**4*a**5*b**3 + 1, Lambda(_t, _t*log(4096*_t**3*a**4*b
**2 + sqrt(x))))/b
```

Giac [A]
time = 1.08, size = 298, normalized size = 1.00

$$\frac{3\,Babx^{\frac{7}{2}}+5\,Ab^{2}x^{\frac{7}{2}}-Ba^{2}x^{\frac{3}{2}}+9\,Aabx^{\frac{3}{2}}}{16\,{\left(bx^{2}+a\right)}^{2}a^{2}b}+\frac{\sqrt{2}\left(3\,{\left(ab^{3}\right)}^{\frac{3}{4}}\,Ba+5\,{\left(ab^{3}\right)}^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,{\left(\frac{a}{b}\right)}^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,{\left(\frac{a}{b}\right)}^{\frac{1}{4}}}\right)}{64\,a^{3}b^{4}}+\frac{\sqrt{2}\left(3\,{\left(ab^{3}\right)}^{\frac{3}{4}}\,Ba+5\,{\left(ab^{3}\right)}^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,{\left(\frac{a}{b}\right)}^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,{\left(\frac{a}{b}\right)}^{\frac{1}{4}}}\right)}{64\,a^{3}b^{4}}-\frac{\sqrt{2}\left(3\,{\left(ab^{3}\right)}^{\frac{3}{4}}\,Ba+5\,{\left(ab^{3}\right)}^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,{\left(\frac{a}{b}\right)}^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{3}b^{4}}+\frac{\sqrt{2}\left(3\,{\left(ab^{3}\right)}^{\frac{3}{4}}\,Ba+5\,{\left(ab^{3}\right)}^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,{\left(\frac{a}{b}\right)}^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{3}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*B*a*b*x^(7/2) + 5*A*b^2*x^(7/2) - B*a^2*x^(3/2) + 9*A*a*b*x^(3/2))/
((b*x^2 + a)^2*a^2*b) + 1/64*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)
*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^
3*b^4) + 1/64*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) - 1/128
*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(a^3*b^4) + 1/128*sqrt(2)*(3*(a*b^3)^(3/4)*B*a +
 5*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^
3*b^4)
```

Mupad [B]
time = 0.18, size = 124, normalized size = 0.42

$$\frac{\frac{x^{7/2}\,\left(5\,A\,b+3\,B\,a\right)}{16\,a^2}+\frac{x^{3/2}\,\left(9\,A\,b-B\,a\right)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(5\,A\,b+3\,B\,a\right)}{32\,{\left(-a\right)}^{9/4}\,b^{7/4}}-\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(5\,A\,b+3\,B\,a\right)}{32\,{\left(-a\right)}^{9/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] ((x^(7/2)*(5*A*b + 3*B*a))/(16*a^2) + (x^(3/2)*(9*A*b - B*a))/(16*a*b))/(a^
2 + b^2*x^4 + 2*a*b*x^2) + (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b + 3*B
*a))/(32*(-a)^(9/4)*b^(7/4)) - (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b
+ 3*B*a))/(32*(-a)^(9/4)*b^(7/4))
```

3.387 $\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx$

Optimal. Leaf size=293

$$\frac{(Ab-aB)\sqrt{x}}{4ab\,(a+bx^2)^2}+\frac{(7Ab+aB)\sqrt{x}}{16a^2b\,(a+bx^2)}-\frac{3(7Ab+aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3(7Ab+aB)\tan^{-1}\left(1+\frac{\sqrt{2}\right.}{32\sqrt{2}\,a^{11/4}b^{5/4}}$$

```
[Out] -3/64*(7*A*b+B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(5/4
)*2^(1/2)+3/64*(7*A*b+B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/
4)/b^(5/4)*2^(1/2)-3/128*(7*A*b+B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+3/128*(7*A*b+B*a)*ln(a^(1/2)+x*b^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/4*(A*b-B*a
)*x^(1/2)/a/b/(b*x^2+a)^2+1/16*(7*A*b+B*a)*x^(1/2)/a^2/b/(b*x^2+a)
```

Rubi [A]
time = 0.15, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3(aB+7Ab)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{3(aB+7Ab)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{\sqrt{x}\,(aB+7Ab)}{16a^2b\,(a+bx^2)}+\frac{\sqrt{x}\,(Ab-aB)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(4*a*b*(a + b*x^2)^2) + ((7*A*b + a*B)*Sqrt[x])/(16*a
^2*b*(a + b*x^2)) - (3*(7*A*b + a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) + (3*(7*A*b + a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) - (3*(7*A*b + a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(11/4)*b^(5/4)) + (3*(7*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{(Ab-aB)\sqrt{x}}{4ab\,(a+bx^2)^2} + \frac{\left(\frac{7Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\,(a+bx^2)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\,(a+bx^2)} + \frac{(3(7Ab+aB))\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx}{32a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\,(a+bx^2)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\,(a+bx^2)} + \frac{(3(7Ab+aB))\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\,(a+bx^2)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\,(a+bx^2)} + \frac{(3(7Ab+aB))\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{}\right.}{32a^{5/2}b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\,(a+bx^2)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\,(a+bx^2)} + \frac{(3(7Ab+aB))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,d\right.}{64a^{5/2}b^{3/2}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\,(a+bx^2)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\,(a+bx^2)} - \frac{3(7Ab+aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{64\sqrt{2}\,a^{11/4}b^{5/4}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\,(a+bx^2)^2} + \frac{(7Ab+aB)\sqrt{x}}{16a^2b\,(a+bx^2)} - \frac{3(7Ab+aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}b^{5/4}} + \frac{3}{}$$

Mathematica [A]
time = 0.61, size = 172, normalized size = 0.59

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}\,(11aAb-3a^2B+7Ab^2x^2+abBx^2)}{(a+bx^2)^2} - 3\sqrt{2}\,(7Ab+aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 3\sqrt{2}\,(7Ab+aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{64a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*Sqrt[x]*(11*a*A*b - 3*a^2*B + 7*A*b^2*x^2 + a*b*B*x^2))
/(a + b*x^2)^2 - 3*Sqrt[2]*(7*A*b + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(7*A*b + a*B)*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(11/4)*b^(5/4))
```

Maple [A]
time = 0.09, size = 166, normalized size = 0.57

method	result
derivativedivides	$\frac{\frac{(7Ab+Ba)x^{\frac{5}{2}}}{16a^2}+\frac{(11Ab-3Ba)\sqrt{x}}{16ab}}{(b x^2+a)^2}+\frac{3(7Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128a^3b}$
default	$\frac{\frac{(7Ab+Ba)x^{\frac{5}{2}}}{16a^2}+\frac{(11Ab-3Ba)\sqrt{x}}{16ab}}{(b x^2+a)^2}+\frac{3(7Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128a^3b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(7*A*b+B*a)/a^2*x^(5/2)+1/32*(11*A*b-3*B*a)/a/b*x^(1/2))/(b*x^2+a)^
2+3/128*(7*A*b+B*a)/a^3/b*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^
(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 276, normalized size = 0.94

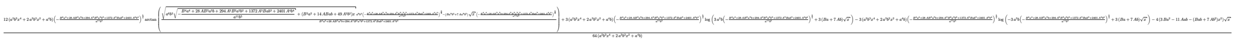
$$\frac{(Bab+7\,Ab^2)x^{\frac{5}{2}}-(3\,Ba^2-11\,Aab)\sqrt{x}}{16\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}+\frac{3\left(\frac{2\,\sqrt{2}\,(Ba+7\,Ab)\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(Ba+7\,Ab)\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba+7\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba+7\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*((B*a*b + 7*A*b^2)*x^(5/2) - (3*B*a^2 - 11*A*a*b)*sqrt(x))/(a^2*b^3*x^
4 + 2*a^3*b^2*x^2 + a^4*b) + 3/128*(2*sqrt(2)*(B*a + 7*A*b)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(B*a + 7*A*b)*arctan(-1/2*sqrt(2)
*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a + 7*A*b)*log(sqrt(2)*a^(1/4)*b^(1
/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a + 7*A*b
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1
/4)))/(a^2*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(213) = 426$.
time = 0.78, size = 793, normalized size = 2.71

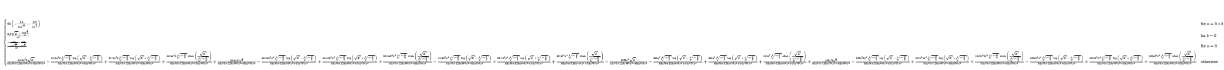


Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b
+ 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*
arctan((sqrt(a^6*b^2*sqrt(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2
+ 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5)) + (B^2*a^2 + 14*A*B*a*b + 49
*A^2*b^2)*x)*a^8*b^4*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 13
72*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(3/4) - (B*a^9*b^4 + 7*A*a^8*b^5
)*sqrt(x)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*
b^3 + 2401*A^4*b^4)/(a^11*b^5))^(3/4))/(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*
B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)) + 3*(a^2*b^3*x^4 + 2*a^3*b^
2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3
*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*log(3*a^3*b*(-(B^4*a^4 + 28*A*B^
3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5)
)^(1/4) + 3*(B*a + 7*A*b)*sqrt(x)) - 3*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b
)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 24
01*A^4*b^4)/(a^11*b^5))^(1/4)*log(-3*a^3*b*(-(B^4*a^4 + 28*A*B^3*a^3*b + 29
4*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4) + 3*
(B*a + 7*A*b)*sqrt(x)) - 4*(3*B*a^2 - 11*A*a*b - (B*a*b + 7*A*b^2)*x^2)*sqr
t(x))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1406 vs. $2(287) = 574$.
time = 141.89, size = 1406, normalized size = 4.80



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0
)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/a**3, Eq(b, 0)), ((-2*A/(11*x**(11/2))
- 2*B/(7*x**(7/2)))/b**3, Eq(a, 0)), (44*A*a**2*b*sqrt(x)/(64*a**5*b + 128*
a**4*b**2*x**2 + 64*a**3*b**3*x**4) - 21*A*a**2*b*(-a/b)**(1/4)*log(sqrt(x)
 - (-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 21
*A*a**2*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5*b + 128*a**4*
b**2*x**2 + 64*a**3*b**3*x**4) + 42*A*a**2*b*(-a/b)**(1/4)*atan(sqrt(x)/(-a
/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 28*A*a*b
```

```
**2*x**(5/2)/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) - 42*A*a*
b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**5*b + 128*a**4*
b**2*x**2 + 64*a**3*b**3*x**4) + 42*A*a*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x)
 + (-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 84
*A*a*b**2*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**5*b + 128*a
**4*b**2*x**2 + 64*a**3*b**3*x**4) - 21*A*b**3*x**4*(-a/b)**(1/4)*log(sqrt(
x) - (-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) +
21*A*b**3*x**4*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5*b + 128*
a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 42*A*b**3*x**4*(-a/b)**(1/4)*atan(sqr
t(x)/(-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) -
12*B*a**3*sqrt(x)/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) - 3*
B*a**3*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**
2*x**2 + 64*a**3*b**3*x**4) + 3*B*a**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**
(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 6*B*a**3*(-a/
b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*
a**3*b**3*x**4) + 4*B*a**2*b*x**(5/2)/(64*a**5*b + 128*a**4*b**2*x**2 + 64*
a**3*b**3*x**4) - 6*B*a**2*b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4)
)/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 6*B*a**2*b*x**2*(-
a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 +
64*a**3*b**3*x**4) + 12*B*a**2*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1
/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) - 3*B*a*b**2*x**4
*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2
 + 64*a**3*b**3*x**4) + 3*B*a*b**2*x**4*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)*
*(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 6*B*a*b**2*x
**4*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x*
*2 + 64*a**3*b**3*x**4), True))
```

Giac [A]
time = 1.27, size = 293, normalized size = 1.00

$$\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba+7(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3b^2}+\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba+7(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3b^2}+\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba+7(ab^3)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^3b^2}-\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba+7(ab^3)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^3b^2}+\frac{Babx^{\frac{5}{2}}+7Ab^2x^{\frac{5}{2}}-3Ba^2\sqrt{x}+11Aab\sqrt{x}}{16\left(bx^2+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 3/64*sqrt(2)*((a*
b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 3/128*sqrt(2)*((a*b^3)^(1/4)*B*a
+ 7*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^
3*b^2) - 3/128*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) + 1/16*(B*a*b*x^(5/2) + 7
*A*b^2*x^(5/2) - 3*B*a^2*sqrt(x) + 11*A*a*b*sqrt(x))/((b*x^2 + a)^2*a^2*b)
```

Mupad [B]

time = 0.27, size = 780, normalized size = 2.66

$$\frac{\frac{x^{5/2}\,(7\,A\,b+B\,a)}{16\,a^2}+\frac{\sqrt{x}\,(11\,A\,b-3\,B\,a)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\mathrm{atan}\left(\frac{\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b)}{64\,a^4}-\frac{9\,(7\,A\,b+B\,a)\,(7\,A\,b^3+B\,a\,b^2)}{64\,{(-a)}^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,{(-a)}^{11/4}\,b^{5/4}}+\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b)}{64\,a^4}+\frac{9\,(7\,A\,b+B\,a)\,(7\,A\,b^3+B\,a\,b^2)}{64\,{(-a)}^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,{(-a)}^{11/4}\,b^{5/4}}}{\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b)}{64\,a^4}-\frac{9\,(7\,A\,b+B\,a)\,(7\,A\,b^3+B\,a\,b^2)}{64\,{(-a)}^{15/4}\,b^{5/4}}\right)}{64\,{(-a)}^{11/4}\,b^{5/4}}-\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b)}{64\,a^4}+\frac{9\,(7\,A\,b+B\,a)\,(7\,A\,b^3+B\,a\,b^2)}{64\,{(-a)}^{15/4}\,b^{5/4}}\right)}{64\,{(-a)}^{11/4}\,b^{5/4}}}\right)(7\,A\,b+B\,a)\,3\mathrm{i}}{32\,{(-a)}^{11/4}\,b^{5/4}}-\frac{3\,\mathrm{atan}\left(\frac{\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b)}{64\,a^4}-\frac{(7\,A\,b+B\,a)\,(7\,A\,b^3+B\,a\,b^2)\,9\mathrm{i}}{64\,{(-a)}^{15/4}\,b^{5/4}}\right)}{64\,{(-a)}^{11/4}\,b^{5/4}}+\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b)}{64\,a^4}+\frac{(7\,A\,b+B\,a)\,(7\,A\,b^3+B\,a\,b^2)\,9\mathrm{i}}{64\,{(-a)}^{15/4}\,b^{5/4}}\right)}{64\,{(-a)}^{11/4}\,b^{5/4}}}{\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b)}{64\,a^4}-\frac{(7\,A\,b+B\,a)\,(7\,A\,b^3+B\,a\,b^2)\,9\mathrm{i}}{64\,{(-a)}^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,{(-a)}^{11/4}\,b^{5/4}}-\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b)}{64\,a^4}+\frac{(7\,A\,b+B\,a)\,(7\,A\,b^3+B\,a\,b^2)\,9\mathrm{i}}{64\,{(-a)}^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,{(-a)}^{11/4}\,b^{5/4}}}\right)(7\,A\,b+B\,a)}{32\,{(-a)}^{11/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(a + b*x^2)^3),x)
```

```
[Out] ((x^(5/2)*(7*A*b + B*a))/(16*a^2) + (x^(1/2)*(11*A*b - 3*B*a))/(16*a*b))/(a
^2 + b^2*x^4 + 2*a*b*x^2) - (atan((((7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 +
 B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) - (9*(7*A*b + B*a)*(7*A*b^3 + B*a*b^2)
)/(64*(-a)^(15/4)*b^(5/4)))*3i)/(64*(-a)^(11/4)*b^(5/4)) + ((7*A*b + B*a)*(
(9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) + (9*(7*A*b +
B*a)*(7*A*b^3 + B*a*b^2))/(64*(-a)^(15/4)*b^(5/4)))*3i)/(64*(-a)^(11/4)*b^(
5/4)))/((3*(7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2
))/(64*a^4) - (9*(7*A*b + B*a)*(7*A*b^3 + B*a*b^2))/(64*(-a)^(15/4)*b^(5/4)
)))/(64*(-a)^(11/4)*b^(5/4)) - (3*(7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B
^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) + (9*(7*A*b + B*a)*(7*A*b^3 + B*a*b^2))/
(64*(-a)^(15/4)*b^(5/4))))/(64*(-a)^(11/4)*b^(5/4))))*(7*A*b + B*a)*3i)/(32
*(-a)^(11/4)*b^(5/4)) - (3*atan(((3*(7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 +
 B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) - ((7*A*b + B*a)*(7*A*b^3 + B*a*b^2)*9
i)/(64*(-a)^(15/4)*b^(5/4))))/(64*(-a)^(11/4)*b^(5/4)) + (3*(7*A*b + B*a)*(
(9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) + ((7*A*b + B*
a)*(7*A*b^3 + B*a*b^2)*9i)/(64*(-a)^(15/4)*b^(5/4))))/(64*(-a)^(11/4)*b^(5/
4)))/(((7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(
64*a^4) - ((7*A*b + B*a)*(7*A*b^3 + B*a*b^2)*9i)/(64*(-a)^(15/4)*b^(5/4)))*
3i)/(64*(-a)^(11/4)*b^(5/4)) - ((7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B^2
*a^2*b + 14*A*B*a*b^2))/(64*a^4) + ((7*A*b + B*a)*(7*A*b^3 + B*a*b^2)*9i)/(
64*(-a)^(15/4)*b^(5/4)))*3i)/(64*(-a)^(11/4)*b^(5/4))))*(7*A*b + B*a))/(32*
(-a)^(11/4)*b^(5/4))
```

3.388 $\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=322

$$-\frac{5(9Ab-aB)}{16a^3b\sqrt{x}}+\frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2}+\frac{9Ab-aB}{16a^2b\sqrt{x}\,\left(a+bx^2\right)}+\frac{5(9Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{5(9Ab}{}$$

```
[Out] 5/64*(9*A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/b^(3/4)
*2^(1/2)-5/64*(9*A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4
)/b^(3/4)*2^(1/2)-5/128*(9*A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^
(1/2)*x^(1/2))/a^(13/4)/b^(3/4)*2^(1/2)+5/128*(9*A*b-B*a)*ln(a^(1/2)+x*b^(1
/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/b^(3/4)*2^(1/2)-5/16*(9*A*b-B
*a)/a^3/b/x^(1/2)+1/4*(A*b-B*a)/a/b/(b*x^2+a)^2/x^(1/2)+1/16*(9*A*b-B*a)/a^
2/b/(b*x^2+a)/x^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {468, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{5(9Ab-aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{5(9Ab-aB)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{5(9Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{5(9Ab-aB)}{16a^3b\sqrt{x}}+\frac{9Ab-aB}{16a^2b\sqrt{x}\,\left(a+bx^2\right)}+\frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-5*(9*A*b - a*B))/(16*a^3*b*Sqrt[x]) + (A*b - a*B)/(4*a*b*Sqrt[x]*(a + b*x
^2)^2) + (9*A*b - a*B)/(16*a^2*b*Sqrt[x]*(a + b*x^2)) + (5*(9*A*b - a*B)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(13/4)*b^(3/4))
- (5*(9*A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[
2]*a^(13/4)*b^(3/4)) - (5*(9*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4)*b^(3/4)) + (5*(9*A*b - a*B)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(1
3/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx = \frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2} + \frac{\left(\frac{9Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\,\left(a+bx^2\right)} + \frac{(5(9Ab-aB))\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{32a^2b}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\,\left(a+bx^2\right)} - \frac{(5(9Ab-aB))\int \frac{\sqrt{\cdots}}{a+\cdots}}{32a^3}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\,\left(a+bx^2\right)} - \frac{(5(9Ab-aB))\text{Subs}\cdots}{1\cdots}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\,\left(a+bx^2\right)} + \frac{(5(9Ab-aB))\text{Subs}\cdots}{\cdots}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\,\left(a+bx^2\right)} - \frac{(5(9Ab-aB))\text{Subs}\cdots}{\cdots}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\,\left(a+bx^2\right)} - \frac{5(9Ab-aB)\log\left(\sqrt{\cdots}\right.}{6\cdots}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\,\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\,\left(a+bx^2\right)} + \frac{5(9Ab-aB)\tan^{-1}\cdots}{32\sqrt{2}\,a\cdots}$$

Mathematica [A]

time = 0.64, size = 186, normalized size = 0.58

$$\frac{-\frac{4\sqrt[4]{a}\,\left(45Ab^2x^4+a^2\left(32A-9Bx^2\right)+abx^2\left(81A-5Bx^2\right)\right)}{\sqrt{x}\,\left(a+bx^2\right)^2}+\frac{5\sqrt{2}\,(9Ab-aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{b^{3/4}}+\frac{5\sqrt{2}\,(9Ab-aB)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{3/4}}}{64a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a^(1/4)*(45*A*b^2*x^4 + a^2*(32*A - 9*B*x^2) + a*b*x^2*(81*A - 5*B*x^2
)))/(Sqrt[x]*(a + b*x^2)^2) + (5*Sqrt[2]*(9*A*b - a*B)*ArcTan[(Sqrt[a] - Sq
rt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) + (5*Sqrt[2]*(9*A*b -
a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3
/4))/(64*a^(13/4))
```

Maple [A]
time = 0.13, size = 173, normalized size = 0.54

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{13}{32}b^{2}A-\frac{5}{32}abB\right)x^{\frac{7}{2}}+\frac{a(17Ab-9Ba)x^{\frac{3}{2}}}{32}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\left(\frac{45Ab}{32}-\frac{5Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.}{a^{3}}$
default	$-\frac{2\left(\frac{\left(\frac{13}{32}b^{2}A-\frac{5}{32}abB\right)x^{\frac{7}{2}}+\frac{a(17Ab-9Ba)x^{\frac{3}{2}}}{32}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\left(\frac{45Ab}{32}-\frac{5Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.}{a^{3}}$
risch	$-\frac{2A}{a^{3}\sqrt{x}}-\frac{13x^{\frac{7}{2}}b^{2}A}{16a^{3}(b\,x^{2}+a)^{2}}+\frac{5x^{\frac{7}{2}}bB}{16a^{2}(b\,x^{2}+a)^{2}}-\frac{17A\,x^{\frac{3}{2}}b}{16a^{2}(b\,x^{2}+a)^{2}}+\frac{9B\,x^{\frac{3}{2}}}{16a(b\,x^{2}+a)^{2}}-\frac{45\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\cdots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\cdots}\right.}{128a^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^3*(((13/32*b^2*A-5/32*a*b*B)*x^(7/2)+1/32*a*(17*A*b-9*B*a)*x^(3/2))/(b
*x^2+a)^2+1/8*(45/32*A*b-5/32*B*a)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+
2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)))-2*A/a^3/x^(1/2)
```

Maxima [A]
time = 0.55, size = 255, normalized size = 0.79

$$\frac{5\,(Bab-9\,Ab^{2})x^{4}-32\,Aa^{2}+9\,(Ba^{2}-9\,Aab)x^{2}}{16\left(a^{3}b^{2}x^{\frac{9}{2}}+2\,a^{4}bx^{\frac{5}{2}}+a^{5}\sqrt{x}\right)}+\frac{5\,(Ba-9\,Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*(B*a*b - 9*A*b^2)*x^4 - 32*A*a^2 + 9*(B*a^2 - 9*A*a*b)*x^2)/(a^3*b^
2*x^(9/2) + 2*a^4*b*x^(5/2) + a^5*sqrt(x)) + 5/128*(B*a - 9*A*b)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sq
```

```
rt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(
1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(232) = 464$.
time = 0.73, size = 988, normalized size = 3.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^3*b +
486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*ar
ctan((sqrt((B^6*a^6 - 54*A*B^5*a^5*b + 1215*A^2*B^4*a^4*b^2 - 14580*A^3*B^3
*a^3*b^3 + 98415*A^4*B^2*a^2*b^4 - 354294*A^5*B*a*b^5 + 531441*A^6*b^6)*x -
 (B^4*a^11*b - 36*A*B^3*a^10*b^2 + 486*A^2*B^2*a^9*b^3 - 2916*A^3*B*a^8*b^4
 + 6561*A^4*a^7*b^5)*sqrt(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2
- 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3)))*a^3*b*(-(B^4*a^4 - 36*A*B^3
*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))
^(1/4) + (B^3*a^6*b - 27*A*B^2*a^5*b^2 + 243*A^2*B*a^4*b^3 - 729*A^3*a^3*b^
4)*sqrt(x)*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a
*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4))/(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2
*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)) - 5*(a^3*b^2*x^5 + 2*a^4*b
*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*
B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*log(125*a^10*b^2*(-(B^4*a^4 - 36*
A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*
b^3))^(3/4) - 125*(B^3*a^3 - 27*A*B^2*a^2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3
)*sqrt(x)) + 5*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^
3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1
/4)*log(-125*a^10*b^2*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2
916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(3/4) - 125*(B^3*a^3 - 27*A*B^2
*a^2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3)*sqrt(x)) + 4*(5*(B*a*b - 9*A*b^2)*x
^4 - 32*A*a^2 + 9*(B*a^2 - 9*A*a*b)*x^2)*sqrt(x))/(a^3*b^2*x^5 + 2*a^4*b*x^
3 + a^5*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]

time = 1.78, size = 300, normalized size = 0.93

$$-\frac{2\,A}{a^3\sqrt{x}}+\frac{5\,Babx^{\frac{7}{2}}-13\,Ab^2x^{\frac{7}{2}}+9\,Ba^2x^{\frac{3}{2}}-17\,Aabx^{\frac{3}{2}}}{16\,(bx^2+a)^2a^3}+\frac{5\,\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^3}+\frac{5\,\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^3}-\frac{5\,\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^3}+\frac{5\,\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -2*A/(a^3*sqrt(x)) + 1/16*(5*B*a*b*x^(7/2) - 13*A*b^2*x^(7/2) + 9*B*a^2*x^(
3/2) - 17*A*a*b*x^(3/2))/((b*x^2 + a)^2*a^3) + 5/64*sqrt(2)*((a*b^3)^(3/4)*
B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^3) + 5/64*sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(
3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a^4*b^3) - 5/128*sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) + 5/128*sqrt(2)*((a*b
^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a^4*b^3)
```

Mupad [B]

time = 0.10, size = 133, normalized size = 0.41

$$\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)\,\left(9\,A\,b-B\,a\right)}{32\,{(-a)}^{13/4}\,b^{3/4}}-\frac{\frac{2\,A}{a}+\frac{9\,x^2\,\left(9\,A\,b-B\,a\right)}{16\,a^2}+\frac{5\,b\,x^4\,\left(9\,A\,b-B\,a\right)}{16\,a^3}}{a^2\,\sqrt{x}+b^2\,x^{9/2}+2\,a\,b\,x^{5/2}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)\,\left(9\,A\,b-B\,a\right)}{32\,{(-a)}^{13/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x)
```

```
[Out] (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(9*A*b - B*a))/(32*(-a)^(13/4)*b^(3/4
)) - ((2*A)/a + (9*x^2*(9*A*b - B*a))/(16*a^2) + (5*b*x^4*(9*A*b - B*a))/(1
6*a^3))/(a^2*x^(1/2) + b^2*x^(9/2) + 2*a*b*x^(5/2)) - (5*atanh((b^(1/4)*x^(
1/2))/(-a)^(1/4))*(9*A*b - B*a))/(32*(-a)^(13/4)*b^(3/4))
```

3.389 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

Optimal. Leaf size=322

$$-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}+\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}+\frac{7(11Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-7$$

```
[Out] -7/48*(11*A*b-3*B*a)/a^3/b/x^(3/2)+1/4*(A*b-B*a)/a/b/x^(3/2)/(b*x^2+a)^2+1/
16*(11*A*b-3*B*a)/a^2/b/x^(3/2)/(b*x^2+a)+7/64*(11*A*b-3*B*a)*arctan(1-b^(1
/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4)/b^(1/4)*2^(1/2)-7/64*(11*A*b-3*B*a)*a
rctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4)/b^(1/4)*2^(1/2)+7/128*(11
*A*b-3*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(15/4)/
b^(1/4)*2^(1/2)-7/128*(11*A*b-3*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {468, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{7(11Ab-3aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{7(11Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] (-7*(11*A*b - 3*a*B))/(48*a^3*b*x^(3/2)) + (A*b - a*B)/(4*a*b*x^(3/2)*(a +
b*x^2)^2) + (11*A*b - 3*a*B)/(16*a^2*b*x^(3/2)*(a + b*x^2)) + (7*(11*A*b -
3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(15/4)*
b^(1/4)) - (7*(11*A*b - 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(32*Sqrt[2]*a^(15/4)*b^(1/4)) + (7*(11*A*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)*b^(1/4)) - (7*
(11*A*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(64*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{\left(\frac{11Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} + \frac{(7(11Ab-3aB))\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} - \frac{(7(11Ab-3aB))\int}{32a^3}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} - \frac{(7(11Ab-3aB))\mathrm{Su}}{}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} - \frac{(7(11Ab-3aB))\mathrm{Su}}{}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} - \frac{(7(11Ab-3aB))\mathrm{Su}}{}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} + \frac{7(11Ab-3aB)\log(}{}$$

$$= -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}} + \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2} + \frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)} + \frac{7(11Ab-3aB)\tan}{32\sqrt{2}}$$

Mathematica [A]
time = 0.54, size = 186, normalized size = 0.58

$$\frac{-\frac{4a^{3/4}\left(77Ab^2x^4+a^2\left(32A-33Bx^2\right)+abx^2\left(121A-21Bx^2\right)\right)}{x^{3/2}\left(a+bx^2\right)^2}+\frac{21\sqrt{2}\,(11Ab-3aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}}+\frac{21\sqrt{2}\,(-11Ab+3aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{b}}}{192a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a^(3/4)*(77*A*b^2*x^4 + a^2*(32*A - 33*B*x^2) + a*b*x^2*(121*A - 21*B*
x^2)))/(x^(3/2)*(a + b*x^2)^2) + (21*Sqrt[2]*(11*A*b - 3*a*B)*ArcTan[(Sqrt[
a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (21*Sqrt[2]*(
-11*A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[
b]*x)])/b^(1/4))/(192*a^(15/4))
```

Maple [A]

time = 0.14, size = 173, normalized size = 0.54

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{15}{32}b^{2}A-\frac{7}{32}abB\right)x^{\frac{5}{2}}+\frac{a(19Ab-11Ba)\sqrt{x}}{32}}{\left(b\,x^{2}+a\right)^{2}}+\frac{7(11Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\cdots\right.}{256a}\cdots\right.}{a^{3}}$
default	$-\frac{2\left(\frac{\left(\frac{15}{32}b^{2}A-\frac{7}{32}abB\right)x^{\frac{5}{2}}+\frac{a(19Ab-11Ba)\sqrt{x}}{32}}{\left(b\,x^{2}+a\right)^{2}}+\frac{7(11Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\cdots\right.}{256a}\cdots\right.}{a^{3}}$
risch	$-\frac{2A}{3a^{3}x^{\frac{3}{2}}}-\frac{15x^{\frac{5}{2}}b^{2}A}{16a^{3}(b\,x^{2}+a)^{2}}+\frac{7x^{\frac{5}{2}}bB}{16a^{2}(b\,x^{2}+a)^{2}}-\frac{19A\sqrt{x}\,b}{16a^{2}(b\,x^{2}+a)^{2}}+\frac{11B\sqrt{x}}{16a(b\,x^{2}+a)^{2}}-\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\sqrt{\cdots}\right.}{64a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^3*(((15/32*b^2*A-7/32*a*b*B)*x^(5/2)+1/32*a*(19*A*b-11*B*a)*x^(1/2))/(
b*x^2+a)^2+7/256*(11*A*b-3*B*a)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1
)))-2/3*A/a^3/x^(3/2)
```

Maxima [A]

time = 0.51, size = 285, normalized size = 0.89

$$\frac{7\,(3\,Bab-11\,Ab^{2})x^{4}-32\,Aa^{2}+11\,(3\,Ba^{2}-11\,Aab)x^{2}}{48\left(a^{3}b^{2}x^{\frac{11}{2}}+2\,a^{4}bx^{\frac{7}{2}}+a^{5}x^{\frac{3}{2}}\right)}+\frac{7\left(\frac{2\sqrt{2}\,(3\,Ba-11\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(3\,Ba-11\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(3\,Ba-11\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,Ba-11\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(7*(3*B*a*b - 11*A*b^2)*x^4 - 32*A*a^2 + 11*(3*B*a^2 - 11*A*a*b)*x^2)/
(a^3*b^2*x^(11/2) + 2*a^4*b*x^(7/2) + a^5*x^(3/2)) + 7/128*(2*sqrt(2)*(3*B*
a - 11*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*B*a
 - 11*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*B*a -
```

```
11*A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4
)*b^(1/4)) - sqrt(2)*(3*B*a - 11*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(238) = 476$.
time = 0.67, size = 809, normalized size = 2.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(84*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*B^3
*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b)
)^(1/4)*arctan((sqrt(a^8*sqrt(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^
2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b)) + (9*B^2*a^2 - 66*
A*B*a*b + 121*A^2*b^2)*x)*a^11*b*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^
2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(3/4) + (3*B*a
^12*b - 11*A*a^11*b^2)*sqrt(x)*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*
B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(3/4))/(81*B^4*a
^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^
4*b^4)) + 21*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*B
^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*
b))^(1/4)*log(7*a^4*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2
 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4) - 7*(3*B*a - 11*A*b)*
sqrt(x)) - 21*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*
B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15
*b))^(1/4)*log(-7*a^4*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b
^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4) - 7*(3*B*a - 11*A*b
)*sqrt(x)) - 4*(7*(3*B*a*b - 11*A*b^2)*x^4 - 32*A*a^2 + 11*(3*B*a^2 - 11*A*
a*b)*x^2)*sqrt(x))/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.91, size = 304, normalized size = 0.94

$$\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b}+\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b}+\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{4}b}-\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{4}b}-\frac{2\,A}{3\,a^{3}x^{\frac{3}{2}}}+\frac{7\,Babx^{\frac{5}{2}}-15\,Ab^{2}x^{\frac{5}{2}}+11\,Ba^{2}\sqrt{x}-19\,Aab\sqrt{x}}{16\left(bx^{2}+a\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 7/64*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 7/64*sqrt(2)*(3*
(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 7/128*sqrt(2)*(3*(a*b^3)^(1/4)*
B*a - 11*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a^4*b) - 7/128*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) - 2/3*A/(a^3*x^(3/2))
 + 1/16*(7*B*a*b*x^(5/2) - 15*A*b^2*x^(5/2) + 11*B*a^2*sqrt(x) - 19*A*a*b*s
qrt(x))/((b*x^2 + a)^2*a^3)
```

Mupad [B]
time = 0.30, size = 888, normalized size = 2.76

$$-\frac{\frac{2A}{3a}+\frac{11x^2(11Ab-3Ba)}{48a^2}+\frac{7bx^4(11Ab-3Ba)}{48a^3}}{a^2x^{3/2}+b^2x^{11/2}+2abx^{7/2}}-\frac{\operatorname{atan}\left(\frac{\frac{(11Ab-3Ba)\left(\sqrt{x}\,(97140736A^2a^9b^5-52985856ABa^{10}b^4+7225344B^2a^{11}b^3)-\frac{7(11Ab-3Ba)(80740352Aa^{13}b^4-22020096Ba^{14}b^3)}{64(-a)^{15/4}b^{1/4}}\right)7i}{64(-a)^{15/4}b^{1/4}}+\frac{(11Ab-3Ba)\left(\sqrt{x}\,(97140736A^2a^9b^5-52985856ABa^{10}b^4+7225344B^2a^{11}b^3)+\frac{7(11Ab-3Ba)(80740352Aa^{13}b^4-22020096Ba^{14}b^3)}{64(-a)^{15/4}b^{1/4}}\right)7i}{64(-a)^{15/4}b^{1/4}}}{\frac{7(11Ab-3Ba)\left(\sqrt{x}\,(97140736A^2a^9b^5-52985856ABa^{10}b^4+7225344B^2a^{11}b^3)-\frac{7(11Ab-3Ba)(80740352Aa^{13}b^4-22020096Ba^{14}b^3)}{64(-a)^{15/4}b^{1/4}}\right)}{64(-a)^{15/4}b^{1/4}}-\frac{7(11Ab-3Ba)\left(\sqrt{x}\,(97140736A^2a^9b^5-52985856ABa^{10}b^4+7225344B^2a^{11}b^3)+\frac{7(11Ab-3Ba)(80740352Aa^{13}b^4-22020096Ba^{14}b^3)}{64(-a)^{15/4}b^{1/4}}\right)}{64(-a)^{15/4}b^{1/4}}}\right)(11Ab-3Ba)\,7i}{32(-a)^{15/4}b^{1/4}}-\frac{7\operatorname{atan}\left(\frac{\frac{7(11Ab-3Ba)\left(\sqrt{x}\,(97140736A^2a^9b^5-52985856ABa^{10}b^4+7225344B^2a^{11}b^3)-\frac{(11Ab-3Ba)(80740352Aa^{13}b^4-22020096Ba^{14}b^3)7i}{64(-a)^{15/4}b^{1/4}}\right)}{64(-a)^{15/4}b^{1/4}}+\frac{7(11Ab-3Ba)\left(\sqrt{x}\,(97140736A^2a^9b^5-52985856ABa^{10}b^4+7225344B^2a^{11}b^3)+\frac{(11Ab-3Ba)(80740352Aa^{13}b^4-22020096Ba^{14}b^3)7i}{64(-a)^{15/4}b^{1/4}}\right)}{64(-a)^{15/4}b^{1/4}}}{\frac{(11Ab-3Ba)\left(\sqrt{x}\,(97140736A^2a^9b^5-52985856ABa^{10}b^4+7225344B^2a^{11}b^3)-\frac{(11Ab-3Ba)(80740352Aa^{13}b^4-22020096Ba^{14}b^3)7i}{64(-a)^{15/4}b^{1/4}}\right)7i}{64(-a)^{15/4}b^{1/4}}-\frac{(11Ab-3Ba)\left(\sqrt{x}\,(97140736A^2a^9b^5-52985856ABa^{10}b^4+7225344B^2a^{11}b^3)+\frac{(11Ab-3Ba)(80740352Aa^{13}b^4-22020096Ba^{14}b^3)7i}{64(-a)^{15/4}b^{1/4}}\right)7i}{64(-a)^{15/4}b^{1/4}}}\right)(11Ab-3Ba)}{32(-a)^{15/4}b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x)
```

```
[Out] - ((2*A)/(3*a) + (11*x^2*(11*A*b - 3*B*a))/(48*a^2) + (7*b*x^4*(11*A*b - 3*
B*a))/(48*a^3))/(a^2*x^(3/2) + b^2*x^(11/2) + 2*a*b*x^(7/2)) - (atan((((11*
A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 529858
56*A*B*a^10*b^4) - (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^
14*b^3))/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*(-a)^(15/4)*b^(1/4)) + ((11*A*b
- 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A
*B*a^10*b^4) + (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b
^3))/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*(-a)^(15/4)*b^(1/4)))/((7*(11*A*b -
3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B
*a^10*b^4) - (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b^3
))/(64*(-a)^(15/4)*b^(1/4))))/(64*(-a)^(15/4)*b^(1/4)) - (7*(11*A*b - 3*B*a
)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10
*b^4) + (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b^3))/(6
4*(-a)^(15/4)*b^(1/4))))/(64*(-a)^(15/4)*b^(1/4))))*(11*A*b - 3*B*a)*7i)/(3
2*(-a)^(15/4)*b^(1/4)) - (7*atan(((7*(11*A*b - 3*B*a)*(x^(1/2)*(97140736*A^
2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) - ((11*A*b - 3*B*
a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(-a)^(15/4)*b^(1/4))
))/(64*(-a)^(15/4)*b^(1/4)) + (7*(11*A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^
9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) + ((11*A*b - 3*B*a)*(
80740352*A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(-a)^(15/4)*b^(1/4))))/(
64*(-a)^(15/4)*b^(1/4)))/(((11*A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5
+ 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) - ((11*A*b - 3*B*a)*(807403
52*A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*
(-a)^(15/4)*b^(1/4)) - ((11*A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7
```

```
225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) + ((11*A*b - 3*B*a)*(80740352*
A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*(-a
)^(15/4)*b^(1/4))))*(11*A*b - 3*B*a))/(32*(-a)^(15/4)*b^(1/4))
```

3.390 $\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}-\frac{9\sqrt[4]{b}\,(13Ab-5aB)\tan^{-1}\left(\right.}{32\sqrt{2}\,a^{17/}}$$

```
[Out] -9/80*(13*A*b-5*B*a)/a^3/b/x^(5/2)+1/4*(A*b-B*a)/a/b/x^(5/2)/(b*x^2+a)^2+1/
16*(13*A*b-5*B*a)/a^2/b/x^(5/2)/(b*x^2+a)-9/64*b^(1/4)*(13*A*b-5*B*a)*arcta
n(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+9/64*b^(1/4)*(13*A*b-
5*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+9/128*b^(
1/4)*(13*A*b-5*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(17/4)*2^(1/2)-9/128*b^(1/4)*(13*A*b-5*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b
^(1/4)*2^(1/2)*x^(1/2))/a^(17/4)*2^(1/2)+9/16*(13*A*b-5*B*a)/a^4/x^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {468, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{9\sqrt[4]{b}\,(13Ab-5aB)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{17/4}}+\frac{9\sqrt[4]{b}\,(13Ab-5aB)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{17/4}}+\frac{9\sqrt[4]{b}\,(13Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-\frac{9\sqrt[4]{b}\,(13Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] (-9*(13*A*b - 5*a*B))/(80*a^3*b*x^(5/2)) + (9*(13*A*b - 5*a*B))/(16*a^4*Sqr
t[x]) + (A*b - a*B)/(4*a*b*x^(5/2)*(a + b*x^2)^2) + (13*A*b - 5*a*B)/(16*a^
2*b*x^(5/2)*(a + b*x^2)) - (9*b^(1/4)*(13*A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (9*b^(1/4)*(13*A*b - 5*a
*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) +
(9*b^(1/4)*(13*A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(64*Sqrt[2]*a^(17/4)) - (9*b^(1/4)*(13*A*b - 5*a*B)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx = \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{\left(\frac{13Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} + \frac{(9(13Ab-5aB))\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} - \frac{(9(13Ab-5aB))\int}{32a^3}$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} + \frac{(}{}$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} + \frac{(}{}$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} - \frac{(}{}$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} + \frac{(}{}$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} + \frac{9}{}$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} - \frac{9}{}$$

<u>Mathematica</u> [A]
time = 0.57, size = 207, normalized size = 0.60

$$\frac{-\frac{4\sqrt[4]{a}\left(-585Ab^3x^6+32a^3\left(A+5Bx^2\right)+9ab^2x^4\left(-117A+25Bx^2\right)+a^2\left(-416Abx^2+405bBx^4\right)\right)}{x^{5/2}\left(a+bx^2\right)^2}+45\sqrt{2}\sqrt[4]{b}\,(-13Ab+5aB)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+45\sqrt{2}\sqrt[4]{b}\,(-13Ab+5aB)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{320a^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a^(1/4)*(-585*A*b^3*x^6 + 32*a^3*(A + 5*B*x^2) + 9*a*b^2*x^4*(-117*A +
 25*B*x^2) + a^2*(-416*A*b*x^2 + 405*b*B*x^4)))/(x^(5/2)*(a + b*x^2)^2) + 4
5*Sqrt[2]*b^(1/4)*(-13*A*b + 5*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a
```

```
^(1/4)*b^(1/4)*Sqrt[x])] + 45*Sqrt[2]*b^(1/4)*(-13*A*b + 5*a*B)*ArcTanh[(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(320*a^(17/4))
```

Maple [A]
time = 0.15, size = 190, normalized size = 0.55

method	result
derivativedivides	$\frac{2b\left(\frac{\left(\frac{21}{32}b^2A-\frac{13}{32}abB\right)x^{\frac{7}{2}}+\frac{a(25Ab-17Ba)x^{\frac{3}{2}}}{32}}{\left(bx^2+a\right)^2}+\frac{\left(\frac{117Ab}{32}-\frac{45Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right.}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^4}$
default	$\frac{2b\left(\frac{\left(\frac{21}{32}b^2A-\frac{13}{32}abB\right)x^{\frac{7}{2}}+\frac{a(25Ab-17Ba)x^{\frac{3}{2}}}{32}}{\left(bx^2+a\right)^2}+\frac{\left(\frac{117Ab}{32}-\frac{45Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right.}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^4}$
risch	$-\frac{2\left(-15Abx^2+5Bax^2+Aa\right)}{5a^4x^{\frac{5}{2}}}+\frac{21b^3x^{\frac{7}{2}}A}{16a^4\left(bx^2+a\right)^2}-\frac{13b^2x^{\frac{7}{2}}B}{16a^3\left(bx^2+a\right)^2}+\frac{25b^2Ax^{\frac{3}{2}}}{16a^3\left(bx^2+a\right)^2}-\frac{17bBx^{\frac{3}{2}}}{16a^2\left(bx^2+a\right)^2}+\frac{117b\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^4*b*(((21/32*b^2*A-13/32*a*b*B)*x^(7/2)+1/32*a*(25*A*b-17*B*a)*x^(3/2))
/(b*x^2+a)^2+1/8*(117/32*A*b-45/32*B*a)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)))-2/5*A/a^3/x^(5/2)-2*(-3*A*b+B*a)/a^4/x^(1/2)
```

Maxima [A]
time = 0.49, size = 285, normalized size = 0.83

$$-\frac{45\,(5\,Bab^2-13\,Ab^3)x^6+81\,(5\,Ba^2b-13\,Aab^2)x^4+32\,Aa^3+32\,(5\,Ba^3-13\,Aa^2b)x^2}{80\left(a^4b^2x^{\frac{13}{2}}+2\,a^5bx^{\frac{9}{2}}+a^6x^{\frac{5}{2}}\right)}-\frac{9\,(5\,Bab-13\,Ab^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/80*(45*(5*B*a*b^2 - 13*A*b^3)*x^6 + 81*(5*B*a^2*b - 13*A*a*b^2)*x^4 + 32
*A*a^3 + 32*(5*B*a^3 - 13*A*a^2*b)*x^2)/(a^4*b^2*x^(13/2) + 2*a^5*b*x^(9/2)
 + a^6*x^(5/2)) - 9/128*(5*B*a*b - 13*A*b^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*
```

```
(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*
b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*
sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a
))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(
b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(255) = 510$.
time = 0.57, size = 1043, normalized size = 3.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(180*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A
*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a
^17)^(1/4)*arctan((sqrt((15625*B^6*a^6*b^2 - 243750*A*B^5*a^5*b^3 + 1584375
*A^2*B^4*a^4*b^4 - 5492500*A^3*B^3*a^3*b^5 + 10710375*A^4*B^2*a^2*b^6 - 111
38790*A^5*B*a*b^7 + 4826809*A^6*b^8)*x - (625*B^4*a^13*b - 6500*A*B^3*a^12*
b^2 + 25350*A^2*B^2*a^11*b^3 - 43940*A^3*B*a^10*b^4 + 28561*A^4*a^9*b^5)*sq
rt(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3
*B*a*b^4 + 28561*A^4*b^5)/a^17))*a^4*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2
+ 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4) +
(125*B^3*a^7*b - 975*A*B^2*a^6*b^2 + 2535*A^2*B*a^5*b^3 - 2197*A^3*a^4*b^4)
*sqrt(x)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43
940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4))/(625*B^4*a^4*b - 6500*A*B^3*a
^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)) - 45*(
a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2
+ 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4)*lo
g(729*a^13*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 -
43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B^3*a^3*b - 975*A
*B^2*a^2*b^2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)) + 45*(a^4*b^2*x^7
+ 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*
B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4)*log(-729*a^13*
(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B
*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B^3*a^3*b - 975*A*B^2*a^2*b^
2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)) + 4*(45*(5*B*a*b^2 - 13*A*b^3
)*x^6 + 81*(5*B*a^2*b - 13*A*a*b^2)*x^4 + 32*A*a^3 + 32*(5*B*a^3 - 13*A*a^2
*b)*x^2)*sqrt(x))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.30, size = 326, normalized size = 0.95

$$-\frac{9\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}Ba-13\,(ab^3)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b^2}-\frac{9\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}Ba-13\,(ab^3)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b^2}+\frac{9\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}Ba-13\,(ab^3)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b^2}-\frac{9\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}Ba-13\,(ab^3)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b^2}-\frac{13\,Bab^2x^{\frac{7}{2}}-21\,Ab^3x^{\frac{7}{2}}+17\,Ba^2bx^{\frac{3}{2}}-25\,Aab^2x^{\frac{3}{2}}}{16\,(bx^2+a)^2a^4}-\frac{2\,(5\,Bax^2-15\,Abx^2+Aa)}{5\,a^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -9/64*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(
2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) - 9/64*sqrt(2)*
(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) + 9/128*sqrt(2)*(5*(a*b^3)^(
3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt
(a/b))/(a^5*b^2) - 9/128*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*
b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) - 1/16*(13*B
*a*b^2*x^(7/2) - 21*A*b^3*x^(7/2) + 17*B*a^2*b*x^(3/2) - 25*A*a*b^2*x^(3/2)
)/((b*x^2 + a)^2*a^4) - 2/5*(5*B*a*x^2 - 15*A*b*x^2 + A*a)/(a^4*x^(5/2))
```

Mupad [B]
time = 0.20, size = 152, normalized size = 0.44

$$\frac{\frac{2\,x^2\,(13\,A\,b-5\,B\,a)}{5\,a^2}-\frac{2\,A}{5\,a}+\frac{9\,b^2\,x^6\,(13\,A\,b-5\,B\,a)}{16\,a^4}+\frac{81\,b\,x^4\,(13\,A\,b-5\,B\,a)}{80\,a^3}}{a^2\,x^{5/2}+b^2\,x^{13/2}+2\,a\,b\,x^{9/2}}+\frac{9\,{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(13\,A\,b-5\,B\,a)}{32\,a^{17/4}}-\frac{9\,{(-b)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(13\,A\,b-5\,B\,a)}{32\,a^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x)
```

```
[Out] ((2*x^2*(13*A*b - 5*B*a))/(5*a^2) - (2*A)/(5*a) + (9*b^2*x^6*(13*A*b - 5*B*
a))/(16*a^4) + (81*b*x^4*(13*A*b - 5*B*a))/(80*a^3))/(a^2*x^(5/2) + b^2*x^(
13/2) + 2*a*b*x^(9/2)) + (9*(-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4))*(
13*A*b - 5*B*a))/(32*a^(17/4)) - (9*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a
^(1/4))*(13*A*b - 5*B*a))/(32*a^(17/4))
```

3.391 $\int x^{7/2}(a+bx^2)^2 (c+dx^2) \, dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2cx^{9/2}+\frac{2}{13}a(2bc+ad)x^{13/2}+\frac{2}{17}b(bc+2ad)x^{17/2}+\frac{2}{21}b^2dx^{21/2}$$

```
[Out] 2/9*a^2*c*x^(9/2)+2/13*a*(a*d+2*b*c)*x^(13/2)+2/17*b*(2*a*d+b*c)*x^(17/2)+2
/21*b^2*d*x^(21/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\frac{2}{9}a^2cx^{9/2}+\frac{2}{17}bx^{17/2}(2ad+bc)+\frac{2}{13}ax^{13/2}(ad+2bc)+\frac{2}{21}b^2dx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(9/2))/9 + (2*a*(2*b*c + a*d)*x^(13/2))/13 + (2*b*(b*c + 2*a*d)*
x^(17/2))/17 + (2*b^2*d*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx &= \int \left(a^2cx^{7/2}+a(2bc+ad)x^{11/2}+b(bc+2ad)x^{15/2}+b^2dx^{19/2}\right)\,dx\\ &= \frac{2}{9}a^2cx^{9/2}+\frac{2}{13}a(2bc+ad)x^{13/2}+\frac{2}{17}b(bc+2ad)x^{17/2}+\frac{2}{21}b^2dx^{21/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.95

$$\frac{2x^{9/2}(119a^2(13c+9dx^2)+126abx^2(17c+13dx^2)+39b^2x^4(21c+17dx^2))}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(9/2)*(119*a^2*(13*c + 9*d*x^2) + 126*a*b*x^2*(17*c + 13*d*x^2) + 39*b
^2*x^4*(21*c + 17*d*x^2)))/13923
```

Maple [A]
time = 0.09, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2 d\,x^{\frac{21}{2}}}{21}+\frac{2(2abd+b^2c)x^{\frac{17}{2}}}{17}+\frac{2(a^2d+2abc)x^{\frac{13}{2}}}{13}+\frac{2a^2c\,x^{\frac{9}{2}}}{9}$	52
default	$\frac{2b^2 d\,x^{\frac{21}{2}}}{21}+\frac{2(2abd+b^2c)x^{\frac{17}{2}}}{17}+\frac{2(a^2d+2abc)x^{\frac{13}{2}}}{13}+\frac{2a^2c\,x^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}(663b^2d\,x^6+1638abd\,x^4+819b^2c\,x^4+1071a^2d\,x^2+2142abc\,x^2+1547a^2c)}{13923}$	56
trager	$\frac{2x^{\frac{9}{2}}(663b^2d\,x^6+1638abd\,x^4+819b^2c\,x^4+1071a^2d\,x^2+2142abc\,x^2+1547a^2c)}{13923}$	56
risch	$\frac{2x^{\frac{9}{2}}(663b^2d\,x^6+1638abd\,x^4+819b^2c\,x^4+1071a^2d\,x^2+2142abc\,x^2+1547a^2c)}{13923}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*b^2*d*x^(21/2)+2/17*(2*a*b*d+b^2*c)*x^(17/2)+2/13*(a^2*d+2*a*b*c)*x^(1
3/2)+2/9*a^2*c*x^(9/2)
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.81

$$\frac{2}{21}\,b^2dx^{\frac{21}{2}}+\frac{2}{17}\left(b^2c+2\,abd\right)x^{\frac{17}{2}}+\frac{2}{9}\,a^2cx^{\frac{9}{2}}+\frac{2}{13}\left(2\,abc+a^2d\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d*x^(21/2) + 2/17*(b^2*c + 2*a*b*d)*x^(17/2) + 2/9*a^2*c*x^(9/2) +
 2/13*(2*a*b*c + a^2*d)*x^(13/2)
```

Fricas [A]
time = 0.48, size = 56, normalized size = 0.89

$$\frac{2}{13923}\left(663\,b^2dx^{10}+819\left(b^2c+2\,abd\right)x^8+1547\,a^2cx^4+1071\left(2\,abc+a^2d\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^2*d*x^10 + 819*(b^2*c + 2*a*b*d)*x^8 + 1547*a^2*c*x^4 + 1071
*(2*a*b*c + a^2*d)*x^6)*sqrt(x)
```

Sympy [A]
time = 1.11, size = 80, normalized size = 1.27

$$\frac{2a^2cx^{\frac{9}{2}}}{9}+\frac{2a^2dx^{\frac{13}{2}}}{13}+\frac{4abcx^{\frac{13}{2}}}{13}+\frac{4abdx^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{17}{2}}}{17}+\frac{2b^2dx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(9/2)/9 + 2*a**2*d*x**(13/2)/13 + 4*a*b*c*x**(13/2)/13 + 4*a*b*
d*x**(17/2)/17 + 2*b**2*c*x**(17/2)/17 + 2*b**2*d*x**(21/2)/21
```

Giac [A]
time = 1.48, size = 53, normalized size = 0.84

$$\frac{2}{21}\,b^2dx^{\frac{21}{2}}+\frac{2}{17}\,b^2cx^{\frac{17}{2}}+\frac{4}{17}\,abdx^{\frac{17}{2}}+\frac{4}{13}\,abcx^{\frac{13}{2}}+\frac{2}{13}\,a^2dx^{\frac{13}{2}}+\frac{2}{9}\,a^2cx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/21*b^2*d*x^(21/2) + 2/17*b^2*c*x^(17/2) + 4/17*a*b*d*x^(17/2) + 4/13*a*b*
c*x^(13/2) + 2/13*a^2*d*x^(13/2) + 2/9*a^2*c*x^(9/2)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,d\,a^2}{13}+\frac{4\,b\,c\,a}{13}\right)+x^{17/2}\left(\frac{2\,c\,b^2}{17}+\frac{4\,a\,d\,b}{17}\right)+\frac{2\,a^2\,c\,x^{9/2}}{9}+\frac{2\,b^2\,d\,x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^(13/2)*((2*a^2*d)/13 + (4*a*b*c)/13) + x^(17/2)*((2*b^2*c)/17 + (4*a*b*d)
/17) + (2*a^2*c*x^(9/2))/9 + (2*b^2*d*x^(21/2))/21
```

3.392 $\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2cx^{7/2}+\frac{2}{11}a(2bc+ad)x^{11/2}+\frac{2}{15}b(bc+2ad)x^{15/2}+\frac{2}{19}b^2dx^{19/2}$$

```
[Out] 2/7*a^2*c*x^(7/2)+2/11*a*(a*d+2*b*c)*x^(11/2)+2/15*b*(2*a*d+b*c)*x^(15/2)+2
/19*b^2*d*x^(19/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\frac{2}{7}a^2cx^{7/2}+\frac{2}{15}bx^{15/2}(2ad+bc)+\frac{2}{11}ax^{11/2}(ad+2bc)+\frac{2}{19}b^2dx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c + 2*a*d)*
x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\int\left(a^2cx^{5/2}+a(2bc+ad)x^{9/2}+b(bc+2ad)x^{13/2}+b^2dx^{17/2}\right)dx\\&=\frac{2}{7}a^2cx^{7/2}+\frac{2}{11}a(2bc+ad)x^{11/2}+\frac{2}{15}b(bc+2ad)x^{15/2}+\frac{2}{19}b^2dx^{19/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.95

$$\frac{2x^{7/2}(285a^2(11c+7dx^2)+266abx^2(15c+11dx^2)+77b^2x^4(19c+15dx^2))}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(7/2)*(285*a^2*(11*c + 7*d*x^2) + 266*a*b*x^2*(15*c + 11*d*x^2) + 77*b
^2*x^4*(19*c + 15*d*x^2)))/21945
```

Maple [A]
time = 0.09, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2 d\,x^{\frac{19}{2}}}{19}+\frac{2(2abd+b^2c)x^{\frac{15}{2}}}{15}+\frac{2(a^2d+2abc)x^{\frac{11}{2}}}{11}+\frac{2a^2c\,x^{\frac{7}{2}}}{7}$	52
default	$\frac{2b^2 d\,x^{\frac{19}{2}}}{19}+\frac{2(2abd+b^2c)x^{\frac{15}{2}}}{15}+\frac{2(a^2d+2abc)x^{\frac{11}{2}}}{11}+\frac{2a^2c\,x^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}(1155b^2d\,x^6+2926abd\,x^4+1463b^2c\,x^4+1995a^2d\,x^2+3990abc\,x^2+3135a^2c)}{21945}$	56
trager	$\frac{2x^{\frac{7}{2}}(1155b^2d\,x^6+2926abd\,x^4+1463b^2c\,x^4+1995a^2d\,x^2+3990abc\,x^2+3135a^2c)}{21945}$	56
risch	$\frac{2x^{\frac{7}{2}}(1155b^2d\,x^6+2926abd\,x^4+1463b^2c\,x^4+1995a^2d\,x^2+3990abc\,x^2+3135a^2c)}{21945}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*b^2*d*x^(19/2)+2/15*(2*a*b*d+b^2*c)*x^(15/2)+2/11*(a^2*d+2*a*b*c)*x^(1
1/2)+2/7*a^2*c*x^(7/2)
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.81

$$\frac{2}{19}\,b^2dx^{\frac{19}{2}}+\frac{2}{15}\left(b^2c+2\,abd\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^2cx^{\frac{7}{2}}+\frac{2}{11}\left(2\,abc+a^2d\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d*x^(19/2) + 2/15*(b^2*c + 2*a*b*d)*x^(15/2) + 2/7*a^2*c*x^(7/2) +
 2/11*(2*a*b*c + a^2*d)*x^(11/2)
```

Fricas [A]
time = 0.44, size = 56, normalized size = 0.89

$$\frac{2}{21945}\left(1155\,b^2dx^9+1463\left(b^2c+2\,abd\right)x^7+3135\,a^2cx^3+1995\left(2\,abc+a^2d\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d*x^9 + 1463*(b^2*c + 2*a*b*d)*x^7 + 3135*a^2*c*x^3 + 199
5*(2*a*b*c + a^2*d)*x^5)*sqrt(x)
```

Sympy [A]
time = 0.79, size = 80, normalized size = 1.27

$$\frac{2a^2cx^{\frac{7}{2}}}{7}+\frac{2a^2dx^{\frac{11}{2}}}{11}+\frac{4abcx^{\frac{11}{2}}}{11}+\frac{4abdx^{\frac{15}{2}}}{15}+\frac{2b^2cx^{\frac{15}{2}}}{15}+\frac{2b^2dx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(7/2)/7 + 2*a**2*d*x**(11/2)/11 + 4*a*b*c*x**(11/2)/11 + 4*a*b*
d*x**(15/2)/15 + 2*b**2*c*x**(15/2)/15 + 2*b**2*d*x**(19/2)/19
```

Giac [A]
time = 0.94, size = 53, normalized size = 0.84

$$\frac{2}{19}b^2dx^{\frac{19}{2}}+\frac{2}{15}b^2cx^{\frac{15}{2}}+\frac{4}{15}abdx^{\frac{15}{2}}+\frac{4}{11}abcx^{\frac{11}{2}}+\frac{2}{11}a^2dx^{\frac{11}{2}}+\frac{2}{7}a^2cx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d*x^(19/2) + 2/15*b^2*c*x^(15/2) + 4/15*a*b*d*x^(15/2) + 4/11*a*b*
c*x^(11/2) + 2/11*a^2*d*x^(11/2) + 2/7*a^2*c*x^(7/2)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,d\,a^2}{11}+\frac{4\,b\,c\,a}{11}\right)+x^{15/2}\left(\frac{2\,c\,b^2}{15}+\frac{4\,a\,d\,b}{15}\right)+\frac{2\,a^2\,c\,x^{7/2}}{7}+\frac{2\,b^2\,d\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^(11/2)*((2*a^2*d)/11 + (4*a*b*c)/11) + x^(15/2)*((2*b^2*c)/15 + (4*a*b*d)
/15) + (2*a^2*c*x^(7/2))/7 + (2*b^2*d*x^(19/2))/19
```

3.393 $\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2cx^{5/2}+\frac{2}{9}a(2bc+ad)x^{9/2}+\frac{2}{13}b(bc+2ad)x^{13/2}+\frac{2}{17}b^2dx^{17/2}$$

```
[Out] 2/5*a^2*c*x^(5/2)+2/9*a*(a*d+2*b*c)*x^(9/2)+2/13*b*(2*a*d+b*c)*x^(13/2)+2/1
7*b^2*d*x^(17/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = $\{459\}$

$$\frac{2}{5}a^2cx^{5/2}+\frac{2}{13}bx^{13/2}(2ad+bc)+\frac{2}{9}ax^{9/2}(ad+2bc)+\frac{2}{17}b^2dx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(5/2))/5 + (2*a*(2*b*c + a*d)*x^(9/2))/9 + (2*b*(b*c + 2*a*d)*x^
(13/2))/13 + (2*b^2*d*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\int\left(a^2cx^{3/2}+a(2bc+ad)x^{7/2}+b(bc+2ad)x^{11/2}+b^2dx^{15/2}\right)\,dx\\&=\frac{2}{5}a^2cx^{5/2}+\frac{2}{9}a(2bc+ad)x^{9/2}+\frac{2}{13}b(bc+2ad)x^{13/2}+\frac{2}{17}b^2dx^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.95

$$\frac{2x^{5/2}\left(221a^2\left(9c+5dx^2\right)+170abx^2\left(13c+9dx^2\right)+45b^2x^4\left(17c+13dx^2\right)\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(5/2)*(221*a^2*(9*c + 5*d*x^2) + 170*a*b*x^2*(13*c + 9*d*x^2) + 45*b^2
*x^4*(17*c + 13*d*x^2)))/9945
```

Maple [A]
time = 0.09, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2 d\,x^{\frac{17}{2}}}{17}+\frac{2(2abd+b^2c)x^{\frac{13}{2}}}{13}+\frac{2(a^2d+2abc)x^{\frac{9}{2}}}{9}+\frac{2a^2c\,x^{\frac{5}{2}}}{5}$	52
default	$\frac{2b^2 d\,x^{\frac{17}{2}}}{17}+\frac{2(2abd+b^2c)x^{\frac{13}{2}}}{13}+\frac{2(a^2d+2abc)x^{\frac{9}{2}}}{9}+\frac{2a^2c\,x^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(585b^2d\,x^6+1530abd\,x^4+765b^2c\,x^4+1105a^2d\,x^2+2210abc\,x^2+1989a^2c)}{9945}$	56
trager	$\frac{2x^{\frac{5}{2}}(585b^2d\,x^6+1530abd\,x^4+765b^2c\,x^4+1105a^2d\,x^2+2210abc\,x^2+1989a^2c)}{9945}$	56
risch	$\frac{2x^{\frac{5}{2}}(585b^2d\,x^6+1530abd\,x^4+765b^2c\,x^4+1105a^2d\,x^2+2210abc\,x^2+1989a^2c)}{9945}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*b^2*d*x^(17/2)+2/13*(2*a*b*d+b^2*c)*x^(13/2)+2/9*(a^2*d+2*a*b*c)*x^(9/
2)+2/5*a^2*c*x^(5/2)
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.81

$$\frac{2}{17}\,b^2dx^{\frac{17}{2}}+\frac{2}{13}\,\left(b^2c+2\,abd\right)x^{\frac{13}{2}}+\frac{2}{5}\,a^2cx^{\frac{5}{2}}+\frac{2}{9}\,\left(2\,abc+a^2d\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d*x^(17/2) + 2/13*(b^2*c + 2*a*b*d)*x^(13/2) + 2/5*a^2*c*x^(5/2) +
 2/9*(2*a*b*c + a^2*d)*x^(9/2)
```

Fricas [A]
time = 1.17, size = 56, normalized size = 0.89

$$\frac{2}{9945}\left(585\,b^2dx^8+765\,\left(b^2c+2\,abd\right)x^6+1989\,a^2cx^2+1105\,\left(2\,abc+a^2d\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d*x^8 + 765*(b^2*c + 2*a*b*d)*x^6 + 1989*a^2*c*x^2 + 1105*(
2*a*b*c + a^2*d)*x^4)*sqrt(x)
```

Sympy [A]
time = 0.58, size = 80, normalized size = 1.27

$$\frac{2a^2cx^{\frac{5}{2}}}{5}+\frac{2a^2dx^{\frac{9}{2}}}{9}+\frac{4abcx^{\frac{9}{2}}}{9}+\frac{4abdx^{\frac{13}{2}}}{13}+\frac{2b^2cx^{\frac{13}{2}}}{13}+\frac{2b^2dx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(5/2)/5 + 2*a**2*d*x**(9/2)/9 + 4*a*b*c*x**(9/2)/9 + 4*a*b*d*x*
*(13/2)/13 + 2*b**2*c*x**(13/2)/13 + 2*b**2*d*x**(17/2)/17
```

Giac [A]
time = 0.97, size = 53, normalized size = 0.84

$$\frac{2}{17}\,b^2dx^{\frac{17}{2}}+\frac{2}{13}\,b^2cx^{\frac{13}{2}}+\frac{4}{13}\,abdx^{\frac{13}{2}}+\frac{4}{9}\,abcx^{\frac{9}{2}}+\frac{2}{9}\,a^2dx^{\frac{9}{2}}+\frac{2}{5}\,a^2cx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d*x^(17/2) + 2/13*b^2*c*x^(13/2) + 4/13*a*b*d*x^(13/2) + 4/9*a*b*c
*x^(9/2) + 2/9*a^2*d*x^(9/2) + 2/5*a^2*c*x^(5/2)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.81

$$x^{9/2}\,\left(\frac{2\,d\,a^2}{9}+\frac{4\,b\,c\,a}{9}\right)+x^{13/2}\,\left(\frac{2\,c\,b^2}{13}+\frac{4\,a\,d\,b}{13}\right)+\frac{2\,a^2\,c\,x^{5/2}}{5}+\frac{2\,b^2\,d\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^(9/2)*((2*a^2*d)/9 + (4*a*b*c)/9) + x^(13/2)*((2*b^2*c)/13 + (4*a*b*d)/13
) + (2*a^2*c*x^(5/2))/5 + (2*b^2*d*x^(17/2))/17
```

3.394 $\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2cx^{3/2}+\frac{2}{7}a(2bc+ad)x^{7/2}+\frac{2}{11}b(bc+2ad)x^{11/2}+\frac{2}{15}b^2dx^{15/2}$$

```
[Out] 2/3*a^2*c*x^(3/2)+2/7*a*(a*d+2*b*c)*x^(7/2)+2/11*b*(2*a*d+b*c)*x^(11/2)+2/1
5*b^2*d*x^(15/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\frac{2}{3}a^2cx^{3/2}+\frac{2}{11}bx^{11/2}(2ad+bc)+\frac{2}{7}ax^{7/2}(ad+2bc)+\frac{2}{15}b^2dx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(3/2))/3 + (2*a*(2*b*c + a*d)*x^(7/2))/7 + (2*b*(b*c + 2*a*d)*x^
(11/2))/11 + (2*b^2*d*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx &= \int \left(a^2c\sqrt{x}+a(2bc+ad)x^{5/2}+b(bc+2ad)x^{9/2}+b^2dx^{13/2}\right)dx\\ &= \frac{2}{3}a^2cx^{3/2}+\frac{2}{7}a(2bc+ad)x^{7/2}+\frac{2}{11}b(bc+2ad)x^{11/2}+\frac{2}{15}b^2dx^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.94

$$\frac{2x^{3/2}\left(385a^2c+330abcx^2+165a^2dx^2+105b^2cx^4+210abdx^4+77b^2dx^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(3/2)*(385*a^2*c + 330*a*b*c*x^2 + 165*a^2*d*x^2 + 105*b^2*c*x^4 + 210
*a*b*d*x^4 + 77*b^2*d*x^6))/1155
```

Maple [A]
time = 0.09, size = 52, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2b^2 d\, x^{\frac{15}{2}}}{15}+\frac{2(2abd+b^2c)\, x^{\frac{11}{2}}}{11}+\frac{2(a^2 d+2abc)\, x^{\frac{7}{2}}}{7}+\frac{2a^2 c\, x^{\frac{3}{2}}}{3}$	52
default	$\frac{2b^2 d\, x^{\frac{15}{2}}}{15}+\frac{2(2abd+b^2c)\, x^{\frac{11}{2}}}{11}+\frac{2(a^2 d+2abc)\, x^{\frac{7}{2}}}{7}+\frac{2a^2 c\, x^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}\left(77b^2 d\, x^6+210abd\, x^4+105b^2 c\, x^4+165a^2 d\, x^2+330abc\, x^2+385a^2 c\right)}{1155}$	56
trager	$\frac{2x^{\frac{3}{2}}\left(77b^2 d\, x^6+210abd\, x^4+105b^2 c\, x^4+165a^2 d\, x^2+330abc\, x^2+385a^2 c\right)}{1155}$	56
risch	$\frac{2x^{\frac{3}{2}}\left(77b^2 d\, x^6+210abd\, x^4+105b^2 c\, x^4+165a^2 d\, x^2+330abc\, x^2+385a^2 c\right)}{1155}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*b^2*d*x^(15/2)+2/11*(2*a*b*d+b^2*c)*x^(11/2)+2/7*(a^2*d+2*a*b*c)*x^(7/
2)+2/3*a^2*c*x^(3/2)
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.81

$$\frac{2}{15}\,b^2dx^{\frac{15}{2}}+\frac{2}{11}\left(b^2c+2\,abd\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^2cx^{\frac{3}{2}}+\frac{2}{7}\left(2\,abc+a^2d\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d*x^(15/2) + 2/11*(b^2*c + 2*a*b*d)*x^(11/2) + 2/3*a^2*c*x^(3/2) +
 2/7*(2*a*b*c + a^2*d)*x^(7/2)
```

Fricas [A]
time = 0.98, size = 54, normalized size = 0.86

$$\frac{2}{1155}\left(77\,b^2dx^7+105\left(b^2c+2\,abd\right)x^5+385\,a^2cx+165\left(2\,abc+a^2d\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d*x^7 + 105*(b^2*c + 2*a*b*d)*x^5 + 385*a^2*c*x + 165*(2*a*b
*c + a^2*d)*x^3)*sqrt(x)
```

Sympy [A]
time = 1.35, size = 66, normalized size = 1.05

$$\frac{2a^2cx^{\frac{3}{2}}}{3} + \frac{2b^2dx^{\frac{15}{2}}}{15} + \frac{2x^{\frac{11}{2}}\cdot(2abd+b^2c)}{11} + \frac{2x^{\frac{7}{2}}(a^2d+2abc)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)*x**(1/2),x)
```

```
[Out] 2*a**2*c*x**(3/2)/3 + 2*b**2*d*x**(15/2)/15 + 2*x**(11/2)*(2*a*b*d + b**2*c
)/11 + 2*x**(7/2)*(a**2*d + 2*a*b*c)/7
```

Giac [A]
time = 1.19, size = 53, normalized size = 0.84

$$\frac{2}{15}b^2dx^{\frac{15}{2}} + \frac{2}{11}b^2cx^{\frac{11}{2}} + \frac{4}{11}abdx^{\frac{11}{2}} + \frac{4}{7}abcx^{\frac{7}{2}} + \frac{2}{7}a^2dx^{\frac{7}{2}} + \frac{2}{3}a^2cx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d*x^(15/2) + 2/11*b^2*c*x^(11/2) + 4/11*a*b*d*x^(11/2) + 4/7*a*b*c
*x^(7/2) + 2/7*a^2*d*x^(7/2) + 2/3*a^2*c*x^(3/2)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.81

$$x^{7/2}\left(\frac{2\,d\,a^2}{7}+\frac{4\,b\,c\,a}{7}\right)+x^{11/2}\left(\frac{2\,c\,b^2}{11}+\frac{4\,a\,d\,b}{11}\right)+\frac{2\,a^2\,c\,x^{3/2}}{3}+\frac{2\,b^2\,d\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^(7/2)*((2*a^2*d)/7 + (4*a*b*c)/7) + x^(11/2)*((2*b^2*c)/11 + (4*a*b*d)/11
) + (2*a^2*c*x^(3/2))/3 + (2*b^2*d*x^(15/2))/15
```

3.395 $\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$2a^2c\sqrt{x}+\frac{2}{5}a(2bc+ad)x^{5/2}+\frac{2}{9}b(bc+2ad)x^{9/2}+\frac{2}{13}b^2dx^{13/2}$$

```
[Out] 2/5*a*(a*d+2*b*c)*x^(5/2)+2/9*b*(2*a*d+b*c)*x^(9/2)+2/13*b^2*d*x^(13/2)+2*a
^2*c*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$2a^2c\sqrt{x}+\frac{2}{9}bx^{9/2}(2ad+bc)+\frac{2}{5}ax^{5/2}(ad+2bc)+\frac{2}{13}b^2dx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^2*c*Sqrt[x] + (2*a*(2*b*c + a*d)*x^(5/2))/5 + (2*b*(b*c + 2*a*d)*x^(9/2
))/9 + (2*b^2*d*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx &= \int \left(\frac{a^2c}{\sqrt{x}}+a(2bc+ad)x^{3/2}+b(bc+2ad)x^{7/2}+b^2dx^{11/2}\right)dx\\ &= 2a^2c\sqrt{x}+\frac{2}{5}a(2bc+ad)x^{5/2}+\frac{2}{9}b(bc+2ad)x^{9/2}+\frac{2}{13}b^2dx^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.97

$$\frac{2}{585}\sqrt{x}\left(117a^2(5c+dx^2)+26abx^2(9c+5dx^2)+5b^2x^4(13c+9dx^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(117*a^2*(5*c + d*x^2) + 26*a*b*x^2*(9*c + 5*d*x^2) + 5*b^2*x^4*
(13*c + 9*d*x^2)))/585
```

Maple [A]
time = 0.09, size = 52, normalized size = 0.85

method	result	size
derivativedivides	$\frac{2b^2d\,x^{\frac{13}{2}}}{13}+\frac{2(2abd+b^2c)x^{\frac{9}{2}}}{9}+\frac{2(a^2d+2abc)x^{\frac{5}{2}}}{5}+2a^2c\sqrt{x}$	52
default	$\frac{2b^2d\,x^{\frac{13}{2}}}{13}+\frac{2(2abd+b^2c)x^{\frac{9}{2}}}{9}+\frac{2(a^2d+2abc)x^{\frac{5}{2}}}{5}+2a^2c\sqrt{x}$	52
trager	$\left(\frac{2}{13}b^2d\,x^6+\frac{4}{9}abd\,x^4+\frac{2}{9}b^2c\,x^4+\frac{2}{5}a^2d\,x^2+\frac{4}{5}abc\,x^2+2a^2c\right)\sqrt{x}$	55
gosper	$\frac{2\sqrt{x}\,(45b^2d\,x^6+130abd\,x^4+65b^2c\,x^4+117a^2d\,x^2+234abc\,x^2+585a^2c)}{585}$	56
risch	$\frac{2\sqrt{x}\,(45b^2d\,x^6+130abd\,x^4+65b^2c\,x^4+117a^2d\,x^2+234abc\,x^2+585a^2c)}{585}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*b^2*d*x^(13/2)+2/9*(2*a*b*d+b^2*c)*x^(9/2)+2/5*(a^2*d+2*a*b*c)*x^(5/2)
+2*a^2*c*x^(1/2)
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.84

$$\frac{2}{13}\,b^2dx^{\frac{13}{2}}+\frac{2}{9}\left(b^2c+2\,abd\right)x^{\frac{9}{2}}+2\,a^2c\sqrt{x}+\frac{2}{5}\left(2\,abc+a^2d\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*b^2*d*x^(13/2) + 2/9*(b^2*c + 2*a*b*d)*x^(9/2) + 2*a^2*c*sqrt(x) + 2/5
*(2*a*b*c + a^2*d)*x^(5/2)
```

Fricas [A]
time = 1.10, size = 53, normalized size = 0.87

$$\frac{2}{585}\left(45\,b^2dx^6+65\left(b^2c+2\,abd\right)x^4+585\,a^2c+117\left(2\,abc+a^2d\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d*x^6 + 65*(b^2*c + 2*a*b*d)*x^4 + 585*a^2*c + 117*(2*a*b*c +
 a^2*d)*x^2)*sqrt(x)
```

Sympy [A]
time = 0.28, size = 78, normalized size = 1.28

$$2a^2c\sqrt{x} + \frac{2a^2dx^{\frac{5}{2}}}{5} + \frac{4abcx^{\frac{5}{2}}}{5} + \frac{4abdx^{\frac{9}{2}}}{9} + \frac{2b^2cx^{\frac{9}{2}}}{9} + \frac{2b^2dx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(1/2),x)
```

```
[Out] 2*a**2*c*sqrt(x) + 2*a**2*d*x**(5/2)/5 + 4*a*b*c*x**(5/2)/5 + 4*a*b*d*x**(9
/2)/9 + 2*b**2*c*x**(9/2)/9 + 2*b**2*d*x**(13/2)/13
```

Giac [A]
time = 1.17, size = 53, normalized size = 0.87

$$\frac{2}{13}\,b^2dx^{\frac{13}{2}} + \frac{2}{9}\,b^2cx^{\frac{9}{2}} + \frac{4}{9}\,abdx^{\frac{9}{2}} + \frac{4}{5}\,abcx^{\frac{5}{2}} + \frac{2}{5}\,a^2dx^{\frac{5}{2}} + 2\,a^2c\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*b^2*d*x^(13/2) + 2/9*b^2*c*x^(9/2) + 4/9*a*b*d*x^(9/2) + 4/5*a*b*c*x^(
5/2) + 2/5*a^2*d*x^(5/2) + 2*a^2*c*sqrt(x)
```

Mupad [B]
time = 0.02, size = 51, normalized size = 0.84

$$x^{5/2}\,\left(\frac{2\,d\,a^2}{5}+\frac{4\,b\,c\,a}{5}\right)+x^{9/2}\,\left(\frac{2\,c\,b^2}{9}+\frac{4\,a\,d\,b}{9}\right)+2\,a^2\,c\,\sqrt{x}+\frac{2\,b^2\,d\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^(1/2),x)
```

```
[Out] x^(5/2)*((2*a^2*d)/5 + (4*a*b*c)/5) + x^(9/2)*((2*b^2*c)/9 + (4*a*b*d)/9) +
 2*a^2*c*x^(1/2) + (2*b^2*d*x^(13/2))/13
```

3.396 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2}{3}a(2bc+ad)x^{3/2}+\frac{2}{7}b(bc+2ad)x^{7/2}+\frac{2}{11}b^2dx^{11/2}$$

```
[Out] 2/3*a*(a*d+2*b*c)*x^(3/2)+2/7*b*(2*a*d+b*c)*x^(7/2)+2/11*b^2*d*x^(11/2)-2*a
^2*c/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2ad+bc)+\frac{2}{3}ax^{3/2}(ad+2bc)+\frac{2}{11}b^2dx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^2*c)/Sqrt[x] + (2*a*(2*b*c + a*d)*x^(3/2))/3 + (2*b*(b*c + 2*a*d)*x^(
7/2))/7 + (2*b^2*d*x^(11/2))/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx &= \int \left(\frac{a^2c}{x^{3/2}}+a(2bc+ad)\sqrt{x}+b(bc+2ad)x^{5/2}+b^2dx^{9/2}\right)dx\\ &= -\frac{2a^2c}{\sqrt{x}}+\frac{2}{3}a(2bc+ad)x^{3/2}+\frac{2}{7}b(bc+2ad)x^{7/2}+\frac{2}{11}b^2dx^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.97

$$-\frac{2(231a^2c-154abcx^2-77a^2dx^2-33b^2cx^4-66abdx^4-21b^2dx^6)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

```
[Out] (-2*(231*a^2*c - 154*a*b*c*x^2 - 77*a^2*d*x^2 - 33*b^2*c*x^4 - 66*a*b*d*x^4
 - 21*b^2*d*x^6))/(231*Sqrt[x])
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2b^2dx^{\frac{11}{2}}}{11}+\frac{4abdx^{\frac{7}{2}}}{7}+\frac{2b^2cx^{\frac{7}{2}}}{7}+\frac{2a^2dx^{\frac{3}{2}}}{3}+\frac{4abcx^{\frac{3}{2}}}{3}-\frac{2a^2c}{\sqrt{x}}$	54
default	$\frac{2b^2dx^{\frac{11}{2}}}{11}+\frac{4abdx^{\frac{7}{2}}}{7}+\frac{2b^2cx^{\frac{7}{2}}}{7}+\frac{2a^2dx^{\frac{3}{2}}}{3}+\frac{4abcx^{\frac{3}{2}}}{3}-\frac{2a^2c}{\sqrt{x}}$	54
gosper	$-\frac{2(-21b^2dx^6-66abdx^4-33b^2cx^4-77a^2dx^2-154abcx^2+231a^2c)}{231\sqrt{x}}$	56
trager	$-\frac{2(-21b^2dx^6-66abdx^4-33b^2cx^4-77a^2dx^2-154abcx^2+231a^2c)}{231\sqrt{x}}$	56
risch	$-\frac{2(-21b^2dx^6-66abdx^4-33b^2cx^4-77a^2dx^2-154abcx^2+231a^2c)}{231\sqrt{x}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*b^2*d*x^(11/2)+4/7*a*b*d*x^(7/2)+2/7*b^2*c*x^(7/2)+2/3*a^2*d*x^(3/2)+4
/3*a*b*c*x^(3/2)-2*a^2*c/x^(1/2)
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.84

$$\frac{2}{11}\,b^2dx^{\frac{11}{2}}+\frac{2}{7}\left(b^2c+2\,abd\right)x^{\frac{7}{2}}-\frac{2\,a^2c}{\sqrt{x}}+\frac{2}{3}\left(2\,abc+a^2d\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*d*x^(11/2) + 2/7*(b^2*c + 2*a*b*d)*x^(7/2) - 2*a^2*c/sqrt(x) + 2/3
*(2*a*b*c + a^2*d)*x^(3/2)
```

Fricas [A]
time = 1.54, size = 53, normalized size = 0.87

$$\frac{2\,(21\,b^2dx^6+33\,(b^2c+2\,abd)x^4-231\,a^2c+77\,(2\,abc+a^2d)x^2)}{231\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*d*x^6 + 33*(b^2*c + 2*a*b*d)*x^4 - 231*a^2*c + 77*(2*a*b*c +
a^2*d)*x^2)/sqrt(x)
```

Sympy [A]
time = 0.46, size = 78, normalized size = 1.28

$$-\frac{2a^2c}{\sqrt{x}} + \frac{2a^2dx^{\frac{3}{2}}}{3} + \frac{4abcx^{\frac{3}{2}}}{3} + \frac{4abdx^{\frac{7}{2}}}{7} + \frac{2b^2cx^{\frac{7}{2}}}{7} + \frac{2b^2dx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(3/2),x)
```

```
[Out] -2*a**2*c/sqrt(x) + 2*a**2*d*x**(3/2)/3 + 4*a*b*c*x**(3/2)/3 + 4*a*b*d*x**(
7/2)/7 + 2*b**2*c*x**(7/2)/7 + 2*b**2*d*x**(11/2)/11
```

Giac [A]
time = 1.22, size = 53, normalized size = 0.87

$$\frac{2}{11}\,b^2dx^{\frac{11}{2}} + \frac{2}{7}\,b^2cx^{\frac{7}{2}} + \frac{4}{7}\,abdx^{\frac{7}{2}} + \frac{4}{3}\,abcx^{\frac{3}{2}} + \frac{2}{3}\,a^2dx^{\frac{3}{2}} - \frac{2\,a^2c}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*d*x^(11/2) + 2/7*b^2*c*x^(7/2) + 4/7*a*b*d*x^(7/2) + 4/3*a*b*c*x^(
3/2) + 2/3*a^2*d*x^(3/2) - 2*a^2*c/sqrt(x)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.84

$$x^{3/2}\left(\frac{2\,d\,a^2}{3}+\frac{4\,b\,c\,a}{3}\right)+x^{7/2}\left(\frac{2\,c\,b^2}{7}+\frac{4\,a\,d\,b}{7}\right)-\frac{2\,a^2\,c}{\sqrt{x}}+\frac{2\,b^2\,d\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x)
```

```
[Out] x^(3/2)*((2*a^2*d)/3 + (4*a*b*c)/3) + x^(7/2)*((2*b^2*c)/7 + (4*a*b*d)/7) -
 (2*a^2*c)/x^(1/2) + (2*b^2*d*x^(11/2))/11
```

3.397 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{3x^{3/2}}+2a(2bc+ad)\sqrt{x}+\frac{2}{5}b(bc+2ad)x^{5/2}+\frac{2}{9}b^2dx^{9/2}$$

```
[Out] -2/3*a^2*c/x^(3/2)+2/5*b*(2*a*d+b*c)*x^(5/2)+2/9*b^2*d*x^(9/2)+2*a*(a*d+2*b
*c)*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$-\frac{2a^2c}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2ad+bc)+2a\sqrt{x}(ad+2bc)+\frac{2}{9}b^2dx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^2*c)/(3*x^(3/2)) + 2*a*(2*b*c + a*d)*Sqrt[x] + (2*b*(b*c + 2*a*d)*x^(
5/2))/5 + (2*b^2*d*x^(9/2))/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx&=\int\left(\frac{a^2c}{x^{5/2}}+\frac{a(2bc+ad)}{\sqrt{x}}+b(bc+2ad)x^{3/2}+b^2dx^{7/2}\right)dx\\&=-\frac{2a^2c}{3x^{3/2}}+2a(2bc+ad)\sqrt{x}+\frac{2}{5}b(bc+2ad)x^{5/2}+\frac{2}{9}b^2dx^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.97

$$-\frac{2(15a^2c-90abcx^2-45a^2dx^2-9b^2cx^4-18abdx^4-5b^2dx^6)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

```
[Out] (-2*(15*a^2*c - 90*a*b*c*x^2 - 45*a^2*d*x^2 - 9*b^2*c*x^4 - 18*a*b*d*x^4 -
5*b^2*d*x^6))/(45*x^(3/2))
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2b^2dx^{\frac{9}{2}}}{9}+\frac{4abdx^{\frac{5}{2}}}{5}+\frac{2b^2cx^{\frac{5}{2}}}{5}+2a^2d\sqrt{x}+4abc\sqrt{x}-\frac{2a^2c}{3x^{\frac{3}{2}}}$	54
default	$\frac{2b^2dx^{\frac{9}{2}}}{9}+\frac{4abdx^{\frac{5}{2}}}{5}+\frac{2b^2cx^{\frac{5}{2}}}{5}+2a^2d\sqrt{x}+4abc\sqrt{x}-\frac{2a^2c}{3x^{\frac{3}{2}}}$	54
gosper	$-\frac{2(-5b^2dx^6-18abdx^4-9b^2cx^4-45a^2dx^2-90abcx^2+15a^2c)}{45x^{\frac{3}{2}}}$	56
trager	$-\frac{2(-5b^2dx^6-18abdx^4-9b^2cx^4-45a^2dx^2-90abcx^2+15a^2c)}{45x^{\frac{3}{2}}}$	56
risch	$-\frac{2(-5b^2dx^6-18abdx^4-9b^2cx^4-45a^2dx^2-90abcx^2+15a^2c)}{45x^{\frac{3}{2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*b^2*d*x^(9/2)+4/5*a*b*d*x^(5/2)+2/5*b^2*c*x^(5/2)+2*a^2*d*x^(1/2)+4*a*b
*c*x^(1/2)-2/3*a^2*c/x^(3/2)
```

Maxima [A]
time = 0.30, size = 51, normalized size = 0.84

$$\frac{2}{9}\,b^2dx^{\frac{9}{2}}+\frac{2}{5}\,\left(b^2c+2\,abd\right)x^{\frac{5}{2}}-\frac{2\,a^2c}{3\,x^{\frac{3}{2}}}+2\,\left(2\,abc+a^2d\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^2*d*x^(9/2) + 2/5*(b^2*c + 2*a*b*d)*x^(5/2) - 2/3*a^2*c/x^(3/2) + 2*(
2*a*b*c + a^2*d)*sqrt(x)
```

Fricas [A]
time = 1.55, size = 53, normalized size = 0.87

$$\frac{2\,(5\,b^2dx^6+9\,(b^2c+2\,abd)x^4-15\,a^2c+45\,(2\,abc+a^2d)x^2)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*d*x^6 + 9*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 45*(2*a*b*c + a^2*
d)*x^2)/x^(3/2)
```

Sympy [A]
time = 0.54, size = 76, normalized size = 1.25

$$-\frac{2a^2c}{3x^{\frac{3}{2}}}+2a^2d\sqrt{x}+4abc\sqrt{x}+\frac{4abdx^{\frac{5}{2}}}{5}+\frac{2b^2cx^{\frac{5}{2}}}{5}+\frac{2b^2dx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(5/2),x)
```

```
[Out] -2*a**2*c/(3*x**(3/2)) + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt(x) + 4*a*b*d*x**(5
/2)/5 + 2*b**2*c*x**(5/2)/5 + 2*b**2*d*x**(9/2)/9
```

Giac [A]
time = 0.92, size = 53, normalized size = 0.87

$$\frac{2}{9}\,b^2dx^{\frac{9}{2}}+\frac{2}{5}\,b^2cx^{\frac{5}{2}}+\frac{4}{5}\,abdx^{\frac{5}{2}}+4\,abc\sqrt{x}+2\,a^2d\sqrt{x}-\frac{2\,a^2c}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*b^2*d*x^(9/2) + 2/5*b^2*c*x^(5/2) + 4/5*a*b*d*x^(5/2) + 4*a*b*c*sqrt(x)
 + 2*a^2*d*sqrt(x) - 2/3*a^2*c/x^(3/2)
```

Mupad [B]
time = 0.03, size = 51, normalized size = 0.84

$$\sqrt{x}\,\left(2\,d\,a^2+4\,b\,c\,a\right)+x^{5/2}\,\left(\frac{2\,c\,b^2}{5}+\frac{4\,a\,d\,b}{5}\right)-\frac{2\,a^2\,c}{3\,x^{3/2}}+\frac{2\,b^2\,d\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x)
```

```
[Out] x^(1/2)*(2*a^2*d + 4*a*b*c) + x^(5/2)*((2*b^2*c)/5 + (4*a*b*d)/5) - (2*a^2*
c)/(3*x^(3/2)) + (2*b^2*d*x^(9/2))/9
```

3.398 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{5x^{5/2}}-\frac{2a(2bc+ad)}{\sqrt{x}}+\frac{2}{3}b(bc+2ad)x^{3/2}+\frac{2}{7}b^2dx^{7/2}$$

```
[Out] -2/5*a^2*c/x^(5/2)+2/3*b*(2*a*d+b*c)*x^(3/2)+2/7*b^2*d*x^(7/2)-2*a*(a*d+2*b
*c)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$-\frac{2a^2c}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2ad+bc)-\frac{2a(ad+2bc)}{\sqrt{x}}+\frac{2}{7}b^2dx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^2*c)/(5*x^(5/2)) - (2*a*(2*b*c + a*d))/Sqrt[x] + (2*b*(b*c + 2*a*d)*x
^(3/2))/3 + (2*b^2*d*x^(7/2))/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx &= \int \left(\frac{a^2c}{x^{7/2}}+\frac{a(2bc+ad)}{x^{3/2}}+b(bc+2ad)\sqrt{x}+b^2dx^{5/2}\right)dx\\ &= -\frac{2a^2c}{5x^{5/2}}-\frac{2a(2bc+ad)}{\sqrt{x}}+\frac{2}{3}b(bc+2ad)x^{3/2}+\frac{2}{7}b^2dx^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.97

$$-\frac{2(21a^2c+210abcx^2+105a^2dx^2-35b^2cx^4-70abdx^4-15b^2dx^6)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

```
[Out] (-2*(21*a^2*c + 210*a*b*c*x^2 + 105*a^2*d*x^2 - 35*b^2*c*x^4 - 70*a*b*d*x^4
 - 15*b^2*d*x^6))/(105*x^(5/2))
```

Maple [A]
time = 0.08, size = 51, normalized size = 0.84

method	result	size
derivativedivides	$\frac{2b^{2}d\,x^{\frac{7}{2}}}{7}+\frac{4abd\,x^{\frac{3}{2}}}{3}+\frac{2b^{2}c\,x^{\frac{3}{2}}}{3}-\frac{2a^{2}c}{5x^{\frac{5}{2}}}-\frac{2a(ad+2bc)}{\sqrt{x}}$	51
default	$\frac{2b^{2}d\,x^{\frac{7}{2}}}{7}+\frac{4abd\,x^{\frac{3}{2}}}{3}+\frac{2b^{2}c\,x^{\frac{3}{2}}}{3}-\frac{2a^{2}c}{5x^{\frac{5}{2}}}-\frac{2a(ad+2bc)}{\sqrt{x}}$	51
gosper	$-\frac{2(-15b^{2}d\,x^{6}-70abd\,x^{4}-35b^{2}c\,x^{4}+105a^{2}d\,x^{2}+210abc\,x^{2}+21a^{2}c)}{105x^{\frac{5}{2}}}$	56
trager	$-\frac{2(-15b^{2}d\,x^{6}-70abd\,x^{4}-35b^{2}c\,x^{4}+105a^{2}d\,x^{2}+210abc\,x^{2}+21a^{2}c)}{105x^{\frac{5}{2}}}$	56
risch	$-\frac{2(-15b^{2}d\,x^{6}-70abd\,x^{4}-35b^{2}c\,x^{4}+105a^{2}d\,x^{2}+210abc\,x^{2}+21a^{2}c)}{105x^{\frac{5}{2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*b^2*d*x^(7/2)+4/3*a*b*d*x^(3/2)+2/3*b^2*c*x^(3/2)-2/5*a^2*c/x^(5/2)-2*a
*(a*d+2*b*c)/x^(1/2)
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.87

$$\frac{2}{7}\,b^{2}dx^{\frac{7}{2}}+\frac{2}{3}\,\left(b^{2}c+2\,abd\right)x^{\frac{3}{2}}-\frac{2\,\left(a^{2}c+5\,\left(2\,abc+a^{2}d\right)x^{2}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^2*d*x^(7/2) + 2/3*(b^2*c + 2*a*b*d)*x^(3/2) - 2/5*(a^2*c + 5*(2*a*b*c
 + a^2*d)*x^2)/x^(5/2)
```

Fricas [A]
time = 1.35, size = 53, normalized size = 0.87

$$\frac{2\,\left(15\,b^{2}dx^{6}+35\,\left(b^{2}c+2\,abd\right)x^{4}-21\,a^{2}c-105\,\left(2\,abc+a^{2}d\right)x^{2}\right)}{105\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*d*x^6 + 35*(b^2*c + 2*a*b*d)*x^4 - 21*a^2*c - 105*(2*a*b*c +
a^2*d)*x^2)/x^(5/2)
```

Sympy [A]
time = 0.57, size = 76, normalized size = 1.25

$$-\frac{2a^2c}{5x^{\frac{5}{2}}}-\frac{2a^2d}{\sqrt{x}}-\frac{4abc}{\sqrt{x}}+\frac{4abdx^{\frac{3}{2}}}{3}+\frac{2b^2cx^{\frac{3}{2}}}{3}+\frac{2b^2dx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(7/2),x)
```

```
[Out] -2*a**2*c/(5*x**(5/2)) - 2*a**2*d/sqrt(x) - 4*a*b*c/sqrt(x) + 4*a*b*d*x**(3
/2)/3 + 2*b**2*c*x**(3/2)/3 + 2*b**2*d*x**(7/2)/7
```

Giac [A]
time = 0.76, size = 55, normalized size = 0.90

$$\frac{2}{7}\,b^2dx^{\frac{7}{2}}+\frac{2}{3}\,b^2cx^{\frac{3}{2}}+\frac{4}{3}\,abdx^{\frac{3}{2}}-\frac{2\,(10\,abcx^2+5\,a^2dx^2+a^2c)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*d*x^(7/2) + 2/3*b^2*c*x^(3/2) + 4/3*a*b*d*x^(3/2) - 2/5*(10*a*b*c*x
^2 + 5*a^2*d*x^2 + a^2*c)/x^(5/2)
```

Mupad [B]
time = 0.03, size = 55, normalized size = 0.90

$$-\frac{210\,d\,a^2\,x^2+42\,c\,a^2-140\,d\,a\,b\,x^4+420\,c\,a\,b\,x^2-30\,d\,b^2\,x^6-70\,c\,b^2\,x^4}{105\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x)
```

```
[Out] -(42*a^2*c + 210*a^2*d*x^2 - 70*b^2*c*x^4 - 30*b^2*d*x^6 + 420*a*b*c*x^2 -
140*a*b*d*x^4)/(105*x^(5/2))
```

3.399 $\int x^{7/2}(a+bx^2)^2(c+dx^2)^2\,dx$

Optimal. Leaf size=97

$$\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{13}ac(bc+ad)x^{13/2}+\frac{2}{17}\left(b^2c^2+4abcd+a^2d^2\right)x^{17/2}+\frac{4}{21}bd(bc+ad)x^{21/2}+\frac{2}{25}b^2d^2x^{25/2}$$

```
[Out] 2/9*a^2*c^2*x^(9/2)+4/13*a*c*(a*d+b*c)*x^(13/2)+2/17*(a^2*d^2+4*a*b*c*d+b^2
*c^2)*x^(17/2)+4/21*b*d*(a*d+b*c)*x^(21/2)+2/25*b^2*d^2*x^(25/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {459}

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(21/2))/21 + (2*b^2*d
^2*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2x^{7/2}+2ac(bc+ad)x^{11/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{15/2}+2bd(bc+\right.\\ &= \frac{2}{9}a^2c^2x^{9/2}+\frac{4}{13}ac(bc+ad)x^{13/2}+\frac{2}{17}\left(b^2c^2+4abcd+a^2d^2\right)x^{17/2}+\frac{4}{21}bd(\end{aligned}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.96

$$\frac{2x^{9/2}\left(175a^2\left(221c^2+306cdx^2+117d^2x^4\right)+150abx^2\left(357c^2+546cdx^2+221d^2x^4\right)+39b^2x^4\left(525c^2+850cdx^2+357d^2x^4\right)\right)}{348075}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]

[Out] (2*x^(9/2)*(175*a^2*(221*c^2 + 306*c*d*x^2 + 117*d^2*x^4) + 150*a*b*x^2*(357*c^2 + 546*c*d*x^2 + 221*d^2*x^4) + 39*b^2*x^4*(525*c^2 + 850*c*d*x^2 + 357*d^2*x^4)))/348075

Maple [A]
time = 0.11, size = 90, normalized size = 0.93

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{25}{2}}}{25}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{21}{2}}}{21}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{17}{2}}}{17}+\frac{2(2a^2cd+2ab\,c^2)x^{\frac{13}{2}}}{13}+\frac{2a^2c^2x^{\frac{9}{2}}}{9}$
default	$\frac{2b^2d^2x^{\frac{25}{2}}}{25}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{21}{2}}}{21}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{17}{2}}}{17}+\frac{2(2a^2cd+2ab\,c^2)x^{\frac{13}{2}}}{13}+\frac{2a^2c^2x^{\frac{9}{2}}}{9}$
gosper	$\frac{2x^{\frac{9}{2}}(13923b^2d^2x^8+33150ab\,d^2x^6+33150b^2cd\,x^6+20475a^2d^2x^4+81900abcd\,x^4+20475b^2c^2x^4+53550a^2cd\,x^2+53550ab}{348075}$
trager	$\frac{2x^{\frac{9}{2}}(13923b^2d^2x^8+33150ab\,d^2x^6+33150b^2cd\,x^6+20475a^2d^2x^4+81900abcd\,x^4+20475b^2c^2x^4+53550a^2cd\,x^2+53550ab}{348075}$
risch	$\frac{2x^{\frac{9}{2}}(13923b^2d^2x^8+33150ab\,d^2x^6+33150b^2cd\,x^6+20475a^2d^2x^4+81900abcd\,x^4+20475b^2c^2x^4+53550a^2cd\,x^2+53550ab}{348075}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] 2/25*b^2*d^2*x^(25/2)+2/21*(2*a*b*d^2+2*b^2*c*d)*x^(21/2)+2/17*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(17/2)+2/13*(2*a^2*c*d+2*a*b*c^2)*x^(13/2)+2/9*a^2*c^2*x^(9/2)

Maxima [A]
time = 0.32, size = 85, normalized size = 0.88

$$\frac{2}{25}\,b^2d^2x^{\frac{25}{2}}+\frac{4}{21}\left(b^2cd+abd^2\right)x^{\frac{21}{2}}+\frac{2}{17}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,a^2c^2x^{\frac{9}{2}}+\frac{4}{13}\left(abc^2+a^2cd\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")

[Out] 2/25*b^2*d^2*x^(25/2) + 4/21*(b^2*c*d + a*b*d^2)*x^(21/2) + 2/17*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(17/2) + 2/9*a^2*c^2*x^(9/2) + 4/13*(a*b*c^2 + a^2*c*d)*x^(13/2)

Fricas [A]
time = 1.00, size = 90, normalized size = 0.93

$$\frac{2}{348075}\left(13923\,b^2d^2x^{12}+33150\left(b^2cd+abd^2\right)x^{10}+20475\left(b^2c^2+4\,abcd+a^2d^2\right)x^8+38675\,a^2c^2x^4+53550\left(abc^2+a^2cd\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")

```
[Out] 2/348075*(13923*b^2*d^2*x^12 + 33150*(b^2*c*d + a*b*d^2)*x^10 + 20475*(b^2*
c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 38675*a^2*c^2*x^4 + 53550*(a*b*c^2 + a^2*c
*d)*x^6)*sqrt(x)
```

Sympy [A]
time = 1.67, size = 136, normalized size = 1.40

$$\frac{2a^2c^2x^{\frac{9}{2}}}{9}+\frac{4a^2cdx^{\frac{13}{2}}}{13}+\frac{2a^2d^2x^{\frac{17}{2}}}{17}+\frac{4abc^2x^{\frac{13}{2}}}{13}+\frac{8abcdx^{\frac{17}{2}}}{17}+\frac{4abd^2x^{\frac{21}{2}}}{21}+\frac{2b^2c^2x^{\frac{17}{2}}}{17}+\frac{4b^2cdx^{\frac{21}{2}}}{21}+\frac{2b^2d^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(9/2)/9 + 4*a**2*c*d*x**(13/2)/13 + 2*a**2*d**2*x**(17/2)/17
 + 4*a*b*c**2*x**(13/2)/13 + 8*a*b*c*d*x**(17/2)/17 + 4*a*b*d**2*x**(21/2)/
21 + 2*b**2*c**2*x**(17/2)/17 + 4*b**2*c*d*x**(21/2)/21 + 2*b**2*d**2*x**(2
5/2)/25
```

Giac [A]
time = 1.05, size = 94, normalized size = 0.97

$$\frac{2}{25}\,b^2d^2x^{\frac{25}{2}}+\frac{4}{21}\,b^2cdx^{\frac{21}{2}}+\frac{4}{21}\,abd^2x^{\frac{21}{2}}+\frac{2}{17}\,b^2c^2x^{\frac{17}{2}}+\frac{8}{17}\,abcdx^{\frac{17}{2}}+\frac{2}{17}\,a^2d^2x^{\frac{17}{2}}+\frac{4}{13}\,abc^2x^{\frac{13}{2}}+\frac{4}{13}\,a^2cdx^{\frac{13}{2}}+\frac{2}{9}\,a^2c^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/25*b^2*d^2*x^(25/2) + 4/21*b^2*c*d*x^(21/2) + 4/21*a*b*d^2*x^(21/2) + 2/1
7*b^2*c^2*x^(17/2) + 8/17*a*b*c*d*x^(17/2) + 2/17*a^2*d^2*x^(17/2) + 4/13*a
*b*c^2*x^(13/2) + 4/13*a^2*c*d*x^(13/2) + 2/9*a^2*c^2*x^(9/2)
```

Mupad [B]
time = 0.03, size = 78, normalized size = 0.80

$$x^{17/2}\left(\frac{2\,a^2\,d^2}{17}+\frac{8\,a\,b\,c\,d}{17}+\frac{2\,b^2\,c^2}{17}\right)+\frac{2\,a^2\,c^2\,x^{9/2}}{9}+\frac{2\,b^2\,d^2\,x^{25/2}}{25}+\frac{4\,a\,c\,x^{13/2}\,\left(a\,d+b\,c\right)}{13}+\frac{4\,b\,d\,x^{21/2}\,\left(a\,d+b\,c\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^(17/2)*((2*a^2*d^2)/17 + (2*b^2*c^2)/17 + (8*a*b*c*d)/17) + (2*a^2*c^2*x^
(9/2))/9 + (2*b^2*d^2*x^(25/2))/25 + (4*a*c*x^(13/2)*(a*d + b*c))/13 + (4*b
*d*x^(21/2)*(a*d + b*c))/21
```

3.400 $\int x^{5/2}(a+bx^2)^2 (c+dx^2)^2\, dx$

Optimal. Leaf size=97

$$\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{11}ac(bc+ad)x^{11/2}+\frac{2}{15}\left(b^2c^2+4abcd+a^2d^2\right)x^{15/2}+\frac{4}{19}bd(bc+ad)x^{19/2}+\frac{2}{23}b^2d^2x^{23/2}$$

```
[Out] 2/7*a^2*c^2*x^(7/2)+4/11*a*c*(a*d+b*c)*x^(11/2)+2/15*(a^2*d^2+4*a*b*c*d+b^2
*c^2)*x^(15/2)+4/19*b*d*(a*d+b*c)*x^(19/2)+2/23*b^2*d^2*x^(23/2)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {459}

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(19/2))/19 + (2*b^2*d
^2*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2x^{5/2}+2ac(bc+ad)x^{9/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{13/2}+2bd(bc\right.\\ &= \frac{2}{7}a^2c^2x^{7/2}+\frac{4}{11}ac(bc+ad)x^{11/2}+\frac{2}{15}\left(b^2c^2+4abcd+a^2d^2\right)x^{15/2}+\frac{4}{19}bc\end{aligned}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.96

$$\frac{2x^{7/2}(437a^2(165c^2+210cdx^2+77d^2x^4)+322abx^2(285c^2+418cdx^2+165d^2x^4)+77b^2x^4(437c^2+690cdx^2+285d^2x^4))}{504735}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*x^(7/2)*(437*a^2*(165*c^2 + 210*c*d*x^2 + 77*d^2*x^4) + 322*a*b*x^2*(285
*c^2 + 418*c*d*x^2 + 165*d^2*x^4) + 77*b^2*x^4*(437*c^2 + 690*c*d*x^2 + 285
*d^2*x^4)))/504735
```

Maple [A]
time = 0.10, size = 90, normalized size = 0.93

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{23}{2}}}{23}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{19}{2}}}{19}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{15}{2}}}{15}+\frac{2(2a^2cd+2ab\,c^2)x^{\frac{11}{2}}}{11}+\frac{2a^2c^2x^{\frac{7}{2}}}{7}$
default	$\frac{2b^2d^2x^{\frac{23}{2}}}{23}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{19}{2}}}{19}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{15}{2}}}{15}+\frac{2(2a^2cd+2ab\,c^2)x^{\frac{11}{2}}}{11}+\frac{2a^2c^2x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}}(21945b^2d^2x^8+53130ab\,d^2x^6+53130b^2cd\,x^6+33649a^2d^2x^4+134596abcd\,x^4+33649b^2c^2x^4+91770a^2cd\,x^2+91770ab\,c}{504735}$
trager	$\frac{2x^{\frac{7}{2}}(21945b^2d^2x^8+53130ab\,d^2x^6+53130b^2cd\,x^6+33649a^2d^2x^4+134596abcd\,x^4+33649b^2c^2x^4+91770a^2cd\,x^2+91770ab\,c}{504735}$
risch	$\frac{2x^{\frac{7}{2}}(21945b^2d^2x^8+53130ab\,d^2x^6+53130b^2cd\,x^6+33649a^2d^2x^4+134596abcd\,x^4+33649b^2c^2x^4+91770a^2cd\,x^2+91770ab\,c}{504735}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*b^2*d^2*x^(23/2)+2/19*(2*a*b*d^2+2*b^2*c*d)*x^(19/2)+2/15*(a^2*d^2+4*a
*b*c*d+b^2*c^2)*x^(15/2)+2/11*(2*a^2*c*d+2*a*b*c^2)*x^(11/2)+2/7*a^2*c^2*x^
(7/2)
```

Maxima [A]
time = 0.29, size = 85, normalized size = 0.88

$$\frac{2}{23}\,b^2d^2x^{\frac{23}{2}}+\frac{4}{19}\,(b^2cd+abd^2)x^{\frac{19}{2}}+\frac{2}{15}\,(b^2c^2+4\,abcd+a^2d^2)x^{\frac{15}{2}}+\frac{2}{7}\,a^2c^2x^{\frac{7}{2}}+\frac{4}{11}\,(abc^2+a^2cd)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 2/23*b^2*d^2*x^(23/2) + 4/19*(b^2*c*d + a*b*d^2)*x^(19/2) + 2/15*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(15/2) + 2/7*a^2*c^2*x^(7/2) + 4/11*(a*b*c^2 + a^2*
c*d)*x^(11/2)
```

Fricas [A]
time = 0.74, size = 90, normalized size = 0.93

$$\frac{2}{504735}\,(21945\,b^2d^2x^{11}+53130\,(b^2cd+abd^2)x^9+33649\,(b^2c^2+4\,abcd+a^2d^2)x^7+72105\,a^2c^2x^3+91770\,(abc^2+a^2cd)x^5)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*b^2*d^2*x^11 + 53130*(b^2*c*d + a*b*d^2)*x^9 + 33649*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^7 + 72105*a^2*c^2*x^3 + 91770*(a*b*c^2 + a^2*c*
d)*x^5)*sqrt(x)
```

Sympy [A]
time = 1.20, size = 136, normalized size = 1.40

$$\frac{2a^2c^2x^{\frac{7}{2}}}{7}+\frac{4a^2cdx^{\frac{11}{2}}}{11}+\frac{2a^2d^2x^{\frac{15}{2}}}{15}+\frac{4abc^2x^{\frac{11}{2}}}{11}+\frac{8abcdx^{\frac{15}{2}}}{15}+\frac{4abd^2x^{\frac{19}{2}}}{19}+\frac{2b^2c^2x^{\frac{15}{2}}}{15}+\frac{4b^2cdx^{\frac{19}{2}}}{19}+\frac{2b^2d^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(7/2)/7 + 4*a**2*c*d*x**(11/2)/11 + 2*a**2*d**2*x**(15/2)/15
 + 4*a*b*c**2*x**(11/2)/11 + 8*a*b*c*d*x**(15/2)/15 + 4*a*b*d**2*x**(19/2)/
19 + 2*b**2*c**2*x**(15/2)/15 + 4*b**2*c*d*x**(19/2)/19 + 2*b**2*d**2*x**(2
3/2)/23
```

Giac [A]
time = 0.76, size = 94, normalized size = 0.97

$$\frac{2}{23}\,b^2d^2x^{\frac{23}{2}}+\frac{4}{19}\,b^2cdx^{\frac{19}{2}}+\frac{4}{19}\,abd^2x^{\frac{19}{2}}+\frac{2}{15}\,b^2c^2x^{\frac{15}{2}}+\frac{8}{15}\,abcdx^{\frac{15}{2}}+\frac{2}{15}\,a^2d^2x^{\frac{15}{2}}+\frac{4}{11}\,abc^2x^{\frac{11}{2}}+\frac{4}{11}\,a^2cdx^{\frac{11}{2}}+\frac{2}{7}\,a^2c^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/23*b^2*d^2*x^(23/2) + 4/19*b^2*c*d*x^(19/2) + 4/19*a*b*d^2*x^(19/2) + 2/1
5*b^2*c^2*x^(15/2) + 8/15*a*b*c*d*x^(15/2) + 2/15*a^2*d^2*x^(15/2) + 4/11*a
*b*c^2*x^(11/2) + 4/11*a^2*c*d*x^(11/2) + 2/7*a^2*c^2*x^(7/2)
```

Mupad [B]
time = 0.02, size = 78, normalized size = 0.80

$$x^{15/2}\left(\frac{2\,a^2\,d^2}{15}+\frac{8\,a\,b\,c\,d}{15}+\frac{2\,b^2\,c^2}{15}\right)+\frac{2\,a^2\,c^2\,x^{7/2}}{7}+\frac{2\,b^2\,d^2\,x^{23/2}}{23}+\frac{4\,a\,c\,x^{11/2}\,\left(a\,d+b\,c\right)}{11}+\frac{4\,b\,d\,x^{19/2}\,\left(a\,d+b\,c\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^(15/2)*((2*a^2*d^2)/15 + (2*b^2*c^2)/15 + (8*a*b*c*d)/15) + (2*a^2*c^2*x^
(7/2))/7 + (2*b^2*d^2*x^(23/2))/23 + (4*a*c*x^(11/2)*(a*d + b*c))/11 + (4*b
*d*x^(19/2)*(a*d + b*c))/19
```

3.401 $\int x^{3/2}(a+bx^2)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=97

$$\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{9}ac(bc+ad)x^{9/2}+\frac{2}{13}\left(b^2c^2+4abcd+a^2d^2\right)x^{13/2}+\frac{4}{17}bd(bc+ad)x^{17/2}+\frac{2}{21}b^2d^2x^{21/2}$$

```
[Out] 2/5*a^2*c^2*x^(5/2)+4/9*a*c*(a*d+b*c)*x^(9/2)+2/13*(a^2*d^2+4*a*b*c*d+b^2*c
^2)*x^(13/2)+4/17*b*d*(a*d+b*c)*x^(17/2)+2/21*b^2*d^2*x^(21/2)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {459}

$$\frac{2}{13}x^{13/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{17}bdx^{17/2}(ad+bc)+\frac{4}{9}acx^{9/2}(ad+bc)+\frac{2}{21}b^2d^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17/2))/17 + (2*b^2*d^2
*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2x^{3/2}+2ac(bc+ad)x^{7/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{11/2}+2bd(bc+\right.\\ &= \frac{2}{5}a^2c^2x^{5/2}+\frac{4}{9}ac(bc+ad)x^{9/2}+\frac{2}{13}\left(b^2c^2+4abcd+a^2d^2\right)x^{13/2}+\frac{4}{17}bd(bc\end{aligned}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.96

$$\frac{2x^{5/2}\left(119a^2\left(117c^2+130cdx^2+45d^2x^4\right)+70abx^2\left(221c^2+306cdx^2+117d^2x^4\right)+15b^2x^4\left(357c^2+546cdx^2+221d^2x^4\right)\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*x^(5/2)*(119*a^2*(117*c^2 + 130*c*d*x^2 + 45*d^2*x^4) + 70*a*b*x^2*(221*
c^2 + 306*c*d*x^2 + 117*d^2*x^4) + 15*b^2*x^4*(357*c^2 + 546*c*d*x^2 + 221*
d^2*x^4)))/69615
```

Maple [A]
time = 0.10, size = 90, normalized size = 0.93

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{21}{2}}}{21}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{17}{2}}}{17}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{13}{2}}}{13}+\frac{2(2a^2cd+2ab\,c^2)x^{\frac{9}{2}}}{9}+\frac{2a^2c^2x^{\frac{5}{2}}}{5}$
default	$\frac{2b^2d^2x^{\frac{21}{2}}}{21}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{17}{2}}}{17}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{13}{2}}}{13}+\frac{2(2a^2cd+2ab\,c^2)x^{\frac{9}{2}}}{9}+\frac{2a^2c^2x^{\frac{5}{2}}}{5}$
gosper	$\frac{2x^{\frac{5}{2}}(3315b^2d^2x^8+8190ab\,d^2x^6+8190b^2cd\,x^6+5355a^2d^2x^4+21420abcd\,x^4+5355b^2c^2x^4+15470a^2cd\,x^2+15470ab\,c^2x^2+}{69615}$
trager	$\frac{2x^{\frac{5}{2}}(3315b^2d^2x^8+8190ab\,d^2x^6+8190b^2cd\,x^6+5355a^2d^2x^4+21420abcd\,x^4+5355b^2c^2x^4+15470a^2cd\,x^2+15470ab\,c^2x^2+}{69615}$
risch	$\frac{2x^{\frac{5}{2}}(3315b^2d^2x^8+8190ab\,d^2x^6+8190b^2cd\,x^6+5355a^2d^2x^4+21420abcd\,x^4+5355b^2c^2x^4+15470a^2cd\,x^2+15470ab\,c^2x^2+}{69615}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*b^2*d^2*x^(21/2)+2/17*(2*a*b*d^2+2*b^2*c*d)*x^(17/2)+2/13*(a^2*d^2+4*a
*b*c*d+b^2*c^2)*x^(13/2)+2/9*(2*a^2*c*d+2*a*b*c^2)*x^(9/2)+2/5*a^2*c^2*x^(5
/2)
```

Maxima [A]
time = 0.31, size = 85, normalized size = 0.88

$$\frac{2}{21}\,b^2d^2x^{\frac{21}{2}}+\frac{4}{17}\left(b^2cd+abd^2\right)x^{\frac{17}{2}}+\frac{2}{13}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{13}{2}}+\frac{2}{5}\,a^2c^2x^{\frac{5}{2}}+\frac{4}{9}\left(abc^2+a^2cd\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d^2*x^(21/2) + 4/17*(b^2*c*d + a*b*d^2)*x^(17/2) + 2/13*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(13/2) + 2/5*a^2*c^2*x^(5/2) + 4/9*(a*b*c^2 + a^2*c
*d)*x^(9/2)
```

Fricas [A]
time = 0.69, size = 90, normalized size = 0.93

$$\frac{2}{69615}\left(3315\,b^2d^2x^{10}+8190\left(b^2cd+abd^2\right)x^8+5355\left(b^2c^2+4\,abcd+a^2d^2\right)x^6+13923\,a^2c^2x^2+15470\left(abc^2+a^2cd\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^2*d^2*x^10 + 8190*(b^2*c*d + a*b*d^2)*x^8 + 5355*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^6 + 13923*a^2*c^2*x^2 + 15470*(a*b*c^2 + a^2*c*d)*x
^4)*sqrt(x)
```

Sympy [A]
time = 0.76, size = 136, normalized size = 1.40

$$\frac{2a^2c^2x^{\frac{5}{2}}}{5}+\frac{4a^2cdx^{\frac{9}{2}}}{9}+\frac{2a^2d^2x^{\frac{13}{2}}}{13}+\frac{4abc^2x^{\frac{9}{2}}}{9}+\frac{8abcdx^{\frac{13}{2}}}{13}+\frac{4abd^2x^{\frac{17}{2}}}{17}+\frac{2b^2c^2x^{\frac{13}{2}}}{13}+\frac{4b^2cdx^{\frac{17}{2}}}{17}+\frac{2b^2d^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(5/2)/5 + 4*a**2*c*d*x**(9/2)/9 + 2*a**2*d**2*x**(13/2)/13 +
 4*a*b*c**2*x**(9/2)/9 + 8*a*b*c*d*x**(13/2)/13 + 4*a*b*d**2*x**(17/2)/17 +
 2*b**2*c**2*x**(13/2)/13 + 4*b**2*c*d*x**(17/2)/17 + 2*b**2*d**2*x**(21/2)
/21
```

Giac [A]
time = 0.95, size = 94, normalized size = 0.97

$$\frac{2}{21}\,b^2d^2x^{\frac{21}{2}}+\frac{4}{17}\,b^2cdx^{\frac{17}{2}}+\frac{4}{17}\,abd^2x^{\frac{17}{2}}+\frac{2}{13}\,b^2c^2x^{\frac{13}{2}}+\frac{8}{13}\,abcdx^{\frac{13}{2}}+\frac{2}{13}\,a^2d^2x^{\frac{13}{2}}+\frac{4}{9}\,abc^2x^{\frac{9}{2}}+\frac{4}{9}\,a^2cdx^{\frac{9}{2}}+\frac{2}{5}\,a^2c^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/21*b^2*d^2*x^(21/2) + 4/17*b^2*c*d*x^(17/2) + 4/17*a*b*d^2*x^(17/2) + 2/1
3*b^2*c^2*x^(13/2) + 8/13*a*b*c*d*x^(13/2) + 2/13*a^2*d^2*x^(13/2) + 4/9*a*
b*c^2*x^(9/2) + 4/9*a^2*c*d*x^(9/2) + 2/5*a^2*c^2*x^(5/2)
```

Mupad [B]
time = 0.02, size = 78, normalized size = 0.80

$$x^{13/2}\left(\frac{2\,a^2\,d^2}{13}+\frac{8\,a\,b\,c\,d}{13}+\frac{2\,b^2\,c^2}{13}\right)+\frac{2\,a^2\,c^2\,x^{5/2}}{5}+\frac{2\,b^2\,d^2\,x^{21/2}}{21}+\frac{4\,a\,c\,x^{9/2}\,\left(a\,d+b\,c\right)}{9}+\frac{4\,b\,d\,x^{17/2}\,\left(a\,d+b\,c\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^(13/2)*((2*a^2*d^2)/13 + (2*b^2*c^2)/13 + (8*a*b*c*d)/13) + (2*a^2*c^2*x^
(5/2))/5 + (2*b^2*d^2*x^(21/2))/21 + (4*a*c*x^(9/2)*(a*d + b*c))/9 + (4*b*d
*x^(17/2)*(a*d + b*c))/17
```

3.402 $\int \sqrt{x} \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{2}{3}a^2c^2x^{3/2}+\frac{4}{7}ac(bc+ad)x^{7/2}+\frac{2}{11}\left(b^2c^2+4abcd+a^2d^2\right)x^{11/2}+\frac{4}{15}bd(bc+ad)x^{15/2}+\frac{2}{19}b^2d^2x^{19/2}$$

```
[Out] 2/3*a^2*c^2*x^(3/2)+4/7*a*c*(a*d+b*c)*x^(7/2)+2/11*(a^2*d^2+4*a*b*c*d+b^2*c
^2)*x^(11/2)+4/15*b*d*(a*d+b*c)*x^(15/2)+2/19*b^2*d^2*x^(19/2)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{459\}$

$$\frac{2}{11}x^{11/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{3}a^2c^2x^{3/2}+\frac{4}{15}bdx^{15/2}(ad+bc)+\frac{4}{7}acx^{7/2}(ad+bc)+\frac{2}{19}b^2d^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(3/2))/3 + (4*a*c*(b*c + a*d)*x^(7/2))/7 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(11/2))/11 + (4*b*d*(b*c + a*d)*x^(15/2))/15 + (2*b^2*d^2
*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2\sqrt{x}+2ac(bc+ad)x^{5/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{9/2}+2bd(bc+\right.\\ &= \frac{2}{3}a^2c^2x^{3/2}+\frac{4}{7}ac(bc+ad)x^{7/2}+\frac{2}{11}\left(b^2c^2+4abcd+a^2d^2\right)x^{11/2}+\frac{4}{15}bd(\end{aligned}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 0.96

$$\frac{2x^{3/2}\left(95a^2\left(77c^2+66cdx^2+21d^2x^4\right)+38abx^2\left(165c^2+210cdx^2+77d^2x^4\right)+7b^2x^4\left(285c^2+418cdx^2+165d^2x^4\right)\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(95*a^2*(77*c^2 + 66*c*d*x^2 + 21*d^2*x^4) + 38*a*b*x^2*(165*c^2
 + 210*c*d*x^2 + 77*d^2*x^4) + 7*b^2*x^4*(285*c^2 + 418*c*d*x^2 + 165*d^2*x
^4)))/21945
```

Maple [A]
time = 0.10, size = 90, normalized size = 0.93

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{19}{2}}}{19}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{15}{2}}}{15}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{11}{2}}}{11}+\frac{2(2a^2cd+2ab\,c^2)x^{\frac{7}{2}}}{7}+\frac{2a^2c^2x^{\frac{3}{2}}}{3}$
default	$\frac{2b^2d^2x^{\frac{19}{2}}}{19}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{15}{2}}}{15}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{11}{2}}}{11}+\frac{2(2a^2cd+2ab\,c^2)x^{\frac{7}{2}}}{7}+\frac{2a^2c^2x^{\frac{3}{2}}}{3}$
gosper	$\frac{2x^{\frac{3}{2}}(1155b^2d^2x^8+2926ab\,d^2x^6+2926b^2cd\,x^6+1995a^2d^2x^4+7980abcd\,x^4+1995b^2c^2x^4+6270a^2cd\,x^2+6270ab\,c^2x^2+7315}{21945}$
trager	$\frac{2x^{\frac{3}{2}}(1155b^2d^2x^8+2926ab\,d^2x^6+2926b^2cd\,x^6+1995a^2d^2x^4+7980abcd\,x^4+1995b^2c^2x^4+6270a^2cd\,x^2+6270ab\,c^2x^2+7315}{21945}$
risch	$\frac{2x^{\frac{3}{2}}(1155b^2d^2x^8+2926ab\,d^2x^6+2926b^2cd\,x^6+1995a^2d^2x^4+7980abcd\,x^4+1995b^2c^2x^4+6270a^2cd\,x^2+6270ab\,c^2x^2+7315}{21945}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*b^2*d^2*x^(19/2)+2/15*(2*a*b*d^2+2*b^2*c*d)*x^(15/2)+2/11*(a^2*d^2+4*a
*b*c*d+b^2*c^2)*x^(11/2)+2/7*(2*a^2*c*d+2*a*b*c^2)*x^(7/2)+2/3*a^2*c^2*x^(3
/2)
```

Maxima [A]
time = 0.31, size = 85, normalized size = 0.88

$$\frac{2}{19}\,b^2d^2x^{\frac{19}{2}}+\frac{4}{15}\left(b^2cd+abd^2\right)x^{\frac{15}{2}}+\frac{2}{11}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^2c^2x^{\frac{3}{2}}+\frac{4}{7}\left(abc^2+a^2cd\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d^2*x^(19/2) + 4/15*(b^2*c*d + a*b*d^2)*x^(15/2) + 2/11*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(11/2) + 2/3*a^2*c^2*x^(3/2) + 4/7*(a*b*c^2 + a^2*c
*d)*x^(7/2)
```

Fricas [A]
time = 0.78, size = 88, normalized size = 0.91

$$\frac{2}{21945}\left(1155\,b^2d^2x^9+2926\left(b^2cd+abd^2\right)x^7+1995\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+7315\,a^2c^2x+6270\left(abc^2+a^2cd\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d^2*x^9 + 2926*(b^2*c*d + a*b*d^2)*x^7 + 1995*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^5 + 7315*a^2*c^2*x + 6270*(a*b*c^2 + a^2*c*d)*x^3)*s
qrt(x)
```

Sympy [A]
time = 1.72, size = 110, normalized size = 1.13

$$\frac{2a^2c^2x^{\frac{3}{2}}}{3}+\frac{2b^2d^2x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{15}{2}}\cdot(2abd^2+2b^2cd)}{15}+\frac{2x^{\frac{11}{2}}(a^2d^2+4abcd+b^2c^2)}{11}+\frac{2x^{\frac{7}{2}}\cdot(2a^2cd+2abc^2)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2*x**(1/2),x)
```

```
[Out] 2*a**2*c**2*x**(3/2)/3 + 2*b**2*d**2*x**(19/2)/19 + 2*x**(15/2)*(2*a*b*d**2
 + 2*b**2*c*d)/15 + 2*x**(11/2)*(a**2*d**2 + 4*a*b*c*d + b**2*c**2)/11 + 2*
x**(7/2)*(2*a**2*c*d + 2*a*b*c**2)/7
```

Giac [A]
time = 1.02, size = 94, normalized size = 0.97

$$\frac{2}{19}b^2d^2x^{\frac{19}{2}}+\frac{4}{15}b^2cdx^{\frac{15}{2}}+\frac{4}{15}abd^2x^{\frac{15}{2}}+\frac{2}{11}b^2c^2x^{\frac{11}{2}}+\frac{8}{11}abcdx^{\frac{11}{2}}+\frac{2}{11}a^2d^2x^{\frac{11}{2}}+\frac{4}{7}abc^2x^{\frac{7}{2}}+\frac{4}{7}a^2cdx^{\frac{7}{2}}+\frac{2}{3}a^2c^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d^2*x^(19/2) + 4/15*b^2*c*d*x^(15/2) + 4/15*a*b*d^2*x^(15/2) + 2/1
1*b^2*c^2*x^(11/2) + 8/11*a*b*c*d*x^(11/2) + 2/11*a^2*d^2*x^(11/2) + 4/7*a*
b*c^2*x^(7/2) + 4/7*a^2*c*d*x^(7/2) + 2/3*a^2*c^2*x^(3/2)
```

Mupad [B]
time = 0.02, size = 78, normalized size = 0.80

$$x^{11/2}\left(\frac{2\,a^2\,d^2}{11}+\frac{8\,a\,b\,c\,d}{11}+\frac{2\,b^2\,c^2}{11}\right)+\frac{2\,a^2\,c^2\,x^{3/2}}{3}+\frac{2\,b^2\,d^2\,x^{19/2}}{19}+\frac{4\,a\,c\,x^{7/2}\,(a\,d+b\,c)}{7}+\frac{4\,b\,d\,x^{15/2}\,(a\,d+b\,c)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^(11/2)*((2*a^2*d^2)/11 + (2*b^2*c^2)/11 + (8*a*b*c*d)/11) + (2*a^2*c^2*x^
(3/2))/3 + (2*b^2*d^2*x^(19/2))/19 + (4*a*c*x^(7/2)*(a*d + b*c))/7 + (4*b*d
*x^(15/2)*(a*d + b*c))/15
```

3.403 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=95

$$2a^2c^2\sqrt{x}+\frac{4}{5}ac(bc+ad)x^{5/2}+\frac{2}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^{9/2}+\frac{4}{13}bd(bc+ad)x^{13/2}+\frac{2}{17}b^2d^2x^{17/2}$$

```
[Out] 4/5*a*c*(a*d+b*c)*x^(5/2)+2/9*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(9/2)+4/13*b*d*
(a*d+b*c)*x^(13/2)+2/17*b^2*d^2*x^(17/2)+2*a^2*c^2*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {459}

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/13 + (2*b^2*d^2*x^(17
/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{\sqrt{x}}\,dx = \int \left(\frac{a^2c^2}{\sqrt{x}}+2ac(bc+ad)x^{3/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{7/2}+2bd(bc+ad)x^{11/2}\right.$$

$$=2a^2c^2\sqrt{x}+\frac{4}{5}ac(bc+ad)x^{5/2}+\frac{2}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^{9/2}+\frac{4}{13}bd(bc+ad)x$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.98

$$\frac{2\sqrt{x}\left(221a^2(45c^2+18cdx^2+5d^2x^4)+34abx^2(117c^2+130cdx^2+45d^2x^4)+5b^2x^4(221c^2+306cdx^2+117d^2x^4)\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(221*a^2*(45*c^2 + 18*c*d*x^2 + 5*d^2*x^4) + 34*a*b*x^2*(117*c^2
 + 130*c*d*x^2 + 45*d^2*x^4) + 5*b^2*x^4*(221*c^2 + 306*c*d*x^2 + 117*d^2*x
^4)))/9945
```

Maple [A]
time = 0.10, size = 90, normalized size = 0.95

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{17}{2}}}{17}+\frac{2(2ab\,d^2+2b^2cd)\,x^{\frac{13}{2}}}{13}+\frac{2(a^2d^2+4abcd+b^2c^2)\,x^{\frac{9}{2}}}{9}+\frac{2(2a^2cd+2ab\,c^2)\,x^{\frac{5}{2}}}{5}+2a^2c^2\sqrt{x}$
default	$\frac{2b^2d^2x^{\frac{17}{2}}}{17}+\frac{2(2ab\,d^2+2b^2cd)\,x^{\frac{13}{2}}}{13}+\frac{2(a^2d^2+4abcd+b^2c^2)\,x^{\frac{9}{2}}}{9}+\frac{2(2a^2cd+2ab\,c^2)\,x^{\frac{5}{2}}}{5}+2a^2c^2\sqrt{x}$
trager	$\left(\frac{2}{17}b^2d^2x^8+\frac{4}{13}ab\,d^2x^6+\frac{4}{13}b^2cd\,x^6+\frac{2}{9}a^2d^2x^4+\frac{8}{9}abcd\,x^4+\frac{2}{9}b^2c^2x^4+\frac{4}{5}a^2cd\,x^2+\frac{4}{5}ab\,c^{2}\right.$
gosper	$\frac{2\sqrt{x}\,(585b^2d^2x^8+1530ab\,d^2x^6+1530b^2cd\,x^6+1105a^2d^2x^4+4420abcd\,x^4+1105b^2c^2x^4+3978a^2cd\,x^2+3978ab\,c^2x^2+9}{9945}$
risch	$\frac{2\sqrt{x}\,(585b^2d^2x^8+1530ab\,d^2x^6+1530b^2cd\,x^6+1105a^2d^2x^4+4420abcd\,x^4+1105b^2c^2x^4+3978a^2cd\,x^2+3978ab\,c^2x^2+9}{9945}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*b^2*d^2*x^(17/2)+2/13*(2*a*b*d^2+2*b^2*c*d)*x^(13/2)+2/9*(a^2*d^2+4*a*
b*c*d+b^2*c^2)*x^(9/2)+2/5*(2*a^2*c*d+2*a*b*c^2)*x^(5/2)+2*a^2*c^2*x^(1/2)
```

Maxima [A]
time = 0.30, size = 85, normalized size = 0.89

$$\frac{2}{17}\,b^2d^2x^{\frac{17}{2}}+\frac{4}{13}\,\left(b^2cd+abd^2\right)x^{\frac{13}{2}}+\frac{2}{9}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{9}{2}}+2\,a^2c^2\sqrt{x}+\frac{4}{5}\,\left(abc^2+a^2cd\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d^2*x^(17/2) + 4/13*(b^2*c*d + a*b*d^2)*x^(13/2) + 2/9*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^(9/2) + 2*a^2*c^2*sqrt(x) + 4/5*(a*b*c^2 + a^2*c*d)*
x^(5/2)
```

Fricas [A]
time = 0.70, size = 87, normalized size = 0.92

$$\frac{2}{9945}\,\left(585\,b^2d^2x^8+1530\,\left(b^2cd+abd^2\right)x^6+1105\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+9945\,a^2c^2+3978\,\left(abc^2+a^2cd\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d^2*x^8 + 1530*(b^2*c*d + a*b*d^2)*x^6 + 1105*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^4 + 9945*a^2*c^2 + 3978*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(
x)
```

Sympy [A]
time = 0.55, size = 134, normalized size = 1.41

$$2a^2c^2\sqrt{x} + \frac{4a^2cdx^{\frac{5}{2}}}{5} + \frac{2a^2d^2x^{\frac{9}{2}}}{9} + \frac{4abc^2x^{\frac{5}{2}}}{5} + \frac{8abcdx^{\frac{9}{2}}}{9} + \frac{4abd^2x^{\frac{13}{2}}}{13} + \frac{2b^2c^2x^{\frac{9}{2}}}{9} + \frac{4b^2cdx^{\frac{13}{2}}}{13} + \frac{2b^2d^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] 2*a**2*c**2*sqrt(x) + 4*a**2*c*d*x**(5/2)/5 + 2*a**2*d**2*x**(9/2)/9 + 4*a*
b*c**2*x**(5/2)/5 + 8*a*b*c*d*x**(9/2)/9 + 4*a*b*d**2*x**(13/2)/13 + 2*b**2
*c**2*x**(9/2)/9 + 4*b**2*c*d*x**(13/2)/13 + 2*b**2*d**2*x**(17/2)/17
```

Giac [A]
time = 1.12, size = 94, normalized size = 0.99

$$\frac{2}{17}\,b^2d^2x^{\frac{17}{2}} + \frac{4}{13}\,b^2cdx^{\frac{13}{2}} + \frac{4}{13}\,abd^2x^{\frac{13}{2}} + \frac{2}{9}\,b^2c^2x^{\frac{9}{2}} + \frac{8}{9}\,abcdx^{\frac{9}{2}} + \frac{2}{9}\,a^2d^2x^{\frac{9}{2}} + \frac{4}{5}\,abc^2x^{\frac{5}{2}} + \frac{4}{5}\,a^2cdx^{\frac{5}{2}} + 2\,a^2c^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d^2*x^(17/2) + 4/13*b^2*c*d*x^(13/2) + 4/13*a*b*d^2*x^(13/2) + 2/9
*b^2*c^2*x^(9/2) + 8/9*a*b*c*d*x^(9/2) + 2/9*a^2*d^2*x^(9/2) + 4/5*a*b*c^2*
x^(5/2) + 4/5*a^2*c*d*x^(5/2) + 2*a^2*c^2*sqrt(x)
```

Mupad [B]
time = 0.02, size = 78, normalized size = 0.82

$$x^{9/2}\left(\frac{2\,a^2\,d^2}{9}+\frac{8\,a\,b\,c\,d}{9}+\frac{2\,b^2\,c^2}{9}\right)+2\,a^2\,c^2\,\sqrt{x}+\frac{2\,b^2\,d^2\,x^{17/2}}{17}+\frac{4\,a\,c\,x^{5/2}\,\left(a\,d+b\,c\right)}{5}+\frac{4\,b\,d\,x^{13/2}\,\left(a\,d+b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(1/2),x)
```

```
[Out] x^(9/2)*((2*a^2*d^2)/9 + (2*b^2*c^2)/9 + (8*a*b*c*d)/9) + 2*a^2*c^2*x^(1/2)
 + (2*b^2*d^2*x^(17/2))/17 + (4*a*c*x^(5/2)*(a*d + b*c))/5 + (4*b*d*x^(13/2
)*(a*d + b*c))/13
```

3.404 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}} dx$

Optimal. Leaf size=95

$$-\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{3}ac(bc+ad)x^{3/2}+\frac{2}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^{7/2}+\frac{4}{11}bd(bc+ad)x^{11/2}+\frac{2}{15}b^2d^2x^{15/2}$$

```
[Out] 4/3*a*c*(a*d+b*c)*x^(3/2)+2/7*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(7/2)+4/11*b*d*
(a*d+b*c)*x^(11/2)+2/15*b^2*d^2*x^(15/2)-2*a^2*c^2/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {459}

$$\frac{2}{7}x^{7/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{11}bdx^{11/2}(ad+bc)+\frac{4}{3}acx^{3/2}(ad+bc)+\frac{2}{15}b^2d^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^2)/Sqrt[x] + (4*a*c*(b*c + a*d)*x^(3/2))/3 + (2*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^(7/2))/7 + (4*b*d*(b*c + a*d)*x^(11/2))/11 + (2*b^2*d^2*x^
(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2c^2}{x^{3/2}}+2ac(bc+ad)\sqrt{x}+\left(b^2c^2+4abcd+a^2d^2\right)x^{5/2}+2bd(bc+ad)x^{9/}\right.\\ &= -\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{3}ac(bc+ad)x^{3/2}+\frac{2}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^{7/2}+\frac{4}{11}bd(bc+ad)x\end{aligned}$$

Mathematica [A]
time = 0.05, size = 92, normalized size = 0.97

$$\frac{2(-55a^2(21c^2-14cdx^2-3d^2x^4)+10abx^2(77c^2+66cdx^2+21d^2x^4)+b^2x^4(165c^2+210cdx^2+77d^2x^4))}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

```
[Out] (2*(-55*a^2*(21*c^2 - 14*c*d*x^2 - 3*d^2*x^4) + 10*a*b*x^2*(77*c^2 + 66*c*d
*x^2 + 21*d^2*x^4) + b^2*x^4*(165*c^2 + 210*c*d*x^2 + 77*d^2*x^4)))/(1155*S
qrt[x])
```

Maple [A]
time = 0.08, size = 95, normalized size = 1.00

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{15}{2}}}{15}+\frac{4ab\,d^2x^{\frac{11}{2}}}{11}+\frac{4b^2cd\,x^{\frac{11}{2}}}{11}+\frac{2a^2d^2x^{\frac{7}{2}}}{7}+\frac{8abcd\,x^{\frac{7}{2}}}{7}+\frac{2b^2c^2x^{\frac{7}{2}}}{7}+\frac{4a^2cd\,x^{\frac{3}{2}}}{3}+\frac{4ab\,c^2x^{\frac{3}{2}}}{3}-\frac{2a^2c^2}{\sqrt{x}}$
default	$\frac{2b^2d^2x^{\frac{15}{2}}}{15}+\frac{4ab\,d^2x^{\frac{11}{2}}}{11}+\frac{4b^2cd\,x^{\frac{11}{2}}}{11}+\frac{2a^2d^2x^{\frac{7}{2}}}{7}+\frac{8abcd\,x^{\frac{7}{2}}}{7}+\frac{2b^2c^2x^{\frac{7}{2}}}{7}+\frac{4a^2cd\,x^{\frac{3}{2}}}{3}+\frac{4ab\,c^2x^{\frac{3}{2}}}{3}-\frac{2a^2c^2}{\sqrt{x}}$
gosper	$-\frac{2\left(-77b^2d^2x^8-210ab\,d^2x^6-210b^2cd\,x^6-165a^2d^2x^4-660abcd\,x^4-165b^2c^2x^4-770a^2cd\,x^2-770ab\,c^2x^2+1155a^2c^2\right)}{1155\sqrt{x}}$
trager	$-\frac{2\left(-77b^2d^2x^8-210ab\,d^2x^6-210b^2cd\,x^6-165a^2d^2x^4-660abcd\,x^4-165b^2c^2x^4-770a^2cd\,x^2-770ab\,c^2x^2+1155a^2c^2\right)}{1155\sqrt{x}}$
risch	$-\frac{2\left(-77b^2d^2x^8-210ab\,d^2x^6-210b^2cd\,x^6-165a^2d^2x^4-660abcd\,x^4-165b^2c^2x^4-770a^2cd\,x^2-770ab\,c^2x^2+1155a^2c^2\right)}{1155\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*b^2*d^2*x^(15/2)+4/11*a*b*d^2*x^(11/2)+4/11*b^2*c*d*x^(11/2)+2/7*a^2*d
^2*x^(7/2)+8/7*a*b*c*d*x^(7/2)+2/7*b^2*c^2*x^(7/2)+4/3*a^2*c*d*x^(3/2)+4/3*
a*b*c^2*x^(3/2)-2*a^2*c^2/x^(1/2)
```

Maxima [A]
time = 0.30, size = 85, normalized size = 0.89

$$\frac{2}{15}\,b^2d^2x^{\frac{15}{2}}+\frac{4}{11}\left(b^2cd+abd^2\right)x^{\frac{11}{2}}+\frac{2}{7}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{7}{2}}-\frac{2\,a^2c^2}{\sqrt{x}}+\frac{4}{3}\left(abc^2+a^2cd\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d^2*x^(15/2) + 4/11*(b^2*c*d + a*b*d^2)*x^(11/2) + 2/7*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^(7/2) - 2*a^2*c^2/sqrt(x) + 4/3*(a*b*c^2 + a^2*c*d)*
x^(3/2)
```

Fricas [A]
time = 0.80, size = 87, normalized size = 0.92

$$\frac{2\left(77\,b^2d^2x^8+210\left(b^2cd+abd^2\right)x^6+165\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-1155\,a^2c^2+770\left(abc^2+a^2cd\right)x^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^2*x^8 + 210*(b^2*c*d + a*b*d^2)*x^6 + 165*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^4 - 1155*a^2*c^2 + 770*(a*b*c^2 + a^2*c*d)*x^2)/sqrt(x)
```

Sympy [A]
time = 0.65, size = 134, normalized size = 1.41

$$-\frac{2a^2c^2}{\sqrt{x}} + \frac{4a^2cdx^{\frac{3}{2}}}{3} + \frac{2a^2d^2x^{\frac{7}{2}}}{7} + \frac{4abc^2x^{\frac{3}{2}}}{3} + \frac{8abcdx^{\frac{7}{2}}}{7} + \frac{4abd^2x^{\frac{11}{2}}}{11} + \frac{2b^2c^2x^{\frac{7}{2}}}{7} + \frac{4b^2cdx^{\frac{11}{2}}}{11} + \frac{2b^2d^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(3/2),x)
```

```
[Out] -2*a**2*c**2/sqrt(x) + 4*a**2*c*d*x**(3/2)/3 + 2*a**2*d**2*x**(7/2)/7 + 4*a
*b*c**2*x**(3/2)/3 + 8*a*b*c*d*x**(7/2)/7 + 4*a*b*d**2*x**(11/2)/11 + 2*b**
2*c**2*x**(7/2)/7 + 4*b**2*c*d*x**(11/2)/11 + 2*b**2*d**2*x**(15/2)/15
```

Giac [A]
time = 0.56, size = 94, normalized size = 0.99

$$\frac{2}{15}\,b^2d^2x^{\frac{15}{2}} + \frac{4}{11}\,b^2cdx^{\frac{11}{2}} + \frac{4}{11}\,abd^2x^{\frac{11}{2}} + \frac{2}{7}\,b^2c^2x^{\frac{7}{2}} + \frac{8}{7}\,abcdx^{\frac{7}{2}} + \frac{2}{7}\,a^2d^2x^{\frac{7}{2}} + \frac{4}{3}\,abc^2x^{\frac{3}{2}} + \frac{4}{3}\,a^2cdx^{\frac{3}{2}} - \frac{2\,a^2c^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d^2*x^(15/2) + 4/11*b^2*c*d*x^(11/2) + 4/11*a*b*d^2*x^(11/2) + 2/7
*b^2*c^2*x^(7/2) + 8/7*a*b*c*d*x^(7/2) + 2/7*a^2*d^2*x^(7/2) + 4/3*a*b*c^2*
x^(3/2) + 4/3*a^2*c*d*x^(3/2) - 2*a^2*c^2/sqrt(x)
```

Mupad [B]
time = 0.02, size = 78, normalized size = 0.82

$$x^{7/2}\left(\frac{2\,a^2\,d^2}{7}+\frac{8\,a\,b\,c\,d}{7}+\frac{2\,b^2\,c^2}{7}\right)-\frac{2\,a^2\,c^2}{\sqrt{x}}+\frac{2\,b^2\,d^2\,x^{15/2}}{15}+\frac{4\,a\,c\,x^{3/2}\,\left(a\,d+b\,c\right)}{3}+\frac{4\,b\,d\,x^{11/2}\,\left(a\,d+b\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x)
```

```
[Out] x^(7/2)*((2*a^2*d^2)/7 + (2*b^2*c^2)/7 + (8*a*b*c*d)/7) - (2*a^2*c^2)/x^(1/
2) + (2*b^2*d^2*x^(15/2))/15 + (4*a*c*x^(3/2)*(a*d + b*c))/3 + (4*b*d*x^(11
/2)*(a*d + b*c))/11
```

3.405 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{2a^2c^2}{3x^{3/2}}+4ac(bc+ad)\sqrt{x}+\frac{2}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^{5/2}+\frac{4}{9}bd(bc+ad)x^{9/2}+\frac{2}{13}b^2d^2x^{13/2}$$

```
[Out] -2/3*a^2*c^2/x^(3/2)+2/5*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(5/2)+4/9*b*d*(a*d+b
*c)*x^(9/2)+2/13*b^2*d^2*x^(13/2)+4*a*c*(a*d+b*c)*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {459}

$$\frac{2}{5}x^{5/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{3x^{3/2}}+\frac{4}{9}bdx^{9/2}(ad+bc)+4ac\sqrt{x}(ad+bc)+\frac{2}{13}b^2d^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^2)/(3*x^(3/2)) + 4*a*c*(b*c + a*d)*Sqrt[x] + (2*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^(5/2))/5 + (4*b*d*(b*c + a*d)*x^(9/2))/9 + (2*b^2*d^2*x^(1
3/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2c^2}{x^{5/2}}+\frac{2ac(bc+ad)}{\sqrt{x}}+\left(b^2c^2+4abcd+a^2d^2\right)x^{3/2}+2bd(bc+ad)x^{7/2}+b^2\right.\\ &= -\frac{2a^2c^2}{3x^{3/2}}+4ac(bc+ad)\sqrt{x}+\frac{2}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^{5/2}+\frac{4}{9}bd(bc+ad)x^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 0.98

$$\frac{-78a^2(5c^2-30cdx^2-3d^2x^4)+52abx^2(45c^2+18cdx^2+5d^2x^4)+2b^2x^4(117c^2+130cdx^2+45d^2x^4)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

```
[Out] (-78*a^2*(5*c^2 - 30*c*d*x^2 - 3*d^2*x^4) + 52*a*b*x^2*(45*c^2 + 18*c*d*x^2
 + 5*d^2*x^4) + 2*b^2*x^4*(117*c^2 + 130*c*d*x^2 + 45*d^2*x^4))/(585*x^(3/2
))
```

Maple [A]
time = 0.08, size = 95, normalized size = 1.00

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{13}{2}}}{13}+\frac{4ab\,d^2x^{\frac{9}{2}}}{9}+\frac{4b^2cd\,x^{\frac{9}{2}}}{9}+\frac{2a^2d^2x^{\frac{5}{2}}}{5}+\frac{8abcd\,x^{\frac{5}{2}}}{5}+\frac{2b^2c^2x^{\frac{5}{2}}}{5}+4a^2cd\sqrt{x}+4ab\,c^2\sqrt{x}-$
default	$\frac{2b^2d^2x^{\frac{13}{2}}}{13}+\frac{4ab\,d^2x^{\frac{9}{2}}}{9}+\frac{4b^2cd\,x^{\frac{9}{2}}}{9}+\frac{2a^2d^2x^{\frac{5}{2}}}{5}+\frac{8abcd\,x^{\frac{5}{2}}}{5}+\frac{2b^2c^2x^{\frac{5}{2}}}{5}+4a^2cd\sqrt{x}+4ab\,c^2\sqrt{x}-$
gosper	$-\frac{2\left(-45b^2d^2x^8-130ab\,d^2x^6-130b^2cd\,x^6-117a^2d^2x^4-468abcd\,x^4-117b^2c^2x^4-1170a^2cd\,x^2-1170ab\,c^2x^2+195a^2c^2\right)}{585x^{\frac{3}{2}}}$
trager	$-\frac{2\left(-45b^2d^2x^8-130ab\,d^2x^6-130b^2cd\,x^6-117a^2d^2x^4-468abcd\,x^4-117b^2c^2x^4-1170a^2cd\,x^2-1170ab\,c^2x^2+195a^2c^2\right)}{585x^{\frac{3}{2}}}$
risch	$-\frac{2\left(-45b^2d^2x^8-130ab\,d^2x^6-130b^2cd\,x^6-117a^2d^2x^4-468abcd\,x^4-117b^2c^2x^4-1170a^2cd\,x^2-1170ab\,c^2x^2+195a^2c^2\right)}{585x^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*b^2*d^2*x^(13/2)+4/9*a*b*d^2*x^(9/2)+4/9*b^2*c*d*x^(9/2)+2/5*a^2*d^2*x
^(5/2)+8/5*a*b*c*d*x^(5/2)+2/5*b^2*c^2*x^(5/2)+4*a^2*c*d*x^(1/2)+4*a*b*c^2*
x^(1/2)-2/3*a^2*c^2/x^(3/2)
```

Maxima [A]
time = 0.29, size = 85, normalized size = 0.89

$$\frac{2}{13}\,b^2d^2x^{\frac{13}{2}}+\frac{4}{9}\left(b^2cd+abd^2\right)x^{\frac{9}{2}}+\frac{2}{5}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{5}{2}}-\frac{2\,a^2c^2}{3\,x^{\frac{3}{2}}}+4\left(abc^2+a^2cd\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*b^2*d^2*x^(13/2) + 4/9*(b^2*c*d + a*b*d^2)*x^(9/2) + 2/5*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^(5/2) - 2/3*a^2*c^2/x^(3/2) + 4*(a*b*c^2 + a^2*c*d)*sq
rt(x)
```

Fricas [A]
time = 0.73, size = 87, normalized size = 0.92

$$\frac{2\,(45\,b^2d^2x^8+130\,(b^2cd+abd^2)x^6+117\,(b^2c^2+4\,abcd+a^2d^2)x^4-195\,a^2c^2+1170\,(abc^2+a^2cd)x^2)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d^2*x^8 + 130*(b^2*c*d + a*b*d^2)*x^6 + 117*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^4 - 195*a^2*c^2 + 1170*(a*b*c^2 + a^2*c*d)*x^2)/x^(3/2)
```

Sympy [A]
time = 0.73, size = 133, normalized size = 1.40

$$-\frac{2a^2c^2}{3x^{\frac{3}{2}}} + 4a^2cd\sqrt{x} + \frac{2a^2d^2x^{\frac{5}{2}}}{5} + 4abc^2\sqrt{x} + \frac{8abcdx^{\frac{5}{2}}}{5} + \frac{4abd^2x^{\frac{9}{2}}}{9} + \frac{2b^2c^2x^{\frac{5}{2}}}{5} + \frac{4b^2cdx^{\frac{9}{2}}}{9} + \frac{2b^2d^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(5/2),x)
```

```
[Out] -2*a**2*c**2/(3*x**(3/2)) + 4*a**2*c*d*sqrt(x) + 2*a**2*d**2*x**(5/2)/5 + 4
*a*b*c**2*sqrt(x) + 8*a*b*c*d*x**(5/2)/5 + 4*a*b*d**2*x**(9/2)/9 + 2*b**2*c
**2*x**(5/2)/5 + 4*b**2*c*d*x**(9/2)/9 + 2*b**2*d**2*x**(13/2)/13
```

Giac [A]
time = 0.75, size = 94, normalized size = 0.99

$$\frac{2}{13}\,b^2d^2x^{\frac{13}{2}} + \frac{4}{9}\,b^2cdx^{\frac{9}{2}} + \frac{4}{9}\,abd^2x^{\frac{9}{2}} + \frac{2}{5}\,b^2c^2x^{\frac{5}{2}} + \frac{8}{5}\,abcdx^{\frac{5}{2}} + \frac{2}{5}\,a^2d^2x^{\frac{5}{2}} + 4\,abc^2\sqrt{x} + 4\,a^2cd\sqrt{x} - \frac{2\,a^2c^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*b^2*d^2*x^(13/2) + 4/9*b^2*c*d*x^(9/2) + 4/9*a*b*d^2*x^(9/2) + 2/5*b^2
*c^2*x^(5/2) + 8/5*a*b*c*d*x^(5/2) + 2/5*a^2*d^2*x^(5/2) + 4*a*b*c^2*sqrt(x
) + 4*a^2*c*d*sqrt(x) - 2/3*a^2*c^2/x^(3/2)
```

Mupad [B]
time = 0.02, size = 78, normalized size = 0.82

$$x^{5/2}\left(\frac{2\,a^2\,d^2}{5} + \frac{8\,a\,b\,c\,d}{5} + \frac{2\,b^2\,c^2}{5}\right) - \frac{2\,a^2\,c^2}{3\,x^{3/2}} + \frac{2\,b^2\,d^2\,x^{13/2}}{13} + 4\,a\,c\,\sqrt{x}\,\left(a\,d + b\,c\right) + \frac{4\,b\,d\,x^{9/2}\,\left(a\,d + b\,c\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x)
```

```
[Out] x^(5/2)*((2*a^2*d^2)/5 + (2*b^2*c^2)/5 + (8*a*b*c*d)/5) - (2*a^2*c^2)/(3*x^
(3/2)) + (2*b^2*d^2*x^(13/2))/13 + 4*a*c*x^(1/2)*(a*d + b*c) + (4*b*d*x^(9/
2)*(a*d + b*c))/9
```

3.406 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{2a^2c^2}{5x^{5/2}}-\frac{4ac(bc+ad)}{\sqrt{x}}+\frac{2}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^{3/2}+\frac{4}{7}bd(bc+ad)x^{7/2}+\frac{2}{11}b^2d^2x^{11/2}$$

```
[Out] -2/5*a^2*c^2/x^(5/2)+2/3*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(3/2)+4/7*b*d*(a*d+b
*c)*x^(7/2)+2/11*b^2*d^2*x^(11/2)-4*a*c*(a*d+b*c)/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {459}

$$\frac{2}{3}x^{3/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{5x^{5/2}}+\frac{4}{7}bdx^{7/2}(ad+bc)-\frac{4ac(ad+bc)}{\sqrt{x}}+\frac{2}{11}b^2d^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

```
[Out] (-2*a^2*c^2)/(5*x^(5/2)) - (4*a*c*(b*c + a*d))/Sqrt[x] + (2*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^(3/2))/3 + (4*b*d*(b*c + a*d)*x^(7/2))/7 + (2*b^2*d^2*x^
(11/2))/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{7/2}}\,dx &= \int \left(\frac{a^2c^2}{x^{7/2}}+\frac{2ac(bc+ad)}{x^{3/2}}+\left(b^2c^2+4abcd+a^2d^2\right)\sqrt{x}+2bd(bc+ad)x^{5/2}+\right.\\ &= -\frac{2a^2c^2}{5x^{5/2}}-\frac{4ac(bc+ad)}{\sqrt{x}}+\frac{2}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^{3/2}+\frac{4}{7}bd(bc+ad)x^{7/2}+\end{aligned}$$

Mathematica [A]
time = 0.07, size = 93, normalized size = 0.98

$$\frac{-154a^2(3c^2+30cdx^2-5d^2x^4)+220abx^2(-21c^2+14cdx^2+3d^2x^4)+10b^2x^4(77c^2+66cdx^2+21d^2x^4)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

```
[Out] (-154*a^2*(3*c^2 + 30*c*d*x^2 - 5*d^2*x^4) + 220*a*b*x^2*(-21*c^2 + 14*c*d*
x^2 + 3*d^2*x^4) + 10*b^2*x^4*(77*c^2 + 66*c*d*x^2 + 21*d^2*x^4))/(1155*x^(
5/2))
```

Maple [A]
time = 0.09, size = 89, normalized size = 0.94

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{11}{2}}}{11}+\frac{4ab\,d^2x^{\frac{7}{2}}}{7}+\frac{4b^2cd\,x^{\frac{7}{2}}}{7}+\frac{2a^2d^2x^{\frac{3}{2}}}{3}+\frac{8abcd\,x^{\frac{3}{2}}}{3}+\frac{2b^2c^2x^{\frac{3}{2}}}{3}-\frac{2a^2c^2}{5x^{\frac{5}{2}}}-\frac{4ac(ad+bc)}{\sqrt{x}}$
default	$\frac{2b^2d^2x^{\frac{11}{2}}}{11}+\frac{4ab\,d^2x^{\frac{7}{2}}}{7}+\frac{4b^2cd\,x^{\frac{7}{2}}}{7}+\frac{2a^2d^2x^{\frac{3}{2}}}{3}+\frac{8abcd\,x^{\frac{3}{2}}}{3}+\frac{2b^2c^2x^{\frac{3}{2}}}{3}-\frac{2a^2c^2}{5x^{\frac{5}{2}}}-\frac{4ac(ad+bc)}{\sqrt{x}}$
gosper	$-\frac{2(-105b^2d^2x^8-330ab\,d^2x^6-330b^2cd\,x^6-385a^2d^2x^4-1540abcd\,x^4-385b^2c^2x^4+2310a^2cd\,x^2+2310ab\,c^2x^2+231a^2c^2)}{1155x^{\frac{5}{2}}}$
trager	$-\frac{2(-105b^2d^2x^8-330ab\,d^2x^6-330b^2cd\,x^6-385a^2d^2x^4-1540abcd\,x^4-385b^2c^2x^4+2310a^2cd\,x^2+2310ab\,c^2x^2+231a^2c^2)}{1155x^{\frac{5}{2}}}$
risch	$-\frac{2(-105b^2d^2x^8-330ab\,d^2x^6-330b^2cd\,x^6-385a^2d^2x^4-1540abcd\,x^4-385b^2c^2x^4+2310a^2cd\,x^2+2310ab\,c^2x^2+231a^2c^2)}{1155x^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*b^2*d^2*x^(11/2)+4/7*a*b*d^2*x^(7/2)+4/7*b^2*c*d*x^(7/2)+2/3*a^2*d^2*x
^(3/2)+8/3*a*b*c*d*x^(3/2)+2/3*b^2*c^2*x^(3/2)-2/5*a^2*c^2/x^(5/2)-4*a*c*(a
*d+b*c)/x^(1/2)
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.92

$$\frac{2}{11}\,b^2d^2x^{\frac{11}{2}}+\frac{4}{7}\left(b^2cd+abd^2\right)x^{\frac{7}{2}}+\frac{2}{3}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{3}{2}}-\frac{2\,(a^2c^2+10\,(abc^2+a^2cd)x^2)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*d^2*x^(11/2) + 4/7*(b^2*c*d + a*b*d^2)*x^(7/2) + 2/3*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^(3/2) - 2/5*(a^2*c^2 + 10*(a*b*c^2 + a^2*c*d)*x^2)/x^(
5/2)
```

Fricas [A]
time = 0.60, size = 87, normalized size = 0.92

$$\frac{2\,(105\,b^2d^2x^8+330\,(b^2cd+abd^2)x^6+385\,(b^2c^2+4\,abcd+a^2d^2)x^4-231\,a^2c^2-2310\,(abc^2+a^2cd)x^2)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^2*d^2*x^8 + 330*(b^2*c*d + a*b*d^2)*x^6 + 385*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^4 - 231*a^2*c^2 - 2310*(a*b*c^2 + a^2*c*d)*x^2)/x^(5/2)
```

Sympy [A]
time = 0.93, size = 133, normalized size = 1.40

$$-\frac{2a^2c^2}{5x^{\frac{5}{2}}}-\frac{4a^2cd}{\sqrt{x}}+\frac{2a^2d^2x^{\frac{3}{2}}}{3}-\frac{4abc^2}{\sqrt{x}}+\frac{8abcdx^{\frac{3}{2}}}{3}+\frac{4abd^2x^{\frac{7}{2}}}{7}+\frac{2b^2c^2x^{\frac{3}{2}}}{3}+\frac{4b^2cdx^{\frac{7}{2}}}{7}+\frac{2b^2d^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(7/2),x)
```

```
[Out] -2*a**2*c**2/(5*x**(5/2)) - 4*a**2*c*d/sqrt(x) + 2*a**2*d**2*x**(3/2)/3 - 4
*a*b*c**2/sqrt(x) + 8*a*b*c*d*x**(3/2)/3 + 4*a*b*d**2*x**(7/2)/7 + 2*b**2*c
**2*x**(3/2)/3 + 4*b**2*c*d*x**(7/2)/7 + 2*b**2*d**2*x**(11/2)/11
```

Giac [A]
time = 0.64, size = 96, normalized size = 1.01

$$\frac{2}{11}\,b^2d^2x^{\frac{11}{2}}+\frac{4}{7}\,b^2cdx^{\frac{7}{2}}+\frac{4}{7}\,abd^2x^{\frac{7}{2}}+\frac{2}{3}\,b^2c^2x^{\frac{3}{2}}+\frac{8}{3}\,abcdx^{\frac{3}{2}}+\frac{2}{3}\,a^2d^2x^{\frac{3}{2}}-\frac{2\,(10\,abc^2x^2+10\,a^2cdx^2+a^2c^2)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*d^2*x^(11/2) + 4/7*b^2*c*d*x^(7/2) + 4/7*a*b*d^2*x^(7/2) + 2/3*b^2
*c^2*x^(3/2) + 8/3*a*b*c*d*x^(3/2) + 2/3*a^2*d^2*x^(3/2) - 2/5*(10*a*b*c^2*
x^2 + 10*a^2*c*d*x^2 + a^2*c^2)/x^(5/2)
```

Mupad [B]
time = 0.03, size = 86, normalized size = 0.91

$$x^{3/2}\left(\frac{2\,a^2\,d^2}{3}+\frac{8\,a\,b\,c\,d}{3}+\frac{2\,b^2\,c^2}{3}\right)-\frac{x^2\,\left(4\,d\,a^2\,c+4\,b\,a\,c^2\right)+\frac{2\,a^2\,c^2}{5}}{x^{5/2}}+\frac{2\,b^2\,d^2\,x^{11/2}}{11}+\frac{4\,b\,d\,x^{7/2}\,\left(a\,d+b\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x)
```

```
[Out] x^(3/2)*((2*a^2*d^2)/3 + (2*b^2*c^2)/3 + (8*a*b*c*d)/3) - (x^2*(4*a*b*c^2 +
 4*a^2*c*d) + (2*a^2*c^2)/5)/x^(5/2) + (2*b^2*d^2*x^(11/2))/11 + (4*b*d*x^(
7/2)*(a*d + b*c))/7
```

3.407 $\int x^{7/2}(a+bx^2)^2 (c+dx^2)^3 \, dx$

Optimal. Leaf size=139

$$\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2(2bc+3ad)x^{13/2}+\frac{2}{17}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{17/2}+\frac{2}{21}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{21/2}+\frac{2}{25}bd^2$$

```
[Out] 2/9*a^2*c^3*x^(9/2)+2/13*a*c^2*(3*a*d+2*b*c)*x^(13/2)+2/17*c*(3*a^2*d^2+6*a
*b*c*d+b^2*c^2)*x^(17/2)+2/21*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(21/2)+2/25
*b*d^2*(2*a*d+3*b*c)*x^(25/2)+2/29*b^2*d^3*x^(29/2)
```

Rubi [A]
time = 0.05, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {459}

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/2}(2ad+3bc)+\frac{2}{29}b^2d^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d)*x^(25/2))/25 + (2*b^2*d^3*x^
(29/2))/29
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^{7/2}+ac^2(2bc+3ad)x^{11/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{15/2}+d(3b\right.\\ &= \frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2(2bc+3ad)x^{13/2}+\frac{2}{17}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{17/2}+\end{aligned}$$

Mathematica [A]
time = 0.09, size = 126, normalized size = 0.91

$$\frac{2x^{9/2}\left(725a^2\left(1547c^3+3213c^2dx^2+2457cd^2x^4+663d^3x^6\right)+522abx^2\left(2975c^3+6825c^2dx^2+5525cd^2x^4+1547d^3x^6\right)+117b^2x^4\left(5075c^3+12325c^2dx^2+10353cd^2x^4+2975d^3x^6\right)\right)}{10094175}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*x^(9/2)*(725*a^2*(1547*c^3 + 3213*c^2*d*x^2 + 2457*c*d^2*x^4 + 663*d^3*x
^6) + 522*a*b*x^2*(2975*c^3 + 6825*c^2*d*x^2 + 5525*c*d^2*x^4 + 1547*d^3*x^
6) + 117*b^2*x^4*(5075*c^3 + 12325*c^2*d*x^2 + 10353*c*d^2*x^4 + 2975*d^3*x
^6)))/10094175
```

Maple [A]
time = 0.12, size = 128, normalized size = 0.92

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{29}{2}}}{29}+\frac{2(2abd^3+3b^2cd^2)x^{\frac{25}{2}}}{25}+\frac{2(a^2d^3+6abcd^2+3b^2c^2d)x^{\frac{21}{2}}}{21}+\frac{2(3a^2cd^2+6abc^2d+b^2c^3)x^{\frac{17}{2}}}{17}+\frac{2(3a^2c^2$
default	$\frac{2b^2d^3x^{\frac{29}{2}}}{29}+\frac{2(2abd^3+3b^2cd^2)x^{\frac{25}{2}}}{25}+\frac{2(a^2d^3+6abcd^2+3b^2c^2d)x^{\frac{21}{2}}}{21}+\frac{2(3a^2cd^2+6abc^2d+b^2c^3)x^{\frac{17}{2}}}{17}+\frac{2(3a^2c^2$
gosper	$\frac{2x^{\frac{9}{2}}(348075b^2d^3x^{10}+807534abd^3x^8+1211301b^2cd^2x^8+480675a^2d^3x^6+2884050abcd^2x^6+1442025b^2c^2dx^6+1781325$ … $}{10094175}$
trager	$\frac{2x^{\frac{9}{2}}(348075b^2d^3x^{10}+807534abd^3x^8+1211301b^2cd^2x^8+480675a^2d^3x^6+2884050abcd^2x^6+1442025b^2c^2dx^6+1781325$ … $}{10094175}$
risch	$\frac{2x^{\frac{9}{2}}(348075b^2d^3x^{10}+807534abd^3x^8+1211301b^2cd^2x^8+480675a^2d^3x^6+2884050abcd^2x^6+1442025b^2c^2dx^6+1781325$ … $}{10094175}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/29*b^2*d^3*x^(29/2)+2/25*(2*a*b*d^3+3*b^2*c*d^2)*x^(25/2)+2/21*(a^2*d^3+6
*a*b*c*d^2+3*b^2*c^2*d)*x^(21/2)+2/17*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^(
17/2)+2/13*(3*a^2*c^2*d+2*a*b*c^3)*x^(13/2)+2/9*a^2*c^3*x^(9/2)
```

Maxima [A]
time = 0.29, size = 127, normalized size = 0.91

$$\frac{2}{29}\,b^2d^3x^{\frac{29}{2}}+\frac{2}{25}\,(3\,b^2cd^2+2\,abd^3)x^{\frac{25}{2}}+\frac{2}{21}\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^{\frac{21}{2}}+\frac{2}{9}\,a^2c^3x^{\frac{9}{2}}+\frac{2}{17}\,(b^2c^3+6\,abc^2d+3\,a^2cd^2)x^{\frac{17}{2}}+\frac{2}{13}\,(2\,abc^3+3\,a^2c^2d)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/29*b^2*d^3*x^(29/2) + 2/25*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(25/2) + 2/21*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(21/2) + 2/9*a^2*c^3*x^(9/2) + 2/17*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(17/2) + 2/13*(2*a*b*c^3 + 3*a^2*c^2*d
)*x^(13/2)
```

Fricas [A]
time = 0.53, size = 132, normalized size = 0.95

$$\frac{2}{10094175}\,(348075\,b^2d^3x^{14}+403767\,(3\,b^2cd^2+2\,abd^3)x^{12}+480675\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^{10}+1121575\,a^2c^3x^4+593775\,(b^2c^3+6\,abc^2d+3\,a^2cd^2)x^8+776475\,(2\,abc^3+3\,a^2c^2d)x^6)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 2/10094175*(348075*b^2*d^3*x^14 + 403767*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 4
80675*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1121575*a^2*c^3*x^4 + 59
3775*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^8 + 776475*(2*a*b*c^3 + 3*a^2*
c^2*d)*x^6)*sqrt(x)
```

Sympy [A]
time = 2.33, size = 192, normalized size = 1.38

$$\frac{2a^2c^3x^{\frac{9}{2}}}{9} + \frac{6a^2c^2dx^{\frac{13}{2}}}{13} + \frac{6a^2cd^2x^{\frac{17}{2}}}{17} + \frac{2a^2d^3x^{\frac{21}{2}}}{21} + \frac{4abc^3x^{\frac{13}{2}}}{13} + \frac{12abc^2dx^{\frac{17}{2}}}{17} + \frac{4abcd^2x^{\frac{21}{2}}}{7} + \frac{4abd^3x^{\frac{25}{2}}}{25} + \frac{2b^2c^3x^{\frac{17}{2}}}{17} + \frac{2b^2c^2dx^{\frac{21}{2}}}{7} + \frac{6b^2cd^2x^{\frac{25}{2}}}{25} + \frac{2b^2d^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(9/2)/9 + 6*a**2*c**2*d*x**(13/2)/13 + 6*a**2*c*d**2*x**(17/
2)/17 + 2*a**2*d**3*x**(21/2)/21 + 4*a*b*c**3*x**(13/2)/13 + 12*a*b*c**2*d*
x**(17/2)/17 + 4*a*b*c*d**2*x**(21/2)/7 + 4*a*b*d**3*x**(25/2)/25 + 2*b**2*
c**3*x**(17/2)/17 + 2*b**2*c**2*d*x**(21/2)/7 + 6*b**2*c*d**2*x**(25/2)/25
+ 2*b**2*d**3*x**(29/2)/29
```

Giac [A]
time = 0.51, size = 135, normalized size = 0.97

$$\frac{2}{29}\,b^2d^3x^{\frac{29}{2}} + \frac{6}{25}\,b^2cd^2x^{\frac{25}{2}} + \frac{4}{25}\,abd^3x^{\frac{25}{2}} + \frac{2}{7}\,b^2c^2dx^{\frac{21}{2}} + \frac{4}{7}\,abcd^2x^{\frac{21}{2}} + \frac{2}{21}\,a^2d^3x^{\frac{21}{2}} + \frac{2}{17}\,b^2c^3x^{\frac{17}{2}} + \frac{12}{17}\,abc^2dx^{\frac{17}{2}} + \frac{6}{17}\,a^2cd^2x^{\frac{17}{2}} + \frac{4}{13}\,abc^3x^{\frac{13}{2}} + \frac{6}{13}\,a^2c^2dx^{\frac{13}{2}} + \frac{2}{9}\,a^2c^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/29*b^2*d^3*x^(29/2) + 6/25*b^2*c*d^2*x^(25/2) + 4/25*a*b*d^3*x^(25/2) + 2
/7*b^2*c^2*d*x^(21/2) + 4/7*a*b*c*d^2*x^(21/2) + 2/21*a^2*d^3*x^(21/2) + 2/
17*b^2*c^3*x^(17/2) + 12/17*a*b*c^2*d*x^(17/2) + 6/17*a^2*c*d^2*x^(17/2) +
4/13*a*b*c^3*x^(13/2) + 6/13*a^2*c^2*d*x^(13/2) + 2/9*a^2*c^3*x^(9/2)
```

Mupad [B]
time = 0.12, size = 119, normalized size = 0.86

$$x^{17/2}\left(\frac{6\,a^2\,c\,d^2}{17}+\frac{12\,a\,b\,c^2\,d}{17}+\frac{2\,b^2\,c^3}{17}\right)+x^{21/2}\left(\frac{2\,a^2\,d^3}{21}+\frac{4\,a\,b\,c\,d^2}{7}+\frac{2\,b^2\,c^2\,d}{7}\right)+\frac{2\,a^2\,c^3\,x^{9/2}}{9}+\frac{2\,b^2\,d^3\,x^{29/2}}{29}+\frac{2\,a\,c^2\,x^{13/2}\,\left(3\,a\,d+2\,b\,c\right)}{13}+\frac{2\,b\,d^2\,x^{25/2}\,\left(2\,a\,d+3\,b\,c\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^(17/2)*((2*b^2*c^3)/17 + (6*a^2*c*d^2)/17 + (12*a*b*c^2*d)/17) + x^(21/2)
*((2*a^2*d^3)/21 + (2*b^2*c^2*d)/7 + (4*a*b*c*d^2)/7) + (2*a^2*c^3*x^(9/2))
/9 + (2*b^2*d^3*x^(29/2))/29 + (2*a*c^2*x^(13/2)*(3*a*d + 2*b*c))/13 + (2*b
*d^2*x^(25/2)*(2*a*d + 3*b*c))/25
```

3.408 $\int x^{5/2}(a+bx^2)^2 (c+dx^2)^3 \, dx$

Optimal. Leaf size=139

$$\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2(2bc+3ad)x^{11/2}+\frac{2}{15}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{15/2}+\frac{2}{19}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{19/2}+\frac{2}{23}b$$

```
[Out] 2/7*a^2*c^3*x^(7/2)+2/11*a*c^2*(3*a*d+2*b*c)*x^(11/2)+2/15*c*(3*a^2*d^2+6*a
*b*c*d+b^2*c^2)*x^(15/2)+2/19*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(19/2)+2/23
*b*d^2*(2*a*d+3*b*c)*x^(23/2)+2/27*b^2*d^3*x^(27/2)
```

Rubi [A]
time = 0.05, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {459}

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2ad+3bc)+\frac{2}{27}b^2d^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d)*x^(23/2))/23 + (2*b^2*d^3*x^
(27/2))/27
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^{5/2}+ac^2(2bc+3ad)x^{9/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{13/2}+d(3\right.\\ &= \frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2(2bc+3ad)x^{11/2}+\frac{2}{15}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{15/2}+\end{aligned}$$

Mathematica [A]
time = 0.08, size = 126, normalized size = 0.91

$$\frac{2x^{7/2}(621a^2(1045c^3+1995c^2dx^2+1463cd^2x^4+385d^3x^6)+378abx^2(2185c^3+4807c^2dx^2+3795cd^2x^4+1045d^3x^6)+77b^2x^4(3933c^3+9315c^2dx^2+7695cd^2x^4+2185d^3x^6))}{4542615}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*x^(7/2)*(621*a^2*(1045*c^3 + 1995*c^2*d*x^2 + 1463*c*d^2*x^4 + 385*d^3*x
^6) + 378*a*b*x^2*(2185*c^3 + 4807*c^2*d*x^2 + 3795*c*d^2*x^4 + 1045*d^3*x^
6) + 77*b^2*x^4*(3933*c^3 + 9315*c^2*d*x^2 + 7695*c*d^2*x^4 + 2185*d^3*x^6)
))/4542615
```

Maple [A]
time = 0.11, size = 128, normalized size = 0.92

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{27}{2}}}{27}+\frac{2\left(2ab\,d^3+3b^2c\,d^2\right)x^{\frac{23}{2}}}{23}+\frac{2\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{\frac{19}{2}}}{19}+\frac{2\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3a^2c^2d-\right.}{}$
default	$\frac{2b^2d^3x^{\frac{27}{2}}}{27}+\frac{2\left(2ab\,d^3+3b^2c\,d^2\right)x^{\frac{23}{2}}}{23}+\frac{2\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{\frac{19}{2}}}{19}+\frac{2\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3a^2c^2d-\right.}{}$
gosper	$\frac{2x^{\frac{7}{2}}\left(168245b^2d^3x^{10}+395010ab\,d^3x^8+592515b^2c\,d^2x^8+239085a^2d^3x^6+1434510abc\,d^2x^6+717255b^2c^2d\,x^6+908523a^2c\,d^2\right.}{4542615}$
trager	$\frac{2x^{\frac{7}{2}}\left(168245b^2d^3x^{10}+395010ab\,d^3x^8+592515b^2c\,d^2x^8+239085a^2d^3x^6+1434510abc\,d^2x^6+717255b^2c^2d\,x^6+908523a^2c\,d^2\right.}{4542615}$
risch	$\frac{2x^{\frac{7}{2}}\left(168245b^2d^3x^{10}+395010ab\,d^3x^8+592515b^2c\,d^2x^8+239085a^2d^3x^6+1434510abc\,d^2x^6+717255b^2c^2d\,x^6+908523a^2c\,d^2\right.}{4542615}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*b^2*d^3*x^(27/2)+2/23*(2*a*b*d^3+3*b^2*c*d^2)*x^(23/2)+2/19*(a^2*d^3+6
*a*b*c*d^2+3*b^2*c^2*d)*x^(19/2)+2/15*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^(
15/2)+2/11*(3*a^2*c^2*d+2*a*b*c^3)*x^(11/2)+2/7*a^2*c^3*x^(7/2)
```

Maxima [A]
time = 0.30, size = 127, normalized size = 0.91

$$\frac{2}{27}\,b^2d^3x^{\frac{27}{2}}+\frac{2}{23}\,\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{23}{2}}+\frac{2}{19}\,\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{19}{2}}+\frac{2}{7}\,a^2c^3x^{\frac{7}{2}}+\frac{2}{15}\,\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{15}{2}}+\frac{2}{11}\,\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/27*b^2*d^3*x^(27/2) + 2/23*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(23/2) + 2/19*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(19/2) + 2/7*a^2*c^3*x^(7/2) + 2/15*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(15/2) + 2/11*(2*a*b*c^3 + 3*a^2*c^2*d
)*x^(11/2)
```

Fricas [A]
time = 0.78, size = 132, normalized size = 0.95

$$\frac{2}{4542615}\,\left(168245\,b^2d^3x^{13}+197505\,\left(3\,b^2cd^2+2\,abd^3\right)x^{11}+239085\,\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^9+648945\,a^2c^3x^3+302841\,\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+412965\,\left(2\,abc^3+3\,a^2c^2d\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 2/4542615*(168245*b^2*d^3*x^13 + 197505*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 23
9085*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^9 + 648945*a^2*c^3*x^3 + 30284
1*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^7 + 412965*(2*a*b*c^3 + 3*a^2*c^2
*d)*x^5)*sqrt(x)
```

Sympy [A]
time = 1.67, size = 192, normalized size = 1.38

$$\frac{2a^2c^3x^{\frac{7}{2}}}{7}+\frac{6a^2c^2dx^{\frac{11}{2}}}{11}+\frac{2a^2cd^2x^{\frac{15}{2}}}{5}+\frac{2a^2d^3x^{\frac{19}{2}}}{19}+\frac{4abc^3x^{\frac{11}{2}}}{11}+\frac{4abc^2dx^{\frac{15}{2}}}{5}+\frac{12abcd^2x^{\frac{19}{2}}}{19}+\frac{4abd^3x^{\frac{23}{2}}}{23}+\frac{2b^2c^3x^{\frac{15}{2}}}{15}+\frac{6b^2c^2dx^{\frac{19}{2}}}{19}+\frac{6b^2cd^2x^{\frac{23}{2}}}{23}+\frac{2b^2d^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(7/2)/7 + 6*a**2*c**2*d*x**(11/2)/11 + 2*a**2*c*d**2*x**(15/
2)/5 + 2*a**2*d**3*x**(19/2)/19 + 4*a*b*c**3*x**(11/2)/11 + 4*a*b*c**2*d*x*
*(15/2)/5 + 12*a*b*c*d**2*x**(19/2)/19 + 4*a*b*d**3*x**(23/2)/23 + 2*b**2*c
**3*x**(15/2)/15 + 6*b**2*c**2*d*x**(19/2)/19 + 6*b**2*c*d**2*x**(23/2)/23
+ 2*b**2*d**3*x**(27/2)/27
```

Giac [A]
time = 0.81, size = 135, normalized size = 0.97

$$\frac{2}{27}b^2d^3x^{\frac{27}{2}}+\frac{6}{23}b^2cd^2x^{\frac{23}{2}}+\frac{4}{23}abd^3x^{\frac{23}{2}}+\frac{6}{19}b^2c^2dx^{\frac{19}{2}}+\frac{12}{19}abcd^2x^{\frac{19}{2}}+\frac{2}{19}a^2d^3x^{\frac{19}{2}}+\frac{2}{15}b^2c^3x^{\frac{15}{2}}+\frac{4}{5}abc^2dx^{\frac{15}{2}}+\frac{2}{5}a^2cd^2x^{\frac{15}{2}}+\frac{4}{11}abc^3x^{\frac{11}{2}}+\frac{6}{11}a^2c^2dx^{\frac{11}{2}}+\frac{2}{7}a^2c^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/27*b^2*d^3*x^(27/2) + 6/23*b^2*c*d^2*x^(23/2) + 4/23*a*b*d^3*x^(23/2) + 6
/19*b^2*c^2*d*x^(19/2) + 12/19*a*b*c*d^2*x^(19/2) + 2/19*a^2*d^3*x^(19/2) +
 2/15*b^2*c^3*x^(15/2) + 4/5*a*b*c^2*d*x^(15/2) + 2/5*a^2*c*d^2*x^(15/2) +
4/11*a*b*c^3*x^(11/2) + 6/11*a^2*c^2*d*x^(11/2) + 2/7*a^2*c^3*x^(7/2)
```

Mupad [B]
time = 0.02, size = 119, normalized size = 0.86

$$x^{15/2}\left(\frac{2\,a^2\,c\,d^2}{5}+\frac{4\,a\,b\,c^2\,d}{5}+\frac{2\,b^2\,c^3}{15}\right)+x^{19/2}\left(\frac{2\,a^2\,d^3}{19}+\frac{12\,a\,b\,c\,d^2}{19}+\frac{6\,b^2\,c^2\,d}{19}\right)+\frac{2\,a^2\,c^3\,x^{7/2}}{7}+\frac{2\,b^2\,d^3\,x^{27/2}}{27}+\frac{2\,a\,c^2\,x^{11/2}\,\left(3\,a\,d+2\,b\,c\right)}{11}+\frac{2\,b\,d^2\,x^{23/2}\,\left(2\,a\,d+3\,b\,c\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^(15/2)*((2*b^2*c^3)/15 + (2*a^2*c*d^2)/5 + (4*a*b*c^2*d)/5) + x^(19/2)*((
2*a^2*d^3)/19 + (6*b^2*c^2*d)/19 + (12*a*b*c*d^2)/19) + (2*a^2*c^3*x^(7/2))
/7 + (2*b^2*d^3*x^(27/2))/27 + (2*a*c^2*x^(11/2)*(3*a*d + 2*b*c))/11 + (2*b
*d^2*x^(23/2)*(2*a*d + 3*b*c))/23
```

3.409 $\int x^{3/2}(a+bx^2)^2 (c+dx^2)^3 \, dx$

Optimal. Leaf size=139

$$\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2(2bc+3ad)x^{9/2}+\frac{2}{13}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{13/2}+\frac{2}{17}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{17/2}+\frac{2}{21}bd^2(3$$

```
[Out] 2/5*a^2*c^3*x^(5/2)+2/9*a*c^2*(3*a*d+2*b*c)*x^(9/2)+2/13*c*(3*a^2*d^2+6*a*b
*c*d+b^2*c^2)*x^(13/2)+2/17*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(17/2)+2/21*b
*d^2*(2*a*d+3*b*c)*x^(21/2)+2/25*b^2*d^3*x^(25/2)
```

Rubi [A]
time = 0.04, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {459}

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}(2ad+3bc)+\frac{2}{25}b^2d^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*x^(21/2))/21 + (2*b^2*d^3*x^(2
5/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^{3/2}+ac^2(2bc+3ad)x^{7/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{11/2}+d\left(3b\right.\right.\\ &= \frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2(2bc+3ad)x^{9/2}+\frac{2}{13}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{13/2}+\frac{2}{17}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 126, normalized size = 0.91

$$\frac{2x^{5/2}\left(105a^2\left(663c^3+1105c^2dx^2+765cd^2x^4+195d^3x^6\right)+50abx^2\left(1547c^3+3213c^2dx^2+2457cd^2x^4+663d^3x^6\right)+9b^2x^4\left(2975c^3+6825c^2dx^2+5525cd^2x^4+1547d^3x^6\right)\right)}{348075}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*x^(5/2)*(105*a^2*(663*c^3 + 1105*c^2*d*x^2 + 765*c*d^2*x^4 + 195*d^3*x^6
) + 50*a*b*x^2*(1547*c^3 + 3213*c^2*d*x^2 + 2457*c*d^2*x^4 + 663*d^3*x^6) +
 9*b^2*x^4*(2975*c^3 + 6825*c^2*d*x^2 + 5525*c*d^2*x^4 + 1547*d^3*x^6)))/34
8075
```

Maple [A]
time = 0.11, size = 128, normalized size = 0.92

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{25}{2}}}{25}+\frac{2\left(2ab\,d^3+3b^2c\,d^2\right)x^{\frac{21}{2}}}{21}+\frac{2\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{\frac{17}{2}}}{17}+\frac{2\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3a^2c^2\right.}{}$
default	$\frac{2b^2d^3x^{\frac{25}{2}}}{25}+\frac{2\left(2ab\,d^3+3b^2c\,d^2\right)x^{\frac{21}{2}}}{21}+\frac{2\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{\frac{17}{2}}}{17}+\frac{2\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3a^2c^2\right.}{}$
gosper	$\frac{2x^{\frac{5}{2}}\left(13923b^2d^3x^{10}+33150ab\,d^3x^8+49725b^2c\,d^2x^8+20475a^2d^3x^6+122850abc\,d^2x^6+61425b^2c^2d\,x^6+80325a^2c\,d^2x^4+1\right.}{348075}$
trager	$\frac{2x^{\frac{5}{2}}\left(13923b^2d^3x^{10}+33150ab\,d^3x^8+49725b^2c\,d^2x^8+20475a^2d^3x^6+122850abc\,d^2x^6+61425b^2c^2d\,x^6+80325a^2c\,d^2x^4+1\right.}{348075}$
risch	$\frac{2x^{\frac{5}{2}}\left(13923b^2d^3x^{10}+33150ab\,d^3x^8+49725b^2c\,d^2x^8+20475a^2d^3x^6+122850abc\,d^2x^6+61425b^2c^2d\,x^6+80325a^2c\,d^2x^4+1\right.}{348075}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*b^2*d^3*x^(25/2)+2/21*(2*a*b*d^3+3*b^2*c*d^2)*x^(21/2)+2/17*(a^2*d^3+6
*a*b*c*d^2+3*b^2*c^2*d)*x^(17/2)+2/13*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^(
13/2)+2/9*(3*a^2*c^2*d+2*a*b*c^3)*x^(9/2)+2/5*a^2*c^3*x^(5/2)
```

Maxima [A]
time = 0.27, size = 127, normalized size = 0.91

$$\frac{2}{25}\,b^2d^3x^{\frac{25}{2}}+\frac{2}{21}\,\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{21}{2}}+\frac{2}{17}\,\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{17}{2}}+\frac{2}{5}\,a^2c^3x^{\frac{5}{2}}+\frac{2}{13}\,\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{13}{2}}+\frac{2}{9}\,\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/25*b^2*d^3*x^(25/2) + 2/21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(21/2) + 2/17*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(17/2) + 2/5*a^2*c^3*x^(5/2) + 2/13*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(13/2) + 2/9*(2*a*b*c^3 + 3*a^2*c^2*d)
*x^(9/2)
```

Fricas [A]
time = 0.50, size = 132, normalized size = 0.95

$$\frac{2}{348075}\,\left(13923\,b^2d^3x^{12}+16575\,\left(3\,b^2cd^2+2\,abd^3\right)x^{10}+20475\,\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^8+69615\,a^2c^3x^2+26775\,\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^6+38675\,\left(2\,abc^3+3\,a^2c^2d\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^2*d^3*x^12 + 16575*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 20475
*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^8 + 69615*a^2*c^3*x^2 + 26775*(b^2
*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^6 + 38675*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
)*sqrt(x)
```

Sympy [A]
time = 1.19, size = 192, normalized size = 1.38

$$\frac{2a^2c^3x^{\frac{5}{2}}}{5}+\frac{2a^2c^2dx^{\frac{9}{2}}}{3}+\frac{6a^2cd^2x^{\frac{13}{2}}}{13}+\frac{2a^2d^3x^{\frac{17}{2}}}{17}+\frac{4abc^3x^{\frac{9}{2}}}{9}+\frac{12abc^2dx^{\frac{13}{2}}}{13}+\frac{12abcd^2x^{\frac{17}{2}}}{17}+\frac{4abd^3x^{\frac{21}{2}}}{21}+\frac{2b^2c^3x^{\frac{13}{2}}}{13}+\frac{6b^2c^2dx^{\frac{17}{2}}}{17}+\frac{2b^2cd^2x^{\frac{21}{2}}}{7}+\frac{2b^2d^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(5/2)/5 + 2*a**2*c**2*d*x**(9/2)/3 + 6*a**2*c*d**2*x**(13/2)
/13 + 2*a**2*d**3*x**(17/2)/17 + 4*a*b*c**3*x**(9/2)/9 + 12*a*b*c**2*d*x**(
13/2)/13 + 12*a*b*c*d**2*x**(17/2)/17 + 4*a*b*d**3*x**(21/2)/21 + 2*b**2*c*
*3*x**(13/2)/13 + 6*b**2*c**2*d*x**(17/2)/17 + 2*b**2*c*d**2*x**(21/2)/7 +
2*b**2*d**3*x**(25/2)/25
```

Giac [A]
time = 0.58, size = 135, normalized size = 0.97

$$\frac{2}{25}b^2d^3x^{\frac{25}{2}}+\frac{2}{7}b^2cd^2x^{\frac{21}{2}}+\frac{4}{21}abd^3x^{\frac{21}{2}}+\frac{6}{17}b^2c^2dx^{\frac{17}{2}}+\frac{12}{17}abcd^2x^{\frac{17}{2}}+\frac{2}{17}a^2d^3x^{\frac{17}{2}}+\frac{2}{13}b^2c^3x^{\frac{13}{2}}+\frac{12}{13}abc^2dx^{\frac{13}{2}}+\frac{6}{13}a^2cd^2x^{\frac{13}{2}}+\frac{4}{9}abc^3x^{\frac{9}{2}}+\frac{2}{3}a^2c^2dx^{\frac{9}{2}}+\frac{2}{5}a^2c^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/25*b^2*d^3*x^(25/2) + 2/7*b^2*c*d^2*x^(21/2) + 4/21*a*b*d^3*x^(21/2) + 6/
17*b^2*c^2*d*x^(17/2) + 12/17*a*b*c*d^2*x^(17/2) + 2/17*a^2*d^3*x^(17/2) +
2/13*b^2*c^3*x^(13/2) + 12/13*a*b*c^2*d*x^(13/2) + 6/13*a^2*c*d^2*x^(13/2)
+ 4/9*a*b*c^3*x^(9/2) + 2/3*a^2*c^2*d*x^(9/2) + 2/5*a^2*c^3*x^(5/2)
```

Mupad [B]
time = 0.02, size = 119, normalized size = 0.86

$$x^{13/2}\left(\frac{6\,a^2\,c\,d^2}{13}+\frac{12\,a\,b\,c^2\,d}{13}+\frac{2\,b^2\,c^3}{13}\right)+x^{17/2}\left(\frac{2\,a^2\,d^3}{17}+\frac{12\,a\,b\,c\,d^2}{17}+\frac{6\,b^2\,c^2\,d}{17}\right)+\frac{2\,a^2\,c^3\,x^{5/2}}{5}+\frac{2\,b^2\,d^3\,x^{25/2}}{25}+\frac{2\,a\,c^2\,x^{9/2}\,\left(3\,a\,d+2\,b\,c\right)}{9}+\frac{2\,b\,d^2\,x^{21/2}\,\left(2\,a\,d+3\,b\,c\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^(13/2)*((2*b^2*c^3)/13 + (6*a^2*c*d^2)/13 + (12*a*b*c^2*d)/13) + x^(17/2)
*((2*a^2*d^3)/17 + (6*b^2*c^2*d)/17 + (12*a*b*c*d^2)/17) + (2*a^2*c^3*x^(5/
2))/5 + (2*b^2*d^3*x^(25/2))/25 + (2*a*c^2*x^(9/2)*(3*a*d + 2*b*c))/9 + (2*
b*d^2*x^(21/2)*(2*a*d + 3*b*c))/21
```

3.410 $\int \sqrt{x}\,(a+bx^2)^2\,(c+dx^2)^3\,dx$

Optimal. Leaf size=139

$$\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2(2bc+3ad)x^{7/2}+\frac{2}{11}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{11/2}+\frac{2}{15}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{15/2}+\frac{2}{19}bd^2$$

```
[Out] 2/3*a^2*c^3*x^(3/2)+2/7*a*c^2*(3*a*d+2*b*c)*x^(7/2)+2/11*c*(3*a^2*d^2+6*a*b
*c*d+b^2*c^2)*x^(11/2)+2/15*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(15/2)+2/19*b
*d^2*(2*a*d+3*b*c)*x^(19/2)+2/23*b^2*d^3*x^(23/2)
```

Rubi [A]
time = 0.04, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {459}

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad+3bc)+\frac{2}{23}b^2d^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*x^(19/2))/19 + (2*b^2*d^3*x^(2
3/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \sqrt{x}\,\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=\int \left(a^2c^3\sqrt{x}+ac^2(2bc+3ad)x^{5/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{9/2}+d(3\right.$$

$$=\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2(2bc+3ad)x^{7/2}+\frac{2}{11}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{11/2}+$$

Mathematica [A]
time = 0.08, size = 126, normalized size = 0.91

$$\frac{2x^{3/2}\left(437a^2\left(385c^3+495c^2dx^2+315cd^2x^4+77d^3x^6\right)+138abx^2\left(1045c^3+1995c^2dx^2+1463cd^2x^4+385d^3x^6\right)+21b^2x^4\left(2185c^3+4807c^2dx^2+3795cd^2x^4+1045d^3x^6\right)\right)}{504735}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(437*a^2*(385*c^3 + 495*c^2*d*x^2 + 315*c*d^2*x^4 + 77*d^3*x^6)
+ 138*a*b*x^2*(1045*c^3 + 1995*c^2*d*x^2 + 1463*c*d^2*x^4 + 385*d^3*x^6) +
21*b^2*x^4*(2185*c^3 + 4807*c^2*d*x^2 + 3795*c*d^2*x^4 + 1045*d^3*x^6)))/50
4735
```

Maple [A]
time = 0.10, size = 128, normalized size = 0.92

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{23}{2}}}{23}+\frac{2\left(2ab\,d^3+3b^2c\,d^2\right)x^{\frac{19}{2}}}{19}+\frac{2\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^{\frac{11}{2}}}{11}+2\left(3a^2c^2d-\right.$
default	$\frac{2b^2d^3x^{\frac{23}{2}}}{23}+\frac{2\left(2ab\,d^3+3b^2c\,d^2\right)x^{\frac{19}{2}}}{19}+\frac{2\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^{\frac{11}{2}}}{11}+2\left(3a^2c^2d-\right.$
gosper	$\frac{2x^{\frac{3}{2}}\left(21945b^2d^3x^{10}+53130ab\,d^3x^8+79695b^2c\,d^2x^8+33649a^2d^3x^6+201894abc\,d^2x^6+100947b^2c^2d\,x^6+137655a^2c\,d^2x^4+2\right.}{504735}$
trager	$\frac{2x^{\frac{3}{2}}\left(21945b^2d^3x^{10}+53130ab\,d^3x^8+79695b^2c\,d^2x^8+33649a^2d^3x^6+201894abc\,d^2x^6+100947b^2c^2d\,x^6+137655a^2c\,d^2x^4+2\right.}{504735}$
risch	$\frac{2x^{\frac{3}{2}}\left(21945b^2d^3x^{10}+53130ab\,d^3x^8+79695b^2c\,d^2x^8+33649a^2d^3x^6+201894abc\,d^2x^6+100947b^2c^2d\,x^6+137655a^2c\,d^2x^4+2\right.}{504735}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*b^2*d^3*x^(23/2)+2/19*(2*a*b*d^3+3*b^2*c*d^2)*x^(19/2)+2/15*(a^2*d^3+6
*a*b*c*d^2+3*b^2*c^2*d)*x^(15/2)+2/11*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^(
11/2)+2/7*(3*a^2*c^2*d+2*a*b*c^3)*x^(7/2)+2/3*a^2*c^3*x^(3/2)
```

Maxima [A]
time = 0.28, size = 127, normalized size = 0.91

$$\frac{2}{23}\,b^2d^3x^{\frac{23}{2}}+\frac{2}{19}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{19}{2}}+\frac{2}{15}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{15}{2}}+\frac{2}{3}\,a^2c^3x^{\frac{3}{2}}+\frac{2}{11}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{11}{2}}+\frac{2}{7}\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/23*b^2*d^3*x^(23/2) + 2/19*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(19/2) + 2/15*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(15/2) + 2/3*a^2*c^3*x^(3/2) + 2/11*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(11/2) + 2/7*(2*a*b*c^3 + 3*a^2*c^2*d)
*x^(7/2)
```

Fricas [A]
time = 0.51, size = 130, normalized size = 0.94

$$\frac{2}{504735}\left(21945\,b^2d^3x^{11}+26565\left(3\,b^2cd^2+2\,abd^3\right)x^9+33649\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+168245\,a^2c^3x+45885\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^5+72105\left(2\,abc^3+3\,a^2c^2d\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*b^2*d^3*x^11 + 26565*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 33649*
(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^7 + 168245*a^2*c^3*x + 45885*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^5 + 72105*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3)*
sqrt(x)
```

Sympy [A]
time = 2.16, size = 155, normalized size = 1.12

$$\frac{2a^2c^3x^{\frac{3}{2}}}{3}+\frac{2b^2d^3x^{\frac{23}{2}}}{23}+\frac{2x^{\frac{19}{2}}\cdot(2abd^3+3b^2cd^2)}{19}+\frac{2x^{\frac{15}{2}}(a^2d^3+6abcd^2+3b^2c^2d)}{15}+\frac{2x^{\frac{11}{2}}\cdot(3a^2cd^2+6abc^2d+b^2c^3)}{11}+\frac{2x^{\frac{7}{2}}\cdot(3a^2c^2d+2abc^3)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3*x**(1/2),x)
```

```
[Out] 2*a**2*c**3*x**(3/2)/3 + 2*b**2*d**3*x**(23/2)/23 + 2*x**(19/2)*(2*a*b*d**3
 + 3*b**2*c*d**2)/19 + 2*x**(15/2)*(a**2*d**3 + 6*a*b*c*d**2 + 3*b**2*c**2*
d)/15 + 2*x**(11/2)*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c**3)/11 + 2*x**(7
/2)*(3*a**2*c**2*d + 2*a*b*c**3)/7
```

Giac [A]
time = 0.72, size = 135, normalized size = 0.97

$$\frac{2}{23}b^2d^3x^{\frac{23}{2}}+\frac{6}{19}b^2cd^2x^{\frac{19}{2}}+\frac{4}{19}abd^3x^{\frac{19}{2}}+\frac{2}{5}b^2c^2dx^{\frac{15}{2}}+\frac{4}{5}abcd^2x^{\frac{15}{2}}+\frac{2}{15}a^2d^3x^{\frac{15}{2}}+\frac{2}{11}b^2c^3x^{\frac{11}{2}}+\frac{12}{11}abc^2dx^{\frac{11}{2}}+\frac{6}{11}a^2cd^2x^{\frac{11}{2}}+\frac{4}{7}abc^3x^{\frac{7}{2}}+\frac{6}{7}a^2c^2dx^{\frac{7}{2}}+\frac{2}{3}a^2c^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/23*b^2*d^3*x^(23/2) + 6/19*b^2*c*d^2*x^(19/2) + 4/19*a*b*d^3*x^(19/2) + 2
/5*b^2*c^2*d*x^(15/2) + 4/5*a*b*c*d^2*x^(15/2) + 2/15*a^2*d^3*x^(15/2) + 2/
11*b^2*c^3*x^(11/2) + 12/11*a*b*c^2*d*x^(11/2) + 6/11*a^2*c*d^2*x^(11/2) +
4/7*a*b*c^3*x^(7/2) + 6/7*a^2*c^2*d*x^(7/2) + 2/3*a^2*c^3*x^(3/2)
```

Mupad [B]
time = 0.02, size = 119, normalized size = 0.86

$$x^{11/2}\left(\frac{6\,a^2\,c\,d^2}{11}+\frac{12\,a\,b\,c^2\,d}{11}+\frac{2\,b^2\,c^3}{11}\right)+x^{15/2}\left(\frac{2\,a^2\,d^3}{15}+\frac{4\,a\,b\,c\,d^2}{5}+\frac{2\,b^2\,c^2\,d}{5}\right)+\frac{2\,a^2\,c^3\,x^{3/2}}{3}+\frac{2\,b^2\,d^3\,x^{23/2}}{23}+\frac{2\,a\,c^2\,x^{7/2}\,(3\,a\,d+2\,b\,c)}{7}+\frac{2\,b\,d^2\,x^{19/2}\,(2\,a\,d+3\,b\,c)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^(11/2)*((2*b^2*c^3)/11 + (6*a^2*c*d^2)/11 + (12*a*b*c^2*d)/11) + x^(15/2)
*((2*a^2*d^3)/15 + (2*b^2*c^2*d)/5 + (4*a*b*c*d^2)/5) + (2*a^2*c^3*x^(3/2))
/3 + (2*b^2*d^3*x^(23/2))/23 + (2*a*c^2*x^(7/2)*(3*a*d + 2*b*c))/7 + (2*b*d
^2*x^(19/2)*(2*a*d + 3*b*c))/19
```

3.411 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=137

$$2a^2c^3\sqrt{x}+\frac{2}{5}ac^2(2bc+3ad)x^{5/2}+\frac{2}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{9/2}+\frac{2}{13}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{13/2}+\frac{2}{17}bd^2(3bc$$

```
[Out] 2/5*a*c^2*(3*a*d+2*b*c)*x^(5/2)+2/9*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^(9/2)
+2/13*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(13/2)+2/17*b*d^2*(2*a*d+3*b*c)*x^(
17/2)+2/21*b^2*d^3*x^(21/2)+2*a^2*c^3*x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {459}

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad+3bc)+\frac{2}{21}b^2d^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x
^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/2))/17 + (2*b^2*d^3*x^(21/2))/
21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^2c^3}{\sqrt{x}}+ac^2(2bc+3ad)x^{3/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{7/2}+d\left(3b^2c^2+6a\right.\right.\\ &= 2a^2c^3\sqrt{x}+\frac{2}{5}ac^2(2bc+3ad)x^{5/2}+\frac{2}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{9/2}+\frac{2}{13}d\left(3b^2c\right.\end{aligned}$$

Mathematica [A]
time = 0.07, size = 126, normalized size = 0.92

$$\frac{2\sqrt{x}\left(357a^2(195c^3+117c^2dx^2+65cd^2x^4+15d^3x^6)+42abx^2(663c^3+1105c^2dx^2+765cd^2x^4+195d^3x^6)+5b^2x^4(1547c^3+3213c^2dx^2+2457cd^2x^4+663d^3x^6)\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(357*a^2*(195*c^3 + 117*c^2*d*x^2 + 65*c*d^2*x^4 + 15*d^3*x^6) +
 42*a*b*x^2*(663*c^3 + 1105*c^2*d*x^2 + 765*c*d^2*x^4 + 195*d^3*x^6) + 5*b^
2*x^4*(1547*c^3 + 3213*c^2*d*x^2 + 2457*c*d^2*x^4 + 663*d^3*x^6)))/69615
```

Maple [A]
time = 0.12, size = 128, normalized size = 0.93

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{21}{2}}}{21}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{17}{2}}}{17}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{13}{2}}}{13}+\frac{2(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^{\frac{9}{2}}}{9}+\frac{2(3a^2c^2d$
default	$\frac{2b^2d^3x^{\frac{21}{2}}}{21}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{17}{2}}}{17}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{13}{2}}}{13}+\frac{2(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^{\frac{9}{2}}}{9}+\frac{2(3a^2c^2d$
trager	$\left(\frac{2}{21}b^2d^3x^{10}+\frac{4}{17}ab\,d^3x^8+\frac{6}{17}b^2c\,d^2x^8+\frac{2}{13}a^2d^3x^6+\frac{12}{13}abc\,d^2x^6+\frac{6}{13}b^2c^2d\,x^6+\frac{2}{3}a^2c\,d^2x^4\right.$
gosper	$\frac{2\sqrt{x}\,(3315b^2d^3x^{10}+8190ab\,d^3x^8+12285b^2c\,d^2x^8+5355a^2d^3x^6+32130abc\,d^2x^6+16065b^2c^2d\,x^6+23205a^2c\,d^2x^4+464}{69615}$
risch	$\frac{2\sqrt{x}\,(3315b^2d^3x^{10}+8190ab\,d^3x^8+12285b^2c\,d^2x^8+5355a^2d^3x^6+32130abc\,d^2x^6+16065b^2c^2d\,x^6+23205a^2c\,d^2x^4+464}{69615}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*b^2*d^3*x^(21/2)+2/17*(2*a*b*d^3+3*b^2*c*d^2)*x^(17/2)+2/13*(a^2*d^3+6
*a*b*c*d^2+3*b^2*c^2*d)*x^(13/2)+2/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^(9
/2)+2/5*(3*a^2*c^2*d+2*a*b*c^3)*x^(5/2)+2*a^2*c^3*x^(1/2)
```

Maxima [A]
time = 0.29, size = 127, normalized size = 0.93

$$\frac{2}{21}\,b^2d^3x^{\frac{21}{2}}+\frac{2}{17}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{17}{2}}+\frac{2}{13}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{13}{2}}+2\,a^2c^3\sqrt{x}+\frac{2}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{9}{2}}+\frac{2}{5}\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d^3*x^(21/2) + 2/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(17/2) + 2/13*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(13/2) + 2*a^2*c^3*sqrt(x) + 2/9*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(9/2) + 2/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(
5/2)
```

Fricas [A]
time = 0.47, size = 129, normalized size = 0.94

$$\frac{2}{69615}\left(3315\,b^2d^3x^{10}+4095\left(3\,b^2cd^2+2\,abd^3\right)x^8+5355\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6+69615\,a^2c^3+7735\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+13923\left(2\,abc^3+3\,a^2c^2d\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")

[Out] 2/69615*(3315*b^2*d^3*x^10 + 4095*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 5355*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 + 69615*a^2*c^3 + 7735*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 13923*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt(x)

Sympy [A]
time = 0.79, size = 190, normalized size = 1.39

$$2a^2c^3\sqrt{x} + \frac{6a^2c^2dx^{\frac{5}{2}}}{5} + \frac{2a^2cd^2x^{\frac{9}{2}}}{3} + \frac{2a^2d^3x^{\frac{13}{2}}}{13} + \frac{4abc^3x^{\frac{5}{2}}}{5} + \frac{4abc^2dx^{\frac{9}{2}}}{3} + \frac{12abcd^2x^{\frac{13}{2}}}{13} + \frac{4abd^3x^{\frac{17}{2}}}{17} + \frac{2b^2c^3x^{\frac{9}{2}}}{9} + \frac{6b^2c^2dx^{\frac{13}{2}}}{13} + \frac{6b^2cd^2x^{\frac{17}{2}}}{17} + \frac{2b^2d^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(1/2),x)

[Out] 2*a**2*c**3*sqrt(x) + 6*a**2*c**2*d*x**(5/2)/5 + 2*a**2*c*d**2*x**(9/2)/3 + 2*a**2*d**3*x**(13/2)/13 + 4*a*b*c**3*x**(5/2)/5 + 4*a*b*c**2*d*x**(9/2)/3 + 12*a*b*c*d**2*x**(13/2)/13 + 4*a*b*d**3*x**(17/2)/17 + 2*b**2*c**3*x**(9/2)/9 + 6*b**2*c**2*d*x**(13/2)/13 + 6*b**2*c*d**2*x**(17/2)/17 + 2*b**2*d**3*x**(21/2)/21

Giac [A]
time = 0.80, size = 135, normalized size = 0.99

$$\frac{2}{21}b^2d^3x^{\frac{21}{2}} + \frac{6}{17}b^2cd^2x^{\frac{17}{2}} + \frac{4}{17}abd^3x^{\frac{17}{2}} + \frac{6}{13}b^2c^2dx^{\frac{13}{2}} + \frac{12}{13}abcd^2x^{\frac{13}{2}} + \frac{2}{13}a^2d^3x^{\frac{13}{2}} + \frac{2}{9}b^2c^3x^{\frac{9}{2}} + \frac{4}{3}abc^2dx^{\frac{9}{2}} + \frac{2}{3}a^2cd^2x^{\frac{9}{2}} + \frac{4}{5}abc^3x^{\frac{5}{2}} + \frac{6}{5}a^2c^2dx^{\frac{5}{2}} + 2a^2c^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="giac")

[Out] 2/21*b^2*d^3*x^(21/2) + 6/17*b^2*c*d^2*x^(17/2) + 4/17*a*b*d^3*x^(17/2) + 6/13*b^2*c^2*d*x^(13/2) + 12/13*a*b*c*d^2*x^(13/2) + 2/13*a^2*d^3*x^(13/2) + 2/9*b^2*c^3*x^(9/2) + 4/3*a*b*c^2*d*x^(9/2) + 2/3*a^2*c*d^2*x^(9/2) + 4/5*a*b*c^3*x^(5/2) + 6/5*a^2*c^2*d*x^(5/2) + 2*a^2*c^3*sqrt(x)

Mupad [B]
time = 0.02, size = 119, normalized size = 0.87

$$x^{9/2}\left(\frac{2a^2cd^2}{3}+\frac{4abc^2d}{3}+\frac{2b^2c^3}{9}\right)+x^{13/2}\left(\frac{2a^2d^3}{13}+\frac{12abcd^2}{13}+\frac{6b^2c^2d}{13}\right)+2a^2c^3\sqrt{x}+\frac{2b^2d^3x^{21/2}}{21}+\frac{2ac^2x^{5/2}(3ad+2bc)}{5}+\frac{2bd^2x^{17/2}(2ad+3bc)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(1/2),x)

[Out] x^(9/2)*((2*b^2*c^3)/9 + (2*a^2*c*d^2)/3 + (4*a*b*c^2*d)/3) + x^(13/2)*((2*a^2*d^3)/13 + (6*b^2*c^2*d)/13 + (12*a*b*c*d^2)/13) + 2*a^2*c^3*x^(1/2) + (2*b^2*d^3*x^(21/2))/21 + (2*a*c^2*x^(5/2)*(3*a*d + 2*b*c))/5 + (2*b*d^2*x^(17/2)*(2*a*d + 3*b*c))/17

3.412 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2(2bc+3ad)x^{3/2}+\frac{2}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{7/2}+\frac{2}{11}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{11/2}+\frac{2}{15}bd^2(3bc$$

```
[Out] 2/3*a*c^2*(3*a*d+2*b*c)*x^(3/2)+2/7*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^(7/2)
+2/11*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(11/2)+2/15*b*d^2*(2*a*d+3*b*c)*x^(
15/2)+2/19*b^2*d^3*x^(19/2)-2*a^2*c^3/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {459}

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc)+\frac{2}{19}b^2d^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(15/2))/15 + (2*b^2*d^3*x^(19/2
))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx=\int \left(\frac{a^2c^3}{x^{3/2}}+ac^2(2bc+3ad)\sqrt{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{5/2}+d\left(3b^2c^2+6\right.\right.$$
$$=-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2(2bc+3ad)x^{3/2}+\frac{2}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{7/2}+\frac{2}{11}d\left(3b^2c^2\right.$$

Mathematica [A]
time = 0.07, size = 126, normalized size = 0.92

$$\frac{-570a^2(77c^3-77c^2dx^2-33cd^2x^4-7d^3x^6)+76abx^2(385c^3+495c^2dx^2+315cd^2x^4+77d^3x^6)+6b^2x^4(1045c^3+1995c^2dx^2+1463cd^2x^4+385d^3x^6)}{21945\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

```
[Out] (-570*a^2*(77*c^3 - 77*c^2*d*x^2 - 33*c*d^2*x^4 - 7*d^3*x^6) + 76*a*b*x^2*(
385*c^3 + 495*c^2*d*x^2 + 315*c*d^2*x^4 + 77*d^3*x^6) + 6*b^2*x^4*(1045*c^3
 + 1995*c^2*d*x^2 + 1463*c*d^2*x^4 + 385*d^3*x^6))/(21945*Sqrt[x])
```

Maple [A]
time = 0.08, size = 136, normalized size = 0.99

method	result
derivativedivides	$\frac{2b^{2}d^{3}x^{\frac{19}{2}}}{19}+\frac{4ab\,d^{3}x^{\frac{15}{2}}}{15}+\frac{2b^{2}c\,d^{2}x^{\frac{15}{2}}}{5}+\frac{2a^{2}d^{3}x^{\frac{11}{2}}}{11}+\frac{12abc\,d^{2}x^{\frac{11}{2}}}{11}+\frac{6b^{2}c^{2}d\,x^{\frac{11}{2}}}{11}+\frac{6a^{2}c\,d^{2}x^{\frac{7}{2}}}{7}+\frac{12ab\,c^{2}d\,x^{\frac{7}{2}}}{7}$
default	$\frac{2b^{2}d^{3}x^{\frac{19}{2}}}{19}+\frac{4ab\,d^{3}x^{\frac{15}{2}}}{15}+\frac{2b^{2}c\,d^{2}x^{\frac{15}{2}}}{5}+\frac{2a^{2}d^{3}x^{\frac{11}{2}}}{11}+\frac{12abc\,d^{2}x^{\frac{11}{2}}}{11}+\frac{6b^{2}c^{2}d\,x^{\frac{11}{2}}}{11}+\frac{6a^{2}c\,d^{2}x^{\frac{7}{2}}}{7}+\frac{12ab\,c^{2}d\,x^{\frac{7}{2}}}{7}$
gosper	$-\frac{2\left(-1155b^{2}d^{3}x^{10}-2926ab\,d^{3}x^{8}-4389b^{2}c\,d^{2}x^{8}-1995a^{2}d^{3}x^{6}-11970abc\,d^{2}x^{6}-5985b^{2}c^{2}d\,x^{6}-9405a^{2}c\,d^{2}x^{4}-18810ab\,c^{2}\right.}{21945\sqrt{x}}$
trager	$-\frac{2\left(-1155b^{2}d^{3}x^{10}-2926ab\,d^{3}x^{8}-4389b^{2}c\,d^{2}x^{8}-1995a^{2}d^{3}x^{6}-11970abc\,d^{2}x^{6}-5985b^{2}c^{2}d\,x^{6}-9405a^{2}c\,d^{2}x^{4}-18810ab\,c^{2}\right.}{21945\sqrt{x}}$
risch	$-\frac{2\left(-1155b^{2}d^{3}x^{10}-2926ab\,d^{3}x^{8}-4389b^{2}c\,d^{2}x^{8}-1995a^{2}d^{3}x^{6}-11970abc\,d^{2}x^{6}-5985b^{2}c^{2}d\,x^{6}-9405a^{2}c\,d^{2}x^{4}-18810ab\,c^{2}\right.}{21945\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*b^2*d^3*x^(19/2)+4/15*a*b*d^3*x^(15/2)+2/5*b^2*c*d^2*x^(15/2)+2/11*a^2
*d^3*x^(11/2)+12/11*a*b*c*d^2*x^(11/2)+6/11*b^2*c^2*d*x^(11/2)+6/7*a^2*c*d^
2*x^(7/2)+12/7*a*b*c^2*d*x^(7/2)+2/7*b^2*c^3*x^(7/2)+2*a^2*c^2*d*x^(3/2)+4/
3*a*b*c^3*x^(3/2)-2*a^2*c^3/x^(1/2)
```

Maxima [A]
time = 0.28, size = 127, normalized size = 0.93

$$\frac{2}{19}\,b^{2}d^{3}x^{\frac{19}{2}}+\frac{2}{15}\,(3\,b^{2}cd^{2}+2\,abd^{3})x^{\frac{15}{2}}+\frac{2}{11}\,(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3})x^{\frac{11}{2}}-\frac{2\,a^{2}c^{3}}{\sqrt{x}}+\frac{2}{7}\,(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2})x^{\frac{7}{2}}+\frac{2}{3}\,(2\,abc^{3}+3\,a^{2}c^{2}d)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d^3*x^(19/2) + 2/15*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(15/2) + 2/11*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(11/2) - 2*a^2*c^3/sqrt(x) + 2/7*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(7/2) + 2/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(
3/2)
```

Fricas [A]
time = 0.46, size = 129, normalized size = 0.94

$$\frac{2\,(1155\,b^{2}d^{3}x^{10}+1463\,(3\,b^{2}cd^{2}+2\,abd^{3})x^{8}+1995\,(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3})x^{6}-21945\,a^{2}c^{3}+3135\,(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2})x^{4}+7315\,(2\,abc^{3}+3\,a^{2}c^{2}d)x^{2})}{21945\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d^3*x^10 + 1463*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1995*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 21945*a^2*c^3 + 3135*(b^2*c^3 + 6*a
*b*c^2*d + 3*a^2*c*d^2)*x^4 + 7315*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/sqrt(x)
```

Sympy [A]
time = 1.03, size = 189, normalized size = 1.38

$$-\frac{2a^2c^3}{\sqrt{x}}+2a^2c^2dx^{\frac{3}{2}}+\frac{6a^2cd^2x^{\frac{7}{2}}}{7}+\frac{2a^2d^3x^{\frac{11}{2}}}{11}+\frac{4abc^3x^{\frac{3}{2}}}{3}+\frac{12abc^2dx^{\frac{7}{2}}}{7}+\frac{12abcd^2x^{\frac{11}{2}}}{11}+\frac{4abd^3x^{\frac{15}{2}}}{15}+\frac{2b^2c^3x^{\frac{7}{2}}}{7}+\frac{6b^2c^2dx^{\frac{11}{2}}}{11}+\frac{2b^2cd^2x^{\frac{15}{2}}}{5}+\frac{2b^2d^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(3/2),x)
```

```
[Out] -2*a**2*c**3/sqrt(x) + 2*a**2*c**2*d*x**(3/2) + 6*a**2*c*d**2*x**(7/2)/7 +
2*a**2*d**3*x**(11/2)/11 + 4*a*b*c**3*x**(3/2)/3 + 12*a*b*c**2*d*x**(7/2)/7
 + 12*a*b*c*d**2*x**(11/2)/11 + 4*a*b*d**3*x**(15/2)/15 + 2*b**2*c**3*x**(7
/2)/7 + 6*b**2*c**2*d*x**(11/2)/11 + 2*b**2*c*d**2*x**(15/2)/5 + 2*b**2*d**
3*x**(19/2)/19
```

Giac [A]
time = 0.69, size = 135, normalized size = 0.99

$$\frac{2}{19}b^2d^3x^{\frac{19}{2}}+\frac{2}{5}b^2cd^2x^{\frac{15}{2}}+\frac{4}{15}abd^3x^{\frac{15}{2}}+\frac{6}{11}b^2c^2dx^{\frac{11}{2}}+\frac{12}{11}abcd^2x^{\frac{11}{2}}+\frac{2}{11}a^2d^3x^{\frac{11}{2}}+\frac{2}{7}b^2c^3x^{\frac{7}{2}}+\frac{12}{7}abc^2dx^{\frac{7}{2}}+\frac{6}{7}a^2cd^2x^{\frac{7}{2}}+\frac{4}{3}abc^3x^{\frac{3}{2}}+2a^2c^2dx^{\frac{3}{2}}-\frac{2a^2c^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d^3*x^(19/2) + 2/5*b^2*c*d^2*x^(15/2) + 4/15*a*b*d^3*x^(15/2) + 6/
11*b^2*c^2*d*x^(11/2) + 12/11*a*b*c*d^2*x^(11/2) + 2/11*a^2*d^3*x^(11/2) +
2/7*b^2*c^3*x^(7/2) + 12/7*a*b*c^2*d*x^(7/2) + 6/7*a^2*c*d^2*x^(7/2) + 4/3*
a*b*c^3*x^(3/2) + 2*a^2*c^2*d*x^(3/2) - 2*a^2*c^3/sqrt(x)
```

Mupad [B]
time = 0.02, size = 119, normalized size = 0.87

$$x^{7/2}\left(\frac{6a^2cd^2}{7}+\frac{12abc^2d}{7}+\frac{2b^2c^3}{7}\right)+x^{11/2}\left(\frac{2a^2d^3}{11}+\frac{12abcd^2}{11}+\frac{6b^2c^2d}{11}\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2b^2d^3x^{19/2}}{19}+\frac{2ac^2x^{3/2}(3ad+2bc)}{3}+\frac{2bd^2x^{15/2}(2ad+3bc)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x)
```

```
[Out] x^(7/2)*((2*b^2*c^3)/7 + (6*a^2*c*d^2)/7 + (12*a*b*c^2*d)/7) + x^(11/2)*((2
*a^2*d^3)/11 + (6*b^2*c^2*d)/11 + (12*a*b*c*d^2)/11) - (2*a^2*c^3)/x^(1/2)
+ (2*b^2*d^3*x^(19/2))/19 + (2*a*c^2*x^(3/2)*(3*a*d + 2*b*c))/3 + (2*b*d^2*
x^(15/2)*(2*a*d + 3*b*c))/15
```

3.413 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{2a^2c^3}{3x^{3/2}}+2ac^2(2bc+3ad)\sqrt{x}+\frac{2}{5}c(b^2c^2+6abcd+3a^2d^2)\,x^{5/2}+\frac{2}{9}d(3b^2c^2+6abcd+a^2d^2)\,x^{9/2}+\frac{2}{13}bd^2(3bc+2a$$

```
[Out] -2/3*a^2*c^3/x^(3/2)+2/5*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^(5/2)+2/9*d*(a^2
*d^2+6*a*b*c*d+3*b^2*c^2)*x^(9/2)+2/13*b*d^2*(2*a*d+3*b*c)*x^(13/2)+2/17*b^
2*d^3*x^(17/2)+2*a*c^2*(3*a*d+2*b*c)*x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {459}

$$\frac{2}{9}dx^{9/2}(a^2d^2+6abcd+3b^2c^2)+\frac{2}{5}cx^{5/2}(3a^2d^2+6abcd+b^2c^2)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}\,(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3bc)+\frac{2}{17}b^2d^3x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13/2))/13 + (2*b^2*d^3*x^(17/2))
/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx=\int\left(\frac{a^2c^3}{x^{5/2}}+\frac{ac^2(2bc+3ad)}{\sqrt{x}}+c(b^2c^2+6abcd+3a^2d^2)\,x^{3/2}+d(3b^2c^2+6abcd$$

$$=-\frac{2a^2c^3}{3x^{3/2}}+2ac^2(2bc+3ad)\sqrt{x}+\frac{2}{5}c(b^2c^2+6abcd+3a^2d^2)\,x^{5/2}+\frac{2}{9}d(3b^2c^2+$$

Mathematica [A]
time = 0.09, size = 126, normalized size = 0.92

$$\frac{-442a^2(15c^3-135c^2dx^2-27cd^2x^4-5d^3x^6)+204abx^2(195c^3+117c^2dx^2+65cd^2x^4+15d^3x^6)+6b^2x^4(663c^3+1105c^2dx^2+765cd^2x^4+195d^3x^6)}{9945x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

```
[Out] (-442*a^2*(15*c^3 - 135*c^2*d*x^2 - 27*c*d^2*x^4 - 5*d^3*x^6) + 204*a*b*x^2
*(195*c^3 + 117*c^2*d*x^2 + 65*c*d^2*x^4 + 15*d^3*x^6) + 6*b^2*x^4*(663*c^3
 + 1105*c^2*d*x^2 + 765*c*d^2*x^4 + 195*d^3*x^6))/(9945*x^(3/2))
```

Maple [A]
time = 0.08, size = 136, normalized size = 0.99

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{17}{2}}}{17}+\frac{4ab\,d^3x^{\frac{13}{2}}}{13}+\frac{6b^2c\,d^2x^{\frac{13}{2}}}{13}+\frac{2a^2d^3x^{\frac{9}{2}}}{9}+\frac{4abc\,d^2x^{\frac{9}{2}}}{3}+\frac{2b^2c^2d\,x^{\frac{9}{2}}}{3}+\frac{6a^2c\,d^2x^{\frac{5}{2}}}{5}+\frac{12ab\,c^2d\,x^{\frac{5}{2}}}{5}+$
default	$\frac{2b^2d^3x^{\frac{17}{2}}}{17}+\frac{4ab\,d^3x^{\frac{13}{2}}}{13}+\frac{6b^2c\,d^2x^{\frac{13}{2}}}{13}+\frac{2a^2d^3x^{\frac{9}{2}}}{9}+\frac{4abc\,d^2x^{\frac{9}{2}}}{3}+\frac{2b^2c^2d\,x^{\frac{9}{2}}}{3}+\frac{6a^2c\,d^2x^{\frac{5}{2}}}{5}+\frac{12ab\,c^2d\,x^{\frac{5}{2}}}{5}+$
gosper	$-\frac{2(-585b^2d^3x^{10}-1530ab\,d^3x^8-2295b^2c\,d^2x^8-1105a^2d^3x^6-6630abc\,d^2x^6-3315b^2c^2d\,x^6-5967a^2c\,d^2x^4-11934ab\,c^2}{9945x^{\frac{3}{2}}}$
trager	$-\frac{2(-585b^2d^3x^{10}-1530ab\,d^3x^8-2295b^2c\,d^2x^8-1105a^2d^3x^6-6630abc\,d^2x^6-3315b^2c^2d\,x^6-5967a^2c\,d^2x^4-11934ab\,c^2}{9945x^{\frac{3}{2}}}$
risch	$-\frac{2(-585b^2d^3x^{10}-1530ab\,d^3x^8-2295b^2c\,d^2x^8-1105a^2d^3x^6-6630abc\,d^2x^6-3315b^2c^2d\,x^6-5967a^2c\,d^2x^4-11934ab\,c^2}{9945x^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*b^2*d^3*x^(17/2)+4/13*a*b*d^3*x^(13/2)+6/13*b^2*c*d^2*x^(13/2)+2/9*a^2
*d^3*x^(9/2)+4/3*a*b*c*d^2*x^(9/2)+2/3*b^2*c^2*d*x^(9/2)+6/5*a^2*c*d^2*x^(5
/2)+12/5*a*b*c^2*d*x^(5/2)+2/5*b^2*c^3*x^(5/2)+6*a^2*c^2*d*x^(1/2)+4*a*b*c^
3*x^(1/2)-2/3*a^2*c^3/x^(3/2)
```

Maxima [A]
time = 0.27, size = 127, normalized size = 0.93

$$\frac{2}{17}\,b^2d^3x^{\frac{17}{2}}+\frac{2}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{13}{2}}+\frac{2}{9}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{9}{2}}-\frac{2\,a^2c^3}{3\,x^{\frac{3}{2}}}+\frac{2}{5}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{5}{2}}+2\left(2\,abc^3+3\,a^2c^2d\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d^3*x^(17/2) + 2/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(13/2) + 2/9*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(9/2) - 2/3*a^2*c^3/x^(3/2) + 2/5*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(5/2) + 2*(2*a*b*c^3 + 3*a^2*c^2*d)*sqrt(
x)
```

Fricas [A]
time = 0.49, size = 129, normalized size = 0.94

$$\frac{2\left(585\,b^2d^3x^{10}+765\left(3\,b^2cd^2+2\,abd^3\right)x^8+1105\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-3315\,a^2c^3+1989\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+9945\left(2\,abc^3+3\,a^2c^2d\right)x^2\right)}{9945\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="fricas")

[Out] 2/9945*(585*b^2*d^3*x^10 + 765*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1105*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 3315*a^2*c^3 + 1989*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 9945*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(3/2)

Sympy [A]
time = 1.12, size = 189, normalized size = 1.38

$$-\frac{2a^2c^3}{3x^{\frac{3}{2}}}+6a^2c^2d\sqrt{x}+\frac{6a^2cd^2x^{\frac{5}{2}}}{5}+\frac{2a^2d^3x^{\frac{9}{2}}}{9}+4abc^3\sqrt{x}+\frac{12abc^2dx^{\frac{5}{2}}}{5}+\frac{4abcd^2x^{\frac{9}{2}}}{3}+\frac{4abd^3x^{\frac{13}{2}}}{13}+\frac{2b^2c^3x^{\frac{5}{2}}}{5}+\frac{2b^2c^2dx^{\frac{9}{2}}}{3}+\frac{6b^2cd^2x^{\frac{13}{2}}}{13}+\frac{2b^2d^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(5/2),x)

[Out] -2*a**2*c**3/(3*x**(3/2)) + 6*a**2*c**2*d*sqrt(x) + 6*a**2*c*d**2*x**(5/2)/5 + 2*a**2*d**3*x**(9/2)/9 + 4*a*b*c**3*sqrt(x) + 12*a*b*c**2*d*x**(5/2)/5 + 4*a*b*c*d**2*x**(9/2)/3 + 4*a*b*d**3*x**(13/2)/13 + 2*b**2*c**3*x**(5/2)/5 + 2*b**2*c**2*d*x**(9/2)/3 + 6*b**2*c*d**2*x**(13/2)/13 + 2*b**2*d**3*x**(17/2)/17

Giac [A]
time = 1.32, size = 135, normalized size = 0.99

$$\frac{2}{17}b^2d^3x^{\frac{17}{2}}+\frac{6}{13}b^2cd^2x^{\frac{13}{2}}+\frac{4}{13}abd^3x^{\frac{13}{2}}+\frac{2}{3}b^2c^2dx^{\frac{9}{2}}+\frac{4}{3}abcd^2x^{\frac{9}{2}}+\frac{2}{9}a^2d^3x^{\frac{9}{2}}+\frac{2}{5}b^2c^3x^{\frac{5}{2}}+\frac{12}{5}abc^2dx^{\frac{5}{2}}+\frac{6}{5}a^2cd^2x^{\frac{5}{2}}+4abc^3\sqrt{x}+6a^2c^2d\sqrt{x}-\frac{2a^2c^3}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="giac")

[Out] 2/17*b^2*d^3*x^(17/2) + 6/13*b^2*c*d^2*x^(13/2) + 4/13*a*b*d^3*x^(13/2) + 2/3*b^2*c^2*d*x^(9/2) + 4/3*a*b*c*d^2*x^(9/2) + 2/9*a^2*d^3*x^(9/2) + 2/5*b^2*c^3*x^(5/2) + 12/5*a*b*c^2*d*x^(5/2) + 6/5*a^2*c*d^2*x^(5/2) + 4*a*b*c^3*sqrt(x) + 6*a^2*c^2*d*sqrt(x) - 2/3*a^2*c^3/x^(3/2)

Mupad [B]
time = 0.02, size = 119, normalized size = 0.87

$$x^{5/2}\left(\frac{6a^2cd^2}{5}+\frac{12abc^2d}{5}+\frac{2b^2c^3}{5}\right)+x^{9/2}\left(\frac{2a^2d^3}{9}+\frac{4abcd^2}{3}+\frac{2b^2c^2d}{3}\right)-\frac{2a^2c^3}{3x^{3/2}}+\frac{2b^2d^3x^{17/2}}{17}+2ac^2\sqrt{x}\,(3ad+2bc)+\frac{2bd^2x^{13/2}(2ad+3bc)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x)

[Out] x^(5/2)*((2*b^2*c^3)/5 + (6*a^2*c*d^2)/5 + (12*a*b*c^2*d)/5) + x^(9/2)*((2*a^2*d^3)/9 + (2*b^2*c^2*d)/3 + (4*a*b*c*d^2)/3) - (2*a^2*c^3)/(3*x^(3/2)) + (2*b^2*d^3*x^(17/2))/17 + 2*a*c^2*x^(1/2)*(3*a*d + 2*b*c) + (2*b*d^2*x^(13/2)*(2*a*d + 3*b*c))/13

3.414 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(2bc+3ad)}{\sqrt{x}}+\frac{2}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{3/2}+\frac{2}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{7/2}+\frac{2}{11}bd^2(3bc+2ac$$

```
[Out] -2/5*a^2*c^3/x^(5/2)+2/3*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^(3/2)+2/7*d*(a^2
*d^2+6*a*b*c*d+3*b^2*c^2)*x^(7/2)+2/11*b*d^2*(2*a*d+3*b*c)*x^(11/2)+2/15*b^
2*d^3*x^(15/2)-2*a*c^2*(3*a*d+2*b*c)/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{459\}$

$$\frac{2}{7}dx^{7/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{3}cx^{3/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(3ad+2bc)}{\sqrt{x}}+\frac{2}{11}bd^2x^{11/2}(2ad+3bc)+\frac{2}{15}b^2d^3x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

```
[Out] (-2*a^2*c^3)/(5*x^(5/2)) - (2*a*c^2*(2*b*c + 3*a*d))/Sqrt[x] + (2*c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^(3/2))/3 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^(7/2))/7 + (2*b*d^2*(3*b*c + 2*a*d)*x^(11/2))/11 + (2*b^2*d^3*x^(15/2
))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx = \int \left(\frac{a^2c^3}{x^{7/2}}+\frac{ac^2(2bc+3ad)}{x^{3/2}}+c\left(b^2c^2+6abcd+3a^2d^2\right)\sqrt{x}+d\left(3b^2c^2+6abc\right.$$

$$= -\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(2bc+3ad)}{\sqrt{x}}+\frac{2}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{3/2}+\frac{2}{7}d\left(3b^2c^2+6a\right.$$

Mathematica [A]
time = 0.07, size = 126, normalized size = 0.92

$$\frac{-66a^2(7c^3+105c^2dx^2-35cd^2x^4-5d^3x^6)+60abx^2(-77c^3+77c^2dx^2+33cd^2x^4+7d^3x^6)+2b^2x^4(385c^3+495c^2dx^2+315cd^2x^4+77d^3x^6)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

```
[Out] (-66*a^2*(7*c^3 + 105*c^2*d*x^2 - 35*c*d^2*x^4 - 5*d^3*x^6) + 60*a*b*x^2*(-
77*c^3 + 77*c^2*d*x^2 + 33*c*d^2*x^4 + 7*d^3*x^6) + 2*b^2*x^4*(385*c^3 + 49
5*c^2*d*x^2 + 315*c*d^2*x^4 + 77*d^3*x^6))/(1155*x^(5/2))
```

Maple [A]
time = 0.08, size = 132, normalized size = 0.96

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{15}{2}}}{15}+\frac{4ab\,d^3x^{\frac{11}{2}}}{11}+\frac{6b^2c\,d^2x^{\frac{11}{2}}}{11}+\frac{2a^2d^3x^{\frac{7}{2}}}{7}+\frac{12abc\,d^2x^{\frac{7}{2}}}{7}+\frac{6b^2c^2d\,x^{\frac{7}{2}}}{7}+2a^2c\,d^2x^{\frac{3}{2}}+4ab\,c^2d\,x^{\frac{3}{2}}$
default	$\frac{2b^2d^3x^{\frac{15}{2}}}{15}+\frac{4ab\,d^3x^{\frac{11}{2}}}{11}+\frac{6b^2c\,d^2x^{\frac{11}{2}}}{11}+\frac{2a^2d^3x^{\frac{7}{2}}}{7}+\frac{12abc\,d^2x^{\frac{7}{2}}}{7}+\frac{6b^2c^2d\,x^{\frac{7}{2}}}{7}+2a^2c\,d^2x^{\frac{3}{2}}+4ab\,c^2d\,x^{\frac{3}{2}}$
gosper	$-\frac{2(-77b^2d^3x^{10}-210ab\,d^3x^8-315b^2c\,d^2x^8-165a^2d^3x^6-990abc\,d^2x^6-495b^2c^2d\,x^6-1155a^2c\,d^2x^4-2310ab\,c^2d\,x^4-385b}{1155x^{\frac{5}{2}}}$
trager	$-\frac{2(-77b^2d^3x^{10}-210ab\,d^3x^8-315b^2c\,d^2x^8-165a^2d^3x^6-990abc\,d^2x^6-495b^2c^2d\,x^6-1155a^2c\,d^2x^4-2310ab\,c^2d\,x^4-385b}{1155x^{\frac{5}{2}}}$
risch	$-\frac{2(-77b^2d^3x^{10}-210ab\,d^3x^8-315b^2c\,d^2x^8-165a^2d^3x^6-990abc\,d^2x^6-495b^2c^2d\,x^6-1155a^2c\,d^2x^4-2310ab\,c^2d\,x^4-385b}{1155x^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*b^2*d^3*x^(15/2)+4/11*a*b*d^3*x^(11/2)+6/11*b^2*c*d^2*x^(11/2)+2/7*a^2
*d^3*x^(7/2)+12/7*a*b*c*d^2*x^(7/2)+6/7*b^2*c^2*d*x^(7/2)+2*a^2*c*d^2*x^(3/
2)+4*a*b*c^2*d*x^(3/2)+2/3*b^2*c^3*x^(3/2)-2/5*a^2*c^3/x^(5/2)-2*a*c^2*(3*a
*d+2*b*c)/x^(1/2)
```

Maxima [A]
time = 0.31, size = 129, normalized size = 0.94

$$\frac{2}{15}\,b^2d^3x^{\frac{15}{2}}+\frac{2}{11}\,(3\,b^2cd^2+2\,abd^3)x^{\frac{11}{2}}+\frac{2}{7}\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^{\frac{7}{2}}+\frac{2}{3}\,(b^2c^3+6\,abc^2d+3\,a^2cd^2)x^{\frac{3}{2}}-\frac{2\,(a^2c^3+5\,(2\,abc^3+3\,a^2c^2d)x^2)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d^3*x^(15/2) + 2/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(11/2) + 2/7*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(7/2) + 2/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a
^2*c*d^2)*x^(3/2) - 2/5*(a^2*c^3 + 5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(5/2)
```

Fricas [A]
time = 0.50, size = 129, normalized size = 0.94

$$\frac{2\,(77\,b^2d^3x^{10}+105\,(3\,b^2cd^2+2\,abd^3)x^8+165\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^6-231\,a^2c^3+385\,(b^2c^3+6\,abc^2d+3\,a^2cd^2)x^4-1155\,(2\,abc^3+3\,a^2c^2d)x^2)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^3*x^10 + 105*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 165*(3*b^2*c^
2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 231*a^2*c^3 + 385*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^4 - 1155*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(5/2)
```

Sympy [A]
time = 1.31, size = 185, normalized size = 1.35

$$-\frac{2a^2c^3}{5x^{\frac{5}{2}}}-\frac{6a^2c^2d}{\sqrt{x}}+2a^2cd^2x^{\frac{3}{2}}+\frac{2a^2d^3x^{\frac{7}{2}}}{7}-\frac{4abc^3}{\sqrt{x}}+4abc^2dx^{\frac{3}{2}}+\frac{12abcd^2x^{\frac{7}{2}}}{7}+\frac{4abd^3x^{\frac{11}{2}}}{11}+\frac{2b^2c^3x^{\frac{3}{2}}}{3}+\frac{6b^2c^2dx^{\frac{7}{2}}}{7}+\frac{6b^2cd^2x^{\frac{11}{2}}}{11}+\frac{2b^2d^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(7/2),x)
```

```
[Out] -2*a**2*c**3/(5*x**(5/2)) - 6*a**2*c**2*d/sqrt(x) + 2*a**2*c*d**2*x**(3/2)
+ 2*a**2*d**3*x**(7/2)/7 - 4*a*b*c**3/sqrt(x) + 4*a*b*c**2*d*x**(3/2) + 12*
a*b*c*d**2*x**(7/2)/7 + 4*a*b*d**3*x**(11/2)/11 + 2*b**2*c**3*x**(3/2)/3 +
6*b**2*c**2*d*x**(7/2)/7 + 6*b**2*c*d**2*x**(11/2)/11 + 2*b**2*d**3*x**(15/
2)/15
```

Giac [A]
time = 0.95, size = 137, normalized size = 1.00

$$\frac{2}{15}b^2d^3x^{\frac{15}{2}}+\frac{6}{11}b^2cd^2x^{\frac{11}{2}}+\frac{4}{11}abd^3x^{\frac{11}{2}}+\frac{6}{7}b^2c^2dx^{\frac{7}{2}}+\frac{12}{7}abcd^2x^{\frac{7}{2}}+\frac{2}{7}a^2d^3x^{\frac{7}{2}}+\frac{2}{3}b^2c^3x^{\frac{3}{2}}+4\,abc^2dx^{\frac{3}{2}}+2\,a^2cd^2x^{\frac{3}{2}}-\frac{2\,(10\,abc^3x^2+15\,a^2c^2dx^2+a^2c^3)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d^3*x^(15/2) + 6/11*b^2*c*d^2*x^(11/2) + 4/11*a*b*d^3*x^(11/2) + 6
/7*b^2*c^2*d*x^(7/2) + 12/7*a*b*c*d^2*x^(7/2) + 2/7*a^2*d^3*x^(7/2) + 2/3*b
^2*c^3*x^(3/2) + 4*a*b*c^2*d*x^(3/2) + 2*a^2*c*d^2*x^(3/2) - 2/5*(10*a*b*c^
3*x^2 + 15*a^2*c^2*d*x^2 + a^2*c^3)/x^(5/2)
```

Mupad [B]
time = 0.02, size = 125, normalized size = 0.91

$$x^{3/2}\left(2\,a^2\,c\,d^2+4\,a\,b\,c^2\,d+\frac{2\,b^2\,c^3}{3}\right)+x^{7/2}\left(\frac{2\,a^2\,d^3}{7}+\frac{12\,a\,b\,c\,d^2}{7}+\frac{6\,b^2\,c^2\,d}{7}\right)-\frac{x^2\,(6\,d\,a^2\,c^2+4\,b\,a\,c^3)+\frac{2\,a^2\,c^3}{5}}{x^{5/2}}+\frac{2\,b^2\,d^3\,x^{15/2}}{15}+\frac{2\,b\,d^2\,x^{11/2}\,(2\,a\,d+3\,b\,c)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x)
```

```
[Out] x^(3/2)*((2*b^2*c^3)/3 + 2*a^2*c*d^2 + 4*a*b*c^2*d) + x^(7/2)*((2*a^2*d^3)/
7 + (6*b^2*c^2*d)/7 + (12*a*b*c*d^2)/7) - (x^2*(6*a^2*c^2*d + 4*a*b*c^3) +
(2*a^2*c^3)/5)/x^(5/2) + (2*b^2*d^3*x^(15/2))/15 + (2*b*d^2*x^(11/2)*(2*a*d
 + 3*b*c))/11
```

3.415 $\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=311

$$-\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{17/4}}$$

```
[Out] 2/5*(-a*d+b*c)^2*x^(5/2)/d^3-2/9*b*(-2*a*d+b*c)*x^(9/2)/d^2+2/13*b^2*x^(13/
2)/d-1/2*c^(5/4)*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(
17/4)*2^(1/2)+1/2*c^(5/4)*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(
1/4))/d^(17/4)*2^(1/2)-1/4*c^(5/4)*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(1/4
)*d^(1/4)*2^(1/2)*x^(1/2))/d^(17/4)*2^(1/2)+1/4*c^(5/4)*(-a*d+b*c)^2*ln(c^(
1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(17/4)*2^(1/2)-2*c*(-a*d+
b*c)^2*x^(1/2)/d^4
```

Rubi [A]
time = 0.22, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {472, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{c^{5/4}(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{17/4}}-\frac{c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{17/4}}-\frac{2c\sqrt{x}\,(bc-ad)^2}{d^4}+\frac{2x^{5/2}(bc-ad)^2}{5d^3}-\frac{2bx^{9/2}(bc-2ad)}{9d^2}+\frac{2b^2x^{13/2}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-2*c*(b*c - a*d)^2*Sqrt[x])/d^4 + (2*(b*c - a*d)^2*x^(5/2))/(5*d^3) - (2*b
*(b*c - 2*a*d)*x^(9/2))/(9*d^2) + (2*b^2*x^(13/2))/(13*d) - (c^(5/4)*(b*c -
 a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(17/4)) +
 (c^(5/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqr
t[2]*d^(17/4)) - (c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(17/4)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)x^{7/2}}{d^2}+\frac{b^2x^{11/2}}{d}+\frac{(b^2c^2-2abcd+a^2d^2)\,x^{7/2}}{d^2\,(c+dx^2)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{(bc-ad)^2\int \frac{x^{7/2}}{c+dx^2}\,dx}{d^2}$$

$$= \frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{(c(bc-ad)^2)\int \frac{x^{3/2}}{c+dx^2}\,dx}{d^3}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{(c^2(bc-ad)^2 \cdots}{\cdots}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{(2c^2(bc-ad \cdots}{\cdots}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{(c^{3/2}(bc-ad \cdots}{\cdots}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{(c^{3/2}(bc-ad \cdots}{\cdots}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{c^{5/4}(bc-ad) \cdots}{\cdots}$$

$$= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{c^{5/4}(bc-ad) \cdots}{\cdots}$$

Mathematica [A]
time = 0.24, size = 219, normalized size = 0.70

$$\frac{2\sqrt{x}\,(117a^2d^2(-5c+dx^2)+26abd(45c^2-9cdx^2+5d^2x^4)+b^2(-585c^3+117c^2dx^2-65cd^2x^4+45d^3x^6))}{585d^4}-\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt{2}\,d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,d^{17/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]

[Out] (2*Sqrt[x]*(117*a^2*d^2*(-5*c + d*x^2) + 26*a*b*d*(45*c^2 - 9*c*d*x^2 + 5*d^2*x^4) + b^2*(-585*c^3 + 117*c^2*d*x^2 - 65*c*d^2*x^4 + 45*d^3*x^6)))/(585*d^4) - (c^(5/4)*(b*c - a*d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*d^(17/4))

Maple [A]

time = 0.13, size = 230, normalized size = 0.74

method	result
derivativedivides	$-\frac{2\left(-\frac{b^{2} x^{\frac{13}{2}} d^{3}}{13}+\frac{\left(-(a d-b c) b\,d^{2}-a b\,d^{3}\right) x^{\frac{9}{2}}}{9}+\frac{\left(-(a d-b c) a\,d^{2}+b d\left(a c d-b\,c^{2}\right)\right) x^{\frac{5}{2}}}{5}+(a d-b c)\left(a c d-b\,c^{2}\right) \sqrt{x}\right)}{d^{4}}+\frac{c\left(a^{2} d^{2}-\right.}{}$
default	$-\frac{2\left(-\frac{b^{2} x^{\frac{13}{2}} d^{3}}{13}+\frac{\left(-(a d-b c) b\,d^{2}-a b\,d^{3}\right) x^{\frac{9}{2}}}{9}+\frac{\left(-(a d-b c) a\,d^{2}+b d\left(a c d-b\,c^{2}\right)\right) x^{\frac{5}{2}}}{5}+(a d-b c)\left(a c d-b\,c^{2}\right) \sqrt{x}\right)}{d^{4}}+\frac{c\left(a^{2} d^{2}-\right.}{}$
risch	$-\frac{2\left(-45 b^{2} d^{3} x^{6}-130 a b\,d^{3} x^{4}+65 b^{2} c\,d^{2} x^{4}-117 a^{2} d^{3} x^{2}+234 a b c\,d^{2} x^{2}-117 b^{2} c^{2} d\,x^{2}+585 a^{2} c\,d^{2}-1170 a b\,c^{2} d+585 b^{2} c^{3}\right) \sqrt{}}{585 d^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)

[Out] -2/d^4*(-1/13*b^2*x^(13/2)*d^3+1/9*(-(a*d-b*c)*b*d^2-a*b*d^3)*x^(9/2)+1/5*(-(a*d-b*c)*a*d^2+b*d*(a*c*d-b*c^2))*x^(5/2)+(a*d-b*c)*(a*c*d-b*c^2)*x^(1/2))+1/4*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))

Maxima [A]

time = 0.52, size = 360, normalized size = 1.16

$$\frac{\left(\frac{2\sqrt{2}\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)c^{2}}{4\,d^{4}}+\frac{2\left(45\,b^{2}d^{3}x^{\frac{13}{2}}-65\left(b^{2}cd^{2}-2\,abd^{3}\right)x^{\frac{9}{2}}+117\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)x^{\frac{5}{2}}-585\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)\sqrt{x}\right)}{585\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")

[Out] 1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/

```
(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4))
 - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c^2/d^4 + 2/585*(45*b^2*d^3*
x^(13/2) - 65*(b^2*c*d^2 - 2*a*b*d^3)*x^(9/2) + 117*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*x^(5/2) - 585*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x))/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1334 vs. $2(230) = 460$.

time = 0.63, size = 1334, normalized size = 4.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/1170*(2340*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^
3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d
^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*arctan((sqrt(d^8*sqrt(-(b^8
*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4
*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 +
a^8*c^5*d^8)/d^17) + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b
*c^3*d^3 + a^4*c^2*d^4)*x)*d^13*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c
^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 2
8*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(3/4) - (b^2*c^3*d
^13 - 2*a*b*c^2*d^14 + a^2*c*d^15)*sqrt(x)*(-(b^8*c^13 - 8*a*b^7*c^12*d + 2
8*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*
c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(3/4))/
(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70
*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^
7 + a^8*c^5*d^8)) + 585*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*
d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^
6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*log(d^4*(-(b^8*c
^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b
^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^
8*c^5*d^8)/d^17)^(1/4) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) - 585
*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d
^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b
*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*log(-d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d +
 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^
3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)
 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) + 4*(45*b^2*d^3*x^6 - 585*b
^2*c^3 + 1170*a*b*c^2*d - 585*a^2*c*d^2 - 65*(b^2*c*d^2 - 2*a*b*d^3)*x^4 +
117*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(x))/d^4
```

Sympy [A]

time = 93.27, size = 561, normalized size = 1.80

$$\begin{cases} \tilde{\infty}\left(\frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{13}{2}}}{13}\right) & \text{for } c = 0 \wedge d = 0 \\ \frac{\frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{13}{2}}}{13}}{d} & \text{for } c = 0 \\ \frac{\frac{2a^2x^{\frac{9}{2}}}{9} + \frac{4abx^{\frac{13}{2}}}{13} + \frac{2b^2x^{\frac{17}{2}}}{17}}{c} & \text{for } d = 0 \\ -\frac{2a^2c\sqrt{x}}{d^2} - \frac{a^2c\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{2d^2} + \frac{a^2c\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{2d^2} + \frac{a^2c\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^2} + \frac{2a^2x^{\frac{5}{2}}}{5d} + \frac{4abc^2\sqrt{x}}{d^3} + \frac{abc^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{d^3} - \frac{abc^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{d^3} - \frac{2abc^2\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^3} - \frac{4abcx^{\frac{5}{2}}}{5d^2} + \frac{4abx^{\frac{9}{2}}}{9d} - \frac{2b^2c^3\sqrt{x}}{d^4} - \frac{b^2c^3\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{2d^4} + \frac{b^2c^3\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{2d^4} + \frac{b^2c^3\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^4} + \frac{2b^2c^2x^{\frac{5}{2}}}{5d^3} - \frac{2b^2cx^{\frac{9}{2}}}{9d^2} + \frac{2b^2x^{\frac{13}{2}}}{13d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13), Eq(c, 0) & Eq(d, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13)/d, Eq(c, 0)), ((2*a**2*x**(9/2)/9 + 4*a*b*x**(13/2)/13 + 2*b**2*x**(17/2)/17)/c, Eq(d, 0)), (-2*a**2*c*sqrt(x)/d**2 - a**2*c*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*d**2) + a**2*c*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d**2) + a**2*c*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d**2 + 2*a**2*x**(5/2)/(5*d) + 4*a*b*c**2*sqrt(x)/d**3 + a*b*c**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/d**3 - a*b*c**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/d**3 - 2*a*b*c**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d**3 - 4*a*b*c*x**(5/2)/(5*d**2) + 4*a*b*x**(9/2)/(9*d) - 2*b**2*c**3*sqrt(x)/d**4 - b**2*c**3*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*d**4) + b**2*c**3*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d**4) + b**2*c**3*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d**4 + 2*b**2*c**2*x**(5/2)/(5*d**3) - 2*b**2*c*x**(9/2)/(9*d**2) + 2*b**2*x**(13/2)/(13*d), True))
```

Giac [A]
time = 1.07, size = 436, normalized size = 1.40

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^3 - 2\left(cd^3\right)^{\frac{1}{4}}abc^2d + \left(cd^3\right)^{\frac{1}{4}}a^2cd^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^5} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^3 - 2\left(cd^3\right)^{\frac{1}{4}}abc^2d + \left(cd^3\right)^{\frac{1}{4}}a^2cd^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^5} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^3 - 2\left(cd^3\right)^{\frac{1}{4}}abc^2d + \left(cd^3\right)^{\frac{1}{4}}a^2cd^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4d^5} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^3 - 2\left(cd^3\right)^{\frac{1}{4}}abc^2d + \left(cd^3\right)^{\frac{1}{4}}a^2cd^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4d^5} + \frac{2\left(45b^2d^{12}x^{\frac{13}{2}} - 65b^2cd^{11}x^{\frac{9}{2}} + 130abd^{12}x^{\frac{9}{2}} + 117b^2c^2d^{10}x^{\frac{5}{2}} - 234abcd^{11}x^{\frac{5}{2}} + 117a^2d^{12}x^{\frac{5}{2}} - 585b^2c^3d^9\sqrt{x} + 1170abc^2d^{10}\sqrt{x} - 585a^2cd^{11}\sqrt{x}\right)}{585d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/d^5 + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/d^5 + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^5 - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^5 + 2/585*(45*b^2*d^12*x^(13/2) - 65*b^2*c*d^11*x^(9/2) + 130*a*b*d^12*x^(9/2) + 117*b^2*c^2*d^10*x^(5/2) - 234*a*b*c*d^11*x^(5/2) + 117*a^2*d^12*x^(5/2) - 585*b^2*c^3*d^9*sqrt(x) + 1170*a*b*c^2*d^10*sqrt(x) - 585*a^2*c*d^11*sqrt(x))/d^13
```

Mupad [B]
time = 0.23, size = 1202, normalized size = 3.86

$$x^{5/2}\left(\frac{2a^2}{5d} + \frac{c\left(\frac{2b^2c}{d^2} - \frac{4ab}{d}\right)}{5d}\right) - x^{9/2}\left(\frac{2b^2c}{9d^2} - \frac{4ab}{9d}\right) + \frac{2b^2x^{13/2}}{13d} - \frac{c\sqrt{x}\left(\frac{2a^2}{d} + \frac{c\left(\frac{2b^2c}{d^2} - \frac{4ab}{d}\right)}{d}\right)}{d} + \frac{(-c)^{5/4}\operatorname{atan}\left(\frac{\frac{(-c)^{5/4}(ad-bc)^2\left(\frac{16\sqrt{x}\left(a^4c^4d^4-4a^3bc^5d^3+6a^2b^2c^6d^2-4ab^3c^7d+b^4c^8\right)}{d^5} - \frac{16(-c)^{5/4}(ad-bc)^2\left(a^2c^3d^2-2abc^4d+b^2c^5\right)}{d^{21/4}}\right)1i}{2d^{17/4}} + \frac{(-c)^{5/4}(ad-bc)^2\left(\frac{16\sqrt{x}\left(a^4c^4d^4-4a^3bc^5d^3+6a^2b^2c^6d^2-4ab^3c^7d+b^4c^8\right)}{d^5} + \frac{16(-c)^{5/4}(ad-bc)^2\left(a^2c^3d^2-2abc^4d+b^2c^5\right)}{d^{21/4}}\right)1i}{2d^{17/4}}}{\frac{(-c)^{5/4}(ad-bc)^2\left(\frac{16\sqrt{x}\left(a^4c^4d^4-4a^3bc^5d^3+6a^2b^2c^6d^2-4ab^3c^7d+b^4c^8\right)}{d^5} - \frac{16(-c)^{5/4}(ad-bc)^2\left(a^2c^3d^2-2abc^4d+b^2c^5\right)}{d^{21/4}}\right)}{2d^{17/4}} - \frac{(-c)^{5/4}(ad-bc)^2\left(\frac{16\sqrt{x}\left(a^4c^4d^4-4a^3bc^5d^3+6a^2b^2c^6d^2-4ab^3c^7d+b^4c^8\right)}{d^5} + \frac{16(-c)^{5/4}(ad-bc)^2\left(a^2c^3d^2-2abc^4d+b^2c^5\right)}{d^{21/4}}\right)}{2d^{17/4}}}\right)(ad-bc)^2\,1i}{d^{17/4}} + \frac{(-c)^{5/4}\operatorname{atan}\left(\frac{\frac{(-c)^{5/4}(ad-bc)^2\left(\frac{16\sqrt{x}\left(a^4c^4d^4-4a^3bc^5d^3+6a^2b^2c^6d^2-4ab^3c^7d+b^4c^8\right)}{d^5} - \frac{(-c)^{5/4}(ad-bc)^2\left(a^2c^3d^2-2abc^4d+b^2c^5\right)16i}{d^{21/4}}\right)}{2d^{17/4}} + \frac{(-c)^{5/4}(ad-bc)^2\left(\frac{16\sqrt{x}\left(a^4c^4d^4-4a^3bc^5d^3+6a^2b^2c^6d^2-4ab^3c^7d+b^4c^8\right)}{d^5} + \frac{(-c)^{5/4}(ad-bc)^2\left(a^2c^3d^2-2abc^4d+b^2c^5\right)16i}{d^{21/4}}\right)}{2d^{17/4}}}{\frac{(-c)^{5/4}(ad-bc)^2\left(\frac{16\sqrt{x}\left(a^4c^4d^4-4a^3bc^5d^3+6a^2b^2c^6d^2-4ab^3c^7d+b^4c^8\right)}{d^5} - \frac{(-c)^{5/4}(ad-bc)^2\left(a^2c^3d^2-2abc^4d+b^2c^5\right)16i}{d^{21/4}}\right)1i}{2d^{17/4}} - \frac{(-c)^{5/4}(ad-bc)^2\left(\frac{16\sqrt{x}\left(a^4c^4d^4-4a^3bc^5d^3+6a^2b^2c^6d^2-4ab^3c^7d+b^4c^8\right)}{d^5} + \frac{(-c)^{5/4}(ad-bc)^2\left(a^2c^3d^2-2abc^4d+b^2c^5\right)16i}{d^{21/4}}\right)1i}{2d^{17/4}}}\right)(ad-bc)^2}{d^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x)

[Out] x^(5/2)*((2*a^2)/(5*d) + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/(5*d)) - x^(9/2)*((2*b^2*c)/(9*d^2) - (4*a*b)/(9*d)) + (2*b^2*x^(13/2))/(13*d) - (c*x^(1/2)*((2*a^2)/d + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/d))/d + ((-c)^(5/4)*atan((((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - (16*(-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4))*1i)/(2*d^(17/4)) + ((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 + (16*(-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4))*1i)/(2*d^(17/4)))/(((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - (16*(-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4)))/(2*d^(17/4)) - ((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 + (16*(-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4)))/(2*d^(17/4))))*(a*d - b*c)^2*1i)/d^(17/4) + ((-c)^(5/4)*atan((((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - ((-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)*16i)/d^(21/4)))/(2*d^(17/4)) + ((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 + ((-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)*16i)/d^(21/4)))/(2*d^(17/4)))/(((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - ((-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)*16i)/d^(21/4))*1i)/(2*d^(17/4)) - ((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 + ((-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)*16i)/d^(21/4))*1i)/(2*d^(17/4))))*(a*d - b*c)^2)/d^(17/4)

3.416 $\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=290

$$\frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{15/4}}-\frac{c^{3/4}(bc-ad)^2\tan}{\sqrt{}}$$

```
[Out] 2/3*(-a*d+b*c)^2*x^(3/2)/d^3-2/7*b*(-2*a*d+b*c)*x^(7/2)/d^2+2/11*b^2*x^(11/
2)/d+1/2*c^(3/4)*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(
15/4)*2^(1/2)-1/2*c^(3/4)*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(
1/4))/d^(15/4)*2^(1/2)-1/4*c^(3/4)*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(1/4
)*d^(1/4)*2^(1/2)*x^(1/2))/d^(15/4)*2^(1/2)+1/4*c^(3/4)*(-a*d+b*c)^2*ln(c^(
1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(15/4)*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {472, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{c^{3/4}(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{15/4}}-\frac{c^{3/4}(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{15/4}}-\frac{c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{15/4}}+\frac{2x^{3/2}(bc-ad)^2}{3d^3}-\frac{2bx^{7/2}(bc-2ad)}{7d^2}+\frac{2b^2x^{11/2}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*(b*c - a*d)^2*x^(3/2))/(3*d^3) - (2*b*(b*c - 2*a*d)*x^(7/2))/(7*d^2) + (
2*b^2*x^(11/2))/(11*d) + (c^(3/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*d)^2*ArcTan[1 +
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*
d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]
*d^(15/4)) + (c^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)x^{5/2}}{d^2}+\frac{b^2x^{9/2}}{d}+\frac{(b^2c^2-2abcd+a^2d^2)\,x^{5/2}}{d^2\,(c+dx^2)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{(bc-ad)^2\int\frac{x^{5/2}}{c+dx^2}\,dx}{d^2}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{(c(bc-ad)^2)\int\frac{\sqrt{x}}{c+dx^2}\,dx}{d^3}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{(2c(bc-ad)^2)\,\mathrm{Subst}\left(\int\frac{x^2}{c+dx^4}\,dx,\right.}{d^3}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{(c(bc-ad)^2)\,\mathrm{Subst}\left(\int\frac{\sqrt{c}-\sqrt{d}\,x}{c+dx^4}\right.}{d^{7/2}}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{(c(bc-ad)^2)\,\mathrm{Subst}\left(\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}}{}}\right.}{2d^4}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\right.}{2\sqrt{2}\,d^{15/4}}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}\right.}{\sqrt{2}\,d^{15/4}}$$

Mathematica [A]
time = 0.21, size = 187, normalized size = 0.64

$$\frac{2x^{3/2}(77a^2d^2+22abd(-7c+3dx^2)+b^2(77c^2-33cdx^2+21d^2x^4))}{231d^3}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt{2}\,d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*x^(3/2)*(77*a^2*d^2 + 22*a*b*d*(-7*c + 3*d*x^2) + b^2*(77*c^2 - 33*c*d*x
^2 + 21*d^2*x^4)))/(231*d^3) + (c^(3/4)*(b*c - a*d)^2*ArcTan[(Sqrt[c] - Sqr
t[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(Sqrt[2]*d^(15/4)) + (c^(3/4)*(
b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x
)])/(Sqrt[2]*d^(15/4))
```

Maple [A]
time = 0.11, size = 192, normalized size = 0.66

method	result
derivativedivides	$\frac{\frac{2b^2d^2x^{\frac{11}{2}}}{11}+\frac{2\left(2ab\,d^2-b^2cd\right)x^{\frac{7}{2}}}{7}+\frac{2\left(a^2d^2-2abcd+b^2c^2\right)x^{\frac{3}{2}}}{3}}{d^3}-\frac{c\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{4d}$
default	$\frac{\frac{2b^2d^2x^{\frac{11}{2}}}{11}+\frac{2\left(2ab\,d^2-b^2cd\right)x^{\frac{7}{2}}}{7}+\frac{2\left(a^2d^2-2abcd+b^2c^2\right)x^{\frac{3}{2}}}{3}}{d^3}-\frac{c\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{4d}$
risch	$\frac{2x^{\frac{3}{2}}\left(21b^2d^2x^4+66ab\,d^2x^2-33b^2cd\,x^2+77a^2d^2-154abcd+77b^2c^2\right)}{231d^3}-\frac{c\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2}{2d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{c^2\sqrt{2}\,a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/11*b^2*d^2*x^(11/2)+1/7*(2*a*b*d^2-b^2*c*d)*x^(7/2)+1/3*(a^2*d^2-2
*a*b*c*d+b^2*c^2)*x^(3/2))-1/4*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4/(c/d)^(1/4
)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 263, normalized size = 0.91

$$-\frac{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\,d^3}+\frac{2\,\left(21\,b^2d^2x^{\frac{11}{2}}-33\,\left(b^2cd-2\,abd^2\right)x^{\frac{7}{2}}+77\,\left(b^2c^2-2\,abcd+a^2d^2\right)x^{\frac{3}{2}}\right)}{231\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqr
t(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt
(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt
(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(
```

```
c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
 + sqrt(c))/(c^(1/4)*d^(3/4)))/d^3 + 2/231*(21*b^2*d^2*x^(11/2) - 33*(b^2*c
*d - 2*a*b*d^2)*x^(7/2) + 77*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^(3/2))/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1701 vs. $2(211) = 422$.
time = 0.65, size = 1701, normalized size = 5.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/462*(924*d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b
^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*arctan((sqrt((b^12*c^16 - 12*a*
b^11*c^15*d + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^1
2*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 +
 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a^11
*b*c^5*d^11 + a^12*c^4*d^12)*x - (b^8*c^11*d^7 - 8*a*b^7*c^10*d^8 + 28*a^2*
b^6*c^9*d^9 - 56*a^3*b^5*c^8*d^10 + 70*a^4*b^4*c^7*d^11 - 56*a^5*b^3*c^6*d^
12 + 28*a^6*b^2*c^5*d^13 - 8*a^7*b*c^4*d^14 + a^8*c^3*d^15)*sqrt(-(b^8*c^11
 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^
7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3
*d^8)/d^15))*d^4*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3
*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6
 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4) - (b^6*c^8*d^4 - 6*a*b^5*c^7*
d^5 + 15*a^2*b^4*c^6*d^6 - 20*a^3*b^3*c^5*d^7 + 15*a^4*b^2*c^4*d^8 - 6*a^5*
b*c^3*d^9 + a^6*c^2*d^10)*sqrt(x)*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6
*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 2
8*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4))/(b^8*c^11 -
 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*
d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d
^8)) - 231*d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b
^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*log(d^11*(-(b^8*c^11 - 8*a*b^7*
c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*
a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)
^(3/4) + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3
 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x)) + 231*d^3*(
-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*
a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7
 + a^8*c^3*d^8)/d^15)^(1/4)*log(-d^11*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2
*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5
 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(3/4) + (b^6*c
```

```
^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c
^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x)) + 4*(21*b^2*d^2*x^5 - 33*(
b^2*c*d - 2*a*b*d^2)*x^3 + 77*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)*sqrt(x))/d
^3
```

Sympy [A]
time = 226.34, size = 440, normalized size = 1.52

$$a^{2}\left(\begin{cases}\tilde{\infty}x^{\frac{3}{2}} & \text{for}\: c = 0 \wedge d = 0\\\frac{2x^{\frac{3}{2}}}{3d} & \text{for}\: c = 0\\\frac{2x^{\frac{7}{2}}}{7c} & \text{for}\: d = 0\\-\frac{c\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d^{2}\sqrt[4]{-\frac{c}{d}}}+\frac{c\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d^{2}\sqrt[4]{-\frac{c}{d}}}-\frac{c\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^{2}\sqrt[4]{-\frac{c}{d}}}+\frac{2x^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+2ab\left(\begin{cases}\tilde{\infty}x^{\frac{7}{2}} & \text{for}\: c = 0 \wedge d = 0\\\frac{2x^{\frac{7}{2}}}{7d} & \text{for}\: c = 0\\\frac{2x^{\frac{11}{2}}}{11c} & \text{for}\: d = 0\\\frac{c^{2}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d^{3}\sqrt[4]{-\frac{c}{d}}}-\frac{c^{2}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d^{3}\sqrt[4]{-\frac{c}{d}}}+\frac{c^{2}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^{3}\sqrt[4]{-\frac{c}{d}}}-\frac{2cx^{\frac{3}{2}}}{3d^{2}}+\frac{2x^{\frac{7}{2}}}{7d} & \text{otherwise}\end{cases}\right)+b^{2}\left(\begin{cases}\tilde{\infty}x^{\frac{11}{2}} & \text{for}\: c = 0 \wedge d = 0\\\frac{2x^{\frac{11}{2}}}{11d} & \text{for}\: c = 0\\\frac{2x^{\frac{15}{2}}}{15c} & \text{for}\: d = 0\\-\frac{c^{3}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d^{4}\sqrt[4]{-\frac{c}{d}}}+\frac{c^{3}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d^{4}\sqrt[4]{-\frac{c}{d}}}-\frac{c^{3}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^{4}\sqrt[4]{-\frac{c}{d}}}+\frac{2c^{2}x^{\frac{3}{2}}}{3d^{3}}-\frac{2cx^{\frac{7}{2}}}{7d^{2}}+\frac{2x^{\frac{11}{2}}}{11d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] a**2*Piecewise((zoo*x**(3/2), Eq(c, 0) & Eq(d, 0)), (2*x**(3/2)/(3*d), Eq(c
, 0)), (2*x**(7/2)/(7*c), Eq(d, 0)), (-c*log(sqrt(x) - (-c/d)**(1/4))/(2*d*
*2*(-c/d)**(1/4)) + c*log(sqrt(x) + (-c/d)**(1/4))/(2*d**2*(-c/d)**(1/4)) -
 c*atan(sqrt(x)/(-c/d)**(1/4))/(d**2*(-c/d)**(1/4)) + 2*x**(3/2)/(3*d), Tru
e)) + 2*a*b*Piecewise((zoo*x**(7/2), Eq(c, 0) & Eq(d, 0)), (2*x**(7/2)/(7*d
), Eq(c, 0)), (2*x**(11/2)/(11*c), Eq(d, 0)), (c**2*log(sqrt(x) - (-c/d)**(
1/4))/(2*d**3*(-c/d)**(1/4)) - c**2*log(sqrt(x) + (-c/d)**(1/4))/(2*d**3*(-
c/d)**(1/4)) + c**2*atan(sqrt(x)/(-c/d)**(1/4))/(d**3*(-c/d)**(1/4)) - 2*c*
x**(3/2)/(3*d**2) + 2*x**(7/2)/(7*d), True)) + b**2*Piecewise((zoo*x**(11/2
), Eq(c, 0) & Eq(d, 0)), (2*x**(11/2)/(11*d), Eq(c, 0)), (2*x**(15/2)/(15*c
), Eq(d, 0)), (-c**3*log(sqrt(x) - (-c/d)**(1/4))/(2*d**4*(-c/d)**(1/4)) +
c**3*log(sqrt(x) + (-c/d)**(1/4))/(2*d**4*(-c/d)**(1/4)) - c**3*atan(sqrt(x
)/(-c/d)**(1/4))/(d**4*(-c/d)**(1/4)) + 2*c**2*x**(3/2)/(3*d**3) - 2*c*x**(
7/2)/(7*d**2) + 2*x**(11/2)/(11*d), True))
```

Giac [A]
time = 0.78, size = 385, normalized size = 1.33

$$-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{6}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{6}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{6}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{6}}+\frac{2\left(21b^{2}d^{10}x^{\frac{11}{2}}-33b^{2}cd^{9}x^{\frac{7}{2}}+66abd^{10}x^{\frac{7}{2}}+77b^{2}c^{2}d^{8}x^{\frac{3}{2}}-154abcd^{9}x^{\frac{3}{2}}+77a^{2}d^{10}x^{\frac{3}{2}}\right)}{231d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4
))/d^6 - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*
d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(
c/d)^(1/4))/d^6 + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*
c*d + (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/d^6 - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*
d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^6 +
```

```
2/231*(21*b^2*d^10*x^(11/2) - 33*b^2*c*d^9*x^(7/2) + 66*a*b*d^10*x^(7/2) +
77*b^2*c^2*d^8*x^(3/2) - 154*a*b*c*d^9*x^(3/2) + 77*a^2*d^10*x^(3/2))/d^11
```

Mupad [B]

time = 0.18, size = 435, normalized size = 1.50

$$x^{3/2}\left(\frac{2a^2}{3d}+\frac{c\left(\frac{2b^2c}{d^2}-\frac{4ab}{d}\right)}{3d}\right)-x^{7/2}\left(\frac{2b^2c}{7d^2}-\frac{4ab}{7d}\right)+\frac{2b^2x^{11/2}}{11d}-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{3/4}d^{1/4}\sqrt{x}\,(ad-bc)^2\,(a^4c^3d^4-4a^3bc^4d^3+6a^2b^2c^5d^2-4ab^3c^6d+b^4c^7)}{a^6c^4d^6-6a^5bc^5d^5+15a^4b^2c^6d^4-20a^3b^3c^7d^3+15a^2b^4c^8d^2-6ab^5c^9d+b^6c^{10}}\right)(ad-bc)^2}{d^{15/4}}-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{3/4}d^{1/4}\sqrt{x}\,(ad-bc)^2\,(a^4c^3d^4-4a^3bc^4d^3+6a^2b^2c^5d^2-4ab^3c^6d+b^4c^7)\,1i}{a^6c^4d^6-6a^5bc^5d^5+15a^4b^2c^6d^4-20a^3b^3c^7d^3+15a^2b^4c^8d^2-6ab^5c^9d+b^6c^{10}}\right)(ad-bc)^2\,1i}{d^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x^(3/2)*((2*a^2)/(3*d) + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/(3*d)) - x^(7/2)*(
(2*b^2*c)/(7*d^2) - (4*a*b)/(7*d)) + (2*b^2*x^(11/2))/(11*d) - ((-c)^(3/4)*
atan(((-c)^(3/4)*d^(1/4)*x^(1/2)*(a*d - b*c)^2*(b^4*c^7 + a^4*c^3*d^4 - 4*a
^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d))/(b^6*c^10 + a^6*c^4*d^6
- 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^
6*d^4 - 6*a*b^5*c^9*d))*(a*d - b*c)^2)/d^(15/4) - ((-c)^(3/4)*atan(((-c)^(3
/4)*d^(1/4)*x^(1/2)*(a*d - b*c)^2*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3
+ 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*1i)/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*
c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*
a*b^5*c^9*d))*(a*d - b*c)^2*1i)/d^(15/4)
```

3.417 $\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=288

$$\frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{13/4}}-\frac{\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}}{\sqrt{2}}$$

```
[Out] -2/5*b*(-2*a*d+b*c)*x^(5/2)/d^2+2/9*b^2*x^(9/2)/d+1/2*c^(1/4)*(-a*d+b*c)^2*
arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(13/4)*2^(1/2)-1/2*c^(1/4)*(-a*
d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(13/4)*2^(1/2)+1/4*c^(
1/4)*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(
13/4)*2^(1/2)-1/4*c^(1/4)*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)
*2^(1/2)*x^(1/2))/d^(13/4)*2^(1/2)+2*(-a*d+b*c)^2*x^(1/2)/d^3
```

Rubi [A]
time = 0.17, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {472, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{c}\,(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{13/4}}-\frac{\sqrt[4]{c}\,(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{13/4}}+\frac{\sqrt[4]{c}\,(bc-ad)^2\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{13/4}}-\frac{\sqrt[4]{c}\,(bc-ad)^2\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{13/4}}+\frac{2\sqrt{x}\,(bc-ad)^2}{d^3}-\frac{2bx^{5/2}(bc-2ad)}{5d^2}+\frac{2b^2x^{9/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[x])/d^3 - (2*b*(b*c - 2*a*d)*x^(5/2))/(5*d^2) + (2*b^
2*x^(9/2))/(9*d) + (c^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) - (c^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) + (c^(1/4)*(b*c - a*d)^2*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(13
/4)) - (c^(1/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*d^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)x^{3/2}}{d^2}+\frac{b^2x^{7/2}}{d}+\frac{(b^2c^2-2abcd+a^2d^2)\,x^{3/2}}{d^2\,(c+dx^2)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{(bc-ad)^2\int \frac{x^{3/2}}{c+dx^2}\,dx}{d^2}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{(c(bc-ad)^2)\int \frac{1}{\sqrt{x}\,(c+dx^2)}\,dx}{d^3}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{(2c(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{c+dx^4}\,dx,x,\right.}{d^3}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(\sqrt{c}\,(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}}{c+dx^4}\right.}{d^3}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(\sqrt{c}\,(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{c}}{\sqrt{d}}-\sqrt{}}\right.}{2d^{7/2}}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{\sqrt[4]{c}\,(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\,\sqrt[4]{c}\right.}{2\sqrt{2}\,d^{13/4}}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}}{\sqrt[4]{c}}\right.}{\sqrt{2}\,d^{13/4}}$$

Mathematica [A]
time = 0.21, size = 187, normalized size = 0.65

$$\frac{4\sqrt[4]{d}\,\sqrt{x}\,(45a^2d^2+18abd(-5c+dx^2)+b^2(45c^2-9cdx^2+5d^2x^4))+45\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)-45\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{90d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (4*d^(1/4)*Sqrt[x]*(45*a^2*d^2 + 18*a*b*d*(-5*c + d*x^2) + b^2*(45*c^2 - 9*
c*d*x^2 + 5*d^2*x^4)) + 45*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*ArcTan[(Sqrt[c] -
```

```
Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - 45*Sqrt[2]*c^(1/4)*(b*c - a
*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(90
*d^(13/4))
```

Maple [A]
time = 0.09, size = 194, normalized size = 0.67

method	result
derivativedivides	$\frac{\frac{2b^2x^{\frac{9}{2}}d^2}{9}+\frac{4ab\,d^2x^{\frac{5}{2}}}{5}-\frac{2b^2cd\,x^{\frac{5}{2}}}{5}+2a^2d^2\sqrt{x}-4abcd\sqrt{x}+2b^2c^2\sqrt{x}}{d^3}-\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+(}{x-(}\right.\right.}{}$
default	$\frac{\frac{2b^2x^{\frac{9}{2}}d^2}{9}+\frac{4ab\,d^2x^{\frac{5}{2}}}{5}-\frac{2b^2cd\,x^{\frac{5}{2}}}{5}+2a^2d^2\sqrt{x}-4abcd\sqrt{x}+2b^2c^2\sqrt{x}}{d^3}-\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+(}{x-(}\right.\right.}{}$
risch	$\frac{2(5b^2d^2x^4+18ab\,d^2x^2-9b^2cd\,x^2+45a^2d^2-90abcd+45b^2c^2)\sqrt{x}}{45d^3}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2}{2d}+\frac{\left(\frac{c}{d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/9*b^2*x^(9/2)*d^2+2/5*a*b*d^2*x^(5/2)-1/5*b^2*c*d*x^(5/2)+a^2*d^2*
x^(1/2)-2*a*b*c*d*x^(1/2)+b^2*c^2*x^(1/2))-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/
d^3*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.49, size = 324, normalized size = 1.12

$$-\frac{\left(\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)c}{4\,d^3}+\frac{2\left(5\,b^2d^2x^{\frac{9}{2}}-9\,(b^2cd-2\,abd^2)x^{\frac{5}{2}}+45\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{x}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(
sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))
/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)
) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sq
rt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/d^3 + 2/45*(5*b^2*d^2*x^(
```

9/2) - 9*(b^2*c*d - 2*a*b*d^2)*x^(5/2) + 45*(b^2*c^2 - 2*a*b*c*d + a^2*d^2) *sqrt(x))/d^3

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(211) = 422$.
time = 0.47, size = 1268, normalized size = 4.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/90*(180*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5
*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8
*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*arctan((sqrt(d^6*sqrt(-(b^8*c^9 - 8
*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4
 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d
^13) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d
^4)*x)*d^10*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^
6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^
7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(3/4) - (b^2*c^2*d^10 - 2*a*b*c*d^11 + a^2*d
^12)*sqrt(x)*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c
^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a
^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(3/4))/(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^
6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 +
28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)) + 45*d^3*(-(b^8*c^9 - 8*
a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4
- 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^
13)^(1/4)*log(d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*
b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6
- 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*sqrt(x)) - 45*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^
3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^
6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*log(-d^3*(-(b^8*c^9 - 8*a*b^7*
c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a
^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1
/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 4*(5*b^2*d^2*x^4 + 45*b^2*
c^2 - 90*a*b*c*d + 45*a^2*d^2 - 9*(b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(x))/d^3
```

Sympy [A]
time = 17.10, size = 488, normalized size = 1.69

$$\begin{cases} \tilde{\infty}\left(2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}\right) & \text{for}\ c = 0 \wedge d = 0 \\ \frac{2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}}{d} & \text{for}\ c = 0 \\ \frac{\frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{13}{2}}}{13}}{c} & \text{for}\ d = 0 \\ \frac{2a^2\sqrt{x}}{d} + \frac{a^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{2d} - \frac{a^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{2d} - \frac{a^2\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d} - \frac{4abc\sqrt{x}}{d^2} - \frac{abc\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{d^2} + \frac{abc\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{d^2} + \frac{2abc\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^2} + \frac{4abx^{\frac{5}{2}}}{5d} + \frac{2b^2c^2\sqrt{x}}{d^3} + \frac{b^2c^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{2d^3} - \frac{b^2c^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{2d^3} - \frac{b^2c^2\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^3} - \frac{2b^2cx^{\frac{5}{2}}}{5d^2} + \frac{2b^2x^{\frac{9}{2}}}{9d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9), Eq(
c, 0) & Eq(d, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9)
/d, Eq(c, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13
)/c, Eq(d, 0)), (2*a**2*sqrt(x)/d + a**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)
**(1/4))/(2*d) - a**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d) - a*
*2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d - 4*a*b*c*sqrt(x)/d**2 - a*b
*c*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/d**2 + a*b*c*(-c/d)**(1/4)*lo
g(sqrt(x) + (-c/d)**(1/4))/d**2 + 2*a*b*c*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)
**(1/4))/d**2 + 4*a*b*x**(5/2)/(5*d) + 2*b**2*c**2*sqrt(x)/d**3 + b**2*c**2
*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*d**3) - b**2*c**2*(-c/d)**(1
/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d**3) - b**2*c**2*(-c/d)**(1/4)*atan(sq
rt(x)/(-c/d)**(1/4))/d**3 - 2*b**2*c*x**(5/2)/(5*d**2) + 2*b**2*x**(9/2)/(9
*d), True))
```

Giac [**A**]
time = 0.88, size = 385, normalized size = 1.34

$$-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{4}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{4}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{4}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{4}}+\frac{2\left(5b^{2}d^{8}x^{\frac{9}{2}}-9b^{2}cd^{7}x^{\frac{5}{2}}+18abd^{8}x^{\frac{5}{2}}+45b^{2}c^{2}d^{6}\sqrt{x}-90abcd^{7}\sqrt{x}+45a^{2}d^{8}\sqrt{x}\right)}{45d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4
))/d^4 - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*
d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(
c/d)^(1/4))/d^4 - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*
c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/d^4 + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*
d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^4 +
 2/45*(5*b^2*d^8*x^(9/2) - 9*b^2*c*d^7*x^(5/2) + 18*a*b*d^8*x^(5/2) + 45*b^
2*c^2*d^6*sqrt(x) - 90*a*b*c*d^7*sqrt(x) + 45*a^2*d^8*sqrt(x))/d^9
```

Mupad [**B**]
time = 0.20, size = 1175, normalized size = 4.08

$$\sqrt{x}\left(\frac{2a^{2}}{d}+\frac{c\left(\frac{2b^{2}c}{d^{2}}-\frac{4ab}{d}\right)}{d}\right)-x^{5/2}\left(\frac{2b^{2}c}{5d^{2}}-\frac{4ab}{5d}\right)+\frac{2b^{2}x^{9/2}}{9d}-\frac{(-c)^{1/4}\,\mathrm{atan}\left(\ldots\right)\ldots}{\ldots}$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x^(1/2)*((2*a^2)/d + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/d) - x^(5/2)*((2*b^2*c
)/(5*d^2) - (4*a*b)/(5*d)) + (2*b^2*x^(9/2))/(9*d) - ((-c)^(1/4)*atan((((-c
)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3
```

```
+ 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 - ((-c)^(1/4)*(a*d - b*c)^2*(16*b
^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4)))*1i)/d^(13/4) + ((-c)
^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 +
 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 + ((-c)^(1/4)*(a*d - b*c)^2*(16*b^
2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4)))*1i)/d^(13/4))/(((-c)^
(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 +
6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 - ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2
*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4))))/d^(13/4) - ((-c)^(1/4
)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^
2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 + ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4
 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4))))/d^(13/4)))*(a*d - b*c)^2*
1i)/d^(13/4) - ((-c)^(1/4)*atan((((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4
*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/
d^3 - ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d
)*1i)/(2*d^(13/4))))/d^(13/4) + ((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*
c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d
^3 + ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d)
*1i)/(2*d^(13/4))))/d^(13/4))/(((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c
^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^
3 - ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d)*
1i)/(2*d^(13/4)))*1i)/d^(13/4) - ((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4
*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/
d^3 + ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d
)*1i)/(2*d^(13/4)))*1i)/d^(13/4)))*(a*d - b*c)^2)/d^(13/4)
```

3.418 $\int \frac{\sqrt{x}\,(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=268

$$-\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}$$

```
[Out] -2/3*b*(-2*a*d+b*c)*x^(3/2)/d^2+2/7*b^2*x^(7/2)/d-1/2*(-a*d+b*c)^2*arctan(1
-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(11/4)*2^(1/2)+1/2*(-a*d+b*c)^2
*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(11/4)*2^(1/2)+1/4*(-a
*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/d^(
11/4)*2^(1/2)-1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)
*x^(1/2))/c^(1/4)/d^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {472, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}+\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{c}\,d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{c}\,d^{11/4}}-\frac{2bx^{3/2}(bc-2ad)}{3d^2}+\frac{2b^2x^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-2*b*(b*c - 2*a*d)*x^(3/2))/(3*d^2) + (2*b^2*x^(7/2))/(7*d) - ((b*c - a*d)
^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*d^(11/4)
) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11/4)) - ((b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11
/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\,(a+bx^2)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)\sqrt{x}}{d^2}+\frac{b^2x^{5/2}}{d}+\frac{(b^2c^2-2abcd+a^2d^2)\sqrt{x}}{d^2(c+dx^2)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(bc-ad)^2\int\frac{\sqrt{x}}{c+dx^2}\,dx}{d^2}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(2(bc-ad)^2)\,\text{Subst}\left(\int\frac{x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}-\frac{(bc-ad)^2\text{Subst}\left(\int\frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^{5/2}}+\frac{(bc}{}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(bc-ad)^2\text{Subst}\left(\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right)}{2d^3}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,\sqrt[4]{c}\,d^{11/4}}-$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,\sqrt[4]{c}\,d^{11/4}}+\frac{(bc-ad)^2}{}$$

Mathematica [A]
time = 0.19, size = 156, normalized size = 0.58

$$\frac{4bd^{3/4}x^{3/2}(-7bc+14ad+3bdx^2)-\frac{21\sqrt{2}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{\sqrt[4]{c}}-\frac{21\sqrt{2}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt[4]{c}}}{42d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (4*b*d^(3/4)*x^(3/2)*(-7*b*c + 14*a*d + 3*b*d*x^2) - (21*Sqrt[2]*(b*c - a*d
)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(1/4
) - (21*Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sq
rt[c] + Sqrt[d]*x)])/c^(1/4))/(42*d^(11/4))
```

Maple [A]
time = 0.09, size = 156, normalized size = 0.58

method	result
derivativedivides	$\frac{2b\left(\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-bc)x^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right.\right.}{4d^3\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
default	$\frac{2b\left(\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-bc)x^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right.\right.}{4d^3\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
risch	$\frac{2(3bd\,x^2+14ad-7bc)b\,x^{\frac{3}{2}}}{21d^2}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2}{2d\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)abc}{d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\text{arct}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b/d^2*(1/7*b*d*x^(7/2)+1/3*(2*a*d-b*c)*x^(3/2))+1/4*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/d^3/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 229, normalized size = 0.85

$$\frac{(b^2c^2-2abcd+a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\,d^2}+\frac{2\left(3\,b^2dx^{\frac{7}{2}}-7\,(b^2c-2\,abd)x^{\frac{3}{2}}\right)}{21\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*s
qrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) -
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
- sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/
4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq
rt(c))/(c^(1/4)*d^(3/4)))/d^2 + 2/21*(3*b^2*d*x^(7/2) - 7*(b^2*c - 2*a*b*d)
*x^(3/2))/d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1629 vs. 2(193) = 386.
time = 0.54, size = 1629, normalized size = 6.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/42*(84*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*
a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*arctan((sqrt((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)*x - (b^8*c^9*d^5 - 8*a*b^7*c^8*d^6 + 28*a^2*b^6*c^7*d^7 - 56*
a^3*b^5*c^6*d^8 + 70*a^4*b^4*c^5*d^9 - 56*a^5*b^3*c^4*d^10 + 28*a^6*b^2*c^3
*d^11 - 8*a^7*b*c^2*d^12 + a^8*c*d^13)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*
a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*
d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11)))*d^3*(-(b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c*d^11))^(1/4) - (b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20
*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8 + a^6*d^9)*sqrt(x)*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(c*d^11))^(1/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*
a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*
d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) - 21*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*log(c*d^8
*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*
a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8)/(c*d^11))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)
) + 21*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5
*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7
*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*log(-c*d^8*(-(b^8*c^8 - 8*a*b^7*c^7*d +
 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*
c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(3/4) + (
b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) - 4*(3*b^2*d*x^3 - 7*(b^2*c
 - 2*a*b*d)*x)*sqrt(x))/d^2
```

Sympy [A]
time = 4.89, size = 87, normalized size = 0.32

$$\frac{4abx^{\frac{3}{2}}}{3d} - \frac{2b^2cx^{\frac{3}{2}}}{3d^2} + \frac{2b^2x^{\frac{7}{2}}}{7d} + \frac{2(ad-bc)^2 \operatorname{RootSum}\left(256t^4cd^3+1, \left(t \mapsto t\log\left(64t^3cd^2+\sqrt{x}\right)\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c),x)
```

```
[Out] 4*a*b*x**(3/2)/(3*d) - 2*b**2*c*x**(3/2)/(3*d**2) + 2*b**2*x**(7/2)/(7*d) +
 2*(a*d - b*c)**2*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*_t**3*
c*d**2 + sqrt(x))))/d**2
```

Giac [A]
time = 1.04, size = 361, normalized size = 1.35

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^{5}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^{5}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^{5}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^{5}}+\frac{2\left(3\,b^{2}d^{6}x^{\frac{7}{2}}-7\,b^{2}cd^{5}x^{\frac{3}{2}}+14\,abd^{6}x^{\frac{3}{2}}\right)}{21\,d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c*d^5) + 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d +
(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x)
)/(c/d)^(1/4))/(c*d^5) - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/
4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c*d^5) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*
b*c*d + (c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(c*d^5) + 2/21*(3*b^2*d^6*x^(7/2) - 7*b^2*c*d^5*x^(3/2) + 14*a*b*d^6*
x^(3/2))/d^7
```

Mupad [B]
time = 0.09, size = 390, normalized size = 1.46

$$\frac{2\,b^2\,x^{7/2}}{7\,d}-x^{3/2}\,\left(\frac{2\,b^2\,c}{3\,d^2}-\frac{4\,a\,b}{3\,d}\right)+\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(a^4\,c\,d^4-4\,a^3\,b\,c^2\,d^3+6\,a^2\,b^2\,c^3\,d^2-4\,a\,b^3\,c^4\,d+b^4\,c^5\right)}{{\left(-c\right)}^{1/4}\,\left(a^6\,c\,d^6-6\,a^5\,b\,c^2\,d^5+15\,a^4\,b^2\,c^3\,d^4-20\,a^3\,b^3\,c^4\,d^3+15\,a^2\,b^4\,c^5\,d^2-6\,a\,b^5\,c^6\,d+b^6\,c^7\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{1/4}\,d^{11/4}}+\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(a^4\,c\,d^4-4\,a^3\,b\,c^2\,d^3+6\,a^2\,b^2\,c^3\,d^2-4\,a\,b^3\,c^4\,d+b^4\,c^5\right)\,1{}\mathrm{i}}{{\left(-c\right)}^{1/4}\,\left(a^6\,c\,d^6-6\,a^5\,b\,c^2\,d^5+15\,a^4\,b^2\,c^3\,d^4-20\,a^3\,b^3\,c^4\,d^3+15\,a^2\,b^4\,c^5\,d^2-6\,a\,b^5\,c^6\,d+b^6\,c^7\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{1/4}\,d^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] (2*b^2*x^(7/2))/(7*d) - x^(3/2)*((2*b^2*c)/(3*d^2) - (4*a*b)/(3*d)) + (atan
((d^(1/4)*x^(1/2)*(a*d - b*c)^2*(b^4*c^5 + a^4*c*d^4 - 4*a^3*b*c^2*d^3 + 6*
a^2*b^2*c^3*d^2 - 4*a*b^3*c^4*d))/((-c)^(1/4)*(b^6*c^7 + a^6*c*d^6 - 6*a^5*
b*c^2*d^5 + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 -
6*a*b^5*c^6*d)))*(a*d - b*c)^2)/((-c)^(1/4)*d^(11/4)) + (atan((d^(1/4)*x^(1
/2)*(a*d - b*c)^2*(b^4*c^5 + a^4*c*d^4 - 4*a^3*b*c^2*d^3 + 6*a^2*b^2*c^3*d^
2 - 4*a*b^3*c^4*d)*1i)/((-c)^(1/4)*(b^6*c^7 + a^6*c*d^6 - 6*a^5*b*c^2*d^5 +
 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a*b^5*c^6
*d)))*(a*d - b*c)^2*1i)/((-c)^(1/4)*d^(11/4))
```

3.419 $\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx$

Optimal. Leaf size=266

$$-\frac{2b(bc-2ad)\sqrt{x}}{d^2}+\frac{2b^2x^{5/2}}{5d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{3/4}d^{9/4}}$$

```
[Out] 2/5*b^2*x^(5/2)/d-1/2*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4)
)/c^(3/4)/d^(9/4)*2^(1/2)+1/2*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)
/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)-1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(
1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*ln(c
^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)-2
*b*(-2*a*d+b*c)*x^(1/2)/d^2
```

Rubi [A]
time = 0.15, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {472, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{2b\sqrt{x}\,(bc-2ad)}{d^2}+\frac{2b^2x^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

```
[Out] (-2*b*(b*c - 2*a*d)*Sqrt[x])/d^2 + (2*b^2*x^(5/2))/(5*d) - ((b*c - a*d)^2*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*d^(9/4)) + (
(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/
4)*d^(9/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx &= \int \left(-\frac{b(bc-2ad)}{d^2\sqrt{x}} + \frac{b^2x^{3/2}}{d} + \frac{b^2c^2-2abcd+a^2d^2}{d^2\sqrt{x}\,(c+dx^2)}\right)dx \\
&= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{x}\,(c+dx^2)}\,dx}{d^2} \\
&= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} + \frac{(2(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{d^2} \\
&= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} + \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}\,d^2} + (bc \\
&= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} + \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{c}\,d^{5/2}} \\
&= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} - \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}} + \\
&= -\frac{2b(bc-2ad)\sqrt{x}}{d^2} + \frac{2b^2x^{5/2}}{5d} - \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{3/4}d^{9/4}} + \frac{(bc-ad)^2}{}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 155, normalized size = 0.58

$$
\frac{4b\sqrt[4]{d}\sqrt{x}\,(-5bc+10ad+bdx^2) - \frac{5\sqrt{2}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{3/4}} + \frac{5\sqrt{2}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{3/4}}}{10d^{9/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

```
[Out] (4*b*d^(1/4)*Sqrt[x]*(-5*b*c + 10*a*d + b*d*x^2) - (5*Sqrt[2]*(b*c - a*d)^2
*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(3/4) +
 (5*Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c
] + Sqrt[d]*x)])/c^(3/4))/(10*d^(9/4))
```

Maple [A]
time = 0.10, size = 159, normalized size = 0.60

method	result
derivativedivides	$\frac{2b\left(\frac{bd\,x^{\frac{5}{2}}}{5}+2ad\sqrt{x}-bc\sqrt{x}\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\right.\right.}{4d^2c}$
default	$\frac{2b\left(\frac{bd\,x^{\frac{5}{2}}}{5}+2ad\sqrt{x}-bc\sqrt{x}\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\right.\right.}{4d^2c}$
risch	$\frac{2(bd\,x^2+10ad-5bc)b\sqrt{x}}{5d^2}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2}{2c}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)ab}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b/d^2*(1/5*b*d*x^(5/2)+2*a*d*x^(1/2)-b*c*x^(1/2))+1/4*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/d^2*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 291, normalized size = 1.09

$$\frac{2\left(b^2dx^{\frac{5}{2}}-5\left(b^2c-2abd\right)\sqrt{x}\right)}{5d^2}+\frac{\frac{2\sqrt{2}(b^2c^2-2abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(b^2c^2-2abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(b^2c^2-2abcd+a^2d^2)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(b^2c^2-2abcd+a^2d^2)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^2*d*x^(5/2) - 5*(b^2*c - 2*a*b*d)*sqrt(x))/d^2 + 1/4*(2*sqrt(2)*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2
*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1
/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(
c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(2)*c^(1/4)*
d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
sqrt(c))/(c^(3/4)*d^(1/4)))/d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. $2(193) = 386$.
time = 0.52, size = 1245, normalized size = 4.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*arctan((sqrt(c^2*d^4*sqrt(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*
d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c
^3*d^9)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*x)*c^2*d^7*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(3/4) - (b^2*c^4*d^7 - 2*a*b*c^3*d^8
 + a^2*c^2*d^9)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 5
6*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^
2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(3/4))/(b^8*c^8 - 8*a*b^7*c^7*d
 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 5*d^2*(-(b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*sqrt(x)) - 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d
^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) + 4*(b^2*d*x^2
 - 5*b^2*c + 10*a*b*d)*sqrt(x))/d^2
```

Sympy [A]
time = 4.69, size = 423, normalized size = 1.59

$$\begin{cases} \tilde{\infty} \left(- \frac{2 a^{2}}{3 x^{\frac{3}{2}}} + 4 a b \sqrt{x} + \frac{2 b^{2} x^{\frac{5}{2}}}{5}\right) & \text{for}\: c = 0 \wedge d = 0 \\ \frac{- \frac{2 a^{2}}{3 x^{\frac{3}{2}}} + 4 a b \sqrt{x} + \frac{2 b^{2} x^{\frac{5}{2}}}{5}}{d} & \text{for}\: c = 0 \\ \frac{2 a^{2} \sqrt{x} + \frac{4 a b x^{\frac{5}{2}}}{5} + \frac{2 b^{2} x^{\frac{9}{2}}}{9}}{c} & \text{for}\: d = 0 \\ - \frac{a^{2} \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{c}{d}} \right)}}{2 c} + \frac{a^{2} \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{c}{d}} \right)}}{2 c} + \frac{a^{2} \sqrt[4]{- \frac{c}{d}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{c}{d}}} \right)}}{c} + \frac{4 a b \sqrt{x}}{d} + \frac{a b \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{c}{d}} \right)}}{d} - \frac{a b \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{c}{d}} \right)}}{d} - \frac{2 a b \sqrt[4]{- \frac{c}{d}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{c}{d}}} \right)}}{d} - \frac{2 b^{2} c \sqrt{x}}{d^{2}} - \frac{b^{2} c \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{c}{d}} \right)}}{2 d^{2}} + \frac{b^{2} c \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{c}{d}} \right)}}{2 d^{2}} + \frac{b^{2} c \sqrt[4]{- \frac{c}{d}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{c}{d}}} \right)}}{d^{2}} + \frac{2 b^{2} x^{\frac{5}{2}}}{5 d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5),
Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5
/2)/5)/d, Eq(c, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/
9)/c, Eq(d, 0)), (-a**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*c) +
a**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*c) + a**2*(-c/d)**(1/4)*
```

```
atan(sqrt(x)/(-c/d)**(1/4))/c + 4*a*b*sqrt(x)/d + a*b*(-c/d)**(1/4)*log(sqr
t(x) - (-c/d)**(1/4))/d - a*b*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/d
- 2*a*b*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d - 2*b**2*c*sqrt(x)/d**2
 - b**2*c*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*d**2) + b**2*c*(-c/
d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d**2) + b**2*c*(-c/d)**(1/4)*atan
(sqrt(x)/(-c/d)**(1/4))/d**2 + 2*b**2*x**(5/2)/(5*d), True))
```

Giac [A]
time = 1.24, size = 360, normalized size = 1.35

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^3}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^3}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^3}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^3}+\frac{2\left(b^2d^4x^{\frac{5}{2}}-5\,b^2cd^3\sqrt{x}+10\,abd^4\sqrt{x}\right)}{5\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c*d^3) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x)
)/(c/d)^(1/4))/(c*d^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/
4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c*d^3) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*
b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(c*d^3) + 2/5*(b^2*d^4*x^(5/2) - 5*b^2*c*d^3*sqrt(x) + 10*a*b*d^4*sqr
t(x))/d^5
```

Mupad [B]
time = 0.20, size = 1107, normalized size = 4.16

$$\frac{2\,b^2\,x^{5/2}}{5\,d}-\sqrt{x}\,\left(\frac{2\,b^2\,c}{d^2}-\frac{4\,a\,b}{d}\right)+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{8\,\sqrt{x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c\,d^3-32\,a\,b\,c^2\,d^2+16\,b^2\,c^3\,d\right)}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}+\frac{\left(\frac{8\,\sqrt{x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c\,d^3-32\,a\,b\,c^2\,d^2+16\,b^2\,c^3\,d\right)}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}}{\frac{\left(\frac{8\,\sqrt{x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c\,d^3-32\,a\,b\,c^2\,d^2+16\,b^2\,c^3\,d\right)}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{3/4}\,d^{9/4}}-\frac{\left(\frac{8\,\sqrt{x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c\,d^3-32\,a\,b\,c^2\,d^2+16\,b^2\,c^3\,d\right)}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{3/4}\,d^{9/4}}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{8\,\sqrt{x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c\,d^3-32\,a\,b\,c^2\,d^2+16\,b^2\,c^3\,d\right)\,1{}\mathrm{i}}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{3/4}\,d^{9/4}}+\frac{\left(\frac{8\,\sqrt{x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c\,d^3-32\,a\,b\,c^2\,d^2+16\,b^2\,c^3\,d\right)\,1{}\mathrm{i}}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{3/4}\,d^{9/4}}}{\frac{\left(\frac{8\,\sqrt{x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c\,d^3-32\,a\,b\,c^2\,d^2+16\,b^2\,c^3\,d\right)\,1{}\mathrm{i}}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}-\frac{\left(\frac{8\,\sqrt{x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c\,d^3-32\,a\,b\,c^2\,d^2+16\,b^2\,c^3\,d\right)\,1{}\mathrm{i}}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{3/4}\,d^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(1/2)*(c + d*x^2)),x)
```

```
[Out] (2*b^2*x^(5/2))/(5*d) - x^(1/2)*((2*b^2*c)/d^2 - (4*a*b)/d) + (atan(((((8*x
^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d
^3))/d - ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*
(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)) + (((8*x^(1/2)*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d
+ ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*(-c)^(3
/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)))/((((8*x^(1/2)*(a^4*d^
4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d
 - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*(-c)^(3/4)*d^(
9/4)))*(a*d - b*c)^2)/((-c)^(3/4)*d^(9/4)) - (((8*x^(1/2)*(a^4*d^4 + b^4*c^
4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*
```

```
(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*(-c)^(3/4)*d^(9/4)))*(a*
d - b*c)^2)/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)) +
 (atan(((((8*x^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
 - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b
*c^2*d^2)*1i)/(2*(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2)/((-c)^(3/4)*d^(9/4)) +
 (((8*x^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^
3*b*c*d^3))/d + ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^
2)*1i)/(2*(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2)/((-c)^(3/4)*d^(9/4)))/((((8*x
^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d
^3))/d - ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2)*1i)/
(2*(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)) - (((8*x^(1/
2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))
/d + ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2)*1i)/(2*(
-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2)
/((-c)^(3/4)*d^(9/4))
```

3.420 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx$

Optimal. Leaf size=260

$$-\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}-\frac{(bc-ad)^2}{}$$

```
[Out] 2/3*b^2*x^(3/2)/d+1/2*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4)
)/c^(5/4)/d^(7/4)*2^(1/2)-1/2*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)
/c^(1/4))/c^(5/4)/d^(7/4)*2^(1/2)-1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(
1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(7/4)*2^(1/2)+1/4*(-a*d+b*c)^2*ln(c
^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(7/4)*2^(1/2)-2
*a^2/c/x^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {473, 470, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{2a^2}{c\sqrt{x}}+\frac{(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}-\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}d^{7/4}}+\frac{2b^2x^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(c*Sqrt[x]) + (2*b^2*x^(3/2))/(3*d) + ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7/4)) - ((b*c - a*d)^
2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7/4))
- ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(2*Sqrt[2]*c^(5/4)*d^(7/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*d^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx = -\frac{2a^2}{c\sqrt{x}} + \frac{2\int \frac{\sqrt{x}\,\left(\frac{1}{2}a(2bc-ad)+\frac{1}{2}b^2cx^2\right)}{c+dx^2}\,dx}{c}$$

$$= -\frac{2a^2}{c\sqrt{x}} + \frac{2b^2x^{3/2}}{3d} - \frac{(bc-ad)^2\int \frac{\sqrt{x}}{c+dx^2}\,dx}{cd}$$

$$= -\frac{2a^2}{c\sqrt{x}} + \frac{2b^2x^{3/2}}{3d} - \frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{cd}$$

$$= -\frac{2a^2}{c\sqrt{x}} + \frac{2b^2x^{3/2}}{3d} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{cd^{3/2}} - \frac{(bc-ad)^2\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{2a^2}{c\sqrt{x}} + \frac{2b^2x^{3/2}}{3d} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{2cd^2} - \frac{(bc-ad)\ldots}{\ldots}$$

$$= -\frac{2a^2}{c\sqrt{x}} + \frac{2b^2x^{3/2}}{3d} - \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}d^{7/4}} + \frac{(bc-ad)^2\,\text{l}\ldots}{\ldots}$$

$$= -\frac{2a^2}{c\sqrt{x}} + \frac{2b^2x^{3/2}}{3d} + \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}d^{7/4}} - \frac{(bc-ad)^2\tan^{-1}\left(1+\ldots\right.}{\sqrt{2}\,c^{5/4}d^{7/\ldots}}$$

Mathematica [A]
time = 0.19, size = 154, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{c}\,d^{3/4}(-3a^2d+b^2cx^2)}{\sqrt{x}} + 3\sqrt{2}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right) + 3\sqrt{2}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{6c^{5/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```

[Out] ((4*c^(1/4)*d^(3/4)*(-3*a^2*d + b^2*c*x^2))/Sqrt[x] + 3*Sqrt[2]*(b*c - a*d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(6*c^(5/4)*d^(7/4))

Maple [A]
time = 0.10, size = 153, normalized size = 0.59

method	result
derivativedivides	$\frac{2b^2x^{\frac{3}{2}}}{3d} - \frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4c\,d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
default	$\frac{2b^2x^{\frac{3}{2}}}{3d} - \frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4c\,d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
risch	$-\frac{2\left(-b^2c\,x^2+3a^2d\right)}{3c\sqrt{x}\,d} - \frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2}{2c\left(\frac{c}{d}\right)^{\frac{1}{4}}} + \frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)ab}{d\left(\frac{c}{d}\right)^{\frac{1}{4}}} - \frac{c\sqrt{2}\,\arctan\left(\right.}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x,method=_RETURNVERBOSE)

[Out] 2/3*b^2*x^(3/2)/d-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^2/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))-2*a^2/c/x^(1/2)

Maxima [A]
time = 0.53, size = 223, normalized size = 0.86

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d} - \frac{2\,a^2}{c\sqrt{x}} - \frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}} + \frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}} - \frac{\sqrt{2}\,\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}} + \frac{\sqrt{2}\,\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="maxima")

[Out] 2/3*b^2*x^(3/2)/d - 2*a^2/(c*sqrt(x)) - 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/

```
4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*
c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(187) = 374$.
time = 0.53, size = 1636, normalized size = 6.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*d*x*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*
a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4)*arctan((sqrt((b^12*c^12 - 12*a*b^11
*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
- 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8
*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^1
1 + a^12*d^12)*x - (b^8*c^11*d^3 - 8*a*b^7*c^10*d^4 + 28*a^2*b^6*c^9*d^5 -
56*a^3*b^5*c^8*d^6 + 70*a^4*b^4*c^7*d^7 - 56*a^5*b^3*c^6*d^8 + 28*a^6*b^2*c
^5*d^9 - 8*a^7*b*c^4*d^10 + a^8*c^3*d^11)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d +
28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7)))*c*d^2*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(c^5*d^7))^(1/4) - (b^6*c^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d
^4 - 20*a^3*b^3*c^4*d^5 + 15*a^4*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)
*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d
^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b
*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) - 3*c*d*x*(-(b^8*c^8 - 8*a*b^7*
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(
1/4)*log(c^4*d^5*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 +
 a^6*d^6)*sqrt(x)) + 3*c*d*x*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4)*log(-c^4*d^5*(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^5*d^7))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3
*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) + 4*(
b^2*c*x^2 - 3*a^2*d)*sqrt(x))/(c*d*x)
```

Sympy [A]
time = 12.52, size = 282, normalized size = 1.08

$$a^2 \left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } c = 0 \wedge d = 0 \\ -\frac{2}{5dx^{\frac{5}{2}}} & \text{for } c = 0 \\ -\frac{2}{c\sqrt{x}} & \text{for } d = 0 \\ -\frac{\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{2c\sqrt[4]{-\frac{c}{d}}} + \frac{\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{2c\sqrt[4]{-\frac{c}{d}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{c\sqrt[4]{-\frac{c}{d}}} - \frac{2}{c\sqrt{x}} & \text{otherwise} \end{cases} \right) + 4ab \operatorname{RootSum}\left(256t^4cd^3 + 1, \left(t \mapsto t\log\left(64t^3cd^2 + \sqrt{x}\right)\right)\right) + b^2 \left(\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } c = 0 \wedge d = 0 \\ \frac{2x^{\frac{3}{2}}}{3d} & \text{for } c = 0 \\ \frac{2x^{\frac{7}{2}}}{7c} & \text{for } d = 0 \\ -\frac{c\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{2d^2\sqrt[4]{-\frac{c}{d}}} + \frac{c\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{2d^2\sqrt[4]{-\frac{c}{d}}} - \frac{c\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^2\sqrt[4]{-\frac{c}{d}}} + \frac{2x^{\frac{3}{2}}}{3d} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c),x)
```

```
[Out] a**2*Piecewise((zoo/x**(5/2), Eq(c, 0) & Eq(d, 0)), (-2/(5*d*x**(5/2)), Eq(
c, 0)), (-2/(c*sqrt(x)), Eq(d, 0)), (-log(sqrt(x) - (-c/d)**(1/4))/(2*c*(-c
/d)**(1/4)) + log(sqrt(x) + (-c/d)**(1/4))/(2*c*(-c/d)**(1/4)) - atan(sqrt(
x)/(-c/d)**(1/4))/(c*(-c/d)**(1/4)) - 2/(c*sqrt(x)), True)) + 4*a*b*RootSum
(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*_t**3*c*d**2 + sqrt(x)))) + b**
2*Piecewise((zoo*x**(3/2), Eq(c, 0) & Eq(d, 0)), (2*x**(3/2)/(3*d), Eq(c, 0
)), (2*x**(7/2)/(7*c), Eq(d, 0)), (-c*log(sqrt(x) - (-c/d)**(1/4))/(2*d**2*
(-c/d)**(1/4)) + c*log(sqrt(x) + (-c/d)**(1/4))/(2*d**2*(-c/d)**(1/4)) - c*
atan(sqrt(x)/(-c/d)**(1/4))/(d**2*(-c/d)**(1/4)) + 2*x**(3/2)/(3*d), True))
```

Giac [A]
time = 0.88, size = 344, normalized size = 1.32

$$\frac{2 b^2 x^{\frac{3}{2}}}{3 d} - \frac{2 a^2}{c \sqrt{x}} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}} b^2 c^2 - 2\left(cd^3\right)^{\frac{3}{4}} abcd + \left(cd^3\right)^{\frac{3}{4}} a^2 d^2\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2 c^2 d^4} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}} b^2 c^2 - 2\left(cd^3\right)^{\frac{3}{4}} abcd + \left(cd^3\right)^{\frac{3}{4}} a^2 d^2\right) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2 c^2 d^4} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}} b^2 c^2 - 2\left(cd^3\right)^{\frac{3}{4}} abcd + \left(cd^3\right)^{\frac{3}{4}} a^2 d^2\right) \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4 c^2 d^4} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}} b^2 c^2 - 2\left(cd^3\right)^{\frac{3}{4}} abcd + \left(cd^3\right)^{\frac{3}{4}} a^2 d^2\right) \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4 c^2 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d - 2*a^2/(c*sqrt(x)) - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2
- 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/2*sqrt(2)*((c*d^3)^
(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) + 1/4*sq
rt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*
d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) - 1/4*sqrt(
2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2
)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4)
```

Mupad [B]
time = 0.19, size = 416, normalized size = 1.60

$$\frac{2\,b^2\,x^{3/2}}{3\,d} - \frac{2\,a^2}{c\,\sqrt{x}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}\,\left(a\,d-b\,c\right)^2\,\left(16\,a^4\,c^4\,d^9-64\,a^3\,b\,c^5\,d^8+96\,a^2\,b^2\,c^6\,d^7-64\,a\,b^3\,c^7\,d^6+16\,b^4\,c^8\,d^5\right)}{{\left(-c\right)}^{5/4}\,d^{7/4}\,\left(16\,a^6\,c^3\,d^9-96\,a^5\,b\,c^4\,d^8+240\,a^4\,b^2\,c^5\,d^7-320\,a^3\,b^3\,c^6\,d^6+240\,a^2\,b^4\,c^7\,d^5-96\,a\,b^5\,c^8\,d^4+16\,b^6\,c^9\,d^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{5/4}\,d^{7/4}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}\,\left(a\,d-b\,c\right)^2\,\left(16\,a^4\,c^4\,d^9-64\,a^3\,b\,c^5\,d^8+96\,a^2\,b^2\,c^6\,d^7-64\,a\,b^3\,c^7\,d^6+16\,b^4\,c^8\,d^5\right)\,1{}\mathrm{i}}{{\left(-c\right)}^{5/4}\,d^{7/4}\,\left(16\,a^6\,c^3\,d^9-96\,a^5\,b\,c^4\,d^8+240\,a^4\,b^2\,c^5\,d^7-320\,a^3\,b^3\,c^6\,d^6+240\,a^2\,b^4\,c^7\,d^5-96\,a\,b^5\,c^8\,d^4+16\,b^6\,c^9\,d^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{5/4}\,d^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x)
```

```
[Out] (2*b^2*x^(3/2))/(3*d) - (2*a^2)/(c*x^(1/2)) - (atan((x^(1/2)*(a*d - b*c)^2*
(16*a^4*c^4*d^9 + 16*b^4*c^8*d^5 - 64*a*b^3*c^7*d^6 - 64*a^3*b*c^5*d^8 + 96
*a^2*b^2*c^6*d^7))/((-c)^(5/4)*d^(7/4)*(16*a^6*c^3*d^9 + 16*b^6*c^9*d^3 - 9
6*a*b^5*c^8*d^4 - 96*a^5*b*c^4*d^8 + 240*a^2*b^4*c^7*d^5 - 320*a^3*b^3*c^6*
d^6 + 240*a^4*b^2*c^5*d^7)))*(a*d - b*c)^2)/((-c)^(5/4)*d^(7/4)) - (atan((x
^(1/2)*(a*d - b*c)^2*(16*a^4*c^4*d^9 + 16*b^4*c^8*d^5 - 64*a*b^3*c^7*d^6 -
64*a^3*b*c^5*d^8 + 96*a^2*b^2*c^6*d^7)*1i)/((-c)^(5/4)*d^(7/4)*(16*a^6*c^3*
d^9 + 16*b^6*c^9*d^3 - 96*a*b^5*c^8*d^4 - 96*a^5*b*c^4*d^8 + 240*a^2*b^4*c^
7*d^5 - 320*a^3*b^3*c^6*d^6 + 240*a^4*b^2*c^5*d^7)))*(a*d - b*c)^2*1i)/((-c
)^(5/4)*d^(7/4))
```

3.421 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx$

Optimal. Leaf size=260

$$-\frac{2a^2}{3cx^{3/2}}+\frac{2b^2\sqrt{x}}{d}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(bc-ac}{}$$

```
[Out] -2/3*a^2/c/x^(3/2)+1/2*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4
))/c^(7/4)/d^(5/4)*2^(1/2)-1/2*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2
)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)+1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^
(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)-1/4*(-a*d+b*c)^2*ln(
c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)+
2*b^2*x^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {473, 470, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{2a^2}{3cx^{3/2}}+\frac{(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{2b^2\sqrt{x}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)) + (2*b^2*Sqrt[x])/d + ((b*c - a*d)^2*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c - a*d)^2*
ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4)) +
((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(2*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx = -\frac{2a^2}{3cx^{3/2}} + \frac{2\int \frac{\frac{3}{2}a(2bc-ad)+\frac{3}{2}b^2cx^2}{\sqrt{x}\,(c+dx^2)}\,dx}{3c}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{x}\,(c+dx^2)}\,dx}{cd}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} - \frac{(2(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{cd}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} - \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}d} - \frac{(bc-ad)^2\mathrm{Subs}}{}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} - \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/2}d^{3/2}} - \frac{(bc-}{}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} + \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}d^{5/4}} - \frac{(bc-ad)}{}$$

$$= -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} + \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}d^{5/4}} - \frac{(bc-ad)^2\tan^{-1}\left(1-\right.}{\sqrt{2}\,c^{7/4}}$$

Mathematica [A]
time = 0.18, size = 155, normalized size = 0.60

$$\frac{\frac{4c^{3/4}\sqrt[4]{d}\,(-a^2d+3b^2cx^2)}{x^{3/2}} + 3\sqrt{2}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right) - 3\sqrt{2}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{6c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

```
[Out] ((4*c^(3/4)*d^(1/4)*(-(a^2*d) + 3*b^2*c*x^2))/x^(3/2) + 3*Sqrt[2]*(b*c - a*
d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - 3*Sq
rt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sq
rt[d]*x)])/(6*c^(7/4)*d^(5/4))
```

Maple [A]
time = 0.09, size = 155, normalized size = 0.60

method	result
derivativedivides	$\frac{2b^2\sqrt{x}}{d}-\frac{2a^2}{3c\,x^{\frac{3}{2}}}+\frac{(-a^2d^2+2abcd-b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\right.\right.}{4c^2d}$
default	$\frac{2b^2\sqrt{x}}{d}-\frac{2a^2}{3c\,x^{\frac{3}{2}}}+\frac{(-a^2d^2+2abcd-b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\right.\right.}{4c^2d}$
risch	$-\frac{2(-3b^2c\,x^2+a^2d)}{3d\,x^{\frac{3}{2}}\,c}-\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2}{2c^2}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)ab}{c}-\frac{\left(\frac{c}{d}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*x^(1/2)/d-2/3*a^2/c/x^(3/2)+1/4/c^2/d*(-a^2*d^2+2*a*b*c*d-b^2*c^2)*(c
/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.51, size = 286, normalized size = 1.10

$$\frac{2\,b^2\sqrt{x}}{d}-\frac{2\,a^2}{3\,cx^{\frac{3}{2}}}-\frac{\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{4\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/d - 2/3*a^2/(c*x^(3/2)) - 1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + s
qrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
+ sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a
```

```
^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4
)*d^(1/4)))/(c*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(187) = 374$.
time = 0.49, size = 1253, normalized size = 4.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(12*c*d*x^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*arctan((sqrt(c^4*d^2*sqrt(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^7*d^5)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*x)*c^5*d^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(3/4) - (b^2*c^7*d^4 - 2*a*b*c^6
*d^5 + a^2*c^5*d^6)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(3/4))/(b^8*c^8 - 8*a*b^7*c
^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^
5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 3*c*d*x^2*
(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6
*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 2
8*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4) + (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt(x)) - 3*c*d*x^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a
^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*log(-c^
2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^
7 + a^8*d^8)/(c^7*d^5))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) -
4*(3*b^2*c*x^2 - a^2*d)*sqrt(x))/(c*d*x^2)
```

Sympy [A]
time = 9.48, size = 408, normalized size = 1.57

$$\begin{cases} \tilde{\infty}\left(-\frac{2a^2}{7x^{\frac{7}{2}}} - \frac{4ab}{3x^{\frac{3}{2}}} + 2b^2\sqrt{x}\right) & \text{for } c = 0 \wedge d = 0 \\ \frac{-\frac{2a^2}{7x^{\frac{7}{2}}} - \frac{4ab}{3x^{\frac{3}{2}}} + 2b^2\sqrt{x}}{d} & \text{for } c = 0 \\ \frac{-\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{5}{2}}}{5}}{c} & \text{for } d = 0 \\ -\frac{2a^2}{3cx^{\frac{3}{2}}} + \frac{a^2d\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{2c^2} - \frac{a^2d\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{2c^2} - \frac{a^2d\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{c^2} - \frac{ab\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{c} + \frac{ab\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{c} + \frac{2ab\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{c} + \frac{2b^2\sqrt{x}}{d} + \frac{b^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{2d} - \frac{b^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{2d} - \frac{b^2\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2*sqrt(x))
, Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**
2*sqrt(x))/d, Eq(c, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x*
*(5/2)/5)/c, Eq(d, 0)), (-2*a**2/(3*c*x**(3/2)) + a**2*d*(-c/d)**(1/4)*log(
sqrt(x) - (-c/d)**(1/4))/(2*c**2) - a**2*d*(-c/d)**(1/4)*log(sqrt(x) + (-c/
d)**(1/4))/(2*c**2) - a**2*d*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/c**2
 - a*b*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/c + a*b*(-c/d)**(1/4)*log
(sqrt(x) + (-c/d)**(1/4))/c + 2*a*b*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4
))/c + 2*b**2*sqrt(x)/d + b**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(
2*d) - b**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d) - b**2*(-c/d)*
*(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d, True))
```

<u>Giac</u> [A]
time = 0.94, size = 344, normalized size = 1.32

$$\frac{2\,b^2\sqrt{x}}{d}-\frac{2\,a^2}{3\,c x^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\left(c d^3\right)^{\frac{1}{4}} b^2 c^2-2\left(c d^3\right)^{\frac{1}{4}} a b c d+\left(c d^3\right)^{\frac{1}{4}} a^2 d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2 d^2}-\frac{\sqrt{2}\left(\left(c d^3\right)^{\frac{1}{4}} b^2 c^2-2\left(c d^3\right)^{\frac{1}{4}} a b c d+\left(c d^3\right)^{\frac{1}{4}} a^2 d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2 d^2}-\frac{\sqrt{2}\left(\left(c d^3\right)^{\frac{1}{4}} b^2 c^2-2\left(c d^3\right)^{\frac{1}{4}} a b c d+\left(c d^3\right)^{\frac{1}{4}} a^2 d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^2 d^2}+\frac{\sqrt{2}\left(\left(c d^3\right)^{\frac{1}{4}} b^2 c^2-2\left(c d^3\right)^{\frac{1}{4}} a b c d+\left(c d^3\right)^{\frac{1}{4}} a^2 d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d - 2/3*a^2/(c*x^(3/2)) - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2
- 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^2) - 1/2*sqrt(2)*((c*d^3)^
(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^2) - 1/4*sq
rt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*
d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^2) + 1/4*sqrt(
2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2
)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^2)
```

<u>Mupad</u> [B]
time = 0.21, size = 1201, normalized size = 4.62

$$\frac{2\,b^2\sqrt{x}}{d}-\frac{2\,a^2}{3\,c\,x^{3/2}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{x}\,\left(16\,a^4\,c^3\,d^{10}-64\,a^3\,b\,c^4\,d^9+96\,a^2\,b^2\,c^5\,d^8-64\,a\,b^3\,c^6\,d^7+16\,b^4\,c^7\,d^6\right)}{2}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c^5\,d^9-32\,a\,b\,c^6\,d^8+16\,b^2\,c^7\,d^7\right)}{2\,{\left(-c\right)}^{7/4}\,d^{5/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{7/4}\,d^{5/4}}+\frac{\left(\frac{\sqrt{x}\,\left(16\,a^4\,c^3\,d^{10}-64\,a^3\,b\,c^4\,d^9+96\,a^2\,b^2\,c^5\,d^8-64\,a\,b^3\,c^6\,d^7+16\,b^4\,c^7\,d^6\right)}{2}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c^5\,d^9-32\,a\,b\,c^6\,d^8+16\,b^2\,c^7\,d^7\right)}{2\,{\left(-c\right)}^{7/4}\,d^{5/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{7/4}\,d^{5/4}}}{\frac{\left(\frac{\sqrt{x}\,\left(16\,a^4\,c^3\,d^{10}-64\,a^3\,b\,c^4\,d^9+96\,a^2\,b^2\,c^5\,d^8-64\,a\,b^3\,c^6\,d^7+16\,b^4\,c^7\,d^6\right)}{2}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c^5\,d^9-32\,a\,b\,c^6\,d^8+16\,b^2\,c^7\,d^7\right)}{2\,{\left(-c\right)}^{7/4}\,d^{5/4}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{7/4}\,d^{5/4}}-\frac{\left(\frac{\sqrt{x}\,\left(16\,a^4\,c^3\,d^{10}-64\,a^3\,b\,c^4\,d^9+96\,a^2\,b^2\,c^5\,d^8-64\,a\,b^3\,c^6\,d^7+16\,b^4\,c^7\,d^6\right)}{2}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c^5\,d^9-32\,a\,b\,c^6\,d^8+16\,b^2\,c^7\,d^7\right)}{2\,{\left(-c\right)}^{7/4}\,d^{5/4}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{7/4}\,d^{5/4}}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{7/4}\,d^{5/4}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{x}\,\left(16\,a^4\,c^3\,d^{10}-64\,a^3\,b\,c^4\,d^9+96\,a^2\,b^2\,c^5\,d^8-64\,a\,b^3\,c^6\,d^7+16\,b^4\,c^7\,d^6\right)}{2}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c^5\,d^9-32\,a\,b\,c^6\,d^8+16\,b^2\,c^7\,d^7\right)\,1{}\mathrm{i}}{2\,{\left(-c\right)}^{7/4}\,d^{5/4}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{7/4}\,d^{5/4}}+\frac{\left(\frac{\sqrt{x}\,\left(16\,a^4\,c^3\,d^{10}-64\,a^3\,b\,c^4\,d^9+96\,a^2\,b^2\,c^5\,d^8-64\,a\,b^3\,c^6\,d^7+16\,b^4\,c^7\,d^6\right)}{2}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c^5\,d^9-32\,a\,b\,c^6\,d^8+16\,b^2\,c^7\,d^7\right)\,1{}\mathrm{i}}{2\,{\left(-c\right)}^{7/4}\,d^{5/4}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{7/4}\,d^{5/4}}}{\frac{\left(\frac{\sqrt{x}\,\left(16\,a^4\,c^3\,d^{10}-64\,a^3\,b\,c^4\,d^9+96\,a^2\,b^2\,c^5\,d^8-64\,a\,b^3\,c^6\,d^7+16\,b^4\,c^7\,d^6\right)}{2}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c^5\,d^9-32\,a\,b\,c^6\,d^8+16\,b^2\,c^7\,d^7\right)\,1{}\mathrm{i}}{2\,{\left(-c\right)}^{7/4}\,d^{5/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{7/4}\,d^{5/4}}-\frac{\left(\frac{\sqrt{x}\,\left(16\,a^4\,c^3\,d^{10}-64\,a^3\,b\,c^4\,d^9+96\,a^2\,b^2\,c^5\,d^8-64\,a\,b^3\,c^6\,d^7+16\,b^4\,c^7\,d^6\right)}{2}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(16\,a^2\,c^5\,d^9-32\,a\,b\,c^6\,d^8+16\,b^2\,c^7\,d^7\right)\,1{}\mathrm{i}}{2\,{\left(-c\right)}^{7/4}\,d^{5/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{7/4}\,d^{5/4}}}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{7/4}\,d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x)
```

```
[Out] (2*b^2*x^(1/2))/d - (2*a^2)/(3*c*x^(3/2)) - (atan(((((x^(1/2)*(16*a^4*c^3*d
^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5
*d^8))/2 - ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8
))/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d^(5/4)) + (((x^(1
/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9
 + 96*a^2*b^2*c^5*d^8))/2 + ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7
```

```
 - 32*a*b*c^6*d^8))/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d
^(5/4)))/((((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 -
 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 - ((a*d - b*c)^2*(16*a^2*c^5*d^9
 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8))/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2)
/((-c)^(7/4)*d^(5/4)) - (((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a
*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 + ((a*d - b*c)^2*(
16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8))/(2*(-c)^(7/4)*d^(5/4)))*
(a*d - b*c)^2)/((-c)^(7/4)*d^(5/4))))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d^(5/4)
) - (atan(((((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7
- 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 - ((a*d - b*c)^2*(16*a^2*c^5*d^
9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8)*1i)/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c
)^2)/((-c)^(7/4)*d^(5/4)) + (((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 -
64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 + ((a*d - b*c)
^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8)*1i)/(2*(-c)^(7/4)*d^(
5/4)))*(a*d - b*c)^2)/((-c)^(7/4)*d^(5/4)))/((((x^(1/2)*(16*a^4*c^3*d^10 +
16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))
/2 - ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8)*1i)/
(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d^(5/4)) - (((x^(1/2)
*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 +
96*a^2*b^2*c^5*d^8))/2 + ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 -
32*a*b*c^6*d^8)*1i)/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d
^(5/4))))*(a*d - b*c)^2)/((-c)^(7/4)*d^(5/4))
```

3.422 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx$

Optimal. Leaf size=267

$$-\frac{2a^2}{5cx^{5/2}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{9/4}d^{3/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{9/4}d^{3/4}}+\frac{(bc-}{}$$

```
[Out] -2/5*a^2/c/x^(5/2)-1/2*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4
))/c^(9/4)/d^(3/4)*2^(1/2)+1/2*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2
)/c^(1/4))/c^(9/4)/d^(3/4)*2^(1/2)+1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^
(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/d^(3/4)*2^(1/2)-1/4*(-a*d+b*c)^2*ln(
c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/d^(3/4)*2^(1/2)-
2*a*(-a*d+2*b*c)/c^2/x^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$,
Rules used = {473, 464, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{2a^2}{5cx^{5/2}}-\frac{(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{9/4}d^{3/4}}+\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{9/4}d^{3/4}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}d^{3/4}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(5*c*x^(5/2)) - (2*a*(2*b*c - a*d))/(c^2*Sqrt[x]) - ((b*c - a*d)^2
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(9/4)*d^(3/4)) +
 ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(
9/4)*d^(3/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4)) - ((b*c - a*d)^2*Log[Sqrt[c] +
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx = -\frac{2a^2}{5cx^{5/2}} + \frac{2\int \frac{\frac{5}{2}a(2bc-ad)+\frac{5}{2}b^2cx^2}{x^{3/2}(c+dx^2)}\,dx}{5c}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} + \frac{(bc-ad)^2\int \frac{\sqrt{x}}{c+dx^2}\,dx}{c^2}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} + \frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^2\sqrt{d}} + \frac{(bc-ad)^2}{}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2d} + \frac{(bc}{}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} + \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}d^{3/4}} - \frac{(bc-}{}$$

$$= -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} - \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{9/4}d^{3/4}} + \frac{(bc-ad)^2\tan^{-1}}{\sqrt{2}}$$

Mathematica [A]
time = 0.19, size = 158, normalized size = 0.59

$$\frac{-\frac{4a\sqrt[4]{c}\left(10bcx^2+a(c-5dx^2)\right)}{x^{5/2}} - \frac{5\sqrt{2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{d^{3/4}} - \frac{5\sqrt{2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{d^{3/4}}}{10c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

[Out] ((-4*a*c^(1/4)*(10*b*c*x^2 + a*(c - 5*d*x^2)))/x^(5/2) - (5*Sqrt[2]*(b*c - a*d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/d^(3/4) - (5*Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(3/4))/(10*c^(9/4))

Maple [A]
time = 0.09, size = 159, normalized size = 0.60

method	result
derivativedivides	$\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-(\frac{c}{d})^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+(\frac{c}{d})^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{(\frac{c}{d})^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{(\frac{c}{d})^{\frac{1}{4}}}-\right.\right.}{4c^2d(\frac{c}{d})^{\frac{1}{4}}}$
default	$\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-(\frac{c}{d})^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+(\frac{c}{d})^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{(\frac{c}{d})^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{(\frac{c}{d})^{\frac{1}{4}}}-\right.\right.}{4c^2d(\frac{c}{d})^{\frac{1}{4}}}$
risch	$-\frac{2(-5adx^2+10cx^2b+ac)a}{5c^2x^{\frac{5}{2}}}+\frac{d\sqrt{2}\ln\left(\frac{x-(\frac{c}{d})^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+(\frac{c}{d})^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)a^2}{4c^2(\frac{c}{d})^{\frac{1}{4}}}-\frac{\sqrt{2}\ln\left(\frac{x-(\frac{c}{d})^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{}}{x+(\frac{c}{d})^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{}}\right.}{2c(\frac{c}{d})^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x,method=_RETURNVERBOSE)

[Out] 1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^2/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))-2/5*a^2/c/x^(5/2)+2*a*(a*d-2*b*c)/c^2/x^(1/2)

Maxima [A]
time = 0.50, size = 229, normalized size = 0.86

$$\frac{(b^2c^2-2abcd+a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4c^2}-\frac{2\left(a^2c+5(2abc-a^2d)x^2\right)}{5c^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="maxima")

[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq

```
rt(c))/(c^(1/4)*d^(3/4)))/c^2 - 2/5*(a^2*c + 5*(2*a*b*c - a^2*d)*x^2)/(c^2*
x^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1648 vs. $2(194) = 388$.
time = 0.50, size = 1648, normalized size = 6.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/10*(20*c^2*x^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
- 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4)*arctan((sqrt((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c
*d^11 + a^12*d^12)*x - (b^8*c^13*d - 8*a*b^7*c^12*d^2 + 28*a^2*b^6*c^11*d^3
 - 56*a^3*b^5*c^10*d^4 + 70*a^4*b^4*c^9*d^5 - 56*a^5*b^3*c^8*d^6 + 28*a^6*b
^2*c^7*d^7 - 8*a^7*b*c^6*d^8 + a^8*c^5*d^9)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3)))*c^2*d*
(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^9*d^3))^(1/4) - (b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d
^3 - 20*a^3*b^3*c^5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^
7)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5
*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7
*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6
*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 2
8*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) - 5*c^2*x^3*(-(b^8*c^8 - 8*a*
b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3
))^(1/4)*log(c^7*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a
^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d
^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d +
 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d
^5 + a^6*d^6)*sqrt(x)) + 5*c^2*x^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4)*log(-c^7*d^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(c^9*d^3))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x))
 + 4*(a^2*c + 5*(2*a*b*c - a^2*d)*x^2)*sqrt(x))/(c^2*x^3)
```

Sympy [A]
time = 42.32, size = 299, normalized size = 1.12

$$a^{2}\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: c = 0 \wedge d = 0 \\ - \frac{2}{9 d x^{\frac{9}{2}}} & \text{for}\: c = 0 \\ - \frac{2}{5 c x^{\frac{5}{2}}} & \text{for}\: d = 0 \\ - \frac{2}{5 c x^{\frac{5}{2}}} + \frac{d \log{\left(\sqrt{x} - \sqrt[4]{- \frac{c}{d}} \right)}}{2 c^{2} \sqrt[4]{- \frac{c}{d}}} - \frac{d \log{\left(\sqrt{x} + \sqrt[4]{- \frac{c}{d}} \right)}}{2 c^{2} \sqrt[4]{- \frac{c}{d}}} + \frac{d \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{c}{d}}} \right)}}{c^{2} \sqrt[4]{- \frac{c}{d}}} + \frac{2 d}{c^{2} \sqrt{x}} & \text{otherwise} \end{cases}\right) + 2 a b \left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: c = 0 \wedge d = 0 \\ - \frac{2}{5 d x^{\frac{5}{2}}} & \text{for}\: c = 0 \\ - \frac{2}{c \sqrt{x}} & \text{for}\: d = 0 \\ - \frac{\log{\left(\sqrt{x} - \sqrt[4]{- \frac{c}{d}} \right)}}{2 c \sqrt[4]{- \frac{c}{d}}} + \frac{\log{\left(\sqrt{x} + \sqrt[4]{- \frac{c}{d}} \right)}}{2 c \sqrt[4]{- \frac{c}{d}}} - \frac{\operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{c}{d}}} \right)}}{c \sqrt[4]{- \frac{c}{d}}} - \frac{2}{c \sqrt{x}} & \text{otherwise} \end{cases}\right) + 2 b^{2} \operatorname{RootSum} {\left(256 t^{4} c d^{3} + 1, \left(t \mapsto t \log{\left(64 t^{3} c d^{2} + \sqrt{x} \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c),x)
```

```
[Out] a**2*Piecewise((zoo/x**(9/2), Eq(c, 0) & Eq(d, 0)), (-2/(9*d*x**(9/2)), Eq(
c, 0)), (-2/(5*c*x**(5/2)), Eq(d, 0)), (-2/(5*c*x**(5/2)) + d*log(sqrt(x) -
 (-c/d)**(1/4))/(2*c**2*(-c/d)**(1/4)) - d*log(sqrt(x) + (-c/d)**(1/4))/(2*
c**2*(-c/d)**(1/4)) + d*atan(sqrt(x)/(-c/d)**(1/4))/(c**2*(-c/d)**(1/4)) +
2*d/(c**2*sqrt(x)), True)) + 2*a*b*Piecewise((zoo/x**(5/2), Eq(c, 0) & Eq(d
, 0)), (-2/(5*d*x**(5/2)), Eq(c, 0)), (-2/(c*sqrt(x)), Eq(d, 0)), (-log(sqr
t(x) - (-c/d)**(1/4))/(2*c*(-c/d)**(1/4)) + log(sqrt(x) + (-c/d)**(1/4))/(2
*c*(-c/d)**(1/4)) - atan(sqrt(x)/(-c/d)**(1/4))/(c*(-c/d)**(1/4)) - 2/(c*sq
rt(x)), True)) + 2*b**2*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*
_t**3*c*d**2 + sqrt(x))))
```

Giac [A]
time = 0.67, size = 353, normalized size = 1.32

$$-\frac{2 \, {\left(10 \, a b c x^{2} - 5 \, a^{2} d x^{2} + a^{2} c\right)}}{5 \, c^{2} x^{\frac{5}{2}}} + \frac{\sqrt{2} {\left(\left(c d^{3}\right)^{\frac{3}{4}} b^{2} c^{2} - 2 \, \left(c d^{3}\right)^{\frac{3}{4}} a b c d + \left(c d^{3}\right)^{\frac{3}{4}} a^{2} d^{2}\right)} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} + 2 \, \sqrt{x}\right)}}{2 \, \left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2 \, c^{3} d^{3}} + \frac{\sqrt{2} {\left(\left(c d^{3}\right)^{\frac{3}{4}} b^{2} c^{2} - 2 \, \left(c d^{3}\right)^{\frac{3}{4}} a b c d + \left(c d^{3}\right)^{\frac{3}{4}} a^{2} d^{2}\right)} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} - 2 \, \sqrt{x}\right)}}{2 \, \left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2 \, c^{3} d^{3}} - \frac{\sqrt{2} {\left(\left(c d^{3}\right)^{\frac{3}{4}} b^{2} c^{2} - 2 \, \left(c d^{3}\right)^{\frac{3}{4}} a b c d + \left(c d^{3}\right)^{\frac{3}{4}} a^{2} d^{2}\right)} \log\left(\sqrt{2} \sqrt{x} \left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4 \, c^{3} d^{3}} + \frac{\sqrt{2} {\left(\left(c d^{3}\right)^{\frac{3}{4}} b^{2} c^{2} - 2 \, \left(c d^{3}\right)^{\frac{3}{4}} a b c d + \left(c d^{3}\right)^{\frac{3}{4}} a^{2} d^{2}\right)} \log\left(-\sqrt{2} \sqrt{x} \left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4 \, c^{3} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -2/5*(10*a*b*c*x^2 - 5*a^2*d*x^2 + a^2*c)/(c^2*x^(5/2)) + 1/2*sqrt(2)*((c*d
^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan
(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) + 1/2
*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a
^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/
(c^3*d^3) - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d +
(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^
3*d^3) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*
d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*
d^3)
```

Mupad [B]
time = 0.19, size = 417, normalized size = 1.56

$$\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^4\,c^7\,d^6-64\,a^3\,b\,c^8\,d^5+96\,a^2\,b^2\,c^9\,d^4-64\,a\,b^3\,c^{10}\,d^3+16\,b^4\,c^{11}\,d^2\right)}{{\left(-c\right)}^{9/4}\,d^{3/4}\,\left(16\,a^6\,c^5\,d^7-96\,a^5\,b\,c^6\,d^6+240\,a^4\,b^2\,c^7\,d^5-320\,a^3\,b^3\,c^8\,d^4+240\,a^2\,b^4\,c^9\,d^3-96\,a\,b^5\,c^{10}\,d^2+16\,b^6\,c^{11}\,d\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{9/4}\,d^{3/4}}-\frac{\frac{2\,a^2}{5\,c}-\frac{2\,a\,x^2\,\left(a\,d-2\,b\,c\right)}{c^2}}{x^{5/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^4\,c^7\,d^6-64\,a^3\,b\,c^8\,d^5+96\,a^2\,b^2\,c^9\,d^4-64\,a\,b^3\,c^{10}\,d^3+16\,b^4\,c^{11}\,d^2\right)}{{\left(-c\right)}^{9/4}\,d^{3/4}\,\left(16\,a^6\,c^5\,d^7-96\,a^5\,b\,c^6\,d^6+240\,a^4\,b^2\,c^7\,d^5-320\,a^3\,b^3\,c^8\,d^4+240\,a^2\,b^4\,c^9\,d^3-96\,a\,b^5\,c^{10}\,d^2+16\,b^6\,c^{11}\,d\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{9/4}\,d^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x)
```

```
[Out] (atan((x^(1/2)*(a*d - b*c)^2*(16*a^4*c^7*d^6 + 16*b^4*c^11*d^2 - 64*a*b^3*c
^10*d^3 - 64*a^3*b*c^8*d^5 + 96*a^2*b^2*c^9*d^4))/((-c)^(9/4)*d^(3/4)*(16*b
^6*c^11*d + 16*a^6*c^5*d^7 - 96*a*b^5*c^10*d^2 - 96*a^5*b*c^6*d^6 + 240*a^2
*b^4*c^9*d^3 - 320*a^3*b^3*c^8*d^4 + 240*a^4*b^2*c^7*d^5)))*(a*d - b*c)^2)/
((-c)^(9/4)*d^(3/4)) - ((2*a^2)/(5*c) - (2*a*x^2*(a*d - 2*b*c))/c^2)/x^(5/2
) - (atanh((x^(1/2)*(a*d - b*c)^2*(16*a^4*c^7*d^6 + 16*b^4*c^11*d^2 - 64*a*
b^3*c^10*d^3 - 64*a^3*b*c^8*d^5 + 96*a^2*b^2*c^9*d^4))/((-c)^(9/4)*d^(3/4)*
(16*b^6*c^11*d + 16*a^6*c^5*d^7 - 96*a*b^5*c^10*d^2 - 96*a^5*b*c^6*d^6 + 24
0*a^2*b^4*c^9*d^3 - 320*a^3*b^3*c^8*d^4 + 240*a^4*b^2*c^7*d^5)))*(a*d - b*c
)^2)/((-c)^(9/4)*d^(3/4))
```

3.423 $\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx$

Optimal. Leaf size=269

$$-\frac{2a^2}{7cx^{7/2}}-\frac{2a(2bc-ad)}{3c^2x^{3/2}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}}-\frac{(bc}{}$$

```
[Out] -2/7*a^2/c/x^(7/2)-2/3*a*(-a*d+2*b*c)/c^2/x^(3/2)-1/2*(-a*d+b*c)^2*arctan(1
-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(1/4)*2^(1/2)+1/2*(-a*d+b*c)^2
*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(1/4)*2^(1/2)-1/4*(-a
*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/d^
(1/4)*2^(1/2)+1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)
*x^(1/2))/c^(11/4)/d^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {473, 464, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{2a^2}{7cx^{7/2}}-\frac{(bc-ad)^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{11/4}\sqrt[4]{d}}-\frac{2a(2bc-ad)}{3c^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(7*c*x^(7/2)) - (2*a*(2*b*c - a*d))/(3*c^2*x^(3/2)) - ((b*c - a*d)
^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(11/4)*d^(1/4)
) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(11/4)*d^(1/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1/4)) + ((b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1
/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx = -\frac{2a^2}{7cx^{7/2}} + \frac{2\int \frac{\frac{7}{2}a(2bc-ad)+\frac{7}{2}b^2cx^2}{x^{5/2}(c+dx^2)}\,dx}{7c}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{x}\,(c+dx^2)}\,dx}{c^2}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(2(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{(bc-ad)\cdots}{\cdots}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}\sqrt{d}} + \cdots$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} - \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{11/4}\sqrt[4]{d}} + \frac{(bc\cdots}{\cdots}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} - \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}} + \frac{(bc-ad)^2\tan^{-1}\cdots}{\sqrt{2}\cdots}$$

Mathematica [A]
time = 0.19, size = 159, normalized size = 0.59

$$\frac{\frac{4ac^{3/4}(-3ac-14bcx^2+7adx^2)}{x^{7/2}} - \frac{21\sqrt{2}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{d}} + \frac{21\sqrt{2}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt[4]{d}}}{42c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

```
[Out] ((4*a*c^(3/4)*(-3*a*c - 14*b*c*x^2 + 7*a*d*x^2))/x^(7/2) - (21*Sqrt[2]*(b*c
 - a*d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/
d^(1/4) + (21*Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
])/(Sqrt[c] + Sqrt[d]*x)])/d^(1/4))/(42*c^(11/4))
```

Maple [A]
time = 0.10, size = 156, normalized size = 0.58

method	result
derivativedivides	$\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{4c^3}$
default	$\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{4c^3}$
risch	$-\frac{2(-7adx^2+14cx^2b+3ac)a}{21c^2x^{\frac{7}{2}}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2d^2}{2c^3}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)c}{c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
-1))-2/7*a^2/c/x^(7/2)+2/3*a*(a*d-2*b*c)/c^2/x^(3/2)
```

Maxima [A]
time = 0.51, size = 293, normalized size = 1.09

$$\frac{\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(b^2c^2-2\,abcd+a^2d^2)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{4\,c^2}-\frac{2\,(3\,a^2c+7\,(2\,abc-a^2d)x^2)}{21\,c^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(s
qrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/
(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4))
 - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqr
```

```
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/c^2 - 2/21*(3*a^2*c + 7*(2*a
*b*c - a^2*d)*x^2)/(c^2*x^(7/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1252 vs. $2(194) = 388$.
time = 0.48, size = 1252, normalized size = 4.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/42*(84*c^2*x^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*arctan((sqrt(c^6*sqrt(-(b^8*c^8 -
 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d
^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^
11*d)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*x)*c^8*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(3/4) - (b^2*c^10*d - 2*a*b*c^9*d^2 + a^2
*c^8*d^3)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(3/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a
^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 21*c^2*x^4*(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(
c^11*d))^(1/4)*log(c^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*sqrt(x)) - 21*c^2*x^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*log(-c^3*(-(b^8*c^8 -
 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d
^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^
11*d))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 4*(3*a^2*c + 7*(2
*a*b*c - a^2*d)*x^2)*sqrt(x))/(c^2*x^4)
```

Sympy [A]
time = 68.54, size = 449, normalized size = 1.67

$$\begin{cases} \tilde{\infty} \left(- \frac{2 a^{2}}{11 x^{\frac{11}{2}}} - \frac{4 a b}{7 x^{\frac{7}{2}}} - \frac{2 b^{2}}{3 x^{\frac{3}{2}}}\right) & \text{for}\: c = 0 \wedge d = 0 \\ \frac{- \frac{2 a^{2}}{11 x^{\frac{11}{2}}} - \frac{4 a b}{7 x^{\frac{7}{2}}} - \frac{2 b^{2}}{3 x^{\frac{3}{2}}}}{d} & \text{for}\: c = 0 \\ \frac{- \frac{2 a^{2}}{7 x^{\frac{7}{2}}} - \frac{4 a b}{3 x^{\frac{3}{2}}} + 2 b^{2} \sqrt{x}}{c} & \text{for}\: d = 0 \\ - \frac{2 a^{2}}{7 c x^{\frac{7}{2}}} + \frac{2 a^{2} d}{3 c^{2} x^{\frac{3}{2}}} - \frac{a^{2} d^{2} \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{c}{d}} \right)}}{2 c^{3}} + \frac{a^{2} d^{2} \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{c}{d}} \right)}}{2 c^{3}} + \frac{a^{2} d^{2} \sqrt[4]{- \frac{c}{d}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{c}{d}}} \right)}}{c^{3}} - \frac{4 a b}{3 c x^{\frac{3}{2}}} + \frac{a b d \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{c}{d}} \right)}}{c^{2}} - \frac{a b d \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{c}{d}} \right)}}{c^{2}} - \frac{2 a b d \sqrt[4]{- \frac{c}{d}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{c}{d}}} \right)}}{c^{2}} - \frac{b^{2} \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{c}{d}} \right)}}{2 c} + \frac{b^{2} \sqrt[4]{- \frac{c}{d}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{c}{d}} \right)}}{2 c} + \frac{b^{2} \sqrt[4]{- \frac{c}{d}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{c}{d}}} \right)}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(9/2)/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/2)) - 2*b**2/(3*x**
(3/2))), Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/2)
) - 2*b**2/(3*x**(3/2)))/d, Eq(c, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x*
*(3/2)) + 2*b**2*sqrt(x))/c, Eq(d, 0)), (-2*a**2/(7*c*x**(7/2)) + 2*a**2*d/
(3*c**2*x**(3/2)) - a**2*d**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2
*c**3) + a**2*d**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*c**3) + a*
*2*d**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/c**3 - 4*a*b/(3*c*x**(3/2
)) + a*b*d*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/c**2 - a*b*d*(-c/d)**
(1/4)*log(sqrt(x) + (-c/d)**(1/4))/c**2 - 2*a*b*d*(-c/d)**(1/4)*atan(sqrt(x
)/(-c/d)**(1/4))/c**2 - b**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*
c) + b**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*c) + b**2*(-c/d)**(
1/4)*atan(sqrt(x)/(-c/d)**(1/4))/c, True))
```

Giac [A]
time = 0.74, size = 354, normalized size = 1.32

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2c^{3}d}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2c^{3}d}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4c^{3}d}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4c^{3}d}-\frac{2\left(14abcx^{2}-7a^{2}dx^{2}+3a^{2}c\right)}{21c^{2}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c^3*d) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x)
)/(c/d)^(1/4))/(c^3*d) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/
4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c^3*d) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*
b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(c^3*d) - 2/21*(14*a*b*c*x^2 - 7*a^2*d*x^2 + 3*a^2*c)/(c^2*x^(7/2))
```

Mupad [B]
time = 0.24, size = 1209, normalized size = 4.49

$$-\frac{\frac{2a^2}{7c}-\frac{2ax^2(ad-2bc)}{3c^2}}{x^{7/2}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{x}\,(16a^4c^6d^7-64a^3bc^7d^6+96a^2b^2c^8d^5-64ab^3c^9d^4+16b^4c^{10}d^3)}{2}-\frac{(ad-bc)^2(16a^2c^9d^5-32abc^{10}d^4+16b^2c^{11}d^3)}{2(-c)^{11/4}d^{1/4}}\right)(ad-bc)^2\,1{}\mathrm{i}}{(-c)^{11/4}d^{1/4}}+\frac{\left(\frac{\sqrt{x}\,(16a^4c^6d^7-64a^3bc^7d^6+96a^2b^2c^8d^5-64ab^3c^9d^4+16b^4c^{10}d^3)}{2}+\frac{(ad-bc)^2(16a^2c^9d^5-32abc^{10}d^4+16b^2c^{11}d^3)}{2(-c)^{11/4}d^{1/4}}\right)(ad-bc)^2\,1{}\mathrm{i}}{(-c)^{11/4}d^{1/4}}}{\frac{\left(\frac{\sqrt{x}\,(16a^4c^6d^7-64a^3bc^7d^6+96a^2b^2c^8d^5-64ab^3c^9d^4+16b^4c^{10}d^3)}{2}-\frac{(ad-bc)^2(16a^2c^9d^5-32abc^{10}d^4+16b^2c^{11}d^3)}{2(-c)^{11/4}d^{1/4}}\right)(ad-bc)^2}{(-c)^{11/4}d^{1/4}}-\frac{\left(\frac{\sqrt{x}\,(16a^4c^6d^7-64a^3bc^7d^6+96a^2b^2c^8d^5-64ab^3c^9d^4+16b^4c^{10}d^3)}{2}+\frac{(ad-bc)^2(16a^2c^9d^5-32abc^{10}d^4+16b^2c^{11}d^3)}{2(-c)^{11/4}d^{1/4}}\right)(ad-bc)^2}{(-c)^{11/4}d^{1/4}}}\right)(ad-bc)^2\,1{}\mathrm{i}}{(-c)^{11/4}d^{1/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{x}\,(16a^4c^6d^7-64a^3bc^7d^6+96a^2b^2c^8d^5-64ab^3c^9d^4+16b^4c^{10}d^3)}{2}-\frac{(ad-bc)^2(16a^2c^9d^5-32abc^{10}d^4+16b^2c^{11}d^3)\,1{}\mathrm{i}}{2(-c)^{11/4}d^{1/4}}\right)(ad-bc)^2}{(-c)^{11/4}d^{1/4}}+\frac{\left(\frac{\sqrt{x}\,(16a^4c^6d^7-64a^3bc^7d^6+96a^2b^2c^8d^5-64ab^3c^9d^4+16b^4c^{10}d^3)}{2}+\frac{(ad-bc)^2(16a^2c^9d^5-32abc^{10}d^4+16b^2c^{11}d^3)\,1{}\mathrm{i}}{2(-c)^{11/4}d^{1/4}}\right)(ad-bc)^2}{(-c)^{11/4}d^{1/4}}}{\frac{\left(\frac{\sqrt{x}\,(16a^4c^6d^7-64a^3bc^7d^6+96a^2b^2c^8d^5-64ab^3c^9d^4+16b^4c^{10}d^3)}{2}-\frac{(ad-bc)^2(16a^2c^9d^5-32abc^{10}d^4+16b^2c^{11}d^3)\,1{}\mathrm{i}}{2(-c)^{11/4}d^{1/4}}\right)(ad-bc)^2\,1{}\mathrm{i}}{(-c)^{11/4}d^{1/4}}-\frac{\left(\frac{\sqrt{x}\,(16a^4c^6d^7-64a^3bc^7d^6+96a^2b^2c^8d^5-64ab^3c^9d^4+16b^4c^{10}d^3)}{2}+\frac{(ad-bc)^2(16a^2c^9d^5-32abc^{10}d^4+16b^2c^{11}d^3)\,1{}\mathrm{i}}{2(-c)^{11/4}d^{1/4}}\right)(ad-bc)^2\,1{}\mathrm{i}}{(-c)^{11/4}d^{1/4}}}\right)(ad-bc)^2}{(-c)^{11/4}d^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x)
```

```
[Out] (atan(((((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64
*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 - ((a*d - b*c)^2*(16*a^2*c^9*d^5 +
16*b^2*c^11*d^3 - 32*a*b*c^10*d^4))/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2*
1i)/((-c)^(11/4)*d^(1/4)) + (((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 -
64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 + ((a*d - b*c)
```

```
^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4))/(2*(-c)^(11/4)*d^(
1/4)))*(a*d - b*c)^2*1i)/((-c)^(11/4)*d^(1/4)))/((((x^(1/2)*(16*a^4*c^6*d^7
 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d
^5))/2 - ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4
))/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*d^(1/4)) - (((x^(1/
2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6
+ 96*a^2*b^2*c^8*d^5))/2 + ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3
 - 32*a*b*c^10*d^4))/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*d
^(1/4))))*(a*d - b*c)^2*1i)/((-c)^(11/4)*d^(1/4)) - ((2*a^2)/(7*c) - (2*a*x
^2*(a*d - 2*b*c))/(3*c^2))/x^(7/2) + (atan(((((x^(1/2)*(16*a^4*c^6*d^7 + 16
*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/
2 - ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4)*1i)
/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*d^(1/4)) + (((x^(1/2)
*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 +
96*a^2*b^2*c^8*d^5))/2 + ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 -
 32*a*b*c^10*d^4)*1i)/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*
d^(1/4)))/((((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4
- 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 - ((a*d - b*c)^2*(16*a^2*c^9*d^
5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4)*1i)/(2*(-c)^(11/4)*d^(1/4)))*(a*d -
b*c)^2*1i)/((-c)^(11/4)*d^(1/4)) - (((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10
*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 + ((a*d
 - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4)*1i)/(2*(-c)^
(11/4)*d^(1/4)))*(a*d - b*c)^2*1i)/((-c)^(11/4)*d^(1/4))))*(a*d - b*c)^2)/(
(-c)^(11/4)*d^(1/4))
```

3.424 $\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx$

Optimal. Leaf size=288

$$-\frac{2c^2}{9ax^{9/2}}+\frac{2c(bc-2ad)}{5a^2x^{5/2}}-\frac{2(bc-ad)^2}{a^3\sqrt{x}}+\frac{\sqrt[4]{b}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{13/4}}-\frac{\sqrt[4]{b}\,(bc-ad)^2\tan^{-1}\left(1+\right.}{\sqrt{2}\,a^{13/4}}$$

```
[Out] -2/9*c^2/a/x^(9/2)+2/5*c*(-2*a*d+b*c)/a^2/x^(5/2)+1/2*b^(1/4)*(-a*d+b*c)^2*
arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-1/2*b^(1/4)*(-a*
d+b*c)^2*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-1/4*b^(
1/4)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(
13/4)*2^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)
*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)-2*(-a*d+b*c)^2/a^3/x^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {473, 464, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2}{\sqrt{2}\,a^{13/4}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2}{\sqrt{2}\,a^{13/4}}-\frac{\sqrt[4]{b}\,(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}}+\frac{\sqrt[4]{b}\,(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}}-\frac{2(bc-ad)^2}{a^3\sqrt{x}}+\frac{2c(bc-2ad)}{5a^2x^{5/2}}-\frac{2c^2}{9ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^2)/(9*a*x^(9/2)) + (2*c*(b*c - 2*a*d))/(5*a^2*x^(5/2)) - (2*(b*c - a*
d)^2)/(a^3*Sqrt[x]) + (b^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^
2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(13/4)) + (b^(1/4)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx = -\frac{2c^2}{9ax^{9/2}} + \frac{2\int \frac{-\frac{9}{2}c(bc-2ad)+\frac{9}{2}ad^2x^2}{x^{7/2}(a+bx^2)}\,dx}{9a}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} + \frac{(bc-ad)^2\int \frac{1}{x^{3/2}(a+bx^2)}\,dx}{a^2}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{(b(bc-ad)^2)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a^3}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{(2b(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^3}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} + \frac{\left(\sqrt{b}\,(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx\right.}{a^3}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,d\right.}{2a^3}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{\sqrt[4]{b}\,(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,\sqrt{\right.}}{2\sqrt{2}\,a^{13/4}}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} + \frac{\sqrt[4]{b}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{13/4}}$$

Mathematica [A]
time = 0.23, size = 192, normalized size = 0.67

$$\frac{-\frac{4\sqrt[4]{a}\,(45b^2c^2x^4-9abcx^2(c+10dx^2)+a^2(5c^2+18cdx^2+45d^2x^4))}{x^{9/2}}+45\sqrt{2}\sqrt[4]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+45\sqrt{2}\sqrt[4]{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{90a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] ((-4*a^(1/4)*(45*b^2*c^2*x^4 - 9*a*b*c*x^2*(c + 10*d*x^2) + a^2*(5*c^2 + 18
*c*d*x^2 + 45*d^2*x^4)))/x^(9/2) + 45*Sqrt[2]*b^(1/4)*(b*c - a*d)^2*ArcTan[
(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 45*Sqrt[2]*b^(1/
4)*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[
b]*x)])/(90*a^(13/4))
```

Maple [A]
time = 0.10, size = 186, normalized size = 0.65

method	result
derivativedivides	$-\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4a^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$-\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4a^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(45a^2d^2x^4-90abcd\,x^4+45b^2c^2x^4+18a^2cd\,x^2-9ab\,c^2x^2+5a^2c^2)}{45a^3x^{\frac{9}{2}}}-\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)d^2}{2a\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\ \text{arc}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+
2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1))-2/9*c^2/a/x^(9/2)-2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^3/x^(1/2)-2/5*c*(2*
a*d-b*c)/a^2/x^(5/2)
```

Maxima [A]
time = 0.50, size = 263, normalized size = 0.91

$$-\frac{(b^3c^2-2\,ab^2cd+a^2bd^2)\left(\frac{2\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\ \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\ \log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\ \log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,a^3}-\frac{2\left(45\left(b^2c^2-2\,abcd+a^2d^2\right)x^4+5\,a^2c^2-9\left(abc^2-2\,a^2cd\right)x^2\right)}{45\,a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqr
t(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt
(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(
a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3 - 2/45*(45*(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*x^4 + 5*a^2*c^2 - 9*(a*b*c^2 - 2*a^2*c*d)*x^2)/(a^3*x^(9/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1686 vs. $2(211) = 422$.

time = 0.54, size = 1686, normalized size = 5.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/90*(180*a^3*x^5*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*
b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6
- 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4)*arctan((sqrt((b^14*c^12 - 12*a*b
^13*c^11*d + 66*a^2*b^12*c^10*d^2 - 220*a^3*b^11*c^9*d^3 + 495*a^4*b^10*c^8
*d^4 - 792*a^5*b^9*c^7*d^5 + 924*a^6*b^8*c^6*d^6 - 792*a^7*b^7*c^5*d^7 + 49
5*a^8*b^6*c^4*d^8 - 220*a^9*b^5*c^3*d^9 + 66*a^10*b^4*c^2*d^10 - 12*a^11*b^
3*c*d^11 + a^12*b^2*d^12)*x - (a^7*b^9*c^8 - 8*a^8*b^8*c^7*d + 28*a^9*b^7*c
^6*d^2 - 56*a^10*b^6*c^5*d^3 + 70*a^11*b^5*c^4*d^4 - 56*a^12*b^4*c^3*d^5 +
28*a^13*b^3*c^2*d^6 - 8*a^14*b^2*c*d^7 + a^15*b*d^8)*sqrt(-(b^9*c^8 - 8*a*b
^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 5
6*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)
)*a^3*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3
+ 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*
c*d^7 + a^8*b*d^8)/a^13)^(1/4) - (a^3*b^7*c^6 - 6*a^4*b^6*c^5*d + 15*a^5*b^
5*c^4*d^2 - 20*a^6*b^4*c^3*d^3 + 15*a^7*b^3*c^2*d^4 - 6*a^8*b^2*c*d^5 + a^9
*b*d^6)*sqrt(x)*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^
6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 -
8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4))/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2
*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5
 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)) - 45*a^3*x^5*(-(b^9*c
^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c
^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*
d^8)/a^13)^(1/4)*log(a^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 -
 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*
c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(3/4) + (b^7*c^6 - 6*a*b^6*c^5
*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b
^2*c*d^5 + a^6*b*d^6)*sqrt(x)) + 45*a^3*x^5*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28
*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3
*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4)*log(-a
^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 +
70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*
d^7 + a^8*b*d^8)/a^13)^(3/4) + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^
2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*
sqrt(x)) - 4*(45*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 + 5*a^2*c^2 - 9*(a*b*c
^2 - 2*a^2*c*d)*x^2)*sqrt(x))/(a^3*x^5)
```

Sympy [A]

time = 184.60, size = 428, normalized size = 1.49

$$c^2\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{13}{2}}} & \text{for}\ a=0 \wedge b=0 \\ -\frac{2}{13bx^{\frac{13}{2}}} & \text{for}\ a=0 \\ -\frac{2}{9ax^{\frac{9}{2}}} & \text{for}\ b=0 \\ -\frac{2}{9ax^{\frac{9}{2}}}+\frac{2b}{5a^2x^{\frac{5}{2}}}-\frac{b^2\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a^3\sqrt[4]{-\frac{a}{b}}}+\frac{b^2\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a^3\sqrt[4]{-\frac{a}{b}}}-\frac{b^2\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^3\sqrt[4]{-\frac{a}{b}}}-\frac{2b^2}{a^3\sqrt{x}} & \text{otherwise} \end{cases}\right)+2cd\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\ a=0 \wedge b=0 \\ -\frac{2}{9bx^{\frac{9}{2}}} & \text{for}\ a=0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for}\ b=0 \\ -\frac{2}{5ax^{\frac{5}{2}}}+\frac{b\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}}-\frac{b\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2\sqrt[4]{-\frac{a}{b}}}+\frac{2b}{a^2\sqrt{x}} & \text{otherwise} \end{cases}\right)+d^2\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\ a=0 \wedge b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\ a=0 \\ -\frac{2}{a\sqrt{x}} & \text{for}\ b=0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}+\frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}-\frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}}-\frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**(11/2)/(b*x**2+a),x)
```

```
[Out] c**2*Piecewise((zoo/x**(13/2), Eq(a, 0) & Eq(b, 0)), (-2/(13*b*x**(13/2)),
Eq(a, 0)), (-2/(9*a*x**(9/2)), Eq(b, 0)), (-2/(9*a*x**(9/2)) + 2*b/(5*a**2*
x**(5/2)) - b**2*log(sqrt(x) - (-a/b)**(1/4))/(2*a**3*(-a/b)**(1/4)) + b**2
*log(sqrt(x) + (-a/b)**(1/4))/(2*a**3*(-a/b)**(1/4)) - b**2*atan(sqrt(x)/(-
a/b)**(1/4))/(a**3*(-a/b)**(1/4)) - 2*b**2/(a**3*sqrt(x)), True)) + 2*c*d*P
iecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(a, 0))
, (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + b*log(sqrt(x) - (-a/b
)**(1/4))/(2*a**2*(-a/b)**(1/4)) - b*log(sqrt(x) + (-a/b)**(1/4))/(2*a**2*(
-a/b)**(1/4)) + b*atan(sqrt(x)/(-a/b)**(1/4))/(a**2*(-a/b)**(1/4)) + 2*b/(a
**2*sqrt(x)), True)) + d**2*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)),
(-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) -
(-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2*a*(-a/
b)**(1/4)) - atan(sqrt(x)/(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*sqrt(x)),
 True))
```

<u>Giac</u> [A]
time = 0.63, size = 390, normalized size = 1.35

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}abcd+\left(ab^3\right)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b^2}-\frac{2\left(45b^2c^2x^4-90abcdx^4+45a^2d^2x^4-9abc^2x^2+18a^2cdx^2+5a^2c^2\right)}{45a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d + (a*b^3)^(3/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4
))/(a^4*b^2) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d
 + (a*b^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^2) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3
)^(3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(a^4*b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(
3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
+ sqrt(a/b))/(a^4*b^2) - 2/45*(45*b^2*c^2*x^4 - 90*a*b*c*d*x^4 + 45*a^2*d^2
*x^4 - 9*a*b*c^2*x^2 + 18*a^2*c*d*x^2 + 5*a^2*c^2)/(a^3*x^(9/2))
```

<u>Mupad</u> [B]
time = 0.20, size = 451, normalized size = 1.57

$$\frac{{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^{14}\,b^4\,d^4-64\,a^{13}\,b^5\,c\,d^3+96\,a^{12}\,b^6\,c^2\,d^2-64\,a^{11}\,b^7\,c^3\,d+16\,a^{10}\,b^8\,c^4\right)}{a^{13/4}\,\left(16\,a^{13}\,b^4\,d^6-96\,a^{12}\,b^5\,c\,d^5+240\,a^{11}\,b^6\,c^2\,d^4-320\,a^{10}\,b^7\,c^3\,d^3+240\,a^9\,b^8\,c^4\,d^2-96\,a^8\,b^9\,c^5\,d+16\,a^7\,b^{10}\,c^6\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{a^{13/4}}-\frac{{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^{14}\,b^4\,d^4-64\,a^{13}\,b^5\,c\,d^3+96\,a^{12}\,b^6\,c^2\,d^2-64\,a^{11}\,b^7\,c^3\,d+16\,a^{10}\,b^8\,c^4\right)}{a^{13/4}\,\left(16\,a^{13}\,b^4\,d^6-96\,a^{12}\,b^5\,c\,d^5+240\,a^{11}\,b^6\,c^2\,d^4-320\,a^{10}\,b^7\,c^3\,d^3+240\,a^9\,b^8\,c^4\,d^2-96\,a^8\,b^9\,c^5\,d+16\,a^7\,b^{10}\,c^6\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{a^{13/4}}-\frac{\frac{2\,c^2}{9\,a}+\frac{2\,x^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{a^3}+\frac{2\,c\,x^2\,\left(2\,a\,d-b\,c\right)}{5\,a^2}}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2)*(a*d - b*c)^2*(16*a^10*b^8*c^4 + 16*a
^14*b^4*d^4 - 64*a^11*b^7*c^3*d - 64*a^13*b^5*c*d^3 + 96*a^12*b^6*c^2*d^2))
/(a^(13/4)*(16*a^7*b^10*c^6 + 16*a^13*b^4*d^6 - 96*a^8*b^9*c^5*d - 96*a^12*
b^5*c*d^5 + 240*a^9*b^8*c^4*d^2 - 320*a^10*b^7*c^3*d^3 + 240*a^11*b^6*c^2*d
^4)))*(a*d - b*c)^2)/a^(13/4) - ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2)*(a*d -
 b*c)^2*(16*a^10*b^8*c^4 + 16*a^14*b^4*d^4 - 64*a^11*b^7*c^3*d - 64*a^13*b^
5*c*d^3 + 96*a^12*b^6*c^2*d^2))/(a^(13/4)*(16*a^7*b^10*c^6 + 16*a^13*b^4*d^
6 - 96*a^8*b^9*c^5*d - 96*a^12*b^5*c*d^5 + 240*a^9*b^8*c^4*d^2 - 320*a^10*b
^7*c^3*d^3 + 240*a^11*b^6*c^2*d^4)))*(a*d - b*c)^2)/a^(13/4) - ((2*c^2)/(9*
a) + (2*x^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/a^3 + (2*c*x^2*(2*a*d - b*c))/
(5*a^2))/x^(9/2)
```

3.425 $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=375

$$\frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4}-\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3}+\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2\,(c+dx^2)}+\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)}{4\sqrt{2}}$$

```
[Out] -1/10*(-5*a*d+13*b*c)*(-a*d+b*c)*x^(5/2)/c/d^3+2/9*b^2*x^(9/2)/d^2+1/2*(-a*
d+b*c)^2*x^(9/2)/c/d^2/(d*x^2+c)+1/8*c^(1/4)*(-5*a*d+13*b*c)*(-a*d+b*c)*arc
tan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(17/4)*2^(1/2)-1/8*c^(1/4)*(-5*a*d
+13*b*c)*(-a*d+b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(17/4)*2^(1
/2)+1/16*c^(1/4)*(-5*a*d+13*b*c)*(-a*d+b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^
(1/4)*2^(1/2)*x^(1/2))/d^(17/4)*2^(1/2)-1/16*c^(1/4)*(-5*a*d+13*b*c)*(-a*d+
b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(17/4)*2^(1/2)
+1/2*(-5*a*d+13*b*c)*(-a*d+b*c)*x^(1/2)/d^4
```

Rubi [A]

time = 0.30, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {474, 470, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,d^{17/4}}-\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,d^{17/4}}+\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{17/4}}-\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{17/4}}+\frac{\sqrt{x}\,(13bc-5ad)(bc-ad)}{2d^4}-\frac{x^{5/2}(13bc-5ad)(bc-ad)}{10cd^3}+\frac{x^{9/2}(bc-ad)^2}{2cd^2\,(c+dx^2)}+\frac{2b^2x^{9/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((13*b*c - 5*a*d)*(b*c - a*d)*Sqrt[x])/(2*d^4) - ((13*b*c - 5*a*d)*(b*c - a
*d)*x^(5/2))/(10*c*d^3) + (2*b^2*x^(9/2))/(9*d^2) + ((b*c - a*d)^2*x^(9/2))
/(2*c*d^2*(c + d*x^2)) + (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*b*c
- 5*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt
[2]*d^(17/4)) + (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*
b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(8*Sqrt[2]*d^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{x^{7/2}\left(\frac{1}{2}(3bc-5ad)(3bc-ad)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} - \frac{((13bc-5ad)(bc-ad))\int \frac{x^{7/2}}{c+dx^2}\,dx}{4cd^2}$$

$$= -\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} + \frac{((13bc-5ad)(bc-ad))}{4d^3}$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)}$$

Mathematica [A]
time = 0.63, size = 255, normalized size = 0.68

$$\frac{\frac{4\sqrt[4]{d}\sqrt{x}\left(45a^2d^2(5c+4dx^2)+18abd(-45c^2-36cdx^2+4d^2x^4)+b^2(585c^3+468c^2dx^2-52cd^2x^4+20d^3x^6)\right)}{c+dx^2}+45\sqrt{2}\sqrt[4]{c}\left(13b^2c^2-18abcd+5a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)-45\sqrt{2}\sqrt[4]{c}\left(13b^2c^2-18abcd+5a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{360d^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((4*d^(1/4)*Sqrt[x]*(45*a^2*d^2*(5*c + 4*d*x^2) + 18*a*b*d*(-45*c^2 - 36*c*
d*x^2 + 4*d^2*x^4) + b^2*(585*c^3 + 468*c^2*d*x^2 - 52*c*d^2*x^4 + 20*d^3*x
^6)))/(c + d*x^2) + 45*Sqrt[2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2
)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - 45*Sqrt
```

```
[2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(360*d^(17/4))
```

Maple [A]
time = 0.12, size = 240, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{2b^2x^{\frac{9}{2}}d^2}{9}+\frac{4abd^2x^{\frac{5}{2}}}{5}-\frac{4b^2cdx^{\frac{5}{2}}}{5}+2a^2d^2\sqrt{x}-8abcd\sqrt{x}+6b^2c^2\sqrt{x}}{d^4}-\frac{2c\left(\frac{\left(-\frac{1}{4}a^2d^2+\frac{1}{2}abcd-\frac{1}{4}b^2c^2\right)\sqrt{x}}{dx^2+c}+\frac{\left(5a^2d^2-\right.}{}\right.}{}$
default	$\frac{\frac{2b^2x^{\frac{9}{2}}d^2}{9}+\frac{4abd^2x^{\frac{5}{2}}}{5}-\frac{4b^2cdx^{\frac{5}{2}}}{5}+2a^2d^2\sqrt{x}-8abcd\sqrt{x}+6b^2c^2\sqrt{x}}{d^4}-\frac{2c\left(\frac{\left(-\frac{1}{4}a^2d^2+\frac{1}{2}abcd-\frac{1}{4}b^2c^2\right)\sqrt{x}}{dx^2+c}+\frac{\left(5a^2d^2-\right.}{}\right.}{}$
risch	$\frac{2\left(5b^2d^2x^4+18abd^2x^2-18b^2cdx^2+45a^2d^2-180abcd+135b^2c^2\right)\sqrt{x}}{45d^4}+\frac{c\sqrt{x}\,a^2}{2d^2\left(dx^2+c\right)}-\frac{c^2\sqrt{x}\,ab}{d^3\left(dx^2+c\right)}+\frac{c^3\sqrt{x}\,b^2}{2d^4\left(dx^2+c\right)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/9*b^2*x^(9/2)*d^2+2/5*a*b*d^2*x^(5/2)-2/5*b^2*c*d*x^(5/2)+a^2*d^2*
x^(1/2)-4*a*b*c*d*x^(1/2)+3*b^2*c^2*x^(1/2))-2*c/d^4*((-1/4*a^2*d^2+1/2*a*b
*c*d-1/4*b^2*c^2)*x^(1/2)/(d*x^2+c)+1/32*(5*a^2*d^2-18*a*b*c*d+13*b^2*c^2)*
(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.53, size = 377, normalized size = 1.01

$$\frac{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{x}}{2\left(d^5x^2+cd^4\right)}-\frac{\left(\frac{2\sqrt{2}\left(13\,b^2c^2-18\,abcd+5\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(13\,b^2c^2-18\,abcd+5\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(13\,b^2c^2-18\,abcd+5\,a^2d^2\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(13\,b^2c^2-18\,abcd+5\,a^2d^2\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)c}{16\,d^4}+\frac{2\left(5\,b^2d^2x^{\frac{9}{2}}-18\left(b^2cd-abd^2\right)x^{\frac{5}{2}}+45\left(3\,b^2c^2-4\,abcd+a^2d^2\right)\sqrt{x}\right)}{45\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)/(d^5*x^2 + c*d^4) - 1/16*(2
*sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(s
qrt(c)*sqrt(d))) + 2*sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*arctan(-
```

```
1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt
(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d +
5*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3
/4)*d^(1/4)) - sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*log(-sqrt(2)*c
^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/d^4 + 2/
45*(5*b^2*d^2*x^(9/2) - 18*(b^2*c*d - a*b*d^2)*x^(5/2) + 45*(3*b^2*c^2 - 4*
a*b*c*d + a^2*d^2)*sqrt(x))/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(287) = 574$.
time = 0.52, size = 1373, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/360*(180*(d^5*x^2 + c*d^4)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 37247
6*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 18684
0*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*
d^8)/d^17)^(1/4)*arctan((sqrt(d^8*sqrt(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d
 + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4
 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 62
5*a^8*c*d^8)/d^17) + (169*b^4*c^4 - 468*a*b^3*c^3*d + 454*a^2*b^2*c^2*d^2 -
 180*a^3*b*c*d^3 + 25*a^4*d^4)*x)*d^13*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*
d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^
4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 6
25*a^8*c*d^8)/d^17)^(3/4) - (13*b^2*c^2*d^13 - 18*a*b*c*d^14 + 5*a^2*d^15)*
sqrt(x)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 48
5784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55
100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(3/4))/(285
61*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c
^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^
3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)) + 45*(d^5*x^2 + c*d^4)*(-(2856
1*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^
6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3
*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/4)*log(d^4*(-(28561*b^8
*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3
 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6
- 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/4) + (13*b^2*c^2 - 18*a*b*c*
d + 5*a^2*d^2)*sqrt(x)) - 45*(d^5*x^2 + c*d^4)*(-(28561*b^8*c^9 - 158184*a*
b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^
4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2
*d^7 + 625*a^8*c*d^8)/d^17)^(1/4)*log(-d^4*(-(28561*b^8*c^9 - 158184*a*b^7*
c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^
```

```
5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7
 + 625*a^8*c*d^8)/d^17)^(1/4) + (13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*sqrt(
x)) - 4*(20*b^2*d^3*x^6 + 585*b^2*c^3 - 810*a*b*c^2*d + 225*a^2*c*d^2 - 4*(
13*b^2*c*d^2 - 18*a*b*d^3)*x^4 + 36*(13*b^2*c^2*d - 18*a*b*c*d^2 + 5*a^2*d^
3)*x^2)*sqrt(x))/(d^5*x^2 + c*d^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 440, normalized size = 1.17

$$-\frac{\sqrt{2}\left(13\,(cd^3)^{\frac{1}{4}}b^2c^2-18\,(cd^3)^{\frac{1}{4}}abcd+5\,(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,d^5}-\frac{\sqrt{2}\left(13\,(cd^3)^{\frac{1}{4}}b^2c^2-18\,(cd^3)^{\frac{1}{4}}abcd+5\,(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,d^5}-\frac{\sqrt{2}\left(13\,(cd^3)^{\frac{1}{4}}b^2c^2-18\,(cd^3)^{\frac{1}{4}}abcd+5\,(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,d^5}+\frac{\sqrt{2}\left(13\,(cd^3)^{\frac{1}{4}}b^2c^2-18\,(cd^3)^{\frac{1}{4}}abcd+5\,(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,d^5}+\frac{b^2c^3\sqrt{x}-2\,abc^2d\sqrt{x}+a^2cd^2\sqrt{x}}{2\,(dx^2+c)d^4}+\frac{2\left(5\,b^2d^{16}x^{\frac{9}{2}}-18\,b^2cd^{15}x^{\frac{5}{2}}+18\,abd^{16}x^{\frac{5}{2}}+135\,b^2c^2d^{14}\sqrt{x}-180\,abcd^{15}\sqrt{x}+45\,a^2d^{16}\sqrt{x}\right)}{45\,d^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d
)^(1/4))/d^5 - 1/8*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*a*b
*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
2*sqrt(x))/(c/d)^(1/4))/d^5 - 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(
c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(
1/4) + x + sqrt(c/d))/d^5 + 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*
d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1
/4) + x + sqrt(c/d))/d^5 + 1/2*(b^2*c^3*sqrt(x) - 2*a*b*c^2*d*sqrt(x) + a^2
*c*d^2*sqrt(x))/((d*x^2 + c)*d^4) + 2/45*(5*b^2*d^16*x^(9/2) - 18*b^2*c*d^1
5*x^(5/2) + 18*a*b*d^16*x^(5/2) + 135*b^2*c^2*d^14*sqrt(x) - 180*a*b*c*d^15
*sqrt(x) + 45*a^2*d^16*sqrt(x))/d^18
```

Mupad [**B**]
time = 0.24, size = 1367, normalized size = 3.65

$$\sqrt{x}\left(\frac{2\,a^2}{d^2}+\frac{2\,c\left(\frac{4\,b^2\,c}{d^3}-\frac{4\,a\,b}{d^2}\right)}{d}-\frac{2\,b^2\,c^2}{d^4}\right)-x^{5/2}\left(\frac{4\,b^2\,c}{5\,d^3}-\frac{4\,a\,b}{5\,d^2}\right)+\frac{\sqrt{x}\left(\frac{a^2\,c\,d^2}{2}-a\,b\,c^2\,d+\frac{b^2\,c^3}{2}\right)}{d^5\,x^2+c\,d^4}+\frac{2\,b^2\,x^{9/2}}{9\,d^2}+\ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] x^(1/2)*((2*a^2)/d^2 + (2*c*((4*b^2*c)/d^3 - (4*a*b)/d^2))/d - (2*b^2*c^2)/
d^4) - x^(5/2)*((4*b^2*c)/(5*d^3) - (4*a*b)/(5*d^2)) + (x^(1/2)*((b^2*c^3)/
```

```
2 + (a^2*c*d^2)/2 - a*b*c^2*d))/(c*d^4 + d^5*x^2) + (2*b^2*x^(9/2))/(9*d^2)
 + ((-c)^(1/4)*atan((((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 -
180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 + ((-c)^(1/
4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d)
)/d^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c)*1i)/(8*d^(17/4)) + ((-c)^(1/4)*((x
^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*
d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*
b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d))/d^(21/4))*(a*d - b*c)*(5*a*d - 13*
b*c)*1i)/(8*d^(17/4)))/(((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4
 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 + ((-c)^
(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3
*d))/d^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c))/(8*d^(17/4)) - ((-c)^(1/4)*((x
^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*
d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*
b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d))/d^(21/4))*(a*d - b*c)*(5*a*d - 13*
b*c))/(8*d^(17/4))))*(a*d - b*c)*(5*a*d - 13*b*c)*1i)/(4*d^(17/4)) - ((-c)^
(1/4)*atan((((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b
*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*d -
 b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d)*1i)/d^(2
1/4))*(a*d - b*c)*(5*a*d - 13*b*c))/(8*d^(17/4)) + ((-c)^(1/4)*((x^(1/2)*(1
69*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468
*a*b^3*c^5*d))/d^5 + ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 +
 5*a^2*c^2*d^2 - 18*a*b*c^3*d)*1i)/d^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c))/
(8*d^(17/4)))/(((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^
3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*
d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d)*1i)/d
^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c)*1i)/(8*d^(17/4)) - ((-c)^(1/4)*((x^(1
/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2
 - 468*a*b^3*c^5*d))/d^5 + ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2
*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d)*1i)/d^(21/4))*(a*d - b*c)*(5*a*d - 13*
b*c)*1i)/(8*d^(17/4))))*(a*d - b*c)*(5*a*d - 13*b*c))/(4*d^(17/4))
```

3.426 $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=346

$$-\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3}+\frac{2b^2x^{7/2}}{7d^2}+\frac{(bc-ad)^2x^{7/2}}{2cd^2\,(c+dx^2)}-\frac{(11bc-3ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,\sqrt[4]{c}\,d^{15/4}}+\ (\ +$$

```
[Out] -1/6*(-3*a*d+11*b*c)*(-a*d+b*c)*x^(3/2)/c/d^3+2/7*b^2*x^(7/2)/d^2+1/2*(-a*d
+b*c)^2*x^(7/2)/c/d^2/(d*x^2+c)-1/8*(-3*a*d+11*b*c)*(-a*d+b*c)*arctan(1-d^(
1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(15/4)*2^(1/2)+1/8*(-3*a*d+11*b*c)*
(-a*d+b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(15/4)*2^(1/
2)+1/16*(-3*a*d+11*b*c)*(-a*d+b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(
1/2)*x^(1/2))/c^(1/4)/d^(15/4)*2^(1/2)-1/16*(-3*a*d+11*b*c)*(-a*d+b*c)*ln(c
^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/d^(15/4)*2^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {474, 470, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{(11bc-3ad)(bc-ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,\sqrt[4]{c}\,d^{15/4}}+\frac{(11bc-3ad)(bc-ad)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,\sqrt[4]{c}\,d^{15/4}}+\frac{(11bc-3ad)(bc-ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{c}\,d^{15/4}}-\frac{(11bc-3ad)(bc-ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{c}\,d^{15/4}}-\frac{x^{3/2}(11bc-3ad)(bc-ad)}{6cd^3}+\frac{x^{7/2}(bc-ad)^2}{2cd^2\,(c+dx^2)}+\frac{2b^2x^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -1/6*((11*b*c - 3*a*d)*(b*c - a*d)*x^(3/2))/(c*d^3) + (2*b^2*x^(7/2))/(7*d^
2) + ((b*c - a*d)^2*x^(7/2))/(2*c*d^2*(c + d*x^2)) - ((11*b*c - 3*a*d)*(b*c
 - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d
^(15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x
])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/
4)*d^(15/4)) - ((11*b*c - 3*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(bc-ad)^2x^{7/2}}{2cd^2(c+dx^2)} - \frac{\int \frac{x^{5/2}\left(\frac{1}{2}(-4a^2d^2+7(bc-ad)^2)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2(c+dx^2)} - \frac{((11bc-3ad)(bc-ad))\int \frac{x^{5/2}}{c+dx^2}\,dx}{4cd^2}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2(c+dx^2)} + \frac{((11bc-3ad)(bc-ad))}{4d^3}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2(c+dx^2)} + \frac{((11bc-3ad)(bc-ad))}{2}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2(c+dx^2)} - \frac{((11bc-3ad)(bc-ad))}{}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2(c+dx^2)} + \frac{((11bc-3ad)(bc-ad))}{}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2(c+dx^2)} + \frac{(11bc-3ad)(bc-ad)\,\mathrm{lo}}{8}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2(c+dx^2)} - \frac{(11bc-3ad)(bc-ad)\,\mathrm{ta}}{4\sqrt{2}}$$

Mathematica [A]
time = 0.57, size = 223, normalized size = 0.64

$$\frac{\frac{4d^{3/4}x^{3/2}(-21a^2d^2+14abd(7c+4dx^2)+b^2(-77c^2-44cdx^2+12d^2x^4))}{c+dx^2} - \frac{21\sqrt{2}(11b^2c^2-14abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{c}} - \frac{21\sqrt{2}(11b^2c^2-14abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt[4]{c}}}{168d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((4*d^(3/4)*x^(3/2)*(-21*a^2*d^2 + 14*a*b*d*(7*c + 4*d*x^2) + b^2*(-77*c^2
- 44*c*d*x^2 + 12*d^2*x^4)))/(c + d*x^2) - (21*Sqrt[2]*(11*b^2*c^2 - 14*a*b
*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x])])/c^(1/4) - (21*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTanh
[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(1/4))/(168*d^
(15/4))
```

Maple [A]

time = 0.12, size = 200, normalized size = 0.58

method	result
derivativedivides	$\frac{2b\left(\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-2bc)x^{\frac{3}{2}}}{3}\right)}{d^{3}}+\frac{\frac{2\left(-\frac{1}{4}a^{2}d^{2}+\frac{1}{2}abcd-\frac{1}{4}b^{2}c^{2}\right)x^{\frac{3}{2}}}{d\,x^{2}+c}+\frac{\left(-\frac{7}{2}abcd+\frac{11}{4}b^{2}c^{2}+\frac{3}{4}a^{2}d^{2}\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}\,\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}\,\cdots}\right)\cdots\right.}{4\cdots}}{d^{3}}$
default	$\frac{2b\left(\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-2bc)x^{\frac{3}{2}}}{3}\right)}{d^{3}}+\frac{\frac{2\left(-\frac{1}{4}a^{2}d^{2}+\frac{1}{2}abcd-\frac{1}{4}b^{2}c^{2}\right)x^{\frac{3}{2}}}{d\,x^{2}+c}+\frac{\left(-\frac{7}{2}abcd+\frac{11}{4}b^{2}c^{2}+\frac{3}{4}a^{2}d^{2}\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}\,\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}\,\cdots}\right)\cdots\right.}{4\cdots}}{d^{3}}$
risch	$\frac{2b\,x^{\frac{3}{2}}\left(3bd\,x^{2}+14ad-14bc\right)}{21d^{3}}-\frac{x^{\frac{3}{2}}a^{2}}{2d(d\,x^{2}+c)}+\frac{x^{\frac{3}{2}}abc}{d^{2}(d\,x^{2}+c)}-\frac{x^{\frac{3}{2}}b^{2}c^{2}}{2d^{3}(d\,x^{2}+c)}-\frac{7\sqrt{2}\,bc\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}\cdots}\right)\cdots}{8d^{3}\left(\frac{c}{d}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b/d^3*(1/7*b*d*x^(7/2)+1/3*(2*a*d-2*b*c)*x^(3/2))+2/d^3*((-1/4*a^2*d^2+1/
2*a*b*c*d-1/4*b^2*c^2)*x^(3/2)/(d*x^2+c)+1/8*(-7/2*a*b*c*d+11/4*b^2*c^2+3/4
*a^2*d^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.52, size = 272, normalized size = 0.79

$$-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x^{\frac{3}{2}}}{2\left(d^{4}x^{2}+cd^{3}\right)}+\frac{\left(11\,b^{2}c^{2}-14\,abcd+3\,a^{2}d^{2}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\,d^{3}}+\frac{2\left(3\,b^{2}dx^{\frac{7}{2}}-14\left(b^{2}c-abd\right)x^{\frac{3}{2}}\right)}{21\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^(3/2)/(d^4*x^2 + c*d^3) + 1/16*(11*b
^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(
1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt
(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - s
qrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*
d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(
c))/(c^(1/4)*d^(3/4)))/d^3 + 2/21*(3*b^2*d*x^(7/2) - 14*(b^2*c - a*b*d)*x^(
3/2))/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1733 vs. $2(262) = 524$.
time = 1.38, size = 1733, normalized size = 5.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/168*(84*(d^4*x^2 + c*d^3)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*
a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a
^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*
d^15))^(1/4)*arctan((sqrt((1771561*b^12*c^12 - 13528284*a*b^11*c^11*d + 459
43458*a^2*b^10*c^10*d^2 - 91492940*a^3*b^9*c^9*d^3 + 118659255*a^4*b^8*c^8*
d^4 - 105323064*a^5*b^7*c^7*d^5 + 65490076*a^6*b^6*c^6*d^6 - 28724472*a^7*b
^5*c^5*d^7 + 8825895*a^8*b^4*c^4*d^8 - 1855980*a^9*b^3*c^3*d^9 + 254178*a^1
0*b^2*c^2*d^10 - 20412*a^11*b*c*d^11 + 729*a^12*d^12)*x - (14641*b^8*c^9*d^
7 - 74536*a*b^7*c^8*d^8 + 158268*a^2*b^6*c^7*d^9 - 181720*a^3*b^5*c^6*d^10
+ 122566*a^4*b^4*c^5*d^11 - 49560*a^5*b^3*c^4*d^12 + 11772*a^6*b^2*c^3*d^13
 - 1512*a^7*b*c^2*d^14 + 81*a^8*c*d^15)*sqrt(-(14641*b^8*c^8 - 74536*a*b^7*
c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^
4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 +
81*a^8*d^8)/(c*d^15)))*d^4*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^
2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5
*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^
15))^(1/4) - (1331*b^6*c^6*d^4 - 5082*a*b^5*c^5*d^5 + 7557*a^2*b^4*c^4*d^6
- 5516*a^3*b^3*c^3*d^7 + 2061*a^4*b^2*c^2*d^8 - 378*a^5*b*c*d^9 + 27*a^6*d^
10)*sqrt(x)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 -
 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 +
11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4))/(14
641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c
^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2
*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)) - 21*(d^4*x^2 + c*d^3)*(-(14641*b^8*
c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 +
 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1
512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4)*log(c*d^11*(-(14641*b^8*c^8 -
 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 1225
66*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a
^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(3/4) + (1331*b^6*c^6 - 5082*a*b^5*c^5*d
 + 7557*a^2*b^4*c^4*d^2 - 5516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378
*a^5*b*c*d^5 + 27*a^6*d^6)*sqrt(x)) + 21*(d^4*x^2 + c*d^3)*(-(14641*b^8*c^8
 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 12
2566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512
*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4)*log(-c*d^11*(-(14641*b^8*c^8 - 7
4536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566
```

```
*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7
*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(3/4) + (1331*b^6*c^6 - 5082*a*b^5*c^5*d +
 7557*a^2*b^4*c^4*d^2 - 5516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378*a
^5*b*c*d^5 + 27*a^6*d^6)*sqrt(x)) - 4*(12*b^2*d^2*x^5 - 4*(11*b^2*c*d - 14*
a*b*d^2)*x^3 - 7*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*x)*sqrt(x))/(d^4*x^2
 + c*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 413, normalized size = 1.19

$$-\frac{b^2c^2x^{\frac{3}{2}}-2\,abcdx^{\frac{3}{2}}+a^2d^2x^{\frac{3}{2}}}{2\,(dx^2+c)d^3}+\frac{\sqrt{2}\left(11\,(cd^3)^{\frac{3}{4}}\,b^2c^2-14\,(cd^3)^{\frac{3}{4}}\,abcd+3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{c}{d})^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,(\frac{c}{d})^{\frac{1}{4}}}\right)}{8\,cd^6}+\frac{\sqrt{2}\left(11\,(cd^3)^{\frac{3}{4}}\,b^2c^2-14\,(cd^3)^{\frac{3}{4}}\,abcd+3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{c}{d})^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,(\frac{c}{d})^{\frac{1}{4}}}\right)}{8\,cd^6}-\frac{\sqrt{2}\left(11\,(cd^3)^{\frac{3}{4}}\,b^2c^2-14\,(cd^3)^{\frac{3}{4}}\,abcd+3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\,\sqrt{x}\,(\frac{c}{d})^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,cd^6}+\frac{\sqrt{2}\left(11\,(cd^3)^{\frac{3}{4}}\,b^2c^2-14\,(cd^3)^{\frac{3}{4}}\,abcd+3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\,\sqrt{x}\,(\frac{c}{d})^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,cd^6}+\frac{2\left(3\,b^2d^{12}x^{\frac{7}{2}}-14\,b^2cd^{11}x^{\frac{3}{2}}+14\,abd^{12}x^{\frac{3}{2}}\right)}{21\,d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*x^2 + c)*d
^3) + 1/8*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*
(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(c*d^6) + 1/8*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3)^
(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^6) - 1/16*sqrt(2)*(11*(c*d^3)^(3/4)*b
^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^6) + 1/16*sqrt(2)*(11*(c*d^3)^(3/4)
*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^6) + 2/21*(3*b^2*d^12*x^(7/2) -
14*b^2*c*d^11*x^(3/2) + 14*a*b*d^12*x^(3/2))/d^14
```

Mupad [B]
time = 0.12, size = 160, normalized size = 0.46

$$\frac{2\,b^2\,x^{7/2}}{7\,d^2}-\frac{x^{3/2}\,\left(\frac{a^2\,d^2}{2}-a\,b\,c\,d+\frac{b^2\,c^2}{2}\right)}{d^4\,x^2+c\,d^3}-x^{3/2}\,\left(\frac{4\,b^2\,c}{3\,d^3}-\frac{4\,a\,b}{3\,d^2}\right)+\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d-11\,b\,c\right)}{4\,{\left(-c\right)}^{1/4}\,d^{15/4}}+\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d-11\,b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-c\right)}^{1/4}\,d^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (2*b^2*x^(7/2))/(7*d^2) - (x^(3/2)*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(
c*d^3 + d^4*x^2) - x^(3/2)*((4*b^2*c)/(3*d^3) - (4*a*b)/(3*d^2)) + (atan((d
^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(3*a*d - 11*b*c))/(4*(-c)^(1/4)*d^(
15/4)) + (atan((d^(1/4)*x^(1/2)*1i)/(-c)^(1/4))*(a*d - b*c)*(3*a*d - 11*b*c
)*1i)/(4*(-c)^(1/4)*d^(15/4))
```

3.427 $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=346

$$-\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3}+\frac{2b^2x^{5/2}}{5d^2}+\frac{(bc-ad)^2x^{5/2}}{2cd^2\,(c+dx^2)}-\frac{(bc-ad)(9bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{3/4}d^{13/4}}+\frac{(bc-$$

```
[Out] 2/5*b^2*x^(5/2)/d^2+1/2*(-a*d+b*c)^2*x^(5/2)/c/d^2/(d*x^2+c)-1/8*(-a*d+b*c)
*(-a*d+9*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(13/4)*2^
(1/2)+1/8*(-a*d+b*c)*(-a*d+9*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))
/c^(3/4)/d^(13/4)*2^(1/2)-1/16*(-a*d+b*c)*(-a*d+9*b*c)*ln(c^(1/2)+x*d^(1/2)
-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(13/4)*2^(1/2)+1/16*(-a*d+b*c)*
(-a*d+9*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/
d^(13/4)*2^(1/2)-1/2*(-a*d+b*c)*(-a*d+9*b*c)*x^(1/2)/c/d^3
```

Rubi [A]
time = 0.21, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$,
Rules used = {474, 470, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(bc-ad)(9bc-ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{3/4}d^{13/4}}-\frac{(bc-ad)(9bc-ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}d^{13/4}}-\frac{\sqrt{x}\,(bc-ad)(9bc-ad)}{2cd^3}+\frac{x^{5/2}(bc-ad)^2}{2cd^2\,(c+dx^2)}+\frac{2b^2x^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*(9*b*c - a*d)*Sqrt[x])/(c*d^3) + (2*b^2*x^(5/2))/(5*d^2)
+ ((b*c - a*d)^2*x^(5/2))/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(9*b*c - a*d
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(13/4
)) + ((b*c - a*d)*(9*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(4*Sqrt[2]*c^(3/4)*d^(13/4)) - ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4))
 + ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(bc-ad)^2x^{5/2}}{2cd^2(c+dx^2)} - \frac{\int \frac{x^{3/2}\left(\frac{1}{2}(-4a^2d^2+5(bc-ad)^2)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(9bc-ad))\int \frac{x^{3/2}}{c+dx^2}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2(c+dx^2)} + \frac{((bc-ad)(9bc-ad))\int \frac{}{\sqrt{x}}}{4d^3}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2(c+dx^2)} + \frac{((bc-ad)(9bc-ad))\text{Subst}}{2d^3}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2(c+dx^2)} + \frac{((bc-ad)(9bc-ad))\text{Subst}}{4\sqrt{}}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2(c+dx^2)} + \frac{((bc-ad)(9bc-ad))\text{Subst}}{8}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2(c+dx^2)} - \frac{(bc-ad)(9bc-ad)\log\left(\sqrt{}\right.}{8\sqrt{2}}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2(c+dx^2)} - \frac{(bc-ad)(9bc-ad)\tan^{-1}\left(\right.}{4\sqrt{2}\,c^{3/4}d^{1}}$$

Mathematica [A]

time = 0.58, size = 221, normalized size = 0.64

$$\frac{\frac{4\sqrt[4]{d}\sqrt{x}\left(-5a^2d^2+10abd(5c+4dx^2)+b^2(-45c^2-36cdx^2+4d^2x^4)\right)}{c+dx^2}-\frac{5\sqrt{2}\left(9b^2c^2-10abcd+a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{3/4}}+\frac{5\sqrt{2}\left(9b^2c^2-10abcd+a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{3/4}}}{40d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((4*d^(1/4)*Sqrt[x]*(-5*a^2*d^2 + 10*a*b*d*(5*c + 4*d*x^2) + b^2*(-45*c^2 -
 36*c*d*x^2 + 4*d^2*x^4)))/(c + d*x^2) - (5*Sqrt[2]*(9*b^2*c^2 - 10*a*b*c*d
 + a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]
)/c^(3/4) + (5*Sqrt[2]*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(3/4))/(40*d^(13/4))
```

Maple [A]

time = 0.12, size = 199, normalized size = 0.58

method	result
derivativedivides	$\frac{2b\left(\frac{bd\,x^{\frac{5}{2}}}{5}+2ad\sqrt{x}-2bc\sqrt{x}\right)}{d^3}+\frac{\frac{2\left(-\frac{1}{4}a^2d^2+\frac{1}{2}abcd-\frac{1}{4}b^2c^2\right)\sqrt{x}}{d\,x^2+c}+\frac{\left(a^2d^2-10abcd+9b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{}}{d^3}$
default	$\frac{2b\left(\frac{bd\,x^{\frac{5}{2}}}{5}+2ad\sqrt{x}-2bc\sqrt{x}\right)}{d^3}+\frac{\frac{2\left(-\frac{1}{4}a^2d^2+\frac{1}{2}abcd-\frac{1}{4}b^2c^2\right)\sqrt{x}}{d\,x^2+c}+\frac{\left(a^2d^2-10abcd+9b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{}}{d^3}$
risch	$\frac{2b\left(bd\,x^2+10ad-10bc\right)\sqrt{x}}{5d^3}-\frac{\sqrt{x}\,a^2}{2d\left(d\,x^2+c\right)}+\frac{\sqrt{x}\,abc}{d^2\left(d\,x^2+c\right)}-\frac{\sqrt{x}\,b^2c^2}{2d^3\left(d\,x^2+c\right)}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8dc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b/d^3*(1/5*b*d*x^(5/2)+2*a*d*x^(1/2)-2*b*c*x^(1/2))+2/d^3*((-1/4*a^2*d^2+
1/2*a*b*c*d-1/4*b^2*c^2)*x^(1/2)/(d*x^2+c)+1/32*(a^2*d^2-10*a*b*c*d+9*b^2*c
^2)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.54, size = 336, normalized size = 0.97

$$-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{x}}{2\left(d^4x^2+cd^3\right)}+\frac{2\left(b^2dx^{\frac{5}{2}}-10\left(b^2c-abd\right)\sqrt{x}\right)}{5\,d^3}+\frac{\frac{2\sqrt{2}\left(9\,b^2c^2-10\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(9\,b^2c^2-10\,abcd+a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(9\,b^2c^2-10\,abcd+a^2d^2\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(9\,b^2c^2-10\,abcd+a^2d^2\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")

[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)/(d^4*x^2 + c*d^3) + 2/5*(b^2*d*x^(5/2) - 10*(b^2*c - a*b*d)*sqrt(x))/d^3 + 1/16*(2*sqrt(2)*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^3

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1334 vs. $2(262) = 524$.
time = 1.53, size = 1334, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

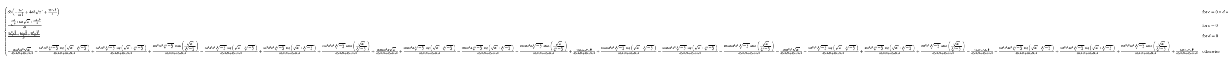
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")

[Out] 1/40*(20*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)*arctan((sqrt(c^2*d^6*sqrt(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13)) + (81*b^4*c^4 - 180*a*b^3*c^3*d + 118*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + a^4*d^4)*x)*c^2*d^10*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(3/4) - (9*b^2*c^4*d^10 - 10*a*b*c^3*d^11 + a^2*c^2*d^12)*sqrt(x)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(3/4))/(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)) + 5*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)*log(c*d^3*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4) + (9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*sqrt(x

```
)) - 5*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^
6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^
3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)*l
og(-c*d^3*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 457
20*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6
*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4) + (9*b^2*c^2 - 1
0*a*b*c*d + a^2*d^2)*sqrt(x)) + 4*(4*b^2*d^2*x^4 - 45*b^2*c^2 + 50*a*b*c*d
- 5*a^2*d^2 - 4*(9*b^2*c*d - 10*a*b*d^2)*x^2)*sqrt(x))/(d^4*x^2 + c*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(321) = 642$.

time = 70.04, size = 1280, normalized size = 3.70



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5),
Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5
/2)/5)/d**2, Eq(c, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**
(13/2)/13)/c**2, Eq(d, 0)), (-20*a**2*c*d**2*sqrt(x)/(40*c**2*d**3 + 40*c*d
**4*x**2) - 5*a**2*c*d**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(40*c*
*2*d**3 + 40*c*d**4*x**2) + 5*a**2*c*d**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d
)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) + 10*a**2*c*d**2*(-c/d)**(1/4)*at
an(sqrt(x)/(-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) - 5*a**2*d**3*x**
2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2
) + 5*a**2*d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(40*c**2*d*
*3 + 40*c*d**4*x**2) + 10*a**2*d**3*x**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)*
*(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) + 200*a*b*c**2*d*sqrt(x)/(40*c**2*d
**3 + 40*c*d**4*x**2) + 50*a*b*c**2*d*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(
1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) - 50*a*b*c**2*d*(-c/d)**(1/4)*log(sqr
t(x) + (-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) - 100*a*b*c**2*d*(-c/
d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) + 160
*a*b*c*d**2*x**(5/2)/(40*c**2*d**3 + 40*c*d**4*x**2) + 50*a*b*c*d**2*x**2*(
-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) -
 50*a*b*c*d**2*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(40*c**2*d**
3 + 40*c*d**4*x**2) - 100*a*b*c*d**2*x**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)
**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) - 180*b**2*c**3*sqrt(x)/(40*c**2*d
**3 + 40*c*d**4*x**2) - 45*b**2*c**3*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1
/4))/(40*c**2*d**3 + 40*c*d**4*x**2) + 45*b**2*c**3*(-c/d)**(1/4)*log(sqrt(
x) + (-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) + 90*b**2*c**3*(-c/d)**
(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) - 144*b**
2*c**2*d*x**(5/2)/(40*c**2*d**3 + 40*c*d**4*x**2) - 45*b**2*c**2*d*x**2*(-c
/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) + 4
```

```
5*b**2*c**2*d*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(40*c**2*d**3
 + 40*c*d**4*x**2) + 90*b**2*c**2*d*x**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)*
*(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) + 16*b**2*c*d**2*x**(9/2)/(40*c**2*
d**3 + 40*c*d**4*x**2), True))
```

Giac [A]
time = 0.62, size = 408, normalized size = 1.18

$$\frac{\sqrt{2}\left(9\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-10\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^{4}}+\frac{\sqrt{2}\left(9\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-10\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^{4}}+\frac{\sqrt{2}\left(9\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-10\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,cd^{4}}-\frac{\sqrt{2}\left(9\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-10\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,cd^{4}}-\frac{b^{2}c^{2}\sqrt{x}-2\,abcd\sqrt{x}+a^{2}d^{2}\sqrt{x}}{2\left(dx^{2}+c\right)d^{3}}+\frac{2\left(b^{2}d^{8}x^{\frac{5}{2}}-10\,b^{2}cd^{7}\sqrt{x}+10\,abd^{8}\sqrt{x}\right)}{5\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(
1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1
/4))/(c*d^4) + 1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*
c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*s
qrt(x))/(c/d)^(1/4))/(c*d^4) + 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(
c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(c*d^4) - 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c
*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(c*d^4) - 1/2*(b^2*c^2*sqrt(x) - 2*a*b*c*d*sqrt(x) + a^
2*d^2*sqrt(x))/((d*x^2 + c)*d^3) + 2/5*(b^2*d^8*x^(5/2) - 10*b^2*c*d^7*sqrt
(x) + 10*a*b*d^8*sqrt(x))/d^10
```

Mupad [B]
time = 0.14, size = 1238, normalized size = 3.58

$$\frac{2\,b^{2}\,x^{5/2}}{5\,d^{2}}-\frac{\sqrt{x}\,\left(\frac{a^{2}\,d^{2}}{2}-a\,b\,c\,d+\frac{b^{2}\,c^{2}}{2}\right)}{d^{4}\,x^{2}+c\,d^{3}}-\sqrt{x}\,\left(\frac{4\,b^{2}\,c}{d^{3}}-\frac{4\,a\,b}{d^{2}}\right)+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{x}\,\left(a^{4}\,d^{4}-20\,a^{3}\,b\,c\,d^{3}+118\,a^{2}\,b^{2}\,c^{2}\,d^{2}-180\,a\,b^{3}\,c^{3}\,d+81\,b^{4}\,c^{4}\right)}{d^{3}}-\frac{\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,\left(8\,a^{2}\,c\,d^{2}-80\,a\,b\,c^{2}\,d+72\,b^{2}\,c^{3}\right)}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}+\frac{\left(\frac{\sqrt{x}\,\left(a^{4}\,d^{4}-20\,a^{3}\,b\,c\,d^{3}+118\,a^{2}\,b^{2}\,c^{2}\,d^{2}-180\,a\,b^{3}\,c^{3}\,d+81\,b^{4}\,c^{4}\right)}{d^{3}}+\frac{\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,\left(8\,a^{2}\,c\,d^{2}-80\,a\,b\,c^{2}\,d+72\,b^{2}\,c^{3}\right)}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}}{\frac{\left(\frac{\sqrt{x}\,\left(a^{4}\,d^{4}-20\,a^{3}\,b\,c\,d^{3}+118\,a^{2}\,b^{2}\,c^{2}\,d^{2}-180\,a\,b^{3}\,c^{3}\,d+81\,b^{4}\,c^{4}\right)}{d^{3}}-\frac{\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,\left(8\,a^{2}\,c\,d^{2}-80\,a\,b\,c^{2}\,d+72\,b^{2}\,c^{3}\right)}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}-\frac{\left(\frac{\sqrt{x}\,\left(a^{4}\,d^{4}-20\,a^{3}\,b\,c\,d^{3}+118\,a^{2}\,b^{2}\,c^{2}\,d^{2}-180\,a\,b^{3}\,c^{3}\,d+81\,b^{4}\,c^{4}\right)}{d^{3}}+\frac{\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,\left(8\,a^{2}\,c\,d^{2}-80\,a\,b\,c^{2}\,d+72\,b^{2}\,c^{3}\right)}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-c\right)}^{3/4}\,d^{13/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{x}\,\left(a^{4}\,d^{4}-20\,a^{3}\,b\,c\,d^{3}+118\,a^{2}\,b^{2}\,c^{2}\,d^{2}-180\,a\,b^{3}\,c^{3}\,d+81\,b^{4}\,c^{4}\right)}{d^{3}}-\frac{\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,\left(8\,a^{2}\,c\,d^{2}-80\,a\,b\,c^{2}\,d+72\,b^{2}\,c^{3}\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}+\frac{\left(\frac{\sqrt{x}\,\left(a^{4}\,d^{4}-20\,a^{3}\,b\,c\,d^{3}+118\,a^{2}\,b^{2}\,c^{2}\,d^{2}-180\,a\,b^{3}\,c^{3}\,d+81\,b^{4}\,c^{4}\right)}{d^{3}}+\frac{\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,\left(8\,a^{2}\,c\,d^{2}-80\,a\,b\,c^{2}\,d+72\,b^{2}\,c^{3}\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}}{\frac{\left(\frac{\sqrt{x}\,\left(a^{4}\,d^{4}-20\,a^{3}\,b\,c\,d^{3}+118\,a^{2}\,b^{2}\,c^{2}\,d^{2}-180\,a\,b^{3}\,c^{3}\,d+81\,b^{4}\,c^{4}\right)}{d^{3}}-\frac{\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,\left(8\,a^{2}\,c\,d^{2}-80\,a\,b\,c^{2}\,d+72\,b^{2}\,c^{3}\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}-\frac{\left(\frac{\sqrt{x}\,\left(a^{4}\,d^{4}-20\,a^{3}\,b\,c\,d^{3}+118\,a^{2}\,b^{2}\,c^{2}\,d^{2}-180\,a\,b^{3}\,c^{3}\,d+81\,b^{4}\,c^{4}\right)}{d^{3}}+\frac{\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,\left(8\,a^{2}\,c\,d^{2}-80\,a\,b\,c^{2}\,d+72\,b^{2}\,c^{3}\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{3/4}\,d^{13/4}}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-9\,b\,c\right)}{4\,{\left(-c\right)}^{3/4}\,d^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (2*b^2*x^(5/2))/(5*d^2) - (x^(1/2)*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(
c*d^3 + d^4*x^2) - x^(1/2)*((4*b^2*c)/d^3 - (4*a*b)/d^2) + (atan(((((x^(1/2
)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3*b*
c*d^3))/d^3 - ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b
*c^2*d))/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(8*(-c)^(3/
4)*d^(13/4)) + (((x^(1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180
*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^
3 + 8*a^2*c*d^2 - 80*a*b*c^2*d))/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d
- 9*b*c)*1i)/(8*(-c)^(3/4)*d^(13/4)))/((((x^(1/2)*(a^4*d^4 + 81*b^4*c^4 + 1
18*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 - ((a*d - b*c)*
(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d))/(8*(-c)^(3/4)*d^(1
3/4)))*(a*d - b*c)*(a*d - 9*b*c))/(8*(-c)^(3/4)*d^(13/4)) - (((x^(1/2)*(a^4
```

```
*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3*b*c*d^3)
)/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d
))/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c))/(8*(-c)^(3/4)*d^(13/
4))))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(4*(-c)^(3/4)*d^(13/4)) + (atan(((((x^(
1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3
*b*c*d^3))/d^3 - ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*
a*b*c^2*d)*1i)/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c))/(8*(-c)^
(3/4)*d^(13/4)) + (((x^(1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 -
180*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2
*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d)*1i)/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)
*(a*d - 9*b*c))/(8*(-c)^(3/4)*d^(13/4)))/((((x^(1/2)*(a^4*d^4 + 81*b^4*c^4
+ 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 - ((a*d - b*
c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d)*1i)/(8*(-c)^(3/4
)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(8*(-c)^(3/4)*d^(13/4)) - (((x^(
1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3
*b*c*d^3))/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*
a*b*c^2*d)*1i)/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(8*(-
c)^(3/4)*d^(13/4))))*(a*d - b*c)*(a*d - 9*b*c))/(4*(-c)^(3/4)*d^(13/4))
```

3.428 $\int \frac{\sqrt{x}\,(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=310

$$\frac{2b^2x^{3/2}}{3d^2}+\frac{(bc-ad)^2x^{3/2}}{2cd^2\,(c+dx^2)}+\frac{(bc-ad)(7bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{5/4}d^{11/4}}-\frac{(bc-ad)(7bc+ad)\tan^{-1}\left(1+\right.}{4\sqrt{2}\,c^{5/4}d^{11/4}}$$

```
[Out] 2/3*b^2*x^(3/2)/d^2+1/2*(-a*d+b*c)^2*x^(3/2)/c/d^2/(d*x^2+c)+1/8*(-a*d+b*c)
*(a*d+7*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/d^(11/4)*2^(
1/2)-1/8*(-a*d+b*c)*(a*d+7*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c
^(5/4)/d^(11/4)*2^(1/2)-1/16*(-a*d+b*c)*(a*d+7*b*c)*ln(c^(1/2)+x*d^(1/2)-c^
(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(11/4)*2^(1/2)+1/16*(-a*d+b*c)*(a*
d+7*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(1
1/4)*2^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {474, 470, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{(bc-ad)(ad+7bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{5/4}d^{11/4}}-\frac{(bc-ad)(ad+7bc)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{5/4}d^{11/4}}-\frac{(bc-ad)(ad+7bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}d^{11/4}}+\frac{x^{3/2}(bc-ad)^2}{2cd^2\,(c+dx^2)}+\frac{2b^2x^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (2*b^2*x^(3/2))/(3*d^2) + ((b*c - a*d)^2*x^(3/2))/(2*c*d^2*(c + d*x^2)) + (
(b*c - a*d)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4
*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*b*c + a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*
b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(5/4)*d^(11/4)) + ((b*c - a*d)*(7*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*d^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{\sqrt{x}\left(\frac{1}{2}\left(-4a^2d^2+3(bc-ad)^2\right)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(7bc+ad))\int \frac{\sqrt{x}}{c+dx^2}\,dx}{4cd^2}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(7bc+ad))\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{2cd^2}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(7bc+ad))\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{4cd^{5/2}}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(7bc+ad))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx, x,\right.}{8cd^3}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(7bc+ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right.}{8\sqrt{2}\,c^{5/4}d^{11/4}}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} + \frac{(bc-ad)(7bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{5/4}d^{11/4}} - \frac{(bc-}{}$$

Mathematica [A]
time = 0.61, size = 204, normalized size = 0.66

$$\frac{\frac{4\sqrt[4]{c}\,d^{3/4}x^{3/2}\left(-6abcd+3a^2d^2+b^2c\left(7c+4dx^2\right)\right)}{c+dx^2}+3\sqrt{2}\left(7b^2c^2-6abcd-a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)+3\sqrt{2}\left(7b^2c^2-6abcd-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{24c^{5/4}d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((4*c^(1/4)*d^(3/4)*x^(3/2)*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(7*c + 4*d*x^2)
))/(c + d*x^2) + 3*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[c
] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(7*b^2*c^2 -
6*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + S
qrt[d]*x)])/(24*c^(5/4)*d^(11/4))
```

Maple [A]
time = 0.11, size = 187, normalized size = 0.60

method	result
derivativedivides	$\frac{2b^2x^{\frac{3}{2}}}{3d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)x^{\frac{3}{2}}}{2c\left(dx^2+c\right)}+\frac{\left(a^2d^2+6abcd-7b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{16cd\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{d^2}$
default	$\frac{2b^2x^{\frac{3}{2}}}{3d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)x^{\frac{3}{2}}}{2c\left(dx^2+c\right)}+\frac{\left(a^2d^2+6abcd-7b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{16cd\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{d^2}$
risch	$\frac{2b^2x^{\frac{3}{2}}}{3d^2}+\frac{x^{\frac{3}{2}}a^2}{2c(dx^2+c)}-\frac{x^{\frac{3}{2}}ab}{d(dx^2+c)}+\frac{x^{\frac{3}{2}}b^2c}{2d^2(dx^2+c)}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2}{8dc\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}}{(\right.}}{4d^2\left(\frac{c}{d}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*b^2*x^(3/2)/d^2+2/d^2*(1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x^(3/2)/(d*x^2
+c)+1/32*(a^2*d^2+6*a*b*c*d-7*b^2*c^2)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 258, normalized size = 0.83

$$\frac{(b^2c^2-2\,abcd+a^2d^2)x^{\frac{3}{2}}}{2\,(cd^3x^2+c^2d^2)}+\frac{2\,b^2x^{\frac{3}{2}}}{3\,d^2}-\frac{(7\,b^2c^2-6\,abcd-a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^(3/2)/(c*d^3*x^2 + c^2*d^2) + 2/3*b^2
*x^(3/2)/d^2 - 1/16*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)
```

```
))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*
sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
 + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x
) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1723 vs. $2(230) = 460$.
time = 1.23, size = 1723, normalized size = 5.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a
^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*
c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4)
*arctan((sqrt((117649*b^12*c^12 - 605052*a*b^11*c^11*d + 1195698*a^2*b^10*c
^10*d^2 - 1049580*a^3*b^9*c^9*d^3 + 247695*a^4*b^8*c^8*d^4 + 184968*a^5*b^7
*c^7*d^5 - 73604*a^6*b^6*c^6*d^6 - 26424*a^7*b^5*c^5*d^7 + 5055*a^8*b^4*c^4
*d^8 + 3060*a^9*b^3*c^3*d^9 + 498*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^
12*d^12)*x - (2401*b^8*c^11*d^5 - 8232*a*b^7*c^10*d^6 + 9212*a^2*b^6*c^9*d^
7 - 2520*a^3*b^5*c^8*d^8 - 1434*a^4*b^4*c^7*d^9 + 360*a^5*b^3*c^6*d^10 + 18
8*a^6*b^2*c^5*d^11 + 24*a^7*b*c^4*d^12 + a^8*c^3*d^13)*sqrt(-(2401*b^8*c^8
- 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4
*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 +
 a^8*d^8)/(c^5*d^11)))*c*d^3*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*
b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3
*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4) +
(343*b^6*c^7*d^3 - 882*a*b^5*c^6*d^4 + 609*a^2*b^4*c^5*d^5 + 36*a^3*b^3*c^4
*d^6 - 87*a^4*b^2*c^3*d^7 - 18*a^5*b*c^2*d^8 - a^6*c*d^9)*sqrt(x)*(-(2401*b
^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1
434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*
c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4))/(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212
*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^
3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)) - 3*(c*d^3*x^2
 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 252
0*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^
2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4)*log(c^4*d^8*(-(2401
*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 -
 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*
b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) - (343*b^6*c^6 - 882*a*b^5*c^5*d + 609
*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 87*a^4*b^2*c^2*d^4 - 18*a^5*b*c*d^5
 - a^6*d^6)*sqrt(x)) + 3*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7
```

```
*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4
 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c
^5*d^11))^(1/4)*log(-c^4*d^8*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*
b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3
*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) -
(343*b^6*c^6 - 882*a*b^5*c^5*d + 609*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 -
 87*a^4*b^2*c^2*d^4 - 18*a^5*b*c*d^5 - a^6*d^6)*sqrt(x)) - 4*(4*b^2*c*d*x^3
 + (7*b^2*c^2 - 6*a*b*c*d + 3*a^2*d^2)*x)*sqrt(x))/(c*d^3*x^2 + c^2*d^2)
```

Sympy [A]
time = 16.35, size = 173, normalized size = 0.56

$$\frac{4ab\operatorname{RootSum}\left(256t^4cd^3+1,\left(t\mapsto t\log\left(64t^3cd^2+\sqrt{x}\right)\right)\right)}{d}-\frac{4b^2c\operatorname{RootSum}\left(256t^4cd^3+1,\left(t\mapsto t\log\left(64t^3cd^2+\sqrt{x}\right)\right)\right)}{d^2}+\frac{2b^2x^{\frac{3}{2}}}{3d^2}+\frac{2x^{\frac{3}{2}}(ad-bc)^2}{4c^2d^2+4cd^3x^2}+\frac{2(ad-bc)^2\operatorname{RootSum}\left(65536t^4c^5d^3+1,\left(t\mapsto t\log\left(4096t^3c^4d^2+\sqrt{x}\right)\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] 4*a*b*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*_t**3*c*d**2 + sqr
t(x))))/d - 4*b**2*c*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*_t*
*3*c*d**2 + sqrt(x))))/d**2 + 2*b**2*x**(3/2)/(3*d**2) + 2*x**(3/2)*(a*d -
b*c)**2/(4*c**2*d**2 + 4*c*d**3*x**2) + 2*(a*d - b*c)**2*RootSum(65536*_t**
4*c**5*d**3 + 1, Lambda(_t, _t*log(4096*_t**3*c**4*d**2 + sqrt(x))))/d**2
```

Giac [A]
time = 0.59, size = 388, normalized size = 1.25

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d^2}+\frac{b^2c^2x^{\frac{3}{2}}-2\,abcdx^{\frac{3}{2}}+a^2d^2x^{\frac{3}{2}}}{2\,(dx^2+c)cd^2}-\frac{\sqrt{2}\left(7\,(cd^3)^{\frac{3}{4}}\,b^2c^2-6\,(cd^3)^{\frac{3}{4}}\,abcd-(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^5}-\frac{\sqrt{2}\left(7\,(cd^3)^{\frac{3}{4}}\,b^2c^2-6\,(cd^3)^{\frac{3}{4}}\,abcd-(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^5}+\frac{\sqrt{2}\left(7\,(cd^3)^{\frac{3}{4}}\,b^2c^2-6\,(cd^3)^{\frac{3}{4}}\,abcd-(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^2d^5}-\frac{\sqrt{2}\left(7\,(cd^3)^{\frac{3}{4}}\,b^2c^2-6\,(cd^3)^{\frac{3}{4}}\,abcd-(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d^2 + 1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^
(3/2))/((d*x^2 + c)*c*d^2) - 1/8*sqrt(2)*(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^
3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^5) - 1/8*sqrt(2)*(7*(c*d^3)^(3/4)*b
^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^5) + 1/16*sqrt(2)*
(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)
*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^5) - 1/16*sqrt(2)*
(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)
*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^5)
```

Mupad [B]
time = 0.22, size = 137, normalized size = 0.44

$$\frac{2\,b^2\,x^{3/2}}{3\,d^2}+\frac{x^{3/2}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c\,\left(d^3\,x^2+c\,d^2\right)}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+7\,b\,c\right)}{4\,{\left(-c\right)}^{5/4}\,d^{11/4}}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+7\,b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-c\right)}^{5/4}\,d^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (2*b^2*x^(3/2))/(3*d^2) + (x^(3/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c
*d^2 + d^3*x^2)) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(a*d + 7
*b*c))/(4*(-c)^(5/4)*d^(11/4)) - (atan((d^(1/4)*x^(1/2)*1i)/(-c)^(1/4))*(a*
d - b*c)*(a*d + 7*b*c)*1i)/(4*(-c)^(5/4)*d^(11/4))
```

3.429 $\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx$

Optimal. Leaf size=312

$$\frac{2b^2\sqrt{x}}{d^2}+\frac{(bc-ad)^2\sqrt{x}}{2cd^2\,(c+dx^2)}+\frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1\right.}{4\sqrt{2}\,c^{7/4}d^{9/4}}$$

```
[Out] 1/8*(-a*d+b*c)*(3*a*d+5*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7
/4)/d^(9/4)*2^(1/2)-1/8*(-a*d+b*c)*(3*a*d+5*b*c)*arctan(1+d^(1/4)*2^(1/2)*x
^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1/2)+1/16*(-a*d+b*c)*(3*a*d+5*b*c)*ln(c^
(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/d^(9/4)*2^(1/2)-1/
16*(-a*d+b*c)*(3*a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^
(1/2))/c^(7/4)/d^(9/4)*2^(1/2)+2*b^2*x^(1/2)/d^2+1/2*(-a*d+b*c)^2*x^(1/2)/c
/d^2/(d*x^2+c)
```

Rubi [A]

time = 0.22, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {474, 470, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{(bc-ad)(3ad+5bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{7/4}d^{9/4}}+\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}+\frac{\sqrt{x}\,(bc-ad)^2}{2cd^2\,(c+dx^2)}+\frac{2b^2\sqrt{x}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

```
[Out] (2*b^2*Sqrt[x])/d^2 + ((b*c - a*d)^2*Sqrt[x])/(2*c*d^2*(c + d*x^2)) + ((b*c
 - a*d)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*
c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx = \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{\int \frac{\frac{1}{2}(bc-3ad)(bc+ad)-2b^2cdx^2}{\sqrt{x}\,(c+dx^2)}\,dx}{2cd^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(5bc+3ad))\int \frac{1}{\sqrt{x}\,(c+dx^2)}\,dx}{4cd^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(5bc+3ad))\mathrm{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{2cd^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(5bc+3ad))\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right.}{4c^{3/2}d^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} - \frac{((bc-ad)(5bc+3ad))\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx\right.}{8c^{3/2}d^{5/2}}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} + \frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\right.}{8\sqrt{2}\,c^{7/4}d^{9/4}}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2(c+dx^2)} + \frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{7/4}d^{9/4}} - \frac{(bc}{}$$

Mathematica [A]
time = 0.59, size = 202, normalized size = 0.65

$$\frac{\frac{4c^{3/4}\sqrt[4]{d}\sqrt{x}\,(-2abcd+a^2d^2+b^2c(5c+4dx^2))}{c+dx^2}+\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)-\sqrt{2}\,(5b^2c^2-2abcd-3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{8c^{7/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

```
[Out] ((4*c^(3/4)*d^(1/4)*Sqrt[x]*(-2*a*b*c*d + a^2*d^2 + b^2*c*(5*c + 4*d*x^2)))
/(c + d*x^2) + Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c]
- Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - Sqrt[2]*(5*b^2*c^2 - 2*a*
b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqr
t[d]*x)])/(8*c^(7/4)*d^(9/4))
```

Maple [A]
time = 0.11, size = 185, normalized size = 0.59

method	result
derivativedivides	$\frac{2b^2\sqrt{x}}{d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{x}}{2c\left(dx^2+c\right)}+\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\right.}{16c^2}}{d^2}$
default	$\frac{2b^2\sqrt{x}}{d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{x}}{2c\left(dx^2+c\right)}+\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\right.}{16c^2}}{d^2}$
risch	$\frac{2b^2\sqrt{x}}{d^2}+\frac{\sqrt{x}\,a^2}{2c(dx^2+c)}-\frac{\sqrt{x}\,ab}{d(dx^2+c)}+\frac{\sqrt{x}\,b^2c}{2d^2(dx^2+c)}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2}{8c^2}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\ a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*x^(1/2)/d^2+2/d^2*(1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x^(1/2)/(d*x^2+c
)+1/32*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)-1)))
```

Maxima [A]
time = 0.52, size = 327, normalized size = 1.05

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{x}}{2\,(cd^3x^2+c^2d^2)}+\frac{2\,b^2\sqrt{x}}{d^2}-\frac{\frac{2\,\sqrt{2}\,(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\,(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(5\,b^2c^2-2\,abcd-3\,a^2d^2)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(5\,b^2c^2-2\,abcd-3\,a^2d^2)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)/(c*d^3*x^2 + c^2*d^2) + 2*b^2*s
qrt(x)/d^2 - 1/16*(2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2
*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a
^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/s
```

```
qrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2
- 2*a*b*c*d - 3*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*lo
g(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4))
)/(c*d^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. $2(234) = 468$.
time = 0.92, size = 1341, normalized size = 4.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^
6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^
5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*ar
ctan((sqrt(c^4*d^4*sqrt(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*
d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 32
4*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9)) + (25*b^4*c^4
- 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + 9*a^4*d^4)*x)*c^5*
d^7*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*
c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 +
 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4) + (5*b^2*c^7*d^7 - 2*a*b*c^
6*d^8 - 3*a^2*c^5*d^9)*sqrt(x)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*
b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*
d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4))
/(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d
^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*
a^7*b*c*d^7 + 81*a^8*d^8)) + (c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*
b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^
4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^
8)/(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2
*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3
*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)
 - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*sqrt(x)) - (c*d^3*x^2 + c^2*d^2)*(-(
625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3
 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^
7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-c^2*d^2*(-(625*b^8*c^8 - 1000
*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4
*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8
*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*sqrt(x)) + 4*(
4*b^2*c*d*x^2 + 5*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x))/(c*d^3*x^2 + c^2*
d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(298) = 596$.
time = 25.56, size = 1248, normalized size = 4.00

$$\begin{cases} \tilde{\infty}\left(-\frac{2a^2}{7x^{\frac{7}{2}}} - \frac{4ab}{3x^{\frac{3}{2}}} + 2b^2\sqrt{x}\right) & \text{for}\: c = 0 \wedge d = 0 \\ \frac{-\frac{2a^2}{7x^{\frac{7}{2}}} - \frac{4ab}{3x^{\frac{3}{2}}} + 2b^2\sqrt{x}}{d^2} & \text{for}\: c = 0 \\ \frac{2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}}{c^2} & \text{for}\: d = 0 \\ \frac{4a^2cd^2\sqrt{x}}{8c^3d^2+8c^2d^3x^2} - \frac{3a^2cd^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{3a^2cd^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{6a^2cd^2\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{8c^3d^2+8c^2d^3x^2} - \frac{3a^2d^3x^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{3a^2d^3x^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{6a^2d^3x^2\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{8c^3d^2+8c^2d^3x^2} - \frac{8abc^2d\sqrt{x}}{8c^3d^2+8c^2d^3x^2} - \frac{2abc^2d\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{2abc^2d\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{4abc^2d\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{8c^3d^2+8c^2d^3x^2} - \frac{2abcd^2x^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{2abcd^2x^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{4abcd^2x^2\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{20b^2c^3\sqrt{x}}{8c^3d^2+8c^2d^3x^2} + \frac{5b^2c^3\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} - \frac{5b^2c^3\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} - \frac{10b^2c^3\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{8c^3d^2+8c^2d^3x^2} + \frac{16b^2c^2dx^{\frac{5}{2}}}{8c^3d^2+8c^2d^3x^2} + \frac{5b^2c^2dx^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} - \frac{5b^2c^2dx^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{8c^3d^2+8c^2d^3x^2} - \frac{10b^2c^2dx^2\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{8c^3d^2+8c^2d^3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2*sqrt(x))
, Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**
2*sqrt(x))/d**2, Eq(c, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x*
*(9/2)/9)/c**2, Eq(d, 0)), (4*a**2*c*d**2*sqrt(x)/(8*c**3*d**2 + 8*c**2*d**
3*x**2) - 3*a**2*c*d**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(8*c**3*
d**2 + 8*c**2*d**3*x**2) + 3*a**2*c*d**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)
**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 6*a**2*c*d**2*(-c/d)**(1/4)*ata
n(sqrt(x)/(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 3*a**2*d**3*x**
2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**
2) + 3*a**2*d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(8*c**3*d*
*2 + 8*c**2*d**3*x**2) + 6*a**2*d**3*x**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)
**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 8*a*b*c**2*d*sqrt(x)/(8*c**3*d*
*2 + 8*c**2*d**3*x**2) - 2*a*b*c**2*d*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(
1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 2*a*b*c**2*d*(-c/d)**(1/4)*log(sqr
t(x) + (-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 4*a*b*c**2*d*(-c/d
)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 2*a
*b*c*d**2*x**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(8*c**3*d**2 + 8*
c**2*d**3*x**2) + 2*a*b*c*d**2*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/
4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 4*a*b*c*d**2*x**2*(-c/d)**(1/4)*atan
(sqrt(x)/(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 20*b**2*c**3*sqr
t(x)/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 5*b**2*c**3*(-c/d)**(1/4)*log(sqrt(
x) - (-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 5*b**2*c**3*(-c/d)**
(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 10*b*
*2*c**3*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**
3*x**2) + 16*b**2*c**2*d*x**(5/2)/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 5*b**2
*c**2*d*x**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(8*c**3*d**2 + 8*c*
*2*d**3*x**2) - 5*b**2*c**2*d*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4
))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 10*b**2*c**2*d*x**2*(-c/d)**(1/4)*ata
n(sqrt(x)/(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2), True))
```

Giac [A]
time = 0.64, size = 388, normalized size = 1.24

$$\frac{2\,b^2\sqrt{x}}{d^2} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^3} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^3} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^2d^3} + \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^2d^3} + \frac{b^2c^2\sqrt{x} - 2\,abcd\sqrt{x} + a^2d^2\sqrt{x}}{2\,(dx^2 + c)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")

[Out] 2*b^2*sqrt(x)/d^2 - 1/8*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/8*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^3) + 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^3) + 1/2*(b^2*c^2*sqrt(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*c*d^2)

Mupad [B]
time = 0.24, size = 1267, normalized size = 4.06

$$\frac{2\,b^2\,\sqrt{x}}{d^2}+\frac{\sqrt{x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c\,\left(d^3\,x^2+c\,d^2\right)}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{x}\,\left(9\,a^4\,d^4+12\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2-20\,a\,b^3\,c^3\,d+25\,b^4\,c^4\right)}{c^2\,d}-\frac{\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,\left(24\,a^2\,d^3+16\,a\,b\,c\,d^2-40\,b^2\,c^2\,d\right)}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}+\frac{\left(\frac{\sqrt{x}\,\left(9\,a^4\,d^4+12\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2-20\,a\,b^3\,c^3\,d+25\,b^4\,c^4\right)}{c^2\,d}+\frac{\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,\left(24\,a^2\,d^3+16\,a\,b\,c\,d^2-40\,b^2\,c^2\,d\right)}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}}{\frac{\left(\frac{\sqrt{x}\,\left(9\,a^4\,d^4+12\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2-20\,a\,b^3\,c^3\,d+25\,b^4\,c^4\right)}{c^2\,d}-\frac{\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,\left(24\,a^2\,d^3+16\,a\,b\,c\,d^2-40\,b^2\,c^2\,d\right)}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}-\frac{\left(\frac{\sqrt{x}\,\left(9\,a^4\,d^4+12\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2-20\,a\,b^3\,c^3\,d+25\,b^4\,c^4\right)}{c^2\,d}+\frac{\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,\left(24\,a^2\,d^3+16\,a\,b\,c\,d^2-40\,b^2\,c^2\,d\right)}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-c\right)}^{7/4}\,d^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{x}\,\left(9\,a^4\,d^4+12\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2-20\,a\,b^3\,c^3\,d+25\,b^4\,c^4\right)}{c^2\,d}-\frac{\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,\left(24\,a^2\,d^3+16\,a\,b\,c\,d^2-40\,b^2\,c^2\,d\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}+\frac{\left(\frac{\sqrt{x}\,\left(9\,a^4\,d^4+12\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2-20\,a\,b^3\,c^3\,d+25\,b^4\,c^4\right)}{c^2\,d}+\frac{\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,\left(24\,a^2\,d^3+16\,a\,b\,c\,d^2-40\,b^2\,c^2\,d\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}}{\frac{\left(\frac{\sqrt{x}\,\left(9\,a^4\,d^4+12\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2-20\,a\,b^3\,c^3\,d+25\,b^4\,c^4\right)}{c^2\,d}-\frac{\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,\left(24\,a^2\,d^3+16\,a\,b\,c\,d^2-40\,b^2\,c^2\,d\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}-\frac{\left(\frac{\sqrt{x}\,\left(9\,a^4\,d^4+12\,a^3\,b\,c\,d^3-26\,a^2\,b^2\,c^2\,d^2-20\,a\,b^3\,c^3\,d+25\,b^4\,c^4\right)}{c^2\,d}+\frac{\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,\left(24\,a^2\,d^3+16\,a\,b\,c\,d^2-40\,b^2\,c^2\,d\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)\,1{}\mathrm{i}}{8\,{\left(-c\right)}^{7/4}\,d^{9/4}}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+5\,b\,c\right)}{4\,{\left(-c\right)}^{7/4}\,d^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^2/(x^(1/2)*(c + d*x^2)^2),x)

[Out] (2*b^2*x^(1/2))/d^2 + (x^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^2 + d^3*x^2)) + (atan(((((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2))/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(-c)^(7/4)*d^(9/4)) + (((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2))/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(-c)^(7/4)*d^(9/4)))/((((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2))/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4)) - (((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2))/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4))))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(4*(-c)^(7/4)*d^(9/4)) + (atan(((((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2)*1i)/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4)) + (((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2)*1i)/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4)))/((((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2)*1i)/(8*(-c

```
)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(-c)^(7/4)*d^(9/4)) -
(((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d +
12*a^3*b*c*d^3))/(c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^
2*c^2*d + 16*a*b*c*d^2)*1i)/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*
b*c)*1i)/(8*(-c)^(7/4)*d^(9/4))))*(a*d - b*c)*(3*a*d + 5*b*c))/(4*(-c)^(7/4
)*d^(9/4))
```

3.430 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2} dx$

Optimal. Leaf size=333

$$-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}-\frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d\,(c+dx^2)}-\frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{9/4}d^{7/4}}+\frac{(bc-ad}{}$$

```
[Out] -1/2*(5*a^2*d^2-2*a*b*c*d+b^2*c^2)*x^(3/2)/c^2/d/(d*x^2+c)-1/8*(-a*d+b*c)*(
5*a*d+3*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/d^(7/4)*2^(1
/2)+1/8*(-a*d+b*c)*(5*a*d+3*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/
c^(9/4)/d^(7/4)*2^(1/2)+1/16*(-a*d+b*c)*(5*a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)-
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/d^(7/4)*2^(1/2)-1/16*(-a*d+b*c)*(5
*a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/d
^(7/4)*2^(1/2)-2*a^2/c/(d*x^2+c)/x^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 331, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {473, 468, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{x^{3/2}\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2c\,(c+dx^2)}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}-\frac{(bc-ad)(5ad+3bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{9/4}d^{7/4}}+\frac{(bc-ad)(5ad+3bc)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{9/4}d^{7/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}d^{7/4}}-\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)) + ((2*a*b - (b^2*c)/d - (5*a^2*d)/c)*x^(3/
2))/(2*c*(c + d*x^2)) - ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*d^(7/4)) + ((b*c - a*d)*(3*b*c
+ 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*
d^(7/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*d^(7/4)) - ((b*c - a*d)*(3*b*c
+ 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqr
t[2]*c^(9/4)*d^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx = -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)} + \frac{2\int \frac{\sqrt{x}\,\left(\frac{1}{2}a(2bc-5ad)+\frac{1}{2}b^2cx^2\right)}{(c+dx^2)^2}\,dx}{c}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d\,(c+dx^2)} + \frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{x}}{c+dx^2}\,d}{4c^2d}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d\,(c+dx^2)} + \frac{((bc-ad)(3bc+5ad))\text{Subst}\Big(\int}{2c^2d}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d\,(c+dx^2)} - \frac{((bc-ad)(3bc+5ad))\text{Subst}\Big(\int}{4c^2d^{3/2}}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d\,(c+dx^2)} + \frac{((bc-ad)(3bc+5ad))\text{Subst}\Bigg(\int}{8c^2}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d\,(c+dx^2)} + \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{c}\;-\right.}{8\sqrt{2}\,c^{9/4}}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d\,(c+dx^2)} - \frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\right.}{4\sqrt{2}\,c^{9/4}d^{7/4}}$$

Mathematica [A]

time = 0.59, size = 209, normalized size = 0.63

$$\frac{-\frac{4\sqrt[4]{c}\,d^{3/4}(b^2c^2x^2-2abcdx^2+a^2d(4c+5dx^2))}{\sqrt{x}\,(c+dx^2)} - \sqrt{2}\,(3b^2c^2+2abcd-5a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right) - \sqrt{2}\,(3b^2c^2+2abcd-5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{8c^{9/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-4*c^(1/4)*d^(3/4)*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d*(4*c + 5*d*x^2)))
/(Sqrt[x]*(c + d*x^2)) - Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan
[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - Sqrt[2]*(3*b^2*
c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqr
t[c] + Sqrt[d]*x)])/(8*c^(9/4)*d^(7/4))
```

Maple [A]
time = 0.11, size = 185, normalized size = 0.56

method	result
derivativedivides	$-\frac{2\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)x^{\frac{3}{2}}}{4d\left(dx^2+c\right)}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)\cdots\right.}{32d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\cdots\right)}{c^2}$
default	$-\frac{2\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)x^{\frac{3}{2}}}{4d\left(dx^2+c\right)}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)\cdots\right.}{32d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\cdots\right)}{c^2}$
risch	$-\frac{2a^2}{c^2\sqrt{x}}-\frac{x^{\frac{3}{2}}a^2d}{2c^2(dx^2+c)}+\frac{x^{\frac{3}{2}}ab}{c(dx^2+c)}-\frac{x^{\frac{3}{2}}b^2}{2d(dx^2+c)}-\frac{5\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2}{8c^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\cdots\right.}{4cd(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^2*(1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*x^(3/2)/(d*x^2+c)+1/32*(5*a^2*d^2
-2*a*b*c*d-3*b^2*c^2)/d^2/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2*a^
2/c^2/x^(1/2)
```

Maxima [A]
time = 0.52, size = 260, normalized size = 0.78

$$-\frac{4a^2cd+(b^2c^2-2abcd+5a^2d^2)x^2}{2\left(c^2d^2x^{\frac{5}{2}}+c^3d\sqrt{x}\right)}+\frac{(3b^2c^2+2abcd-5a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^2)/(c^2*d^2*x^(5/2) +
 c^3*d*sqrt(x)) + 1/16*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*(2*sqrt(2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sq
rt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqr
t(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt
(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*s
qrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1739 vs. $2(255) = 510$.

time = 1.08, size = 1739, normalized size = 5.22

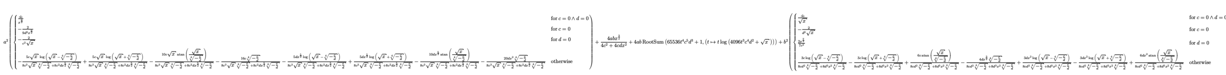
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^2*d^2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^
6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^
5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4)*
arctan((sqrt((729*b^12*c^12 + 2916*a*b^11*c^11*d - 2430*a^2*b^10*c^10*d^2 -
 19980*a^3*b^9*c^9*d^3 + 135*a^4*b^8*c^8*d^4 + 59976*a^5*b^7*c^7*d^5 + 6364
*a^6*b^6*c^6*d^6 - 99960*a^7*b^5*c^5*d^7 + 375*a^8*b^4*c^4*d^8 + 92500*a^9*
b^3*c^3*d^9 - 18750*a^10*b^2*c^2*d^10 - 37500*a^11*b*c*d^11 + 15625*a^12*d^
12)*x - (81*b^8*c^13*d^3 + 216*a*b^7*c^12*d^4 - 324*a^2*b^6*c^11*d^5 - 984*
a^3*b^5*c^10*d^6 + 646*a^4*b^4*c^9*d^7 + 1640*a^5*b^3*c^8*d^8 - 900*a^6*b^2
*c^7*d^9 - 1000*a^7*b*c^6*d^10 + 625*a^8*c^5*d^11)*sqrt(-(81*b^8*c^8 + 216*
a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d
^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^
8*d^8)/(c^9*d^7)))*c^2*d^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^
6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 -
900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4) + (2
7*b^6*c^8*d^2 + 54*a*b^5*c^7*d^3 - 99*a^2*b^4*c^6*d^4 - 172*a^3*b^3*c^5*d^5
 + 165*a^4*b^2*c^4*d^6 + 150*a^5*b*c^3*d^7 - 125*a^6*c^2*d^8)*sqrt(x)*(-(81
*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 64
6*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b
*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4))/(81*b^8*c^8 + 216*a*b^7*c^7*d - 324
*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3
*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)) - (c^2*d^
2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 98
4*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^
```

```
2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4)*log(c^7*d^5*(-
(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 +
 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^
7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(3/4) - (27*b^6*c^6 + 54*a*b^5*c^5*d -
99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165*a^4*b^2*c^2*d^4 + 150*a^5*b*
c*d^5 - 125*a^6*d^6)*sqrt(x)) + (c^2*d^2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216
*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*
d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a
^8*d^8)/(c^9*d^7))^(1/4)*log(-c^7*d^5*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324
*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3
*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))
^(3/4) - (27*b^6*c^6 + 54*a*b^5*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^
3*d^3 + 165*a^4*b^2*c^2*d^4 + 150*a^5*b*c*d^5 - 125*a^6*d^6)*sqrt(x)) - 4*(
4*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^2)*sqrt(x))/(c^2*d^2*x^3 +
c^3*d*x)
```

Sympy [A]
time = 129.42, size = 976, normalized size = 2.93



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] a**2*Piecewise((zoo/x**(9/2), Eq(c, 0) & Eq(d, 0)), (-2/(9*d**2*x**(9/2)),
Eq(c, 0)), (-2/(c**2*sqrt(x)), Eq(d, 0)), (-5*c*sqrt(x)*log(sqrt(x) - (-c/d
)**(1/4))/(8*c**3*sqrt(x)*(-c/d)**(1/4) + 8*c**2*d*x**(5/2)*(-c/d)**(1/4))
+ 5*c*sqrt(x)*log(sqrt(x) + (-c/d)**(1/4))/(8*c**3*sqrt(x)*(-c/d)**(1/4) +
8*c**2*d*x**(5/2)*(-c/d)**(1/4)) - 10*c*sqrt(x)*atan(sqrt(x)/(-c/d)**(1/4))
/(8*c**3*sqrt(x)*(-c/d)**(1/4) + 8*c**2*d*x**(5/2)*(-c/d)**(1/4)) - 16*c*(-
c/d)**(1/4)/(8*c**3*sqrt(x)*(-c/d)**(1/4) + 8*c**2*d*x**(5/2)*(-c/d)**(1/4)
) - 5*d*x**(5/2)*log(sqrt(x) - (-c/d)**(1/4))/(8*c**3*sqrt(x)*(-c/d)**(1/4)
 + 8*c**2*d*x**(5/2)*(-c/d)**(1/4)) + 5*d*x**(5/2)*log(sqrt(x) + (-c/d)**(1
/4))/(8*c**3*sqrt(x)*(-c/d)**(1/4) + 8*c**2*d*x**(5/2)*(-c/d)**(1/4)) - 10*
d*x**(5/2)*atan(sqrt(x)/(-c/d)**(1/4))/(8*c**3*sqrt(x)*(-c/d)**(1/4) + 8*c*
*2*d*x**(5/2)*(-c/d)**(1/4)) - 20*d*x**2*(-c/d)**(1/4)/(8*c**3*sqrt(x)*(-c/
d)**(1/4) + 8*c**2*d*x**(5/2)*(-c/d)**(1/4)), True)) + 4*a*b*x**(3/2)/(4*c*
*2 + 4*c*d*x**2) + 4*a*b*RootSum(65536*_t**4*c**5*d**3 + 1, Lambda(_t, _t*l
og(4096*_t**3*c**4*d**2 + sqrt(x)))) + b**2*Piecewise((zoo/sqrt(x), Eq(c, 0
) & Eq(d, 0)), (-2/(d**2*sqrt(x)), Eq(c, 0)), (2*x**(7/2)/(7*c**2), Eq(d, 0
)), (3*c*log(sqrt(x) - (-c/d)**(1/4))/(8*c*d**2*(-c/d)**(1/4) + 8*d**3*x**2
*(-c/d)**(1/4)) - 3*c*log(sqrt(x) + (-c/d)**(1/4))/(8*c*d**2*(-c/d)**(1/4)
+ 8*d**3*x**2*(-c/d)**(1/4)) + 6*c*atan(sqrt(x)/(-c/d)**(1/4))/(8*c*d**2*(-
c/d)**(1/4) + 8*d**3*x**2*(-c/d)**(1/4)) - 4*d*x**(3/2)*(-c/d)**(1/4)/(8*c*
d**2*(-c/d)**(1/4) + 8*d**3*x**2*(-c/d)**(1/4)) + 3*d*x**2*log(sqrt(x) - (-
```

```
c/d)**(1/4))/(8*c*d**2*(-c/d)**(1/4) + 8*d**3*x**2*(-c/d)**(1/4)) - 3*d*x**
2*log(sqrt(x) + (-c/d)**(1/4))/(8*c*d**2*(-c/d)**(1/4) + 8*d**3*x**2*(-c/d)
**(1/4)) + 6*d*x**2*atan(sqrt(x)/(-c/d)**(1/4))/(8*c*d**2*(-c/d)**(1/4) + 8
*d**3*x**2*(-c/d)**(1/4)), True))
```

Giac [A]
time = 0.63, size = 389, normalized size = 1.17

$$-\frac{b^2c^2x^2 - 2\,abcdx^2 + 5\,a^2d^2x^2 + 4\,a^2cd}{2\left(dx^{\frac{5}{2}} + c\sqrt{x}\right)c^2d} + \frac{\sqrt{2}\left(3\,(cd^3)^{\frac{3}{4}}\,b^2c^2 + 2\,(cd^3)^{\frac{3}{4}}\,abcd - 5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^4} + \frac{\sqrt{2}\left(3\,(cd^3)^{\frac{3}{4}}\,b^2c^2 + 2\,(cd^3)^{\frac{3}{4}}\,abcd - 5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^4} - \frac{\sqrt{2}\left(3\,(cd^3)^{\frac{3}{4}}\,b^2c^2 + 2\,(cd^3)^{\frac{3}{4}}\,abcd - 5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^3d^4} + \frac{\sqrt{2}\left(3\,(cd^3)^{\frac{3}{4}}\,b^2c^2 + 2\,(cd^3)^{\frac{3}{4}}\,abcd - 5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 5*a^2*d^2*x^2 + 4*a^2*c*d)/((d*x^(5/2)
+ c*sqrt(x))*c^2*d) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4
)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^4) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^
2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^4) - 1/16*sqrt(2)*(3*
(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*
log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^4) + 1/16*sqrt(2)*(
3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2
)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^4)
```

Mupad [B]
time = 0.22, size = 138, normalized size = 0.41

$$\frac{\mathrm{atanh}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,(a\,d-b\,c)\,(5\,a\,d+3\,b\,c)}{4\,(-c)^{9/4}\,d^{7/4}}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,(a\,d-b\,c)\,(5\,a\,d+3\,b\,c)}{4\,(-c)^{9/4}\,d^{7/4}}-\frac{\frac{2\,a^2}{c}+\frac{x^2\,(5\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,c^2\,d}}{c\,\sqrt{x}+d\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x)
```

```
[Out] (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(5*a*d + 3*b*c))/(4*(-c)^(
9/4)*d^(7/4)) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(5*a*d + 3*
b*c))/(4*(-c)^(9/4)*d^(7/4)) - ((2*a^2)/c + (x^2*(5*a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))/(2*c^2*d))/(c*x^(1/2) + d*x^(5/2))
```

3.431 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=332

$$-\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}-\frac{(bc-ad)(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{(bc-ad}{}$$

```
[Out] -2/3*a^2/c/x^(3/2)/(d*x^2+c)-1/8*(-a*d+b*c)*(7*a*d+b*c)*arctan(1-d^(1/4)*2^
(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/2)+1/8*(-a*d+b*c)*(7*a*d+b*c)*
arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/2)-1/16*(-a
*d+b*c)*(7*a*d+b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c
^(11/4)/d^(5/4)*2^(1/2)+1/16*(-a*d+b*c)*(7*a*d+b*c)*ln(c^(1/2)+x*d^(1/2)+c^
(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/d^(5/4)*2^(1/2)-1/6*(7*a^2*d^2-6*a*
b*c*d+3*b^2*c^2)*x^(1/2)/c^2/d/(d*x^2+c)
```

Rubi [A]
time = 0.22, antiderivative size = 329, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {473, 468, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{x}\left(-\frac{7a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{6c(c+dx^2)}-\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(bc-ad)(7ad+bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad+bc)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{11/4}d^{5/4}}-\frac{(bc-ad)(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)) + ((6*a*b - (3*b^2*c)/d - (7*a^2*d)/c)*S
qrt[x])/(6*c*(c + d*x^2)) - ((b*c - a*d)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*(b*c
 + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4
)*d^(5/4)) - ((b*c - a*d)*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*(b*c +
 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt
[2]*c^(11/4)*d^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx = -\frac{2a^2}{3cx^{3/2}(c+dx^2)}+\frac{2\int \frac{\frac{1}{2}a(6bc-7ad)+\frac{3}{2}b^2cx^2}{\sqrt{x}\,(c+dx^2)^2}\,dx}{3c}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}+\frac{((bc-ad)(bc+7ad))\int \frac{1}{\sqrt{x}\,(c+\cdots}}{4c^2d}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}+\frac{((bc-ad)(bc+7ad))\text{Subst}\left(\int \cdots\right.}{2c^2d}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}+\frac{((bc-ad)(bc+7ad))\text{Subst}\left(\int \cdots\right.}{4c^{5/2}d}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}+\frac{((bc-ad)(bc+7ad))\text{Subst}\left(\int \cdots\right.}{8c^{5/\cdots}}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}-\frac{(bc-ad)(bc+7ad)\log\left(\sqrt{c}-\cdots\right.}{8\sqrt{2}\,c^{11/\cdots}}$$

$$= -\frac{2a^2}{3cx^{3/2}(c+dx^2)}-\frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)}-\frac{(bc-ad)(bc+7ad)\tan^{-1}\left(1-\cdots\right.}{4\sqrt{2}\,c^{11/4}d^{5/4}}$$

Mathematica [A]
time = 0.61, size = 208, normalized size = 0.63

$$\frac{-\frac{4c^{3/4}\sqrt[4]{d}\,(3b^2c^2x^2-6abcdx^2+a^2d(4c+7dx^2))}{x^{3/2}(c+dx^2)}-3\sqrt{2}\,(b^2c^2+6abcd-7a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)+3\sqrt{2}\,(b^2c^2+6abcd-7a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{24c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-4*c^(3/4)*d^(1/4)*(3*b^2*c^2*x^2 - 6*a*b*c*d*x^2 + a^2*d*(4*c + 7*d*x^2)
))/(x^(3/2)*(c + d*x^2)) - 3*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*ArcT
an[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(b^
2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(S
qrt[c] + Sqrt[d]*x)])/(24*c^(11/4)*d^(5/4))
```

Maple [A]
time = 0.12, size = 188, normalized size = 0.57

method	result
derivativedivides	$-\frac{2\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{x}}{4d\left(dx^2+c\right)}+\frac{\left(7a^2d^2-6abcd-b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\ \cdots}{\left(\frac{c}{d}\right)\cdots}\right)\cdots\right)}{32dc}\cdots\right)}{c^2}\cdots$
default	$-\frac{2\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{x}}{4d\left(dx^2+c\right)}+\frac{\left(7a^2d^2-6abcd-b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\ \cdots}{\left(\frac{c}{d}\right)\cdots}\right)\cdots\right)}{32dc}\cdots\right)}{c^2}\cdots$
risch	$-\frac{2a^2}{3c^2x^{\frac{3}{2}}}-\frac{\sqrt{x}\,a^2d}{2c^2\left(dx^2+c\right)}+\frac{\sqrt{x}\,ab}{c\left(dx^2+c\right)}-\frac{\sqrt{x}\,b^2}{2d\left(dx^2+c\right)}-\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)a^2d}{8c^3}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^2*(1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*x^(1/2)/(d*x^2+c)+1/32*(7*a^2*d^2
-6*a*b*c*d-b^2*c^2)/d*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/3*a^
2/c^2/x^(3/2)
```

Maxima [A]
time = 0.52, size = 326, normalized size = 0.98

$$-\frac{4a^2cd+\left(3b^2c^2-6abcd+7a^2d^2\right)x^2}{6\left(c^2d^2x^{\frac{7}{2}}+c^3dx^{\frac{3}{2}}\right)}+\frac{\frac{2\sqrt{2}\left(b^2c^2+6abcd-7a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(b^2c^2+6abcd-7a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(b^2c^2+6abcd-7a^2d^2\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^2c^2+6abcd-7a^2d^2\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(4*a^2*c*d + (3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^2)/(c^2*d^2*x^(7/2)
 + c^3*d*x^(3/2)) + 1/16*(2*sqrt(2)*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sq
rt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 + 6*a*b*c*d -
7*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2
 + 6*a*b*c*d - 7*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
 sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*log
(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))
/(c^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. $2(252) = 504$.
time = 1.65, size = 1340, normalized size = 4.04

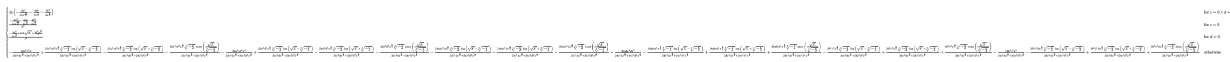
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(c^2*d^2*x^4 + c^3*d*x^2)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b
^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*
d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(
1/4)*arctan((sqrt(c^6*d^2*sqrt(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6
*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 +
9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5)) + (b^4*
c^4 + 12*a*b^3*c^3*d + 22*a^2*b^2*c^2*d^2 - 84*a^3*b*c*d^3 + 49*a^4*d^4)*x)
*c^8*d^4*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^
5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6
- 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(3/4) + (b^2*c^10*d^4 + 6*a*
b*c^9*d^5 - 7*a^2*c^8*d^6)*sqrt(x)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^
6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d
^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(3
/4))/(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3
- 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232
*a^7*b*c*d^7 + 2401*a^8*d^8)) + 3*(c^2*d^2*x^4 + c^3*d*x^2)*(-(b^8*c^8 + 24
*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4
*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 240
1*a^8*d^8)/(c^11*d^5))^(1/4)*log(c^3*d*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^
2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c
^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5)
)^(1/4) - (b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*sqrt(x)) - 3*(c^2*d^2*x^4 + c^3
*d*x^2)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5
*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 -
 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4)*log(-c^3*d*(-(b^8*c^8 +
```

24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*
c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 +
2401*a^8*d^8)/(c^11*d^5))^(1/4) - (b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*sqrt(x)
) + 4*(4*a^2*c*d + (3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^2)*sqrt(x))/(c^2*d
^2*x^4 + c^3*d*x^2)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. $2(313) = 626$.
time = 118.60, size = 1418, normalized size = 4.27



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**2,x)

[Out] Piecewise((zoo*(-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/2)) - 2*b**2/(3*x**
(3/2))), Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/2)
) - 2*b**2/(3*x**(3/2)))/d**2, Eq(c, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sq
rt(x) + 2*b**2*x**(5/2)/5)/c**2, Eq(d, 0)), (-16*a**2*c**2*d/(24*c**4*d*x**
(3/2) + 24*c**3*d**2*x**(7/2)) + 21*a**2*c*d**2*x**(3/2)*(-c/d)**(1/4)*log(
sqrt(x) - (-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) - 21*
a**2*c*d**2*x**(3/2)*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(24*c**4*d*
x**(3/2) + 24*c**3*d**2*x**(7/2)) - 42*a**2*c*d**2*x**(3/2)*(-c/d)**(1/4)*a
tan(sqrt(x)/(-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) - 2
8*a**2*c*d**2*x**2/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) + 21*a**2*d
3*x(7/2)*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(24*c**4*d*x**(3/2)
 + 24*c**3*d**2*x**(7/2)) - 21*a**2*d**3*x**(7/2)*(-c/d)**(1/4)*log(sqrt(x)
 + (-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) - 42*a**2*d*
*3*x**(7/2)*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(24*c**4*d*x**(3/2) +
 24*c**3*d**2*x**(7/2)) - 18*a*b*c**2*d*x**(3/2)*(-c/d)**(1/4)*log(sqrt(x)
- (-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) + 18*a*b*c**2
*d*x**(3/2)*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(24*c**4*d*x**(3/2)
+ 24*c**3*d**2*x**(7/2)) + 36*a*b*c**2*d*x**(3/2)*(-c/d)**(1/4)*atan(sqrt(x
)/(-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) + 24*a*b*c**2
*d*x**2/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) - 18*a*b*c*d**2*x**(7/
2)*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3
*d**2*x**(7/2)) + 18*a*b*c*d**2*x**(7/2)*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)
(1/4))/(24*c4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) + 36*a*b*c*d**2*x**(7
/2)*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3
*d**2*x**(7/2)) - 3*b**2*c**3*x**(3/2)*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**
(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) + 3*b**2*c**3*x**(3/2)*
(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d*
*2*x**(7/2)) + 6*b**2*c**3*x**(3/2)*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4
))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) - 12*b**2*c**3*x**2/(24*c**
4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) - 3*b**2*c**2*d*x**(7/2)*(-c/d)**(1/4

```
)*log(sqrt(x) - (-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2))
 + 3*b**2*c**2*d*x**(7/2)*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(24*c*
*4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) + 6*b**2*c**2*d*x**(7/2)*(-c/d)**(1/
4)*atan(sqrt(x)/(-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2))
, True))
```

Giac [A]
time = 0.62, size = 384, normalized size = 1.16

$$-\frac{2a^2}{3c^2x^{\frac{3}{2}}}+\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2+6(cd^3)^{\frac{1}{4}}abcd-7(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8c^3d^2}+\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2+6(cd^3)^{\frac{1}{4}}abcd-7(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8c^3d^2}+\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2+6(cd^3)^{\frac{1}{4}}abcd-7(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16c^3d^2}-\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2+6(cd^3)^{\frac{1}{4}}abcd-7(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16c^3d^2}-\frac{b^2c^2\sqrt{x}-2abcd\sqrt{x}+a^2d^2\sqrt{x}}{2(dx^2+c)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -2/3*a^2/(c^2*x^(3/2)) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/
4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2
 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/16*sqrt(2)*((c*
d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*log
(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^2) - 1/16*sqrt(2)*((c*
d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*log
(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^2) - 1/2*(b^2*c^2*sqr
t(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*c^2*d)
```

Mupad [B]
time = 0.31, size = 1340, normalized size = 4.04

$$-\frac{\frac{2a^2}{3c}+\frac{x^2\,(7a^2d^2-6abcd+3b^2c^2)}{6c^2d}}{c\,x^{3/2}+d\,x^{7/2}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\sqrt{x}\,(1568a^4c^6d^{10}-2688a^3bc^7d^9+704a^2b^2c^8d^8+384ab^3c^9d^7+32b^4c^{10}d^6)-\frac{(ad-bc)(7ad+bc)(-1792a^2c^9d^9+1536abc^{10}d^8+256b^2c^{11}d^7)}{8(-c)^{11/4}d^{5/4}}\right)(ad-bc)(7ad+bc)\,1i}{8(-c)^{11/4}d^{5/4}}+\frac{\left(\sqrt{x}\,(1568a^4c^6d^{10}-2688a^3bc^7d^9+704a^2b^2c^8d^8+384ab^3c^9d^7+32b^4c^{10}d^6)+\frac{(ad-bc)(7ad+bc)(-1792a^2c^9d^9+1536abc^{10}d^8+256b^2c^{11}d^7)}{8(-c)^{11/4}d^{5/4}}\right)(ad-bc)(7ad+bc)\,1i}{8(-c)^{11/4}d^{5/4}}}{\frac{\left(\sqrt{x}\,(1568a^4c^6d^{10}-2688a^3bc^7d^9+704a^2b^2c^8d^8+384ab^3c^9d^7+32b^4c^{10}d^6)-\frac{(ad-bc)(7ad+bc)(-1792a^2c^9d^9+1536abc^{10}d^8+256b^2c^{11}d^7)}{8(-c)^{11/4}d^{5/4}}\right)(ad-bc)(7ad+bc)}{8(-c)^{11/4}d^{5/4}}-\frac{\left(\sqrt{x}\,(1568a^4c^6d^{10}-2688a^3bc^7d^9+704a^2b^2c^8d^8+384ab^3c^9d^7+32b^4c^{10}d^6)+\frac{(ad-bc)(7ad+bc)(-1792a^2c^9d^9+1536abc^{10}d^8+256b^2c^{11}d^7)}{8(-c)^{11/4}d^{5/4}}\right)(ad-bc)(7ad+bc)}{8(-c)^{11/4}d^{5/4}}}\right)(ad-bc)\,(7ad+bc)\,1i}{4(-c)^{11/4}d^{5/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\sqrt{x}\,(1568a^4c^6d^{10}-2688a^3bc^7d^9+704a^2b^2c^8d^8+384ab^3c^9d^7+32b^4c^{10}d^6)-\frac{(ad-bc)(7ad+bc)(-1792a^2c^9d^9+1536abc^{10}d^8+256b^2c^{11}d^7)\,1i}{8(-c)^{11/4}d^{5/4}}\right)(ad-bc)(7ad+bc)}{8(-c)^{11/4}d^{5/4}}+\frac{\left(\sqrt{x}\,(1568a^4c^6d^{10}-2688a^3bc^7d^9+704a^2b^2c^8d^8+384ab^3c^9d^7+32b^4c^{10}d^6)+\frac{(ad-bc)(7ad+bc)(-1792a^2c^9d^9+1536abc^{10}d^8+256b^2c^{11}d^7)\,1i}{8(-c)^{11/4}d^{5/4}}\right)(ad-bc)(7ad+bc)}{8(-c)^{11/4}d^{5/4}}}{\frac{\left(\sqrt{x}\,(1568a^4c^6d^{10}-2688a^3bc^7d^9+704a^2b^2c^8d^8+384ab^3c^9d^7+32b^4c^{10}d^6)-\frac{(ad-bc)(7ad+bc)(-1792a^2c^9d^9+1536abc^{10}d^8+256b^2c^{11}d^7)\,1i}{8(-c)^{11/4}d^{5/4}}\right)(ad-bc)(7ad+bc)\,1i}{8(-c)^{11/4}d^{5/4}}-\frac{\left(\sqrt{x}\,(1568a^4c^6d^{10}-2688a^3bc^7d^9+704a^2b^2c^8d^8+384ab^3c^9d^7+32b^4c^{10}d^6)+\frac{(ad-bc)(7ad+bc)(-1792a^2c^9d^9+1536abc^{10}d^8+256b^2c^{11}d^7)\,1i}{8(-c)^{11/4}d^{5/4}}\right)(ad-bc)(7ad+bc)\,1i}{8(-c)^{11/4}d^{5/4}}}\right)(ad-bc)\,(7ad+bc)}{4(-c)^{11/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x)
```

```
[Out] (atan((((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 -
 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) - ((a*d - b*c)*(7*a*d + b*c)*(25
6*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8))/(8*(-c)^(11/4)*d^(5
/4)))*(a*d - b*c)*(7*a*d + b*c)*1i)/(8*(-c)^(11/4)*d^(5/4)) + ((x^(1/2)*(15
68*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9
+ 704*a^2*b^2*c^8*d^8) + ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 179
2*a^2*c^9*d^9 + 1536*a*b*c^10*d^8))/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7
*a*d + b*c)*1i)/(8*(-c)^(11/4)*d^(5/4)))/(((x^(1/2)*(1568*a^4*c^6*d^10 + 32
*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^
8) - ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536
*a*b*c^10*d^8))/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*c))/(8*(-c)
^(11/4)*d^(5/4)) - ((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b
^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) + ((a*d - b*c)*(7*a*
```

```
d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8))/(8*(-c)
^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*c))/(8*(-c)^(11/4)*d^(5/4))))*(a*d
 - b*c)*(7*a*d + b*c)*1i)/(4*(-c)^(11/4)*d^(5/4)) - ((2*a^2)/(3*c) + (x^2*(
7*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d))/(6*c^2*d))/(c*x^(3/2) + d*x^(7/2)) + (a
tan((((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2
688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) - ((a*d - b*c)*(7*a*d + b*c)*(256*
b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8)*1i)/(8*(-c)^(11/4)*d^(
5/4)))*(a*d - b*c)*(7*a*d + b*c))/(8*(-c)^(11/4)*d^(5/4)) + ((x^(1/2)*(1568
*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 +
704*a^2*b^2*c^8*d^8) + ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*
a^2*c^9*d^9 + 1536*a*b*c^10*d^8)*1i)/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(
7*a*d + b*c))/(8*(-c)^(11/4)*d^(5/4)))/(((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b
^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8)
 - ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a
*b*c^10*d^8)*1i)/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*c)*1i)/(8*
(-c)^(11/4)*d^(5/4)) - ((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384
*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) + ((a*d - b*c)*(
7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8)*1i)/
(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*c)*1i)/(8*(-c)^(11/4)*d^(5/
4))))*(a*d - b*c)*(7*a*d + b*c))/(4*(-c)^(11/4)*d^(5/4))
```

3.432 $\int \frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=363

$$\frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}-\frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}\,\left(c+dx^2\right)}-\frac{(bc-9ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{}}{\sqrt[4]{c}}\right.}{4\sqrt{2}\,c^{13/4}d^{3/4}}$$

```
[Out] -2/5*a^2/c/x^(5/2)/(d*x^2+c)-1/8*(-9*a*d+b*c)*(-a*d+b*c)*arctan(1-d^(1/4)*2
^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/d^(3/4)*2^(1/2)+1/8*(-9*a*d+b*c)*(-a*d+b*c
)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/d^(3/4)*2^(1/2)+1/16*(
-9*a*d+b*c)*(-a*d+b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2)
)/c^(13/4)/d^(3/4)*2^(1/2)-1/16*(-9*a*d+b*c)*(-a*d+b*c)*ln(c^(1/2)+x*d^(1/2
)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/d^(3/4)*2^(1/2)+1/2*(-9*a*d+b*c
)*(-a*d+b*c)/c^3/d/x^(1/2)+1/10*(-9*a^2*d^2+10*a*b*c*d-5*b^2*c^2)/c^2/d/(d*
x^2+c)/x^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 360, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {473, 468, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{10c\sqrt{x}\,\left(c+dx^2\right)}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}-\frac{(bc-9ad)(bc-ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{13/4}d^{3/4}}+\frac{(bc-9ad)(bc-ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{13/4}d^{3/4}}+\frac{(bc-9ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{13/4}d^{3/4}}-\frac{(bc-9ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{13/4}d^{3/4}}+\frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```

```
[Out] ((b*c - 9*a*d)*(b*c - a*d))/(2*c^3*d*Sqrt[x]) - (2*a^2)/(5*c*x^(5/2)*(c + d
*x^2)) + (10*a*b - (5*b^2*c)/d - (9*a^2*d)/c)/(10*c*Sqrt[x]*(c + d*x^2)) -
((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a*d)*
(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*
Sqrt[2]*c^(13/4)*d^(3/4)) - ((b*c - 9*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*d^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx = -\frac{2a^2}{5cx^{5/2}(c+dx^2)} + \frac{2\int \frac{\frac{1}{2}a(10bc-9ad)+\frac{5}{2}b^2cx^2}{x^{3/2}(c+dx^2)^2}\,dx}{5c}$$

$$= -\frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}\,(c+dx^2)} - \frac{((bc-9ad)(bc-ad))\int \frac{1}{x^{3/2}(c+dx^2)}\,d}{4c^2d}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}\,(c+dx^2)} + \frac{((bc-9ad)(bc}{}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}\,(c+dx^2)} + \frac{((bc-9ad)(bc}{}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}\,(c+dx^2)} - \frac{((bc-9ad)(bc}{}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}\,(c+dx^2)} + \frac{((bc-9ad)(bc}{}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}\,(c+dx^2)} + \frac{(bc-9ad)(bc}{}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} - \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}\,(c+dx^2)} - \frac{(bc-9ad)(bc}{}$$

Mathematica [A]
time = 0.61, size = 227, normalized size = 0.63

$$\frac{\frac{4\sqrt[4]{c}\,(5b^2c^2x^4-10abcx^2(4c+5dx^2)+a^2(-4c^2+36cdx^2+45d^2x^4))}{x^{5/2}(c+dx^2)} - \frac{5\sqrt{2}\,(b^2c^2-10abcd+9a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{d^{3/4}} - \frac{5\sqrt{2}\,(b^2c^2-10abcd+9a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{d^{3/4}}}{40c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```

```
[Out] ((4*c^(1/4)*(5*b^2*c^2*x^4 - 10*a*b*c*x^2*(4*c + 5*d*x^2) + a^2*(-4*c^2 + 3
6*c*d*x^2 + 45*d^2*x^4)))/(x^(5/2)*(c + d*x^2)) - (5*Sqrt[2]*(b^2*c^2 - 10*
a*b*c*d + 9*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x])])/d^(3/4) - (5*Sqrt[2]*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*ArcTanh[
```

```
(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(3/4))/(40*c^(1
3/4))
```

Maple [A]
time = 0.13, size = 200, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}a^2d^2-\frac{1}{2}abcd+\frac{1}{4}b^2c^2\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{9}{4}a^2d^2-\frac{5}{2}abcd+\frac{1}{4}b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)\right.}{4d\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{c^3}$
default	$\frac{\frac{2\left(\frac{1}{4}a^2d^2-\frac{1}{2}abcd+\frac{1}{4}b^2c^2\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{9}{4}a^2d^2-\frac{5}{2}abcd+\frac{1}{4}b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)\right.}{4d\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{c^3}$
risch	$-\frac{2a\left(-10ad\,x^2+10c\,x^2b+ac\right)}{5c^3x^{\frac{5}{2}}}+\frac{x^{\frac{3}{2}}a^2d^2}{2c^3\left(d\,x^2+c\right)}-\frac{x^{\frac{3}{2}}adb}{c^2\left(d\,x^2+c\right)}+\frac{x^{\frac{3}{2}}b^2}{2c\left(d\,x^2+c\right)}+\frac{9\sqrt{2}\,a^2\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}\right.}{16c^3\left(\frac{c}{d}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^3*((1/4*a^2*d^2-1/2*a*b*c*d+1/4*b^2*c^2)*x^(3/2)/(d*x^2+c)+1/8*(9/4*a^2
*d^2-5/2*a*b*c*d+1/4*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
)-2/5*a^2/c^2/x^(5/2)+4*a*(a*d-b*c)/c^3/x^(1/2)
```

Maxima [A]
time = 0.52, size = 275, normalized size = 0.76

$$\frac{5\left(b^2c^2-10\,abcd+9\,a^2d^2\right)x^4-4\,a^2c^2-4\left(10\,abc^2-9\,a^2cd\right)x^2}{10\left(c^3dx^{\frac{9}{2}}+c^4x^{\frac{5}{2}}\right)}+\frac{\left(b^2c^2-10\,abcd+9\,a^2d^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(5*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*x^4 - 4*a^2*c^2 - 4*(10*a*b*c^2
- 9*a^2*c*d)*x^2)/(c^3*d*x^(9/2) + c^4*x^(5/2)) + 1/16*(b^2*c^2 - 10*a*b*c*
d + 9*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*s
qrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/s
qrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)
```

```
*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)
*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/
4)))/c^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1737 vs. $2(279) = 558$.
time = 2.32, size = 1737, normalized size = 4.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c
^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d
^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^
(1/4)*arctan((sqrt((b^12*c^12 - 60*a*b^11*c^11*d + 1554*a^2*b^10*c^10*d^2 -
 22700*a^3*b^9*c^9*d^3 + 205215*a^4*b^8*c^8*d^4 - 1188600*a^5*b^7*c^7*d^5 +
 4443580*a^6*b^6*c^6*d^6 - 10697400*a^7*b^5*c^5*d^7 + 16622415*a^8*b^4*c^4*
d^8 - 16548300*a^9*b^3*c^3*d^9 + 10195794*a^10*b^2*c^2*d^10 - 3542940*a^11*
b*c*d^11 + 531441*a^12*d^12)*x - (b^8*c^15*d - 40*a*b^7*c^14*d^2 + 636*a^2*
b^6*c^13*d^3 - 5080*a^3*b^5*c^12*d^4 + 21286*a^4*b^4*c^11*d^5 - 45720*a^5*b
^3*c^10*d^6 + 51516*a^6*b^2*c^9*d^7 - 29160*a^7*b*c^8*d^8 + 6561*a^8*c^7*d^
9)*sqrt(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5
*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^
6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3)))*c^3*d*(-(b^8*c^8 - 40*a*
b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*
d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6
561*a^8*d^8)/(c^13*d^3))^(1/4) - (b^6*c^9*d - 30*a*b^5*c^8*d^2 + 327*a^2*b^
4*c^7*d^3 - 1540*a^3*b^3*c^6*d^4 + 2943*a^4*b^2*c^5*d^5 - 2430*a^5*b*c^4*d^
6 + 729*a^6*c^3*d^7)*sqrt(x)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*
d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5
+ 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(1/
4))/(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3
+ 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 2
9160*a^7*b*c*d^7 + 6561*a^8*d^8)) - 5*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40
*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c
^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7
+ 6561*a^8*d^8)/(c^13*d^3))^(1/4)*log(c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d
+ 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 4572
0*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^
8)/(c^13*d^3))^(3/4) + (b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*c^4*d^2 - 15
40*a^3*b^3*c^3*d^3 + 2943*a^4*b^2*c^2*d^4 - 2430*a^5*b*c*d^5 + 729*a^6*d^6)
*sqrt(x)) + 5*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b
^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c
```

```
^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^
3))^(1/4)*log(-c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 -
 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 515
16*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(3/4) +
(b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*c^4*d^2 - 1540*a^3*b^3*c^3*d^3 + 29
43*a^4*b^2*c^2*d^4 - 2430*a^5*b*c*d^5 + 729*a^6*d^6)*sqrt(x)) - 4*(5*(b^2*c
^2 - 10*a*b*c*d + 9*a^2*d^2)*x^4 - 4*a^2*c^2 - 4*(10*a*b*c^2 - 9*a^2*c*d)*x
^2)*sqrt(x))/(c^3*d*x^5 + c^4*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 401, normalized size = 1.10

$$\frac{b^2c^2x^{\frac{3}{2}} - 2\,abcdx^{\frac{3}{2}} + a^2d^2x^{\frac{3}{2}}}{2\,(dx^2+c)c^3} - \frac{2\,(10\,abcx^2 - 10\,a^2dx^2 + a^2c)}{5\,c^3x^{\frac{5}{2}}} + \frac{\sqrt{2}\,\left((cd^3)^{\frac{3}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{3}{4}}\,abcd + 9\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,(\frac{c}{d})^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,(\frac{c}{d})^{\frac{1}{4}}}\right)}{8\,c^4d^3} + \frac{\sqrt{2}\,\left((cd^3)^{\frac{3}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{3}{4}}\,abcd + 9\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,(\frac{c}{d})^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,(\frac{c}{d})^{\frac{1}{4}}}\right)}{8\,c^4d^3} - \frac{\sqrt{2}\,\left((cd^3)^{\frac{3}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{3}{4}}\,abcd + 9\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\,\sqrt{x}\,(\frac{c}{d})^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^4d^3} + \frac{\sqrt{2}\,\left((cd^3)^{\frac{3}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{3}{4}}\,abcd + 9\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\,\sqrt{x}\,(\frac{c}{d})^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*x^2 + c)*c^
3) - 2/5*(10*a*b*c*x^2 - 10*a^2*d*x^2 + a^2*c)/(c^3*x^(5/2)) + 1/8*sqrt(2)*
((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2
)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^
3) + 1/8*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9*(c*d
^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c
/d)^(1/4))/(c^4*d^3) - 1/16*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/
4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
 sqrt(c/d))/(c^4*d^3) + 1/16*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3
/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(c^4*d^3)
```

Mupad [B]
time = 0.12, size = 152, normalized size = 0.42

$$\frac{\frac{x^4\,(9\,a^2\,d^2-10\,a\,b\,c\,d+b^2\,c^2)}{2\,c^3}-\frac{2\,a^2}{5\,c}+\frac{2\,a\,x^2\,(9\,a\,d-10\,b\,c)}{5\,c^2}}{c\,x^{5/2}+d\,x^{9/2}}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{(-c)}^{1/4}}\right)\,(a\,d-b\,c)\,(9\,a\,d-b\,c)}{4\,{(-c)}^{13/4}\,d^{3/4}}+\frac{\mathrm{atanh}\left(\frac{d^{1/4}\,\sqrt{x}}{{(-c)}^{1/4}}\right)\,(a\,d-b\,c)\,(9\,a\,d-b\,c)}{4\,{(-c)}^{13/4}\,d^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x)
```

```
[Out] ((x^4*(9*a^2*d^2 + b^2*c^2 - 10*a*b*c*d))/(2*c^3) - (2*a^2)/(5*c) + (2*a*x^
2*(9*a*d - 10*b*c))/(5*c^2))/(c*x^(5/2) + d*x^(9/2)) - (atan((d^(1/4)*x^(1/
2))/(-c)^(1/4))*(a*d - b*c)*(9*a*d - b*c))/(4*(-c)^(13/4)*d^(3/4)) + (atanh
((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(9*a*d - b*c))/(4*(-c)^(13/4)*d^
(3/4))
```

3.433 $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=440

$$-\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4}+\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3}+\frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2}-\frac{(bc-ad)(17bc-ad)}{16c^2d^2\left(c+dx^2\right)}$$

```
[Out] 1/80*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*x^(5/2)/c^2/d^3+1/4*(-a*d+b*c)^2*x^
(9/2)/c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(-a*d+17*b*c)*x^(9/2)/c^2/d^2/(d*x^
2+c)-1/64*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/
2)/c^(1/4))/c^(3/4)/d^(17/4)*2^(1/2)+1/64*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2
)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(17/4)*2^(1/2)-1/128*
(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(
1/2)*x^(1/2))/c^(3/4)/d^(17/4)*2^(1/2)+1/128*(5*a^2*d^2-90*a*b*c*d+117*b^2*
c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(17/4)
*2^(1/2)-1/16*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*x^(1/2)/c/d^4
```

Rubi [A]
time = 0.26, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {474, 468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{17/4}}+\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{3/4}d^{17/4}}-\frac{\sqrt{x}\left(5a^2d^2-90abcd+117b^2c^2\right)}{16cd^4}+\frac{x^{5/2}\left(5a^2d^2-90abcd+117b^2c^2\right)}{80c^2d^3}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{17/4}}+\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{17/4}}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -1/16*((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Sqrt[x])/(c*d^4) + ((117*b^2*
c^2 - 90*a*b*c*d + 5*a^2*d^2)*x^(5/2))/(80*c^2*d^3) + ((b*c - a*d)^2*x^(9/2
))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(17*b*c - a*d)*x^(9/2))/(16*c^2*d
^2*(c + d*x^2)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(17/4)) + ((117*b^2*c^2
 - 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(3/4)*d^(17/4)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)
*d^(17/4)) + ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(bc-ad)^2x^{9/2}}{4cd^2\,(c+dx^2)^2} - \frac{\int \frac{x^{7/2}\left(\frac{1}{2}\left(-8a^2d^2+9(bc-ad)^2\right)-4b^2cdx^2\right)}{(c+dx^2)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{9/2}}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(17bc-ad)x^{9/2}}{16c^2d^2\,(c+dx^2)} + \frac{(117b^2c^2-90abcd+5a^2d^2)\int \frac{x^{7/2}}{c+dx^2}\,d}{32c^2d^2}$$

$$= \frac{(117b^2c^2-90abcd+5a^2d^2)\,x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(17bc-ad)x^{9/2}}{16c^2d^2\,(c+dx^2)} - \frac{(1}{}$$

$$= -\frac{(117b^2c^2-90abcd+5a^2d^2)\sqrt{x}}{16cd^4} + \frac{(117b^2c^2-90abcd+5a^2d^2)\,x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2}{4cd^2\,(c+d}$$

$$= -\frac{(117b^2c^2-90abcd+5a^2d^2)\sqrt{x}}{16cd^4} + \frac{(117b^2c^2-90abcd+5a^2d^2)\,x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2}{4cd^2\,(c+d}$$

$$= -\frac{(117b^2c^2-90abcd+5a^2d^2)\sqrt{x}}{16cd^4} + \frac{(117b^2c^2-90abcd+5a^2d^2)\,x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2}{4cd^2\,(c+d}$$

$$= -\frac{(117b^2c^2-90abcd+5a^2d^2)\sqrt{x}}{16cd^4} + \frac{(117b^2c^2-90abcd+5a^2d^2)\,x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2}{4cd^2\,(c+d}$$

$$= -\frac{(117b^2c^2-90abcd+5a^2d^2)\sqrt{x}}{16cd^4} + \frac{(117b^2c^2-90abcd+5a^2d^2)\,x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2}{4cd^2\,(c+d}$$

$$= -\frac{(117b^2c^2-90abcd+5a^2d^2)\sqrt{x}}{16cd^4} + \frac{(117b^2c^2-90abcd+5a^2d^2)\,x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2}{4cd^2\,(c+d}$$

Mathematica [A]
time = 0.74, size = 256, normalized size = 0.58

$$\frac{\frac{4\sqrt[4]{d}\sqrt{x}\left(-5a^2d^2(5c+9dx^2)+10abd(45c^2+81cdx^2+32d^2x^4)-b^2(585c^3+1053c^2dx^2+416cd^2x^4-32d^3x^6)\right)}{(c+dx^2)^2} - \frac{5\sqrt{2}\,(117b^2c^2-90abcd+5a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{3/4}} + \frac{5\sqrt{2}\,(117b^2c^2-90abcd+5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{3/4}}}{320d^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((4*d^(1/4)*Sqrt[x]*(-5*a^2*d^2*(5*c + 9*d*x^2) + 10*a*b*d*(45*c^2 + 81*c*d
*x^2 + 32*d^2*x^4) - b^2*(585*c^3 + 1053*c^2*d*x^2 + 416*c*d^2*x^4 - 32*d^3
*x^6)))/(c + d*x^2)^2 - (5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*A
```

```
rcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(3/4) + (
5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(3/4))/(320*d^(17/4))
```

Maple [A]
time = 0.15, size = 234, normalized size = 0.53

method	result
derivativedivides	$\frac{2b\left(\frac{bd\,x^{\frac{5}{2}}}{5}+2ad\sqrt{x}-3bc\sqrt{x}\right)}{d^4}+\frac{2\left(\left(-\frac{9}{32}a^2d^3+\frac{17}{16}abc\,d^2-\frac{25}{32}b^2c^2d\right)x^{\frac{5}{2}}-\frac{c\left(5a^2d^2-26abcd+21b^2c^2\right)\sqrt{x}}{32}\right)}{\left(d\,x^2+c\right)^2}+\frac{\left(5a^2d^2-9\right.}{}$
default	$\frac{2b\left(\frac{bd\,x^{\frac{5}{2}}}{5}+2ad\sqrt{x}-3bc\sqrt{x}\right)}{d^4}+\frac{2\left(\left(-\frac{9}{32}a^2d^3+\frac{17}{16}abc\,d^2-\frac{25}{32}b^2c^2d\right)x^{\frac{5}{2}}-\frac{c\left(5a^2d^2-26abcd+21b^2c^2\right)\sqrt{x}}{32}\right)}{\left(d\,x^2+c\right)^2}+\frac{\left(5a^2d^2-9\right.}{}$
risch	$\frac{2b(bd\,x^2+10ad-15bc)\sqrt{x}}{5d^4}-\frac{9x^{\frac{5}{2}}a^2}{16d(d\,x^2+c)^2}+\frac{17x^{\frac{5}{2}}abc}{8d^2(d\,x^2+c)^2}-\frac{25x^{\frac{5}{2}}b^2c^2}{16d^3(d\,x^2+c)^2}-\frac{5\sqrt{x}\,a^2c}{16d^2(d\,x^2+c)^2}+\frac{13\sqrt{x}\,ab\,c^2}{8d^3(d\,x^2+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b/d^4*(1/5*b*d*x^(5/2)+2*a*d*x^(1/2)-3*b*c*x^(1/2))+2/d^4*(((-9/32*a^2*d^
3+17/16*a*b*c*d^2-25/32*b^2*c^2*d)*x^(5/2)-1/32*c*(5*a^2*d^2-26*a*b*c*d+21*
b^2*c^2)*x^(1/2))/(d*x^2+c)^2+1/256*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*(c/d
)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.52, size = 389, normalized size = 0.88

$$-\frac{(25\,b^2c^2d-34\,abcd^2+9\,a^2d^3)x^{\frac{5}{2}}+(21\,b^2c^3-26\,abc^2d+5\,a^2cd^2)\sqrt{x}}{16\,(d^6x^4+2\,cd^5x^2+c^2d^4)}+\frac{2\left(b^2dx^{\frac{5}{2}}-5\,(3\,b^2c-2\,abd)\sqrt{x}\right)}{5\,d^4}+\frac{\frac{2\sqrt{2}\,(117\,b^2c^2-90\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(117\,b^2c^2-90\,abcd+5\,a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(117\,b^2c^2-90\,abcd+5\,a^2d^2)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(117\,b^2c^2-90\,abcd+5\,a^2d^2)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((25*b^2*c^2*d - 34*a*b*c*d^2 + 9*a^2*d^3)*x^(5/2) + (21*b^2*c^3 - 26
*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(x))/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) + 2/5*
(b^2*d*x^(5/2) - 5*(3*b^2*c - 2*a*b*d)*sqrt(x))/d^4 + 1/128*(2*sqrt(2)*(117
*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(
d))) + 2*sqrt(2)*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*arctan(-1/2*sqrt(2)
```

```
*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)
*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4
)) - sqrt(2)*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^
(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1427 vs. $2(352) = 704$.
time = 2.12, size = 1427, normalized size = 3.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(20*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8*c^8 - 57658068
0*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124
525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6
 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)*arctan((sqrt(c^2*d^8*
sqrt(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^
2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^
3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3
*d^17)) + (13689*b^4*c^4 - 21060*a*b^3*c^3*d + 9270*a^2*b^2*c^2*d^2 - 900*a
^3*b*c*d^3 + 25*a^4*d^4)*x)*c^2*d^13*(-(187388721*b^8*c^8 - 576580680*a*b^7
*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*
a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 4500
0*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(3/4) - (117*b^2*c^4*d^13 - 90*a*b
*c^3*d^14 + 5*a^2*c^2*d^15)*sqrt(x)*(-(187388721*b^8*c^8 - 576580680*a*b^7*
c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a
^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000
*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(3/4))/(187388721*b^8*c^8 - 5765806
80*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 12
4525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^
6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)) + 5*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
*(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 -
 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c
^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^
17))^(1/4)*log(c*d^4*(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 6973176
60*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4
- 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 +
625*a^8*d^8)/(c^3*d^17))^(1/4) + (117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqr
t(x)) - 5*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8*c^8 - 57658068
0*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124
525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6
 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)*log(-c*d^4*(-(1873887
```

```
21*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*
a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 12
73500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)
+ (117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqrt(x)) + 4*(32*b^2*d^3*x^6 - 585
*b^2*c^3 + 450*a*b*c^2*d - 25*a^2*c*d^2 - 32*(13*b^2*c*d^2 - 10*a*b*d^3)*x^
4 - 9*(117*b^2*c^2*d - 90*a*b*c*d^2 + 5*a^2*d^3)*x^2)*sqrt(x))/(d^6*x^4 + 2
*c*d^5*x^2 + c^2*d^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 451, normalized size = 1.02

$$\frac{\sqrt{2}\left(117\,(cd^3)^{\frac14} b^2c^2 - 90\,(cd^3)^{\frac14} abcd + 5\,(cd^3)^{\frac14} a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{64\,cd^5} + \frac{\sqrt{2}\left(117\,(cd^3)^{\frac14} b^2c^2 - 90\,(cd^3)^{\frac14} abcd + 5\,(cd^3)^{\frac14} a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{64\,cd^5} + \frac{\sqrt{2}\left(117\,(cd^3)^{\frac14} b^2c^2 - 90\,(cd^3)^{\frac14} abcd + 5\,(cd^3)^{\frac14} a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14} + x + \sqrt{\frac{c}{d}}\right)}{128\,cd^5} - \frac{\sqrt{2}\left(117\,(cd^3)^{\frac14} b^2c^2 - 90\,(cd^3)^{\frac14} abcd + 5\,(cd^3)^{\frac14} a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14} + x + \sqrt{\frac{c}{d}}\right)}{128\,cd^5} - \frac{25\,b^2c^2dx^{\frac52} - 34\,abcd^2x^{\frac52} + 9\,a^2d^3x^{\frac52} + 21\,b^2c^3\sqrt{x} - 26\,abc^2d\sqrt{x} + 5\,a^2cd^2\sqrt{x}}{16\,(dx^2+c)^2d^4} + \frac{2\left(b^2d^{12}x^{\frac52} - 15\,b^2cd^{11}\sqrt{x} + 10\,abd^{12}\sqrt{x}\right)}{5\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d
^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(c*d^5) + 1/64*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*d^3)^(1
/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^5) + 1/128*sqrt(2)*(117*(c*d^3)^(1/4)*b
^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/128*sqrt(2)*(117*(c*d^3)^(1/
4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(
2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/16*(25*b^2*c^2*d*x^(5/2
) - 34*a*b*c*d^2*x^(5/2) + 9*a^2*d^3*x^(5/2) + 21*b^2*c^3*sqrt(x) - 26*a*b*
c^2*d*sqrt(x) + 5*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*d^4) + 2/5*(b^2*d^12*x^
(5/2) - 15*b^2*c*d^11*sqrt(x) + 10*a*b*d^12*sqrt(x))/d^15
```

Mupad [B]
time = 0.25, size = 1426, normalized size = 3.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (2*b^2*x^(5/2))/(5*d^3) - (x^(1/2)*((21*b^2*c^3)/16 + (5*a^2*c*d^2)/16 - (1
3*a*b*c^2*d)/8) + x^(5/2)*((9*a^2*d^3)/16 + (25*b^2*c^2*d)/16 - (17*a*b*c*d
^2)/8))/(c^2*d^4 + d^6*x^4 + 2*c*d^5*x^2) - x^(1/2)*((6*b^2*c)/d^4 - (4*a*b
)/d^3) + (atan(((((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d
^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) - ((5*a^2*d^2 + 117*b^2
*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/
4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*1i)/(64*(-c)^(3/4)*d^(
17/4)) + (((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21
060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) + ((5*a^2*d^2 + 117*b^2*c^2 -
90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/4)*d^(2
1/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*1i)/(64*(-c)^(3/4)*d^(17/4)))
/((((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b
^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) - ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*
c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/4)*d^(21/4)))*
(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17/4)) - (((x^(1/
2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d -
 900*a^3*b*c*d^3))/(64*d^5) + ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*
b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2
 + 117*b^2*c^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17/4))))*(5*a^2*d^2 + 117*b
^2*c^2 - 90*a*b*c*d)*1i)/(32*(-c)^(3/4)*d^(17/4)) + (atan(((((x^(1/2)*(25*a
^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d - 900*a^3
*b*c*d^3))/(64*d^5) - ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3
+ 5*a^2*c*d^2 - 90*a*b*c^2*d)*1i)/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2 + 11
7*b^2*c^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17/4)) + (((x^(1/2)*(25*a^4*d^4
+ 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^
3))/(64*d^5) + ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2
*c*d^2 - 90*a*b*c^2*d)*1i)/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c
^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17/4)))/((((x^(1/2)*(25*a^4*d^4 + 13689
*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64
*d^5) - ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2
- 90*a*b*c^2*d)*1i)/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90
*a*b*c*d)*1i)/(64*(-c)^(3/4)*d^(17/4)) - (((x^(1/2)*(25*a^4*d^4 + 13689*b^4
*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5
) + ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90
*a*b*c^2*d)*1i)/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b
*c*d)*1i)/(64*(-c)^(3/4)*d^(17/4))))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)
)/(32*(-c)^(3/4)*d^(17/4))
```

3.434 $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=401

$$-\frac{\left(42ab-\frac{77b^2c}{d}+\frac{3a^2d}{c}\right)x^{3/2}}{48cd^2}+\frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2}-\frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)}+\frac{\left(77b^2c^2-42abcd-3a^2d^2\right)\tan^{-1}\left(\right.}{32\sqrt{2}\,c^{5/4}d^{15/4}}$$

```
[Out] -1/48*(42*a*b-77*b^2*c/d+3*a^2*d/c)*x^(3/2)/c/d^2+1/4*(-a*d+b*c)^2*x^(7/2)/
c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(a*d+15*b*c)*x^(7/2)/c^2/d^2/(d*x^2+c)+1/
64*(-3*a^2*d^2-42*a*b*c*d+77*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1
/4))/c^(5/4)/d^(15/4)*2^(1/2)-1/64*(-3*a^2*d^2-42*a*b*c*d+77*b^2*c^2)*arcta
n(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/d^(15/4)*2^(1/2)-1/128*(-3*a^2
*d^2-42*a*b*c*d+77*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^
(1/2))/c^(5/4)/d^(15/4)*2^(1/2)+1/128*(-3*a^2*d^2-42*a*b*c*d+77*b^2*c^2)*ln
(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(15/4)*2^(1/2
)
```

Rubi [A]

time = 0.23, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {474, 468, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{5/4}d^{15/4}}-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{5/4}d^{15/4}}-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}\,c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}\,c^{5/4}d^{15/4}}-\frac{x^{3/2}\left(\frac{3a^2d}{c}+42ab-\frac{77b^2c}{d}\right)}{48cd^2}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -1/48*((42*a*b - (77*b^2*c)/d + (3*a^2*d)/c)*x^(3/2))/(c*d^2) + ((b*c - a*d
)^2*x^(7/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(15*b*c + a*d)*x^(7/2))
/(16*c^2*d^2*(c + d*x^2)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77
*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^
2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*
c^(5/4)*d^(15/4)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(bc-ad)^2x^{7/2}}{4cd^2(c+dx^2)^2} - \frac{\int \frac{x^{5/2}\left(\frac{1}{2}(-8a^2d^2+7(bc-ad)^2)-4b^2cdx^2\right)}{(c+dx^2)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{7/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2(c+dx^2)} + \frac{(77b^2c^2-42abcd-3a^2d^2)\int \frac{x^{5/2}}{c+dx^2}\,dx}{32c^2d^2}$$

$$= \frac{(77b^2c^2-42abcd-3a^2d^2)\,x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2(c+dx^2)} - \frac{(77}{}$$

$$= \frac{(77b^2c^2-42abcd-3a^2d^2)\,x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2(c+dx^2)} - \frac{(77}{}$$

$$= \frac{(77b^2c^2-42abcd-3a^2d^2)\,x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2(c+dx^2)} + \frac{(77}{}$$

$$(77$$

$$= \frac{(77b^2c^2-42abcd-3a^2d^2)\,x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2(c+dx^2)} - \frac{}{}$$

$$= \frac{(77b^2c^2-42abcd-3a^2d^2)\,x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2(c+dx^2)} - \frac{(77}{}$$

$$= \frac{(77b^2c^2-42abcd-3a^2d^2)\,x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2(c+dx^2)} + \frac{(77}{}$$

Mathematica [A]
time = 0.77, size = 235, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{c}\,d^{3/4}x^{3/2}\left(3a^2d^2(-c+3dx^2)-6abcd(7c+11dx^2)+b^2c(77c^2+121cdx^2+32d^2x^4)\right)}{(c+dx^2)^2}+3\sqrt{2}\,(77b^2c^2-42abcd-3a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)+3\sqrt{2}\,(77b^2c^2-42abcd-3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{192c^{5/4}d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((4*c^(1/4)*d^(3/4)*x^(3/2)*(3*a^2*d^2*(-c + 3*d*x^2) - 6*a*b*c*d*(7*c + 11
*d*x^2) + b^2*c*(77*c^2 + 121*c*d*x^2 + 32*d^2*x^4)))/(c + d*x^2)^2 + 3*Sqr
t[2]*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3*a^
2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(1
92*c^(5/4)*d^(15/4))
```

Maple [A]

time = 0.15, size = 220, normalized size = 0.55

method	result
derivativedivides	$\frac{2b^2x^{\frac{3}{2}}}{3d^3}+\frac{\frac{2\left(\frac{d\left(3a^2d^2-22abcd+19b^2c^2\right)x^{\frac{7}{2}}}{32c}+\left(-\frac{1}{32}a^2d^2-\frac{7}{16}abcd+\frac{15}{32}b^2c^2\right)x^{\frac{3}{2}}\right)}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2+42abcd-77b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}}{d^3}$
default	$\frac{2b^2x^{\frac{3}{2}}}{3d^3}+\frac{\frac{2\left(\frac{d\left(3a^2d^2-22abcd+19b^2c^2\right)x^{\frac{7}{2}}}{32c}+\left(-\frac{1}{32}a^2d^2-\frac{7}{16}abcd+\frac{15}{32}b^2c^2\right)x^{\frac{3}{2}}\right)}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2+42abcd-77b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}}{d^3}$
risch	$\frac{2b^2x^{\frac{3}{2}}}{3d^3}+\frac{3x^{\frac{7}{2}}a^2}{16(dx^2+c)^2c}-\frac{11x^{\frac{7}{2}}ab}{8d(dx^2+c)^2}+\frac{19cx^{\frac{7}{2}}b^2}{16d^2(dx^2+c)^2}-\frac{x^{\frac{3}{2}}a^2}{16d(dx^2+c)^2}-\frac{7x^{\frac{3}{2}}abc}{8d^2(dx^2+c)^2}+\frac{15x^{\frac{3}{2}}b^2c^2}{16d^3(dx^2+c)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*b^2*x^(3/2)/d^3+2/d^3*((1/32*d*(3*a^2*d^2-22*a*b*c*d+19*b^2*c^2)/c*x^(7
/2)+(-1/32*a^2*d^2-7/16*a*b*c*d+15/32*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+1/256*(
3*a^2*d^2+42*a*b*c*d-77*b^2*c^2)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)
*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+
2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)))
```

Maxima [A]

time = 0.50, size = 306, normalized size = 0.76

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d^3}+\frac{(19\,b^2c^2d-22\,abcd^2+3\,a^2d^3)x^{\frac{7}{2}}+(15\,b^2c^3-14\,abc^2d-a^2cd^2)x^{\frac{3}{2}}}{16\,(cd^5x^4+2\,c^2d^4x^2+c^3d^3)}-\frac{(77\,b^2c^2-42\,abcd-3\,a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x^(3/2)/d^3 + 1/16*((19*b^2*c^2*d - 22*a*b*c*d^2 + 3*a^2*d^3)*x^(7/
2) + (15*b^2*c^3 - 14*a*b*c^2*d - a^2*c*d^2)*x^(3/2))/(c*d^5*x^4 + 2*c^2*d^
4*x^2 + c^3*d^3) - 1/128*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*(2*sqrt(2)*a
rctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c
)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) +
sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/
4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1813 vs. $2(317) = 634$.
time = 1.20, size = 1813, normalized size = 4.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(12*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c^8 - 7669
7544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 14
57946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 45
36*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*arctan((sqrt((208422380089*b
^12*c^12 - 682109607564*a*b^11*c^11*d + 881427350034*a^2*b^10*c^10*d^2 - 54
3593843100*a^3*b^9*c^9*d^3 + 136525986135*a^4*b^8*c^8*d^4 + 8334677736*a^5*
b^7*c^7*d^5 - 7849956996*a^6*b^6*c^6*d^6 - 324727704*a^7*b^5*c^5*d^7 + 2072
41335*a^8*b^4*c^4*d^8 + 32148900*a^9*b^3*c^3*d^9 + 2030994*a^10*b^2*c^2*d^1
0 + 61236*a^11*b*c*d^11 + 729*a^12*d^12)*x - (35153041*b^8*c^11*d^7 - 76697
544*a*b^7*c^10*d^8 + 57274140*a^2*b^6*c^9*d^9 - 13854456*a^3*b^5*c^8*d^10 -
 1457946*a^4*b^4*c^7*d^11 + 539784*a^5*b^3*c^6*d^12 + 86940*a^6*b^2*c^5*d^1
3 + 4536*a^7*b*c^4*d^14 + 81*a^8*c^3*d^15)*sqrt(-(35153041*b^8*c^8 - 766975
44*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457
946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536
*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15)))*c*d^4*(-(35153041*b^8*c^8 - 7669754
4*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 14579
46*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536*
a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4) + (456533*b^6*c^7*d^4 - 747054*
a*b^5*c^6*d^5 + 354123*a^2*b^4*c^5*d^6 - 15876*a^3*b^3*c^4*d^7 - 13797*a^4*
b^2*c^3*d^8 - 1134*a^5*b*c^2*d^9 - 27*a^6*c*d^10)*sqrt(x)*(-(35153041*b^8*c
^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5
*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2
*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4))/(35153041*b^8*c^8
- 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^
3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^
6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)) - 3*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^
3)*(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 -
13854456*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5
 + 86940*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)
*log(c^4*d^11*(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6
*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*
b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d
^15))^(3/4) - (456533*b^6*c^6 - 747054*a*b^5*c^5*d + 354123*a^2*b^4*c^4*d^2
 - 15876*a^3*b^3*c^3*d^3 - 13797*a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^
6*d^6)*sqrt(x)) + 3*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c
^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5
```

```
*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2
*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*log(-c^4*d^11*(-(35
153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456
*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940
*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(3/4) - (4565
33*b^6*c^6 - 747054*a*b^5*c^5*d + 354123*a^2*b^4*c^4*d^2 - 15876*a^3*b^3*c^
3*d^3 - 13797*a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^6*d^6)*sqrt(x)) - 4
*(32*b^2*c*d^2*x^5 + (121*b^2*c^2*d - 66*a*b*c*d^2 + 9*a^2*d^3)*x^3 + (77*b
^2*c^3 - 42*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2
 + c^3*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.62, size = 427, normalized size = 1.06

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d^3}+\frac{19\,b^2c^2dx^{\frac{7}{2}}-22\,abcd^2x^{\frac{7}{2}}+3\,a^2d^3x^{\frac{7}{2}}+15\,b^2c^3x^{\frac{3}{2}}-14\,abc^2dx^{\frac{3}{2}}-a^2cd^2x^{\frac{3}{2}}}{16\,(dx^2+c)^2cd^3}-\frac{\sqrt{2}\left(77\,(cd^3)^{\frac{3}{4}}b^2c^2-42\,(cd^3)^{\frac{3}{4}}abcd-3\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^6}-\frac{\sqrt{2}\left(77\,(cd^3)^{\frac{3}{4}}b^2c^2-42\,(cd^3)^{\frac{3}{4}}abcd-3\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^6}+\frac{\sqrt{2}\left(77\,(cd^3)^{\frac{3}{4}}b^2c^2-42\,(cd^3)^{\frac{3}{4}}abcd-3\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^2d^6}-\frac{\sqrt{2}\left(77\,(cd^3)^{\frac{3}{4}}b^2c^2-42\,(cd^3)^{\frac{3}{4}}abcd-3\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d^3 + 1/16*(19*b^2*c^2*d*x^(7/2) - 22*a*b*c*d^2*x^(7/2) + 3
*a^2*d^3*x^(7/2) + 15*b^2*c^3*x^(3/2) - 14*a*b*c^2*d*x^(3/2) - a^2*c*d^2*x^
(3/2))/((d*x^2 + c)^2*c*d^3) - 1/64*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*
(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^6) - 1/64*sqrt(2)*(77*(c*d^3
)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^6) + 1
/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^
3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^6
) - 1/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*
(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c
^2*d^6)
```

Mupad [B]
time = 0.22, size = 197, normalized size = 0.49

$$\frac{2\,b^2\,x^{3/2}}{3\,d^3}-\frac{x^{3/2}\,\left(\frac{a^2\,d^2}{16}+\frac{7\,a\,b\,c\,d}{8}-\frac{15\,b^2\,c^2}{16}\right)-\frac{x^{7/2}\,\left(3\,a^2\,d^3-22\,a\,b\,c\,d^2+19\,b^2\,c^2\,d\right)}{16\,c}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(3\,a^2\,d^2+42\,a\,b\,c\,d-77\,b^2\,c^2\right)}{32\,{\left(-c\right)}^{5/4}\,d^{15/4}}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-c\right)}^{1/4}}\right)\,\left(3\,a^2\,d^2+42\,a\,b\,c\,d-77\,b^2\,c^2\right)\,1{}\mathrm{i}}{32\,{\left(-c\right)}^{5/4}\,d^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (2*b^2*x^(3/2))/(3*d^3) - (x^(3/2)*((a^2*d^2)/16 - (15*b^2*c^2)/16 + (7*a*b
*c*d)/8) - (x^(7/2)*(3*a^2*d^3 + 19*b^2*c^2*d - 22*a*b*c*d^2))/(16*c))/(c^2
*d^3 + d^5*x^4 + 2*c*d^4*x^2) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(3*a^2*
d^2 - 77*b^2*c^2 + 42*a*b*c*d))/(32*(-c)^(5/4)*d^(15/4)) - (atan((d^(1/4)*x
^(1/2)*1i)/(-c)^(1/4))*(3*a^2*d^2 - 77*b^2*c^2 + 42*a*b*c*d)*1i)/(32*(-c)^(
5/4)*d^(15/4))
```

3.435 $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=402

$$-\frac{\left(10ab-\frac{45b^2c}{d}+\frac{3a^2d}{c}\right)\sqrt{x}}{16cd^2}+\frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2}-\frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)}+\frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\tan^{-1}}{32\sqrt{2}\,c^{7/4}d^{13}}$$

```
[Out] 1/4*(-a*d+b*c)^2*x^(5/2)/c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(3*a*d+13*b*c)*x
^(5/2)/c^2/d^2/(d*x^2+c)+1/64*(-3*a^2*d^2-10*a*b*c*d+45*b^2*c^2)*arctan(1-d
^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/d^(13/4)*2^(1/2)-1/64*(-3*a^2*d^2-1
0*a*b*c*d+45*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/d^(
13/4)*2^(1/2)+1/128*(-3*a^2*d^2-10*a*b*c*d+45*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)
-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/d^(13/4)*2^(1/2)-1/128*(-3*a^2*d^
2-10*a*b*c*d+45*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/
2))/c^(7/4)/d^(13/4)*2^(1/2)-1/16*(10*a*b-45*b^2*c/d+3*a^2*d/c)*x^(1/2)/c/d
^2
```

Rubi [A]
time = 0.22, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {474, 468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{7/4}d^{13/4}}-\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{7/4}d^{13/4}}+\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{7/4}d^{13/4}}-\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{7/4}d^{13/4}}-\frac{\sqrt{x}\left(\frac{3a^2d}{c}+10ab-\frac{45b^2c}{d}\right)}{16cd^2}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -1/16*((10*a*b - (45*b^2*c)/d + (3*a^2*d)/c)*Sqrt[x])/(c*d^2) + ((b*c - a*d
)^2*x^(5/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(13*b*c + 3*a*d)*x^(5/2
))/(16*c^2*d^2*(c + d*x^2)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan
[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) - ((
45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*
d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2
]*c^(7/4)*d^(13/4)) - ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] +
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(bc-ad)^2x^{5/2}}{4cd^2(c+dx^2)^2} - \frac{\int \frac{x^{3/2}\left(\frac{1}{2}\left(-8a^2d^2+5(bc-ad)^2\right)-4b^2cdx^2\right)}{(c+dx^2)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{5/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2(c+dx^2)} + \frac{(45b^2c^2-10abcd-3a^2d^2)\int \frac{x^{3/2}}{c+dx^2}\,d}{32c^2d^2}$$

$$= \frac{(45b^2c^2-10abcd-3a^2d^2)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2(c+dx^2)} - \cdots$$

$$= \frac{(45b^2c^2-10abcd-3a^2d^2)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2(c+dx^2)} - \cdots$$

$$= \frac{(45b^2c^2-10abcd-3a^2d^2)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2(c+dx^2)} - \cdots$$

$$= \frac{(45b^2c^2-10abcd-3a^2d^2)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2(c+dx^2)} - \cdots$$

$$= \frac{(45b^2c^2-10abcd-3a^2d^2)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2(c+dx^2)} + \cdots$$

$$= \frac{(45b^2c^2-10abcd-3a^2d^2)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2(c+dx^2)} + \cdots$$

Mathematica [A]
time = 0.82, size = 232, normalized size = 0.58

$$\frac{\frac{4c^{3/4}\sqrt[4]{d}\sqrt{x}\left(a^2d^2(-3c+dx^2)-2abcd(5c+9dx^2)+b^2c(45c^2+81cdx^2+32d^2x^4)\right)}{(c+dx^2)^2}+\sqrt{2}\left(45b^2c^2-10abcd-3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)-\sqrt{2}\left(45b^2c^2-10abcd-3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{64c^{7/4}d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((4*c^(3/4)*d^(1/4)*Sqrt[x]*(a^2*d^2*(-3*c + d*x^2) - 2*a*b*c*d*(5*c + 9*d*
x^2) + b^2*c*(45*c^2 + 81*c*d*x^2 + 32*d^2*x^4)))/(c + d*x^2)^2 + Sqrt[2]*(
45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x])] - Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*A
rcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(64*c^(7/4
)*d^(13/4))
```

Maple [A]

time = 0.13, size = 216, normalized size = 0.54

method	result
derivativedivides	$\frac{2b^2\sqrt{x}}{d^3}+\frac{\frac{2\left(\frac{d\left(a^2d^2-18abcd+17b^2c^2\right)x^{\frac{5}{2}}}{32c}+\left(-\frac{3}{32}a^2d^2-\frac{5}{16}abcd+\frac{13}{32}b^2c^2\right)\sqrt{x}\right)}{\left(d\,x^2+c\right)^2}+\frac{\left(3a^2d^2+10abcd-45b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\right.}{}}{d^3}$
default	$\frac{2b^2\sqrt{x}}{d^3}+\frac{\frac{2\left(\frac{d\left(a^2d^2-18abcd+17b^2c^2\right)x^{\frac{5}{2}}}{32c}+\left(-\frac{3}{32}a^2d^2-\frac{5}{16}abcd+\frac{13}{32}b^2c^2\right)\sqrt{x}\right)}{\left(d\,x^2+c\right)^2}+\frac{\left(3a^2d^2+10abcd-45b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\right.}{}}{d^3}$
risch	$\frac{2b^2\sqrt{x}}{d^3}+\frac{x^{\frac{5}{2}}a^2}{16(d\,x^2+c)^2c}-\frac{9x^{\frac{5}{2}}ab}{8d(d\,x^2+c)^2}+\frac{17c\,x^{\frac{5}{2}}b^2}{16d^2(d\,x^2+c)^2}-\frac{3\sqrt{x}\,a^2}{16d(d\,x^2+c)^2}-\frac{5\sqrt{x}\,abc}{8d^2(d\,x^2+c)^2}+\frac{13\sqrt{x}\,b^2c^2}{16d^3(d\,x^2+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/d^3*x^(1/2)+2/d^3*((1/32*d*(a^2*d^2-18*a*b*c*d+17*b^2*c^2)/c*x^(5/2)+
(-3/32*a^2*d^2-5/16*a*b*c*d+13/32*b^2*c^2)*x^(1/2))/(d*x^2+c)^2+1/256*(3*a^
2*d^2+10*a*b*c*d-45*b^2*c^2)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
))
```

Maxima [A]

time = 0.51, size = 374, normalized size = 0.93

$$\frac{(17b^2c^2d-18abcd^2+a^2d^3)x^{\frac{5}{2}}+(13b^2c^3-10abc^2d-3a^2cd^2)\sqrt{x}}{16(cd^5x^4+2c^2d^4x^2+c^3d^3)}+\frac{2b^2\sqrt{x}}{d^3}-\frac{\frac{2\sqrt{2}(45b^2c^2-10abcd-3a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(45b^2c^2-10abcd-3a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(45b^2c^2-10abcd-3a^2d^2)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(45b^2c^2-10abcd-3a^2d^2)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((17*b^2*c^2*d - 18*a*b*c*d^2 + a^2*d^3)*x^(5/2) + (13*b^2*c^3 - 10*a*
b*c^2*d - 3*a^2*c*d^2)*sqrt(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3) + 2*b
^2*sqrt(x)/d^3 - 1/128*(2*sqrt(2)*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(45*b^2*c^2 - 10*a*b*
c*d - 3*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*s
qrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(4
5*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + s
qrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(45*b^2*c^2 - 10*a*b*c*d -
```

```
3*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(
3/4)*d^(1/4)))/(c*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. $2(318) = 636$.
time = 1.18, size = 1420, normalized size = 3.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*
a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b
^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7
 + 81*a^8*d^8)/(c^7*d^13))^(1/4)*arctan((sqrt(c^4*d^6*sqrt(-(4100625*b^8*c^
8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 -
 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080
*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13)) + (2025*b^4*c^4 - 900*a*b^3*c^3*d -
170*a^2*b^2*c^2*d^2 + 60*a^3*b*c*d^3 + 9*a^4*d^4)*x)*c^5*d^10*(-(4100625*b^
8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d
^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 +
1080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13))^(3/4) + (45*b^2*c^7*d^10 - 10*a*
b*c^6*d^11 - 3*a^2*c^5*d^12)*sqrt(x)*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7
*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^
4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8
*d^8)/(c^7*d^13))^(3/4))/(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^
2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*
b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)) + (c*d^
5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 -
36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8
)/(c^7*d^13))^(1/4)*log(c^2*d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 3
6600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)
/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*sqrt(x)) - (c*d^
5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 -
36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8
)/(c^7*d^13))^(1/4)*log(-c^2*d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 -
36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8
)/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*sqrt(x)) + 4*(3
2*b^2*c*d^2*x^4 + 45*b^2*c^3 - 10*a*b*c^2*d - 3*a^2*c*d^2 + (81*b^2*c^2*d -
 18*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2302 vs. $2(403) = 806$.

time = 160.53, size = 2302, normalized size = 5.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2*sqrt(x))
, Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**
2*sqrt(x))/d**3, Eq(c, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2
*x**(13/2)/13)/c**3, Eq(d, 0)), (-12*a**2*c**2*d**2*sqrt(x)/(64*c**4*d**3 +
 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 3*a**2*c**2*d**2*(-c/d)**(1/4)*l
og(sqrt(x) - (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d*
*5*x**4) + 3*a**2*c**2*d**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*
c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 6*a**2*c**2*d**2*(-c/
d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 +
64*c**2*d**5*x**4) + 4*a**2*c*d**3*x**(5/2)/(64*c**4*d**3 + 128*c**3*d**4*x
**2 + 64*c**2*d**5*x**4) - 6*a**2*c*d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) - (
-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 6*a
**2*c*d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**4*d**3 +
128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 12*a**2*c*d**3*x**2*(-c/d)**(1/4)
*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d
**5*x**4) - 3*a**2*d**4*x**4*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64
*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 3*a**2*d**4*x**4*(-c
/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2
+ 64*c**2*d**5*x**4) + 6*a**2*d**4*x**4*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**
(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 40*a*b*c**
3*d*sqrt(x)/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 10*a*
b*c**3*d*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**
3*d**4*x**2 + 64*c**2*d**5*x**4) + 10*a*b*c**3*d*(-c/d)**(1/4)*log(sqrt(x)
+ (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) +
20*a*b*c**3*d*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**4*d**3 + 128
*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 72*a*b*c**2*d**2*x**(5/2)/(64*c**4*d
**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 20*a*b*c**2*d**2*x**2*(-c/d
)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 +
64*c**2*d**5*x**4) + 20*a*b*c**2*d**2*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/
d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 40*a*b
*c**2*d**2*x**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**4*d**3 + 1
28*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 10*a*b*c*d**3*x**4*(-c/d)**(1/4)*l
og(sqrt(x) - (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d*
*5*x**4) + 10*a*b*c*d**3*x**4*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(6
4*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 20*a*b*c*d**3*x**4*
(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**
```

```
2 + 64*c**2*d**5*x**4) + 180*b**2*c**4*sqrt(x)/(64*c**4*d**3 + 128*c**3*d**
4*x**2 + 64*c**2*d**5*x**4) + 45*b**2*c**4*(-c/d)**(1/4)*log(sqrt(x) - (-c/
d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 45*b**
2*c**4*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*
d**4*x**2 + 64*c**2*d**5*x**4) - 90*b**2*c**4*(-c/d)**(1/4)*atan(sqrt(x)/(-
c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 324*
b**2*c**3*d*x**(5/2)/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4
) + 90*b**2*c**3*d*x**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**4
*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 90*b**2*c**3*d*x**2*(-c/d
)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 +
64*c**2*d**5*x**4) - 180*b**2*c**3*d*x**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)
**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 128*b**2
*c**2*d**2*x**(9/2)/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4)
 + 45*b**2*c**2*d**2*x**4*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c*
*4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 45*b**2*c**2*d**2*x**4*
(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x*
*2 + 64*c**2*d**5*x**4) - 90*b**2*c**2*d**2*x**4*(-c/d)**(1/4)*atan(sqrt(x)
/(-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4), Tr
ue))
```

Giac [**A**]
time = 0.56, size = 426, normalized size = 1.06

$$\frac{2b^2\sqrt{x}}{d^3}-\frac{\sqrt{2}\left(45(cd^3)^{\frac14}b^2c^2-10(cd^3)^{\frac14}abcd-3(cd^3)^{\frac14}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{64c^2d^4}-\frac{\sqrt{2}\left(45(cd^3)^{\frac14}b^2c^2-10(cd^3)^{\frac14}abcd-3(cd^3)^{\frac14}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{64c^2d^4}-\frac{\sqrt{2}\left(45(cd^3)^{\frac14}b^2c^2-10(cd^3)^{\frac14}abcd-3(cd^3)^{\frac14}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{128c^2d^4}+\frac{\sqrt{2}\left(45(cd^3)^{\frac14}b^2c^2-10(cd^3)^{\frac14}abcd-3(cd^3)^{\frac14}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{128c^2d^4}+\frac{17b^2c^2dx^{\frac52}-18abcd^2x^{\frac52}+a^2d^3x^{\frac52}+13b^2c^3\sqrt{x}-10abc^2d\sqrt{x}-3a^2cd^2\sqrt{x}}{16(dx^2+c)^2cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d^3 - 1/64*sqrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/
4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/64*sqrt(2)*(45*(c*d^3)^(1/4)*b^2
*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/128*sqrt(2
)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^
2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) + 1/128*s
qrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/
4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) + 1
/16*(17*b^2*c^2*d*x^(5/2) - 18*a*b*c*d^2*x^(5/2) + a^2*d^3*x^(5/2) + 13*b^2
*c^3*sqrt(x) - 10*a*b*c^2*d*sqrt(x) - 3*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c
*d^3)
```

Mupad [**B**]
time = 0.25, size = 1236, normalized size = 3.07

$$\frac{2b^2\sqrt{x}}{d^3}-\frac{\sqrt{x}\left(\frac{3a^2d^2}{16}+\frac{5abcd}{8}-\frac{13b^2c^2}{16}\right)-\frac{x^{5/2}\left(a^2d^3-18abcd^2+17b^2c^2d\right)}{16c}}{c^2d^3+2cd^4x^2+d^5x^4}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{(3a^2d^2+10abcd-45b^2c^2)^2}{64(-c)^{7/4}d^{13/4}}-\frac{\sqrt{x}\,(9a^4d^4+60a^3bcd^3-170a^2b^2c^2d^2-900ab^3c^3d+2025b^4c^4)}{64c^2d^3}\right)(3a^2d^2+10abcd-45b^2c^2)\,1\mathrm{i}}{64(-c)^{7/4}d^{13/4}}-\frac{\left(\frac{(3a^2d^2+10abcd-45b^2c^2)^2}{64(-c)^{7/4}d^{13/4}}+\frac{\sqrt{x}\,(9a^4d^4+60a^3bcd^3-170a^2b^2c^2d^2-900ab^3c^3d+2025b^4c^4)}{64c^2d^3}\right)(3a^2d^2+10abcd-45b^2c^2)\,1\mathrm{i}}{64(-c)^{7/4}d^{13/4}}}{\frac{\left(\frac{(3a^2d^2+10abcd-45b^2c^2)^2}{64(-c)^{7/4}d^{13/4}}-\frac{\sqrt{x}\,(9a^4d^4+60a^3bcd^3-170a^2b^2c^2d^2-900ab^3c^3d+2025b^4c^4)}{64c^2d^3}\right)(3a^2d^2+10abcd-45b^2c^2)}{64(-c)^{7/4}d^{13/4}}+\frac{\left(\frac{(3a^2d^2+10abcd-45b^2c^2)^2}{64(-c)^{7/4}d^{13/4}}+\frac{\sqrt{x}\,(9a^4d^4+60a^3bcd^3-170a^2b^2c^2d^2-900ab^3c^3d+2025b^4c^4)}{64c^2d^3}\right)(3a^2d^2+10abcd-45b^2c^2)}{64(-c)^{7/4}d^{13/4}}}\right)(3a^2d^2+10abcd-45b^2c^2)\,1\mathrm{i}}{32(-c)^{7/4}d^{13/4}}+\frac{\mathrm{atan}\left(\frac{\frac{\left(-\frac{\sqrt{x}\,(9a^4d^4+60a^3bcd^3-170a^2b^2c^2d^2-900ab^3c^3d+2025b^4c^4)}{64c^2d^3}+\frac{(3a^2d^2+10abcd-45b^2c^2)^2\,1\mathrm{i}}{64(-c)^{7/4}d^{13/4}}\right)(3a^2d^2+10abcd-45b^2c^2)}{64(-c)^{7/4}d^{13/4}}-\frac{\left(\frac{\sqrt{x}\,(9a^4d^4+60a^3bcd^3-170a^2b^2c^2d^2-900ab^3c^3d+2025b^4c^4)}{64c^2d^3}+\frac{(3a^2d^2+10abcd-45b^2c^2)^2\,1\mathrm{i}}{64(-c)^{7/4}d^{13/4}}\right)(3a^2d^2+10abcd-45b^2c^2)}{64(-c)^{7/4}d^{13/4}}}{\frac{\left(-\frac{\sqrt{x}\,(9a^4d^4+60a^3bcd^3-170a^2b^2c^2d^2-900ab^3c^3d+2025b^4c^4)}{64c^2d^3}+\frac{(3a^2d^2+10abcd-45b^2c^2)^2\,1\mathrm{i}}{64(-c)^{7/4}d^{13/4}}\right)(3a^2d^2+10abcd-45b^2c^2)\,1\mathrm{i}}{64(-c)^{7/4}d^{13/4}}+\frac{\left(\frac{\sqrt{x}\,(9a^4d^4+60a^3bcd^3-170a^2b^2c^2d^2-900ab^3c^3d+2025b^4c^4)}{64c^2d^3}+\frac{(3a^2d^2+10abcd-45b^2c^2)^2\,1\mathrm{i}}{64(-c)^{7/4}d^{13/4}}\right)(3a^2d^2+10abcd-45b^2c^2)\,1\mathrm{i}}{64(-c)^{7/4}d^{13/4}}}\right)(3a^2d^2+10abcd-45b^2c^2)}{32(-c)^{7/4}d^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (2*b^2*x^(1/2))/d^3 - (x^(1/2)*((3*a^2*d^2)/16 - (13*b^2*c^2)/16 + (5*a*b*c
*d)/8) - (x^(5/2)*(a^2*d^3 + 17*b^2*c^2*d - 18*a*b*c*d^2))/(16*c))/(c^2*d^3
 + d^5*x^4 + 2*c*d^4*x^2) + (atan(((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^
2/(64*(-c)^(7/4)*d^(13/4)) - (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b
^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 -
45*b^2*c^2 + 10*a*b*c*d)*1i)/(64*(-c)^(7/4)*d^(13/4)) - (((3*a^2*d^2 - 45*b
^2*c^2 + 10*a*b*c*d)^2/(64*(-c)^(7/4)*d^(13/4)) + (x^(1/2)*(9*a^4*d^4 + 202
5*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^
2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)*1i)/(64*(-c)^(7/4)*d^(13/4)))
/((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^2/(64*(-c)^(7/4)*d^(13/4)) - (x^(
1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60
*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d))/(64*(-c
)^(7/4)*d^(13/4)) + (((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^2/(64*(-c)^(7/4
)*d^(13/4)) + (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 90
0*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10
*a*b*c*d))/(64*(-c)^(7/4)*d^(13/4))))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)
*1i)/(32*(-c)^(7/4)*d^(13/4)) + (atan((((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*
c*d)^2*1i)/(64*(-c)^(7/4)*d^(13/4)) - (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 -
170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a
^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d))/(64*(-c)^(7/4)*d^(13/4)) - ((((3*a^2*d^2
 - 45*b^2*c^2 + 10*a*b*c*d)^2*1i)/(64*(-c)^(7/4)*d^(13/4)) + (x^(1/2)*(9*a^
4*d^4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d
^3))/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d))/(64*(-c)^(7/4)*d^
(13/4)))/((((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^2*1i)/(64*(-c)^(7/4)*d^(
13/4)) - (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b
^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*
c*d)*1i)/(64*(-c)^(7/4)*d^(13/4)) + ((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d
)^2*1i)/(64*(-c)^(7/4)*d^(13/4)) + (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170
*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*
d^2 - 45*b^2*c^2 + 10*a*b*c*d)*1i)/(64*(-c)^(7/4)*d^(13/4))))*(3*a^2*d^2 -
45*b^2*c^2 + 10*a*b*c*d))/(32*(-c)^(7/4)*d^(13/4))
```

3.436 $\int \frac{\sqrt{x}\,(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=364

$$\frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2}-\frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)}-\frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{9/4}d^{11/4}}+\frac{(21b^2c^2$$

```
[Out] 1/4*(-a*d+b*c)^2*x^(3/2)/c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(5*a*d+11*b*c)*x
^(3/2)/c^2/d^2/(d*x^2+c)-1/64*(5*a^2*d^2+6*a*b*c*d+21*b^2*c^2)*arctan(1-d^(
1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/d^(11/4)*2^(1/2)+1/64*(5*a^2*d^2+6*a*
b*c*d+21*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/d^(11/4
)*2^(1/2)+1/128*(5*a^2*d^2+6*a*b*c*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/
4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/d^(11/4)*2^(1/2)-1/128*(5*a^2*d^2+6*a*b
*c*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9
/4)/d^(11/4)*2^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {474, 468, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{9/4}d^{11/4}}+\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{9/4}d^{11/4}}+\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}d^{11/4}}-\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}d^{11/4}}-\frac{x^{3/2}(5ad+11bc)(bc-ad)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{3/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^2*x^(3/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(11*b*c + 5*
a*d)*x^(3/2))/(16*c^2*d^2*(c + d*x^2)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*d^(1
1/4)) + ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*d^(11/4)) + ((21*b^2*c^2 + 6*a*b*c*d
+ 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(6
4*Sqrt[2]*c^(9/4)*d^(11/4)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*d^
(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{\sqrt{x}\left(\frac{1}{2}\left(-8a^2d^2+3(bc-ad)^2\right)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\int \frac{\sqrt{x}}{c+dx^2}\,dx}{32c^2d^2}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\operatorname{Subst}\left(\int \ldots\right)}{16c^2d^2}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\operatorname{Subst}\left(\int \ldots\right)}{32c^2d^{5/2}}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\operatorname{Subst}\left(\int \ldots\right)}{64c^2 \ldots}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\log\left(\sqrt{c} \ldots\right)}{64\sqrt{2}\,c^{9/4} \ldots}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\tan^{-1}\left(1 \ldots\right)}{32\sqrt{2}\,c^{9/4}d^{11/4}}$$

Mathematica [A]

time = 0.69, size = 224, normalized size = 0.62

$$\frac{-\frac{4\sqrt[4]{c}\,d^{3/4}x^{3/2}\left(2abcd\left(c-3dx^2\right)-a^2d^2\left(9c+5dx^2\right)+b^2c^2\left(7c+11dx^2\right)\right)}{\left(c+dx^2\right)^2}-\sqrt{2}\left(21b^2c^2+6abcd+5a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)-\sqrt{2}\left(21b^2c^2+6abcd+5a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{64c^{9/4}d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((-4*c^(1/4)*d^(3/4)*x^(3/2)*(2*a*b*c*d*(c - 3*d*x^2) - a^2*d^2*(9*c + 5*d*
x^2) + b^2*c^2*(7*c + 11*d*x^2)))/(c + d*x^2)^2 - Sqrt[2]*(21*b^2*c^2 + 6*a
*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x])] - Sqrt[2]*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(64*c^(9/4)*d^(11/4))
```

Maple [A]
time = 0.09, size = 213, normalized size = 0.59

method	result
derivativedivides	$\frac{\frac{\left(5a^2d^2+6abcd-11b^2c^2\right)x^{\frac{7}{2}}}{16c^2d}+\frac{\left(9a^2d^2-2abcd-7b^2c^2\right)x^{\frac{3}{2}}}{16c\,d^2}}{\left(d\,x^2+c\right)^2}+\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right.\right.}{128d^3c}$
default	$\frac{\frac{\left(5a^2d^2+6abcd-11b^2c^2\right)x^{\frac{7}{2}}}{16c^2d}+\frac{\left(9a^2d^2-2abcd-7b^2c^2\right)x^{\frac{3}{2}}}{16c\,d^2}}{\left(d\,x^2+c\right)^2}+\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right.\right.}{128d^3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(5*a^2*d^2+6*a*b*c*d-11*b^2*c^2)/c^2/d*x^(7/2)+1/32*(9*a^2*d^2-2*a*
b*c*d-7*b^2*c^2)/c/d^2*x^(3/2))/(d*x^2+c)^2+1/128*(5*a^2*d^2+6*a*b*c*d+21*b
^2*c^2)/d^3/c^2/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.52, size = 297, normalized size = 0.82

$$-\frac{\left(11\,b^2c^2d-6\,abcd^2-5\,a^2d^3\right)x^{\frac{7}{2}}+\left(7\,b^2c^3+2\,abc^2d-9\,a^2cd^2\right)x^{\frac{3}{2}}}{16\left(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2\right)}+\frac{\left(21\,b^2c^2+6\,abcd+5\,a^2d^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((11*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*x^(7/2) + (7*b^2*c^3 + 2*a*
b*c^2*d - 9*a^2*c*d^2)*x^(3/2))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1
/128*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqr
t(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
```

1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/
(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*
x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c^2*d^2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1811 vs. $2(284) = 568$.
time = 1.20, size = 1811, normalized size = 4.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(194481*b^8*c^8 + 222264
*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4
*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c
*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4)*arctan((sqrt((85766121*b^12*c^12 + 14
7027636*a*b^11*c^11*d + 227542770*a^2*b^10*c^10*d^2 + 215040420*a^3*b^9*c^9
*d^3 + 181522215*a^4*b^8*c^8*d^4 + 112905576*a^5*b^7*c^7*d^5 + 63002556*a^6
*b^6*c^6*d^6 + 26882280*a^7*b^5*c^5*d^7 + 10290375*a^8*b^4*c^4*d^8 + 290250
0*a^9*b^3*c^3*d^9 + 731250*a^10*b^2*c^2*d^10 + 112500*a^11*b*c*d^11 + 15625
*a^12*d^12)*x - (194481*b^8*c^13*d^5 + 222264*a*b^7*c^12*d^6 + 280476*a^2*b
^6*c^11*d^7 + 176904*a^3*b^5*c^10*d^8 + 112806*a^4*b^4*c^9*d^9 + 42120*a^5*
b^3*c^8*d^10 + 15900*a^6*b^2*c^7*d^11 + 3000*a^7*b*c^6*d^12 + 625*a^8*c^5*d
^13)*sqrt(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 +
176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 1
5900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11)))*c^2*d^3
*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a
^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6
*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4) - (9261*b^
6*c^8*d^3 + 7938*a*b^5*c^7*d^4 + 8883*a^2*b^4*c^6*d^5 + 3996*a^3*b^3*c^5*d^
6 + 2115*a^4*b^2*c^4*d^7 + 450*a^5*b*c^3*d^8 + 125*a^6*c^2*d^9)*sqrt(x)*(-(
194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b
^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2
*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4))/(194481*b^8*c
^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 +
 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3
000*a^7*b*c*d^7 + 625*a^8*d^8)) - (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(
-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3
*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b
^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4)*log(c^7*d^8*
(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^
3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*
b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(3/4) + (9261*b^6
```

```
*c^6 + 7938*a*b^5*c^5*d + 8883*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d^3 + 211
5*a^4*b^2*c^2*d^4 + 450*a^5*b*c*d^5 + 125*a^6*d^6)*sqrt(x)) + (c^2*d^4*x^4
+ 2*c^3*d^3*x^2 + c^4*d^2)*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*
a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a
^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c
^9*d^11))^(1/4)*log(-c^7*d^8*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 28047
6*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120
*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/
(c^9*d^11))^(3/4) + (9261*b^6*c^6 + 7938*a*b^5*c^5*d + 8883*a^2*b^4*c^4*d^2
 + 3996*a^3*b^3*c^3*d^3 + 2115*a^4*b^2*c^2*d^4 + 450*a^5*b*c*d^5 + 125*a^6*
d^6)*sqrt(x)) + 4*((11*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*x^3 + (7*b^2*c^
3 + 2*a*b*c^2*d - 9*a^2*c*d^2)*x)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c
^4*d^2)
```

Sympy [A]
time = 58.56, size = 325, normalized size = 0.89

$$\frac{4abx^{\frac{3}{2}}}{4c^2d+4cd^2x^2}+\frac{4ab\operatorname{RootSum}\left(65536t^4c^5d^3+1,\left(t\mapsto t\log\left(4096t^3c^4d^2+\sqrt{x}\right)\right)\right)}{d}-\frac{4b^2cx^{\frac{3}{2}}}{4c^2d^2+4cd^3x^2}-\frac{4b^2c\operatorname{RootSum}\left(65536t^4c^5d^3+1,\left(t\mapsto t\log\left(4096t^3c^4d^2+\sqrt{x}\right)\right)\right)}{d^2}+\frac{2b^2\operatorname{RootSum}\left(256t^4cd^3+1,\left(t\mapsto t\log\left(64t^3cd^2+\sqrt{x}\right)\right)\right)}{d^2}+\frac{18cx^{\frac{3}{2}}(ad-bc)^2}{32c^4d^2+64c^3d^3x^2+32c^2d^4x^4}+\frac{10x^{\frac{7}{2}}(ad-bc)^2}{32c^4d+64c^3d^2x^2+32c^2d^3x^4}+\frac{2(ad-bc)^2\operatorname{RootSum}\left(268435456t^4c^9d^3+625,\left(t\mapsto t\log\left(\frac{2097152t^3c^7d^2}{125}+\sqrt{x}\right)\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] 4*a*b*x**(3/2)/(4*c**2*d + 4*c*d**2*x**2) + 4*a*b*RootSum(65536*_t**4*c**5*
d**3 + 1, Lambda(_t, _t*log(4096*_t**3*c**4*d**2 + sqrt(x))))/d - 4*b**2*c*
x**(3/2)/(4*c**2*d**2 + 4*c*d**3*x**2) - 4*b**2*c*RootSum(65536*_t**4*c**5*
d**3 + 1, Lambda(_t, _t*log(4096*_t**3*c**4*d**2 + sqrt(x))))/d**2 + 2*b**2
*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*_t**3*c*d**2 + sqrt(x))
))/d**2 + 18*c*x**(3/2)*(a*d - b*c)**2/(32*c**4*d**2 + 64*c**3*d**3*x**2 +
32*c**2*d**4*x**4) + 10*x**(7/2)*(a*d - b*c)**2/(32*c**4*d + 64*c**3*d**2*x
**2 + 32*c**2*d**3*x**4) + 2*(a*d - b*c)**2*RootSum(268435456*_t**4*c**9*d*
*3 + 625, Lambda(_t, _t*log(2097152*_t**3*c**7*d**2/125 + sqrt(x))))/d**2
```

Giac [A]
time = 0.55, size = 416, normalized size = 1.14

$$-\frac{11b^2c^2dx^{\frac{7}{2}}-6abcd^2x^{\frac{7}{2}}-5a^2d^3x^{\frac{7}{2}}+7b^2c^3x^{\frac{3}{2}}+2abc^2dx^{\frac{3}{2}}-9a^2cd^2x^{\frac{3}{2}}}{16(dx^2+c)^2c^2d^2}+\frac{\sqrt{2}\left(21(cd^3)^{\frac{3}{4}}b^2c^2+6(cd^3)^{\frac{3}{4}}abcd+5(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^3d^5}+\frac{\sqrt{2}\left(21(cd^3)^{\frac{3}{4}}b^2c^2+6(cd^3)^{\frac{3}{4}}abcd+5(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^3d^5}-\frac{\sqrt{2}\left(21(cd^3)^{\frac{3}{4}}b^2c^2+6(cd^3)^{\frac{3}{4}}abcd+5(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128c^3d^5}+\frac{\sqrt{2}\left(21(cd^3)^{\frac{3}{4}}b^2c^2+6(cd^3)^{\frac{3}{4}}abcd+5(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128c^3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*b^2*c^2*d*x^(7/2) - 6*a*b*c*d^2*x^(7/2) - 5*a^2*d^3*x^(7/2) + 7*b
^2*c^3*x^(3/2) + 2*a*b*c^2*d*x^(3/2) - 9*a^2*c*d^2*x^(3/2))/((d*x^2 + c)^2*
c^2*d^2) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d
 + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqr
t(x))/(c/d)^(1/4))/(c^3*d^5) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(
c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2
```

```
)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^5) - 1/128*sqrt(2)*(21*(c*d^
3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(s
qrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^5) + 1/128*sqrt(2)*(21*(
c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*l
og(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^5)
```

Mupad [B]
time = 0.22, size = 184, normalized size = 0.51

$$\frac{\operatorname{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,\left(5\,a^2\,d^2+6\,a\,b\,c\,d+21\,b^2\,c^2\right)}{32\,(-c)^{9/4}\,d^{11/4}}-\frac{\frac{x^{3/2}\,\left(-9\,a^2\,d^2+2\,a\,b\,c\,d+7\,b^2\,c^2\right)}{16\,c\,d^2}-\frac{x^{7/2}\,\left(5\,a^2\,d^2+6\,a\,b\,c\,d-11\,b^2\,c^2\right)}{16\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}-\frac{\operatorname{atanh}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,\left(5\,a^2\,d^2+6\,a\,b\,c\,d+21\,b^2\,c^2\right)}{32\,(-c)^{9/4}\,d^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(5*a^2*d^2 + 21*b^2*c^2 + 6*a*b*c*d))/(
32*(-c)^(9/4)*d^(11/4)) - ((x^(3/2)*(7*b^2*c^2 - 9*a^2*d^2 + 2*a*b*c*d))/(1
6*c*d^2) - (x^(7/2)*(5*a^2*d^2 - 11*b^2*c^2 + 6*a*b*c*d))/(16*c^2*d))/(c^2
+ d^2*x^4 + 2*c*d*x^2) - (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(5*a^2*d^2 +
21*b^2*c^2 + 6*a*b*c*d))/(32*(-c)^(9/4)*d^(11/4))
```

3.437 $\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx$

Optimal. Leaf size=364

$$\frac{(bc-ad)^2\sqrt{x}}{4cd^2(c+dx^2)^2}-\frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2(c+dx^2)}-\frac{(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{(5b^2c^2+\cdots}{}$$

```
[Out] -1/64*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(
1/4))/c^(11/4)/d^(9/4)*2^(1/2)+1/64*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arctan
(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)-1/128*(21*a^2*
d^2+6*a*b*c*d+5*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/
2))/c^(11/4)/d^(9/4)*2^(1/2)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*ln(c^(1
/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)+1/4
*(-a*d+b*c)^2*x^(1/2)/c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(7*a*d+9*b*c)*x^(1/
2)/c^2/d^2/(d*x^2+c)
```

Rubi [A]
time = 0.21, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {474, 468, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{11/4}d^{9/4}}-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}d^{9/4}}-\frac{\sqrt{x}\,(7ad+9bc)(bc-ad)}{16c^2d^2(c+dx^2)}+\frac{\sqrt{x}\,(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[x])/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9*b*c + 7*a
*d)*Sqrt[x])/(16*c^2*d^2*(c + d*x^2)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^
2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*d^(9
/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d +
21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64
*Sqrt[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*d^
(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx = \frac{(bc-ad)^2\sqrt{x}}{4cd^2\,(c+dx^2)^2} - \frac{\int \frac{\frac{1}{2}(-8a^2d^2+(bc-ad)^2)-4b^2cdx^2}{\sqrt{x}\,(c+dx^2)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\,(c+dx^2)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{1}{\sqrt{x}\,(c+dx^2 \cdots}}{32c^2d^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\,(c+dx^2)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\,\mathrm{Subst}\left(\int \frac{1}{c+ \cdots}\right.}{16c^2d^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\,(c+dx^2)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\,\mathrm{Subst}\left(\int \cdots\right.}{32c^{5/2}d^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\,(c+dx^2)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\,\mathrm{Subst}\left(\int - \cdots\right.}{64c^{5/2}d^{5/ \cdots}}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\,(c+dx^2)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c} - \cdots\right.}{64\sqrt{2}\,c^{11/4}d^{9 \cdots}}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\,(c+dx^2)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1 - \cdots\right.}{32\sqrt{2}\,c^{11/4}d^{9/4}}$$

Mathematica [A]

time = 0.65, size = 225, normalized size = 0.62

$$\frac{-\frac{4c^{3/4}\sqrt[4]{d}\,\sqrt{x}\,(2abcd(3c-dx^2)-a^2d^2(11c+7dx^2)+b^2c^2(5c+9dx^2))}{(c+dx^2)^2} - \sqrt{2}\,(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right) + \sqrt{2}\,(5b^2c^2+6abcd+21a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{64c^{11/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

```
[Out] ((-4*c^(3/4)*d^(1/4)*Sqrt[x]*(2*a*b*c*d*(3*c - d*x^2) - a^2*d^2*(11*c + 7*d
*x^2) + b^2*c^2*(5*c + 9*d*x^2)))/(c + d*x^2)^2 - Sqrt[2]*(5*b^2*c^2 + 6*a*
b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x])] + Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTanh[(Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(64*c^(11/4)*d^(9/4))
```

Maple [A]

time = 0.09, size = 213, normalized size = 0.59

method	result
derivativedivides	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^{\frac{5}{2}}}{16c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)\sqrt{x}}{16d^2c}}{\left(d\,x^2+c\right)^2}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{}}\right.\right.}{}$
default	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^{\frac{5}{2}}}{16c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)\sqrt{x}}{16d^2c}}{\left(d\,x^2+c\right)^2}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^(5/2)+1/32*(11*a^2*d^2-6*a*
b*c*d-5*b^2*c^2)/d^2/c*x^(1/2))/(d*x^2+c)^2+1/128*(21*a^2*d^2+6*a*b*c*d+5*b
^2*c^2)/c^3/d^2*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]

time = 0.53, size = 366, normalized size = 1.01

$$-\frac{\left(9\,b^2c^2d-2\,abcd^2-7\,a^2d^3\right)x^{\frac{5}{2}}+\left(5\,b^2c^3+6\,abc^2d-11\,a^2cd^2\right)\sqrt{x}}{16\left(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2\right)}+\frac{\frac{2\sqrt{2}\left(5\,b^2c^2+6\,abcd+21\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(5\,b^2c^2+6\,abcd+21\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(5\,b^2c^2+6\,abcd+21\,a^2d^2\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(5\,b^2c^2+6\,abcd+21\,a^2d^2\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^(5/2) + (5*b^2*c^3 + 6*a*b
*c^2*d - 11*a^2*c*d^2)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1
/128*(2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*
sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)
```

```
*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d
 + 21*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(
c^(3/4)*d^(1/4)) - sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(-sqrt(2
)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d^
2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(284) = 568$.

time = 0.87, size = 1416, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7
*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*arctan((sqrt(c^6*d^4*sqrt(-(625*b^8*c^
8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 1128
06*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 2222
64*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9)) + (25*b^4*c^4 + 60*a*b^3*c^3*d
 + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4)*x)*c^8*d^7*(-(625*b
^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 +
 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 +
 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(3/4) - (5*b^2*c^10*d^7 +
 6*a*b*c^9*d^8 + 21*a^2*c^8*d^9)*sqrt(x)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d
+ 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 +
176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 1944
81*a^8*d^8)/(c^11*d^9))^(3/4))/(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*
b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b
^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8))
 + (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*
d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
+ 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 19
4481*a^8*d^8)/(c^11*d^9))^(1/4)*log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7
*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 1
94481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*sqr
t(x)) - (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7
*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(-c^3*d^2*(-(625*b^8*c^8 + 3000*a*b
^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c
^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d
```

```
^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^
2)*sqrt(x)) - 4*(5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2 + (9*b^2*c^2*d - 2*
a*b*c*d^2 - 7*a^2*d^3)*x^2)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2258 vs. $2(367) = 734$.
time = 144.21, size = 2258, normalized size = 6.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/2)) - 2*b**2/(3*x**
(3/2))), Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/2)
) - 2*b**2/(3*x**(3/2)))/d**3, Eq(c, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)
/5 + 2*b**2*x**(9/2)/9)/c**3, Eq(d, 0)), (44*a**2*c**2*d**2*sqrt(x)/(64*c**
5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) - 21*a**2*c**2*d**2*(-c/d)
**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 6
4*c**3*d**4*x**4) + 21*a**2*c**2*d**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(
1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 42*a**2*c**
2*d**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d
**3*x**2 + 64*c**3*d**4*x**4) + 28*a**2*c*d**3*x**(5/2)/(64*c**5*d**2 + 128
*c**4*d**3*x**2 + 64*c**3*d**4*x**4) - 42*a**2*c*d**3*x**2*(-c/d)**(1/4)*lo
g(sqrt(x) - (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**
4*x**4) + 42*a**2*c*d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(6
4*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 84*a**2*c*d**3*x**2
*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x*
*2 + 64*c**3*d**4*x**4) - 21*a**2*d**4*x**4*(-c/d)**(1/4)*log(sqrt(x) - (-c
/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 21*a*
*2*d**4*x**4*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**5*d**2 + 128
*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 42*a**2*d**4*x**4*(-c/d)**(1/4)*atan
(sqrt(x)/(-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x
**4) - 24*a*b*c**3*d*sqrt(x)/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d
**4*x**4) - 6*a*b*c**3*d*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**
5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 6*a*b*c**3*d*(-c/d)**(1/
4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**
3*d**4*x**4) + 12*a*b*c**3*d*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*
c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 8*a*b*c**2*d**2*x**(5
/2)/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) - 12*a*b*c**2*d
**2*x**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**
4*d**3*x**2 + 64*c**3*d**4*x**4) + 12*a*b*c**2*d**2*x**2*(-c/d)**(1/4)*log(
sqrt(x) + (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*
x**4) + 24*a*b*c**2*d**2*x**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64
```

```
*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) - 6*a*b*c*d**3*x**4*(-
c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2
 + 64*c**3*d**4*x**4) + 6*a*b*c*d**3*x**4*(-c/d)**(1/4)*log(sqrt(x) + (-c/d
)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 12*a*b*
c*d**3*x**4*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**5*d**2 + 128*c
**4*d**3*x**2 + 64*c**3*d**4*x**4) - 20*b**2*c**4*sqrt(x)/(64*c**5*d**2 + 1
28*c**4*d**3*x**2 + 64*c**3*d**4*x**4) - 5*b**2*c**4*(-c/d)**(1/4)*log(sqrt
(x) - (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4
) + 5*b**2*c**4*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**5*d**2 +
128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 10*b**2*c**4*(-c/d)**(1/4)*atan(s
qrt(x)/(-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**
4) - 36*b**2*c**3*d*x**(5/2)/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d
**4*x**4) - 10*b**2*c**3*d*x**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/
(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 10*b**2*c**3*d*x*
*2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3
*x**2 + 64*c**3*d**4*x**4) + 20*b**2*c**3*d*x**2*(-c/d)**(1/4)*atan(sqrt(x)
/(-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) - 5
*b**2*c**2*d**2*x**4*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**5*d*
*2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 5*b**2*c**2*d**2*x**4*(-c/d)
**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 6
4*c**3*d**4*x**4) + 10*b**2*c**2*d**2*x**4*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d
)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4), True))
```

Giac [A]
time = 0.59, size = 416, normalized size = 1.14

$$\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\left(cd^3\right)^{\frac{1}{4}}abcd+21\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^3}+\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\left(cd^3\right)^{\frac{1}{4}}abcd+21\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^3}+\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\left(cd^3\right)^{\frac{1}{4}}abcd+21\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^3d^3}-\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\left(cd^3\right)^{\frac{1}{4}}abcd+21\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^3d^3}-\frac{9\,b^2c^2dx^{\frac{5}{2}}-2\,abcd^2x^{\frac{5}{2}}-7\,a^2d^3x^{\frac{5}{2}}+5\,b^2c^3\sqrt{x}+6\,abc^2d\sqrt{x}-11\,a^2cd^2\sqrt{x}}{16\left(dx^2+c\right)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)
^(1/4))/(c^3*d^3) + 1/64*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)
*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2
*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt
(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/128*sqrt(2)*(5*(c*d^3)^(1/4)
*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/16*(9*b^2*c^2*d*x^(5/2)
 - 2*a*b*c*d^2*x^(5/2) - 7*a^2*d^3*x^(5/2) + 5*b^2*c^3*sqrt(x) + 6*a*b*c^2*
d*sqrt(x) - 11*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c^2*d^2)
```

Mupad [B]
time = 0.33, size = 1419, normalized size = 3.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^2/(x^(1/2)*(c + d*x^2)^3),x)

[Out] - ((x^(1/2)*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(16*c*d^2) - (x^(5/2)*(7*a^2*d^2 - 9*b^2*c^2 + 2*a*b*c*d))/(16*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2) - (atan((((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(64*(-c)^(15/4)*d^(9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(64*(-c)^(15/4)*d^(9/4)) + (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(64*(-c)^(15/4)*d^(9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(64*(-c)^(15/4)*d^(9/4)) + (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(32*(-c)^(11/4)*d^(9/4)) - (atan((((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(64*(-c)^(15/4)*d^(9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(64*(-c)^(15/4)*d^(9/4)) + (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(64*(-c)^(15/4)*d^(9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(64*(-c)^(15/4)*d^(9/4)) + (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(32*(-c)^(11/4)*d^(9/4))

3.438 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3} \, dx$

Optimal. Leaf size=399

$$-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}-\frac{(b^2c^2-2abcd+9a^2d^2)\,x^{3/2}}{4c^2d\,(c+dx^2)^2}+\frac{(3b^2c^2+5ad(2bc-9ad))\,x^{3/2}}{16c^3d\,(c+dx^2)}-\frac{(3b^2c^2+5ad(2bc-9ad))\,\text{ta}}{32\sqrt{2}\,c^{1}}$$

```
[Out] -1/4*(9*a^2*d^2-2*a*b*c*d+b^2*c^2)*x^(3/2)/c^2/d/(d*x^2+c)^2+1/16*(3*b^2*c^
2+5*a*d*(-9*a*d+2*b*c))*x^(3/2)/c^3/d/(d*x^2+c)-1/64*(3*b^2*c^2+5*a*d*(-9*a
*d+2*b*c))*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/d^(7/4)*2^(1/
2)+1/64*(3*b^2*c^2+5*a*d*(-9*a*d+2*b*c))*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c
^(1/4))/c^(13/4)/d^(7/4)*2^(1/2)+1/128*(3*b^2*c^2+5*a*d*(-9*a*d+2*b*c))*ln(
c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/d^(7/4)*2^(1/2)
-1/128*(3*b^2*c^2+5*a*d*(-9*a*d+2*b*c))*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4
)*2^(1/2)*x^(1/2))/c^(13/4)/d^(7/4)*2^(1/2)-2*a^2/c/(d*x^2+c)^2/x^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 396, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {473, 468, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{x^{3/2}\left(-\frac{9a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4c\,(c+dx^2)^2}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}-\frac{(5ad(2bc-9ad)+3b^2c^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{13/4}d^{7/4}}+\frac{(5ad(2bc-9ad)+3b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{13/4}d^{7/4}}+\frac{x^{3/2}\left(\frac{5a(2bc-9ad)}{c^2}+\frac{3b^2}{d}\right)}{16c\,(c+dx^2)}+\frac{(5ad(2bc-9ad)+3b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{13/4}d^{7/4}}-\frac{(5ad(2bc-9ad)+3b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{13/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)^2) + ((2*a*b - (b^2*c)/d - (9*a^2*d)/c)*x^(
3/2))/(4*c*(c + d*x^2)^2) + (((3*b^2)/d + (5*a*(2*b*c - 9*a*d))/c^2)*x^(3/2
))/(16*c*(c + d*x^2)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^2*c^
2 + 5*a*d*(2*b*c - 9*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*d
^(7/4)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*d^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx = -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2} + \frac{2\int \frac{\sqrt{x}\left(\frac{1}{2}a(2bc-9ad)+\frac{1}{2}b^2cx^2\right)}{(c+dx^2)^3}dx}{c}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2} - \frac{(b^2c^2-2abcd+9a^2d^2)\,x^{3/2}}{4c^2d\,(c+dx^2)^2} + \frac{1}{8}\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)\int \frac{}{(c}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2} - \frac{(b^2c^2-2abcd+9a^2d^2)\,x^{3/2}}{4c^2d\,(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\,(c+dx^2)} + \frac{\left(\frac{3b^2}{d}\right.}{}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2} - \frac{(b^2c^2-2abcd+9a^2d^2)\,x^{3/2}}{4c^2d\,(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\,(c+dx^2)} + \frac{\left(\frac{3b^2}{d}\right.}{}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2} - \frac{(b^2c^2-2abcd+9a^2d^2)\,x^{3/2}}{4c^2d\,(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\,(c+dx^2)} - \frac{(3b^2c}{}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2} - \frac{(b^2c^2-2abcd+9a^2d^2)\,x^{3/2}}{4c^2d\,(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\,(c+dx^2)} + \frac{(3b^2c}{}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2} - \frac{(b^2c^2-2abcd+9a^2d^2)\,x^{3/2}}{4c^2d\,(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\,(c+dx^2)} + \frac{(3b^2c}{}$$

$$= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2} - \frac{(b^2c^2-2abcd+9a^2d^2)\,x^{3/2}}{4c^2d\,(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\,(c+dx^2)} - \frac{(3b^2c}{}$$

Mathematica [A]
time = 0.78, size = 238, normalized size = 0.60

$$\frac{-\frac{4\sqrt[4]{c}\,d^{3/4}\left(b^2c^2x^2(c-3dx^2)-2abcdx^2(9c+5dx^2)+a^2d(32c^2+81cdx^2+45d^2x^4)\right)}{\sqrt{x}\,(c+dx^2)^2}-\sqrt{2}\,(3b^2c^2+10abcd-45a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)-\sqrt{2}\,(3b^2c^2+10abcd-45a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{64c^{13/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-4*c^(1/4)*d^(3/4)*(b^2*c^2*x^2*(c - 3*d*x^2) - 2*a*b*c*d*x^2*(9*c + 5*d*
x^2) + a^2*d*(32*c^2 + 81*c*d*x^2 + 45*d^2*x^4)))/(Sqrt[x]*(c + d*x^2)^2) -
 Sqrt[2]*(3*b^2*c^2 + 10*a*b*c*d - 45*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)
/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - Sqrt[2]*(3*b^2*c^2 + 10*a*b*c*d - 45*
a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/
(64*c^(13/4)*d^(7/4))
```

Maple [A]

time = 0.14, size = 216, normalized size = 0.54

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{13}{32}a^2d^2-\frac{5}{16}abcd-\frac{3}{32}b^2c^2\right)x^{\frac{7}{2}}+\frac{c\left(17a^2d^2-18abcd+b^2c^2\right)x^{\frac{3}{2}}}{32d}}{\left(dx^2+c\right)^2}+\frac{\left(45a^2d^2-10abcd-3b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}\right.\right.}{}\right.}{c^3}$
default	$-\frac{2\left(\frac{\left(\frac{13}{32}a^2d^2-\frac{5}{16}abcd-\frac{3}{32}b^2c^2\right)x^{\frac{7}{2}}+\frac{c\left(17a^2d^2-18abcd+b^2c^2\right)x^{\frac{3}{2}}}{32d}}{\left(dx^2+c\right)^2}+\frac{\left(45a^2d^2-10abcd-3b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}\right.\right.}{}\right.}{c^3}$
risch	$-\frac{2a^2}{c^3\sqrt{x}}-\frac{13x^{\frac{7}{2}}a^2d^2}{16c^3(dx^2+c)^2}+\frac{5x^{\frac{7}{2}}abd}{8c^2(dx^2+c)^2}+\frac{3x^{\frac{7}{2}}b^2}{16c(dx^2+c)^2}-\frac{17dx^{\frac{3}{2}}a^2}{16c^2(dx^2+c)^2}+\frac{9x^{\frac{3}{2}}ab}{8c(dx^2+c)^2}-\frac{x^{\frac{3}{2}}b^2}{16(dx^2+c)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^3*(((13/32*a^2*d^2-5/16*a*b*c*d-3/32*b^2*c^2)*x^(7/2)+1/32*c*(17*a^2*d
^2-18*a*b*c*d+b^2*c^2)/d*x^(3/2))/(d*x^2+c)^2+1/256*(45*a^2*d^2-10*a*b*c*d-
3*b^2*c^2)/d^2/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2*a^2/c^3/x^(1/
2)
```

Maxima [A]

time = 0.50, size = 307, normalized size = 0.77

$$-\frac{32\,a^2c^2d-(3\,b^2c^2d+10\,abcd^2-45\,a^2d^3)x^4+(b^2c^3-18\,abc^2d+81\,a^2cd^2)x^2}{16\left(c^3d^3x^{\frac{9}{2}}+2\,c^4d^2x^{\frac{5}{2}}+c^5d\sqrt{x}\right)}+\frac{(3\,b^2c^2+10\,abcd-45\,a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\,c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(32*a^2*c^2*d - (3*b^2*c^2*d + 10*a*b*c*d^2 - 45*a^2*d^3)*x^4 + (b^2*
c^3 - 18*a*b*c^2*d + 81*a^2*c*d^2)*x^2)/(c^3*d^3*x^(9/2) + 2*c^4*d^2*x^(5/2
) + c^5*d*sqrt(x)) + 1/128*(3*b^2*c^2 + 10*a*b*c*d - 45*a^2*d^2)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt
(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(
```

```
2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
+ sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(
1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(317) = 634$.
time = 0.77, size = 1819, normalized size = 4.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*(-(81*b^8*c^8 + 1080*a*b^7*
c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4
 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 +
4100625*a^8*d^8)/(c^13*d^7))^(1/4)*arctan((sqrt((729*b^12*c^12 + 14580*a*b^
11*c^11*d + 55890*a^2*b^10*c^10*d^2 - 553500*a^3*b^9*c^9*d^3 - 3479625*a^4*
b^8*c^8*d^4 + 10305000*a^5*b^7*c^7*d^5 + 75317500*a^6*b^6*c^6*d^6 - 1545750
00*a^7*b^5*c^5*d^7 - 782915625*a^8*b^4*c^4*d^8 + 1868062500*a^9*b^3*c^3*d^9
 + 2829431250*a^10*b^2*c^2*d^10 - 11071687500*a^11*b*c*d^11 + 8303765625*a^
12*d^12)*x - (81*b^8*c^15*d^3 + 1080*a*b^7*c^14*d^4 + 540*a^2*b^6*c^13*d^5
- 36600*a^3*b^5*c^12*d^6 - 42650*a^4*b^4*c^11*d^7 + 549000*a^5*b^3*c^10*d^8
 + 121500*a^6*b^2*c^9*d^9 - 3645000*a^7*b*c^8*d^10 + 4100625*a^8*c^7*d^11)*
sqrt(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*
c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c
^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7)))*c^3*d^2*(-(81*
b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 -
42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3
645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4) + (27*b^6*c^9*d^2 +
 270*a*b^5*c^8*d^3 - 315*a^2*b^4*c^7*d^4 - 7100*a^3*b^3*c^6*d^5 + 4725*a^4*
b^2*c^5*d^6 + 60750*a^5*b*c^4*d^7 - 91125*a^6*c^3*d^8)*sqrt(x)*(-(81*b^8*c^
8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*
a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000
*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4))/(81*b^8*c^8 + 1080*a*b^7
*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^
4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 +
 4100625*a^8*d^8)) - (c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*(-(81*b^8*c^8
+ 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^
4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a
^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4)*log(c^10*d^5*(-(81*b^8*c^8
+ 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^
4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a
^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(3/4) - (27*b^6*c^6 + 270*a*b^5*c
```

```
^5*d - 315*a^2*b^4*c^4*d^2 - 7100*a^3*b^3*c^3*d^3 + 4725*a^4*b^2*c^2*d^4 +
60750*a^5*b*c*d^5 - 91125*a^6*d^6)*sqrt(x)) + (c^3*d^3*x^5 + 2*c^4*d^2*x^3
+ c^5*d*x)*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a
^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^
6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4)*lo
g(-c^10*d^5*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*
a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a
^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(3/4) -
 (27*b^6*c^6 + 270*a*b^5*c^5*d - 315*a^2*b^4*c^4*d^2 - 7100*a^3*b^3*c^3*d^3
 + 4725*a^4*b^2*c^2*d^4 + 60750*a^5*b*c*d^5 - 91125*a^6*d^6)*sqrt(x)) - 4*(
32*a^2*c^2*d - (3*b^2*c^2*d + 10*a*b*c*d^2 - 45*a^2*d^3)*x^4 + (b^2*c^3 - 1
8*a*b*c^2*d + 81*a^2*c*d^2)*x^2)*sqrt(x))/(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^
5*d*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.62, size = 427, normalized size = 1.07

$$-\frac{2a^2}{c^3\sqrt{x}}+\frac{3b^2c^2dx^{\frac{7}{2}}+10abcd^2x^{\frac{7}{2}}-13a^2d^3x^{\frac{7}{2}}-b^2c^3x^{\frac{3}{2}}+18abc^2dx^{\frac{3}{2}}-17a^2cd^2x^{\frac{3}{2}}}{16(dx^2+c)^2c^3d}+\frac{\sqrt{2}\left(3(cd^3)^{\frac{3}{4}}b^2c^2+10(cd^3)^{\frac{3}{4}}abcd-45(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^4d^4}+\frac{\sqrt{2}\left(3(cd^3)^{\frac{3}{4}}b^2c^2+10(cd^3)^{\frac{3}{4}}abcd-45(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^4d^4}-\frac{\sqrt{2}\left(3(cd^3)^{\frac{3}{4}}b^2c^2+10(cd^3)^{\frac{3}{4}}abcd-45(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128c^4d^4}+\frac{\sqrt{2}\left(3(cd^3)^{\frac{3}{4}}b^2c^2+10(cd^3)^{\frac{3}{4}}abcd-45(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^2/(c^3*sqrt(x)) + 1/16*(3*b^2*c^2*d*x^(7/2) + 10*a*b*c*d^2*x^(7/2) - 1
3*a^2*d^3*x^(7/2) - b^2*c^3*x^(3/2) + 18*a*b*c^2*d*x^(3/2) - 17*a^2*c*d^2*x
^(3/2))/((d*x^2 + c)^2*c^3*d) + 1/64*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*
(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(
2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^4) + 1/64*sqrt(2)*(3*(c*d^3
)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^4) -
1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d
^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^
4) + 1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45
*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(
c^4*d^4)
```

Mupad [B]
time = 0.22, size = 192, normalized size = 0.48

$$\frac{\operatorname{atanh}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)\left(-45\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{32\,(-c)^{13/4}\,d^{7/4}}-\frac{\operatorname{atan}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)\left(-45\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{32\,(-c)^{13/4}\,d^{7/4}}-\frac{\frac{2\,a^2}{c}-\frac{x^4\,\left(-45\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{16\,c^3}+\frac{x^2\,\left(81\,a^2\,d^2-18\,a\,b\,c\,d+b^2\,c^2\right)}{16\,c^2\,d}}{c^2\,\sqrt{x}+d^2\,x^{9/2}+2\,c\,d\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x)
```

```
[Out] (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(3*b^2*c^2 - 45*a^2*d^2 + 10*a*b*c*d))
/(32*(-c)^(13/4)*d^(7/4)) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(3*b^2*c^2
- 45*a^2*d^2 + 10*a*b*c*d))/(32*(-c)^(13/4)*d^(7/4)) - ((2*a^2)/c - (x^4*(3
*b^2*c^2 - 45*a^2*d^2 + 10*a*b*c*d))/(16*c^3) + (x^2*(81*a^2*d^2 + b^2*c^2
- 18*a*b*c*d))/(16*c^2*d))/(c^2*x^(1/2) + d^2*x^(9/2) + 2*c*d*x^(5/2))
```

3.439 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=402

$$-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(3b^2c^2-6abcd+11a^2d^2\right)\sqrt{x}}{12c^2d\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2+7ad(6bc-11ad)\right)\sqrt{x}}{48c^3d\left(c+dx^2\right)}-\frac{\left(3b^2c^2+7ad(6bc-11\right.}{32}$$

```
[Out] -2/3*a^2/c/x^(3/2)/(d*x^2+c)^2-1/64*(3*b^2*c^2+7*a*d*(-11*a*d+6*b*c))*arcta
n(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(15/4)/d^(5/4)*2^(1/2)+1/64*(3*b^2*c
^2+7*a*d*(-11*a*d+6*b*c))*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(15/4
)/d^(5/4)*2^(1/2)-1/128*(3*b^2*c^2+7*a*d*(-11*a*d+6*b*c))*ln(c^(1/2)+x*d^(1
/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)/d^(5/4)*2^(1/2)+1/128*(3*b^2*
c^2+7*a*d*(-11*a*d+6*b*c))*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(
1/2))/c^(15/4)/d^(5/4)*2^(1/2)-1/12*(11*a^2*d^2-6*a*b*c*d+3*b^2*c^2)*x^(1/2
)/c^2/d/(d*x^2+c)^2+1/48*(3*b^2*c^2+7*a*d*(-11*a*d+6*b*c))*x^(1/2)/c^3/d/(d
*x^2+c)
```

Rubi [A]

time = 0.26, antiderivative size = 398, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {473, 468, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{x}\left(-\frac{11a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{12c\left(c+dx^2\right)^2}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}d^{5/4}}+\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{15/4}d^{5/4}}+\frac{\sqrt{x}\left(\frac{7a(6bc-11ad)}{c^2}+\frac{3b^2}{d}\right)}{48c\left(c+dx^2\right)}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}}+\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)^2) + ((6*a*b - (3*b^2*c)/d - (11*a^2*d)/c
)*Sqrt[x])/(12*c*(c + d*x^2)^2) + (((3*b^2)/d + (7*a*(6*b*c - 11*a*d))/c^2)
*Sqrt[x])/(48*c*(c + d*x^2)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*ArcTan
[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4)) + ((
3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d
))*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*
c^(15/4)*d^(5/4)) + ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*d^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx &= -\frac{2a^2}{3cx^{3/2}(c+dx^2)^2} + \frac{2\int \frac{\frac{1}{2}a(6bc-11ad)+\frac{3}{2}b^2cx^2}{\sqrt{x}\,(c+dx^2)^3}\,dx}{3c} \\
&= -\frac{2a^2}{3cx^{3/2}(c+dx^2)^2} - \frac{(3b^2c^2-6abcd+11a^2d^2)\sqrt{x}}{12c^2d(c+dx^2)^2} + \frac{1}{24}\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right. \\
&= -\frac{2a^2}{3cx^{3/2}(c+dx^2)^2} - \frac{(3b^2c^2-6abcd+11a^2d^2)\sqrt{x}}{12c^2d(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c(c+dx^2)} + \\
&= -\frac{2a^2}{3cx^{3/2}(c+dx^2)^2} - \frac{(3b^2c^2-6abcd+11a^2d^2)\sqrt{x}}{12c^2d(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c(c+dx^2)} + \\
&= -\frac{2a^2}{3cx^{3/2}(c+dx^2)^2} - \frac{(3b^2c^2-6abcd+11a^2d^2)\sqrt{x}}{12c^2d(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c(c+dx^2)} + \\
&= -\frac{2a^2}{3cx^{3/2}(c+dx^2)^2} - \frac{(3b^2c^2-6abcd+11a^2d^2)\sqrt{x}}{12c^2d(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c(c+dx^2)} + \\
&= -\frac{2a^2}{3cx^{3/2}(c+dx^2)^2} - \frac{(3b^2c^2-6abcd+11a^2d^2)\sqrt{x}}{12c^2d(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c(c+dx^2)} - \\
&= -\frac{2a^2}{3cx^{3/2}(c+dx^2)^2} - \frac{(3b^2c^2-6abcd+11a^2d^2)\sqrt{x}}{12c^2d(c+dx^2)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c(c+dx^2)} -
\end{aligned}
$$

Mathematica [**A**]
time = 0.77, size = 241, normalized size = 0.60

$$
\frac{-\frac{4c^{3/4}\sqrt[4]{d}\left(3b^2c^2x^2(3c-dx^2)-6abcdx^2(11c+7dx^2)+a^2d(32c^2+121cdx^2+77d^2x^4)\right)}{x^{3/2}(c+dx^2)^2} - 3\sqrt{2}\,(3b^2c^2+42abcd-77a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right) + 3\sqrt{2}\,(3b^2c^2+42abcd-77a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{192c^{15/4}d^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
[Out] ((-4*c^(3/4)*d^(1/4)*(3*b^2*c^2*x^2*(3*c - d*x^2) - 6*a*b*c*d*x^2*(11*c + 7
*d*x^2) + a^2*d*(32*c^2 + 121*c*d*x^2 + 77*d^2*x^4)))/(x^(3/2)*(c + d*x^2)^
2) - 3*Sqrt[2]*(3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt
[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(3*b^2*c^2 + 42*a*b*c
*d - 77*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[
d]*x)])/(192*c^(15/4)*d^(5/4))
```

Maple [A]

time = 0.15, size = 220, normalized size = 0.55

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{15}{32}a^{2}d^{2}-\frac{7}{16}abcd-\frac{1}{32}b^{2}c^{2}\right)x^{\frac{5}{2}}+\frac{c\left(19a^{2}d^{2}-22abcd+3b^{2}c^{2}\right)\sqrt{x}}{32d}}{\left(dx^{2}+c\right)^{2}}+\frac{\left(77a^{2}d^{2}-42abcd-3b^{2}c^{2}\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{}}\right.\right.}{}\right.}{c^{3}}$
default	$-\frac{2\left(\frac{\left(\frac{15}{32}a^{2}d^{2}-\frac{7}{16}abcd-\frac{1}{32}b^{2}c^{2}\right)x^{\frac{5}{2}}+\frac{c\left(19a^{2}d^{2}-22abcd+3b^{2}c^{2}\right)\sqrt{x}}{32d}}{\left(dx^{2}+c\right)^{2}}+\frac{\left(77a^{2}d^{2}-42abcd-3b^{2}c^{2}\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{}}\right.\right.}{}\right.}{c^{3}}$
risch	$-\frac{2a^{2}}{3c^{3}x^{\frac{3}{2}}}-\frac{15x^{\frac{5}{2}}a^{2}d^{2}}{16c^{3}\left(dx^{2}+c\right)^{2}}+\frac{7x^{\frac{5}{2}}abd}{8c^{2}\left(dx^{2}+c\right)^{2}}+\frac{x^{\frac{5}{2}}b^{2}}{16c\left(dx^{2}+c\right)^{2}}-\frac{19d\sqrt{x}\,a^{2}}{16c^{2}\left(dx^{2}+c\right)^{2}}+\frac{11\sqrt{x}\,ab}{8c\left(dx^{2}+c\right)^{2}}-\frac{3\sqrt{x}\,b^{2}}{16\left(dx^{2}+c\right)^{2}d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^3*(((15/32*a^2*d^2-7/16*a*b*c*d-1/32*b^2*c^2)*x^(5/2)+1/32*c*(19*a^2*d
^2-22*a*b*c*d+3*b^2*c^2)/d*x^(1/2))/(d*x^2+c)^2+1/256*(77*a^2*d^2-42*a*b*c*
d-3*b^2*c^2)/d*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/3*a^2/c^3/x
^(3/2)
```

Maxima [A]

time = 0.52, size = 377, normalized size = 0.94

$$-\frac{32a^{2}c^{2}d-\left(3b^{2}c^{2}d+42abcd^{2}-77a^{2}d^{3}\right)x^{4}+\left(9b^{2}c^{3}-66abc^{2}d+121a^{2}cd^{2}\right)x^{2}}{48\left(c^{3}d^{3}x^{\frac{11}{2}}+2c^{4}d^{2}x^{\frac{7}{2}}+c^{5}dx^{\frac{3}{2}}\right)}+\frac{\frac{2\sqrt{2}\left(3b^{2}c^{2}+42abcd-77a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(3b^{2}c^{2}+42abcd-77a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(3b^{2}c^{2}+42abcd-77a^{2}d^{2}\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(3b^{2}c^{2}+42abcd-77a^{2}d^{2}\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128c^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(32*a^2*c^2*d - (3*b^2*c^2*d + 42*a*b*c*d^2 - 77*a^2*d^3)*x^4 + (9*b^
2*c^3 - 66*a*b*c^2*d + 121*a^2*c*d^2)*x^2)/(c^3*d^3*x^(11/2) + 2*c^4*d^2*x^
(7/2) + c^5*d*x^(3/2)) + 1/128*(2*sqrt(2)*(3*b^2*c^2 + 42*a*b*c*d - 77*a^2*
d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b^2*c^2 +
42*a*b*c*d - 77*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*s
```

```
qrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sq
rt(2)*(3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(3*b^2*c^2 + 42*a*b
*c*d - 77*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(
c))/(c^(3/4)*d^(1/4)))/(c^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1433 vs. $2(318) = 636$.
time = 1.18, size = 1433, normalized size = 3.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(12*(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536*a
*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b
^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544
*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)*arctan((sqrt(c^8*d^2*sqr
t(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*
c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6
*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5)) + (9*b^
4*c^4 + 252*a*b^3*c^3*d + 1302*a^2*b^2*c^2*d^2 - 6468*a^3*b*c*d^3 + 5929*a^
4*d^4)*x)*c^11*d^4*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2
 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*
d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(
c^15*d^5))^(3/4) + (3*b^2*c^13*d^4 + 42*a*b*c^12*d^5 - 77*a^2*c^11*d^6)*sqr
t(x)*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*
b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140
*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(3/
4))/(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5
*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^
6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)) + 3*(c^3*d^3*x^6
+ 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b
^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^
5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*
a^8*d^8)/(c^15*d^5))^(1/4)*log(c^4*d*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 869
40*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 138
54456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 3
5153041*a^8*d^8)/(c^15*d^5))^(1/4) - (3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*
sqrt(x)) - 3*(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536
*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4
*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 766975
44*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)*log(-c^4*d*(-(81*b^8*c
^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 14
```

57946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4) - (3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*sqrt(x)) + 4*(32*a^2*c^2*d - (3*b^2*c^2*d + 42*a*b*c*d^2 - 77*a^2*d^3)*x^4 + (9*b^2*c^3 - 66*a*b*c^2*d + 121*a^2*c*d^2)*x^2)*sqrt(x))/(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**3,x)

[Out] Timed out

Giac [A]
time = 1.41, size = 426, normalized size = 1.06

$$-\frac{2a^2}{3c^3x^{\frac{3}{2}}}+\frac{\sqrt{2}\left(3\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\left(cd^3\right)^{\frac{1}{4}}abcd-77\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^4d^2}+\frac{\sqrt{2}\left(3\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\left(cd^3\right)^{\frac{1}{4}}abcd-77\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^4d^2}+\frac{\sqrt{2}\left(3\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\left(cd^3\right)^{\frac{1}{4}}abcd-77\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128c^4d^2}-\frac{\sqrt{2}\left(3\left(cd^3\right)^{\frac{1}{4}}b^2c^2+42\left(cd^3\right)^{\frac{1}{4}}abcd-77\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128c^4d^2}+\frac{b^2c^2dx^{\frac{5}{2}}+14abcd^2x^{\frac{5}{2}}-15a^2d^3x^{\frac{5}{2}}-3b^2c^3\sqrt{x}+22abc^2d\sqrt{x}-19a^2cd^2\sqrt{x}}{16\left(dx^2+c\right)^2c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="giac")

[Out] -2/3*a^2/(c^3*x^(3/2)) + 1/64*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) + 1/64*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) + 1/128*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^2) - 1/128*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^2) + 1/16*(b^2*c^2*d*x^(5/2) + 14*a*b*c*d^2*x^(5/2) - 15*a^2*d^3*x^(5/2) - 3*b^2*c^3*sqrt(x) + 22*a*b*c^2*d*sqrt(x) - 19*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c^3*d)

Mupad [B]
time = 0.35, size = 1508, normalized size = 3.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x)

[Out] (atan((((x^(1/2)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) - ((3*b

```
^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13
*d^9 + 44040192*a*b*c^14*d^8))/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^
2*d^2 + 42*a*b*c*d)*1i)/(64*(-c)^(15/4)*d^(5/4)) + ((x^(1/2)*(97140736*a^4*
c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c
^10*d^9 + 21331968*a^2*b^2*c^11*d^8) + ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*
d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8))/
(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*1i)/(64*(-c
)^(15/4)*d^(5/4)))/(((x^(1/2)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6
+ 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11
*d^8) - ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 8074
0352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8))/(64*(-c)^(15/4)*d^(5/4)))*(3*b^
2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/(64*(-c)^(15/4)*d^(5/4)) - ((x^(1/2)*(971
40736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 1059717
12*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) + ((3*b^2*c^2 - 77*a^2*d^2 +
 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c
^14*d^8))/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/
(64*(-c)^(15/4)*d^(5/4))))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*1i)/(32*(-
c)^(15/4)*d^(5/4)) - ((2*a^2)/(3*c) - (x^4*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b
*c*d))/(48*c^3) + (x^2*(121*a^2*d^2 + 9*b^2*c^2 - 66*a*b*c*d))/(48*c^2*d))/
(c^2*x^(3/2) + d^2*x^(11/2) + 2*c*d*x^(7/2)) + (atan((((x^(1/2)*(97140736*a
^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*
b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) - ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b
*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8
)*1i)/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/(64*
(-c)^(15/4)*d^(5/4)) + ((x^(1/2)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d
^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c
^11*d^8) + ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 8
0740352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8)*1i)/(64*(-c)^(15/4)*d^(5/4)))
*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/(64*(-c)^(15/4)*d^(5/4)))/(((x^(1/2
)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 1
05971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) - ((3*b^2*c^2 - 77*a^2
*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192
*a*b*c^14*d^8)*1i)/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a
*b*c*d)*1i)/(64*(-c)^(15/4)*d^(5/4)) - ((x^(1/2)*(97140736*a^4*c^9*d^10 + 1
47456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21
331968*a^2*b^2*c^11*d^8) + ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*
b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8)*1i)/(64*(-c)^
(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*1i)/(64*(-c)^(15/4)*
d^(5/4))))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/(32*(-c)^(15/4)*d^(5/4))
```

3.440 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=439

$$\frac{5b^2c^2-9ad(10bc-13ad)}{16c^4d\sqrt{x}}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}-\frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}\left(c+dx^2\right)^2}-\frac{5b^2c^2-9ad(10bc-13ad)}{80c^3d\sqrt{x}\left(c+dx^2\right)}-\frac{(5b^2c^2$$

```
[Out] -2/5*a^2/c/x^(5/2)/(d*x^2+c)^2-1/64*(5*b^2*c^2-9*a*d*(-13*a*d+10*b*c))*arct
an(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(17/4)/d^(3/4)*2^(1/2)+1/64*(5*b^2*
c^2-9*a*d*(-13*a*d+10*b*c))*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(17
/4)/d^(3/4)*2^(1/2)+1/128*(5*b^2*c^2-9*a*d*(-13*a*d+10*b*c))*ln(c^(1/2)+x*d
^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)/d^(3/4)*2^(1/2)-1/128*(5*b
^2*c^2-9*a*d*(-13*a*d+10*b*c))*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)
*x^(1/2))/c^(17/4)/d^(3/4)*2^(1/2)+1/16*(5*b^2*c^2-9*a*d*(-13*a*d+10*b*c))/
c^4/d/x^(1/2)+1/20*(-13*a^2*d^2+10*a*b*c*d-5*b^2*c^2)/c^2/d/(d*x^2+c)^2/x^(
1/2)+1/80*(-5*b^2*c^2+9*a*d*(-13*a*d+10*b*c))/c^3/d/(d*x^2+c)/x^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 435, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {473, 468, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{20c\sqrt{x}\left(c+dx^2\right)^2}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}-\frac{\left(5b^2c^2-9ad(10bc-13ad)\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}d^{3/4}}+\frac{\left(5b^2c^2-9ad(10bc-13ad)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{17/4}d^{3/4}}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\left(c+dx^2\right)}+\frac{\left(5b^2c^2-9ad(10bc-13ad)\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}}-\frac{\left(5b^2c^2-9ad(10bc-13ad)\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}}+\frac{5b^2c^2-9ad(10bc-13ad)}{16c^4d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

```
[Out] (5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))/(16*c^4*d*Sqrt[x]) - (2*a^2)/(5*c*x^(
5/2)*(c + d*x^2)^2) + (10*a*b - (5*b^2*c)/d - (13*a^2*d)/c)/(20*c*Sqrt[x]*(
c + d*x^2)^2) - ((5*b^2)/d - (9*a*(10*b*c - 13*a*d))/c^2)/(80*c*Sqrt[x]*(c
+ d*x^2)) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*a*d*
(10*b*c - 13*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[
2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*d^(3/4)
) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*d^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
```

```
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx = -\frac{2a^2}{5cx^{5/2}(c+dx^2)^2} + \frac{2\int \frac{\frac{1}{2}a(10bc-13ad)+\frac{5}{2}b^2cx^2}{x^{3/2}(c+dx^2)^3}\,dx}{5c}$$

$$= -\frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}\,(c+dx^2)^2} + \frac{1}{40}\left(-\frac{5b^2}{d}+\frac{9a(10bc-13ad)}{c^2}\right)\int$$

$$= -\frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}\,(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\,(c+dx^2)} - \frac{\left(\frac{5b^2}{d}-\frac{9a(10b}{}\right.}{}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}\,(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-1}{c^2}}{80c\sqrt{x}\,(c+d}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}\,(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-1}{c^2}}{80c\sqrt{x}\,(c+d}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}\,(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-1}{c^2}}{80c\sqrt{x}\,(c+d}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}\,(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-1}{c^2}}{80c\sqrt{x}\,(c+d}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}\,(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-1}{c^2}}{80c\sqrt{x}\,(c+d}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} - \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}\,(c+dx^2)^2} - \frac{\frac{5b^2}{d}-\frac{9a(10bc-1}{c^2}}{80c\sqrt{x}\,(c+d}$$

Mathematica [A]

time = 0.83, size = 261, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{c}\,(5b^2c^2x^4(9c+5dx^2)-10abcx^2(32c^2+81cdx^2+45d^2x^4)+a^2(-32c^3+416c^2dx^2+1053cd^2x^4+585d^3x^6))}{x^{5/2}(c+dx^2)^2} - \frac{5\sqrt{2}\,(5b^2c^2-90abcd+117a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{d^{3/4}} - \frac{5\sqrt{2}\,(5b^2c^2-90abcd+117a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{d^{3/4}}}{320c^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

```
[Out] ((4*c^(1/4)*(5*b^2*c^2*x^4*(9*c + 5*d*x^2) - 10*a*b*c*x^2*(32*c^2 + 81*c*d*
x^2 + 45*d^2*x^4) + a^2*(-32*c^3 + 416*c^2*d*x^2 + 1053*c*d^2*x^4 + 585*d^3
```

```
*x^6)))/(x^(5/2)*(c + d*x^2)^2) - (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117*
a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/d
^(3/4) - (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117*a^2*d^2)*ArcTanh[(Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(3/4))/(320*c^(17/4))
```

Maple [A]
time = 0.17, size = 235, normalized size = 0.54

method	result
derivativedivides	$\frac{\frac{2\left(\left(\frac{21}{32}a^2d^3-\frac{13}{16}abc\,d^2+\frac{5}{32}b^2c^2d\right)x^{\frac{7}{2}}+\frac{c\left(25a^2d^2-34abcd+9b^2c^2\right)x^{\frac{3}{2}}}{32}\right)}{\left(d\,x^2+c\right)^2}+\frac{\left(\frac{117}{32}a^2d^2-\frac{45}{16}abcd+\frac{5}{32}b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}}\right.\right.}{}}{c^4}$
default	$\frac{\frac{2\left(\left(\frac{21}{32}a^2d^3-\frac{13}{16}abc\,d^2+\frac{5}{32}b^2c^2d\right)x^{\frac{7}{2}}+\frac{c\left(25a^2d^2-34abcd+9b^2c^2\right)x^{\frac{3}{2}}}{32}\right)}{\left(d\,x^2+c\right)^2}+\frac{\left(\frac{117}{32}a^2d^2-\frac{45}{16}abcd+\frac{5}{32}b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}}\right.\right.}{}}{c^4}$
risch	$-\frac{2a\left(-15ad\,x^2+10c\,x^2b+ac\right)}{5c^4x^{\frac{5}{2}}}+\frac{21x^{\frac{7}{2}}a^2d^3}{16c^4\left(d\,x^2+c\right)^2}-\frac{13x^{\frac{7}{2}}ab\,d^2}{8c^3\left(d\,x^2+c\right)^2}+\frac{5x^{\frac{7}{2}}b^2d}{16c^2\left(d\,x^2+c\right)^2}+\frac{25x^{\frac{3}{2}}a^2d^2}{16c^3\left(d\,x^2+c\right)^2}-\frac{17x^{\frac{3}{2}}abd}{8c^2\left(d\,x^2+c\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^4*(((21/32*a^2*d^3-13/16*a*b*c*d^2+5/32*b^2*c^2*d)*x^(7/2)+1/32*c*(25*a
^2*d^2-34*a*b*c*d+9*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+1/8*(117/32*a^2*d^2-45/16
*a*b*c*d+5/32*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/5*a
^2/c^3/x^(5/2)+2*a*(3*a*d-2*b*c)/c^4/x^(1/2)
```

Maxima [A]
time = 0.52, size = 324, normalized size = 0.74

$$\frac{5\left(5b^2c^2d-90abcd^2+117a^2d^3\right)x^6-32a^2c^3+9\left(5b^2c^3-90abc^2d+117a^2cd^2\right)x^4-32\left(10abc^3-13a^2c^2d\right)x^2}{80\left(c^4d^2x^{\frac{13}{2}}+2c^5dx^{\frac{9}{2}}+c^6x^{\frac{5}{2}}\right)}+\frac{\left(5b^2c^2-90abcd+117a^2d^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(5*(5*b^2*c^2*d - 90*a*b*c*d^2 + 117*a^2*d^3)*x^6 - 32*a^2*c^3 + 9*(5*
b^2*c^3 - 90*a*b*c^2*d + 117*a^2*c*d^2)*x^4 - 32*(10*a*b*c^3 - 13*a^2*c^2*d
)*x^2)/(c^4*d^2*x^(13/2) + 2*c^5*d*x^(9/2) + c^6*x^(5/2)) + 1/128*(5*b^2*c^
```

```
2 - 90*a*b*c*d + 117*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4
)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d)
)*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqr
t(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt
(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(
3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))
/(c^(1/4)*d^(3/4)))/c^4
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1832 vs. $2(351) = 702$.
time = 1.14, size = 1832, normalized size = 4.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(20*(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b
^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a
^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 57
6580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*arctan((sqrt((15
625*b^12*c^12 - 1687500*a*b^11*c^11*d + 78131250*a^2*b^10*c^10*d^2 - 201993
7500*a^3*b^9*c^9*d^3 + 31839834375*a^4*b^8*c^8*d^4 - 314326575000*a^5*b^7*c
^7*d^5 + 1936382557500*a^6*b^6*c^6*d^6 - 7355241855000*a^7*b^5*c^5*d^7 + 17
434219710375*a^8*b^4*c^4*d^8 - 25881265273500*a^9*b^3*c^3*d^9 + 23425464012
210*a^10*b^2*c^2*d^10 - 11839219392780*a^11*b*c*d^11 + 2565164201769*a^12*d
^12)*x - (625*b^8*c^17*d - 45000*a*b^7*c^16*d^2 + 1273500*a^2*b^6*c^15*d^3
 - 17739000*a^3*b^5*c^14*d^4 + 124525350*a^4*b^4*c^13*d^5 - 415092600*a^5*b^
3*c^12*d^6 + 697317660*a^6*b^2*c^11*d^7 - 576580680*a^7*b*c^10*d^8 + 187388
721*a^8*c^9*d^9)*sqrt(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c
^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a
^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388
721*a^8*d^8)/(c^17*d^3)))*c^4*d*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 127350
0*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 -
415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d
^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4) - (125*b^6*c^10*d - 6750*a*b^5*c^
9*d^2 + 130275*a^2*b^4*c^8*d^3 - 1044900*a^3*b^3*c^7*d^4 + 3048435*a^4*b^2*
c^6*d^5 - 3696030*a^5*b*c^5*d^6 + 1601613*a^6*c^4*d^7)*sqrt(x)*(-(625*b^8*c
^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3
 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^
2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4))/(
625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^
5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 6973176
60*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)) - 5*(c^4*d
^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 127350
```

```
0*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 -
415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d
^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*log(c^13*d^2*(-(625*b^8*c^8 - 450
00*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 12452
5350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^
6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(3/4) + (125*b^6
*c^6 - 6750*a*b^5*c^5*d + 130275*a^2*b^4*c^4*d^2 - 1044900*a^3*b^3*c^3*d^3
+ 3048435*a^4*b^2*c^2*d^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d^6)*sqrt(x))
 + 5*(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b^7*c^7
*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4
*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 57658068
0*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*log(-c^13*d^2*(-(625*b
^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5
*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^
6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(3/4
) + (125*b^6*c^6 - 6750*a*b^5*c^5*d + 130275*a^2*b^4*c^4*d^2 - 1044900*a^3*
b^3*c^3*d^3 + 3048435*a^4*b^2*c^2*d^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d
^6)*sqrt(x)) - 4*(5*(5*b^2*c^2*d - 90*a*b*c*d^2 + 117*a^2*d^3)*x^6 - 32*a^2
*c^3 + 9*(5*b^2*c^3 - 90*a*b*c^2*d + 117*a^2*c*d^2)*x^4 - 32*(10*a*b*c^3 -
13*a^2*c^2*d)*x^2)*sqrt(x))/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.63, size = 444, normalized size = 1.01

$$\frac{5\,b^2c^2dx^{\frac{7}{2}}-26\,abcd^2x^{\frac{7}{2}}+21\,a^2d^3x^{\frac{7}{2}}+9\,b^2c^3x^{\frac{3}{2}}-34\,abc^2dx^{\frac{3}{2}}+25\,a^2cd^2x^{\frac{3}{2}}}{16\,(dx^2+c)^2c^4}-\frac{2\,(10\,abcx^2-15\,a^2dx^2+a^2c)}{5\,c^4x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2-90\,(cd^3)^{\frac{3}{4}}abcd+117\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^5d^3}+\frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2-90\,(cd^3)^{\frac{3}{4}}abcd+117\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^5d^3}-\frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2-90\,(cd^3)^{\frac{3}{4}}abcd+117\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^5d^3}+\frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2-90\,(cd^3)^{\frac{3}{4}}abcd+117\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*b^2*c^2*d*x^(7/2) - 26*a*b*c*d^2*x^(7/2) + 21*a^2*d^3*x^(7/2) + 9*b
^2*c^3*x^(3/2) - 34*a*b*c^2*d*x^(3/2) + 25*a^2*c*d^2*x^(3/2))/((d*x^2 + c)^
2*c^4) - 2/5*(10*a*b*c*x^2 - 15*a^2*d*x^2 + a^2*c)/(c^4*x^(5/2)) + 1/64*sqr
t(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c^5*d^3) + 1/64*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*
c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
```

```
 2*sqrt(x))/(c/d)^(1/4))/(c^5*d^3) - 1/128*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2
 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(c^5*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(3/4)*b
^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^5*d^3)
```

Mupad [B]
time = 0.13, size = 208, normalized size = 0.47

$$\frac{\frac{9x^4\left(117a^2d^2-90abcd+5b^2c^2\right)}{80c^3}-\frac{2a^2}{5c}+\frac{2ax^2\left(13ad-10bc\right)}{5c^2}+\frac{dx^6\left(117a^2d^2-90abcd+5b^2c^2\right)}{16c^4}}{c^2x^{5/2}+d^2x^{13/2}+2cdx^{9/2}}+\frac{\mathrm{atan}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)\left(117a^2d^2-90abcd+5b^2c^2\right)}{32\,(-c)^{17/4}\,d^{3/4}}-\frac{\mathrm{atanh}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)\left(117a^2d^2-90abcd+5b^2c^2\right)}{32\,(-c)^{17/4}\,d^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x)
```

```
[Out] ((9*x^4*(117*a^2*d^2 + 5*b^2*c^2 - 90*a*b*c*d))/(80*c^3) - (2*a^2)/(5*c) +
(2*a*x^2*(13*a*d - 10*b*c))/(5*c^2) + (d*x^6*(117*a^2*d^2 + 5*b^2*c^2 - 90*
a*b*c*d))/(16*c^4))/(c^2*x^(5/2) + d^2*x^(13/2) + 2*c*d*x^(9/2)) + (atan((d
^(1/4)*x^(1/2))/(-c)^(1/4))*(117*a^2*d^2 + 5*b^2*c^2 - 90*a*b*c*d))/(32*(-c
)^(17/4)*d^(3/4)) - (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(117*a^2*d^2 + 5*b
^2*c^2 - 90*a*b*c*d))/(32*(-c)^(17/4)*d^(3/4))
```

3.441 $\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=328

$$\frac{2(bc-ad)^3x^{3/2}}{3b^4}+\frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{7/2}}{7b^3}+\frac{2d^2(3bc-ad)x^{11/2}}{11b^2}+\frac{2d^3x^{15/2}}{15b}+\frac{a^{3/4}(bc-ad)^3\tan^{-1}\left(1-\right.}{\sqrt{2}\,b^{19/4}}$$

```
[Out] 2/3*(-a*d+b*c)^3*x^(3/2)/b^4+2/7*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^(7/2)/b^
3+2/11*d^2*(-a*d+3*b*c)*x^(11/2)/b^2+2/15*d^3*x^(15/2)/b+1/2*a^(3/4)*(-a*d+
b*c)^3*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(19/4)*2^(1/2)-1/2*a^(3/
4)*(-a*d+b*c)^3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(19/4)*2^(1/2)-
1/4*a^(3/4)*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/
2))/b^(19/4)*2^(1/2)+1/4*a^(3/4)*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*
b^(1/4)*2^(1/2)*x^(1/2))/b^(19/4)*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {472, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{a^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,b^{19/4}}-\frac{a^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,b^{19/4}}-\frac{a^{3/4}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{19/4}}+\frac{2dx^{7/2}(a^2d^2-3abcd+3b^2c^2)}{7b^3}+\frac{2x^{3/2}(bc-ad)^3}{3b^4}+\frac{2d^2x^{11/2}(3bc-ad)}{11b^2}+\frac{2d^3x^{15/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*(b*c - a*d)^3*x^(3/2))/(3*b^4) + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*
x^(7/2))/(7*b^3) + (2*d^2*(3*b*c - a*d)*x^(11/2))/(11*b^2) + (2*d^3*x^(15/2
))/(15*b) + (a^(3/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(19/4)) +
(a^(3/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(2*Sqrt[2]*b^(19/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{b^3} + \frac{d^2(3bc-ad)x^{9/2}}{b^2} + \frac{d^3x^{13/2}}{b} + \frac{(b^3c^3-3ab^2c^2d+}{b^3\,(a}\right.$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} + \frac{(bc-ad)^3\int\frac{x^{5/2}}{a+bx^2}}{b^3}$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} -$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} -$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} +$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} -$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} -$$

$$= \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} +$$

Mathematica [A]
time = 0.25, size = 230, normalized size = 0.70

$$\frac{2x^{3/2}(-385a^3d^3+165a^2bd^2(7c+dx^2)-15ab^2d(77c^2+33cdx^2+7d^2x^4)+b^3(385c^3+495c^2dx^2+315cd^2x^4+77d^3x^6))}{1155b^4} - \frac{a^{3/4}(-bc+ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,b^{19/4}} + \frac{a^{3/4}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{19/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2)*(-385*a^3*d^3 + 165*a^2*b*d^2*(7*c + d*x^2) - 15*a*b^2*d*(77*c^2
 + 33*c*d*x^2 + 7*d^2*x^4) + b^3*(385*c^3 + 495*c^2*d*x^2 + 315*c*d^2*x^4 +
 77*d^3*x^6)))/(1155*b^4) - (a^(3/4)*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a] - Sqr
t[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(19/4)) + (a^(3/4)*(
b*c - a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x
)])/(Sqrt[2]*b^(19/4))
```

Maple [A]
time = 0.09, size = 259, normalized size = 0.79

method	result
derivativedivides	$-\frac{2\left(-\frac{d^3 x^{\frac{15}{2}} b^3}{15}+\frac{\left(a\,b^2 d^3-3b^3 c\,d^2\right)x^{\frac{11}{2}}}{11}+\frac{\left(-a^2 b\,d^3+3a\,b^2 c\,d^2-3b^3 c^2 d\right)x^{\frac{7}{2}}}{7}+\frac{\left(a^3 d^3-3a^2 bc\,d^2+3a\,b^2 c^2 d-b^3 c^3\right)x^{\frac{3}{2}}}{3}\right)}{b^4}+\frac{a(a}{}$
default	$-\frac{2\left(-\frac{d^3 x^{\frac{15}{2}} b^3}{15}+\frac{\left(a\,b^2 d^3-3b^3 c\,d^2\right)x^{\frac{11}{2}}}{11}+\frac{\left(-a^2 b\,d^3+3a\,b^2 c\,d^2-3b^3 c^2 d\right)x^{\frac{7}{2}}}{7}+\frac{\left(a^3 d^3-3a^2 bc\,d^2+3a\,b^2 c^2 d-b^3 c^3\right)x^{\frac{3}{2}}}{3}\right)}{b^4}+\frac{a(a}{}$
risch	$-\frac{2x^{\frac{3}{2}}\left(-77b^3 d^3 x^6+105a\,b^2 d^3 x^4-315b^3 c\,d^2 x^4-165a^2 b\,d^3 x^2+495a\,b^2 c\,d^2 x^2-495b^3 c^2 d\,x^2+385a^3 d^3-1155a^2 bc\,d^2+115\right.}{1155b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^4*(-1/15*d^3*x^(15/2)*b^3+1/11*(a*b^2*d^3-3*b^3*c*d^2)*x^(11/2)+1/7*(-
a^2*b*d^3+3*a*b^2*c*d^2-3*b^3*c^2*d)*x^(7/2)+1/3*(a^3*d^3-3*a^2*b*c*d^2+3*a
*b^2*c^2*d-b^3*c^3)*x^(3/2))+1/4*a*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)/b^5/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.51, size = 331, normalized size = 1.01

$$-\frac{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,b^4}+\frac{2\left(77\,b^3d^3x^{\frac{15}{2}}+105\left(3\,b^3cd^2-ab^2d^3\right)x^{\frac{11}{2}}+165\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x^{\frac{7}{2}}+385\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^{\frac{3}{2}}\right)}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(2*sqrt(2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(
```

```
sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(s
qrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 + 2/1155*(77*b^3*d^3
*x^(15/2) + 105*(3*b^3*c*d^2 - a*b^2*d^3)*x^(11/2) + 165*(3*b^3*c^2*d - 3*a
*b^2*c*d^2 + a^2*b*d^3)*x^(7/2) + 385*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3)*x^(3/2))/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2528 vs. $2(245) = 490$.
time = 0.82, size = 2528, normalized size = 7.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

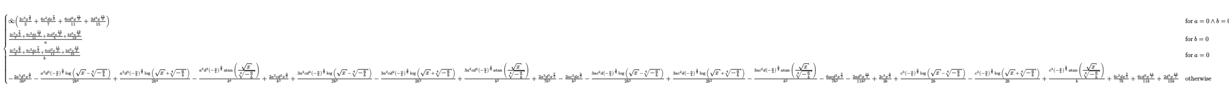
```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2310*(4620*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*
d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924
*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b
^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/
4)*arctan((sqrt((a^4*b^18*c^18 - 18*a^5*b^17*c^17*d + 153*a^6*b^16*c^16*d^2
 - 816*a^7*b^15*c^15*d^3 + 3060*a^8*b^14*c^14*d^4 - 8568*a^9*b^13*c^13*d^5
+ 18564*a^10*b^12*c^12*d^6 - 31824*a^11*b^11*c^11*d^7 + 43758*a^12*b^10*c^1
0*d^8 - 48620*a^13*b^9*c^9*d^9 + 43758*a^14*b^8*c^8*d^10 - 31824*a^15*b^7*c
^7*d^11 + 18564*a^16*b^6*c^6*d^12 - 8568*a^17*b^5*c^5*d^13 + 3060*a^18*b^4*
c^4*d^14 - 816*a^19*b^3*c^3*d^15 + 153*a^20*b^2*c^2*d^16 - 18*a^21*b*c*d^17
 + a^22*d^18)*x - (a^3*b^21*c^12 - 12*a^4*b^20*c^11*d + 66*a^5*b^19*c^10*d^
2 - 220*a^6*b^18*c^9*d^3 + 495*a^7*b^17*c^8*d^4 - 792*a^8*b^16*c^7*d^5 + 92
4*a^9*b^15*c^6*d^6 - 792*a^10*b^14*c^5*d^7 + 495*a^11*b^13*c^4*d^8 - 220*a^
12*b^12*c^3*d^9 + 66*a^13*b^11*c^2*d^10 - 12*a^14*b^10*c*d^11 + a^15*b^9*d^
12)*sqrt(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*
a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c
^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9
 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19))*b^5*(-(a^3*b
^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3
+ 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^1
0*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c
^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4) + (a^2*b^14*c^9 - 9*a^3
*b^13*c^8*d + 36*a^4*b^12*c^7*d^2 - 84*a^5*b^11*c^6*d^3 + 126*a^6*b^10*c^5*
d^4 - 126*a^7*b^9*c^4*d^5 + 84*a^8*b^8*c^3*d^6 - 36*a^9*b^7*c^2*d^7 + 9*a^1
0*b^6*c*d^8 - a^11*b^5*d^9)*sqrt(x)*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d +
 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8
*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^
4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^
```

```
15*d^12)/b^19)^(1/4))/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^1
0*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 9
24*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12
*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)) - 1155
*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6
*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*
d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 +
66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4)*log(b^14*(
-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c
^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 -
792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^1
3*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(3/4) - (a^2*b^9*c^9 -
 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5*c^
5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^7 + 9*a
^10*b*c*d^8 - a^11*d^9)*sqrt(x)) + 1155*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*
c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 -
 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^1
1*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d
^11 + a^15*d^12)/b^19)^(1/4)*log(-b^14*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*
d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*
a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4
*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 +
 a^15*d^12)/b^19)^(3/4) - (a^2*b^9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d
^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5*c^5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^
8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^7 + 9*a^10*b*c*d^8 - a^11*d^9)*sqrt(x)) -
4*(77*b^3*d^3*x^7 + 105*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 165*(3*b^3*c^2*d -
3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 + 385*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)*x)*sqrt(x))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(311) = 622$.

time = 87.96, size = 743, normalized size = 2.27



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(2*c**3*x**(3/2)/3 + 6*c**2*d*x**(7/2)/7 + 6*c*d**2*x**(11/2
)/11 + 2*d**3*x**(15/2)/15), Eq(a, 0) & Eq(b, 0)), ((2*c**3*x**(7/2)/7 + 6*
c**2*d*x**(11/2)/11 + 2*c*d**2*x**(15/2)/5 + 2*d**3*x**(19/2)/19)/a, Eq(b,
0)), ((2*c**3*x**(3/2)/3 + 6*c**2*d*x**(7/2)/7 + 6*c*d**2*x**(11/2)/11 + 2*
d**3*x**(15/2)/15)/b, Eq(a, 0)), (-2*a**3*d**3*x**(3/2)/(3*b**4) - a**3*d**
3*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**4) + a**3*d**3*(-a/b)**(
3/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b**4) - a**3*d**3*(-a/b)**(3/4)*atan(s
```

```
qrt(x)/(-a/b)**(1/4))/b**4 + 2*a**2*c*d**2*x**(3/2)/b**3 + 3*a**2*c*d**2*(-
a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**3) - 3*a**2*c*d**2*(-a/b)**(
3/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b**3) + 3*a**2*c*d**2*(-a/b)**(3/4)*at
an(sqrt(x)/(-a/b)**(1/4))/b**3 + 2*a**2*d**3*x**(7/2)/(7*b**3) - 2*a*c**2*d
*x**(3/2)/b**2 - 3*a*c**2*d*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b
**2) + 3*a*c**2*d*(-a/b)**(3/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b**2) - 3*a
*c**2*d*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 - 6*a*c*d**2*x**(7/2
)/(7*b**2) - 2*a*d**3*x**(11/2)/(11*b**2) + 2*c**3*x**(3/2)/(3*b) + c**3*(-
a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b) - c**3*(-a/b)**(3/4)*log(sqr
t(x) + (-a/b)**(1/4))/(2*b) + c**3*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4)
)/b + 6*c**2*d*x**(7/2)/(7*b) + 6*c*d**2*x**(11/2)/(11*b) + 2*d**3*x**(15/2
)/(15*b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(245) = 490.

time = 2.07, size = 531, normalized size = 1.62

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{7}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{7}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{7}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{7}}+\frac{2\left(77b^{14}d^{3}x^{\frac{15}{2}}+315b^{14}cd^{2}x^{\frac{11}{2}}-105ab^{13}d^{3}x^{\frac{11}{2}}+495b^{14}c^{2}dx^{\frac{7}{2}}-495ab^{13}cd^{2}x^{\frac{7}{2}}+165a^{2}b^{12}d^{3}x^{\frac{7}{2}}+385b^{14}c^{3}x^{\frac{3}{2}}-1155ab^{13}c^{2}dx^{\frac{3}{2}}+1155a^{2}b^{12}cd^{2}x^{\frac{3}{2}}-385a^{3}b^{11}d^{3}x^{\frac{3}{2}}\right)}{1155b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^
3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^7 - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c
^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3
/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/b^7 + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/b^7 - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*
(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3
*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^7 + 2/1155*(77*b^
14*d^3*x^(15/2) + 315*b^14*c*d^2*x^(11/2) - 105*a*b^13*d^3*x^(11/2) + 495*b
^14*c^2*d*x^(7/2) - 495*a*b^13*c*d^2*x^(7/2) + 165*a^2*b^12*d^3*x^(7/2) + 3
85*b^14*c^3*x^(3/2) - 1155*a*b^13*c^2*d*x^(3/2) + 1155*a^2*b^12*c*d^2*x^(3/
2) - 385*a^3*b^11*d^3*x^(3/2))/b^15
```

Mupad [B]

time = 0.21, size = 634, normalized size = 1.93

$$x^{3/2}\left(\frac{2c^3}{3b}-\frac{a\left(\frac{6c^2d}{b}+\frac{a\left(\frac{2ad^3}{b^2}-\frac{6cd^2}{b}\right)}{b}\right)}{3b}\right)-x^{11/2}\left(\frac{2ad^3}{11b^2}-\frac{6cd^2}{11b}\right)+x^{7/2}\left(\frac{6c^2d}{7b}+\frac{a\left(\frac{2ad^3}{b^2}-\frac{6cd^2}{b}\right)}{7b}\right)+\frac{2d^3x^{15/2}}{15b}+\frac{(-a)^{3/4}\,\mathrm{atan}\left(\frac{(-a)^{3/4}b^{1/4}\sqrt{x}\,(ad-bc)^3\left(a^9d^6-6a^8bcd^5+15a^7b^2c^2d^4-20a^6b^3c^3d^3+15a^5b^4c^4d^2-6a^4b^5c^5d+a^3b^6c^6\right)}{a^{13}d^9-9a^{12}bcd^8+36a^{11}b^2c^2d^7-84a^{10}b^3c^3d^6+126a^9b^4c^4d^5-126a^8b^5c^5d^4+84a^7b^6c^6d^3-36a^6b^7c^7d^2+9a^5b^8c^8d-a^4b^9c^9}\right)(ad-bc)^3}{b^{19/4}}+\frac{(-a)^{3/4}\,\mathrm{atan}\left(\frac{(-a)^{3/4}b^{1/4}\sqrt{x}\,(ad-bc)^3\left(a^9d^6-6a^8bcd^5+15a^7b^2c^2d^4-20a^6b^3c^3d^3+15a^5b^4c^4d^2-6a^4b^5c^5d+a^3b^6c^6\right)1\mathrm{i}}{a^{13}d^9-9a^{12}bcd^8+36a^{11}b^2c^2d^7-84a^{10}b^3c^3d^6+126a^9b^4c^4d^5-126a^8b^5c^5d^4+84a^7b^6c^6d^3-36a^6b^7c^7d^2+9a^5b^8c^8d-a^4b^9c^9}\right)(ad-bc)^3\,1\mathrm{i}}{b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^(3/2)*((2*c^3)/(3*b) - (a*((6*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b)
)/b))/(3*b)) - x^(11/2)*((2*a*d^3)/(11*b^2) - (6*c*d^2)/(11*b)) + x^(7/2)*(
(6*c^2*d)/(7*b) + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/(7*b)) + (2*d^3*x^(15/2
))/(15*b) + ((-a)^(3/4)*atan(((-a)^(3/4)*b^(1/4)*x^(1/2)*(a*d - b*c)^3*(a^9
*d^6 + a^3*b^6*c^6 - 6*a^4*b^5*c^5*d + 15*a^5*b^4*c^4*d^2 - 20*a^6*b^3*c^3*
d^3 + 15*a^7*b^2*c^2*d^4 - 6*a^8*b*c*d^5))/(a^13*d^9 - a^4*b^9*c^9 + 9*a^5*
b^8*c^8*d - 36*a^6*b^7*c^7*d^2 + 84*a^7*b^6*c^6*d^3 - 126*a^8*b^5*c^5*d^4 +
 126*a^9*b^4*c^4*d^5 - 84*a^10*b^3*c^3*d^6 + 36*a^11*b^2*c^2*d^7 - 9*a^12*b
*c*d^8))*(a*d - b*c)^3)/b^(19/4) + ((-a)^(3/4)*atan(((-a)^(3/4)*b^(1/4)*x^(
1/2)*(a*d - b*c)^3*(a^9*d^6 + a^3*b^6*c^6 - 6*a^4*b^5*c^5*d + 15*a^5*b^4*c^
4*d^2 - 20*a^6*b^3*c^3*d^3 + 15*a^7*b^2*c^2*d^4 - 6*a^8*b*c*d^5)*1i)/(a^13*
d^9 - a^4*b^9*c^9 + 9*a^5*b^8*c^8*d - 36*a^6*b^7*c^7*d^2 + 84*a^7*b^6*c^6*d
^3 - 126*a^8*b^5*c^5*d^4 + 126*a^9*b^4*c^4*d^5 - 84*a^10*b^3*c^3*d^6 + 36*a
^11*b^2*c^2*d^7 - 9*a^12*b*c*d^8))*(a*d - b*c)^3*1i)/b^(19/4)
```

3.442 $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=326

$$\frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}+\frac{\sqrt[4]{a}\,(bc-ad)^3\tan^{-1}\left(1-\right.}{\sqrt{2}\,b^{17/4}}$$

```
[Out] 2/5*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^(5/2)/b^3+2/9*d^2*(-a*d+3*b*c)*x^(9/2
)/b^2+2/13*d^3*x^(13/2)/b+1/2*a^(1/4)*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1/2)
*x^(1/2)/a^(1/4))/b^(17/4)*2^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*arctan(1+b^(1/4
)*2^(1/2)*x^(1/2)/a^(1/4))/b^(17/4)*2^(1/2)+1/4*a^(1/4)*(-a*d+b*c)^3*ln(a^(
1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2)-1/4*a^(1/4
)*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(17/
4)*2^(1/2)+2*(-a*d+b*c)^3*x^(1/2)/b^4
```

Rubi [A]
time = 0.19, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {472, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{2dx^{5/2}(a^2d^2-3abcd+3b^2c^2)}{5b^3}+\frac{\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,b^{17/4}}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,b^{17/4}}+\frac{\sqrt[4]{a}\,(bc-ad)^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{17/4}}-\frac{\sqrt[4]{a}\,(bc-ad)^3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{17/4}}+\frac{2\sqrt{x}\,(bc-ad)^3}{b^4}+\frac{2d^2x^{9/2}(3bc-ad)}{9b^2}+\frac{2d^3x^{13/2}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*(b*c - a*d)^3*Sqrt[x])/b^4 + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(5
/2))/(5*b^3) + (2*d^2*(3*b*c - a*d)*x^(9/2))/(9*b^2) + (2*d^3*x^(13/2))/(13
*b) + (a^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(17/4)) - (a^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*b^(17/4)) + (a^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(17/4)) - (a^(1/
4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^{3/2}}{b^3} + \frac{d^2(3bc-ad)x^{7/2}}{b^2} + \frac{d^3x^{11/2}}{b} + \frac{(b^3c^3-3ab^2c^2d+}{b^3\,(a}\right.$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{5b^3} + \frac{2d^2(3bc-ad)x^{9/2}}{9b^2} + \frac{2d^3x^{13/2}}{13b} + \frac{(bc-ad)^3\int \frac{x^{3/2}}{a+bx^2}\,d}{b^3}$$

$$= \frac{2(bc-ad)^3\sqrt{x}}{b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{5b^3} + \frac{2d^2(3bc-ad)x^{9/2}}{9b^2} + \frac{2d^3x^{13/2}}{13b} -$$

$$= \frac{2(bc-ad)^3\sqrt{x}}{b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{5b^3} + \frac{2d^2(3bc-ad)x^{9/2}}{9b^2} + \frac{2d^3x^{13/2}}{13b} -$$

$$= \frac{2(bc-ad)^3\sqrt{x}}{b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{5b^3} + \frac{2d^2(3bc-ad)x^{9/2}}{9b^2} + \frac{2d^3x^{13/2}}{13b} -$$

$$= \frac{2(bc-ad)^3\sqrt{x}}{b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{5b^3} + \frac{2d^2(3bc-ad)x^{9/2}}{9b^2} + \frac{2d^3x^{13/2}}{13b} -$$

$$= \frac{2(bc-ad)^3\sqrt{x}}{b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{5b^3} + \frac{2d^2(3bc-ad)x^{9/2}}{9b^2} + \frac{2d^3x^{13/2}}{13b} +$$

$$= \frac{2(bc-ad)^3\sqrt{x}}{b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{5b^3} + \frac{2d^2(3bc-ad)x^{9/2}}{9b^2} + \frac{2d^3x^{13/2}}{13b} +$$

Mathematica [A]
time = 0.25, size = 231, normalized size = 0.71

$$\frac{2\sqrt{x}\,(-585a^3d^3+117a^2bd^2(15c+dx^2)-13ab^2d(135c^2+27cdx^2+5d^2x^4)+3b^3(195c^3+117c^2dx^2+65cd^2x^4+15d^3x^6))}{585b^4} - \frac{\sqrt[4]{a}\,(-bc+ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,b^{17/4}} + \frac{\sqrt[4]{a}\,(-bc+ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

[Out] (2*Sqrt[x]*(-585*a^3*d^3 + 117*a^2*b*d^2*(15*c + d*x^2) - 13*a*b^2*d*(135*c
^2 + 27*c*d*x^2 + 5*d^2*x^4) + 3*b^3*(195*c^3 + 117*c^2*d*x^2 + 65*c*d^2*x^
4 + 15*d^3*x^6)))/(585*b^4) - (a^(1/4)*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a] - S
qrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(17/4)) + (a^(1/4)
*(-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)])/(Sqrt[2]*b^(17/4))

Maple [A]
time = 0.09, size = 268, normalized size = 0.82

method	result
derivativedivides	$-\frac{2\left(-\frac{d^{3} x^{\frac{13}{2}} b^{3}}{13}+\frac{a\,b^{2} d^{3} x^{\frac{9}{2}}}{9}-\frac{b^{3} c\,d^{2} x^{\frac{9}{2}}}{3}-\frac{a^{2} b\,d^{3} x^{\frac{5}{2}}}{5}+\frac{3 a\,b^{2} c\,d^{2} x^{\frac{5}{2}}}{5}-\frac{3 b^{3} c^{2} d\,x^{\frac{5}{2}}}{5}+a^{3} d^{3} \sqrt{x}-3 a^{2} b c\,d^{2} \sqrt{x}+3 a\,b^{2} c^{2} d\sqrt{x}}{b^{4}}$
default	$-\frac{2\left(-\frac{d^{3} x^{\frac{13}{2}} b^{3}}{13}+\frac{a\,b^{2} d^{3} x^{\frac{9}{2}}}{9}-\frac{b^{3} c\,d^{2} x^{\frac{9}{2}}}{3}-\frac{a^{2} b\,d^{3} x^{\frac{5}{2}}}{5}+\frac{3 a\,b^{2} c\,d^{2} x^{\frac{5}{2}}}{5}-\frac{3 b^{3} c^{2} d\,x^{\frac{5}{2}}}{5}+a^{3} d^{3} \sqrt{x}-3 a^{2} b c\,d^{2} \sqrt{x}+3 a\,b^{2} c^{2} d\sqrt{x}}{b^{4}}$
risch	$-\frac{2\left(-45 b^{3} d^{3} x^{6}+65 a\,b^{2} d^{3} x^{4}-195 b^{3} c\,d^{2} x^{4}-117 a^{2} b\,d^{3} x^{2}+351 a\,b^{2} c\,d^{2} x^{2}-351 b^{3} c^{2} d\,x^{2}+585 a^{3} d^{3}-1755 a^{2} b c\,d^{2}+1755 a\,b}{585 b^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -2/b^4*(-1/13*d^3*x^(13/2)*b^3+1/9*a*b^2*d^3*x^(9/2)-1/3*b^3*c*d^2*x^(9/2)-
1/5*a^2*b*d^3*x^(5/2)+3/5*a*b^2*c*d^2*x^(5/2)-3/5*b^3*c^2*d*x^(5/2)+a^3*d^3
*x^(1/2)-3*a^2*b*c*d^2*x^(1/2)+3*a*b^2*c^2*d*x^(1/2)-b^3*c^3*x^(1/2))+1/4*(
a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^4*(a/b)^(1/4)*2^(1/2)*(ln((x
+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)-1))

Maxima [A]
time = 0.55, size = 437, normalized size = 1.34

$$-\frac{\left(\frac{2 \sqrt{2} \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}+\frac{2 \sqrt{2} \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}+\frac{\sqrt{2} \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{1}{4}}}-\frac{\sqrt{2} \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{1}{4}}}\right) a}{4 b^{4}}+\frac{2 \left(45 b^{3} d^{3} x^{\frac{13}{2}}+65 \left(3 b^{3} c d^{2}-a b^{2} d^{3}\right) x^{\frac{9}{2}}+117 \left(3 b^{3} c^{2} d-3 a b^{2} c d^{2}+a^{2} b d^{3}\right) x^{\frac{5}{2}}+585 \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) \sqrt{x}\right)}{585 b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")

[Out] -1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(
1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2

```
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^4 + 2/585*(45*b^3*
d^3*x^(13/2) + 65*(3*b^3*c*d^2 - a*b^2*d^3)*x^(9/2) + 117*(3*b^3*c^2*d - 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^(5/2) + 585*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*sqrt(x))/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1898 vs. $2(245) = 490$.
time = 0.63, size = 1898, normalized size = 5.82

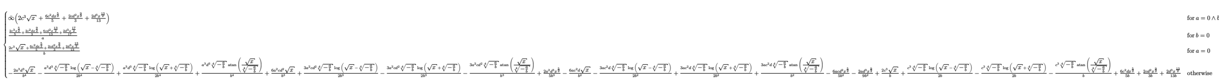
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/1170*(2340*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2
 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^
7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^
3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*ar
ctan((sqrt(b^8*sqrt(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d
^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*
a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*
c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17) + (b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*b^13*(-(a*b^12*c^12 - 12*a^2*b^11*c^
11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 7
92*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^
4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11
+ a^13*d^12)/b^17)^(3/4) + (b^16*c^3 - 3*a*b^15*c^2*d + 3*a^2*b^14*c*d^2 -
a^3*b^13*d^3)*sqrt(x)*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^1
0*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 9
24*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b
^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(3/
4))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*
c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 -
 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11
*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)) + 585*b^4*(-(a*b^12*c^12 - 1
2*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^
8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7
 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a
^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*log(b^4*(-(a*b^12*c^12 - 12*a^2*b^11*
c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 -
```

```
 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*
b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^1
1 + a^13*d^12)/b^17)^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(x)) - 585*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*
c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5
+ 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^1
0*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^
(1/4)*log(-b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 -
 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*
b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*
d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4) - (b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 4*(45*b^3*d^3*
x^6 + 585*b^3*c^3 - 1755*a*b^2*c^2*d + 1755*a^2*b*c*d^2 - 585*a^3*d^3 + 65*
(3*b^3*c*d^2 - a*b^2*d^3)*x^4 + 117*(3*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^
3)*x^2)*sqrt(x))/b^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(309) = 618$.
time = 37.78, size = 736, normalized size = 2.26



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(2*c**3*sqrt(x) + 6*c**2*d*x**(5/2)/5 + 2*c*d**2*x**(9/2)/3
+ 2*d**3*x**(13/2)/13), Eq(a, 0) & Eq(b, 0)), ((2*c**3*x**(5/2)/5 + 2*c**2*
d*x**(9/2)/3 + 6*c*d**2*x**(13/2)/13 + 2*d**3*x**(17/2)/17)/a, Eq(b, 0)), (
(2*c**3*sqrt(x) + 6*c**2*d*x**(5/2)/5 + 2*c*d**2*x**(9/2)/3 + 2*d**3*x**(13
/2)/13)/b, Eq(a, 0)), (-2*a**3*d**3*sqrt(x)/b**4 - a**3*d**3*(-a/b)**(1/4)*
log(sqrt(x) - (-a/b)**(1/4))/(2*b**4) + a**3*d**3*(-a/b)**(1/4)*log(sqrt(x)
 + (-a/b)**(1/4))/(2*b**4) + a**3*d**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(
1/4))/b**4 + 6*a**2*c*d**2*sqrt(x)/b**3 + 3*a**2*c*d**2*(-a/b)**(1/4)*log(s
qrt(x) - (-a/b)**(1/4))/(2*b**3) - 3*a**2*c*d**2*(-a/b)**(1/4)*log(sqrt(x)
+ (-a/b)**(1/4))/(2*b**3) - 3*a**2*c*d**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)
**(1/4))/b**3 + 2*a**2*d**3*x**(5/2)/(5*b**3) - 6*a*c**2*d*sqrt(x)/b**2 - 3
*a*c**2*d*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**2) + 3*a*c**2*d*
(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b**2) + 3*a*c**2*d*(-a/b)**(1
/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 - 6*a*c*d**2*x**(5/2)/(5*b**2) - 2*a*d
**3*x**(9/2)/(9*b**2) + 2*c**3*sqrt(x)/b + c**3*(-a/b)**(1/4)*log(sqrt(x) -
 (-a/b)**(1/4))/(2*b) - c**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*
b) - c**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b + 6*c**2*d*x**(5/2)/(
5*b) + 2*c*d**2*x**(9/2)/(3*b) + 2*d**3*x**(13/2)/(13*b), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(245) = 490$.

time = 1.21, size = 531, normalized size = 1.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c
^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1
/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/b^5 - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*
(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3
*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 2/585*(45*b^1
2*d^3*x^(13/2) + 195*b^12*c*d^2*x^(9/2) - 65*a*b^11*d^3*x^(9/2) + 351*b^12*
c^2*d*x^(5/2) - 351*a*b^11*c*d^2*x^(5/2) + 117*a^2*b^10*d^3*x^(5/2) + 585*b
^12*c^3*sqrt(x) - 1755*a*b^11*c^2*d*sqrt(x) + 1755*a^2*b^10*c*d^2*sqrt(x) -
 585*a^3*b^9*d^3*sqrt(x))/b^13
```

Mupad [B]
time = 0.22, size = 1564, normalized size = 4.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^(1/2)*((2*c^3)/b - (a*((6*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/b)
)/b) - x^(9/2)*((2*a*d^3)/(9*b^2) - (2*c*d^2)/(3*b)) + x^(5/2)*((6*c^2*d)/(
5*b) + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/(5*b)) + (2*d^3*x^(13/2))/(13*b) +
 ((-a)^(1/4)*atan((((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*
b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 -
6*a^7*b*c*d^5))/b^5 - (16*(-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 +
 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))/b^(21/4))*(a*d - b*c)^3*1i)/(2*b^(17/4))
 + ((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a
^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/
b^5 + (16*(-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d
 - 3*a^4*b*c*d^2))/b^(21/4))*(a*d - b*c)^3*1i)/(2*b^(17/4)))/(((-a)^(1/4)*(
(16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 -
 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 - (16*(-a)^(
1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2
))/b^(21/4))*(a*d - b*c)^3)/(2*b^(17/4)) - ((-a)^(1/4)*((16*x^(1/2)*(a^8*d^
```

```
6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3
 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 + (16*(-a)^(1/4)*(a*d - b*c)^3*
(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))/b^(21/4))*(a*d -
 b*c)^3)/(2*b^(17/4))))*(a*d - b*c)^3*1i)/b^(17/4) + ((-a)^(1/4)*atan((((-a
)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*
c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 - (
(-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b
*c*d^2)*16i)/b^(21/4))*(a*d - b*c)^3)/(2*b^(17/4)) + ((-a)^(1/4)*((16*x^(1/
2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b
^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 + ((-a)^(1/4)*(a*d -
b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)*16i)/b^(21
/4))*(a*d - b*c)^3)/(2*b^(17/4)))/(((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*
b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^
6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 - ((-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 -
a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)*16i)/b^(21/4))*(a*d - b*c)^3
*1i)/(2*b^(17/4)) - ((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3
*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 -
 6*a^7*b*c*d^5))/b^5 + ((-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3
*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)*16i)/b^(21/4))*(a*d - b*c)^3*1i)/(2*b^(17/4
))))*(a*d - b*c)^3)/b^(17/4)
```

3.443 $\int \frac{\sqrt{x}\,(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=306

$$\frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{3/2}}{3b^3}+\frac{2d^2(3bc-ad)x^{7/2}}{7b^2}+\frac{2d^3x^{11/2}}{11b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}+\frac{(bc-a}{}$$

```
[Out] 2/3*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^(3/2)/b^3+2/7*d^2*(-a*d+3*b*c)*x^(7/2
)/b^2+2/11*d^3*x^(11/2)/b-1/2*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)
/a^(1/4))/a^(1/4)/b^(15/4)*2^(1/2)+1/2*(-a*d+b*c)^3*arctan(1+b^(1/4)*2^(1/2
)*x^(1/2)/a^(1/4))/a^(1/4)/b^(15/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b
^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(15/4)*2^(1/2)-1/4*(-a*d+
b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(15/
4)*2^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {472, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{2dx^{3/2}(a^2d^2-3abcd+3b^2c^2)}{3b^3}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}+\frac{2d^2x^{7/2}(3bc-ad)}{7b^2}+\frac{2d^3x^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(3/2))/(3*b^3) + (2*d^2*(3*b*c - a
*d)*x^(7/2))/(7*b^2) + (2*d^3*x^(11/2))/(11*b) - ((b*c - a*d)^3*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/4)) + ((b*c - a*
d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/
4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\,(c+dx^2)^3}{a+bx^2}\,dx = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)\sqrt{x}}{b^3} + \frac{d^2(3bc-ad)x^{5/2}}{b^2} + \frac{d^3x^{9/2}}{b} + \frac{(b^3c^3-3ab^2c^2d+}{b^3(a}\right.$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} + \frac{(bc-ad)^3\int\frac{\sqrt{x}}{a+bx^2}\,d}{b^3}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} + \frac{(2(bc-ad)^3)\,\text{Subst}}{}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} - \frac{(bc-ad)^3\text{Subst}\Big(\int}{}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} + \frac{(bc-ad)^3\text{Subst}\Bigg(\int}{}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} + \frac{(bc-ad)^3\log\left(\sqrt{a}\right.}{2}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{3/2}}{3b^3} + \frac{2d^2(3bc-ad)x^{7/2}}{7b^2} + \frac{2d^3x^{11/2}}{11b} - \frac{(bc-ad)^3\tan^{-1}\left(\right.}{\sqrt{2}\,\sqrt[4]{a}}$$

Mathematica [A]
time = 0.22, size = 188, normalized size = 0.61

$$\frac{2dx^{3/2}(77a^2d^2-33abd(7c+dx^2)+3b^2(77c^2+33cdx^2+7d^2x^4))}{231b^3} + \frac{(-bc+ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}} + \frac{(-bc+ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*d*x^(3/2)*(77*a^2*d^2 - 33*a*b*d*(7*c + d*x^2) + 3*b^2*(77*c^2 + 33*c*d*
x^2 + 7*d^2*x^4)))/(231*b^3) + ((-(b*c) + a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b]*
x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(1/4)*b^(15/4)) + ((-(b*c
) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
)/(Sqrt[2]*a^(1/4)*b^(15/4))
```

Maple [A]
time = 0.10, size = 208, normalized size = 0.68

method	result
derivativedivides	$\frac{2d\left(\frac{b^2d^2x^{\frac{11}{2}}}{11}+\frac{\left(-ab\,d^2+3b^2cd\right)x^{\frac{7}{2}}}{7}+\frac{\left(a^2d^2-3abcd+3b^2c^2\right)x^{\frac{3}{2}}}{3}\right)}{b^3}-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)}{x+\left(\frac{a}{b}\right)}\right.\right.}{}$
default	$\frac{2d\left(\frac{b^2d^2x^{\frac{11}{2}}}{11}+\frac{\left(-ab\,d^2+3b^2cd\right)x^{\frac{7}{2}}}{7}+\frac{\left(a^2d^2-3abcd+3b^2c^2\right)x^{\frac{3}{2}}}{3}\right)}{b^3}-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)}{x+\left(\frac{a}{b}\right)}\right.\right.}{}$
risch	$\frac{2\left(21b^2d^2x^4-33ab\,d^2x^2+99b^2cd\,x^2+77a^2d^2-231abcd+231b^2c^2\right)d\,x^{\frac{3}{2}}}{231b^3}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)a^3d^3}{2b^4\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{3\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/b^3*(1/11*b^2*d^2*x^(11/2)+1/7*(-a*b*d^2+3*b^2*c*d)*x^(7/2)+1/3*(a^2*d^
2-3*a*b*c*d+3*b^2*c^2)*x^(3/2))-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)/b^4/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.54, size = 282, normalized size = 0.92

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,b^3}+\frac{2\left(21\,b^2d^3x^{\frac{11}{2}}+33\left(3\,b^2cd^2-abd^3\right)x^{\frac{7}{2}}+77\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x^{\frac{3}{2}}\right)}{231\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(2*sqrt(2)*arctan(1
/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(
2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a
)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^3 + 2/231*(21*b^2*d^3*x^(11
/2) + 33*(3*b^2*c*d^2 - a*b*d^3)*x^(7/2) + 77*(3*b^2*c^2*d - 3*a*b*c*d^2 +
a^2*d^3)*x^(3/2))/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2441 vs. $2(227) = 454$.

time = 0.65, size = 2441, normalized size = 7.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/462*(924*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220
*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*
c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4)*arcta
n((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^1
5*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^
12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9
*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^
12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^1
5*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)*x -
(a*b^19*c^12 - 12*a^2*b^18*c^11*d + 66*a^3*b^17*c^10*d^2 - 220*a^4*b^16*c^9
*d^3 + 495*a^5*b^15*c^8*d^4 - 792*a^6*b^14*c^7*d^5 + 924*a^7*b^13*c^6*d^6 -
 792*a^8*b^12*c^5*d^7 + 495*a^9*b^11*c^4*d^8 - 220*a^10*b^10*c^3*d^9 + 66*a
^11*b^9*c^2*d^10 - 12*a^12*b^8*c*d^11 + a^13*b^7*d^12)*sqrt(-(b^12*c^12 - 1
2*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11
*b*c*d^11 + a^12*d^12)/(a*b^15)))*b^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a*b^15))^(1/4) + (b^13*c^9 - 9*a*b^12*c^8*d + 36*a^2*b^11*c^7*d^2 - 84*a
^3*b^10*c^6*d^3 + 126*a^4*b^9*c^5*d^4 - 126*a^5*b^8*c^4*d^5 + 84*a^6*b^7*c^
3*d^6 - 36*a^7*b^6*c^2*d^7 + 9*a^8*b^5*c*d^8 - a^9*b^4*d^9)*sqrt(x)*(-(b^12
*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495
*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*
c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
- 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4))/(b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)) - 231*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4
)*log(a*b^11*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
```

```
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(3/4) - (b^9*
c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5
*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 +
9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 231*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(a*b^15))^(1/4)*log(-a*b^11*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(a*b^15))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^
6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6
- 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 4*(21*b^2*d^3*x^
5 + 33*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 77*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^
3)*x)*sqrt(x))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(291) = 582$.

time = 18.28, size = 649, normalized size = 2.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/sqrt(x) + 2*c**2*d*x**(3/2) + 6*c*d**2*x**(7/2)/7 +
 2*d**3*x**(11/2)/11), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/sqrt(x) + 2*c**2*d*x
**(3/2) + 6*c*d**2*x**(7/2)/7 + 2*d**3*x**(11/2)/11)/b, Eq(a, 0)), ((2*c**3
*x**(3/2)/3 + 6*c**2*d*x**(7/2)/7 + 6*c*d**2*x**(11/2)/11 + 2*d**3*x**(15/2
)/15)/a, Eq(b, 0)), (2*a**2*d**3*x**(3/2)/(3*b**3) + a**2*d**3*(-a/b)**(3/4
)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**3) - a**2*d**3*(-a/b)**(3/4)*log(sqrt(
x) + (-a/b)**(1/4))/(2*b**3) + a**2*d**3*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)*
*(1/4))/b**3 - 2*a*c*d**2*x**(3/2)/b**2 - 3*a*c*d**2*(-a/b)**(3/4)*log(sqrt
(x) - (-a/b)**(1/4))/(2*b**2) + 3*a*c*d**2*(-a/b)**(3/4)*log(sqrt(x) + (-a/
b)**(1/4))/(2*b**2) - 3*a*c*d**2*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))/
b**2 - 2*a*d**3*x**(7/2)/(7*b**2) + 2*c**2*d*x**(3/2)/b + 3*c**2*d*(-a/b)**
(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b) - 3*c**2*d*(-a/b)**(3/4)*log(sqrt(
x) + (-a/b)**(1/4))/(2*b) + 3*c**2*d*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/
4))/b + 6*c*d**2*x**(7/2)/(7*b) + 2*d**3*x**(11/2)/(11*b) - c**3*(-a/b)**(3
/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) + c**3*(-a/b)**(3/4)*log(sqrt(x) + (
-a/b)**(1/4))/(2*a) - c**3*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))/a, Tru
e))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(227) = 454$.

time = 1.61, size = 490, normalized size = 1.60

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^{6}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^{6}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab^{6}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab^{6}}+\frac{2\left(21b^{10}d^{3}x^{\frac{11}{2}}+99b^{10}cd^{2}x^{\frac{7}{2}}-33ab^{9}d^{3}x^{\frac{7}{2}}+231b^{10}c^{2}dx^{\frac{3}{2}}-231ab^{9}cd^{2}x^{\frac{3}{2}}+77a^{2}b^{8}d^{3}x^{\frac{3}{2}}\right)}{231b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3
)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^6) + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^
3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)
^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)
^(1/4))/(a*b^6) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^
2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^6) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b
^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3
)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^6)
+ 2/231*(21*b^10*d^3*x^(11/2) + 99*b^10*c*d^2*x^(7/2) - 33*a*b^9*d^3*x^(7/2
) + 231*b^10*c^2*d*x^(3/2) - 231*a*b^9*c*d^2*x^(3/2) + 77*a^2*b^8*d^3*x^(3/
2))/b^11
```

Mupad [B]
time = 0.09, size = 574, normalized size = 1.88

$$x^{3/2}\left(\frac{2c^{2}d}{b}+\frac{a\left(\frac{2ad^{3}}{b^{2}}-\frac{6cd^{2}}{b}\right)}{3b}\right)-x^{7/2}\left(\frac{2ad^{3}}{7b^{2}}-\frac{6cd^{2}}{7b}\right)+\frac{2d^{3}x^{11/2}}{11b}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}\,(ad-bc)^{3}\,(a^{7}d^{6}-6a^{6}bcd^{5}+15a^{5}b^{2}c^{2}d^{4}-20a^{4}b^{3}c^{3}d^{3}+15a^{3}b^{4}c^{4}d^{2}-6a^{2}b^{5}c^{5}d+ab^{6}c^{6})}{(-a)^{1/4}\,(a^{10}d^{9}-9a^{9}bcd^{8}+36a^{8}b^{2}c^{2}d^{7}-84a^{7}b^{3}c^{3}d^{6}+126a^{6}b^{4}c^{4}d^{5}-126a^{5}b^{5}c^{5}d^{4}+84a^{4}b^{6}c^{6}d^{3}-36a^{3}b^{7}c^{7}d^{2}+9a^{2}b^{8}c^{8}d-ab^{9}c^{9})}\right)(ad-bc)^{3}}{(-a)^{1/4}\,b^{15/4}}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}\,(ad-bc)^{3}\,(a^{7}d^{6}-6a^{6}bcd^{5}+15a^{5}b^{2}c^{2}d^{4}-20a^{4}b^{3}c^{3}d^{3}+15a^{3}b^{4}c^{4}d^{2}-6a^{2}b^{5}c^{5}d+ab^{6}c^{6})\,1\mathrm{i}}{(-a)^{1/4}\,(a^{10}d^{9}-9a^{9}bcd^{8}+36a^{8}b^{2}c^{2}d^{7}-84a^{7}b^{3}c^{3}d^{6}+126a^{6}b^{4}c^{4}d^{5}-126a^{5}b^{5}c^{5}d^{4}+84a^{4}b^{6}c^{6}d^{3}-36a^{3}b^{7}c^{7}d^{2}+9a^{2}b^{8}c^{8}d-ab^{9}c^{9})}\right)(ad-bc)^{3}\,1\mathrm{i}}{(-a)^{1/4}\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^(3/2)*((2*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/(3*b)) - x^(7/2)*(
(2*a*d^3)/(7*b^2) - (6*c*d^2)/(7*b)) + (2*d^3*x^(11/2))/(11*b) - (atan((b^(
1/4)*x^(1/2)*(a*d - b*c)^3*(a^7*d^6 + a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*
b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5))/((-
a)^(1/4)*(a^10*d^9 - a*b^9*c^9 + 9*a^2*b^8*c^8*d - 36*a^3*b^7*c^7*d^2 + 84*
a^4*b^6*c^6*d^3 - 126*a^5*b^5*c^5*d^4 + 126*a^6*b^4*c^4*d^5 - 84*a^7*b^3*c^
3*d^6 + 36*a^8*b^2*c^2*d^7 - 9*a^9*b*c*d^8)))*(a*d - b*c)^3)/((-a)^(1/4)*b^
(15/4)) - (atan((b^(1/4)*x^(1/2)*(a*d - b*c)^3*(a^7*d^6 + a*b^6*c^6 - 6*a^2
*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 -
 6*a^6*b*c*d^5)*1i)/((-a)^(1/4)*(a^10*d^9 - a*b^9*c^9 + 9*a^2*b^8*c^8*d - 3
6*a^3*b^7*c^7*d^2 + 84*a^4*b^6*c^6*d^3 - 126*a^5*b^5*c^5*d^4 + 126*a^6*b^4*
c^4*d^5 - 84*a^7*b^3*c^3*d^6 + 36*a^8*b^2*c^2*d^7 - 9*a^9*b*c*d^8)))*(a*d -
 b*c)^3*1i)/((-a)^(1/4)*b^(15/4))
```

3.444 $\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx$

Optimal. Leaf size=304

$$\frac{2d(3b^2c^2-3abcd+a^2d^2)\sqrt{x}}{b^3}+\frac{2d^2(3bc-ad)x^{5/2}}{5b^2}+\frac{2d^3x^{9/2}}{9b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc-$$

```
[Out] 2/5*d^2*(-a*d+3*b*c)*x^(5/2)/b^2+2/9*d^3*x^(9/2)/b-1/2*(-a*d+b*c)^3*arctan(
1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(13/4)*2^(1/2)+1/2*(-a*d+b*c)^
3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(13/4)*2^(1/2)-1/4*(-
a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^
(13/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2
)*x^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)+2*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^(1/
2)/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {472, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{2d\sqrt{x}\,(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{2d^2x^{5/2}(3bc-ad)}{5b^2}+\frac{2d^3x^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[x])/b^3 + (2*d^2*(3*b*c - a*d)*
x^(5/2))/(5*b^2) + (2*d^3*x^(9/2))/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) -
((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx = \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)}{b^3\sqrt{x}} + \frac{d^2(3bc-ad)x^{3/2}}{b^2} + \frac{d^3x^{7/2}}{b} + \frac{b^3c^3-3ab^2c^2d+3a^2bc\cdots}{b^3\sqrt{x}\,(a+bx\cdots}\right.$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(bc-ad)^3\int\frac{1}{\sqrt{x}\,(a\cdots}}{b^3}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(2(bc-ad)^3)\,\text{Subs}\cdots}{\cdots}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(bc-ad)^3\text{Subst}\Big(\cdots}{\cdots}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(bc-ad)^3\text{Subst}\Bigg(\cdots}{\cdots}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} - \frac{(bc-ad)^3\log\left(\sqrt{a}\cdots\right.}{2\cdots}$$

$$= \frac{2d(3b^2c^2-3abcd+a^2d^2)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} - \frac{(bc-ad)^3\tan^{-1}\Big(\cdots}{\sqrt{2}\,a^{3\cdots}}$$

Mathematica [A]
time = 0.23, size = 188, normalized size = 0.62

$$\frac{4\sqrt[4]{b}\,d\sqrt{x}\,(45a^2d^2-9abd(15c+dx^2)+b^2(135c^2+27cdx^2+5d^2x^4)) + \frac{45\sqrt{2}\,(-bc+ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{3/4}} + \frac{45\sqrt{2}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}}}{90b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (4*b^(1/4)*d*Sqrt[x]*(45*a^2*d^2 - 9*a*b*d*(15*c + d*x^2) + b^2*(135*c^2 +
27*c*d*x^2 + 5*d^2*x^4)) + (45*Sqrt[2]*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a] - S
qrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (45*Sqrt[2]*(b*c -
a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^
(3/4))/(90*b^(13/4))
```

Maple [A]
time = 0.10, size = 214, normalized size = 0.70

method	result
derivativedivides	$\frac{2d\left(\frac{b^2 x^{\frac{9}{2}} d^2}{9}-\frac{ab\,d^2 x^{\frac{5}{2}}}{5}+\frac{3b^2 cd\,x^{\frac{5}{2}}}{5}+a^2d^2\sqrt{x}-3abcd\sqrt{x}+3b^2c^2\sqrt{x}\right)}{b^3}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}{}$
default	$\frac{2d\left(\frac{b^2 x^{\frac{9}{2}} d^2}{9}-\frac{ab\,d^2 x^{\frac{5}{2}}}{5}+\frac{3b^2 cd\,x^{\frac{5}{2}}}{5}+a^2d^2\sqrt{x}-3abcd\sqrt{x}+3b^2c^2\sqrt{x}\right)}{b^3}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}{}$
risch	$\frac{2\left(5b^2d^2x^4-9ab\,d^2x^2+27b^2cd\,x^2+45a^2d^2-135abcd+135b^2c^2\right)d\sqrt{x}}{45b^3}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)d^3}{2b^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/b^3*(1/9*b^2*x^(9/2)*d^2-1/5*a*b*d^2*x^(5/2)+3/5*b^2*c*d*x^(5/2)+a^2*d^
2*x^(1/2)-3*a*b*c*d*x^(1/2)+3*b^2*c^2*x^(1/2))+1/4*(-a^3*d^3+3*a^2*b*c*d^2-
3*a*b^2*c^2*d+b^3*c^3)/b^3*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.51, size = 390, normalized size = 1.28

$$\frac{2\left(5\,b^2d^3x^{\frac{9}{2}}+9\,(3\,b^2cd^2-abd^3)x^{\frac{5}{2}}+45\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)\sqrt{x}\right)}{45\,b^3}+\frac{\frac{2\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*b^2*d^3*x^(9/2) + 9*(3*b^2*c*d^2 - a*b*d^3)*x^(5/2) + 45*(3*b^2*c^2
*d - 3*a*b*c*d^2 + a^2*d^3)*sqrt(x))/b^3 + 1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))
/(a^(3/4)*b^(1/4)))/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1862 vs. 2(227) = 454.
time = 0.58, size = 1862, normalized size = 6.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/90*(180*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220
*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*
c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*arc
tan((sqrt(a^2*b^6*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13)) +
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^2*b^10*(-(b^12*c^12 - 12*a*b^1
1*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^
11 + a^12*d^12)/(a^3*b^13))^(3/4) + (a^2*b^13*c^3 - 3*a^3*b^12*c^2*d + 3*a^
4*b^11*c*d^2 - a^5*b^10*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(a^3*b^13))^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^
9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 45*b^3*(-(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(a*b^3*(-(b^12*c^12 - 1
2*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11
*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*sqrt(x)) - 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a^3*b^13))^(1/4)*log(-a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*
b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x))
 - 4*(5*b^2*d^3*x^4 + 135*b^2*c^2*d - 135*a*b*c*d^2 + 45*a^2*d^3 + 9*(3*b^2
*c*d^2 - a*b*d^3)*x^2)*sqrt(x))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(289) = 578.

time = 13.44, size = 649, normalized size = 2.13

$$\begin{cases} \tilde{\infty}\left(-\frac{2c^3}{3x^{\frac{3}{2}}}+6c^2d\sqrt{x}+\frac{6cd^2x^{\frac{5}{2}}}{5}+\frac{2d^3x^{\frac{9}{2}}}{9}\right) & \text{for } a=0 \wedge b=0 \\ \frac{-\frac{2c^3}{3x^{\frac{3}{2}}}+6c^2d\sqrt{x}+\frac{6cd^2x^{\frac{5}{2}}}{5}+\frac{2d^3x^{\frac{9}{2}}}{9}}{b} & \text{for } a=0 \\ \frac{2c^3\sqrt{x}+\frac{6c^2dx^{\frac{5}{2}}}{5}+\frac{2cd^2x^{\frac{9}{2}}}{3}+\frac{2d^3x^{\frac{13}{2}}}{13}}{a} & \text{for } b=0 \\ \frac{2a^2d^3\sqrt{x}}{b^3}+\frac{a^2d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^3}-\frac{a^2d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^3}-\frac{a^2d^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^3}-\frac{6acd^2\sqrt{x}}{b^2}-\frac{3acd^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^2}+\frac{3acd^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^2}+\frac{3acd^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2}-\frac{2ad^3x^{\frac{5}{2}}}{5b^2}+\frac{6c^2d\sqrt{x}}{b}+\frac{3c^2d\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{3c^2d\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{3c^2d\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b}+\frac{6cd^2x^{\frac{5}{2}}}{5b}+\frac{2d^3x^{\frac{9}{2}}}{9b}-\frac{c^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a}+\frac{c^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a}+\frac{c^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)
/5 + 2*d**3*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(3*x**(3/2)) + 6*c
**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/2)/9)/b, Eq(a, 0)), ((2*
c**3*sqrt(x) + 6*c**2*d*x**(5/2)/5 + 2*c*d**2*x**(9/2)/3 + 2*d**3*x**(13/2)
/13)/a, Eq(b, 0)), (2*a**2*d**3*sqrt(x)/b**3 + a**2*d**3*(-a/b)**(1/4)*log(
sqrt(x) - (-a/b)**(1/4))/(2*b**3) - a**2*d**3*(-a/b)**(1/4)*log(sqrt(x) + (
-a/b)**(1/4))/(2*b**3) - a**2*d**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4)
)/b**3 - 6*a*c*d**2*sqrt(x)/b**2 - 3*a*c*d**2*(-a/b)**(1/4)*log(sqrt(x) - (
-a/b)**(1/4))/(2*b**2) + 3*a*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/
4))/(2*b**2) + 3*a*c*d**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 -
2*a*d**3*x**(5/2)/(5*b**2) + 6*c**2*d*sqrt(x)/b + 3*c**2*d*(-a/b)**(1/4)*lo
g(sqrt(x) - (-a/b)**(1/4))/(2*b) - 3*c**2*d*(-a/b)**(1/4)*log(sqrt(x) + (-a
/b)**(1/4))/(2*b) - 3*c**2*d*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b +
6*c*d**2*x**(5/2)/(5*b) + 2*d**3*x**(9/2)/(9*b) - c**3*(-a/b)**(1/4)*log(sq
rt(x) - (-a/b)**(1/4))/(2*a) + c**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/
4))/(2*a) + c**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(227) = 454.

time = 1.30, size = 490, normalized size = 1.61

$$\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3(ab^3)^{\frac{1}{4}}ab^2c^2d+3(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^4}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3(ab^3)^{\frac{1}{4}}ab^2c^2d+3(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^4}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3(ab^3)^{\frac{1}{4}}ab^2c^2d+3(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab^4}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3(ab^3)^{\frac{1}{4}}ab^2c^2d+3(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4ab^4}+\frac{2\left(5b^8d^3x^{\frac{9}{2}}+27b^8cd^2x^{\frac{5}{2}}-9ab^7d^3x^{\frac{5}{2}}+135b^8c^2d\sqrt{x}-135ab^7cd^2\sqrt{x}+45a^2b^6d^3\sqrt{x}\right)}{45b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 1/2*sqrt(2)*((a*b^3)^(1/4)*b^
3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)
^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)
^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^
2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3
```

)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4)
+ 2/45*(5*b^8*d^3*x^(9/2) + 27*b^8*c*d^2*x^(5/2) - 9*a*b^7*d^3*x^(5/2) + 13
5*b^8*c^2*d*sqrt(x) - 135*a*b^7*c*d^2*sqrt(x) + 45*a^2*b^6*d^3*sqrt(x))/b^9

Mupad [B]
time = 0.21, size = 1460, normalized size = 4.80

$$\sqrt{x}\left(\frac{6c^2d}{b}+\frac{a\left(\frac{2ad^3}{b^2}-\frac{6cd^2}{b}\right)}{b}\right)-x^{5/2}\left(\frac{2ad^3}{5b^2}-\frac{6cd^2}{5b}\right)+\frac{2d^3x^{9/2}}{9b}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{8\sqrt{x}\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}-\frac{(ad-bc)^3\left(16a^4d^3-48a^3bcd^2+48a^2b^2c^2d-16ab^3c^3\right)}{2(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3\,1\mathrm{i}}{(-a)^{3/4}b^{13/4}}+\frac{\left(\frac{8\sqrt{x}\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}+\frac{(ad-bc)^3\left(16a^4d^3-48a^3bcd^2+48a^2b^2c^2d-16ab^3c^3\right)}{2(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3\,1\mathrm{i}}{(-a)^{3/4}b^{13/4}}}{\frac{\left(\frac{8\sqrt{x}\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}-\frac{(ad-bc)^3\left(16a^4d^3-48a^3bcd^2+48a^2b^2c^2d-16ab^3c^3\right)}{2(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3}{(-a)^{3/4}b^{13/4}}-\frac{\left(\frac{8\sqrt{x}\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}+\frac{(ad-bc)^3\left(16a^4d^3-48a^3bcd^2+48a^2b^2c^2d-16ab^3c^3\right)}{2(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3}{(-a)^{3/4}b^{13/4}}}\right)(ad-bc)^3\,1\mathrm{i}}{(-a)^{3/4}b^{13/4}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{8\sqrt{x}\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}-\frac{(ad-bc)^3\left(16a^4d^3-48a^3bcd^2+48a^2b^2c^2d-16ab^3c^3\right)1\mathrm{i}}{2(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3}{(-a)^{3/4}b^{13/4}}+\frac{\left(\frac{8\sqrt{x}\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}+\frac{(ad-bc)^3\left(16a^4d^3-48a^3bcd^2+48a^2b^2c^2d-16ab^3c^3\right)1\mathrm{i}}{2(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3}{(-a)^{3/4}b^{13/4}}}{\frac{\left(\frac{8\sqrt{x}\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}-\frac{(ad-bc)^3\left(16a^4d^3-48a^3bcd^2+48a^2b^2c^2d-16ab^3c^3\right)1\mathrm{i}}{2(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3\,1\mathrm{i}}{(-a)^{3/4}b^{13/4}}-\frac{\left(\frac{8\sqrt{x}\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{b^3}+\frac{(ad-bc)^3\left(16a^4d^3-48a^3bcd^2+48a^2b^2c^2d-16ab^3c^3\right)1\mathrm{i}}{2(-a)^{3/4}b^{13/4}}\right)(ad-bc)^3\,1\mathrm{i}}{(-a)^{3/4}b^{13/4}}}\right)(ad-bc)^3}{(-a)^{3/4}b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^2)^3/(x^(1/2)*(a + b*x^2)),x)

[Out] x^(1/2)*((6*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/b) - x^(5/2)*((2*a
*d^3)/(5*b^2) - (6*c*d^2)/(5*b)) + (2*d^3*x^(9/2))/(9*b) - (atan(((((8*x^(1
/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b
^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(16*a^4*d
^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2))/(2*(-a)^(3/4)*b^(13
/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) + (((8*x^(1/2)*(a^6*d^6 + b^6
*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b
^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3
+ 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2))/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^
3*1i)/((-a)^(3/4)*b^(13/4)))/((((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*
c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b
*c*d^5))/b^3 - ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d
 - 48*a^3*b*c*d^2))/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(
13/4)) - (((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*
c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d
- b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2))/(
2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4))))*(a*d - b*c)^
3*1i)/((-a)^(3/4)*b^(13/4)) - (atan(((((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a
^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d -
6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^
2*c^2*d - 48*a^3*b*c*d^2)*1i)/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)
^(3/4)*b^(13/4)) + (((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^
3 + ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b
*c*d^2)*1i)/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)))/
((((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3
*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2)*1i)/(2*(-a
)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) - (((8*x^(1/2)*(
a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(16*a^4*d^3 -
16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2)*1i)/(2*(-a)^(3/4)*b^(13/4
)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4))))*(a*d - b*c)^3)/((-a)^(3/4)*b^
(13/4))

3.445 $\int \frac{\left(c+dx^2\right)^3}{x^{3/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=284

$$-\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{11/4}}$$

```
[Out] 2/3*d^2*(-a*d+3*b*c)*x^(3/2)/b^2+2/7*d^3*x^(7/2)/b+1/2*(-a*d+b*c)^3*arctan(
1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(11/4)*2^(1/2)-1/2*(-a*d+b*c)^
3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(11/4)*2^(1/2)-1/4*(-
a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^
(11/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2
)*x^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)-2*c^3/a/x^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 472, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{2d^2x^{3/2}(3bc-ad)}{3b^2}-\frac{2c^3}{a\sqrt{x}}+\frac{2d^3x^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(a*Sqrt[x]) + (2*d^2*(3*b*c - a*d)*x^(3/2))/(3*b^2) + (2*d^3*x^(7/
2))/(7*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(11/4)) +
((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^2(a+bx^4)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{c^3}{ax^2} + \frac{d^2(3bc-ad)x^2}{b^2} + \frac{d^3x^6}{b} + \frac{(-bc+ad)^3x^2}{ab^2(a+bx^4)}\right)dx, x, \sqrt{x}\right)$$

$$= -\frac{2c^3}{a\sqrt{x}} + \frac{2d^2(3bc-ad)x^{3/2}}{3b^2} + \frac{2d^3x^{7/2}}{7b} - \frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab^2}$$

$$= -\frac{2c^3}{a\sqrt{x}} + \frac{2d^2(3bc-ad)x^{3/2}}{3b^2} + \frac{2d^3x^{7/2}}{7b} + \frac{(bc-ad)^3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{ab^{5/2}}$$

$$= -\frac{2c^3}{a\sqrt{x}} + \frac{2d^2(3bc-ad)x^{3/2}}{3b^2} + \frac{2d^3x^{7/2}}{7b} - \frac{(bc-ad)^3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx,\right.}{2ab^3}$$

$$= -\frac{2c^3}{a\sqrt{x}} + \frac{2d^2(3bc-ad)x^{3/2}}{3b^2} + \frac{2d^3x^{7/2}}{7b} - \frac{(bc-ad)^3\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \right.}{2\sqrt{2}\,a^{5/4}b^{11/4}}$$

$$= -\frac{2c^3}{a\sqrt{x}} + \frac{2d^2(3bc-ad)x^{3/2}}{3b^2} + \frac{2d^3x^{7/2}}{7b} + \frac{(bc-ad)^3\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{11/4}} - \frac{(b}{}$$

Mathematica [A]
time = 0.24, size = 178, normalized size = 0.63

$$\frac{\frac{4\sqrt[4]{a}\,b^{3/4}(-21b^2c^3-7a^2d^3x^2+3abd^2x^2(7c+dx^2))}{\sqrt{x}} + 21\sqrt{2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 21\sqrt{2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{42a^{5/4}b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] ((4*a^(1/4)*b^(3/4)*(-21*b^2*c^3 - 7*a^2*d^3*x^2 + 3*a*b*d^2*x^2*(7*c + d*x
^2)))/Sqrt[x] + 21*Sqrt[2]*(b*c - a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 21*Sqrt[2]*(b*c - a*d)^3*ArcTanh[(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(42*a^(5/4)*b^(11/4))
```

Maple [A]
time = 0.10, size = 186, normalized size = 0.65

---

method	result
derivativedivides	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(ad-3bc)x^{\frac{3}{2}}}{3}\right)}{b^2}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\operatorname{arc}\right.}{4a\,b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(ad-3bc)x^{\frac{3}{2}}}{3}\right)}{b^2}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\operatorname{arc}\right.}{4a\,b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(-3ab\,d^3x^4+7a^2d^3x^2-21abc\,d^2x^2+21b^2c^3)}{21a\sqrt{x}\,b^2}+\frac{a^2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)d^3}{2b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{3a\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right.}\right.}{2b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*d^2/b^2*(-1/7*b*d*x^(7/2)+1/3*(a*d-3*b*c)*x^(3/2))+1/4*(a^3*d^3-3*a^2*b*
c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a/b^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1))-2*c^3/a/x^(1/2)
```

<u>Maxima</u> [A]
time = 0.52, size = 261, normalized size = 0.92

$$-\frac{2\,c^3}{a\sqrt{x}}+\frac{2\left(3\,bd^3x^{\frac{7}{2}}+7\,(3\,bcd^2-ad^3)x^{\frac{3}{2}}\right)}{21\,b^2}-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2*c^3/(a*sqrt(x)) + 2/21*(3*b*d^3*x^(7/2) + 7*(3*b*c*d^2 - a*d^3)*x^(3/2))
/b^2 - 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(2*sqrt(2)*a
rctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b^2)
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 2442 vs. $2(207) = 414$.
time = 0.60, size = 2442, normalized size = 8.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="fricas")

[Out] -1/42*(84*a*b^2*x*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*arctan((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)*x - (a^3*b^17*c^12 - 12*a^4*b^16*c^11*d + 66*a^5*b^15*c^10*d^2 - 220*a^6*b^14*c^9*d^3 + 495*a^7*b^13*c^8*d^4 - 792*a^8*b^12*c^7*d^5 + 924*a^9*b^11*c^6*d^6 - 792*a^10*b^10*c^5*d^7 + 495*a^11*b^9*c^4*d^8 - 220*a^12*b^8*c^3*d^9 + 66*a^13*b^7*c^2*d^10 - 12*a^14*b^6*c*d^11 + a^15*b^5*d^12)*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11)))*a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4) + (a*b^12*c^9 - 9*a^2*b^11*c^8*d + 36*a^3*b^10*c^7*d^2 - 84*a^4*b^9*c^6*d^3 + 126*a^5*b^8*c^5*d^4 - 126*a^6*b^7*c^4*d^5 + 84*a^7*b^6*c^3*d^6 - 36*a^8*b^5*c^2*d^7 + 9*a^9*b^4*c*d^8 - a^10*b^3*d^9)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) - 21*a*b^2*x*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*log(a^4*b^8*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*

```
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 21*a*b^2*x*(
-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^
7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2
*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*log(-a^4*b^8*(-(b^1
2*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 49
5*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5
*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8
*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^
5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a
^9*d^9)*sqrt(x)) - 4*(3*a*b*d^3*x^4 - 21*b^2*c^3 + 7*(3*a*b*c*d^2 - a^2*d^3
)*x^2)*sqrt(x))/(a*b^2*x)
```

Sympy [A]
time = 32.07, size = 435, normalized size = 1.53

$$c^3 \left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{5 b x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{a \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{\log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 a \sqrt[4]{- \frac{a}{b}}} + \frac{\log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 a \sqrt[4]{- \frac{a}{b}}} - \frac{\operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{a \sqrt[4]{- \frac{a}{b}}} - \frac{2}{a \sqrt{x}} & \text{otherwise} \end{cases}\right) + 6 c^2 d \operatorname{RootSum}{\left(256 t^{4} a b^{3} + 1, \left(t \mapsto t \log{\left(64 t^{3} a b^{2} + \sqrt{x} \right)} \right)\right)} + 3 c d^2 \left(\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{7}{2}}}{7 a} & \text{for}\: b = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 b} & \text{for}\: a = 0 \\ - \frac{a \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{2} \sqrt[4]{- \frac{a}{b}}} + \frac{a \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{2} \sqrt[4]{- \frac{a}{b}}} - \frac{a \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{2} \sqrt[4]{- \frac{a}{b}}} + \frac{2 x^{\frac{3}{2}}}{3 b} & \text{otherwise} \end{cases}\right) + d^3 \left(\begin{cases} \tilde{\infty} x^{\frac{7}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{11}{2}}}{11 a} & \text{for}\: b = 0 \\ \frac{2 x^{\frac{7}{2}}}{7 b} & \text{for}\: a = 0 \\ \frac{a^{2} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{3} \sqrt[4]{- \frac{a}{b}}} - \frac{a^{2} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 b^{3} \sqrt[4]{- \frac{a}{b}}} + \frac{a^{2} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{b^{3} \sqrt[4]{- \frac{a}{b}}} - \frac{2 a x^{\frac{3}{2}}}{3 b^{2}} + \frac{2 x^{\frac{7}{2}}}{7 b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a),x)
```

```
[Out] c**3*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(
a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) - (-a/b)**(1/4))/(2*a*(-a
/b)**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) - atan(sqrt(
x)/(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*sqrt(x)), True)) + 6*c**2*d*Root
Sum(256*_t**4*a*b**3 + 1, Lambda(_t, _t*log(64*_t**3*a*b**2 + sqrt(x)))) +
3*c*d**2*Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a),
Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(a, 0)), (-a*log(sqrt(x) - (-a/b)**(1/4))/(
2*b**2*(-a/b)**(1/4)) + a*log(sqrt(x) + (-a/b)**(1/4))/(2*b**2*(-a/b)**(1/4
)) - a*atan(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b)**(1/4)) + 2*x**(3/2)/(3*b),
 True)) + d**3*Piecewise((zoo*x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*x**(11/2)/
(11*a), Eq(b, 0)), (2*x**(7/2)/(7*b), Eq(a, 0)), (a**2*log(sqrt(x) - (-a/b)
**(1/4))/(2*b**3*(-a/b)**(1/4)) - a**2*log(sqrt(x) + (-a/b)**(1/4))/(2*b**3
*(-a/b)**(1/4)) + a**2*atan(sqrt(x)/(-a/b)**(1/4))/(b**3*(-a/b)**(1/4)) - 2
*a*x**(3/2)/(3*b**2) + 2*x**(7/2)/(7*b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(207) = 414.
time = 1.30, size = 462, normalized size = 1.63

$$-\frac{2 c^3}{a \sqrt{x}} - \frac{\sqrt{2} \left((ab^3)^{\frac{3}{4}} b^3 c^3 - 3 (ab^3)^{\frac{3}{4}} a b^2 c^2 d + 3 (ab^3)^{\frac{3}{4}} a^2 b c d^2 - (ab^3)^{\frac{3}{4}} a^3 d^3 \right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x} \right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}} \right)}{2 a^2 b^5} - \frac{\sqrt{2} \left((ab^3)^{\frac{3}{4}} b^3 c^3 - 3 (ab^3)^{\frac{3}{4}} a b^2 c^2 d + 3 (ab^3)^{\frac{3}{4}} a^2 b c d^2 - (ab^3)^{\frac{3}{4}} a^3 d^3 \right) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x} \right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}} \right)}{2 a^2 b^5} + \frac{\sqrt{2} \left((ab^3)^{\frac{3}{4}} b^3 c^3 - 3 (ab^3)^{\frac{3}{4}} a b^2 c^2 d + 3 (ab^3)^{\frac{3}{4}} a^2 b c d^2 - (ab^3)^{\frac{3}{4}} a^3 d^3 \right) \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}} \right)}{4 a^2 b^5} - \frac{\sqrt{2} \left((ab^3)^{\frac{3}{4}} b^3 c^3 - 3 (ab^3)^{\frac{3}{4}} a b^2 c^2 d + 3 (ab^3)^{\frac{3}{4}} a^2 b c d^2 - (ab^3)^{\frac{3}{4}} a^3 d^3 \right) \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}} \right)}{4 a^2 b^5} + \frac{2 \left(3 b^6 d^3 x^{\frac{7}{2}} + 21 b^6 c d^2 x^{\frac{3}{2}} - 7 a b^5 d^3 x^{\frac{3}{2}} \right)}{21 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*c^3/(a*sqrt(x)) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a
*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) - 1/2*sq
rt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4
)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c
^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3
/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^5) - 1
/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)
^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4
) + x + sqrt(a/b))/(a^2*b^5) + 2/21*(3*b^6*d^3*x^(7/2) + 21*b^6*c*d^2*x^(3/
2) - 7*a*b^5*d^3*x^(3/2))/b^7
```

Mupad [B]
time = 0.10, size = 580, normalized size = 2.04

$$\frac{2\,d^3\,x^{7/2}}{7\,b}-\frac{2\,c^3}{a\,\sqrt{x}}-x^{3/2}\,\left(\frac{2\,a\,d^3}{3\,b^2}-\frac{2\,c\,d^2}{b}\right)-\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^3\,\left(16\,a^{10}\,b^8\,d^6-96\,a^9\,b^9\,c\,d^5+240\,a^8\,b^{10}\,c^2\,d^4-320\,a^7\,b^{11}\,c^3\,d^3+240\,a^6\,b^{12}\,c^4\,d^2-96\,a^5\,b^{13}\,c^5\,d+16\,a^4\,b^{14}\,c^6\right)}{{\left(-a\right)}^{5/4}\,b^{11/4}\,\left(-16\,a^{12}\,b^5\,d^9+144\,a^{11}\,b^6\,c\,d^8-576\,a^{10}\,b^7\,c^2\,d^7+1344\,a^9\,b^8\,c^3\,d^6-2016\,a^8\,b^9\,c^4\,d^5+2016\,a^7\,b^{10}\,c^5\,d^4-1344\,a^6\,b^{11}\,c^6\,d^3+576\,a^5\,b^{12}\,c^7\,d^2-144\,a^4\,b^{13}\,c^8\,d+16\,a^3\,b^{14}\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^3}{{\left(-a\right)}^{5/4}\,b^{11/4}}-\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^3\,\left(16\,a^{10}\,b^8\,d^6-96\,a^9\,b^9\,c\,d^5+240\,a^8\,b^{10}\,c^2\,d^4-320\,a^7\,b^{11}\,c^3\,d^3+240\,a^6\,b^{12}\,c^4\,d^2-96\,a^5\,b^{13}\,c^5\,d+16\,a^4\,b^{14}\,c^6\right)\,1{}\mathrm{i}}{{\left(-a\right)}^{5/4}\,b^{11/4}\,\left(-16\,a^{12}\,b^5\,d^9+144\,a^{11}\,b^6\,c\,d^8-576\,a^{10}\,b^7\,c^2\,d^7+1344\,a^9\,b^8\,c^3\,d^6-2016\,a^8\,b^9\,c^4\,d^5+2016\,a^7\,b^{10}\,c^5\,d^4-1344\,a^6\,b^{11}\,c^6\,d^3+576\,a^5\,b^{12}\,c^7\,d^2-144\,a^4\,b^{13}\,c^8\,d+16\,a^3\,b^{14}\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^3\,1{}\mathrm{i}}{{\left(-a\right)}^{5/4}\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x)
```

```
[Out] (2*d^3*x^(7/2))/(7*b) - (2*c^3)/(a*x^(1/2)) - x^(3/2)*((2*a*d^3)/(3*b^2) -
(2*c*d^2)/b) - (atan((x^(1/2)*(a*d - b*c)^3*(16*a^4*b^14*c^6 + 16*a^10*b^8*
d^6 - 96*a^5*b^13*c^5*d - 96*a^9*b^9*c*d^5 + 240*a^6*b^12*c^4*d^2 - 320*a^7
*b^11*c^3*d^3 + 240*a^8*b^10*c^2*d^4))/((-a)^(5/4)*b^(11/4)*(16*a^3*b^14*c^
9 - 16*a^12*b^5*d^9 - 144*a^4*b^13*c^8*d + 144*a^11*b^6*c*d^8 + 576*a^5*b^1
2*c^7*d^2 - 1344*a^6*b^11*c^6*d^3 + 2016*a^7*b^10*c^5*d^4 - 2016*a^8*b^9*c^
4*d^5 + 1344*a^9*b^8*c^3*d^6 - 576*a^10*b^7*c^2*d^7)))*(a*d - b*c)^3)/((-a)
^(5/4)*b^(11/4)) - (atan((x^(1/2)*(a*d - b*c)^3*(16*a^4*b^14*c^6 + 16*a^10*
b^8*d^6 - 96*a^5*b^13*c^5*d - 96*a^9*b^9*c*d^5 + 240*a^6*b^12*c^4*d^2 - 320
*a^7*b^11*c^3*d^3 + 240*a^8*b^10*c^2*d^4)*1i)/((-a)^(5/4)*b^(11/4)*(16*a^3*
b^14*c^9 - 16*a^12*b^5*d^9 - 144*a^4*b^13*c^8*d + 144*a^11*b^6*c*d^8 + 576*
a^5*b^12*c^7*d^2 - 1344*a^6*b^11*c^6*d^3 + 2016*a^7*b^10*c^5*d^4 - 2016*a^8
*b^9*c^4*d^5 + 1344*a^9*b^8*c^3*d^6 - 576*a^10*b^7*c^2*d^7)))*(a*d - b*c)^3
*1i)/((-a)^(5/4)*b^(11/4))
```

3.446 $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx$

Optimal. Leaf size=284

$$-\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,a^{7/4}b^{9/4}}$$

```
[Out] -2/3*c^3/a/x^(3/2)+2/5*d^3*x^(5/2)/b+1/2*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)-1/2*(-a*d+b*c)^3*arctan(1+b^(1
/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)-1/
4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4
)/b^(9/4)*2^(1/2)+2*d^2*(-a*d+3*b*c)*x^(1/2)/b^2
```

Rubi [A]
time = 0.18, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 472, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{2d^2\sqrt{x}(3bc-ad)}{b^2}-\frac{2c^3}{3ax^{3/2}}+\frac{2d^3x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(3*a*x^(3/2)) + (2*d^2*(3*b*c - a*d)*Sqrt[x])/b^2 + (2*d^3*x^(5/2)
)/(5*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(9/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c
 - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(7/4)*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^4(a+bx^4)}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \left(\frac{d^2(3bc-ad)}{b^2} + \frac{c^3}{ax^4} + \frac{d^3x^4}{b} + \frac{(-bc+ad)^3}{ab^2(a+bx^4)}\right) dx, x, \sqrt{x}\right)$$
$$= -\frac{2c^3}{3ax^{3/2}} + \frac{2d^2(3bc-ad)\sqrt{x}}{b^2} + \frac{2d^3x^{5/2}}{5b} - \frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab^2}$$
$$= -\frac{2c^3}{3ax^{3/2}} + \frac{2d^2(3bc-ad)\sqrt{x}}{b^2} + \frac{2d^3x^{5/2}}{5b} - \frac{(bc-ad)^3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{\right.}}{a^{3/2}b^2}$$
$$= -\frac{2c^3}{3ax^{3/2}} + \frac{2d^2(3bc-ad)\sqrt{x}}{b^2} + \frac{2d^3x^{5/2}}{5b} - \frac{(bc-ad)^3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,d\right.}{2a^{3/2}b^{5/2}}$$
$$= -\frac{2c^3}{3ax^{3/2}} + \frac{2d^2(3bc-ad)\sqrt{x}}{b^2} + \frac{2d^3x^{5/2}}{5b} + \frac{(bc-ad)^3\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,a^{7/4}b^{9/4}}$$
$$= -\frac{2c^3}{3ax^{3/2}} + \frac{2d^2(3bc-ad)\sqrt{x}}{b^2} + \frac{2d^3x^{5/2}}{5b} + \frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}b^{9/4}} -$$

Mathematica [A]
time = 0.24, size = 178, normalized size = 0.63

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\,(-5b^2c^3-15a^2d^3x^2+3abd^2x^2(15c+dx^2))}{x^{3/2}} + 15\sqrt{2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 15\sqrt{2}(-bc+ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{30a^{7/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(-5*b^2*c^3 - 15*a^2*d^3*x^2 + 3*a*b*d^2*x^2*(15*c + d*
x^2)))/x^(3/2) + 15*Sqrt[2]*(b*c - a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 15*Sqrt[2]*(-(b*c) + a*d)^3*ArcTanh[(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(30*a^(7/4)*b^(9/4))
```

Maple [A]
time = 0.09, size = 186, normalized size = 0.65

method	result

derivativedivides	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{5}{2}}}{5}+ad\sqrt{x}-3bc\sqrt{x}\right)}{b^2}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}\right.\right.}{4a^2b^2}$
default	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{5}{2}}}{5}+ad\sqrt{x}-3bc\sqrt{x}\right)}{b^2}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}\right.\right.}{4a^2b^2}$
risch	$-\frac{2(-3ab\,d^3x^4+15a^2d^3x^2-45abc\,d^2x^2+5b^2c^3)}{15b^2x^{\frac{3}{2}}a}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)d^3}{2b^2}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*d^2/b^2*(-1/5*b*d*x^(5/2)+a*d*x^(1/2)-3*b*c*x^(1/2))+1/4/a^2/b^2*(a^3*d^
3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1))-2/3*c^3/a/x^(3/2)
```

Maxima [A]
time = 0.54, size = 368, normalized size = 1.30

$$-\frac{2\,c^3}{3\,a x^{\frac{3}{2}}}+\frac{2\left(bd^3x^{\frac{5}{2}}+5\,(3\,bcd^2-ad^3)\sqrt{x}\right)}{5\,b^2}-\frac{\frac{2\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/3*c^3/(a*x^(3/2)) + 2/5*(b*d^3*x^(5/2) + 5*(3*b*c*d^2 - a*d^3)*sqrt(x))/
b^2 - 1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2
)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1866 vs. 2(207) = 414.
time = 0.55, size = 1866, normalized size = 6.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/30*(60*a*b^2*x^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)*
arctan((sqrt(a^4*b^4*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10
*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 92
4*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3
*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))
+ (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^5*b^7*(-(b^12*c^12 - 12*a*b^
11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d
^11 + a^12*d^12)/(a^7*b^9))^(3/4) + (a^5*b^10*c^3 - 3*a^6*b^9*c^2*d + 3*a^7
*b^8*c*d^2 - a^8*b^7*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a
^7*b^9))^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 15*a*b^2*x^2*(-(b^1
2*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 49
5*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5
*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)*log(a^2*b^2*(-(b^12*c^12
- 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^
7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a
^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 15*a*b^2*x^2*(-(b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(a^7*b^9))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
sqrt(x)) + 4*(3*a*b*d^3*x^4 - 5*b^2*c^3 + 15*(3*a*b*c*d^2 - a^2*d^3)*x^2)*s
qrt(x))/(a*b^2*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(265) = 530$.
time = 21.60, size = 604, normalized size = 2.13

$$\begin{cases} \tilde{\infty}\left(-\frac{2c^3}{7x^{\frac{7}{2}}} - \frac{2c^2d}{x^{\frac{3}{2}}} + 6cd^2\sqrt{x} + \frac{2d^3x^{\frac{5}{2}}}{5}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{-\frac{2c^3}{7x^{\frac{7}{2}}} - \frac{2c^2d}{x^{\frac{3}{2}}} + 6cd^2\sqrt{x} + \frac{2d^3x^{\frac{5}{2}}}{5}}{b} & \text{for } a = 0 \\ \frac{-\frac{2c^3}{3x^{\frac{3}{2}}} + 6c^2d\sqrt{x} + \frac{6cd^2x^{\frac{5}{2}}}{5} + \frac{2d^3x^{\frac{9}{2}}}{9}}{a} & \text{for } b = 0 \\ -\frac{2ad^3\sqrt{x}}{b^2} - \frac{ad^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b^2} + \frac{ad^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b^2} + \frac{ad^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2} + \frac{6cd^2\sqrt{x}}{b} + \frac{3cd^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{3cd^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{3cd^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b} + \frac{2d^3x^{\frac{5}{2}}}{5b} - \frac{2c^3}{3ax^{\frac{3}{2}}} - \frac{3c^2d\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{3c^2d\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{3c^2d\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a} + \frac{bc^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a^2} - \frac{bc^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a^2} - \frac{bc^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x)
 + 2*d**3*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(7*x**(7/2)) - 2*c**
2*d/x**(3/2) + 6*c*d**2*sqrt(x) + 2*d**3*x**(5/2)/5)/b, Eq(a, 0)), ((-2*c**
3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/2)/9
)/a, Eq(b, 0)), (-2*a*d**3*sqrt(x)/b**2 - a*d**3*(-a/b)**(1/4)*log(sqrt(x)
- (-a/b)**(1/4))/(2*b**2) + a*d**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4
))/(2*b**2) + a*d**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 + 6*c*d
**2*sqrt(x)/b + 3*c*d**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b) -
 3*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - 3*c*d**2*(-a/b
)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b + 2*d**3*x**(5/2)/(5*b) - 2*c**3/(3*
a*x**(3/2)) - 3*c**2*d*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) + 3
*c**2*d*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a) + 3*c**2*d*(-a/b)*
*(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a + b*c**3*(-a/b)**(1/4)*log(sqrt(x) - (
-a/b)**(1/4))/(2*a**2) - b*c**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/
(2*a**2) - b*c**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(207) = 414$.
time = 2.19, size = 461, normalized size = 1.62

$$-\frac{2\,c^3}{3\,ax^{\frac{3}{2}}} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^2b^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^2b^3} + \frac{2\left(b^4d^3x^{\frac{5}{2}} + 15\,b^4cd^2\sqrt{x} - 5\,ab^3d^3\sqrt{x}\right)}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2/3*c^3/(a*x^(3/2)) - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)
*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/2*
sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1
/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3
*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^
(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) +
 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1
```

```
/4) + x + sqrt(a/b))/(a^2*b^3) + 2/5*(b^4*d^3*x^(5/2) + 15*b^4*c*d^2*sqrt(x
) - 5*a*b^3*d^3*sqrt(x))/b^5
```

Mupad [B]
time = 0.12, size = 1561, normalized size = 5.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x)
```

```
[Out] (2*d^3*x^(5/2))/(5*b) - (2*c^3)/(3*a*x^(3/2)) - x^(1/2)*((2*a*d^3)/b^2 - (6
*c*d^2)/b) - (atan(((((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b
^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3
 + 240*a^7*b^11*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11
*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2))/(2*(-a)^(7/4)*b^(9/4)))*(a*d
 - b*c)^3*1i)/((-a)^(7/4)*b^(9/4)) + (((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b
^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320
*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^5*b^14*
c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2))/(2*(-a)^(7/
4)*b^(9/4)))*(a*d - b*c)^3*1i)/((-a)^(7/4)*b^(9/4)))/((((x^(1/2)*(16*a^3*b^
15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b
^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 - ((a*d - b*c
)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*
d^2))/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3)/((-a)^(7/4)*b^(9/4)) - (((x^(1
/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d
^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2
 + ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d +
48*a^7*b^12*c*d^2))/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3)/((-a)^(7/4)*b^(9
/4))))*(a*d - b*c)^3*1i)/((-a)^(7/4)*b^(9/4)) - (atan(((((x^(1/2)*(16*a^3*b
^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*
b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 - ((a*d - b*
c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c
*d^2)*1i)/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3)/((-a)^(7/4)*b^(9/4)) + (((
x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10
*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4
))/2 + ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*
d + 48*a^7*b^12*c*d^2)*1i)/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3)/((-a)^(7/
4)*b^(9/4)))/((((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^
5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240
*a^7*b^11*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 -
 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2)*1i)/(2*(-a)^(7/4)*b^(9/4)))*(a*d -
b*c)^3*1i)/((-a)^(7/4)*b^(9/4)) - (((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*
d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^
6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^5*b^14*c^3
```

```
 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2)*1i)/(2*(-a)^(7/
4)*b^(9/4)))*(a*d - b*c)^3*1i)/((-a)^(7/4)*b^(9/4))))*(a*d - b*c)^3)/((-a)^
(7/4)*b^(9/4))
```

3.447 $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx$

Optimal. Leaf size=283

$$-\frac{2c^3}{5ax^{5/2}}+\frac{2c^2(bc-3ad)}{a^2\sqrt{x}}+\frac{2d^3x^{3/2}}{3b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}b^{7/4}}$$

```
[Out] -2/5*c^3/a/x^(5/2)+2/3*d^3*x^(3/2)/b-1/2*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(7/4)*2^(1/2)+1/2*(-a*d+b*c)^3*arctan(1+b^(1
/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(7/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)-1/
4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4
)/b^(7/4)*2^(1/2)+2*c^2*(-3*a*d+b*c)/a^2/x^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 472, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{2c^2(bc-3ad)}{a^2\sqrt{x}}-\frac{2c^3}{5ax^{5/2}}+\frac{2d^3x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(5*a*x^(5/2)) + (2*c^2*(b*c - 3*a*d))/(a^2*Sqrt[x]) + (2*d^3*x^(3/
2))/(3*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*b^(7/4)) - ((b
*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*
Sqrt[2]*a^(9/4)*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^6(a+bx^4)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{c^3}{ax^6} + \frac{c^2(-bc+3ad)}{a^2x^2} + \frac{d^3x^2}{b} - \frac{(-bc+ad)^3x^2}{a^2b(a+bx^4)}\right)\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} + \frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2b}$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} - \frac{(bc-ad)^3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2b^{3/2}}$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} + \frac{(bc-ad)^3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x\right.}{2a^2b^2}$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} + \frac{(bc-ad)^3\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{\right.}}{2\sqrt{2}\,a^{9/4}b^{7/4}}$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} - \frac{(bc-ad)^3\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}b^{7/4}} + \frac{(bc-}{}$$

Mathematica [A]
time = 0.23, size = 177, normalized size = 0.63

$$\frac{\frac{4\sqrt[4]{a}\,b^{3/4}(15b^2c^3x^2+5a^2d^3x^4-3abc^2(c+15dx^2))}{x^{5/2}} + 15\sqrt{2}(-bc+ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 15\sqrt{2}(-bc+ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{30a^{9/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] ((4*a^(1/4)*b^(3/4)*(15*b^2*c^3*x^2 + 5*a^2*d^3*x^4 - 3*a*b*c^2*(c + 15*d*x
^2)))/x^(5/2) + 15*Sqrt[2]*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 15*Sqrt[2]*(-(b*c) + a*d)^3*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(30*a^(9/4)*b^(7/4))
```

Maple [A]
time = 0.10, size = 188, normalized size = 0.66

method	result
derivativedivides	$\frac{2d^{3}x^{\frac{3}{2}}}{3b}-\frac{(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\right.}{4a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{2d^{3}x^{\frac{3}{2}}}{3b}-\frac{(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\right.}{4a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$\frac{-6abc^{2}dx^{2}+2b^{2}c^{3}x^{2}-\frac{2}{5}abc^{3}+\frac{2}{3}a^{2}d^{3}x^{4}}{a^{2}x^{\frac{5}{2}}b}-\frac{a\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)d^{3}}{2b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)c}{2b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d^3*x^(3/2)/b-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^2/b^2
/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/5*c^3/a/x^(5/2)-2*c^2*(3*a*d
-b*c)/a^2/x^(1/2)
```

Maxima [A]
time = 0.50, size = 259, normalized size = 0.92

$$\frac{2\,d^{3}x^{\frac{3}{2}}}{3\,b}+\frac{(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,a^{2}b}-\frac{2\,(ac^{3}-5\,(bc^{3}-3\,ac^{2}d)x^{2})}{5\,a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/3*d^3*x^(3/2)/b + 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b) -
 2/5*(a*c^3 - 5*(b*c^3 - 3*a*c^2*d)*x^2)/(a^2*x^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2451 vs. $2(206) = 412$.
time = 0.59, size = 2451, normalized size = 8.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="fricas")

[Out] 1/30*(60*a^2*b*x^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*arctan((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)*x - (a^5*b^15*c^12 - 12*a^6*b^14*c^11*d + 66*a^7*b^13*c^10*d^2 - 220*a^8*b^12*c^9*d^3 + 495*a^9*b^11*c^8*d^4 - 792*a^10*b^10*c^7*d^5 + 924*a^11*b^9*c^6*d^6 - 792*a^12*b^8*c^5*d^7 + 495*a^13*b^7*c^4*d^8 - 220*a^14*b^6*c^3*d^9 + 66*a^15*b^5*c^2*d^10 - 12*a^16*b^4*c*d^11 + a^17*b^3*d^12)*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7)))*a^2*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4) + (a^2*b^11*c^9 - 9*a^3*b^10*c^8*d + 36*a^4*b^9*c^7*d^2 - 84*a^5*b^8*c^6*d^3 + 126*a^6*b^7*c^5*d^4 - 126*a^7*b^6*c^4*d^5 + 84*a^8*b^5*c^3*d^6 - 36*a^9*b^4*c^2*d^7 + 9*a^10*b^3*c*d^8 - a^11*b^2*d^9)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) - 15*a^2*b*x^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*log(a^7*b^5*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3

```
*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 15*a^2*b*x^
3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792
*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*
c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*log(-a^7*b^5*(-(b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(3/4) - (b^9*c^9 - 9*a*b^8*c^
8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a
^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 -
a^9*d^9)*sqrt(x)) + 4*(5*a^2*d^3*x^4 - 3*a*b*c^3 + 15*(b^2*c^3 - 3*a*b*c^2*
d)*x^2)*sqrt(x))/(a^2*b*x^3)
```

Sympy [**A**]
time = 59.73, size = 432, normalized size = 1.53

$$c^3\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } a=0 \wedge b=0\\ -\frac{2}{9bx^{\frac{9}{2}}} & \text{for } a=0\\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for } b=0\\ -\frac{2}{5ax^{\frac{5}{2}}}+\frac{b\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}}-\frac{b\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2\sqrt[4]{-\frac{a}{b}}}+\frac{2b}{a^2\sqrt{x}} & \text{otherwise}\end{cases}\right)+3c^2d\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a=0 \wedge b=0\\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a=0\\ -\frac{2}{a\sqrt{x}} & \text{for } b=0\\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}+\frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}-\frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}}-\frac{2}{a\sqrt{x}} & \text{otherwise}\end{cases}\right)+6cd^2\operatorname{RootSum}\left(256t^4ab^3+1,\left(t\mapsto t\log\left(64t^3ab^2+\sqrt{x}\right)\right)\right)+d^3\left(\begin{cases}\tilde{\infty}x^{\frac{3}{2}} & \text{for } a=0 \wedge b=0\\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b=0\\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a=0\\ -\frac{a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}}+\frac{a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}}-\frac{a\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2\sqrt[4]{-\frac{a}{b}}}+\frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a),x)
```

```
[Out] c**3*Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(
a, 0)), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + b*log(sqrt(x) -
 (-a/b)**(1/4))/(2*a**2*(-a/b)**(1/4)) - b*log(sqrt(x) + (-a/b)**(1/4))/(2*
a**2*(-a/b)**(1/4)) + b*atan(sqrt(x)/(-a/b)**(1/4))/(a**2*(-a/b)**(1/4)) +
2*b/(a**2*sqrt(x)), True)) + 3*c**2*d*Piecewise((zoo/x**(5/2), Eq(a, 0) & E
q(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(
sqrt(x) - (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))
/(2*a*(-a/b)**(1/4)) - atan(sqrt(x)/(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a
*sqrt(x)), True)) + 6*c*d**2*RootSum(256*_t**4*a*b**3 + 1, Lambda(_t, _t*lo
g(64*_t**3*a*b**2 + sqrt(x)))) + d**3*Piecewise((zoo*x**(3/2), Eq(a, 0) & E
q(b, 0)), (2*x**(7/2)/(7*a), Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(a, 0)), (-a*l
og(sqrt(x) - (-a/b)**(1/4))/(2*b**2*(-a/b)**(1/4)) + a*log(sqrt(x) + (-a/b)
**(1/4))/(2*b**2*(-a/b)**(1/4)) - a*atan(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b
)**(1/4)) + 2*x**(3/2)/(3*b), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(206) = 412.
time = 1.41, size = 455, normalized size = 1.61

$$\frac{2\,d^3x^{\frac{3}{2}}}{3\,b}+\frac{2\left(5\,bc^3x^2-15\,ac^2dx^2-ac^3\right)}{5\,a^2x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^4}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*d^3*x^(3/2)/b + 2/5*(5*b*c^3*x^2 - 15*a*c^2*d*x^2 - a*c^3)/(a^2*x^(5/2)
) + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a
*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) + 1/2*sqrt(2)*((a*b^3)^(3
/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (
a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^3*b^4) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3
/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^4) + 1/4*sqrt(2)*((a*b^
3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^
2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a^3*b^4)
```

Mupad [B]

time = 0.20, size = 583, normalized size = 2.06

$$\frac{2\,d^3\,x^{3/2}}{3\,b}-\frac{\frac{2\,b\,c^3}{5\,a}+\frac{2\,b\,c^2\,x^2\,\left(3\,a\,d-b\,c\right)}{a^2}}{b\,x^{5/2}}+\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^3\,\left(16\,a^{13}\,b^5\,d^6-96\,a^{12}\,b^6\,c\,d^5+240\,a^{11}\,b^7\,c^2\,d^4-320\,a^{10}\,b^8\,c^3\,d^3+240\,a^9\,b^9\,c^4\,d^2-96\,a^8\,b^{10}\,c^5\,d+16\,a^7\,b^{11}\,c^6\right)}{{\left(-a\right)}^{9/4}\,b^{7/4}\,\left(-16\,a^{14}\,b^3\,d^9+144\,a^{13}\,b^4\,c\,d^8-576\,a^{12}\,b^5\,c^2\,d^7+1344\,a^{11}\,b^6\,c^3\,d^6-2016\,a^{10}\,b^7\,c^4\,d^5+2016\,a^9\,b^8\,c^5\,d^4-1344\,a^8\,b^9\,c^6\,d^3+576\,a^7\,b^{10}\,c^7\,d^2-144\,a^6\,b^{11}\,c^8\,d+16\,a^5\,b^{12}\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^3}{{\left(-a\right)}^{9/4}\,b^{7/4}}+\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^3\,\left(16\,a^{13}\,b^5\,d^6-96\,a^{12}\,b^6\,c\,d^5+240\,a^{11}\,b^7\,c^2\,d^4-320\,a^{10}\,b^8\,c^3\,d^3+240\,a^9\,b^9\,c^4\,d^2-96\,a^8\,b^{10}\,c^5\,d+16\,a^7\,b^{11}\,c^6\right)\,1{}\mathrm{i}}{{\left(-a\right)}^{9/4}\,b^{7/4}\,\left(-16\,a^{14}\,b^3\,d^9+144\,a^{13}\,b^4\,c\,d^8-576\,a^{12}\,b^5\,c^2\,d^7+1344\,a^{11}\,b^6\,c^3\,d^6-2016\,a^{10}\,b^7\,c^4\,d^5+2016\,a^9\,b^8\,c^5\,d^4-1344\,a^8\,b^9\,c^6\,d^3+576\,a^7\,b^{10}\,c^7\,d^2-144\,a^6\,b^{11}\,c^8\,d+16\,a^5\,b^{12}\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^3\,1{}\mathrm{i}}{{\left(-a\right)}^{9/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x)
```

```
[Out] (2*d^3*x^(3/2))/(3*b) - ((2*b*c^3)/(5*a) + (2*b*c^2*x^2*(3*a*d - b*c))/a^2)
/(b*x^(5/2)) + (atan((x^(1/2)*(a*d - b*c)^3*(16*a^7*b^11*c^6 + 16*a^13*b^5*
d^6 - 96*a^8*b^10*c^5*d - 96*a^12*b^6*c*d^5 + 240*a^9*b^9*c^4*d^2 - 320*a^1
0*b^8*c^3*d^3 + 240*a^11*b^7*c^2*d^4))/((-a)^(9/4)*b^(7/4)*(16*a^5*b^12*c^9
 - 16*a^14*b^3*d^9 - 144*a^6*b^11*c^8*d + 144*a^13*b^4*c*d^8 + 576*a^7*b^10
*c^7*d^2 - 1344*a^8*b^9*c^6*d^3 + 2016*a^9*b^8*c^5*d^4 - 2016*a^10*b^7*c^4*
d^5 + 1344*a^11*b^6*c^3*d^6 - 576*a^12*b^5*c^2*d^7)))*(a*d - b*c)^3)/((-a)^
(9/4)*b^(7/4)) + (atan((x^(1/2)*(a*d - b*c)^3*(16*a^7*b^11*c^6 + 16*a^13*b^
5*d^6 - 96*a^8*b^10*c^5*d - 96*a^12*b^6*c*d^5 + 240*a^9*b^9*c^4*d^2 - 320*a
^10*b^8*c^3*d^3 + 240*a^11*b^7*c^2*d^4)*1i)/((-a)^(9/4)*b^(7/4)*(16*a^5*b^1
2*c^9 - 16*a^14*b^3*d^9 - 144*a^6*b^11*c^8*d + 144*a^13*b^4*c*d^8 + 576*a^7
*b^10*c^7*d^2 - 1344*a^8*b^9*c^6*d^3 + 2016*a^9*b^8*c^5*d^4 - 2016*a^10*b^7
*c^4*d^5 + 1344*a^11*b^6*c^3*d^6 - 576*a^12*b^5*c^2*d^7)))*(a*d - b*c)^3*1i
)/((-a)^(9/4)*b^(7/4))
```

3.448 $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx$

Optimal. Leaf size=283

$$-\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{}}{\sqrt[4]{a}}\right.}{\sqrt{2}\,a^{11/4}b^{5/4}}$$

```
[Out] -2/7*c^3/a/x^(7/2)+2/3*c^2*(-3*a*d+b*c)/a^2/x^(3/2)-1/2*(-a*d+b*c)^3*arctan
(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(5/4)*2^(1/2)+1/2*(-a*d+b*c)
^3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(5/4)*2^(1/2)-1/4*(
-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/
b^(5/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/
2)*x^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+2*d^3*x^(1/2)/b
```

Rubi [A]
time = 0.18, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 472, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}-\frac{2c^3}{7ax^{7/2}}+\frac{2d^3\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(7*a*x^(7/2)) + (2*c^2*(b*c - 3*a*d))/(3*a^2*x^(3/2)) + (2*d^3*Sqr
t[x])/b - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(11/4)*b^(5/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(11/4)*b^(5/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(11/4)*b^(5/4)) + ((
b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^8(a+bx^4)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{d^3}{b}+\frac{c^3}{ax^8}+\frac{c^2(-bc+3ad)}{a^2x^4}-\frac{(-bc+ad)^3}{a^2b(a+bx^4)}\right)\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}+\frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2b}$$

$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}+\frac{(bc-ad)^3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{5/2}b}+$$

$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}+\frac{(bc-ad)^3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x,\right.}{2a^{5/2}b^{3/2}}$$

$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\right.}{2\sqrt{2}\,a^{11/4}b^{5/4}}$$

$$= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{(bc-}{}$$

Mathematica [A]
time = 0.23, size = 177, normalized size = 0.63

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\,(7b^2c^3x^2+21a^2d^3x^4-3abc^2(c+7dx^2))}{x^{7/2}}+21\sqrt{2}\,(-bc+ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+21\sqrt{2}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{42a^{11/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(7*b^2*c^3*x^2 + 21*a^2*d^3*x^4 - 3*a*b*c^2*(c + 7*d*x^
2)))/x^(7/2) + 21*Sqrt[2]*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 21*Sqrt[2]*(b*c - a*d)^3*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(42*a^(11/4)*b^(5/4))
```

Maple [A]
time = 0.10, size = 188, normalized size = 0.66

method	result

derivativedivides	$\frac{2d^3\sqrt{x}}{b}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4a^3b}$
default	$\frac{2d^3\sqrt{x}}{b}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4a^3b}$
risch	$\frac{2a^2d^3x^4-2abc^2d\,x^2+\frac{2}{3}b^2c^3x^2-\frac{2}{7}abc^3}{b\,x^{\frac{7}{2}}a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)d^3}{2b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*x^(1/2)/b+1/4/a^3/b*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)*(a
/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/7*c^3/a/x^(7/2)-2/3*c^2*(3*a*d-
b*c)/a^2/x^(3/2)
```

Maxima [A]
time = 0.51, size = 368, normalized size = 1.30

$$\frac{2\,d^3\sqrt{x}}{b}+\frac{\frac{2\,\sqrt{2}\,\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,a^2b}-\frac{2\,\left(3\,ac^3-7\,\left(bc^3-3\,ac^2d\right)x^2\right)}{21\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2*d^3*sqrt(x)/b + 1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^
(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/
4)) - sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*
b) - 2/21*(3*a*c^3 - 7*(b*c^3 - 3*a*c^2*d)*x^2)/(a^2*x^(7/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1861 vs. $2(206) = 412$.
time = 0.55, size = 1861, normalized size = 6.58

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="fricas")

[Out] -1/42*(84*a^2*b*x^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4)*arctan((sqrt(a^6*b^2*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5)) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^8*b^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(3/4) + (a^8*b^7*c^3 - 3*a^9*b^6*c^2*d + 3*a^10*b^5*c*d^2 - a^11*b^4*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 21*a^2*b*x^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4)*log(a^3*b*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 21*a^2*b*x^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4)*log(-a^3*b*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 4*(21*a^2*d^3*x^4 - 3*a*b*c^3 + 7*(b^2*c^3 - 3*a*b*c^2*d)*x^2)*sqrt(x))/(a^2*b*x^4)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(265) = 530$.
time = 66.49, size = 607, normalized size = 2.14

$$\begin{cases} \tilde{\infty}\left(-\frac{2c^3}{11x^{\frac{11}{2}}} - \frac{6c^2d}{7x^{\frac{7}{2}}} - \frac{2cd^2}{x^{\frac{3}{2}}} + 2d^3\sqrt{x}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{-\frac{2c^3}{11x^{\frac{11}{2}}} - \frac{6c^2d}{7x^{\frac{7}{2}}} - \frac{2cd^2}{x^{\frac{3}{2}}} + 2d^3\sqrt{x}}{b} & \text{for } a = 0 \\ \frac{-\frac{2c^3}{7x^{\frac{7}{2}}} - \frac{2c^2d}{x^{\frac{3}{2}}} + 6cd^2\sqrt{x} + \frac{2d^3x^{\frac{5}{2}}}{5}}{a} & \text{for } b = 0 \\ \frac{2d^3\sqrt{x}}{b} + \frac{d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{d^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b} - \frac{2c^3}{7ax^{\frac{7}{2}}} - \frac{2c^2d}{ax^{\frac{3}{2}}} - \frac{3cd^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{3cd^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{3cd^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a} + \frac{2bc^3}{3a^2x^{\frac{3}{2}}} + \frac{3bc^2d\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a^2} - \frac{3bc^2d\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a^2} - \frac{3bc^2d\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2} - \frac{b^2c^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2a^3} + \frac{b^2c^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2a^3} + \frac{b^2c^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(11*x**(11/2)) - 6*c**2*d/(7*x**(7/2)) - 2*c*d**2/x
**(3/2) + 2*d**3*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(11*x**(11/2)) -
 6*c**2*d/(7*x**(7/2)) - 2*c*d**2/x**(3/2) + 2*d**3*sqrt(x))/b, Eq(a, 0)),
((-2*c**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x) + 2*d**3*x**(
5/2)/5)/a, Eq(b, 0)), (2*d**3*sqrt(x)/b + d**3*(-a/b)**(1/4)*log(sqrt(x) -
(-a/b)**(1/4))/(2*b) - d**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b
) - d**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b - 2*c**3/(7*a*x**(7/2)
) - 2*c**2*d/(a*x**(3/2)) - 3*c*d**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1
/4))/(2*a) + 3*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a) + 3*
c*d**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a + 2*b*c**3/(3*a**2*x**(3
/2)) + 3*b*c**2*d*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a**2) - 3*b
*c**2*d*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a**2) - 3*b*c**2*d*(-
a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a**2 - b**2*c**3*(-a/b)**(1/4)*log(
sqrt(x) - (-a/b)**(1/4))/(2*a**3) + b**2*c**3*(-a/b)**(1/4)*log(sqrt(x) + (
-a/b)**(1/4))/(2*a**3) + b**2*c**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4)
)/a**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(206) = 412$.
time = 1.61, size = 455, normalized size = 1.61

$$\frac{2\,d^3\sqrt{x}}{b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^3b^2} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^3b^2} + \frac{2\,(7\,bc^3x^2 - 21\,ac^2dx^2 - 3\,ac^3)}{21\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2*d^3*sqrt(x)/b + 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^
2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*s
qrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/2*sqrt(
2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a
^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
- 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)
*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 1/4*
sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1
```

```
/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(a^3*b^2) + 2/21*(7*b*c^3*x^2 - 21*a*c^2*d*x^2 - 3*a*c^3)/(
a^2*x^(7/2))
```

Mupad [B]
time = 0.13, size = 1564, normalized size = 5.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x)
```

```
[Out] (2*d^3*x^(1/2))/b - ((2*b*c^3)/(7*a) + (2*b*c^2*x^2*(3*a*d - b*c))/(3*a^2))
/(b*x^(7/2)) + (atan(((((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^
7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d
^3 + 240*a^10*b^8*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b
^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2))/(2*(-a)^(11/4)*b^(5/4)))*(
a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)) + (((x^(1/2)*(16*a^6*b^12*c^6 + 16*a
^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2
- 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^9*b
^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2))/(2*(-a)
^(11/4)*b^(5/4)))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)))/((((x^(1/2)*(16*
a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 24
0*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 - ((a*d
 - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*
b^8*c*d^2))/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3)/((-a)^(11/4)*b^(5/4)) -
 (((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^1
1*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2
*d^4))/2 + ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*
c^2*d + 48*a^11*b^8*c*d^2))/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3)/((-a)^(
11/4)*b^(5/4))))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)) + (atan(((((x^(1/2
)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^
5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 -
 ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48
*a^11*b^8*c*d^2)*1i)/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3)/((-a)^(11/4)*b
^(5/4)) + (((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d
 - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^1
0*b^8*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*
a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2)*1i)/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c
)^3)/((-a)^(11/4)*b^(5/4)))/((((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6
- 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^
9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16
*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2)*1i)/(2*(-a)^(11/4)*b
^(5/4)))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)) - (((x^(1/2)*(16*a^6*b^12*
c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^1
```

```
0*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 + ((a*d - b*c)^3
*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2
)*1i)/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4))))*(a
*d - b*c)^3)/((-a)^(11/4)*b^(5/4))
```

3.449 $\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx$

Optimal. Leaf size=303

$$-\frac{2c^3}{9ax^{9/2}}+\frac{2c^2(bc-3ad)}{5a^2x^{5/2}}-\frac{2c(b^2c^2-3abcd+3a^2d^2)}{a^3\sqrt{x}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{(bc-ad)^3\tan^{-}}{\sqrt{2}}$$

```
[Out] -2/9*c^3/a/x^(9/2)+2/5*c^2*(-3*a*d+b*c)/a^2/x^(5/2)+1/2*(-a*d+b*c)^3*arctan
(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/b^(3/4)*2^(1/2)-1/2*(-a*d+b*c)
^3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/b^(3/4)*2^(1/2)-1/4*(
-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/
b^(3/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/
2)*x^(1/2))/a^(13/4)/b^(3/4)*2^(1/2)-2*c*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/a^3/
x^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 472, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{2c^2(bc-3ad)}{5a^2x^{5/2}}-\frac{2c(3a^2d^2-3abcd+b^2c^2)}{a^3\sqrt{x}}-\frac{2c^3}{9ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(9*a*x^(9/2)) + (2*c^2*(b*c - 3*a*d))/(5*a^2*x^(5/2)) - (2*c*(b^2*
c^2 - 3*a*b*c*d + 3*a^2*d^2))/(a^3*Sqrt[x]) + ((b*c - a*d)^3*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4)) - ((b*c - a*d)^
3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4))
 - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(2*Sqrt[2]*a^(13/4)*b^(3/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^{10}(a+bx^4)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{c^3}{ax^{10}} + \frac{c^2(-bc+3ad)}{a^2x^6} + \frac{c(b^2c^2-3abcd+3a^2d^2)}{a^3x^2} + \frac{(-bc+ad)^3x^2}{a^3(a+bx^4)}\right) d$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{a^3\sqrt{x}} - \frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\right.}{a^3}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{a^3\sqrt{x}} + \frac{(bc-ad)^3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{}}{a+bx^4}\right.}{a^3\sqrt{b}}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{a^3\sqrt{x}} - \frac{(bc-ad)^3\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}-}\right.}{2a^3}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{a^3\sqrt{x}} - \frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\right.}{2\sqrt{2}\,a^{13/4}}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{a^3\sqrt{x}} + \frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,a^{13/4}b^{3/4}}$$

Mathematica [A]
time = 0.24, size = 193, normalized size = 0.64

$$\frac{-\frac{4\sqrt[4]{a}\,c(45b^2c^2x^4-9abcx^2(c+15dx^2)+a^2(5c^2+27cdx^2+135d^2x^4))}{x^{9/2}} + \frac{45\sqrt{2}\,(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{b^{3/4}} + \frac{45\sqrt{2}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{3/4}}}{90a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] ((-4*a^(1/4)*c*(45*b^2*c^2*x^4 - 9*a*b*c*x^2*(c + 15*d*x^2) + a^2*(5*c^2 +
27*c*d*x^2 + 135*d^2*x^4)))/x^(9/2) + (45*Sqrt[2]*(b*c - a*d)^3*ArcTan[(Sqr
t[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) + (45*Sqrt[2]
*(b*c - a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]
*x)])/b^(3/4))/(90*a^(13/4))
```

Maple [A]
time = 0.09, size = 208, normalized size = 0.69

method	result
derivativedivides	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4a^3b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4a^3b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(135a^2d^2x^4-135abcd\,x^4+45b^2c^2x^4+27a^2cd\,x^2-9ab\,c^2x^2+5a^2c^2)c}{45a^3x^{\frac{9}{2}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)d^3}{2b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{3\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^3/b/(a/b)^(1/4)*2^(1/2)
*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1))-2/9*c^3/a/x^(9/2)-2*c*(3*a^2*d^2-3*a*b*c*d+b^2*c^2
)/a^3/x^(1/2)-2/5*c^2*(3*a*d-b*c)/a^2/x^(5/2)
```

Maxima [A]
time = 0.53, size = 281, normalized size = 0.93

$$-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,a^3}-\frac{2\,(5\,a^2c^3+45\,(b^2c^3-3\,abc^2d+3\,a^2cd^2)x^4-9\,(abc^3-3\,a^2c^2d)x^2)}{45\,a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(2*sqrt(2)*arctan(
1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(
a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3 - 2/45*(5*a^2*c^3 + 45*(
b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 9*(a*b*c^3 - 3*a^2*c^2*d)*x^2)/(
a^3*x^(9/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2459 vs. $2(226) = 452$.

time = 0.66, size = 2459, normalized size = 8.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/90*(180*a^3*x^5*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4)
*arctan((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a
^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*
a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 486
20*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18
564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 8
16*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18
)*x - (a^7*b^13*c^12 - 12*a^8*b^12*c^11*d + 66*a^9*b^11*c^10*d^2 - 220*a^10
*b^10*c^9*d^3 + 495*a^11*b^9*c^8*d^4 - 792*a^12*b^8*c^7*d^5 + 924*a^13*b^7*
c^6*d^6 - 792*a^14*b^6*c^5*d^7 + 495*a^15*b^5*c^4*d^8 - 220*a^16*b^4*c^3*d^
9 + 66*a^17*b^3*c^2*d^10 - 12*a^18*b^2*c*d^11 + a^19*b*d^12)*sqrt(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3)))*a^3*b*(-(b^12*c^12 - 12*a*b^11*c^
11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 7
92*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
 a^12*d^12)/(a^13*b^3))^(1/4) + (a^3*b^10*c^9 - 9*a^4*b^9*c^8*d + 36*a^5*b^
8*c^7*d^2 - 84*a^6*b^7*c^6*d^3 + 126*a^7*b^6*c^5*d^4 - 126*a^8*b^5*c^4*d^5
+ 84*a^9*b^4*c^3*d^6 - 36*a^10*b^3*c^2*d^7 + 9*a^11*b^2*c*d^8 - a^12*b*d^9)
*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b
^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^
6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^
10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4))/(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)) - 45*a^3*x^5*(-(b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(a^13*b^3))^(1/4)*log(a^10*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
```

```
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a^13*b^3))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 45*a^3*x^5*(
-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^
7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2
*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4)*log(-a^10*b^2*(-(b^
12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^1
0 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(3/4) - (b^9*c^9 - 9*a*b^8*c^
8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a
^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 -
a^9*d^9)*sqrt(x)) + 4*(5*a^2*c^3 + 45*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)
*x^4 - 9*(a*b*c^3 - 3*a^2*c^2*d)*x^2)*sqrt(x))/(a^3*x^5)
```

Sympy [A]
time = 179.31, size = 469, normalized size = 1.55

$$c^{3}\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{13}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{13 b x^{\frac{13}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{9 a x^{\frac{9}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{9 a x^{\frac{9}{2}}} + \frac{2 b}{5 a^{2} x^{\frac{5}{2}}} - \frac{b^{2} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 a^{3} \sqrt[4]{- \frac{a}{b}}} + \frac{b^{2} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 a^{3} \sqrt[4]{- \frac{a}{b}}} - \frac{b^{2} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{a^{3} \sqrt[4]{- \frac{a}{b}}} - \frac{2 b^{2}}{a^{3} \sqrt{x}} & \text{otherwise} \end{cases}\right) + 3 c^{2} d \left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{9 b x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{5 a x^{\frac{5}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{5 a x^{\frac{5}{2}}} + \frac{b \log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 a^{2} \sqrt[4]{- \frac{a}{b}}} - \frac{b \log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 a^{2} \sqrt[4]{- \frac{a}{b}}} + \frac{b \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{a^{2} \sqrt[4]{- \frac{a}{b}}} + \frac{2 b}{a^{2} \sqrt{x}} & \text{otherwise} \end{cases}\right) + 3 c d^{2} \left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{5 b x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{a \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{\log{\left(\sqrt{x} - \sqrt[4]{- \frac{a}{b}} \right)}}{2 a \sqrt[4]{- \frac{a}{b}}} + \frac{\log{\left(\sqrt{x} + \sqrt[4]{- \frac{a}{b}} \right)}}{2 a \sqrt[4]{- \frac{a}{b}}} - \frac{\operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{a}{b}}} \right)}}{a \sqrt[4]{- \frac{a}{b}}} - \frac{2}{a \sqrt{x}} & \text{otherwise} \end{cases}\right) + 2 d^{3} \operatorname{RootSum} {\left(256 t^{4} a b^{3} + 1, \left(t \mapsto t \log{\left(64 t^{3} a b^{2} + \sqrt{x} \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(11/2)/(b*x**2+a),x)
```

```
[Out] c**3*Piecewise((zoo/x**(13/2), Eq(a, 0) & Eq(b, 0)), (-2/(13*b*x**(13/2)),
Eq(a, 0)), (-2/(9*a*x**(9/2)), Eq(b, 0)), (-2/(9*a*x**(9/2)) + 2*b/(5*a**2*
x**(5/2)) - b**2*log(sqrt(x) - (-a/b)**(1/4))/(2*a**3*(-a/b)**(1/4)) + b**2
*log(sqrt(x) + (-a/b)**(1/4))/(2*a**3*(-a/b)**(1/4)) - b**2*atan(sqrt(x)/(-
a/b)**(1/4))/(a**3*(-a/b)**(1/4)) - 2*b**2/(a**3*sqrt(x)), True)) + 3*c**2*
d*Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(a,
0)), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + b*log(sqrt(x) - (-
a/b)**(1/4))/(2*a**2*(-a/b)**(1/4)) - b*log(sqrt(x) + (-a/b)**(1/4))/(2*a**
2*(-a/b)**(1/4)) + b*atan(sqrt(x)/(-a/b)**(1/4))/(a**2*(-a/b)**(1/4)) + 2*b
/(a**2*sqrt(x)), True)) + 3*c*d**2*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b
, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqr
t(x) - (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2
*a*(-a/b)**(1/4)) - atan(sqrt(x)/(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*sq
rt(x)), True)) + 2*d**3*RootSum(256*_t**4*a*b**3 + 1, Lambda(_t, _t*log(64*
_t**3*a*b**2 + sqrt(x))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(226) = 452.
time = 1.60, size = 483, normalized size = 1.59

$$-\frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{3}{4}} b^{3} c^{3} - 3 \left(ab^{3}\right)^{\frac{3}{4}} a b^{2} c^{2} d + 3 \left(ab^{3}\right)^{\frac{3}{4}} a^{2} b c d^{2} - \left(ab^{3}\right)^{\frac{3}{4}} a^{3} d^{3}\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 a^{4} b^{3}} - \frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{3}{4}} b^{3} c^{3} - 3 \left(ab^{3}\right)^{\frac{3}{4}} a b^{2} c^{2} d + 3 \left(ab^{3}\right)^{\frac{3}{4}} a^{2} b c d^{2} - \left(ab^{3}\right)^{\frac{3}{4}} a^{3} d^{3}\right) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 a^{4} b^{3}} + \frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{3}{4}} b^{3} c^{3} - 3 \left(ab^{3}\right)^{\frac{3}{4}} a b^{2} c^{2} d + 3 \left(ab^{3}\right)^{\frac{3}{4}} a^{2} b c d^{2} - \left(ab^{3}\right)^{\frac{3}{4}} a^{3} d^{3}\right) \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 a^{4} b^{3}} - \frac{\sqrt{2} \left(\left(ab^{3}\right)^{\frac{3}{4}} b^{3} c^{3} - 3 \left(ab^{3}\right)^{\frac{3}{4}} a b^{2} c^{2} d + 3 \left(ab^{3}\right)^{\frac{3}{4}} a^{2} b c d^{2} - \left(ab^{3}\right)^{\frac{3}{4}} a^{3} d^{3}\right) \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 a^{4} b^{3}} - \frac{2 \left(45 b^{2} c^{3} x^{4} - 135 a b c^{2} d x^{4} + 135 a^{2} c d^{2} x^{4} - 9 a b c^{3} x^{2} + 27 a^{2} c^{2} d x^{2} + 5 a^{2} c^{3}\right)}{45 a^{3} x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^
3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^3) - 1/2*sqrt(2)*((a*b^3)^(3/4)
*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b
^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a
/b)^(1/4))/(a^4*b^3) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)
*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) - 1/4*sqrt(2)*((a*b^3)^
(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 -
 (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
a^4*b^3) - 2/45*(45*b^2*c^3*x^4 - 135*a*b*c^2*d*x^4 + 135*a^2*c*d^2*x^4 - 9
*a*b*c^3*x^2 + 27*a^2*c^2*d*x^2 + 5*a^2*c^3)/(a^3*x^(9/2))
```

Mupad [B]
time = 0.21, size = 591, normalized size = 1.95

$$\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,(16\,a^{16}\,b^2\,d^6-96\,a^{15}\,b^3\,c\,d^5+240\,a^{14}\,b^4\,c^2\,d^4-320\,a^{13}\,b^5\,c^3\,d^3+240\,a^{12}\,b^6\,c^4\,d^2-96\,a^{11}\,b^7\,c^5\,d+16\,a^{10}\,b^8\,c^6)}{(-a)^{13/4}\,b^{3/4}\,(16\,a^{16}\,b\,d^9-144\,a^{15}\,b^2\,c\,d^8+576\,a^{14}\,b^3\,c^2\,d^7-1344\,a^{13}\,b^4\,c^3\,d^6+2016\,a^{12}\,b^5\,c^4\,d^5-2016\,a^{11}\,b^6\,c^5\,d^4+1344\,a^{10}\,b^7\,c^6\,d^3-576\,a^9\,b^8\,c^7\,d^2+144\,a^8\,b^9\,c^8\,d-16\,a^7\,b^{10}\,c^9)}\right)\,(a\,d-b\,c)^3}{(-a)^{13/4}\,b^{3/4}}-\frac{\frac{2\,c^3}{9\,a}+\frac{2\,c^2\,x^2\,(3\,a\,d-b\,c)}{5\,a^2}+\frac{2\,c\,x^4\,(3\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2)}{a^3}}{x^{9/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,(16\,a^{16}\,b^2\,d^6-96\,a^{15}\,b^3\,c\,d^5+240\,a^{14}\,b^4\,c^2\,d^4-320\,a^{13}\,b^5\,c^3\,d^3+240\,a^{12}\,b^6\,c^4\,d^2-96\,a^{11}\,b^7\,c^5\,d+16\,a^{10}\,b^8\,c^6)}{(-a)^{13/4}\,b^{3/4}\,(16\,a^{16}\,b\,d^9-144\,a^{15}\,b^2\,c\,d^8+576\,a^{14}\,b^3\,c^2\,d^7-1344\,a^{13}\,b^4\,c^3\,d^6+2016\,a^{12}\,b^5\,c^4\,d^5-2016\,a^{11}\,b^6\,c^5\,d^4+1344\,a^{10}\,b^7\,c^6\,d^3-576\,a^9\,b^8\,c^7\,d^2+144\,a^8\,b^9\,c^8\,d-16\,a^7\,b^{10}\,c^9)}\right)\,(a\,d-b\,c)^3}{(-a)^{13/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x)
```

```
[Out] (atan((x^(1/2)*(a*d - b*c)^3*(16*a^10*b^8*c^6 + 16*a^16*b^2*d^6 - 96*a^11*b
^7*c^5*d - 96*a^15*b^3*c*d^5 + 240*a^12*b^6*c^4*d^2 - 320*a^13*b^5*c^3*d^3
+ 240*a^14*b^4*c^2*d^4))/((-a)^(13/4)*b^(3/4)*(16*a^16*b*d^9 - 16*a^7*b^10*
c^9 + 144*a^8*b^9*c^8*d - 144*a^15*b^2*c*d^8 - 576*a^9*b^8*c^7*d^2 + 1344*a
^10*b^7*c^6*d^3 - 2016*a^11*b^6*c^5*d^4 + 2016*a^12*b^5*c^4*d^5 - 1344*a^13
*b^4*c^3*d^6 + 576*a^14*b^3*c^2*d^7)))*(a*d - b*c)^3)/((-a)^(13/4)*b^(3/4))
 - ((2*c^3)/(9*a) + (2*c^2*x^2*(3*a*d - b*c))/(5*a^2) + (2*c*x^4*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/a^3)/x^(9/2) - (atanh((x^(1/2)*(a*d - b*c)^3*(16*a
^10*b^8*c^6 + 16*a^16*b^2*d^6 - 96*a^11*b^7*c^5*d - 96*a^15*b^3*c*d^5 + 240
*a^12*b^6*c^4*d^2 - 320*a^13*b^5*c^3*d^3 + 240*a^14*b^4*c^2*d^4))/((-a)^(13
/4)*b^(3/4)*(16*a^16*b*d^9 - 16*a^7*b^10*c^9 + 144*a^8*b^9*c^8*d - 144*a^15
*b^2*c*d^8 - 576*a^9*b^8*c^7*d^2 + 1344*a^10*b^7*c^6*d^3 - 2016*a^11*b^6*c^
5*d^4 + 2016*a^12*b^5*c^4*d^5 - 1344*a^13*b^4*c^3*d^6 + 576*a^14*b^3*c^2*d^
7)))*(a*d - b*c)^3)/((-a)^(13/4)*b^(3/4))
```

3.450 $\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx$

Optimal. Leaf size=305

$$-\frac{2c^3}{11ax^{11/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}-\frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\,\mathrm{ta}}{}$$

```
[Out] -2/11*c^3/a/x^(11/2)+2/7*c^2*(-3*a*d+b*c)/a^2/x^(7/2)-2/3*c*(3*a^2*d^2-3*a*
b*c*d+b^2*c^2)/a^3/x^(3/2)+1/2*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1/2)*x^(1/2
)/a^(1/4))/a^(15/4)/b^(1/4)*2^(1/2)-1/2*(-a*d+b*c)^3*arctan(1+b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/a^(15/4)/b^(1/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*
b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)-1/4*(-a*d
+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(15/4)/b^(1
/4)*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 472, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}-\frac{2c(3a^2d^2-3abcd+b^2c^2)}{3a^3x^{3/2}}-\frac{2c^3}{11ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(11*a*x^(11/2)) + (2*c^2*(b*c - 3*a*d))/(7*a^2*x^(7/2)) - (2*c*(b^
2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(3*a^3*x^(3/2)) + ((b*c - a*d)^3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c - a
*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1
/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^{12}(a+bx^4)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{c^3}{ax^{12}} + \frac{c^2(-bc+3ad)}{a^2x^8} + \frac{c(b^2c^2-3abcd+3a^2d^2)}{a^3x^4} + \frac{(-bc+ad)^3}{a^3(a+bx^4)}\right) dx$$

$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} - \frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{}{a}\right.}{a^3}$$

$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} - \frac{(bc-ad)^3\text{Subst}\left(\int \frac{\sqrt{a}}{a}\right.}{a^{7/2}}$$

$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} - \frac{(bc-ad)^3\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}}\right.}{2a}$$

$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} + \frac{(bc-ad)^3\log\left(\sqrt{a} - \right.}{2\sqrt{2}\,a}$$

$$= -\frac{2c^3}{11ax^{11/2}} + \frac{2c^2(bc-3ad)}{7a^2x^{7/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}} + \frac{(bc-ad)^3\tan^{-1}\left(1 - \right.}{\sqrt{2}\,a^{15/4}\sqrt[4]{b}}$$

Mathematica [A]
time = 0.25, size = 194, normalized size = 0.64

$$\frac{-\frac{4a^{3/4}c(77b^2c^2x^4-33abcx^2(c+7dx^2)+3a^2(7c^2+33cdx^2+77d^2x^4))}{x^{11/2}} + \frac{231\sqrt{2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{231\sqrt{2}(-bc+ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{b}}}{462a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

```
[Out] ((-4*a^(3/4)*c*(77*b^2*c^2*x^4 - 33*a*b*c*x^2*(c + 7*d*x^2) + 3*a^2*(7*c^2
+ 33*c*d*x^2 + 77*d^2*x^4)))/x^(11/2) + (231*Sqrt[2]*(b*c - a*d)^3*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (231*Sqr
t[2]*(-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
Sqrt[b]*x)])/b^(1/4))/(462*a^(15/4))
```

Maple [A]
time = 0.09, size = 205, normalized size = 0.67

method	result
derivativedivides	$\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d-b^{3} c^{3}\right) \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan\right.}{4 a^{4}}$
default	$\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d-b^{3} c^{3}\right) \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan\right.}{4 a^{4}}$
risch	$-\frac{2 \left(231 a^{2} d^{2} x^{4}-231 a b c d \,x^{4}+77 b^{2} c^{2} x^{4}+99 a^{2} c d \,x^{2}-33 a b \,c^{2} x^{2}+21 a^{2} c^{2}\right) c}{231 a^{3} x^{\frac{11}{2}}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right) d^{3}}{2 a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^4*(a/b)^(1/4)*2^(1/2)*(
ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1))-2/11*c^3/a/x^(11/2)-2/3*c*(3*a^2*d^2-3*a*b*c*d+b^2*c
^2)/a^3/x^(3/2)-2/7*c^2*(3*a*d-b*c)/a^2/x^(7/2)
```

Maxima [A]
time = 0.49, size = 389, normalized size = 1.28

$$-\frac{\frac{2 \sqrt{2} (b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2 \sqrt{2} (b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2} (b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3) \log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2} (b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3) \log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4a^3}-\frac{2\left(21a^2c^3+77(b^2c^3-3abc^2d+3a^2cd^2)x^4-33(abc^3-3a^2c^2d)x^2\right)}{231a^3x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(
1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^3 - 2/231*(21*a^2*c^
3 + 77*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 33*(a*b*c^3 - 3*a^2*c^2*
d)*x^2)/(a^3*x^(11/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1866 vs. $2(226) = 452$.

time = 0.76, size = 1866, normalized size = 6.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/462*(924*a^3*x^6*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4)*a
rctan((sqrt(a^8*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6
*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b)) + (b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^11*b*(-(b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(a^15*b))^(3/4) + (a^11*b^4*c^3 - 3*a^12*b^3*c^2*d + 3*a^13*b^2*c
*d^2 - a^14*b*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))
^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 231*a^3*x^6*(-(b^12*c^12 -
12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8
*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
+ 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^1
1*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4)*log(a^4*(-(b^12*c^12 - 12*a*b^11*c^
11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 7
92*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
 a^12*d^12)/(a^15*b))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(x)) - 231*a^3*x^6*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*
b))^(1/4)*log(-a^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4) -
```

```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 4*(21*a^2*c
^3 + 77*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 33*(a*b*c^3 - 3*a^2*c^2
*d)*x^2)*sqrt(x))/(a^3*x^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(13/2)/(b*x**2+a),x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(226) = 452.
time = 1.11, size = 483, normalized size = 1.58

$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b} - \frac{2\left(77b^2c^3x^4 - 231abc^2dx^4 + 231a^2cd^2x^4 - 33abc^3x^2 + 99a^2c^2dx^2 + 21a^2c^3\right)}{231a^3x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="giac")
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) - 1/2*sqrt(2)*((a*b^3)^(1/4)*b
^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3
)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(a^4*b) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b
^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 1/4*sqrt(2)*((a*b^3)^(1/4)*
b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^
3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b)
 - 2/231*(77*b^2*c^3*x^4 - 231*a*b*c^2*d*x^4 + 231*a^2*c*d^2*x^4 - 33*a*b*c
^3*x^2 + 99*a^2*c^2*d*x^2 + 21*a^2*c^3)/(a^3*x^(11/2))
```

Mupad [**B**]
time = 0.29, size = 1580, normalized size = 5.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x)
[Out] - ((2*c^3)/(11*a) + (2*c^2*x^2*(3*a*d - b*c))/(7*a^2) + (2*c*x^4*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/(3*a^3))/x^(11/2) - (atan(((((x^(1/2)*(16*a^9*b^9*
```

c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2))/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*1i)/((-a)^(15/4)*b^(1/4)) + (((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2))/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*1i)/((-a)^(15/4)*b^(1/4)))/((((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2))/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4)) - (((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2))/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4))))*(a*d - b*c)^3*1i)/((-a)^(15/4)*b^(1/4)) - (atan(((((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2)*1i)/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4)) + (((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2)*1i)/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4)))/((((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2)*1i)/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*1i)/((-a)^(15/4)*b^(1/4)) - (((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2)*1i)/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*1i)/((-a)^(15/4)*b^(1/4))))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4))

3.451 $\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx$

Optimal. Leaf size=325

$$-\frac{2c^3}{13ax^{13/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}-\frac{2c(b^2c^2-3abcd+3a^2d^2)}{5a^3x^{5/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}-\frac{\sqrt[4]{b}\,(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right.}{\sqrt{2}\,a^{17/4}}$$

```
[Out] -2/13*c^3/a/x^(13/2)+2/9*c^2*(-3*a*d+b*c)/a^2/x^(9/2)-2/5*c*(3*a^2*d^2-3*a*
b*c*d+b^2*c^2)/a^3/x^(5/2)-1/2*b^(1/4)*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1/2
)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+1/2*b^(1/4)*(-a*d+b*c)^3*arctan(1+b^(1/
4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^3*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(17/4)*2^(1/2)-1/4*b^(1/
4)*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(17
/4)*2^(1/2)+2*(-a*d+b*c)^3/a^4/x^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 472, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\,a^{17/4}}+\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\,a^{17/4}}+\frac{\sqrt[4]{b}\,(bc-ad)^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{17/4}}-\frac{\sqrt[4]{b}\,(bc-ad)^3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{17/4}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}-\frac{2c(3a^2d^2-3abcd+b^2c^2)}{5a^3x^{5/2}}-\frac{2c^3}{13ax^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(13*a*x^(13/2)) + (2*c^2*(b*c - 3*a*d))/(9*a^2*x^(9/2)) - (2*c*(b^
2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(5*a^3*x^(5/2)) + (2*(b*c - a*d)^3)/(a^4*Sq
rt[x]) - (b^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(17/4)) - (b^
(1/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(2*Sqrt[2]*a^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^{14}(a+bx^4)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{c^3}{ax^{14}} + \frac{c^2(-bc+3ad)}{a^2x^{10}} + \frac{c(b^2c^2-3abcd+3a^2d^2)}{a^3x^6} + \frac{(-bc+ad)^3}{a^4x^2} - \frac{b(-}{a}\right.\right.$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} + \frac{(2b(bc-ad}{}$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} - \frac{\left(\sqrt{b}\,(bc-\right.}{}$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} + \frac{(bc-ad)^3S}{}$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} + \frac{\sqrt[4]{b}\,(bc-a}{}$$

$$= -\frac{2c^3}{13ax^{13/2}} + \frac{2c^2(bc-3ad)}{9a^2x^{9/2}} - \frac{2c(b^2c^2-3abcd+3a^2d^2)}{5a^3x^{5/2}} + \frac{2(bc-ad)^3}{a^4\sqrt{x}} - \frac{\sqrt[4]{b}\,(bc-a}{}$$

Mathematica [A]
time = 0.26, size = 238, normalized size = 0.73

$$\frac{1170b^3c^3x^6 - 234ab^2c^2x^4(c+15dx^2) + 26a^2bcx^2(5c^2+27cdx^2+135d^2x^4) - 6a^3(15c^3+65c^2dx^2+117cd^2x^4+195d^3x^6)}{585a^4x^{13/2}} + \frac{\sqrt[4]{b}\,(-bc+ad)^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,a^{17/4}} + \frac{\sqrt[4]{b}\,(-bc+ad)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```

```
[Out] (1170*b^3*c^3*x^6 - 234*a*b^2*c^2*x^4*(c + 15*d*x^2) + 26*a^2*b*c*x^2*(5*c^
2 + 27*c*d*x^2 + 135*d^2*x^4) - 6*a^3*(15*c^3 + 65*c^2*d*x^2 + 117*c*d^2*x^
4 + 195*d^3*x^6))/(585*a^4*x^(13/2)) + (b^(1/4)*(-(b*c) + a*d)^3*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(17/4)) +
 (b^(1/4)*(-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[
a] + Sqrt[b]*x)])/(Sqrt[2]*a^(17/4))
```

Maple [A]
time = 0.10, size = 249, normalized size = 0.77

method	result
derivativedivides	$-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{4a^4\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{4a^4\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(585a^3d^3x^6-1755a^2bc\,d^2x^6+1755a\,b^2c^2d\,x^6-585b^3c^3x^6+351a^3c\,d^2x^4-351a^2b\,c^2d\,x^4+117a\,b^2c^3x^4+195a^3c^2d\,x^2-}{585a^4x^{\frac{13}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^4/(a/b)^(1/4)*2^(1/2)*
(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1))-2/13*c^3/a/x^(13/2)-2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^
2*c^2*d-b^3*c^3)/a^4/x^(1/2)-2/5*c*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/a^3/x^(5/2
)-2/9*c^2*(3*a*d-b*c)/a^2/x^(9/2)
```

Maxima [A]

time = 0.52, size = 330, normalized size = 1.02

$$\frac{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,a^4}+\frac{2\,(585\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^6-45\,a^3c^3-117\,(ab^2c^3-3\,a^2bc^2d+3\,a^3cd^2)x^4+65\,(a^2bc^3-3\,a^3c^2d)x^2)}{585\,a^4x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4 + 2/585*(585*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^6 - 45*a^3*c^3 - 117*(a*b^2*c
^3 - 3*a^2*b*c^2*d + 3*a^3*c*d^2)*x^4 + 65*(a^2*b*c^3 - 3*a^3*c^2*d)*x^2)/(
a^4*x^(13/2))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2512 vs. $2(244) = 488$.
time = 0.88, size = 2512, normalized size = 7.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/1170*(2340*a^4*x^7*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2
 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a
^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^
3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4
)*arctan((sqrt((b^20*c^18 - 18*a*b^19*c^17*d + 153*a^2*b^18*c^16*d^2 - 816*
a^3*b^17*c^15*d^3 + 3060*a^4*b^16*c^14*d^4 - 8568*a^5*b^15*c^13*d^5 + 18564
*a^6*b^14*c^12*d^6 - 31824*a^7*b^13*c^11*d^7 + 43758*a^8*b^12*c^10*d^8 - 48
620*a^9*b^11*c^9*d^9 + 43758*a^10*b^10*c^8*d^10 - 31824*a^11*b^9*c^7*d^11 +
 18564*a^12*b^8*c^6*d^12 - 8568*a^13*b^7*c^5*d^13 + 3060*a^14*b^6*c^4*d^14
- 816*a^15*b^5*c^3*d^15 + 153*a^16*b^4*c^2*d^16 - 18*a^17*b^3*c*d^17 + a^18
*b^2*d^18)*x - (a^9*b^13*c^12 - 12*a^10*b^12*c^11*d + 66*a^11*b^11*c^10*d^2
 - 220*a^12*b^10*c^9*d^3 + 495*a^13*b^9*c^8*d^4 - 792*a^14*b^8*c^7*d^5 + 92
4*a^15*b^7*c^6*d^6 - 792*a^16*b^6*c^5*d^7 + 495*a^17*b^5*c^4*d^8 - 220*a^18
*b^4*c^3*d^9 + 66*a^19*b^3*c^2*d^10 - 12*a^20*b^2*c*d^11 + a^21*b*d^12)*sqr
t(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*
d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792
*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*
c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17))*a^4*(-(b^13*c^12 - 12*a
*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^
8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 4
95*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b
^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4) + (a^4*b^10*c^9 - 9*a^5*b^9*c^8*d + 36
*a^6*b^8*c^7*d^2 - 84*a^7*b^7*c^6*d^3 + 126*a^8*b^6*c^5*d^4 - 126*a^9*b^5*c
^4*d^5 + 84*a^10*b^4*c^3*d^6 - 36*a^11*b^3*c^2*d^7 + 9*a^12*b^2*c*d^8 - a^1
3*b*d^9)*sqrt(x)*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 2
20*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b
^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^
9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4))/(
b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3
+ 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7
*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*
d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)) - 585*a^4*x^7*(-(b^13*c^12 - 12*a
*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^
8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 4
95*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b
^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*log(a^13*(-(b^13*c^12 - 12*a*b^12*c^11
```

```
*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 79
2*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5
*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11
+ a^12*b*d^12)/a^17)^(3/4) - (b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2
 - 84*a^3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*
b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9)*sqrt(x)) +
585*a^4*x^7*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^
3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^
6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 6
6*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*log(-a^
13*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^
9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 7
92*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^
3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(3/4) - (b^10*c^9 - 9*
a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4
 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^
2*c*d^8 - a^9*b*d^9)*sqrt(x)) + 4*(585*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*x^6 - 45*a^3*c^3 - 117*(a*b^2*c^3 - 3*a^2*b*c^2*d + 3*a^3*c
*d^2)*x^4 + 65*(a^2*b*c^3 - 3*a^3*c^2*d)*x^2)*sqrt(x))/(a^4*x^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(15/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(244) = 488.
time = 2.34, size = 536, normalized size = 1.65

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{5}b^{2}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{5}b^{2}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{5}b^{2}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{5}b^{2}}+\frac{2\left(585b^{3}c^{3}x^{6}-1755ab^{2}c^{2}dx^{6}+1755a^{2}bcd^{2}x^{6}-585a^{3}d^{3}x^{6}-117ab^{2}c^{3}x^{4}+351a^{2}bc^{2}dx^{4}-351a^{3}cd^{2}x^{4}+65a^{2}bc^{3}x^{2}-195a^{3}c^{2}dx^{2}-45a^{3}c^{3}\right)}{585a^{4}x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3
)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) + 1/2*sqrt(2)*((a*b^3)^(3/4)*
b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^
3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
b)^(1/4))/(a^5*b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*
a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt
```

```
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) + 1/4*sqrt(2)*((a*b^3)^(
3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 -
(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a
^5*b^2) + 2/585*(585*b^3*c^3*x^6 - 1755*a*b^2*c^2*d*x^6 + 1755*a^2*b*c*d^2*
x^6 - 585*a^3*d^3*x^6 - 117*a*b^2*c^3*x^4 + 351*a^2*b*c^2*d*x^4 - 351*a^3*c
*d^2*x^4 + 65*a^2*b*c^3*x^2 - 195*a^3*c^2*d*x^2 - 45*a^3*c^3)/(a^4*x^(13/2)
)
```

Mupad [B]

time = 0.22, size = 639, normalized size = 1.97

$$\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^3\,(16\,a^{19}\,b^4\,d^6-96\,a^{18}\,b^5\,c\,d^5+240\,a^{17}\,b^6\,c^2\,d^4-320\,a^{16}\,b^7\,c^3\,d^3+240\,a^{15}\,b^8\,c^4\,d^2-96\,a^{14}\,b^9\,c^5\,d+16\,a^{13}\,b^{10}\,c^6)}{a^{17/4}\,(-16\,a^{18}\,b^4\,d^9+144\,a^{17}\,b^5\,c\,d^8-576\,a^{16}\,b^6\,c^2\,d^7+1344\,a^{15}\,b^7\,c^3\,d^6-2016\,a^{14}\,b^8\,c^4\,d^5+2016\,a^{13}\,b^9\,c^5\,d^4-1344\,a^{12}\,b^{10}\,c^6\,d^3+576\,a^{11}\,b^{11}\,c^7\,d^2-144\,a^{10}\,b^{12}\,c^8\,d+16\,a^9\,b^{13}\,c^9)}\right)\,(a\,d-b\,c)^3}{a^{17/4}}-\frac{\frac{2\,c^3}{13\,a}+\frac{2\,x^6\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{a^4}+\frac{2\,c^2\,x^2\,(3\,a\,d-b\,c)}{9\,a^2}+\frac{2\,c\,x^4\,(3\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2)}{5\,a^3}}{x^{13/2}}-\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^3\,(16\,a^{19}\,b^4\,d^6-96\,a^{18}\,b^5\,c\,d^5+240\,a^{17}\,b^6\,c^2\,d^4-320\,a^{16}\,b^7\,c^3\,d^3+240\,a^{15}\,b^8\,c^4\,d^2-96\,a^{14}\,b^9\,c^5\,d+16\,a^{13}\,b^{10}\,c^6)}{a^{17/4}\,(-16\,a^{18}\,b^4\,d^9+144\,a^{17}\,b^5\,c\,d^8-576\,a^{16}\,b^6\,c^2\,d^7+1344\,a^{15}\,b^7\,c^3\,d^6-2016\,a^{14}\,b^8\,c^4\,d^5+2016\,a^{13}\,b^9\,c^5\,d^4-1344\,a^{12}\,b^{10}\,c^6\,d^3+576\,a^{11}\,b^{11}\,c^7\,d^2-144\,a^{10}\,b^{12}\,c^8\,d+16\,a^9\,b^{13}\,c^9)}\right)\,(a\,d-b\,c)^3}{a^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2)*(a*d - b*c)^3*(16*a^13*b^10*c^6 + 16*a
^19*b^4*d^6 - 96*a^14*b^9*c^5*d - 96*a^18*b^5*c*d^5 + 240*a^15*b^8*c^4*d^2
- 320*a^16*b^7*c^3*d^3 + 240*a^17*b^6*c^2*d^4))/(a^(17/4)*(16*a^9*b^13*c^9
- 16*a^18*b^4*d^9 - 144*a^10*b^12*c^8*d + 144*a^17*b^5*c*d^8 + 576*a^11*b^1
1*c^7*d^2 - 1344*a^12*b^10*c^6*d^3 + 2016*a^13*b^9*c^5*d^4 - 2016*a^14*b^8*
c^4*d^5 + 1344*a^15*b^7*c^3*d^6 - 576*a^16*b^6*c^2*d^7)))*(a*d - b*c)^3)/a^
(17/4) - ((2*c^3)/(13*a) + (2*x^6*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^
2*b*c*d^2))/a^4 + (2*c^2*x^2*(3*a*d - b*c))/(9*a^2) + (2*c*x^4*(3*a^2*d^2 +
 b^2*c^2 - 3*a*b*c*d))/(5*a^3))/x^(13/2) - ((-b)^(1/4)*atanh(((-b)^(1/4)*x^
(1/2)*(a*d - b*c)^3*(16*a^13*b^10*c^6 + 16*a^19*b^4*d^6 - 96*a^14*b^9*c^5*d
 - 96*a^18*b^5*c*d^5 + 240*a^15*b^8*c^4*d^2 - 320*a^16*b^7*c^3*d^3 + 240*a^
17*b^6*c^2*d^4))/(a^(17/4)*(16*a^9*b^13*c^9 - 16*a^18*b^4*d^9 - 144*a^10*b^
12*c^8*d + 144*a^17*b^5*c*d^8 + 576*a^11*b^11*c^7*d^2 - 1344*a^12*b^10*c^6*
d^3 + 2016*a^13*b^9*c^5*d^4 - 2016*a^14*b^8*c^4*d^5 + 1344*a^15*b^7*c^3*d^6
 - 576*a^16*b^6*c^2*d^7)))*(a*d - b*c)^3)/a^(17/4)
```

3.452 $\int \frac{x^{7/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=409

$$\frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5}+\frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4}+\frac{d^2(39bc-17ad)x^{9/2}}{18b^3}+\frac{17d^3x^{13/2}}{26b^2}-\frac{x^{5/2}(c+}{2b\,(a+}$$

```
[Out] 1/10*d*(17*a^2*d^2-39*a*b*c*d+27*b^2*c^2)*x^(5/2)/b^4+1/18*d^2*(-17*a*d+39*
b*c)*x^(9/2)/b^3+17/26*d^3*x^(13/2)/b^2-1/2*x^(5/2)*(d*x^2+c)^3/b/(b*x^2+a)
+1/8*a^(1/4)*(-17*a*d+5*b*c)*(-a*d+b*c)^2*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/
a^(1/4))/b^(21/4)*2^(1/2)-1/8*a^(1/4)*(-17*a*d+5*b*c)*(-a*d+b*c)^2*arctan(1
+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(21/4)*2^(1/2)+1/16*a^(1/4)*(-17*a*d+5*
b*c)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(
21/4)*2^(1/2)-1/16*a^(1/4)*(-17*a*d+5*b*c)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(21/4)*2^(1/2)+1/2*(-17*a*d+5*b*c)*(-a
*d+b*c)^2*x^(1/2)/b^5
```

Rubi [A]

time = 0.31, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 478, 584, 217, 1179, 642, 1176, 631, 210}

$$\frac{dx^{5/2}\left(17a^2d^2-39abcd+27b^2c^2\right)}{10b^4}+\frac{\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(5bc-17ad)(bc-ad)^2}{4\sqrt{2}\,b^{21/4}}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(5bc-17ad)(bc-ad)^2}{4\sqrt{2}\,b^{21/4}}+\frac{\sqrt[4]{a}\,(5bc-17ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{21/4}}-\frac{\sqrt[4]{a}\,(5bc-17ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{21/4}}+\frac{\sqrt{x}\,(5bc-17ad)(bc-ad)^2}{2b^5}+\frac{d^2x^{9/2}(39bc-17ad)}{18b^3}-\frac{x^{5/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{17d^3x^{13/2}}{26b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((5*b*c - 17*a*d)*(b*c - a*d)^2*Sqrt[x])/(2*b^5) + (d*(27*b^2*c^2 - 39*a*b*
c*d + 17*a^2*d^2)*x^(5/2))/(10*b^4) + (d^2*(39*b*c - 17*a*d)*x^(9/2))/(18*b
^3) + (17*d^3*x^(13/2))/(26*b^2) - (x^(5/2)*(c + d*x^2)^3)/(2*b*(a + b*x^2)
) + (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d
)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) + (
a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b
*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*
Sqrt[2]*b^(21/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^8(c+dx^4)^3}{(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{5/2}(c+dx^2)^3}{2b\,(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{x^4(c+dx^4)^2(5c+17dx^4)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= -\frac{x^{5/2}(c+dx^2)^3}{2b\,(a+bx^2)} + \frac{\text{Subst}\left(\int \left(\frac{(5bc-17ad)(bc-ad)^2}{b^4} + \frac{d(27b^2c^2-39abcd+17a^2d^2)x^4}{b^3} + \frac{d^2(39bc-17a}{b^2}\right.\right.}{2b}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d(27b^2c^2-39abcd+17a^2d^2)\,x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)}{18b^3}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d(27b^2c^2-39abcd+17a^2d^2)\,x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)}{18b^3}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d(27b^2c^2-39abcd+17a^2d^2)\,x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)}{18b^3}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d(27b^2c^2-39abcd+17a^2d^2)\,x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)}{18b^3}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d(27b^2c^2-39abcd+17a^2d^2)\,x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)}{18b^3}$$

Mathematica [A]
time = 0.57, size = 301, normalized size = 0.74

$$\frac{\frac{4\sqrt[4]{b}\sqrt{x}\left(-9945a^4d^3+117a^3bd^2(195c-68dx^2)+13a^2b^2d(-1215c^2+1404cdx^2+68d^2x^4)+ab^3(2925c^3-12636c^2dx^2-2028cd^2x^4-340d^3x^6)+12b^4x^2(195c^3+117c^2dx^2+65cd^2x^4+15d^3x^6)\right)}{a+bx^2}-585\sqrt{2}\sqrt[4]{a}(bc-ad)^2(-5bc+17ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+585\sqrt{2}\sqrt[4]{a}(bc-ad)^2(-5bc+17ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4680b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((4*b^(1/4)*Sqrt[x]*(-9945*a^4*d^3 + 117*a^3*b*d^2*(195*c - 68*d*x^2) + 13*
a^2*b^2*d*(-1215*c^2 + 1404*c*d*x^2 + 68*d^2*x^4) + a*b^3*(2925*c^3 - 12636
*c^2*d*x^2 - 2028*c*d^2*x^4 - 340*d^3*x^6) + 12*b^4*x^2*(195*c^3 + 117*c^2*
d*x^2 + 65*c*d^2*x^4 + 15*d^3*x^6)))/(a + b*x^2) - 585*Sqrt[2]*a^(1/4)*(b*c
 - a*d)^2*(-5*b*c + 17*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x])] + 585*Sqrt[2]*a^(1/4)*(b*c - a*d)^2*(-5*b*c + 17*a*d)*ArcT
anh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4680*b^(21/4
))
```

Maple [A]
time = 0.15, size = 327, normalized size = 0.80 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^5*(-1/13*d^3*x^(13/2)*b^3+2/9*a*b^2*d^3*x^(9/2)-1/3*b^3*c*d^2*x^(9/2)-
3/5*a^2*b*d^3*x^(5/2)+6/5*a*b^2*c*d^2*x^(5/2)-3/5*b^3*c^2*d*x^(5/2)+4*a^3*d
^3*x^(1/2)-9*a^2*b*c*d^2*x^(1/2)+6*a*b^2*c^2*d*x^(1/2)-b^3*c^3*x^(1/2))+2*a
/b^5*((-1/4*a^3*d^3+3/4*a^2*b*c*d^2-3/4*a*b^2*c^2*d+1/4*b^3*c^3)*x^(1/2)/(b
*x^2+a)+1/32*(17*a^3*d^3-39*a^2*b*c*d^2+27*a*b^2*c^2*d-5*b^3*c^3)*(a/b)^(1/
4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.50, size = 499, normalized size = 1.22

$$\frac{(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3)\sqrt{x}}{2(b^6x^2+ab^5)}-\frac{\left(\frac{2\sqrt{2}(5b^3c^3-27ab^2c^2d+39a^2bcd^2-17a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(5b^3c^3-27ab^2c^2d+39a^2bcd^2-17a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(5b^3c^3-27ab^2c^2d+39a^2bcd^2-17a^3d^3)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(5b^3c^3-27ab^2c^2d+39a^2bcd^2-17a^3d^3)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a}{16b^5}+\frac{2\left(45b^3d^3x^{\frac{13}{2}}+65(3b^3cd^2-2ab^2d^3)x^{\frac{9}{2}}+351(b^3c^2d-2ab^2cd^2+a^2bd^3)x^{\frac{5}{2}}+585(b^3c^3-6ab^2c^2d+9a^2bcd^2-4a^3d^3)\sqrt{x}\right)}{585b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(x)/(b^6*x^
2 + a*b^5) - 1/16*(2*sqrt(2)*(5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2 -
 17*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(5*b^
3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2 - 17*a^3*d^3)*arctan(-1/2*sqrt(2)*(
```

```
sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c
*d^2 - 17*a^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a
))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2
 - 17*a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(3/4)*b^(1/4)))*a/b^5 + 2/585*(45*b^3*d^3*x^(13/2) + 65*(3*b^3*c*d^2 - 2
*a*b^2*d^3)*x^(9/2) + 351*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^(5/2) +
 585*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*sqrt(x))/b^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2014 vs. $2(317) = 634$.
time = 0.79, size = 2014, normalized size = 4.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4680*(2340*(b^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d +
 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^
4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c
^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11
*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4)*arctan(
(sqrt(b^10*sqrt(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10
*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*
b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 1263145
5*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 -
766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21) + (25*b^6*c^6 - 270*a*b^5*c^5
*d + 1119*a^2*b^4*c^4*d^2 - 2276*a^3*b^3*c^3*d^3 + 2439*a^4*b^2*c^2*d^4 - 1
326*a^5*b*c*d^5 + 289*a^6*d^6)*x)*b^16*(-(625*a*b^12*c^12 - 13500*a^2*b^11*
c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^
8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a
^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168
018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(3/4)
 + (5*b^19*c^3 - 27*a*b^18*c^2*d + 39*a^2*b^17*c*d^2 - 17*a^3*b^16*d^3)*sqr
t(x)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2
- 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^
5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*
c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^1
2*b*c*d^11 + 83521*a^13*d^12)/b^21)^(3/4))/(625*a*b^12*c^12 - 13500*a^2*b^1
1*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*
b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976
*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 31
68018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)) + 585*(b
^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^1
```

```
0*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6
*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 126314
55*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 -
 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4)*log(b^5*(-(625*a*b^12*
c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^
9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^
6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a
^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*
a^13*d^12)/b^21)^(1/4) - (5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2 - 17*
a^3*d^3)*sqrt(x)) - 585*(b^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^
11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5
*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 1422597
6*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3
168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1
/4)*log(-b^5*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c
^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^
7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*
a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 76
6428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4) - (5*b^3*c^3 - 27*a*b^2*c
^2*d + 39*a^2*b*c*d^2 - 17*a^3*d^3)*sqrt(x)) + 4*(180*b^4*d^3*x^8 + 2925*a*
b^3*c^3 - 15795*a^2*b^2*c^2*d + 22815*a^3*b*c*d^2 - 9945*a^4*d^3 + 20*(39*b
^4*c*d^2 - 17*a*b^3*d^3)*x^6 + 52*(27*b^4*c^2*d - 39*a*b^3*c*d^2 + 17*a^2*b
^2*d^3)*x^4 + 468*(5*b^4*c^3 - 27*a*b^3*c^2*d + 39*a^2*b^2*c*d^2 - 17*a^3*b
*d^3)*x^2)*sqrt(x))/(b^6*x^2 + a*b^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.95, size = 600, normalized size = 1.47

$$-\frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-27\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+39\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-17\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8b^{6}}-\frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-27\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+39\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-17\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8b^{6}}-\frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-27\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+39\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-17\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16b^{6}}+\frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-27\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+39\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-17\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16b^{6}}+\frac{ab^{3}c^{3}\sqrt{x}-3a^{2}b^{2}c^{2}d\sqrt{x}+3a^{3}bcd^{2}\sqrt{x}-a^{4}d^{3}\sqrt{x}}{2\left(bx^{2}+a\right)b^{5}}+\frac{2\left(45b^{24}d^{3}x^{\frac{13}{2}}+195b^{24}cd^{2}x^{\frac{9}{2}}-130ab^{23}d^{3}x^{\frac{9}{2}}+351b^{24}c^{2}dx^{\frac{5}{2}}-702ab^{23}cd^{2}x^{\frac{5}{2}}+351a^{2}b^{22}d^{3}x^{\frac{5}{2}}+585b^{24}c^{3}\sqrt{x}-3510ab^{23}c^{2}d\sqrt{x}+5265a^{2}b^{22}cd^{2}\sqrt{x}-2340a^{3}b^{21}d^{3}\sqrt{x}\right)}{585b^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(
a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^6 - 1/8*sqrt(2)*(5*(a*b^3)^(1
```

```
/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 -
 17*(a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqr
t(x))/(a/b)^(1/4))/b^6 - 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)
^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^
3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^6 + 1/16*sqrt(2)*(5*(
a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*
b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
sqrt(a/b))/b^6 + 1/2*(a*b^3*c^3*sqrt(x) - 3*a^2*b^2*c^2*d*sqrt(x) + 3*a^3*b
*c*d^2*sqrt(x) - a^4*d^3*sqrt(x))/((b*x^2 + a)*b^5) + 2/585*(45*b^24*d^3*x^
(13/2) + 195*b^24*c*d^2*x^(9/2) - 130*a*b^23*d^3*x^(9/2) + 351*b^24*c^2*d*x
^(5/2) - 702*a*b^23*c*d^2*x^(5/2) + 351*a^2*b^22*d^3*x^(5/2) + 585*b^24*c^3
*sqrt(x) - 3510*a*b^23*c^2*d*sqrt(x) + 5265*a^2*b^22*c*d^2*sqrt(x) - 2340*a
^3*b^21*d^3*sqrt(x))/b^26
```

Mupad [B]
time = 0.16, size = 1850, normalized size = 4.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x^(1/2)*((2*c^3)/b^2 - (2*a*((6*c^2*d)/b^2 + (2*a*((4*a*d^3)/b^3 - (6*c*d^2
)/b^2))/b - (2*a^2*d^3)/b^4))/b + (a^2*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/b^2
) - x^(9/2)*((4*a*d^3)/(9*b^3) - (2*c*d^2)/(3*b^2)) + x^(5/2)*((6*c^2*d)/(5
*b^2) + (2*a*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/(5*b) - (2*a^2*d^3)/(5*b^4))
- (x^(1/2)*((a^4*d^3)/2 - (a*b^3*c^3)/2 + (3*a^2*b^2*c^2*d)/2 - (3*a^3*b*c*
d^2)/2))/(a*b^5 + b^6*x^2) + (2*d^3*x^(13/2))/(13*b^2) - ((-a)^(1/4)*atan((
((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*
b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 +
 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((-a)^(1/4)*(a*d - b*c)^2*
(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*
c*d^2))/b^(29/4))*1i)/(8*b^(21/4)) + ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*
b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4
*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d
^5))/b^7 - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b
^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2))/b^(29/4))*1i)/(8*b^(21/4)))/((
(-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b
^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 +
2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((-a)^(1/4)*(a*d - b*c)^2*(
17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c
*d^2))/b^(29/4)))/(8*b^(21/4)) - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)
*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4
*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))
/b^7 - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c
```

^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2))/b^(29/4)))/(8*b^(21/4))))*(a*d - b*c)^2*(17*a*d - 5*b*c)*1i)/(4*b^(21/4)) + ((-a)^(1/4)*atan((((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2)*1i)/b^(29/4)))/(8*b^(21/4)) + ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2)*1i)/b^(29/4)))/(8*b^(21/4)))/(((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2)*1i)/b^(29/4))*1i)/(8*b^(21/4)) - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2)*1i)/b^(29/4))*1i)/(8*b^(21/4))))*(a*d - b*c)^2*(17*a*d - 5*b*c))/(4*b^(21/4))

3.453 $\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=374

$$\frac{d(7b^2c^2-11abcd+5a^2d^2)\,x^{3/2}}{2b^4}+\frac{3d^2(11bc-5ad)x^{7/2}}{14b^3}+\frac{15d^3x^{11/2}}{22b^2}-\frac{x^{3/2}(c+dx^2)^3}{2b\,(a+bx^2)}-\frac{3(bc-5ad)(bc-ad)^2\,\mathrm{t}}{4\sqrt{2}}$$

```
[Out] 1/2*d*(5*a^2*d^2-11*a*b*c*d+7*b^2*c^2)*x^(3/2)/b^4+3/14*d^2*(-5*a*d+11*b*c)
*x^(7/2)/b^3+15/22*d^3*x^(11/2)/b^2-1/2*x^(3/2)*(d*x^2+c)^3/b/(b*x^2+a)-3/8
*(-5*a*d+b*c)*(-a*d+b*c)^2*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4
)/b^(19/4)*2^(1/2)+3/8*(-5*a*d+b*c)*(-a*d+b*c)^2*arctan(1+b^(1/4)*2^(1/2)*x
^(1/2)/a^(1/4))/a^(1/4)/b^(19/4)*2^(1/2)+3/16*(-5*a*d+b*c)*(-a*d+b*c)^2*ln(
a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(19/4)*2^(1/2)
-3/16*(-5*a*d+b*c)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2
)*x^(1/2))/a^(1/4)/b^(19/4)*2^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 478, 584, 303, 1176, 631, 210, 1179, 642}

$$\frac{dx^{3/2}(5a^2d^2-11abcd+7b^2c^2)}{2b^4}-\frac{3\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-5ad)(bc-ad)^2}{4\sqrt{2}\sqrt[4]{a}\,b^{19/4}}+\frac{3\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-5ad)(bc-ad)^2}{4\sqrt{2}\sqrt[4]{a}\,b^{19/4}}+\frac{3(bc-5ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{19/4}}-\frac{3(bc-5ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{19/4}}+\frac{3d^2x^{7/2}(11bc-5ad)}{14b^3}-\frac{x^{3/2}(c+dx^2)^3}{2b\,(a+bx^2)}+\frac{15d^3x^{11/2}}{22b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d*(7*b^2*c^2 - 11*a*b*c*d + 5*a^2*d^2)*x^(3/2))/(2*b^4) + (3*d^2*(11*b*c -
 5*a*d)*x^(7/2))/(14*b^3) + (15*d^3*x^(11/2))/(22*b^2) - (x^(3/2)*(c + d*x^
2)^3)/(2*b*(a + b*x^2)) - (3*(b*c - 5*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d
)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a
^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4)) - (3*(b*c -
 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^6(c+dx^4)^3}{(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}(c+dx^2)^3}{2b\,(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{x^2(c+dx^4)^2(3c+15dx^4)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= -\frac{x^{3/2}(c+dx^2)^3}{2b\,(a+bx^2)} + \frac{\text{Subst}\left(\int \left(\frac{3d(7b^2c^2-11abcd+5a^2d^2)x^2}{b^3} + \frac{3d^2(11bc-5ad)x^6}{b^2} + \frac{15d^3x^{10}}{b} + \frac{3(b^3}{}\right.\right.}{2b}$$

$$= \frac{d(7b^2c^2-11abcd+5a^2d^2)\,x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}(c+dx^2)}{2b\,(a+bx^2)}$$

$$= \frac{d(7b^2c^2-11abcd+5a^2d^2)\,x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}(c+dx^2)}{2b\,(a+bx^2)}$$

$$= \frac{d(7b^2c^2-11abcd+5a^2d^2)\,x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}(c+dx^2)}{2b\,(a+bx^2)}$$

$$= \frac{d(7b^2c^2-11abcd+5a^2d^2)\,x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}(c+dx^2)}{2b\,(a+bx^2)}$$

$$= \frac{d(7b^2c^2-11abcd+5a^2d^2)\,x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}(c+dx^2)}{2b\,(a+bx^2)}$$

Mathematica [A]

time = 0.52, size = 257, normalized size = 0.69

$$\frac{\frac{4b^{3/4}x^{3/2}(385a^3d^3+11a^2bd^2(-77c+20dx^2)+ab^2d(539c^2-484cdx^2-60d^2x^4)+b^3(-77c^3+308c^2dx^2+132cd^2x^4+28d^3x^6))}{a+bx^2}+\frac{231\sqrt{2}(bc-ad)^2(-bc+5ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}}+\frac{231\sqrt{2}(bc-ad)^2(-bc+5ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{a}}}{616b^{19/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((4*b^(3/4)*x^(3/2)*(385*a^3*d^3 + 11*a^2*b*d^2*(-77*c + 20*d*x^2) + a*b^2*
d*(539*c^2 - 484*c*d*x^2 - 60*d^2*x^4) + b^3*(-77*c^3 + 308*c^2*d*x^2 + 132
*c*d^2*x^4 + 28*d^3*x^6)))/(a + b*x^2) + (231*Sqrt[2]*(b*c - a*d)^2*(-(b*c)
 + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/
a^(1/4) + (231*Sqrt[2]*(b*c - a*d)^2*(-(b*c) + 5*a*d)*ArcTanh[(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4))/(616*b^(19/4))
```

Maple [A]

time = 0.13, size = 266, normalized size = 0.71

method	result
derivativedivides	$\frac{2d\left(\frac{b^2d^2x^{\frac{11}{2}}}{11}+\frac{(-2ab\,d^2+3b^2cd)x^{\frac{7}{2}}}{7}+\frac{(3a^2d^2-6abcd+3b^2c^2)x^{\frac{3}{2}}}{3}\right)}{b^4}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{15}{4}a^3\right.}{}\right.}{}$
default	$\frac{2d\left(\frac{b^2d^2x^{\frac{11}{2}}}{11}+\frac{(-2ab\,d^2+3b^2cd)x^{\frac{7}{2}}}{7}+\frac{(3a^2d^2-6abcd+3b^2c^2)x^{\frac{3}{2}}}{3}\right)}{b^4}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{15}{4}a^3\right.}{}\right.}{}$
risch	$\frac{2d\,x^{\frac{3}{2}}(7b^2d^2x^4-22ab\,d^2x^2+33b^2cd\,x^2+77a^2d^2-154abcd+77b^2c^2)}{77b^4}+\frac{x^{\frac{3}{2}}a^3d^3}{2b^4(b\,x^2+a)}-\frac{3x^{\frac{3}{2}}a^2d^2c}{2b^3(b\,x^2+a)}+\frac{3x^{\frac{3}{2}}ad\,c^2}{2b^2(b\,x^2+a)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/b^4*(1/11*b^2*d^2*x^(11/2)+1/7*(-2*a*b*d^2+3*b^2*c*d)*x^(7/2)+1/3*(3*a^
2*d^2-6*a*b*c*d+3*b^2*c^2)*x^(3/2))-2/b^4*((-1/4*a^3*d^3+3/4*a^2*b*c*d^2-3/
4*a*b^2*c^2*d+1/4*b^3*c^3)*x^(3/2)/(b*x^2+a)+1/8*(15/4*a^3*d^3-33/4*a^2*b*c
*d^2+21/4*a*b^2*c^2*d-3/4*b^3*c^3)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+
2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)))
```

Maxima [A]
time = 0.53, size = 337, normalized size = 0.90

$$-\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^{\frac{3}{2}}}{2\,(b^5x^2 + ab^4)} + \frac{3\,(b^3c^3 - 7\,ab^2c^2d + 11\,a^2bcd^2 - 5\,a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} - 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}x + \sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}x + \sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,b^4} + \frac{2\left(7\,b^2d^3x^{\frac{11}{2}} + 11\,(3\,b^2cd^2 - 2\,abd^3)x^{\frac{7}{2}} + 77\,(b^2c^2d - 2\,abcd^2 + a^2d^3)x^{\frac{3}{2}}\right)}{77\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^(3/2)/(b^5*x^2 +
 a*b^4) + 3/16*(b^3*c^3 - 7*a*b^2*c^2*d + 11*a^2*b*c*d^2 - 5*a^3*d^3)*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 + 2/77*(7*b
^2*d^3*x^(11/2) + 11*(3*b^2*c*d^2 - 2*a*b*d^3)*x^(7/2) + 77*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x^(3/2))/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2542 vs. $2(286) = 572$.
time = 0.65, size = 2542, normalized size = 6.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/616*(924*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10
*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^
7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d
^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 +
 625*a^12*d^12)/(a*b^19))^(1/4)*arctan((sqrt((b^18*c^18 - 42*a*b^17*c^17*d
+ 801*a^2*b^16*c^16*d^2 - 9200*a^3*b^15*c^15*d^3 + 71220*a^4*b^14*c^14*d^4
- 394392*a^5*b^13*c^13*d^5 + 1619684*a^6*b^12*c^12*d^6 - 5050512*a^7*b^11*c
^11*d^7 + 12147630*a^8*b^10*c^10*d^8 - 22765820*a^9*b^9*c^9*d^9 + 33419166*
a^10*b^8*c^8*d^10 - 38446992*a^11*b^7*c^7*d^11 + 34503236*a^12*b^6*c^6*d^12
 - 23888280*a^13*b^5*c^5*d^13 + 12508500*a^14*b^4*c^4*d^14 - 4790000*a^15*b
^3*c^3*d^15 + 1265625*a^16*b^2*c^2*d^16 - 206250*a^17*b*c*d^17 + 15625*a^18
*d^18)*x - (a*b^21*c^12 - 28*a^2*b^20*c^11*d + 338*a^3*b^19*c^10*d^2 - 2316
*a^4*b^18*c^9*d^3 + 10015*a^5*b^17*c^8*d^4 - 28856*a^6*b^16*c^7*d^5 + 57148
*a^7*b^15*c^6*d^6 - 78968*a^8*b^14*c^5*d^7 + 76111*a^9*b^13*c^4*d^8 - 50220
*a^10*b^12*c^3*d^9 + 21650*a^11*b^11*c^2*d^10 - 5500*a^12*b^10*c*d^11 + 625
*a^13*b^9*d^12)*sqrt(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2
```

- 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19)))*b^5*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(1/4) + (b^14*c^9 - 21*a*b^13*c^8*d + 180*a^2*b^12*c^7*d^2 - 820*a^3*b^11*c^6*d^3 + 2190*a^4*b^10*c^5*d^4 - 3606*a^5*b^9*c^4*d^5 + 3716*a^6*b^8*c^3*d^6 - 2340*a^7*b^7*c^2*d^7 + 825*a^8*b^6*c*d^8 - 125*a^9*b^5*d^9)*sqrt(x)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(1/4))/(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)) - 231*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(1/4)*log(27*a*b^14*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(3/4) - 27*(b^9*c^9 - 21*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 820*a^3*b^6*c^6*d^3 + 2190*a^4*b^5*c^5*d^4 - 3606*a^5*b^4*c^4*d^5 + 3716*a^6*b^3*c^3*d^6 - 2340*a^7*b^2*c^2*d^7 + 825*a^8*b*c*d^8 - 125*a^9*d^9)*sqrt(x)) + 231*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(1/4)*log(-27*a*b^14*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(3/4) - 27*(b^9*c^9 - 21*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 820*a^3*b^6*c^6*d^3 + 2190*a^4*b^5*c^5*d^4 - 3606*a^5*b^4*c^4*d^5 + 3716*a^6*b^3*c^3*d^6 - 2340*a^7*b^2*c^2*d^7 + 825*a^8*b*c*d^8 - 125*a^9*d^9)*sqrt(x)) + 4*(28*b^3*d^3*x^7 + 12*(11*b^3*c*d^2 - 5*a*b^2*d^3)*x^5 + 44*(7*b^3*c^2*d - 11*a*b^2*c*d^2 + 5*a^2*b*d^3)*x^3 - 77*(b^3*c^3 - 7*a*b^2*c^2*d + 11*a^2*b*c*d^2 - 5*a^3*d^3)*x)*sqrt(x))/(b^5*x^2 + a*b^4)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.54, size = 552, normalized size = 1.48

$$-\frac{b^3c^3x^{\frac{3}{2}} - 3\,ab^2c^2dx^{\frac{3}{2}} + 3\,a^2bcd^2x^{\frac{3}{2}} - a^3d^3x^{\frac{3}{2}}}{2\,(bx^2+a)b^4} + \frac{3\,\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 7\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 11\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - 5\,(ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^7} + \frac{3\,\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 7\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 11\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - 5\,(ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^7} - \frac{3\,\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 7\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 11\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - 5\,(ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,ab^7} + \frac{3\,\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 7\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 11\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - 5\,(ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,ab^7} + \frac{2\left(7\,b^{20}d^3x^{\frac{11}{2}} + 33\,b^{20}cd^2x^{\frac{7}{2}} - 22\,ab^{19}d^3x^{\frac{7}{2}} + 77\,b^{20}c^2dx^{\frac{3}{2}} - 154\,ab^{19}cd^2x^{\frac{3}{2}} + 77\,a^2b^{18}d^3x^{\frac{3}{2}}\right)}{77\,b^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3
*d^3*x^(3/2))/((b*x^2 + a)*b^4) + 3/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a
*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^
3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a
*b^7) + 3/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d +
11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^7) - 3/16*sqrt(2)*((a*
b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c
*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(a*b^7) + 3/16*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2
*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^7) + 2/77*(7*b^20*d^3*x^(11/2)
 + 33*b^20*c*d^2*x^(7/2) - 22*a*b^19*d^3*x^(7/2) + 77*b^20*c^2*d*x^(3/2) -
154*a*b^19*c*d^2*x^(3/2) + 77*a^2*b^18*d^3*x^(3/2))/b^22
```

Mupad [**B**]
time = 0.22, size = 681, normalized size = 1.82

$$x^{3/2}\left(\frac{2\,c^2\,d}{b^2} + \frac{2\,a\left(\frac{4\,a\,d^3}{b^3} - \frac{6\,c\,d^2}{b^2}\right)}{3\,b} - \frac{2\,a^2\,d^3}{3\,b^4}\right) - x^{7/2}\left(\frac{4\,a\,d^3}{7\,b^3} - \frac{6\,c\,d^2}{7\,b^2}\right) + \frac{2\,d^3\,x^{11/2}}{11\,b^2} + \frac{x^{3/2}\left(\frac{a^3\,d^3}{2} - \frac{3\,a^2\,b\,c\,d^2}{2} + \frac{3\,a\,b^2\,c^2\,d}{2} - \frac{b^3\,c^3}{2}\right)}{b^5\,x^2 + a\,b^4} - \frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^2\,(5\,a\,d-b\,c)\,(25\,a^7\,d^6-110\,a^6\,b\,c\,d^5+191\,a^5\,b^2\,c^2\,d^4-164\,a^4\,b^3\,c^3\,d^3+71\,a^3\,b^4\,c^4\,d^2-14\,a^2\,b^5\,c^5\,d+a\,b^6\,c^6)}{(-a)^{1/4}\,(125\,a^{10}\,d^9-825\,a^9\,b\,c\,d^8+2340\,a^8\,b^2\,c^2\,d^7-3716\,a^7\,b^3\,c^3\,d^6+3606\,a^6\,b^4\,c^4\,d^5-2190\,a^5\,b^5\,c^5\,d^4+820\,a^4\,b^6\,c^6\,d^3-180\,a^3\,b^7\,c^7\,d^2+21\,a^2\,b^8\,c^8\,d-a\,b^9\,c^9)}\right)\,(a\,d-b\,c)^2\,(5\,a\,d-b\,c)}{4\,(-a)^{1/4}\,b^{19/4}} - \frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^2\,(5\,a\,d-b\,c)\,(25\,a^7\,d^6-110\,a^6\,b\,c\,d^5+191\,a^5\,b^2\,c^2\,d^4-164\,a^4\,b^3\,c^3\,d^3+71\,a^3\,b^4\,c^4\,d^2-14\,a^2\,b^5\,c^5\,d+a\,b^6\,c^6)\,1{}\mathrm{i}}{(-a)^{1/4}\,(125\,a^{10}\,d^9-825\,a^9\,b\,c\,d^8+2340\,a^8\,b^2\,c^2\,d^7-3716\,a^7\,b^3\,c^3\,d^6+3606\,a^6\,b^4\,c^4\,d^5-2190\,a^5\,b^5\,c^5\,d^4+820\,a^4\,b^6\,c^6\,d^3-180\,a^3\,b^7\,c^7\,d^2+21\,a^2\,b^8\,c^8\,d-a\,b^9\,c^9)}\right)\,(a\,d-b\,c)^2\,(5\,a\,d-b\,c)\,3{}\mathrm{i}}{4\,(-a)^{1/4}\,b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x^(3/2)*((2*c^2*d)/b^2 + (2*a*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/(3*b) - (2*a
^2*d^3)/(3*b^4)) - x^(7/2)*((4*a*d^3)/(7*b^3) - (6*c*d^2)/(7*b^2)) + (2*d^3
*x^(11/2))/(11*b^2) + (x^(3/2)*((a^3*d^3)/2 - (b^3*c^3)/2 + (3*a*b^2*c^2*d)
/2 - (3*a^2*b*c*d^2)/2))/(a*b^4 + b^5*x^2) - (3*atan((b^(1/4)*x^(1/2)*(a*d
```

```
- b*c)^2*(5*a*d - b*c)*(25*a^7*d^6 + a*b^6*c^6 - 14*a^2*b^5*c^5*d + 71*a^3*
b^4*c^4*d^2 - 164*a^4*b^3*c^3*d^3 + 191*a^5*b^2*c^2*d^4 - 110*a^6*b*c*d^5))
/((-a)^(1/4)*(125*a^10*d^9 - a*b^9*c^9 + 21*a^2*b^8*c^8*d - 180*a^3*b^7*c^7
*d^2 + 820*a^4*b^6*c^6*d^3 - 2190*a^5*b^5*c^5*d^4 + 3606*a^6*b^4*c^4*d^5 -
3716*a^7*b^3*c^3*d^6 + 2340*a^8*b^2*c^2*d^7 - 825*a^9*b*c*d^8)))*(a*d - b*c
)^2*(5*a*d - b*c))/(4*(-a)^(1/4)*b^(19/4)) - (atan((b^(1/4)*x^(1/2)*(a*d -
b*c)^2*(5*a*d - b*c)*(25*a^7*d^6 + a*b^6*c^6 - 14*a^2*b^5*c^5*d + 71*a^3*b^
4*c^4*d^2 - 164*a^4*b^3*c^3*d^3 + 191*a^5*b^2*c^2*d^4 - 110*a^6*b*c*d^5)*1i
)/((-a)^(1/4)*(125*a^10*d^9 - a*b^9*c^9 + 21*a^2*b^8*c^8*d - 180*a^3*b^7*c^
7*d^2 + 820*a^4*b^6*c^6*d^3 - 2190*a^5*b^5*c^5*d^4 + 3606*a^6*b^4*c^4*d^5 -
 3716*a^7*b^3*c^3*d^6 + 2340*a^8*b^2*c^2*d^7 - 825*a^9*b*c*d^8)))*(a*d - b*
c)^2*(5*a*d - b*c)*3i)/(4*(-a)^(1/4)*b^(19/4))
```

3.454 $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=386

$$\frac{d(497b^2c^2-1098abcd+585a^2d^2)\sqrt{x}}{90b^4}+\frac{d(113bc-117ad)\sqrt{x}\,(c+dx^2)}{90b^3}+\frac{13d\sqrt{x}\,(c+dx^2)^2}{18b^2}-\frac{\sqrt{x}\,(c+dx^2)}{2b\,(a+bx^2)}$$

```
[Out] -1/8*(-13*a*d+b*c)*(-a*d+b*c)^2*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a
^(3/4)/b^(17/4)*2^(1/2)+1/8*(-13*a*d+b*c)*(-a*d+b*c)^2*arctan(1+b^(1/4)*2^(
1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/2)-1/16*(-13*a*d+b*c)*(-a*d+b*c
)^2*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(17/4)*
2^(1/2)+1/16*(-13*a*d+b*c)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4
)*2^(1/2)*x^(1/2))/a^(3/4)/b^(17/4)*2^(1/2)+1/90*d*(585*a^2*d^2-1098*a*b*c*
d+497*b^2*c^2)*x^(1/2)/b^4+1/90*d*(-117*a*d+113*b*c)*(d*x^2+c)*x^(1/2)/b^3+
13/18*d*(d*x^2+c)^2*x^(1/2)/b^2-1/2*(d*x^2+c)^3*x^(1/2)/b/(b*x^2+a)
```

Rubi [A]

time = 0.36, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 478, 542, 396, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-13ad)(bc-ad)^2}{4\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-13ad)(bc-ad)^2}{4\sqrt{2}\,a^{3/4}b^{17/4}}-\frac{(bc-13ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{d\sqrt{x}\,(585a^2d^2-1098abcd+497b^2c^2)}{90b^4}+\frac{d\sqrt{x}\,(c+dx^2)(113bc-117ad)}{90b^3}-\frac{\sqrt{x}\,(c+dx^2)^3}{2b\,(a+bx^2)}+\frac{13d\sqrt{x}\,(c+dx^2)^2}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d*(497*b^2*c^2 - 1098*a*b*c*d + 585*a^2*d^2)*Sqrt[x])/(90*b^4) + (d*(113*b
*c - 117*a*d)*Sqrt[x]*(c + d*x^2))/(90*b^3) + (13*d*Sqrt[x]*(c + d*x^2)^2)/
(18*b^2) - (Sqrt[x]*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - ((b*c - 13*a*d)*(b*c
 - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)
*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - 13*a*d)*(b*c - a*d)^2
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(
3/4)*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = 2\text{Subst}\left(\int \frac{x^4\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)^2\left(c+13dx^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)\left(c(9bc-13ad)+d(113bc-117ad)x^4\right)}{a+bx^4}\,dx,\right.}{18b^2}$$

$$= \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{c(}{}\right.}{}$$

$$= \frac{d(497b^2c^2-1098abcd+585a^2d^2)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\,(}{18}$$

$$= \frac{d(497b^2c^2-1098abcd+585a^2d^2)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\,(}{18}$$

$$= \frac{d(497b^2c^2-1098abcd+585a^2d^2)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\,(}{18}$$

$$= \frac{d(497b^2c^2-1098abcd+585a^2d^2)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\,(}{18}$$

$$= \frac{d(497b^2c^2-1098abcd+585a^2d^2)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\,(}{18}$$

Mathematica [A]
time = 0.54, size = 256, normalized size = 0.66

$$\frac{\frac{4\sqrt[4]{b}\sqrt{x}\left(585a^3d^3+9a^2bd^2\left(-135c+52dx^2\right)+ab^2d\left(675c^2-972cdx^2-52d^2x^4\right)+b^3\left(-45c^3+540c^2dx^2+108cd^2x^4+20d^3x^6\right)\right)}{a+bx^2}+\frac{45\sqrt{2}\,(bc-ad)^2(-bc+13ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}}+\frac{45\sqrt{2}\,(bc-13ad)(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}}}{360b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((4*b^(1/4)*Sqrt[x]*(585*a^3*d^3 + 9*a^2*b*d^2*(-135*c + 52*d*x^2) + a*b^2*
d*(675*c^2 - 972*c*d*x^2 - 52*d^2*x^4) + b^3*(-45*c^3 + 540*c^2*d*x^2 + 108
*c*d^2*x^4 + 20*d^3*x^6)))/(a + b*x^2) + (45*Sqrt[2]*(b*c - a*d)^2*(-(b*c)
```

```
+ 13*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/
a^(3/4) + (45*Sqrt[2]*(b*c - 13*a*d)*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4))/(360*b^(17/4))
```

Maple [A]
time = 0.12, size = 269, normalized size = 0.70

method	result
derivativedivides	$\frac{2d\left(\frac{b^2 x^{\frac{9}{2}} d^2}{9}-\frac{2ab\,d^2 x^{\frac{5}{2}}}{5}+\frac{3b^2 cd\,x^{\frac{5}{2}}}{5}+3a^2d^2\sqrt{x}-6abcd\sqrt{x}+3b^2c^2\sqrt{x}\right)}{b^4}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3\right.}{b\,x^2+a}\right.}{}$
default	$\frac{2d\left(\frac{b^2 x^{\frac{9}{2}} d^2}{9}-\frac{2ab\,d^2 x^{\frac{5}{2}}}{5}+\frac{3b^2 cd\,x^{\frac{5}{2}}}{5}+3a^2d^2\sqrt{x}-6abcd\sqrt{x}+3b^2c^2\sqrt{x}\right)}{b^4}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3\right.}{b\,x^2+a}\right.}{}$
risch	$\frac{2d\left(5b^2d^2x^4-18ab\,d^2x^2+27b^2cd\,x^2+135a^2d^2-270abcd+135b^2c^2\right)\sqrt{x}}{45b^4}+\frac{\sqrt{x}\,a^3d^3}{2b^4\left(b\,x^2+a\right)}-\frac{3\sqrt{x}\,a^2d^2c}{2b^3\left(b\,x^2+a\right)}+\frac{3\sqrt{x}}{2b^2\left(b\,x^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/b^4*(1/9*b^2*x^(9/2)*d^2-2/5*a*b*d^2*x^(5/2)+3/5*b^2*c*d*x^(5/2)+3*a^2*
d^2*x^(1/2)-6*a*b*c*d*x^(1/2)+3*b^2*c^2*x^(1/2))-2/b^4*((-1/4*a^3*d^3+3/4*a
^2*b*c*d^2-3/4*a*b^2*c^2*d+1/4*b^3*c^3)*x^(1/2)/(b*x^2+a)+1/32*(13*a^3*d^3-
27*a^2*b*c*d^2+15*a*b^2*c^2*d-b^3*c^3)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(
1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)-1)))
```

Maxima [A]
time = 0.55, size = 445, normalized size = 1.15

$$-\frac{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{x}}{2\left(b^5x^2+ab^4\right)}+\frac{2\left(5b^2d^3x^{\frac{9}{2}}+9\left(3b^2cd^2-2abd^3\right)x^{\frac{5}{2}}+135\left(b^2c^2d-2abcd^2+a^2d^3\right)\sqrt{x}\right)}{45b^4}+\frac{\frac{2\sqrt{2}\left(b^3c^3-15ab^2c^2d+27a^2bcd^2-13a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(b^3c^3-15ab^2c^2d+27a^2bcd^2-13a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\left(b^3c^3-15ab^2c^2d+27a^2bcd^2-13a^3d^3\right)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^3c^3-15ab^2c^2d+27a^2bcd^2-13a^3d^3\right)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)/(b^5*x^2 +
 a*b^4) + 2/45*(5*b^2*d^3*x^(9/2) + 9*(3*b^2*c*d^2 - 2*a*b*d^3)*x^(5/2) + 1
35*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(x))/b^4 + 1/16*(2*sqrt(2)*(b^3*
```

```
c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*arctan(1/2*sqrt(2)*(sqr
t(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^
2 - 13*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^
3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3
- 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1961 vs. $2(298) = 596$.
time = 0.54, size = 1961, normalized size = 5.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/360*(180*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^
10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b
^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^
8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276
*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4)*arctan((sqrt(a^2*b^8*sq
rt(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*
c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6
*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*
b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*
d^12)/(a^3*b^17)) + (b^6*c^6 - 30*a*b^5*c^5*d + 279*a^2*b^4*c^4*d^2 - 836*a
^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 702*a^5*b*c*d^5 + 169*a^6*d^6)*x)*a
^2*b^13*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^
3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a
^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 185364
4*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561
*a^12*d^12)/(a^3*b^17))^(3/4) + (a^2*b^16*c^3 - 15*a^3*b^15*c^2*d + 27*a^4*
b^14*c*d^2 - 13*a^5*b^13*d^3)*sqrt(x)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 145
8*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 5350
32*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 25
20207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10
- 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(3/4))/(b^12*c^12 - 6
0*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a
^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824
*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 8710
26*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)) + 45*(b^5*x
^2 + a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1841
```

```
2*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 13657
56*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 18
53644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 2
8561*a^12*d^12)/(a^3*b^17))^(1/4)*log(a*b^4*(-(b^12*c^12 - 60*a*b^11*c^11*d
 + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4
- 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^
7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2
*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4) - (b^3*c^
3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*sqrt(x)) - 45*(b^5*x^2 +
a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3
*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^
6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644
*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*
a^12*d^12)/(a^3*b^17))^(1/4)*log(-a*b^4*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1
458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 53
5032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 +
2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^1
0 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4) - (b^3*c^3 -
15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*sqrt(x)) - 4*(20*b^3*d^3*x^6
- 45*b^3*c^3 + 675*a*b^2*c^2*d - 1215*a^2*b*c*d^2 + 585*a^3*d^3 + 4*(27*b^3
*c*d^2 - 13*a*b^2*d^3)*x^4 + 36*(15*b^3*c^2*d - 27*a*b^2*c*d^2 + 13*a^2*b*d
^3)*x^2)*sqrt(x))/(b^5*x^2 + a*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(369) = 738$.
time = 158.91, size = 1833, normalized size = 4.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)
/5 + 2*d**3*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((2*c**3*x**(5/2)/5 + 2*c**2
*d*x**(9/2)/3 + 6*c*d**2*x**(13/2)/13 + 2*d**3*x**(17/2)/17)/a**2, Eq(b, 0)
), ((-2*c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3
*x**(9/2)/9)/b**2, Eq(a, 0)), (2340*a**4*d**3*sqrt(x)/(360*a**2*b**4 + 360*
a*b**5*x**2) + 585*a**4*d**3*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(36
0*a**2*b**4 + 360*a*b**5*x**2) - 585*a**4*d**3*(-a/b)**(1/4)*log(sqrt(x) +
(-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) - 1170*a**4*d**3*(-a/b)**(
1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) - 4860*a
**3*b*c*d**2*sqrt(x)/(360*a**2*b**4 + 360*a*b**5*x**2) - 1215*a**3*b*c*d**2
*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**
2) + 1215*a**3*b*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(360*a**
2*b**4 + 360*a*b**5*x**2) + 2430*a**3*b*c*d**2*(-a/b)**(1/4)*atan(sqrt(x)/(
```

```
-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) + 1872*a**3*b*d**3*x**(5/2)
/(360*a**2*b**4 + 360*a*b**5*x**2) + 585*a**3*b*d**3*x**2*(-a/b)**(1/4)*log
(sqrt(x) - (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) - 585*a**3*b*d*
*3*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b
**5*x**2) - 1170*a**3*b*d**3*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))
/(360*a**2*b**4 + 360*a*b**5*x**2) + 2700*a**2*b**2*c**2*d*sqrt(x)/(360*a**
2*b**4 + 360*a*b**5*x**2) + 675*a**2*b**2*c**2*d*(-a/b)**(1/4)*log(sqrt(x)
- (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) - 675*a**2*b**2*c**2*d*(
-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2)
 - 1350*a**2*b**2*c**2*d*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(360*a**
2*b**4 + 360*a*b**5*x**2) - 3888*a**2*b**2*c*d**2*x**(5/2)/(360*a**2*b**4 +
 360*a*b**5*x**2) - 1215*a**2*b**2*c*d**2*x**2*(-a/b)**(1/4)*log(sqrt(x) -
(-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) + 1215*a**2*b**2*c*d**2*x*
*2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x
**2) + 2430*a**2*b**2*c*d**2*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))
/(360*a**2*b**4 + 360*a*b**5*x**2) - 208*a**2*b**2*d**3*x**(9/2)/(360*a**2*
b**4 + 360*a*b**5*x**2) - 180*a*b**3*c**3*sqrt(x)/(360*a**2*b**4 + 360*a*b*
*5*x**2) - 45*a*b**3*c**3*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(360*a
**2*b**4 + 360*a*b**5*x**2) + 45*a*b**3*c**3*(-a/b)**(1/4)*log(sqrt(x) + (-
a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) + 90*a*b**3*c**3*(-a/b)**(1/
4)*atan(sqrt(x)/(-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) + 2160*a*b
**3*c**2*d*x**(5/2)/(360*a**2*b**4 + 360*a*b**5*x**2) + 675*a*b**3*c**2*d*x
**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*
x**2) - 675*a*b**3*c**2*d*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(
360*a**2*b**4 + 360*a*b**5*x**2) - 1350*a*b**3*c**2*d*x**2*(-a/b)**(1/4)*at
an(sqrt(x)/(-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) + 432*a*b**3*c*
d**2*x**(9/2)/(360*a**2*b**4 + 360*a*b**5*x**2) + 80*a*b**3*d**3*x**(13/2)/
(360*a**2*b**4 + 360*a*b**5*x**2) - 45*b**4*c**3*x**2*(-a/b)**(1/4)*log(sqr
t(x) - (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) + 45*b**4*c**3*x**2
*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**
2) + 90*b**4*c**3*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(360*a**2*
b**4 + 360*a*b**5*x**2), True))
```

Giac [A]
time = 1.55, size = 552, normalized size = 1.43

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}} b^{3} c^{3}-15 \left(ab^{3}\right)^{\frac{1}{4}} a b^{2} c^{2} d+27 \left(ab^{3}\right)^{\frac{1}{4}} a^{2} b c d^{2}-13 \left(ab^{3}\right)^{\frac{1}{4}} a^{3} d^{3}\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^{5}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}} b^{3} c^{3}-15 \left(ab^{3}\right)^{\frac{1}{4}} a b^{2} c^{2} d+27 \left(ab^{3}\right)^{\frac{1}{4}} a^{2} b c d^{2}-13 \left(ab^{3}\right)^{\frac{1}{4}} a^{3} d^{3}\right) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^{5}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}} b^{3} c^{3}-15 \left(ab^{3}\right)^{\frac{1}{4}} a b^{2} c^{2} d+27 \left(ab^{3}\right)^{\frac{1}{4}} a^{2} b c d^{2}-13 \left(ab^{3}\right)^{\frac{1}{4}} a^{3} d^{3}\right) \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^{5}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}} b^{3} c^{3}-15 \left(ab^{3}\right)^{\frac{1}{4}} a b^{2} c^{2} d+27 \left(ab^{3}\right)^{\frac{1}{4}} a^{2} b c d^{2}-13 \left(ab^{3}\right)^{\frac{1}{4}} a^{3} d^{3}\right) \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^{5}}-\frac{b^{3} c^{3}\sqrt{x}-3 a b^{2} c^{2} d\sqrt{x}+3 a^{2} b c d^{2}\sqrt{x}-a^{3} d^{3}\sqrt{x}}{2\left(bx^{2}+a\right)b^{4}}+\frac{2\left(5 b^{16} d^{3} x^{\frac{9}{2}}+27 b^{16} c d^{2} x^{\frac{5}{2}}-18 a b^{15} d^{3} x^{\frac{5}{2}}+135 b^{16} c^{2} d\sqrt{x}-270 a b^{15} c d^{2}\sqrt{x}+135 a^{2} b^{14} d^{3}\sqrt{x}\right)}{45\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b
^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(
2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) + 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 -
```

```
13*(a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a*b^5) + 1/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3
)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d
^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5) - 1/16*sqrt(2)
*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a
^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a*b^5) - 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^
2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*b^4) + 2/45*(5*b^16*d^3*x
^(9/2) + 27*b^16*c*d^2*x^(5/2) - 18*a*b^15*d^3*x^(5/2) + 135*b^16*c^2*d*sqr
t(x) - 270*a*b^15*c*d^2*sqrt(x) + 135*a^2*b^14*d^3*sqrt(x))/b^18
```

Mupad [B]
time = 0.24, size = 1691, normalized size = 4.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x^(1/2)*((6*c^2*d)/b^2 + (2*a*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/b - (2*a^2*d
^3)/b^4) - x^(5/2)*((4*a*d^3)/(5*b^3) - (6*c*d^2)/(5*b^2)) + (2*d^3*x^(9/2)
)/(9*b^2) + (x^(1/2)*((a^3*d^3)/2 - (b^3*c^3)/2 + (3*a*b^2*c^2*d)/2 - (3*a^
2*b*c*d^2)/2))/(a*b^4 + b^5*x^2) + (atan(((((x^(1/2)*(169*a^6*d^6 + b^6*c^6
 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*
b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 + ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d
^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2))/((-a)^(3/4)*b^(21/4)))
*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(8*(-a)^(3/4)*b^(17/4)) + (((x^(1/2)*(169
*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b
^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 - ((a*d - b*c)^2*(13*a*
d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2))/((-a
)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(8*(-a)^(3/4)*b^(17/4))
)/((((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^
3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 + ((a
*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*
a^3*b*c*d^2))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c))/(8*(-a)^
(3/4)*b^(17/4)) - (((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 -
 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*
d^5))/b^5 - ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*
b^2*c^2*d - 27*a^3*b*c*d^2))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d -
 b*c))/(8*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(4*(-a)^(
3/4)*b^(17/4)) - (atan(((((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4
*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^
5*b*c*d^5))/b^5 - ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 1
5*a^2*b^2*c^2*d - 27*a^3*b*c*d^2)*1i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*
(13*a*d - b*c))/(8*(-a)^(3/4)*b^(17/4)) + (((x^(1/2)*(169*a^6*d^6 + b^6*c^6
```

+ 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 + ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2)*1i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c))/(8*(-a)^(3/4)*b^(17/4)))/((((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 - ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2)*1i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(8*(-a)^(3/4)*b^(17/4)) - (((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 + ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2)*1i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(8*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^2*(13*a*d - b*c))/(4*(-a)^(3/4)*b^(17/4))

3.455 $\int \frac{\sqrt{x}\,(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=376

$$-\frac{d(6b^2c^2-21abcd+11a^2d^2)\,x^{3/2}}{6ab^3}-\frac{d^2(7bc-11ad)x^{7/2}}{14ab^2}+\frac{(bc-ad)x^{3/2}(c+dx^2)^2}{2ab\,(a+bx^2)}-\frac{(bc-ad)^2(bc+11ad)\,\mathrm{ta}}{4\sqrt{2}\,a}$$

```
[Out] -1/6*d*(11*a^2*d^2-21*a*b*c*d+6*b^2*c^2)*x^(3/2)/a/b^3-1/14*d^2*(-11*a*d+7*
b*c)*x^(7/2)/a/b^2+1/2*(-a*d+b*c)*x^(3/2)*(d*x^2+c)^2/a/b/(b*x^2+a)-1/8*(-a
*d+b*c)^2*(11*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^
(15/4)*2^(1/2)+1/8*(-a*d+b*c)^2*(11*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/
2)/a^(1/4))/a^(5/4)/b^(15/4)*2^(1/2)+1/16*(-a*d+b*c)^2*(11*a*d+b*c)*ln(a^(1
/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(15/4)*2^(1/2)-1/1
6*(-a*d+b*c)^2*(11*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^
(1/2))/a^(5/4)/b^(15/4)*2^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 479, 584, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(11ad+bc)}{4\sqrt{2}\,a^{5/4}b^{15/4}}+\frac{\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(11ad+bc)}{4\sqrt{2}\,a^{5/4}b^{15/4}}+\frac{(bc-ad)^2(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{15/4}}-\frac{(bc-ad)^2(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{15/4}}-\frac{dx^{3/2}(11a^2d^2-21abcd+6b^2c^2)}{6ab^3}-\frac{d^2x^{7/2}(7bc-11ad)}{14ab^2}+\frac{x^{3/2}(c+dx^2)^2(bc-ad)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] -1/6*(d*(6*b^2*c^2 - 21*a*b*c*d + 11*a^2*d^2)*x^(3/2))/(a*b^3) - (d^2*(7*b*
c - 11*a*d)*x^(7/2))/(14*a*b^2) + ((b*c - a*d)*x^(3/2)*(c + d*x^2)^2)/(2*a*
b*(a + b*x^2)) - ((b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(15/4)) + ((b*c - a*d)^2*(b*c + 11*
a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(1
5/4)) + ((b*c - a*d)^2*(b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(15/4)) - ((b*c - a*d)^2*(b*c +
 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqr
t[2]*a^(5/4)*b^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\,(c+dx^2)^3}{(a+bx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^2(c+dx^4)^3}{(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)x^{3/2}(c+dx^2)^2}{2ab\,(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{x^2(c+dx^4)(-c(bc+3ad)+d(7bc-11ad)x^4)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)x^{3/2}(c+dx^2)^2}{2ab\,(a+bx^2)} - \frac{\text{Subst}\left(\int \left(\frac{d(6b^2c^2-21abcd+11a^2d^2)x^2}{b^2} + \frac{d^2(7bc-11ad)x^6}{b} - \frac{(b^3c^3+9}{}\right.\right.}{2ab}$$

$$= -\frac{d(6b^2c^2-21abcd+11a^2d^2)\,x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}(c+dx^2)^2}{2ab\,(a+bx^2)}$$

$$= -\frac{d(6b^2c^2-21abcd+11a^2d^2)\,x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}(c+dx^2)^2}{2ab\,(a+bx^2)}$$

$$= -\frac{d(6b^2c^2-21abcd+11a^2d^2)\,x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}(c+dx^2)^2}{2ab\,(a+bx^2)}$$

$$= -\frac{d(6b^2c^2-21abcd+11a^2d^2)\,x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}(c+dx^2)^2}{2ab\,(a+bx^2)}$$

$$= -\frac{d(6b^2c^2-21abcd+11a^2d^2)\,x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}(c+dx^2)^2}{2ab\,(a+bx^2)}$$

Mathematica [A]

time = 0.47, size = 227, normalized size = 0.60

$$\frac{\frac{4\sqrt[4]{a}\,b^{3/4}x^{3/2}(21b^3c^3-77a^3d^3+a^2bd^2(147c-44dx^2)+3ab^2d(-21c^2+28cdx^2+4d^2x^4))}{a+bx^2}-21\sqrt{2}\,(bc-ad)^2(bc+11ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-21\sqrt{2}\,(bc-ad)^2(bc+11ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{168a^{5/4}b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((4*a^(1/4)*b^(3/4)*x^(3/2)*(21*b^3*c^3 - 77*a^3*d^3 + a^2*b*d^2*(147*c - 4
4*d*x^2) + 3*a*b^2*d*(-21*c^2 + 28*c*d*x^2 + 4*d^2*x^4)))/(a + b*x^2) - 21*
Sqrt[2]*(b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])] - 21*Sqrt[2]*(b*c - a*d)^2*(b*c + 11*a*d)*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(168*a^(5/4)*b^(
15/4))
```

Maple [A]

time = 0.11, size = 235, normalized size = 0.62

method	result
derivativedivides	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-3bc)x^{\frac{3}{2}}}{3}\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^{\frac{3}{2}}}{2a\left(b\,x^2+a\right)}+\frac{\left(11a^3d^3-21a^2bc\,d^2+9a\,b^2c^2d+b^3c^3\right)\sqrt{2}\left(\ln\left(\right.\right.}{}}{}$
default	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-3bc)x^{\frac{3}{2}}}{3}\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^{\frac{3}{2}}}{2a\left(b\,x^2+a\right)}+\frac{\left(11a^3d^3-21a^2bc\,d^2+9a\,b^2c^2d+b^3c^3\right)\sqrt{2}\left(\ln\left(\right.\right.}{}}{}$
risch	$-\frac{2d^2x^{\frac{3}{2}}\left(-3bd\,x^2+14ad-21bc\right)}{21b^3}-\frac{x^{\frac{3}{2}}d^3a^2}{2b^3(b\,x^2+a)}+\frac{3x^{\frac{3}{2}}d^2ac}{2b^2(b\,x^2+a)}-\frac{3x^{\frac{3}{2}}d\,c^2}{2b(b\,x^2+a)}+\frac{x^{\frac{3}{2}}c^3}{2a(b\,x^2+a)}+\frac{11\sqrt{2}\,\arctan\left(\right.}{8b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*d^2/b^3*(-1/7*b*d*x^(7/2)+1/3*(2*a*d-3*b*c)*x^(3/2))+2/b^3*(-1/4*(a^3*d^
3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a*x^(3/2)/(b*x^2+a)+1/32*(11*a^3*d^3
-21*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)/a/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)
^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)-1)))
```

Maxima [A]

time = 0.52, size = 309, normalized size = 0.82

$$\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^{\frac{3}{2}}}{2\,(ab^4x^2+a^2b^3)}+\frac{2\left(3\,bd^3x^{\frac{7}{2}}+7\,(3\,bcd^2-2\,ad^3)x^{\frac{3}{2}}\right)}{21\,b^3}+\frac{(b^3c^3+9\,ab^2c^2d-21\,a^2bcd^2+11\,a^3d^3)\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^(3/2)/(a*b^4*x^2
+ a^2*b^3) + 2/21*(3*b*d^3*x^(7/2) + 7*(3*b*c*d^2 - 2*a*d^3)*x^(3/2))/b^3 +
 1/16*(b^3*c^3 + 9*a*b^2*c^2*d - 21*a^2*b*c*d^2 + 11*a^3*d^3)*(2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2531 vs. $2(292) = 584$.
time = 0.58, size = 2531, normalized size = 6.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/168*(84*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*
b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*
c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*
c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b
*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/4)*arctan((sqrt((b^18*c^18 + 54*a
*b^17*c^17*d + 1089*a^2*b^16*c^16*d^2 + 8976*a^3*b^15*c^15*d^3 + 5940*a^4*b
^14*c^14*d^4 - 279576*a^5*b^13*c^13*d^5 - 338844*a^6*b^12*c^12*d^6 + 600177
6*a^7*b^11*c^11*d^7 - 6412626*a^8*b^10*c^10*d^8 - 62165180*a^9*b^9*c^9*d^9
+ 294333534*a^10*b^8*c^8*d^10 - 671362704*a^11*b^7*c^7*d^11 + 974580036*a^1
2*b^6*c^6*d^12 - 971334936*a^13*b^5*c^5*d^13 + 678512340*a^14*b^4*c^4*d^14
- 328575984*a^15*b^3*c^3*d^15 + 105546969*a^16*b^2*c^2*d^16 - 20292426*a^17
*b*c*d^17 + 1771561*a^18*d^18)*x - (a^3*b^19*c^12 + 36*a^4*b^18*c^11*d + 40
2*a^5*b^17*c^10*d^2 + 692*a^6*b^16*c^9*d^3 - 10017*a^7*b^15*c^8*d^4 - 5688*
a^8*b^14*c^7*d^5 + 160188*a^9*b^13*c^6*d^6 - 486648*a^10*b^12*c^5*d^7 + 746
703*a^11*b^11*c^4*d^8 - 676588*a^12*b^10*c^3*d^9 + 368082*a^13*b^9*c^2*d^10
 - 111804*a^14*b^8*c*d^11 + 14641*a^15*b^7*d^12)*sqrt(-(b^12*c^12 + 36*a*b^
11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8
*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d
^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*
d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15)))*a*b^4*(-(b^12*c
^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 1001
7*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*
a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*
```

```
a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/
4) - (a*b^13*c^9 + 27*a^2*b^12*c^8*d + 180*a^3*b^11*c^7*d^2 - 372*a^4*b^10*
c^6*d^3 - 3186*a^5*b^9*c^5*d^4 + 13194*a^6*b^8*c^4*d^5 - 21372*a^7*b^7*c^3*
d^6 + 17820*a^8*b^6*c^2*d^7 - 7623*a^9*b^5*c*d^8 + 1331*a^10*b^4*d^9)*sqrt(
x)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^
9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d
^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d
^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^
5*b^15))^(1/4))/(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692
*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^
6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^
9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^1
2*d^12)) - 21*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a
^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b
^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b
^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^1
1*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/4)*log(a^4*b^11*(-(b^12*c^12 +
 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4
*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b
^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*
b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(3/4) +
(b^9*c^9 + 27*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 318
6*a^4*b^5*c^5*d^4 + 13194*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a
^7*b^2*c^2*d^7 - 7623*a^8*b*c*d^8 + 1331*a^9*d^9)*sqrt(x)) + 21*(a*b^4*x^2
+ a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^
3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b
^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b
^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d
^12)/(a^5*b^15))^(1/4)*log(-a^4*b^11*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*
a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*
b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*
b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^
11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(3/4) + (b^9*c^9 + 27*a*b^8*c^8*
d + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 1319
4*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 - 7623*a^
8*b*c*d^8 + 1331*a^9*d^9)*sqrt(x)) - 4*(12*a*b^2*d^3*x^5 + 4*(21*a*b^2*c*d^
2 - 11*a^2*b*d^3)*x^3 + 7*(3*b^3*c^3 - 9*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 11*
a^3*d^3)*x)*sqrt(x))/(a*b^4*x^2 + a^2*b^3)
```

Sympy [A]
time = 40.66, size = 173, normalized size = 0.46

$$- \frac{4 a d^{3} x^{\frac{3}{2}}}{3 b^{3}} - \frac{2 x^{\frac{3}{2}} \left(a d - b c\right)^{3}}{4 a^{2} b^{3} + 4 a b^{4} x^{2}} + \frac{2 c d^{2} x^{\frac{3}{2}}}{b^{2}} + \frac{2 d^{3} x^{\frac{7}{2}}}{7 b^{2}} + \frac{6 d \left(a d - b c\right)^{2} \operatorname{RootSum} {\left(256 t^{4} a b^{3} + 1, \left(t \mapsto t \log {\left(64 t^{3} a b^{2} + \sqrt{x} \right)} \right)\right)}}{b^{3}} - \frac{2 \left(a d - b c\right)^{3} \operatorname{RootSum} {\left(65536 t^{4} a^{5} b^{3} + 1, \left(t \mapsto t \log {\left(4096 t^{3} a^{4} b^{2} + \sqrt{x} \right)} \right)\right)}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] -4*a*d**3*x**(3/2)/(3*b**3) - 2*x**(3/2)*(a*d - b*c)**3/(4*a**2*b**3 + 4*a*
b**4*x**2) + 2*c*d**2*x**(3/2)/b**2 + 2*d**3*x**(7/2)/(7*b**2) + 6*d*(a*d -
 b*c)**2*RootSum(256*_t**4*a*b**3 + 1, Lambda(_t, _t*log(64*_t**3*a*b**2 +
sqrt(x))))/b**3 - 2*(a*d - b*c)**3*RootSum(65536*_t**4*a**5*b**3 + 1, Lambd
a(_t, _t*log(4096*_t**3*a**4*b**2 + sqrt(x))))/b**3
```

Giac [A]
time = 1.10, size = 516, normalized size = 1.37

$$\frac{b^{3}c^{3}x^{\frac{3}{2}} - 3\,ab^{2}c^{2}dx^{\frac{3}{2}} + 3\,a^{2}bcd^{2}x^{\frac{3}{2}} - a^{3}d^{3}x^{\frac{3}{2}}}{2\,(bx^{2}+a)ab^{3}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} + 9\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d - 21\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} + 11\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{2}b^{6}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} + 9\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d - 21\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} + 11\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{2}b^{6}} - \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} + 9\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d - 21\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} + 11\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^{2}b^{6}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} + 9\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d - 21\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} + 11\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^{2}b^{6}} + \frac{2\left(3\,b^{12}d^{3}x^{\frac{7}{2}} + 21\,b^{12}cd^{2}x^{\frac{3}{2}} - 14\,ab^{11}d^{3}x^{\frac{3}{2}}\right)}{21\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3*
d^3*x^(3/2))/((b*x^2 + a)*a*b^3) + 1/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(
a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*
a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/
(a^2*b^6) + 1/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*
d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^6) - 1/16*sqrt(
2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*
a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^6) + 1/16*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(
3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)
*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^6) + 2/21*(3*b^12
*d^3*x^(7/2) + 21*b^12*c*d^2*x^(3/2) - 14*a*b^11*d^3*x^(3/2))/b^14
```

Mupad [B]
time = 0.12, size = 616, normalized size = 1.64

$$\frac{2\,d^3\,x^{7/2}}{7\,b^2}-x^{3/2}\,\left(\frac{4\,a\,d^3}{3\,b^3}-\frac{2\,c\,d^2}{b^2}\right)-\frac{x^{3/2}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a\,\left(b^4\,x^2+a\,b^3\right)}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(11\,a\,d+b\,c\right)\,\left(121\,a^6\,d^6-462\,a^5\,b\,c\,d^5+639\,a^4\,b^2\,c^2\,d^4-356\,a^3\,b^3\,c^3\,d^3+39\,a^2\,b^4\,c^4\,d^2+18\,a\,b^5\,c^5\,d+b^6\,c^6\right)}{{\left(-a\right)}^{1/4}\,\left(1331\,a^9\,d^9-7623\,a^8\,b\,c\,d^8+17820\,a^7\,b^2\,c^2\,d^7-21372\,a^6\,b^3\,c^3\,d^6+13194\,a^5\,b^4\,c^4\,d^5-3186\,a^4\,b^5\,c^5\,d^4-372\,a^3\,b^6\,c^6\,d^3+180\,a^2\,b^7\,c^7\,d^2+27\,a\,b^8\,c^8\,d+b^9\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(11\,a\,d+b\,c\right)}{4\,{\left(-a\right)}^{5/4}\,b^{15/4}}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(11\,a\,d+b\,c\right)\,\left(121\,a^6\,d^6-462\,a^5\,b\,c\,d^5+639\,a^4\,b^2\,c^2\,d^4-356\,a^3\,b^3\,c^3\,d^3+39\,a^2\,b^4\,c^4\,d^2+18\,a\,b^5\,c^5\,d+b^6\,c^6\right)\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\left(1331\,a^9\,d^9-7623\,a^8\,b\,c\,d^8+17820\,a^7\,b^2\,c^2\,d^7-21372\,a^6\,b^3\,c^3\,d^6+13194\,a^5\,b^4\,c^4\,d^5-3186\,a^4\,b^5\,c^5\,d^4-372\,a^3\,b^6\,c^6\,d^3+180\,a^2\,b^7\,c^7\,d^2+27\,a\,b^8\,c^8\,d+b^9\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(11\,a\,d+b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{5/4}\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] (2*d^3*x^(7/2))/(7*b^2) - x^(3/2)*((4*a*d^3)/(3*b^3) - (2*c*d^2)/b^2) - (x^
(3/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^
4*x^2)) - (atan((b^(1/4)*x^(1/2)*(a*d - b*c)^2*(11*a*d + b*c)*(121*a^6*d^6
+ b^6*c^6 + 39*a^2*b^4*c^4*d^2 - 356*a^3*b^3*c^3*d^3 + 639*a^4*b^2*c^2*d^4
+ 18*a*b^5*c^5*d - 462*a^5*b*c*d^5))/((-a)^(1/4)*(1331*a^9*d^9 + b^9*c^9 +
180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 13194*a^
5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 + 27*a*b^8*c^
8*d - 7623*a^8*b*c*d^8)))*(a*d - b*c)^2*(11*a*d + b*c))/(4*(-a)^(5/4)*b^(15
```

```
/4)) - (atan((b^(1/4)*x^(1/2)*(a*d - b*c)^2*(11*a*d + b*c)*(121*a^6*d^6 + b
^6*c^6 + 39*a^2*b^4*c^4*d^2 - 356*a^3*b^3*c^3*d^3 + 639*a^4*b^2*c^2*d^4 + 1
8*a*b^5*c^5*d - 462*a^5*b*c*d^5)*1i)/((-a)^(1/4)*(1331*a^9*d^9 + b^9*c^9 +
180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 13194*a^
5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 + 27*a*b^8*c^
8*d - 7623*a^8*b*c*d^8)))*(a*d - b*c)^2*(11*a*d + b*c)*1i)/(4*(-a)^(5/4)*b^
(15/4))
```

3.456 $\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)^2}\,dx$

Optimal. Leaf size=340

$$\frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\,(a+bx^2)}-\frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}b^{13/4}}+\frac{3(bc-a}{}$$

```
[Out] 2/5*d^3*x^(5/2)/b^2-3/8*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x
^(1/2)/a^(1/4))/a^(7/4)/b^(13/4)*2^(1/2)+3/8*(-a*d+b*c)^2*(3*a*d+b*c)*arcta
n(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(13/4)*2^(1/2)-3/16*(-a*d+b*
c)^2*(3*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7
/4)/b^(13/4)*2^(1/2)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)+2*d^2*(-2*a*d+3*b*c)
*x^(1/2)/b^3+1/2*(-a*d+b*c)^3*x^(1/2)/a/b^3/(b*x^2+a)
```

Rubi [A]
time = 0.26, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 398, 393, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(3ad+bc)}{4\sqrt{2}\,a^{7/4}b^{13/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(3ad+bc)}{4\sqrt{2}\,a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{13/4}}+\frac{2d^2\sqrt{x}\,(3bc-2ad)}{b^3}+\frac{\sqrt{x}\,(bc-ad)^3}{2ab^3\,(a+bx^2)}+\frac{2d^3x^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] (2*d^2*(3*b*c - 2*a*d)*Sqrt[x])/b^3 + (2*d^3*x^(5/2))/(5*b^2) + ((b*c - a*d
)^3*Sqrt[x])/(2*a*b^3*(a + b*x^2)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(
b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
4*Sqrt[2]*a^(7/4)*b^(13/4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
+ (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)^2}\,dx &= 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^4}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{b^3\,(a+bx^4)^2}\right)\,dx\right.\\
&= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{2\text{Subst}\left(\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{(a+bx^4)^2}\,dx, x, \sqrt{x}\right)}{b^3}\\
&= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\,(a+bx^2)}+\frac{(3(bc-ad)^2(bc+3ad))\,\text{Subst}\left(\right.}{2ab^3}\\
&= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\,(a+bx^2)}+\frac{(3(bc-ad)^2(bc+3ad))\,\text{Subst}\left(\right.}{4a^{3/2}b}\\
&= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\,(a+bx^2)}+\frac{(3(bc-ad)^2(bc+3ad))\,\text{Subst}\left(\right.}{8a^3}\\
&= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\,(a+bx^2)}-\frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}\right.}{8\sqrt{2}\,a^{7/}}\\
&= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\,(a+bx^2)}-\frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1\right.}{4\sqrt{2}\,a^{7/4}b^{13/4}}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 227, normalized size = 0.67

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\,\sqrt{x}\,(5b^3c^3-45a^3d^3+3a^2bd^2(25c-12dx^2)+ab^2d(-15c^2+60cdx^2+4d^2x^4))}{a+bx^2}-15\sqrt{2}\,(bc-ad)^2(bc+3ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+15\sqrt{2}\,(bc-ad)^2(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{40a^{7/4}b^{13/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]

[Out] ((4*a^(3/4)*b^(1/4)*Sqrt[x]*(5*b^3*c^3 - 45*a^3*d^3 + 3*a^2*b*d^2*(25*c - 12*d*x^2) + a*b^2*d*(-15*c^2 + 60*c*d*x^2 + 4*d^2*x^4)))/(a + b*x^2) - 15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(40*a^(7/4)*b^(13/4))

Maple [A]
time = 0.11, size = 231, normalized size = 0.68

method	result
derivativedivides	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{5}{2}}}{5}+2ad\sqrt{x}-3bc\sqrt{x}\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\sqrt{x}}{2a\left(b\,x^2+a\right)}+\frac{3\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right.}{}}{}$
default	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{5}{2}}}{5}+2ad\sqrt{x}-3bc\sqrt{x}\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\sqrt{x}}{2a\left(b\,x^2+a\right)}+\frac{3\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right.}{}}{}$
risch	$-\frac{2d^2\left(-bd\,x^2+10ad-15bc\right)\sqrt{x}}{5b^3}-\frac{\sqrt{x}\,d^3a^2}{2b^3\left(b\,x^2+a\right)}+\frac{3\sqrt{x}\,d^2ac}{2b^2\left(b\,x^2+a\right)}-\frac{3\sqrt{x}\,dc^2}{2b\left(b\,x^2+a\right)}+\frac{\sqrt{x}\,c^3}{2a\left(b\,x^2+a\right)}+\frac{9a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\mathrm{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*d^2/b^3*(-1/5*b*d*x^(5/2)+2*a*d*x^(1/2)-3*b*c*x^(1/2))+2/b^3*(-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a*x^(1/2)/(b*x^2+a)+3/32*(3*a^3*d^3-5*a^2*b*c*d^2+a*b^2*c^2*d+b^3*c^3)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))

Maxima [A]
time = 0.50, size = 412, normalized size = 1.21

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{x}}{2\left(ab^4x^2+a^2b^3\right)}+\frac{2\left(bd^3x^{\frac{5}{2}}+5\left(3\,bcd^2-2\,ad^3\right)\sqrt{x}\right)}{5\,b^3}+\frac{3\left(\frac{2\sqrt{2}\left(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\left(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{16\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")

[Out] 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)/(a*b^4*x^2 + a^2*b^3) + 2/5*(b*d^3*x^(5/2) + 5*(3*b*c*d^2 - 2*a*d^3)*sqrt(x))/b^3 + 3/

```
16*(2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(1/
2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*
a^2*b*c*d^2 + 3*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*s
qrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sq
rt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*
c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1944 vs. $2(258) = 516$.
time = 0.55, size = 1944, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(60*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10
*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5
- 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a
^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)
/(a^7*b^13))^(1/4)*arctan((sqrt(a^4*b^6*sqrt(-(b^12*c^12 + 4*a*b^11*c^11*d
- 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5
*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4
*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 +
81*a^12*d^12)/(a^7*b^13)) + (b^6*c^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 -
4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6)*x)*a^5
*b^10*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^
9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 3
28*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^1
0*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(3/4) - (a^5
*b^13*c^3 + a^6*b^12*c^2*d - 5*a^7*b^11*c*d^2 + 3*a^8*b^10*d^3)*sqrt(x)*(-(
b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 1
27*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^
5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2
*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(3/4))/(b^12*c^12 + 4
*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^
8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1
039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a
^11*b*c*d^11 + 81*a^12*d^12)) + 15*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 4*a
*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*
d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 103
9*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^1
```

```
1*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(3*a^2*b^3*(-(b^12*c^12 + 4
*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^
8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1
039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a
^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d -
 5*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x)) - 15*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^1
2 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b
^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^
7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 -
540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(-3*a^2*b^3*(-(b^12*
c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^
4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5
*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10
 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2
*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x)) + 4*(4*a*b^2*d^3*x^4 + 5*b^3*c
^3 - 15*a*b^2*c^2*d + 75*a^2*b*c*d^2 - 45*a^3*d^3 + 12*(5*a*b^2*c*d^2 - 3*a
^2*b*d^3)*x^2)*sqrt(x))/(a*b^4*x^2 + a^2*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1775 vs. $2(323) = 646$.
time = 56.92, size = 1775, normalized size = 5.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x)
 + 2*d**3*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(7*x**(7/2)) - 2*c**
2*d/x**(3/2) + 6*c*d**2*sqrt(x) + 2*d**3*x**(5/2)/5)/b**2, Eq(a, 0)), ((2*c
**3*sqrt(x) + 6*c**2*d*x**(5/2)/5 + 2*c*d**2*x**(9/2)/3 + 2*d**3*x**(13/2)/
13)/a**2, Eq(b, 0)), (-180*a**4*d**3*sqrt(x)/(40*a**3*b**3 + 40*a**2*b**4*x
**2) - 45*a**4*d**3*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a**3*b**
3 + 40*a**2*b**4*x**2) + 45*a**4*d**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(
1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 90*a**4*d**3*(-a/b)**(1/4)*atan(
sqrt(x)/(-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 300*a**3*b*c*d*
*2*sqrt(x)/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 75*a**3*b*c*d**2*(-a/b)**(1
/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) - 75*a*
*3*b*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a**3*b**3 + 40*a
**2*b**4*x**2) - 150*a**3*b*c*d**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4)
)/(40*a**3*b**3 + 40*a**2*b**4*x**2) - 144*a**3*b*d**3*x**(5/2)/(40*a**3*b*
*3 + 40*a**2*b**4*x**2) - 45*a**3*b*d**3*x**2*(-a/b)**(1/4)*log(sqrt(x) - (
-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 45*a**3*b*d**3*x**2*(-a/
b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) +
 90*a**3*b*d**3*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(40*a**3*b**
```

```
3 + 40*a**2*b**4*x**2) - 60*a**2*b**2*c**2*d*sqrt(x)/(40*a**3*b**3 + 40*a**
2*b**4*x**2) - 15*a**2*b**2*c**2*d*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4
))/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 15*a**2*b**2*c**2*d*(-a/b)**(1/4)*l
og(sqrt(x) + (-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 30*a**2*b*
*2*c**2*d*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2
*b**4*x**2) + 240*a**2*b**2*c*d**2*x**(5/2)/(40*a**3*b**3 + 40*a**2*b**4*x*
*2) + 75*a**2*b**2*c*d**2*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(
40*a**3*b**3 + 40*a**2*b**4*x**2) - 75*a**2*b**2*c*d**2*x**2*(-a/b)**(1/4)*
log(sqrt(x) + (-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) - 150*a**2*
b**2*c*d**2*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(40*a**3*b**3 +
40*a**2*b**4*x**2) + 16*a**2*b**2*d**3*x**(9/2)/(40*a**3*b**3 + 40*a**2*b**
4*x**2) + 20*a*b**3*c**3*sqrt(x)/(40*a**3*b**3 + 40*a**2*b**4*x**2) - 15*a*
b**3*c**3*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a**3*b**3 + 40*a**
2*b**4*x**2) + 15*a*b**3*c**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(4
0*a**3*b**3 + 40*a**2*b**4*x**2) + 30*a*b**3*c**3*(-a/b)**(1/4)*atan(sqrt(x
)/(-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) - 15*a*b**3*c**2*d*x**2
*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x*
*2) + 15*a*b**3*c**2*d*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*
a**3*b**3 + 40*a**2*b**4*x**2) + 30*a*b**3*c**2*d*x**2*(-a/b)**(1/4)*atan(s
qrt(x)/(-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) - 15*b**4*c**3*x**
2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x
**2) + 15*b**4*c**3*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a**
3*b**3 + 40*a**2*b**4*x**2) + 30*b**4*c**3*x**2*(-a/b)**(1/4)*atan(sqrt(x)/
(-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2), True))
```

Giac [A]
time = 1.70, size = 511, normalized size = 1.50

$$\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3+(ab^3)^{\frac{1}{4}}ab^2c^2d-5(ab^3)^{\frac{1}{4}}a^2bcd^2+3(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^4}+\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3+(ab^3)^{\frac{1}{4}}ab^2c^2d-5(ab^3)^{\frac{1}{4}}a^2bcd^2+3(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^4}+\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3+(ab^3)^{\frac{1}{4}}ab^2c^2d-5(ab^3)^{\frac{1}{4}}a^2bcd^2+3(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b^4}-\frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3+(ab^3)^{\frac{1}{4}}ab^2c^2d-5(ab^3)^{\frac{1}{4}}a^2bcd^2+3(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b^4}+\frac{b^3c^3\sqrt{x}-3ab^2c^2d\sqrt{x}+3a^2bcd^2\sqrt{x}-a^3d^3\sqrt{x}}{2(bx^2+a)ab^3}+\frac{2\left(b^8d^3x^{\frac{5}{2}}+15b^8cd^2\sqrt{x}-10ab^7d^3\sqrt{x}\right)}{5b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^
(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) + 3/8*sqrt(2)*((a*b^3)^(1/4)*
b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^
3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a
*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) - 3/16*sqrt(2)*((a*b^3)
^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 +
3*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
(a^2*b^4) + 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sq
rt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*a*b^3) + 2/5*(b^8*d^3*x^(5/2) + 15*b^
8*c*d^2*sqrt(x) - 10*a*b^7*d^3*sqrt(x))/b^10
```

Mupad [B]

time = 0.13, size = 1636, normalized size = 4.81

$$\frac{2\,d^3\,x^{5/2}}{5\,b^2}-\sqrt{x}\,\left(\frac{4\,a\,d^3}{b^3}-\frac{6\,c\,d^2}{b^2}\right)-\frac{\sqrt{x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a\,\left(b^4\,x^2+a\,b^3\right)}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{9\,\sqrt{x}\,\left(9\,a^6\,d^6-30\,a^5\,b\,c\,d^5+31\,a^4\,b^2\,c^2\,d^4-4\,a^3\,b^3\,c^3\,d^3-9\,a^2\,b^4\,c^4\,d^2+2\,a\,b^5\,c^5\,d+b^6\,c^6\right)}{a^2\,b^3}-\frac{3\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,\left(72\,a^3\,d^3-120\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+24\,b^3\,c^3\right)}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,3{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}+\frac{\left(\frac{9\,\sqrt{x}\,\left(9\,a^6\,d^6-30\,a^5\,b\,c\,d^5+31\,a^4\,b^2\,c^2\,d^4-4\,a^3\,b^3\,c^3\,d^3-9\,a^2\,b^4\,c^4\,d^2+2\,a\,b^5\,c^5\,d+b^6\,c^6\right)}{a^2\,b^3}+\frac{3\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,\left(72\,a^3\,d^3-120\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+24\,b^3\,c^3\right)}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,3{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}}{\frac{3\,\left(\frac{9\,\sqrt{x}\,\left(9\,a^6\,d^6-30\,a^5\,b\,c\,d^5+31\,a^4\,b^2\,c^2\,d^4-4\,a^3\,b^3\,c^3\,d^3-9\,a^2\,b^4\,c^4\,d^2+2\,a\,b^5\,c^5\,d+b^6\,c^6\right)}{a^2\,b^3}-\frac{3\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,\left(72\,a^3\,d^3-120\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+24\,b^3\,c^3\right)}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}-\frac{3\,\left(\frac{9\,\sqrt{x}\,\left(9\,a^6\,d^6-30\,a^5\,b\,c\,d^5+31\,a^4\,b^2\,c^2\,d^4-4\,a^3\,b^3\,c^3\,d^3-9\,a^2\,b^4\,c^4\,d^2+2\,a\,b^5\,c^5\,d+b^6\,c^6\right)}{a^2\,b^3}+\frac{3\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,\left(72\,a^3\,d^3-120\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+24\,b^3\,c^3\right)}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,3{}\mathrm{i}}{4\,{\left(-a\right)}^{7/4}\,b^{13/4}}+\frac{3\,\mathrm{atan}\left(\frac{\frac{3\,\left(\frac{9\,\sqrt{x}\,\left(9\,a^6\,d^6-30\,a^5\,b\,c\,d^5+31\,a^4\,b^2\,c^2\,d^4-4\,a^3\,b^3\,c^3\,d^3-9\,a^2\,b^4\,c^4\,d^2+2\,a\,b^5\,c^5\,d+b^6\,c^6\right)}{a^2\,b^3}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,\left(72\,a^3\,d^3-120\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+24\,b^3\,c^3\right)\,3{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}+\frac{3\,\left(\frac{9\,\sqrt{x}\,\left(9\,a^6\,d^6-30\,a^5\,b\,c\,d^5+31\,a^4\,b^2\,c^2\,d^4-4\,a^3\,b^3\,c^3\,d^3-9\,a^2\,b^4\,c^4\,d^2+2\,a\,b^5\,c^5\,d+b^6\,c^6\right)}{a^2\,b^3}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,\left(72\,a^3\,d^3-120\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+24\,b^3\,c^3\right)\,3{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}}{\frac{\left(\frac{9\,\sqrt{x}\,\left(9\,a^6\,d^6-30\,a^5\,b\,c\,d^5+31\,a^4\,b^2\,c^2\,d^4-4\,a^3\,b^3\,c^3\,d^3-9\,a^2\,b^4\,c^4\,d^2+2\,a\,b^5\,c^5\,d+b^6\,c^6\right)}{a^2\,b^3}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,\left(72\,a^3\,d^3-120\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+24\,b^3\,c^3\right)\,3{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,3{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}-\frac{\left(\frac{9\,\sqrt{x}\,\left(9\,a^6\,d^6-30\,a^5\,b\,c\,d^5+31\,a^4\,b^2\,c^2\,d^4-4\,a^3\,b^3\,c^3\,d^3-9\,a^2\,b^4\,c^4\,d^2+2\,a\,b^5\,c^5\,d+b^6\,c^6\right)}{a^2\,b^3}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,\left(72\,a^3\,d^3-120\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+24\,b^3\,c^3\right)\,3{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)\,3{}\mathrm{i}}{8\,{\left(-a\right)}^{7/4}\,b^{13/4}}}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d+b\,c\right)}{4\,{\left(-a\right)}^{7/4}\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(1/2)*(a + b*x^2)^2),x)
```

```
[Out] (2*d^3*x^(5/2))/(5*b^2) - x^(1/2)*((4*a*d^3)/b^3 - (6*c*d^2)/b^2) - (x^(1/2
)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*x^
2)) + (atan(((((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*
b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^
3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2
*d - 120*a^2*b*c*d^2))/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c)
*3i)/(8*(-a)^(7/4)*b^(13/4)) + (((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^
4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5
*b*c*d^5))/(a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*
c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2))/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*
c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)))/((3*((9*x^(1/2)*(9*a^6*d^6
+ b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*
a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(
72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2))/(8*(-a)^(7/4)*
b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^(7/4)*b^(13/4)) - (3*((9*x^
(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4
*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3) + (3*(a*d - b*c)^
2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2
))/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^(7/4)*b^(1
3/4))))*(a*d - b*c)^2*(3*a*d + b*c)*3i)/(4*(-a)^(7/4)*b^(13/4)) + (3*atan((
(3*((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3
 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3) - ((a*d
- b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*
b*c*d^2)*3i)/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^
(7/4)*b^(13/4)) + (3*((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 -
 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/
(a^2*b^3) + ((a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^
2*c^2*d - 120*a^2*b*c*d^2)*3i)/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*
d + b*c))/(8*(-a)^(7/4)*b^(13/4)))/((((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a
^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 3
0*a^5*b*c*d^5))/(a^2*b^3) - ((a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b
^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2)*3i)/(8*(-a)^(7/4)*b^(13/4)))*(a*
d - b*c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)) - (((9*x^(1/2)*(9*a^6*
d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4
+ 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3) + ((a*d - b*c)^2*(3*a*d + b*c)
*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2)*3i)/(8*(-a)^(
7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4))))*(
a*d - b*c)^2*(3*a*d + b*c))/(4*(-a)^(7/4)*b^(13/4))
```

3.457 $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=368

$$-\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}}-\frac{d^2(3bc-7ad)x^{3/2}}{6ab^2}+\frac{(bc-ad)\left(c+dx^2\right)^2}{2ab\sqrt{x}\,\left(a+bx^2\right)}+\frac{(bc-ad)^2(5bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}b^{11/4}}$$

```
[Out] -1/6*d^2*(-7*a*d+3*b*c)*x^(3/2)/a/b^2+1/8*(-a*d+b*c)^2*(7*a*d+5*b*c)*arctan
(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(11/4)*2^(1/2)-1/8*(-a*d+b*c)
^2*(7*a*d+5*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(11/4)
*2^(1/2)-1/16*(-a*d+b*c)^2*(7*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/
4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(11/4)*2^(1/2)+1/16*(-a*d+b*c)^2*(7*a*d+5*b*c
)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(11/4)*2^
(1/2)-1/2*c^2*(-a*d+5*b*c)/a^2/b/x^(1/2)+1/2*(-a*d+b*c)*(d*x^2+c)^2/a/b/(b*
x^2+a)/x^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 479, 584, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(7ad+5bc)}{4\sqrt{2}\,a^{9/4}b^{11/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(7ad+5bc)}{4\sqrt{2}\,a^{9/4}b^{11/4}}-\frac{(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{11/4}}-\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}}-\frac{d^2x^{3/2}(3bc-7ad)}{6ab^2}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2ab\sqrt{x}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/2*(c^2*(5*b*c - a*d))/(a^2*b*Sqrt[x]) - (d^2*(3*b*c - 7*a*d)*x^(3/2))/(6
*a*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*Sqrt[x]*(a + b*x^2)) + ((b*c -
 a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*S
qrt[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2
*(5*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(8*Sqrt[2]*a^(9/4)*b^(11/4)) + ((b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(11/4
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}\,(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{(c+dx^4)(-c(5bc-ad)+d(3bc-7ad)x^4)}{x^2(a+bx^4)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}\,(a+bx^2)} - \frac{\text{Subst}\left(\int \left(\frac{c^2(-5bc+ad)}{ax^2} + \frac{d^2(3bc-7ad)x^2}{b} + \frac{(-bc+ad)^2(5bc+7ad)x^2}{ab(a+bx^4)}\right) d\right.}{2ab}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}\,(a+bx^2)} - \frac{((bc-ad)^2(5bc+7a}{}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}\,(a+bx^2)} + \frac{((bc-ad)^2(5bc+7a}{}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}\,(a+bx^2)} - \frac{((bc-ad)^2(5bc+7a}{}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}\,(a+bx^2)} - \frac{(bc-ad)^2(5bc+7ad}{}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}\,(a+bx^2)} + \frac{(bc-ad)^2(5bc+7ad}{4\sqrt{}}$$

Mathematica [A]

time = 0.48, size = 229, normalized size = 0.62

$$\frac{\frac{4\sqrt[4]{a}\,b^{3/4}(-15b^3c^3x^2+7a^3d^3x^2+3ab^2c^2(-4c+3dx^2)+a^2bd^2x^2(-9c+4dx^2))}{\sqrt{x}\,(a+bx^2)}+3\sqrt{2}\,(bc-ad)^2(5bc+7ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+3\sqrt{2}\,(bc-ad)^2(5bc+7ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{24a^{9/4}b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] ((4*a^(1/4)*b^(3/4)*(-15*b^3*c^3*x^2 + 7*a^3*d^3*x^2 + 3*a*b^2*c^2*(-4*c +
3*d*x^2) + a^2*b*d^2*x^2*(-9*c + 4*d*x^2)))/(Sqrt[x]*(a + b*x^2)) + 3*Sqrt[
2]*(b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTanh[(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(24*a^(9/4)*b^(11/4)
)
```

Maple [A]

time = 0.15, size = 225, normalized size = 0.61

method	result
derivativedivides	$\frac{2d^3x^{\frac{3}{2}}}{3b^2}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bcd^2-\frac{3}{4}ab^2c^2d+\frac{1}{4}b^3c^3\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(\frac{7}{4}a^3d^3-\frac{3}{4}ab^2c^2d+\frac{5}{4}b^3c^3-\frac{9}{4}a^2bcd^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\ldots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\ldots}\right.\right.\ldots}{8\ldots}\right.\ldots}{a^2b^2}$
default	$\frac{2d^3x^{\frac{3}{2}}}{3b^2}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bcd^2-\frac{3}{4}ab^2c^2d+\frac{1}{4}b^3c^3\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(\frac{7}{4}a^3d^3-\frac{3}{4}ab^2c^2d+\frac{5}{4}b^3c^3-\frac{9}{4}a^2bcd^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\ldots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\ldots}\right.\right.\ldots}{8\ldots}\right.\ldots}{a^2b^2}$
risch	$\frac{-2b^2c^3+\frac{2a^2d^3x^2}{3}}{a^2\sqrt{x}\,b^2}+\frac{ax^{\frac{3}{2}}d^3}{2b^2(bx^2+a)}-\frac{3x^{\frac{3}{2}}d^2c}{2b(bx^2+a)}+\frac{3x^{\frac{3}{2}}dc^2}{2a(bx^2+a)}-\frac{bx^{\frac{3}{2}}c^3}{2a^2(bx^2+a)}-\frac{7a\sqrt{2}\,d^3\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\ldots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\ldots}\right.\ldots}{16b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d^3*x^(3/2)/b^2-2/a^2/b^2*((-1/4*a^3*d^3+3/4*a^2*b*c*d^2-3/4*a*b^2*c^2*
d+1/4*b^3*c^3)*x^(3/2)/(b*x^2+a)+1/8*(7/4*a^3*d^3-3/4*a*b^2*c^2*d+5/4*b^3*c
^3-9/4*a^2*b*c*d^2)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2*c^3/a^
2/x^(1/2)
```

Maxima [A]
time = 0.51, size = 304, normalized size = 0.83

$$\frac{2\,d^3x^{\frac{3}{2}}}{3\,b^2}-\frac{4\,ab^2c^3+\left(5\,b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^2}{2\,\left(a^2b^3x^{\frac{5}{2}}+a^3b^2\sqrt{x}\right)}-\frac{\left(5\,b^3c^3-3\,ab^2c^2d-9\,a^2bcd^2+7\,a^3d^3\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/3*d^3*x^(3/2)/b^2 - 1/2*(4*a*b^2*c^3 + (5*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*x^2)/(a^2*b^3*x^(5/2) + a^3*b^2*sqrt(x)) - 1/16*(5*b^3*
c^3 - 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 7*a^3*d^3)*(2*sqrt(2)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1
/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(
b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2547 vs. 2(284) = 568.
time = 0.56, size = 2547, normalized size = 6.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d -
3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 287
28*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*
a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^
11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*arctan((sqrt((15625*b^18*c^
18 - 56250*a*b^17*c^17*d - 84375*a^2*b^16*c^16*d^2 + 570000*a^3*b^15*c^15*d
^3 - 211500*a^4*b^14*c^14*d^4 - 2174040*a^5*b^13*c^13*d^5 + 2720004*a^6*b^1
2*c^12*d^6 + 3321072*a^7*b^11*c^11*d^7 - 8368866*a^8*b^10*c^10*d^8 + 640420
*a^9*b^9*c^9*d^9 + 11255310*a^10*b^8*c^8*d^10 - 8509968*a^11*b^7*c^7*d^11 -
 4831644*a^12*b^6*c^6*d^12 + 9537192*a^13*b^5*c^5*d^13 - 3095820*a^14*b^4*c
^4*d^14 - 2551920*a^15*b^3*c^3*d^15 + 2614689*a^16*b^2*c^2*d^16 - 907578*a^
17*b*c*d^17 + 117649*a^18*d^18)*x - (625*a^5*b^17*c^12 - 1500*a^6*b^16*c^11
*d - 3150*a^7*b^15*c^10*d^2 + 11060*a^8*b^14*c^9*d^3 + 1071*a^9*b^13*c^8*d^
4 - 28728*a^10*b^12*c^7*d^5 + 19068*a^11*b^11*c^6*d^6 + 27144*a^12*b^10*c^5
*d^7 - 37665*a^13*b^9*c^4*d^8 + 2324*a^14*b^8*c^3*d^9 + 19698*a^15*b^7*c^2*
d^10 - 12348*a^16*b^6*c*d^11 + 2401*a^17*b^5*d^12)*sqrt(-(625*b^12*c^12 - 1
```

500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11)))*a^2*b^3*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4) - (125*a^2*b^12*c^9 - 225*a^3*b^11*c^8*d - 540*a^4*b^10*c^7*d^2 + 1308*a^5*b^9*c^6*d^3 + 342*a^6*b^8*c^5*d^4 - 2430*a^7*b^7*c^4*d^5 + 1140*a^8*b^6*c^3*d^6 + 1260*a^9*b^5*c^2*d^7 - 1323*a^10*b^4*c*d^8 + 343*a^11*b^3*d^9)*sqrt(x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4))/(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)) - 3*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(a^7*b^8*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(3/4) + (125*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 1308*a^3*b^6*c^6*d^3 + 342*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 + 1140*a^6*b^3*c^3*d^6 + 1260*a^7*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 343*a^9*d^9)*sqrt(x)) + 3*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(-a^7*b^8*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(3/4) + (125*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 1308*a^3*b^6*c^6*d^3 + 342*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 + 1140*a^6*b^3*c^3*d^6 + 1260*a^7*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 343*a^9*d^9)*sqrt(x)) + 4*(4*a^2*b*d^3*x^4 - 12*a*b^2*c^3 - (15*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 7*a^3*d^3)*x^2)*sqrt(x))/(a^2*b^3*x^3 + a^3*b^2*x)

Sympy [A]
time = 202.38, size = 1443, normalized size = 3.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] c**3*Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b**2*x**(9/2)),
Eq(a, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-5*a*sqrt(x)*log(sqrt(x) - (-a/b
)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4))
+ 5*a*sqrt(x)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) +
8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 10*a*sqrt(x)*atan(sqrt(x)/(-a/b)**(1/4))
/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 16*a*(-
a/b)**(1/4)/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)
) - 5*b*x**(5/2)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4)
 + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) + 5*b*x**(5/2)*log(sqrt(x) + (-a/b)**(1
/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 10*
b*x**(5/2)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a*
*2*b*x**(5/2)*(-a/b)**(1/4)) - 20*b*x**2*(-a/b)**(1/4)/(8*a**3*sqrt(x)*(-a/
b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)), True)) + 6*c**2*d*x**(3/2)/(4
*a**2 + 4*a*b*x**2) + 6*c**2*d*RootSum(65536*_t**4*a**5*b**3 + 1, Lambda(_t
, _t*log(4096*_t**3*a**4*b**2 + sqrt(x)))) + 3*c*d**2*Piecewise((zoo/sqrt(x
), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b, 0)), (-2/(b**2*sqrt(x)
), Eq(a, 0)), (3*a*log(sqrt(x) - (-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8
*b**3*x**2*(-a/b)**(1/4)) - 3*a*log(sqrt(x) + (-a/b)**(1/4))/(8*a*b**2*(-a/
b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) + 6*a*atan(sqrt(x)/(-a/b)**(1/4))/(8
*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) - 4*b*x**(3/2)*(-a/b)**(
1/4)/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) + 3*b*x**2*log(sq
rt(x) - (-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4))
 - 3*b*x**2*log(sqrt(x) + (-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x
**2*(-a/b)**(1/4)) + 6*b*x**2*atan(sqrt(x)/(-a/b)**(1/4))/(8*a*b**2*(-a/b)*
*(1/4) + 8*b**3*x**2*(-a/b)**(1/4)), True)) + d**3*Piecewise((zoo*x**(3/2),
 Eq(a, 0) & Eq(b, 0)), (2*x**(11/2)/(11*a**2), Eq(b, 0)), (2*x**(3/2)/(3*b*
*2), Eq(a, 0)), (-21*a**2*log(sqrt(x) - (-a/b)**(1/4))/(24*a*b**3*(-a/b)**(
1/4) + 24*b**4*x**2*(-a/b)**(1/4)) + 21*a**2*log(sqrt(x) + (-a/b)**(1/4))/(
24*a*b**3*(-a/b)**(1/4) + 24*b**4*x**2*(-a/b)**(1/4)) - 42*a**2*atan(sqrt(x
)/(-a/b)**(1/4))/(24*a*b**3*(-a/b)**(1/4) + 24*b**4*x**2*(-a/b)**(1/4)) + 2
8*a*b*x**(3/2)*(-a/b)**(1/4)/(24*a*b**3*(-a/b)**(1/4) + 24*b**4*x**2*(-a/b)
**(1/4)) - 21*a*b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(24*a*b**3*(-a/b)**(1/4
) + 24*b**4*x**2*(-a/b)**(1/4)) + 21*a*b*x**2*log(sqrt(x) + (-a/b)**(1/4))/
(24*a*b**3*(-a/b)**(1/4) + 24*b**4*x**2*(-a/b)**(1/4)) - 42*a*b*x**2*atan(s
qrt(x)/(-a/b)**(1/4))/(24*a*b**3*(-a/b)**(1/4) + 24*b**4*x**2*(-a/b)**(1/4)
) + 16*b**2*x**(7/2)*(-a/b)**(1/4)/(24*a*b**3*(-a/b)**(1/4) + 24*b**4*x**2*
(-a/b)**(1/4)), True))
```

Giac [A]
time = 1.82, size = 504, normalized size = 1.37

$$\frac{2\,d^{3}x^{\frac{3}{2}}}{3\,b^{2}}-\frac{5\,b^{3}c^{3}x^{2}-3\,ab^{2}c^{2}dx^{2}+3\,a^{2}bcd^{2}x^{2}-a^{3}d^{3}x^{2}+4\,ab^{2}c^{3}}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^{2}b^{2}}-\frac{\sqrt{2}\left(5\,(ab^{3})^{\frac{3}{4}}b^{3}c^{3}-3\,(ab^{3})^{\frac{3}{4}}ab^{2}c^{2}d-9\,(ab^{3})^{\frac{3}{4}}a^{2}bcd^{2}+7\,(ab^{3})^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{5}}-\frac{\sqrt{2}\left(5\,(ab^{3})^{\frac{3}{4}}b^{3}c^{3}-3\,(ab^{3})^{\frac{3}{4}}ab^{2}c^{2}d-9\,(ab^{3})^{\frac{3}{4}}a^{2}bcd^{2}+7\,(ab^{3})^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{5}}+\frac{\sqrt{2}\left(5\,(ab^{3})^{\frac{3}{4}}b^{3}c^{3}-3\,(ab^{3})^{\frac{3}{4}}ab^{2}c^{2}d-9\,(ab^{3})^{\frac{3}{4}}a^{2}bcd^{2}+7\,(ab^{3})^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{3}b^{5}}-\frac{\sqrt{2}\left(5\,(ab^{3})^{\frac{3}{4}}b^{3}c^{3}-3\,(ab^{3})^{\frac{3}{4}}ab^{2}c^{2}d-9\,(ab^{3})^{\frac{3}{4}}a^{2}bcd^{2}+7\,(ab^{3})^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{3}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*d^3*x^(3/2)/b^2 - 1/2*(5*b^3*c^3*x^2 - 3*a*b^2*c^2*d*x^2 + 3*a^2*b*c*d^
2*x^2 - a^3*d^3*x^2 + 4*a*b^2*c^3)/((b*x^(5/2) + a*sqrt(x))*a^2*b^2) - 1/8*
sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^
(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^5) - 1/8*sqrt(2)*(5*(a*b^3)^(3/4
)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(
a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^3*b^5) + 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)
^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^5) - 1/16*sqrt(2)*
(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^
2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a^3*b^5)
```

Mupad [B]
time = 0.22, size = 657, normalized size = 1.79

$$\frac{\frac{x^{2}\left(a^{3}d^{3}-3\,a^{2}b\,c\,d^{2}+3\,a\,b^{2}c^{2}d-5\,b^{3}c^{3}\right)}{2\,a^{2}}-\frac{2\,b^{2}c^{3}}{a}}{b^{3}x^{5/2}+a\,b^{2}\sqrt{x}}+\frac{2\,d^{3}x^{3/2}}{3\,b^{2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^{2}\,(7\,a\,d+5\,b\,c)\,(1568\,a^{13}b^{8}d^{6}-4032\,a^{12}b^{9}c\,d^{5}+1248\,a^{11}b^{10}c^{2}d^{4}+3968\,a^{10}b^{11}c^{3}d^{3}-2592\,a^{9}b^{12}c^{4}d^{2}-960\,a^{8}b^{13}c^{5}d+800\,a^{7}b^{14}c^{6})}{4\,(-a)^{9/4}b^{11/4}\,(2744\,a^{14}b^{5}d^{9}-10584\,a^{13}b^{6}c\,d^{8}+10080\,a^{12}b^{7}c^{2}d^{7}+9120\,a^{11}b^{8}c^{3}d^{6}-19440\,a^{10}b^{9}c^{4}d^{5}+2736\,a^{9}b^{10}c^{5}d^{4}+10464\,a^{8}b^{11}c^{6}d^{3}-4320\,a^{7}b^{12}c^{7}d^{2}-1800\,a^{6}b^{13}c^{8}d+1000\,a^{5}b^{14}c^{9})}\right)(a\,d-b\,c)^{2}\,(7\,a\,d+5\,b\,c)}{4\,(-a)^{9/4}\,b^{11/4}}-\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^{2}\,(7\,a\,d+5\,b\,c)\,(1568\,a^{13}b^{8}d^{6}-4032\,a^{12}b^{9}c\,d^{5}+1248\,a^{11}b^{10}c^{2}d^{4}+3968\,a^{10}b^{11}c^{3}d^{3}-2592\,a^{9}b^{12}c^{4}d^{2}-960\,a^{8}b^{13}c^{5}d+800\,a^{7}b^{14}c^{6})\,1\mathrm{i}}{4\,(-a)^{9/4}b^{11/4}\,(2744\,a^{14}b^{5}d^{9}-10584\,a^{13}b^{6}c\,d^{8}+10080\,a^{12}b^{7}c^{2}d^{7}+9120\,a^{11}b^{8}c^{3}d^{6}-19440\,a^{10}b^{9}c^{4}d^{5}+2736\,a^{9}b^{10}c^{5}d^{4}+10464\,a^{8}b^{11}c^{6}d^{3}-4320\,a^{7}b^{12}c^{7}d^{2}-1800\,a^{6}b^{13}c^{8}d+1000\,a^{5}b^{14}c^{9})}\right)(a\,d-b\,c)^{2}\,(7\,a\,d+5\,b\,c)\,1\mathrm{i}}{4\,(-a)^{9/4}\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(a^3*d^3 - 5*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2) - (2*b
^2*c^3)/a)/(b^3*x^(5/2) + a*b^2*x^(1/2)) + (2*d^3*x^(3/2))/(3*b^2) - (atan(
(x^(1/2)*(a*d - b*c)^2*(7*a*d + 5*b*c)*(800*a^7*b^14*c^6 + 1568*a^13*b^8*d^
6 - 960*a^8*b^13*c^5*d - 4032*a^12*b^9*c*d^5 - 2592*a^9*b^12*c^4*d^2 + 3968
*a^10*b^11*c^3*d^3 + 1248*a^11*b^10*c^2*d^4))/(4*(-a)^(9/4)*b^(11/4)*(1000*
a^5*b^14*c^9 + 2744*a^14*b^5*d^9 - 1800*a^6*b^13*c^8*d - 10584*a^13*b^6*c*d
^8 - 4320*a^7*b^12*c^7*d^2 + 10464*a^8*b^11*c^6*d^3 + 2736*a^9*b^10*c^5*d^4
 - 19440*a^10*b^9*c^4*d^5 + 9120*a^11*b^8*c^3*d^6 + 10080*a^12*b^7*c^2*d^7)
))*(a*d - b*c)^2*(7*a*d + 5*b*c))/(4*(-a)^(9/4)*b^(11/4)) - (atan((x^(1/2)*
(a*d - b*c)^2*(7*a*d + 5*b*c)*(800*a^7*b^14*c^6 + 1568*a^13*b^8*d^6 - 960*a
^8*b^13*c^5*d - 4032*a^12*b^9*c*d^5 - 2592*a^9*b^12*c^4*d^2 + 3968*a^10*b^1
1*c^3*d^3 + 1248*a^11*b^10*c^2*d^4)*1i)/(4*(-a)^(9/4)*b^(11/4)*(1000*a^5*b^
14*c^9 + 2744*a^14*b^5*d^9 - 1800*a^6*b^13*c^8*d - 10584*a^13*b^6*c*d^8 - 4
320*a^7*b^12*c^7*d^2 + 10464*a^8*b^11*c^6*d^3 + 2736*a^9*b^10*c^5*d^4 - 194
40*a^10*b^9*c^4*d^5 + 9120*a^11*b^8*c^3*d^6 + 10080*a^12*b^7*c^2*d^7)))*(a*
d - b*c)^2*(7*a*d + 5*b*c)*1i)/(4*(-a)^(9/4)*b^(11/4))
```

3.458 $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=367

$$-\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}}-\frac{d^2(bc-5ad)\sqrt{x}}{2ab^2}+\frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{3/2}\left(a+bx^2\right)}+\frac{(bc-ad)^2(7bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}b^{9/4}}$$

```
[Out] -1/6*c^2*(-3*a*d+7*b*c)/a^2/b/x^(3/2)+1/2*(-a*d+b*c)*(d*x^2+c)^2/a/b/x^(3/2
)/(b*x^2+a)+1/8*(-a*d+b*c)^2*(5*a*d+7*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)
/a^(1/4))/a^(11/4)/b^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*(5*a*d+7*b*c)*arctan(1+
b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(9/4)*2^(1/2)+1/16*(-a*d+b*c)^2
*(5*a*d+7*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/
4)/b^(9/4)*2^(1/2)-1/16*(-a*d+b*c)^2*(5*a*d+7*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/b^(9/4)*2^(1/2)-1/2*d^2*(-5*a*d+b*c)
*x^(1/2)/a/b^2
```

Rubi [A]
time = 0.29, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 479, 584, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(5ad+7bc)}{4\sqrt{2}\,a^{11/4}b^{9/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(5ad+7bc)}{4\sqrt{2}\,a^{11/4}b^{9/4}}+\frac{(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\,a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\,a^{11/4}b^{9/4}}-\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}}-\frac{d^2\sqrt{x}(bc-5ad)}{2ab^2}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/6*(c^2*(7*b*c - 3*a*d))/(a^2*b*x^(3/2)) - (d^2*(b*c - 5*a*d)*Sqrt[x])/(2
*a*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(3/2)*(a + b*x^2)) + ((b*c -
 a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*S
qrt[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(9/4)) + ((b*c - a*d)^2
*(7*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(8*Sqrt[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(9/4
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{(c+dx^4)(-c(7bc-3ad)+d(bc-5ad)x^4)}{x^4(a+bx^4)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \left(\frac{d^2(bc-5ad)}{b} + \frac{c^2(-7bc+3ad)}{ax^4} + \frac{(-bc+ad)^2(7bc+5ad)}{ab(a+bx^4)}\right)\,dx, x\right.}{2ab}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{((bc-ad)^2(7bc+5a}{}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{((bc-ad)^2(7bc+5a}{}$$

$$((bc-ad)^2(7bc+5a$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} - \frac{}{}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} + \frac{(bc-ad)^2(7bc+5ad}{}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)} + \frac{(bc-ad)^2(7bc+5ad}{4\sqrt{}}$$

Mathematica [A]

time = 0.49, size = 229, normalized size = 0.62

$$\frac{\frac{4a^{3/4}\sqrt[4]{b}\left(-7b^3c^3x^2+15a^3d^3x^2+3a^2bd^2x^2(-3c+4dx^2)+ab^2c^2(-4c+9dx^2)\right)}{x^{3/2}(a+bx^2)}+3\sqrt{2}\,(bc-ad)^2(7bc+5ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-3\sqrt{2}\,(bc-ad)^2(7bc+5ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{24a^{11/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] ((4*a^(3/4)*b^(1/4)*(-7*b^3*c^3*x^2 + 15*a^3*d^3*x^2 + 3*a^2*b*d^2*x^2*(-3*
c + 4*d*x^2) + a*b^2*c^2*(-4*c + 9*d*x^2)))/(x^(3/2)*(a + b*x^2)) + 3*Sqrt[
2]*(b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*(b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTanh[(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(24*a^(11/4)*b^(9/4)
)
```

Maple [A]

time = 0.12, size = 225, normalized size = 0.61

method	result
derivativedivides	$\frac{2d^3\sqrt{x}}{b^2}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)\sqrt{x}}{b\,x^2+a}+\frac{\left(5a^3d^3-3a^2bc\,d^2-9a\,b^2c^2d+7b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)}{x-\left(\frac{a}{b}\right)}\right.\right.}{}\right.}{a^2b^2}$
default	$\frac{2d^3\sqrt{x}}{b^2}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)\sqrt{x}}{b\,x^2+a}+\frac{\left(5a^3d^3-3a^2bc\,d^2-9a\,b^2c^2d+7b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)}{x-\left(\frac{a}{b}\right)}\right.\right.}{}\right.}{a^2b^2}$
risch	$\frac{2a^2d^3x^2-\frac{2b^2c^3}{3}}{b^2x^{\frac{3}{2}}a^2}+\frac{a\sqrt{x}\,d^3}{2b^2(b\,x^2+a)}-\frac{3\sqrt{x}\,d^2c}{2b(b\,x^2+a)}+\frac{3\sqrt{x}\,d\,c^2}{2a(b\,x^2+a)}-\frac{b\sqrt{x}\,c^3}{2a^2(b\,x^2+a)}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{8b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3/b^2*x^(1/2)-2/a^2/b^2*((-1/4*a^3*d^3+3/4*a^2*b*c*d^2-3/4*a*b^2*c^2*d+
1/4*b^3*c^3)*x^(1/2)/(b*x^2+a)+1/32*(5*a^3*d^3-3*a^2*b*c*d^2-9*a*b^2*c^2*d+
7*b^3*c^3)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(
1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/3*c^3/a^2/x^(3/
2)
```

Maxima [A]
time = 0.50, size = 415, normalized size = 1.13

$$\frac{2\,d^3\sqrt{x}}{b^2}-\frac{4\,ab^2c^3+(7\,b^3c^3-9\,ab^2c^2d+9\,a^2bcd^2-3\,a^3d^3)x^2}{6\left(a^2b^3x^{\frac{7}{2}}+a^3b^2x^{\frac{3}{2}}\right)}-\frac{\frac{2\sqrt{2}\,(7\,b^3c^3-9\,ab^2c^2d-3\,a^2bcd^2+5\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(7\,b^3c^3-9\,ab^2c^2d-3\,a^2bcd^2+5\,a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(7\,b^3c^3-9\,ab^2c^2d-3\,a^2bcd^2+5\,a^3d^3)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(7\,b^3c^3-9\,ab^2c^2d-3\,a^2bcd^2+5\,a^3d^3)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2*d^3*sqrt(x)/b^2 - 1/6*(4*a*b^2*c^3 + (7*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 - 3*a^3*d^3)*x^2)/(a^2*b^3*x^(7/2) + a^3*b^2*x^(3/2)) - 1/16*(2*sqrt
(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^
2*b*c*d^2 + 5*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt
(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(7*
b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*log(-sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1967 vs. $2(283) = 566$.
time = 0.51, size = 1967, normalized size = 5.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 - 12348*a*b^11*c^11
*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4
 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 +
1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 150
0*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*arctan((sqrt(a^6*b^4*sqr
t(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a
^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*
b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*
c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^1
1*b^9)) + (49*b^6*c^6 - 126*a*b^5*c^5*d + 39*a^2*b^4*c^4*d^2 + 124*a^3*b^3*
c^3*d^3 - 81*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 25*a^6*d^6)*x)*a^8*b^7*(-(2
401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^
9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c
^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d
^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9
```

```
))^(3/4) - (7*a^8*b^10*c^3 - 9*a^9*b^9*c^2*d - 3*a^10*b^8*c*d^2 + 5*a^11*b^
7*d^3)*sqrt(x)*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^1
0*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^
5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 +
11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a
^12*d^12)/(a^11*b^9))^(3/4))/(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*
a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^
5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^
4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*
d^11 + 625*a^12*d^12)) + 3*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 -
12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 3766
5*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a
^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b
^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*log(a^3
*b^2*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 23
24*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*
a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*
b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/
(a^11*b^9))^(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)
*sqrt(x)) - 3*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 - 12348*a*b^11*
c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8
*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^
7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 -
 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*log(-a^3*b^2*(-(2401
*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c
^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*
d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9
- 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^
(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*sqrt(x)) -
4*(12*a^2*b*d^3*x^4 - 4*a*b^2*c^3 - (7*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b*c*
d^2 - 15*a^3*d^3)*x^2)*sqrt(x))/(a^2*b^3*x^4 + a^3*b^2*x^2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2008 vs. $2(340) = 680$.
time = 116.77, size = 2008, normalized size = 5.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(11*x**(11/2)) - 6*c**2*d/(7*x**(7/2)) - 2*c*d**2/x
**(3/2) + 2*d**3*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(3*x**(3/2)) + 6
*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/2)/9)/a**2, Eq(b, 0)),
 ((-2*c**3/(11*x**(11/2)) - 6*c**2*d/(7*x**(7/2)) - 2*c*d**2/x**(3/2) + 2*d
```

```
**3*sqrt(x))/b**2, Eq(a, 0)), (15*a**4*d**3*x**(3/2)*(-a/b)**(1/4)*log(sqrt
(x) - (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 15*a
**4*d**3*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*b**2*
x**(3/2) + 24*a**3*b**3*x**(7/2)) - 30*a**4*d**3*x**(3/2)*(-a/b)**(1/4)*ata
n(sqrt(x)/(-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) +
60*a**4*d**3*x**2/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 9*a**3*
b*c*d**2*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24*a**4*b**2*
x**(3/2) + 24*a**3*b**3*x**(7/2)) + 9*a**3*b*c*d**2*x**(3/2)*(-a/b)**(1/4)*
log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)
) + 18*a**3*b*c*d**2*x**(3/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24
*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 36*a**3*b*c*d**2*x**2/(24*a*
*4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) + 15*a**3*b*d**3*x**(7/2)*(-a/b)*
*(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x
**(7/2)) - 15*a**3*b*d**3*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4
))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 30*a**3*b*d**3*x**(7/2
)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**
3*b**3*x**(7/2)) + 48*a**3*b*d**3*x**4/(24*a**4*b**2*x**(3/2) + 24*a**3*b**
3*x**(7/2)) - 16*a**2*b**2*c**3/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7
/2)) - 27*a**2*b**2*c**2*d*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/
4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) + 27*a**2*b**2*c**2*d*x
**(3/2)*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) +
 24*a**3*b**3*x**(7/2)) + 54*a**2*b**2*c**2*d*x**(3/2)*(-a/b)**(1/4)*atan(s
qrt(x)/(-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) + 36*
a**2*b**2*c**2*d*x**2/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 9*a
**2*b**2*c*d**2*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24*a**
4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) + 9*a**2*b**2*c*d**2*x**(7/2)*(-a/
b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**
3*x**(7/2)) + 18*a**2*b**2*c*d**2*x**(7/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b
)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) + 21*a*b**3*c**3*
x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2)
+ 24*a**3*b**3*x**(7/2)) - 21*a*b**3*c**3*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x
) + (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 42*a*b
**3*c**3*x**(3/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*b**2*x
**(3/2) + 24*a**3*b**3*x**(7/2)) - 28*a*b**3*c**3*x**2/(24*a**4*b**2*x**(3/
2) + 24*a**3*b**3*x**(7/2)) - 27*a*b**3*c**2*d*x**(7/2)*(-a/b)**(1/4)*log(s
qrt(x) - (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) + 2
7*a*b**3*c**2*d*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**
4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) + 54*a*b**3*c**2*d*x**(7/2)*(-a/b)
**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x
**(7/2)) + 21*b**4*c**3*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))
/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 21*b**4*c**3*x**(7/2)*(-
a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b
**3*x**(7/2)) - 42*b**4*c**3*x**(7/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1
/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)), True))
```

Giac [A]

time = 0.95, size = 501, normalized size = 1.37

$$\frac{2\,d^{3}\sqrt{x}}{b^{2}}-\frac{2\,c^{3}}{3\,a^{2}x^{\frac{3}{2}}}-\frac{\sqrt{2}\left(7\,(ab^{3})^{\frac{1}{4}}\,b^{3}c^{3}-9\,(ab^{3})^{\frac{1}{4}}\,ab^{2}c^{2}d-3\,(ab^{3})^{\frac{1}{4}}\,a^{2}bcd^{2}+5\,(ab^{3})^{\frac{1}{4}}\,a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{3}}-\frac{\sqrt{2}\left(7\,(ab^{3})^{\frac{1}{4}}\,b^{3}c^{3}-9\,(ab^{3})^{\frac{1}{4}}\,ab^{2}c^{2}d-3\,(ab^{3})^{\frac{1}{4}}\,a^{2}bcd^{2}+5\,(ab^{3})^{\frac{1}{4}}\,a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{3}}-\frac{\sqrt{2}\left(7\,(ab^{3})^{\frac{1}{4}}\,b^{3}c^{3}-9\,(ab^{3})^{\frac{1}{4}}\,ab^{2}c^{2}d-3\,(ab^{3})^{\frac{1}{4}}\,a^{2}bcd^{2}+5\,(ab^{3})^{\frac{1}{4}}\,a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{3}b^{3}}+\frac{\sqrt{2}\left(7\,(ab^{3})^{\frac{1}{4}}\,b^{3}c^{3}-9\,(ab^{3})^{\frac{1}{4}}\,ab^{2}c^{2}d-3\,(ab^{3})^{\frac{1}{4}}\,a^{2}bcd^{2}+5\,(ab^{3})^{\frac{1}{4}}\,a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{3}b^{3}}-\frac{b^{3}c^{3}\sqrt{x}-3\,ab^{2}c^{2}d\sqrt{x}+3\,a^{2}bcd^{2}\sqrt{x}-a^{3}d^{3}\sqrt{x}}{2\,(bx^{2}+a)a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2*d^3*sqrt(x)/b^2 - 2/3*c^3/(a^2*x^(3/2)) - 1/8*sqrt(2)*(7*(a*b^3)^(1/4)*b^
3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^
3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(a^3*b^3) - 1/8*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)
*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 1
/16*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b
^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(a^3*b^3) + 1/16*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 -
9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)
*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) - 1/2
*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sqrt(x) - a^3*d^3
*sqrt(x))/((b*x^2 + a)*a^2*b^2)
```

Mupad [B]

time = 0.25, size = 1759, normalized size = 4.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(3*a^3*d^3 - 7*b^3*c^3 + 9*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(6*a^2) - (2
*b^2*c^3)/(3*a))/(b^3*x^(7/2) + a*b^2*x^(3/2)) + (2*d^3*x^(1/2))/b^2 - (ata
n((((x^(1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d -
960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*
a^10*b^11*c^2*d^4) - ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 12
80*a^12*b^11*d^3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(-a)^(11
/4)*b^(9/4)))*(a*d - b*c)^2*(5*a*d + 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9/4)) + (
(x^(1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*
a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10
*b^11*c^2*d^4) + ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a
^12*b^11*d^3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(-a)^(11/4)*
b^(9/4)))*(a*d - b*c)^2*(5*a*d + 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9/4)))/(((x^(
1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11
*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^1
1*c^2*d^4) - ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*
```

```
b^11*d^3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(-a)^(11/4)*b^(9
/4)))*(a*d - b*c)^2*(5*a*d + 7*b*c))/(8*(-a)^(11/4)*b^(9/4)) - ((x^(1/2)*(1
568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c
*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d
^4) + ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^
3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(-a)^(11/4)*b^(9/4)))*(
a*d - b*c)^2*(5*a*d + 7*b*c))/(8*(-a)^(11/4)*b^(9/4))))*(a*d - b*c)^2*(5*a*
d + 7*b*c)*1i)/(4*(-a)^(11/4)*b^(9/4)) - (atan((((x^(1/2)*(1568*a^6*b^15*c^
6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c*d^5 + 1248*a^8
*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d^4) - ((a*d - b
*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^3 - 2304*a^10*b
^13*c^2*d - 768*a^11*b^12*c*d^2)*1i)/(8*(-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2
*(5*a*d + 7*b*c))/(8*(-a)^(11/4)*b^(9/4)) + ((x^(1/2)*(1568*a^6*b^15*c^6 +
800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c*d^5 + 1248*a^8*b^1
3*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d^4) + ((a*d - b*c)^
2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^3 - 2304*a^10*b^13*
c^2*d - 768*a^11*b^12*c*d^2)*1i)/(8*(-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2*(5*
a*d + 7*b*c))/(8*(-a)^(11/4)*b^(9/4)))/(((x^(1/2)*(1568*a^6*b^15*c^6 + 800*
a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^
4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d^4) - ((a*d - b*c)^2*(5
*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^3 - 2304*a^10*b^13*c^2*
d - 768*a^11*b^12*c*d^2)*1i)/(8*(-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2*(5*a*d
+ 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9/4)) - ((x^(1/2)*(1568*a^6*b^15*c^6 + 800*a
^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4
*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d^4) + ((a*d - b*c)^2*(5*
a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^3 - 2304*a^10*b^13*c^2*d
 - 768*a^11*b^12*c*d^2)*1i)/(8*(-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2*(5*a*d +
 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9/4))))*(a*d - b*c)^2*(5*a*d + 7*b*c))/(4*(-a
)^(11/4)*b^(9/4))
```

3.459 $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2} \, dx$

Optimal. Leaf size=376

$$-\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}}+\frac{c(9b^2c^2-15abcd+2a^2d^2)}{2a^3b\sqrt{x}}+\frac{(bc-ad)\,(c+dx^2)^2}{2abx^{5/2}\,(a+bx^2)}-\frac{3(bc-ad)^2(3bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}\right.}{4\sqrt{2}\,a^{13/4}b^{7/4}}$$

[Out] -1/10*c^2*(-5*a*d+9*b*c)/a^2/b/x^(5/2)+1/2*(-a*d+b*c)*(d*x^2+c)^2/a/b/x^(5/2)/(b*x^2+a)-3/8*(-a*d+b*c)^2*(a*d+3*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/b^(7/4)*2^(1/2)+3/8*(-a*d+b*c)^2*(a*d+3*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/b^(7/4)*2^(1/2)+3/16*(-a*d+b*c)^2*(a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/b^(7/4)*2^(1/2)-3/16*(-a*d+b*c)^2*(a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/b^(7/4)*2^(1/2)+1/2*c*(2*a^2*d^2-15*a*b*c*d+9*b^2*c^2)/a^3/b/x^(1/2)

Rubi [A]

time = 0.29, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 479, 584, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(ad+3bc)}{4\sqrt{2}\,a^{13/4}b^{7/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(ad+3bc)}{4\sqrt{2}\,a^{13/4}b^{7/4}}+\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}b^{7/4}}-\frac{3(bc-ad)^2(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}b^{7/4}}-\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}}+\frac{c(2a^2d^2-15abcd+9b^2c^2)}{2a^3b\sqrt{x}}+\frac{(c+dx^2)^2(bc-ad)}{2abx^{5/2}(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]

[Out] -1/10*(c^2*(9*b*c - 5*a*d))/(a^2*b*x^(5/2)) + (c*(9*b^2*c^2 - 15*a*b*c*d + 2*a^2*d^2))/(2*a^3*b*Sqrt[x]) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(5/2)*(a + b*x^2)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/4)) + (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/4)) + (3*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*b^(7/4)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*b^(7/4))

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^6(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{5/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{(c+dx^4)(-c(9bc-5ad)-d(bc+3ad)x^4)}{x^6(a+bx^4)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{5/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \left(\frac{c^2(-9bc+5ad)}{ax^6} + \frac{c(9b^2c^2-15abcd+2a^2d^2)}{a^2x^2} - \frac{3(-bc+ad)^2(3bc+ad}{a^2(a+bx^4)}\right.\right.}{2ab}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c(9b^2c^2-15abcd+2a^2d^2)}{2a^3b\sqrt{x}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{5/2}(a+bx^2)} + \frac{(3(bc-ad)^2(}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c(9b^2c^2-15abcd+2a^2d^2)}{2a^3b\sqrt{x}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{5/2}(a+bx^2)} - \frac{(3(bc-ad)^2(}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c(9b^2c^2-15abcd+2a^2d^2)}{2a^3b\sqrt{x}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{5/2}(a+bx^2)} + \frac{(3(bc-ad)^2(}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c(9b^2c^2-15abcd+2a^2d^2)}{2a^3b\sqrt{x}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{5/2}(a+bx^2)} + \frac{3(bc-ad)^2(3}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c(9b^2c^2-15abcd+2a^2d^2)}{2a^3b\sqrt{x}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{5/2}(a+bx^2)} - \frac{3(bc-ad)^2(3}{}$$

Mathematica [A]

time = 0.56, size = 236, normalized size = 0.63

$$\frac{-\frac{4\sqrt[4]{a}\,b^{3/4}\left(-45b^3c^3x^4+5a^3d^3x^4+3ab^2c^2x^2(-12c+25dx^2)+a^2bc(4c^2+60cdx^2-15d^2x^4)\right)}{x^{5/2}(a+bx^2)}-15\sqrt{2}\,(bc-ad)^2(3bc+ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-15\sqrt{2}\,(bc-ad)^2(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{40a^{13/4}b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-4*a^(1/4)*b^(3/4)*(-45*b^3*c^3*x^4 + 5*a^3*d^3*x^4 + 3*a*b^2*c^2*x^2*(-1
2*c + 25*d*x^2) + a^2*b*c*(4*c^2 + 60*c*d*x^2 - 15*d^2*x^4)))/(x^(5/2)*(a +
 b*x^2)) - 15*Sqrt[2]*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[(Sqrt[a] - Sqrt[b]
*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 15*Sqrt[2]*(b*c - a*d)^2*(3*b*c +
a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(40*
a^(13/4)*b^(7/4))
```

Maple [A]

time = 0.12, size = 232, normalized size = 0.62

method	result
derivativedivides	$\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^{\frac{3}{2}}}{2b\left(b\,x^2+a\right)}+\frac{3\left(a^3d^3+a^2bc\,d^2-5a\,b^2c^2d+3b^3c^3\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arct}\cdots\right.}{16b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^3}$
default	$\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^{\frac{3}{2}}}{2b\left(b\,x^2+a\right)}+\frac{3\left(a^3d^3+a^2bc\,d^2-5a\,b^2c^2d+3b^3c^3\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arct}\cdots\right.}{16b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^3}$
risch	$-\frac{2c^2\left(15ad\,x^2-10c\,x^2b+ac\right)}{5a^3x^{\frac{5}{2}}}-\frac{x^{\frac{3}{2}}d^3}{2b(b\,x^2+a)}+\frac{3x^{\frac{3}{2}}d^2c}{2a(b\,x^2+a)}-\frac{3b\,x^{\frac{3}{2}}d\,c^2}{2a^2(b\,x^2+a)}+\frac{x^{\frac{3}{2}}c^3b^2}{2a^3(b\,x^2+a)}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{\cdots}\right.}{8b^2\left(\frac{a}{b}\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^3*(-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b*x^(3/2)/(b*x^2+
a)+3/32*(a^3*d^3+a^2*b*c*d^2-5*a*b^2*c^2*d+3*b^3*c^3)/b^2/(a/b)^(1/4)*2^(1/
2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2
^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)-1)))-2/5*c^3/a^2/x^(5/2)-2*c^2*(3*a*d-2*b*c)/a^3/x^(
1/2)
```

Maxima [A]

time = 0.55, size = 315, normalized size = 0.84

$$-\frac{4\,a^2bc^3-5\,(9\,b^3c^3-15\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^4-12\,(3\,ab^2c^3-5\,a^2bc^2d)x^2}{10\left(a^3b^2x^{\frac{9}{2}}+a^4bx^{\frac{5}{2}}\right)}+\frac{3\,(3\,b^3c^3-5\,ab^2c^2d+a^2bcd^2+a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/10*(4*a^2*b*c^3 - 5*(9*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*x^4 - 12*(3*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)/(a^3*b^2*x^(9/2) + a^4*b*x^(
5/2)) + 3/16*(3*b^3*c^3 - 5*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(
1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^3*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2549 vs. $2(292) = 584$.
time = 0.58, size = 2549, normalized size = 6.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(60*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1
458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a
^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^
4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*
d^12)/(a^13*b^7))^(1/4)*arctan((sqrt((729*b^18*c^18 - 7290*a*b^17*c^17*d +
31833*a^2*b^16*c^16*d^2 - 78192*a^3*b^15*c^15*d^3 + 113940*a^4*b^14*c^14*d^
4 - 88920*a^5*b^13*c^13*d^5 + 10180*a^6*b^12*c^12*d^6 + 46320*a^7*b^11*c^11
*d^7 - 35970*a^8*b^10*c^10*d^8 - 220*a^9*b^9*c^9*d^9 + 12078*a^10*b^8*c^8*d
^10 - 3600*a^11*b^7*c^7*d^11 - 1884*a^12*b^6*c^6*d^12 + 936*a^13*b^5*c^5*d^
13 + 180*a^14*b^4*c^4*d^14 - 112*a^15*b^3*c^3*d^15 - 15*a^16*b^2*c^2*d^16 +
 6*a^17*b*c*d^17 + a^18*d^18)*x - (81*a^7*b^15*c^12 - 540*a^8*b^14*c^11*d +
 1458*a^9*b^13*c^10*d^2 - 1932*a^10*b^12*c^9*d^3 + 1039*a^11*b^11*c^8*d^4 +
 328*a^12*b^10*c^7*d^5 - 644*a^13*b^9*c^6*d^6 + 136*a^14*b^8*c^5*d^7 + 127*
a^15*b^7*c^4*d^8 - 44*a^16*b^6*c^3*d^9 - 14*a^17*b^5*c^2*d^10 + 4*a^18*b^4*
c*d^11 + a^19*b^3*d^12)*sqrt(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*
b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c
^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 -
44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a
^13*b^7)))*a^3*b^2*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10
*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 -
644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^
3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))
^(1/4) - (27*a^3*b^11*c^9 - 135*a^4*b^10*c^8*d + 252*a^5*b^9*c^7*d^2 - 188*
```

```
a^6*b^8*c^6*d^3 - 6*a^7*b^7*c^5*d^4 + 78*a^8*b^6*c^4*d^5 - 20*a^9*b^5*c^3*d
^6 - 12*a^10*b^4*c^2*d^7 + 3*a^11*b^3*c*d^8 + a^12*b^2*d^9)*sqrt(x)*(-(81*b
^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^
3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*
a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^
2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4))/(81*b^12*c^12 - 54
0*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*
b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d
^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^
11*b*c*d^11 + a^12*d^12)) - 15*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 -
540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^
4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5
*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*
a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4)*log(27*a^10*b^5*(-(81*b^12*c^1
2 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 103
9*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5
*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10
+ 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(3/4) + 27*(27*b^9*c^9 - 135*a*b
^8*c^8*d + 252*a^2*b^7*c^7*d^2 - 188*a^3*b^6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 +
78*a^5*b^4*c^4*d^5 - 20*a^6*b^3*c^3*d^6 - 12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^
8 + a^9*d^9)*sqrt(x)) + 15*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 - 540*
a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^
8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7
 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11
*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4)*log(-27*a^10*b^5*(-(81*b^12*c^12 -
 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a
^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^
5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4
*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(3/4) + 27*(27*b^9*c^9 - 135*a*b^8*
c^8*d + 252*a^2*b^7*c^7*d^2 - 188*a^3*b^6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 + 78*
a^5*b^4*c^4*d^5 - 20*a^6*b^3*c^3*d^6 - 12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^8 +
 a^9*d^9)*sqrt(x)) + 4*(4*a^2*b*c^3 - 5*(9*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*x^4 - 12*(3*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqrt(x))/(a
^3*b^2*x^5 + a^4*b*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 1.34, size = 505, normalized size = 1.34

$$\frac{b^3c^3x^{\frac{3}{2}} - 3\,ab^2c^2dx^{\frac{3}{2}} + 3\,a^2bcd^2x^{\frac{3}{2}} - a^3d^3x^{\frac{3}{2}}}{2\,(bx^2+a)a^3b} + \frac{2\,(10\,bc^3x^2 - 15\,ac^2dx^2 - ac^3)}{5\,a^3x^{\frac{5}{2}}} + \frac{3\,\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^4} + \frac{3\,\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^4} - \frac{3\,\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^4} + \frac{3\,\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3*
d^3*x^(3/2))/((b*x^2 + a)*a^3*b) + 2/5*(10*b*c^3*x^2 - 15*a*c^2*d*x^2 - a*c
^3)/(a^3*x^(5/2)) + 3/8*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*
a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) + 3/8*sqr
t(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)
*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) - 3/16*sqrt(2)*(3*(a*b^3)^(3/4)*b^3
*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3
/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^4) + 3
/16*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3
)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/
4) + x + sqrt(a/b))/(a^4*b^4)
```

Mupad [B]
time = 0.14, size = 656, normalized size = 1.74

$$\frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{x}\,(a\,d-b\,c)^2\,(a\,d+3\,b\,c)\,(288\,a^{16}\,b^5\,d^6+576\,a^{15}\,b^6\,c\,d^5-2592\,a^{14}\,b^7\,c^2\,d^4-1152\,a^{13}\,b^8\,c^3\,d^3+8928\,a^{12}\,b^9\,c^4\,d^2-8640\,a^{11}\,b^{10}\,c^5\,d+2592\,a^{10}\,b^{11}\,c^6)}{4\,(-a)^{13/4}\,b^{7/4}\,(216\,a^{16}\,b^3\,d^9+648\,a^{15}\,b^4\,c\,d^8-2592\,a^{14}\,b^5\,c^2\,d^7-4320\,a^{13}\,b^6\,c^3\,d^6+16848\,a^{12}\,b^7\,c^4\,d^5-1296\,a^{11}\,b^8\,c^5\,d^4-40608\,a^{10}\,b^9\,c^6\,d^3+54432\,a^9\,b^{10}\,c^7\,d^2-29160\,a^8\,b^{11}\,c^8\,d+5832\,a^7\,b^{12}\,c^9)}\right)\,(a\,d-b\,c)^2\,(a\,d+3\,b\,c)}{4\,(-a)^{13/4}\,b^{7/4}} - \frac{\frac{2\,c^3}{5\,a}+\frac{x^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d-9\,b^3\,c^3)}{2\,a^3\,b}+\frac{6\,c^2\,x^2\,(5\,a\,d-3\,b\,c)}{5\,a^2}}{a\,x^{5/2}+b\,x^{9/2}} - \frac{3\,\mathrm{atanh}\left(\frac{3\,\sqrt{x}\,(a\,d-b\,c)^2\,(a\,d+3\,b\,c)\,(288\,a^{16}\,b^5\,d^6+576\,a^{15}\,b^6\,c\,d^5-2592\,a^{14}\,b^7\,c^2\,d^4-1152\,a^{13}\,b^8\,c^3\,d^3+8928\,a^{12}\,b^9\,c^4\,d^2-8640\,a^{11}\,b^{10}\,c^5\,d+2592\,a^{10}\,b^{11}\,c^6)}{4\,(-a)^{13/4}\,b^{7/4}\,(216\,a^{16}\,b^3\,d^9+648\,a^{15}\,b^4\,c\,d^8-2592\,a^{14}\,b^5\,c^2\,d^7-4320\,a^{13}\,b^6\,c^3\,d^6+16848\,a^{12}\,b^7\,c^4\,d^5-1296\,a^{11}\,b^8\,c^5\,d^4-40608\,a^{10}\,b^9\,c^6\,d^3+54432\,a^9\,b^{10}\,c^7\,d^2-29160\,a^8\,b^{11}\,c^8\,d+5832\,a^7\,b^{12}\,c^9)}\right)\,(a\,d-b\,c)^2\,(a\,d+3\,b\,c)}{4\,(-a)^{13/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x)
```

```
[Out] (3*atan((3*x^(1/2)*(a*d - b*c)^2*(a*d + 3*b*c)*(2592*a^10*b^11*c^6 + 288*a^
16*b^5*d^6 - 8640*a^11*b^10*c^5*d + 576*a^15*b^6*c*d^5 + 8928*a^12*b^9*c^4*
d^2 - 1152*a^13*b^8*c^3*d^3 - 2592*a^14*b^7*c^2*d^4))/(4*(-a)^(13/4)*b^(7/4
)*(5832*a^7*b^12*c^9 + 216*a^16*b^3*d^9 - 29160*a^8*b^11*c^8*d + 648*a^15*b
^4*c*d^8 + 54432*a^9*b^10*c^7*d^2 - 40608*a^10*b^9*c^6*d^3 - 1296*a^11*b^8*
c^5*d^4 + 16848*a^12*b^7*c^4*d^5 - 4320*a^13*b^6*c^3*d^6 - 2592*a^14*b^5*c^
2*d^7)))*(a*d - b*c)^2*(a*d + 3*b*c))/(4*(-a)^(13/4)*b^(7/4)) - ((2*c^3)/(5
*a) + (x^4*(a^3*d^3 - 9*b^3*c^3 + 15*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^3*b
) + (6*c^2*x^2*(5*a*d - 3*b*c))/(5*a^2))/(a*x^(5/2) + b*x^(9/2)) - (3*atanh
((3*x^(1/2)*(a*d - b*c)^2*(a*d + 3*b*c)*(2592*a^10*b^11*c^6 + 288*a^16*b^5*
d^6 - 8640*a^11*b^10*c^5*d + 576*a^15*b^6*c*d^5 + 8928*a^12*b^9*c^4*d^2 - 1
152*a^13*b^8*c^3*d^3 - 2592*a^14*b^7*c^2*d^4))/(4*(-a)^(13/4)*b^(7/4)*(5832
*a^7*b^12*c^9 + 216*a^16*b^3*d^9 - 29160*a^8*b^11*c^8*d + 648*a^15*b^4*c*d^
8 + 54432*a^9*b^10*c^7*d^2 - 40608*a^10*b^9*c^6*d^3 - 1296*a^11*b^8*c^5*d^4
 + 16848*a^12*b^7*c^4*d^5 - 4320*a^13*b^6*c^3*d^6 - 2592*a^14*b^5*c^2*d^7))
)*(a*d - b*c)^2*(a*d + 3*b*c))/(4*(-a)^(13/4)*b^(7/4))
```

3.460 $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=376

$$-\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}}+\frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}}+\frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{7/2}\left(a+bx^2\right)}-\frac{(bc-ad)^2(11bc+ad)\tan^{-1}\left(1-\sqrt{}\right.}{4\sqrt{2}\,a^{15/4}b^{5/4}}$$

```
[Out] -1/14*c^2*(-7*a*d+11*b*c)/a^2/b/x^(7/2)+1/6*c*(6*a^2*d^2-21*a*b*c*d+11*b^2*
c^2)/a^3/b/x^(3/2)+1/2*(-a*d+b*c)*(d*x^2+c)^2/a/b/x^(7/2)/(b*x^2+a)-1/8*(-a
*d+b*c)^2*(a*d+11*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4)/b
^(5/4)*2^(1/2)+1/8*(-a*d+b*c)^2*(a*d+11*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/
2)/a^(1/4))/a^(15/4)/b^(5/4)*2^(1/2)-1/16*(-a*d+b*c)^2*(a*d+11*b*c)*ln(a^(1
/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)+1/1
6*(-a*d+b*c)^2*(a*d+11*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^
(1/2))/a^(15/4)/b^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 479, 584, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(ad+11bc)}{4\sqrt{2}\,a^{15/4}b^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(ad+11bc)}{4\sqrt{2}\,a^{15/4}b^{5/4}}-\frac{(bc-ad)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\,a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\,a^{15/4}b^{5/4}}-\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}}+\frac{c(6a^2d^2-21abcd+11b^2c^2)}{6a^3bx^{3/2}}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/14*(c^2*(11*b*c - 7*a*d))/(a^2*b*x^(7/2)) + (c*(11*b^2*c^2 - 21*a*b*c*d
+ 6*a^2*d^2))/(6*a^3*b*x^(3/2)) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(7/2
)*(a + b*x^2)) - ((b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11*b*c +
a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(
5/4)) - ((b*c - a*d)^2*(11*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11*b*
c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqr
t[2]*a^(15/4)*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{(c+dx^4)^3}{x^8(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{(c+dx^4)(-c(11bc-7ad)-d(3bc+ad)x^4)}{x^8(a+bx^4)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{\text{Subst}\left(\int \left(\frac{c^2(-11bc+7ad)}{ax^8} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{a^2x^4} - \frac{(-bc+ad)^2(11bc}{a^2(a+bx^4)}\right.\right.}{2ab}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} + \frac{((bc-ad}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} + \frac{((bc-ad}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} + \frac{((bc-ad}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{(bc-ad)}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{(bc-ad)}{}$$

Mathematica [A]

time = 0.55, size = 236, normalized size = 0.63

$$-\frac{\frac{4a^{3/4}\sqrt[4]{b}\left(-77b^3c^3x^4+21a^3d^3x^4+ab^2c^2x^2(-44c+147dx^2)+3a^2bc(4c^2+28cdx^2-21d^2x^4)\right)}{x^{7/2}(a+bx^2)}-21\sqrt{2}\,(bc-ad)^2(11bc+ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+21\sqrt{2}\,(bc-ad)^2(11bc+ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{168a^{15/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-4*a^(3/4)*b^(1/4)*(-77*b^3*c^3*x^4 + 21*a^3*d^3*x^4 + a*b^2*c^2*x^2*(-44
*c + 147*d*x^2) + 3*a^2*b*c*(4*c^2 + 28*c*d*x^2 - 21*d^2*x^4)))/(x^(7/2)*(a
 + b*x^2)) - 21*Sqrt[2]*(b*c - a*d)^2*(11*b*c + a*d)*ArcTan[(Sqrt[a] - Sqrt
[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 21*Sqrt[2]*(b*c - a*d)^2*(11*b*
c + a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
(168*a^(15/4)*b^(5/4))
```

Maple [A]
time = 0.12, size = 236, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\sqrt{x}}{2b\left(b\,x^2+a\right)}+\frac{\left(a^3d^3+9a^2bc\,d^2-21a\,b^2c^2d+11b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{16ba}}{a^3}$
default	$\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\sqrt{x}}{2b\left(b\,x^2+a\right)}+\frac{\left(a^3d^3+9a^2bc\,d^2-21a\,b^2c^2d+11b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{16ba}}{a^3}$
risch	$-\frac{2c^2\left(21ad\,x^2-14c\,x^2b+3ac\right)}{21a^3x^{\frac{7}{2}}}-\frac{\sqrt{x}\,d^3}{2b\left(b\,x^2+a\right)}+\frac{3\sqrt{x}\,d^2c}{2a\left(b\,x^2+a\right)}-\frac{3b\sqrt{x}\,d\,c^2}{2a^2\left(b\,x^2+a\right)}+\frac{\sqrt{x}\,c^3b^2}{2a^3\left(b\,x^2+a\right)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\right.}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^3*(-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b*x^(1/2)/(b*x^2+
a)+1/32*(a^3*d^3+9*a^2*b*c*d^2-21*a*b^2*c^2*d+11*b^3*c^3)/b*(a/b)^(1/4)/a*2
^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/7*c^3/a^2/x^(7/2)-2/3*c^2*(3*a*d-2*b*c)/a
^3/x^(3/2)
```

Maxima [A]
time = 0.55, size = 424, normalized size = 1.13

$$-\frac{12\,a^2bc^3-7\,(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2-3\,a^3d^3)x^4-4\,(11\,ab^2c^3-21\,a^2bc^2d)x^2}{42\left(a^3b^2x^{\frac{11}{2}}+a^4bx^{\frac{7}{2}}\right)}+\frac{\frac{2\sqrt{2}\,(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/42*(12*a^2*b*c^3 - 7*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 3*a^
3*d^3)*x^4 - 4*(11*a*b^2*c^3 - 21*a^2*b*c^2*d)*x^2)/(a^3*b^2*x^(11/2) + a^4
*b*x^(7/2)) + 1/16*(2*sqrt(2)*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2
+ a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(11*b^3
*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(
2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))) + sqrt(2)*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2
+ a^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3
/4)*b^(1/4)) - sqrt(2)*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d
^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^
(1/4)))/(a^3*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1955 vs. $2(292) = 584$.

time = 0.51, size = 1955, normalized size = 5.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(84*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11
*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8
*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d
^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 +
36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*arctan((sqrt(a^8*b^2*sqrt(-
(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588
*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188
*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b
^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^
5)) + (121*b^6*c^6 - 462*a*b^5*c^5*d + 639*a^2*b^4*c^4*d^2 - 356*a^3*b^3*c^
3*d^3 + 39*a^4*b^2*c^2*d^4 + 18*a^5*b*c*d^5 + a^6*d^6)*x)*a^11*b^4*(-(14641
*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b
^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b
^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3
*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(3
/4) - (11*a^11*b^7*c^3 - 21*a^12*b^6*c^2*d + 9*a^13*b^5*c*d^2 + a^14*b^4*d^
3)*sqrt(x)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10
*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7
*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^
8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d
```

```
^12)/(a^15*b^5))^(3/4))/(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^
2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*
a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8
*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^
11 + a^12*d^12)) + 21*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804
*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703
*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a
^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2
*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*log(a^4*b*(-(14
641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^
3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^
6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*
c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))
^(1/4) + (11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(x)) -
 21*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 3
68082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 -
 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 1
0017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^1
1*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*log(-a^4*b*(-(14641*b^12*c^12 - 1
11804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 7
46703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5
688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^1
0*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4) + (11*b^3*
c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(x)) - 4*(12*a^2*b*c^3
- 7*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 3*a^3*d^3)*x^4 - 4*(11*a
*b^2*c^3 - 21*a^2*b*c^2*d)*x^2)*sqrt(x))/(a^3*b^2*x^6 + a^4*b*x^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.02, size = 509, normalized size = 1.35

$$\frac{\sqrt{2}\left(11\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-21\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+9\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{4}b^{2}}+\frac{\sqrt{2}\left(11\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-21\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+9\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{4}b^{2}}+\frac{\sqrt{2}\left(11\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-21\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+9\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{4}b^{2}}-\frac{\sqrt{2}\left(11\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-21\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+9\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{4}b^{2}}+\frac{b^{3}c^{3}\sqrt{x}-3ab^{2}c^{2}d\sqrt{x}+3a^{2}bcd^{2}\sqrt{x}-a^{3}d^{3}\sqrt{x}}{2\left(bx^{2}+a\right)a^{3}b}+\frac{2\left(14bc^{3}x^{2}-21ac^{2}dx^{2}-3ac^{3}\right)}{21a^{3}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a
*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) + 1/8*sqrt(2)*(11*(a*b^3)
^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2
 + (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^2) + 1/16*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(
a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*
d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*sqrt
(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1
/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(a^4*b^2) + 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) +
3*a^2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*a^3*b) + 2/21*(14*b*c
^3*x^2 - 21*a*c^2*d*x^2 - 3*a*c^3)/(a^3*x^(7/2))
```

Mupad [B]
time = 0.33, size = 1746, normalized size = 4.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x)
```

```
[Out] (atan((((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5
*d + 576*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3
+ 1248*a^13*b^8*c^2*d^4) - ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^
3 + 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a)^
(15/4)*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*b*c)*1i)/(8*(-a)^(15/4)*b^(5/4)) +
 ((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 5
76*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248
*a^13*b^8*c^2*d^4) + ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 25
6*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a)^(15/4)
*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*b*c)*1i)/(8*(-a)^(15/4)*b^(5/4)))/(((x^(
1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^1
4*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*
b^8*c^2*d^4) - ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16
*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a)^(15/4)*b^(5/
4)))*(a*d - b*c)^2*(a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4)) - ((x^(1/2)*(387
2*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d
^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d
^4) + ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3
- 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a)^(15/4)*b^(5/4)))*(a*d
 - b*c)^2*(a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4))))*(a*d - b*c)^2*(a*d + 11
*b*c)*1i)/(4*(-a)^(15/4)*b^(5/4)) - ((2*c^3)/(7*a) + (x^4*(3*a^3*d^3 - 11*b
^3*c^3 + 21*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(6*a^3*b) + (2*c^2*x^2*(21*a*d -
11*b*c))/(21*a^2))/(a*x^(7/2) + b*x^(11/2)) + (atan((((x^(1/2)*(3872*a^9*b^
```

```
12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d^5 + 204
48*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d^4) - ((
a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3 - 5376*a
^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2)*1i)/(8*(-a)^(15/4)*b^(5/4)))*(a*d - b*
c)^2*(a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4)) + ((x^(1/2)*(3872*a^9*b^12*c^6
 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d^5 + 20448*a^1
1*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d^4) + ((a*d -
b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3 - 5376*a^14*b^
9*c^2*d + 2304*a^15*b^8*c*d^2)*1i)/(8*(-a)^(15/4)*b^(5/4)))*(a*d - b*c)^2*(
a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4)))/(((x^(1/2)*(3872*a^9*b^12*c^6 + 32*
a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d^5 + 20448*a^11*b^10
*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d^4) - ((a*d - b*c)^2
*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*
d + 2304*a^15*b^8*c*d^2)*1i)/(8*(-a)^(15/4)*b^(5/4)))*(a*d - b*c)^2*(a*d +
11*b*c)*1i)/(8*(-a)^(15/4)*b^(5/4)) - ((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^1
5*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^
4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d^4) + ((a*d - b*c)^2*(a
*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d +
 2304*a^15*b^8*c*d^2)*1i)/(8*(-a)^(15/4)*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*
b*c)*1i)/(8*(-a)^(15/4)*b^(5/4))))*(a*d - b*c)^2*(a*d + 11*b*c))/(4*(-a)^(1
5/4)*b^(5/4))
```

3.461 $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=478

$$\frac{2x^{3/2}}{3bd}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}(bc-ad)}+\frac{a^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}(bc-ad)}+\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{7/4}(bc-ad)}-$$

```
[Out] 2/3*x^(3/2)/b/d-1/2*a^(7/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(7/
4)/(-a*d+b*c)*2^(1/2)+1/2*a^(7/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))
/b^(7/4)/(-a*d+b*c)*2^(1/2)+1/2*c^(7/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^
(1/4))/d^(7/4)/(-a*d+b*c)*2^(1/2)-1/2*c^(7/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1
/2)/c^(1/4))/d^(7/4)/(-a*d+b*c)*2^(1/2)+1/4*a^(7/4)*ln(a^(1/2)+x*b^(1/2)-a^
(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/(-a*d+b*c)*2^(1/2)-1/4*a^(7/4)*ln(a^
(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/(-a*d+b*c)*2^(1/2)
-1/4*c^(7/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(7/4)/
(-a*d+b*c)*2^(1/2)+1/4*c^(7/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)
*x^(1/2))/d^(7/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 490, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{a^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}(bc-ad)}+\frac{a^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{7/4}(bc-ad)}+\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,b^{7/4}(bc-ad)}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,b^{7/4}(bc-ad)}+\frac{c^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{7/4}(bc-ad)}-\frac{c^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{7/4}(bc-ad)}-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\,d^{7/4}(bc-ad)}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\,d^{7/4}(bc-ad)}+\frac{2x^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*x^(3/2))/(3*b*d) - (a^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (a^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (c^(7/4)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) - (c^(7/4)*ArcTa
n[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) + (
a^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt
[2]*b^(7/4)*(b*c - a*d)) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)*(b*c - a*d)) - (c^(7/4)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(7/4)*(b*c -
a*d)) + (c^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(2*Sqrt[2]*d^(7/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx = 2\text{Subst}\left(\int \frac{x^{10}}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2x^{3/2}}{3bd} - \frac{2\text{Subst}\left(\int \frac{x^2(3ac+3(bc+ad)x^4)}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{3bd}$$

$$= \frac{2x^{3/2}}{3bd} - \frac{2\text{Subst}\left(\int \left(\frac{3a^2dx^2}{(-bc+ad)(a+bx^4)} + \frac{3bc^2x^2}{(bc-ad)(c+dx^4)}\right)dx, x, \sqrt{x}\right)}{3bd}$$

$$= \frac{2x^{3/2}}{3bd} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{d(bc-ad)}$$

$$= \frac{2x^{3/2}}{3bd} - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}(bc-ad)} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{\right.}{b^{3/2}(bc-ad)}$$

$$= \frac{2x^{3/2}}{3bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^2(bc-ad)} + \frac{a^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{\ }}{\sqrt[4]{b}}}\right.}{2b^2(bc-}$$

$$= \frac{2x^{3/2}}{3bd} + \frac{a^{7/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}(bc-ad)} - \frac{a^{7/4}\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\,b^{7/4}(bc-}$$

$$= \frac{2x^{3/2}}{3bd} - \frac{a^{7/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}(bc-ad)} + \frac{a^{7/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}(bc-ad)} + \frac{c^{7/4}}{}$$

Mathematica [A]
time = 0.51, size = 249, normalized size = 0.52

$$\frac{-\frac{4ax^{3/2}}{b} + \frac{4cx^{3/2}}{d} - \frac{3\sqrt{2}\,a^{7/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{7/4}} + \frac{3\sqrt{2}\,c^{7/4}\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{d^{7/4}} - \frac{3\sqrt{2}\,a^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{7/4}} + \frac{3\sqrt{2}\,c^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{d^{7/4}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((-4*a*x^(3/2))/b + (4*c*x^(3/2))/d - (3*Sqrt[2]*a^(7/4)*ArcTan[(Sqrt[a] -
Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(7/4) + (3*Sqrt[2]*c^(7/4)
*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/d^(7/4) -
 (3*Sqrt[2]*a^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sq
rt[b]*x)])/b^(7/4) + (3*Sqrt[2]*c^(7/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(7/4))/(6*b*c - 6*a*d)
```

Maple [A]

time = 0.11, size = 249, normalized size = 0.52

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3bd}-\frac{a^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+$
default	$\frac{2x^{\frac{3}{2}}}{3bd}-\frac{a^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+$
risch	$\frac{2x^{\frac{3}{2}}}{3bd}-\frac{a^{2}\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{a^{2}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{a^{2}\sqrt{2}\,\arctan\left(\sqrt{2}\right.}{2b^{2}(ad-bc)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)/b/d-1/4*a^2/b^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)-1))+1/4*c^2/d^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]

time = 0.54, size = 390, normalized size = 0.82

$$\frac{a^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(b^{2}c-abd\right)}-\frac{c^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\left(bcd-ad^{2}\right)}+\frac{2x^{\frac{3}{2}}}{3bd}$$

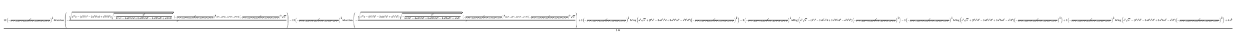
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b
^2*c - a*b*d) - 1/4*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt
(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*s
```

```
qrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*lo
g(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4))
+ sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1
/4)*d^(3/4)))/(b*c*d - a*d^2) + 2/3*x^(3/2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1422 vs. 2(340) = 680.
time = 1.81, size = 1422, normalized size = 2.97



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(12*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*
d^3 + a^4*b^7*d^4))^(1/4)*b*d*arctan(-(sqrt(a^10*x - (a^7*b^5*c^2 - 2*a^8*b
^4*c*d + a^9*b^3*d^2)*sqrt(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*
d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)))*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6
*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*(b^3*c - a*b^2*d)
- (a^5*b^3*c - a^6*b^2*d)*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*
d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*sqrt(x))/a^7) - 12*(-c^7/(b^4*c
^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^
(1/4)*b*d*arctan(-(sqrt(c^10*x - (b^2*c^9*d^3 - 2*a*b*c^8*d^4 + a^2*c^7*d^5
)*sqrt(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*
d^10 + a^4*d^11)))*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9
 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4)*(b*c*d^2 - a*d^3) - (b*c^6*d^2 - a*c^5
*d^3)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*
d^10 + a^4*d^11))^(1/4)*sqrt(x))/c^7) + 3*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d
+ 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqr
t(x) + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*(-a^7/(b^1
1*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)
)^(3/4)) - 3*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b
^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqrt(x) - (b^8*c^3 - 3*a*b^7*c^2
*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^
2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(3/4)) - 3*(-c^7/(b^4*c^4*d
^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4
)*b*d*log(c^5*sqrt(x) + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^
3*d^8)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c
*d^10 + a^4*d^11))^(3/4)) + 3*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*
b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4)*b*d*log(c^5*sqrt(x) - (b^3*
c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*(-c^7/(b^4*c^4*d^7 - 4
*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(3/4)) + 4
*x^(3/2))/(b*d)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.77, size = 476, normalized size = 1.00

$$\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{5}c-\sqrt{2}\,ab^{4}d}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{5}c-\sqrt{2}\,ab^{4}d}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} c \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^{4}-\sqrt{2}\,ad^{5}}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} c \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^{4}-\sqrt{2}\,ad^{5}}-\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{5}c-\sqrt{2}\,ab^{4}d\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{5}c-\sqrt{2}\,ab^{4}d\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}} c \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^{4}-\sqrt{2}\,ad^{5}\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} c \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^{4}-\sqrt{2}\,ad^{5}\right)}+\frac{2\,x^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^5*c - sqrt(2)*a*b^4*d) + (a*b^3)^(3/4)*a*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^5*c - sqrt(2)
*a*b^4*d) - (c*d^3)^(3/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqr
t(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) - (c*d^3)^(3/4)*c*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c
*d^4 - sqrt(2)*a*d^5) - 1/2*(a*b^3)^(3/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*b^5*c - sqrt(2)*a*b^4*d) + 1/2*(a*b^3)^(3/4)*a*l
og(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c - sqrt(2)*a
*b^4*d) + 1/2*(c*d^3)^(3/4)*c*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) - 1/2*(c*d^3)^(3/4)*c*log(-sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) + 2/3*
x^(3/2)/(b*d)
```

Mupad [B]
time = 1.08, size = 2500, normalized size = 5.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] atan(((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c
^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*
a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*
b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^
6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*
b^3*c^3*d^10))/(b^3*d^3) - (256*x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7
```

- 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*
b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*
d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^
3*d^3)) - (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3))*1i - (-c^7
/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64
*a^3*b*c*d^10))^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d
^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d
^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*
b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^1
0))/(b^3*d^3) + (256*x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3
*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^
5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a
^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^3*d^3)) + (
256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3))*1i)/((-c^7/(16*a^4*d^
11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^
10))^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2
*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4
*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7
 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^
3) - (256*x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 +
96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*
b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*
d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^3*d^3)) - (256*x^(1/2)
*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3)) + (-c^7/(16*a^4*d^11 + 16*b^4*c^
4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((-
c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 -
 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 +
 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*
c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) + (256*x^(1
/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2
*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 +
 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b
^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^3*d^3)) + (256*x^(1/2)*(a^5*b^5*c^10
 + a^10*c^5*d^5))/(b^3*d^3)) - (256*(a^7*b^2*c^9 + a^9*c^7*d^2 + a^8*b*c^8*
d))/(b^3*d^3)))*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96
*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*2i + 2*atan(((-c^7/(16*a^4*d^11
+ 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10)
)^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^
2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^
9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 +
48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3)
- (x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*
b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^
8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 6
4*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11)*256i)/(b^3*d^3))*1i + (256*x^(1/2

```
)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3)) - (-c^7/(16*a^4*d^11 + 16*b^4*c
^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((
-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9
- 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4
+ 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5
*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) + (x^(1/2)
*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^
9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 11
2*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*
c^4*d^10 + 16*a^9*b^5*c^3*d^11)*256i)/(b^3*d^3))*1i - (256*x^(1/2)*(a^5*b^5
*c^10 + a^10*c^5*d^5))/(b^3*d^3)))/((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 6
4*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((-c^7/(16*a
^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b
*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b
^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 -
 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b...
```

3.462 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=476

$$\frac{2\sqrt{x}}{bd}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5}}{}$$

```
[Out] -1/2*a^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)*2
^(1/2)+1/2*a^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)/(-a*d+
b*c)*2^(1/2)+1/2*c^(5/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(5/4)/
(-a*d+b*c)*2^(1/2)-1/2*c^(5/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^
(5/4)/(-a*d+b*c)*2^(1/2)-1/4*a^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/b^(5/4)/(-a*d+b*c)*2^(1/2)+1/4*a^(5/4)*ln(a^(1/2)+x*b^(1/2)
+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/(-a*d+b*c)*2^(1/2)+1/4*c^(5/4)*ln
(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(5/4)/(-a*d+b*c)*2^(1
/2)-1/4*c^(5/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(5/
4)/(-a*d+b*c)*2^(1/2)+2*x^(1/2)/b/d
```

Rubi [A]
time = 0.33, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 490, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{a^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{5/4}(bc-ad)}-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{c^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{5/4}(bc-ad)}+\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}+\frac{2\sqrt{x}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*Sqrt[x])/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^
(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2
]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(5/4)*(b*c - a*
d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(2*Sqrt[2]*d^(5/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx = 2\text{Subst}\left(\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\sqrt{x}}{bd} - \frac{2\text{Subst}\left(\int \frac{ac+(bc+ad)x^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{bd}$$

$$= \frac{2\sqrt{x}}{bd} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{d(bc-ad)}$$

$$= \frac{2\sqrt{x}}{bd} + \frac{a^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} + \frac{a^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x\right.}{b(bc-ad)}$$

$$= \frac{2\sqrt{x}}{bd} + \frac{a^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}(bc-ad)} + \frac{a^{3/2}\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{}}{}}\right.}{2b^{3/2}(b}$$

$$= \frac{2\sqrt{x}}{bd} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\,b^{5/4}(bc}$$

$$= \frac{2\sqrt{x}}{bd} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{c^{5/4}}{}$$

Mathematica [A]

time = 0.46, size = 247, normalized size = 0.52

$$\frac{-\frac{4a\sqrt{x}}{b}+\frac{4c\sqrt{x}}{d}-\frac{\sqrt{2}\,a^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{b^{5/4}}+\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{d^{5/4}}+\frac{\sqrt{2}\,a^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{5/4}}-\frac{\sqrt{2}\,c^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{d^{5/4}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((-4*a*Sqrt[x])/b + (4*c*Sqrt[x])/d - (Sqrt[2]*a^(5/4)*ArcTan[(Sqrt[a] - Sq
rt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])]])/b^(5/4) + (Sqrt[2]*c^(5/4)*Arc
Tan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]])/d^(5/4) + (Sq
rt[2]*a^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*
x)]])/b^(5/4) - (Sqrt[2]*c^(5/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(
Sqrt[c] + Sqrt[d]*x)]])/d^(5/4))/(2*b*c - 2*a*d)
```

Maple [A]
time = 0.11, size = 245, normalized size = 0.51

method	result
derivativedivides	$\frac{2\sqrt{x}}{bd}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.\right.}{4b(ad-bc)}$
default	$\frac{2\sqrt{x}}{bd}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.\right.}{4b(ad-bc)}$
risch	$\frac{2\sqrt{x}}{bd}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4b(ad-bc)}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b(ad-bc)}-a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{\ }$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b/d-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
+1/4/d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.57, size = 384, normalized size = 0.81

$$\frac{\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4\left(b^{2}c-abd\right)}-\frac{\frac{2\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,c^{\frac{5}{4}}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}\,c^{\frac{5}{4}}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{d^{\frac{1}{4}}}}{4\left(bcd-ad^{2}\right)}+\frac{2\sqrt{x}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/(b^2*c - a*b*d
) - 1/4*(2*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*c
^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sq
rt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + sqrt(2)*c^(5/4)*log(sqrt(2)*c^
(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4) - sqrt(2)*c^(5/4)*log(
-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4))/(b*c*d - a
*d^2) + 2*sqrt(x)/(b*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. $2(340) = 680$.
time = 0.69, size = 1388, normalized size = 2.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^
3 + a^4*b^5*d^4))^(1/4)*b*d*arctan(-((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c
*d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*
a^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt(a^2*x + (b^4*c^2 - 2*a*b^3*c*d + a
^2*b^2*d^2)*sqrt(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*
b^6*c*d^3 + a^4*b^5*d^4))) - (a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2
 - a^4*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*
b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt(x))/a^5) - 4*(-c^5/(b^4*c^4*d^5 - 4*a*
b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*arcta
n(-((b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^
4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/
4)*sqrt(c^2*x + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*sqrt(-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))) - (b^3
*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*(-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4)*s
qrt(x))/c^5) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b
^6*c*d^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) + (-a^5/(b^9*c^4 - 4*a*b^8
*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c -
 a*b*d)) + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c
```

```
*d^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3
*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b
*d)) + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c
*d^8 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3
*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2))
 - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8
 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6
 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - 4
*sqrt(x))/(b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.64, size = 476, normalized size = 1.00

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{3}c-\sqrt{2}\,ab^{2}d}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{3}c-\sqrt{2}\,ab^{2}d}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^{2}-\sqrt{2}\,ad^{3}}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} c \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^{2}-\sqrt{2}\,ad^{3}}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{3}c-\sqrt{2}\,ab^{2}d\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{3}c-\sqrt{2}\,ab^{2}d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} c \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^{2}-\sqrt{2}\,ad^{3}\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} c \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^{2}-\sqrt{2}\,ad^{3}\right)}+\frac{2\sqrt{x}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + (a*b^3)^(1/4)*a*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)
*a*b^2*d) - (c*d^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqr
t(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - (c*d^3)^(1/4)*c*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c
*d^2 - sqrt(2)*a*d^3) + 1/2*(a*b^3)^(1/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/2*(a*b^3)^(1/4)*a*l
og(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a
*b^2*d) - 1/2*(c*d^3)^(1/4)*c*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + 1/2*(c*d^3)^(1/4)*c*log(-sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + 2*sq
rt(x)/(b*d)
```

Mupad [**B**]
time = 0.86, size = 2500, normalized size = 5.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] atan(((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(
b*d) - (256*x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 +
 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b
^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 +
 16*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^
6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) - (256*x^(1/2)*(a^4*b
^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b
^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i - (((512*(a^3*b^6
*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (256*x^(1/2)*(
-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 -
 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b
^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9))
/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*
c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) + (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4)
)/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7
*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i)/((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 -
a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (256*x^(1/2)*(-a^5/(16*b^9*c^4 + 16
*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/
4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b
^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(16*b^9
*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^
3*d))^(1/4) - (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^
9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c
^3*d))^(1/4) + (((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^
4*d^5))/(b*d) + (256*x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^
6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 -
 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*
c^4*d^8 + 16*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 -
64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) + (256*x^(1/
2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 -
 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)))*(-a^5/(16
*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^
8*c^3*d))^(1/4)*2i - 2*atan(((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^
8*d - a^8*b*c^4*d^5))/(b*d) - (x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 -
 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9
*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 4
8*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-a^5/(16*b^9*c^4 + 16
*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/
4)*1i + (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4
+ 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))
^(1/4) - (((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5)
```

)/(b*d) + (x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i - (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4))/((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i + (((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i - (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) + atan(((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (256*x^(1/2)*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3...

3.463 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\tan^{-1}}{\sqrt{2}}$$

```
[Out] 1/2*a^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2^
(1/2)-1/2*a^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b
*c)*2^(1/2)-1/2*c^(3/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(3/4)/(
-a*d+b*c)*2^(1/2)+1/2*c^(3/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(
3/4)/(-a*d+b*c)*2^(1/2)-1/4*a^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^
(1/2)*x^(1/2))/b^(3/4)/(-a*d+b*c)*2^(1/2)+1/4*a^(3/4)*ln(a^(1/2)+x*b^(1/2)+
a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/(-a*d+b*c)*2^(1/2)+1/4*c^(3/4)*ln(
c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(3/4)/(-a*d+b*c)*2^(1/
2)-1/4*c^(3/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(3/4
)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {477, 492, 303, 1176, 631, 210, 1179, 642}

$$\frac{a^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b
*c - a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(3/4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d
)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(
b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx = 2\text{Subst}\left(\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{(2a)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{(2c)\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}$$

$$= \frac{a\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,(bc-ad)} - \frac{a\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,(bc-ad)} - \frac{c\text{Su}}{}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)} - \frac{a\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,d\right.}{2b(bc-ad)}$$

$$= -\frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,b^{3/4}(bc-ad)}$$

$$= \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)} - \frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)} - \frac{c^{3/4}\tan^{-1}\left(\right.}{\sqrt{2}\,d}$$

Mathematica [A]
time = 0.39, size = 219, normalized size = 0.47

$$\frac{a^{3/4}d^{3/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-b^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)+a^{3/4}d^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)-b^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,b^{3/4}d^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(3/4)*d^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])] - b^(3/4)*c^(3/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x])] + a^(3/4)*d^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(S
```

```
qrt[a] + Sqrt[b]*x)] - b^(3/4)*c^(3/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))
```

Maple [A]
time = 0.10, size = 234, normalized size = 0.51

method	result
derivativedivides	$\frac{a\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{c\sqrt{2}\,\left(\right.}{}$
default	$\frac{a\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{c\sqrt{2}\,\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-1/4*c/(a*d-b*
c)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x
+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 369, normalized size = 0.80

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4(bc-ad)}+\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4(bc-ad)}$$

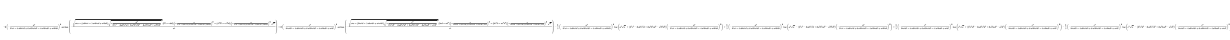
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*
c - a*d) + 1/4*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2
*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))
/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(
2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(
```

```
2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(
3/4)))/(b*c - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1385 vs. $2(329) = 658$.
time = 0.55, size = 1385, normalized size = 2.99



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -2*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a
^4*b^3*d^4))^(1/4)*arctan(-(sqrt(a^4*x - (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5
*b*d^2)*sqrt(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*
c*d^3 + a^4*b^3*d^4)))*(b^2*c - a*b*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a
^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4) - (a^2*b^2*c - a^3*b
*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 +
a^4*b^3*d^4))^(1/4)*sqrt(x))/a^3) + 2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4
+ 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*arctan(-(sqrt(c^4*x -
 (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*sqrt(-c^3/(b^4*c^4*d^3 - 4*a*b^3
*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7)))*(b*c*d - a*d^2)*(
-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a
^4*d^7))^(1/4) - (b*c^3*d - a*c^2*d^2)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4
 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*sqrt(x))/c^3) - 1/2*
(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*
b^3*d^4))^(1/4)*log(a^2*sqrt(x) + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^
2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3
*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/2*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*
a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*sqrt(x) - (
b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4
*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) +
 1/2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d
^6 + a^4*d^7))^(1/4)*log(c^2*sqrt(x) + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a
^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*
d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c
^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*sqrt(x
) - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^
4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/
4))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.12, size = 457, normalized size = 0.99

$$-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^4c-\sqrt{2}\,ab^3d}-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^4c-\sqrt{2}\,ab^3d}+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^3-\sqrt{2}\,ad^4}+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^3-\sqrt{2}\,ad^4}+\frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^4c-\sqrt{2}\,ab^3d\right)}-\frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^4c-\sqrt{2}\,ab^3d\right)}-\frac{(cd^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^3-\sqrt{2}\,ad^4\right)}+\frac{(cd^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^3-\sqrt{2}\,ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*
b^3*d) + (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + (c*d^3)^(3/4)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 -
sqrt(2)*a*d^4) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sq
rt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2*(a*b^3)^(3/4)*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1
/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*
b*c*d^3 - sqrt(2)*a*d^4) + 1/2*(c*d^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4)
```

Mupad [B]
time = 0.68, size = 2609, normalized size = 5.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - 2*atan((2*b^4*c^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4
*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4) + 64*a^4*b^4*d^7*x^(1/
2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d
^2 - 64*a*b^6*c^3*d))^(5/4) + 64*b^8*c^4*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16
*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/
4) + 2*a^3*b*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*
d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4) + 384*a^2*b^6*c^2*d^5*x^(
1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2
*d^2 - 64*a*b^6*c^3*d))^(5/4) - 256*a*b^7*c^3*d^4*x^(1/2)*(-a^3/(16*b^7*c^4
 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d)
)^(5/4) - 256*a^3*b^5*c*d^6*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64
*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4))/(a^3*d^2 + a*
b^2*c^2 + a^2*b*c*d))*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3
```

+ 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4) - atan((b^4*c^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4)*2i + a^4*b^4*d^7*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*64i + b^8*c^4*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*64i + a^3*b*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4)*2i + a^2*b^6*c^2*d^5*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*384i - a*b^7*c^3*d^4*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*256i - a^3*b^5*c*d^6*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*256i)/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4)*2i - 2*atan((2*a^3*d^4*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4) + 2*b^3*c^3*d*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4) + 64*a^4*b^3*d^8*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4) + 64*b^7*c^4*d^4*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4) + 384*a^2*b^5*c^2*d^6*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4) - 256*a*b^6*c^3*d^5*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4) - 256*a^3*b^4*c*d^7*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4))/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4) - atan((a^3*d^4*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4)*2i + b^3*c^3*d*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4)*2i + a^4*b^3*d^8*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4)*64i + b^7*c^4*d^4*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4)*64i + a^2*b^5*c^2*d^6*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4)*384i - a*b^6*c^3*d^5*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4)*256i - a^3*b^4*c*d^7*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4)*256i)/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4)*2i

3.464 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\tan}{}$$

```
[Out] 1/2*a^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2^
(1/2)-1/2*a^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b
*c)*2^(1/2)-1/2*c^(1/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(1/4)/(
-a*d+b*c)*2^(1/2)+1/2*c^(1/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(
1/4)/(-a*d+b*c)*2^(1/2)+1/4*a^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^
(1/2)*x^(1/2))/b^(1/4)/(-a*d+b*c)*2^(1/2)-1/4*a^(1/4)*ln(a^(1/2)+x*b^(1/2)+
a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/(-a*d+b*c)*2^(1/2)-1/4*c^(1/4)*ln(
c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(1/4)/(-a*d+b*c)*2^(1/
2)+1/4*c^(1/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(1/4
)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 492, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b
*c - a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d
)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(
b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx = 2\text{Subst}\left(\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{(2a)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{(2c)\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}$$
$$= -\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{bc-ad}$$
$$= -\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\,(bc-ad)} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}\right.}{2\sqrt{b}\,(bc-}$$
$$= \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)}$$
$$= \frac{\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{a}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{c}\,\tan^{-1}}{\sqrt{2}}$$

Mathematica [A]
time = 0.35, size = 219, normalized size = 0.47

$$\frac{\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)-\sqrt[4]{a}\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)+\sqrt[4]{b}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*d^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])] - b^(1/4)*c^(1/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x])] - a^(1/4)*d^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(S
```

```
qrt[a] + Sqrt[b]*x)] + b^(1/4)*c^(1/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))
```

Maple [A]
time = 0.09, size = 226, normalized size = 0.49

method	result
derivativedivides	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ad-4bc} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}{}$
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ad-4bc} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^
(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-1/4/(a*d-b*c)*(c/
d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+
2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 367, normalized size = 0.79

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4(bc-ad)}+\frac{\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\sqrt{c}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}}{4(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt
(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/(b*c - a*d) +
 1/4*(2*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqr
t(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*sqrt
(c)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + sqrt(2)*c^(1/4)*log(sqrt(2)*c^(1/
4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4) - sqrt(2)*c^(1/4)*log(-sq
rt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4))/(b*c - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1249 vs. $2(329) = 658$.
time = 0.50, size = 1249, normalized size = 2.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4))^(1/4)*arctan(-((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d
^3)*sqrt((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)) + x)*(-a/(b^5*c^4 - 4*a*
b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4) - (b^4*
c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(x)*(-a/(b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4))/a)
- 2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 +
a^4*d^5))^(1/4)*arctan(-((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3
*d^4)*sqrt((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)) + x)*(-c/(b^4*c^4*d -
4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4) - (b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)*(-c/(b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4))/c
) - 1/2*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3
+ a^4*b*d^4))^(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-a/(b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4
)*log(-(b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-c/(b^4*c^4*d - 4*a*b^3*c^
3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)
*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4
*d^5))^(1/4) + sqrt(x)) - 1/2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*
c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d -
 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + sq
rt(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.44, size = 441, normalized size = 0.95

$$-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^2c-\sqrt{2}\,abd}-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^2c-\sqrt{2}\,abd}+\frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd-\sqrt{2}\,ad^2}+\frac{(cd^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd-\sqrt{2}\,ad^2}-\frac{(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}+\frac{(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}+\frac{(cd^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}-\frac{(cd^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) - (a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*
d) + (c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) + (c*d^3)^(1/4)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*
a*d^2) - 1/2*(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/2*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(
a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/2*(c*d^3)^(
1/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt
(2)*a*d^2) - 1/2*(c*d^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2)
```

Mupad [B]
time = 0.77, size = 2500, normalized size = 5.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] 2*atan(-((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2
*d^2 - 64*a*b^4*c^3*d))^(1/4)*((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*
c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*b^9
*c^7*d^4 - 12288*a^3*b^8*c^6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c^4*
d^7 - 12288*a^6*b^5*c^3*d^8 + 4096*a^7*b^4*c^2*d^9) - (-a/(16*b^5*c^4 + 16*
a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*
(8192*a^2*b^10*c^8*d^4 - 49152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6 - 1
63840*a^5*b^7*c^5*d^7 + 122880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 + 81
92*a^8*b^4*c^2*d^10)*1i)*(-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3
+ 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(3/4)*1i + 512*a^2*b^6*c^5*d^3 - 51
2*a^3*b^5*c^4*d^4 - 512*a^4*b^4*c^3*d^5 + 512*a^5*b^3*c^2*d^6)*1i - x^(1/2)
*(256*a^2*b^5*c^4*d^3 + 256*a^4*b^3*c^2*d^5)) + (-a/(16*b^5*c^4 + 16*a^4*b*
d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((-a/(
16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^
4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*b^9*c^7*d^4 - 12288*a^3*b^8*c^6*d^5 + 8
192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c^4*d^7 - 12288*a^6*b^5*c^3*d^8 + 4096*a
```

^7*b^4*c^2*d^9) + (-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^
2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*(8192*a^2*b^10*c^8*d^4 - 49152*a^3*b
^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6 - 163840*a^5*b^7*c^5*d^7 + 122880*a^6*b
^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 + 8192*a^8*b^4*c^2*d^10)*1i)*(-a/(16*b^5
*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*
d))^(3/4)*1i - 512*a^2*b^6*c^5*d^3 + 512*a^3*b^5*c^4*d^4 + 512*a^4*b^4*c^3*
d^5 - 512*a^5*b^3*c^2*d^6)*1i - x^(1/2)*(256*a^2*b^5*c^4*d^3 + 256*a^4*b^3*
c^2*d^5)))/((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*
c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b
^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*
b^9*c^7*d^4 - 12288*a^3*b^8*c^6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c
^4*d^7 - 12288*a^6*b^5*c^3*d^8 + 4096*a^7*b^4*c^2*d^9) - (-a/(16*b^5*c^4 +
16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/
4)*(8192*a^2*b^10*c^8*d^4 - 49152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6
- 163840*a^5*b^7*c^5*d^7 + 122880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 +
8192*a^8*b^4*c^2*d^10)*1i)*(-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d
^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(3/4)*1i + 512*a^2*b^6*c^5*d^3 -
512*a^3*b^5*c^4*d^4 - 512*a^4*b^4*c^3*d^5 + 512*a^5*b^3*c^2*d^6)*1i - x^(1
/2)*(256*a^2*b^5*c^4*d^3 + 256*a^4*b^3*c^2*d^5))*1i - (-a/(16*b^5*c^4 + 16*
a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*
((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 6
4*a*b^4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*b^9*c^7*d^4 - 12288*a^3*b^8*c^6*d
^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c^4*d^7 - 12288*a^6*b^5*c^3*d^8 +
4096*a^7*b^4*c^2*d^9) + (-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 +
96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*(8192*a^2*b^10*c^8*d^4 - 49152
*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6 - 163840*a^5*b^7*c^5*d^7 + 122880
*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 + 8192*a^8*b^4*c^2*d^10)*1i)*(-a/(
16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^
4*c^3*d))^(3/4)*1i - 512*a^2*b^6*c^5*d^3 + 512*a^3*b^5*c^4*d^4 + 512*a^4*b^
4*c^3*d^5 - 512*a^5*b^3*c^2*d^6)*1i - x^(1/2)*(256*a^2*b^5*c^4*d^3 + 256*a^
4*b^3*c^2*d^5))*1i))*(-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96
*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4) - atan((a^2*d^2*x^(1/2)*1i + b^2*
c^2*x^(1/2)*1i - (b^6*c^6*d*x^(1/2)*16i)/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*
b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4) - (a^2*b^4*c^4*d^3*x^(1/
2)*32i)/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3
- 64*a^3*b*c*d^4) - (a^3*b^3*c^3*d^4*x^(1/2)*32i)/(16*a^4*d^5 + 16*b^4*c^4*
d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4) + (a^4*b^2*c^2*
d^5*x^(1/2)*48i)/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2
*c^2*d^3 - 64*a^3*b*c*d^4) - (a^5*b*c*d^6*x^(1/2)*16i)/(16*a^4*d^5 + 16*b^4
*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4) + (a*b^5*c
^5*d^2*x^(1/2)*48i)/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*
b^2*c^2*d^3 - 64*a^3*b*c*d^4))/((-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c
^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*((c*(32*a^6*b*d^7 + 32
*b^7*c^6*d - 192*a*b^6*c^5*d^2 - 192*a^5*b^2*c*d^6 + 480*a^2*b^5*c^4*d^3 -
640*a^3*b^4*c^3*d^4 + 480*a^4*b^3*c^2*d^5))/(16*a^4*d^5 + 16*b^4*c^4*d - 64

```
*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4) - 2*b^3*c^3 - 2*a^3*d
^3 + 2*a*b^2*c^2*d + 2*a^2*b*c*d^2)))*(-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a
*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*2i - atan((a^2*d
^2*x^(1/2)*1i + b^2*c^2*x^(1/2)*1i - (a^6*b*d^6*x^(1/2)*16i)/(16*b^5*c^4 +
16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d) + (a
^2*b^5*c^4*d^2*x^(1/2)*48i)/(16*b^5*c^4 + 16*a^...
```

3.465 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\ \mathrm{t}}{}$$

```
[Out] -1/2*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*2
^(1/2)+1/2*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+
b*c)*2^(1/2)+1/2*d^(1/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/
(-a*d+b*c)*2^(1/2)-1/2*d^(1/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^
(1/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(1/4)*ln(a^(1/2)+x*b^(1/2)
+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(1/4)*ln
(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/(-a*d+b*c)*2^(1
/2)+1/4*d^(1/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/
4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 493, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{b}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*
(b*c - a*d))) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(1/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c -
a*d)) - (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4
)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx = 2\text{Subst}\left(\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$
$$= \frac{(2b)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} - \frac{(2d)\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}$$
$$= -\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{bc-ad}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x\right.}{2(bc-ad)}$$
$$= \frac{\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc-ad)} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\sqrt[4]{a}\,(bc-ad)}$$
$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{d}\,\tan}{\sqrt{}}$$

Mathematica [A]
time = 0.41, size = 219, normalized size = 0.47

$$\frac{-\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)-\sqrt[4]{b}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)+\sqrt[4]{a}\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-(b^(1/4)*c^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])]) + a^(1/4)*d^(1/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x])] - b^(1/4)*c^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)] + a^(1/4)*d^(1/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*a^(1/4)*c^(1/4)*(b*c - a*d))
```

Maple [A]

time = 0.09, size = 226, normalized size = 0.49

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\left(\ln\right.}{}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\left(\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)
^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+1/4/(a*d-b*c)/(c
/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]

time = 0.52, size = 369, normalized size = 0.80

$$\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4(bc-ad)}-\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c
 - a*d) - 1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/
sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2
)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2
)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3
/4)))/(b*c - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1285 vs. $2(329) = 658$.
time = 0.51, size = 1285, normalized size = 2.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^
5*d^4))^(1/4)*arctan(-(sqrt(b^2*x - (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)
*sqrt(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4)))*(b*c - a*d)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^
2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4) - (b^2*c - a*b*d)*sqrt(x)*(-b/(a*b^4*c^
4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4))/
b) - 2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 +
 a^4*c*d^4))^(1/4)*arctan(-(sqrt(d^2*x - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c
*d^3)*sqrt(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^
3 + a^4*c*d^4)))*(b*c - a*d)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d
^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4) - (b*c*d - a*d^2)*sqrt(x)*(-d/(b^4
*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1
/4))/d) + 1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*
b*c*d^3 + a^5*d^4))^(1/4)*log((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
- a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c
*d^3 + a^5*d^4))^(3/4) + b*sqrt(x)) - 1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d
+ 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(-(a*b^3*c^3 - 3*a
^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d +
6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4) + b*sqrt(x)) - 1/2*(-d/
(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)
)^(1/4)*log((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^
4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(
3/4) + d*sqrt(x)) + 1/2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 -
4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(-(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b
*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*
a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4) + d*sqrt(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.58, size = 481, normalized size = 1.04

$$\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d}+\frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d}-\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3}-\frac{(cd^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3}-\frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}+\frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}+\frac{(cd^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}-\frac{(cd^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1
/4))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + (a*b^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c - sqrt(
2)*a^2*b^2*d) - (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*s
qrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - (c*d^3)^(3/4)*
arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)
*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2*(a*b^3)
^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c -
 sqrt(2)*a^2*b^2*d) + 1/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(c*d^3)^(3/4)*log
(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)
*a*c*d^3)
```

Mupad [B]
time = 0.64, size = 2500, normalized size = 5.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] atan((((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d
^2 - 64*a^4*b*c*d^3))^(3/4)*(x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^
2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*a*b^10*c^7*
d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 -
 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9) + 2
048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^
7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) + x^(1/2)*(256*a*b
^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^
3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*1i - ((-b/(16*a^5*d^4
 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^
(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2
*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*a*b^10*c^7*d
^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 -
32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9) + 20
48*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b
```

```
^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) - x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b
^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c
^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*1i)/(((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a
^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(3/4)*(x^(1/2)*(-b/(16
*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*
c*d^3))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^
6*d^5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d
^8 - 16384*a^6*b^5*c^2*d^9) + 2048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 614
4*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*
b^5*c^2*d^8) + x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5
*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^
3))^(1/4) + ((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2
*c^2*d^2 - 64*a^4*b*c*d^3))^(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(-b/(16*a^5
*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^
3))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^
5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 -
 16384*a^6*b^5*c^2*d^9) + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*
a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) - x^(1/2)*(2
56*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*
a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4) + 256*a*b^5*c*d
^5))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2
 - 64*a^4*b*c*d^3))^(1/4)*2i - 2*atan((((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64
*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(3/4)*(2048*a*b^9*c^
6*d^4 - x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*
b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*
d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^
7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9)*1i + 2048*a^6*b^4*c*d^9
- 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144
*a^5*b^5*c^2*d^8)*1i + x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b
/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^
4*b*c*d^3))^(1/4) - ((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96
*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(3/4)*(x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b
^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(40
96*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3
*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^
5*c^2*d^9)*1i + 2048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*
d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8)*1
i - x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a
*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4))/
(((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 -
64*a^4*b*c*d^3))^(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(-b/(16*a^5*d^4 + 16*a
*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(
4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a
^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*
b^5*c^2*d^9)*1i + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*
```

c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8)*1i + x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*1i + ((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(3/4)*(x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^...

3.466 $\int \frac{1}{\sqrt{x}\,(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\,\mathrm{ta}}{}$$

```
[Out] -1/2*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2
^(1/2)+1/2*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-a*d+
b*c)*2^(1/2)+1/2*d^(3/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/
(-a*d+b*c)*2^(1/2)-1/2*d^(3/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^
(3/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(3/4)*ln(a^(1/2)+x*b^(1/2)
+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/4*d^(3/4)*ln
(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1
/2)-1/4*d^(3/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/
4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 400, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{3/4}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}(bc-ad)}-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{d^{3/4}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*
(b*c - a*d))) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c -
a*d)) + (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4
)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)(c+dx^2)}\,dx = 2\text{Subst}\left(\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(2b)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} - \frac{(2d)\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}$$

$$= \frac{b\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\,(bc-ad)} + \frac{b\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\,(bc-ad)}$$

$$= \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\,(bc-ad)} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}}{}}\right.}{2\sqrt{a}\,(bc}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\,a^{3/4}(bc}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3/4}}{}$$

Mathematica [A]
time = 0.40, size = 219, normalized size = 0.47

$$\frac{-b^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+a^{3/4}d^{3/4}\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)+b^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)-a^{3/4}d^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,a^{3/4}c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-(b^(3/4)*c^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])]) + a^(3/4)*d^(3/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x])] + b^(3/4)*c^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)] - a^(3/4)*d^(3/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

Maple [A]
time = 0.11, size = 234, normalized size = 0.51

method	result
derivativedivides	$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)a}+ -$
default	$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)a}+ -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+1/4*d/(a*d-b
*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 371, normalized size = 0.80

$$\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4(bc-ad)}-\frac{\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,d^{\frac{3}{4}}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}\,d^{\frac{3}{4}}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{4(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(b*c -
 a*d) - 1/4*(2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sq
rt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*s
qrt(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/
sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*lo
g(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/c^(3/4) - sqrt(2)*
d^(3/4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/c^(3/4)
)/(b*c - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(329) = 658$.
time = 0.75, size = 1365, normalized size = 2.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3
 + a^7*d^4))^(1/4)*arctan(-((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*sqrt(b^2*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^3/(a^
3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))
)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3
+ a^7*d^4))^(3/4) - (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*
b*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c
*d^3 + a^7*d^4))^(3/4)*sqrt(x))/b^3) + 2*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(-((b^3*c^5
- 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(d^2*x + (b^2*c^4 - 2*
a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5
*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*
a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4) - (b^3*c^5*d - 3*a*
b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
 + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*sqrt(x))/d^3)
+ 1/2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*
d^3 + a^7*d^4))^(1/4)*log(b*sqrt(x) + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 -
4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/
2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3
+ a^7*d^4))^(1/4)*log(b*sqrt(x) - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^
4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/2*(-
d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^
3*d^4))^(1/4)*log(d*sqrt(x) + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*
d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/2*(-d^3/
(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^
4))^(1/4)*log(d*sqrt(x) - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.46, size = 441, normalized size = 0.95

$$\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,abc-\sqrt{2}\,a^2d}+\frac{(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,abc-\sqrt{2}\,a^2d}-\frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^2-\sqrt{2}\,acd}-\frac{(cd^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^2-\sqrt{2}\,acd}+\frac{(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,abc-\sqrt{2}\,a^2d\right)}-\frac{(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,abc-\sqrt{2}\,a^2d\right)}-\frac{(cd^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^2-\sqrt{2}\,acd\right)}+\frac{(cd^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^2-\sqrt{2}\,acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1
/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + (a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d
) - (c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - (c*d^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a
*c*d) + 1/2*(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/2*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/2*(c*d^3)^(1
/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(
2)*a*c*d) + 1/2*(c*d^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d)
```

Mupad [B]
time = 0.88, size = 2500, normalized size = 5.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - atan(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*
c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a
^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*
c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6
*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*
d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8
 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) + x^(1/2)*(4096*a^7*b^4*
d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24
576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576
*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 +
96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4) - 512*a^2*b^6*d^8 - 512*b^8*c^2
*d^6 + 1024*a*b^7*c*d^7) + 512*b^7*d^7*x^(1/2))*1i - (-d^3/(16*b^4*c^7 + 16
*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/
4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*
d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*
```

b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) - x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4) - 512*a^2*b^6*d^8 - 512*b^8*c^2*d^6 + 1024*a*b^7*c*d^7) - 512*b^7*d^7*x^(1/2))*1i)/((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) + x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4) - 512*a^2*b^6*d^8 - 512*b^8*c^2*d^6 + 1024*a*b^7*c*d^7) + 512*b^7*d^7*x^(1/2)) + (-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) - x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4) - 512*a^2*b^6*d^8 - 512*b^8*c^2*d^6 + 1024*a*b^7*c*d^7) - 512*b^7*d^7*x^(1/2))))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*2i - 2*atan(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10)*1i + x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4)*1i + 512*a^2*b^6*d^8 + 512*b^8*c^2*d^6 - 1024*a*b^7*c*d^7)*1i - 512*b^7*d^7*x^(1/2)) - (-d^3/(16*b^4*c^7 + 16*a^4*c^

```
3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d
^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 6
4*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d
^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 819
2*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a
^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*
b^5*c^2*d^10)*1i - x^(1/2)*(4096*a^7*b^4*d^11 +...
```

3.467 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=476

$$-\frac{2}{ac\sqrt{x}}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}(bc-ad)}$$

```
[Out] 1/2*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)*2^
(1/2)-1/2*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b
*c)*2^(1/2)-1/2*d^(5/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/(
-a*d+b*c)*2^(1/2)+1/2*d^(5/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(
5/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^
(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)+
a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)*2^(1/2)+1/4*d^(5/4)*ln(
c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a*d+b*c)*2^(1/
2)-1/4*d^(5/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4
)/(-a*d+b*c)*2^(1/2)-2/a/c/x^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 491, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{d^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}(bc-ad)}+\frac{d^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{5/4}(bc-ad)}+\frac{d^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{d^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{2}{ac\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(a*c*Sqrt[x]) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(
5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d
)) - (d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
2*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)}\,dx = 2\text{Subst}\left(\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{ac\sqrt{x}} + \frac{2\text{Subst}\left(\int \frac{x^2(-bc-ad-bdx^4)}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{ac}$$

$$= -\frac{2}{ac\sqrt{x}} + \frac{2\text{Subst}\left(\int \left(-\frac{b^2cx^2}{(bc-ad)(a+bx^4)} - \frac{ad^2x^2}{(-bc+ad)(c+dx^4)}\right)dx, x, \sqrt{x}\right)}{ac}$$

$$= -\frac{2}{ac\sqrt{x}} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{\right.}{c(bc-ad)}$$

$$= -\frac{2}{ac\sqrt{x}} + \frac{b^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)} - \frac{b^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\right.}{a(bc-ad}$$

$$= -\frac{2}{ac\sqrt{x}} - \frac{b\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)} - \frac{b\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+\right.}{2a}$$

$$= -\frac{2}{ac\sqrt{x}} - \frac{b^{5/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)} + \frac{b^{5/4}\log\left(\sqrt{a} + \sqrt{\right.}{2\sqrt{2}\,a}$$

$$= -\frac{2}{ac\sqrt{x}} + \frac{b^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)} - \frac{b^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)}$$

<u>Mathematica</u> [A]
time = 0.53, size = 247, normalized size = 0.52

$$\frac{\frac{4b}{a\sqrt{x}} - \frac{4d}{c\sqrt{x}} - \frac{\sqrt{2}\,b^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{5/4}} + \frac{\sqrt{2}\,d^{5/4}\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{5/4}} - \frac{\sqrt{2}\,b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{5/4}} + \frac{\sqrt{2}\,d^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{5/4}}}{-2bc+2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((4*b)/(a*Sqrt[x]) - (4*d)/(c*Sqrt[x]) - (Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[a] -
 Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(5/4) + (Sqrt[2]*d^(5/4)*
ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(5/4) -
(Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[
b]*x)])/a^(5/4) + (Sqrt[2]*d^(5/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
)/(Sqrt[c] + Sqrt[d]*x)])/c^(5/4))/(-2*b*c + 2*a*d)
```

Maple [A]
time = 0.11, size = 245, normalized size = 0.51

method	result
derivativedivides	$\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{d\sqrt{2}\,(}{}$
default	$\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{d\sqrt{2}\,(}{}$
risch	$-\frac{2}{ac\sqrt{x}}+\frac{b\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4a(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan\left(\right.}{2a(ad-bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-1/4*d/c/(a*d-
b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x
+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))-2/a/c/x^(1/2)
```

Maxima [A]
time = 0.52, size = 390, normalized size = 0.82

$$-\frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}\right)}{4\left(abc-a^2d\right)}+\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac14}d^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac14}d^{\frac34}}\right)}{4\left(bc^2-acd\right)}-\frac{2}{ac\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(
a*b*c - a^2*d) + 1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*
sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*l
```

```
og(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4))
 + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(
1/4)*d^(3/4)))/(b*c^2 - a*c*d) - 2/(a*c*sqrt(x))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. $2(340) = 680$.
time = 0.89, size = 1421, normalized size = 2.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(1/4)*a*c*x*arctan(-(sqrt(b^8*x - (a^3*b^7*c^2 - 2*a^4*b^
6*c*d + a^5*b^5*d^2)*sqrt(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c
^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4)))*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d +
6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*(a*b*c - a^2*d) - (a*b^
5*c - a^2*b^4*d)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*sqrt(x))/b^5) - 4*(-d^5/(b^4*c^9 - 4*a*b^3
*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*ar
ctan(-(sqrt(d^8*x - (b^2*c^5*d^5 - 2*a*b*c^4*d^6 + a^2*c^3*d^7)*sqrt(-d^5/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
)))*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
a^4*c^5*d^4))^(1/4)*(b*c^2 - a*c*d) - (b*c^2*d^4 - a*c*d^5)*(-d^5/(b^4*c^9
- 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)
*sqrt(x))/d^5) + (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*sqrt(x) + (a^4*b^3*c^3 - 3*a
^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*
d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c
^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*
a*c*x*log(b^4*sqrt(x) - (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^
7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c
*d^3 + a^9*d^4))^(3/4)) - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^
2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*sqrt(x) + (b^3*c^7
- 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c
^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + (-d^5/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
))^(1/4)*a*c*x*log(d^4*sqrt(x) - (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2
 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*
b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + 4*sqrt(x))/(a*c*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.37, size = 492, normalized size = 1.03

$$-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^2c-\sqrt{2}\,a^3bd}-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^2c-\sqrt{2}\,a^3bd}+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^3d-\sqrt{2}\,ac^2d^2}+\frac{(cd^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^3d-\sqrt{2}\,ac^2d^2}+\frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2b^2c-\sqrt{2}\,a^3bd\right)}-\frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2b^2c-\sqrt{2}\,a^3bd\right)}-\frac{(cd^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^3d-\sqrt{2}\,ac^2d^2\right)}+\frac{(cd^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^3d-\sqrt{2}\,ac^2d^2\right)}-\frac{2}{ac\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - (a*b^3)^(3/4)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c - sq
rt(2)*a^3*b*d) + (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*
sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + (c*d^3)^(3/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2
)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/2*(a*b^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*
c - sqrt(2)*a^3*b*d) - 1/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
x + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(c*d^3)^(3/4)*lo
g(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*
a*c^2*d^2) - 2/(a*c*sqrt(x))
```

Mupad [B]
time = 0.90, size = 2500, normalized size = 5.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] atan((a^6*b^8*c^9*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c
^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*32i + a^6*b^4*d^5*x^(1/2
)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^
2 - 64*a^8*b*c*d^3))^(1/4)*2i + a^14*c*d^8*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a
^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)
*32i + a^8*b^6*c^7*d^2*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*
b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*192i - a^9*b^5*c^6*
d^3*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*
b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*128i + a^10*b^4*c^5*d^4*x^(1/2)*(-b^5/
(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a
```

^8*b*c*d^3))^(5/4)*64i - a^11*b^3*c^4*d^5*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*128i + a^12*b^2*c^3*d^6*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*192i + a^5*b^5*c*d^4*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*2i - a^7*b^7*c^8*d*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*128i - a^13*b*c^2*d^7*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*128i)/(b^9*c^4 + a^4*b^5*d^4 + a^3*b^6*c*d^3 + a^2*b^7*c^2*d^2 + a*b^8*c^3*d))*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*2i + atan((a^9*c^6*d^8*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*32i + b^5*c^6*d^4*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*2i + a*b^8*c^14*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*32i + a^3*b^6*c^12*d^2*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*192i - a^4*b^5*c^11*d^3*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*128i + a^5*b^4*c^10*d^4*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*64i - a^6*b^3*c^9*d^5*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*128i + a^7*b^2*c^8*d^6*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*192i + a*b^4*c^5*d^5*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*2i - a^2*b^7*c^13*d*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*128i - a^8*b*c^7*d^7*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*128i)/(a^4*d^9 + b^4*c^4*d^5 + a*b^3*c^3*d^6 + a^2*b^2*c^2*d^7 + a^3*b*c*d^8))*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*2i + 2*atan(((-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12*b^8*c^11*d^9) - (-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(3/4)*(x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*(4096*a^12*b^12*c^20*d^4 - 16384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*c^18*d^6 - 16384*a^15*b^9*c^17*d^7 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^7*c^15*d^9 + 24576*a^18*b^6*c^14*d^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^20*b^4*c^12*d^12)*1i - 2048*a^11*b^12*c^19*d^4 + 6144*a^12*b^11*c^18*d^5 - 6144*a^13*b^10*c^17*d^6 + 2048*a^14*b^9*c^16*d^7 + 2048*a^16*b^7*c^14*d^9 - 6144*a^17*b^6*c^13*d^10 + 6144*a^18*b^5*c^12*d^11 - 2048*a^19*b^4*c^11*d^12)*1i) + (-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3)

```
)^(1/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12*b^8*c^11*d^9) - (-b^5/(1
6*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8
*b*c*d^3))^(3/4)*(x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c
^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*(4096*a^12*b^12*c^20*d^4
 - 16384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*c^18*d^6 - 16384*a^15*b^9*c^1
7*d^7 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^7*c^15*d^9 + 24576*a^18*b^6*c
^14*d^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^20*b^4*c^12*d^12)*1i + 2048*a^
11*b^12*c^19*d^4 - 6144*a^12*b^11*c^18*d^5 + 6144*a^13*b^10*c^17*d^6 - 2048
*a^14*b^9*c^16*d^7 - 2048*a^16*b^7*c^14*d^9 + 6144*a^17*b^6*c^13*d^10 - 614
4*a^18*b^5*c^12*d^11 + 2048*a^19*b^4*c^11*d^12)*1i))/((-b^5/(16*a^9*d^4 + 1
6*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1
/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12*...
```

3.468 $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=478

$$-\frac{2}{3acx^{3/2}}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}(bc-ad)}$$

```
[Out] -2/3/a/c/x^(3/2)+1/2*b^(7/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7
/4)/(-a*d+b*c)*2^(1/2)-1/2*b^(7/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4)
)/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/2*d^(7/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c
^(1/4))/c^(7/4)/(-a*d+b*c)*2^(1/2)+1/2*d^(7/4)*arctan(1+d^(1/4)*2^(1/2)*x^(
1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(7/4)*ln(a^(1/2)+x*b^(1/2)-a
^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(7/4)*ln(a
^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(7/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)
/(-a*d+b*c)*2^(1/2)+1/4*d^(7/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2
)*x^(1/2))/c^(7/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 491, 536, 217, 1179, 642, 1176, 631, 210}

$$\frac{b^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}(bc-ad)}+\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{7/4}(bc-ad)}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}(bc-ad)}-\frac{2}{3acx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(3*a*c*x^(3/2)) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b
^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*(b*c - a
*d)) + (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(2*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx = 2\text{Subst}\left(\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{3acx^{3/2}} + \frac{2\text{Subst}\left(\int \frac{-3(bc+ad)-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{3ac}$$

$$= -\frac{2}{3acx^{3/2}} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{\right.}{c(bc-ad)}$$

$$= -\frac{2}{3acx^{3/2}} - \frac{b^2\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}(bc-ad)} - \frac{b^2\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\right.}{a^{3/2}(bc-ad}$$

$$= -\frac{2}{3acx^{3/2}} - \frac{b^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}(bc-ad)} - \frac{b^{3/2}\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2}{3acx^{3/2}} + \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)} - \frac{b^{7/4}\log\left(\sqrt{a}+\right.}{2\sqrt{2}}$$

$$= -\frac{2}{3acx^{3/2}} + \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)} - \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{\sqrt{2}\,a^{7/4}(bc-ad)}$$

Mathematica [A]

time = 0.53, size = 249, normalized size = 0.52

$$\frac{\frac{4b}{ax^{3/2}}-\frac{4d}{cx^{3/2}}-\frac{3\sqrt{2}\,b^{7/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{7/4}}+\frac{3\sqrt{2}\,d^{7/4}\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{7/4}}+\frac{3\sqrt{2}\,b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{7/4}}-\frac{3\sqrt{2}\,d^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{7/4}}}{-6bc+6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((4*b)/(a*x^(3/2)) - (4*d)/(c*x^(3/2)) - (3*Sqrt[2]*b^(7/4)*ArcTan[(Sqrt[a]
 - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(7/4) + (3*Sqrt[2]*d^(7
/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(7/4
) + (3*Sqrt[2]*b^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
 Sqrt[b]*x)])/a^(7/4) - (3*Sqrt[2]*d^(7/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(7/4))/(-6*b*c + 6*a*d)
```

Maple [A]
time = 0.11, size = 249, normalized size = 0.52

method	result
derivativedivides	$\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2(ad-bc)}-$
default	$\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2(ad-bc)}-$
risch	$-\frac{2}{3ac\,x^{\frac{3}{2}}}+\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4a^2(ad-bc)}+\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^2(ad-bc)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/3/a/c/x
^(3/2)-1/4/c^2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.53, size = 396, normalized size = 0.83

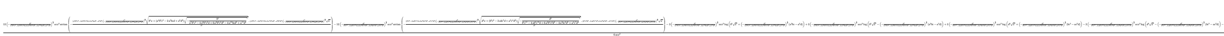
$$-\frac{\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\left(abc-a^2d\right)}+\frac{\frac{2\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,d^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,d^{\frac{7}{4}}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}\,d^{\frac{7}{4}}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{4\left(bc^2-acd\right)}-\frac{2}{3\,acx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2
)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(
7/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(
a*b*c - a^2*d) + 1/4*(2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt
(d))) + 2*sqrt(2)*d^2*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt
(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(
2)*d^(7/4)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/c^(3/
4) - sqrt(2)*d^(7/4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqr
t(c))/c^(3/4))/(b*c^2 - a*c*d) - 2/3/(a*c*x^(3/2))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1431 vs. $2(340) = 680$.
time = 3.53, size = 1431, normalized size = 2.99



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(12*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b
*c*d^3 + a^11*d^4))^(1/4)*a*c*x^2*arctan(-((a^5*b^3*c^3 - 3*a^6*b^2*c^2*d +
 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*
c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt(b^4*x + (a^4*b^2*c^2 - 2*a
^5*b*c*d + a^6*d^2)*sqrt(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^
2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))) - (a^5*b^5*c^3 - 3*a^6*b^4*c^2*d + 3*a
^7*b^3*c*d^2 - a^8*b^2*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^
2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt(x))/b^7) - 12*(-d^7/(b^4
*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4)
)^(1/4)*a*c*x^2*arctan(-((b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c
^5*d^3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(3/4)*sqrt(d^4*x + (b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2
)*sqrt(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^
3 + a^4*c^7*d^4))) - (b^3*c^8*d^2 - 3*a*b^2*c^7*d^3 + 3*a^2*b*c^6*d^4 - a^3
*c^5*d^5)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^
8*d^3 + a^4*c^7*d^4))^(3/4)*sqrt(x))/d^7) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^
3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^2*log
(b^2*sqrt(x) + (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4
*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-b^7/(a^7*b^4*c^4
```

```
- 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a
*c*x^2*log(b^2*sqrt(x) - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c
^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-d^7/(b^
4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4
))^(1/4)*a*c*x^2*log(d^2*sqrt(x) + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2
*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 3
*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a
^4*c^7*d^4))^(1/4)*a*c*x^2*log(d^2*sqrt(x) - (-d^7/(b^4*c^11 - 4*a*b^3*c^10
*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c
^2*d)) - 4*sqrt(x))/(a*c*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.39, size = 476, normalized size = 1.00

$$-\frac{(ab^3)^{\frac{1}{4}}\, b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2bc-\sqrt{2}\,a^3d}-\frac{(ab^3)^{\frac{1}{4}}\, b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2bc-\sqrt{2}\,a^3d}+\frac{(cd^3)^{\frac{1}{4}}\, d \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^3-\sqrt{2}\,ac^2d}+\frac{(cd^3)^{\frac{1}{4}}\, d \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^3-\sqrt{2}\,ac^2d}-\frac{(ab^3)^{\frac{1}{4}}\, b \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2bc-\sqrt{2}\,a^3d\right)}+\frac{(ab^3)^{\frac{1}{4}}\, b \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2bc-\sqrt{2}\,a^3d\right)}+\frac{(cd^3)^{\frac{1}{4}}\, d \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^3-\sqrt{2}\,ac^2d\right)}-\frac{(cd^3)^{\frac{1}{4}}\, d \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^3-\sqrt{2}\,ac^2d\right)}-\frac{2}{3\,acx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - (a*b^3)^(1/4)*b*arctan(-1/2*sqr
t(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt
(2)*a^3*d) + (c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sq
rt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + (c*d^3)^(1/4)*d*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*
c^3 - sqrt(2)*a*c^2*d) - 1/2*(a*b^3)^(1/4)*b*log(sqrt(2)*sqrt(x)*(a/b)^(1/4
) + x + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/2*(a*b^3)^(1/4)*b*
log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2
)*a^3*d) + 1/2*(c*d^3)^(1/4)*d*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/2*(c*d^3)^(1/4)*d*log(-sqrt(2)*s
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 2/3
/(a*c*x^(3/2))
```

Mupad [**B**]
time = 1.31, size = 2500, normalized size = 5.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] 2*atan(((x^(1/2)*(256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11) - (-d^7/(1
6*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*
b^3*c^10*d))^(1/4)*(((-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3
 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*(8192*a^13*b^12*c^21*d^4 -
40960*a^14*b^11*c^20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d
^7 + 81920*a^17*b^8*c^17*d^8 - 90112*a^18*b^7*c^16*d^9 + 81920*a^19*b^6*c^1
5*d^10 - 40960*a^20*b^5*c^14*d^11 + 8192*a^21*b^4*c^13*d^12)*1i + x^(1/2)*(
4096*a^11*b^13*c^20*d^4 - 16384*a^12*b^12*c^19*d^5 + 24576*a^13*b^11*c^18*d
^6 - 16384*a^14*b^10*c^17*d^7 + 4096*a^15*b^9*c^16*d^8 + 4096*a^16*b^8*c^15
*d^9 - 16384*a^17*b^7*c^14*d^10 + 24576*a^18*b^6*c^13*d^11 - 16384*a^19*b^5
*c^12*d^12 + 4096*a^20*b^4*c^11*d^13))*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4
- 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(3/4)*1i + 512*
a^9*b^12*c^14*d^7 - 512*a^10*b^11*c^13*d^8 - 512*a^13*b^8*c^10*d^11 + 512*a
^14*b^7*c^9*d^12)*1i)*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^
3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4) + (x^(1/2)*(256*a^9*b^11*c
^11*d^9 + 256*a^11*b^9*c^9*d^11) + (-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64
*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*(((-d^7/(16*b
^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3
*c^10*d))^(1/4)*(8192*a^13*b^12*c^21*d^4 - 40960*a^14*b^11*c^20*d^5 + 81920
*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d^7 + 81920*a^17*b^8*c^17*d^8 - 9
0112*a^18*b^7*c^16*d^9 + 81920*a^19*b^6*c^15*d^10 - 40960*a^20*b^5*c^14*d^1
1 + 8192*a^21*b^4*c^13*d^12)*1i - x^(1/2)*(4096*a^11*b^13*c^20*d^4 - 16384*
a^12*b^12*c^19*d^5 + 24576*a^13*b^11*c^18*d^6 - 16384*a^14*b^10*c^17*d^7 +
4096*a^15*b^9*c^16*d^8 + 4096*a^16*b^8*c^15*d^9 - 16384*a^17*b^7*c^14*d^10
+ 24576*a^18*b^6*c^13*d^11 - 16384*a^19*b^5*c^12*d^12 + 4096*a^20*b^4*c^11*
d^13))*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*
c^9*d^2 - 64*a*b^3*c^10*d))^(3/4)*1i + 512*a^9*b^12*c^14*d^7 - 512*a^10*b^1
1*c^13*d^8 - 512*a^13*b^8*c^10*d^11 + 512*a^14*b^7*c^9*d^12)*1i)*(-d^7/(16*
b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^
3*c^10*d))^(1/4))/((x^(1/2)*(256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11)
 - (-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*
d^2 - 64*a*b^3*c^10*d))^(1/4)*(((-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^
3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*(8192*a^13*b^12*
c^21*d^4 - 40960*a^14*b^11*c^20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16
*b^9*c^18*d^7 + 81920*a^17*b^8*c^17*d^8 - 90112*a^18*b^7*c^16*d^9 + 81920*a
^19*b^6*c^15*d^10 - 40960*a^20*b^5*c^14*d^11 + 8192*a^21*b^4*c^13*d^12)*1i
+ x^(1/2)*(4096*a^11*b^13*c^20*d^4 - 16384*a^12*b^12*c^19*d^5 + 24576*a^13*
b^11*c^18*d^6 - 16384*a^14*b^10*c^17*d^7 + 4096*a^15*b^9*c^16*d^8 + 4096*a^
16*b^8*c^15*d^9 - 16384*a^17*b^7*c^14*d^10 + 24576*a^18*b^6*c^13*d^11 - 163
84*a^19*b^5*c^12*d^12 + 4096*a^20*b^4*c^11*d^13))*(-d^7/(16*b^4*c^11 + 16*a
^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(3/4
```

```
)*1i + 512*a^9*b^12*c^14*d^7 - 512*a^10*b^11*c^13*d^8 - 512*a^13*b^8*c^10*d
^11 + 512*a^14*b^7*c^9*d^12)*1i)*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a
^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*1i - (x^(1/2)*(
256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11) + (-d^7/(16*b^4*c^11 + 16*a^
4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)
*(((-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*
d^2 - 64*a*b^3*c^10*d))^(1/4)*(8192*a^13*b^12*c^21*d^4 - 40960*a^14*b^11*c^
20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d^7 + 81920*a^17*b^
8*c^17*d^8 - 90112*a^18*b^7*c^16*d^9 + 81920*a^19*b^6*c^15*d^10 - 40960*a^2
0*b^5*c^14*d^11 + 8192*a^21*b^4*c^13*d^12)*1i - x^(1/2)*(4096*a^11*b^13*c^2
0*d^4 - 16384*a^12*b^12*c^19*d^5 + 24576*a^13*b^11*c^18*d^6 - 16384*a^14*b^
10*c^17*d^7 + 4096*a^15*b^9*c^16*d^8 + 4096*a^16*b^8*c^15*d^9 - 16384*a^17*
b^7*c^14*d^10 + 24576*a^18*b^6*c^13*d^11 - 16384*a^19*b^5*c^12*d^12 + 4096*
a^20*b^4*c^11*d^13))*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3
 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(3/4)*1i + 512*a^9*b^12*c^14*d^7
- 512*a^10*b^11*c^13*d^8 - 512*a^13*b^8*c^10*d^11 + 512*a^14*b^7*c^9*d^12)*
1i)*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9
*d^2 - 64*a*b^3*c^10*d))^(1/4)*1i))*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 6
4*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4) - atan((a^2*
b^5*d^7*x^(1/2)*1i + b^7*c^2*d^5*x^(1/2)*1i - (a^2*b^16*c^11*x^(1/2)*16i)/(
16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a
^10*b*c*d^3) + (a^3*b^15*c^10*d*x^(1/2)*64i)/(16*a^11*d^4 + 16*a^7*b^4*c^4
- 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3) - (a^4*b^14*c^9*
d^2*x^(1/2)*96i)/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*
b^2*c^2*d^2 - 64*a^10*b*c*d^3) + (a^5*b^13*c^8*d^3*x^(1/2)*64i)/(16*a^11*d^
4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*...
```

3.469 $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=498

$$-\frac{2}{5acx^{5/2}}+\frac{2(bc+ad)}{a^2c^2\sqrt{x}}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{d^{9/4}\tan^{-1}\left(1-\sqrt{\ldots}\right.}{\sqrt{2}\,c^{9/4}(bc\ldots}$$

```
[Out] -2/5/a/c/x^(5/2)-1/2*b^(9/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9
/4)/(-a*d+b*c)*2^(1/2)+1/2*b^(9/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4)
)/a^(9/4)/(-a*d+b*c)*2^(1/2)+1/2*d^(9/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c
^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)-1/2*d^(9/4)*arctan(1+d^(1/4)*2^(1/2)*x^(
1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(9/4)*ln(a^(1/2)+x*b^(1/2)-a
^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(9/4)*ln(a
^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(9/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)
/(-a*d+b*c)*2^(1/2)+1/4*d^(9/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2
)*x^(1/2))/c^(9/4)/(-a*d+b*c)*2^(1/2)+2*(a*d+b*c)/a^2/c^2/x^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 491, 597, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{2(ad+bc)}{a^2c^2\sqrt{x}}+\frac{d^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{d^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}(bc-ad)}+\frac{d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{2}{5acx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(5*a*c*x^(5/2)) + (2*(b*c + a*d))/(a^2*c^2*Sqrt[x]) - (b^(9/4)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c -
a*d)) + (d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^
(9/4)*(b*c - a*d)) - (d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(Sqrt[2]*c^(9/4)*(b*c - a*d)) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*
(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) + (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)}\,dx = 2\text{Subst}\left(\int \frac{1}{x^6(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2\text{Subst}\left(\int \frac{-5(bc+ad)-5bdx^4}{x^2(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{5ac}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{2\text{Subst}\left(\int \frac{x^2(-5(b^2c^2+abcd+a^2d^2)-5bd(bc+ad)x^4)}{(a+bx^4)(c+dx^4)}\,dx, x,\right.}{5a^2c^2}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{2\text{Subst}\left(\int \left(-\frac{5b^3c^2x^2}{(bc-ad)(a+bx^4)} - \frac{5a^2d^3x^2}{(-bc+ad)(c+dx^4)}\right) dx\right.}{5a^2c^2}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2(bc-ad)} - \frac{(2d^3)\,\text{Subst}\left(\right.}{c^2}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{b^{5/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2(bc-ad)} + \frac{b^{5/2}\text{Sub}}{}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} + \frac{b^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a^2(bc-ad)} + \frac{b}{}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} + \frac{b^{9/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)} - \frac{b^{9}}{}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{b^{9/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)} + \frac{b^{9/4}\tan^{-1}\left(1\right.}{\sqrt{2}\,a^{9/4}}$$

Mathematica [A]
time = 0.61, size = 285, normalized size = 0.57

$$\frac{2(-ac+5bcx^2+5adx^2)}{5a^2c^2x^{5/2}} + \frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\,a^{9/4}(-bc+ad)} + \frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt{2}\,c^{9/4}(bc-ad)} + \frac{b^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{9/4}(-bc+ad)} + \frac{d^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,c^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*(-(a*c) + 5*b*c*x^2 + 5*a*d*x^2))/(5*a^2*c^2*x^(5/2)) + (b^(9/4)*ArcTan[
(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(9/4)*
(-(b*c) + a*d)) + (d^(9/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^
```

```
(1/4)*Sqrt[x])])/(Sqrt[2]*c^(9/4)*(b*c - a*d)) + (b^(9/4)*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(9/4)*(-(b*c) +
 a*d)) + (d^(9/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt
[d]*x)])/(Sqrt[2]*c^(9/4)*(b*c - a*d))
```

Maple [A]

time = 0.13, size = 269, normalized size = 0.54

method	result
derivativedivides	$-\frac{b^2\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{d^2\sqrt{2}}{}$
default	$-\frac{b^2\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{d^2\sqrt{2}}{}$
risch	$-\frac{2(-5ad\,x^2-5c\,x^2b+ac)}{5a^2c^2x^{\frac{5}{2}}}-\frac{b^2\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4a^2(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{b^2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^2(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+1/4*d^2/
c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))-2/5/a/c/x^(5/2)-2
/a^2/c^2*(-a*d-b*c)/x^(1/2)
```

Maxima [A]

time = 0.56, size = 411, normalized size = 0.83

$$\frac{b^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(a^2bc-a^3d\right)}-\frac{d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\left(bc^3-ac^2d\right)}+\frac{2\left(5\left(bc+ad\right)x^2-ac\right)}{5a^2c^2x^{\frac{5}{2}}}$$

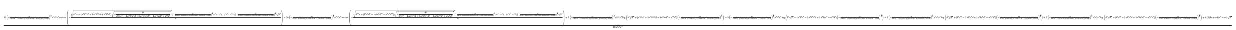
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
```

```
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a
^2*b*c - a^3*d) - 1/4*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^
(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sq
rt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)
*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*
log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)
) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^
(1/4)*d^(3/4)))/(b*c^3 - a*c^2*d) + 2/5*(5*(b*c + a*d)*x^2 - a*c)/(a^2*c^2*
x^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1484 vs. $2(358) = 716$.
time = 10.09, size = 1484, normalized size = 2.98



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/10*(20*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^3*arctan(-(sqrt(b^14*x - (a^5*b^11*c
^2 - 2*a^6*b^10*c*d + a^7*b^9*d^2)*sqrt(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*
d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4)))*(-b^9/(a^9*b^4*c^4 -
4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(
a^2*b*c - a^3*d) - (a^2*b^8*c - a^3*b^7*d)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*
c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*sqrt(x))/b^9
) - 20*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10
*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^3*arctan(-(sqrt(d^14*x - (b^2*c^7*d^9
- 2*a*b*c^6*d^10 + a^2*c^5*d^11)*sqrt(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a
^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4)))*(-d^9/(b^4*c^13 - 4*a*b
^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*(b*
c^3 - a*c^2*d) - (b*c^3*d^7 - a*c^2*d^8)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d +
 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*sqrt(x))/d^9)
+ 5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c
*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) + (a^7*b^3*c^3 - 3*a^8*
b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d
 + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9*b
^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4)
)^(1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) - (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^
9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^
2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^
12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*
x^3*log(d^7*sqrt(x) + (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7
*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*
```

```
d^3 + a^4*c^9*d^4))^(3/4)) + 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2
*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^3*log(d^7*sqrt
(x) - (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4
*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^
4))^(3/4)) + 4*(5*(b*c + a*d)*x^2 - a*c)*sqrt(x))/(a^2*c^2*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.38, size = 487, normalized size = 0.98

$$\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^3bc-\sqrt{2}\,a^4d}+\frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^3bc-\sqrt{2}\,a^4d}-\frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^4-\sqrt{2}\,ac^3d}-\frac{(cd^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^4-\sqrt{2}\,ac^3d}-\frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^3bc-\sqrt{2}\,a^4d\right)}+\frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^3bc-\sqrt{2}\,a^4d\right)}+\frac{(cd^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^4-\sqrt{2}\,ac^3d\right)}-\frac{(cd^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^4-\sqrt{2}\,ac^3d\right)}+\frac{2\,(5\,bcx^2+5\,adx^2-ac)}{5\,a^2c^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1
/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a
^4*d) - (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - (c*d^3)^(3/4)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqr
t(2)*a*c^3*d) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/
2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b
*c^4 - sqrt(2)*a*c^3*d) - 1/2*(c*d^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4
) + x + sqrt(c/d))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) + 2/5*(5*b*c*x^2 + 5*a
*d*x^2 - a*c)/(a^2*c^2*x^(5/2))
```

Mupad [B]
time = 1.40, size = 2500, normalized size = 5.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - 2*atan((32*a^11*b^10*c^13*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 6
4*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) + 2*a^11*b
^6*d^9*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96
*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(1/4) + 32*a^21*c^3*d^10*x^(1/2)*(-b^
9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 -
 64*a^12*b*c*d^3))^(5/4) + 2*a^8*b^9*c^3*d^6*x^(1/2)*(-b^9/(16*a^13*d^4 + 1
6*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))
^(1/4) + 192*a^13*b^8*c^11*d^2*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4
- 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) - 128*a
^14*b^7*c^10*d^3*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*
c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) + 32*a^15*b^6*c^9*d^4
*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*
b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) + 32*a^17*b^4*c^7*d^6*x^(1/2)*(-b^9/(
16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64
*a^12*b*c*d^3))^(5/4) - 128*a^18*b^3*c^6*d^7*x^(1/2)*(-b^9/(16*a^13*d^4 + 1
6*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))
^(5/4) + 192*a^19*b^2*c^5*d^8*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 -
 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) - 128*a^
12*b^9*c^12*d*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3
*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) - 128*a^20*b*c^4*d^9*x^(
1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*
c^2*d^2 - 64*a^12*b*c*d^3))^(5/4))/(b^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7
+ a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5
 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 -
64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(1/4) - atan((a
^11*b^10*c^13*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3
*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*32i + a^11*b^6*d^9*x^(1/
2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^
2*d^2 - 64*a^12*b*c*d^3))^(1/4)*2i + a^21*c^3*d^10*x^(1/2)*(-b^9/(16*a^13*d
^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c
*d^3))^(5/4)*32i + a^8*b^9*c^3*d^6*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*
c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(1/4)*2i
+ a^13*b^8*c^11*d^2*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b
^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*192i - a^14*b^7*c^
10*d^3*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96
*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*128i + a^15*b^6*c^9*d^4*x^(1/2)
*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*
d^2 - 64*a^12*b*c*d^3))^(5/4)*32i + a^17*b^4*c^7*d^6*x^(1/2)*(-b^9/(16*a^13
*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b
*c*d^3))^(5/4)*32i - a^18*b^3*c^6*d^7*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b
^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*
128i + a^19*b^2*c^5*d^8*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^
10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*192i - a^12*b^
9*c^12*d*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d +
96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*128i - a^20*b*c^4*d^9*x^(1/2)
```

*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*128i)/(b^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(1/4)*2i - (2/(5*a*c) - (2*x^2*(a*d + b*c))/(a^2*c^2))/x^(5/2) - 2*atan((32*a^3*b^10*c^21*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) + 32*a^13*c^11*d^10*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) + 2*b^9*c^11*d^6*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(1/4) + 2*a^3*b^6*c^8*d^9*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(1/4) + 192*a^5*b^8*c^19*d^2*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) - 128*a^6*b^7*c^18*d^3*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) + 32*a^7*b^6*c^17*d^4*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) + 32*a^9*b^4*c^15*d^6*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) - 128*a^10*b^3*c^14*d^7*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d)...

3.470 $\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=570

$$\frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)}-\frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)}+\frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2}+$$

```
[Out] -1/2*c*x^(5/2)/d/(-a*d+b*c)/(d*x^2+c)+1/2*a^(9/4)*arctan(1-b^(1/4)*2^(1/2)*
x^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)^2*2^(1/2)-1/2*a^(9/4)*arctan(1+b^(1/4)*
2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/8*c^(5/4)*(-9*a*d+5
*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(9/4)/(-a*d+b*c)^2*2^(1/2
)-1/8*c^(5/4)*(-9*a*d+5*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(9
/4)/(-a*d+b*c)^2*2^(1/2)+1/4*a^(9/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/b^(5/4)/(-a*d+b*c)^2*2^(1/2)-1/4*a^(9/4)*ln(a^(1/2)+x*b^(1/
2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/16*c^(5/
4)*(-9*a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(
9/4)/(-a*d+b*c)^2*2^(1/2)-1/16*c^(5/4)*(-9*a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)+
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(9/4)/(-a*d+b*c)^2*2^(1/2)+1/2*(-4*a*d+5
*b*c)*x^(1/2)/b/d^2/(-a*d+b*c)
```

Rubi [A]
time = 0.55, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 481, 596, 536, 217, 1179, 642, 1176, 631, 210}

$$\frac{a^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2}+\frac{a^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)^2}+\frac{c^{5/4}(5bc-9ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,d^{9/4}(bc-ad)^2}-\frac{c^{5/4}(5bc-9ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,d^{9/4}(bc-ad)^2}+\frac{c^{5/4}(5bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{9/4}(bc-ad)^2}-\frac{c^{5/4}(5bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{9/4}(bc-ad)^2}+\frac{\sqrt{x}(5bc-4ad)}{2bd^2(bc-ad)}-\frac{cx^{5/2}}{2d(c+dx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((5*b*c - 4*a*d)*Sqrt[x])/(2*b*d^2*(b*c - a*d)) - (c*x^(5/2))/(2*d*(b*c - a
*d)*(c + d*x^2)) + (a^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d
)^2) - (c^(5/4)*(5*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) + (a^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*
b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) - (c^(
5/4)*(5*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
```

```
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^{12}}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{x^4(5ac+(5bc-4ad)x^4)}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} - \frac{\text{Subst}\left(\int \frac{ac(5bc-4ad)+(5b^2c^2-4abcd-4a^2d}{(a+bx^4)(c+dx^4)}\right.}{2bd^2(bc-ad)}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)^2} -$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} - \frac{a^{5/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{b(bc-ad)^2}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} - \frac{a^{5/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx\right.}{2b^{3/2}(bc-ad)^2}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} + \frac{a^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{2\sqrt{2}\,b^{5/4}(bc-ad)^2}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} + \frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2} - \frac{a}{-}$$

Mathematica [A]
time = 1.09, size = 301, normalized size = 0.53

$$\frac{\frac{4(bc-ad)\sqrt{x}\,(-4ad(c+dx^2)+bc(5c+4dx^2))}{bd^2(c+dx^2)} + \frac{4\sqrt{2}\,a^{9/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{5/4}} + \frac{\sqrt{2}\,c^{5/4}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{d^{9/4}} - \frac{4\sqrt{2}\,a^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{5/4}} - \frac{\sqrt{2}\,c^{5/4}(5bc-9ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{d^{9/4}}}{8(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((4*(b*c - a*d)*Sqrt[x]*(-4*a*d*(c + d*x^2) + b*c*(5*c + 4*d*x^2)))/(b*d^2*
(c + d*x^2)) + (4*Sqrt[2]*a^(9/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x])])/b^(5/4) + (Sqrt[2]*c^(5/4)*(5*b*c - 9*a*d)*ArcTan[(
Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/d^(9/4) - (4*Sqrt[
2]*a^(9/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
```

```
)/b^(5/4) - (Sqrt[2]*c^(5/4)*(5*b*c - 9*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(9/4))/(8*(b*c - a*d)^2)
```

Maple [A]

time = 0.12, size = 286, normalized size = 0.50

method	result
derivativedivides	$\frac{2\sqrt{x}}{b\,d^{2}}-\frac{a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\right.\right.}{4b\left(ad-bc\right)^{2}}$
default	$\frac{2\sqrt{x}}{b\,d^{2}}-\frac{a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\right.\right.}{4b\left(ad-bc\right)^{2}}$
risch	$\frac{2\sqrt{x}}{b\,d^{2}}-\frac{a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4b\left(ad-bc\right)^{2}}-\frac{a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b\left(ad-bc\right)^{2}}-\frac{a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b/d^2*x^(1/2)-1/4/b*a^2/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))
+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)-1))+2*c^2/d^2/(a*d-b*c)^2*((-1/4*a*d+1/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(9*
a*d-5*b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(
1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.56, size = 494, normalized size = 0.87

$$-\frac{\left(\frac{2\sqrt{2}(5bc-9ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(5bc-9ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(5bc-9ad)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(5bc-9ad)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)c^{2}}{16\left(b^{2}c^{2}d^{2}-2abcd^{3}+a^{2}d^{4}\right)}+\frac{c^{2}\sqrt{x}}{2\left(bc^{2}d^{2}-acd^{3}+(bcd^{3}-ad^{4})x^{2}\right)}-\frac{\frac{2\sqrt{2}a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}a^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{9}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{9}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4\left(b^{3}c^{2}-2ab^{2}cd+a^{2}bd^{2}\right)}+\frac{2\sqrt{x}}{bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(5*b*c - 9*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4
) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)
)) + 2*sqrt(2)*(5*b*c - 9*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
```

```
 - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))
) + sqrt(2)*(5*b*c - 9*a*d)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
 + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(5*b*c - 9*a*d)*log(-sqrt(2)*c^(1/4
)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c^2/(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4) + 1/2*c^2*sqrt(x)/(b*c^2*d^2 - a*c*d^3 + (b*c*d^
3 - a*d^4)*x^2) - 1/4*(2*sqrt(2)*a^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b))
 + 2*sqrt(2)*a^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(9/4)*
log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2
)*a^(9/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/
4))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + 2*sqrt(x)/(b*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3393 vs. $2(423) = 846$.
time = 69.52, size = 3393, normalized size = 5.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9
 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^
4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b
^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^
15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)*arctan(((b^6*c^6*d^7 - 6*a*b^5*c^5*d
^8 + 15*a^2*b^4*c^4*d^9 - 20*a^3*b^3*c^3*d^10 + 15*a^4*b^2*c^2*d^11 - 6*a^5
*b*c*d^12 + a^6*d^13)*sqrt((25*b^2*c^4 - 90*a*b*c^3*d + 81*a^2*c^2*d^2)*x +
 (b^4*c^4*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d
^8)*sqrt(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a
^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b
^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^
14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17)))*(-(625*b^4*c^9 - 45
00*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5
*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^
5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 -
8*a^7*b*c*d^16 + a^8*d^17))^(3/4) + (5*b^7*c^8*d^7 - 39*a*b^6*c^7*d^8 + 129
*a^2*b^5*c^6*d^9 - 235*a^3*b^4*c^5*d^10 + 255*a^4*b^3*c^4*d^11 - 165*a^5*b^
2*c^3*d^12 + 59*a^6*b*c^2*d^13 - 9*a^7*c*d^14)*sqrt(x)*(-(625*b^4*c^9 - 450
0*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*
d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5
*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8
*a^7*b*c*d^16 + a^8*d^17))^(3/4))/(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a
^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)) - 16*(-a^9/(b^13*
```

```
c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b
^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^
8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*arc
tan(((b^10*c^6 - 6*a*b^9*c^5*d + 15*a^2*b^8*c^4*d^2 - 20*a^3*b^7*c^3*d^3 +
15*a^4*b^6*c^2*d^4 - 6*a^5*b^5*c*d^5 + a^6*b^4*d^6)*(-a^9/(b^13*c^8 - 8*a*b
^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4
- 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))
^(3/4)*sqrt(a^4*x + (b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^
3*c*d^3 + a^4*b^2*d^4)*sqrt(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c
^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28
*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))) - (a^2*b^10*c^6 - 6*a^3
*b^9*c^5*d + 15*a^4*b^8*c^4*d^2 - 20*a^5*b^7*c^3*d^3 + 15*a^6*b^6*c^2*d^4 -
 6*a^7*b^5*c*d^5 + a^8*b^4*d^6)*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b
^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5
 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(3/4)*sqrt(x))/a^9)
 - 4*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c
^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a
^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 -
a*b*d^4)*x^2)*log(a^2*sqrt(x) + (-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b
^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5
 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)) + 4*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*
c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 2
8*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*
b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*log(a^2*sqrt(x) - (-a^9/(b^13*c^8 - 8*
a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d
^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^
8))^(1/4)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)) + (b^2*c^2*d^2 - a*b*c*d^3 +
 (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b
^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7
*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13
 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^
(1/4)*log(-(5*b*c^2 - 9*a*c*d)*sqrt(x) + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b
*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c
^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 +
 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)) - (b^2*c^2*d^2 -
a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d +
12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^
9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b
^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 +
a^8*d^17))^(1/4)*log(-(5*b*c^2 - 9*a*c*d)*sqrt(x) - (b^2*c^2*d^2 - 2*a*b*c*
d^3 + a^2*d^4)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 -
14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.18, size = 718, normalized size = 1.26

$$-\frac{(ab^3)^{\frac{1}{4}}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^4c^2-2\sqrt{2}\,ab^3cd+\sqrt{2}\,a^2b^2d^2}-\frac{(ab^3)^{\frac{1}{4}}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^4c^2-2\sqrt{2}\,ab^3cd+\sqrt{2}\,a^2b^2d^2}-\frac{(ab^3)^{\frac{1}{4}}a^2\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^4c^2-2\sqrt{2}\,ab^3cd+\sqrt{2}\,a^2b^2d^2\right)}+\frac{(ab^3)^{\frac{1}{4}}a^2\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^4c^2-2\sqrt{2}\,ab^3cd+\sqrt{2}\,a^2b^2d^2\right)}-\frac{\left(5(cd^3)^{\frac{1}{4}}bc^2-9(cd^3)^{\frac{1}{4}}acd\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^2d^3-2\sqrt{2}\,abcd^4+\sqrt{2}\,a^2d^5\right)}-\frac{\left(5(cd^3)^{\frac{1}{4}}bc^2-9(cd^3)^{\frac{1}{4}}acd\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^2d^3-2\sqrt{2}\,abcd^4+\sqrt{2}\,a^2d^5\right)}-\frac{\left(5(cd^3)^{\frac{1}{4}}bc^2-9(cd^3)^{\frac{1}{4}}acd\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^2d^3-2\sqrt{2}\,abcd^4+\sqrt{2}\,a^2d^5\right)}+\frac{\left(5(cd^3)^{\frac{1}{4}}bc^2-9(cd^3)^{\frac{1}{4}}acd\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^2d^3-2\sqrt{2}\,abcd^4+\sqrt{2}\,a^2d^5\right)}+\frac{c^2\sqrt{x}}{2(bcd^2-ad^3)(dx^2+c)}+\frac{2\sqrt{x}}{bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - (
a*b^3)^(1/4)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/
2*(a*b^3)^(1/4)*a^2*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) + 1/2*(a*b^3)^(1/4)
*a^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^2 - 2
*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/4*(5*(c*d^3)^(1/4)*b*c^2 - 9*
(c*d^3)^(1/4)*a*c*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(
c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) -
 1/4*(5*(c*d^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1/4)*a*c*d)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(
2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) - 1/8*(5*(c*d^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1
/4)*a*c*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^
2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) + 1/8*(5*(c*d^3)^(1/4)*b*c^2
 - 9*(c*d^3)^(1/4)*a*c*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) + 1/2*c^2*sq
rt(x)/((b*c*d^2 - a*d^3)*(d*x^2 + c)) + 2*sqrt(x)/(b*d^2)
```

Mupad [B]
time = 1.62, size = 2500, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

[Out] atan((((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) - (2*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) - (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9))*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*1i + ((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) + (2*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d

^14 - 40960*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9))*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*1i)/(((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3...

3.471 $\int \frac{x^{9/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=536

$$-\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2}+\frac{a^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2}-\frac{c^{3/4}(3bc-7ad)}{4\sqrt{2}}$$

```
[Out] -1/2*c*x^(3/2)/d/(-a*d+b*c)/(d*x^2+c)-1/2*a^(7/4)*arctan(1-b^(1/4)*2^(1/2)*
x^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/2*a^(7/4)*arctan(1+b^(1/4)*
2^(1/2)*x^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/8*c^(3/4)*(-7*a*d+3
*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(7/4)/(-a*d+b*c)^2*2^(1/2
)+1/8*c^(3/4)*(-7*a*d+3*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(7
/4)/(-a*d+b*c)^2*2^(1/2)+1/4*a^(7/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/b^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/4*a^(7/4)*ln(a^(1/2)+x*b^(1/
2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/16*c^(3/
4)*(-7*a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(
7/4)/(-a*d+b*c)^2*2^(1/2)-1/16*c^(3/4)*(-7*a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)+
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(7/4)/(-a*d+b*c)^2*2^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 481, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{a^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2}+\frac{a^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2}+\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)^2}-\frac{c^{3/4}(3bc-7ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,d^{7/4}(bc-ad)^2}+\frac{c^{3/4}(3bc-7ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,d^{7/4}(bc-ad)^2}+\frac{c^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{7/4}(bc-ad)^2}-\frac{c^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{7/4}(bc-ad)^2}-\frac{cx^{3/2}}{2d\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(c*x^(3/2))/(d*(b*c - a*d)*(c + d*x^2)) - (a^(7/4)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^2) + (a^(7/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^2)
- (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
4*Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (a^(7/4
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^
(3/4)*(b*c - a*d)^2) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)^2) + (c^(3/4)*(3*b*c - 7*a*
d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d
^(7/4)*(b*c - a*d)^2) - (c^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(7/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^{10}}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{x^2(3ac+(3bc-4ad)x^4)}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)(c+dx^2)} + \frac{\text{Subst}\left(\int \left(-\frac{4a^2dx^2}{(-bc+ad)(a+bx^4)} + \frac{c(3bc-7ad)x^2}{(bc-ad)(c+dx^4)}\right)dx, x, \sqrt{\cdots}\right.}{2d(bc-ad)}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)(c+dx^2)} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(c(3bc-7ad))\text{Su}\cdots}{2d\cdots}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)(c+dx^2)} - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,(bc-ad)^2} + \frac{a^2\text{Subst}\left(\int \cdots\right.}{\sqrt{\cdots}}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)(c+dx^2)} + \frac{a^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)^2} + \frac{a^2\text{Sub}\cdots}{\cdots}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)(c+dx^2)} + \frac{a^{7/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)^2} - \frac{a^{7/4}\log\cdots}{\cdots}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)(c+dx^2)} - \frac{a^{7/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2} + \frac{a^{7/4}\tan^{-1}\left(1+\frac{\sqrt{\cdots}}{\cdots}\right.}{\sqrt{2}\,b^{3/4}(bc-\cdots}$$

<u>Mathematica</u> [A]
time = 0.89, size = 307, normalized size = 0.57

$$\frac{1}{8}\left(\frac{4cx^{3/2}}{d(-bc+ad)(c+dx^2)} - \frac{4\sqrt{2}\,a^{7/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}(bc-ad)^2} - \frac{\sqrt{2}\,c^{3/4}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{d^{7/4}(bc-ad)^2} - \frac{4\sqrt{2}\,a^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{3/4}(bc-ad)^2} - \frac{\sqrt{2}\,c^{3/4}(3bc-7ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{d^{7/4}(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((4*c*x^(3/2))/(d*(-(b*c) + a*d)*(c + d*x^2)) - (4*Sqrt[2]*a^(7/4)*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(b^(3/4)*(b*c - a*
d)^2) - (Sqrt[2]*c^(3/4)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(d^(7/4)*(b*c - a*d)^2) - (4*Sqrt[2]*a^(7/4)
*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(b^(3/4)
```

```
*(b*c - a*d)^2) - (Sqrt[2]*c^(3/4)*(3*b*c - 7*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(d^(7/4)*(b*c - a*d)^2))/8
```

Maple [A]

time = 0.09, size = 273, normalized size = 0.51

method	result
derivativedivides	$\frac{a^{2}\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4\left(ad-bc\right)^{2}b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2c\left(-\frac{(\cdots}{4\cdots}\cdots\right.}{\cdots}$
default	$\frac{a^{2}\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4\left(ad-bc\right)^{2}b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2c\left(-\frac{(\cdots}{4\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2/(a*d-b*c)^2/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*c/(a*d-
b*c)^2*(-1/4*(a*d-b*c)/d*x^(3/2)/(d*x^2+c)+1/32*(7*a*d-3*b*c)/d^2/(c/d)^(1/
4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x
^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.52, size = 450, normalized size = 0.84

$$\frac{a^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)}-\frac{cx^{\frac{3}{2}}}{2\left(bc^{2}d-acd^{2}+\left(bcd^{2}-ad^{3}\right)x^{2}\right)}+\frac{\left(3bc^{2}-7acd\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{2}c^{2}d-2abcd^{2}+a^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b
```

```
^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c*x^(3/2)/(b*c^2*d - a*c*d^2 + (b*c*d^2
 - a*d^3)*x^2) + 1/16*(3*b*c^2 - 7*a*c*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqr
t(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/
(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*
x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3524 vs. $2(393) = 786$.
time = 39.89, size = 3524, normalized size = 6.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*c*x^(3/2) - 4*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4
*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a
^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^
5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^1
3 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4)*arctan(((b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*sqrt((729*b^6*c^10 - 10206*a*b^5*c^9*d + 59535*a^2*b^4*c^8*d^2 - 1
85220*a^3*b^3*c^7*d^3 + 324135*a^4*b^2*c^6*d^4 - 302526*a^5*b*c^5*d^5 + 117
649*a^6*c^4*d^6)*x - (81*b^8*c^11*d^3 - 1080*a*b^7*c^10*d^4 + 6156*a^2*b^6*
c^9*d^5 - 19560*a^3*b^5*c^8*d^6 + 37846*a^4*b^4*c^7*d^7 - 45640*a^5*b^3*c^6
*d^8 + 33516*a^6*b^2*c^5*d^9 - 13720*a^7*b*c^4*d^10 + 2401*a^8*c^3*d^11)*sq
rt(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d
^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9
- 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*
b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15)))*(-(81*b^4*c^7 - 756*a*b^3*c^6*d
 + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d
^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^
4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a
^8*d^15))^(1/4) + (27*b^5*c^7*d^2 - 243*a*b^4*c^6*d^3 + 846*a^2*b^3*c^5*d^4
 - 1414*a^3*b^2*c^4*d^5 + 1127*a^4*b*c^3*d^6 - 343*a^5*c^2*d^7)*sqrt(x)*(-(
81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 +
2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*
a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c
^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4))/(81*b^4*c^7 - 756*a*b^3*c^6*d
+ 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)) + 16*(-a^7
/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*
a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7
 + a^8*b^3*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*arctan((
```

```
sqrt(a^10*x - (a^7*b^5*c^4 - 4*a^8*b^4*c^3*d + 6*a^9*b^3*c^2*d^2 - 4*a^10*b
^2*c*d^3 + a^11*b*d^4)*sqrt(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^
6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a
^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8)))*(-a^7/(b^11*c^8 - 8*a*b^1
0*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56
*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/
4)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - (a^5*b^3*c^2 - 2*a^6*b^2*c*d + a^7
*b*d^2)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*
c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*
a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/4)*sqrt(x))/a^7) - 4*(-a^7/(b^11*c^8 - 8*a
*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4
- 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))
^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a^5*sqrt(x) + (b^8*c
^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c
^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d +
28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c
^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(3/4)) + 4*(-
a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 +
70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*
d^7 + a^8*b^3*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a
^5*sqrt(x) - (b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3
*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*(-a^7/(b^11*c^8
- 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4
*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*
d^8))^(3/4)) + (b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4*c^7 -
756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*
d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d
^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a
^7*b*c*d^14 + a^8*d^15))^(1/4)*log((b^6*c^6*d^5 - 6*a*b^5*c^5*d^6 + 15*a^2*
b^4*c^4*d^7 - 20*a^3*b^3*c^3*d^8 + 15*a^4*b^2*c^2*d^9 - 6*a^5*b*c*d^10 + a^
6*d^11)*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b
*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^
6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 2
8*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(3/4) - (27*b^3*c^5 - 189*
a*b^2*c^4*d + 441*a^2*b*c^3*d^2 - 343*a^3*c^2*d^3)*sqrt(x)) - (b*c^2*d - a*
c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b
^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*
c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 -
56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/
4)*log(-(b^6*c^6*d^5 - 6*a*b^5*c^5*d^6 + 15*a^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.59, size = 681, normalized size = 1.27

$$\frac{\left(ab^3\right)^{\frac{3}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^5c^2-2\sqrt{2}\,ab^4cd+\sqrt{2}\,a^2b^3d^2}+\frac{\left(ab^3\right)^{\frac{3}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^5c^2-2\sqrt{2}\,ab^4cd+\sqrt{2}\,a^2b^3d^2}-\frac{\left(ab^3\right)^{\frac{3}{4}} a \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^5c^2-2\sqrt{2}\,ab^4cd+\sqrt{2}\,a^2b^3d^2\right)}+\frac{\left(ab^3\right)^{\frac{3}{4}} a \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^5c^2-2\sqrt{2}\,ab^4cd+\sqrt{2}\,a^2b^3d^2\right)}+\frac{\left(3\left(cd^3\right)^{\frac{3}{4}}bc-7\left(cd^3\right)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^2d^4-2\sqrt{2}\,abcd^5+\sqrt{2}\,a^2d^6\right)}+\frac{\left(3\left(cd^3\right)^{\frac{3}{4}}bc-7\left(cd^3\right)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^2d^4-2\sqrt{2}\,abcd^5+\sqrt{2}\,a^2d^6\right)}-\frac{\left(3\left(cd^3\right)^{\frac{3}{4}}bc-7\left(cd^3\right)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^2d^4-2\sqrt{2}\,abcd^5+\sqrt{2}\,a^2d^6\right)}+\frac{\left(3\left(cd^3\right)^{\frac{3}{4}}bc-7\left(cd^3\right)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^2d^4-2\sqrt{2}\,abcd^5+\sqrt{2}\,a^2d^6\right)}-\frac{cx^{\frac{3}{2}}}{2\left(bcd-ad^2\right)\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + (a*b
^3)^(3/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/
4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) - 1/2*(a*
b^3)^(3/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*
c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/2*(a*b^3)^(3/4)*a*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*
a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/4*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/
4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) + 1/4*(3*(c*d^
3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sq
rt(2)*a^2*d^6) - 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)*log(sqrt(2
)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b
*c*d^5 + sqrt(2)*a^2*d^6) + 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)
*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^4 - 2
*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) - 1/2*c*x^(3/2)/((b*c*d - a*d^2)*(d*x
^2 + c))
```

Mupad [B]
time = 1.34, size = 2500, normalized size = 4.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] 2*atan(((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b
^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d
^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8 + 1
6*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d
^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128
```

*a*b^10*c^7*d))^(3/4)*(((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) + (x^(1/2)*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) + (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) - (-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(3/4)*(((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) - (x^(1/2)*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) - (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)))/((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b

```
^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7
+ 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^
5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(3/4)*(((864*a^3*b
^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a
^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054
144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 -
 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^1
4)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*
a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9)
+ (x^(1/2)*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^
2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^
3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*(2304*a^3*b^14*c^13*
d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*
c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^
9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143
872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 -
6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b...
```

3.472 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=532

$$-\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}-\frac{\sqrt[4]{c}(bc-5ad)\,\mathrm{t}}{4\sqrt{2}}$$

```
[Out] -1/2*a^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)^2
*2^(1/2)+1/2*a^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4)/(-a*
d+b*c)^2*2^(1/2)-1/8*c^(1/4)*(-5*a*d+b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/
c^(1/4))/d^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/8*c^(1/4)*(-5*a*d+b*c)*arctan(1+d^(
1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b*c)^2*2^(1/2)-1/4*a^(5/4)*ln(a
^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/(-a*d+b*c)^2*2^(1
/2)+1/4*a^(5/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(1/
4)/(-a*d+b*c)^2*2^(1/2)-1/16*c^(1/4)*(-5*a*d+b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1
/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/16*c^(1/4)*(-5*
a*d+b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(5/4)/(-a*
d+b*c)^2*2^(1/2)-1/2*c*x^(1/2)/d/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A]
time = 0.35, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 481, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{a^{5/4}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}-\frac{\sqrt[4]{c}(bc-5ad)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{c}(bc-5ad)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}d^{5/4}(bc-ad)^2}-\frac{\sqrt[4]{c}(bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{c}(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}d^{5/4}(bc-ad)^2}-\frac{c\sqrt{x}}{2d(c+dx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(c*Sqrt[x])/(d*(b*c - a*d)*(c + d*x^2)) - (a^(5/4)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^2) + (a^(5/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^2)
- (c^(1/4)*(b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*
Sqrt[2]*d^(5/4)*(b*c - a*d)^2) + (c^(1/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)^2) - (a^(5/4)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4
)*(b*c - a*d)^2) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)^2) - (c^(1/4)*(b*c - 5*a*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)
*(b*c - a*d)^2) + (c^(1/4)*(b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^8}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{ac+(bc-4ad)x^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(c(bc-5ad))\text{Sul}}{2d(}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} + \frac{a^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{a^{3/2}\text{Subst}}{}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} + \frac{a^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\,(bc-ad)^2} + \frac{a^{3}}{}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^2} + \frac{a^{5/4}\,\text{l}}{}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^2} + \frac{a^{5/4}\tan^{-1}\left(1+\right.}{\sqrt{2}\,\sqrt[4]{b}\,(bc}$$

2388

Mathematica [A]
time = 0.86, size = 304, normalized size = 0.57

$$\frac{1}{8}\left(\frac{4c\sqrt{x}}{d(-bc+ad)\left(c+dx^2\right)}-\frac{4\sqrt{2}\,a^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt[4]{b}\,(bc-ad)^2}-\frac{\sqrt{2}\,\sqrt[4]{c}\,(bc-5ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{d^{5/4}(bc-ad)^2}+\frac{4\sqrt{2}\,a^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{b}\,(bc-ad)^2}+\frac{\sqrt{2}\,\sqrt[4]{c}\,(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{d^{5/4}(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((4*c*Sqrt[x])/(d*(-(b*c) + a*d)*(c + d*x^2)) - (4*Sqrt[2]*a^(5/4)*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(b^(1/4)*(b*c - a*
d)^2) - (Sqrt[2]*c^(1/4)*(b*c - 5*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x])])/(d^(5/4)*(b*c - a*d)^2) + (4*Sqrt[2]*a^(5/4)*A
rcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(b^(1/4)*(
b*c - a*d)^2) + (Sqrt[2]*c^(1/4)*(b*c - 5*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(d^(5/4)*(b*c - a*d)^2))/8
```

Maple [A]
time = 0.10, size = 271, normalized size = 0.51

method	result
derivativedivides	$\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2} - 2c\left(\ldots\right.$
default	$\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2} - 2c\left(\ldots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*c/(a*d-b*c)
^2*(-1/4*(a*d-b*c)/d*x^(1/2)/(d*x^2+c)+1/32*(5*a*d-b*c)/d*(c/d)^(1/4)/c*2^(
1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.53, size = 468, normalized size = 0.88

$$\frac{\left(\frac{2\,\sqrt{2}\,(bc-5\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\,(bc-5\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(bc-5\,ad)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(bc-5\,ad)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)c}{16\,(b^2c^2d-2\,abcd^2+a^2d^3)}-\frac{c\sqrt{x}}{2\,(bc^2d-acd^2+(bcd^2-ad^3)x^2)}+\frac{\frac{2\,\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4\,(b^2c^2-2\,abcd+a^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*(b*c - 5*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) +
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)))
+ 2*sqrt(2)*(b*c - 5*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + s
qrt(2)*(b*c - 5*a*d)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt
(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c - 5*a*d)*log(-sqrt(2)*c^(1/4)*d^(1/4)
*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/(b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3) - 1/2*c*sqrt(x)/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2) +
 1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3
/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3224 vs. $2(389) = 778$.
time = 10.40, size = 3224, normalized size = 6.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c
^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^
5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*
c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*
d^13))^(1/4)*arctan(((b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4*c^4*d^6 -
20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^10)*sqrt((b
^2*c^2 - 10*a*b*c*d + 25*a^2*d^2)*x + (b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^
2*b^2*c^2*d^4 - 4*a^3*b*c*d^5 + a^4*d^6)*sqrt(-(b^4*c^5 - 20*a*b^3*c^4*d +
150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a
*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9
 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13)))
```

```
*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 62
5*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b
^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11
 - 8*a^7*b*c*d^12 + a^8*d^13))^(3/4) + (b^7*c^7*d^4 - 11*a*b^6*c^6*d^5 + 45
*a^2*b^5*c^5*d^6 - 95*a^3*b^4*c^4*d^7 + 115*a^4*b^3*c^3*d^8 - 81*a^5*b^2*c^
2*d^9 + 31*a^6*b*c*d^10 - 5*a^7*d^11)*sqrt(x)*(-(b^4*c^5 - 20*a*b^3*c^4*d +
 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*
a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^
9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))
^(3/4))/(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3
 + 625*a^4*c*d^4)) - 16*(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2
 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^
3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d
^2 - a*d^3)*x^2)*arctan(((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*(-a^5/
(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4
*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 +
a^8*b*d^8))^(3/4)*sqrt(a^2*x + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7
*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 2
8*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))) - (a*b^7*c^6 - 6*a^2*b^6
*c^5*d + 15*a^3*b^5*c^4*d^2 - 20*a^4*b^4*c^3*d^3 + 15*a^5*b^3*c^2*d^4 - 6*a
^6*b^2*c*d^5 + a^7*b*d^6)*(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d
^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*
b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(3/4)*sqrt(x))/a^5) + (b*c^2*d
 - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^
2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d
^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*
b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))^(1/4)*log(
-(b*c - 5*a*d)*sqrt(x) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b^4*c^5 - 2
0*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b
^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70
*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^
12 + a^8*d^13))^(1/4)) - (b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4
*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c
*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*
d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^
7*b*c*d^12 + a^8*d^13))^(1/4)*log(-(b*c - 5*a*d)*sqrt(x) - (b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500
*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6
*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 +
28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))^(1/4)) - 4*(-a^5/(b^9*c^8
 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4
*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^
8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a*sqrt(x) + (-a^5
```

```
/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^
4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 +
 a^8*b*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 4*(-a^5/(b^9*c^8 - 8*
a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4
- 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(
1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a*sqrt(x) - (-a^5/(b^9
*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5
*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*
b*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 4*c*sqrt(x))/(b*c^2*d - a*
c*d^2 + (b*c*d^2 - a*d^3)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.32, size = 669, normalized size = 1.26

$$\frac{(ab^3)^{\frac14} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{\sqrt{2}\, b^3c^2 - 2\sqrt{2}\, ab^2cd + \sqrt{2}\, a^2bd^2} + \frac{(ab^3)^{\frac14} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{\sqrt{2}\, b^3c^2 - 2\sqrt{2}\, ab^2cd + \sqrt{2}\, a^2bd^2} + \frac{(ab^3)^{\frac14} a \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14} + x + \sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\, b^3c^2 - 2\sqrt{2}\, ab^2cd + \sqrt{2}\, a^2bd^2\right)} - \frac{(ab^3)^{\frac14} a \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14} + x + \sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\, b^3c^2 - 2\sqrt{2}\, ab^2cd + \sqrt{2}\, a^2bd^2\right)} + \frac{\left((cd^3)^{\frac14} bc - 5(cd^3)^{\frac14} ad\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{4\left(\sqrt{2}\, b^2c^2d^2 - 2\sqrt{2}\, abcd^3 + \sqrt{2}\, a^2d^4\right)} + \frac{\left((cd^3)^{\frac14} bc - 5(cd^3)^{\frac14} ad\right) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{4\left(\sqrt{2}\, b^2c^2d^2 - 2\sqrt{2}\, abcd^3 + \sqrt{2}\, a^2d^4\right)} + \frac{\left((cd^3)^{\frac14} bc - 5(cd^3)^{\frac14} ad\right) \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\, b^2c^2d^2 - 2\sqrt{2}\, abcd^3 + \sqrt{2}\, a^2d^4\right)} - \frac{\left((cd^3)^{\frac14} bc - 5(cd^3)^{\frac14} ad\right) \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\, b^2c^2d^2 - 2\sqrt{2}\, abcd^3 + \sqrt{2}\, a^2d^4\right)} - \frac{c\sqrt{x}}{2(bcd - ad^2)(dx^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) + (a*b^3
)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) + 1/2*(a*b^3)
^(1/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^2
- 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) - 1/2*(a*b^3)^(1/4)*a*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*
c*d + sqrt(2)*a^2*b*d^2) + 1/4*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^
2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^4) + 1/4*((c*d^3)^(1/4)*b*c
 - 5*(c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x
))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^
4) + 1/8*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d
)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(
2)*a^2*d^4) - 1/8*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*log(-sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d
^3 + sqrt(2)*a^2*d^4) - 1/2*c*sqrt(x)/((b*c*d - a*d^2)*(d*x^2 + c))
```

Mupad [B]
time = 1.29, size = 2500, normalized size = 4.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] atan((((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c
^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 +
 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*((2*(a^3*b^8*c^7 - 19*a^4*b^
7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 +
 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d
^3) + ((2*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^
7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^
5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*a^3*b^12*c^10*d^5 -
 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8
+ 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^1
1 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13))/(a^3*d^4 - b^3*c^3*d
 + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512
*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 1111
04*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 31
7440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13
 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2
*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-
a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 -
896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*
b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(3/4)) + (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8
*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 -
160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5
*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6
*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a
^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c
^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*1i - ((-a^5/(16*b^9*
c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*
c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6
- 128*a*b^8*c^7*d))^(1/4)*((2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6
*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5)
)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + ((2*(-a^5/(16*b
^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b
^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d
^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11*c^9*d
^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^8*c^6*
d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*b^5*c^
3*d^12 + 5120*a^11*b^4*c^2*d^13))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
```

3*a^2*b*c*d^3) - (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(3/4)) - (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*1i)/(((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*((2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + ((2*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a^13*b...

3.473 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=528

$$\frac{x^{3/2}}{2(bc-ad)(c+dx^2)}+\frac{a^{3/4}\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{a^{3/4}\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{(bc+3ad)\,\mathrm{ta}}{4\sqrt{2}}$$

```
[Out] 1/2*x^(3/2)/(-a*d+b*c)/(d*x^2+c)+1/2*a^(3/4)*b^(1/4)*arctan(1-b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^2*2^(1/2)-1/2*a^(3/4)*b^(1/4)*arctan(1+b^(1/
4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^2*2^(1/2)-1/8*(3*a*d+b*c)*arctan(1-d
^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/8*(3
*a*d+b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(3/4)/(-a*d+b
*c)^2*2^(1/2)-1/4*a^(3/4)*b^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1
/2)*x^(1/2))/(-a*d+b*c)^2*2^(1/2)+1/4*a^(3/4)*b^(1/4)*ln(a^(1/2)+x*b^(1/2)+
a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^2*2^(1/2)+1/16*(3*a*d+b*c)*ln(c
^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/d^(3/4)/(-a*d+b*c
)^2*2^(1/2)-1/16*(3*a*d+b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x
^(1/2))/c^(1/4)/d^(3/4)/(-a*d+b*c)^2*2^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 482, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{a^{3/4}\sqrt[4]{b}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{a^{3/4}\sqrt[4]{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{a^{3/4}\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}-\frac{(3ad+bc)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,\sqrt[4]{c}\,d^{3/4}(bc-ad)^2}+\frac{(3ad+bc)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,\sqrt[4]{c}\,d^{3/4}(bc-ad)^2}+\frac{(3ad+bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{c}\,d^{3/4}(bc-ad)^2}-\frac{(3ad+bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{c}\,d^{3/4}(bc-ad)^2}+\frac{x^{3/2}}{2\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] x^(3/2)/(2*(b*c - a*d)*(c + d*x^2)) + (a^(3/4)*b^(1/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - ((b*c +
 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d
^(3/4)*(b*c - a*d)^2) + ((b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c -
a*d)^2) + (a^(3/4)*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) + ((b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a
*d)^2) - ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^6}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$
$$= \frac{x^{3/2}}{2(bc-ad)(c+dx^2)} - \frac{\text{Subst}\left(\int \frac{x^2(3a-bx^4)}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$
$$= \frac{x^{3/2}}{2(bc-ad)(c+dx^2)} - \frac{\text{Subst}\left(\int \left(\frac{4abx^2}{(bc-ad)(a+bx^4)} - \frac{(bc+3ad)x^2}{(bc-ad)(c+dx^4)}\right)dx, x, \sqrt{x}\right)}{2(bc-ad)}$$
$$= \frac{x^{3/2}}{2(bc-ad)(c+dx^2)} - \frac{(2ab)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(bc+3ad)\text{Subst}\left(\int\right.}{2(bc-}$$
$$= \frac{x^{3/2}}{2(bc-ad)(c+dx^2)} + \frac{\left(a\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{\left(a\sqrt{b}\right)}{}$$
$$= \frac{x^{3/2}}{2(bc-ad)(c+dx^2)} - \frac{a\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^2} - \frac{a\text{Subst}}{}$$
$$= \frac{x^{3/2}}{2(bc-ad)(c+dx^2)} - \frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2} + \frac{a^{3/4}}{}$$
$$= \frac{x^{3/2}}{2(bc-ad)(c+dx^2)} + \frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2} - \frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(\right.}{\sqrt{2}(b}$$

<u>Mathematica</u> [A]
time = 0.79, size = 269, normalized size = 0.51

$$\frac{\frac{4(bc-ad)x^{3/2}}{c+dx^2} + 4\sqrt{2}a^{3/4}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - \frac{\sqrt{2}(bc+3ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{c}d^{3/4}} + 4\sqrt{2}a^{3/4}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right) - \frac{\sqrt{2}(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt[4]{c}d^{3/4}}}{8(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((4*(b*c - a*d)*x^(3/2))/(c + d*x^2) + 4*Sqrt[2]*a^(3/4)*b^(1/4)*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(b*c + 3*a
*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(1/
4)*d^(3/4)) + 4*Sqrt[2]*a^(3/4)*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])/(Sqrt[a] + Sqrt[b]*x)] - (Sqrt[2]*(b*c + 3*a*d)*ArcTanh[(Sqrt[2]*c^(
```

```
1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(1/4)*d^(3/4)))/(8*(b*c -
a*d)^2)
```

Maple [A]
time = 0.09, size = 264, normalized size = 0.50

method	result
derivativedivides	$-\frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2\left(-\frac{ad}{4}+\right.}{dx^{2}}$
default	$-\frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2\left(-\frac{ad}{4}+\right.}{dx^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2/(a*d-b*c)^
2*((-1/4*a*d+1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(3/4*a*d+1/4*b*c)/d/(c/d)^(1/4)
*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 436, normalized size = 0.83

$$-\frac{ab\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)}+\frac{(bc+3ad)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)}+\frac{x^{\frac{3}{2}}}{2\left(bc^{2}-acd+\left(bcd-ad^{2}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(
b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/16*(b*c + 3*a*d)*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))
```

```
/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c
^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sq
rt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
 sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
+ sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) +
 1/2*x^(3/2)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3393 vs. $2(385) = 770$.
time = 12.19, size = 3393, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d +
 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*
c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56
*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/
4)*arctan(((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt((b^6*c^6 + 18*a*b^5*c^5
*d + 135*a^2*b^4*c^4*d^2 + 540*a^3*b^3*c^3*d^3 + 1215*a^4*b^2*c^2*d^4 + 145
8*a^5*b*c*d^5 + 729*a^6*d^6)*x - (b^8*c^9*d + 8*a*b^7*c^8*d^2 + 12*a^2*b^6*
c^7*d^3 - 40*a^3*b^5*c^6*d^4 - 74*a^4*b^4*c^5*d^5 + 120*a^5*b^3*c^4*d^6 + 1
08*a^6*b^2*c^3*d^7 - 216*a^7*b*c^2*d^8 + 81*a^8*c*d^9)*sqrt(-(b^4*c^4 + 12*
a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d
^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4
*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8
*c*d^11)))*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d
^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a
^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d
^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4) - (b^5*c^5*d + 7*a*b^4*c^4*d^2 +
 10*a^2*b^3*c^3*d^3 - 18*a^3*b^2*c^2*d^4 - 27*a^4*b*c*d^5 + 27*a^5*d^6)*sqr
t(x)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 8
1*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5
*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8
*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4))/(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^
2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)) - 16*(-a^3*b/(b^8*c^8 - 8*a*b^7*
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)*arctan((sqrt(a^4*b^2*x - (a^3*b^5*c^4 - 4*
a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*sqrt(-a^3*
b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)))*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d
```

```
 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4) - (a^2*b^3
*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*sqrt(x))/(a^3*b)) +
 4*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d
^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b
*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(a^2*b*sq
rt(x) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a^3*b/(b^8*c^8 - 8*a*b^7*
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)) - 4*
(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3
+ 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*
d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(a^2*b*sqrt(
x) - (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)) - (b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c
^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*
a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*
d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4)*log((b^6*c
^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d^4 - 20*a^3*b^3*c^4*d^5 + 15*a^4
*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 5
4*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^
8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a
^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(3/4)
 + (b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)*sqrt(x)) + (b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c
^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*
a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*
d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4)*log(-(b^6*
c^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d^4 - 20*a^3*b^3*c^4*d^5 + 15*a^
4*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)*(-(b^4*c^4 + 12*a*b^3*c^3*d +
54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c
^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*
a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.10, size = 683, normalized size = 1.29

$$\frac{\left((cd^3)^{\frac{3}{4}}bc + 3(cd^3)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^3d^3 - 2\sqrt{2}\,abc^2d^4 + \sqrt{2}\,a^2cd^5\right)} + \frac{\left((cd^3)^{\frac{3}{4}}bc + 3(cd^3)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^3d^3 - 2\sqrt{2}\,abc^2d^4 + \sqrt{2}\,a^2cd^5\right)} - \frac{\left((cd^3)^{\frac{3}{4}}bc + 3(cd^3)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^3d^3 - 2\sqrt{2}\,abc^2d^4 + \sqrt{2}\,a^2cd^5\right)} + \frac{\left((cd^3)^{\frac{3}{4}}bc + 3(cd^3)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^3d^3 - 2\sqrt{2}\,abc^2d^4 + \sqrt{2}\,a^2cd^5\right)} - \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^4c^2 - 2\sqrt{2}\,ab^3cd + \sqrt{2}\,a^2b^2d^2} - \frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^4c^2 - 2\sqrt{2}\,ab^3cd + \sqrt{2}\,a^2b^2d^2} + \frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^4c^2 - 2\sqrt{2}\,ab^3cd + \sqrt{2}\,a^2b^2d^2\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^4c^2 - 2\sqrt{2}\,ab^3cd + \sqrt{2}\,a^2b^2d^2\right)} + \frac{x^{\frac{3}{2}}}{2(dx^2+c)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c
^2*d^4 + sqrt(2)*a^2*c*d^5) + 1/4*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2
)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) - 1/8*((c*d^3)^(
3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) + 1
/8*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*
a^2*c*d^5) - (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*
d^2) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/
(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2)
+ 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/2*(a*b^3)^(3/4)
*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^2 - 2*sqr
t(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) + 1/2*x^(3/2)/((d*x^2 + c)*(b*c - a*d
))
```

Mupad [B]
time = 1.27, size = 2500, normalized size = 4.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] atan(((((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^
4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^
7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 +
 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 - 21*
a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*
d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + (x^(1/2)*(-(a^3*b)/(16*a^8*d^8 + 16*
b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4
```

- 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c
d^7))^(1/4)(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^1
3*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7
*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a
^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6
*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d
^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 4
48*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b
^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4
)*1i + (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 +
54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5)*1i)/(a^6*d^
6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*
a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*
c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4) -
(((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^4 - 1
3856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^7 + 7
4368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 + 1033
6*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 - 21*a^2*b
^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 +
7*a*b^6*c^6*d - 7*a^6*b*c*d^6) - (x^(1/2)*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c
^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896
*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7)
)^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^1
0*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10
*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a^10*b
^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6
+ b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^
2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^
3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4)*1i
- (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^
5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5)*1i)/(a^6*d^6 + b
^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b
^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d
^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4))/((a^4
*b^9*c^5*d + 108*a^8*b^5*c*d^5 + 13*a^5*b^8*c^4*d^2 + 63*a^6*b^7*c^3*d^3 +
135*a^7*b^6*c^2*d^4)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c
^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*
c*d^6) + (((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10
*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7
*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^1
0 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 -
21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c

```
^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + (x^(1/2)*(-(a^3*b)/(16*a^8*d^8 +
16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d
^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*
b*c*d^7))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*
b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*
a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 12083
2*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(
a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8
+ 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^
5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(
3/4) + (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 +
54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5))/(a^6*d^6 +
 b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^...
```

3.474 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=528

$$\frac{\sqrt{x}}{2(bc-ad)(c+dx^2)}+\frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{(3bc+ad)\,\mathrm{ta}}{4\sqrt{2}}$$

```
[Out] 1/2*a^(1/4)*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^2*
2^(1/2)-1/2*a^(1/4)*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d
+b*c)^2*2^(1/2)-1/8*(a*d+3*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c
^(3/4)/d^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/8*(a*d+3*b*c)*arctan(1+d^(1/4)*2^(1/2
)*x^(1/2)/c^(1/4))/c^(3/4)/d^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/4*a^(1/4)*b^(3/4)
*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^2*2^(1/2)
-1/4*a^(1/4)*b^(3/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/
(-a*d+b*c)^2*2^(1/2)-1/16*(a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*
2^(1/2)*x^(1/2))/c^(3/4)/d^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/16*(a*d+3*b*c)*ln(c
^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(1/4)/(-a*d+b*c
)^2*2^(1/2)+1/2*x^(1/2)/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A]
time = 0.31, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 482, 536, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{a}\,b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{\sqrt[4]{a}\,b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{(ad+3bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}+\frac{(ad+3bc)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}+\frac{\sqrt[4]{a}\,b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}-\frac{\sqrt[4]{a}\,b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}-\frac{(ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}+\frac{(ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}+\frac{\sqrt{x}}{2\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*(b*c - a*d)*(c + d*x^2)) + (a^(1/4)*b^(3/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a^(1/4)*b^(3/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - ((3*b*c
 + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d
^(1/4)*(b*c - a*d)^2) + ((3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + (a^(1/4)*b^(3/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c -
a*d)^2) - (a^(1/4)*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) - ((3*b*c + a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a
*d)^2) + ((3*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^4}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}}{2(bc-ad)(c+dx^2)} - \frac{\text{Subst}\left(\int \frac{a-3bx^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= \frac{\sqrt{x}}{2(bc-ad)(c+dx^2)} - \frac{(2ab)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(3bc+ad)\text{Subst}\left(\int \frac{}{c+}\right.}{2(bc-a}$$

$$= \frac{\sqrt{x}}{2(bc-ad)(c+dx^2)} - \frac{\left(\sqrt{a}\,b\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{\left(\sqrt{a}\,b\right)\text{Su}}{}$$

$$= \frac{\sqrt{x}}{2(bc-ad)(c+dx^2)} - \frac{\left(\sqrt{a}\,\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^2} -$$

$$= \frac{\sqrt{x}}{2(bc-ad)(c+dx^2)} + \frac{\sqrt[4]{a}\,b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2} - \frac{\sqrt[4]{a}\,b^{3}}{}$$

$$= \frac{\sqrt{x}}{2(bc-ad)(c+dx^2)} + \frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2} - \frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(1\right.}{\sqrt{2}\,(bc}$$

Mathematica [A]
time = 0.79, size = 268, normalized size = 0.51

$$\frac{\frac{4(bc-ad)\sqrt{x}}{c+dx^2}+4\sqrt{2}\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\frac{\sqrt{2}\,(3bc+ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{3/4}\sqrt[4]{d}}-4\sqrt{2}\sqrt[4]{a}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)+\frac{\sqrt{2}\,(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{3/4}\sqrt[4]{d}}}{8(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((4*(b*c - a*d)*Sqrt[x])/(c + d*x^2) + 4*Sqrt[2]*a^(1/4)*b^(3/4)*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(3*b*c + a
*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(3/
4)*d^(1/4)) - 4*Sqrt[2]*a^(1/4)*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])/(Sqrt[a] + Sqrt[b]*x)] + (Sqrt[2]*(3*b*c + a*d)*ArcTanh[(Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(3/4)*d^(1/4)))/(8*(b*c -
a*d)^2)
```

Maple [A]
time = 0.08, size = 263, normalized size = 0.50

method	result
derivativedivides	$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2}+$
default	$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2/(a*d-b*c)^
2*((-1/4*a*d+1/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(a*d+3*b*c)*(c/d)^(1/4)/c*2^(1
/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.52, size = 461, normalized size = 0.87

$$-\frac{\left(\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)a}{4\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{\frac{2\sqrt{2}\,(3\,bc+ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(3\,bc+ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(3\,bc+ad)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,bc+ad)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{\sqrt{x}}{2\left(bc^2-acd+(bcd-ad^2)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*
b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)
*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))*a/(b^
2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/16*(2*sqrt(2)*(3*b*c + a*d)*arctan(1/2*sqr
t(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(
sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b*c + a*d)*arctan(-1/2*sqrt(2
)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqr
t(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(3*b*c + a*d)*log(sqrt(2)*c^(1/4)*d^(
1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(3*b*c + a*
d)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(
1/4)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*sqrt(x)/(b*c^2 - a*c*d + (b*c*
d - a*d^2)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3171 vs. $2(385) = 770$.
time = 4.53, size = 3171, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*
d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^
10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*
a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4
)*arctan(((b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^3*b^3*c^
5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7)*sqrt((9*b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 -
4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2
*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 2
8*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^
6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9)))*(-(81*b^4*c^4
 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^
11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*
b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a
^8*c^3*d^9))^(3/4) - (3*b^7*c^9*d - 17*a*b^6*c^8*d^2 + 39*a^2*b^5*c^7*d^3 -
 45*a^3*b^4*c^6*d^4 + 25*a^4*b^3*c^5*d^5 - 3*a^5*b^2*c^4*d^6 - 3*a^6*b*c^3*
d^7 + a^7*c^2*d^8)*sqrt(x)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2
```

*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(3/4))/(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)) - 16*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*arctan(((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)*sqrt(b^2*x + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))) - (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)*sqrt(x))/(a*b^3)) + (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)*log((3*b*c + a*d)*sqrt(x) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)) - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)*log((3*b*c + a*d)*sqrt(x) - (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)) - 4*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(b*sqrt(x) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)) + 4*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(b*sqrt(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^

```
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8))^(1/4)) + 4*sqrt(x))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.42, size = 655, normalized size = 1.24

$$\frac{\left(3\,(cd^3)^{\frac{1}{4}}bc+(cd^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^3d-2\sqrt{2}\,abc^2d^2+\sqrt{2}\,a^2cd^3\right)}+\frac{\left(3\,(cd^3)^{\frac{1}{4}}bc+(cd^3)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^3d-2\sqrt{2}\,abc^2d^2+\sqrt{2}\,a^2cd^3\right)}+\frac{\left(3\,(cd^3)^{\frac{1}{4}}bc+(cd^3)^{\frac{1}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^3d-2\sqrt{2}\,abc^2d^2+\sqrt{2}\,a^2cd^3\right)}-\frac{\left(3\,(cd^3)^{\frac{1}{4}}bc+(cd^3)^{\frac{1}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^3d-2\sqrt{2}\,abc^2d^2+\sqrt{2}\,a^2cd^3\right)}-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^2c^2-2\sqrt{2}\,abcd+\sqrt{2}\,a^2d^2}-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^2c^2-2\sqrt{2}\,abcd+\sqrt{2}\,a^2d^2}-\frac{(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^2c^2-2\sqrt{2}\,abcd+\sqrt{2}\,a^2d^2\right)}+\frac{(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^2c^2-2\sqrt{2}\,abcd+\sqrt{2}\,a^2d^2\right)}+\frac{\sqrt{x}}{2\,(dx^2+c)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2
*d^2 + sqrt(2)*a^2*c*d^3) + 1/4*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*
b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) + 1/8*(3*(c*d^3)^(1/
4)*b*c + (c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) - 1/8*(3*
(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^
3) - (a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - (a*b^3)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(
sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - 1/2*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c^2 - 2*sqrt(
2)*a*b*c*d + sqrt(2)*a^2*d^2) + 1/2*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2
*d^2) + 1/2*sqrt(x)/((d*x^2 + c)*(b*c - a*d))
```

Mupad [B]
time = 1.26, size = 2500, normalized size = 4.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] - atan(((((2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3
*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((x^(1
/2)*(256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 304
64*a^3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 1
98656*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 -
 36352*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13)
)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (2*(-(a*b^3)/(16*a^8*d^8 + 16*b
^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 -
 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*
d^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12
*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b
^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6
*c^3*d^11 - 4096*a^10*b^5*c^2*d^12))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 8
96*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b
^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4))*(-(a*b^3)/(16*a^8*d
^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*
c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128
*a^7*b*c*d^7))^(1/4) + (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^
2*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^
6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*
b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^
6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^
5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*1i - ((
(2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2*d
^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - ((x^(1/2)*(256*a
^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b^1
4*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^6*
b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a^9
*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d^6
 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (2*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 4
48*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b
^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4
)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 +
 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8
 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11
- 4096*a^10*b^5*c^2*d^12))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5
*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6
 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4))*(-(a*b^3)/(16*a^8*d^8 + 16*b^
8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 -
896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d
^7))^(1/4) - (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^2*b^11*c^4
```

```
*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
- 6*a^5*b*c*d^5))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2
- 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^
6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*1i)/((((2*(51*a^4
*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2*d^4))/(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((x^(1/2)*(256*a^13*b^4*d^
15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b^14*c^10*d^5
 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^6*b^11*c^7*d
^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a^9*b^8*c^4*d
^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d^6 + b^6*c^6
 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c
^5*d - 6*a^5*b*c*d^5) + (2*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6
*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5
 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*(1024*a^
11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 + 106496*a^
4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8 - 157696*
a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11 - 4096*a^1
0*b^5*c^2*d^12))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a*
b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 +
 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b
^7*c^7*d - 128*a^7*b*c*d^7))^(3/4))*(-(a*b^3)/(...
```

3.475 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=536

$$-\frac{dx^{3/2}}{2c(bc-ad)(c+dx^2)}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}+\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}+\frac{\sqrt[4]{d}(5bc-ad)\,\mathrm{t}}{4\sqrt{2}}$$

```
[Out] -1/2*d*x^(3/2)/c/(-a*d+b*c)/(d*x^2+c)-1/2*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)*
x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/2*b^(5/4)*arctan(1+b^(1/4)*
2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/8*d^(1/4)*(-a*d+5*b
*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*c)^2*2^(1/2)-
1/8*d^(1/4)*(-a*d+5*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/
(-a*d+b*c)^2*2^(1/2)+1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/
2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^2*2^(1/2)-1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)+a
^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(1/4)*(
-a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(
-a*d+b*c)^2*2^(1/2)+1/16*d^(1/4)*(-a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*
d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a*d+b*c)^2*2^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 483, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}+\frac{b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2}+\frac{\sqrt[4]{d}(5bc-ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}(bc-ad)^2}-\frac{\sqrt[4]{d}(5bc-ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{5/4}(bc-ad)^2}+\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2}-\frac{\sqrt[4]{d}(5bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{d}(5bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^2}-\frac{dx^{3/2}}{2c(c+dx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)) - (b^(5/4)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^2) + (b^(5/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^2)
+ (d^(1/4)*(5*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*
Sqrt[2]*c^(5/4)*(b*c - a*d)^2) - (d^(1/4)*(5*b*c - a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^2) + (b^(5/4)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4
)*(b*c - a*d)^2) - (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^2) - (d^(1/4)*(5*b*c - a*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)
*(b*c - a*d)^2) + (d^(1/4)*(5*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^2}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{x^2(4bc-ad-bdx^4)}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)(c+dx^2)} + \frac{\text{Subst}\left(\int \left(\frac{4b^2cx^2}{(bc-ad)(a+bx^4)} + \frac{d(-5bc+ad)x^2}{(bc-ad)(c+dx^4)}\right)dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)(c+dx^2)} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{(d(5bc-ad))\text{Sub}}{2c(b}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)(c+dx^2)} - \frac{b^{3/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{b^{3/2}\text{Subst}\Big(\int}{}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)(c+dx^2)} + \frac{b\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^2} + \frac{b\text{Subst}}{}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)(c+dx^2)} + \frac{b^{5/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2} - \frac{b^{5/4}\log}{}$$

$$= -\frac{dx^{3/2}}{2c(bc-ad)(c+dx^2)} - \frac{b^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2} + \frac{b^{5/4}\tan^{-1}\Big(1+\frac{\sqrt{2}}{}}{\sqrt{2}\sqrt[4]{a}(bc-}$$

Mathematica [A]
time = 1.01, size = 273, normalized size = 0.51

$$\frac{\frac{4d(-bc+ad)x^{3/2}}{c(c+dx^2)} - \frac{4\sqrt{2}\,b^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}} + \frac{\sqrt{2}\sqrt[4]{d}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{5/4}} - \frac{4\sqrt{2}\,b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{a}} + \frac{\sqrt{2}\sqrt[4]{d}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{5/4}}}{8(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((4*d*(-(b*c) + a*d)*x^(3/2))/(c*(c + d*x^2)) - (4*Sqrt[2]*b^(5/4)*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(1/4) + (Sqrt[2]
*d^(1/4)*(5*b*c - a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x])])/c^(5/4) - (4*Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4) + (Sqrt[2]*d^(1/4)*(5*b*c - a*d)*A
```

```
rcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(5/4))/(
8*(b*c - a*d)^2)
```

Maple [A]
time = 0.09, size = 270, normalized size = 0.50

method	result
derivativedivides	$\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2d\left(\frac{(ad-}{4c(d}\right.}{}$
default	$\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2d\left(\frac{(ad-}{4c(d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2*d/(a*d-b*c)
^2*(1/4*(a*d-b*c)/c*x^(3/2)/(d*x^2+c)+1/32*(a*d-5*b*c)/c/d/(c/d)^(1/4)*2^(1
/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.54, size = 450, normalized size = 0.84

$$\frac{b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)}-\frac{dx^{\frac{3}{2}}}{2\left(bc^{3}-ac^{2}d+\left(bc^{2}d-acd^{2}\right)x^{2}\right)}-\frac{\left(5bcd-ad^{2}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{2}c^{3}-2abc^{2}d+a^{2}cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b
```

```
^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*d*x^(3/2)/(b*c^3 - a*c^2*d + (b*c^2*d -
 a*c*d^2)*x^2) - 1/16*(5*b*c*d - a*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt
(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(
c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(
d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c
^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
+ sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3457 vs. $2(393) = 786$.
time = 12.24, size = 3457, normalized size = 6.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*d*x^(3/2) + 4*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(625*b^
4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^
5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 +
 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6
*d^7 + a^8*c^5*d^8))^(1/4)*arctan(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt
((15625*b^6*c^6*d^2 - 18750*a*b^5*c^5*d^3 + 9375*a^2*b^4*c^4*d^4 - 2500*a^3
*b^3*c^3*d^5 + 375*a^4*b^2*c^2*d^6 - 30*a^5*b*c*d^7 + a^6*d^8)*x - (625*b^8
*c^11*d - 3000*a*b^7*c^10*d^2 + 5900*a^2*b^6*c^9*d^3 - 6120*a^3*b^5*c^8*d^4
 + 3606*a^4*b^4*c^7*d^5 - 1224*a^5*b^3*c^6*d^6 + 236*a^6*b^2*c^5*d^7 - 24*a
^7*b*c^4*d^8 + a^8*c^3*d^9)*sqrt(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*
a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28
*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c
^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)))*(-(625*b^4*c
^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/
(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70
*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^
7 + a^8*c^5*d^8))^(1/4) + (125*b^5*c^6*d - 325*a*b^4*c^5*d^2 + 290*a^2*b^3*
c^4*d^3 - 106*a^3*b^2*c^3*d^4 + 17*a^4*b*c^2*d^5 - a^5*c*d^6)*sqrt(x)*(-(62
5*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^
4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d
^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b
*c^6*d^7 + a^8*c^5*d^8))^(1/4))/(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^
2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)) + 16*(-b^5/(a*b^8*c^8 - 8*a^2*b^
7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56
*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*
c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*arctan((sqrt(b^8*x - (a*b^9*c^4 -
4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*sqrt(-
```

```
b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3
+ 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*
d^7 + a^9*d^8)))*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 -
56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c
^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2) -
(b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d +
28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c
^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*sqrt(x))/b^5)
 - 4*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c
^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a
^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*lo
g(b^4*sqrt(x) + (a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*
b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6)*(-b^5/(a*b^8*c^
8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*
c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8
))^(3/4)) + 4*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*
a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*
d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2
)*x^2)*log(b^4*sqrt(x) - (a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2
- 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6)*(-b^5/
(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70
*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7
+ a^9*d^8))^(3/4)) - (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(625*b^4
*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5
)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 +
70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*
d^7 + a^8*c^5*d^8))^(1/4)*log((b^6*c^10 - 6*a*b^5*c^9*d + 15*a^2*b^4*c^8*d^
2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a^5*b*c^5*d^5 + a^6*c^4*d^6
)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d
^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5
*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 -
8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(3/4) - (125*b^3*c^3*d - 75*a*b^2*c^2*d^2 +
 15*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)) + (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2
)*x^2)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*
b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^
3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d
^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(1/4)*log(-(b^6*c^10 - 6*a*b^5*c^9*d +
 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a^5*b*c^5
*d^5 + a^6*c^4*d^6)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*
d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.60, size = 701, normalized size = 1.31

$$-\frac{\left(5\,(cd^3)^{\frac{3}{4}}bc-(cd^3)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^4d^2-2\sqrt{2}\,abc^3d^3+\sqrt{2}\,a^2c^2d^4\right)}-\frac{\left(5\,(cd^3)^{\frac{3}{4}}bc-(cd^3)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^4d^2-2\sqrt{2}\,abc^3d^3+\sqrt{2}\,a^2c^2d^4\right)}+\frac{\left(5\,(cd^3)^{\frac{3}{4}}bc-(cd^3)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^4d^2-2\sqrt{2}\,abc^3d^3+\sqrt{2}\,a^2c^2d^4\right)}-\frac{\left(5\,(cd^3)^{\frac{3}{4}}bc-(cd^3)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^4d^2-2\sqrt{2}\,abc^3d^3+\sqrt{2}\,a^2c^2d^4\right)}+\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3c^2-2\sqrt{2}\,a^2b^2cd+\sqrt{2}\,a^3bd^2}+\frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3c^2-2\sqrt{2}\,a^2b^2cd+\sqrt{2}\,a^3bd^2}-\frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^3c^2-2\sqrt{2}\,a^2b^2cd+\sqrt{2}\,a^3bd^2\right)}+\frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^3c^2-2\sqrt{2}\,a^2b^2cd+\sqrt{2}\,a^3bd^2\right)}-\frac{dx^{\frac{3}{2}}}{2\,(bc^2-acd)(dx^2+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*
(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*
c^3*d^3 + sqrt(2)*a^2*c^2*d^4) - 1/4*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqr
t(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d^4) + 1/8*(5*(c
*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
sqrt(c/d))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d
^4) - 1/8*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c
/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + s
qrt(2)*a^2*c^2*d^4) + (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqr
t(2)*a^3*b*d^2) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2
)*a^3*b*d^2) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt
(a/b))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*a^3*b*d^2) + 1/
2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*
a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*a^3*b*d^2) - 1/2*d*x^(3/2)/((b*
c^2 - a*c*d)*(d*x^2 + c))
```

Mupad [B]
time = 1.23, size = 2500, normalized size = 4.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] 2*atan(((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*
c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5
+ 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16*a^9*d^8 + 16*a*b
^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 11
20*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*
```

c*d^7))^(3/4)*(((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) + (x^(1/2)*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(3/4)*(((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) - (x^(1/2)*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)))/((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6* c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(3/4)*(((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) + (x^(1/2)*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d...

3.476 $\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx$

Optimal. Leaf size=536

$$-\frac{d\sqrt{x}}{2c(bc-ad)\,(c+dx^2)}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\,\mathrm{t}}{4\sqrt{2}}$$

```
[Out] -1/2*b^(7/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)^2
*2^(1/2)+1/2*b^(7/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-a*
d+b*c)^2*2^(1/2)+1/8*d^(3/4)*(-3*a*d+7*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2
)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/8*d^(3/4)*(-3*a*d+7*b*c)*arctan(1
+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/4*b^(7/4)*
ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^2*
2^(1/2)+1/4*b^(7/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/16*d^(3/4)*(-3*a*d+7*b*c)*ln(c^(1/2)+x*d^(1/2
)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(3/4
)*(-3*a*d+7*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7
/4)/(-a*d+b*c)^2*2^(1/2)-1/2*d*x^(1/2)/c/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A]
time = 0.36, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {477, 425, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d\sqrt{x}}{2c\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^2)
+ (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
4*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(3/4)*(b*c - a*d)^2) + (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*
d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c
^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{(a+bx^4)\,(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\,(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{4bc-3ad-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\,(c+dx^2)} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{(d(7bc-3a\ldots}{\ldots}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\,(c+dx^2)} + \frac{b^2\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\,(bc-ad)^2} + \frac{b^2\text{Subst}\ldots}{\ldots}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\,(c+dx^2)} + \frac{b^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\,(bc-ad)^2} \ldots$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\,(c+dx^2)} - \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2} + \ldots$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\,(c+dx^2)} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\tan^{-1}\left(1\ldots\right)}{\sqrt{2}\,a^{3/\ldots}}$$

Mathematica [A]

time = 1.01, size = 273, normalized size = 0.51

$$\frac{\frac{4d(-bc+ad)\sqrt{x}}{c(c+dx^2)}-\frac{4\sqrt{2}\,b^{7/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{3/4}}+\frac{\sqrt{2}\,d^{3/4}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{7/4}}+\frac{4\sqrt{2}\,b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}}+\frac{\sqrt{2}\,d^{3/4}(-7bc+3ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{7/4}}}{8(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((4*d*(-(b*c) + a*d)*Sqrt[x])/(c*(c + d*x^2)) - (4*Sqrt[2]*b^(7/4)*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (Sqrt[2]
*d^(3/4)*(7*b*c - 3*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x])])/c^(7/4) + (4*Sqrt[2]*b^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4) + (Sqrt[2]*d^(3/4)*(-7*b*c + 3*a
*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(7/
4))/(8*(b*c - a*d)^2)
```

Maple [A]

time = 0.09, size = 273, normalized size = 0.51

method	result
derivativedivides	$\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2a}+\frac{2d}{}$
default	$\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2a}+\frac{2d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2*d/(a*d-
b*c)^2*(1/4*(a*d-b*c)/c*x^(1/2)/(d*x^2+c)+1/32*(3*a*d-7*b*c)/c^2*(c/d)^(1/4
)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.56, size = 489, normalized size = 0.91

$$-\frac{d\sqrt{x}}{2\,(bc^3-ac^2d+(bc^2d-acd^2)x^2)}+\frac{\frac{2\,\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\,(b^2c^2-2\,abcd+a^2d^2)}-\frac{\frac{2\,\sqrt{2}\,(7\,bcd-3\,ad^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\,(7\,bcd-3\,ad^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(7\,bcd-3\,ad^2)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(7\,bcd-3\,ad^2)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\,(b^2c^3-2\,abc^2d+a^2cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*d*sqrt(x)/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) + 1/4*(2*sqrt(2)
*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan(-1
/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(-sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(b^2*c^2 - 2*a*b
*c*d + a^2*d^2) - 1/16*(2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)
*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(2)*c^(1/4
)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(7*b*c
*d - 3*a*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(
c^(3/4)*d^(1/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3310 vs. $2(393) = 786$.
time = 12.91, size = 3310, normalized size = 6.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(2401*b^4*c^4*d^3 - 41
16*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^
8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^
4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7
 + a^8*c^7*d^8))^(1/4)*arctan(((b^6*c^11 - 6*a*b^5*c^10*d + 15*a^2*b^4*c^9*
d^2 - 20*a^3*b^3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*a^5*b*c^6*d^5 + a^6*c^5*d
^6)*sqrt((49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)*x + (b^4*c^8 - 4*a*b^3
*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(-(2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81
*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^1
2*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*
a^7*b*c^8*d^7 + a^8*c^7*d^8)))*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2
```

646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4) + (7*b^7*c^12*d - 45*a*b^6*c^11*d^2 + 123*a^2*b^5*c^10*d^3 - 185*a^3*b^4*c^9*d^4 + 165*a^4*b^3*c^8*d^5 - 87*a^5*b^2*c^7*d^6 + 25*a^6*b*c^6*d^7 - 3*a^7*c^5*d^8)*sqrt(x)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4))/(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)) + 16*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*arctan(((a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(3/4)*sqrt(b^4*x + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))) - (a^2*b^8*c^6 - 6*a^3*b^7*c^5*d + 15*a^4*b^6*c^4*d^2 - 20*a^5*b^5*c^3*d^3 + 15*a^6*b^4*c^2*d^4 - 6*a^7*b^3*c*d^5 + a^8*b^2*d^6)*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(3/4)*sqrt(x))/b^7) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*log(b^2*sqrt(x) + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) - 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*log(b^2*sqrt(x) - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) + (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*sqrt(x) + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4

```
*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^
8*c^7*d^8))^(1/4)) - (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(2401*b^
4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 8
1*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^
12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8
*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*sqrt(x) - (b^
2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7
*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 -
 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2/x**(1/2),x)
[Out] Timed out
```

Giac [**A**]
time = 1.78, size = 673, normalized size = 1.26

$$\frac{(ab^3)^{\frac{1}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^2c^2-2\sqrt{2}\,a^2bcd+\sqrt{2}\,a^3d^2}+\frac{(ab^3)^{\frac{1}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^2c^2-2\sqrt{2}\,a^2bcd+\sqrt{2}\,a^3d^2}+\frac{(ab^3)^{\frac{1}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^2c^2-2\sqrt{2}\,a^2bcd+\sqrt{2}\,a^3d^2\right)}-\frac{(ab^3)^{\frac{1}{4}}b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^2c^2-2\sqrt{2}\,a^2bcd+\sqrt{2}\,a^3d^2\right)}-\frac{\left(7(cd^3)^{\frac{1}{4}}bc-3(cd^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^4-2\sqrt{2}\,abc^3d+\sqrt{2}\,a^2c^2d^2\right)}-\frac{\left(7(cd^3)^{\frac{1}{4}}bc-3(cd^3)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^4-2\sqrt{2}\,abc^3d+\sqrt{2}\,a^2c^2d^2\right)}-\frac{\left(7(cd^3)^{\frac{1}{4}}bc-3(cd^3)^{\frac{1}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^4-2\sqrt{2}\,abc^3d+\sqrt{2}\,a^2c^2d^2\right)}+\frac{\left(7(cd^3)^{\frac{1}{4}}bc-3(cd^3)^{\frac{1}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^4-2\sqrt{2}\,abc^3d+\sqrt{2}\,a^2c^2d^2\right)}-\frac{d\sqrt{x}}{2(bc^2-acd)(dx^2+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
[Out] (a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + (a*b^3
)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*b^3)
^(1/4)*b*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^2*c^
2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/2*(a*b^3)^(1/4)*b*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*
b*c*d + sqrt(2)*a^3*d^2) - 1/4*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*
arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*
b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/4*(7*(c*d^3)^(1/4)
*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sq
rt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^
2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqr
t(2)*a^2*c^2*d^2) + 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(-sq
rt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b
*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/2*d*sqrt(x)/((b*c^2 - a*c*d)*(d*x^2 + c))
```

Mupad [B]
time = 1.42, size = 2500, normalized size = 4.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] atan(((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6
*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5
 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((-b^7/(16*a^11*d^8 + 16*
a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3
 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a
^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2145*a*b^10*c
^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c^4*d^3
+ 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 -
128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^
4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^
(3/4)*((2*(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5
*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3
*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*
d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*
d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8
*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*
c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^
2 - 3*a*b^2*c^6*d) + (x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^16*c^14*d^5 +
 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a^4*b^13*c^11
*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 1511936*a^7*b^
10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^13 - 530432*
a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c^3*d^16 + 23
04*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b
^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + (
x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^
3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c
^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a
*b^5*c^9*d))*1i - (-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d +
 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8
*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((-b^7/(16*a^
11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6
*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2
*d^6 - 128*a^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2
145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 -
a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-b^7/(16*a^11*d^8 + 16*a^
3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 +
 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^1
```

0*b*c*d^7))^(3/4)*((2*(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7 *d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896 *a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*(28672*a^ 2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^ 4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728 *a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 286 72*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3* a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^1 6*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a ^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 15 11936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^ 13 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c ^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^ 5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5* c^9*d))) - (x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3* d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c ^6*d^4 - 6*a*b^5*c^9*d))*1i)/((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4 *b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d ^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d ^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448 *a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 + 448*b^11 *c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8)) /(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-b^7/(16*a^11 *d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b ^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d ^6 - 128*a^10*b*c*d^7))^(3/4)*((2*(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128 *a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c ^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*...

3.477 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=570

$$-\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}}-\frac{d}{2c(bc-ad)\sqrt{x}\,(c+dx^2)}+\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^2}-\frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^2}$$

```
[Out] 1/2*b^(9/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^2*
2^(1/2)-1/2*b^(9/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d
+b*c)^2*2^(1/2)-1/8*d^(5/4)*(-5*a*d+9*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)
/c^(1/4))/c^(9/4)/(-a*d+b*c)^2*2^(1/2)+1/8*d^(5/4)*(-5*a*d+9*b*c)*arctan(1+
d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^2*2^(1/2)-1/4*b^(9/4)*l
n(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)^2*2
^(1/2)+1/4*b^(9/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^
(5/4)/(-a*d+b*c)^2*2^(1/2)+1/16*d^(5/4)*(-5*a*d+9*b*c)*ln(c^(1/2)+x*d^(1/2)
-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(5/4)
*(-5*a*d+9*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/
4)/(-a*d+b*c)^2*2^(1/2)+1/2*(5*a*d-4*b*c)/a/c^2/(-a*d+b*c)/x^(1/2)-1/2*d/c/
(-a*d+b*c)/(d*x^2+c)/x^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 483, 597, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{b^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^2}-\frac{b^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^2}-\frac{b^{9/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^2}-\frac{d^{5/4}(9bc-5ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^2}+\frac{d^{5/4}(9bc-5ad)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^2}+\frac{d^{5/4}(9bc-5ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}(bc-ad)^2}-\frac{d^{5/4}(9bc-5ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}(bc-ad)^2}-\frac{4bc-5ad}{2ac^2\sqrt{x}\,(bc-ad)}-\frac{d}{2c\sqrt{x}\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(4*b*c - 5*a*d)/(a*c^2*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*d)*Sqrt[
x]*(c + d*x^2)) + (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)*(b*c - a*d)^2) - (b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^2) - (d^(5/4)*(9*b*c - 5*a*d)*Ar
cTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)
^2) + (d^(5/4)*(9*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^2) + (b^(9/
4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a
^(5/4)*(b*c - a*d)^2) + (d^(5/4)*(9*b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (d^(5
/4)*(9*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{2c(bc-ad)\sqrt{x}\,\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4bc-5ad-5bdx^4}{x^2(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\,\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(4b^2c^2+4abc\cdots\right.}{(a+\cdots}\right.}{2c\cdots}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\,\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int\left(\frac{4b^3c^2x^2}{(bc-ad)(a+bx\cdots}\right.\right.}{2\cdots}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\,\left(c+dx^2\right)} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,d\cdots\right.}{a(bc-ad)^2\cdots}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\,\left(c+dx^2\right)} + \frac{b^{5/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{\cdots}}{a+bx^4\cdots}\right.}{a(bc-a\cdots}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\,\left(c+dx^2\right)} - \frac{b^2\text{Subst}\left(\int \frac{\cdots}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\cdots}{\cdots}}\right.}{2a(bc\cdots}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\,\left(c+dx^2\right)} - \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\cdots\right.}{2\sqrt{2}\,a^{5/4}\cdots}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\,\left(c+dx^2\right)} + \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right.}{\sqrt{2}\,a^{5/4}(bc-a\cdots}$$

Mathematica [A]
time = 1.17, size = 332, normalized size = 0.58

$$\frac{1}{8}\left(\frac{16bc(c+dx^2)-4ad(4c+5dx^2)}{ac^2(-bc+ad)\sqrt{x}\,(c+dx^2)} + \frac{4\sqrt{2}\,b^{9/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{5/4}(bc-ad)^2} + \frac{\sqrt{2}\,d^{5/4}(-9bc+5ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{9/4}(bc-ad)^2} + \frac{4\sqrt{2}\,b^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{5/4}(bc-ad)^2} + \frac{\sqrt{2}\,d^{5/4}(-9bc+5ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{9/4}(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((16*b*c*(c + d*x^2) - 4*a*d*(4*c + 5*d*x^2))/(a*c^2*(-(b*c) + a*d)*Sqrt[x]
*(c + d*x^2)) + (4*Sqrt[2]*b^(9/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^
```

```
(1/4)*b^(1/4)*Sqrt[x])])/(a^(5/4)*(b*c - a*d)^2) + (Sqrt[2]*d^(5/4)*(-9*b*c
 + 5*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/
(c^(9/4)*(b*c - a*d)^2) + (4*Sqrt[2]*b^(9/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(5/4)*(b*c - a*d)^2) + (Sqrt[2]*d^(5
/4)*(-9*b*c + 5*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + S
qrt[d]*x)])/(c^(9/4)*(b*c - a*d)^2))/8
```

Maple [A]
time = 0.13, size = 286, normalized size = 0.50

method	result
derivativedivides	$-\frac{b^{2} \sqrt{2}\, \left(\ln \left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4 a \left(a d -b c \right)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2 d^{2} \left(\left(\right.\right.}{}$
default	$-\frac{b^{2} \sqrt{2}\, \left(\ln \left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4 a \left(a d -b c \right)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2 d^{2} \left(\left(\right.\right.}{}$
risch	$-\frac{2}{c^{2} a \sqrt{x}}-\frac{b^{2} \sqrt{2}\, \ln \left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4 a \left(a d -b c \right)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{b^{2} \sqrt{2}\, \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 a \left(a d -b c \right)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{b^{2} \sqrt{2}\, \arctan \left(\right.}{2 a \left(a d - \right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b^2/a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*d^2/c^
2/(a*d-b*c)^2*((1/4*a*d-1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(5/4*a*d-9/4*b*c)/d/
(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/c^2/a/x^(1/2)
```

Maxima [A]
time = 0.54, size = 494, normalized size = 0.87

$$-\frac{b^{3} \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} + 2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}} + \frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} - 2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}} - \frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{4 \left(a b^{2} c^{2} - 2 a^{2} b c d + a^{3} d^{2}\right)} + \frac{\left(9 b c d^{2} - 5 a d^{3}\right) \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} + 2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}} + \frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} - 2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}} - \frac{\sqrt{2} \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x} + \sqrt{d} x + \sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x} + \sqrt{d} x + \sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}\right)}{16 \left(b^{2} c^{4} - 2 a b c^{3} d + a^{2} c^{2} d^{2}\right)} - \frac{4 b c^{2} - 4 a c d + \left(4 b c d - 5 a d^{2}\right) x^{2}}{2 \left(\left(a b c^{3} d - a^{2} c^{2} d^{2}\right) x^{\frac{5}{2}} + \left(a b c^{4} - a^{2} c^{3} d\right) \sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(
a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) + 1/16*(9*b*c*d^2 - 5*a*d^3)*(2*sqrt(2)*
arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(
c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqr
t(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) +
 sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^4 - 2*a*b*c^3*
d + a^2*c^2*d^2) - 1/2*(4*b*c^2 - 4*a*c*d + (4*b*c*d - 5*a*d^2)*x^2)/((a*b*
c^3*d - a^2*c^2*d^2)*x^(5/2) + (a*b*c^4 - a^2*c^3*d)*sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3630 vs. $2(423) = 846$.
time = 24.13, size = 3630, normalized size = 6.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*(-(6561*b^
4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8
+ 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^
5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^
6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4)*arctan(((b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2)*sqrt((531441*b^6*c^6*d^8 - 1771470*a*b^5*c^5*d^9 + 2460375*a^
2*b^4*c^4*d^10 - 1822500*a^3*b^3*c^3*d^11 + 759375*a^4*b^2*c^2*d^12 - 16875
0*a^5*b*c*d^13 + 15625*a^6*d^14)*x - (6561*b^8*c^13*d^5 - 40824*a*b^7*c^12*
d^6 + 109836*a^2*b^6*c^11*d^7 - 166824*a^3*b^5*c^10*d^8 + 156406*a^4*b^4*c^
9*d^9 - 92680*a^5*b^3*c^8*d^10 + 33900*a^6*b^2*c^7*d^11 - 7000*a^7*b*c^6*d^
12 + 625*a^8*c^5*d^13)*sqrt(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 1215
0*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^1
6*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*
a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8)))*
(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^
3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 -
 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b
^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4) + (729*b^5*c^7*d^4 - 2
673*a*b^4*c^6*d^5 + 3834*a^2*b^3*c^5*d^6 - 2690*a^3*b^2*c^4*d^7 + 925*a^4*b
```

*c^3*d^8 - 125*a^5*c^2*d^9)*sqrt(x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4))/(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)) + 16*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*arctan((sqrt(b^14*x - (a^3*b^13*c^4 - 4*a^4*b^12*c^3*d + 6*a^5*b^11*c^2*d^2 - 4*a^6*b^10*c*d^3 + a^7*b^9*d^4)*sqrt(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8)))*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) - (a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*sqrt(x))/b^9) - 4*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*log(b^7*sqrt(x) + (a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5 + a^10*d^6)*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(3/4)) + 4*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*log(b^7*sqrt(x) - (a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5 + a^10*d^6)*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(3/4)) - ((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4)*log((b^6*c^13 - 6*a*b^5*c^12*d + 15*a^2*b^4*c^11*d^2 - 20*a^3*b^3*c^10*d^3 + 15*a^4*b^2*c^9*d^4 - 6*a^5*b*c^8*d^5 + a^6*c^7*d^6)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(3/4) - (729*b^3*c^3*d^4 - 1215*a*b^2*c^2*d^5 + 675*a^2*b*c*d^6 - 125*a^3*d^7)*sqrt(x)) + ((a*b*c^3*d - a^2*c^

```
2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3
*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 -
8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.79, size = 725, normalized size = 1.27

$$\frac{\left(9\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^5d-2\sqrt{2}\,abc^4d^2+\sqrt{2}\,a^2c^3d^3\right)}+\frac{\left(9\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^5d-2\sqrt{2}\,abc^4d^2+\sqrt{2}\,a^2c^3d^3\right)}-\frac{\left(9\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^5d-2\sqrt{2}\,abc^4d^2+\sqrt{2}\,a^2c^3d^3\right)}+\frac{\left(9\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^5d-2\sqrt{2}\,abc^4d^2+\sqrt{2}\,a^2c^3d^3\right)}-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^2c^2-2\sqrt{2}\,a^3bcd+\sqrt{2}\,a^4d^2}-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^2c^2-2\sqrt{2}\,a^3bcd+\sqrt{2}\,a^4d^2}+\frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2b^2c^2-2\sqrt{2}\,a^3bcd+\sqrt{2}\,a^4d^2\right)}-\frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2b^2c^2-2\sqrt{2}\,a^3bcd+\sqrt{2}\,a^4d^2\right)}-\frac{4\,bcdx^2-5\,ad^2x^2+4\,bc^2-4\,acd}{2\,(abc^3-a^2c^2d)\left(dx^{\frac{5}{2}}+c\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)
*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c
^4*d^2 + sqrt(2)*a^2*c^3*d^3) + 1/4*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*
a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sq
rt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^3) - 1/8*(9*(c*
d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
 sqrt(c/d))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^
3) + 1/8*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sq
rt(2)*a^2*c^3*d^3) - (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
+ 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt
(2)*a^4*d^2) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*s
qrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a
^4*d^2) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/2*(a*b^
3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2
*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/2*(4*b*c*d*x^2 - 5*a*d^2*
x^2 + 4*b*c^2 - 4*a*c*d)/((a*b*c^3 - a^2*c^2*d)*(d*x^(5/2) + c*sqrt(x)))
```

Mupad [**B**]
time = 2.34, size = 2500, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x)

[Out] atan(((-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6
*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^
5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*((-b^9/(16*a^13*d^8 + 1
6*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d
^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 1
28*a^12*b*c*d^7))^(3/4)*(x^(1/2)*(-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*
a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^
4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1
/4)*(33554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 + 352321536
0*a^14*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 + 45971668992*a^16*b^
21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38
*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12
 - 959547703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 2
241016627200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 2405
664030720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 - 1061108
580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 175279964160
*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*
c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25) - 1
6777216*a^11*b^25*c^42*d^4 + 218103808*a^12*b^24*c^41*d^5 - 1308622848*a^13
*b^23*c^40*d^6 + 4798283776*a^14*b^22*c^39*d^7 - 11995709440*a^15*b^21*c^38
*d^8 + 21783379968*a^16*b^20*c^37*d^9 - 31592546304*a^17*b^19*c^36*d^10 + 4
8013246464*a^18*b^18*c^35*d^11 - 103424196608*a^19*b^17*c^34*d^12 + 2539546
21440*a^20*b^16*c^33*d^13 - 531641663488*a^21*b^15*c^32*d^14 + 875046109184
*a^22*b^14*c^31*d^15 - 1125865488384*a^23*b^13*c^30*d^16 + 1138334629888*a^
24*b^12*c^29*d^17 - 906425794560*a^25*b^11*c^28*d^18 + 566347431936*a^26*b^
10*c^27*d^19 - 274688114688*a^27*b^9*c^26*d^20 + 101363744768*a^28*b^8*c^25
*d^21 - 27505197056*a^29*b^7*c^24*d^22 + 5174722560*a^30*b^6*c^23*d^23 - 60
2931200*a^31*b^5*c^22*d^24 + 32768000*a^32*b^4*c^21*d^25) - x^(1/2)*(323665
92*a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c
^30*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12
 - 7664386048*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863
552*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*
b^13*c^23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^
19 + 5120000*a^23*b^10*c^20*d^20))*1i + (-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8
 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9
*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d
^7))^(1/4)*((-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a
^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*
c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(3/4)*(x^(1/2)*(-b^9/(1
6*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896
*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b
^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 503316
480*a^13*b^24*c^43*d^5 + 3523215360*a^14*b^23*c^42*d^6 - 15267266560*a^15*b
```

^22*c^41*d^7 + 45971668992*a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12 - 959547703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 2241016627200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 2405664030720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 - 1061108580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 175279964160*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25) + 16777216*a^11*b^25*c^42*d^4 - 218103808*a^12*b^24*c^41*d^5 + 1308622848*a^13*b^23*c^40*d^6 - 4798283776*a^14*b^22*c^39*d^7 + 11995709440*a^15*b^21*c^38*d^8 - 21783379968*a^16*b^20*c^37*d^9 + 31592546304*a^17*b^19*c^36*d^10 - 48013246464*a^18*b^18*c^35*d^11 + 103424196608*a^19*b^17*c^34*d^12 - 253954621440*a^20*b^16*c^33*d^13 + 531641663488*a^21*b^15*c^32*d^14 - 875046109184*a^22*b^14*c^31*d^15 + 1125865488384*a^23*b^13*c^30*d^16 - 1138334629888*a^24*b^12*c^29*d^17 + 906425794560*a^25*b^11*c^28*d^18 - 566347431936*a^26*b^10*c^27*d^19 + 274688114688*a^27*b^9*c^26*d^20 - 101363744768*a^28*b^8*c^25*d^21 + 27505197056*a^29*b^7*c^24*d^22 - 5174722560*a^30*b^6*c^23*d^23 + 602931200*a^31*b^5*c^22*d^24 - 32768000*a^32*b^4*c^21*d^25) - x^(1/2)*(32366592*a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c^30*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12 - 7664386048*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863552*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*b^13*c^23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^19 + 5120000*...

3.478 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=570

$$-\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}}-\frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)}+\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\ldots}{\sqrt[4]{a}}\right.}{\sqrt{2}\,a^{7/4}(bc-ad)^2}$$

```
[Out] 1/6*(7*a*d-4*b*c)/a/c^2/(-a*d+b*c)/x^(3/2)-1/2*d/c/(-a*d+b*c)/x^(3/2)/(d*x^
2+c)+1/2*b^(11/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b
*c)^2*2^(1/2)-1/2*b^(11/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4
)/(-a*d+b*c)^2*2^(1/2)-1/8*d^(7/4)*(-7*a*d+11*b*c)*arctan(1-d^(1/4)*2^(1/2)
*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^2*2^(1/2)+1/8*d^(7/4)*(-7*a*d+11*b*c)
*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^2*2^(1/2)+1/
4*b^(11/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-
a*d+b*c)^2*2^(1/2)-1/4*b^(11/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2
)*x^(1/2))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(7/4)*(-7*a*d+11*b*c)*ln(c^(
1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^2*2^(1/
2)+1/16*d^(7/4)*(-7*a*d+11*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2
)*x^(1/2))/c^(11/4)/(-a*d+b*c)^2*2^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 483, 597, 536, 217, 1179, 642, 1176, 631, 210}

$$\frac{b^{11/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^2}+\frac{b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{d^{7/4}(11bc-7ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{11/4}(bc-ad)^2}+\frac{d^{7/4}(11bc-7ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{11/4}(bc-ad)^2}-\frac{d^{7/4}(11bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\,c^{11/4}(bc-ad)^2}+\frac{d^{7/4}(11bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\,c^{11/4}(bc-ad)^2}-\frac{4bc-7ad}{6ac^2x^{3/2}(bc-ad)}-\frac{d}{2cx^{3/2}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/6*(4*b*c - 7*a*d)/(a*c^2*(b*c - a*d)*x^(3/2)) - d/(2*c*(b*c - a*d)*x^(3/
2)*(c + d*x^2)) + (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (b^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c -
a*d)^2) + (d^(7/4)*(11*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (b^(11/4)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) -
 (b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*S
qrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] - Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^
2) + (d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4bc-7ad-7bdx^4}{x^4(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{3(4b^2c^2+4abcd-}{(a+}\right.}{6a}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx\right.}{a(bc-ad)^2}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{b^3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x}{a+bx^4}\right.}{a^{3/2}(bc-a}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{b^{5/2}\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}-\sqrt{}}\right.}{2a^{3/2}(b}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} + \frac{b^{11/4}\log\left(\sqrt{a}-\sqrt{2}\right.}{2\sqrt{2}\,a^{7/4}(}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} + \frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,a^{7/4}(bc-a}$$

Mathematica [A]
time = 1.21, size = 334, normalized size = 0.59

$$\frac{1}{24}\left(\frac{16bc(c+dx^2)-4ad(4c+7dx^2)}{ac^2(-bc+ad)x^{3/2}\left(c+dx^2\right)}+\frac{12\sqrt{2}\,b^{11/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{7/4}(bc-ad)^2}+\frac{3\sqrt{2}\,d^{7/4}(-11bc+7ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{11/4}(bc-ad)^2}-\frac{12\sqrt{2}\,b^{11/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{7/4}(bc-ad)^2}+\frac{3\sqrt{2}\,d^{7/4}(11bc-7ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{11/4}(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((16*b*c*(c + d*x^2) - 4*a*d*(4*c + 7*d*x^2))/(a*c^2*(-(b*c) + a*d)*x^(3/2)
*(c + d*x^2)) + (12*Sqrt[2]*b^(11/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(7/4)*(b*c - a*d)^2) + (3*Sqrt[2]*d^(7/4)*(-1
1*b*c + 7*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
])])/(c^(11/4)*(b*c - a*d)^2) - (12*Sqrt[2]*b^(11/4)*ArcTanh[(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(7/4)*(b*c - a*d)^2) + (3*Sq
```

```
rt[2]*d^(7/4)*(11*b*c - 7*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(S
qrt[c] + Sqrt[d]*x)])/(c^(11/4)*(b*c - a*d)^2))/24
```

Maple [A]
time = 0.14, size = 286, normalized size = 0.50

method	result
derivativedivides	$-\frac{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}(ad-bc)^{2}}-$
default	$-\frac{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}(ad-bc)^{2}}-$
risch	$-\frac{2}{3c^{2}a\,x^{\frac{3}{2}}}-\frac{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4a^{2}(ad-bc)^{2}}-\frac{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^{2}(ad-bc)^{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2*b^3/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*d^2/
c^2/(a*d-b*c)^2*((1/4*a*d-1/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(7*a*d-11*b*c)*(c
/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/3/c^2/a/x^(3/2)
```

Maxima [A]
time = 0.52, size = 539, normalized size = 0.95

$$-\frac{\frac{2\sqrt{2}\,b^{3}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b^{3}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{11}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{11}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\left(ab^{2}c^{2}-2a^{2}bcd+a^{3}d^{2}\right)}-\frac{4bc^{2}-4acd+\left(4bcd-7ad^{2}\right)x^{2}}{6\left(\left(abc^{3}d-a^{2}c^{2}d^{2}\right)x^{\frac{7}{2}}+\left(abc^{4}-a^{2}c^{3}d\right)x^{\frac{3}{2}}\right)}+\frac{\frac{2\sqrt{2}\left(11bcd^{2}-7ad^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\left(11bcd^{2}-7ad^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(11bcd^{2}-7ad^{3}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(11bcd^{2}-7ad^{3}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^{2}c^{4}-2abc^{3}d+a^{2}c^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2
)*b^3*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
```

```
t(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(11/4)*log(
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^
(11/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))
/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) - 1/6*(4*b*c^2 - 4*a*c*d + (4*b*c*d -
7*a*d^2)*x^2)/((a*b*c^3*d - a^2*c^2*d^2)*x^(7/2) + (a*b*c^4 - a^2*c^3*d)*x^
(3/2)) + 1/16*(2*sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(
sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*arctan(-1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)
*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*log(sqrt(2)*c^(1/4
)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(11*b*
c*d^2 - 7*a*d^3)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c)
)/(c^(3/4)*d^(1/4)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3439 vs. $2(423) = 846$.
time = 71.49, size = 3439, normalized size = 6.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b*c^4 - a^2*c^3*d)*x^2)*(-(14
641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b
*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 28*a^2*b^6*c^17*d^2 -
 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56*a^5*b^3*c^14*d^5 + 28*a^6*b
^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*d^8))^(1/4)*arctan(((b^6*c^14 - 6
*a*b^5*c^13*d + 15*a^2*b^4*c^12*d^2 - 20*a^3*b^3*c^11*d^3 + 15*a^4*b^2*c^10
*d^4 - 6*a^5*b*c^9*d^5 + a^6*c^8*d^6)*sqrt((121*b^2*c^2*d^4 - 154*a*b*c*d^5
 + 49*a^2*d^6)*x + (b^4*c^10 - 4*a*b^3*c^9*d + 6*a^2*b^2*c^8*d^2 - 4*a^3*b*
c^7*d^3 + a^4*c^6*d^4)*sqrt(-(14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 355
74*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^
7*c^18*d + 28*a^2*b^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4
- 56*a^5*b^3*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*d
^8)))*(-(14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 -
15092*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 28*a^2*b^6
*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56*a^5*b^3*c^14*d^5
 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*d^8))^(3/4) + (11*b^7*
c^15*d^2 - 73*a*b^6*c^14*d^3 + 207*a^2*b^5*c^13*d^4 - 325*a^3*b^4*c^12*d^5
+ 305*a^4*b^3*c^11*d^6 - 171*a^5*b^2*c^10*d^7 + 53*a^6*b*c^9*d^8 - 7*a^7*c^
8*d^9)*sqrt(x)*(-(14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c
^2*d^9 - 15092*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 2
8*a^2*b^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56*a^5*b^3
*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*d^8))^(3/4))/
```

(14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10 + 2401*a^4*d^11)) + 48*(-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b*c^4 - a^2*c^3*d)*x^2)*arctan(((a^5*b^6*c^6 - 6*a^6*b^5*c^5*d + 15*a^7*b^4*c^4*d^2 - 20*a^8*b^3*c^3*d^3 + 15*a^9*b^2*c^2*d^4 - 6*a^10*b*c*d^5 + a^11*d^6)*(-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(3/4)*sqrt(b^6*x + (a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4)*sqrt(-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))) - (a^5*b^9*c^6 - 6*a^6*b^8*c^5*d + 15*a^7*b^7*c^4*d^2 - 20*a^8*b^6*c^3*d^3 + 15*a^9*b^5*c^2*d^4 - 6*a^10*b^4*c*d^5 + a^11*b^3*d^6)*(-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(3/4)*sqrt(x))/b^11) + 12*(-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b*c^4 - a^2*c^3*d)*x^2)*log(b^3*sqrt(x) + (-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)) - 12*(-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b*c^4 - a^2*c^3*d)*x^2)*log(b^3*sqrt(x) - (-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)) + 3*((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b*c^4 - a^2*c^3*d)*x^2)*(-(14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 28*a^2*b^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56*a^5*b^3*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*d^8))^(1/4)*log(-(11*b*c*d^2 - 7*a*d^3)*sqrt(x) + (b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(-(14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 28*a^2*b^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56*a^5*b^3*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*d^8))^(1/4)) - 3*((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b*c^4 - a^2*c^3*d)*x^2)*(-(14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10 + 2401*a^4*d^11)/(b^8*c^19 - 8*a*b^7*c^18*d + 28*a^2*b^6*c^17*d^2 - 56*a^3*b^5*c^16*d^3 + 70*a^4*b^4*c^15*d^4 - 56*a^5*b^3*c^14*d^5 + 28*a^6*b^2*c^13*d^6 - 8*a^7*b*c^12*d^7 + a^8*c^11*d^8))^(1/4)*log(-(11*b*c*d^2 - 7*a*d^3)*sqrt(x) - (b^2*c^5...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.64, size = 718, normalized size = 1.26

$$-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b^{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\, a^{2}b^{2}c^{2}-2\sqrt{2}\, a^{3}bcd+\sqrt{2}\, a^{4}d^{2}}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b^{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\, a^{2}b^{2}c^{2}-2\sqrt{2}\, a^{3}bcd+\sqrt{2}\, a^{4}d^{2}}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b^{2} \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\, a^{2}b^{2}c^{2}-2\sqrt{2}\, a^{3}bcd+\sqrt{2}\, a^{4}d^{2}\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} b^{2} \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\, a^{2}b^{2}c^{2}-2\sqrt{2}\, a^{3}bcd+\sqrt{2}\, a^{4}d^{2}\right)}+\frac{\left(11\left(cd^{3}\right)^{\frac{1}{4}} bcd-7\left(cd^{3}\right)^{\frac{1}{4}} ad^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\, b^{2}c^{5}-2\sqrt{2}\, abc^{4}d+\sqrt{2}\, a^{2}c^{3}d^{2}\right)}+\frac{\left(11\left(cd^{3}\right)^{\frac{1}{4}} bcd-7\left(cd^{3}\right)^{\frac{1}{4}} ad^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\, b^{2}c^{5}-2\sqrt{2}\, abc^{4}d+\sqrt{2}\, a^{2}c^{3}d^{2}\right)}+\frac{\left(11\left(cd^{3}\right)^{\frac{1}{4}} bcd-7\left(cd^{3}\right)^{\frac{1}{4}} ad^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\, b^{2}c^{5}-2\sqrt{2}\, abc^{4}d+\sqrt{2}\, a^{2}c^{3}d^{2}\right)}-\frac{\left(11\left(cd^{3}\right)^{\frac{1}{4}} bcd-7\left(cd^{3}\right)^{\frac{1}{4}} ad^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\, b^{2}c^{5}-2\sqrt{2}\, abc^{4}d+\sqrt{2}\, a^{2}c^{3}d^{2}\right)}+\frac{d^{2}\sqrt{x}}{2\left(bc^{3}-ac^{2}d\right)\left(dx^{2}+c\right)}-\frac{2}{3\, ac^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - (
a*b^3)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/
2*(a*b^3)^(1/4)*b^2*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/2*(a*b^3)^(1/4)
*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2
 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/4*(11*(c*d^3)^(1/4)*b*c*d - 7
*(c*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2)
+ 1/4*(11*(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)^(1/4)*a*d^2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)
*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)
^(1/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2
*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) - 1/8*(11*(c*d^3)^(1/4)*b
*c*d - 7*(c*d^3)^(1/4)*a*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
/d))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) + 1/2*d^
2*sqrt(x)/((b*c^3 - a*c^2*d)*(d*x^2 + c)) - 2/3/(a*c^2*x^(3/2))
```

Mupad [**B**]
time = 3.83, size = 2500, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

[Out] 2*atan(((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(x^(1/2)*(15859712*a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*d^11 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 14037065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 26429997056*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a^18*b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^20*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) - (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(3/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(67108864*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 - 442364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 1824220250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - 4642121449472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 7600917708800*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 6603814207488*a^26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 2604562120704*a^28*b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104*a^30*b^8*c^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^27*d^23 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25)*1i + x^(1/2)*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215360*a^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*b^22*c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^38*d^10 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^12 + 336173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 + 2334365057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 4221965434880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 3039679217664*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 864890650624*a^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30*b^7*c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*d^25 - 102760448*a^33*b^4*c^22*d^26))*1i + 11534336*a^9*b^25*c^35*d^7 - 111149056*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^22*c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*d^12 - 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 - 104394047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 192702906368*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 109211664384*a^21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23*b^11*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25

```
*b^9*c^19*d^23 + 39337984*a^26*b^8*c^18*d^24)*1i) + (-b^11/(16*a^15*d^8 + 1
6*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*
d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 -
 128*a^14*b*c*d^7))^(1/4)*(x^(1/2)*(15859712*a^9*b^24*c^31*d^9 - 131203072*
a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*d^11 - 2168807424*a^12*b^21*
c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 14037065728*a^14*b^19*c^26*d^1
4 + 22648012800*a^15*b^18*c^25*d^15 - 26429997056*a^16*b^17*c^24*d^16 + 222
56009216*a^17*b^16*c^23*d^17 - 13398917120*a^18*b^15*c^22*d^18 + 5629976576
*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^20*d^20 + 261316608*a^21*b^12
*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) + (-b^11/(16*a^15*d^8 + 16*a^7*b
^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1
120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^
14*b*c*d^7))^(1/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*
d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 89
6*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(3/4)*((-b^1
1/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 -
 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*
a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(67108864*a^13*b^25*c^46*d^4 -
1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^44*d^6 - 45818576896
*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 - 442364854272*a^18*b
^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 1824220250112*a^20*b^18*c
^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - ...
```

3.479 $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=618

$$-\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}}+\frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}}-\frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)}-\frac{b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)^2}+$$

```
[Out] 1/10*(9*a*d-4*b*c)/a/c^2/(-a*d+b*c)/x^(5/2)-1/2*d/c/(-a*d+b*c)/x^(5/2)/(d*x
^2+c)-1/2*b^(13/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/(-a*d+
b*c)^2*2^(1/2)+1/2*b^(13/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/
4)/(-a*d+b*c)^2*2^(1/2)+1/8*d^(9/4)*(-9*a*d+13*b*c)*arctan(1-d^(1/4)*2^(1/2
)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^2*2^(1/2)-1/8*d^(9/4)*(-9*a*d+13*b*c
)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^2*2^(1/2)+1
/4*b^(13/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(
-a*d+b*c)^2*2^(1/2)-1/4*b^(13/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/
2)*x^(1/2))/a^(9/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(9/4)*(-9*a*d+13*b*c)*ln(c^
(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^2*2^(1
/2)+1/16*d^(9/4)*(-9*a*d+13*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/
2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^2*2^(1/2)+1/2*(-9*a^2*d^2+4*a*b*c*d+4*b^2*c
^2)/a^2/c^3/(-a*d+b*c)/x^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 483, 597, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{13/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)^2}+\frac{b^{13/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{9/4}(bc-ad)^2}+\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^2}+\frac{-9a^2d^2+4abcd+4b^2c^2}{2a^2c^3\sqrt{x}(bc-ad)}+\frac{d^{9/4}(13bc-9ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{13/4}(bc-ad)^2}-\frac{d^{9/4}(13bc-9ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{13/4}(bc-ad)^2}-\frac{d^{9/4}(13bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\,c^{13/4}(bc-ad)^2}+\frac{d^{9/4}(13bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\,c^{13/4}(bc-ad)^2}-\frac{4bc-9ad}{10ac^2x^{5/2}(bc-ad)}-\frac{d}{2cx^{5/2}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/10*(4*b*c - 9*a*d)/(a*c^2*(b*c - a*d)*x^(5/2)) + (4*b^2*c^2 + 4*a*b*c*d
- 9*a^2*d^2)/(2*a^2*c^3*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*d)*x^(5/2)*(
c + d*x^2)) - (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqr
t[2]*a^(9/4)*(b*c - a*d)^2) + (b^(13/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^2) + (d^(9/4)*(13*b*c - 9*a*d)*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)
^2) - (d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^2) - (b^
(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(9/4)*(b*c - a*d)^2) - (d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) +
```

```
 (d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4bc-9ad-9bdx^4}{x^6(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{5\left(4b^2c^2+4abcd\right.}{x^2(\right.}\right.}{10}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right.}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right.}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right.}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right.}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right.}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right.}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right.}$$

Mathematica [A]
time = 1.28, size = 378, normalized size = 0.61

$$\frac{1}{40}\left(-\frac{4\left(20b^2c^2x^2\left(c+dx^2\right)+a^2d\left(4c^2-36cdx^2-45d^2x^4\right)-4abc\left(c^2-4cdx^2-5d^2x^4\right)\right)}{a^2c^3(-bc+ad)x^{5/2}\left(c+dx^2\right)}-\frac{20\sqrt{2}\,b^{13/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{9/4}(bc-ad)^2}+\frac{5\sqrt{2}\,d^{9/4}(13bc-9ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{13/4}(bc-ad)^2}-\frac{20\sqrt{2}\,b^{13/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{9/4}(bc-ad)^2}+\frac{5\sqrt{2}\,d^{9/4}(13bc-9ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{13/4}(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((-4*(20*b^2*c^2*x^2*(c + d*x^2) + a^2*d*(4*c^2 - 36*c*d*x^2 - 45*d^2*x^4)
- 4*a*b*c*(c^2 - 4*c*d*x^2 - 5*d^2*x^4)))/(a^2*c^3*(-(b*c) + a*d)*x^(5/2)*(
c + d*x^2)) - (20*Sqrt[2]*b^(13/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])])/(a^(9/4)*(b*c - a*d)^2) + (5*Sqrt[2]*d^(9/4)*(13*b
*c - 9*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]
)/(c^(13/4)*(b*c - a*d)^2) - (20*Sqrt[2]*b^(13/4)*ArcTanh[(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(9/4)*(b*c - a*d)^2) + (5*Sqrt[
2]*d^(9/4)*(13*b*c - 9*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt
[c] + Sqrt[d]*x)])/(c^(13/4)*(b*c - a*d)^2))/40
```

Maple [A]
time = 0.14, size = 306, normalized size = 0.50

method	result
derivativedivides	$\frac{b^{3}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2d^{3}\left(\frac{\left(\frac{ad}{4}\right.}{d}\right.}{}$
default	$\frac{b^{3}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2d^{3}\left(\frac{\left(\frac{ad}{4}\right.}{d}\right.}{}$
risch	$-\frac{2\left(-10ad\,x^{2}-5c\,x^{2}b+ac\right)}{5a^{2}c^{3}x^{\frac{5}{2}}}+\frac{b^{3}\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)}{4a^{2}(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{b^{3}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^{2}(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^3/a^2/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2*d^3/c
^3/(a*d-b*c)^2*((1/4*a*d-1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(9/4*a*d-13/4*b*c)/
d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/5/c^2/a/x^(5/2)-2*(-2*a*d-
b*c)/a^2/c^3/x^(1/2)
```

Maxima [A]
time = 0.52, size = 551, normalized size = 0.89

$$\frac{b^{4}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(a^{2}b^{2}c^{2}-2a^{3}bcd+a^{4}d^{2}\right)}-\frac{\left(13bcd^{3}-9ad^{4}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{2}c^{5}-2abc^{4}d+a^{2}c^{3}d^{2}\right)}-\frac{4abc^{3}-4a^{2}c^{2}d-5\left(4b^{2}c^{2}d+4abcd^{2}-9a^{2}d^{3}\right)x^{4}-4\left(5b^{2}c^{3}+4abc^{2}d-9a^{2}cd^{2}\right)x^{2}}{10\left(\left(a^{2}bc^{4}d-a^{3}c^{3}d^{2}\right)x^{\frac{9}{2}}+\left(a^{2}bc^{5}-a^{3}c^{4}d\right)x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a
^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) - 1/16*(13*b*c*d^3 - 9*a*d^4)*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqr
t(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
 + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^
(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^5 - 2*a*b*c^
4*d + a^2*c^3*d^2) - 1/10*(4*a*b*c^3 - 4*a^2*c^2*d - 5*(4*b^2*c^2*d + 4*a*b
*c*d^2 - 9*a^2*d^3)*x^4 - 4*(5*b^2*c^3 + 4*a*b*c^2*d - 9*a^2*c*d^2)*x^2)/((
a^2*b*c^4*d - a^3*c^3*d^2)*x^(9/2) + (a^2*b*c^5 - a^3*c^4*d)*x^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3728 vs. $2(467) = 934$.
time = 100.56, size = 3728, normalized size = 6.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*((a^2*b*c^4*d - a^3*c^3*d^2)*x^5 + (a^2*b*c^5 - a^3*c^4*d)*x^3)*(
-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908
*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19
*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28
*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8))^(1/4)*arctan(((b^2*c^
5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt((4826809*b^6*c^6*d^14 - 20049822*a*b^5*
c^5*d^15 + 34701615*a^2*b^4*c^4*d^16 - 32032260*a^3*b^3*c^3*d^17 + 16632135
*a^4*b^2*c^2*d^18 - 4605822*a^5*b*c*d^19 + 531441*a^6*d^20)*x - (28561*b^8*
c^15*d^9 - 193336*a*b^7*c^14*d^10 + 569868*a^2*b^6*c^13*d^11 - 955240*a^3*b
^5*c^12*d^12 + 995926*a^4*b^4*c^11*d^13 - 661320*a^5*b^3*c^10*d^14 + 273132
*a^6*b^2*c^9*d^15 - 64152*a^7*b*c^8*d^16 + 6561*a^8*c^7*d^17)*sqrt(-(28561*
b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c
*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 5
6*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28*a^6*b^2
*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8)))*(-(28561*b^4*c^4*d^9 - 79092
*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^
```

```
13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3
+ 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b
*c^14*d^7 + a^8*c^13*d^8))^(1/4) + (2197*b^5*c^8*d^7 - 8957*a*b^4*c^7*d^8 +
 14482*a^2*b^3*c^6*d^9 - 11610*a^3*b^2*c^5*d^10 + 4617*a^4*b*c^4*d^11 - 729
*a^5*c^3*d^12)*sqrt(x)*(-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*
a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*
c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 -
56*a^5*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8
))^(1/4))/(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^1
1 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)) + 80*(-b^13/(a^9*b^8*c^8 - 8*a^10*
b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4
 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^
(1/4)*((a^2*b*c^4*d - a^3*c^3*d^2)*x^5 + (a^2*b*c^5 - a^3*c^4*d)*x^3)*arcta
n((sqrt(b^20*x - (a^5*b^17*c^4 - 4*a^6*b^16*c^3*d + 6*a^7*b^15*c^2*d^2 - 4*
a^8*b^14*c*d^3 + a^9*b^13*d^4)*sqrt(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d +
 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*
b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8)))*(-b^13/(a^
9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 +
70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*
c*d^7 + a^17*d^8))^(1/4)*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) - (a^2*b^12*
c^2 - 2*a^3*b^11*c*d + a^4*b^10*d^2)*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d
 + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^1
4*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(1/4)*sqr
t(x))/b^13) - 20*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d
^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a
^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(1/4)*((a^2*b*c^4*d - a^3*c^3
*d^2)*x^5 + (a^2*b*c^5 - a^3*c^4*d)*x^3)*log(b^10*sqrt(x) + (a^7*b^6*c^6 -
6*a^8*b^5*c^5*d + 15*a^9*b^4*c^4*d^2 - 20*a^10*b^3*c^3*d^3 + 15*a^11*b^2*c^
2*d^4 - 6*a^12*b*c*d^5 + a^13*d^6)*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d +
 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*
b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(3/4)) + 20
*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5
*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6
 - 8*a^16*b*c*d^7 + a^17*d^8))^(1/4)*((a^2*b*c^4*d - a^3*c^3*d^2)*x^5 + (a^2
*b*c^5 - a^3*c^4*d)*x^3)*log(b^10*sqrt(x) - (a^7*b^6*c^6 - 6*a^8*b^5*c^5*d
+ 15*a^9*b^4*c^4*d^2 - 20*a^10*b^3*c^3*d^3 + 15*a^11*b^2*c^2*d^4 - 6*a^12*b
*c*d^5 + a^13*d^6)*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6
*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28
*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(3/4)) - 5*((a^2*b*c^4*d -
a^3*c^3*d^2)*x^5 + (a^2*b*c^5 - a^3*c^4*d)*x^3)*(-(28561*b^4*c^4*d^9 - 7909
2*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d
^13)/(b^8*c^21 - 8*a*b^7*c^20*d + 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3
 + 70*a^4*b^4*c^17*d^4 - 56*a^5*b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*
b*c^14*d^7 + a^8*c^13*d^8))^(1/4)*log((b^6*c^16 - 6*a*b^5*c^15*d + 15*a^2*b
^4*c^14*d^2 - 20*a^3*b^3*c^13*d^3 + 15*a^4*b^2*c^12*d^4 - 6*a^5*b*c^11*d^5
```

```
+ a^6*c^10*d^6)*(-(28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2
*c^2*d^11 - 37908*a^3*b*c*d^12 + 6561*a^4*d^13)/(b^8*c^21 - 8*a*b^7*c^20*d
+ 28*a^2*b^6*c^19*d^2 - 56*a^3*b^5*c^18*d^3 + 70*a^4*b^4*c^17*d^4 - 56*a^5*
b^3*c^16*d^5 + 28*a^6*b^2*c^15*d^6 - 8*a^7*b*c^14*d^7 + a^8*c^13*d^8))^(3/4
) - (2197*b^3*c^3*d^7 - 4563*a*b^2*c^2*d^8 + 3159*a^2*b*c*d^9 - 729*a^3*d^1
0)*sqrt(x)) + 5*((a^2*b*c^4*d - a^3*c^3*d^2)*x^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.81, size = 715, normalized size = 1.16

$$-\frac{d^{3} x^{\frac{3}{2}}}{2 \, {\left(b c^{4} - a c^{3} d\right)} {\left(d x^{2} + c\right)}} + \frac{\left(a b^{3}\right)^{\frac{3}{4}} b \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \, \sqrt{x}\right)}{2 \, \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2} a^{3} b^{2} c^{2} - 2 \, \sqrt{2} a^{4} b c d + \sqrt{2} a^{5} d^{2}} + \frac{\left(a b^{3}\right)^{\frac{3}{4}} b \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \, \sqrt{x}\right)}{2 \, \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2} a^{3} b^{2} c^{2} - 2 \, \sqrt{2} a^{4} b c d + \sqrt{2} a^{5} d^{2}} - \frac{\left(a b^{3}\right)^{\frac{3}{4}} b \log\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{2 \, \left(\sqrt{2} a^{3} b^{2} c^{2} - 2 \, \sqrt{2} a^{4} b c d + \sqrt{2} a^{5} d^{2}\right)} + \frac{\left(a b^{3}\right)^{\frac{3}{4}} b \log\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{2 \, \left(\sqrt{2} a^{3} b^{2} c^{2} - 2 \, \sqrt{2} a^{4} b c d + \sqrt{2} a^{5} d^{2}\right)} - \frac{\left(13 \, \left(c d^{3}\right)^{\frac{3}{4}} b c - 9 \, \left(c d^{3}\right)^{\frac{3}{4}} a d\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} + 2 \, \sqrt{x}\right)}{2 \, \left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4 \, \left(\sqrt{2} b^{2} c^{6} - 2 \, \sqrt{2} a b c^{5} d + \sqrt{2} a^{2} c^{4} d^{2}\right)} - \frac{\left(13 \, \left(c d^{3}\right)^{\frac{3}{4}} b c - 9 \, \left(c d^{3}\right)^{\frac{3}{4}} a d\right) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} - 2 \, \sqrt{x}\right)}{2 \, \left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4 \, \left(\sqrt{2} b^{2} c^{6} - 2 \, \sqrt{2} a b c^{5} d + \sqrt{2} a^{2} c^{4} d^{2}\right)} + \frac{\left(13 \, \left(c d^{3}\right)^{\frac{3}{4}} b c - 9 \, \left(c d^{3}\right)^{\frac{3}{4}} a d\right) \log\left(\sqrt{2} \sqrt{x} \left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{8 \, \left(\sqrt{2} b^{2} c^{6} - 2 \, \sqrt{2} a b c^{5} d + \sqrt{2} a^{2} c^{4} d^{2}\right)} - \frac{\left(13 \, \left(c d^{3}\right)^{\frac{3}{4}} b c - 9 \, \left(c d^{3}\right)^{\frac{3}{4}} a d\right) \log\left(-\sqrt{2} \sqrt{x} \left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{8 \, \left(\sqrt{2} b^{2} c^{6} - 2 \, \sqrt{2} a b c^{5} d + \sqrt{2} a^{2} c^{4} d^{2}\right)} + \frac{2 \, \left(5 \, b c x^{2} + 10 \, a d x^{2} - a c\right)}{5 \, a^{2} c^{3} x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*x^(3/2)/((b*c^4 - a*c^3*d)*(d*x^2 + c)) + (a*b^3)^(3/4)*b*arctan(1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2*
c^2 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*a^5*d^2) + (a*b^3)^(3/4)*b*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2*c^2
 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*a^5*d^2) - 1/2*(a*b^3)^(3/4)*b*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*
b*c*d + sqrt(2)*a^5*d^2) + 1/2*(a*b^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*(a/b)^(
1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*
a^5*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt
(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^6 - 2*sqr
t(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^
3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(
1/4))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) + 1/8*(
13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4)
 + x + sqrt(c/d))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*
d^2) - 1/8*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqr
t(2)*a^2*c^4*d^2) + 2/5*(5*b*c*x^2 + 10*a*d*x^2 - a*c)/(a^2*c^3*x^(5/2))
```

Mupad [**B**]
time = 3.00, size = 2500, normalized size = 4.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] - (2/(5*a*c) - (2*x^2*(9*a*d + 5*b*c))/(5*a^2*c^2) + (d*x^4*(4*b^2*c^2 - 9*
a^2*d^2 + 4*a*b*c*d))/(2*a^2*c^3*(a*d - b*c)))/(c*x^(5/2) + d*x^(9/2)) - 2*
atan((524288*a^3*b^16*c^32*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 7
9092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^
8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2
 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*
d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 2654208*a^19*c
^16*d^16*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^1
0 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*
c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*
c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b
^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 346112*b^15*c^18*d^6*x^(1/2)*(-(
6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^
2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7
*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^
4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*
a*b^7*c^20*d))^(1/4) - 479232*a*b^14*c^17*d^7*x^(1/2)*(-(6561*a^4*d^13 + 28
561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3
*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 1146
88*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 2
29376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/
4) - 4194304*a^4*b^15*c^31*d*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 -
 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*
b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d
^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^1
6*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) - 28901376*a^1
8*b*c^17*d^15*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^
3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096
*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3
*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*
a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 165888*a^2*b^13*c^16*d^8*x^
(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a
^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 -
32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 +
286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6
 - 32768*a*b^7*c^20*d))^(1/4) + 3655808*a^3*b^12*c^15*d^9*x^(1/2)*(-(6561*a
^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11
 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^1
4*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*
c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*
c^20*d))^(1/4) - 10123776*a^4*b^11*c^14*d^10*x^(1/2)*(-(6561*a^4*d^13 + 285
```

61*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4) + 10513152*a^5*b^10*c^13*d^11*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4) - 4852224*a^6*b^9*c^12*d^12*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4) + 839808*a^7*b^8*c^11*d^13*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4) + 14680064*a^5*b^14*c^30*d^2*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) - 29360128*a^6*b^13*c^29*d^3*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8...

3.480 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=631

$$-\frac{c\sqrt{x}}{4d(bc-ad)(c+dx^2)^2}+\frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2(c+dx^2)}-\frac{a^{5/4}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\tan^{-1}\left(1+\right.}{\sqrt{2}(bc-}$$

```
[Out] -1/2*a^(5/4)*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^3
*2^(1/2)+1/2*a^(5/4)*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*
d+b*c)^3*2^(1/2)-1/64*(-5*a^2*d^2-30*a*b*c*d+3*b^2*c^2)*arctan(1-d^(1/4)*2^
(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/64*(-5*a^2*d^
2-30*a*b*c*d+3*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d
^(5/4)/(-a*d+b*c)^3*2^(1/2)-1/4*a^(5/4)*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4
)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2)+1/4*a^(5/4)*b^(3/4)*ln(a^(1
/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2)-1/128*(
-5*a^2*d^2-30*a*b*c*d+3*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/
2)*x^(1/2))/c^(3/4)/d^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/128*(-5*a^2*d^2-30*a*b*c
*d+3*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)
/d^(5/4)/(-a*d+b*c)^3*2^(1/2)-1/4*c*x^(1/2)/d/(-a*d+b*c)/(d*x^2+c)^2+1/16*(
-9*a*d+b*c)*x^(1/2)/d/(-a*d+b*c)^2/(d*x^2+c)
```

Rubi [A]
time = 0.55, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 481, 541, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{a^{5/4}b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^3}-\frac{a^{5/4}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}-\frac{(-5a^2d^2-30abcd+3b^2c^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{(-5a^2d^2-30abcd+3b^2c^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}-\frac{(-5a^2d^2-30abcd+3b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{(-5a^2d^2-30abcd+3b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{\sqrt{x}(bc-9ad)}{16d(c+dx^2)(bc-ad)^2}-\frac{c\sqrt{x}}{4d(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(c*Sqrt[x])/(d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 9*a*d)*Sqrt[x])/(1
6*d*(b*c - a*d)^2*(c + d*x^2)) - (a^(5/4)*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) + (a^(5/4)*b^(3/4)*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((3*b^2*c^2
- 30*a*b*c*d - 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(3
2*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) + ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2
*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^
(5/4)*(b*c - a*d)^3) - (a^(5/4)*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(5/4)*b^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*
d)^3) - ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) +
```

```
((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx = 2\text{Subst}\left(\int \frac{x^8}{(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)(c+dx^2)^2} + \frac{\text{Subst}\left(\int \frac{ac+(bc-8ad)x^4}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{4d(bc-ad)}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)(c+dx^2)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{ac(3bc+5ad)+3bc(b\ldots}{(a+bx^4)(c+\ldots}\right.}{16cd(bc-\ldots}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)(c+dx^2)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2(c+dx^2)} + \frac{(2a^2b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx\ldots\right.}{(bc-ad)^3}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)(c+dx^2)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2(c+dx^2)} + \frac{(a^{3/2}b)\,\text{Subst}\left(\int \frac{\sqrt{a}-\ldots}{a+b\ldots}\right.}{(bc-ad\ldots}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)(c+dx^2)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2(c+dx^2)} + \frac{\left(a^{3/2}\sqrt{b}\right)\text{Subst}\left(\int \ldots\right.}{2(b\ldots}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)(c+dx^2)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2(c+dx^2)} - \frac{a^{5/4}b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\ldots\right.}{2\sqrt{2}(bc\ldots}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)(c+dx^2)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2(c+dx^2)} - \frac{a^{5/4}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{\ldots}\right.}{\sqrt{2}(bc-ad)\ldots}$$

<u>Mathematica</u> [A]
time = 1.61, size = 328, normalized size = 0.52

$$\frac{\frac{4(-bc+ad)\sqrt{x}\,(bc(3c-dx^2)+ad(5c+9dx^2))}{d(c+dx^2)^2} - 32\sqrt{2}\,a^{5/4}b^{3/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right) - \frac{\sqrt{2}\,(3b^2c^2-30abcd-5a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{3/4}d^{5/4}} + 32\sqrt{2}\,a^{5/4}b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right) + \frac{\sqrt{2}\,(3b^2c^2-30abcd-5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{3/4}d^{5/4}}}{64(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((4*(-(b*c) + a*d)*Sqrt[x]*(b*c*(3*c - d*x^2) + a*d*(5*c + 9*d*x^2)))/(d*(c
 + d*x^2)^2) - 32*Sqrt[2]*a^(5/4)*b^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d
^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(3/
4)*d^(5/4)) + 32*Sqrt[2]*a^(5/4)*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*S
```

```
qrt[x])/(Sqrt[a] + Sqrt[b]*x)] + (Sqrt[2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d
^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(3
/4)*d^(5/4)))/(64*(b*c - a*d)^3)
```

Maple [A]
time = 0.09, size = 330, normalized size = 0.52

method	result
derivativedivides	$-\frac{ab\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}}+ \cdots$
default	$-\frac{ab\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}}+ \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2/(a*d-b*c
)^3*(((-9/32*a^2*d^2+5/16*a*b*c*d-1/32*b^2*c^2)*x^(5/2)-1/32*c*(5*a^2*d^2-2
*a*b*c*d-3*b^2*c^2)/d*x^(1/2))/(d*x^2+c)^2+1/256*(5*a^2*d^2+30*a*b*c*d-3*b^
2*c^2)/d*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.54, size = 653, normalized size = 1.03

$$\frac{\left(\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)a^{2}}{4\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{(bcd-9\,ad^{2})x^{\frac{5}{2}}-(3\,bc^{2}+5\,acd)\sqrt{x}}{16\left(b^{2}c^{4}d-2\,abc^{3}d^{2}+a^{2}c^{2}d^{3}+(b^{2}c^{2}d^{3}-2\,abcd^{4}+a^{2}d^{5})x^{4}+2\,(b^{2}c^{3}d^{2}-2\,abc^{2}d^{3}+a^{2}cd^{4})x^{2}\right)}+\frac{\frac{2\sqrt{2}\,(3\,b^{2}c^{2}-30\,abcd-5\,a^{2}d^{2})\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,(3\,b^{2}c^{2}-30\,abcd-5\,a^{2}d^{2})\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(3\,b^{2}c^{2}-30\,abcd-5\,a^{2}d^{2})\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,b^{2}c^{2}-30\,abcd-5\,a^{2}d^{2})\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\left(b^{3}c^{3}d-3\,ab^{2}c^{2}d^{2}+3\,a^{2}bcd^{3}-a^{3}d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))*a^2/(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/16*((b*c*d - 9*a*d^2)
```

```
*x^(5/2) - (3*b*c^2 + 5*a*c*d)*sqrt(x))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*
c^2*d^3 + a^2*c*d^4)*x^2) + 1/128*(2*sqrt(2)*(3*b^2*c^2 - 30*a*b*c*d - 5*a^
2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqr
t(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b^2*c^2
- 30*a*b*c*d - 5*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + s
qrt(2)*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(3*b^2*c^2 - 30*a*b
*c*d - 5*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c
))/(c^(3/4)*d^(1/4)))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^
4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5496 vs. $2(484) = 968$.
time = 100.80, size = 5496, normalized size = 8.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d
^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)*(-(81*
b^8*c^8 - 3240*a*b^7*c^7*d + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3
 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6
 + 15000*a^7*b*c*d^7 + 625*a^8*d^8)/(b^12*c^15*d^5 - 12*a*b^11*c^14*d^6 + 6
6*a^2*b^10*c^13*d^7 - 220*a^3*b^9*c^12*d^8 + 495*a^4*b^8*c^11*d^9 - 792*a^5
*b^7*c^10*d^10 + 924*a^6*b^6*c^9*d^11 - 792*a^7*b^5*c^8*d^12 + 495*a^8*b^4*
c^7*d^13 - 220*a^9*b^3*c^6*d^14 + 66*a^10*b^2*c^5*d^15 - 12*a^11*b*c^4*d^16
 + a^12*c^3*d^17))^(1/4)*arctan(-((b^9*c^11*d^4 - 9*a*b^8*c^10*d^5 + 36*a^2
*b^7*c^9*d^6 - 84*a^3*b^6*c^8*d^7 + 126*a^4*b^5*c^7*d^8 - 126*a^5*b^4*c^6*d
^9 + 84*a^6*b^3*c^5*d^10 - 36*a^7*b^2*c^4*d^11 + 9*a^8*b*c^3*d^12 - a^9*c^2
*d^13)*sqrt((9*b^4*c^4 - 180*a*b^3*c^3*d + 870*a^2*b^2*c^2*d^2 + 300*a^3*b*
c*d^3 + 25*a^4*d^4)*x + (b^6*c^8*d^2 - 6*a*b^5*c^7*d^3 + 15*a^2*b^4*c^6*d^4
 - 20*a^3*b^3*c^5*d^5 + 15*a^4*b^2*c^4*d^6 - 6*a^5*b*c^3*d^7 + a^6*c^2*d^8)
*sqrt(-(81*b^8*c^8 - 3240*a*b^7*c^7*d + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*
b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*
b^2*c^2*d^6 + 15000*a^7*b*c*d^7 + 625*a^8*d^8)/(b^12*c^15*d^5 - 12*a*b^11*c
^14*d^6 + 66*a^2*b^10*c^13*d^7 - 220*a^3*b^9*c^12*d^8 + 495*a^4*b^8*c^11*d^
9 - 792*a^5*b^7*c^10*d^10 + 924*a^6*b^6*c^9*d^11 - 792*a^7*b^5*c^8*d^12 + 4
95*a^8*b^4*c^7*d^13 - 220*a^9*b^3*c^6*d^14 + 66*a^10*b^2*c^5*d^15 - 12*a^11
*b*c^4*d^16 + a^12*c^3*d^17)))*(-(81*b^8*c^8 - 3240*a*b^7*c^7*d + 48060*a^2
*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5
*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 + 15000*a^7*b*c*d^7 + 625*a^8*d^8)/(b
```

^12*c^15*d^5 - 12*a*b^11*c^14*d^6 + 66*a^2*b^10*c^13*d^7 - 220*a^3*b^9*c^12
*d^8 + 495*a^4*b^8*c^11*d^9 - 792*a^5*b^7*c^10*d^10 + 924*a^6*b^6*c^9*d^11
- 792*a^7*b^5*c^8*d^12 + 495*a^8*b^4*c^7*d^13 - 220*a^9*b^3*c^6*d^14 + 66*a
^10*b^2*c^5*d^15 - 12*a^11*b*c^4*d^16 + a^12*c^3*d^17))^(3/4) + (3*b^11*c^1
3*d^4 - 57*a*b^10*c^12*d^5 + 373*a^2*b^9*c^11*d^6 - 1287*a^3*b^8*c^10*d^7 +
2718*a^4*b^7*c^9*d^8 - 3738*a^5*b^6*c^8*d^9 + 3402*a^6*b^5*c^7*d^10 - 1998
*a^7*b^4*c^6*d^11 + 687*a^8*b^3*c^5*d^12 - 93*a^9*b^2*c^4*d^13 - 15*a^10*b*
c^3*d^14 + 5*a^11*c^2*d^15)*sqrt(x)*(-(81*b^8*c^8 - 3240*a*b^7*c^7*d + 4806
0*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 51300
0*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 + 15000*a^7*b*c*d^7 + 625*a^8*d^
8)/(b^12*c^15*d^5 - 12*a*b^11*c^14*d^6 + 66*a^2*b^10*c^13*d^7 - 220*a^3*b^9
*c^12*d^8 + 495*a^4*b^8*c^11*d^9 - 792*a^5*b^7*c^10*d^10 + 924*a^6*b^6*c^9*
d^11 - 792*a^7*b^5*c^8*d^12 + 495*a^8*b^4*c^7*d^13 - 220*a^9*b^3*c^6*d^14 +
66*a^10*b^2*c^5*d^15 - 12*a^11*b*c^4*d^16 + a^12*c^3*d^17))^(3/4))/(81*b^8
*c^8 - 3240*a*b^7*c^7*d + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 +
649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 +
15000*a^7*b*c*d^7 + 625*a^8*d^8)) - 128*(-a^5*b^3/(b^12*c^12 - 12*a*b^11*c^
11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 7
92*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12))^(1/4)*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3
- 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*
x^2)*arctan(-((b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^
6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36
*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*(-a^5*b^3/(b^12*c^12 - 12*a*b^1
1*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
- 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^
11 + a^12*d^12))^(3/4)*sqrt(a^2*b^2*x + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b
^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*
d^6)*sqrt(-a^5*b^3/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))) - (a*b^10*c^9 - 9
*a^2*b^9*c^8*d + 36*a^3*b^8*c^7*d^2 - 84*a^4*b^7*c^6*d^3 + 126*a^5*b^6*c^5*
d^4 - 126*a^6*b^5*c^4*d^5 + 84*a^7*b^4*c^3*d^6 - 36*a^8*b^3*c^2*d^7 + 9*a^9
*b^2*c*d^8 - a^10*b*d^9)*(-a^5*b^3/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b
^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*
d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220
*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(3
/4)*sqrt(x))/(a^5*b^3)) + 32*(-a^5*b^3/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.29, size = 944, normalized size = 1.50

$$\frac{(ab^3)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^3c^3-3\sqrt{2}\,ab^2c^2d+3\sqrt{2}\,a^2bcd^2-\sqrt{2}\,a^3d^3}+\frac{(ab^3)^{\frac{1}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^3c^3-3\sqrt{2}\,ab^2c^2d+3\sqrt{2}\,a^2bcd^2-\sqrt{2}\,a^3d^3}+\frac{(ab^3)^{\frac{1}{4}} a \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^3c^3-3\sqrt{2}\,ab^2c^2d+3\sqrt{2}\,a^2bcd^2-\sqrt{2}\,a^3d^3\right)}-\frac{(ab^3)^{\frac{1}{4}} a \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^3c^3-3\sqrt{2}\,ab^2c^2d+3\sqrt{2}\,a^2bcd^2-\sqrt{2}\,a^3d^3\right)}+\frac{\left(3(cd^3)^{\frac{1}{4}}b^2c^2-30(cd^3)^{\frac{1}{4}}abcd-5(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^4d^2-3\sqrt{2}\,ab^2c^3d^3+3\sqrt{2}\,a^2bc^2d^4-\sqrt{2}\,a^3cd^5\right)}+\frac{\left(3(cd^3)^{\frac{1}{4}}b^2c^2-30(cd^3)^{\frac{1}{4}}abcd-5(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^4d^2-3\sqrt{2}\,ab^2c^3d^3+3\sqrt{2}\,a^2bc^2d^4-\sqrt{2}\,a^3cd^5\right)}+\frac{\left(3(cd^3)^{\frac{1}{4}}b^2c^2-30(cd^3)^{\frac{1}{4}}abcd-5(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^4d^2-3\sqrt{2}\,ab^2c^3d^3+3\sqrt{2}\,a^2bc^2d^4-\sqrt{2}\,a^3cd^5\right)}-\frac{\left(3(cd^3)^{\frac{1}{4}}b^2c^2-30(cd^3)^{\frac{1}{4}}abcd-5(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^4d^2-3\sqrt{2}\,ab^2c^3d^3+3\sqrt{2}\,a^2bc^2d^4-\sqrt{2}\,a^3cd^5\right)}+\frac{bcdx^{\frac{5}{2}}-9ad^2x^{\frac{5}{2}}-3bc^2\sqrt{x}-5acd\sqrt{x}}{16\left(b^2c^2d-2abcd^2+a^2d^3\right)\left(dx^2+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2 - s
qrt(2)*a^3*d^3) + (a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
- 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt
(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/2*(a*b^3)^(1/4)*a*log(sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3
*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) - 1/2*(a*b^3)^(1/4)*a*log(-sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2
*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/32*(3*(c*d^3)^(1/4)*b^2*c
^2 - 30*(c*d^3)^(1/4)*a*b*c*d - 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)
*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^4*d^2 - 3*sq
rt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d^5) + 1/32*(
3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*d - 5*(c*d^3)^(1/4)*a^2*d^
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt
(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(
2)*a^3*c*d^5) + 1/64*(3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*d -
5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(
sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - s
qrt(2)*a^3*c*d^5) - 1/64*(3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*
d - 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^
4 - sqrt(2)*a^3*c*d^5) + 1/16*(b*c*d*x^(5/2) - 9*a*d^2*x^(5/2) - 3*b*c^2*sq
rt(x) - 5*a*c*d*sqrt(x))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(d*x^2 + c)^2
)
```

Mupad [**B**]
time = 2.32, size = 2500, normalized size = 3.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x)

[Out] atan((((((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^5)/2048)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (((-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) - (x^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 259522560*a^12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17 - 17694720*a^17*b^6*c^2*d^18))/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*1i - (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9*c^2*d^6)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9* b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - ((((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^5)/2048)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (((-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (x^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 25...

3.481 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=628

$$\frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2}+\frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)}+\frac{a^{3/4}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}-\frac{a^{3/4}b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\cdots}{\cdots}\right.}{\sqrt{2}(bc-ad\cdots}$$

```
[Out] 1/4*x^(3/2)/(-a*d+b*c)/(d*x^2+c)^2+1/16*(3*a*d+5*b*c)*x^(3/2)/c/(-a*d+b*c)^
2/(d*x^2+c)+1/2*a^(3/4)*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(
-a*d+b*c)^3*2^(1/2)-1/2*a^(3/4)*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^
(1/4))/(-a*d+b*c)^3*2^(1/2)-1/64*(-3*a^2*d^2+30*a*b*c*d+5*b^2*c^2)*arctan(1
-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/d^(3/4)/(-a*d+b*c)^3*2^(1/2)+1/64
*(-3*a^2*d^2+30*a*b*c*d+5*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4)
)/c^(5/4)/d^(3/4)/(-a*d+b*c)^3*2^(1/2)-1/4*a^(3/4)*b^(5/4)*ln(a^(1/2)+x*b^(
1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2)+1/4*a^(3/4)*b^(5
/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1
/2)+1/128*(-3*a^2*d^2+30*a*b*c*d+5*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^
(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(3/4)/(-a*d+b*c)^3*2^(1/2)-1/128*(-3*a^2*d
^2+30*a*b*c*d+5*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/
2))/c^(5/4)/d^(3/4)/(-a*d+b*c)^3*2^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 482, 593, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{a^{3/4}b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}-\frac{a^{3/4}b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^3}-\frac{a^{3/4}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{a^{3/4}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}+\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}+\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}+\frac{x^{3/2}(3ad+5bc)}{16c\left(c+dx^2\right)(bc-ad)^2}+\frac{x^{3/2}}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] x^(3/2)/(4*(b*c - a*d)*(c + d*x^2)^2) + ((5*b*c + 3*a*d)*x^(3/2))/(16*c*(b*
c - a*d)^2*(c + d*x^2)) + (a^(3/4)*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(3/4)*b^(5/4)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((5*b^2*c^2 + 30*a*
b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[
2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) + ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(3/4)*(
b*c - a*d)^3) - (a^(3/4)*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(3/4)*b^(5/4)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) +
 ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) - ((5*b^
```

```
2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx = 2\text{Subst}\left(\int \frac{x^6}{(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}}{4(bc-ad)(c+dx^2)^2} - \frac{\text{Subst}\left(\int \frac{x^2(3a-5bx^4)}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{4(bc-ad)}$$

$$= \frac{x^{3/2}}{4(bc-ad)(c+dx^2)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2(c+dx^2)} - \frac{\text{Subst}\left(\int \frac{x^2(3a(9bc-ad)-b(5bc+}{(a+bx^4)(c+dx^4}\right.}{16c(bc-a}$$

$$= \frac{x^{3/2}}{4(bc-ad)(c+dx^2)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2(c+dx^2)} - \frac{\text{Subst}\left(\int \left(\frac{32ab^2cx^2}{(bc-ad)(a+bx^4)} - \right.\right.}{16c}$$

$$= \frac{x^{3/2}}{4(bc-ad)(c+dx^2)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2(c+dx^2)} - \frac{(2ab^2)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x\right.}{(bc-ad)^3}$$

$$= \frac{x^{3/2}}{4(bc-ad)(c+dx^2)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2(c+dx^2)} + \frac{(ab^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}}{a+bx^4}\right.}{(bc-ad)^3}$$

$$= \frac{x^{3/2}}{4(bc-ad)(c+dx^2)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2(c+dx^2)} - \frac{(ab)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{b}}}\right.}{2(bc-ad}$$

$$= \frac{x^{3/2}}{4(bc-ad)(c+dx^2)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2(c+dx^2)} - \frac{a^{3/4}b^{5/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{}\right.}{2\sqrt{2}(bc-}$$

$$= \frac{x^{3/2}}{4(bc-ad)(c+dx^2)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2(c+dx^2)} + \frac{a^{3/4}b^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}(bc-ad)^3}$$

Mathematica [A]
time = 1.17, size = 329, normalized size = 0.52

$$\frac{\frac{4(bc-ad)x^{3/2}(ad(-c+3dx^2)+bc(9c+5dx^2))}{c(c+dx^2)^2} + 32\sqrt{2}\,a^{3/4}b^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - \frac{\sqrt{2}(5b^2c^2+30abcd-3a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{5/4}d^{3/4}} + 32\sqrt{2}\,a^{3/4}b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right) - \frac{\sqrt{2}(5b^2c^2+30abcd-3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{5/4}d^{3/4}}}{64(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((4*(b*c - a*d)*x^(3/2)*(a*d*(-c + 3*d*x^2) + b*c*(9*c + 5*d*x^2)))/(c*(c +
 d*x^2)^2) + 32*Sqrt[2]*a^(3/4)*b^(5/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
```

```
2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2
)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(5/4)
*d^(3/4)) + 32*Sqrt[2]*a^(3/4)*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])/(Sqrt[a] + Sqrt[b]*x)] - (Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2
)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(5/4
)*d^(3/4)))/(64*(b*c - a*d)^3)
```

Maple [A]

time = 0.09, size = 330, normalized size = 0.53

method	result
derivativedivides	$\frac{ab\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2\left(\frac{d\left(3a^{2}d\right.}{\ldots}\right.}{\ldots}$
default	$\frac{ab\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2\left(\frac{d\left(3a^{2}d\right.}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2/(a*d-b*c)
^3*((1/32*d*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c*x^(7/2)+(-1/32*a^2*d^2+5/16*a
*b*c*d-9/32*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+1/256*(3*a^2*d^2-30*a*b*c*d-5*b^2
*c^2)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.52, size = 583, normalized size = 0.93

$$-\frac{ab^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(b^{3}c^{3}-3ab^{2}c^{2}d+3a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{\left(5b^{2}c^{2}+30abcd-3a^{2}d^{2}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^{3}c^{4}-3ab^{2}c^{3}d+3a^{2}bc^{2}d^{2}-a^{3}cd^{3}\right)}+\frac{\left(5bcd+3ad^{2}\right)x^{\frac{7}{2}}+\left(9bc^{2}-acd\right)x^{\frac{3}{2}}}{16\left(b^{2}c^{5}-2abc^{4}d+a^{2}c^{3}d^{2}+\left(b^{2}c^{3}d^{2}-2abc^{2}d^{3}+a^{2}cd^{4}\right)x^{4}+2\left(b^{2}c^{4}d-2abc^{3}d^{2}+a^{2}c^{2}d^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
```

```
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))
/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/128*(5*b^2*c^2 + 3
0*a*b*c*d - 3*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(
d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sq
rt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log
(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) +
 sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/
4)*d^(3/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3) + 1/16
*((5*b*c*d + 3*a*d^2)*x^(7/2) + (9*b*c^2 - a*c*d)*x^(3/2))/(b^2*c^5 - 2*a*b
*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b
^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 963 vs. 2(481) = 962.
time = 1.36, size = 963, normalized size = 1.53

$$\frac{\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2+30\,(cd^3)^{\frac{3}{4}}abcd-3\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^5d^3-3\sqrt{2}\,ab^2c^4d^4+3\sqrt{2}\,a^2bc^3d^5-\sqrt{2}\,a^3c^2d^6\right)}+\frac{\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2+30\,(cd^3)^{\frac{3}{4}}abcd-3\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^5d^3-3\sqrt{2}\,ab^2c^4d^4+3\sqrt{2}\,a^2bc^3d^5-\sqrt{2}\,a^3c^2d^6\right)}-\frac{\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2+30\,(cd^3)^{\frac{3}{4}}abcd-3\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^5d^3-3\sqrt{2}\,ab^2c^4d^4+3\sqrt{2}\,a^2bc^3d^5-\sqrt{2}\,a^3c^2d^6\right)}+\frac{\left(5\,(cd^3)^{\frac{3}{4}}b^2c^2+30\,(cd^3)^{\frac{3}{4}}abcd-3\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^5d^3-3\sqrt{2}\,ab^2c^4d^4+3\sqrt{2}\,a^2bc^3d^5-\sqrt{2}\,a^3c^2d^6\right)}-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^4c^3-3\sqrt{2}\,ab^3c^2d+3\sqrt{2}\,a^2b^2cd^2-\sqrt{2}\,a^3bd^3}-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^4c^3-3\sqrt{2}\,ab^3c^2d+3\sqrt{2}\,a^2b^2cd^2-\sqrt{2}\,a^3bd^3}+\frac{(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^4c^3-3\sqrt{2}\,ab^3c^2d+3\sqrt{2}\,a^2b^2cd^2-\sqrt{2}\,a^3bd^3\right)}-\frac{(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^4c^3-3\sqrt{2}\,ab^3c^2d+3\sqrt{2}\,a^2b^2cd^2-\sqrt{2}\,a^3bd^3\right)}+\frac{5\,bcdx^{\frac{7}{2}}+3\,ad^2x^{\frac{7}{2}}+9\,bc^2x^{\frac{3}{2}}-acdx^{\frac{3}{2}}}{16\,(b^2c^3-2\,abc^2d+a^2cd^2)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*(c*d^3)^(3/4)*b^2*c^2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*
a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/
```

```
(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^4 + 3*sqrt(2)*a^2*b*c^3*d^5 -
sqrt(2)*a^3*c^2*d^6) + 1/32*(5*(c*d^3)^(3/4)*b^2*c^2 + 30*(c*d^3)^(3/4)*a*b
*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^4 + 3*
sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) - 1/64*(5*(c*d^3)^(3/4)*b^2*c^
2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)
*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^
4 + 3*sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) + 1/64*(5*(c*d^3)^(3/4)*
b^2*c^2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*
sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2
*c^4*d^4 + 3*sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) - (a*b^3)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b
^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^
3) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a
/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d
^2 - sqrt(2)*a^3*b*d^3) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*
b^2*c*d^2 - sqrt(2)*a^3*b*d^3) - 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt
(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/16*(5*b*c*d*x^(7/2) + 3*a*d^2*x^
(7/2) + 9*b*c^2*x^(3/2) - a*c*d*x^(3/2))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*(d*x^2 + c)^2)
```

Mupad [B]
time = 2.30, size = 2500, normalized size = 3.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] 2*atan((((((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^
21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/
2 - (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535
*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*
c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12
)/8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (688
489*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c
^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)*1i)/(b^14*c^16 + a^14*c^2*d^14 - 1
4*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4
*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b
^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^
6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) -
 (x^(1/2)*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2
 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14
```

784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*d^11 + 20194099200*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4) - (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - (((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)*1i)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) + (x^(1/2)*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 108960

```
1536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16
*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10
- 21851799552*a^10*b^13*c^10*d^11 + 20194099200*a^11*b^12*c^9*d^12 - 154790
78912*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*
b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 +
 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14
+ a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c
^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^...
```

3.482 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=627

$$\frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2}+\frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)}+\frac{\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(1+\cdots\right.}{\sqrt{2}\,(bc-a\cdots}$$

```
[Out] 1/2*a^(1/4)*b^(7/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^3*
2^(1/2)-1/2*a^(1/4)*b^(7/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d
+b*c)^3*2^(1/2)-1/64*(-3*a^2*d^2+14*a*b*c*d+21*b^2*c^2)*arctan(1-d^(1/4)*2^
(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/64*(-3*a^2*d^
2+14*a*b*c*d+21*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/
d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/4*a^(1/4)*b^(7/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/
4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2)-1/4*a^(1/4)*b^(7/4)*ln(a^(
1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2)-1/128*
(-3*a^2*d^2+14*a*b*c*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(
1/2)*x^(1/2))/c^(7/4)/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/128*(-3*a^2*d^2+14*a*b
*c*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7
/4)/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/4*x^(1/2)/(-a*d+b*c)/(d*x^2+c)^2+1/16*(a
*d+7*b*c)*x^(1/2)/c/(-a*d+b*c)^2/(d*x^2+c)
```

Rubi [A]
time = 0.48, antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 482, 541, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}+\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}+\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,b^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,b^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,b^{7/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,b^{7/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^3}+\frac{\sqrt{x}\,(ad+7bc)}{16c\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt{x}}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*(b*c - a*d)*(c + d*x^2)^2) + ((7*b*c + a*d)*Sqrt[x])/(16*c*(b*c
- a*d)^2*(c + d*x^2)) + (a^(1/4)*b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(1/4)*b^(7/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((21*b^2*c^2 + 14*a*b
*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2
]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(1/4)*(
b*c - a*d)^3) + (a^(1/4)*b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - (a^(1/4)*b^(7/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) -
 ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*
```

```
b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
```

```
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx = 2\text{Subst}\left(\int \frac{x^4}{(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}}{4(bc-ad)(c+dx^2)^2} - \frac{\text{Subst}\left(\int \frac{a-7bx^4}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{4(bc-ad)}$$

$$= \frac{\sqrt{x}}{4(bc-ad)(c+dx^2)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2(c+dx^2)} - \frac{\text{Subst}\left(\int \frac{a(11bc-3ad)-3b(7bc+ad)}{(a+bx^4)(c+dx^4)}\right.}{16c(bc-ad)^2}$$

$$= \frac{\sqrt{x}}{4(bc-ad)(c+dx^2)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2(c+dx^2)} - \frac{(2ab^2)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \right.}{(bc-ad)^3}$$

$$= \frac{\sqrt{x}}{4(bc-ad)(c+dx^2)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2(c+dx^2)} - \frac{\left(\sqrt{a}\,b^2\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}}{a+bx^4}\right.}{(bc-ad)^3}$$

$$= \frac{\sqrt{x}}{4(bc-ad)(c+dx^2)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2(c+dx^2)} - \frac{\left(\sqrt{a}\,b^{3/2}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}-}\right.}{2(bc-}$$

$$= \frac{\sqrt{x}}{4(bc-ad)(c+dx^2)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2(c+dx^2)} + \frac{\sqrt[4]{a}\,b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}(bc-a}$$

$$= \frac{\sqrt{x}}{4(bc-ad)(c+dx^2)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2(c+dx^2)} + \frac{\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}(bc-ad)^3}$$

Mathematica [A]
time = 1.18, size = 327, normalized size = 0.52

$$\frac{\frac{4(bc-ad)\sqrt{x}\left(ad(-3c+dx^2)+bc(11c+7dx^2)\right)}{c(c+dx^2)^2}+32\sqrt{2}\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\frac{\sqrt{2}\,(21b^2c^2+14abcd-3a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{7/4}\sqrt[4]{d}}-32\sqrt{2}\sqrt[4]{a}\,b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)+\frac{\sqrt{2}\,(21b^2c^2+14abcd-3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{7/4}\sqrt[4]{d}}}{64(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((4*(b*c - a*d)*Sqrt[x]*(a*d*(-3*c + d*x^2) + b*c*(11*c + 7*d*x^2)))/(c*(c
+ d*x^2)^2) + 32*Sqrt[2]*a^(1/4)*b^(7/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d
^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(7/
4)*d^(1/4)) - 32*Sqrt[2]*a^(1/4)*b^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*S
```

```
qrt[x])/(Sqrt[a] + Sqrt[b]*x)] + (Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*
d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(
7/4)*d^(1/4)))/(64*(b*c - a*d)^3)
```

Maple [A]
time = 0.09, size = 327, normalized size = 0.52

method	result
derivativedivides	$\frac{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}}+\ldots$
default	$\frac{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}}+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2/(a*d-b*c)
^3*((1/32*d*(a^2*d^2+6*a*b*c*d-7*b^2*c^2)/c*x^(5/2)+(7/16*a*b*c*d-11/32*b^2
*c^2-3/32*a^2*d^2)*x^(1/2))/(d*x^2+c)^2+1/256*(3*a^2*d^2-14*a*b*c*d-21*b^2*
c^2)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
)/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 654, normalized size = 1.04

$$-\frac{\left(\frac{2\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)a}{4\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{(7\,bcd+ad^{2})x^{\frac{5}{2}}+(11\,bc^{2}-3\,acd)\sqrt{x}}{16\left(b^{2}c^{5}-2\,abc^{4}d+a^{2}c^{3}d^{2}+(b^{2}c^{3}d^{2}-2\,abc^{2}d^{3}+a^{2}cd^{4})x^{4}+2\,(b^{2}c^{4}d-2\,abc^{3}d^{2}+a^{2}c^{2}d^{3})x^{2}\right)}+\frac{\frac{2\sqrt{2}\,(21\,b^{2}c^{2}+14\,abcd-3\,a^{2}d^{2})\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,(21\,b^{2}c^{2}+14\,abcd-3\,a^{2}d^{2})\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,(21\,b^{2}c^{2}+14\,abcd-3\,a^{2}d^{2})\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(21\,b^{2}c^{2}+14\,abcd-3\,a^{2}d^{2})\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\left(b^{3}c^{4}-3\,ab^{2}c^{3}d+3\,a^{2}bc^{2}d^{2}-a^{3}cd^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2
)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(
7/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))*a
/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/16*((7*b*c*d + a*d
```

```
^2)*x^(5/2) + (11*b*c^2 - 3*a*c*d)*sqrt(x))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^
3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*
b*c^3*d^2 + a^2*c^2*d^3)*x^2) + 1/128*(2*sqrt(2)*(21*b^2*c^2 + 14*a*b*c*d -
 3*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(21*b^
2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4
) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)
)) + sqrt(2)*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(21*b^2*c^2
+ 14*a*b*c*d - 3*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
+ sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 -
a^3*c*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5450 vs. $2(480) = 960$.
time = 88.78, size = 5450, normalized size = 8.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3
 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)*(-(194
481*b^8*c^8 + 518616*a*b^7*c^7*d + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^
5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6
 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(b^12*c^19*d - 12*a*b^11*c^18*d^2 + 66*a^
2*b^10*c^17*d^3 - 220*a^3*b^9*c^16*d^4 + 495*a^4*b^8*c^15*d^5 - 792*a^5*b^7
*c^14*d^6 + 924*a^6*b^6*c^13*d^7 - 792*a^7*b^5*c^12*d^8 + 495*a^8*b^4*c^11*
d^9 - 220*a^9*b^3*c^10*d^10 + 66*a^10*b^2*c^9*d^11 - 12*a^11*b*c^8*d^12 + a
^12*c^7*d^13))^(1/4)*arctan(-((b^9*c^14*d - 9*a*b^8*c^13*d^2 + 36*a^2*b^7*c
^12*d^3 - 84*a^3*b^6*c^11*d^4 + 126*a^4*b^5*c^10*d^5 - 126*a^5*b^4*c^9*d^6
+ 84*a^6*b^3*c^8*d^7 - 36*a^7*b^2*c^7*d^8 + 9*a^8*b*c^6*d^9 - a^9*c^5*d^10)
*sqrt((441*b^4*c^4 + 588*a*b^3*c^3*d + 70*a^2*b^2*c^2*d^2 - 84*a^3*b*c*d^3
+ 9*a^4*d^4)*x + (b^6*c^10 - 6*a*b^5*c^9*d + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^
3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a^5*b*c^5*d^5 + a^6*c^4*d^6)*sqrt(-(1944
81*b^8*c^8 + 518616*a*b^7*c^7*d + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5
*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6
- 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(b^12*c^19*d - 12*a*b^11*c^18*d^2 + 66*a^2
*b^10*c^17*d^3 - 220*a^3*b^9*c^16*d^4 + 495*a^4*b^8*c^15*d^5 - 792*a^5*b^7*
c^14*d^6 + 924*a^6*b^6*c^13*d^7 - 792*a^7*b^5*c^12*d^8 + 495*a^8*b^4*c^11*d
^9 - 220*a^9*b^3*c^10*d^10 + 66*a^10*b^2*c^9*d^11 - 12*a^11*b*c^8*d^12 + a^
12*c^7*d^13)))*(-(194481*b^8*c^8 + 518616*a*b^7*c^7*d + 407484*a^2*b^6*c^6*
d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 +
 8316*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(b^12*c^19*d - 12*a*
```

b^11*c^18*d^2 + 66*a^2*b^10*c^17*d^3 - 220*a^3*b^9*c^16*d^4 + 495*a^4*b^8*c^15*d^5 - 792*a^5*b^7*c^14*d^6 + 924*a^6*b^6*c^13*d^7 - 792*a^7*b^5*c^12*d^8 + 495*a^8*b^4*c^11*d^9 - 220*a^9*b^3*c^10*d^10 + 66*a^10*b^2*c^9*d^11 - 12*a^11*b*c^8*d^12 + a^12*c^7*d^13))^(3/4) + (21*b^11*c^16*d - 175*a*b^10*c^15*d^2 + 627*a^2*b^9*c^14*d^3 - 1233*a^3*b^8*c^13*d^4 + 1362*a^4*b^7*c^12*d^5 - 630*a^5*b^6*c^11*d^6 - 378*a^6*b^5*c^10*d^7 + 798*a^7*b^4*c^9*d^8 - 567*a^8*b^3*c^8*d^9 + 213*a^9*b^2*c^7*d^10 - 41*a^10*b*c^6*d^11 + 3*a^11*c^5*d^12)*sqrt(x)*(-(194481*b^8*c^8 + 518616*a*b^7*c^7*d + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(b^12*c^19*d - 12*a*b^11*c^18*d^2 + 66*a^2*b^10*c^17*d^3 - 220*a^3*b^9*c^16*d^4 + 495*a^4*b^8*c^15*d^5 - 792*a^5*b^7*c^14*d^6 + 924*a^6*b^6*c^13*d^7 - 792*a^7*b^5*c^12*d^8 + 495*a^8*b^4*c^11*d^9 - 220*a^9*b^3*c^10*d^10 + 66*a^10*b^2*c^9*d^11 - 12*a^11*b*c^8*d^12 + a^12*c^7*d^13))^(3/4))/(194481*b^8*c^8 + 518616*a*b^7*c^7*d + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)) + 128*(-a*b^7/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(1/4)*(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)*arctan(-((b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*(-a*b^7/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(3/4)*sqrt(b^4*x + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(-a*b^7/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))) - (b^11*c^9 - 9*a*b^10*c^8*d + 36*a^2*b^9*c^7*d^2 - 84*a^3*b^8*c^6*d^3 + 126*a^4*b^7*c^5*d^4 - 126*a^5*b^6*c^4*d^5 + 84*a^6*b^5*c^3*d^6 - 36*a^7*b^4*c^2*d^7 + 9*a^8*b^3*c*d^8 - a^9*b^2*d^9)*(-a*b^7/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(3/4)*sqrt(x))/(a*b^7)) - 32*(-a*b^7/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12))^(1/4)*(b^2*c^5 -...

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.81, size = 946, normalized size = 1.51

$$-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{3}c^{3}-3\sqrt{2}\,ab^{2}c^{2}d+3\sqrt{2}\,a^{2}bcd^{2}-\sqrt{2}\,a^{3}d^{3}}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{3}c^{3}-3\sqrt{2}\,ab^{2}c^{2}d+3\sqrt{2}\,a^{2}bcd^{2}-\sqrt{2}\,a^{3}d^{3}}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{3}c^{3}-3\sqrt{2}\,ab^{2}c^{2}d+3\sqrt{2}\,a^{2}bcd^{2}-\sqrt{2}\,a^{3}d^{3}\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{3}c^{3}-3\sqrt{2}\,ab^{2}c^{2}d+3\sqrt{2}\,a^{2}bcd^{2}-\sqrt{2}\,a^{3}d^{3}\right)}+\frac{\left(21\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+14\left(cd^{3}\right)^{\frac{1}{4}}abcd-3\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^{3}c^{5}d-3\sqrt{2}\,ab^{2}c^{4}d^{2}+3\sqrt{2}\,a^{2}bc^{3}d^{3}-\sqrt{2}\,a^{3}c^{2}d^{4}\right)}+\frac{\left(21\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+14\left(cd^{3}\right)^{\frac{1}{4}}abcd-3\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^{3}c^{5}d-3\sqrt{2}\,ab^{2}c^{4}d^{2}+3\sqrt{2}\,a^{2}bc^{3}d^{3}-\sqrt{2}\,a^{3}c^{2}d^{4}\right)}+\frac{\left(21\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+14\left(cd^{3}\right)^{\frac{1}{4}}abcd-3\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^{3}c^{5}d-3\sqrt{2}\,ab^{2}c^{4}d^{2}+3\sqrt{2}\,a^{2}bc^{3}d^{3}-\sqrt{2}\,a^{3}c^{2}d^{4}\right)}-\frac{\left(21\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+14\left(cd^{3}\right)^{\frac{1}{4}}abcd-3\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^{3}c^{5}d-3\sqrt{2}\,ab^{2}c^{4}d^{2}+3\sqrt{2}\,a^{2}bc^{3}d^{3}-\sqrt{2}\,a^{3}c^{2}d^{4}\right)}+\frac{7\,bcdx^{\frac{5}{2}}+ad^{2}x^{\frac{5}{2}}+11\,bc^{2}\sqrt{x}-3\,acd\sqrt{x}}{16\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2 -
sqrt(2)*a^3*d^3) - (a*b^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqr
t(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) - 1/2*(a*b^3)^(1/4)*b*log(sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d +
3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/2*(a*b^3)^(1/4)*b*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^
2*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/32*(21*(c*d^3)^(1/4)*b^2
*c^2 + 14*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5*d - 3*sq
rt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/32
*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2
*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(s
qrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt
(2)*a^3*c^2*d^4) + 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a*b*c*
d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 -
 sqrt(2)*a^3*c^2*d^4) - 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a
*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sq
rt(c/d))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3
*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/16*(7*b*c*d*x^(5/2) + a*d^2*x^(5/2) + 11*b*
c^2*sqrt(x) - 3*a*c*d*sqrt(x))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(d*x^2
+ c)^2)
```

Mupad [B]
time = 2.35, size = 2500, normalized size = 3.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] 2*atan(((((((81*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^
2*b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^
5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048
+ (6885*a^7*b^9*c^2*d^8)/2048)*1i)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^
7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*
b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (((-(a*b^7)/(16*a^12*d
^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a
^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*
b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c
^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(8192*a^2*b^18*c^18
*d^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^1
5*c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 495897
6*a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*
d^12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^
7*c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b
^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d
^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*
b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 1941
01248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^1
8*c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 +
 14203486208*a^8*b^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 98618572
80*a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b
^11*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 +
343080960*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b
^6*c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21)*1i)/(40
96*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 -
 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a
^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c
^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12
 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*
b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5
*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*
d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4)*1i)*(-(a*b^7)/(16*a^12
*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920
*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^
7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2
*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - (x^(1/2)*(81*a^
10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*b
^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^6
*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2*d^9))/(4096*(b^
12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a
```

```
^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6
*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9
 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12 + 16*
b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^
8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d
^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 -
 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - (((((81*a^9*b^7*d^10)/2048
 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048 - (713097*a^3
*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c^4*
d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)*1i)
/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b
^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 -
 8*a*b^7*c^11*d) - (((-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10
*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7
*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8
 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*
a^11*b*c*d^11))^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 + 5
06112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^14
*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d^10 - 3277824*a^9
*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^11*b^9*c^9*d^13 -
459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*d^
16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*
d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b
^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) + (x
^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^18*d^5 + 10302259
20*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*c^16...
```

3.483 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=633

$$-\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{\ }}{\sqrt[4]{\ }}\right.}{\sqrt{2}\sqrt[4]{a}(bc-a}$$

```
[Out] -1/4*d*x^(3/2)/c/(-a*d+b*c)/(d*x^2+c)^2-1/16*d*(-5*a*d+13*b*c)*x^(3/2)/c^2/
(-a*d+b*c)^2/(d*x^2+c)-1/2*b^(9/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4)
)/a^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/2*b^(9/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)
/a^(1/4))/a^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/64*d^(1/4)*(5*a^2*d^2-18*a*b*c*d+4
5*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^3*2
^(1/2)-1/64*d^(1/4)*(5*a^2*d^2-18*a*b*c*d+45*b^2*c^2)*arctan(1+d^(1/4)*2^(1
/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(9/4)*ln(a^(1/2)+x*
b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^3*2^(1/2)-1/4*b
^(9/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+
b*c)^3*2^(1/2)-1/128*d^(1/4)*(5*a^2*d^2-18*a*b*c*d+45*b^2*c^2)*ln(c^(1/2)+x
*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/12
8*d^(1/4)*(5*a^2*d^2-18*a*b*c*d+45*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^
(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^3*2^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 633, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 483, 593, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^3}-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{9/4}(bc-ad)^3}-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3}-\frac{b^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{b^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{dx^{3/2}(13bc-5ad)}{16c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)^2) - (d*(13*b*c - 5*a*d)*x^(3/2
))/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (b^(9/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (d^(1
/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 1
8*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*S
qrt[2]*c^(9/4)*(b*c - a*d)^3) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b^(9/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*
(b*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c]
```

```
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c -
 a*d)^3) + (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a*d)^
3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx = 2\text{Subst}\left(\int \frac{x^2}{(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)(c+dx^2)^2} + \frac{\text{Subst}\left(\int \frac{x^2(8bc-5ad-5bdx^4)}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{x^2(32b^2c^2-13abcd+}{(a+bx}\right.}{16c^2}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2(c+dx^2)} + \frac{\text{Subst}\left(\int \left(\frac{32b^3c^2x^2}{(bc-ad)(a+bx^4)} \right.\right.}{1}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2(c+dx^2)} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, \right.}{(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2(c+dx^2)} - \frac{b^{5/2}\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4} \right.}{(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2(c+dx^2)} + \frac{b^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{b}}} \right.}{2(bc-a}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2(c+dx^2)} + \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\sqrt[4]{a}(bc}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2(c+dx^2)} - \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}$$

Mathematica [A]
time = 1.38, size = 361, normalized size = 0.57

$$\frac{1}{64}\left(\frac{4dx^{3/2}(ad(9c+5dx^2)-bc(17c+13dx^2))}{c^2(bc-ad)^2(c+dx^2)^2} + \frac{32\sqrt{2}\,b^{9/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}(-bc+ad)^3} + \frac{\sqrt{2}\sqrt[4]{d}(45b^2c^2-18abcd+5a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{9/4}(bc-ad)^3} + \frac{32\sqrt{2}\,b^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{a}(-bc+ad)^3} + \frac{\sqrt{2}\sqrt[4]{d}(45b^2c^2-18abcd+5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{9/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((4*d*x^(3/2)*(a*d*(9*c + 5*d*x^2) - b*c*(17*c + 13*d*x^2)))/(c^2*(b*c - a*
d)^2*(c + d*x^2)^2) + (32*Sqrt[2]*b^(9/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqr
```

```
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/
4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(9/4)*(b*c - a*d)^3) + (32*Sqrt[2]*b^(9/4
)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(1/4
)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2
)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(9/4
)*(b*c - a*d)^3))/64
```

Maple [A]
time = 0.09, size = 336, normalized size = 0.53

method	result
derivativedivides	$-\frac{b^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+$ 2d …
default	$-\frac{b^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+$ 2d …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2*d/(a*d-b
*c)^3*((1/32*d*(5*a^2*d^2-18*a*b*c*d+13*b^2*c^2)/c^2*x^(7/2)+1/32*(9*a^2*d^
2-26*a*b*c*d+17*b^2*c^2)/c*x^(3/2))/(d*x^2+c)^2+1/256*(5*a^2*d^2-18*a*b*c*d
+45*b^2*c^2)/c^2/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.52, size = 594, normalized size = 0.94

$$\frac{b^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{\left(45\,b^{2}c^{2}d-18\,abcd^{2}+5\,a^{2}d^{3}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)}-\frac{\left(13\,bcd^{2}-5\,ad^{3}\right)x^{\frac{7}{2}}+\left(17\,bc^{2}d-9\,acd^{2}\right)x^{\frac{3}{2}}}{16\left(b^{2}c^{6}-2\,abc^{5}d+a^{2}c^{4}d^{2}+\left(b^{2}c^{4}d^{2}-2\,abc^{3}d^{3}+a^{2}c^{2}d^{4}\right)x^{4}+2\left(b^{2}c^{5}d-2\,abc^{4}d^{2}+a^{2}c^{3}d^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/128*(45*b^2*c^2*d - 1
8*a*b*c*d^2 + 5*a^2*d^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*
sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*l
og(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4))
 + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(
1/4)*d^(3/4)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) -
1/16*((13*b*c*d^2 - 5*a*d^3)*x^(7/2) + (17*b*c^2*d - 9*a*c*d^2)*x^(3/2))/(b
^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2
*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.05, size = 968, normalized size = 1.53

$$-\frac{\left(45\,(cd^3)^{\frac{3}{4}}b^2c^2-18\,(cd^3)^{\frac{3}{4}}abcd+5\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^6d^2-3\sqrt{2}\,ab^2c^5d^3+3\sqrt{2}\,a^2bc^4d^4-\sqrt{2}\,a^3c^3d^5\right)}-\frac{\left(45\,(cd^3)^{\frac{3}{4}}b^2c^2-18\,(cd^3)^{\frac{3}{4}}abcd+5\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^6d^2-3\sqrt{2}\,ab^2c^5d^3+3\sqrt{2}\,a^2bc^4d^4-\sqrt{2}\,a^3c^3d^5\right)}+\frac{\left(45\,(cd^3)^{\frac{3}{4}}b^2c^2-18\,(cd^3)^{\frac{3}{4}}abcd+5\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^6d^2-3\sqrt{2}\,ab^2c^5d^3+3\sqrt{2}\,a^2bc^4d^4-\sqrt{2}\,a^3c^3d^5\right)}-\frac{\left(45\,(cd^3)^{\frac{3}{4}}b^2c^2-18\,(cd^3)^{\frac{3}{4}}abcd+5\,(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^6d^2-3\sqrt{2}\,ab^2c^5d^3+3\sqrt{2}\,a^2bc^4d^4-\sqrt{2}\,a^3c^3d^5\right)}+\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3c^3-3\sqrt{2}\,a^2b^2c^2d+3\sqrt{2}\,a^3bcd^2-\sqrt{2}\,a^4d^3}+\frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3c^3-3\sqrt{2}\,a^2b^2c^2d+3\sqrt{2}\,a^3bcd^2-\sqrt{2}\,a^4d^3}-\frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^3c^3-3\sqrt{2}\,a^2b^2c^2d+3\sqrt{2}\,a^3bcd^2-\sqrt{2}\,a^4d^3\right)}+\frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^3c^3-3\sqrt{2}\,a^2b^2c^2d+3\sqrt{2}\,a^3bcd^2-\sqrt{2}\,a^4d^3\right)}-\frac{13\,bcd^2x^{\frac{7}{2}}-5\,ad^3x^{\frac{7}{2}}+17\,bc^2dx^{\frac{3}{2}}-9\,acd^2x^{\frac{3}{2}}}{16\,(b^2c^4-2\,abc^3d+a^2c^2d^2)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/32*(45*(c*d^3)^(3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4
- sqrt(2)*a^3*c^3*d^5) - 1/32*(45*(c*d^3)^(3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*
a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4)
 - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^5*d^3 +
 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) + 1/64*(45*(c*d^3)^(3/4)*b^
2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqr
t(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^
5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) - 1/64*(45*(c*d^3)^(
3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(-sqr
t(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*
a*b^2*c^5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) + (a*b^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt
(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a
^4*d^3) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x
))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^
3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqr
t(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d
 + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/16*(13*b*c*d^2*x^(7/2) - 5*
a*d^3*x^(7/2) + 17*b*c^2*d*x^(3/2) - 9*a*c*d^2*x^(3/2))/((b^2*c^4 - 2*a*b*c
^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

Mupad [B]
time = 2.37, size = 2500, normalized size = 3.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] 2*atan((((((2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^2
2*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/1
6 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*
a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^1
5*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^1
0*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/
2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (28062
3*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d
^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)*1i)/(b^14*c^20 + a^14*c^6*d^14 - 14*a
^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^
10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*
c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^
```

10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d)
- (x^(1/2)*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 105
6*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a
^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b
^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*
d^11))^(1/4)*(16777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140
473856*a^3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b
^18*c^17*d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^
10 - 55691968512*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 6608
5978112*a^10*b^13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344
*a^12*b^11*c^10*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9
*c^8*d^17 + 2040201216*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 75
104256*a^17*b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*
d^22))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*
c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d
^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 2
20*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a
^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 -
3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784
*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10
*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(3/4) - (x^(1/2
)*(625*a^9*b^10*d^13 + 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487
400*a^2*b^17*c^7*d^6 + 4100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8
+ 1099206*a^5*b^14*c^4*d^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*
d^11))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*
c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d
^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 2
20*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a
^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 -
3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784
*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10
*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4) - ((((204
8*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (1
711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/16 + (565575*a^5
*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d
^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12)/2
+ (15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^10*d^14)/8 + (32
835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/2 + (1717635*a^
14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*
d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*
a^19*b^5*c^2*d^22)/16)*1i)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13
+ 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2
002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003
*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a
^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d) + (x^(1/2)*(-b^

```
9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10
*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5
+ 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 35
20*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4)*(1
6777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^3*b^20
*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*d^8 -
22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 5569196851
2*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 66085978112*a^10*b^
13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^11*c^10
*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 672399...
```

3.484 $\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^3}\,dx$

Optimal. Leaf size=633

$$-\frac{d\sqrt{x}}{4c(bc-ad)\,(c+dx^2)^2}-\frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\,(c+dx^2)}-\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^3}$$

```
[Out] -1/2*b^(11/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)^
3*2^(1/2)+1/2*b^(11/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-
a*d+b*c)^3*2^(1/2)+1/64*d^(3/4)*(21*a^2*d^2-66*a*b*c*d+77*b^2*c^2)*arctan(1
-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/64*d^(3/4
)*(21*a^2*d^2-66*a*b*c*d+77*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/
4))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/4*b^(11/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)
*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(11/4)*ln(a^(1
/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^3*2^(1/2)
+1/128*d^(3/4)*(21*a^2*d^2-66*a*b*c*d+77*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1
/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/128*d^(3/4)*(2
1*a^2*d^2-66*a*b*c*d+77*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/
2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/4*d*x^(1/2)/c/(-a*d+b*c)/(d*x^2
+c)^2-1/16*d*(-7*a*d+15*b*c)*x^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c)
```

Rubi [A]
time = 0.57, antiderivative size = 633, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 425, 541, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{11/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^3}-\frac{b^{11/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{d^{3/4}(21a^2d^2-66abcd+77b^2c^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^3}-\frac{d^{3/4}(21a^2d^2-66abcd+77b^2c^2)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^3}+\frac{d^{3/4}(21a^2d^2-66abcd+77b^2c^2)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}(bc-ad)^3}-\frac{d^{3/4}(21a^2d^2-66abcd+77b^2c^2)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}(bc-ad)^3}-\frac{d\sqrt{x}\,(15bc-7ad)}{16c^2\,(c+dx^2)\,(bc-ad)^2}-\frac{d\sqrt{x}}{4c\,(c+dx^2)^2\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)^2) - (d*(15*b*c - 7*a*d)*Sqrt[x
])/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(11/4)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^
(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2
 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/
(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (b^(11/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(11
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*
a^(3/4)*(b*c - a*d)^3) + (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Lo
g[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11
```

```
/4)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*
(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)(c+dx^2)^3}\,dx = 2\text{Subst}\left(\int \frac{1}{(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)(c+dx^2)^2} + \frac{\text{Subst}\left(\int \frac{8bc-7ad-7bdx^4}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{32b^2c^2-45a}{}\right.}{}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2(c+dx^2)} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{a+bx}\right.}{(bc-ac}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2(c+dx^2)} + \frac{b^3\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{}}{a+b}\right.}{\sqrt{a}\,(bc}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2(c+dx^2)} + \frac{b^{5/2}\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}}\right.}{2\sqrt{}}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2(c+dx^2)} - \frac{b^{11/4}\log\left(\sqrt{a}-\right.}{2\sqrt{2}\,a}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2(c+dx^2)} - \frac{b^{11/4}\tan^{-1}\left(1-\right.}{\sqrt{2}\,a^{3/4}(bc}$$

Mathematica [A]
time = 1.41, size = 362, normalized size = 0.57

$$\frac{1}{64}\left(\frac{4d\sqrt{x}\,(ad(11c+7dx^2)-bc(19c+15dx^2))}{c^2(bc-ad)^2(c+dx^2)^2}+\frac{32\sqrt{2}\,b^{11/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}(-bc+ad)^3}+\frac{\sqrt{2}\,d^{3/4}(77b^2c^2-66abcd+21a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{11/4}(bc-ad)^3}-\frac{32\sqrt{2}\,b^{11/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}(-bc+ad)^3}-\frac{\sqrt{2}\,d^{3/4}(77b^2c^2-66abcd+21a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{11/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((4*d*Sqrt[x]*(a*d*(11*c + 7*d*x^2) - b*c*(19*c + 15*d*x^2)))/(c^2*(b*c - a
*d)^2*(c + d*x^2)^2) + (32*Sqrt[2]*b^(11/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(3/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(
3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(11/4)*(b*c - a*d)^3) - (32*Sqrt[2]*b^
(11/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a
```

```
^(3/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a
^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(
c^(11/4)*(b*c - a*d)^3))/64
```

Maple [A]
time = 0.09, size = 336, normalized size = 0.53

method	result
derivativedivides	$-\frac{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4\left(ad-bc\right)^{3}a}+ -$
default	$-\frac{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4\left(ad-bc\right)^{3}a}+ -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b^3/(a*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2*d/(a*d
-b*c)^3*((1/32*d*(7*a^2*d^2-22*a*b*c*d+15*b^2*c^2)/c^2*x^(5/2)+1/32*(11*a^2
*d^2-30*a*b*c*d+19*b^2*c^2)/c*x^(1/2))/(d*x^2+c)^2+1/256*(21*a^2*d^2-66*a*b
*c*d+77*b^2*c^2)/c^3*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.52, size = 675, normalized size = 1.07

$$-\frac{(15\,bcd^2-7\,ad^3)x^{\frac{5}{2}}+(19\,bc^2d-11\,acd^2)\sqrt{x}}{16\,(b^2c^6-2\,abc^5d+a^2c^4d^2+(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4)x^4+2\,(b^2c^5d-2\,abc^4d^2+a^2c^3d^3)x^2)}+\frac{\frac{2\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{11}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{11}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}-\frac{\frac{2\sqrt{2}\,(77\,b^2c^2d-66\,abcd^2+21\,a^2d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(77\,b^2c^2d-66\,abcd^2+21\,a^2d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(77\,b^2c^2d-66\,abcd^2+21\,a^2d^3)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(77\,b^2c^2d-66\,abcd^2+21\,a^2d^3)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*((15*b*c*d^2 - 7*a*d^3)*x^(5/2) + (19*b*c^2*d - 11*a*c*d^2)*sqrt(x))/
(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c
^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2) + 1/4*(2*sqr
t(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^3*arcta
```

```
n(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(11/4)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(11/4)*log(
-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/128*(2*sqrt(2)*(77*b^2*c^2*d
- 66*a*b*c*d^2 + 21*a^2*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) +
2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
 2*sqrt(2)*(77*b^2*c^2*d - 66*a*b*c*d^2 + 21*a^2*d^3)*arctan(-1/2*sqrt(2)*(
sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c
)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(77*b^2*c^2*d - 66*a*b*c*d^2 + 21*a^2*d^
3)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1
/4)) - sqrt(2)*(77*b^2*c^2*d - 66*a*b*c*d^2 + 21*a^2*d^3)*log(-sqrt(2)*c^(1
/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^5 - 3*
a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5548 vs. $2(486) = 972$.
time = 257.05, size = 5548, normalized size = 8.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(4*(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3
 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)*(-(3
5153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 -
 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c
^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10 + 194481*a^8*d^11)
/(b^12*c^23 - 12*a*b^11*c^22*d + 66*a^2*b^10*c^21*d^2 - 220*a^3*b^9*c^20*d^
3 + 495*a^4*b^8*c^19*d^4 - 792*a^5*b^7*c^18*d^5 + 924*a^6*b^6*c^17*d^6 - 79
2*a^7*b^5*c^16*d^7 + 495*a^8*b^4*c^15*d^8 - 220*a^9*b^3*c^14*d^9 + 66*a^10*
b^2*c^13*d^10 - 12*a^11*b*c^12*d^11 + a^12*c^11*d^12))^(1/4)*arctan(-((b^9*
c^17 - 9*a*b^8*c^16*d + 36*a^2*b^7*c^15*d^2 - 84*a^3*b^6*c^14*d^3 + 126*a^4
*b^5*c^13*d^4 - 126*a^5*b^4*c^12*d^5 + 84*a^6*b^3*c^11*d^6 - 36*a^7*b^2*c^1
0*d^7 + 9*a^8*b*c^9*d^8 - a^9*c^8*d^9)*sqrt((5929*b^4*c^4*d^2 - 10164*a*b^3
*c^3*d^3 + 7590*a^2*b^2*c^2*d^4 - 2772*a^3*b*c*d^5 + 441*a^4*d^6)*x + (b^6*
c^12 - 6*a*b^5*c^11*d + 15*a^2*b^4*c^10*d^2 - 20*a^3*b^3*c^9*d^3 + 15*a^4*b
^2*c^8*d^4 - 6*a^5*b*c^7*d^5 + a^6*c^6*d^6)*sqrt(-(35153041*b^8*c^8*d^3 - 1
20524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*
d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b
^2*c^2*d^9 - 2444904*a^7*b*c*d^10 + 194481*a^8*d^11)/(b^12*c^23 - 12*a*b^11
*c^22*d + 66*a^2*b^10*c^21*d^2 - 220*a^3*b^9*c^20*d^3 + 495*a^4*b^8*c^19*d^
4 - 792*a^5*b^7*c^18*d^5 + 924*a^6*b^6*c^17*d^6 - 792*a^7*b^5*c^16*d^7 + 49
5*a^8*b^4*c^15*d^8 - 220*a^9*b^3*c^14*d^9 + 66*a^10*b^2*c^13*d^10 - 12*a^11
```

*b*c^12*d^11 + a^12*c^11*d^12)))*(-(35153041*b^8*c^8*d^3 - 120524712*a*b^7*
c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694
*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 24
44904*a^7*b*c*d^10 + 194481*a^8*d^11)/(b^12*c^23 - 12*a*b^11*c^22*d + 66*a^
2*b^10*c^21*d^2 - 220*a^3*b^9*c^20*d^3 + 495*a^4*b^8*c^19*d^4 - 792*a^5*b^7
*c^18*d^5 + 924*a^6*b^6*c^17*d^6 - 792*a^7*b^5*c^16*d^7 + 495*a^8*b^4*c^15*
d^8 - 220*a^9*b^3*c^14*d^9 + 66*a^10*b^2*c^13*d^10 - 12*a^11*b*c^12*d^11 +
a^12*c^11*d^12))^(3/4) - (77*b^11*c^19*d - 759*a*b^10*c^18*d^2 + 3387*a^2*b
^9*c^17*d^3 - 9033*a^3*b^8*c^16*d^4 + 16002*a^4*b^7*c^15*d^5 - 19782*a^5*b^
6*c^14*d^6 + 17430*a^6*b^5*c^13*d^7 - 10962*a^7*b^4*c^12*d^8 + 4833*a^8*b^3
*c^11*d^9 - 1427*a^9*b^2*c^10*d^10 + 255*a^10*b*c^9*d^11 - 21*a^11*c^8*d^12
)*sqrt(x)*(-(35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2
*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 5104
3608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10 + 19
4481*a^8*d^11)/(b^12*c^23 - 12*a*b^11*c^22*d + 66*a^2*b^10*c^21*d^2 - 220*a
^3*b^9*c^20*d^3 + 495*a^4*b^8*c^19*d^4 - 792*a^5*b^7*c^18*d^5 + 924*a^6*b^6
*c^17*d^6 - 792*a^7*b^5*c^16*d^7 + 495*a^8*b^4*c^15*d^8 - 220*a^9*b^3*c^14*
d^9 + 66*a^10*b^2*c^13*d^10 - 12*a^11*b*c^12*d^11 + a^12*c^11*d^12))^(3/4))
/(35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^
5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^
3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10 + 194481*a^8*d^
11)) - 128*(-b^11/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^
2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a
^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3
*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(1/4)*(b^2
*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d
^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)*arctan(-((a^2*b^
9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6
*b^5*c^5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^
7 + 9*a^10*b*c*d^8 - a^11*d^9)*(-b^11/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d +
66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8
*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^
4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^
15*d^12))^(3/4)*sqrt(b^6*x + (a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^
4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*
sqrt(-b^11/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220
*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*
c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^
9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))) - (a^2*b^12*c^9
- 9*a^3*b^11*c^8*d + 36*a^4*b^10*c^7*d^2 - 84*a^5*b^9*c^6*d^3 + 126*a^6*b^8
*c^5*d^4 - 126*a^7*b^7*c^4*d^5 + 84*a^8*b^6*c^3*d^6 - 36*a^9*b^5*c^2*d^7 +
9*a^10*b^4*c*d^8 - a^11*b^3*d^9)*(-b^11/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d
+ 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a
^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*
c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 +

```
a^15*d^12))^(3/4)*sqrt(x))/b^11) + 32*(-b^11/(a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.85, size = 960, normalized size = 1.52

$$\frac{(ab^3)^{\frac{1}{4}} b^2 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3c^3-3\sqrt{2}\,a^2b^2c^2d+3\sqrt{2}\,a^3bcd^2-\sqrt{2}\,a^4d^3}+\frac{(ab^3)^{\frac{1}{4}} b^2 \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3c^3-3\sqrt{2}\,a^2b^2c^2d+3\sqrt{2}\,a^3bcd^2-\sqrt{2}\,a^4d^3}+\frac{(ab^3)^{\frac{1}{4}} b^2 \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^3c^3-3\sqrt{2}\,a^2b^2c^2d+3\sqrt{2}\,a^3bcd^2-\sqrt{2}\,a^4d^3\right)}-\frac{(ab^3)^{\frac{1}{4}} b^2 \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^3c^3-3\sqrt{2}\,a^2b^2c^2d+3\sqrt{2}\,a^3bcd^2-\sqrt{2}\,a^4d^3\right)}-\frac{\left(77(cd^3)^{\frac{1}{4}}b^2c^2-66(cd^3)^{\frac{1}{4}}abcd+21(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}-\frac{\left(77(cd^3)^{\frac{1}{4}}b^2c^2-66(cd^3)^{\frac{1}{4}}abcd+21(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}-\frac{\left(77(cd^3)^{\frac{1}{4}}b^2c^2-66(cd^3)^{\frac{1}{4}}abcd+21(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}+\frac{\left(77(cd^3)^{\frac{1}{4}}b^2c^2-66(cd^3)^{\frac{1}{4}}abcd+21(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}-\frac{15\,bcd^2x^{\frac{5}{2}}-7\,ad^3x^{\frac{5}{2}}+19\,bc^2d\sqrt{x}-11\,acd^2\sqrt{x}}{16\,(b^2c^4-2\,abc^3d+a^2c^2d^2)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d
^2 - sqrt(2)*a^4*d^3) + (a*b^3)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^
2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/2*(a*b^3)^(1/4)*b^2*log(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)
*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/2*(a*b^3)^(1/
4)*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3
 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/3
2*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a
^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)
*a^3*c^3*d^3) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*d^3)^(1/4)*a*b*c*d +
 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqr
t(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2
*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*
d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1
/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a
^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) + 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(
c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)
^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2
)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/16*(15*b*c*d^2*x^(5/2) - 7*a*d^3
*x^(5/2) + 19*b*c^2*d*sqrt(x) - 11*a*c*d^2*sqrt(x))/((b^2*c^4 - 2*a*b*c^3*d
 + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

Mupad [B]
time = 2.33, size = 2500, normalized size = 3.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)*(c + d*x^2)^3),x)
[Out] atan(((-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*
a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8
*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b
^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*
d^11))^(1/4)*((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*
a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c
^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d
^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12
)/2048)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 5
6*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2
*c^10*d^6 - 8*a*b^7*c^15*d) + ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592
*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*
d^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 1839726
5920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8
*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*
c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*
d^16 + 147248775168*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 2
3871029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^
17*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23))
/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d
^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 9
24*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9
*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) - ((-b^11/(16*a^
15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 -
 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 1478
4*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a
^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*(8192*
a*b^19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117
952*a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^
9 - 5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a^9
*b^11*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^1
4 + 5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14*b
^6*c^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16
+ a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3
 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^
7*c^15*d))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d +
1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 1267
```

2*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(3/4))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*1i + (x^(1/2)*(194481*a^8*b^11*d^15 + 41224337*b^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 201081276*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13)*1i)/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d))) - (-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12)/2048)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d) - ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*d^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 18397265920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*d...

3.485 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=681

$$-\frac{32b^2c^2-85abcd+45a^2d^2}{16ac^3(bc-ad)^2\sqrt{x}}-\frac{d}{4c(bc-ad)\sqrt{x}\,\left(c+dx^2\right)^2}-\frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right)}+\frac{b^{13/4}\tan^{-1}\left(1-\sqrt{}\right.}{\sqrt{2}\,a^{5/4}(bc-}$$

```
[Out] 1/2*b^(13/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^3
*2^(1/2)-1/2*b^(13/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a
*d+b*c)^3*2^(1/2)-1/64*d^(5/4)*(45*a^2*d^2-130*a*b*c*d+117*b^2*c^2)*arctan(
1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/64*d^(5/
4)*(45*a^2*d^2-130*a*b*c*d+117*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^
(1/4))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)-1/4*b^(13/4)*ln(a^(1/2)+x*b^(1/2)-a^(1
/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(13/4)*ln(a
^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)^3*2^(1
/2)+1/128*d^(5/4)*(45*a^2*d^2-130*a*b*c*d+117*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)
-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)-1/128*d^(5/
4)*(45*a^2*d^2-130*a*b*c*d+117*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4
)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/16*(-45*a^2*d^2+85*a*b*c
*d-32*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x^(1/2)-1/4*d/c/(-a*d+b*c)/(d*x^2+c)^2/x^
(1/2)-1/16*d*(-9*a*d+17*b*c)/c^2/(-a*d+b*c)^2/(d*x^2+c)/x^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {477, 483, 593, 597, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{b^{13/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{b^{13/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{d^{5/4}(45a^2d^2-130abcd+117b^2c^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{13/4}(bc-ad)^3}+\frac{d^{5/4}(45a^2d^2-130abcd+117b^2c^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{13/4}(bc-ad)^3}+\frac{d^{5/4}(45a^2d^2-130abcd+117b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{13/4}(bc-ad)^3}-\frac{d^{5/4}(45a^2d^2-130abcd+117b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{13/4}(bc-ad)^3}-\frac{45a^2d^2-85abcd+32b^2c^2}{16ac^3\sqrt{x}\,(bc-ad)^2}-\frac{d(17bc-9ad)}{16c^2\sqrt{x}\,(c+dx^2)(bc-ad)^2}-\frac{d}{4c\sqrt{x}\,(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/16*(32*b^2*c^2 - 85*a*b*c*d + 45*a^2*d^2)/(a*c^3*(b*c - a*d)^2*Sqrt[x])
- d/(4*c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)^2) - (d*(17*b*c - 9*a*d))/(16*c^2*
(b*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (b^(13/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/
4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*c^2
 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (b^(1
3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
```

```
*a^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(13/4)*(b*c - a*d)^3) - (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(1
3/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-9ad-9bdx^4}{x^2(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{\text{Subst}\Big(\int$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9a}{16c^2(bc-ad)^2\sqrt{x}}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9a}{16c^2(bc-ad)^2\sqrt{x}}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9a}{16c^2(bc-ad)^2\sqrt{x}}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9a}{16c^2(bc-ad)^2\sqrt{x}}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9a}{16c^2(bc-ad)^2\sqrt{x}}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9a}{16c^2(bc-ad)^2\sqrt{x}}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9a}{16c^2(bc-ad)^2\sqrt{x}}$$

Mathematica [A]
time = 1.15, size = 410, normalized size = 0.60

$$\frac{1}{64}\left(-\frac{4\left(32b^2c^2\left(c+dx^2\right)^2+a^2d^2\left(32c^2+81cdx^2+45d^2x^4\right)-abcd\left(64c^2+153cdx^2+85d^2x^4\right)\right)}{ac^3(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2}-\frac{32\sqrt{2}\,b^{13/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{5/4}(-bc+ad)^3}-\frac{\sqrt{2}\,d^{5/4}\left(117b^2c^2-130abcd+45a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{13/4}(bc-ad)^3}-\frac{32\sqrt{2}\,b^{13/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{5/4}(-bc+ad)^3}-\frac{\sqrt{2}\,d^{5/4}\left(117b^2c^2-130abcd+45a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{13/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-4*(32*b^2*c^2*(c + d*x^2)^2 + a^2*d^2*(32*c^2 + 81*c*d*x^2 + 45*d^2*x^4)
 - a*b*c*d*(64*c^2 + 153*c*d*x^2 + 85*d^2*x^4)))/(a*c^3*(b*c - a*d)^2*Sqrt[
x]*(c + d*x^2)^2) - (32*Sqrt[2]*b^(13/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(5/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*d^(5/4
)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(13/4)*(b*c - a*d)^3) - (32*Sqrt[2]*b^(
13/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^
(5/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*
a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/
(c^(13/4)*(b*c - a*d)^3))/64
```

Maple [A]

time = 0.18, size = 348, normalized size = 0.51 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^3/a/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*d^2/c^3
/(a*d-b*c)^3*(((13/32*a^2*d^3-17/16*a*b*c*d^2+21/32*b^2*c^2*d)*x^(7/2)+1/32
*c*(17*a^2*d^2-42*a*b*c*d+25*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+1/8*(45/32*a^2*d
^2-65/16*a*b*c*d+117/32*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x
^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-
1)))-2/a/c^3/x^(1/2)
```

Maxima [A]

time = 0.54, size = 668, normalized size = 0.98

$$-\frac{b^{4}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(ab^{3}c^{3}-3a^{2}b^{2}c^{2}d+3a^{3}bcd^{2}-a^{4}d^{3}\right)}+\frac{\left(117b^{2}c^{2}d^{2}-130abcd^{3}+45a^{2}d^{4}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^{3}c^{6}-3ab^{2}c^{5}d+3a^{2}bc^{4}d^{2}-a^{3}c^{3}d^{3}\right)}-\frac{32b^{2}c^{4}-64abc^{3}d+32a^{2}c^{2}d^{2}+\left(32b^{2}c^{2}d^{2}-85abcd^{3}+45a^{2}d^{4}\right)x^{4}+\left(64b^{2}c^{3}d-153abc^{2}d^{2}+81a^{2}cd^{3}\right)x^{2}}{16\left(\left(ab^{2}c^{5}d^{2}-2a^{2}bc^{4}d^{3}+a^{3}c^{3}d^{4}\right)x^{\frac{9}{2}}+2\left(ab^{2}c^{6}d-2a^{2}bc^{5}d^{2}+a^{3}c^{4}d^{3}\right)x^{\frac{5}{2}}+\left(ab^{2}c^{7}-2a^{2}bc^{6}d+a^{3}c^{5}d^{2}\right)\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(
a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/128*(117*b^2*c^2
*d^2 - 130*a*b*c*d^3 + 45*a^2*d^4)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c
^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sq
```

```
rt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) -
2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) -
 sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4
)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqr
t(c))/(c^(1/4)*d^(3/4)))/(b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c
^3*d^3) - 1/16*(32*b^2*c^4 - 64*a*b*c^3*d + 32*a^2*c^2*d^2 + (32*b^2*c^2*d^
2 - 85*a*b*c*d^3 + 45*a^2*d^4)*x^4 + (64*b^2*c^3*d - 153*a*b*c^2*d^2 + 81*a
^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^(9/2) + 2
*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^(5/2) + (a*b^2*c^7 - 2*a^2
*b*c^6*d + a^3*c^5*d^2)*sqrt(x))
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.99, size = 987, normalized size = 1.45

$$-\frac{(ab^3)^{\frac{3}{4}} b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^3c^3-3\sqrt{2}\,a^3b^2c^2d+3\sqrt{2}\,a^4bcd^2-\sqrt{2}\,a^5d^3}-\frac{(ab^3)^{\frac{3}{4}} b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^3c^3-3\sqrt{2}\,a^3b^2c^2d+3\sqrt{2}\,a^4bcd^2-\sqrt{2}\,a^5d^3}+\frac{(ab^3)^{\frac{3}{4}} b \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2b^3c^3-3\sqrt{2}\,a^3b^2c^2d+3\sqrt{2}\,a^4bcd^2-\sqrt{2}\,a^5d^3\right)}-\frac{(ab^3)^{\frac{3}{4}} b \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2b^3c^3-3\sqrt{2}\,a^3b^2c^2d+3\sqrt{2}\,a^4bcd^2-\sqrt{2}\,a^5d^3\right)}+\frac{\left(117(cd^3)^{\frac{3}{4}}b^2c^2-130(cd^3)^{\frac{3}{4}}abcd+45(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^7d-3\sqrt{2}\,ab^2c^6d^2+3\sqrt{2}\,a^2bc^5d^3-\sqrt{2}\,a^3c^4d^4\right)}+\frac{\left(117(cd^3)^{\frac{3}{4}}b^2c^2-130(cd^3)^{\frac{3}{4}}abcd+45(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^3c^7d-3\sqrt{2}\,ab^2c^6d^2+3\sqrt{2}\,a^2bc^5d^3-\sqrt{2}\,a^3c^4d^4\right)}-\frac{\left(117(cd^3)^{\frac{3}{4}}b^2c^2-130(cd^3)^{\frac{3}{4}}abcd+45(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^7d-3\sqrt{2}\,ab^2c^6d^2+3\sqrt{2}\,a^2bc^5d^3-\sqrt{2}\,a^3c^4d^4\right)}+\frac{\left(117(cd^3)^{\frac{3}{4}}b^2c^2-130(cd^3)^{\frac{3}{4}}abcd+45(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^3c^7d-3\sqrt{2}\,ab^2c^6d^2+3\sqrt{2}\,a^2bc^5d^3-\sqrt{2}\,a^3c^4d^4\right)}+\frac{21\,bcd^3x^{\frac{7}{2}}-13\,ad^4x^{\frac{7}{2}}+25\,bc^2d^2x^{\frac{3}{2}}-17\,acd^3x^{\frac{3}{2}}}{16\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)\left(dx^2+c\right)^2}-\frac{2}{ac^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*
d^2 - sqrt(2)*a^5*d^3) - (a*b^3)^(3/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c
^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/2*(a*b^3)^(3/4)*b*log(s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2
)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/2*(a*b^3)^(3
```

```
/4)*b*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^
3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/
32*(117*(c*d^3)^(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 -
sqrt(2)*a^3*c^4*d^4) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*
a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^6*d^2 +
3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) - 1/64*(117*(c*d^3)^(3/4)*b^
2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*s
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^
6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) + 1/64*(117*(c*d^3)^
(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*log(-
sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)
*a*b^2*c^6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) + 1/16*(21*
b*c*d^3*x^(7/2) - 13*a*d^4*x^(7/2) + 25*b*c^2*d^2*x^(3/2) - 17*a*c*d^3*x^(3
/2))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 2/(a*c^3*sqrt(
x))
```

Mupad [B]
time = 6.01, size = 2500, normalized size = 3.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] atan((a^21*c^16*d^20*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 -
832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c
^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 24798150
0*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*
a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2
- 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a
^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d
^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256
*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*2174327193600i + b^17
*c^20*d^4*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a
*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 15
19673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c
^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d
^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 369098752
0*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20
*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 830472
1920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c
^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*918653239296i + a*b^20*c^36*x^(1
```

/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*1099511627776i + a*b^16*c^19*d^5*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*10239255576576i - a^2*b^19*c^35*d*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*13194139533312i - a^20*b*c^17*d^19*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*38654705664000i - a^2*b^15*c^18*d^6*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*52740124835840i + a^3*b^14*c^17*d^7*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*

```
b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 198916
3800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d
^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^
25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b...
```

3.486 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=681

$$-\frac{32b^2c^2-133abcd+77a^2d^2}{48ac^3(bc-ad)^2x^{3/2}}-\frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2}-\frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)}+\frac{b^{15/4}\tan^{-1}\left(1-\right.}{\sqrt{2}\,a^{7/4}(bc}$$

[Out] 1/48*(-77*a^2*d^2+133*a*b*c*d-32*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x^(3/2)-1/4*d/c/(-a*d+b*c)/x^(3/2)/(d*x^2+c)^2-1/16*d*(-11*a*d+19*b*c)/c^2/(-a*d+b*c)^2/x^(3/2)/(d*x^2+c)+1/2*b^(15/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/2*b^(15/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/64*d^(7/4)*(77*a^2*d^2-210*a*b*c*d+165*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(15/4)/(-a*d+b*c)^3*2^(1/2)+1/64*d^(7/4)*(77*a^2*d^2-210*a*b*c*d+165*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(15/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(15/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/4*b^(15/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/128*d^(7/4)*(77*a^2*d^2-210*a*b*c*d+165*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)/(-a*d+b*c)^3*2^(1/2)+1/128*d^(7/4)*(77*a^2*d^2-210*a*b*c*d+165*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)/(-a*d+b*c)^3*2^(1/2)

Rubi [A]
time = 0.62, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {477, 483, 593, 597, 536, 217, 1179, 642, 1176, 631, 210}

$$\frac{b^{15/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{15/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{15/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{15/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{d^{7/4}(77a^2d^2-210abcd+165b^2c^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{15/4}(bc-ad)^3}+\frac{d^{7/4}(77a^2d^2-210abcd+165b^2c^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{15/4}(bc-ad)^3}-\frac{d^{7/4}(77a^2d^2-210abcd+165b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{15/4}(bc-ad)^3}+\frac{d^{7/4}(77a^2d^2-210abcd+165b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{15/4}(bc-ad)^3}-\frac{77a^2d^2-133abcd+32b^2c^2}{48ac^3x^{3/2}(bc-ad)^2}-\frac{d(19bc-11ad)}{16c^2x^{3/2}\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]

[Out] -1/48*(32*b^2*c^2 - 133*a*b*c*d + 77*a^2*d^2)/(a*c^3*(b*c - a*d)^2*x^(3/2)) - d/(4*c*(b*c - a*d)*x^(3/2)*(c + d*x^2)^2) - (d*(19*b*c - 11*a*d))/(16*c^2*(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + (b^(15/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(15/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (b^(15/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(15/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[

```
2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^
2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*
c^(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(15/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-11ad-11bdx^4}{x^4(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int 32b\right.}{}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11a}{16c^2(bc-ad)^2x^{3/2}}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11a}{16c^2(bc-ad)^2x^{3/2}}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11a}{16c^2(bc-ad)^2x^{3/2}}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11a}{16c^2(bc-ad)^2x^{3/2}}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11a}{16c^2(bc-ad)^2x^{3/2}}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11a}{16c^2(bc-ad)^2x^{3/2}}$$

Mathematica [A]
time = 1.62, size = 410, normalized size = 0.60

$$\frac{1}{192}\left(-\frac{4\left(32b^2c^2(c+dx^2)^2+a^2d^2(32c^2+121cdx^2+77d^2x^4)-abcd(64c^2+209cdx^2+133d^2x^4)\right)}{ac^3(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} - \frac{96\sqrt{2}\,b^{15/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{7/4}(-bc+ad)^3} - \frac{3\sqrt{2}\,d^{7/4}(165b^2c^2-210abcd+77a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{15/4}(bc-ad)^3} + \frac{96\sqrt{2}\,b^{15/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{7/4}(-bc+ad)^3} + \frac{3\sqrt{2}\,d^{7/4}(165b^2c^2-210abcd+77a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{15/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-4*(32*b^2*c^2*(c + d*x^2)^2 + a^2*d^2*(32*c^2 + 121*c*d*x^2 + 77*d^2*x^4
) - a*b*c*d*(64*c^2 + 209*c*d*x^2 + 133*d^2*x^4)))/(a*c^3*(b*c - a*d)^2*x^(
3/2)*(c + d*x^2)^2) - (96*Sqrt[2]*b^(15/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
```

```
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(7/4)*(-(b*c) + a*d)^3) - (3*Sqrt[2]*d^
(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)
/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(15/4)*(b*c - a*d)^3) + (96*Sqrt[2]
*b^(15/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])
/(a^(7/4)*(-(b*c) + a*d)^3) + (3*Sqrt[2]*d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d
 + 77*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]
*x)])/(c^(15/4)*(b*c - a*d)^3))/192
```

Maple [A]
time = 0.19, size = 348, normalized size = 0.51 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*b^4/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*d^2/c
^3/(a*d-b*c)^3*(((15/32*a^2*d^3-19/16*a*b*c*d^2+23/32*b^2*c^2*d)*x^(5/2)+1/
32*c*(19*a^2*d^2-46*a*b*c*d+27*b^2*c^2)*x^(1/2))/(d*x^2+c)^2+1/256*(77*a^2*
d^2-210*a*b*c*d+165*b^2*c^2)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
-2/3/a/c^3/x^(3/2)
```

Maxima [A]
time = 0.53, size = 755, normalized size = 1.11

$$-\frac{32\,b^2c^4-64\,abc^3d+32\,a^2c^2d^2+(32\,b^2c^2d^2-133\,abcd^3+77\,a^2d^4)x^4+(64\,b^2c^3d-209\,abc^2d^2+121\,a^2cd^3)x^2}{48\left((ab^2c^5d^2-2\,a^2bc^4d^3+a^3c^3d^4)x^{\frac{11}{2}}+2\,(ab^2c^6d-2\,a^2bc^5d^2+a^3c^4d^3)x^{\frac{7}{2}}+(ab^2c^7-2\,a^2bc^6d+a^3c^5d^2)x^{\frac{3}{2}}\right)}-\frac{\frac{2\sqrt{2}\,b^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b^4\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{15}{4}}\log\left(\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac34}}-\frac{\sqrt{2}\,b^{\frac{15}{4}}\log\left(-\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac34}}}{4\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)}+\frac{\frac{2\sqrt{2}\,(165\,b^2c^2d^2-210\,abcd^3+77\,a^2d^4)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(165\,b^2c^2d^2-210\,abcd^3+77\,a^2d^4)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(165\,b^2c^2d^2-210\,abcd^3+77\,a^2d^4)\log\left(\sqrt{2}\,c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac34}d^{\frac14}}-\frac{\sqrt{2}\,(165\,b^2c^2d^2-210\,abcd^3+77\,a^2d^4)\log\left(-\sqrt{2}\,c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac34}d^{\frac14}}}{128\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(32*b^2*c^4 - 64*a*b*c^3*d + 32*a^2*c^2*d^2 + (32*b^2*c^2*d^2 - 133*a
*b*c*d^3 + 77*a^2*d^4)*x^4 + (64*b^2*c^3*d - 209*a*b*c^2*d^2 + 121*a^2*c*d^
3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^(11/2) + 2*(a*b^
2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^(7/2) + (a*b^2*c^7 - 2*a^2*b*c^6
*d + a^3*c^5*d^2)*x^(3/2)) - 1/4*(2*sqrt(2)*b^4*arctan(1/2*sqrt(2)*(sqrt(2)
*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^4*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))) + sqrt(2)*b^(15/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/a^(3/4) - sqrt(2)*b^(15/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/a^(3/4))/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
 a^4*d^3) + 1/128*(2*sqrt(2)*(165*b^2*c^2*d^2 - 210*a*b*c*d^3 + 77*a^2*d^4)
*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt
(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(165*b^2*c^2*d^2
```

```
- 210*a*b*c*d^3 + 77*a^2*d^4)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
- 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)))
 + sqrt(2)*(165*b^2*c^2*d^2 - 210*a*b*c*d^3 + 77*a^2*d^4)*log(sqrt(2)*c^(1/
4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(165*
b^2*c^2*d^2 - 210*a*b*c*d^3 + 77*a^2*d^4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt
(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^6 - 3*a*b^2*c^5*d + 3*
a^2*b*c^4*d^2 - a^3*c^3*d^3)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.73, size = 995, normalized size = 1.46

$$-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b^{3} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^{2}b^{3}c^{3}-3\sqrt{2}\,a^{3}b^{2}c^{2}d+3\sqrt{2}\,a^{4}bcd^{2}-\sqrt{2}\,a^{5}d^{3}}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b^{3} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^{2}b^{3}c^{3}-3\sqrt{2}\,a^{3}b^{2}c^{2}d+3\sqrt{2}\,a^{4}bcd^{2}-\sqrt{2}\,a^{5}d^{3}}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b^{3} \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^{2}b^{3}c^{3}-3\sqrt{2}\,a^{3}b^{2}c^{2}d+3\sqrt{2}\,a^{4}bcd^{2}-\sqrt{2}\,a^{5}d^{3}\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} b^{3} \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^{2}b^{3}c^{3}-3\sqrt{2}\,a^{3}b^{2}c^{2}d+3\sqrt{2}\,a^{4}bcd^{2}-\sqrt{2}\,a^{5}d^{3}\right)}+\frac{\left(165\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}d-210\left(cd^{3}\right)^{\frac{1}{4}}abcd^{2}+77\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^{3}c^{7}-3\sqrt{2}\,ab^{2}c^{6}d+3\sqrt{2}\,a^{2}bc^{5}d^{2}-\sqrt{2}\,a^{3}c^{4}d^{3}\right)}+\frac{\left(165\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}d-210\left(cd^{3}\right)^{\frac{1}{4}}abcd^{2}+77\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}\,b^{3}c^{7}-3\sqrt{2}\,ab^{2}c^{6}d+3\sqrt{2}\,a^{2}bc^{5}d^{2}-\sqrt{2}\,a^{3}c^{4}d^{3}\right)}+\frac{\left(165\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}d-210\left(cd^{3}\right)^{\frac{1}{4}}abcd^{2}+77\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^{3}c^{7}-3\sqrt{2}\,ab^{2}c^{6}d+3\sqrt{2}\,a^{2}bc^{5}d^{2}-\sqrt{2}\,a^{3}c^{4}d^{3}\right)}-\frac{\left(165\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}d-210\left(cd^{3}\right)^{\frac{1}{4}}abcd^{2}+77\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}\,b^{3}c^{7}-3\sqrt{2}\,ab^{2}c^{6}d+3\sqrt{2}\,a^{2}bc^{5}d^{2}-\sqrt{2}\,a^{3}c^{4}d^{3}\right)}+\frac{23\,bcd^{3}x^{\frac{7}{2}}-15\,ad^{4}x^{\frac{7}{2}}+27\,bc^{2}d^{2}\sqrt{x}-19\,acd^{3}\sqrt{x}}{16\left(b^{2}c^{5}-2\,abc^{4}d+a^{2}c^{3}d^{2}\right)\left(dx^{2}+c\right)^{2}}-\frac{2}{3\,ac^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*
c*d^2 - sqrt(2)*a^5*d^3) - (a*b^3)^(1/4)*b^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b
^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/2*(a*b^3)^(1/4)*b^3
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*
sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/2*(a*b
^3)^(1/4)*b^3*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^
2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d
^3) + 1/32*(165*(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*
```

```
(c*d^3)^(1/4)*a^2*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^
5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/32*(165*(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d
^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*
c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/64*(165*(c*d^3)^
(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^2*d^3)*l
og(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7 - 3*sqrt(2
)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) - 1/64*(165*
(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^
2*d^3)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7 -
 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1
/16*(23*b*c*d^3*x^(5/2) - 15*a*d^4*x^(5/2) + 27*b*c^2*d^2*sqrt(x) - 19*a*c*
d^3*sqrt(x))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 2/3/(a
*c^3*x^(3/2))
```

Mupad [B]
time = 5.86, size = 2500, normalized size = 3.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] atan(((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d
^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150
*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^
13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12
- 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^
3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5
 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920
*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*
d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200625*b^
8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 1139499
9000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^
3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^
12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256
*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*
d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 1328755
5072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^
18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((
-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 85
87309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^
4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 38
3487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 20132
```

6592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c
^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 1550
2147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^
4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 -
201326592*a*b^11*c^26*d))^(3/4)*(x^(1/2)*(18446744073709551616*a^11*b^39*c^
68*d^4 - 479615345916448342016*a^12*b^38*c^67*d^5 + 5995191823955604275200*
a^13*b^37*c^66*d^6 - 47961534591644834201600*a^14*b^36*c^65*d^7 + 275778823
901957796659200*a^15*b^35*c^64*d^8 - 1212936383169193658286080*a^16*b^34*c^
63*d^9 + 4232993998288506144686080*a^17*b^33*c^62*d^10 - 11941164077799654041845760*a^18*b^32*c^61*d^11 + 27104869321333056471040000*a^19*b^31*c^60*d^
12 - 46637619173392487079215104*a^20*b^30*c^59*d^13 + 4361160653855789513336
4224*a^21*b^29*c^58*d^14 + 72781112360087761599856640*a^22*b^28*c^57*d^15
- 523234066593179210717593600*a^23*b^27*c^56*d^16 + 172375300102079718474383
3600*a^24*b^26*c^55*d^17 - 4269437167365872814842183680*a^25*b^25*c^54*d^1
8 + 8727322757849829186700574720*a^26*b^24*c^53*d^19 - 15215326043975142249
374679040*a^27*b^23*c^52*d^20 + 22962658463246519625580544000*a^28*b^22*c^5
1*d^21 - 30231538828274701475145318400*a^29*b^21*c^50*d^22 + 34870163031766
389952882933760*a^30*b^20*c^49*d^23 - 35316718238336158489724846080*a^31*b^
19*c^48*d^24 + 31433146498544749041648926720*a^32*b^18*c^47*d^25 - 24575140
799491012895231180800*a^33*b^17*c^46*d^26 + 16850754961433442876234137600*a
^34*b^16*c^45*d^27 - 10105200492115418262179676160*a^35*b^15*c^44*d^28 + 52
78011312905736232783314944*a^36*b^14*c^43*d^29 - 238724839940591616616982111
84*a^37*b^13*c^42*d^30 + 927828632312674738870681600*a^38*b^12*c^41*d^31 -
306693733103726739901644800*a^39*b^11*c^40*d^32 + 850380759594460460666060 8
0*a^40*b^10*c^39*d^33 - 19409595119210898894356480*a^41*b^9*c^38*d^34 + 355
1400405635812871372800*a^42*b^8*c^37*d^35 - 500844593983932480880640*a^43*b
^7*c^36*d^36 + 51111802530990496153600*a^44*b^6*c^35*d^37 - 335957723562733
3124096*a^45*b^5*c^34*d^38 + 106807368762718683136*a^46*b^4*c^33*d^39) + (-
(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 858
7309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4
*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383
487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326
592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^
24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502
147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4
*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 2
01326592*a*b^11*c^26*d))^(1/4)*(36893488147419103232*a^13*b^38*c^71*d^4 - 1
069911156275153993728*a^14*b^37*c^70*d^5 + 14978756187852155912192*a^15*b^3
6*c^69*d^6 - 134999037738929532960768*a^16*b^35*c^68*d^7 + 8820160799048623
21508352*a^17*b^34*c^67*d^8 - 4465630463459278708539392*a^18*b^33*c^66*d^9
+ 18321125205332103390035968*a^19*b^32*c^65*d^10 - 630215452283771668681195
52*a^20*b^31*c^64*d^11 + 187018029382071665408606208*a^21*b^30*c^63*d^12 -
490713180393588600090918912*a^22*b^29*c^62*d^13 + 1161438545048511890042388
480*a^23*b^28*c^61*d^14 - 2512974056309066269898833920*a^24*b^27*c^60*d^15
+ 4997541469898172697285754880*a^25*b^26*c^59*d^16 - 9119889428539397211967

```
979520*a^26*b^25*c^58*d^17 + 151815443064610397...
```

3.487 $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=743

$$-\frac{32b^2c^2-189abcd+117a^2d^2}{80ac^3(bc-ad)^2x^{5/2}}+\frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}}-\frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2}-\frac{}{16c^2}$$

```
[Out] 1/80*(-117*a^2*d^2+189*a*b*c*d-32*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x^(5/2)-1/4*d
/c/(-a*d+b*c)/x^(5/2)/(d*x^2+c)^2-1/16*d*(-13*a*d+21*b*c)/c^2/(-a*d+b*c)^2/
x^(5/2)/(d*x^2+c)-1/2*b^(17/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^
(9/4)/(-a*d+b*c)^3*2^(1/2)+1/2*b^(17/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^
(1/4))/a^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/64*d^(9/4)*(117*a^2*d^2-306*a*b*c*d+2
21*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(17/4)/(-a*d+b*c)^3
*2^(1/2)-1/64*d^(9/4)*(117*a^2*d^2-306*a*b*c*d+221*b^2*c^2)*arctan(1+d^(1/4
)*2^(1/2)*x^(1/2)/c^(1/4))/c^(17/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(17/4)*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)^3*2^(1/
2)-1/4*b^(17/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/
4)/(-a*d+b*c)^3*2^(1/2)-1/128*d^(9/4)*(117*a^2*d^2-306*a*b*c*d+221*b^2*c^2)
*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)/(-a*d+b*c)^
3*2^(1/2)+1/128*d^(9/4)*(117*a^2*d^2-306*a*b*c*d+221*b^2*c^2)*ln(c^(1/2)+x*
d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)/(-a*d+b*c)^3*2^(1/2)+1/16
*(117*a^3*d^3-189*a^2*b*c*d^2+32*a*b^2*c^2*d+32*b^3*c^3)/a^2/c^4/(-a*d+b*c)
^2/x^(1/2)
```

Rubi [A]
time = 0.83, antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {477, 483, 593, 597, 598, 303, 1176, 631, 210, 1179, 642}

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/80*(32*b^2*c^2 - 189*a*b*c*d + 117*a^2*d^2)/(a*c^3*(b*c - a*d)^2*x^(5/2)
) + (32*b^3*c^3 + 32*a*b^2*c^2*d - 189*a^2*b*c*d^2 + 117*a^3*d^3)/(16*a^2*c
^4*(b*c - a*d)^2*Sqrt[x]) - d/(4*c*(b*c - a*d)*x^(5/2)*(c + d*x^2)^2) - (d*
(21*b*c - 13*a*d))/(16*c^2*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)) - (b^(17/4)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^
3) + (b^(17/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9
/4)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d
)^3) - (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTan[1 + (Sqrt[
```

```
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (b^(17/
4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a
^(9/4)*(b*c - a*d)^3) - (b^(17/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(221*b^2*c^
2 - 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2
 - 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
```

```
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-13ad-13bdx^4}{x^6(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2} - \frac{d(21bc-13ad)}{16c^2(bc-ad)^2x^{5/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int 3\right.}{}$$

$$= -\frac{\frac{32b^2c}{a} - 189bd + \frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} - \frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2} - \frac{d(21bc-1}{16c^2(bc-ad)^2x^5}$$

$$= -\frac{\frac{32b^2c}{a} - 189bd + \frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4}$$

$$= -\frac{\frac{32b^2c}{a} - 189bd + \frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4}$$

$$= -\frac{\frac{32b^2c}{a} - 189bd + \frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4}$$

$$= -\frac{\frac{32b^2c}{a} - 189bd + \frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4}$$

$$= -\frac{\frac{32b^2c}{a} - 189bd + \frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4}$$

$$= -\frac{\frac{32b^2c}{a} - 189bd + \frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4}$$

$$= -\frac{\frac{32b^2c}{a} - 189bd + \frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4}$$

Mathematica [A]

time = 1.18, size = 462, normalized size = 0.62

$$\frac{1}{320}\left(\frac{4\left(160b^3c^3x^2(c+dx^2)^2-32ab^2c^2(c-5dx^2)(c+dx^2)^2+a^2bcd(64c^3-672c^2dx^2-1701cd^2x^4-945d^3x^6)+a^3d^2(-32c^3+416c^2dx^2+1053cd^2x^4+585d^3x^6)\right)}{a^2c^4(bc-ad)^2x^{5/2}(c+dx^2)^2}+\frac{160\sqrt{2}b^{17/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{9/4}(-bc+ad)^3}+\frac{5\sqrt{2}d^{9/4}(221b^2c^2-306abcd+117a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{17/4}(bc-ad)^3}+\frac{160\sqrt{2}b^{17/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{9/4}(-bc+ad)^3}+\frac{5\sqrt{2}d^{9/4}(221b^2c^2-306abcd+117a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{17/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]

[Out] ((4*(160*b^3*c^3*x^2*(c + d*x^2)^2 - 32*a*b^2*c^2*(c - 5*d*x^2)*(c + d*x^2)^2 + a^2*b*c*d*(64*c^3 - 672*c^2*d*x^2 - 1701*c*d^2*x^4 - 945*d^3*x^6) + a^3*d^2*(-32*c^3 + 416*c^2*d*x^2 + 1053*c*d^2*x^4 + 585*d^3*x^6)))/(a^2*c^4*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)^2) + (160*Sqrt[2]*b^(17/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(9/4)*(-(b*c) + a*d)^3) + (5*Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(17/4)*(b*c - a*d)^3) + (160*Sqrt[2]*b^(17/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(9/4)*(-(b*c) + a*d)^3) + (5*Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(17/4)*(b*c - a*d)^3))/320

Maple [A]
time = 0.23, size = 368, normalized size = 0.50 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] -1/4*b^4/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2*d^3/c^4/(a*d-b*c)^3*((1/32*d*(21*a^2*d^2-50*a*b*c*d+29*b^2*c^2)*x^(7/2)+(25/32*a^2*c*d^2-29/16*a*b*c^2*d+33/32*b^2*c^3)*x^(3/2))/(d*x^2+c)^2+1/8*(117/32*a^2*d^2-153/16*a*b*c*d+221/32*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/5/a/c^3/x^(5/2)-2*(-3*a*d-b*c)/a^2/c^4/x^(1/2)

Maxima [A]
time = 0.61, size = 756, normalized size = 1.02

$$\frac{b^5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2-a^5d^3\right)}-\frac{\left(221b^2c^2d^3-306abcd^4+117a^2d^5\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^3c^7-3ab^2c^6d+3a^2bc^5d^2-a^3c^4d^3\right)}-\frac{32ab^2c^5-64a^2bc^4d+32a^3c^3d^2-5\left(32b^3c^3d^2+32ab^2c^2d^3-189a^2bcd^4+117a^3d^5\right)x^6-\left(320b^3c^4d+288ab^2c^3d^2-1701a^2bc^2d^3+1053a^3cd^4\right)x^4-32\left(5b^3c^5+3ab^2c^4d-21a^2bc^3d^2+13a^3c^2d^3\right)x^2}{80\left(\left(a^2b^2c^6d^2-2a^3bc^5d^3+a^4c^4d^4\right)x^{\frac{13}{2}}+2\left(a^2b^2c^7d-2a^3bc^6d^2+a^4c^5d^3\right)x^{\frac{9}{2}}+\left(a^2b^2c^8-2a^3bc^7d+a^4c^6d^2\right)x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")

[Out] 1/4*b^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)

```
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a
^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 1/128*(221*b^2*c^
2*d^3 - 306*a*b*c*d^4 + 117*a^2*d^5)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*
sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
- 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
 - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1
/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + s
qrt(c))/(c^(1/4)*d^(3/4)))/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3
*c^4*d^3) - 1/80*(32*a*b^2*c^5 - 64*a^2*b*c^4*d + 32*a^3*c^3*d^2 - 5*(32*b^
3*c^3*d^2 + 32*a*b^2*c^2*d^3 - 189*a^2*b*c*d^4 + 117*a^3*d^5)*x^6 - (320*b^
3*c^4*d + 288*a*b^2*c^3*d^2 - 1701*a^2*b*c^2*d^3 + 1053*a^3*c*d^4)*x^4 - 32
*(5*b^3*c^5 + 3*a*b^2*c^4*d - 21*a^2*b*c^3*d^2 + 13*a^3*c^2*d^3)*x^2)/((a^2
*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^(13/2) + 2*(a^2*b^2*c^7*d -
 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^(9/2) + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^
4*c^6*d^2)*x^(5/2))
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.92, size = 1000, normalized size = 1.35

$$\frac{\left(ab^{3}\right)^{\frac{3}{4}} b^{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^{3}b^{3}c^{3}-3\sqrt{2}a^{4}b^{2}c^{2}d+3\sqrt{2}a^{5}bcd^{2}-\sqrt{2}a^{6}d^{3}}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} b^{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^{3}b^{3}c^{3}-3\sqrt{2}a^{4}b^{2}c^{2}d+3\sqrt{2}a^{5}bcd^{2}-\sqrt{2}a^{6}d^{3}}-\frac{\left(ab^{3}\right)^{\frac{3}{4}} b^{2} \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^{3}b^{3}c^{3}-3\sqrt{2}a^{4}b^{2}c^{2}d+3\sqrt{2}a^{5}bcd^{2}-\sqrt{2}a^{6}d^{3}\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} b^{2} \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^{3}b^{3}c^{3}-3\sqrt{2}a^{4}b^{2}c^{2}d+3\sqrt{2}a^{5}bcd^{2}-\sqrt{2}a^{6}d^{3}\right)}-\frac{\left(221\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-306\left(cd^{3}\right)^{\frac{3}{4}}abcd+117\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}b^{3}c^{8}-3\sqrt{2}ab^{2}c^{7}d+3\sqrt{2}a^{2}bc^{6}d^{2}-\sqrt{2}a^{3}c^{5}d^{3}\right)}-\frac{\left(221\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-306\left(cd^{3}\right)^{\frac{3}{4}}abcd+117\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}b^{3}c^{8}-3\sqrt{2}ab^{2}c^{7}d+3\sqrt{2}a^{2}bc^{6}d^{2}-\sqrt{2}a^{3}c^{5}d^{3}\right)}+\frac{\left(221\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-306\left(cd^{3}\right)^{\frac{3}{4}}abcd+117\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^{3}c^{8}-3\sqrt{2}ab^{2}c^{7}d+3\sqrt{2}a^{2}bc^{6}d^{2}-\sqrt{2}a^{3}c^{5}d^{3}\right)}-\frac{\left(221\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-306\left(cd^{3}\right)^{\frac{3}{4}}abcd+117\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^{3}c^{8}-3\sqrt{2}ab^{2}c^{7}d+3\sqrt{2}a^{2}bc^{6}d^{2}-\sqrt{2}a^{3}c^{5}d^{3}\right)}-\frac{29bcd^{4}x^{\frac{7}{2}}-21ad^{5}x^{\frac{7}{2}}+33bc^{2}d^{3}x^{\frac{3}{2}}-25acd^{4}x^{\frac{3}{2}}}{16\left(b^{2}c^{6}-2abc^{5}d+a^{2}c^{4}d^{2}\right)\left(dx^{2}+c\right)^{2}}+\frac{2\left(5bcx^{2}+15adx^{2}-ac\right)}{5a^{2}c^{4}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c
*d^2 - sqrt(2)*a^6*d^3) + (a*b^3)^(3/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^
2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/2*(a*b^3)^(3/4)*b^2*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*s
qrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) + 1/2*(a*b^
3)^(3/4)*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3
*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^
3) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d
^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^7*d + 3*sqrt(2)*a^2*b*c^6*d^
2 - sqrt(2)*a^3*c^5*d^3) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 306*(c*d^3)^(3
/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^7*d +
 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) + 1/64*(221*(c*d^3)^(3/4)*b
^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^
7*d + 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) - 1/64*(221*(c*d^3)^(3
/4)*b^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(-s
qrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*
b^2*c^7*d + 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) - 1/16*(29*b*c*d
^4*x^(7/2) - 21*a*d^5*x^(7/2) + 33*b*c^2*d^3*x^(3/2) - 25*a*c*d^4*x^(3/2))/
((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*(d*x^2 + c)^2) + 2/5*(5*b*c*x^2 + 15
*a*d*x^2 - a*c)/(a^2*c^4*x^(5/2))
```

Mupad [B]
time = 7.03, size = 2500, normalized size = 3.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((2*x^2*(13*a*d + 5*b*c))/(5*a^2*c^2) - 2/(5*a*c) + (x^4*(1053*a^3*d^4 + 32
0*b^3*c^3*d + 288*a*b^2*c^2*d^2 - 1701*a^2*b*c*d^3))/(80*a^2*c^2*(b^2*c^3 +
 a^2*c*d^2 - 2*a*b*c^2*d)) + (d^2*x^6*(117*a^3*d^3 + 32*b^3*c^3 + 32*a*b^2*
c^2*d - 189*a^2*b*c*d^2))/(16*a^2*c^3*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))
/(c^2*x^(5/2) + d^2*x^(13/2) + 2*c*d*x^(9/2)) - atan((a^11*b^22*c^29*x^(1/2
)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^1
1*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^1
4*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17
*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*
c*d^11))^(5/4)*33554432i + a^19*b^10*d^17*x^(1/2)*(-b^17/(16*a^21*d^12 + 16
```

*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*374777442i + a^33*c^7*d^22*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*448561152i + a^8*b^21*c^11*d^6*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*100026368i - a^9*b^20*c^10*d^7*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*276996096i + a^10*b^19*c^9*d^8*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*297676800i + a^11*b^18*c^8*d^9*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*4624241570i - a^12*b^17*c^7*d^10*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*26395336656i + a^13*b^16*c^6*d^11*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*64982364408i - a^14*b^15*c^5*d^12*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*92624356656i + a^15*b^14*c^4*d^13*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b

```
^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*
d^11))^(1/4)*83665919628i - a^16*b^13*c^3*d^14*x^(1/2)*(-b^17/(16*a^21*d^12
 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520
*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*
a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^
18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*4903642
4112i + a^17*b^12*c^2*d^15*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12
- 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 +
7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 1
2672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 105
6*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*18213050232i + a^13*b^20*c^
27*d^2*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^...
```

3.488 $\int \frac{x^{7/2}}{(a+bx^2)^2 (c+dx^2)^2} \, dx$

Optimal. Leaf size=624

$$\frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2(c+dx^2)}+\frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)}+\frac{\sqrt[4]{a}\,(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,(}{}$$

```
[Out] 1/8*a^(1/4)*(3*a*d+5*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4)
/(-a*d+b*c)^3*2^(1/2)-1/8*a^(1/4)*(3*a*d+5*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^
(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)^3*2^(1/2)-1/8*c^(1/4)*(5*a*d+3*b*c)*arcta
n(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/8*c^(1/
4)*(5*a*d+3*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*
c)^3*2^(1/2)+1/16*a^(1/4)*(3*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4
)*2^(1/2)*x^(1/2))/b^(1/4)/(-a*d+b*c)^3*2^(1/2)-1/16*a^(1/4)*(3*a*d+5*b*c)*
ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/(-a*d+b*c)^3*
2^(1/2)-1/16*c^(1/4)*(5*a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(
1/2)*x^(1/2))/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/16*c^(1/4)*(5*a*d+3*b*c)*ln(c^
(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(1/4)/(-a*d+b*c)^3*2^(1/
2)+1/2*(a*d+b*c)*x^(1/2)/b/(-a*d+b*c)^2/(d*x^2+c)+1/2*a*x^(1/2)/b/(-a*d+b*c
)/(b*x^2+a)/(d*x^2+c)
```

Rubi [A]
time = 0.49, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 481, 541, 536, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(3ad+5bc)}{4\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(3ad+5bc)}{4\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\sqrt[4]{c}\,(5ad+3bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,\sqrt[4]{d}\,(bc-ad)^3}+\frac{\sqrt[4]{c}\,(5ad+3bc)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,\sqrt[4]{d}\,(bc-ad)^3}+\frac{a\sqrt{x}}{2b(a+bx^2)(c+dx^2)(bc-ad)}+\frac{\sqrt{x}\,(ad+bc)}{2b(c+dx^2)(bc-ad)^2}+\frac{\sqrt[4]{a}\,(3ad+5bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,(3ad+5bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\sqrt[4]{c}\,(5ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{d}\,(bc-ad)^3}+\frac{\sqrt[4]{c}\,(5ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{d}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*c + a*d)*Sqrt[x])/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*Sqrt[x])/(2*b*(b
*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (a^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (a^(1
/4)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*b^(1/4)*(b*c - a*d)^3) - (c^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d)^3) + (c^(1/4)*(3*b*
c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(1/4
)*(b*c - a*d)^3) + (a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (a^(1/4)*(
5*b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d
```

```
)^3) + (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{ac+(-4bc-3ad)x^4}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{8abc^2-12bc(}{(a+bx^4)(}\right.}{8bc(bc}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(a(5bc+3ad))\text{Sub}}{2(bc}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}\,(5bc+3ad)\right)}{}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}\,(5bc+3ad)\right)}{}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}\,(5bc+3ad)\log}{8}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}\,(5bc+3ad)\tan}{4\sqrt{2}\,\sqrt[4]{}}$$

Mathematica [A]
time = 1.31, size = 344, normalized size = 0.55

$$\frac{1}{8}\left(\frac{4\sqrt{x}\left(2ac+bcx^2+adx^2\right)}{(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt{2}\sqrt[4]{a}\,(5bc+3ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}\,(bc-ad)^3} + \frac{\sqrt{2}\sqrt[4]{c}\,(3bc+5ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{d}\,(-bc+ad)^3} - \frac{\sqrt{2}\sqrt[4]{a}\,(5bc+3ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{b}\,(bc-ad)^3} + \frac{\sqrt{2}\sqrt[4]{c}\,(3bc+5ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt[4]{d}\,(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((4*Sqrt[x]*(2*a*c + b*c*x^2 + a*d*x^2))/((b*c - a*d)^2*(a + b*x^2)*(c + d*
x^2)) + (Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])]])/(b^(1/4)*(b*c - a*d)^3) + (Sqrt[2]*c^(1/4)*(
3*b*c + 5*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
])]])/(d^(1/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*ArcTanh[
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(b^(1/4)*(b*c - a
```

```
*d)^3) + (Sqrt[2]*c^(1/4)*(3*b*c + 5*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(d^(1/4)*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.11, size = 302, normalized size = 0.48

method	result
derivativedivides	$\frac{2a\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^{2}+a}+\frac{(3ad+5bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{32a}\right.}{(ad-bc)^{3}}$
default	$\frac{2a\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^{2}+a}+\frac{(3ad+5bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\sqrt{\right.\right.}{32a}\right.}{(ad-bc)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(3*a*d+5*b*c)*(a/
b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)
^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*c/(a*d-b*c)^3*((-1/4*a*d+1/4*
b*c)*x^(1/2)/(d*x^2+c)+1/32*(5*a*d+3*b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(
1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)-1)))
```

Maxima [A]
time = 0.54, size = 617, normalized size = 0.99

$$-\frac{\left(\frac{2\sqrt{2}(5bc+3ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(5bc+3ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(5bc+3ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(5bc+3ad)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a}{16\left(b^{3}c^{3}-3ab^{2}c^{2}d+3a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{\left(\frac{2\sqrt{2}(3bc+5ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(3bc+5ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(3bc+5ad)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(3bc+5ad)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)c}{16\left(b^{3}c^{3}-3ab^{2}c^{2}d+3a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{(bc+ad)x^{\frac{5}{2}}+2ac\sqrt{x}}{2\left(ab^{2}c^{3}-2a^{2}bc^{2}d+a^{3}cd^{2}+\left(b^{3}c^{2}d-2ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{4}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(5*b*c + 3*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 2*sqrt(2)*(5*b*c + 3*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
) + sqrt(2)*(5*b*c + 3*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*b*c + 3*a*d)*log(-sqrt(2)*a^(1/4
```

```
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/(b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/16*(2*sqrt(2)*(3*b*c + 5*a*d)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)
*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b*c + 5*a*d)*arct
an(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(3*b*c + 5*a*d)*log(sqr
t(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqr
t(2)*(3*b*c + 5*a*d)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqr
t(c))/(c^(3/4)*d^(1/4)))*c/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3) + 1/2*((b*c + a*d)*x^(5/2) + 2*a*c*sqrt(x))/(a*b^2*c^3 - 2*a^2*b*c^2*d
+ a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5375 vs. $2(472) = 944$.

time = 32.22, size = 5375, normalized size = 8.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
+ a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(
625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3
 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a
^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c
^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 +
66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)*arctan(-((b
^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^7*c^6*d^3 + 126*a^4
*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^
7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9)*sqrt((25*b^2*c^2 + 30*a*b*c*d + 9*a^2*d^2)
*x + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(-(625*a*b^4*c^4 + 1500*a^
2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^
12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a
^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^
5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 -
12*a^11*b^2*c*d^11 + a^12*b*d^12)))*(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d +
 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^1
2*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^
4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a
^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c
*d^11 + a^12*b*d^12))^(3/4) - (5*b^11*c^10 - 42*a*b^10*c^9*d + 153*a^2*b^9*
c^8*d^2 - 312*a^3*b^8*c^7*d^3 + 378*a^4*b^7*c^6*d^4 - 252*a^5*b^6*c^5*d^5 +
```

42*a^6*b^5*c^4*d^6 + 72*a^7*b^4*c^3*d^7 - 63*a^8*b^3*c^2*d^8 + 22*a^9*b^2*c*d^9 - 3*a^10*b*d^10)*sqrt(x)*(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(3/4))/(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13))^(1/4)*arctan(-((b^9*c^9*d - 9*a*b^8*c^8*d^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*a^5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 - a^9*d^10)*sqrt((9*b^2*c^2 + 30*a*b*c*d + 25*a^2*d^2)*x + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)))*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13))^(3/4) - (3*b^10*c^10*d - 22*a*b^9*c^9*d^2 + 63*a^2*b^8*c^8*d^3 - 72*a^3*b^7*c^7*d^4 - 42*a^4*b^6*c^6*d^5 + 252*a^5*b^5*c^5*d^6 - 378*a^6*b^4*c^4*d^7 + 312*a^7*b^3*c^3*d^8 - 153*a^8*b^2*c^2*d^9 + 42*a^9*b*c*d^10 - 5*a^10*d^11)*sqrt(x)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13))^(3/4))/(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)) - (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d...

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.92, size = 912, normalized size = 1.46

$$-\frac{\left(5\,(ab^3)^{\frac14} bc+3\,(ab^3)^{\frac14} ad\right)\arctan\left(\frac{\sqrt2\left(\sqrt2\left(\frac ab\right)^{\frac14}+2\sqrt x\right)}{2\left(\frac ab\right)^{\frac14}}\right)}{4\left(\sqrt2 b^4c^3-3\sqrt2 ab^3c^2d+3\sqrt2 a^2b^2cd^2-\sqrt2 a^3bd^3\right)}-\frac{\left(5\,(ab^3)^{\frac14} bc+3\,(ab^3)^{\frac14} ad\right)\arctan\left(-\frac{\sqrt2\left(\sqrt2\left(\frac ab\right)^{\frac14}-2\sqrt x\right)}{2\left(\frac ab\right)^{\frac14}}\right)}{4\left(\sqrt2 b^4c^3-3\sqrt2 ab^3c^2d+3\sqrt2 a^2b^2cd^2-\sqrt2 a^3bd^3\right)}+\frac{\left(3\,(cd^3)^{\frac14} bc+5\,(cd^3)^{\frac14} ad\right)\arctan\left(\frac{\sqrt2\left(\sqrt2\left(\frac cd\right)^{\frac14}+2\sqrt x\right)}{2\left(\frac cd\right)^{\frac14}}\right)}{4\left(\sqrt2 b^3c^3d-3\sqrt2 ab^2c^2d^2+3\sqrt2 a^2bcd^3-\sqrt2 a^3d^4\right)}+\frac{\left(3\,(cd^3)^{\frac14} bc+5\,(cd^3)^{\frac14} ad\right)\arctan\left(-\frac{\sqrt2\left(\sqrt2\left(\frac cd\right)^{\frac14}-2\sqrt x\right)}{2\left(\frac cd\right)^{\frac14}}\right)}{4\left(\sqrt2 b^3c^3d-3\sqrt2 ab^2c^2d^2+3\sqrt2 a^2bcd^3-\sqrt2 a^3d^4\right)}-\frac{\left(5\,(ab^3)^{\frac14} bc+3\,(ab^3)^{\frac14} ad\right)\log\left(\sqrt2\sqrt x\left(\frac ab\right)^{\frac14}+x+\sqrt{\frac ab}\right)}{8\left(\sqrt2 b^4c^3-3\sqrt2 ab^3c^2d+3\sqrt2 a^2b^2cd^2-\sqrt2 a^3bd^3\right)}+\frac{\left(5\,(ab^3)^{\frac14} bc+3\,(ab^3)^{\frac14} ad\right)\log\left(-\sqrt2\sqrt x\left(\frac ab\right)^{\frac14}+x+\sqrt{\frac ab}\right)}{8\left(\sqrt2 b^4c^3-3\sqrt2 ab^3c^2d+3\sqrt2 a^2b^2cd^2-\sqrt2 a^3bd^3\right)}+\frac{\left(3\,(cd^3)^{\frac14} bc+5\,(cd^3)^{\frac14} ad\right)\log\left(\sqrt2\sqrt x\left(\frac cd\right)^{\frac14}+x+\sqrt{\frac cd}\right)}{8\left(\sqrt2 b^3c^3d-3\sqrt2 ab^2c^2d^2+3\sqrt2 a^2bcd^3-\sqrt2 a^3d^4\right)}-\frac{\left(3\,(cd^3)^{\frac14} bc+5\,(cd^3)^{\frac14} ad\right)\log\left(-\sqrt2\sqrt x\left(\frac cd\right)^{\frac14}+x+\sqrt{\frac cd}\right)}{8\left(\sqrt2 b^3c^3d-3\sqrt2 ab^2c^2d^2+3\sqrt2 a^2bcd^3-\sqrt2 a^3d^4\right)}+\frac{bcx^{\frac52}+adx^{\frac52}+2ac\sqrt x}{2\left(bdx^4+bcx^2+adx^2+ac\right)\left(b^2c^2-2abcd+a^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(1/4)*b*c + 3*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*
c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) - 1/4*(5*(a*b^3)^(1/4)
*b*c + 3*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sq
rt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^
2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/4*(3*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^3*c^3*d - 3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^2*b*c*d^3 - sqrt(
2)*a^3*d^4) + 1/4*(3*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/4)*a*d)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^3*d -
3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^2*b*c*d^3 - sqrt(2)*a^3*d^4) - 1/8*(5
*(a*b^3)^(1/4)*b*c + 3*(a*b^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^
2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/8*(5*(a*b^3)^(1/4)*b*c + 3*(a*b^3)^(1/4)*a
*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*
sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/8*(3
*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/(sqrt(2)*b^3*c^3*d - 3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^
2*b*c*d^3 - sqrt(2)*a^3*d^4) - 1/8*(3*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/4)*a
*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^3*d -
3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^2*b*c*d^3 - sqrt(2)*a^3*d^4) + 1/2*(b
*c*x^(5/2) + a*d*x^(5/2) + 2*a*c*sqrt(x))/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a
*c)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))
```

Mupad [B]
time = 2.17, size = 2500, normalized size = 4.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] atan(((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3
*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c
^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b
^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^
7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*
a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((-(81*b^4*c^5 + 625*a^4*c*
d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^
12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d
^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7
*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*
c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b
*c*d^12))^(1/4)*(((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^
4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a
^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (-(81*b^4*c^5 + 625*a^4*
c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*
a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10
*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c
^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^
4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11
*b*c*d^12))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^
16*d^5 + 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*
a^6*b^15*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^1
0 - 9326592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11
*b^10*c^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 -
13799424*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c
^3*d^18 + 102400*a^17*b^4*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^
10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d
^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*
a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^1
1)) + ((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^
3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*
c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*
b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a
^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336
*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*(10240*a^2*b^17*c^15*d^4 -
 112640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^1
2*d^7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8
*b^11*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 28
16000*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d
```

```
^15 - 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c
^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)))*1i + (x
^(1/2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8
*d^5 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5
*d^8 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d
^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) - (-(81*b^4*c^5 + 625*a
^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(40
96*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c
^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^
7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8
*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a
^11*b*c*d^12))^(1/4)*((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 +
1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d
- 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4
 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*
d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^
3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*(((405*a^2*
b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5
*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b
^3*c^2*d^9)/2)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3
 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c
^7*d - 8*a^7*b*c*d^7) - (-(81*b^4*c^5 + 625*a^4...
```

3.489 $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=609

$$-\frac{dx^{3/2}}{(bc-ad)^2\,(c+dx^2)}-\frac{x^{3/2}}{2(bc-ad)\,(a+bx^2)\,(c+dx^2)}-\frac{\sqrt[4]{b}\,(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}+\frac{\sqrt[4]{b}\,(3\cdots}{\cdots}$$

```
[Out] -d*x^(3/2)/(-a*d+b*c)^2/(d*x^2+c)-1/2*x^(3/2)/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c
)-1/8*b^(1/4)*(5*a*d+3*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/
4)/(-a*d+b*c)^3*2^(1/2)+1/8*b^(1/4)*(5*a*d+3*b*c)*arctan(1+b^(1/4)*2^(1/2)*
x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/8*d^(1/4)*(3*a*d+5*b*c)*arc
tan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)^3*2^(1/2)-1/8*d^(
1/4)*(3*a*d+5*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/(-a*d+
b*c)^3*2^(1/2)+1/16*b^(1/4)*(5*a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^3*2^(1/2)-1/16*b^(1/4)*(5*a*d+3*b*c
)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^
3*2^(1/2)-1/16*d^(1/4)*(3*a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2
^(1/2)*x^(1/2))/c^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(1/4)*(3*a*d+5*b*c)*ln(
c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/(-a*d+b*c)^3*2^(
1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 482, 593, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(5ad+3bc)}{4\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}+\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(5ad+3bc)}{4\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}+\frac{\sqrt[4]{d}\,(3ad+5bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}-\frac{\sqrt[4]{d}\,(3ad+5bc)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}-\frac{x^{3/2}}{2\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}-\frac{dx^{3/2}}{(c+dx^2)\,(bc-ad)^2}+\frac{\sqrt[4]{b}\,(5ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}-\frac{\sqrt[4]{b}\,(5ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}-\frac{\sqrt[4]{d}\,(3ad+5bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}+\frac{\sqrt[4]{d}\,(3ad+5bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -((d*x^(3/2))/((b*c - a*d)^2*(c + d*x^2))) - x^(3/2)/(2*(b*c - a*d)*(a + b*
x^2)*(c + d*x^2)) - (b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (b^(1/4)*(3*b*c + 5*a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c -
 a*d)^3) + (d^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
+ (b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b^(1/4)*(3*b*c + 5*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4
)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(
```

```
5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^6}{(a+bx^4)^2(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{x^2(3c-5dx^4)}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2(c+dx^2)} - \frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{x^2(12c(bc+ad}{(a+bx^4)(c}\right.}{8c(bc}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2(c+dx^2)} - \frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)} + \frac{\text{Subst}\left(\int \left(\frac{4bc(3bc+5ad}{(bc-ad)(a+}\right.\right.}{}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2(c+dx^2)} - \frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)} - \frac{(d(5bc+3ad))\text{Subst}}{2(bc}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2(c+dx^2)} - \frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)} + \frac{\left(\sqrt{d}(5bc+3ad)\right)\text{S}}{}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2(c+dx^2)} - \frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)} - \frac{(5bc+3ad)\text{Subst}\left(\right.}{}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2(c+dx^2)} - \frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)} + \frac{\sqrt[4]{b}(3bc+5ad)\log(}{8\sqrt{}}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2(c+dx^2)} - \frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\sqrt[4]{b}(3bc+5ad)\tan}{4\sqrt{2}\sqrt[4]{a}}$$

<u>Mathematica</u> [A]

time = 1.61, size = 340, normalized size = 0.56

$$\frac{1}{8}\left(-\frac{4x^{3/2}(ad+b(c+2dx^2))}{(bc-ad)^2(a+bx^2)(c+dx^2)} + \frac{\sqrt{2}\sqrt[4]{b}(3bc+5ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}(-bc+ad)^3} + \frac{\sqrt{2}\sqrt[4]{d}(5bc+3ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{c}(bc-ad)^3} + \frac{\sqrt{2}\sqrt[4]{b}(3bc+5ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{a}(-bc+ad)^3} + \frac{\sqrt{2}\sqrt[4]{d}(5bc+3ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt[4]{c}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-4*x^(3/2)*(a*d + b*(c + 2*d*x^2)))/((b*c - a*d)^2*(a + b*x^2)*(c + d*x^2
)) + (Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(
```

```
5*b*c + 3*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
])])/(c^(1/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*ArcTanh[(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(1/4)*(-(b*c) + a
*d)^3) + (Sqrt[2]*d^(1/4)*(5*b*c + 3*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(1/4)*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.12, size = 302, normalized size = 0.50

method	result
derivativedivides	$-\frac{2b\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^{2}+a}+\frac{\left(\frac{5ad}{4}+\frac{3bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{a}{b}\right)\cdots}\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right)}{(ad-bc)^{3}}$
default	$-\frac{2b\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^{2}+a}+\frac{\left(\frac{5ad}{4}+\frac{3bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{a}{b}\right)\cdots}\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right)}{(ad-bc)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(5/4*a*d+3/4*b*c)
/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(
a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d/(a*d-b*c)^3*((-1/4*a*d+
1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(5/4*b*c+3/4*a*d)/d/(c/d)^(1/4)*2^(1/2)*(ln(
(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+
(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 567, normalized size = 0.93

$$\frac{\left(3\,b^{2}c+5\,abd\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{\left(5\,bcd+3\,ad^{2}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{2\,bdx^{\frac{7}{2}}+\left(bc+ad\right)x^{\frac{3}{2}}}{2\left(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{4}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*b^2*c + 5*a*b*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
```

```
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(
1/4)*b^(3/4)))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/16*(
5*b*c*d + 3*a*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) +
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
+ 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x
))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqr
t(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqr
t(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d
^(3/4)))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/2*(2*b*d*x
^(7/2) + (b*c + a*d)*x^(3/2))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3
*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*
d^2 + a^3*d^3)*x^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5814 vs. $2(459) = 918$.
time = 63.14, size = 5814, normalized size = 9.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
+ a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(
81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 +
625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 2
20*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^
6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^
9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(1/4)*arctan(-((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt((729*b^8*c^6 + 7290*
a*b^7*c^5*d + 30375*a^2*b^6*c^4*d^2 + 67500*a^3*b^5*c^3*d^3 + 84375*a^4*b^4
*c^2*d^4 + 56250*a^5*b^3*c*d^5 + 15625*a^6*b^2*d^6)*x - (81*a*b^11*c^10 + 5
4*a^2*b^10*c^9*d - 675*a^3*b^9*c^8*d^2 - 120*a^4*b^8*c^7*d^3 + 2290*a^5*b^7
*c^6*d^4 - 636*a^6*b^6*c^5*d^5 - 3534*a^7*b^5*c^4*d^6 + 2440*a^8*b^4*c^3*d^
7 + 1725*a^9*b^3*c^2*d^8 - 2250*a^10*b^2*c*d^9 + 625*a^11*b*d^10)*sqrt(-(81
*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 62
5*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220
*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*
c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9
+ 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)))*(-(81*b^5*c^4 + 54
0*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/
(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*
d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792
```

*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(1/4) - (27*b^7*c^6 + 54*a*b^6*c^5*d - 99*a^2*b^5*c^4*d^2 - 172*a^3*b^4*c^3*d^3 + 165*a^4*b^3*c^2*d^4 + 150*a^5*b^2*c*d^5 - 125*a^6*b*d^6)*sqrt(x)*(-(81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(1/4))/(81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(1/4)*arctan(-((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt((15625*b^6*c^6*d^2 + 56250*a*b^5*c^5*d^3 + 84375*a^2*b^4*c^4*d^4 + 67500*a^3*b^3*c^3*d^5 + 30375*a^4*b^2*c^2*d^6 + 7290*a^5*b*c*d^7 + 729*a^6*d^8)*x - (625*b^10*c^11*d - 2250*a*b^9*c^10*d^2 + 1725*a^2*b^8*c^9*d^3 + 2440*a^3*b^7*c^8*d^4 - 3534*a^4*b^6*c^7*d^5 - 636*a^5*b^5*c^6*d^6 + 2290*a^6*b^4*c^5*d^7 - 120*a^7*b^3*c^4*d^8 - 675*a^8*b^2*c^3*d^9 + 54*a^9*b*c^2*d^10 + 81*a^10*c*d^11)*sqrt(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12)))*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(1/4) - (125*b^6*c^6*d - 150*a*b^5*c^5*d^2 - 165*a^2*b^4*c^4*d^3 + 172*a^3*b^3*c^3*d^4 + 99*a^4*b^2*c^2*d^5 - 54*a^5*b*c*d^6 - 27*a^6*d^7)*sqrt(x)*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(1/4))/(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)) + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7

```
*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(459) = 918.
time = 1.11, size = 952, normalized size = 1.56

$$\frac{\left(3\left(ab^{3}\right)^{\frac{3}{4}}bc+5\left(ab^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}ab^{5}c^{3}-3\sqrt{2}a^{2}b^{4}c^{2}d+3\sqrt{2}a^{3}b^{3}cd^{2}-\sqrt{2}a^{4}b^{2}d^{3}\right)}+\frac{\left(3\left(ab^{3}\right)^{\frac{3}{4}}bc+5\left(ab^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}ab^{5}c^{3}-3\sqrt{2}a^{2}b^{4}c^{2}d+3\sqrt{2}a^{3}b^{3}cd^{2}-\sqrt{2}a^{4}b^{2}d^{3}\right)}-\frac{\left(5\left(cd^{3}\right)^{\frac{3}{4}}bc+3\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{3}c^{4}d^{2}-3\sqrt{2}ab^{2}c^{3}d^{3}+3\sqrt{2}a^{2}bc^{2}d^{4}-\sqrt{2}a^{3}cd^{5}\right)}-\frac{\left(5\left(cd^{3}\right)^{\frac{3}{4}}bc+3\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{3}c^{4}d^{2}-3\sqrt{2}ab^{2}c^{3}d^{3}+3\sqrt{2}a^{2}bc^{2}d^{4}-\sqrt{2}a^{3}cd^{5}\right)}-\frac{\left(3\left(ab^{3}\right)^{\frac{3}{4}}bc+5\left(ab^{3}\right)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}ab^{5}c^{3}-3\sqrt{2}a^{2}b^{4}c^{2}d+3\sqrt{2}a^{3}b^{3}cd^{2}-\sqrt{2}a^{4}b^{2}d^{3}\right)}+\frac{\left(3\left(ab^{3}\right)^{\frac{3}{4}}bc+5\left(ab^{3}\right)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}ab^{5}c^{3}-3\sqrt{2}a^{2}b^{4}c^{2}d+3\sqrt{2}a^{3}b^{3}cd^{2}-\sqrt{2}a^{4}b^{2}d^{3}\right)}+\frac{\left(5\left(cd^{3}\right)^{\frac{3}{4}}bc+3\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{3}c^{4}d^{2}-3\sqrt{2}ab^{2}c^{3}d^{3}+3\sqrt{2}a^{2}bc^{2}d^{4}-\sqrt{2}a^{3}cd^{5}\right)}-\frac{\left(5\left(cd^{3}\right)^{\frac{3}{4}}bc+3\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{3}c^{4}d^{2}-3\sqrt{2}ab^{2}c^{3}d^{3}+3\sqrt{2}a^{2}bc^{2}d^{4}-\sqrt{2}a^{3}cd^{5}\right)}-\frac{2\,bdx^{\frac{7}{2}}+bcx^{\frac{3}{2}}+adx^{\frac{3}{2}}}{2\left(bdx^{4}+bcx^{2}+adx^{2}+ac\right)\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*(a*b^3)^(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b
^4*c^2*d + 3*sqrt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) + 1/4*(3*(a*b^3)^
(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b^4*c^2*d + 3*s
qrt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) - 1/4*(5*(c*d^3)^(3/4)*b*c + 3*
(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*
c^2*d^4 - sqrt(2)*a^3*c*d^5) - 1/4*(5*(c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqr
t(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt
(2)*a^3*c*d^5) - 1/8*(3*(a*b^3)^(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*log(sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b
^4*c^2*d + 3*sqrt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) + 1/8*(3*(a*b^3)^
(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b^4*c^2*d + 3*sqrt(2)*a^3*b^3*c*
d^2 - sqrt(2)*a^4*b^2*d^3) + 1/8*(5*(c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d
)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^4*d^2 - 3
*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d^5) - 1/8
*(5*(c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt
(2)*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d^5) - 1/2*(2*b*d*x^(7/2) + b*c*x^(3/2) +
 a*d*x^(3/2))/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2))
```

Mupad [B]
time = 2.11, size = 2500, normalized size = 4.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] 2*atan((((-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*
c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b
^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^
5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032
*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 2703
36*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(3/4)*(((864*a*b^20*c^17*d^4 +
 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 56
640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9
 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*
c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a
^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 320
0*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)*1i)/(a^14*d^14 + b^14*c^14 +
91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 200
2*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*
b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*
c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13) - (x
^(1/2)*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^
2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^1
1*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*
b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a
^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336
*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4)*(36864*a*b^20*c^17*d^4 + 3
6864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6
 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 5758976*a^6*b^15
*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^10*d^11 + 310
88640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8*d^13 + 13516800*a^11*b^10*
c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16 - 2334720*
a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*c^2*d^19))/
(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^1
0 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) - (x^(1/2)*(5625*a*b^12*c^8*d^5
+ 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7 + 1
33539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10 + 3
4275*a^7*b^6*c^2*d^11))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
```

^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4) - ((-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(3/4)*(((864*a*b^20*c^17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)*1i)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13) + (x^(1/2)*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4)*(36864*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^17...

3.490 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=601

$$-\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)}-\frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{b^{3/4}(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(bc}{}$$

```
[Out] -1/8*b^(3/4)*(7*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/
(-a*d+b*c)^3*2^(1/2)+1/8*b^(3/4)*(7*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/
2)/a^(1/4))/a^(3/4)/(-a*d+b*c)^3*2^(1/2)+1/8*d^(3/4)*(a*d+7*b*c)*arctan(1-d
^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^3*2^(1/2)-1/8*d^(3/4)*(a
*d+7*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^3*2^
(1/2)-1/16*b^(3/4)*(7*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)
*x^(1/2))/a^(3/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(3/4)*(7*a*d+b*c)*ln(a^(1/2)+
x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^3*2^(1/2)+1/1
6*d^(3/4)*(a*d+7*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))
/c^(3/4)/(-a*d+b*c)^3*2^(1/2)-1/16*d^(3/4)*(a*d+7*b*c)*ln(c^(1/2)+x*d^(1/2)
+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/(-a*d+b*c)^3*2^(1/2)-d*x^(1/2)/(-
a*d+b*c)^2/(d*x^2+c)-1/2*x^(1/2)/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)
```

Rubi [A]
time = 0.45, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 482, 541, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(7ad+bc)}{4\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(7ad+bc)}{4\sqrt{2}\,a^{3/4}(bc-ad)^3}-\frac{b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{d^{3/4}(ad+7bc)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)^3}-\frac{d^{3/4}(ad+7bc)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)^3}+\frac{d^{3/4}(ad+7bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}(bc-ad)^3}-\frac{d^{3/4}(ad+7bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}(bc-ad)^3}-\frac{d\sqrt{x}}{\left(c+dx^2\right)(bc-ad)^2}-\frac{\sqrt{x}}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -((d*Sqrt[x])/((b*c - a*d)^2*(c + d*x^2))) - Sqrt[x]/(2*(b*c - a*d)*(a + b*
x^2)*(c + d*x^2)) - (b^(3/4)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*c + 7*a*d)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d
)^3) + (d^(3/4)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (b^(3/
4)*(b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d
)^3) + (d^(3/4)*(7*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*L
```

```
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/
4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
```

```
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^4}{(a+bx^4)^2(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{2(bc-ad)(a+bx^2)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{c-7dx^4}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2(c+dx^2)} - \frac{\sqrt{x}}{2(bc-ad)(a+bx^2)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{4c(bc+ad)-24}{(a+bx^4)(c+}\right.}{8c(bc-}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2(c+dx^2)} - \frac{\sqrt{x}}{2(bc-ad)(a+bx^2)(c+dx^2)} - \frac{(d(7bc+ad))\text{Subst}(}{2(bc-}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2(c+dx^2)} - \frac{\sqrt{x}}{2(bc-ad)(a+bx^2)(c+dx^2)} - \frac{(d(7bc+ad))\text{Subst}(}{4\sqrt{c}}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2(c+dx^2)} - \frac{\sqrt{x}}{2(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\left(\sqrt{d}\,(7bc+ad)\right)\text{Su}}{}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2(c+dx^2)} - \frac{\sqrt{x}}{2(bc-ad)(a+bx^2)(c+dx^2)} - \frac{b^{3/4}(bc+7ad)\log\left(\right.}{8\sqrt{}}$$

$$= -\frac{d\sqrt{x}}{(bc-ad)^2(c+dx^2)} - \frac{\sqrt{x}}{2(bc-ad)(a+bx^2)(c+dx^2)} - \frac{b^{3/4}(bc+7ad)\tan^{-1}}{4\sqrt{2}\,a^{3/4}}$$

Mathematica [A]
time = 1.73, size = 338, normalized size = 0.56

$$\frac{1}{8}\left(-\frac{4\sqrt{x}\,(ad+b(c+2dx^2))}{(bc-ad)^2(a+bx^2)(c+dx^2)} + \frac{\sqrt{2}\,b^{3/4}(bc+7ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{3/4}(-bc+ad)^3} + \frac{\sqrt{2}\,d^{3/4}(7bc+ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{3/4}(bc-ad)^3} - \frac{\sqrt{2}\,b^{3/4}(bc+7ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}(-bc+ad)^3} - \frac{\sqrt{2}\,d^{3/4}(7bc+ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{3/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-4*Sqrt[x]*(a*d + b*(c + 2*d*x^2)))/((b*c - a*d)^2*(a + b*x^2)*(c + d*x^2
)) + (Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])]])/(a^(3/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(3/4)*(7*
b*c + a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])
/(c^(3/4)*(b*c - a*d)^3) - (Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(3/4)*(-(b*c) + a*d)^3)
```

```
 - (Sqrt[2]*d^(3/4)*(7*b*c + a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])
/(Sqrt[c] + Sqrt[d]*x)])/(c^(3/4)*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.11, size = 300, normalized size = 0.50

method	result
derivativedivides	$-\frac{2b\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(7ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{32a}\right.}{(ad-bc)^3}$
default	$-\frac{2b\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(7ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{32a}\right.}{(ad-bc)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(7*a*d+b*c)*(a/b
)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d/(a*d-b*c)^3*((-1/4*a*d+1/4*b
*c)*x^(1/2)/(d*x^2+c)+1/32*(a*d+7*b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 620, normalized size = 1.03

$$\frac{\left(\frac{2\sqrt{2}(bc+7ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(bc+7ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(bc+7ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(bc+7ad)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b}{16\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)}-\frac{2bdx^{\frac{5}{2}}+(bc+ad)\sqrt{x}}{2\left(ab^2c^3-2a^2bc^2d+a^3cd^2+\left(b^3c^2d-2ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^2\right)}-\frac{\frac{2\sqrt{2}(7bcd+ad^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(7bcd+ad^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(7bcd+ad^2)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(7bcd+ad^2)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) +
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ 2*sqrt(2)*(b*c + 7*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + s
qrt(2)*(b*c + 7*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(-sqrt(2)*a^(1/4)*b^(1/4)
```

```
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b/(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/2*(2*b*d*x^(5/2) + (b*c + a*d)*sqrt(x))/(
a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*
d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2) - 1/16*(2*s
qrt(2)*(7*b*c*d + a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sq
rt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*s
qrt(2)*(7*b*c*d + a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*s
qrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sq
rt(2)*(7*b*c*d + a*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(7*b*c*d + a*d^2)*log(-sqrt(2)*c^(1/4)*
d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5474 vs. $2(451) = 902$.
time = 83.04, size = 5474, normalized size = 9.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-
(b^7*c^4 + 28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401
*a^4*b^3*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 -
220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b
^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3
*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(1/4)*arctan(-
((a^2*b^9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d^2 - 84*a^5*b^6*c^6*d^3 +
 126*a^6*b^5*c^5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^8*b^3*c^3*d^6 - 36*a^9*b^
2*c^2*d^7 + 9*a^10*b*c*d^8 - a^11*d^9)*sqrt((b^4*c^2 + 14*a*b^3*c*d + 49*a^
2*b^2*d^2)*x + (a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5
*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*sqrt(-(b^7*c^4
 + 28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3
*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*
b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d
^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 6
6*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)))*(-(b^7*c^4 + 28*a*b^6
*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^3*
b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3
 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^
10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*
c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(3/4) - (a^2*b^11*c^10 - 2*a^3*b^
10*c^9*d - 27*a^4*b^9*c^8*d^2 + 168*a^5*b^8*c^7*d^3 - 462*a^6*b^7*c^6*d^4 +
```

756*a^7*b^6*c^5*d^5 - 798*a^8*b^5*c^4*d^6 + 552*a^9*b^4*c^3*d^7 - 243*a^10
*b^3*c^2*d^8 + 62*a^11*b^2*c*d^9 - 7*a^12*b*d^10)*sqrt(x)*(-(b^7*c^4 + 28*a
*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(
a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9
*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 79
2*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*
b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(3/4))/(b^7*c^4 + 28*a*b^6*c^
3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)) - 4*(a*
b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(2401*b^4*c
^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^
7)/(b^12*c^15 - 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*
d^3 + 495*a^4*b^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 7
92*a^7*b^5*c^8*d^7 + 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^
2*c^5*d^10 - 12*a^11*b*c^4*d^11 + a^12*c^3*d^12))^(1/4)*arctan(-((b^9*c^11
- 9*a*b^8*c^10*d + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^
7*d^4 - 126*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 + 9*a
^8*b*c^3*d^8 - a^9*c^2*d^9)*sqrt((49*b^2*c^2*d^2 + 14*a*b*c*d^3 + a^2*d^4)*
x + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15
*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(-(2401*b^4*c^4*d^3 +
1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12
*c^15 - 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 49
5*a^4*b^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b
^5*c^8*d^7 + 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^
10 - 12*a^11*b*c^4*d^11 + a^12*c^3*d^12)))*(-(2401*b^4*c^4*d^3 + 1372*a*b^3
*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 - 12*
a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b^8*c
^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7
+ 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^1
1*b*c^4*d^11 + a^12*c^3*d^12))^(3/4) - (7*b^10*c^12*d - 62*a*b^9*c^11*d^2 +
243*a^2*b^8*c^10*d^3 - 552*a^3*b^7*c^9*d^4 + 798*a^4*b^6*c^8*d^5 - 756*a^5
*b^5*c^7*d^6 + 462*a^6*b^4*c^6*d^7 - 168*a^7*b^3*c^5*d^8 + 27*a^8*b^2*c^4*d
^9 + 2*a^9*b*c^3*d^10 - a^10*c^2*d^11)*sqrt(x)*(-(2401*b^4*c^4*d^3 + 1372*a
*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 -
12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b
^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*
d^7 + 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12
*a^11*b*c^4*d^11 + a^12*c^3*d^12))^(3/4))/(2401*b^4*c^4*d^3 + 1372*a*b^3*c^
3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)) - (a*b^2*c^3 - 2*a
^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3
*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(b^7*c^4 + 28*a*b^6*c^3*
d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^3*b^12*
c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 49
5*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 904 vs. 2(451) = 902.
time = 2.56, size = 904, normalized size = 1.50

$$\frac{\left((ab^3)^{\frac{1}{4}}bc+7(ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}ab^3c^3-3\sqrt{2}a^2b^2c^2d+3\sqrt{2}a^3bcd^2-\sqrt{2}a^4d^3\right)}+\frac{\left((ab^3)^{\frac{1}{4}}bc+7(ab^3)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}ab^3c^3-3\sqrt{2}a^2b^2c^2d+3\sqrt{2}a^3bcd^2-\sqrt{2}a^4d^3\right)}-\frac{\left(7(cd^3)^{\frac{1}{4}}bc+(cd^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^3c^4-3\sqrt{2}ab^2c^3d+3\sqrt{2}a^2bc^2d^2-\sqrt{2}a^3cd^3\right)}-\frac{\left(7(cd^3)^{\frac{1}{4}}bc+(cd^3)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^3c^4-3\sqrt{2}ab^2c^3d+3\sqrt{2}a^2bc^2d^2-\sqrt{2}a^3cd^3\right)}+\frac{\left((ab^3)^{\frac{1}{4}}bc+7(ab^3)^{\frac{1}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}ab^3c^3-3\sqrt{2}a^2b^2c^2d+3\sqrt{2}a^3bcd^2-\sqrt{2}a^4d^3\right)}-\frac{\left((ab^3)^{\frac{1}{4}}bc+7(ab^3)^{\frac{1}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}ab^3c^3-3\sqrt{2}a^2b^2c^2d+3\sqrt{2}a^3bcd^2-\sqrt{2}a^4d^3\right)}-\frac{\left(7(cd^3)^{\frac{1}{4}}bc+(cd^3)^{\frac{1}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^3c^4-3\sqrt{2}ab^2c^3d+3\sqrt{2}a^2bc^2d^2-\sqrt{2}a^3cd^3\right)}+\frac{\left(7(cd^3)^{\frac{1}{4}}bc+(cd^3)^{\frac{1}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^3c^4-3\sqrt{2}ab^2c^3d+3\sqrt{2}a^2bc^2d^2-\sqrt{2}a^3cd^3\right)}-\frac{2bdx^{\frac{5}{2}}+bc\sqrt{x}+ad\sqrt{x}}{2(bdx^4+bcx^2+adx^2+ac)(b^2c^2-2abcd+a^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2
*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/4*((a*b^3)^(1/4)*b*c
+ 7*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x)
)/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3
*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/4*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)
*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)
*b^3*c^4 - 3*sqrt(2)*a*b^2*c^3*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(2)*a^3*c*
d^3) - 1/4*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^4 - 3*sqrt(2)*a
*b^2*c^3*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(2)*a^3*c*d^3) + 1/8*((a*b^3)^(1
/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a
/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 -
 sqrt(2)*a^4*d^3) - 1/8*((a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*log(-sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2
*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/8*(7*(c*d^3)^(1/4
)*b*c + (c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(sqrt(2)*b^3*c^4 - 3*sqrt(2)*a*b^2*c^3*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(
2)*a^3*c*d^3) + 1/8*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*log(-sqrt(2)*
sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^4 - 3*sqrt(2)*a*b^2*c^3
*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(2)*a^3*c*d^3) - 1/2*(2*b*d*x^(5/2) + b*
c*sqrt(x) + a*d*sqrt(x))/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2))
```

Mupad [**B**]
time = 2.07, size = 2500, normalized size = 4.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)

[Out] 2*atan(-(((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)) + ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*(8192*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^11*b^8*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*a^14*b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(3/4)*1i - (((2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (2197*a^4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)*1i)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4) - (x^(1/2)*(1225*a^6*b^7*d^13 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11))/(8

*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 137
2*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4
096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 9011
20*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 37
84704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8
 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11
))^(1/4) + ((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^
20*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*
b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648
704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9
*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a
^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 +
337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12 + b^
12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
- 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8
*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*
d - 12*a^11*b*c*d^11)) - ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^
3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c
^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9
*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*
c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12
*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*(8192
*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 204...

3.491 $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=624

$$\frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)}+\frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{b^{5/4}(bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{b^{5/4}(\ldots}{\ldots}$$

```
[Out] 1/2*d*(a*d+b*c)*x^(3/2)/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*b*x^(3/2)/a/(-a*d+b*
c)/(b*x^2+a)/(d*x^2+c)-1/8*b^(5/4)*(-9*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^
(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/8*b^(5/4)*(-9*a*d+b*c)*arctan
(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^3*2^(1/2)-1/8*d^(5/4
)*(-a*d+9*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*c)
^3*2^(1/2)+1/8*d^(5/4)*(-a*d+9*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4
))/c^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(5/4)*(-9*a*d+b*c)*ln(a^(1/2)+x*b^(1
/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)^3*2^(1/2)-1/16*b^(5
/4)*(-9*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5
/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(5/4)*(-a*d+9*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(
1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a*d+b*c)^3*2^(1/2)-1/16*d^(5/4)*(-a
*d+9*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a
*d+b*c)^3*2^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 483, 593, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-9ad)}{4\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-9ad)}{4\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{b^{5/4}(bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{d^{5/4}(9bc-ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^3}+\frac{d^{5/4}(9bc-ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^3}+\frac{d^{5/4}(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\,c^{5/4}(bc-ad)^3}-\frac{d^{5/4}(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}\,c^{5/4}(bc-ad)^3}+\frac{dx^{3/2}(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}+\frac{bx^{3/2}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x^(3/2))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x^(3/2))/(2*
a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(5/4)*(b*c - 9*a*d)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (b^
(5/4)*(b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(9*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c
- a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b
*c - a*d)^3) + (b^(5/4)*(b*c - 9*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (b^(5/4)*(b*c -
9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^
```

```
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) - (d^
(5/4)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}\,dx = 2\text{Subst}\left(\int \frac{x^2}{(a+bx^4)^2(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\text{Subst}\left(\int \frac{x^2(-bc+4ad-5bdx^4)}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \text{Subst}\Big(\int \frac{x^2(-4(b^2c \ldots$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \text{Subst}\Big(\int \Big(-\frac{4b^2c(b \ldots}{(bc-a \ldots} \ldots$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{(b^2(bc-9ad))\,\text{Su}\ldots}{2a\ldots}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \frac{(b^{3/2}(bc-9ad))\ldots}{\ldots}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{(b(bc-9ad))\text{Sub}\ldots}{\ldots}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{b^{5/4}(bc-9ad)\log\ldots}{8\ldots}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \frac{b^{5/4}(bc-9ad)\tan\ldots}{4\sqrt{2}\,a\ldots}$$

Mathematica [A]
time = 1.42, size = 357, normalized size = 0.57

$$\frac{1}{8}\left(\frac{4x^{3/2}(a^2d^2+abd^2x^2+b^2c(c+dx^2))}{ac(bc-ad)^2(a+bx^2)(c+dx^2)} + \frac{\sqrt{2}\,b^{5/4}(bc-9ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{5/4}(-bc+ad)^3} + \frac{\sqrt{2}\,d^{5/4}(-9bc+ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{5/4}(bc-ad)^3} + \frac{\sqrt{2}\,b^{5/4}(bc-9ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{5/4}(-bc+ad)^3} + \frac{\sqrt{2}\,d^{5/4}(-9bc+ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{5/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((4*x^(3/2)*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)))/(a*c*(b*c - a*d)^2
*(a + b*x^2)*(c + d*x^2)) + (Sqrt[2]*b^(5/4)*(b*c - 9*a*d)*ArcTan[(Sqrt[a]
- Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(5/4)*(-(b*c) + a*d)^3)
```

```
 + (Sqrt[2]*d^(5/4)*(-9*b*c + a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x])])/(c^(5/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(5/4)*(b*c -
9*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a
^(5/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(5/4)*(-9*b*c + a*d)*ArcTanh[(Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(5/4)*(b*c - a*d)^3))/
8
```

Maple [A]
time = 0.12, size = 317, normalized size = 0.51

method	result
derivativedivides	$\frac{2b^{2}\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4a\left(b\,x^{2}+a\right)}+\frac{(9ad-bc)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{32ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{(ad-bc)^{3}}$
default	$\frac{2b^{2}\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4a\left(b\,x^{2}+a\right)}+\frac{(9ad-bc)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{32ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{(ad-bc)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/a*x^(3/2)/(b*x^2+a)+1/32*(9*a*d-b*c)/a/b/(
a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)
^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d^2/(a*d-b*c)^3*(1/4*(a*d-b*c
)/c*x^(3/2)/(d*x^2+c)+1/32*(a*d-9*b*c)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)-1)))
```

Maxima [A]
time = 0.54, size = 610, normalized size = 0.98

$$\frac{(b^3c-9ab^2d)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)}+\frac{(9bcd^2-ad^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3\right)}+\frac{(b^2cd+abd^2)x^{\frac{7}{2}}+(b^2c^2+a^2d^2)x^{\frac{3}{2}}}{2\left(a^2b^2c^4-2a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2a^2b^2c^2d^2+a^3bcd^3)x^4+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(b^3*c - 9*a*b^2*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
```

```
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(
1/4)*b^(3/4)))/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/
16*(9*b*c*d^2 - a*d^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(
d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sq
rt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log
(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) +
 sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/
4)*d^(3/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3) + 1/2*
((b^2*c*d + a*b*d^2)*x^(7/2) + (b^2*c^2 + a^2*d^2)*x^(3/2))/(a^2*b^2*c^4 -
2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^
3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6028 vs. $2(472) = 944$.
time = 84.73, size = 6028, normalized size = 9.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b
^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2
+ a^4*c*d^3)*x^2)*(-(b^9*c^4 - 36*a*b^8*c^3*d + 486*a^2*b^7*c^2*d^2 - 2916*
a^3*b^6*c*d^3 + 6561*a^4*b^5*d^4)/(a^5*b^12*c^12 - 12*a^6*b^11*c^11*d + 66*
a^7*b^10*c^10*d^2 - 220*a^8*b^9*c^9*d^3 + 495*a^9*b^8*c^8*d^4 - 792*a^10*b^
7*c^7*d^5 + 924*a^11*b^6*c^6*d^6 - 792*a^12*b^5*c^5*d^7 + 495*a^13*b^4*c^4*
d^8 - 220*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^10 - 12*a^16*b*c*d^11 + a^17
*d^12))^(1/4)*arctan(-((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d
^3)*sqrt((b^14*c^6 - 54*a*b^13*c^5*d + 1215*a^2*b^12*c^4*d^2 - 14580*a^3*b^
11*c^3*d^3 + 98415*a^4*b^10*c^2*d^4 - 354294*a^5*b^9*c*d^5 + 531441*a^6*b^8
*d^6)*x - (a^3*b^15*c^10 - 42*a^4*b^14*c^9*d + 717*a^5*b^13*c^8*d^2 - 6392*
a^6*b^12*c^7*d^3 + 32082*a^7*b^11*c^6*d^4 - 93372*a^8*b^10*c^5*d^5 + 164242
*a^9*b^9*c^4*d^6 - 177912*a^10*b^8*c^3*d^7 + 116397*a^11*b^7*c^2*d^8 - 4228
2*a^12*b^6*c*d^9 + 6561*a^13*b^5*d^10)*sqrt(-(b^9*c^4 - 36*a*b^8*c^3*d + 48
6*a^2*b^7*c^2*d^2 - 2916*a^3*b^6*c*d^3 + 6561*a^4*b^5*d^4)/(a^5*b^12*c^12 -
 12*a^6*b^11*c^11*d + 66*a^7*b^10*c^10*d^2 - 220*a^8*b^9*c^9*d^3 + 495*a^9*
b^8*c^8*d^4 - 792*a^10*b^7*c^7*d^5 + 924*a^11*b^6*c^6*d^6 - 792*a^12*b^5*c^
5*d^7 + 495*a^13*b^4*c^4*d^8 - 220*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^10
- 12*a^16*b*c*d^11 + a^17*d^12)))*(-(b^9*c^4 - 36*a*b^8*c^3*d + 486*a^2*b^7
*c^2*d^2 - 2916*a^3*b^6*c*d^3 + 6561*a^4*b^5*d^4)/(a^5*b^12*c^12 - 12*a^6*b
```

^11*c^11*d + 66*a^7*b^10*c^10*d^2 - 220*a^8*b^9*c^9*d^3 + 495*a^9*b^8*c^8*d^4 - 792*a^10*b^7*c^7*d^5 + 924*a^11*b^6*c^6*d^6 - 792*a^12*b^5*c^5*d^7 + 495*a^13*b^4*c^4*d^8 - 220*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^10 - 12*a^16*b*c*d^11 + a^17*d^12))^(1/4) + (a*b^10*c^6 - 30*a^2*b^9*c^5*d + 327*a^3*b^8*c^4*d^2 - 1540*a^4*b^7*c^3*d^3 + 2943*a^5*b^6*c^2*d^4 - 2430*a^6*b^5*c*d^5 + 729*a^7*b^4*d^6)*sqrt(x)*(-(b^9*c^4 - 36*a*b^8*c^3*d + 486*a^2*b^7*c^2*d^2 - 2916*a^3*b^6*c*d^3 + 6561*a^4*b^5*d^4)/(a^5*b^12*c^12 - 12*a^6*b^11*c^11*d + 66*a^7*b^10*c^10*d^2 - 220*a^8*b^9*c^9*d^3 + 495*a^9*b^8*c^8*d^4 - 792*a^10*b^7*c^7*d^5 + 924*a^11*b^6*c^6*d^6 - 792*a^12*b^5*c^5*d^7 + 495*a^13*b^4*c^4*d^8 - 220*a^14*b^3*c^3*d^9 + 66*a^15*b^2*c^2*d^10 - 12*a^16*b*c*d^11 + a^17*d^12))^(1/4))/(b^9*c^4 - 36*a*b^8*c^3*d + 486*a^2*b^7*c^2*d^2 - 2916*a^3*b^6*c*d^3 + 6561*a^4*b^5*d^4)) + 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)*(-(6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8 + a^4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66*a^2*b^10*c^15*d^2 - 220*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4 - 792*a^5*b^7*c^12*d^5 + 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*d^7 + 495*a^8*b^4*c^9*d^8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7*d^10 - 12*a^11*b*c^6*d^11 + a^12*c^5*d^12))^(1/4)*arctan(-((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt((531441*b^6*c^6*d^8 - 354294*a*b^5*c^5*d^9 + 98415*a^2*b^4*c^4*d^10 - 14580*a^3*b^3*c^3*d^11 + 1215*a^4*b^2*c^2*d^12 - 54*a^5*b*c*d^13 + a^6*d^14)*x - (6561*b^10*c^13*d^5 - 42282*a*b^9*c^12*d^6 + 116397*a^2*b^8*c^11*d^7 - 177912*a^3*b^7*c^10*d^8 + 164242*a^4*b^6*c^9*d^9 - 93372*a^5*b^5*c^8*d^10 + 32082*a^6*b^4*c^7*d^11 - 6392*a^7*b^3*c^6*d^12 + 717*a^8*b^2*c^5*d^13 - 42*a^9*b*c^4*d^14 + a^10*c^3*d^15)*sqrt(-(6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8 + a^4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66*a^2*b^10*c^15*d^2 - 220*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4 - 792*a^5*b^7*c^12*d^5 + 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*d^7 + 495*a^8*b^4*c^9*d^8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7*d^10 - 12*a^11*b*c^6*d^11 + a^12*c^5*d^12)))*(-(6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8 + a^4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66*a^2*b^10*c^15*d^2 - 220*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4 - 792*a^5*b^7*c^12*d^5 + 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*d^7 + 495*a^8*b^4*c^9*d^8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7*d^10 - 12*a^11*b*c^6*d^11 + a^12*c^5*d^12))^(1/4) + (729*b^6*c^7*d^4 - 2430*a*b^5*c^6*d^5 + 2943*a^2*b^4*c^5*d^6 - 1540*a^3*b^3*c^4*d^7 + 327*a^4*b^2*c^3*d^8 - 30*a^5*b*c^2*d^9 + a^6*c*d^10)*sqrt(x)*(-(6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8 + a^4*d^9)/(b^12*c^17 - 12*a*b^11*c^16*d + 66*a^2*b^10*c^15*d^2 - 220*a^3*b^9*c^14*d^3 + 495*a^4*b^8*c^13*d^4 - 792*a^5*b^7*c^12*d^5 + 924*a^6*b^6*c^11*d^6 - 792*a^7*b^5*c^10*d^7 + 495*a^8*b^4*c^9*d^8 - 220*a^9*b^3*c^8*d^9 + 66*a^10*b^2*c^7*d^10 - 12*a^11*b*c^6*d^11 + a^12*c^5*d^12))^(1/4))/(6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8 + a^4*d^9)) + (a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a

```
^4*c*d^3)*x^2)*(-(b^9*c^4 - 36*a*b^8*c^3*d + 486*a^2*b^7*c^2*d^2 - 2916*a^3
*b^6*c*d^3 + 6561*a^4*b^5*d^4)/(a^5*b^12*c^12 -...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 973 vs. 2(472) = 944.
time = 1.64, size = 973, normalized size = 1.56

$$\frac{\left((ab^3)^{\frac{3}{4}}bc - 9(ab^3)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a^2b^4c^3 - 3\sqrt{2}a^3b^3c^2d + 3\sqrt{2}a^4b^2cd^2 - \sqrt{2}a^5bd^3\right)} + \frac{\left((ab^3)^{\frac{3}{4}}bc - 9(ab^3)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a^2b^4c^3 - 3\sqrt{2}a^3b^3c^2d + 3\sqrt{2}a^4b^2cd^2 - \sqrt{2}a^5bd^3\right)} + \frac{\left(9(cd^3)^{\frac{3}{4}}bc - (cd^3)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^3c^5d - 3\sqrt{2}ab^2c^4d^2 + 3\sqrt{2}a^2bc^3d^3 - \sqrt{2}a^3c^2d^4\right)} + \frac{\left(9(cd^3)^{\frac{3}{4}}bc - (cd^3)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^3c^5d - 3\sqrt{2}ab^2c^4d^2 + 3\sqrt{2}a^2bc^3d^3 - \sqrt{2}a^3c^2d^4\right)} - \frac{\left((ab^3)^{\frac{3}{4}}bc - 9(ab^3)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}a^2b^4c^3 - 3\sqrt{2}a^3b^3c^2d + 3\sqrt{2}a^4b^2cd^2 - \sqrt{2}a^5bd^3\right)} + \frac{\left((ab^3)^{\frac{3}{4}}bc - 9(ab^3)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}a^2b^4c^3 - 3\sqrt{2}a^3b^3c^2d + 3\sqrt{2}a^4b^2cd^2 - \sqrt{2}a^5bd^3\right)} - \frac{\left(9(cd^3)^{\frac{3}{4}}bc - (cd^3)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^3c^5d - 3\sqrt{2}ab^2c^4d^2 + 3\sqrt{2}a^2bc^3d^3 - \sqrt{2}a^3c^2d^4\right)} + \frac{\left(9(cd^3)^{\frac{3}{4}}bc - (cd^3)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^3c^5d - 3\sqrt{2}ab^2c^4d^2 + 3\sqrt{2}a^2bc^3d^3 - \sqrt{2}a^3c^2d^4\right)} + \frac{b^2cdx^{\frac{7}{2}} + abd^2x^{\frac{7}{2}} + b^2c^2x^{\frac{3}{2}} + a^2d^2x^{\frac{3}{2}}}{2(ab^2c^3 - 2a^2bc^2d + a^3cd^2)(bdx^4 + bcx^2 + adx^2 + ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((a*b^3)^(3/4)*b*c - 9*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b
^3*c^2*d + 3*sqrt(2)*a^4*b^2*c*d^2 - sqrt(2)*a^5*b*d^3) + 1/4*((a*b^3)^(3/4
)*b*c - 9*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*s
qrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b^3*c^2*d + 3*sqr
t(2)*a^4*b^2*c*d^2 - sqrt(2)*a^5*b*d^3) + 1/4*(9*(c*d^3)^(3/4)*b*c - (c*d^3
)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/
4))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3
- sqrt(2)*a^3*c^2*d^4) + 1/4*(9*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3
*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^
2*d^4) - 1/8*((a*b^3)^(3/4)*b*c - 9*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b^3*c^2*d
 + 3*sqrt(2)*a^4*b^2*c*d^2 - sqrt(2)*a^5*b*d^3) + 1/8*((a*b^3)^(3/4)*b*c -
9*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqr
t(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b^3*c^2*d + 3*sqrt(2)*a^4*b^2*c*d^2 - sqrt
(2)*a^5*b*d^3) - 1/8*(9*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(sqrt(2)*
sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c
^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/8*(9*(c*d^3)^(3
/4)*b*c - (c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3
- sqrt(2)*a^3*c^2*d^4) + 1/2*(b^2*c*d*x^(7/2) + a*b*d^2*x^(7/2) + b^2*c^2*x
^(3/2) + a^2*d^2*x^(3/2))/((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4
 + b*c*x^2 + a*d*x^2 + a*c))
```

Mupad [B]
time = 2.45, size = 2500, normalized size = 4.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] 2*atan((((((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 12
16*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 8
37408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13
*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^11*d^12 - 4458784
*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8*d
^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d^17 - 2842656*a^
14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 190
40*a^17*b^6*c^2*d^21)*1i)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15
*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 10
01*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432
*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^1
2*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12) - (x^(1/2)*(
-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 3
6*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11
 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^1
3*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b
^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^1
0*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a
^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*c^17*d^6 - 974
8480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 - 116785152*a^6*b^17*c^
14*d^9 + 249192448*a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^12*d^11 + 5477
00736*a^9*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10*d^13 + 547700736*a^11*b
^12*c^9*d^14 - 412041216*a^12*b^11*c^8*d^15 + 249192448*a^13*b^10*c^7*d^16
 - 116785152*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 - 9748480*a^16*b
^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2*d^21))/(16*(a
^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66
*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*
b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*
d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d^9 + 6561*b^4*
c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*
b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^
15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b
^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520
*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 4915
2*a*b^11*c^16*d))^(3/4) - (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 362
7*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a
^3*b^12*c^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10*c^2*d^13))/(16
```

```
*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 +
 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a
^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c
^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d^9 + 6561*b
^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(40
96*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10
*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^
5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027
520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 4
9152*a*b^11*c^16*d))^(1/4) - ((((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216
*a*b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*
a^3*b^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 +
6564768*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^1
5*c^11*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 103
74112*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*
c^6*d^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^
16*b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)*1i)/(a^2*b^14*c^16 + a^16*c^2*d^
14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a
^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^
8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b
^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c
^4*d^12) + (x^(1/2)*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 48
6*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 -
49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3
+ 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^1
1*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3
*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)*(4096*a*b
^22*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^
3*b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 -
116785152*a^6*b^17*c^14*d^9 + 249192448*a^7*b^1...
```

3.492 $\int \frac{1}{\sqrt{x}\,(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=628

$$\frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\,(c+dx^2)}+\frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)}-\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/}}{}$$

```
[Out] -1/8*b^(7/4)*(-11*a*d+3*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7
/4)/(-a*d+b*c)^3*2^(1/2)+1/8*b^(7/4)*(-11*a*d+3*b*c)*arctan(1+b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/8*d^(7/4)*(-3*a*d+11*b*c
)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/
8*d^(7/4)*(-3*a*d+11*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)
/(-a*d+b*c)^3*2^(1/2)-1/16*b^(7/4)*(-11*a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(
1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(7/4)*(-1
1*a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/
(-a*d+b*c)^3*2^(1/2)-1/16*d^(7/4)*(-3*a*d+11*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1
/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(7/4)*(-3*
a*d+11*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/(
-a*d+b*c)^3*2^(1/2)+1/2*d*(a*d+b*c)*x^(1/2)/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*
b*x^(1/2)/a/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)
```

Rubi [A]

time = 0.58, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = $\{477, 425, 541, 536, 217, 1179, 642, 1176, 631, 210\}$

$$-\frac{b^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(3bc-11ad)}{4\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(3bc-11ad)}{4\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{b\sqrt{x}}{2a\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{d\sqrt{x}\,(ad+bc)}{2ac\,(c+dx^2)\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*Sqrt[x])/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*Sqrt[x])/(2*
a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) +
(b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4
*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4
)*(11*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2
]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (
b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)
```

```
*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
```

```
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-7bdx^4}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-4(\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{(b^2(3bc-11\ldots}{\ldots}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{(b^2(3bc-11\ldots}{\ldots}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{(b^{3/2}(3bc-1\ldots}{\ldots}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{7/4}(3bc-1\ldots}{\ldots}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{7/4}(3bc-1\ldots}{4\ldots}$$

Mathematica [A]

time = 1.67, size = 361, normalized size = 0.57

$$\frac{1}{8}\left(\frac{4\sqrt{x}\left(a^2d^2+abd^2x^2+b^2c(c+dx^2)\right)}{ac(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt{2}\,b^{7/4}(-3bc+11ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{7/4}(bc-ad)^3} + \frac{\sqrt{2}\,d^{7/4}(-11bc+3ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{7/4}(bc-ad)^3} + \frac{\sqrt{2}\,b^{7/4}(-3bc+11ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{7/4}(-bc+ad)^3} + \frac{\sqrt{2}\,d^{7/4}(11bc-3ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{7/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((4*Sqrt[x]*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)))/(a*c*(b*c - a*d)^2
*(a + b*x^2)*(c + d*x^2)) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[(Sqrt
[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(7/4)*(b*c - a*d)^3
) + (Sqrt[2]*d^(7/4)*(-11*b*c + 3*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3
*b*c + 11*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]
```

```
*x)])/(a^(7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*ArcTan
h[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(7/4)*(b*c -
 a*d)^3))/8
```

Maple [A]

time = 0.11, size = 312, normalized size = 0.50

method	result
derivativedivides	$\frac{2b^2\left(\frac{(ad-bc)\sqrt{x}}{4a\left(b\,x^2+a\right)}+\frac{(11ad-3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\sqrt{}\right.\right.}{32a^2}\right)}{(ad-bc)^3}$
default	$\frac{2b^2\left(\frac{(ad-bc)\sqrt{x}}{4a\left(b\,x^2+a\right)}+\frac{(11ad-3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\sqrt{}\right.\right.}{32a^2}\right)}{(ad-bc)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/a*x^(1/2)/(b*x^2+a)+1/32*(11*a*d-3*b*c)/a^
2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d^2/(a*d-b*c)^3*(1/4*(a*d-
b*c)/c*x^(1/2)/(d*x^2+c)+1/32*(3*a*d-11*b*c)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x
+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.53, size = 678, normalized size = 1.08

$$\frac{\left(\frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc-11ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc-11ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(3bc-11ad)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b^2}{16\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)}+\frac{\left(b^2cd+abd^2\right)x^{\frac{5}{2}}+\left(b^2c^2+a^2d^2\right)\sqrt{x}}{2\left(a^2b^2c^4-2a^3bc^3d+a^4c^2d^2+\left(ab^3c^3d-2a^2b^2c^2d^2+a^3bcd^3\right)x^4+\left(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3\right)x^2\right)}+\frac{\frac{2\sqrt{2}\left(11bcd^2-3ad^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(11bcd^2-3ad^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(11bcd^2-3ad^3\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(11bcd^2-3ad^3\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 2*sqrt(2)*(3*b*c - 11*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
```

```
*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 11*a*d)*log(-sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(a*b^3*c
^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/2*((b^2*c*d + a*b*d^2)*
x^(5/2) + (b^2*c^2 + a^2*d^2)*sqrt(x))/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c
^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 -
 a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2) + 1/16*(2*sqrt(2)*(11*b*c*
d^2 - 3*a*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt
(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11
*b*c*d^2 - 3*a*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d
)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)
*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(-sqrt(2)*c^(
1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^4 - 3
*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 977 vs. 2(476) = 952.
time = 1.10, size = 977, normalized size = 1.56

$$\frac{\left(3\,(ab^3)^{\frac14} b^2c - 11\,(ab^3)^{\frac14} abd\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\left(\sqrt{2}\,a^2b^3c^3 - 3\sqrt{2}\,a^3b^2c^2d + 3\sqrt{2}\,a^4bcd^2 - \sqrt{2}\,a^5d^3\right)} + \frac{\left(3\,(ab^3)^{\frac14} b^2c - 11\,(ab^3)^{\frac14} abd\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\left(\sqrt{2}\,a^2b^3c^3 - 3\sqrt{2}\,a^3b^2c^2d + 3\sqrt{2}\,a^4bcd^2 - \sqrt{2}\,a^5d^3\right)} + \frac{\left(11\,(cd^3)^{\frac14} bcd - 3\,(cd^3)^{\frac14} ad^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{4\left(\sqrt{2}\,b^3c^5 - 3\sqrt{2}\,ab^2c^4d + 3\sqrt{2}\,a^2bc^3d^2 - \sqrt{2}\,a^3c^2d^3\right)} + \frac{\left(11\,(cd^3)^{\frac14} bcd - 3\,(cd^3)^{\frac14} ad^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{4\left(\sqrt{2}\,b^3c^5 - 3\sqrt{2}\,ab^2c^4d + 3\sqrt{2}\,a^2bc^3d^2 - \sqrt{2}\,a^3c^2d^3\right)} + \frac{\left(3\,(ab^3)^{\frac14} b^2c - 11\,(ab^3)^{\frac14} abd\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14} + x + \sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}\,a^2b^3c^3 - 3\sqrt{2}\,a^3b^2c^2d + 3\sqrt{2}\,a^4bcd^2 - \sqrt{2}\,a^5d^3\right)} - \frac{\left(3\,(ab^3)^{\frac14} b^2c - 11\,(ab^3)^{\frac14} abd\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14} + x + \sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}\,a^2b^3c^3 - 3\sqrt{2}\,a^3b^2c^2d + 3\sqrt{2}\,a^4bcd^2 - \sqrt{2}\,a^5d^3\right)} + \frac{\left(11\,(cd^3)^{\frac14} bcd - 3\,(cd^3)^{\frac14} ad^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^3c^5 - 3\sqrt{2}\,ab^2c^4d + 3\sqrt{2}\,a^2bc^3d^2 - \sqrt{2}\,a^3c^2d^3\right)} - \frac{\left(11\,(cd^3)^{\frac14} bcd - 3\,(cd^3)^{\frac14} ad^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^3c^5 - 3\sqrt{2}\,ab^2c^4d + 3\sqrt{2}\,a^2bc^3d^2 - \sqrt{2}\,a^3c^2d^3\right)} + \frac{b^2cdx^{\frac52} + abd^2x^{\frac52} + b^2c^2\sqrt{x} + a^2d^2\sqrt{x}}{2\,(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2)(bdx^4 + bcx^2 + adx^2 + ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2
```

```
)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/4*(3*(a*b^3)
^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2
*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/4*(11*(c*d^3)^(1/4)*b*c*d
 - 3*(c*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(
x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b
*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/4*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(
1/4)*a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/
4))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sq
rt(2)*a^3*c^2*d^3) + 1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sq
rt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/8*(3*(a*
b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2
)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^
(1/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*
c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3
) - 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*log(-sqrt(2)*sqrt(
x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d +
3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/2*(b^2*c*d*x^(5/2) + a*b
*d^2*x^(5/2) + b^2*c^2*sqrt(x) + a^2*d^2*sqrt(x))/((a*b^2*c^3 - 2*a^2*b*c^2
*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
```

Mupad [B]
time = 2.33, size = 2500, normalized size = 3.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((x^(1/2)*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b
*d*x^(5/2)*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2
*(a*d + b*c) + b*d*x^4) - atan((((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 7127
04*a^3*b^22*c^20*d^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d
^7 + 110432256*a^6*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*
a^8*b^17*c^15*d^10 - 635523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^1
3*d^12 - 227217408*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 56
2982912*a^13*b^12*c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15
*b^10*c^8*d^17 - 271552512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19
- 31899648*a^18*b^7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*
c^3*d^22 + 36864*a^21*b^4*c^2*d^23))/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 1
2*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9
*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10
*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 +
```

66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*(6144*a^4*b^19*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230272*a^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d^9 + 9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^12*b^11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14 + 5490688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(3/4) + ((891*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7*b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*b^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*1i + (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4) + ((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d

^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6
*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10
 - 635523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 227217408
*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*
c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 2
71552512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^
7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a
^21*b^4*c^2*d^23))/(16*(a^4*b^12*c^16 + a^16*c^...

3.493 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=676

$$\frac{-5b^2c^2+8abcd-5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}}+\frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right)}+\frac{b}{2a(bc-ad)\sqrt{x}\,\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{b^{9/4}(5bc-13a}{4\sqrt{}}$$

```
[Out] 1/8*b^(9/4)*(-13*a*d+5*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/
4)/(-a*d+b*c)^3*2^(1/2)-1/8*b^(9/4)*(-13*a*d+5*b*c)*arctan(1+b^(1/4)*2^(1/2
)*x^(1/2)/a^(1/4))/a^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/8*d^(9/4)*(-5*a*d+13*b*c)
*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^3*2^(1/2)-1/8
*d^(9/4)*(-5*a*d+13*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/
(-a*d+b*c)^3*2^(1/2)-1/16*b^(9/4)*(-13*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1
/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(9/4)*(-13
*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(
-a*d+b*c)^3*2^(1/2)-1/16*d^(9/4)*(-5*a*d+13*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/
4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(9/4)*(-5*a
*d+13*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-
a*d+b*c)^3*2^(1/2)+1/2*(-5*a^2*d^2+8*a*b*c*d-5*b^2*c^2)/a^2/c^2/(-a*d+b*c)^
2/x^(1/2)+1/2*d*(a*d+b*c)/a/c/(-a*d+b*c)^2/(d*x^2+c)/x^(1/2)+1/2*b/a/(-a*d+
b*c)/(b*x^2+a)/(d*x^2+c)/x^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {477, 483, 593, 597, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{b^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(5bc-13ad)}{4\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(5bc-13ad)}{4\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{9/4}(5bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{5a^2d^2-8abcd+5b^2c^2}{2a^2c^2\sqrt{x}(bc-ad)^2}+\frac{d^{9/4}(13bc-5ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(bc-ad)^3}-\frac{d^{9/4}(13bc-5ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{9/4}(bc-ad)^3}-\frac{d^{9/4}(13bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{d^{9/4}(13bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^3}+\frac{b}{2a\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2ac\sqrt{x}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(5*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2)/(a^2*c^2*(b*c - a*d)^2*Sqrt[x]) +
(d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + b/(2*a*(b*c - a
*d)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)) + (b^(9/4)*(5*b*c - 13*a*d)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (b
^(9/4)*(5*b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*S
qrt[2]*a^(9/4)*(b*c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(9/4)*
(13*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*
c^(9/4)*(b*c - a*d)^3) - (b^(9/4)*(5*b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) + (b^
(9/4)*(5*b*c - 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
```

```
[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(13*b*c - 5*a*d)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(
b*c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)\sqrt{x}\,(a+bx^2)(c+dx^2)} - \frac{\text{Subst}\left(\int \frac{-5bc+4ad-9bdx^4}{x^2(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\,(c+dx^2)} + \frac{b}{2a(bc-ad)\sqrt{x}\,(a+bx^2)(c+dx^2)} - \text{Su}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\,(c+dx^2)} + \frac{}{2a(bc-ad)\sqrt{}}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\,(c+dx^2)} + \frac{}{2a(bc-ad)\sqrt{}}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\,(c+dx^2)} + \frac{}{2a(bc-ad)\sqrt{}}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\,(c+dx^2)} + \frac{}{2a(bc-ad)\sqrt{}}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\,(c+dx^2)} + \frac{}{2a(bc-ad)\sqrt{}}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\,(c+dx^2)} + \frac{}{2a(bc-ad)\sqrt{}}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\,(c+dx^2)} + \frac{}{2a(bc-ad)\sqrt{}}$$

<u>Mathematica</u> [A]
time = 1.20, size = 421, normalized size = 0.62

$$\frac{1}{8}\left(-\frac{4\left(5b^3c^2x^2(c+dx^2)+a^3d^2(4c+5dx^2)+4ab^2c(c^2-cdx^2-2d^2x^4)+a^2bd(-8c^2-4cdx^2+5d^2x^4)\right)}{a^2c^2(bc-ad)^2\sqrt{x}\,(a+bx^2)(c+dx^2)}+\frac{\sqrt{2}\,b^{9/4}(-5bc+13ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{9/4}(-bc+ad)^3}+\frac{\sqrt{2}\,d^{9/4}(13bc-5ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{9/4}(bc-ad)^3}+\frac{\sqrt{2}\,b^{9/4}(-5bc+13ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{9/4}(-bc+ad)^3}+\frac{\sqrt{2}\,d^{9/4}(13bc-5ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{9/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-4*(5*b^3*c^2*x^2*(c + d*x^2) + a^3*d^2*(4*c + 5*d*x^2) + 4*a*b^2*c*(c^2
- c*d*x^2 - 2*d^2*x^4) + a^2*b*d*(-8*c^2 - 4*c*d*x^2 + 5*d^2*x^4)))/(a^2*c^
2*(b*c - a*d)^2*Sqrt[x]*(a + b*x^2)*(c + d*x^2)) + (Sqrt[2]*b^(9/4)*(-5*b*c
 + 13*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])
/(a^(9/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(9/4)*(13*b*c - 5*a*d)*ArcTan[(Sqr
t[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(9/4)*(b*c - a*d)^
3) + (Sqrt[2]*b^(9/4)*(-5*b*c + 13*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(9/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(9/4
)*(13*b*c - 5*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqr
t[d]*x)])/(c^(9/4)*(b*c - a*d)^3))/8
```

Maple [A]
time = 0.19, size = 323, normalized size = 0.48 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3/a^2/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(13/4*a*d-5
/4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2*d^3/c^2/(a*d-b*c)^
3*((1/4*a*d-1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(5/4*a*d-13/4*b*c)/d/(c/d)^(1/4)
*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/a^2/c^2/x^(1/2)
```

Maxima [A]
time = 0.56, size = 694, normalized size = 1.03

$$-\frac{(5b^4c-13ab^3d)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\left(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2-a^5d^3\right)}-\frac{(13bcd^3-5ad^4)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3\right)}-\frac{4ab^2c^3-8a^2bc^2d+4a^3cd^2+\left(5b^3c^2d-8ab^2cd^2+5a^2bd^3\right)x^4+\left(5b^3c^3-4ab^2c^2d-4a^2bcd^2+5a^3d^3\right)x^2}{2\left(\left(a^2b^3c^4d-2a^3b^2c^3d^2+a^4bc^2d^3\right)x^{\frac{9}{2}}+\left(a^2b^3c^5-a^3b^2c^4d-a^4bc^3d^2+a^5c^2d^3\right)x^{\frac{5}{2}}+\left(a^3b^2c^5-2a^4bc^4d+a^5c^3d^2\right)\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(5*b^4*c - 13*a*b^3*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))
*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(
2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3
/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(1/4)*b^(3/4)))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3
) - 1/16*(13*b*c*d^3 - 5*a*d^4)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1
/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(
d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*s
qrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sq
rt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d
```

```
^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c
))/(c^(1/4)*d^(3/4)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*
d^3) - 1/2*(4*a*b^2*c^3 - 8*a^2*b*c^2*d + 4*a^3*c*d^2 + (5*b^3*c^2*d - 8*a*
b^2*c*d^2 + 5*a^2*b*d^3)*x^4 + (5*b^3*c^3 - 4*a*b^2*c^2*d - 4*a^2*b*c*d^2 +
 5*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^(9/
2) + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^(5/2) +
(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6207 vs. $2(520) = 1040$.
time = 270.75, size = 6207, normalized size = 9.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^5 + (a^2*b^3*
c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3 + (a^3*b^2*c^5 - 2*a
^4*b*c^4*d + a^5*c^3*d^2)*x)*(-(625*b^13*c^4 - 6500*a*b^12*c^3*d + 25350*a^
2*b^11*c^2*d^2 - 43940*a^3*b^10*c*d^3 + 28561*a^4*b^9*d^4)/(a^9*b^12*c^12 -
 12*a^10*b^11*c^11*d + 66*a^11*b^10*c^10*d^2 - 220*a^12*b^9*c^9*d^3 + 495*a
^13*b^8*c^8*d^4 - 792*a^14*b^7*c^7*d^5 + 924*a^15*b^6*c^6*d^6 - 792*a^16*b^
5*c^5*d^7 + 495*a^17*b^4*c^4*d^8 - 220*a^18*b^3*c^3*d^9 + 66*a^19*b^2*c^2*d
^10 - 12*a^20*b*c*d^11 + a^21*d^12))^(1/4)*arctan(-((a^2*b^3*c^3 - 3*a^3*b^
2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt((15625*b^20*c^6 - 243750*a*b^19*c^5
*d + 1584375*a^2*b^18*c^4*d^2 - 5492500*a^3*b^17*c^3*d^3 + 10710375*a^4*b^1
6*c^2*d^4 - 11138790*a^5*b^15*c*d^5 + 4826809*a^6*b^14*d^6)*x - (625*a^5*b^
19*c^10 - 10250*a^6*b^18*c^9*d + 73725*a^7*b^17*c^8*d^2 - 306040*a^8*b^16*c
^7*d^3 + 811826*a^9*b^15*c^6*d^4 - 1438716*a^10*b^14*c^5*d^5 + 1727090*a^11
*b^13*c^4*d^6 - 1388920*a^12*b^12*c^3*d^7 + 717405*a^13*b^11*c^2*d^8 - 2153
06*a^14*b^10*c*d^9 + 28561*a^15*b^9*d^10)*sqrt(-(625*b^13*c^4 - 6500*a*b^12
*c^3*d + 25350*a^2*b^11*c^2*d^2 - 43940*a^3*b^10*c*d^3 + 28561*a^4*b^9*d^4)
/(a^9*b^12*c^12 - 12*a^10*b^11*c^11*d + 66*a^11*b^10*c^10*d^2 - 220*a^12*b^
9*c^9*d^3 + 495*a^13*b^8*c^8*d^4 - 792*a^14*b^7*c^7*d^5 + 924*a^15*b^6*c^6*
d^6 - 792*a^16*b^5*c^5*d^7 + 495*a^17*b^4*c^4*d^8 - 220*a^18*b^3*c^3*d^9 +
66*a^19*b^2*c^2*d^10 - 12*a^20*b*c*d^11 + a^21*d^12)))*(-(625*b^13*c^4 - 65
00*a*b^12*c^3*d + 25350*a^2*b^11*c^2*d^2 - 43940*a^3*b^10*c*d^3 + 28561*a^4
*b^9*d^4)/(a^9*b^12*c^12 - 12*a^10*b^11*c^11*d + 66*a^11*b^10*c^10*d^2 - 22
0*a^12*b^9*c^9*d^3 + 495*a^13*b^8*c^8*d^4 - 792*a^14*b^7*c^7*d^5 + 924*a^15
*b^6*c^6*d^6 - 792*a^16*b^5*c^5*d^7 + 495*a^17*b^4*c^4*d^8 - 220*a^18*b^3*c
^3*d^9 + 66*a^19*b^2*c^2*d^10 - 12*a^20*b*c*d^11 + a^21*d^12))^(1/4) + (125
*a^2*b^13*c^6 - 1350*a^3*b^12*c^5*d + 5835*a^4*b^11*c^4*d^2 - 12852*a^5*b^1
0*c^3*d^3 + 15171*a^6*b^9*c^2*d^4 - 9126*a^7*b^8*c*d^5 + 2197*a^8*b^7*d^6)*
sqrt(x)*(-(625*b^13*c^4 - 6500*a*b^12*c^3*d + 25350*a^2*b^11*c^2*d^2 - 4394
```

```
0*a^3*b^10*c*d^3 + 28561*a^4*b^9*d^4)/(a^9*b^12*c^12 - 12*a^10*b^11*c^11*d
+ 66*a^11*b^10*c^10*d^2 - 220*a^12*b^9*c^9*d^3 + 495*a^13*b^8*c^8*d^4 - 792
*a^14*b^7*c^7*d^5 + 924*a^15*b^6*c^6*d^6 - 792*a^16*b^5*c^5*d^7 + 495*a^17*
b^4*c^4*d^8 - 220*a^18*b^3*c^3*d^9 + 66*a^19*b^2*c^2*d^10 - 12*a^20*b*c*d^1
1 + a^21*d^12))^(1/4))/(625*b^13*c^4 - 6500*a*b^12*c^3*d + 25350*a^2*b^11*c
^2*d^2 - 43940*a^3*b^10*c*d^3 + 28561*a^4*b^9*d^4)) + 4*((a^2*b^3*c^4*d - 2
*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^5 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*
b*c^3*d^2 + a^5*c^2*d^3)*x^3 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*
x)*(-(28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6
500*a^3*b*c*d^12 + 625*a^4*d^13)/(b^12*c^21 - 12*a*b^11*c^20*d + 66*a^2*b^1
0*c^19*d^2 - 220*a^3*b^9*c^18*d^3 + 495*a^4*b^8*c^17*d^4 - 792*a^5*b^7*c^16
*d^5 + 924*a^6*b^6*c^15*d^6 - 792*a^7*b^5*c^14*d^7 + 495*a^8*b^4*c^13*d^8 -
 220*a^9*b^3*c^12*d^9 + 66*a^10*b^2*c^11*d^10 - 12*a^11*b*c^10*d^11 + a^12*
c^9*d^12))^(1/4)*arctan(-((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*
c^2*d^3)*sqrt((4826809*b^6*c^6*d^14 - 11138790*a*b^5*c^5*d^15 + 10710375*a^
2*b^4*c^4*d^16 - 5492500*a^3*b^3*c^3*d^17 + 1584375*a^4*b^2*c^2*d^18 - 2437
50*a^5*b*c*d^19 + 15625*a^6*d^20)*x - (28561*b^10*c^15*d^9 - 215306*a*b^9*c
^14*d^10 + 717405*a^2*b^8*c^13*d^11 - 1388920*a^3*b^7*c^12*d^12 + 1727090*a
^4*b^6*c^11*d^13 - 1438716*a^5*b^5*c^10*d^14 + 811826*a^6*b^4*c^9*d^15 - 30
6040*a^7*b^3*c^8*d^16 + 73725*a^8*b^2*c^7*d^17 - 10250*a^9*b*c^6*d^18 + 625
*a^10*c^5*d^19)*sqrt(-(28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2
*b^2*c^2*d^11 - 6500*a^3*b*c*d^12 + 625*a^4*d^13)/(b^12*c^21 - 12*a*b^11*c^
20*d + 66*a^2*b^10*c^19*d^2 - 220*a^3*b^9*c^18*d^3 + 495*a^4*b^8*c^17*d^4 -
 792*a^5*b^7*c^16*d^5 + 924*a^6*b^6*c^15*d^6 - 792*a^7*b^5*c^14*d^7 + 495*a
^8*b^4*c^13*d^8 - 220*a^9*b^3*c^12*d^9 + 66*a^10*b^2*c^11*d^10 - 12*a^11*b*
c^10*d^11 + a^12*c^9*d^12)))*(-(28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 +
25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12 + 625*a^4*d^13)/(b^12*c^21 - 12*
a*b^11*c^20*d + 66*a^2*b^10*c^19*d^2 - 220*a^3*b^9*c^18*d^3 + 495*a^4*b^8*c
^17*d^4 - 792*a^5*b^7*c^16*d^5 + 924*a^6*b^6*c^15*d^6 - 792*a^7*b^5*c^14*d^
7 + 495*a^8*b^4*c^13*d^8 - 220*a^9*b^3*c^12*d^9 + 66*a^10*b^2*c^11*d^10 - 1
2*a^11*b*c^10*d^11 + a^12*c^9*d^12))^(1/4) + (2197*b^6*c^8*d^7 - 9126*a*b^5
*c^7*d^8 + 15171*a^2*b^4*c^6*d^9 - 12852*a^3*b^3*c^5*d^10 + 5835*a^4*b^2*c^
4*d^11 - 1350*a^5*b*c^3*d^12 + 125*a^6*c^2*d^13)*sqrt(x)*(-(28561*b^4*c^4*d
^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12 + 62
5*a^4*d^13)/(b^12*c^21 - 12*a*b^11*c^20*d + 66*a^2*b^10*c^19*d^2 - 220*a^3*
b^9*c^18*d^3 + 495*a^4*b^8*c^17*d^4 - 792*a^5*b^7*c^16*d^5 + 924*a^6*b^6*c^
15*d^6 - 792*a^7*b^5*c^14*d^7 + 495*a^8*b^4*c^13*d^8 - 220*a^9*b^3*c^12*d^9
 + 66*a^10*b^2*c^11*d^10 - 12*a^11*b*c^10*d^11 + a^12*c^9*d^12))^(1/4))/(28
561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.30, size = 1035, normalized size = 1.53

$$-\frac{\left(5\,(ab^3)^{\frac{3}{4}}\,bc-13\,(ab^3)^{\frac{3}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,a^3b^3c^3-3\sqrt{2}\,a^4b^2c^2d+3\sqrt{2}\,a^5bcd^2-\sqrt{2}\,a^6d^3\right)}-\frac{\left(5\,(ab^3)^{\frac{3}{4}}\,bc-13\,(ab^3)^{\frac{3}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,a^3b^3c^3-3\sqrt{2}\,a^4b^2c^2d+3\sqrt{2}\,a^5bcd^2-\sqrt{2}\,a^6d^3\right)}-\frac{\left(13\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}-\frac{\left(13\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}+\frac{\left(5\,(ab^3)^{\frac{3}{4}}\,bc-13\,(ab^3)^{\frac{3}{4}}\,ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}\,a^3b^3c^3-3\sqrt{2}\,a^4b^2c^2d+3\sqrt{2}\,a^5bcd^2-\sqrt{2}\,a^6d^3\right)}-\frac{\left(5\,(ab^3)^{\frac{3}{4}}\,bc-13\,(ab^3)^{\frac{3}{4}}\,ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}\,a^3b^3c^3-3\sqrt{2}\,a^4b^2c^2d+3\sqrt{2}\,a^5bcd^2-\sqrt{2}\,a^6d^3\right)}+\frac{\left(13\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}-\frac{\left(13\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}-\frac{5\,b^3c^2dx^4-8\,ab^2cd^2x^4+5\,a^2bd^3x^4+5\,b^3c^3x^2-4\,ab^2c^2dx^2-4\,a^2bcd^2x^2+5\,a^3d^3x^2+4\,ab^2c^3-8\,a^2bc^2d+4\,a^3cd^2}{2\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(bdx^{\frac{9}{2}}+bcx^{\frac{5}{2}}+adx^{\frac{5}{2}}+ac\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(
2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a
^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(5*(a*b^3)^(3
/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*
sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(13*(c*d^3)^(3/4)*b*c - 5*(c*d
^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(
1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 -
sqrt(2)*a^3*c^3*d^3) - 1/4*(13*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^
3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d
^3) + 1/8*(5*(a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d
 + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/8*(5*(a*b^3)^(3/4)*b*c - 13
*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt
(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)
*a^6*d^3) + 1/8*(13*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d
 + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/8*(13*(c*d^3)^(3/4)*b
*c - 5*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(
2)*a^3*c^3*d^3) - 1/2*(5*b^3*c^2*d*x^4 - 8*a*b^2*c*d^2*x^4 + 5*a^2*b*d^3*x^
4 + 5*b^3*c^3*x^2 - 4*a*b^2*c^2*d*x^2 - 4*a^2*b*c*d^2*x^2 + 5*a^3*d^3*x^2 +
 4*a*b^2*c^3 - 8*a^2*b*c^2*d + 4*a^3*c*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d +
 a^4*c^2*d^2)*(b*d*x^(9/2) + b*c*x^(5/2) + a*d*x^(5/2) + a*c*sqrt(x)))
```

Mupad [B]
time = 5.96, size = 2500, normalized size = 3.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

[Out] atan((((-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a
^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 -
 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^
3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c
^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*
b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(3/4)*(x^(
1/2)*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2
*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 4
9152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3
+ 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^1
5*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^
3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4)*(52428
800*a^23*b^38*c^57*d^4 - 1635778560*a^24*b^37*c^56*d^5 + 24482152448*a^25*b
^36*c^55*d^6 - 234134437888*a^26*b^35*c^54*d^7 + 1607834009600*a^27*b^34*c^
53*d^8 - 8446069964800*a^28*b^33*c^52*d^9 + 35303182041088*a^29*b^32*c^51*d
^10 - 120578363097088*a^30*b^31*c^50*d^11 + 342964201062400*a^31*b^30*c^49*
d^12 - 823887134720000*a^32*b^29*c^48*d^13 + 1690057100492800*a^33*b^28*c^4
7*d^14 - 2988135038320640*a^34*b^27*c^46*d^15 + 4595616128696320*a^35*b^26*
c^45*d^16 - 6215915829985280*a^36*b^25*c^44*d^17 + 7509830061260800*a^37*b^
24*c^43*d^18 - 8292025971507200*a^38*b^23*c^42*d^19 + 8624070071418880*a^39
*b^22*c^41*d^20 - 8700497871503360*a^40*b^21*c^40*d^21 + 8624070071418880*a
^41*b^20*c^39*d^22 - 8292025971507200*a^42*b^19*c^38*d^23 + 750983006126080
0*a^43*b^18*c^37*d^24 - 6215915829985280*a^44*b^17*c^36*d^25 + 459561612869
6320*a^45*b^16*c^35*d^26 - 2988135038320640*a^46*b^15*c^34*d^27 + 169005710
0492800*a^47*b^14*c^33*d^28 - 823887134720000*a^48*b^13*c^32*d^29 + 3429642
01062400*a^49*b^12*c^31*d^30 - 120578363097088*a^50*b^11*c^30*d^31 + 353031
82041088*a^51*b^10*c^29*d^32 - 8446069964800*a^52*b^9*c^28*d^33 + 160783400
9600*a^53*b^8*c^27*d^34 - 234134437888*a^54*b^7*c^26*d^35 + 24482152448*a^5
5*b^6*c^25*d^36 - 1635778560*a^56*b^5*c^24*d^37 + 52428800*a^57*b^4*c^23*d^
38) - 32768000*a^21*b^38*c^55*d^4 + 1009254400*a^22*b^37*c^54*d^5 - 1483341
8240*a^23*b^36*c^53*d^6 + 138556735488*a^24*b^35*c^52*d^7 - 924185001984*a^
25*b^34*c^51*d^8 + 4688465362944*a^26*b^33*c^50*d^9 - 18812623126528*a^27*b
^32*c^49*d^10 + 61295191654400*a^28*b^31*c^48*d^11 - 165189260410880*a^29*b
^30*c^47*d^12 + 373165003898880*a^30*b^29*c^46*d^13 - 713540118773760*a^31*
b^28*c^45*d^14 + 1163349301657600*a^32*b^27*c^44*d^15 - 1627141704253440*a^
33*b^26*c^43*d^16 + 1966197351383040*a^34*b^25*c^42*d^17 - 2079216623943680
*a^35*b^24*c^41*d^18 + 1981073955225600*a^36*b^23*c^40*d^19 - 1807512431493
120*a^37*b^22*c^39*d^20 + 1724885956034560*a^38*b^21*c^38*d^21 - 1807512431
493120*a^39*b^20*c^37*d^22 + 1981073955225600*a^40*b^19*c^36*d^23 - 2079216
623943680*a^41*b^18*c^35*d^24 + 1966197351383040*a^42*b^17*c^34*d^25 - 1627
141704253440*a^43*b^16*c^33*d^26 + 1163349301657600*a^44*b^15*c^32*d^27 - 7
13540118773760*a^45*b^14*c^31*d^28 + 373165003898880*a^46*b^13*c^30*d^29 -
165189260410880*a^47*b^12*c^29*d^30 + 61295191654400*a^48*b^11*c^28*d^31 -
18812623126528*a^49*b^10*c^27*d^32 + 4688465362944*a^50*b^9*c^26*d^33 - 924
185001984*a^51*b^8*c^25*d^34 + 138556735488*a^52*b^7*c^24*d^35 - 1483341824

0*a^53*b^6*c^23*d^36 + 1009254400*a^54*b^5*c^22*d^37 - 32768000*a^55*b^4*c^21*d^38) + x^(1/2)*(54080000*a^20*b^33*c^43*d^10 - 1361152000*a^21*b^32*c^42*d^11 + 16011852800*a^22*b^31*c^41*d^12 - 116736734720*a^23*b^30*c^40*d^13 + 589861462528*a^24*b^29*c^39*d^14 - 2187899577344*a^25*b^28*c^38*d^15 + 6149347117056*a^26*b^27*c^37*d^16 - 13298820601344*a^27*b^26*c^36*d^17 + 22133436343296*a^28*b^25*c^35*d^18 - 27715689750528*a^29*b^24*c^34*d^19 + 24077503776768*a^30*b^23*c^33*d^20 - 9645706816512*a^31*b^22*c^32*d^21 - 9645706816512*a^32*b^21*c^31*d^22 + 24077503776768*a^33*b^20*c^30*d^23 - 27715689750528*a^34*b^19*c^29*d^24 + 22133436343296*a^35*b^18*c^28*d^25 - 13298820601344*a^36*b^17*c^27*d^26 + 6149347117056*a^37*b^16*c^26*d^27 - 2187899577344*a^38*b^15*c^25*d^28 + 589861462528*a^39*b^14*c^24*d^29 - 116736734720*a^40*b^13*c^23*d^30 + 16011852800*a^41*b^12*c^22*d^31 - 1361152000*a^42*b^11*c^21*d^32 + 54080000*a^43*b^10*c^20*d^33))*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4)*1i + ((-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*...

3.494 $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=676

$$\frac{-7b^2c^2+8abcd-7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}}+\frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)}+\frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{b^{11/4}(7bc-15a}{4\sqrt{}}$$

```
[Out] 1/6*(-7*a^2*d^2+8*a*b*c*d-7*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/x^(3/2)+1/2*d*(a*
d+b*c)/a/c/(-a*d+b*c)^2/x^(3/2)/(d*x^2+c)+1/2*b/a/(-a*d+b*c)/x^(3/2)/(b*x^2
+a)/(d*x^2+c)+1/8*b^(11/4)*(-15*a*d+7*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)
/a^(1/4))/a^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/8*b^(11/4)*(-15*a*d+7*b*c)*arctan
(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/(-a*d+b*c)^3*2^(1/2)+1/8*d^(11
/4)*(-7*a*d+15*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*
d+b*c)^3*2^(1/2)-1/8*d^(11/4)*(-7*a*d+15*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1
/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(11/4)*(-15*a*d+7*b*c)*ln
(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/(-a*d+b*c)^3*2
^(1/2)-1/16*b^(11/4)*(-15*a*d+7*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(11/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(11/4)*(-7*a*d+15*b*c)
*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^
3*2^(1/2)-1/16*d^(11/4)*(-7*a*d+15*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4
)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {477, 483, 593, 597, 536, 217, 1179, 642, 1176, 631, 210}

$$\frac{b^{11/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(7bc-15ad)}{4\sqrt{2}a^{11/4}(bc-ad)^3}-\frac{b^{11/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(7bc-15ad)}{4\sqrt{2}a^{11/4}(bc-ad)^3}+\frac{b^{11/4}(7bc-15ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}-\frac{b^{11/4}(7bc-15ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}-\frac{7a^2d^2-8abcd+7b^2c^2}{6a^2c^2x^{3/2}(bc-ad)^2}+\frac{d^{11/4}(15bc-7ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{11/4}(15bc-7ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3}+\frac{d^{11/4}(15bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{11/4}(15bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3}+\frac{b}{2ax^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2acx^{3/2}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/6*(7*b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2)/(a^2*c^2*(b*c - a*d)^2*x^(3/2)) +
(d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + b/(2*a*(b*c - a
*d)*x^(3/2)*(a + b*x^2)*(c + d*x^2)) + (b^(11/4)*(7*b*c - 15*a*d)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) -
(b^(11/4)*(7*b*c - 15*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
4*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7*a*d)*ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^
(11/4)*(15*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(11/4)*(b*c - a*d)^3) + (b^(11/4)*(7*b*c - 15*a*d)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d
)^3) - (b^(11/4)*(7*b*c - 15*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
```

```
t[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c -
 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt
[2]*c^(11/4)*(b*c - a*d)^3) - (d^(11/4)*(15*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^3
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-7bc+4ad-11bdx^4}{x^4(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right.}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Sub}}{}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{}{2a(bc-ad)x^{3/}}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{}{2a(bc-ad)x^{3/}}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{}{2a(bc-ad)x^{3/}}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{}{2a(bc-ad)x^{3/}}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{}{2a(bc-ad)x^{3/}}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{}{2a(bc-ad)x^{3/}}$$

Mathematica [A]
time = 1.28, size = 425, normalized size = 0.63

$$\frac{1}{24}\left(-\frac{4\left(7b^3c^2x^2\left(c+dx^2\right)+a^3d^2\left(4c+7dx^2\right)+4ab^2c\left(c^2-cdx^2-2d^2x^4\right)+a^2bd\left(-8c^2-4cdx^2+7d^2x^4\right)\right)}{a^2c^2(bc-ad)^2x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{3\sqrt{2}\,b^{11/4}(-7bc+15ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{11/4}(-bc+ad)^3}+\frac{3\sqrt{2}\,d^{11/4}(15bc-7ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{11/4}(bc-ad)^3}+\frac{3\sqrt{2}\,b^{11/4}(-7bc+15ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{11/4}(bc-ad)^3}+\frac{3\sqrt{2}\,d^{11/4}(-15bc+7ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{11/4}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-4*(7*b^3*c^2*x^2*(c + d*x^2) + a^3*d^2*(4*c + 7*d*x^2) + 4*a*b^2*c*(c^2
- c*d*x^2 - 2*d^2*x^4) + a^2*b*d*(-8*c^2 - 4*c*d*x^2 + 7*d^2*x^4)))/(a^2*c^
2*(b*c - a*d)^2*x^(3/2)*(a + b*x^2)*(c + d*x^2)) + (3*Sqrt[2]*b^(11/4)*(-7*
```

```
b*c + 15*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
)])/(a^(11/4)*(-(b*c) + a*d)^3) + (3*Sqrt[2]*d^(11/4)*(15*b*c - 7*a*d)*ArcT
an[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(11/4)*(b*c
 - a*d)^3) + (3*Sqrt[2]*b^(11/4)*(-7*b*c + 15*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(11/4)*(b*c - a*d)^3) + (3*Sqr
t[2]*d^(11/4)*(-15*b*c + 7*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(
Sqrt[c] + Sqrt[d]*x)])/(c^(11/4)*(b*c - a*d)^3))/24
```

Maple [A]
time = 0.19, size = 323, normalized size = 0.48 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3/a^2/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(15*a*d-7*
b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/
(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2*d^3/c^2/(a*d-b*c)^3*(
(1/4*a*d-1/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(7*a*d-15*b*c)*(c/d)^(1/4)/c*2^(1/
2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2
^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)))-2/3/a^2/c^2/x^(3/2)
```

Maxima [A]
time = 0.53, size = 761, normalized size = 1.13

$$-\frac{\left(\frac{2\sqrt{2}(7bc-15ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(7bc-15ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(7bc-15ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(7bc-15ad)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b^3}{16\left(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2-a^5d^3\right)}-\frac{4ab^2c^3-8a^2bc^2d+4a^3cd^2+(7b^3c^2d-8ab^2cd^2+7a^2bd^3)x^4+(7b^3c^3-4ab^2c^2d-4a^2bcd^2+7a^3d^3)x^2}{6\left((a^2b^3c^4d-2a^3b^2c^3d^2+a^4bc^2d^3)x^{\frac{11}{2}}+(a^2b^3c^5-a^3b^2c^4d-a^4bc^3d^2+a^5c^2d^3)x^{\frac{7}{2}}+(a^3b^2c^5-2a^4bc^4d+a^5c^3d^2)x^{\frac{3}{2}}\right)}-\frac{\frac{2\sqrt{2}(15bcd^3-7ad^4)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(15bcd^3-7ad^4)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(15bcd^3-7ad^4)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(15bcd^3-7ad^4)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(7*b*c - 15*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + 2*sqrt(2)*(7*b*c - 15*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + sqrt(2)*(7*b*c - 15*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(7*b*c - 15*a*d)*log(-sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^3/(a^2*b^
3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 1/6*(4*a*b^2*c^3 - 8*a
^2*b*c^2*d + 4*a^3*c*d^2 + (7*b^3*c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*x^4
+ (7*b^3*c^3 - 4*a*b^2*c^2*d - 4*a^2*b*c*d^2 + 7*a^3*d^3)*x^2)/((a^2*b^3*c^
4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^(11/2) + (a^2*b^3*c^5 - a^3*b^2*
c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^(7/2) + (a^3*b^2*c^5 - 2*a^4*b*c^4*d
 + a^5*c^3*d^2)*x^(3/2)) - 1/16*(2*sqrt(2)*(15*b*c*d^3 - 7*a*d^4)*arctan(1/
2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d
)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(15*b*c*d^3 - 7*a*d^4)*arct
```

```
an(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(15*b*c*d^3 - 7*a*d^4)*
log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)
) - sqrt(2)*(15*b*c*d^3 - 7*a*d^4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + s
qrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c
^3*d^2 - a^3*c^2*d^3)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.52, size = 1012, normalized size = 1.50

$$-\frac{\left(7\,(ab^3)^{\frac{1}{4}}b^3c-15\,(ab^3)^{\frac{1}{4}}ab^2d\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,a^3b^3c^3-3\sqrt{2}\,a^4b^2c^2d+3\sqrt{2}\,a^5bcd^2-\sqrt{2}\,a^6d^3\right)}-\frac{\left(7\,(ab^3)^{\frac{1}{4}}b^3c-15\,(ab^3)^{\frac{1}{4}}ab^2d\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,a^3b^3c^3-3\sqrt{2}\,a^4b^2c^2d+3\sqrt{2}\,a^5bcd^2-\sqrt{2}\,a^6d^3\right)}-\frac{\left(15\,(cd^3)^{\frac{1}{4}}bcd^2-7\,(cd^3)^{\frac{1}{4}}ad^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}-\frac{\left(15\,(cd^3)^{\frac{1}{4}}bcd^2-7\,(cd^3)^{\frac{1}{4}}ad^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}-\frac{\left(7\,(ab^3)^{\frac{1}{4}}b^3c-15\,(ab^3)^{\frac{1}{4}}ab^2d\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}\,a^3b^3c^3-3\sqrt{2}\,a^4b^2c^2d+3\sqrt{2}\,a^5bcd^2-\sqrt{2}\,a^6d^3\right)}+\frac{\left(7\,(ab^3)^{\frac{1}{4}}b^3c-15\,(ab^3)^{\frac{1}{4}}ab^2d\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}\,a^3b^3c^3-3\sqrt{2}\,a^4b^2c^2d+3\sqrt{2}\,a^5bcd^2-\sqrt{2}\,a^6d^3\right)}-\frac{\left(15\,(cd^3)^{\frac{1}{4}}bcd^2-7\,(cd^3)^{\frac{1}{4}}ad^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}+\frac{\left(15\,(cd^3)^{\frac{1}{4}}bcd^2-7\,(cd^3)^{\frac{1}{4}}ad^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^3c^6-3\sqrt{2}\,ab^2c^5d+3\sqrt{2}\,a^2bc^4d^2-\sqrt{2}\,a^3c^3d^3\right)}-\frac{b^3c^2dx^{\frac{5}{2}}+a^2bd^3x^{\frac{5}{2}}+b^3c^3\sqrt{x}+a^3d^3\sqrt{x}}{2\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2)(bdx^4+bcx^2+adx^2+ac)}-\frac{2}{3\,a^2c^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(7*(a*b^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)*a*b^2*d)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqr
t(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(7*(a*b
^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)*a*b^2*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^
2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(15*(c*d^3)^(1/4)*
b*c*d^2 - 7*(c*d^3)^(1/4)*a*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) +
2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2
)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/4*(15*(c*d^3)^(1/4)*b*c*d^2 - 7*
(c*d^3)^(1/4)*a*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4
```

```
*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/8*(7*(a*b^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)
*a*b^2*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3
*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) +
 1/8*(7*(a*b^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)*a*b^2*d)*log(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2
*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/8*(15*(c*d^3)^(1/4)*b*c*d
^2 - 7*(c*d^3)^(1/4)*a*d^3)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt
(2)*a^3*c^3*d^3) + 1/8*(15*(c*d^3)^(1/4)*b*c*d^2 - 7*(c*d^3)^(1/4)*a*d^3)*l
og(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(
2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/2*(b^3*
c^2*d*x^(5/2) + a^2*b*d^3*x^(5/2) + b^3*c^3*sqrt(x) + a^3*d^3*sqrt(x))/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c
)) - 2/3/(a^2*c^2*x^(3/2))
```

Mupad [B]
time = 5.61, size = 2500, normalized size = 3.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] atan((((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150
*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^
12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c
^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17
*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120
*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*
(((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*
b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 -
49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^
3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*
c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20
*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(1174
40512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^58*d^5 + 54978936832*a^27
*b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 + 3709140467712*a^29*b^34*
c^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^5
3*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c^
51*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 5006182506823680*a^35*b^28
*c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15 + 16064830746132480*a^37*
b^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*d^17 + 32403938271559680*
a^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*c^44*d^19 + 4461143707877
3760*a^41*b^22*c^43*d^20 - 46346397171056640*a^42*b^21*c^42*d^21 + 44611437
078773760*a^43*b^20*c^41*d^22 - 39685869262602240*a^44*b^19*c^40*d^23 + 324
```

03938271559680*a^45*b^18*c^39*d^24 - 24054442827448320*a^46*b^17*c^38*d^25 + 16064830746132480*a^47*b^16*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*d^27 + 5006182506823680*a^49*b^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^34*d^29 + 903261116694528*a^51*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c^32*d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^30*d^33 + 3709140467712*a^55*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^27*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38) + x^(1/2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^36*c^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^52*d^9 + 59682471280640*a^28*b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50*d^11 + 558859896750080*a^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^48*d^13 + 2657695282757632*a^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28*c^46*d^15 + 6863546220544000*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b^26*c^44*d^17 + 9711406085570560*a^36*b^25*c^43*d^18 - 8904303328624640*a^37*b^24*c^42*d^19 + 6275554166702080*a^38*b^23*c^41*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 2263049201254400*a^40*b^21*c^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 - 8904303328624640*a^42*b^19*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^25 - 8828557564313600*a^44*b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*d^27 - 4599356881633280*a^46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^32*d^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*c^30*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*c^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(3/4) + 147517440*a^18*b^37*c^47*d^8 - 3841073152*a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^22*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c...

3.495 $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=731

$$\frac{-9b^2c^2+8abcd-9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}}+\frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}}+\frac{d(bc+ad)}{2ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)}+\frac{}{2a(bc-ad)x^{5/2}}$$

[Out] 1/10*(-9*a^2*d^2+8*a*b*c*d-9*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/x^(5/2)+1/2*d*(a*d+b*c)/a/c/(-a*d+b*c)^2/x^(5/2)/(d*x^2+c)+1/2*b/a/(-a*d+b*c)/x^(5/2)/(b*x^2+a)/(d*x^2+c)-1/8*b^(13/4)*(-17*a*d+9*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/8*b^(13/4)*(-17*a*d+9*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/(-a*d+b*c)^3*2^(1/2)-1/8*d^(13/4)*(-9*a*d+17*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/8*d^(13/4)*(-9*a*d+17*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(13/4)*(-17*a*d+9*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/(-a*d+b*c)^3*2^(1/2)-1/16*b^(13/4)*(-17*a*d+9*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(13/4)*(-9*a*d+17*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)-1/16*d^(13/4)*(-9*a*d+17*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/2*(a*d+b*c)*(9*a^2*d^2-17*a*b*c*d+9*b^2*c^2)/a^3/c^3/(-a*d+b*c)^2/x^(1/2)

Rubi [A]

time = 0.86, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {477, 483, 593, 597, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{13/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(9bc-17ad)}{4\sqrt{2}a^{13/4}(bc-ad)^3}+\frac{b^{13/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(9bc-17ad)}{4\sqrt{2}a^{13/4}(bc-ad)^3}+\frac{b^{13/4}(9bc-17ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3}-\frac{b^{13/4}(9bc-17ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3}-\frac{9a^2d^2-8abcd+9b^2c^2}{10a^2c^2x^{5/2}(bc-ad)^2}+\frac{(ad+bc)\left(9a^2d^2-17abcd+9b^2c^2\right)}{2a^3c^3\sqrt{x}(bc-ad)^2}-\frac{d^{13/4}(17bc-9ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3}+\frac{d^{13/4}(17bc-9ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3}+\frac{d^{13/4}(17bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{d^{13/4}(17bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^3}+\frac{b}{2ax^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2acx^{5/2}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/10*(9*b^2*c^2 - 8*a*b*c*d + 9*a^2*d^2)/(a^2*c^2*(b*c - a*d)^2*x^(5/2)) + ((b*c + a*d)*(9*b^2*c^2 - 17*a*b*c*d + 9*a^2*d^2))/(2*a^3*c^3*(b*c - a*d)^2*Sqrt[x]) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)) - (b^(13/4)*(9*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (b^(13/4)*(9*b*c - 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (d^(13/4)*(17*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (b^(13/4)*(9*b*c - 17*a*d)*Lo
```

```
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/
4)*(b*c - a*d)^3) - (b^(13/4)*(9*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (d^(13
/4)*(17*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*
(b*c - a*d)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
```

```
a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-9bc+4ad-13bdx^4}{x^6(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right.}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Sub}}{}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)} + \frac{}{2a(bc-ad)x^{5/}}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{}{2ac(bc}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{}{2ac(bc}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{}{2ac(bc}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{}{2ac(bc}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{}{2ac(bc}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{}{2ac(bc}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{}{2ac(bc}$$

Mathematica [A]
time = 1.33, size = 460, normalized size = 0.63

$$\frac{\frac{4(bc-ad)\left(45b^4c^3x^4\left(c+dx^2\right)-4ab^3c^2x^2\left(-9c^2+cdx^2+10d^2x^4\right)+a^4d^2\left(-4c^2+36cdx^2+45d^2x^4\right)-4a^2b^2c\left(c^3+9c^2dx^2+18cd^2x^4+10d^3x^6\right)+a^3bd\left(8c^3-36c^2dx^2-4cd^2x^4+45d^3x^6\right)\right)}{a^3c^3x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{5\sqrt{2}\,b^{13/4}(-9bc+17ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{13/4}}+\frac{5\sqrt{2}\,d^{13/4}(-17bc+9ad)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{13/4}}+\frac{5\sqrt{2}\,b^{13/4}(-9bc+17ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{13/4}}+\frac{5\sqrt{2}\,d^{13/4}(-17bc+9ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{13/4}}}{40(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((4*(b*c - a*d)*(45*b^4*c^3*x^4*(c + d*x^2) - 4*a*b^3*c^2*x^2*(-9*c^2 + c*d
*x^2 + 10*d^2*x^4) + a^4*d^2*(-4*c^2 + 36*c*d*x^2 + 45*d^2*x^4) - 4*a^2*b^2
*c*(c^3 + 9*c^2*d*x^2 + 18*c*d^2*x^4 + 10*d^3*x^6) + a^3*b*d*(8*c^3 - 36*c^
2*d*x^2 - 4*c*d^2*x^4 + 45*d^3*x^6)))/(a^3*c^3*x^(5/2)*(a + b*x^2)*(c + d*x
^2)) + (5*Sqrt[2]*b^(13/4)*(-9*b*c + 17*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(13/4) + (5*Sqrt[2]*d^(13/4)*(-17*b*c
+ 9*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c
^(13/4) + (5*Sqrt[2]*b^(13/4)*(-9*b*c + 17*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(13/4) + (5*Sqrt[2]*d^(13/4)*(-17*
b*c + 9*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x
)])/c^(13/4))/(40*(b*c - a*d)^3)
```

Maple [A]
time = 0.20, size = 343, normalized size = 0.47 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4/a^3/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(17/4*a*d-9/
4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
)/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d^4/c^3/(a*d-b*c)^3
*((1/4*a*d-1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(9/4*a*d-17/4*b*c)/d/(c/d)^(1/4)*
2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/5/a^2/c^2/x^(5/2)-2*(-2*a*d-2*b*c)/a^3/c
^3/x^(1/2)
```

Maxima [A]
time = 0.58, size = 774, normalized size = 1.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(9*b^5*c - 17*a*b^4*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*
b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*
sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2
)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/
4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(
a^(1/4)*b^(3/4)))/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)
```

```
 + 1/16*(17*b*c*d^4 - 9*a*d^5)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/
4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d
))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sq
rt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqr
t(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^
(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c)
)/(c^(1/4)*d^(3/4)))/(b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d
^3) - 1/10*(4*a^2*b^2*c^4 - 8*a^3*b*c^3*d + 4*a^4*c^2*d^2 - 5*(9*b^4*c^3*d
- 8*a*b^3*c^2*d^2 - 8*a^2*b^2*c*d^3 + 9*a^3*b*d^4)*x^6 - (45*b^4*c^4 - 4*a*
b^3*c^3*d - 72*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 45*a^4*d^4)*x^4 - 36*(a*b^
3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)/((a^3*b^3*c^5*d - 2
*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^(13/2) + (a^3*b^3*c^6 - a^4*b^2*c^5*d -
 a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^(9/2) + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*
c^4*d^2)*x^(5/2))
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.16, size = 1015, normalized size = 1.39

$$\frac{\left(9\,(ab^3)^{\frac{3}{4}}b^2c-17\,(ab^3)^{\frac{3}{4}}abd\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{a}{b})^{\frac{1}{4}}+2\sqrt{x}\right)}{2\,(\frac{a}{b})^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,a^4b^3c^3-3\sqrt{2}\,a^5b^2c^2d+3\sqrt{2}\,a^6bcd^2-\sqrt{2}\,a^7d^3\right)}+\frac{\left(9\,(ab^3)^{\frac{3}{4}}b^2c-17\,(ab^3)^{\frac{3}{4}}abd\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{a}{b})^{\frac{1}{4}}-2\sqrt{x}\right)}{2\,(\frac{a}{b})^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,a^4b^3c^3-3\sqrt{2}\,a^5b^2c^2d+3\sqrt{2}\,a^6bcd^2-\sqrt{2}\,a^7d^3\right)}+\frac{\left(17\,(cd^3)^{\frac{3}{4}}bcd-9\,(cd^3)^{\frac{3}{4}}ad^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{c}{d})^{\frac{1}{4}}+2\sqrt{x}\right)}{2\,(\frac{c}{d})^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^3c^7-3\sqrt{2}\,ab^2c^6d+3\sqrt{2}\,a^2bc^5d^2-\sqrt{2}\,a^3c^4d^3\right)}+\frac{\left(17\,(cd^3)^{\frac{3}{4}}bcd-9\,(cd^3)^{\frac{3}{4}}ad^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{c}{d})^{\frac{1}{4}}-2\sqrt{x}\right)}{2\,(\frac{c}{d})^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^3c^7-3\sqrt{2}\,ab^2c^6d+3\sqrt{2}\,a^2bc^5d^2-\sqrt{2}\,a^3c^4d^3\right)}-\frac{\left(9\,(ab^3)^{\frac{3}{4}}b^2c-17\,(ab^3)^{\frac{3}{4}}abd\right)\log\left(\sqrt{2}\sqrt{x}\,(\frac{a}{b})^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}\,a^4b^3c^3-3\sqrt{2}\,a^5b^2c^2d+3\sqrt{2}\,a^6bcd^2-\sqrt{2}\,a^7d^3\right)}+\frac{\left(9\,(ab^3)^{\frac{3}{4}}b^2c-17\,(ab^3)^{\frac{3}{4}}abd\right)\log\left(-\sqrt{2}\sqrt{x}\,(\frac{a}{b})^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}\,a^4b^3c^3-3\sqrt{2}\,a^5b^2c^2d+3\sqrt{2}\,a^6bcd^2-\sqrt{2}\,a^7d^3\right)}-\frac{\left(17\,(cd^3)^{\frac{3}{4}}bcd-9\,(cd^3)^{\frac{3}{4}}ad^2\right)\log\left(\sqrt{2}\sqrt{x}\,(\frac{c}{d})^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^3c^7-3\sqrt{2}\,ab^2c^6d+3\sqrt{2}\,a^2bc^5d^2-\sqrt{2}\,a^3c^4d^3\right)}+\frac{\left(17\,(cd^3)^{\frac{3}{4}}bcd-9\,(cd^3)^{\frac{3}{4}}ad^2\right)\log\left(-\sqrt{2}\sqrt{x}\,(\frac{c}{d})^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^3c^7-3\sqrt{2}\,ab^2c^6d+3\sqrt{2}\,a^2bc^5d^2-\sqrt{2}\,a^3c^4d^3\right)}+\frac{b^4c^3dx^{\frac{7}{2}}+a^3bd^4x^{\frac{7}{2}}+b^4c^4x^{\frac{3}{2}}+a^4d^4x^{\frac{3}{2}}}{2\,(a^3b^2c^5-2\,a^4bc^4d+a^5c^3d^2)(bdx^4+bcx^2+adx^2+ac)}+\frac{2\,(10\,bcx^2+10\,adx^2-ac)}{5\,a^3c^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(9*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4*b^3*c^3 - 3*sqrt(2
)*a^5*b^2*c^2*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) + 1/4*(9*(a*b^3)
```

```
^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4*b^3*c^3 - 3*sqrt(2)*a^5*b^2*c^2
*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) + 1/4*(17*(c*d^3)^(3/4)*b*c*d
 - 9*(c*d^3)^(3/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(
x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b
*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/4*(17*(c*d^3)^(3/4)*b*c*d - 9*(c*d^3)^(
3/4)*a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/
4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sq
rt(2)*a^3*c^4*d^3) - 1/8*(9*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^3*c^3 - 3*sq
rt(2)*a^5*b^2*c^2*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) + 1/8*(9*(a*
b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*a^4*b^3*c^3 - 3*sqrt(2)*a^5*b^2*c^2*d + 3*sqrt(2
)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) - 1/8*(17*(c*d^3)^(3/4)*b*c*d - 9*(c*d^3)^
(3/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*
c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3
) + 1/8*(17*(c*d^3)^(3/4)*b*c*d - 9*(c*d^3)^(3/4)*a*d^2)*log(-sqrt(2)*sqrt(
x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d +
3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/2*(b^4*c^3*d*x^(7/2) + a
^3*b*d^4*x^(7/2) + b^4*c^4*x^(3/2) + a^4*d^4*x^(3/2))/((a^3*b^2*c^5 - 2*a^4
*b*c^4*d + a^5*c^3*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)) + 2/5*(10*b*c*
x^2 + 10*a*d*x^2 - a*c)/(a^3*c^3*x^(5/2))
```

Mupad [B]
time = 6.62, size = 2500, normalized size = 3.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] 2*atan((2654208*a^16*b^22*c^27*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^
4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(
4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15
*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 32440
32*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 +
 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d
^10 - 49152*a^24*b*c*d^11))^(5/4) + 15169032*a^22*b^8*d^19*x^(1/2)*(-(6561*
b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2
*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^
14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027
520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6
- 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3
*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) + 2654208*a^3
8*c^5*d^22*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*
```

c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) - 130671792*a^21*b^9*c*d^18*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) - 41877504*a^17*b^21*c^26*d*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) - 41877504*a^37*b*c^6*d^21*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 15169032*a^11*b^19*c^11*d^8*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) - 130671792*a^12*b^18*c^10*d^9*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) + 450333432*a^13*b^17*c^9*d^10*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) - 784872864*a^14*b^16*c^8*d^11*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^1

4*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) + 717087608*a^15*b^15*c^7*d^12*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12...

3.496 $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=718

$$\frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2}+\frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}+\frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\left(c+dx^2\right)}+\frac{\sqrt[4]{a}\,b^{3/4}(5bc+7ad)\tan}{4\sqrt{2}\,(bc}$$

```
[Out] 1/8*a^(1/4)*b^(3/4)*(7*a*d+5*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))
/(-a*d+b*c)^4*2^(1/2)-1/8*a^(1/4)*b^(3/4)*(7*a*d+5*b*c)*arctan(1+b^(1/4)*2^
(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^4*2^(1/2)-1/64*(5*a^2*d^2+70*a*b*c*d+21*b
^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(1/4)/(-a*d+b*c
)^4*2^(1/2)+1/64*(5*a^2*d^2+70*a*b*c*d+21*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)
*x^(1/2)/c^(1/4))/c^(3/4)/d^(1/4)/(-a*d+b*c)^4*2^(1/2)+1/16*a^(1/4)*b^(3/4)
*(7*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+
b*c)^4*2^(1/2)-1/16*a^(1/4)*b^(3/4)*(7*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1
/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^4*2^(1/2)-1/128*(5*a^2*d^2+70*a*b*c
*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4
)/d^(1/4)/(-a*d+b*c)^4*2^(1/2)+1/128*(5*a^2*d^2+70*a*b*c*d+21*b^2*c^2)*ln(c
^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(1/4)/(-a*d+b*c
)^4*2^(1/2)+1/4*(2*a*d+b*c)*x^(1/2)/b/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*a*x^(1/2
)/b/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^2+1/16*(17*a*d+7*b*c)*x^(1/2)/(-a*d+b*c)
^3/(d*x^2+c)
```

Rubi [A]
time = 0.68, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 481, 541, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}+\frac{\sqrt[4]{a}\,b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(7ad+5bc)}{4\sqrt{2}\,(bc-ad)^4}-\frac{\sqrt[4]{a}\,b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(7ad+5bc)}{4\sqrt{2}\,(bc-ad)^4}+\frac{\sqrt[4]{a}\,b^{3/4}(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,(bc-ad)^4}-\frac{\sqrt[4]{a}\,b^{3/4}(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,(bc-ad)^4}+\frac{a\sqrt{x}}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{\sqrt{x}\,(17ad+7bc)}{16\left(c+dx^2\right)(bc-ad)^3}+\frac{\sqrt{x}\,(2ad+bc)}{4b\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((b*c + 2*a*d)*Sqrt[x])/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*Sqrt[x])/(2*
b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((7*b*c + 17*a*d)*Sqrt[x])/(16*(
b*c - a*d)^3*(c + d*x^2)) + (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/
4)*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*(b*c - a*d)^4) - ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) +
((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) + (a^(1/4)*b^(3/4)*(5
*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
```

```
8*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*(b*c - a*d)^4) - ((
21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) + ((21*b^2
*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2)^2 (c+dx^2)^3}\,dx = 2\text{Subst}\left(\int \frac{x^8}{(a+bx^4)^2 (c+dx^4)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\text{Subst}\left(\int \frac{ac+(-4bc-7ad)x^4}{(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2(c+dx^2)^2} + \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\text{Subst}\left(\int \frac{12abc^2-2\cdots}{(a+bx\cdots}\right.}{16bc\cdots}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2(c+dx^2)^2} + \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{(7bc+17ad)\cdots}{16(bc-ad)^3(c+\cdots}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2(c+dx^2)^2} + \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{(7bc+17ad)\cdots}{16(bc-ad)^3(c+\cdots}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2(c+dx^2)^2} + \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{(7bc+17ad)\cdots}{16(bc-ad)^3(c+\cdots}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2(c+dx^2)^2} + \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{(7bc+17ad)\cdots}{16(bc-ad)^3(c+\cdots}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2(c+dx^2)^2} + \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{(7bc+17ad)\cdots}{16(bc-ad)^3(c+\cdots}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2(c+dx^2)^2} + \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{(7bc+17ad)\cdots}{16(bc-ad)^3(c+\cdots}$$

Mathematica [A]
time = 1.78, size = 383, normalized size = 0.53

$$\frac{\frac{4(bc-ad)\sqrt{x}\left(b^2cx^2(11c+7dx^2)+a^2d(5c+9dx^2)+ab(19c^2+28cdx^2+17d^2x^4)\right)}{(a+bx^2)(c+dx^2)^2} + 8\sqrt{2}\sqrt[4]{a}\,b^{3/4}(5bc+7ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - \frac{\sqrt{2}\,(21b^2c^2+70abcd+5a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{3/4}\sqrt[4]{d}} - 8\sqrt{2}\sqrt[4]{a}\,b^{3/4}(5bc+7ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right) + \frac{\sqrt{2}\,(21b^2c^2+70abcd+5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{3/4}\sqrt[4]{d}}}{64(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((4*(b*c - a*d)*Sqrt[x]*(b^2*c*x^2*(11*c + 7*d*x^2) + a^2*d*(5*c + 9*d*x^2)
 + a*b*(19*c^2 + 28*c*d*x^2 + 17*d^2*x^4)))/((a + b*x^2)*(c + d*x^2)^2) + 8
*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt
```

```
[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d
^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(3/
4)*d^(1/4)) - 8*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)] + (Sqrt[2]*(21*b^2*c^2 + 70*a
*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sq
rt[d]*x)])/(c^(3/4)*d^(1/4)))/(64*(b*c - a*d)^4)
```

Maple [A]

time = 0.18, size = 364, normalized size = 0.51

method	result
derivativedivides	$-\frac{2ab\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^{2}+a}+\frac{(7ad+5bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{32a}\right.}{(ad-bc)^{4}}$
default	$-\frac{2ab\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^{2}+a}+\frac{(7ad+5bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{32a}\right.}{(ad-bc)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*b/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(7*a*d+5*b*c)*
(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2/(a*d-b*c)^4*(((-9/32*a^2*d
^3+1/16*a*b*c*d^2+7/32*b^2*c^2*d)*x^(5/2)-1/32*c*(5*a^2*d^2+6*a*b*c*d-11*b^
2*c^2)*x^(1/2))/(d*x^2+c)^2+1/256*(5*a^2*d^2+70*a*b*c*d+21*b^2*c^2)*(c/d)^(
1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.58, size = 855, normalized size = 1.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(5*b*c + 7*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
```

```
)) + 2*sqrt(2)*(5*b*c + 7*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
) + sqrt(2)*(5*b*c + 7*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*b*c + 7*a*d)*log(-sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a*b/(b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/16*((7*b^2
*c*d + 17*a*b*d^2)*x^(9/2) + (11*b^2*c^2 + 28*a*b*c*d + 9*a^2*d^2)*x^(5/2)
+ (19*a*b*c^2 + 5*a^2*c*d)*sqrt(x))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*
c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 -
a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b
*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*
b*c^2*d^3 - 2*a^4*c*d^4)*x^2) + 1/128*(2*sqrt(2)*(21*b^2*c^2 + 70*a*b*c*d +
 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(21*b^
2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4
) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)
)) + sqrt(2)*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(21*b^2*c^2
+ 70*a*b*c*d + 5*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
+ sqrt(c))/(c^(3/4)*d^(1/4)))/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. 2(562) = 1124.
time = 1.62, size = 1193, normalized size = 1.66

$$-\frac{\left(5\left(ab^{3}\right)^{\frac{1}{4}}bc+7\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{4}c^{4}-4\sqrt{2}ab^{3}c^{3}d+6\sqrt{2}a^{2}b^{2}c^{2}d^{2}-4\sqrt{2}a^{3}bcd^{3}+\sqrt{2}a^{4}d^{4}\right)}-\frac{\left(5\left(ab^{3}\right)^{\frac{1}{4}}bc+7\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{4}c^{4}-4\sqrt{2}ab^{3}c^{3}d+6\sqrt{2}a^{2}b^{2}c^{2}d^{2}-4\sqrt{2}a^{3}bcd^{3}+\sqrt{2}a^{4}d^{4}\right)}+\frac{\left(21\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+70\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}b^{4}c^{5}d-4\sqrt{2}ab^{3}c^{4}d^{2}+6\sqrt{2}a^{2}b^{2}c^{3}d^{3}-4\sqrt{2}a^{3}bc^{2}d^{4}+\sqrt{2}a^{4}cd^{5}\right)}+\frac{\left(21\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+70\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}b^{4}c^{5}d-4\sqrt{2}ab^{3}c^{4}d^{2}+6\sqrt{2}a^{2}b^{2}c^{3}d^{3}-4\sqrt{2}a^{3}bc^{2}d^{4}+\sqrt{2}a^{4}cd^{5}\right)}-\frac{\left(5\left(ab^{3}\right)^{\frac{1}{4}}bc+7\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}b^{4}c^{4}-4\sqrt{2}ab^{3}c^{3}d+6\sqrt{2}a^{2}b^{2}c^{2}d^{2}-4\sqrt{2}a^{3}bcd^{3}+\sqrt{2}a^{4}d^{4}\right)}+\frac{\left(5\left(ab^{3}\right)^{\frac{1}{4}}bc+7\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}b^{4}c^{4}-4\sqrt{2}ab^{3}c^{3}d+6\sqrt{2}a^{2}b^{2}c^{2}d^{2}-4\sqrt{2}a^{3}bcd^{3}+\sqrt{2}a^{4}d^{4}\right)}+\frac{\left(21\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+70\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^{4}c^{5}d-4\sqrt{2}ab^{3}c^{4}d^{2}+6\sqrt{2}a^{2}b^{2}c^{3}d^{3}-4\sqrt{2}a^{3}bc^{2}d^{4}+\sqrt{2}a^{4}cd^{5}\right)}-\frac{\left(21\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+70\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^{4}c^{5}d-4\sqrt{2}ab^{3}c^{4}d^{2}+6\sqrt{2}a^{2}b^{2}c^{3}d^{3}-4\sqrt{2}a^{3}bc^{2}d^{4}+\sqrt{2}a^{4}cd^{5}\right)}+\frac{ab\sqrt{x}}{2\left(b^{3}c^{3}-3ab^{2}c^{2}d+3a^{2}bcd^{2}-a^{3}d^{3}\right)\left(bx^{2}+a\right)}+\frac{7bcdx^{\frac{5}{2}}+9ad^{2}x^{\frac{5}{2}}+11bc^{2}\sqrt{x}+5acd\sqrt{x}}{16\left(b^{3}c^{3}-3ab^{2}c^{2}d+3a^{2}bcd^{2}-a^{3}d^{3}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*b^3*
c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4*d^4
) - 1/4*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*
b^3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4
*d^4) + 1/32*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^4*c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c
^3*d^3 - 4*sqrt(2)*a^3*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) + 1/32*(21*(c*d^3)^(1
/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^5*
d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3*b*c
^2*d^4 + sqrt(2)*a^4*c*d^5) - 1/8*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*
d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^4 - 4*sq
rt(2)*a*b^3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqr
t(2)*a^4*d^4) + 1/8*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*log(-sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*b^3*c
^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4*d^4)
 + 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1
/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*
c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3
*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) - 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^
3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4
) + x + sqrt(c/d))/(sqrt(2)*b^4*c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)
*a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) + 1/2*a*b*s
qrt(x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a)) +
1/16*(7*b*c*d*x^(5/2) + 9*a*d^2*x^(5/2) + 11*b*c^2*sqrt(x) + 5*a*c*d*sqrt(x
))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x^2 + c)^2)
```

Mupad [B]
time = 3.34, size = 2500, normalized size = 3.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] 2*atan(-((((((1473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (9
72405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (11560479*
a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*
b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2
```

*d^9)/512)*1i)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10
*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d
^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 28
6*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c
*d^12) + (-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664
200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 +
745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216
*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 2684354
56*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^1
6*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134
351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*
a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c
^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9
395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(3/4)*(((-(625
*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5
*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*
c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d +
16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d
^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 3053453
3120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b
^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^
9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282
879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*
b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 +
10240*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*
d^6 - 3450880*a^5*b^19*c^16*d^7 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*
b^17*c^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 7
7608960*a^10*b^14*c^11*d^12 + 115315200*a^11*b^13*c^10*d^13 - 121172480*a^1
2*b^12*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16
+ 23618560*a^15*b^9*c^6*d^17 - 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^
7*c^4*d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13
- b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c
^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7
- 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*
a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) - (x^(1/2)*(655360
0*a^23*b^4*d^25 + 78643200*a^22*b^5*c*d^24 + 419430400*a^2*b^25*c^21*d^4 -
5420875776*a^3*b^24*c^20*d^5 + 31284264960*a^4*b^23*c^19*d^6 - 104224784384
*a^5*b^22*c^18*d^7 + 210842419200*a^6*b^21*c^17*d^8 - 218396098560*a^7*b^20
*c^16*d^9 - 105331556352*a^8*b^19*c^15*d^10 + 910845542400*a^9*b^18*c^14*d^
11 - 2125492912128*a^10*b^17*c^13*d^12 + 3520229539840*a^11*b^16*c^12*d^13
- 4783425454080*a^12*b^15*c^11*d^14 + 5470166188032*a^13*b^14*c^10*d^15 - 5
154201927680*a^14*b^13*c^9*d^16 + 3867903787008*a^15*b^12*c^8*d^17 - 222988
0750080*a^16*b^11*c^7*d^18 + 945071063040*a^17*b^10*c^6*d^19 - 273892245504
*a^18*b^9*c^5*d^20 + 45719224320*a^19*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^
3*d^22 - 810024960*a^21*b^6*c^2*d^23)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 1

53*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 856
8*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 4
3758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 -
31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13
+ 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 -
18*a*b^17*c^17*d - 18*a^17*b*c*d^17)))*1i)*(-(625*a^8*d^8 + 194481*b^8*c^8
+ 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^
4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^
7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 2
68435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*
c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 7
3282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 19193135104
0*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c
^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 +
 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*
a^14*b^2*c^5*d^15))^(1/4) + (x^(1/2)*(3872225*a^12*b^7*d^13 + 120299550*a^1
1*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^16*c^9*d^4 + 5374
50669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^14*c^...

3.497 $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=703

$$-\frac{3dx^{3/2}}{4(bc-ad)^2\,(c+dx^2)^2}-\frac{x^{3/2}}{2(bc-ad)\,(a+bx^2)\,(c+dx^2)^2}-\frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\,(c+dx^2)}-\frac{3b^{5/4}(bc+3ad)\tan^{-1}\Big(}{4\sqrt{2}\,\sqrt[4]{a}\,(bc}$$

```
[Out] -3/4*d*x^(3/2)/(-a*d+b*c)^2/(d*x^2+c)^2-1/2*x^(3/2)/(-a*d+b*c)/(b*x^2+a)/(d
*x^2+c)^2-3/16*d*(a*d+7*b*c)*x^(3/2)/c/(-a*d+b*c)^3/(d*x^2+c)-3/8*b^(5/4)*(
3*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)^4*2
^(1/2)+3/8*b^(5/4)*(3*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^
(1/4)/(-a*d+b*c)^4*2^(1/2)+3/64*d^(1/4)*(-a^2*d^2+18*a*b*c*d+15*b^2*c^2)*ar
ctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*c)^4*2^(1/2)-3/64*d
^(1/4)*(-a^2*d^2+18*a*b*c*d+15*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^
(1/4))/c^(5/4)/(-a*d+b*c)^4*2^(1/2)+3/16*b^(5/4)*(3*a*d+b*c)*ln(a^(1/2)+x*b
^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^4*2^(1/2)-3/16*b
^(5/4)*(3*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^
(1/4)/(-a*d+b*c)^4*2^(1/2)-3/128*d^(1/4)*(-a^2*d^2+18*a*b*c*d+15*b^2*c^2)*l
n(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a*d+b*c)^4*2
^(1/2)+3/128*d^(1/4)*(-a^2*d^2+18*a*b*c*d+15*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a*d+b*c)^4*2^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 703, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 482, 593, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{3\sqrt[4]{d}\,(-a^2d^2+18abcd+15b^2c^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{5/4}(bc-ad)^4}-\frac{3\sqrt[4]{d}\,(-a^2d^2+18abcd+15b^2c^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{5/4}(bc-ad)^4}-\frac{3\sqrt[4]{d}\,(-a^2d^2+18abcd+15b^2c^2)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\,(-a^2d^2+18abcd+15b^2c^2)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}(bc-ad)^4}-\frac{3b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)(3ad+bc)}{4\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^4}+\frac{3b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)(3ad+bc)}{4\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^4}+\frac{3b^{5/4}(3ad+bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^4}-\frac{3b^{5/4}(3ad+bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^4}-\frac{3dx^{3/2}(ad+7bc)}{16c\,(c+dx^2)\,(bc-ad)^3}-\frac{x^{3/2}}{2\,(a+bx^2)\,(c+dx^2)^2\,(bc-ad)}-\frac{3dx^{3/2}}{4\,(c+dx^2)^2\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*x^(3/2))/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x^(3/2)/(2*(b*c - a*d)*(a
 + b*x^2)*(c + d*x^2)^2) - (3*d*(7*b*c + a*d)*x^(3/2))/(16*c*(b*c - a*d)^3*(
c + d*x^2)) - (3*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^
4) + (3*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15
*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c
 - a*d)^4) - (3*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
```

```
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15*b
^2*c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*d^(1/4)*(15*b^2*c^2
 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3c-9dx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(12c(}{(a+}\right.}{1}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad}{16c(bc-ad)^3(c}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad}{16c(bc-ad)^3(c}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad}{16c(bc-ad)^3(c}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad}{16c(bc-ad)^3(c}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad}{16c(bc-ad)^3(c}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad}{16c(bc-ad)^3(c}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad}{16c(bc-ad)^3(c}$$

Mathematica [A]

time = 2.59, size = 396, normalized size = 0.56

$$\frac{-\frac{4(bc-ad)x^{3/2}\left(a^2d^2\left(-c+3dx^2\right)+abd\left(17c^2+12cdx^2+3d^2x^4\right)+b^2c\left(8c^2+33cdx^2+21d^2x^4\right)\right)}{c\left(a+bx^2\right)\left(c+dx^2\right)^2}-\frac{24\sqrt{2}\,b^{5/4}(bc+3ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt[4]{a}}+\frac{3\sqrt{2}\,\sqrt[4]{d}\left(15b^2c^2+18abcd-a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{5/4}}-\frac{24\sqrt{2}\,b^{5/4}(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{a}}+\frac{3\sqrt{2}\,\sqrt[4]{d}\left(15b^2c^2+18abcd-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{5/4}}}{64(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

[Out] ((-4*(b*c - a*d)*x^(3/2)*(a^2*d^2*(-c + 3*d*x^2) + a*b*d*(17*c^2 + 12*c*d*x^2 + 3*d^2*x^4) + b^2*c*(8*c^2 + 33*c*d*x^2 + 21*d^2*x^4)))/(c*(a + b*x^2)*(c + d*x^2)^2) - (24*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(1/4) + (3*Sqrt[2]*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(5/4) - (24*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4) + (3*Sqrt[2]*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(5/4))/(64*(b*c - a*d)^4)

Maple [A]
time = 0.18, size = 368, normalized size = 0.52

method	result
derivativedivides	$\frac{2b^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{9ad}{4}+\frac{3bc}{4}\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^4}$
default	$\frac{2b^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{9ad}{4}+\frac{3bc}{4}\right)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] 2*b^2/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(9/4*a*d+3/4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d/(a*d-b*c)^4*((1/32*d*(3*a^2*d^2+10*a*b*c*d-13*b^2*c^2)/c*x^(7/2)+(-1/32*a^2*d^2+9/16*a*b*c*d-17/32*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+3/256*(a^2*d^2-18*a*b*c*d-15*b^2*c^2)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))

Maxima [A]
time = 0.51, size = 791, normalized size = 1.13

$$\frac{3\,(b^3c+3\,ab^2d)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}\right)}{16\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)}-\frac{3\,(15\,b^2c^2d+18\,abcd^2-a^2d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac14}d^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac14}d^{\frac34}}\right)}{128\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)}-\frac{3\,(7\,b^2cd^2+abd^3)x^{\frac{11}{2}}+3\,(11\,b^2c^2d+4\,abcd^2+a^2d^3)x^{\frac72}+(8\,b^2c^3+17\,abc^2d-a^2cd^2)x^{\frac32}}{16\,(ab^3c^6-3\,a^2b^2c^5d+3\,a^3bc^4d^2-a^4c^3d^3+(b^4c^4d^2-3\,ab^3c^3d^3+3\,a^2b^2c^2d^4-a^3bcd^5)x^6+(2\,b^4c^5d-5\,ab^3c^4d^2+3\,a^2b^2c^3d^3+a^3bc^2d^4-a^4cd^5)x^4+(b^4c^6-ab^3c^5d-3\,a^2b^2c^4d^2+5\,a^3bc^3d^3-2\,a^4c^2d^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/16*(b^3*c + 3*a*b^2*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(
1/4)*b^(3/4)))/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4) - 3/128*(15*b^2*c^2*d + 18*a*b*c*d^2 - a^2*d^3)*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*s
qrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sq
rt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqr
t(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*
sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^4*c^5 - 4*a*b^3*c^4*d
+ 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4) - 1/16*(3*(7*b^2*c*d^2 +
 a*b*d^3)*x^(11/2) + 3*(11*b^2*c^2*d + 4*a*b*c*d^2 + a^2*d^3)*x^(7/2) + (8*
b^2*c^3 + 17*a*b*c^2*d - a^2*c*d^2)*x^(3/2))/(a*b^3*c^6 - 3*a^2*b^2*c^5*d +
 3*a^3*b*c^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c^3*d^3 + 3*a^2*b^2
*c^2*d^4 - a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3*c^4*d^2 + 3*a^2*b^2*c^
3*d^3 + a^3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2
*c^4*d^2 + 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(547) = 1094$.
time = 1.97, size = 1238, normalized size = 1.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")

[Out] -1/2*b^2*x^(3/2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a)) + 3/4*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) + 3/4*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) - 3/32*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sqrt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 3/32*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sqrt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 3/8*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) + 3/8*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) + 3/64*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sqrt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 3/64*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sqrt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 1/16*(13*b*c*d^2*x^(7/2) + 3*a*d^3*x^(7/2) + 17*b*c^2*d*x^(3/2) - a*c*d^2*x^(3/2))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2 + c)^2)

Mupad [B]
time = 3.63, size = 2500, normalized size = 3.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)

[Out] 2*atan((((((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587113*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22

*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d
^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/
2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 -
(274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (1
66924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637
*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^
9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24
)/8 - (24867*a^22*b^6*c^2*d^25)/8)*1i)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20
*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^1
7*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7
*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 3527
16*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*
d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^
6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b
^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) - (9*x^(1/2)*(-(81*
a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6
*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3
*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 +
16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c
^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73
282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040
*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^
12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 +
30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*
a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(1/4)*(16777216*a*b^26*c^23*d
^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*b^25*c^22*d^5 + 612630528*a^3*b
^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 - 2827485184*a^5*b^22*c^19*d^8 +
8081375232*a^6*b^21*c^18*d^9 + 6940786688*a^7*b^20*c^17*d^10 - 89661636608
*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*c^15*d^12 - 518906707968*a^10*b
^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*d^14 - 805866307584*a^12*b^15*c
^12*d^15 + 754870910976*a^13*b^14*c^11*d^16 - 615914668032*a^14*b^13*c^10*d
^17 + 437990719488*a^15*b^12*c^9*d^18 - 263356153856*a^16*b^11*c^8*d^19 + 1
27919980544*a^17*b^10*c^7*d^20 - 47752151040*a^18*b^9*c^6*d^21 + 1295541862
4*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^4*d^23 + 259522560*a^21*b^6*c^3
*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18
*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4
*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a
^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758
*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8
568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 15
3*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d
+ 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5
*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c
^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 2
68435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^

13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(3/4) - (9*x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 505671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 13218147*a^9*b^10*c^2*d^13))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16...

3.498 $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=703

$$-\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2}-\frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}-\frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx^2\right)}-\frac{b^{7/4}(bc+11ad)\tan^{-1}}{4\sqrt{2}\,a^{3/4}(b}$$

```
[Out] -1/8*b^(7/4)*(11*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)
/(-a*d+b*c)^4*2^(1/2)+1/8*b^(7/4)*(11*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(
1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)^4*2^(1/2)+1/64*d^(3/4)*(-3*a^2*d^2+22*a*b*
c*d+77*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c
)^4*2^(1/2)-1/64*d^(3/4)*(-3*a^2*d^2+22*a*b*c*d+77*b^2*c^2)*arctan(1+d^(1/4
)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^4*2^(1/2)-1/16*b^(7/4)*(11*a*
d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+
b*c)^4*2^(1/2)+1/16*b^(7/4)*(11*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/
4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^4*2^(1/2)+1/128*d^(3/4)*(-3*a^2*d^2+
22*a*b*c*d+77*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2)
)/c^(7/4)/(-a*d+b*c)^4*2^(1/2)-1/128*d^(3/4)*(-3*a^2*d^2+22*a*b*c*d+77*b^2*
c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/(-a*d+b*
c)^4*2^(1/2)-3/4*d*x^(1/2)/(-a*d+b*c)^2/(d*x^2+c)^2-1/2*x^(1/2)/(-a*d+b*c)/
(b*x^2+a)/(d*x^2+c)^2-1/16*d*(a*d+23*b*c)*x^(1/2)/c/(-a*d+b*c)^3/(d*x^2+c)
```

Rubi [A]
time = 0.68, antiderivative size = 703, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 482, 541, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(11ad+bc)}{4\sqrt{2}\,a^{3/4}(bc-ad)^4}+\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(11ad+bc)}{4\sqrt{2}\,a^{3/4}(bc-ad)^4}-\frac{b^{7/4}(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\,a^{3/4}(bc-ad)^4}+\frac{b^{7/4}(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\,a^{3/4}(bc-ad)^4}+\frac{d^{3/4}(-3a^2d^2+22abcd+77b^2c^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{7/4}(bc-ad)^4}-\frac{d^{3/4}(-3a^2d^2+22abcd+77b^2c^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{7/4}(bc-ad)^4}+\frac{d^{3/4}(-3a^2d^2+22abcd+77b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}\,c^{7/4}(bc-ad)^4}-\frac{d^{3/4}(-3a^2d^2+22abcd+77b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}\,c^{7/4}(bc-ad)^4}-\frac{d\sqrt{x}(ad+23bc)}{16c\left(c+dx^2\right)(bc-ad)^3}-\frac{3d\sqrt{x}}{4\left(c+dx^2\right)^2(bc-ad)^2}-\frac{\sqrt{x}}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*Sqrt[x])/(4*(b*c - a*d)^2*(c + d*x^2)^2) - Sqrt[x]/(2*(b*c - a*d)*(a
+ b*x^2)*(c + d*x^2)^2) - (d*(23*b*c + a*d)*Sqrt[x])/(16*c*(b*c - a*d)^3*(c
 + d*x^2)) - (b^(7/4)*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (b^(7/4)*(b*c + 11*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4)
+ (d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*
c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a
*d)^4) + (b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
```

```
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (d^(3/4)*(77*b^2*c^2
+ 22*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*c^2 + 22*a
*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{c-11dx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{4c(2bc+a\cdots}{\left(a+bx^4\right)\cdots}\cdots\right.}{16c(b\cdots}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\cdots}{16c(bc-ad)^3(c\cdots}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\cdots}{16c(bc-ad)^3(c\cdots}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\cdots}{16c(bc-ad)^3(c\cdots}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\cdots}{16c(bc-ad)^3(c\cdots}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\cdots}{16c(bc-ad)^3(c\cdots}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\cdots}{16c(bc-ad)^3(c\cdots}$$

Mathematica [A]
time = 2.80, size = 392, normalized size = 0.56

$$\frac{-\frac{4(bc-ad)\sqrt{x}\left(a^2d^2\left(-3c+dx^2\right)+abd\left(19c^2+12cdx^2+d^2x^4\right)+b^2c\left(8c^2+35cdx^2+23d^2x^4\right)\right)}{c\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{8\sqrt{2}\,b^{7/4}(bc+11ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}} + \frac{\sqrt{2}\,d^{3/4}\left(77b^2c^2+22abcd-3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{7/4}} + \frac{8\sqrt{2}\,b^{7/4}(bc+11ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}} + \frac{\sqrt{2}\,d^{3/4}\left(-77b^2c^2-22abcd+3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{7/4}}}{64(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*(b*c - a*d)*Sqrt[x]*(a^2*d^2*(-3*c + d*x^2) + a*b*d*(19*c^2 + 12*c*d*x
^2 + d^2*x^4) + b^2*c*(8*c^2 + 35*c*d*x^2 + 23*d^2*x^4)))/(c*(a + b*x^2)*(c
 + d*x^2)^2) - (8*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*
```

```
x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (Sqrt[2]*d^(3/4)*(77*b^2*c
^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x])])/c^(7/4) + (8*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4) + (Sqrt[2]*d^
(3/4)*(-77*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(7/4))/(64*(b*c - a*d)^4)
```

Maple [A]
time = 0.18, size = 364, normalized size = 0.52

method	result
derivativedivides	$\frac{2b^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(11ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right)\right)}{32a}\cdots\right)}{(ad-bc)^4}$
default	$\frac{2b^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(11ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right)\right)}{32a}\cdots\right)}{(ad-bc)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(11*a*d+b*c)*(a
/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b
)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+
1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d/(a*d-b*c)^4*((1/32*d*(a^2*
d^2+14*a*b*c*d-15*b^2*c^2)/c*x^(5/2)+(11/16*a*b*c*d-19/32*b^2*c^2-3/32*a^2*
d^2)*x^(1/2))/(d*x^2+c)^2+1/256*(3*a^2*d^2-22*a*b*c*d-77*b^2*c^2)/c^2*(c/d)
^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.56, size = 889, normalized size = 1.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*(b*c + 11*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
```

```
 + 2*sqrt(2)*(b*c + 11*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 sqrt(2)*(b*c + 11*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b*c + 11*a*d)*log(-sqrt(2)*a^(1/4)*b^(
1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/16*((23*b^2*c*d
^2 + a*b*d^3)*x^(9/2) + (35*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*x^(5/2) + (
8*b^2*c^3 + 19*a*b*c^2*d - 3*a^2*c*d^2)*sqrt(x))/(a*b^3*c^6 - 3*a^2*b^2*c^5
*d + 3*a^3*b*c^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c^3*d^3 + 3*a^2
*b^2*c^2*d^4 - a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3*c^4*d^2 + 3*a^2*b^
2*c^3*d^3 + a^3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2
*b^2*c^4*d^2 + 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2) - 1/128*(2*sqrt(2)*(77
*b^2*c^2*d + 22*a*b*c*d^2 - 3*a^2*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*
d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*s
qrt(d))) + 2*sqrt(2)*(77*b^2*c^2*d + 22*a*b*c*d^2 - 3*a^2*d^3)*arctan(-1/2*
sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d))
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(77*b^2*c^2*d + 22*a*b*c*d^2 -
3*a^2*d^3)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3
/4)*d^(1/4)) - sqrt(2)*(77*b^2*c^2*d + 22*a*b*c*d^2 - 3*a^2*d^3)*log(-sqrt(
2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^4*c
^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(547) = 1094$.
time = 1.88, size = 1217, normalized size = 1.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")

[Out] 1/4*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c^4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c*d^3 + sqrt(2)*a^5*d^4) + 1/4*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c^4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c*d^3 + sqrt(2)*a^5*d^4) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*a^2*b^2*c^4*d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*a^2*b^2*c^4*d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) + 1/8*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^4*c^4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c*d^3 + sqrt(2)*a^5*d^4) - 1/8*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^4*c^4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c*d^3 + sqrt(2)*a^5*d^4) - 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*a^2*b^2*c^4*d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) + 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*a^2*b^2*c^4*d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) - 1/2*b^2*sqrt(x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a)) - 1/16*(15*b*c*d^2*x^(5/2) + a*d^3*x^(5/2) + 19*b*c^2*d*sqrt(x) - 3*a*c*d^2*sqrt(x))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2 + c)^2)

Mupad [B]
time = 3.54, size = 2500, normalized size = 3.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)

[Out] ((x^(1/2)*(8*b^2*c^2 - 3*a^2*d^2 + 19*a*b*c*d))/(16*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x^(5/2)*(a^2*d^3 + 35*b^2*c^2*d + 12*a*b*c*d^2))/(16*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d*x^

(9/2)*(a*d^2 + 23*b*c*d))/(16*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*
b*c*d^2)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x
^6) + 2*atan(((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^
4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4
*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(
16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2
013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^
4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^1
7*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191
931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*
a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c
^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*(
(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a
^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a
^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*
c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*
b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^
4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931
351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*
b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*
d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 201
3265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*((((891*a^9*b^7
*d^15)/8192 + (77*b^16*c^9*d^6)/16 - (33367697*a*b^15*c^8*d^7)/8192 - (6291
*a^8*b^8*c*d^14)/2048 - (107777537*a^2*b^14*c^7*d^8)/2048 - (83346257*a^3*b
^13*c^6*d^9)/1024 - (39606577*a^4*b^12*c^5*d^10)/2048 + (7338751*a^5*b^11*c
^4*d^11)/4096 + (198309*a^6*b^10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/2
56)*1i)/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*
d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5
+ 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 71
5*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^1
2*c^16*d) + (-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4
+ 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d
^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16
777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 201
3265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*
b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*
d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 19193
1351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^
11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^1
0*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(3/4)*(((
-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^
2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^
5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c
^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b
^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4

- 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 1919313
51040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b
^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d
^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013
265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*(8192*a^2*b^22*c
^22*d^5 - 2048*a*b^23*c^23*d^4 + 142592*a^3*b^21*c^21*d^6 - 1723648*a^4*b^2
0*c^20*d^7 + 9439232*a^5*b^19*c^19*d^8 - 32966656*a^6*b^18*c^18*d^9 + 81665
024*a^7*b^17*c^17*d^10 - 150731776*a^8*b^16*c^16*d^11 + 212486144*a^9*b^15*
c^15*d^12 - 231069696*a^10*b^14*c^14*d^13 + 193363456*a^11*b^13*c^13*d^14 -
122330624*a^12*b^12*c^12*d^15 + 55883776*a^13*b^11*c^11*d^16 - 16185344*a^
14*b^10*c^10*d^17 + 1309696*a^15*b^9*c^9*d^18 + 1205248*a^16*b^8*c^8*d^19 -
622592*a^17*b^7*c^7*d^20 + 145408*a^18*b^6*c^6*d^21 - 17152*a^19*b^5*c^5*d
^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^
12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 -
1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 128
7*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b
^2*c^6*d^11 - 13*a*b^12*c^16*d) - (x^(1/2)*(16777216*b^27*c^25*d^4 + 100663
296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970170880*a^3*b^24*c^
22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 366041...

3.499 $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=739

$$\frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2}+\frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}+\frac{d\left(8b^2c^2+21abcd-5a^2d^2\right)x^{3/2}}{16ac^2(bc-ad)^3\left(c+dx^2\right)}-\frac{b^{9/4}(bc-13ad}{4\sqrt{}}$$

```
[Out] 1/4*d*(a*d+2*b*c)*x^(3/2)/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x^(3/2)/a/(-a*
d+b*c)/(b*x^2+a)/(d*x^2+c)^2+1/16*d*(-5*a^2*d^2+21*a*b*c*d+8*b^2*c^2)*x^(3/
2)/a/c^2/(-a*d+b*c)^3/(d*x^2+c)-1/8*b^(9/4)*(-13*a*d+b*c)*arctan(1-b^(1/4)*
2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^4*2^(1/2)+1/8*b^(9/4)*(-13*a*d+
b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^4*2^(1/2)
-1/64*d^(5/4)*(5*a^2*d^2-26*a*b*c*d+117*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x
^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^4*2^(1/2)+1/64*d^(5/4)*(5*a^2*d^2-26*a*b
*c*d+117*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b
*c)^4*2^(1/2)+1/16*b^(9/4)*(-13*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/
4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)^4*2^(1/2)-1/16*b^(9/4)*(-13*a*d+b*c)
*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)^4
*2^(1/2)+1/128*d^(5/4)*(5*a^2*d^2-26*a*b*c*d+117*b^2*c^2)*ln(c^(1/2)+x*d^(1
/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^4*2^(1/2)-1/128*d^(
5/4)*(5*a^2*d^2-26*a*b*c*d+117*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4
)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^4*2^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 483, 593, 598, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-13ad)}{4\sqrt{2}a^{5/4}(bc-ad)^4}+\frac{b^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-13ad)}{4\sqrt{2}a^{5/4}(bc-ad)^4}+\frac{b^{9/4}(bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4}-\frac{b^{9/4}(bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4}-\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4}+\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4}+\frac{dx^{3/2}\left(-5a^2d^2+21abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4}-\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4}+\frac{bx^{3/2}}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx^{3/2}(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x^(3/2))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x^(3/2))
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 21*a*b*c*d -
 5*a^2*d^2)*x^(3/2))/(16*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (b^(9/4)*(b*c -
 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*
(b*c - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26
*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sq
rt[2]*c^(9/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*(b*c
```

```
 - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (b^(9/4)*(b*c - 1
3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(9/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)
*(b*c - a*d)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
```

```
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}\,dx = 2\text{Subst}\left(\int \frac{x^2}{(a+bx^4)^2(c+dx^4)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\text{Subst}\left(\int \frac{x^2(-bc+4ad-9bdx^4)}{(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{\text{Subst}\left(\int \frac{x^2(-4}{\cdots}\right.}{\cdots}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(8b^2c^2+21a\cdots}{16ac^2(bc-\cdots}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(8b^2c^2+21a\cdots}{16ac^2(bc-\cdots}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(8b^2c^2+21a\cdots}{16ac^2(bc-\cdots}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(8b^2c^2+21a\cdots}{16ac^2(bc-\cdots}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(8b^2c^2+21a\cdots}{16ac^2(bc-\cdots}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(8b^2c^2+21a\cdots}{16ac^2(bc-\cdots}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(8b^2c^2+21a\cdots}{16ac^2(bc-\cdots}$$

Mathematica [A]
time = 2.05, size = 451, normalized size = 0.61

$$\frac{1}{64}\left(-\frac{4x^{3/2}\left(8b^3c^2(c+dx^2)^2-a^3d^3(9c+5dx^2)+ab^2cd^2x^2(25c+21dx^2)+a^2bd^2(25c^2+12cdx^2-5d^2x^4)\right)}{ac^2(-bc+ad)^3(a+bx^2)(c+dx^2)^2}+\frac{8\sqrt{2}\,b^{9/4}(-bc+13ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{5/4}(bc-ad)^4}-\frac{\sqrt{2}\,d^{5/4}(117b^2c^2-26abcd+5a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{9/4}(bc-ad)^4}+\frac{8\sqrt{2}\,b^{9/4}(-bc+13ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{5/4}(bc-ad)^4}-\frac{\sqrt{2}\,d^{5/4}(117b^2c^2-26abcd+5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{9/4}(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*x^(3/2)*(8*b^3*c^2*(c + d*x^2)^2 - a^3*d^3*(9*c + 5*d*x^2) + a*b^2*c*d
^2*x^2*(25*c + 21*d*x^2) + a^2*b*d^2*(25*c^2 + 12*c*d*x^2 - 5*d^2*x^4)))/(a
*c^2*(-(b*c) + a*d)^3*(a + b*x^2)*(c + d*x^2)^2) + (8*Sqrt[2]*b^(9/4)*(-(b*
c) + 13*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
])/(a^(5/4)*(b*c - a*d)^4) - (Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5
*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/
(c^(9/4)*(b*c - a*d)^4) + (8*Sqrt[2]*b^(9/4)*(-(b*c) + 13*a*d)*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(5/4)*(b*c - a*d)^
4) - (Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(9/4)*(b*c - a*d)^4)
)/64
```

Maple [A]

time = 0.18, size = 381, normalized size = 0.52

method	result
derivativedivides	$-\frac{2b^3\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4a\left(b\,x^2+a\right)}+\frac{(13ad-bc)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{32ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{(ad-bc)^4}$
default	$-\frac{2b^3\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4a\left(b\,x^2+a\right)}+\frac{(13ad-bc)\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{32ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{(ad-bc)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3/(a*d-b*c)^4*(1/4*(a*d-b*c)/a*x^(3/2)/(b*x^2+a)+1/32*(13*a*d-b*c)/a/b
/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d^2/(a*d-b*c)^4*((1/32*d*(5
*a^2*d^2-26*a*b*c*d+21*b^2*c^2)/c^2*x^(7/2)+1/32*(9*a^2*d^2-34*a*b*c*d+25*b
^2*c^2)/c*x^(3/2))/(d*x^2+c)^2+1/256*(5*a^2*d^2-26*a*b*c*d+117*b^2*c^2)/c^2
/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/
2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.54, size = 845, normalized size = 1.14

$$\frac{(b^4c-13ab^3d)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{16\left(ab^4c^4-4a^2b^3c^3d+6a^3b^2c^2d^2-4a^4bcd^3+a^5d^4\right)}+\frac{(117b^2c^2d^2-26abcd^3+5a^2d^4)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)}{128\left(b^4c^6-4ab^3c^5d+6a^2b^2c^4d^2-4a^3bc^3d^3+a^4c^2d^4\right)}+\frac{(8b^3c^2d^2+21ab^2cd^3-5a^2bd^4)x^{\frac{11}{2}}+(16b^3c^3d+25ab^2c^2d^2+12a^2bcd^3-5a^3d^4)x^{\frac{7}{2}}+(8b^3c^4+25a^2bc^2d^2-9a^3cd^3)x^{\frac{3}{2}}}{16\left(a^2b^3c^7-3a^3b^2c^6d+3a^4bc^5d^2-a^5c^4d^3+(ab^4c^5d^2-3a^2b^3c^4d^3+3a^3b^2c^3d^4-a^4bc^2d^5)x^6+(2ab^4c^6d-5a^2b^3c^5d^2+3a^3b^2c^4d^3+a^4bc^3d^4-a^5c^2d^5)x^4+(ab^4c^7-a^2b^3c^6d-3a^3b^2c^5d^2+5a^4bc^4d^3-2a^5c^3d^4)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b^4*c - 13*a*b^3*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sq
rt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*
log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^
(1/4)*b^(3/4)))/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*
c*d^3 + a^5*d^4) + 1/128*(117*b^2*c^2*d^2 - 26*a*b*c*d^3 + 5*a^2*d^4)*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt
(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d))
)/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4
)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^4*c^6 - 4*a*
b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4) + 1/16*((8*b
^3*c^2*d^2 + 21*a*b^2*c*d^3 - 5*a^2*b*d^4)*x^(11/2) + (16*b^3*c^3*d + 25*a*
b^2*c^2*d^2 + 12*a^2*b*c*d^3 - 5*a^3*d^4)*x^(7/2) + (8*b^3*c^4 + 25*a^2*b*c
^2*d^2 - 9*a^3*c*d^3)*x^(3/2))/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5
*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4
 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*
d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3
*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1233 vs. $2(583) = 1166$.
time = 2.35, size = 1233, normalized size = 1.67

$$\frac{b^{3}x^{\frac{3}{2}}}{2\left(ab^{3}c^{3}-3a^{2}b^{2}c^{2}d+3a^{3}bcd^{2}-a^{4}d^{3}\right)\left(bx^{2}+a\right)}+\frac{\left(\left(ab^{3}\right)^{\frac{3}{4}}bc-13\left(ab^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a^{2}b^{4}c^{4}-4\sqrt{2}a^{3}b^{3}c^{3}d+6\sqrt{2}a^{4}b^{2}c^{2}d^{2}-4\sqrt{2}a^{5}bcd^{3}+\sqrt{2}a^{6}d^{4}\right)}+\frac{\left(\left(ab^{3}\right)^{\frac{3}{4}}bc-13\left(ab^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a^{2}b^{4}c^{4}-4\sqrt{2}a^{3}b^{3}c^{3}d+6\sqrt{2}a^{4}b^{2}c^{2}d^{2}-4\sqrt{2}a^{5}bcd^{3}+\sqrt{2}a^{6}d^{4}\right)}+\frac{\left(117\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-26\left(cd^{3}\right)^{\frac{3}{4}}abcd+5\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}b^{4}c^{7}d-4\sqrt{2}ab^{3}c^{6}d^{2}+6\sqrt{2}a^{2}b^{2}c^{5}d^{3}-4\sqrt{2}a^{3}bc^{4}d^{4}+\sqrt{2}a^{4}c^{3}d^{5}\right)}+\frac{\left(117\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-26\left(cd^{3}\right)^{\frac{3}{4}}abcd+5\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{32\left(\sqrt{2}b^{4}c^{7}d-4\sqrt{2}ab^{3}c^{6}d^{2}+6\sqrt{2}a^{2}b^{2}c^{5}d^{3}-4\sqrt{2}a^{3}bc^{4}d^{4}+\sqrt{2}a^{4}c^{3}d^{5}\right)}-\frac{\left(\left(ab^{3}\right)^{\frac{3}{4}}bc-13\left(ab^{3}\right)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}a^{2}b^{4}c^{4}-4\sqrt{2}a^{3}b^{3}c^{3}d+6\sqrt{2}a^{4}b^{2}c^{2}d^{2}-4\sqrt{2}a^{5}bcd^{3}+\sqrt{2}a^{6}d^{4}\right)}+\frac{\left(\left(ab^{3}\right)^{\frac{3}{4}}bc-13\left(ab^{3}\right)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}a^{2}b^{4}c^{4}-4\sqrt{2}a^{3}b^{3}c^{3}d+6\sqrt{2}a^{4}b^{2}c^{2}d^{2}-4\sqrt{2}a^{5}bcd^{3}+\sqrt{2}a^{6}d^{4}\right)}-\frac{\left(117\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-26\left(cd^{3}\right)^{\frac{3}{4}}abcd+5\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^{4}c^{7}d-4\sqrt{2}ab^{3}c^{6}d^{2}+6\sqrt{2}a^{2}b^{2}c^{5}d^{3}-4\sqrt{2}a^{3}bc^{4}d^{4}+\sqrt{2}a^{4}c^{3}d^{5}\right)}+\frac{\left(117\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-26\left(cd^{3}\right)^{\frac{3}{4}}abcd+5\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^{4}c^{7}d-4\sqrt{2}ab^{3}c^{6}d^{2}+6\sqrt{2}a^{2}b^{2}c^{5}d^{3}-4\sqrt{2}a^{3}bc^{4}d^{4}+\sqrt{2}a^{4}c^{3}d^{5}\right)}+\frac{21bcd^{3}x^{\frac{7}{2}}-5ad^{4}x^{\frac{7}{2}}+25bc^{2}d^{2}x^{\frac{3}{2}}-9acd^{3}x^{\frac{3}{2}}}{16\left(b^{3}c^{5}-3ab^{2}c^{4}d+3a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^(3/2)/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b
*x^2 + a)) + 1/4*((a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^4 - 4
*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3
+ sqrt(2)*a^6*d^4) + 1/4*((a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^
4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5
*b*c*d^3 + sqrt(2)*a^6*d^4) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^
(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^
(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^
2 + 6*sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d
^5) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3
)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*sqrt(2)*a^2*b^2*c
^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) - 1/8*((a*b^3)^(3/4
)*b*c - 13*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/
b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*
d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 1/8*((a*b^3)^(3/4)*b*c - 1
3*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqr
t(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*
sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) - 1/64*(117*(c*d^3)^(3/4)*b^2*c^2 -
26*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/
d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*
sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) +
1/64*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4
)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c
^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*
b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) + 1/16*(21*b*c*d^3*x^(7/2) - 5*a*d^4*x^(7/
2) + 25*b*c^2*d^2*x^(3/2) - 9*a*c*d^3*x^(3/2))/((b^3*c^5 - 3*a*b^2*c^4*d +
3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

Mupad [**B**]
time = 4.25, size = 2500, normalized size = 3.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] ((x^(7/2)*(16*b^3*c^3*d - 5*a^3*d^4 + 25*a*b^2*c^2*d^2 + 12*a^2*b*c*d^3))/(
16*a*c*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) - (x^(3/2)*
(8*b^3*c^3 - 9*a^3*d^3 + 25*a^2*b*c*d^2))/(16*a*c*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d^2*x^(11/2)*(8*b^2*c^2 - 5*a^2*d^2 + 21*a
*b*c*d))/(16*a*c*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))/
(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x^6) - atan(
((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 875548
44*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 1
264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(16
777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 20
13265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4
*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19
*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 1919
31351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a
^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^
12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(1/4)*(
(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 8755484
4*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 12
64120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(167
77216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 201
3265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4*
b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19*
d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 19193
1351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a^
11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^1
2*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(3/4)*((
32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 + 38304*
a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*d^8 -
(263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16 - (111
6788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (471127
4035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (409977
699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (80023416
93*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (331589514
3*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (2088923057
*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (69181515*a^19
*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*b^9*c^
6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^27)/8
- (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)/(a^2*b^21*c^27
 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c
^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*
c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10
*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 3
52716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^1
```

4*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21*b^2*c^8*d^19) - (x^(1/2)*(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7 + 618641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 7656663678976*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 35394969403392*a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275600384*a^11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 93255551680512*a^13*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 59448946065408*a^15*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 18584022024192*a^17*b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584*a^19*b^10*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^24 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 173015040*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28...

3.500 $\int \frac{1}{\sqrt{x}\,(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=739

$$\frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\,(c+dx^2)^2}+\frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2}+\frac{d(8b^2c^2+23abcd-7a^2d^2)\,\sqrt{x}}{16ac^2(bc-ad)^3\,(c+dx^2)}-\frac{3b^{11/4}(bc-5}{4}$$

[Out] -3/8*b^(11/4)*(-5*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^4*2^(1/2)+3/8*b^(11/4)*(-5*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^4*2^(1/2)-3/64*d^(7/4)*(7*a^2*d^2-30*a*b*c*d+55*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^4*2^(1/2)+3/64*d^(7/4)*(7*a^2*d^2-30*a*b*c*d+55*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^4*2^(1/2)-3/16*b^(11/4)*(-5*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)^4*2^(1/2)+3/16*b^(11/4)*(-5*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)^4*2^(1/2)-3/128*d^(7/4)*(7*a^2*d^2-30*a*b*c*d+55*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^4*2^(1/2)+3/128*d^(7/4)*(7*a^2*d^2-30*a*b*c*d+55*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^4*2^(1/2)+1/4*d*(a*d+2*b*c)*x^(1/2)/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x^(1/2)/a/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^2+1/16*d*(-7*a^2*d^2+23*a*b*c*d+8*b^2*c^2)*x^(1/2)/a/c^2/(-a*d+b*c)^3/(d*x^2+c)

Rubi [A]

time = 0.67, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {477, 425, 541, 536, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3b^{11/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-5ad)}{4\sqrt{2}\,a^{7/4}(bc-ad)^4}+\frac{3b^{11/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-5ad)}{4\sqrt{2}\,a^{7/4}(bc-ad)^4}-\frac{3b^{11/4}(bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^4}+\frac{3b^{11/4}(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^4}-\frac{3d^{7/4}(7a^2d^2-30abcd+55b^2c^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}(7a^2d^2-30abcd+55b^2c^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^4}+\frac{d\sqrt{x}\,(-7a^2d^2+23abcd+8b^2c^2)}{16ac^2\,(c+dx^2)\,(bc-ad)^3}-\frac{3d^{7/4}(7a^2d^2-30abcd+55b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}(7a^2d^2-30abcd+55b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}(bc-ad)^4}+\frac{b\sqrt{x}}{2a\,(a+bx^2)\,(c+dx^2)^2\,(bc-ad)}+\frac{d\sqrt{x}\,(ad+2bc)}{4ac\,(c+dx^2)^2\,(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]

[Out] (d*(2*b*c + a*d)*Sqrt[x])/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*Sqrt[x])/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 23*a*b*c*d - 7*a^2*d^2)*Sqrt[x])/(16*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (3*b^(11/4)*(b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*b^(11/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) - (3*b^(11/4)*(b*c - 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4

```
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*b^(1
1/4)*(b*c - 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d
 + 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
64*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d +
7*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
Sqrt[2]*c^(11/4)*(b*c - a*d)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\,(a+bx^2)^2\,(c+dx^2)^3}\,dx &= 2\text{Subst}\left(\int \frac{1}{(a+bx^4)^2\,(c+dx^4)^3}\,dx, x, \sqrt{x}\right) \\
&= \frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-11bdx^4}{(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)} \\
&= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\,(c+dx^2)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\,(c+dx^2)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2} + \frac{d(8b^2c^2+\cdots}{16ac^2(b\cdots} \\
&= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\,(c+dx^2)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2} + \frac{d(8b^2c^2+\cdots}{16ac^2(b\cdots} \\
&= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\,(c+dx^2)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2} + \frac{d(8b^2c^2+\cdots}{16ac^2(b\cdots} \\
&= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\,(c+dx^2)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2} + \frac{d(8b^2c^2+\cdots}{16ac^2(b\cdots} \\
&= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\,(c+dx^2)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2} + \frac{d(8b^2c^2+\cdots}{16ac^2(b\cdots} \\
&= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\,(c+dx^2)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^2} + \frac{d(8b^2c^2+\cdots}{16ac^2(b\cdots}
\end{aligned}
$$

Mathematica [A]
time = 3.20, size = 450, normalized size = 0.61

$$
\frac{1}{64}\left(-\frac{4\sqrt{x}\left(8b^3c^2(c+dx^2)^2-a^3d^3(11c+7dx^2)+ab^2cd^2x^2(27c+23dx^2)+a^2bd^2(27c^2+12cdx^2-7d^2x^4)\right)}{ac^2(-bc+ad)^3\,(a+bx^2)\,(c+dx^2)^2}+\frac{24\sqrt{2}\,b^{11/4}(-bc+5ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{7/4}(bc-ad)^4}-\frac{3\sqrt{2}\,d^{7/4}(55b^2c^2-30abcd+7a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{11/4}(bc-ad)^4}+\frac{24\sqrt{2}\,b^{11/4}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{7/4}(bc-ad)^4}+\frac{3\sqrt{2}\,d^{7/4}(55b^2c^2-30abcd+7a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{11/4}(bc-ad)^4}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*Sqrt[x]*(8*b^3*c^2*(c + d*x^2)^2 - a^3*d^3*(11*c + 7*d*x^2) + a*b^2*c*
d^2*x^2*(27*c + 23*d*x^2) + a^2*b*d^2*(27*c^2 + 12*c*d*x^2 - 7*d^2*x^4)))/(
a*c^2*(-(b*c) + a*d)^3*(a + b*x^2)*(c + d*x^2)^2) + (24*Sqrt[2]*b^(11/4)*(-
```

```
(b*c) + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])])/(a^(7/4)*(b*c - a*d)^4) - (3*Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d
+ 7*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])
])/(c^(11/4)*(b*c - a*d)^4) + (24*Sqrt[2]*b^(11/4)*(b*c - 5*a*d)*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(7/4)*(b*c - a*d
)^4) + (3*Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*ArcTanh[(Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(11/4)*(b*c - a*d
)^4))/64
```

Maple [A]

time = 0.19, size = 375, normalized size = 0.51

method	result
derivativedivides	$-\frac{2b^{3}\left(\frac{(ad-bc)\sqrt{x}}{4a\left(b\,x^{2}+a\right)}+\frac{3(5ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right.}{32a^{2}}\right)}{(ad-bc)^{4}}$
default	$-\frac{2b^{3}\left(\frac{(ad-bc)\sqrt{x}}{4a\left(b\,x^{2}+a\right)}+\frac{3(5ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right.}{32a^{2}}\right)}{(ad-bc)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3/(a*d-b*c)^4*(1/4*(a*d-b*c)/a*x^(1/2)/(b*x^2+a)+3/32*(5*a*d-b*c)/a^2*
(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b
)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+
1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d^2/(a*d-b*c)^4*((1/32*d*(7*
a^2*d^2-30*a*b*c*d+23*b^2*c^2)/c^2*x^(5/2)+1/32*(11*a^2*d^2-38*a*b*c*d+27*b
^2*c^2)/c*x^(1/2))/(d*x^2+c)^2+3/256*(7*a^2*d^2-30*a*b*c*d+55*b^2*c^2)/c^3*
(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d
)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.54, size = 951, normalized size = 1.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*(b*c - 5*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) +
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ 2*sqrt(2)*(b*c - 5*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + s
qrt(2)*(b*c - 5*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b*c - 5*a*d)*log(-sqrt(2)*a^(1/4)*b^(1/4)
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^3/(a*b^4*c^4 - 4*a^2*b
^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4) + 1/16*((8*b^3*c^2*
d^2 + 23*a*b^2*c*d^3 - 7*a^2*b*d^4)*x^(9/2) + (16*b^3*c^3*d + 27*a*b^2*c^2*
d^2 + 12*a^2*b*c*d^3 - 7*a^3*d^4)*x^(5/2) + (8*b^3*c^4 + 27*a^2*b*c^2*d^2 -
 11*a^3*c*d^3)*sqrt(x))/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^2 -
a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*
b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a
^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^
5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2) + 3/128*(2*sqrt(2)*(55*b^2*c^
2*d^2 - 30*a*b*c*d^3 + 7*a^2*d^4)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d
))) + 2*sqrt(2)*(55*b^2*c^2*d^2 - 30*a*b*c*d^3 + 7*a^2*d^4)*arctan(-1/2*sqr
t(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(
sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(55*b^2*c^2*d^2 - 30*a*b*c*d^3 + 7
*a^2*d^4)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/
4)*d^(1/4)) - sqrt(2)*(55*b^2*c^2*d^2 - 30*a*b*c*d^3 + 7*a^2*d^4)*log(-sqrt
(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^4*
c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(583) = 1166$.
time = 2.59, size = 1253, normalized size = 1.70

$$\frac{b^3\sqrt{x}}{2\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)\left(bx^2+a\right)}+\frac{3\left(\left(ab^3\right)^{\frac14}b^3c-5\left(ab^3\right)^{\frac14}ab^2d\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\left(\sqrt{2}a^2b^4c^4-4\sqrt{2}a^3b^3c^3d+6\sqrt{2}a^4b^2c^2d^2-4\sqrt{2}a^5bcd^3+\sqrt{2}a^6d^4\right)}+\frac{3\left(\left(ab^3\right)^{\frac14}b^3c-5\left(ab^3\right)^{\frac14}ab^2d\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\left(\sqrt{2}a^2b^4c^4-4\sqrt{2}a^3b^3c^3d+6\sqrt{2}a^4b^2c^2d^2-4\sqrt{2}a^5bcd^3+\sqrt{2}a^6d^4\right)}+\frac{3\left(55\left(cd^3\right)^{\frac14}b^2c^2d-30\left(cd^3\right)^{\frac14}abcd^2+7\left(cd^3\right)^{\frac14}a^2d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{32\left(\sqrt{2}b^4c^7-4\sqrt{2}ab^3c^6d+6\sqrt{2}a^2b^2c^5d^2-4\sqrt{2}a^3bc^4d^3+\sqrt{2}a^4c^3d^4\right)}+\frac{3\left(55\left(cd^3\right)^{\frac14}b^2c^2d-30\left(cd^3\right)^{\frac14}abcd^2+7\left(cd^3\right)^{\frac14}a^2d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{32\left(\sqrt{2}b^4c^7-4\sqrt{2}ab^3c^6d+6\sqrt{2}a^2b^2c^5d^2-4\sqrt{2}a^3bc^4d^3+\sqrt{2}a^4c^3d^4\right)}+\frac{3\left(\left(ab^3\right)^{\frac14}b^3c-5\left(ab^3\right)^{\frac14}ab^2d\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}a^2b^4c^4-4\sqrt{2}a^3b^3c^3d+6\sqrt{2}a^4b^2c^2d^2-4\sqrt{2}a^5bcd^3+\sqrt{2}a^6d^4\right)}-\frac{3\left(\left(ab^3\right)^{\frac14}b^3c-5\left(ab^3\right)^{\frac14}ab^2d\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}a^2b^4c^4-4\sqrt{2}a^3b^3c^3d+6\sqrt{2}a^4b^2c^2d^2-4\sqrt{2}a^5bcd^3+\sqrt{2}a^6d^4\right)}+\frac{3\left(55\left(cd^3\right)^{\frac14}b^2c^2d-30\left(cd^3\right)^{\frac14}abcd^2+7\left(cd^3\right)^{\frac14}a^2d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^4c^7-4\sqrt{2}ab^3c^6d+6\sqrt{2}a^2b^2c^5d^2-4\sqrt{2}a^3bc^4d^3+\sqrt{2}a^4c^3d^4\right)}-\frac{3\left(55\left(cd^3\right)^{\frac14}b^2c^2d-30\left(cd^3\right)^{\frac14}abcd^2+7\left(cd^3\right)^{\frac14}a^2d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^4c^7-4\sqrt{2}ab^3c^6d+6\sqrt{2}a^2b^2c^5d^2-4\sqrt{2}a^3bc^4d^3+\sqrt{2}a^4c^3d^4\right)}+\frac{23bcd^3x^{\frac52}-7ad^4x^{\frac52}+27bc^2d^2\sqrt{x}-11acd^3\sqrt{x}}{16\left(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3\right)\left(dx^2+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b^3*sqrt(x)/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b
*x^2 + a)) + 3/4*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^
4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c
*d^3 + sqrt(2)*a^6*d^4) + 3/4*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*
d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt
(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*s
qrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 3/32*(55*(c*d^3)^(1/4)*b^2*c^2*d -
30*(c*d^3)^(1/4)*a*b*c*d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*arctan(1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a
*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*
a^4*c^3*d^4) + 3/32*(55*(c*d^3)^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1/4)*a*b*c*d^
2 + 7*(c*d^3)^(1/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*s
qrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a*b^3*c^6*d + 6*sqrt(2)*a
^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*a^4*c^3*d^4) + 3/8*((a*b
^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)
*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) - 3/8*((a*b^3)^
(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^
4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 3/64*(55*(c*d^3)
^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1/4)*a*b*c*d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*lo
g(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)
*a*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2
)*a^4*c^3*d^4) - 3/64*(55*(c*d^3)^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1/4)*a*b*c*
d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2
- 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*a^4*c^3*d^4) + 1/16*(23*b*c*d^3*x^(5/2)
 - 7*a*d^4*x^(5/2) + 27*b*c^2*d^2*sqrt(x) - 11*a*c*d^3*sqrt(x))/((b^3*c^5 -
 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

Mupad [**B**]
time = 8.10, size = 2500, normalized size = 3.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] atan(-((((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 114
3142782440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^
59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 469019849064388682475
1104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346
602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a
^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312
40117103771404688424960*a^10*b^52*c^52*d^10 - 23802252231371417628822208512
0*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12
- 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367
585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^
47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701
324613150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061982551585754
13329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*
c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 1234226
492432831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144
979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^3
9*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084
025233921418993848385529708544*a^25*b^37*c^37*d^25 + 1351791876832068562487
1901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^2
7*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28
- 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222
80459690440055062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634
496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^3
0*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 73965997
892283818508917976575508480*a^34*b^28*c^28*d^34 - 6833570476198873825279649
5977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b
^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3
3004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822
79521941129789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088
*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^
41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 1845284865146
033724645937218846720*a^43*b^19*c^19*d^43 + 8022426954872914969051201221427
20*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d
^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 3860660847444
8543697499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*
a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 +
 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821
717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^1
0*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 96372229934943254
3100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^5
5 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147600953241373491226214
4*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 74129824
69913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60
```

- 3324163179957505228 8*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^
(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c
^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^
4 + 132340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 +
 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 +
5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 +
 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^1
2 - 26292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^1
7*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^
15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*
a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 5560978211448
4224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000
583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 16999671
06662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 9791654790
7584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a
^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30
*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^
39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^
31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3
+ 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^
5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^3
6*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^
23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280
*a^18*b^21*c^32*d^11 + 15516365815535370240*a^1...

3.501 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=805

$$\frac{-40b^3c^3+96ab^2c^2d-125a^2bcd^2+45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}}+\frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right)^2}+\frac{b}{2a(bc-ad)\sqrt{x}\,\left(a+bx^2\right)\left(c+dx^2\right)^2}$$

```
[Out] 1/8*b^(13/4)*(-17*a*d+5*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9
/4)/(-a*d+b*c)^4*2^(1/2)-1/8*b^(13/4)*(-17*a*d+5*b*c)*arctan(1+b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/a^(9/4)/(-a*d+b*c)^4*2^(1/2)+1/64*d^(9/4)*(45*a^2*d^2-
170*a*b*c*d+221*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)
/(-a*d+b*c)^4*2^(1/2)-1/64*d^(9/4)*(45*a^2*d^2-170*a*b*c*d+221*b^2*c^2)*arc
tan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^4*2^(1/2)-1/16*b
^(13/4)*(-17*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2
))/a^(9/4)/(-a*d+b*c)^4*2^(1/2)+1/16*b^(13/4)*(-17*a*d+5*b*c)*ln(a^(1/2)+x*
b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)^4*2^(1/2)-1/128
*d^(9/4)*(45*a^2*d^2-170*a*b*c*d+221*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*
d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^4*2^(1/2)+1/128*d^(9/4)*(45*a^
2*d^2-170*a*b*c*d+221*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)
*x^(1/2))/c^(13/4)/(-a*d+b*c)^4*2^(1/2)+1/16*(45*a^3*d^3-125*a^2*b*c*d^2+96
*a*b^2*c^2*d-40*b^3*c^3)/a^2/c^3/(-a*d+b*c)^3/x^(1/2)+1/4*d*(a*d+2*b*c)/a/c
/(-a*d+b*c)^2/(d*x^2+c)^2/x^(1/2)+1/2*b/a/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^2/
x^(1/2)+1/16*d*(-9*a^2*d^2+25*a*b*c*d+8*b^2*c^2)/a/c^2/(-a*d+b*c)^3/(d*x^2+
c)/x^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 805, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {477, 483, 593, 597, 598, 303, 1176, 631, 210, 1179, 642}

$$\frac{(5bc-17ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{13/4}}{8\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{(5bc-17ad)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{13/4}}{8\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)\sqrt{x}\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}-\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}-\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}+\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}-\frac{40b^3c^3-96ab^2dc^2+125a^2bd^2c-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}}+\frac{d\left(8b^2c^2+25abdc-9a^2d^2\right)}{16ac^2(bc-ad)^3\sqrt{x}\left(dx^2+c\right)}+\frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(dx^2+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/16*(40*b^3*c^3 - 96*a*b^2*c^2*d + 125*a^2*b*c*d^2 - 45*a^3*d^3)/(a^2*c^3
*(b*c - a*d)^3*Sqrt[x]) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*Sqrt[x]*(c
 + d*x^2)^2) + b/(2*a*(b*c - a*d)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2) + (d*(
8*b^2*c^2 + 25*a*b*c*d - 9*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*Sqrt[x]*(c + d
*x^2)) + (b^(13/4)*(5*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^4
) + (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d
```

```
^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) - (d^(9/4)*(2
21*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(
9/4)*(b*c - a*d)^4) + (b^(13/4)*(5*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (d^(9
/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) + (d^(9/4
)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
```

```
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)\sqrt{x}\,\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-5bc+4ad-13bdx^4}{x^2(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{}\right.}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)\sqrt{x}\,\left(a+bx^2\right)\left(c+dx^2\right)^2} -$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)\sqrt{x}\,\left(a+bx^2\right)\left(c+dx^2\right)^2} +$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right.}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right.}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right.}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right.}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right.}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right.}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\,\left(c+dx^2\right.}$$

Mathematica [A]

time = 2.11, size = 518, normalized size = 0.64

$$\frac{1}{64}\left(-\frac{4\left(-40b^4c^3x^2(c+dx^2)^2-32ab^3c^2(c-3dx^2)\left(c+dx^2\right)^2+a^4d^3(32c^2+81cdx^2+45d^2x^4)+a^2b^2cd(96c^3+96c^2dx^2-129cd^2x^4-125d^3x^6)+a^3bd^2(-96c^3-193c^2dx^2-44cd^2x^4+45d^3x^6)\right)}{a^2c^3(-bc+ad)^3\sqrt{x}\,(a+bx^2)\left(c+dx^2\right)^2}+\frac{8\sqrt{2}\,b^{13/4}(5bc-17ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{9/4}(bc-ad)^4}+\frac{\sqrt{2}\,d^{9/4}(221b^2c^2-170abcd+45a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{13/4}(bc-ad)^4}+\frac{8\sqrt{2}\,b^{13/4}(5bc-17ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{9/4}(bc-ad)^4}+\frac{\sqrt{2}\,d^{9/4}(221b^2c^2-170abcd+45a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{13/4}(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*(-40*b^4*c^3*x^2*(c + d*x^2)^2 - 32*a*b^3*c^2*(c - 3*d*x^2)*(c + d*x^2
)^2 + a^4*d^3*(32*c^2 + 81*c*d*x^2 + 45*d^2*x^4) + a^2*b^2*c*d*(96*c^3 + 96
*c^2*d*x^2 - 129*c*d^2*x^4 - 125*d^3*x^6) + a^3*b*d^2*(-96*c^3 - 193*c^2*d*
x^2 - 44*c*d^2*x^4 + 45*d^3*x^6)))/(a^2*c^3*(-(b*c) + a*d)^3*Sqrt[x]*(a + b
*x^2)*(c + d*x^2)^2) + (8*Sqrt[2]*b^(13/4)*(5*b*c - 17*a*d)*ArcTan[(Sqrt[a]
 - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(9/4)*(b*c - a*d)^4) +
 (Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[(Sqrt[c]
- Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(13/4)*(b*c - a*d)^4) +
 (8*Sqrt[2]*b^(13/4)*(5*b*c - 17*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(9/4)*(b*c - a*d)^4) + (Sqrt[2]*d^(9/4)*(22
1*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(13/4)*(b*c - a*d)^4))/64
```

Maple [A]

time = 0.27, size = 385, normalized size = 0.48

method	result
derivativedivides	$\frac{2b^4\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{17ad}{4}-\frac{5bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^2(ad-bc)^4}$
default	$\frac{2b^4\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{17ad}{4}-\frac{5bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^2(ad-bc)^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4/a^2/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(17/4*a*d-5/
4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
)/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2*d^3/c^3/(a*d-b*c)^4
```

```
*((1/32*d*(13*a^2*d^2-42*a*b*c*d+29*b^2*c^2)*x^(7/2)+(17/32*a^2*c*d^2-25/16
*a*b*c^2*d+33/32*b^2*c^3)*x^(3/2))/(d*x^2+c)^2+1/8*(45/32*a^2*d^2-85/16*a*b
*c*d+221/32*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/a^2/c
^3/x^(1/2)
```

Maxima [**A**]
time = 0.59, size = 955, normalized size = 1.19

$$-\frac{(5b^5c-17ab^4d)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\left(a^2b^4c^4-4a^3b^3c^3d+6a^4b^2c^2d^2-4a^5bcd^3+a^6d^4\right)}-\frac{(221b^2c^2d^3-170abcd^4+45a^2d^5)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^4c^7-4ab^3c^6d+6a^2b^2c^5d^2-4a^3bc^4d^3+a^4c^3d^4\right)}-\frac{32ab^3c^5-96a^2b^2c^4d+96a^3bc^3d^2-32a^4c^2d^3+(40b^4c^3d^2-96ab^3c^2d^3+125a^2b^2cd^4-45a^3bd^5)x^6+(80b^4c^4d-160ab^3c^3d^2+129a^2b^2c^2d^3+44a^3bcd^4-45a^4d^5)x^4+(40b^4c^5-32ab^3c^4d-96a^2b^2c^3d^2+193a^3bc^2d^3-81a^4cd^4)x^2}{16\left((a^2b^4c^6d^2-3a^3b^3c^5d^3+3a^4b^2c^4d^4-a^5bc^3d^5)x^{\frac{13}{2}}+(2a^2b^4c^7d-5a^3b^3c^6d^2+3a^4b^2c^5d^3+a^5bc^4d^4-a^6c^3d^5)x^{\frac{9}{2}}+(a^2b^4c^8-a^3b^3c^7d-3a^4b^2c^6d^2+5a^5bc^5d^3-2a^6c^4d^4)x^{\frac{5}{2}}+(a^3b^3c^8-3a^4b^2c^7d+3a^5bc^6d^2-a^6c^5d^3)\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(5*b^5*c - 17*a*b^4*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))
*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(
2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3
/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(1/4)*b^(3/4)))/(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a
^5*b*c*d^3 + a^6*d^4) - 1/128*(221*b^2*c^2*d^3 - 170*a*b*c*d^4 + 45*a^2*d^5
)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x
))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*s
qrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)
*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^4*c^7
 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4) - 1/1
6*(32*a*b^3*c^5 - 96*a^2*b^2*c^4*d + 96*a^3*b*c^3*d^2 - 32*a^4*c^2*d^3 + (4
0*b^4*c^3*d^2 - 96*a*b^3*c^2*d^3 + 125*a^2*b^2*c*d^4 - 45*a^3*b*d^5)*x^6 +
(80*b^4*c^4*d - 160*a*b^3*c^3*d^2 + 129*a^2*b^2*c^2*d^3 + 44*a^3*b*c*d^4 -
45*a^4*d^5)*x^4 + (40*b^4*c^5 - 32*a*b^3*c^4*d - 96*a^2*b^2*c^3*d^2 + 193*a
^3*b*c^2*d^3 - 81*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3
*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^(13/2) + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c
^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^(9/2) + (a^2*b^
4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4
)*x^(5/2) + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3)
*sqrt(x))
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. 2(645) = 1290.
time = 2.90, size = 1333, normalized size = 1.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(
2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt
(2)*a^7*d^4) - 1/4*(5*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^
4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6
*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)
^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d
)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d
+ 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4
) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3
)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d
^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) + 1/8*(5*(a*b^3)^(3/4)*
b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^
2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/8*(5*(a*b^3)^(3/4)*b^2
*c - 17*(a*b^3)^(3/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/
b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*
d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) + 1/64*(221*(c*d^3)^(3/4)*b^
2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*s
```

```
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*
d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d
^4) - 1/64*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d
^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(
2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*
a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) - 1/2*(5*b^4*c^3*x^2 - 12*a*b^3*c^2*d*
x^2 + 12*a^2*b^2*c*d^2*x^2 - 4*a^3*b*d^3*x^2 + 4*a*b^3*c^3 - 12*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 - 4*a^4*d^3)/((a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*
c^4*d^2 - a^5*c^3*d^3)*(b*x^(5/2) + a*sqrt(x))) - 1/16*(29*b*c*d^4*x^(7/2)
- 13*a*d^5*x^(7/2) + 33*b*c^2*d^3*x^(3/2) - 17*a*c*d^4*x^(3/2))/((b^3*c^6 -
 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^2)
```

Mupad [B]
time = 11.67, size = 2500, normalized size = 3.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] 2*atan((((-(8398080000*a^33*d^33 - (7052774768640000000*a^66*d^66 + 274877
90694400000000*b^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 8523
959496286928896000000*a^3*b^63*c^63*d^3 + 1130310047981633536000000*a^4*b^6
2*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 94660933391357
8145788723200*a^6*b^60*c^60*d^6 - 63988382063497445934681292800*a^7*b^59*c^5
9*d^7 + 363943805075927975134589091840*a^8*b^58*c^58*d^8 - 17682391555307866
7757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*
c^56*d^10 - 272041584290086689049656950784*a^11*b^55*c^55*d^11 + 876084883
86430107181928939520000*a^12*b^54*c^54*d^12 - 249552350040826187070412286853
12*a^13*b^53*c^53*d^13 + 6321444674258436379964151850598 4*a^14*b^52*c^52*d^
14 - 14313378011069462050587268035379 2*a^15*b^51*c^51*d^15 + 29143271303237
7964853953403289600*a^16*b^50*c^50*d^16 - 53837688933932732209219051192320 0
*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^1
8 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 23701242613793
32590916233678815232*a^20*b^46*c^46*d^20 - 394568205038255080146693645139 96
80*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*
d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814
015359003912068230073548 8*a^24*b^42*c^42*d^24 - 390224984604071598537729189
44169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^4
0*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129
947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012 8
4104911182626816 0*a^29*b^37*c^37*d^29 + 2431372713606781682807248874425548 8
0*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35
*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807
3329926280441232590010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050 56
```

4698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a
^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d
^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093
9745327887207329057621409792*a^38*b^28*c^28*d^38 - 273939082748055449346653
4979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a
^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d
^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380
5867374689790053020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361
710073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^4
5*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46
 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217
1953043067855358132224*a^48*b^18*c^18*d^48 - 323873725819524777875553934355
98848*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c
^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965
466014670506117809664163840*a^52*b^14*c^14*d^52 - 4763713185671452589806061
61715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*
c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836
15003612481702592512000*a^56*b^10*c^10*d^56 - 13472186556040911549104128000
00*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 3
3942156347965157513625600000*a^59*b^7*c^7*d^59 + 42954568799822401241088000
00*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 3950
4294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b
^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 1627277209108480
000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 524288
0000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*
c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^2
8*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26
*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24
*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*
c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b
^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*
a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173
068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 79006
5381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20
+ 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^1
1*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b
^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*
b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b
^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c
^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 26
1273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^1
3*d^32 - 2199023255552*a^10*b^31*c^44*d - 21990...

3.502 $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=805

$$\frac{-56b^3c^3+96ab^2c^2d-189a^2bcd^2+77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}}+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2}+\frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+d\right.}$$

```
[Out] 1/48*(77*a^3*d^3-189*a^2*b*c*d^2+96*a*b^2*c^2*d-56*b^3*c^3)/a^2/c^3/(-a*d+b
*c)^3/x^(3/2)+1/4*d*(a*d+2*b*c)/a/c/(-a*d+b*c)^2/x^(3/2)/(d*x^2+c)^2+1/2*b/
a/(-a*d+b*c)/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2+1/16*d*(-11*a^2*d^2+27*a*b*c*d+8
*b^2*c^2)/a/c^2/(-a*d+b*c)^3/x^(3/2)/(d*x^2+c)+1/8*b^(15/4)*(-19*a*d+7*b*c)
*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/(-a*d+b*c)^4*2^(1/2)-1/
8*b^(15/4)*(-19*a*d+7*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/
4)/(-a*d+b*c)^4*2^(1/2)+1/64*d^(11/4)*(77*a^2*d^2-266*a*b*c*d+285*b^2*c^2)*
arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(15/4)/(-a*d+b*c)^4*2^(1/2)-1/6
4*d^(11/4)*(77*a^2*d^2-266*a*b*c*d+285*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^
(1/2)/c^(1/4))/c^(15/4)/(-a*d+b*c)^4*2^(1/2)+1/16*b^(15/4)*(-19*a*d+7*b*c)*
ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/(-a*d+b*c)^4
*2^(1/2)-1/16*b^(15/4)*(-19*a*d+7*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)
*2^(1/2)*x^(1/2))/a^(11/4)/(-a*d+b*c)^4*2^(1/2)+1/128*d^(11/4)*(77*a^2*d^2-
266*a*b*c*d+285*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/
2))/c^(15/4)/(-a*d+b*c)^4*2^(1/2)-1/128*d^(11/4)*(77*a^2*d^2-266*a*b*c*d+28
5*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)/(
-a*d+b*c)^4*2^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 805, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {477, 483, 593, 597, 536, 217, 1179, 642, 1176, 631, 210}

$$\frac{(7bc-19ad)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{15/4}}{4\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{(7bc-19ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{15/4}}{8\sqrt{2}a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{15/4}}{8\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)x^{3/2}(bx^2+a)(dx^2+c)^2}+\frac{d^{11/4}(285b^2c^2-266abdc+77a^2d^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4}-\frac{d^{11/4}(285b^2c^2-266abdc+77a^2d^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4}+\frac{d^{11/4}(285b^2c^2-266abdc+77a^2d^2)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{15/4}(bc-ad)^4}-\frac{d^{11/4}(285b^2c^2-266abdc+77a^2d^2)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{15/4}(bc-ad)^4}+\frac{d(8b^2c^2+27abdc-11a^2d^2)}{16ac^2(bc-ad)^3x^{3/2}(dx^2+c)}-\frac{56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}}+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}(dx^2+c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/48*(56*b^3*c^3 - 96*a*b^2*c^2*d + 189*a^2*b*c*d^2 - 77*a^3*d^3)/(a^2*c^3
*(b*c - a*d)^3*x^(3/2)) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^(3/2)*(c
 + d*x^2)^2) + b/(2*a*(b*c - a*d)*x^(3/2)*(a + b*x^2)*(c + d*x^2)^2) + (d*(
8*b^2*c^2 + 27*a*b*c*d - 11*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*x^(3/2)*(c +
d*x^2)) + (b^(15/4)*(7*b*c - 19*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d
)^4) + (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[
```

```
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) - (d^(11/
4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) + (b^(15/4)*(7*b*c - 19
*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2
]*a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^4)
+ (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) -
 (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-7bc+4ad-15bdx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \text{Su}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(}{16a}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2}$$

Mathematica [A]
time = 6.12, size = 521, normalized size = 0.65

$$\frac{1}{192}\left(-\frac{4\left(-56b^4c^3x^2\left(c+dx^2\right)^2-32ab^3c^2\left(c-3dx^2\right)\left(c+dx^2\right)^2+a^4d^3\left(32c^2+121cdx^2+77d^2x^4\right)+3a^2b^2cd\left(32c^3+32c^2dx^2-67cd^2x^4-63d^3x^6\right)+a^3bd^2\left(-96c^3-265c^2dx^2-68cd^2x^4+77d^3x^6\right)\right)}{a^2c^3(-bc+ad)^3x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}+\frac{24\sqrt{2}\,b^{15/4}(7bc-19ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{11/4}(bc-ad)^4}+\frac{3\sqrt{2}\,d^{11/4}\left(285b^2c^2-266abcd+77a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{15/4}(bc-ad)^4}+\frac{24\sqrt{2}\,b^{15/4}(-7bc+19ad)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{11/4}(bc-ad)^4}-\frac{3\sqrt{2}\,d^{11/4}\left(285b^2c^2-266abcd+77a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{15/4}(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]

[Out] ((-4*(-56*b^4*c^3*x^2*(c + d*x^2)^2 - 32*a*b^3*c^2*(c - 3*d*x^2)*(c + d*x^2
)^2 + a^4*d^3*(32*c^2 + 121*c*d*x^2 + 77*d^2*x^4) + 3*a^2*b^2*c*d*(32*c^3 +
 32*c^2*d*x^2 - 67*c*d^2*x^4 - 63*d^3*x^6) + a^3*b*d^2*(-96*c^3 - 265*c^2*d
*x^2 - 68*c*d^2*x^4 + 77*d^3*x^6)))/(a^2*c^3*(-(b*c) + a*d)^3*x^(3/2)*(a +
b*x^2)*(c + d*x^2)^2) + (24*Sqrt[2]*b^(15/4)*(7*b*c - 19*a*d)*ArcTan[(Sqrt[
a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(11/4)*(b*c - a*d)^4
) + (3*Sqrt[2]*d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[(Sq
rt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(15/4)*(b*c - a*d
)^4) + (24*Sqrt[2]*b^(15/4)*(-7*b*c + 19*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(11/4)*(b*c - a*d)^4) - (3*Sqrt[2]*
d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(15/4)*(b*c - a*d)^4))/192

Maple [A]
time = 0.35, size = 385, normalized size = 0.48

method	result
derivativedivides	$\frac{2b^4\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(19ad-7bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right)\right)}{32a}\cdots\right)}{a^2(ad-bc)^4}$
default	$\frac{2b^4\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(19ad-7bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right)\right)}{32a}\cdots\right)}{a^2(ad-bc)^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] 2*b^4/a^2/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(19*a*d-7*b
c)(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2*d^3/c^3/(a*d-b*c)^4*((
(15/32*a^2*d^3-23/16*a*b*c*d^2+31/32*b^2*c^2*d)*x^(5/2)+1/32*c*(19*a^2*d^2-
54*a*b*c*d+35*b^2*c^2)*x^(1/2))/(d*x^2+c)^2+1/256*(77*a^2*d^2-266*a*b*c*d+2
85*b^2*c^2)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^
(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^

```
(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/3/a^2/c^3/x^(3
/2)
```

Maxima [A]
time = 0.54, size = 1064, normalized size = 1.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(7*b*c - 19*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + 2*sqrt(2)*(7*b*c - 19*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + sqrt(2)*(7*b*c - 19*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(7*b*c - 19*a*d)*log(-sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^4/(a^2*b^
4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4) - 1/
48*(32*a*b^3*c^5 - 96*a^2*b^2*c^4*d + 96*a^3*b*c^3*d^2 - 32*a^4*c^2*d^3 + (
56*b^4*c^3*d^2 - 96*a*b^3*c^2*d^3 + 189*a^2*b^2*c*d^4 - 77*a^3*b*d^5)*x^6 +
 (112*b^4*c^4*d - 160*a*b^3*c^3*d^2 + 201*a^2*b^2*c^2*d^3 + 68*a^3*b*c*d^4
- 77*a^4*d^5)*x^4 + (56*b^4*c^5 - 32*a*b^3*c^4*d - 96*a^2*b^2*c^3*d^2 + 265
*a^3*b*c^2*d^3 - 121*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3
+ 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^(15/2) + (2*a^2*b^4*c^7*d - 5*a^3*b^
3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^(11/2) + (a^
2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4
*d^4)*x^(7/2) + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*
d^3)*x^(3/2)) - 1/128*(2*sqrt(2)*(285*b^2*c^2*d^3 - 266*a*b*c*d^4 + 77*a^2*
d^5)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(285*b^2*c^2*
d^3 - 266*a*b*c*d^4 + 77*a^2*d^5)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(
d))) + sqrt(2)*(285*b^2*c^2*d^3 - 266*a*b*c*d^4 + 77*a^2*d^5)*log(sqrt(2)*c
^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(
285*b^2*c^2*d^3 - 266*a*b*c*d^4 + 77*a^2*d^5)*log(-sqrt(2)*c^(1/4)*d^(1/4)*
sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^4*c^7 - 4*a*b^3*c^6*d
+ 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)

[Out] Timed out

Giac [**A**]
time = 1.80, size = 1278, normalized size = 1.59

$$-\frac{b^4\sqrt{x}}{2\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)(bx^2+a)}-\frac{\left(7\,(ab^3)^{\frac14}b^4c-19\,(ab^3)^{\frac14}ab^3d\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\left(\sqrt{2}a^3b^4c^4-4\sqrt{2}a^4b^3c^3d+6\sqrt{2}a^5b^2c^2d^2-4\sqrt{2}a^6bcd^3+\sqrt{2}a^7d^4\right)}-\frac{\left(7\,(ab^3)^{\frac14}b^4c-19\,(ab^3)^{\frac14}ab^3d\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{4\left(\sqrt{2}a^3b^4c^4-4\sqrt{2}a^4b^3c^3d+6\sqrt{2}a^5b^2c^2d^2-4\sqrt{2}a^6bcd^3+\sqrt{2}a^7d^4\right)}-\frac{\left(285\,(cd^3)^{\frac14}b^2c^2d^2-266\,(cd^3)^{\frac14}abcd^3+77\,(cd^3)^{\frac14}a^2d^4\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{32\left(\sqrt{2}b^4c^8-4\sqrt{2}ab^3c^7d+6\sqrt{2}a^2b^2c^6d^2-4\sqrt{2}a^3bc^5d^3+\sqrt{2}a^4c^4d^4\right)}-\frac{\left(285\,(cd^3)^{\frac14}b^2c^2d^2-266\,(cd^3)^{\frac14}abcd^3+77\,(cd^3)^{\frac14}a^2d^4\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{32\left(\sqrt{2}b^4c^8-4\sqrt{2}ab^3c^7d+6\sqrt{2}a^2b^2c^6d^2-4\sqrt{2}a^3bc^5d^3+\sqrt{2}a^4c^4d^4\right)}-\frac{\left(7\,(ab^3)^{\frac14}b^4c-19\,(ab^3)^{\frac14}ab^3d\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}a^3b^4c^4-4\sqrt{2}a^4b^3c^3d+6\sqrt{2}a^5b^2c^2d^2-4\sqrt{2}a^6bcd^3+\sqrt{2}a^7d^4\right)}+\frac{\left(7\,(ab^3)^{\frac14}b^4c-19\,(ab^3)^{\frac14}ab^3d\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{8\left(\sqrt{2}a^3b^4c^4-4\sqrt{2}a^4b^3c^3d+6\sqrt{2}a^5b^2c^2d^2-4\sqrt{2}a^6bcd^3+\sqrt{2}a^7d^4\right)}-\frac{\left(285\,(cd^3)^{\frac14}b^2c^2d^2-266\,(cd^3)^{\frac14}abcd^3+77\,(cd^3)^{\frac14}a^2d^4\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^4c^8-4\sqrt{2}ab^3c^7d+6\sqrt{2}a^2b^2c^6d^2-4\sqrt{2}a^3bc^5d^3+\sqrt{2}a^4c^4d^4\right)}+\frac{\left(285\,(cd^3)^{\frac14}b^2c^2d^2-266\,(cd^3)^{\frac14}abcd^3+77\,(cd^3)^{\frac14}a^2d^4\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{64\left(\sqrt{2}b^4c^8-4\sqrt{2}ab^3c^7d+6\sqrt{2}a^2b^2c^6d^2-4\sqrt{2}a^3bc^5d^3+\sqrt{2}a^4c^4d^4\right)}-\frac{31\,bcd^4x^{\frac52}-15\,ad^5x^{\frac52}+35\,bc^2d^3\sqrt{x}-19\,acd^4\sqrt{x}}{16\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)(dx^2+c)^2}-\frac{2}{3\,a^2c^3x^{\frac32}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")

[Out] -1/2*b^4*sqrt(x)/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(b*x^2 + a)) - 1/4*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/4*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/32*(285*(c*d^3)^(1/4)*b^2*c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) - 1/32*(285*(c*d^3)^(1/4)*b^2*c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) - 1/8*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) + 1/8*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/64*(285*(c*d^3)^(1/4)*b^2*c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*

sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) + 1/64*(285*(c*d^3)^(1/4)*b^2*
c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*log(-sqrt
(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3
*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*
c^4*d^4) - 1/16*(31*b*c*d^4*x^(5/2) - 15*a*d^5*x^(5/2) + 35*b*c^2*d^3*sqrt(
x) - 19*a*c*d^4*sqrt(x))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*
c^3*d^3)*(d*x^2 + c)^2) - 2/3/(a^2*c^3*x^(3/2))

Mupad [B]
time = 8.73, size = 2500, normalized size = 3.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] atan(((x^(1/2)*(857712418202478182400*a^18*b^48*c^62*d^11 - 289253302176664
30894080*a^19*b^47*c^61*d^12 + 465808355868544602210304*a^20*b^46*c^60*d^13
 - 4772189938359453553262592*a^21*b^45*c^59*d^14 + 349820765298262334012129
28*a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^23*b^43*c^57*d^16 +
873231122236416493313064960*a^24*b^42*c^56*d^17 - 3201588318340888739356606
464*a^25*b^41*c^55*d^18 + 9904866981547362725832687616*a^26*b^40*c^54*d^19
- 26475613142538536817178705920*a^27*b^39*c^53*d^20 + 625280040368754051508
57986048*a^28*b^38*c^52*d^21 - 133143680796215491474489344000*a^29*b^37*c^5
1*d^22 + 259595474982835164713400139776*a^30*b^36*c^50*d^23 - 4671065777388
76991145070559232*a^31*b^35*c^49*d^24 + 775321096823109302674935250944*a^32
*b^34*c^48*d^25 - 1179424943892680059222782640128*a^33*b^33*c^47*d^26 + 162
9690593600095833823295569920*a^34*b^32*c^46*d^27 - 202814334571931467607479
5761664*a^35*b^31*c^45*d^28 + 2257905973104023956972306956288*a^36*b^30*c^4
4*d^29 - 2237449183565830435563494178816*a^37*b^29*c^43*d^30 + 196620485445
7469918399988498432*a^38*b^28*c^42*d^31 - 1527649406048366621262568488960*a
^39*b^27*c^41*d^32 + 1046409458758522347995126562816*a^40*b^26*c^40*d^33 -
629956523592774331698776113152*a^41*b^25*c^39*d^34 + 3320657643355840042301
53764864*a^42*b^24*c^38*d^35 - 152543196968133650922715742208*a^43*b^23*c^3
7*d^36 + 60699171433471101739298979840*a^44*b^22*c^36*d^37 - 20757436699772
395749793333248*a^45*b^21*c^35*d^38 + 6037825951797032255320227840*a^46*b^2
0*c^34*d^39 - 1473449639082715479512449024*a^47*b^19*c^33*d^40 + 2960843394
24033093684559872*a^48*b^18*c^32*d^41 - 47717950421254308290887680*a^49*b^1
7*c^31*d^42 + 5931528400797457427988480*a^50*b^16*c^30*d^43 - 5340378611857
24002336768*a^51*b^15*c^29*d^44 + 31006369751209579905024*a^52*b^14*c^28*d^
45 - 872067188534894657536*a^53*b^13*c^27*d^46) + (-(((143986855936*a^35*d^
35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 10675201612
1856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 23907154306007
04*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 188295341785743
36*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110
```

464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771
167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841
37459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11
2491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^
16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^1
7*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328
*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677
746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4
293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10
*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8
*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b
^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b
^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c
^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 -
(4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 23112188256
1500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*
a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271
*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 1649241388010969
2*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 594157271624297
5*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*
a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^3
2 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084
860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152
383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273
040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7
22812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d
^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^2
1*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 238713320239005696
00*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616
9296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^1
6 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^
14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370
240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247
375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 +
 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d
^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^2
0*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^1
8*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1...

3.503 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=881

$$-\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}}+\frac{3(24b^4c^4-32ab^3c^3d-32a^2b^2c^2d^2+87a^3bcd^3-39a^4d^4)}{16a^3c^4(bc-ad)^3\sqrt{x}}+\frac{\ldots}{4ac\ldots}$$

```
[Out] -3/80*(-39*a^3*d^3+87*a^2*b*c*d^2-32*a*b^2*c^2*d+24*b^3*c^3)/a^2/c^3/(-a*d+
b*c)^3/x^(5/2)+1/4*d*(a*d+2*b*c)/a/c/(-a*d+b*c)^2/x^(5/2)/(d*x^2+c)^2+1/2*b
/a/(-a*d+b*c)/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2+1/16*d*(-13*a^2*d^2+29*a*b*c*d+
8*b^2*c^2)/a/c^2/(-a*d+b*c)^3/x^(5/2)/(d*x^2+c)-3/8*b^(17/4)*(-7*a*d+3*b*c)
*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/(-a*d+b*c)^4*2^(1/2)+3/
8*b^(17/4)*(-7*a*d+3*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4
)/(-a*d+b*c)^4*2^(1/2)-3/64*d^(13/4)*(39*a^2*d^2-126*a*b*c*d+119*b^2*c^2)*a
rctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(17/4)/(-a*d+b*c)^4*2^(1/2)+3/64
*d^(13/4)*(39*a^2*d^2-126*a*b*c*d+119*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(
1/2)/c^(1/4))/c^(17/4)/(-a*d+b*c)^4*2^(1/2)+3/16*b^(17/4)*(-7*a*d+3*b*c)*ln
(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/(-a*d+b*c)^4*2
^(1/2)-3/16*b^(17/4)*(-7*a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^
(1/2)*x^(1/2))/a^(13/4)/(-a*d+b*c)^4*2^(1/2)+3/128*d^(13/4)*(39*a^2*d^2-126
*a*b*c*d+119*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))
/c^(17/4)/(-a*d+b*c)^4*2^(1/2)-3/128*d^(13/4)*(39*a^2*d^2-126*a*b*c*d+119*b
^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)/(-a*
d+b*c)^4*2^(1/2)+3/16*(-39*a^4*d^4+87*a^3*b*c*d^3-32*a^2*b^2*c^2*d^2-32*a*b
^3*c^3*d+24*b^4*c^4)/a^3/c^4/(-a*d+b*c)^3/x^(1/2)
```

Rubi [A]
time = 1.14, antiderivative size = 881, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {477, 483, 593, 597, 598, 303, 1176, 631, 210, 1179, 642}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*(24*b^3*c^3 - 32*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 39*a^3*d^3))/(80*a^2*c^
3*(b*c - a*d)^3*x^(5/2)) + (3*(24*b^4*c^4 - 32*a*b^3*c^3*d - 32*a^2*b^2*c^2
*d^2 + 87*a^3*b*c*d^3 - 39*a^4*d^4))/(16*a^3*c^4*(b*c - a*d)^3*Sqrt[x]) + (
d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)^2) + b/(2*a*(b*c
- a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 29*a*b*c*d - 13
*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*x^(5/2)*(c + d*x^2)) - (3*b^(17/4)*(3*b*
c - 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/
```

```
4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119*b
^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*b^2*c^2 - 126
*a*b*c*d + 39*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*S
qrt[2]*c^(17/4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*
d)^4) - (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*
b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119
*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-9bc+4ad-17bdx^4}{x^6(a+bx^4)(c+dx^4)^3}\,dx, x, \sqrt{x}\right.}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \text{Su}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(}{16a}$$

$$= -\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(c+dx^2\right.}$$

$$= -\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3(24b^4c^4-32ab^3c^3d-32a}{16a^3c^4(bc}$$

$$= -\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3(24b^4c^4-32ab^3c^3d-32a}{16a^3c^4(bc}$$

$$= -\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3(24b^4c^4-32ab^3c^3d-32a}{16a^3c^4(bc}$$

$$= -\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3(24b^4c^4-32ab^3c^3d-32a}{16a^3c^4(bc}$$

$$= -\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3(24b^4c^4-32ab^3c^3d-32a}{16a^3c^4(bc}$$

$$= -\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3(24b^4c^4-32ab^3c^3d-32a}{16a^3c^4(bc}$$

$$= -\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3(24b^4c^4-32ab^3c^3d-32a}{16a^3c^4(bc}$$

Mathematica [A]

time = 1.97, size = 598, normalized size = 0.68

$$\frac{1}{320}\left(\frac{4\left(-360b^5c^4x^4(c+dx^2)^2-96ab^4c^3x^2(3c-5dx^2)(c+dx^2)^2+32a^2b^3c^2(c+dx^2)^2(c^2+12cdx^2+15d^2x^4)+a^5d^3(-32c^3+416c^2dx^2+1053cd^2x^4+585d^3x^6)+a^4bd^2(96c^4-960c^3dx^2-1933c^2d^2x^4-252cd^3x^6+585d^4x^8)-a^3b^2cd(96c^4-384c^3dx^2+64c^2d^2x^4+1869cd^3x^6+1305d^4x^8)\right)}{a^3c^4(-bc+ad)^3x^{5/2}(a+bx^2)(c+dx^2)^2}+\frac{120\sqrt{2}\,b^{17/4}(-3bc+7ad)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{13/4}(bc-ad)^4}-\frac{15\sqrt{2}\,d^{13/4}(119b^2c^2-126abcd+39a^2d^2)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{c^{17/4}(bc-ad)^4}+\frac{120\sqrt{2}\,b^{17/4}(-3bc+7ad)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{13/4}(bc-ad)^4}-\frac{15\sqrt{2}\,d^{13/4}(119b^2c^2-126abcd+39a^2d^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{c^{17/4}(bc-ad)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((4*(-360*b^5*c^4*x^4*(c + d*x^2)^2 - 96*a*b^4*c^3*x^2*(3*c - 5*d*x^2)*(c +
 d*x^2)^2 + 32*a^2*b^3*c^2*(c + d*x^2)^2*(c^2 + 12*c*d*x^2 + 15*d^2*x^4) +
a^5*d^3*(-32*c^3 + 416*c^2*d*x^2 + 1053*c*d^2*x^4 + 585*d^3*x^6) + a^4*b*d^
2*(96*c^4 - 960*c^3*d*x^2 - 1933*c^2*d^2*x^4 - 252*c*d^3*x^6 + 585*d^4*x^8)
 - a^3*b^2*c*d*(96*c^4 - 384*c^3*d*x^2 + 64*c^2*d^2*x^4 + 1869*c*d^3*x^6 +
1305*d^4*x^8)))/(a^3*c^4*(-(b*c) + a*d)^3*x^(5/2)*(a + b*x^2)*(c + d*x^2)^2
) + (120*Sqrt[2]*b^(17/4)*(-3*b*c + 7*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(13/4)*(b*c - a*d)^4) - (15*Sqrt[2]*d^(
13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)
/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(17/4)*(b*c - a*d)^4) + (120*Sqrt[2
]*b^(17/4)*(-3*b*c + 7*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt
[a] + Sqrt[b]*x)])/(a^(13/4)*(b*c - a*d)^4) - (15*Sqrt[2]*d^(13/4)*(119*b^2
*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/
(Sqrt[c] + Sqrt[d]*x)])/(c^(17/4)*(b*c - a*d)^4))/320
```

Maple [A]

time = 0.41, size = 405, normalized size = 0.46

method	result
derivativedivides	$-\frac{2b^5\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{21ad}{4}-\frac{9bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{(\cdots}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^3(ad-bc)^4}$
default	$-\frac{2b^5\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{21ad}{4}-\frac{9bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{(\cdots}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^3(ad-bc)^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^5/a^3/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(21/4*a*d-9
/4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
```

```
))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d^4/c^4/(a*d-b*c)^
4*(((21/32*a^2*d^3-29/16*a*b*c*d^2+37/32*b^2*c^2*d)*x^(7/2)+1/32*c*(25*a^2*
d^2-66*a*b*c*d+41*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+1/8*(117/32*a^2*d^2-189/16*
a*b*c*d+357/32*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/5/
a^2/c^3/x^(5/2)-2*(-3*a*d-2*b*c)/a^3/c^4/x^(1/2)
```

Maxima [A]
time = 0.54, size = 1066, normalized size = 1.21

$$\frac{3\,(3\,b^6c-7\,ab^5d)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac14}b^{\frac14}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}\right)}{16\,(a^3b^4c^4-4\,a^4b^3c^3d+6\,a^5b^2c^2d^2-4\,a^6bcd^3+a^7d^4)}+\frac{3\,(119\,b^2c^2d^4-126\,abcd^5+39\,a^2d^6)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac14}d^{\frac14}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac14}d^{\frac14}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac14}d^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac14}d^{\frac34}}\right)}{128\,(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4)}-\frac{32\,a^2b^3c^6-96\,a^3b^2c^5d+96\,a^4bc^4d^2-32\,a^5c^3d^3-15\,(24\,b^5c^4d^2-32\,ab^4c^3d^3-32\,a^2b^3c^2d^4+87\,a^3b^2cd^5-39\,a^4bd^6)x^8-3\,(240\,b^5c^5d-224\,ab^4c^4d^2-448\,a^2b^3c^3d^3+623\,a^3b^2c^2d^4+84\,a^4bcd^5-195\,a^5d^6)x^6-(360\,b^5c^6+96\,ab^4c^5d-1280\,a^2b^3c^4d^2+64\,a^3b^2c^3d^3+1933\,a^4bc^2d^4-1053\,a^5cd^5)x^4-32\,(9\,ab^4c^6-14\,a^2b^3c^5d-12\,a^3b^2c^4d^2+30\,a^4bc^3d^3-13\,a^5c^2d^4)x^2}{80\left((a^3b^4c^7d^2-3\,a^4b^3c^6d^3+3\,a^5b^2c^5d^4-a^6bc^4d^5)x^{\frac{17}{2}}+(2\,a^3b^4c^8d-5\,a^4b^3c^7d^2+3\,a^5b^2c^6d^3+a^6bc^5d^4-a^7c^4d^5)x^{\frac{13}{2}}+(a^3b^4c^9-a^4b^3c^8d-3\,a^5b^2c^7d^2+5\,a^6bc^6d^3-2\,a^7c^5d^4)x^{\frac92}+(a^4b^3c^9-3\,a^5b^2c^8d+3\,a^6bc^7d^2-a^7c^6d^3)x^{\frac52}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/16*(3*b^6*c - 7*a*b^5*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(1/4)*b^(3/4)))/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6
*b*c*d^3 + a^7*d^4) + 3/128*(119*b^2*c^2*d^4 - 126*a*b*c*d^5 + 39*a^2*d^6)*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))
/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4
)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c
^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^4*c^8 -
 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4) - 1/80*
(32*a^2*b^3*c^6 - 96*a^3*b^2*c^5*d + 96*a^4*b*c^4*d^2 - 32*a^5*c^3*d^3 - 15
*(24*b^5*c^4*d^2 - 32*a*b^4*c^3*d^3 - 32*a^2*b^3*c^2*d^4 + 87*a^3*b^2*c*d^5
 - 39*a^4*b*d^6)*x^8 - 3*(240*b^5*c^5*d - 224*a*b^4*c^4*d^2 - 448*a^2*b^3*c
^3*d^3 + 623*a^3*b^2*c^2*d^4 + 84*a^4*b*c*d^5 - 195*a^5*d^6)*x^6 - (360*b^5
*c^6 + 96*a*b^4*c^5*d - 1280*a^2*b^3*c^4*d^2 + 64*a^3*b^2*c^3*d^3 + 1933*a^
4*b*c^2*d^4 - 1053*a^5*c*d^5)*x^4 - 32*(9*a*b^4*c^6 - 14*a^2*b^3*c^5*d - 12
*a^3*b^2*c^4*d^2 + 30*a^4*b*c^3*d^3 - 13*a^5*c^2*d^4)*x^2)/((a^3*b^4*c^7*d^
2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^(17/2) + (2*a^
3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c
^4*d^5)*x^(13/2) + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6
*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^(9/2) + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^
6*b*c^7*d^2 - a^7*c^6*d^3)*x^(5/2))
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.61, size = 1289, normalized size = 1.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^(3/2)/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*
(b*x^2 + a)) + 3/4*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4*b^
4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7
*b*c*d^3 + sqrt(2)*a^8*d^4) + 3/4*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*
a*b^2*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*d + 6*sqrt(2)*a^6*b^2*c^2*d^2
 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) + 3/32*(119*(c*d^3)^(3/4)*b^2*c
^2*d - 126*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*arctan(1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^9 - 4*
sqrt(2)*a*b^3*c^8*d + 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 +
 sqrt(2)*a^4*c^5*d^4) + 3/32*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126*(c*d^3)^(3/
4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^9 - 4*sqrt(2)*a*b^3*c^8*d +
 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4)
 - 3/8*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*log(sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3
*d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) +
```

```
3/8*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*log(-sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*
d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) -
3/64*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3
)^(3/4)*a^2*d^3)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*
b^4*c^9 - 4*sqrt(2)*a*b^3*c^8*d + 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3
*b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4) + 3/64*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126
*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*log(-sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^9 - 4*sqrt(2)*a*b^3*c^8*d + 6*sq
rt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4) + 1/
16*(37*b*c*d^5*x^(7/2) - 21*a*d^6*x^(7/2) + 41*b*c^2*d^4*x^(3/2) - 25*a*c*d
^5*x^(3/2))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(d*x
^2 + c)^2) + 2/5*(10*b*c*x^2 + 15*a*d*x^2 - a*c)/(a^3*c^4*x^(5/2))
```

Mupad [B]
time = 10.13, size = 2500, normalized size = 2.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] atan(((((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152
*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a
^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a
^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*
a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 1490305003825397
76*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023
528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 563
47698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15
- 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20
*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*
b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 1183226127125708
3904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743
319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d
^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*
b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 559713091980460
0320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963
087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 842002
49113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 27334
33701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574
625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*
a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c
^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^1
0*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c
```

^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c...

3.504 $\int x^5 \sqrt{a+bx^2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=103

$$\frac{a^2(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^4}$$

```
[Out] 1/3*a^2*(A*b-B*a)*(b*x^2+a)^(3/2)/b^4-1/5*a*(2*A*b-3*B*a)*(b*x^2+a)^(5/2)/b
^4+1/7*(A*b-3*B*a)*(b*x^2+a)^(7/2)/b^4+1/9*B*(b*x^2+a)^(9/2)/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{a^2\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^4}+\frac{\left(a+bx^2\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{a\left(a+bx^2\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(5/2))/(5*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(7/2))/(7*b^4) + (B*(a + b*x^
2)^(9/2))/(9*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\sqrt{a+bx^2}\,(A+Bx^2)\ dx = \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx}\,(A+Bx)\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\Bigg(\int\Bigg(-\frac{a^2(-Ab+aB)\sqrt{a+bx}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{3/2}}{b^3}+$$
$$= \frac{a^2(Ab-aB)\,(a+bx^2)^{3/2}}{3b^4}-\frac{a(2Ab-3aB)\,(a+bx^2)^{5/2}}{5b^4}+\frac{(Ab-3aB)\,(a}{7b^4}$$

Mathematica [A]
time = 0.06, size = 75, normalized size = 0.73

$$\frac{(a+bx^2)^{3/2}\,(-16a^3B+24a^2b(A+Bx^2)-6ab^2x^2(6A+5Bx^2)+5b^3x^4(9A+7Bx^2))}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^2) - 6*a*b^2*x^2*(6*A + 5
*B*x^2) + 5*b^3*x^4*(9*A + 7*B*x^2)))/(315*b^4)
```

Maple [A]
time = 0.08, size = 144, normalized size = 1.40

method	result
gosper	$\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(35B\,x^6b^3+45A\,b^3x^4-30Ba\,b^2x^4-36Aa\,b^2x^2+24B\,a^2b\,x^2+24A\,a^2b-16B\,a^3\right)}{315b^4}$
trager	$\frac{\left(35B\,b^4x^8+45A\,b^4x^6+5Ba\,b^3x^6+9Aa\,b^3x^4-6B\,a^2b^2x^4-12a^2A\,b^2x^2+8B\,a^3b\,x^2+24A\,a^3b-16B\,a^4\right)\sqrt{b\,x^2+a}}{315b^4}$
risch	$\frac{\left(35B\,b^4x^8+45A\,b^4x^6+5Ba\,b^3x^6+9Aa\,b^3x^4-6B\,a^2b^2x^4-12a^2A\,b^2x^2+8B\,a^3b\,x^2+24A\,a^3b-16B\,a^4\right)\sqrt{b\,x^2+a}}{315b^4}$
default	$B\left(\frac{x^6\left(b\,x^2+a\right)^{\frac{3}{2}}}{9b}-\frac{2a\left(\frac{x^4\left(b\,x^2+a\right)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^2\left(b\,x^2+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^2+a\right)^{\frac{3}{2}}}{15b^2}\right)}{7b}\right)}{3b}\right)+A\Bigg(\frac{x^4\left(b\,x^2+a\right)^{\frac{3}{2}}}{7b}-\frac{4a\Big(\frac{x^2\left(b\,x^2+a\right)^{\frac{3}{2}}}{5b}-\frac{2}{}}{7b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] B*(1/9*x^6*(b*x^2+a)^(3/2)/b-2/3*a/b*(1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))))+A*(1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2)))

Maxima [A]
time = 0.33, size = 132, normalized size = 1.28

$$\frac{(bx^2+a)^{\frac{3}{2}}Bx^6}{9\,b}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Bax^4}{21\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ax^4}{7\,b}+\frac{8\,(bx^2+a)^{\frac{3}{2}}Ba^2x^2}{105\,b^3}-\frac{4\,(bx^2+a)^{\frac{3}{2}}Aax^2}{35\,b^2}-\frac{16\,(bx^2+a)^{\frac{3}{2}}Ba^3}{315\,b^4}+\frac{8\,(bx^2+a)^{\frac{3}{2}}Aa^2}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] 1/9*(b*x^2 + a)^(3/2)*B*x^6/b - 2/21*(b*x^2 + a)^(3/2)*B*a*x^4/b^2 + 1/7*(b*x^2 + a)^(3/2)*A*x^4/b + 8/105*(b*x^2 + a)^(3/2)*B*a^2*x^2/b^3 - 4/35*(b*x^2 + a)^(3/2)*A*a*x^2/b^2 - 16/315*(b*x^2 + a)^(3/2)*B*a^3/b^4 + 8/105*(b*x^2 + a)^(3/2)*A*a^2/b^3

Fricas [A]
time = 3.27, size = 99, normalized size = 0.96

$$\frac{(35\,Bb^4x^8+5\,(Bab^3+9\,Ab^4)x^6-16\,Ba^4+24\,Aa^3b-3\,(2\,Ba^2b^2-3\,Aab^3)x^4+4\,(2\,Ba^3b-3\,Aa^2b^2)x^2)\sqrt{bx^2+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] 1/315*(35*B*b^4*x^8 + 5*(B*a*b^3 + 9*A*b^4)*x^6 - 16*B*a^4 + 24*A*a^3*b - 3*(2*B*a^2*b^2 - 3*A*a*b^3)*x^4 + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/b^4

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(94) = 188.
time = 0.23, size = 212, normalized size = 2.06

$$\begin{cases} \frac{8Aa^3\sqrt{a+bx^2}}{105b^3}-\frac{4Aa^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{Aax^4\sqrt{a+bx^2}}{35b}+\frac{Ax^6\sqrt{a+bx^2}}{7}-\frac{16Ba^4\sqrt{a+bx^2}}{315b^4}+\frac{8Ba^3x^2\sqrt{a+bx^2}}{315b^3}-\frac{2Ba^2x^4\sqrt{a+bx^2}}{105b^2}+\frac{Bax^6\sqrt{a+bx^2}}{63b}+\frac{Bx^8\sqrt{a+bx^2}}{9} & \text{for } b\neq 0\\ \sqrt{a}\left(\frac{Ax^6}{6}+\frac{Bx^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(B*x**2+A)*(b*x**2+a)**(1/2),x)

[Out] Piecewise((8*A*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*A*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + A*a*x**4*sqrt(a + b*x**2)/(35*b) + A*x**6*sqrt(a + b*x**2)/7 - 16*B*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*B*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*B*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + B*a*x**6*sqrt(a + b*x**2)/(63*b) + B*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*(A*x**6/6 + B*x**8/8), True))

Giac [A]
time = 1.44, size = 104, normalized size = 1.01

$$\frac{35\left(bx^2+a\right)^{\frac{9}{2}}B-135\left(bx^2+a\right)^{\frac{7}{2}}Ba+189\left(bx^2+a\right)^{\frac{5}{2}}Ba^2-105\left(bx^2+a\right)^{\frac{3}{2}}Ba^3+45\left(bx^2+a\right)^{\frac{7}{2}}Ab-126\left(bx^2+a\right)^{\frac{5}{2}}Aab+105\left(bx^2+a\right)^{\frac{3}{2}}Aa^2b}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2)*B - 135*(b*x^2 + a)^(7/2)*B*a + 189*(b*x^2 + a)
^(5/2)*B*a^2 - 105*(b*x^2 + a)^(3/2)*B*a^3 + 45*(b*x^2 + a)^(7/2)*A*b - 126
*(b*x^2 + a)^(5/2)*A*a*b + 105*(b*x^2 + a)^(3/2)*A*a^2*b)/b^4
```

Mupad [B]
time = 0.30, size = 96, normalized size = 0.93

$$\sqrt{b\,x^2+a}\,\left(\frac{B\,x^8}{9}-\frac{16\,B\,a^4-24\,A\,a^3\,b}{315\,b^4}+\frac{x^6\,\left(45\,A\,b^4+5\,B\,a\,b^3\right)}{315\,b^4}-\frac{4\,a^2\,x^2\,\left(3\,A\,b-2\,B\,a\right)}{315\,b^3}+\frac{a\,x^4\,\left(3\,A\,b-2\,B\,a\right)}{105\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((B*x^8)/9 - (16*B*a^4 - 24*A*a^3*b)/(315*b^4) + (x^6*(45
*A*b^4 + 5*B*a*b^3))/(315*b^4) - (4*a^2*x^2*(3*A*b - 2*B*a))/(315*b^3) + (a
*x^4*(3*A*b - 2*B*a))/(105*b^2))
```

3.505 $\int x^4\sqrt{a+bx^2}\,(A+Bx^2)\,dx$

Optimal. Leaf size=155

$$-\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3}+\frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2}+\frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b}+\frac{Bx^5(a+bx^2)^{3/2}}{8b}+\dots$$

```
[Out] 1/8*B*x^5*(b*x^2+a)^(3/2)/b+1/128*a^3*(8*A*b-5*B*a)*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))/b^(7/2)-1/128*a^2*(8*A*b-5*B*a)*x*(b*x^2+a)^(1/2)/b^3+1/192*a*(
8*A*b-5*B*a)*x^3*(b*x^2+a)^(1/2)/b^2+1/48*(8*A*b-5*B*a)*x^5*(b*x^2+a)^(1/2)
/b
```

Rubi [A]
time = 0.05, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 285, 327, 223, 212}

$$\frac{a^3(8Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}-\frac{a^2x\sqrt{a+bx^2}\,(8Ab-5aB)}{128b^3}+\frac{ax^3\sqrt{a+bx^2}\,(8Ab-5aB)}{192b^2}+\frac{x^5\sqrt{a+bx^2}\,(8Ab-5aB)}{48b}+\frac{Bx^5(a+bx^2)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] -1/128*(a^2*(8*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/b^3 + (a*(8*A*b - 5*a*B)*x^3
*Sqrt[a + b*x^2])/(192*b^2) + ((8*A*b - 5*a*B)*x^5*Sqrt[a + b*x^2])/(48*b)
+ (B*x^5*(a + b*x^2)^(3/2))/(8*b) + (a^3*(8*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(128*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{Bx^5(a+bx^2)^{3/2}}{8b} - \frac{(-8Ab+5aB)\int x^4\sqrt{a+bx^2}\,dx}{8b}$$

$$= \frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b} + \frac{Bx^5(a+bx^2)^{3/2}}{8b} + \frac{(a(8Ab-5aB))\int \frac{x^4}{\sqrt{a+}}}{48b}$$

$$= \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b} + \frac{Bx^5(a+bx^2)^3}{8b}$$

$$= -\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3} + \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB}{}$$

$$= -\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3} + \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB}{}$$

$$= -\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3} + \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB}{}$$

Mathematica [A]
time = 0.17, size = 124, normalized size = 0.80

$$\frac{x\sqrt{a+bx^2}\left(-24a^2Ab+15a^3B+16aAb^2x^2-10a^2bBx^2+64Ab^3x^4+8ab^2Bx^4+48b^3Bx^6\right)}{384b^3}+\frac{a^3(-8Ab+5aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{128b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(-24*a^2*A*b + 15*a^3*B + 16*a*A*b^2*x^2 - 10*a^2*b*B*x^
2 + 64*A*b^3*x^4 + 8*a*b^2*B*x^4 + 48*b^3*B*x^6))/(384*b^3) + (a^3*(-8*A*b
+ 5*a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(7/2))
```

Maple [A]
time = 0.09, size = 192, normalized size = 1.24

method	result
risch	$-\frac{x\left(-48B\,x^{6}b^{3}-64A\,b^{3}x^{4}-8Ba\,b^{2}x^{4}-16Aa\,b^{2}x^{2}+10B\,a^{2}b\,x^{2}+24A\,a^{2}b-15B\,a^{3}\right)\sqrt{b\,x^{2}+a}}{384b^{3}}+\frac{a^{3}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{5}{2}}}$
default	$B\left(\frac{x^{5}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{8b}-\frac{5a\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)}{8b}\right)+A\left(\frac{x^{3}(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/8*x^5*(b*x^2+a)^(3/2)/b-5/8*a/b*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/
4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))))))+A*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^
2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x
^2+a)^(1/2)))))
```

Maxima [A]
time = 0.33, size = 166, normalized size = 1.07

$$\frac{(bx^2+a)^{\frac{3}{2}}Bx^5}{8\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}}Bax^3}{48\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ax^3}{6\,b}+\frac{5\,(bx^2+a)^{\frac{3}{2}}Ba^2x}{64\,b^3}-\frac{5\,\sqrt{bx^2+a}\,Ba^3x}{128\,b^3}-\frac{(bx^2+a)^{\frac{3}{2}}Aax}{8\,b^2}+\frac{\sqrt{bx^2+a}\,Aa^2x}{16\,b^2}-\frac{5\,Ba^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}}+\frac{Aa^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(3/2)*B*x^5/b - 5/48*(b*x^2 + a)^(3/2)*B*a*x^3/b^2 + 1/6*(b
*x^2 + a)^(3/2)*A*x^3/b + 5/64*(b*x^2 + a)^(3/2)*B*a^2*x/b^3 - 5/128*sqrt(b
*x^2 + a)*B*a^3*x/b^3 - 1/8*(b*x^2 + a)^(3/2)*A*a*x/b^2 + 1/16*sqrt(b*x^2 +
 a)*A*a^2*x/b^2 - 5/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 1/16*A*a^3*a
rcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 3.11, size = 257, normalized size = 1.66

$$\left[-\frac{3\,(5\,Ba^4-8\,Aa^3b)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(48\,Bb^4x^7+8\,(Bab^3+8\,Ab^4)x^5-2\,(5\,Ba^2b^2-8\,Aab^3)x^3+3\,(5\,Ba^3b-8\,Aa^2b^2)x)\sqrt{bx^2+a}}{768\,b^4},\ \frac{3\,(5\,Ba^4-8\,Aa^3b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(48\,Bb^4x^7+8\,(Bab^3+8\,Ab^4)x^5-2\,(5\,Ba^2b^2-8\,Aab^3)x^3+3\,(5\,Ba^3b-8\,Aa^2b^2)x)\sqrt{bx^2+a}}{384\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(B*a*b^3 + 8*A*b^4)*x^5 - 2*(5*B*a^2*b^
2 - 8*A*a*b^3)*x^3 + 3*(5*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1
/384*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) +
 (48*B*b^4*x^7 + 8*(B*a*b^3 + 8*A*b^4)*x^5 - 2*(5*B*a^2*b^2 - 8*A*a*b^3)*x^
3 + 3*(5*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A]
time = 25.57, size = 286, normalized size = 1.85

$$-\frac{Aa^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5A\sqrt{a}\,x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{Aa^3\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Abx^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{7}{2}}x}{128b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{5}{2}}x^3}{384b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{3}{2}}x^5}{192b\sqrt{1+\frac{bx^2}{a}}}+\frac{7B\sqrt{a}\,x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^4\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{Bbx^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] -A*a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - A*a**(3/2)*x**3/(48*b*sqrt(1 +
 b*x**2/a)) + 5*A*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(5/2)) + A*b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + 5*B
*a**(7/2)*x/(128*b**3*sqrt(1 + b*x**2/a)) + 5*B*a**(5/2)*x**3/(384*b**2*sqr
t(1 + b*x**2/a)) - B*a**(3/2)*x**5/(192*b*sqrt(1 + b*x**2/a)) + 7*B*sqrt(a)
*x**7/(48*sqrt(1 + b*x**2/a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(
7/2)) + B*b*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.17, size = 132, normalized size = 0.85

$$\frac{1}{384}\left(2\left(4\left(6\,Bx^2+\frac{Bab^5+8\,Ab^6}{b^6}\right)x^2-\frac{5\,Ba^2b^4-8\,Aab^5}{b^6}\right)x^2+\frac{3\,(5\,Ba^3b^3-8\,Aa^2b^4)}{b^6}\right)\sqrt{bx^2+a}\,x+\frac{(5\,Ba^4-8\,Aa^3b)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*x^2 + (B*a*b^5 + 8*A*b^6)/b^6)*x^2 - (5*B*a^2*b^4 - 8*A*a*
b^5)/b^6)*x^2 + 3*(5*B*a^3*b^3 - 8*A*a^2*b^4)/b^6)*sqrt(b*x^2 + a)*x + 1/12
8*(5*B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4*(A + B*x^2)*(a + b*x^2)^(1/2), x)
```

3.506 $\int x^3 \sqrt{a+bx^2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=73

$$-\frac{a(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}$$

```
[Out] -1/3*a*(A*b-B*a)*(b*x^2+a)^(3/2)/b^3+1/5*(A*b-2*B*a)*(b*x^2+a)^(5/2)/b^3+1/
7*B*(b*x^2+a)^(7/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{a\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] -1/3*(a*(A*b - a*B)*(a + b*x^2)^(3/2))/b^3 + ((A*b - 2*a*B)*(a + b*x^2)^(5/
2))/(5*b^3) + (B*(a + b*x^2)^(7/2))/(7*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\sqrt{a+bx^2}\,(A+Bx^2)\,dx=\frac{1}{2}\text{Subst}\left(\int x\sqrt{a+bx}\,(A+Bx)\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a(-Ab+aB)\sqrt{a+bx}}{b^2}+\frac{(Ab-2aB)(a+bx)^{3/2}}{b^2}+\frac{B(a+bx}{b^2}\right.\right.$$
$$=-\frac{a(Ab-aB)\,(a+bx^2)^{3/2}}{3b^3}+\frac{(Ab-2aB)\,(a+bx^2)^{5/2}}{5b^3}+\frac{B(a+bx^2)^{7/2}}{7b^3}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.77

$$\frac{(a+bx^2)^{3/2}\,(-14aAb+8a^2B+21Ab^2x^2-12abBx^2+15b^2Bx^4)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-14*a*A*b + 8*a^2*B + 21*A*b^2*x^2 - 12*a*b*B*x^2 + 15*
b^2*B*x^4))/(105*b^3)
```

Maple [A]
time = 0.08, size = 96, normalized size = 1.32

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}\,(-15b^2B\,x^4-21A\,b^2x^2+12Bab\,x^2+14abA-8a^2B)}{105b^3}$	53
trager	$-\frac{(-15B\,x^6b^3-21A\,b^3x^4-3Ba\,b^2x^4-7Aa\,b^2x^2+4B\,a^2b\,x^2+14A\,a^2b-8B\,a^3)\sqrt{b\,x^2+a}}{105b^3}$	77
risch	$-\frac{(-15B\,x^6b^3-21A\,b^3x^4-3Ba\,b^2x^4-7Aa\,b^2x^2+4B\,a^2b\,x^2+14A\,a^2b-8B\,a^3)\sqrt{b\,x^2+a}}{105b^3}$	77
default	$B\left(\frac{x^4(b\,x^2+a)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)}{7b}\right)+A\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*
(b*x^2+a)^(3/2)))+A*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))
```

Maxima [A]
time = 0.32, size = 90, normalized size = 1.23

$$\frac{(bx^2+a)^{\frac{3}{2}}Bx^4}{7\,b}-\frac{4\,(bx^2+a)^{\frac{3}{2}}Bax^2}{35\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ax^2}{5\,b}+\frac{8\,(bx^2+a)^{\frac{3}{2}}Ba^2}{105\,b^3}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Aa}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(3/2)*B*x^4/b - 4/35*(b*x^2 + a)^(3/2)*B*a*x^2/b^2 + 1/5*(b
*x^2 + a)^(3/2)*A*x^2/b + 8/105*(b*x^2 + a)^(3/2)*B*a^2/b^3 - 2/15*(b*x^2 +
 a)^(3/2)*A*a/b^2
```

Fricas [A]
time = 2.35, size = 75, normalized size = 1.03

$$\frac{(15\,Bb^3x^6+3\,(Bab^2+7\,Ab^3)x^4+8\,Ba^3-14\,Aa^2b-(4\,Ba^2b-7\,Aab^2)x^2)\sqrt{bx^2+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^3*x^6 + 3*(B*a*b^2 + 7*A*b^3)*x^4 + 8*B*a^3 - 14*A*a^2*b - (4
*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(65) = 130.
time = 0.15, size = 162, normalized size = 2.22

$$\begin{cases} -\frac{2Aa^2\sqrt{a+bx^2}}{15b^2}+\frac{Aax^2\sqrt{a+bx^2}}{15b}+\frac{Ax^4\sqrt{a+bx^2}}{5}+\frac{8Ba^3\sqrt{a+bx^2}}{105b^3}-\frac{4Ba^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{Bax^4\sqrt{a+bx^2}}{35b}+\frac{Bx^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^4}{4}+\frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*A*a**2*sqrt(a + b*x**2)/(15*b**2) + A*a*x**2*sqrt(a + b*x**2)
/(15*b) + A*x**4*sqrt(a + b*x**2)/5 + 8*B*a**3*sqrt(a + b*x**2)/(105*b**3)
- 4*B*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + B*a*x**4*sqrt(a + b*x**2)/(35
*b) + B*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*(A*x**4/4 + B*x**6/6),
 True))
```

Giac [A]
time = 1.19, size = 73, normalized size = 1.00

$$\frac{15\,(bx^2+a)^{\frac{7}{2}}B-42\,(bx^2+a)^{\frac{5}{2}}Ba+35\,(bx^2+a)^{\frac{3}{2}}Ba^2+21\,(bx^2+a)^{\frac{5}{2}}Ab-35\,(bx^2+a)^{\frac{3}{2}}Aab}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(b*x^2 + a)^(7/2)*B - 42*(b*x^2 + a)^(5/2)*B*a + 35*(b*x^2 + a)^(
3/2)*B*a^2 + 21*(b*x^2 + a)^(5/2)*A*b - 35*(b*x^2 + a)^(3/2)*A*a*b)/b^3
```

Mupad [B]
time = 0.29, size = 76, normalized size = 1.04

$$\sqrt{b\,x^2+a}\,\left(\frac{B\,x^6}{7}+\frac{8\,B\,a^3-14\,A\,a^2\,b}{105\,b^3}+\frac{x^4\,\left(21\,A\,b^3+3\,B\,a\,b^2\right)}{105\,b^3}+\frac{a\,x^2\,\left(7\,A\,b-4\,B\,a\right)}{105\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((B*x^6)/7 + (8*B*a^3 - 14*A*a^2*b)/(105*b^3) + (x^4*(21*
A*b^3 + 3*B*a*b^2))/(105*b^3) + (a*x^2*(7*A*b - 4*B*a))/(105*b^2))
```

3.507 $\int x^2 \sqrt{a+bx^2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=122

$$\frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2}+\frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}-\frac{a^2(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

```
[Out] 1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/16*a^2*(2*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+a
)^(1/2))/b^(5/2)+1/16*a*(2*A*b-B*a)*x*(b*x^2+a)^(1/2)/b^2+1/8*(2*A*b-B*a)*x
^3*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 285, 327, 223, 212}

$$-\frac{a^2(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{ax\sqrt{a+bx^2}\,(2Ab-aB)}{16b^2}+\frac{x^3\sqrt{a+bx^2}\,(2Ab-aB)}{8b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (a*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b^2) + ((2*A*b - a*B)*x^3*Sqrt[a +
b*x^2])/(8*b) + (B*x^3*(a + b*x^2)^(3/2))/(6*b) - (a^2*(2*A*b - a*B)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx &= \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{(-6Ab+3aB)\int x^2\sqrt{a+bx^2}\,dx}{6b}\\
&= \frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}+\frac{(a(2Ab-aB))\int \frac{x^2}{\sqrt{a+bx^2}}}{8b}\\
&= \frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2}+\frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\\
&= \frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2}+\frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\\
&= \frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2}+\frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 100, normalized size = 0.82

$$
\frac{\sqrt{a+bx^2}\left(6aAbx-3a^2Bx+12Ab^2x^3+2abBx^3+8b^2Bx^5\right)}{48b^2}-\frac{a^2(-2Ab+aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(6*a*A*b*x - 3*a^2*B*x + 12*A*b^2*x^3 + 2*a*b*B*x^3 + 8*b^
2*B*x^5))/(48*b^2) - (a^2*(-2*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
])/(16*b^(5/2))
```

Maple [A]
time = 0.08, size = 144, normalized size = 1.18

method	result
risch	$\frac{x\left(8b^{2}B\,x^{4}+12A\,b^{2}x^{2}+2Bab\,x^{2}+6abA-3a^{2}B\right)\sqrt{b\,x^{2}+a}}{48b^{2}}-\frac{a^{2}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)A}{8b^{\frac{3}{2}}}+\frac{a^{3}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+\cdots\right.}}{16b^{\frac{5}{2}}}$ …
default	$B\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)+A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*
x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(1/4*x*(
b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+
(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.29, size = 124, normalized size = 1.02

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^3}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{8\,b^2}+\frac{\sqrt{bx^2+a}\,Ba^2x}{16\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax}{4\,b}-\frac{\sqrt{bx^2+a}\,Aax}{8\,b}+\frac{Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*B*x^3/b - 1/8*(b*x^2 + a)^(3/2)*B*a*x/b^2 + 1/16*sqrt
(b*x^2 + a)*B*a^2*x/b^2 + 1/4*(b*x^2 + a)^(3/2)*A*x/b - 1/8*sqrt(b*x^2 + a)
*A*a*x/b + 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*A*a^2*arcsinh(b*
x/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 2.19, size = 206, normalized size = 1.69

$$\left[-\frac{3\,\left(Ba^3-2\,Aa^2b\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,\left(8\,Bb^3x^5+2\,\left(Bab^2+6\,Ab^3\right)x^3-3\,\left(Ba^2b-2\,Aab^2\right)x\right)\sqrt{bx^2+a}}{96\,b^3},-\frac{3\,\left(Ba^3-2\,Aa^2b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(8\,Bb^3x^5+2\,\left(Bab^2+6\,Ab^3\right)x^3-3\,\left(Ba^2b-2\,Aab^2\right)x\right)\sqrt{bx^2+a}}{48\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*a^3 - 2*A*a^2*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*B*b^3*x^5 + 2*(B*a*b^2 + 6*A*b^3)*x^3 - 3*(B*a^2*b - 2*A*
a*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*(B*a^3 - 2*A*a^2*b)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*B*b^3*x^5 + 2*(B*a*b^2 + 6*A*b^3)*x^3 -
3*(B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(107) = 214$.
time = 7.35, size = 226, normalized size = 1.85

$$\frac{Aa^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Abx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}\,x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Bbx^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*x**3/(8*sqrt(1 + b*x**2
/a)) - A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + A*b*x**5/(4*sqrt(a)*s
qrt(1 + b*x**2/a)) - B*a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - B*a**(3/2)
*x**3/(48*b*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a))
+ B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) + B*b*x**7/(6*sqrt(a)*sqrt(
1 + b*x**2/a))
```

Giac [A]
time = 2.96, size = 100, normalized size = 0.82

$$\frac{1}{48}\left(2\left(4\,Bx^2+\frac{Bab^3+6\,Ab^4}{b^4}\right)x^2-\frac{3\,(Ba^2b^2-2\,Aab^3)}{b^4}\right)\sqrt{bx^2+a}\,x-\frac{(Ba^3-2\,Aa^2b)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2 + (B*a*b^3 + 6*A*b^4)/b^4)*x^2 - 3*(B*a^2*b^2 - 2*A*a*b^3)
/b^4)*sqrt(b*x^2 + a)*x - 1/16*(B*a^3 - 2*A*a^2*b)*log(abs(-sqrt(b)*x + sqr
t(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^2*(A + B*x^2)*(a + b*x^2)^(1/2), x)
```

3.508 $\int x\sqrt{a+bx^2}\,(A+Bx^2)\,dx$

Optimal. Leaf size=46

$$\frac{(Ab-aB)\,(a+bx^2)^{3/2}}{3b^2}+\frac{B(a+bx^2)^{5/2}}{5b^2}$$

```
[Out] 1/3*(A*b-B*a)*(b*x^2+a)^(3/2)/b^2+1/5*B*(b*x^2+a)^(5/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\frac{(a+bx^2)^{3/2}\,(Ab-aB)}{3b^2}+\frac{B(a+bx^2)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^2) + (B*(a + b*x^2)^(5/2))/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^2}\,(A+Bx^2)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{a+bx}\,(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(Ab-aB)\sqrt{a+bx}}{b}+\frac{B(a+bx)^{3/2}}{b}\right)\,dx,x,x^2\right)\\ &= \frac{(Ab-aB)\,(a+bx^2)^{3/2}}{3b^2}+\frac{B(a+bx^2)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.74

$$\frac{(a+bx^2)^{3/2}(5Ab-2aB+3bBx^2)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
[Out] ((a + b*x^2)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^2))/(15*b^2)
```

Maple [A]
time = 0.08, size = 52, normalized size = 1.13

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{3}{2}}(3bB\,x^2+5Ab-2Ba)}{15b^2}$	31
default	$B\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)+\frac{A(b\,x^2+a)^{\frac{3}{2}}}{3b}$	52
trager	$\frac{(3b^2B\,x^4+5A\,b^2x^2+Bab\,x^2+5abA-2a^2B)\sqrt{b\,x^2+a}}{15b^2}$	52
risch	$\frac{(3b^2B\,x^4+5A\,b^2x^2+Bab\,x^2+5abA-2a^2B)\sqrt{b\,x^2+a}}{15b^2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] B*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))+1/3*A*(b*x^2+a)^(3
/2)/b
```

Maxima [A]
time = 0.30, size = 50, normalized size = 1.09

$$\frac{(bx^2+a)^{\frac{3}{2}}Bx^2}{5\,b}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Ba}{15\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}A}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/5*(b*x^2 + a)^(3/2)*B*x^2/b - 2/15*(b*x^2 + a)^(3/2)*B*a/b^2 + 1/3*(b*x^2
 + a)^(3/2)*A/b
```

Fricas [A]
time = 1.77, size = 50, normalized size = 1.09

$$\frac{(3\,Bb^2x^4-2\,Ba^2+5\,Aab+(Bab+5\,Ab^2)x^2)\sqrt{bx^2+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^4 - 2*B*a^2 + 5*A*a*b + (B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2 +
 a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(39) = 78$.
time = 0.10, size = 110, normalized size = 2.39

$$\begin{cases} \frac{Aa\sqrt{a+bx^2}}{3b} + \frac{Ax^2\sqrt{a+bx^2}}{3} - \frac{2Ba^2\sqrt{a+bx^2}}{15b^2} + \frac{Bax^2\sqrt{a+bx^2}}{15b} + \frac{Bx^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{Ax^2}{2} + \frac{Bx^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((A*a*sqrt(a + b*x**2)/(3*b) + A*x**2*sqrt(a + b*x**2)/3 - 2*B*a**
2*sqrt(a + b*x**2)/(15*b**2) + B*a*x**2*sqrt(a + b*x**2)/(15*b) + B*x**4*sq
rt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*(A*x**2/2 + B*x**4/4), True))
```

Giac [A]
time = 1.44, size = 44, normalized size = 0.96

$$\frac{3\,(bx^2+a)^{\frac{5}{2}}B - 5\,(bx^2+a)^{\frac{3}{2}}Ba + 5\,(bx^2+a)^{\frac{3}{2}}Ab}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2)*B - 5*(b*x^2 + a)^(3/2)*B*a + 5*(b*x^2 + a)^(3/2)
*A*b)/b^2
```

Mupad [B]
time = 0.26, size = 53, normalized size = 1.15

$$\sqrt{b\,x^2+a}\,\left(\frac{B\,x^4}{5} - \frac{2\,B\,a^2-5\,A\,a\,b}{15\,b^2} + \frac{x^2\,\left(5\,A\,b^2+B\,a\,b\right)}{15\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((B*x^4)/5 - (2*B*a^2 - 5*A*a*b)/(15*b^2) + (x^2*(5*A*b^2
 + B*a*b))/(15*b^2))
```

3.509 $\int \sqrt{a+bx^2}\,(A+Bx^2)\,dx$

Optimal. Leaf size=87

$$\frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b}+\frac{Bx(a+bx^2)^{3/2}}{4b}+\frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

```
[Out] 1/4*B*x*(b*x^2+a)^(3/2)/b+1/8*a*(4*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/
2))/b^(3/2)+1/8*(4*A*b-B*a)*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {396, 201, 223, 212}

$$\frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}\,(4Ab-aB)}{8b}+\frac{Bx(a+bx^2)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((4*A*b - a*B)*x*Sqrt[a + b*x^2])/(8*b) + (B*x*(a + b*x^2)^(3/2))/(4*b) + (
a*(4*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{Bx\left(a+bx^2\right)^{3/2}}{4b} - \frac{(-4Ab+aB)\int \sqrt{a+bx^2}\,dx}{4b}$$
$$= \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b} + \frac{Bx\left(a+bx^2\right)^{3/2}}{4b} + \frac{(a(4Ab-aB))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b}$$
$$= \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b} + \frac{Bx\left(a+bx^2\right)^{3/2}}{4b} + \frac{(a(4Ab-aB))\text{Subst}\left(\int \frac{1}{1-bx^2}\,d$$
$$= \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b} + \frac{Bx\left(a+bx^2\right)^{3/2}}{4b} + \frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+b}$$

Mathematica [A]
time = 0.09, size = 74, normalized size = 0.85

$$\frac{x\sqrt{a+bx^2}\left(4Ab+aB+2bBx^2\right)}{8b} + \frac{a(-4Ab+aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(A + B*x^2),x]
[Out] (x*Sqrt[a + b*x^2]*(4*A*b + a*B + 2*b*B*x^2))/(8*b) + (a*(-4*A*b + a*B)*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Maple [A]
time = 0.08, size = 98, normalized size = 1.13

method	result
risch	$\frac{x(2bB\,x^2+4Ab+Ba)\sqrt{b\,x^2+a}}{8b} + \frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)A}{2\sqrt{b}} - \frac{a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)B}{8b^{\frac{3}{2}}}$

default	$B\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+A\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+\right.}}{2\sqrt{b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] B*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))))+A*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 81, normalized size = 0.93

$$\frac{1}{2}\sqrt{bx^2+a}\,Ax+\frac{(bx^2+a)^{\frac{3}{2}}Bx}{4\,b}-\frac{\sqrt{bx^2+a}\,Bax}{8\,b}-\frac{Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b*x^2 + a)*A*x + 1/4*(b*x^2 + a)^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a)
*B*a*x/b - 1/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*A*a*arcsinh(b*x/s
qrt(a*b))/sqrt(b)
```

Fricas [A]
time = 2.62, size = 155, normalized size = 1.78

$$\left[-\frac{(Ba^2-4\,Aab)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(2\,Bb^2x^3+(Bab+4\,Ab^2)x)\sqrt{bx^2+a}}{16\,b^2},\ \frac{(Ba^2-4\,Aab)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(2\,Bb^2x^3+(Bab+4\,Ab^2)x)\sqrt{bx^2+a}}{8\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [-1/16*((B*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(2*B*b^2*x^3 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/8*((
B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b^2*x^3
 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A]
time = 3.29, size = 144, normalized size = 1.66

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{Ba^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Bbx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + A*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
 + B*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*x**3/(8*sqrt(1 + b*x
**2/a)) - B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + B*b*x**5/(4*sqrt(a
)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.88, size = 69, normalized size = 0.79

$$\frac{1}{8}\left(2\,Bx^2+\frac{Bab+4\,Ab^2}{b^2}\right)\sqrt{bx^2+a}\,x+\frac{(Ba^2-4\,Aab)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*B*x^2 + (B*a*b + 4*A*b^2)/b^2)*sqrt(b*x^2 + a)*x + 1/8*(B*a^2 - 4*A*
a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int((A + B*x^2)*(a + b*x^2)^(1/2), x)
```

3.510 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=59

$$A\sqrt{a+bx^2}+\frac{B(a+bx^2)^{3/2}}{3b}-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+A*(b*x^2
+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 81, 52, 65, 214}

$$A\sqrt{a+bx^2}-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{B(a+bx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

```
[Out] A*Sqrt[a + b*x^2] + (B*(a + b*x^2)^(3/2))/(3*b) - Sqrt[a]*A*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x}\,dx, x, x^2\right) \\
&= \frac{B(a+bx^2)^{3/2}}{3b} + \frac{1}{2}A\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right) \\
&= A\sqrt{a+bx^2} + \frac{B(a+bx^2)^{3/2}}{3b} + \frac{1}{2}(aA)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= A\sqrt{a+bx^2} + \frac{B(a+bx^2)^{3/2}}{3b} + \frac{(aA)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= A\sqrt{a+bx^2} + \frac{B(a+bx^2)^{3/2}}{3b} - \sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\,(3Ab+aB+bBx^2)}{3b} - \sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

[Out] (Sqrt[a + b*x^2]*(3*A*b + a*B + b*B*x^2))/(3*b) - Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]

Maple [A]
time = 0.08, size = 57, normalized size = 0.97

method	result	size
default	$\frac{B\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}+A\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x,x,method=_RETURNVERBOSE)

[Out] 1/3*B*(b*x^2+a)^(3/2)/b+A*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))

Maxima [A]
time = 0.28, size = 45, normalized size = 0.76

$$-A\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\sqrt{bx^2+a}\,A+\frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")

[Out] -A*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*A + 1/3*(b*x^2 + a)^(3/2)*B/b

Fricas [A]
time = 1.78, size = 123, normalized size = 2.08

$$\left[\frac{3\,A\sqrt{a}\,b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(Bbx^2+Ba+3\,Ab)\sqrt{bx^2+a}}{6\,b},\frac{3\,A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(Bbx^2+Ba+3\,Ab)\sqrt{bx^2+a}}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")

[Out] [1/6*(3*A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B*b*x^2 + B*a + 3*A*b)*sqrt(b*x^2 + a))/b, 1/3*(3*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (B*b*x^2 + B*a + 3*A*b)*sqrt(b*x^2 + a))/b]

Sympy [A]
time = 10.93, size = 76, normalized size = 1.29

$$-\frac{A\left(-\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+bx^2}\right)}{2}-\frac{B\left(\begin{cases}-\sqrt{a}\,x^2 & \text{for } b=0\\ -\frac{2(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*(-2*a*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) - 2*sqrt(a + b*x**2))/2 -
 B*Piecewise((-sqrt(a)*x**2, Eq(b, 0)), (-2*(a + b*x**2)**(3/2)/(3*b), True
))/2
```

Giac [A]
time = 1.61, size = 60, normalized size = 1.02

$$\frac{Aa\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{(bx^2+a)^{\frac{3}{2}}Bb^2+3\sqrt{bx^2+a}\,Ab^3}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] A*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*((b*x^2 + a)^(3/2)*B*b^
2 + 3*sqrt(b*x^2 + a)*A*b^3)/b^3
```

Mupad [B]
time = 0.42, size = 47, normalized size = 0.80

$$A\sqrt{b\,x^2+a}+\frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{3\,b}-A\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x,x)
```

```
[Out] A*(a + b*x^2)^(1/2) + (B*(a + b*x^2)^(3/2))/(3*b) - A*a^(1/2)*atanh((a + b*
x^2)^(1/2)/a^(1/2))
```

3.511 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=84

$$\frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a}-\frac{A(a+bx^2)^{3/2}}{ax}+\frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] -A*(b*x^2+a)^(3/2)/a/x+1/2*(2*A*b+B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b
^(1/2)+1/2*(2*A*b+B*a)*x*(b*x^2+a)^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 201, 223, 212}

$$\frac{x\sqrt{a+bx^2}\,(aB+2Ab)}{2a}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{A(a+bx^2)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

```
[Out] ((2*A*b + a*B)*x*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(a*x) + ((2
*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^2}\,dx &= -\frac{A(a+bx^2)^{3/2}}{ax}-\frac{(-2Ab-aB)\int\sqrt{a+bx^2}\,dx}{a} \\
&= \frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a}-\frac{A(a+bx^2)^{3/2}}{ax}-\frac{1}{2}(-2Ab-aB)\int\frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a}-\frac{A(a+bx^2)^{3/2}}{ax}-\frac{1}{2}(-2Ab-aB)\text{Subst}\left(\int\frac{1}{1-bx} \right. \\
&= \frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a}-\frac{A(a+bx^2)^{3/2}}{ax}+\frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 67, normalized size = 0.80

$$\frac{\sqrt{a+bx^2}\,(-2A+Bx^2)}{2x}+\frac{(-2Ab-aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*A + B*x^2))/(2*x) + ((-2*A*b - a*B)*Log[-(Sqrt[b]*x) +
 Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Maple [A]
time = 0.08, size = 100, normalized size = 1.19

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-B\,x^2+2A)}{2x}+\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)\sqrt{b}\,A+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)Ba}{2\sqrt{b}}$

default	$B\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+A\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+ \right.}{2\sqrt{b}}\right.}{a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+A*(-1
/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.70

$$\frac{1}{2}\sqrt{bx^{2}+a}\,Bx+\frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}+A\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\sqrt{bx^{2}+a}\,A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + A*sqrt(b
)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*A/x
```

Fricas [A]
time = 2.26, size = 134, normalized size = 1.60

$$\left[\frac{(Ba+2\,Ab)\sqrt{b}\,x\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,(Bbx^{2}-2\,Ab)\sqrt{bx^{2}+a}}{4\,bx},\,-\frac{(Ba+2\,Ab)\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(Bbx^{2}-2\,Ab)\sqrt{bx^{2}+a}}{2\,bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((B*a + 2*A*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(B*b*x^2 - 2*A*b)*sqrt(b*x^2 + a))/(b*x), -1/2*((B*a + 2*A*b)*sqrt(-
b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - 2*A*b)*sqrt(b*x^2 + a)
)/(b*x)]
```

Sympy [A]
time = 2.05, size = 107, normalized size = 1.27

$$-\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^{2}}{a}}}+A\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}+\frac{B\sqrt{a}\,x\sqrt{1+\frac{bx^{2}}{a}}}{2}+\frac{Ba\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A*
b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + B*a*a
sinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Giac [A]
time = 1.49, size = 84, normalized size = 1.00

$$\frac{1}{2}\sqrt{bx^2+a}\,Bx+\frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}-\frac{\left(Ba\sqrt{b}+2\,Ab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 2*A*a*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a) - 1/4*(B*a*sqrt(b) + 2*A*b^(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b
```

Mupad [B]
time = 0.56, size = 94, normalized size = 1.12

$$\frac{B\,x\,\sqrt{b\,x^2+a}}{2}-\frac{A\,\sqrt{b\,x^2+a}}{x}+\frac{B\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}-\frac{A\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^2,x)
```

```
[Out] (B*x*(a + b*x^2)^(1/2))/2 - (A*(a + b*x^2)^(1/2))/x + (B*a*log(b^(1/2)*x +
(a + b*x^2)^(1/2)))/(2*b^(1/2)) - (A*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(
a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

3.512 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{(Ab+2aB)\sqrt{a+bx^2}}{2a}-\frac{A(a+bx^2)^{3/2}}{2ax^2}-\frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*A*(b*x^2+a)^(3/2)/a/x^2-1/2*(A*b+2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2
))/a^(1/2)+1/2*(A*b+2*B*a)*(b*x^2+a)^(1/2)/a
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 52, 65, 214}

$$\frac{\sqrt{a+bx^2}\,(2aB+Ab)}{2a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{A(a+bx^2)^{3/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(2*a*x^2) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{A(a+bx^2)^{3/2}}{2ax^2} + \frac{(Ab+2aB)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)}{4a}$$

$$= \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a} - \frac{A(a+bx^2)^{3/2}}{2ax^2} + \frac{1}{4}(Ab+2aB)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.$$

$$= \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a} - \frac{A(a+bx^2)^{3/2}}{2ax^2} + \frac{(Ab+2aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{2b}$$

$$= \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a} - \frac{A(a+bx^2)^{3/2}}{2ax^2} - \frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Mathematica [A]
time = 0.11, size = 65, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\,(-A+2Bx^2)}{2x^2} + \frac{(-Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-A + 2*B*x^2))/(2*x^2) + ((-(A*b) - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Maple [A]
time = 0.10, size = 106, normalized size = 1.26

method	result
risch	$-\frac{A\sqrt{b\,x^2+a}}{2x^2}+B\sqrt{b\,x^2+a}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)Ab}{2\sqrt{a}}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)B$
default	$A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)+B\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^
(1/2)*(b*x^2+a)^(1/2))/x)))+B*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.30, size = 83, normalized size = 0.99

$$-B\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,\sqrt{a}}+\sqrt{bx^2+a}\,B+\frac{\sqrt{bx^2+a}\,Ab}{2\,a}-\frac{(bx^2+a)^{\frac{3}{2}}A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -B*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) - 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs
(x)))/sqrt(a) + sqrt(b*x^2 + a)*B + 1/2*sqrt(b*x^2 + a)*A*b/a - 1/2*(b*x^2
+ a)^(3/2)*A/(a*x^2)
```

Fricas [A]
time = 1.18, size = 141, normalized size = 1.68

$$\left[\frac{(2\,Ba+Ab)\sqrt{a}\,x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Bax^2-Aa)\sqrt{bx^2+a}}{4\,ax^2},\ \frac{(2\,Ba+Ab)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Bax^2-Aa)\sqrt{bx^2+a}}{2\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*B*a + A*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(2*B*a*x^2 - A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*((2*B*a + A*b)
*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x^2 - A*a)*sqrt(b*x
^2 + a))/(a*x^2)]
```

Sympy [A]
time = 15.65, size = 107, normalized size = 1.27

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2\sqrt{a}}-B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*s
qrt(a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a/(sqrt(b)*x*sqrt(a/(b*x
**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A]
time = 2.36, size = 68, normalized size = 0.81

$$\frac{2\sqrt{bx^2+a}\,Bb+\frac{(2\,Bab+Ab^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^2+a}\,Ab}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(b*x^2 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(
-a))/sqrt(-a) - sqrt(b*x^2 + a)*A*b/x^2)/b
```

Mupad [B]
time = 0.62, size = 68, normalized size = 0.81

$$B\sqrt{b\,x^2+a}-\frac{A\sqrt{b\,x^2+a}}{2\,x^2}-B\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)-\frac{A\,b\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^3,x)
```

```
[Out] B*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/(2*x^2) - B*a^(1/2)*atanh((a +
b*x^2)^(1/2)/a^(1/2)) - (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2))
```

3.513 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=66

$$-\frac{B\sqrt{a+bx^2}}{x}-\frac{A(a+bx^2)^{3/2}}{3ax^3}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/3*A*(b*x^2+a)^(3/2)/a/x^3+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)-B
*(b*x^2+a)^(1/2)/x
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {462, 283, 223, 212}

$$-\frac{A(a+bx^2)^{3/2}}{3ax^3}-\frac{B\sqrt{a+bx^2}}{x}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

```
[Out] -((B*Sqrt[a + b*x^2])/x) - (A*(a + b*x^2)^(3/2))/(3*a*x^3) + Sqrt[b]*B*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^4}\,dx &= -\frac{A(a+bx^2)^{3/2}}{3ax^3}+B\int \frac{\sqrt{a+bx^2}}{x^2}\,dx\\ &= -\frac{B\sqrt{a+bx^2}}{x}-\frac{A(a+bx^2)^{3/2}}{3ax^3}+(bB)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= -\frac{B\sqrt{a+bx^2}}{x}-\frac{A(a+bx^2)^{3/2}}{3ax^3}+(bB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= -\frac{B\sqrt{a+bx^2}}{x}-\frac{A(a+bx^2)^{3/2}}{3ax^3}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 70, normalized size = 1.06

$$\frac{\sqrt{a+bx^2}\,(-aA-Abx^2-3aBx^2)}{3ax^3}-\sqrt{b}\,B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-(a*A) - A*b*x^2 - 3*a*B*x^2))/(3*a*x^3) - Sqrt[b]*B*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [A]
time = 0.08, size = 81, normalized size = 1.23

method	result	size
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(Ab\,x^2+3Ba\,x^2+Aa\right)}{3x^3a}+B\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)$	57

default	$-\frac{A(b\,x^{2}+a)^{\frac{3}{2}}}{3a\,x^{3}}+B\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{a}\right)$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A*(b*x^2+a)^(3/2)/a/x^3+B*(-1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.73

$$B\sqrt{b}\ \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\sqrt{bx^{2}+a}\ B}{x}-\frac{(bx^{2}+a)^{\frac{3}{2}}A}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] B*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*B/x - 1/3*(b*x^2 + a)^(3
/2)*A/(a*x^3)
```

Fricas [A]
time = 2.13, size = 137, normalized size = 2.08

$$\left[\frac{3\,Ba\sqrt{b}\,x^{3}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,((3\,Ba+Ab)x^{2}+Aa)\sqrt{bx^{2}+a}}{6\,ax^{3}},-\frac{3\,Ba\sqrt{-b}\,x^{3}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+((3\,Ba+Ab)x^{2}+Aa)\sqrt{bx^{2}+a}}{3\,ax^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*a*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2
*((3*B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a))/(a*x^3), -1/3*(3*B*a*sqrt(-b)*x
^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((3*B*a + A*b)*x^2 + A*a)*sqrt(b*x^
2 + a))/(a*x^3)]
```

Sympy [A]
time = 1.51, size = 107, normalized size = 1.62

$$-\frac{A\sqrt{b}\,\sqrt{\frac{a}{bx^{2}}+1}}{3x^{2}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^{2}}{a}}}+B\sqrt{b}\ \operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt(a
)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(54) = 108.
time = 1.38, size = 151, normalized size = 2.29

$$-\frac{1}{2}B\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+3\,Ba^3\sqrt{b}+Aa^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*B*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*B*a*sqrt(b) + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2)
 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 3*B*a^3*sqrt(b) + A*a^
2*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [B]
time = 0.68, size = 76, normalized size = 1.15

$$-\frac{B\,\sqrt{b\,x^2+a}}{x}-\frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{3\,a\,x^3}-\frac{B\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^4,x)
```

```
[Out] - (B*(a + b*x^2)^(1/2))/x - (A*(a + b*x^2)^(3/2))/(3*a*x^3) - (B*b^(1/2)*as
in((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(
1/2))
```

3.514 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=88

$$\frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2}-\frac{A(a+bx^2)^{3/2}}{4ax^4}+\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

```
[Out] -1/4*A*(b*x^2+a)^(3/2)/a/x^4+1/8*b*(A*b-4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1
/2))/a^(3/2)+1/8*(A*b-4*B*a)*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 43, 65, 214}

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx^2}\,(Ab-4aB)}{8ax^2}-\frac{A(a+bx^2)^{3/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a*x^2) - (A*(a + b*x^2)^(3/2))/(4*a*x^4)
 + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{A(a+bx^2)^{3/2}}{4ax^4} + \frac{\left(-\frac{Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx,x,x^2\right)}{4a}$$

$$= \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2} - \frac{A(a+bx^2)^{3/2}}{4ax^4} - \frac{(b(Ab-4aB))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} \cdots\right.}{16a}$$

$$= \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2} - \frac{A(a+bx^2)^{3/2}}{4ax^4} - \frac{(Ab-4aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x, \cdots\right.}{8a}$$

$$= \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2} - \frac{A(a+bx^2)^{3/2}}{4ax^4} + \frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

Mathematica [A]
time = 0.14, size = 78, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\,(-2aA-Abx^2-4aBx^2)}{8ax^4} - \frac{b(-Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]

[Out] (Sqrt[a + b*x^2]*(-2*a*A - A*b*x^2 - 4*a*B*x^2))/(8*a*x^4) - (b*(-(A*b) + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(72) = 144$.
time = 0.09, size = 154, normalized size = 1.75

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(A b x^2+4 B a x^2+2 A a)}{8 x^4 a}+\frac{b^2 \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right) A}{8 a^{\frac{3}{2}}}-\frac{b \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right) B}{2\sqrt{a}}$
default	$A\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{4 a x^4}-\frac{b\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{2 a x^2}+\frac{b\left(\sqrt{b x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)+B\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{2 a x^2}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)

[Out] A*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+B*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))

Maxima [A]
time = 0.27, size = 130, normalized size = 1.48

$$-\frac{Bb\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,\sqrt{a}}+\frac{Ab^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{3}{2}}}+\frac{\sqrt{bx^2+a}\,Bb}{2\,a}-\frac{\sqrt{bx^2+a}\,Ab^2}{8\,a^2}-\frac{(bx^2+a)^{\frac{3}{2}}B}{2\,ax^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ab}{8\,a^2x^2}-\frac{(bx^2+a)^{\frac{3}{2}}A}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")

[Out] -1/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*A*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 1/2*sqrt(b*x^2 + a)*B*b/a - 1/8*sqrt(b*x^2 + a)*A*b^2/a^2 - 1/2*(b*x^2 + a)^(3/2)*B/(a*x^2) + 1/8*(b*x^2 + a)^(3/2)*A*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(3/2)*A/(a*x^4)

Fricas [A]
time = 1.18, size = 170, normalized size = 1.93

$$\left[-\frac{(4\,Bab-Ab^2)\sqrt{a}\,x^4\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Aa^2+(4\,Ba^2+Aab)x^2)\sqrt{bx^2+a}}{16\,a^2x^4}, \frac{(4\,Bab-Ab^2)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(2\,Aa^2+(4\,Ba^2+Aab)x^2)\sqrt{bx^2+a}}{8\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*a*b - A*b^2)*sqrt(a)*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(2*A*a^2 + (4*B*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*
x^4), 1/8*((4*B*a*b - A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a))
- (2*A*a^2 + (4*B*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^4)]
```

Sympy [A]
time = 41.41, size = 144, normalized size = 1.64

$$-\frac{Aa}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3A\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Ab^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{3}{2}}}-\frac{B\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -A*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*sqrt(b)/(8*x**3*sqrt(a/(b*
x**2) + 1)) - A*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + A*b**2*asinh(sqrt(a
)/(sqrt(b)*x))/(8*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - B*b*as
inh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a))
```

Giac [A]
time = 1.26, size = 120, normalized size = 1.36

$$\frac{\frac{(4\,Bab^2-Ab^3)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{4\,(bx^2+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^2+a}\,Ba^2b^2+(bx^2+a)^{\frac{3}{2}}Ab^3+\sqrt{bx^2+a}\,Aab^3}{ab^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (4
*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^2*b^2 + (b*x^2 + a)^(3/2
)*A*b^3 + sqrt(b*x^2 + a)*A*a*b^3)/(a*b^2*x^4))/b
```

Mupad [B]
time = 0.78, size = 93, normalized size = 1.06

$$\frac{A\,b^2\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{A\,\sqrt{b\,x^2+a}}{8\,x^4}-\frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a\,x^4}-\frac{B\,b\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^5,x)
```

```
[Out] (A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (B*(a + b*x^2)^(1/2)
)/(2*x^2) - (A*(a + b*x^2)^(1/2))/(8*x^4) - (A*(a + b*x^2)^(3/2))/(8*a*x^4)
 - (B*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2))
```

3.515 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=53

$$-\frac{A(a+bx^2)^{3/2}}{5ax^5}+\frac{(2Ab-5aB)\,(a+bx^2)^{3/2}}{15a^2x^3}$$

```
[Out] -1/5*A*(b*x^2+a)^(3/2)/a/x^5+1/15*(2*A*b-5*B*a)*(b*x^2+a)^(3/2)/a^2/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {464, 270}

$$\frac{(a+bx^2)^{3/2}\,(2Ab-5aB)}{15a^2x^3}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(A*(a + b*x^2)^(3/2))/(a*x^5) + ((2*A*b - 5*a*B)*(a + b*x^2)^(3/2))/(1
5*a^2*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^6}\,dx &= -\frac{A(a+bx^2)^{3/2}}{5ax^5}-\frac{(2Ab-5aB)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{5a}\\ &= -\frac{A(a+bx^2)^{3/2}}{5ax^5}+\frac{(2Ab-5aB)\,(a+bx^2)^{3/2}}{15a^2x^3}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 40, normalized size = 0.75

$$\frac{(a+bx^2)^{3/2}\left(-3aA+2Abx^2-5aBx^2\right)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-3*a*A + 2*A*b*x^2 - 5*a*B*x^2))/(15*a^2*x^5)
```

Maple [A]
time = 0.08, size = 58, normalized size = 1.09

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}(-2Ab\,x^2+5Ba\,x^2+3Aa)}{15a^2x^5}$	37
default	$A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}\right)-\frac{B(b\,x^2+a)^{\frac{3}{2}}}{3a\,x^3}$	58
trager	$-\frac{(-2A\,b^2x^4+5Bab\,x^4+aAb\,x^2+5B\,a^2x^2+3a^2A)\sqrt{b\,x^2+a}}{15a^2x^5}$	58
risch	$-\frac{(-2A\,b^2x^4+5Bab\,x^4+aAb\,x^2+5B\,a^2x^2+3a^2A)\sqrt{b\,x^2+a}}{15a^2x^5}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a^2*(b*x^2+a)^(3/2)/x^3)-1/3*B*(b*x^2+
a)^(3/2)/a/x^3
```

Maxima [A]
time = 0.28, size = 56, normalized size = 1.06

$$-\frac{(bx^2+a)^{\frac{3}{2}}B}{3\,ax^3}+\frac{2\,(bx^2+a)^{\frac{3}{2}}Ab}{15\,a^2x^3}-\frac{(bx^2+a)^{\frac{3}{2}}A}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/3*(b*x^2 + a)^(3/2)*B/(a*x^3) + 2/15*(b*x^2 + a)^(3/2)*A*b/(a^2*x^3) - 1
/5*(b*x^2 + a)^(3/2)*A/(a*x^5)
```

Fricas [A]
time = 1.21, size = 55, normalized size = 1.04

$$-\frac{((5\,Bab-2\,Ab^2)x^4+3\,Aa^2+(5\,Ba^2+Aab)x^2)\sqrt{bx^2+a}}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*((5*B*a*b - 2*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + A*a*b)*x^2)*sqrt(b*x^
2 + a)/(a^2*x^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(46) = 92$.
time = 1.31, size = 119, normalized size = 2.25

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(15*a*x**2) + 2*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*sqrt(b)*sqrt(
a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(45) = 90$.
time = 1.84, size = 232, normalized size = 4.38

$$\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bb^{\frac{3}{2}}-30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bab^{\frac{3}{2}}+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ab^{\frac{5}{2}}+20\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^2b^{\frac{3}{2}}+10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aab^{\frac{5}{2}}-10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^3b^{\frac{3}{2}}+10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^2b^{\frac{5}{2}}+5Ba^4b^{\frac{3}{2}}-2Aa^3b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(3/2) - 30*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*B*a*b^(3/2) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(5/2) + 2
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2) + 10*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*A*a*b^(5/2) - 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2) +
10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(5/2) + 5*B*a^4*b^(3/2) - 2*A*a^
3*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [B]
time = 0.43, size = 97, normalized size = 1.83

$$\frac{\left(A\,b^2+B\,a\,b\right)\,\sqrt{b\,x^2+a}}{5\,a^2\,x}-\frac{\left(5\,B\,a^2+A\,b\,a\right)\,\sqrt{b\,x^2+a}}{15\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^2+a}}{5\,x^5}-\frac{b\,\sqrt{b\,x^2+a}\,\left(A\,b+8\,B\,a\right)}{15\,a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^6,x)
```

```
[Out] ((A*b^2 + B*a*b)*(a + b*x^2)^(1/2))/(5*a^2*x) - ((5*B*a^2 + A*a*b)*(a + b*x
^2)^(1/2))/(15*a^2*x^3) - (A*(a + b*x^2)^(1/2))/(5*x^5) - (b*(a + b*x^2)^(1
/2)*(A*b + 8*B*a))/(15*a^2*x)
```

3.516 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=120

$$\frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4}+\frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2}-\frac{A(a+bx^2)^{3/2}}{6ax^6}-\frac{b^2(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

```
[Out] -1/6*A*(b*x^2+a)^(3/2)/a/x^6-1/16*b^2*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a
^(1/2))/a^(5/2)+1/8*(A*b-2*B*a)*(b*x^2+a)^(1/2)/a/x^4+1/16*b*(A*b-2*B*a)*(b
*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 43, 44, 65, 214}

$$-\frac{b^2(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b\sqrt{a+bx^2}\,(Ab-2aB)}{16a^2x^2}+\frac{\sqrt{a+bx^2}\,(Ab-2aB)}{8ax^4}-\frac{A(a+bx^2)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

```
[Out] ((A*b - 2*a*B)*Sqrt[a + b*x^2])/(8*a*x^4) + (b*(A*b - 2*a*B)*Sqrt[a + b*x^2
])/(16*a^2*x^2) - (A*(a + b*x^2)^(3/2))/(6*a*x^6) - (b^2*(A*b - 2*a*B)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{A(a+bx^2)^{3/2}}{6ax^6} + \frac{\left(-\frac{3Ab}{2}+3aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right)}{6a}$$

$$= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4} - \frac{A(a+bx^2)^{3/2}}{6ax^6} - \frac{(b(Ab-2aB))\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\right.}{16a}$$

$$= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4} + \frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2} - \frac{A(a+bx^2)^{3/2}}{6ax^6} + \frac{(b^2(Ab-}{}$$

$$= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4} + \frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2} - \frac{A(a+bx^2)^{3/2}}{6ax^6} + \frac{(b(Ab-}{}$$

$$= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4} + \frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2} - \frac{A(a+bx^2)^{3/2}}{6ax^6} - \frac{b^2(Ab-}{}$$

Mathematica [A]
time = 0.16, size = 102, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\,(-8a^2A-2aAbx^2-12a^2Bx^2+3Ab^2x^4-6abBx^4)}{48a^2x^6} + \frac{b^2(-Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A - 2*a*A*b*x^2 - 12*a^2*B*x^2 + 3*A*b^2*x^4 - 6*a
*b*B*x^4))/(48*a^2*x^6) + (b^2*(-(A*b) + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/(16*a^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(100) = 200$.
time = 0.09, size = 202, normalized size = 1.68

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-3A\,b^2x^4+6Bab\,x^4+2aAb\,x^2+12B\,a^2x^2+8a^2A)}{48x^6a^2} - \frac{b^3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)A}{16a^{\frac{5}{2}}} + \frac{b^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{}\right.}{8a^{\frac{3}{2}}}$

default	$B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)+A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{6a\,x^6}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*(
(b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+A*(-1/6/a/
x^6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2
*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+
a)^(1/2))/x)))))
```

Maxima [A]

time = 0.29, size = 174, normalized size = 1.45

$$\frac{Bb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{3}{2}}}-\frac{Ab^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{5}{2}}}-\frac{\sqrt{bx^2+a}\,Bb^2}{8\,a^2}+\frac{\sqrt{bx^2+a}\,Ab^3}{16\,a^3}+\frac{(bx^2+a)^{\frac{3}{2}}Bb}{8\,a^2x^2}-\frac{(bx^2+a)^{\frac{3}{2}}Ab^2}{16\,a^3x^2}-\frac{(bx^2+a)^{\frac{3}{2}}B}{4\,ax^4}+\frac{(bx^2+a)^{\frac{3}{2}}Ab}{8\,a^2x^4}-\frac{(bx^2+a)^{\frac{3}{2}}A}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/16*A*b^3*arcsinh(a/(sqr
t(a*b)*abs(x)))/a^(5/2) - 1/8*sqrt(b*x^2 + a)*B*b^2/a^2 + 1/16*sqrt(b*x^2 +
 a)*A*b^3/a^3 + 1/8*(b*x^2 + a)^(3/2)*B*b/(a^2*x^2) - 1/16*(b*x^2 + a)^(3/2
)*A*b^2/(a^3*x^2) - 1/4*(b*x^2 + a)^(3/2)*B/(a*x^4) + 1/8*(b*x^2 + a)^(3/2)
*A*b/(a^2*x^4) - 1/6*(b*x^2 + a)^(3/2)*A/(a*x^6)
```

Fricas [A]

time = 1.50, size = 221, normalized size = 1.84

$$\left[-\frac{3\,(2\,Bab^2-Ab^3)\sqrt{a}\,x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,(2\,Ba^2b-Aab^2)x^4+8\,Aa^3+2\,(6\,Ba^3+Aa^2b)x^2)\sqrt{bx^2+a}}{96\,a^3x^6},-\frac{3\,(2\,Bab^2-Ab^3)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(2\,Ba^2b-Aab^2)x^4+8\,Aa^3+2\,(6\,Ba^3+Aa^2b)x^2)\sqrt{bx^2+a}}{48\,a^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(2*B*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(3*(2*B*a^2*b - A*a*b^2)*x^4 + 8*A*a^3 + 2*(6*B*a^3
+ A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^6), -1/48*(3*(2*B*a*b^2 - A*b^3)*sq
rt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(2*B*a^2*b - A*a*b^2)*x^4
+ 8*A*a^3 + 2*(6*B*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(107) = 214$.
time = 54.83, size = 226, normalized size = 1.88

$$-\frac{Aa}{6\sqrt{b}\,x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5A\sqrt{b}}{24x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Ab^{\frac{3}{2}}}{48ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Ab^{\frac{5}{2}}}{16a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Ab^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{5}{2}}}-\frac{Ba}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3B\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Bb^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Bb^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -A*a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*A*sqrt(b)/(24*x**5*sqrt(a/(b
*x**2) + 1)) + A*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + A*b**(5/2)/(16
*a**2*x*sqrt(a/(b*x**2) + 1)) - A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5
/2)) - B*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*sqrt(b)/(8*x**3*sqrt
(a/(b*x**2) + 1)) - B*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + B*b**2*asinh(
sqrt(a)/(sqrt(b)*x))/(8*a**(3/2))
```

Giac [A]
time = 1.77, size = 140, normalized size = 1.17

$$-\frac{\frac{3\,(2\,Bab^{3}-Ab^{4})\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{6\,(bx^{2}+a)^{\frac{5}{2}}Bab^{3}-6\,\sqrt{bx^{2}+a}\,Ba^{3}b^{3}-3\,(bx^{2}+a)^{\frac{5}{2}}Ab^{4}+8\,(bx^{2}+a)^{\frac{3}{2}}Aab^{4}+3\,\sqrt{bx^{2}+a}\,Aa^{2}b^{4}}{a^{2}b^{3}x^{6}}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*(2*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2
) + (6*(b*x^2 + a)^(5/2)*B*a*b^3 - 6*sqrt(b*x^2 + a)*B*a^3*b^3 - 3*(b*x^2 +
 a)^(5/2)*A*b^4 + 8*(b*x^2 + a)^(3/2)*A*a*b^4 + 3*sqrt(b*x^2 + a)*A*a^2*b^4
)/(a^2*b^3*x^6))/b
```

Mupad [B]
time = 1.02, size = 134, normalized size = 1.12

$$\frac{B\,b^{2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8\,a^{3/2}}-\frac{B\,\sqrt{b\,x^{2}+a}}{8\,x^{4}}-\frac{A\,\sqrt{b\,x^{2}+a}}{16\,x^{6}}-\frac{A\,{\left(b\,x^{2}+a\right)}^{3/2}}{6\,a\,x^{6}}+\frac{A\,{\left(b\,x^{2}+a\right)}^{5/2}}{16\,a^{2}\,x^{6}}-\frac{B\,{\left(b\,x^{2}+a\right)}^{3/2}}{8\,a\,x^{4}}+\frac{A\,b^{3}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{16\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^7,x)
```

```
[Out] (A*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (B*(a + b*x^
2)^(1/2))/(8*x^4) - (A*(a + b*x^2)^(1/2))/(16*x^6) + (B*b^2*atanh((a + b*x^
2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (A*(a + b*x^2)^(3/2))/(6*a*x^6) + (A*(a +
b*x^2)^(5/2))/(16*a^2*x^6) - (B*(a + b*x^2)^(3/2))/(8*a*x^4)
```

3.517 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=84

$$-\frac{A(a+bx^2)^{3/2}}{7ax^7}+\frac{(4Ab-7aB)\,(a+bx^2)^{3/2}}{35a^2x^5}-\frac{2b(4Ab-7aB)\,(a+bx^2)^{3/2}}{105a^3x^3}$$

```
[Out] -1/7*A*(b*x^2+a)^(3/2)/a/x^7+1/35*(4*A*b-7*B*a)*(b*x^2+a)^(3/2)/a^2/x^5-2/1
05*b*(4*A*b-7*B*a)*(b*x^2+a)^(3/2)/a^3/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$-\frac{2b(a+bx^2)^{3/2}\,(4Ab-7aB)}{105a^3x^3}+\frac{(a+bx^2)^{3/2}\,(4Ab-7aB)}{35a^2x^5}-\frac{A(a+bx^2)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

```
[Out] -1/7*(A*(a + b*x^2)^(3/2))/(a*x^7) + ((4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(3
5*a^2*x^5) - (2*b*(4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(105*a^3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^8}\,dx = -\frac{A(a+bx^2)^{3/2}}{7ax^7} - \frac{(4Ab-7aB)\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{7a}$$
$$= -\frac{A(a+bx^2)^{3/2}}{7ax^7} + \frac{(4Ab-7aB)\,(a+bx^2)^{3/2}}{35a^2x^5} + \frac{(2b(4Ab-7aB))\int \frac{\sqrt{a+bx^2}}{x^4}}{35a^2}$$
$$= -\frac{A(a+bx^2)^{3/2}}{7ax^7} + \frac{(4Ab-7aB)\,(a+bx^2)^{3/2}}{35a^2x^5} - \frac{2b(4Ab-7aB)\,(a+bx^2)^{3/2}}{105a^3x^3}$$

Mathematica [A]
time = 0.12, size = 62, normalized size = 0.74

$$\frac{(a+bx^2)^{3/2}\,(-15a^2A+12aAbx^2-21a^2Bx^2-8Ab^2x^4+14abBx^4)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-15*a^2*A + 12*a*A*b*x^2 - 21*a^2*B*x^2 - 8*A*b^2*x^4 +
 14*a*b*B*x^4))/(105*a^3*x^7)
```

Maple [A]
time = 0.09, size = 102, normalized size = 1.21

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}\,(8A\,b^2x^4-14Bab\,x^4-12aAb\,x^2+21B\,a^2x^2+15a^2A)}{105a^3x^7}$	59
trager	$-\frac{(8x^6A\,b^3-14x^6Ba\,b^2-4Aa\,b^2x^4+7x^4B\,a^2b+3x^2A\,a^2b+21B\,a^3x^2+15A\,a^3)\sqrt{b\,x^2+a}}{105a^3x^7}$	83
risch	$-\frac{(8x^6A\,b^3-14x^6Ba\,b^2-4Aa\,b^2x^4+7x^4B\,a^2b+3x^2A\,a^2b+21B\,a^3x^2+15A\,a^3)\sqrt{b\,x^2+a}}{105a^3x^7}$	83
default	$B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}\right)+A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a^2*(b*x^2+a)^(3/2)/x^3)+A*(-1/7/a/x^7
*(b*x^2+a)^(3/2)-4/7*b/a*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a^2*(b*x^2+a)^(
3/2)/x^3))
```

Maxima [A]
time = 0.32, size = 96, normalized size = 1.14

$$\frac{2\,(bx^2+a)^{\frac{3}{2}}Bb}{15\,a^2x^3}-\frac{8\,(bx^2+a)^{\frac{3}{2}}Ab^2}{105\,a^3x^3}-\frac{(bx^2+a)^{\frac{3}{2}}B}{5\,ax^5}+\frac{4\,(bx^2+a)^{\frac{3}{2}}Ab}{35\,a^2x^5}-\frac{(bx^2+a)^{\frac{3}{2}}A}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^2 + a)^(3/2)*B*b/(a^2*x^3) - 8/105*(b*x^2 + a)^(3/2)*A*b^2/(a^3*x
^3) - 1/5*(b*x^2 + a)^(3/2)*B/(a*x^5) + 4/35*(b*x^2 + a)^(3/2)*A*b/(a^2*x^5
) - 1/7*(b*x^2 + a)^(3/2)*A/(a*x^7)
```

Fricas [A]
time = 1.51, size = 81, normalized size = 0.96

$$\frac{(2\,(7\,Bab^2-4\,Ab^3)x^6-(7\,Ba^2b-4\,Aab^2)x^4-15\,Aa^3-3\,(7\,Ba^3+Aa^2b)x^2)\sqrt{bx^2+a}}{105\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(7*B*a*b^2 - 4*A*b^3)*x^6 - (7*B*a^2*b - 4*A*a*b^2)*x^4 - 15*A*a^3
 - 3*(7*B*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^3*x^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(78) = 156$.
time = 1.62, size = 442, normalized size = 5.26

$$-\frac{15Aa^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Aa^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Aa^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3Aa^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12Aab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8Ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**8,x)
```

```
[Out] -15*A*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a
**3*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 3*A*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a*b
**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
 + 105*a**3*b**6*x**10) - 8*A*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - B*sqrt(b)*sqrt(a/
(b*x**2) + 1)/(5*x**4) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*B*
b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(72) = 144.
time = 0.88, size = 288, normalized size = 3.43

$$\frac{4\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bb^{\frac{5}{2}}-175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Bab^{\frac{5}{2}}+280\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ab^{\frac{7}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^2b^{\frac{5}{2}}+140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Aab^{\frac{7}{2}}-42\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Ba^3b^{\frac{5}{2}}+84\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Aa^2b^{\frac{7}{2}}+49\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Ba^4b^{\frac{5}{2}}-28\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Aa^3b^{\frac{7}{2}}-7Ba^5b^{\frac{5}{2}}+4Aa^4b^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*b^(5/2) - 175*(sqrt(b)*x - sq
rt(b*x^2 + a))^8*B*a*b^(5/2) + 280*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(7/2
) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(5/2) + 140*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*A*a*b^(7/2) - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(5
/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(7/2) + 49*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*B*a^4*b^(5/2) - 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b
^(7/2) - 7*B*a^5*b^(5/2) + 4*A*a^4*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^7
```

Mupad [B]
time = 0.55, size = 132, normalized size = 1.57

$$\frac{4\,A\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3}-\frac{B\,\sqrt{b\,x^2+a}}{5\,x^5}-\frac{A\,b\,\sqrt{b\,x^2+a}}{35\,a\,x^5}-\frac{B\,b\,\sqrt{b\,x^2+a}}{15\,a\,x^3}-\frac{A\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{8\,A\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x}+\frac{2\,B\,b^2\,\sqrt{b\,x^2+a}}{15\,a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^8,x)
```

```
[Out] (4*A*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^3) - (B*(a + b*x^2)^(1/2))/(5*x^5) -
 (A*b*(a + b*x^2)^(1/2))/(35*a*x^5) - (B*b*(a + b*x^2)^(1/2))/(15*a*x^3) -
(A*(a + b*x^2)^(1/2))/(7*x^7) - (8*A*b^3*(a + b*x^2)^(1/2))/(105*a^3*x) + (
2*B*b^2*(a + b*x^2)^(1/2))/(15*a^2*x)
```

3.518 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=156

$$\frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6}+\frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4}-\frac{b^2(5Ab-8aB)\sqrt{a+bx^2}}{128a^3x^2}-\frac{A(a+bx^2)^{3/2}}{8ax^8}+\frac{b^3(5Ab-8a}{}$$

```
[Out] -1/8*A*(b*x^2+a)^(3/2)/a/x^8+1/128*b^3*(5*A*b-8*B*a)*arctanh((b*x^2+a)^(1/2
)/a^(1/2))/a^(7/2)+1/48*(5*A*b-8*B*a)*(b*x^2+a)^(1/2)/a/x^6+1/192*b*(5*A*b-
8*B*a)*(b*x^2+a)^(1/2)/a^2/x^4-1/128*b^2*(5*A*b-8*B*a)*(b*x^2+a)^(1/2)/a^3/
x^2
```

Rubi [A]
time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 43, 44, 65, 214}

$$\frac{b^3(5Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}-\frac{b^2\sqrt{a+bx^2}\,(5Ab-8aB)}{128a^3x^2}+\frac{b\sqrt{a+bx^2}\,(5Ab-8aB)}{192a^2x^4}+\frac{\sqrt{a+bx^2}\,(5Ab-8aB)}{48ax^6}-\frac{A(a+bx^2)^{3/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

```
[Out] ((5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(48*a*x^6) + (b*(5*A*b - 8*a*B)*Sqrt[a +
b*x^2])/(192*a^2*x^4) - (b^2*(5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a^3*x^2)
 - (A*(a + b*x^2)^(3/2))/(8*a*x^8) + (b^3*(5*A*b - 8*a*B)*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/(128*a^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{A(a+bx^2)^{3/2}}{8ax^8} + \frac{\left(-\frac{5Ab}{2}+4aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^2\right)}{8a}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} - \frac{A(a+bx^2)^{3/2}}{8ax^8} - \frac{(b(5Ab-8aB))\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+}}\right.}{96a}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{A(a+bx^2)^{3/2}}{8ax^8} + \frac{(b^2(5A}{}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx}}{128a^3x^2}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx}}{128a^3x^2}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx}}{128a^3x^2}$$

Mathematica [A]
time = 0.22, size = 126, normalized size = 0.81

$$\frac{\sqrt{a+bx^2}\,(-48a^3A-8a^2Abx^2-64a^3Bx^2+10aAb^2x^4-16a^2bBx^4-15Ab^3x^6+24ab^2Bx^6)}{384a^3x^8} - \frac{b^3(-5Ab+8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*A - 8*a^2*A*b*x^2 - 64*a^3*B*x^2 + 10*a*A*b^2*x^4
 - 16*a^2*b*B*x^4 - 15*A*b^3*x^6 + 24*a*b^2*B*x^6))/(384*a^3*x^8) - (b^3*(-
5*A*b + 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(7/2))
```

Maple [A]
time = 0.10, size = 250, normalized size = 1.60

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(15x^6Ab^3-24x^6Bab^2-10Aab^2x^4+16x^4Ba^2b+8x^2Aa^2b+64Ba^3x^2+48Aa^3)}{384x^8a^3}+\frac{5b^4\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right.}{128a^{\frac{7}{2}}}$

default	$B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)}{2a}\right)+A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/6/a/x^6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(
-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)))))+A*(-1/8/a/x^8*(b*x^2+a)^(3/2)-5/8*b/a*(-1/6/a/x^
6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(
b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)
^(1/2))/x))))))
```

Maxima [A]

time = 0.30, size = 216, normalized size = 1.38

$$-\frac{Bb^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{5}{2}}}+\frac{5\,Ab^4\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{7}{2}}}+\frac{\sqrt{bx^2+a}\,Bb^3}{16\,a^3}-\frac{5\,\sqrt{bx^2+a}\,Ab^4}{128\,a^4}-\frac{(bx^2+a)^{\frac{3}{2}}Bb^2}{16\,a^3x^2}+\frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^3}{128\,a^4x^2}+\frac{(bx^2+a)^{\frac{3}{2}}Bb}{8\,a^2x^4}-\frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^2}{64\,a^3x^4}-\frac{(bx^2+a)^{\frac{3}{2}}B}{6\,ax^6}+\frac{5\,(bx^2+a)^{\frac{3}{2}}Ab}{48\,a^2x^6}-\frac{(bx^2+a)^{\frac{3}{2}}A}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/128*A*b^4*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(7/2) + 1/16*sqrt(b*x^2 + a)*B*b^3/a^3 - 5/128*sqrt(b*
x^2 + a)*A*b^4/a^4 - 1/16*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^2) + 5/128*(b*x^2
+ a)^(3/2)*A*b^3/(a^4*x^2) + 1/8*(b*x^2 + a)^(3/2)*B*b/(a^2*x^4) - 5/64*(b*
x^2 + a)^(3/2)*A*b^2/(a^3*x^4) - 1/6*(b*x^2 + a)^(3/2)*B/(a*x^6) + 5/48*(b*
x^2 + a)^(3/2)*A*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(3/2)*A/(a*x^8)
```

Fricas [A]
time = 1.38, size = 269, normalized size = 1.72

$$\left[-\frac{3\,(8\,Bab^3-5\,Ab^4)\sqrt{a}\,x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(3\,(8\,Ba^2b^2-5\,Aab^3)x^6-48\,Aa^4-2\,(8\,Ba^3b-5\,Aa^2b^2)x^4-8\,(8\,Ba^4+Aa^3b)x^2)\sqrt{bx^2+a}}{768\,a^4x^8},\ \frac{3\,(8\,Bab^3-5\,Ab^4)\sqrt{-a}\,x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(8\,Ba^2b^2-5\,Aab^3)x^6-48\,Aa^4-2\,(8\,Ba^3b-5\,Aa^2b^2)x^4-8\,(8\,Ba^4+Aa^3b)x^2)\sqrt{bx^2+a}}{384\,a^4x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 48*A*a^4 - 2*(
8*B*a^3*b - 5*A*a^2*b^2)*x^4 - 8*(8*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a))/
(a^4*x^8), 1/384*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)/sqrt
(b*x^2 + a)) + (3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 48*A*a^4 - 2*(8*B*a^3*b -
 5*A*a^2*b^2)*x^4 - 8*(8*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^8)]
```

Sympy [A]
time = 99.14, size = 286, normalized size = 1.83

$$-\frac{Aa}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{7A\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}}{192ax^5\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ab^{\frac{5}{2}}}{384a^2x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ab^{\frac{7}{2}}}{128a^3x\sqrt{\frac{a}{bx^2}+1}}+\frac{5Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{7}{2}}}-\frac{Ba}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{5B\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{Bb^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{Bb^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] -A*a/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 7*A*sqrt(b)/(48*x**7*sqrt(a/(b
*x**2) + 1)) + A*b**(3/2)/(192*a*x**5*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5/2)/
(384*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(7/2)/(128*a**3*x*sqrt(a/(b*x
**2) + 1)) + 5*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(7/2)) - B*a/(6*sq
rt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*B*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1
)) + B*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + B*b**(5/2)/(16*a**2*x*sq
rt(a/(b*x**2) + 1)) - B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2))
```

Giac [A]
time = 0.85, size = 194, normalized size = 1.24

$$\frac{\frac{3\,(8\,Bab^4-5\,Ab^5)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{24\,(bx^2+a)^{\frac{7}{2}}Bab^4-88\,(bx^2+a)^{\frac{5}{2}}Ba^2b^4+40\,(bx^2+a)^{\frac{3}{2}}Ba^3b^4+24\,\sqrt{bx^2+a}\,Ba^4b^4-15\,(bx^2+a)^{\frac{7}{2}}Ab^5+55\,(bx^2+a)^{\frac{5}{2}}Aab^5-73\,(bx^2+a)^{\frac{3}{2}}Aa^2b^5-15\,\sqrt{bx^2+a}\,Aa^3b^5}{a^3b^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/384*(3*(8*B*a*b^4 - 5*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a
^3) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 - 88*(b*x^2 + a)^(5/2)*B*a^2*b^4 + 40*(
b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 + a)*B*a^4*b^4 - 15*(b*x^2 + a)^
(7/2)*A*b^5 + 55*(b*x^2 + a)^(5/2)*A*a*b^5 - 73*(b*x^2 + a)^(3/2)*A*a^2*b^5
 - 15*sqrt(b*x^2 + a)*A*a^3*b^5)/(a^3*b^4*x^8))/b
```

Mupad [B]
time = 1.24, size = 173, normalized size = 1.11

$$\frac{55\,A\,{\left(b\,x^2+a\right)}^{5/2}}{384\,a^2\,x^8}-\frac{B\,\sqrt{b\,x^2+a}}{16\,x^6}-\frac{73\,A\,{\left(b\,x^2+a\right)}^{3/2}}{384\,a\,x^8}-\frac{5\,A\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{5\,A\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^3\,x^8}-\frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a\,x^6}+\frac{B\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^2\,x^6}-\frac{A\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{7/2}}+\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{16\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^9,x)
```

```
[Out] (B*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (B*(a + b*x^
2)^(1/2))/(16*x^6) - (A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(128*a
^(7/2)) - (5*A*(a + b*x^2)^(1/2))/(128*x^8) - (73*A*(a + b*x^2)^(3/2))/(384
*a*x^8) + (55*A*(a + b*x^2)^(5/2))/(384*a^2*x^8) - (5*A*(a + b*x^2)^(7/2))/
(128*a^3*x^8) - (B*(a + b*x^2)^(3/2))/(6*a*x^6) + (B*(a + b*x^2)^(5/2))/(16
*a^2*x^6)
```

3.519 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{10}}\,dx$

Optimal. Leaf size=117

$$-\frac{A(a+bx^2)^{3/2}}{9ax^9}+\frac{(2Ab-3aB)\,(a+bx^2)^{3/2}}{21a^2x^7}-\frac{4b(2Ab-3aB)\,(a+bx^2)^{3/2}}{105a^3x^5}+\frac{8b^2(2Ab-3aB)\,(a+bx^2)^{3/2}}{315a^4x^3}$$

```
[Out] -1/9*A*(b*x^2+a)^(3/2)/a/x^9+1/21*(2*A*b-3*B*a)*(b*x^2+a)^(3/2)/a^2/x^7-4/1
05*b*(2*A*b-3*B*a)*(b*x^2+a)^(3/2)/a^3/x^5+8/315*b^2*(2*A*b-3*B*a)*(b*x^2+a
)^(3/2)/a^4/x^3
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$\frac{8b^2(a+bx^2)^{3/2}\,(2Ab-3aB)}{315a^4x^3}-\frac{4b(a+bx^2)^{3/2}\,(2Ab-3aB)}{105a^3x^5}+\frac{(a+bx^2)^{3/2}\,(2Ab-3aB)}{21a^2x^7}-\frac{A(a+bx^2)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

```
[Out] -1/9*(A*(a + b*x^2)^(3/2))/(a*x^9) + ((2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(2
1*a^2*x^7) - (4*b*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(105*a^3*x^5) + (8*b^2
*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(315*a^4*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{10}}\,dx = -\frac{A(a+bx^2)^{3/2}}{9ax^9} - \frac{(6Ab-9aB)\int \frac{\sqrt{a+bx^2}}{x^8}\,dx}{9a}$$
$$= -\frac{A(a+bx^2)^{3/2}}{9ax^9} + \frac{(2Ab-3aB)(a+bx^2)^{3/2}}{21a^2x^7} + \frac{(4b(2Ab-3aB))\int \frac{\sqrt{a+bx^2}}{x^6}}{21a^2}$$
$$= -\frac{A(a+bx^2)^{3/2}}{9ax^9} + \frac{(2Ab-3aB)(a+bx^2)^{3/2}}{21a^2x^7} - \frac{4b(2Ab-3aB)(a+bx^2)^{3/2}}{105a^3x^5}$$
$$= -\frac{A(a+bx^2)^{3/2}}{9ax^9} + \frac{(2Ab-3aB)(a+bx^2)^{3/2}}{21a^2x^7} - \frac{4b(2Ab-3aB)(a+bx^2)^{3/2}}{105a^3x^5}$$

Mathematica [A]
time = 0.15, size = 81, normalized size = 0.69

$$\frac{(a+bx^2)^{3/2}\left(16Ab^3x^6-24ab^2x^4(A+Bx^2)+6a^2bx^2(5A+6Bx^2)-5a^3(7A+9Bx^2)\right)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
[Out] ((a + b*x^2)^(3/2)*(16*A*b^3*x^6 - 24*a*b^2*x^4*(A + B*x^2) + 6*a^2*b*x^2*(
5*A + 6*B*x^2) - 5*a^3*(7*A + 9*B*x^2)))/(315*a^4*x^9)
```

Maple [A]
time = 0.09, size = 150, normalized size = 1.28

method	result
gosper	$-\frac{(b\,x^2+a)^{\frac{3}{2}}\left(-16x^6Ab^3+24x^6Bab^2+24Aab^2x^4-36x^4Ba^2b-30x^2Aa^2b+45Ba^3x^2+35Aa^3\right)}{315x^9a^4}$
trager	$-\frac{\left(-16Ab^4x^8+24Bab^3x^8+8Aab^3x^6-12Ba^2b^2x^6-6Aa^2b^2x^4+9Ba^3bx^4+5Aa^3bx^2+45Ba^4x^2+35Aa^4\right)\sqrt{b\,x^2+a}}{315x^9a^4}$
risch	$-\frac{\left(-16Ab^4x^8+24Bab^3x^8+8Aab^3x^6-12Ba^2b^2x^6-6Aa^2b^2x^4+9Ba^3bx^4+5Aa^3bx^2+45Ba^4x^2+35Aa^4\right)\sqrt{b\,x^2+a}}{315x^9a^4}$

default	$A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{9a\,x^9}-\frac{2b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)}{3a}\right)+B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x \ldots}{15a \ldots}\right.}{7a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/9/a/x^9*(b*x^2+a)^(3/2)-2/3*b/a*(-1/7/a/x^7*(b*x^2+a)^(3/2)-4/7*b/a*(
-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a^2*(b*x^2+a)^(3/2)/x^3)))+B*(-1/7/a/x^7*
(b*x^2+a)^(3/2)-4/7*b/a*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a^2*(b*x^2+a)^(3
/2)/x^3))
```

Maxima [A]
time = 0.33, size = 138, normalized size = 1.18

$$-\frac{8\,(bx^2+a)^{\frac{3}{2}}Bb^2}{105\,a^3x^3}+\frac{16\,(bx^2+a)^{\frac{3}{2}}Ab^3}{315\,a^4x^3}+\frac{4\,(bx^2+a)^{\frac{3}{2}}Bb}{35\,a^2x^5}-\frac{8\,(bx^2+a)^{\frac{3}{2}}Ab^2}{105\,a^3x^5}-\frac{(bx^2+a)^{\frac{3}{2}}B}{7\,ax^7}+\frac{2\,(bx^2+a)^{\frac{3}{2}}Ab}{21\,a^2x^7}-\frac{(bx^2+a)^{\frac{3}{2}}A}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -8/105*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^3) + 16/315*(b*x^2 + a)^(3/2)*A*b^3/(
a^4*x^3) + 4/35*(b*x^2 + a)^(3/2)*B*b/(a^2*x^5) - 8/105*(b*x^2 + a)^(3/2)*A
*b^2/(a^3*x^5) - 1/7*(b*x^2 + a)^(3/2)*B/(a*x^7) + 2/21*(b*x^2 + a)^(3/2)*A
*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(3/2)*A/(a*x^9)
```

Fricas [A]
time = 1.50, size = 105, normalized size = 0.90

$$-\frac{(8\,(3\,Bab^3-2\,Ab^4)x^8-4\,(3\,Ba^2b^2-2\,Aab^3)x^6+35\,Aa^4+3\,(3\,Ba^3b-2\,Aa^2b^2)x^4+5\,(9\,Ba^4+Aa^3b)x^2)\sqrt{bx^2+a}}{315\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(8*(3*B*a*b^3 - 2*A*b^4)*x^8 - 4*(3*B*a^2*b^2 - 2*A*a*b^3)*x^6 + 35*
A*a^4 + 3*(3*B*a^3*b - 2*A*a^2*b^2)*x^4 + 5*(9*B*a^4 + A*a^3*b)*x^2)*sqrt(b
*x^2 + a)/(a^4*x^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(112) = 224$.
time = 2.11, size = 957, normalized size = 8.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**10,x)
```

```
[Out] -35*A*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b*
*10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**6*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**5*b**(23/2)*x**4*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**4*b**(25/2)*x**6*sqrt(a/(b*x**
2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) + 5*A*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*
a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**
12*x**14) + 30*A*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x
**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) +
 40*A*a*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6
*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 16*A*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 9
45*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 15*B*a**5*b**(9/2)*sqrt(a/(b*
x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
- 33*B*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a
**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*B*a**3*b**(13/2)*x**4*sqrt(a/(b*x
**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) -
 3*B*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**
4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*B*a*b**(17/2)*x**8*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*
b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x*
*8 + 105*a**3*b**6*x**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(101) = 202.
time = 1.31, size = 344, normalized size = 2.94

$$\frac{16\left(210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bb^{\frac{7}{2}}-315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bab^{\frac{7}{2}}+630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ab^{\frac{9}{2}}+63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ba^2b^{\frac{7}{2}}+378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Aab^{\frac{9}{2}}-42\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^3b^{\frac{7}{2}}+168\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Aa^2b^{\frac{9}{2}}+108\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Ba^4b^{\frac{7}{2}}-72\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Aa^3b^{\frac{9}{2}}-27\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Ba^5b^{\frac{7}{2}}+18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Aa^4b^{\frac{9}{2}}+3Ba^6b^{\frac{7}{2}}-2Aa^5b^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(7/2) - 315*(sqrt(b)*x - s
qrt(b*x^2 + a))^10*B*a*b^(7/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*b^(
9/2) + 63*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(7/2) + 378*(sqrt(b)*x -
sqrt(b*x^2 + a))^8*A*a*b^(9/2) - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b
^(7/2) + 168*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(9/2) + 108*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*B*a^4*b^(7/2) - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*
a^3*b^(9/2) - 27*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(7/2) + 18*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(9/2) + 3*B*a^6*b^(7/2) - 2*A*a^5*b^(9/2))
/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

Mupad [B]
time = 0.68, size = 174, normalized size = 1.49

$$\frac{2\,A\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^5}-\frac{B\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{A\,b\,\sqrt{b\,x^2+a}}{63\,a\,x^7}-\frac{B\,b\,\sqrt{b\,x^2+a}}{35\,a\,x^5}-\frac{A\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{8\,A\,b^3\,\sqrt{b\,x^2+a}}{315\,a^3\,x^3}+\frac{16\,A\,b^4\,\sqrt{b\,x^2+a}}{315\,a^4\,x}+\frac{4\,B\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3}-\frac{8\,B\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^10,x)
```

```
[Out] (2*A*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^5) - (B*(a + b*x^2)^(1/2))/(7*x^7) -
 (A*b*(a + b*x^2)^(1/2))/(63*a*x^7) - (B*b*(a + b*x^2)^(1/2))/(35*a*x^5) -
(A*(a + b*x^2)^(1/2))/(9*x^9) - (8*A*b^3*(a + b*x^2)^(1/2))/(315*a^3*x^3) +
 (16*A*b^4*(a + b*x^2)^(1/2))/(315*a^4*x) + (4*B*b^2*(a + b*x^2)^(1/2))/(10
5*a^2*x^3) - (8*B*b^3*(a + b*x^2)^(1/2))/(105*a^3*x)
```

3.520 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{11}}\,dx$

Optimal. Leaf size=189

$$\frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8}+\frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6}-\frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4}+\frac{b^3(7Ab-10aB)\sqrt{a+b}}{256a^4x^2}$$

```
[Out] -1/10*A*(b*x^2+a)^(3/2)/a/x^10-1/256*b^4*(7*A*b-10*B*a)*arctanh((b*x^2+a)^(
1/2)/a^(1/2))/a^(9/2)+1/80*(7*A*b-10*B*a)*(b*x^2+a)^(1/2)/a/x^8+1/480*b*(7*
A*b-10*B*a)*(b*x^2+a)^(1/2)/a^2/x^6-1/384*b^2*(7*A*b-10*B*a)*(b*x^2+a)^(1/2
)/a^3/x^4+1/256*b^3*(7*A*b-10*B*a)*(b*x^2+a)^(1/2)/a^4/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 79, 43, 44, 65, 214}

$$-\frac{b^4(7Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{9/2}}+\frac{b^3\sqrt{a+bx^2}\,(7Ab-10aB)}{256a^4x^2}-\frac{b^2\sqrt{a+bx^2}\,(7Ab-10aB)}{384a^3x^4}+\frac{b\sqrt{a+bx^2}\,(7Ab-10aB)}{480a^2x^6}+\frac{\sqrt{a+bx^2}\,(7Ab-10aB)}{80ax^8}-\frac{A(a+bx^2)^{3/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

```
[Out] ((7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(80*a*x^8) + (b*(7*A*b - 10*a*B)*Sqrt[a
+ b*x^2])/(480*a^2*x^6) - (b^2*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(384*a^3*x
^4) + (b^3*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(256*a^4*x^2) - (A*(a + b*x^2)
^(3/2))/(10*a*x^10) - (b^4*(7*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/(256*a^(9/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^6}\,dx,x,x^2\right)$$

$$= -\frac{A(a+bx^2)^{3/2}}{10ax^{10}} + \frac{\left(-\frac{7Ab}{2}+5aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^5}\,dx,x,x^2\right)}{10a}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} - \frac{A(a+bx^2)^{3/2}}{10ax^{10}} - \frac{(b(7Ab-10aB))\text{Subst}\left(\int \frac{}{x^4\sqrt{a}}\right.}{160a}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{A(a+bx^2)^{3/2}}{10ax^{10}} + \frac{(b}{}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a}}{384a^3x^4}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a}}{384a^3x^4}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a}}{384a^3x^4}$$

$$= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a}}{384a^3x^4}$$

Mathematica [A]
time = 0.26, size = 146, normalized size = 0.77

$$\frac{\frac{\sqrt{a}\sqrt{a+bx^2}\,(105Ab^4x^8-16a^3bx^2(3A+5Bx^2)-96a^4(4A+5Bx^2)-10ab^3x^6(7A+15Bx^2)+4a^2b^2x^4(14A+25Bx^2))}{x^{10}} - 15b^4(7Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{3840a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + b*x^2]*(105*A*b^4*x^8 - 16*a^3*b*x^2*(3*A + 5*B*x^2) - 9
6*a^4*(4*A + 5*B*x^2) - 10*a*b^3*x^6*(7*A + 15*B*x^2) + 4*a^2*b^2*x^4*(14*A
 + 25*B*x^2)))/x^10 - 15*b^4*(7*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]])/(3840*a^(9/2))
```

Maple [A]
time = 0.10, size = 298, normalized size = 1.58

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-105A\,b^{4}x^{8}+150Ba\,b^{3}x^{8}+70Aa\,b^{3}x^{6}-100B\,a^{2}b^{2}x^{6}-56A\,a^{2}b^{2}x^{4}+80B\,a^{3}b\,x^{4}+48A\,a^{3}b\,x^{2}+480B\,a^{4}x^{2}+384A\,a^{4}\right)}{3840x^{10}a^{4}}$

default	$B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{8a\,x^8}-\frac{5b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right.\right.}{2a}\right.}{4a}\right.}{2a}\right.}{8a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/8/a/x^8*(b*x^2+a)^(3/2)-5/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(3/2)-1/2*b/a*(
-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*
x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))+A*(-1/10/a/
x^10*(b*x^2+a)^(3/2)-7/10*b/a*(-1/8/a/x^8*(b*x^2+a)^(3/2)-5/8*b/a*(-1/6/a/x
^6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*
(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a
)^(1/2))/x)))))))
```

Maxima [A]
time = 0.31, size = 258, normalized size = 1.37

$$\frac{5\,Bb^{4}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{7}{2}}}-\frac{7\,Ab^{5}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,a^{\frac{9}{2}}}-\frac{5\,\sqrt{bx^{2}+a}\,Bb^{4}}{128\,a^{4}}+\frac{7\,\sqrt{bx^{2}+a}\,Ab^{5}}{256\,a^{5}}+\frac{5\,(bx^{2}+a)^{\frac{3}{2}}Bb^{3}}{128\,a^{4}x^{2}}-\frac{7\,(bx^{2}+a)^{\frac{3}{2}}Ab^{4}}{256\,a^{5}x^{2}}-\frac{5\,(bx^{2}+a)^{\frac{3}{2}}Bb^{2}}{64\,a^{3}x^{4}}+\frac{7\,(bx^{2}+a)^{\frac{3}{2}}Ab^{3}}{128\,a^{4}x^{4}}+\frac{5\,(bx^{2}+a)^{\frac{3}{2}}Bb}{48\,a^{2}x^{6}}-\frac{7\,(bx^{2}+a)^{\frac{3}{2}}Ab^{2}}{96\,a^{3}x^{6}}-\frac{(bx^{2}+a)^{\frac{3}{2}}B}{8\,ax^{8}}+\frac{7\,(bx^{2}+a)^{\frac{3}{2}}Ab}{80\,a^{2}x^{8}}-\frac{(bx^{2}+a)^{\frac{3}{2}}A}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 5/128*B*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 7/256*A*b^5*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(9/2) - 5/128*sqrt(b*x^2 + a)*B*b^4/a^4 + 7/256*sqrt(b
*x^2 + a)*A*b^5/a^5 + 5/128*(b*x^2 + a)^(3/2)*B*b^3/(a^4*x^2) - 7/256*(b*x^
2 + a)^(3/2)*A*b^4/(a^5*x^2) - 5/64*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^4) + 7/1
28*(b*x^2 + a)^(3/2)*A*b^3/(a^4*x^4) + 5/48*(b*x^2 + a)^(3/2)*B*b/(a^2*x^6)
 - 7/96*(b*x^2 + a)^(3/2)*A*b^2/(a^3*x^6) - 1/8*(b*x^2 + a)^(3/2)*B/(a*x^8)
 + 7/80*(b*x^2 + a)^(3/2)*A*b/(a^2*x^8) - 1/10*(b*x^2 + a)^(3/2)*A/(a*x^10)
```

Fricas [A]
time = 1.22, size = 317, normalized size = 1.68

$$\left[-\frac{15\,(10\,Bab^{4}-7\,Ab^{5})\sqrt{a}\,x^{10}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(15\,(10\,Ba^{2}b^{3}-7\,Aab^{4})x^{8}-10\,(10\,Ba^{3}b^{2}-7\,Aa^{2}b^{3})x^{6}+384\,Aa^{5}+8\,(10\,Ba^{4}b-7\,Aa^{3}b^{2})x^{4}+48\,(10\,Ba^{5}+Aa^{4}b)x^{2})\sqrt{bx^{2}+a}}{7680\,a^{5}x^{10}},\ -\frac{15\,(10\,Bab^{4}-7\,Ab^{5})\sqrt{-a}\,x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(15\,(10\,Ba^{2}b^{3}-7\,Aab^{4})x^{8}-10\,(10\,Ba^{3}b^{2}-7\,Aa^{2}b^{3})x^{6}+384\,Aa^{5}+8\,(10\,Ba^{4}b-7\,Aa^{3}b^{2})x^{4}+48\,(10\,Ba^{5}+Aa^{4}b)x^{2})\sqrt{bx^{2}+a}}{3840\,a^{5}x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(10*B*a*b^4 - 7*A*b^5)*sqrt(a)*x^10*log(-(b*x^2 - 2*sqrt(b*x^2
 + a)*sqrt(a) + 2*a)/x^2) + 2*(15*(10*B*a^2*b^3 - 7*A*a*b^4)*x^8 - 10*(10*B
*a^3*b^2 - 7*A*a^2*b^3)*x^6 + 384*A*a^5 + 8*(10*B*a^4*b - 7*A*a^3*b^2)*x^4
+ 48*(10*B*a^5 + A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^5*x^10), -1/3840*(15*(10
*B*a*b^4 - 7*A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(1
0*B*a^2*b^3 - 7*A*a*b^4)*x^8 - 10*(10*B*a^3*b^2 - 7*A*a^2*b^3)*x^6 + 384*A*
a^5 + 8*(10*B*a^4*b - 7*A*a^3*b^2)*x^4 + 48*(10*B*a^5 + A*a^4*b)*x^2)*sqrt(
b*x^2 + a))/(a^5*x^10)]
```

Sympy [A]
time = 209.28, size = 347, normalized size = 1.84

$$-\frac{Aa}{10\sqrt{b}\,x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{9A\sqrt{b}}{80x^9\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}}{480ax^7\sqrt{\frac{a}{bx^2}+1}}-\frac{7Ab^{\frac{5}{2}}}{1920a^2x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{7Ab^{\frac{7}{2}}}{768a^3x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{7Ab^{\frac{9}{2}}}{256a^4x\sqrt{\frac{a}{bx^2}+1}}-\frac{7Ab^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{256a^{\frac{9}{2}}}-\frac{Ba}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{7B\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{Bb^{\frac{3}{2}}}{192ax^5\sqrt{\frac{a}{bx^2}+1}}-\frac{5Bb^{\frac{5}{2}}}{384a^2x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5Bb^{\frac{7}{2}}}{128a^3x\sqrt{\frac{a}{bx^2}+1}}+\frac{5Bb^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] -A*a/(10*sqrt(b)*x**11*sqrt(a/(b*x**2) + 1)) - 9*A*sqrt(b)/(80*x**9*sqrt(a/
(b*x**2) + 1)) + A*b**(3/2)/(480*a*x**7*sqrt(a/(b*x**2) + 1)) - 7*A*b**(5/2
)/(1920*a**2*x**5*sqrt(a/(b*x**2) + 1)) + 7*A*b**(7/2)/(768*a**3*x**3*sqrt(
a/(b*x**2) + 1)) + 7*A*b**(9/2)/(256*a**4*x*sqrt(a/(b*x**2) + 1)) - 7*A*b**
5*asinh(sqrt(a)/(sqrt(b)*x))/(256*a**(9/2)) - B*a/(8*sqrt(b)*x**9*sqrt(a/(b
*x**2) + 1)) - 7*B*sqrt(b)/(48*x**7*sqrt(a/(b*x**2) + 1)) + B*b**(3/2)/(192
*a*x**5*sqrt(a/(b*x**2) + 1)) - 5*B*b**(5/2)/(384*a**2*x**3*sqrt(a/(b*x**2)
 + 1)) - 5*B*b**(7/2)/(128*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*B*b**4*asinh(sq
rt(a)/(sqrt(b)*x))/(128*a**(7/2))
```

Giac [A]
time = 1.42, size = 230, normalized size = 1.22

$$-\frac{\frac{15\left(10Bab^5-7Ab^6\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}+\frac{150\left(bx^2+a\right)^{\frac{9}{2}}Bab^5-700\left(bx^2+a\right)^{\frac{7}{2}}Ba^2b^5+1280\left(bx^2+a\right)^{\frac{5}{2}}Ba^3b^5-580\left(bx^2+a\right)^{\frac{3}{2}}Ba^4b^5-150\sqrt{bx^2+a}\,Ba^5b^5-105\left(bx^2+a\right)^{\frac{9}{2}}Ab^6+490\left(bx^2+a\right)^{\frac{7}{2}}Aab^6-896\left(bx^2+a\right)^{\frac{5}{2}}Aa^2b^6+790\left(bx^2+a\right)^{\frac{3}{2}}Aa^3b^6+105\sqrt{bx^2+a}\,Aa^4b^6}{a^4b^5x^{10}}}{3840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/3840*(15*(10*B*a*b^5 - 7*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-
a)*a^4) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 - 700*(b*x^2 + a)^(7/2)*B*a^2*b^5
+ 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 - 580*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 150*
sqrt(b*x^2 + a)*B*a^5*b^5 - 105*(b*x^2 + a)^(9/2)*A*b^6 + 490*(b*x^2 + a)^(
7/2)*A*a*b^6 - 896*(b*x^2 + a)^(5/2)*A*a^2*b^6 + 790*(b*x^2 + a)^(3/2)*A*a^
3*b^6 + 105*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^4*b^5*x^10))/b
```

Mupad [B]
time = 1.59, size = 209, normalized size = 1.11

$$\frac{7A\left(bx^2+a\right)^{5/2}}{30\,a^2x^{10}}-\frac{5B\sqrt{bx^2+a}}{128\,x^8}-\frac{79A\left(bx^2+a\right)^{3/2}}{384\,a\,x^{10}}-\frac{7A\sqrt{bx^2+a}}{256\,x^{10}}-\frac{49A\left(bx^2+a\right)^{7/2}}{384\,a^3x^{10}}+\frac{7A\left(bx^2+a\right)^{9/2}}{256\,a^4x^{10}}-\frac{73B\left(bx^2+a\right)^{3/2}}{384\,a\,x^8}+\frac{55B\left(bx^2+a\right)^{5/2}}{384\,a^2x^8}-\frac{5B\left(bx^2+a\right)^{7/2}}{128\,a^3x^8}+\frac{Ab^5\operatorname{atan}\left(\frac{\sqrt{bx^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)7\mathrm{i}}{256\,a^{9/2}}-\frac{Bb^4\operatorname{atan}\left(\frac{\sqrt{bx^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{128\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^11,x)
```

```
[Out] (A*b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*7i)/(256*a^(9/2)) - (5*B*(a + b
*x^2)^(1/2))/(128*x^8) - (7*A*(a + b*x^2)^(1/2))/(256*x^10) - (B*b^4*atan((
(a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(128*a^(7/2)) - (79*A*(a + b*x^2)^(3/2))
/(384*a*x^10) + (7*A*(a + b*x^2)^(5/2))/(30*a^2*x^10) - (49*A*(a + b*x^2)^(
7/2))/(384*a^3*x^10) + (7*A*(a + b*x^2)^(9/2))/(256*a^4*x^10) - (73*B*(a +
b*x^2)^(3/2))/(384*a*x^8) + (55*B*(a + b*x^2)^(5/2))/(384*a^2*x^8) - (5*B*(
a + b*x^2)^(7/2))/(128*a^3*x^8)
```

3.521 $\int x^5(a+bx^2)^{3/2}(A+Bx^2)\,dx$

Optimal. Leaf size=103

$$\frac{a^2(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{9/2}}{9b^4}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^4}$$

```
[Out] 1/5*a^2*(A*b-B*a)*(b*x^2+a)^(5/2)/b^4-1/7*a*(2*A*b-3*B*a)*(b*x^2+a)^(7/2)/b
^4+1/9*(A*b-3*B*a)*(b*x^2+a)^(9/2)/b^4+1/11*B*(b*x^2+a)^(11/2)/b^4
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{a^2\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^4}+\frac{\left(a+bx^2\right)^{9/2}(Ab-3aB)}{9b^4}-\frac{a\left(a+bx^2\right)^{7/2}(2Ab-3aB)}{7b^4}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(7/2))/(7*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(9/2))/(9*b^4) + (B*(a + b*x^
2)^(11/2))/(11*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx = \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{3/2}(A+Bx)\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^{3/2}}{b^3} + \frac{a(-2Ab+3aB)(a+bx)^{5/2}}{b^3}\right.\right.$$
$$= \frac{a^2(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^4} - \frac{a(2Ab-3aB)\left(a+bx^2\right)^{7/2}}{7b^4} + \frac{(Ab-3aB)\,(}{9b}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{5/2}\left(88a^2Ab-48a^3B-220aAb^2x^2+120a^2bBx^2+385Ab^3x^4-210ab^2Bx^4+315b^3Bx^6\right)}{3465b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(88*a^2*A*b - 48*a^3*B - 220*a*A*b^2*x^2 + 120*a^2*b*B*x
^2 + 385*A*b^3*x^4 - 210*a*b^2*B*x^4 + 315*b^3*B*x^6))/(3465*b^4)
```

Maple [A]
time = 0.08, size = 144, normalized size = 1.40

method	result
gosper	$\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(315B\,x^6b^3+385A\,b^3x^4-210Ba\,b^2x^4-220Aa\,b^2x^2+120B\,a^2b\,x^2+88A\,a^2b-48B\,a^3\right)}{3465b^4}$
trager	$\frac{(315b^5B\,x^{10}+385A\,b^5x^8+420Ba\,b^4x^8+550Aa\,b^4x^6+15B\,a^2b^3x^6+33A\,a^2b^3x^4-18B\,a^3b^2x^4-44A\,a^3b^2x^2+24B\,a^4b\,x^2+88a^4bA-48}{3465b^4}$
risch	$\frac{(315b^5B\,x^{10}+385A\,b^5x^8+420Ba\,b^4x^8+550Aa\,b^4x^6+15B\,a^2b^3x^6+33A\,a^2b^3x^4-18B\,a^3b^2x^4-44A\,a^3b^2x^2+24B\,a^4b\,x^2+88a^4bA-48}{3465b^4}$
default	$B\left(\frac{x^6\left(b\,x^2+a\right)^{\frac{5}{2}}}{11b}-\frac{6a\left(\frac{x^4\left(b\,x^2+a\right)^{\frac{5}{2}}}{9b}-\frac{4a\left(\frac{x^2\left(b\,x^2+a\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(b\,x^2+a\right)^{\frac{5}{2}}}{35b^2}\right)}{9b}\right)}{11b}\right)+A\left(\frac{x^4\left(b\,x^2+a\right)^{\frac{5}{2}}}{9b}-\frac{4a\left(\frac{x^2\left(b\,x^2+a\right)^{\frac{5}{2}}}{7b}-2\right.}{9b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/11*x^6*(b*x^2+a)^(5/2)/b-6/11*a/b*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(
1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))))+A*(1/9*x^4*(b*x^2+a
)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2)))
```

Maxima [A]
time = 0.28, size = 132, normalized size = 1.28

$$\frac{(bx^2+a)^{\frac{5}{2}}Bx^6}{11\,b} - \frac{2\,(bx^2+a)^{\frac{5}{2}}Bax^4}{33\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ax^4}{9\,b} + \frac{8\,(bx^2+a)^{\frac{5}{2}}Ba^2x^2}{231\,b^3} - \frac{4\,(bx^2+a)^{\frac{5}{2}}Aax^2}{63\,b^2} - \frac{16\,(bx^2+a)^{\frac{5}{2}}Ba^3}{1155\,b^4} + \frac{8\,(bx^2+a)^{\frac{5}{2}}Aa^2}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(5/2)*B*x^6/b - 2/33*(b*x^2 + a)^(5/2)*B*a*x^4/b^2 + 1/9*(
b*x^2 + a)^(5/2)*A*x^4/b + 8/231*(b*x^2 + a)^(5/2)*B*a^2*x^2/b^3 - 4/63*(b*
x^2 + a)^(5/2)*A*a*x^2/b^2 - 16/1155*(b*x^2 + a)^(5/2)*B*a^3/b^4 + 8/315*(b
*x^2 + a)^(5/2)*A*a^2/b^3
```

Fricas [A]
time = 1.26, size = 124, normalized size = 1.20

$$\frac{(315\,Bb^5x^{10} + 35\,(12\,Bab^4 + 11\,Ab^5)x^8 + 5\,(3\,Ba^2b^3 + 110\,Aab^4)x^6 - 48\,Ba^5 + 88\,Aa^4b - 3\,(6\,Ba^3b^2 - 11\,Aa^2b^3)x^4 + 4\,(6\,Ba^4b - 11\,Aa^3b^2)x^2)\sqrt{bx^2+a}}{3465\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*B*b^5*x^10 + 35*(12*B*a*b^4 + 11*A*b^5)*x^8 + 5*(3*B*a^2*b^3 +
110*A*a*b^4)*x^6 - 48*B*a^5 + 88*A*a^4*b - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x
^4 + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(92) = 184$.
time = 0.40, size = 260, normalized size = 2.52

$$\begin{cases} \frac{8Aa^4\sqrt{a+bx^2}}{315b^3} - \frac{4Aa^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{Aa^2x^4\sqrt{a+bx^2}}{105b} + \frac{10Aax^6\sqrt{a+bx^2}}{63} + \frac{Abx^8\sqrt{a+bx^2}}{9} - \frac{16Ba^5\sqrt{a+bx^2}}{1155b^4} + \frac{8Ba^4x^2\sqrt{a+bx^2}}{1155b^3} - \frac{2Ba^3x^4\sqrt{a+bx^2}}{385b^2} + \frac{Ba^2x^6\sqrt{a+bx^2}}{231b} + \frac{4Bax^8\sqrt{a+bx^2}}{33} + \frac{Bbx^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((8*A*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*A*a**3*x**2*sqrt(a + b*
x**2)/(315*b**2) + A*a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*A*a*x**6*sqrt(
a + b*x**2)/63 + A*b*x**8*sqrt(a + b*x**2)/9 - 16*B*a**5*sqrt(a + b*x**2)/(
1155*b**4) + 8*B*a**4*x**2*sqrt(a + b*x**2)/(1155*b**3) - 2*B*a**3*x**4*sqr
t(a + b*x**2)/(385*b**2) + B*a**2*x**6*sqrt(a + b*x**2)/(231*b) + 4*B*a*x**
8*sqrt(a + b*x**2)/33 + B*b*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(3/2)
*(A*x**6/6 + B*x**8/8), True))
```

Giac [A]
time = 1.52, size = 104, normalized size = 1.01

$$\frac{315\,(bx^2+a)^{\frac{11}{2}}B-1155\,(bx^2+a)^{\frac{9}{2}}Ba+1485\,(bx^2+a)^{\frac{7}{2}}Ba^2-693\,(bx^2+a)^{\frac{5}{2}}Ba^3+385\,(bx^2+a)^{\frac{9}{2}}Ab-990\,(bx^2+a)^{\frac{7}{2}}Aab+693\,(bx^2+a)^{\frac{5}{2}}Aa^2b}{3465\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/3465*(315*(b*x^2 + a)^(11/2)*B - 1155*(b*x^2 + a)^(9/2)*B*a + 1485*(b*x^2
 + a)^(7/2)*B*a^2 - 693*(b*x^2 + a)^(5/2)*B*a^3 + 385*(b*x^2 + a)^(9/2)*A*b
 - 990*(b*x^2 + a)^(7/2)*A*a*b + 693*(b*x^2 + a)^(5/2)*A*a^2*b)/b^4
```

Mupad [B]
time = 0.35, size = 117, normalized size = 1.14

$$\sqrt{b\,x^2+a}\,\left(\frac{x^8\,\left(385\,A\,b^5+420\,B\,a\,b^4\right)}{3465\,b^4}-\frac{48\,B\,a^5-88\,A\,a^4\,b}{3465\,b^4}+\frac{B\,b\,x^{10}}{11}+\frac{a^2\,x^4\,\left(11\,A\,b-6\,B\,a\right)}{1155\,b^2}-\frac{4\,a^3\,x^2\,\left(11\,A\,b-6\,B\,a\right)}{3465\,b^3}+\frac{a\,x^6\,\left(110\,A\,b+3\,B\,a\right)}{693\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^8*(385*A*b^5 + 420*B*a*b^4))/(3465*b^4) - (48*B*a^5 -
 88*A*a^4*b)/(3465*b^4) + (B*b*x^10)/11 + (a^2*x^4*(11*A*b - 6*B*a))/(1155*
b^2) - (4*a^3*x^2*(11*A*b - 6*B*a))/(3465*b^3) + (a*x^6*(110*A*b + 3*B*a))/
(693*b))
```

3.522 $\int x^4 (a+bx^2)^{3/2} (A+Bx^2)\, dx$

Optimal. Leaf size=188

$$-\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3}+\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b}+\frac{(2Ab-aB)x^5(a+b$$

```
[Out] 1/16*(2*A*b-B*a)*x^5*(b*x^2+a)^(3/2)/b+1/10*B*x^5*(b*x^2+a)^(5/2)/b+3/256*a
^4*(2*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)-3/256*a^3*(2*A*b-
B*a)*x*(b*x^2+a)^(1/2)/b^3+1/128*a^2*(2*A*b-B*a)*x^3*(b*x^2+a)^(1/2)/b^2+1/
32*a*(2*A*b-B*a)*x^5*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 285, 327, 223, 212}

$$\frac{3a^4(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}-\frac{3a^3x\sqrt{a+bx^2}\,(2Ab-aB)}{256b^3}+\frac{a^2x^3\sqrt{a+bx^2}\,(2Ab-aB)}{128b^2}+\frac{x^5(a+bx^2)^{3/2}(2Ab-aB)}{16b}+\frac{ax^5\sqrt{a+bx^2}\,(2Ab-aB)}{32b}+\frac{Bx^5(a+bx^2)^{5/2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (-3*a^3*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(256*b^3) + (a^2*(2*A*b - a*B)*x^3
*Sqrt[a + b*x^2])/(128*b^2) + (a*(2*A*b - a*B)*x^5*Sqrt[a + b*x^2])/(32*b)
+ ((2*A*b - a*B)*x^5*(a + b*x^2)^(3/2))/(16*b) + (B*x^5*(a + b*x^2)^(5/2))/
(10*b) + (3*a^4*(2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^
(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}-\frac{(-10Ab+5aB)\int x^4\left(a+bx^2\right)^{3/2}\,dx}{10b}$$
$$=\frac{(2Ab-aB)x^5\left(a+bx^2\right)^{3/2}}{16b}+\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}+\frac{(3a(2Ab-aB))\int x^4\sqrt{}}{16b}$$
$$=\frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b}+\frac{(2Ab-aB)x^5\left(a+bx^2\right)^{3/2}}{16b}+\frac{Bx^5\left(a+bx^2\right.}{10b}$$
$$=\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b}+\frac{(2Ab-aB)}{1}$$
$$=-\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3}+\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-}{}$$
$$=-\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3}+\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-}{}$$
$$=-\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3}+\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-}{}$$

Mathematica [A]

time = 0.18, size = 142, normalized size = 0.76

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\,\left(15a^4B-10a^3b(3A+Bx^2)+4a^2b^2x^2(5A+2Bx^2)+32b^4x^6(5A+4Bx^2)+16ab^3x^4(15A+11Bx^2)\right)+15a^4(-2Ab+aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{1280b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^4*B - 10*a^3*b*(3*A + B*x^2) + 4*a^2*b^2*x
^2*(5*A + 2*B*x^2) + 32*b^4*x^6*(5*A + 4*B*x^2) + 16*a*b^3*x^4*(15*A + 11*B
*x^2)) + 15*a^4*(-2*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1280*b
^(7/2))
```

Maple [A]
time = 0.08, size = 224, normalized size = 1.19

method	result
risch	$-\frac{x\left(-128B\,b^4x^8-160A\,b^4x^6-176Ba\,b^3x^6-240Aa\,b^3x^4-8B\,a^2b^2x^4-20a^2A\,b^2x^2+10B\,a^3b\,x^2+30A\,a^3b-15B\,a^4\right)\sqrt{b\,x^2+a}}{1280b^3}+$

default	B	$\left(\frac{x^{5}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{10b}-\frac{a\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/10*x^5*(b*x^2+a)^(5/2)/b-1/2*a/b*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1
/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a
)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+A*(1/8*x^3*(b*x^2+
a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+
3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))
)
```

Maxima [A]
time = 0.28, size = 204, normalized size = 1.09

$$\frac{(bx^2+a)^{\frac{5}{2}}Bx^5}{10\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Bax^3}{16\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Ax^3}{8\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Ba^2x}{32\,b^3}-\frac{(bx^2+a)^{\frac{3}{2}}Ba^3x}{128\,b^3}-\frac{3\,\sqrt{bx^2+a}\,Ba^4x}{256\,b^3}-\frac{(bx^2+a)^{\frac{5}{2}}Aax}{16\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Aa^2x}{64\,b^2}+\frac{3\,\sqrt{bx^2+a}\,Aa^3x}{128\,b^2}-\frac{3\,Ba^5\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}}+\frac{3\,Aa^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(5/2)*B*x^5/b - 1/16*(b*x^2 + a)^(5/2)*B*a*x^3/b^2 + 1/8*(
b*x^2 + a)^(5/2)*A*x^3/b + 1/32*(b*x^2 + a)^(5/2)*B*a^2*x/b^3 - 1/128*(b*x^
2 + a)^(3/2)*B*a^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*B*a^4*x/b^3 - 1/16*(b*x^2
+ a)^(5/2)*A*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*A*a^2*x/b^2 + 3/128*sqrt(b*x^
2 + a)*A*a^3*x/b^2 - 3/256*B*a^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/128*A*a
^4*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 1.02, size = 299, normalized size = 1.59

$$\left[-\frac{15\,(Ba^5-2\,Aa^4b)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(128\,Bb^5x^9+16\,(11\,Bab^4+10\,Ab^5)x^7+8\,(Ba^2b^3+30\,Aab^4)x^5-10\,(Ba^3b^2-2\,Aa^2b^3)x^3+15\,(Ba^4b-2\,Aa^3b^2)x)\sqrt{bx^2+a}}{2560\,b^4},\ \frac{15\,(Ba^5-2\,Aa^4b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(128\,Bb^5x^9+16\,(11\,Bab^4+10\,Ab^5)x^7+8\,(Ba^2b^3+30\,Aab^4)x^5-10\,(Ba^3b^2-2\,Aa^2b^3)x^3+15\,(Ba^4b-2\,Aa^3b^2)x)\sqrt{bx^2+a}}{1280\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(B*a^5 - 2*A*a^4*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(128*B*b^5*x^9 + 16*(11*B*a*b^4 + 10*A*b^5)*x^7 + 8*(B*a^
2*b^3 + 30*A*a*b^4)*x^5 - 10*(B*a^3*b^2 - 2*A*a^2*b^3)*x^3 + 15*(B*a^4*b -
2*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/1280*(15*(B*a^5 - 2*A*a^4*b)*sqrt(-
b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (128*B*b^5*x^9 + 16*(11*B*a*b^4 + 1
0*A*b^5)*x^7 + 8*(B*a^2*b^3 + 30*A*a*b^4)*x^5 - 10*(B*a^3*b^2 - 2*A*a^2*b^3
)*x^3 + 15*(B*a^4*b - 2*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(170) = 340$.
time = 164.25, size = 345, normalized size = 1.84

$$-\frac{3Aa^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13Aa^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5A\sqrt{a}\,bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Ab^2x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^{\frac{9}{2}}x}{256b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^{\frac{7}{2}}x^3}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{5}{2}}x^5}{640b\sqrt{1+\frac{bx^2}{a}}}+\frac{23Ba^{\frac{3}{2}}x^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{19B\sqrt{a}\,bx^9}{80\sqrt{1+\frac{bx^2}{a}}}-\frac{3Ba^5\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{7}{2}}}+\frac{Bb^2x^{11}}{10\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] -3*A*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - A*a**(5/2)*x**3/(128*b*sqrt
(1 + b*x**2/a)) + 13*A*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*A*sqrt(a)*
b*x**7/(16*sqrt(1 + b*x**2/a)) + 3*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**
(5/2)) + A*b**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + 3*B*a**(9/2)*x/(256*b
**3*sqrt(1 + b*x**2/a)) + B*a**(7/2)*x**3/(256*b**2*sqrt(1 + b*x**2/a)) - B
*a**(5/2)*x**5/(640*b*sqrt(1 + b*x**2/a)) + 23*B*a**(3/2)*x**7/(160*sqrt(1
+ b*x**2/a)) + 19*B*sqrt(a)*b*x**9/(80*sqrt(1 + b*x**2/a)) - 3*B*a**5*asinh
(sqrt(b)*x/sqrt(a))/(256*b**(7/2)) + B*b**2*x**11/(10*sqrt(a)*sqrt(1 + b*x*
*2/a))
```

Giac [A]
time = 2.35, size = 159, normalized size = 0.85

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,Bbx^2+\frac{11\,Bab^8+10\,Ab^9}{b^8}\right)x^2+\frac{Ba^2b^7+30\,Aab^8}{b^8}\right)x^2-\frac{5\,(Ba^3b^6-2\,Aa^2b^7)}{b^8}\right)x^2+\frac{15\,(Ba^4b^5-2\,Aa^3b^6)}{b^8}\right)\sqrt{bx^2+a}\,x+\frac{3\,(Ba^5-2\,Aa^4b)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*B*b*x^2 + (11*B*a*b^8 + 10*A*b^9)/b^8)*x^2 + (B*a^2*b^7
+ 30*A*a*b^8)/b^8)*x^2 - 5*(B*a^3*b^6 - 2*A*a^2*b^7)/b^8)*x^2 + 15*(B*a^4*b
^5 - 2*A*a^3*b^6)/b^8)*sqrt(b*x^2 + a)*x + 3/256*(B*a^5 - 2*A*a^4*b)*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^4*(A + B*x^2)*(a + b*x^2)^(3/2), x)
```

3.523 $\int x^3(a+bx^2)^{3/2}(A+Bx^2)\,dx$

Optimal. Leaf size=73

$$-\frac{a(Ab-aB)(a+bx^2)^{5/2}}{5b^3}+\frac{(Ab-2aB)(a+bx^2)^{7/2}}{7b^3}+\frac{B(a+bx^2)^{9/2}}{9b^3}$$

```
[Out] -1/5*a*(A*b-B*a)*(b*x^2+a)^(5/2)/b^3+1/7*(A*b-2*B*a)*(b*x^2+a)^(7/2)/b^3+1/
9*B*(b*x^2+a)^(9/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{(a+bx^2)^{7/2}(Ab-2aB)}{7b^3}-\frac{a(a+bx^2)^{5/2}(Ab-aB)}{5b^3}+\frac{B(a+bx^2)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] -1/5*(a*(A*b - a*B)*(a + b*x^2)^(5/2))/b^3 + ((A*b - 2*a*B)*(a + b*x^2)^(7/
2))/(7*b^3) + (B*(a + b*x^2)^(9/2))/(9*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx = \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{3/2}(A+Bx) \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^{3/2}}{b^2} + \frac{(Ab-2aB)(a+bx)^{5/2}}{b^2} + \frac{B(a$$
$$= -\frac{a(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^3} + \frac{(Ab-2aB)\left(a+bx^2\right)^{7/2}}{7b^3} + \frac{B\left(a+bx^2\right)^{9/2}}{9b^3}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.77

$$\frac{\left(a+bx^2\right)^{5/2}\left(-18aAb+8a^2B+45Ab^2x^2-20abBx^2+35b^2Bx^4\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-18*a*A*b + 8*a^2*B + 45*A*b^2*x^2 - 20*a*b*B*x^2 + 35*
b^2*B*x^4))/(315*b^3)
```

Maple [A]
time = 0.08, size = 96, normalized size = 1.32

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-35b^{2}B\,x^{4}-45A\,b^{2}x^{2}+20Bab\,x^{2}+18abA-8a^{2}B\right)}{315b^{3}}$	53
default	$B\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{9b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35b^{2}}\right)}{9b}\right)+A\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35b^{2}}\right)$	96
trager	$-\frac{\left(-35B\,b^{4}x^{8}-45A\,b^{4}x^{6}-50Ba\,b^{3}x^{6}-72Aa\,b^{3}x^{4}-3B\,a^{2}b^{2}x^{4}-9a^{2}A\,b^{2}x^{2}+4B\,a^{3}b\,x^{2}+18A\,a^{3}b-8B\,a^{4}\right)\sqrt{b\,x^{2}+a}}{315b^{3}}$	101
risch	$-\frac{\left(-35B\,b^{4}x^{8}-45A\,b^{4}x^{6}-50Ba\,b^{3}x^{6}-72Aa\,b^{3}x^{4}-3B\,a^{2}b^{2}x^{4}-9a^{2}A\,b^{2}x^{2}+4B\,a^{3}b\,x^{2}+18A\,a^{3}b-8B\,a^{4}\right)\sqrt{b\,x^{2}+a}}{315b^{3}}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*
(b*x^2+a)^(5/2)))+A*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))
```

Maxima [A]
time = 0.27, size = 90, normalized size = 1.23

$$\frac{(bx^2+a)^{\frac{5}{2}}Bx^4}{9\,b}-\frac{4\,(bx^2+a)^{\frac{5}{2}}Bax^2}{63\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Ax^2}{7\,b}+\frac{8\,(bx^2+a)^{\frac{5}{2}}Ba^2}{315\,b^3}-\frac{2\,(bx^2+a)^{\frac{5}{2}}Aa}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(5/2)*B*x^4/b - 4/63*(b*x^2 + a)^(5/2)*B*a*x^2/b^2 + 1/7*(b
*x^2 + a)^(5/2)*A*x^2/b + 8/315*(b*x^2 + a)^(5/2)*B*a^2/b^3 - 2/35*(b*x^2 +
 a)^(5/2)*A*a/b^2
```

Fricas [A]
time = 1.54, size = 99, normalized size = 1.36

$$\frac{\left(35\,Bb^4x^8+5\,(10\,Bab^3+9\,Ab^4)x^6+8\,Ba^4-18\,Aa^3b+3\,(Ba^2b^2+24\,Aab^3)x^4-(4\,Ba^3b-9\,Aa^2b^2)x^2\right)\sqrt{bx^2+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*b^4*x^8 + 5*(10*B*a*b^3 + 9*A*b^4)*x^6 + 8*B*a^4 - 18*A*a^3*b +
 3*(B*a^2*b^2 + 24*A*a*b^3)*x^4 - (4*B*a^3*b - 9*A*a^2*b^2)*x^2)*sqrt(b*x^2
 + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(65) = 130$.
time = 0.29, size = 209, normalized size = 2.86

$$\begin{cases} -\frac{2Aa^3\sqrt{a+bx^2}}{35b^2}+\frac{Aa^2x^2\sqrt{a+bx^2}}{35b}+\frac{8Aax^4\sqrt{a+bx^2}}{35}+\frac{Abx^6\sqrt{a+bx^2}}{7}+\frac{8Ba^4\sqrt{a+bx^2}}{315b^3}-\frac{4Ba^3x^2\sqrt{a+bx^2}}{315b^2}+\frac{Ba^2x^4\sqrt{a+bx^2}}{105b}+\frac{10Bax^6\sqrt{a+bx^2}}{63}+\frac{Bbx^8\sqrt{a+bx^2}}{9} & \text{for } b\neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((-2*A*a**3*sqrt(a + b*x**2)/(35*b**2) + A*a**2*x**2*sqrt(a + b*x*
*2)/(35*b) + 8*A*a*x**4*sqrt(a + b*x**2)/35 + A*b*x**6*sqrt(a + b*x**2)/7 +
 8*B*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*B*a**3*x**2*sqrt(a + b*x**2)/(315
*b**2) + B*a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*B*a*x**6*sqrt(a + b*x**2
)/63 + B*b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*(A*x**4/4 + B*x**6
/6), True))
```

Giac [A]
time = 1.37, size = 73, normalized size = 1.00

$$\frac{35\,(bx^2+a)^{\frac{9}{2}}B-90\,(bx^2+a)^{\frac{7}{2}}Ba+63\,(bx^2+a)^{\frac{5}{2}}Ba^2+45\,(bx^2+a)^{\frac{7}{2}}Ab-63\,(bx^2+a)^{\frac{5}{2}}Aab}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2)*B - 90*(b*x^2 + a)^(7/2)*B*a + 63*(b*x^2 + a)^(
5/2)*B*a^2 + 45*(b*x^2 + a)^(7/2)*A*b - 63*(b*x^2 + a)^(5/2)*A*a*b)/b^3
```

Mupad [B]
time = 0.31, size = 96, normalized size = 1.32

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,B\,a^4-18\,A\,a^3\,b}{315\,b^3}+\frac{x^6\,\left(45\,A\,b^4+50\,B\,a\,b^3\right)}{315\,b^3}+\frac{B\,b\,x^8}{9}+\frac{a^2\,x^2\,\left(9\,A\,b-4\,B\,a\right)}{315\,b^2}+\frac{a\,x^4\,\left(24\,A\,b+B\,a\right)}{105\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*B*a^4 - 18*A*a^3*b)/(315*b^3) + (x^6*(45*A*b^4 + 50*B
*a*b^3))/(315*b^3) + (B*b*x^8)/9 + (a^2*x^2*(9*A*b - 4*B*a))/(315*b^2) + (a
*x^4*(24*A*b + B*a))/(105*b))
```

3.524 $\int x^2(a+bx^2)^{3/2}(A+Bx^2)\,dx$

Optimal. Leaf size=155

$$\frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3(a+bx^2)^{3/2}}{48b}+\frac{Bx^3(a+bx^2)^{5/2}}{8b}-\frac{a^3\ldots}{\ldots}$$

```
[Out] 1/48*(8*A*b-3*B*a)*x^3*(b*x^2+a)^(3/2)/b+1/8*B*x^3*(b*x^2+a)^(5/2)/b-1/128*
a^3*(8*A*b-3*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/128*a^2*(8*A
*b-3*B*a)*x*(b*x^2+a)^(1/2)/b^2+1/64*a*(8*A*b-3*B*a)*x^3*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 285, 327, 223, 212}

$$-\frac{a^3(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{a^2x\sqrt{a+bx^2}\,(8Ab-3aB)}{128b^2}+\frac{ax^3\sqrt{a+bx^2}\,(8Ab-3aB)}{64b}+\frac{x^3(a+bx^2)^{3/2}(8Ab-3aB)}{48b}+\frac{Bx^3(a+bx^2)^{5/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(8*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(128*b^2) + (a*(8*A*b - 3*a*B)*x^3*
Sqrt[a + b*x^2])/(64*b) + ((8*A*b - 3*a*B)*x^3*(a + b*x^2)^(3/2))/(48*b) +
(B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a^3*(8*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{(-8Ab+3aB)\int x^2\left(a+bx^2\right)^{3/2}\,dx}{8b}$$
$$=\frac{(8Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{48b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{(a(8Ab-3aB))\int x^2}{16b}$$
$$=\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{48b}+\frac{Bx^3(a+b}{8b}$$
$$=\frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3a}{}$$
$$=\frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3a}{}$$
$$=\frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3a}{}$$

Mathematica [A]
time = 0.16, size = 122, normalized size = 0.79

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(-9a^3B+6a^2b(4A+Bx^2)+16b^3x^4(4A+3Bx^2)+8ab^2x^2(14A+9Bx^2)\right)+3a^3(8Ab-3aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{384b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-9*a^3*B + 6*a^2*b*(4*A + B*x^2) + 16*b^3*x^4*(
4*A + 3*B*x^2) + 8*a*b^2*x^2*(14*A + 9*B*x^2)) + 3*a^3*(8*A*b - 3*a*B)*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(384*b^(5/2))

Maple [A]

time = 0.08, size = 176, normalized size = 1.14

method	result
risch	$\frac{x\left(48Bx^6b^3+64Ab^3x^4+72Bab^2x^4+112Aab^2x^2+6Ba^2bx^2+24Aa^2b-9Ba^3\right)\sqrt{bx^2+a}}{384b^2}-\frac{a^3\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)A}{16b^{\frac{3}{2}}}+$
default	$B\left(\frac{x^3\left(bx^2+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(bx^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(bx^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)+A\left(\frac{x(b}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)

[Out] B*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*
x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(
b*x^2+a)^(1/2))))))+A*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/
2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
)))

Maxima [A]

time = 0.29, size = 162, normalized size = 1.05

$$\frac{(bx^2+a)^{\frac{5}{2}}Bx^3}{8b}-\frac{(bx^2+a)^{\frac{5}{2}}Bax}{16b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ba^2x}{64b^2}+\frac{3\sqrt{bx^2+a}\,Ba^3x}{128b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Ax}{6b}-\frac{(bx^2+a)^{\frac{3}{2}}Aax}{24b}-\frac{\sqrt{bx^2+a}\,Aa^2x}{16b}+\frac{3Ba^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128b^{\frac{5}{2}}}-\frac{Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*B*x^3/b - 1/16*(b*x^2 + a)^(5/2)*B*a*x/b^2 + 1/64*(b*
x^2 + a)^(3/2)*B*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*B*a^3*x/b^2 + 1/6*(b*x^2
 + a)^(5/2)*A*x/b - 1/24*(b*x^2 + a)^(3/2)*A*a*x/b - 1/16*sqrt(b*x^2 + a)*A
*a^2*x/b + 3/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*A*a^3*arcsinh(
b*x/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 1.19, size = 260, normalized size = 1.68

$$\left[-\frac{3\,(3\,Ba^4-8\,Aa^3b)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(48\,Bb^4x^7+8\,(9\,Bab^3+8\,Ab^4)x^5+2\,(3\,Ba^2b^2+56\,Aab^3)x^3-3\,(3\,Ba^3b-8\,Aa^2b^2)x)\sqrt{bx^2+a}}{768\,b^3},\ -\frac{3\,(3\,Ba^4-8\,Aa^3b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(48\,Bb^4x^7+8\,(9\,Bab^3+8\,Ab^4)x^5+2\,(3\,Ba^2b^2+56\,Aab^3)x^3-3\,(3\,Ba^3b-8\,Aa^2b^2)x)\sqrt{bx^2+a}}{384\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(9*B*a*b^3 + 8*A*b^4)*x^5 + 2*(3*B*a^2*
b^2 + 56*A*a*b^3)*x^3 - 3*(3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^3
, -1/384*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - (48*B*b^4*x^7 + 8*(9*B*a*b^3 + 8*A*b^4)*x^5 + 2*(3*B*a^2*b^2 + 56*A*a*
b^3)*x^3 - 3*(3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(141) = 282$.
time = 33.49, size = 287, normalized size = 1.85

$$\frac{Aa^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Aa^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11A\sqrt{a}\,bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Ab^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-\frac{3Ba^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13Ba^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}\,bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Bb^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*A*a**(3/2)*x**3/(48*sqrt(1 + b*
x**2/a)) + 11*A*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(3/2)) + A*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) -
3*B*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - B*a**(5/2)*x**3/(128*b*sqrt(
1 + b*x**2/a)) + 13*B*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*b
*x**7/(16*sqrt(1 + b*x**2/a)) + 3*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(
5/2)) + B*b**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.02, size = 133, normalized size = 0.86

$$\frac{1}{384}\left(2\left(4\left(6\,Bbx^2+\frac{9\,Bab^6+8\,Ab^7}{b^6}\right)x^2+\frac{3\,Ba^2b^5+56\,Aab^6}{b^6}\right)x^2-\frac{3\,(3\,Ba^3b^4-8\,Aa^2b^5)}{b^6}\right)\sqrt{bx^2+a}\,x-\frac{(3\,Ba^4-8\,Aa^3b)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*b*x^2 + (9*B*a*b^6 + 8*A*b^7)/b^6)*x^2 + (3*B*a^2*b^5 + 56
*A*a*b^6)/b^6)*x^2 - 3*(3*B*a^3*b^4 - 8*A*a^2*b^5)/b^6)*sqrt(b*x^2 + a)*x -
 1/128*(3*B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^2*(A + B*x^2)*(a + b*x^2)^(3/2), x)
```

3.525 $\int x(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}$$

```
[Out] 1/5*(A*b-B*a)*(b*x^2+a)^(5/2)/b^2+1/7*B*(b*x^2+a)^(7/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^2) + (B*(a + b*x^2)^(7/2))/(7*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^{3/2}(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^{3/2}}{b}+\frac{B(a+bx)^{5/2}}{b}\right)\,dx,x,x^2\right)\\ &= \frac{(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.74

$$\frac{(a+bx^2)^{5/2}\,(7Ab-2aB+5bBx^2)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
[Out] ((a + b*x^2)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^2))/(35*b^2)
```

Maple [A]
time = 0.08, size = 52, normalized size = 1.13

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{5}{2}}\,(5bB\,x^2+7Ab-2Ba)}{35b^2}$	31
default	$B\left(\frac{x^2\,(b\,x^2+a)^{\frac{5}{2}}}{7b}-\frac{2a\,(b\,x^2+a)^{\frac{5}{2}}}{35b^2}\right)+\frac{A\,(b\,x^2+a)^{\frac{5}{2}}}{5b}$	52
trager	$\frac{(5B\,x^6b^3+7A\,b^3x^4+8Ba\,b^2x^4+14Aa\,b^2x^2+B\,a^2b\,x^2+7A\,a^2b-2B\,a^3)\sqrt{b\,x^2+a}}{35b^2}$	76
risch	$\frac{(5B\,x^6b^3+7A\,b^3x^4+8Ba\,b^2x^4+14Aa\,b^2x^2+B\,a^2b\,x^2+7A\,a^2b-2B\,a^3)\sqrt{b\,x^2+a}}{35b^2}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
[Out] B*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+1/5*A/b*(b*x^2+a)^
(5/2)
```

Maxima [A]
time = 0.29, size = 50, normalized size = 1.09

$$\frac{(bx^2+a)^{\frac{5}{2}}Bx^2}{7\,b}-\frac{2\,(bx^2+a)^{\frac{5}{2}}Ba}{35\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}A}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
[Out] 1/7*(b*x^2 + a)^(5/2)*B*x^2/b - 2/35*(b*x^2 + a)^(5/2)*B*a/b^2 + 1/5*(b*x^2
 + a)^(5/2)*A/b
```

Fricas [A]
time = 1.38, size = 73, normalized size = 1.59

$$\frac{(5\,Bb^3x^6+(8\,Bab^2+7\,Ab^3)x^4-2\,Ba^3+7\,Aa^2b+(Ba^2b+14\,Aab^2)x^2)\sqrt{bx^2+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*b^3*x^6 + (8*B*a*b^2 + 7*A*b^3)*x^4 - 2*B*a^3 + 7*A*a^2*b + (B*a^
2*b + 14*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(39) = 78$.
time = 0.19, size = 158, normalized size = 3.43

$$\begin{cases} \frac{Aa^2\sqrt{a+bx^2}}{5b} + \frac{2Aax^2\sqrt{a+bx^2}}{5} + \frac{Abx^4\sqrt{a+bx^2}}{5} - \frac{2Ba^3\sqrt{a+bx^2}}{35b^2} + \frac{Ba^2x^2\sqrt{a+bx^2}}{35b} + \frac{8Bax^4\sqrt{a+bx^2}}{35} + \frac{Bbx^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^2}{2} + \frac{Bx^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((A*a**2*sqrt(a + b*x**2)/(5*b) + 2*A*a*x**2*sqrt(a + b*x**2)/5 +
A*b*x**4*sqrt(a + b*x**2)/5 - 2*B*a**3*sqrt(a + b*x**2)/(35*b**2) + B*a**2*
x**2*sqrt(a + b*x**2)/(35*b) + 8*B*a*x**4*sqrt(a + b*x**2)/35 + B*b*x**6*sq
rt(a + b*x**2)/7, Ne(b, 0)), (a**(3/2)*(A*x**2/2 + B*x**4/4), True))
```

Giac [A]
time = 2.27, size = 44, normalized size = 0.96

$$\frac{5\,(bx^2+a)^{\frac{7}{2}}B - 7\,(bx^2+a)^{\frac{5}{2}}Ba + 7\,(bx^2+a)^{\frac{5}{2}}Ab}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/35*(5*(b*x^2 + a)^(7/2)*B - 7*(b*x^2 + a)^(5/2)*B*a + 7*(b*x^2 + a)^(5/2)
*A*b)/b^2
```

Mupad [B]
time = 0.29, size = 76, normalized size = 1.65

$$\sqrt{b\,x^2+a}\,\left(\frac{x^4\,(7\,A\,b^3+8\,B\,a\,b^2)}{35\,b^2} - \frac{2\,B\,a^3-7\,A\,a^2\,b}{35\,b^2} + \frac{B\,b\,x^6}{7} + \frac{a\,x^2\,(14\,A\,b+B\,a)}{35\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^4*(7*A*b^3 + 8*B*a*b^2))/(35*b^2) - (2*B*a^3 - 7*A*a^
2*b)/(35*b^2) + (B*b*x^6)/7 + (a*x^2*(14*A*b + B*a))/(35*b))
```

3.526 $\int (a+bx^2)^{3/2} (A+Bx^2)\, dx$

Optimal. Leaf size=118

$$\frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b}+\frac{(6Ab-aB)x(a+bx^2)^{3/2}}{24b}+\frac{Bx(a+bx^2)^{5/2}}{6b}+\frac{a^2(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

```
[Out] 1/24*(6*A*b-B*a)*x*(b*x^2+a)^(3/2)/b+1/6*B*x*(b*x^2+a)^(5/2)/b+1/16*a^2*(6*
A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/16*a*(6*A*b-B*a)*x*(b
*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {396, 201, 223, 212}

$$\frac{a^2(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x(a+bx^2)^{3/2}(6Ab-aB)}{24b}+\frac{ax\sqrt{a+bx^2}(6Ab-aB)}{16b}+\frac{Bx(a+bx^2)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a*(6*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b) + ((6*A*b - a*B)*x*(a + b*x^2)^(
3/2))/(24*b) + (B*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*A*b - a*B)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx = \frac{Bx(a+bx^2)^{5/2}}{6b} - \frac{(-6Ab+aB)\int (a+bx^2)^{3/2}\,dx}{6b}$$
$$= \frac{(6Ab-aB)x(a+bx^2)^{3/2}}{24b} + \frac{Bx(a+bx^2)^{5/2}}{6b} + \frac{(a(6Ab-aB))\int \sqrt{a+bx^2}$$
$$= \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aB)x(a+bx^2)^{3/2}}{24b} + \frac{Bx(a+bx^2)^{5/2}}{6b} +$$
$$= \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aB)x(a+bx^2)^{3/2}}{24b} + \frac{Bx(a+bx^2)^{5/2}}{6b} +$$
$$= \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aB)x(a+bx^2)^{3/2}}{24b} + \frac{Bx(a+bx^2)^{5/2}}{6b} +$$

Mathematica [A]
time = 0.14, size = 99, normalized size = 0.84

$$\frac{x\sqrt{a+bx^2}\,(30aAb+3a^2B+12Ab^2x^2+14abBx^2+8b^2Bx^4)}{48b}+\frac{a^2(-6Ab+aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(30*a*A*b + 3*a^2*B + 12*A*b^2*x^2 + 14*a*b*B*x^2 + 8*b^
2*B*x^4))/(48*b) + (a^2*(-6*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/(16*b^(3/2))
```

Maple [A]
time = 0.08, size = 130, normalized size = 1.10

method	result
risch	$\frac{x(8b^2Bx^4+12Ab^2x^2+14Babx^2+30abA+3a^2B)\sqrt{bx^2+a}}{48b}+\frac{3a^2\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)A}{8\sqrt{b}}-\frac{a^3\ln\left(x\sqrt{b}+\sqrt{bx^2}\right.}{16b^{\frac{3}{2}}}$

default	$B\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)+A\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{}}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x
^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(1/4*x*(b*x^2+
a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))))
```

Maxima [A]
time = 0.31, size = 116, normalized size = 0.98

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\sqrt{bx^2+a}\,Aax+\frac{(bx^2+a)^{\frac{5}{2}}Bx}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}Bax}{24\,b}-\frac{\sqrt{bx^2+a}\,Ba^2x}{16\,b}-\frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{3\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 1/6*(b*x^2 + a)^(5/
2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b*x^2 + a)*B*a^2*x/b
- 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*
b))/sqrt(b)
```

Fricas [A]
time = 1.52, size = 207, normalized size = 1.75

$$\left[-\frac{3\,(Ba^3-6\,Aa^2b)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(8\,Bb^3x^5+2\,(7\,Bab^2+6\,Ab^3)x^3+3\,(Ba^2b+10\,Aab^2)x)\sqrt{bx^2+a}}{96\,b^2},\ \frac{3\,(Ba^3-6\,Aa^2b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(8\,Bb^3x^5+2\,(7\,Bab^2+6\,Ab^3)x^3+3\,(Ba^2b+10\,Aab^2)x)\sqrt{bx^2+a}}{48\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3)*x^3 + 3*(B*a^2*b + 10
*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-b)*arc
```

```
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3)*x^
3 + 3*(B*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(102) = 204$.
time = 10.28, size = 253, normalized size = 2.14

$$\frac{Aa^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{Aa^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{3A\sqrt{a}\,bx^3}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}} + \frac{Ab^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{Ba^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}} + \frac{17Ba^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}} + \frac{11B\sqrt{a}\,bx^5}{24\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + \frac{Bb^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + A*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3
*A*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a)
)/(8*sqrt(b)) + A*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**(5/2)*x/(
16*b*sqrt(1 + b*x**2/a)) + 17*B*a**(3/2)*x**3/(48*sqrt(1 + b*x**2/a)) + 11*
B*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - B*a**3*asinh(sqrt(b)*x/sqrt(a))/
(16*b**(3/2)) + B*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.12, size = 102, normalized size = 0.86

$$\frac{1}{48}\left(2\left(4\,Bbx^2 + \frac{7\,Bab^4 + 6\,Ab^5}{b^4}\right)x^2 + \frac{3\left(Ba^2b^3 + 10\,Aab^4\right)}{b^4}\right)\sqrt{bx^2+a}\,x + \frac{\left(Ba^3 - 6\,Aa^2b\right)\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*b*x^2 + (7*B*a*b^4 + 6*A*b^5)/b^4)*x^2 + 3*(B*a^2*b^3 + 10*A*a
*b^4)/b^4)*sqrt(b*x^2 + a)*x + 1/16*(B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int((A + B*x^2)*(a + b*x^2)^(3/2), x)
```

3.527 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=76

$$aA\sqrt{a+bx^2}+\frac{1}{3}A(a+bx^2)^{3/2}+\frac{B(a+bx^2)^{5/2}}{5b}-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/3*A*(b*x^2+a)^(3/2)+1/5*B*(b*x^2+a)^(5/2)/b-a^(3/2)*A*arctanh((b*x^2+a)^(
1/2)/a^(1/2))+a*A*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 81, 52, 65, 214}

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{3}A(a+bx^2)^{3/2}+aA\sqrt{a+bx^2}+\frac{B(a+bx^2)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

```
[Out] a*A*Sqrt[a + b*x^2] + (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5/2))/(5*b)
 - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx, x, x^2\right) \\
&= \frac{B(a+bx^2)^{5/2}}{5b} + \frac{1}{2}A\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right) \\
&= \frac{1}{3}A(a+bx^2)^{3/2} + \frac{B(a+bx^2)^{5/2}}{5b} + \frac{1}{2}(aA)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right) \\
&= aA\sqrt{a+bx^2} + \frac{1}{3}A(a+bx^2)^{3/2} + \frac{B(a+bx^2)^{5/2}}{5b} + \frac{1}{2}(a^2A)\,\text{Subst}\left(\int \frac{}{x\sqrt{}}\right. \\
&= aA\sqrt{a+bx^2} + \frac{1}{3}A(a+bx^2)^{3/2} + \frac{B(a+bx^2)^{5/2}}{5b} + \frac{(a^2A)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{b} \\
&= aA\sqrt{a+bx^2} + \frac{1}{3}A(a+bx^2)^{3/2} + \frac{B(a+bx^2)^{5/2}}{5b} - a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 83, normalized size = 1.09

$$\frac{\sqrt{a+bx^2}\,(20aAb+3a^2B+5Ab^2x^2+6abBx^2+3b^2Bx^4)}{15b} - a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(20*a*A*b + 3*a^2*B + 5*A*b^2*x^2 + 6*a*b*B*x^2 + 3*b^2*B*
x^4))/(15*b) - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Maple [A]
time = 0.08, size = 71, normalized size = 0.93

method	result	size
default	$\frac{B\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5b}+A\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*B*(b*x^2+a)^(5/2)/b+A*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Maxima [A]
time = 0.35, size = 58, normalized size = 0.76

$$-Aa^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}A+\sqrt{bx^{2}+a}\,Aa+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}B}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] -A*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*A + sqrt(b
*x^2 + a)*A*a + 1/5*(b*x^2 + a)^(5/2)*B/b
```

Fricas [A]
time = 1.37, size = 170, normalized size = 2.24

$$\left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(3\,Bb^{2}x^{4}+3\,Ba^{2}+20\,Aab+(6\,Bab+5\,Ab^{2})x^{2})\sqrt{bx^{2}+a}}{30\,b},\ \frac{15\,A\sqrt{-a}\,ab\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(3\,Bb^{2}x^{4}+3\,Ba^{2}+20\,Aab+(6\,Bab+5\,Ab^{2})x^{2})\sqrt{bx^{2}+a}}{15\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(3*B*b^2*x^4 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2
+ a))/b, 1/15*(15*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*B*b^
2*x^4 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 24.40, size = 71, normalized size = 0.93

$$\frac{Aa^2 \operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} + Aa\sqrt{a+bx^2} + \frac{A(a+bx^2)^{\frac{3}{2}}}{3} + \frac{B(a+bx^2)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x,x)
```

```
[Out] A*a**2*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + A*a*sqrt(a + b*x**2) + A*
(a + b*x**2)**(3/2)/3 + B*(a + b*x**2)**(5/2)/(5*b)
```

Giac [A]
time = 1.51, size = 79, normalized size = 1.04

$$\frac{Aa^2 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{3\,(bx^2+a)^{\frac{5}{2}}Bb^4 + 5\,(bx^2+a)^{\frac{3}{2}}Ab^5 + 15\,\sqrt{bx^2+a}\,Aab^5}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] A*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(3*(b*x^2 + a)^(5/2)
*B*b^4 + 5*(b*x^2 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^2 + a)*A*a*b^5)/b^5
```

Mupad [B]
time = 0.45, size = 60, normalized size = 0.79

$$\frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{3}+\frac{B\,{\left(b\,x^2+a\right)}^{5/2}}{5\,b}-A\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+A\,a\,\sqrt{b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x,x)
```

```
[Out] (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5/2))/(5*b) - A*a^(3/2)*atanh((a
+ b*x^2)^(1/2)/a^(1/2)) + A*a*(a + b*x^2)^(1/2)
```

3.528 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=109

$$\frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2}+\frac{(4Ab+aB)x(a+bx^2)^{3/2}}{4a}-\frac{A(a+bx^2)^{5/2}}{ax}+\frac{3a(4Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

```
[Out] 1/4*(4*A*b+B*a)*x*(b*x^2+a)^(3/2)/a-A*(b*x^2+a)^(5/2)/a/x+3/8*a*(4*A*b+B*a)
*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+3/8*(4*A*b+B*a)*x*(b*x^2+a)^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 201, 223, 212}

$$\frac{x(a+bx^2)^{3/2}(aB+4Ab)}{4a}+\frac{3}{8}x\sqrt{a+bx^2}(aB+4Ab)+\frac{3a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}-\frac{A(a+bx^2)^{5/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (3*(4*A*b + a*B)*x*Sqrt[a + b*x^2])/8 + ((4*A*b + a*B)*x*(a + b*x^2)^(3/2))
/(4*a) - (A*(a + b*x^2)^(5/2))/(a*x) + (3*a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx = -\frac{A(a+bx^2)^{5/2}}{ax} - \frac{(-4Ab-aB)\int (a+bx^2)^{3/2}\,dx}{a}$$

$$= \frac{(4Ab+aB)x(a+bx^2)^{3/2}}{4a} - \frac{A(a+bx^2)^{5/2}}{ax} + \frac{1}{4}(3(4Ab+aB))\int \sqrt{a+bx^2}$$

$$= \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x(a+bx^2)^{3/2}}{4a} - \frac{A(a+bx^2)^{5/2}}{ax} + \frac{1}{8}$$

$$= \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x(a+bx^2)^{3/2}}{4a} - \frac{A(a+bx^2)^{5/2}}{ax} + \frac{1}{8}$$

$$= \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x(a+bx^2)^{3/2}}{4a} - \frac{A(a+bx^2)^{5/2}}{ax} + \frac{3}{}$$

Mathematica [A]
time = 0.17, size = 84, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\,(-8aA+4Abx^2+5aBx^2+2bBx^4)}{8x} - \frac{3a(4Ab+aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a*A + 4*A*b*x^2 + 5*a*B*x^2 + 2*b*B*x^4))/(8*x) - (3*a
*(4*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Maple [A]
time = 0.09, size = 132, normalized size = 1.21

method	result

risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-2bB\,x^4-4Ab\,x^2-5Ba\,x^2+8Aa\right)}{8x}+\frac{3a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)\sqrt{b}\,A}{2}+\frac{3a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}$
default	$B\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(x\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))))+A*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(
3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
)))))
```

Maxima [A]
time = 0.31, size = 91, normalized size = 0.83

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bx+\frac{3}{8}\sqrt{bx^2+a}\,Bax+\frac{3}{2}\sqrt{bx^2+a}\,Abx+\frac{3\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}+\frac{3}{2}\,Aa\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2 + a)*B*a*x + 3/2*sqrt(b*x^2 + a)
*A*b*x + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/2*A*a*sqrt(b)*arcsinh
(b*x/sqrt(a*b)) - (b*x^2 + a)^(3/2)*A/x
```

Fricas [A]
time = 1.31, size = 182, normalized size = 1.67

$$\left[\frac{3\,(Ba^2+4\,Aab)\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(2\,Bb^2x^4-8\,Aab+(5\,Bab+4\,Ab^2)x^2)\sqrt{bx^2+a}}{16\,bx},-\frac{3\,(Ba^2+4\,Aab)\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2\,Bb^2x^4-8\,Aab+(5\,Bab+4\,Ab^2)x^2)\sqrt{bx^2+a}}{8\,bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(B*a^2 + 4*A*a*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + 2*(2*B*b^2*x^4 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x^2)*sqrt(b*x^2
+ a))/(b*x), -1/8*(3*(B*a^2 + 4*A*a*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) - (2*B*b^2*x^4 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x^2)*sqrt(b*x^2 +
a))/(b*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(99) = 198$.
time = 5.18, size = 216, normalized size = 1.98

$$-\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{A\sqrt{a}\,bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{A\sqrt{a}\,bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}+\frac{Ba^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\,bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Bb^2x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - A
*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*A*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
+ B*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + B*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) +
 3*B*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(
a))/(8*sqrt(b)) + B*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.90, size = 114, normalized size = 1.05

$$\frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{8}\left(2\,Bbx^2+\frac{5\,Bab^2+4\,Ab^3}{b^2}\right)\sqrt{bx^2+a}\,x-\frac{3\left(Ba^2\sqrt{b}+4\,Aab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 2*A*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*B*b*x^2 + (5
*B*a*b^2 + 4*A*b^3)/b^2)*sqrt(b*x^2 + a)*x - 3/16*(B*a^2*sqrt(b) + 4*A*a*b^
(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b
```

Mupad [B]
time = 0.70, size = 80, normalized size = 0.73

$$\frac{B\,x\,{\left(b\,x^2+a\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}-\frac{A\,{\left(b\,x^2+a\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^2,x)
```

```
[Out] (B*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a
+ 1)^(3/2) - (A*(a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x^2)/a))
/(x*((b*x^2)/a + 1)^(3/2))
```

3.529 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=110

$$\frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2}+\frac{(3Ab+2aB)\left(a+bx^2\right)^{3/2}}{6a}-\frac{A(a+bx^2)^{5/2}}{2ax^2}-\frac{1}{2}\sqrt{a}\,(3Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/6*(3*A*b+2*B*a)*(b*x^2+a)^(3/2)/a-1/2*A*(b*x^2+a)^(5/2)/a/x^2-1/2*(3*A*b+
2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*(3*A*b+2*B*a)*(b*x^2+a)
^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 52, 65, 214}

$$\frac{\left(a+bx^2\right)^{3/2}(2aB+3Ab)}{6a}+\frac{1}{2}\sqrt{a+bx^2}\,(2aB+3Ab)-\frac{1}{2}\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((3*A*b + 2*a*B)*(a + b*x^2)^(3/2))/(
6*a) - (A*(a + b*x^2)^(5/2))/(2*a*x^2) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{A(a+bx^2)^{5/2}}{2ax^2} + \frac{\left(\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)}{2a}$$

$$= \frac{(3Ab+2aB)(a+bx^2)^{3/2}}{6a} - \frac{A(a+bx^2)^{5/2}}{2ax^2} + \frac{1}{4}(3Ab+2aB)\text{Subst}\left(\int \frac{\sqrt{a}}{}\right.$$

$$= \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2} + \frac{(3Ab+2aB)(a+bx^2)^{3/2}}{6a} - \frac{A(a+bx^2)^{5/2}}{2ax^2} + \frac{1}{4}$$

$$= \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2} + \frac{(3Ab+2aB)(a+bx^2)^{3/2}}{6a} - \frac{A(a+bx^2)^{5/2}}{2ax^2} + $$

$$= \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2} + \frac{(3Ab+2aB)(a+bx^2)^{3/2}}{6a} - \frac{A(a+bx^2)^{5/2}}{2ax^2} - \frac{1}{2}$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-3aA+6Abx^2+8aBx^2+2bBx^4\right)}{6x^2} - \frac{1}{2}\sqrt{a}\,(3Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a*A + 6*A*b*x^2 + 8*a*B*x^2 + 2*b*B*x^4))/(6*x^2) - (S
qrt[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Maple [A]
time = 0.10, size = 134, normalized size = 1.22

method	result
risch	$-\frac{aA\sqrt{b\,x^{2}+a}}{2x^{2}}+\frac{Bb\,x^{2}\sqrt{b\,x^{2}+a}}{3}+\frac{4aB\sqrt{b\,x^{2}+a}}{3}+bA\sqrt{b\,x^{2}+a}-\frac{3A\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+}}{x}\right.}{2}$
default	$A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2a}\right)+B\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/
2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+B*(1/3*(b*x^2+a)^(3/2)+
a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Maxima [A]
time = 0.27, size = 109, normalized size = 0.99

$$-Ba^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{3}{2}\,A\sqrt{a}\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\,(bx^{2}+a)^{\frac{3}{2}}B+\sqrt{bx^{2}+a}\,Ba+\frac{3}{2}\,\sqrt{bx^{2}+a}\,Ab+\frac{(bx^{2}+a)^{\frac{3}{2}}Ab}{2\,a}-\frac{(bx^{2}+a)^{\frac{5}{2}}A}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] -B*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) - 3/2*A*sqrt(a)*b*arcsinh(a/(sqrt(
a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*B + sqrt(b*x^2 + a)*B*a + 3/2*sqrt(b*
x^2 + a)*A*b + 1/2*(b*x^2 + a)^(3/2)*A*b/a - 1/2*(b*x^2 + a)^(5/2)*A/(a*x^2
)
```

Fricas [A]
time = 1.58, size = 167, normalized size = 1.52

$$\left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}\,x^{2}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(2\,Bbx^{4}+2\,(4\,Ba+3\,Ab)x^{2}-3\,Aa)\sqrt{bx^{2}+a}}{12\,x^{2}},\ \frac{3\,(2\,Ba+3\,Ab)\sqrt{-a}\,x^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(2\,Bbx^{4}+2\,(4\,Ba+3\,Ab)x^{2}-3\,Aa)\sqrt{bx^{2}+a}}{6\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*B*a + 3*A*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a
) + 2*a)/x^2) + 2*(2*B*b*x^4 + 2*(4*B*a + 3*A*b)*x^2 - 3*A*a)*sqrt(b*x^2 +
a))/x^2, 1/6*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a
)) + (2*B*b*x^4 + 2*(4*B*a + 3*A*b)*x^2 - 3*A*a)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A]
time = 19.81, size = 184, normalized size = 1.67

$$-\frac{3A\sqrt{a}\,b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Aa\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}-Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba^2}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+Bb\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2) +
 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b*x
**2) + 1) - B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(
a/(b*x**2) + 1)) + B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((sqrt
(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

Giac [A]
time = 1.51, size = 103, normalized size = 0.94

$$\frac{2\,(bx^2+a)^{\frac{3}{2}}Bb+6\,\sqrt{bx^2+a}\,Bab+6\,\sqrt{bx^2+a}\,Ab^2-\frac{3\,\sqrt{bx^2+a}\,Aab}{x^2}+\frac{3\,(2\,Ba^2b+3\,Aab^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b*x^2 + a)^(3/2)*B*b + 6*sqrt(b*x^2 + a)*B*a*b + 6*sqrt(b*x^2 + a)*
A*b^2 - 3*sqrt(b*x^2 + a)*A*a*b/x^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sqrt
(b*x^2 + a)/sqrt(-a))/sqrt(-a))/b
```

Mupad [B]
time = 0.75, size = 94, normalized size = 0.85

$$\frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{3}-B\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+A\,b\,\sqrt{b\,x^2+a}+B\,a\,\sqrt{b\,x^2+a}-\frac{A\,a\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{3\,A\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^3,x)
```

```
[Out] (B*(a + b*x^2)^(3/2))/3 - B*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + A*b*
(a + b*x^2)^(1/2) + B*a*(a + b*x^2)^(1/2) - (A*a*(a + b*x^2)^(1/2))/(2*x^2)
 - (3*A*a^(1/2)*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/2
```

3.530 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4} dx$

Optimal. Leaf size=119

$$\frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a}-\frac{(2Ab+3aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}+\frac{1}{2}\sqrt{b}\,(2Ab+3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/3*(2*A*b+3*B*a)*(b*x^2+a)^(3/2)/a/x-1/3*A*(b*x^2+a)^(5/2)/a/x^3+1/2*(2*A
*b+3*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)+1/2*b*(2*A*b+3*B*a)*x*
(b*x^2+a)^(1/2)/a
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 283, 201, 223, 212}

$$-\frac{\left(a+bx^2\right)^{3/2}(3aB+2Ab)}{3ax}+\frac{bx\sqrt{a+bx^2}\,(3aB+2Ab)}{2a}+\frac{1}{2}\sqrt{b}\,(3aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (b*(2*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/(2*a) - ((2*A*b + 3*a*B)*(a + b*x^2)^
(3/2))/(3*a*x) - (A*(a + b*x^2)^(5/2))/(3*a*x^3) + (Sqrt[b]*(2*A*b + 3*a*B)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^4}\,dx &= -\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}-\frac{(-2Ab-3aB)\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx}{3a}\\ &= -\frac{(2Ab+3aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}+\frac{(b(2Ab+3aB))\int \sqrt{a+bx}\\ &= \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a}-\frac{(2Ab+3aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}+\\ &= \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a}-\frac{(2Ab+3aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}+\\ &= \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a}-\frac{(2Ab+3aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}+\end{aligned}$$

Mathematica [A]
time = 0.18, size = 84, normalized size = 0.71

$$\frac{\sqrt{a+bx^2}\left(-2aA-8Abx^2-6aBx^2+3bBx^4\right)}{6x^3}-\frac{1}{2}\sqrt{b}\,(2Ab+3aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
[Out] (Sqrt[a + b*x^2]*(-2*a*A - 8*A*b*x^2 - 6*a*B*x^2 + 3*b*B*x^4))/(6*x^3) - (S
qrt[b]*(2*A*b + 3*a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/2
```

Maple [A]
time = 0.09, size = 180, normalized size = 1.51

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-3bB\,x^{4}+8Ab\,x^{2}+6Ba\,x^{2}+2Aa\right)}{6x^{3}}+A\,b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)+\frac{3B\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+\right.}{2}$
default	$A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{3a\,x^{3}}+\frac{2b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)}{3a}\right)+B\left(-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/3/a/x^3*(b*x^2+a)^(5/2)+2/3*b/a*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*
(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*
x^2+a)^(1/2))))))+B*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/
4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))
```

Maxima [A]
time = 0.27, size = 115, normalized size = 0.97

$$\frac{3}{2}\sqrt{bx^2+a}\,Bbx+\frac{\sqrt{bx^2+a}\,Ab^2x}{a}+\frac{3}{2}\,Ba\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)+Ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{(bx^2+a)^{\frac{3}{2}}B}{x}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Ab}{3\,ax}-\frac{(bx^2+a)^{\frac{5}{2}}A}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*B*b*x + sqrt(b*x^2 + a)*A*b^2*x/a + 3/2*B*a*sqrt(b)*arc
sinh(b*x/sqrt(a*b)) + A*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(3/2)*
B/x - 2/3*(b*x^2 + a)^(3/2)*A*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)*A/(a*x^3)
```

Fricas [A]
time = 1.12, size = 166, normalized size = 1.39

$$\left[\frac{3\,(3\,Ba+2\,Ab)\sqrt{b}\,x^{3}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,(3\,Bbx^{4}-2\,(3\,Ba+4\,Ab)x^{2}-2\,Aa)\sqrt{bx^{2}+a}}{12\,x^{3}},\,-\frac{3\,(3\,Ba+2\,Ab)\sqrt{-b}\,x^{3}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(3\,Bbx^{4}-2\,(3\,Ba+4\,Ab)x^{2}-2\,Aa)\sqrt{bx^{2}+a}}{6\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*B*a + 2*A*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + 2*(3*B*b*x^4 - 2*(3*B*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/
x^3, -1/6*(3*(3*B*a + 2*A*b)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) - (3*B*b*x^4 - 2*(3*B*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A]
time = 3.21, size = 202, normalized size = 1.70

$$-\frac{A\sqrt{a}\,b}{x\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa\sqrt{b}\,\sqrt{\frac{a}{bx^{2}}+1}}{3x^{2}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3}+Ab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Ab^{2}x}{\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^{2}}{a}}}+\frac{B\sqrt{a}\,bx\sqrt{1+\frac{bx^{2}}{a}}}{2}-\frac{B\sqrt{a}\,bx}{\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Ba\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -A*sqrt(a)*b/(x*sqrt(1 + b*x**2/a)) - A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x
**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + A*b**(3/2)*asinh(sqrt(b)*x/sqrt(
a)) - A*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*a**(3/2)/(x*sqrt(1 + b*x**2
/a)) + B*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a
) + 3*B*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(99) = 198$.
time = 1.23, size = 207, normalized size = 1.74

$$\frac{1}{2}\sqrt{bx^{2}+a}\,Bbx-\frac{1}{4}\left(3\,Ba\sqrt{b}+2\,Ab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{4}Ba^{2}\sqrt{b}+6\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{4}Aab^{\frac{3}{2}}-6\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}Ba^{3}\sqrt{b}-6\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}Aa^{2}b^{\frac{3}{2}}+3\,Ba^{4}\sqrt{b}+4\,Aa^{3}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*b*x - 1/4*(3*B*a*sqrt(b) + 2*A*b^(3/2))*log((sqrt(b)*
x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*sqrt
(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(3/2) - 6*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*B*a^3*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(3/2
```

```
) + 3*B*a^4*sqrt(b) + 4*A*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a
)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^2+A)\,\left(b\,x^2+a\right)^{3/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^4, x)
```

3.531 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=115

$$\frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a}-\frac{(Ab+4aB)\left(a+bx^2\right)^{3/2}}{8ax^2}-\frac{A(a+bx^2)^{5/2}}{4ax^4}-\frac{3b(Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

```
[Out] -1/8*(A*b+4*B*a)*(b*x^2+a)^(3/2)/a/x^2-1/4*A*(b*x^2+a)^(5/2)/a/x^4-3/8*b*(A
*b+4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+3/8*b*(A*b+4*B*a)*(b*x^2
+a)^(1/2)/a
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 43, 52, 65, 214}

$$-\frac{\left(a+bx^2\right)^{3/2}(4aB+Ab)}{8ax^2}+\frac{3b\sqrt{a+bx^2}\,(4aB+Ab)}{8a}-\frac{3b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (3*b*(A*b + 4*a*B)*Sqrt[a + b*x^2])/(8*a) - ((A*b + 4*a*B)*(a + b*x^2)^(3/2
))/(8*a*x^2) - (A*(a + b*x^2)^(5/2))/(4*a*x^4) - (3*b*(A*b + 4*a*B)*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{A(a+bx^2)^{5/2}}{4ax^4} + \frac{(Ab+4aB)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx,x,x^2\right)}{8a}$$

$$= -\frac{(Ab+4aB)(a+bx^2)^{3/2}}{8ax^2} - \frac{A(a+bx^2)^{5/2}}{4ax^4} + \frac{(3b(Ab+4aB))\text{Subst}\left(\int \sqrt{\cdots}\right.}{16a}$$

$$= \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a} - \frac{(Ab+4aB)(a+bx^2)^{3/2}}{8ax^2} - \frac{A(a+bx^2)^{5/2}}{4ax^4} + \frac{1}{16\cdots}$$

$$= \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a} - \frac{(Ab+4aB)(a+bx^2)^{3/2}}{8ax^2} - \frac{A(a+bx^2)^{5/2}}{4ax^4} + \frac{1}{8}(\cdots$$

$$= \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a} - \frac{(Ab+4aB)(a+bx^2)^{3/2}}{8ax^2} - \frac{A(a+bx^2)^{5/2}}{4ax^4} - \frac{3b\cdots}{\cdots}$$

Mathematica [**A**]
time = 0.17, size = 81, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\,(-2aA-5Abx^2-4aBx^2+8bBx^4)}{8x^4} - \frac{3b(Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
[Out] (Sqrt[a + b*x^2]*(-2*a*A - 5*A*b*x^2 - 4*a*B*x^2 + 8*b*B*x^4))/(8*x^4) - (3
*b*(A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Maple [**A**]
time = 0.10, size = 182, normalized size = 1.58

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(5Ab\,x^2+4Ba\,x^2+2Aa)}{8x^4}+bB\sqrt{b\,x^2+a}-\frac{3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)A\,b^2}{8\sqrt{a}}-\frac{3b\sqrt{a}\,\ln\left(\frac{2a+2\cdots}{\cdots}\right.}{\cdots}$

default	$A\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right)+B\Big($

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)

[Out] A*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))+B*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))

Maxima [**A**]
time = 0.31, size = 161, normalized size = 1.40

$$-\frac{3}{2}\,B\sqrt{a}\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{3\,Ab^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,\sqrt{a}}+\frac{3}{2}\,\sqrt{bx^2+a}\,Bb+\frac{(bx^2+a)^{\frac{3}{2}}Bb}{2\,a}+\frac{(bx^2+a)^{\frac{3}{2}}Ab^2}{8\,a^2}+\frac{3\,\sqrt{bx^2+a}\,Ab^2}{8\,a}-\frac{(bx^2+a)^{\frac{5}{2}}B}{2\,ax^2}-\frac{(bx^2+a)^{\frac{5}{2}}Ab}{8\,a^2x^2}-\frac{(bx^2+a)^{\frac{5}{2}}A}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="maxima")

[Out] -3/2*B*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) - 3/8*A*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 3/2*sqrt(b*x^2 + a)*B*b + 1/2*(b*x^2 + a)^(3/2)*B*b/a + 1/8*(b*x^2 + a)^(3/2)*A*b^2/a^2 + 3/8*sqrt(b*x^2 + a)*A*b^2/a - 1/2*(b*x^2 + a)^(5/2)*B/(a*x^2) - 1/8*(b*x^2 + a)^(5/2)*A*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(5/2)*A/(a*x^4)

Fricas [**A**]
time = 1.28, size = 189, normalized size = 1.64

$$\left[\frac{3\,(4\,Bab+Ab^2)\sqrt{a}\,x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,Babx^4-2\,Aa^2-(4\,Ba^2+5\,Aab)x^2)\sqrt{bx^2+a}}{16\,ax^4},\ \frac{3\,(4\,Bab+Ab^2)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(8\,Babx^4-2\,Aa^2-(4\,Ba^2+5\,Aab)x^2)\sqrt{bx^2+a}}{8\,ax^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="fricas")

[Out] [1/16*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*B*a*b*x^4 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a*x^4), 1/8*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*B*a*b*x^4 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a*x^4)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(105) = 210$.

time = 50.51, size = 216, normalized size = 1.88

$$-\frac{Aa^{2}}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Aa\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{2x}-\frac{Ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Ab^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8\sqrt{a}}-\frac{3B\sqrt{a}\,b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{2x}+\frac{Ba\sqrt{b}}{x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Bb^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -A*a**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*a*sqrt(b)/(8*x**3*sqrt(
a/(b*x**2) + 1)) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b**(3/2)/(8*x*
sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a)) - 3
*B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1
)/(2*x) + B*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + B*b**(3/2)*x/sqrt(a/(b*x**
2) + 1)
```

Giac [A]

time = 1.47, size = 131, normalized size = 1.14

$$\frac{8\sqrt{bx^{2}+a}\,Bb^{2}+\frac{3\,(4\,Bab^{2}+Ab^{3})\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{4\,(bx^{2}+a)^{\frac{3}{2}}Bab^{2}-4\sqrt{bx^{2}+a}\,Ba^{2}b^{2}+5\,(bx^{2}+a)^{\frac{3}{2}}Ab^{3}-3\sqrt{bx^{2}+a}\,Aab^{3}}{b^{2}x^{4}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^2 + a)
/sqrt(-a))/sqrt(-a) - (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^
2*b^2 + 5*(b*x^2 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^2 + a)*A*a*b^3)/(b^2*x^4))/b
```

Mupad [B]

time = 0.97, size = 104, normalized size = 0.90

$$B\,b\,\sqrt{b\,x^{2}+a}-\frac{5\,A\,{\left(b\,x^{2}+a\right)}^{3/2}}{8\,x^{4}}-\frac{3\,A\,b^{2}\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}}+\frac{3\,A\,a\,\sqrt{b\,x^{2}+a}}{8\,x^{4}}-\frac{B\,a\,\sqrt{b\,x^{2}+a}}{2\,x^{2}}-\frac{3\,B\,\sqrt{a}\,\,b\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^5,x)
```

```
[Out] B*b*(a + b*x^2)^(1/2) - (5*A*(a + b*x^2)^(3/2))/(8*x^4) - (3*A*b^2*atanh((a
 + b*x^2)^(1/2)/a^(1/2)))/(8*a^(1/2)) + (3*A*a*(a + b*x^2)^(1/2))/(8*x^4) -
 (B*a*(a + b*x^2)^(1/2))/(2*x^2) - (3*B*a^(1/2)*b*atanh((a + b*x^2)^(1/2)/a
^(1/2)))/2
```

3.532 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6} \, dx$

Optimal. Leaf size=86

$$-\frac{bB\sqrt{a+bx^2}}{x}-\frac{B(a+bx^2)^{3/2}}{3x^3}-\frac{A(a+bx^2)^{5/2}}{5ax^5}+b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/3*B*(b*x^2+a)^(3/2)/x^3-1/5*A*(b*x^2+a)^(5/2)/a/x^5+b^(3/2)*B*arctanh(x*
b^(1/2)/(b*x^2+a)^(1/2))-b*B*(b*x^2+a)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {462, 283, 223, 212}

$$-\frac{A(a+bx^2)^{5/2}}{5ax^5}+b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{bB\sqrt{a+bx^2}}{x}-\frac{B(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

```
[Out] -((b*B*Sqrt[a + b*x^2])/x) - (B*(a + b*x^2)^(3/2))/(3*x^3) - (A*(a + b*x^2)
^(5/2))/(5*a*x^5) + b^(3/2)*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx &= -\frac{A(a+bx^2)^{5/2}}{5ax^5} + B\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx \\
&= -\frac{B(a+bx^2)^{3/2}}{3x^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} + (bB)\int \frac{\sqrt{a+bx^2}}{x^2}\,dx \\
&= -\frac{bB\sqrt{a+bx^2}}{x} - \frac{B(a+bx^2)^{3/2}}{3x^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} + \left(b^2B\right)\int \frac{1}{\sqrt{a+bx^2}}\,d \\
&= -\frac{bB\sqrt{a+bx^2}}{x} - \frac{B(a+bx^2)^{3/2}}{3x^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} + \left(b^2B\right)\text{Subst}\left(\int \frac{1}{1-b} \right. \\
&= -\frac{bB\sqrt{a+bx^2}}{x} - \frac{B(a+bx^2)^{3/2}}{3x^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} + b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+}} \right.
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 92, normalized size = 1.07

$$\frac{\sqrt{a+bx^2}\,(-3a^2A-6aAbx^2-5a^2Bx^2-3Ab^2x^4-20abBx^4)}{15ax^5} - b^{3/2}B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2*A - 6*a*A*b*x^2 - 5*a^2*B*x^2 - 3*A*b^2*x^4 - 20*a
*b*B*x^4))/(15*a*x^5) - b^(3/2)*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [A]
time = 0.09, size = 121, normalized size = 1.41

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(3A\,b^2x^4+20Bab\,x^4+6aAb\,x^2+5B\,a^2x^2+3a^2A)}{15x^5a} + B\,b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)$

default	$-\frac{A\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5a\,x^{5}}+B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{3a\,x^{3}}+\frac{2b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{a x}+\frac{4b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)}{3a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*A*(b*x^2+a)^(5/2)/a/x^5+B*(-1/3/a/x^3*(b*x^2+a)^(5/2)+2/3*b/a*(-1/a/x*
(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1
/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.28, size = 88, normalized size = 1.02

$$\frac{\sqrt{bx^2+a}\,Bb^2x}{a}+Bb^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Bb}{3\,ax}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{3\,ax^3}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*B*b^2*x/a + B*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 2/3*(b*x^2 +
 a)^(3/2)*B*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)*B/(a*x^3) - 1/5*(b*x^2 + a)^(5/
2)*A/(a*x^5)
```

Fricas [A]
time = 1.35, size = 184, normalized size = 2.14

$$\left[\frac{15\,Bab^{\frac{3}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,\left((20\,Bab+3\,Ab^2)x^4+3\,Aa^2+(5\,Ba^2+6\,Aab)x^2\right)\sqrt{bx^2+a}}{30\,ax^5},-\frac{15\,Ba\sqrt{-b}\,bx^5\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left((20\,Bab+3\,Ab^2)x^4+3\,Aa^2+(5\,Ba^2+6\,Aab)x^2\right)\sqrt{bx^2+a}}{15\,ax^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*B*a*b^(3/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
 2*((20*B*a*b + 3*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + 6*A*a*b)*x^2)*sqrt(b*x^
2 + a))/(a*x^5), -1/15*(15*B*a*sqrt(-b)*b*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)) + ((20*B*a*b + 3*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + 6*A*a*b)*x^2)*sqrt
(b*x^2 + a))/(a*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(75) = 150$.
time = 2.59, size = 184, normalized size = 2.14

$$-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a}-\frac{B\sqrt{a}\,b}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}+Bb^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Bb^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*A*b**(3/2)*sqrt(a/(b*x**2) +
 1)/(5*x**2) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) - B*sqrt(a)*b/(x*sqrt(
1 + b*x**2/a)) - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*sqr
t(a/(b*x**2) + 1)/3 + B*b**(3/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**2*x/(sqrt(
a)*sqrt(1 + b*x**2/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(70) = 140$.
time = 1.29, size = 236, normalized size = 2.74

$$-\frac{1}{2}Bb^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(30\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}+15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-90\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6Ba^2b^{\frac{3}{2}}+110\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ba^3b^{\frac{3}{2}}+30\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Aa^2b^{\frac{5}{2}}-70\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^4b^{\frac{3}{2}}+20\,Ba^5b^{\frac{3}{2}}+3\,Aa^4b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*B*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(30*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*B*a*b^(3/2) + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5
/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 110*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*B*a^3*b^(3/2) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*
b^(5/2) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3/2) + 20*B*a^5*b^(3/
2) + 3*A*a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^6,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^6, x)
```

3.533 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=120

$$\frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2}+\frac{(Ab-6aB)\left(a+bx^2\right)^{3/2}}{24ax^4}-\frac{A(a+bx^2)^{5/2}}{6ax^6}+\frac{b^2(Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

```
[Out] 1/24*(A*b-6*B*a)*(b*x^2+a)^(3/2)/a/x^4-1/6*A*(b*x^2+a)^(5/2)/a/x^6+1/16*b^2
*(A*b-6*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+1/16*b*(A*b-6*B*a)*(b
*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 43, 65, 214}

$$\frac{b^2(Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{b\sqrt{a+bx^2}\,(Ab-6aB)}{16ax^2}+\frac{\left(a+bx^2\right)^{3/2}(Ab-6aB)}{24ax^4}-\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (b*(A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a*x^2) + ((A*b - 6*a*B)*(a + b*x^2)^(
3/2))/(24*a*x^4) - (A*(a + b*x^2)^(5/2))/(6*a*x^6) + (b^2*(A*b - 6*a*B)*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{A(a+bx^2)^{5/2}}{6ax^6} + \frac{\left(-\frac{Ab}{2}+3aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^2\right)}{6a}$$

$$= \frac{(Ab-6aB)(a+bx^2)^{3/2}}{24ax^4} - \frac{A(a+bx^2)^{5/2}}{6ax^6} - \frac{(b(Ab-6aB))\text{Subst}\left(\int \frac{\sqrt{a+b}}{x^2}\right.}{16a}$$

$$= \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)(a+bx^2)^{3/2}}{24ax^4} - \frac{A(a+bx^2)^{5/2}}{6ax^6} - \frac{(b^2(A}{}$$

$$= \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)(a+bx^2)^{3/2}}{24ax^4} - \frac{A(a+bx^2)^{5/2}}{6ax^6} - \frac{(b(A}{}$$

$$= \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)(a+bx^2)^{3/2}}{24ax^4} - \frac{A(a+bx^2)^{5/2}}{6ax^6} + \frac{b^2(A}{}$$

Mathematica [A]
time = 0.18, size = 102, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\,(-8a^2A-14aAbx^2-12a^2Bx^2-3Ab^2x^4-30abBx^4)}{48ax^6} - \frac{b^2(-Ab+6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A - 14*a*A*b*x^2 - 12*a^2*B*x^2 - 3*A*b^2*x^4 - 30
*a*b*B*x^4))/(48*a*x^6) - (b^2*(-(A*b) + 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/(16*a^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(100) = 200$.
time = 0.10, size = 230, normalized size = 1.92

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(3A b^2 x^4+30Bab x^4+14aAb x^2+12B a^2 x^2+8a^2 A)}{48x^6 a}+\frac{b^3 \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right) A}{16a^{\frac{3}{2}}}-\frac{3b^2 \ln\left(\frac{2a+2\sqrt{a}\cdots}{\cdots}\right.}{8\cdots}$
default	$B\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{4a x^4}+\frac{b\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{2a x^2}+\frac{3b\left(\frac{(b x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right)+A\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(
1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x)))))+A*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(
5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b
*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(103) = 206$.
time = 0.31, size = 210, normalized size = 1.75

$$-\frac{3\,Bb^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,\sqrt{a}}+\frac{Ab^3 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{3}{2}}}+\frac{(bx^2+a)^{\frac{3}{2}} Bb^2}{8\,a^2}+\frac{3\,\sqrt{bx^2+a}\,Bb^2}{8\,a}-\frac{(bx^2+a)^{\frac{3}{2}} Ab^3}{48\,a^3}-\frac{\sqrt{bx^2+a}\,Ab^3}{16\,a^2}-\frac{(bx^2+a)^{\frac{5}{2}} Bb}{8\,a^2x^2}+\frac{(bx^2+a)^{\frac{5}{2}} Ab^2}{48\,a^3x^2}-\frac{(bx^2+a)^{\frac{5}{2}} B}{4\,ax^4}+\frac{(bx^2+a)^{\frac{5}{2}} Ab}{24\,a^2x^4}-\frac{(bx^2+a)^{\frac{5}{2}} A}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] -3/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*A*b^3*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(3/2) + 1/8*(b*x^2 + a)^(3/2)*B*b^2/a^2 + 3/8*sqrt(b*x^2
 + a)*B*b^2/a - 1/48*(b*x^2 + a)^(3/2)*A*b^3/a^3 - 1/16*sqrt(b*x^2 + a)*A*b
^3/a^2 - 1/8*(b*x^2 + a)^(5/2)*B*b/(a^2*x^2) + 1/48*(b*x^2 + a)^(5/2)*A*b^2
/(a^3*x^2) - 1/4*(b*x^2 + a)^(5/2)*B/(a*x^4) + 1/24*(b*x^2 + a)^(5/2)*A*b/(
a^2*x^4) - 1/6*(b*x^2 + a)^(5/2)*A/(a*x^6)
```

Fricas [A]
time = 1.45, size = 222, normalized size = 1.85

$$\left[-\frac{3\,(6\,Bab^2 - Ab^3)\sqrt{a}\,x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right) + 2\,(3\,(10\,Ba^2b + Aab^2)x^4 + 8\,Aa^3 + 2\,(6\,Ba^3 + 7\,Aa^2b)x^2)\sqrt{bx^2+a}}{96\,a^2x^6}, \frac{3\,(6\,Bab^2 - Ab^3)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - (3\,(10\,Ba^2b + Aab^2)x^4 + 8\,Aa^3 + 2\,(6\,Ba^3 + 7\,Aa^2b)x^2)\sqrt{bx^2+a}}{48\,a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(3*(10*B*a^2*b + A*a*b^2)*x^4 + 8*A*a^3 + 2*(6*B*a^3
 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6), 1/48*(3*(6*B*a*b^2 - A*b^3)*
sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (3*(10*B*a^2*b + A*a*b^2)*x
^4 + 8*A*a^3 + 2*(6*B*a^3 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(105) = 210$.
time = 73.37, size = 253, normalized size = 2.11

$$-\frac{Aa^2}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{11Aa\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{17Ab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{3}{2}}} - \frac{Ba^2}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ba\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**7,x)
```

```
[Out] -A*a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*A*a*sqrt(b)/(24*x**5*sqr
t(a/(b*x**2) + 1)) - 17*A*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - A*b**(5
/2)/(16*a*x*sqrt(a/(b*x**2) + 1)) + A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a
**(3/2)) - B*a**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*a*sqrt(b)/(8*
x**3*sqrt(a/(b*x**2) + 1)) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - B*b**(
3/2)/(8*x*sqrt(a/(b*x**2) + 1)) - 3*B*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sq
rt(a))
```

Giac [A]
time = 2.42, size = 159, normalized size = 1.32

$$\frac{\frac{3\,(6\,Bab^3 - Ab^4)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} - \frac{30\,(bx^2+a)^{\frac{5}{2}}Bab^3 - 48\,(bx^2+a)^{\frac{3}{2}}Ba^2b^3 + 18\,\sqrt{bx^2+a}\,Ba^3b^3 + 3\,(bx^2+a)^{\frac{5}{2}}Ab^4 + 8\,(bx^2+a)^{\frac{3}{2}}Aab^4 - 3\,\sqrt{bx^2+a}\,Aa^2b^4}{ab^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) -
 (30*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/2)*B*a^2*b^3 + 18*sqrt(b
*x^2 + a)*B*a^3*b^3 + 3*(b*x^2 + a)^(5/2)*A*b^4 + 8*(b*x^2 + a)^(3/2)*A*a*b
^4 - 3*sqrt(b*x^2 + a)*A*a^2*b^4)/(a*b^3*x^6))/b
```

Mupad [B]
time = 1.27, size = 130, normalized size = 1.08

$$\frac{A\,a\,\sqrt{b\,x^2+a}}{16\,x^6}-\frac{5\,B\,{\left(b\,x^2+a\right)}^{3/2}}{8\,x^4}-\frac{3\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{6\,x^6}+\frac{3\,B\,a\,\sqrt{b\,x^2+a}}{8\,x^4}-\frac{A\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a\,x^6}-\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{16\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^7,x)
```

```
[Out] (A*a*(a + b*x^2)^(1/2))/(16*x^6) - (5*B*(a + b*x^2)^(3/2))/(8*x^4) - (A*b^3
*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(3/2)) - (3*B*b^2*atanh((a
+ b*x^2)^(1/2)/a^(1/2)))/(8*a^(1/2)) - (A*(a + b*x^2)^(3/2))/(6*x^6) + (3*B
*a*(a + b*x^2)^(1/2))/(8*x^4) - (A*(a + b*x^2)^(5/2))/(16*a*x^6)
```

3.534 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=53

$$-\frac{A(a+bx^2)^{5/2}}{7ax^7}+\frac{(2Ab-7aB)\,(a+bx^2)^{5/2}}{35a^2x^5}$$

```
[Out] -1/7*A*(b*x^2+a)^(5/2)/a/x^7+1/35*(2*A*b-7*B*a)*(b*x^2+a)^(5/2)/a^2/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {464, 270}

$$\frac{(a+bx^2)^{5/2}\,(2Ab-7aB)}{35a^2x^5}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -1/7*(A*(a + b*x^2)^(5/2))/(a*x^7) + ((2*A*b - 7*a*B)*(a + b*x^2)^(5/2))/(3
5*a^2*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}\,(A+Bx^2)}{x^8}\,dx &= -\frac{A(a+bx^2)^{5/2}}{7ax^7}-\frac{(2Ab-7aB)\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx}{7a}\\ &= -\frac{A(a+bx^2)^{5/2}}{7ax^7}+\frac{(2Ab-7aB)\,(a+bx^2)^{5/2}}{35a^2x^5}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 40, normalized size = 0.75

$$\frac{(a+bx^2)^{5/2}\left(-5aA+2Abx^2-7aBx^2\right)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
[Out] ((a + b*x^2)^(5/2)*(-5*a*A + 2*A*b*x^2 - 7*a*B*x^2))/(35*a^2*x^7)
```

Maple [A]
time = 0.09, size = 58, normalized size = 1.09

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{5}{2}}(-2Ab\,x^2+7Ba\,x^2+5Aa)}{35x^7a^2}$	37
default	$-\frac{B(b\,x^2+a)^{\frac{5}{2}}}{5a\,x^5}+A\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{7a\,x^7}+\frac{2b(b\,x^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)$	58
trager	$-\frac{(-2x^6A\,b^3+7x^6Ba\,b^2+Aa\,b^2x^4+14x^4B\,a^2b+8x^2A\,a^2b+7B\,a^3x^2+5A\,a^3)\sqrt{b\,x^2+a}}{35x^7a^2}$	82
risch	$-\frac{(-2x^6A\,b^3+7x^6Ba\,b^2+Aa\,b^2x^4+14x^4B\,a^2b+8x^2A\,a^2b+7B\,a^3x^2+5A\,a^3)\sqrt{b\,x^2+a}}{35x^7a^2}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/5*B/a/x^5*(b*x^2+a)^(5/2)+A*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/35*b/a^2/x^5*(
b*x^2+a)^(5/2))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 1.06

$$-\frac{(bx^2+a)^{\frac{5}{2}}B}{5\,ax^5}+\frac{2\,(bx^2+a)^{\frac{5}{2}}Ab}{35\,a^2x^5}-\frac{(bx^2+a)^{\frac{5}{2}}A}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="maxima")
[Out] -1/5*(b*x^2 + a)^(5/2)*B/(a*x^5) + 2/35*(b*x^2 + a)^(5/2)*A*b/(a^2*x^5) - 1
/7*(b*x^2 + a)^(5/2)*A/(a*x^7)
```

Fricas [A]
time = 1.20, size = 78, normalized size = 1.47

$$-\frac{((7\,Bab^2-2\,Ab^3)x^6+(14\,Ba^2b+Aab^2)x^4+5\,Aa^3+(7\,Ba^3+8\,Aa^2b)x^2)\sqrt{bx^2+a}}{35\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*((7*B*a*b^2 - 2*A*b^3)*x^6 + (14*B*a^2*b + A*a*b^2)*x^4 + 5*A*a^3 + (
7*B*a^3 + 8*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^2*x^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(46) = 92$.
time = 2.75, size = 518, normalized size = 9.77

$$-\frac{15Aa^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Aa^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Aa^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3Aa^3b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12Aa^2b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8Aab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2}-\frac{Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -15*A*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a
**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 3*A*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a**
2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x
**8 + 105*a**3*b**6*x**10) - 8*A*a*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(10
5*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*b**(3/2)*s
qrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2)
+ 2*A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*a*sqrt(b)*sqrt(a/(b*x**2)
 + 1)/(5*x**4) - 2*B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**2) - B*b**(5/2)*sq
rt(a/(b*x**2) + 1)/(5*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(45) = 90$.
time = 3.26, size = 344, normalized size = 6.49

$$\frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bb^{\frac{5}{2}}-70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bab^{\frac{5}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ab^{\frac{7}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ba^2b^{\frac{5}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Aab^{\frac{7}{2}}-140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^3b^{\frac{5}{2}}+140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Aa^2b^{\frac{7}{2}}+77\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Ba^4b^{\frac{5}{2}}+28\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Aa^3b^{\frac{7}{2}}-14\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Ba^5b^{\frac{5}{2}}+14\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Aa^4b^{\frac{7}{2}}+7Ba^6b^{\frac{5}{2}}-2Aa^5b^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(5/2) - 70*(sqrt(b)*x - sqrt(
b*x^2 + a))^10*B*a*b^(5/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*b^(7/2)
+ 105*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(5/2) + 70*(sqrt(b)*x - sqrt(
b*x^2 + a))^8*A*a*b^(7/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(5/
2) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(7/2) + 77*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*B*a^4*b^(5/2) + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*b
^(7/2) - 14*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(5/2) + 14*(sqrt(b)*x -
```

```
sqrt(b*x^2 + a))^2*A*a^4*b^(7/2) + 7*B*a^6*b^(5/2) - 2*A*a^5*b^(7/2))/((sq
rt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

Mupad [B]
time = 0.73, size = 128, normalized size = 2.42

$$\frac{2\,A\,b^3\,\sqrt{b\,x^2+a}}{35\,a^2\,x}-\frac{8\,A\,b\,\sqrt{b\,x^2+a}}{35\,x^5}-\frac{B\,a\,\sqrt{b\,x^2+a}}{5\,x^5}-\frac{2\,B\,b\,\sqrt{b\,x^2+a}}{5\,x^3}-\frac{A\,b^2\,\sqrt{b\,x^2+a}}{35\,a\,x^3}-\frac{A\,a\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{B\,b^2\,\sqrt{b\,x^2+a}}{5\,a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^8,x)
```

```
[Out] (2*A*b^3*(a + b*x^2)^(1/2))/(35*a^2*x) - (8*A*b*(a + b*x^2)^(1/2))/(35*x^5)
 - (B*a*(a + b*x^2)^(1/2))/(5*x^5) - (2*B*b*(a + b*x^2)^(1/2))/(5*x^3) - (A
*b^2*(a + b*x^2)^(1/2))/(35*a*x^3) - (A*a*(a + b*x^2)^(1/2))/(7*x^7) - (B*b
^2*(a + b*x^2)^(1/2))/(5*a*x)
```

3.535 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9} dx$

Optimal. Leaf size=156

$$\frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4}+\frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2}+\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}-\frac{A(a+bx^2)^{5/2}}{8ax^8}-\frac{b^3(3Ab-8}{}$$

```
[Out] 1/48*(3*A*b-8*B*a)*(b*x^2+a)^(3/2)/a/x^6-1/8*A*(b*x^2+a)^(5/2)/a/x^8-1/128*
b^3*(3*A*b-8*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/64*b*(3*A*b-8*
B*a)*(b*x^2+a)^(1/2)/a/x^4+1/128*b^2*(3*A*b-8*B*a)*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 43, 44, 65, 214}

$$-\frac{b^3(3Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{b^2\sqrt{a+bx^2}\,(3Ab-8aB)}{128a^2x^2}+\frac{\left(a+bx^2\right)^{3/2}(3Ab-8aB)}{48ax^6}+\frac{b\sqrt{a+bx^2}\,(3Ab-8aB)}{64ax^4}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (b*(3*A*b - 8*a*B)*Sqrt[a + b*x^2])/(64*a*x^4) + (b^2*(3*A*b - 8*a*B)*Sqrt[
a + b*x^2])/(128*a^2*x^2) + ((3*A*b - 8*a*B)*(a + b*x^2)^(3/2))/(48*a*x^6)
- (A*(a + b*x^2)^(5/2))/(8*a*x^8) - (b^3*(3*A*b - 8*a*B)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{A(a+bx^2)^{5/2}}{8ax^8} + \frac{\left(-\frac{3Ab}{2}+4aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx, x, x^2\right)}{8a}$$

$$= \frac{(3Ab-8aB)(a+bx^2)^{3/2}}{48ax^6} - \frac{A(a+bx^2)^{5/2}}{8ax^8} - \frac{(b(3Ab-8aB))\text{Subst}\left(\int \frac{\sqrt{a-}}{}\right.}{32a}$$

$$= \frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4} + \frac{(3Ab-8aB)(a+bx^2)^{3/2}}{48ax^6} - \frac{A(a+bx^2)^{5/2}}{8ax^8} - \frac{(b^2}{}$$

$$= \frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4} + \frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2} + \frac{(3Ab-8aB)(a+b}{48ax^6}$$

$$= \frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4} + \frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2} + \frac{(3Ab-8aB)(a+b}{48ax^6}$$

$$= \frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4} + \frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2} + \frac{(3Ab-8aB)(a+b}{48ax^6}$$

Mathematica [A]
time = 0.23, size = 126, normalized size = 0.81

$$\frac{\sqrt{a+bx^2}\,(-48a^3A-72a^2Abx^2-64a^3Bx^2-6aAb^2x^4-112a^2bBx^4+9Ab^3x^6-24ab^2Bx^6)}{384a^2x^8} + \frac{b^3(-3Ab+8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*A - 72*a^2*A*b*x^2 - 64*a^3*B*x^2 - 6*a*A*b^2*x^4
 - 112*a^2*b*B*x^4 + 9*A*b^3*x^6 - 24*a*b^2*B*x^6))/(384*a^2*x^8) + (b^3*(-
3*A*b + 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(132) = 264.
time = 0.10, size = 278, normalized size = 1.78

method	result

risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-9x^{6}A\,b^{3}+24x^{6}Ba\,b^{2}+6Aa\,b^{2}x^{4}+112x^{4}B\,a^{2}b+72x^{2}A\,a^{2}b+64B\,a^{3}x^{2}+48A\,a^{3}\right)}{384x^{8}a^{2}}-\frac{3b^{4}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}}}{x}\right.}{128a^{\frac{5}{2}}}$
default	$B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6a\,x^{6}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{4a\,x^{4}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right.\right.}{2a}\right.}{4a}\right.}{6a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(
-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-
a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))+A*(-1/8/a/x^8*(b*x^2+a)^
(5/2)-3/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/
```

```
2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x
^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))
```

Maxima [A]
time = 0.27, size = 252, normalized size = 1.62

$$\frac{Bb^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{3}{2}}}-\frac{3\,Ab^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{5}{2}}}-\frac{(bx^2+a)^{\frac{3}{2}}Bb^3}{48\,a^3}-\frac{\sqrt{bx^2+a}\,Bb^3}{16\,a^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ab^4}{128\,a^4}+\frac{3\,\sqrt{bx^2+a}\,Ab^4}{128\,a^3}+\frac{(bx^2+a)^{\frac{5}{2}}Bb^2}{48\,a^3x^2}-\frac{(bx^2+a)^{\frac{5}{2}}Ab^3}{128\,a^4x^2}+\frac{(bx^2+a)^{\frac{5}{2}}Bb}{24\,a^2x^4}-\frac{(bx^2+a)^{\frac{5}{2}}Ab^2}{64\,a^3x^4}-\frac{(bx^2+a)^{\frac{5}{2}}B}{6\,ax^6}+\frac{(bx^2+a)^{\frac{5}{2}}Ab}{16\,a^2x^6}-\frac{(bx^2+a)^{\frac{5}{2}}A}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/128*A*b^4*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(5/2) - 1/48*(b*x^2 + a)^(3/2)*B*b^3/a^3 - 1/16*sqrt(b*
x^2 + a)*B*b^3/a^2 + 1/128*(b*x^2 + a)^(3/2)*A*b^4/a^4 + 3/128*sqrt(b*x^2 +
 a)*A*b^4/a^3 + 1/48*(b*x^2 + a)^(5/2)*B*b^2/(a^3*x^2) - 1/128*(b*x^2 + a)^
(5/2)*A*b^3/(a^4*x^2) + 1/24*(b*x^2 + a)^(5/2)*B*b/(a^2*x^4) - 1/64*(b*x^2
+ a)^(5/2)*A*b^2/(a^3*x^4) - 1/6*(b*x^2 + a)^(5/2)*B/(a*x^6) + 1/16*(b*x^2
+ a)^(5/2)*A*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(5/2)*A/(a*x^8)
```

Fricas [A]
time = 1.68, size = 271, normalized size = 1.74

$$\left[-\frac{3\,(8\,Bab^3-3\,Ab^4)\sqrt{a}\,x^8\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,(8\,Ba^2b^2-3\,Aab^3)x^6+48\,Aa^4+2\,(56\,Ba^3b+3\,Aa^2b^2)x^4+8\,(8\,Ba^4+9\,Aa^3b)x^2)\sqrt{bx^2+a}}{768\,a^3x^8},-\frac{3\,(8\,Bab^3-3\,Ab^4)\sqrt{-a}\,x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(8\,Ba^2b^2-3\,Aab^3)x^6+48\,Aa^4+2\,(56\,Ba^3b+3\,Aa^2b^2)x^4+8\,(8\,Ba^4+9\,Aa^3b)x^2)\sqrt{bx^2+a}}{384\,a^3x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^6 + 48*A*a^4 + 2*(
56*B*a^3*b + 3*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a
))/(a^3*x^8), -1/384*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)/
sqrt(b*x^2 + a)) + (3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^6 + 48*A*a^4 + 2*(56*B*a^
3*b + 3*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^3
*x^8)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(141) = 282$.
time = 135.61, size = 287, normalized size = 1.84

$$-\frac{Aa^2}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{5Aa\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{13Ab^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{5}{2}}}{128ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3Ab^{\frac{7}{2}}}{128a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3Ab^4\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{5}{2}}}-\frac{Ba^2}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{11Ba\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{17Bb^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Bb^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}}+\frac{Bb^3\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**9,x)
```

```
[Out] -A*a**2/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 5*A*a*sqrt(b)/(16*x**7*sqrt
(a/(b*x**2) + 1)) - 13*A*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + A*b**(5/
```

```
2)/(128*a*x**3*sqrt(a/(b*x**2) + 1)) + 3*A*b**(7/2)/(128*a**2*x*sqrt(a/(b*x
**2) + 1)) - 3*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(5/2)) - B*a**2/(6
*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*B*a*sqrt(b)/(24*x**5*sqrt(a/(b*x**
2) + 1)) - 17*B*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - B*b**(5/2)/(16*a*
x*sqrt(a/(b*x**2) + 1)) + B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2))
```

Giac [A]
time = 1.36, size = 194, normalized size = 1.24

$$-\frac{\frac{3\,(8\,Bab^4-3\,Ab^5)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{24\,(bx^2+a)^{\frac{7}{2}}Bab^4+40\,(bx^2+a)^{\frac{5}{2}}Ba^2b^4-88\,(bx^2+a)^{\frac{3}{2}}Ba^3b^4+24\,\sqrt{bx^2+a}\,Ba^4b^4-9\,(bx^2+a)^{\frac{7}{2}}Ab^5+33\,(bx^2+a)^{\frac{5}{2}}Aab^5+33\,(bx^2+a)^{\frac{3}{2}}Aa^2b^5-9\,\sqrt{bx^2+a}\,Aa^3b^5}{a^2b^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] -1/384*(3*(8*B*a*b^4 - 3*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*
a^2) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 + 40*(b*x^2 + a)^(5/2)*B*a^2*b^4 - 88*
(b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 + a)*B*a^4*b^4 - 9*(b*x^2 + a)^
(7/2)*A*b^5 + 33*(b*x^2 + a)^(5/2)*A*a*b^5 + 33*(b*x^2 + a)^(3/2)*A*a^2*b^5
 - 9*sqrt(b*x^2 + a)*A*a^3*b^5)/(a^2*b^4*x^8))/b
```

Mupad [B]
time = 1.72, size = 169, normalized size = 1.08

$$\frac{3\,A\,a\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{6\,x^6}-\frac{11\,A\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^8}+\frac{B\,a\,\sqrt{b\,x^2+a}}{16\,x^6}-\frac{11\,A\,{\left(b\,x^2+a\right)}^{5/2}}{128\,a\,x^8}+\frac{3\,A\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^2\,x^8}-\frac{B\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a\,x^6}+\frac{A\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{128\,a^{5/2}}-\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{16\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^9,x)
```

```
[Out] (A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (B*(a + b*x
^2)^(3/2))/(6*x^6) - (11*A*(a + b*x^2)^(3/2))/(128*x^8) - (B*b^3*atan(((a +
 b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(3/2)) + (3*A*a*(a + b*x^2)^(1/2))/(12
8*x^8) + (B*a*(a + b*x^2)^(1/2))/(16*x^6) - (11*A*(a + b*x^2)^(5/2))/(128*a
*x^8) + (3*A*(a + b*x^2)^(7/2))/(128*a^2*x^8) - (B*(a + b*x^2)^(5/2))/(16*a
*x^6)
```

3.536 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}} dx$

Optimal. Leaf size=84

$$-\frac{A(a+bx^2)^{5/2}}{9ax^9}+\frac{(4Ab-9aB)(a+bx^2)^{5/2}}{63a^2x^7}-\frac{2b(4Ab-9aB)(a+bx^2)^{5/2}}{315a^3x^5}$$

```
[Out] -1/9*A*(b*x^2+a)^(5/2)/a/x^9+1/63*(4*A*b-9*B*a)*(b*x^2+a)^(5/2)/a^2/x^7-2/3
15*b*(4*A*b-9*B*a)*(b*x^2+a)^(5/2)/a^3/x^5
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$-\frac{2b(a+bx^2)^{5/2}(4Ab-9aB)}{315a^3x^5}+\frac{(a+bx^2)^{5/2}(4Ab-9aB)}{63a^2x^7}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -1/9*(A*(a + b*x^2)^(5/2))/(a*x^9) + ((4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(6
3*a^2*x^7) - (2*b*(4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(315*a^3*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx = -\frac{A(a+bx^2)^{5/2}}{9ax^9} - \frac{(4Ab-9aB)\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx}{9a}$$

$$= -\frac{A(a+bx^2)^{5/2}}{9ax^9} + \frac{(4Ab-9aB)(a+bx^2)^{5/2}}{63a^2x^7} + \frac{(2b(4Ab-9aB))\int \frac{(a+bx^2)^{3}}{x^6}}{63a^2}$$

$$= -\frac{A(a+bx^2)^{5/2}}{9ax^9} + \frac{(4Ab-9aB)(a+bx^2)^{5/2}}{63a^2x^7} - \frac{2b(4Ab-9aB)(a+bx^2)^{5/}}{315a^3x^5}$$

Mathematica [A]
time = 0.20, size = 62, normalized size = 0.74

$$\frac{(a+bx^2)^{5/2}(-35a^2A+20aAbx^2-45a^2Bx^2-8Ab^2x^4+18abBx^4)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-35*a^2*A + 20*a*A*b*x^2 - 45*a^2*B*x^2 - 8*A*b^2*x^4 +
 18*a*b*B*x^4))/(315*a^3*x^9)
```

Maple [A]
time = 0.09, size = 102, normalized size = 1.21

method	result	size
gosper	$-\frac{(bx^2+a)^{\frac{5}{2}}(8Ab^2x^4-18Babx^4-20aAbx^2+45Ba^2x^2+35a^2A)}{315x^9a^3}$	59
default	$A\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{9ax^9}-\frac{4b\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{7ax^7}+\frac{2b(bx^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)+B\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{7ax^7}+\frac{2b(bx^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)$	102
trager	$-\frac{(8Ab^4x^8-18Bab^3x^8-4Aab^3x^6+9Ba^2b^2x^6+3Aa^2b^2x^4+72Ba^3bx^4+50Aa^3bx^2+45Ba^4x^2+35Aa^4)\sqrt{bx^2+a}}{315x^9a^3}$	107
risch	$-\frac{(8Ab^4x^8-18Bab^3x^8-4Aab^3x^6+9Ba^2b^2x^6+3Aa^2b^2x^4+72Ba^3bx^4+50Aa^3bx^2+45Ba^4x^2+35Aa^4)\sqrt{bx^2+a}}{315x^9a^3}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/9/a/x^9*(b*x^2+a)^(5/2)-4/9*b/a*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/35*b/a^
2/x^5*(b*x^2+a)^(5/2)))+B*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/35*b/a^2/x^5*(b*x^2
+a)^(5/2))
```

Maxima [A]
time = 0.28, size = 96, normalized size = 1.14

$$\frac{2\,(bx^2+a)^{\frac{5}{2}}Bb}{35\,a^2x^5}-\frac{8\,(bx^2+a)^{\frac{5}{2}}Ab^2}{315\,a^3x^5}-\frac{(bx^2+a)^{\frac{5}{2}}B}{7\,ax^7}+\frac{4\,(bx^2+a)^{\frac{5}{2}}Ab}{63\,a^2x^7}-\frac{(bx^2+a)^{\frac{5}{2}}A}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="maxima")
```

```
[Out] 2/35*(b*x^2 + a)^(5/2)*B*b/(a^2*x^5) - 8/315*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x
^5) - 1/7*(b*x^2 + a)^(5/2)*B/(a*x^7) + 4/63*(b*x^2 + a)^(5/2)*A*b/(a^2*x^7
) - 1/9*(b*x^2 + a)^(5/2)*A/(a*x^9)
```

Fricas [A]
time = 1.23, size = 105, normalized size = 1.25

$$\frac{(2\,(9\,Bab^3-4\,Ab^4)x^8-(9\,Ba^2b^2-4\,Aab^3)x^6-35\,Aa^4-3\,(24\,Ba^3b+Aa^2b^2)x^4-5\,(9\,Ba^4+10\,Aa^3b)x^2)\sqrt{bx^2+a}}{315\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(9*B*a*b^3 - 4*A*b^4)*x^8 - (9*B*a^2*b^2 - 4*A*a*b^3)*x^6 - 35*A*a
^4 - 3*(24*B*a^3*b + A*a^2*b^2)*x^4 - 5*(9*B*a^4 + 10*A*a^3*b)*x^2)*sqrt(b*
x^2 + a)/(a^3*x^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1408 vs. 2(78) = 156.
time = 3.42, size = 1408, normalized size = 16.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -35*A*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b*
*10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**7*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**6*b**(23/2)*x**4*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**5*b**(25/2)*x**6*sqrt(a/(b*x**
2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) - 15*A*a**5*b**(11/2)*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**4*b**(27/2)
*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 33*A*a**4*b**(13/2)*x**2*sqrt(a
```

```
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) + 30*A*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 +
945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 17*A*
a**3*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**2*b**(31/2)*x**12*sqrt(a/(b*x**2) +
 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315
*a**4*b**12*x**14) - 3*A*a**2*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 16*A*a*b**(33/2)*x
**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*
a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 12*A*a*b**(19/2)*x**8*sqrt(a/(b*
x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
- 8*A*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b
**5*x**8 + 105*a**3*b**6*x**10) - 15*B*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**5*
b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**
8 + 105*a**3*b**6*x**10) - 17*B*a**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(1
05*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*B*a**3*b*
*(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
+ 105*a**3*b**6*x**10) - 12*B*a**2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105
*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*a*b**(19/
2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 10
5*a**3*b**6*x**10) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - B*b**(5/2)*
sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*B*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a*
*2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(72) = 144.

time = 0.93, size = 400, normalized size = 4.76

$$\frac{4\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bb^{\frac{7}{2}}-315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{7}{2}}+840\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ab^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ba^2b^{\frac{7}{2}}+1260\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Aab^{\frac{9}{2}}-819\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ba^3b^{\frac{7}{2}}+1764\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Aa^2b^{\frac{9}{2}}+441\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^4b^{\frac{7}{2}}+504\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Aa^3b^{\frac{9}{2}}-9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Ba^5b^{\frac{7}{2}}+144\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Aa^4b^{\frac{9}{2}}+81\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Ba^6b^{\frac{7}{2}}-36\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Aa^5b^{\frac{9}{2}}-9Ba^7b^{\frac{7}{2}}+4Aa^6b^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*b^(7/2) - 315*(sqrt(b)*x - sq
rt(b*x^2 + a))^12*B*a*b^(7/2) + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(9
/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(7/2) + 1260*(sqrt(b)*x
- sqrt(b*x^2 + a))^10*A*a*b^(9/2) - 819*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a
^3*b^(7/2) + 1764*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(9/2) + 441*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(7/2) + 504*(sqrt(b)*x - sqrt(b*x^2 + a)
)^6*A*a^3*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(7/2) + 144*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(9/2) + 81*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*B*a^6*b^(7/2) - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^(9/2) - 9
*B*a^7*b^(7/2) + 4*A*a^6*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

Mupad [**B**]

time = 0.95, size = 170, normalized size = 2.02

$$\frac{4 A b^3 \sqrt{b x^2+a}}{315 a^2 x^3}-\frac{10 A b \sqrt{b x^2+a}}{63 x^7}-\frac{B a \sqrt{b x^2+a}}{7 x^7}-\frac{8 B b \sqrt{b x^2+a}}{35 x^5}-\frac{A b^2 \sqrt{b x^2+a}}{105 a x^5}-\frac{A a \sqrt{b x^2+a}}{9 x^9}-\frac{8 A b^4 \sqrt{b x^2+a}}{315 a^3 x}-\frac{B b^2 \sqrt{b x^2+a}}{35 a x^3}+\frac{2 B b^3 \sqrt{b x^2+a}}{35 a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^10,x)
```

```
[Out] (4*A*b^3*(a + b*x^2)^(1/2))/(315*a^2*x^3) - (10*A*b*(a + b*x^2)^(1/2))/(63*
x^7) - (B*a*(a + b*x^2)^(1/2))/(7*x^7) - (8*B*b*(a + b*x^2)^(1/2))/(35*x^5)
 - (A*b^2*(a + b*x^2)^(1/2))/(105*a*x^5) - (A*a*(a + b*x^2)^(1/2))/(9*x^9)
- (8*A*b^4*(a + b*x^2)^(1/2))/(315*a^3*x) - (B*b^2*(a + b*x^2)^(1/2))/(35*a
*x^3) + (2*B*b^3*(a + b*x^2)^(1/2))/(35*a^2*x)
```

3.537 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^{11}}\,dx$

Optimal. Leaf size=184

$$\frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6}+\frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4}-\frac{3b^3(Ab-2aB)\sqrt{a+bx^2}}{256a^3x^2}+\frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{16ax^8}-$$

```
[Out] 1/16*(A*b-2*B*a)*(b*x^2+a)^(3/2)/a/x^8-1/10*A*(b*x^2+a)^(5/2)/a/x^10+3/256*
b^4*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)+1/32*b*(A*b-2*B*a)
*(b*x^2+a)^(1/2)/a/x^6+1/128*b^2*(A*b-2*B*a)*(b*x^2+a)^(1/2)/a^2/x^4-3/256*
b^3*(A*b-2*B*a)*(b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 43, 44, 65, 214}

$$\frac{3b^4(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}-\frac{3b^3\sqrt{a+bx^2}\,(Ab-2aB)}{256a^3x^2}+\frac{b^2\sqrt{a+bx^2}\,(Ab-2aB)}{128a^2x^4}+\frac{\left(a+bx^2\right)^{3/2}(Ab-2aB)}{16ax^8}+\frac{b\sqrt{a+bx^2}\,(Ab-2aB)}{32ax^6}-\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (b*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(32*a*x^6) + (b^2*(A*b - 2*a*B)*Sqrt[a +
b*x^2])/(128*a^2*x^4) - (3*b^3*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(256*a^3*x^2)
 + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/(16*a*x^8) - (A*(a + b*x^2)^(5/2))/(10
*a*x^10) + (3*b^4*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(7
/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{A(a+bx^2)^{5/2}}{10ax^{10}} + \frac{\left(-\frac{5Ab}{2}+5aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x, x^2\right)}{10a}$$

$$= \frac{(Ab-2aB)(a+bx^2)^{3/2}}{16ax^8} - \frac{A(a+bx^2)^{5/2}}{10ax^{10}} - \frac{(3b(Ab-2aB))\text{Subst}\left(\int \sqrt{a \ldots\right.}{32a}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{(Ab-2aB)(a+bx^2)^{3/2}}{16ax^8} - \frac{A(a+bx^2)^{5/2}}{10ax^{10}} - \ldots$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} + \frac{(Ab-2aB)(a+bx^2 \ldots}{16ax^8}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+\ldots}}{256a^3x^2}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+\ldots}}{256a^3x^2}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+\ldots}}{256a^3x^2}$$

Mathematica [A]
time = 0.28, size = 142, normalized size = 0.77

$$-\frac{\sqrt{a+bx^2}\left(15Ab^4x^8-10ab^3x^6(A+3Bx^2)+4a^2b^2x^4(2A+5Bx^2)+32a^4(4A+5Bx^2)+16a^3bx^2(11A+15Bx^2)\right)}{1280a^3x^{10}}+\frac{3b^4(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

```
[Out] -1/1280*(Sqrt[a + b*x^2]*(15*A*b^4*x^8 - 10*a*b^3*x^6*(A + 3*B*x^2) + 4*a^2
*b^2*x^4*(2*A + 5*B*x^2) + 32*a^4*(4*A + 5*B*x^2) + 16*a^3*b*x^2*(11*A + 15
*B*x^2)))/(a^3*x^10) + (3*b^4*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/(256*a^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(156) = 312.
time = 0.10, size = 326, normalized size = 1.77

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(15A\,b^{4}x^{8}-30Ba\,b^{3}x^{8}-10Aa\,b^{3}x^{6}+20B\,a^{2}b^{2}x^{6}+8A\,a^{2}b^{2}x^{4}+240B\,a^{3}b\,x^{4}+176A\,a^{3}b\,x^{2}+160B\,a^{4}x^{2}+128A\,a^{4}\right)}{1280x^{10}a^{3}}+$

default $B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{8a\,x^{8}}-\frac{3b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6a\,x^{6}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{4a\,x^{4}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}}{}\right.\right.\right.}{2a}}{4a}}{6a}}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/8/a/x^8*(b*x^2+a)^(5/2)-3/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(
-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3
*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/
2))/x)))))))+A*(-1/10/a/x^10*(b*x^2+a)^(5/2)-1/2*b/a*(-1/8/a/x^8*(b*x^2+a)^
(5/2)-3/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/
2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x
^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))))
```

Maxima [A]
time = 0.28, size = 294, normalized size = 1.60

$$-\frac{3\,Bb^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{5}{2}}}+\frac{3\,Ab^5\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,a^{\frac{7}{2}}}+\frac{(bx^2+a)^{\frac{3}{2}}Bb^4}{128\,a^4}+\frac{3\,\sqrt{bx^2+a}\,Bb^4}{128\,a^3}-\frac{(bx^2+a)^{\frac{3}{2}}Ab^5}{256\,a^5}-\frac{3\,\sqrt{bx^2+a}\,Ab^5}{256\,a^4}-\frac{(bx^2+a)^{\frac{5}{2}}Bb^3}{128\,a^4x^2}+\frac{(bx^2+a)^{\frac{5}{2}}Ab^4}{256\,a^5x^2}-\frac{(bx^2+a)^{\frac{5}{2}}Bb^2}{64\,a^3x^4}+\frac{(bx^2+a)^{\frac{5}{2}}Ab^3}{128\,a^4x^4}+\frac{(bx^2+a)^{\frac{5}{2}}Bb}{16\,a^2x^6}-\frac{(bx^2+a)^{\frac{5}{2}}Ab^2}{32\,a^3x^6}-\frac{(bx^2+a)^{\frac{5}{2}}B}{8\,ax^8}+\frac{(bx^2+a)^{\frac{5}{2}}Ab}{16\,a^2x^8}-\frac{(bx^2+a)^{\frac{5}{2}}A}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="maxima")
```

```
[Out] -3/128*B*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/256*A*b^5*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(7/2) + 1/128*(b*x^2 + a)^(3/2)*B*b^4/a^4 + 3/128*sqr
t(b*x^2 + a)*B*b^4/a^3 - 1/256*(b*x^2 + a)^(3/2)*A*b^5/a^5 - 3/256*sqrt(b*x
^2 + a)*A*b^5/a^4 - 1/128*(b*x^2 + a)^(5/2)*B*b^3/(a^4*x^2) + 1/256*(b*x^2
+ a)^(5/2)*A*b^4/(a^5*x^2) - 1/64*(b*x^2 + a)^(5/2)*B*b^2/(a^3*x^4) + 1/128
*(b*x^2 + a)^(5/2)*A*b^3/(a^4*x^4) + 1/16*(b*x^2 + a)^(5/2)*B*b/(a^2*x^6) -
 1/32*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x^6) - 1/8*(b*x^2 + a)^(5/2)*B/(a*x^8) +
 1/16*(b*x^2 + a)^(5/2)*A*b/(a^2*x^8) - 1/10*(b*x^2 + a)^(5/2)*A/(a*x^10)
```

Fricas [A]
time = 1.59, size = 317, normalized size = 1.72

$$\left[-\frac{15\,(2\,Bab^4-Ab^5)\sqrt{a}\,x^{10}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(15\,(2\,Ba^2b^3-Aab^4)x^8-10\,(2\,Ba^3b^2-Aa^2b^3)x^6-128\,Aa^5-8\,(30\,Ba^4b+Aa^3b^2)x^4-16\,(10\,Ba^5+11\,Aa^4b)x^2)\sqrt{bx^2+a}}{2560\,a^4x^{10}},\ \frac{15\,(2\,Bab^4-Ab^5)\sqrt{-a}\,x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(15\,(2\,Ba^2b^3-Aab^4)x^8-10\,(2\,Ba^3b^2-Aa^2b^3)x^6-128\,Aa^5-8\,(30\,Ba^4b+Aa^3b^2)x^4-16\,(10\,Ba^5+11\,Aa^4b)x^2)\sqrt{bx^2+a}}{1280\,a^4x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(2*B*a*b^4 - A*b^5)*sqrt(a)*x^10*log(-(b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(a) + 2*a)/x^2) - 2*(15*(2*B*a^2*b^3 - A*a*b^4)*x^8 - 10*(2*B*a^3*b^
2 - A*a^2*b^3)*x^6 - 128*A*a^5 - 8*(30*B*a^4*b + A*a^3*b^2)*x^4 - 16*(10*B*
a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^10), 1/1280*(15*(2*B*a*b^4 -
 A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(2*B*a^2*b^3 -
 A*a*b^4)*x^8 - 10*(2*B*a^3*b^2 - A*a^2*b^3)*x^6 - 128*A*a^5 - 8*(30*B*a^4*
b + A*a^3*b^2)*x^4 - 16*(10*B*a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^4*
x^10)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**11,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.61, size = 212, normalized size = 1.15

$$\frac{\frac{15\,(2\,Bab^5-Ab^6)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{30\,(bx^2+a)^{\frac{9}{2}}Bab^5-140\,(bx^2+a)^{\frac{7}{2}}Ba^2b^5+140\,(bx^2+a)^{\frac{3}{2}}Ba^4b^5-30\,\sqrt{bx^2+a}\,Ba^5b^5-15\,(bx^2+a)^{\frac{9}{2}}Ab^6+70\,(bx^2+a)^{\frac{7}{2}}Aab^6-128\,(bx^2+a)^{\frac{5}{2}}Aa^2b^6-70\,(bx^2+a)^{\frac{3}{2}}Aa^3b^6+15\,\sqrt{bx^2+a}\,Aa^4b^6}{a^3b^5x^{10}}}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(15*(2*B*a*b^5 - A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a
^3) + (30*(b*x^2 + a)^(9/2)*B*a*b^5 - 140*(b*x^2 + a)^(7/2)*B*a^2*b^5 + 140
*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 30*sqrt(b*x^2 + a)*B*a^5*b^5 - 15*(b*x^2 + a
)^(9/2)*A*b^6 + 70*(b*x^2 + a)^(7/2)*A*a*b^6 - 128*(b*x^2 + a)^(5/2)*A*a^2*
b^6 - 70*(b*x^2 + a)^(3/2)*A*a^3*b^6 + 15*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^3*b
^5*x^10))/b
```

Mupad [**B**]
time = 2.14, size = 205, normalized size = 1.11

$$\frac{3\,A\,a\,\sqrt{b\,x^2+a}}{256\,x^{10}}-\frac{11\,B\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^8}-\frac{7\,A\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^{10}}+\frac{3\,B\,a\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{A\,{\left(b\,x^2+a\right)}^{5/2}}{10\,a\,x^{10}}+\frac{7\,A\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^2\,x^{10}}-\frac{3\,A\,{\left(b\,x^2+a\right)}^{9/2}}{256\,a^3\,x^{10}}-\frac{11\,B\,{\left(b\,x^2+a\right)}^{5/2}}{128\,a\,x^8}+\frac{3\,B\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^2\,x^8}-\frac{A\,b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{256\,a^{7/2}}+\frac{B\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{128\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^11,x)
```

```
[Out] (B*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (11*B*(a +
b*x^2)^(3/2))/(128*x^8) - (A*b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(
256*a^(7/2)) - (7*A*(a + b*x^2)^(3/2))/(128*x^10) + (3*A*a*(a + b*x^2)^(1/2
))/(256*x^10) + (3*B*a*(a + b*x^2)^(1/2))/(128*x^8) - (A*(a + b*x^2)^(5/2))
/(10*a*x^10) + (7*A*(a + b*x^2)^(7/2))/(128*a^2*x^10) - (3*A*(a + b*x^2)^(9
/2))/(256*a^3*x^10) - (11*B*(a + b*x^2)^(5/2))/(128*a*x^8) + (3*B*(a + b*x^
2)^(7/2))/(128*a^2*x^8)
```

3.538 $\int x^5(a+bx^2)^{5/2}(A+Bx^2)\,dx$

Optimal. Leaf size=103

$$\frac{a^2(Ab-aB)(a+bx^2)^{7/2}}{7b^4}-\frac{a(2Ab-3aB)(a+bx^2)^{9/2}}{9b^4}+\frac{(Ab-3aB)(a+bx^2)^{11/2}}{11b^4}+\frac{B(a+bx^2)^{13/2}}{13b^4}$$

```
[Out] 1/7*a^2*(A*b-B*a)*(b*x^2+a)^(7/2)/b^4-1/9*a*(2*A*b-3*B*a)*(b*x^2+a)^(9/2)/b
^4+1/11*(A*b-3*B*a)*(b*x^2+a)^(11/2)/b^4+1/13*B*(b*x^2+a)^(13/2)/b^4
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{a^2(a+bx^2)^{7/2}(Ab-aB)}{7b^4}+\frac{(a+bx^2)^{11/2}(Ab-3aB)}{11b^4}-\frac{a(a+bx^2)^{9/2}(2Ab-3aB)}{9b^4}+\frac{B(a+bx^2)^{13/2}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(9/2))/(9*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(11/2))/(11*b^4) + (B*(a + b*
x^2)^(13/2))/(13*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) \, dx = \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{5/2}(A+Bx) \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^{5/2}}{b^3} + \frac{a(-2Ab+3aB)(a+bx)^{7/2}}{b^3}\right.\right.$$
$$= \frac{a^2(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^4} - \frac{a(2Ab-3aB)\left(a+bx^2\right)^{9/2}}{9b^4} + \frac{(Ab-3aB)\left(\right.}{11b}$$

Mathematica [A]
time = 0.07, size = 80, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{7/2}\left(104a^2Ab-48a^3B-364aAb^2x^2+168a^2bBx^2+819Ab^3x^4-378ab^2Bx^4+693b^3Bx^6\right)}{9009b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(104*a^2*A*b - 48*a^3*B - 364*a*A*b^2*x^2 + 168*a^2*b*B*
x^2 + 819*A*b^3*x^4 - 378*a*b^2*B*x^4 + 693*b^3*B*x^6))/(9009*b^4)
```

Maple [A]
time = 0.08, size = 144, normalized size = 1.40

method	result
gosper	$\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(693B\,x^6b^3+819A\,b^3x^4-378Ba\,b^2x^4-364Aa\,b^2x^2+168B\,a^2b\,x^2+104A\,a^2b-48B\,a^3\right)}{9009b^4}$
default	$B\left(\frac{x^6\left(b\,x^2+a\right)^{\frac{7}{2}}}{13b}-\frac{6a\left(\frac{x^4\left(b\,x^2+a\right)^{\frac{7}{2}}}{11b}-\frac{4a\left(\frac{x^2\left(b\,x^2+a\right)^{\frac{7}{2}}}{9b}-\frac{2a\left(b\,x^2+a\right)^{\frac{7}{2}}}{63b^2}\right)}{11b}\right)}{13b}\right)+A\left(\frac{x^4\left(b\,x^2+a\right)^{\frac{7}{2}}}{11b}-\frac{4a\left(\frac{x^2\left(b\,x^2+a\right)^{\frac{7}{2}}}{9b}-\frac{2}{}\right.}{11b}\right.$
trager	$\frac{(693B\,b^6x^{12}+819A\,b^6x^{10}+1701Ba\,b^5x^{10}+2093Aa\,b^5x^8+1113B\,a^2b^4x^8+1469A\,a^2b^4x^6+15B\,a^3b^3x^6+39A\,a^3b^3x^4-18B\,a^4b^2x^4-52}{9009b^4}$
risch	$\frac{(693B\,b^6x^{12}+819A\,b^6x^{10}+1701Ba\,b^5x^{10}+2093Aa\,b^5x^8+1113B\,a^2b^4x^8+1469A\,a^2b^4x^6+15B\,a^3b^3x^6+39A\,a^3b^3x^4-18B\,a^4b^2x^4-52}{9009b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/13*x^6*(b*x^2+a)^(7/2)/b-6/13*a/b*(1/11*x^4*(b*x^2+a)^(7/2)/b-4/11*a/b
*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))))+A*(1/11*x^4*(b*x^
2+a)^(7/2)/b-4/11*a/b*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2)
))
```

Maxima [A]
time = 0.28, size = 132, normalized size = 1.28

$$\frac{(bx^2+a)^{\frac{7}{2}}Bx^6}{13\,b}-\frac{6\,(bx^2+a)^{\frac{7}{2}}Bax^4}{143\,b^2}+\frac{(bx^2+a)^{\frac{7}{2}}Ax^4}{11\,b}+\frac{8\,(bx^2+a)^{\frac{7}{2}}Ba^2x^2}{429\,b^3}-\frac{4\,(bx^2+a)^{\frac{7}{2}}Aax^2}{99\,b^2}-\frac{16\,(bx^2+a)^{\frac{7}{2}}Ba^3}{3003\,b^4}+\frac{8\,(bx^2+a)^{\frac{7}{2}}Aa^2}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^2 + a)^(7/2)*B*x^6/b - 6/143*(b*x^2 + a)^(7/2)*B*a*x^4/b^2 + 1/11
*(b*x^2 + a)^(7/2)*A*x^4/b + 8/429*(b*x^2 + a)^(7/2)*B*a^2*x^2/b^3 - 4/99*(
b*x^2 + a)^(7/2)*A*a*x^2/b^2 - 16/3003*(b*x^2 + a)^(7/2)*B*a^3/b^4 + 8/693*
(b*x^2 + a)^(7/2)*A*a^2/b^3
```

Fricas [A]
time = 1.19, size = 147, normalized size = 1.43

$$\frac{(693\,Bb^6x^{12}+63\,(27\,Bab^5+13\,Ab^6)x^{10}+7\,(159\,Ba^2b^4+299\,Aab^5)x^8-48\,Ba^6+104\,Aa^5b+(15\,Ba^3b^3+1469\,Aa^2b^4)x^6-3\,(6\,Ba^4b^2-13\,Aa^3b^3)x^4+4\,(6\,Ba^5b-13\,Aa^4b^2)x^2)\sqrt{bx^2+a}}{9009\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9009*(693*B*b^6*x^12 + 63*(27*B*a*b^5 + 13*A*b^6)*x^10 + 7*(159*B*a^2*b^4
 + 299*A*a*b^5)*x^8 - 48*B*a^6 + 104*A*a^5*b + (15*B*a^3*b^3 + 1469*A*a^2*b
^4)*x^6 - 3*(6*B*a^4*b^2 - 13*A*a^3*b^3)*x^4 + 4*(6*B*a^5*b - 13*A*a^4*b^2)
*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(94) = 188$.
time = 0.72, size = 313, normalized size = 3.04

$$\begin{cases}\frac{8Aa^5\sqrt{a+bx^2}}{693b^3}-\frac{4Aa^4x^2\sqrt{a+bx^2}}{693b^2}+\frac{Aa^3x^4\sqrt{a+bx^2}}{231b}+\frac{113Aa^2x^6\sqrt{a+bx^2}}{693}+\frac{23Aabx^8\sqrt{a+bx^2}}{99}+\frac{Ab^2x^{10}\sqrt{a+bx^2}}{11}-\frac{16Ba^6\sqrt{a+bx^2}}{3003b^4}+\frac{8Ba^5x^2\sqrt{a+bx^2}}{3003b^3}-\frac{2Ba^4x^4\sqrt{a+bx^2}}{1001b^2}+\frac{5Ba^3x^6\sqrt{a+bx^2}}{3003b}+\frac{53Ba^2x^8\sqrt{a+bx^2}}{429}+\frac{27Babx^{10}\sqrt{a+bx^2}}{143}+\frac{Bb^2x^{12}\sqrt{a+bx^2}}{13} & \text{for } b\neq 0\\ a^{\frac{5}{2}}\left(\frac{Ax^6}{6}+\frac{Bx^8}{8}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((8*A*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*A*a**4*x**2*sqrt(a + b*
x**2)/(693*b**2) + A*a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*A*a**2*x**6*s
qrt(a + b*x**2)/693 + 23*A*a*b*x**8*sqrt(a + b*x**2)/99 + A*b**2*x**10*sqrt
(a + b*x**2)/11 - 16*B*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*B*a**5*x**2*sq
rt(a + b*x**2)/(3003*b**3) - 2*B*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5
```

```
*B*a**3*x**6*sqrt(a + b*x**2)/(3003*b) + 53*B*a**2*x**8*sqrt(a + b*x**2)/42
9 + 27*B*a*b*x**10*sqrt(a + b*x**2)/143 + B*b**2*x**12*sqrt(a + b*x**2)/13,
 Ne(b, 0)), (a**(5/2)*(A*x**6/6 + B*x**8/8), True))
```

Giac [A]
time = 0.63, size = 104, normalized size = 1.01

$$\frac{693\,(bx^2+a)^{\frac{13}{2}}B-2457\,(bx^2+a)^{\frac{11}{2}}Ba+3003\,(bx^2+a)^{\frac{9}{2}}Ba^2-1287\,(bx^2+a)^{\frac{7}{2}}Ba^3+819\,(bx^2+a)^{\frac{11}{2}}Ab-2002\,(bx^2+a)^{\frac{9}{2}}Aab+1287\,(bx^2+a)^{\frac{7}{2}}Aa^2b}{9009\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9009*(693*(b*x^2 + a)^(13/2)*B - 2457*(b*x^2 + a)^(11/2)*B*a + 3003*(b*x^
2 + a)^(9/2)*B*a^2 - 1287*(b*x^2 + a)^(7/2)*B*a^3 + 819*(b*x^2 + a)^(11/2)*
A*b - 2002*(b*x^2 + a)^(9/2)*A*a*b + 1287*(b*x^2 + a)^(7/2)*A*a^2*b)/b^4
```

Mupad [B]
time = 0.38, size = 136, normalized size = 1.32

$$\sqrt{b\,x^2+a}\,\left(\frac{B\,b^2\,x^{12}}{13}-\frac{48\,B\,a^6-104\,A\,a^5\,b}{9009\,b^4}+\frac{x^{10}\,\left(819\,A\,b^6+1701\,B\,a\,b^5\right)}{9009\,b^4}+\frac{a\,x^8\,\left(299\,A\,b+159\,B\,a\right)}{1287}+\frac{a^3\,x^4\,\left(13\,A\,b-6\,B\,a\right)}{3003\,b^2}-\frac{4\,a^4\,x^2\,\left(13\,A\,b-6\,B\,a\right)}{9009\,b^3}+\frac{a^2\,x^6\,\left(1469\,A\,b+15\,B\,a\right)}{9009\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((B*b^2*x^12)/13 - (48*B*a^6 - 104*A*a^5*b)/(9009*b^4) +
(x^10*(819*A*b^6 + 1701*B*a*b^5))/(9009*b^4) + (a*x^8*(299*A*b + 159*B*a))/
1287 + (a^3*x^4*(13*A*b - 6*B*a))/(3003*b^2) - (4*a^4*x^2*(13*A*b - 6*B*a))
/(9009*b^3) + (a^2*x^6*(1469*A*b + 15*B*a))/(9009*b))
```

3.539 $\int x^4(a+bx^2)^{5/2}(A+Bx^2)\,dx$

Optimal. Leaf size=221

$$-\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3}+\frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12Ab-5aB)x^5\sqrt{a+bx^2}}{384b}+\frac{a(12Ab-5a}{1}$$

```
[Out] 1/192*a*(12*A*b-5*B*a)*x^5*(b*x^2+a)^(3/2)/b+1/120*(12*A*b-5*B*a)*x^5*(b*x^
2+a)^(5/2)/b+1/12*B*x^5*(b*x^2+a)^(7/2)/b+1/1024*a^5*(12*A*b-5*B*a)*arctanh
(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)-1/1024*a^4*(12*A*b-5*B*a)*x*(b*x^2+a)^(
1/2)/b^3+1/1536*a^3*(12*A*b-5*B*a)*x^3*(b*x^2+a)^(1/2)/b^2+1/384*a^2*(12*A*
b-5*B*a)*x^5*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 285, 327, 223, 212}

$$\frac{a^5(12Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}-\frac{a^4x\sqrt{a+bx^2}\,(12Ab-5aB)}{1024b^3}+\frac{a^3x^3\sqrt{a+bx^2}\,(12Ab-5aB)}{1536b^2}+\frac{a^2x^5\sqrt{a+bx^2}\,(12Ab-5aB)}{384b}+\frac{ax^5(a+bx^2)^{3/2}(12Ab-5aB)}{192b}+\frac{x^5(a+bx^2)^{5/2}(12Ab-5aB)}{120b}+\frac{Bx^5(a+bx^2)^{7/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] -1/1024*(a^4*(12*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/b^3 + (a^3*(12*A*b - 5*a*B
)*x^3*Sqrt[a + b*x^2])/(1536*b^2) + (a^2*(12*A*b - 5*a*B)*x^5*Sqrt[a + b*x^
2])/(384*b) + (a*(12*A*b - 5*a*B)*x^5*(a + b*x^2)^(3/2))/(192*b) + ((12*A*b
 - 5*a*B)*x^5*(a + b*x^2)^(5/2))/(120*b) + (B*x^5*(a + b*x^2)^(7/2))/(12*b)
 + (a^5*(12*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
```

```
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}-\frac{(-12Ab+5aB)\int x^4\left(a+bx^2\right)^{5/2}\,dx}{12b}$$

$$=\frac{(12Ab-5aB)x^5\left(a+bx^2\right)^{5/2}}{120b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}+\frac{(a(12Ab-5aB))\int x}{24b}$$

$$=\frac{a(12Ab-5aB)x^5\left(a+bx^2\right)^{3/2}}{192b}+\frac{(12Ab-5aB)x^5\left(a+bx^2\right)^{5/2}}{120b}+\frac{Bx^5(a}{1}$$

$$=\frac{a^2(12Ab-5aB)x^5\sqrt{a+bx^2}}{384b}+\frac{a(12Ab-5aB)x^5\left(a+bx^2\right)^{3/2}}{192b}+\frac{(12Ab}{}$$

$$=\frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12Ab-5aB)x^5\sqrt{a+bx^2}}{384b}+\frac{a(12Ab}{}$$

$$=-\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3}+\frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12}{}$$

$$=-\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3}+\frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12}{}$$

$$=-\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3}+\frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12}{}$$

Mathematica [A]
time = 0.26, size = 164, normalized size = 0.74

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\,(75a^5B+40a^3b^2x^2(3A+Bx^2)+256b^5x^8(6A+5Bx^2)-10a^4b(18A+5Bx^2)+48a^2b^3x^4(62A+45Bx^2)+64ab^4x^6(63A+50Bx^2))+15a^5(-12Ab+5aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{15360b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(75*a^5*B + 40*a^3*b^2*x^2*(3*A + B*x^2) + 256*b
^5*x^8*(6*A + 5*B*x^2) - 10*a^4*b*(18*A + 5*B*x^2) + 48*a^2*b^3*x^4*(62*A +
 45*B*x^2) + 64*a*b^4*x^6*(63*A + 50*B*x^2)) + 15*a^5*(-12*A*b + 5*a*B)*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(15360*b^(7/2))
```

Maple [A]
time = 0.09, size = 256, normalized size = 1.16

method	result
risch	$-\frac{x(-1280b^5B\,x^{10}-1536A\,b^5x^8-3200Ba\,b^4x^8-4032Aa\,b^4x^6-2160B\,a^2b^3x^6-2976A\,a^2b^3x^4-40B\,a^3b^2x^4-120A\,a^3b^2x^2+50B\,a^4b\,x^2}{15360b^3}$

default B

$$B\left(\frac{x^{5}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{12b}-\frac{5a\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{10b}-\frac{3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\cdots\right.}{2\cdots}\right.}{4}\right.}{6}\right.}{8b}\right.}{10b}\right.}{12b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/12*x^5*(b*x^2+a)^(7/2)/b-5/12*a/b*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b
*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^
2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))))))))+A*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b*(1/8*x*(b*x^2+a)^(7/2
)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*
x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))
```

Maxima [A]
time = 0.29, size = 242, normalized size = 1.10

$$\frac{(bx^2+a)^{\frac{7}{2}}Bx^5}{12\,b}-\frac{(bx^2+a)^{\frac{7}{2}}Bax^3}{24\,b^2}+\frac{(bx^2+a)^{\frac{7}{2}}Ax^3}{10\,b}+\frac{(bx^2+a)^{\frac{7}{2}}Ba^2x}{64\,b^3}-\frac{(bx^2+a)^{\frac{5}{2}}Ba^3x}{384\,b^3}-\frac{5\,(bx^2+a)^{\frac{3}{2}}Ba^4x}{1536\,b^3}-\frac{5\,\sqrt{bx^2+a}\,Ba^5x}{1024\,b^3}-\frac{3\,(bx^2+a)^{\frac{7}{2}}Aax}{80\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Aa^2x}{160\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Aa^3x}{128\,b^2}+\frac{3\,\sqrt{bx^2+a}\,Aa^4x}{256\,b^2}-\frac{5\,Ba^6\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{7}{2}}}+\frac{3\,Aa^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^(7/2)*B*x^5/b - 1/24*(b*x^2 + a)^(7/2)*B*a*x^3/b^2 + 1/10*
(b*x^2 + a)^(7/2)*A*x^3/b + 1/64*(b*x^2 + a)^(7/2)*B*a^2*x/b^3 - 1/384*(b*x
^2 + a)^(5/2)*B*a^3*x/b^3 - 5/1536*(b*x^2 + a)^(3/2)*B*a^4*x/b^3 - 5/1024*s
qrt(b*x^2 + a)*B*a^5*x/b^3 - 3/80*(b*x^2 + a)^(7/2)*A*a*x/b^2 + 1/160*(b*x^
2 + a)^(5/2)*A*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*A*a^3*x/b^2 + 3/256*sqrt
(b*x^2 + a)*A*a^4*x/b^2 - 5/1024*B*a^6*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/2
56*A*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 1.38, size = 355, normalized size = 1.61

$$\left[-\frac{15\,(5\,Ba^6-12\,Aa^5b)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(1280\,Bb^6x^{11}+128\,(25\,Bab^5+12\,Ab^6)x^9+144\,(15\,Ba^2b^4+28\,Aab^5)x^7+8\,(5\,Ba^3b^3+372\,Aa^2b^4)x^5-10\,(5\,Ba^4b^2-12\,Aa^3b^3)x^3+15\,(5\,Ba^5b-12\,Aa^4b^2)x)\sqrt{bx^2+a}}{30720\,b^4},\ \frac{15\,(5\,Ba^6-12\,Aa^5b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(1280\,Bb^6x^{11}+128\,(25\,Bab^5+12\,Ab^6)x^9+144\,(15\,Ba^2b^4+28\,Aab^5)x^7+8\,(5\,Ba^3b^3+372\,Aa^2b^4)x^5-10\,(5\,Ba^4b^2-12\,Aa^3b^3)x^3+15\,(5\,Ba^5b-12\,Aa^4b^2)x)\sqrt{bx^2+a}}{15360\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(1280*B*b^6*x^11 + 128*(25*B*a*b^5 + 12*A*b^6)*x^9 +
144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^7 + 8*(5*B*a^3*b^3 + 372*A*a^2*b^4)*x^5 -
 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x^3 + 15*(5*B*a^5*b - 12*A*a^4*b^2)*x)*sqr
t(b*x^2 + a))/b^4, 1/15360*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) + (1280*B*b^6*x^11 + 128*(25*B*a*b^5 + 12*A*b^6)*x^9
 + 144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^7 + 8*(5*B*a^3*b^3 + 372*A*a^2*b^4)*x^
5 - 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x^3 + 15*(5*B*a^5*b - 12*A*a^4*b^2)*x)*
sqrt(b*x^2 + a))/b^4]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2)*(B*x**2+A),x)
[Out] Timed out
```

Giac [**A**]
time = 0.64, size = 195, normalized size = 0.88

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bb^{2}x^{2}+\frac{25\,Bab^{11}+12\,Ab^{12}}{b^{10}}\right)x^{2}+\frac{9\,(15\,Ba^{2}b^{10}+28\,Aab^{11})}{b^{10}}\right)x^{2}+\frac{5\,Ba^{3}b^{9}+372\,Aa^{2}b^{10}}{b^{10}}\right)x^{2}-\frac{5\,(5\,Ba^{4}b^{8}-12\,Aa^{3}b^{9})}{b^{10}}\right)x^{2}+\frac{15\,(5\,Ba^{5}b^{7}-12\,Aa^{4}b^{8})}{b^{10}}\right)\sqrt{bx^{2}+a}\,x+\frac{(5\,Ba^{6}-12\,Aa^{5}b)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{1024\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
[Out] 1/15360*(2*(4*(2*(8*(10*B*b^2*x^2 + (25*B*a*b^11 + 12*A*b^12)/b^10)*x^2 + 9
*(15*B*a^2*b^10 + 28*A*a*b^11)/b^10)*x^2 + (5*B*a^3*b^9 + 372*A*a^2*b^10)/b
^10)*x^2 - 5*(5*B*a^4*b^8 - 12*A*a^3*b^9)/b^10)*x^2 + 15*(5*B*a^5*b^7 - 12*
A*a^4*b^8)/b^10)*sqrt(b*x^2 + a)*x + 1/1024*(5*B*a^6 - 12*A*a^5*b)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{4}\left(B\,x^{2}+A\right)\,\left(b\,x^{2}+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(a + b*x^2)^(5/2),x)
[Out] int(x^4*(A + B*x^2)*(a + b*x^2)^(5/2), x)
```

3.540 $\int x^3(a+bx^2)^{5/2}(A+Bx^2)\,dx$

Optimal. Leaf size=73

$$-\frac{a(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{9/2}}{9b^3}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^3}$$

```
[Out] -1/7*a*(A*b-B*a)*(b*x^2+a)^(7/2)/b^3+1/9*(A*b-2*B*a)*(b*x^2+a)^(9/2)/b^3+1/
11*B*(b*x^2+a)^(11/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {457, 78}

$$\frac{\left(a+bx^2\right)^{9/2}(Ab-2aB)}{9b^3}-\frac{a\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^3}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] -1/7*(a*(A*b - a*B)*(a + b*x^2)^(7/2))/b^3 + ((A*b - 2*a*B)*(a + b*x^2)^(9/
2))/(9*b^3) + (B*(a + b*x^2)^(11/2))/(11*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{5/2}(A+Bx)\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^{5/2}}{b^2} + \frac{(Ab-2aB)(a+bx)^{7/2}}{b^2} + \frac{B(a$$

$$= -\frac{a(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^3} + \frac{(Ab-2aB)\left(a+bx^2\right)^{9/2}}{9b^3} + \frac{B\left(a+bx^2\right)^{11/2}}{11b^3}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.77

$$\frac{\left(a+bx^2\right)^{7/2}\left(-22aAb+8a^2B+77Ab^2x^2-28abBx^2+63b^2Bx^4\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-22*a*A*b + 8*a^2*B + 77*A*b^2*x^2 - 28*a*b*B*x^2 + 63*
b^2*B*x^4))/(693*b^3)
```

Maple [A]
time = 0.08, size = 96, normalized size = 1.32

method	result
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(-63b^{2}B\,x^{4}-77A\,b^{2}x^{2}+28Bab\,x^{2}+22abA-8a^{2}B\right)}{693b^{3}}$
default	$B\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{11b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63b^{2}}\right)}{11b}\right)+A\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63b^{2}}\right)$
trager	$-\frac{\left(-63b^{5}B\,x^{10}-77A\,b^{5}x^{8}-161Ba\,b^{4}x^{8}-209Aa\,b^{4}x^{6}-113B\,a^{2}b^{3}x^{6}-165A\,a^{2}b^{3}x^{4}-3B\,a^{3}b^{2}x^{4}-11A\,a^{3}b^{2}x^{2}+4B\,a^{4}b\,x^{2}+22a^{4}bA-8\right.}{693b^{3}}$
risch	$-\frac{\left(-63b^{5}B\,x^{10}-77A\,b^{5}x^{8}-161Ba\,b^{4}x^{8}-209Aa\,b^{4}x^{6}-113B\,a^{2}b^{3}x^{6}-165A\,a^{2}b^{3}x^{4}-3B\,a^{3}b^{2}x^{4}-11A\,a^{3}b^{2}x^{2}+4B\,a^{4}b\,x^{2}+22a^{4}bA-8\right.}{693b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/11*x^4*(b*x^2+a)^(7/2)/b-4/11*a/b*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^
2*(b*x^2+a)^(7/2)))+A*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2)
)
```

Maxima [A]
time = 0.32, size = 90, normalized size = 1.23

$$\frac{(bx^2+a)^{\frac{7}{2}}Bx^4}{11\,b}-\frac{4\,(bx^2+a)^{\frac{7}{2}}Bax^2}{99\,b^2}+\frac{(bx^2+a)^{\frac{7}{2}}Ax^2}{9\,b}+\frac{8\,(bx^2+a)^{\frac{7}{2}}Ba^2}{693\,b^3}-\frac{2\,(bx^2+a)^{\frac{7}{2}}Aa}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(7/2)*B*x^4/b - 4/99*(b*x^2 + a)^(7/2)*B*a*x^2/b^2 + 1/9*(
b*x^2 + a)^(7/2)*A*x^2/b + 8/693*(b*x^2 + a)^(7/2)*B*a^2/b^3 - 2/63*(b*x^2
+ a)^(7/2)*A*a/b^2
```

Fricas [A]
time = 1.19, size = 122, normalized size = 1.67

$$\frac{\left(63\,Bb^5x^{10}+7\,(23\,Bab^4+11\,Ab^5)x^8+(113\,Ba^2b^3+209\,Aab^4)x^6+8\,Ba^5-22\,Aa^4b+3\,(Ba^3b^2+55\,Aa^2b^3)x^4-(4\,Ba^4b-11\,Aa^3b^2)x^2\right)\sqrt{bx^2+a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/693*(63*B*b^5*x^10 + 7*(23*B*a*b^4 + 11*A*b^5)*x^8 + (113*B*a^2*b^3 + 209
*A*a*b^4)*x^6 + 8*B*a^5 - 22*A*a^4*b + 3*(B*a^3*b^2 + 55*A*a^2*b^3)*x^4 - (
4*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(65) = 130$.
time = 0.53, size = 260, normalized size = 3.56

$$\begin{cases} -\frac{2Aa^4\sqrt{a+bx^2}}{63b^2}+\frac{Aa^3x^2\sqrt{a+bx^2}}{63b}+\frac{5Aa^2x^4\sqrt{a+bx^2}}{21}+\frac{19Aabx^6\sqrt{a+bx^2}}{63}+\frac{Ab^2x^8\sqrt{a+bx^2}}{9}+\frac{8Ba^5\sqrt{a+bx^2}}{693b^3}-\frac{4Ba^4x^2\sqrt{a+bx^2}}{693b^2}+\frac{Ba^3x^4\sqrt{a+bx^2}}{231b}+\frac{113Ba^2x^6\sqrt{a+bx^2}}{693}+\frac{23Babx^8\sqrt{a+bx^2}}{99}+\frac{Bb^2x^{10}\sqrt{a+bx^2}}{11} & \text{for } b\neq 0 \\ a^{\frac{5}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((-2*A*a**4*sqrt(a + b*x**2)/(63*b**2) + A*a**3*x**2*sqrt(a + b*x*
*2)/(63*b) + 5*A*a**2*x**4*sqrt(a + b*x**2)/21 + 19*A*a*b*x**6*sqrt(a + b*x
**2)/63 + A*b**2*x**8*sqrt(a + b*x**2)/9 + 8*B*a**5*sqrt(a + b*x**2)/(693*b
**3) - 4*B*a**4*x**2*sqrt(a + b*x**2)/(693*b**2) + B*a**3*x**4*sqrt(a + b*x
**2)/(231*b) + 113*B*a**2*x**6*sqrt(a + b*x**2)/693 + 23*B*a*b*x**8*sqrt(a
+ b*x**2)/99 + B*b**2*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(5/2)*(A*x*
*4/4 + B*x**6/6), True))
```

Giac [A]
time = 0.66, size = 73, normalized size = 1.00

$$\frac{63\,(bx^2+a)^{\frac{11}{2}}B-154\,(bx^2+a)^{\frac{9}{2}}Ba+99\,(bx^2+a)^{\frac{7}{2}}Ba^2+77\,(bx^2+a)^{\frac{9}{2}}Ab-99\,(bx^2+a)^{\frac{7}{2}}Aab}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/693*(63*(b*x^2 + a)^(11/2)*B - 154*(b*x^2 + a)^(9/2)*B*a + 99*(b*x^2 + a)
^(7/2)*B*a^2 + 77*(b*x^2 + a)^(9/2)*A*b - 99*(b*x^2 + a)^(7/2)*A*a*b)/b^3
```

Mupad [B]
time = 0.37, size = 115, normalized size = 1.58

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,B\,a^5-22\,A\,a^4\,b}{693\,b^3}+\frac{B\,b^2\,x^{10}}{11}+\frac{x^8\,\left(77\,A\,b^5+161\,B\,a\,b^4\right)}{693\,b^3}+\frac{a\,x^6\,\left(209\,A\,b+113\,B\,a\right)}{693}+\frac{a^3\,x^2\,\left(11\,A\,b-4\,B\,a\right)}{693\,b^2}+\frac{a^2\,x^4\,\left(55\,A\,b+B\,a\right)}{231\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*B*a^5 - 22*A*a^4*b)/(693*b^3) + (B*b^2*x^10)/11 + (x^
8*(77*A*b^5 + 161*B*a*b^4))/(693*b^3) + (a*x^6*(209*A*b + 113*B*a))/693 + (
a^3*x^2*(11*A*b - 4*B*a))/(693*b^2) + (a^2*x^4*(55*A*b + B*a))/(231*b))
```

3.541 $\int x^2(a+bx^2)^{5/2}(A+Bx^2)\,dx$

Optimal. Leaf size=188

$$\frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2}+\frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab-3aB)x^3(a+bx^2)^{3/2}}{96b}+\frac{(10Ab-3aB)x^3 \ldots}{80 \ldots}$$

```
[Out] 1/96*a*(10*A*b-3*B*a)*x^3*(b*x^2+a)^(3/2)/b+1/80*(10*A*b-3*B*a)*x^3*(b*x^2+
a)^(5/2)/b+1/10*B*x^3*(b*x^2+a)^(7/2)/b-1/256*a^4*(10*A*b-3*B*a)*arctanh(x*
b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/256*a^3*(10*A*b-3*B*a)*x*(b*x^2+a)^(1/2)
/b^2+1/128*a^2*(10*A*b-3*B*a)*x^3*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 285, 327, 223, 212}

$$-\frac{a^4(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{a^3x\sqrt{a+bx^2}\,(10Ab-3aB)}{256b^2}+\frac{a^2x^3\sqrt{a+bx^2}\,(10Ab-3aB)}{128b}+\frac{ax^3(a+bx^2)^{3/2}(10Ab-3aB)}{96b}+\frac{x^3(a+bx^2)^{5/2}(10Ab-3aB)}{80b}+\frac{Bx^3(a+bx^2)^{7/2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (a^3*(10*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(256*b^2) + (a^2*(10*A*b - 3*a*B)*
x^3*Sqrt[a + b*x^2])/(128*b) + (a*(10*A*b - 3*a*B)*x^3*(a + b*x^2)^(3/2))/(
96*b) + ((10*A*b - 3*a*B)*x^3*(a + b*x^2)^(5/2))/(80*b) + (B*x^3*(a + b*x^2
)^(7/2))/(10*b) - (a^4*(10*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(256*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx &= \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{(-10Ab+3aB)\int x^2\left(a+bx^2\right)^{5/2}\,dx}{10b}\\
&= \frac{(10Ab-3aB)x^3\left(a+bx^2\right)^{5/2}}{80b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}+\frac{(a(10Ab-3aB))\int x}{16b}\\
&= \frac{a(10Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{96b}+\frac{(10Ab-3aB)x^3\left(a+bx^2\right)^{5/2}}{80b}+\frac{Bx^3(a}{}\\
&= \frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{96b}+\frac{(10Ab}{}\\
&= \frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2}+\frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab}{}\\
&= \frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2}+\frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab}{}\\
&= \frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2}+\frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab}{}
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 143, normalized size = 0.76

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(-45a^4B+30a^3b(5A+Bx^2)+96b^4x^6(5A+4Bx^2)+16ab^3x^4(85A+63Bx^2)+4a^2b^2x^2(295A+186Bx^2)\right)+15a^4(10Ab-3aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{3840b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]

[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-45*a^4*B + 30*a^3*b*(5*A + B*x^2) + 96*b^4*x^6*(5*A + 4*B*x^2) + 16*a*b^3*x^4*(85*A + 63*B*x^2) + 4*a^2*b^2*x^2*(295*A + 186*B*x^2)) + 15*a^4*(10*A*b - 3*a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(3840*b^(5/2))

Maple [A]
time = 0.08, size = 208, normalized size = 1.11

method	result
risch	$\frac{x\left(384 B\,b^{4} x^{8}+480 A\,b^{4} x^{6}+1008 B a\,b^{3} x^{6}+1360 A a\,b^{3} x^{4}+744 B\,a^{2} b^{2} x^{4}+1180 a^{2} A\,b^{2} x^{2}+30 B\,a^{3} b\,x^{2}+150 A\,a^{3} b-45 B\,a^{4}\right) \sqrt{b\,x^{2}+a}}{3840 b^{2}}$

default	$B\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{10b}-\frac{3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right.}{6}\right.}{8b}\right.}{10b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/
6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/
2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+A*(1/8*x*(b*x^2+a)^(7/2
)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*
x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.30, size = 200, normalized size = 1.06

$$\frac{(bx^2+a)^{\frac{7}{2}}Bx^3}{10\,b} - \frac{3\,(bx^2+a)^{\frac{7}{2}}Bax}{80\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ba^2x}{160\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ba^3x}{128\,b^2} + \frac{3\,\sqrt{bx^2+a}\,Ba^4x}{256\,b^2} + \frac{(bx^2+a)^{\frac{7}{2}}Ax}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Aax}{48\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Aa^2x}{192\,b} - \frac{5\,\sqrt{bx^2+a}\,Aa^3x}{128\,b} + \frac{3\,Ba^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}} - \frac{5\,Aa^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*B*x^3/b - 3/80*(b*x^2 + a)^(7/2)*B*a*x/b^2 + 1/160*(
b*x^2 + a)^(5/2)*B*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*B*a^3*x/b^2 + 3/256*
sqrt(b*x^2 + a)*B*a^4*x/b^2 + 1/8*(b*x^2 + a)^(7/2)*A*x/b - 1/48*(b*x^2 + a
)^(5/2)*A*a*x/b - 5/192*(b*x^2 + a)^(3/2)*A*a^2*x/b - 5/128*sqrt(b*x^2 + a)
*A*a^3*x/b + 3/256*B*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/128*A*a^4*arcsi
nh(b*x/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 1.30, size = 308, normalized size = 1.64

$$\left[-\frac{15\,(3\,Ba^5-10\,Aa^4b)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(384\,Bb^5x^9+48\,(21\,Bab^4+10\,Ab^5)x^7+8\,(93\,Ba^2b^3+170\,Aab^4)x^5+10\,(3\,Ba^3b^2+118\,Aa^2b^3)x^3-15\,(3\,Ba^4b-10\,Aa^3b^2)x)\sqrt{bx^2+a}}{7680\,b^3},\ -\frac{15\,(3\,Ba^5-10\,Aa^4b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(384\,Bb^5x^9+48\,(21\,Bab^4+10\,Ab^5)x^7+8\,(93\,Ba^2b^3+170\,Aab^4)x^5+10\,(3\,Ba^3b^2+118\,Aa^2b^3)x^3-15\,(3\,Ba^4b-10\,Aa^3b^2)x)\sqrt{bx^2+a}}{3840\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(3*B*a^5 - 10*A*a^4*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(384*B*b^5*x^9 + 48*(21*B*a*b^4 + 10*A*b^5)*x^7 + 8*(9
3*B*a^2*b^3 + 170*A*a*b^4)*x^5 + 10*(3*B*a^3*b^2 + 118*A*a^2*b^3)*x^3 - 15*
(3*B*a^4*b - 10*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/3840*(15*(3*B*a^5 -
10*A*a^4*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*B*b^5*x^9 +
48*(21*B*a*b^4 + 10*A*b^5)*x^7 + 8*(93*B*a^2*b^3 + 170*A*a*b^4)*x^5 + 10*(3
*B*a^3*b^2 + 118*A*a^2*b^3)*x^3 - 15*(3*B*a^4*b - 10*A*a^3*b^2)*x)*sqrt(b*x
^2 + a))/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(173) = 346$.
time = 168.71, size = 348, normalized size = 1.85

$$\frac{5Aa^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}} + \frac{133Aa^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}} + \frac{127Aa^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}} + \frac{23A\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}} - \frac{5Aa^4\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}} + \frac{Ab^3x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}} - \frac{3Ba^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}} + \frac{129Ba^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}} + \frac{73Ba^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}} + \frac{29B\sqrt{a}\,b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}} + \frac{3Ba^5\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}} + \frac{Bb^3x^{11}}{10\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] 5*A*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*A*a**(5/2)*x**3/(384*sqrt(1
 + b*x**2/a)) + 127*A*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*A*sqrt(
a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(3/2)) + A*b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) - 3*B*a**(9/2)*x/
(256*b**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**3/(256*b*sqrt(1 + b*x**2/a))
+ 129*B*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*B*a**(3/2)*b*x**7/(160*
sqrt(1 + b*x**2/a)) + 29*B*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a)) + 3*B*
a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + B*b**3*x**11/(10*sqrt(a)*sqr
t(1 + b*x**2/a))
```

Giac [A]
time = 0.71, size = 165, normalized size = 0.88

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,Bb^2x^2+\frac{21\,Bab^9+10\,Ab^{10}}{b^8}\right)x^2+\frac{93\,Ba^2b^8+170\,Aab^9}{b^8}\right)x^2+\frac{5\,(3\,Ba^3b^7+118\,Aa^2b^8)}{b^8}\right)x^2-\frac{15\,(3\,Ba^4b^6-10\,Aa^3b^7)}{b^8}\right)\sqrt{bx^2+a}\,x-\frac{(3\,Ba^5-10\,Aa^4b)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*B*b^2*x^2 + (21*B*a*b^9 + 10*A*b^10)/b^8)*x^2 + (93*B*a^
2*b^8 + 170*A*a*b^9)/b^8)*x^2 + 5*(3*B*a^3*b^7 + 118*A*a^2*b^8)/b^8)*x^2 -
15*(3*B*a^4*b^6 - 10*A*a^3*b^7)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(3*B*a^5 - 1
0*A*a^4*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^2*(A + B*x^2)*(a + b*x^2)^(5/2), x)
```

3.542 $\int x(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}$$

```
[Out] 1/7*(A*b-B*a)*(b*x^2+a)^(7/2)/b^2+1/9*B*(b*x^2+a)^(9/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\frac{\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^2) + (B*(a + b*x^2)^(9/2))/(9*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^{5/2}(A+Bx)\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^{5/2}}{b}+\frac{B(a+bx)^{7/2}}{b}\right)\,dx,x,x^2\right)\\
&= \frac{(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.74

$$\frac{(a+bx^2)^{7/2}(9Ab-2aB+7bBx^2)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(9*A*b - 2*a*B + 7*b*B*x^2))/(63*b^2)
```

Maple [A]
time = 0.08, size = 52, normalized size = 1.13

method	result	size
gosper	$\frac{(bx^2+a)^{\frac{7}{2}}(7bBx^2+9Ab-2Ba)}{63b^2}$	31
default	$B\left(\frac{x^2(bx^2+a)^{\frac{7}{2}}}{9b}-\frac{2a(bx^2+a)^{\frac{7}{2}}}{63b^2}\right)+\frac{A(bx^2+a)^{\frac{7}{2}}}{7b}$	52
trager	$\frac{(7Bb^4x^8+9Ab^4x^6+19Bab^3x^6+27Aab^3x^4+15Ba^2b^2x^4+27a^2Ab^2x^2+Ba^3bx^2+9Aa^3b-2Ba^4)\sqrt{bx^2+a}}{63b^2}$	100
risch	$\frac{(7Bb^4x^8+9Ab^4x^6+19Bab^3x^6+27Aab^3x^4+15Ba^2b^2x^4+27a^2Ab^2x^2+Ba^3bx^2+9Aa^3b-2Ba^4)\sqrt{bx^2+a}}{63b^2}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))+1/7*A/b*(b*x^2+a)^
(7/2)
```

Maxima [A]
time = 0.33, size = 50, normalized size = 1.09

$$\frac{(bx^2+a)^{\frac{7}{2}}Bx^2}{9\,b}-\frac{2\,(bx^2+a)^{\frac{7}{2}}Ba}{63\,b^2}+\frac{(bx^2+a)^{\frac{7}{2}}A}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(7/2)*B*x^2/b - 2/63*(b*x^2 + a)^(7/2)*B*a/b^2 + 1/7*(b*x^2
 + a)^(7/2)*A/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(38) = 76.
time = 1.25, size = 97, normalized size = 2.11

$$\frac{(7\,Bb^4x^8+(19\,Bab^3+9\,Ab^4)x^6-2\,Ba^4+9\,Aa^3b+3\,(5\,Ba^2b^2+9\,Aab^3)x^4+(Ba^3b+27\,Aa^2b^2)x^2)\sqrt{bx^2+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/63*(7*B*b^4*x^8 + (19*B*a*b^3 + 9*A*b^4)*x^6 - 2*B*a^4 + 9*A*a^3*b + 3*(5
*B*a^2*b^2 + 9*A*a*b^3)*x^4 + (B*a^3*b + 27*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)
/b^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(39) = 78$.
time = 0.37, size = 209, normalized size = 4.54

$$\begin{cases} \frac{Aa^3\sqrt{a+bx^2}}{7b} + \frac{3Aa^2x^2\sqrt{a+bx^2}}{7} + \frac{3Aabx^4\sqrt{a+bx^2}}{7} + \frac{Ab^2x^6\sqrt{a+bx^2}}{7} - \frac{2Ba^4\sqrt{a+bx^2}}{63b^2} + \frac{Ba^3x^2\sqrt{a+bx^2}}{63b} + \frac{5Ba^2x^4\sqrt{a+bx^2}}{21} + \frac{19Babx^6\sqrt{a+bx^2}}{63} + \frac{Bb^2x^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ a^{\frac{5}{2}}\left(\frac{Ax^2}{2} + \frac{Bx^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((A*a**3*sqrt(a + b*x**2)/(7*b) + 3*A*a**2*x**2*sqrt(a + b*x**2)/7
 + 3*A*a*b*x**4*sqrt(a + b*x**2)/7 + A*b**2*x**6*sqrt(a + b*x**2)/7 - 2*B*a
**4*sqrt(a + b*x**2)/(63*b**2) + B*a**3*x**2*sqrt(a + b*x**2)/(63*b) + 5*B*
a**2*x**4*sqrt(a + b*x**2)/21 + 19*B*a*b*x**6*sqrt(a + b*x**2)/63 + B*b**2*
x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(5/2)*(A*x**2/2 + B*x**4/4), True))
```

Giac [**A**]
time = 0.70, size = 44, normalized size = 0.96

$$\frac{7\,(bx^2+a)^{\frac{9}{2}}B - 9\,(bx^2+a)^{\frac{7}{2}}Ba + 9\,(bx^2+a)^{\frac{7}{2}}Ab}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/63*(7*(b*x^2 + a)^(9/2)*B - 9*(b*x^2 + a)^(7/2)*B*a + 9*(b*x^2 + a)^(7/2)
*A*b)/b^2
```

Mupad [**B**]
time = 0.33, size = 44, normalized size = 0.96

$$\frac{7\,B\,{\left(b\,x^2+a\right)}^{9/2}+9\,A\,b\,{\left(b\,x^2+a\right)}^{7/2}-9\,B\,a\,{\left(b\,x^2+a\right)}^{7/2}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] (7*B*(a + b*x^2)^(9/2) + 9*A*b*(a + b*x^2)^(7/2) - 9*B*a*(a + b*x^2)^(7/2))
/(63*b^2)
```

3.543 $\int (a+bx^2)^{5/2} (A+Bx^2)\, dx$

Optimal. Leaf size=149

$$\frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b}+\frac{5a(8Ab-aB)x(a+bx^2)^{3/2}}{192b}+\frac{(8Ab-aB)x(a+bx^2)^{5/2}}{48b}+\frac{Bx(a+bx^2)^{7/2}}{8b}+\frac{5a^3}{}$$

```
[Out] 5/192*a*(8*A*b-B*a)*x*(b*x^2+a)^(3/2)/b+1/48*(8*A*b-B*a)*x*(b*x^2+a)^(5/2)/
b+1/8*B*x*(b*x^2+a)^(7/2)/b+5/128*a^3*(8*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+
a)^(1/2))/b^(3/2)+5/128*a^2*(8*A*b-B*a)*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]

time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {396, 201, 223, 212}

$$\frac{5a^3(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}\,(8Ab-aB)}{128b}+\frac{x(a+bx^2)^{5/2}(8Ab-aB)}{48b}+\frac{5ax(a+bx^2)^{3/2}(8Ab-aB)}{192b}+\frac{Bx(a+bx^2)^{7/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (5*a^2*(8*A*b - a*B)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*A*b - a*B)*x*(a +
 b*x^2)^(3/2))/(192*b) + ((8*A*b - a*B)*x*(a + b*x^2)^(5/2))/(48*b) + (B*x*
(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(128*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx = \frac{Bx(a+bx^2)^{7/2}}{8b} - \frac{(-8Ab+aB)\int (a+bx^2)^{5/2}\,dx}{8b}$$
$$= \frac{(8Ab-aB)x(a+bx^2)^{5/2}}{48b} + \frac{Bx(a+bx^2)^{7/2}}{8b} + \frac{(5a(8Ab-aB))\int (a+bx^2)^{3}}{48b}$$
$$= \frac{5a(8Ab-aB)x(a+bx^2)^{3/2}}{192b} + \frac{(8Ab-aB)x(a+bx^2)^{5/2}}{48b} + \frac{Bx(a+bx^2)^{7/2}}{8b}$$
$$= \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x(a+bx^2)^{3/2}}{192b} + \frac{(8Ab-aB)x(}{48b}$$
$$= \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x(a+bx^2)^{3/2}}{192b} + \frac{(8Ab-aB)x(}{48b}$$
$$= \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x(a+bx^2)^{3/2}}{192b} + \frac{(8Ab-aB)x(}{48b}$$

Mathematica [A]
time = 0.19, size = 123, normalized size = 0.83

$$\frac{x\sqrt{a+bx^2}\,(264a^2Ab+15a^3B+208aAb^2x^2+118a^2bBx^2+64Ab^3x^4+136ab^2Bx^4+48b^3Bx^6)}{384b} + \frac{5a^3(-8Ab+aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{128b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(264*a^2*A*b + 15*a^3*B + 208*a*A*b^2*x^2 + 118*a^2*b*B*
x^2 + 64*A*b^3*x^4 + 136*a*b^2*B*x^4 + 48*b^3*B*x^6))/(384*b) + (5*a^3*(-8*
A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Maple [A]
time = 0.08, size = 162, normalized size = 1.09

method	result

risch	$\frac{x\left(48B\,x^{6}b^{3}+64A\,b^{3}x^{4}+136Ba\,b^{2}x^{4}+208Aa\,b^{2}x^{2}+118B\,a^{2}b\,x^{2}+264A\,a^{2}b+15B\,a^{3}\right)\sqrt{b\,x^{2}+a}}{384b}+\frac{5a^{3}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+\right.}{16\sqrt{b}}$
default	$B\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)+A\left(\frac{x(b\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x
^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))))))+A*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(
1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))
```

Maxima [A]
time = 0.31, size = 151, normalized size = 1.01

$$\frac{1}{6}\left(bx^{2}+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(bx^{2}+a\right)^{\frac{3}{2}}Aax+\frac{5}{16}\sqrt{bx^{2}+a}\,Aa^{2}x+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Bx}{8\,b}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bax}{48\,b}-\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{2}x}{192\,b}-\frac{5\sqrt{bx^{2}+a}\,Ba^{3}x}{128\,b}-\frac{5\,Ba^{4}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}+\frac{5\,Aa^{3}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
+ a)*A*a^2*x + 1/8*(b*x^2 + a)^(7/2)*B*x/b - 1/48*(b*x^2 + a)^(5/2)*B*a*x/b
 - 5/192*(b*x^2 + a)^(3/2)*B*a^2*x/b - 5/128*sqrt(b*x^2 + a)*B*a^3*x/b - 5/
```

```
128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 5/16*A*a^3*arcsinh(b*x/sqrt(a*b)
)/sqrt(b)
```

Fricas [A]
time = 1.59, size = 257, normalized size = 1.72

$$\left[-\frac{15\,(Ba^4-8\,Aa^3b)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(48\,Bb^4x^7+8\,(17\,Bab^3+8\,Ab^4)x^5+2\,(59\,Ba^2b^2+104\,Aab^3)x^3+3\,(5\,Ba^3b+88\,Aa^2b^2)x)\sqrt{bx^2+a}}{768\,b^2},\ \frac{15\,(Ba^4-8\,Aa^3b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(48\,Bb^4x^7+8\,(17\,Bab^3+8\,Ab^4)x^5+2\,(59\,Ba^2b^2+104\,Aab^3)x^3+3\,(5\,Ba^3b+88\,Aa^2b^2)x)\sqrt{bx^2+a}}{384\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a^2
*b^2 + 104*A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/
b^2, 1/384*(15*(B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a^2*b^2 + 104*
A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(134) = 268$.
time = 35.81, size = 316, normalized size = 2.12

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17A\sqrt{a}\,b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Ab^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Ba^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Ba^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23B\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Bb^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + b*x**2/a))
+ 35*A*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*A*sqrt(a)*b**2*x**5/(24
*sqrt(1 + b*x**2/a)) + 5*A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + A*b
**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(7/2)*x/(128*b*sqrt(1 + b*
x**2/a)) + 133*B*a**(5/2)*x**3/(384*sqrt(1 + b*x**2/a)) + 127*B*a**(3/2)*b*
x**5/(192*sqrt(1 + b*x**2/a)) + 23*B*sqrt(a)*b**2*x**7/(48*sqrt(1 + b*x**2/
a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + B*b**3*x**9/(8*sqr
t(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.68, size = 134, normalized size = 0.90

$$\frac{1}{384}\left(2\left(4\left(6\,Bb^2x^2+\frac{17\,Bab^7+8\,Ab^8}{b^6}\right)x^2+\frac{59\,Ba^2b^6+104\,Aab^7}{b^6}\right)x^2+\frac{3\,(5\,Ba^3b^5+88\,Aa^2b^6)}{b^6}\right)\sqrt{bx^2+a}\,x+\frac{5\,(Ba^4-8\,Aa^3b)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*b^2*x^2 + (17*B*a*b^7 + 8*A*b^8)/b^6)*x^2 + (59*B*a^2*b^6
+ 104*A*a*b^7)/b^6)*x^2 + 3*(5*B*a^3*b^5 + 88*A*a^2*b^6)/b^6)*sqrt(b*x^2 +
```

```
a)*x + 5/128*(B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
3/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] int((A + B*x^2)*(a + b*x^2)^(5/2), x)
```

3.544 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x} dx$

Optimal. Leaf size=95

$$a^2A\sqrt{a+bx^2}+\frac{1}{3}aA(a+bx^2)^{3/2}+\frac{1}{5}A(a+bx^2)^{5/2}+\frac{B(a+bx^2)^{7/2}}{7b}-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/3*a*A*(b*x^2+a)^(3/2)+1/5*A*(b*x^2+a)^(5/2)+1/7*B*(b*x^2+a)^(7/2)/b-a^(5/
2)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))+a^2*A*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 81, 52, 65, 214}

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+a^2A\sqrt{a+bx^2}+\frac{1}{5}A(a+bx^2)^{5/2}+\frac{1}{3}aA(a+bx^2)^{3/2}+\frac{B(a+bx^2)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

```
[Out] a^2*A*Sqrt[a + b*x^2] + (a*A*(a + b*x^2)^(3/2))/3 + (A*(a + b*x^2)^(5/2))/5
 + (B*(a + b*x^2)^(7/2))/(7*b) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x}\,dx, x, x^2\right)\\
&= \frac{B(a+bx^2)^{7/2}}{7b} + \frac{1}{2}A\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{5}A(a+bx^2)^{5/2} + \frac{B(a+bx^2)^{7/2}}{7b} + \frac{1}{2}(aA)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{3}aA(a+bx^2)^{3/2} + \frac{1}{5}A(a+bx^2)^{5/2} + \frac{B(a+bx^2)^{7/2}}{7b} + \frac{1}{2}(a^2A)\,\text{Subst}\Bigg(\int\\
&= a^2A\sqrt{a+bx^2} + \frac{1}{3}aA(a+bx^2)^{3/2} + \frac{1}{5}A(a+bx^2)^{5/2} + \frac{B(a+bx^2)^{7/2}}{7b} + \frac{1}{2}\\
&= a^2A\sqrt{a+bx^2} + \frac{1}{3}aA(a+bx^2)^{3/2} + \frac{1}{5}A(a+bx^2)^{5/2} + \frac{B(a+bx^2)^{7/2}}{7b} + \\
&= a^2A\sqrt{a+bx^2} + \frac{1}{3}aA(a+bx^2)^{3/2} + \frac{1}{5}A(a+bx^2)^{5/2} + \frac{B(a+bx^2)^{7/2}}{7b} - a^5
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 103, normalized size = 1.08

$$\frac{\sqrt{a+bx^2}\,(15a^3B+3b^3x^4(7A+5Bx^2)+ab^2x^2(77A+45Bx^2)+a^2b(161A+45Bx^2))}{105b} - a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(15*a^3*B + 3*b^3*x^4*(7*A + 5*B*x^2) + a*b^2*x^2*(77*A +
45*B*x^2) + a^2*b*(161*A + 45*B*x^2)))/(105*b) - a^(5/2)*A*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]]
```

Maple [A]
time = 0.08, size = 85, normalized size = 0.89

method	result	s
default	$\frac{B\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7b}+A\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)\right)$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*B*(b*x^2+a)^(7/2)/b+A*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b
*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
```

Maxima [A]
time = 0.37, size = 73, normalized size = 0.77

$$-Aa^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^{2}+a\right)^{\frac{5}{2}}A+\frac{1}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}Aa+\sqrt{bx^{2}+a}\,Aa^{2}+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}B}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] -A*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*A + 1/3*(b
*x^2 + a)^(3/2)*A*a + sqrt(b*x^2 + a)*A*a^2 + 1/7*(b*x^2 + a)^(7/2)*B/b
```

Fricas [A]
time = 1.28, size = 220, normalized size = 2.32

$$\left[\frac{105\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(15\,Bb^{3}x^{6}+3\,(15\,Bab^{2}+7\,Ab^{3})x^{4}+15\,Ba^{3}+161\,Aa^{2}b+(45\,Ba^{2}b+77\,Aab^{2})x^{2})\sqrt{bx^{2}+a}}{210\,b},\ \frac{105\,A\sqrt{-a}\,a^{2}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(15\,Bb^{3}x^{6}+3\,(15\,Bab^{2}+7\,Ab^{3})x^{4}+15\,Ba^{3}+161\,Aa^{2}b+(45\,Ba^{2}b+77\,Aab^{2})x^{2})\sqrt{bx^{2}+a}}{105\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] [1/210*(105*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(15*B*b^3*x^6 + 3*(15*B*a*b^2 + 7*A*b^3)*x^4 + 15*B*a^3 + 161*A*a^2*b
+ (45*B*a^2*b + 77*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/b, 1/105*(105*A*sqrt(-a)*
a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*B*b^3*x^6 + 3*(15*B*a*b^2 + 7*
```

```
A*b^3)*x^4 + 15*B*a^3 + 161*A*a^2*b + (45*B*a^2*b + 77*A*a*b^2)*x^2)*sqrt(b
*x^2 + a))/b]
```

Sympy [A]
time = 33.90, size = 88, normalized size = 0.93

$$\frac{Aa^3\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}+Aa^2\sqrt{a+bx^2}+\frac{Aa(a+bx^2)^{\frac{3}{2}}}{3}+\frac{A(a+bx^2)^{\frac{5}{2}}}{5}+\frac{B(a+bx^2)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x,x)
```

```
[Out] A*a**3*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + A*a**2*sqrt(a + b*x**2) +
 A*a*(a + b*x**2)**(3/2)/3 + A*(a + b*x**2)**(5/2)/5 + B*(a + b*x**2)**(7/2
)/(7*b)
```

Giac [A]
time = 0.94, size = 97, normalized size = 1.02

$$\frac{Aa^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{15\,(bx^2+a)^{\frac{7}{2}}Bb^6+21\,(bx^2+a)^{\frac{5}{2}}Ab^7+35\,(bx^2+a)^{\frac{3}{2}}Aab^7+105\,\sqrt{bx^2+a}\,Aa^2b^7}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] A*a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/105*(15*(b*x^2 + a)^(7/
2)*B*b^6 + 21*(b*x^2 + a)^(5/2)*A*b^7 + 35*(b*x^2 + a)^(3/2)*A*a*b^7 + 105*
sqrt(b*x^2 + a)*A*a^2*b^7)/b^7
```

Mupad [B]
time = 0.48, size = 78, normalized size = 0.82

$$\frac{A\,{\left(b\,x^2+a\right)}^{5/2}}{5}+A\,a^2\,\sqrt{b\,x^2+a}+\frac{B\,{\left(b\,x^2+a\right)}^{7/2}}{7\,b}+\frac{A\,a\,{\left(b\,x^2+a\right)}^{3/2}}{3}+A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x,x)
```

```
[Out] (A*(a + b*x^2)^(5/2))/5 + A*a^2*(a + b*x^2)^(1/2) + (B*(a + b*x^2)^(7/2))/(
7*b) + A*a^(5/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (A*a*(a + b*x^2)
^(3/2))/3
```

3.545 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=136

$$\frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2}+\frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2}+\frac{(6Ab+aB)x(a+bx^2)^{5/2}}{6a}-\frac{A(a+bx^2)^{7/2}}{ax}+\frac{5a^2(6A}{}$$

```
[Out] 5/24*(6*A*b+B*a)*x*(b*x^2+a)^(3/2)+1/6*(6*A*b+B*a)*x*(b*x^2+a)^(5/2)/a-A*(b
*x^2+a)^(7/2)/a/x+5/16*a^2*(6*A*b+B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b
^(1/2)+5/16*a*(6*A*b+B*a)*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 201, 223, 212}

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{x\left(a+bx^2\right)^{5/2}(aB+6Ab)}{6a}+\frac{5}{24}x\left(a+bx^2\right)^{3/2}(aB+6Ab)+\frac{5}{16}ax\sqrt{a+bx^2}\,(aB+6Ab)-\frac{A\left(a+bx^2\right)^{7/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (5*a*(6*A*b + a*B)*x*Sqrt[a + b*x^2])/16 + (5*(6*A*b + a*B)*x*(a + b*x^2)^(
3/2))/24 + ((6*A*b + a*B)*x*(a + b*x^2)^(5/2))/(6*a) - (A*(a + b*x^2)^(7/2)
)/(a*x) + (5*a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sq
rt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx = -\frac{A(a+bx^2)^{7/2}}{ax} - \frac{(-6Ab-aB)\int (a+bx^2)^{5/2}\,dx}{a}$$

$$= \frac{(6Ab+aB)x(a+bx^2)^{5/2}}{6a} - \frac{A(a+bx^2)^{7/2}}{ax} + \frac{1}{6}(5(6Ab+aB))\int (a+bx^2)$$

$$= \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x(a+bx^2)^{5/2}}{6a} - \frac{A(a+bx^2)^{7/2}}{ax}$$

$$= \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2} + \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x}{6}$$

$$= \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2} + \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x}{6}$$

$$= \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2} + \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x}{6}$$

Mathematica [A]
time = 0.23, size = 110, normalized size = 0.81

$$\frac{\sqrt{a+bx^2}\,(-48a^2A+54aAbx^2+33a^2Bx^2+12Ab^2x^4+26abBx^4+8b^2Bx^6)}{48x} - \frac{5a^2(6Ab+aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^2*A + 54*a*A*b*x^2 + 33*a^2*B*x^2 + 12*A*b^2*x^4 +
26*a*b*B*x^4 + 8*b^2*B*x^6))/(48*x) - (5*a^2*(6*A*b + a*B)*Log[-(Sqrt[b]*x)
 + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Maple [A]
time = 0.09, size = 164, normalized size = 1.21

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-8 b^{2} B\,x^{6}-12 A\,b^{2} x^{4}-26 B a b\,x^{4}-54 a A b\,x^{2}-33 B\,a^{2} x^{2}+48 a^{2} A\right)}{48 x}+\frac{15 a^{2} \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right) \sqrt{b}\, A}{8}+$
default	$B \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6}\right)+A \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{a x}+\frac{6 b \left(\ldots\right)}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a
)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(-1/a/x*(b*x^2+a)^
(7/2)+6*b/a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*
x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]

time = 0.37, size = 124, normalized size = 0.91

$$\frac{1}{6}\left(bx^{2}+a\right)^{\frac{5}{2}}Bx+\frac{5}{24}\left(bx^{2}+a\right)^{\frac{3}{2}}Bax+\frac{5}{16}\sqrt{bx^{2}+a}\,Ba^{2}x+\frac{5}{4}\left(bx^{2}+a\right)^{\frac{3}{2}}Abx+\frac{15}{8}\sqrt{bx^{2}+a}\,Aabx+\frac{5\,Ba^{3}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}+\frac{15}{8}Aa^{2}\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x + 5/24*(b*x^2 + a)^(3/2)*B*a*x + 5/16*sqrt(b*x^2
+ a)*B*a^2*x + 5/4*(b*x^2 + a)^(3/2)*A*b*x + 15/8*sqrt(b*x^2 + a)*A*a*b*x +
```

```
 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 15/8*A*a^2*sqrt(b)*arcsinh(b*x
/sqrt(a*b)) - (b*x^2 + a)^(5/2)*A/x
```

Fricas [A]
time = 1.87, size = 236, normalized size = 1.74

$$\left[\frac{15\,(Ba^3 + 6\,Aa^2b)\sqrt{b}\,x\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\,\sqrt{b}\,x - a\right) + 2\,(8\,Bb^3x^6 + 2\,(13\,Bab^2 + 6\,Ab^3)x^4 - 48\,Aa^2b + 3\,(11\,Ba^2b + 18\,Aab^2)x^2)\sqrt{bx^2 + a}}{96\,bx}, -\frac{15\,(Ba^3 + 6\,Aa^2b)\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2 + a}}\right) - (8\,Bb^3x^6 + 2\,(13\,Bab^2 + 6\,Ab^3)x^4 - 48\,Aa^2b + 3\,(11\,Ba^2b + 18\,Aab^2)x^2)\sqrt{bx^2 + a}}{48\,bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + 2*(8*B*b^3*x^6 + 2*(13*B*a*b^2 + 6*A*b^3)*x^4 - 48*A*a^2*b +
 3*(11*B*a^2*b + 18*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(b*x), -1/48*(15*(B*a^3
+ 6*A*a^2*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*B*b^3*x^6 +
 2*(13*B*a*b^2 + 6*A*b^3)*x^4 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*a*b^2)*x^
2)*sqrt(b*x^2 + a))/(b*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(126) = 252$.
time = 12.62, size = 306, normalized size = 2.25

$$-\frac{Aa^{\frac{5}{2}}}{x\sqrt{1 + \frac{bx^2}{a}}} + Aa^{\frac{3}{2}}bx\sqrt{1 + \frac{bx^2}{a}} - \frac{7Aa^{\frac{3}{2}}bx}{8\sqrt{1 + \frac{bx^2}{a}}} + \frac{3A\sqrt{a}\,b^2x^3}{8\sqrt{1 + \frac{bx^2}{a}}} + \frac{15Aa^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8} + \frac{Ab^3x^5}{4\sqrt{a}\,\sqrt{1 + \frac{bx^2}{a}}} + \frac{Ba^{\frac{5}{2}}x\sqrt{1 + \frac{bx^2}{a}}}{2} + \frac{3Ba^{\frac{5}{2}}x}{16\sqrt{1 + \frac{bx^2}{a}}} + \frac{35Ba^{\frac{3}{2}}bx^3}{48\sqrt{1 + \frac{bx^2}{a}}} + \frac{17B\sqrt{a}\,b^2x^5}{24\sqrt{1 + \frac{bx^2}{a}}} + \frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}} + \frac{Bb^3x^7}{6\sqrt{a}\,\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + A*a**(3/2)*b*x*sqrt(1 + b*x**2/a) - 7*
A*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*b**2*x**3/(8*sqrt(1 + b
*x**2/a)) + 15*A*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/8 + A*b**3*x**5/(4*s
qrt(a)*sqrt(1 + b*x**2/a)) + B*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*B*a**(5/
2)*x/(16*sqrt(1 + b*x**2/a)) + 35*B*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a))
 + 17*B*sqrt(a)*b**2*x**5/(24*sqrt(1 + b*x**2/a)) + 5*B*a**3*asinh(sqrt(b)*
x/sqrt(a))/(16*sqrt(b)) + B*b**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.89, size = 146, normalized size = 1.07

$$\frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2 - a} + \frac{1}{48}\left(2\left(4\,Bb^2x^2 + \frac{13\,Bab^5 + 6\,Ab^6}{b^4}\right)x^2 + \frac{3\,(11\,Ba^2b^4 + 18\,Aab^5)}{b^4}\right)\sqrt{bx^2 + a}\,x - \frac{5\left(Ba^3\sqrt{b} + 6\,Aa^2b^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 2*A*a^3*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/48*(2*(4*B*b^2*x^
2 + (13*B*a*b^5 + 6*A*b^6)/b^4)*x^2 + 3*(11*B*a^2*b^4 + 18*A*a*b^5)/b^4)*sq
```

```
rt(b*x^2 + a)*x - 5/32*(B*a^3*sqrt(b) + 6*A*a^2*b^(3/2))*log((sqrt(b)*x - s
qrt(b*x^2 + a))^2)/b
```

Mupad [B]
time = 0.87, size = 80, normalized size = 0.59

$$\frac{B\,x\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}-\frac{A\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^2,x)
```

```
[Out] (B*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a
+ 1)^(5/2) - (A*(a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))
/(x*((b*x^2)/a + 1)^(5/2))
```

3.546 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=135

$$\frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2}+\frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2}+\frac{(5Ab+2aB)\left(a+bx^2\right)^{5/2}}{10a}-\frac{A(a+bx^2)^{7/2}}{2ax^2}-\frac{1}{2}a^{3/2}(5A$$

```
[Out] 1/6*(5*A*b+2*B*a)*(b*x^2+a)^(3/2)+1/10*(5*A*b+2*B*a)*(b*x^2+a)^(5/2)/a-1/2*
A*(b*x^2+a)^(7/2)/a/x^2-1/2*a^(3/2)*(5*A*b+2*B*a)*arctanh((b*x^2+a)^(1/2)/a
^(1/2))+1/2*a*(5*A*b+2*B*a)*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 52, 65, 214}

$$-\frac{1}{2}a^{3/2}(2aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{(a+bx^2)^{5/2}(2aB+5Ab)}{10a}+\frac{1}{6}(a+bx^2)^{3/2}(2aB+5Ab)+\frac{1}{2}a\sqrt{a+bx^2}(2aB+5Ab)-\frac{A(a+bx^2)^{7/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (a*(5*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((5*A*b + 2*a*B)*(a + b*x^2)^(3/2))
/6 + ((5*A*b + 2*a*B)*(a + b*x^2)^(5/2))/(10*a) - (A*(a + b*x^2)^(7/2))/(2*
a*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{A(a+bx^2)^{7/2}}{2ax^2} + \frac{\left(\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)}{2a}$$
$$= \frac{(5Ab+2aB)(a+bx^2)^{5/2}}{10a} - \frac{A(a+bx^2)^{7/2}}{2ax^2} + \frac{1}{4}(5Ab+2aB)\text{Subst}\left(\int \frac{(a+}{}\right.$$
$$= \frac{1}{6}(5Ab+2aB)(a+bx^2)^{3/2} + \frac{(5Ab+2aB)(a+bx^2)^{5/2}}{10a} - \frac{A(a+bx^2)^{7/2}}{2ax^2} +$$
$$= \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2} + \frac{1}{6}(5Ab+2aB)(a+bx^2)^{3/2} + \frac{(5Ab+2aB)(a}{10a}$$
$$= \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2} + \frac{1}{6}(5Ab+2aB)(a+bx^2)^{3/2} + \frac{(5Ab+2aB)(a}{10a}$$
$$= \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2} + \frac{1}{6}(5Ab+2aB)(a+bx^2)^{3/2} + \frac{(5Ab+2aB)(a}{10a}$$

Mathematica [A]
time = 0.18, size = 105, normalized size = 0.78

$$\frac{\sqrt{a+bx^2}\,(-15a^2A+70aAbx^2+46a^2Bx^2+10Ab^2x^4+22abBx^4+6b^2Bx^6)}{30x^2} - \frac{1}{2}a^{3/2}(5Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^2*A + 70*a*A*b*x^2 + 46*a^2*B*x^2 + 10*A*b^2*x^4 +
22*a*b*B*x^4 + 6*b^2*B*x^6))/(30*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]])/2
```

Maple [A]
time = 0.10, size = 162, normalized size = 1.20

method	result
risch	$-\frac{a^{2} A \sqrt{b\,x^{2}+a}}{2 x^{2}}+\frac{b^{2} B\,x^{4} \sqrt{b\,x^{2}+a}}{5}+\frac{11 b B a\,x^{2} \sqrt{b\,x^{2}+a}}{15}+\frac{23 B\,a^{2} \sqrt{b\,x^{2}+a}}{15}+\frac{A\,b^{2} x^{2} \sqrt{b\,x^{2}+a}}{3}$ −
default	$A \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2 a\,x^{2}}+\frac{5 b \left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a \left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln \left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)\right)}{2 a}\right)+B$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)
^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
+B*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
```

Maxima [A]
time = 0.29, size = 138, normalized size = 1.02

$$-Ba^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right) - \frac{5}{2}\,Aa^{\frac{3}{2}}b \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right) + \frac{1}{5}\,(bx^2+a)^{\frac{5}{2}}B + \frac{1}{3}\,(bx^2+a)^{\frac{3}{2}}Ba + \sqrt{bx^2+a}\,Ba^2 + \frac{5}{6}\,(bx^2+a)^{\frac{3}{2}}Ab + \frac{(bx^2+a)^{\frac{5}{2}}Ab}{2\,a} + \frac{5}{2}\,\sqrt{bx^2+a}\,Aab - \frac{(bx^2+a)^{\frac{7}{2}}A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] -B*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) - 5/2*A*a^(3/2)*b*arcsinh(a/(sqrt(
a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*B + 1/3*(b*x^2 + a)^(3/2)*B*a + sqrt(
b*x^2 + a)*B*a^2 + 5/6*(b*x^2 + a)^(3/2)*A*b + 1/2*(b*x^2 + a)^(5/2)*A*b/a
+ 5/2*sqrt(b*x^2 + a)*A*a*b - 1/2*(b*x^2 + a)^(7/2)*A/(a*x^2)
```

Fricas [A]
time = 1.10, size = 221, normalized size = 1.64

$$\left[\frac{15\,(2\,Ba^2+5\,Aab)\sqrt{a}\,x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(6\,Bb^2x^6+2\,(11\,Bab+5\,Ab^2)x^4-15\,Aa^2+2\,(23\,Ba^2+35\,Aab)x^2)\sqrt{bx^2+a}}{60\,x^2},\ \frac{15\,(2\,Ba^2+5\,Aab)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(6\,Bb^2x^6+2\,(11\,Bab+5\,Ab^2)x^4-15\,Aa^2+2\,(23\,Ba^2+35\,Aab)x^2)\sqrt{bx^2+a}}{30\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(2*B*a^2 + 5*A*a*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(6*B*b^2*x^6 + 2*(11*B*a*b + 5*A*b^2)*x^4 - 15*A*a^2
 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^2, 1/30*(15*(2*B*a^2 + 5
*A*a*b)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (6*B*b^2*x^6 + 2*(1
1*B*a*b + 5*A*b^2)*x^4 - 15*A*a^2 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(b*x^2
 + a))/x^2]
```

Sympy [A]
time = 23.28, size = 296, normalized size = 2.19

$$-\frac{5Aa^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-Ba^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba^3}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba^2\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+2Bab\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Bb^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x**
2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2)*
x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a +
 b*x**2)**(3/2)/(3*b), True)) - B*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a
**3/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1
) + 2*B*a*b*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b
), True)) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*s
qrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/
4, True))
```

Giac [A]
time = 1.02, size = 139, normalized size = 1.03

$$\frac{6\,(bx^2+a)^{\frac{5}{2}}Bb+10\,(bx^2+a)^{\frac{3}{2}}Bab+30\,\sqrt{bx^2+a}\,Ba^2b+10\,(bx^2+a)^{\frac{3}{2}}Ab^2+60\,\sqrt{bx^2+a}\,Aab^2-\frac{15\,\sqrt{bx^2+a}\,Aa^2b}{x^2}+\frac{15\,(2\,Ba^3b+5\,Aa^2b^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*(b*x^2 + a)^(5/2)*B*b + 10*(b*x^2 + a)^(3/2)*B*a*b + 30*sqrt(b*x^2
+ a)*B*a^2*b + 10*(b*x^2 + a)^(3/2)*A*b^2 + 60*sqrt(b*x^2 + a)*A*a*b^2 - 15
*sqrt(b*x^2 + a)*A*a^2*b/x^2 + 15*(2*B*a^3*b + 5*A*a^2*b^2)*arctan(sqrt(b*x
^2 + a)/sqrt(-a))/sqrt(-a))/b
```

Mupad [B]
time = 0.85, size = 132, normalized size = 0.98

$$\frac{B\,{\left(b\,x^2+a\right)}^{5/2}}{5}+B\,a^2\,\sqrt{b\,x^2+a}+B\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}+\frac{A\,b\,{\left(b\,x^2+a\right)}^{3/2}}{3}+\frac{B\,a\,{\left(b\,x^2+a\right)}^{3/2}}{3}+2\,A\,a\,b\,\sqrt{b\,x^2+a}-\frac{A\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2}+\frac{A\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^3,x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/5 + B*a^2*(a + b*x^2)^(1/2) + B*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (A*b*(a + b*x^2)^(3/2))/3 + (B*a*(a + b*x^2)^(
3/2))/3 + 2*A*a*b*(a + b*x^2)^(1/2) - (A*a^2*(a + b*x^2)^(1/2))/(2*x^2) + (
A*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2
```

3.547 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=146

$$\frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2}+\frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a}-\frac{(4Ab+3aB)\,(a+bx^2)^{5/2}}{3ax}-\frac{A(a+bx^2)^{7/2}}{3ax^3}+\frac{5}{8}a\sqrt{b}$$

```
[Out] 5/12*b*(4*A*b+3*B*a)*x*(b*x^2+a)^(3/2)/a-1/3*(4*A*b+3*B*a)*(b*x^2+a)^(5/2)/
a/x-1/3*A*(b*x^2+a)^(7/2)/a/x^3+5/8*a*(4*A*b+3*B*a)*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))*b^(1/2)+5/8*b*(4*A*b+3*B*a)*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 283, 201, 223, 212}

$$-\frac{(a+bx^2)^{5/2}(3aB+4Ab)}{3ax}+\frac{5bx(a+bx^2)^{3/2}(3aB+4Ab)}{12a}+\frac{5}{8}bx\sqrt{a+bx^2}\,(3aB+4Ab)+\frac{5}{8}a\sqrt{b}\,(3aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{A(a+bx^2)^{7/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (5*b*(4*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/8 + (5*b*(4*A*b + 3*a*B)*x*(a + b*x
^2)^(3/2))/(12*a) - ((4*A*b + 3*a*B)*(a + b*x^2)^(5/2))/(3*a*x) - (A*(a + b
*x^2)^(7/2))/(3*a*x^3) + (5*a*Sqrt[b]*(4*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/8
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx &= -\frac{A(a+bx^2)^{7/2}}{3ax^3}-\frac{(-4Ab-3aB)\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx}{3a}\\
&= -\frac{(4Ab+3aB)(a+bx^2)^{5/2}}{3ax}-\frac{A(a+bx^2)^{7/2}}{3ax^3}+\frac{(5b(4Ab+3aB))\int (a+bx\\
&= \frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a}-\frac{(4Ab+3aB)(a+bx^2)^{5/2}}{3ax}-\frac{A(a+bx^2)^{7/2}}{3ax^3}\\
&= \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2}+\frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a}-\frac{(4Ab+3aB}{3a}\\
&= \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2}+\frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a}-\frac{(4Ab+3aB}{3a}\\
&= \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2}+\frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a}-\frac{(4Ab+3aB}{3a}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 109, normalized size = 0.75

$$\frac{\sqrt{a+bx^2}\left(-8a^2A-56aAbx^2-24a^2Bx^2+12Ab^2x^4+27abBx^4+6b^2Bx^6\right)}{24x^3}-\frac{5}{8}a\sqrt{b}\,(4Ab+3aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

[Out] (Sqrt[a + b*x^2]*(-8*a^2*A - 56*a*A*b*x^2 - 24*a^2*B*x^2 + 12*A*b^2*x^4 + 2
7*a*b*B*x^4 + 6*b^2*B*x^6))/(24*x^3) - (5*a*Sqrt[b]*(4*A*b + 3*a*B)*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]])/8

Maple [A]
time = 0.09, size = 212, normalized size = 1.45

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-6 b^{2} B\,x^{6}-12 A\,b^{2} x^{4}-27 B a b\,x^{4}+56 a A b\,x^{2}+24 B\,a^{2} x^{2}+8 a^{2} A\right)}{24 x^{3}}+\frac{5 A\,b^{\frac{3}{2}} \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right) a}{2}+\frac{15 B \sqrt{}}{}$

default	$A\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{3a\,x^3}+\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{a}\right)}{3a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/3/a/x^3*(b*x^2+a)^(7/2)+4/3*b/a*(-1/a/x*(b*x^2+a)^(7/2)+6*b/a*(1/6*x*
(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1
/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+B*(-1/a/x*(b*x^2+a)^(7/2)+6
*b/a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^
2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.33, size = 151, normalized size = 1.03

$$\frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bbx+\frac{15}{8}\sqrt{bx^2+a}\,Babx+\frac{5}{2}\sqrt{bx^2+a}\,Ab^2x+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ab^2x}{3\,a}+\frac{15}{8}Ba^2\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)+\frac{5}{2}Aab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{x}-\frac{4\left(bx^2+a\right)^{\frac{5}{2}}Ab}{3\,ax}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*B*b*x + 15/8*sqrt(b*x^2 + a)*B*a*b*x + 5/2*sqrt(b*x^2
 + a)*A*b^2*x + 5/3*(b*x^2 + a)^(3/2)*A*b^2*x/a + 15/8*B*a^2*sqrt(b)*arcsin
h(b*x/sqrt(a*b)) + 5/2*A*a*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(5/
2)*B/x - 4/3*(b*x^2 + a)^(5/2)*A*b/(a*x) - 1/3*(b*x^2 + a)^(7/2)*A/(a*x^3)
```

Fricas [A]
time = 1.71, size = 220, normalized size = 1.51

$$\left[\frac{15\left(3\,Ba^2+4\,Aab\right)\sqrt{b}\,x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(6\,Bb^2x^6+3\left(9\,Bab+4\,Ab^2\right)x^4-8\,Aa^2-8\left(3\,Ba^2+7\,Aab\right)x^2\right)\sqrt{bx^2+a}}{48\,x^3},\,-\frac{15\left(3\,Ba^2+4\,Aab\right)\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(6\,Bb^2x^6+3\left(9\,Bab+4\,Ab^2\right)x^4-8\,Aa^2-8\left(3\,Ba^2+7\,Aab\right)x^2\right)\sqrt{bx^2+a}}{24\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(3*B*a^2 + 4*A*a*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) + 2*(6*B*b^2*x^6 + 3*(9*B*a*b + 4*A*b^2)*x^4 - 8*A*a^2 - 8*(
3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^3, -1/24*(15*(3*B*a^2 + 4*A*a*b)
*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^6 + 3*(9*B*a*
b + 4*A*b^2)*x^4 - 8*A*a^2 - 8*(3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^
3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(136) = 272$.
time = 6.56, size = 299, normalized size = 2.05

$$-\frac{2Aa^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{A\sqrt{a}\,b^2x\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{2A\sqrt{a}\,b^2x}{\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}+\frac{5Aab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}-\frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Ba^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Ba^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\,b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Ba^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{Bb^3x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -2*A*a**(3/2)*b/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b**2*x*sqrt(1 + b*x**2/a
)/2 - 2*A*sqrt(a)*b**2*x/sqrt(1 + b*x**2/a) - A*a**2*sqrt(b)*sqrt(a/(b*x**2
```

```
) + 1)/(3*x**2) - A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + 5*A*a*b**(3/2)*asin
h(sqrt(b)*x/sqrt(a))/2 - B*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + B*a**(3/2)*b*x
*sqrt(1 + b*x**2/a) - 7*B*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)
*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*B*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(
a))/8 + B*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.52, size = 238, normalized size = 1.63

$$\frac{1}{8}\left(2\,Bb^2x^2+\frac{9\,Bab^3+4\,Ab^4}{b^2}\right)\sqrt{bx^2+a}\,x-\frac{5}{16}\left(3\,Ba^2\sqrt{b}+4\,Aab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ba^3\sqrt{b}+9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Aa^2b^{\frac{3}{2}}-6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^4\sqrt{b}-12\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Aa^3b^{\frac{3}{2}}+3\,Ba^5\sqrt{b}+7\,Aa^4b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/8*(2*B*b^2*x^2 + (9*B*a*b^3 + 4*A*b^4)/b^2)*sqrt(b*x^2 + a)*x - 5/16*(3*B
*a^2*sqrt(b) + 4*A*a*b^(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*(3
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*sqrt(b) + 9*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*A*a^2*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*sqrt(b) - 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(3/2) + 3*B*a^5*sqrt(b) + 7*A*a^4
*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^4, x)
```

3.548 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5} \, dx$

Optimal. Leaf size=143

$$\frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2}+\frac{5b(3Ab+4aB)\left(a+bx^2\right)^{3/2}}{24a}-\frac{(3Ab+4aB)\left(a+bx^2\right)^{5/2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{7/2}}{4ax^4}-\frac{5}{8}\sqrt{a}\,b(3$$

```
[Out] 5/24*b*(3*A*b+4*B*a)*(b*x^2+a)^(3/2)/a-1/8*(3*A*b+4*B*a)*(b*x^2+a)^(5/2)/a/
x^2-1/4*A*(b*x^2+a)^(7/2)/a/x^4-5/8*b*(3*A*b+4*B*a)*arctanh((b*x^2+a)^(1/2)
/a^(1/2))*a^(1/2)+5/8*b*(3*A*b+4*B*a)*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 43, 52, 65, 214}

$$-\frac{\left(a+bx^2\right)^{5/2}(4aB+3Ab)}{8ax^2}+\frac{5b\left(a+bx^2\right)^{3/2}(4aB+3Ab)}{24a}+\frac{5}{8}b\sqrt{a+bx^2}\,(4aB+3Ab)-\frac{5}{8}\sqrt{a}\,b(4aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A\left(a+bx^2\right)^{7/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (5*b*(3*A*b + 4*a*B)*Sqrt[a + b*x^2])/8 + (5*b*(3*A*b + 4*a*B)*(a + b*x^2)^
(3/2))/(24*a) - ((3*A*b + 4*a*B)*(a + b*x^2)^(5/2))/(8*a*x^2) - (A*(a + b*x
^2)^(7/2))/(4*a*x^4) - (5*Sqrt[a]*b*(3*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{A(a+bx^2)^{7/2}}{4ax^4} + \frac{\left(\frac{3Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{(3Ab+4aB)(a+bx^2)^{5/2}}{8ax^2} - \frac{A(a+bx^2)^{7/2}}{4ax^4} + \frac{(5b(3Ab+4aB))\text{Subst}\left(\int \frac{(a\cdot}{}\right.}{16a}$$
$$= \frac{5b(3Ab+4aB)(a+bx^2)^{3/2}}{24a} - \frac{(3Ab+4aB)(a+bx^2)^{5/2}}{8ax^2} - \frac{A(a+bx^2)^{7/2}}{4ax^4} +$$
$$= \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2} + \frac{5b(3Ab+4aB)(a+bx^2)^{3/2}}{24a} - \frac{(3Ab+4aB)(a}{8ax^2}$$
$$= \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2} + \frac{5b(3Ab+4aB)(a+bx^2)^{3/2}}{24a} - \frac{(3Ab+4aB)(a}{8ax^2}$$
$$= \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2} + \frac{5b(3Ab+4aB)(a+bx^2)^{3/2}}{24a} - \frac{(3Ab+4aB)(a}{8ax^2}$$

Mathematica [A]
time = 0.20, size = 106, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-6a^2A-27aAbx^2-12a^2Bx^2+24Ab^2x^4+56abBx^4+8b^2Bx^6)}{24x^4} - \frac{5}{8}\sqrt{a}\,b(3Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
[Out] (Sqrt[a + b*x^2]*(-6*a^2*A - 27*a*A*b*x^2 - 12*a^2*B*x^2 + 24*A*b^2*x^4 + 5
6*a*b*B*x^4 + 8*b^2*B*x^6))/(24*x^4) - (5*Sqrt[a]*b*(3*A*b + 4*a*B)*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Maple [A]
time = 0.10, size = 210, normalized size = 1.47

method	result
risch	$-\frac{a\sqrt{b\,x^2+a}\,(9Ab\,x^2+4Ba\,x^2+2Aa)}{8x^4} + \frac{b^2B\,x^2\sqrt{b\,x^2+a}}{3} + \frac{7bBa\sqrt{b\,x^2+a}}{3} + b^2A\sqrt{b\,x^2+a} - \frac{15A\sqrt{a}}{}$

default	$A\left(-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{4a\,x^4}+\frac{3b\left(-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right.\right.\right.}{2a}\right.}{4a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(
1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2
*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))+B*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a
*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

Maxima [A]
time = 0.29, size = 190, normalized size = 1.33

$$-\frac{5}{2}\,Ba^{\frac{3}{2}}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{15}{8}\,A\sqrt{a}\,b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{5}{6}\,(bx^2+a)^{\frac{3}{2}}Bb+\frac{(bx^2+a)^{\frac{5}{2}}Bb}{2\,a}+\frac{5}{2}\,\sqrt{bx^2+a}\,Bab+\frac{15}{8}\,\sqrt{bx^2+a}\,Ab^2+\frac{3\,(bx^2+a)^{\frac{5}{2}}Ab^2}{8\,a^2}+\frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^2}{8\,a}-\frac{(bx^2+a)^{\frac{7}{2}}B}{2\,ax^2}-\frac{3\,(bx^2+a)^{\frac{7}{2}}Ab}{8\,a^2x^2}-\frac{(bx^2+a)^{\frac{7}{2}}A}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] -5/2*B*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) - 15/8*A*sqrt(a)*b^2*arcsinh
(a/(sqrt(a*b)*abs(x))) + 5/6*(b*x^2 + a)^(3/2)*B*b + 1/2*(b*x^2 + a)^(5/2)*
B*b/a + 5/2*sqrt(b*x^2 + a)*B*a*b + 15/8*sqrt(b*x^2 + a)*A*b^2 + 3/8*(b*x^2
 + a)^(5/2)*A*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*A*b^2/a - 1/2*(b*x^2 + a)^(7/
2)*B/(a*x^2) - 3/8*(b*x^2 + a)^(7/2)*A*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(7/2)*
A/(a*x^4)
```

Fricas [A]
time = 1.90, size = 221, normalized size = 1.55

$$\left[\frac{15\,(4\,Bab+3\,Ab^2)\sqrt{a}\,x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,Bb^2x^6+8\,(7\,Bab+3\,Ab^2)x^4-6\,Aa^2-3\,(4\,Ba^2+9\,Aab)x^2)\sqrt{bx^2+a}}{48\,x^4},\ \frac{15\,(4\,Bab+3\,Ab^2)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(8\,Bb^2x^6+8\,(7\,Bab+3\,Ab^2)x^4-6\,Aa^2-3\,(4\,Ba^2+9\,Aab)x^2)\sqrt{bx^2+a}}{24\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(4*B*a*b + 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(8*B*b^2*x^6 + 8*(7*B*a*b + 3*A*b^2)*x^4 - 6*A*a^2 -
 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^4, 1/24*(15*(4*B*a*b + 3*A*b
```

```
^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*B*b^2*x^6 + 8*(7*B*a
*b + 3*A*b^2)*x^4 - 6*A*a^2 - 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 + a))/x
^4]
```

Sympy [A]
time = 54.51, size = 279, normalized size = 1.95

$$-\frac{15A\sqrt{a}\,b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8}-\frac{Aa^3}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3Aa^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{x}+\frac{7Aab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ba^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{2Ba^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{2Bab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}+Bb^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -15*A*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - A*a**3/(4*sqrt(b)*x**5*sq
rt(a/(b*x**2) + 1)) - 3*A*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - A*a*
b**(3/2)*sqrt(a/(b*x**2) + 1)/x + 7*A*a*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1))
 + A*b**(5/2)*x/sqrt(a/(b*x**2) + 1) - 5*B*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b
)*x))/2 - B*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) + 2*B*a**2*sqrt(b)/(x*s
qrt(a/(b*x**2) + 1)) + 2*B*a*b**(3/2)*x/sqrt(a/(b*x**2) + 1) + B*b**2*Piece
wise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

Giac [A]
time = 1.29, size = 171, normalized size = 1.20

$$\frac{8\,(bx^2+a)^{\frac{3}{2}}Bb^2+48\,\sqrt{bx^2+a}\,Bab^2+24\,\sqrt{bx^2+a}\,Ab^3+\frac{15\,(4\,Ba^2b^2+3\,Aab^3)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\left(4\,(bx^2+a)^{\frac{3}{2}}Ba^2b^2-4\,\sqrt{bx^2+a}\,Ba^3b^2+9\,(bx^2+a)^{\frac{3}{2}}Aab^3-7\,\sqrt{bx^2+a}\,Aa^2b^3\right)}{b^2x^4}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(8*(b*x^2 + a)^(3/2)*B*b^2 + 48*sqrt(b*x^2 + a)*B*a*b^2 + 24*sqrt(b*x^
2 + a)*A*b^3 + 15*(4*B*a^2*b^2 + 3*A*a*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a)
)/sqrt(-a) - 3*(4*(b*x^2 + a)^(3/2)*B*a^2*b^2 - 4*sqrt(b*x^2 + a)*B*a^3*b^2
 + 9*(b*x^2 + a)^(3/2)*A*a*b^3 - 7*sqrt(b*x^2 + a)*A*a^2*b^3)/(b^2*x^4))/b
```

Mupad [B]
time = 1.16, size = 144, normalized size = 1.01

$$A\,b^2\,\sqrt{b\,x^2+a}+\frac{B\,b\,{\left(b\,x^2+a\right)}^{3/2}}{3}+2\,B\,a\,b\,\sqrt{b\,x^2+a}+\frac{A\,\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,15{}\mathrm{i}}{8}-\frac{9\,A\,a\,{\left(b\,x^2+a\right)}^{3/2}}{8\,x^4}+\frac{7\,A\,a^2\,\sqrt{b\,x^2+a}}{8\,x^4}-\frac{B\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2}+\frac{B\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^5,x)
```

```
[Out] A*b^2*(a + b*x^2)^(1/2) + (B*b*(a + b*x^2)^(3/2))/3 + 2*B*a*b*(a + b*x^2)^(
1/2) + (A*a^(1/2)*b^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*15i)/8 - (9*A*a*
(a + b*x^2)^(3/2))/(8*x^4) + (7*A*a^2*(a + b*x^2)^(1/2))/(8*x^4) - (B*a^2*(
a + b*x^2)^(1/2))/(2*x^2) + (B*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2
))*5i)/2
```

3.549 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=152

$$\frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a}-\frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{(2Ab+5aB)\left(a+bx^2\right)^{5/2}}{15ax^3}-\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}+\frac{1}{2}b^{3/2}($$

```
[Out] -1/3*b*(2*A*b+5*B*a)*(b*x^2+a)^(3/2)/a/x-1/15*(2*A*b+5*B*a)*(b*x^2+a)^(5/2)
/a/x^3-1/5*A*(b*x^2+a)^(7/2)/a/x^5+1/2*b^(3/2)*(2*A*b+5*B*a)*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))+1/2*b^2*(2*A*b+5*B*a)*x*(b*x^2+a)^(1/2)/a
```

Rubi [A]
time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 283, 201, 223, 212}

$$\frac{1}{2}b^{3/2}(5aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{b^2x\sqrt{a+bx^2}\,(5aB+2Ab)}{2a}-\frac{b\left(a+bx^2\right)^{3/2}(5aB+2Ab)}{3ax}-\frac{\left(a+bx^2\right)^{5/2}(5aB+2Ab)}{15ax^3}-\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

```
[Out] (b^2*(2*A*b + 5*a*B)*x*Sqrt[a + b*x^2])/(2*a) - (b*(2*A*b + 5*a*B)*(a + b*x
^2)^(3/2))/(3*a*x) - ((2*A*b + 5*a*B)*(a + b*x^2)^(5/2))/(15*a*x^3) - (A*(a
 + b*x^2)^(7/2))/(5*a*x^5) + (b^(3/2)*(2*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx = -\frac{A(a+bx^2)^{7/2}}{5ax^5} - \frac{(-2Ab-5aB)\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx}{5a}$$

$$= -\frac{(2Ab+5aB)(a+bx^2)^{5/2}}{15ax^3} - \frac{A(a+bx^2)^{7/2}}{5ax^5} + \frac{(b(2Ab+5aB))\int \frac{(a+bx^2)^{3/2}}{x^2}\,d}{3a}$$

$$= -\frac{b(2Ab+5aB)(a+bx^2)^{3/2}}{3ax} - \frac{(2Ab+5aB)(a+bx^2)^{5/2}}{15ax^3} - \frac{A(a+bx^2)^{7/2}}{5ax^5} +$$

$$= \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a} - \frac{b(2Ab+5aB)(a+bx^2)^{3/2}}{3ax} - \frac{(2Ab+5aB)(a}{15ax^3}$$

$$= \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a} - \frac{b(2Ab+5aB)(a+bx^2)^{3/2}}{3ax} - \frac{(2Ab+5aB)(a}{15ax^3}$$

$$= \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a} - \frac{b(2Ab+5aB)(a+bx^2)^{3/2}}{3ax} - \frac{(2Ab+5aB)(a}{15ax^3}$$

Mathematica [A]
time = 0.24, size = 108, normalized size = 0.71

$$\frac{\sqrt{a+bx^2}\,(-6a^2A-22aAbx^2-10a^2Bx^2-46Ab^2x^4-70abBx^4+15b^2Bx^6)}{30x^5} - \frac{1}{2}b^{3/2}(2Ab+5aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

[Out] (Sqrt[a + b*x^2]*(-6*a^2*A - 22*a*A*b*x^2 - 10*a^2*B*x^2 - 46*A*b^2*x^4 - 7
0*a*b*B*x^4 + 15*b^2*B*x^6))/(30*x^5) - (b^(3/2)*(2*A*b + 5*a*B)*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/2

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(128) = 256$.
time = 0.09, size = 260, normalized size = 1.71

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-15b^2 B\,x^6+46A\,b^2x^4+70Bab\,x^4+22aAb\,x^2+10B\,a^2x^2+6a^2A\right)}{30x^5}+A\,b^{\frac{5}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)+$

default | $A\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{5a\,x^5}+\frac{2b\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{3a\,x^3}+\frac{4b\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{\vphantom{x}}\right.}{2\sqrt{\vphantom{x}}}\right.}{4}\right.}{6}\right.}{a}\right.}{3a}\right.}{5a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5/a/x^5*(b*x^2+a)^(7/2)+2/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(7/2)+4/3*b/a*(
-1/a/x*(b*x^2+a)^(7/2)+6*b/a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^
(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/
2))))))))+B*(-1/3/a/x^3*(b*x^2+a)^(7/2)+4/3*b/a*(-1/a/x*(b*x^2+a)^(7/2)+6*b
/a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.30, size = 198, normalized size = 1.30

$$\frac{5}{2}\sqrt{bx^2+a}\,Bb^2x+\frac{5\,(bx^2+a)^{\frac{3}{2}}Bb^2x}{3\,a}+\frac{2\,(bx^2+a)^{\frac{3}{2}}Ab^3x}{3\,a^2}+\frac{\sqrt{bx^2+a}\,Ab^3x}{a}+\frac{5}{2}\,Bab^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)+Ab^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{4\,(bx^2+a)^{\frac{5}{2}}Bb}{3\,ax}-\frac{8\,(bx^2+a)^{\frac{5}{2}}Ab^2}{15\,a^2x}-\frac{(bx^2+a)^{\frac{7}{2}}B}{3\,ax^3}-\frac{2\,(bx^2+a)^{\frac{7}{2}}Ab}{15\,a^2x^3}-\frac{(bx^2+a)^{\frac{7}{2}}A}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(b*x^2 + a)*B*b^2*x + 5/3*(b*x^2 + a)^(3/2)*B*b^2*x/a + 2/3*(b*x^2
+ a)^(3/2)*A*b^3*x/a^2 + sqrt(b*x^2 + a)*A*b^3*x/a + 5/2*B*a*b^(3/2)*arcsin
h(b*x/sqrt(a*b)) + A*b^(5/2)*arcsinh(b*x/sqrt(a*b)) - 4/3*(b*x^2 + a)^(5/2)
*B*b/(a*x) - 8/15*(b*x^2 + a)^(5/2)*A*b^2/(a^2*x) - 1/3*(b*x^2 + a)^(7/2)*B
/(a*x^3) - 2/15*(b*x^2 + a)^(7/2)*A*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2)*A/(
a*x^5)
```

Fricas [A]
time = 1.13, size = 220, normalized size = 1.45

$$\left[\frac{15\,(5\,Bab+2\,Ab^2)\sqrt{b}\,x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(15\,Bb^2x^6-2\,(35\,Bab+23\,Ab^2)x^4-6\,Aa^2-2\,(5\,Ba^2+11\,Aab)x^2)\sqrt{bx^2+a}}{60\,x^5},\,-\frac{15\,(5\,Bab+2\,Ab^2)\sqrt{-b}\,x^5\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(15\,Bb^2x^6-2\,(35\,Bab+23\,Ab^2)x^4-6\,Aa^2-2\,(5\,Ba^2+11\,Aab)x^2)\sqrt{bx^2+a}}{30\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(5*B*a*b + 2*A*b^2)*sqrt(b)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) + 2*(15*B*b^2*x^6 - 2*(35*B*a*b + 23*A*b^2)*x^4 - 6*A*a^2 -
2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^5, -1/30*(15*(5*B*a*b + 2*A*
b^2)*sqrt(-b)*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (15*B*b^2*x^6 - 2*(3
5*B*a*b + 23*A*b^2)*x^4 - 6*A*a^2 - 2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b*x^2
+ a))/x^5]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(136) = 272$.
time = 4.36, size = 292, normalized size = 1.92

$$-\frac{A\sqrt{a}\,b^2}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{11Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{8Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15}+Ab^{\frac{5}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Ab^3x}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}-\frac{2Ba^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}\,b^2x\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{2B\sqrt{a}\,b^2x}{\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}+\frac{5Bab^{\frac{3}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*sqrt(a)*b**2/(x*sqrt(1 + b*x**2/a)) - A*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1
)/(5*x**4) - 11*A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - 8*A*b**(5/2)*
sqrt(a/(b*x**2) + 1)/15 + A*b**(5/2)*asinh(sqrt(b)*x/sqrt(a)) - A*b**3*x/(s
qrt(a)*sqrt(1 + b*x**2/a)) - 2*B*a**(3/2)*b/(x*sqrt(1 + b*x**2/a)) + B*sqrt
(a)*b**2*x*sqrt(1 + b*x**2/a)/2 - 2*B*sqrt(a)*b**2*x/sqrt(1 + b*x**2/a) - B
*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/3 + 5*B*a*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(128) = 256.
time = 1.46, size = 321, normalized size = 2.11

$$\frac{1}{2}\sqrt{bx^2+a}\,Bb^2x-\frac{1}{4}\left(5\,Bab^{\frac{3}{2}}+2\,Ab^{\frac{5}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ba^2b^{\frac{3}{2}}+45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Aab^{\frac{5}{2}}-150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^3b^{\frac{3}{2}}-90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Aa^2b^{\frac{5}{2}}+200\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^4b^{\frac{3}{2}}+140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aa^3b^{\frac{5}{2}}-130\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^5b^{\frac{3}{2}}-70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^4b^{\frac{5}{2}}+35\,Ba^6b^{\frac{3}{2}}+23\,Aa^5b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*b^2*x - 1/4*(5*B*a*b^(3/2) + 2*A*b^(5/2))*log((sqrt(b
)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*
b^(3/2) + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^(5/2) - 150*(sqrt(b)*x -
 sqrt(b*x^2 + a))^6*B*a^3*b^(3/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^
2*b^(5/2) + 200*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^4*b^(3/2) + 140*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*A*a^3*b^(5/2) - 130*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*B*a^5*b^(3/2) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(5/2) + 35*B*a
^6*b^(3/2) + 23*A*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{5/2}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^6,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^6, x)
```

3.550 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7} dx$

Optimal. Leaf size=149

$$\frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a} - \frac{5b(Ab+6aB)\left(a+bx^2\right)^{3/2}}{48ax^2} - \frac{(Ab+6aB)\left(a+bx^2\right)^{5/2}}{24ax^4} - \frac{A(a+bx^2)^{7/2}}{6ax^6} - \frac{5b^2(Ab-$$

```
[Out] -5/48*b*(A*b+6*B*a)*(b*x^2+a)^(3/2)/a/x^2-1/24*(A*b+6*B*a)*(b*x^2+a)^(5/2)/
a/x^4-1/6*A*(b*x^2+a)^(7/2)/a/x^6-5/16*b^2*(A*b+6*B*a)*arctanh((b*x^2+a)^(1
/2)/a^(1/2))/a^(1/2)+5/16*b^2*(A*b+6*B*a)*(b*x^2+a)^(1/2)/a
```

Rubi [A]
time = 0.07, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 43, 52, 65, 214}

$$\frac{5b^2\sqrt{a+bx^2}\,(6aB+Ab)}{16a} - \frac{5b^2(6aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{5b(a+bx^2)^{3/2}(6aB+Ab)}{48ax^2} - \frac{(a+bx^2)^{5/2}(6aB+Ab)}{24ax^4} - \frac{A(a+bx^2)^{7/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (5*b^2*(A*b + 6*a*B)*Sqrt[a + b*x^2])/(16*a) - (5*b*(A*b + 6*a*B)*(a + b*x^
2)^(3/2))/(48*a*x^2) - ((A*b + 6*a*B)*(a + b*x^2)^(5/2))/(24*a*x^4) - (A*(a
 + b*x^2)^(7/2))/(6*a*x^6) - (5*b^2*(A*b + 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(16*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{A(a+bx^2)^{7/2}}{6ax^6} + \frac{(Ab+6aB)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, x^2\right)}{12a}$$
$$= -\frac{(Ab+6aB)(a+bx^2)^{5/2}}{24ax^4} - \frac{A(a+bx^2)^{7/2}}{6ax^6} + \frac{(5b(Ab+6aB))\text{Subst}\left(\int \frac{(a+}{}\right.}{48a}$$
$$= -\frac{5b(Ab+6aB)(a+bx^2)^{3/2}}{48ax^2} - \frac{(Ab+6aB)(a+bx^2)^{5/2}}{24ax^4} - \frac{A(a+bx^2)^{7/2}}{6ax^6} +$$
$$= \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a} - \frac{5b(Ab+6aB)(a+bx^2)^{3/2}}{48ax^2} - \frac{(Ab+6aB)(a+}{24ax^4}$$
$$= \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a} - \frac{5b(Ab+6aB)(a+bx^2)^{3/2}}{48ax^2} - \frac{(Ab+6aB)(a+}{24ax^4}$$
$$= \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a} - \frac{5b(Ab+6aB)(a+bx^2)^{3/2}}{48ax^2} - \frac{(Ab+6aB)(a+}{24ax^4}$$

Mathematica [A]
time = 0.21, size = 107, normalized size = 0.72

$$\frac{\sqrt{a+bx^2}\,(-8a^2A-26aAbx^2-12a^2Bx^2-33Ab^2x^4-54abBx^4+48b^2Bx^6)}{48x^6} - \frac{5b^2(Ab+6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A - 26*a*A*b*x^2 - 12*a^2*B*x^2 - 33*A*b^2*x^4 - 5
4*a*b*B*x^4 + 48*b^2*B*x^6))/(48*x^6) - (5*b^2*(A*b + 6*a*B)*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]])/(16*Sqrt[a])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(125) = 250$.
time = 0.10, size = 258, normalized size = 1.73

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(33A\,b^2x^4+54Bab\,x^4+26aAb\,x^2+12B\,a^2x^2+8a^2A)}{48x^6} + b^2B\sqrt{b\,x^2+a} - \frac{5b^3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right.}{16\sqrt{a}}$

default	$B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{4a\,x^{4}}+\frac{3b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2a\,x^{2}}+\frac{5b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)\right.}{2a}}{4a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(
1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2
*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))+A*(-1/6/a/x^6*(b*x^2+a)^(7/2)+1/6*b/a
*(-1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1
/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*
a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))))
```

Maxima [A]

time = 0.35, size = 243, normalized size = 1.63

$$-\frac{15}{8}\,B\sqrt{a}\,b^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{5\,Ab^{3}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,\sqrt{a}}+\frac{15}{8}\,\sqrt{bx^{2}+a}\,Bb^{2}+\frac{3\,(bx^{2}+a)^{\frac{5}{2}}Bb^{2}}{8\,a^{2}}+\frac{5\,(bx^{2}+a)^{\frac{3}{2}}Bb^{2}}{8\,a}+\frac{(bx^{2}+a)^{\frac{5}{2}}Ab^{3}}{16\,a^{3}}+\frac{5\,(bx^{2}+a)^{\frac{3}{2}}Ab^{3}}{48\,a^{2}}+\frac{5\,\sqrt{bx^{2}+a}\,Ab^{3}}{16\,a}-\frac{3\,(bx^{2}+a)^{\frac{7}{2}}Bb}{8\,a^{2}x^{2}}-\frac{(bx^{2}+a)^{\frac{7}{2}}Ab^{2}}{16\,a^{3}x^{2}}-\frac{(bx^{2}+a)^{\frac{7}{2}}B}{4\,ax^{4}}-\frac{(bx^{2}+a)^{\frac{7}{2}}Ab}{24\,a^{2}x^{4}}-\frac{(bx^{2}+a)^{\frac{7}{2}}A}{6\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] -15/8*B*sqrt(a)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) - 5/16*A*b^3*arcsinh(a/(s
qrt(a*b)*abs(x)))/sqrt(a) + 15/8*sqrt(b*x^2 + a)*B*b^2 + 3/8*(b*x^2 + a)^(5
/2)*B*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*B*b^2/a + 1/16*(b*x^2 + a)^(5/2)*A*b^
3/a^3 + 5/48*(b*x^2 + a)^(3/2)*A*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*A*b^3/a - 3
/8*(b*x^2 + a)^(7/2)*B*b/(a^2*x^2) - 1/16*(b*x^2 + a)^(7/2)*A*b^2/(a^3*x^2)
 - 1/4*(b*x^2 + a)^(7/2)*B/(a*x^4) - 1/24*(b*x^2 + a)^(7/2)*A*b/(a^2*x^4) -
 1/6*(b*x^2 + a)^(7/2)*A/(a*x^6)
```

Fricas [A]

time = 1.29, size = 241, normalized size = 1.62

$$\left[\frac{15\,(6\,Bab^{2}+Ab^{3})\sqrt{a}\,x^{6}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(48\,Bab^{2}x^{6}-3\,(18\,Ba^{2}b+11\,Aab^{2})x^{4}-8\,Aa^{3}-2\,(6\,Ba^{3}+13\,Aa^{2}b)x^{2})\sqrt{bx^{2}+a}}{96\,ax^{6}},\ \frac{15\,(6\,Bab^{2}+Ab^{3})\sqrt{-a}\,x^{6}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(48\,Bab^{2}x^{6}-3\,(18\,Ba^{2}b+11\,Aab^{2})x^{4}-8\,Aa^{3}-2\,(6\,Ba^{3}+13\,Aa^{2}b)x^{2})\sqrt{bx^{2}+a}}{48\,ax^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(6*B*a*b^2 + A*b^3)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(48*B*a*b^2*x^6 - 3*(18*B*a^2*b + 11*A*a*b^2)*x^4 -
8*A*a^3 - 2*(6*B*a^3 + 13*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^6), 1/48*(15*
(6*B*a*b^2 + A*b^3)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (48*B*a
*b^2*x^6 - 3*(18*B*a^2*b + 11*A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*B*a^3 + 13*A*a^
2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(136) = 272$.
time = 79.27, size = 306, normalized size = 2.05

$$-\frac{Aa^3}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{17Aa^2\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{35Aab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{3Ab^{\frac{5}{2}}}{16x\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16\sqrt{a}}-\frac{15B\sqrt{a}\,b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8}-\frac{Ba^3}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3Ba^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{x}+\frac{7Bab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}}+\frac{Bb^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**7,x)
```

```
[Out] -A*a**3/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 17*A*a**2*sqrt(b)/(24*x**5*
sqrt(a/(b*x**2) + 1)) - 35*A*a*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - A*
b**(5/2)*sqrt(a/(b*x**2) + 1)/(2*x) - 3*A*b**(5/2)/(16*x*sqrt(a/(b*x**2) +
1)) - 5*A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*sqrt(a)) - 15*B*sqrt(a)*b**2*
asinh(sqrt(a)/(sqrt(b)*x))/8 - B*a**3/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1))
 - 3*B*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - B*a*b**(3/2)*sqrt(a/(b*
x**2) + 1)/x + 7*B*a*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1)) + B*b**(5/2)*x/sqr
t(a/(b*x**2) + 1)
```

Giac [A]
time = 1.31, size = 167, normalized size = 1.12

$$\frac{48\sqrt{bx^2+a}\,Bb^3+\frac{15\,(6\,Bab^3+Ab^4)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{54\,(bx^2+a)^{\frac{5}{2}}\,Bab^3-96\,(bx^2+a)^{\frac{3}{2}}\,Ba^2b^3+42\,\sqrt{bx^2+a}\,Ba^3b^3+33\,(bx^2+a)^{\frac{5}{2}}\,Ab^4-40\,(bx^2+a)^{\frac{3}{2}}\,Aab^4+15\,\sqrt{bx^2+a}\,Aa^2b^4}{b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(48*sqrt(b*x^2 + a)*B*b^3 + 15*(6*B*a*b^3 + A*b^4)*arctan(sqrt(b*x^2 +
 a)/sqrt(-a))/sqrt(-a) - (54*(b*x^2 + a)^(5/2)*B*a*b^3 - 96*(b*x^2 + a)^(3/
2)*B*a^2*b^3 + 42*sqrt(b*x^2 + a)*B*a^3*b^3 + 33*(b*x^2 + a)^(5/2)*A*b^4 -
40*(b*x^2 + a)^(3/2)*A*a*b^4 + 15*sqrt(b*x^2 + a)*A*a^2*b^4)/(b^3*x^6))/b
```

Mupad [B]
time = 1.49, size = 150, normalized size = 1.01

$$B\,b^2\,\sqrt{b\,x^2+a}-\frac{11\,A\,{\left(b\,x^2+a\right)}^{5/2}}{16\,x^6}+\frac{5\,A\,a\,{\left(b\,x^2+a\right)}^{3/2}}{6\,x^6}-\frac{9\,B\,a\,{\left(b\,x^2+a\right)}^{3/2}}{8\,x^4}-\frac{5\,A\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6}+\frac{7\,B\,a^2\,\sqrt{b\,x^2+a}}{8\,x^4}+\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,\sqrt{a}}+\frac{B\,\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,15{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^7,x)
```

```
[Out] B*b^2*(a + b*x^2)^(1/2) - (11*A*(a + b*x^2)^(5/2))/(16*x^6) + (A*b^3*atan((
(a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) + (B*a^(1/2)*b^2*atan(((a +
 b*x^2)^(1/2)*1i)/a^(1/2))*15i)/8 + (5*A*a*(a + b*x^2)^(3/2))/(6*x^6) - (9*
B*a*(a + b*x^2)^(3/2))/(8*x^4) - (5*A*a^2*(a + b*x^2)^(1/2))/(16*x^6) + (7*
B*a^2*(a + b*x^2)^(1/2))/(8*x^4)
```

3.551 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=108

$$-\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{bB(a+bx^2)^{3/2}}{3x^3}-\frac{B(a+bx^2)^{5/2}}{5x^5}-\frac{A(a+bx^2)^{7/2}}{7ax^7}+b^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -1/3*b*B*(b*x^2+a)^(3/2)/x^3-1/5*B*(b*x^2+a)^(5/2)/x^5-1/7*A*(b*x^2+a)^(7/2
)/a/x^7+b^(5/2)*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))-b^2*B*(b*x^2+a)^(1/2)/
x
```

Rubi [A]
time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {462, 283, 223, 212}

$$-\frac{A(a+bx^2)^{7/2}}{7ax^7}+b^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{B(a+bx^2)^{5/2}}{5x^5}-\frac{bB(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -((b^2*B*Sqrt[a + b*x^2])/x) - (b*B*(a + b*x^2)^(3/2))/(3*x^3) - (B*(a + b*
x^2)^(5/2))/(5*x^5) - (A*(a + b*x^2)^(7/2))/(7*a*x^7) + b^(5/2)*B*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx &= -\frac{A(a+bx^2)^{7/2}}{7ax^7}+B\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx\\ &= -\frac{B(a+bx^2)^{5/2}}{5x^5}-\frac{A(a+bx^2)^{7/2}}{7ax^7}+(bB)\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx\\ &= -\frac{bB(a+bx^2)^{3/2}}{3x^3}-\frac{B(a+bx^2)^{5/2}}{5x^5}-\frac{A(a+bx^2)^{7/2}}{7ax^7}+\left(b^2B\right)\int \frac{\sqrt{a+bx^2}}{x^2}\,d\\ &= -\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{bB(a+bx^2)^{3/2}}{3x^3}-\frac{B(a+bx^2)^{5/2}}{5x^5}-\frac{A(a+bx^2)^{7/2}}{7ax^7}+\left(b^3B\right.\\ &= -\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{bB(a+bx^2)^{3/2}}{3x^3}-\frac{B(a+bx^2)^{5/2}}{5x^5}-\frac{A(a+bx^2)^{7/2}}{7ax^7}+\left(b^3B\right.\\ &= -\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{bB(a+bx^2)^{3/2}}{3x^3}-\frac{B(a+bx^2)^{5/2}}{5x^5}-\frac{A(a+bx^2)^{7/2}}{7ax^7}+b^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 112, normalized size = 1.04

$$-\frac{\sqrt{a+bx^2}\,(15Ab^3x^6+3a^3(5A+7Bx^2)+a^2bx^2(45A+77Bx^2)+ab^2x^4(45A+161Bx^2))}{105ax^7}-b^{5/2}B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -1/105*(Sqrt[a + b*x^2]*(15*A*b^3*x^6 + 3*a^3*(5*A + 7*B*x^2) + a^2*b*x^2*(
45*A + 77*B*x^2) + a*b^2*x^4*(45*A + 161*B*x^2)))/(a*x^7) - b^(5/2)*B*Log[-
(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [A]
time = 0.09, size = 161, normalized size = 1.49

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(15x^{6}A\,b^{3}+161x^{6}Ba\,b^{2}+45Aa\,b^{2}x^{4}+77x^{4}B\,a^{2}b+45x^{2}A\,a^{2}b+21B\,a^{3}x^{2}+15A\,a^{3}\right)}{105x^{7}a}+B\,b^{\frac{5}{2}}\ln\left(x\sqrt{b}+\sqrt{b\right.$

default | $B\left(-\frac{(b x^2+a)^{\frac{7}{2}}}{5a x^5}+\frac{2b\left(-\frac{(b x^2+a)^{\frac{7}{2}}}{3a x^3}+\frac{4b\left(-\frac{(b x^2+a)^{\frac{7}{2}}}{a x}+\frac{6b\left(\frac{x(b x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{\cdots}\right.}{2\sqrt{\cdots}}\right.}{4}\right.}{6}\right.}{a}\right.}{3a}\right.}{5a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/5/a/x^5*(b*x^2+a)^(7/2)+2/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(7/2)+4/3*b/a*(
-1/a/x*(b*x^2+a)^(7/2)+6*b/a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^
(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/
2))))))))-1/7*A*(b*x^2+a)^(7/2)/a/x^7
```

Maxima [A]
time = 0.29, size = 128, normalized size = 1.19

$$\frac{2\,(bx^2+a)^{\frac{3}{2}}Bb^3x}{3\,a^2}+\frac{\sqrt{bx^2+a}\,Bb^3x}{a}+Bb^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{8\,(bx^2+a)^{\frac{5}{2}}Bb^2}{15\,a^2x}-\frac{2\,(bx^2+a)^{\frac{7}{2}}Bb}{15\,a^2x^3}-\frac{(bx^2+a)^{\frac{7}{2}}B}{5\,ax^5}-\frac{(bx^2+a)^{\frac{7}{2}}A}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x^2 + a)^(3/2)*B*b^3*x/a^2 + sqrt(b*x^2 + a)*B*b^3*x/a + B*b^(5/2)*a
rcsinh(b*x/sqrt(a*b)) - 8/15*(b*x^2 + a)^(5/2)*B*b^2/(a^2*x) - 2/15*(b*x^2
+ a)^(7/2)*B*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2)*B/(a*x^5) - 1/7*(b*x^2 + a
)^(7/2)*A/(a*x^7)
```

Fricas [A]
time = 1.42, size = 234, normalized size = 2.17

$$\left[\frac{105\,Bab^{\frac{5}{2}}x^7\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,((161\,Bab^2+15\,Ab^3)x^6+(77\,Ba^2b+45\,Aab^2)x^4+15\,Aa^3+3\,(7\,Ba^3+15\,Aa^2b)x^2)\sqrt{bx^2+a}}{210\,ax^7},-\frac{105\,Ba\sqrt{-b}\,b^2x^7\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+((161\,Bab^2+15\,Ab^3)x^6+(77\,Ba^2b+45\,Aab^2)x^4+15\,Aa^3+3\,(7\,Ba^3+15\,Aa^2b)x^2)\sqrt{bx^2+a}}{105\,ax^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] [1/210*(105*B*a*b^(5/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 - 2*((161*B*a*b^2 + 15*A*b^3)*x^6 + (77*B*a^2*b + 45*A*a*b^2)*x^4 + 15*A*a
^3 + 3*(7*B*a^3 + 15*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^7), -1/105*(105*B*
a*sqrt(-b)*b^2*x^7*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((161*B*a*b^2 + 15*
A*b^3)*x^6 + (77*B*a^2*b + 45*A*a*b^2)*x^4 + 15*A*a^3 + 3*(7*B*a^3 + 15*A*a
^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^7)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(95) = 190$.
time = 3.75, size = 592, normalized size = 5.48

$$-\frac{15Aa^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Aa^6b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Aa^5b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3Aa^4b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12Aa^3b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8Aa^2b^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{2Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{7Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a}-\frac{B\sqrt{a}\,b^2}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{11Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{8Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15}+Bb^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Bb^3x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -15*A*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**6*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a
**5*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 3*A*a**4*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a**
3*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x
**8 + 105*a**3*b**6*x**10) - 8*A*a**2*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/
(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 2*A*a*b**
(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 7*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15
*x**2) - A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a) - B*sqrt(a)*b**2/(x*sqrt(1
+ b*x**2/a)) - B*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*B*a*b**(3/
2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - 8*B*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 +
B*b**(5/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**3*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(88) = 176.
time = 1.76, size = 320, normalized size = 2.96

$$-\frac{1}{2}\,Bb^{\frac{5}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(315\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{5}{2}}+105\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}Ab^{\frac{7}{2}}-1260\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}Ba^2b^{\frac{5}{2}}+2555\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}Ba^3b^{\frac{5}{2}}+525\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}Aa^2b^{\frac{7}{2}}-3080\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{6}Ba^4b^{\frac{5}{2}}+2121\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}Ba^5b^{\frac{5}{2}}+315\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}Aa^4b^{\frac{7}{2}}-812\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{2}Ba^6b^{\frac{5}{2}}+161\,Ba^7b^{\frac{5}{2}}+15\,Aa^6b^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] -1/2*B*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/105*(315*(sqrt(b)*x
 - sqrt(b*x^2 + a))^12*B*a*b^(5/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A
*b^(7/2) - 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) + 2555*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) + 525*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*A*a^2*b^(7/2) - 3080*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 21
21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) + 315*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*A*a^4*b^(7/2) - 812*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^6*b^(5/
2) + 161*B*a^7*b^(5/2) + 15*A*a^6*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^7
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{5/2}}{x^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^8,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^8, x)
```

3.552 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx$

Optimal. Leaf size=152

$$\frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2}+\frac{5b(Ab-8aB)\left(a+bx^2\right)^{3/2}}{192ax^4}+\frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6}-\frac{A\left(a+bx^2\right)^{7/2}}{8ax^8}+\frac{5b^3(Ab-$$

```
[Out] 5/192*b*(A*b-8*B*a)*(b*x^2+a)^(3/2)/a/x^4+1/48*(A*b-8*B*a)*(b*x^2+a)^(5/2)/
a/x^6-1/8*A*(b*x^2+a)^(7/2)/a/x^8+5/128*b^3*(A*b-8*B*a)*arctanh((b*x^2+a)^(
1/2)/a^(1/2))/a^(3/2)+5/128*b^2*(A*b-8*B*a)*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 43, 65, 214}

$$\frac{5b^3(Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}+\frac{5b^2\sqrt{a+bx^2}\,(Ab-8aB)}{128ax^2}+\frac{\left(a+bx^2\right)^{5/2}(Ab-8aB)}{48ax^6}+\frac{5b\left(a+bx^2\right)^{3/2}(Ab-8aB)}{192ax^4}-\frac{A\left(a+bx^2\right)^{7/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (5*b^2*(A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a*x^2) + (5*b*(A*b - 8*a*B)*(a +
 b*x^2)^(3/2))/(192*a*x^4) + ((A*b - 8*a*B)*(a + b*x^2)^(5/2))/(48*a*x^6) -
 (A*(a + b*x^2)^(7/2))/(8*a*x^8) + (5*b^3*(A*b - 8*a*B)*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{A(a+bx^2)^{7/2}}{8ax^8} + \frac{\left(-\frac{Ab}{2}+4aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx,x,x^2\right)}{8a}$$

$$= \frac{(Ab-8aB)(a+bx^2)^{5/2}}{48ax^6} - \frac{A(a+bx^2)^{7/2}}{8ax^8} - \frac{(5b(Ab-8aB))\text{Subst}\left(\int \frac{(a+bx)^3}{x^3}\right.}{96a}$$

$$= \frac{5b(Ab-8aB)(a+bx^2)^{3/2}}{192ax^4} + \frac{(Ab-8aB)(a+bx^2)^{5/2}}{48ax^6} - \frac{A(a+bx^2)^{7/2}}{8ax^8} - \frac{(5}{}$$

$$= \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2} + \frac{5b(Ab-8aB)(a+bx^2)^{3/2}}{192ax^4} + \frac{(Ab-8aB)(a+}{48ax^6}$$

$$= \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2} + \frac{5b(Ab-8aB)(a+bx^2)^{3/2}}{192ax^4} + \frac{(Ab-8aB)(a+}{48ax^6}$$

$$= \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2} + \frac{5b(Ab-8aB)(a+bx^2)^{3/2}}{192ax^4} + \frac{(Ab-8aB)(a+}{48ax^6}$$

Mathematica [A]

time = 0.22, size = 123, normalized size = 0.81

$$-\frac{\sqrt{a+bx^2}\,(15Ab^3x^6+16a^3(3A+4Bx^2)+8a^2bx^2(17A+26Bx^2)+2ab^2x^4(59A+132Bx^2))}{384ax^8}+\frac{5b^3(Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

```
[Out] -1/384*(Sqrt[a + b*x^2]*(15*A*b^3*x^6 + 16*a^3*(3*A + 4*B*x^2) + 8*a^2*b*x^
2*(17*A + 26*B*x^2) + 2*a*b^2*x^4*(59*A + 132*B*x^2)))/(a*x^8) + (5*b^3*(A*
b - 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(128) = 256$.
time = 0.10, size = 306, normalized size = 2.01

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(15x^6A\,b^3+264x^6Ba\,b^2+118Aa\,b^2x^4+208x^4B\,a^2b+136x^2A\,a^2b+64B\,a^3x^2+48A\,a^3)}{384x^8a}+\frac{5b^4\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,}}{x}\right.}{128a^{\frac{3}{2}}}$

default	$B\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{6a\,x^6}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{4a\,x^4}+\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{\cdots}}{x}\right.\right.\right.\right.}{2a}\right.}{4a}\right.}{6a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/6/a/x^6*(b*x^2+a)^(7/2)+1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(
-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3
/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))))+
A*(-1/8/a/x^8*(b*x^2+a)^(7/2)-1/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(7/2)+1/6*b/a*(
-1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5
*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+
2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(132) = 264.

time = 0.29, size = 288, normalized size = 1.89

$$-\frac{5\,Bb^{3}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,\sqrt{a}}+\frac{5\,Ab^{4}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{3}{2}}}+\frac{(bx^{2}+a)^{\frac{5}{2}}Bb^{3}}{16\,a^{3}}+\frac{5\,(bx^{2}+a)^{\frac{3}{2}}Bb^{3}}{48\,a^{2}}+\frac{5\,\sqrt{bx^{2}+a}\,Bb^{3}}{16\,a}-\frac{(bx^{2}+a)^{\frac{5}{2}}Ab^{4}}{128\,a^{4}}-\frac{5\,(bx^{2}+a)^{\frac{3}{2}}Ab^{4}}{384\,a^{3}}-\frac{5\,\sqrt{bx^{2}+a}\,Ab^{4}}{128\,a^{2}}-\frac{(bx^{2}+a)^{\frac{7}{2}}Bb^{2}}{16\,a^{3}x^{2}}+\frac{(bx^{2}+a)^{\frac{7}{2}}Ab^{3}}{128\,a^{4}x^{2}}-\frac{(bx^{2}+a)^{\frac{7}{2}}Bb}{24\,a^{2}x^{4}}+\frac{(bx^{2}+a)^{\frac{7}{2}}Ab^{2}}{192\,a^{3}x^{4}}-\frac{(bx^{2}+a)^{\frac{7}{2}}B}{6\,ax^{6}}+\frac{(bx^{2}+a)^{\frac{7}{2}}Ab}{48\,a^{2}x^{6}}-\frac{(bx^{2}+a)^{\frac{7}{2}}A}{8\,ax^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] -5/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 5/128*A*b^4*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(3/2) + 1/16*(b*x^2 + a)^(5/2)*B*b^3/a^3 + 5/48*(b*x^2
 + a)^(3/2)*B*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*B*b^3/a - 1/128*(b*x^2 + a)^(5
/2)*A*b^4/a^4 - 5/384*(b*x^2 + a)^(3/2)*A*b^4/a^3 - 5/128*sqrt(b*x^2 + a)*A
*b^4/a^2 - 1/16*(b*x^2 + a)^(7/2)*B*b^2/(a^3*x^2) + 1/128*(b*x^2 + a)^(7/2)
*A*b^3/(a^4*x^2) - 1/24*(b*x^2 + a)^(7/2)*B*b/(a^2*x^4) + 1/192*(b*x^2 + a)
^(7/2)*A*b^2/(a^3*x^4) - 1/6*(b*x^2 + a)^(7/2)*B/(a*x^6) + 1/48*(b*x^2 + a)
^(7/2)*A*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(7/2)*A/(a*x^8)
```

Fricas [A]

time = 1.82, size = 272, normalized size = 1.79

$$\left[-\frac{15\,(8\,Bab^{3}-Ab^{4})\sqrt{a}\,x^{8}\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(3\,(88\,Ba^{2}b^{2}+5\,Aab^{3})x^{6}+48\,Aa^{4}+2\,(104\,Ba^{3}b+59\,Aa^{2}b^{2})x^{4}+8\,(8\,Ba^{4}+17\,Aa^{3}b)x^{2})\sqrt{bx^{2}+a}}{768\,a^{2}x^{8}},\ \frac{15\,(8\,Bab^{3}-Ab^{4})\sqrt{-a}\,x^{8}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-(3\,(88\,Ba^{2}b^{2}+5\,Aab^{3})x^{6}+48\,Aa^{4}+2\,(104\,Ba^{3}b+59\,Aa^{2}b^{2})x^{4}+8\,(8\,Ba^{4}+17\,Aa^{3}b)x^{2})\sqrt{bx^{2}+a}}{384\,a^{2}x^{8}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(8*B*a*b^3 - A*b^4)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^6 + 48*A*a^4 + 2*(
104*B*a^3*b + 59*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 17*A*a^3*b)*x^2)*sqrt(b*x^2
+ a))/(a^2*x^8), 1/384*(15*(8*B*a*b^3 - A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)
/sqrt(b*x^2 + a)) - (3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^6 + 48*A*a^4 + 2*(104*B
*a^3*b + 59*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 17*A*a^3*b)*x^2)*sqrt(b*x^2 + a))
/(a^2*x^8)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(139) = 278.

time = 158.08, size = 316, normalized size = 2.08

$$-\frac{Aa^{3}}{8\sqrt{b}\,x^{9}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{23Aa^{2}\sqrt{b}}{48x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{127Aab^{\frac{3}{2}}}{192x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{133Ab^{\frac{5}{2}}}{384x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ab^{\frac{7}{2}}}{128ax\sqrt{\frac{a}{bx^{2}}+1}}+\frac{5Ab^{4}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{3}{2}}}-\frac{Ba^{3}}{6\sqrt{b}\,x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{17Ba^{2}\sqrt{b}}{24x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{35Bab^{\frac{3}{2}}}{48x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{2x}-\frac{3Bb^{\frac{5}{2}}}{16x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Bb^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**9,x)
```

```
[Out] -A*a**3/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 23*A*a**2*sqrt(b)/(48*x**7*
sqrt(a/(b*x**2) + 1)) - 127*A*a*b**(3/2)/(192*x**5*sqrt(a/(b*x**2) + 1)) -
```

```
133*A*b**(5/2)/(384*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(7/2)/(128*a*x*sqrt
(a/(b*x**2) + 1)) + 5*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(3/2)) - B*
a**3/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 17*B*a**2*sqrt(b)/(24*x**5*sqr
t(a/(b*x**2) + 1)) - 35*B*a*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - B*b**
(5/2)*sqrt(a/(b*x**2) + 1)/(2*x) - 3*B*b**(5/2)/(16*x*sqrt(a/(b*x**2) + 1))
 - 5*B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*sqrt(a))
```

Giac [A]
time = 1.38, size = 195, normalized size = 1.28

$$\frac{\frac{15\,(8\,Bab^4-Ab^5)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{264\,(bx^2+a)^{\frac{7}{2}}Bab^4-584\,(bx^2+a)^{\frac{5}{2}}Ba^2b^4+440\,(bx^2+a)^{\frac{3}{2}}Ba^3b^4-120\,\sqrt{bx^2+a}\,Ba^4b^4+15\,(bx^2+a)^{\frac{7}{2}}Ab^5+73\,(bx^2+a)^{\frac{5}{2}}Aab^5-55\,(bx^2+a)^{\frac{3}{2}}Aa^2b^5+15\,\sqrt{bx^2+a}\,Aa^3b^5}{ab^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/384*(15*(8*B*a*b^4 - A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a)
 - (264*(b*x^2 + a)^(7/2)*B*a*b^4 - 584*(b*x^2 + a)^(5/2)*B*a^2*b^4 + 440*(
b*x^2 + a)^(3/2)*B*a^3*b^4 - 120*sqrt(b*x^2 + a)*B*a^4*b^4 + 15*(b*x^2 + a)
^(7/2)*A*b^5 + 73*(b*x^2 + a)^(5/2)*A*a*b^5 - 55*(b*x^2 + a)^(3/2)*A*a^2*b^
5 + 15*sqrt(b*x^2 + a)*A*a^3*b^5)/(a*b^4*x^8))/b
```

Mupad [B]
time = 2.05, size = 169, normalized size = 1.11

$$\frac{55\,A\,a\,{\left(b\,x^2+a\right)}^{3/2}}{384\,x^8}-\frac{11\,B\,{\left(b\,x^2+a\right)}^{5/2}}{16\,x^6}-\frac{73\,A\,{\left(b\,x^2+a\right)}^{5/2}}{384\,x^8}+\frac{5\,B\,a\,{\left(b\,x^2+a\right)}^{3/2}}{6\,x^6}-\frac{5\,A\,a^2\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{5\,A\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^8}-\frac{5\,B\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6}-\frac{A\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{3/2}}+\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^9,x)
```

```
[Out] (B*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) - (11*B*(a + b
*x^2)^(5/2))/(16*x^6) - (A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(12
8*a^(3/2)) - (73*A*(a + b*x^2)^(5/2))/(384*x^8) + (55*A*a*(a + b*x^2)^(3/2)
)/(384*x^8) + (5*B*a*(a + b*x^2)^(3/2))/(6*x^6) - (5*A*a^2*(a + b*x^2)^(1/2
))/(128*x^8) - (5*A*(a + b*x^2)^(7/2))/(128*a*x^8) - (5*B*a^2*(a + b*x^2)^(
1/2))/(16*x^6)
```

3.553 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx$

Optimal. Leaf size=53

$$-\frac{A(a+bx^2)^{7/2}}{9ax^9}+\frac{(2Ab-9aB)\,(a+bx^2)^{7/2}}{63a^2x^7}$$

```
[Out] -1/9*A*(b*x^2+a)^(7/2)/a/x^9+1/63*(2*A*b-9*B*a)*(b*x^2+a)^(7/2)/a^2/x^7
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {464, 270}

$$\frac{(a+bx^2)^{7/2}\,(2Ab-9aB)}{63a^2x^7}-\frac{A(a+bx^2)^{7/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -1/9*(A*(a + b*x^2)^(7/2))/(a*x^9) + ((2*A*b - 9*a*B)*(a + b*x^2)^(7/2))/(6
3*a^2*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}\,(A+Bx^2)}{x^{10}}\,dx &= -\frac{A(a+bx^2)^{7/2}}{9ax^9}-\frac{(2Ab-9aB)\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx}{9a}\\ &= -\frac{A(a+bx^2)^{7/2}}{9ax^9}+\frac{(2Ab-9aB)\,(a+bx^2)^{7/2}}{63a^2x^7}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 40, normalized size = 0.75

$$\frac{\left(a+bx^2\right)^{7/2}\left(-7aA+2Abx^2-9aBx^2\right)}{63a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
[Out] ((a + b*x^2)^(7/2)*(-7*a*A + 2*A*b*x^2 - 9*a*B*x^2))/(63*a^2*x^9)
```

Maple [A]
time = 0.10, size = 58, normalized size = 1.09

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(-2Ab\,x^{2}+9Ba\,x^{2}+7Aa\right)}{63x^{9}a^{2}}$	37
default	$A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9a\,x^{9}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63a^{2}x^{7}}\right)-\frac{B\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7a\,x^{7}}$	58
trager	$-\frac{\left(-2A\,b^{4}x^{8}+9Ba\,b^{3}x^{8}+Aa\,b^{3}x^{6}+27B\,a^{2}b^{2}x^{6}+15A\,a^{2}b^{2}x^{4}+27B\,a^{3}b\,x^{4}+19A\,a^{3}b\,x^{2}+9B\,a^{4}x^{2}+7A\,a^{4}\right)\sqrt{b\,x^{2}+a}}{63x^{9}a^{2}}$	106
risch	$-\frac{\left(-2A\,b^{4}x^{8}+9Ba\,b^{3}x^{8}+Aa\,b^{3}x^{6}+27B\,a^{2}b^{2}x^{6}+15A\,a^{2}b^{2}x^{4}+27B\,a^{3}b\,x^{4}+19A\,a^{3}b\,x^{2}+9B\,a^{4}x^{2}+7A\,a^{4}\right)\sqrt{b\,x^{2}+a}}{63x^{9}a^{2}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x,method=_RETURNVERBOSE)
[Out] A*(-1/9/a/x^9*(b*x^2+a)^(7/2)+2/63*b/a^2/x^7*(b*x^2+a)^(7/2))-1/7*B/a/x^7*(
b*x^2+a)^(7/2)
```

Maxima [A]
time = 0.32, size = 56, normalized size = 1.06

$$-\frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{7\,ax^7}+\frac{2\left(bx^2+a\right)^{\frac{7}{2}}Ab}{63\,a^2x^7}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="maxima")
[Out] -1/7*(b*x^2 + a)^(7/2)*B/(a*x^7) + 2/63*(b*x^2 + a)^(7/2)*A*b/(a^2*x^7) - 1
/9*(b*x^2 + a)^(7/2)*A/(a*x^9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(45) = 90.
time = 1.48, size = 102, normalized size = 1.92

$$-\frac{\left((9\,Bab^3-2\,Ab^4)x^8+(27\,Ba^2b^2+Aab^3)x^6+7\,Aa^4+3\,(9\,Ba^3b+5\,Aa^2b^2)x^4+(9\,Ba^4+19\,Aa^3b)x^2\right)\sqrt{bx^2+a}}{63\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="fricas")
```

```
[Out] -1/63*((9*B*a*b^3 - 2*A*b^4)*x^8 + (27*B*a^2*b^2 + A*a*b^3)*x^6 + 7*A*a^4 +
 3*(9*B*a^3*b + 5*A*a^2*b^2)*x^4 + (9*B*a^4 + 19*A*a^3*b)*x^2)*sqrt(b*x^2 +
 a)/(a^2*x^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1489 vs. $2(46) = 92$.
time = 4.35, size = 1489, normalized size = 28.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -35*A*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b*
*10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**8*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**7*b**(23/2)*x**4*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**6*b**(25/2)*x**6*sqrt(a/(b*x**
2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) - 30*A*a**6*b**(11/2)*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**5*b**(27/2)
*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 66*A*a**5*b**(13/2)*x**2*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) + 30*A*a**4*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 +
945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 34*A*
a**4*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**3*b**(31/2)*x**12*sqrt(a/(b*x**2) +
 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315
*a**4*b**12*x**14) - 6*A*a**3*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 16*A*a**2*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 9
45*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 24*A*a**2*b**(19/2)*x**8*sqrt
(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x
**10) - 16*A*a*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 2
10*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/
(5*x**4) - A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*A*b**(9/2)*sqrt(
a/(b*x**2) + 1)/(15*a**2) - 15*B*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**6*b**(11
/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 10
5*a**3*b**6*x**10) - 17*B*a**5*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**
```

```
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*B*a**4*b**(15/2
)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*
a**3*b**6*x**10) - 12*B*a**3*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*
b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*a**2*b**(19/2)*
x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) - 2*B*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 7*B*b**(5/
2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - B*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(45) = 90.
time = 1.20, size = 456, normalized size = 8.60

$$\frac{2\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}Bb^{\frac{7}{2}}-126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bab^{\frac{7}{2}}+126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Ab^{\frac{9}{2}}+378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ba^2b^{\frac{7}{2}}+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Aab^{\frac{9}{2}}-630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ba^3b^{\frac{7}{2}}+630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Aa^2b^{\frac{9}{2}}+504\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ba^4b^{\frac{7}{2}}+378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Aa^3b^{\frac{9}{2}}-378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^5b^{\frac{7}{2}}+378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Aa^4b^{\frac{9}{2}}+198\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Ba^6b^{\frac{7}{2}}+54\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Aa^5b^{\frac{9}{2}}-18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Ba^7b^{\frac{7}{2}}+18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Aa^6b^{\frac{9}{2}}+9Ba^8b^{\frac{7}{2}}-2Aa^7b^{\frac{9}{2}}\right)}{63\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="giac")
```

```
[Out] 2/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*b^(7/2) - 126*(sqrt(b)*x - sqrt
(b*x^2 + a))^14*B*a*b^(7/2) + 126*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*b^(9/2
) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^2*b^(7/2) + 210*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*A*a*b^(9/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^3
*b^(7/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^2*b^(9/2) + 504*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*B*a^4*b^(7/2) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^
8*A*a^3*b^(9/2) - 378*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5*b^(7/2) + 378*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^4*b^(9/2) + 198*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a^6*b^(7/2) + 54*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^5*b^(9/2) -
18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^7*b^(7/2) + 18*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*A*a^6*b^(9/2) + 9*B*a^8*b^(7/2) - 2*A*a^7*b^(9/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^9
```

Mupad [B]
time = 1.28, size = 170, normalized size = 3.21

$$\frac{2\,A\,b^4\,\sqrt{b\,x^2+a}}{63\,a^2\,x}-\frac{5\,A\,b^2\,\sqrt{b\,x^2+a}}{21\,x^5}-\frac{B\,a^2\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{3\,B\,b^2\,\sqrt{b\,x^2+a}}{7\,x^3}-\frac{A\,b^3\,\sqrt{b\,x^2+a}}{63\,a\,x^3}-\frac{A\,a^2\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{B\,b^3\,\sqrt{b\,x^2+a}}{7\,a\,x}-\frac{19\,A\,a\,b\,\sqrt{b\,x^2+a}}{63\,x^7}-\frac{3\,B\,a\,b\,\sqrt{b\,x^2+a}}{7\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^10,x)
```

```
[Out] (2*A*b^4*(a + b*x^2)^(1/2))/(63*a^2*x) - (5*A*b^2*(a + b*x^2)^(1/2))/(21*x^
5) - (B*a^2*(a + b*x^2)^(1/2))/(7*x^7) - (3*B*b^2*(a + b*x^2)^(1/2))/(7*x^3
) - (A*b^3*(a + b*x^2)^(1/2))/(63*a*x^3) - (A*a^2*(a + b*x^2)^(1/2))/(9*x^9
) - (B*b^3*(a + b*x^2)^(1/2))/(7*a*x) - (19*A*a*b*(a + b*x^2)^(1/2))/(63*x^
7) - (3*B*a*b*(a + b*x^2)^(1/2))/(7*x^5)
```

3.554 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx$

Optimal. Leaf size=189

$$\frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4}+\frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2}+\frac{b(3Ab-10aB)\left(a+bx^2\right)^{3/2}}{96ax^6}+\frac{(3Ab-10aB)\left(a+\right.}{80ax^8}$$

```
[Out] 1/96*b*(3*A*b-10*B*a)*(b*x^2+a)^(3/2)/a/x^6+1/80*(3*A*b-10*B*a)*(b*x^2+a)^(
5/2)/a/x^8-1/10*A*(b*x^2+a)^(7/2)/a/x^10-1/256*b^4*(3*A*b-10*B*a)*arctanh((
b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/128*b^2*(3*A*b-10*B*a)*(b*x^2+a)^(1/2)/a/
x^4+1/256*b^3*(3*A*b-10*B*a)*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 43, 44, 65, 214}

$$-\frac{b^4(3Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{b^3\sqrt{a+bx^2}\,(3Ab-10aB)}{256a^2x^2}+\frac{b^2\sqrt{a+bx^2}\,(3Ab-10aB)}{128ax^4}+\frac{\left(a+bx^2\right)^{5/2}(3Ab-10aB)}{80ax^8}+\frac{b\left(a+bx^2\right)^{3/2}(3Ab-10aB)}{96ax^6}-\frac{A\left(a+bx^2\right)^{7/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (b^2*(3*A*b - 10*a*B)*Sqrt[a + b*x^2])/(128*a*x^4) + (b^3*(3*A*b - 10*a*B)*
Sqrt[a + b*x^2])/(256*a^2*x^2) + (b*(3*A*b - 10*a*B)*(a + b*x^2)^(3/2))/(96
*a*x^6) + ((3*A*b - 10*a*B)*(a + b*x^2)^(5/2))/(80*a*x^8) - (A*(a + b*x^2)^
(7/2))/(10*a*x^10) - (b^4*(3*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/(256*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx, x, x^2\right)$$
$$= -\frac{A(a+bx^2)^{7/2}}{10ax^{10}} + \frac{\left(-\frac{3Ab}{2}+5aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx, x, x^2\right)}{10a}$$
$$= \frac{(3Ab-10aB)(a+bx^2)^{5/2}}{80ax^8} - \frac{A(a+bx^2)^{7/2}}{10ax^{10}} - \frac{(b(3Ab-10aB))\text{Subst}\left(\int \frac{(}{}\right.}{32a}$$
$$= \frac{b(3Ab-10aB)(a+bx^2)^{3/2}}{96ax^6} + \frac{(3Ab-10aB)(a+bx^2)^{5/2}}{80ax^8} - \frac{A(a+bx^2)^{7/2}}{10ax^{10}}$$
$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b(3Ab-10aB)(a+bx^2)^{3/2}}{96ax^6} + \frac{(3Ab-10aB)}{80a}$$
$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)}{96a}$$
$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)}{96a}$$
$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)}{96a}$$

Mathematica [**A**]
time = 0.31, size = 143, normalized size = 0.76

$$-\frac{\sqrt{a+bx^2}\,(-45Ab^4x^8+30ab^3x^6(A+5Bx^2)+96a^4(4A+5Bx^2)+16a^3bx^2(63A+85Bx^2)+4a^2b^2x^4(186A+295Bx^2))}{3840a^2x^{10}} + \frac{b^4(-3Ab+10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

```
[Out] -1/3840*(Sqrt[a + b*x^2]*(-45*A*b^4*x^8 + 30*a*b^3*x^6*(A + 5*B*x^2) + 96*a
^4*(4*A + 5*B*x^2) + 16*a^3*b*x^2*(63*A + 85*B*x^2) + 4*a^2*b^2*x^4*(186*A
+ 295*B*x^2)))/(a^2*x^10) + (b^4*(-3*A*b + 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]])/(256*a^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(161) = 322$.
time = 0.12, size = 354, normalized size = 1.87

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-45A\,b^4x^8+150Ba\,b^3x^8+30Aa\,b^3x^6+1180B\,a^2b^2x^6+744A\,a^2b^2x^4+1360B\,a^3b\,x^4+1008A\,a^3b\,x^2+480B\,a^4x^2+384\right.}{3840x^{10}a^2}$

default	$B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8a\,x^{8}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{6a\,x^{6}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{4a\,x^{4}}+\frac{3b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2a\,x^{2}}+\frac{5b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\right.\right.\right.}{2a}}{4a}}{6a}}{8a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/8/a/x^8*(b*x^2+a)^(7/2)-1/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(7/2)+1/6*b/a*(
-1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5
*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+
2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))))+A*(-1/10/a/x^10*(b*x^2+a)^(7/2)-3/10*b
/a*(-1/8/a/x^8*(b*x^2+a)^(7/2)-1/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(7/2)+1/6*b/a*
(-1/4/a/x^4*(b*x^2+a)^(7/2)+3/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/
5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a
+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(161) = 322$.
time = 0.31, size = 330, normalized size = 1.75

$$\frac{5\,Bb^4 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{3}{2}}} - \frac{3\,Ab^5 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,a^{\frac{5}{2}}} - \frac{(bx^2+a)^{\frac{5}{2}}Bb^4}{128\,a^4} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Bb^4}{384\,a^3} - \frac{5\,\sqrt{bx^2+a}\,Bb^4}{128\,a^2} + \frac{3\,(bx^2+a)^{\frac{5}{2}}Ab^5}{1280\,a^5} + \frac{(bx^2+a)^{\frac{3}{2}}Ab^5}{256\,a^4} + \frac{3\,\sqrt{bx^2+a}\,Ab^5}{256\,a^3} + \frac{(bx^2+a)^{\frac{7}{2}}Bb^3}{128\,a^4x^2} - \frac{3\,(bx^2+a)^{\frac{7}{2}}Ab^4}{1280\,a^5x^2} + \frac{(bx^2+a)^{\frac{7}{2}}Bb^2}{192\,a^3x^4} - \frac{(bx^2+a)^{\frac{7}{2}}Ab^3}{640\,a^4x^4} + \frac{(bx^2+a)^{\frac{7}{2}}Bb}{48\,a^2x^6} - \frac{(bx^2+a)^{\frac{7}{2}}Ab^2}{160\,a^3x^6} - \frac{(bx^2+a)^{\frac{7}{2}}B}{8\,ax^8} + \frac{3\,(bx^2+a)^{\frac{7}{2}}Ab}{80\,a^2x^8} - \frac{(bx^2+a)^{\frac{7}{2}}A}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="maxima")
```

```
[Out] 5/128*B*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/256*A*b^5*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(5/2) - 1/128*(b*x^2 + a)^(5/2)*B*b^4/a^4 - 5/384*(b*x
^2 + a)^(3/2)*B*b^4/a^3 - 5/128*sqrt(b*x^2 + a)*B*b^4/a^2 + 3/1280*(b*x^2 +
 a)^(5/2)*A*b^5/a^5 + 1/256*(b*x^2 + a)^(3/2)*A*b^5/a^4 + 3/256*sqrt(b*x^2
+ a)*A*b^5/a^3 + 1/128*(b*x^2 + a)^(7/2)*B*b^3/(a^4*x^2) - 3/1280*(b*x^2 +
a)^(7/2)*A*b^4/(a^5*x^2) + 1/192*(b*x^2 + a)^(7/2)*B*b^2/(a^3*x^4) - 1/640*
(b*x^2 + a)^(7/2)*A*b^3/(a^4*x^4) + 1/48*(b*x^2 + a)^(7/2)*B*b/(a^2*x^6) -
1/160*(b*x^2 + a)^(7/2)*A*b^2/(a^3*x^6) - 1/8*(b*x^2 + a)^(7/2)*B/(a*x^8) +
 3/80*(b*x^2 + a)^(7/2)*A*b/(a^2*x^8) - 1/10*(b*x^2 + a)^(7/2)*A/(a*x^10)
```

Fricas [**A**]
time = 1.35, size = 319, normalized size = 1.69

$$\left[-\frac{15\,(10\,Bab^4 - 3\,Ab^5)\sqrt{a}\,x^{10}\log\left(-\frac{bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{a} + 2\,a}{x^2}\right) + 2\,(15\,(10\,Ba^2b^3 - 3\,Aab^4)x^8 + 10\,(118\,Ba^3b^2 + 3\,Aa^2b^3)x^6 + 384\,Aa^5 + 8\,(170\,Ba^4b + 93\,Aa^3b^2)x^4 + 48\,(10\,Ba^5 + 21\,Aa^4b)x^2)\sqrt{bx^2+a}}{7680\,a^3x^{10}}, -\frac{15\,(10\,Bab^4 - 3\,Ab^5)\sqrt{-a}\,x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + (15\,(10\,Ba^2b^3 - 3\,Aab^4)x^8 + 10\,(118\,Ba^3b^2 + 3\,Aa^2b^3)x^6 + 384\,Aa^5 + 8\,(170\,Ba^4b + 93\,Aa^3b^2)x^4 + 48\,(10\,Ba^5 + 21\,Aa^4b)x^2)\sqrt{bx^2+a}}{3840\,a^3x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(10*B*a*b^4 - 3*A*b^5)*sqrt(a)*x^10*log(-(b*x^2 - 2*sqrt(b*x^2
 + a)*sqrt(a) + 2*a)/x^2) + 2*(15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^8 + 10*(118*
B*a^3*b^2 + 3*A*a^2*b^3)*x^6 + 384*A*a^5 + 8*(170*B*a^4*b + 93*A*a^3*b^2)*x
^4 + 48*(10*B*a^5 + 21*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^10), -1/3840*(
```

```
15*(10*B*a*b^4 - 3*A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
(15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^8 + 10*(118*B*a^3*b^2 + 3*A*a^2*b^3)*x^6 +
 384*A*a^5 + 8*(170*B*a^4*b + 93*A*a^3*b^2)*x^4 + 48*(10*B*a^5 + 21*A*a^4*b
)*x^2)*sqrt(b*x^2 + a))/(a^3*x^10)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**11,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.24, size = 230, normalized size = 1.22

$$-\frac{\frac{15\,(10\,Bab^5-3\,Ab^6)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{150\,(bx^2+a)^{\frac{9}{2}}Bab^5+580\,(bx^2+a)^{\frac{7}{2}}Ba^2b^5-1280\,(bx^2+a)^{\frac{5}{2}}Ba^3b^5+700\,(bx^2+a)^{\frac{3}{2}}Ba^4b^5-150\,\sqrt{bx^2+a}\,Ba^5b^5-45\,(bx^2+a)^{\frac{9}{2}}Ab^6+210\,(bx^2+a)^{\frac{7}{2}}Aab^6+384\,(bx^2+a)^{\frac{5}{2}}Aa^2b^6-210\,(bx^2+a)^{\frac{3}{2}}Aa^3b^6+45\,\sqrt{bx^2+a}\,Aa^4b^6}{a^2b^5x^{10}}}{3840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="giac")
```

```
[Out] -1/3840*(15*(10*B*a*b^5 - 3*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-
a)*a^2) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 + 580*(b*x^2 + a)^(7/2)*B*a^2*b^5
- 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 + 700*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 150*
sqrt(b*x^2 + a)*B*a^5*b^5 - 45*(b*x^2 + a)^(9/2)*A*b^6 + 210*(b*x^2 + a)^(7
/2)*A*a*b^6 + 384*(b*x^2 + a)^(5/2)*A*a^2*b^6 - 210*(b*x^2 + a)^(3/2)*A*a^3
*b^6 + 45*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^2*b^5*x^10))/b
```

Mupad [**B**]
time = 2.73, size = 205, normalized size = 1.08

$$\frac{7\,A\,a\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^{10}}-\frac{73\,B\,{\left(b\,x^2+a\right)}^{5/2}}{384\,x^8}-\frac{A\,{\left(b\,x^2+a\right)}^{5/2}}{10\,x^{10}}+\frac{55\,B\,a\,{\left(b\,x^2+a\right)}^{3/2}}{384\,x^8}-\frac{3\,A\,a^2\,\sqrt{b\,x^2+a}}{256\,x^{10}}-\frac{7\,A\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^{10}}+\frac{3\,A\,{\left(b\,x^2+a\right)}^{9/2}}{256\,a^2\,x^{10}}-\frac{5\,B\,a^2\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{5\,B\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^8}+\frac{A\,b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{256\,a^{5/2}}-\frac{B\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^11,x)
```

```
[Out] (A*b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(256*a^(5/2)) - (73*B*(a +
b*x^2)^(5/2))/(384*x^8) - (A*(a + b*x^2)^(5/2))/(10*x^10) - (B*b^4*atan(((a
 + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(128*a^(3/2)) + (7*A*a*(a + b*x^2)^(3/2))/
(128*x^10) + (55*B*a*(a + b*x^2)^(3/2))/(384*x^8) - (3*A*a^2*(a + b*x^2)^(1
/2))/(256*x^10) - (7*A*(a + b*x^2)^(7/2))/(128*a*x^10) + (3*A*(a + b*x^2)^(
9/2))/(256*a^2*x^10) - (5*B*a^2*(a + b*x^2)^(1/2))/(128*x^8) - (5*B*(a + b*
x^2)^(7/2))/(128*a*x^8)
```

3.555 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}} dx$

Optimal. Leaf size=100

$$\frac{a^2(Ab-aB)\sqrt{a+bx^2}}{b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^4}$$

```
[Out] -1/3*a*(2*A*b-3*B*a)*(b*x^2+a)^(3/2)/b^4+1/5*(A*b-3*B*a)*(b*x^2+a)^(5/2)/b^
4+1/7*B*(b*x^2+a)^(7/2)/b^4+a^2*(A*b-B*a)*(b*x^2+a)^(1/2)/b^4
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{a^2\sqrt{a+bx^2}\,(Ab-aB)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(Ab-3aB)}{5b^4}-\frac{a\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{3b^4}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(A*b - a*B)*Sqrt[a + b*x^2])/b^4 - (a*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2
))/(3*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(5/2))/(5*b^4) + (B*(a + b*x^2)^(7/
2))/(7*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3\sqrt{a+bx}} + \frac{a(-2Ab+3aB)\sqrt{a+bx}}{b^3} + \frac{(Ab-3aB)(a+bx)^{3/}}{b^3}\right.\right.$$
$$= \frac{a^2(Ab-aB)\sqrt{a+bx^2}}{b^4} - \frac{a(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{3b^4} + \frac{(Ab-3aB)\left(a+bx^2\right)^{5/2}}{5b^4} + $$

Mathematica [A]
time = 0.05, size = 80, normalized size = 0.80

$$\frac{\sqrt{a+bx^2}\,(56a^2Ab-48a^3B-28aAb^2x^2+24a^2bBx^2+21Ab^3x^4-18ab^2Bx^4+15b^3Bx^6)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(56*a^2*A*b - 48*a^3*B - 28*a*A*b^2*x^2 + 24*a^2*b*B*x^2 +
 21*A*b^3*x^4 - 18*a*b^2*B*x^4 + 15*b^3*B*x^6))/(105*b^4)
```

Maple [A]
time = 0.08, size = 144, normalized size = 1.44

method	result
gosper	$\frac{\sqrt{b\,x^2+a}\,(15B\,x^6b^3+21A\,b^3x^4-18Ba\,b^2x^4-28Aa\,b^2x^2+24B\,a^2b\,x^2+56A\,a^2b-48B\,a^3)}{105b^4}$
trager	$\frac{\sqrt{b\,x^2+a}\,(15B\,x^6b^3+21A\,b^3x^4-18Ba\,b^2x^4-28Aa\,b^2x^2+24B\,a^2b\,x^2+56A\,a^2b-48B\,a^3)}{105b^4}$
risch	$\frac{\sqrt{b\,x^2+a}\,(15B\,x^6b^3+21A\,b^3x^4-18Ba\,b^2x^4-28Aa\,b^2x^2+24B\,a^2b\,x^2+56A\,a^2b-48B\,a^3)}{105b^4}$
default	$B\left(\frac{x^6\sqrt{b\,x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)}{7b}\right)+A\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/
3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))))+A*(1/5*x^4/b*(b*x^2+a)
^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.29, size = 132, normalized size = 1.32

$$\frac{\sqrt{bx^2+a}\,Bx^6}{7\,b}-\frac{6\,\sqrt{bx^2+a}\,Bax^4}{35\,b^2}+\frac{\sqrt{bx^2+a}\,Ax^4}{5\,b}+\frac{8\,\sqrt{bx^2+a}\,Ba^2x^2}{35\,b^3}-\frac{4\,\sqrt{bx^2+a}\,Aax^2}{15\,b^2}-\frac{16\,\sqrt{bx^2+a}\,Ba^3}{35\,b^4}+\frac{8\,\sqrt{bx^2+a}\,Aa^2}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*B*x^6/b - 6/35*sqrt(b*x^2 + a)*B*a*x^4/b^2 + 1/5*sqrt(b
*x^2 + a)*A*x^4/b + 8/35*sqrt(b*x^2 + a)*B*a^2*x^2/b^3 - 4/15*sqrt(b*x^2 +
a)*A*a*x^2/b^2 - 16/35*sqrt(b*x^2 + a)*B*a^3/b^4 + 8/15*sqrt(b*x^2 + a)*A*a
^2/b^3
```

Fricas [A]
time = 1.34, size = 76, normalized size = 0.76

$$\frac{\left(15\,Bb^3x^6-3\,(6\,Bab^2-7\,Ab^3)x^4-48\,Ba^3+56\,Aa^2b+4\,(6\,Ba^2b-7\,Aab^2)x^2\right)\sqrt{bx^2+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^3*x^6 - 3*(6*B*a*b^2 - 7*A*b^3)*x^4 - 48*B*a^3 + 56*A*a^2*b +
 4*(6*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

Sympy [A]
time = 0.35, size = 172, normalized size = 1.72

$$\begin{cases} \frac{8Aa^2\sqrt{a+bx^2}}{15b^3}-\frac{4Aax^2\sqrt{a+bx^2}}{15b^2}+\frac{Ax^4\sqrt{a+bx^2}}{5b}-\frac{16Ba^3\sqrt{a+bx^2}}{35b^4}+\frac{8Ba^2x^2\sqrt{a+bx^2}}{35b^3}-\frac{6Bax^4\sqrt{a+bx^2}}{35b^2}+\frac{Bx^6\sqrt{a+bx^2}}{7b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*A*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*A*a*x**2*sqrt(a + b*x**2
)/(15*b**2) + A*x**4*sqrt(a + b*x**2)/(5*b) - 16*B*a**3*sqrt(a + b*x**2)/(3
5*b**4) + 8*B*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*B*a*x**4*sqrt(a + b*
x**2)/(35*b**2) + B*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)), ((A*x**6/6 + B*
x**8/8)/sqrt(a), True))
```

Giac [A]
time = 1.41, size = 101, normalized size = 1.01

$$-\frac{(Ba^3-Aa^2b)\sqrt{bx^2+a}}{b^4}+\frac{15\,(bx^2+a)^{\frac{7}{2}}B-63\,(bx^2+a)^{\frac{5}{2}}Ba+105\,(bx^2+a)^{\frac{3}{2}}Ba^2+21\,(bx^2+a)^{\frac{5}{2}}Ab-70\,(bx^2+a)^{\frac{3}{2}}Aab}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(B*a^3 - A*a^2*b)*sqrt(b*x^2 + a)/b^4 + 1/105*(15*(b*x^2 + a)^(7/2)*B - 63
*(b*x^2 + a)^(5/2)*B*a + 105*(b*x^2 + a)^(3/2)*B*a^2 + 21*(b*x^2 + a)^(5/2)
*A*b - 70*(b*x^2 + a)^(3/2)*A*a*b)/b^4
```

Mupad [B]
time = 0.32, size = 80, normalized size = 0.80

$$-\sqrt{b\,x^2+a}\,\left(\frac{48\,B\,a^3-56\,A\,a^2\,b}{105\,b^4}-\frac{B\,x^6}{7\,b}-\frac{x^4\,\left(21\,A\,b^3-18\,B\,a\,b^2\right)}{105\,b^4}+\frac{4\,a\,x^2\,\left(7\,A\,b-6\,B\,a\right)}{105\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

```
[Out] -(a + b*x^2)^(1/2)*((48*B*a^3 - 56*A*a^2*b)/(105*b^4) - (B*x^6)/(7*b) - (x^
4*(21*A*b^3 - 18*B*a*b^2))/(105*b^4) + (4*a*x^2*(7*A*b - 6*B*a))/(105*b^3))
```

3.556 $\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3}+\frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2}+\frac{Bx^5\sqrt{a+bx^2}}{6b}+\frac{a^2(6Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

```
[Out] 1/16*a^2*(6*A*b-5*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)-1/16*a*(6
*A*b-5*B*a)*x*(b*x^2+a)^(1/2)/b^3+1/24*(6*A*b-5*B*a)*x^3*(b*x^2+a)^(1/2)/b^
2+1/6*B*x^5*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {470, 327, 223, 212}

$$\frac{a^2(6Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}-\frac{ax\sqrt{a+bx^2}\,(6Ab-5aB)}{16b^3}+\frac{x^3\sqrt{a+bx^2}\,(6Ab-5aB)}{24b^2}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] -1/16*(a*(6*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/b^3 + ((6*A*b - 5*a*B)*x^3*Sqrt
[a + b*x^2])/(24*b^2) + (B*x^5*Sqrt[a + b*x^2])/(6*b) + (a^2*(6*A*b - 5*a*B
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{Bx^5\sqrt{a+bx^2}}{6b} - \frac{(-6Ab+5aB)\int \frac{x^4}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} - \frac{(a(6Ab-5aB))\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{8b^2}$$

$$= -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} + \frac{(a^2(6A}{}$$

$$= -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} + \frac{(a^2(6A}{}$$

$$= -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} + \frac{a^2(6A}{}$$

Mathematica [A]
time = 0.13, size = 100, normalized size = 0.82

$$\frac{x\sqrt{a+bx^2}\,(-18aAb+15a^2B+12Ab^2x^2-10abBx^2+8b^2Bx^4)}{48b^3} + \frac{a^2(-6Ab+5aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(-18*a*A*b + 15*a^2*B + 12*A*b^2*x^2 - 10*a*b*B*x^2 + 8*
b^2*B*x^4))/(48*b^3) + (a^2*(-6*A*b + 5*a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*
x^2]])/(16*b^(7/2))
```

Maple [A]
time = 0.09, size = 154, normalized size = 1.26

method	result

risch	$-\frac{x\left(-8b^{2}B\,x^{4}-12A\,b^{2}x^{2}+10Bab\,x^{2}+18abA-15a^{2}B\right)\sqrt{b\,x^{2}+a}}{48b^{3}}+\frac{3a^{2}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)A}{8b^{\frac{5}{2}}}-\frac{5a^{3}\ln\left(x\sqrt{b}+\sqrt{b\right.}}{16b^{\frac{7}{2}}}$
default	$B\left(\frac{x^{5}\sqrt{b\,x^{2}+a}}{6b}-\frac{5a\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+A\left(\frac{x^{3}\sqrt{b\,x^{2}+\right.}}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x^5/b*(b*x^2+a)^(1/2)-5/6*a/b*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/
2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(1/4
*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.29, size = 128, normalized size = 1.05

$$\frac{\sqrt{bx^{2}+a}\,Bx^{5}}{6\,b}-\frac{5\,\sqrt{bx^{2}+a}\,Bax^{3}}{24\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,Ax^{3}}{4\,b}+\frac{5\,\sqrt{bx^{2}+a}\,Ba^{2}x}{16\,b^{3}}-\frac{3\,\sqrt{bx^{2}+a}\,Aax}{8\,b^{2}}-\frac{5\,Ba^{3}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}+\frac{3\,Aa^{2}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*B*x^5/b - 5/24*sqrt(b*x^2 + a)*B*a*x^3/b^2 + 1/4*sqrt(b
*x^2 + a)*A*x^3/b + 5/16*sqrt(b*x^2 + a)*B*a^2*x/b^3 - 3/8*sqrt(b*x^2 + a)*
A*a*x/b^2 - 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/8*A*a^2*arcsinh(b
*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 1.21, size = 211, normalized size = 1.73

$$\left[-\frac{3\,(5\,Ba^{3}-6\,Aa^{2}b)\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,(8\,Bb^{3}x^{5}-2\,(5\,Bab^{2}-6\,Ab^{3})x^{3}+3\,(5\,Ba^{2}b-6\,Aab^{2})x)\sqrt{bx^{2}+a}}{96\,b^{4}},\ \frac{3\,(5\,Ba^{3}-6\,Aa^{2}b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+(8\,Bb^{3}x^{5}-2\,(5\,Bab^{2}-6\,Ab^{3})x^{3}+3\,(5\,Ba^{2}b-6\,Aab^{2})x)\sqrt{bx^{2}+a}}{48\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*B*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(8*B*b^3*x^5 - 2*(5*B*a*b^2 - 6*A*b^3)*x^3 + 3*(5*B*a^2*b
- 6*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/48*(3*(5*B*a^3 - 6*A*a^2*b)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^3*x^5 - 2*(5*B*a*b^2 - 6*A*b^
3)*x^3 + 3*(5*B*a^2*b - 6*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(114) = 228$.
time = 10.88, size = 235, normalized size = 1.93

$$-\frac{3Aa^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{A\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Ax^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{5}{2}}x}{16b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{3}{2}}x^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}\,x^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{Bx^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*A*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - A*sqrt(a)*x**3/(8*b*sqrt(1 +
b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + A*x**5/(4*sqr
t(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(5/2)*x/(16*b**3*sqrt(1 + b*x**2/a)) + 5*
B*a**(3/2)*x**3/(48*b**2*sqrt(1 + b*x**2/a)) - B*sqrt(a)*x**5/(24*b*sqrt(1
+ b*x**2/a)) - 5*B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + B*x**7/(6*
sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.33, size = 107, normalized size = 0.88

$$\frac{1}{48}\left(2\left(\frac{4\,Bx^2}{b}-\frac{5\,Bab^3-6\,Ab^4}{b^5}\right)x^2+\frac{3\,(5\,Ba^2b^2-6\,Aab^3)}{b^5}\right)\sqrt{bx^2+a}\;x+\frac{(5\,Ba^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2/b - (5*B*a*b^3 - 6*A*b^4)/b^5)*x^2 + 3*(5*B*a^2*b^2 - 6*A*
a*b^3)/b^5)*sqrt(b*x^2 + a)*x + 1/16*(5*B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(B\,x^2+A\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^4*(A + B*x^2))/(a + b*x^2)^(1/2), x)
```

3.557 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=71

$$-\frac{a(Ab-aB)\sqrt{a+bx^2}}{b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{B(a+bx^2)^{5/2}}{5b^3}$$

```
[Out] 1/3*(A*b-2*B*a)*(b*x^2+a)^(3/2)/b^3+1/5*B*(b*x^2+a)^(5/2)/b^3-a*(A*b-B*a)*(
b*x^2+a)^(1/2)/b^3
```

Rubi [A]

time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-2aB)}{3b^3}-\frac{a\sqrt{a+bx^2}\,(Ab-aB)}{b^3}+\frac{B(a+bx^2)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(A*b - a*B)*Sqrt[a + b*x^2])/b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/
(3*b^3) + (B*(a + b*x^2)^(5/2))/(5*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2\sqrt{a+bx}}+\frac{(Ab-2aB)\sqrt{a+bx}}{b^2}+\frac{B(a+bx)^{3/2}}{b^2}\right)\,dx,x,x^2\right)\\ &= -\frac{a(Ab-aB)\sqrt{a+bx^2}}{b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\left(-10aAb+8a^2B+5Ab^2x^2-4abBx^2+3b^2Bx^4\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(-10*a*A*b + 8*a^2*B + 5*A*b^2*x^2 - 4*a*b*B*x^2 + 3*b^2*B
*x^4))/(15*b^3)
```

Maple [A]
time = 0.08, size = 96, normalized size = 1.35

method	result	siz
gosper	$-\frac{\sqrt{b\,x^2+a}\,\left(-3b^2B\,x^4-5A\,b^2x^2+4Bab\,x^2+10abA-8a^2B\right)}{15b^3}$	53
trager	$-\frac{\sqrt{b\,x^2+a}\,\left(-3b^2B\,x^4-5A\,b^2x^2+4Bab\,x^2+10abA-8a^2B\right)}{15b^3}$	53
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-3b^2B\,x^4-5A\,b^2x^2+4Bab\,x^2+10abA-8a^2B\right)}{15b^3}$	53
default	$B\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)+A\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] B*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(
b*x^2+a)^(1/2)))+A*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.30, size = 90, normalized size = 1.27

$$\frac{\sqrt{bx^2+a}\,Bx^4}{5\,b}-\frac{4\,\sqrt{bx^2+a}\,Bax^2}{15\,b^2}+\frac{\sqrt{bx^2+a}\,Ax^2}{3\,b}+\frac{8\,\sqrt{bx^2+a}\,Ba^2}{15\,b^3}-\frac{2\,\sqrt{bx^2+a}\,Aa}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(b*x^2 + a)*B*x^4/b - 4/15*sqrt(b*x^2 + a)*B*a*x^2/b^2 + 1/3*sqrt(b
*x^2 + a)*A*x^2/b + 8/15*sqrt(b*x^2 + a)*B*a^2/b^3 - 2/3*sqrt(b*x^2 + a)*A*
a/b^2
```

Fricas [A]
time = 1.51, size = 52, normalized size = 0.73

$$\frac{\left(3\,Bb^2x^4+8\,Ba^2-10\,Aab-\left(4\,Bab-5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^4 + 8*B*a^2 - 10*A*a*b - (4*B*a*b - 5*A*b^2)*x^2)*sqrt(b*x^
2 + a)/b^3
```

Sympy [A]
time = 0.30, size = 121, normalized size = 1.70

$$\begin{cases} -\frac{2Aa\sqrt{a+bx^2}}{3b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{8Ba^2\sqrt{a+bx^2}}{15b^3}-\frac{4Bax^2\sqrt{a+bx^2}}{15b^2}+\frac{Bx^4\sqrt{a+bx^2}}{5b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*A*a*sqrt(a + b*x**2)/(3*b**2) + A*x**2*sqrt(a + b*x**2)/(3*b)
 + 8*B*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*B*a*x**2*sqrt(a + b*x**2)/(15*b*
*2) + B*x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/sqrt
(a), True))
```

Giac [A]
time = 1.28, size = 69, normalized size = 0.97

$$\frac{\left(Ba^2-Aab\right)\sqrt{bx^2+a}}{b^3}+\frac{3\,\left(bx^2+a\right)^{\frac{5}{2}}B-10\,\left(bx^2+a\right)^{\frac{3}{2}}Ba+5\,\left(bx^2+a\right)^{\frac{3}{2}}Ab}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (B*a^2 - A*a*b)*sqrt(b*x^2 + a)/b^3 + 1/15*(3*(b*x^2 + a)^(5/2)*B - 10*(b*x
^2 + a)^(3/2)*B*a + 5*(b*x^2 + a)^(3/2)*A*b)/b^3
```

Mupad [B]
time = 0.30, size = 57, normalized size = 0.80

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,B\,a^2-10\,A\,a\,b}{15\,b^3}+\frac{x^2\,\left(5\,A\,b^2-4\,B\,a\,b\right)}{15\,b^3}+\frac{B\,x^4}{5\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*B*a^2 - 10*A*a*b)/(15*b^3) + (x^2*(5*A*b^2 - 4*B*a*b)
)/(15*b^3) + (B*x^4)/(5*b))
```

3.558 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

```
[Out] -1/8*a*(4*A*b-3*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/8*(4*A*b-
3*B*a)*x*(b*x^2+a)^(1/2)/b^2+1/4*B*x^3*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {470, 327, 223, 212}

$$-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}\,(4Ab-3aB)}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] ((4*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(8*b^2) + (B*x^3*Sqrt[a + b*x^2])/(4*b)
 - (a*(4*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx &= \frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{(-4Ab+3aB)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b}\\ &= \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{(a(4Ab-3aB))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2}\\ &= \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{(a(4Ab-3aB))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \ldots\right)}{8b^2}\\ &= \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 76, normalized size = 0.85

$$\frac{x\sqrt{a+bx^2}\,(4Ab-3aB+2bBx^2)}{8b^2}-\frac{a(-4Ab+3aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(4*A*b - 3*a*B + 2*b*B*x^2))/(8*b^2) - (a*(-4*A*b + 3*a*
B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A]
time = 0.09, size = 106, normalized size = 1.19

method	result
risch	$\frac{x(2bB\,x^2+4Ab-3Ba)\sqrt{b\,x^2+a}}{8b^2}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)A}{2b^{\frac{3}{2}}}+\frac{3a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)B}{8b^{\frac{5}{2}}}$

default	$B\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+A\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\right.}}{2b^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))+A*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*l
n(x*b^(1/2)+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.30, size = 86, normalized size = 0.97

$$\frac{\sqrt{bx^{2}+a}\,Bx^{3}}{4\,b}-\frac{3\,\sqrt{bx^{2}+a}\,Bax}{8\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,Ax}{2\,b}+\frac{3\,Ba^{2}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*B*x^3/b - 3/8*sqrt(b*x^2 + a)*B*a*x/b^2 + 1/2*sqrt(b*x^
2 + a)*A*x/b + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/2*A*a*arcsinh(b
*x/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 1.12, size = 162, normalized size = 1.82

$$\left[-\frac{(3\,Ba^{2}-4\,Aab)\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,(2\,Bb^{2}x^{3}-(3\,Bab-4\,Ab^{2})x)\sqrt{bx^{2}+a}}{16\,b^{3}},\,-\frac{(3\,Ba^{2}-4\,Aab)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(2\,Bb^{2}x^{3}-(3\,Bab-4\,Ab^{2})x)\sqrt{bx^{2}+a}}{8\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*B*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) - 2*(2*B*b^2*x^3 - (3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1
/8*((3*B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*
b^2*x^3 - (3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 4.21, size = 150, normalized size = 1.69

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^{2}}{a}}}{2b}-\frac{Aa\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{3Ba^{\frac{3}{2}}x}{8b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{B\sqrt{a}\,x^{3}}{8b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Ba^{2}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^{5}}{4\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - A*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) - 3*B*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - B*sqrt(a)*x**3/(8*b*sq
rt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + B*x**5
/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.93, size = 75, normalized size = 0.84

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,Bx^2}{b}-\frac{3\,Bab-4\,Ab^2}{b^3}\right)x-\frac{(3\,Ba^2-4\,Aab)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*B*x^2/b - (3*B*a*b - 4*A*b^2)/b^3)*x - 1/8*(3*B*a^2
- 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(B\,x^2+A)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^(1/2),x)
[Out] int((x^2*(A + B*x^2))/(a + b*x^2)^(1/2), x)
```

3.559 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{(Ab-aB)\sqrt{a+bx^2}}{b^2}+\frac{B(a+bx^2)^{3/2}}{3b^2}$$

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b^2+(A*b-B*a)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\frac{\sqrt{a+bx^2}\,(Ab-aB)}{b^2}+\frac{B(a+bx^2)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
[Out] ((A*b - a*B)*Sqrt[a + b*x^2])/b^2 + (B*(a + b*x^2)^(3/2))/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab-aB}{b\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b}\right)dx,x,x^2\right)\\ &= \frac{(Ab-aB)\sqrt{a+bx^2}}{b^2}+\frac{B(a+bx^2)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(3Ab-2aB+bBx^2\right)}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]

[Out] (Sqrt[a + b*x^2]*(3*A*b - 2*a*B + b*B*x^2))/(3*b^2)

Maple [A]
time = 0.08, size = 51, normalized size = 1.19

method	result	size
gosper	$\frac{\sqrt{b\,x^2+a}\,\left(bB\,x^2+3Ab-2Ba\right)}{3b^2}$	30
trager	$\frac{\sqrt{b\,x^2+a}\,\left(bB\,x^2+3Ab-2Ba\right)}{3b^2}$	30
risch	$\frac{\sqrt{b\,x^2+a}\,\left(bB\,x^2+3Ab-2Ba\right)}{3b^2}$	30
default	$B\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+\frac{A\sqrt{b\,x^2+a}}{b}$	51

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] B*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+A*(b*x^2+a)^(1/2)/b

Maxima [A]
time = 0.32, size = 49, normalized size = 1.14

$$\frac{\sqrt{bx^2+a}\,Bx^2}{3\,b}-\frac{2\,\sqrt{bx^2+a}\,Ba}{3\,b^2}+\frac{\sqrt{bx^2+a}\,A}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(b*x^2 + a)*B*x^2/b - 2/3*sqrt(b*x^2 + a)*B*a/b^2 + sqrt(b*x^2 + a)*A/b

Fricas [A]
time = 1.33, size = 29, normalized size = 0.67

$$\frac{\left(Bbx^2-2\,Ba+3\,Ab\right)\sqrt{bx^2+a}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^2 - 2*B*a + 3*A*b)*sqrt(b*x^2 + a)/b^2
```

Sympy [A]
time = 0.23, size = 70, normalized size = 1.63

$$\begin{cases} \frac{A\sqrt{a+bx^2}}{b} - \frac{2Ba\sqrt{a+bx^2}}{3b^2} + \frac{Bx^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((A*sqrt(a + b*x**2)/b - 2*B*a*sqrt(a + b*x**2)/(3*b**2) + B*x**2*
sqrt(a + b*x**2)/(3*b), Ne(b, 0)), ((A*x**2/2 + B*x**4/4)/sqrt(a), True))
```

Giac [A]
time = 1.27, size = 38, normalized size = 0.88

$$\frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b^2} - \frac{\sqrt{bx^2+a}\,(Ba-Ab)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)*B/b^2 - sqrt(b*x^2 + a)*(B*a - A*b)/b^2
```

Mupad [B]
time = 0.28, size = 34, normalized size = 0.79

$$\left(\frac{3\,A\,b-2\,B\,a}{3\,b^2}+\frac{B\,x^2}{3\,b}\right)\,\sqrt{b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

```
[Out] ((3*A*b - 2*B*a)/(3*b^2) + (B*x^2)/(3*b))*(a + b*x^2)^(1/2)
```

3.560 $\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{Bx\sqrt{a+bx^2}}{2b}+\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] 1/2*(2*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*B*x*(b*x^2+a
)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {396, 223, 212}

$$\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx = \frac{Bx\sqrt{a+bx^2}}{2b} - \frac{(-2Ab+aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$

$$= \frac{Bx\sqrt{a+bx^2}}{2b} - \frac{(-2Ab+aB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$

$$= \frac{Bx\sqrt{a+bx^2}}{2b} + \frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 1.02

$$\frac{Bx\sqrt{a+bx^2}}{2b} + \frac{(-2Ab+aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) + ((-2*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Maple [A]
time = 0.08, size = 63, normalized size = 1.09

method	result	size
risch	$\frac{Bx\sqrt{b\,x^2+a}}{2b}+\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)Ba}{2b^{\frac{3}{2}}}$	62
default	$B\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+A*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.31, size = 47, normalized size = 0.81

$$\frac{\sqrt{bx^2+a}\,Bx}{2\,b} - \frac{Ba\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{A\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + A*arcs
inh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 1.27, size = 110, normalized size = 1.90

$$\left[\frac{2\sqrt{bx^2+a}\,Bbx-(Ba-2\,Ab)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b^2},\ \frac{\sqrt{bx^2+a}\,Bbx+(Ba-2\,Ab)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*B*b*x - (B*a - 2*A*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*B*b*x + (B*a - 2*A*b)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

Sympy [A]
time = 1.52, size = 126, normalized size = 2.17

$$A\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\,\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\,\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\,\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\: b>0\wedge a<0\end{cases}\right)+\frac{B\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sq
rt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x
*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)
/(2*b) - B*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2))
```

Giac [A]
time = 1.73, size = 48, normalized size = 0.83

$$\frac{\sqrt{bx^2+a}\,Bx}{2\,b}+\frac{(Ba-2\,Ab)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b + 1/2*(B*a - 2*A*b)*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

Mupad [B]
time = 0.50, size = 86, normalized size = 1.48

$$\begin{cases} \frac{B\,x^3+3\,A\,x}{3\,\sqrt{a}} & \text{if}\ \ b=0\\ \frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{B\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{B\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if}\ \ b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (3*A*x + B*x^3)/(3*a^(1/2)), b ~= 0, (A*log(b^(1/2)*x + (
a + b*x^2)^(1/2)))/b^(1/2) - (B*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(
2*b^(3/2)) + (B*x*(a + b*x^2)^(1/2))/(2*b))
```

3.561 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 81, 65, 214}

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{B\sqrt{a+bx^2}}{b}+\frac{1}{2}A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{B\sqrt{a+bx^2}}{b}+\frac{A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= \frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 1.00

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.08, size = 45, normalized size = 1.05

method	result	size
default	$\frac{B\sqrt{b\,x^2+a}}{b}-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(b*x^2+a)^(1/2)/b-A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [A]
time = 0.33, size = 33, normalized size = 0.77

$$-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*B/b
```

Fricas [A]
time = 2.90, size = 102, normalized size = 2.37

$$\left[\frac{A\sqrt{a}\,b\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)+2\sqrt{bx^2+a}\,Ba}{2\,ab},\frac{A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\,Ba}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*s
qrt(b*x^2 + a)*B*a)/(a*b), (A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
 sqrt(b*x^2 + a)*B*a)/(a*b)]
```

Sympy [A]
time = 4.74, size = 61, normalized size = 1.42

$$\frac{A\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+bx^2}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{B\left(\begin{cases}-\frac{x^2}{\sqrt{a}} & \text{for } b=0\\ -\frac{2\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] A*atan(1/(sqrt(-1/a)*sqrt(a + b*x**2)))/(a*sqrt(-1/a)) - B*Piecewise((-x**2
/sqrt(a), Eq(b, 0)), (-2*sqrt(a + b*x**2)/b, True))/2
```

Giac [A]
time = 1.27, size = 38, normalized size = 0.88

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{bx^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)*B/b
```

Mupad [B]
time = 0.52, size = 35, normalized size = 0.81

$$\frac{B\sqrt{b\,x^2+a}}{b}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (B*(a + b*x^2)^(1/2))/b - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
```

3.562 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{A\sqrt{a+bx^2}}{ax}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {462, 223, 212}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqr
t[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{ax}+B\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= -\frac{A\sqrt{a+bx^2}}{ax}+B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 1.06

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt
[b]
```

Maple [A]
time = 0.08, size = 41, normalized size = 0.87

method	result	size
default	$\frac{B\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{A\sqrt{b\,x^2+a}}{ax}$	41
risch	$\frac{B\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{A\sqrt{b\,x^2+a}}{ax}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.70

$$\frac{B\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{\sqrt{bx^2+a}\,A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] B*arcsinh(b*x/sqrt(a*b))/sqrt(b) - sqrt(b*x^2 + a)*A/(a*x)
```

Fricas [A]
time = 1.58, size = 109, normalized size = 2.32

$$\left[\frac{Ba\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,\sqrt{bx^2+a}\,Ab}{2\,abx},-\frac{Ba\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\,Ab}{abx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sqr
t(b*x^2 + a)*A*b)/(a*b*x), -(B*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + sqrt(b*x^2 + a)*A*b)/(a*b*x)]
```

Sympy [A]
time = 0.76, size = 99, normalized size = 2.11

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}+B\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a + B*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/
a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a
> 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0
)))
```

Giac [A]
time = 0.93, size = 58, normalized size = 1.23

$$-\frac{B\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{2\,\sqrt{b}}+\frac{2\,A\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*B*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2*A*sqrt(b)/((sqrt(b)
*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B]
time = 0.36, size = 40, normalized size = 0.85

$$\frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{A\,\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (A*(a + b*x^2)^(1/2))/(a*x
)
```

3.563 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$-\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[Out] 1/2*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*A*(b*x^2+a)^(1
/2)/a/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {457, 79, 65, 214}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^2])/(a*x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(2*a^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 1.00

$$-\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^2])/(a*x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(2*a^(3/2))
```

Maple [A]
time = 0.10, size = 80, normalized size = 1.38

method	result	size

risch	$-\frac{A\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)Ab}{2a^{\frac{3}{2}}}-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}$	79
default	$A\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/
2))/x))-B/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.97

$$-\frac{B\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}\,A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs
(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

Fricas [A]
time = 1.45, size = 124, normalized size = 2.14

$$\left[-\frac{(2\,Ba-Ab)\sqrt{a}\,x^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}\,Aa}{4\,a^2x^2},\ \frac{(2\,Ba-Ab)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,Aa}{2\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*sqrt(a)*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) + 2*sqrt(b*x^2 + a)*A*a)/(a^2*x^2), 1/2*((2*B*a - A*b)*sqrt(-a)*x
^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*A*a)/(a^2*x^2)]
```

Sympy [A]
time = 11.30, size = 66, normalized size = 1.14

$$-\frac{A\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}-\frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2
*a**(3/2)) - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Giac [A]
time = 1.23, size = 62, normalized size = 1.07

$$\frac{\frac{(2\,Bab-Ab^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{\sqrt{bx^2+a}\,Ab}{ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt
(b*x^2 + a)*A*b/(a*x^2))/b
```

Mupad [B]
time = 0.60, size = 60, normalized size = 1.03

$$\frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (A*(a + b*x^2)^(1/2))/
(2*a*x^2) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
```

3.564 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=53

$$-\frac{A\sqrt{a+bx^2}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2}}{3a^2x}$$

```
[Out] -1/3*A*(b*x^2+a)^(1/2)/a/x^3+1/3*(2*A*b-3*B*a)*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {464, 270}

$$\frac{\sqrt{a+bx^2}\,(2Ab-3aB)}{3a^2x}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/3*(A*Sqrt[a + b*x^2])/(a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2])/(3*a^2
*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{3ax^3}-\frac{(2Ab-3aB)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\ &= -\frac{A\sqrt{a+bx^2}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2}}{3a^2x}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.75

$$\frac{\sqrt{a+bx^2}\,(-aA+2Abx^2-3aBx^2)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-(a*A) + 2*A*b*x^2 - 3*a*B*x^2))/(3*a^2*x^3)
```

Maple [A]
time = 0.08, size = 58, normalized size = 1.09

method	result	size
gosper	$-\frac{\sqrt{b\,x^2+a}\,\left(-2Ab\,x^2+3Ba\,x^2+Aa\right)}{3a^2x^3}$	36
trager	$-\frac{\sqrt{b\,x^2+a}\,\left(-2Ab\,x^2+3Ba\,x^2+Aa\right)}{3a^2x^3}$	36
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-2Ab\,x^2+3Ba\,x^2+Aa\right)}{3a^2x^3}$	36
default	$A\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)-\frac{B\sqrt{b\,x^2+a}}{ax}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2*(b*x^2+a)^(1/2)/x)-B*(b*x^2+a)^(1/2
)/a/x
```

Maxima [A]
time = 0.29, size = 56, normalized size = 1.06

$$-\frac{\sqrt{bx^2+a}\,B}{ax}+\frac{2\,\sqrt{bx^2+a}\,Ab}{3\,a^2x}-\frac{\sqrt{bx^2+a}\,A}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x^2 + a)*B/(a*x) + 2/3*sqrt(b*x^2 + a)*A*b/(a^2*x) - 1/3*sqrt(b*x^2
 + a)*A/(a*x^3)
```

Fricas [A]
time = 1.86, size = 34, normalized size = 0.64

$$-\frac{((3\,Ba-2\,Ab)x^2+Aa)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*B*a - 2*A*b)*x^2 + A*a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A]
time = 0.96, size = 70, normalized size = 1.32

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*A*b**(3/2)*sqrt(a/(b*x**2) +
 1)/(3*a**2) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(45) = 90.
time = 1.23, size = 120, normalized size = 2.26

$$\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4B\sqrt{b}-6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ab^{\frac{3}{2}}+3\,Ba^2\sqrt{b}-2\,Aab^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a*sqrt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(3/2) + 3*B*a
^2*sqrt(b) - 2*A*a*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [B]
time = 0.29, size = 35, normalized size = 0.66

$$-\frac{\sqrt{b\,x^2+a}\,\left(A\,a-2\,A\,b\,x^2+3\,B\,a\,x^2\right)}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(A*a - 2*A*b*x^2 + 3*B*a*x^2))/(3*a^2*x^3)
```

3.565 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=90

$$-\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

```
[Out] -1/8*b*(3*A*b-4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)-1/4*A*(b*x^2+
a)^(1/2)/a/x^4+1/8*(3*A*b-4*B*a)*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 44, 65, 214}

$$-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{\sqrt{a+bx^2}\,(3Ab-4aB)}{8a^2x^2}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/4*(A*Sqrt[a + b*x^2])/(a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^2
*x^2) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{\left(-\frac{3Ab}{2}+2aB\right)\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{4a}\\
&= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}+\frac{(b(3Ab-4aB))\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x\right.}{16a^2}\\
&= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}+\frac{(3Ab-4aB)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\right.}{8a^2}\\
&= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 78, normalized size = 0.87

$$
\frac{\sqrt{a+bx^2}\left(-2aA+3Abx^2-4aBx^2\right)}{8a^2x^4}+\frac{b(-3Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*A + 3*A*b*x^2 - 4*a*B*x^2))/(8*a^2*x^4) + (b*(-3*A*b
 + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Maple [A]
time = 0.09, size = 124, normalized size = 1.38

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-3Ab\,x^{2}+4Ba\,x^{2}+2Aa\right)}{8a^{2}x^{4}}-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)A}{8a^{\frac{5}{2}}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)B}{2a^{\frac{3}{2}}}$
default	$A\left(-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)+B\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{}}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4/a/x^4*(b*x^2+a)^(1/2)-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(
3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+B*(-1/2*(b*x^2+a)^(1/2)/a/x^2+
1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.30, size = 96, normalized size = 1.07

$$\frac{Bb\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{3\,Ab^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}-\frac{\sqrt{bx^{2}+a}\,B}{2\,ax^{2}}+\frac{3\,\sqrt{bx^{2}+a}\,Ab}{8\,a^{2}x^{2}}-\frac{\sqrt{bx^{2}+a}\,A}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/8*A*b^2*arcsinh(a/(sqrt(a
*b)*abs(x)))/a^(5/2) - 1/2*sqrt(b*x^2 + a)*B/(a*x^2) + 3/8*sqrt(b*x^2 + a)*
A*b/(a^2*x^2) - 1/4*sqrt(b*x^2 + a)*A/(a*x^4)
```

Fricas [A]
time = 1.97, size = 171, normalized size = 1.90

$$\left[-\frac{(4\,Bab-3\,Ab^{2})\sqrt{a}\,x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(2\,Aa^{2}+(4\,Ba^{2}-3\,Aab)x^{2})\sqrt{bx^{2}+a}}{16\,a^{3}x^{4}},-\frac{(4\,Bab-3\,Ab^{2})\sqrt{-a}\,x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(2\,Aa^{2}+(4\,Ba^{2}-3\,Aab)x^{2})\sqrt{bx^{2}+a}}{8\,a^{3}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) + 2*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(
a^3*x^4), -1/8*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2
 + a)) + (2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^4)]
```

Sympy [A]
time = 25.88, size = 150, normalized size = 1.67

$$-\frac{A}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3Ab^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] -A/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(8*a*x**3*sqrt(a/(b*x*
*2) + 1)) + 3*A*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh(s
qrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) +
 B*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2))
```

Giac [A]
time = 1.41, size = 121, normalized size = 1.34

$$-\frac{\frac{(4\,Bab^2-3\,Ab^3)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{4\,(bx^2+a)^{\frac{3}{2}}Bab^2-4\sqrt{bx^2+a}\,Ba^2b^2-3\,(bx^2+a)^{\frac{3}{2}}Ab^3+5\sqrt{bx^2+a}\,Aab^3}{a^2b^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2)
 + (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^2*b^2 - 3*(b*x^2 +
a)^(3/2)*A*b^3 + 5*sqrt(b*x^2 + a)*A*a*b^3)/(a^2*b^2*x^4))/b
```

Mupad [B]
time = 0.70, size = 99, normalized size = 1.10

$$\frac{3\,A\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{5\,A\,\sqrt{b\,x^2+a}}{8\,a\,x^4}-\frac{3\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{B\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^(1/2)),x)
```

```
[Out] (3*A*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (5*A*(a + b*x^2)^(1/2))/(8*a*x^4) - (
3*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2)) - (B*(a + b*x^2)^(1/2
))/(2*a*x^2) + (B*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2))
```

3.566 $\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=84

$$-\frac{A\sqrt{a+bx^2}}{5ax^5}+\frac{(4Ab-5aB)\sqrt{a+bx^2}}{15a^2x^3}-\frac{2b(4Ab-5aB)\sqrt{a+bx^2}}{15a^3x}$$

```
[Out] -1/5*A*(b*x^2+a)^(1/2)/a/x^5+1/15*(4*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^2/x^3-2/1
5*b*(4*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$-\frac{2b\sqrt{a+bx^2}\,(4Ab-5aB)}{15a^3x}+\frac{\sqrt{a+bx^2}\,(4Ab-5aB)}{15a^2x^3}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/5*(A*Sqrt[a + b*x^2])/(a*x^5) + ((4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^
2*x^3) - (2*b*(4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^3*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{5ax^5}-\frac{(4Ab-5aB)\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx}{5a}\\
&= -\frac{A\sqrt{a+bx^2}}{5ax^5}+\frac{(4Ab-5aB)\sqrt{a+bx^2}}{15a^2x^3}+\frac{(2b(4Ab-5aB))\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{15a^2}\\
&= -\frac{A\sqrt{a+bx^2}}{5ax^5}+\frac{(4Ab-5aB)\sqrt{a+bx^2}}{15a^2x^3}-\frac{2b(4Ab-5aB)\sqrt{a+bx^2}}{15a^3x}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 62, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-3a^2A+4aAbx^2-5a^2Bx^2-8Ab^2x^4+10abBx^4)}{15a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2*A + 4*a*A*b*x^2 - 5*a^2*B*x^2 - 8*A*b^2*x^4 + 10*a
*b*B*x^4))/(15*a^3*x^5)
```

Maple [A]
time = 0.09, size = 102, normalized size = 1.21

method	result	size
gosper	$-\frac{\sqrt{b\,x^2+a}\,(8A\,b^2x^4-10Bab\,x^4-4aAb\,x^2+5B\,a^2x^2+3a^2A)}{15a^3x^5}$	59
trager	$-\frac{\sqrt{b\,x^2+a}\,(8A\,b^2x^4-10Bab\,x^4-4aAb\,x^2+5B\,a^2x^2+3a^2A)}{15a^3x^5}$	59
risch	$-\frac{\sqrt{b\,x^2+a}\,(8A\,b^2x^4-10Bab\,x^4-4aAb\,x^2+5B\,a^2x^2+3a^2A)}{15a^3x^5}$	59
default	$A\left(-\frac{\sqrt{b\,x^2+a}}{5a\,x^5}-\frac{4b\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)}{5a}\right)+B\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5/a/x^5*(b*x^2+a)^(1/2)-4/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2
*(b*x^2+a)^(1/2)/x))+B*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2*(b*x^2+a)^(1/2
)/x)
```

Maxima [A]
time = 0.32, size = 96, normalized size = 1.14

$$\frac{2\sqrt{bx^2+a}\,Bb}{3\,a^2x}-\frac{8\sqrt{bx^2+a}\,Ab^2}{15\,a^3x}-\frac{\sqrt{bx^2+a}\,B}{3\,ax^3}+\frac{4\sqrt{bx^2+a}\,Ab}{15\,a^2x^3}-\frac{\sqrt{bx^2+a}\,A}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^2 + a)*B*b/(a^2*x) - 8/15*sqrt(b*x^2 + a)*A*b^2/(a^3*x) - 1/3*
sqrt(b*x^2 + a)*B/(a*x^3) + 4/15*sqrt(b*x^2 + a)*A*b/(a^2*x^3) - 1/5*sqrt(b
*x^2 + a)*A/(a*x^5)
```

Fricas [A]
time = 1.68, size = 58, normalized size = 0.69

$$\frac{\left(2\left(5\,Bab-4\,Ab^2\right)x^4-3\,Aa^2-\left(5\,Ba^2-4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 - (5*B*a^2 - 4*A*a*b)*x^2)*sqrt(b
*x^2 + a)/(a^3*x^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(76) = 152.
time = 1.29, size = 355, normalized size = 4.23

$$-\frac{3Aa^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{2Aa^3b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{3Aa^2b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{12Aab^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{8Ab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*A*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x
**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*A*a**2*b**(13/
2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a*
*3*b**6*x**8) - 12*A*a*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*A*b**(17/2)*x**8*sqrt(a/(b*
x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - B*
sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*B*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(3*a**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(72) = 144.

time = 1.43, size = 176, normalized size = 2.10

$$\frac{4\left(15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6Bb^{\frac{3}{2}}-35\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Bab^{\frac{3}{2}}+40\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ab^{\frac{5}{2}}+25\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^2b^{\frac{3}{2}}-20\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Aab^{\frac{5}{2}}-5\,Ba^3b^{\frac{3}{2}}+4\,Aa^2b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*b^(3/2) - 35*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*B*a*b^(3/2) + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(5/2) + 2
5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*b^(3/2) - 20*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*A*a*b^(5/2) - 5*B*a^3*b^(3/2) + 4*A*a^2*b^(5/2))/((sqrt(b)*x - sq
rt(b*x^2 + a))^2 - a)^5
```

Mupad [B]
time = 0.34, size = 58, normalized size = 0.69

$$-\frac{\sqrt{b\,x^2+a}\,\left(5\,B\,a^2\,x^2+3\,A\,a^2-10\,B\,a\,b\,x^4-4\,A\,a\,b\,x^2+8\,A\,b^2\,x^4\right)}{15\,a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(3*A*a^2 + 5*B*a^2*x^2 + 8*A*b^2*x^4 - 4*A*a*b*x^2 - 10
*B*a*b*x^4))/(15*a^3*x^5)
```

3.567 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=123

$$-\frac{A\sqrt{a+bx^2}}{6ax^6}+\frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4}-\frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2}+\frac{b^2(5Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

```
[Out] 1/16*b^2*(5*A*b-6*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)-1/6*A*(b*x^
2+a)^(1/2)/a/x^6+1/24*(5*A*b-6*B*a)*(b*x^2+a)^(1/2)/a^2/x^4-1/16*b*(5*A*b-6
*B*a)*(b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 44, 65, 214}

$$\frac{b^2(5Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{b\sqrt{a+bx^2}\,(5Ab-6aB)}{16a^3x^2}+\frac{\sqrt{a+bx^2}\,(5Ab-6aB)}{24a^2x^4}-\frac{A\sqrt{a+bx^2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/6*(A*Sqrt[a + b*x^2])/(a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(24*a^
2*x^4) - (b*(5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a^3*x^2) + (b^2*(5*A*b - 6
*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{\left(-\frac{5Ab}{2}+3aB\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} + \frac{(b(5Ab-6aB))\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,\right.}{16a^2}$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2} - \frac{(b^2(5Ab-6a}{}$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2} - \frac{(b(5Ab-6a}{}$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2} + \frac{b^2(5Ab-6a}{}$$

Mathematica [A]

time = 0.20, size = 102, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\,(-8a^2A+10aAbx^2-12a^2Bx^2-15Ab^2x^4+18abBx^4)}{48a^3x^6}-\frac{b^2(-5Ab+6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A + 10*a*A*b*x^2 - 12*a^2*B*x^2 - 15*A*b^2*x^4 + 1
8*a*b*B*x^4))/(48*a^3*x^6) - (b^2*(-5*A*b + 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]])/(16*a^(7/2))
```

Maple [A]
time = 0.09, size = 172, normalized size = 1.40

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(15A\,b^2x^4-18Bab\,x^4-10aAb\,x^2+12B\,a^2x^2+8a^2A)}{48a^3x^6}+\frac{5b^3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)A}{16a^{\frac{7}{2}}}-\frac{3b^2\ln\left(\frac{2a+2\sqrt{a}\ldots}{}\right.}{}$
default	$B\left(-\frac{\sqrt{b\,x^2+a}}{4a\,x^4}-\frac{3b\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)+A\left(-\frac{\sqrt{b\,x^2+a}}{6a\,x^6}-\frac{5b\left(-\frac{\sqrt{b\,x\ldots}}{4\ldots}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4/a/x^4*(b*x^2+a)^(1/2)-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(
3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+A*(-1/6/a/x^6*(b*x^2+a)^(1/2)-
5/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(1/2)-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2
*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
```

Maxima [A]
time = 0.29, size = 138, normalized size = 1.12

$$-\frac{3\,Bb^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{5\,Ab^3 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{7}{2}}} + \frac{3\,\sqrt{bx^2+a}\,Bb}{8\,a^2x^2} - \frac{5\,\sqrt{bx^2+a}\,Ab^2}{16\,a^3x^2} - \frac{\sqrt{bx^2+a}\,B}{4\,ax^4} + \frac{5\,\sqrt{bx^2+a}\,Ab}{24\,a^2x^4} - \frac{\sqrt{bx^2+a}\,A}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/16*A*b^3*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(7/2) + 3/8*sqrt(b*x^2 + a)*B*b/(a^2*x^2) - 5/16*sqrt(b*
x^2 + a)*A*b^2/(a^3*x^2) - 1/4*sqrt(b*x^2 + a)*B/(a*x^4) + 5/24*sqrt(b*x^2
+ a)*A*b/(a^2*x^4) - 1/6*sqrt(b*x^2 + a)*A/(a*x^6)
```

Fricas [A]
time = 1.70, size = 223, normalized size = 1.81

$$\left[-\frac{3\,(6\,Bab^2-5\,Ab^3)\sqrt{a}\,x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(3\,(6\,Ba^2b-5\,Aab^2)x^4-8\,Aa^3-2\,(6\,Ba^3-5\,Aa^2b)x^2)\sqrt{bx^2+a}}{96\,a^4x^6},\ \frac{3\,(6\,Bab^2-5\,Ab^3)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(6\,Ba^2b-5\,Aab^2)x^4-8\,Aa^3-2\,(6\,Ba^3-5\,Aa^2b)x^2)\sqrt{bx^2+a}}{48\,a^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*a*b^2 - 5*A*b^3)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) - 2*(3*(6*B*a^2*b - 5*A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*B*
a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6), 1/48*(3*(6*B*a*b^2 - 5*A*
b^3)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(6*B*a^2*b - 5*A*a*
b^2)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6
)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(114) = 228$.
time = 46.84, size = 235, normalized size = 1.91

$$-\frac{A}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}}{24ax^5\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^{\frac{3}{2}}}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^{\frac{5}{2}}}{16a^3x\sqrt{\frac{a}{bx^2}+1}} + \frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{7}{2}}} - \frac{B}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3Bb^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -A/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(24*a*x**5*sqrt(a/(b*x
**2) + 1)) - 5*A*b**(3/2)/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5/2
)/(16*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(1
6*a**(7/2)) - B/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)/(8*a*x**3
*sqrt(a/(b*x**2) + 1)) + 3*B*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*B
*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2))
```

Giac [A]
time = 1.20, size = 158, normalized size = 1.28

$$\frac{\frac{3\,(6\,Bab^3-5\,Ab^4)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{18\,(bx^2+a)^{\frac{5}{2}}Bab^3-48\,(bx^2+a)^{\frac{3}{2}}Ba^2b^3+30\,\sqrt{bx^2+a}\,Ba^3b^3-15\,(bx^2+a)^{\frac{5}{2}}Ab^4+40\,(bx^2+a)^{\frac{3}{2}}Aab^4-33\,\sqrt{bx^2+a}\,Aa^2b^4}{a^3b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*a*b^3 - 5*A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^
3) + (18*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/2)*B*a^2*b^3 + 30*sq
rt(b*x^2 + a)*B*a^3*b^3 - 15*(b*x^2 + a)^(5/2)*A*b^4 + 40*(b*x^2 + a)^(3/2)
*A*a*b^4 - 33*sqrt(b*x^2 + a)*A*a^2*b^4)/(a^3*b^3*x^6))/b
```

Mupad [B]
time = 0.81, size = 140, normalized size = 1.14

$$\frac{5\,A\,(b\,x^2+a)^{3/2}}{6\,a^2\,x^6}-\frac{11\,A\,\sqrt{b\,x^2+a}}{16\,a\,x^6}-\frac{3\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{5\,A\,(b\,x^2+a)^{5/2}}{16\,a^3\,x^6}-\frac{5\,B\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,B\,(b\,x^2+a)^{3/2}}{8\,a^2\,x^4}-\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{16\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)^(1/2)),x)
```

```
[Out] (5*A*(a + b*x^2)^(3/2))/(6*a^2*x^6) - (3*B*b^2*atanh((a + b*x^2)^(1/2)/a^(1
/2)))/(8*a^(5/2)) - (11*A*(a + b*x^2)^(1/2))/(16*a*x^6) - (A*b^3*atan(((a +
 b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (5*A*(a + b*x^2)^(5/2))/(16*a
^3*x^6) - (5*B*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*B*(a + b*x^2)^(3/2))/(8*a^
2*x^4)
```

3.568 $\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$-\frac{A\sqrt{a+bx^2}}{7ax^7}+\frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5}-\frac{4b(6Ab-7aB)\sqrt{a+bx^2}}{105a^3x^3}+\frac{8b^2(6Ab-7aB)\sqrt{a+bx^2}}{105a^4x}$$

```
[Out] -1/7*A*(b*x^2+a)^(1/2)/a/x^7+1/35*(6*A*b-7*B*a)*(b*x^2+a)^(1/2)/a^2/x^5-4/1
05*b*(6*A*b-7*B*a)*(b*x^2+a)^(1/2)/a^3/x^3+8/105*b^2*(6*A*b-7*B*a)*(b*x^2+a
)^(1/2)/a^4/x
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 277, 270}

$$\frac{8b^2\sqrt{a+bx^2}\,(6Ab-7aB)}{105a^4x}-\frac{4b\sqrt{a+bx^2}\,(6Ab-7aB)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\,(6Ab-7aB)}{35a^2x^5}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/7*(A*Sqrt[a + b*x^2])/(a*x^7) + ((6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(35*a^
2*x^5) - (4*b*(6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(105*a^3*x^3) + (8*b^2*(6*A*
b - 7*a*B)*Sqrt[a + b*x^2])/(105*a^4*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{7ax^7} - \frac{(6Ab-7aB)\int \frac{1}{x^6\sqrt{a+bx^2}}\,dx}{7a}$$

$$= -\frac{A\sqrt{a+bx^2}}{7ax^7} + \frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5} + \frac{(4b(6Ab-7aB))\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx}{35a^2}$$

$$= -\frac{A\sqrt{a+bx^2}}{7ax^7} + \frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5} - \frac{4b(6Ab-7aB)\sqrt{a+bx^2}}{105a^3x^3} - \frac{(8b^2(6Ab-7}{}$$

$$= -\frac{A\sqrt{a+bx^2}}{7ax^7} + \frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5} - \frac{4b(6Ab-7aB)\sqrt{a+bx^2}}{105a^3x^3} + \frac{8b^2(6Ab-7a}{105}$$

Mathematica [A]
time = 0.12, size = 86, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-15a^3A+18a^2Abx^2-21a^3Bx^2-24aAb^2x^4+28a^2bBx^4+48Ab^3x^6-56ab^2Bx^6)}{105a^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^3*A + 18*a^2*A*b*x^2 - 21*a^3*B*x^2 - 24*a*A*b^2*x^
4 + 28*a^2*b*B*x^4 + 48*A*b^3*x^6 - 56*a*b^2*B*x^6))/(105*a^4*x^7)
```

Maple [A]
time = 0.09, size = 150, normalized size = 1.28

method	result
gosper	$-\frac{\sqrt{b\,x^2+a}\,\left(-48x^6Ab^3+56x^6Ba\,b^2+24Aa\,b^2x^4-28x^4Ba^2b-18x^2Aa^2b+21Ba^3x^2+15Aa^3\right)}{105x^7a^4}$
trager	$-\frac{\sqrt{b\,x^2+a}\,\left(-48x^6Ab^3+56x^6Ba\,b^2+24Aa\,b^2x^4-28x^4Ba^2b-18x^2Aa^2b+21Ba^3x^2+15Aa^3\right)}{105x^7a^4}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-48x^6Ab^3+56x^6Ba\,b^2+24Aa\,b^2x^4-28x^4Ba^2b-18x^2Aa^2b+21Ba^3x^2+15Aa^3\right)}{105x^7a^4}$

default	$B\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)}{5a}\right)+A\left(-\frac{\sqrt{b\,x^{2}+a}}{7a\,x^{7}}-\frac{6b\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b(\ldots}{\ldots}\right.}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/5/a/x^5*(b*x^2+a)^(1/2)-4/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2
*(b*x^2+a)^(1/2)/x))+A*(-1/7/a/x^7*(b*x^2+a)^(1/2)-6/7*b/a*(-1/5/a/x^5*(b*x
^2+a)^(1/2)-4/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2*(b*x^2+a)^(1/2)/x
)))
```

Maxima [**A**]
time = 0.30, size = 138, normalized size = 1.18

$$-\frac{8\sqrt{bx^{2}+a}\,Bb^{2}}{15\,a^{3}x}+\frac{16\sqrt{bx^{2}+a}\,Ab^{3}}{35\,a^{4}x}+\frac{4\sqrt{bx^{2}+a}\,Bb}{15\,a^{2}x^{3}}-\frac{8\sqrt{bx^{2}+a}\,Ab^{2}}{35\,a^{3}x^{3}}-\frac{\sqrt{bx^{2}+a}\,B}{5\,ax^{5}}+\frac{6\sqrt{bx^{2}+a}\,Ab}{35\,a^{2}x^{5}}-\frac{\sqrt{bx^{2}+a}\,A}{7\,ax^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/15*sqrt(b*x^2 + a)*B*b^2/(a^3*x) + 16/35*sqrt(b*x^2 + a)*A*b^3/(a^4*x) +
 4/15*sqrt(b*x^2 + a)*B*b/(a^2*x^3) - 8/35*sqrt(b*x^2 + a)*A*b^2/(a^3*x^3)
- 1/5*sqrt(b*x^2 + a)*B/(a*x^5) + 6/35*sqrt(b*x^2 + a)*A*b/(a^2*x^5) - 1/7*
sqrt(b*x^2 + a)*A/(a*x^7)
```

Fricas [**A**]
time = 1.20, size = 82, normalized size = 0.70

$$-\frac{\left(8\,(7\,Bab^{2}-6\,Ab^{3})x^{6}-4\,(7\,Ba^{2}b-6\,Aab^{2})x^{4}+15\,Aa^{3}+3\,(7\,Ba^{3}-6\,Aa^{2}b)x^{2}\right)\sqrt{bx^{2}+a}}{105\,a^{4}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(8*(7*B*a*b^2 - 6*A*b^3)*x^6 - 4*(7*B*a^2*b - 6*A*a*b^2)*x^4 + 15*A*
a^3 + 3*(7*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*x^7)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(110) = 220$.
time = 1.66, size = 819, normalized size = 7.00

$$-\frac{5Aa^{6}b^{\frac{19}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{7}b^{9}x^{6}+105a^{6}b^{10}x^{8}+105a^{5}b^{11}x^{10}+35a^{4}b^{12}x^{12}}-\frac{9Aa^{5}b^{\frac{21}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{7}b^{9}x^{6}+105a^{6}b^{10}x^{8}+105a^{5}b^{11}x^{10}+35a^{4}b^{12}x^{12}}-\frac{5Aa^{4}b^{\frac{23}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{7}b^{9}x^{6}+105a^{6}b^{10}x^{8}+105a^{5}b^{11}x^{10}+35a^{4}b^{12}x^{12}}+\frac{5Aa^{3}b^{\frac{25}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{7}b^{9}x^{6}+105a^{6}b^{10}x^{8}+105a^{5}b^{11}x^{10}+35a^{4}b^{12}x^{12}}+\frac{30Aa^{2}b^{\frac{27}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{7}b^{9}x^{6}+105a^{6}b^{10}x^{8}+105a^{5}b^{11}x^{10}+35a^{4}b^{12}x^{12}}+\frac{40Aab^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{7}b^{9}x^{6}+105a^{6}b^{10}x^{8}+105a^{5}b^{11}x^{10}+35a^{4}b^{12}x^{12}}+\frac{16Ab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{7}b^{9}x^{6}+105a^{6}b^{10}x^{8}+105a^{5}b^{11}x^{10}+35a^{4}b^{12}x^{12}}-\frac{3Ba^{4}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{2Ba^{3}b^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{3Ba^{2}b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{12Bab^{\frac{15}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{8Bb^{\frac{17}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*A*a**6*b**(19/2)*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**1
0*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*A*a**5*b**(21/2)*x
**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**
5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*A*a**4*b**(23/2)*x**4*sqrt(a/(b*x*
*2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 +
35*a**4*b**12*x**12) + 5*A*a**3*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**
7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x*
*12) + 30*A*a**2*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 1
05*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) + 40*A*a*b
**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**
8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) + 16*A*b**(31/2)*x**12*sqrt
(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*
x**10 + 35*a**4*b**12*x**12) - 3*B*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a
**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 2*B*a**3*b**(11/2)
*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3
*b**6*x**8) - 3*B*a**2*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 12*B*a*b**(15/2)*x**6*sqrt(a/
(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) -
 8*B*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(101) = 202$.
time = 1.17, size = 232, normalized size = 1.98

$$\frac{16\left(70\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8Bb^{\frac{5}{2}}-175\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6Bab^{\frac{5}{2}}+210\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6Ab^{\frac{7}{2}}+147\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ba^2b^{\frac{5}{2}}-126\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Aab^{\frac{7}{2}}-49\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^3b^{\frac{5}{2}}+42\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Aa^2b^{\frac{7}{2}}+7\,Ba^4b^{\frac{5}{2}}-6\,Aa^3b^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(5/2) - 175*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*B*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(7/2)
 + 147*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(5/2) - 126*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*A*a*b^(7/2) - 49*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(5
/2) + 42*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(7/2) + 7*B*a^4*b^(5/2) -
6*A*a^3*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

Mupad [B]
time = 0.36, size = 105, normalized size = 0.90

$$\frac{\sqrt{b\,x^2+a}\,\left(6\,A\,b-7\,B\,a\right)}{35\,a^2\,x^5}+\frac{\sqrt{b\,x^2+a}\,\left(48\,A\,b^3-56\,B\,a\,b^2\right)}{105\,a^4\,x}-\frac{\left(24\,A\,b^2-28\,B\,a\,b\right)\,\sqrt{b\,x^2+a}}{105\,a^3\,x^3}-\frac{A\,\sqrt{b\,x^2+a}}{7\,a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^8*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(6*A*b - 7*B*a))/(35*a^2*x^5) + ((a + b*x^2)^(1/2)*(48*A
*b^3 - 56*B*a*b^2))/(105*a^4*x) - ((24*A*b^2 - 28*B*a*b)*(a + b*x^2)^(1/2))
/(105*a^3*x^3) - (A*(a + b*x^2)^(1/2))/(7*a*x^7)
```

3.569 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}}+\frac{Bx^7}{6b\sqrt{a+bx^2}}-\frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4}+\frac{5(6Ab-7aB)x^3\sqrt{a+bx^2}}{24b^3}+\frac{5a^2(6Ab-7aB}{}$$

```
[Out] 5/16*a^2*(6*A*b-7*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-1/6*(6*A*
b-7*B*a)*x^5/b^2/(b*x^2+a)^(1/2)+1/6*B*x^7/b/(b*x^2+a)^(1/2)-5/16*a*(6*A*b-
7*B*a)*x*(b*x^2+a)^(1/2)/b^4+5/24*(6*A*b-7*B*a)*x^3*(b*x^2+a)^(1/2)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 294, 327, 223, 212}

$$\frac{5a^2(6Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{5ax\sqrt{a+bx^2}\,(6Ab-7aB)}{16b^4}+\frac{5x^3\sqrt{a+bx^2}\,(6Ab-7aB)}{24b^3}-\frac{x^5(6Ab-7aB)}{6b^2\sqrt{a+bx^2}}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/6*((6*A*b - 7*a*B)*x^5)/(b^2*Sqrt[a + b*x^2]) + (B*x^7)/(6*b*Sqrt[a + b*
x^2]) - (5*a*(6*A*b - 7*a*B)*x*Sqrt[a + b*x^2])/(16*b^4) + (5*(6*A*b - 7*a*
B)*x^3*Sqrt[a + b*x^2])/(24*b^3) + (5*a^2*(6*A*b - 7*a*B)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{(-6Ab+7aB)\int \frac{x^6}{(a+bx^2)^{3/2}}\,dx}{6b}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} + \frac{(5(6Ab-7aB))\int \frac{x^4}{\sqrt{a+bx^2}}\,dx}{6b^2}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} + \frac{5(6Ab-7aB)x^3\sqrt{a+bx^2}}{24b^3} - \frac{(5a(6Ab-7aB))}{8b}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4} + \frac{5(6Ab-7aB)x^3}{24b^3}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4} + \frac{5(6Ab-7aB)x^3}{24b^3}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4} + \frac{5(6Ab-7aB)x^3}{24b^3}$$

Mathematica [A]
time = 0.20, size = 124, normalized size = 0.82

$$\frac{x(-90a^2Ab+105a^3B-30aAb^2x^2+35a^2bBx^2+12Ab^3x^4-14ab^2Bx^4+8b^3Bx^6)}{48b^4\sqrt{a+bx^2}} + \frac{5a^2(-6Ab+7aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*(-90*a^2*A*b + 105*a^3*B - 30*a*A*b^2*x^2 + 35*a^2*b*B*x^2 + 12*A*b^3*x^
4 - 14*a*b^2*B*x^4 + 8*b^3*B*x^6))/(48*b^4*Sqrt[a + b*x^2]) + (5*a^2*(-6*A*
b + 7*a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Maple [A]
time = 0.10, size = 198, normalized size = 1.30

method	result
risch	$-\frac{x\left(-8b^2Bx^4-12Ab^2x^2+22Babx^2+42abA-57a^2B\right)\sqrt{bx^2+a}}{48b^4}-\frac{a^2xA}{b^3\sqrt{bx^2+a}}+\frac{a^3xB}{b^4\sqrt{bx^2+a}}+\frac{15a^2\ln\left(x\sqrt{b}\right.}{8}$
default	$B\left(\frac{x^7}{6b\sqrt{bx^2+a}}-\frac{7a\left(\frac{x^5}{4b\sqrt{bx^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{bx^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)}{6b}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x^7/b/(b*x^2+a)^(1/2)-7/6*a/b*(1/4*x^5/b/(b*x^2+a)^(1/2)-5/4*a/b*(1/
2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))))))+A*(1/4*x^5/b/(b*x^2+a)^(1/2)-5/4*a/b*(1/2*x^3/b/(b*x
^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2)))))
```

Maxima [A]
time = 0.29, size = 170, normalized size = 1.12

$$\frac{Bx^7}{6\sqrt{bx^2+a}\,b}-\frac{7\,Bax^5}{24\sqrt{bx^2+a}\,b^2}+\frac{Ax^5}{4\sqrt{bx^2+a}\,b}+\frac{35\,Ba^2x^3}{48\sqrt{bx^2+a}\,b^3}-\frac{5\,Aax^3}{8\sqrt{bx^2+a}\,b^2}+\frac{35\,Ba^3x}{16\sqrt{bx^2+a}\,b^4}-\frac{15\,Aa^2x}{8\sqrt{bx^2+a}\,b^3}-\frac{35\,Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}}+\frac{15\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*B*x^7/(sqrt(b*x^2 + a)*b) - 7/24*B*a*x^5/(sqrt(b*x^2 + a)*b^2) + 1/4*A*
x^5/(sqrt(b*x^2 + a)*b) + 35/48*B*a^2*x^3/(sqrt(b*x^2 + a)*b^3) - 5/8*A*a*x
^3/(sqrt(b*x^2 + a)*b^2) + 35/16*B*a^3*x/(sqrt(b*x^2 + a)*b^4) - 15/8*A*a^2
*x/(sqrt(b*x^2 + a)*b^3) - 35/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(9/2) + 15/
8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Fricas [A]
time = 1.97, size = 325, normalized size = 2.14

$$\left[-\frac{15\,(7\,Ba^4-6\,Aa^3b+(7\,Ba^3b-6\,Aa^2b^2)x^2)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(8\,Bb^4x^7-2\,(7\,Bab^3-6\,Ab^4)x^5+5\,(7\,Ba^2b^2-6\,Aab^3)x^3+15\,(7\,Ba^3b-6\,Aa^2b^2)x)\sqrt{bx^2+a}}{96\,(b^6x^2+ab^5)},\ \frac{15\,(7\,Ba^4-6\,Aa^3b+(7\,Ba^3b-6\,Aa^2b^2)x^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(8\,Bb^4x^7-2\,(7\,Bab^3-6\,Ab^4)x^5+5\,(7\,Ba^2b^2-6\,Aab^3)x^3+15\,(7\,Ba^3b-6\,Aa^2b^2)x)\sqrt{bx^2+a}}{48\,(b^6x^2+ab^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(7*B*a^4 - 6*A*a^3*b + (7*B*a^3*b - 6*A*a^2*b^2)*x^2)*sqrt(b)*lo
g(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*B*b^4*x^7 - 2*(7*B*a*b
^3 - 6*A*b^4)*x^5 + 5*(7*B*a^2*b^2 - 6*A*a*b^3)*x^3 + 15*(7*B*a^3*b - 6*A*a
^2*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^2 + a*b^5), 1/48*(15*(7*B*a^4 - 6*A*a^3*
b + (7*B*a^3*b - 6*A*a^2*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (8*B*b^4*x^7 - 2*(7*B*a*b^3 - 6*A*b^4)*x^5 + 5*(7*B*a^2*b^2 - 6*A*a*b
^3)*x^3 + 15*(7*B*a^3*b - 6*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^2 + a*b^5
)]
```

Sympy [A]
time = 20.20, size = 233, normalized size = 1.53

$$A\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{5\sqrt{a}\,x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^5}{4\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\frac{35a^{\frac{5}{2}}x}{16b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}x^3}{48b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{7\sqrt{a}\,x^5}{24b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^3\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{9}{2}}}+\frac{x^7}{6\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqrt
(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4*s
qrt(a)*b*sqrt(1 + b*x**2/a))) + B*(35*a**(5/2)*x/(16*b**4*sqrt(1 + b*x**2/a
)) + 35*a**(3/2)*x**3/(48*b**3*sqrt(1 + b*x**2/a)) - 7*sqrt(a)*x**5/(24*b**
```

```
2*sqrt(1 + b*x**2/a)) - 35*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(9/2)) + x*
*7/(6*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Giac [A]
time = 1.04, size = 136, normalized size = 0.89

$$\frac{\left(\left(2\left(\frac{4\,Bx^2}{b}-\frac{7\,Bab^5-6\,Ab^6}{b^7}\right)x^2+\frac{5\,(7\,Ba^2b^4-6\,Aab^5)}{b^7}\right)x^2+\frac{15\,(7\,Ba^3b^3-6\,Aa^2b^4)}{b^7}\right)x}{48\,\sqrt{bx^2+a}}+\frac{5\,(7\,Ba^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{16\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*B*x^2/b - (7*B*a*b^5 - 6*A*b^6)/b^7)*x^2 + 5*(7*B*a^2*b^4 - 6*A
*a*b^5)/b^7)*x^2 + 15*(7*B*a^3*b^3 - 6*A*a^2*b^4)/b^7)*x/sqrt(b*x^2 + a) +
5/16*(7*B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\,(B\,x^2+A)}{(b\,x^2+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] int((x^6*(A + B*x^2))/(a + b*x^2)^(3/2), x)
```

3.570 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}-\frac{a(2Ab-3aB)\sqrt{a+bx^2}}{b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^4}$$

```
[Out] 1/3*(A*b-3*B*a)*(b*x^2+a)^(3/2)/b^4+1/5*B*(b*x^2+a)^(5/2)/b^4-a^2*(A*b-B*a)
/b^4/(b*x^2+a)^(1/2)-a*(2*A*b-3*B*a)*(b*x^2+a)^(1/2)/b^4
```

Rubi [A]

time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}+\frac{\left(a+bx^2\right)^{3/2}(Ab-3aB)}{3b^4}-\frac{a\sqrt{a+bx^2}\,(2Ab-3aB)}{b^4}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x^2])) - (a*(2*A*b - 3*a*B)*Sqrt[a + b*
x^2])/b^4 + ((A*b - 3*a*B)*(a + b*x^2)^(3/2))/(3*b^4) + (B*(a + b*x^2)^(5/2
))/(5*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{3/2}}+\frac{a(-2Ab+3aB)}{b^3\sqrt{a+bx}}+\frac{(Ab-3aB)\sqrt{a+bx}}{b^3}+\frac{B(a+b\right.\right.$$
$$= -\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}-\frac{a(2Ab-3aB)\sqrt{a+bx^2}}{b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{B(a+bx^2)^{5}}{5b^4}$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 0.78

$$\frac{48a^3B-8a^2b(5A-3Bx^2)+b^3x^4(5A+3Bx^2)-2ab^2x^2(10A+3Bx^2)}{15b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
[Out] (48*a^3*B - 8*a^2*b*(5*A - 3*B*x^2) + b^3*x^4*(5*A + 3*B*x^2) - 2*a*b^2*x^2
*(10*A + 3*B*x^2))/(15*b^4*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.09, size = 142, normalized size = 1.43

method	result
gosper	$-\frac{-3B\,x^6b^3-5A\,b^3x^4+6Ba\,b^2x^4+20Aa\,b^2x^2-24B\,a^2b\,x^2+40A\,a^2b-48B\,a^3}{15\sqrt{b\,x^2+a}\;b^4}$
trager	$-\frac{-3B\,x^6b^3-5A\,b^3x^4+6Ba\,b^2x^4+20Aa\,b^2x^2-24B\,a^2b\,x^2+40A\,a^2b-48B\,a^3}{15\sqrt{b\,x^2+a}\;b^4}$
risch	$-\frac{(-3b^2B\,x^4-5A\,b^2x^2+9Bab\,x^2+25abA-33a^2B)\sqrt{b\,x^2+a}}{15b^4}-\frac{a^2(Ab-Ba)}{b^4\sqrt{b\,x^2+a}}$
default	$B\left(\frac{x^6}{5b\sqrt{b\,x^2+a}}-\frac{6a\left(\frac{x^4}{3b\sqrt{b\,x^2+a}}-\frac{4a\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)}{3b}\right)}{5b}\right)+A\left(\frac{x^4}{3b\sqrt{b\,x^2+a}}-\frac{4a(}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/5*x^6/b/(b*x^2+a)^(1/2)-6/5*a/b*(1/3*x^4/b/(b*x^2+a)^(1/2)-4/3*a/b*(x^
2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))))+A*(1/3*x^4/b/(b*x^2+a)^(1/2)
-4/3*a/b*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.32, size = 132, normalized size = 1.33

$$\frac{Bx^6}{5\sqrt{bx^2+a}\,b}-\frac{2\,Bax^4}{5\sqrt{bx^2+a}\,b^2}+\frac{Ax^4}{3\sqrt{bx^2+a}\,b}+\frac{8\,Ba^2x^2}{5\sqrt{bx^2+a}\,b^3}-\frac{4\,Aax^2}{3\sqrt{bx^2+a}\,b^2}+\frac{16\,Ba^3}{5\sqrt{bx^2+a}\,b^4}-\frac{8\,Aa^2}{3\sqrt{bx^2+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*B*x^6/(sqrt(b*x^2 + a)*b) - 2/5*B*a*x^4/(sqrt(b*x^2 + a)*b^2) + 1/3*A*x
^4/(sqrt(b*x^2 + a)*b) + 8/5*B*a^2*x^2/(sqrt(b*x^2 + a)*b^3) - 4/3*A*a*x^2/
(sqrt(b*x^2 + a)*b^2) + 16/5*B*a^3/(sqrt(b*x^2 + a)*b^4) - 8/3*A*a^2/(sqrt(
b*x^2 + a)*b^3)
```

Fricas [A]
time = 1.68, size = 88, normalized size = 0.89

$$\frac{\left(3\,Bb^3x^6-(6\,Bab^2-5\,Ab^3)x^4+48\,Ba^3-40\,Aa^2b+4\,(6\,Ba^2b-5\,Aab^2)x^2\right)\sqrt{bx^2+a}}{15\,(b^5x^2+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^3*x^6 - (6*B*a*b^2 - 5*A*b^3)*x^4 + 48*B*a^3 - 40*A*a^2*b + 4*(
6*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^5*x^2 + a*b^4)
```

Sympy [A]
time = 0.44, size = 172, normalized size = 1.74

$$\begin{cases} -\frac{8Aa^2}{3b^3\sqrt{a+bx^2}}-\frac{4Aax^2}{3b^2\sqrt{a+bx^2}}+\frac{Ax^4}{3b\sqrt{a+bx^2}}+\frac{16Ba^3}{5b^4\sqrt{a+bx^2}}+\frac{8Ba^2x^2}{5b^3\sqrt{a+bx^2}}-\frac{2Bax^4}{5b^2\sqrt{a+bx^2}}+\frac{Bx^6}{5b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-8*A*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*A*a*x**2/(3*b**2*sqrt(a
+ b*x**2)) + A*x**4/(3*b*sqrt(a + b*x**2)) + 16*B*a**3/(5*b**4*sqrt(a + b*x
**2)) + 8*B*a**2*x**2/(5*b**3*sqrt(a + b*x**2)) - 2*B*a*x**4/(5*b**2*sqrt(a
 + b*x**2)) + B*x**6/(5*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**6/6 + B*x**8
/8)/a**(3/2), True))
```

Giac [A]
time = 0.85, size = 113, normalized size = 1.14

$$\frac{Ba^3 - Aa^2b}{\sqrt{bx^2 + a}\,b^4} + \frac{3\,(bx^2 + a)^{\frac{5}{2}}Bb^{16} - 15\,(bx^2 + a)^{\frac{3}{2}}Bab^{16} + 45\,\sqrt{bx^2 + a}\,Ba^2b^{16} + 5\,(bx^2 + a)^{\frac{3}{2}}Ab^{17} - 30\,\sqrt{bx^2 + a}\,Aab^{17}}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*a^3 - A*a^2*b)/(sqrt(b*x^2 + a)*b^4) + 1/15*(3*(b*x^2 + a)^(5/2)*B*b^16
- 15*(b*x^2 + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x^2 + a)*B*a^2*b^16 + 5*(b*x^2
+ a)^(3/2)*A*b^17 - 30*sqrt(b*x^2 + a)*A*a*b^17)/b^20
```

Mupad [B]
time = 0.41, size = 89, normalized size = 0.90

$$\frac{\frac{B\,{\left(b\,x^2+a\right)}^3}{5} + B\,a^3 + \frac{A\,b\,{\left(b\,x^2+a\right)}^2}{3} - B\,a\,{\left(b\,x^2+a\right)}^2 + 3\,B\,a^2\,\left(b\,x^2+a\right) - A\,a^2\,b - 2\,A\,a\,b\,\left(b\,x^2+a\right)}{b^4\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] ((B*(a + b*x^2)^3)/5 + B*a^3 + (A*b*(a + b*x^2)^2)/3 - B*a*(a + b*x^2)^2 +
3*B*a^2*(a + b*x^2) - A*a^2*b - 2*A*a*b*(a + b*x^2))/(b^4*(a + b*x^2)^(1/2)
)
```

3.571 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3}-\frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

```
[Out] -3/8*a*(4*A*b-5*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)-1/4*(4*A*b-
5*B*a)*x^3/b^2/(b*x^2+a)^(1/2)+1/4*B*x^5/b/(b*x^2+a)^(1/2)+3/8*(4*A*b-5*B*a
)*x*(b*x^2+a)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 294, 327, 223, 212}

$$-\frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}+\frac{3x\sqrt{a+bx^2}\,(4Ab-5aB)}{8b^3}-\frac{x^3(4Ab-5aB)}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/4*((4*A*b - 5*a*B)*x^3)/(b^2*Sqrt[a + b*x^2]) + (B*x^5)/(4*b*Sqrt[a + b*
x^2]) + (3*(4*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(8*b^3) - (3*a*(4*A*b - 5*a*B
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx &= \frac{Bx^5}{4b\sqrt{a+bx^2}}-\frac{(-4Ab+5aB)\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx}{4b}\\ &= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{(3(4Ab-5aB))\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b^2}\\ &= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3}-\frac{(3a(4Ab-5aB))\int}{8b^3}\\ &= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3}-\frac{(3a(4Ab-5aB))\mathrm{Su}}{}\\ &= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3}-\frac{3a(4Ab-5aB)\tanh}{8b^{7/}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 99, normalized size = 0.83

$$\frac{12aAbx-15a^2Bx+4Ab^2x^3-5abBx^3+2b^2Bx^5}{8b^3\sqrt{a+bx^2}}-\frac{3a(-4Ab+5aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]

[Out] (12*a*A*b*x - 15*a^2*B*x + 4*A*b^2*x^3 - 5*a*b*B*x^3 + 2*b^2*B*x^5)/(8*b^3*Sqrt[a + b*x^2]) - (3*a*(-4*A*b + 5*a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(7/2))

Maple [A]
time = 0.10, size = 150, normalized size = 1.26

method	result
risch	$\frac{x(2bBx^2+4Ab-7Ba)\sqrt{bx^2+a}}{8b^3}+\frac{axA}{b^2\sqrt{bx^2+a}}-\frac{a^2xB}{b^3\sqrt{bx^2+a}}-\frac{3a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)A}{2b^{\frac{5}{2}}}+\frac{15a^2\ln\left(\right.}{}$
default	$B\left(\frac{x^5}{4b\sqrt{bx^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{bx^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)+A\left(\frac{x^3}{2b\sqrt{bx^2-}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] B*(1/4*x^5/b/(b*x^2+a)^(1/2)-5/4*a/b*(1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))

Maxima [A]
time = 0.31, size = 126, normalized size = 1.06

$$\frac{Bx^5}{4\sqrt{bx^2+a}\,b}-\frac{5\,Bax^3}{8\sqrt{bx^2+a}\,b^2}+\frac{Ax^3}{2\sqrt{bx^2+a}\,b}-\frac{15\,Ba^2x}{8\sqrt{bx^2+a}\,b^3}+\frac{3\,Aax}{2\sqrt{bx^2+a}\,b^2}+\frac{15\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}}-\frac{3\,Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")

[Out] 1/4*B*x^5/(sqrt(b*x^2 + a)*b) - 5/8*B*a*x^3/(sqrt(b*x^2 + a)*b^2) + 1/2*A*x^3/(sqrt(b*x^2 + a)*b) - 15/8*B*a^2*x/(sqrt(b*x^2 + a)*b^3) + 3/2*A*a*x/(sqrt(b*x^2 + a)*b^2) + 15/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/2*A*a*arcsinh(b*x/sqrt(a*b))/b^(5/2)

Fricas [A]
time = 1.55, size = 274, normalized size = 2.30

$$\left[-\frac{3\,(5\,Ba^{3}-4\,Aa^{2}b+(5\,Ba^{2}b-4\,Aab^{2})x^{2})\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,(2\,Bb^{3}x^{5}-(5\,Bab^{2}-4\,Ab^{3})x^{3}-3\,(5\,Ba^{2}b-4\,Aab^{2})x)\sqrt{bx^{2}+a}}{16\,(b^{5}x^{2}+ab^{4})},\ -\frac{3\,(5\,Ba^{3}-4\,Aa^{2}b+(5\,Ba^{2}b-4\,Aab^{2})x^{2})\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(2\,Bb^{3}x^{5}-(5\,Bab^{2}-4\,Ab^{3})x^{3}-3\,(5\,Ba^{2}b-4\,Aab^{2})x)\sqrt{bx^{2}+a}}{8\,(b^{5}x^{2}+ab^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*sqrt(b)*log(-
2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(2*B*b^3*x^5 - (5*B*a*b^2 -
4*A*b^3)*x^3 - 3*(5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*b
^4), -1/8*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b^3*x^5 - (5*B*a*b^2 - 4*A*b^3)*x^
3 - 3*(5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

Sympy [A]
time = 7.34, size = 177, normalized size = 1.49

$$A\left(\frac{3\sqrt{a}\,x}{2b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^{3}}{2\sqrt{a}\,b\sqrt{1+\frac{bx^{2}}{a}}}\right)+B\left(-\frac{15a^{\frac{3}{2}}x}{8b^{3}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{5\sqrt{a}\,x^{3}}{8b^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{15a^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{5}}{4\sqrt{a}\,b\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(
2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) + B*(-15*a**(3/2)*x/(8
*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqrt(1 + b*x**2/a)) + 15
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1 + b*
x**2/a)))
```

Giac [A]
time = 1.26, size = 104, normalized size = 0.87

$$\frac{\left(\left(\frac{2\,Bx^{2}}{b}-\frac{5\,Bab^{3}-4\,Ab^{4}}{b^{5}}\right)x^{2}-\frac{3\,(5\,Ba^{2}b^{2}-4\,Aab^{3})}{b^{5}}\right)x}{8\,\sqrt{bx^{2}+a}}-\frac{3\,(5\,Ba^{2}-4\,Aab)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*B*x^2/b - (5*B*a*b^3 - 4*A*b^4)/b^5)*x^2 - 3*(5*B*a^2*b^2 - 4*A*a*b
^3)/b^5)*x/sqrt(b*x^2 + a) - 3/8*(5*B*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + s
qrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(B\,x^2+A\right)}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x)
[Out] int((x^4*(A + B*x^2))/(a + b*x^2)^(3/2), x)
```

3.572 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{(Ab-2aB)\sqrt{a+bx^2}}{b^3}+\frac{B(a+bx^2)^{3/2}}{3b^3}$$

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b^3+a*(A*b-B*a)/b^3/(b*x^2+a)^(1/2)+(A*b-2*B*a)*(b*x^
2+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\frac{\sqrt{a+bx^2}\,(Ab-2aB)}{b^3}+\frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{B(a+bx^2)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (a*(A*b - a*B))/(b^3*Sqrt[a + b*x^2]) + ((A*b - 2*a*B)*Sqrt[a + b*x^2])/b^3
 + (B*(a + b*x^2)^(3/2))/(3*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{3/2}} + \frac{Ab-2aB}{b^2\sqrt{a+bx}} + \frac{B\sqrt{a+bx}}{b^2}\right)\,dx, x, x^2\right)$$
$$= \frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}} + \frac{(Ab-2aB)\sqrt{a+bx^2}}{b^3} + \frac{B(a+bx^2)^{3/2}}{3b^3}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.82

$$\frac{6aAb-8a^2B+3Ab^2x^2-4abBx^2+b^2Bx^4}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (6*a*A*b - 8*a^2*B + 3*A*b^2*x^2 - 4*a*b*B*x^2 + b^2*B*x^4)/(3*b^3*Sqrt[a +
 b*x^2])
```

Maple [A]
time = 0.09, size = 94, normalized size = 1.40

method	result	size
gosper	$\frac{b^2Bx^4+3Ab^2x^2-4Babx^2+6abA-8a^2B}{3\sqrt{bx^2+a}\,b^3}$	52
trager	$\frac{b^2Bx^4+3Ab^2x^2-4Babx^2+6abA-8a^2B}{3\sqrt{bx^2+a}\,b^3}$	52
risch	$\frac{(bBx^2+3Ab-5Ba)\sqrt{bx^2+a}}{3b^3}+\frac{a(Ab-Ba)}{b^3\sqrt{bx^2+a}}$	53
default	$B\left(\frac{x^4}{3b\sqrt{bx^2+a}}-\frac{4a\left(\frac{x^2}{b\sqrt{bx^2+a}}+\frac{2a}{b^2\sqrt{bx^2+a}}\right)}{3b}\right)+A\left(\frac{x^2}{b\sqrt{bx^2+a}}+\frac{2a}{b^2\sqrt{bx^2+a}}\right)$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/3*x^4/b/(b*x^2+a)^(1/2)-4/3*a/b*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+
a)^(1/2)))+A*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.30, size = 89, normalized size = 1.33

$$\frac{Bx^4}{3\sqrt{bx^2+a}\,b} - \frac{4\,Bax^2}{3\sqrt{bx^2+a}\,b^2} + \frac{Ax^2}{\sqrt{bx^2+a}\,b} - \frac{8\,Ba^2}{3\sqrt{bx^2+a}\,b^3} + \frac{2\,Aa}{\sqrt{bx^2+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*B*x^4/(sqrt(b*x^2 + a)*b) - 4/3*B*a*x^2/(sqrt(b*x^2 + a)*b^2) + A*x^2/(
sqrt(b*x^2 + a)*b) - 8/3*B*a^2/(sqrt(b*x^2 + a)*b^3) + 2*A*a/(sqrt(b*x^2 +
a)*b^2)
```

Fricas [A]
time = 2.23, size = 63, normalized size = 0.94

$$\frac{\left(Bb^2x^4 - 8\,Ba^2 + 6\,Aab - (4\,Bab - 3\,Ab^2)x^2\right)\sqrt{bx^2+a}}{3\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^4 - 8*B*a^2 + 6*A*a*b - (4*B*a*b - 3*A*b^2)*x^2)*sqrt(b*x^2 +
a)/(b^4*x^2 + a*b^3)
```

Sympy [A]
time = 0.33, size = 117, normalized size = 1.75

$$\begin{cases} \frac{2Aa}{b^2\sqrt{a+bx^2}} + \frac{Ax^2}{b\sqrt{a+bx^2}} - \frac{8Ba^2}{3b^3\sqrt{a+bx^2}} - \frac{4Bax^2}{3b^2\sqrt{a+bx^2}} + \frac{Bx^4}{3b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^4}{4} + \frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((2*A*a/(b**2*sqrt(a + b*x**2)) + A*x**2/(b*sqrt(a + b*x**2)) - 8*
B*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*B*a*x**2/(3*b**2*sqrt(a + b*x**2)) + B
*x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/a**(3/2), T
rue))
```

Giac [A]
time = 1.23, size = 77, normalized size = 1.15

$$-\frac{Ba^2 - Aab}{\sqrt{bx^2+a}\,b^3} + \frac{(bx^2+a)^{\frac{3}{2}}Bb^6 - 6\sqrt{bx^2+a}\,Bab^6 + 3\sqrt{bx^2+a}\,Ab^7}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*a^2 - A*a*b)/(sqrt(b*x^2 + a)*b^3) + 1/3*((b*x^2 + a)^(3/2)*B*b^6 - 6*s
qrt(b*x^2 + a)*B*a*b^6 + 3*sqrt(b*x^2 + a)*A*b^7)/b^9
```

Mupad [B]
time = 0.33, size = 59, normalized size = 0.88

$$\frac{B\,{\left(b\,x^2+a\right)}^2-3\,B\,a^2+3\,A\,b\,\left(b\,x^2+a\right)-6\,B\,a\,\left(b\,x^2+a\right)+3\,A\,a\,b}{3\,b^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] (B*(a + b*x^2)^2 - 3*B*a^2 + 3*A*b*(a + b*x^2) - 6*B*a*(a + b*x^2) + 3*A*a*
b)/(3*b^3*(a + b*x^2)^(1/2))
```

3.573 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}+\frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[Out] 1/2*(2*A*b-3*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-(A*b-B*a)*x/b^
2/(b*x^2+a)^(1/2)+1/2*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {466, 396, 223, 212}

$$\frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -(((A*b - a*B)*x)/(b^2*Sqrt[a + b*x^2])) + (B*x*Sqrt[a + b*x^2])/(2*b^2) +
((2*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx &= -\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}} - \frac{\int \frac{-Ab+aB-bBx^2}{\sqrt{a+bx^2}}\,dx}{b^2} \\
&= -\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^2} + \frac{(2Ab-3aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2} \\
&= -\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^2} + \frac{(2Ab-3aB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^2} \\
&= -\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^2} + \frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 75, normalized size = 0.90

$$
\frac{-2Abx+3aBx+bBx^3}{2b^2\sqrt{a+bx^2}} + \frac{(-2Ab+3aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-2*A*b*x + 3*a*B*x + b*B*x^3)/(2*b^2*Sqrt[a + b*x^2]) + ((-2*A*b + 3*a*B)*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Maple [A]
time = 0.10, size = 102, normalized size = 1.23

method	result
risch	$\frac{Bx\sqrt{b\,x^2+a}}{2b^2} - \frac{xA}{b\sqrt{b\,x^2+a}} + \frac{xaB}{b^2\sqrt{b\,x^2+a}} + \frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)A}{b^{\frac{3}{2}}} - \frac{3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)B}{2b^{\frac{5}{2}}}$

default	$B\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+A\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\right.}{b^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] B*(1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))))+A*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b
*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.31, size = 82, normalized size = 0.99

$$\frac{Bx^3}{2\sqrt{bx^2+a}\,b}+\frac{3\,Bax}{2\sqrt{bx^2+a}\,b^2}-\frac{Ax}{\sqrt{bx^2+a}\,b}-\frac{3\,Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}+\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] 1/2*B*x^3/(sqrt(b*x^2 + a)*b) + 3/2*B*a*x/(sqrt(b*x^2 + a)*b^2) - A*x/(sqrt
(b*x^2 + a)*b) - 3/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + A*arcsinh(b*x/sqr
t(a*b))/b^(3/2)
```

Fricas [A]
time = 1.43, size = 213, normalized size = 2.57

$$\left[-\frac{(3\,Ba^2-2\,Aab+(3\,Bab-2\,Ab^2)x^2)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(Bb^2x^3+(3\,Bab-2\,Ab^2)x)\sqrt{bx^2+a}}{4\,(b^4x^2+ab^3)},\ \frac{(3\,Ba^2-2\,Aab+(3\,Bab-2\,Ab^2)x^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(Bb^2x^3+(3\,Bab-2\,Ab^2)x)\sqrt{bx^2+a}}{2\,(b^4x^2+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] [-1/4*((3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(B*b^2*x^3 + (3*B*a*b - 2*A*b^2)*x)*s
qrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2*((3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A
*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b^2*x^3 + (3*B*
a*b - 2*A*b^2)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [A]
time = 3.90, size = 114, normalized size = 1.37

$$A\left(\frac{\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\frac{3\sqrt{a}\,x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(
2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Giac [A]
time = 1.36, size = 70, normalized size = 0.84

$$\frac{\left(\frac{Bx^2}{b}+\frac{3\,Bab-2\,Ab^2}{b^3}\right)x}{2\,\sqrt{bx^2+a}}+\frac{(3\,Ba-2\,Ab)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(B*x^2/b + (3*B*a*b - 2*A*b^2)/b^3)*x/sqrt(b*x^2 + a) + 1/2*(3*B*a - 2*
A*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(B\,x^2+A\right)}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] int((x^2*(A + B*x^2))/(a + b*x^2)^(3/2), x)
```

3.574 $\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{Ab-aB}{b^2\sqrt{a+bx^2}}+\frac{B\sqrt{a+bx^2}}{b^2}$$

```
[Out] (-A*b+B*a)/b^2/(b*x^2+a)^(1/2)+B*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]

time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 45}

$$\frac{B\sqrt{a+bx^2}}{b^2}-\frac{Ab-aB}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]
[Out] -((A*b - a*B)/(b^2*Sqrt[a + b*x^2])) + (B*Sqrt[a + b*x^2])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}}+\frac{B}{b\sqrt{a+bx}}\right)\,dx,x,x^2\right)\\ &= -\frac{Ab-aB}{b^2\sqrt{a+bx^2}}+\frac{B\sqrt{a+bx^2}}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.73

$$\frac{-Ab+2aB+bBx^2}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]
[Out] (-(A*b) + 2*a*B + b*B*x^2)/(b^2*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.09, size = 51, normalized size = 1.24

method	result	size
gosper	$-\frac{-bB\,x^2+Ab-2Ba}{\sqrt{b\,x^2+a}\,b^2}$	30
trager	$-\frac{-bB\,x^2+Ab-2Ba}{\sqrt{b\,x^2+a}\,b^2}$	30
risch	$\frac{B\sqrt{b\,x^2+a}}{b^2}-\frac{Ab-Ba}{\sqrt{b\,x^2+a}\,b^2}$	38
default	$B\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)-\frac{A}{b\sqrt{b\,x^2+a}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] B*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))-A/b/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.32, size = 49, normalized size = 1.20

$$\frac{Bx^2}{\sqrt{bx^2+a}\,b}+\frac{2\,Ba}{\sqrt{bx^2+a}\,b^2}-\frac{A}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] B*x^2/(sqrt(b*x^2 + a)*b) + 2*B*a/(sqrt(b*x^2 + a)*b^2) - A/(sqrt(b*x^2 + a
)*b)
```

Fricas [A]
time = 3.55, size = 40, normalized size = 0.98

$$\frac{(Bbx^2+2\,Ba-Ab)\sqrt{bx^2+a}}{b^3x^2+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (B*b*x^2 + 2*B*a - A*b)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

Sympy [A]
time = 0.28, size = 66, normalized size = 1.61

$$\begin{cases} -\frac{A}{b\sqrt{a+bx^2}} + \frac{2Ba}{b^2\sqrt{a+bx^2}} + \frac{Bx^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2} + \frac{Bx^4}{4}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-A/(b*sqrt(a + b*x**2)) + 2*B*a/(b**2*sqrt(a + b*x**2)) + B*x**2
/(b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**2/2 + B*x**4/4)/a**(3/2), True))
```

Giac [A]
time = 0.93, size = 36, normalized size = 0.88

$$\frac{\sqrt{bx^2+a}\,B}{b^2} + \frac{Ba - Ab}{\sqrt{bx^2+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)*B/b^2 + (B*a - A*b)/(sqrt(b*x^2 + a)*b^2)
```

Mupad [B]
time = 0.29, size = 30, normalized size = 0.73

$$\frac{B\,a - A\,b + B\,(b\,x^2 + a)}{b^2\,\sqrt{b\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] (B*a - A*b + B*(a + b*x^2))/(b^2*(a + b*x^2)^(1/2))
```

3.575 $\int \frac{A+Bx^2}{(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=54

$$\frac{(Ab-aB)x}{ab\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[Out] B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+(A*b-B*a)*x/a/b/(b*x^2+a)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {393, 223, 212}

$$\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((A*b - a*B)*x)/(a*b*Sqrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}} + \frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}$$
$$= \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}} + \frac{B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}$$
$$= \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.09, size = 58, normalized size = 1.07

$$\frac{Abx-aBx}{ab\sqrt{a+bx^2}} - \frac{B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (A*b*x - a*B*x)/(a*b*Sqrt[a + b*x^2]) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/b^(3/2)
```

Maple [A]
time = 0.09, size = 55, normalized size = 1.02

method	result	size
default	$B\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)+\frac{Ax}{a\sqrt{b\,x^2+a}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+A*x/a/(b*x
^2+a)^(1/2)
```

Maxima [A]
time = 0.30, size = 46, normalized size = 0.85

$$\frac{Ax}{\sqrt{bx^2+a}\,a} - \frac{Bx}{\sqrt{bx^2+a}\,b} + \frac{B\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] A*x/(sqrt(b*x^2 + a)*a) - B*x/(sqrt(b*x^2 + a)*b) + B*arcsinh(b*x/sqrt(a*b)
)/b^(3/2)
```

Fricas [A]
time = 1.58, size = 168, normalized size = 3.11

$$\left[-\frac{2\,(Bab-Ab^2)\sqrt{bx^2+a}\,x-(Babx^2+Ba^2)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,(ab^3x^2+a^2b^2)},\,-\frac{(Bab-Ab^2)\sqrt{bx^2+a}\,x+(Babx^2+Ba^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{ab^3x^2+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(B*a*b - A*b^2)*sqrt(b*x^2 + a)*x - (B*a*b*x^2 + B*a^2)*sqrt(b)*lo
g(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), -((B*
a*b - A*b^2)*sqrt(b*x^2 + a)*x + (B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A]
time = 2.43, size = 60, normalized size = 1.11

$$\frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) -
x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Giac [A]
time = 1.33, size = 51, normalized size = 0.94

$$-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}-\frac{(Ba-Ab)x}{\sqrt{bx^2+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - (B*a - A*b)*x/(sqrt(b*x
^2 + a)*a*b)
```

Mupad [B]
time = 0.37, size = 53, normalized size = 0.98

$$\frac{B\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}+\frac{A\,x}{a\,\sqrt{b\,x^2+a}}-\frac{B\,x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^(3/2),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) + (A*x)/(a*(a + b*x^2)^(1/2)
) - (B*x)/(b*(a + b*x^2)^(1/2))
```

3.576 $\int \frac{A+Bx^2}{x(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(A*b-B*a)/a/b/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {457, 79, 65, 214}

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^
(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{Ab-aB}{ab\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\ &= \frac{Ab-aB}{ab\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\ &= \frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 53, normalized size = 1.00

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^
(3/2)
```

Maple [A]
time = 0.09, size = 61, normalized size = 1.15

method	result	size

default	$-\frac{B}{b\sqrt{b\,x^2+a}}+A\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -B/b/(b*x^2+a)^(1/2)+A*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*
x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.91

$$-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{bx^2+a}\,a}-\frac{B}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqrt(b*x^2 + a)*a) - B/(sqrt(
b*x^2 + a)*b)
```

Fricas [A]
time = 1.59, size = 167, normalized size = 3.15

$$\left[\frac{(Ab^2x^2+Aab)\sqrt{a}\,\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)-2\,(Ba^2-Aab)\sqrt{bx^2+a}}{2\,(a^2b^2x^2+a^3b)},\ \frac{(Ab^2x^2+Aab)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(Ba^2-Aab)\sqrt{bx^2+a}}{a^2b^2x^2+a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b^2*x^2 + A*a*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) - 2*(B*a^2 - A*a*b)*sqrt(b*x^2 + a))/(a^2*b^2*x^2 + a^3*b), ((A*
b^2*x^2 + A*a*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*a^2 - A*a*b
)*sqrt(b*x^2 + a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [A]
time = 7.57, size = 48, normalized size = 0.91

$$\frac{A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{a\sqrt{-a}}-\frac{-Ab+Ba}{ab\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*atan(sqrt(a + b*x**2)/sqrt(-a))/(a*sqrt(-a)) - (-A*b + B*a)/(a*b*sqrt(a +
 b*x**2))
```

Giac [A]
time = 1.14, size = 52, normalized size = 0.98

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{Ba-Ab}{\sqrt{bx^2+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (B*a - A*b)/(sqrt(b*x^2 +
 a)*a*b)
```

Mupad [B]
time = 0.48, size = 50, normalized size = 0.94

$$\frac{A}{a\sqrt{b\,x^2+a}}-\frac{B}{b\sqrt{b\,x^2+a}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^(3/2)),x)
```

```
[Out] A/(a*(a + b*x^2)^(1/2)) - B/(b*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/a^(3/2)
```

3.577 $\int \frac{A+Bx^2}{x^2(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{A}{ax\sqrt{a+bx^2}}-\frac{(2Ab-aB)x}{a^2\sqrt{a+bx^2}}$$

```
[Out] -A/a/x/(b*x^2+a)^(1/2)-(2*A*b-B*a)*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {464, 197}

$$-\frac{x(2Ab-aB)}{a^2\sqrt{a+bx^2}}-\frac{A}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(A/(a*x*Sqrt[a + b*x^2])) - ((2*A*b - a*B)*x)/(a^2*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{A}{ax\sqrt{a+bx^2}}-\frac{(2Ab-aB)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{a}\\ &= -\frac{A}{ax\sqrt{a+bx^2}}-\frac{(2Ab-aB)x}{a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 36, normalized size = 0.77

$$\frac{-aA-2Abx^2+aBx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
[Out] (-(a*A) - 2*A*b*x^2 + a*B*x^2)/(a^2*x*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.09, size = 53, normalized size = 1.13

method	result	size
gosper	$-\frac{2Ab\,x^2-Ba\,x^2+Aa}{x\sqrt{b\,x^2+a}\;a^2}$	36
trager	$-\frac{2Ab\,x^2-Ba\,x^2+Aa}{x\sqrt{b\,x^2+a}\;a^2}$	36
risch	$-\frac{A\sqrt{b\,x^2+a}}{a^2x}-\frac{x(Ab-Ba)}{\sqrt{b\,x^2+a}\;a^2}$	43
default	$\frac{Bx}{a\sqrt{b\,x^2+a}}+A\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] B*x/a/(b*x^2+a)^(1/2)+A*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.32, size = 51, normalized size = 1.09

$$\frac{Bx}{\sqrt{bx^2+a}\,a}-\frac{2\,Abx}{\sqrt{bx^2+a}\,a^2}-\frac{A}{\sqrt{bx^2+a}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] B*x/(sqrt(b*x^2 + a)*a) - 2*A*b*x/(sqrt(b*x^2 + a)*a^2) - A/(sqrt(b*x^2 + a
)*a*x)
```

Fricas [A]
time = 2.12, size = 43, normalized size = 0.91

$$\frac{((Ba-2\,Ab)x^2-Aa)\sqrt{bx^2+a}}{a^2bx^3+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] ((B*a - 2*A*b)*x^2 - A*a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

Sympy [A]
time = 2.80, size = 68, normalized size = 1.45

$$A\left(-\frac{1}{a\sqrt{b}\,x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**
2) + 1))) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.82, size = 57, normalized size = 1.21

$$\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a}+\frac{(Ba-Ab)x}{\sqrt{bx^2+a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a) + (B*a - A*b)*x/(sqrt
(b*x^2 + a)*a^2)
```

Mupad [B]
time = 0.27, size = 46, normalized size = 0.98

$$-\frac{\sqrt{b\,x^2+a}\,\left(\frac{A}{a}-x^2\,\left(\frac{B}{a}-\frac{2\,A\,b}{a^2}\right)\right)}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(A/a - x^2*(B/a - (2*A*b)/a^2)))/(a*x + b*x^3)
```

3.578 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=86

$$-\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] 1/2*(3*A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/2*(-3*A*b+2*B*
a)/a^2/(b*x^2+a)^(1/2)-1/2*A/a/x^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 53, 65, 214}

$$\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/2*(3*A*b - 2*a*B)/(a^2*Sqrt[a + b*x^2]) - A/(2*a*x^2*Sqrt[a + b*x^2]) +
((3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{A}{2ax^2\sqrt{a+bx^2}} + \frac{\left(-\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}} - \frac{A}{2ax^2\sqrt{a+bx^2}} - \frac{(3Ab-2aB)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^2}$$
$$= -\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}} - \frac{A}{2ax^2\sqrt{a+bx^2}} - \frac{(3Ab-2aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^2b}$$
$$= -\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}} - \frac{A}{2ax^2\sqrt{a+bx^2}} + \frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A]
time = 0.14, size = 77, normalized size = 0.90

$$\frac{-aA-3Abx^2+2aBx^2}{2a^2x^2\sqrt{a+bx^2}} + \frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) - 3*A*b*x^2 + 2*a*B*x^2)/(2*a^2*x^2*Sqrt[a + b*x^2]) + ((3*A*b - 2*
a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Maple [A]
time = 0.10, size = 114, normalized size = 1.33

method	result
risch	$-\frac{A\sqrt{b\,x^2+a}}{2a^2x^2}-\frac{bA}{a^2\sqrt{b\,x^2+a}}+\frac{B}{a\sqrt{b\,x^2+a}}+\frac{3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)Ab}{2a^{\frac{5}{2}}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+}}{x}\right.}{a^{\frac{3}{2}}}$
default	$A\left(-\frac{1}{2a\,x^2\sqrt{b\,x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)+B\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*
a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+B*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+
2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.29, size = 86, normalized size = 1.00

$$-\frac{B\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{3\,Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a}-\frac{3\,Ab}{2\,\sqrt{bx^2+a}\,a^2}-\frac{A}{2\,\sqrt{bx^2+a}\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 3/2*A*b*arcsinh(a/(sqrt(a*b)*abs
(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a) - 3/2*A*b/(sqrt(b*x^2 + a)*a^2) - 1/2
*A/(sqrt(b*x^2 + a)*a*x^2)
```

Fricas [A]
time = 2.01, size = 232, normalized size = 2.70

$$\left[-\frac{((2\,Bab-3\,Ab^2)x^4+(2\,Ba^2-3\,Aab)x^2)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(Aa^2-(2\,Ba^2-3\,Aab)x^2)\sqrt{bx^2+a}}{4\,(a^3bx^4+a^4x^2)},\ \frac{((2\,Bab-3\,Ab^2)x^4+(2\,Ba^2-3\,Aab)x^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(Aa^2-(2\,Ba^2-3\,Aab)x^2)\sqrt{bx^2+a}}{2\,(a^3bx^4+a^4x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*B*a*b - 3*A*b^2)*x^4 + (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(a)*log(-(b*
x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(A*a^2 - (2*B*a^2 - 3*A*a*b
)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2), 1/2*(((2*B*a*b - 3*A*b^2)*x^
4 + (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (A
*a^2 - (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(73) = 146$.
time = 15.97, size = 262, normalized size = 3.05

$$A\left(-\frac{1}{2a\sqrt{b}\,x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/
(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(2*a**3*s
qrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*
a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9
/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/
2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(
7/2)*b*x**2))
```

Giac [A]
time = 0.84, size = 99, normalized size = 1.15

$$\frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}\,a^2}+\frac{2\,(bx^2+a)Ba-2\,Ba^2-3\,(bx^2+a)Ab+2\,Aab}{2\left((bx^2+a)^{\frac{3}{2}}-\sqrt{bx^2+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - 3*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/2*(
2*(b*x^2 + a)*B*a - 2*B*a^2 - 3*(b*x^2 + a)*A*b + 2*A*a*b)/(((b*x^2 + a)^(3
/2) - sqrt(b*x^2 + a)*a)*a^2)
```

Mupad [B]
time = 0.71, size = 90, normalized size = 1.05

$$\frac{B}{a\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{3\,A\,b}{2\,a^2\sqrt{b\,x^2+a}}-\frac{A}{2\,a\,x^2\sqrt{b\,x^2+a}}+\frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x)
```

```
[Out] B/(a*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) - (3
*A*b)/(2*a^2*(a + b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*A*b*at
anh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2))
```

3.579 $\int \frac{A+Bx^2}{x^4(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{A}{3ax^3\sqrt{a+bx^2}}+\frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}}+\frac{2b(4Ab-3aB)x}{3a^3\sqrt{a+bx^2}}$$

```
[Out] -1/3*A/a/x^3/(b*x^2+a)^(1/2)+1/3*(4*A*b-3*B*a)/a^2/x/(b*x^2+a)^(1/2)+2/3*b*
(4*A*b-3*B*a)*x/a^3/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 197}

$$\frac{2bx(4Ab-3aB)}{3a^3\sqrt{a+bx^2}}+\frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/3*A/(a*x^3*Sqrt[a + b*x^2]) + (4*A*b - 3*a*B)/(3*a^2*x*Sqrt[a + b*x^2])
+ (2*b*(4*A*b - 3*a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4(a+bx^2)^{3/2}}\,dx = -\frac{A}{3ax^3\sqrt{a+bx^2}} - \frac{(4Ab-3aB)\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx}{3a}$$
$$= -\frac{A}{3ax^3\sqrt{a+bx^2}} + \frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}} + \frac{(2b(4Ab-3aB))\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{3a^2}$$
$$= -\frac{A}{3ax^3\sqrt{a+bx^2}} + \frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}} + \frac{2b(4Ab-3aB)x}{3a^3\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.10, size = 62, normalized size = 0.76

$$\frac{-a^2A+4aAbx^2-3a^2Bx^2+8Ab^2x^4-6abBx^4}{3a^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*A) + 4*a*A*b*x^2 - 3*a^2*B*x^2 + 8*A*b^2*x^4 - 6*a*b*B*x^4)/(3*a^3*x
^3*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.10, size = 98, normalized size = 1.20

method	result
gosper	$-\frac{-8A\,b^2x^4+6Bab\,x^4-4aAb\,x^2+3B\,a^2x^2+a^2A}{3x^3\sqrt{b\,x^2+a}\;a^3}$
trager	$-\frac{-8A\,b^2x^4+6Bab\,x^4-4aAb\,x^2+3B\,a^2x^2+a^2A}{3x^3\sqrt{b\,x^2+a}\;a^3}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-5Ab\,x^2+3Ba\,x^2+Aa)}{3a^3x^3}+\frac{x(Ab-Ba)b}{\sqrt{b\,x^2+a}\;a^3}$
default	$A\left(-\frac{1}{3a\,x^3\sqrt{b\,x^2+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)}{3a}\right)+B\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/3/a/x^3/(b*x^2+a)^(1/2)-4/3*b/a*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*
x^2+a)^(1/2)))+B*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.29, size = 92, normalized size = 1.12

$$-\frac{2\,Bbx}{\sqrt{bx^2+a}\,a^2}+\frac{8\,Ab^2x}{3\,\sqrt{bx^2+a}\,a^3}-\frac{B}{\sqrt{bx^2+a}\,ax}+\frac{4\,Ab}{3\,\sqrt{bx^2+a}\,a^2x}-\frac{A}{3\,\sqrt{bx^2+a}\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*B*b*x/(sqrt(b*x^2 + a)*a^2) + 8/3*A*b^2*x/(sqrt(b*x^2 + a)*a^3) - B/(sqr
t(b*x^2 + a)*a*x) + 4/3*A*b/(sqrt(b*x^2 + a)*a^2*x) - 1/3*A/(sqrt(b*x^2 + a
)*a*x^3)
```

Fricas [A]
time = 1.73, size = 68, normalized size = 0.83

$$-\frac{\left(2\,(3\,Bab-4\,Ab^2)x^4+Aa^2+(3\,Ba^2-4\,Aab)x^2\right)\sqrt{bx^2+a}}{3\,(a^3bx^5+a^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*B*a*b - 4*A*b^2)*x^4 + A*a^2 + (3*B*a^2 - 4*A*a*b)*x^2)*sqrt(b*x
^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(73) = 146.
time = 3.76, size = 284, normalized size = 3.46

$$A\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right)+B\left(-\frac{1}{a\sqrt{b}\,x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4
 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b
**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(
a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) +
8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4
+ 3*a**3*b**6*x**6)) + B*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt
(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(70) = 140.
time = 1.09, size = 181, normalized size = 2.21

$$-\frac{(Bab-Ab^2)x}{\sqrt{bx^2+a}\,a^3}+\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+12\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Aab^{\frac{3}{2}}+3\,Ba^3\sqrt{b}-5\,Aa^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*x/(sqrt(b*x^2 + a)*a^3) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 +
 a))^4*B*a*sqrt(b) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 5*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3*a^2)
```

Mupad [B]
time = 0.33, size = 57, normalized size = 0.70

$$-\frac{3\,B\,a^2\,x^2+A\,a^2+6\,B\,a\,b\,x^4-4\,A\,a\,b\,x^2-8\,A\,b^2\,x^4}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x)
```

```
[Out] -(A*a^2 + 3*B*a^2*x^2 - 8*A*b^2*x^4 - 4*A*a*b*x^2 + 6*B*a*b*x^4)/(3*a^3*x^3
*(a + b*x^2)^(1/2))
```

3.580 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{3b(5Ab-4aB)}{8a^3\sqrt{a+bx^2}}-\frac{A}{4ax^4\sqrt{a+bx^2}}+\frac{5Ab-4aB}{8a^2x^2\sqrt{a+bx^2}}-\frac{3b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}$$

```
[Out] -3/8*b*(5*A*b-4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)+3/8*b*(5*A*b-
4*B*a)/a^3/(b*x^2+a)^(1/2)-1/4*A/a/x^4/(b*x^2+a)^(1/2)+1/8*(5*A*b-4*B*a)/a^
2/x^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 44, 53, 65, 214}

$$-\frac{3b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{3b(5Ab-4aB)}{8a^3\sqrt{a+bx^2}}+\frac{5Ab-4aB}{8a^2x^2\sqrt{a+bx^2}}-\frac{A}{4ax^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

```
[Out] (3*b*(5*A*b - 4*a*B))/(8*a^3*Sqrt[a + b*x^2]) - A/(4*a*x^4*Sqrt[a + b*x^2])
 + (5*A*b - 4*a*B)/(8*a^2*x^2*Sqrt[a + b*x^2]) - (3*b*(5*A*b - 4*a*B)*ArcTa
nh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx,x,x^2\right)$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} + \frac{\left(-\frac{5Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^2\right)}{4a}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} - \frac{(3(5Ab-4aB))\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,\right.}{8a^2}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} + \frac{3(5Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2} + \frac{(3b(5Ab-4aB}{}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} + \frac{3(5Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2} + \frac{(3(5Ab-4aB)}{}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} + \frac{3(5Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2} - \frac{3b(5Ab-4aB)}{}$$

Mathematica [A]
time = 0.18, size = 100, normalized size = 0.85

$$\frac{-2a^2A+5aAbx^2-4a^2Bx^2+15Ab^2x^4-12abBx^4}{8a^3x^4\sqrt{a+bx^2}} + \frac{3b(-5Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2*A + 5*a*A*b*x^2 - 4*a^2*B*x^2 + 15*A*b^2*x^4 - 12*a*b*B*x^4)/(8*a^3
*x^4*Sqrt[a + b*x^2]) + (3*b*(-5*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]])/(8*a^(7/2))
```

Maple [A]
time = 0.10, size = 162, normalized size = 1.37

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-7Ab\,x^2+4Ba\,x^2+2Aa\right)}{8a^3x^4} + \frac{b^2A}{a^3\sqrt{b\,x^2+a}} - \frac{bB}{a^2\sqrt{b\,x^2+a}} - \frac{15b^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)A}{8a^{\frac{7}{2}}}$

default	$A\left(-\frac{1}{4a\,x^{4}\sqrt{b\,x^{2}+a}}-\frac{5b\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{4a}\right)+B\Bigg(-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4/a/x^4/(b*x^2+a)^(1/2)-5/4*b/a*(-1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(
1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+B*(-
1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*
a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Maxima [A]
time = 0.29, size = 130, normalized size = 1.10

$$\frac{3\,Bb\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{15\,Ab^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{7}{2}}}-\frac{3\,Bb}{2\,\sqrt{bx^{2}+a}\,a^{2}}+\frac{15\,Ab^{2}}{8\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{B}{2\,\sqrt{bx^{2}+a}\,ax^{2}}+\frac{5\,Ab}{8\,\sqrt{bx^{2}+a}\,a^{2}x^{2}}-\frac{A}{4\,\sqrt{bx^{2}+a}\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 15/8*A*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(7/2) - 3/2*B*b/(sqrt(b*x^2 + a)*a^2) + 15/8*A*b^2/(sqrt(b*
x^2 + a)*a^3) - 1/2*B/(sqrt(b*x^2 + a)*a*x^2) + 5/8*A*b/(sqrt(b*x^2 + a)*a^
2*x^2) - 1/4*A/(sqrt(b*x^2 + a)*a*x^4)
```

Fricas [A]
time = 1.27, size = 287, normalized size = 2.43

$$\left[-\frac{3\,((4\,Bab^{2}-5\,Ab^{3})x^{6}+(4\,Ba^{2}b-5\,Aab^{2})x^{4})\sqrt{a}\,\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(3\,(4\,Ba^{2}b-5\,Aab^{2})x^{4}+2\,Aa^{3}+(4\,Ba^{3}-5\,Aa^{2}b)x^{2})\sqrt{bx^{2}+a}}{16\,(a^{4}bx^{6}+a^{5}x^{4})},\ -\frac{3\,((4\,Bab^{2}-5\,Ab^{3})x^{6}+(4\,Ba^{2}b-5\,Aab^{2})x^{4})\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(3\,(4\,Ba^{2}b-5\,Aab^{2})x^{4}+2\,Aa^{3}+(4\,Ba^{3}-5\,Aa^{2}b)x^{2})\sqrt{bx^{2}+a}}{8\,(a^{4}bx^{6}+a^{5}x^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((4*B*a*b^2 - 5*A*b^3)*x^6 + (4*B*a^2*b - 5*A*a*b^2)*x^4)*sqrt(a)
*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(4*B*a^2*b - 5*
A*a*b^2)*x^4 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b
*x^6 + a^5*x^4), -1/8*(3*((4*B*a*b^2 - 5*A*b^3)*x^6 + (4*B*a^2*b - 5*A*a*b^
2)*x^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(4*B*a^2*b - 5*A*a*b
^2)*x^4 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b*x^6
+ a^5*x^4)]
```

Sympy [A]
time = 32.94, size = 180, normalized size = 1.53

$$A\left(-\frac{1}{4a\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{2a\sqrt{b}\,x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(b)/(8*a**2*x**3*sqrt
(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**2) + 1)) - 15*b**2*a
sinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2))) + B*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b
*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a
)/(sqrt(b)*x))/(2*a**(5/2)))
```

Giac [A]
time = 1.06, size = 137, normalized size = 1.16

$$-\frac{3\,(4\,Bab-5\,Ab^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^3}-\frac{Bab-Ab^2}{\sqrt{bx^2+a}\,a^3}-\frac{4\,(bx^2+a)^{\frac{3}{2}}Bab-4\,\sqrt{bx^2+a}\,Ba^2b-7\,(bx^2+a)^{\frac{3}{2}}Ab^2+9\,\sqrt{bx^2+a}\,Aab^2}{8\,a^3b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
(B*a*b - A*b^2)/(sqrt(b*x^2 + a)*a^3) - 1/8*(4*(b*x^2 + a)^(3/2)*B*a*b - 4*
sqrt(b*x^2 + a)*B*a^2*b - 7*(b*x^2 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^2 + a)*A*a
*b^2)/(a^3*b^2*x^4)
```

Mupad [B]
time = 0.91, size = 134, normalized size = 1.14

$$\frac{15\,A\,b^2}{8\,a^3\,\sqrt{b\,x^2+a}}-\frac{3\,B\,b}{2\,a^2\,\sqrt{b\,x^2+a}}-\frac{15\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{A}{4\,a\,x^4\,\sqrt{b\,x^2+a}}-\frac{B}{2\,a\,x^2\,\sqrt{b\,x^2+a}}+\frac{3\,B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}+\frac{5\,A\,b}{8\,a^2\,x^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x)
```

```
[Out] (15*A*b^2)/(8*a^3*(a + b*x^2)^(1/2)) - (3*B*b)/(2*a^2*(a + b*x^2)^(1/2)) -
(15*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(7/2)) - A/(4*a*x^4*(a + b
*x^2)^(1/2)) - B/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*B*b*atanh((a + b*x^2)^(1/
2)/a^(1/2)))/(2*a^(5/2)) + (5*A*b)/(8*a^2*x^2*(a + b*x^2)^(1/2))
```

3.581 $\int \frac{A+Bx^2}{x^6(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{A}{5ax^5\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}-\frac{8b^2(6Ab-5aB)x}{15a^4\sqrt{a+bx^2}}$$

```
[Out] -1/5*A/a/x^5/(b*x^2+a)^(1/2)+1/15*(6*A*b-5*B*a)/a^2/x^3/(b*x^2+a)^(1/2)-4/1
5*b*(6*A*b-5*B*a)/a^3/x/(b*x^2+a)^(1/2)-8/15*b^2*(6*A*b-5*B*a)*x/a^4/(b*x^2
+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 197}

$$-\frac{8b^2x(6Ab-5aB)}{15a^4\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/5*A/(a*x^5*Sqrt[a + b*x^2]) + (6*A*b - 5*a*B)/(15*a^2*x^3*Sqrt[a + b*x^2
]) - (4*b*(6*A*b - 5*a*B))/(15*a^3*x*Sqrt[a + b*x^2]) - (8*b^2*(6*A*b - 5*a
*B)*x)/(15*a^4*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{A}{5ax^5\sqrt{a+bx^2}}-\frac{(6Ab-5aB)\int \frac{1}{x^4(a+bx^2)^{3/2}}\,dx}{5a}\\
&= -\frac{A}{5ax^5\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}+\frac{(4b(6Ab-5aB))\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx}{15a^2}\\
&= -\frac{A}{5ax^5\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}-\frac{(8b^2(6Ab-5aB))\int \frac{}{(a+}}{15a^3}\\
&= -\frac{A}{5ax^5\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}-\frac{8b^2(6Ab-5aB)x}{15a^4\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 86, normalized size = 0.75

$$\frac{-3a^3A+6a^2Abx^2-5a^3Bx^2-24aAb^2x^4+20a^2bBx^4-48Ab^3x^6+40ab^2Bx^6}{15a^4x^5\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
[Out] (-3*a^3*A + 6*a^2*A*b*x^2 - 5*a^3*B*x^2 - 24*a*A*b^2*x^4 + 20*a^2*b*B*x^4 -
 48*A*b^3*x^6 + 40*a*b^2*B*x^6)/(15*a^4*x^5*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.10, size = 146, normalized size = 1.27

method	result
gosper	$-\frac{48x^6Ab^3-40x^6Bab^2+24Aab^2x^4-20x^4Ba^2b-6x^2Aa^2b+5Ba^3x^2+3Aa^3}{15x^5\sqrt{bx^2+a}\,a^4}$
trager	$-\frac{48x^6Ab^3-40x^6Bab^2+24Aab^2x^4-20x^4Ba^2b-6x^2Aa^2b+5Ba^3x^2+3Aa^3}{15x^5\sqrt{bx^2+a}\,a^4}$
risch	$-\frac{\sqrt{bx^2+a}\,(33Ab^2x^4-25Babx^4-9aAbx^2+5Ba^2x^2+3a^2A)}{15a^4x^5}-\frac{xb^2(Ab-Ba)}{\sqrt{bx^2+a}\,a^4}$

default	$A\left(-\frac{1}{5a\,x^5\sqrt{b\,x^2+a}}-\frac{6b\left(-\frac{1}{3a\,x^3\sqrt{b\,x^2+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)}{3a}\right)}{5a}\right)+B\left(-\frac{}{3a\,x^3\sqrt{}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5/a/x^5/(b*x^2+a)^(1/2)-6/5*b/a*(-1/3/a/x^3/(b*x^2+a)^(1/2)-4/3*b/a*(
-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))))+B*(-1/3/a/x^3/(b*x^2+a)
^(1/2)-4/3*b/a*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.29, size = 134, normalized size = 1.17

$$\frac{8\,Bb^2x}{3\,\sqrt{bx^2+a}\,a^3}-\frac{16\,Ab^3x}{5\,\sqrt{bx^2+a}\,a^4}+\frac{4\,Bb}{3\,\sqrt{bx^2+a}\,a^2x}-\frac{8\,Ab^2}{5\,\sqrt{bx^2+a}\,a^3x}-\frac{B}{3\,\sqrt{bx^2+a}\,ax^3}+\frac{2\,Ab}{5\,\sqrt{bx^2+a}\,a^2x^3}-\frac{A}{5\,\sqrt{bx^2+a}\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*B*b^2*x/(sqrt(b*x^2 + a)*a^3) - 16/5*A*b^3*x/(sqrt(b*x^2 + a)*a^4) + 4/
3*B*b/(sqrt(b*x^2 + a)*a^2*x) - 8/5*A*b^2/(sqrt(b*x^2 + a)*a^3*x) - 1/3*B/(
sqrt(b*x^2 + a)*a*x^3) + 2/5*A*b/(sqrt(b*x^2 + a)*a^2*x^3) - 1/5*A/(sqrt(b*
x^2 + a)*a*x^5)
```

Fricas [A]
time = 1.56, size = 94, normalized size = 0.82

$$\frac{(8\,(5\,Bab^2-6\,Ab^3)x^6+4\,(5\,Ba^2b-6\,Aab^2)x^4-3\,Aa^3-(5\,Ba^3-6\,Aa^2b)x^2)\sqrt{bx^2+a}}{15\,(a^4bx^7+a^5x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*(5*B*a*b^2 - 6*A*b^3)*x^6 + 4*(5*B*a^2*b - 6*A*a*b^2)*x^4 - 3*A*a^3
 - (5*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b*x^7 + a^5*x^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(109) = 218$.
time = 4.87, size = 593, normalized size = 5.16

$$A\left(-\frac{a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{5a^3b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{30a^2b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{40ab^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{16b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}\right)+B\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-a**5*b**(19/2)*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x
**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sqrt
(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**
8 + 5*a**4*b**12*x**10) - 30*a**2*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(5*a*
*7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10
) - 40*a*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b*
*10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 16*b**(29/2)*x**10*sq
rt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x
**8 + 5*a**4*b**12*x**10)) + B*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5
*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*s
qrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6
) + 12*a*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**
5*x**4 + 3*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*
b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(99) = 198.

time = 0.90, size = 294, normalized size = 2.56

$$\frac{(Bab^2-Ab^3)x}{\sqrt{bx^2+a}\,a^4}-\frac{2\left(15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}-15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-90\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6Ba^2b^{\frac{3}{2}}+90\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6Aab^{\frac{5}{2}}+160\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ba^3b^{\frac{3}{2}}-240\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Aa^2b^{\frac{5}{2}}-110\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^4b^{\frac{3}{2}}+150\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Aa^3b^{\frac{5}{2}}+25\,Ba^5b^{\frac{3}{2}}-33\,Aa^4b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*a*b^2 - A*b^3)*x/(sqrt(b*x^2 + a)*a^4) - 2/15*(15*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*B*a*b^(3/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 90*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*A*a*b^(5/2) + 160*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) - 24
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 110*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*B*a^4*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2
) + 25*B*a^5*b^(3/2) - 33*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)^5*a^3)
```

Mupad [B]

time = 0.44, size = 82, normalized size = 0.71

$$-\frac{5\,B\,a^3\,x^2+3\,A\,a^3-20\,B\,a^2\,b\,x^4-6\,A\,a^2\,b\,x^2-40\,B\,a\,b^2\,x^6+24\,A\,a\,b^2\,x^4+48\,A\,b^3\,x^6}{15\,a^4\,x^5\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x)
```

```
[Out] -(3*A*a^3 + 5*B*a^3*x^2 + 48*A*b^3*x^6 - 6*A*a^2*b*x^2 + 24*A*a*b^2*x^4 - 2
0*B*a^2*b*x^4 - 40*B*a*b^2*x^6)/(15*a^4*x^5*(a + b*x^2)^(1/2))
```

3.582 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{5b^2(7Ab-6aB)}{16a^4\sqrt{a+bx^2}}-\frac{A}{6ax^6\sqrt{a+bx^2}}+\frac{7Ab-6aB}{24a^2x^4\sqrt{a+bx^2}}-\frac{5b(7Ab-6aB)}{48a^3x^2\sqrt{a+bx^2}}+\frac{5b^2(7Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}$$

```
[Out] 5/16*b^2*(7*A*b-6*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)-5/16*b^2*(7
*A*b-6*B*a)/a^4/(b*x^2+a)^(1/2)-1/6*A/a/x^6/(b*x^2+a)^(1/2)+1/24*(7*A*b-6*B
*a)/a^2/x^4/(b*x^2+a)^(1/2)-5/48*b*(7*A*b-6*B*a)/a^3/x^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 44, 53, 65, 214}

$$\frac{5b^2(7Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{5b^2(7Ab-6aB)}{16a^4\sqrt{a+bx^2}}-\frac{5b(7Ab-6aB)}{48a^3x^2\sqrt{a+bx^2}}+\frac{7Ab-6aB}{24a^2x^4\sqrt{a+bx^2}}-\frac{A}{6ax^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-5*b^2*(7*A*b - 6*a*B))/(16*a^4*Sqrt[a + b*x^2]) - A/(6*a*x^6*Sqrt[a + b*x
^2]) + (7*A*b - 6*a*B)/(24*a^2*x^4*Sqrt[a + b*x^2]) - (5*b*(7*A*b - 6*a*B))
/(48*a^3*x^2*Sqrt[a + b*x^2]) + (5*b^2*(7*A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/(16*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7 (a+bx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} + \frac{\left(-\frac{7Ab}{2}+3aB\right)\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/2}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} - \frac{(5(7Ab-6aB))\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x,\right.}{12a^2}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} + \frac{(5b(7Ab-6aB}{}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{5b(7Ab-6aB)}{16a^4x}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{5b(7Ab-6aB)}{16a^4x}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{5b(7Ab-6aB)}{16a^4x}$$

Mathematica [A]
time = 0.23, size = 126, normalized size = 0.82

$$\frac{-8a^3A+14a^2Abx^2-12a^3Bx^2-35aAb^2x^4+30a^2bBx^4-105Ab^3x^6+90ab^2Bx^6}{48a^4x^6\sqrt{a+bx^2}} - \frac{5b^2(-7Ab+6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
[Out] (-8*a^3*A + 14*a^2*A*b*x^2 - 12*a^3*B*x^2 - 35*a*A*b^2*x^4 + 30*a^2*b*B*x^4
 - 105*A*b^3*x^6 + 90*a*b^2*B*x^6)/(48*a^4*x^6*Sqrt[a + b*x^2]) - (5*b^2*(-
7*A*b + 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(9/2))
```

Maple [A]
time = 0.12, size = 210, normalized size = 1.37

method	result

risch	$-\frac{\sqrt{b x^2+a}\,(57A b^2x^4-42Bab x^4-22aAb x^2+12B a^2x^2+8a^2A)}{48a^4x^6}-\frac{b^3A}{a^4\sqrt{b x^2+a}}+\frac{b^2B}{a^3\sqrt{b x^2+a}}+\frac{35b^3\ln\left(\frac{2a+2\sqrt{}\ldots}{}\right)}{}$
default	$B\left(-\frac{1}{4a x^4\sqrt{b x^2+a}}-\frac{5b\left(-\frac{1}{2a x^2\sqrt{b x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{b x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{4a}\right)+A\left(-\ldots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4/a/x^4/(b*x^2+a)^(1/2)-5/4*b/a*(-1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(
1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+A*(-
1/6/a/x^6/(b*x^2+a)^(1/2)-7/6*b/a*(-1/4/a/x^4/(b*x^2+a)^(1/2)-5/4*b/a*(-1/2
/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(
1/2)*(b*x^2+a)^(1/2))/x)))))
```

Maxima [A]
time = 0.29, size = 174, normalized size = 1.14

$$-\frac{15\,Bb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{7}{2}}}+\frac{35\,Ab^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{9}{2}}}+\frac{15\,Bb^2}{8\,\sqrt{bx^2+a}\,a^3}-\frac{35\,Ab^3}{16\,\sqrt{bx^2+a}\,a^4}+\frac{5\,Bb}{8\,\sqrt{bx^2+a}\,a^2x^2}-\frac{35\,Ab^2}{48\,\sqrt{bx^2+a}\,a^3x^2}-\frac{B}{4\,\sqrt{bx^2+a}\,ax^4}+\frac{7\,Ab}{24\,\sqrt{bx^2+a}\,a^2x^4}-\frac{A}{6\,\sqrt{bx^2+a}\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -15/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) + 35/16*A*b^3*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(9/2) + 15/8*B*b^2/(sqrt(b*x^2 + a)*a^3) - 35/16*A*b^3
/(sqrt(b*x^2 + a)*a^4) + 5/8*B*b/(sqrt(b*x^2 + a)*a^2*x^2) - 35/48*A*b^2/(s
qrt(b*x^2 + a)*a^3*x^2) - 1/4*B/(sqrt(b*x^2 + a)*a*x^4) + 7/24*A*b/(sqrt(b*
x^2 + a)*a^2*x^4) - 1/6*A/(sqrt(b*x^2 + a)*a*x^6)
```

Fricas [A]
time = 2.34, size = 341, normalized size = 2.23

$$\left[-\frac{15\left((6\,Bab^3-7\,Ab^4)x^8+(6\,Ba^2b^2-7\,Aab^3)x^6\right)\sqrt{a}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(15\,(6\,Ba^2b^2-7\,Aab^3)x^6-8\,Aa^4+5\,(6\,Ba^3b-7\,Aa^2b^2)x^4-2\,(6\,Ba^4-7\,Aa^3b)x^2\right)\sqrt{bx^2+a}}{96\,(a^5bx^8+a^6x^6)},\ \frac{15\left((6\,Bab^3-7\,Ab^4)x^8+(6\,Ba^2b^2-7\,Aab^3)x^6\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(15\,(6\,Ba^2b^2-7\,Aab^3)x^6-8\,Aa^4+5\,(6\,Ba^3b-7\,Aa^2b^2)x^4-2\,(6\,Ba^4-7\,Aa^3b)x^2\right)\sqrt{bx^2+a}}{48\,(a^5bx^8+a^6x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*((6*B*a*b^3 - 7*A*b^4)*x^8 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^6)*sqrt
(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(15*(6*B*a^2*b^
2 - 7*A*a*b^3)*x^6 - 8*A*a^4 + 5*(6*B*a^3*b - 7*A*a^2*b^2)*x^4 - 2*(6*B*a^4
 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b*x^8 + a^6*x^6), 1/48*(15*((6*B*a
*b^3 - 7*A*b^4)*x^8 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^6)*sqrt(-a)*arctan(sqrt(-
a)/sqrt(b*x^2 + a)) + (15*(6*B*a^2*b^2 - 7*A*a*b^3)*x^6 - 8*A*a^4 + 5*(6*B*
a^3*b - 7*A*a^2*b^2)*x^4 - 2*(6*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a
^5*b*x^8 + a^6*x^6)]
```

Sympy [A]
time = 60.83, size = 236, normalized size = 1.54

$$A\left(-\frac{1}{6a\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{7\sqrt{b}}{24a^2x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{35b^{\frac{3}{2}}}{48a^3x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{35b^{\frac{5}{2}}}{16a^4x\sqrt{\frac{a}{bx^2}+1}}+\frac{35b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{9}{2}}}\right)+B\left(-\frac{1}{4a\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(6*a*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + 7*sqrt(b)/(24*a**2*x**5*sqr
t(a/(b*x**2) + 1)) - 35*b**(3/2)/(48*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 35*b
**(5/2)/(16*a**4*x*sqrt(a/(b*x**2) + 1)) + 35*b**3*asinh(sqrt(a)/(sqrt(b)*x
))/(16*a**(9/2))) + B*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(
b)/(8*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**
2) + 1)) - 15*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2)))
```

Giac [A]
time = 0.67, size = 180, normalized size = 1.18

$$\frac{5\,(6\,Bab^2-7\,Ab^3)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a^4}+\frac{Bab^2-Ab^3}{\sqrt{bx^2+a}\,a^4}+\frac{42\,(bx^2+a)^{\frac{5}{2}}Bab^2-96\,(bx^2+a)^{\frac{3}{2}}Ba^2b^2+54\,\sqrt{bx^2+a}\,Ba^3b^2-57\,(bx^2+a)^{\frac{5}{2}}Ab^3+136\,(bx^2+a)^{\frac{3}{2}}Aab^3-87\,\sqrt{bx^2+a}\,Aa^2b^3}{48\,a^4b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 5/16*(6*B*a*b^2 - 7*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4)
+ (B*a*b^2 - A*b^3)/(sqrt(b*x^2 + a)*a^4) + 1/48*(42*(b*x^2 + a)^(5/2)*B*a*
b^2 - 96*(b*x^2 + a)^(3/2)*B*a^2*b^2 + 54*sqrt(b*x^2 + a)*B*a^3*b^2 - 57*(b
*x^2 + a)^(5/2)*A*b^3 + 136*(b*x^2 + a)^(3/2)*A*a*b^3 - 87*sqrt(b*x^2 + a)*
A*a^2*b^3)/(a^4*b^3*x^6)
```

Mupad [B]
time = 1.08, size = 178, normalized size = 1.16

$$\frac{35\,A\,b^3\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{16\,a^{9/2}}-\frac{15\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{35\,A\,b^3}{16\,a^4\,\sqrt{b\,x^2+a}}+\frac{15\,B\,b^2}{8\,a^3\,\sqrt{b\,x^2+a}}-\frac{A}{6\,a\,x^6\,\sqrt{b\,x^2+a}}-\frac{B}{4\,a\,x^4\,\sqrt{b\,x^2+a}}+\frac{7\,A\,b}{24\,a^2\,x^4\,\sqrt{b\,x^2+a}}+\frac{5\,B\,b}{8\,a^2\,x^2\,\sqrt{b\,x^2+a}}-\frac{35\,A\,b^2}{48\,a^3\,x^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x)
```

```
[Out] (35*A*b^3*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(16*a^(9/2)) - (15*B*b^2*atanh(
(a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(7/2)) - (35*A*b^3)/(16*a^4*(a + b*x^2)^(1
/2)) + (15*B*b^2)/(8*a^3*(a + b*x^2)^(1/2)) - A/(6*a*x^6*(a + b*x^2)^(1/2))
 - B/(4*a*x^4*(a + b*x^2)^(1/2)) + (7*A*b)/(24*a^2*x^4*(a + b*x^2)^(1/2)) +
 (5*B*b)/(8*a^2*x^2*(a + b*x^2)^(1/2)) - (35*A*b^2)/(48*a^3*x^2*(a + b*x^2)
^(1/2))
```

3.583 $\int \frac{A+Bx^2}{x^8 (a+bx^2)^{3/2}} \, dx$

Optimal. Leaf size=148

$$-\frac{A}{7ax^7\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}+\frac{16b^3(8Ab-7aB)x}{35a^5\sqrt{a+bx^2}}$$

```
[Out] -1/7*A/a/x^7/(b*x^2+a)^(1/2)+1/35*(8*A*b-7*B*a)/a^2/x^5/(b*x^2+a)^(1/2)-2/3
5*b*(8*A*b-7*B*a)/a^3/x^3/(b*x^2+a)^(1/2)+8/35*b^2*(8*A*b-7*B*a)/a^4/x/(b*x
^2+a)^(1/2)+16/35*b^3*(8*A*b-7*B*a)*x/a^5/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 277, 197}

$$\frac{16b^3x(8Ab-7aB)}{35a^5\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/7*A/(a*x^7*Sqrt[a + b*x^2]) + (8*A*b - 7*a*B)/(35*a^2*x^5*Sqrt[a + b*x^2
]) - (2*b*(8*A*b - 7*a*B))/(35*a^3*x^3*Sqrt[a + b*x^2]) + (8*b^2*(8*A*b - 7
*a*B))/(35*a^4*x*Sqrt[a + b*x^2]) + (16*b^3*(8*A*b - 7*a*B)*x)/(35*a^5*Sqrt
[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A}{7ax^7\sqrt{a+bx^2}} - \frac{(8Ab-7aB)\int \frac{1}{x^6(a+bx^2)^{3/2}}\,dx}{7a}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^2}} + \frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}} + \frac{(6b(8Ab-7aB))\int \frac{1}{x^4(a+bx^2)^{3/2}}\,dx}{35a^2}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^2}} + \frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}} - \frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}} - \frac{(8b^2(8Ab-7aB))\int \frac{}{x^2}}{35a^3}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^2}} + \frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}} - \frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}} + \frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}} + \frac{(16}{}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^2}} + \frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}} - \frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}} + \frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}} + \frac{16b}{3}$$

Mathematica [A]
time = 0.17, size = 105, normalized size = 0.71

$$\frac{128Ab^4x^8+16ab^3x^6(4A-7Bx^2)-8a^2b^2x^4(2A+7Bx^2)+2a^3bx^2(4A+7Bx^2)-a^4(5A+7Bx^2)}{35a^5x^7\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

```
[Out] (128*A*b^4*x^8 + 16*a*b^3*x^6*(4*A - 7*B*x^2) - 8*a^2*b^2*x^4*(2*A + 7*B*x^
2) + 2*a^3*b*x^2*(4*A + 7*B*x^2) - a^4*(5*A + 7*B*x^2))/(35*a^5*x^7*Sqrt[a
+ b*x^2])
```

Maple [A]
time = 0.11, size = 194, normalized size = 1.31

method	result
gosper	$-\frac{-128A\,b^4x^8+112Ba\,b^3x^8-64Aa\,b^3x^6+56B\,a^2b^2x^6+16A\,a^2b^2x^4-14B\,a^3b\,x^4-8A\,a^3b\,x^2+7B\,a^4x^2+5A\,a^4}{35x^7\sqrt{b\,x^2+a}\;a^5}$
trager	$-\frac{-128A\,b^4x^8+112Ba\,b^3x^8-64Aa\,b^3x^6+56B\,a^2b^2x^6+16A\,a^2b^2x^4-14B\,a^3b\,x^4-8A\,a^3b\,x^2+7B\,a^4x^2+5A\,a^4}{35x^7\sqrt{b\,x^2+a}\;a^5}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-93x^6A\,b^3+77x^6Ba\,b^2+29Aa\,b^2x^4-21x^4B\,a^2b-13x^2A\,a^2b+7B\,a^3x^2+5A\,a^3\right)}{35a^5x^7}+\frac{x(Ab-Ba)b^3}{\sqrt{b\,x^2+a}\;a^5}$

default	$B\left(-\frac{1}{5a\,x^{5}\sqrt{b\,x^{2}+a}}-\frac{6b\left(-\frac{1}{3a\,x^{3}\sqrt{b\,x^{2}+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}\right)}{3a}\right)}{5a}\right)+A\left(-\frac{1}{7a\,x^{7}\sqrt{\ldots}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/5/a/x^5/(b*x^2+a)^(1/2)-6/5*b/a*(-1/3/a/x^3/(b*x^2+a)^(1/2)-4/3*b/a*(
-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))))+A*(-1/7/a/x^7/(b*x^2+a)
^(1/2)-8/7*b/a*(-1/5/a/x^5/(b*x^2+a)^(1/2)-6/5*b/a*(-1/3/a/x^3/(b*x^2+a)^(1
/2)-4/3*b/a*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2)))))
```

Maxima [A]
time = 0.32, size = 176, normalized size = 1.19

$$-\frac{16\,Bb^{3}x}{5\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{128\,Ab^{4}x}{35\,\sqrt{bx^{2}+a}\,a^{5}}-\frac{8\,Bb^{2}}{5\,\sqrt{bx^{2}+a}\,a^{3}x}+\frac{64\,Ab^{3}}{35\,\sqrt{bx^{2}+a}\,a^{4}x}+\frac{2\,Bb}{5\,\sqrt{bx^{2}+a}\,a^{2}x^{3}}-\frac{16\,Ab^{2}}{35\,\sqrt{bx^{2}+a}\,a^{3}x^{3}}-\frac{B}{5\,\sqrt{bx^{2}+a}\,ax^{5}}+\frac{8\,Ab}{35\,\sqrt{bx^{2}+a}\,a^{2}x^{5}}-\frac{A}{7\,\sqrt{bx^{2}+a}\,ax^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/5*B*b^3*x/(sqrt(b*x^2 + a)*a^4) + 128/35*A*b^4*x/(sqrt(b*x^2 + a)*a^5)
- 8/5*B*b^2/(sqrt(b*x^2 + a)*a^3*x) + 64/35*A*b^3/(sqrt(b*x^2 + a)*a^4*x) +
 2/5*B*b/(sqrt(b*x^2 + a)*a^2*x^3) - 16/35*A*b^2/(sqrt(b*x^2 + a)*a^3*x^3)
- 1/5*B/(sqrt(b*x^2 + a)*a*x^5) + 8/35*A*b/(sqrt(b*x^2 + a)*a^2*x^5) - 1/7*
A/(sqrt(b*x^2 + a)*a*x^7)
```

Fricas [A]
time = 1.48, size = 117, normalized size = 0.79

$$-\frac{\left(16\,(7\,Bab^{3}-8\,Ab^{4})x^{8}+8\,(7\,Ba^{2}b^{2}-8\,Aab^{3})x^{6}+5\,Aa^{4}-2\,(7\,Ba^{3}b-8\,Aa^{2}b^{2})x^{4}+(7\,Ba^{4}-8\,Aa^{3}b)x^{2}\right)\sqrt{bx^{2}+a}}{35\,(a^{5}bx^{9}+a^{6}x^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*(7*B*a*b^3 - 8*A*b^4)*x^8 + 8*(7*B*a^2*b^2 - 8*A*a*b^3)*x^6 + 5*A
*a^4 - 2*(7*B*a^3*b - 8*A*a^2*b^2)*x^4 + (7*B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*
x^2 + a)/(a^5*b*x^9 + a^6*x^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(143) = 286$.
time = 6.74, size = 1030, normalized size = 6.96

$$A\left(-\frac{5a^{7}b^{\frac{33}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}-\frac{7a^{6}b^{\frac{35}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}-\frac{7a^{5}b^{\frac{37}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{35a^{4}b^{\frac{39}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{280a^{3}b^{\frac{41}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{560a^{2}b^{\frac{43}{2}}x^{10}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{448ab^{\frac{45}{2}}x^{12}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{128b^{\frac{47}{2}}x^{14}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}\right)+B\left(-\frac{a^{5}b^{\frac{19}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{5a^{3}b^{\frac{23}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{30a^{2}b^{\frac{25}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{40ab^{\frac{27}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{16b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**8/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-5*a**7*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b*
*17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**1
4) - 7*a**6*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a
**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**2
0*x**14) - 7*a**5*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 +
 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**
5*b**20*x**14) + 35*a**4*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16
*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 +
 35*a**5*b**20*x**14) + 280*a**3*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a*
*9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19
*x**12 + 35*a**5*b**20*x**14) + 560*a**2*b**(43/2)*x**10*sqrt(a/(b*x**2) +
1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a
**6*b**19*x**12 + 35*a**5*b**20*x**14) + 448*a*b**(45/2)*x**12*sqrt(a/(b*x*
*2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 +
 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 128*b**(47/2)*x**14*sqrt(a/(
b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**
10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14)) + B*(-a**5*b**(19/2)*sqrt
(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**
8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(5*a**
7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10)
 - 30*a**2*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*
b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 40*a*b**(27/2)*x**8
*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**1
1*x**8 + 5*a**4*b**12*x**10) - 16*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(5*a
**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**1
0))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(128) = 256$.
time = 0.57, size = 407, normalized size = 2.75

$$-\frac{(Bab^{3}-Ab^{4})x}{\sqrt{bx^{2}+a}\,a^{5}}+\frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}Bab^{\frac{5}{2}}-35\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}Ab^{\frac{7}{2}}-280\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{10}Ba^{2}b^{\frac{5}{2}}+280\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{10}Aab^{\frac{7}{2}}+1015\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Ba^{3}b^{\frac{5}{2}}-1015\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Aa^{2}b^{\frac{7}{2}}-1680\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Ba^{4}b^{\frac{5}{2}}+2240\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Aa^{3}b^{\frac{7}{2}}+1337\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ba^{5}b^{\frac{5}{2}}-1673\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Aa^{4}b^{\frac{7}{2}}-504\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba^{6}b^{\frac{5}{2}}+616\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Aa^{5}b^{\frac{7}{2}}+77Ba^{7}b^{\frac{5}{2}}-93Aa^{6}b^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{7}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*a*b^3 - A*b^4)*x/(sqrt(b*x^2 + a)*a^5) + 2/35*(35*(sqrt(b)*x - sqrt(b*x
^2 + a))^12*B*a*b^(5/2) - 35*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 2
80*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) + 280*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*A*a*b^(7/2) + 1015*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5
/2) - 1015*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 1680*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 2240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A
*a^3*b^(7/2) + 1337*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 1673*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2) - 504*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*B*a^6*b^(5/2) + 616*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^(7/2) +
77*B*a^7*b^(5/2) - 93*A*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
^7*a^4)
```

Mupad [B]
time = 0.51, size = 148, normalized size = 1.00

$$-\frac{x^2\left(\frac{58\,A\,b^4-42\,B\,a\,b^3}{35\,a^5}-\frac{2\,b^3\,(93\,A\,b-77\,B\,a)}{35\,a^5}\right)-\frac{b^2\,(93\,A\,b-77\,B\,a)}{35\,a^4}}{x\,\sqrt{b\,x^2+a}}-\frac{(7\,B\,a^2-13\,A\,a\,b)\,\sqrt{b\,x^2+a}}{35\,a^4\,x^5}-\frac{A\,\sqrt{b\,x^2+a}}{7\,a^2\,x^7}-\frac{b\,\sqrt{b\,x^2+a}\,(29\,A\,b-21\,B\,a)}{35\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x)
```

```
[Out] - (x^2*((58*A*b^4 - 42*B*a*b^3)/(35*a^5) - (2*b^3*(93*A*b - 77*B*a))/(35*a^
5)) - (b^2*(93*A*b - 77*B*a))/(35*a^4))/(x*(a + b*x^2)^(1/2)) - ((7*B*a^2 -
 13*A*a*b)*(a + b*x^2)^(1/2))/(35*a^4*x^5) - (A*(a + b*x^2)^(1/2))/(7*a^2*x
^7) - (b*(a + b*x^2)^(1/2)*(29*A*b - 21*B*a))/(35*a^4*x^3)
```

3.584 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{a^3(Ab-aB)}{3b^5(a+bx^2)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a(Ab-2aB)\sqrt{a+bx^2}}{b^5}+\frac{(Ab-4aB)(a+bx^2)^{3/2}}{3b^5}+\frac{B(a+bx^2)^{5/2}}{5b^5}$$

```
[Out] 1/3*a^3*(A*b-B*a)/b^5/(b*x^2+a)^(3/2)+1/3*(A*b-4*B*a)*(b*x^2+a)^(3/2)/b^5+1
/5*B*(b*x^2+a)^(5/2)/b^5-a^2*(3*A*b-4*B*a)/b^5/(b*x^2+a)^(1/2)-3*a*(A*b-2*B
*a)*(b*x^2+a)^(1/2)/b^5
```

Rubi [A]
time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\frac{a^3(Ab-aB)}{3b^5(a+bx^2)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a\sqrt{a+bx^2}(Ab-2aB)}{b^5}+\frac{(a+bx^2)^{3/2}(Ab-4aB)}{3b^5}+\frac{B(a+bx^2)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a^3*(A*b - a*B))/(3*b^5*(a + b*x^2)^(3/2)) - (a^2*(3*A*b - 4*a*B))/(b^5*Sq
rt[a + b*x^2]) - (3*a*(A*b - 2*a*B)*Sqrt[a + b*x^2])/b^5 + ((A*b - 4*a*B)*(
a + b*x^2)^(3/2))/(3*b^5) + (B*(a + b*x^2)^(5/2))/(5*b^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3(-Ab+aB)}{b^4(a+bx)^{5/2}} - \frac{a^2(-3Ab+4aB)}{b^4(a+bx)^{3/2}} + \frac{3a(-Ab+2aB)}{b^4\sqrt{a+bx}} + \frac{(Ab-4aB)}{b^4}\right.\right.$$

$$= \frac{a^3(Ab-aB)}{3b^5\left(a+bx^2\right)^{3/2}} - \frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}} - \frac{3a(Ab-2aB)\sqrt{a+bx^2}}{b^5} + \frac{(Ab-4aB)\left(a+b\right.}{3b^5}$$

Mathematica [A]
time = 0.07, size = 98, normalized size = 0.77

$$\frac{128a^4B+24a^2b^2x^2(-5A+2Bx^2)+b^4x^6(5A+3Bx^2)-2ab^3x^4(15A+4Bx^2)+a^3(-80Ab+192bBx^2)}{15b^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (128*a^4*B + 24*a^2*b^2*x^2*(-5*A + 2*B*x^2) + b^4*x^6*(5*A + 3*B*x^2) - 2*
a*b^3*x^4*(15*A + 4*B*x^2) + a^3*(-80*A*b + 192*b*B*x^2))/(15*b^5*(a + b*x^
2)^(3/2))
```

Maple [A]
time = 0.10, size = 190, normalized size = 1.48

method	result
gosper	$-\frac{-3B\,b^4x^8-5A\,b^4x^6+8Ba\,b^3x^6+30Aa\,b^3x^4-48B\,a^2b^2x^4+120a^2A\,b^2x^2-192B\,a^3b\,x^2+80A\,a^3b-128B\,a^4}{15(b\,x^2+a)^{\frac{3}{2}}b^5}$
trager	$-\frac{-3B\,b^4x^8-5A\,b^4x^6+8Ba\,b^3x^6+30Aa\,b^3x^4-48B\,a^2b^2x^4+120a^2A\,b^2x^2-192B\,a^3b\,x^2+80A\,a^3b-128B\,a^4}{15(b\,x^2+a)^{\frac{3}{2}}b^5}$
risch	$-\frac{(-3b^2B\,x^4-5A\,b^2x^2+14Bab\,x^2+40abA-73a^2B)\sqrt{b\,x^2+a}}{15b^5}-\frac{\sqrt{b\,x^2+a}\,(9A\,b^2x^2-12Bab\,x^2+8abA-11a^2B)a^2}{3b^5(b^2x^4+2ab\,x^2+a^2)}$

default	$B\left(\frac{x^8}{5b(bx^2+a)^{\frac{3}{2}}}-\frac{8a\left(\frac{x^6}{3b(bx^2+a)^{\frac{3}{2}}}-\frac{2a\left(\frac{x^4}{b(bx^2+a)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^2}{b(bx^2+a)^{\frac{3}{2}}}-\frac{2a}{3b^2(bx^2+a)^{\frac{3}{2}}}\right)}{b}\right)}{b}\right)}{5b}\right)+A\left(\frac{x^6}{3b(bx^2+a)^{\frac{3}{2}}}-\frac{2a(}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/5*x^8/b/(b*x^2+a)^(3/2)-8/5*a/b*(1/3*x^6/b/(b*x^2+a)^(3/2)-2*a/b*(x^4/
b/(b*x^2+a)^(3/2)-4*a/b*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2)))
))+A*(1/3*x^6/b/(b*x^2+a)^(3/2)-2*a/b*(x^4/b/(b*x^2+a)^(3/2)-4*a/b*(-x^2/b/
(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))))
```

Maxima [A]
time = 0.30, size = 174, normalized size = 1.36

$$\frac{Bx^8}{5\,(bx^2+a)^{\frac{3}{2}}b}-\frac{8\,Bax^6}{15\,(bx^2+a)^{\frac{3}{2}}b^2}+\frac{Ax^6}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{16\,Ba^2x^4}{5\,(bx^2+a)^{\frac{3}{2}}b^3}-\frac{2\,Aax^4}{(bx^2+a)^{\frac{3}{2}}b^2}+\frac{64\,Ba^3x^2}{5\,(bx^2+a)^{\frac{3}{2}}b^4}-\frac{8\,Aa^2x^2}{(bx^2+a)^{\frac{3}{2}}b^3}+\frac{128\,Ba^4}{15\,(bx^2+a)^{\frac{3}{2}}b^5}-\frac{16\,Aa^3}{3\,(bx^2+a)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*B*x^8/((b*x^2 + a)^(3/2)*b) - 8/15*B*a*x^6/((b*x^2 + a)^(3/2)*b^2) + 1/
3*A*x^6/((b*x^2 + a)^(3/2)*b) + 16/5*B*a^2*x^4/((b*x^2 + a)^(3/2)*b^3) - 2*
A*a*x^4/((b*x^2 + a)^(3/2)*b^2) + 64/5*B*a^3*x^2/((b*x^2 + a)^(3/2)*b^4) -
8*A*a^2*x^2/((b*x^2 + a)^(3/2)*b^3) + 128/15*B*a^4/((b*x^2 + a)^(3/2)*b^5)
- 16/3*A*a^3/((b*x^2 + a)^(3/2)*b^4)
```

Fricas [A]
time = 1.77, size = 123, normalized size = 0.96

$$\frac{(3\,Bb^4x^8-(8\,Bab^3-5\,Ab^4)x^6+128\,Ba^4-80\,Aa^3b+6\,(8\,Ba^2b^2-5\,Aab^3)x^4+24\,(8\,Ba^3b-5\,Aa^2b^2)x^2)\sqrt{bx^2+a}}{15\,(b^7x^4+2\,ab^6x^2+a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^4*x^8 - (8*B*a*b^3 - 5*A*b^4)*x^6 + 128*B*a^4 - 80*A*a^3*b + 6*
(8*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 24*(8*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(b*x^
2 + a)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(119) = 238$.
time = 0.58, size = 437, normalized size = 3.41

$$\begin{cases} -\frac{80Aa^3b}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} - \frac{120Aa^2b^2x^2}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} - \frac{30Aab^3x^4}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} + \frac{5Ab^4x^6}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} + \frac{128Ba^4}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} + \frac{192Ba^3bx^2}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} + \frac{48Ba^2b^2x^4}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} - \frac{8Bab^3x^6}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} + \frac{3Bb^4x^8}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^8}{8}+\frac{Bx^{10}}{10}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-80*A*a**3*b/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a +
 b*x**2)) - 120*A*a**2*b**2*x**2/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2
*sqrt(a + b*x**2)) - 30*A*a*b**3*x**4/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6
*x**2*sqrt(a + b*x**2)) + 5*A*b**4*x**6/(15*a*b**5*sqrt(a + b*x**2) + 15*b*
*6*x**2*sqrt(a + b*x**2)) + 128*B*a**4/(15*a*b**5*sqrt(a + b*x**2) + 15*b**
6*x**2*sqrt(a + b*x**2)) + 192*B*a**3*b*x**2/(15*a*b**5*sqrt(a + b*x**2) +
15*b**6*x**2*sqrt(a + b*x**2)) + 48*B*a**2*b**2*x**4/(15*a*b**5*sqrt(a + b*
x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) - 8*B*a*b**3*x**6/(15*a*b**5*sqrt(a
+ b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) + 3*B*b**4*x**8/(15*a*b**5*sqrt(
a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**8/8 + B*x**
10/10)/a**(5/2), True))
```

Giac [A]
time = 0.51, size = 141, normalized size = 1.10

$$\frac{12\,(bx^2+a)Ba^3 - Ba^4 - 9\,(bx^2+a)Aa^2b + Aa^3b}{3\,(bx^2+a)^{\frac{3}{2}}b^5} + \frac{3\,(bx^2+a)^{\frac{5}{2}}Bb^{20} - 20\,(bx^2+a)^{\frac{3}{2}}Bab^{20} + 90\,\sqrt{bx^2+a}\,Ba^2b^{20} + 5\,(bx^2+a)^{\frac{3}{2}}Ab^{21} - 45\,\sqrt{bx^2+a}\,Aab^{21}}{15\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(12*(b*x^2 + a)*B*a^3 - B*a^4 - 9*(b*x^2 + a)*A*a^2*b + A*a^3*b)/((b*x^
2 + a)^(3/2)*b^5) + 1/15*(3*(b*x^2 + a)^(5/2)*B*b^20 - 20*(b*x^2 + a)^(3/2)
*B*a*b^20 + 90*sqrt(b*x^2 + a)*B*a^2*b^20 + 5*(b*x^2 + a)^(3/2)*A*b^21 - 45
*sqrt(b*x^2 + a)*A*a*b^21)/b^25
```

Mupad [B]
time = 0.48, size = 122, normalized size = 0.95

$$\frac{3\,B\,{\left(b\,x^2+a\right)}^4-5\,B\,a^4+90\,B\,a^2\,{\left(b\,x^2+a\right)}^2+5\,A\,b\,{\left(b\,x^2+a\right)}^3-20\,B\,a\,{\left(b\,x^2+a\right)}^3+60\,B\,a^3\,\left(b\,x^2+a\right)+5\,A\,a^3\,b-45\,A\,a\,b\,{\left(b\,x^2+a\right)}^2-45\,A\,a^2\,b\,\left(b\,x^2+a\right)}{15\,b^5\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] (3*B*(a + b*x^2)^4 - 5*B*a^4 + 90*B*a^2*(a + b*x^2)^2 + 5*A*b*(a + b*x^2)^3
 - 20*B*a*(a + b*x^2)^3 + 60*B*a^3*(a + b*x^2) + 5*A*a^3*b - 45*A*a*b*(a +
b*x^2)^2 - 45*A*a^2*b*(a + b*x^2))/(15*b^5*(a + b*x^2)^(3/2))
```

3.585 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{(4Ab-7aB)x^5}{12b^2(a+bx^2)^{3/2}}+\frac{Bx^7}{4b(a+bx^2)^{3/2}}-\frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}}+\frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4}-\frac{5a(4Ab-7aB)\tanh^{-1}}{8b^{9/2}}$$

```
[Out] -1/12*(4*A*b-7*B*a)*x^5/b^2/(b*x^2+a)^(3/2)+1/4*B*x^7/b/(b*x^2+a)^(3/2)-5/8
*a*(4*A*b-7*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-5/12*(4*A*b-7*B
*a)*x^3/b^3/(b*x^2+a)^(1/2)+5/8*(4*A*b-7*B*a)*x*(b*x^2+a)^(1/2)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 294, 327, 223, 212}

$$-\frac{5a(4Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}+\frac{5x\sqrt{a+bx^2}\,(4Ab-7aB)}{8b^4}-\frac{5x^3(4Ab-7aB)}{12b^3\sqrt{a+bx^2}}-\frac{x^5(4Ab-7aB)}{12b^2(a+bx^2)^{3/2}}+\frac{Bx^7}{4b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/12*((4*A*b - 7*a*B)*x^5)/(b^2*(a + b*x^2)^(3/2)) + (B*x^7)/(4*b*(a + b*x
^2)^(3/2)) - (5*(4*A*b - 7*a*B)*x^3)/(12*b^3*Sqrt[a + b*x^2]) + (5*(4*A*b -
 7*a*B)*x*Sqrt[a + b*x^2])/(8*b^4) - (5*a*(4*A*b - 7*a*B)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{Bx^7}{4b\,(a+bx^2)^{3/2}} - \frac{(-4Ab+7aB)\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx}{4b}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\,(a+bx^2)^{3/2}} + \frac{Bx^7}{4b\,(a+bx^2)^{3/2}} + \frac{(5(4Ab-7aB))\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx}{12b^2}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\,(a+bx^2)^{3/2}} + \frac{Bx^7}{4b\,(a+bx^2)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{(5(4Ab-7aB))\int \frac{x^2}{\sqrt{a+b}}}{4b^3}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\,(a+bx^2)^{3/2}} + \frac{Bx^7}{4b\,(a+bx^2)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\,(a+bx^2)^{3/2}} + \frac{Bx^7}{4b\,(a+bx^2)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\,(a+bx^2)^{3/2}} + \frac{Bx^7}{4b\,(a+bx^2)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4}$$

Mathematica [A]
time = 0.23, size = 119, normalized size = 0.80

$$\frac{-105a^3Bx+ab^2x^3(80A-21Bx^2)+20a^2bx(3A-7Bx^2)+6b^3x^5(2A+Bx^2)}{24b^4\,(a+bx^2)^{3/2}} + \frac{5a(4Ab-7aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-105*a^3*B*x + a*b^2*x^3*(80*A - 21*B*x^2) + 20*a^2*b*x*(3*A - 7*B*x^2) +
6*b^3*x^5*(2*A + B*x^2))/(24*b^4*(a + b*x^2)^(3/2)) + (5*a*(4*A*b - 7*a*B)*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Maple [A]
time = 0.13, size = 194, normalized size = 1.30

method	result
default	$B\left(\frac{x^{7}}{4b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{7a\left(\frac{x^{5}}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2b}\right)}{4b}\right)+A\left(\frac{\cdots}{2b(b\cdots}\right.$
risch	$\frac{x\left(2bB\,x^{2}+4Ab-11Ba\right)\sqrt{b\,x^{2}+a}}{8b^{4}}-\frac{5a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)A}{2b^{\frac{7}{2}}}+\frac{35a^{2}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)B}{8b^{\frac{9}{2}}}+\frac{a^{2}\sqrt{\left(x\cdots\right.}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/4*x^7/b/(b*x^2+a)^(3/2)-7/4*a/b*(1/2*x^5/b/(b*x^2+a)^(3/2)-5/2*a/b*(-1
/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(
b*x^2+a)^(1/2))))))+A*(1/2*x^5/b/(b*x^2+a)^(3/2)-5/2*a/b*(-1/3*x^3/b/(b*x^2
+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
)))
```

Maxima [A]

time = 0.32, size = 210, normalized size = 1.41

$$\frac{Bx^{7}}{4\,(bx^{2}+a)^{\frac{3}{2}}b}-\frac{7\,Bax^{5}}{8\,(bx^{2}+a)^{\frac{3}{2}}b^{2}}+\frac{Ax^{5}}{2\,(bx^{2}+a)^{\frac{3}{2}}b}-\frac{35\,Ba^{2}x\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{24\,b^{2}}+\frac{5\,Aax\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{6\,b}-\frac{35\,Ba^{2}x}{24\,\sqrt{bx^{2}+a}\,b^{4}}+\frac{5\,Aax}{6\,\sqrt{bx^{2}+a}\,b^{3}}+\frac{35\,Ba^{2}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{9}{2}}}-\frac{5\,Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*B*x^7/((b*x^2 + a)^(3/2)*b) - 7/8*B*a*x^5/((b*x^2 + a)^(3/2)*b^2) + 1/2
*A*x^5/((b*x^2 + a)^(3/2)*b) - 35/24*B*a^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) +
 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 5/6*A*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
+ 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 35/24*B*a^2*x/(sqrt(b*x^2 + a)*b^4) + 5/
6*A*a*x/(sqrt(b*x^2 + a)*b^3) + 35/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(9/2) -
 5/2*A*a*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Fricas [**A**]
time = 2.12, size = 392, normalized size = 2.63

$$\left[-\frac{15\,(7\,Ba^{4}-4\,Aa^{3}b+(7\,Ba^{2}b^{2}-4\,Aab^{3})x^{4}+2\,(7\,Ba^{3}b-4\,Aa^{2}b^{2})x^{2})\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,(6\,Bb^{4}x^{7}-3\,(7\,Bab^{3}-4\,Ab^{4})x^{5}-20\,(7\,Ba^{2}b^{2}-4\,Aab^{3})x^{3}-15\,(7\,Ba^{3}b-4\,Aa^{2}b^{2})x)\sqrt{bx^{2}+a}}{48\,(b^{7}x^{4}+2\,ab^{6}x^{2}+a^{2}b^{5})},\ -\frac{15\,(7\,Ba^{4}-4\,Aa^{3}b+(7\,Ba^{2}b^{2}-4\,Aab^{3})x^{4}+2\,(7\,Ba^{3}b-4\,Aa^{2}b^{2})x^{2})\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(6\,Bb^{4}x^{7}-3\,(7\,Bab^{3}-4\,Ab^{4})x^{5}-20\,(7\,Ba^{2}b^{2}-4\,Aab^{3})x^{3}-15\,(7\,Ba^{3}b-4\,Aa^{2}b^{2})x)\sqrt{bx^{2}+a}}{24\,(b^{7}x^{4}+2\,ab^{6}x^{2}+a^{2}b^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*a*b^3)*x^4 + 2*(7*B*a^
3*b - 4*A*a^2*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) - 2*(6*B*b^4*x^7 - 3*(7*B*a*b^3 - 4*A*b^4)*x^5 - 20*(7*B*a^2*b^2 - 4*A
*a*b^3)*x^3 - 15*(7*B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^7*x^4 + 2
*a*b^6*x^2 + a^2*b^5), -1/24*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*
a*b^3)*x^4 + 2*(7*B*a^3*b - 4*A*a^2*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (6*B*b^4*x^7 - 3*(7*B*a*b^3 - 4*A*b^4)*x^5 - 20*(7*B*a^2*b
^2 - 4*A*a*b^3)*x^3 - 15*(7*B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^7
*x^4 + 2*a*b^6*x^2 + a^2*b^5)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 804 vs. 2(144) = 288.
time = 16.14, size = 804, normalized size = 5.40

$$A\left(-\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{2}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^{3}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{38}b^{\frac{49}{2}}x^{5}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)+B\left(\frac{105a^{\frac{157}{2}}b^{41}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^{2}}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{105a^{\frac{155}{2}}b^{42}x^{2}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^{2}}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{105a^{78}b^{\frac{83}{2}}x}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^{2}}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{140a^{77}b^{\frac{85}{2}}x^{3}}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^{2}}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{21a^{76}b^{\frac{87}{2}}x^{5}}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^{2}}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{6a^{75}b^{\frac{89}{2}}x^{7}}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^{2}}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(7
9/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x
**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a
))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*s
```

```
qrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b
*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47
/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*
x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sq
rt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a))) + B*(105
*a**(157/2)*b**41*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(24*a**(153/2
)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x*
*2/a)) + 105*a**(155/2)*b**42*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(
a))/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x
**2*sqrt(1 + b*x**2/a)) - 105*a**78*b**(83/2)*x/(24*a**(153/2)*b**(91/2)*sq
rt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) - 140*a
**77*b**(85/2)*x**3/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(15
1/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) - 21*a**76*b**(87/2)*x**5/(24*a**(1
53/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 +
b*x**2/a)) + 6*a**75*b**(89/2)*x**7/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**
2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)))
```

Giac [A]
time = 0.47, size = 148, normalized size = 0.99

$$\frac{\left(\left(3\left(\frac{2Bx^2}{b}-\frac{7Ba^2b^5-4Aab^6}{ab^7}\right)x^2-\frac{20\left(7Ba^3b^4-4Aa^2b^5\right)}{ab^7}\right)x^2-\frac{15\left(7Ba^4b^3-4Aa^3b^4\right)}{ab^7}\right)x}{24\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{5\left(7Ba^2-4Aab\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*B*x^2/b - (7*B*a^2*b^5 - 4*A*a*b^6)/(a*b^7))*x^2 - 20*(7*B*a^3*
b^4 - 4*A*a^2*b^5)/(a*b^7))*x^2 - 15*(7*B*a^4*b^3 - 4*A*a^3*b^4)/(a*b^7))*x
/(b*x^2 + a)^(3/2) - 5/8*(7*B*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\left(B\,x^2+A\right)}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^6*(A + B*x^2))/(a + b*x^2)^(5/2), x)
```

3.586 $\int \frac{x^5 (A+Bx^2)}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=97

$$-\frac{a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{(Ab-3aB)\sqrt{a+bx^2}}{b^4}+\frac{B(a+bx^2)^{3/2}}{3b^4}$$

```
[Out] -1/3*a^2*(A*b-B*a)/b^4/(b*x^2+a)^(3/2)+1/3*B*(b*x^2+a)^(3/2)/b^4+a*(2*A*b-3
*B*a)/b^4/(b*x^2+a)^(1/2)+(A*b-3*B*a)*(b*x^2+a)^(1/2)/b^4
```

Rubi [A]

time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$-\frac{a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\,(Ab-3aB)}{b^4}+\frac{B(a+bx^2)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(a^2*(A*b - a*B))/(b^4*(a + b*x^2)^(3/2)) + (a*(2*A*b - 3*a*B))/(b^4*S
qrt[a + b*x^2]) + ((A*b - 3*a*B)*Sqrt[a + b*x^2])/b^4 + (B*(a + b*x^2)^(3/2
))/(3*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{5/2}}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^{3/2}}+\frac{Ab-3aB}{b^3\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^3}\right) dx\right.\\
&= -\frac{a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{(Ab-3aB)\sqrt{a+bx^2}}{b^4}+\frac{B(a+bx^2)^{3/2}}{3b^4}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 73, normalized size = 0.75

$$\frac{-16a^3B+8a^2b(A-3Bx^2)-6ab^2x^2(-2A+Bx^2)+b^3x^4(3A+Bx^2)}{3b^4(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
[Out] (-16*a^3*B + 8*a^2*b*(A - 3*B*x^2) - 6*a*b^2*x^2*(-2*A + B*x^2) + b^3*x^4*(
3*A + B*x^2))/(3*b^4*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 142, normalized size = 1.46

method	result
gosper	$\frac{B\,x^6b^3+3A\,b^3x^4-6Ba\,b^2x^4+12Aa\,b^2x^2-24B\,a^2b\,x^2+8A\,a^2b-16B\,a^3}{3(b\,x^2+a)^{\frac{3}{2}}b^4}$
trager	$\frac{B\,x^6b^3+3A\,b^3x^4-6Ba\,b^2x^4+12Aa\,b^2x^2-24B\,a^2b\,x^2+8A\,a^2b-16B\,a^3}{3(b\,x^2+a)^{\frac{3}{2}}b^4}$
risch	$\frac{(bB\,x^2+3Ab-8Ba)\sqrt{b\,x^2+a}}{3b^4}+\frac{\sqrt{b\,x^2+a}\,(6A\,b^2x^2-9Bab\,x^2+5abA-8a^2B)a}{3b^4(b^2x^4+2ab\,x^2+a^2)}$
default	$B\left(\frac{x^6}{3b(b\,x^2+a)^{\frac{3}{2}}}-\frac{2a\left(\frac{x^4}{b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^2}{b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^2\left(b\,x^2+a\right)^{\frac{3}{2}}}\right)}{b}\right)}{b}\right)+A\left(\frac{x^4}{b(b\,x^2+a)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^2}{b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{}{3b^2}\right.}{b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/3*x^6/b/(b*x^2+a)^(3/2)-2*a/b*(x^4/b/(b*x^2+a)^(3/2)-4*a/b*(-x^2/b/(b*
x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))))+A*(x^4/b/(b*x^2+a)^(3/2)-4*a/b*(-
x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2)))
```

Maxima [A]
time = 0.28, size = 131, normalized size = 1.35

$$\frac{Bx^6}{3\,(bx^2+a)^{\frac{3}{2}}b}-\frac{2\,Bax^4}{(bx^2+a)^{\frac{3}{2}}b^2}+\frac{Ax^4}{(bx^2+a)^{\frac{3}{2}}b}-\frac{8\,Ba^2x^2}{(bx^2+a)^{\frac{3}{2}}b^3}+\frac{4\,Aax^2}{(bx^2+a)^{\frac{3}{2}}b^2}-\frac{16\,Ba^3}{3\,(bx^2+a)^{\frac{3}{2}}b^4}+\frac{8\,Aa^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*B*x^6/((b*x^2 + a)^(3/2)*b) - 2*B*a*x^4/((b*x^2 + a)^(3/2)*b^2) + A*x^4
/((b*x^2 + a)^(3/2)*b) - 8*B*a^2*x^2/((b*x^2 + a)^(3/2)*b^3) + 4*A*a*x^2/((
b*x^2 + a)^(3/2)*b^2) - 16/3*B*a^3/((b*x^2 + a)^(3/2)*b^4) + 8/3*A*a^2/((b*
x^2 + a)^(3/2)*b^3)
```

Fricas [A]
time = 1.28, size = 98, normalized size = 1.01

$$\frac{(Bb^3x^6-3\,(2\,Bab^2-Ab^3)x^4-16\,Ba^3+8\,Aa^2b-12\,(2\,Ba^2b-Aab^2)x^2)\sqrt{bx^2+a}}{3\,(b^6x^4+2\,ab^5x^2+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^3*x^6 - 3*(2*B*a*b^2 - A*b^3)*x^4 - 16*B*a^3 + 8*A*a^2*b - 12*(2*B
*a^2*b - A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(88) = 176.
time = 0.56, size = 337, normalized size = 3.47

$$\begin{cases} \frac{8Aa^2b}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}}+\frac{12Aab^2x^2}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}}+\frac{3Ab^3x^4}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}}-\frac{16Ba^3}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}}-\frac{24Ba^2bx^2}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}}-\frac{6Bab^2x^4}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}}+\frac{Bb^3x^6}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*A*a**2*b/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*x
**2)) + 12*A*a*b**2*x**2/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
b*x**2)) + 3*A*b**3*x**4/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
b*x**2)) - 16*B*a**3/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*x*
*2)) - 24*B*a**2*b*x**2/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b
*x**2)) - 6*B*a*b**2*x**4/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
```

```
b*x**2)) + B*b**3*x**6/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b
*x**2)), Ne(b, 0)), ((A*x**6/6 + B*x**8/8)/a**(5/2), True))
```

Giac [A]
time = 0.94, size = 104, normalized size = 1.07

$$-\frac{9\,(bx^2+a)Ba^2 - Ba^3 - 6\,(bx^2+a)Aab + Aa^2b}{3\,(bx^2+a)^{\frac{3}{2}}b^4} + \frac{(bx^2+a)^{\frac{3}{2}}Bb^8 - 9\,\sqrt{bx^2+a}\,Bab^8 + 3\,\sqrt{bx^2+a}\,Ab^9}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(9*(b*x^2 + a)*B*a^2 - B*a^3 - 6*(b*x^2 + a)*A*a*b + A*a^2*b)/((b*x^2
+ a)^(3/2)*b^4) + 1/3*((b*x^2 + a)^(3/2)*B*b^8 - 9*sqrt(b*x^2 + a)*B*a*b^8
+ 3*sqrt(b*x^2 + a)*A*b^9)/b^12
```

Mupad [B]
time = 0.40, size = 89, normalized size = 0.92

$$\frac{B\,{\left(b\,x^2+a\right)}^3+B\,a^3+3\,A\,b\,{\left(b\,x^2+a\right)}^2-9\,B\,a\,{\left(b\,x^2+a\right)}^2-9\,B\,a^2\,\left(b\,x^2+a\right)-A\,a^2\,b+6\,A\,a\,b\,\left(b\,x^2+a\right)}{3\,b^4\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] (B*(a + b*x^2)^3 + B*a^3 + 3*A*b*(a + b*x^2)^2 - 9*B*a*(a + b*x^2)^2 - 9*B*
a^2*(a + b*x^2) - A*a^2*b + 6*A*a*b*(a + b*x^2))/(3*b^4*(a + b*x^2)^(3/2))
```

3.587 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{a(Ab-aB)x}{3b^3(a+bx^2)^{3/2}}-\frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}+\frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

```
[Out] 1/3*a*(A*b-B*a)*x/b^3/(b*x^2+a)^(3/2)+1/2*(2*A*b-5*B*a)*arctanh(x*b^(1/2)/(
b*x^2+a)^(1/2))/b^(7/2)-1/3*(4*A*b-7*B*a)*x/b^3/(b*x^2+a)^(1/2)+1/2*B*x*(b*
x^2+a)^(1/2)/b^3
```

Rubi [A]

time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.227$, Rules used = {466, 1171, 396, 223, 212}

$$\frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}-\frac{x(4Ab-7aB)}{3b^3\sqrt{a+bx^2}}+\frac{ax(Ab-aB)}{3b^3(a+bx^2)^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a*(A*b - a*B)*x)/(3*b^3*(a + b*x^2)^(3/2)) - ((4*A*b - 7*a*B)*x)/(3*b^3*Sq
rt[a + b*x^2]) + (B*x*Sqrt[a + b*x^2])/(2*b^3) + ((2*A*b - 5*a*B)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{a(Ab-aB)x}{3b^3(a+bx^2)^{3/2}} - \frac{\int \frac{a(Ab-aB)-3b(Ab-aB)x^2-3b^2Bx^4}{(a+bx^2)^{3/2}}\,dx}{3b^3}$$

$$= \frac{a(Ab-aB)x}{3b^3(a+bx^2)^{3/2}} - \frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}} + \frac{\int \frac{3a(Ab-2aB)+3abBx^2}{\sqrt{a+bx^2}}\,dx}{3ab^3}$$

$$= \frac{a(Ab-aB)x}{3b^3(a+bx^2)^{3/2}} - \frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^3} + \frac{(2Ab-5aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^3}$$

$$= \frac{a(Ab-aB)x}{3b^3(a+bx^2)^{3/2}} - \frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^3} + \frac{(2Ab-5aB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx\right.}{2b^3}$$

$$= \frac{a(Ab-aB)x}{3b^3(a+bx^2)^{3/2}} - \frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^3} + \frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right.}{2b^{7/2}}$$

Mathematica [A]
time = 0.17, size = 97, normalized size = 0.85

$$\frac{x(-6aAb+15a^2B-8Ab^2x^2+20abBx^2+3b^2Bx^4)}{6b^3(a+bx^2)^{3/2}} + \frac{(-2Ab+5aB)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(-6*a*A*b + 15*a^2*B - 8*A*b^2*x^2 + 20*a*b*B*x^2 + 3*b^2*B*x^4))/(6*b^3
*(a + b*x^2)^(3/2)) + ((-2*A*b + 5*a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/(2*b^(7/2))
```

Maple [A]
time = 0.11, size = 146, normalized size = 1.28

method	result
default	$B\left(\frac{x^{5}}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2b}\right)+A\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{\ }{b\sqrt{\ }}}{\ }\right.$
risch	$\frac{Bx\sqrt{b\,x^{2}+a}}{2b^{3}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)A}{b^{\frac{5}{2}}}-\frac{5\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)Ba}{2b^{\frac{7}{2}}}-\frac{a\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}b-2\sqrt{\ }}}{12b^{3}\sqrt{-ab}\left(x+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/2*x^5/b/(b*x^2+a)^(3/2)-5/2*a/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/
(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(-1/3*x^3/b/(b
*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))))
```

Maxima [A]
time = 0.32, size = 160, normalized size = 1.40

$$\frac{Bx^{5}}{2\,(bx^{2}+a)^{\frac{3}{2}}b}-\frac{1}{3}\,Ax\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)+\frac{5\,Bax\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{6\,b}+\frac{5\,Bax}{6\,\sqrt{bx^{2}+a}\,b^{3}}-\frac{Ax}{3\,\sqrt{bx^{2}+a}\,b^{2}}-\frac{5\,Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}+\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^5/((b*x^2 + a)^(3/2)*b) - 1/3*A*x*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*
a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*B*a*x*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/(
```

(b*x^2 + a)^(3/2)*b^2))/b + 5/6*B*a*x/(sqrt(b*x^2 + a)*b^3) - 1/3*A*x/(sqrt
(b*x^2 + a)*b^2) - 5/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(7/2) + A*arcsinh(b*x/s
qrt(a*b))/b^(5/2)

Fricas [A]
time = 1.77, size = 333, normalized size = 2.92

$$\left[-\frac{3\,\left(\left(5\,Bab^{2}-2\,Ab^{3}\right)x^{4}+5\,Ba^{3}-2\,Aa^{2}b+2\,\left(5\,Ba^{2}b-2\,Aab^{2}\right)x^{2}\right)\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,\left(3\,Bb^{3}x^{5}+4\,\left(5\,Bab^{2}-2\,Ab^{3}\right)x^{3}+3\,\left(5\,Ba^{2}b-2\,Aab^{2}\right)x\right)\sqrt{bx^{2}+a}}{12\,\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)},\ \frac{3\,\left(\left(5\,Bab^{2}-2\,Ab^{3}\right)x^{4}+5\,Ba^{3}-2\,Aa^{2}b+2\,\left(5\,Ba^{2}b-2\,Aab^{2}\right)x^{2}\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(3\,Bb^{3}x^{5}+4\,\left(5\,Bab^{2}-2\,Ab^{3}\right)x^{3}+3\,\left(5\,Ba^{2}b-2\,Aab^{2}\right)x\right)\sqrt{bx^{2}+a}}{6\,\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")

[Out] [-1/12*(3*((5*B*a*b^2 - 2*A*b^3)*x^4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b -
 2*A*a*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
2*(3*B*b^3*x^5 + 4*(5*B*a*b^2 - 2*A*b^3)*x^3 + 3*(5*B*a^2*b - 2*A*a*b^2)*x)
*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(3*((5*B*a*b^2 - 2
*A*b^3)*x^4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^2)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*B*b^3*x^5 + 4*(5*B*a*b^2 - 2*A*b^3
)*x^3 + 3*(5*B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^
2 + a^2*b^4)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(105) = 210$.
time = 8.94, size = 675, normalized size = 5.92

$$A\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^{2}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^{3}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)+B\left(-\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{2}\sqrt{1+\frac{bx^{2}}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^{3}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{38}b^{\frac{49}{2}}x^{5}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**(5/2),x)

[Out] A*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/
2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**
2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/
(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt
(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x
3/(3*a(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*
sqrt(1 + b*x**2/a))) + B*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt
(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(
53/2)*x**2*sqrt(1 + b*x**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)
*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**
(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/
2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**
2/a)) + 20*a**39*b**(47/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) +
 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6

```
*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1
 + b*x**2/a)))
```

Giac [A]
time = 0.81, size = 112, normalized size = 0.98

$$\frac{\left(\left(\frac{3 Bx^2}{b} + \frac{4 (5 Ba^2b^3 - 2 Aab^4)}{ab^5}\right)x^2 + \frac{3 (5 Ba^3b^2 - 2 Aa^2b^3)}{ab^5}\right)x}{6 (bx^2 + a)^{\frac{3}{2}}} + \frac{(5 Ba - 2 Ab) \log\left(\left|-\sqrt{b}\, x + \sqrt{bx^2 + a}\right|\right)}{2 b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*B*x^2/b + 4*(5*B*a^2*b^3 - 2*A*a*b^4)/(a*b^5))*x^2 + 3*(5*B*a^3*b^2
 - 2*A*a^2*b^3)/(a*b^5))*x/(b*x^2 + a)^(3/2) + 1/2*(5*B*a - 2*A*b)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(B\,x^2+A\right)}{{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^4*(A + B*x^2))/(a + b*x^2)^(5/2), x)
```

3.588 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{a(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{B\sqrt{a+bx^2}}{b^3}$$

```
[Out] 1/3*a*(A*b-B*a)/b^3/(b*x^2+a)^(3/2)+(-A*b+2*B*a)/b^3/(b*x^2+a)^(1/2)+B*(b*x
^2+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{a(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{B\sqrt{a+bx^2}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a*(A*b - a*B))/(3*b^3*(a + b*x^2)^(3/2)) - (A*b - 2*a*B)/(b^3*Sqrt[a + b*x
^2]) + (B*Sqrt[a + b*x^2])/b^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{5/2}} + \frac{Ab-2aB}{b^2(a+bx)^{3/2}} + \frac{B}{b^2\sqrt{a+bx}}\right) dx, x, x^2\right)$$
$$= \frac{a(Ab-aB)}{3b^3(a+bx^2)^{3/2}} - \frac{Ab-2aB}{b^3\sqrt{a+bx^2}} + \frac{B\sqrt{a+bx^2}}{b^3}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.82

$$\frac{-2aAb+8a^2B-3Ab^2x^2+12abBx^2+3b^2Bx^4}{3b^3(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a*A*b + 8*a^2*B - 3*A*b^2*x^2 + 12*a*b*B*x^2 + 3*b^2*B*x^4)/(3*b^3*(a +
 b*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 95, normalized size = 1.40

method	result	size
gosper	$-\frac{-3b^2Bx^4+3Ab^2x^2-12Babx^2+2abA-8a^2B}{3(bx^2+a)^{\frac{3}{2}}b^3}$	53
trager	$-\frac{-3b^2Bx^4+3Ab^2x^2-12Babx^2+2abA-8a^2B}{3(bx^2+a)^{\frac{3}{2}}b^3}$	53
risch	$\frac{B\sqrt{bx^2+a}}{b^3} - \frac{\sqrt{bx^2+a}\,(3Ab^2x^2-6Babx^2+2abA-5a^2B)}{3b^3(b^2x^4+2abx^2+a^2)}$	79
default	$B\left(\frac{x^4}{b(bx^2+a)^{\frac{3}{2}}} - \frac{4a\left(-\frac{x^2}{b(bx^2+a)^{\frac{3}{2}}} - \frac{2a}{3b^2(bx^2+a)^{\frac{3}{2}}}\right)}{b}\right) + A\left(-\frac{x^2}{b(bx^2+a)^{\frac{3}{2}}} - \frac{2a}{3b^2(bx^2+a)^{\frac{3}{2}}}\right)$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(x^4/b/(b*x^2+a)^(3/2)-4*a/b*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^
(3/2)))+A*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))
```

Maxima [A]
time = 0.31, size = 89, normalized size = 1.31

$$\frac{Bx^4}{(bx^2+a)^{\frac{3}{2}}b}+\frac{4\,Bax^2}{(bx^2+a)^{\frac{3}{2}}b^2}-\frac{Ax^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{8\,Ba^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3}-\frac{2\,Aa}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] B*x^4/((b*x^2 + a)^(3/2)*b) + 4*B*a*x^2/((b*x^2 + a)^(3/2)*b^2) - A*x^2/((b
*x^2 + a)^(3/2)*b) + 8/3*B*a^2/((b*x^2 + a)^(3/2)*b^3) - 2/3*A*a/((b*x^2 +
a)^(3/2)*b^2)
```

Fricas [A]
time = 1.86, size = 75, normalized size = 1.10

$$\frac{(3\,Bb^2x^4+8\,Ba^2-2\,Aab+3\,(4\,Bab-Ab^2)x^2)\sqrt{bx^2+a}}{3\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*B*b^2*x^4 + 8*B*a^2 - 2*A*a*b + 3*(4*B*a*b - A*b^2)*x^2)*sqrt(b*x^2
+ a)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(60) = 120.
time = 0.39, size = 240, normalized size = 3.53

$$\begin{cases} -\frac{2Aab}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}}-\frac{3Ab^2x^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}}+\frac{8Ba^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}}+\frac{12Babx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}}+\frac{3Bb^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*A*a*b/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**
2)) - 3*A*b**2*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**
2)) + 8*B*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)) +
 12*B*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)) +
 3*B*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)),
Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/a**(5/2), True))
```

Giac [A]
time = 1.34, size = 62, normalized size = 0.91

$$\frac{\sqrt{bx^2+a}\,B}{b^3}+\frac{6\,(bx^2+a)Ba-Ba^2-3\,(bx^2+a)Ab+Aab}{3\,(bx^2+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)*B/b^3 + 1/3*(6*(b*x^2 + a)*B*a - B*a^2 - 3*(b*x^2 + a)*A*b
+ A*a*b)/((b*x^2 + a)^(3/2)*b^3)
```

Mupad [B]
time = 0.34, size = 59, normalized size = 0.87

$$\frac{3\,B\,{\left(b\,x^2+a\right)}^2-B\,a^2-3\,A\,b\,\left(b\,x^2+a\right)+6\,B\,a\,\left(b\,x^2+a\right)+A\,a\,b}{3\,b^3\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] (3*B*(a + b*x^2)^2 - B*a^2 - 3*A*b*(a + b*x^2) + 6*B*a*(a + b*x^2) + A*a*b)
/(3*b^3*(a + b*x^2)^(3/2))
```

3.589 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=77

$$\frac{(Ab-aB)x^3}{3ab(a+bx^2)^{3/2}} - \frac{Bx}{b^2\sqrt{a+bx^2}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] 1/3*(A*b-B*a)*x^3/a/b/(b*x^2+a)^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/
b^(5/2)-B*x/b^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {463, 294, 223, 212}

$$\frac{x^3(Ab-aB)}{3ab(a+bx^2)^{3/2}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}} - \frac{Bx}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*x^3)/(3*a*b*(a + b*x^2)^(3/2)) - (B*x)/(b^2*Sqrt[a + b*x^2]) +
 (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx &= \frac{(Ab-aB)x^3}{3ab\,(a+bx^2)^{3/2}}+\frac{B\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx}{b}\\
&= \frac{(Ab-aB)x^3}{3ab\,(a+bx^2)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}\\
&= \frac{(Ab-aB)x^3}{3ab\,(a+bx^2)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^2}\\
&= \frac{(Ab-aB)x^3}{3ab\,(a+bx^2)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.12, size = 75, normalized size = 0.97

$$
\frac{-3a^2Bx+Ab^2x^3-4abBx^3}{3ab^2\,(a+bx^2)^{3/2}}-\frac{B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a^2*B*x + A*b^2*x^3 - 4*a*b*B*x^3)/(3*a*b^2*(a + b*x^2)^(3/2)) - (B*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2)
```

Maple [**A**]
time = 0.09, size = 117, normalized size = 1.52

method	result

default	$B\left(-\frac{x^{3}}{3b(b\,x^{2}+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+A\left(-\frac{x}{2b(b\,x^{2}+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{}{3a^{2}\sqrt{}}\right.}{2b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2))))+A*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+
a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

Maxima [**A**]
time = 0.30, size = 103, normalized size = 1.34

$$-\frac{1}{3}\,Bx\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)-\frac{Bx}{3\,\sqrt{bx^{2}+a}\,b^{2}}-\frac{Ax}{3\,(bx^{2}+a)^{\frac{3}{2}}b}+\frac{Ax}{3\,\sqrt{bx^{2}+a}\,ab}+\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - 1/3*
B*x/(sqrt(b*x^2 + a)*b^2) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/3*A*x/(sqrt(b
*x^2 + a)*a*b) + B*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Fricas [**A**]
time = 1.96, size = 245, normalized size = 3.18

$$\left[\frac{3\,(Bab^{2}x^{4}+2\,Ba^{2}bx^{2}+Ba^{3})\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,(3\,Ba^{2}bx+(4\,Bab^{2}-Ab^{3})x^{3})\sqrt{bx^{2}+a}}{6\,(ab^{5}x^{4}+2\,a^{2}b^{4}x^{2}+a^{3}b^{3})},-\frac{3\,(Bab^{2}x^{4}+2\,Ba^{2}bx^{2}+Ba^{3})\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+(3\,Ba^{2}bx+(4\,Bab^{2}-Ab^{3})x^{3})\sqrt{bx^{2}+a}}{3\,(ab^{5}x^{4}+2\,a^{2}b^{4}x^{2}+a^{3}b^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*a*b^2*x^4 + 2*B*a^2*b*x^2 + B*a^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a) - 2*(3*B*a^2*b*x + (4*B*a*b^2 - A*b^3)*x^3)*sqrt
(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3), -1/3*(3*(B*a*b^2*x^4 +
2*B*a^2*b*x^2 + B*a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*B*a
^2*b*x + (4*B*a*b^2 - A*b^3)*x^3)*sqrt(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*b^4*x
^2 + a^3*b^3)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(68) = 136$.
time = 5.75, size = 352, normalized size = 4.57

$$\frac{Ax^{3}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{3}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}+B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^{2}}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^{2}\sqrt{1+\frac{bx^{2}}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^{3}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a
)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a*
*(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 +
b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt
(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2
*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 +
b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25
/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*
x**2*sqrt(1 + b*x**2/a)))
```

Giac [A]
time = 1.39, size = 69, normalized size = 0.90

$$-\frac{x\left(\frac{3\,Ba}{b^2}+\frac{(4\,Bab^2-Ab^3)x^2}{ab^3}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(3*B*a/b^2 + (4*B*a*b^2 - A*b^3)*x^2/(a*b^3))/(b*x^2 + a)^(3/2) - B*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(B\,x^2+A)}{(b\,x^2+a)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^2*(A + B*x^2))/(a + b*x^2)^(5/2), x)
```

3.590 $\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=44

$$\frac{-Ab+aB}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}$$

```
[Out] 1/3*(-A*b+B*a)/b^2/(b*x^2+a)^(3/2)-B/b^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$-\frac{Ab-aB}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(A*b - a*B)/(b^2*(a + b*x^2)^(3/2)) - B/(b^2*Sqrt[a + b*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{5/2}}+\frac{B}{b(a+bx)^{3/2}}\right)\,dx, x, x^2\right)\\
&= -\frac{Ab-aB}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.77

$$\frac{-Ab-2aB-3bBx^2}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
[Out] (-(A*b) - 2*a*B - 3*b*B*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.09, size = 52, normalized size = 1.18

method	result	size
gosper	$-\frac{3bB\,x^2+Ab+2Ba}{3(b\,x^2+a)^{\frac{3}{2}}\,b^2}$	30
trager	$-\frac{3bB\,x^2+Ab+2Ba}{3(b\,x^2+a)^{\frac{3}{2}}\,b^2}$	30
default	$B\left(-\frac{x^2}{b(b\,x^2+a)^{\frac{3}{2}}}-\frac{2a}{3b^2(b\,x^2+a)^{\frac{3}{2}}}\right)-\frac{A}{3b(b\,x^2+a)^{\frac{3}{2}}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] B*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))-1/3*A/b/(b*x^2+a)^(3/2
)
```

Maxima [A]
time = 0.27, size = 50, normalized size = 1.14

$$-\frac{Bx^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{2\,Ba}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}-\frac{A}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2) - 1/3*A/((b*
x^2 + a)^(3/2)*b)
```

Fricas [A]
time = 1.40, size = 52, normalized size = 1.18

$$-\frac{\left(3\,Bbx^2+2\,Ba+Ab\right)\sqrt{bx^2+a}}{3\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*B*b*x^2 + 2*B*a + A*b)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2
*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(37) = 74$.
time = 0.39, size = 143, normalized size = 3.25

$$\begin{cases} -\frac{Ab}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{2Ba}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3Bbx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^4}{4}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-A*b/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2))
- 2*B*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)) - 3*B*b*
x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b, 0)),
 ((A*x**2/2 + B*x**4/4)/a**(5/2), True))
```

Giac [A]
time = 1.42, size = 32, normalized size = 0.73

$$-\frac{3\,(bx^2+a)B - Ba + Ab}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(b*x^2 + a)*B - B*a + A*b)/((b*x^2 + a)^(3/2)*b^2)
```

Mupad [B]
time = 0.28, size = 32, normalized size = 0.73

$$-\frac{A\,b - B\,a + 3\,B\,(b\,x^2+a)}{3\,b^2\,(b\,x^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] -(A*b - B*a + 3*B*(a + b*x^2))/(3*b^2*(a + b*x^2)^(3/2))
```

3.591 $\int \frac{A+Bx^2}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x(A+Bx^2)}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*x*(B*x^2+A)/a/(b*x^2+a)^(3/2)+2/3*A*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {386, 197}

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x(A+Bx^2)}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*A*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(A + B*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{x(A+Bx^2)}{3a\left(a+bx^2\right)^{3/2}}+\frac{(2A)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\ &= \frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x(A+Bx^2)}{3a\left(a+bx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 37, normalized size = 0.79

$$\frac{x(3aA+2Abx^2+aBx^2)}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(3*a*A + 2*A*b*x^2 + a*B*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(39) = 78.
time = 0.08, size = 90, normalized size = 1.91

method	result	size
gosper	$\frac{x\left(2Ab\,x^{2}+Ba\,x^{2}+3Aa\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	34
trager	$\frac{x\left(2Ab\,x^{2}+Ba\,x^{2}+3Aa\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	34
default	$B\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}\right)+A\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x
^2+a)^(1/2)))+A*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.29, size = 68, normalized size = 1.45

$$\frac{2\,Ax}{3\,\sqrt{bx^2+a}\,a^2}+\frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{Bx}{3\,\sqrt{bx^2+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B*x/((b
*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b)
```

Fricas [A]
time = 1.66, size = 54, normalized size = 1.15

$$\frac{((Ba+2\,Ab)x^3+3\,Aax)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((B*a + 2*A*b)*x^3 + 3*A*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^
2 + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(41) = 82$.
time = 4.36, size = 144, normalized size = 3.06

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sq
rt(1 + b*x**2/a))
```

Giac [A]
time = 1.00, size = 40, normalized size = 0.85

$$\frac{x\left(\frac{3\,A}{a}+\frac{(Bab+2\,Ab^2)x^2}{a^2b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(3*A/a + (B*a*b + 2*A*b^2)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

Mupad [B]
time = 0.28, size = 33, normalized size = 0.70

$$\frac{3\,A\,a\,x+2\,A\,b\,x^3+B\,a\,x^3}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^(5/2),x)
```

```
[Out] (3*A*a*x + 2*A*b*x^3 + B*a*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

3.592 $\int \frac{A+Bx^2}{x(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 1/3*(A*b-B*a)/a/b/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2
)+A/a^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 53, 65, 214}

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^2)^(3/2)) + A/(a^2*Sqrt[a + b*x^2]) - (A*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 69, normalized size = 0.96

$$\frac{4aAb-a^2B+3Ab^2x^2}{3a^2b\left(a+bx^2\right)^{3/2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (4*a*A*b - a^2*B + 3*A*b^2*x^2)/(3*a^2*b*(a + b*x^2)^(3/2)) - (A*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Maple [A]
time = 0.09, size = 80, normalized size = 1.11

method	result	size
default	$-\frac{B}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+A\left(\frac{1}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*B/b/(b*x^2+a)^(3/2)+A*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-
1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Maxima [A]
time = 0.39, size = 63, normalized size = 0.88

$$-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{A}{\sqrt{bx^2+a}\,a^2}+\frac{A}{3\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{B}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + A/(sqrt(b*x^2 + a)*a^2) + 1/3*A/
((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*x^2 + a)^(3/2)*b)
```

Fricas [A]
time = 1.64, size = 241, normalized size = 3.35

$$\left[\frac{3\left(Ab^3x^4+2\,Aab^2x^2+Aa^2b\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,Aab^2x^2-Ba^3+4\,Aa^2b\right)\sqrt{bx^2+a}}{6\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)},\ \frac{3\left(Ab^3x^4+2\,Aab^2x^2+Aa^2b\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3\,Aab^2x^2-Ba^3+4\,Aa^2b\right)\sqrt{bx^2+a}}{3\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*A*a*b^2*x^2 - B*a^3 + 4*A*a^2*b)*sqrt
(b*x^2 + a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b), 1/3*(3*(A*b^3*x^4 + 2*A
*a*b^2*x^2 + A*a^2*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*A*a*b^
2*x^2 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^2 + a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 +
a^5*b)]
```

Sympy [A]
time = 14.69, size = 66, normalized size = 0.92

$$\frac{A}{a^2\sqrt{a+bx^2}}+\frac{A\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{a^2\sqrt{-a}}-\frac{-Ab+Ba}{3ab\left(a+bx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A/(a**2*sqrt(a + b*x**2)) + A*atan(sqrt(a + b*x**2)/sqrt(-a))/(a**2*sqrt(-a
)) - (-A*b + B*a)/(3*a*b*(a + b*x**2)**(3/2))
```

Giac [A]
time = 1.37, size = 66, normalized size = 0.92

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}-\frac{Ba^2-3\left(bx^2+a\right)Ab-Aab}{3\left(bx^2+a\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/3*(B*a^2 - 3*(b*x^2 +
 a)*A*b - A*a*b)/((b*x^2 + a)^(3/2)*a^2*b)
```

Mupad [B]
time = 0.54, size = 65, normalized size = 0.90

$$\frac{\frac{A}{3\,a}+\frac{A\,\left(b\,x^2+a\right)}{a^2}}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{B}{3\,b\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^(5/2)),x)
```

```
[Out] (A/(3*a) + (A*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - B/(3*b*(a + b*x^2)^(3/2
)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(5/2)
```

3.593 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{A}{ax\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-aB)x}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{2(4Ab-aB)x}{3a^3\sqrt{a+bx^2}}$$

```
[Out] -A/a/x/(b*x^2+a)^(3/2)-1/3*(4*A*b-B*a)*x/a^2/(b*x^2+a)^(3/2)-2/3*(4*A*b-B*a
)*x/a^3/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 198, 197}

$$-\frac{2x(4Ab-aB)}{3a^3\sqrt{a+bx^2}}-\frac{x(4Ab-aB)}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(3/2))) - ((4*A*b - a*B)*x)/(3*a^2*(a + b*x^2)^(3/2))
- (2*(4*A*b - a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{A}{ax\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-aB)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a}\\ &= -\frac{A}{ax\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-aB)x}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{(2(4Ab-aB))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}\\ &= -\frac{A}{ax\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-aB)x}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{2(4Ab-aB)x}{3a^3\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 62, normalized size = 0.81

$$\frac{-3a^2A-12aAbx^2+3a^2Bx^2-8Ab^2x^4+2abBx^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*A - 12*a*A*b*x^2 + 3*a^2*B*x^2 - 8*A*b^2*x^4 + 2*a*b*B*x^4)/(3*a^3*
x*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 92, normalized size = 1.19

method	result	size
gosper	$-\frac{8A\,b^2x^4-2Bab\,x^4+12aAb\,x^2-3B\,a^2x^2+3a^2A}{3x\left(b\,x^2+a\right)^{\frac{3}{2}}a^3}$	59
trager	$-\frac{8A\,b^2x^4-2Bab\,x^4+12aAb\,x^2-3B\,a^2x^2+3a^2A}{3x\left(b\,x^2+a\right)^{\frac{3}{2}}a^3}$	59
risch	$-\frac{A\sqrt{b\,x^2+a}}{a^3x}-\frac{\sqrt{b\,x^2+a}\,x\left(5A\,b^2x^2-2Bab\,x^2+6abA-3a^2B\right)}{3a^3\left(b^2x^4+2ab\,x^2+a^2\right)}$	84
default	$B\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+A\left(-\frac{1}{ax\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}\right)$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))+A*(-1/a/x/(b*x^2+a)^(
3/2)-4*b/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 85, normalized size = 1.10

$$\frac{2\,Bx}{3\,\sqrt{bx^2+a}\,a^2}+\frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{8\,Abx}{3\,\sqrt{bx^2+a}\,a^3}-\frac{4\,Abx}{3\,(bx^2+a)^{\frac{3}{2}}a^2}-\frac{A}{(bx^2+a)^{\frac{3}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*x/(sqrt(b*x^2 + a)*a^2) + 1/3*B*x/((b*x^2 + a)^(3/2)*a) - 8/3*A*b*x/(
sqrt(b*x^2 + a)*a^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a^2) - A/((b*x^2 + a)^(
3/2)*a*x)
```

Fricas [A]
time = 1.61, size = 77, normalized size = 1.00

$$\frac{\left(2\,(Bab-4\,Ab^2)x^4-3\,Aa^2+3\,(Ba^2-4\,Aab)x^2\right)\sqrt{bx^2+a}}{3\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 + 3*(B*a^2 - 4*A*a*b)*x^2)*sqrt(b*x^
2 + a)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(71) = 142$.
time = 7.36, size = 265, normalized size = 3.44

$$A\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)+B\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 +
 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) +
1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(3*a*x/(3*a**(7
/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x**3/(
3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)))
```

Giac [A]
time = 1.18, size = 101, normalized size = 1.31

$$\frac{x\left(\frac{(2\,Ba^3b^2-5\,Aa^2b^3)x^2}{a^5b}+\frac{3\,(Ba^4b-2\,Aa^3b^2)}{a^5b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*((2*B*a^3*b^2 - 5*A*a^2*b^3)*x^2/(a^5*b) + 3*(B*a^4*b - 2*A*a^3*b^2)/
(a^5*b))/(b*x^2 + a)^(3/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)*a^2)
```

Mupad [B]
time = 0.32, size = 68, normalized size = 0.88

$$\frac{A\,a^2-8\,A\,{\left(b\,x^2+a\right)}^2+B\,a^2\,x^2+4\,A\,a\,\left(b\,x^2+a\right)+2\,B\,a\,x^2\,\left(b\,x^2+a\right)}{3\,a^3\,x\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x)
```

```
[Out] (A*a^2 - 8*A*(a + b*x^2)^2 + B*a^2*x^2 + 4*A*a*(a + b*x^2) + 2*B*a*x^2*(a +
 b*x^2))/(3*a^3*x*(a + b*x^2)^(3/2))
```

3.594 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{-5Ab+2aB}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}}-\frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] 1/6*(-5*A*b+2*B*a)/a^2/(b*x^2+a)^(3/2)-1/2*A/a/x^2/(b*x^2+a)^(3/2)+1/2*(5*A
*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)+1/2*(-5*A*b+2*B*a)/a^3/(
b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 53, 65, 214}

$$\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}}-\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/6*(5*A*b - 2*a*B)/(a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2)
) - (5*A*b - 2*a*B)/(2*a^3*Sqrt[a + b*x^2]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx,x,x^2\right)$$

$$= -\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} + \frac{\left(-\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,x^2\right)}{2a}$$

$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{(5Ab-2aB)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)}{4a^2}$$

$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}} - \frac{(5Ab-2aB)\text{Subst}\left(\int \frac{\cdots}{x\sqrt{\cdots}}\cdots\right.}{4a^3}$$

$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}} - \frac{(5Ab-2aB)\text{Subst}\left(\int \frac{\cdots}{-\frac{a}{b}\cdots}\cdots\right.}{2a^3b\cdots}$$

$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}} + \frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a\cdots}}{\cdots}\right.}{2a^{7/2}}$$

Mathematica [A]
time = 0.14, size = 99, normalized size = 0.88

$$\frac{-3a^2A - 20aAbx^2 + 8a^2Bx^2 - 15Ab^2x^4 + 6abBx^4}{6a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*A - 20*a*A*b*x^2 + 8*a^2*B*x^2 - 15*A*b^2*x^4 + 6*a*b*B*x^4)/(6*a^3
*x^2*(a + b*x^2)^(3/2)) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/(2*a^(7/2))
```

Maple [A]
time = 0.11, size = 152, normalized size = 1.35

method	result
default	$A\left(-\frac{1}{2a\,x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)}{2a}\right)+B\left(\frac{1}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\right.$
risch	$-\frac{A\sqrt{b\,x^{2}+a}}{2a^{3}x^{2}}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}\,A}{12a^{3}\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}b+2}}{12a^{2}b\left(x-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2/(b*x^2+a)^(3/2)-5/2*b/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^
2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+B*(1/3/a/(b*x
^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x)))
```

Maxima [A]
time = 0.29, size = 117, normalized size = 1.04

$$-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{5\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{7}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a^2}+\frac{B}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{5\,Ab}{2\,\sqrt{bx^2+a}\,a^3}-\frac{5\,Ab}{6\,(bx^2+a)^{\frac{3}{2}}a^2}-\frac{A}{2\,(bx^2+a)^{\frac{3}{2}}ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/2*A*b*arcsinh(a/(sqrt(a*b)*abs
(x)))/a^(7/2) + B/(sqrt(b*x^2 + a)*a^2) + 1/3*B/((b*x^2 + a)^(3/2)*a) - 5/2
*A*b/(sqrt(b*x^2 + a)*a^3) - 5/6*A*b/((b*x^2 + a)^(3/2)*a^2) - 1/2*A/((b*x^
2 + a)^(3/2)*a*x^2)
```

Fricas [A]
time = 1.26, size = 349, normalized size = 3.09

$$\left[-\frac{3\,((2\,Bab^2-5\,Ab^3)x^6+2\,(2\,Ba^2b-5\,Aab^2)x^4+(2\,Ba^3-5\,Aa^2b)x^2)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(3\,(2\,Ba^2b-5\,Aab^2)x^4-3\,Aa^3+4\,(2\,Ba^3-5\,Aa^2b)x^2)\sqrt{bx^2+a}}{12\,(a^4b^2x^6+2\,a^5bx^4+a^6x^2)},\ \frac{3\,((2\,Bab^2-5\,Ab^3)x^6+2\,(2\,Ba^2b-5\,Aab^2)x^4+(2\,Ba^3-5\,Aa^2b)x^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(2\,Ba^2b-5\,Aab^2)x^4-3\,Aa^3+4\,(2\,Ba^3-5\,Aa^2b)x^2)\sqrt{bx^2+a}}{6\,(a^4b^2x^6+2\,a^5bx^4+a^6x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*B*a*b^2 - 5*A*b^3)*x^6 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^4 + (2*B
*a^3 - 5*A*a^2*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^4 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2
*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), 1/6*(3*((2
*B*a*b^2 - 5*A*b^3)*x^6 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^4 + (2*B*a^3 - 5*A*a^
2*b)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(2*B*a^2*b - 5*A*a
*b^2)*x^4 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b^
2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(99) = 198$.
time = 25.55, size = 1608, normalized size = 14.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x*
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
```

```
 + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(
37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*
b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*
a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x*
*2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12
*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sq
rt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/
2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(
12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(3
3/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39
/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3
*x**8)) + B*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*
a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3
*x**6) - 6*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x
**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt
(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4
+ 6*a**(13/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a*
*(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*
b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*
a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*
x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17
/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2
*x**4*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a
)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)
*b**3*x**6) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 1
8*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

Giac [A]
time = 1.38, size = 101, normalized size = 0.89

$$\frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^3}+\frac{3\,(bx^2+a)Ba+Ba^2-6\,(bx^2+a)Ab-Aab}{3\,(bx^2+a)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx^2+a}\,A}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - 5*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 1/3*(
3*(b*x^2 + a)*B*a + B*a^2 - 6*(b*x^2 + a)*A*b - A*a*b)/((b*x^2 + a)^(3/2)*a
^3) - 1/2*sqrt(b*x^2 + a)*A/(a^3*x^2)
```

Mupad [B]
time = 0.72, size = 126, normalized size = 1.12

$$\frac{\frac{B}{3\,a}+\frac{B\,\left(b\,x^2+a\right)}{a^2}}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{10\,A\,b}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{A}{2\,a\,x^2\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{5\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}-\frac{5\,A\,b^2\,x^2}{2\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B/(3*a) + (B*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - (B*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/a^(5/2) - (10*A*b)/(3*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a
+ b*x^2)^(3/2)) + (5*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - (5
*A*b^2*x^2)/(2*a^3*(a + b*x^2)^(3/2))
```

3.595 $\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{4b(2Ab-aB)x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{8b(2Ab-aB)x}{3a^4\sqrt{a+bx^2}}$$

```
[Out] -1/3*A/a/x^3/(b*x^2+a)^(3/2)+(2*A*b-B*a)/a^2/x/(b*x^2+a)^(3/2)+4/3*b*(2*A*b
-B*a)*x/a^3/(b*x^2+a)^(3/2)+8/3*b*(2*A*b-B*a)*x/a^4/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 277, 198, 197}

$$\frac{8bx(2Ab-aB)}{3a^4\sqrt{a+bx^2}}+\frac{4bx(2Ab-aB)}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/3*A/(a*x^3*(a + b*x^2)^(3/2)) + (2*A*b - a*B)/(a^2*x*(a + b*x^2)^(3/2))
+ (4*b*(2*A*b - a*B)*x)/(3*a^3*(a + b*x^2)^(3/2)) + (8*b*(2*A*b - a*B)*x)/(
3*a^4*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}-\frac{(6Ab-3aB)\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{3a}\\ &= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{(4b(2Ab-aB))\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a^2}\\ &= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{4b(2Ab-aB)x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{(8b(2Ab-aB))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^3}\\ &= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{4b(2Ab-aB)x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{8b(2Ab-aB)x}{3a^4\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 79, normalized size = 0.73

$$\frac{16Ab^3x^6+6a^2bx^2(A-2Bx^2)-8ab^2x^4(-3A+Bx^2)-a^3(A+3Bx^2)}{3a^4x^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (16*A*b^3*x^6 + 6*a^2*b*x^2*(A - 2*B*x^2) - 8*a*b^2*x^4*(-3*A + B*x^2) - a^
3*(A + 3*B*x^2))/(3*a^4*x^3*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 140, normalized size = 1.30

method	result
gosper	$-\frac{-16x^6Ab^3+8x^6Bab^2-24Aab^2x^4+12x^4Ba^2b-6x^2Aa^2b+3Ba^3x^2+Aa^3}{3x^3\left(bx^2+a\right)^{\frac{3}{2}}a^4}$
trager	$-\frac{-16x^6Ab^3+8x^6Bab^2-24Aab^2x^4+12x^4Ba^2b-6x^2Aa^2b+3Ba^3x^2+Aa^3}{3x^3\left(bx^2+a\right)^{\frac{3}{2}}a^4}$
risch	$-\frac{\sqrt{bx^2+a}\,(-8Abx^2+3Bax^2+Aa)}{3a^4x^3}+\frac{\sqrt{bx^2+a}\,x(8Ab^2x^2-5Babx^2+9abA-6a^2B)b}{3a^4(b^2x^4+2abx^2+a^2)}$

default	$A\left(-\frac{1}{3a\,x^{3}(b\,x^{2}+a)^{\frac{3}{2}}}-\frac{2b\left(-\frac{1}{ax\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{a}\right)}{a}\right)+B\left(-\frac{1}{ax(b\,x^{2}+a)^{\frac{3}{2}}}-\frac{4b\left(\frac{}{3a}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/3/a/x^3/(b*x^2+a)^(3/2)-2*b/a*(-1/a/x/(b*x^2+a)^(3/2)-4*b/a*(1/3*x/a/
(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))+B*(-1/a/x/(b*x^2+a)^(3/2)-4*b/
a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.30, size = 128, normalized size = 1.19

$$-\frac{8\,Bbx}{3\,\sqrt{bx^2+a}\,a^3}-\frac{4\,Bbx}{3\,(bx^2+a)^{\frac{3}{2}}a^2}+\frac{16\,Ab^2x}{3\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Ab^2x}{3\,(bx^2+a)^{\frac{3}{2}}a^3}-\frac{B}{(bx^2+a)^{\frac{3}{2}}ax}+\frac{2\,Ab}{(bx^2+a)^{\frac{3}{2}}a^2x}-\frac{A}{3\,(bx^2+a)^{\frac{3}{2}}ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*B*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*B*b*x/((b*x^2 + a)^(3/2)*a^2) + 16/3
*A*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*A*b^2*x/((b*x^2 + a)^(3/2)*a^3) - B/((
b*x^2 + a)^(3/2)*a*x) + 2*A*b/((b*x^2 + a)^(3/2)*a^2*x) - 1/3*A/((b*x^2 + a
)^(3/2)*a*x^3)
```

Fricas [A]
time = 1.24, size = 101, normalized size = 0.94

$$-\frac{\left(8\,(Bab^2-2\,Ab^3)x^6+12\,(Ba^2b-2\,Aab^2)x^4+Aa^3+3\,(Ba^3-2\,Aa^2b)x^2\right)\sqrt{bx^2+a}}{3\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*(B*a*b^2 - 2*A*b^3)*x^6 + 12*(B*a^2*b - 2*A*a*b^2)*x^4 + A*a^3 + 3*
(B*a^3 - 2*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x
^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(99) = 198.
time = 10.72, size = 524, normalized size = 4.85

$$A\left(-\frac{a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{5a^3b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{30a^2b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{40ab^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{16b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\right)+B\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x*
*4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(21/2)*x**2*sqrt(a/
(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3
*a**4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7*b**
9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 40*a*
b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 +
 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(b*x**2)
 + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b*
*12*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4
*b**5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(
a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(92) = 184$.
time = 1.24, size = 224, normalized size = 2.07

$$-\frac{x\left(\frac{(5\,Ba^4b^3-8\,Aa^3b^4)x^2}{a^7b}+\frac{3\,(2\,Ba^5b^2-3\,Aa^4b^3)}{a^7b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+18\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Aab^{\frac{3}{2}}+3\,Ba^3\sqrt{b}-8\,Aa^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*((5*B*a^4*b^3 - 8*A*a^3*b^4)*x^2/(a^7*b) + 3*(2*B*a^5*b^2 - 3*A*a^4*
b^3)/(a^7*b))/(b*x^2 + a)^(3/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*
a*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*B*a^2*sqrt(b) + 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(
3/2) + 3*B*a^3*sqrt(b) - 8*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^3*a^3)
```

Mupad [B]
time = 0.37, size = 123, normalized size = 1.14

$$-\frac{16\,A\,{\left(b\,x^2+a\right)}^3+A\,a^3+B\,a^3\,x^2-24\,A\,a\,{\left(b\,x^2+a\right)}^2+6\,A\,a^2\,\left(b\,x^2+a\right)-8\,B\,a\,x^2\,{\left(b\,x^2+a\right)}^2+4\,B\,a^2\,x^2\,\left(b\,x^2+a\right)}{{\left(b\,x^2+a\right)}^{3/2}\,\left(\frac{3\,a^5\,x}{b}-\frac{3\,a^4\,x\,\left(b\,x^2+a\right)}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x)
```

```
[Out] -(16*A*(a + b*x^2)^3 + A*a^3 + B*a^3*x^2 - 24*A*a*(a + b*x^2)^2 + 6*A*a^2*(
a + b*x^2) - 8*B*a*x^2*(a + b*x^2)^2 + 4*B*a^2*x^2*(a + b*x^2))/((a + b*x^2
)^(3/2)*((3*a^5*x)/b - (3*a^4*x*(a + b*x^2))/b))
```

3.596 $\int \frac{A+Bx^2}{x^5 \left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=146

$$\frac{5b(7Ab-4aB)}{24a^3 (a+bx^2)^{3/2}}-\frac{A}{4ax^4 (a+bx^2)^{3/2}}+\frac{7Ab-4aB}{8a^2x^2 (a+bx^2)^{3/2}}+\frac{5b(7Ab-4aB)}{8a^4\sqrt{a+bx^2}}-\frac{5b(7Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}$$

```
[Out] 5/24*b*(7*A*b-4*B*a)/a^3/(b*x^2+a)^(3/2)-1/4*A/a/x^4/(b*x^2+a)^(3/2)+1/8*(7
*A*b-4*B*a)/a^2/x^2/(b*x^2+a)^(3/2)-5/8*b*(7*A*b-4*B*a)*arctanh((b*x^2+a)^(
1/2)/a^(1/2))/a^(9/2)+5/8*b*(7*A*b-4*B*a)/a^4/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 44, 53, 65, 214}

$$-\frac{5b(7Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{5b(7Ab-4aB)}{8a^4\sqrt{a+bx^2}}+\frac{5b(7Ab-4aB)}{24a^3 (a+bx^2)^{3/2}}+\frac{7Ab-4aB}{8a^2x^2 (a+bx^2)^{3/2}}-\frac{A}{4ax^4 (a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

```
[Out] (5*b*(7*A*b - 4*a*B))/(24*a^3*(a + b*x^2)^(3/2)) - A/(4*a*x^4*(a + b*x^2)^(
3/2)) + (7*A*b - 4*a*B)/(8*a^2*x^2*(a + b*x^2)^(3/2)) + (5*b*(7*A*b - 4*a*B
))/(8*a^4*Sqrt[a + b*x^2]) - (5*b*(7*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(8*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5(a+bx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx, x, x^2\right)$$
$$= -\frac{A}{4ax^4(a+bx^2)^{3/2}} + \frac{\left(-\frac{7Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{A}{4ax^4(a+bx^2)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2(a+bx^2)^{3/2}} - \frac{(5(7Ab-4aB))\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx\right.}{24a^2}$$
$$= -\frac{A}{4ax^4(a+bx^2)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2(a+bx^2)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} - \frac{(5(7Ab-4aB))\text{S}}{}$$
$$= -\frac{A}{4ax^4(a+bx^2)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2(a+bx^2)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} + \frac{5(7Ab-4aB)\sqrt{a}}{8a^4x^2}$$
$$= -\frac{A}{4ax^4(a+bx^2)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2(a+bx^2)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} + \frac{5(7Ab-4aB)\sqrt{a}}{8a^4x^2}$$
$$= -\frac{A}{4ax^4(a+bx^2)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2(a+bx^2)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} + \frac{5(7Ab-4aB)\sqrt{a}}{8a^4x^2}$$

Mathematica [A]
time = 0.20, size = 119, normalized size = 0.82

$$\frac{105Ab^3x^6+a^2bx^2(21A-80Bx^2)+20ab^2x^4(7A-3Bx^2)-6a^3(A+2Bx^2)}{24a^4x^4(a+bx^2)^{3/2}} + \frac{5b(-7Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

```
[Out] (105*A*b^3*x^6 + a^2*b*x^2*(21*A - 80*B*x^2) + 20*a*b^2*x^4*(7*A - 3*B*x^2)
 - 6*a^3*(A + 2*B*x^2))/(24*a^4*x^4*(a + b*x^2)^(3/2)) + (5*b*(-7*A*b + 4*a
*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(9/2))
```

Maple [A]
time = 0.12, size = 200, normalized size = 1.37

method	result

default	$A\left(-\frac{1}{4a\,x^4(b\,x^2+a)^{\frac{3}{2}}}-\frac{7b\left(-\frac{1}{2a\,x^2\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)}{2a}\right)}{4a}\right)+ B$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-11Ab\,x^2+4Ba\,x^2+2Aa)}{8a^4x^4}-\frac{b\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}\,A}{12a^4\left(x+\frac{\sqrt{-ab}}{b}\right)^2}+\frac{\sqrt{\left(x+\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4/a/x^4/(b*x^2+a)^(3/2)-7/4*b/a*(-1/2/a/x^2/(b*x^2+a)^(3/2)-5/2*b/a*(
1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*
(b*x^2+a)^(1/2))/x)))))+B*(-1/2/a/x^2/(b*x^2+a)^(3/2)-5/2*b/a*(1/3/a/(b*x^2
+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1
/2))/x))))
```

Maxima [A]

time = 0.28, size = 164, normalized size = 1.12

$$\frac{5\,Bb\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{7}{2}}}-\frac{35\,Ab^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{9}{2}}}-\frac{5\,Bb}{2\,\sqrt{bx^2+a}\,a^3}-\frac{5\,Bb}{6\,(bx^2+a)^{\frac{3}{2}}a^2}+\frac{35\,Ab^2}{8\,\sqrt{bx^2+a}\,a^4}+\frac{35\,Ab^2}{24\,(bx^2+a)^{\frac{3}{2}}a^3}-\frac{B}{2\,(bx^2+a)^{\frac{3}{2}}ax^2}+\frac{7\,Ab}{8\,(bx^2+a)^{\frac{3}{2}}a^2x^2}-\frac{A}{4\,(bx^2+a)^{\frac{3}{2}}ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 35/8*A*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(9/2) - 5/2*B*b/(sqrt(b*x^2 + a)*a^3) - 5/6*B*b/((b*x^2 + a
)^(3/2)*a^2) + 35/8*A*b^2/(sqrt(b*x^2 + a)*a^4) + 35/24*A*b^2/((b*x^2 + a)^
(3/2)*a^3) - 1/2*B/((b*x^2 + a)^(3/2)*a*x^2) + 7/8*A*b/((b*x^2 + a)^(3/2)*a
^2*x^2) - 1/4*A/((b*x^2 + a)^(3/2)*a*x^4)
```

Fricas [A]
time = 1.82, size = 407, normalized size = 2.79

$$\left[-\frac{15\left((4Bab^3-7Ab^4)x^8+2(4Ba^2b^2-7Aab^3)x^6+(4Ba^3b-7Aa^2b^2)x^4\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(15(4Ba^2b^2-7Aab^3)x^6+6Aa^4+20(4Ba^3b-7Aa^2b^2)x^4+3(4Ba^4-7Aa^3b)x^2\right)\sqrt{bx^2+a}}{48(a^5b^2x^8+2a^6bx^6+a^7x^4)}, -\frac{15\left((4Bab^3-7Ab^4)x^8+2(4Ba^2b^2-7Aab^3)x^6+(4Ba^3b-7Aa^2b^2)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(15(4Ba^2b^2-7Aab^3)x^6+6Aa^4+20(4Ba^3b-7Aa^2b^2)x^4+3(4Ba^4-7Aa^3b)x^2\right)\sqrt{bx^2+a}}{24(a^5b^2x^8+2a^6bx^6+a^7x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*((4*B*a*b^3 - 7*A*b^4)*x^8 + 2*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + (
4*B*a^3*b - 7*A*a^2*b^2)*x^4)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + 6*A*a^4 + 20*(4*B*a^
3*b - 7*A*a^2*b^2)*x^4 + 3*(4*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5
*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4), -1/24*(15*((4*B*a*b^3 - 7*A*b^4)*x^8 + 2
*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + (4*B*a^3*b - 7*A*a^2*b^2)*x^4)*sqrt(-a)*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + 6*A*a^
4 + 20*(4*B*a^3*b - 7*A*a^2*b^2)*x^4 + 3*(4*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*
x^2 + a))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. $2(141) = 282$.
time = 49.69, size = 1323, normalized size = 9.06

$$A\left(-\frac{6a^{\frac{89}{2}}b^{75}}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^5\sqrt{\frac{a}{bx^2}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{21a^{\frac{87}{2}}b^{76}x^2}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^5\sqrt{\frac{a}{bx^2}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{140a^{\frac{85}{2}}b^{77}x^4}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^5\sqrt{\frac{a}{bx^2}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{105a^{\frac{83}{2}}b^{78}x^6}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^5\sqrt{\frac{a}{bx^2}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{105a^{42}b^{\frac{155}{2}}x^5\sqrt{\frac{a}{bx^2}+1}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^5\sqrt{\frac{a}{bx^2}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{105a^{41}b^{\frac{157}{2}}x^7\sqrt{\frac{a}{bx^2}+1}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^5\sqrt{\frac{a}{bx^2}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}x^7\sqrt{\frac{a}{bx^2}+1}}\right)+B\left(\ldots\right)$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**(89/2)*b**75/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) +
24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1)) + 21*a**(87/2)*b**76*x**
2/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153
/2)*x**7*sqrt(a/(b*x**2) + 1)) + 140*a**(85/2)*b**77*x**4/(24*a**(93/2)*b**
(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*
x**2) + 1)) + 105*a**(83/2)*b**78*x**6/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a
/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1)) - 105*a
**42*b**(155/2)*x**5*sqrt(a/(b*x**2) + 1)*asinh(sqrt(a)/(sqrt(b)*x))/(24*a*
*(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7
*sqrt(a/(b*x**2) + 1)) - 105*a**41*b**(157/2)*x**7*sqrt(a/(b*x**2) + 1)*asi
nh(sqrt(a)/(sqrt(b)*x))/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1)
```

```
+ 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1))) + B*(-6*a**17*sqrt(1
+ b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x*
*6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 + b*x**2/a)/(12*a**(3
9/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**
3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b
*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**16*b*x**2*
log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*b**2*x**4*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x**2/a)/(12*a**(39/2)
*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x*
*8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 3
6*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 30*
a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4
 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**14*b**3*x**6*lo
g(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x*
*6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sqrt(1 + b*x**2/a) +
1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a
**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*a**(39/2)*x**2 +
 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 3
0*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(3
7/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8))
```

Giac [A]

time = 1.46, size = 165, normalized size = 1.13

$$-\frac{5\,(4\,Bab-7\,Ab^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^4}-\frac{6\,(bx^2+a)Bab+Ba^2b-9\,(bx^2+a)Ab^2-Aab^2}{3\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{4\,(bx^2+a)^{\frac{3}{2}}Bab-4\,\sqrt{bx^2+a}\,Ba^2b-11\,(bx^2+a)^{\frac{3}{2}}Ab^2+13\,\sqrt{bx^2+a}\,Aab^2}{8\,a^4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/8*(4*B*a*b - 7*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) -
1/3*(6*(b*x^2 + a)*B*a*b + B*a^2*b - 9*(b*x^2 + a)*A*b^2 - A*a*b^2)/((b*x^2
 + a)^(3/2)*a^4) - 1/8*(4*(b*x^2 + a)^(3/2)*B*a*b - 4*sqrt(b*x^2 + a)*B*a^2
*b - 11*(b*x^2 + a)^(3/2)*A*b^2 + 13*sqrt(b*x^2 + a)*A*a*b^2)/(a^4*b^2*x^4)
```

Mupad [B]

time = 0.92, size = 176, normalized size = 1.21

$$\frac{35\,A\,b^2}{6\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{10\,B\,b}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{35\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{9/2}}-\frac{A}{4\,a\,x^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{B}{2\,a\,x^2\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{5\,B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}+\frac{7\,A\,b}{8\,a^2\,x^2\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{35\,A\,b^3\,x^2}{8\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{5\,B\,b^2\,x^2}{2\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x)
```

```
[Out] (35*A*b^2)/(6*a^3*(a + b*x^2)^(3/2)) - (10*B*b)/(3*a^2*(a + b*x^2)^(3/2)) -
 (35*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(9/2)) - A/(4*a*x^4*(a +
```

```
b*x^2)^(3/2)) - B/(2*a*x^2*(a + b*x^2)^(3/2)) + (5*B*b*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/(2*a^(7/2)) + (7*A*b)/(8*a^2*x^2*(a + b*x^2)^(3/2)) + (35*A*b
^3*x^2)/(8*a^4*(a + b*x^2)^(3/2)) - (5*B*b^2*x^2)/(2*a^3*(a + b*x^2)^(3/2))
```

3.597 $\int \frac{A+Bx^2}{x^6(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}}+\frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}}-\frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}}-\frac{8b^2(8Ab-5aB)x}{15a^4\left(a+bx^2\right)^{3/2}}-\frac{16b^2(8Ab-5aB)x}{15a^5\sqrt{a+bx^2}}$$

```
[Out] -1/5*A/a/x^5/(b*x^2+a)^(3/2)+1/15*(8*A*b-5*B*a)/a^2/x^3/(b*x^2+a)^(3/2)-2/5
*b*(8*A*b-5*B*a)/a^3/x/(b*x^2+a)^(3/2)-8/15*b^2*(8*A*b-5*B*a)*x/a^4/(b*x^2+
a)^(3/2)-16/15*b^2*(8*A*b-5*B*a)*x/a^5/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 277, 198, 197}

$$-\frac{16b^2x(8Ab-5aB)}{15a^5\sqrt{a+bx^2}}-\frac{8b^2x(8Ab-5aB)}{15a^4\left(a+bx^2\right)^{3/2}}-\frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}}+\frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/5*A/(a*x^5*(a + b*x^2)^(3/2)) + (8*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)^
(3/2)) - (2*b*(8*A*b - 5*a*B))/(5*a^3*x*(a + b*x^2)^(3/2)) - (8*b^2*(8*A*b
- 5*a*B)*x)/(15*a^4*(a + b*x^2)^(3/2)) - (16*b^2*(8*A*b - 5*a*B)*x)/(15*a^5
*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx = -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} - \frac{(8Ab-5aB)\int \frac{1}{x^4(a+bx^2)^{5/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} + \frac{(2b(8Ab-5aB))\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx}{5a^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{(8b^2(8Ab-5aB))}{5a^3}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{8b^2(8Ab-5aB)x}{15a^4\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{8b^2(8Ab-5aB)x}{15a^4\left(a+bx^2\right)^{3/2}}$$

Mathematica [A]
time = 0.17, size = 105, normalized size = 0.72

$$\frac{-128Ab^4x^8+16ab^3x^6(-12A+5Bx^2)+24a^2b^2x^4(-2A+5Bx^2)-a^4(3A+5Bx^2)+a^3(8Abx^2+30bBx^4)}{15a^5x^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-128*A*b^4*x^8 + 16*a*b^3*x^6*(-12*A + 5*B*x^2) + 24*a^2*b^2*x^4*(-2*A + 5
*B*x^2) - a^4*(3*A + 5*B*x^2) + a^3*(8*A*b*x^2 + 30*b*B*x^4))/(15*a^5*x^5*(
a + b*x^2)^(3/2))
```

Maple [A]
time = 0.11, size = 188, normalized size = 1.29

method	result

gosper	$-\frac{128A\,b^4x^8-80Ba\,b^3x^8+192Aa\,b^3x^6-120B\,a^2b^2x^6+48A\,a^2b^2x^4-30B\,a^3b\,x^4-8A\,a^3b\,x^2+5B\,a^4x^2+3A\,a^4}{15x^5(b\,x^2+a)^{\frac{3}{2}}a^5}$
trager	$-\frac{128A\,b^4x^8-80Ba\,b^3x^8+192Aa\,b^3x^6-120B\,a^2b^2x^6+48A\,a^2b^2x^4-30B\,a^3b\,x^4-8A\,a^3b\,x^2+5B\,a^4x^2+3A\,a^4}{15x^5(b\,x^2+a)^{\frac{3}{2}}a^5}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(73A\,b^2x^4-40Bab\,x^4-14aAb\,x^2+5B\,a^2x^2+3a^2A)}{15a^5x^5}-\frac{\sqrt{b\,x^2+a}\,x(11A\,b^2x^2-8Bab\,x^2+12abA-9a^2B)b^2}{3a^5(b^2x^4+2ab\,x^2+a^2)}$
default	$A\left(-\frac{1}{5a\,x^5(b\,x^2+a)^{\frac{3}{2}}}-\frac{8b\left(-\frac{1}{3a\,x^3\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{2b\left(-\frac{1}{ax\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}\right)}{a}\right)}{5a}\right)+B\left(-\frac{}{3a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/5/a/x^5/(b*x^2+a)^(3/2)-8/5*b/a*(-1/3/a/x^3/(b*x^2+a)^(3/2)-2*b/a*(-1
/a/x/(b*x^2+a)^(3/2)-4*b/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/
2)))))+B*(-1/3/a/x^3/(b*x^2+a)^(3/2)-2*b/a*(-1/a/x/(b*x^2+a)^(3/2)-4*b/a*(1
/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))
```

Maxima [A]

time = 0.35, size = 172, normalized size = 1.18

$$\frac{16\,Bb^2x}{3\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Bb^2x}{3\,(bx^2+a)^{\frac{3}{2}}a^3}-\frac{128\,Ab^3x}{15\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Ab^3x}{15\,(bx^2+a)^{\frac{3}{2}}a^4}+\frac{2\,Bb}{(bx^2+a)^{\frac{3}{2}}a^2x}-\frac{16\,Ab^2}{5\,(bx^2+a)^{\frac{3}{2}}a^3x}-\frac{B}{3\,(bx^2+a)^{\frac{3}{2}}ax^3}+\frac{8\,Ab}{15\,(bx^2+a)^{\frac{3}{2}}a^2x^3}-\frac{A}{5\,(bx^2+a)^{\frac{3}{2}}ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/3*B*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*B*b^2*x/((b*x^2 + a)^(3/2)*a^3) -
128/15*A*b^3*x/(sqrt(b*x^2 + a)*a^5) - 64/15*A*b^3*x/((b*x^2 + a)^(3/2)*a^4
) + 2*B*b/((b*x^2 + a)^(3/2)*a^2*x) - 16/5*A*b^2/((b*x^2 + a)^(3/2)*a^3*x)
- 1/3*B/((b*x^2 + a)^(3/2)*a*x^3) + 8/15*A*b/((b*x^2 + a)^(3/2)*a^2*x^3) -
1/5*A/((b*x^2 + a)^(3/2)*a*x^5)
```

Fricas [A]
time = 1.95, size = 129, normalized size = 0.88

$$\frac{\left(16\,\left(5\,Bab^{3}-8\,Ab^{4}\right)x^{8}+24\,\left(5\,Ba^{2}b^{2}-8\,Aab^{3}\right)x^{6}-3\,Aa^{4}+6\,\left(5\,Ba^{3}b-8\,Aa^{2}b^{2}\right)x^{4}-\left(5\,Ba^{4}-8\,Aa^{3}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{15\,\left(a^{5}b^{2}x^{9}+2\,a^{6}bx^{7}+a^{7}x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(16*(5*B*a*b^3 - 8*A*b^4)*x^8 + 24*(5*B*a^2*b^2 - 8*A*a*b^3)*x^6 - 3*A
*a^4 + 6*(5*B*a^3*b - 8*A*a^2*b^2)*x^4 - (5*B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*
x^2 + a)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 944 vs. 2(141) = 282.
time = 16.30, size = 944, normalized size = 6.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**6*b**(33/2)*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**
17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**12) +
 2*a**5*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**8*b
**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**12)
 - 35*a**4*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**
8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**
12) - 280*a**3*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60
*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20
*x**12) - 560*a**2*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4
+ 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b
**20*x**12) - 448*a*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**
4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5
*b**20*x**12) - 128*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**
4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5
*b**20*x**12)) + B*(-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2
+ 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a*
*5*b**11*x**6 + 3*a**4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2)
 + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b*
*12*x**8) + 40*a*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*
a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**
8*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11
*x**6 + 3*a**4*b**12*x**8))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(126) = 252.
time = 1.63, size = 336, normalized size = 2.30

$$\frac{x\left(\frac{(8\,Ba^5b^4-11\,Aa^4b^5)x^2}{a^9b}+\frac{3\,(3\,Ba^6b^3-4\,Aa^5b^4)}{a^9b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{2\left(30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}-45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^2b^{\frac{3}{2}}+240\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Aab^{\frac{5}{2}}+250\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^3b^{\frac{3}{2}}-490\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aa^2b^{\frac{5}{2}}-170\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^4b^{\frac{3}{2}}+320\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^3b^{\frac{5}{2}}+40\,Ba^5b^{\frac{3}{2}}-73\,Aa^4b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*((8*B*a^5*b^4 - 11*A*a^4*b^5)*x^2/(a^9*b) + 3*(3*B*a^6*b^3 - 4*A*a^5*
b^4)/(a^9*b))/(b*x^2 + a)^(3/2) - 2/15*(30*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
B*a*b^(3/2) - 45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 150*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A
*a*b^(5/2) + 250*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) - 490*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 170*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*B*a^4*b^(3/2) + 320*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2) + 40*B
*a^5*b^(3/2) - 73*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a
^4)
```

Mupad [B]
time = 0.49, size = 231, normalized size = 1.58

$$\frac{\frac{a\left(\frac{b^2\,(73\,A\,b-40\,B\,a)}{18\,a^4}+\frac{b^2\,(86\,A\,b-35\,B\,a)}{30\,a^4}+\frac{a\left(\frac{28\,A\,b^4-10\,B\,a\,b^3}{45\,a^5}-\frac{b^3\,(86\,A\,b-35\,B\,a)}{18\,a^5}\right)}{b}\right)}{b}-\frac{b\,(73\,A\,b-40\,B\,a)}{30\,a^3}}{x\,(b\,x^2+a)^{3/2}}+\frac{x^2\left(\frac{28\,A\,b^3-10\,B\,a\,b^2}{15\,a^5}-\frac{2\,b^2\,(26\,A\,b-15\,B\,a)}{5\,a^5}\right)-\frac{b\,(26\,A\,b-15\,B\,a)}{5\,a^4}}{x\,\sqrt{b\,x^2+a}}-\frac{\sqrt{b\,x^2+a}\,\left(5\,B\,a^3-14\,A\,a^2\,b\right)}{15\,a^6\,x^3}-\frac{A\,\sqrt{b\,x^2+a}}{5\,a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x)
```

```
[Out] ((a*((b^2*(73*A*b - 40*B*a))/(18*a^4) + (b^2*(86*A*b - 35*B*a))/(30*a^4) +
(a*((28*A*b^4 - 10*B*a*b^3)/(45*a^5) - (b^3*(86*A*b - 35*B*a))/(18*a^5)))/b
))/b - (b*(73*A*b - 40*B*a))/(30*a^3))/(x*(a + b*x^2)^(3/2)) + (x^2*((28*A*
b^3 - 10*B*a*b^2)/(15*a^5) - (2*b^2*(26*A*b - 15*B*a))/(5*a^5)) - (b*(26*A*
b - 15*B*a))/(5*a^4))/(x*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(5*B*a^3 -
 14*A*a^2*b))/(15*a^6*x^3) - (A*(a + b*x^2)^(1/2))/(5*a^3*x^5)
```

3.598 $\int x^5(a+bx^2)^2\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=157

$$\frac{c^2(bc-ad)^2(c+dx^2)^{3/2}}{3d^5}-\frac{2c(bc-ad)(2bc-ad)(c+dx^2)^{5/2}}{5d^5}+\frac{(6b^2c^2-6abcd+a^2d^2)(c+dx^2)^{7/2}}{7d^5}-\frac{2b(2bc$$

```
[Out] 1/3*c^2*(-a*d+b*c)^2*(d*x^2+c)^(3/2)/d^5-2/5*c*(-a*d+b*c)*(-a*d+2*b*c)*(d*x
^2+c)^(5/2)/d^5+1/7*(a^2*d^2-6*a*b*c*d+6*b^2*c^2)*(d*x^2+c)^(7/2)/d^5-2/9*b
*(-a*d+2*b*c)*(d*x^2+c)^(9/2)/d^5+1/11*b^2*(d*x^2+c)^(11/2)/d^5
```

Rubi [A]
time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {457, 90}

$$\frac{(c+dx^2)^{7/2}(a^2d^2-6abcd+6b^2c^2)}{7d^5}+\frac{c^2(c+dx^2)^{3/2}(bc-ad)^2}{3d^5}-\frac{2b(c+dx^2)^{9/2}(2bc-ad)}{9d^5}-\frac{2c(c+dx^2)^{5/2}(bc-ad)(2bc-ad)}{5d^5}+\frac{b^2(c+dx^2)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^5) - (2*c*(b*c - a*d)*(2*b*c - a
*d)*(c + d*x^2)^(5/2))/(5*d^5) + ((6*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(c + d*
x^2)^(7/2))/(7*d^5) - (2*b*(2*b*c - a*d)*(c + d*x^2)^(9/2))/(9*d^5) + (b^2*
(c + d*x^2)^(11/2))/(11*d^5)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^2\sqrt{c+dx}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{c^2(bc-ad)^2\sqrt{c+dx}}{d^4}+\frac{2c(bc-ad)(-2bc+ad)(c+dx)^{3/2}}{d^4}+\right.\right.$$
$$=\frac{c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^5}-\frac{2c(bc-ad)(2bc-ad)\left(c+dx^2\right)^{5/2}}{5d^5}+\frac{(6b^2c^2-6$$

Mathematica [A]
time = 0.08, size = 132, normalized size = 0.84

$$\frac{(c+dx^2)^{3/2}\left(33a^2d^2(8c^2-12cdx^2+15d^2x^4)+22abd(-16c^3+24c^2dx^2-30cd^2x^4+35d^3x^6)+b^2(128c^4-192c^3dx^2+240c^2d^2x^4-280cd^3x^6+315d^4x^8)\right)}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(33*a^2*d^2*(8*c^2 - 12*c*d*x^2 + 15*d^2*x^4) + 22*a*b*d
*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d^2*x^4 + 35*d^3*x^6) + b^2*(128*c^4 - 192*
c^3*d*x^2 + 240*c^2*d^2*x^4 - 280*c*d^3*x^6 + 315*d^4*x^8)))/(3465*d^5)
```

Maple [A]
time = 0.08, size = 257, normalized size = 1.64

method	result
gosper	$\frac{(d\,x^2+c)^{\frac{3}{2}}(315b^2x^8d^4+770ab\,d^4x^6-280b^2c\,d^3x^6+495a^2x^4d^4-660x^4abc\,d^3+240b^2c^2x^4d^2-396a^2c\,d^3x^2+528ab\,c^2d^2x^2-192b^2c^3d\,x^2-}{3465d^5}$
trager	$\frac{(315b^2d^5x^{10}+770ab\,d^5x^8+35b^2c\,d^4x^8+495a^2d^5x^6+110abc\,d^4x^6-40b^2c^2d^3x^6+99a^2c\,d^4x^4-132ab\,c^2d^3x^4+48b^2c^3d^2x^4-132a^2c^2d^3x^2}{3465d^5}$
risch	$\frac{(315b^2d^5x^{10}+770ab\,d^5x^8+35b^2c\,d^4x^8+495a^2d^5x^6+110abc\,d^4x^6-40b^2c^2d^3x^6+99a^2c\,d^4x^4-132ab\,c^2d^3x^4+48b^2c^3d^2x^4-132a^2c^2d^3x^2}{3465d^5}$

default	$b^2\left(\frac{x^8(dx^2+c)^{\frac{3}{2}}}{11d}-\frac{8c\left(\frac{x^6(dx^2+c)^{\frac{3}{2}}}{9d}-\frac{2c\left(\frac{x^4(dx^2+c)^{\frac{3}{2}}}{7d}-\frac{4c\left(\frac{x^2(dx^2+c)^{\frac{3}{2}}}{5d}-\frac{2c(dx^2+c)^{\frac{3}{2}}}{15d^2}\right)}{7d}\right)}{3d}\right)}{11d}\right)+2ab\left(\frac{x^6(dx^2+c)^{\frac{3}{2}}}{9d}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/11*x^8*(d*x^2+c)^(3/2)/d-8/11*c/d*(1/9*x^6*(d*x^2+c)^(3/2)/d-2/3*c/d
*(1/7*x^4*(d*x^2+c)^(3/2)/d-4/7*c/d*(1/5*x^2*(d*x^2+c)^(3/2)/d-2/15*c/d^2*(
d*x^2+c)^(3/2)))))+2*a*b*(1/9*x^6*(d*x^2+c)^(3/2)/d-2/3*c/d*(1/7*x^4*(d*x^2
+c)^(3/2)/d-4/7*c/d*(1/5*x^2*(d*x^2+c)^(3/2)/d-2/15*c/d^2*(d*x^2+c)^(3/2)))
)+a^2*(1/7*x^4*(d*x^2+c)^(3/2)/d-4/7*c/d*(1/5*x^2*(d*x^2+c)^(3/2)/d-2/15*c/
d^2*(d*x^2+c)^(3/2)))
```

Maxima [A]

time = 0.35, size = 249, normalized size = 1.59

$$\frac{(dx^2+c)^{\frac{3}{2}}b^2x^8}{11\,d}-\frac{8\,(dx^2+c)^{\frac{3}{2}}b^2cx^6}{99\,d^2}+\frac{2\,(dx^2+c)^{\frac{3}{2}}abx^6}{9\,d}+\frac{16\,(dx^2+c)^{\frac{3}{2}}b^2c^2x^4}{231\,d^3}-\frac{4\,(dx^2+c)^{\frac{3}{2}}abcx^4}{21\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}a^2x^4}{7\,d}-\frac{64\,(dx^2+c)^{\frac{3}{2}}b^2c^3x^2}{1155\,d^4}+\frac{16\,(dx^2+c)^{\frac{3}{2}}abc^2x^2}{105\,d^3}-\frac{4\,(dx^2+c)^{\frac{3}{2}}a^2cx^2}{35\,d^2}+\frac{128\,(dx^2+c)^{\frac{3}{2}}b^2c^4}{3465\,d^5}-\frac{32\,(dx^2+c)^{\frac{3}{2}}abc^3}{315\,d^4}+\frac{8\,(dx^2+c)^{\frac{3}{2}}a^2c^2}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*(d*x^2 + c)^(3/2)*b^2*x^8/d - 8/99*(d*x^2 + c)^(3/2)*b^2*c*x^6/d^2 + 2
/9*(d*x^2 + c)^(3/2)*a*b*x^6/d + 16/231*(d*x^2 + c)^(3/2)*b^2*c^2*x^4/d^3 -
 4/21*(d*x^2 + c)^(3/2)*a*b*c*x^4/d^2 + 1/7*(d*x^2 + c)^(3/2)*a^2*x^4/d - 6
4/1155*(d*x^2 + c)^(3/2)*b^2*c^3*x^2/d^4 + 16/105*(d*x^2 + c)^(3/2)*a*b*c^2
*x^2/d^3 - 4/35*(d*x^2 + c)^(3/2)*a^2*c*x^2/d^2 + 128/3465*(d*x^2 + c)^(3/2
)*b^2*c^4/d^5 - 32/315*(d*x^2 + c)^(3/2)*a*b*c^3/d^4 + 8/105*(d*x^2 + c)^(3
/2)*a^2*c^2/d^3
```

Fricas [A]

time = 1.36, size = 179, normalized size = 1.14

$$\frac{(315\,b^2d^5x^{10}+35\,(b^2cd^4+22\,abd^5)x^8+128\,b^2c^5-352\,abc^4d+264\,a^2c^3d^2-5\,(8\,b^2c^2d^3-22\,abcd^4-99\,a^2d^5)x^6+3\,(16\,b^2c^3d^2-44\,abc^2d^3+33\,a^2cd^4)x^4-4\,(16\,b^2c^4d-44\,abc^3d^2+33\,a^2c^2d^3)x^2)\sqrt{dx^2+c}}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^2*d^5*x^10 + 35*(b^2*c*d^4 + 22*a*b*d^5)*x^8 + 128*b^2*c^5 -
352*a*b*c^4*d + 264*a^2*c^3*d^2 - 5*(8*b^2*c^2*d^3 - 22*a*b*c*d^4 - 99*a^2*
d^5)*x^6 + 3*(16*b^2*c^3*d^2 - 44*a*b*c^2*d^3 + 33*a^2*c*d^4)*x^4 - 4*(16*b
^2*c^4*d - 44*a*b*c^3*d^2 + 33*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(148) = 296$.
time = 0.37, size = 389, normalized size = 2.48

$$\begin{cases} \frac{8a^2c^3\sqrt{c+dx^2}}{105d^3} - \frac{4a^2c^2x^2\sqrt{c+dx^2}}{105d^2} + \frac{a^2cx^4\sqrt{c+dx^2}}{35d} + \frac{a^2x^6\sqrt{c+dx^2}}{7} - \frac{32abc^4\sqrt{c+dx^2}}{315d^4} + \frac{16abc^3x^2\sqrt{c+dx^2}}{315d^3} - \frac{4abc^2x^4\sqrt{c+dx^2}}{105d^2} + \frac{2abcx^6\sqrt{c+dx^2}}{63d} + \frac{2abx^8\sqrt{c+dx^2}}{9} + \frac{128b^2c^5\sqrt{c+dx^2}}{3465d^5} - \frac{64b^2c^4x^2\sqrt{c+dx^2}}{3465d^4} + \frac{16b^2c^3x^4\sqrt{c+dx^2}}{1155d^3} - \frac{8b^2c^2x^6\sqrt{c+dx^2}}{693d^2} + \frac{b^2cx^8\sqrt{c+dx^2}}{99d} + \frac{b^2x^{10}\sqrt{c+dx^2}}{11} & \text{for } d \neq 0 \\ \sqrt{c}\left(\frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((8*a**2*c**3*sqrt(c + d*x**2)/(105*d**3) - 4*a**2*c**2*x**2*sqrt(
c + d*x**2)/(105*d**2) + a**2*c*x**4*sqrt(c + d*x**2)/(35*d) + a**2*x**6*sq
rt(c + d*x**2)/7 - 32*a*b*c**4*sqrt(c + d*x**2)/(315*d**4) + 16*a*b*c**3*x*
*2*sqrt(c + d*x**2)/(315*d**3) - 4*a*b*c**2*x**4*sqrt(c + d*x**2)/(105*d**2
) + 2*a*b*c*x**6*sqrt(c + d*x**2)/(63*d) + 2*a*b*x**8*sqrt(c + d*x**2)/9 +
128*b**2*c**5*sqrt(c + d*x**2)/(3465*d**5) - 64*b**2*c**4*x**2*sqrt(c + d*x
**2)/(3465*d**4) + 16*b**2*c**3*x**4*sqrt(c + d*x**2)/(1155*d**3) - 8*b**2*
c**2*x**6*sqrt(c + d*x**2)/(693*d**2) + b**2*c*x**8*sqrt(c + d*x**2)/(99*d)
 + b**2*x**10*sqrt(c + d*x**2)/11, Ne(d, 0)), (sqrt(c)*(a**2*x**6/6 + a*b*x
**8/4 + b**2*x**10/10), True))
```

Giac [A]
time = 1.41, size = 204, normalized size = 1.30

$$\frac{315\,(dx^2+c)^{\frac{11}{2}}b^2 - 1540\,(dx^2+c)^{\frac{9}{2}}b^2c + 2970\,(dx^2+c)^{\frac{7}{2}}b^2c^2 - 2772\,(dx^2+c)^{\frac{5}{2}}b^2c^3 + 1155\,(dx^2+c)^{\frac{3}{2}}b^2c^4 + 770\,(dx^2+c)^{\frac{9}{2}}abd - 2970\,(dx^2+c)^{\frac{7}{2}}abcd + 4158\,(dx^2+c)^{\frac{5}{2}}abc^2d - 2310\,(dx^2+c)^{\frac{3}{2}}abc^3d + 495\,(dx^2+c)^{\frac{7}{2}}a^2d^2 - 1386\,(dx^2+c)^{\frac{5}{2}}a^2cd^2 + 1155\,(dx^2+c)^{\frac{3}{2}}a^2c^2d^2}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3465*(315*(d*x^2 + c)^(11/2)*b^2 - 1540*(d*x^2 + c)^(9/2)*b^2*c + 2970*(d
*x^2 + c)^(7/2)*b^2*c^2 - 2772*(d*x^2 + c)^(5/2)*b^2*c^3 + 1155*(d*x^2 + c)
^(3/2)*b^2*c^4 + 770*(d*x^2 + c)^(9/2)*a*b*d - 2970*(d*x^2 + c)^(7/2)*a*b*c
*d + 4158*(d*x^2 + c)^(5/2)*a*b*c^2*d - 2310*(d*x^2 + c)^(3/2)*a*b*c^3*d +
495*(d*x^2 + c)^(7/2)*a^2*d^2 - 1386*(d*x^2 + c)^(5/2)*a^2*c*d^2 + 1155*(d*
x^2 + c)^(3/2)*a^2*c^2*d^2)/d^5
```

Mupad [B]
time = 0.37, size = 171, normalized size = 1.09

$$\sqrt{d\,x^2+c}\,\left(\frac{264\,a^2\,c^3\,d^2-352\,a\,b\,c^4\,d+128\,b^2\,c^5}{3465\,d^5}+\frac{b^2\,x^{10}}{11}+\frac{x^6\,\left(495\,a^2\,d^5+110\,a\,b\,c\,d^4-40\,b^2\,c^2\,d^3\right)}{3465\,d^5}+\frac{b\,x^8\,\left(22\,a\,d+b\,c\right)}{99\,d}+\frac{c\,x^4\,\left(33\,a^2\,d^2-44\,a\,b\,c\,d+16\,b^2\,c^2\right)}{1155\,d^3}-\frac{4\,c^2\,x^2\,\left(33\,a^2\,d^2-44\,a\,b\,c\,d+16\,b^2\,c^2\right)}{3465\,d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((128*b^2*c^5 + 264*a^2*c^3*d^2 - 352*a*b*c^4*d)/(3465*d^
5) + (b^2*x^10)/11 + (x^6*(495*a^2*d^5 - 40*b^2*c^2*d^3 + 110*a*b*c*d^4))/(
3465*d^5) + (b*x^8*(22*a*d + b*c))/(99*d) + (c*x^4*(33*a^2*d^2 + 16*b^2*c^2
 - 44*a*b*c*d))/(1155*d^3) - (4*c^2*x^2*(33*a^2*d^2 + 16*b^2*c^2 - 44*a*b*c
*d))/(3465*d^4))
```

3.599 $\int x^3(a+bx^2)^2\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=114

$$-\frac{c(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{5/2}}{5d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^4}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^4}$$

```
[Out] -1/3*c*(-a*d+b*c)^2*(d*x^2+c)^(3/2)/d^4+1/5*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2+
c)^(5/2)/d^4-1/7*b*(-2*a*d+3*b*c)*(d*x^2+c)^(7/2)/d^4+1/9*b^2*(d*x^2+c)^(9/
2)/d^4
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {457, 78}

$$-\frac{b\left(c+dx^2\right)^{7/2}(3bc-2ad)}{7d^4}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)(3bc-ad)}{5d^4}-\frac{c\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^4}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] -1/3*(c*(b*c - a*d)^2*(c + d*x^2)^(3/2))/d^4 + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(5/2))/(5*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^4) +
 (b^2*(c + d*x^2)^(9/2))/(9*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^2\sqrt{c+dx}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{c(bc-ad)^2\sqrt{c+dx}}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^{3/2}}{d^3}-\right.\right.$$

$$=-\frac{c(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{5/2}}{5d^4}-\frac{b(3bc-2a}{}$$

Mathematica [A]
time = 0.07, size = 99, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{3/2}\left(21a^2d^2\left(-2c+3dx^2\right)+6abd\left(8c^2-12cdx^2+15d^2x^4\right)+b^2\left(-16c^3+24c^2dx^2-30cd^2x^4+35d^3x^6\right)\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(21*a^2*d^2*(-2*c + 3*d*x^2) + 6*a*b*d*(8*c^2 - 12*c*d*x
^2 + 15*d^2*x^4) + b^2*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d^2*x^4 + 35*d^3*x^6)
))/(315*d^4)
```

Maple [A]
time = 0.09, size = 185, normalized size = 1.62

method	result
gosper	$-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}\left(-35b^2x^6d^3-90ab\,d^3x^4+30b^2c\,d^2x^4-63a^2d^3x^2+72abc\,d^2x^2-24b^2c^2d\,x^2+42a^2c\,d^2-48ab\,c^2d+16b^2c^3\right)}{315d^4}$
trager	$-\frac{\left(-35b^2x^8d^4-90ab\,d^4x^6-5b^2c\,d^3x^6-63a^2x^4d^4-18x^4abc\,d^3+6b^2c^2x^4d^2-21a^2c\,d^3x^2+24ab\,c^2d^2x^2-8b^2c^3d\,x^2+42a^2c^2d^2-48ab\,c^3\right.}{315d^4}$
risch	$-\frac{\left(-35b^2x^8d^4-90ab\,d^4x^6-5b^2c\,d^3x^6-63a^2x^4d^4-18x^4abc\,d^3+6b^2c^2x^4d^2-21a^2c\,d^3x^2+24ab\,c^2d^2x^2-8b^2c^3d\,x^2+42a^2c^2d^2-48ab\,c^3\right.}{315d^4}$
default	$b^2\left(\frac{x^6\left(d\,x^2+c\right)^{\frac{3}{2}}}{9d}-\frac{2c\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{3}{2}}}{7d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{3}{2}}}{5d}-\frac{2c\left(d\,x^2+c\right)^{\frac{3}{2}}}{15d^2}\right)}{7d}\right)}{3d}\right)+2ab\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{3}{2}}}{7d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{3}{2}}}{5d}-\right.}{7d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/9*x^6*(d*x^2+c)^(3/2)/d-2/3*c/d*(1/7*x^4*(d*x^2+c)^(3/2)/d-4/7*c/d*(
1/5*x^2*(d*x^2+c)^(3/2)/d-2/15*c/d^2*(d*x^2+c)^(3/2))))+2*a*b*(1/7*x^4*(d*x
^2+c)^(3/2)/d-4/7*c/d*(1/5*x^2*(d*x^2+c)^(3/2)/d-2/15*c/d^2*(d*x^2+c)^(3/2)
))+a^2*(1/5*x^2*(d*x^2+c)^(3/2)/d-2/15*c/d^2*(d*x^2+c)^(3/2))
```

Maxima [**A**]
time = 0.31, size = 181, normalized size = 1.59

$$\frac{(dx^2+c)^{\frac{3}{2}}b^2x^6}{9\,d}-\frac{2\,(dx^2+c)^{\frac{3}{2}}b^2cx^4}{21\,d^2}+\frac{2\,(dx^2+c)^{\frac{3}{2}}abx^4}{7\,d}+\frac{8\,(dx^2+c)^{\frac{3}{2}}b^2c^2x^2}{105\,d^3}-\frac{8\,(dx^2+c)^{\frac{3}{2}}abcx^2}{35\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}a^2x^2}{5\,d}-\frac{16\,(dx^2+c)^{\frac{3}{2}}b^2c^3}{315\,d^4}+\frac{16\,(dx^2+c)^{\frac{3}{2}}abc^2}{105\,d^3}-\frac{2\,(dx^2+c)^{\frac{3}{2}}a^2c}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(d*x^2 + c)^(3/2)*b^2*x^6/d - 2/21*(d*x^2 + c)^(3/2)*b^2*c*x^4/d^2 + 2/
7*(d*x^2 + c)^(3/2)*a*b*x^4/d + 8/105*(d*x^2 + c)^(3/2)*b^2*c^2*x^2/d^3 - 8
/35*(d*x^2 + c)^(3/2)*a*b*c*x^2/d^2 + 1/5*(d*x^2 + c)^(3/2)*a^2*x^2/d - 16/
315*(d*x^2 + c)^(3/2)*b^2*c^3/d^4 + 16/105*(d*x^2 + c)^(3/2)*a*b*c^2/d^3 -
2/15*(d*x^2 + c)^(3/2)*a^2*c/d^2
```

Fricas [**A**]
time = 1.34, size = 140, normalized size = 1.23

$$\frac{(35\,b^2d^4x^8+5\,(b^2cd^3+18\,abd^4)x^6-16\,b^2c^4+48\,abc^3d-42\,a^2c^2d^2-3\,(2\,b^2c^2d^2-6\,abcd^3-21\,a^2d^4)x^4+(8\,b^2c^3d-24\,abc^2d^2+21\,a^2cd^3)x^2)\sqrt{dx^2+c}}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^4*x^8 + 5*(b^2*c*d^3 + 18*a*b*d^4)*x^6 - 16*b^2*c^4 + 48*a*
b*c^3*d - 42*a^2*c^2*d^2 - 3*(2*b^2*c^2*d^2 - 6*a*b*c*d^3 - 21*a^2*d^4)*x^4
 + (8*b^2*c^3*d - 24*a*b*c^2*d^2 + 21*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(102) = 204$.
time = 0.27, size = 308, normalized size = 2.70

$$\begin{cases} -\frac{2a^2c^2\sqrt{c+dx^2}}{15d^2}+\frac{a^2cx^2\sqrt{c+dx^2}}{15d}+\frac{a^2x^4\sqrt{c+dx^2}}{5}+\frac{16abc^3\sqrt{c+dx^2}}{105d^3}-\frac{8abc^2x^2\sqrt{c+dx^2}}{105d^2}+\frac{2abcx^4\sqrt{c+dx^2}}{35d}+\frac{2abx^6\sqrt{c+dx^2}}{7}-\frac{16b^2c^4\sqrt{c+dx^2}}{315d^4}+\frac{8b^2c^3x^2\sqrt{c+dx^2}}{315d^3}-\frac{2b^2c^2x^4\sqrt{c+dx^2}}{105d^2}+\frac{b^2cx^6\sqrt{c+dx^2}}{63d}+\frac{b^2x^8\sqrt{c+dx^2}}{9} & \text{for } d \neq 0 \\ \sqrt{c}\left(\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*c**2*sqrt(c + d*x**2)/(15*d**2) + a**2*c*x**2*sqrt(c + d
*x**2)/(15*d) + a**2*x**4*sqrt(c + d*x**2)/5 + 16*a*b*c**3*sqrt(c + d*x**2)
/(105*d**3) - 8*a*b*c**2*x**2*sqrt(c + d*x**2)/(105*d**2) + 2*a*b*c*x**4*sq
rt(c + d*x**2)/(35*d) + 2*a*b*x**6*sqrt(c + d*x**2)/7 - 16*b**2*c**4*sqrt(c
 + d*x**2)/(315*d**4) + 8*b**2*c**3*x**2*sqrt(c + d*x**2)/(315*d**3) - 2*b*
*2*c**2*x**4*sqrt(c + d*x**2)/(105*d**2) + b**2*c*x**6*sqrt(c + d*x**2)/(63
```

```
*d) + b**2*x**8*sqrt(c + d*x**2)/9, Ne(d, 0)), (sqrt(c)*(a**2*x**4/4 + a*b*
x**6/3 + b**2*x**8/8), True))
```

Giac [A]
time = 1.33, size = 150, normalized size = 1.32

$$\frac{35\,(dx^2+c)^{\frac{9}{2}}b^2-135\,(dx^2+c)^{\frac{7}{2}}b^2c+189\,(dx^2+c)^{\frac{5}{2}}b^2c^2-105\,(dx^2+c)^{\frac{3}{2}}b^2c^3+90\,(dx^2+c)^{\frac{7}{2}}abd-252\,(dx^2+c)^{\frac{5}{2}}abcd+210\,(dx^2+c)^{\frac{3}{2}}abc^2d+63\,(dx^2+c)^{\frac{5}{2}}a^2d^2-105\,(dx^2+c)^{\frac{3}{2}}a^2cd^2}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(d*x^2 + c)^(9/2)*b^2 - 135*(d*x^2 + c)^(7/2)*b^2*c + 189*(d*x^2
+ c)^(5/2)*b^2*c^2 - 105*(d*x^2 + c)^(3/2)*b^2*c^3 + 90*(d*x^2 + c)^(7/2)*a
*b*d - 252*(d*x^2 + c)^(5/2)*a*b*c*d + 210*(d*x^2 + c)^(3/2)*a*b*c^2*d + 63
*(d*x^2 + c)^(5/2)*a^2*d^2 - 105*(d*x^2 + c)^(3/2)*a^2*c*d^2)/d^4
```

Mupad [B]
time = 0.33, size = 137, normalized size = 1.20

$$\sqrt{d\,x^2+c}\,\left(\frac{b^2\,x^8}{9}-\frac{42\,a^2\,c^2\,d^2-48\,a\,b\,c^3\,d+16\,b^2\,c^4}{315\,d^4}+\frac{x^4\,\left(63\,a^2\,d^4+18\,a\,b\,c\,d^3-6\,b^2\,c^2\,d^2\right)}{315\,d^4}+\frac{b\,x^6\,\left(18\,a\,d+b\,c\right)}{63\,d}+\frac{c\,x^2\,\left(21\,a^2\,d^2-24\,a\,b\,c\,d+8\,b^2\,c^2\right)}{315\,d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((b^2*x^8)/9 - (16*b^2*c^4 + 42*a^2*c^2*d^2 - 48*a*b*c^3*
d)/(315*d^4) + (x^4*(63*a^2*d^4 - 6*b^2*c^2*d^2 + 18*a*b*c*d^3))/(315*d^4)
+ (b*x^6*(18*a*d + b*c))/(63*d) + (c*x^2*(21*a^2*d^2 + 8*b^2*c^2 - 24*a*b*c
*d))/(315*d^3))
```

3.600 $\int x(a+bx^2)^2 \sqrt{c+dx^2}\, dx$

Optimal. Leaf size=77

$$\frac{(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{5/2}}{5d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}$$

```
[Out] 1/3*(-a*d+b*c)^2*(d*x^2+c)^(3/2)/d^3-2/5*b*(-a*d+b*c)*(d*x^2+c)^(5/2)/d^3+1
/7*b^2*(d*x^2+c)^(7/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$-\frac{2b\left(c+dx^2\right)^{5/2}(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(5
/2))/(5*d^3) + (b^2*(c + d*x^2)^(7/2))/(7*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^2\sqrt{c+dx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{(-bc+ad)^2\sqrt{c+dx}}{d^2}-\frac{2b(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{b^2(c+dx)^{5/}}{d^2}\right.\right.\\ &= \frac{(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{5/2}}{5d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 0.87

$$\frac{(c+dx^2)^{3/2}\left(35a^2d^2+14abd(-2c+3dx^2)+b^2(8c^2-12cdx^2+15d^2x^4)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x^2) + b^2*(8*c^2 - 1
2*c*d*x^2 + 15*d^2*x^4)))/(105*d^3)
```

Maple [A]
time = 0.09, size = 117, normalized size = 1.52

method	result	siz
gosper	$\frac{(d\,x^2+c)^{\frac{3}{2}}(15b^2x^4d^2+42ab\,d^2x^2-12b^2cd\,x^2+35a^2d^2-28abcd+8b^2c^2)}{105d^3}$	69
trager	$\frac{(15b^2x^6d^3+42ab\,d^3x^4+3b^2c\,d^2x^4+35a^2d^3x^2+14abc\,d^2x^2-4b^2c^2d\,x^2+35a^2c\,d^2-28ab\,c^2d+8b^2c^3)\sqrt{d\,x^2+c}}{105d^3}$	10
risch	$\frac{(15b^2x^6d^3+42ab\,d^3x^4+3b^2c\,d^2x^4+35a^2d^3x^2+14abc\,d^2x^2-4b^2c^2d\,x^2+35a^2c\,d^2-28ab\,c^2d+8b^2c^3)\sqrt{d\,x^2+c}}{105d^3}$	10
default	$b^2\left(\frac{x^4(d\,x^2+c)^{\frac{3}{2}}}{7d}-\frac{4c\left(\frac{x^2(d\,x^2+c)^{\frac{3}{2}}}{5d}-\frac{2c(d\,x^2+c)^{\frac{3}{2}}}{15d^2}\right)}{7d}\right)+2ab\left(\frac{x^2(d\,x^2+c)^{\frac{3}{2}}}{5d}-\frac{2c(d\,x^2+c)^{\frac{3}{2}}}{15d^2}\right)+\frac{a^2(d\,x^2+c)^{\frac{3}{2}}}{3d}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/7*x^4*(d*x^2+c)^(3/2)/d-4/7*c/d*(1/5*x^2*(d*x^2+c)^(3/2)/d-2/15*c/d^
2*(d*x^2+c)^(3/2)))+2*a*b*(1/5*x^2*(d*x^2+c)^(3/2)/d-2/15*c/d^2*(d*x^2+c)^(
3/2))+1/3*a^2/d*(d*x^2+c)^(3/2)
```

Maxima [A]
time = 0.31, size = 115, normalized size = 1.49

$$\frac{(dx^2+c)^{\frac{3}{2}}b^2x^4}{7\,d}-\frac{4\,(dx^2+c)^{\frac{3}{2}}b^2cx^2}{35\,d^2}+\frac{2\,(dx^2+c)^{\frac{3}{2}}abx^2}{5\,d}+\frac{8\,(dx^2+c)^{\frac{3}{2}}b^2c^2}{105\,d^3}-\frac{4\,(dx^2+c)^{\frac{3}{2}}abc}{15\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}a^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(d*x^2 + c)^(3/2)*b^2*x^4/d - 4/35*(d*x^2 + c)^(3/2)*b^2*c*x^2/d^2 + 2/
5*(d*x^2 + c)^(3/2)*a*b*x^2/d + 8/105*(d*x^2 + c)^(3/2)*b^2*c^2/d^3 - 4/15*
(d*x^2 + c)^(3/2)*a*b*c/d^2 + 1/3*(d*x^2 + c)^(3/2)*a^2/d
```

Fricas [A]
time = 1.60, size = 103, normalized size = 1.34

$$\frac{\left(15\,b^{2}d^{3}x^{6}+8\,b^{2}c^{3}-28\,abc^{2}d+35\,a^{2}cd^{2}+3\,\left(b^{2}cd^{2}+14\,abd^{3}\right)x^{4}-\left(4\,b^{2}c^{2}d-14\,abcd^{2}-35\,a^{2}d^{3}\right)x^{2}\right)\sqrt{dx^{2}+c}}{105\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^6 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 + 3*(b^2*c*
d^2 + 14*a*b*d^3)*x^4 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*x^2)*sqrt
(d*x^2 + c)/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(66) = 132$.
time = 0.16, size = 226, normalized size = 2.94

$$\begin{cases} \frac{a^{2}c\sqrt{c+dx^{2}}}{3d}+\frac{a^{2}x^{2}\sqrt{c+dx^{2}}}{3}-\frac{4abc^{2}\sqrt{c+dx^{2}}}{15d^{2}}+\frac{2abcx^{2}\sqrt{c+dx^{2}}}{15d}+\frac{2abx^{4}\sqrt{c+dx^{2}}}{5}+\frac{8b^{2}c^{3}\sqrt{c+dx^{2}}}{105d^{3}}-\frac{4b^{2}c^{2}x^{2}\sqrt{c+dx^{2}}}{105d^{2}}+\frac{b^{2}cx^{4}\sqrt{c+dx^{2}}}{35d}+\frac{b^{2}x^{6}\sqrt{c+dx^{2}}}{7} & \text{for}\: d\neq 0 \\ \sqrt{c}\left(\frac{a^{2}x^{2}}{2}+\frac{abx^{4}}{2}+\frac{b^{2}x^{6}}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((a**2*c*sqrt(c + d*x**2)/(3*d) + a**2*x**2*sqrt(c + d*x**2)/3 - 4
*a*b*c**2*sqrt(c + d*x**2)/(15*d**2) + 2*a*b*c*x**2*sqrt(c + d*x**2)/(15*d)
 + 2*a*b*x**4*sqrt(c + d*x**2)/5 + 8*b**2*c**3*sqrt(c + d*x**2)/(105*d**3)
- 4*b**2*c**2*x**2*sqrt(c + d*x**2)/(105*d**2) + b**2*c*x**4*sqrt(c + d*x**
2)/(35*d) + b**2*x**6*sqrt(c + d*x**2)/7, Ne(d, 0)), (sqrt(c)*(a**2*x**2/2
+ a*b*x**4/2 + b**2*x**6/6), True))
```

Giac [A]
time = 1.23, size = 98, normalized size = 1.27

$$\frac{15\,\left(dx^{2}+c\right)^{\frac{7}{2}}b^{2}-42\,\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}c+35\,\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}c^{2}+42\,\left(dx^{2}+c\right)^{\frac{5}{2}}abd-70\,\left(dx^{2}+c\right)^{\frac{3}{2}}abcd+35\,\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}}{105\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(d*x^2 + c)^(7/2)*b^2 - 42*(d*x^2 + c)^(5/2)*b^2*c + 35*(d*x^2 +
c)^(3/2)*b^2*c^2 + 42*(d*x^2 + c)^(5/2)*a*b*d - 70*(d*x^2 + c)^(3/2)*a*b*c*
d + 35*(d*x^2 + c)^(3/2)*a^2*d^2)/d^3
```

Mupad [B]
time = 0.31, size = 101, normalized size = 1.31

$$\sqrt{d\,x^2+c}\,\left(\frac{35\,a^2\,c\,d^2-28\,a\,b\,c^2\,d+8\,b^2\,c^3}{105\,d^3}+\frac{b^2\,x^6}{7}+\frac{x^2\,\left(35\,a^2\,d^3+14\,a\,b\,c\,d^2-4\,b^2\,c^2\,d\right)}{105\,d^3}+\frac{b\,x^4\,\left(14\,a\,d+b\,c\right)}{35\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((8*b^2*c^3 + 35*a^2*c*d^2 - 28*a*b*c^2*d)/(105*d^3) + (b
^2*x^6)/7 + (x^2*(35*a^2*d^3 - 4*b^2*c^2*d + 14*a*b*c*d^2))/(105*d^3) + (b*
x^4*(14*a*d + b*c))/(35*d))
```

3.601 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx$

Optimal. Leaf size=92

$$a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}-a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

```
[Out] -1/3*b*(-2*a*d+b*c)*(d*x^2+c)^(3/2)/d^2+1/5*b^2*(d*x^2+c)^(5/2)/d^2-a^2*arc
tanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+a^2*(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 90, 52, 65, 214}

$$a^2\sqrt{c+dx^2}-a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{b\left(c+dx^2\right)^{3/2}(bc-2ad)}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

```
[Out] a^2*Sqrt[c + d*x^2] - (b*(b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^2) + (b^2*(c
 + d*x^2)^(5/2))/(5*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b(bc-2ad)\sqrt{c+dx}}{d}+\frac{a^2\sqrt{c+dx}}{x}+\frac{b^2(c+dx)^{3/2}}{d}\right)\,dx, x,$$

$$= -\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{1}{2}a^2\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x,$$

$$= a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{1}{2}\left(a^2c\right)\text{Subst}\Bigg(\int$$

$$= a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{\left(a^2c\right)\text{Subst}\Big(\int -$$

$$= a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}-a^2\sqrt{c}\tanh^{-1}\Bigg($$

Mathematica [A]
time = 0.11, size = 92, normalized size = 1.00

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2+10abd(c+dx^2)+b^2(-2c^2+cdx^2+3d^2x^4)\right)}{15d^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(c + d*x^2) + b^2*(-2*c^2 + c*d*x^2
 + 3*d^2*x^4)))/(15*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Maple [A]
time = 0.10, size = 97, normalized size = 1.05

method	result
default	$b^{2}\left(\frac{x^{2}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{5d}-\frac{2c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{15d^{2}}\right)+\frac{2ab\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{3d}+a^{2}\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/5*x^2*(d*x^2+c)^(3/2)/d-2/15*c/d^2*(d*x^2+c)^(3/2))+2/3*a*b/d*(d*x^2
+c)^(3/2)+a^2*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x
))
```

Maxima [A]
time = 0.29, size = 88, normalized size = 0.96

$$\frac{(dx^2+c)^{\frac{3}{2}}b^2x^2}{5\,d}-a^2\sqrt{c}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\sqrt{dx^2+c}\,a^2-\frac{2\,(dx^2+c)^{\frac{3}{2}}b^2c}{15\,d^2}+\frac{2\,(dx^2+c)^{\frac{3}{2}}ab}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/5*(d*x^2 + c)^(3/2)*b^2*x^2/d - a^2*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x)))
 + sqrt(d*x^2 + c)*a^2 - 2/15*(d*x^2 + c)^(3/2)*b^2*c/d^2 + 2/3*(d*x^2 + c)
^(3/2)*a*b/d
```

Fricas [A]
time = 1.46, size = 207, normalized size = 2.25

$$\left[\frac{15\,a^2\sqrt{c}\,d^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,(3\,b^2d^2x^4-2\,b^2c^2+10\,abcd+15\,a^2d^2+(b^2cd+10\,abd^2)x^2)\sqrt{dx^2+c}}{30\,d^2},\ \frac{15\,a^2\sqrt{-c}\,d^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(3\,b^2d^2x^4-2\,b^2c^2+10\,abcd+15\,a^2d^2+(b^2cd+10\,abd^2)x^2)\sqrt{dx^2+c}}{15\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^2*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^
2) + 2*(3*b^2*d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2 + (b^2*c*d + 10
*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/d^2, 1/15*(15*a^2*sqrt(-c)*d^2*arctan(sqrt(
-c)/sqrt(d*x^2 + c)) + (3*b^2*d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2
 + (b^2*c*d + 10*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/d^2]
```

Sympy [A]
time = 17.68, size = 90, normalized size = 0.98

$$\frac{a^2 c \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}} + a^2\sqrt{c+dx^2} + \frac{b^2(c+dx^2)^{\frac{5}{2}}}{5d^2} + \frac{(c+dx^2)^{\frac{3}{2}}\cdot(4abd-2b^2c)}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] a**2*c*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + a**2*sqrt(c + d*x**2) + b
**2*(c + d*x**2)**(5/2)/(5*d**2) + (c + d*x**2)**(3/2)*(4*a*b*d - 2*b**2*c)
/(6*d**2)
```

Giac [A]
time = 2.73, size = 101, normalized size = 1.10

$$\frac{a^2 c \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{3\,(dx^2+c)^{\frac{5}{2}}b^2d^8 - 5\,(dx^2+c)^{\frac{3}{2}}b^2cd^8 + 10\,(dx^2+c)^{\frac{3}{2}}abd^9 + 15\,\sqrt{dx^2+c}\,a^2d^{10}}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/15*(3*(d*x^2 + c)^(5/2)
*b^2*d^8 - 5*(d*x^2 + c)^(3/2)*b^2*c*d^8 + 10*(d*x^2 + c)^(3/2)*a*b*d^9 + 1
5*sqrt(d*x^2 + c)*a^2*d^10)/d^10
```

Mupad [B]
time = 0.34, size = 135, normalized size = 1.47

$$\sqrt{d\,x^2+c}\,\left(\frac{{\left(a\,d-b\,c\right)}^2}{d^2}-c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\right)-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{3\,d^2}-\frac{b^2\,c}{3\,d^2}\right)\,{\left(d\,x^2+c\right)}^{3/2}+\frac{b^2\,{\left(d\,x^2+c\right)}^{5/2}}{5\,d^2}+a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x,x)
```

```
[Out] (c + d*x^2)^(1/2)*((a*d - b*c)^2/d^2 - c*((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)
/d^2)) - ((2*b^2*c - 2*a*b*d)/(3*d^2) - (b^2*c)/(3*d^2))*(c + d*x^2)^(3/2)
+ a^2*c^(1/2)*atan(((c + d*x^2)^(1/2)*1i)/c^(1/2))*1i + (b^2*(c + d*x^2)^(5
/2))/(5*d^2)
```

3.602 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx$

Optimal. Leaf size=109

$$\frac{a(4bc+ad)\sqrt{c+dx^2}}{2c}+\frac{b^2(c+dx^2)^{3/2}}{3d}-\frac{a^2(c+dx^2)^{3/2}}{2cx^2}-\frac{a(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

```
[Out] 1/3*b^2*(d*x^2+c)^(3/2)/d-1/2*a^2*(d*x^2+c)^(3/2)/c/x^2-1/2*a*(a*d+4*b*c)*a
rctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+1/2*a*(a*d+4*b*c)*(d*x^2+c)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 81, 52, 65, 214}

$$-\frac{a^2(c+dx^2)^{3/2}}{2cx^2}+\frac{a\sqrt{c+dx^2}\,(ad+4bc)}{2c}-\frac{a(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{b^2(c+dx^2)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] (a*(4*b*c + a*d)*Sqrt[c + d*x^2])/(2*c) + (b^2*(c + d*x^2)^(3/2))/(3*d) - (
a^2*(c + d*x^2)^(3/2))/(2*c*x^2) - (a*(4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]])/(2*Sqrt[c])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^3} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x^2} dx, x, x^2\right)$$

$$= -\frac{a^2(c+dx^2)^{3/2}}{2cx^2} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(4bc+ad)+b^2cx\right)\sqrt{c+dx}}{x} dx, x, x^2\right)}{2c}$$

$$= \frac{b^2(c+dx^2)^{3/2}}{3d} - \frac{a^2(c+dx^2)^{3/2}}{2cx^2} + \frac{(a(4bc+ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x} dx, x, x^2\right)}{4c}$$

$$= \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c} + \frac{b^2(c+dx^2)^{3/2}}{3d} - \frac{a^2(c+dx^2)^{3/2}}{2cx^2} + \frac{1}{4}(a(4bc+ad))\text{Su}$$

$$= \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c} + \frac{b^2(c+dx^2)^{3/2}}{3d} - \frac{a^2(c+dx^2)^{3/2}}{2cx^2} + \frac{(a(4bc+ad))\text{Subs}}{}$$

$$= \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c} + \frac{b^2(c+dx^2)^{3/2}}{3d} - \frac{a^2(c+dx^2)^{3/2}}{2cx^2} - \frac{a(4bc+ad)\tanh^{-}}{2\sqrt{}}$$

Mathematica [A]
time = 0.16, size = 87, normalized size = 0.80

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^2}\,(-3a^2d+12abdx^2+2b^2x^2(c+dx^2))}{dx^2} - \frac{3a(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-3*a^2*d + 12*a*b*d*x^2 + 2*b^2*x^2*(c + d*x^2)))/(d*x^2
) - (3*a*(4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/6
```

Maple [A]
time = 0.11, size = 127, normalized size = 1.17

method	result
default	$\frac{b^2(dx^2+c)^{\frac{3}{2}}}{3d} + a^2\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{2cx^2} + \frac{d\left(\sqrt{dx^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{dx^2+c}}{x}\right)\right)}{2c}\right) + 2ab\left(\sqrt{dx^2+c}\right.$ −

risch	$-\frac{a^2\sqrt{d\,x^2+c}}{2x^2}+\frac{b^2x^2\sqrt{d\,x^2+c}}{3}+\frac{b^2c\sqrt{d\,x^2+c}}{3d}+2ab\sqrt{d\,x^2+c}-\frac{a^2\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)d}{2\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*(d*x^2+c)^(3/2)/d+a^2*(-1/2/c/x^2*(d*x^2+c)^(3/2)+1/2*d/c*((d*x^2+c
)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))+2*a*b*((d*x^2+c)^(1
/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))
```

Maxima [A]
time = 0.28, size = 109, normalized size = 1.00

$$-2\,ab\sqrt{c}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{a^2d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,\sqrt{c}}+2\,\sqrt{dx^2+c}\,ab+\frac{(dx^2+c)^{\frac{3}{2}}b^2}{3\,d}+\frac{\sqrt{dx^2+c}\,a^2d}{2\,c}-\frac{(dx^2+c)^{\frac{3}{2}}a^2}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*a*b*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - 1/2*a^2*d*arcsinh(c/(sqrt(c*
d)*abs(x)))/sqrt(c) + 2*sqrt(d*x^2 + c)*a*b + 1/3*(d*x^2 + c)^(3/2)*b^2/d +
 1/2*sqrt(d*x^2 + c)*a^2*d/c - 1/2*(d*x^2 + c)^(3/2)*a^2/(c*x^2)
```

Fricas [A]
time = 1.46, size = 211, normalized size = 1.94

$$\left[\frac{3\,(4\,abcd+a^2d^2)\sqrt{c}\,x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,(2\,b^2cdx^4-3\,a^2cd+2\,(b^2c^2+6\,abcd)x^2)\sqrt{dx^2+c}}{12\,cdx^2},\ \frac{3\,(4\,abcd+a^2d^2)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(2\,b^2cdx^4-3\,a^2cd+2\,(b^2c^2+6\,abcd)x^2)\sqrt{dx^2+c}}{6\,cdx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(4*a*b*c*d + a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*
sqrt(c) + 2*c)/x^2) + 2*(2*b^2*c*d*x^4 - 3*a^2*c*d + 2*(b^2*c^2 + 6*a*b*c*d
)*x^2)*sqrt(d*x^2 + c))/(c*d*x^2), 1/6*(3*(4*a*b*c*d + a^2*d^2)*sqrt(-c)*x^
2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b^2*c*d*x^4 - 3*a^2*c*d + 2*(b^2*c^
2 + 6*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(c*d*x^2)]
```

Sympy [A]
time = 25.21, size = 148, normalized size = 1.36

$$-\frac{a^2\sqrt{d}\,\sqrt{\frac{c}{dx^2}+1}}{2x}-\frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\sqrt{c}}-2ab\sqrt{c}\,\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)+\frac{2abc}{\sqrt{d}\,x\sqrt{\frac{c}{dx^2}+1}}+\frac{2ab\sqrt{d}\,x}{\sqrt{\frac{c}{dx^2}+1}}+b^2\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\ d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) - a**2*d*asinh(sqrt(c)/(sqrt(d)*x)
)/(2*sqrt(c)) - 2*a*b*sqrt(c)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*b*c/(sqrt(d)
*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*sqrt(d)*x/sqrt(c/(d*x**2) + 1) + b**2*Piec
ewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True))
```

Giac [A]
time = 1.18, size = 89, normalized size = 0.82

$$\frac{2\,(dx^2+c)^{\frac{3}{2}}b^2+12\,\sqrt{dx^2+c}\,abd-\frac{3\,\sqrt{dx^2+c}\,a^2d}{x^2}+\frac{3\,(4\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*(d*x^2 + c)^(3/2)*b^2 + 12*sqrt(d*x^2 + c)*a*b*d - 3*sqrt(d*x^2 + c)
*a^2*d/x^2 + 3*(4*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(
-c))/d
```

Mupad [B]
time = 0.51, size = 103, normalized size = 0.94

$$\frac{b^2\,{\left(d\,x^2+c\right)}^{3/2}}{3\,d}-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)\,\sqrt{d\,x^2+c}-\frac{a^2\,\sqrt{d\,x^2+c}}{2\,x^2}+\frac{a\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,\left(a\,d+4\,b\,c\right)\,1{}\mathrm{i}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^3,x)
```

```
[Out] (b^2*(c + d*x^2)^(3/2))/(3*d) - ((2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d)*(c +
d*x^2)^(1/2) - (a^2*(c + d*x^2)^(1/2))/(2*x^2) + (a*atan(((c + d*x^2)^(1/2)
*1i)/c^(1/2))*(a*d + 4*b*c)*1i)/(2*c^(1/2))
```

3.603 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^5} \, dx$

Optimal. Leaf size=143

$$\frac{(8b^2c^2+ad(8bc-ad))\sqrt{c+dx^2}}{8c^2}-\frac{a^2(c+dx^2)^{3/2}}{4cx^4}-\frac{a(8bc-ad)(c+dx^2)^{3/2}}{8c^2x^2}-\frac{(8b^2c^2+ad(8bc-ad))\tanh^{-1}}{8c^{3/2}}$$

```
[Out] -1/4*a^2*(d*x^2+c)^(3/2)/c/x^4-1/8*a*(-a*d+8*b*c)*(d*x^2+c)^(3/2)/c^2/x^2-1
/8*(8*b^2*c^2+a*d*(-a*d+8*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+1/
8*(8*b^2*c^2+a*d*(-a*d+8*b*c))*(d*x^2+c)^(1/2)/c^2
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 79, 52, 65, 214}

$$-\frac{a^2(c+dx^2)^{3/2}}{4cx^4}+\frac{1}{8}\sqrt{c+dx^2}\left(\frac{ad(8bc-ad)}{c^2}+8b^2\right)-\frac{(ad(8bc-ad)+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}-\frac{a(c+dx^2)^{3/2}(8bc-ad)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

```
[Out] ((8*b^2 + (a*d*(8*b*c - a*d))/c^2)*Sqrt[c + d*x^2])/8 - (a^2*(c + d*x^2)^(3
/2))/(4*c*x^4) - (a*(8*b*c - a*d)*(c + d*x^2)^(3/2))/(8*c^2*x^2) - ((8*b^2*
c^2 + a*d*(8*b*c - a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{a^2(c+dx^2)^{3/2}}{4cx^4} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(8bc-ad)+2b^2cx\right)\sqrt{c+dx}}{x^2}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a^2(c+dx^2)^{3/2}}{4cx^4} - \frac{a(8bc-ad)\,(c+dx^2)^{3/2}}{8c^2x^2} + \frac{1}{16}\left(8b^2 + \frac{ad(8bc-ad)}{c^2}\right)\text{Sub}$$

$$= \frac{1}{8}\left(8b^2 + \frac{ad(8bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{a^2(c+dx^2)^{3/2}}{4cx^4} - \frac{a(8bc-ad)\,(c+dx^2}{8c^2x^2}$$

$$= \frac{1}{8}\left(8b^2 + \frac{ad(8bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{a^2(c+dx^2)^{3/2}}{4cx^4} - \frac{a(8bc-ad)\,(c+dx^2}{8c^2x^2}$$

$$= \frac{1}{8}\left(8b^2 + \frac{ad(8bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{a^2(c+dx^2)^{3/2}}{4cx^4} - \frac{a(8bc-ad)\,(c+dx^2}{8c^2x^2}$$

Mathematica [A]
time = 0.22, size = 104, normalized size = 0.73

$$\frac{\sqrt{c+dx^2}\,(-2a^2c - 8abcx^2 - a^2dx^2 + 8b^2cx^4)}{8cx^4} + \frac{(-8b^2c^2 - 8abcd + a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
[Out] (Sqrt[c + d*x^2]*(-2*a^2*c - 8*a*b*c*x^2 - a^2*d*x^2 + 8*b^2*c*x^4))/(8*c*x
^4) + ((-8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])
/(8*c^(3/2))
```

Maple [A]
time = 0.11, size = 200, normalized size = 1.40

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\;a\left(a d\,x^2+8c\,x^2 b+2ac\right)}{8x^4 c} + b^2\sqrt{d\,x^2+c} + \frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)a^2d^2}{8c^{\frac{3}{2}}} - \frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2}}{x}\right.}{\sqrt{c}}$

default	$a^2\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{4c\,x^4}-\frac{d\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}+\frac{d\left(\sqrt{d\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)\right)}{2c}\right)}{4c}\right)+2ab\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/4/c/x^4*(d*x^2+c)^(3/2)-1/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(3/2)+1/2*d/c
*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))+2*a*b*(-
1/2/c/x^2*(d*x^2+c)^(3/2)+1/2*d/c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2
)*(d*x^2+c)^(1/2))/x)))+b^2*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x
^2+c)^(1/2))/x))
```

Maxima [A]
time = 0.29, size = 173, normalized size = 1.21

$$-b^2\sqrt{c}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{\sqrt{c}}+\frac{a^2d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{8\,c^{\frac{3}{2}}}+\sqrt{dx^2+c}\,b^2+\frac{\sqrt{dx^2+c}\,abd}{c}-\frac{\sqrt{dx^2+c}\,a^2d^2}{8\,c^2}-\frac{(dx^2+c)^{\frac{3}{2}}ab}{cx^2}+\frac{(dx^2+c)^{\frac{3}{2}}a^2d}{8\,c^2x^2}-\frac{(dx^2+c)^{\frac{3}{2}}a^2}{4\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -b^2*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - a*b*d*arcsinh(c/(sqrt(c*d)*abs
(x)))/sqrt(c) + 1/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + sqrt(d*
x^2 + c)*b^2 + sqrt(d*x^2 + c)*a*b*d/c - 1/8*sqrt(d*x^2 + c)*a^2*d^2/c^2 -
(d*x^2 + c)^(3/2)*a*b/(c*x^2) + 1/8*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^2) - 1/4
*(d*x^2 + c)^(3/2)*a^2/(c*x^4)
```

Fricas [A]
time = 1.73, size = 225, normalized size = 1.57

$$\left[-\frac{(8\,b^2c^2+8\,abcd-a^2d^2)\sqrt{c}\,x^4\log\left(-\frac{dx^2+2\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,(8\,b^2c^2x^4-2\,a^2c^2-(8\,abc^2+a^2cd)x^2)\sqrt{dx^2+c}}{16\,c^2x^4},\ \frac{(8\,b^2c^2+8\,abcd-a^2d^2)\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(8\,b^2c^2x^4-2\,a^2c^2-(8\,abc^2+a^2cd)x^2)\sqrt{dx^2+c}}{8\,c^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/16*((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 + 2*sqrt(
d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*b^2*c^2*x^4 - 2*a^2*c^2 - (8*a*b*c^2
+ a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^4), 1/8*((8*b^2*c^2 + 8*a*b*c*d - a
```

```
^2*d^2)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (8*b^2*c^2*x^4 - 2*
a^2*c^2 - (8*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^4)]
```

Sympy [A]
time = 67.57, size = 219, normalized size = 1.53

$$-\frac{a^2c}{4\sqrt{d}\,x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^{\frac{3}{2}}}{8cx\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{8c^{\frac{3}{2}}}-\frac{ab\sqrt{d}\,\sqrt{\frac{c}{dx^2}+1}}{x}-\frac{abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{\sqrt{c}}-b^2\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)+\frac{b^2c}{\sqrt{d}\,x\sqrt{\frac{c}{dx^2}+1}}+\frac{b^2\sqrt{d}\,x}{\sqrt{\frac{c}{dx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**5,x)
```

```
[Out] -a**2*c/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*sqrt(d)/(8*x**3*sqrt
(c/(d*x**2) + 1)) - a**2*d**(3/2)/(8*c*x*sqrt(c/(d*x**2) + 1)) + a**2*d**2*
asinh(sqrt(c)/(sqrt(d)*x))/(8*c**(3/2)) - a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/
x - a*b*d*asinh(sqrt(c)/(sqrt(d)*x))/sqrt(c) - b**2*sqrt(c)*asinh(sqrt(c)/(
sqrt(d)*x)) + b**2*c/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*sqrt(d)*x/sqrt
(c/(d*x**2) + 1)
```

Giac [A]
time = 1.39, size = 153, normalized size = 1.07

$$\frac{8\sqrt{dx^2+c}\,b^2d+\frac{(8\,b^2c^2d+8\,abcd^2-a^2d^3)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}-\frac{8\,(dx^2+c)^{\frac{3}{2}}abcd^2-8\sqrt{dx^2+c}\,abc^2d^2+(dx^2+c)^{\frac{3}{2}}a^2d^3+\sqrt{dx^2+c}\,a^2cd^3}{cd^2x^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(d*x^2 + c)*b^2*d + (8*b^2*c^2*d + 8*a*b*c*d^2 - a^2*d^3)*arctan
(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (8*(d*x^2 + c)^(3/2)*a*b*c*d^2 -
8*sqrt(d*x^2 + c)*a*b*c^2*d^2 + (d*x^2 + c)^(3/2)*a^2*d^3 + sqrt(d*x^2 + c)
*a^2*c*d^3)/(c*d^2*x^4))/d
```

Mupad [B]
time = 0.62, size = 137, normalized size = 0.96

$$b^2\sqrt{d\,x^2+c}-\frac{\left(\frac{a^2\,d^2}{8}-a\,b\,c\,d\right)\sqrt{d\,x^2+c}+\frac{(a^2\,d^2+8\,b\,c\,a\,d)\,(d\,x^2+c)^{3/2}}{8\,c}}{(d\,x^2+c)^2-2\,c\,(d\,x^2+c)+c^2}-\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(-a^2\,d^2+8\,a\,b\,c\,d+8\,b^2\,c^2)}{8\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^5,x)
```

```
[Out] b^2*(c + d*x^2)^(1/2) - (((a^2*d^2)/8 - a*b*c*d)*(c + d*x^2)^(1/2) + ((a^2*
d^2 + 8*a*b*c*d)*(c + d*x^2)^(3/2))/(8*c))/((c + d*x^2)^2 - 2*c*(c + d*x^2)
 + c^2) - (atanh((c + d*x^2)^(1/2)/c^(1/2))*(8*b^2*c^2 - a^2*d^2 + 8*a*b*c*
d))/(8*c^(3/2))
```

3.604 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx$

Optimal. Leaf size=149

$$-\frac{(8b^2c^2-4abcd+a^2d^2)\sqrt{c+dx^2}}{16c^2x^2}-\frac{a^2(c+dx^2)^{3/2}}{6cx^6}-\frac{a(4bc-ad)\,(c+dx^2)^{3/2}}{8c^2x^4}-\frac{d(8b^2c^2-4abcd+a^2d^2)\tan}{16c^{5/2}}$$

```
[Out] -1/6*a^2*(d*x^2+c)^(3/2)/c/x^6-1/8*a*(-a*d+4*b*c)*(d*x^2+c)^(3/2)/c^2/x^4-1
/16*d*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(5/2
)-1/16*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*(d*x^2+c)^(1/2)/c^2/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 79, 43, 65, 214}

$$-\frac{\sqrt{c+dx^2}\,(a^2d^2-4abcd+8b^2c^2)}{16c^2x^2}-\frac{d(a^2d^2-4abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}-\frac{a^2(c+dx^2)^{3/2}}{6cx^6}-\frac{a(c+dx^2)^{3/2}\,(4bc-ad)}{8c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

```
[Out] -1/16*((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2])/(c^2*x^2) - (a^2*
(c + d*x^2)^(3/2))/(6*c*x^6) - (a*(4*b*c - a*d)*(c + d*x^2)^(3/2))/(8*c^2*x
^4) - (d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
)/(16*c^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{a^2(c+dx^2)^{3/2}}{6cx^6} + \frac{\text{Subst}\left(\int \frac{\left(\frac{3}{2}a(4bc-ad)+3b^2cx\right)\sqrt{c+dx}}{x^3}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{a^2(c+dx^2)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\,(c+dx^2)^{3/2}}{8c^2x^4} + \frac{(8b^2c^2-4abcd+a^2d^2)\,\text{Subst}\Big(}{16c^2}$$

$$= -\frac{(8b^2c^2-4abcd+a^2d^2)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2(c+dx^2)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\,(c+dx^2)^{3/2}}{8c^2x^4}$$

$$= -\frac{(8b^2c^2-4abcd+a^2d^2)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2(c+dx^2)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\,(c+dx^2)^{3/2}}{8c^2x^4}$$

$$= -\frac{(8b^2c^2-4abcd+a^2d^2)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2(c+dx^2)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\,(c+dx^2)^{3/2}}{8c^2x^4}$$

Mathematica [A]
time = 0.22, size = 126, normalized size = 0.85

$$-\frac{\sqrt{c+dx^2}\,(24b^2c^2x^4+12abcx^2(2c+dx^2)+a^2(8c^2+2cdx^2-3d^2x^4))}{48c^2x^6} - \frac{d(8b^2c^2-4abcd+a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

```
[Out] -1/48*(Sqrt[c + d*x^2]*(24*b^2*c^2*x^4 + 12*a*b*c*x^2*(2*c + d*x^2) + a^2*(
8*c^2 + 2*c*d*x^2 - 3*d^2*x^4)))/(c^2*x^6) - (d*(8*b^2*c^2 - 4*a*b*c*d + a^
2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(129) = 258.
time = 0.10, size = 272, normalized size = 1.83

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\,(-3a^2d^2x^4+12abcd\,x^4+24b^2c^2x^4+2a^2cd\,x^2+24ab\,c^2x^2+8a^2c^2)}{48x^6c^2} - \frac{d^3\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)a^2}{16c^{\frac{5}{2}}} + \frac{d^2\ln}{}$

default	$2ab\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{4cx^4}-\frac{d\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{2cx^2}+\frac{d\left(\sqrt{dx^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{dx^2+c}}{x}\right)\right)}{2c}\right)}{4c}\right)+a^2\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{6cx^6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b*(-1/4/c/x^4*(d*x^2+c)^(3/2)-1/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(3/2)+1/2*d
/c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))+a^2*(-
1/6/c/x^6*(d*x^2+c)^(3/2)-1/2*d/c*(-1/4/c/x^4*(d*x^2+c)^(3/2)-1/4*d/c*(-1/2
/c/x^2*(d*x^2+c)^(3/2)+1/2*d/c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(
d*x^2+c)^(1/2))/x)))))+b^2*(-1/2/c/x^2*(d*x^2+c)^(3/2)+1/2*d/c*((d*x^2+c)^(
1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))
```

Maxima [A]

time = 0.28, size = 247, normalized size = 1.66

$$-\frac{b^2d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\sqrt{c}}+\frac{abd^2\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{4c^{\frac{3}{2}}}-\frac{a^2d^3\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16c^{\frac{5}{2}}}+\frac{\sqrt{dx^2+c}\,b^2d}{2c}-\frac{\sqrt{dx^2+c}\,abd^2}{4c^2}+\frac{\sqrt{dx^2+c}\,a^2d^3}{16c^3}-\frac{(dx^2+c)^{\frac{3}{2}}b^2}{2cx^2}+\frac{(dx^2+c)^{\frac{3}{2}}abd}{4c^2x^2}-\frac{(dx^2+c)^{\frac{3}{2}}a^2d^2}{16c^3x^2}-\frac{(dx^2+c)^{\frac{3}{2}}ab}{2cx^4}+\frac{(dx^2+c)^{\frac{3}{2}}a^2d}{8c^2x^4}-\frac{(dx^2+c)^{\frac{3}{2}}a^2}{6cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*d*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 1/4*a*b*d^2*arcsinh(c/(s
qrt(c*d)*abs(x)))/c^(3/2) - 1/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5
/2) + 1/2*sqrt(d*x^2 + c)*b^2*d/c - 1/4*sqrt(d*x^2 + c)*a*b*d^2/c^2 + 1/16*
sqrt(d*x^2 + c)*a^2*d^3/c^3 - 1/2*(d*x^2 + c)^(3/2)*b^2/(c*x^2) + 1/4*(d*x^
2 + c)^(3/2)*a*b*d/(c^2*x^2) - 1/16*(d*x^2 + c)^(3/2)*a^2*d^2/(c^3*x^2) - 1
/2*(d*x^2 + c)^(3/2)*a*b/(c*x^4) + 1/8*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^4) -
1/6*(d*x^2 + c)^(3/2)*a^2/(c*x^6)
```

Fricas [A]

time = 1.50, size = 276, normalized size = 1.85

$$\left[\frac{3\,(8\,b^2c^2d-4\,abcd^2+a^2d^3)\sqrt{c}\,x^6\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,(8\,a^2c^3+3\,(8\,b^2c^3+4\,abc^2d-a^2cd^2)x^4+2\,(12\,abc^3+a^2c^2d)x^2)\sqrt{dx^2+c}}{96\,c^3x^6},\ \frac{3\,(8\,b^2c^2d-4\,abcd^2+a^2d^3)\sqrt{-c}\,x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-(8\,a^2c^3+3\,(8\,b^2c^3+4\,abc^2d-a^2cd^2)x^4+2\,(12\,abc^3+a^2c^2d)x^2)\sqrt{dx^2+c}}{48\,c^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*a^2*c^3 + 3*(8*b^2*c^3 + 4*a*b*c
^2*d - a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c
^3*x^6), 1/48*(3*(8*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*sqrt(-c)*x^6*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) - (8*a^2*c^3 + 3*(8*b^2*c^3 + 4*a*b*c^2*d - a^2*c
*d^2)*x^4 + 2*(12*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^3*x^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(141) = 282$.
time = 80.62, size = 291, normalized size = 1.95

$$-\frac{a^2 c}{6\sqrt{d}\, x^7 \sqrt{\frac{c}{dx^2}+1}} - \frac{5a^2\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2 d^{\frac{3}{2}}}{48cx^3\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2 d^{\frac{5}{2}}}{16c^2 x\sqrt{\frac{c}{dx^2}+1}} - \frac{a^2 d^3 \operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{16c^{\frac{5}{2}}} - \frac{abc}{2\sqrt{d}\,x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{3ab\sqrt{d}}{4x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{abd^{\frac{3}{2}}}{4cx\sqrt{\frac{c}{dx^2}+1}} + \frac{abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{4c^{\frac{3}{2}}} - \frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x} - \frac{b^2 d \operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**7,x)
```

```
[Out] -a**2*c/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 5*a**2*sqrt(d)/(24*x**5*sqr
t(c/(d*x**2) + 1)) + a**2*d**(3/2)/(48*c*x**3*sqrt(c/(d*x**2) + 1)) + a**2*
d**(5/2)/(16*c**2*x*sqrt(c/(d*x**2) + 1)) - a**2*d**3*asinh(sqrt(c)/(sqrt(d
)*x))/(16*c**(5/2)) - a*b*c/(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a*b*s
qrt(d)/(4*x**3*sqrt(c/(d*x**2) + 1)) - a*b*d**(3/2)/(4*c*x*sqrt(c/(d*x**2)
+ 1)) + a*b*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(4*c**(3/2)) - b**2*sqrt(d)*sqr
t(c/(d*x**2) + 1)/(2*x) - b**2*d*asinh(sqrt(c)/(sqrt(d)*x))/(2*sqrt(c))
```

Giac [A]
time = 0.94, size = 222, normalized size = 1.49

$$\frac{\frac{3\,(8\,b^2c^2d^2-4\,abcd^3+a^2d^4)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^2} - \frac{24\,(dx^2+c)^{\frac{5}{2}}b^2c^2d^2-48\,(dx^2+c)^{\frac{3}{2}}b^2c^3d^2+24\,\sqrt{dx^2+c}\,b^2c^4d^2+12\,(dx^2+c)^{\frac{5}{2}}abcd^3-12\,\sqrt{dx^2+c}\,abc^3d^3-3\,(dx^2+c)^{\frac{5}{2}}a^2d^4+8\,(dx^2+c)^{\frac{3}{2}}a^2cd^4+3\,\sqrt{dx^2+c}\,a^2c^2d^4}{c^2d^3x^6}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(8*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*arctan(sqrt(d*x^2 + c)/sqrt
(-c))/(sqrt(-c)*c^2) - (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2 + c)^(
3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 + 12*(d*x^2 + c)^(5/2)*a*
b*c*d^3 - 12*sqrt(d*x^2 + c)*a*b*c^3*d^3 - 3*(d*x^2 + c)^(5/2)*a^2*d^4 + 8*
(d*x^2 + c)^(3/2)*a^2*c*d^4 + 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c^2*d^3*x^6))
/d
```

Mupad [B]
time = 0.86, size = 193, normalized size = 1.30

$$\frac{\sqrt{d\,x^2+c}\,\left(\frac{a^2\,d^3}{16}-\frac{a\,b\,c\,d^2}{4}+\frac{b^2\,c^2\,d}{2}\right)+\frac{{\left(d\,x^2+c\right)}^{3/2}\,\left(a^2\,d^3-6\,b^2\,c^2\,d\right)}{6\,c}+\frac{{\left(d\,x^2+c\right)}^{5/2}\,\left(-a^2\,d^3+4\,a\,b\,c\,d^2+8\,b^2\,c^2\,d\right)}{16\,c^2}}{3\,c\,{\left(d\,x^2+c\right)}^2-3\,c^2\,\left(d\,x^2+c\right)-{\left(d\,x^2+c\right)}^3+c^3}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(a^2\,d^2-4\,a\,b\,c\,d+8\,b^2\,c^2\right)}{16\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^7,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((a^2*d^3)/16 + (b^2*c^2*d)/2 - (a*b*c*d^2)/4) + ((c + d
*x^2)^(3/2)*(a^2*d^3 - 6*b^2*c^2*d))/(6*c) + ((c + d*x^2)^(5/2)*(8*b^2*c^2*
d - a^2*d^3 + 4*a*b*c*d^2))/(16*c^2))/(3*c*(c + d*x^2)^2 - 3*c^2*(c + d*x^2
) - (c + d*x^2)^3 + c^3) - (d*atanh((c + d*x^2)^(1/2)/c^(1/2))*(a^2*d^2 + 8
*b^2*c^2 - 4*a*b*c*d))/(16*c^(5/2))
```

3.605 $\int x^2(a+bx^2)^2\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=191

$$\frac{c(16a^2d^2+bc(5bc-16ad))\,x\sqrt{c+dx^2}}{128d^3}+\frac{(16a^2d^2+bc(5bc-16ad))\,x^3\sqrt{c+dx^2}}{64d^2}-\frac{b(5bc-16ad)x^3(c+dx^2)}{48d^2}$$

```
[Out] -1/48*b*(-16*a*d+5*b*c)*x^3*(d*x^2+c)^(3/2)/d^2+1/8*b^2*x^5*(d*x^2+c)^(3/2)
/d-1/128*c^2*(16*a^2*d^2+b*c*(-16*a*d+5*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(
1/2))/d^(7/2)+1/128*c*(16*a^2*d^2+b*c*(-16*a*d+5*b*c))*x*(d*x^2+c)^(1/2)/d^
3+1/64*(16*a^2*d^2+b*c*(-16*a*d+5*b*c))*x^3*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.13, antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {475, 470, 285, 327, 223, 212}

$$-\frac{c^2(16a^2d^2+bc(5bc-16ad))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{7/2}}+\frac{1}{64}x^3\sqrt{c+dx^2}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)+\frac{cx\sqrt{c+dx^2}\,(16a^2d^2+bc(5bc-16ad))}{128d^3}-\frac{bx^3(c+dx^2)^{3/2}(5bc-16ad)}{48d^2}+\frac{b^2x^5(c+dx^2)^{3/2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (c*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128*d^3) + ((16*
a^2 + (b*c*(5*b*c - 16*a*d))/d^2)*x^3*Sqrt[c + d*x^2])/64 - (b*(5*b*c - 16*
a*d)*x^3*(c + d*x^2)^(3/2))/(48*d^2) + (b^2*x^5*(c + d*x^2)^(3/2))/(8*d) -
(c^2*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(128*d^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx=\frac{b^2x^5(c+dx^2)^{3/2}}{8d}+\frac{\int x^2\sqrt{c+dx^2}\,(8a^2d-b(5bc-16ad)x^2)\,dx}{8d}$$

$$=-\frac{b(5bc-16ad)x^3(c+dx^2)^{3/2}}{48d^2}+\frac{b^2x^5(c+dx^2)^{3/2}}{8d}+\frac{1}{16}\left(16a^2+\frac{bc(5bc-16\right.$$

$$=\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}-\frac{b(5bc-16ad)x^3(c+dx^2)^{3/2}}{48d^2}+$$

$$=\frac{c\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d}+\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+d}$$

$$=\frac{c\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d}+\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+d}$$

$$=\frac{c\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d}+\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+d}$$

Mathematica [A]
time = 0.19, size = 156, normalized size = 0.82

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\,(48a^2d^2(c+2dx^2)+16abd(-3c^2+2cdx^2+8d^2x^4)+b^2(15c^3-10c^2dx^2+8cd^2x^4+48d^3x^6))+3c^2(5b^2c^2-16abcd+16a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{384d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(c + 2*d*x^2) + 16*a*b*d*(-3*c^2 + 2
*c*d*x^2 + 8*d^2*x^4) + b^2*(15*c^3 - 10*c^2*d*x^2 + 8*c*d^2*x^4 + 48*d^3*x
^6)) + 3*c^2*(5*b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[
c + d*x^2]])/(384*d^(7/2))
```

Maple [A]
time = 0.09, size = 257, normalized size = 1.35

method	result
risch	$\frac{x\left(48b^2x^6d^3+128ab\,d^3x^4+8b^2c\,d^2x^4+96a^2d^3x^2+32abc\,d^2x^2-10b^2c^2d\,x^2+48a^2c\,d^2-48ab\,c^2d+15b^2c^3\right)\sqrt{d\,x^2+c}}{384d^3}-\frac{c^2\ln\left(x\sqrt{c\right.}{}$

default	$b^{2}\left(\frac{x^{5}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{8d}-\frac{5c\left(\frac{x^{3}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{6d}-\frac{c\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{4d}-\frac{c\left(\frac{x\sqrt{d\,x^{2}+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2\sqrt{d}}\right)}{4d}\right)}{2d}\right)}{8d}\right)+2ab\Bigg($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/8*x^5*(d*x^2+c)^(3/2)/d-5/8*c/d*(1/6*x^3*(d*x^2+c)^(3/2)/d-1/2*c/d*(
1/4*x*(d*x^2+c)^(3/2)/d-1/4*c/d*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d
^(1/2)+(d*x^2+c)^(1/2))))))+2*a*b*(1/6*x^3*(d*x^2+c)^(3/2)/d-1/2*c/d*(1/4*x
*(d*x^2+c)^(3/2)/d-1/4*c/d*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2
)+(d*x^2+c)^(1/2)))))+a^2*(1/4*x*(d*x^2+c)^(3/2)/d-1/4*c/d*(1/2*x*(d*x^2+c)
^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))
```

<u>Maxima</u> [A]
time = 0.27, size = 237, normalized size = 1.24

$$\frac{(dx^2+c)^{\frac{3}{2}}b^2x^5}{8\,d}-\frac{5\,(dx^2+c)^{\frac{3}{2}}b^2cx^3}{48\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}abx^3}{3\,d}+\frac{5\,(dx^2+c)^{\frac{3}{2}}b^2c^2x}{64\,d^3}-\frac{5\,\sqrt{dx^2+c}\,b^2c^3x}{128\,d^3}-\frac{(dx^2+c)^{\frac{3}{2}}abcx}{4\,d^2}+\frac{\sqrt{dx^2+c}\,abc^2x}{8\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}a^2x}{4\,d}-\frac{\sqrt{dx^2+c}\,a^2cx}{8\,d}-\frac{5\,b^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{7}{2}}}+\frac{abc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{5}{2}}}-\frac{a^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^2 + c)^(3/2)*b^2*x^5/d - 5/48*(d*x^2 + c)^(3/2)*b^2*c*x^3/d^2 + 1/
3*(d*x^2 + c)^(3/2)*a*b*x^3/d + 5/64*(d*x^2 + c)^(3/2)*b^2*c^2*x/d^3 - 5/12
8*sqrt(d*x^2 + c)*b^2*c^3*x/d^3 - 1/4*(d*x^2 + c)^(3/2)*a*b*c*x/d^2 + 1/8*s
qrt(d*x^2 + c)*a*b*c^2*x/d^2 + 1/4*(d*x^2 + c)^(3/2)*a^2*x/d - 1/8*sqrt(d*x
^2 + c)*a^2*c*x/d - 5/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 1/8*a*b*
```

```
c^3*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(
3/2)
```

Fricas [A]
time = 1.94, size = 341, normalized size = 1.79

$$\left[\frac{3\,(5\,b^2c^4 - 16\,abc^3d + 16\,a^2c^2d^2)\sqrt{d}\,\log\left(-2\,dx^2 + 2\,\sqrt{dx^2+c}\,\sqrt{d}\,x - c\right) + 2\,(48\,b^2d^4x^7 + 8\,(b^2cd^3 + 16\,abd^4)x^5 - 2\,(5\,b^2c^2d^2 - 16\,abcd^3 - 48\,a^2d^4)x^3 + 3\,(5\,b^2c^3d - 16\,abc^2d^2 + 16\,a^2cd^3)x)\sqrt{dx^2+c}}{768\,d^4}, \frac{3\,(5\,b^2c^4 - 16\,abc^3d + 16\,a^2c^2d^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right) + (48\,b^2d^4x^7 + 8\,(b^2cd^3 + 16\,abd^4)x^5 - 2\,(5\,b^2c^2d^2 - 16\,abcd^3 - 48\,a^2d^4)x^3 + 3\,(5\,b^2c^3d - 16\,abc^2d^2 + 16\,a^2cd^3)x)\sqrt{dx^2+c}}{384\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
+ 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 + 8*(b^2*c*d^3 + 16*
a*b*d^4)*x^5 - 2*(5*b^2*c^2*d^2 - 16*a*b*c*d^3 - 48*a^2*d^4)*x^3 + 3*(5*b^2
*c^3*d - 16*a*b*c^2*d^2 + 16*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/384*(3*(
5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) + (48*b^2*d^4*x^7 + 8*(b^2*c*d^3 + 16*a*b*d^4)*x^5 - 2*(5*b^2*c
^2*d^2 - 16*a*b*c*d^3 - 48*a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 16*a*b*c^2*d^2 +
 16*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(182) = 364$.
time = 28.59, size = 411, normalized size = 2.15

$$\frac{a^2c^{\frac{3}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}} + \frac{3a^2\sqrt{c}\,x^3}{8\sqrt{1+\frac{dx^2}{c}}} - \frac{a^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}} + \frac{a^2dx^5}{4\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}} - \frac{abc^{\frac{5}{2}}x}{8d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{abc^{\frac{3}{2}}x^3}{24d\sqrt{1+\frac{dx^2}{c}}} + \frac{5ab\sqrt{c}\,x^5}{12\sqrt{1+\frac{dx^2}{c}}} + \frac{abc^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{5}{2}}} + \frac{abdx^7}{3\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}} + \frac{5b^2c^{\frac{7}{2}}x}{128d^3\sqrt{1+\frac{dx^2}{c}}} + \frac{5b^2c^{\frac{5}{2}}x^3}{384d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{b^2c^{\frac{3}{2}}x^5}{192d\sqrt{1+\frac{dx^2}{c}}} + \frac{7b^2\sqrt{c}\,x^7}{48\sqrt{1+\frac{dx^2}{c}}} - \frac{5b^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{7}{2}}} + \frac{b^2dx^9}{8\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*x/(8*d*sqrt(1 + d*x**2/c)) + 3*a**2*sqrt(c)*x**3/(8*sqrt(1 +
d*x**2/c)) - a**2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(3/2)) + a**2*d*x**5/
(4*sqrt(c)*sqrt(1 + d*x**2/c)) - a*b*c**(5/2)*x/(8*d**2*sqrt(1 + d*x**2/c))
 - a*b*c**(3/2)*x**3/(24*d*sqrt(1 + d*x**2/c)) + 5*a*b*sqrt(c)*x**5/(12*sqr
t(1 + d*x**2/c)) + a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + a*b*d*x
**7/(3*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*b**2*c**(7/2)*x/(128*d**3*sqrt(1 + d
*x**2/c)) + 5*b**2*c**(5/2)*x**3/(384*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(3
/2)*x**5/(192*d*sqrt(1 + d*x**2/c)) + 7*b**2*sqrt(c)*x**7/(48*sqrt(1 + d*x*
*2/c)) - 5*b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**(7/2)) + b**2*d*x**9/
(8*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A]
time = 1.56, size = 174, normalized size = 0.91

$$\frac{1}{384}\left(2\left(4\left(6\,b^2x^2 + \frac{b^2cd^5 + 16\,abd^6}{d^6}\right)x^2 - \frac{5\,b^2c^2d^4 - 16\,abcd^5 - 48\,a^2d^6}{d^6}\right)x^2 + \frac{3\,(5\,b^2c^3d^3 - 16\,abc^2d^4 + 16\,a^2cd^5)}{d^6}\right)\sqrt{dx^2+c}\,x + \frac{(5\,b^2c^4 - 16\,abc^3d + 16\,a^2c^2d^2)\log\left(\left|-\sqrt{d}\,x + \sqrt{dx^2+c}\right|\right)}{128\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] 1/384*(2*(4*(6*b^2*x^2 + (b^2*c*d^5 + 16*a*b*d^6)/d^6)*x^2 - (5*b^2*c^2*d^4
 - 16*a*b*c*d^5 - 48*a^2*d^6)/d^6)*x^2 + 3*(5*b^2*c^3*d^3 - 16*a*b*c^2*d^4
+ 16*a^2*c*d^5)/d^6)*sqrt(d*x^2 + c)*x + 1/128*(5*b^2*c^4 - 16*a*b*c^3*d +
16*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)

[Out] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

3.606 $\int \left(a+bx^2\right)^2 \sqrt{c+dx^2}\, dx$

Optimal. Leaf size=149

$$\frac{\left(b^2c^2-4abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16d^2}-\frac{b(3bc-8ad)x\left(c+dx^2\right)^{3/2}}{24d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}+\frac{c\left(b^2c^2-4abcd+\right.}{}$$

```
[Out] -1/24*b*(-8*a*d+3*b*c)*x*(d*x^2+c)^(3/2)/d^2+1/6*b*x*(b*x^2+a)*(d*x^2+c)^(3
/2)/d+1/16*c*(8*a^2*d^2-4*a*b*c*d+b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2
))/d^(5/2)+1/16*(8*a^2*d^2-4*a*b*c*d+b^2*c^2)*x*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {427, 396, 201, 223, 212}

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-4abcd+b^2c^2\right)}{16d^2}+\frac{c\left(8a^2d^2-4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}}-\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{24d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^2) - (b*(3*b*c
- 8*a*d)*x*(c + d*x^2)^(3/2))/(24*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^(3/2)
)/(6*d) + (c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(16*d^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx = \frac{bx(a+bx^2)\left(c+dx^2\right)^{3/2}}{6d} + \frac{\int \sqrt{c+dx^2}\left(-a(bc-6ad)-b(3bc-8ad)x^2\right)\,dx}{6d}$$

$$= -\frac{b(3bc-8ad)x(c+dx^2)^{3/2}}{24d^2} + \frac{bx(a+bx^2)\left(c+dx^2\right)^{3/2}}{6d} + \frac{(b^2c^2-4abcd+8a}{8}$$

$$= \frac{(b^2c^2-4abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16d^2} - \frac{b(3bc-8ad)x(c+dx^2)^{3/2}}{24d^2} + \frac{bx(a+b}{}$$

$$= \frac{(b^2c^2-4abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16d^2} - \frac{b(3bc-8ad)x(c+dx^2)^{3/2}}{24d^2} + \frac{bx(a+b}{}$$

$$= \frac{(b^2c^2-4abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16d^2} - \frac{b(3bc-8ad)x(c+dx^2)^{3/2}}{24d^2} + \frac{bx(a+b}{}$$

Mathematica [A]
time = 0.16, size = 121, normalized size = 0.81

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\left(24a^2d^2+12abd(c+2dx^2)+b^2(-3c^2+2cdx^2+8d^2x^4)\right)-3c(b^2c^2-4abcd+8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{48d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(c + 2*d*x^2) + b^2*(-3*c
^2 + 2*c*d*x^2 + 8*d^2*x^4)) - 3*c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*Log[-(
Sqrt[d]*x) + Sqrt[c + d*x^2]])/(48*d^(5/2))
```

Maple [A]
time = 0.09, size = 187, normalized size = 1.26

method	result
risch	$\frac{x(8b^2x^4d^2+24abd^2x^2+2b^2cdx^2+24a^2d^2+12abcd-3b^2c^2)\sqrt{dx^2+c}}{48d^2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)a^2}{2\sqrt{d}}-\frac{c^2\ln\left(x\sqrt{d}+\sqrt{\cdots}\right.}{4d^{\frac{3}{2}}\cdots}$
default	$b^2\left(\frac{x^3(dx^2+c)^{\frac{3}{2}}}{6d}-\frac{c\left(\frac{x(dx^2+c)^{\frac{3}{2}}}{4d}-\frac{c\left(\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{2\sqrt{d}}\right)}{4d}\right)}{2d}\right)+2ab\left(\frac{x(dx^2+c)^{\frac{3}{2}}}{4d}-\frac{c\left(\frac{x\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/6*x^3*(d*x^2+c)^(3/2)/d-1/2*c/d*(1/4*x*(d*x^2+c)^(3/2)/d-1/4*c/d*(1/
2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+2*a*b*(1
/4*x*(d*x^2+c)^(3/2)/d-1/4*c/d*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^
(1/2)+(d*x^2+c)^(1/2))))+a^2*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1
/2)+(d*x^2+c)^(1/2)))
```

Maxima [A]
time = 0.30, size = 168, normalized size = 1.13

$$\frac{(dx^2+c)^{\frac{3}{2}}b^2x^3}{6\,d}+\frac{1}{2}\sqrt{dx^2+c}\,a^2x-\frac{(dx^2+c)^{\frac{3}{2}}b^2cx}{8\,d^2}+\frac{\sqrt{dx^2+c}\,b^2c^2x}{16\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}abx}{2\,d}-\frac{\sqrt{dx^2+c}\,abcx}{4\,d}+\frac{b^2c^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{5}{2}}}-\frac{abc^2\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,d^{\frac{3}{2}}}+\frac{a^2c\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^2 + c)^(3/2)*b^2*x^3/d + 1/2*sqrt(d*x^2 + c)*a^2*x - 1/8*(d*x^2 +
c)^(3/2)*b^2*c*x/d^2 + 1/16*sqrt(d*x^2 + c)*b^2*c^2*x/d^2 + 1/2*(d*x^2 + c)
^(3/2)*a*b*x/d - 1/4*sqrt(d*x^2 + c)*a*b*c*x/d + 1/16*b^2*c^3*arcsinh(d*x/s
qrt(c*d))/d^(5/2) - 1/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 1/2*a^2*c*
arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

Fricas [A]
time = 1.51, size = 262, normalized size = 1.76

$$\left[\frac{3\,(b^2c^3-4\,abc^2d+8\,a^2cd^2)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(8\,b^2d^3x^5+2\,(b^2cd^2+12\,abd^3)x^3-3\,(b^2c^2d-4\,abcd^2-8\,a^2d^3)x)\sqrt{dx^2+c}}{96\,d^3},\,-\frac{3\,(b^2c^3-4\,abc^2d+8\,a^2cd^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(8\,b^2d^3x^5+2\,(b^2cd^2+12\,abd^3)x^3-3\,(b^2c^2d-4\,abcd^2-8\,a^2d^3)x)\sqrt{dx^2+c}}{48\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqr
t(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^5 + 2*(b^2*c*d^2 + 12*a*b*d^3)
*x^3 - 3*(b^2*c^2*d - 4*a*b*c*d^2 - 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/
48*(3*(b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - (8*b^2*d^3*x^5 + 2*(b^2*c*d^2 + 12*a*b*d^3)*x^3 - 3*(b^2*c^2
*d - 4*a*b*c*d^2 - 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(143) = 286$.
time = 8.41, size = 291, normalized size = 1.95

$$\frac{a^2\sqrt{c}\,x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{a^2c\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{d}}+\frac{abc^{\frac{3}{2}}x}{4d\sqrt{1+\frac{dx^2}{c}}}+\frac{3ab\sqrt{c}\,x^3}{4\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^2\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{4d^{\frac{3}{2}}}+\frac{abdx^5}{2\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{5}{2}}x}{16d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{3}{2}}x^3}{48d\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2\sqrt{c}\,x^5}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16d^{\frac{5}{2}}}+\frac{b^2dx^7}{6\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/2 + a**2*c*asinh(sqrt(d)*x/sqrt(c))/(2*sq
rt(d)) + a*b*c**(3/2)*x/(4*d*sqrt(1 + d*x**2/c)) + 3*a*b*sqrt(c)*x**3/(4*sq
rt(1 + d*x**2/c)) - a*b*c**2*asinh(sqrt(d)*x/sqrt(c))/(4*d**(3/2)) + a*b*d*
x**5/(2*sqrt(c)*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x/(16*d**2*sqrt(1 + d*x
**2/c)) - b**2*c**(3/2)*x**3/(48*d*sqrt(1 + d*x**2/c)) + 5*b**2*sqrt(c)*x**
5/(24*sqrt(1 + d*x**2/c)) + b**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*d**(5/2)
) + b**2*d*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A]
time = 1.40, size = 128, normalized size = 0.86

$$\frac{1}{48}\left(2\left(4\,b^2x^2+\frac{b^2cd^3+12\,abd^4}{d^4}\right)x^2-\frac{3\,(b^2c^2d^2-4\,abcd^3-8\,a^2d^4)}{d^4}\right)\sqrt{dx^2+c}\,x-\frac{(b^2c^3-4\,abc^2d+8\,a^2cd^2)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*x^2 + (b^2*c*d^3 + 12*a*b*d^4)/d^4)*x^2 - 3*(b^2*c^2*d^2 - 4
*a*b*c*d^3 - 8*a^2*d^4)/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b^2*c^3 - 4*a*b*c^2*
d + 8*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

3.607 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx$

Optimal. Leaf size=133

$$-\frac{(b^2c^2-8ad(bc+ad))\,x\sqrt{c+dx^2}}{8cd}-\frac{a^2(c+dx^2)^{3/2}}{cx}+\frac{b^2x(c+dx^2)^{3/2}}{4d}-\frac{(b^2c^2-8ad(bc+ad))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}}$$

```
[Out] -a^2*(d*x^2+c)^(3/2)/c/x+1/4*b^2*x*(d*x^2+c)^(3/2)/d-1/8*(b^2*c^2-8*a*d*(a*
d+b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-1/8*(b^2*c^2-8*a*d*(a*d+
b*c))*x*(d*x^2+c)^(1/2)/c/d
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {473, 396, 201, 223, 212}

$$-\frac{a^2(c+dx^2)^{3/2}}{cx}-\frac{(b^2c^2-8ad(ad+bc))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}}-\frac{1}{8}x\sqrt{c+dx^2}\left(\frac{b^2c}{d}-\frac{8a(ad+bc)}{c}\right)+\frac{b^2x(c+dx^2)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] -1/8*(((b^2*c)/d - (8*a*(b*c + a*d))/c)*x*Sqrt[c + d*x^2]) - (a^2*(c + d*x^
2)^(3/2))/(c*x) + (b^2*x*(c + d*x^2)^(3/2))/(4*d) - ((b^2*c^2 - 8*a*d*(b*c
+ a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx &= -\frac{a^2(c+dx^2)^{3/2}}{cx}+\frac{\int (2a(bc+ad)+b^2cx^2)\sqrt{c+dx^2}\,dx}{c}\\
&= -\frac{a^2(c+dx^2)^{3/2}}{cx}+\frac{b^2x(c+dx^2)^{3/2}}{4d}-\frac{1}{4}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)\int\sqrt{c+dx^2}\,dx\\
&= -\frac{1}{8}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)x\sqrt{c+dx^2}-\frac{a^2(c+dx^2)^{3/2}}{cx}+\frac{b^2x(c+dx^2)^{3/2}}{4d}-\frac{1}{8}\\
&= -\frac{1}{8}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)x\sqrt{c+dx^2}-\frac{a^2(c+dx^2)^{3/2}}{cx}+\frac{b^2x(c+dx^2)^{3/2}}{4d}-\frac{1}{8}\\
&= -\frac{1}{8}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)x\sqrt{c+dx^2}-\frac{a^2(c+dx^2)^{3/2}}{cx}+\frac{b^2x(c+dx^2)^{3/2}}{4d}-
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 106, normalized size = 0.80

$$
\frac{\sqrt{c+dx^2}\,(-8a^2d+b^2cx^2+8abdx^2+2b^2dx^4)}{8dx}+\frac{(b^2c^2-8abcd-8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{8d^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-8*a^2*d + b^2*c*x^2 + 8*a*b*d*x^2 + 2*b^2*d*x^4))/(8*d*x
) + ((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])
/(8*d^(3/2))
```

Maple [A]
time = 0.09, size = 165, normalized size = 1.24

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\,\left(-2b^2d\,x^4-8abd\,x^2-b^2c\,x^2+8a^2d\right)}{8dx}+\sqrt{d}\,\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)a^2+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c\right.}}{\sqrt{d}}$
default	$b^2\left(\frac{x\left(d\,x^2+c\right)^{\frac{3}{2}}}{4d}-\frac{c\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{4d}\right)+2ab\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x\right.}}{2\sqrt{d}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/4*x*(d*x^2+c)^(3/2)/d-1/4*c/d*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*l
n(x*d^(1/2)+(d*x^2+c)^(1/2))))+2*a*b*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*l
n(x*d^(1/2)+(d*x^2+c)^(1/2)))+a^2*(-1/c/x*(d*x^2+c)^(3/2)+2*d/c*(1/2*x*(d*x
^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))
```

Maxima [A]
time = 0.31, size = 120, normalized size = 0.90

$$\sqrt{dx^2+c}\,abx+\frac{(dx^2+c)^{\frac{3}{2}}b^2x}{4\,d}-\frac{\sqrt{dx^2+c}\,b^2cx}{8\,d}-\frac{b^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}+\frac{abc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}}+a^2\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\sqrt{dx^2+c}\,a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] sqrt(d*x^2 + c)*a*b*x + 1/4*(d*x^2 + c)^(3/2)*b^2*x/d - 1/8*sqrt(d*x^2 + c)
*b^2*c*x/d - 1/8*b^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + a*b*c*arcsinh(d*x
/sqrt(c*d))/sqrt(d) + a^2*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*
a^2/x
```

Fricas [A]
time = 1.40, size = 215, normalized size = 1.62

$$\left[-\frac{(b^2c^2-8\,abcd-8\,a^2d^2)\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\,(2\,b^2d^2x^4-8\,a^2d^2+(b^2cd+8\,abd^2)x^2)\sqrt{dx^2+c}}{16\,d^2x},\ \frac{(b^2c^2-8\,abcd-8\,a^2d^2)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(2\,b^2d^2x^4-8\,a^2d^2+(b^2cd+8\,abd^2)x^2)\sqrt{dx^2+c}}{8\,d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(d)*x - c) - 2*(2*b^2*d^2*x^4 - 8*a^2*d^2 + (b^2*c*d + 8*a*b*
```

d^2)*x^2)*sqrt(d*x^2 + c))/(d^2*x), 1/8*((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*
sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b^2*d^2*x^4 - 8*a^2*d^2
+ (b^2*c*d + 8*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(d^2*x)]

Sympy [A]
time = 4.17, size = 219, normalized size = 1.65

$$-\frac{a^2\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}}+a^2\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{a^2dx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+ab\sqrt{c}\,x\sqrt{1+\frac{dx^2}{c}}+\frac{abc\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}}+\frac{b^2c^{\frac{3}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2\sqrt{c}\,x^3}{8\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}}+\frac{b^2dx^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**2,x)

[Out] -a**2*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)
) - a**2*d*x/(sqrt(c)*sqrt(1 + d*x**2/c)) + a*b*sqrt(c)*x*sqrt(1 + d*x**2/c
) + a*b*c*asinh(sqrt(d)*x/sqrt(c))/sqrt(d) + b**2*c**(3/2)*x/(8*d*sqrt(1 +
d*x**2/c)) + 3*b**2*sqrt(c)*x**3/(8*sqrt(1 + d*x**2/c)) - b**2*c**2*asinh(s
qrt(d)*x/sqrt(c))/(8*d**(3/2)) + b**2*d*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))

Giac [A]
time = 1.25, size = 126, normalized size = 0.95

$$\frac{2\,a^2c\sqrt{d}}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}+\frac{1}{8}\left(2\,b^2x^2+\frac{b^2cd+8\,abd^2}{d^2}\right)\sqrt{dx^2+c}\,x+\frac{\left(b^2c^2\sqrt{d}\,-8\,abcd^{\frac{3}{2}}-8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{16\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")

[Out] 2*a^2*c*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/8*(2*b^2*x^2 + (b
^2*c*d + 8*a*b*d^2)/d^2)*sqrt(d*x^2 + c)*x + 1/16*(b^2*c^2*sqrt(d) - 8*a*b*
c*d^(3/2) - 8*a^2*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^2,x)

[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^2, x)

3.608 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx$

Optimal. Leaf size=111

$$\frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{b(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

```
[Out] -1/3*a^2*(d*x^2+c)^(3/2)/c/x^3-2*a*b*(d*x^2+c)^(3/2)/c/x+1/2*b*(4*a*d+b*c)*
arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)+1/2*b*(4*a*d+b*c)*x*(d*x^2+c)^(1
/2)/c
```

Rubi [A]

time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.208$, Rules used = {473, 464, 201, 223, 212}

$$-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{bx\sqrt{c+dx^2}\,(4ad+bc)}{2c}+\frac{b(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

```
[Out] (b*(b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (a^2*(c + d*x^2)^(3/2))/(3*c*x^
3) - (2*a*b*(c + d*x^2)^(3/2))/(c*x) + (b*(b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]])/(2*Sqrt[d])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx &= -\frac{a^2(c+dx^2)^{3/2}}{3cx^3}+\frac{\int \frac{(6abc+3b^2cx^2)\sqrt{c+dx^2}}{x^2}\,dx}{3c}\\
&= -\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{(b(bc+4ad))\int \sqrt{c+dx^2}\,dx}{c}\\
&= \frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{1}{2}(b(bc+4ad))\\
&= \frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{1}{2}(b(bc+4ad))\mathrm{S}\\
&= \frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{b(bc+4ad)\tan}{2}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 90, normalized size = 0.81

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^2}\left(-12abcx^2+3b^2cx^4-2a^2(c+dx^2)\right)}{cx^3}-\frac{3b(bc+4ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-12*a*b*c*x^2 + 3*b^2*c*x^4 - 2*a^2*(c + d*x^2)))/(c*x^3
) - (3*b*(b*c + 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[d])/6
```

Maple [A]
time = 0.09, size = 124, normalized size = 1.12

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\,\left(-3b^2c\,x^4+2a^2d\,x^2+12abc\,x^2+2a^2c\right)}{6x^3c}+2b\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)\sqrt{d}\;a+\frac{b^2\ln\left(x\sqrt{d}+\sqrt{d\,x\right.}}{2\sqrt{d}}$
default	$b^2\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)-\frac{a^2\left(d\,x^2+c\right)^{\frac{3}{2}}}{3c\,x^3}+2ab\left(-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}}{cx}+\frac{2d\left(\frac{x\sqrt{d\,x^2+c}}{2}+\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))-1/3
*a^2*(d*x^2+c)^(3/2)/c/x^3+2*a*b*(-1/c/x*(d*x^2+c)^(3/2)+2*d/c*(1/2*x*(d*x^
2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))
```

Maxima [A]
time = 0.33, size = 86, normalized size = 0.77

$$\frac{1}{2}\sqrt{dx^2+c}\,b^2x+\frac{b^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}}+2\,ab\sqrt{d}\;\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{2\sqrt{dx^2+c}\,ab}{x}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x + 1/2*b^2*c*arcsinh(d*x/sqrt(c*d))/sqrt(d) + 2*a*
b*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - 2*sqrt(d*x^2 + c)*a*b/x - 1/3*(d*x^2 + c
)^(3/2)*a^2/(c*x^3)
```

Fricas [A]
time = 1.30, size = 210, normalized size = 1.89

$$\left[\frac{3\,(b^2c^2+4\,abcd)\sqrt{d}\,x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(3\,b^2cdx^4-2\,a^2cd-2\,(6\,abcd+a^2d^2)x^2)\sqrt{dx^2+c}}{12\,cdx^3},\ -\frac{3\,(b^2c^2+4\,abcd)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(3\,b^2cdx^4-2\,a^2cd-2\,(6\,abcd+a^2d^2)x^2)\sqrt{dx^2+c}}{6\,cdx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(d)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)
*sqrt(d)*x - c) + 2*(3*b^2*c*d*x^4 - 2*a^2*c*d - 2*(6*a*b*c*d + a^2*d^2)*x^
2)*sqrt(d*x^2 + c))/(c*d*x^3), -1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-d)*x^3*a
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (3*b^2*c*d*x^4 - 2*a^2*c*d - 2*(6*a*b*c
*d + a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c*d*x^3)]
```

Sympy [A]
time = 2.51, size = 170, normalized size = 1.53

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c}-\frac{2ab\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}}+2ab\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{2abdx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2\sqrt{c}\,x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**4,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - a**2*d**(3/2)*sqrt(c/(d*x**2)
 + 1)/(3*c) - 2*a*b*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + 2*a*b*sqrt(d)*asinh(sq
rt(d)*x/sqrt(c)) - 2*a*b*d*x/(sqrt(c)*sqrt(1 + d*x**2/c)) + b**2*sqrt(c)*x*
sqrt(1 + d*x**2/c)/2 + b**2*c*asinh(sqrt(d)*x/sqrt(c))/(2*sqrt(d))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(93) = 186.
time = 1.44, size = 188, normalized size = 1.69

$$\frac{1}{2}\sqrt{dx^2+c}\,b^2x-\frac{\left(b^2c\sqrt{d}+4\,abd^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{4\,d}+\frac{2\left(6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc\sqrt{d}+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{3}{2}}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^2\sqrt{d}+6\,abc^3\sqrt{d}+a^2c^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x - 1/4*(b^2*c*sqrt(d) + 4*a*b*d^(3/2))*log((sqrt(d
)*x - sqrt(d*x^2 + c))^2)/d + 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*
sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(3/2) - 12*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 6*a*b*c^3*sqrt(d) + a^2*c^2*d^(3/2))/((
sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^4, x)
```

3.609 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx$

Optimal. Leaf size=103

$$-\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\,(c+dx^2)^{3/2}}{15c^2x^3}+b^2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)$$

```
[Out] -1/5*a^2*(d*x^2+c)^(3/2)/c/x^5-2/15*a*(-a*d+5*b*c)*(d*x^2+c)^(3/2)/c^2/x^3+
b^2*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)-b^2*(d*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {473, 462, 283, 223, 212}

$$-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}-\frac{2a(c+dx^2)^{3/2}\,(5bc-ad)}{15c^2x^3}-\frac{b^2\sqrt{c+dx^2}}{x}+b^2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

```
[Out] -((b^2*Sqrt[c + d*x^2])/x) - (a^2*(c + d*x^2)^(3/2))/(5*c*x^5) - (2*a*(5*b*
c - a*d)*(c + d*x^2)^(3/2))/(15*c^2*x^3) + b^2*Sqrt[d]*ArcTanh[(Sqrt[d]*x)/
Sqrt[c + d*x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx &= -\frac{a^2(c+dx^2)^{3/2}}{5cx^5}+\frac{\int \frac{(2a(5bc-ad)+5b^2cx^2)\sqrt{c+dx^2}}{x^4}\,dx}{5c}\\
&= -\frac{a^2(c+dx^2)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+b^2\int \frac{\sqrt{c+dx^2}}{x^2}\,dx\\
&= -\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+\left(b^2d\right)\int \frac{1}{\sqrt{c+\ldots}}\\
&= -\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+\left(b^2d\right)\text{Subst}\Big(\int\ldots\\
&= -\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+b^2\sqrt{d}\,\tanh^{-1}\ldots
\end{aligned}
$$

Mathematica [**A**]
time = 0.18, size = 104, normalized size = 1.01

$$-\frac{\sqrt{c+dx^2}\left(15b^2c^2x^4+10abcx^2(c+dx^2)+a^2(3c^2+cdx^2-2d^2x^4)\right)}{15c^2x^5}-b^2\sqrt{d}\,\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

```
[Out] -1/15*(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c + d*x^2) + a^2*(3*
c^2 + c*d*x^2 - 2*d^2*x^4)))/(c^2*x^5) - b^2*Sqrt[d]*Log[-(Sqrt[d]*x) + Sqr
t[c + d*x^2]]
```

Maple [A]
time = 0.10, size = 124, normalized size = 1.20

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-2 a^{2} d^{2} x^{4}+10 a b c d\,x^{4}+15 b^{2} c^{2} x^{4}+a^{2} c d\,x^{2}+10 a b\,c^{2} x^{2}+3 a^{2} c^{2}\right)}{15 x^{5} c^{2}}+b^{2} \sqrt{d}\,\ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)$
default	$a^{2} \left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{5 c\,x^{5}}+\frac{2 d \left(d\,x^{2}+c\right)^{\frac{3}{2}}}{15 c^{2} x^{3}}\right)-\frac{2 a b \left(d\,x^{2}+c\right)^{\frac{3}{2}}}{3 c\,x^{3}}+b^{2} \left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{c x}+\frac{2 d \left(\frac{x \sqrt{d\,x^{2}+c}}{2}+\frac{c \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}}\right.}{2 \sqrt{d}}\right.}{c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/5/c/x^5*(d*x^2+c)^(3/2)+2/15*d/c^2/x^3*(d*x^2+c)^(3/2))-2/3*a*b/c/x
^3*(d*x^2+c)^(3/2)+b^2*(-1/c/x*(d*x^2+c)^(3/2)+2*d/c*(1/2*x*(d*x^2+c)^(1/2)
+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))
```

Maxima [A]
time = 0.30, size = 94, normalized size = 0.91

$$b^{2} \sqrt{d} \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{\sqrt{dx^{2} + c}\, b^{2}}{x} - \frac{2\,{\left(dx^{2} + c\right)}^{\frac{3}{2}} ab}{3\,cx^{3}} + \frac{2\,{\left(dx^{2} + c\right)}^{\frac{3}{2}} a^{2} d}{15\,c^{2} x^{3}} - \frac{{\left(dx^{2} + c\right)}^{\frac{3}{2}} a^{2}}{5\,cx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] b^2*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*b^2/x - 2/3*(d*x^2 + c
)^(3/2)*a*b/(c*x^3) + 2/15*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^3) - 1/5*(d*x^2 +
 c)^(3/2)*a^2/(c*x^5)
```

Fricas [A]
time = 1.75, size = 221, normalized size = 2.15

$$\left[\frac{15\,b^{2}c^{2}\sqrt{d}\,x^{5}\log\left(-2\,dx^{2} - 2\,\sqrt{dx^{2} + c}\,\sqrt{d}\,x - c\right) - 2\,{\left({\left(15\,b^{2}c^{2} + 10\,abcd - 2\,a^{2}d^{2}\right)}x^{4} + 3\,a^{2}c^{2} + {\left(10\,abc^{2} + a^{2}cd\right)}x^{2}\right)}\sqrt{dx^{2} + c}}{30\,c^{2}x^{5}},\ -\frac{15\,b^{2}c^{2}\sqrt{-d}\,x^{5}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2} + c}}\right) + {\left({\left(15\,b^{2}c^{2} + 10\,abcd - 2\,a^{2}d^{2}\right)}x^{4} + 3\,a^{2}c^{2} + {\left(10\,abc^{2} + a^{2}cd\right)}x^{2}\right)}\sqrt{dx^{2} + c}}{15\,c^{2}x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^2*c^2*sqrt(d)*x^5*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) - 2*((15*b^2*c^2 + 10*a*b*c*d - 2*a^2*d^2)*x^4 + 3*a^2*c^2 + (10*a*b*c^2
 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^5), -1/15*(15*b^2*c^2*sqrt(-d)*x^5
```

```
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((15*b^2*c^2 + 10*a*b*c*d - 2*a^2*d^2
)*x^4 + 3*a^2*c^2 + (10*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(92) = 184$.

time = 2.13, size = 199, normalized size = 1.93

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15cx^2}+\frac{2a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^2}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c}-\frac{b^2\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}}+b^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{b^2dx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**6,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - a**2*d**(3/2)*sqrt(c/(d*x**2)
 + 1)/(15*c*x**2) + 2*a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2) - 2*a*b*
sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)
/(3*c) - b**2*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + b**2*sqrt(d)*asinh(sqrt(d)*x
/sqrt(c)) - b**2*d*x/(sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(87) = 174$.

time = 1.20, size = 403, normalized size = 3.91

$$-\frac{1}{2}b^2\sqrt{d}\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)+\frac{2\left(15\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8b^2c\sqrt{d}+30\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8abd^{\frac{3}{2}}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6b^2c^2\sqrt{d}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6abcd^{\frac{3}{2}}+30\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6a^2d^{\frac{5}{2}}+90\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b^2c^3\sqrt{d}+40\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc^2d^{\frac{3}{2}}+10\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2cd^{\frac{5}{2}}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c^4\sqrt{d}-20\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^3d^{\frac{3}{2}}+10\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2c^2d^{\frac{5}{2}}+15\,b^2c^5\sqrt{d}+10\,abc^4d^{\frac{3}{2}}-2\,a^2c^3d^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*b^2*sqrt(d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*b^2*c*sqrt(d) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*
b*d^(3/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^2*sqrt(d) - 60*(sqrt(d
)*x - sqrt(d*x^2 + c))^6*a*b*c*d^(3/2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*a^2*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^3*sqrt(d) + 40*(sqr
t(d)*x - sqrt(d*x^2 + c))^4*a*b*c^2*d^(3/2) + 10*(sqrt(d)*x - sqrt(d*x^2 +
c))^4*a^2*c*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^4*sqrt(d) -
20*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^3*d^(3/2) + 10*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*a^2*c^2*d^(5/2) + 15*b^2*c^5*sqrt(d) + 10*a*b*c^4*d^(3/2) - 2*
a^2*c^3*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^6,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^6, x)
```

3.610 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^8} \, dx$

Optimal. Leaf size=99

$$-\frac{a^2(c+dx^2)^{3/2}}{7cx^7}-\frac{2a(7bc-2ad)\,(c+dx^2)^{3/2}}{35c^2x^5}-\frac{(35b^2c^2-4ad(7bc-2ad))\,(c+dx^2)^{3/2}}{105c^3x^3}$$

```
[Out] -1/7*a^2*(d*x^2+c)^(3/2)/c/x^7-2/35*a*(-2*a*d+7*b*c)*(d*x^2+c)^(3/2)/c^2/x^
5-1/105*(35*b^2*c^2-4*a*d*(-2*a*d+7*b*c))*(d*x^2+c)^(3/2)/c^3/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {473, 464, 270}

$$-\frac{(c+dx^2)^{3/2}\,(8a^2d^2-28abcd+35b^2c^2)}{105c^3x^3}-\frac{a^2(c+dx^2)^{3/2}}{7cx^7}-\frac{2a(c+dx^2)^{3/2}\,(7bc-2ad)}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

```
[Out] -1/7*(a^2*(c + d*x^2)^(3/2))/(c*x^7) - (2*a*(7*b*c - 2*a*d)*(c + d*x^2)^(3/
2))/(35*c^2*x^5) - ((35*b^2*c^2 - 28*a*b*c*d + 8*a^2*d^2)*(c + d*x^2)^(3/2)
)/(105*c^3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
```

```
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx = -\frac{a^2(c+dx^2)^{3/2}}{7cx^7} + \frac{\int \frac{(2a(7bc-2ad)+7b^2cx^2)\sqrt{c+dx^2}}{x^6}\,dx}{7c}$$
$$= -\frac{a^2(c+dx^2)^{3/2}}{7cx^7} - \frac{2a(7bc-2ad)(c+dx^2)^{3/2}}{35c^2x^5} - \frac{1}{35}\left(-35b^2+\frac{4ad(7bc-2ad)}{c^2}\right.$$
$$= -\frac{a^2(c+dx^2)^{3/2}}{7cx^7} - \frac{2a(7bc-2ad)(c+dx^2)^{3/2}}{35c^2x^5} - \frac{\left(35b^2-\frac{4ad(7bc-2ad)}{c^2}\right)(c+dx}{105cx^3}$$

Mathematica [A]
time = 0.16, size = 76, normalized size = 0.77

$$-\frac{(c+dx^2)^{3/2}(35b^2c^2x^4+14abcx^2(3c-2dx^2)+a^2(15c^2-12cdx^2+8d^2x^4))}{105c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
[Out] -1/105*((c + d*x^2)^(3/2)*(35*b^2*c^2*x^4 + 14*a*b*c*x^2*(3*c - 2*d*x^2) +
a^2*(15*c^2 - 12*c*d*x^2 + 8*d^2*x^4)))/(c^3*x^7)
```

Maple [A]
time = 0.10, size = 126, normalized size = 1.27

method	result	size
gosper	$-\frac{(dx^2+c)^{\frac{3}{2}}(8a^2d^2x^4-28abcdx^4+35b^2c^2x^4-12a^2cdx^2+42abc^2x^2+15a^2c^2)}{105x^7c^3}$	78
trager	$-\frac{(8a^2d^3x^6-28abcd^2x^6+35b^2c^2dx^6-4a^2cd^2x^4+14abc^2dx^4+35b^2c^3x^4+3a^2c^2dx^2+42abc^3x^2+15a^2c^3)\sqrt{dx^2+c}}{105x^7c^3}$	117
risch	$-\frac{(8a^2d^3x^6-28abcd^2x^6+35b^2c^2dx^6-4a^2cd^2x^4+14abc^2dx^4+35b^2c^3x^4+3a^2c^2dx^2+42abc^3x^2+15a^2c^3)\sqrt{dx^2+c}}{105x^7c^3}$	117
default	$2ab\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{5cx^5}+\frac{2d(dx^2+c)^{\frac{3}{2}}}{15c^2x^3}\right)+a^2\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{7cx^7}-\frac{4d\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{5cx^5}+\frac{2d(dx^2+c)^{\frac{3}{2}}}{15c^2x^3}\right)}{7c}\right)-\frac{b^2(dx^2+c)^{\frac{3}{2}}}{3cx^3}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b*(-1/5/c/x^5*(d*x^2+c)^(3/2)+2/15*d/c^2/x^3*(d*x^2+c)^(3/2))+a^2*(-1/7
/c/x^7*(d*x^2+c)^(3/2)-4/7*d/c*(-1/5/c/x^5*(d*x^2+c)^(3/2)+2/15*d/c^2/x^3*(
d*x^2+c)^(3/2)))-1/3*b^2/c/x^3*(d*x^2+c)^(3/2)
```

Maxima [A]
time = 0.30, size = 124, normalized size = 1.25

$$-\frac{(dx^2+c)^{\frac{3}{2}}b^2}{3\,cx^3}+\frac{4\,(dx^2+c)^{\frac{3}{2}}abd}{15\,c^2x^3}-\frac{8\,(dx^2+c)^{\frac{3}{2}}a^2d^2}{105\,c^3x^3}-\frac{2\,(dx^2+c)^{\frac{3}{2}}ab}{5\,cx^5}+\frac{4\,(dx^2+c)^{\frac{3}{2}}a^2d}{35\,c^2x^5}-\frac{(dx^2+c)^{\frac{3}{2}}a^2}{7\,cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/3*(d*x^2 + c)^(3/2)*b^2/(c*x^3) + 4/15*(d*x^2 + c)^(3/2)*a*b*d/(c^2*x^3)
 - 8/105*(d*x^2 + c)^(3/2)*a^2*d^2/(c^3*x^3) - 2/5*(d*x^2 + c)^(3/2)*a*b/(c
*x^5) + 4/35*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^5) - 1/7*(d*x^2 + c)^(3/2)*a^2/
(c*x^7)
```

Fricas [A]
time = 1.77, size = 107, normalized size = 1.08

$$-\frac{((35\,b^2c^2d-28\,abcd^2+8\,a^2d^3)x^6+15\,a^2c^3+(35\,b^2c^3+14\,abc^2d-4\,a^2cd^2)x^4+3\,(14\,abc^3+a^2c^2d)x^2)\sqrt{dx^2+c}}{105\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*((35*b^2*c^2*d - 28*a*b*c*d^2 + 8*a^2*d^3)*x^6 + 15*a^2*c^3 + (35*b^
2*c^3 + 14*a*b*c^2*d - 4*a^2*c*d^2)*x^4 + 3*(14*a*b*c^3 + a^2*c^2*d)*x^2)*s
qrt(d*x^2 + c)/(c^3*x^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(95) = 190$.
time = 2.12, size = 510, normalized size = 5.15

$$-\frac{15a^2c^5d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{33a^2c^4d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{17a^2c^3d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{3a^2c^2d^{\frac{15}{2}}x^6\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{12a^2cd^{\frac{17}{2}}x^8\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{8a^2d^{\frac{19}{2}}x^{10}\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15cx^2}+\frac{4abd^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^2}-\frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{b^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**8,x)
```

```
[Out] -15*a**2*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*
d**5*x**8 + 105*c**3*d**6*x**10) - 33*a**2*c**4*d**(11/2)*x**2*sqrt(c/(d*x*
*2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) -
17*a**2*c**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*
c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 3*a**2*c**2*d**(15/2)*x**6*sqrt(c/(
```

```
d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10
) - 12*a**2*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210
*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 8*a**2*d**(19/2)*x**10*sqrt(c/(d*x
**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) -
 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - 2*a*b*d**(3/2)*sqrt(c/(d*x**
2) + 1)/(15*c*x**2) + 4*a*b*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2) - b**2*
sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - b**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/
(3*c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(87) = 174.

time = 1.28, size = 490, normalized size = 4.95

$$\frac{2\left(105\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2d^{\frac{3}{2}}-420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^2cd^{\frac{3}{2}}+420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}abd^{\frac{5}{2}}+665\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}b^2c^2d^{\frac{3}{2}}-700\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}abcd^{\frac{5}{2}}+560\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}a^2d^{\frac{7}{2}}-560\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}b^2c^3d^{\frac{3}{2}}+280\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}abc^2d^{\frac{5}{2}}+280\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}a^2cd^{\frac{7}{2}}+315\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}b^2c^4d^{\frac{3}{2}}-168\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}abc^3d^{\frac{5}{2}}+168\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}a^2c^2d^{\frac{7}{2}}-140\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}b^2c^5d^{\frac{3}{2}}+196\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}abc^4d^{\frac{5}{2}}-56\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}a^2c^3d^{\frac{7}{2}}+35b^2c^6d^{\frac{3}{2}}-28abc^5d^{\frac{5}{2}}+8a^2c^4d^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(d)*x - sqrt(d*x^2 + c))^12*b^2*d^(3/2) - 420*(sqrt(d)*x -
sqrt(d*x^2 + c))^10*b^2*c*d^(3/2) + 420*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*
b*d^(5/2) + 665*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*d^(3/2) - 700*(sqrt
(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(5/2) + 560*(sqrt(d)*x - sqrt(d*x^2 + c)
)^8*a^2*d^(7/2) - 560*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*d^(3/2) + 280
*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(5/2) + 280*(sqrt(d)*x - sqrt(d*
x^2 + c))^6*a^2*c*d^(7/2) + 315*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*d^(
3/2) - 168*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(5/2) + 168*(sqrt(d)*x
 - sqrt(d*x^2 + c))^4*a^2*c^2*d^(7/2) - 140*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*b^2*c^5*d^(3/2) + 196*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(5/2) - 56
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(7/2) + 35*b^2*c^6*d^(3/2) - 28*
a*b*c^5*d^(5/2) + 8*a^2*c^4*d^(7/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^
7
```

Mupad [B]

time = 0.78, size = 181, normalized size = 1.83

$$\frac{4a^2d^2\sqrt{dx^2+c}}{105c^2x^3}-\frac{b^2\sqrt{dx^2+c}}{3x^3}-\frac{2ab\sqrt{dx^2+c}}{5x^5}-\frac{a^2\sqrt{dx^2+c}}{7x^7}-\frac{8a^2d^3\sqrt{dx^2+c}}{105c^3x}-\frac{a^2d\sqrt{dx^2+c}}{35cx^5}-\frac{b^2d\sqrt{dx^2+c}}{3cx}+\frac{4abd^2\sqrt{dx^2+c}}{15c^2x}-\frac{2abd\sqrt{dx^2+c}}{15cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^8,x)
```

```
[Out] (4*a^2*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^3) - (b^2*(c + d*x^2)^(1/2))/(3*x^
3) - (2*a*b*(c + d*x^2)^(1/2))/(5*x^5) - (a^2*(c + d*x^2)^(1/2))/(7*x^7) -
(8*a^2*d^3*(c + d*x^2)^(1/2))/(105*c^3*x) - (a^2*d*(c + d*x^2)^(1/2))/(35*c
*x^5) - (b^2*d*(c + d*x^2)^(1/2))/(3*c*x) + (4*a*b*d^2*(c + d*x^2)^(1/2))/(
15*c^2*x) - (2*a*b*d*(c + d*x^2)^(1/2))/(15*c*x^3)
```

3.611 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{10}} \, dx$

Optimal. Leaf size=143

$$-\frac{a^2(c+dx^2)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\,(c+dx^2)^{3/2}}{21c^2x^7}-\frac{(21b^2c^2-8ad(3bc-ad))\,(c+dx^2)^{3/2}}{105c^3x^5}+\frac{2d(21b^2c^2-8ad(3bc-}{315c^4}$$

[Out] -1/9*a^2*(d*x^2+c)^(3/2)/c/x^9-2/21*a*(-a*d+3*b*c)*(d*x^2+c)^(3/2)/c^2/x^7-1/105*(21*b^2*c^2-8*a*d*(-a*d+3*b*c))*(d*x^2+c)^(3/2)/c^3/x^5+2/315*d*(21*b^2*c^2-8*a*d*(-a*d+3*b*c))*(d*x^2+c)^(3/2)/c^4/x^3

Rubi [A]
time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {473, 464, 277, 270}

$$-\frac{(c+dx^2)^{3/2}(8a^2d^2-24abcd+21b^2c^2)}{105c^3x^5}-\frac{a^2(c+dx^2)^{3/2}}{9cx^9}+\frac{2d(c+dx^2)^{3/2}(21b^2c^2-8ad(3bc-ad))}{315c^4x^3}-\frac{2a(c+dx^2)^{3/2}(3bc-ad)}{21c^2x^7}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]

[Out] -1/9*(a^2*(c + d*x^2)^(3/2))/(c*x^9) - (2*a*(3*b*c - a*d)*(c + d*x^2)^(3/2))/(21*c^2*x^7) - ((21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*(c + d*x^2)^(3/2))/(105*c^3*x^5) + (2*d*(21*b^2*c^2 - 8*a*d*(3*b*c - a*d))*(c + d*x^2)^(3/2))/(315*c^4*x^3)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx &= -\frac{a^2(c+dx^2)^{3/2}}{9cx^9}+\frac{\int \frac{(6a(3bc-ad)+9b^2cx^2)\sqrt{c+dx^2}}{x^8}\,dx}{9c}\\ &= -\frac{a^2(c+dx^2)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\,(c+dx^2)^{3/2}}{21c^2x^7}-\frac{1}{21}\left(-21b^2+\frac{8ad(3bc-ad)}{c^2}\right)\\ &= -\frac{a^2(c+dx^2)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\,(c+dx^2)^{3/2}}{21c^2x^7}-\frac{\left(21b^2-\frac{8ad(3bc-ad)}{c^2}\right)(c+dx^2)^{3}}{105cx^5}\\ &= -\frac{a^2(c+dx^2)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\,(c+dx^2)^{3/2}}{21c^2x^7}-\frac{\left(21b^2-\frac{8ad(3bc-ad)}{c^2}\right)(c+dx^2)^{3}}{105cx^5}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 108, normalized size = 0.76

$$-\frac{(c+dx^2)^{3/2}\left(21b^2c^2x^4(3c-2dx^2)+6abcx^2(15c^2-12cdx^2+8d^2x^4)+a^2(35c^3-30c^2dx^2+24cd^2x^4-16d^3x^6)\right)}{315c^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

```
[Out] -1/315*((c + d*x^2)^(3/2)*(21*b^2*c^2*x^4*(3*c - 2*d*x^2) + 6*a*b*c*x^2*(15
*c^2 - 12*c*d*x^2 + 8*d^2*x^4) + a^2*(35*c^3 - 30*c^2*d*x^2 + 24*c*d^2*x^4
- 16*d^3*x^6)))/(c^4*x^9)
```

Maple [A]
time = 0.11, size = 194, normalized size = 1.36

method	result

gosper	$-\frac{(dx^2+c)^{\frac{3}{2}}(-16a^2d^3x^6+48abcd^2x^6-42b^2c^2dx^6+24a^2cd^2x^4-72abc^2dx^4+63b^2c^3x^4-30a^2c^2dx^2+90abc^3x^2+35a^2c^3)}{315x^9c^4}$
trager	$-\frac{(-16a^2d^4x^8+48abcd^3x^8-42b^2c^2d^2x^8+8a^2cd^3x^6-24abc^2d^2x^6+21b^2c^3dx^6-6a^2c^2d^2x^4+18abc^3dx^4+63b^2c^4x^4+5a^2c^3dx^2+90}{315x^9c^4}$
risch	$-\frac{(-16a^2d^4x^8+48abcd^3x^8-42b^2c^2d^2x^8+8a^2cd^3x^6-24abc^2d^2x^6+21b^2c^3dx^6-6a^2c^2d^2x^4+18abc^3dx^4+63b^2c^4x^4+5a^2c^3dx^2+90}{315x^9c^4}$
default	$b^2\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{5cx^5}+\frac{2d(dx^2+c)^{\frac{3}{2}}}{15c^2x^3}\right)+a^2\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{9cx^9}-\frac{2d\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{7cx^7}-\frac{4d\left(-\frac{(dx^2+c)^{\frac{3}{2}}}{5cx^5}+\frac{2d(dx^2+c)^{\frac{3}{2}}}{15c^2x^3}\right)}{7c}\right)}{3c}\right)+2a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/5/c/x^5*(d*x^2+c)^(3/2)+2/15*d/c^2/x^3*(d*x^2+c)^(3/2))+a^2*(-1/9/c
/x^9*(d*x^2+c)^(3/2)-2/3*d/c*(-1/7/c/x^7*(d*x^2+c)^(3/2)-4/7*d/c*(-1/5/c/x^
5*(d*x^2+c)^(3/2)+2/15*d/c^2/x^3*(d*x^2+c)^(3/2))))+2*a*b*(-1/7/c/x^7*(d*x^
2+c)^(3/2)-4/7*d/c*(-1/5/c/x^5*(d*x^2+c)^(3/2)+2/15*d/c^2/x^3*(d*x^2+c)^(3/
2)))
```

Maxima [A]
time = 0.29, size = 190, normalized size = 1.33

$$\frac{2\,(dx^2+c)^{\frac{3}{2}}b^2d}{15\,c^2x^3}-\frac{16\,(dx^2+c)^{\frac{3}{2}}abd^2}{105\,c^3x^3}+\frac{16\,(dx^2+c)^{\frac{3}{2}}a^2d^3}{315\,c^4x^3}-\frac{(dx^2+c)^{\frac{3}{2}}b^2}{5\,cx^5}+\frac{8\,(dx^2+c)^{\frac{3}{2}}abd}{35\,c^2x^5}-\frac{8\,(dx^2+c)^{\frac{3}{2}}a^2d^2}{105\,c^3x^5}-\frac{2\,(dx^2+c)^{\frac{3}{2}}ab}{7\,cx^7}+\frac{2\,(dx^2+c)^{\frac{3}{2}}a^2d}{21\,c^2x^7}-\frac{(dx^2+c)^{\frac{3}{2}}a^2}{9\,cx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] 2/15*(d*x^2 + c)^(3/2)*b^2*d/(c^2*x^3) - 16/105*(d*x^2 + c)^(3/2)*a*b*d^2/(
c^3*x^3) + 16/315*(d*x^2 + c)^(3/2)*a^2*d^3/(c^4*x^3) - 1/5*(d*x^2 + c)^(3/
2)*b^2/(c*x^5) + 8/35*(d*x^2 + c)^(3/2)*a*b*d/(c^2*x^5) - 8/105*(d*x^2 + c)
^(3/2)*a^2*d^2/(c^3*x^5) - 2/7*(d*x^2 + c)^(3/2)*a*b/(c*x^7) + 2/21*(d*x^2
+ c)^(3/2)*a^2*d/(c^2*x^7) - 1/9*(d*x^2 + c)^(3/2)*a^2/(c*x^9)
```

Fricas [A]
time = 2.01, size = 147, normalized size = 1.03

$$\frac{(2\,(21\,b^2c^2d^2-24\,abcd^3+8\,a^2d^4)x^8-(21\,b^2c^3d-24\,abc^2d^2+8\,a^2cd^3)x^6-35\,a^2c^4-3\,(21\,b^2c^4+6\,abc^3d-2\,a^2c^2d^2)x^4-5\,(18\,abc^4+a^2c^3d)x^2)\sqrt{dx^2+c}}{315\,c^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(21*b^2*c^2*d^2 - 24*a*b*c*d^3 + 8*a^2*d^4)*x^8 - (21*b^2*c^3*d -
24*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^6 - 35*a^2*c^4 - 3*(21*b^2*c^4 + 6*a*b*c^3*
d - 2*a^2*c^2*d^2)*x^4 - 5*(18*a*b*c^4 + a^2*c^3*d)*x^2)*sqrt(d*x^2 + c)/(c
^4*x^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(134) = 268$.
time = 2.70, size = 1061, normalized size = 7.42

$$- \frac{35 a^{2} c^{7} d^{\frac{19}{2}} \sqrt{\frac{c}{d x^{2}} + 1}}{315 c^{7} d^{9} x^{8} + 945 c^{6} d^{10} x^{10} + 945 c^{5} d^{11} x^{12} + 315 c^{4} d^{12} x^{14}} - \frac{110 a^{2} c^{6} d^{\frac{21}{2}} x^{2} \sqrt{\frac{c}{d x^{2}} + 1}}{315 c^{7} d^{9} x^{8} + 945 c^{6} d^{10} x^{10} + 945 c^{5} d^{11} x^{12} + 315 c^{4} d^{12} x^{14}} - \frac{114 a^{2} c^{5} d^{\frac{23}{2}} x^{4} \sqrt{\frac{c}{d x^{2}} + 1}}{315 c^{7} d^{9} x^{8} + 945 c^{6} d^{10} x^{10} + 945 c^{5} d^{11} x^{12} + 315 c^{4} d^{12} x^{14}} - \frac{40 a^{2} c^{4} d^{\frac{25}{2}} x^{6} \sqrt{\frac{c}{d x^{2}} + 1}}{315 c^{7} d^{9} x^{8} + 945 c^{6} d^{10} x^{10} + 945 c^{5} d^{11} x^{12} + 315 c^{4} d^{12} x^{14}} + \frac{5 a^{2} c^{3} d^{\frac{27}{2}} x^{8} \sqrt{\frac{c}{d x^{2}} + 1}}{315 c^{7} d^{9} x^{8} + 945 c^{6} d^{10} x^{10} + 945 c^{5} d^{11} x^{12} + 315 c^{4} d^{12} x^{14}} + \frac{30 a^{2} c^{2} d^{\frac{29}{2}} x^{10} \sqrt{\frac{c}{d x^{2}} + 1}}{315 c^{7} d^{9} x^{8} + 945 c^{6} d^{10} x^{10} + 945 c^{5} d^{11} x^{12} + 315 c^{4} d^{12} x^{14}} + \frac{40 a^{2} c d^{\frac{31}{2}} x^{12} \sqrt{\frac{c}{d x^{2}} + 1}}{315 c^{7} d^{9} x^{8} + 945 c^{6} d^{10} x^{10} + 945 c^{5} d^{11} x^{12} + 315 c^{4} d^{12} x^{14}} + \frac{16 a^{2} d^{\frac{33}{2}} x^{14} \sqrt{\frac{c}{d x^{2}} + 1}}{315 c^{7} d^{9} x^{8} + 945 c^{6} d^{10} x^{10} + 945 c^{5} d^{11} x^{12} + 315 c^{4} d^{12} x^{14}} - \frac{30 a b c^{5} d^{\frac{9}{2}} \sqrt{\frac{c}{d x^{2}} + 1}}{105 c^{5} d^{4} x^{6} + 210 c^{4} d^{5} x^{8} + 105 c^{3} d^{6} x^{10}} - \frac{66 a b c^{4} d^{\frac{11}{2}} x^{2} \sqrt{\frac{c}{d x^{2}} + 1}}{105 c^{5} d^{4} x^{6} + 210 c^{4} d^{5} x^{8} + 105 c^{3} d^{6} x^{10}} - \frac{34 a b c^{3} d^{\frac{13}{2}} x^{4} \sqrt{\frac{c}{d x^{2}} + 1}}{105 c^{5} d^{4} x^{6} + 210 c^{4} d^{5} x^{8} + 105 c^{3} d^{6} x^{10}} - \frac{6 a b c^{2} d^{\frac{15}{2}} x^{6} \sqrt{\frac{c}{d x^{2}} + 1}}{105 c^{5} d^{4} x^{6} + 210 c^{4} d^{5} x^{8} + 105 c^{3} d^{6} x^{10}} - \frac{24 a b c d^{\frac{17}{2}} x^{8} \sqrt{\frac{c}{d x^{2}} + 1}}{105 c^{5} d^{4} x^{6} + 210 c^{4} d^{5} x^{8} + 105 c^{3} d^{6} x^{10}} - \frac{16 a b d^{\frac{19}{2}} x^{10} \sqrt{\frac{c}{d x^{2}} + 1}}{105 c^{5} d^{4} x^{6} + 210 c^{4} d^{5} x^{8} + 105 c^{3} d^{6} x^{10}} - \frac{b^{2} \sqrt{d} \sqrt{\frac{c}{d x^{2}} + 1}}{5 x^{4}} - \frac{b^{2} d^{\frac{3}{2}} \sqrt{\frac{c}{d x^{2}} + 1}}{15 c x^{2}} + \frac{2 b^{2} d^{\frac{5}{2}} \sqrt{\frac{c}{d x^{2}} + 1}}{15 c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**10,x)
```

```
[Out] -35*a**2*c**7*d**(19/2)*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6
*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 110*a**2*c**6
*d**(21/2)*x**2*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x
**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 114*a**2*c**5*d**(23/
2)*x**4*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 9
45*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 40*a**2*c**4*d**(25/2)*x**6*s
qrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d
**11*x**12 + 315*c**4*d**12*x**14) + 5*a**2*c**3*d**(27/2)*x**8*sqrt(c/(d*x
**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12
 + 315*c**4*d**12*x**14) + 30*a**2*c**2*d**(29/2)*x**10*sqrt(c/(d*x**2) + 1
)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c
**4*d**12*x**14) + 40*a**2*c*d**(31/2)*x**12*sqrt(c/(d*x**2) + 1)/(315*c**7
*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x
**14) + 16*a**2*d**(33/2)*x**14*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 +
945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 30*a*
b*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x*
*8 + 105*c**3*d**6*x**10) - 66*a*b*c**4*d**(11/2)*x**2*sqrt(c/(d*x**2) + 1)
/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 34*a*b*c
**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5
*x**8 + 105*c**3*d**6*x**10) - 6*a*b*c**2*d**(15/2)*x**6*sqrt(c/(d*x**2) +
1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 24*a*b
*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*
x**8 + 105*c**3*d**6*x**10) - 16*a*b*d**(19/2)*x**10*sqrt(c/(d*x**2) + 1)/(
105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - b**2*sqrt(
d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - b**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/(15*c
*x**2) + 2*b**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(127) = 254$.
time = 1.34, size = 579, normalized size = 4.05

$$\frac{4 \left(315 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{14} b^{2} d^{\frac{9}{2}} - 1155 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{12} b^{2} c d^{\frac{9}{2}} + 1680 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{12} a b d^{\frac{11}{2}} + 1575 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{10} b^{2} c^{2} d^{\frac{9}{2}} - 2520 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{10} a b c d^{\frac{11}{2}} + 2520 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{10} a^{2} d^{\frac{13}{2}} - 1071 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{8} b^{2} c^{3} d^{\frac{9}{2}} + 504 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{8} a b c^{2} d^{\frac{11}{2}} + 1512 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{8} a^{2} c d^{\frac{13}{2}} + 609 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{6} b^{2} c^{4} d^{\frac{9}{2}} - 336 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{6} a b c^{3} d^{\frac{11}{2}} + 672 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{6} a^{2} c^{2} d^{\frac{13}{2}} - 441 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{4} b^{2} c^{5} d^{\frac{9}{2}} + 864 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{4} a b c^{4} d^{\frac{11}{2}} - 288 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{4} a^{2} c^{3} d^{\frac{13}{2}} + 189 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{2} b^{2} c^{6} d^{\frac{9}{2}} - 216 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{2} a b c^{5} d^{\frac{11}{2}} + 72 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{2} a^{2} c^{4} d^{\frac{13}{2}} - 21 b^{2} c^{7} d^{\frac{9}{2}} + 24 a b c^{6} d^{\frac{11}{2}} - 8 a^{2} c^{5} d^{\frac{13}{2}}\right)}{315 \left({\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{2} - c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(d)*x - sqrt(d*x^2 + c))^14*b^2*d^(5/2) - 1155*(sqrt(d)*x -
 sqrt(d*x^2 + c))^12*b^2*c*d^(5/2) + 1680*(sqrt(d)*x - sqrt(d*x^2 + c))^12*
a*b*d^(7/2) + 1575*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^2*c^2*d^(5/2) - 2520*
(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b*c*d^(7/2) + 2520*(sqrt(d)*x - sqrt(d*x
^2 + c))^10*a^2*d^(9/2) - 1071*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^3*d^(5
/2) + 504*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^2*d^(7/2) + 1512*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*a^2*c*d^(9/2) + 609*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b
^2*c^4*d^(5/2) - 336*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3*d^(7/2) + 672*
(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^2*d^(9/2) - 441*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*b^2*c^5*d^(5/2) + 864*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^4*d^
(7/2) - 288*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^3*d^(9/2) + 189*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*b^2*c^6*d^(5/2) - 216*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*a*b*c^5*d^(7/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^4*d^(9/2) - 21
*b^2*c^7*d^(5/2) + 24*a*b*c^6*d^(7/2) - 8*a^2*c^5*d^(9/2))/((sqrt(d)*x - sq
rt(d*x^2 + c))^2 - c)^9
```

Mupad [B]
time = 1.05, size = 249, normalized size = 1.74

$$\frac{2\,a^2\,d^2\,\sqrt{d\,x^2+c}}{105\,c^2\,x^5}-\frac{b^2\,\sqrt{d\,x^2+c}}{5\,x^5}-\frac{2\,a\,b\,\sqrt{d\,x^2+c}}{7\,x^7}-\frac{a^2\,\sqrt{d\,x^2+c}}{9\,x^9}-\frac{8\,a^2\,d^3\,\sqrt{d\,x^2+c}}{315\,c^3\,x^3}+\frac{16\,a^2\,d^4\,\sqrt{d\,x^2+c}}{315\,c^4\,x}+\frac{2\,b^2\,d^2\,\sqrt{d\,x^2+c}}{15\,c^2\,x}-\frac{a^2\,d\,\sqrt{d\,x^2+c}}{63\,c\,x^7}-\frac{b^2\,d\,\sqrt{d\,x^2+c}}{15\,c\,x^3}+\frac{8\,a\,b\,d^2\,\sqrt{d\,x^2+c}}{105\,c^2\,x^3}-\frac{16\,a\,b\,d^3\,\sqrt{d\,x^2+c}}{105\,c^3\,x}-\frac{2\,a\,b\,d\,\sqrt{d\,x^2+c}}{35\,c\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^10,x)
```

```
[Out] (2*a^2*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^5) - (b^2*(c + d*x^2)^(1/2))/(5*x^
5) - (2*a*b*(c + d*x^2)^(1/2))/(7*x^7) - (a^2*(c + d*x^2)^(1/2))/(9*x^9) -
(8*a^2*d^3*(c + d*x^2)^(1/2))/(315*c^3*x^3) + (16*a^2*d^4*(c + d*x^2)^(1/2)
)/(315*c^4*x) + (2*b^2*d^2*(c + d*x^2)^(1/2))/(15*c^2*x) - (a^2*d*(c + d*x^
2)^(1/2))/(63*c*x^7) - (b^2*d*(c + d*x^2)^(1/2))/(15*c*x^3) + (8*a*b*d^2*(c
 + d*x^2)^(1/2))/(105*c^2*x^3) - (16*a*b*d^3*(c + d*x^2)^(1/2))/(105*c^3*x)
 - (2*a*b*d*(c + d*x^2)^(1/2))/(35*c*x^5)
```

3.612 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{12}} \, dx$

Optimal. Leaf size=189

$$-\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}}-\frac{2a(11bc-4ad)\,(c+dx^2)^{3/2}}{99c^2x^9}-\frac{(33b^2c^2-4ad(11bc-4ad))\,(c+dx^2)^{3/2}}{231c^3x^7}+\frac{4d(33b^2c^2-4ad(1}{11}$$

```
[Out] -1/11*a^2*(d*x^2+c)^(3/2)/c/x^11-2/99*a*(-4*a*d+11*b*c)*(d*x^2+c)^(3/2)/c^2
/x^9-1/231*(33*b^2*c^2-4*a*d*(-4*a*d+11*b*c))*(d*x^2+c)^(3/2)/c^3/x^7+4/115
5*d*(33*b^2*c^2-4*a*d*(-4*a*d+11*b*c))*(d*x^2+c)^(3/2)/c^4/x^5-8/3465*d^2*(
33*b^2*c^2-4*a*d*(-4*a*d+11*b*c))*(d*x^2+c)^(3/2)/c^5/x^3
```

Rubi [A]
time = 0.11, antiderivative size = 190, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {473, 464, 277, 270}

$$-\frac{(c+dx^2)^{3/2}\,(16a^2d^2-44abcd+33b^2c^2)}{231c^3x^7}-\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}}-\frac{8d^2(c+dx^2)^{3/2}\,(33b^2c^2-4ad(11bc-4ad))}{3465c^5x^3}+\frac{4d(c+dx^2)^{3/2}\,(33b^2c^2-4ad(11bc-4ad))}{1155c^4x^5}-\frac{2a(c+dx^2)^{3/2}\,(11bc-4ad)}{99c^2x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

```
[Out] -1/11*(a^2*(c + d*x^2)^(3/2))/(c*x^11) - (2*a*(11*b*c - 4*a*d)*(c + d*x^2)^
(3/2))/(99*c^2*x^9) - ((33*b^2*c^2 - 44*a*b*c*d + 16*a^2*d^2)*(c + d*x^2)^(
3/2))/(231*c^3*x^7) + (4*d*(33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d*x^2
)^(3/2))/(1155*c^4*x^5) - (8*d^2*(33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c +
 d*x^2)^(3/2))/(3465*c^5*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx = -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} + \frac{\int \frac{(2a(11bc-4ad)+11b^2cx^2)\sqrt{c+dx^2}}{x^{10}}\,dx}{11c}$$
$$= -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} - \frac{2a(11bc-4ad)(c+dx^2)^{3/2}}{99c^2x^9} - \frac{1}{33}\left(-33b^2+\frac{4ad(11bc-4\right.$$
$$= -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} - \frac{2a(11bc-4ad)(c+dx^2)^{3/2}}{99c^2x^9} - \frac{\left(33b^2-\frac{4ad(11bc-4ad)}{c^2}\right)(c+}{231cx^7}$$
$$= -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} - \frac{2a(11bc-4ad)(c+dx^2)^{3/2}}{99c^2x^9} - \frac{\left(33b^2-\frac{4ad(11bc-4ad)}{c^2}\right)(c+}{231cx^7}$$
$$= -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} - \frac{2a(11bc-4ad)(c+dx^2)^{3/2}}{99c^2x^9} - \frac{\left(33b^2-\frac{4ad(11bc-4ad)}{c^2}\right)(c+}{231cx^7}$$

Mathematica [A]
time = 0.24, size = 141, normalized size = 0.75

$$-\frac{(c+dx^2)^{3/2}(33b^2c^2x^4(15c^2-12cdx^2+8d^2x^4)+22abcx^2(35c^3-30c^2dx^2+24cd^2x^4-16d^3x^6)+a^2(315c^4-280c^3dx^2+240c^2d^2x^4-192cd^3x^6+128d^4x^8))}{3465c^5x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

```
[Out] -1/3465*((c + d*x^2)^(3/2)*(33*b^2*c^2*x^4*(15*c^2 - 12*c*d*x^2 + 8*d^2*x^4
) + 22*a*b*c*x^2*(35*c^3 - 30*c^2*d*x^2 + 24*c*d^2*x^4 - 16*d^3*x^6) + a^2*
(315*c^4 - 280*c^3*d*x^2 + 240*c^2*d^2*x^4 - 192*c*d^3*x^6 + 128*d^4*x^8)))
/(c^5*x^11)
```

Maple [A]

time = 0.13, size = 266, normalized size = 1.41

method	result
gosper	$-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}\left(128a^{2}d^{4}x^{8}-352abc\,d^{3}x^{8}+264b^{2}c^{2}d^{2}x^{8}-192a^{2}c\,d^{3}x^{6}+528ab\,c^{2}d^{2}x^{6}-396b^{2}c^{3}d\,x^{6}+240a^{2}c^{2}d^{2}x^{4}-660ab\,c^{3}d\,x^{4}+495b^{2}c\right.}{3465x^{11}c^{5}}$
trager	$-\frac{\left(128a^{2}d^{5}x^{10}-352abc\,d^{4}x^{10}+264b^{2}c^{2}d^{3}x^{10}-64a^{2}c\,d^{4}x^{8}+176ab\,c^{2}d^{3}x^{8}-132b^{2}c^{3}d^{2}x^{8}+48a^{2}c^{2}d^{3}x^{6}-132ab\,c^{3}d^{2}x^{6}+99b^{2}c^{4}d\,x^{6}-40\right.}{3465x^{11}c^{5}}$
risch	$-\frac{\left(128a^{2}d^{5}x^{10}-352abc\,d^{4}x^{10}+264b^{2}c^{2}d^{3}x^{10}-64a^{2}c\,d^{4}x^{8}+176ab\,c^{2}d^{3}x^{8}-132b^{2}c^{3}d^{2}x^{8}+48a^{2}c^{2}d^{3}x^{6}-132ab\,c^{3}d^{2}x^{6}+99b^{2}c^{4}d\,x^{6}-40\right.}{3465x^{11}c^{5}}$
default	$a^{2}\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{11c\,x^{11}}-\frac{8d\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{9c\,x^{9}}-\frac{2d\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{7c\,x^{7}}-\frac{4d\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{5c\,x^{5}}+\frac{2d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{15c^{2}x^{3}}\right)}{7c}\right)}{3c}\right)}{11c}\right)+2ab\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{9c\,x^{9}}-\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/11/c/x^11*(d*x^2+c)^(3/2)-8/11*d/c*(-1/9/c/x^9*(d*x^2+c)^(3/2)-2/3*
d/c*(-1/7/c/x^7*(d*x^2+c)^(3/2)-4/7*d/c*(-1/5/c/x^5*(d*x^2+c)^(3/2)+2/15*d/
c^2/x^3*(d*x^2+c)^(3/2)))))+2*a*b*(-1/9/c/x^9*(d*x^2+c)^(3/2)-2/3*d/c*(-1/7
/c/x^7*(d*x^2+c)^(3/2)-4/7*d/c*(-1/5/c/x^5*(d*x^2+c)^(3/2)+2/15*d/c^2/x^3*(
d*x^2+c)^(3/2))))+b^2*(-1/7/c/x^7*(d*x^2+c)^(3/2)-4/7*d/c*(-1/5/c/x^5*(d*x^
2+c)^(3/2)+2/15*d/c^2/x^3*(d*x^2+c)^(3/2)))
```

Maxima [A]

time = 0.31, size = 258, normalized size = 1.37

$$-\frac{8\,(dx^2+c)^{\frac{3}{2}}b^2d^2}{105\,c^3x^3}+\frac{32\,(dx^2+c)^{\frac{3}{2}}abd^3}{315\,c^4x^3}-\frac{128\,(dx^2+c)^{\frac{3}{2}}a^2d^4}{3465\,c^5x^3}+\frac{4\,(dx^2+c)^{\frac{3}{2}}b^2d}{35\,c^2x^5}-\frac{16\,(dx^2+c)^{\frac{3}{2}}abd^2}{105\,c^3x^5}+\frac{64\,(dx^2+c)^{\frac{3}{2}}a^2d^3}{1155\,c^4x^5}-\frac{(dx^2+c)^{\frac{3}{2}}b^2}{7\,cx^7}+\frac{4\,(dx^2+c)^{\frac{3}{2}}abd}{21\,c^2x^7}-\frac{16\,(dx^2+c)^{\frac{3}{2}}a^2d^2}{231\,c^3x^7}-\frac{2\,(dx^2+c)^{\frac{3}{2}}ab}{9\,cx^9}+\frac{8\,(dx^2+c)^{\frac{3}{2}}a^2d}{99\,c^2x^9}-\frac{(dx^2+c)^{\frac{3}{2}}a^2}{11\,cx^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="maxima")
```

```
[Out] -8/105*(d*x^2 + c)^(3/2)*b^2*d^2/(c^3*x^3) + 32/315*(d*x^2 + c)^(3/2)*a*b*d
^3/(c^4*x^3) - 128/3465*(d*x^2 + c)^(3/2)*a^2*d^4/(c^5*x^3) + 4/35*(d*x^2 +
 c)^(3/2)*b^2*d/(c^2*x^5) - 16/105*(d*x^2 + c)^(3/2)*a*b*d^2/(c^3*x^5) + 64
/1155*(d*x^2 + c)^(3/2)*a^2*d^3/(c^4*x^5) - 1/7*(d*x^2 + c)^(3/2)*b^2/(c*x^
7) + 4/21*(d*x^2 + c)^(3/2)*a*b*d/(c^2*x^7) - 16/231*(d*x^2 + c)^(3/2)*a^2*
d^2/(c^3*x^7) - 2/9*(d*x^2 + c)^(3/2)*a*b/(c*x^9) + 8/99*(d*x^2 + c)^(3/2)*
a^2*d/(c^2*x^9) - 1/11*(d*x^2 + c)^(3/2)*a^2/(c*x^11)
```

Fricas [**A**]
time = 1.83, size = 185, normalized size = 0.98

$$-\frac{\left(8\left(33\,b^2c^2d^3-44\,abcd^4+16\,a^2d^5\right)x^{10}-4\left(33\,b^2c^3d^2-44\,abc^2d^3+16\,a^2cd^4\right)x^8+315\,a^2c^5+3\left(33\,b^2c^4d-44\,abc^3d^2+16\,a^2c^2d^3\right)x^6+5\left(99\,b^2c^5+22\,abc^4d-8\,a^2c^3d^2\right)x^4+35\left(22\,abc^5+a^2c^4d\right)x^2\right)\sqrt{dx^2+c}}{3465\,c^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(8*(33*b^2*c^2*d^3 - 44*a*b*c*d^4 + 16*a^2*d^5)*x^10 - 4*(33*b^2*c^
3*d^2 - 44*a*b*c^2*d^3 + 16*a^2*c*d^4)*x^8 + 315*a^2*c^5 + 3*(33*b^2*c^4*d
- 44*a*b*c^3*d^2 + 16*a^2*c^2*d^3)*x^6 + 5*(99*b^2*c^5 + 22*a*b*c^4*d - 8*a
^2*c^3*d^2)*x^4 + 35*(22*a*b*c^5 + a^2*c^4*d)*x^2)*sqrt(d*x^2 + c)/(c^5*x^1
1)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1856 vs. $2(187) = 374$.
time = 3.38, size = 1856, normalized size = 9.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**12,x)
```

```
[Out] -315*a**2*c**9*d**(33/2)*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 1386
0*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465
*c**5*d**20*x**18) - 1295*a**2*c**8*d**(35/2)*x**2*sqrt(c/(d*x**2) + 1)/(34
65*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 138
60*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 1990*a**2*c**7*d**(37/2)*x**
4*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20
790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 13
58*a**2*c**6*d**(39/2)*x**6*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 1
3860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3
465*c**5*d**20*x**18) - 343*a**2*c**5*d**(41/2)*x**8*sqrt(c/(d*x**2) + 1)/(
3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 1
3860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 35*a**2*c**4*d**(43/2)*x**
10*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 2
0790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 2
80*a**2*c**3*d**(45/2)*x**12*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 +
```

```
13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 +
3465*c**5*d**20*x**18) - 560*a**2*c**2*d**(47/2)*x**14*sqrt(c/(d*x**2) + 1)
/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 +
 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 448*a**2*c*d**(49/2)*x**
16*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 2
0790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 1
28*a**2*d**(51/2)*x**18*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860
*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*
c**5*d**20*x**18) - 70*a*b*c**7*d**(19/2)*sqrt(c/(d*x**2) + 1)/(315*c**7*d*
*9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**1
4) - 220*a*b*c**6*d**(21/2)*x**2*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 +
 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 228*
a*b*c**5*d**(23/2)*x**4*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6
*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 80*a*b*c**4*d
**(25/2)*x**6*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**
10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) + 10*a*b*c**3*d**(27/2)*x
**8*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c
**5*d**11*x**12 + 315*c**4*d**12*x**14) + 60*a*b*c**2*d**(29/2)*x**10*sqrt(
c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11
*x**12 + 315*c**4*d**12*x**14) + 80*a*b*c*d**(31/2)*x**12*sqrt(c/(d*x**2) +
 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315
*c**4*d**12*x**14) + 32*a*b*d**(33/2)*x**14*sqrt(c/(d*x**2) + 1)/(315*c**7*
d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x*
*14) - 15*b**2*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210
*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 33*b**2*c**4*d**(11/2)*x**2*sqrt(c
/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**
10) - 17*b**2*c**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6
+ 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 3*b**2*c**2*d**(15/2)*x**6*sq
rt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6
*x**10) - 12*b**2*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6
 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 8*b**2*d**(19/2)*x**10*sqrt(
c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x*
*10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(169) = 338.

time = 1.32, size = 668, normalized size = 3.53

$$\frac{16\left(2310\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{16}b^2d^{\frac{7}{2}}-8085\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}b^2cd^{\frac{7}{2}}+13860\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}abd^{\frac{9}{2}}+9933\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2c^2d^{\frac{7}{2}}-19404\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}abcd^{\frac{9}{2}}+22176\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}a^2d^{\frac{11}{2}}-5313\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^2c^3d^{\frac{7}{2}}+924\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}abc^2d^{\frac{9}{2}}+14784\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}a^2cd^{\frac{11}{2}}+2805\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}b^2c^4d^{\frac{7}{2}}-660\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}abc^3d^{\frac{9}{2}}+5280\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}a^2c^2d^{\frac{11}{2}}-3135\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}b^2c^5d^{\frac{7}{2}}+7260\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}abc^4d^{\frac{9}{2}}-2640\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}a^2c^3d^{\frac{11}{2}}+1815\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}b^2c^6d^{\frac{7}{2}}-2420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}abc^5d^{\frac{9}{2}}+880\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}a^2c^4d^{\frac{11}{2}}-363\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}b^2c^7d^{\frac{7}{2}}+484\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}abc^6d^{\frac{9}{2}}-176\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}a^2c^5d^{\frac{11}{2}}+33b^2c^8d^{\frac{7}{2}}-44abc^7d^{\frac{9}{2}}+16a^2c^6d^{\frac{11}{2}}\right)}{3465\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(d)*x - sqrt(d*x^2 + c))^16*b^2*d^(7/2) - 8085*(sqrt(d)*
x - sqrt(d*x^2 + c))^14*b^2*c*d^(7/2) + 13860*(sqrt(d)*x - sqrt(d*x^2 + c))
```

```
^14*a*b*d^(9/2) + 9933*(sqrt(d)*x - sqrt(d*x^2 + c))^12*b^2*c^2*d^(7/2) - 1
9404*(sqrt(d)*x - sqrt(d*x^2 + c))^12*a*b*c*d^(9/2) + 22176*(sqrt(d)*x - sq
rt(d*x^2 + c))^12*a^2*d^(11/2) - 5313*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^2*
c^3*d^(7/2) + 924*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b*c^2*d^(9/2) + 14784*
(sqrt(d)*x - sqrt(d*x^2 + c))^10*a^2*c*d^(11/2) + 2805*(sqrt(d)*x - sqrt(d*
x^2 + c))^8*b^2*c^4*d^(7/2) - 660*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^3*d
^(9/2) + 5280*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*c^2*d^(11/2) - 3135*(sqrt
(d)*x - sqrt(d*x^2 + c))^6*b^2*c^5*d^(7/2) + 7260*(sqrt(d)*x - sqrt(d*x^2 +
 c))^6*a*b*c^4*d^(9/2) - 2640*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^3*d^(11
/2) + 1815*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^6*d^(7/2) - 2420*(sqrt(d)*
x - sqrt(d*x^2 + c))^4*a*b*c^5*d^(9/2) + 880*(sqrt(d)*x - sqrt(d*x^2 + c))^
4*a^2*c^4*d^(11/2) - 363*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^7*d^(7/2) +
484*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^6*d^(9/2) - 176*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a^2*c^5*d^(11/2) + 33*b^2*c^8*d^(7/2) - 44*a*b*c^7*d^(9/2) +
 16*a^2*c^6*d^(11/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^11
```

Mupad [B]
time = 1.38, size = 317, normalized size = 1.68

$$\frac{8a^2d^2\sqrt{dx^2+c}}{693c^2x^7}-\frac{b^2\sqrt{dx^2+c}}{7x^7}-\frac{2ab\sqrt{dx^2+c}}{9x^9}-\frac{a^2\sqrt{dx^2+c}}{11x^{11}}-\frac{16a^2d^3\sqrt{dx^2+c}}{1155c^3x^5}+\frac{64a^2d^4\sqrt{dx^2+c}}{3465c^4x^3}-\frac{128a^2d^5\sqrt{dx^2+c}}{3465c^5x}+\frac{4b^2d^2\sqrt{dx^2+c}}{105c^2x^3}-\frac{8b^2d^3\sqrt{dx^2+c}}{105c^3x}-\frac{a^2d\sqrt{dx^2+c}}{99cx^9}-\frac{b^2d\sqrt{dx^2+c}}{35cx^5}+\frac{4abd^2\sqrt{dx^2+c}}{105c^2x^5}-\frac{16abd^3\sqrt{dx^2+c}}{315c^3x^3}+\frac{32abd^4\sqrt{dx^2+c}}{315c^4x}-\frac{2abd\sqrt{dx^2+c}}{63cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^12,x)
```

```
[Out] (8*a^2*d^2*(c + d*x^2)^(1/2))/(693*c^2*x^7) - (b^2*(c + d*x^2)^(1/2))/(7*x^
7) - (2*a*b*(c + d*x^2)^(1/2))/(9*x^9) - (a^2*(c + d*x^2)^(1/2))/(11*x^11)
- (16*a^2*d^3*(c + d*x^2)^(1/2))/(1155*c^3*x^5) + (64*a^2*d^4*(c + d*x^2)^(
1/2))/(3465*c^4*x^3) - (128*a^2*d^5*(c + d*x^2)^(1/2))/(3465*c^5*x) + (4*b^
2*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^3) - (8*b^2*d^3*(c + d*x^2)^(1/2))/(105
*c^3*x) - (a^2*d*(c + d*x^2)^(1/2))/(99*c*x^9) - (b^2*d*(c + d*x^2)^(1/2))/
(35*c*x^5) + (4*a*b*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^5) - (16*a*b*d^3*(c +
 d*x^2)^(1/2))/(315*c^3*x^3) + (32*a*b*d^4*(c + d*x^2)^(1/2))/(315*c^4*x) -
 (2*a*b*d*(c + d*x^2)^(1/2))/(63*c*x^7)
```

3.613 $\int x^4(a+bx^2)^2\,(c+dx^2)^{3/2}\,dx$

Optimal. Leaf size=281

$$-\frac{c^3(24a^2d^2+bc(7bc-24ad))\,x\sqrt{c+dx^2}}{1024d^4}+\frac{c^2(24a^2d^2+bc(7bc-24ad))\,x^3\sqrt{c+dx^2}}{1536d^3}+\frac{c(24a^2d^2+bc(7bc-}{384}$$

```
[Out] 1/192*(24*a^2*d^2+b*c*(-24*a*d+7*b*c))*x^5*(d*x^2+c)^(3/2)/d^2-1/120*b*(-24
*a*d+7*b*c)*x^5*(d*x^2+c)^(5/2)/d^2+1/12*b^2*x^7*(d*x^2+c)^(5/2)/d+1/1024*c
^4*(24*a^2*d^2+b*c*(-24*a*d+7*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(9
/2)-1/1024*c^3*(24*a^2*d^2+b*c*(-24*a*d+7*b*c))*x*(d*x^2+c)^(1/2)/d^4+1/153
6*c^2*(24*a^2*d^2+b*c*(-24*a*d+7*b*c))*x^3*(d*x^2+c)^(1/2)/d^3+1/384*c*(24*
a^2*d^2+b*c*(-24*a*d+7*b*c))*x^5*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.18, antiderivative size = 278, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {475, 470, 285, 327, 223, 212}

$$\frac{c^4(24a^2d^2+bc(7bc-24ad))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{1024d^{9/2}}-\frac{c^3x\sqrt{c+dx^2}\,(24a^2d^2+bc(7bc-24ad))}{1024d^4}+\frac{c^2x^3\sqrt{c+dx^2}\,(24a^2d^2+bc(7bc-24ad))}{1536d^3}+\frac{1}{192}x^5(c+dx^2)^{3/2}\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)+\frac{cx^5\sqrt{c+dx^2}\,(24a^2d^2+bc(7bc-24ad))}{384d^2}-\frac{bx^5(c+dx^2)^{5/2}(7bc-24ad)}{120d^2}+\frac{b^2x^7(c+dx^2)^{5/2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] -1/1024*(c^3*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/d^4 + (
c^2*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^3*Sqrt[c + d*x^2])/(1536*d^3) + (
c*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^5*Sqrt[c + d*x^2])/(384*d^2) + ((24
*a^2 + (b*c*(7*b*c - 24*a*d))/d^2)*x^5*(c + d*x^2)^(3/2))/192 - (b*(7*b*c -
 24*a*d)*x^5*(c + d*x^2)^(5/2))/(120*d^2) + (b^2*x^7*(c + d*x^2)^(5/2))/(12
*d) + (c^4*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(1024*d^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int x^4(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \frac{b^2x^7(c+dx^2)^{5/2}}{12d} + \frac{\int x^4(c+dx^2)^{3/2}\,(12a^2d-b(7bc-24ad)x^2)\,dx}{12d}$$

$$= -\frac{b(7bc-24ad)x^5(c+dx^2)^{5/2}}{120d^2} + \frac{b^2x^7(c+dx^2)^{5/2}}{12d} + \frac{1}{24}\left(24a^2 + \frac{bc(7bc-}{d^2}\right.$$

$$= \frac{1}{192}\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x^5(c+dx^2)^{3/2} - \frac{b(7bc-24ad)x^5(c+dx^2)}{120d^2}$$

$$= \frac{1}{384}c\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x^5\sqrt{c+dx^2} + \frac{1}{192}\left(24a^2 + \frac{bc(7bc-24a}{d^2}\right.$$

$$= \frac{c^2\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{1536d} + \frac{1}{384}c\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x^5$$

$$= -\frac{c^3\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d^2} + \frac{c^2\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx}}{1536d}$$

$$= -\frac{c^3\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d^2} + \frac{c^2\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx}}{1536d}$$

$$= -\frac{c^3\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d^2} + \frac{c^2\left(24a^2 + \frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx}}{1536d}$$

Mathematica [A]
time = 0.31, size = 224, normalized size = 0.80

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\,(120a^2d^2(-3c^3+2c^2dx^2+24cd^2x^4+16d^3x^6)+24abd(15c^4-10c^3dx^2+8c^2d^2x^4+176cd^3x^6+128d^4x^8)+b^2(-105c^5+70c^4dx^2-56c^3d^2x^4+48c^2d^3x^6+1664cd^4x^8+1280d^5x^{10}))-15c^4(7b^2c^2-24abcd+24a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{15360d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(120*a^2*d^2*(-3*c^3 + 2*c^2*d*x^2 + 24*c*d^2*x^
4 + 16*d^3*x^6) + 24*a*b*d*(15*c^4 - 10*c^3*d*x^2 + 8*c^2*d^2*x^4 + 176*c*d
^3*x^6 + 128*d^4*x^8) + b^2*(-105*c^5 + 70*c^4*d*x^2 - 56*c^3*d^2*x^4 + 48*
c^2*d^3*x^6 + 1664*c*d^4*x^8 + 1280*d^5*x^10)) - 15*c^4*(7*b^2*c^2 - 24*a*b
*c*d + 24*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(15360*d^(9/2))
```

Maple [A]
time = 0.10, size = 377, normalized size = 1.34

method	result

risch	$-\frac{x\left(-1280b^2d^5x^{10}-3072ab\,d^5x^8-1664b^2c\,d^4x^8-1920a^2d^5x^6-4224abc\,d^4x^6-48b^2c^2d^3x^6-2880a^2c\,d^4x^4-192ab\,c^2d^3x^4+56b^2c^3d^2\right.}{15360d^4}$

default | b^2 | $$\frac{x^7(dx^2+c)^{\frac{5}{2}}}{12d} - \frac{7c\left(\frac{x^5(dx^2+c)^{\frac{5}{2}}}{10d} - \frac{c\left(\frac{x^3(dx^2+c)^{\frac{5}{2}}}{8d} - \frac{3c\left(\frac{x(dx^2+c)^{\frac{5}{2}}}{6d} - \frac{c\left(\frac{x(dx^2+c)^{\frac{3}{2}}}{4} + \frac{3c\left(\frac{x\sqrt{dx^2+c}}{2} + \frac{c\ln\left(x\sqrt{d}+\sqrt{c}\right.}{2\sqrt{d}}\right.}{4}\right.}{6d}\right.}{8d}\right.}{2d}\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/12*x^7*(d*x^2+c)^(5/2)/d-7/12*c/d*(1/10*x^5*(d*x^2+c)^(5/2)/d-1/2*c/
d*(1/8*x^3*(d*x^2+c)^(5/2)/d-3/8*c/d*(1/6*x*(d*x^2+c)^(5/2)/d-1/6*c/d*(1/4*
x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(
d*x^2+c)^(1/2))))))))+2*a*b*(1/10*x^5*(d*x^2+c)^(5/2)/d-1/2*c/d*(1/8*x^3*(d
*x^2+c)^(5/2)/d-3/8*c/d*(1/6*x*(d*x^2+c)^(5/2)/d-1/6*c/d*(1/4*x*(d*x^2+c)^(
3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2
)))))))+a^2*(1/8*x^3*(d*x^2+c)^(5/2)/d-3/8*c/d*(1/6*x*(d*x^2+c)^(5/2)/d-1/6
*c/d*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x
*d^(1/2)+(d*x^2+c)^(1/2))))))
```

Maxima [A]
time = 0.30, size = 367, normalized size = 1.31

$$\frac{(dx^2+c)^{\frac{5}{2}}b^2x^7}{12\,d}-\frac{7\,(dx^2+c)^{\frac{5}{2}}b^2cx^5}{120\,d^2}+\frac{(dx^2+c)^{\frac{5}{2}}abx^5}{5\,d}+\frac{7\,(dx^2+c)^{\frac{5}{2}}b^2c^2x^3}{192\,d^3}-\frac{(dx^2+c)^{\frac{5}{2}}abcx^3}{8\,d^2}+\frac{(dx^2+c)^{\frac{5}{2}}a^2x^3}{8\,d}-\frac{7\,(dx^2+c)^{\frac{5}{2}}b^2c^3x}{384\,d^4}+\frac{7\,(dx^2+c)^{\frac{3}{2}}b^2c^4x}{1536\,d^4}+\frac{7\,\sqrt{dx^2+c}\,b^2c^5x}{1024\,d^4}+\frac{(dx^2+c)^{\frac{5}{2}}abc^2x}{16\,d^3}-\frac{(dx^2+c)^{\frac{3}{2}}abc^3x}{64\,d^3}-\frac{3\,\sqrt{dx^2+c}\,abc^4x}{128\,d^3}-\frac{(dx^2+c)^{\frac{5}{2}}a^2cx}{16\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}a^2c^2x}{64\,d^2}+\frac{3\,\sqrt{dx^2+c}\,a^2c^3x}{128\,d^2}+\frac{7\,b^2c^6\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{1024\,d^{\frac{9}{2}}}-\frac{3\,abc^5\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{7}{2}}}+\frac{3\,a^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(d*x^2 + c)^(5/2)*b^2*x^7/d - 7/120*(d*x^2 + c)^(5/2)*b^2*c*x^5/d^2 +
1/5*(d*x^2 + c)^(5/2)*a*b*x^5/d + 7/192*(d*x^2 + c)^(5/2)*b^2*c^2*x^3/d^3 -
 1/8*(d*x^2 + c)^(5/2)*a*b*c*x^3/d^2 + 1/8*(d*x^2 + c)^(5/2)*a^2*x^3/d - 7/
384*(d*x^2 + c)^(5/2)*b^2*c^3*x/d^4 + 7/1536*(d*x^2 + c)^(3/2)*b^2*c^4*x/d^
4 + 7/1024*sqrt(d*x^2 + c)*b^2*c^5*x/d^4 + 1/16*(d*x^2 + c)^(5/2)*a*b*c^2*x
/d^3 - 1/64*(d*x^2 + c)^(3/2)*a*b*c^3*x/d^3 - 3/128*sqrt(d*x^2 + c)*a*b*c^4
*x/d^3 - 1/16*(d*x^2 + c)^(5/2)*a^2*c*x/d^2 + 1/64*(d*x^2 + c)^(3/2)*a^2*c^
2*x/d^2 + 3/128*sqrt(d*x^2 + c)*a^2*c^3*x/d^2 + 7/1024*b^2*c^6*arcsinh(d*x/
sqrt(c*d))/d^(9/2) - 3/128*a*b*c^5*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 3/128*a
^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

Fricas [A]
time = 2.58, size = 494, normalized size = 1.76

$$\left[\frac{15\,(7\,b^2c^6-24\,abc^5d+24\,a^2c^4d^2)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(1280\,b^2d^6x^{11}+128\,(13\,b^2cd^5+24\,abd^6)x^9+48\,(b^2c^2d^4+88\,abcd^5+40\,a^2d^6)x^7-8\,(7\,b^2c^3d^3-24\,abc^2d^4-360\,a^2cd^5)x^5+10\,(7\,b^2c^4d^2-24\,abc^3d^3+24\,a^2c^2d^4)x^3-15\,(7\,b^2c^5d-24\,abc^4d^2+24\,a^2c^3d^3)x)\sqrt{dx^2+c}}{30720\,d^5},-\frac{15\,(7\,b^2c^6-24\,abc^5d+24\,a^2c^4d^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(1280\,b^2d^6x^{11}+128\,(13\,b^2cd^5+24\,abd^6)x^9+48\,(b^2c^2d^4+88\,abcd^5+40\,a^2d^6)x^7-8\,(7\,b^2c^3d^3-24\,abc^2d^4-360\,a^2cd^5)x^5+10\,(7\,b^2c^4d^2-24\,abc^3d^3+24\,a^2c^2d^4)x^3-15\,(7\,b^2c^5d-24\,abc^4d^2+24\,a^2c^3d^3)x)\sqrt{dx^2+c}}{15360\,d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(7*b^2*c^6 - 24*a*b*c^5*d + 24*a^2*c^4*d^2)*sqrt(d)*log(-2*d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(1280*b^2*d^6*x^11 + 128*(13*b^2*
c*d^5 + 24*a*b*d^6)*x^9 + 48*(b^2*c^2*d^4 + 88*a*b*c*d^5 + 40*a^2*d^6)*x^7
```

```
- 8*(7*b^2*c^3*d^3 - 24*a*b*c^2*d^4 - 360*a^2*c*d^5)*x^5 + 10*(7*b^2*c^4*d^
2 - 24*a*b*c^3*d^3 + 24*a^2*c^2*d^4)*x^3 - 15*(7*b^2*c^5*d - 24*a*b*c^4*d^2
 + 24*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^5, -1/15360*(15*(7*b^2*c^6 - 24*a*
b*c^5*d + 24*a^2*c^4*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (12
80*b^2*d^6*x^11 + 128*(13*b^2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(b^2*c^2*d^4 + 8
8*a*b*c*d^5 + 40*a^2*d^6)*x^7 - 8*(7*b^2*c^3*d^3 - 24*a*b*c^2*d^4 - 360*a^2
*c*d^5)*x^5 + 10*(7*b^2*c^4*d^2 - 24*a*b*c^3*d^3 + 24*a^2*c^2*d^4)*x^3 - 15
*(7*b^2*c^5*d - 24*a*b*c^4*d^2 + 24*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^5]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.15, size = 263, normalized size = 0.94

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^2dx^2+\frac{13\,b^2cd^{10}+24\,abd^{11}}{d^{10}}\right)x^2+\frac{3\,(b^2c^2d^9+88\,abcd^{10}+40\,a^2d^{11})}{d^{10}}\right)x^2-\frac{7\,b^2c^3d^8-24\,abc^2d^9-360\,a^2cd^{10}}{d^{10}}\right)x^2+\frac{5\,(7\,b^2c^4d^7-24\,abc^3d^8+24\,a^2c^2d^9)}{d^{10}}\right)x^2-\frac{15\,(7\,b^2c^5d^6-24\,abc^4d^7+24\,a^2c^3d^8)}{d^{10}}\right)\sqrt{dx^2+c}\,x-\frac{(7\,b^2c^6-24\,abc^5d+24\,a^2c^4d^2)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{1024\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d*x^2 + (13*b^2*c*d^10 + 24*a*b*d^11)/d^10)*x^2
 + 3*(b^2*c^2*d^9 + 88*a*b*c*d^10 + 40*a^2*d^11)/d^10)*x^2 - (7*b^2*c^3*d^8
 - 24*a*b*c^2*d^9 - 360*a^2*c*d^10)/d^10)*x^2 + 5*(7*b^2*c^4*d^7 - 24*a*b*c
^3*d^8 + 24*a^2*c^2*d^9)/d^10)*x^2 - 15*(7*b^2*c^5*d^6 - 24*a*b*c^4*d^7 + 2
4*a^2*c^3*d^8)/d^10)*sqrt(d*x^2 + c)*x - 1/1024*(7*b^2*c^6 - 24*a*b*c^5*d +
 24*a^2*c^4*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.614 $\int x^3(a+bx^2)^2 (c+dx^2)^{3/2} dx$

Optimal. Leaf size=114

$$-\frac{c(bc-ad)^2 (c+dx^2)^{5/2}}{5d^4}+\frac{(bc-ad)(3bc-ad) (c+dx^2)^{7/2}}{7d^4}-\frac{b(3bc-2ad) (c+dx^2)^{9/2}}{9d^4}+\frac{b^2(c+dx^2)^{11/2}}{11d^4}$$

```
[Out] -1/5*c*(-a*d+b*c)^2*(d*x^2+c)^(5/2)/d^4+1/7*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2+
c)^(7/2)/d^4-1/9*b*(-2*a*d+3*b*c)*(d*x^2+c)^(9/2)/d^4+1/11*b^2*(d*x^2+c)^(1
1/2)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {457, 78}

$$-\frac{b(c+dx^2)^{9/2} (3bc-2ad)}{9d^4}+\frac{(c+dx^2)^{7/2} (bc-ad)(3bc-ad)}{7d^4}-\frac{c(c+dx^2)^{5/2} (bc-ad)^2}{5d^4}+\frac{b^2(c+dx^2)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] -1/5*(c*(b*c - a*d)^2*(c + d*x^2)^(5/2))/d^4 + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(7/2))/(7*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(9/2))/(9*d^4) +
 (b^2*(c + d*x^2)^(11/2))/(11*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx = \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)^{3/2} \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2(c+dx)^{3/2}}{d^3} + \frac{(bc-ad)(3bc-ad)(c+dx)^{5/2}}{d^3} - \right.\right.$$
$$= -\frac{c(bc-ad)^2 \left(c+dx^2\right)^{5/2}}{5d^4} + \frac{(bc-ad)(3bc-ad) \left(c+dx^2\right)^{7/2}}{7d^4} - \frac{b(3bc-2}{}$$

Mathematica [A]
time = 0.07, size = 100, normalized size = 0.88

$$\frac{\left(c+dx^2\right)^{5/2} \left(99a^2d^2(-2c+5dx^2)+22abd(8c^2-20cdx^2+35d^2x^4)-3b^2(16c^3-40c^2dx^2+70cd^2x^4-105d^3x^6)\right)}{3465d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(99*a^2*d^2*(-2*c + 5*d*x^2) + 22*a*b*d*(8*c^2 - 20*c*d*
x^2 + 35*d^2*x^4) - 3*b^2*(16*c^3 - 40*c^2*d*x^2 + 70*c*d^2*x^4 - 105*d^3*x
^6)))/(3465*d^4)
```

Maple [A]
time = 0.09, size = 185, normalized size = 1.62

method	result
gosper	$-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}} \left(-315b^2x^6d^3-770ab\,d^3x^4+210b^2c\,d^2x^4-495a^2d^3x^2+440abc\,d^2x^2-120b^2c^2d\,x^2+198a^2c\,d^2-176ab\,c^2d+48b^2c^3\right)}{3465d^4}$
default	$b^2\left(\frac{x^6\left(d\,x^2+c\right)^{\frac{5}{2}}}{11d}-\frac{6c\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{5}{2}}}{9d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{5}{2}}}{7d}-\frac{2c\left(d\,x^2+c\right)^{\frac{5}{2}}}{35d^2}\right)}{9d}\right)}{11d}\right)+2ab\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{5}{2}}}{9d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{5}{2}}}{7d}-\frac{2c}{}\right.}{9d}\right.$
trager	$-\frac{\left(-315b^2d^5x^{10}-770ab\,d^5x^8-420b^2c\,d^4x^8-495a^2d^5x^6-1100abc\,d^4x^6-15b^2c^2d^3x^6-792a^2c\,d^4x^4-66ab\,c^2d^3x^4+18b^2c^3d^2x^4-99a^2c\right.}{3465d^4}$
risch	$-\frac{\left(-315b^2d^5x^{10}-770ab\,d^5x^8-420b^2c\,d^4x^8-495a^2d^5x^6-1100abc\,d^4x^6-15b^2c^2d^3x^6-792a^2c\,d^4x^4-66ab\,c^2d^3x^4+18b^2c^3d^2x^4-99a^2c\right.}{3465d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/11*x^6*(d*x^2+c)^(5/2)/d-6/11*c/d*(1/9*x^4*(d*x^2+c)^(5/2)/d-4/9*c/d
*(1/7*x^2*(d*x^2+c)^(5/2)/d-2/35*c/d^2*(d*x^2+c)^(5/2))))+2*a*b*(1/9*x^4*(d
*x^2+c)^(5/2)/d-4/9*c/d*(1/7*x^2*(d*x^2+c)^(5/2)/d-2/35*c/d^2*(d*x^2+c)^(5/
2)))+a^2*(1/7*x^2*(d*x^2+c)^(5/2)/d-2/35*c/d^2*(d*x^2+c)^(5/2))
```

Maxima [A]
time = 0.29, size = 181, normalized size = 1.59

$$\frac{(dx^2+c)^{\frac{5}{2}}b^2x^6}{11\,d}-\frac{2\,(dx^2+c)^{\frac{5}{2}}b^2cx^4}{33\,d^2}+\frac{2\,(dx^2+c)^{\frac{5}{2}}abx^4}{9\,d}+\frac{8\,(dx^2+c)^{\frac{5}{2}}b^2c^2x^2}{231\,d^3}-\frac{8\,(dx^2+c)^{\frac{5}{2}}abcx^2}{63\,d^2}+\frac{(dx^2+c)^{\frac{5}{2}}a^2x^2}{7\,d}-\frac{16\,(dx^2+c)^{\frac{5}{2}}b^2c^3}{1155\,d^4}+\frac{16\,(dx^2+c)^{\frac{5}{2}}abc^2}{315\,d^3}-\frac{2\,(dx^2+c)^{\frac{5}{2}}a^2c}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*(d*x^2 + c)^(5/2)*b^2*x^6/d - 2/33*(d*x^2 + c)^(5/2)*b^2*c*x^4/d^2 + 2
/9*(d*x^2 + c)^(5/2)*a*b*x^4/d + 8/231*(d*x^2 + c)^(5/2)*b^2*c^2*x^2/d^3 -
8/63*(d*x^2 + c)^(5/2)*a*b*c*x^2/d^2 + 1/7*(d*x^2 + c)^(5/2)*a^2*x^2/d - 16
/1155*(d*x^2 + c)^(5/2)*b^2*c^3/d^4 + 16/315*(d*x^2 + c)^(5/2)*a*b*c^2/d^3
- 2/35*(d*x^2 + c)^(5/2)*a^2*c/d^2
```

Fricas [A]
time = 1.52, size = 179, normalized size = 1.57

$$\frac{\left(315\,b^2d^5x^{10}+70\,(6\,b^2cd^4+11\,abd^5)x^8-48\,b^2c^5+176\,abc^4d-198\,a^2c^3d^2+5\,(3\,b^2c^2d^3+220\,abcd^4+99\,a^2d^5)x^6-6\,(3\,b^2c^3d^2-11\,abc^2d^3-132\,a^2cd^4)x^4+(24\,b^2c^4d-88\,abc^3d^2+99\,a^2c^2d^3)x^2\right)\sqrt{dx^2+c}}{3465\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^2*d^5*x^10 + 70*(6*b^2*c*d^4 + 11*a*b*d^5)*x^8 - 48*b^2*c^5 +
 176*a*b*c^4*d - 198*a^2*c^3*d^2 + 5*(3*b^2*c^2*d^3 + 220*a*b*c*d^4 + 99*a^
2*d^5)*x^6 - 6*(3*b^2*c^3*d^2 - 11*a*b*c^2*d^3 - 132*a^2*c*d^4)*x^4 + (24*b
^2*c^4*d - 88*a*b*c^3*d^2 + 99*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(102) = 204$.
time = 0.43, size = 384, normalized size = 3.37

$$\begin{cases} -\frac{2a^2c^3\sqrt{c+dx^2}}{35d^2}+\frac{a^2c^2x^2\sqrt{c+dx^2}}{35d}+\frac{8a^2cx^4\sqrt{c+dx^2}}{35}+\frac{a^2dx^6\sqrt{c+dx^2}}{7}+\frac{16abc^4\sqrt{c+dx^2}}{315d^3}-\frac{8abc^3x^2\sqrt{c+dx^2}}{315d^2}+\frac{2abc^2x^4\sqrt{c+dx^2}}{105d}+\frac{20abcx^6\sqrt{c+dx^2}}{63}+\frac{2abdx^8\sqrt{c+dx^2}}{9}-\frac{16b^2c^5\sqrt{c+dx^2}}{1155d^4}+\frac{8b^2c^4x^2\sqrt{c+dx^2}}{1155d^3}-\frac{2b^2c^3x^4\sqrt{c+dx^2}}{385d^2}+\frac{b^2c^2x^6\sqrt{c+dx^2}}{231d}+\frac{4b^2cx^8\sqrt{c+dx^2}}{33}+\frac{b^2dx^{10}\sqrt{c+dx^2}}{11} & \text{for}\ d\neq 0 \\ c^{\frac{3}{2}}\left(\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((-2*a**2*c**3*sqrt(c + d*x**2)/(35*d**2) + a**2*c**2*x**2*sqrt(c
+ d*x**2)/(35*d) + 8*a**2*c*x**4*sqrt(c + d*x**2)/35 + a**2*d*x**6*sqrt(c +
 d*x**2)/7 + 16*a*b*c**4*sqrt(c + d*x**2)/(315*d**3) - 8*a*b*c**3*x**2*sqrt
(c + d*x**2)/(315*d**2) + 2*a*b*c**2*x**4*sqrt(c + d*x**2)/(105*d) + 20*a*b
*c*x**6*sqrt(c + d*x**2)/63 + 2*a*b*d*x**8*sqrt(c + d*x**2)/9 - 16*b**2*c**
```

```
5*sqrt(c + d*x**2)/(1155*d**4) + 8*b**2*c**4*x**2*sqrt(c + d*x**2)/(1155*d*
*3) - 2*b**2*c**3*x**4*sqrt(c + d*x**2)/(385*d**2) + b**2*c**2*x**6*sqrt(c
+ d*x**2)/(231*d) + 4*b**2*c*x**8*sqrt(c + d*x**2)/33 + b**2*d*x**10*sqrt(c
 + d*x**2)/11, Ne(d, 0)), (c**(3/2)*(a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
), True))
```

Giac [A]
time = 1.21, size = 150, normalized size = 1.32

$$\frac{315\,(dx^2+c)^{\frac{11}{2}}b^2 - 1155\,(dx^2+c)^{\frac{9}{2}}b^2c + 1485\,(dx^2+c)^{\frac{7}{2}}b^2c^2 - 693\,(dx^2+c)^{\frac{5}{2}}b^2c^3 + 770\,(dx^2+c)^{\frac{9}{2}}abd - 1980\,(dx^2+c)^{\frac{7}{2}}abcd + 1386\,(dx^2+c)^{\frac{5}{2}}abc^2d + 495\,(dx^2+c)^{\frac{7}{2}}a^2d^2 - 693\,(dx^2+c)^{\frac{5}{2}}a^2cd^2}{3465\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3465*(315*(d*x^2 + c)^(11/2)*b^2 - 1155*(d*x^2 + c)^(9/2)*b^2*c + 1485*(d
*x^2 + c)^(7/2)*b^2*c^2 - 693*(d*x^2 + c)^(5/2)*b^2*c^3 + 770*(d*x^2 + c)^(
9/2)*a*b*d - 1980*(d*x^2 + c)^(7/2)*a*b*c*d + 1386*(d*x^2 + c)^(5/2)*a*b*c^
2*d + 495*(d*x^2 + c)^(7/2)*a^2*d^2 - 693*(d*x^2 + c)^(5/2)*a^2*c*d^2)/d^4
```

Mupad [B]
time = 0.39, size = 170, normalized size = 1.49

$$\sqrt{d\,x^2+c}\,\left(\frac{x^6\,\left(495\,a^2\,d^5+1100\,a\,b\,c\,d^4+15\,b^2\,c^2\,d^3\right)}{3465\,d^4}-\frac{198\,a^2\,c^3\,d^2-176\,a\,b\,c^4\,d+48\,b^2\,c^5}{3465\,d^4}+\frac{2\,b\,x^8\,\left(11\,a\,d+6\,b\,c\right)}{99}+\frac{b^2\,d\,x^{10}}{11}+\frac{2\,c\,x^4\,\left(132\,a^2\,d^2+11\,a\,b\,c\,d-3\,b^2\,c^2\right)}{1155\,d^2}+\frac{c^2\,x^2\,\left(99\,a^2\,d^2-88\,a\,b\,c\,d+24\,b^2\,c^2\right)}{3465\,d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((x^6*(495*a^2*d^5 + 15*b^2*c^2*d^3 + 1100*a*b*c*d^4))/(3
465*d^4) - (48*b^2*c^5 + 198*a^2*c^3*d^2 - 176*a*b*c^4*d)/(3465*d^4) + (2*b
*x^8*(11*a*d + 6*b*c))/99 + (b^2*d*x^10)/11 + (2*c*x^4*(132*a^2*d^2 - 3*b^2
*c^2 + 11*a*b*c*d))/(1155*d^2) + (c^2*x^2*(99*a^2*d^2 + 24*b^2*c^2 - 88*a*b
*c*d))/(3465*d^3))
```

3.615 $\int x^2(a+bx^2)^2(c+dx^2)^{3/2}\,dx$

Optimal. Leaf size=235

$$\frac{c^2(16a^2d^2+3bc(bc-4ad))\,x\sqrt{c+dx^2}}{256d^3}+\frac{c(16a^2d^2+3bc(bc-4ad))\,x^3\sqrt{c+dx^2}}{128d^2}+\frac{(16a^2d^2+3bc(bc-4ad)}{96d^2}$$

```
[Out] 1/96*(16*a^2*d^2+3*b*c*(-4*a*d+b*c))*x^3*(d*x^2+c)^(3/2)/d^2-1/16*b*(-4*a*d
+b*c)*x^3*(d*x^2+c)^(5/2)/d^2+1/10*b^2*x^5*(d*x^2+c)^(5/2)/d-1/256*c^3*(16*
a^2*d^2+3*b*c*(-4*a*d+b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(7/2)+1/25
6*c^2*(16*a^2*d^2+3*b*c*(-4*a*d+b*c))*x*(d*x^2+c)^(1/2)/d^3+1/128*c*(16*a^2
*d^2+3*b*c*(-4*a*d+b*c))*x^3*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.15, antiderivative size = 232, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {475, 470, 285, 327, 223, 212}

$$-\frac{c^3(16a^2d^2+3bc(bc-4ad))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{256d^{7/2}}+\frac{c^2x\sqrt{c+dx^2}\,(16a^2d^2+3bc(bc-4ad))}{256d^3}+\frac{1}{96}x^3(c+dx^2)^{3/2}\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)+\frac{cx^3\sqrt{c+dx^2}\,(16a^2d^2+3bc(bc-4ad))}{128d^2}-\frac{bx^3(c+dx^2)^{5/2}(bc-4ad)}{16d^2}+\frac{b^2x^5(c+dx^2)^{5/2}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (c^2*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x*Sqrt[c + d*x^2])/(256*d^3) + (c*(
16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x^3*Sqrt[c + d*x^2])/(128*d^2) + ((16*a^2
 + (3*b*c*(b*c - 4*a*d))/d^2)*x^3*(c + d*x^2)^(3/2))/96 - (b*(b*c - 4*a*d)*
x^3*(c + d*x^2)^(5/2))/(16*d^2) + (b^2*x^5*(c + d*x^2)^(5/2))/(10*d) - (c^3
*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
256*d^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
```

```
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx = \frac{b^2 x^5 (c+dx^2)^{5/2}}{10d} + \frac{\int x^2 (c+dx^2)^{3/2} \left(10a^2 d - 5b(bc-4ad)x^2\right) \, dx}{10d}$$

$$= -\frac{b(bc-4ad)x^3 (c+dx^2)^{5/2}}{16d^2} + \frac{b^2 x^5 (c+dx^2)^{5/2}}{10d} + \frac{1}{16}\left(16a^2 + \frac{3bc(bc-4\right.$$

$$= \frac{1}{96}\left(16a^2 + \frac{3bc(bc-4ad)}{d^2}\right) x^3 \left(c+dx^2\right)^{3/2} - \frac{b(bc-4ad)x^3 (c+dx^2)^{5/2}}{16d^2}$$

$$= \frac{1}{128} c \left(16a^2 + \frac{3bc(bc-4ad)}{d^2}\right) x^3 \sqrt{c+dx^2} + \frac{1}{96}\left(16a^2 + \frac{3bc(bc-4ad)}{d^2}\right.$$

$$= \frac{c^2 \left(16a^2 + \frac{3bc(bc-4ad)}{d^2}\right) x \sqrt{c+dx^2}}{256d} + \frac{1}{128} c \left(16a^2 + \frac{3bc(bc-4ad)}{d^2}\right) x^3 \sqrt{}$$

$$= \frac{c^2 \left(16a^2 + \frac{3bc(bc-4ad)}{d^2}\right) x \sqrt{c+dx^2}}{256d} + \frac{1}{128} c \left(16a^2 + \frac{3bc(bc-4ad)}{d^2}\right) x^3 \sqrt{}$$

$$= \frac{c^2 \left(16a^2 + \frac{3bc(bc-4ad)}{d^2}\right) x \sqrt{c+dx^2}}{256d} + \frac{1}{128} c \left(16a^2 + \frac{3bc(bc-4ad)}{d^2}\right) x^3 \sqrt{}$$

Mathematica [A]
time = 0.27, size = 192, normalized size = 0.82

$$\frac{\sqrt{d}\, x \sqrt{c+dx^2} \left(80a^2d^2(3c^2+14cdx^2+8d^2x^4)+60abd(-3c^3+2c^2dx^2+24cd^2x^4+16d^3x^6)+3b^2(15c^4-10c^3dx^2+8c^2d^2x^4+176cd^3x^6+128d^4x^8)\right)+15c^3(3b^2c^2-12abcd+16a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{3840d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(3*c^2 + 14*c*d*x^2 + 8*d^2*x^4) + 6
0*a*b*d*(-3*c^3 + 2*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6) + 3*b^2*(15*c^4
- 10*c^3*d*x^2 + 8*c^2*d^2*x^4 + 176*c*d^3*x^6 + 128*d^4*x^8)) + 15*c^3*(3*
b^2*c^2 - 12*a*b*c*d + 16*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(38
40*d^(7/2))
```

Maple [A]
time = 0.09, size = 305, normalized size = 1.30

method	result
risch	$\frac{x\left(384b^2x^8d^4+960ab\,d^4x^6+528b^2c\,d^3x^6+640a^2x^4d^4+1440x^4abc\,d^3+24b^2c^2x^4d^2+1120a^2c\,d^3x^2+120ab\,c^2d^2x^2-30b^2c^3d\,x^2+240a^2c\right.}{3840d^3}$

default	b^2	$$\left(\frac{x^5(dx^2+c)^{\frac{5}{2}}}{10d}-\frac{c\left(\frac{x^3(dx^2+c)^{\frac{5}{2}}}{8d}-\frac{3c\left(\frac{x(dx^2+c)^{\frac{5}{2}}}{6d}-\frac{c\left(\frac{x(dx^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6d}\right)}{8d}\right)}{2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/10*x^5*(d*x^2+c)^(5/2)/d-1/2*c/d*(1/8*x^3*(d*x^2+c)^(5/2)/d-3/8*c/d*
(1/6*x*(d*x^2+c)^(5/2)/d-1/6*c/d*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2
+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))))+2*a*b*(1/8*x^3*(
d*x^2+c)^(5/2)/d-3/8*c/d*(1/6*x*(d*x^2+c)^(5/2)/d-1/6*c/d*(1/4*x*(d*x^2+c)^
(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/
2))))))+a^2*(1/6*x*(d*x^2+c)^(5/2)/d-1/6*c/d*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(
1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))
```

Maxima [A]
time = 0.32, size = 299, normalized size = 1.27

$$\frac{(dx^2+c)^{\frac{5}{2}}b^2x^5}{10\,d} - \frac{(dx^2+c)^{\frac{5}{2}}b^2cx^3}{16\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}}abx^3}{4\,d} + \frac{(dx^2+c)^{\frac{5}{2}}b^2c^2x}{32\,d^3} - \frac{(dx^2+c)^{\frac{3}{2}}b^2c^3x}{128\,d^3} - \frac{3\sqrt{dx^2+c}\,b^2c^4x}{256\,d^3} - \frac{(dx^2+c)^{\frac{5}{2}}abcx}{8\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}abc^2x}{32\,d^2} + \frac{3\sqrt{dx^2+c}\,abc^3x}{64\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}}a^2x}{6\,d} - \frac{(dx^2+c)^{\frac{3}{2}}a^2cx}{24\,d} - \frac{\sqrt{dx^2+c}\,a^2c^2x}{16\,d} - \frac{3\,b^2c^5\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{256\,d^{\frac{7}{2}}} + \frac{3\,abc^4\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{64\,d^{\frac{5}{2}}} - \frac{a^2c^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*(d*x^2 + c)^(5/2)*b^2*x^5/d - 1/16*(d*x^2 + c)^(5/2)*b^2*c*x^3/d^2 + 1
/4*(d*x^2 + c)^(5/2)*a*b*x^3/d + 1/32*(d*x^2 + c)^(5/2)*b^2*c^2*x/d^3 - 1/1
28*(d*x^2 + c)^(3/2)*b^2*c^3*x/d^3 - 3/256*sqrt(d*x^2 + c)*b^2*c^4*x/d^3 -
1/8*(d*x^2 + c)^(5/2)*a*b*c*x/d^2 + 1/32*(d*x^2 + c)^(3/2)*a*b*c^2*x/d^2 +
3/64*sqrt(d*x^2 + c)*a*b*c^3*x/d^2 + 1/6*(d*x^2 + c)^(5/2)*a^2*x/d - 1/24*(
d*x^2 + c)^(3/2)*a^2*c*x/d - 1/16*sqrt(d*x^2 + c)*a^2*c^2*x/d - 3/256*b^2*c
^5*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 3/64*a*b*c^4*arcsinh(d*x/sqrt(c*d))/d^(
5/2) - 1/16*a^2*c^3*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

Fricas [A]
time = 1.42, size = 419, normalized size = 1.78

$$\left[\frac{15\,(3\,b^2c^5 - 12\,abc^4d + 16\,a^2c^3d^2)\sqrt{d}\,\log\left(-2\,dx^2 + 2\,\sqrt{dx^2+c}\,\sqrt{d}\,x - c\right) + 2\,(384\,b^2d^5x^9 + 48\,(11\,b^2cd^4 + 20\,abd^5)x^7 + 8\,(3\,b^2c^2d^3 + 180\,abcd^4 + 80\,a^2d^5)x^5 - 10\,(3\,b^2c^3d^2 - 12\,abc^2d^3 - 112\,a^2cd^4)x^3 + 15\,(3\,b^2c^4d - 12\,abc^3d^2 + 16\,a^2c^2d^3)x)\sqrt{dx^2+c}}{7680\,d^4}, \frac{15\,(3\,b^2c^5 - 12\,abc^4d + 16\,a^2c^3d^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right) + (384\,b^2d^5x^9 + 48\,(11\,b^2cd^4 + 20\,abd^5)x^7 + 8\,(3\,b^2c^2d^3 + 180\,abcd^4 + 80\,a^2d^5)x^5 - 10\,(3\,b^2c^3d^2 - 12\,abc^2d^3 - 112\,a^2cd^4)x^3 + 15\,(3\,b^2c^4d - 12\,abc^3d^2 + 16\,a^2c^2d^3)x)\sqrt{dx^2+c}}{3840\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(3*b^2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*sqrt(d)*log(-2*d*x^
2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(384*b^2*d^5*x^9 + 48*(11*b^2*c*d^
4 + 20*a*b*d^5)*x^7 + 8*(3*b^2*c^2*d^3 + 180*a*b*c*d^4 + 80*a^2*d^5)*x^5 -
10*(3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 - 112*a^2*c*d^4)*x^3 + 15*(3*b^2*c^4*d -
 12*a*b*c^3*d^2 + 16*a^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/3840*(15*(3*b^
2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^
2 + c)) + (384*b^2*d^5*x^9 + 48*(11*b^2*c*d^4 + 20*a*b*d^5)*x^7 + 8*(3*b^2*
c^2*d^3 + 180*a*b*c*d^4 + 80*a^2*d^5)*x^5 - 10*(3*b^2*c^3*d^2 - 12*a*b*c^2*
d^3 - 112*a^2*c*d^4)*x^3 + 15*(3*b^2*c^4*d - 12*a*b*c^3*d^2 + 16*a^2*c^2*d^
3)*x)*sqrt(d*x^2 + c))/d^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(223) = 446$.

time = 169.31, size = 505, normalized size = 2.15

$$\frac{a^2c^{\frac{5}{2}}x}{16d\sqrt{1+\frac{dx^2}{c}}}+\frac{17a^2c^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{dx^2}{c}}}+\frac{11a^2\sqrt{c}\,dx^5}{24\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16d^{\frac{3}{2}}}+\frac{a^2d^2x^7}{6\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}-\frac{3abc^{\frac{7}{2}}x}{64d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{5}{2}}x^3}{64d\sqrt{1+\frac{dx^2}{c}}}+\frac{13abc^{\frac{3}{2}}x^5}{32\sqrt{1+\frac{dx^2}{c}}}+\frac{5ab\sqrt{c}\,dx^7}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3abc^4\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{64d^{\frac{5}{2}}}+\frac{abd^2x^9}{4\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c^{\frac{9}{2}}x}{256d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2c^{\frac{7}{2}}x^3}{256d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{5}{2}}x^5}{640d\sqrt{1+\frac{dx^2}{c}}}+\frac{23b^2c^{\frac{3}{2}}x^7}{160\sqrt{1+\frac{dx^2}{c}}}+\frac{19b^2\sqrt{c}\,dx^9}{80\sqrt{1+\frac{dx^2}{c}}}-\frac{3b^2c^5\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{256d^{\frac{7}{2}}}+\frac{b^2d^2x^{11}}{10\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(5/2)*x/(16*d*sqrt(1 + d*x**2/c)) + 17*a**2*c**(3/2)*x**3/(48*sqrt(
1 + d*x**2/c)) + 11*a**2*sqrt(c)*d*x**5/(24*sqrt(1 + d*x**2/c)) - a**2*c**3
*asinh(sqrt(d)*x/sqrt(c))/(16*d**(3/2)) + a**2*d**2*x**7/(6*sqrt(c)*sqrt(1
+ d*x**2/c)) - 3*a*b*c**(7/2)*x/(64*d**2*sqrt(1 + d*x**2/c)) - a*b*c**(5/2)
*x**3/(64*d*sqrt(1 + d*x**2/c)) + 13*a*b*c**(3/2)*x**5/(32*sqrt(1 + d*x**2/
c)) + 5*a*b*sqrt(c)*d*x**7/(8*sqrt(1 + d*x**2/c)) + 3*a*b*c**4*asinh(sqrt(d
)*x/sqrt(c))/(64*d**(5/2)) + a*b*d**2*x**9/(4*sqrt(c)*sqrt(1 + d*x**2/c)) +
 3*b**2*c**(9/2)*x/(256*d**3*sqrt(1 + d*x**2/c)) + b**2*c**(7/2)*x**3/(256*
d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x**5/(640*d*sqrt(1 + d*x**2/c)) +
23*b**2*c**(3/2)*x**7/(160*sqrt(1 + d*x**2/c)) + 19*b**2*sqrt(c)*d*x**9/(80
*sqrt(1 + d*x**2/c)) - 3*b**2*c**5*asinh(sqrt(d)*x/sqrt(c))/(256*d**(7/2))
+ b**2*d**2*x**11/(10*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A]
time = 2.15, size = 219, normalized size = 0.93

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2dx^2+\frac{11\,b^2cd^8+20\,abd^9}{d^8}\right)x^2+\frac{3\,b^2c^2d^7+180\,abcd^8+80\,a^2d^9}{d^8}\right)x^2-\frac{5\,(3\,b^2c^3d^6-12\,abc^2d^7-112\,a^2cd^8)}{d^8}\right)x^2+\frac{15\,(3\,b^2c^4d^5-12\,abc^3d^6+16\,a^2c^2d^7)}{d^8}\right)\sqrt{dx^2+c}\,x+\frac{(3\,b^2c^5-12\,abc^4d+16\,a^2c^3d^2)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{256\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d*x^2 + (11*b^2*c*d^8 + 20*a*b*d^9)/d^8)*x^2 + (3*b^
2*c^2*d^7 + 180*a*b*c*d^8 + 80*a^2*d^9)/d^8)*x^2 - 5*(3*b^2*c^3*d^6 - 12*a*
b*c^2*d^7 - 112*a^2*c*d^8)/d^8)*x^2 + 15*(3*b^2*c^4*d^5 - 12*a*b*c^3*d^6 +
16*a^2*c^2*d^7)/d^8)*sqrt(d*x^2 + c)*x + 1/256*(3*b^2*c^5 - 12*a*b*c^4*d +
16*a^2*c^3*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.616 $\int x(a+bx^2)^2\left(c+dx^2\right)^{3/2}\,dx$

Optimal. Leaf size=77

$$\frac{(bc-ad)^2\left(c+dx^2\right)^{5/2}}{5d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{7/2}}{7d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}$$

```
[Out] 1/5*(-a*d+b*c)^2*(d*x^2+c)^(5/2)/d^3-2/7*b*(-a*d+b*c)*(d*x^2+c)^(7/2)/d^3+1
/9*b^2*(d*x^2+c)^(9/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {455, 45}

$$-\frac{2b\left(c+dx^2\right)^{7/2}(bc-ad)}{7d^3}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(7
/2))/(7*d^3) + (b^2*(c + d*x^2)^(9/2))/(9*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx&=\frac{1}{2}\text{Subst}\left(\int(a+bx)^2(c+dx)^{3/2}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{(-bc+ad)^2(c+dx)^{3/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{5/2}}{d^2}+\frac{b^2(c+\right.\right.\\&=\frac{(bc-ad)^2\left(c+dx^2\right)^{5/2}}{5d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{7/2}}{7d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 0.87

$$\frac{(c+dx^2)^{5/2}\left(63a^2d^2+18abd(-2c+5dx^2)+b^2(8c^2-20cdx^2+35d^2x^4)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x^2) + b^2*(8*c^2 - 2
0*c*d*x^2 + 35*d^2*x^4)))/(315*d^3)
```

Maple [A]
time = 0.09, size = 117, normalized size = 1.52

method	result
gosper	$\frac{(d\,x^2+c)^{\frac{5}{2}}(35b^2x^4d^2+90ab\,d^2x^2-20b^2cd\,x^2+63a^2d^2-36abcd+8b^2c^2)}{315d^3}$
default	$b^2\left(\frac{x^4(d\,x^2+c)^{\frac{5}{2}}}{9d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{5}{2}}}{7d}-\frac{2c\left(d\,x^2+c\right)^{\frac{5}{2}}}{35d^2}\right)}{9d}\right)+2ab\left(\frac{x^2(d\,x^2+c)^{\frac{5}{2}}}{7d}-\frac{2c(d\,x^2+c)^{\frac{5}{2}}}{35d^2}\right)+\frac{a^2(d\,x^2+c)^{\frac{5}{2}}}{5d}$
trager	$\frac{(35b^2x^8d^4+90ab\,d^4x^6+50b^2c\,d^3x^6+63a^2x^4d^4+144x^4abc\,d^3+3b^2c^2x^4d^2+126a^2c\,d^3x^2+18ab\,c^2d^2x^2-4b^2c^3d\,x^2+63a^2c^2d^2-36ab\,c^3d+}{315d^3}$
risch	$\frac{(35b^2x^8d^4+90ab\,d^4x^6+50b^2c\,d^3x^6+63a^2x^4d^4+144x^4abc\,d^3+3b^2c^2x^4d^2+126a^2c\,d^3x^2+18ab\,c^2d^2x^2-4b^2c^3d\,x^2+63a^2c^2d^2-36ab\,c^3d+}{315d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/9*x^4*(d*x^2+c)^(5/2)/d-4/9*c/d*(1/7*x^2*(d*x^2+c)^(5/2)/d-2/35*c/d^
2*(d*x^2+c)^(5/2)))+2*a*b*(1/7*x^2*(d*x^2+c)^(5/2)/d-2/35*c/d^2*(d*x^2+c)^(
5/2))+1/5*a^2/d*(d*x^2+c)^(5/2)
```

Maxima [A]
time = 0.29, size = 115, normalized size = 1.49

$$\frac{(dx^2+c)^{\frac{5}{2}}b^2x^4}{9\,d}-\frac{4\,(dx^2+c)^{\frac{5}{2}}b^2cx^2}{63\,d^2}+\frac{2\,(dx^2+c)^{\frac{5}{2}}abx^2}{7\,d}+\frac{8\,(dx^2+c)^{\frac{5}{2}}b^2c^2}{315\,d^3}-\frac{4\,(dx^2+c)^{\frac{5}{2}}abc}{35\,d^2}+\frac{(dx^2+c)^{\frac{5}{2}}a^2}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*(d*x^2 + c)^(5/2)*b^2*x^4/d - 4/63*(d*x^2 + c)^(5/2)*b^2*c*x^2/d^2 + 2/
7*(d*x^2 + c)^(5/2)*a*b*x^2/d + 8/315*(d*x^2 + c)^(5/2)*b^2*c^2/d^3 - 4/35*
(d*x^2 + c)^(5/2)*a*b*c/d^2 + 1/5*(d*x^2 + c)^(5/2)*a^2/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.
time = 1.09, size = 141, normalized size = 1.83

$$\frac{\left(35 b^2 d^4 x^8 + 10 \left(5 b^2 c d^3 + 9 a b d^4\right) x^6 + 8 b^2 c^4 - 36 a b c^3 d + 63 a^2 c^2 d^2 + 3 \left(b^2 c^2 d^2 + 48 a b c d^3 + 21 a^2 d^4\right) x^4 - 2 \left(2 b^2 c^3 d - 9 a b c^2 d^2 - 63 a^2 c d^3\right) x^2\right) \sqrt{d x^2 + c}}{315 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^4*x^8 + 10*(5*b^2*c*d^3 + 9*a*b*d^4)*x^6 + 8*b^2*c^4 - 36*a
*b*c^3*d + 63*a^2*c^2*d^2 + 3*(b^2*c^2*d^2 + 48*a*b*c*d^3 + 21*a^2*d^4)*x^4
 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(66) = 132$.
time = 0.30, size = 303, normalized size = 3.94

$$\begin{cases} \frac{a^2 c^2 \sqrt{c + d x^2}}{5 d} + \frac{2 a^2 c x^2 \sqrt{c + d x^2}}{5} + \frac{a^2 d x^4 \sqrt{c + d x^2}}{5} - \frac{4 a b c^3 \sqrt{c + d x^2}}{35 d^2} + \frac{2 a b c^2 x^2 \sqrt{c + d x^2}}{35 d} + \frac{16 a b c x^4 \sqrt{c + d x^2}}{35} + \frac{2 a b d x^6 \sqrt{c + d x^2}}{7} + \frac{8 b^2 c^4 \sqrt{c + d x^2}}{315 d^3} - \frac{4 b^2 c^3 x^2 \sqrt{c + d x^2}}{315 d^2} + \frac{b^2 c^2 x^4 \sqrt{c + d x^2}}{105 d} + \frac{10 b^2 c x^6 \sqrt{c + d x^2}}{63} + \frac{b^2 d x^8 \sqrt{c + d x^2}}{9} & \text{for}\ d \neq 0 \\ c^{\frac{3}{2}} \left(\frac{a^2 x^2}{2} + \frac{a b x^4}{2} + \frac{b^2 x^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((a**2*c**2*sqrt(c + d*x**2)/(5*d) + 2*a**2*c*x**2*sqrt(c + d*x**2
)/5 + a**2*d*x**4*sqrt(c + d*x**2)/5 - 4*a*b*c**3*sqrt(c + d*x**2)/(35*d**2
) + 2*a*b*c**2*x**2*sqrt(c + d*x**2)/(35*d) + 16*a*b*c*x**4*sqrt(c + d*x**2
)/35 + 2*a*b*d*x**6*sqrt(c + d*x**2)/7 + 8*b**2*c**4*sqrt(c + d*x**2)/(315*
d**3) - 4*b**2*c**3*x**2*sqrt(c + d*x**2)/(315*d**2) + b**2*c**2*x**4*sqrt(
c + d*x**2)/(105*d) + 10*b**2*c*x**6*sqrt(c + d*x**2)/63 + b**2*d*x**8*sqrt
(c + d*x**2)/9, Ne(d, 0)), (c**(3/2)*(a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/
6), True))
```

Giac [A]
time = 1.47, size = 98, normalized size = 1.27

$$\frac{35 \left(d x^2 + c\right)^{\frac{9}{2}} b^2 - 90 \left(d x^2 + c\right)^{\frac{7}{2}} b^2 c + 63 \left(d x^2 + c\right)^{\frac{5}{2}} b^2 c^2 + 90 \left(d x^2 + c\right)^{\frac{7}{2}} a b d - 126 \left(d x^2 + c\right)^{\frac{5}{2}} a b c d + 63 \left(d x^2 + c\right)^{\frac{5}{2}} a^2 d^2}{315 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(d*x^2 + c)^(9/2)*b^2 - 90*(d*x^2 + c)^(7/2)*b^2*c + 63*(d*x^2 +
c)^(5/2)*b^2*c^2 + 90*(d*x^2 + c)^(7/2)*a*b*d - 126*(d*x^2 + c)^(5/2)*a*b*c
*d + 63*(d*x^2 + c)^(5/2)*a^2*d^2)/d^3
```

Mupad [B]
time = 0.37, size = 136, normalized size = 1.77

$$\sqrt{d x^2 + c} \left(\frac{63 a^2 c^2 d^2 - 36 a b c^3 d + 8 b^2 c^4}{315 d^3} + \frac{x^4 \left(63 a^2 d^4 + 144 a b c d^3 + 3 b^2 c^2 d^2\right)}{315 d^3} + \frac{2 b x^6 \left(9 a d + 5 b c\right)}{63} + \frac{b^2 d x^8}{9} + \frac{2 c x^2 \left(63 a^2 d^2 + 9 a b c d - 2 b^2 c^2\right)}{315 d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((8*b^2*c^4 + 63*a^2*c^2*d^2 - 36*a*b*c^3*d)/(315*d^3) +
(x^4*(63*a^2*d^4 + 3*b^2*c^2*d^2 + 144*a*b*c*d^3))/(315*d^3) + (2*b*x^6*(9*
a*d + 5*b*c))/63 + (b^2*d*x^8)/9 + (2*c*x^2*(63*a^2*d^2 - 2*b^2*c^2 + 9*a*b
*c*d))/(315*d^2))
```

3.617 $\int (a+bx^2)^2 (c+dx^2)^{3/2} \, dx$

Optimal. Leaf size=196

$$\frac{c(3b^2c^2-16abcd+48a^2d^2)\,x\sqrt{c+dx^2}}{128d^2}+\frac{(3b^2c^2-16abcd+48a^2d^2)\,x(c+dx^2)^{3/2}}{192d^2}-\frac{b(3bc-10ad)x(c+dx^2}{48d^2}$$

```
[Out] 1/192*(48*a^2*d^2-16*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(3/2)/d^2-1/48*b*(-10*a
*d+3*b*c)*x*(d*x^2+c)^(5/2)/d^2+1/8*b*x*(b*x^2+a)*(d*x^2+c)^(5/2)/d+1/128*c
^2*(48*a^2*d^2-16*a*b*c*d+3*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(
5/2)+1/128*c*(48*a^2*d^2-16*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.08, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {427, 396, 201, 223, 212}

$$\frac{x(c+dx^2)^{3/2}(48a^2d^2-16abcd+3b^2c^2)}{192d^2}+\frac{cx\sqrt{c+dx^2}\,(48a^2d^2-16abcd+3b^2c^2)}{128d^2}+\frac{c^2(48a^2d^2-16abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{5/2}}-\frac{bx(c+dx^2)^{5/2}(3bc-10ad)}{48d^2}+\frac{bx(a+bx^2)(c+dx^2)^{5/2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (c*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*Sqrt[c + d*x^2])/(128*d^2) + ((3
*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*(c + d*x^2)^(3/2))/(192*d^2) - (b*(3*
b*c - 10*a*d)*x*(c + d*x^2)^(5/2))/(48*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^
(5/2))/(8*d) + (c^2*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/(128*d^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}\, dx = \frac{bx(a+bx^2)\left(c+dx^2\right)^{5/2}}{8d} + \frac{\int \left(c+dx^2\right)^{3/2}\left(-a(bc-8ad)-b(3bc-10ad)x^2\right)}{8d}$$

$$= -\frac{b(3bc-10ad)x(c+dx^2)^{5/2}}{48d^2} + \frac{bx(a+bx^2)\left(c+dx^2\right)^{5/2}}{8d} - \frac{(-bc(3bc-10ac}{}$$

$$= \frac{\left(3b^2c^2-16abcd+48a^2d^2\right)x(c+dx^2)^{3/2}}{192d^2} - \frac{b(3bc-10ad)x(c+dx^2)^{5/2}}{48d^2} + \frac{bx}{}$$

$$= \frac{c(3b^2c^2-16abcd+48a^2d^2)\,x\sqrt{c+dx^2}}{128d^2} + \frac{(3b^2c^2-16abcd+48a^2d^2)\,x(c+}{192d^2}$$

$$= \frac{c(3b^2c^2-16abcd+48a^2d^2)\,x\sqrt{c+dx^2}}{128d^2} + \frac{(3b^2c^2-16abcd+48a^2d^2)\,x(c+}{192d^2}$$

$$= \frac{c(3b^2c^2-16abcd+48a^2d^2)\,x\sqrt{c+dx^2}}{128d^2} + \frac{(3b^2c^2-16abcd+48a^2d^2)\,x(c+}{192d^2}$$

Mathematica [A]

time = 0.23, size = 158, normalized size = 0.81

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\left(48a^2d^2(5c+2dx^2)+16abd(3c^2+14cdx^2+8d^2x^4)+b^2(-9c^3+6c^2dx^2+72cd^2x^4+48d^3x^6)\right)-3c^2(3b^2c^2-16abcd+48a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{384d^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]

[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(5*c + 2*d*x^2) + 16*a*b*d*(3*c^2 + 14*c*d*x^2 + 8*d^2*x^4) + b^2*(-9*c^3 + 6*c^2*d*x^2 + 72*c*d^2*x^4 + 48*d^3*x^6)) - 3*c^2*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(384*d^(5/2))

Maple [A]

time = 0.09, size = 235, normalized size = 1.20

method	result
risch	$\frac{x\left(48b^2x^6d^3+128ab\,d^3x^4+72b^2c\,d^2x^4+96a^2d^3x^2+224abc\,d^2x^2+6b^2c^2d\,x^2+240a^2c\,d^2+48ab\,c^2d-9b^2c^3\right)\sqrt{d\,x^2+c}}{384d^2}+\frac{3c^2\ln\left(x\right.}{}$
default	$b^2\left(\frac{x^3(d\,x^2+c)^{\frac{5}{2}}}{8d}-\frac{3c\left(\frac{x(d\,x^2+c)^{\frac{5}{2}}}{6d}-\frac{c\left(\frac{x(d\,x^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6d}\right)}{8d}\right)+2ab\left(\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] b^2*(1/8*x^3*(d*x^2+c)^(5/2)/d-3/8*c/d*(1/6*x*(d*x^2+c)^(5/2)/d-1/6*c/d*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))))+2*a*b*(1/6*x*(d*x^2+c)^(5/2)/d-1/6*c/d*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+a^2*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))

Maxima [A]
time = 0.28, size = 227, normalized size = 1.16

$$\frac{(dx^2+c)^{\frac{5}{2}}b^2x^3}{8\,d}+\frac{1}{4}\,(dx^2+c)^{\frac{3}{2}}a^2x+\frac{3}{8}\,\sqrt{dx^2+c}\,a^2cx-\frac{(dx^2+c)^{\frac{5}{2}}b^2cx}{16\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}b^2c^2x}{64\,d^2}+\frac{3\,\sqrt{dx^2+c}\,b^2c^3x}{128\,d^2}+\frac{(dx^2+c)^{\frac{5}{2}}abx}{3\,d}-\frac{(dx^2+c)^{\frac{3}{2}}abcx}{12\,d}-\frac{\sqrt{dx^2+c}\,abc^2x}{8\,d}+\frac{3\,b^2c^4\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{5}{2}}}-\frac{abc^3\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}+\frac{3\,a^2c^2\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^2 + c)^(5/2)*b^2*x^3/d + 1/4*(d*x^2 + c)^(3/2)*a^2*x + 3/8*sqrt(d*
x^2 + c)*a^2*c*x - 1/16*(d*x^2 + c)^(5/2)*b^2*c*x/d^2 + 1/64*(d*x^2 + c)^(3
/2)*b^2*c^2*x/d^2 + 3/128*sqrt(d*x^2 + c)*b^2*c^3*x/d^2 + 1/3*(d*x^2 + c)^(
5/2)*a*b*x/d - 1/12*(d*x^2 + c)^(3/2)*a*b*c*x/d - 1/8*sqrt(d*x^2 + c)*a*b*c
^2*x/d + 3/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a*b*c^3*arcsinh
(d*x/sqrt(c*d))/d^(3/2) + 3/8*a^2*c^2*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

Fricas [A]
time = 1.94, size = 344, normalized size = 1.76

$$\left[\frac{3\,(3\,b^2c^4-16\,abc^3d+48\,a^2c^2d^2)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(48\,b^2d^4x^7+8\,(9\,b^2cd^3+16\,abd^4)x^5+2\,(3\,b^2c^2d^2+112\,abcd^3+48\,a^2d^4)x^3-3\,(3\,b^2c^3d-16\,abc^2d^2-80\,a^2cd^3)x)\sqrt{dx^2+c}}{768\,d^3},\ -\frac{3\,(3\,b^2c^4-16\,abc^3d+48\,a^2c^2d^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(48\,b^2d^4x^7+8\,(9\,b^2cd^3+16\,abd^4)x^5+2\,(3\,b^2c^2d^2+112\,abcd^3+48\,a^2d^4)x^3-3\,(3\,b^2c^3d-16\,abc^2d^2-80\,a^2cd^3)x)\sqrt{dx^2+c}}{384\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 + 8*(9*b^2*c*d^3 + 1
6*a*b*d^4)*x^5 + 2*(3*b^2*c^2*d^2 + 112*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3*
b^2*c^3*d - 16*a*b*c^2*d^2 - 80*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/384*
(3*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/s
qrt(d*x^2 + c)) - (48*b^2*d^4*x^7 + 8*(9*b^2*c*d^3 + 16*a*b*d^4)*x^5 + 2*(3
*b^2*c^2*d^2 + 112*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3*b^2*c^3*d - 16*a*b*c^
2*d^2 - 80*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(190) = 380.
time = 35.65, size = 440, normalized size = 2.24

$$\frac{a^2c^{\frac{3}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{a^2c^{\frac{3}{2}}x}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2\sqrt{c}\,dx^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{a^2d^2x^5}{4\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}+\frac{abc^{\frac{5}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{17abc^{\frac{3}{2}}x^3}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{11ab\sqrt{c}\,dx^5}{12\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}}+\frac{abd^2x^7}{3\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}-\frac{3b^2c^{\frac{7}{2}}x}{128d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{5}{2}}x^3}{128d\sqrt{1+\frac{dx^2}{c}}}+\frac{13b^2c^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2\sqrt{c}\,dx^7}{16\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{5}{2}}}+\frac{b^2d^2x^9}{8\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*x*sqrt(1 + d*x**2/c)/2 + a**2*c**(3/2)*x/(8*sqrt(1 + d*x**2/c
)) + 3*a**2*sqrt(c)*d*x**3/(8*sqrt(1 + d*x**2/c)) + 3*a**2*c**2*asinh(sqrt(
d)*x/sqrt(c))/(8*sqrt(d)) + a**2*d**2*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) +
 a*b*c**(5/2)*x/(8*d*sqrt(1 + d*x**2/c)) + 17*a*b*c**(3/2)*x**3/(24*sqrt(1
```

```
+ d*x**2/c)) + 11*a*b*sqrt(c)*d*x**5/(12*sqrt(1 + d*x**2/c)) - a*b*c**3*asi
nh(sqrt(d)*x/sqrt(c))/(8*d**(3/2)) + a*b*d**2*x**7/(3*sqrt(c)*sqrt(1 + d*x*
*2/c)) - 3*b**2*c**(7/2)*x/(128*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x*
*3/(128*d*sqrt(1 + d*x**2/c)) + 13*b**2*c**(3/2)*x**5/(64*sqrt(1 + d*x**2/c
)) + 5*b**2*sqrt(c)*d*x**7/(16*sqrt(1 + d*x**2/c)) + 3*b**2*c**4*asinh(sqrt
(d)*x/sqrt(c))/(128*d**(5/2)) + b**2*d**2*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c
))
```

Giac [A]
time = 1.59, size = 175, normalized size = 0.89

$$\frac{1}{384}\left(2\left(4\left(6\,b^{2}dx^{2}+\frac{9\,b^{2}cd^{6}+16\,abd^{7}}{d^{6}}\right)x^{2}+\frac{3\,b^{2}c^{2}d^{5}+112\,abcd^{6}+48\,a^{2}d^{7}}{d^{6}}\right)x^{2}-\frac{3\,(3\,b^{2}c^{3}d^{4}-16\,abc^{2}d^{5}-80\,a^{2}cd^{6})}{d^{6}}\right)\sqrt{dx^{2}+c}\,x-\frac{(3\,b^{2}c^{4}-16\,abc^{3}d+48\,a^{2}c^{2}d^{2})\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^{2}+c}\right|\right)}{128\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*d*x^2 + (9*b^2*c*d^6 + 16*a*b*d^7)/d^6)*x^2 + (3*b^2*c^2
*d^5 + 112*a*b*c*d^6 + 48*a^2*d^7)/d^6)*x^2 - 3*(3*b^2*c^3*d^4 - 16*a*b*c^2
*d^5 - 80*a^2*c*d^6)/d^6)*sqrt(d*x^2 + c)*x - 1/128*(3*b^2*c^4 - 16*a*b*c^3
*d + 48*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.618 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x} \, dx$

Optimal. Leaf size=111

$$a^2c\sqrt{c+dx^2}+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

```
[Out] 1/3*a^2*(d*x^2+c)^(3/2)-1/5*b*(-2*a*d+b*c)*(d*x^2+c)^(5/2)/d^2+1/7*b^2*(d*x
^2+c)^(7/2)/d^2-a^2*c^(3/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))+a^2*c*(d*x^2+c
)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 90, 52, 65, 214}

$$-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}+a^2c\sqrt{c+dx^2}-\frac{b\left(c+dx^2\right)^{5/2}(bc-2ad)}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

```
[Out] a^2*c*Sqrt[c + d*x^2] + (a^2*(c + d*x^2)^(3/2))/3 - (b*(b*c - 2*a*d)*(c + d
*x^2)^(5/2))/(5*d^2) + (b^2*(c + d*x^2)^(7/2))/(7*d^2) - a^2*c^(3/2)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x}dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x}dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b(bc-2ad)(c+dx)^{3/2}}{d} + \frac{a^2(c+dx)^{3/2}}{x} + \frac{b^2(c+dx)^{5/2}}{d}\right) d$$

$$= -\frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7/2}}{7d^2} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}dx,\right.$$

$$= \frac{1}{3}a^2(c+dx^2)^{3/2} - \frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7/2}}{7d^2} + \frac{1}{2}(a^2c)\text{Subs}$$

$$= a^2c\sqrt{c+dx^2} + \frac{1}{3}a^2(c+dx^2)^{3/2} - \frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7}}{7d^2}$$

$$= a^2c\sqrt{c+dx^2} + \frac{1}{3}a^2(c+dx^2)^{3/2} - \frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7}}{7d^2}$$

$$= a^2c\sqrt{c+dx^2} + \frac{1}{3}a^2(c+dx^2)^{3/2} - \frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7}}{7d^2}$$

Mathematica [A]
time = 0.14, size = 103, normalized size = 0.93

$$\frac{\sqrt{c+dx^2}\left(42abd(c+dx^2)^2 - 3b^2(2c-5dx^2)(c+dx^2)^2 + 35a^2d^2(4c+dx^2)\right)}{105d^2} - a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[c + d*x^2]*(42*a*b*d*(c + d*x^2)^2 - 3*b^2*(2*c - 5*d*x^2)*(c + d*x^2
)^2 + 35*a^2*d^2*(4*c + d*x^2)))/(105*d^2) - a^2*c^(3/2)*ArcTanh[Sqrt[c + d
*x^2]/Sqrt[c]]
```

Maple [A]
time = 0.10, size = 111, normalized size = 1.00

method	result
default	$b^{2}\left(\frac{x^{2}\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{7d}-\frac{2c\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{35d^{2}}\right)+\frac{2ab\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{5d}+a^{2}\left(\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{3}+c\left(\sqrt{d\,x^{2}+c}\,-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d}}{x}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/7*x^2*(d*x^2+c)^(5/2)/d-2/35*c/d^2*(d*x^2+c)^(5/2))+2/5*a*b/d*(d*x^2
+c)^(5/2)+a^2*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(
1/2)*(d*x^2+c)^(1/2))/x)))
```

Maxima [A]
time = 0.30, size = 103, normalized size = 0.93

$$\frac{(dx^2+c)^{\frac{5}{2}}b^2x^2}{7\,d}-a^2c^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\frac{1}{3}\,(dx^2+c)^{\frac{3}{2}}a^2+\sqrt{dx^2+c}\,a^2c-\frac{2\,(dx^2+c)^{\frac{5}{2}}b^2c}{35\,d^2}+\frac{2\,(dx^2+c)^{\frac{5}{2}}ab}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/7*(d*x^2 + c)^(5/2)*b^2*x^2/d - a^2*c^(3/2)*arcsinh(c/(sqrt(c*d)*abs(x)))
 + 1/3*(d*x^2 + c)^(3/2)*a^2 + sqrt(d*x^2 + c)*a^2*c - 2/35*(d*x^2 + c)^(5/
2)*b^2*c/d^2 + 2/5*(d*x^2 + c)^(5/2)*a*b/d
```

Fricas [A]
time = 1.27, size = 282, normalized size = 2.54

$$\left[\frac{105\,a^2c^{\frac{3}{2}}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,(15\,b^2d^3x^6-6\,b^2c^3+42\,abc^2d+140\,a^2cd^2+6\,(4\,b^2cd^2+7\,abd^3)x^4+(3\,b^2c^2d+84\,abcd^2+35\,a^2d^3)x^2)\sqrt{dx^2+c}}{210\,d^2},\ \frac{105\,a^2\sqrt{-c}\,cd^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(15\,b^2d^3x^6-6\,b^2c^3+42\,abc^2d+140\,a^2cd^2+6\,(4\,b^2cd^2+7\,abd^3)x^4+(3\,b^2c^2d+84\,abcd^2+35\,a^2d^3)x^2)\sqrt{dx^2+c}}{105\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/210*(105*a^2*c^(3/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2) + 2*(15*b^2*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^2 + 6*(4*
b^2*c*d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 84*a*b*c*d^2 + 35*a^2*d^3)*x^2)
```

```
*sqrt(d*x^2 + c))/d^2, 1/105*(105*a^2*sqrt(-c)*c*d^2*arctan(sqrt(-c)/sqrt(d
*x^2 + c)) + (15*b^2*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^2 + 6
*(4*b^2*c*d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 84*a*b*c*d^2 + 35*a^2*d^3)*
x^2)*sqrt(d*x^2 + c))/d^2]
```

Sympy [A]
time = 41.81, size = 109, normalized size = 0.98

$$\frac{a^2c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}}+a^2c\sqrt{c+dx^2}+\frac{a^2(c+dx^2)^{\frac{3}{2}}}{3}+\frac{b^2(c+dx^2)^{\frac{7}{2}}}{7d^2}+\frac{(c+dx^2)^{\frac{5}{2}}\cdot(4abd-2b^2c)}{10d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x,x)
```

```
[Out] a**2*c**2*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + a**2*c*sqrt(c + d*x**2
) + a**2*(c + d*x**2)**(3/2)/3 + b**2*(c + d*x**2)**(7/2)/(7*d**2) + (c + d
*x**2)**(5/2)*(4*a*b*d - 2*b**2*c)/(10*d**2)
```

Giac [A]
time = 1.08, size = 121, normalized size = 1.09

$$\frac{a^2c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{15\,(dx^2+c)^{\frac{7}{2}}b^2d^{12}-21\,(dx^2+c)^{\frac{5}{2}}b^2cd^{12}+42\,(dx^2+c)^{\frac{5}{2}}abd^{13}+35\,(dx^2+c)^{\frac{3}{2}}a^2d^{14}+105\,\sqrt{dx^2+c}\,a^2cd^{14}}{105\,d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/105*(15*(d*x^2 + c)^(
7/2)*b^2*d^12 - 21*(d*x^2 + c)^(5/2)*b^2*c*d^12 + 42*(d*x^2 + c)^(5/2)*a*b*
d^13 + 35*(d*x^2 + c)^(3/2)*a^2*d^14 + 105*sqrt(d*x^2 + c)*a^2*c*d^14)/d^14
```

Mupad [B]
time = 0.34, size = 191, normalized size = 1.72

$$(d\,x^2+c)^{3/2}\,\left(\frac{(a\,d-b\,c)^2}{3\,d^2}-\frac{c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)}{3}\right)-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{5\,d^2}-\frac{b^2\,c}{5\,d^2}\right)\,(d\,x^2+c)^{5/2}+\frac{b^2\,(d\,x^2+c)^{7/2}}{7\,d^2}+c\,\sqrt{d\,x^2+c}\,\left(\frac{(a\,d-b\,c)^2}{d^2}-c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\right)+a^2\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x)
```

```
[Out] (c + d*x^2)^(3/2)*((a*d - b*c)^2/(3*d^2) - (c*((2*b^2*c - 2*a*b*d)/d^2 - (b
^2*c)/d^2))/3) - ((2*b^2*c - 2*a*b*d)/(5*d^2) - (b^2*c)/(5*d^2))*(c + d*x^2
)^(5/2) + a^2*c^(3/2)*atan(((c + d*x^2)^(1/2)*1i)/c^(1/2))*1i + (b^2*(c + d
*x^2)^(7/2))/(7*d^2) + c*(c + d*x^2)^(1/2)*((a*d - b*c)^2/d^2 - c*((2*b^2*c
 - 2*a*b*d)/d^2 - (b^2*c)/d^2))
```

3.619 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=175

$$-\frac{(b^2c^2-12ad(bc+2ad))\,x\sqrt{c+dx^2}}{16d}-\frac{(b^2c^2-12ad(bc+2ad))\,x(c+dx^2)^{3/2}}{24cd}-\frac{a^2(c+dx^2)^{5/2}}{cx}+\frac{b^2x(c+dx^2)^{5/2}}{6d}$$

```
[Out] -1/24*(b^2*c^2-12*a*d*(2*a*d+b*c))*x*(d*x^2+c)^(3/2)/c/d-a^2*(d*x^2+c)^(5/2
)/c/x+1/6*b^2*x*(d*x^2+c)^(5/2)/d-1/16*c*(b^2*c^2-12*a*d*(2*a*d+b*c))*arcta
nh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-1/16*(b^2*c^2-12*a*d*(2*a*d+b*c))*x*(
d*x^2+c)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 172, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {473, 396, 201, 223, 212}

$$-\frac{a^2(c+dx^2)^{5/2}}{cx}-\frac{c(b^2c^2-12ad(2ad+bc))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{3/2}}-\frac{x\sqrt{c+dx^2}\,(b^2c^2-12ad(2ad+bc))}{16d}-\frac{1}{24}x(c+dx^2)^{3/2}\left(\frac{b^2c}{d}-\frac{12a(2ad+bc)}{c}\right)+\frac{b^2x(c+dx^2)^{5/2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

```
[Out] -1/16*((b^2*c^2 - 12*a*d*(b*c + 2*a*d))*x*Sqrt[c + d*x^2])/d - (((b^2*c)/d
- (12*a*(b*c + 2*a*d))/c)*x*(c + d*x^2)^(3/2))/24 - (a^2*(c + d*x^2)^(5/2))
/(c*x) + (b^2*x*(c + d*x^2)^(5/2))/(6*d) - (c*(b^2*c^2 - 12*a*d*(b*c + 2*a*
d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^2}\,dx &= -\frac{a^2(c+dx^2)^{5/2}}{cx} + \frac{\int (2a(bc+2ad)+b^2cx^2)(c+dx^2)^{3/2}\,dx}{c} \\
&= -\frac{a^2(c+dx^2)^{5/2}}{cx} + \frac{b^2x(c+dx^2)^{5/2}}{6d} - \frac{(b^2c^2-12ad(bc+2ad))\int (c+dx^2)^{3/}}{6cd} \\
&= -\frac{1}{24}\left(\frac{b^2c}{d} - \frac{12a(bc+2ad)}{c}\right)x(c+dx^2)^{3/2} - \frac{a^2(c+dx^2)^{5/2}}{cx} + \frac{b^2x(c+dx^2}{6d} \\
&= -\frac{(b^2c^2-12ad(bc+2ad))\,x\sqrt{c+dx^2}}{16d} - \frac{1}{24}\left(\frac{b^2c}{d} - \frac{12a(bc+2ad)}{c}\right)x(c+ \\
&= -\frac{(b^2c^2-12ad(bc+2ad))\,x\sqrt{c+dx^2}}{16d} - \frac{1}{24}\left(\frac{b^2c}{d} - \frac{12a(bc+2ad)}{c}\right)x(c+ \\
&= -\frac{(b^2c^2-12ad(bc+2ad))\,x\sqrt{c+dx^2}}{16d} - \frac{1}{24}\left(\frac{b^2c}{d} - \frac{12a(bc+2ad)}{c}\right)x(c+
\end{aligned}
$$

Mathematica [A]

time = 0.25, size = 139, normalized size = 0.79

$$
\frac{\sqrt{d}\,\sqrt{c+dx^2}\,(24a^2d(-2c+dx^2)+12abdx^2(5c+2dx^2)+b^2x^2(3c^2+14cdx^2+8d^2x^4))+3c(b^2c^2-12abcd-24a^2d^2)\,x\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{48d^{3/2}x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

[Out] (Sqrt[d]*Sqrt[c + d*x^2]*(24*a^2*d*(-2*c + d*x^2) + 12*a*b*d*x^2*(5*c + 2*d
*x^2) + b^2*x^2*(3*c^2 + 14*c*d*x^2 + 8*d^2*x^4)) + 3*c*(b^2*c^2 - 12*a*b*c
*d - 24*a^2*d^2)*x*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(48*d^(3/2)*x)

Maple [A]
time = 0.10, size = 213, normalized size = 1.22

method	result
risch	$-\frac{\sqrt{dx^2+c}\,\left(-8b^2d^2x^6-24ab\,d^2x^4-14b^2cd\,x^4-24a^2d^2x^2-60abcd\,x^2-3b^2c^2x^2+48a^2cd\right)}{48dx}+\frac{3c\sqrt{d}\,\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right.}{2}$
default	$b^2\left(\frac{x\left(dx^2+c\right)^{\frac{5}{2}}}{6d}-\frac{c\left(\frac{x\left(dx^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6d}\right)+2ab\left(\frac{x\left(dx^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x,method=_RETURNVERBOSE)

[Out] b^2*(1/6*x*(d*x^2+c)^(5/2)/d-1/6*c/d*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d
*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+2*a*b*(1/4*x*(
d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x
^2+c)^(1/2))))+a^2*(-1/c/x*(d*x^2+c)^(5/2)+4*d/c*(1/4*x*(d*x^2+c)^(3/2)+3/4
c(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))

Maxima [A]
time = 0.32, size = 178, normalized size = 1.02

$$\frac{1}{2}\left(dx^2+c\right)^{\frac{3}{2}}abx+\frac{3}{4}\sqrt{dx^2+c}\,abcx+\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2x}{6\,d}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2cx}{24\,d}-\frac{\sqrt{dx^2+c}\,b^2c^2x}{16\,d}+\frac{3}{2}\sqrt{dx^2+c}\,a^2dx-\frac{b^2c^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{3}{2}}}+\frac{3\,abc^2\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,\sqrt{d}}+\frac{3}{2}a^2c\sqrt{d}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="maxima")

[Out] 1/2*(d*x^2 + c)^(3/2)*a*b*x + 3/4*sqrt(d*x^2 + c)*a*b*c*x + 1/6*(d*x^2 + c)
^(5/2)*b^2*x/d - 1/24*(d*x^2 + c)^(3/2)*b^2*c*x/d - 1/16*sqrt(d*x^2 + c)*b^
2*c^2*x/d + 3/2*sqrt(d*x^2 + c)*a^2*d*x - 1/16*b^2*c^3*arcsinh(d*x/sqrt(c*d
))/d^(3/2) + 3/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/sqrt(d) + 3/2*a^2*c*sqrt(d)
*arcsinh(d*x/sqrt(c*d)) - (d*x^2 + c)^(3/2)*a^2/x

Fricas [A]
time = 1.20, size = 293, normalized size = 1.67

$$\left[-\frac{3\left(b^{2}c^{3}-12\,abc^{2}d-24\,a^{2}cd^{2}\right)\sqrt{d}\,x\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)-2\left(8\,b^{2}d^{3}x^{6}-48\,a^{2}cd^{2}+2\left(7\,b^{2}cd^{2}+12\,abd^{3}\right)x^{4}+3\left(b^{2}c^{2}d+20\,abcd^{2}+8\,a^{2}d^{3}\right)x^{2}\right)\sqrt{dx^{2}+c}}{96\,d^{2}x},\ \frac{3\left(b^{2}c^{3}-12\,abc^{2}d-24\,a^{2}cd^{2}\right)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)+\left(8\,b^{2}d^{3}x^{6}-48\,a^{2}cd^{2}+2\left(7\,b^{2}cd^{2}+12\,abd^{3}\right)x^{4}+3\left(b^{2}c^{2}d+20\,abcd^{2}+8\,a^{2}d^{3}\right)x^{2}\right)\sqrt{dx^{2}+c}}{48\,d^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*sqrt(d)*x*log(-2*d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^2*d^3*x^6 - 48*a^2*c*d^2 + 2*(7*b
^2*c*d^2 + 12*a*b*d^3)*x^4 + 3*(b^2*c^2*d + 20*a*b*c*d^2 + 8*a^2*d^3)*x^2)*
sqrt(d*x^2 + c))/(d^2*x), 1/48*(3*(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*s
qrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^3*x^6 - 48*a^2*c*d^
2 + 2*(7*b^2*c*d^2 + 12*a*b*d^3)*x^4 + 3*(b^2*c^2*d + 20*a*b*c*d^2 + 8*a^2*
d^3)*x^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(156) = 312$.
time = 12.77, size = 367, normalized size = 2.10

$$-\frac{a^{2}c^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^{2}}{c}}}+\frac{a^{2}\sqrt{c}\,dx\sqrt{1+\frac{dx^{2}}{c}}}{2}-\frac{a^{2}\sqrt{c}\,dx}{\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3a^{2}c\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2}+abc^{\frac{3}{2}}x\sqrt{1+\frac{dx^{2}}{c}}+\frac{abc^{\frac{3}{2}}x}{4\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3ab\sqrt{c}\,dx^{3}}{4\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3abc^{2}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{4\sqrt{d}}+\frac{abd^{2}x^{5}}{2\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{b^{2}c^{\frac{5}{2}}x}{16d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{17b^{2}c^{\frac{3}{2}}x^{3}}{48\sqrt{1+\frac{dx^{2}}{c}}}+\frac{11b^{2}\sqrt{c}\,dx^{5}}{24\sqrt{1+\frac{dx^{2}}{c}}}-\frac{b^{2}c^{3}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16d^{\frac{3}{2}}}+\frac{b^{2}d^{2}x^{7}}{6\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**2,x)
```

```
[Out] -a**2*c**(3/2)/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(c)*d*x*sqrt(1 + d*x**2/c)
/2 - a**2*sqrt(c)*d*x/sqrt(1 + d*x**2/c) + 3*a**2*c*sqrt(d)*asinh(sqrt(d)*x
/sqrt(c))/2 + a*b*c**(3/2)*x*sqrt(1 + d*x**2/c) + a*b*c**(3/2)*x/(4*sqrt(1
+ d*x**2/c)) + 3*a*b*sqrt(c)*d*x**3/(4*sqrt(1 + d*x**2/c)) + 3*a*b*c**2*asi
nh(sqrt(d)*x/sqrt(c))/(4*sqrt(d)) + a*b*d**2*x**5/(2*sqrt(c)*sqrt(1 + d*x**
2/c)) + b**2*c**(5/2)*x/(16*d*sqrt(1 + d*x**2/c)) + 17*b**2*c**(3/2)*x**3/(
48*sqrt(1 + d*x**2/c)) + 11*b**2*sqrt(c)*d*x**5/(24*sqrt(1 + d*x**2/c)) - b
**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*d**(3/2)) + b**2*d**2*x**7/(6*sqrt(c)
*sqrt(1 + d*x**2/c))
```

Giac [A]
time = 1.63, size = 173, normalized size = 0.99

$$\frac{2\,a^{2}c^{2}\sqrt{d}}{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c}+\frac{1}{48}\left(2\left(4\,b^{2}dx^{2}+\frac{7\,b^{2}cd^{4}+12\,abd^{5}}{d^{4}}\right)x^{2}+\frac{3\left(b^{2}c^{2}d^{3}+20\,abcd^{4}+8\,a^{2}d^{5}\right)}{d^{4}}\right)\sqrt{dx^{2}+c}\,x+\frac{\left(b^{2}c^{3}\sqrt{d}-12\,abc^{2}d^{\frac{3}{2}}-24\,a^{2}cd^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}\right)}{32\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c^2*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/48*(2*(4*b^2*d*
x^2 + (7*b^2*c*d^4 + 12*a*b*d^5)/d^4)*x^2 + 3*(b^2*c^2*d^3 + 20*a*b*c*d^4 +
```

```
 8*a^2*d^5)/d^4)*sqrt(d*x^2 + c)*x + 1/32*(b^2*c^3*sqrt(d) - 12*a*b*c^2*d^(
3/2) - 24*a^2*c*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^2\,(d\,x^2+c)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2, x)
```

3.620 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=136

$$\frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2}+\frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d}-\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2}-\frac{1}{2}a\sqrt{c}\,(4bc+3ad)\tanh$$

```
[Out] 1/6*a*(3*a*d+4*b*c)*(d*x^2+c)^(3/2)/c+1/5*b^2*(d*x^2+c)^(5/2)/d-1/2*a^2*(d*
x^2+c)^(5/2)/c/x^2-1/2*a*(3*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(
1/2)+1/2*a*(3*a*d+4*b*c)*(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 81, 52, 65, 214}

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2}+\frac{a\left(c+dx^2\right)^{3/2}(3ad+4bc)}{6c}+\frac{1}{2}a\sqrt{c+dx^2}\,(3ad+4bc)-\frac{1}{2}a\sqrt{c}\,(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

```
[Out] (a*(4*b*c + 3*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 3*a*d)*(c + d*x^2)^(3/2
))/(6*c) + (b^2*(c + d*x^2)^(5/2))/(5*d) - (a^2*(c + d*x^2)^(5/2))/(2*c*x^2
) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{a^2(c+dx^2)^{5/2}}{2cx^2} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(4bc+3ad)+b^2cx\right)(c+dx)^{3/2}}{x}\,dx, x, x^2\right)}{2c}$$

$$= \frac{b^2(c+dx^2)^{5/2}}{5d} - \frac{a^2(c+dx^2)^{5/2}}{2cx^2} + \frac{(a(4bc+3ad))\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, x^2 \right.}{4c}$$

$$= \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2(c+dx^2)^{5/2}}{5d} - \frac{a^2(c+dx^2)^{5/2}}{2cx^2} + \frac{1}{4}(a(4bc+3$$

$$= \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2} + \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2(c+dx^2)^{5/2}}{5d} - $$

$$= \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2} + \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2(c+dx^2)^{5/2}}{5d} - $$

$$= \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2} + \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2(c+dx^2)^{5/2}}{5d} - $$

Mathematica [A]
time = 0.19, size = 108, normalized size = 0.79

$$\frac{\sqrt{c+dx^2}\left(-15a^2d(c-2dx^2)+6b^2x^2(c+dx^2)^2+20abdx^2(4c+dx^2)\right)}{30dx^2} - \frac{1}{2}a\sqrt{c}\,(4bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*d*(c - 2*d*x^2) + 6*b^2*x^2*(c + d*x^2)^2 + 20*a*
b*d*x^2*(4*c + d*x^2)))/(30*d*x^2) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[Sqr
t[c + d*x^2]/Sqrt[c]])/2
```

Maple [A]
time = 0.12, size = 155, normalized size = 1.14

method	result
default	$\frac{b^2(dx^2+c)^{\frac{5}{2}}}{5d} + a^2\left(-\frac{(dx^2+c)^{\frac{5}{2}}}{2cx^2} + \frac{3d\left(\frac{(dx^2+c)^{\frac{3}{2}}}{3}+c\left(\sqrt{dx^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{dx^2+c}}{x}\right)\right)\right)}{2c}\right) + 2a$

risch	$-\frac{c a^2 \sqrt{d x^2+c}}{2x^2}+\frac{b^2 d x^4 \sqrt{d x^2+c}}{5}+\frac{2b^2 c x^2 \sqrt{d x^2+c}}{5}+\frac{b^2 c^2 \sqrt{d x^2+c}}{5d}+\frac{2abd x^2 \sqrt{d x^2+c}}{3}+\frac{8abc$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*(d*x^2+c)^(5/2)/d+a^2*(-1/2/c/x^2*(d*x^2+c)^(5/2)+3/2*d/c*(1/3*(d*x
^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x
))))+2*a*b*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2
)*(d*x^2+c)^(1/2))/x)))
```

Maxima [A]
time = 0.30, size = 138, normalized size = 1.01

$$-2\,abc^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{3}{2}\,a^2\sqrt{c}\,d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\frac{2}{3}\,(dx^2+c)^{\frac{3}{2}}ab+2\,\sqrt{dx^2+c}\,abc+\frac{(dx^2+c)^{\frac{5}{2}}b^2}{5\,d}+\frac{3}{2}\,\sqrt{dx^2+c}\,a^2d+\frac{(dx^2+c)^{\frac{3}{2}}a^2d}{2\,c}-\frac{(dx^2+c)^{\frac{5}{2}}a^2}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*a*b*c^(3/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 3/2*a^2*sqrt(c)*d*arcsinh(c/
(sqrt(c*d)*abs(x))) + 2/3*(d*x^2 + c)^(3/2)*a*b + 2*sqrt(d*x^2 + c)*a*b*c +
 1/5*(d*x^2 + c)^(5/2)*b^2/d + 3/2*sqrt(d*x^2 + c)*a^2*d + 1/2*(d*x^2 + c)^
(3/2)*a^2*d/c - 1/2*(d*x^2 + c)^(5/2)*a^2/(c*x^2)
```

Fricas [A]
time = 1.32, size = 267, normalized size = 1.96

$$\left[\frac{15\,(4\,abcd+3\,a^2d^2)\sqrt{c}\,x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2c}{x^2}\right)+2\,(6\,b^2d^2x^6+4\,(3\,b^2cd+5\,abd^2)x^4-15\,a^2cd+2\,(3\,b^2c^2+40\,abcd+15\,a^2d^2)x^2)\sqrt{dx^2+c}}{60\,dx^2},\ \frac{15\,(4\,abcd+3\,a^2d^2)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(6\,b^2d^2x^6+4\,(3\,b^2cd+5\,abd^2)x^4-15\,a^2cd+2\,(3\,b^2c^2+40\,abcd+15\,a^2d^2)x^2)\sqrt{dx^2+c}}{30\,dx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 +
c)*sqrt(c) + 2*c)/x^2) + 2*(6*b^2*d^2*x^6 + 4*(3*b^2*c*d + 5*a*b*d^2)*x^4 -
 15*a^2*c*d + 2*(3*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*x^2)*sqrt(d*x^2 + c))
/(d*x^2), 1/30*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqr
t(d*x^2 + c)) + (6*b^2*d^2*x^6 + 4*(3*b^2*c*d + 5*a*b*d^2)*x^4 - 15*a^2*c*d
 + 2*(3*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2)]
```

Sympy [A]
time = 31.32, size = 303, normalized size = 2.23

$$-\frac{3a^2\sqrt{c}\,d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2}-\frac{a^2c\sqrt{d}\,\sqrt{\frac{c}{dx^2}+1}}{2x}+\frac{a^2c\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}-2abc^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)+\frac{2abc^2}{\sqrt{d}\,x\sqrt{\frac{c}{dx^2}+1}}+\frac{2abc\sqrt{d}\,x}{\sqrt{\frac{c}{dx^2}+1}}+2abd\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\ d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+b^2c\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\ d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+b^2d\left(\begin{cases}-\frac{2c^2\sqrt{c+dx^2}}{15d^2}+\frac{cx^2\sqrt{c+dx^2}}{15d}+\frac{x^4\sqrt{c+dx^2}}{5} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}\,x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**3,x)
```

```
[Out] -3*a**2*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c*sqrt(d)*sqrt(c/(d*x
**2) + 1)/(2*x) + a**2*c*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + a**2*d**(3/2)*x
/sqrt(c/(d*x**2) + 1) - 2*a*b*c**(3/2)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*b*c
**2/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*c*sqrt(d)*x/sqrt(c/(d*x**2) +
1) + 2*a*b*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*
d), True)) + b**2*c*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3
/2)/(3*d), True)) + b**2*d*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*d**2) +
c*x**2*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(
c)*x**4/4, True))
```

Giac [A]
time = 1.62, size = 126, normalized size = 0.93

$$\frac{6\,(dx^2+c)^{\frac{5}{2}}b^2+20\,(dx^2+c)^{\frac{3}{2}}abd+60\,\sqrt{dx^2+c}\,abcd+30\,\sqrt{dx^2+c}\,a^2d^2-\frac{15\,\sqrt{dx^2+c}\,a^2cd}{x^2}+\frac{15\,(4\,abc^2d+3\,a^2cd^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*(d*x^2 + c)^(5/2)*b^2 + 20*(d*x^2 + c)^(3/2)*a*b*d + 60*sqrt(d*x^2
+ c)*a*b*c*d + 30*sqrt(d*x^2 + c)*a^2*d^2 - 15*sqrt(d*x^2 + c)*a^2*c*d/x^2
+ 15*(4*a*b*c^2*d + 3*a^2*c*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c))
/d
```

Mupad [B]
time = 0.55, size = 201, normalized size = 1.48

$$\frac{b^2\,{\left(d\,x^2+c\right)}^{5/2}}{5\,d}-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{3\,d}-\frac{2\,b^2\,c}{3\,d}\right)\,{\left(d\,x^2+c\right)}^{3/2}-\sqrt{d\,x^2+c}\,\left(2\,c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)-\frac{{\left(a\,d-b\,c\right)}^2}{d}+\frac{b^2\,c^2}{d}\right)-\frac{a^2\,c\,\sqrt{d\,x^2+c}}{2\,x^2}+2\,a\,\mathrm{atan}\left(\frac{2\,a\,\sqrt{d\,x^2+c}\,\left(3\,a\,d+4\,b\,c\right)\,\sqrt{-\frac{c}{16}}}{\frac{3\,d\,a^2\,c}{2}+2\,b\,a\,c^2}\right)\,\left(3\,a\,d+4\,b\,c\right)\,\sqrt{-\frac{c}{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x)
```

```
[Out] (b^2*(c + d*x^2)^(5/2))/(5*d) - ((2*b^2*c - 2*a*b*d)/(3*d) - (2*b^2*c)/(3*d
))*(c + d*x^2)^(3/2) - (c + d*x^2)^(1/2)*(2*c*((2*b^2*c - 2*a*b*d)/d - (2*b
^2*c)/d) - (a*d - b*c)^2/d + (b^2*c^2)/d) - (a^2*c*(c + d*x^2)^(1/2))/(2*x^
2) + 2*a*atan((2*a*(c + d*x^2)^(1/2)*(3*a*d + 4*b*c)*(-c/16)^(1/2))/(2*a*b*
c^2 + (3*a^2*c*d)/2))*(3*a*d + 4*b*c)*(-c/16)^(1/2)
```

3.621 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=184

$$\frac{(3b^2c^2+8ad(3bc+ad))\,x\sqrt{c+dx^2}}{8c}+\frac{(3b^2c^2+8ad(3bc+ad))\,x(c+dx^2)^{3/2}}{12c^2}-\frac{a^2(c+dx^2)^{5/2}}{3cx^3}-\frac{2a(3bc+ad)\,(}{3c^2}$$

```
[Out] 1/12*(3*b^2*c^2+8*a*d*(a*d+3*b*c))*x*(d*x^2+c)^(3/2)/c^2-1/3*a^2*(d*x^2+c)^
(5/2)/c/x^3-2/3*a*(a*d+3*b*c)*(d*x^2+c)^(5/2)/c^2/x+1/8*(3*b^2*c^2+8*a*d*(a
*d+3*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)+1/8*(3*b^2*c^2+8*a*d*
(a*d+3*b*c))*x*(d*x^2+c)^(1/2)/c
```

Rubi [A]
time = 0.09, antiderivative size = 181, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {473, 464, 201, 223, 212}

$$-\frac{a^2(c+dx^2)^{5/2}}{3cx^3}+\frac{1}{12}x(c+dx^2)^{3/2}\left(\frac{8ad(ad+3bc)}{c^2}+3b^2\right)+\frac{x\sqrt{c+dx^2}\,(8ad(ad+3bc)+3b^2c^2)}{8c}+\frac{(8ad(ad+3bc)+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8\sqrt{d}}-\frac{2a(c+dx^2)^{5/2}(ad+3bc)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

```
[Out] ((3*b^2*c^2 + 8*a*d*(3*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + ((3*b^2 + (8*
a*d*(3*b*c + a*d))/c^2)*x*(c + d*x^2)^(3/2))/12 - (a^2*(c + d*x^2)^(5/2))/(
3*c*x^3) - (2*a*(3*b*c + a*d)*(c + d*x^2)^(5/2))/(3*c^2*x) + ((3*b^2*c^2 +
8*a*d*(3*b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*Sqrt[d])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4}\,dx = -\frac{a^2(c+dx^2)^{5/2}}{3cx^3} + \frac{\int \frac{(2a(3bc+ad)+3b^2cx^2)(c+dx^2)^{3/2}}{x^2}\,dx}{3c}$$

$$= -\frac{a^2(c+dx^2)^{5/2}}{3cx^3} - \frac{2a(3bc+ad)(c+dx^2)^{5/2}}{3c^2x} - \frac{1}{3}\left(-3b^2 - \frac{8ad(3bc+ad)}{c^2}\right)$$

$$= \frac{1}{12}\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right) x(c+dx^2)^{3/2} - \frac{a^2(c+dx^2)^{5/2}}{3cx^3} - \frac{2a(3bc+ad)(}{3c^2}$$

$$= \frac{1}{8}c\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right) x\sqrt{c+dx^2} + \frac{1}{12}\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right) x(c+$$

$$= \frac{1}{8}c\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right) x\sqrt{c+dx^2} + \frac{1}{12}\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right) x(c+$$

$$= \frac{1}{8}c\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right) x\sqrt{c+dx^2} + \frac{1}{12}\left(3b^2 + \frac{8ad(3bc+ad)}{c^2}\right) x(c+$$

Mathematica [A]
time = 0.27, size = 119, normalized size = 0.65

$$\frac{1}{24}\left(\frac{\sqrt{c+dx^2}\,(24abx^2(-2c+dx^2)+3b^2x^4(5c+2dx^2)-8a^2(c+4dx^2))}{x^3} - \frac{3(3b^2c^2+24abcd+8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(24*a*b*x^2*(-2*c + d*x^2) + 3*b^2*x^4*(5*c + 2*d*x^2) -
8*a^2*(c + 4*d*x^2)))/x^3 - (3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*Log[-(S
qrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[d])/24
```

Maple [A]
time = 0.10, size = 239, normalized size = 1.30

method	result
risch	$-\frac{\sqrt{d x^2+c}\,\left(-6b^2 d x^6-24abd x^4-15b^2 c x^4+32a^2 d x^2+48abc x^2+8a^2 c\right)}{24x^3}+\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right) d^{\frac{3}{2}} a^2+3\ln\Big($
default	$b^2\left(\frac{x\left(d x^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)+a^2\Bigg(-\frac{\left(d x^2+c\right)^{\frac{5}{2}}}{3c x^3}+\frac{2d\left(-\frac{\left(d x^2+c\right)^{\frac{5}{2}}}{cx}+\frac{4d\left(\frac{x\ldots}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*
d^(1/2)+(d*x^2+c)^(1/2))))+a^2*(-1/3/c/x^3*(d*x^2+c)^(5/2)+2/3*d/c*(-1/c/x*
(d*x^2+c)^(5/2)+4*d/c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1
/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))))+2*a*b*(-1/c/x*(d*x^2+c)^(5/2
)+4*d/c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*l
n(x*d^(1/2)+(d*x^2+c)^(1/2)))))
```

Maxima [A]

time = 0.27, size = 177, normalized size = 0.96

$$\frac{1}{4}\left(dx^2+c\right)^{\frac{3}{2}}b^2x+\frac{3}{8}\sqrt{dx^2+c}\,b^2cx+3\sqrt{dx^2+c}\,abdx+\frac{\sqrt{dx^2+c}\,a^2d^2x}{c}+\frac{3\,b^2c^2\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{d}}+3\,abc\sqrt{d}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+a^2d^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}ab}{x}-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}a^2d}{3\,cx}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*(d*x^2 + c)^(3/2)*b^2*x + 3/8*sqrt(d*x^2 + c)*b^2*c*x + 3*sqrt(d*x^2 +
c)*a*b*d*x + sqrt(d*x^2 + c)*a^2*d^2*x/c + 3/8*b^2*c^2*arcsinh(d*x/sqrt(c*d
))/sqrt(d) + 3*a*b*c*sqrt(d)*arcsinh(d*x/sqrt(c*d)) + a^2*d^(3/2)*arcsinh(d
*x/sqrt(c*d)) - 2*(d*x^2 + c)^(3/2)*a*b/x - 2/3*(d*x^2 + c)^(3/2)*a^2*d/(c*
x) - 1/3*(d*x^2 + c)^(5/2)*a^2/(c*x^3)
```

Fricas [A]
time = 2.01, size = 266, normalized size = 1.45

$$\left[\frac{3\left(3\,b^2c^2+24\,abcd+8\,a^2d^2\right)\sqrt{d}\,x^3\log\left(-2\,dx^2-2\sqrt{dx^2+c}\sqrt{d}\,x-c\right)+2\left(6\,b^2d^2x^6+3\left(5\,b^2cd+8\,abd^2\right)x^4-8\,a^2cd-16\left(3\,abcd+2\,a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{48\,dx^3},-\frac{3\left(3\,b^2c^2+24\,abcd+8\,a^2d^2\right)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(6\,b^2d^2x^6+3\left(5\,b^2cd+8\,abd^2\right)x^4-8\,a^2cd-16\left(3\,abcd+2\,a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{24\,dx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*sqrt(d)*x^3*log(-2*d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b^2*d^2*x^6 + 3*(5*b^2*c*d + 8*a*b*d^
2)*x^4 - 8*a^2*c*d - 16*(3*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^
3), -1/24*(3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*x^3*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) - (6*b^2*d^2*x^6 + 3*(5*b^2*c*d + 8*a*b*d^2)*x^4 - 8
*a^2*c*d - 16*(3*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(168) = 336$.
time = 6.30, size = 352, normalized size = 1.91

$$-\frac{a^2\sqrt{c}\,d}{x\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+a^2d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{a^2d^2x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{2abc^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+ab\sqrt{c}\,dx\sqrt{1+\frac{dx^2}{c}}-\frac{2ab\sqrt{c}\,dx}{\sqrt{1+\frac{dx^2}{c}}}+3abc\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)+\frac{b^2c^{\frac{3}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{b^2c^{\frac{3}{2}}x}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2\sqrt{c}\,dx^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{b^2d^2x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**4,x)
```

```
[Out] -a**2*sqrt(c)*d/(x*sqrt(1 + d*x**2/c)) - a**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1
)/(3*x**2) - a**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + a**2*d**(3/2)*asinh(sqr
t(d)*x/sqrt(c)) - a**2*d**2*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - 2*a*b*c**(3/2)
/(x*sqrt(1 + d*x**2/c)) + a*b*sqrt(c)*d*x*sqrt(1 + d*x**2/c) - 2*a*b*sqrt(c
)*d*x/sqrt(1 + d*x**2/c) + 3*a*b*c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) + b**2*
c**(3/2)*x*sqrt(1 + d*x**2/c)/2 + b**2*c**(3/2)*x/(8*sqrt(1 + d*x**2/c)) +
3*b**2*sqrt(c)*d*x**3/(8*sqrt(1 + d*x**2/c)) + 3*b**2*c**2*asinh(sqrt(d)*x/
sqrt(c))/(8*sqrt(d)) + b**2*d**2*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A]
time = 1.06, size = 262, normalized size = 1.42

$$\frac{1}{8}\left(2\,b^{2}dx^{2}+\frac{5\,b^{2}cd^{2}+8\,abd^{3}}{d^{2}}\right)\sqrt{dx^{2}+c}\,x-\frac{\left(3\,b^{2}c^{2}\sqrt{d}+24\,abcd^{\frac{3}{2}}+8\,a^{2}d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}\right)}{16\,d}+\frac{4\left(3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}abc^{2}\sqrt{d}+3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}a^{2}cd^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}abc^{3}\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}a^{2}c^{2}d^{\frac{3}{2}}+3\,abc^{4}\sqrt{d}+2\,a^{2}c^{3}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*d*x^2 + (5*b^2*c*d^2 + 8*a*b*d^3)/d^2)*sqrt(d*x^2 + c)*x - 1/16*
(3*b^2*c^2*sqrt(d) + 24*a*b*c*d^(3/2) + 8*a^2*d^(5/2))*log((sqrt(d)*x - sqr
t(d*x^2 + c))^2)/d + 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^2*sqrt(d)
 + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*a*b*c^3*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^2*d^(
3/2) + 3*a*b*c^4*sqrt(d) + 2*a^2*c^3*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c)
)^2 - c)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4, x)
```

3.622 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^5} \, dx$

Optimal. Leaf size=181

$$\frac{(8b^2c^2 + 3ad(8bc + ad))\sqrt{c + dx^2}}{8c} + \frac{(8b^2c^2 + 3ad(8bc + ad))(c + dx^2)^{3/2}}{24c^2} - \frac{a^2(c + dx^2)^{5/2}}{4cx^4} - \frac{a(8bc + ad)(c + \cdots}{8c^2x^2}$$

```
[Out] 1/24*(8*b^2*c^2+3*a*d*(a*d+8*b*c))*(d*x^2+c)^(3/2)/c^2-1/4*a^2*(d*x^2+c)^(5
/2)/c/x^4-1/8*a*(a*d+8*b*c)*(d*x^2+c)^(5/2)/c^2/x^2-1/8*(8*b^2*c^2+3*a*d*(a
*d+8*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+1/8*(8*b^2*c^2+3*a*d*(a
*d+8*b*c))*(d*x^2+c)^(1/2)/c
```

Rubi [A]
time = 0.14, antiderivative size = 178, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 79, 52, 65, 214}

$$-\frac{a^2(c+dx^2)^{5/2}}{4cx^4} + \frac{1}{24}(c+dx^2)^{3/2}\left(\frac{3ad(ad+8bc)}{c^2} + 8b^2\right) + \frac{\sqrt{c+dx^2}\,(3ad(ad+8bc)+8b^2c^2)}{8c} - \frac{(3ad(ad+8bc)+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8\sqrt{c}} - \frac{a(c+dx^2)^{5/2}(ad+8bc)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

```
[Out] ((8*b^2*c^2 + 3*a*d*(8*b*c + a*d))*Sqrt[c + d*x^2])/(8*c) + ((8*b^2 + (3*a*
d*(8*b*c + a*d))/c^2)*(c + d*x^2)^(3/2))/24 - (a^2*(c + d*x^2)^(5/2))/(4*c*
x^4) - (a*(8*b*c + a*d)*(c + d*x^2)^(5/2))/(8*c^2*x^2) - ((8*b^2*c^2 + 3*a*
d*(8*b*c + a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^5} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x^3} dx, x, x^2\right)$$
$$= -\frac{a^2(c+dx^2)^{5/2}}{4cx^4} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(8bc+ad)+2b^2cx\right)(c+dx)^{3/2}}{x^2} dx, x, x^2\right)}{4c}$$
$$= -\frac{a^2(c+dx^2)^{5/2}}{4cx^4} - \frac{a(8bc+ad)(c+dx^2)^{5/2}}{8c^2x^2} + \frac{1}{16}\left(8b^2 + \frac{3ad(8bc+ad)}{c^2}\right) \text{S}$$
$$= \frac{1}{24}\left(8b^2 + \frac{3ad(8bc+ad)}{c^2}\right)(c+dx^2)^{3/2} - \frac{a^2(c+dx^2)^{5/2}}{4cx^4} - \frac{a(8bc+ad)(c}{8c^2x^2}$$
$$= \frac{1}{8}c\left(8b^2 + \frac{3ad(8bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{1}{24}\left(8b^2 + \frac{3ad(8bc+ad)}{c^2}\right)(c+dx$$
$$= \frac{1}{8}c\left(8b^2 + \frac{3ad(8bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{1}{24}\left(8b^2 + \frac{3ad(8bc+ad)}{c^2}\right)(c+dx$$
$$= \frac{1}{8}c\left(8b^2 + \frac{3ad(8bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{1}{24}\left(8b^2 + \frac{3ad(8bc+ad)}{c^2}\right)(c+dx$$

Mathematica [A]
time = 0.23, size = 116, normalized size = 0.64

$$\frac{1}{24}\left(\frac{\sqrt{c+dx^2}\,(-24abx^2(c-2dx^2)+8b^2x^4(4c+dx^2)-3a^2(2c+5dx^2))}{x^4} - \frac{3(8b^2c^2+24abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-24*a*b*x^2*(c - 2*d*x^2) + 8*b^2*x^4*(4*c + d*x^2) - 3*
a^2*(2*c + 5*d*x^2)))/x^4 - (3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*ArcTanh
[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/24
```

Maple [A]
time = 0.12, size = 242, normalized size = 1.34

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\;a(5ad\,x^2+8c\,x^2b+2ac)}{8x^4} + \frac{b^2d\,x^2\sqrt{d\,x^2+c}}{3} + \frac{4b^2c\sqrt{d\,x^2+c}}{3} + 2abd\sqrt{d\,x^2+c} - \frac{3\ln\left(\frac{2c+2}{}\right.}{}$

default	$a^{2}\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{4c\,x^{4}}+\frac{d\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{2c\,x^{2}}+\frac{3d\left(\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{3}+c\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)\right)\right)}{2c}\right)}{4c}\right)+2ab\Big($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/4/c/x^4*(d*x^2+c)^(5/2)+1/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(5/2)+3/2*d/c
*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c
)^(1/2))/x)))))+2*a*b*(-1/2/c/x^2*(d*x^2+c)^(5/2)+3/2*d/c*(1/3*(d*x^2+c)^(3
/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))+b^2
*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c
)^(1/2))/x)))
```

Maxima [A]

time = 0.27, size = 222, normalized size = 1.23

$$-b^{2}c^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-3\,ab\sqrt{c}\,d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{3\,a^{2}d^{2}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{8\sqrt{c}}+\frac{1}{3}\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}+\sqrt{dx^{2}+c}\,b^{2}c+3\sqrt{dx^{2}+c}\,abd+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}abd}{c}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}}{8\,c^{2}}+\frac{3\sqrt{dx^{2}+c}\,a^{2}d^{2}}{8\,c}-\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}ab}{cx^{2}}-\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d}{8\,c^{2}x^{2}}-\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}}{4\,cx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -b^2*c^(3/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 3*a*b*sqrt(c)*d*arcsinh(c/(sqr
t(c*d)*abs(x))) - 3/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 1/3*(
d*x^2 + c)^(3/2)*b^2 + sqrt(d*x^2 + c)*b^2*c + 3*sqrt(d*x^2 + c)*a*b*d + (d
*x^2 + c)^(3/2)*a*b*d/c + 1/8*(d*x^2 + c)^(3/2)*a^2*d^2/c^2 + 3/8*sqrt(d*x^
2 + c)*a^2*d^2/c - (d*x^2 + c)^(5/2)*a*b/(c*x^2) - 1/8*(d*x^2 + c)^(5/2)*a^
2*d/(c^2*x^2) - 1/4*(d*x^2 + c)^(5/2)*a^2/(c*x^4)
```

Fricas [A]

time = 1.74, size = 267, normalized size = 1.48

$$\left[\frac{3\left(8\,b^{2}c^{2}+24\,abcd+3\,a^{2}d^{2}\right)\sqrt{c}\,x^{4}\log\left(-\frac{dx^{2}-2\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+2\left(8\,b^{2}cdx^{6}+16\left(2\,b^{2}c^{2}+3\,abcd\right)x^{4}-6\,a^{2}c^{2}-3\left(8\,abc^{2}+5\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{48\,cx^{4}},\ \frac{3\left(8\,b^{2}c^{2}+24\,abcd+3\,a^{2}d^{2}\right)\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(8\,b^{2}cdx^{6}+16\left(2\,b^{2}c^{2}+3\,abcd\right)x^{4}-6\,a^{2}c^{2}-3\left(8\,abc^{2}+5\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{24\,cx^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(8*b^2*c*d*x^6 + 16*(2*b^2*c^2 + 3*a
```

```
*b*c*d)*x^4 - 6*a^2*c^2 - 3*(8*a*b*c^2 + 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(
c*x^4), 1/24*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(-c)*x^4*arctan(sq
rt(-c)/sqrt(d*x^2 + c)) + (8*b^2*c*d*x^6 + 16*(2*b^2*c^2 + 3*a*b*c*d)*x^4 -
 6*a^2*c^2 - 3*(8*a*b*c^2 + 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^4)]
```

Sympy [A]
time = 77.68, size = 332, normalized size = 1.83

$$-\frac{a^{2}c^{2}}{4\sqrt{d}\,x^{5}\sqrt{\frac{c}{dx^{2}}+1}}-\frac{3a^{2}c\sqrt{d}}{8x^{3}\sqrt{\frac{c}{dx^{2}}+1}}-\frac{a^{2}d^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{2x}-\frac{a^{2}d^{\frac{3}{2}}}{8x\sqrt{\frac{c}{dx^{2}}+1}}-\frac{3a^{2}d^{2}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{8\sqrt{c}}-3ab\sqrt{c}\,d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)-\frac{abc\sqrt{d}\sqrt{\frac{c}{dx^{2}}+1}}{x}+\frac{2abc\sqrt{d}}{x\sqrt{\frac{c}{dx^{2}}+1}}+\frac{2abd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^{2}}+1}}-b^{2}c^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)+\frac{b^{2}c^{2}}{\sqrt{d}\,x\sqrt{\frac{c}{dx^{2}}+1}}+\frac{b^{2}c\sqrt{d}\,x}{\sqrt{\frac{c}{dx^{2}}+1}}+b^{2}d\left(\begin{cases}\frac{\sqrt{c}\,x^{2}}{2} & \text{for}\ d=0\\ \frac{(c+dx^{2})^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**5,x)
```

```
[Out] -a**2*c**2/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*c*sqrt(d)/(8*x**3
*sqrt(c/(d*x**2) + 1)) - a**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/(2*x) - a**2*d*
*(3/2)/(8*x*sqrt(c/(d*x**2) + 1)) - 3*a**2*d**2*asinh(sqrt(c)/(sqrt(d)*x))/
(8*sqrt(c)) - 3*a*b*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x)) - a*b*c*sqrt(d)*sq
rt(c/(d*x**2) + 1)/x + 2*a*b*c*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*a*b*d**
(3/2)*x/sqrt(c/(d*x**2) + 1) - b**2*c**(3/2)*asinh(sqrt(c)/(sqrt(d)*x)) + b
**2*c**2/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*c*sqrt(d)*x/sqrt(c/(d*x**2
) + 1) + b**2*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/
(3*d), True))
```

Giac [A]
time = 1.07, size = 182, normalized size = 1.01

$$\frac{8\,(dx^{2}+c)^{\frac{3}{2}}b^{2}d+24\,\sqrt{dx^{2}+c}\,b^{2}cd+48\,\sqrt{dx^{2}+c}\,abd^{2}+\frac{3\,(8\,b^{2}c^{2}d+24\,abcd^{2}+3\,a^{2}d^{3})\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{3\left(8\,(dx^{2}+c)^{\frac{3}{2}}abcd^{2}-8\,\sqrt{dx^{2}+c}\,abc^{2}d^{2}+5\,(dx^{2}+c)^{\frac{3}{2}}a^{2}d^{3}-3\,\sqrt{dx^{2}+c}\,a^{2}cd^{3}\right)}{d^{2}x^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(8*(d*x^2 + c)^(3/2)*b^2*d + 24*sqrt(d*x^2 + c)*b^2*c*d + 48*sqrt(d*x^
2 + c)*a*b*d^2 + 3*(8*b^2*c^2*d + 24*a*b*c*d^2 + 3*a^2*d^3)*arctan(sqrt(d*x
^2 + c)/sqrt(-c))/sqrt(-c) - 3*(8*(d*x^2 + c)^(3/2)*a*b*c*d^2 - 8*sqrt(d*x^
2 + c)*a*b*c^2*d^2 + 5*(d*x^2 + c)^(3/2)*a^2*d^3 - 3*sqrt(d*x^2 + c)*a^2*c*
d^3)/(d^2*x^4))/d
```

Mupad [B]
time = 0.76, size = 208, normalized size = 1.15

$$\frac{\sqrt{d\,x^{2}+c}\,\left(\frac{3\,a^{2}\,c\,d^{2}}{8}+b\,a\,c^{2}\,d\right)-\left(\frac{5\,a^{2}\,d^{2}}{8}+b\,c\,a\,d\right)\,{\left(d\,x^{2}+c\right)}^{3/2}}{{\left(d\,x^{2}+c\right)}^{2}-2\,c\,\left(d\,x^{2}+c\right)+c^{2}}+\sqrt{d\,x^{2}+c}\,\left(c\,b^{2}+2\,a\,d\,b\right)+\frac{b^{2}\,{\left(d\,x^{2}+c\right)}^{3/2}}{3}+\frac{\mathrm{atan}\left(\frac{\sqrt{d\,x^{2}+c}\,\left(3\,a^{2}\,d^{2}+24\,a\,b\,c\,d+8\,b^{2}\,c^{2}\right)\,1{}\mathrm{i}}{4\,\sqrt{c}\,\left(\frac{3\,a^{2}\,d^{2}}{4}+6\,a\,b\,c\,d+2\,b^{2}\,c^{2}\right)}\right)\,\left(3\,a^{2}\,d^{2}+24\,a\,b\,c\,d+8\,b^{2}\,c^{2}\right)\,1{}\mathrm{i}}{8\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((3*a^2*c*d^2)/8 + a*b*c^2*d) - ((5*a^2*d^2)/8 + a*b*c*d
)*(c + d*x^2)^(3/2))/((c + d*x^2)^2 - 2*c*(c + d*x^2) + c^2) + (c + d*x^2)^
(1/2)*(b^2*c + 2*a*b*d) + (b^2*(c + d*x^2)^(3/2))/3 + (atan(((c + d*x^2)^(1
/2)*(3*a^2*d^2 + 8*b^2*c^2 + 24*a*b*c*d)*1i)/(4*c^(1/2)*((3*a^2*d^2)/4 + 2*
b^2*c^2 + 6*a*b*c*d)))*(3*a^2*d^2 + 8*b^2*c^2 + 24*a*b*c*d)*1i)/(8*c^(1/2))
```

3.623 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^6} dx$

Optimal. Leaf size=147

$$\frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{b(3bc+4ad)\left(c+dx^2\right)^{3/2}}{3cx}-\frac{a^2(c+dx^2)^{5/2}}{5cx^5}-\frac{2ab(c+dx^2)^{5/2}}{3cx^3}+\frac{1}{2}b\sqrt{d}\,(3bc+4ad)$$

```
[Out] -1/3*b*(4*a*d+3*b*c)*(d*x^2+c)^(3/2)/c/x-1/5*a^2*(d*x^2+c)^(5/2)/c/x^5-2/3*
a*b*(d*x^2+c)^(5/2)/c/x^3+1/2*b*(4*a*d+3*b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(
1/2))*d^(1/2)+1/2*b*d*(4*a*d+3*b*c)*x*(d*x^2+c)^(1/2)/c
```

Rubi [A]

time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 464, 283, 201, 223, 212}

$$-\frac{a^2(c+dx^2)^{5/2}}{5cx^5}-\frac{b(c+dx^2)^{3/2}(4ad+3bc)}{3cx}+\frac{bdx\sqrt{c+dx^2}\,(4ad+3bc)}{2c}+\frac{1}{2}b\sqrt{d}\,(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)-\frac{2ab(c+dx^2)^{5/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

```
[Out] (b*d*(3*b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (b*(3*b*c + 4*a*d)*(c + d*x
^2)^(3/2))/(3*c*x) - (a^2*(c + d*x^2)^(5/2))/(5*c*x^5) - (2*a*b*(c + d*x^2)
^(5/2))/(3*c*x^3) + (b*Sqrt[d]*(3*b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^{3/2}}{x^6}\,dx = -\frac{a^2(c+dx^2)^{5/2}}{5cx^5} + \frac{\int \frac{(10abc+5b^2cx^2)(c+dx^2)^{3/2}}{x^4}\,dx}{5c}$$

$$= -\frac{a^2(c+dx^2)^{5/2}}{5cx^5} - \frac{2ab(c+dx^2)^{5/2}}{3cx^3} + \frac{(b(3bc+4ad))\int \frac{(c+dx^2)^{3/2}}{x^2}\,dx}{3c}$$

$$= -\frac{b(3bc+4ad)\,(c+dx^2)^{3/2}}{3cx} - \frac{a^2(c+dx^2)^{5/2}}{5cx^5} - \frac{2ab(c+dx^2)^{5/2}}{3cx^3} + \frac{(bd(3bc+}{}$$

$$= \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c} - \frac{b(3bc+4ad)\,(c+dx^2)^{3/2}}{3cx} - \frac{a^2(c+dx^2)^{5/2}}{5cx^5} - \frac{2a}{}$$

$$= \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c} - \frac{b(3bc+4ad)\,(c+dx^2)^{3/2}}{3cx} - \frac{a^2(c+dx^2)^{5/2}}{5cx^5} - \frac{2a}{}$$

$$= \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c} - \frac{b(3bc+4ad)\,(c+dx^2)^{3/2}}{3cx} - \frac{a^2(c+dx^2)^{5/2}}{5cx^5} - \frac{2a}{}$$

Mathematica [A]

time = 0.27, size = 112, normalized size = 0.76

$$-\frac{\sqrt{c+dx^2}\left(15b^2cx^4(2c-dx^2)+6a^2(c+dx^2)^2+20abcx^2(c+4dx^2)\right)}{30cx^5}-\frac{1}{2}b\sqrt{d}\,(3bc+4ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

```
[Out] -1/30*(Sqrt[c + d*x^2]*(15*b^2*c*x^4*(2*c - d*x^2) + 6*a^2*(c + d*x^2)^2 +
20*a*b*c*x^2*(c + 4*d*x^2)))/(c*x^5) - (b*Sqrt[d]*(3*b*c + 4*a*d)*Log[-(Sqr
t[d]*x) + Sqrt[c + d*x^2]])/2
```

Maple [A]

time = 0.10, size = 204, normalized size = 1.39

method	result
risch	$-\frac{\sqrt{dx^2+c}\,(-15b^2cd\,x^6+6a^2d^2x^4+80abcd\,x^4+30b^2c^2x^4+12a^2cd\,x^2+20ab\,c^2x^2+6a^2c^2)}{30x^5c}+2d^{\frac{3}{2}}\ln\left(x\sqrt{d}+\sqrt{dx^2}\right.$
default	$-\frac{a^2(dx^2+c)^{\frac{5}{2}}}{5c\,x^5}+2ab\left(-\frac{(dx^2+c)^{\frac{5}{2}}}{3c\,x^3}+\frac{2d\left(-\frac{(dx^2+c)^{\frac{5}{2}}}{cx}+\frac{4d\left(\frac{x(dx^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right.}{2\sqrt{d}}\right.}{4}\right.}{c}\right.}{3c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^2*(d*x^2+c)^(5/2)/c/x^5+2*a*b*(-1/3/c/x^3*(d*x^2+c)^(5/2)+2/3*d/c*(-
1/c/x*(d*x^2+c)^(5/2)+4*d/c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(
```

```
1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))))+b^2*(-1/c/x*(d*x^2+c)^
(5/2)+4*d/c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/
2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))
```

Maxima [A]
time = 0.29, size = 147, normalized size = 1.00

$$\frac{3}{2}\sqrt{dx^2+c}\,b^2dx+\frac{2\sqrt{dx^2+c}\,abd^2x}{c}+\frac{3}{2}b^2c\sqrt{d}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+2\,abd^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{(dx^2+c)^{\frac{3}{2}}b^2}{x}-\frac{4\,(dx^2+c)^{\frac{3}{2}}abd}{3\,cx}-\frac{2\,(dx^2+c)^{\frac{5}{2}}ab}{3\,cx^3}-\frac{(dx^2+c)^{\frac{5}{2}}a^2}{5\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(d*x^2 + c)*b^2*d*x + 2*sqrt(d*x^2 + c)*a*b*d^2*x/c + 3/2*b^2*c*sqr
t(d)*arcsinh(d*x/sqrt(c*d)) + 2*a*b*d^(3/2)*arcsinh(d*x/sqrt(c*d)) - (d*x^2
 + c)^(3/2)*b^2/x - 4/3*(d*x^2 + c)^(3/2)*a*b*d/(c*x) - 2/3*(d*x^2 + c)^(5/
2)*a*b/(c*x^3) - 1/5*(d*x^2 + c)^(5/2)*a^2/(c*x^5)
```

Fricas [A]
time = 1.73, size = 266, normalized size = 1.81

$$\left[\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{d}\,x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(15\,b^2cdx^6-2\,(15\,b^2c^2+40\,abcd+3\,a^2d^2)x^4-6\,a^2c^2-4\,(5\,abc^2+3\,a^2cd)x^2)\sqrt{dx^2+c}}{60\,cx^5},-\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{-d}\,x^5\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(15\,b^2cdx^6-2\,(15\,b^2c^2+40\,abcd+3\,a^2d^2)x^4-6\,a^2c^2-4\,(5\,abc^2+3\,a^2cd)x^2)\sqrt{dx^2+c}}{30\,cx^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(d)*x^5*log(-2*d*x^2 - 2*sqrt(d*x^2 +
 c)*sqrt(d)*x - c) + 2*(15*b^2*c*d*x^6 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2
*d^2)*x^4 - 6*a^2*c^2 - 4*(5*a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*
x^5), -1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - (15*b^2*c*d*x^6 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2*d^2)*x^
4 - 6*a^2*c^2 - 4*(5*a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(134) = 268$.
time = 4.25, size = 304, normalized size = 2.07

$$-\frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{2a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{5x^2}-\frac{a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{5c}-\frac{2ab\sqrt{c}\,d}{x\sqrt{1+\frac{dx^2}{c}}}-\frac{2abc\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+2abd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{2abd^2x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2\sqrt{c}\,dx\sqrt{1+\frac{dx^2}{c}}}{2}-\frac{b^2\sqrt{c}\,dx}{\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**6,x)
```

```
[Out] -a**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - 2*a**2*d**(3/2)*sqrt(c/(d*x
**2) + 1)/(5*x**2) - a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(5*c) - 2*a*b*sqrt(
c)*d/(x*sqrt(1 + d*x**2/c)) - 2*a*b*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2)
 - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 2*a*b*d**(3/2)*asinh(sqrt(d)*x/s
```

```
qrt(c)) - 2*a*b*d**2*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - b**2*c**(3/2)/(x*sqrt
(1 + d*x**2/c)) + b**2*sqrt(c)*d*x*sqrt(1 + d*x**2/c)/2 - b**2*sqrt(c)*d*x/
sqrt(1 + d*x**2/c) + 3*b**2*c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(123) = 246.
time = 1.16, size = 407, normalized size = 2.77

$$\frac{1}{2} \sqrt{dx^2+c}\, b^2 dx - \frac{1}{4} \left(3 b^2 c \sqrt{d} + 4 abd^{\frac{3}{2}}\right) \log\left(\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2\right) + \frac{2 \left(15 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^8 b^2 c^2 \sqrt{d} + 60 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^8 abcd^{\frac{3}{2}} + 15 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^8 a^2 d^{\frac{5}{2}} - 60 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^6 b^2 c^3 \sqrt{d} - 180 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^6 abc^2 d^{\frac{3}{2}} + 90 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^4 b^2 c^4 \sqrt{d} + 220 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^4 abc^3 d^{\frac{3}{2}} + 30 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^4 a^2 c^2 d^{\frac{5}{2}} - 60 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 b^2 c^5 \sqrt{d} - 140 \left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 abc^4 d^{\frac{3}{2}} + 15 b^2 c^6 \sqrt{d} + 40 abc^5 d^{\frac{3}{2}} + 3 a^2 c^4 d^{\frac{5}{2}}\right)}{15 \left(\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 - c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*d*x - 1/4*(3*b^2*c*sqrt(d) + 4*a*b*d^(3/2))*log((sq
rt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^
2*c^2*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 15*(sqrt
(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*b^2*c^3*sqrt(d) - 180*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2) + 90
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) + 220*(sqrt(d)*x - sqrt(d*
x^2 + c))^4*a*b*c^3*d^(3/2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^2*d^
(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt(d) - 140*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) + 15*b^2*c^6*sqrt(d) + 40*a*b*c^5*d^(
3/2) + 3*a^2*c^4*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6, x)
```

3.624 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=187

$$\frac{d(24b^2c^2+ad(12bc-ad))\sqrt{c+dx^2}}{16c^2}-\frac{(24b^2c^2+ad(12bc-ad))(c+dx^2)^{3/2}}{48c^2x^2}-\frac{a^2(c+dx^2)^{5/2}}{6cx^6}-\frac{a(12bc-ad)}{24c}$$

```
[Out] -1/48*(24*b^2*c^2+a*d*(-a*d+12*b*c))*(d*x^2+c)^(3/2)/c^2/x^2-1/6*a^2*(d*x^2
+c)^(5/2)/c/x^6-1/24*a*(-a*d+12*b*c)*(d*x^2+c)^(5/2)/c^2/x^4-1/16*d*(24*b^2
*c^2+a*d*(-a*d+12*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+1/16*d*(24
*b^2*c^2+a*d*(-a*d+12*b*c))*(d*x^2+c)^(1/2)/c^2
```

Rubi [A]
time = 0.15, antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 91, 79, 43, 52, 65, 214}

$$-\frac{a^2(c+dx^2)^{5/2}}{6cx^6}-\frac{(c+dx^2)^{3/2}\left(\frac{ad(12bc-ad)}{c^2}+24b^2\right)}{48x^2}+\frac{d\sqrt{c+dx^2}\,(ad(12bc-ad)+24b^2c^2)}{16c^2}-\frac{d(ad(12bc-ad)+24b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{3/2}}-\frac{a(c+dx^2)^{5/2}(12bc-ad)}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

```
[Out] (d*(24*b^2*c^2 + a*d*(12*b*c - a*d))*Sqrt[c + d*x^2])/(16*c^2) - ((24*b^2 +
 (a*d*(12*b*c - a*d))/c^2)*(c + d*x^2)^(3/2))/(48*x^2) - (a^2*(c + d*x^2)^(
5/2))/(6*c*x^6) - (a*(12*b*c - a*d)*(c + d*x^2)^(5/2))/(24*c^2*x^4) - (d*(2
4*b^2*c^2 + a*d*(12*b*c - a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(3/
2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^7} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x^4} dx, x, x^2\right)$$

$$= -\frac{a^2(c+dx^2)^{5/2}}{6cx^6} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(12bc-ad)+3b^2cx\right)(c+dx)^{3/2}}{x^3} dx, x, x^2\right)}{6c}$$

$$= -\frac{a^2(c+dx^2)^{5/2}}{6cx^6} - \frac{a(12bc-ad)(c+dx^2)^{5/2}}{24c^2x^4} + \frac{1}{48}\left(24b^2 + \frac{ad(12bc-ad)}{c^2}\right) \text{S}$$

$$= -\frac{\left(24b^2 + \frac{ad(12bc-ad)}{c^2}\right)(c+dx^2)^{3/2}}{48x^2} - \frac{a^2(c+dx^2)^{5/2}}{6cx^6} - \frac{a(12bc-ad)(c+dx^2}{24c^2x^4}$$

$$= \frac{1}{16}d\left(24b^2 + \frac{ad(12bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{\left(24b^2 + \frac{ad(12bc-ad)}{c^2}\right)(c+dx^2)^{3/2}}{48x^2}$$

$$= \frac{1}{16}d\left(24b^2 + \frac{ad(12bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{\left(24b^2 + \frac{ad(12bc-ad)}{c^2}\right)(c+dx^2)^{3/2}}{48x^2}$$

$$= \frac{1}{16}d\left(24b^2 + \frac{ad(12bc-ad)}{c^2}\right)\sqrt{c+dx^2} - \frac{\left(24b^2 + \frac{ad(12bc-ad)}{c^2}\right)(c+dx^2)^{3/2}}{48x^2}$$

Mathematica [A]
time = 0.23, size = 133, normalized size = 0.71

$$-\frac{\sqrt{c+dx^2}\,(24b^2cx^4(c-2dx^2)+12abcx^2(2c+5dx^2)+a^2(8c^2+14cdx^2+3d^2x^4))}{48cx^6} + \frac{d(-24b^2c^2-12abcd+a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

```
[Out] -1/48*(Sqrt[c + d*x^2]*(24*b^2*c*x^4*(c - 2*d*x^2) + 12*a*b*c*x^2*(2*c + 5*
d*x^2) + a^2*(8*c^2 + 14*c*d*x^2 + 3*d^2*x^4)))/(c*x^6) + (d*(-24*b^2*c^2 -
 12*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(3/2))
```

Maple [A]
time = 0.12, size = 314, normalized size = 1.68

method	result
risch	$-\frac{\sqrt{dx^2+c}\,(3a^2d^2x^4+60abcd\,x^4+24b^2c^2x^4+14a^2cd\,x^2+24ab\,c^2x^2+8a^2c^2)}{48x^6c} + d\,b^2\sqrt{dx^2+c} + \frac{d^3\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{dx}}{x}\right.}{16c^{\frac{3}{2}}}$

default	$2ab\left(-\frac{(d\,x^2+c)^{\frac{5}{2}}}{4c\,x^4}+\frac{d\left(-\frac{(d\,x^2+c)^{\frac{5}{2}}}{2c\,x^2}+\frac{3d\left(\frac{(d\,x^2+c)^{\frac{3}{2}}}{3}+c\left(\sqrt{d\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)\right)\right)}{2c}\right)}{4c}\right)+a^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b*(-1/4/c/x^4*(d*x^2+c)^(5/2)+1/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(5/2)+3/2*d
/c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2
+c)^(1/2))/x)))))+a^2*(-1/6/c/x^6*(d*x^2+c)^(5/2)-1/6*d/c*(-1/4/c/x^4*(d*x^
2+c)^(5/2)+1/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(5/2)+3/2*d/c*(1/3*(d*x^2+c)^(3/2)
+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))))+b^2*
(-1/2/c/x^2*(d*x^2+c)^(5/2)+3/2*d/c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)
-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))
```

Maxima [A]
time = 0.30, size = 301, normalized size = 1.61

$$-\frac{3}{2}\,b^2\sqrt{c}\,d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{3\,abd^2\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{4\,\sqrt{c}}+\frac{a^2d^3\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16\,c^{\frac{3}{2}}}+\frac{3}{2}\,\sqrt{dx^2+c}\,b^2d+\frac{(dx^2+c)^{\frac{3}{2}}b^2d}{2\,c}+\frac{(dx^2+c)^{\frac{3}{2}}abd^2}{4\,c^2}+\frac{3\,\sqrt{dx^2+c}\,abd^2}{4\,c}-\frac{(dx^2+c)^{\frac{3}{2}}a^2d^3}{48\,c^3}-\frac{\sqrt{dx^2+c}\,a^2d^3}{16\,c^2}-\frac{(dx^2+c)^{\frac{5}{2}}b^2}{2\,cx^2}-\frac{(dx^2+c)^{\frac{5}{2}}abd}{4\,c^2x^2}+\frac{(dx^2+c)^{\frac{5}{2}}a^2d^2}{48\,c^3x^2}-\frac{(dx^2+c)^{\frac{5}{2}}ab}{2\,cx^4}+\frac{(dx^2+c)^{\frac{5}{2}}a^2d}{24\,c^2x^4}-\frac{(dx^2+c)^{\frac{5}{2}}a^2}{6\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -3/2*b^2*sqrt(c)*d*arcsinh(c/(sqrt(c*d)*abs(x))) - 3/4*a*b*d^2*arcsinh(c/(s
qrt(c*d)*abs(x)))/sqrt(c) + 1/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3
/2) + 3/2*sqrt(d*x^2 + c)*b^2*d + 1/2*(d*x^2 + c)^(3/2)*b^2*d/c + 1/4*(d*x^
2 + c)^(3/2)*a*b*d^2/c^2 + 3/4*sqrt(d*x^2 + c)*a*b*d^2/c - 1/48*(d*x^2 + c)
^(3/2)*a^2*d^3/c^3 - 1/16*sqrt(d*x^2 + c)*a^2*d^3/c^2 - 1/2*(d*x^2 + c)^(5/
2)*b^2/(c*x^2) - 1/4*(d*x^2 + c)^(5/2)*a*b*d/(c^2*x^2) + 1/48*(d*x^2 + c)^(
5/2)*a^2*d^2/(c^3*x^2) - 1/2*(d*x^2 + c)^(5/2)*a*b/(c*x^4) + 1/24*(d*x^2 +
c)^(5/2)*a^2*d/(c^2*x^4) - 1/6*(d*x^2 + c)^(5/2)*a^2/(c*x^6)
```

Fricas [A]
time = 1.12, size = 301, normalized size = 1.61

$$\left[-\frac{3\,(24\,b^2c^2d+12\,abcd^2-a^2d^3)\sqrt{c}\,x^6\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,(48\,b^2c^2dx^6-8\,a^2c^3-3\,(8\,b^2c^3+20\,abc^2d+a^2cd^2)x^4-2\,(12\,abc^3+7\,a^2c^2d)x^2)\sqrt{dx^2+c}}{96\,c^2x^6},\ \frac{3\,(24\,b^2c^2d+12\,abcd^2-a^2d^3)\sqrt{-c}\,x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(48\,b^2c^2dx^6-8\,a^2c^3-3\,(8\,b^2c^3+20\,abc^2d+a^2cd^2)x^4-2\,(12\,abc^3+7\,a^2c^2d)x^2)\sqrt{dx^2+c}}{48\,c^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(24*b^2*c^2*d + 12*a*b*c*d^2 - a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 +
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(48*b^2*c^2*d*x^6 - 8*a^2*c^3 -
3*(8*b^2*c^3 + 20*a*b*c^2*d + a^2*c*d^2)*x^4 - 2*(12*a*b*c^3 + 7*a^2*c^2*d)
*x^2)*sqrt(d*x^2 + c))/(c^2*x^6), 1/48*(3*(24*b^2*c^2*d + 12*a*b*c*d^2 - a^
2*d^3)*sqrt(-c)*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (48*b^2*c^2*d*x^6 -
8*a^2*c^3 - 3*(8*b^2*c^3 + 20*a*b*c^2*d + a^2*c*d^2)*x^4 - 2*(12*a*b*c^3 +
7*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(172) = 344$.
time = 104.80, size = 367, normalized size = 1.96

$$-\frac{a^2c^2}{6\sqrt{d}\,x^7\sqrt{\frac{c}{dx^2}+1}}-\frac{11a^2c\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{17a^2d^{\frac{3}{2}}}{48x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2d^{\frac{5}{2}}}{16cx\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{16c^{\frac{3}{2}}}-\frac{abc^2}{2\sqrt{d}\,x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3abc\sqrt{d}}{4x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x}-\frac{abd^{\frac{3}{2}}}{4x\sqrt{\frac{c}{dx^2}+1}}-\frac{3abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{4\sqrt{c}}-\frac{3b^2\sqrt{c}\,d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2}-\frac{b^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}+\frac{b^2c\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{b^2d^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**7,x)
```

```
[Out] -a**2*c**2/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 11*a**2*c*sqrt(d)/(24*x*
*5*sqrt(c/(d*x**2) + 1)) - 17*a**2*d**(3/2)/(48*x**3*sqrt(c/(d*x**2) + 1))
- a**2*d**(5/2)/(16*c*x*sqrt(c/(d*x**2) + 1)) + a**2*d**3*asinh(sqrt(c)/(sq
rt(d)*x))/(16*c**(3/2)) - a*b*c**2/(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) -
3*a*b*c*sqrt(d)/(4*x**3*sqrt(c/(d*x**2) + 1)) - a*b*d**(3/2)*sqrt(c/(d*x**2
) + 1)/x - a*b*d**(3/2)/(4*x*sqrt(c/(d*x**2) + 1)) - 3*a*b*d**2*asinh(sqrt(
c)/(sqrt(d)*x))/(4*sqrt(c)) - 3*b**2*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x))/2
 - b**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) + b**2*c*sqrt(d)/(x*sqrt(c/(d*
x**2) + 1)) + b**2*d**(3/2)*x/sqrt(c/(d*x**2) + 1)
```

Giac [A]
time = 1.01, size = 259, normalized size = 1.39

$$\frac{48\,\sqrt{dx^2+c}\,b^2d^2+\frac{3\,(24\,b^2c^2d^2+12\,abcd^3-a^2d^4)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}-\frac{24\,(dx^2+c)^{\frac{5}{2}}b^2c^2d^2-48\,(dx^2+c)^{\frac{3}{2}}b^2c^3d^2+24\,\sqrt{dx^2+c}\,b^2c^4d^2+60\,(dx^2+c)^{\frac{5}{2}}abcd^3-96\,(dx^2+c)^{\frac{3}{2}}abc^2d^3+36\,\sqrt{dx^2+c}\,abc^3d^3+3\,(dx^2+c)^{\frac{5}{2}}a^2d^4+8\,(dx^2+c)^{\frac{3}{2}}a^2cd^4-3\,\sqrt{dx^2+c}\,a^2c^2d^4}{cd^3x^6}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(48*sqrt(d*x^2 + c)*b^2*d^2 + 3*(24*b^2*c^2*d^2 + 12*a*b*c*d^3 - a^2*d
^4)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (24*(d*x^2 + c)^(5/2)*b
```

```
^2*c^2*d^2 - 48*(d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*
d^2 + 60*(d*x^2 + c)^(5/2)*a*b*c*d^3 - 96*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 + 3
6*sqrt(d*x^2 + c)*a*b*c^3*d^3 + 3*(d*x^2 + c)^(5/2)*a^2*d^4 + 8*(d*x^2 + c)
^(3/2)*a^2*c*d^4 - 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c*d^3*x^6))/d
```

Mupad [B]
time = 1.09, size = 215, normalized size = 1.15

$$\frac{\sqrt{d\,x^2+c}\,\left(-\frac{a^2\,c\,d^3}{16}+\frac{3\,a\,b\,c^2\,d^2}{4}+\frac{b^2\,c^3\,d}{2}\right)-{\left(d\,x^2+c\right)}^{3/2}\,\left(-\frac{a^2\,d^3}{6}+2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)+\frac{{\left(d\,x^2+c\right)}^{5/2}\,\left(a^2\,d^3+20\,a\,b\,c\,d^2+8\,b^2\,c^2\,d\right)}{16\,c}}{3\,c\,{\left(d\,x^2+c\right)}^2-3\,c^2\,\left(d\,x^2+c\right)-{\left(d\,x^2+c\right)}^3+c^3}+b^2\,d\,\sqrt{d\,x^2+c}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(-a^2\,d^2+12\,a\,b\,c\,d+24\,b^2\,c^2\right)}{16\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((b^2*c^3*d)/2 - (a^2*c*d^3)/16 + (3*a*b*c^2*d^2)/4) - (
c + d*x^2)^(3/2)*(b^2*c^2*d - (a^2*d^3)/6 + 2*a*b*c*d^2) + ((c + d*x^2)^(5/
2)*(a^2*d^3 + 8*b^2*c^2*d + 20*a*b*c*d^2))/(16*c))/(3*c*(c + d*x^2)^2 - 3*c
^2*(c + d*x^2) - (c + d*x^2)^3 + c^3) + b^2*d*(c + d*x^2)^(1/2) - (d*atanh(
(c + d*x^2)^(1/2)/c^(1/2))*(24*b^2*c^2 - a^2*d^2 + 12*a*b*c*d))/(16*c^(3/2)
)
```

3.625 $\int x^3(a+bx^2)^2\left(c+dx^2\right)^{5/2}\,dx$

Optimal. Leaf size=114

$$-\frac{c(bc-ad)^2\left(c+dx^2\right)^{7/2}}{7d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{9/2}}{9d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{11/2}}{11d^4}+\frac{b^2\left(c+dx^2\right)^{13/2}}{13d^4}$$

```
[Out] -1/7*c*(-a*d+b*c)^2*(d*x^2+c)^(7/2)/d^4+1/9*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2+
c)^(9/2)/d^4-1/11*b*(-2*a*d+3*b*c)*(d*x^2+c)^(11/2)/d^4+1/13*b^2*(d*x^2+c)^
(13/2)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {457, 78}

$$-\frac{b\left(c+dx^2\right)^{11/2}(3bc-2ad)}{11d^4}+\frac{\left(c+dx^2\right)^{9/2}(bc-ad)(3bc-ad)}{9d^4}-\frac{c\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^4}+\frac{b^2\left(c+dx^2\right)^{13/2}}{13d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] -1/7*(c*(b*c - a*d)^2*(c + d*x^2)^(7/2))/d^4 + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(9/2))/(9*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(11/2))/(11*d^4)
 + (b^2*(c + d*x^2)^(13/2))/(13*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)^{5/2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{c(bc-ad)^2(c+dx)^{5/2}}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^{7/2}}{d^3}\right.\right.$$
$$=-\frac{c(bc-ad)^2\left(c+dx^2\right)^{7/2}}{7d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{9/2}}{9d^4}-\frac{b(3bc-}{}$$

Mathematica [A]
time = 0.08, size = 99, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{7/2}\left(143a^2d^2(-2c+7dx^2)+26abd(8c^2-28cdx^2+63d^2x^4)+b^2(-48c^3+168c^2dx^2-378cd^2x^4+693d^3x^6)\right)}{9009d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(143*a^2*d^2*(-2*c + 7*d*x^2) + 26*a*b*d*(8*c^2 - 28*c*d
*x^2 + 63*d^2*x^4) + b^2*(-48*c^3 + 168*c^2*d*x^2 - 378*c*d^2*x^4 + 693*d^3
*x^6)))/(9009*d^4)
```

Maple [A]
time = 0.09, size = 185, normalized size = 1.62

method	result
gosper	$-\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}\left(-693b^2x^6d^3-1638ab\,d^3x^4+378b^2c\,d^2x^4-1001a^2d^3x^2+728abc\,d^2x^2-168b^2c^2d\,x^2+286a^2c\,d^2-208ab\,c^2d+48b^2c^3\right)}{9009d^4}$
default	$b^2\left(\frac{x^6\left(d\,x^2+c\right)^{\frac{7}{2}}}{13d}-\frac{6c\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{7}{2}}}{11d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{7}{2}}}{9d}-\frac{2c\left(d\,x^2+c\right)^{\frac{7}{2}}}{63d^2}\right)}{11d}\right)}{13d}\right)+2ab\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{7}{2}}}{11d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{7}{2}}}{9d}-\right.}{11d}\right.$
trager	$-\frac{\left(-693b^2d^6x^{12}-1638ab\,d^6x^{10}-1701b^2c\,d^5x^{10}-1001a^2d^6x^8-4186abc\,d^5x^8-1113b^2c^2d^4x^8-2717a^2c\,d^5x^6-2938ab\,c^2d^4x^6-15b^2c^3\right.}{}$
risch	$-\frac{\left(-693b^2d^6x^{12}-1638ab\,d^6x^{10}-1701b^2c\,d^5x^{10}-1001a^2d^6x^8-4186abc\,d^5x^8-1113b^2c^2d^4x^8-2717a^2c\,d^5x^6-2938ab\,c^2d^4x^6-15b^2c^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/13*x^6*(d*x^2+c)^(7/2)/d-6/13*c/d*(1/11*x^4*(d*x^2+c)^(7/2)/d-4/11*c
/d*(1/9*x^2*(d*x^2+c)^(7/2)/d-2/63*c/d^2*(d*x^2+c)^(7/2))))+2*a*b*(1/11*x^4
*(d*x^2+c)^(7/2)/d-4/11*c/d*(1/9*x^2*(d*x^2+c)^(7/2)/d-2/63*c/d^2*(d*x^2+c)
^(7/2)))+a^2*(1/9*x^2*(d*x^2+c)^(7/2)/d-2/63*c/d^2*(d*x^2+c)^(7/2))
```

Maxima [A]
time = 0.30, size = 181, normalized size = 1.59

$$\frac{(dx^2+c)^{\frac{7}{2}}b^2x^6}{13\,d}-\frac{6\,(dx^2+c)^{\frac{7}{2}}b^2cx^4}{143\,d^2}+\frac{2\,(dx^2+c)^{\frac{7}{2}}abx^4}{11\,d}+\frac{8\,(dx^2+c)^{\frac{7}{2}}b^2c^2x^2}{429\,d^3}-\frac{8\,(dx^2+c)^{\frac{7}{2}}abcx^2}{99\,d^2}+\frac{(dx^2+c)^{\frac{7}{2}}a^2x^2}{9\,d}-\frac{16\,(dx^2+c)^{\frac{7}{2}}b^2c^3}{3003\,d^4}+\frac{16\,(dx^2+c)^{\frac{7}{2}}abc^2}{693\,d^3}-\frac{2\,(dx^2+c)^{\frac{7}{2}}a^2c}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/13*(d*x^2 + c)^(7/2)*b^2*x^6/d - 6/143*(d*x^2 + c)^(7/2)*b^2*c*x^4/d^2 +
2/11*(d*x^2 + c)^(7/2)*a*b*x^4/d + 8/429*(d*x^2 + c)^(7/2)*b^2*c^2*x^2/d^3
- 8/99*(d*x^2 + c)^(7/2)*a*b*c*x^2/d^2 + 1/9*(d*x^2 + c)^(7/2)*a^2*x^2/d -
16/3003*(d*x^2 + c)^(7/2)*b^2*c^3/d^4 + 16/693*(d*x^2 + c)^(7/2)*a*b*c^2/d^
3 - 2/63*(d*x^2 + c)^(7/2)*a^2*c/d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(98) = 196.
time = 1.19, size = 216, normalized size = 1.89

$$\frac{\left(693\,b^2d^6x^{12}+63\,(27\,b^2cd^5+26\,abd^6)x^{10}+7\,(159\,b^2c^2d^4+598\,abcd^5+143\,a^2d^6)x^8-48\,b^2c^6+208\,abc^5d-286\,a^2c^4d^2+(15\,b^2c^3d^3+2938\,abc^2d^4+2717\,a^2cd^5)x^6-3\,(6\,b^2c^4d^2-26\,abc^3d^3-715\,a^2c^2d^4)x^4+(24\,b^2c^5d-104\,abc^4d^2+143\,a^2c^3d^3)x^2\right)\sqrt{dx^2+c}}{9009\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9009*(693*b^2*d^6*x^12 + 63*(27*b^2*c*d^5 + 26*a*b*d^6)*x^10 + 7*(159*b^2
*c^2*d^4 + 598*a*b*c*d^5 + 143*a^2*d^6)*x^8 - 48*b^2*c^6 + 208*a*b*c^5*d -
286*a^2*c^4*d^2 + (15*b^2*c^3*d^3 + 2938*a*b*c^2*d^4 + 2717*a^2*c*d^5)*x^6
- 3*(6*b^2*c^4*d^2 - 26*a*b*c^3*d^3 - 715*a^2*c^2*d^4)*x^4 + (24*b^2*c^5*d
- 104*a*b*c^4*d^2 + 143*a^2*c^3*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(102) = 204.
time = 0.69, size = 468, normalized size = 4.11

$$\begin{cases} -\frac{2a^2c^4\sqrt{c+dx^2}}{63d^2}+\frac{a^2c^3x^2\sqrt{c+dx^2}}{63d}+\frac{5a^2c^2x^4\sqrt{c+dx^2}}{21}+\frac{19a^2cdx^6\sqrt{c+dx^2}}{63}+\frac{a^2d^2x^8\sqrt{c+dx^2}}{9}+\frac{16abc^5\sqrt{c+dx^2}}{693d^3}-\frac{8abc^4x^2\sqrt{c+dx^2}}{693d^2}+\frac{2abc^3x^4\sqrt{c+dx^2}}{231d}+\frac{226abc^2x^6\sqrt{c+dx^2}}{693}+\frac{46abcdx^8\sqrt{c+dx^2}}{99}+\frac{2abd^2x^{10}\sqrt{c+dx^2}}{11}-\frac{16b^2c^6\sqrt{c+dx^2}}{3003d^4}+\frac{8b^2c^5x^2\sqrt{c+dx^2}}{3003d^3}-\frac{2b^2c^4x^4\sqrt{c+dx^2}}{1001d^2}+\frac{5b^2c^3x^6\sqrt{c+dx^2}}{3003d}+\frac{53b^2c^2x^8\sqrt{c+dx^2}}{429}+\frac{27b^2cdx^{10}\sqrt{c+dx^2}}{143}+\frac{b^2d^2x^{12}\sqrt{c+dx^2}}{13} & \text{for}\ d\neq 0 \\ c^{\frac{5}{2}}\left(\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*c**4*sqrt(c + d*x**2)/(63*d**2) + a**2*c**3*x**2*sqrt(c
+ d*x**2)/(63*d) + 5*a**2*c**2*x**4*sqrt(c + d*x**2)/21 + 19*a**2*c*d*x**6*
sqrt(c + d*x**2)/63 + a**2*d**2*x**8*sqrt(c + d*x**2)/9 + 16*a*b*c**5*sqrt(
```

```
c + d*x**2)/(693*d**3) - 8*a*b*c**4*x**2*sqrt(c + d*x**2)/(693*d**2) + 2*a*
b*c**3*x**4*sqrt(c + d*x**2)/(231*d) + 226*a*b*c**2*x**6*sqrt(c + d*x**2)/6
93 + 46*a*b*c*d*x**8*sqrt(c + d*x**2)/99 + 2*a*b*d**2*x**10*sqrt(c + d*x**2
)/11 - 16*b**2*c**6*sqrt(c + d*x**2)/(3003*d**4) + 8*b**2*c**5*x**2*sqrt(c
+ d*x**2)/(3003*d**3) - 2*b**2*c**4*x**4*sqrt(c + d*x**2)/(1001*d**2) + 5*b
**2*c**3*x**6*sqrt(c + d*x**2)/(3003*d) + 53*b**2*c**2*x**8*sqrt(c + d*x**2
)/429 + 27*b**2*c*d*x**10*sqrt(c + d*x**2)/143 + b**2*d**2*x**12*sqrt(c + d
*x**2)/13, Ne(d, 0)), (c**(5/2)*(a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8), T
rue))
```

Giac [A]
time = 0.95, size = 150, normalized size = 1.32

$$\frac{693\,(dx^2+c)^{\frac{13}{2}}b^2-2457\,(dx^2+c)^{\frac{11}{2}}b^2c+3003\,(dx^2+c)^{\frac{9}{2}}b^2c^2-1287\,(dx^2+c)^{\frac{7}{2}}b^2c^3+1638\,(dx^2+c)^{\frac{11}{2}}abd-4004\,(dx^2+c)^{\frac{9}{2}}abcd+2574\,(dx^2+c)^{\frac{7}{2}}abc^2d+1001\,(dx^2+c)^{\frac{9}{2}}a^2d^2-1287\,(dx^2+c)^{\frac{7}{2}}a^2cd^2}{9009\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/9009*(693*(d*x^2 + c)^(13/2)*b^2 - 2457*(d*x^2 + c)^(11/2)*b^2*c + 3003*(
d*x^2 + c)^(9/2)*b^2*c^2 - 1287*(d*x^2 + c)^(7/2)*b^2*c^3 + 1638*(d*x^2 + c
)^(11/2)*a*b*d - 4004*(d*x^2 + c)^(9/2)*a*b*c*d + 2574*(d*x^2 + c)^(7/2)*a*
b*c^2*d + 1001*(d*x^2 + c)^(9/2)*a^2*d^2 - 1287*(d*x^2 + c)^(7/2)*a^2*c*d^2
)/d^4
```

Mupad [B]
time = 0.45, size = 207, normalized size = 1.82

$$\sqrt{d\,x^2+c}\,\left(\frac{x^8\,(1001\,a^2\,d^6+4186\,a\,b\,c\,d^5+1113\,b^2\,c^2\,d^4)}{9009\,d^4}-\frac{286\,a^2\,c^4\,d^2-208\,a\,b\,c^5\,d+48\,b^2\,c^6}{9009\,d^4}+\frac{b^2\,d^2\,x^{12}}{13}+\frac{c\,x^6\,(2717\,a^2\,d^2+2938\,a\,b\,c\,d+15\,b^2\,c^2)}{9009\,d}+\frac{b\,d\,x^{10}\,(26\,a\,d+27\,b\,c)}{143}+\frac{c^3\,x^2\,(143\,a^2\,d^2-104\,a\,b\,c\,d+24\,b^2\,c^2)}{9009\,d^3}+\frac{c^2\,x^4\,(715\,a^2\,d^2+26\,a\,b\,c\,d-6\,b^2\,c^2)}{3003\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((x^8*(1001*a^2*d^6 + 1113*b^2*c^2*d^4 + 4186*a*b*c*d^5))
/(9009*d^4) - (48*b^2*c^6 + 286*a^2*c^4*d^2 - 208*a*b*c^5*d)/(9009*d^4) + (
b^2*d^2*x^12)/13 + (c*x^6*(2717*a^2*d^2 + 15*b^2*c^2 + 2938*a*b*c*d))/(9009
*d) + (b*d*x^10*(26*a*d + 27*b*c))/143 + (c^3*x^2*(143*a^2*d^2 + 24*b^2*c^2
 - 104*a*b*c*d))/(9009*d^3) + (c^2*x^4*(715*a^2*d^2 - 6*b^2*c^2 + 26*a*b*c*
d))/(3003*d^2))
```

3.626 $\int x^2(a+bx^2)^2 (c+dx^2)^{5/2} \, dx$

Optimal. Leaf size=281

$$\frac{c^3(40a^2d^2+bc(5bc-24ad))\,x\sqrt{c+dx^2}}{1024d^3}+\frac{c^2(40a^2d^2+bc(5bc-24ad))\,x^3\sqrt{c+dx^2}}{512d^2}+\frac{c(40a^2d^2+bc(5bc-2\ldots}{384\ldots}$$

```
[Out] 1/384*c*(40*a^2*d^2+b*c*(-24*a*d+5*b*c))*x^3*(d*x^2+c)^(3/2)/d^2+1/320*(40*
a^2*d^2+b*c*(-24*a*d+5*b*c))*x^3*(d*x^2+c)^(5/2)/d^2-1/120*b*(-24*a*d+5*b*c
)*x^3*(d*x^2+c)^(7/2)/d^2+1/12*b^2*x^5*(d*x^2+c)^(7/2)/d-1/1024*c^4*(40*a^2
*d^2+b*c*(-24*a*d+5*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(7/2)+1/1024
*c^3*(40*a^2*d^2+b*c*(-24*a*d+5*b*c))*x*(d*x^2+c)^(1/2)/d^3+1/512*c^2*(40*a
^2*d^2+b*c*(-24*a*d+5*b*c))*x^3*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 278, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {475, 470, 285, 327, 223, 212}

$$-\frac{c^4(40a^2d^2+bc(5bc-24ad))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{1024d^{7/2}}+\frac{c^3x\sqrt{c+dx^2}\,(40a^2d^2+bc(5bc-24ad))}{1024d^3}+\frac{c^2x^3\sqrt{c+dx^2}\,(40a^2d^2+bc(5bc-24ad))}{512d^2}+\frac{1}{320}x^3(c+dx^2)^{5/2}\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)+\frac{cx^3(c+dx^2)^{3/2}\,(40a^2d^2+bc(5bc-24ad))}{384d^2}-\frac{bx^3(c+dx^2)^{7/2}\,(5bc-24ad)}{120d^2}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (c^3*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(1024*d^3) + (c
^2*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*Sqrt[c + d*x^2])/(512*d^2) + (c*
(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*(c + d*x^2)^(3/2))/(384*d^2) + ((40
*a^2 + (b*c*(5*b*c - 24*a*d))/d^2)*x^3*(c + d*x^2)^(5/2))/320 - (b*(5*b*c -
 24*a*d)*x^3*(c + d*x^2)^(7/2))/(120*d^2) + (b^2*x^5*(c + d*x^2)^(7/2))/(12
*d) - (c^4*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(1024*d^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} \, dx = \frac{b^2x^5(c+dx^2)^{7/2}}{12d} + \frac{\int x^2(c+dx^2)^{5/2}\left(12a^2d-b(5bc-24ad)x^2\right) \, dx}{12d}$$

$$= -\frac{b(5bc-24ad)x^3(c+dx^2)^{7/2}}{120d^2} + \frac{b^2x^5(c+dx^2)^{7/2}}{12d} + \frac{1}{40}\left(40a^2 + \frac{bc(5bc-\right.$$

$$= \frac{1}{320}\left(40a^2 + \frac{bc(5bc-24ad)}{d^2}\right)x^3\left(c+dx^2\right)^{5/2} - \frac{b(5bc-24ad)x^3(c+dx^2)}{120d^2}$$

$$= \frac{1}{384}c\left(40a^2 + \frac{bc(5bc-24ad)}{d^2}\right)x^3\left(c+dx^2\right)^{3/2} + \frac{1}{320}\left(40a^2 + \frac{bc(5bc-24\right.$$

$$= \frac{1}{512}c^2\left(40a^2 + \frac{bc(5bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2} + \frac{1}{384}c\left(40a^2 + \frac{bc(5bc-24\right.$$

$$= \frac{c^3\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d} + \frac{1}{512}c^2\left(40a^2 + \frac{bc(5bc-24ad)}{d^2}\right)x^3$$

$$= \frac{c^3\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d} + \frac{1}{512}c^2\left(40a^2 + \frac{bc(5bc-24ad)}{d^2}\right)x^3$$

$$= \frac{c^3\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d} + \frac{1}{512}c^2\left(40a^2 + \frac{bc(5bc-24ad)}{d^2}\right)x^3$$

Mathematica [A]
time = 0.35, size = 225, normalized size = 0.80

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\,(40a^2d^2(15c^3+118c^2dx^2+136cd^2x^4+48d^3x^6)+24abd(-15c^4+10c^3dx^2+248c^2d^2x^4+336cd^3x^6+128d^4x^8)+5b^2(15c^5-10c^4dx^2+8c^3d^2x^4+432c^2d^3x^6+640cd^4x^8+256d^5x^{10}))+15c^4(5b^2c^2-24abcd+40a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{15360d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(40*a^2*d^2*(15*c^3 + 118*c^2*d*x^2 + 136*c*d^2*
x^4 + 48*d^3*x^6) + 24*a*b*d*(-15*c^4 + 10*c^3*d*x^2 + 248*c^2*d^2*x^4 + 33
6*c*d^3*x^6 + 128*d^4*x^8) + 5*b^2*(15*c^5 - 10*c^4*d*x^2 + 8*c^3*d^2*x^4 +
 432*c^2*d^3*x^6 + 640*c*d^4*x^8 + 256*d^5*x^10)) + 15*c^4*(5*b^2*c^2 - 24*
a*b*c*d + 40*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(15360*d^(7/2))
```

Maple [A]
time = 0.09, size = 353, normalized size = 1.26

method	result

risch	$\frac{x\left(1280b^2d^5x^{10}+3072ab\,d^5x^8+3200b^2c\,d^4x^8+1920a^2d^5x^6+8064abc\,d^4x^6+2160b^2c^2d^3x^6+5440a^2c\,d^4x^4+5952ab\,c^2d^3x^4+40b^2c^3d^2x\right.}{15360d^3}$

default	$b^2\left(\frac{x^5(dx^2+c)^{\frac{7}{2}}}{12d} - \frac{5c\left(\frac{x^3(dx^2+c)^{\frac{7}{2}}}{10d} - \frac{3c\left(\frac{x(dx^2+c)^{\frac{7}{2}}}{8d} - \frac{c\left(\frac{x(dx^2+c)^{\frac{5}{2}}}{6} + \frac{5c\left(\frac{x(dx^2+c)^{\frac{3}{2}}}{4} + \frac{3c\left(\frac{x\sqrt{dx^2+c}}{2} + \frac{c\ln\left(x\sqrt{d}+\sqrt{c\right.}}{2\sqrt{d}}\right.}{4}\right.}{6}\right.}{8d}\right.}{10d}\right.}{12d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/12*x^5*(d*x^2+c)^(7/2)/d-5/12*c/d*(1/10*x^3*(d*x^2+c)^(7/2)/d-3/10*c
/d*(1/8*x*(d*x^2+c)^(7/2)/d-1/8*c/d*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*
x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2
+c)^(1/2))))))))+2*a*b*(1/10*x^3*(d*x^2+c)^(7/2)/d-3/10*c/d*(1/8*x*(d*x^2+c
)^(7/2)/d-1/8*c/d*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c
*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))))+a^
2*(1/8*x*(d*x^2+c)^(7/2)/d-1/8*c/d*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x
^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+
c)^(1/2))))))
```

Maxima [A]
time = 0.28, size = 361, normalized size = 1.28

$$\frac{(dx^2+c)^{\frac{7}{2}}b^2x^5}{12\,d} - \frac{(dx^2+c)^{\frac{7}{2}}b^2cx^3}{24\,d^2} + \frac{(dx^2+c)^{\frac{7}{2}}abx^3}{5\,d} + \frac{(dx^2+c)^{\frac{7}{2}}b^2c^2x}{64\,d^3} - \frac{(dx^2+c)^{\frac{5}{2}}b^2c^3x}{384\,d^3} - \frac{5\,(dx^2+c)^{\frac{3}{2}}b^2c^4x}{1536\,d^3} - \frac{5\,\sqrt{dx^2+c}\,b^2c^5x}{1024\,d^3} - \frac{3\,(dx^2+c)^{\frac{7}{2}}abcx}{40\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}}abc^2x}{80\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}abc^3x}{64\,d^2} + \frac{3\,\sqrt{dx^2+c}\,abc^4x}{128\,d^2} + \frac{(dx^2+c)^{\frac{7}{2}}a^2x}{8\,d} - \frac{(dx^2+c)^{\frac{5}{2}}a^2cx}{48\,d} - \frac{5\,(dx^2+c)^{\frac{3}{2}}a^2c^2x}{192\,d} - \frac{5\,\sqrt{dx^2+c}\,a^2c^3x}{128\,d} - \frac{5\,b^2c^6\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{1024\,d^{\frac{7}{2}}} + \frac{3\,abc^5\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{5}{2}}} - \frac{5\,a^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(d*x^2 + c)^(7/2)*b^2*x^5/d - 1/24*(d*x^2 + c)^(7/2)*b^2*c*x^3/d^2 + 1
/5*(d*x^2 + c)^(7/2)*a*b*x^3/d + 1/64*(d*x^2 + c)^(7/2)*b^2*c^2*x/d^3 - 1/3
84*(d*x^2 + c)^(5/2)*b^2*c^3*x/d^3 - 5/1536*(d*x^2 + c)^(3/2)*b^2*c^4*x/d^3
 - 5/1024*sqrt(d*x^2 + c)*b^2*c^5*x/d^3 - 3/40*(d*x^2 + c)^(7/2)*a*b*c*x/d^
2 + 1/80*(d*x^2 + c)^(5/2)*a*b*c^2*x/d^2 + 1/64*(d*x^2 + c)^(3/2)*a*b*c^3*x
/d^2 + 3/128*sqrt(d*x^2 + c)*a*b*c^4*x/d^2 + 1/8*(d*x^2 + c)^(7/2)*a^2*x/d
- 1/48*(d*x^2 + c)^(5/2)*a^2*c*x/d - 5/192*(d*x^2 + c)^(3/2)*a^2*c^2*x/d -
5/128*sqrt(d*x^2 + c)*a^2*c^3*x/d - 5/1024*b^2*c^6*arcsinh(d*x/sqrt(c*d))/d
^(7/2) + 3/128*a*b*c^5*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 5/128*a^2*c^4*arcsi
nh(d*x/sqrt(c*d))/d^(3/2)
```

Fricas [A]
time = 2.01, size = 495, normalized size = 1.76

$$\left[\frac{15\,(5\,b^2c^6 - 24\,abc^5d + 40\,a^2c^4d^2)\sqrt{d}\,\log\left(-2\,dx^2 + 2\,\sqrt{dx^2+c}\,\sqrt{d}\,x - c\right) + 2\,(1280\,b^2d^6x^{11} + 128\,(25\,b^2cd^5 + 24\,abd^6)x^9 + 48\,(45\,b^2c^2d^4 + 168\,abcd^5 + 40\,a^2d^6)x^7 + 8\,(5\,b^2c^3d^3 + 744\,abc^2d^4 + 680\,a^2cd^5)x^5 - 10\,(5\,b^2c^4d^2 - 24\,abc^3d^3 - 472\,a^2c^2d^4)x^3 + 15\,(5\,b^2c^5d - 24\,abc^4d^2 + 40\,a^2c^3d^3)x)\sqrt{dx^2+c}}{30720\,d^4}, \frac{15\,(5\,b^2c^6 - 24\,abc^5d + 40\,a^2c^4d^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right) + (1280\,b^2d^6x^{11} + 128\,(25\,b^2cd^5 + 24\,abd^6)x^9 + 48\,(45\,b^2c^2d^4 + 168\,abcd^5 + 40\,a^2d^6)x^7 + 8\,(5\,b^2c^3d^3 + 744\,abc^2d^4 + 680\,a^2cd^5)x^5 - 10\,(5\,b^2c^4d^2 - 24\,abc^3d^3 - 472\,a^2c^2d^4)x^3 + 15\,(5\,b^2c^5d - 24\,abc^4d^2 + 40\,a^2c^3d^3)x)\sqrt{dx^2+c}}{15360\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(5*b^2*c^6 - 24*a*b*c^5*d + 40*a^2*c^4*d^2)*sqrt(d)*log(-2*d*x
^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(1280*b^2*d^6*x^11 + 128*(25*b^2*
c*d^5 + 24*a*b*d^6)*x^9 + 48*(45*b^2*c^2*d^4 + 168*a*b*c*d^5 + 40*a^2*d^6)*
```

```
x^7 + 8*(5*b^2*c^3*d^3 + 744*a*b*c^2*d^4 + 680*a^2*c*d^5)*x^5 - 10*(5*b^2*c
^4*d^2 - 24*a*b*c^3*d^3 - 472*a^2*c^2*d^4)*x^3 + 15*(5*b^2*c^5*d - 24*a*b*c
^4*d^2 + 40*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/15360*(15*(5*b^2*c^6 -
24*a*b*c^5*d + 40*a^2*c^4*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
+ (1280*b^2*d^6*x^11 + 128*(25*b^2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(45*b^2*c^2
*d^4 + 168*a*b*c*d^5 + 40*a^2*d^6)*x^7 + 8*(5*b^2*c^3*d^3 + 744*a*b*c^2*d^4
 + 680*a^2*c*d^5)*x^5 - 10*(5*b^2*c^4*d^2 - 24*a*b*c^3*d^3 - 472*a^2*c^2*d^
4)*x^3 + 15*(5*b^2*c^5*d - 24*a*b*c^4*d^2 + 40*a^2*c^3*d^3)*x)*sqrt(d*x^2 +
 c))/d^4]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.16, size = 265, normalized size = 0.94

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^2d^2x^2+\frac{25\,b^2cd^{11}+24\,abd^{12}}{d^{10}}\right)x^2+\frac{3\,(45\,b^2c^2d^{10}+168\,abcd^{11}+40\,a^2d^{12})}{d^{10}}\right)x^2+\frac{5\,b^2c^3d^9+744\,abc^2d^{10}+680\,a^2cd^{11}}{d^{10}}\right)x^2-\frac{5\,(5\,b^2c^4d^8-24\,abc^3d^9-472\,a^2c^2d^{10})}{d^{10}}\right)x^2+\frac{15\,(5\,b^2c^5d^7-24\,abc^4d^8+40\,a^2c^3d^9)}{d^{10}}\right)\sqrt{dx^2+c}\,x+\frac{(5\,b^2c^6-24\,abc^5d+40\,a^2c^4d^2)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{1024\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d^2*x^2 + (25*b^2*c*d^11 + 24*a*b*d^12)/d^10)*x
^2 + 3*(45*b^2*c^2*d^10 + 168*a*b*c*d^11 + 40*a^2*d^12)/d^10)*x^2 + (5*b^2*
c^3*d^9 + 744*a*b*c^2*d^10 + 680*a^2*c*d^11)/d^10)*x^2 - 5*(5*b^2*c^4*d^8 -
 24*a*b*c^3*d^9 - 472*a^2*c^2*d^10)/d^10)*x^2 + 15*(5*b^2*c^5*d^7 - 24*a*b*
c^4*d^8 + 40*a^2*c^3*d^9)/d^10)*sqrt(d*x^2 + c)*x + 1/1024*(5*b^2*c^6 - 24*
a*b*c^5*d + 40*a^2*c^4*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2), x)
```

3.627 $\int x(a+bx^2)^2 (c+dx^2)^{5/2} \, dx$

Optimal. Leaf size=77

$$\frac{(bc-ad)^2 (c+dx^2)^{7/2}}{7d^3} - \frac{2b(bc-ad) (c+dx^2)^{9/2}}{9d^3} + \frac{b^2(c+dx^2)^{11/2}}{11d^3}$$

[Out] 1/7*(-a*d+b*c)^2*(d*x^2+c)^(7/2)/d^3-2/9*b*(-a*d+b*c)*(d*x^2+c)^(9/2)/d^3+1
/11*b^2*(d*x^2+c)^(11/2)/d^3

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$-\frac{2b(c+dx^2)^{9/2} (bc-ad)}{9d^3} + \frac{(c+dx^2)^{7/2} (bc-ad)^2}{7d^3} + \frac{b^2(c+dx^2)^{11/2}}{11d^3}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]

[Out] ((b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(9
/2))/(9*d^3) + (b^2*(c + d*x^2)^(11/2))/(11*d^3)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x(a+bx^2)^2 (c+dx^2)^{5/2} \, dx &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^2(c+dx)^{5/2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2(c+dx)^{5/2}}{d^2} - \frac{2b(bc-ad)(c+dx)^{7/2}}{d^2} + \frac{b^2(c+\cdots}{d^2}\right.\right. \\
&= \frac{(bc-ad)^2 (c+dx^2)^{7/2}}{7d^3} - \frac{2b(bc-ad) (c+dx^2)^{9/2}}{9d^3} + \frac{b^2(c+dx^2)^{11/2}}{11d^3}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 0.87

$$\frac{(c+dx^2)^{7/2}\left(99a^2d^2+22abd(-2c+7dx^2)+b^2(8c^2-28cdx^2+63d^2x^4)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x^2) + b^2*(8*c^2 - 2
8*c*d*x^2 + 63*d^2*x^4)))/(693*d^3)
```

Maple [A]
time = 0.10, size = 117, normalized size = 1.52

method	result
gosper	$\frac{(d\,x^2+c)^{\frac{7}{2}}\left(63b^2x^4d^2+154ab\,d^2x^2-28b^2cd\,x^2+99a^2d^2-44abcd+8b^2c^2\right)}{693d^3}$
default	$b^2\left(\frac{x^4(d\,x^2+c)^{\frac{7}{2}}}{11d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{7}{2}}}{9d}-\frac{2c\left(d\,x^2+c\right)^{\frac{7}{2}}}{63d^2}\right)}{11d}\right)+2ab\left(\frac{x^2(d\,x^2+c)^{\frac{7}{2}}}{9d}-\frac{2c(d\,x^2+c)^{\frac{7}{2}}}{63d^2}\right)+\frac{a^2(d\,x^2+c)^{\frac{7}{2}}}{7d}$
trager	$\frac{(63b^2d^5x^{10}+154ab\,d^5x^8+161b^2c\,d^4x^8+99a^2d^5x^6+418abc\,d^4x^6+113b^2c^2d^3x^6+297a^2c\,d^4x^4+330ab\,c^2d^3x^4+3b^2c^3d^2x^4+297a^2c^2d^3x^2}{693d^3}$
risch	$\frac{(63b^2d^5x^{10}+154ab\,d^5x^8+161b^2c\,d^4x^8+99a^2d^5x^6+418abc\,d^4x^6+113b^2c^2d^3x^6+297a^2c\,d^4x^4+330ab\,c^2d^3x^4+3b^2c^3d^2x^4+297a^2c^2d^3x^2}{693d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/11*x^4*(d*x^2+c)^(7/2)/d-4/11*c/d*(1/9*x^2*(d*x^2+c)^(7/2)/d-2/63*c/
d^2*(d*x^2+c)^(7/2)))+2*a*b*(1/9*x^2*(d*x^2+c)^(7/2)/d-2/63*c/d^2*(d*x^2+c)
^(7/2))+1/7*a^2/d*(d*x^2+c)^(7/2)
```

Maxima [A]
time = 0.28, size = 115, normalized size = 1.49

$$\frac{(dx^2+c)^{\frac{7}{2}}b^2x^4}{11\,d}-\frac{4\,(dx^2+c)^{\frac{7}{2}}b^2cx^2}{99\,d^2}+\frac{2\,(dx^2+c)^{\frac{7}{2}}abx^2}{9\,d}+\frac{8\,(dx^2+c)^{\frac{7}{2}}b^2c^2}{693\,d^3}-\frac{4\,(dx^2+c)^{\frac{7}{2}}abc}{63\,d^2}+\frac{(dx^2+c)^{\frac{7}{2}}a^2}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11*(d*x^2 + c)^(7/2)*b^2*x^4/d - 4/99*(d*x^2 + c)^(7/2)*b^2*c*x^2/d^2 + 2
/9*(d*x^2 + c)^(7/2)*a*b*x^2/d + 8/693*(d*x^2 + c)^(7/2)*b^2*c^2/d^3 - 4/63
*(d*x^2 + c)^(7/2)*a*b*c/d^2 + 1/7*(d*x^2 + c)^(7/2)*a^2/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(65) = 130$.
time = 1.24, size = 178, normalized size = 2.31

$$\frac{\left(63\,b^{2}d^{5}x^{10}+7\,\left(23\,b^{2}cd^{4}+22\,abd^{5}\right)x^{8}+8\,b^{2}c^{5}-44\,abc^{4}d+99\,a^{2}c^{3}d^{2}+\left(113\,b^{2}c^{2}d^{3}+418\,abcd^{4}+99\,a^{2}d^{5}\right)x^{6}+3\,\left(b^{2}c^{3}d^{2}+110\,abc^{2}d^{3}+99\,a^{2}cd^{4}\right)x^{4}-\left(4\,b^{2}c^{4}d-22\,abc^{3}d^{2}-297\,a^{2}c^{2}d^{3}\right)x^{2}\right)\sqrt{dx^{2}+c}}{693\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/693*(63*b^2*d^5*x^10 + 7*(23*b^2*c*d^4 + 22*a*b*d^5)*x^8 + 8*b^2*c^5 - 44
*a*b*c^4*d + 99*a^2*c^3*d^2 + (113*b^2*c^2*d^3 + 418*a*b*c*d^4 + 99*a^2*d^5
)*x^6 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99*a^2*c*d^4)*x^4 - (4*b^2*c^4*d
 - 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(66) = 132$.
time = 0.57, size = 384, normalized size = 4.99

$$\begin{cases} \frac{a^{2}c^{3}\sqrt{c+dx^{2}}}{7d}+\frac{3a^{2}c^{2}x^{2}\sqrt{c+dx^{2}}}{7}+\frac{3a^{2}cdx^{4}\sqrt{c+dx^{2}}}{7}+\frac{a^{2}d^{2}x^{6}\sqrt{c+dx^{2}}}{7}-\frac{4abc^{4}\sqrt{c+dx^{2}}}{63d^{2}}+\frac{2abc^{3}x^{2}\sqrt{c+dx^{2}}}{63d}+\frac{10abc^{2}x^{4}\sqrt{c+dx^{2}}}{21}+\frac{38abcdx^{6}\sqrt{c+dx^{2}}}{63}+\frac{2abd^{2}x^{8}\sqrt{c+dx^{2}}}{9}+\frac{8b^{2}c^{5}\sqrt{c+dx^{2}}}{693d^{3}}-\frac{4b^{2}c^{4}x^{2}\sqrt{c+dx^{2}}}{693d^{2}}+\frac{b^{2}c^{3}x^{4}\sqrt{c+dx^{2}}}{231d}+\frac{113b^{2}c^{2}x^{6}\sqrt{c+dx^{2}}}{693}+\frac{23b^{2}cdx^{8}\sqrt{c+dx^{2}}}{99}+\frac{b^{2}d^{2}x^{10}\sqrt{c+dx^{2}}}{11} & \text{for}\: d\neq 0 \\ c^{\frac{5}{2}}\left(\frac{a^{2}x^{2}}{2}+\frac{abx^{4}}{2}+\frac{b^{2}x^{6}}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((a**2*c**3*sqrt(c + d*x**2)/(7*d) + 3*a**2*c**2*x**2*sqrt(c + d*x
**2)/7 + 3*a**2*c*d*x**4*sqrt(c + d*x**2)/7 + a**2*d**2*x**6*sqrt(c + d*x**
2)/7 - 4*a*b*c**4*sqrt(c + d*x**2)/(63*d**2) + 2*a*b*c**3*x**2*sqrt(c + d*x
**2)/(63*d) + 10*a*b*c**2*x**4*sqrt(c + d*x**2)/21 + 38*a*b*c*d*x**6*sqrt(c
 + d*x**2)/63 + 2*a*b*d**2*x**8*sqrt(c + d*x**2)/9 + 8*b**2*c**5*sqrt(c + d
*x**2)/(693*d**3) - 4*b**2*c**4*x**2*sqrt(c + d*x**2)/(693*d**2) + b**2*c**
3*x**4*sqrt(c + d*x**2)/(231*d) + 113*b**2*c**2*x**6*sqrt(c + d*x**2)/693 +
 23*b**2*c*d*x**8*sqrt(c + d*x**2)/99 + b**2*d**2*x**10*sqrt(c + d*x**2)/11
, Ne(d, 0)), (c**(5/2)*(a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6), True))
```

Giac [A]
time = 0.88, size = 98, normalized size = 1.27

$$\frac{63\,\left(dx^{2}+c\right)^{\frac{11}{2}}b^{2}-154\,\left(dx^{2}+c\right)^{\frac{9}{2}}b^{2}c+99\,\left(dx^{2}+c\right)^{\frac{7}{2}}b^{2}c^{2}+154\,\left(dx^{2}+c\right)^{\frac{9}{2}}abd-198\,\left(dx^{2}+c\right)^{\frac{7}{2}}abcd+99\,\left(dx^{2}+c\right)^{\frac{7}{2}}a^{2}d^{2}}{693\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/693*(63*(d*x^2 + c)^(11/2)*b^2 - 154*(d*x^2 + c)^(9/2)*b^2*c + 99*(d*x^2
+ c)^(7/2)*b^2*c^2 + 154*(d*x^2 + c)^(9/2)*a*b*d - 198*(d*x^2 + c)^(7/2)*a*
b*c*d + 99*(d*x^2 + c)^(7/2)*a^2*d^2)/d^3
```

Mupad [B]
time = 0.39, size = 98, normalized size = 1.27

$$\frac{d\left(\frac{2\,a\,b\,{\left(d\,x^2+c\right)}^{9/2}}{9}-\frac{2\,a\,b\,c\,{\left(d\,x^2+c\right)}^{7/2}}{7}\right)+\frac{b^2\,{\left(d\,x^2+c\right)}^{11/2}}{11}-\frac{2\,b^2\,c\,{\left(d\,x^2+c\right)}^{9/2}}{9}+\frac{a^2\,d^2\,{\left(d\,x^2+c\right)}^{7/2}}{7}+\frac{b^2\,c^2\,{\left(d\,x^2+c\right)}^{7/2}}{7}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

```
[Out] (d*((2*a*b*(c + d*x^2)^(9/2))/9 - (2*a*b*c*(c + d*x^2)^(7/2))/7) + (b^2*(c
+ d*x^2)^(11/2))/11 - (2*b^2*c*(c + d*x^2)^(9/2))/9 + (a^2*d^2*(c + d*x^2)^
(7/2))/7 + (b^2*c^2*(c + d*x^2)^(7/2))/7)/d^3
```

3.628 $\int (a+bx^2)^2 (c+dx^2)^{5/2} \, dx$

Optimal. Leaf size=240

$$\frac{c^2(3b^2c^2-20abcd+80a^2d^2)\,x\sqrt{c+dx^2}}{256d^2}+\frac{c(3b^2c^2-20abcd+80a^2d^2)\,x(c+dx^2)^{3/2}}{384d^2}+\frac{(3b^2c^2-20abcd+80}{480}$$

```
[Out] 1/384*c*(80*a^2*d^2-20*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(3/2)/d^2+1/480*(80*a
^2*d^2-20*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(5/2)/d^2-3/80*b*(-4*a*d+b*c)*x*(d
*x^2+c)^(7/2)/d^2+1/10*b*x*(b*x^2+a)*(d*x^2+c)^(7/2)/d+1/256*c^3*(80*a^2*d^
2-20*a*b*c*d+3*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(5/2)+1/256*c^
2*(80*a^2*d^2-20*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.10, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {427, 396, 201, 223, 212}

$$\frac{x(c+dx^2)^{5/2}(80a^2d^2-20abcd+3b^2c^2)}{480d^2}+\frac{cx(c+dx^2)^{3/2}(80a^2d^2-20abcd+3b^2c^2)}{384d^2}+\frac{c^2x\sqrt{c+dx^2}(80a^2d^2-20abcd+3b^2c^2)}{256d^2}+\frac{c^3(80a^2d^2-20abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{256d^{5/2}}-\frac{3bx(c+dx^2)^{7/2}(bc-4ad)}{80d^2}+\frac{bx(a+bx^2)(c+dx^2)^{7/2}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (c^2*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*Sqrt[c + d*x^2])/(256*d^2) + (
c*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(3/2))/(384*d^2) + ((
3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(5/2))/(480*d^2) - (3*b*
(b*c - 4*a*d)*x*(c + d*x^2)^(7/2))/(80*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^
(7/2))/(10*d) + (c^3*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(256*d^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} \, dx = \frac{bx(a+bx^2)\left(c+dx^2\right)^{7/2}}{10d} + \frac{\int \left(c+dx^2\right)^{5/2} \left(-a(bc-10ad)-3b(bc-4ad)x^2\right)}{10d}$$

$$= -\frac{3b(bc-4ad)x\left(c+dx^2\right)^{7/2}}{80d^2} + \frac{bx(a+bx^2)\left(c+dx^2\right)^{7/2}}{10d} - \frac{(8ad(bc-10ad)}{}$$

$$= \frac{\left(3b^2c^2-20abcd+80a^2d^2\right)x\left(c+dx^2\right)^{5/2}}{480d^2} - \frac{3b(bc-4ad)x\left(c+dx^2\right)^{7/2}}{80d^2} + \frac{bx(}{}$$

$$= \frac{c\left(3b^2c^2-20abcd+80a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{384d^2} + \frac{\left(3b^2c^2-20abcd+80a^2d^2\right)x(c+}{480d^2}$$

$$= \frac{c^2\left(3b^2c^2-20abcd+80a^2d^2\right)x\sqrt{c+dx^2}}{256d^2} + \frac{c\left(3b^2c^2-20abcd+80a^2d^2\right)x(c+}{384d^2}$$

$$= \frac{c^2\left(3b^2c^2-20abcd+80a^2d^2\right)x\sqrt{c+dx^2}}{256d^2} + \frac{c\left(3b^2c^2-20abcd+80a^2d^2\right)x(c+}{384d^2}$$

$$= \frac{c^2\left(3b^2c^2-20abcd+80a^2d^2\right)x\sqrt{c+dx^2}}{256d^2} + \frac{c\left(3b^2c^2-20abcd+80a^2d^2\right)x(c+}{384d^2}$$

Mathematica [A]

time = 0.30, size = 191, normalized size = 0.80

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\,(80a^2d^2(33c^2+26cdx^2+8d^2x^4)+20abd(15c^3+118c^2dx^2+136cd^2x^4+48d^3x^6)+b^2(-45c^4+30c^3dx^2+744c^2d^2x^4+1008cd^3x^6+384d^4x^8))-15c^3(3b^2c^2-20abcd+80a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{3840d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) +
20*a*b*d*(15*c^3 + 118*c^2*d*x^2 + 136*c*d^2*x^4 + 48*d^3*x^6) + b^2*(-45*c
^4 + 30*c^3*d*x^2 + 744*c^2*d^2*x^4 + 1008*c*d^3*x^6 + 384*d^4*x^8)) - 15*c
^3*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]
])/(3840*d^(5/2))
```

Maple [A]
time = 0.10, size = 283, normalized size = 1.18

method	result
risch	$\frac{x\left(384b^2x^8d^4+960ab\,d^4x^6+1008b^2c\,d^3x^6+640a^2x^4d^4+2720x^4abc\,d^3+744b^2c^2x^4d^2+2080a^2c\,d^3x^2+2360ab\,c^2d^2x^2+30b^2c^3d\,x^2+264\right.}{3840d^2}$

default	$b^{2}\left(\frac{x^{3}\left(d\,x^{2}+c\right)^{\frac{7}{2}}}{10d}-\frac{3c\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{7}{2}}}{8d}-\frac{c\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^{2}+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6}\right)}{8d}\right)}{10d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/10*x^3*(d*x^2+c)^(7/2)/d-3/10*c/d*(1/8*x*(d*x^2+c)^(7/2)/d-1/8*c/d*(
1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(
1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))))+2*a*b*(1/8*x*(d*x^2+c
)^(7/2)/d-1/8*c/d*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c
*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))))+a^2
*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)
^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))
```

Maxima [**A**]
time = 0.29, size = 286, normalized size = 1.19

$$\frac{(dx^2+c)^{\frac{7}{2}}b^2x^3}{10\,d}+\frac{1}{6}\,(dx^2+c)^{\frac{5}{2}}a^2x+\frac{5}{24}\,(dx^2+c)^{\frac{3}{2}}a^2cx+\frac{5}{16}\,\sqrt{dx^2+c}\,a^2c^2x-\frac{3\,(dx^2+c)^{\frac{7}{2}}b^2cx}{80\,d^2}+\frac{(dx^2+c)^{\frac{5}{2}}b^2c^2x}{160\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}b^2c^3x}{128\,d^2}+\frac{3\,\sqrt{dx^2+c}\,b^2c^4x}{256\,d^2}+\frac{(dx^2+c)^{\frac{7}{2}}abx}{4\,d}-\frac{(dx^2+c)^{\frac{5}{2}}abcx}{24\,d}-\frac{5\,(dx^2+c)^{\frac{3}{2}}abc^2x}{96\,d}-\frac{5\,\sqrt{dx^2+c}\,abc^3x}{64\,d}+\frac{3\,b^2c^5\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{256\,d^{\frac{5}{2}}}-\frac{5\,abc^4\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{64\,d^{\frac{3}{2}}}+\frac{5\,a^2c^3\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/10*(d*x^2 + c)^(7/2)*b^2*x^3/d + 1/6*(d*x^2 + c)^(5/2)*a^2*x + 5/24*(d*x^
2 + c)^(3/2)*a^2*c*x + 5/16*sqrt(d*x^2 + c)*a^2*c^2*x - 3/80*(d*x^2 + c)^(7
/2)*b^2*c*x/d^2 + 1/160*(d*x^2 + c)^(5/2)*b^2*c^2*x/d^2 + 1/128*(d*x^2 + c)
^(3/2)*b^2*c^3*x/d^2 + 3/256*sqrt(d*x^2 + c)*b^2*c^4*x/d^2 + 1/4*(d*x^2 + c
)^(7/2)*a*b*x/d - 1/24*(d*x^2 + c)^(5/2)*a*b*c*x/d - 5/96*(d*x^2 + c)^(3/2)
*a*b*c^2*x/d - 5/64*sqrt(d*x^2 + c)*a*b*c^3*x/d + 3/256*b^2*c^5*arcsinh(d*x
/sqrt(c*d))/d^(5/2) - 5/64*a*b*c^4*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 5/16*a^
2*c^3*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

Fricas [**A**]
time = 1.98, size = 420, normalized size = 1.75

$$\left[\frac{15\,(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(384\,b^2d^5x^9+48\,(21\,b^2cd^4+20\,abd^5)x^7+8\,(93\,b^2c^2d^3+340\,abcd^4+80\,a^2d^5)x^5+10\,(3\,b^2c^3d^2+236\,abc^2d^3+208\,a^2cd^4)x^3-15\,(3\,b^2c^4d-20\,abc^3d^2-176\,a^2c^2d^3)x)\sqrt{dx^2+c}}{7680\,d^3},\ -\frac{15\,(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(384\,b^2d^5x^9+48\,(21\,b^2cd^4+20\,abd^5)x^7+8\,(93\,b^2c^2d^3+340\,abcd^4+80\,a^2d^5)x^5+10\,(3\,b^2c^3d^2+236\,abc^2d^3+208\,a^2cd^4)x^3-15\,(3\,b^2c^4d-20\,abc^3d^2-176\,a^2c^2d^3)x)\sqrt{dx^2+c}}{3840\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*sqrt(d)*log(-2*d*x^
2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(384*b^2*d^5*x^9 + 48*(21*b^2*c*d^
4 + 20*a*b*d^5)*x^7 + 8*(93*b^2*c^2*d^3 + 340*a*b*c*d^4 + 80*a^2*d^5)*x^5 +
 10*(3*b^2*c^3*d^2 + 236*a*b*c^2*d^3 + 208*a^2*c*d^4)*x^3 - 15*(3*b^2*c^4*d
 - 20*a*b*c^3*d^2 - 176*a^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/3840*(15*(
3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) - (384*b^2*d^5*x^9 + 48*(21*b^2*c*d^4 + 20*a*b*d^5)*x^7 + 8*(93
*b^2*c^2*d^3 + 340*a*b*c*d^4 + 80*a^2*d^5)*x^5 + 10*(3*b^2*c^3*d^2 + 236*a*
b*c^2*d^3 + 208*a^2*c*d^4)*x^3 - 15*(3*b^2*c^4*d - 20*a*b*c^3*d^2 - 176*a^2
*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(236) = 472$.

time = 179.91, size = 537, normalized size = 2.24

$$\frac{a^2c^{\frac{5}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{3a^2c^{\frac{5}{2}}x}{16\sqrt{1+\frac{dx^2}{c}}}+\frac{35a^2c^{\frac{3}{2}}dx^3}{48\sqrt{1+\frac{dx^2}{c}}}+\frac{17a^2\sqrt{c}\,d^2x^5}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{5a^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16\sqrt{d}}+\frac{a^2d^3x^7}{6\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{5abc^{\frac{7}{2}}x}{64d\sqrt{1+\frac{dx^2}{c}}}+\frac{133abc^{\frac{5}{2}}x^3}{192\sqrt{1+\frac{dx^2}{c}}}+\frac{127abc^{\frac{3}{2}}dx^5}{96\sqrt{1+\frac{dx^2}{c}}}+\frac{23ab\sqrt{c}\,d^2x^7}{24\sqrt{1+\frac{dx^2}{c}}}-\frac{5abc^4\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{64d^{\frac{3}{2}}}+\frac{abd^3x^9}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{3b^2c^{\frac{9}{2}}x}{256d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{7}{2}}x^3}{256d\sqrt{1+\frac{dx^2}{c}}}+\frac{129b^2c^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{dx^2}{c}}}+\frac{73b^2c^{\frac{3}{2}}dx^7}{160\sqrt{1+\frac{dx^2}{c}}}+\frac{29b^2\sqrt{c}\,d^2x^9}{80\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c^5\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{256d^{\frac{5}{2}}}+\frac{b^2d^3x^{11}}{10\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2),x)

[Out] a**2*c**(5/2)*x*sqrt(1 + d*x**2/c)/2 + 3*a**2*c**(5/2)*x/(16*sqrt(1 + d*x**2/c)) + 35*a**2*c**(3/2)*d*x**3/(48*sqrt(1 + d*x**2/c)) + 17*a**2*sqrt(c)*d**2*x**5/(24*sqrt(1 + d*x**2/c)) + 5*a**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*sqrt(d)) + a**2*d**3*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*a*b*c**(7/2)*x/(64*d*sqrt(1 + d*x**2/c)) + 133*a*b*c**(5/2)*x**3/(192*sqrt(1 + d*x**2/c)) + 127*a*b*c**(3/2)*d*x**5/(96*sqrt(1 + d*x**2/c)) + 23*a*b*sqrt(c)*d**2*x**7/(24*sqrt(1 + d*x**2/c)) - 5*a*b*c**4*asinh(sqrt(d)*x/sqrt(c))/(64*d**(3/2)) + a*b*d**3*x**9/(4*sqrt(c)*sqrt(1 + d*x**2/c)) - 3*b**2*c**(9/2)*x/(256*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(7/2)*x**3/(256*d*sqrt(1 + d*x**2/c)) + 129*b**2*c**(5/2)*x**5/(640*sqrt(1 + d*x**2/c)) + 73*b**2*c**(3/2)*d*x**7/(160*sqrt(1 + d*x**2/c)) + 29*b**2*sqrt(c)*d**2*x**9/(80*sqrt(1 + d*x**2/c)) + 3*b**2*c**5*asinh(sqrt(d)*x/sqrt(c))/(256*d**(5/2)) + b**2*d**3*x**11/(10*sqrt(c)*sqrt(1 + d*x**2/c))

Giac [A]
time = 0.67, size = 221, normalized size = 0.92

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2+\frac{21\,b^2cd^9+20\,abd^{10}}{d^8}\right)x^2+\frac{93\,b^2c^2d^8+340\,abcd^9+80\,a^2d^{10}}{d^8}\right)x^2+\frac{5\left(3\,b^2c^3d^7+236\,abc^2d^8+208\,a^2cd^9\right)}{d^8}\right)x^2-\frac{15\left(3\,b^2c^4d^6-20\,abc^3d^7-176\,a^2c^2d^8\right)}{d^8}\right)\sqrt{dx^2+c}\,x-\frac{\left(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{256\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")

[Out] 1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (21*b^2*c*d^9 + 20*a*b*d^10)/d^8)*x^2 + (93*b^2*c^2*d^8 + 340*a*b*c*d^9 + 80*a^2*d^10)/d^8)*x^2 + 5*(3*b^2*c^3*d^7 + 236*a*b*c^2*d^8 + 208*a^2*c*d^9)/d^8)*x^2 - 15*(3*b^2*c^4*d^6 - 20*a*b*c^3*d^7 - 176*a^2*c^2*d^8)/d^8)*sqrt(d*x^2 + c)*x - 1/256*(3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^2*(c + d*x^2)^(5/2),x)

[Out] int((a + b*x^2)^2*(c + d*x^2)^(5/2), x)

3.629 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x}\,dx$

Optimal. Leaf size=132

$$a^2c^2\sqrt{c+dx^2}+\frac{1}{3}a^2c(c+dx^2)^{3/2}+\frac{1}{5}a^2(c+dx^2)^{5/2}-\frac{b(bc-2ad)(c+dx^2)^{7/2}}{7d^2}+\frac{b^2(c+dx^2)^{9/2}}{9d^2}-a^2c^{5/2}\tanh$$

```
[Out] 1/3*a^2*c*(d*x^2+c)^(3/2)+1/5*a^2*(d*x^2+c)^(5/2)-1/7*b*(-2*a*d+b*c)*(d*x^2
+c)^(7/2)/d^2+1/9*b^2*(d*x^2+c)^(9/2)/d^2-a^2*c^(5/2)*arctanh((d*x^2+c)^(1/
2)/c^(1/2))+a^2*c^2*(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 90, 52, 65, 214}

$$-a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+a^2c^2\sqrt{c+dx^2}+\frac{1}{5}a^2(c+dx^2)^{5/2}+\frac{1}{3}a^2c(c+dx^2)^{3/2}-\frac{b(c+dx^2)^{7/2}(bc-2ad)}{7d^2}+\frac{b^2(c+dx^2)^{9/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

```
[Out] a^2*c^2*Sqrt[c + d*x^2] + (a^2*c*(c + d*x^2)^(3/2))/3 + (a^2*(c + d*x^2)^(5
/2))/5 - (b*(b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^2) + (b^2*(c + d*x^2)^(9/
2))/(9*d^2) - a^2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b(bc-2ad)(c+dx)^{5/2}}{d}+\frac{a^2(c+dx)^{5/2}}{x}+\frac{b^2(c+dx)^{7/2}}{d}\right) dx\right.\\
&= -\frac{b(bc-2ad)(c+dx^2)^{7/2}}{7d^2}+\frac{b^2(c+dx^2)^{9/2}}{9d^2}+\frac{1}{2}a^2\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x}\,dx, x\right.\\
&= \frac{1}{5}a^2(c+dx^2)^{5/2}-\frac{b(bc-2ad)(c+dx^2)^{7/2}}{7d^2}+\frac{b^2(c+dx^2)^{9/2}}{9d^2}+\frac{1}{2}(a^2c)\,\text{Subst}\\
&= \frac{1}{3}a^2c(c+dx^2)^{3/2}+\frac{1}{5}a^2(c+dx^2)^{5/2}-\frac{b(bc-2ad)(c+dx^2)^{7/2}}{7d^2}+\frac{b^2(c+dx^2}{9d^2}\\
&= a^2c^2\sqrt{c+dx^2}+\frac{1}{3}a^2c(c+dx^2)^{3/2}+\frac{1}{5}a^2(c+dx^2)^{5/2}-\frac{b(bc-2ad)(c+dx^2}{7d^2}\\
&= a^2c^2\sqrt{c+dx^2}+\frac{1}{3}a^2c(c+dx^2)^{3/2}+\frac{1}{5}a^2(c+dx^2)^{5/2}-\frac{b(bc-2ad)(c+dx^2}{7d^2}\\
&= a^2c^2\sqrt{c+dx^2}+\frac{1}{3}a^2c(c+dx^2)^{3/2}+\frac{1}{5}a^2(c+dx^2)^{5/2}-\frac{b(bc-2ad)(c+dx^2}{7d^2}
\end{aligned}$$

Mathematica [A]

time = 0.15, size = 115, normalized size = 0.87

$$\frac{\sqrt{c+dx^2}\left(90abd(c+dx^2)^3-5b^2(2c-7dx^2)\left(c+dx^2\right)^3+21a^2d^2(23c^2+11cdx^2+3d^2x^4)\right)}{315d^2}-a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[c + d*x^2]*(90*a*b*d*(c + d*x^2)^3 - 5*b^2*(2*c - 7*d*x^2)*(c + d*x^2
)^3 + 21*a^2*d^2*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)))/(315*d^2) - a^2*c^(5/2
)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Maple [A]
time = 0.09, size = 125, normalized size = 0.95

method	result
default	$b^2\left(\frac{x^2(dx^2+c)^{\frac{7}{2}}}{9d}-\frac{2c(dx^2+c)^{\frac{7}{2}}}{63d^2}\right)+\frac{2ab(dx^2+c)^{\frac{7}{2}}}{7d}+a^2\left(\frac{(dx^2+c)^{\frac{5}{2}}}{5}+c\left(\frac{(dx^2+c)^{\frac{3}{2}}}{3}+c\left(\sqrt{dx^2+c}-\sqrt{c}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/9*x^2*(d*x^2+c)^(7/2)/d-2/63*c/d^2*(d*x^2+c)^(7/2))+2/7*a*b/d*(d*x^2
+c)^(7/2)+a^2*(1/5*(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2
)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))
```

Maxima [A]
time = 0.30, size = 120, normalized size = 0.91

$$\frac{(dx^2+c)^{\frac{7}{2}}b^2x^2}{9\,d}-a^2c^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\frac{1}{5}\,(dx^2+c)^{\frac{5}{2}}a^2+\frac{1}{3}\,(dx^2+c)^{\frac{3}{2}}a^2c+\sqrt{dx^2+c}\,a^2c^2-\frac{2\,(dx^2+c)^{\frac{7}{2}}b^2c}{63\,d^2}+\frac{2\,(dx^2+c)^{\frac{7}{2}}ab}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/9*(d*x^2 + c)^(7/2)*b^2*x^2/d - a^2*c^(5/2)*arcsinh(c/(sqrt(c*d)*abs(x)))
 + 1/5*(d*x^2 + c)^(5/2)*a^2 + 1/3*(d*x^2 + c)^(3/2)*a^2*c + sqrt(d*x^2 + c
)*a^2*c^2 - 2/63*(d*x^2 + c)^(7/2)*b^2*c/d^2 + 2/7*(d*x^2 + c)^(7/2)*a*b/d
```

Fricas [A]
time = 1.32, size = 360, normalized size = 2.73

$$\left[\frac{315\,a^2c^{\frac{5}{2}}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,(35\,b^2d^4x^8+5\,(19\,b^2cd^3+18\,abd^4)x^6-10\,b^2c^4+90\,abc^3d+483\,a^2c^2d^2+3\,(25\,b^2c^2d^2+90\,abcd^3+21\,a^2d^4)x^4+(5\,b^2c^3d+270\,abc^2d^2+231\,a^2cd^3)x^2)\sqrt{dx^2+c}}{630\,d^2},\ \frac{315\,a^2\sqrt{-c}\,c^2d^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(35\,b^2d^4x^8+5\,(19\,b^2cd^3+18\,abd^4)x^6-10\,b^2c^4+90\,abc^3d+483\,a^2c^2d^2+3\,(25\,b^2c^2d^2+90\,abcd^3+21\,a^2d^4)x^4+(5\,b^2c^3d+270\,abc^2d^2+231\,a^2cd^3)x^2)\sqrt{dx^2+c}}{315\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*a^2*c^(5/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2) + 2*(35*b^2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4)*x^6 - 10*b^2*c^4 +
 90*a*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c^2*d^2 + 90*a*b*c*d^3 + 21*a^2
*d^4)*x^4 + (5*b^2*c^3*d + 270*a*b*c^2*d^2 + 231*a^2*c*d^3)*x^2)*sqrt(d*x^2
 + c))/d^2, 1/315*(315*a^2*sqrt(-c)*c^2*d^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + (35*b^2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4)*x^6 - 10*b^2*c^4 + 90*a
*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c^2*d^2 + 90*a*b*c*d^3 + 21*a^2*d^4)
*x^4 + (5*b^2*c^3*d + 270*a*b*c^2*d^2 + 231*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)
)/d^2]
```

Sympy [A]
time = 59.13, size = 128, normalized size = 0.97

$$\frac{a^2c^3 \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}} + a^2c^2\sqrt{c+dx^2} + \frac{a^2c(c+dx^2)^{\frac{3}{2}}}{3} + \frac{a^2(c+dx^2)^{\frac{5}{2}}}{5} + \frac{b^2(c+dx^2)^{\frac{9}{2}}}{9d^2} + \frac{(c+dx^2)^{\frac{7}{2}} \cdot (4abd-2b^2c)}{14d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x,x)
```

```
[Out] a**2*c**3*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + a**2*c**2*sqrt(c + d*x
**2) + a**2*c*(c + d*x**2)**(3/2)/3 + a**2*(c + d*x**2)**(5/2)/5 + b**2*(c
+ d*x**2)**(9/2)/(9*d**2) + (c + d*x**2)**(7/2)*(4*a*b*d - 2*b**2*c)/(14*d*
*2)
```

Giac [A]
time = 0.66, size = 141, normalized size = 1.07

$$\frac{a^2c^3 \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{35\,(dx^2+c)^{\frac{9}{2}}b^2d^{16} - 45\,(dx^2+c)^{\frac{7}{2}}b^2cd^{16} + 90\,(dx^2+c)^{\frac{7}{2}}abd^{17} + 63\,(dx^2+c)^{\frac{5}{2}}a^2d^{18} + 105\,(dx^2+c)^{\frac{3}{2}}a^2cd^{18} + 315\,\sqrt{dx^2+c}\,a^2c^2d^{18}}{315\,d^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/315*(35*(d*x^2 + c)^(
9/2)*b^2*d^16 - 45*(d*x^2 + c)^(7/2)*b^2*c*d^16 + 90*(d*x^2 + c)^(7/2)*a*b*
d^17 + 63*(d*x^2 + c)^(5/2)*a^2*d^18 + 105*(d*x^2 + c)^(3/2)*a^2*c*d^18 + 3
15*sqrt(d*x^2 + c)*a^2*c^2*d^18)/d^18
```

Mupad [B]
time = 0.37, size = 249, normalized size = 1.89

$$(d\,x^2+c)^{5/2}\left(\frac{(a\,d-b\,c)^2}{5\,d^2}-\frac{c\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)}{5}\right)-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{7\,d^2}-\frac{b^2\,c}{7\,d^2}\right)\,(d\,x^2+c)^{7/2}+c^2\,\sqrt{d\,x^2+c}\,\left(\frac{(a\,d-b\,c)^2}{d^2}-c\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\right)+\frac{b^2\,(d\,x^2+c)^{9/2}}{9\,d^2}+\frac{c\,(d\,x^2+c)^{3/2}\,\left(\frac{(a\,d-b\,c)^2}{d^2}-c\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\right)}{3}+a^2\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x)
```

```
[Out] (c + d*x^2)^(5/2)*((a*d - b*c)^2/(5*d^2) - (c*((2*b^2*c - 2*a*b*d)/d^2 - (b
^2*c)/d^2))/5) - ((2*b^2*c - 2*a*b*d)/(7*d^2) - (b^2*c)/(7*d^2))*(c + d*x^2
)^(7/2) + a^2*c^(5/2)*atan(((c + d*x^2)^(1/2)*1i)/c^(1/2))*1i + c^2*(c + d*
x^2)^(1/2)*((a*d - b*c)^2/d^2 - c*((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^2))
+ (b^2*(c + d*x^2)^(9/2))/(9*d^2) + (c*(c + d*x^2)^(3/2)*((a*d - b*c)^2/d^2
 - c*((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^2)))/3
```

3.630 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^2} \, dx$

Optimal. Leaf size=217

$$-\frac{5c(b^2c^2-16ad(bc+3ad))\,x\sqrt{c+dx^2}}{128d}-\frac{5(b^2c^2-16ad(bc+3ad))\,x(c+dx^2)^{3/2}}{192d}-\frac{(b^2c^2-16ad(bc+3ad))\,x}{48cd}$$

```
[Out] -5/192*(b^2*c^2-16*a*d*(3*a*d+b*c))*x*(d*x^2+c)^(3/2)/d-1/48*(b^2*c^2-16*a*
d*(3*a*d+b*c))*x*(d*x^2+c)^(5/2)/c/d-a^2*(d*x^2+c)^(7/2)/c/x+1/8*b^2*x*(d*x
^2+c)^(7/2)/d-5/128*c^2*(b^2*c^2-16*a*d*(3*a*d+b*c))*arctanh(x*d^(1/2)/(d*x
^2+c)^(1/2))/d^(3/2)-5/128*c*(b^2*c^2-16*a*d*(3*a*d+b*c))*x*(d*x^2+c)^(1/2)
/d
```

Rubi [A]
time = 0.09, antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {473, 396, 201, 223, 212}

$$-\frac{a^2(c+dx^2)^{7/2}}{cx}-\frac{5c^2(b^2c^2-16ad(3ad+bc))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{3/2}}-\frac{5x(c+dx^2)^{3/2}(b^2c^2-16ad(3ad+bc))}{192d}-\frac{5cx\sqrt{c+dx^2}\,(b^2c^2-16ad(3ad+bc))}{128d}-\frac{1}{48}x(c+dx^2)^{5/2}\left(\frac{b^2c}{d}-\frac{16a(3ad+bc)}{c}\right)+\frac{b^2x(c+dx^2)^{7/2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

```
[Out] (-5*c*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*x*Sqrt[c + d*x^2])/(128*d) - (5*(b^2
*c^2 - 16*a*d*(b*c + 3*a*d))*x*(c + d*x^2)^(3/2))/(192*d) - (((b^2*c)/d - (
16*a*(b*c + 3*a*d))/c)*x*(c + d*x^2)^(5/2))/48 - (a^2*(c + d*x^2)^(7/2))/(c
*x) + (b^2*x*(c + d*x^2)^(7/2))/(8*d) - (5*c^2*(b^2*c^2 - 16*a*d*(b*c + 3*a
*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128*d^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^2} dx &= -\frac{a^2(c+dx^2)^{7/2}}{cx} + \frac{\int (2a(bc+3ad)+b^2cx^2)(c+dx^2)^{5/2}\,dx}{c} \\
&= -\frac{a^2(c+dx^2)^{7/2}}{cx} + \frac{b^2x(c+dx^2)^{7/2}}{8d} - \frac{(b^2c^2-16ad(bc+3ad))\int (c+dx^2)^{5/}}{8cd} \\
&= -\frac{1}{48}\left(\frac{b^2c}{d} - \frac{16a(bc+3ad)}{c}\right)x(c+dx^2)^{5/2} - \frac{a^2(c+dx^2)^{7/2}}{cx} + \frac{b^2x(c+dx^2}{8d} \\
&= -\frac{5(b^2c^2-16ad(bc+3ad))\,x(c+dx^2)^{3/2}}{192d} - \frac{1}{48}\left(\frac{b^2c}{d} - \frac{16a(bc+3ad)}{c}\right)x(c \\
&= -\frac{5c(b^2c^2-16ad(bc+3ad))\,x\sqrt{c+dx^2}}{128d} - \frac{5(b^2c^2-16ad(bc+3ad))\,x(c+}{192d} \\
&= -\frac{5c(b^2c^2-16ad(bc+3ad))\,x\sqrt{c+dx^2}}{128d} - \frac{5(b^2c^2-16ad(bc+3ad))\,x(c+}{192d} \\
&= -\frac{5c(b^2c^2-16ad(bc+3ad))\,x\sqrt{c+dx^2}}{128d} - \frac{5(b^2c^2-16ad(bc+3ad))\,x(c+}{192d}
\end{aligned}$$

Mathematica [A]

time = 0.34, size = 175, normalized size = 0.81

$$\frac{\sqrt{d}\,\sqrt{c+dx^2}\,(48a^2d(-8c^2+9cdx^2+2d^2x^4)+16abdx^2(33c^2+26cdx^2+8d^2x^4)+b^2x^2(15c^3+118c^2dx^2+136cd^2x^4+48d^3x^6))+15c^2(b^2c^2-16abcd-48a^2d^2)\,x\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{384d^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

```
[Out] (Sqrt[d]*Sqrt[c + d*x^2]*(48*a^2*d*(-8*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + 16*a*
b*d*x^2*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) + b^2*x^2*(15*c^3 + 118*c^2*d*x^2
 + 136*c*d^2*x^4 + 48*d^3*x^6)) + 15*c^2*(b^2*c^2 - 16*a*b*c*d - 48*a^2*d^2
)*x*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(384*d^(3/2)*x)
```

Maple [A]
time = 0.10, size = 261, normalized size = 1.20

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\,(-48b^2d^3x^8-128ab\,d^3x^6-136b^2c\,d^2x^6-96a^2d^3x^4-416abc\,d^2x^4-118b^2c^2d\,x^4-432a^2c\,d^2x^2-528ab\,c^2d\,x^2-15b^2c^3x}{384dx}$
default	$b^2\left(\frac{x\left(d\,x^2+c\right)^{\frac{7}{2}}}{8d}-\frac{c\left(\frac{x\left(d\,x^2+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(d\,x^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6}\right)}{8d}\right)+2ab\left(\frac{x(d\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/8*x*(d*x^2+c)^(7/2)/d-1/8*c/d*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d
*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^
```

```
2+c)^(1/2))))))+2*a*b*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3
/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+
a^2*(-1/c/x*(d*x^2+c)^(7/2)+6*d/c*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^
2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c
)^(1/2))))))
```

Maxima [A]
time = 0.33, size = 235, normalized size = 1.08

$$\frac{1}{3}\left(dx^2+c\right)^{\frac{5}{2}}abx+\frac{5}{12}\left(dx^2+c\right)^{\frac{3}{2}}abcx+\frac{5}{8}\sqrt{dx^2+c}\,abc^2x+\frac{\left(dx^2+c\right)^{\frac{7}{2}}b^2x}{8\,d}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2cx}{48\,d}-\frac{5\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2x}{192\,d}-\frac{5\sqrt{dx^2+c}\,b^2c^3x}{128\,d}+\frac{5}{4}\left(dx^2+c\right)^{\frac{3}{2}}a^2dx+\frac{15}{8}\sqrt{dx^2+c}\,a^2cdx-\frac{5\,b^2c^4\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{3}{2}}}+\frac{5\,abc^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{d}}+\frac{15}{8}\,a^2c^2\sqrt{d}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d*x^2 + c)^(5/2)*a*b*x + 5/12*(d*x^2 + c)^(3/2)*a*b*c*x + 5/8*sqrt(d*x
^2 + c)*a*b*c^2*x + 1/8*(d*x^2 + c)^(7/2)*b^2*x/d - 1/48*(d*x^2 + c)^(5/2)*
b^2*c*x/d - 5/192*(d*x^2 + c)^(3/2)*b^2*c^2*x/d - 5/128*sqrt(d*x^2 + c)*b^2
*c^3*x/d + 5/4*(d*x^2 + c)^(3/2)*a^2*d*x + 15/8*sqrt(d*x^2 + c)*a^2*c*d*x -
 5/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 5/8*a*b*c^3*arcsinh(d*x/sqr
t(c*d))/sqrt(d) + 15/8*a^2*c^2*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - (d*x^2 + c)
^(5/2)*a^2/x
```

Fricas [A]
time = 1.23, size = 375, normalized size = 1.73

$$\left[-\frac{15\left(b^2c^4-16\,abc^3d-48\,a^2c^2d^2\right)\sqrt{d}\,x\log\left(-2\,dx^2-2\sqrt{dx^2+c}\sqrt{d}\,x-c\right)-2\left(48\,b^2d^4x^8+8\left(17\,b^2cd^3+16\,abd^4\right)x^6-384\,a^2c^2d^2+2\left(59\,b^2c^2d^2+208\,abcd^3+48\,a^2d^4\right)x^4+3\left(5\,b^2c^3d+176\,abc^2d^2+144\,a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{768\,d^2x},\ \frac{15\left(b^2c^4-16\,abc^3d-48\,a^2c^2d^2\right)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(48\,b^2d^4x^8+8\left(17\,b^2cd^3+16\,abd^4\right)x^6-384\,a^2c^2d^2+2\left(59\,b^2c^2d^2+208\,abcd^3+48\,a^2d^4\right)x^4+3\left(5\,b^2c^3d+176\,abc^2d^2+144\,a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{384\,d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*sqrt(d)*x*log(-2*d*x^
2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(48*b^2*d^4*x^8 + 8*(17*b^2*c*d^3
+ 16*a*b*d^4)*x^6 - 384*a^2*c^2*d^2 + 2*(59*b^2*c^2*d^2 + 208*a*b*c*d^3 + 4
8*a^2*d^4)*x^4 + 3*(5*b^2*c^3*d + 176*a*b*c^2*d^2 + 144*a^2*c*d^3)*x^2)*sqr
t(d*x^2 + c))/(d^2*x), 1/384*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*
sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (48*b^2*d^4*x^8 + 8*(17*b^2
*c*d^3 + 16*a*b*d^4)*x^6 - 384*a^2*c^2*d^2 + 2*(59*b^2*c^2*d^2 + 208*a*b*c*
d^3 + 48*a^2*d^4)*x^4 + 3*(5*b^2*c^3*d + 176*a*b*c^2*d^2 + 144*a^2*c*d^3)*x
^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(201) = 402$.
time = 41.46, size = 496, normalized size = 2.29

$$-\frac{a^2c^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+a^2c^{\frac{3}{2}}dx\sqrt{1+\frac{dx^2}{c}}-\frac{7a^2c^{\frac{3}{2}}dx}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2\sqrt{c}\,d^2x^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{15a^2c^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8}+\frac{a^2d^3x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+abc^{\frac{5}{2}}x\sqrt{1+\frac{dx^2}{c}}+\frac{3abc^{\frac{5}{2}}x}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{35abc^{\frac{3}{2}}dx^3}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{17ab\sqrt{c}\,d^2x^5}{12\sqrt{1+\frac{dx^2}{c}}}+\frac{5abc^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{abd^3x^7}{3\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{7}{2}}x}{128d\sqrt{1+\frac{dx^2}{c}}}+\frac{133b^2c^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{dx^2}{c}}}+\frac{127b^2c^{\frac{3}{2}}dx^5}{192\sqrt{1+\frac{dx^2}{c}}}+\frac{23b^2\sqrt{c}\,d^2x^7}{48\sqrt{1+\frac{dx^2}{c}}}-\frac{5b^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{3}{2}}}+\frac{b^2d^3x^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**2,x)
```

```
[Out] -a**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) + a**2*c**(3/2)*d*x*sqrt(1 + d*x**2/c
) - 7*a**2*c**(3/2)*d*x/(8*sqrt(1 + d*x**2/c)) + 3*a**2*sqrt(c)*d**2*x**3/(
8*sqrt(1 + d*x**2/c)) + 15*a**2*c**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/8 + a
**2*d**3*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) + a*b*c**(5/2)*x*sqrt(1 + d*x*
*2/c) + 3*a*b*c**(5/2)*x/(8*sqrt(1 + d*x**2/c)) + 35*a*b*c**(3/2)*d*x**3/(2
4*sqrt(1 + d*x**2/c)) + 17*a*b*sqrt(c)*d**2*x**5/(12*sqrt(1 + d*x**2/c)) +
5*a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*sqrt(d)) + a*b*d**3*x**7/(3*sqrt(c)*
sqrt(1 + d*x**2/c)) + 5*b**2*c**(7/2)*x/(128*d*sqrt(1 + d*x**2/c)) + 133*b*
*2*c**(5/2)*x**3/(384*sqrt(1 + d*x**2/c)) + 127*b**2*c**(3/2)*d*x**5/(192*s
qrt(1 + d*x**2/c)) + 23*b**2*sqrt(c)*d**2*x**7/(48*sqrt(1 + d*x**2/c)) - 5*
b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**(3/2)) + b**2*d**3*x**9/(8*sqrt(
c)*sqrt(1 + d*x**2/c))
```

Giac [A]
time = 0.58, size = 219, normalized size = 1.01

$$\frac{2\,a^2c^3\sqrt{d}}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}+\frac{1}{384}\left(2\left(4\left(6\,b^2d^2x^2+\frac{17\,b^2cd^7+16\,abd^8}{d^6}\right)x^2+\frac{59\,b^2c^2d^6+208\,abcd^7+48\,a^2d^8}{d^6}\right)x^2+\frac{3\left(5\,b^2c^3d^5+176\,abc^2d^6+144\,a^2cd^7\right)}{d^6}\right)\sqrt{dx^2+c}\,x+\frac{5\left(b^2c^4\sqrt{d}-16\,abc^3d^{\frac{3}{2}}-48\,a^2c^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{256\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c^3*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/384*(2*(4*(6*b^
2*d^2*x^2 + (17*b^2*c*d^7 + 16*a*b*d^8)/d^6)*x^2 + (59*b^2*c^2*d^6 + 208*a*
b*c*d^7 + 48*a^2*d^8)/d^6)*x^2 + 3*(5*b^2*c^3*d^5 + 176*a*b*c^2*d^6 + 144*a
^2*c*d^7)/d^6)*sqrt(d*x^2 + c)*x + 5/256*(b^2*c^4*sqrt(d) - 16*a*b*c^3*d^(3
/2) - 48*a^2*c^2*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{5/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2, x)
```

3.631 $\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^3} \, dx$

Optimal. Leaf size=162

$$\frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2}+\frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2}+\frac{a(4bc+5ad)\left(c+dx^2\right)^{5/2}}{10c}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d}-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2}$$

```
[Out] 1/6*a*(5*a*d+4*b*c)*(d*x^2+c)^(3/2)+1/10*a*(5*a*d+4*b*c)*(d*x^2+c)^(5/2)/c+
1/7*b^2*(d*x^2+c)^(7/2)/d-1/2*a^2*(d*x^2+c)^(7/2)/c/x^2-1/2*a*c^(3/2)*(5*a*
d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))+1/2*a*c*(5*a*d+4*b*c)*(d*x^2+c)^(
1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 81, 52, 65, 214}

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2}-\frac{1}{2}ac^{3/2}(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{a\left(c+dx^2\right)^{5/2}(5ad+4bc)}{10c}+\frac{1}{6}a\left(c+dx^2\right)^{3/2}(5ad+4bc)+\frac{1}{2}ac\sqrt{c+dx^2}(5ad+4bc)+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

```
[Out] (a*c*(4*b*c + 5*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(3
/2))/6 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(5/2))/(10*c) + (b^2*(c + d*x^2)^(7
/2))/(7*d) - (a^2*(c + d*x^2)^(7/2))/(2*c*x^2) - (a*c^(3/2)*(4*b*c + 5*a*d)
*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^3} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x^2} dx, x, x^2\right)$$
$$= -\frac{a^2(c+dx^2)^{7/2}}{2cx^2} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(4bc+5ad)+b^2cx\right)(c+dx)^{5/2}}{x} dx, x, x^2\right)}{2c}$$
$$= \frac{b^2(c+dx^2)^{7/2}}{7d} - \frac{a^2(c+dx^2)^{7/2}}{2cx^2} + \frac{(a(4bc+5ad))\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x} dx, x, x^2\right.}{4c}$$
$$= \frac{a(4bc+5ad)(c+dx^2)^{5/2}}{10c} + \frac{b^2(c+dx^2)^{7/2}}{7d} - \frac{a^2(c+dx^2)^{7/2}}{2cx^2} + \frac{1}{4}(a(4bc+5$$
$$= \frac{1}{6}a(4bc+5ad)(c+dx^2)^{3/2} + \frac{a(4bc+5ad)(c+dx^2)^{5/2}}{10c} + \frac{b^2(c+dx^2)^{7/2}}{7d} -$$
$$= \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)(c+dx^2)^{3/2} + \frac{a(4bc+5ad)(c}{10c}$$
$$= \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)(c+dx^2)^{3/2} + \frac{a(4bc+5ad)(c}{10c}$$
$$= \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)(c+dx^2)^{3/2} + \frac{a(4bc+5ad)(c}{10c}$$

Mathematica [A]
time = 0.21, size = 133, normalized size = 0.82

$$\frac{\sqrt{c+dx^2}\left(30b^2x^2(c+dx^2)^3+35a^2d(-3c^2+14cdx^2+2d^2x^4)+28abdx^2(23c^2+11cdx^2+3d^2x^4)\right)}{210dx^2} - \frac{1}{2}ac^{3/2}(4bc+5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

```
[Out] (Sqrt[c + d*x^2]*(30*b^2*x^2*(c + d*x^2)^3 + 35*a^2*d*(-3*c^2 + 14*c*d*x^2
+ 2*d^2*x^4) + 28*a*b*d*x^2*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)))/(210*d*x^2)
 - (a*c^(3/2)*(4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

Maple [A]
time = 0.12, size = 183, normalized size = 1.13

method	result

default	$\frac{b^{2} \left(d\,x^{2}+c \right)^{\frac{7}{2}}}{7 d}+a^{2} \left(-\frac{\left(d\,x^{2}+c \right)^{\frac{7}{2}}}{2 c\,x^{2}}+\frac{5 d \left(\frac{\left(d\,x^{2}+c \right)^{\frac{5}{2}}}{5}+c \left(\frac{\left(d\,x^{2}+c \right)^{\frac{3}{2}}}{3}+c \left(\sqrt{d\,x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d\,x^{2}+c}}{x}\right)\right)\right)}{2 c}\right.$
risch	$-\frac{c^{2} a^{2} \sqrt{d\,x^{2}+c}}{2 x^{2}}+\frac{b^{2} d^{2} x^{6} \sqrt{d\,x^{2}+c}}{7}+\frac{3 b^{2} d c\,x^{4} \sqrt{d\,x^{2}+c}}{7}+\frac{3 b^{2} c^{2} x^{2} \sqrt{d\,x^{2}+c}}{7}+\frac{b^{2} c^{3} \sqrt{d\,x^{2}+c}}{7 d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^2*(d*x^2+c)^(7/2)/d+a^2*(-1/2/c/x^2*(d*x^2+c)^(7/2)+5/2*d/c*(1/5*(d*x
^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(
1/2)*(d*x^2+c)^(1/2))/x)))))+2*a*b*(1/5*(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3
/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))
```

Maxima [A]
time = 0.36, size = 170, normalized size = 1.05

$$-2\,abc^{\frac{5}{2}}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{5}{2}\,a^{2}c^{\frac{3}{2}}d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\frac{2}{5}\,(dx^{2}+c)^{\frac{5}{2}}ab+\frac{2}{3}\,(dx^{2}+c)^{\frac{3}{2}}abc+2\,\sqrt{dx^{2}+c}\,abc^{2}+\frac{(dx^{2}+c)^{\frac{7}{2}}b^{2}}{7\,d}+\frac{5}{6}\,(dx^{2}+c)^{\frac{3}{2}}a^{2}d+\frac{(dx^{2}+c)^{\frac{5}{2}}a^{2}d}{2\,c}+\frac{5}{2}\,\sqrt{dx^{2}+c}\,a^{2}cd-\frac{(dx^{2}+c)^{\frac{7}{2}}a^{2}}{2\,cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*a*b*c^(5/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 5/2*a^2*c^(3/2)*d*arcsinh(c/
(sqrt(c*d)*abs(x))) + 2/5*(d*x^2 + c)^(5/2)*a*b + 2/3*(d*x^2 + c)^(3/2)*a*b
*c + 2*sqrt(d*x^2 + c)*a*b*c^2 + 1/7*(d*x^2 + c)^(7/2)*b^2/d + 5/6*(d*x^2 +
 c)^(3/2)*a^2*d + 1/2*(d*x^2 + c)^(5/2)*a^2*d/c + 5/2*sqrt(d*x^2 + c)*a^2*c
*d - 1/2*(d*x^2 + c)^(7/2)*a^2/(c*x^2)
```

Fricas [A]
time = 1.21, size = 349, normalized size = 2.15

$$\left[\frac{105\,(4\,abc^{2}d+5\,a^{2}cd^{2})\sqrt{c}\,x^{2}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+2\,(30\,b^{2}d^{3}x^{8}+6\,(15\,b^{2}cd^{2}+14\,abd^{3})x^{6}-105\,a^{2}c^{2}d+2\,(45\,b^{2}c^{2}d+154\,abcd^{2}+35\,a^{2}d^{3})x^{4}+2\,(15\,b^{2}c^{3}+322\,abc^{2}d+245\,a^{2}cd^{2})x^{2})\sqrt{dx^{2}+c}}{420\,dx^{2}},\ \frac{105\,(4\,abc^{2}d+5\,a^{2}cd^{2})\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+(30\,b^{2}d^{3}x^{8}+6\,(15\,b^{2}cd^{2}+14\,abd^{3})x^{6}-105\,a^{2}c^{2}d+2\,(45\,b^{2}c^{2}d+154\,abcd^{2}+35\,a^{2}d^{3})x^{4}+2\,(15\,b^{2}c^{3}+322\,abc^{2}d+245\,a^{2}cd^{2})x^{2})\sqrt{dx^{2}+c}}{210\,dx^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/420*(105*(4*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*
x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(30*b^2*d^3*x^8 + 6*(15*b^2*c*d^2 + 14*a*b
*d^3)*x^6 - 105*a^2*c^2*d + 2*(45*b^2*c^2*d + 154*a*b*c*d^2 + 35*a^2*d^3)*x
^4 + 2*(15*b^2*c^3 + 322*a*b*c^2*d + 245*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(
d*x^2), 1/210*(105*(4*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)
```

```
/sqrt(d*x^2 + c)) + (30*b^2*d^3*x^8 + 6*(15*b^2*c*d^2 + 14*a*b*d^3)*x^6 - 1
05*a^2*c^2*d + 2*(45*b^2*c^2*d + 154*a*b*c*d^2 + 35*a^2*d^3)*x^4 + 2*(15*b^
2*c^3 + 322*a*b*c^2*d + 245*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2)]
```

Sympy [A]
time = 39.16, size = 518, normalized size = 3.20

$$-\frac{5a^2c^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2}-\frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x}+\frac{2a^2c^2\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{2a^2cd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}+a^2d^2\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\ d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)-2abc^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)+\frac{2abc^3}{\sqrt{d}\,x\sqrt{\frac{c}{dx^2}+1}}+\frac{2abc^2\sqrt{d}\,x}{\sqrt{\frac{c}{dx^2}+1}}+4abcd\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\ d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+2abd^2\left(\begin{cases}-\frac{2c^2\sqrt{c+dx^2}}{15d^2}+\frac{cx^2\sqrt{c+dx^2}}{15d}+\frac{x^4\sqrt{c+dx^2}}{5} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}\,x^4}{4} & \text{otherwise}\end{cases}\right)+b^2c^2\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\ d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+2b^2cd\left(\begin{cases}-\frac{2c^2\sqrt{c+dx^2}}{15d^2}+\frac{cx^2\sqrt{c+dx^2}}{15d}+\frac{x^4\sqrt{c+dx^2}}{5} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}\,x^4}{4} & \text{otherwise}\end{cases}\right)+b^2d^2\left(\begin{cases}\frac{8c^3\sqrt{c+dx^2}}{105d^3}-\frac{4c^2x^2\sqrt{c+dx^2}}{105d^2}+\frac{cx^4\sqrt{c+dx^2}}{35d}+\frac{x^6\sqrt{c+dx^2}}{7} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}\,x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**3,x)
```

```
[Out] -5*a**2*c**(3/2)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c**2*sqrt(d)*sqrt(c/
(d*x**2) + 1)/(2*x) + 2*a**2*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*a**2
*c*d**(3/2)*x/sqrt(c/(d*x**2) + 1) + a**2*d**2*Piecewise((sqrt(c)*x**2/2, E
q(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True)) - 2*a*b*c**(5/2)*asinh(sqrt(c)
/(sqrt(d)*x)) + 2*a*b*c**3/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*c**2*sq
rt(d)*x/sqrt(c/(d*x**2) + 1) + 4*a*b*c*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0
)), ((c + d*x**2)**(3/2)/(3*d), True)) + 2*a*b*d**2*Piecewise((-2*c**2*sqrt
(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x
**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)) + b**2*c**2*Piecewise((sqrt(c)*x
**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True)) + 2*b**2*c*d*Piecewise
((-2*c**2*sqrt(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x**
4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)) + b**2*d**2*Piecew
ise((8*c**3*sqrt(c + d*x**2)/(105*d**3) - 4*c**2*x**2*sqrt(c + d*x**2)/(105
*d**2) + c*x**4*sqrt(c + d*x**2)/(35*d) + x**6*sqrt(c + d*x**2)/7, Ne(d, 0)
), (sqrt(c)*x**6/6, True))
```

Giac [A]
time = 0.62, size = 165, normalized size = 1.02

$$\frac{30\,(dx^2+c)^{\frac{7}{2}}b^2+84\,(dx^2+c)^{\frac{5}{2}}abd+140\,(dx^2+c)^{\frac{3}{2}}abcd+420\,\sqrt{dx^2+c}\,abc^2d+70\,(dx^2+c)^{\frac{3}{2}}a^2d^2+420\,\sqrt{dx^2+c}\,a^2cd^2-\frac{105\,\sqrt{dx^2+c}\,a^2c^2d}{x^2}+\frac{105\,(4\,abc^3d+5\,a^2c^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/210*(30*(d*x^2 + c)^(7/2)*b^2 + 84*(d*x^2 + c)^(5/2)*a*b*d + 140*(d*x^2 +
 c)^(3/2)*a*b*c*d + 420*sqrt(d*x^2 + c)*a*b*c^2*d + 70*(d*x^2 + c)^(3/2)*a^
2*d^2 + 420*sqrt(d*x^2 + c)*a^2*c*d^2 - 105*sqrt(d*x^2 + c)*a^2*c^2*d/x^2 +
 105*(4*a*b*c^3*d + 5*a^2*c^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c
))/d
```

Mupad [B]
time = 0.70, size = 274, normalized size = 1.69

$$\sqrt{d\,x^2+c}\,\left(c^2\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)-2\,c\,\left(2\,c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)-\frac{{\left(a\,d-b\,c\right)}^2}{d}+\frac{b^2\,c^2}{d}\right)\right)-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{5\,d}-\frac{2\,b^2\,c}{5\,d}\right)\,{\left(d\,x^2+c\right)}^{5/2}-{\left(d\,x^2+c\right)}^{3/2}\,\left(\frac{2\,c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)}{3}-\frac{{\left(a\,d-b\,c\right)}^2}{3\,d}+\frac{b^2\,c^2}{3\,d}\right)+\frac{b^2\,{\left(d\,x^2+c\right)}^{7/2}}{7\,d}-\frac{a^2\,c^2\,\sqrt{d\,x^2+c}}{2\,x^2}+\frac{a\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,\left(5\,a\,d+4\,b\,c\right)\,1{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x)
```

```
[Out] (c + d*x^2)^(1/2)*(c^2*((2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d) - 2*c*(2*c*((2
*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d) - (a*d - b*c)^2/d + (b^2*c^2)/d)) - ((2*
b^2*c - 2*a*b*d)/(5*d) - (2*b^2*c)/(5*d))*(c + d*x^2)^(5/2) - (c + d*x^2)^(
3/2)*((2*c*((2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d))/3 - (a*d - b*c)^2/(3*d) +
 (b^2*c^2)/(3*d)) + (b^2*(c + d*x^2)^(7/2))/(7*d) + (a*c^(3/2)*atan(((c + d
*x^2)^(1/2)*1i)/c^(1/2))*(5*a*d + 4*b*c)*1i)/2 - (a^2*c^2*(c + d*x^2)^(1/2)
)/(2*x^2)
```

3.632 $\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^4} \, dx$

Optimal. Leaf size=223

$$\frac{5}{16}\left(b^2c^2+4ad(3bc+2ad)\right)x\sqrt{c+dx^2}+\frac{5(b^2c^2+4ad(3bc+2ad))\,x(c+dx^2)^{3/2}}{24c}+\frac{(b^2c^2+4ad(3bc+2ad))\,x}{6c^2}$$

[Out] 5/24*(b^2*c^2+4*a*d*(2*a*d+3*b*c))*x*(d*x^2+c)^(3/2)/c+1/6*(b^2*c^2+4*a*d*(2*a*d+3*b*c))*x*(d*x^2+c)^(5/2)/c^2-1/3*a^2*(d*x^2+c)^(7/2)/c/x^3-2/3*a*(2*a*d+3*b*c)*(d*x^2+c)^(7/2)/c^2/x+5/16*c*(b^2*c^2+4*a*d*(2*a*d+3*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)+5/16*(b^2*c^2+4*a*d*(2*a*d+3*b*c))*x*(d*x^2+c)^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 219, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {473, 464, 201, 223, 212}

$$-\frac{a^2(c+dx^2)^{7/2}}{3cx^3}+\frac{1}{6}x(c+dx^2)^{5/2}\left(\frac{4ad(2ad+3bc)}{c^2}+b^2\right)+\frac{5x(c+dx^2)^{3/2}(4ad(2ad+3bc)+b^2c^2)}{24c}+\frac{5}{16}x\sqrt{c+dx^2}\,(4ad(2ad+3bc)+b^2c^2)+\frac{5c(4ad(2ad+3bc)+b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16\sqrt{d}}-\frac{2a(c+dx^2)^{7/2}(2ad+3bc)}{3c^2x}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]

[Out] (5*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*Sqrt[c + d*x^2])/16 + (5*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*(c + d*x^2)^(3/2))/(24*c) + ((b^2 + (4*a*d*(3*b*c + 2*a*d))/c^2)*x*(c + d*x^2)^(5/2))/6 - (a^2*(c + d*x^2)^(7/2))/(3*c*x^3) - (2*a*(3*b*c + 2*a*d)*(c + d*x^2)^(7/2))/(3*c^2*x) + (5*c*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*Sqrt[d])

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^4}\,dx = -\frac{a^2(c+dx^2)^{7/2}}{3cx^3} + \frac{\int \frac{(2a(3bc+2ad)+3b^2cx^2)(c+dx^2)^{5/2}}{x^2}\,dx}{3c}$$

$$= -\frac{a^2(c+dx^2)^{7/2}}{3cx^3} - \frac{2a(3bc+2ad)(c+dx^2)^{7/2}}{3c^2x} + \left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)\int$$

$$= \frac{1}{6}\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+dx^2)^{5/2} - \frac{a^2(c+dx^2)^{7/2}}{3cx^3} - \frac{2a(3bc+2ad)(c}{3c^2x}$$

$$= \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+dx^2)^{3/2} + \frac{1}{6}\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+$$

$$= \frac{5}{16}(b^2c^2+12abcd+8a^2d^2)x\sqrt{c+dx^2} + \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+$$

$$= \frac{5}{16}(b^2c^2+12abcd+8a^2d^2)x\sqrt{c+dx^2} + \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+$$

$$= \frac{5}{16}(b^2c^2+12abcd+8a^2d^2)x\sqrt{c+dx^2} + \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x(c+$$

Mathematica [A]

time = 0.31, size = 154, normalized size = 0.69

$$\frac{1}{48}\left(\frac{\sqrt{c+dx^2}\,(-8a^2(2c^2+14cdx^2-3d^2x^4)+12abx^2(-8c^2+9cdx^2+2d^2x^4)+b^2x^4(33c^2+26cdx^2+8d^2x^4))}{x^3}-\frac{15c(b^2c^2+12abcd+8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-8*a^2*(2*c^2 + 14*c*d*x^2 - 3*d^2*x^4) + 12*a*b*x^2*(-8
*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + b^2*x^4*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4)))
/x^3 - (15*c*(b^2*c^2 + 12*a*b*c*d + 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c +
 d*x^2]])/Sqrt[d])/48
```

Maple [A]
time = 0.11, size = 287, normalized size = 1.29

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\,(-8b^2d^2x^8-24ab\,d^2x^6-26b^2cd\,x^6-24a^2d^2x^4-108abcd\,x^4-33b^2c^2x^4+112a^2cd\,x^2+96ab\,c^2x^2+16a^2c^2)}{48x^3}+\frac{5c\ln\Big(}{}$

default	$b^2\left(\frac{x(dx^2+c)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x(dx^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6}\right)+a^2\left(-\frac{(dx^2+c)^{\frac{7}{2}}}{3cx^3}+\frac{4d\left(- \right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2
+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+a^2*(-1/3/c/x^3*(d
*x^2+c)^(7/2)+4/3*d/c*(-1/c/x*(d*x^2+c)^(7/2)+6*d/c*(1/6*x*(d*x^2+c)^(5/2)+
5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(
x*d^(1/2)+(d*x^2+c)^(1/2)))))))+2*a*b*(-1/c/x*(d*x^2+c)^(7/2)+6*d/c*(1/6*x*
(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1
/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))))
```

Maxima [A]
time = 0.31, size = 234, normalized size = 1.05

$$\frac{1}{6}\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}x+\frac{5}{24}\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}cx+\frac{5}{16}\sqrt{dx^{2}+c}\,b^{2}c^{2}x+\frac{5}{2}\left(dx^{2}+c\right)^{\frac{3}{2}}abdx+\frac{15}{4}\sqrt{dx^{2}+c}\,abcdx+\frac{5}{2}\sqrt{dx^{2}+c}\,a^{2}d^{2}x+\frac{5\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}x}{3\,c}+\frac{5\,b^{2}c^{3}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,\sqrt{d}}+\frac{15}{4}\,abc^{2}\sqrt{d}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+\frac{5}{2}\,a^{2}cd^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{2\left(dx^{2}+c\right)^{\frac{5}{2}}ab}{x}-\frac{4\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d}{3\,cx}-\frac{\left(dx^{2}+c\right)^{\frac{7}{2}}a^{2}}{3\,cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^2 + c)^(5/2)*b^2*x + 5/24*(d*x^2 + c)^(3/2)*b^2*c*x + 5/16*sqrt(d*
x^2 + c)*b^2*c^2*x + 5/2*(d*x^2 + c)^(3/2)*a*b*d*x + 15/4*sqrt(d*x^2 + c)*a
*b*c*d*x + 5/2*sqrt(d*x^2 + c)*a^2*d^2*x + 5/3*(d*x^2 + c)^(3/2)*a^2*d^2*x/
c + 5/16*b^2*c^3*arcsinh(d*x/sqrt(c*d))/sqrt(d) + 15/4*a*b*c^2*sqrt(d)*arcs
inh(d*x/sqrt(c*d)) + 5/2*a^2*c*d^(3/2)*arcsinh(d*x/sqrt(c*d)) - 2*(d*x^2 +
c)^(5/2)*a*b/x - 4/3*(d*x^2 + c)^(5/2)*a^2*d/(c*x) - 1/3*(d*x^2 + c)^(7/2)*
a^2/(c*x^3)
```

Fricas [A]
time = 1.21, size = 346, normalized size = 1.55

$$\left[\frac{15\left(b^{2}c^{3}+12\,abc^{2}d+8\,a^{2}cd^{2}\right)\sqrt{d}\,x^{3}\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)+2\left(8\,b^{2}d^{3}x^{8}+2\left(13\,b^{2}cd^{2}+12\,abd^{3}\right)x^{6}-16\,a^{2}c^{2}d+3\left(11\,b^{2}c^{2}d+36\,abcd^{2}+8\,a^{2}d^{3}\right)x^{4}-16\left(6\,abc^{2}d+7\,a^{2}cd^{2}\right)x^{2}\right)\sqrt{dx^{2}+c}}{96\,dx^{3}},\ -\frac{15\left(b^{2}c^{3}+12\,abc^{2}d+8\,a^{2}cd^{2}\right)\sqrt{-d}\,x^{3}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)-\left(8\,b^{2}d^{3}x^{8}+2\left(13\,b^{2}cd^{2}+12\,abd^{3}\right)x^{6}-16\,a^{2}c^{2}d+3\left(11\,b^{2}c^{2}d+36\,abcd^{2}+8\,a^{2}d^{3}\right)x^{4}-16\left(6\,abc^{2}d+7\,a^{2}cd^{2}\right)x^{2}\right)\sqrt{dx^{2}+c}}{48\,dx^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*x^3*log(-2*d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^8 + 2*(13*b^2*c*d^2 + 12
*a*b*d^3)*x^6 - 16*a^2*c^2*d + 3*(11*b^2*c^2*d + 36*a*b*c*d^2 + 8*a^2*d^3)*
x^4 - 16*(6*a*b*c^2*d + 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3), -1/48*(
15*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*x^3*arctan(sqrt(-d)*x/sq
rt(d*x^2 + c)) - (8*b^2*d^3*x^8 + 2*(13*b^2*c*d^2 + 12*a*b*d^3)*x^6 - 16*a^
2*c^2*d + 3*(11*b^2*c^2*d + 36*a*b*c*d^2 + 8*a^2*d^3)*x^4 - 16*(6*a*b*c^2*d
 + 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(221) = 442$.
time = 15.15, size = 490, normalized size = 2.20

$$-\frac{2a^{2}c^{\frac{3}{2}}d}{x\sqrt{1+\frac{dx^{2}}{c}}}+\frac{a^{2}\sqrt{c}\,d^{2}x\sqrt{1+\frac{dx^{2}}{c}}}{2}-\frac{2a^{2}\sqrt{c}\,d^{2}x}{\sqrt{1+\frac{dx^{2}}{c}}}-\frac{a^{2}c^{2}\sqrt{d}\,\sqrt{\frac{c}{dx^{2}}+1}}{3x^{2}}-\frac{a^{2}cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{3}+\frac{5a^{2}cd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2}-\frac{2abc^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^{2}}{c}}}+2abc^{\frac{3}{2}}dx\sqrt{1+\frac{dx^{2}}{c}}-\frac{7abc^{\frac{3}{2}}dx}{4\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3ab\sqrt{c}\,d^{2}x^{3}}{4\sqrt{1+\frac{dx^{2}}{c}}}+\frac{15abc^{2}\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{4}+\frac{abd^{3}x^{5}}{2\sqrt{c}\,\sqrt{1+\frac{dx^{2}}{c}}}+\frac{b^{2}c^{\frac{5}{2}}x\sqrt{1+\frac{dx^{2}}{c}}}{2}+\frac{3b^{2}c^{\frac{5}{2}}x}{16\sqrt{1+\frac{dx^{2}}{c}}}+\frac{35b^{2}c^{\frac{3}{2}}dx^{3}}{48\sqrt{1+\frac{dx^{2}}{c}}}+\frac{17b^{2}\sqrt{c}\,d^{2}x^{5}}{24\sqrt{1+\frac{dx^{2}}{c}}}+\frac{5b^{2}c^{3}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16\sqrt{d}}+\frac{b^{2}d^{3}x^{7}}{6\sqrt{c}\,\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**4,x)
```

```
[Out] -2*a**2*c**(3/2)*d/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(c)*d**2*x*sqrt(1 + d*
x**2/c)/2 - 2*a**2*sqrt(c)*d**2*x/sqrt(1 + d*x**2/c) - a**2*c**2*sqrt(d)*sq
rt(c/(d*x**2) + 1)/(3*x**2) - a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 5*a*
*2*c*d**(3/2)*asinh(sqrt(d)*x/sqrt(c))/2 - 2*a*b*c**(5/2)/(x*sqrt(1 + d*x**
2/c)) + 2*a*b*c**(3/2)*d*x*sqrt(1 + d*x**2/c) - 7*a*b*c**(3/2)*d*x/(4*sqrt(
1 + d*x**2/c)) + 3*a*b*sqrt(c)*d**2*x**3/(4*sqrt(1 + d*x**2/c)) + 15*a*b*c*
*2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/4 + a*b*d**3*x**5/(2*sqrt(c)*sqrt(1 + d
*x**2/c)) + b**2*c**(5/2)*x*sqrt(1 + d*x**2/c)/2 + 3*b**2*c**(5/2)*x/(16*sq
rt(1 + d*x**2/c)) + 35*b**2*c**(3/2)*d*x**3/(48*sqrt(1 + d*x**2/c)) + 17*b*
*2*sqrt(c)*d**2*x**5/(24*sqrt(1 + d*x**2/c)) + 5*b**2*c**3*asinh(sqrt(d)*x/
sqrt(c))/(16*sqrt(d)) + b**2*d**3*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A]
time = 0.69, size = 307, normalized size = 1.38

$$\frac{1}{48}\left(2\left(4\,b^2d^2x^2+\frac{13\,b^2cd^5+12\,abd^6}{d^4}\right)x^2+\frac{3\,(11\,b^2c^2d^4+36\,abcd^5+8\,a^2d^6)}{d^4}\right)\sqrt{dx^2+c}\,x-\frac{5\left(b^2c^3\sqrt{d}+12\,abc^2d^{\frac{3}{2}}+8\,a^2cd^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{32\,d}+\frac{2\left(6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc^3\sqrt{d}+9\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2c^2d^{\frac{3}{2}}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^4\sqrt{d}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2c^3d^{\frac{3}{2}}+6\,abc^5\sqrt{d}+7\,a^2c^4d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*d^2*x^2 + (13*b^2*c*d^5 + 12*a*b*d^6)/d^4)*x^2 + 3*(11*b^2*c
^2*d^4 + 36*a*b*c*d^5 + 8*a^2*d^6)/d^4)*sqrt(d*x^2 + c)*x - 5/32*(b^2*c^3*s
qrt(d) + 12*a*b*c^2*d^(3/2) + 8*a^2*c*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2
+ c))^2)/d + 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*sqrt(d) + 9*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 +
 c))^2*a*b*c^4*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(3/2)
 + 6*a*b*c^5*sqrt(d) + 7*a^2*c^4*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2
- c)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{5/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4, x)
```

3.633 $\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^5} dx$

Optimal. Leaf size=222

$$\frac{1}{8}\left(8b^2c^2+5ad(8bc+3ad)\right)\sqrt{c+dx^2}+\frac{\left(8b^2c^2+5ad(8bc+3ad)\right)\left(c+dx^2\right)^{3/2}}{24c}+\frac{\left(8b^2c^2+5ad(8bc+3ad)\right)(c}{40c^2}$$

```
[Out] 1/24*(8*b^2*c^2+5*a*d*(3*a*d+8*b*c))*(d*x^2+c)^(3/2)/c+1/40*(8*b^2*c^2+5*a*
d*(3*a*d+8*b*c))*(d*x^2+c)^(5/2)/c^2-1/4*a^2*(d*x^2+c)^(7/2)/c/x^4-1/8*a*(3
*a*d+8*b*c)*(d*x^2+c)^(7/2)/c^2/x^2-1/8*(8*b^2*c^2+5*a*d*(3*a*d+8*b*c))*arc
tanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/8*(8*b^2*c^2+5*a*d*(3*a*d+8*b*c))*(
d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 219, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 79, 52, 65, 214}

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{4cx^4}+\frac{1}{40}\left(c+dx^2\right)^{5/2}\left(\frac{5ad(3ad+8bc)}{c^2}+8b^2\right)+\frac{\left(c+dx^2\right)^{3/2}\left(5ad(3ad+8bc)+8b^2c^2\right)}{24c}+\frac{1}{8}\sqrt{c+dx^2}\left(5ad(3ad+8bc)+8b^2c^2\right)-\frac{1}{8}\sqrt{c}\left(5ad(3ad+8bc)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{a\left(c+dx^2\right)^{7/2}(3ad+8bc)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

```
[Out] ((8*b^2*c^2 + 5*a*d*(8*b*c + 3*a*d))*Sqrt[c + d*x^2])/8 + ((8*b^2*c^2 + 5*a
*d*(8*b*c + 3*a*d))*(c + d*x^2)^(3/2))/(24*c) + ((8*b^2 + (5*a*d*(8*b*c + 3
*a*d))/c^2)*(c + d*x^2)^(5/2))/40 - (a^2*(c + d*x^2)^(7/2))/(4*c*x^4) - (a*
(8*b*c + 3*a*d)*(c + d*x^2)^(7/2))/(8*c^2*x^2) - (Sqrt[c]*(8*b^2*c^2 + 5*a*
d*(8*b*c + 3*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^5} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x^3} dx, x, x^2\right)$$
$$= -\frac{a^2(c+dx^2)^{7/2}}{4cx^4} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(8bc+3ad)+2b^2cx\right)(c+dx)^{5/2}}{x^2} dx, x, x^2\right)}{4c}$$
$$= -\frac{a^2(c+dx^2)^{7/2}}{4cx^4} - \frac{a(8bc+3ad)(c+dx^2)^{7/2}}{8c^2x^2} + \frac{1}{16}\left(8b^2 + \frac{5ad(8bc+3ad)}{c^2}\right.$$
$$= \frac{1}{40}\left(8b^2 + \frac{5ad(8bc+3ad)}{c^2}\right)(c+dx^2)^{5/2} - \frac{a^2(c+dx^2)^{7/2}}{4cx^4} - \frac{a(8bc+3ad)(}{8c^2x}$$
$$= \frac{1}{24}c\left(8b^2 + \frac{5ad(8bc+3ad)}{c^2}\right)(c+dx^2)^{3/2} + \frac{1}{40}\left(8b^2 + \frac{5ad(8bc+3ad)}{c^2}\right)(c$$
$$= \frac{1}{8}(8b^2c^2+40abcd+15a^2d^2)\sqrt{c+dx^2} + \frac{1}{24}c\left(8b^2 + \frac{5ad(8bc+3ad)}{c^2}\right)(c+$$
$$= \frac{1}{8}(8b^2c^2+40abcd+15a^2d^2)\sqrt{c+dx^2} + \frac{1}{24}c\left(8b^2 + \frac{5ad(8bc+3ad)}{c^2}\right)(c+$$
$$= \frac{1}{8}(8b^2c^2+40abcd+15a^2d^2)\sqrt{c+dx^2} + \frac{1}{24}c\left(8b^2 + \frac{5ad(8bc+3ad)}{c^2}\right)(c+$$

Mathematica [A]
time = 0.22, size = 153, normalized size = 0.69

$$\frac{\sqrt{c+dx^2}\,(-15a^2(2c^2+9cdx^2-8d^2x^4)+40abx^2(-3c^2+14cdx^2+2d^2x^4)+8b^2x^4(23c^2+11cdx^2+3d^2x^4))}{120x^4} - \frac{1}{8}\sqrt{c}\,(8b^2c^2+40abcd+15a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*(2*c^2 + 9*c*d*x^2 - 8*d^2*x^4) + 40*a*b*x^2*(-3*
c^2 + 14*c*d*x^2 + 2*d^2*x^4) + 8*b^2*x^4*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)
))/(120*x^4) - (Sqrt[c]*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]])/8
```

Maple [A]
time = 0.13, size = 284, normalized size = 1.28

method	result

risch	$-\frac{ca\sqrt{d\,x^{2}+c}\,\left(9ad\,x^{2}+8c\,x^{2}b+2ac\right)}{8x^{4}}+\frac{b^{2}d^{2}x^{4}\sqrt{d\,x^{2}+c}}{5}+\frac{11b^{2}dc\,x^{2}\sqrt{d\,x^{2}+c}}{15}+\frac{23b^{2}c^{2}\sqrt{d\,x^{2}+c}}{15}+\frac{2ab\,d^{2}}{}$
default	$a^{2}\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}}{4c\,x^{4}}+\frac{3d\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}}{2c\,x^{2}}+\frac{5d\left(\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{5}+c\left(\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{3}+c\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)\right)\right)\right)}{2c}\right)}{4c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/4/c/x^4*(d*x^2+c)^(7/2)+3/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(7/2)+5/2*d/c
*(1/5*(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln(
(2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))))+2*a*b*(-1/2/c/x^2*(d*x^2+c)^(7/2)+5
/2*d/c*(1/5*(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/
2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))))+b^2*(1/5*(d*x^2+c)^(5/2)+c*(1/
3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1
/2))/x))))
```

Maxima [A]

time = 0.30, size = 271, normalized size = 1.22

$$-b^{2}c^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-5\,abc^{\frac{3}{2}}d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{15}{8}\,a^{2}\sqrt{c}\,d^{2}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\frac{1}{5}\,(dx^{2}+c)^{\frac{5}{2}}b^{2}+\frac{1}{3}\,(dx^{2}+c)^{\frac{3}{2}}b^{2}c+\sqrt{dx^{2}+c}\,b^{2}c^{2}+\frac{5}{3}\,(dx^{2}+c)^{\frac{3}{2}}abd+\frac{(dx^{2}+c)^{\frac{5}{2}}abd}{c}+5\,\sqrt{dx^{2}+c}\,abcd+\frac{15}{8}\,\sqrt{dx^{2}+c}\,a^{2}d^{2}+\frac{3\,(dx^{2}+c)^{\frac{5}{2}}a^{2}d^{2}}{8\,c^{2}}+\frac{5\,(dx^{2}+c)^{\frac{3}{2}}a^{2}d^{2}}{8\,c}-\frac{(dx^{2}+c)^{\frac{7}{2}}ab}{cx^{2}}-\frac{3\,(dx^{2}+c)^{\frac{7}{2}}a^{2}d}{8\,c^{2}x^{2}}-\frac{(dx^{2}+c)^{\frac{7}{2}}a^{2}}{4\,cx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -b^2*c^(5/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 5*a*b*c^(3/2)*d*arcsinh(c/(sqr
t(c*d)*abs(x))) - 15/8*a^2*sqrt(c)*d^2*arcsinh(c/(sqrt(c*d)*abs(x))) + 1/5*
(d*x^2 + c)^(5/2)*b^2 + 1/3*(d*x^2 + c)^(3/2)*b^2*c + sqrt(d*x^2 + c)*b^2*c
^2 + 5/3*(d*x^2 + c)^(3/2)*a*b*d + (d*x^2 + c)^(5/2)*a*b*d/c + 5*sqrt(d*x^2
 + c)*a*b*c*d + 15/8*sqrt(d*x^2 + c)*a^2*d^2 + 3/8*(d*x^2 + c)^(5/2)*a^2*d^
2/c^2 + 5/8*(d*x^2 + c)^(3/2)*a^2*d^2/c - (d*x^2 + c)^(7/2)*a*b/(c*x^2) - 3
/8*(d*x^2 + c)^(7/2)*a^2*d/(c^2*x^2) - 1/4*(d*x^2 + c)^(7/2)*a^2/(c*x^4)
```

Fricas [A]

time = 2.06, size = 319, normalized size = 1.44

$$\left[\frac{15\,(8\,b^{2}c^{2}+40\,abcd+15\,a^{2}d^{2})\sqrt{c}\,x^{4}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+2\,(24\,b^{2}d^{2}x^{8}+8\,(11\,b^{2}cd+10\,abd^{2})x^{6}+8\,(23\,b^{2}c^{2}+70\,abcd+15\,a^{2}d^{2})x^{4}-30\,a^{2}c^{2}-15\,(8\,abc^{2}+9\,a^{2}cd)x^{2})\sqrt{dx^{2}+c}}{240\,x^{4}},\ \frac{15\,(8\,b^{2}c^{2}+40\,abcd+15\,a^{2}d^{2})\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+(24\,b^{2}d^{2}x^{8}+8\,(11\,b^{2}cd+10\,abd^{2})x^{6}+8\,(23\,b^{2}c^{2}+70\,abcd+15\,a^{2}d^{2})x^{4}-30\,a^{2}c^{2}-15\,(8\,abc^{2}+9\,a^{2}cd)x^{2})\sqrt{dx^{2}+c}}{120\,x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(24*b^2*d^2*x^8 + 8*(11*b^2*c*d +
 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2 + 70*a*b*c*d + 15*a^2*d^2)*x^4 - 30*a^2*c^
2 - 15*(8*a*b*c^2 + 9*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/x^4, 1/120*(15*(8*b^2*
c^2 + 40*a*b*c*d + 15*a^2*d^2)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + (24*b^2*d^2*x^8 + 8*(11*b^2*c*d + 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2 + 70*
a*b*c*d + 15*a^2*d^2)*x^4 - 30*a^2*c^2 - 15*(8*a*b*c^2 + 9*a^2*c*d)*x^2)*sq
rt(d*x^2 + c))/x^4]
```

Sympy [A]
time = 84.67, size = 473, normalized size = 2.13

$$-\frac{15a^2\sqrt{c}\,d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{8}-\frac{a^2c^3}{4\sqrt{d}\,x^5\sqrt{\frac{c}{dx^2}+1}}-\frac{3a^2c^2\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}}-\frac{a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x}+\frac{7a^2cd^{\frac{3}{2}}}{8x\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{5}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}-5abc^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)-\frac{abc^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{x}+\frac{4abc^2\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{4abcd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}+2abd^2\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\: d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)-b^2c^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)+\frac{b^2c^3}{\sqrt{d}\,x\sqrt{\frac{c}{dx^2}+1}}+\frac{b^2c^2\sqrt{d}\,x}{\sqrt{\frac{c}{dx^2}+1}}+2b^2cd\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\: d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+b^2d^2\left(\begin{cases}-\frac{2c^2\sqrt{c+dx^2}}{15d^2}+\frac{cx^2\sqrt{c+dx^2}}{15d}+\frac{x^4\sqrt{c+dx^2}}{5} & \text{for}\: d\neq 0\\ \frac{\sqrt{c}\,x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**5,x)
```

```
[Out] -15*a**2*sqrt(c)*d**2*asinh(sqrt(c)/(sqrt(d)*x))/8 - a**2*c**3/(4*sqrt(d)*x
**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*c**2*sqrt(d)/(8*x**3*sqrt(c/(d*x**2) + 1
)) - a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/x + 7*a**2*c*d**(3/2)/(8*x*sqrt(c
/(d*x**2) + 1)) + a**2*d**(5/2)*x/sqrt(c/(d*x**2) + 1) - 5*a*b*c**(3/2)*d*a
sinh(sqrt(c)/(sqrt(d)*x)) - a*b*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/x + 4*a*b
*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 4*a*b*c*d**(3/2)*x/sqrt(c/(d*x**2)
 + 1) + 2*a*b*d**2*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/
2)/(3*d), True)) - b**2*c**(5/2)*asinh(sqrt(c)/(sqrt(d)*x)) + b**2*c**3/(sq
rt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*c**2*sqrt(d)*x/sqrt(c/(d*x**2) + 1) +
2*b**2*c*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d)
, True)) + b**2*d**2*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*d**2) + c*x**2
*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**
4/4, True))
```

Giac [A]
time = 0.65, size = 242, normalized size = 1.09

$$\frac{24\,(dx^2+c)^{\frac{5}{2}}b^2d+40\,(dx^2+c)^{\frac{3}{2}}b^2cd+120\,\sqrt{dx^2+c}\,b^2c^2d+80\,(dx^2+c)^{\frac{3}{2}}abd^2+480\,\sqrt{dx^2+c}\,abcd^2+120\,\sqrt{dx^2+c}\,a^2d^3+\frac{15\,(8\,b^2c^3d+40\,abc^2d^2+15\,a^2cd^3)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{15\left(8\,(dx^2+c)^{\frac{3}{2}}abc^2d^2-8\,\sqrt{dx^2+c}\,abc^3d^2+9\,(dx^2+c)^{\frac{3}{2}}a^2cd^3-7\,\sqrt{dx^2+c}\,a^2c^2d^3\right)}{d^2x^4}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/120*(24*(d*x^2 + c)^(5/2)*b^2*d + 40*(d*x^2 + c)^(3/2)*b^2*c*d + 120*sqrt
(d*x^2 + c)*b^2*c^2*d + 80*(d*x^2 + c)^(3/2)*a*b*d^2 + 480*sqrt(d*x^2 + c)*
a*b*c*d^2 + 120*sqrt(d*x^2 + c)*a^2*d^3 + 15*(8*b^2*c^3*d + 40*a*b*c^2*d^2
```

```
+ 15*a^2*c*d^3)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - 15*(8*(d*x^2 +
c)^(3/2)*a*b*c^2*d^2 - 8*sqrt(d*x^2 + c)*a*b*c^3*d^2 + 9*(d*x^2 + c)^(3/2)*
a^2*c*d^3 - 7*sqrt(d*x^2 + c)*a^2*c^2*d^3)/(d^2*x^4))/d
```

Mupad [B]
time = 0.88, size = 262, normalized size = 1.18

$$(d\,x^2+c)^{3/2}\,\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right)-\frac{(d\,x^2+c)^{3/2}\,\left(\frac{9\,a^2\,c\,d^2}{8}+b\,a\,c^2\,d\right)-\left(\frac{7\,a^2\,c^2\,d^2}{8}+b\,a\,c^3\,d\right)\,\sqrt{d\,x^2+c}}{(d\,x^2+c)^2-2\,c\,(d\,x^2+c)+c^2}+\sqrt{d\,x^2+c}\,\left((a\,d-b\,c)^2+3\,c\,(c\,b^2+2\,a\,d\,b)-3\,b^2\,c^2\right)+\frac{b^2\,(d\,x^2+c)^{5/2}}{5}+2\,\mathrm{atan}\left(\frac{2\,\sqrt{d\,x^2+c}\,\sqrt{-\frac{c}{256}}\,(15\,a^2\,d^2+40\,a\,b\,c\,d+8\,b^2\,c^2)}{\frac{15\,a^2\,c\,d^2}{8}+5\,a\,b\,c^2\,d+b^2\,c^3}\right)\,\sqrt{-\frac{c}{256}}\,(15\,a^2\,d^2+40\,a\,b\,c\,d+8\,b^2\,c^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x)
```

```
[Out] (c + d*x^2)^(3/2)*((b^2*c)/3 + (2*a*b*d)/3) - ((c + d*x^2)^(3/2)*((9*a^2*c*
d^2)/8 + a*b*c^2*d) - ((7*a^2*c^2*d^2)/8 + a*b*c^3*d)*(c + d*x^2)^(1/2))/((
c + d*x^2)^2 - 2*c*(c + d*x^2) + c^2) + (c + d*x^2)^(1/2)*((a*d - b*c)^2 +
3*c*(b^2*c + 2*a*b*d) - 3*b^2*c^2) + (b^2*(c + d*x^2)^(5/2))/5 + 2*atan((2*
(c + d*x^2)^(1/2)*(-c/256)^(1/2)*(15*a^2*d^2 + 8*b^2*c^2 + 40*a*b*c*d))/(b^
2*c^3 + (15*a^2*c*d^2)/8 + 5*a*b*c^2*d))*(-c/256)^(1/2)*(15*a^2*d^2 + 8*b^2
*c^2 + 40*a*b*c*d)
```

3.634 $\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^6} dx$

Optimal. Leaf size=228

$$\frac{d(15b^2c^2 + 8ad(5bc + ad))\, x\sqrt{c + dx^2}}{8c} + \frac{d(15b^2c^2 + 8ad(5bc + ad))\, x(c + dx^2)^{3/2}}{12c^2} - \frac{(15b^2c^2 + 8ad(5bc + ad)}{15c^2x}$$

```
[Out] 1/12*d*(15*b^2*c^2+8*a*d*(a*d+5*b*c))*x*(d*x^2+c)^(3/2)/c^2-1/15*(15*b^2*c^
2+8*a*d*(a*d+5*b*c))*(d*x^2+c)^(5/2)/c^2/x-1/5*a^2*(d*x^2+c)^(7/2)/c/x^5-2/
15*a*(a*d+5*b*c)*(d*x^2+c)^(7/2)/c^2/x^3+1/8*(15*b^2*c^2+8*a*d*(a*d+5*b*c))
*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)+1/8*d*(15*b^2*c^2+8*a*d*(a*d+5*
b*c))*x*(d*x^2+c)^(1/2)/c
```

Rubi [A]

time = 0.11, antiderivative size = 225, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 464, 283, 201, 223, 212}

$$-\frac{a^2(c+dx^2)^{7/2}}{5cx^5} - \frac{(c+dx^2)^{5/2}\left(\frac{8ad(ad+5bc)}{c^2} + 15b^2\right)}{15x} + \frac{dx(c+dx^2)^{3/2}(8ad(ad+5bc)+15b^2c^2)}{12c^2} + \frac{dx\sqrt{c+dx^2}\,(8ad(ad+5bc)+15b^2c^2)}{8c} + \frac{1}{8}\sqrt{d}\,(8ad(ad+5bc)+15b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right) - \frac{2a(c+dx^2)^{7/2}(ad+5bc)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

```
[Out] (d*(15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + (d*(15*b^2
*c^2 + 8*a*d*(5*b*c + a*d))*x*(c + d*x^2)^(3/2))/(12*c^2) - ((15*b^2 + (8*a
*d*(5*b*c + a*d))/c^2)*(c + d*x^2)^(5/2))/(15*x) - (a^2*(c + d*x^2)^(7/2))/
(5*c*x^5) - (2*a*(5*b*c + a*d)*(c + d*x^2)^(7/2))/(15*c^2*x^3) + (Sqrt[d]*(
15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/8
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^6}\,dx = -\frac{a^2(c+dx^2)^{7/2}}{5cx^5} + \frac{\int \frac{(2a(5bc+ad)+5b^2cx^2)(c+dx^2)^{5/2}}{x^4}\,dx}{5c}$$

$$= -\frac{a^2(c+dx^2)^{7/2}}{5cx^5} - \frac{2a(5bc+ad)(c+dx^2)^{7/2}}{15c^2x^3} - \frac{1}{15}\left(-15b^2 - \frac{8ad(5bc+ad)}{c^2}\right.$$

$$= -\frac{\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)(c+dx^2)^{5/2}}{15x} - \frac{a^2(c+dx^2)^{7/2}}{5cx^5} - \frac{2a(5bc+ad)(c+dx}{15c^2x^3}$$

$$= \frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x(c+dx^2)^{3/2} - \frac{\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)(c+dx^2}{15x}$$

$$= \frac{1}{8}cd\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\sqrt{c+dx^2} + \frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x$$

$$= \frac{1}{8}cd\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\sqrt{c+dx^2} + \frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x$$

$$= \frac{1}{8}cd\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\sqrt{c+dx^2} + \frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x$$

Mathematica [A]
time = 0.39, size = 156, normalized size = 0.68

$$\frac{\sqrt{c+dx^2}\,(15b^2x^4(-8c^2+9cdx^2+2d^2x^4)+40abx^2(-2c^2-14cdx^2+3d^2x^4)-8a^2(3c^2+11cdx^2+23d^2x^4))}{120x^5} - \frac{1}{8}\sqrt{d}\,(15b^2c^2+40abcd+8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^2*x^4*(-8*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + 40*a*b*x^2*
(-2*c^2 - 14*c*d*x^2 + 3*d^2*x^4) - 8*a^2*(3*c^2 + 11*c*d*x^2 + 23*d^2*x^4)
))/(120*x^5) - (Sqrt[d]*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*Log[-(Sqrt[d]
*x) + Sqrt[c + d*x^2]])/8
```

Maple [A]
time = 0.10, size = 359, normalized size = 1.57

method	result
risch	$-\frac{\sqrt{dx^2+c}\,(-30b^2d^2x^8-120ab\,d^2x^6-135b^2cd\,x^6+184a^2d^2x^4+560abcd\,x^4+120b^2c^2x^4+88a^2cd\,x^2+80ab\,c^2x^2+24a^2c^2)}{120x^5}+d$

default	a^2	$\left(-\frac{(dx^2+c)^{\frac{7}{2}}}{5c\,x^5}+\frac{2d\left(-\frac{(dx^2+c)^{\frac{7}{2}}}{3c\,x^3}+\frac{4d\left(-\frac{(dx^2+c)^{\frac{7}{2}}}{cx}+\frac{6d\left(\frac{x(dx^2+c)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x(dx^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{\cdots}\right.}{2\sqrt{\cdots}}\right.}{4}\right.}{6}\right.}{c}\right.}{3c}\right.}{5c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/5/c/x^5*(d*x^2+c)^(7/2)+2/5*d/c*(-1/3/c/x^3*(d*x^2+c)^(7/2)+4/3*d/c
*(-1/c/x*(d*x^2+c)^(7/2)+6*d/c*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c
)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(
1/2))))))))+2*a*b*(-1/3/c/x^3*(d*x^2+c)^(7/2)+4/3*d/c*(-1/c/x*(d*x^2+c)^(7/
2)+6*d/c*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(
d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))))+b^2*(-1/c/x
*(d*x^2+c)^(7/2)+6*d/c*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+
3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))
)
```

Maxima [A]
time = 0.31, size = 285, normalized size = 1.25

$$\frac{5}{4}\,(dx^2+c)^{\frac{3}{2}}b^2dx+\frac{15}{8}\,\sqrt{dx^2+c}\,b^2cdx+5\,\sqrt{dx^2+c}\,abd^2x+\frac{10\,(dx^2+c)^{\frac{3}{2}}abd^2x}{3\,c}+\frac{2\,(dx^2+c)^{\frac{3}{2}}a^2d^3x}{3\,c^2}+\frac{\sqrt{dx^2+c}\,a^2d^3x}{c}+\frac{15}{8}\,b^2c^2\sqrt{d}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+5\,abcd^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+a^2d^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{(dx^2+c)^{\frac{5}{2}}b^2}{x}-\frac{8\,(dx^2+c)^{\frac{5}{2}}abd}{3\,cx}-\frac{8\,(dx^2+c)^{\frac{5}{2}}a^2d^2}{15\,c^2x}-\frac{2\,(dx^2+c)^{\frac{7}{2}}ab}{3\,cx^3}-\frac{2\,(dx^2+c)^{\frac{7}{2}}a^2d}{15\,c^2x^3}-\frac{(dx^2+c)^{\frac{7}{2}}a^2}{5\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 5/4*(d*x^2 + c)^(3/2)*b^2*d*x + 15/8*sqrt(d*x^2 + c)*b^2*c*d*x + 5*sqrt(d*x
^2 + c)*a*b*d^2*x + 10/3*(d*x^2 + c)^(3/2)*a*b*d^2*x/c + 2/3*(d*x^2 + c)^(3
/2)*a^2*d^3*x/c^2 + sqrt(d*x^2 + c)*a^2*d^3*x/c + 15/8*b^2*c^2*sqrt(d)*arcs
inh(d*x/sqrt(c*d)) + 5*a*b*c*d^(3/2)*arcsinh(d*x/sqrt(c*d)) + a^2*d^(5/2)*a
rcsinh(d*x/sqrt(c*d)) - (d*x^2 + c)^(5/2)*b^2/x - 8/3*(d*x^2 + c)^(5/2)*a*b
*d/(c*x) - 8/15*(d*x^2 + c)^(5/2)*a^2*d^2/(c^2*x) - 2/3*(d*x^2 + c)^(7/2)*a
*b/(c*x^3) - 2/15*(d*x^2 + c)^(7/2)*a^2*d/(c^2*x^3) - 1/5*(d*x^2 + c)^(7/2)
*a^2/(c*x^5)
```

Fricas [A]
time = 1.32, size = 318, normalized size = 1.39

$$\left[\frac{15\,(15\,b^2c^2+40\,abcd+8\,a^2d^2)\sqrt{d}\,x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(30\,b^2d^2x^8+15\,(9\,b^2cd+8\,abd^2)x^6-8\,(15\,b^2c^2+70\,abcd+23\,a^2d^2)x^4-24\,a^2c^2-8\,(10\,abc^2+11\,a^2cd)x^2)\sqrt{dx^2+c}}{240\,x^5},\,-\frac{15\,(15\,b^2c^2+40\,abcd+8\,a^2d^2)\sqrt{-d}\,x^5\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(30\,b^2d^2x^8+15\,(9\,b^2cd+8\,abd^2)x^6-8\,(15\,b^2c^2+70\,abcd+23\,a^2d^2)x^4-24\,a^2c^2-8\,(10\,abc^2+11\,a^2cd)x^2)\sqrt{dx^2+c}}{120\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*sqrt(d)*x^5*log(-2*d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(30*b^2*d^2*x^8 + 15*(9*b^2*c*d + 8*a
*b*d^2)*x^6 - 8*(15*b^2*c^2 + 70*a*b*c*d + 23*a^2*d^2)*x^4 - 24*a^2*c^2 - 8
*(10*a*b*c^2 + 11*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/x^5, -1/120*(15*(15*b^2*c^
2 + 40*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*x^5*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
```

```
 - (30*b^2*d^2*x^8 + 15*(9*b^2*c*d + 8*a*b*d^2)*x^6 - 8*(15*b^2*c^2 + 70*a*
b*c*d + 23*a^2*d^2)*x^4 - 24*a^2*c^2 - 8*(10*a*b*c^2 + 11*a^2*c*d)*x^2)*sqr
t(d*x^2 + c))/x^5]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(212) = 424$.
time = 8.28, size = 474, normalized size = 2.08

$$-\frac{a^2\sqrt{c}\,d^2}{x\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{11a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15x^2}-\frac{8a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15}+a^2d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{a^2d^3x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{4abc^{\frac{3}{2}}d}{x\sqrt{1+\frac{dx^2}{c}}}+ab\sqrt{c}\,d^2x\sqrt{1+\frac{dx^2}{c}}-\frac{4ab\sqrt{c}\,d^2x}{\sqrt{1+\frac{dx^2}{c}}}-\frac{2abc^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{2abcd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+5abcd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{b^2c^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+b^2c^{\frac{3}{2}}dx\sqrt{1+\frac{dx^2}{c}}-\frac{7b^2c^{\frac{3}{2}}dx}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2\sqrt{c}\,d^2x^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{15b^2c^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8}+\frac{b^2d^3x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**6,x)
```

```
[Out] -a**2*sqrt(c)*d**2/(x*sqrt(1 + d*x**2/c)) - a**2*c**2*sqrt(d)*sqrt(c/(d*x**
2) + 1)/(5*x**4) - 11*a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/(15*x**2) - 8*a*
*2*d**(5/2)*sqrt(c/(d*x**2) + 1)/15 + a**2*d**(5/2)*asinh(sqrt(d)*x/sqrt(c)
) - a**2*d**3*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - 4*a*b*c**(3/2)*d/(x*sqrt(1 +
 d*x**2/c)) + a*b*sqrt(c)*d**2*x*sqrt(1 + d*x**2/c) - 4*a*b*sqrt(c)*d**2*x/
sqrt(1 + d*x**2/c) - 2*a*b*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a
*b*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 5*a*b*c*d**(3/2)*asinh(sqrt(d)*x/sqr
t(c)) - b**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) + b**2*c**(3/2)*d*x*sqrt(1 + d
*x**2/c) - 7*b**2*c**(3/2)*d*x/(8*sqrt(1 + d*x**2/c)) + 3*b**2*sqrt(c)*d**2
*x**3/(8*sqrt(1 + d*x**2/c)) + 15*b**2*c**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)
)/8 + b**2*d**3*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(200) = 400$.
time = 0.57, size = 510, normalized size = 2.24

$$\frac{1}{8}\left(2b^2d^2x^2+\frac{9b^2cd^3+8abd^4}{d^2}\right)\sqrt{dx^2+c}\,x-\frac{1}{16}\left(15b^2c^2\sqrt{d}+40abcd^{\frac{3}{2}}+8a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2c^3\sqrt{d}+90\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8abc^2d^{\frac{3}{2}}+45\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8a^2cd^{\frac{5}{2}}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^2c^4\sqrt{d}-300\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6abc^3d^{\frac{3}{2}}-90\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6a^2c^2d^{\frac{5}{2}}+90\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^2c^5\sqrt{d}+400\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc^4d^{\frac{3}{2}}+140\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2c^3d^{\frac{5}{2}}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c^6\sqrt{d}-260\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc^5d^{\frac{3}{2}}-70\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2c^4d^{\frac{5}{2}}+15b^2c^7\sqrt{d}+70abc^6d^{\frac{3}{2}}+23a^2c^5d^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*d^2*x^2 + (9*b^2*c*d^3 + 8*a*b*d^4)/d^2)*sqrt(d*x^2 + c)*x - 1/1
6*(15*b^2*c^2*sqrt(d) + 40*a*b*c*d^(3/2) + 8*a^2*d^(5/2))*log((sqrt(d)*x -
sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^3*sqrt
(d) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^2*d^(3/2) + 45*(sqrt(d)*x -
sqrt(d*x^2 + c))^8*a^2*c*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c
^4*sqrt(d) - 300*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3*d^(3/2) - 90*(sqrt
(d)*x - sqrt(d*x^2 + c))^6*a^2*c^2*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c
))^4*b^2*c^5*sqrt(d) + 400*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^4*d^(3/2)
+ 140*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^3*d^(5/2) - 60*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*b^2*c^6*sqrt(d) - 260*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c
^5*d^(3/2) - 70*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^4*d^(5/2) + 15*b^2*c^
```

```
7*sqrt(d) + 70*a*b*c^6*d^(3/2) + 23*a^2*c^5*d^(5/2))/((sqrt(d)*x - sqrt(d*x
^2 + c))^2 - c)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^2\,(d\,x^2+c)^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x)
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6, x)
```

3.635 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=222

$$\frac{5d(8b^2c^2+ad(12bc+ad))\sqrt{c+dx^2}}{16c}+\frac{5d(8b^2c^2+ad(12bc+ad))\,(c+dx^2)^{3/2}}{48c^2}-\frac{(8b^2c^2+ad(12bc+ad))\,(c+}{16c^2x^2}$$

```
[Out] 5/48*d*(8*b^2*c^2+a*d*(a*d+12*b*c))*(d*x^2+c)^(3/2)/c^2-1/16*(8*b^2*c^2+a*d
*(a*d+12*b*c))*(d*x^2+c)^(5/2)/c^2/x^2-1/6*a^2*(d*x^2+c)^(7/2)/c/x^6-1/24*a
*(a*d+12*b*c)*(d*x^2+c)^(7/2)/c^2/x^4-5/16*d*(8*b^2*c^2+a*d*(a*d+12*b*c))*a
rctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+5/16*d*(8*b^2*c^2+a*d*(a*d+12*b*c))
*(d*x^2+c)^(1/2)/c
```

Rubi [A]
time = 0.17, antiderivative size = 219, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 91, 79, 43, 52, 65, 214}

$$-\frac{a^2(c+dx^2)^{7/2}}{6cx^6}-\frac{(c+dx^2)^{5/2}\left(\frac{ad(ad+12bc)}{c^2}+8b^2\right)}{16x^2}+\frac{5d(c+dx^2)^{3/2}(ad(ad+12bc)+8b^2c^2)}{48c^2}+\frac{5d\sqrt{c+dx^2}\,(ad(ad+12bc)+8b^2c^2)}{16c}-\frac{5d(ad(ad+12bc)+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16\sqrt{c}}-\frac{a(c+dx^2)^{7/2}(ad+12bc)}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

```
[Out] (5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))*Sqrt[c + d*x^2])/(16*c) + (5*d*(8*b^2
*c^2 + a*d*(12*b*c + a*d))*(c + d*x^2)^(3/2))/(48*c^2) - ((8*b^2 + (a*d*(12
*b*c + a*d))/c^2)*(c + d*x^2)^(5/2))/(16*x^2) - (a^2*(c + d*x^2)^(7/2))/(6*
c*x^6) - (a*(12*b*c + a*d)*(c + d*x^2)^(7/2))/(24*c^2*x^4) - (5*d*(8*b^2*c^
2 + a*d*(12*b*c + a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*Sqrt[c])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^7} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x^4} dx, x, x^2\right)$$
$$= -\frac{a^2(c+dx^2)^{7/2}}{6cx^6} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(12bc+ad)+3b^2cx\right)(c+dx)^{5/2}}{x^3} dx, x, x^2\right)}{6c}$$
$$= -\frac{a^2(c+dx^2)^{7/2}}{6cx^6} - \frac{a(12bc+ad)(c+dx^2)^{7/2}}{24c^2x^4} + \frac{1}{16}\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right) \text{S}$$
$$= -\frac{\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+dx^2)^{5/2}}{16x^2} - \frac{a^2(c+dx^2)^{7/2}}{6cx^6} - \frac{a(12bc+ad)(c+dx^2)}{24c^2x^4}$$
$$= \frac{5}{48}d\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+dx^2)^{3/2} - \frac{\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+dx^2)^{5/2}}{16x^2} -$$
$$= \frac{5}{16}cd\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{5}{48}d\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+$$
$$= \frac{5}{16}cd\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{5}{48}d\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+$$
$$= \frac{5}{16}cd\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{5}{48}d\left(8b^2 + \frac{ad(12bc+ad)}{c^2}\right)(c+$$

Mathematica [A]
time = 0.25, size = 152, normalized size = 0.68

$$-\frac{\sqrt{c+dx^2}\,(12abx^2(2c^2+9cdx^2-8d^2x^4)-8b^2x^4(-3c^2+14cdx^2+2d^2x^4)+a^2(8c^2+26cdx^2+33d^2x^4))}{48x^6} - \frac{5d(8b^2c^2+12abcd+a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

```
[Out] -1/48*(Sqrt[c + d*x^2]*(12*a*b*x^2*(2*c^2 + 9*c*d*x^2 - 8*d^2*x^4) - 8*b^2*
x^4*(-3*c^2 + 14*c*d*x^2 + 2*d^2*x^4) + a^2*(8*c^2 + 26*c*d*x^2 + 33*d^2*x^
4)))/x^6 - (5*d*(8*b^2*c^2 + 12*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/
Sqrt[c]])/(16*Sqrt[c])
```

Maple [A]
time = 0.12, size = 356, normalized size = 1.60

method	result

risch	$-\frac{\sqrt{d x^2+c}\,(33a^2d^2x^4+108abcd\,x^4+24b^2c^2x^4+26a^2cd\,x^2+24ab\,c^2x^2+8a^2c^2)}{48x^6}+\frac{b^2d^2x^2\sqrt{d x^2+c}}{3}+\frac{7d\,b^2c\sqrt{d x^2+}}{3}$
default	$2ab\left(-\frac{(d x^2+c)^{\frac{7}{2}}}{4c\,x^4}+\frac{3d\left(-\frac{(d x^2+c)^{\frac{7}{2}}}{2c\,x^2}+\frac{5d\left(\frac{(d x^2+c)^{\frac{5}{2}}}{5}+c\left(\frac{(d x^2+c)^{\frac{3}{2}}}{3}+c\left(\sqrt{d x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^2+c}}{x}\right)\right.\right.\right.}{2c}\right.}{4c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x,method=_RETURNVERBOSE)

[Out] 2*a*b*(-1/4/c/x^4*(d*x^2+c)^(7/2)+3/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(7/2)+5/2*d/c*(1/5*(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))))+a^2*(-1/6/c/x^6*(d*x^2+c)^(7/2)+1/6*d/c*(-1/4/c/x^4*(d*x^2+c)^(7/2)+3/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(7/2)+5/2*d/c*(1/5*(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))))))+b^2*(-1/2/c/x^2*(d*x^2+c)^(7/2)+5/2*d/c*(1/5*(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))))

Maxima [A]
time = 0.31, size = 353, normalized size = 1.59

$$-\frac{5}{2}\,b^2c^{\frac{3}{2}}d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{15}{4}\,ab\sqrt{c}\,d^2\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{5\,a^2d^3\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16\,\sqrt{c}}+\frac{5}{6}\,(dx^2+c)^{\frac{3}{2}}b^2d+\frac{(dx^2+c)^{\frac{5}{2}}b^2d}{2\,c}+\frac{5}{2}\,\sqrt{dx^2+c}\,b^2cd+\frac{15}{4}\,\sqrt{dx^2+c}\,abd^2+\frac{3\,(dx^2+c)^{\frac{5}{2}}abd^2}{4\,c^2}+\frac{5\,(dx^2+c)^{\frac{3}{2}}abd^2}{4\,c}+\frac{(dx^2+c)^{\frac{5}{2}}a^2d^3}{16\,c^3}+\frac{5\,(dx^2+c)^{\frac{3}{2}}a^2d^3}{48\,c^2}+\frac{5\,\sqrt{dx^2+c}\,a^2d^3}{16\,c}-\frac{(dx^2+c)^{\frac{7}{2}}b^2}{2\,cx^2}-\frac{3\,(dx^2+c)^{\frac{7}{2}}abd}{4\,c^2x^2}-\frac{(dx^2+c)^{\frac{7}{2}}a^2d^2}{16\,c^3x^2}-\frac{(dx^2+c)^{\frac{7}{2}}ab}{2\,cx^4}-\frac{(dx^2+c)^{\frac{7}{2}}a^2d}{24\,c^2x^4}-\frac{(dx^2+c)^{\frac{7}{2}}a^2}{6\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="maxima")

[Out] -5/2*b^2*c^(3/2)*d*arcsinh(c/(sqrt(c*d)*abs(x))) - 15/4*a*b*sqrt(c)*d^2*arcsinh(c/(sqrt(c*d)*abs(x))) - 5/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 5/6*(d*x^2 + c)^(3/2)*b^2*d + 1/2*(d*x^2 + c)^(5/2)*b^2*d/c + 5/2*sq

```
rt(d*x^2 + c)*b^2*c*d + 15/4*sqrt(d*x^2 + c)*a*b*d^2 + 3/4*(d*x^2 + c)^(5/2
)*a*b*d^2/c^2 + 5/4*(d*x^2 + c)^(3/2)*a*b*d^2/c + 1/16*(d*x^2 + c)^(5/2)*a^
2*d^3/c^3 + 5/48*(d*x^2 + c)^(3/2)*a^2*d^3/c^2 + 5/16*sqrt(d*x^2 + c)*a^2*d
^3/c - 1/2*(d*x^2 + c)^(7/2)*b^2/(c*x^2) - 3/4*(d*x^2 + c)^(7/2)*a*b*d/(c^2
*x^2) - 1/16*(d*x^2 + c)^(7/2)*a^2*d^2/(c^3*x^2) - 1/2*(d*x^2 + c)^(7/2)*a*
b/(c*x^4) - 1/24*(d*x^2 + c)^(7/2)*a^2*d/(c^2*x^4) - 1/6*(d*x^2 + c)^(7/2)*
a^2/(c*x^6)
```

Fricas [A]
time = 1.84, size = 347, normalized size = 1.56

$$\left[\frac{15\,(8\,b^2c^2d + 12\,abcd^2 + a^2d^3)\sqrt{c}\,x^6\log\left(-\frac{dx^2 - 2\,\sqrt{dx^2+c}\,\sqrt{c} + 2c}{x^2}\right) + 2\,(16\,b^2cd^2x^8 + 16\,(7\,b^2c^2d + 6\,abcd^2)x^6 - 8\,a^2c^3 - 3\,(8\,b^2c^3 + 36\,abc^2d + 11\,a^2cd^2)x^4 - 2\,(12\,abc^3 + 13\,a^2c^2d)x^2)\sqrt{dx^2+c}}{96\,cx^6}, \frac{15\,(8\,b^2c^2d + 12\,abcd^2 + a^2d^3)\sqrt{-c}\,x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right) + (16\,b^2cd^2x^8 + 16\,(7\,b^2c^2d + 6\,abcd^2)x^6 - 8\,a^2c^3 - 3\,(8\,b^2c^3 + 36\,abc^2d + 11\,a^2cd^2)x^4 - 2\,(12\,abc^3 + 13\,a^2c^2d)x^2)\sqrt{dx^2+c}}{48\,cx^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(16*b^2*c*d^2*x^8 + 16*(7*b^2*c^2
*d + 6*a*b*c*d^2)*x^6 - 8*a^2*c^3 - 3*(8*b^2*c^3 + 36*a*b*c^2*d + 11*a^2*c*
d^2)*x^4 - 2*(12*a*b*c^3 + 13*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c*x^6), 1/4
8*(15*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*sqrt(-c)*x^6*arctan(sqrt(-c)/s
qrt(d*x^2 + c)) + (16*b^2*c*d^2*x^8 + 16*(7*b^2*c^2*d + 6*a*b*c*d^2)*x^6 -
8*a^2*c^3 - 3*(8*b^2*c^3 + 36*a*b*c^2*d + 11*a^2*c*d^2)*x^4 - 2*(12*a*b*c^3
 + 13*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c*x^6)]
```

Sympy [A]
time = 117.70, size = 468, normalized size = 2.11

$$-\frac{a^2c^3}{6\sqrt{d}\,x^7\sqrt{\frac{c}{dx^2}+1}} - \frac{17a^2c^2\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{35a^2cd^{\frac{3}{2}}}{48x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{2x} - \frac{3a^2d^{\frac{5}{2}}}{16x\sqrt{\frac{c}{dx^2}+1}} - \frac{5a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{16\sqrt{c}} - \frac{15ab\sqrt{c}\,d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{4} - \frac{abc^3}{2\sqrt{d}\,x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{3abc^2\sqrt{d}}{4x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{2abcd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x} + \frac{7abcd^{\frac{3}{2}}}{4x\sqrt{\frac{c}{dx^2}+1}} + \frac{2abd^{\frac{5}{2}}x}{\sqrt{\frac{c}{dx^2}+1}} - \frac{5b^2c^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2} - \frac{b^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x} + \frac{2b^2c^2\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}} + \frac{2b^2cd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}} + b^2d^2\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\ d=0\\ \frac{(c+dx^2)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**7,x)
```

```
[Out] -a**2*c**3/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 17*a**2*c**2*sqrt(d)/(24
*x**5*sqrt(c/(d*x**2) + 1)) - 35*a**2*c*d**(3/2)/(48*x**3*sqrt(c/(d*x**2) +
 1)) - a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(2*x) - 3*a**2*d**(5/2)/(16*x*sqr
t(c/(d*x**2) + 1)) - 5*a**2*d**3*asinh(sqrt(c)/(sqrt(d)*x))/(16*sqrt(c)) -
15*a*b*sqrt(c)*d**2*asinh(sqrt(c)/(sqrt(d)*x))/4 - a*b*c**3/(2*sqrt(d)*x**5
*sqrt(c/(d*x**2) + 1)) - 3*a*b*c**2*sqrt(d)/(4*x**3*sqrt(c/(d*x**2) + 1)) -
 2*a*b*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/x + 7*a*b*c*d**(3/2)/(4*x*sqrt(c/(d*
x**2) + 1)) + 2*a*b*d**(5/2)*x/sqrt(c/(d*x**2) + 1) - 5*b**2*c**(3/2)*d*asi
nh(sqrt(c)/(sqrt(d)*x))/2 - b**2*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) +
2*b**2*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*b**2*c*d**(3/2)*x/sqrt(c/(
d*x**2) + 1) + b**2*d**2*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2
)**(3/2)/(3*d), True))
```

Giac [A]
time = 0.57, size = 286, normalized size = 1.29

$$\frac{16\,(dx^2+c)^{\frac{3}{2}}b^2d^2+96\,\sqrt{dx^2+c}\,b^2cd^2+96\,\sqrt{dx^2+c}\,abd^3+\frac{15\,(8\,b^2c^2d^2+12\,abcd^3+a^2d^4)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{24\,(dx^2+c)^{\frac{5}{2}}b^2c^2d^2-48\,(dx^2+c)^{\frac{3}{2}}b^2c^3d^2+24\,\sqrt{dx^2+c}\,b^2c^4d^2+108\,(dx^2+c)^{\frac{5}{2}}abcd^3-192\,(dx^2+c)^{\frac{3}{2}}abc^2d^3+84\,\sqrt{dx^2+c}\,abc^3d^3+33\,(dx^2+c)^{\frac{5}{2}}a^2d^4-40\,(dx^2+c)^{\frac{3}{2}}a^2cd^4+15\,\sqrt{dx^2+c}\,a^2c^2d^4}{d^3x^6}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(16*(d*x^2 + c)^(3/2)*b^2*d^2 + 96*sqrt(d*x^2 + c)*b^2*c*d^2 + 96*sqrt
(d*x^2 + c)*a*b*d^3 + 15*(8*b^2*c^2*d^2 + 12*a*b*c*d^3 + a^2*d^4)*arctan(sq
rt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(
d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 + 108*(d*x^2
+ c)^(5/2)*a*b*c*d^3 - 192*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 + 84*sqrt(d*x^2 +
c)*a*b*c^3*d^3 + 33*(d*x^2 + c)^(5/2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*a^2*c*
d^4 + 15*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(d^3*x^6))/d
```

Mupad [B]
time = 1.31, size = 301, normalized size = 1.36

$$\frac{\sqrt{d\,x^2+c}\,\left(\frac{5\,a^2\,c^2\,d^3}{16}+\frac{7\,a\,b\,c^3\,d^2}{4}+\frac{b^2\,c^4\,d}{2}\right)-{\left(d\,x^2+c\right)}^{3/2}\,\left(\frac{5\,a^2\,c\,d^3}{6}+4\,a\,b\,c^2\,d^2+b^2\,c^3\,d\right)+{\left(d\,x^2+c\right)}^{5/2}\,\left(\frac{11\,a^2\,d^3}{16}+\frac{9\,a\,b\,c\,d^2}{4}+\frac{b^2\,c^2\,d}{2}\right)}{3\,c\,{\left(d\,x^2+c\right)}^2-3\,c^2\,\left(d\,x^2+c\right)-{\left(d\,x^2+c\right)}^3+c^3}+\left(2\,b\,d\,\left(a\,d-b\,c\right)+4\,b^2\,c\,d\right)\,\sqrt{d\,x^2+c}+\frac{b^2\,d\,{\left(d\,x^2+c\right)}^{3/2}}{3}+\frac{d\,\mathrm{atan}\left(\frac{d\,\sqrt{d\,x^2+c}\,\left(a^2\,d^2+12\,a\,b\,c\,d+8\,b^2\,c^2\right)\,5{}\mathrm{i}}{8\,\sqrt{c}\,\left(\frac{5\,a^2\,d^3}{8}+\frac{15\,a\,b\,c\,d^2}{2}+5\,b^2\,c^2\,d\right)}\right)\,\left(a^2\,d^2+12\,a\,b\,c\,d+8\,b^2\,c^2\right)\,5{}\mathrm{i}}{16\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((b^2*c^4*d)/2 + (5*a^2*c^2*d^3)/16 + (7*a*b*c^3*d^2)/4)
 - (c + d*x^2)^(3/2)*((5*a^2*c*d^3)/6 + b^2*c^3*d + 4*a*b*c^2*d^2) + (c + d
*x^2)^(5/2)*((11*a^2*d^3)/16 + (b^2*c^2*d)/2 + (9*a*b*c*d^2)/4))/(3*c*(c +
d*x^2)^2 - 3*c^2*(c + d*x^2) - (c + d*x^2)^3 + c^3) + (2*b*d*(a*d - b*c) +
4*b^2*c*d)*(c + d*x^2)^(1/2) + (b^2*d*(c + d*x^2)^(3/2))/3 + (d*atan((d*(c
+ d*x^2)^(1/2)*(a^2*d^2 + 8*b^2*c^2 + 12*a*b*c*d)*5i)/(8*c^(1/2)*((5*a^2*d^
3)/8 + 5*b^2*c^2*d + (15*a*b*c*d^2)/2)))*(a^2*d^2 + 8*b^2*c^2 + 12*a*b*c*d)
*5i)/(16*c^(1/2))
```

3.636 $\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$-\frac{c(48a^2d^2+5bc(7bc-16ad))\,x\sqrt{c+dx^2}}{128d^4}+\frac{(48a^2d^2+5bc(7bc-16ad))\,x^3\sqrt{c+dx^2}}{192d^3}-\frac{b(7bc-16ad)x^5\sqrt{c+}}{48d^2}$$

```
[Out] 1/128*c^2*(48*a^2*d^2+5*b*c*(-16*a*d+7*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1
/2))/d^(9/2)-1/128*c*(48*a^2*d^2+5*b*c*(-16*a*d+7*b*c))*x*(d*x^2+c)^(1/2)/d
^4+1/192*(48*a^2*d^2+5*b*c*(-16*a*d+7*b*c))*x^3*(d*x^2+c)^(1/2)/d^3-1/48*b*
(-16*a*d+7*b*c)*x^5*(d*x^2+c)^(1/2)/d^2+1/8*b^2*x^7*(d*x^2+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {475, 470, 327, 223, 212}

$$\frac{c^2(48a^2d^2+5bc(7bc-16ad))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{9/2}}+\frac{x^3\sqrt{c+dx^2}\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)}{192d}-\frac{cx\sqrt{c+dx^2}\,(48a^2d^2+5bc(7bc-16ad))}{128d^4}-\frac{bx^5\sqrt{c+dx^2}\,(7bc-16ad)}{48d^2}+\frac{b^2x^7\sqrt{c+dx^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] -1/128*(c*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/d^4 + ((
48*a^2 + (5*b*c*(7*b*c - 16*a*d))/d^2)*x^3*Sqrt[c + d*x^2])/(192*d) - (b*(7
*b*c - 16*a*d)*x^5*Sqrt[c + d*x^2])/(48*d^2) + (b^2*x^7*Sqrt[c + d*x^2])/(8
*d) + (c^2*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(128*d^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{b^2x^7\sqrt{c+dx^2}}{8d} + \frac{\int \frac{x^4\left(8a^2d-b(7bc-16ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{8d}$$

$$= -\frac{b(7bc-16ad)x^5\sqrt{c+dx^2}}{48d^2} + \frac{b^2x^7\sqrt{c+dx^2}}{8d} - \frac{1}{48}\left(-48a^2 - \frac{5bc(7bc-16ad)}{d^2}\right)\int$$

$$= \frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d} - \frac{b(7bc-16ad)x^5\sqrt{c+dx^2}}{48d^2} + \frac{b^2x^7\sqrt{c+dx^2}}{8d} -$$

$$= -\frac{c\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d} - \frac{b(7bc-}{}$$

$$= -\frac{c\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d} - \frac{b(7bc-}{}$$

$$= -\frac{c\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d} - \frac{b(7bc-}{}$$

Mathematica [A]
time = 0.23, size = 158, normalized size = 0.81

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\,\left(48a^2d^2(-3c+2dx^2)+16abd(15c^2-10cdx^2+8d^2x^4)+b^2(-105c^3+70c^2dx^2-56cd^2x^4+48d^3x^6)\right)-3c^2(35b^2c^2-80abcd+48a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{384d^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]

[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(-3*c + 2*d*x^2) + 16*a*b*d*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4) + b^2*(-105*c^3 + 70*c^2*d*x^2 - 56*c*d^2*x^4 + 48*d^3*x^6)) - 3*c^2*(35*b^2*c^2 - 80*a*b*c*d + 48*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(384*d^(9/2))

Maple [A]
time = 0.10, size = 272, normalized size = 1.40

method	result
risch	$-\frac{x\left(-48b^2x^6d^3-128ab\,d^3x^4+56b^2c\,d^2x^4-96a^2d^3x^2+160abc\,d^2x^2-70b^2c^2d\,x^2+144a^2c\,d^2-240ab\,c^2d+105b^2c^3\right)\sqrt{d\,x^2+c}}{384d^4}+$
default	$b^2\left(\frac{x^7\sqrt{d\,x^2+c}}{8d}-\frac{7c\left(\frac{x^5\sqrt{d\,x^2+c}}{6d}-\frac{5c\left(\frac{x^3\sqrt{d\,x^2+c}}{4d}-\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2d}-\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2d^{\frac{3}{2}}}\right)}{4d}\right)}{6d}\right)}{8d}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] b^2*(1/8*x^7/d*(d*x^2+c)^(1/2)-7/8*c/d*(1/6*x^5/d*(d*x^2+c)^(1/2)-5/6*c/d*(1/4*x^3/d*(d*x^2+c)^(1/2)-3/4*c/d*(1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))))+2*a*b*(1/6*x^5/d*(d*x^2+c)^(1/2)-5/6*c/d*(1/4*x^3/d*(d*x^2+c)^(1/2)-3/4*c/d*(1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+a^2*(1/4*x^3/d*(d*x^2+c)^(1/2)-3/4*c/d*(1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))

Maxima [A]
time = 0.30, size = 243, normalized size = 1.25

$$\frac{\sqrt{dx^2+c}\,b^2x^7}{8\,d}-\frac{7\,\sqrt{dx^2+c}\,b^2cx^5}{48\,d^2}+\frac{\sqrt{dx^2+c}\,abx^5}{3\,d}+\frac{35\,\sqrt{dx^2+c}\,b^2c^2x^3}{192\,d^3}-\frac{5\,\sqrt{dx^2+c}\,abcx^3}{12\,d^2}+\frac{\sqrt{dx^2+c}\,a^2x^3}{4\,d}-\frac{35\,\sqrt{dx^2+c}\,b^2c^3x}{128\,d^4}+\frac{5\,\sqrt{dx^2+c}\,abc^2x}{8\,d^3}-\frac{3\,\sqrt{dx^2+c}\,a^2cx}{8\,d^2}+\frac{35\,b^2c^4\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{9}{2}}}-\frac{5\,abc^3\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{7}{2}}}+\frac{3\,a^2c^2\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*b^2*x^7/d - 7/48*sqrt(d*x^2 + c)*b^2*c*x^5/d^2 + 1/3*sq
rt(d*x^2 + c)*a*b*x^5/d + 35/192*sqrt(d*x^2 + c)*b^2*c^2*x^3/d^3 - 5/12*sqr
t(d*x^2 + c)*a*b*c*x^3/d^2 + 1/4*sqrt(d*x^2 + c)*a^2*x^3/d - 35/128*sqrt(d*
x^2 + c)*b^2*c^3*x/d^4 + 5/8*sqrt(d*x^2 + c)*a*b*c^2*x/d^3 - 3/8*sqrt(d*x^2
 + c)*a^2*c*x/d^2 + 35/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(9/2) - 5/8*a*b
*c^3*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 3/8*a^2*c^2*arcsinh(d*x/sqrt(c*d))/d^
(5/2)
```

Fricas [A]
time = 1.15, size = 344, normalized size = 1.77

$$\left[\frac{3\,(35\,b^2c^4-80\,abc^3d+48\,a^2c^2d^2)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(48\,b^2d^4x^7-8\,(7\,b^2cd^3-16\,abd^4)x^5+2\,(35\,b^2c^2d^2-80\,abcd^3+48\,a^2d^4)x^3-3\,(35\,b^2c^3d-80\,abc^2d^2+48\,a^2cd^3)x)\sqrt{dx^2+c}}{768\,d^5},\ -\frac{3\,(35\,b^2c^4-80\,abc^3d+48\,a^2c^2d^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(48\,b^2d^4x^7-8\,(7\,b^2cd^3-16\,abd^4)x^5+2\,(35\,b^2c^2d^2-80\,abcd^3+48\,a^2d^4)x^3-3\,(35\,b^2c^3d-80\,abc^2d^2+48\,a^2cd^3)x)\sqrt{dx^2+c}}{384\,d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 - 8*(7*b^2*c*d^3 -
16*a*b*d^4)*x^5 + 2*(35*b^2*c^2*d^2 - 80*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3
5*b^2*c^3*d - 80*a*b*c^2*d^2 + 48*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^5, -1/38
4*(3*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*
x/sqrt(d*x^2 + c)) - (48*b^2*d^4*x^7 - 8*(7*b^2*c*d^3 - 16*a*b*d^4)*x^5 + 2
*(35*b^2*c^2*d^2 - 80*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 80*a*
b*c^2*d^2 + 48*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^5]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(187) = 374$.
time = 49.87, size = 422, normalized size = 2.18

$$-\frac{3a^2c^{\frac{3}{2}}x}{8d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2\sqrt{c}\,x^3}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2c^2\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{5}{2}}}+\frac{a^2x^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{5abc^{\frac{5}{2}}x}{8d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{5abc^{\frac{3}{2}}x^3}{24d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{ab\sqrt{c}\,x^5}{12d\sqrt{1+\frac{dx^2}{c}}}-\frac{5abc^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{7}{2}}}+\frac{abx^7}{3\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{35b^2c^{\frac{7}{2}}x}{128d^4\sqrt{1+\frac{dx^2}{c}}}-\frac{35b^2c^{\frac{5}{2}}x^3}{384d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{7b^2c^{\frac{3}{2}}x^5}{192d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2\sqrt{c}\,x^7}{48d\sqrt{1+\frac{dx^2}{c}}}+\frac{35b^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{9}{2}}}+\frac{b^2x^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -3*a**2*c**(3/2)*x/(8*d**2*sqrt(1 + d*x**2/c)) - a**2*sqrt(c)*x**3/(8*d*sqr
t(1 + d*x**2/c)) + 3*a**2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + a**2
*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*a*b*c**(5/2)*x/(8*d**3*sqrt(1 + d*
```

```
x**2/c)) + 5*a*b*c**(3/2)*x**3/(24*d**2*sqrt(1 + d*x**2/c)) - a*b*sqrt(c)*x
**5/(12*d*sqrt(1 + d*x**2/c)) - 5*a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*d**(
7/2)) + a*b*x**7/(3*sqrt(c)*sqrt(1 + d*x**2/c)) - 35*b**2*c**(7/2)*x/(128*d
**4*sqrt(1 + d*x**2/c)) - 35*b**2*c**(5/2)*x**3/(384*d**3*sqrt(1 + d*x**2/c
)) + 7*b**2*c**(3/2)*x**5/(192*d**2*sqrt(1 + d*x**2/c)) - b**2*sqrt(c)*x**7
/(48*d*sqrt(1 + d*x**2/c)) + 35*b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**
(9/2)) + b**2*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A]
time = 0.54, size = 178, normalized size = 0.92

$$\frac{1}{384} \left(2 \left(4 \left(\frac{6 b^2 x^2}{d} - \frac{7 b^2 c d^5 - 16 a b d^6}{d^7}\right) x^2 + \frac{35 b^2 c^2 d^4 - 80 a b c d^5 + 48 a^2 d^6}{d^7}\right) x^2 - \frac{3 (35 b^2 c^3 d^3 - 80 a b c^2 d^4 + 48 a^2 c d^5)}{d^7}\right) \sqrt{d x^2 + c}\, x - \frac{(35 b^2 c^4 - 80 a b c^3 d + 48 a^2 c^2 d^2) \log \left(\left|-\sqrt{d} x + \sqrt{d x^2 + c}\right|\right)}{128 d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*x^2/d - (7*b^2*c*d^5 - 16*a*b*d^6)/d^7)*x^2 + (35*b^2*c^
2*d^4 - 80*a*b*c*d^5 + 48*a^2*d^6)/d^7)*x^2 - 3*(35*b^2*c^3*d^3 - 80*a*b*c^
2*d^4 + 48*a^2*c*d^5)/d^7)*sqrt(d*x^2 + c)*x - 1/128*(35*b^2*c^4 - 80*a*b*c
^3*d + 48*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 \left(b x^2 + a\right)^2}{\sqrt{d x^2 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.637 $\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=112

$$-\frac{c(bc-ad)^2\sqrt{c+dx^2}}{d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{3/2}}{3d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^4}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^4}$$

```
[Out] 1/3*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2+c)^(3/2)/d^4-1/5*b*(-2*a*d+3*b*c)*(d*x^2
+c)^(5/2)/d^4+1/7*b^2*(d*x^2+c)^(7/2)/d^4-c*(-a*d+b*c)^2*(d*x^2+c)^(1/2)/d^
4
```

Rubi [A]
time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {457, 78}

$$-\frac{b\left(c+dx^2\right)^{5/2}(3bc-2ad)}{5d^4}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)(3bc-ad)}{3d^4}-\frac{c\sqrt{c+dx^2}\,(bc-ad)^2}{d^4}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] -((c*(b*c - a*d)^2*Sqrt[c + d*x^2])/d^4) + ((b*c - a*d)*(3*b*c - a*d)*(c +
d*x^2)^(3/2))/(3*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(5/2))/(5*d^4) + (b^
2*(c + d*x^2)^(7/2))/(7*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^3\sqrt{c+dx}} + \frac{(bc-ad)(3bc-ad)\sqrt{c+dx}}{d^3} - \frac{b(3bc-2ad)(c+dx)^{3/2}}{d^3}\right.\right.$$
$$= -\frac{c(bc-ad)^2\sqrt{c+dx^2}}{d^4} + \frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{3/2}}{3d^4} - \frac{b(3bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^4}$$

Mathematica [A]
time = 0.07, size = 99, normalized size = 0.88

$$\frac{\sqrt{c+dx^2}\,(35a^2d^2(-2c+dx^2)+14abd(8c^2-4cdx^2+3d^2x^4)-3b^2(16c^3-8c^2dx^2+6cd^2x^4-5d^3x^6))}{105d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(35*a^2*d^2*(-2*c + d*x^2) + 14*a*b*d*(8*c^2 - 4*c*d*x^2 +
 3*d^2*x^4) - 3*b^2*(16*c^3 - 8*c^2*d*x^2 + 6*c*d^2*x^4 - 5*d^3*x^6)))/(105
*d^4)
```

Maple [A]
time = 0.09, size = 185, normalized size = 1.65

method	result
gosper	$-\frac{\sqrt{d\,x^2+c}\,\left(-15b^2x^6d^3-42ab\,d^3x^4+18b^2c\,d^2x^4-35a^2d^3x^2+56abc\,d^2x^2-24b^2c^2d\,x^2+70a^2c\,d^2-112ab\,c^2d+48b^2c^3\right)}{105d^4}$
trager	$-\frac{\sqrt{d\,x^2+c}\,\left(-15b^2x^6d^3-42ab\,d^3x^4+18b^2c\,d^2x^4-35a^2d^3x^2+56abc\,d^2x^2-24b^2c^2d\,x^2+70a^2c\,d^2-112ab\,c^2d+48b^2c^3\right)}{105d^4}$
risch	$-\frac{\sqrt{d\,x^2+c}\,\left(-15b^2x^6d^3-42ab\,d^3x^4+18b^2c\,d^2x^4-35a^2d^3x^2+56abc\,d^2x^2-24b^2c^2d\,x^2+70a^2c\,d^2-112ab\,c^2d+48b^2c^3\right)}{105d^4}$
default	$b^2\left(\frac{x^6\sqrt{d\,x^2+c}}{7d}-\frac{6c\left(\frac{x^4\sqrt{d\,x^2+c}}{5d}-\frac{4c\left(\frac{x^2\sqrt{d\,x^2+c}}{3d}-\frac{2c\sqrt{d\,x^2+c}}{3d^2}\right)}{5d}\right)}{7d}\right)+2ab\left(\frac{x^4\sqrt{d\,x^2+c}}{5d}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/7*x^6/d*(d*x^2+c)^(1/2)-6/7*c/d*(1/5*x^4/d*(d*x^2+c)^(1/2)-4/5*c/d*(
1/3*x^2/d*(d*x^2+c)^(1/2)-2/3*c/d^2*(d*x^2+c)^(1/2))))+2*a*b*(1/5*x^4/d*(d*
x^2+c)^(1/2)-4/5*c/d*(1/3*x^2/d*(d*x^2+c)^(1/2)-2/3*c/d^2*(d*x^2+c)^(1/2)))
+a^2*(1/3*x^2/d*(d*x^2+c)^(1/2)-2/3*c/d^2*(d*x^2+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 181, normalized size = 1.62

$$\frac{\sqrt{dx^2+c}\,b^2x^6}{7\,d}-\frac{6\,\sqrt{dx^2+c}\,b^2cx^4}{35\,d^2}+\frac{2\,\sqrt{dx^2+c}\,abx^4}{5\,d}+\frac{8\,\sqrt{dx^2+c}\,b^2c^2x^2}{35\,d^3}-\frac{8\,\sqrt{dx^2+c}\,abcx^2}{15\,d^2}+\frac{\sqrt{dx^2+c}\,a^2x^2}{3\,d}-\frac{16\,\sqrt{dx^2+c}\,b^2c^3}{35\,d^4}+\frac{16\,\sqrt{dx^2+c}\,abc^2}{15\,d^3}-\frac{2\,\sqrt{dx^2+c}\,a^2c}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(d*x^2 + c)*b^2*x^6/d - 6/35*sqrt(d*x^2 + c)*b^2*c*x^4/d^2 + 2/5*sq
rt(d*x^2 + c)*a*b*x^4/d + 8/35*sqrt(d*x^2 + c)*b^2*c^2*x^2/d^3 - 8/15*sqrt(
d*x^2 + c)*a*b*c*x^2/d^2 + 1/3*sqrt(d*x^2 + c)*a^2*x^2/d - 16/35*sqrt(d*x^2
 + c)*b^2*c^3/d^4 + 16/15*sqrt(d*x^2 + c)*a*b*c^2/d^3 - 2/3*sqrt(d*x^2 + c)
*a^2*c/d^2
```

Fricas [A]
time = 1.31, size = 103, normalized size = 0.92

$$\frac{\left(15\,b^2d^3x^6-48\,b^2c^3+112\,abc^2d-70\,a^2cd^2-6\,(3\,b^2cd^2-7\,abd^3)x^4+(24\,b^2c^2d-56\,abcd^2+35\,a^2d^3)x^2\right)\sqrt{dx^2+c}}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^6 - 48*b^2*c^3 + 112*a*b*c^2*d - 70*a^2*c*d^2 - 6*(3*b^
2*c*d^2 - 7*a*b*d^3)*x^4 + (24*b^2*c^2*d - 56*a*b*c*d^2 + 35*a^2*d^3)*x^2)*
sqrt(d*x^2 + c)/d^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(100) = 200$.
time = 0.38, size = 240, normalized size = 2.14

$$\begin{cases} -\frac{2a^2c\sqrt{c+dx^2}}{3d^2}+\frac{a^2x^2\sqrt{c+dx^2}}{3d}+\frac{16abc^2\sqrt{c+dx^2}}{15d^3}-\frac{8abcx^2\sqrt{c+dx^2}}{15d^2}+\frac{2abx^4\sqrt{c+dx^2}}{5d}-\frac{16b^2c^3\sqrt{c+dx^2}}{35d^4}+\frac{8b^2c^2x^2\sqrt{c+dx^2}}{35d^3}-\frac{6b^2cx^4\sqrt{c+dx^2}}{35d^2}+\frac{b^2x^6\sqrt{c+dx^2}}{7d} & \text{for } d\neq 0 \\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*c*sqrt(c + d*x**2)/(3*d**2) + a**2*x**2*sqrt(c + d*x**2)
/(3*d) + 16*a*b*c**2*sqrt(c + d*x**2)/(15*d**3) - 8*a*b*c*x**2*sqrt(c + d*x
**2)/(15*d**2) + 2*a*b*x**4*sqrt(c + d*x**2)/(5*d) - 16*b**2*c**3*sqrt(c +
d*x**2)/(35*d**4) + 8*b**2*c**2*x**2*sqrt(c + d*x**2)/(35*d**3) - 6*b**2*c*
```

```
x**4*sqrt(c + d*x**2)/(35*d**2) + b**2*x**6*sqrt(c + d*x**2)/(7*d), Ne(d, 0
)), ((a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8)/sqrt(c), True))
```

Giac [A]
time = 0.59, size = 137, normalized size = 1.22

$$-\frac{(b^2c^3 - 2\,abc^2d + a^2cd^2)\sqrt{dx^2+c}}{d^4} + \frac{15\,(dx^2+c)^{\frac{7}{2}}b^2 - 63\,(dx^2+c)^{\frac{5}{2}}b^2c + 105\,(dx^2+c)^{\frac{3}{2}}b^2c^2 + 42\,(dx^2+c)^{\frac{5}{2}}abd - 140\,(dx^2+c)^{\frac{3}{2}}abcd + 35\,(dx^2+c)^{\frac{3}{2}}a^2d^2}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(d*x^2 + c)/d^4 + 1/105*(15*(d*x^2
 + c)^(7/2)*b^2 - 63*(d*x^2 + c)^(5/2)*b^2*c + 105*(d*x^2 + c)^(3/2)*b^2*c^
2 + 42*(d*x^2 + c)^(5/2)*a*b*d - 140*(d*x^2 + c)^(3/2)*a*b*c*d + 35*(d*x^2
+ c)^(3/2)*a^2*d^2)/d^4
```

Mupad [B]
time = 0.36, size = 105, normalized size = 0.94

$$\sqrt{d\,x^2+c}\,\left(\frac{b^2\,x^6}{7\,d} - \frac{70\,a^2\,c\,d^2 - 112\,a\,b\,c^2\,d + 48\,b^2\,c^3}{105\,d^4} + \frac{x^2\,(35\,a^2\,d^3 - 56\,a\,b\,c\,d^2 + 24\,b^2\,c^2\,d)}{105\,d^4} + \frac{2\,b\,x^4\,(7\,a\,d - 3\,b\,c)}{35\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((b^2*x^6)/(7*d) - (48*b^2*c^3 + 70*a^2*c*d^2 - 112*a*b*c
^2*d)/(105*d^4) + (x^2*(35*a^2*d^3 + 24*b^2*c^2*d - 56*a*b*c*d^2))/(105*d^4
) + (2*b*x^4*(7*a*d - 3*b*c))/(35*d^2))
```

3.638 $\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=146

$$\frac{(8a^2d^2+bc(5bc-12ad))\,x\sqrt{c+dx^2}}{16d^3}-\frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}-\frac{c(8a^2d^2+bc(5bc-12$$

```
[Out] -1/16*c*(8*a^2*d^2+b*c*(-12*a*d+5*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/
d^(7/2)+1/16*(8*a^2*d^2+b*c*(-12*a*d+5*b*c))*x*(d*x^2+c)^(1/2)/d^3-1/24*b*(
-12*a*d+5*b*c)*x^3*(d*x^2+c)^(1/2)/d^2+1/6*b^2*x^5*(d*x^2+c)^(1/2)/d
```

Rubi [A]

time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {475, 470, 327, 223, 212}

$$\frac{x\sqrt{c+dx^2}\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)}{16d}-\frac{c(8a^2d^2+bc(5bc-12ad))\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{7/2}}-\frac{bx^3\sqrt{c+dx^2}\,(5bc-12ad)}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((8*a^2 + (b*c*(5*b*c - 12*a*d))/d^2)*x*Sqrt[c + d*x^2])/(16*d) - (b*(5*b*c
 - 12*a*d)*x^3*Sqrt[c + d*x^2])/(24*d^2) + (b^2*x^5*Sqrt[c + d*x^2])/(6*d)
- (c*(8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]
])/(16*d^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx &= \frac{b^2x^5\sqrt{c+dx^2}}{6d} + \frac{\int \frac{x^2(6a^2d-b(5bc-12ad)x^2)}{\sqrt{c+dx^2}}\,dx}{6d}\\
&= -\frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2} + \frac{b^2x^5\sqrt{c+dx^2}}{6d} + \frac{1}{8}\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)\int \frac{x^2}{\sqrt{c+d}}\\
&= \frac{\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)x\sqrt{c+dx^2}}{16d} - \frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2} + \frac{b^2x^5\sqrt{c+dx^2}}{6d} - \frac{(c(5}{}\\
&= \frac{\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)x\sqrt{c+dx^2}}{16d} - \frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2} + \frac{b^2x^5\sqrt{c+dx^2}}{6d} - \frac{(c(5}{}\\
&= \frac{\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)x\sqrt{c+dx^2}}{16d} - \frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2} + \frac{b^2x^5\sqrt{c+dx^2}}{6d} - \frac{c(5}{}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 124, normalized size = 0.85

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\,(24a^2d^2+12abd(-3c+2dx^2)+b^2(15c^2-10cdx^2+8d^2x^4))+3c(5b^2c^2-12abcd+8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{48d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(-3*c + 2*d*x^2) + b^2*(1
5*c^2 - 10*c*d*x^2 + 8*d^2*x^4)) + 3*c*(5*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)
*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(48*d^(7/2))
```

Maple [A]
time = 0.09, size = 200, normalized size = 1.37

method	result
risch	$\frac{x\left(8b^2x^4d^2+24ab\,d^2x^2-10b^2cd\,x^2+24a^2d^2-36abcd+15b^2c^2\right)\sqrt{d\,x^2+c}}{48d^3}-\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)a^2}{2d^{\frac{3}{2}}}+\frac{3c^2\ln\left(x\sqrt{d}\right.}{}$
default	$b^2\left(\frac{x^5\sqrt{d\,x^2+c}}{6d}-\frac{5c\left(\frac{x^3\sqrt{d\,x^2+c}}{4d}-\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2d}-\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2d^{\frac{3}{2}}}\right)}{4d}\right)}{6d}\right)+2ab\left(\frac{x^3\sqrt{d\,x}}{4d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/6*x^5/d*(d*x^2+c)^(1/2)-5/6*c/d*(1/4*x^3/d*(d*x^2+c)^(1/2)-3/4*c/d*(
1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+2*a*
b*(1/4*x^3/d*(d*x^2+c)^(1/2)-3/4*c/d*(1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(3/2)
*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))+a^2*(1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(3/2)
*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))
```

Maxima [A]
time = 0.30, size = 175, normalized size = 1.20

$$\frac{\sqrt{dx^2+c}\,b^2x^5}{6\,d}-\frac{5\sqrt{dx^2+c}\,b^2cx^3}{24\,d^2}+\frac{\sqrt{dx^2+c}\,abx^3}{2\,d}+\frac{5\sqrt{dx^2+c}\,b^2c^2x}{16\,d^3}-\frac{3\sqrt{dx^2+c}\,abcx}{4\,d^2}+\frac{\sqrt{dx^2+c}\,a^2x}{2\,d}-\frac{5\,b^2c^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{7}{2}}}+\frac{3\,abc^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,d^{\frac{5}{2}}}-\frac{a^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(d*x^2 + c)*b^2*x^5/d - 5/24*sqrt(d*x^2 + c)*b^2*c*x^3/d^2 + 1/2*sq
rt(d*x^2 + c)*a*b*x^3/d + 5/16*sqrt(d*x^2 + c)*b^2*c^2*x/d^3 - 3/4*sqrt(d*x
^2 + c)*a*b*c*x/d^2 + 1/2*sqrt(d*x^2 + c)*a^2*x/d - 5/16*b^2*c^3*arcsinh(d*
x/sqrt(c*d))/d^(7/2) + 3/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/2*a^2
*c*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

Fricas [A]
time = 2.41, size = 267, normalized size = 1.83

$$\left[\frac{3\,(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2)\sqrt{d}\,\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(8\,b^2d^3x^5-2\,(5\,b^2cd^2-12\,abd^3)x^3+3\,(5\,b^2c^2d-12\,abcd^2+8\,a^2d^3)x)\sqrt{dx^2+c}}{96\,d^4},\ \frac{3\,(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(8\,b^2d^3x^5-2\,(5\,b^2cd^2-12\,abd^3)x^3+3\,(5\,b^2c^2d-12\,abcd^2+8\,a^2d^3)x)\sqrt{dx^2+c}}{48\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*log(-2*d*x^2 + 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^5 - 2*(5*b^2*c*d^2 - 12*a*b
*d^3)*x^3 + 3*(5*b^2*c^2*d - 12*a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/
d^4, 1/48*(3*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^3*x^5 - 2*(5*b^2*c*d^2 - 12*a*b*d^3)*x^3
+ 3*(5*b^2*c^2*d - 12*a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(138) = 276$.
time = 12.53, size = 301, normalized size = 2.06

$$\frac{a^2\sqrt{c}\,x\sqrt{1+\frac{dx^2}{c}}}{2d}-\frac{a^2c\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}}-\frac{3abc^{\frac{3}{2}}x}{4d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{ab\sqrt{c}\,x^3}{4d\sqrt{1+\frac{dx^2}{c}}}+\frac{3abc^2\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{4d^{\frac{5}{2}}}+\frac{abx^5}{2\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{5}{2}}x}{16d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{3}{2}}x^3}{48d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2\sqrt{c}\,x^5}{24d\sqrt{1+\frac{dx^2}{c}}}-\frac{5b^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16d^{\frac{7}{2}}}+\frac{b^2x^7}{6\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/(2*d) - a**2*c*asinh(sqrt(d)*x/sqrt(c))/(
2*d**(3/2)) - 3*a*b*c**(3/2)*x/(4*d**2*sqrt(1 + d*x**2/c)) - a*b*sqrt(c)*x*
*3/(4*d*sqrt(1 + d*x**2/c)) + 3*a*b*c**2*asinh(sqrt(d)*x/sqrt(c))/(4*d**(5/
2)) + a*b*x**5/(2*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*b**2*c**(5/2)*x/(16*d**3*
sqrt(1 + d*x**2/c)) + 5*b**2*c**(3/2)*x**3/(48*d**2*sqrt(1 + d*x**2/c)) - b
**2*sqrt(c)*x**5/(24*d*sqrt(1 + d*x**2/c)) - 5*b**2*c**3*asinh(sqrt(d)*x/sq
rt(c))/(16*d**(7/2)) + b**2*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

Giac [A]
time = 0.77, size = 135, normalized size = 0.92

$$\frac{1}{48}\left(2\left(\frac{4\,b^2x^2}{d}-\frac{5\,b^2cd^3-12\,abd^4}{d^5}\right)x^2+\frac{3\,(5\,b^2c^2d^2-12\,abcd^3+8\,a^2d^4)}{d^5}\right)\sqrt{dx^2+c}\,x+\frac{(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*x^2/d - (5*b^2*c*d^3 - 12*a*b*d^4)/d^5)*x^2 + 3*(5*b^2*c^2*d
^2 - 12*a*b*c*d^3 + 8*a^2*d^4)/d^5)*sqrt(d*x^2 + c)*x + 1/16*(5*b^2*c^3 - 1
2*a*b*c^2*d + 8*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(b\,x^2+a\right)}^2}{\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
[Out] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.639 $\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{(bc-ad)^2\sqrt{c+dx^2}}{d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{3/2}}{3d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}$$

```
[Out] -2/3*b*(-a*d+b*c)*(d*x^2+c)^(3/2)/d^3+1/5*b^2*(d*x^2+c)^(5/2)/d^3+(-a*d+b*c
)^2*(d*x^2+c)^(1/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$-\frac{2b\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^3}+\frac{\sqrt{c+dx^2}\,(bc-ad)^2}{d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/d^3 - (2*b*(b*c - a*d)*(c + d*x^2)^(3/2))/(
3*d^3) + (b^2*(c + d*x^2)^(5/2))/(5*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2\sqrt{c+dx}} - \frac{2b(bc-ad)\sqrt{c+dx}}{d^2} + \frac{b^2(c+dx)^{3/2}}{d^2}\right) dx, x, x^2\right)$$
$$= \frac{(bc-ad)^2\sqrt{c+dx^2}}{d^3} - \frac{2b(bc-ad)\left(c+dx^2\right)^{3/2}}{3d^3} + \frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\,(15a^2d^2+10abd(-2c+dx^2)+b^2(8c^2-4cdx^2+3d^2x^4))}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x^2) + b^2*(8*c^2 - 4*c*d
*x^2 + 3*d^2*x^4)))/(15*d^3)
```

Maple [A]
time = 0.09, size = 116, normalized size = 1.57

method	result
gosper	$\frac{\sqrt{d\,x^2+c}\,(3b^2x^4d^2+10ab\,d^2x^2-4b^2cd\,x^2+15a^2d^2-20abcd+8b^2c^2)}{15d^3}$
trager	$\frac{\sqrt{d\,x^2+c}\,(3b^2x^4d^2+10ab\,d^2x^2-4b^2cd\,x^2+15a^2d^2-20abcd+8b^2c^2)}{15d^3}$
risch	$\frac{\sqrt{d\,x^2+c}\,(3b^2x^4d^2+10ab\,d^2x^2-4b^2cd\,x^2+15a^2d^2-20abcd+8b^2c^2)}{15d^3}$
default	$b^2\left(\frac{x^4\sqrt{d\,x^2+c}}{5d}-\frac{4c\left(\frac{x^2\sqrt{d\,x^2+c}}{3d}-\frac{2c\sqrt{d\,x^2+c}}{3d^2}\right)}{5d}\right)+2ab\left(\frac{x^2\sqrt{d\,x^2+c}}{3d}-\frac{2c\sqrt{d\,x^2+c}}{3d^2}\right)+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/5*x^4/d*(d*x^2+c)^(1/2)-4/5*c/d*(1/3*x^2/d*(d*x^2+c)^(1/2)-2/3*c/d^2
*(d*x^2+c)^(1/2)))+2*a*b*(1/3*x^2/d*(d*x^2+c)^(1/2)-2/3*c/d^2*(d*x^2+c)^(1/
2))+a^2/d*(d*x^2+c)^(1/2)
```

Maxima [A]
time = 0.30, size = 114, normalized size = 1.54

$$\frac{\sqrt{dx^2+c}\,b^2x^4}{5\,d}-\frac{4\,\sqrt{dx^2+c}\,b^2cx^2}{15\,d^2}+\frac{2\,\sqrt{dx^2+c}\,abx^2}{3\,d}+\frac{8\,\sqrt{dx^2+c}\,b^2c^2}{15\,d^3}-\frac{4\,\sqrt{dx^2+c}\,abc}{3\,d^2}+\frac{\sqrt{dx^2+c}\,a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(d*x^2 + c)*b^2*x^4/d - 4/15*sqrt(d*x^2 + c)*b^2*c*x^2/d^2 + 2/3*sq
rt(d*x^2 + c)*a*b*x^2/d + 8/15*sqrt(d*x^2 + c)*b^2*c^2/d^3 - 4/3*sqrt(d*x^2
 + c)*a*b*c/d^2 + sqrt(d*x^2 + c)*a^2/d
```

Fricas [A]
time = 1.83, size = 68, normalized size = 0.92

$$\frac{(3\,b^2d^2x^4+8\,b^2c^2-20\,abcd+15\,a^2d^2-2\,(2\,b^2cd-5\,abd^2)x^2)\sqrt{dx^2+c}}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*b^2*c*d -
5*a*b*d^2)*x^2)*sqrt(d*x^2 + c)/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(65) = 130$.
time = 0.30, size = 158, normalized size = 2.14

$$\begin{cases}\frac{a^2\sqrt{c+dx^2}}{d}-\frac{4abc\sqrt{c+dx^2}}{3d^2}+\frac{2abx^2\sqrt{c+dx^2}}{3d}+\frac{8b^2c^2\sqrt{c+dx^2}}{15d^3}-\frac{4b^2cx^2\sqrt{c+dx^2}}{15d^2}+\frac{b^2x^4\sqrt{c+dx^2}}{5d} & \text{for } d\neq 0\\ \frac{\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}}{\sqrt{c}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((a**2*sqrt(c + d*x**2)/d - 4*a*b*c*sqrt(c + d*x**2)/(3*d**2) + 2*
a*b*x**2*sqrt(c + d*x**2)/(3*d) + 8*b**2*c**2*sqrt(c + d*x**2)/(15*d**3) -
4*b**2*c*x**2*sqrt(c + d*x**2)/(15*d**2) + b**2*x**4*sqrt(c + d*x**2)/(5*d)
, Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/sqrt(c), True))
```

Giac [A]
time = 1.29, size = 84, normalized size = 1.14

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{dx^2+c}}{d^3}+\frac{3\,(dx^2+c)^{\frac{5}{2}}b^2-10\,(dx^2+c)^{\frac{3}{2}}b^2c+10\,(dx^2+c)^{\frac{3}{2}}abd}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)/d^3 + 1/15*(3*(d*x^2 + c)^(
5/2)*b^2 - 10*(d*x^2 + c)^(3/2)*b^2*c + 10*(d*x^2 + c)^(3/2)*a*b*d)/d^3
```

Mupad [B]
time = 0.32, size = 68, normalized size = 0.92

$$\sqrt{d\,x^2+c}\,\left(\frac{15\,a^2\,d^2-20\,a\,b\,c\,d+8\,b^2\,c^2}{15\,d^3}+\frac{b^2\,x^4}{5\,d}+\frac{2\,b\,x^2\,\left(5\,a\,d-2\,b\,c\right)}{15\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((15*a^2*d^2 + 8*b^2*c^2 - 20*a*b*c*d)/(15*d^3) + (b^2*x^
4)/(5*d) + (2*b*x^2*(5*a*d - 2*b*c))/(15*d^2))
```

3.640 $\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2}+\frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d}+\frac{(3b^2c^2-8abcd+8a^2d^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}$$

```
[Out] 1/8*(8*a^2*d^2-8*a*b*c*d+3*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(5
/2)-3/8*b*(-2*a*d+b*c)*x*(d*x^2+c)^(1/2)/d^2+1/4*b*x*(b*x^2+a)*(d*x^2+c)^(1
/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {427, 396, 223, 212}

$$\frac{(8a^2d^2-8abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{3bx\sqrt{c+dx^2}\,(bc-2ad)}{8d^2}+\frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

```
[Out] (-3*b*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(8*d^2) + (b*x*(a + b*x^2)*Sqrt[c +
d*x^2])/(4*d) + ((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/(8*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeqQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d} + \frac{\int \frac{-a(bc-4ad)-3b(bc-2ad)x^2}{\sqrt{c+dx^2}}\,dx}{4d}$$

$$= -\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2} + \frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d} - \frac{(2ad(bc-4ad)-3bc(bc-2ad)}{8d^2}$$

$$= -\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2} + \frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d} - \frac{(2ad(bc-4ad)-3bc(bc-2ad)}{}$$

$$= -\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2} + \frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d} + \frac{(3b^2c^2-8abcd+8a^2d^2)\tanh^{-1}}{8d^{5/2}}$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 0.84

$$\frac{bx\sqrt{c+dx^2}\,(-3bc+8ad+2bdx^2)}{8d^2} + \frac{(-3b^2c^2+8abcd-8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{8d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(-3*b*c + 8*a*d + 2*b*d*x^2))/(8*d^2) + ((-3*b^2*c^2 +
 8*a*b*c*d - 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*d^(5/2))
```

Maple [A]
time = 0.09, size = 133, normalized size = 1.24

method	result

risch	$\frac{bx(2bd\,x^2+8ad-3bc)\sqrt{d\,x^2+c}}{8d^2}+\frac{a^2\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{\sqrt{d}}-\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)abc}{d^{\frac{3}{2}}}+\frac{3\ln\left(x\sqrt{d}+\sqrt{d\right.}}{8d^{\frac{5}{2}}}$
default	$b^2\left(\frac{x^3\sqrt{d\,x^2+c}}{4d}-\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2d}-\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2d^{\frac{3}{2}}}\right)}{4d}\right)+2ab\left(\frac{x\sqrt{d\,x^2+c}}{2d}-\frac{c\ln\left(x\sqrt{d}+\sqrt{\right.}}{2d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^2+c)^(1/2)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/4*x^3/d*(d*x^2+c)^(1/2)-3/4*c/d*(1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(3/
2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))+2*a*b*(1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(
3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))+a^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/
2)
```

Maxima [A]
time = 0.33, size = 109, normalized size = 1.02

$$\frac{\sqrt{dx^2+c}\,b^2x^3}{4\,d}-\frac{3\,\sqrt{dx^2+c}\,b^2cx}{8\,d^2}+\frac{\sqrt{dx^2+c}\,abx}{d}+\frac{3\,b^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{5}{2}}}-\frac{abc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}}+\frac{a^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(d*x^2 + c)*b^2*x^3/d - 3/8*sqrt(d*x^2 + c)*b^2*c*x/d^2 + sqrt(d*x^
2 + c)*a*b*x/d + 3/8*b^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(5/2) - a*b*c*arcsinh
(d*x/sqrt(c*d))/d^(3/2) + a^2*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

Fricas [A]
time = 1.78, size = 194, normalized size = 1.81

$$\left[\frac{(3\,b^2c^2-8\,abcd+8\,a^2d^2)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(2\,b^2d^2x^3-(3\,b^2cd-8\,abd^2)x)\sqrt{dx^2+c}}{16\,d^3},-\frac{(3\,b^2c^2-8\,abcd+8\,a^2d^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(2\,b^2d^2x^3-(3\,b^2cd-8\,abd^2)x)\sqrt{dx^2+c}}{8\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*
x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 - (3*b^2*c*d - 8*a*b*d^2)*x)*sqr
t(d*x^2 + c))/d^3, -1/8*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^2*x^3 - (3*b^2*c*d - 8*a*b*d^2)*x)
*sqrt(d*x^2 + c))/d^3]
```

Sympy [A]
time = 4.38, size = 238, normalized size = 2.22

$$a^{2}\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d<0\\ \frac{\sqrt{\frac{c}{d}}\operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d>0\\ \frac{\sqrt{-\frac{c}{d}}\operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for}\ d>0\wedge c<0\end{cases}\right)+\frac{ab\sqrt{c}\,x\sqrt{1+\frac{dx^{2}}{c}}}{d}-\frac{abc\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{\frac{3}{2}}}-\frac{3b^{2}c^{\frac{3}{2}}x}{8d^{2}\sqrt{1+\frac{dx^{2}}{c}}}-\frac{b^{2}\sqrt{c}\,x^{3}}{8d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3b^{2}c^{2}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{5}{2}}}+\frac{b^{2}x^{5}}{4\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*Piecewise((sqrt(-c/d)*asin(x*sqrt(-d/c))/sqrt(c), (c > 0) & (d < 0)),
(sqrt(c/d)*asinh(x*sqrt(d/c))/sqrt(c), (c > 0) & (d > 0)), (sqrt(-c/d)*acos
h(x*sqrt(-d/c))/sqrt(-c), (d > 0) & (c < 0))) + a*b*sqrt(c)*x*sqrt(1 + d*x*
*2/c)/d - a*b*c*asinh(sqrt(d)*x/sqrt(c))/d**(3/2) - 3*b**2*c**(3/2)*x/(8*d*
*2*sqrt(1 + d*x**2/c)) - b**2*sqrt(c)*x**3/(8*d*sqrt(1 + d*x**2/c)) + 3*b**
2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + b**2*x**5/(4*sqrt(c)*sqrt(1
+ d*x**2/c))
```

Giac [A]
time = 1.06, size = 91, normalized size = 0.85

$$\frac{1}{8}\left(\frac{2\,b^{2}x^{2}}{d}-\frac{3\,b^{2}cd-8\,abd^{2}}{d^{3}}\right)\sqrt{dx^{2}+c}\,x-\frac{\left(3\,b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^{2}+c}\right|\right)}{8\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*x^2/d - (3*b^2*c*d - 8*a*b*d^2)/d^3)*sqrt(d*x^2 + c)*x - 1/8*(3*
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(
5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^{2}+a\right)^{2}}{\sqrt{d\,x^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2)^2/(c + d*x^2)^(1/2), x)
```

3.641 $\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=75

$$-\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2}+\frac{b^2(c+dx^2)^{3/2}}{3d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] 1/3*b^2*(d*x^2+c)^(3/2)/d^2-a^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(1/2)-b*
(-2*a*d+b*c)*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 90, 65, 214}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+dx^2}\,(bc-2ad)}{d^2}+\frac{b^2(c+dx^2)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*Sqrt[c + d*x^2])/d^2) + (b^2*(c + d*x^2)^(3/2))/(3*d^2)
- (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x\sqrt{c+dx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b(bc-2ad)}{d\sqrt{c+dx}}+\frac{a^2}{x\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d}\right)\,dx, x, x^2\right) \\
&= -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2}+\frac{b^2(c+dx^2)^{3/2}}{3d^2}+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right) \\
&= -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2}+\frac{b^2(c+dx^2)^{3/2}}{3d^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d} \\
&= -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2}+\frac{b^2(c+dx^2)^{3/2}}{3d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.84

$$\frac{b\sqrt{c+dx^2}\,(-2bc+6ad+bdx^2)}{3d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
[Out] (b*Sqrt[c + d*x^2]*(-2*b*c + 6*a*d + b*d*x^2))/(3*d^2) - (a^2*ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

Maple [A]
time = 0.09, size = 86, normalized size = 1.15

method	result	size

default	$b^{2}\left(\frac{x^{2}\sqrt{d\,x^{2}+c}}{3d}-\frac{2c\sqrt{d\,x^{2}+c}}{3d^{2}}\right)+\frac{2ab\sqrt{d\,x^{2}+c}}{d}-\frac{a^{2}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{\sqrt{c}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/3*x^2/d*(d*x^2+c)^(1/2)-2/3*c/d^2*(d*x^2+c)^(1/2))+2*a*b/d*(d*x^2+c)
^(1/2)-a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [A]
time = 0.33, size = 75, normalized size = 1.00

$$\frac{\sqrt{dx^2+c}\,b^2x^2}{3\,d}-\frac{a^2\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{\sqrt{c}}-\frac{2\,\sqrt{dx^2+c}\,b^2c}{3\,d^2}+\frac{2\,\sqrt{dx^2+c}\,ab}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(d*x^2 + c)*b^2*x^2/d - a^2*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) -
 2/3*sqrt(d*x^2 + c)*b^2*c/d^2 + 2*sqrt(d*x^2 + c)*a*b/d
```

Fricas [A]
time = 1.33, size = 157, normalized size = 2.09

$$\left[\frac{3\,a^2\sqrt{c}\,d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,(b^2cdx^2-2\,b^2c^2+6\,abcd)\sqrt{dx^2+c}}{6\,cd^2},\ \frac{3\,a^2\sqrt{-c}\,d^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(b^2cdx^2-2\,b^2c^2+6\,abcd)\sqrt{dx^2+c}}{3\,cd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2)
 + 2*(b^2*c*d*x^2 - 2*b^2*c^2 + 6*a*b*c*d)*sqrt(d*x^2 + c))/(c*d^2), 1/3*(3
*a^2*sqrt(-c)*d^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b^2*c*d*x^2 - 2*b^2*c
^2 + 6*a*b*c*d)*sqrt(d*x^2 + c))/(c*d^2)]
```

Sympy [A]
time = 18.96, size = 76, normalized size = 1.01

$$\frac{a^2\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{c}}\,\sqrt{c+dx^2}}\right)}{c\sqrt{-\frac{1}{c}}}+\frac{b^2(c+dx^2)^{\frac{3}{2}}}{3d^2}+\frac{b\sqrt{c+dx^2}\cdot(2ad-bc)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*atan(1/(sqrt(-1/c)*sqrt(c + d*x**2)))/(c*sqrt(-1/c)) + b**2*(c + d*x**
2)**(3/2)/(3*d**2) + b*sqrt(c + d*x**2)*(2*a*d - b*c)/d**2
```

Giac [A]
time = 1.03, size = 82, normalized size = 1.09

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{(dx^2+c)^{\frac{3}{2}}b^2d^4-3\sqrt{dx^2+c}\,b^2cd^4+6\sqrt{dx^2+c}\,abd^5}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/3*((d*x^2 + c)^(3/2)*b^2*
d^4 - 3*sqrt(d*x^2 + c)*b^2*c*d^4 + 6*sqrt(d*x^2 + c)*a*b*d^5)/d^6
```

Mupad [B]
time = 0.37, size = 77, normalized size = 1.03

$$\frac{b^2\,{\left(d\,x^2+c\right)}^{3/2}}{3\,d^2}-\frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\,\sqrt{d\,x^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^(1/2)),x)
```

```
[Out] (b^2*(c + d*x^2)^(3/2))/(3*d^2) - (a^2*atanh((c + d*x^2)^(1/2)/c^(1/2)))/c^
(1/2) - ((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^2)*(c + d*x^2)^(1/2)
```

3.642 $\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}$$

```
[Out] -1/2*b*(-4*a*d+b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-a^2*(d*x^2+c
)^(1/2)/c/x+1/2*b^2*x*(d*x^2+c)^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {473, 396, 223, 212}

$$-\frac{a^2\sqrt{c+dx^2}}{cx}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((a^2*Sqrt[c + d*x^2])/(c*x)) + (b^2*x*Sqrt[c + d*x^2])/(2*d) - (b*(b*c -
4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx &= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{\int \frac{2abc+b^2cx^2}{\sqrt{c+dx^2}}\,dx}{c}\\
&= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{(b(bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2d}\\
&= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{(b(bc-4ad))\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2d}\\
&= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 79, normalized size = 0.96

$$\frac{(-2a^2d+b^2cx^2)\sqrt{c+dx^2}}{2cdx}+\frac{b(bc-4ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
[Out] ((-2*a^2*d + b^2*c*x^2)*Sqrt[c + d*x^2])/(2*c*d*x) + (b*(b*c - 4*a*d)*Log[-
(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*d^(3/2))
```

Maple [A]
time = 0.10, size = 87, normalized size = 1.06

method	result	size
risch	$-\frac{\sqrt{d\,x^2+c}\,(-b^2c\,x^2+2a^2d)}{2dcx}+\frac{2ab\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{\sqrt{d}}-\frac{b^2\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)c}{2d^{\frac{3}{2}}}$	86

default	$b^2\left(\frac{x\sqrt{d\,x^2+c}}{2d}-\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2d^{\frac{3}{2}}}\right)+\frac{2ab\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{\sqrt{d}}-\frac{a^2\sqrt{d\,x^2+c}}{cx}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))+2
*a*b*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-a^2*(d*x^2+c)^(1/2)/c/x
```

Maxima [A]
time = 0.35, size = 73, normalized size = 0.89

$$\frac{\sqrt{dx^2+c}\,b^2x}{2\,d}-\frac{b^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{3}{2}}}+\frac{2\,ab\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}}-\frac{\sqrt{dx^2+c}\,a^2}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x/d - 1/2*b^2*c*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 2*
a*b*arcsinh(d*x/sqrt(c*d))/sqrt(d) - sqrt(d*x^2 + c)*a^2/(c*x)
```

Fricas [A]
time = 1.27, size = 165, normalized size = 2.01

$$\left[-\frac{(b^2c^2-4\,abcd)\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\,(b^2cdx^2-2\,a^2d^2)\sqrt{dx^2+c}}{4\,cd^2x},\ \frac{(b^2c^2-4\,abcd)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(b^2cdx^2-2\,a^2d^2)\sqrt{dx^2+c}}{2\,cd^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^2 - 4*a*b*c*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqr
t(d)*x - c) - 2*(b^2*c*d*x^2 - 2*a^2*d^2)*sqrt(d*x^2 + c))/(c*d^2*x), 1/2*(
(b^2*c^2 - 4*a*b*c*d)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*
c*d*x^2 - 2*a^2*d^2)*sqrt(d*x^2 + c))/(c*d^2*x)]
```

Sympy [A]
time = 2.17, size = 155, normalized size = 1.89

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}+2ab\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\,\mathrm{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for } c>0\wedge d<0\\ \frac{\sqrt{\frac{c}{d}}\,\mathrm{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for } c>0\wedge d>0\\ \frac{\sqrt{-\frac{c}{d}}\,\mathrm{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for } d>0\wedge c<0\end{cases}\right)+\frac{b^2\sqrt{c}\,x\sqrt{1+\frac{dx^2}{c}}}{2d}-\frac{b^2c\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/c + 2*a*b*Piecewise((sqrt(-c/d)*asin(x*s
qrt(-d/c))/sqrt(c), (c > 0) & (d < 0)), (sqrt(c/d)*asinh(x*sqrt(d/c))/sqrt(
c), (c > 0) & (d > 0)), (sqrt(-c/d)*acosh(x*sqrt(-d/c))/sqrt(-c), (d > 0) &
 (c < 0))) + b**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/(2*d) - b**2*c*asinh(sqrt(d)
*x/sqrt(c))/(2*d**(3/2))
```

Giac [A]
time = 1.02, size = 93, normalized size = 1.13

$$\frac{\sqrt{dx^2+c}\,b^2x}{2\,d}+\frac{2\,a^2\sqrt{d}}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}+\frac{\left(b^2c\sqrt{d}\,-4\,abd^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x/d + 2*a^2*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^
2 - c) + 1/4*(b^2*c*sqrt(d) - 4*a*b*d^(3/2))*log((sqrt(d)*x - sqrt(d*x^2 +
c))^2)/d^2
```

Mupad [B]
time = 0.73, size = 125, normalized size = 1.52

$$\begin{cases}\frac{-a^2+2\,a\,b\,x^2+\frac{b^2\,x^4}{3}}{\sqrt{c}\,x} & \text{if } d=0\\ \frac{2\,a\,b\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{\sqrt{d}}+\frac{b^2\,x\,\sqrt{d\,x^2+c}}{2\,d}-\frac{a^2\,\sqrt{d\,x^2+c}}{c\,x}-\frac{b^2\,c\,\ln\left(2\,\sqrt{d}\,x+2\,\sqrt{d\,x^2+c}\right)}{2\,d^{3/2}} & \text{if } d\neq 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] piecewise(d == 0, (- a^2 + (b^2*x^4)/3 + 2*a*b*x^2)/(c^(1/2)*x), d ~= 0, (2
*a*b*log(d^(1/2)*x + (c + d*x^2)^(1/2)))/d^(1/2) + (b^2*x*(c + d*x^2)^(1/2)
)/(2*d) - (a^2*(c + d*x^2)^(1/2))/(c*x) - (b^2*c*log(2*d^(1/2)*x + 2*(c + d
*x^2)^(1/2)))/(2*d^(3/2)))
```

3.643 $\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=80

$$\frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}$$

```
[Out] -1/2*a*(-a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+b^2*(d*x^2+c)^
(1/2)/d-1/2*a^2*(d*x^2+c)^(1/2)/c/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 91, 81, 65, 214}

$$-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (b^2*Sqrt[c + d*x^2])/d - (a^2*Sqrt[c + d*x^2])/(2*c*x^2) - (a*(4*b*c - a*d
)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
```

```
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4bc-ad)+b^2cx}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2c}\\
&= \frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{(a(4bc-ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4c}\\
&= \frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{(a(4bc-ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2cd}\\
&= \frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 77, normalized size = 0.96

$$
\frac{(-a^2d+2b^2cx^2)\sqrt{c+dx^2}}{2cdx^2}+\frac{a(-4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-(a^2*d) + 2*b^2*c*x^2)*Sqrt[c + d*x^2])/(2*c*d*x^2) + (a*(-4*b*c + a*d)*
ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Maple [A]
time = 0.11, size = 99, normalized size = 1.24

method	result	size
default	$\frac{b^{2} \sqrt{d \,x^{2}+c}}{d}+a^{2} \left(-\frac{\sqrt{d \,x^{2}+c}}{2 c \,x^{2}}+\frac{d \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)}{2 c^{\frac{3}{2}}}\right)-\frac{2 a b \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)}{\sqrt{c}}$	99
risch	$-\frac{a^{2} \sqrt{d \,x^{2}+c}}{2 c \,x^{2}}+\frac{b^{2} \sqrt{d \,x^{2}+c}}{d}+\frac{a^{2} \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right) d}{2 c^{\frac{3}{2}}}-\frac{2 a b \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)}{\sqrt{c}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(d*x^2+c)^(1/2)/d+a^2*(-1/2/c/x^2*(d*x^2+c)^(1/2)+1/2*d/c^(3/2)*ln((2*c
+2*c^(1/2)*(d*x^2+c)^(1/2))/x))-2*a*b/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(
1/2))/x)
```

Maxima [A]
time = 0.27, size = 77, normalized size = 0.96

$$-\frac{2 \, a b \operatorname{arsinh}\left(\frac{c}{\sqrt{c d} {\left| x \right|}}\right)}{\sqrt{c}} + \frac{a^{2} d \operatorname{arsinh}\left(\frac{c}{\sqrt{c d} {\left| x \right|}}\right)}{2 \, c^{\frac{3}{2}}} + \frac{\sqrt{d x^{2} + c} b^{2}}{d} - \frac{\sqrt{d x^{2} + c} a^{2}}{2 \, c x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*b*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 1/2*a^2*d*arcsinh(c/(sqrt(c*
d)*abs(x)))/c^(3/2) + sqrt(d*x^2 + c)*b^2/d - 1/2*sqrt(d*x^2 + c)*a^2/(c*x^
2)
```

Fricas [A]
time = 1.63, size = 175, normalized size = 2.19

$$\left[-\frac{{\left(4 \, a b c d - a^{2} d^{2}\right)} \sqrt{c} x^{2} \log\left(-\frac{d x^{2} + 2 \, \sqrt{d x^{2} + c} \sqrt{c} + 2 \, c}{x^{2}}\right) - 2 \, {\left(2 \, b^{2} c^{2} x^{2} - a^{2} c d\right)} \sqrt{d x^{2} + c}}{4 \, c^{2} d x^{2}}, \frac{{\left(4 \, a b c d - a^{2} d^{2}\right)} \sqrt{-c} x^{2} \arctan\left(\frac{\sqrt{-c}}{\sqrt{d x^{2} + c}}\right) + {\left(2 \, b^{2} c^{2} x^{2} - a^{2} c d\right)} \sqrt{d x^{2} + c}}{2 \, c^{2} d x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a*b*c*d - a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sq
rt(c) + 2*c)/x^2) - 2*(2*b^2*c^2*x^2 - a^2*c*d)*sqrt(d*x^2 + c))/(c^2*d*x^2
), 1/2*((4*a*b*c*d - a^2*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + (2*b^2*c^2*x^2 - a^2*c*d)*sqrt(d*x^2 + c))/(c^2*d*x^2)]
```

Sympy [A]
time = 39.45, size = 99, normalized size = 1.24

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2cx}+\frac{a^2d\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2c^{\frac{3}{2}}}-\frac{2ab\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{\sqrt{c}}+b^2\left(\begin{cases}\frac{x^2}{2\sqrt{c}} & \text{for } d=0\\ \frac{\sqrt{c+dx^2}}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*c*x) + a**2*d*asinh(sqrt(c)/(sqrt(d)*
x))/(2*c**(3/2)) - 2*a*b*asinh(sqrt(c)/(sqrt(d)*x))/sqrt(c) + b**2*Piecewis
e((x**2/(2*sqrt(c)), Eq(d, 0)), (sqrt(c + d*x**2)/d, True))
```

Giac [A]
time = 0.92, size = 81, normalized size = 1.01

$$\frac{2\sqrt{dx^2+c}\,b^2-\frac{\sqrt{dx^2+c}\,a^2d}{cx^2}+\frac{(4\,abcd-a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b^2 - sqrt(d*x^2 + c)*a^2*d/(c*x^2) + (4*a*b*c*d - a
^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c))/d
```

Mupad [B]
time = 0.45, size = 65, normalized size = 0.81

$$\frac{b^2\sqrt{d\,x^2+c}}{d}+\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(a\,d-4\,b\,c)}{2\,c^{3/2}}-\frac{a^2\sqrt{d\,x^2+c}}{2\,c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^(1/2)),x)
```

```
[Out] (b^2*(c + d*x^2)^(1/2))/d + (a*atanh((c + d*x^2)^(1/2)/c^(1/2))*(a*d - 4*b*
c))/(2*c^(3/2)) - (a^2*(c + d*x^2)^(1/2))/(2*c*x^2)
```

3.644 $\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=84

$$-\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

```
[Out] b^2*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)-1/3*a^2*(d*x^2+c)^(1/2)/c/x^
3-2/3*a*(-a*d+3*b*c)*(d*x^2+c)^(1/2)/c^2/x
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {473, 462, 223, 212}

$$-\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a\sqrt{c+dx^2}\,(3bc-ad)}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/3*(a^2*Sqrt[c + d*x^2])/(c*x^3) - (2*a*(3*b*c - a*d)*Sqrt[c + d*x^2])/(3
*c^2*x) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/Sqrt[d]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx &= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}+\frac{\int \frac{2a(3bc-ad)+3b^2cx^2}{x^2\sqrt{c+dx^2}}\,dx}{3c}\\
&= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+b^2\int \frac{1}{\sqrt{c+dx^2}}\,dx\\
&= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+b^2\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 73, normalized size = 0.87

$$\frac{a\sqrt{c+dx^2}\,(-ac-6bcx^2+2adx^2)}{3c^2x^3}-\frac{b^2\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2]*(-(a*c) - 6*b*c*x^2 + 2*a*d*x^2))/(3*c^2*x^3) - (b^2*Log
[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[d]
```

Maple [A]
time = 0.10, size = 84, normalized size = 1.00

method	result	size
risch	$-\frac{\sqrt{d\,x^2+c}\;a(-2ad\,x^2+6c\,x^2b+ac)}{3c^2x^3}+\frac{b^2\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{\sqrt{d}}$	61

default	$\frac{b^{2} \ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{\sqrt{d}}+a^{2} \left(-\frac{\sqrt{d\,x^{2}+c}}{3 c\,x^{3}}+\frac{2 d \sqrt{d\,x^{2}+c}}{3 c^{2} x}\right)-\frac{2 a b \sqrt{d\,x^{2}+c}}{c x}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)+a^2*(-1/3/c/x^3*(d*x^2+c)^(1/2)+2
/3*d/c^2/x*(d*x^2+c)^(1/2))-2*a*b/c/x*(d*x^2+c)^(1/2)
```

Maxima [A]
time = 0.28, size = 77, normalized size = 0.92

$$\frac{b^{2} \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}}-\frac{2 \sqrt{dx^{2}+c}\,ab}{cx}+\frac{2 \sqrt{dx^{2}+c}\,a^{2} d}{3\,c^{2} x}-\frac{\sqrt{dx^{2}+c}\,a^{2}}{3\,cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*arcsinh(d*x/sqrt(c*d))/sqrt(d) - 2*sqrt(d*x^2 + c)*a*b/(c*x) + 2/3*sqrt
(d*x^2 + c)*a^2*d/(c^2*x) - 1/3*sqrt(d*x^2 + c)*a^2/(c*x^3)
```

Fricas [A]
time = 1.61, size = 173, normalized size = 2.06

$$\left[\frac{3\,b^{2} c^{2} \sqrt{d}\,x^{3} \log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)-2\,\left(a^{2} cd+2\,\left(3\,abcd-a^{2} d^{2}\right) x^{2}\right) \sqrt{dx^{2}+c}}{6\,c^{2} dx^{3}},\,-\frac{3\,b^{2} c^{2} \sqrt{-d}\,x^{3} \arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)+\left(a^{2} cd+2\,\left(3\,abcd-a^{2} d^{2}\right) x^{2}\right) \sqrt{dx^{2}+c}}{3\,c^{2} dx^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^2*c^2*sqrt(d)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
 - 2*(a^2*c*d + 2*(3*a*b*c*d - a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c^2*d*x^3),
-1/3*(3*b^2*c^2*sqrt(-d)*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a^2*c*d
+ 2*(3*a*b*c*d - a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c^2*d*x^3)]
```

Sympy [A]
time = 1.43, size = 158, normalized size = 1.88

$$-\frac{a^{2} \sqrt{d}\,\sqrt{\frac{c}{dx^{2}}+1}}{3cx^{2}}+\frac{2a^{2} d^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{3c^{2}}-\frac{2ab\sqrt{d}\,\sqrt{\frac{c}{dx^{2}}+1}}{c}+b^{2}\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\: c>0\wedge d<0\\\frac{\sqrt{\frac{c}{d}}\operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\: c>0\wedge d>0\\\frac{\sqrt{-\frac{c}{d}}\operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for}\: d>0\wedge c<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) + 2*a**2*d**(3/2)*sqrt(c/(d*x
**2) + 1)/(3*c**2) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/c + b**2*Piecewise(
(sqrt(-c/d)*asin(x*sqrt(-d/c))/sqrt(c), (c > 0) & (d < 0)), (sqrt(c/d)*asin
h(x*sqrt(d/c))/sqrt(c), (c > 0) & (d > 0)), (sqrt(-c/d)*acosh(x*sqrt(-d/c))
/sqrt(-c), (d > 0) & (c < 0)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(70) = 140$.
time = 0.93, size = 156, normalized size = 1.86

$$-\frac{b^2 \log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{2\sqrt{d}}+\frac{4\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4 ab\sqrt{d}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 abc\sqrt{d}+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 a^2 d^{\frac{3}{2}}+3\,abc^2\sqrt{d}-a^2 c d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b^2*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/sqrt(d) + 4/3*(3*(sqrt(d)*x -
 sqrt(d*x^2 + c))^4*a*b*sqrt(d) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*s
qrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) + 3*a*b*c^2*sqrt(d)
- a^2*c*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^2}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^2/(x^4*(c + d*x^2)^(1/2)), x)
```

3.645 $\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=106

$$-\frac{a^2\sqrt{c+dx^2}}{4cx^4}-\frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2}-\frac{(8b^2c^2-8abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}$$

```
[Out] -1/8*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(5/
2)-1/4*a^2*(d*x^2+c)^(1/2)/c/x^4-1/8*a*(-3*a*d+8*b*c)*(d*x^2+c)^(1/2)/c^2/x
^2
```

Rubi [A]

time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 91, 79, 65, 214}

$$-\frac{(3a^2d^2-8abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{a^2\sqrt{c+dx^2}}{4cx^4}-\frac{a\sqrt{c+dx^2}\,(8bc-3ad)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/4*(a^2*Sqrt[c + d*x^2])/(c*x^4) - (a*(8*b*c - 3*a*d)*Sqrt[c + d*x^2])/(8
*c^2*x^2) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]])/(8*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(8bc-3ad)+2b^2cx}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)}{4c}\\
&= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} - \frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2} + \frac{1}{16}\left(8b^2-\frac{ad(8bc-3ad)}{c^2}\right)\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.\\
&= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} - \frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2} + \frac{\left(8b^2-\frac{ad(8bc-3ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x\right.}{8d}\\
&= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} - \frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2} - \frac{(8b^2c^2-8abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+}}{\sqrt{c}}\right.}{8c^{5/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.18, size = 92, normalized size = 0.87

$$-\frac{a\sqrt{c+dx^2}\,(2ac+8bcx^2-3adx^2)}{8c^2x^4}+\frac{(-8b^2c^2+8abcd-3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/8*(a*Sqrt[c + d*x^2]*(2*a*c + 8*b*c*x^2 - 3*a*d*x^2))/(c^2*x^4) + ((-8*b
^2*c^2 + 8*a*b*c*d - 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(5/2
))
```

Maple [A]
time = 0.10, size = 159, normalized size = 1.50

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\;a(-3ad\,x^2+8c\,x^2b+2ac)}{8c^2x^4}-\frac{3\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)a^2d^2}{8c^{\frac{5}{2}}}+\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)abd}{c^{\frac{3}{2}}}-\frac{b^2\ln}{}$
default	$a^2\left(-\frac{\sqrt{d\,x^2+c}}{4c\,x^4}-\frac{3d\left(-\frac{\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)+2ab\left(-\frac{\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{d\ln\left(\frac{2c+2}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/4/c/x^4*(d*x^2+c)^(1/2)-3/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(1/2)+1/2*d/c
^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))+2*a*b*(-1/2/c/x^2*(d*x^2+c)^
(1/2)+1/2*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))-b^2/c^(1/2)*ln((
2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [A]
time = 0.30, size = 123, normalized size = 1.16

$$-\frac{b^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{\sqrt{c}}+\frac{abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{3}{2}}}-\frac{3\,a^2d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{8\,c^{\frac{5}{2}}}-\frac{\sqrt{dx^2+c}\,ab}{cx^2}+\frac{3\,\sqrt{dx^2+c}\,a^2d}{8\,c^2x^2}-\frac{\sqrt{dx^2+c}\,a^2}{4\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + a*b*d*arcsinh(c/(sqrt(c*d)*abs
(x)))/c^(3/2) - 3/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) - sqrt(d*
x^2 + c)*a*b/(c*x^2) + 3/8*sqrt(d*x^2 + c)*a^2*d/(c^2*x^2) - 1/4*sqrt(d*x^2
 + c)*a^2/(c*x^4)
```

Fricas [A]
time = 1.64, size = 204, normalized size = 1.92

$$\left[\frac{\left(8\,b^{2}c^{2}-8\,abcd+3\,a^{2}d^{2}\right)\sqrt{c}\,x^{4}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)-2\,\left(2\,a^{2}c^{2}+\left(8\,abc^{2}-3\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{16\,c^{3}x^{4}},\ \frac{\left(8\,b^{2}c^{2}-8\,abcd+3\,a^{2}d^{2}\right)\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)-\left(2\,a^{2}c^{2}+\left(8\,abc^{2}-3\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{8\,c^{3}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a^2*c^2 + (8*a*b*c^2 - 3*a^2*c*d)*x^
2)*sqrt(d*x^2 + c))/(c^3*x^4), 1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqr
t(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a^2*c^2 + (8*a*b*c^2 - 3*a^
2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^3*x^4)]
```

Sympy [A]
time = 74.23, size = 178, normalized size = 1.68

$$-\frac{a^{2}}{4\sqrt{d}\,x^{5}\sqrt{\frac{c}{dx^{2}}+1}}+\frac{a^{2}\sqrt{d}}{8cx^{3}\sqrt{\frac{c}{dx^{2}}+1}}+\frac{3a^{2}d^{\frac{3}{2}}}{8c^{2}x\sqrt{\frac{c}{dx^{2}}+1}}-\frac{3a^{2}d^{2}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{8c^{\frac{5}{2}}}-\frac{ab\sqrt{d}\,\sqrt{\frac{c}{dx^{2}}+1}}{cx}+\frac{abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{c^{\frac{3}{2}}}-\frac{b^{2}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) + a**2*sqrt(d)/(8*c*x**3*sqrt(c
/(d*x**2) + 1)) + 3*a**2*d**(3/2)/(8*c**2*x*sqrt(c/(d*x**2) + 1)) - 3*a**2*
d**2*asinh(sqrt(c)/(sqrt(d)*x))/(8*c**(5/2)) - a*b*sqrt(d)*sqrt(c/(d*x**2)
+ 1)/(c*x) + a*b*d*asinh(sqrt(c)/(sqrt(d)*x))/c**(3/2) - b**2*asinh(sqrt(c)
/(sqrt(d)*x))/sqrt(c)
```

Giac [A]
time = 0.79, size = 140, normalized size = 1.32

$$\frac{\frac{\left(8\,b^{2}c^{2}d-8\,abcd^{2}+3\,a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^{2}}-\frac{8\,\left(dx^{2}+c\right)^{\frac{3}{2}}abcd^{2}-8\,\sqrt{dx^{2}+c}\,abc^{2}d^{2}-3\,\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{3}+5\,\sqrt{dx^{2}+c}\,a^{2}cd^{3}}{c^{2}d^{2}x^{4}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((8*b^2*c^2*d - 8*a*b*c*d^2 + 3*a^2*d^3)*arctan(sqrt(d*x^2 + c)/sqrt(-c
))/(sqrt(-c)*c^2) - (8*(d*x^2 + c)^(3/2)*a*b*c*d^2 - 8*sqrt(d*x^2 + c)*a*b*
c^2*d^2 - 3*(d*x^2 + c)^(3/2)*a^2*d^3 + 5*sqrt(d*x^2 + c)*a^2*c*d^3)/(c^2*d
^2*x^4))/d
```

Mupad [B]
time = 0.51, size = 129, normalized size = 1.22

$$-\frac{\frac{(5a^2d^2-8abcd)\sqrt{dx^2+c}}{8c}-\frac{(3a^2d^2-8abcd)(dx^2+c)^{3/2}}{8c^2}}{(dx^2+c)^2-2c(dx^2+c)+c^2}-\frac{\operatorname{atanh}\left(\frac{\sqrt{dx^2+c}}{\sqrt{c}}\right)(3a^2d^2-8abcd+8b^2c^2)}{8c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^5*(c + d*x^2)^(1/2)),x)
```

```
[Out] - (((5*a^2*d^2 - 8*a*b*c*d)*(c + d*x^2)^(1/2))/(8*c) - ((3*a^2*d^2 - 8*a*b*
c*d)*(c + d*x^2)^(3/2))/(8*c^2))/((c + d*x^2)^2 - 2*c*(c + d*x^2) + c^2) -
(atanh((c + d*x^2)^(1/2)/c^(1/2))*(3*a^2*d^2 + 8*b^2*c^2 - 8*a*b*c*d))/(8*c
^(5/2))
```

3.646 $\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{2a(5bc-2ad)\sqrt{c+dx^2}}{15c^2x^3}-\frac{(15b^2c^2-4ad(5bc-2ad))\sqrt{c+dx^2}}{15c^3x}$$

```
[Out] -1/5*a^2*(d*x^2+c)^(1/2)/c/x^5-2/15*a*(-2*a*d+5*b*c)*(d*x^2+c)^(1/2)/c^2/x^
3-1/15*(15*b^2*c^2-4*a*d*(-2*a*d+5*b*c))*(d*x^2+c)^(1/2)/c^3/x
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {473, 464, 270}

$$-\frac{\sqrt{c+dx^2}\,(8a^2d^2-20abcd+15b^2c^2)}{15c^3x}-\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{2a\sqrt{c+dx^2}\,(5bc-2ad)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/5*(a^2*Sqrt[c + d*x^2])/(c*x^5) - (2*a*(5*b*c - 2*a*d)*Sqrt[c + d*x^2])/
(15*c^2*x^3) - ((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*Sqrt[c + d*x^2])/(15*
c^3*x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
```

```
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{c+dx^2}}{5cx^5} + \frac{\int \frac{2a(5bc-2ad)+5b^2cx^2}{x^4\sqrt{c+dx^2}}\,dx}{5c}$$
$$= -\frac{a^2\sqrt{c+dx^2}}{5cx^5} - \frac{2a(5bc-2ad)\sqrt{c+dx^2}}{15c^2x^3} - \frac{1}{15}\left(-15b^2+\frac{4ad(5bc-2ad)}{c^2}\right)\int \frac{1}{x^2\sqrt{c}}$$
$$= -\frac{a^2\sqrt{c+dx^2}}{5cx^5} - \frac{2a(5bc-2ad)\sqrt{c+dx^2}}{15c^2x^3} - \frac{\left(15b^2-\frac{4ad(5bc-2ad)}{c^2}\right)\sqrt{c+dx^2}}{15cx}$$

Mathematica [A]
time = 0.12, size = 74, normalized size = 0.75

$$-\frac{\sqrt{c+dx^2}\,(15b^2c^2x^4+10abcx^2(c-2dx^2)+a^2(3c^2-4cdx^2+8d^2x^4))}{15c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
[Out] -1/15*(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c - 2*d*x^2) + a^2*(
3*c^2 - 4*c*d*x^2 + 8*d^2*x^4)))/(c^3*x^5)
```

Maple [A]
time = 0.09, size = 126, normalized size = 1.27

method	result
gosper	$-\frac{\sqrt{d\,x^2+c}\,(8a^2d^2x^4-20abcd\,x^4+15b^2c^2x^4-4a^2cd\,x^2+10ab\,c^2x^2+3a^2c^2)}{15x^5c^3}$
trager	$-\frac{\sqrt{d\,x^2+c}\,(8a^2d^2x^4-20abcd\,x^4+15b^2c^2x^4-4a^2cd\,x^2+10ab\,c^2x^2+3a^2c^2)}{15x^5c^3}$
risch	$-\frac{\sqrt{d\,x^2+c}\,(8a^2d^2x^4-20abcd\,x^4+15b^2c^2x^4-4a^2cd\,x^2+10ab\,c^2x^2+3a^2c^2)}{15x^5c^3}$
default	$a^2\left(-\frac{\sqrt{d\,x^2+c}}{5c\,x^5}-\frac{4d\left(-\frac{\sqrt{d\,x^2+c}}{3c\,x^3}+\frac{2d\sqrt{d\,x^2+c}}{3c^2x}\right)}{5c}\right)+2ab\left(-\frac{\sqrt{d\,x^2+c}}{3c\,x^3}+\frac{2d\sqrt{d\,x^2+c}}{3c^2x}\right)-b^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/5/c/x^5*(d*x^2+c)^(1/2)-4/5*d/c*(-1/3/c/x^3*(d*x^2+c)^(1/2)+2/3*d/c
^2/x*(d*x^2+c)^(1/2)))+2*a*b*(-1/3/c/x^3*(d*x^2+c)^(1/2)+2/3*d/c^2/x*(d*x^2
+c)^(1/2))-b^2/c/x*(d*x^2+c)^(1/2)
```

Maxima [A]
time = 0.33, size = 124, normalized size = 1.25

$$-\frac{\sqrt{dx^2+c}\,b^2}{cx}+\frac{4\,\sqrt{dx^2+c}\,abd}{3\,c^2x}-\frac{8\,\sqrt{dx^2+c}\,a^2d^2}{15\,c^3x}-\frac{2\,\sqrt{dx^2+c}\,ab}{3\,cx^3}+\frac{4\,\sqrt{dx^2+c}\,a^2d}{15\,c^2x^3}-\frac{\sqrt{dx^2+c}\,a^2}{5\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(d*x^2 + c)*b^2/(c*x) + 4/3*sqrt(d*x^2 + c)*a*b*d/(c^2*x) - 8/15*sqrt(
d*x^2 + c)*a^2*d^2/(c^3*x) - 2/3*sqrt(d*x^2 + c)*a*b/(c*x^3) + 4/15*sqrt(d*
x^2 + c)*a^2*d/(c^2*x^3) - 1/5*sqrt(d*x^2 + c)*a^2/(c*x^5)
```

Fricas [A]
time = 1.35, size = 73, normalized size = 0.74

$$-\frac{\left((15\,b^2c^2-20\,abcd+8\,a^2d^2)x^4+3\,a^2c^2+2\,(5\,abc^2-2\,a^2cd)x^2\right)\sqrt{dx^2+c}}{15\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*x^4 + 3*a^2*c^2 + 2*(5*a*b*c^2
 - 2*a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*x^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(92) = 184.
time = 1.94, size = 391, normalized size = 3.95

$$-\frac{3a^2c^4d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{2a^2c^3d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{3a^2c^2d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{12a^2cd^{\frac{15}{2}}x^6\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{8a^2d^{\frac{17}{2}}x^8\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{2ab\sqrt{d}\,\sqrt{\frac{c}{dx^2}+1}}{3cx^2}+\frac{4abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c^2}-\frac{b^2\sqrt{d}\,\sqrt{\frac{c}{dx^2}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(1/2),x)
```

```
[Out] -3*a**2*c**4*d**(9/2)*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**
5*x**6 + 15*c**3*d**6*x**8) - 2*a**2*c**3*d**(11/2)*x**2*sqrt(c/(d*x**2) +
1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 3*a**2*c**
2*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**
6 + 15*c**3*d**6*x**8) - 12*a**2*c*d**(15/2)*x**6*sqrt(c/(d*x**2) + 1)/(15*
c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 8*a**2*d**(17/2)*
x**8*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*
```

```
d**6*x**8) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) + 4*a*b*d**(3/2)
*sqrt(c/(d*x**2) + 1)/(3*c**2) - b**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(87) = 174.
time = 1.03, size = 312, normalized size = 3.15

$$\frac{2\left(15\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8b^2\sqrt{d}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6b^2c\sqrt{d}+60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6abd^{\frac{3}{2}}+90\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b^2c^2\sqrt{d}-140\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abcd^{\frac{3}{2}}+80\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{5}{2}}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c^3\sqrt{d}+100\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^2d^{\frac{3}{2}}-40\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2cd^{\frac{5}{2}}+15\,b^2c^4\sqrt{d}-20\,abc^3d^{\frac{3}{2}}+8\,a^2c^2d^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*sqrt(d) - 60*(sqrt(d)*x - sqrt
(d*x^2 + c))^6*b^2*c*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*d^(3/
2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^2*sqrt(d) - 140*(sqrt(d)*x -
sqrt(d*x^2 + c))^4*a*b*c*d^(3/2) + 80*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d
^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^3*sqrt(d) + 100*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*b*c^2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a^2*c*d^(5/2) + 15*b^2*c^4*sqrt(d) - 20*a*b*c^3*d^(3/2) + 8*a^2*c^2*d^(5/2
))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

Mupad [B]
time = 0.39, size = 77, normalized size = 0.78

$$-\frac{\sqrt{d\,x^2+c}\,\left(3\,a^2\,c^2-4\,a^2\,c\,d\,x^2+8\,a^2\,d^2\,x^4+10\,a\,b\,c^2\,x^2-20\,a\,b\,c\,d\,x^4+15\,b^2\,c^2\,x^4\right)}{15\,c^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^6*(c + d*x^2)^(1/2)),x)
```

```
[Out] -((c + d*x^2)^(1/2)*(3*a^2*c^2 + 8*a^2*d^2*x^4 + 15*b^2*c^2*x^4 + 10*a*b*c^
2*x^2 - 4*a^2*c*d*x^2 - 20*a*b*c*d*x^4))/(15*c^3*x^5)
```

3.647 $\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=151

$$-\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4}-\frac{(8b^2c^2-12abcd+5a^2d^2)\sqrt{c+dx^2}}{16c^3x^2}+\frac{d(8b^2c^2-12abcd+5a^2d^2}{16c^7}$$

```
[Out] 1/16*d*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^
(7/2)-1/6*a^2*(d*x^2+c)^(1/2)/c/x^6-1/24*a*(-5*a*d+12*b*c)*(d*x^2+c)^(1/2)/
c^2/x^4-1/16*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*(d*x^2+c)^(1/2)/c^3/x^2
```

Rubi [A]

time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 79, 44, 65, 214}

$$\frac{d(5a^2d^2-12abcd+8b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}-\frac{\sqrt{c+dx^2}\,(5a^2d^2-12abcd+8b^2c^2)}{16c^3x^2}-\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a\sqrt{c+dx^2}\,(12bc-5ad)}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/6*(a^2*Sqrt[c + d*x^2])/(c*x^6) - (a*(12*b*c - 5*a*d)*Sqrt[c + d*x^2])/(
24*c^2*x^4) - ((8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[c + d*x^2])/(16*c^
3*x^2) + (d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]])/(16*c^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^4\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(12bc-5ad)+3b^2cx}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} - \frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4} + \frac{1}{16}\left(8b^2 - \frac{ad(12bc-5ad)}{c^2}\right)\text{Subst}\left(\int \right.$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} - \frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4} - \frac{(8b^2c^2-12abcd+5a^2d^2)\sqrt{c+dx^2}}{16c^3x^2} +$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} - \frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4} - \frac{(8b^2c^2-12abcd+5a^2d^2)\sqrt{c+dx^2}}{16c^3x^2} +$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} - \frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4} - \frac{(8b^2c^2-12abcd+5a^2d^2)\sqrt{c+dx^2}}{16c^3x^2} +$$

Mathematica [A]
time = 0.29, size = 128, normalized size = 0.85

$$-\frac{\sqrt{c+dx^2}\,(24b^2c^2x^4+12abcx^2(2c-3dx^2)+a^2(8c^2-10cdx^2+15d^2x^4))}{48c^3x^6} + \frac{d(8b^2c^2-12abcd+5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
[Out] -1/48*(Sqrt[c + d*x^2]*(24*b^2*c^2*x^4 + 12*a*b*c*x^2*(2*c - 3*d*x^2) + a^2
*(8*c^2 - 10*c*d*x^2 + 15*d^2*x^4)))/(c^3*x^6) + (d*(8*b^2*c^2 - 12*a*b*c*d
 + 5*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(7/2))
```

Maple [A]
time = 0.11, size = 227, normalized size = 1.50

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\,(15a^2d^2x^4-36abcd\,x^4+24b^2c^2x^4-10a^2cd\,x^2+24ab\,c^2x^2+8a^2c^2)}{48c^3x^6}+\frac{5d^3\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)a^2}{16c^{\frac{7}{2}}}-$

default	$2ab\left(-\frac{\sqrt{d\,x^{2}+c}}{4c\,x^{4}}-\frac{3d\left(-\frac{\sqrt{d\,x^{2}+c}}{2c\,x^{2}}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)+a^{2}\left(-\frac{\sqrt{d\,x^{2}+c}}{6c\,x^{6}}-\frac{5d\left(-\frac{\sqrt{c}}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b*(-1/4/c/x^4*(d*x^2+c)^(1/2)-3/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(1/2)+1/2*d
/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))+a^2*(-1/6/c/x^6*(d*x^2+c)^
(1/2)-5/6*d/c*(-1/4/c/x^4*(d*x^2+c)^(1/2)-3/4*d/c*(-1/2/c/x^2*(d*x^2+c)^(1/
2)+1/2*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))+b^2*(-1/2/c/x^2*(
d*x^2+c)^(1/2)+1/2*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))
```

Maxima [A]
time = 0.28, size = 190, normalized size = 1.26

$$\frac{b^{2}d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,c^{\frac{3}{2}}}-\frac{3\,abd^{2}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{4\,c^{\frac{5}{2}}}+\frac{5\,a^{2}d^{3}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16\,c^{\frac{7}{2}}}-\frac{\sqrt{dx^{2}+c}\,b^{2}}{2\,cx^{2}}+\frac{3\,\sqrt{dx^{2}+c}\,abd}{4\,c^{2}x^{2}}-\frac{5\,\sqrt{dx^{2}+c}\,a^{2}d^{2}}{16\,c^{3}x^{2}}-\frac{\sqrt{dx^{2}+c}\,ab}{2\,cx^{4}}+\frac{5\,\sqrt{dx^{2}+c}\,a^{2}d}{24\,c^{2}x^{4}}-\frac{\sqrt{dx^{2}+c}\,a^{2}}{6\,cx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*d*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) - 3/4*a*b*d^2*arcsinh(c/(sq
rt(c*d)*abs(x)))/c^(5/2) + 5/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(7/
2) - 1/2*sqrt(d*x^2 + c)*b^2/(c*x^2) + 3/4*sqrt(d*x^2 + c)*a*b*d/(c^2*x^2)
- 5/16*sqrt(d*x^2 + c)*a^2*d^2/(c^3*x^2) - 1/2*sqrt(d*x^2 + c)*a*b/(c*x^4)
+ 5/24*sqrt(d*x^2 + c)*a^2*d/(c^2*x^4) - 1/6*sqrt(d*x^2 + c)*a^2/(c*x^6)
```

Fricas [A]
time = 1.64, size = 279, normalized size = 1.85

$$\left[\frac{3\,(8\,b^{2}c^{2}d-12\,abcd^{2}+5\,a^{2}d^{3})\sqrt{c}\,x^{6}\log\left(-\frac{dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)-2\,(8\,a^{2}c^{3}+3\,(8\,b^{2}c^{3}-12\,abc^{2}d+5\,a^{2}cd^{2})x^{4}+2\,(12\,abc^{3}-5\,a^{2}c^{2}d)x^{2})\sqrt{dx^{2}+c}}{96\,c^{4}x^{6}},\ -\frac{3\,(8\,b^{2}c^{2}d-12\,abcd^{2}+5\,a^{2}d^{3})\sqrt{-c}\,x^{6}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+(8\,a^{2}c^{3}+3\,(8\,b^{2}c^{3}-12\,abc^{2}d+5\,a^{2}cd^{2})x^{4}+2\,(12\,abc^{3}-5\,a^{2}c^{2}d)x^{2})\sqrt{dx^{2}+c}}{48\,c^{4}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 +
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*a^2*c^3 + 3*(8*b^2*c^3 - 12*a
*b*c^2*d + 5*a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2
+ c))/(c^4*x^6), -1/48*(3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*sqrt(-c)
*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (8*a^2*c^3 + 3*(8*b^2*c^3 - 12*a*b*
c^2*d + 5*a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c
))/(c^4*x^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(146) = 292$.
time = 132.56, size = 301, normalized size = 1.99

$$-\frac{a^2}{6\sqrt{d}\,x^7\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2\sqrt{d}}{24cx^5\sqrt{\frac{c}{dx^2}+1}} - \frac{5a^2d^{\frac{3}{2}}}{48c^2x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{5a^2d^{\frac{5}{2}}}{16c^3x\sqrt{\frac{c}{dx^2}+1}} + \frac{5a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{16c^{\frac{7}{2}}} - \frac{ab}{2\sqrt{d}\,x^5\sqrt{\frac{c}{dx^2}+1}} + \frac{ab\sqrt{d}}{4cx^3\sqrt{\frac{c}{dx^2}+1}} + \frac{3abd^{\frac{3}{2}}}{4c^2x\sqrt{\frac{c}{dx^2}+1}} - \frac{3abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{4c^{\frac{5}{2}}} - \frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2cx} + \frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) + a**2*sqrt(d)/(24*c*x**5*sqrt(
c/(d*x**2) + 1)) - 5*a**2*d**(3/2)/(48*c**2*x**3*sqrt(c/(d*x**2) + 1)) - 5*
a**2*d**(5/2)/(16*c**3*x*sqrt(c/(d*x**2) + 1)) + 5*a**2*d**3*asinh(sqrt(c)/
(sqrt(d)*x))/(16*c**(7/2)) - a*b/(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) + a*
b*sqrt(d)/(4*c*x**3*sqrt(c/(d*x**2) + 1)) + 3*a*b*d**(3/2)/(4*c**2*x*sqrt(c
/(d*x**2) + 1)) - 3*a*b*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(4*c**(5/2)) - b**2
*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*c*x) + b**2*d*asinh(sqrt(c)/(sqrt(d)*x))/(
2*c**(3/2))
```

Giac [A]
time = 1.42, size = 241, normalized size = 1.60

$$-\frac{\frac{3\left(8b^2c^2d^2-12abcd^3+5a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^3} + \frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3d^2+24\sqrt{dx^2+c}\,b^2c^4d^2-36\left(dx^2+c\right)^{\frac{5}{2}}abcd^3+96\left(dx^2+c\right)^{\frac{3}{2}}abc^2d^3-60\sqrt{dx^2+c}\,abc^3d^3+15\left(dx^2+c\right)^{\frac{5}{2}}a^2d^4-40\left(dx^2+c\right)^{\frac{3}{2}}a^2cd^4+33\sqrt{dx^2+c}\,a^2c^2d^4}{c^3d^3x^6}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(8*b^2*c^2*d^2 - 12*a*b*c*d^3 + 5*a^2*d^4)*arctan(sqrt(d*x^2 + c)/
sqrt(-c))/(sqrt(-c)*c^3) + (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2 +
c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 - 36*(d*x^2 + c)^(5/2
)*a*b*c*d^3 + 96*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 - 60*sqrt(d*x^2 + c)*a*b*c^3
*d^3 + 15*(d*x^2 + c)^(5/2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*a^2*c*d^4 + 33*s
qrt(d*x^2 + c)*a^2*c^2*d^4)/(c^3*d^3*x^6))/d
```

Mupad [B]
time = 0.56, size = 207, normalized size = 1.37

$$\frac{\frac{(d x^2+c)^{5/2}(5 a^2 d^3-12 a b c d^2+8 b^2 c^2 d)}{16 c^3}-\frac{(d x^2+c)^{3/2}(5 a^2 d^3-12 a b c d^2+6 b^2 c^2 d)}{6 c^2}+\frac{\sqrt{d x^2+c}\,(11 a^2 d^3-20 a b c d^2+8 b^2 c^2 d)}{16 c}}{3 c (d x^2+c)^2-3 c^2 (d x^2+c)-(d x^2+c)^3+c^3}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d x^2+c}}{\sqrt{c}}\right)(5 a^2 d^2-12 a b c d+8 b^2 c^2)}{16 c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^7*(c + d*x^2)^(1/2)),x)
```

```
[Out] (((c + d*x^2)^(5/2)*(5*a^2*d^3 + 8*b^2*c^2*d - 12*a*b*c*d^2))/(16*c^3) - ((
c + d*x^2)^(3/2)*(5*a^2*d^3 + 6*b^2*c^2*d - 12*a*b*c*d^2))/(6*c^2) + ((c +
d*x^2)^(1/2)*(11*a^2*d^3 + 8*b^2*c^2*d - 20*a*b*c*d^2))/(16*c))/(3*c*(c + d
*x^2)^2 - 3*c^2*(c + d*x^2) - (c + d*x^2)^3 + c^3) + (d*atanh((c + d*x^2)^(
1/2)/c^(1/2))*(5*a^2*d^2 + 8*b^2*c^2 - 12*a*b*c*d))/(16*c^(7/2))
```

3.648 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}}+\frac{(35b^2c^2-60abcd+24a^2d^2)\,x\sqrt{c+dx^2}}{16d^4}-\frac{(35b^2c^2-60abcd+24a^2d^2)\,x^3\sqrt{c+dx^2}}{24cd^3}+\frac{b^2x^5\sqrt{}}{6}$$

```
[Out] -1/16*c*(24*a^2*d^2-60*a*b*c*d+35*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2
))/d^(9/2)+(-a*d+b*c)^2*x^5/c/d^2/(d*x^2+c)^(1/2)+1/16*(24*a^2*d^2-60*a*b*c
*d+35*b^2*c^2)*x*(d*x^2+c)^(1/2)/d^4-1/24*(24*a^2*d^2-60*a*b*c*d+35*b^2*c^2
)*x^3*(d*x^2+c)^(1/2)/c/d^3+1/6*b^2*x^5*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.10, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {474, 470, 327, 223, 212}

$$-\frac{c(24a^2d^2-60abcd+35b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{9/2}}+\frac{x\sqrt{c+dx^2}\,(24a^2d^2-60abcd+35b^2c^2)}{16d^4}-\frac{x^3\sqrt{c+dx^2}\,(24a^2d^2-60abcd+35b^2c^2)}{24cd^3}+\frac{x^5(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^5\sqrt{c+dx^2}}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*x^5)/(c*d^2*Sqrt[c + d*x^2]) + ((35*b^2*c^2 - 60*a*b*c*d + 2
4*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^4) - ((35*b^2*c^2 - 60*a*b*c*d + 24*a^2
*d^2)*x^3*Sqrt[c + d*x^2])/(24*c*d^3) + (b^2*x^5*Sqrt[c + d*x^2])/(6*d^2) -
 (c*(35*b^2*c^2 - 60*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(16*d^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx &= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} - \frac{\int \frac{x^4(-a^2d^2+5(bc-ad)^2-b^2cdx^2)}{\sqrt{c+dx^2}}\,dx}{cd^2} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{b^2x^5\sqrt{c+dx^2}}{6d^2} - \frac{(35b^2c^2-60abcd+24a^2d^2)\int \frac{x^4}{\sqrt{c+dx^2}}\,dx}{6cd^2} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} - \frac{(35b^2c^2-60abcd+24a^2d^2)\,x^3\sqrt{c+dx^2}}{24cd^3} + \frac{b^2x^5\sqrt{c+dx^2}}{6d^2} + \frac{(35b^2c^2}{} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{(35b^2c^2-60abcd+24a^2d^2)\,x\sqrt{c+dx^2}}{16d^4} - \frac{(35b^2c^2-60abcd+24a^2d^2}{24cd^3} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{(35b^2c^2-60abcd+24a^2d^2)\,x\sqrt{c+dx^2}}{16d^4} - \frac{(35b^2c^2-60abcd+24a^2d^2}{24cd^3} \\
&= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{(35b^2c^2-60abcd+24a^2d^2)\,x\sqrt{c+dx^2}}{16d^4} - \frac{(35b^2c^2-60abcd+24a^2d^2}{24cd^3}
\end{aligned}$$

Mathematica [A]

time = 0.26, size = 155, normalized size = 0.79

$$\frac{\frac{\sqrt{d}\, x(24a^2d^2(3c+dx^2)+12abd(-15c^2-5cdx^2+2d^2x^4)+b^2(105c^3+35c^2dx^2-14cd^2x^4+8d^3x^6))}{\sqrt{c+dx^2}}+3c(35b^2c^2-60abcd+24a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{48d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((Sqrt[d]*x*(24*a^2*d^2*(3*c + d*x^2) + 12*a*b*d*(-15*c^2 - 5*c*d*x^2 + 2*d
^2*x^4) + b^2*(105*c^3 + 35*c^2*d*x^2 - 14*c*d^2*x^4 + 8*d^3*x^6)))/Sqrt[c
+ d*x^2] + 3*c*(35*b^2*c^2 - 60*a*b*c*d + 24*a^2*d^2)*Log[-(Sqrt[d]*x) + Sq
rt[c + d*x^2]])/(48*d^(9/2))
```

Maple [A]
time = 0.11, size = 266, normalized size = 1.35

method	result
risch	$\frac{x\left(8b^2x^4d^2+24ab\,d^2x^2-22b^2cd\,x^2+24a^2d^2-84abcd+57b^2c^2\right)\sqrt{d\,x^2+c}}{48d^4}+\frac{cx\,a^2}{d^2\sqrt{d\,x^2+c}}-\frac{2c^2xab}{d^3\sqrt{d\,x^2+c}}+\frac{c^3}{d^4\sqrt{d}}$
default	$b^2\left(\frac{x^7}{6d\sqrt{d\,x^2+c}}-\frac{7c\left(\frac{x^5}{4d\sqrt{d\,x^2+c}}-\frac{5c\left(\frac{x^3}{2d\sqrt{d\,x^2+c}}-\frac{3c\left(-\frac{x}{d\sqrt{d\,x^2+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{d^{\frac{3}{2}}}\right)}{2d}\right)}{4d}\right)}{6d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/6*x^7/d/(d*x^2+c)^(1/2)-7/6*c/d*(1/4*x^5/d/(d*x^2+c)^(1/2)-5/4*c/d*(
1/2*x^3/d/(d*x^2+c)^(1/2)-3/2*c/d*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1
/2)+(d*x^2+c)^(1/2))))))+2*a*b*(1/4*x^5/d/(d*x^2+c)^(1/2)-5/4*c/d*(1/2*x^3/
d/(d*x^2+c)^(1/2)-3/2*c/d*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+(d*x
^2+c)^(1/2)))))+a^2*(1/2*x^3/d/(d*x^2+c)^(1/2)-3/2*c/d*(-x/d/(d*x^2+c)^(1/2
)+1/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))
```

Maxima [A]
time = 0.28, size = 241, normalized size = 1.22

$$\frac{b^2x^7}{6\sqrt{dx^2+c}\,d} - \frac{7\,b^2cx^5}{24\sqrt{dx^2+c}\,d^2} + \frac{abx^5}{2\sqrt{dx^2+c}\,d} + \frac{35\,b^2c^2x^3}{48\sqrt{dx^2+c}\,d^3} - \frac{5\,abcx^3}{4\sqrt{dx^2+c}\,d^2} + \frac{a^2x^3}{2\sqrt{dx^2+c}\,d} + \frac{35\,b^2c^3x}{16\sqrt{dx^2+c}\,d^4} - \frac{15\,abc^2x}{4\sqrt{dx^2+c}\,d^3} + \frac{3\,a^2cx}{2\sqrt{dx^2+c}\,d^2} - \frac{35\,b^2c^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{9}{2}}} + \frac{15\,abc^2\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,d^{\frac{7}{2}}} - \frac{3\,a^2c\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^7/(sqrt(d*x^2 + c)*d) - 7/24*b^2*c*x^5/(sqrt(d*x^2 + c)*d^2) + 1/
2*a*b*x^5/(sqrt(d*x^2 + c)*d) + 35/48*b^2*c^2*x^3/(sqrt(d*x^2 + c)*d^3) - 5
/4*a*b*c*x^3/(sqrt(d*x^2 + c)*d^2) + 1/2*a^2*x^3/(sqrt(d*x^2 + c)*d) + 35/1
6*b^2*c^3*x/(sqrt(d*x^2 + c)*d^4) - 15/4*a*b*c^2*x/(sqrt(d*x^2 + c)*d^3) +
3/2*a^2*c*x/(sqrt(d*x^2 + c)*d^2) - 35/16*b^2*c^3*arcsinh(d*x/sqrt(c*d))/d^
(9/2) + 15/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(7/2) - 3/2*a^2*c*arcsinh(d*x
/sqrt(c*d))/d^(5/2)
```

Fricas [A]
time = 2.12, size = 431, normalized size = 2.19

$$\left[\frac{3\,(35\,b^2c^4 - 60\,abc^3d + 24\,a^2c^2d^2 + (35\,b^2c^3d - 60\,abc^2d^2 + 24\,a^2cd^3)x^2)\sqrt{d}\log\left(-2\,dx^2 + 2\sqrt{dx^2+c}\sqrt{d}x - c\right) + 2\,(8\,b^2d^4x^7 - 2\,(7\,b^2cd^3 - 12\,abd^4)x^5 + (35\,b^2c^2d^2 - 60\,abcd^3 + 24\,a^2d^4)x^3 + 3\,(35\,b^2c^3d - 60\,abc^2d^2 + 24\,a^2cd^3)x)\sqrt{dx^2+c}}{96\,(d^6x^2 + cd^5)}, \frac{3\,(35\,b^2c^4 - 60\,abc^3d + 24\,a^2c^2d^2 + (35\,b^2c^3d - 60\,abc^2d^2 + 24\,a^2cd^3)x^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right) + (8\,b^2d^4x^7 - 2\,(7\,b^2cd^3 - 12\,abd^4)x^5 + (35\,b^2c^2d^2 - 60\,abcd^3 + 24\,a^2d^4)x^3 + 3\,(35\,b^2c^3d - 60\,abc^2d^2 + 24\,a^2cd^3)x)\sqrt{dx^2+c}}{48\,(d^6x^2 + cd^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(35*b^2*c^4 - 60*a*b*c^3*d + 24*a^2*c^2*d^2 + (35*b^2*c^3*d - 60*a
*b*c^2*d^2 + 24*a^2*c*d^3)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sq
rt(d)*x - c) + 2*(8*b^2*d^4*x^7 - 2*(7*b^2*c*d^3 - 12*a*b*d^4)*x^5 + (35*b^
2*c^2*d^2 - 60*a*b*c*d^3 + 24*a^2*d^4)*x^3 + 3*(35*b^2*c^3*d - 60*a*b*c^2*d
^2 + 24*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(d^6*x^2 + c*d^5), 1/48*(3*(35*b^2*c
^4 - 60*a*b*c^3*d + 24*a^2*c^2*d^2 + (35*b^2*c^3*d - 60*a*b*c^2*d^2 + 24*a^
2*c*d^3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^4*x^7
- 2*(7*b^2*c*d^3 - 12*a*b*d^4)*x^5 + (35*b^2*c^2*d^2 - 60*a*b*c*d^3 + 24*a^
2*d^4)*x^3 + 3*(35*b^2*c^3*d - 60*a*b*c^2*d^2 + 24*a^2*c*d^3)*x)*sqrt(d*x^2
 + c))/(d^6*x^2 + c*d^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [A]
time = 1.18, size = 175, normalized size = 0.89

$$\frac{\left(\left(2\left(\frac{4\,b^2x^2}{d}-\frac{7\,b^2cd^5-12\,abd^6}{d^7}\right)x^2+\frac{35\,b^2c^2d^4-60\,abcd^5+24\,a^2d^6}{d^7}\right)x^2+\frac{3\,(35\,b^2c^3d^3-60\,abc^2d^4+24\,a^2cd^5)}{d^7}\right)x}{48\,\sqrt{dx^2+c}}+\frac{(35\,b^2c^3-60\,abc^2d+24\,a^2cd^2)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*b^2*x^2/d - (7*b^2*c*d^5 - 12*a*b*d^6)/d^7)*x^2 + (35*b^2*c^2*d
^4 - 60*a*b*c*d^5 + 24*a^2*d^6)/d^7)*x^2 + 3*(35*b^2*c^3*d^3 - 60*a*b*c^2*d
^4 + 24*a^2*c*d^5)/d^7)*x/sqrt(d*x^2 + c) + 1/16*(35*b^2*c^3 - 60*a*b*c^2*d
 + 24*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.649 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=108

$$\frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{(bc-ad)(3bc-ad)\sqrt{c+dx^2}}{d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^4}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^4}$$

```
[Out] -1/3*b*(-2*a*d+3*b*c)*(d*x^2+c)^(3/2)/d^4+1/5*b^2*(d*x^2+c)^(5/2)/d^4+c*(-a
*d+b*c)^2/d^4/(d*x^2+c)^(1/2)+(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2+c)^(1/2)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {457, 78}

$$-\frac{b\left(c+dx^2\right)^{3/2}(3bc-2ad)}{3d^4}+\frac{\sqrt{c+dx^2}\,(bc-ad)(3bc-ad)}{d^4}+\frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (c*(b*c - a*d)^2)/(d^4*Sqrt[c + d*x^2]) + ((b*c - a*d)*(3*b*c - a*d)*Sqrt[c
 + d*x^2])/d^4 - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^4) + (b^2*(c +
d*x^2)^(5/2))/(5*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^3(c+dx)^{3/2}} + \frac{(bc-ad)(3bc-ad)}{d^3\sqrt{c+dx}} - \frac{b(3bc-2ad)\sqrt{c+dx}}{d^3} + \frac{b^2(c\right.\right.$$

$$= \frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}} + \frac{(bc-ad)(3bc-ad)\sqrt{c+dx^2}}{d^4} - \frac{b(3bc-2ad)\,(c+dx^2)^{3/2}}{3d^4} + \frac{b^2(c+}{5d}$$

Mathematica [A]
time = 0.07, size = 97, normalized size = 0.90

$$\frac{15a^2d^2(2c+dx^2)+10abd(-8c^2-4cdx^2+d^2x^4)+3b^2(16c^3+8c^2dx^2-2cd^2x^4+d^3x^6)}{15d^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (15*a^2*d^2*(2*c + d*x^2) + 10*a*b*d*(-8*c^2 - 4*c*d*x^2 + d^2*x^4) + 3*b^2
*(16*c^3 + 8*c^2*d*x^2 - 2*c*d^2*x^4 + d^3*x^6))/(15*d^4*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.11, size = 182, normalized size = 1.69

method	result
risch	$\frac{(3b^2x^4d^2+10ab\,d^2x^2-9b^2cd\,x^2+15a^2d^2-50abcd+33b^2c^2)\sqrt{d\,x^2+c}}{15d^4}+\frac{c(a^2d^2-2abcd+b^2c^2)}{\sqrt{d\,x^2+c}\;d^4}$
gosper	$\frac{3b^2x^6d^3+10ab\,d^3x^4-6b^2c\,d^2x^4+15a^2d^3x^2-40abc\,d^2x^2+24b^2c^2d\,x^2+30a^2c\,d^2-80ab\,c^2d+48b^2c^3}{15\sqrt{d\,x^2+c}\;d^4}$
trager	$\frac{3b^2x^6d^3+10ab\,d^3x^4-6b^2c\,d^2x^4+15a^2d^3x^2-40abc\,d^2x^2+24b^2c^2d\,x^2+30a^2c\,d^2-80ab\,c^2d+48b^2c^3}{15\sqrt{d\,x^2+c}\;d^4}$
default	$b^2\left(\frac{x^6}{5d\sqrt{d\,x^2+c}}-\frac{6c\left(\frac{x^4}{3d\sqrt{d\,x^2+c}}-\frac{4c\left(\frac{x^2}{d\sqrt{d\,x^2+c}}+\frac{2c}{d^2\sqrt{d\,x^2+c}}\right)}{3d}\right)}{5d}\right)+2ab\left(\frac{x^4}{3d\sqrt{d\,x^2+c}}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/5*x^6/d/(d*x^2+c)^(1/2)-6/5*c/d*(1/3*x^4/d/(d*x^2+c)^(1/2)-4/3*c/d*(
x^2/d/(d*x^2+c)^(1/2)+2*c/d^2/(d*x^2+c)^(1/2))))+2*a*b*(1/3*x^4/d/(d*x^2+c)
^(1/2)-4/3*c/d*(x^2/d/(d*x^2+c)^(1/2)+2*c/d^2/(d*x^2+c)^(1/2)))+a^2*(x^2/d/
(d*x^2+c)^(1/2)+2*c/d^2/(d*x^2+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 180, normalized size = 1.67

$$\frac{b^2x^6}{5\sqrt{dx^2+c}\,d}-\frac{2\,b^2cx^4}{5\sqrt{dx^2+c}\,d^2}+\frac{2\,abx^4}{3\sqrt{dx^2+c}\,d}+\frac{8\,b^2c^2x^2}{5\sqrt{dx^2+c}\,d^3}-\frac{8\,abcx^2}{3\sqrt{dx^2+c}\,d^2}+\frac{a^2x^2}{\sqrt{dx^2+c}\,d}+\frac{16\,b^2c^3}{5\sqrt{dx^2+c}\,d^4}-\frac{16\,abc^2}{3\sqrt{dx^2+c}\,d^3}+\frac{2\,a^2c}{\sqrt{dx^2+c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^6/(sqrt(d*x^2 + c)*d) - 2/5*b^2*c*x^4/(sqrt(d*x^2 + c)*d^2) + 2/3
*a*b*x^4/(sqrt(d*x^2 + c)*d) + 8/5*b^2*c^2*x^2/(sqrt(d*x^2 + c)*d^3) - 8/3*
a*b*c*x^2/(sqrt(d*x^2 + c)*d^2) + a^2*x^2/(sqrt(d*x^2 + c)*d) + 16/5*b^2*c^
3/(sqrt(d*x^2 + c)*d^4) - 16/3*a*b*c^2/(sqrt(d*x^2 + c)*d^3) + 2*a^2*c/(sqr
t(d*x^2 + c)*d^2)
```

Fricas [A]
time = 1.35, size = 115, normalized size = 1.06

$$\frac{\left(3\,b^2d^3x^6+48\,b^2c^3-80\,abc^2d+30\,a^2cd^2-2\,(3\,b^2cd^2-5\,abd^3)x^4+(24\,b^2c^2d-40\,abcd^2+15\,a^2d^3)x^2\right)\sqrt{dx^2+c}}{15\,(d^5x^2+cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^3*x^6 + 48*b^2*c^3 - 80*a*b*c^2*d + 30*a^2*c*d^2 - 2*(3*b^2*c
*d^2 - 5*a*b*d^3)*x^4 + (24*b^2*c^2*d - 40*a*b*c*d^2 + 15*a^2*d^3)*x^2)*sqr
t(d*x^2 + c)/(d^5*x^2 + c*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(97) = 194$.
time = 0.48, size = 236, normalized size = 2.19

$$\begin{cases}\frac{2a^2c}{d^2\sqrt{c+dx^2}}+\frac{a^2x^2}{d\sqrt{c+dx^2}}-\frac{16abc^2}{3d^3\sqrt{c+dx^2}}-\frac{8abcx^2}{3d^2\sqrt{c+dx^2}}+\frac{2abx^4}{3d\sqrt{c+dx^2}}+\frac{16b^2c^3}{5d^4\sqrt{c+dx^2}}+\frac{8b^2c^2x^2}{5d^3\sqrt{c+dx^2}}-\frac{2b^2cx^4}{5d^2\sqrt{c+dx^2}}+\frac{b^2x^6}{5d\sqrt{c+dx^2}} & \text{for}\ d\neq 0\\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((2*a**2*c/(d**2*sqrt(c + d*x**2)) + a**2*x**2/(d*sqrt(c + d*x**2)
) - 16*a*b*c**2/(3*d**3*sqrt(c + d*x**2)) - 8*a*b*c*x**2/(3*d**2*sqrt(c + d
*x**2)) + 2*a*b*x**4/(3*d*sqrt(c + d*x**2)) + 16*b**2*c**3/(5*d**4*sqrt(c +
 d*x**2)) + 8*b**2*c**2*x**2/(5*d**3*sqrt(c + d*x**2)) - 2*b**2*c*x**4/(5*d
```

```
**2*sqrt(c + d*x**2)) + b**2*x**6/(5*d*sqrt(c + d*x**2)), Ne(d, 0)), ((a**2
*x**4/4 + a*b*x**6/3 + b**2*x**8/8)/c**(3/2), True))
```

Giac [A]
time = 0.73, size = 149, normalized size = 1.38

$$\frac{b^2c^3 - 2\,abc^2d + a^2cd^2}{\sqrt{dx^2 + c}\,d^4} + \frac{3\,(dx^2 + c)^{\frac{5}{2}}b^2d^{16} - 15\,(dx^2 + c)^{\frac{3}{2}}b^2cd^{16} + 45\,\sqrt{dx^2 + c}\,b^2c^2d^{16} + 10\,(dx^2 + c)^{\frac{3}{2}}abd^{17} - 60\,\sqrt{dx^2 + c}\,abcd^{17} + 15\,\sqrt{dx^2 + c}\,a^2d^{18}}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(sqrt(d*x^2 + c)*d^4) + 1/15*(3*(d*x^2
+ c)^(5/2)*b^2*d^16 - 15*(d*x^2 + c)^(3/2)*b^2*c*d^16 + 45*sqrt(d*x^2 + c)*
b^2*c^2*d^16 + 10*(d*x^2 + c)^(3/2)*a*b*d^17 - 60*sqrt(d*x^2 + c)*a*b*c*d^1
7 + 15*sqrt(d*x^2 + c)*a^2*d^18)/d^20
```

Mupad [B]
time = 0.42, size = 107, normalized size = 0.99

$$\frac{30\,a^2\,c\,d^2 + 15\,a^2\,d^3\,x^2 - 80\,a\,b\,c^2\,d - 40\,a\,b\,c\,d^2\,x^2 + 10\,a\,b\,d^3\,x^4 + 48\,b^2\,c^3 + 24\,b^2\,c^2\,d\,x^2 - 6\,b^2\,c\,d^2\,x^4 + 3\,b^2\,d^3\,x^6}{15\,d^4\,\sqrt{d\,x^2 + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

```
[Out] (48*b^2*c^3 + 30*a^2*c*d^2 + 15*a^2*d^3*x^2 + 3*b^2*d^3*x^6 + 24*b^2*c^2*d*
x^2 - 6*b^2*c*d^2*x^4 - 80*a*b*c^2*d + 10*a*b*d^3*x^4 - 40*a*b*c*d^2*x^2)/(
15*d^4*(c + d*x^2)^(1/2))
```

3.650 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}}-\frac{(15b^2c^2-24abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{8cd^3}+\frac{b^2x^3\sqrt{c+dx^2}}{4d^2}+\frac{(15b^2c^2-24abcd+8a^2d^2)\tanh^{-1}}{8d^{7/2}}$$

```
[Out] 1/8*(8*a^2*d^2-24*a*b*c*d+15*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^
(7/2)+(-a*d+b*c)^2*x^3/c/d^2/(d*x^2+c)^(1/2)-1/8*(8*a^2*d^2-24*a*b*c*d+15*b
^2*c^2)*x*(d*x^2+c)^(1/2)/c/d^3+1/4*b^2*x^3*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {474, 470, 327, 223, 212}

$$\frac{(8a^2d^2-24abcd+15b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{7/2}}-\frac{x\sqrt{c+dx^2}\,(8a^2d^2-24abcd+15b^2c^2)}{8cd^3}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^3\sqrt{c+dx^2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*x^3)/(c*d^2*Sqrt[c + d*x^2]) - ((15*b^2*c^2 - 24*a*b*c*d + 8
*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^3) + (b^2*x^3*Sqrt[c + d*x^2])/(4*d^2)
+ ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(8*d^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{\int \frac{x^2(-a^2d^2+3(bc-ad)^2-b^2cdx^2)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} - \frac{(15b^2c^2-24abcd+8a^2d^2)\int \frac{x^2}{\sqrt{c+dx^2}}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{(15b^2c^2-24abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{(15b^2c^2}{}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{(15b^2c^2-24abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{(15b^2c^2}{}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{(15b^2c^2-24abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{(15b^2c^2}{}$$

Mathematica [A]
time = 0.20, size = 121, normalized size = 0.80

$$\frac{\frac{\sqrt{d}\,x(-8a^2d^2+8abd(3c+dx^2)+b^2(-15c^2-5cdx^2+2d^2x^4))}{\sqrt{c+dx^2}} + (-15b^2c^2+24abcd-8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{8d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((Sqrt[d]*x*(-8*a^2*d^2 + 8*a*b*d*(3*c + d*x^2) + b^2*(-15*c^2 - 5*c*d*x^2
+ 2*d^2*x^4)))/Sqrt[c + d*x^2] + (-15*b^2*c^2 + 24*a*b*c*d - 8*a^2*d^2)*Log
[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*d^(7/2))
```

Maple [A]
time = 0.11, size = 194, normalized size = 1.28

method	result
risch	$\frac{bx\left(2bd\,x^{2}+8ad-7bc\right)\sqrt{d\,x^{2}+c}}{8d^{3}}-\frac{x\,a^{2}}{d\sqrt{d\,x^{2}+c}}+\frac{2xabc}{d^{2}\sqrt{d\,x^{2}+c}}-\frac{x\,b^{2}c^{2}}{d^{3}\sqrt{d\,x^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}$
default	$b^{2}\left(\frac{x^{5}}{4d\sqrt{d\,x^{2}+c}}-\frac{5c\left(\frac{x^{3}}{2d\sqrt{d\,x^{2}+c}}-\frac{3c\left(-\frac{x}{d\sqrt{d\,x^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}\right)}{2d}\right)}{4d}\right)+2ab\left(\frac{x^{3}}{2d\sqrt{d\,x^{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/4*x^5/d/(d*x^2+c)^(1/2)-5/4*c/d*(1/2*x^3/d/(d*x^2+c)^(1/2)-3/2*c/d*(
-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+2*a*b*(1/2*
x^3/d/(d*x^2+c)^(1/2)-3/2*c/d*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+
(d*x^2+c)^(1/2))))+a^2*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+(d*x^2+
c)^(1/2)))
```

Maxima [A]
time = 0.30, size = 170, normalized size = 1.12

$$\frac{b^{2}x^{5}}{4\sqrt{dx^{2}+c}\,d}-\frac{5\,b^{2}cx^{3}}{8\sqrt{dx^{2}+c}\,d^{2}}+\frac{abx^{3}}{\sqrt{dx^{2}+c}\,d}-\frac{15\,b^{2}c^{2}x}{8\sqrt{dx^{2}+c}\,d^{3}}+\frac{3\,abcx}{\sqrt{dx^{2}+c}\,d^{2}}-\frac{a^{2}x}{\sqrt{dx^{2}+c}\,d}+\frac{15\,b^{2}c^{2}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{7}{2}}}-\frac{3\,abc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}+\frac{a^{2}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^5/(sqrt(d*x^2 + c)*d) - 5/8*b^2*c*x^3/(sqrt(d*x^2 + c)*d^2) + a*b
*x^3/(sqrt(d*x^2 + c)*d) - 15/8*b^2*c^2*x/(sqrt(d*x^2 + c)*d^3) + 3*a*b*c*x
```

```
/(sqrt(d*x^2 + c)*d^2) - a^2*x/(sqrt(d*x^2 + c)*d) + 15/8*b^2*c^2*arcsinh(d
*x/sqrt(c*d))/d^(7/2) - 3*a*b*c*arcsinh(d*x/sqrt(c*d))/d^(5/2) + a^2*arcsin
h(d*x/sqrt(c*d))/d^(3/2)
```

Fricas [A]
time = 1.49, size = 350, normalized size = 2.30

$$\left[\frac{(15\,b^2c^3 - 24\,abc^2d + 8\,a^2cd^2 + (15\,b^2c^2d - 24\,abcd^2 + 8\,a^2d^3)x^2)\sqrt{d}\,\log\left(-2\,dx^2 - 2\,\sqrt{dx^2+c}\,\sqrt{d}\,x - c\right) + 2\,(2\,b^2d^3x^5 - (5\,b^2cd^2 - 8\,abd^3)x^3 - (15\,b^2c^2d - 24\,abcd^2 + 8\,a^2d^3)x)\sqrt{dx^2+c}}{16\,(d^5x^2 + cd^4)}, -\frac{(15\,b^2c^3 - 24\,abc^2d + 8\,a^2cd^2 + (15\,b^2c^2d - 24\,abcd^2 + 8\,a^2d^3)x^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right) - (2\,b^2d^3x^5 - (5\,b^2cd^2 - 8\,abd^3)x^3 - (15\,b^2c^2d - 24\,abcd^2 + 8\,a^2d^3)x)\sqrt{dx^2+c}}{8\,(d^5x^2 + cd^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((15*b^2*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2 + (15*b^2*c^2*d - 24*a*b*c*
d^2 + 8*a^2*d^3)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(2*b^2*d^3*x^5 - (5*b^2*c*d^2 - 8*a*b*d^3)*x^3 - (15*b^2*c^2*d - 24*
a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/(d^5*x^2 + c*d^4), -1/8*((15*b^2
*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2 + (15*b^2*c^2*d - 24*a*b*c*d^2 + 8*a^2*d^
3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^3*x^5 - (5*b
^2*c*d^2 - 8*a*b*d^3)*x^3 - (15*b^2*c^2*d - 24*a*b*c*d^2 + 8*a^2*d^3)*x)*sq
rt(d*x^2 + c))/(d^5*x^2 + c*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + bx^2)^2}{(c + dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [A]
time = 0.56, size = 131, normalized size = 0.86

$$\frac{\left(\left(\frac{2\,b^2x^2}{d} - \frac{5\,b^2cd^3 - 8\,abd^4}{d^5}\right)x^2 - \frac{15\,b^2c^2d^2 - 24\,abcd^3 + 8\,a^2d^4}{d^5}\right)x}{8\,\sqrt{dx^2+c}} - \frac{(15\,b^2c^2 - 24\,abcd + 8\,a^2d^2)\log\left(\left|-\sqrt{d}\,x + \sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*b^2*x^2/d - (5*b^2*c*d^3 - 8*a*b*d^4)/d^5)*x^2 - (15*b^2*c^2*d^2 -
24*a*b*c*d^3 + 8*a^2*d^4)/d^5)*x/sqrt(d*x^2 + c) - 1/8*(15*b^2*c^2 - 24*a*b
*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(b\,x^2+a\right)^2}{\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
[Out] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.651 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=73

$$-\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}-\frac{2b(bc-ad)\sqrt{c+dx^2}}{d^3}+\frac{b^2(c+dx^2)^{3/2}}{3d^3}$$

```
[Out] 1/3*b^2*(d*x^2+c)^(3/2)/d^3-(-a*d+b*c)^2/d^3/(d*x^2+c)^(1/2)-2*b*(-a*d+b*c)
*(d*x^2+c)^(1/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$-\frac{2b\sqrt{c+dx^2}(bc-ad)}{d^3}-\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}+\frac{b^2(c+dx^2)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -((b*c - a*d)^2/(d^3*Sqrt[c + d*x^2])) - (2*b*(b*c - a*d)*Sqrt[c + d*x^2])/
d^3 + (b^2*(c + d*x^2)^(3/2))/(3*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{3/2}} - \frac{2b(bc-ad)}{d^2\sqrt{c+dx}} + \frac{b^2\sqrt{c+dx}}{d^2}\right)dx, x, x^2\right)$$
$$= -\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}} - \frac{2b(bc-ad)\sqrt{c+dx^2}}{d^3} + \frac{b^2(c+dx^2)^{3/2}}{3d^3}$$

Mathematica [A]
time = 0.05, size = 65, normalized size = 0.89

$$\frac{-3a^2d^2+6abd(2c+dx^2)+b^2(-8c^2-4cdx^2+d^2x^4)}{3d^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (-3*a^2*d^2 + 6*a*b*d*(2*c + d*x^2) + b^2*(-8*c^2 - 4*c*d*x^2 + d^2*x^4))/(
3*d^3*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.10, size = 115, normalized size = 1.58

method	result
risch	$\frac{b(bd\,x^2+6ad-5bc)\sqrt{d\,x^2+c}}{3d^3} - \frac{a^2d^2-2abcd+b^2c^2}{\sqrt{d\,x^2+c}\,d^3}$
gosper	$-\frac{-b^2x^4d^2-6ab\,d^2x^2+4b^2cd\,x^2+3a^2d^2-12abcd+8b^2c^2}{3\sqrt{d\,x^2+c}\,d^3}$
trager	$-\frac{-b^2x^4d^2-6ab\,d^2x^2+4b^2cd\,x^2+3a^2d^2-12abcd+8b^2c^2}{3\sqrt{d\,x^2+c}\,d^3}$
default	$b^2\left(\frac{x^4}{3d\sqrt{d\,x^2+c}} - \frac{4c\left(\frac{x^2}{d\sqrt{d\,x^2+c}}+\frac{2c}{d^2\sqrt{d\,x^2+c}}\right)}{3d}\right) + 2ab\left(\frac{x^2}{d\sqrt{d\,x^2+c}} + \frac{2c}{d^2\sqrt{d\,x^2+c}}\right) - \frac{}{d\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/3*x^4/d/(d*x^2+c)^(1/2)-4/3*c/d*(x^2/d/(d*x^2+c)^(1/2)+2*c/d^2/(d*x^
2+c)^(1/2)))+2*a*b*(x^2/d/(d*x^2+c)^(1/2)+2*c/d^2/(d*x^2+c)^(1/2))-a^2/d/(d
*x^2+c)^(1/2)
```

Maxima [A]
time = 0.32, size = 115, normalized size = 1.58

$$\frac{b^2x^4}{3\sqrt{dx^2+c}\,d}-\frac{4\,b^2cx^2}{3\sqrt{dx^2+c}\,d^2}+\frac{2\,abx^2}{\sqrt{dx^2+c}\,d}-\frac{8\,b^2c^2}{3\sqrt{dx^2+c}\,d^3}+\frac{4\,abc}{\sqrt{dx^2+c}\,d^2}-\frac{a^2}{\sqrt{dx^2+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^4/(sqrt(d*x^2 + c)*d) - 4/3*b^2*c*x^2/(sqrt(d*x^2 + c)*d^2) + 2*a
*b*x^2/(sqrt(d*x^2 + c)*d) - 8/3*b^2*c^2/(sqrt(d*x^2 + c)*d^3) + 4*a*b*c/(s
qrt(d*x^2 + c)*d^2) - a^2/(sqrt(d*x^2 + c)*d)
```

Fricas [A]
time = 0.99, size = 79, normalized size = 1.08

$$\frac{\left(b^2d^2x^4-8\,b^2c^2+12\,abcd-3\,a^2d^2-2\,(2\,b^2cd-3\,abd^2)x^2\right)\sqrt{dx^2+c}}{3\,(d^4x^2+cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^2*x^4 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*c*d - 3*a*
b*d^2)*x^2)*sqrt(d*x^2 + c)/(d^4*x^2 + c*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(63) = 126.
time = 0.40, size = 155, normalized size = 2.12

$$\begin{cases} -\frac{a^2}{d\sqrt{c+dx^2}}+\frac{4abc}{d^2\sqrt{c+dx^2}}+\frac{2abx^2}{d\sqrt{c+dx^2}}-\frac{8b^2c^2}{3d^3\sqrt{c+dx^2}}-\frac{4b^2cx^2}{3d^2\sqrt{c+dx^2}}+\frac{b^2x^4}{3d\sqrt{c+dx^2}} & \text{for}\ d\neq 0 \\ \frac{\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((-a**2/(d*sqrt(c + d*x**2)) + 4*a*b*c/(d**2*sqrt(c + d*x**2)) + 2
*a*b*x**2/(d*sqrt(c + d*x**2)) - 8*b**2*c**2/(3*d**3*sqrt(c + d*x**2)) - 4*
b**2*c*x**2/(3*d**2*sqrt(c + d*x**2)) + b**2*x**4/(3*d*sqrt(c + d*x**2)), N
e(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/c**(3/2), True))
```

Giac [A]
time = 0.53, size = 92, normalized size = 1.26

$$-\frac{b^2c^2-2\,abcd+a^2d^2}{\sqrt{dx^2+c}\,d^3}+\frac{(dx^2+c)^{\frac{3}{2}}b^2d^6-6\sqrt{dx^2+c}\,b^2cd^6+6\sqrt{dx^2+c}\,abd^7}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*d^3) + 1/3*((d*x^2 + c)^(
3/2)*b^2*d^6 - 6*sqrt(d*x^2 + c)*b^2*c*d^6 + 6*sqrt(d*x^2 + c)*a*b*d^7)/d^9
```

Mupad [B]
time = 0.36, size = 75, normalized size = 1.03

$$\frac{b^2\,{\left(d\,x^2+c\right)}^2-3\,a^2\,d^2-3\,b^2\,c^2-6\,b^2\,c\,\left(d\,x^2+c\right)+6\,a\,b\,d\,\left(d\,x^2+c\right)+6\,a\,b\,c\,d}{3\,d^3\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

```
[Out] (b^2*(c + d*x^2)^2 - 3*a^2*d^2 - 3*b^2*c^2 - 6*b^2*c*(c + d*x^2) + 6*a*b*d*
(c + d*x^2) + 6*a*b*c*d)/(3*d^3*(c + d*x^2)^(1/2))
```

3.652 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{(bc-ad)x(a+bx^2)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}$$

```
[Out] -1/2*b*(-4*a*d+3*b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(5/2)-(-a*d+b*c)
*x*(b*x^2+a)/c/d/(d*x^2+c)^(1/2)+1/2*b*(-2*a*d+3*b*c)*x*(d*x^2+c)^(1/2)/c/d
^2
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {424, 396, 223, 212}

$$-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}+\frac{bx\sqrt{c+dx^2}\,(3bc-2ad)}{2cd^2}-\frac{x(a+bx^2)\,(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

```
[Out] -(((b*c - a*d)*x*(a + b*x^2))/(c*d*Sqrt[c + d*x^2])) + (b*(3*b*c - 2*a*d)*x
*Sqrt[c + d*x^2])/(2*c*d^2) - (b*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(2*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx &= -\frac{(bc-ad)x(a+bx^2)}{cd\sqrt{c+dx^2}}+\frac{\int \frac{abc+b(3bc-2ad)x^2}{\sqrt{c+dx^2}}\,dx}{cd}\\
&= -\frac{(bc-ad)x(a+bx^2)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{(b(3bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2d^2}\\
&= -\frac{(bc-ad)x(a+bx^2)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{(b(3bc-4ad))\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x\right.}{2d^2}\\
&= -\frac{(bc-ad)x(a+bx^2)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 92, normalized size = 0.87

$$\frac{\frac{\sqrt{d}\,x(-4abcd+2a^2d^2+b^2c(3c+dx^2))}{c\sqrt{c+dx^2}}+b(3bc-4ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

```
[Out] ((Sqrt[d]*x*(-4*a*b*c*d + 2*a^2*d^2 + b^2*c*(3*c + d*x^2)))/(c*Sqrt[c + d*x
^2]) + b*(3*b*c - 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*d^(5/2))
```

Maple [A]
time = 0.11, size = 123, normalized size = 1.16

method	result

risch	$\frac{b^2 x \sqrt{d x^2+c}}{2 d^2}-\frac{2 x a b}{d \sqrt{d x^2+c}}+\frac{x b^2 c}{d^2 \sqrt{d x^2+c}}+\frac{2 b \ln \left(x \sqrt{d}+\sqrt{d x^2+c}\right) a}{d^{\frac{3}{2}}}-\frac{3 b^2 \ln \left(x \sqrt{d}+\sqrt{d x^2+\right.}}{2 d^{\frac{5}{2}}}$
default	$b^2\left(\frac{x^3}{2 d \sqrt{d x^2+c}}-\frac{3 c\left(-\frac{x}{d \sqrt{d x^2+c}}+\frac{\ln \left(x \sqrt{d}+\sqrt{d x^2+c}\right)}{d^{\frac{3}{2}}}\right)}{2 d}\right)+2 a b\left(-\frac{x}{d \sqrt{d x^2+c}}+\frac{\ln \left(x \sqrt{d}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/2*x^3/d/(d*x^2+c)^(1/2)-3/2*c/d*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x
*d^(1/2)+(d*x^2+c)^(1/2))))+2*a*b*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1
/2)+(d*x^2+c)^(1/2)))+a^2*x/c/(d*x^2+c)^(1/2)
```

Maxima [A]
time = 0.29, size = 108, normalized size = 1.02

$$\frac{b^2 x^3}{2 \sqrt{d x^2+c}\, d}+\frac{a^2 x}{\sqrt{d x^2+c}\, c}+\frac{3 b^2 c x}{2 \sqrt{d x^2+c}\, d^2}-\frac{2 a b x}{\sqrt{d x^2+c}\, d}-\frac{3 b^2 c \operatorname{arsinh}\left(\frac{d x}{\sqrt{c d}}\right)}{2 d^{\frac{5}{2}}}+\frac{2 a b \operatorname{arsinh}\left(\frac{d x}{\sqrt{c d}}\right)}{d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^3/(sqrt(d*x^2 + c)*d) + a^2*x/(sqrt(d*x^2 + c)*c) + 3/2*b^2*c*x/(
sqrt(d*x^2 + c)*d^2) - 2*a*b*x/(sqrt(d*x^2 + c)*d) - 3/2*b^2*c*arcsinh(d*x/
sqrt(c*d))/d^(5/2) + 2*a*b*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

Fricas [A]
time = 1.50, size = 275, normalized size = 2.59

$$\left[-\frac{\left(3 b^2 c^3-4 a b c^2 d+\left(3 b^2 c^2 d-4 a b c d^2\right) x^2\right) \sqrt{d} \log \left(-2 d x^2-2 \sqrt{d x^2+c} \sqrt{d} x-c\right)-2\left(b^2 c d^2 x^3+\left(3 b^2 c^2 d-4 a b c d^2+2 a^2 d^3\right) x\right) \sqrt{d x^2+c}}{4\left(c d^4 x^2+c^2 d^3\right)}, \frac{\left(3 b^2 c^3-4 a b c^2 d+\left(3 b^2 c^2 d-4 a b c d^2\right) x^2\right) \sqrt{-d} \arctan \left(\frac{\sqrt{-d} x}{\sqrt{d x^2+c}}\right)+\left(b^2 c d^2 x^3+\left(3 b^2 c^2 d-4 a b c d^2+2 a^2 d^3\right) x\right) \sqrt{d x^2+c}}{2\left(c d^4 x^2+c^2 d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^2)*sqrt(d)*
log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(b^2*c*d^2*x^3 + (3*b^2
*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)*x)*sqrt(d*x^2 + c))/(c*d^4*x^2 + c^2*d^3)
, 1/2*((3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^2)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*c*d^2*x^3 + (3*b^2*c^2*d - 4*a*b
*c*d^2 + 2*a^2*d^3)*x)*sqrt(d*x^2 + c))/(c*d^4*x^2 + c^2*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [A]
time = 0.56, size = 92, normalized size = 0.87

$$\frac{\left(\frac{b^2x^2}{d}+\frac{3\,b^2c^2d-4\,abcd^2+2\,a^2d^3}{cd^3}\right)x}{2\,\sqrt{dx^2+c}}+\frac{(3\,b^2c-4\,abd)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\,\right|\right)}{2\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2/d + (3*b^2*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)/(c*d^3))*x/sqrt(d*
x^2 + c) + 1/2*(3*b^2*c - 4*a*b*d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d
^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2)^2/(c + d*x^2)^(3/2), x)
```

3.653 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

```
[Out] -a^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+(-a*d+b*c)^2/c/d^2/(d*x^2+c)^
(1/2)+b^2*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 89, 65, 214}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*c - a*d)^2/(c*d^2*Sqrt[c + d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^2 - (a^2*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x\,(c+dx^2)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{(bc-ad)^2}{cd(c+dx)^{3/2}}+\frac{b^2}{d\sqrt{c+dx}}+\frac{a^2}{cx\sqrt{c+dx}}\right)dx,x,x^2\right)\\
&= \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2c}\\
&= \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{cd}\\
&= \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 75, normalized size = 1.00

$$\frac{-2abcd+a^2d^2+b^2c(2c+dx^2)}{cd^2\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a*b*c*d + a^2*d^2 + b^2*c*(2*c + d*x^2))/(c*d^2*Sqrt[c + d*x^2]) - (a^2
*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)
```

Maple [A]
time = 0.10, size = 100, normalized size = 1.33

method	result

default	$b^2\left(\frac{x^2}{d\sqrt{dx^2+c}}+\frac{2c}{d^2\sqrt{dx^2+c}}\right)-\frac{2ab}{d\sqrt{dx^2+c}}+a^2\left(\frac{1}{c\sqrt{dx^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{dx^2+c}}{x}\right)}{c^{\frac{3}{2}}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(x^2/d/(d*x^2+c)^(1/2)+2*c/d^2/(d*x^2+c)^(1/2))-2*a*b/d/(d*x^2+c)^(1/2)
+a^2*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))
```

Maxima [A]
time = 0.31, size = 90, normalized size = 1.20

$$\frac{b^2x^2}{\sqrt{dx^2+c}\,d}-\frac{a^2\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{3}{2}}}+\frac{a^2}{\sqrt{dx^2+c}\,c}+\frac{2\,b^2c}{\sqrt{dx^2+c}\,d^2}-\frac{2\,ab}{\sqrt{dx^2+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*x^2/(sqrt(d*x^2 + c)*d) - a^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + a
^2/(sqrt(d*x^2 + c)*c) + 2*b^2*c/(sqrt(d*x^2 + c)*d^2) - 2*a*b/(sqrt(d*x^2
+ c)*d)
```

Fricas [A]
time = 1.48, size = 232, normalized size = 3.09

$$\left[\frac{(a^2d^3x^2+a^2cd^2)\sqrt{c}\,\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\,\sqrt{c}+2c}{x^2}\right)+2\,(b^2c^2dx^2+2\,b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{dx^2+c}}{2\,(c^2d^3x^2+c^3d^2)},\ \frac{(a^2d^3x^2+a^2cd^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(b^2c^2dx^2+2\,b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{dx^2+c}}{c^2d^3x^2+c^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*d^3*x^2 + a^2*c*d^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqr
t(c) + 2*c)/x^2) + 2*(b^2*c^2*d*x^2 + 2*b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*
sqrt(d*x^2 + c))/(c^2*d^3*x^2 + c^3*d^2), ((a^2*d^3*x^2 + a^2*c*d^2)*sqrt(-
c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b^2*c^2*d*x^2 + 2*b^2*c^3 - 2*a*b*c^
2*d + a^2*c*d^2)*sqrt(d*x^2 + c))/(c^2*d^3*x^2 + c^3*d^2)]
```

Sympy [A]
time = 14.81, size = 70, normalized size = 0.93

$$\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{c\sqrt{-c}}+\frac{b^2\sqrt{c+dx^2}}{d^2}+\frac{(ad-bc)^2}{cd^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*atan(sqrt(c + d*x**2)/sqrt(-c))/(c*sqrt(-c)) + b**2*sqrt(c + d*x**2)/d
**2 + (a*d - b*c)**2/(c*d**2*sqrt(c + d*x**2))
```

Giac [A]
time = 0.59, size = 82, normalized size = 1.09

$$\frac{a^2 \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c} + \frac{\sqrt{dx^2+c}\,b^2}{d^2} + \frac{b^2c^2 - 2\,abcd + a^2d^2}{\sqrt{dx^2+c}\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) + sqrt(d*x^2 + c)*b^2/d^2
 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*c*d^2)
```

Mupad [B]
time = 0.41, size = 76, normalized size = 1.01

$$\frac{b^2\,\sqrt{d\,x^2+c}}{d^2} - \frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{c^{3/2}} + \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{c\,d^2\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x)
```

```
[Out] (b^2*(c + d*x^2)^(1/2))/d^2 - (a^2*atanh((c + d*x^2)^(1/2)/c^(1/2)))/c^(3/2
) + (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(c*d^2*(c + d*x^2)^(1/2))
```

3.654 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{(b^2c^2-2ad(bc-ad))\,x}{c^2d\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

```
[Out] b^2*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-a^2/c/x/(d*x^2+c)^(1/2)-(b^2
*c^2-2*a*d*(-a*d+b*c))*x/c^2/d/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {473, 393, 223, 212}

$$-\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{x\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)}{\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(a^2/(c*x*Sqrt[c + d*x^2])) - ((b^2/d - (2*a*(b*c - a*d))/c^2)*x)/Sqrt[c +
 d*x^2] + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/d^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx &= -\frac{a^2}{cx\sqrt{c+dx^2}}+\frac{\int \frac{2a(bc-ad)+b^2cx^2}{(c+dx^2)^{3/2}}\,dx}{c}\\
&= -\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)x}{\sqrt{c+dx^2}}+\frac{b^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{d}\\
&= -\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)x}{\sqrt{c+dx^2}}+\frac{b^2\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{d}\\
&= -\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)x}{\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 88, normalized size = 0.97

$$
\frac{-b^2c^2x^2+2abcdx^2-a^2d(c+2dx^2)}{c^2dx\sqrt{c+dx^2}}-\frac{b^2\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{d^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(b^2*c^2*x^2) + 2*a*b*c*d*x^2 - a^2*d*(c + 2*d*x^2))/(c^2*d*x*Sqrt[c + d*
x^2]) - (b^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/d^(3/2)
```

Maple [A]
time = 0.11, size = 97, normalized size = 1.07

method	result

default	$b^{2}\left(-\frac{x}{d\sqrt{d\,x^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}\right)+\frac{2abx}{c\sqrt{d\,x^{2}+c}}+a^{2}\left(-\frac{1}{cx\sqrt{d\,x^{2}+c}}-\frac{2dx}{c^{2}\sqrt{d\,x^{2}+c}}\right.$
risch	$-\frac{a^{2}\sqrt{d\,x^{2}+c}}{c^{2}x}-\frac{b^{2}x}{d\sqrt{d\,x^{2}+c}}+\frac{b^{2}\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}-\frac{a^{2}dx}{c^{2}\sqrt{d\,x^{2}+c}}+\frac{2abx}{c\sqrt{d\,x^{2}+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))+2*a*b*x/
c/(d*x^2+c)^(1/2)+a^2*(-1/c/x/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2))
```

Maxima [A]
time = 0.30, size = 91, normalized size = 1.00

$$\frac{2\,abx}{\sqrt{dx^{2}+c}\,c}-\frac{b^{2}x}{\sqrt{dx^{2}+c}\,d}-\frac{2\,a^{2}dx}{\sqrt{dx^{2}+c}\,c^{2}}+\frac{b^{2}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}}-\frac{a^{2}}{\sqrt{dx^{2}+c}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*a*b*x/(sqrt(d*x^2 + c)*c) - b^2*x/(sqrt(d*x^2 + c)*d) - 2*a^2*d*x/(sqrt(d
*x^2 + c)*c^2) + b^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) - a^2/(sqrt(d*x^2 + c)*
c*x)
```

Fricas [A]
time = 1.29, size = 239, normalized size = 2.63

$$\left[\frac{\left(b^{2}c^{2}dx^{3}+b^{2}c^{3}x\right)\sqrt{d}\,\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)-2\,\left(a^{2}cd^{2}+\left(b^{2}c^{2}d-2\,abcd^{2}+2\,a^{2}d^{3}\right)x^{2}\right)\sqrt{dx^{2}+c}}{2\,\left(c^{2}d^{3}x^{3}+c^{3}d^{2}x\right)},-\frac{\left(b^{2}c^{2}dx^{3}+b^{2}c^{3}x\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)+\left(a^{2}cd^{2}+\left(b^{2}c^{2}d-2\,abcd^{2}+2\,a^{2}d^{3}\right)x^{2}\right)\sqrt{dx^{2}+c}}{c^{2}d^{3}x^{3}+c^{3}d^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*
sqrt(d)*x - c) - 2*(a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + 2*a^2*d^3)*x^2)*
sqrt(d*x^2 + c))/(c^2*d^3*x^3 + c^3*d^2*x), -((b^2*c^2*d*x^3 + b^2*c^3*x)*s
qrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a^2*c*d^2 + (b^2*c^2*d - 2*a*
b*c*d^2 + 2*a^2*d^3)*x^2)*sqrt(d*x^2 + c))/(c^2*d^3*x^3 + c^3*d^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^{2}\right)^{2}}{x^{2}\left(c+dx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 0.57, size = 104, normalized size = 1.14

$$-\frac{b^2\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{2\,d^{\frac{3}{2}}}+\frac{2\,a^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)c}-\frac{(b^2c^2-2\,abcd+a^2d^2)x}{\sqrt{dx^2+c}\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*b^2*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^(3/2) + 2*a^2*sqrt(d)/(((sq
rt(d)*x - sqrt(d*x^2 + c))^2 - c)*c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(s
qrt(d*x^2 + c)*c^2*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2}{x^2\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)), x)
```

3.655 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{4ab-\frac{2b^2c}{d}-\frac{3a^2d}{c}}{2c\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}-\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*a*(-3*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(5/2)+1/2*(4*a*b-2
*b^2*c/d-3*a^2*d/c)/c/(d*x^2+c)^(1/2)-1/2*a^2/c/x^2/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 91, 79, 65, 214}

$$-\frac{3a^2d^2-4abcd+2b^2c^2}{2c^2d\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}-\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)/(c^2*d*Sqrt[c + d*x^2]) - a^2/(2*c
*x^2*Sqrt[c + d*x^2]) - (a*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
)/(2*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{a^2}{2cx^2\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4bc-3ad)+b^2cx}{x(c+dx)^{3/2}}\,dx, x, x^2\right)}{2c}\\
&= \frac{4ab-\frac{2b^2c}{d}-\frac{3a^2d}{c}}{2c\sqrt{c+dx^2}} - \frac{a^2}{2cx^2\sqrt{c+dx^2}} + \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4c^2}\\
&= \frac{4ab-\frac{2b^2c}{d}-\frac{3a^2d}{c}}{2c\sqrt{c+dx^2}} - \frac{a^2}{2cx^2\sqrt{c+dx^2}} + \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right.}{2c^2d}\\
&= \frac{4ab-\frac{2b^2c}{d}-\frac{3a^2d}{c}}{2c\sqrt{c+dx^2}} - \frac{a^2}{2cx^2\sqrt{c+dx^2}} - \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 97, normalized size = 0.94

$$\frac{-\frac{\sqrt{c}\,(2b^2c^2x^2-4abcdx^2+a^2d(c+3dx^2))}{dx^2\sqrt{c+dx^2}}+a(-4bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-((Sqrt[c]*(2*b^2*c^2*x^2 - 4*a*b*c*d*x^2 + a^2*d*(c + 3*d*x^2)))/(d*x^2*S
qrt[c + d*x^2])) + a*(-4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*
c^(5/2))
```

Maple [A]
time = 0.12, size = 135, normalized size = 1.31

method	result
default	$-\frac{b^2}{d\sqrt{d\,x^2+c}}+a^2\left(-\frac{1}{2c\,x^2\sqrt{d\,x^2+c}}-\frac{3d\left(\frac{1}{c\sqrt{d\,x^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right)+2ab\left(\frac{\ }{c\sqrt{\ }}\right.$
risch	$-\frac{a^2\sqrt{d\,x^2+c}}{2c^2x^2}-\frac{d\,a^2}{c^2\sqrt{d\,x^2+c}}-\frac{b^2}{d\sqrt{d\,x^2+c}}+\frac{2ab}{c\sqrt{d\,x^2+c}}+\frac{3a^2\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)d}{2c^{\frac{5}{2}}}-\frac{2a}{\ }$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b^2/d/(d*x^2+c)^(1/2)+a^2*(-1/2/c/x^2/(d*x^2+c)^(1/2)-3/2*d/c*(1/c/(d*x^2+
c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))+2*a*b*(1/c/(d*x^
2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))
```

Maxima [A]
time = 0.28, size = 112, normalized size = 1.09

$$-\frac{2\,ab\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{3}{2}}}+\frac{3\,a^2d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,c^{\frac{5}{2}}}+\frac{2\,ab}{\sqrt{dx^2+c}\,c}-\frac{b^2}{\sqrt{dx^2+c}\,d}-\frac{3\,a^2d}{2\,\sqrt{dx^2+c}\,c^2}-\frac{a^2}{2\,\sqrt{dx^2+c}\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*a*b*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + 3/2*a^2*d*arcsinh(c/(sqrt(c*
d)*abs(x)))/c^(5/2) + 2*a*b/(sqrt(d*x^2 + c)*c) - b^2/(sqrt(d*x^2 + c)*d) -
 3/2*a^2*d/(sqrt(d*x^2 + c)*c^2) - 1/2*a^2/(sqrt(d*x^2 + c)*c*x^2)
```

Fricas [A]
time = 1.48, size = 292, normalized size = 2.83

$$\left[-\frac{\left(\left(4\,abcd^2-3\,a^2d^3\right)x^4+\left(4\,abc^2d-3\,a^2cd^2\right)x^2\right)\sqrt{c}\,\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\left(a^2c^2d+\left(2\,b^2c^3-4\,abc^2d+3\,a^2cd^2\right)x^2\right)\sqrt{dx^2+c}}{4\left(c^3d^2x^4+c^4dx^2\right)},\ \frac{\left(\left(4\,abcd^2-3\,a^2d^3\right)x^4+\left(4\,abc^2d-3\,a^2cd^2\right)x^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(a^2c^2d+\left(2\,b^2c^3-4\,abc^2d+3\,a^2cd^2\right)x^2\right)\sqrt{dx^2+c}}{2\left(c^3d^2x^4+c^4dx^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sq
rt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*c^2*d +
(2*b^2*c^3 - 4*a*b*c^2*d + 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(c^3*d^2*x^4
+ c^4*d*x^2), 1/2*(((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2*d - 3*a^2*c*
d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (a^2*c^2*d + (2*b^2*c
^3 - 4*a*b*c^2*d + 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(c^3*d^2*x^4 + c^4*d*
x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 0.53, size = 140, normalized size = 1.36

$$\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,\sqrt{-c}\,c^2}-\frac{2\left(dx^2+c\right)b^2c^2-2\,b^2c^3-4\left(dx^2+c\right)abcd+4\,abc^2d+3\left(dx^2+c\right)a^2d^2-2\,a^2cd^2}{2\left(\left(dx^2+c\right)^{\frac{3}{2}}-\sqrt{dx^2+c}\,c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1
/2*(2*(d*x^2 + c)*b^2*c^2 - 2*b^2*c^3 - 4*(d*x^2 + c)*a*b*c*d + 4*a*b*c^2*d
 + 3*(d*x^2 + c)*a^2*d^2 - 2*a^2*c*d^2)/(((d*x^2 + c)^(3/2) - sqrt(d*x^2 +
c)*c)*c^2*d)
```

Mupad [B]
time = 0.52, size = 119, normalized size = 1.16

$$\frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{c}-\frac{\left(d\,x^2+c\right)\,\left(3\,a^2\,d^2-4\,a\,b\,c\,d+2\,b^2\,c^2\right)}{2\,c^2}}{d\,{\left(d\,x^2+c\right)}^{3/2}-c\,d\,\sqrt{d\,x^2+c}}+\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{2\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/c - ((c + d*x^2)*(3*a^2*d^2 + 2*b^2*c^2 -
4*a*b*c*d))/(2*c^2))/(d*(c + d*x^2)^(3/2) - c*d*(c + d*x^2)^(1/2)) + (a*ata
nh((c + d*x^2)^(1/2)/c^(1/2))*(3*a*d - 4*b*c))/(2*c^(5/2))
```

3.656 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{a^2}{3cx^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}+\frac{(3b^2c^2-4ad(3bc-2ad))\,x}{3c^3\sqrt{c+dx^2}}$$

```
[Out] -1/3*a^2/c/x^3/(d*x^2+c)^(1/2)-2/3*a*(-2*a*d+3*b*c)/c^2/x/(d*x^2+c)^(1/2)+1
/3*(3*b^2*c^2-4*a*d*(-2*a*d+3*b*c))*x/c^3/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {473, 464, 197}

$$\frac{x(8a^2d^2-12abcd+3b^2c^2)}{3c^3\sqrt{c+dx^2}}-\frac{a^2}{3cx^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/3*a^2/(c*x^3*Sqrt[c + d*x^2]) - (2*a*(3*b*c - 2*a*d))/(3*c^2*x*Sqrt[c +
d*x^2]) + ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x)/(3*c^3*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx &= -\frac{a^2}{3cx^3\sqrt{c+dx^2}}+\frac{\int \frac{2a(3bc-2ad)+3b^2cx^2}{x^2(c+dx^2)^{3/2}}\,dx}{3c}\\ &= -\frac{a^2}{3cx^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}-\frac{1}{3}\left(-3b^2+\frac{4ad(3bc-2ad)}{c^2}\right)\int \frac{1}{(c+dx^2)^{3/2}}\\ &= -\frac{a^2}{3cx^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}+\frac{\left(3b^2-\frac{4ad(3bc-2ad)}{c^2}\right)x}{3c\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 74, normalized size = 0.76

$$\frac{3b^2c^2x^4-6abcx^2(c+2dx^2)+a^2(-c^2+4cdx^2+8d^2x^4)}{3c^3x^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
[Out] (3*b^2*c^2*x^4 - 6*a*b*c*x^2*(c + 2*d*x^2) + a^2*(-c^2 + 4*c*d*x^2 + 8*d^2*
x^4))/(3*c^3*x^3*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.10, size = 119, normalized size = 1.23

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\;a(-5ad\,x^2+6c\,x^2b+ac)}{3c^3x^3}+\frac{x(a^2d^2-2abcd+b^2c^2)}{\sqrt{d\,x^2+c}\;c^3}$
gosper	$-\frac{-8a^2d^2x^4+12abcd\,x^4-3b^2c^2x^4-4a^2cd\,x^2+6ab\,c^2x^2+a^2c^2}{3x^3\sqrt{d\,x^2+c}\;c^3}$
trager	$-\frac{-8a^2d^2x^4+12abcd\,x^4-3b^2c^2x^4-4a^2cd\,x^2+6ab\,c^2x^2+a^2c^2}{3x^3\sqrt{d\,x^2+c}\;c^3}$
default	$\frac{b^2x}{c\sqrt{d\,x^2+c}}+a^2\left(-\frac{1}{3c\,x^3\sqrt{d\,x^2+c}}-\frac{4d\left(-\frac{1}{cx\sqrt{d\,x^2+c}}-\frac{2dx}{c^2\sqrt{d\,x^2+c}}\right)}{3c}\right)+2ab\left(-\frac{1}{cx\sqrt{d\,x^2+c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/c/(d*x^2+c)^(1/2)+a^2*(-1/3/c/x^3/(d*x^2+c)^(1/2)-4/3*d/c*(-1/c/x/(d*
x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2)))+2*a*b*(-1/c/x/(d*x^2+c)^(1/2)-2*d/
c^2*x/(d*x^2+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 117, normalized size = 1.21

$$\frac{b^2x}{\sqrt{dx^2+c}\,c} - \frac{4\,abdx}{\sqrt{dx^2+c}\,c^2} + \frac{8\,a^2d^2x}{3\,\sqrt{dx^2+c}\,c^3} - \frac{2\,ab}{\sqrt{dx^2+c}\,cx} + \frac{4\,a^2d}{3\,\sqrt{dx^2+c}\,c^2x} - \frac{a^2}{3\,\sqrt{dx^2+c}\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*x/(sqrt(d*x^2 + c)*c) - 4*a*b*d*x/(sqrt(d*x^2 + c)*c^2) + 8/3*a^2*d^2*x
/(sqrt(d*x^2 + c)*c^3) - 2*a*b/(sqrt(d*x^2 + c)*c*x) + 4/3*a^2*d/(sqrt(d*x^
2 + c)*c^2*x) - 1/3*a^2/(sqrt(d*x^2 + c)*c*x^3)
```

Fricas [A]
time = 1.15, size = 85, normalized size = 0.88

$$\frac{((3\,b^2c^2 - 12\,abcd + 8\,a^2d^2)x^4 - a^2c^2 - 2\,(3\,abc^2 - 2\,a^2cd)x^2)\sqrt{dx^2+c}}{3\,(c^3dx^5 + c^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x^4 - a^2*c^2 - 2*(3*a*b*c^2 - 2*
a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d*x^5 + c^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^2}{x^4\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**4*(c + d*x**2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.
time = 0.56, size = 199, normalized size = 2.05

$$\frac{(b^2c^2 - 2\,abcd + a^2d^2)x}{\sqrt{dx^2+c}\,c^3} + \frac{2\left(6\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^4 abc\sqrt{d} - 3\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^4 a^2d^{\frac{3}{2}} - 12\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 abc^2\sqrt{d} + 12\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 a^2cd^{\frac{3}{2}} + 6\,abc^3\sqrt{d} - 5\,a^2c^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 - c\right)^3 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(sqrt(d*x^2 + c)*c^3) + 2/3*(6*(sqrt(d)*x
 - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2
*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 12*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a^2*c*d^(3/2) + 6*a*b*c^3*sqrt(d) - 5*a^2*c^2*d^(3/
2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*c^2)
```

Mupad [B]
time = 0.39, size = 76, normalized size = 0.78

$$-\frac{a^2\,c^2-4\,a^2\,c\,d\,x^2-8\,a^2\,d^2\,x^4+6\,a\,b\,c^2\,x^2+12\,a\,b\,c\,d\,x^4-3\,b^2\,c^2\,x^4}{3\,c^3\,x^3\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x)
```

```
[Out] -(a^2*c^2 - 8*a^2*d^2*x^4 - 3*b^2*c^2*x^4 + 6*a*b*c^2*x^2 - 4*a^2*c*d*x^2 +
 12*a*b*c*d*x^4)/(3*c^3*x^3*(c + d*x^2)^(1/2))
```

3.657 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{8b^2c^2-3ad(8bc-5ad)}{8c^3\sqrt{c+dx^2}}-\frac{a^2}{4cx^4\sqrt{c+dx^2}}-\frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}}-\frac{(8b^2c^2-3ad(8bc-5ad))\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}$$

```
[Out] -1/8*(8*b^2*c^2-3*a*d*(-5*a*d+8*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(7
/2)+1/8*(8*b^2*c^2-3*a*d*(-5*a*d+8*b*c))/c^3/(d*x^2+c)^(1/2)-1/4*a^2/c/x^4/
(d*x^2+c)^(1/2)-1/8*a*(-5*a*d+8*b*c)/c^2/x^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 91, 79, 53, 65, 214}

$$-\frac{a^2}{4cx^4\sqrt{c+dx^2}}+\frac{8b^2-\frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}}-\frac{(8b^2c^2-3ad(8bc-5ad))\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}-\frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

```
[Out] (8*b^2 - (3*a*d*(8*b*c - 5*a*d))/c^2)/(8*c*Sqrt[c + d*x^2]) - a^2/(4*c*x^4*
Sqrt[c + d*x^2]) - (a*(8*b*c - 5*a*d))/(8*c^2*x^2*Sqrt[c + d*x^2]) - ((8*b^
2*c^2 - 3*a*d*(8*b*c - 5*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(7/2)
)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^3(c+dx)^{3/2}}\,dx,x,x^2\right)$$

$$= -\frac{a^2}{4cx^4\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(8bc-5ad)+2b^2cx}{x^2(c+dx)^{3/2}}\,dx,x,x^2\right)}{4c}$$

$$= -\frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} + \frac{1}{16}\left(8b^2 - \frac{3ad(8bc-5ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{x(c+\right.$$

$$= \frac{8b^2-\frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}} - \frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} + \frac{\left(8b^2-\frac{3ad(8bc-5ad)}{c^2}\right)\text{Subst}\left(\right.}{16c}$$

$$= \frac{8b^2-\frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}} - \frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} + \frac{\left(8b^2-\frac{3ad(8bc-5ad)}{c^2}\right)\text{Subst}\left(\right.}{8}$$

$$= \frac{8b^2-\frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}} - \frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} - \frac{\left(8b^2-\frac{3ad(8bc-5ad)}{c^2}\right)\tanh^{-1}}{8c^{3/2}}$$

Mathematica [A]
time = 0.28, size = 125, normalized size = 0.86

$$\frac{\frac{\sqrt{c}\,(8b^2c^2x^4-8abcx^2(c+3dx^2)+a^2(-2c^2+5cdx^2+15d^2x^4))}{x^4\sqrt{c+dx^2}} + (-8b^2c^2+24abcd-15a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
[Out] ((Sqrt[c]*(8*b^2*c^2*x^4 - 8*a*b*c*x^2*(c + 3*d*x^2) + a^2*(-2*c^2 + 5*c*d*
x^2 + 15*d^2*x^4)))/(x^4*Sqrt[c + d*x^2]) + (-8*b^2*c^2 + 24*a*b*c*d - 15*a
^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(7/2))
```

Maple [A]
time = 0.12, size = 212, normalized size = 1.46

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\,a(-7ad\,x^2+8c\,x^2b+2ac)}{8c^3x^4} + \frac{a^2d^2}{c^3\sqrt{d\,x^2+c}} - \frac{2adb}{c^2\sqrt{d\,x^2+c}} + \frac{b^2}{c\sqrt{d\,x^2+c}} - \frac{15\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d}}{x}\right.}{8c^{\frac{7}{2}}}$

default	$a^2\left(-\frac{1}{4c\,x^4\sqrt{d\,x^2+c}}-\frac{5d\left(-\frac{1}{2c\,x^2\sqrt{d\,x^2+c}}-\frac{3d\left(\frac{1}{c\sqrt{d\,x^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right)}{4c}\right)+2ab$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/4/c/x^4/(d*x^2+c)^(1/2)-5/4*d/c*(-1/2/c/x^2/(d*x^2+c)^(1/2)-3/2*d/c
*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))+2*
a*b*(-1/2/c/x^2/(d*x^2+c)^(1/2)-3/2*d/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((
2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))+b^2*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((
2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))
```

Maxima [A]
time = 0.27, size = 177, normalized size = 1.22

$$-\frac{b^2\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{3}{2}}}+\frac{3\,abd\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{5}{2}}}-\frac{15\,a^2d^2\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{8\,c^{\frac{7}{2}}}+\frac{b^2}{\sqrt{dx^2+c}\,c}-\frac{3\,abd}{\sqrt{dx^2+c}\,c^2}+\frac{15\,a^2d^2}{8\,\sqrt{dx^2+c}\,c^3}-\frac{ab}{\sqrt{dx^2+c}\,cx^2}+\frac{5\,a^2d}{8\,\sqrt{dx^2+c}\,c^2x^2}-\frac{a^2}{4\,\sqrt{dx^2+c}\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -b^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + 3*a*b*d*arcsinh(c/(sqrt(c*d)*a
bs(x)))/c^(5/2) - 15/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(7/2) + b^2/
(sqrt(d*x^2 + c)*c) - 3*a*b*d/(sqrt(d*x^2 + c)*c^2) + 15/8*a^2*d^2/(sqrt(d*
x^2 + c)*c^3) - a*b/(sqrt(d*x^2 + c)*c*x^2) + 5/8*a^2*d/(sqrt(d*x^2 + c)*c^
2*x^2) - 1/4*a^2/(sqrt(d*x^2 + c)*c*x^4)
```

Fricas [A]
time = 1.20, size = 364, normalized size = 2.51

$$\left[\frac{((8\,b^2c^2d-24\,abcd^2+15\,a^2d^3)x^6+(8\,b^2c^3-24\,abc^2d+15\,a^2cd^2)x^4)\sqrt{c}\,\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,(2\,a^2c^3-(8\,b^2c^3-24\,abc^2d+15\,a^2cd^2)x^4+(8\,abc^3-5\,a^2c^2d)x^2)\sqrt{dx^2+c}}{16\,(c^4dx^6+c^5x^4)},\ \frac{((8\,b^2c^2d-24\,abcd^2+15\,a^2d^3)x^6+(8\,b^2c^3-24\,abc^2d+15\,a^2cd^2)x^4)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-(2\,a^2c^3-(8\,b^2c^3-24\,abc^2d+15\,a^2cd^2)x^4+(8\,abc^3-5\,a^2c^2d)x^2)\sqrt{dx^2+c}}{8\,(c^4dx^6+c^5x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*b^2*c^2*d - 24*a*b*c*d^2 + 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a*b
*c^2*d + 15*a^2*c*d^2)*x^4)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
 + 2*c)/x^2) - 2*(2*a^2*c^3 - (8*b^2*c^3 - 24*a*b*c^2*d + 15*a^2*c*d^2)*x^4
 + (8*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^4*d*x^6 + c^5*x^4), 1
/8*(((8*b^2*c^2*d - 24*a*b*c*d^2 + 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a*b*c^
2*d + 15*a^2*c*d^2)*x^4)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a^2
*c^3 - (8*b^2*c^3 - 24*a*b*c^2*d + 15*a^2*c*d^2)*x^4 + (8*a*b*c^3 - 5*a^2*c
^2*d)*x^2)*sqrt(d*x^2 + c))/(c^4*d*x^6 + c^5*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{x^5\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**5*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 0.55, size = 163, normalized size = 1.12

$$\frac{(8\,b^2c^2-24\,abcd+15\,a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}\,c^3}+\frac{b^2c^2-2\,abcd+a^2d^2}{\sqrt{dx^2+c}\,c^3}-\frac{8\,(dx^2+c)^{\frac{3}{2}}abcd-8\,\sqrt{dx^2+c}\,abc^2d-7\,(dx^2+c)^{\frac{3}{2}}a^2d^2+9\,\sqrt{dx^2+c}\,a^2cd^2}{8\,c^3d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/
(sqrt(-c)*c^3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*c^3) - 1/
8*(8*(d*x^2 + c)^(3/2)*a*b*c*d - 8*sqrt(d*x^2 + c)*a*b*c^2*d - 7*(d*x^2 + c
)^(3/2)*a^2*d^2 + 9*sqrt(d*x^2 + c)*a^2*c*d^2)/(c^3*d^2*x^4)
```

Mupad [B]
time = 0.58, size = 179, normalized size = 1.23

$$\frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{c}-\frac{(d\,x^2+c)\,(25\,a^2\,d^2-40\,a\,b\,c\,d+16\,b^2\,c^2)}{8\,c^2}+\frac{(d\,x^2+c)^2\,(15\,a^2\,d^2-24\,a\,b\,c\,d+8\,b^2\,c^2)}{8\,c^3}}{(d\,x^2+c)^{5/2}-2\,c\,(d\,x^2+c)^{3/2}+c^2\,\sqrt{d\,x^2+c}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(15\,a^2\,d^2-24\,a\,b\,c\,d+8\,b^2\,c^2)}{8\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/c - ((c + d*x^2)*(25*a^2*d^2 + 16*b^2*c^2
- 40*a*b*c*d))/(8*c^2) + ((c + d*x^2)^2*(15*a^2*d^2 + 8*b^2*c^2 - 24*a*b*c*
d))/(8*c^3))/((c + d*x^2)^(5/2) - 2*c*(c + d*x^2)^(3/2) + c^2*(c + d*x^2)^(
1/2)) - (atanh((c + d*x^2)^(1/2)/c^(1/2))*(15*a^2*d^2 + 8*b^2*c^2 - 24*a*b*
c*d))/(8*c^(7/2))
```

3.658 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{a^2}{5cx^5\sqrt{c+dx^2}}-\frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}}-\frac{15b^2c^2-8ad(5bc-3ad)}{15c^3x\sqrt{c+dx^2}}-\frac{2d(15b^2c^2-8ad(5bc-3ad))\,x}{15c^4\sqrt{c+dx^2}}$$

```
[Out] -1/5*a^2/c/x^5/(d*x^2+c)^(1/2)-2/15*a*(-3*a*d+5*b*c)/c^2/x^3/(d*x^2+c)^(1/2
)+1/15*(-15*b^2*c^2+8*a*d*(-3*a*d+5*b*c))/c^3/x/(d*x^2+c)^(1/2)-2/15*d*(15*
b^2*c^2-8*a*d*(-3*a*d+5*b*c))*x/c^4/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {473, 464, 277, 197}

$$-\frac{a^2}{5cx^5\sqrt{c+dx^2}}-\frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}}-\frac{2dx(15b^2c^2-8ad(5bc-3ad))}{15c^4\sqrt{c+dx^2}}-\frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/5*a^2/(c*x^5*Sqrt[c + d*x^2]) - (2*a*(5*b*c - 3*a*d))/(15*c^2*x^3*Sqrt[c
 + d*x^2]) - (15*b^2 - (8*a*d*(5*b*c - 3*a*d))/c^2)/(15*c*x*Sqrt[c + d*x^2]
) - (2*d*(15*b^2*c^2 - 8*a*d*(5*b*c - 3*a*d))*x)/(15*c^4*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx = -\frac{a^2}{5cx^5\sqrt{c+dx^2}} + \frac{\int \frac{2a(5bc-3ad)+5b^2cx^2}{x^4(c+dx^2)^{3/2}}\,dx}{5c}$$

$$= -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{1}{15}\left(-15b^2+\frac{8ad(5bc-3ad)}{c^2}\right)\int \frac{1}{x^2(c+ \ldots}$$

$$= -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}} - \frac{\left(2d\left(15b^2-\frac{8ad(5bc-3a\ldots}{c^2}\right.\right.}{15\ldots}$$

$$= -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}} - \frac{2d\left(15b^2-\frac{8ad(5bc-3ad\ldots}{c^2}\right.}{15c^2\sqrt{c+dx^2}\ldots}$$

Mathematica [A]
time = 0.16, size = 103, normalized size = 0.73

$$\frac{-15b^2c^2x^4(c+2dx^2)-10abcx^2(c^2-4cdx^2-8d^2x^4)-3a^2(c^3-2c^2dx^2+8cd^2x^4+16d^3x^6)}{15c^4x^5\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-15*b^2*c^2*x^4*(c + 2*d*x^2) - 10*a*b*c*x^2*(c^2 - 4*c*d*x^2 - 8*d^2*x^4)
 - 3*a^2*(c^3 - 2*c^2*d*x^2 + 8*c*d^2*x^4 + 16*d^3*x^6))/(15*c^4*x^5*Sqrt[c
 + d*x^2])
```

Maple [A]
time = 0.11, size = 188, normalized size = 1.33

method	result

risch	$-\frac{\sqrt{d x^2+c}\,(33a^2d^2x^4-50abcd\,x^4+15b^2c^2x^4-9a^2cd\,x^2+10ab\,c^2x^2+3a^2c^2)}{15c^4x^5}-\frac{x(a^2d^2-2abcd+b^2c^2)d}{\sqrt{d x^2+c}\,c^4}$
gosper	$-\frac{48a^2d^3x^6-80abc\,d^2x^6+30b^2c^2d\,x^6+24a^2c\,d^2x^4-40ab\,c^2d\,x^4+15b^2c^3x^4-6a^2c^2d\,x^2+10ab\,c^3x^2+3a^2c^3}{15x^5\sqrt{d x^2+c}\,c^4}$
trager	$-\frac{48a^2d^3x^6-80abc\,d^2x^6+30b^2c^2d\,x^6+24a^2c\,d^2x^4-40ab\,c^2d\,x^4+15b^2c^3x^4-6a^2c^2d\,x^2+10ab\,c^3x^2+3a^2c^3}{15x^5\sqrt{d x^2+c}\,c^4}$
default	$a^2\left(-\frac{1}{5c\,x^5\sqrt{d x^2+c}}-\frac{6d\left(-\frac{1}{3c\,x^3\sqrt{d x^2+c}}-\frac{4d\left(-\frac{1}{cx\sqrt{d x^2+c}}-\frac{2dx}{c^2\sqrt{d x^2+c}}\right)}{3c}\right)}{5c}\right)+2ab\left(-\frac{}{3c\,x^3\sqrt{}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/5/c/x^5/(d*x^2+c)^(1/2)-6/5*d/c*(-1/3/c/x^3/(d*x^2+c)^(1/2)-4/3*d/c
*(-1/c/x/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2))))+2*a*b*(-1/3/c/x^3/(d*
x^2+c)^(1/2)-4/3*d/c*(-1/c/x/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2)))+b^
2*(-1/c/x/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2))
```

Maxima [A]
time = 0.27, size = 184, normalized size = 1.30

$$-\frac{2\,b^2dx}{\sqrt{dx^2+c}\,c^2}+\frac{16\,abd^2x}{3\,\sqrt{dx^2+c}\,c^3}-\frac{16\,a^2d^3x}{5\,\sqrt{dx^2+c}\,c^4}-\frac{b^2}{\sqrt{dx^2+c}\,cx}+\frac{8\,abd}{3\,\sqrt{dx^2+c}\,c^2x}-\frac{8\,a^2d^2}{5\,\sqrt{dx^2+c}\,c^3x}-\frac{2\,ab}{3\,\sqrt{dx^2+c}\,cx^3}+\frac{2\,a^2d}{5\,\sqrt{dx^2+c}\,c^2x^3}-\frac{a^2}{5\,\sqrt{dx^2+c}\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^2*d*x/(sqrt(d*x^2 + c)*c^2) + 16/3*a*b*d^2*x/(sqrt(d*x^2 + c)*c^3) - 1
6/5*a^2*d^3*x/(sqrt(d*x^2 + c)*c^4) - b^2/(sqrt(d*x^2 + c)*c*x) + 8/3*a*b*d
/(sqrt(d*x^2 + c)*c^2*x) - 8/5*a^2*d^2/(sqrt(d*x^2 + c)*c^3*x) - 2/3*a*b/(s
qrt(d*x^2 + c)*c*x^3) + 2/5*a^2*d/(sqrt(d*x^2 + c)*c^2*x^3) - 1/5*a^2/(sqrt
(d*x^2 + c)*c*x^5)
```

Fricas [A]
time = 1.38, size = 121, normalized size = 0.86

$$-\frac{(2\,(15\,b^2c^2d-40\,abcd^2+24\,a^2d^3)x^6+3\,a^2c^3+(15\,b^2c^3-40\,abc^2d+24\,a^2cd^2)x^4+2\,(5\,abc^3-3\,a^2c^2d)x^2)\sqrt{dx^2+c}}{15\,(c^4dx^7+c^5x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(15*b^2*c^2*d - 40*a*b*c*d^2 + 24*a^2*d^3)*x^6 + 3*a^2*c^3 + (15*b
^2*c^3 - 40*a*b*c^2*d + 24*a^2*c*d^2)*x^4 + 2*(5*a*b*c^3 - 3*a^2*c^2*d)*x^2
)*sqrt(d*x^2 + c)/(c^4*d*x^7 + c^5*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{x^6\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**6*(c + d*x**2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(125) = 250.
time = 0.90, size = 452, normalized size = 3.21

$$-\frac{(b^2c^2d-2\,abcd^2+a^2d^3)x}{\sqrt{dx^2+c}\,c^4}+\frac{2\left(15\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8b^2c^2\sqrt{d}-30\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8abcd^{\frac{3}{2}}+15\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8a^2d^{\frac{5}{2}}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6b^2c^3\sqrt{d}+180\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6abc^2d^{\frac{3}{2}}-90\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6a^2cd^{\frac{5}{2}}+90\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b^2c^4\sqrt{d}-320\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc^3d^{\frac{3}{2}}+240\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2c^2d^{\frac{5}{2}}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c^5\sqrt{d}+220\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^4d^{\frac{3}{2}}-150\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2c^3d^{\frac{5}{2}}+15\,b^2c^6\sqrt{d}-50\,abc^5d^{\frac{3}{2}}+33\,a^2c^4d^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^5c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x/(sqrt(d*x^2 + c)*c^4) + 2/15*(15*(sq
rt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*sqrt(d) - 30*(sqrt(d)*x - sqrt(d*x^2 +
 c))^8*a*b*c*d^(3/2) + 15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/2) - 60*
(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) + 180*(sqrt(d)*x - sqrt(d*x
^2 + c))^6*a*b*c^2*d^(3/2) - 90*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c*d^(5/
2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) - 320*(sqrt(d)*x -
sqrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 240*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^
2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt(d) + 220*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) - 150*(sqrt(d)*x - sqrt(d*x^2
 + c))^2*a^2*c^3*d^(5/2) + 15*b^2*c^6*sqrt(d) - 50*a*b*c^5*d^(3/2) + 33*a^2
*c^4*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5*c^3)
```

Mupad [B]
time = 0.48, size = 116, normalized size = 0.82

$$-\frac{3\,a^2\,c^3-6\,a^2\,c^2\,d\,x^2+24\,a^2\,c\,d^2\,x^4+48\,a^2\,d^3\,x^6+10\,a\,b\,c^3\,x^2-40\,a\,b\,c^2\,d\,x^4-80\,a\,b\,c\,d^2\,x^6+15\,b^2\,c^3\,x^4+30\,b^2\,c^2\,d\,x^6}{15\,c^4\,x^5\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x)
```

```
[Out] -(3*a^2*c^3 + 15*b^2*c^3*x^4 + 48*a^2*d^3*x^6 - 6*a^2*c^2*d*x^2 + 24*a^2*c*
d^2*x^4 + 30*b^2*c^2*d*x^6 + 10*a*b*c^3*x^2 - 40*a*b*c^2*d*x^4 - 80*a*b*c*d
^2*x^6)/(15*c^4*x^5*(c + d*x^2)^(1/2))
```

3.659 $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{d(24b^2c^2-5ad(12bc-7ad))}{16c^4\sqrt{c+dx^2}}-\frac{a^2}{6cx^6\sqrt{c+dx^2}}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}-\frac{24b^2c^2-5ad(12bc-7ad)}{48c^3x^2\sqrt{c+dx^2}}+\frac{d(24b^2c^2-$$

```
[Out] 1/16*d*(24*b^2*c^2-5*a*d*(-7*a*d+12*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/
c^(9/2)-1/16*d*(24*b^2*c^2-5*a*d*(-7*a*d+12*b*c))/c^4/(d*x^2+c)^(1/2)-1/6*a
^2/c/x^6/(d*x^2+c)^(1/2)-1/24*a*(-7*a*d+12*b*c)/c^2/x^4/(d*x^2+c)^(1/2)+1/4
8*(-24*b^2*c^2+5*a*d*(-7*a*d+12*b*c))/c^3/x^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 191, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 91, 79, 44, 53, 65, 214}

$$-\frac{35a^2d^2-60abcd+24b^2c^2}{48c^3x^2\sqrt{c+dx^2}}-\frac{a^2}{6cx^6\sqrt{c+dx^2}}+\frac{d(24b^2c^2-5ad(12bc-7ad))\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}-\frac{d(24b^2c^2-5ad(12bc-7ad))}{16c^4\sqrt{c+dx^2}}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/16*(d*(24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d)))/(c^4*Sqrt[c + d*x^2]) - a^2
/(6*c*x^6*Sqrt[c + d*x^2]) - (a*(12*b*c - 7*a*d))/(24*c^2*x^4*Sqrt[c + d*x^
2]) - (24*b^2*c^2 - 60*a*b*c*d + 35*a^2*d^2)/(48*c^3*x^2*Sqrt[c + d*x^2]) +
 (d*(24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])
/(16*c^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^4(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(12bc-7ad)+3b^2cx}{x^3(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{1}{48}\left(24b^2 - \frac{5ad(12bc-7ad)}{c^2}\right)\text{Subst}\left(\int \frac{}{x^2}\right.$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}} + \frac{(24b^2c^2-60ab}{}$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}} - \frac{(24b^2c^2-60ab}{}$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}} - \frac{(24b^2c^2-60ab}{}$$

$$= -\frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} + \frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}} - \frac{(24b^2c^2-60ab}{}$$

Mathematica [A]
time = 0.27, size = 158, normalized size = 0.83

$$-\frac{24b^2c^2x^4(c+3dx^2)+12abcx^2(2c^2-5cdx^2-15d^2x^4)+a^2(8c^3-14c^2dx^2+35cd^2x^4+105d^3x^6)}{48c^4x^6\sqrt{c+dx^2}} + \frac{d(24b^2c^2-60abcd+35a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/48*(24*b^2*c^2*x^4*(c + 3*d*x^2) + 12*a*b*c*x^2*(2*c^2 - 5*c*d*x^2 - 15*
d^2*x^4) + a^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))/(c^4*x^
6*Sqrt[c + d*x^2]) + (d*(24*b^2*c^2 - 60*a*b*c*d + 35*a^2*d^2)*ArcTanh[Sqrt
[c + d*x^2]/Sqrt[c]])/(16*c^(9/2))
```

Maple [A]
time = 0.13, size = 284, normalized size = 1.49

method	result

risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(57a^{2}d^{2}x^{4}-84abcd\,x^{4}+24b^{2}c^{2}x^{4}-22a^{2}cd\,x^{2}+24ab\,c^{2}x^{2}+8a^{2}c^{2}\right)}{48c^{4}x^{6}}-\frac{d^{3}a^{2}}{c^{4}\sqrt{d\,x^{2}+c}}+\frac{2d^{2}ab}{c^{3}\sqrt{d\,x^{2}+c}}-$
default	$2ab\left(-\frac{1}{4c\,x^{4}\sqrt{d\,x^{2}+c}}-\frac{5d\left(-\frac{1}{2c\,x^{2}\sqrt{d\,x^{2}+c}}-\frac{3d\left(\frac{1}{c\sqrt{d\,x^{2}+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right)}{4c}\right)+a^{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b*(-1/4/c/x^4/(d*x^2+c)^(1/2)-5/4*d/c*(-1/2/c/x^2/(d*x^2+c)^(1/2)-3/2*d
/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))+
a^2*(-1/6/c/x^6/(d*x^2+c)^(1/2)-7/6*d/c*(-1/4/c/x^4/(d*x^2+c)^(1/2)-5/4*d/c
*(-1/2/c/x^2/(d*x^2+c)^(1/2)-3/2*d/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c
```

+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))))+b^2*(-1/2/c/x^2/(d*x^2+c)^(1/2)-3/2*d/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))

Maxima [A]
time = 0.29, size = 247, normalized size = 1.30

$$\frac{3\,b^2d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,c^{\frac{5}{2}}}-\frac{15\,abd^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{4\,c^{\frac{7}{2}}}+\frac{35\,a^2d^3\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16\,c^{\frac{9}{2}}}-\frac{3\,b^2d}{2\,\sqrt{dx^2+c}\,c^2}+\frac{15\,abd^2}{4\,\sqrt{dx^2+c}\,c^3}-\frac{35\,a^2d^3}{16\,\sqrt{dx^2+c}\,c^4}-\frac{b^2}{2\,\sqrt{dx^2+c}\,cx^2}+\frac{5\,abd}{4\,\sqrt{dx^2+c}\,c^2x^2}-\frac{35\,a^2d^2}{48\,\sqrt{dx^2+c}\,c^3x^2}-\frac{ab}{2\,\sqrt{dx^2+c}\,cx^4}+\frac{7\,a^2d}{24\,\sqrt{dx^2+c}\,c^2x^4}-\frac{a^2}{6\,\sqrt{dx^2+c}\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="maxima")

[Out] 3/2*b^2*d*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) - 15/4*a*b*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(7/2) + 35/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(9/2) - 3/2*b^2*d/(sqrt(d*x^2 + c)*c^2) + 15/4*a*b*d^2/(sqrt(d*x^2 + c)*c^3) - 35/16*a^2*d^3/(sqrt(d*x^2 + c)*c^4) - 1/2*b^2/(sqrt(d*x^2 + c)*c*x^2) + 5/4*a*b*d/(sqrt(d*x^2 + c)*c^2*x^2) - 35/48*a^2*d^2/(sqrt(d*x^2 + c)*c^3*x^2) - 1/2*a*b/(sqrt(d*x^2 + c)*c*x^4) + 7/24*a^2*d/(sqrt(d*x^2 + c)*c^2*x^4) - 1/6*a^2/(sqrt(d*x^2 + c)*c*x^6)

Fricas [A]
time = 1.19, size = 447, normalized size = 2.35

$$\left[\frac{3\left((24\,b^2c^2d^2-60\,abcd^3+35\,a^2d^4)x^8+(24\,b^2c^3d-60\,abc^2d^2+35\,a^2cd^3)x^6\right)\sqrt{c}\log\left(-\frac{dx^2+2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)-2\left(3(24\,b^2c^3d-60\,abc^2d^2+35\,a^2cd^3)x^6+8\,a^2c^4+(24\,b^2c^4-60\,abc^3d+35\,a^2c^2d^2)x^4+2(12\,abc^4-7\,a^2c^3d)x^2\right)\sqrt{dx^2+c}}{96\,(c^5dx^8+c^6x^6)},-\frac{3\left((24\,b^2c^2d^2-60\,abcd^3+35\,a^2d^4)x^8+(24\,b^2c^3d-60\,abc^2d^2+35\,a^2cd^3)x^6\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(3(24\,b^2c^3d-60\,abc^2d^2+35\,a^2cd^3)x^6+8\,a^2c^4+(24\,b^2c^4-60\,abc^3d+35\,a^2c^2d^2)x^4+2(12\,abc^4-7\,a^2c^3d)x^2\right)\sqrt{dx^2+c}}{48\,(c^5dx^8+c^6x^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="fricas")

[Out] [1/96*(3*((24*b^2*c^2*d^2 - 60*a*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*(24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 + 8*a^2*c^4 + (24*b^2*c^4 - 60*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 + 2*(12*a*b*c^4 - 7*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d*x^8 + c^6*x^6), -1/48*(3*((24*b^2*c^2*d^2 - 60*a*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (3*(24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 + 8*a^2*c^4 + (24*b^2*c^4 - 60*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 + 2*(12*a*b*c^4 - 7*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d*x^8 + c^6*x^6)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{x^7\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**7*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 0.74, size = 267, normalized size = 1.41

$$-\frac{\left(24\,b^2c^2d-60\,abcd^2+35\,a^2d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{16\,\sqrt{-c}\,c^4}-\frac{b^2c^2d-2\,abcd^2+a^2d^3}{\sqrt{dx^2+c}\,c^4}-\frac{24\,(dx^2+c)^{\frac{5}{2}}b^2c^2d-48\,(dx^2+c)^{\frac{3}{2}}b^2c^3d+24\,\sqrt{dx^2+c}\,b^2c^4d-84\,(dx^2+c)^{\frac{5}{2}}abcd^2+192\,(dx^2+c)^{\frac{3}{2}}abc^2d^2-108\,\sqrt{dx^2+c}\,abc^3d^2+57\,(dx^2+c)^{\frac{5}{2}}a^2d^3-136\,(dx^2+c)^{\frac{3}{2}}a^2cd^3+87\,\sqrt{dx^2+c}\,a^2c^2d^3}{48\,c^4d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(24*b^2*c^2*d - 60*a*b*c*d^2 + 35*a^2*d^3)*arctan(sqrt(d*x^2 + c)/sqr
t(-c))/(sqrt(-c)*c^4) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)/(sqrt(d*x^2 + c
)*c^4) - 1/48*(24*(d*x^2 + c)^(5/2)*b^2*c^2*d - 48*(d*x^2 + c)^(3/2)*b^2*c^
3*d + 24*sqrt(d*x^2 + c)*b^2*c^4*d - 84*(d*x^2 + c)^(5/2)*a*b*c*d^2 + 192*(
d*x^2 + c)^(3/2)*a*b*c^2*d^2 - 108*sqrt(d*x^2 + c)*a*b*c^3*d^2 + 57*(d*x^2
+ c)^(5/2)*a^2*d^3 - 136*(d*x^2 + c)^(3/2)*a^2*c*d^3 + 87*sqrt(d*x^2 + c)*a
^2*c^2*d^3)/(c^4*d^3*x^6)
```

Mupad [B]
time = 0.71, size = 246, normalized size = 1.29

$$\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(35\,a^2\,d^2-60\,a\,b\,c\,d+24\,b^2\,c^2\right)}{16\,c^{9/2}}-\frac{\frac{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}{c}-\frac{\left(d\,x^2+c\right)\,\left(77\,a^2\,d^3-132\,a\,b\,c\,d^2+56\,b^2\,c^2\,d\right)}{16\,c^2}+\frac{{\left(d\,x^2+c\right)}^2\,\left(35\,a^2\,d^3-60\,a\,b\,c\,d^2+24\,b^2\,c^2\,d\right)}{6\,c^3}-\frac{{\left(d\,x^2+c\right)}^3\,\left(35\,a^2\,d^3-60\,a\,b\,c\,d^2+24\,b^2\,c^2\,d\right)}{16\,c^4}}{3\,c\,{\left(d\,x^2+c\right)}^{5/2}-{\left(d\,x^2+c\right)}^{7/2}+c^3\,\sqrt{d\,x^2+c}-3\,c^2\,{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x)
```

```
[Out] (d*atanh((c + d*x^2)^(1/2)/c^(1/2))*(35*a^2*d^2 + 24*b^2*c^2 - 60*a*b*c*d))
/(16*c^(9/2)) - ((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/c - ((c + d*x^2)*(77*a
^2*d^3 + 56*b^2*c^2*d - 132*a*b*c*d^2))/(16*c^2) + ((c + d*x^2)^2*(35*a^2*d
^3 + 24*b^2*c^2*d - 60*a*b*c*d^2))/(6*c^3) - ((c + d*x^2)^3*(35*a^2*d^3 + 2
4*b^2*c^2*d - 60*a*b*c*d^2))/(16*c^4))/(3*c*(c + d*x^2)^(5/2) - (c + d*x^2)
^(7/2) + c^3*(c + d*x^2)^(1/2) - 3*c^2*(c + d*x^2)^(3/2))
```

3.660 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=202

$$\frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x^3}{12cd^3\sqrt{c+dx^2}}+\frac{b^2x^5}{4d^2\sqrt{c+dx^2}}-\frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{8cd^4}+\cdots$$

```
[Out] 1/3*(-a*d+b*c)^2*x^5/c/d^2/(d*x^2+c)^(3/2)+1/8*(8*a^2*d^2-40*a*b*c*d+35*b^2
*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(9/2)+1/12*(8*a^2*d^2-40*a*b*c*d
+35*b^2*c^2)*x^3/c/d^3/(d*x^2+c)^(1/2)+1/4*b^2*x^5/d^2/(d*x^2+c)^(1/2)-1/8*
(8*a^2*d^2-40*a*b*c*d+35*b^2*c^2)*x*(d*x^2+c)^(1/2)/c/d^4
```

Rubi [A]
time = 0.10, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {474, 470, 294, 327, 223, 212}

$$\frac{\left(8a^2d^2-40abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{9/2}}-\frac{x\sqrt{c+dx^2}\left(8a^2d^2-40abcd+35b^2c^2\right)}{8cd^4}+\frac{x^3\left(8a^2d^2-40abcd+35b^2c^2\right)}{12cd^3\sqrt{c+dx^2}}+\frac{x^5(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{b^2x^5}{4d^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*x^5)/(3*c*d^2*(c + d*x^2)^(3/2)) + ((35*b^2*c^2 - 40*a*b*c*d
 + 8*a^2*d^2)*x^3)/(12*c*d^3*Sqrt[c + d*x^2]) + (b^2*x^5)/(4*d^2*Sqrt[c + d
*x^2]) - ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^4
) + ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(8*d^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2x^5}{3cd^2(c+dx^2)^{3/2}} - \frac{\int \frac{x^4(-3a^2d^2+5(bc-ad)^2-3b^2cdx^2)}{(c+dx^2)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2(c+dx^2)^{3/2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{(35b^2c^2-40abcd+8a^2d^2)\int \frac{x^4}{(c+dx^2)^{3/2}}\,dx}{12cd^2}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2(c+dx^2)^{3/2}} + \frac{(35b^2c^2-40abcd+8a^2d^2)\,x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{(35b^2c^2-40abcd}{}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2(c+dx^2)^{3/2}} + \frac{(35b^2c^2-40abcd+8a^2d^2)\,x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{(35b^2c^2-40abcd}{}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2(c+dx^2)^{3/2}} + \frac{(35b^2c^2-40abcd+8a^2d^2)\,x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{(35b^2c^2-40abcd}{}$$

$$= \frac{(bc-ad)^2x^5}{3cd^2(c+dx^2)^{3/2}} + \frac{(35b^2c^2-40abcd+8a^2d^2)\,x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{(35b^2c^2-40abcd}{}$$

Mathematica [A]
time = 0.28, size = 155, normalized size = 0.77

$$\frac{x(-8a^2d^2(3c+4dx^2)+8abd(15c^2+20cdx^2+3d^2x^4)-b^2(105c^3+140c^2dx^2+21cd^2x^4-6d^3x^6))}{24d^4(c+dx^2)^{3/2}} - \frac{(35b^2c^2-40abcd+8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{8d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(-8*a^2*d^2*(3*c + 4*d*x^2) + 8*a*b*d*(15*c^2 + 20*c*d*x^2 + 3*d^2*x^4)
- b^2*(105*c^3 + 140*c^2*d*x^2 + 21*c*d^2*x^4 - 6*d^3*x^6)))/(24*d^4*(c + d
*x^2)^(3/2)) - ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sq
rt[c + d*x^2]])/(8*d^(9/2))
```

Maple [A]
time = 0.17, size = 260, normalized size = 1.29

method	result

default	$b^{2}\left(\frac{x^{7}}{4d\left(dx^{2}+c\right)^{\frac{3}{2}}}-\frac{7c\left(\frac{x^{5}}{2d\left(dx^{2}+c\right)^{\frac{3}{2}}}-\frac{5c\left(-\frac{x^{3}}{3d\left(dx^{2}+c\right)^{\frac{3}{2}}}+\frac{-\frac{x}{d\sqrt{dx^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{dx^{2}+c}\right)}{d^{\frac{3}{2}}}}{d}\right)}{2d}\right)}{4d}\right)+2ab\left(\cdots\right.$
risch	$\frac{bx\left(2bdx^{2}+8ad-11bc\right)\sqrt{dx^{2}+c}}{8d^{4}}+\frac{\ln\left(x\sqrt{d}+\sqrt{dx^{2}+c}\right)a^{2}}{d^{\frac{5}{2}}}-\frac{5\ln\left(x\sqrt{d}+\sqrt{dx^{2}+c}\right)abc}{d^{\frac{7}{2}}}+\frac{35\ln\left(x\sqrt{d}+\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/4*x^7/d/(d*x^2+c)^(3/2)-7/4*c/d*(1/2*x^5/d/(d*x^2+c)^(3/2)-5/2*c/d*(
-1/3*x^3/d/(d*x^2+c)^(3/2)+1/d*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)
+(d*x^2+c)^(1/2))))))+2*a*b*(1/2*x^5/d/(d*x^2+c)^(3/2)-5/2*c/d*(-1/3*x^3/d/
(d*x^2+c)^(3/2)+1/d*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^
(1/2)))))+a^2*(-1/3*x^3/d/(d*x^2+c)^(3/2)+1/d*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/
2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))
```

Maxima [A]

time = 0.30, size = 296, normalized size = 1.47

$$\frac{b^{2}x^{7}}{4\left(dx^{2}+c\right)^{\frac{3}{2}}d}-\frac{7b^{2}cx^{5}}{8\left(dx^{2}+c\right)^{\frac{3}{2}}d^{2}}+\frac{abx^{5}}{\left(dx^{2}+c\right)^{\frac{3}{2}}d}-\frac{1}{3}a^{2}x\left(\frac{3x^{2}}{\left(dx^{2}+c\right)^{\frac{3}{2}}d}+\frac{2c}{\left(dx^{2}+c\right)^{\frac{3}{2}}d^{2}}\right)-\frac{35b^{2}c^{2}x\left(\frac{3x^{2}}{\left(dx^{2}+c\right)^{\frac{3}{2}}d}+\frac{2c}{\left(dx^{2}+c\right)^{\frac{3}{2}}d^{2}}\right)}{24d^{2}}+\frac{5abcx\left(\frac{3x^{2}}{\left(dx^{2}+c\right)^{\frac{3}{2}}d}+\frac{2c}{\left(dx^{2}+c\right)^{\frac{3}{2}}d^{2}}\right)}{3d}-\frac{35b^{2}c^{2}x}{24\sqrt{dx^{2}+c}\,d^{4}}+\frac{5abcx}{3\sqrt{dx^{2}+c}\,d^{3}}-\frac{a^{2}x}{3\sqrt{dx^{2}+c}\,d^{2}}+\frac{35b^{2}c^{2}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8d^{\frac{9}{2}}}-\frac{5abc\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{7}{2}}}+\frac{a^{2}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^7/((d*x^2 + c)^(3/2)*d) - 7/8*b^2*c*x^5/((d*x^2 + c)^(3/2)*d^2) +
 a*b*x^5/((d*x^2 + c)^(3/2)*d) - 1/3*a^2*x*(3*x^2/((d*x^2 + c)^(3/2)*d) + 2
```

```
*c/((d*x^2 + c)^(3/2)*d^2)) - 35/24*b^2*c^2*x*(3*x^2/((d*x^2 + c)^(3/2)*d)
+ 2*c/((d*x^2 + c)^(3/2)*d^2))/d^2 + 5/3*a*b*c*x*(3*x^2/((d*x^2 + c)^(3/2)*
d) + 2*c/((d*x^2 + c)^(3/2)*d^2))/d - 35/24*b^2*c^2*x/(sqrt(d*x^2 + c)*d^4)
 + 5/3*a*b*c*x/(sqrt(d*x^2 + c)*d^3) - 1/3*a^2*x/(sqrt(d*x^2 + c)*d^2) + 35
/8*b^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(9/2) - 5*a*b*c*arcsinh(d*x/sqrt(c*d))/
d^(7/2) + a^2*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

Fricas [**A**]
time = 1.94, size = 522, normalized size = 2.58

$$\left[\frac{3\,{\left(35\,b^{2} c^{4} - 40\,a b c^{3} d + 8\,a^{2} c^{2} d^{2} + {\left(35\,b^{2} c^{2} d^{2} - 40\,a b c d^{3} + 8\,a^{2} d^{4}\right)} x^{4} + 2\,{\left(35\,b^{2} c^{3} d - 40\,a b c^{2} d^{2} + 8\,a^{2} c d^{3}\right)} x^{2}\right)} \sqrt{d} \log\left(-2\,d x^{2} - 2\,\sqrt{d x^{2} + c} \sqrt{d} x - c\right) + 2\,{\left(6\,b^{2} d^{4} x^{7} - 3\,{\left(7\,b^{2} c d^{3} - 8\,a b d^{4}\right)} x^{5} - 4\,{\left(35\,b^{2} c^{2} d^{2} - 40\,a b c d^{3} + 8\,a^{2} d^{4}\right)} x^{3} - 3\,{\left(35\,b^{2} c^{3} d - 40\,a b c^{2} d^{2} + 8\,a^{2} c d^{3}\right)} x\right)} \sqrt{d x^{2} + c}}{48\,{\left(d^{7} x^{4} + 2\,c d^{6} x^{2} + c^{2} d^{5}\right)}}, -\frac{3\,{\left(35\,b^{2} c^{4} - 40\,a b c^{3} d + 8\,a^{2} c^{2} d^{2} + {\left(35\,b^{2} c^{2} d^{2} - 40\,a b c d^{3} + 8\,a^{2} d^{4}\right)} x^{4} + 2\,{\left(35\,b^{2} c^{3} d - 40\,a b c^{2} d^{2} + 8\,a^{2} c d^{3}\right)} x^{2}\right)} \sqrt{-d} \arctan\left(\frac{\sqrt{-d} x}{\sqrt{d x^{2} + c}}\right) - {\left(6\,b^{2} d^{4} x^{7} - 3\,{\left(7\,b^{2} c d^{3} - 8\,a b d^{4}\right)} x^{5} - 4\,{\left(35\,b^{2} c^{2} d^{2} - 40\,a b c d^{3} + 8\,a^{2} d^{4}\right)} x^{3} - 3\,{\left(35\,b^{2} c^{3} d - 40\,a b c^{2} d^{2} + 8\,a^{2} c d^{3}\right)} x\right)} \sqrt{d x^{2} + c}}{24\,{\left(d^{7} x^{4} + 2\,c d^{6} x^{2} + c^{2} d^{5}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(35*b^2*c^4 - 40*a*b*c^3*d + 8*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 40*
a*b*c*d^3 + 8*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3
)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b^2*d
^4*x^7 - 3*(7*b^2*c*d^3 - 8*a*b*d^4)*x^5 - 4*(35*b^2*c^2*d^2 - 40*a*b*c*d^3
 + 8*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt
(d*x^2 + c))/(d^7*x^4 + 2*c*d^6*x^2 + c^2*d^5), -1/24*(3*(35*b^2*c^4 - 40*a
*b*c^3*d + 8*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 40*a*b*c*d^3 + 8*a^2*d^4)*x^4
+ 2*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^2)*sqrt(-d)*arctan(sqrt
(-d)*x/sqrt(d*x^2 + c)) - (6*b^2*d^4*x^7 - 3*(7*b^2*c*d^3 - 8*a*b*d^4)*x^5
- 4*(35*b^2*c^2*d^2 - 40*a*b*c*d^3 + 8*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 40*
a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(d^7*x^4 + 2*c*d^6*x^2 + c^2
*d^5)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + b x^{2}\right)^{2}}{\left(c + d x^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

Giac [**A**]
time = 0.66, size = 190, normalized size = 0.94

$$\frac{{\left({\left(3 \, {\left(\frac{2 \, b^{2} x^{2}}{d} - \frac{7 \, b^{2} c^{2} d^{5} - 8 \, a b c d^{6}}{c d^{7}}\right)} x^{2} - \frac{4 \, {\left(35 \, b^{2} c^{3} d^{4} - 40 \, a b c^{2} d^{5} + 8 \, a^{2} c d^{6}\right)}}{c d^{7}}\right)} x^{2} - \frac{3 \, {\left(35 \, b^{2} c^{4} d^{3} - 40 \, a b c^{3} d^{4} + 8 \, a^{2} c^{2} d^{5}\right)}}{c d^{7}}\right)} x}{24 \, {\left(d x^{2} + c\right)}^{\frac{3}{2}}} - \frac{{\left(35 \, b^{2} c^{2} - 40 \, a b c d + 8 \, a^{2} d^{2}\right)} \log\left({\left| -\sqrt{d} x + \sqrt{d x^{2} + c} \right|}\right)}{8 \, d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*b^2*x^2/d - (7*b^2*c^2*d^5 - 8*a*b*c*d^6)/(c*d^7))*x^2 - 4*(35*
b^2*c^3*d^4 - 40*a*b*c^2*d^5 + 8*a^2*c*d^6)/(c*d^7))*x^2 - 3*(35*b^2*c^4*d^
3 - 40*a*b*c^3*d^4 + 8*a^2*c^2*d^5)/(c*d^7))*x/(d*x^2 + c)^(3/2) - 1/8*(35*
b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^
(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.661 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{c(bc-ad)^2}{3d^4(c+dx^2)^{3/2}}-\frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}-\frac{b(3bc-2ad)\sqrt{c+dx^2}}{d^4}+\frac{b^2(c+dx^2)^{3/2}}{3d^4}$$

```
[Out] 1/3*c*(-a*d+b*c)^2/d^4/(d*x^2+c)^(3/2)+1/3*b^2*(d*x^2+c)^(3/2)/d^4-(-a*d+b*
c)*(-a*d+3*b*c)/d^4/(d*x^2+c)^(1/2)-b*(-2*a*d+3*b*c)*(d*x^2+c)^(1/2)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {457, 78}

$$-\frac{b\sqrt{c+dx^2}(3bc-2ad)}{d^4}-\frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}+\frac{c(bc-ad)^2}{3d^4(c+dx^2)^{3/2}}+\frac{b^2(c+dx^2)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (c*(b*c - a*d)^2)/(3*d^4*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*c - a*d))/(
d^4*Sqrt[c + d*x^2]) - (b*(3*b*c - 2*a*d)*Sqrt[c + d*x^2])/d^4 + (b^2*(c +
d*x^2)^(3/2))/(3*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^3(c+dx)^{5/2}} + \frac{(bc-ad)(3bc-ad)}{d^3(c+dx)^{3/2}} - \frac{b(3bc-2ad)}{d^3\sqrt{c+dx}} + \frac{b^2\sqrt{c+dx}}{d^3}\right)\right.$$
$$= \frac{c(bc-ad)^2}{3d^4(c+dx^2)^{3/2}} - \frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}} - \frac{b(3bc-2ad)\sqrt{c+dx^2}}{d^4} + \frac{b^2(c+dx^2)^{3/2}}{3d^4}$$

Mathematica [A]
time = 0.07, size = 98, normalized size = 0.89

$$\frac{-a^2d^2(2c+3dx^2)+2abd(8c^2+12cdx^2+3d^2x^4)+b^2(-16c^3-24c^2dx^2-6cd^2x^4+d^3x^6)}{3d^4(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
[Out] (-(a^2*d^2*(2*c + 3*d*x^2)) + 2*a*b*d*(8*c^2 + 12*c*d*x^2 + 3*d^2*x^4) + b^
2*(-16*c^3 - 24*c^2*d*x^2 - 6*c*d^2*x^4 + d^3*x^6))/(3*d^4*(c + d*x^2)^(3/2
))
```

Maple [A]
time = 0.11, size = 183, normalized size = 1.66

method	result
risch	$\frac{b(bd\,x^2+6ad-8bc)\sqrt{d\,x^2+c}}{3d^4}-\frac{(ad-bc)(3a\,d^2x^2-9bcd\,x^2+2acd-8b\,c^2)\sqrt{d\,x^2+c}}{3d^4(d^2x^4+2x^2dc+c^2)}$
gosper	$-\frac{-b^2x^6d^3-6ab\,d^3x^4+6b^2c\,d^2x^4+3a^2d^3x^2-24abc\,d^2x^2+24b^2c^2d\,x^2+2a^2c\,d^2-16ab\,c^2d+16b^2c^3}{3(d\,x^2+c)^{\frac{3}{2}}d^4}$
trager	$-\frac{-b^2x^6d^3-6ab\,d^3x^4+6b^2c\,d^2x^4+3a^2d^3x^2-24abc\,d^2x^2+24b^2c^2d\,x^2+2a^2c\,d^2-16ab\,c^2d+16b^2c^3}{3(d\,x^2+c)^{\frac{3}{2}}d^4}$
default	$b^2\left(\frac{x^6}{3d(d\,x^2+c)^{\frac{3}{2}}}-\frac{2c\left(\frac{x^4}{d(d\,x^2+c)^{\frac{3}{2}}}-\frac{4c\left(-\frac{x^2}{d(d\,x^2+c)^{\frac{3}{2}}}-\frac{2c}{3d^2(d\,x^2+c)^{\frac{3}{2}}}\right)}{d}\right)}{d}\right)+2ab\left(\frac{x^4}{d(d\,x^2+c)^{\frac{3}{2}}}-\frac{4c\left(-\frac{x^2}{d(d\,x^2+c)^{\frac{3}{2}}}-\right.}{d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/3*x^6/d/(d*x^2+c)^(3/2)-2*c/d*(x^4/d/(d*x^2+c)^(3/2)-4*c/d*(-x^2/d/(
d*x^2+c)^(3/2)-2/3*c/d^2/(d*x^2+c)^(3/2))))+2*a*b*(x^4/d/(d*x^2+c)^(3/2)-4*
c/d*(-x^2/d/(d*x^2+c)^(3/2)-2/3*c/d^2/(d*x^2+c)^(3/2)))+a^2*(-x^2/d/(d*x^2+
c)^(3/2)-2/3*c/d^2/(d*x^2+c)^(3/2))
```

Maxima [A]
time = 0.30, size = 181, normalized size = 1.65

$$\frac{b^2x^6}{3\,(dx^2+c)^{\frac{3}{2}}d}-\frac{2\,b^2cx^4}{(dx^2+c)^{\frac{3}{2}}d^2}+\frac{2\,abx^4}{(dx^2+c)^{\frac{3}{2}}d}-\frac{8\,b^2c^2x^2}{(dx^2+c)^{\frac{3}{2}}d^3}+\frac{8\,abcx^2}{(dx^2+c)^{\frac{3}{2}}d^2}-\frac{a^2x^2}{(dx^2+c)^{\frac{3}{2}}d}-\frac{16\,b^2c^3}{3\,(dx^2+c)^{\frac{3}{2}}d^4}+\frac{16\,abc^2}{3\,(dx^2+c)^{\frac{3}{2}}d^3}-\frac{2\,a^2c}{3\,(dx^2+c)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^6/((d*x^2 + c)^(3/2)*d) - 2*b^2*c*x^4/((d*x^2 + c)^(3/2)*d^2) + 2
*a*b*x^4/((d*x^2 + c)^(3/2)*d) - 8*b^2*c^2*x^2/((d*x^2 + c)^(3/2)*d^3) + 8*
a*b*c*x^2/((d*x^2 + c)^(3/2)*d^2) - a^2*x^2/((d*x^2 + c)^(3/2)*d) - 16/3*b^
2*c^3/((d*x^2 + c)^(3/2)*d^4) + 16/3*a*b*c^2/((d*x^2 + c)^(3/2)*d^3) - 2/3*
a^2*c/((d*x^2 + c)^(3/2)*d^2)
```

Fricas [A]
time = 1.85, size = 124, normalized size = 1.13

$$\frac{(b^2d^3x^6-16\,b^2c^3+16\,abc^2d-2\,a^2cd^2-6\,(b^2cd^2-abd^3)x^4-3\,(8\,b^2c^2d-8\,abcd^2+a^2d^3)x^2)\sqrt{dx^2+c}}{3\,(d^6x^4+2\,cd^5x^2+c^2d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^3*x^6 - 16*b^2*c^3 + 16*a*b*c^2*d - 2*a^2*c*d^2 - 6*(b^2*c*d^2 -
 a*b*d^3)*x^4 - 3*(8*b^2*c^2*d - 8*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(d*x^2 + c
)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(97) = 194$.
time = 0.52, size = 454, normalized size = 4.13

$$\begin{cases} -\frac{2a^2cd^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}}-\frac{3a^2d^3x^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}}+\frac{16abc^2d}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}}+\frac{24abcd^2x^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}}+\frac{6abd^3x^4}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}}-\frac{16b^2c^3}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}}-\frac{24b^2c^2dx^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}}-\frac{6b^2cd^2x^4}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}}+\frac{b^2d^3x^6}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} & \text{for}\ d\neq 0 \\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*c*d**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(c +
 d*x**2)) - 3*a**2*d**3*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(
c + d*x**2)) + 16*a*b*c**2*d/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(
```

```
c + d*x**2)) + 24*a*b*c*d**2*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*
sqrt(c + d*x**2)) + 6*a*b*d**3*x**4/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**
2*sqrt(c + d*x**2)) - 16*b**2*c**3/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2
*sqrt(c + d*x**2)) - 24*b**2*c**2*d*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**
5*x**2*sqrt(c + d*x**2)) - 6*b**2*c*d**2*x**4/(3*c*d**4*sqrt(c + d*x**2) +
3*d**5*x**2*sqrt(c + d*x**2)) + b**2*d**3*x**6/(3*c*d**4*sqrt(c + d*x**2) +
 3*d**5*x**2*sqrt(c + d*x**2)), Ne(d, 0)), ((a**2*x**4/4 + a*b*x**6/3 + b**
2*x**8/8)/c**(5/2), True))
```

Giac [A]
time = 1.03, size = 140, normalized size = 1.27

$$-\frac{9\,(dx^2+c)b^2c^2-b^2c^3-12\,(dx^2+c)abcd+2\,abc^2d+3\,(dx^2+c)a^2d^2-a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}d^4}+\frac{(dx^2+c)^{\frac{3}{2}}b^2d^8-9\,\sqrt{dx^2+c}\,b^2cd^8+6\,\sqrt{dx^2+c}\,abd^9}{3\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(9*(d*x^2 + c)*b^2*c^2 - b^2*c^3 - 12*(d*x^2 + c)*a*b*c*d + 2*a*b*c^2*
d + 3*(d*x^2 + c)*a^2*d^2 - a^2*c*d^2)/((d*x^2 + c)^(3/2)*d^4) + 1/3*((d*x^
2 + c)^(3/2)*b^2*d^8 - 9*sqrt(d*x^2 + c)*b^2*c*d^8 + 6*sqrt(d*x^2 + c)*a*b*
d^9)/d^12
```

Mupad [B]
time = 0.44, size = 107, normalized size = 0.97

$$-\frac{2\,a^2\,c\,d^2+3\,a^2\,d^3\,x^2-16\,a\,b\,c^2\,d-24\,a\,b\,c\,d^2\,x^2-6\,a\,b\,d^3\,x^4+16\,b^2\,c^3+24\,b^2\,c^2\,d\,x^2+6\,b^2\,c\,d^2\,x^4-b^2\,d^3\,x^6}{3\,d^4\,{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] -(16*b^2*c^3 + 2*a^2*c*d^2 + 3*a^2*d^3*x^2 - b^2*d^3*x^6 + 24*b^2*c^2*d*x^2
 + 6*b^2*c*d^2*x^4 - 16*a*b*c^2*d - 6*a*b*d^3*x^4 - 24*a*b*c*d^2*x^2)/(3*d^
4*(c + d*x^2)^(3/2))
```

3.662 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{(bc-ad)^2x^3}{3cd^2\,(c+dx^2)^{3/2}}+\frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}$$

```
[Out] 1/3*(-a*d+b*c)^2*x^3/c/d^2/(d*x^2+c)^(3/2)-1/2*b*(-4*a*d+5*b*c)*arctanh(x*d
^(1/2)/(d*x^2+c)^(1/2))/d^(7/2)+2*b*(-a*d+b*c)*x/d^3/(d*x^2+c)^(1/2)+1/2*b^
2*x*(d*x^2+c)^(1/2)/d^3
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {474, 466, 396, 223, 212}

$$-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}+\frac{2bx(bc-ad)}{d^3\sqrt{c+dx^2}}+\frac{x^3(bc-ad)^2}{3cd^2\,(c+dx^2)^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*x^3)/(3*c*d^2*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d)*x)/(d^3*
Sqrt[c + d*x^2]) + (b^2*x*Sqrt[c + d*x^2])/(2*d^3) - (b*(5*b*c - 4*a*d)*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2x^3}{3cd^2(c+dx^2)^{3/2}} - \frac{\int \frac{x^2(3bc(bc-2ad)-3b^2cdx^2)}{(c+dx^2)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2x^3}{3cd^2(c+dx^2)^{3/2}} + \frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}} + \frac{\int \frac{-6bcd(bc-ad)+3b^2cd^2x^2}{\sqrt{c+dx^2}}\,dx}{3cd^4}$$

$$= \frac{(bc-ad)^2x^3}{3cd^2(c+dx^2)^{3/2}} + \frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}} + \frac{b^2x\sqrt{c+dx^2}}{2d^3} - \frac{(b(5bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2d^3}$$

$$= \frac{(bc-ad)^2x^3}{3cd^2(c+dx^2)^{3/2}} + \frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}} + \frac{b^2x\sqrt{c+dx^2}}{2d^3} - \frac{(b(5bc-4ad))\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx\right.}{2d^3}$$

$$= \frac{(bc-ad)^2x^3}{3cd^2(c+dx^2)^{3/2}} + \frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}} + \frac{b^2x\sqrt{c+dx^2}}{2d^3} - \frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right.}{2d^{7/2}}$$

Mathematica [A]

time = 0.21, size = 117, normalized size = 0.97

$$\frac{x(2a^2d^3x^2 - 4abcd(3c + 4dx^2) + b^2c(15c^2 + 20cdx^2 + 3d^2x^4))}{6cd^3\left(c + dx^2\right)^{3/2}} + \frac{b(5bc - 4ad)\log\left(-\sqrt{d}\,x + \sqrt{c + dx^2}\right)}{2d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(2*a^2*d^3*x^2 - 4*a*b*c*d*(3*c + 4*d*x^2) + b^2*c*(15*c^2 + 20*c*d*x^2
+ 3*d^2*x^4)))/(6*c*d^3*(c + d*x^2)^(3/2)) + (b*(5*b*c - 4*a*d)*Log[-(Sqrt[
d]*x) + Sqrt[c + d*x^2]])/(2*d^(7/2))
```

Maple [A]
time = 0.13, size = 207, normalized size = 1.71

method	result
default	$b^2\left(\frac{x^5}{2d(dx^2+c)^{\frac{3}{2}}} - \frac{5c\left(-\frac{x^3}{3d\left(dx^2+c\right)^{\frac{3}{2}}} + \frac{-\frac{x}{d\sqrt{dx^2+c}} + \frac{\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{d^{\frac{3}{2}}}}{d}\right)}{2d}\right) + 2ab\left(-\frac{x^3}{3d(dx^2+c)^{\frac{3}{2}}} + \ldots\right.$
risch	$\frac{b^2x\sqrt{dx^2+c}}{2d^3} + \frac{2b\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)a}{d^{\frac{5}{2}}} - \frac{5b^2\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)c}{2d^{\frac{7}{2}}} + \frac{\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 d + 2\ldots}}{6d^2c\left(x-\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/2*x^5/d/(d*x^2+c)^(3/2)-5/2*c/d*(-1/3*x^3/d/(d*x^2+c)^(3/2)+1/d*(-x/
d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+2*a*b*(-1/3*x^
3/d/(d*x^2+c)^(3/2)+1/d*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+(d*x^2
+c)^(1/2))))+a^2*(-1/2*x/d/(d*x^2+c)^(3/2)+1/2*c/d*(1/3*x/c/(d*x^2+c)^(3/2)
+2/3*x/c^2/(d*x^2+c)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(103) = 206$.
time = 0.31, size = 211, normalized size = 1.74

$$\frac{b^2x^5}{2\,(dx^2+c)^{\frac{3}{2}}d} - \frac{2}{3}\,abx\left(\frac{3\,x^2}{(dx^2+c)^{\frac{3}{2}}d} + \frac{2\,c}{(dx^2+c)^{\frac{3}{2}}d^2}\right) + \frac{5\,b^2cx\left(\frac{3\,x^2}{(dx^2+c)^{\frac{3}{2}}d} + \frac{2\,c}{(dx^2+c)^{\frac{3}{2}}d^2}\right)}{6\,d} + \frac{5\,b^2cx}{6\,\sqrt{dx^2+c}\,d^3} - \frac{2\,abx}{3\,\sqrt{dx^2+c}\,d^2} - \frac{a^2x}{3\,(dx^2+c)^{\frac{3}{2}}d} + \frac{a^2x}{3\,\sqrt{dx^2+c}\,cd} - \frac{5\,b^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{7}{2}}} + \frac{2\,ab\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")

[Out] 1/2*b^2*x^5/((d*x^2 + c)^(3/2)*d) - 2/3*a*b*x*(3*x^2/((d*x^2 + c)^(3/2)*d) + 2*c/((d*x^2 + c)^(3/2)*d^2)) + 5/6*b^2*c*x*(3*x^2/((d*x^2 + c)^(3/2)*d) + 2*c/((d*x^2 + c)^(3/2)*d^2))/d + 5/6*b^2*c*x/(sqrt(d*x^2 + c)*d^3) - 2/3*a*b*x/(sqrt(d*x^2 + c)*d^2) - 1/3*a^2*x/((d*x^2 + c)^(3/2)*d) + 1/3*a^2*x/(sqrt(d*x^2 + c)*c*d) - 5/2*b^2*c*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 2*a*b*arcsinh(d*x/sqrt(c*d))/d^(5/2)

Fricas [**A**]
time = 2.75, size = 409, normalized size = 3.38

$$\left[-\frac{3\,{\left(5\,b^{2}c^{4}-4\,abc^{3}d+{\left(5\,b^{2}c^{2}d^{2}-4\,abcd^{3}\right)}x^{4}+2\,{\left(5\,b^{2}c^{3}d-4\,abc^{2}d^{2}\right)}x^{2}\right)}\sqrt{d}\,\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)-2\,{\left(3\,b^{2}cd^{3}x^{5}+2\,{\left(10\,b^{2}c^{2}d^{2}-8\,abcd^{3}+a^{2}d^{4}\right)}x^{3}+3\,{\left(5\,b^{2}c^{3}d-4\,abc^{2}d^{2}\right)}x\right)}\sqrt{dx^{2}+c}}{12\,{\left(cd^{6}x^{4}+2\,c^{2}d^{5}x^{2}+c^{3}d^{4}\right)}},\ \frac{3\,{\left(5\,b^{2}c^{4}-4\,abc^{3}d+{\left(5\,b^{2}c^{2}d^{2}-4\,abcd^{3}\right)}x^{4}+2\,{\left(5\,b^{2}c^{3}d-4\,abc^{2}d^{2}\right)}x^{2}\right)}\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)+{\left(3\,b^{2}cd^{3}x^{5}+2\,{\left(10\,b^{2}c^{2}d^{2}-8\,abcd^{3}+a^{2}d^{4}\right)}x^{3}+3\,{\left(5\,b^{2}c^{3}d-4\,abc^{2}d^{2}\right)}x\right)}\sqrt{dx^{2}+c}}{6\,{\left(cd^{6}x^{4}+2\,c^{2}d^{5}x^{2}+c^{3}d^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")

[Out] [-1/12*(3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*c^2*d^2 - 4*a*b*c*d^3)*x^4 + 2*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(3*b^2*c*d^3*x^5 + 2*(10*b^2*c^2*d^2 - 8*a*b*c*d^3 + a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x)*sqrt(d*x^2 + c))/(c*d^6*x^4 + 2*c^2*d^5*x^2 + c^3*d^4), 1/6*(3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*c^2*d^2 - 4*a*b*c*d^3)*x^4 + 2*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (3*b^2*c*d^3*x^5 + 2*(10*b^2*c^2*d^2 - 8*a*b*c*d^3 + a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x)*sqrt(d*x^2 + c))/(c*d^6*x^4 + 2*c^2*d^5*x^2 + c^3*d^4)]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(a+bx^{2}\right)^{2}}{\left(c+dx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)

[Out] Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)

Giac [**A**]
time = 1.26, size = 130, normalized size = 1.07

$$\frac{{\left({\left(\frac{3\,b^{2}x^{2}}{d}+\frac{2\,{\left(10\,b^{2}c^{2}d^{3}-8\,abcd^{4}+a^{2}d^{5}\right)}}{cd^{5}}\right)}x^{2}+\frac{3\,{\left(5\,b^{2}c^{3}d^{2}-4\,abc^{2}d^{3}\right)}}{cd^{5}}\right)}x}{6\,{\left(dx^{2}+c\right)}^{\frac{3}{2}}}+\frac{{\left(5\,b^{2}c-4\,abd\right)}\log\left({\left|-\sqrt{d}\,x+\sqrt{dx^{2}+c}\right|}\right)}{2\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*b^2*x^2/d + 2*(10*b^2*c^2*d^3 - 8*a*b*c*d^4 + a^2*d^5)/(c*d^5))*x^2
 + 3*(5*b^2*c^3*d^2 - 4*a*b*c^2*d^3)/(c*d^5))*x/(d*x^2 + c)^(3/2) + 1/2*(5*
b^2*c - 4*a*b*d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.663 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}} dx$

Optimal. Leaf size=72

$$-\frac{(bc-ad)^2}{3d^3(c+dx^2)^{3/2}}+\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}$$

```
[Out] -1/3*(-a*d+b*c)^2/d^3/(d*x^2+c)^(3/2)+2*b*(-a*d+b*c)/d^3/(d*x^2+c)^(1/2)+b^
2*(d*x^2+c)^(1/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{(bc-ad)^2}{3d^3(c+dx^2)^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c - a*d)^2/(d^3*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d))/(d^3*Sqrt[c
+ d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^{5/2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{5/2}} - \frac{2b(bc-ad)}{d^2(c+dx)^{3/2}} + \frac{b^2}{d^2\sqrt{c+dx}}\right)dx,x,x^2\right)$$
$$= -\frac{(bc-ad)^2}{3d^3(c+dx^2)^{3/2}} + \frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}} + \frac{b^2\sqrt{c+dx^2}}{d^3}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 0.93

$$\frac{-a^2d^2-2abd(2c+3dx^2)+b^2(8c^2+12cdx^2+3d^2x^4)}{3d^3(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*d^2) - 2*a*b*d*(2*c + 3*d*x^2) + b^2*(8*c^2 + 12*c*d*x^2 + 3*d^2*x^4
))/(3*d^3*(c + d*x^2)^(3/2))
```

Maple [A]
time = 0.11, size = 116, normalized size = 1.61

method	result	siz
gosper	$-\frac{-3b^2x^4d^2+6ab\,d^2x^2-12b^2cd\,x^2+a^2d^2+4abcd-8b^2c^2}{3(d\,x^2+c)^{\frac{3}{2}}d^3}$	68
trager	$-\frac{-3b^2x^4d^2+6ab\,d^2x^2-12b^2cd\,x^2+a^2d^2+4abcd-8b^2c^2}{3(d\,x^2+c)^{\frac{3}{2}}d^3}$	68
risch	$\frac{b^2\sqrt{d\,x^2+c}}{d^3}-\frac{(ad-bc)(6bd\,x^2+ad+5bc)\sqrt{d\,x^2+c}}{3d^3(d^2x^4+2x^2dc+c^2)}$	75
default	$b^2\left(\frac{x^4}{d(d\,x^2+c)^{\frac{3}{2}}}-\frac{4c\left(-\frac{x^2}{d(d\,x^2+c)^{\frac{3}{2}}}-\frac{2c}{3d^2(d\,x^2+c)^{\frac{3}{2}}}\right)}{d}\right)+2ab\left(-\frac{x^2}{d(d\,x^2+c)^{\frac{3}{2}}}-\frac{2c}{3d^2(d\,x^2+c)^{\frac{3}{2}}}\right)-\frac{a^2}{3d(d\,x^2+c)^{\frac{3}{2}}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(x^4/d/(d*x^2+c)^(3/2)-4*c/d*(-x^2/d/(d*x^2+c)^(3/2)-2/3*c/d^2/(d*x^2+c
)^(3/2)))+2*a*b*(-x^2/d/(d*x^2+c)^(3/2)-2/3*c/d^2/(d*x^2+c)^(3/2))-1/3*a^2/
d/(d*x^2+c)^(3/2)
```

Maxima [A]
time = 0.28, size = 114, normalized size = 1.58

$$\frac{b^2x^4}{(dx^2+c)^{\frac{3}{2}}d}+\frac{4\,b^2cx^2}{(dx^2+c)^{\frac{3}{2}}d^2}-\frac{2\,abx^2}{(dx^2+c)^{\frac{3}{2}}d}+\frac{8\,b^2c^2}{3\,(dx^2+c)^{\frac{3}{2}}d^3}-\frac{4\,abc}{3\,(dx^2+c)^{\frac{3}{2}}d^2}-\frac{a^2}{3\,(dx^2+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*x^4/((d*x^2 + c)^(3/2)*d) + 4*b^2*c*x^2/((d*x^2 + c)^(3/2)*d^2) - 2*a*b
*x^2/((d*x^2 + c)^(3/2)*d) + 8/3*b^2*c^2/((d*x^2 + c)^(3/2)*d^3) - 4/3*a*b*
c/((d*x^2 + c)^(3/2)*d^2) - 1/3*a^2/((d*x^2 + c)^(3/2)*d)
```

Fricas [A]
time = 1.99, size = 91, normalized size = 1.26

$$\frac{(3\,b^2d^2x^4+8\,b^2c^2-4\,abcd-a^2d^2+6\,(2\,b^2cd-abd^2)x^2)\sqrt{dx^2+c}}{3\,(d^5x^4+2\,cd^4x^2+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)/(d^5*x^4 + 2*c*d^4*x^2 + c^2*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(63) = 126.
time = 0.44, size = 303, normalized size = 4.21

$$\begin{cases} -\frac{a^2d^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}}-\frac{4abcd}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}}-\frac{6abd^2x^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}}+\frac{8b^2c^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}}+\frac{12b^2cdx^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}}+\frac{3b^2d^2x^4}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} & \text{for}\ d\neq 0 \\ \frac{\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-a**2*d**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x
**2)) - 4*a*b*c*d/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**2)
) - 6*a*b*d**2*x**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**
2)) + 8*b**2*c**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**2)
) + 12*b**2*c*d*x**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x*
*2)) + 3*b**2*d**2*x**4/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d
*x**2)), Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/c**(5/2), Tru
e))
```

Giac [A]
time = 1.26, size = 79, normalized size = 1.10

$$\frac{\sqrt{dx^2+c}\,b^2}{d^3}+\frac{6\,(dx^2+c)b^2c-b^2c^2-6\,(dx^2+c)abd+2\,abcd-a^2d^2}{3\,(dx^2+c)^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(d*x^2 + c)*b^2/d^3 + 1/3*(6*(d*x^2 + c)*b^2*c - b^2*c^2 - 6*(d*x^2 + c
)*a*b*d + 2*a*b*c*d - a^2*d^2)/((d*x^2 + c)^(3/2)*d^3)
```

Mupad [B]
time = 0.36, size = 76, normalized size = 1.06

$$\frac{3\,b^2\,{\left(d\,x^2+c\right)}^2-a^2\,d^2-b^2\,c^2+6\,b^2\,c\,\left(d\,x^2+c\right)-6\,a\,b\,d\,\left(d\,x^2+c\right)+2\,a\,b\,c\,d}{3\,d^3\,{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] (3*b^2*(c + d*x^2)^2 - a^2*d^2 - b^2*c^2 + 6*b^2*c*(c + d*x^2) - 6*a*b*d*(c
 + d*x^2) + 2*a*b*c*d)/(3*d^3*(c + d*x^2)^(3/2))
```

3.664 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{(bc-ad)x(a+bx^2)}{3cd\,(c+dx^2)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}$$

```
[Out] -1/3*(-a*d+b*c)*x*(b*x^2+a)/c/d/(d*x^2+c)^(3/2)+b^2*arctanh(x*d^(1/2)/(d*x^
2+c)^(1/2))/d^(5/2)-1/3*(-a*d+b*c)*(2*a*d+3*b*c)*x/c^2/d^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {424, 393, 223, 212}

$$-\frac{x(bc-ad)(2ad+3bc)}{3c^2d^2\sqrt{c+dx^2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{3cd\,(c+dx^2)^{3/2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

```
[Out] -1/3*((b*c - a*d)*x*(a + b*x^2))/(c*d*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*
b*c + 2*a*d)*x)/(3*c^2*d^2*Sqrt[c + d*x^2]) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/d^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx &= -\frac{(bc-ad)x(a+bx^2)}{3cd\,(c+dx^2)^{3/2}}+\frac{\int \frac{a(bc+2ad)+3b^2cx^2}{(c+dx^2)^{3/2}}\,dx}{3cd}\\
&= -\frac{(bc-ad)x(a+bx^2)}{3cd\,(c+dx^2)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{d^2}\\
&= -\frac{(bc-ad)x(a+bx^2)}{3cd\,(c+dx^2)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{d^2}\\
&= -\frac{(bc-ad)x(a+bx^2)}{3cd\,(c+dx^2)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 102, normalized size = 0.97

$$\frac{2abcd^2x^3+a^2d^2x(3c+2dx^2)-b^2c^2x(3c+4dx^2)}{3c^2d^2\,(c+dx^2)^{3/2}}-\frac{b^2\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

```
[Out] (2*a*b*c*d^2*x^3 + a^2*d^2*x*(3*c + 2*d*x^2) - b^2*c^2*x*(3*c + 4*d*x^2))/(
3*c^2*d^2*(c + d*x^2)^(3/2)) - (b^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/d^
(5/2)
```

Maple [A]
time = 0.10, size = 156, normalized size = 1.49

method	result
default	$b^2\left(-\frac{x^3}{3d(dx^2+c)^{\frac{3}{2}}}+\frac{-\frac{x}{d\sqrt{dx^2+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{d^{\frac{3}{2}}}}{d}\right)+2ab\left(-\frac{x}{2d(dx^2+c)^{\frac{3}{2}}}+\frac{c\left(\frac{x}{3c\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{}{3c^2}\right.}{2d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/3*x^3/d/(d*x^2+c)^(3/2)+1/d*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^
(1/2)+(d*x^2+c)^(1/2))))+2*a*b*(-1/2*x/d/(d*x^2+c)^(3/2)+1/2*c/d*(1/3*x/c/(
d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)))+a^2*(1/3*x/c/(d*x^2+c)^(3/2)+2/3
*x/c^2/(d*x^2+c)^(1/2))
```

Maxima [A]

time = 0.27, size = 147, normalized size = 1.40

$$-\frac{1}{3}b^2x\left(\frac{3x^2}{(dx^2+c)^{\frac{3}{2}}d}+\frac{2c}{(dx^2+c)^{\frac{3}{2}}d^2}\right)+\frac{2a^2x}{3\sqrt{dx^2+c}\,c^2}+\frac{a^2x}{3(dx^2+c)^{\frac{3}{2}}c}-\frac{b^2x}{3\sqrt{dx^2+c}\,d^2}-\frac{2abx}{3(dx^2+c)^{\frac{3}{2}}d}+\frac{2abx}{3\sqrt{dx^2+c}\,cd}+\frac{b^2\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b^2*x*(3*x^2/((d*x^2 + c)^(3/2)*d) + 2*c/((d*x^2 + c)^(3/2)*d^2)) + 2/
3*a^2*x/(sqrt(d*x^2 + c)*c^2) + 1/3*a^2*x/((d*x^2 + c)^(3/2)*c) - 1/3*b^2*x
/(sqrt(d*x^2 + c)*d^2) - 2/3*a*b*x/((d*x^2 + c)^(3/2)*d) + 2/3*a*b*x/(sqrt(
d*x^2 + c)*c*d) + b^2*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

Fricas [A]

time = 2.49, size = 321, normalized size = 3.06

$$\left[\frac{3\left(b^2c^2d^2x^4+2b^2c^3dx^2+b^2c^4\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}\,x-c\right)-2\left(2\left(2b^2c^2d^2-abcd^3-a^2d^4\right)x^3+3\left(b^2c^3d-a^2cd^3\right)x\right)\sqrt{dx^2+c}}{6\left(c^2d^5x^4+2c^3d^4x^2+c^4d^3\right)},-\frac{3\left(b^2c^2d^2x^4+2b^2c^3dx^2+b^2c^4\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(2\left(2b^2c^2d^2-abcd^3-a^2d^4\right)x^3+3\left(b^2c^3d-a^2cd^3\right)x\right)\sqrt{dx^2+c}}{3\left(c^2d^5x^4+2c^3d^4x^2+c^4d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(d)*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(2*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*
d^4)*x^3 + 3*(b^2*c^3*d - a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(c^2*d^5*x^4 + 2*c
^3*d^4*x^2 + c^4*d^3), -1/3*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4
)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*(2*b^2*c^2*d^2 - a*b*c*d
^3 - a^2*d^4)*x^3 + 3*(b^2*c^3*d - a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(c^2*d^5*
x^4 + 2*c^3*d^4*x^2 + c^4*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

Giac [A]
time = 1.22, size = 105, normalized size = 1.00

$$-\frac{x\left(\frac{2\,(2\,b^2c^2d^2-abcd^3-a^2d^4)x^2}{c^2d^3}+\frac{3\,(b^2c^3d-a^2cd^3)}{c^2d^3}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}-\frac{b^2\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(2*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^2/(c^2*d^3) + 3*(b^2*c^3*
d - a^2*c*d^3)/(c^2*d^3))/(d*x^2 + c)^(3/2) - b^2*log(abs(-sqrt(d)*x + sqrt
(d*x^2 + c)))/d^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^2}{(d\,x^2+c)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^(5/2),x)
```

```
[Out] int((a + b*x^2)^2/(c + d*x^2)^(5/2), x)
```

3.665 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}$$

```
[Out] 1/3*(-a*d+b*c)^2/c/d^2/(d*x^2+c)^(3/2)-a^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))
/c^(5/2)+(a^2/c^2-b^2/d^2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 89, 65, 214}

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}+\frac{(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b*c - a*d)^2/(3*c*d^2*(c + d*x^2)^(3/2)) + (a^2/c^2 - b^2/d^2)/Sqrt[c + d*
x^2] - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^{5/2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{(bc-ad)^2}{cd(c+dx)^{5/2}}+\frac{b^2c^2-a^2d^2}{c^2d(c+dx)^{3/2}}+\frac{a^2}{c^2x\sqrt{c+dx}}\right)dx, x, x^2\right)\\
&= \frac{(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}+\frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2c^2}\\
&= \frac{(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}+\frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{c^2d}\\
&= \frac{(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 87, normalized size = 0.99

$$-\frac{(bc-ad)(2bc^2+4acd+3bcdx^2+3ad^2x^2)}{3c^2d^2(c+dx^2)^{3/2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
[Out] -1/3*((b*c - a*d)*(2*b*c^2 + 4*a*c*d + 3*b*c*d*x^2 + 3*a*d^2*x^2))/(c^2*d^2
*(c + d*x^2)^(3/2)) - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2)
```

Maple [A]
time = 0.11, size = 120, normalized size = 1.36

method	result

default	$b^{2}\left(-\frac{x^{2}}{d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{2c}{3d^{2}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}\right)-\frac{2ab}{3d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+a^{2}\left(\frac{1}{3c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{\frac{1}{c\sqrt{d\,x^{2}+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{c^{\frac{3}{2}}}}{c}\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-x^2/d/(d*x^2+c)^(3/2)-2/3*c/d^2/(d*x^2+c)^(3/2))-2/3*a*b/d/(d*x^2+c)^
(3/2)+a^2*(1/3/c/(d*x^2+c)^(3/2)+1/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c
+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 1.23

$$-\frac{b^{2}x^{2}}{\left(dx^{2}+c\right)^{\frac{3}{2}}d}-\frac{a^{2}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{5}{2}}}+\frac{a^{2}}{\sqrt{dx^{2}+c}\,c^{2}}+\frac{a^{2}}{3\left(dx^{2}+c\right)^{\frac{3}{2}}c}-\frac{2\,b^{2}c}{3\left(dx^{2}+c\right)^{\frac{3}{2}}d^{2}}-\frac{2\,ab}{3\left(dx^{2}+c\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -b^2*x^2/((d*x^2 + c)^(3/2)*d) - a^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2)
+ a^2/(sqrt(d*x^2 + c)*c^2) + 1/3*a^2/((d*x^2 + c)^(3/2)*c) - 2/3*b^2*c/((d
*x^2 + c)^(3/2)*d^2) - 2/3*a*b/((d*x^2 + c)^(3/2)*d)
```

Fricas [A]
time = 1.60, size = 316, normalized size = 3.59

$$\left[\frac{3\left(a^{2}d^{4}x^{4}+2\,a^{2}cd^{3}x^{2}+a^{2}c^{2}d^{2}\right)\sqrt{c}\,\log\left(-\frac{dx^{2}-2\sqrt{dx^{2}+c}\,\sqrt{c}+2c}{x^{2}}\right)-2\left(2\,b^{2}c^{4}+2\,abc^{3}d-4\,a^{2}c^{2}d^{2}+3\left(b^{2}c^{3}d-a^{2}cd^{3}\right)x^{2}\right)\sqrt{dx^{2}+c}}{6\left(c^{3}d^{4}x^{4}+2\,c^{4}d^{3}x^{2}+c^{5}d^{2}\right)},\ \frac{3\left(a^{2}d^{4}x^{4}+2\,a^{2}cd^{3}x^{2}+a^{2}c^{2}d^{2}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)-\left(2\,b^{2}c^{4}+2\,abc^{3}d-4\,a^{2}c^{2}d^{2}+3\left(b^{2}c^{3}d-a^{2}cd^{3}\right)x^{2}\right)\sqrt{dx^{2}+c}}{3\left(c^{3}d^{4}x^{4}+2\,c^{4}d^{3}x^{2}+c^{5}d^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*sqrt(c)*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*b^2*c^4 + 2*a*b*c^3*d - 4*a^2
*c^2*d^2 + 3*(b^2*c^3*d - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(c^3*d^4*x^4 + 2
*c^4*d^3*x^2 + c^5*d^2), 1/3*(3*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^
2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*b^2*c^4 + 2*a*b*c^3*d - 4
*a^2*c^2*d^2 + 3*(b^2*c^3*d - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(c^3*d^4*x^4
 + 2*c^4*d^3*x^2 + c^5*d^2)]
```

Sympy [A]
time = 18.87, size = 87, normalized size = 0.99

$$\frac{a^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{c^2\sqrt{-c}} + \frac{(ad-bc)^2}{3cd^2\left(c+dx^2\right)^{\frac{3}{2}}} + \frac{(ad-bc)\,(ad+bc)}{c^2d^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**(5/2),x)
```

```
[Out] a**2*atan(sqrt(c + d*x**2)/sqrt(-c))/(c**2*sqrt(-c)) + (a*d - b*c)**2/(3*c*
d**2*(c + d*x**2)**(3/2)) + (a*d - b*c)*(a*d + b*c)/(c**2*d**2*sqrt(c + d*x
**2))
```

Giac [A]
time = 1.01, size = 102, normalized size = 1.16

$$\frac{a^2 \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^2} - \frac{3\,(dx^2+c)b^2c^2 - b^2c^3 + 2\,abc^2d - 3\,(dx^2+c)a^2d^2 - a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/3*(3*(d*x^2 + c)*b^
2*c^2 - b^2*c^3 + 2*a*b*c^2*d - 3*(d*x^2 + c)*a^2*d^2 - a^2*c*d^2)/((d*x^2
+ c)^(3/2)*c^2*d^2)
```

Mupad [B]
time = 0.45, size = 90, normalized size = 1.02

$$\frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{3\,c}+\frac{\left(a^2\,d^2-b^2\,c^2\right)\,\left(d\,x^2+c\right)}{c^2}}{d^2\,{\left(d\,x^2+c\right)}^{3/2}} - \frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(3*c) + ((a^2*d^2 - b^2*c^2)*(c + d*x^2))/
c^2)/(d^2*(c + d*x^2)^(3/2)) - (a^2*atanh((c + d*x^2)^(1/2)/c^(1/2)))/c^(5/
2)
```

3.666 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x(2a(bc-2ad)+b^2cx^2)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{4a(bc-2ad)x}{3c^3\sqrt{c+dx^2}}$$

```
[Out] -a^2/c/x/(d*x^2+c)^(3/2)+1/3*x*(2*a*(-2*a*d+b*c)+b^2*c*x^2)/c^2/(d*x^2+c)^(
3/2)+4/3*a*(-2*a*d+b*c)*x/c^3/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {473, 386, 197}

$$-\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x(2a(bc-2ad)+b^2cx^2)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{4ax(bc-2ad)}{3c^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2)^(3/2))) + (x*(2*a*(b*c - 2*a*d) + b^2*c*x^2))/(3*c^2
*(c + d*x^2)^(3/2)) + (4*a*(b*c - 2*a*d)*x)/(3*c^3*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx &= -\frac{a^2}{cx(c+dx^2)^{3/2}}+\frac{\int \frac{2a(bc-2ad)+b^2cx^2}{(c+dx^2)^{5/2}}\,dx}{c}\\ &= -\frac{a^2}{cx(c+dx^2)^{3/2}}+\frac{x(2a(bc-2ad)+b^2cx^2)}{3c^2(c+dx^2)^{3/2}}+\frac{(4a(bc-2ad))\int \frac{1}{(c+dx^2)^{3/2}}\,dx}{3c^2}\\ &= -\frac{a^2}{cx(c+dx^2)^{3/2}}+\frac{x(2a(bc-2ad)+b^2cx^2)}{3c^2(c+dx^2)^{3/2}}+\frac{4a(bc-2ad)x}{3c^3\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 76, normalized size = 0.84

$$\frac{b^2c^2x^4+2abcx^2(3c+2dx^2)-a^2(3c^2+12cdx^2+8d^2x^4)}{3c^3x(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x^4 + 2*a*b*c*x^2*(3*c + 2*d*x^2) - a^2*(3*c^2 + 12*c*d*x^2 + 8*d^
2*x^4))/(3*c^3*x*(c + d*x^2)^(3/2))
```

Maple [A]
time = 0.11, size = 153, normalized size = 1.70

method	result
gosper	$-\frac{8a^2d^2x^4-4abcd\,x^4-b^2c^2x^4+12a^2cd\,x^2-6ab\,c^2x^2+3a^2c^2}{3x(d\,x^2+c)^{\frac{3}{2}}c^3}$
trager	$-\frac{8a^2d^2x^4-4abcd\,x^4-b^2c^2x^4+12a^2cd\,x^2-6ab\,c^2x^2+3a^2c^2}{3x(d\,x^2+c)^{\frac{3}{2}}c^3}$
risch	$-\frac{a^2\sqrt{d\,x^2+c}}{c^3x}-\frac{(ad-bc)(5ad\,x^2+c\,x^2b+6ac)x\sqrt{d\,x^2+c}}{3(d^2x^4+2x^2dc+c^2)c^3}$
default	$b^2\left(-\frac{x}{2d(d\,x^2+c)^{\frac{3}{2}}}+\frac{c\left(\frac{x}{3c(d\,x^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{d\,x^2+c}}\right)}{2d}\right)+2ab\left(\frac{x}{3c(d\,x^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{d\,x^2+c}}\right)+a^2\left(-\frac{}{cx(}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*x/d/(d*x^2+c)^(3/2)+1/2*c/d*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d
*x^2+c)^(1/2)))+2*a*b*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))+a
```

```
^2*(-1/c/x/(d*x^2+c)^(3/2)-4*d/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+
c)^(1/2)))
```

Maxima [A]
time = 0.35, size = 132, normalized size = 1.47

$$\frac{4\,abx}{3\,\sqrt{dx^2+c}\,c^2}+\frac{2\,abx}{3\,(dx^2+c)^{\frac{3}{2}}c}-\frac{b^2x}{3\,(dx^2+c)^{\frac{3}{2}}d}+\frac{b^2x}{3\,\sqrt{dx^2+c}\,cd}-\frac{8\,a^2dx}{3\,\sqrt{dx^2+c}\,c^3}-\frac{4\,a^2dx}{3\,(dx^2+c)^{\frac{3}{2}}c^2}-\frac{a^2}{(dx^2+c)^{\frac{3}{2}}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 4/3*a*b*x/(sqrt(d*x^2 + c)*c^2) + 2/3*a*b*x/((d*x^2 + c)^(3/2)*c) - 1/3*b^2
*x/((d*x^2 + c)^(3/2)*d) + 1/3*b^2*x/(sqrt(d*x^2 + c)*c*d) - 8/3*a^2*d*x/(s
qrt(d*x^2 + c)*c^3) - 4/3*a^2*d*x/((d*x^2 + c)^(3/2)*c^2) - a^2/((d*x^2 + c
)^(3/2)*c*x)
```

Fricas [A]
time = 2.02, size = 92, normalized size = 1.02

$$\frac{((b^2c^2+4\,abcd-8\,a^2d^2)x^4-3\,a^2c^2+6\,(abc^2-2\,a^2cd)x^2)\sqrt{dx^2+c}}{3\,(c^3d^2x^5+2\,c^4dx^3+c^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*x^4 - 3*a^2*c^2 + 6*(a*b*c^2 - 2*a^2
*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d^2*x^5 + 2*c^4*d*x^3 + c^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx^2)^2}{x^2\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(5/2)), x)
```

Giac [A]
time = 0.79, size = 117, normalized size = 1.30

$$\frac{x\left(\frac{(b^2c^4d+4\,abc^3d^2-5\,a^2c^2d^3)x^2}{c^5d}+\frac{6\,(abc^4d-a^2c^3d^2)}{c^5d}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}+\frac{2\,a^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*((b^2*c^4*d + 4*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x^2/(c^5*d) + 6*(a*b*c^4
*d - a^2*c^3*d^2)/(c^5*d))/(d*x^2 + c)^(3/2) + 2*a^2*sqrt(d)/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^2 - c)*c^2)
```

Mupad [B]
time = 0.35, size = 77, normalized size = 0.86

$$-\frac{3\,a^2\,c^2+12\,a^2\,c\,d\,x^2+8\,a^2\,d^2\,x^4-6\,a\,b\,c^2\,x^2-4\,a\,b\,c\,d\,x^4-b^2\,c^2\,x^4}{3\,c^3\,x\,{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] -(3*a^2*c^2 + 8*a^2*d^2*x^4 - b^2*c^2*x^4 - 6*a*b*c^2*x^2 + 12*a^2*c*d*x^2
- 4*a*b*c*d*x^4)/(3*c^3*x*(c + d*x^2)^(3/2))
```

3.667 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c\left(c+dx^2\right)^{3/2}}-\frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}}+\frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}}-\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}$$

```
[Out] 1/6*(4*a*b-2*b^2*c/d-5*a^2*d/c)/c/(d*x^2+c)^(3/2)-1/2*a^2/c/x^2/(d*x^2+c)^(
3/2)-1/2*a*(-5*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(7/2)+1/2*a*(-
5*a*d+4*b*c)/c^3/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 91, 79, 53, 65, 214}

$$-\frac{5a^2d^2-4abcd+2b^2c^2}{6c^2d\left(c+dx^2\right)^{3/2}}-\frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}}-\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}+\frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(2*b^2*c^2 - 4*a*b*c*d + 5*a^2*d^2)/(c^2*d*(c + d*x^2)^(3/2)) - a^2/(2
*c*x^2*(c + d*x^2)^(3/2)) + (a*(4*b*c - 5*a*d))/(2*c^3*Sqrt[c + d*x^2]) - (
a*(4*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(7/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^{5/2}}\,dx, x, x^2\right)$$
$$= -\frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4bc-5ad)+b^2cx}{x(c+dx)^{5/2}}\,dx, x, x^2\right)}{2c}$$
$$= \frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c(c+dx^2)^{3/2}} - \frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{(a(4bc-5ad))\text{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, x^2\right)}{4c^2}$$
$$= \frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c(c+dx^2)^{3/2}} - \frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} + \frac{(a(4bc-5ad))\text{Subst}\left(\int -\right.}{4c^3}$$
$$= \frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c(c+dx^2)^{3/2}} - \frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} + \frac{(a(4bc-5ad))\text{Subst}\left(\int -\right.}{2c^3}$$
$$= \frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c(c+dx^2)^{3/2}} - \frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} - \frac{a(4bc-5ad)\tanh^{-1}\left(\sqrt{c}\right.}{2c^{7/2}}$$

Mathematica [A]
time = 0.20, size = 120, normalized size = 0.92

$$\frac{-2b^2c^3x^2+4abcdx^2(4c+3dx^2)-a^2d(3c^2+20cdx^2+15d^2x^4)}{6c^3dx^2(c+dx^2)^{3/2}} + \frac{a(-4bc+5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-2*b^2*c^3*x^2 + 4*a*b*c*d*x^2*(4*c + 3*d*x^2) - a^2*d*(3*c^2 + 20*c*d*x^2
 + 15*d^2*x^4))/(6*c^3*d*x^2*(c + d*x^2)^(3/2)) + (a*(-4*b*c + 5*a*d)*ArcTa
nh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(7/2))
```

Maple [A]
time = 0.13, size = 173, normalized size = 1.32

method	result

default	$-\frac{b^{2}}{3d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+a^{2}\left(-\frac{1}{2c\,x^{2}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{5d\left(\frac{1}{3c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{\frac{1}{c\sqrt{d\,x^{2}+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{c^{\frac{3}{2}}}}{c}\right)}{2c}\right)+2ab$
risch	$-\frac{a^{2}\sqrt{d\,x^{2}+c}}{2c^{3}x^{2}}-\frac{13d\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^{2}d+2\sqrt{-cd}\,\left(x-\frac{\sqrt{-cd}}{d}\right)}\,a^{2}}{12c^{3}\sqrt{-cd}\,\left(x-\frac{\sqrt{-cd}}{d}\right)}+\frac{7\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^{2}d+}}{6c^{2}\sqrt{-cd}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^2/d/(d*x^2+c)^(3/2)+a^2*(-1/2/c/x^2/(d*x^2+c)^(3/2)-5/2*d/c*(1/3/c/(
d*x^2+c)^(3/2)+1/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+
c)^(1/2))/x))))+2*a*b*(1/3/c/(d*x^2+c)^(3/2)+1/c*(1/c/(d*x^2+c)^(1/2)-1/c^(
3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))
```

Maxima [A]
time = 0.27, size = 146, normalized size = 1.11

$$-\frac{2\,ab\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{5}{2}}}+\frac{5\,a^{2}d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,c^{\frac{7}{2}}}+\frac{2\,ab}{\sqrt{dx^{2}+c}\,c^{2}}+\frac{2\,ab}{3\,(dx^{2}+c)^{\frac{3}{2}}c}-\frac{b^{2}}{3\,(dx^{2}+c)^{\frac{3}{2}}d}-\frac{5\,a^{2}d}{2\,\sqrt{dx^{2}+c}\,c^{3}}-\frac{5\,a^{2}d}{6\,(dx^{2}+c)^{\frac{3}{2}}c^{2}}-\frac{a^{2}}{2\,(dx^{2}+c)^{\frac{3}{2}}cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2*a*b*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) + 5/2*a^2*d*arcsinh(c/(sqrt(c*
d)*abs(x)))/c^(7/2) + 2*a*b/(sqrt(d*x^2 + c)*c^2) + 2/3*a*b/((d*x^2 + c)^(3
/2)*c) - 1/3*b^2/((d*x^2 + c)^(3/2)*d) - 5/2*a^2*d/(sqrt(d*x^2 + c)*c^3) -
5/6*a^2*d/((d*x^2 + c)^(3/2)*c^2) - 1/2*a^2/((d*x^2 + c)^(3/2)*c*x^2)
```

Fricas [A]
time = 1.37, size = 426, normalized size = 3.25

$$\left[-\frac{3\,((4\,abcd^{3}-5\,a^{2}d^{4})x^{6}+2\,(4\,abc^{2}d^{2}-5\,a^{2}cd^{3})x^{4}+(4\,abc^{3}d-5\,a^{2}c^{2}d^{2})x^{2})\sqrt{c}\,\log\left(-\frac{dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+2\,(3\,a^{2}c^{3}d-3\,(4\,abc^{2}d^{2}-5\,a^{2}cd^{3})x^{4}+2\,(b^{2}c^{4}-8\,abc^{3}d+10\,a^{2}c^{2}d^{2})x^{2})\sqrt{dx^{2}+c}}{12\,(c^{4}d^{3}x^{6}+2\,c^{5}d^{2}x^{4}+c^{6}dx^{2})},\ \frac{3\,((4\,abcd^{3}-5\,a^{2}d^{4})x^{6}+2\,(4\,abc^{2}d^{2}-5\,a^{2}cd^{3})x^{4}+(4\,abc^{3}d-5\,a^{2}c^{2}d^{2})x^{2})\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)-(3\,a^{2}c^{3}d-3\,(4\,abc^{2}d^{2}-5\,a^{2}cd^{3})x^{4}+2\,(b^{2}c^{4}-8\,abc^{3}d+10\,a^{2}c^{2}d^{2})x^{2})\sqrt{dx^{2}+c}}{6\,(c^{4}d^{3}x^{6}+2\,c^{5}d^{2}x^{4}+c^{6}dx^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((4*a*b*c*d^3 - 5*a^2*d^4)*x^6 + 2*(4*a*b*c^2*d^2 - 5*a^2*c*d^3)*
x^4 + (4*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2
 + c)*sqrt(c) + 2*c)/x^2) + 2*(3*a^2*c^3*d - 3*(4*a*b*c^2*d^2 - 5*a^2*c*d^3
)*x^4 + 2*(b^2*c^4 - 8*a*b*c^3*d + 10*a^2*c^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c
^4*d^3*x^6 + 2*c^5*d^2*x^4 + c^6*d*x^2), 1/6*(3*((4*a*b*c*d^3 - 5*a^2*d^4)*
x^6 + 2*(4*a*b*c^2*d^2 - 5*a^2*c*d^3)*x^4 + (4*a*b*c^3*d - 5*a^2*c^2*d^2)*x
^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (3*a^2*c^3*d - 3*(4*a*b*c^2
*d^2 - 5*a^2*c*d^3)*x^4 + 2*(b^2*c^4 - 8*a*b*c^3*d + 10*a^2*c^2*d^2)*x^2)*s
qrt(d*x^2 + c))/(c^4*d^3*x^6 + 2*c^5*d^2*x^4 + c^6*d*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{x^3\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(5/2)), x)
```

Giac [A]
time = 0.99, size = 128, normalized size = 0.98

$$\frac{(4\,abc-5\,a^2d)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,\sqrt{-c}\,c^3}-\frac{\sqrt{dx^2+c}\,a^2}{2\,c^3x^2}-\frac{b^2c^3-6\,(dx^2+c)abcd-2\,abc^2d+6\,(dx^2+c)a^2d^2+a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*c - 5*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1
/2*sqrt(d*x^2 + c)*a^2/(c^3*x^2) - 1/3*(b^2*c^3 - 6*(d*x^2 + c)*a*b*c*d - 2
*a*b*c^2*d + 6*(d*x^2 + c)*a^2*d^2 + a^2*c*d^2)/((d*x^2 + c)^(3/2)*c^3*d)
```

Mupad [B]
time = 0.49, size = 147, normalized size = 1.12

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(5\,a\,d-4\,b\,c)}{2\,c^{7/2}}-\frac{\frac{(d\,x^2+c)\,(-5\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2)}{3\,c^2}-\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{3\,c}+\frac{d\,(d\,x^2+c)^2\,(5\,a^2\,d-4\,a\,b\,c)}{2\,c^3}}{d\,(d\,x^2+c)^{5/2}-c\,d\,(d\,x^2+c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x)
```

```
[Out] (a*atanh((c + d*x^2)^(1/2)/c^(1/2))*(5*a*d - 4*b*c))/(2*c^(7/2)) - (((c + d
*x^2)*(b^2*c^2 - 5*a^2*d^2 + 4*a*b*c*d))/(3*c^2) - (a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)/(3*c) + (d*(c + d*x^2)^2*(5*a^2*d - 4*a*b*c))/(2*c^3))/(d*(c + d*x^
2)^(5/2) - c*d*(c + d*x^2)^(3/2))
```

3.668 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{a^2}{3cx^3\,(c+dx^2)^{3/2}}-\frac{2a(bc-ad)}{c^2x\,(c+dx^2)^{3/2}}+\frac{(b^2c^2-8ad(bc-ad))\,x}{3c^3\,(c+dx^2)^{3/2}}+\frac{2(b^2c^2-8ad(bc-ad))\,x}{3c^4\sqrt{c+dx^2}}$$

```
[Out] -1/3*a^2/c/x^3/(d*x^2+c)^(3/2)-2*a*(-a*d+b*c)/c^2/x/(d*x^2+c)^(3/2)+1/3*(b^
2*c^2-8*a*d*(-a*d+b*c))*x/c^3/(d*x^2+c)^(3/2)+2/3*(b^2*c^2-8*a*d*(-a*d+b*c)
)*x/c^4/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 130, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {473, 464, 198, 197}

$$-\frac{a^2}{3cx^3\,(c+dx^2)^{3/2}}+\frac{x\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)}{3c\,(c+dx^2)^{3/2}}+\frac{2x(b^2c^2-8ad(bc-ad))}{3c^4\sqrt{c+dx^2}}-\frac{2a(bc-ad)}{c^2x\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*a^2/(c*x^3*(c + d*x^2)^(3/2)) - (2*a*(b*c - a*d))/(c^2*x*(c + d*x^2)^(
3/2)) + ((b^2 - (8*a*d*(b*c - a*d))/c^2)*x)/(3*c*(c + d*x^2)^(3/2)) + (2*(b
^2*c^2 - 8*a*d*(b*c - a*d))*x)/(3*c^4*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx &= -\frac{a^2}{3cx^3(c+dx^2)^{3/2}}+\frac{\int \frac{6a(bc-ad)+3b^2cx^2}{x^2(c+dx^2)^{5/2}}\,dx}{3c}\\
&= -\frac{a^2}{3cx^3(c+dx^2)^{3/2}}-\frac{2a(bc-ad)}{c^2x(c+dx^2)^{3/2}}-\left(-b^2+\frac{8ad(bc-ad)}{c^2}\right)\int \frac{1}{(c+dx^2)^{5/2}}\,dx\\
&= -\frac{a^2}{3cx^3(c+dx^2)^{3/2}}-\frac{2a(bc-ad)}{c^2x(c+dx^2)^{3/2}}+\frac{\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)x}{3c(c+dx^2)^{3/2}}+\frac{\left(2\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)\right)}{3c}\\
&= -\frac{a^2}{3cx^3(c+dx^2)^{3/2}}-\frac{2a(bc-ad)}{c^2x(c+dx^2)^{3/2}}+\frac{\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)x}{3c(c+dx^2)^{3/2}}+\frac{2\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)x}{3c^2\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 107, normalized size = 0.82

$$\frac{b^2c^2x^4(3c+2dx^2)-2abcx^2(3c^2+12cdx^2+8d^2x^4)+a^2(-c^3+6c^2dx^2+24cd^2x^4+16d^3x^6)}{3c^4x^3(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x^4*(3*c + 2*d*x^2) - 2*a*b*c*x^2*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4)
 + a^2*(-c^3 + 6*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6))/(3*c^4*x^3*(c + d*
x^2)^(3/2))
```

Maple [A]
time = 0.12, size = 179, normalized size = 1.37

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\;a\left(-8ad\,x^{2}+6c\,x^{2}b+ac\right)}{3c^{4}x^{3}}+\frac{(ad-bc)\left(8a\,d^{2}x^{2}-2bcd\,x^{2}+9acd-3b\,c^{2}\right)x\sqrt{d\,x^{2}+c}}{3\left(d^{2}x^{4}+2x^{2}dc+c^{2}\right)c^{4}}$
gosper	$-\frac{-16a^{2}d^{3}x^{6}+16abc\,d^{2}x^{6}-2b^{2}c^{2}d\,x^{6}-24a^{2}c\,d^{2}x^{4}+24ab\,c^{2}d\,x^{4}-3b^{2}c^{3}x^{4}-6a^{2}c^{2}d\,x^{2}+6ab\,c^{3}x^{2}+a^{2}c^{3}}{3x^{3}\left(d\,x^{2}+c\right)^{\frac{3}{2}}c^{4}}$
trager	$-\frac{-16a^{2}d^{3}x^{6}+16abc\,d^{2}x^{6}-2b^{2}c^{2}d\,x^{6}-24a^{2}c\,d^{2}x^{4}+24ab\,c^{2}d\,x^{4}-3b^{2}c^{3}x^{4}-6a^{2}c^{2}d\,x^{2}+6ab\,c^{3}x^{2}+a^{2}c^{3}}{3x^{3}\left(d\,x^{2}+c\right)^{\frac{3}{2}}c^{4}}$
default	$b^{2}\left(\frac{x}{3c(d\,x^{2}+c)^{\frac{3}{2}}}+\frac{2x}{3c^{2}\sqrt{d\,x^{2}+c}}\right)+a^{2}\left(-\frac{1}{3c\,x^{3}(d\,x^{2}+c)^{\frac{3}{2}}}-\frac{2d\left(-\frac{1}{cx\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{4d\left(\frac{x}{3c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{2x}{3c^{2}\sqrt{d\,x^{2}+}}\right.}{c}\right.}{c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))+a^2*(-1/3/c/x^3/(d*
x^2+c)^(3/2)-2*d/c*(-1/c/x/(d*x^2+c)^(3/2)-4*d/c*(1/3*x/c/(d*x^2+c)^(3/2)+2
/3*x/c^2/(d*x^2+c)^(1/2))))+2*a*b*(-1/c/x/(d*x^2+c)^(3/2)-4*d/c*(1/3*x/c/(d
*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)))
```

Maxima [A]
time = 0.33, size = 175, normalized size = 1.34

$$\frac{2\,b^{2}x}{3\,\sqrt{dx^{2}+c}\,c^{2}}+\frac{b^{2}x}{3\,(dx^{2}+c)^{\frac{3}{2}}c}-\frac{16\,abdx}{3\,\sqrt{dx^{2}+c}\,c^{3}}-\frac{8\,abdx}{3\,(dx^{2}+c)^{\frac{3}{2}}c^{2}}+\frac{16\,a^{2}d^{2}x}{3\,\sqrt{dx^{2}+c}\,c^{4}}+\frac{8\,a^{2}d^{2}x}{3\,(dx^{2}+c)^{\frac{3}{2}}c^{3}}-\frac{2\,ab}{(dx^{2}+c)^{\frac{3}{2}}cx}+\frac{2\,a^{2}d}{(dx^{2}+c)^{\frac{3}{2}}c^{2}x}-\frac{a^{2}}{3\,(dx^{2}+c)^{\frac{3}{2}}cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x/(sqrt(d*x^2 + c)*c^2) + 1/3*b^2*x/((d*x^2 + c)^(3/2)*c) - 16/3*a*
b*d*x/(sqrt(d*x^2 + c)*c^3) - 8/3*a*b*d*x/((d*x^2 + c)^(3/2)*c^2) + 16/3*a^
2*d^2*x/(sqrt(d*x^2 + c)*c^4) + 8/3*a^2*d^2*x/((d*x^2 + c)^(3/2)*c^3) - 2*a
*b/((d*x^2 + c)^(3/2)*c*x) + 2*a^2*d/((d*x^2 + c)^(3/2)*c^2*x) - 1/3*a^2/((
d*x^2 + c)^(3/2)*c*x^3)
```

Fricas [A]
time = 1.68, size = 130, normalized size = 0.99

$$\frac{\left(2\,(b^{2}c^{2}d-8\,abcd^{2}+8\,a^{2}d^{3})x^{6}-a^{2}c^{3}+3\,(b^{2}c^{3}-8\,abc^{2}d+8\,a^{2}cd^{2})x^{4}-6\,(abc^{3}-a^{2}c^{2}d)x^{2}\right)\sqrt{dx^{2}+c}}{3\,(c^{4}d^{2}x^{7}+2\,c^{5}dx^{5}+c^{6}x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(b^2*c^2*d - 8*a*b*c*d^2 + 8*a^2*d^3)*x^6 - a^2*c^3 + 3*(b^2*c^3 - 8
*a*b*c^2*d + 8*a^2*c*d^2)*x^4 - 6*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt(d*x^2 + c
)/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{x^4\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**4*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(117) = 234.
time = 0.88, size = 258, normalized size = 1.97

$$\frac{x\left(\frac{2\,(b^2c^5d^2-5\,abc^4d^3+4\,a^2c^3d^4)x^2}{c^7d}+\frac{3\,(b^2c^6d-4\,abc^5d^2+3\,a^2c^4d^3)}{c^7d}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}+\frac{4\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^2\sqrt{d}+9\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2cd^{\frac{3}{2}}+3\,abc^3\sqrt{d}-4\,a^2c^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*(b^2*c^5*d^2 - 5*a*b*c^4*d^3 + 4*a^2*c^3*d^4)*x^2/(c^7*d) + 3*(b^2
*c^6*d - 4*a*b*c^5*d^2 + 3*a^2*c^4*d^3)/(c^7*d))/(d*x^2 + c)^(3/2) + 4/3*(3
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2
+ c))^4*a^2*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 9
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c*d^(3/2) + 3*a*b*c^3*sqrt(d) - 4*a^2*
c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*c^3)
```

Mupad [B]
time = 0.42, size = 187, normalized size = 1.43

$$\frac{b^2\,c^4\,x^2-a^2\,c^3\,d-16\,a^2\,d\,{\left(d\,x^2+c\right)}^3+2\,a\,b\,c^4+b^2\,c^3\,x^2\,\left(d\,x^2+c\right)+16\,a\,b\,c\,{\left(d\,x^2+c\right)}^3+6\,a\,b\,c^3\,\left(d\,x^2+c\right)-2\,b^2\,c^2\,x^2\,{\left(d\,x^2+c\right)}^2-24\,a\,b\,c^2\,{\left(d\,x^2+c\right)}^2+24\,a^2\,c\,d\,{\left(d\,x^2+c\right)}^2-6\,a^2\,c^2\,d\,\left(d\,x^2+c\right)}{{\left(d\,x^2+c\right)}^{3/2}\,\left(3\,c^5\,x-3\,c^4\,x\,\left(d\,x^2+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x)
```

```
[Out] (b^2*c^4*x^2 - a^2*c^3*d - 16*a^2*d*(c + d*x^2)^3 + 2*a*b*c^4 + b^2*c^3*x^2
*(c + d*x^2) + 16*a*b*c*(c + d*x^2)^3 + 6*a*b*c^3*(c + d*x^2) - 2*b^2*c^2*x
^2*(c + d*x^2)^2 - 24*a*b*c^2*(c + d*x^2)^2 + 24*a^2*c*d*(c + d*x^2)^2 - 6*
a^2*c^2*d*(c + d*x^2))/((c + d*x^2)^(3/2)*(3*c^5*x - 3*c^4*x*(c + d*x^2)))
```

3.669 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{8b^2c^2-5ad(8bc-7ad)}{24c^3\left(c+dx^2\right)^{3/2}}-\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}}-\frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{8c^4\sqrt{c+dx^2}}-\frac{(8b^2c^2-5ad(8bc$$

```
[Out] 1/24*(8*b^2*c^2-5*a*d*(-7*a*d+8*b*c))/c^3/(d*x^2+c)^(3/2)-1/4*a^2/c/x^4/(d*
x^2+c)^(3/2)-1/8*a*(-7*a*d+8*b*c)/c^2/x^2/(d*x^2+c)^(3/2)-1/8*(8*b^2*c^2-5*
a*d*(-7*a*d+8*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(9/2)+1/8*(8*b^2*c^2
-5*a*d*(-7*a*d+8*b*c))/c^4/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 91, 79, 53, 65, 214}

$$-\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}}+\frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}}-\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{8c^4\sqrt{c+dx^2}}-\frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

```
[Out] (8*b^2 - (5*a*d*(8*b*c - 7*a*d))/c^2)/(24*c*(c + d*x^2)^(3/2)) - a^2/(4*c*x
^4*(c + d*x^2)^(3/2)) - (a*(8*b*c - 7*a*d))/(8*c^2*x^2*(c + d*x^2)^(3/2)) +
 (8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))/(8*c^4*Sqrt[c + d*x^2]) - ((8*b^2*c^2
- 5*a*d*(8*b*c - 7*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(9/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^3(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{a^2}{4cx^4(c+dx^2)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(8bc-7ad)+2b^2cx}{x^2(c+dx)^{5/2}}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a^2}{4cx^4(c+dx^2)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2(c+dx^2)^{3/2}} + \frac{1}{16}\left(8b^2 - \frac{5ad(8bc-7ad)}{c^2}\right)\text{Subst}\left(\int \right.$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c(c+dx^2)^{3/2}} - \frac{a^2}{4cx^4(c+dx^2)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2(c+dx^2)^{3/2}} + \frac{\left(8b^2-\frac{5ad(8bc-7ad)}{c^2}\right)\text{S}}{}$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c(c+dx^2)^{3/2}} - \frac{a^2}{4cx^4(c+dx^2)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2(c+dx^2)^{3/2}} + \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{8c^2\sqrt{c+dx^2}} +$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c(c+dx^2)^{3/2}} - \frac{a^2}{4cx^4(c+dx^2)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2(c+dx^2)^{3/2}} + \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{8c^2\sqrt{c+dx^2}} +$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c(c+dx^2)^{3/2}} - \frac{a^2}{4cx^4(c+dx^2)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2(c+dx^2)^{3/2}} + \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{8c^2\sqrt{c+dx^2}} -$$

Mathematica [A]
time = 0.21, size = 159, normalized size = 0.86

$$\frac{8b^2c^2x^4(4c+3dx^2) - 8abcx^2(3c^2+20cdx^2+15d^2x^4) + a^2(-6c^3+21c^2dx^2+140cd^2x^4+105d^3x^6)}{24c^4x^4(c+dx^2)^{3/2}} - \frac{(8b^2c^2-40abcd+35a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

```
[Out] (8*b^2*c^2*x^4*(4*c + 3*d*x^2) - 8*a*b*c*x^2*(3*c^2 + 20*c*d*x^2 + 15*d^2*x
^4) + a^2*(-6*c^3 + 21*c^2*d*x^2 + 140*c*d^2*x^4 + 105*d^3*x^6))/(24*c^4*x^
4*(c + d*x^2)^(3/2)) - ((8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]])/(8*c^(9/2))
```

Maple [A]
time = 0.16, size = 269, normalized size = 1.45

method	result

default	$a^{2} \left(-\frac{1}{4 c \,x^{4} \left(d \,x^{2}+c \right)^{\frac{3}{2}}}-\frac{7 d \left(-\frac{1}{2 c \,x^{2} \left(d \,x^{2}+c \right)^{\frac{3}{2}}}-\frac{5 d \left(\frac{1}{3 c \left(d \,x^{2}+c \right)^{\frac{3}{2}}}+\frac{\frac{1}{c \sqrt{d \,x^{2}+c}}-\frac{\ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)}{c^{\frac{3}{2}}}}{c}\right)}{2 c}\right)}{4 c}\right)+2$
risch	$-\frac{\sqrt{d \,x^{2}+c}\, a \left(-11 a d \,x^{2}+8 c \,x^{2} b +2 a c \right)}{8 c^{4} x^{4}}+\frac{19 \sqrt{\left(x -\frac{\sqrt{-c d}}{d}\right)^{2} d +2 \sqrt{-c d}\, \left(x -\frac{\sqrt{-c d}}{d}\right)}\, a^{2} d^{2}}{12 c^{4} \sqrt{-c d}\, \left(x -\frac{\sqrt{-c d}}{d}\right)}-13 \sqrt{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/4/c/x^4/(d*x^2+c)^(3/2)-7/4*d/c*(-1/2/c/x^2/(d*x^2+c)^(3/2)-5/2*d/c
*(1/3/c/(d*x^2+c)^(3/2)+1/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2
)*(d*x^2+c)^(1/2))/x)))))+2*a*b*(-1/2/c/x^2/(d*x^2+c)^(3/2)-5/2*d/c*(1/3/c/
(d*x^2+c)^(3/2)+1/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2
+c)^(1/2))/x))))+b^2*(1/3/c/(d*x^2+c)^(3/2)+1/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3
/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))
```

Maxima [A]

time = 0.30, size = 231, normalized size = 1.25

$$-\frac{b^{2} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{5}{2}}}+\frac{5\,abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{7}{2}}}-\frac{35\,a^{2}d^{2}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{8\,c^{\frac{9}{2}}}+\frac{b^{2}}{\sqrt{dx^{2}+c}\,c^{2}}+\frac{b^{2}}{3\,(dx^{2}+c)^{\frac{3}{2}}c}-\frac{5\,abd}{\sqrt{dx^{2}+c}\,c^{3}}-\frac{5\,abd}{3\,(dx^{2}+c)^{\frac{3}{2}}c^{2}}+\frac{35\,a^{2}d^{2}}{8\,\sqrt{dx^{2}+c}\,c^{4}}+\frac{35\,a^{2}d^{2}}{24\,(dx^{2}+c)^{\frac{3}{2}}c^{3}}-\frac{ab}{(dx^{2}+c)^{\frac{3}{2}}cx^{2}}+\frac{7\,a^{2}d}{8\,(dx^{2}+c)^{\frac{3}{2}}c^{2}x^{2}}-\frac{a^{2}}{4\,(dx^{2}+c)^{\frac{3}{2}}cx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -b^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) + 5*a*b*d*arcsinh(c/(sqrt(c*d)*a
bs(x)))/c^(7/2) - 35/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(9/2) + b^2/
(sqrt(d*x^2 + c)*c^2) + 1/3*b^2/((d*x^2 + c)^(3/2)*c) - 5*a*b*d/(sqrt(d*x^2
 + c)*c^3) - 5/3*a*b*d/((d*x^2 + c)^(3/2)*c^2) + 35/8*a^2*d^2/(sqrt(d*x^2 +
 c)*c^4) + 35/24*a^2*d^2/((d*x^2 + c)^(3/2)*c^3) - a*b/((d*x^2 + c)^(3/2)*c
*x^2) + 7/8*a^2*d/((d*x^2 + c)^(3/2)*c^2*x^2) - 1/4*a^2/((d*x^2 + c)^(3/2)*
c*x^4)
```

Fricas [**A**]
time = 0.88, size = 537, normalized size = 2.90

$$\left[\frac{3\,\left((8\,b^2c^2d^2 - 40\,abcd^3 + 35\,a^2d^4)x^8 + 2\,(8\,b^2c^3d - 40\,abc^2d^2 + 35\,a^2cd^3)x^6 + (8\,b^2c^4 - 40\,abc^3d + 35\,a^2c^2d^2)x^4\right)\sqrt{c}\,\log\left(-\frac{dx^2 - 2\,\sqrt{dx^2+c}\,\sqrt{c} + 2c}{x^2}\right) + 2\,\left(3\,(8\,b^2c^3d - 40\,abc^2d^2 + 35\,a^2cd^3)x^6 - 6\,a^2c^4 + 4\,(8\,b^2c^4 - 40\,abc^3d + 35\,a^2c^2d^2)x^4 - 3\,(8\,abc^4 - 7\,a^2c^3d)x^2\right)\sqrt{dx^2+c}}{48\,(c^5d^2x^8 + 2\,c^6dx^6 + c^7x^4)}, \frac{3\,\left((8\,b^2c^2d^2 - 40\,abcd^3 + 35\,a^2d^4)x^8 + 2\,(8\,b^2c^3d - 40\,abc^2d^2 + 35\,a^2cd^3)x^6 + (8\,b^2c^4 - 40\,abc^3d + 35\,a^2c^2d^2)x^4\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right) + \left(3\,(8\,b^2c^3d - 40\,abc^2d^2 + 35\,a^2cd^3)x^6 - 6\,a^2c^4 + 4\,(8\,b^2c^4 - 40\,abc^3d + 35\,a^2c^2d^2)x^4 - 3\,(8\,abc^4 - 7\,a^2c^3d)x^2\right)\sqrt{dx^2+c}}{24\,(c^5d^2x^8 + 2\,c^6dx^6 + c^7x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*b^2*c^2*d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d
- 40*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c
^2*d^2)*x^4)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
2*(3*(8*b^2*c^3*d - 40*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 - 6*a^2*c^4 + 4*(8*b
^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 - 3*(8*a*b*c^4 - 7*a^2*c^3*d)*x
^2)*sqrt(d*x^2 + c))/(c^5*d^2*x^8 + 2*c^6*d*x^6 + c^7*x^4), 1/24*(3*((8*b^2
*c^2*d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d - 40*a*b*c^2*d^2
 + 35*a^2*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4)*sqr
t(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (3*(8*b^2*c^3*d - 40*a*b*c^2*d^2 +
 35*a^2*c*d^3)*x^6 - 6*a^2*c^4 + 4*(8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d
^2)*x^4 - 3*(8*a*b*c^4 - 7*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d^2*x^8 +
2*c^6*d*x^6 + c^7*x^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^2}{x^5\left(c + dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**5*(c + d*x**2)**(5/2)), x)
```

Giac [**A**]
time = 1.06, size = 210, normalized size = 1.14

$$\frac{(8\,b^2c^2 - 40\,abcd + 35\,a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}\,c^4} + \frac{3\,(dx^2+c)b^2c^2 + b^2c^3 - 12\,(dx^2+c)abcd - 2\,abc^2d + 9\,(dx^2+c)a^2d^2 + a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}c^4} - \frac{8\,(dx^2+c)^{\frac{3}{2}}abcd - 8\,\sqrt{dx^2+c}\,abc^2d - 11\,(dx^2+c)^{\frac{3}{2}}a^2d^2 + 13\,\sqrt{dx^2+c}\,a^2cd^2}{8\,c^4d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/
(sqrt(-c)*c^4) + 1/3*(3*(d*x^2 + c)*b^2*c^2 + b^2*c^3 - 12*(d*x^2 + c)*a*b*
c*d - 2*a*b*c^2*d + 9*(d*x^2 + c)*a^2*d^2 + a^2*c*d^2)/((d*x^2 + c)^(3/2)*c
^4) - 1/8*(8*(d*x^2 + c)^(3/2)*a*b*c*d - 8*sqrt(d*x^2 + c)*a*b*c^2*d - 11*(
d*x^2 + c)^(3/2)*a^2*d^2 + 13*sqrt(d*x^2 + c)*a^2*c*d^2)/(c^4*d^2*x^4)
```

Mupad [B]
time = 0.62, size = 216, normalized size = 1.17

$$\frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{3\,c}+\frac{\left(d\,x^2+c\right)\,\left(7\,a^2\,d^2-8\,a\,b\,c\,d+b^2\,c^2\right)}{3\,c^2}-\frac{5\,{\left(d\,x^2+c\right)}^2\,\left(35\,a^2\,d^2-40\,a\,b\,c\,d+8\,b^2\,c^2\right)}{24\,c^3}+\frac{{\left(d\,x^2+c\right)}^3\,\left(35\,a^2\,d^2-40\,a\,b\,c\,d+8\,b^2\,c^2\right)}{8\,c^4}}{{\left(d\,x^2+c\right)}^{7/2}-2\,c\,{\left(d\,x^2+c\right)}^{5/2}+c^2\,{\left(d\,x^2+c\right)}^{3/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(35\,a^2\,d^2-40\,a\,b\,c\,d+8\,b^2\,c^2\right)}{8\,c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(3*c) + ((c + d*x^2)*(7*a^2*d^2 + b^2*c^2
- 8*a*b*c*d))/(3*c^2) - (5*(c + d*x^2)^2*(35*a^2*d^2 + 8*b^2*c^2 - 40*a*b*c
*d))/(24*c^3) + ((c + d*x^2)^3*(35*a^2*d^2 + 8*b^2*c^2 - 40*a*b*c*d))/(8*c^
4))/((c + d*x^2)^(7/2) - 2*c*(c + d*x^2)^(5/2) + c^2*(c + d*x^2)^(3/2)) - (
atanh((c + d*x^2)^(1/2)/c^(1/2))*(35*a^2*d^2 + 8*b^2*c^2 - 40*a*b*c*d))/(8*
c^(9/2))
```

3.670 $\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3\left(c+dx^2\right)^{3/2}}-\frac{5b^2c^2-4ad(5bc-4ad)}{5c^3x\left(c+dx^2\right)^{3/2}}-\frac{4d(5b^2c^2-4ad(5bc-4ad))\,x}{15c^4\left(c+dx^2\right)^{3/2}}-\frac{8d(5b^2c$$

```
[Out] -1/5*a^2/c/x^5/(d*x^2+c)^(3/2)-2/15*a*(-4*a*d+5*b*c)/c^2/x^3/(d*x^2+c)^(3/2
)+1/5*(-5*b^2*c^2+4*a*d*(-4*a*d+5*b*c))/c^3/x/(d*x^2+c)^(3/2)-4/15*d*(5*b^2
*c^2-4*a*d*(-4*a*d+5*b*c))*x/c^4/(d*x^2+c)^(3/2)-8/15*d*(5*b^2*c^2-4*a*d*(-
4*a*d+5*b*c))*x/c^5/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {473, 464, 277, 198, 197}

$$-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx\left(c+dx^2\right)^{3/2}}-\frac{8dx(5b^2c^2-4ad(5bc-4ad))}{15c^5\sqrt{c+dx^2}}-\frac{4dx(5b^2c^2-4ad(5bc-4ad))}{15c^4\left(c+dx^2\right)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/5*a^2/(c*x^5*(c + d*x^2)^(3/2)) - (2*a*(5*b*c - 4*a*d))/(15*c^2*x^3*(c +
 d*x^2)^(3/2)) - (5*b^2 - (4*a*d*(5*b*c - 4*a*d))/c^2)/(5*c*x*(c + d*x^2)^(
3/2)) - (4*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))*x)/(15*c^4*(c + d*x^2)^(3/
2)) - (8*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))*x)/(15*c^5*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx = -\frac{a^2}{5cx^5(c+dx^2)^{3/2}}+\frac{\int \frac{2a(5bc-4ad)+5b^2cx^2}{x^4(c+dx^2)^{5/2}}\,dx}{5c}$$
$$= -\frac{a^2}{5cx^5(c+dx^2)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}}-\frac{1}{5}\left(-5b^2+\frac{4ad(5bc-4ad)}{c^2}\right)\int \frac{1}{x^2(c+$$
$$= -\frac{a^2}{5cx^5(c+dx^2)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx(c+dx^2)^{3/2}}-\frac{\left(4d\left(5b^2-\frac{4ad(5bc-4a}{c^2}\right.\right.}{5}$$
$$= -\frac{a^2}{5cx^5(c+dx^2)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx(c+dx^2)^{3/2}}-\frac{4d\left(5b^2-\frac{4ad(5bc-4ad}{c^2}\right.}{15c^2(c+dx^2)^{3/2}}$$
$$= -\frac{a^2}{5cx^5(c+dx^2)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx(c+dx^2)^{3/2}}-\frac{4d\left(5b^2-\frac{4ad(5bc-4ad}{c^2}\right.}{15c^2(c+dx^2)^{3/2}}$$

Mathematica [A]
time = 0.22, size = 142, normalized size = 0.78

$$\frac{-5b^2c^2x^4(3c^2+12cdx^2+8d^2x^4)+10abcx^2(-c^3+6c^2dx^2+24cd^2x^4+16d^3x^6)-a^2(3c^4-8c^3dx^2+48c^2d^2x^4+192cd^3x^6+128d^4x^8)}{15c^5x^5(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]

[Out] (-5*b^2*c^2*x^4*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4) + 10*a*b*c*x^2*(-c^3 + 6*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6) - a^2*(3*c^4 - 8*c^3*d*x^2 + 48*c^2*d^2*x^4 + 192*c*d^3*x^6 + 128*d^4*x^8))/(15*c^5*x^5*(c + d*x^2)^(3/2))

Maple [A]

time = 0.13, size = 251, normalized size = 1.37

method	result
risch	$-\frac{\sqrt{d x^2+c}\,(73a^2d^2x^4-80abcd\,x^4+15b^2c^2x^4-14a^2cd\,x^2+10ab\,c^2x^2+3a^2c^2)}{15c^5x^5}-\frac{d(ad-bc)(11a\,d^2x^2-5bcd\,x^2+12acd-6b\,c^2)x}{3(d^2x^4+2x^2dc+c^2)c^5}$
gosper	$-\frac{128a^2d^4x^8-160abc\,d^3x^8+40b^2c^2d^2x^8+192a^2c\,d^3x^6-240ab\,c^2d^2x^6+60b^2c^3d\,x^6+48a^2c^2d^2x^4-60ab\,c^3d\,x^4+15b^2c^4x^4-8a^2c^3d\,x^2}{15x^5(d\,x^2+c)^{\frac{3}{2}}c^5}$
trager	$-\frac{128a^2d^4x^8-160abc\,d^3x^8+40b^2c^2d^2x^8+192a^2c\,d^3x^6-240ab\,c^2d^2x^6+60b^2c^3d\,x^6+48a^2c^2d^2x^4-60ab\,c^3d\,x^4+15b^2c^4x^4-8a^2c^3d\,x^2}{15x^5(d\,x^2+c)^{\frac{3}{2}}c^5}$
default	$a^2\left(-\frac{1}{5c\,x^5(d\,x^2+c)^{\frac{3}{2}}}-\frac{8d\left(-\frac{1}{3c\,x^3(d\,x^2+c)^{\frac{3}{2}}}-\frac{2d\left(-\frac{1}{cx(d\,x^2+c)^{\frac{3}{2}}}-\frac{4d\left(\frac{x}{3c(d\,x^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{d\,x^2+c}}\right)}{c}\right)}{c}\right)}{5c}\right)+2ab\Bigg($

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] a^2*(-1/5/c/x^5/(d*x^2+c)^(3/2)-8/5*d/c*(-1/3/c/x^3/(d*x^2+c)^(3/2)-2*d/c*(-1/c/x/(d*x^2+c)^(3/2)-4*d/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)))))+2*a*b*(-1/3/c/x^3/(d*x^2+c)^(3/2)-2*d/c*(-1/c/x/(d*x^2+c)^(3/2)-4*d/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))))+b^2*(-1/c/x/(d*x^2+c)^(3/2)-4*d/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)))

Maxima [A]

time = 0.29, size = 244, normalized size = 1.33

$$-\frac{8\,b^2dx}{3\,\sqrt{dx^2+c}\,c^3}-\frac{4\,b^2dx}{3\,(dx^2+c)^{\frac{3}{2}}c^2}+\frac{32\,abd^2x}{3\,\sqrt{dx^2+c}\,c^4}+\frac{16\,abd^2x}{3\,(dx^2+c)^{\frac{3}{2}}c^3}-\frac{128\,a^2d^3x}{15\,\sqrt{dx^2+c}\,c^5}-\frac{64\,a^2d^3x}{15\,(dx^2+c)^{\frac{3}{2}}c^4}-\frac{b^2}{(dx^2+c)^{\frac{3}{2}}cx}+\frac{4\,abd}{(dx^2+c)^{\frac{3}{2}}c^2x}-\frac{16\,a^2d^2}{5\,(dx^2+c)^{\frac{3}{2}}c^3x}-\frac{2\,ab}{3\,(dx^2+c)^{\frac{3}{2}}cx^3}+\frac{8\,a^2d}{15\,(dx^2+c)^{\frac{3}{2}}c^2x^3}-\frac{a^2}{5\,(dx^2+c)^{\frac{3}{2}}cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*b^2*d*x/(sqrt(d*x^2 + c)*c^3) - 4/3*b^2*d*x/((d*x^2 + c)^(3/2)*c^2) +
32/3*a*b*d^2*x/(sqrt(d*x^2 + c)*c^4) + 16/3*a*b*d^2*x/((d*x^2 + c)^(3/2)*c^
3) - 128/15*a^2*d^3*x/(sqrt(d*x^2 + c)*c^5) - 64/15*a^2*d^3*x/((d*x^2 + c)^
(3/2)*c^4) - b^2/((d*x^2 + c)^(3/2)*c*x) + 4*a*b*d/((d*x^2 + c)^(3/2)*c^2*x
) - 16/5*a^2*d^2/((d*x^2 + c)^(3/2)*c^3*x) - 2/3*a*b/((d*x^2 + c)^(3/2)*c*x
^3) + 8/15*a^2*d/((d*x^2 + c)^(3/2)*c^2*x^3) - 1/5*a^2/((d*x^2 + c)^(3/2)*c
*x^5)
```

Fricas [A]
time = 2.05, size = 171, normalized size = 0.93

$$-\frac{\left(8\,\left(5\,b^{2}c^{2}d^{2}-20\,abcd^{3}+16\,a^{2}d^{4}\right)x^{8}+12\,\left(5\,b^{2}c^{3}d-20\,abc^{2}d^{2}+16\,a^{2}cd^{3}\right)x^{6}+3\,a^{2}c^{4}+3\,\left(5\,b^{2}c^{4}-20\,abc^{3}d+16\,a^{2}c^{2}d^{2}\right)x^{4}+2\,\left(5\,abc^{4}-4\,a^{2}c^{3}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{15\,\left(c^{5}d^{2}x^{9}+2\,c^{6}dx^{7}+c^{7}x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(5*b^2*c^2*d^2 - 20*a*b*c*d^3 + 16*a^2*d^4)*x^8 + 12*(5*b^2*c^3*d
- 20*a*b*c^2*d^2 + 16*a^2*c*d^3)*x^6 + 3*a^2*c^4 + 3*(5*b^2*c^4 - 20*a*b*c^
3*d + 16*a^2*c^2*d^2)*x^4 + 2*(5*a*b*c^4 - 4*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c
)/(c^5*d^2*x^9 + 2*c^6*d*x^7 + c^7*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{2}}{x^{6}\left(c+dx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**6*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(163) = 326.
time = 1.36, size = 509, normalized size = 2.78

$$-\frac{x\left(\frac{\left(5\,b^{2}c^{6}d^{3}-16\,abc^{5}d^{4}+11\,a^{2}c^{4}d^{5}\right)x^{2}}{c^{9}d}+\frac{6\,\left(b^{2}c^{7}d^{2}-3\,abc^{6}d^{3}+2\,a^{2}c^{5}d^{4}\right)}{c^{9}d}\right)}{3\,\left(dx^{2}+c\right)^{\frac{3}{2}}}+\frac{2\,\left(15\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}b^{2}c^{2}\sqrt{d}-60\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}abcd^{\frac{3}{2}}+45\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}a^{2}d^{\frac{5}{2}}-60\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}b^{2}c^{3}\sqrt{d}+300\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}abc^{2}d^{\frac{3}{2}}-240\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}a^{2}cd^{\frac{5}{2}}+90\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b^{2}c^{4}\sqrt{d}-500\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}abc^{3}d^{\frac{3}{2}}+490\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{2}c^{2}d^{\frac{5}{2}}-60\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c^{5}\sqrt{d}+340\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}abc^{4}d^{\frac{3}{2}}-320\,\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{2}c^{3}d^{\frac{5}{2}}+15\,b^{2}c^{6}\sqrt{d}-80\,abc^{5}d^{\frac{3}{2}}+73\,a^{2}c^{4}d^{\frac{5}{2}}\right)}{15\,\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{5}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*((5*b^2*c^6*d^3 - 16*a*b*c^5*d^4 + 11*a^2*c^4*d^5)*x^2/(c^9*d) + 6*(
b^2*c^7*d^2 - 3*a*b*c^6*d^3 + 2*a^2*c^5*d^4)/(c^9*d))/(d*x^2 + c)^(3/2) + 2
```

```
/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*sqrt(d) - 60*(sqrt(d)*x - s
qrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 45*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^
(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) + 300*(sqrt(d)*x
 - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2) - 240*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*a^2*c*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) - 500*(
sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 490*(sqrt(d)*x - sqrt(d*x^
2 + c))^4*a^2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt
(d) + 340*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) - 320*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*a^2*c^3*d^(5/2) + 15*b^2*c^6*sqrt(d) - 80*a*b*c^5*d^(3
/2) + 73*a^2*c^4*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5*c^4)
```

Mupad [B]
time = 0.54, size = 298, normalized size = 1.63

$$\frac{2\,a\,\sqrt{d\,x^2+c}\,\left(7\,a\,d-5\,b\,c\right)}{15\,c^4\,x^3}-\frac{\frac{73\,a^2\,c^2\,d^2-80\,a\,b\,c^3\,d+15\,b^2\,c^4}{30\,c^5}-\frac{c\,\left(\frac{d\,\left(73\,a^2\,c^2\,d^2-80\,a\,b\,c^3\,d+15\,b^2\,c^4\right)}{18\,c^6}+\frac{c\,\left(\frac{4\,a\,d^3\,\left(7\,a\,d-5\,b\,c\right)}{45\,c^5}-\frac{a\,d^3\,\left(43\,a\,d-35\,b\,c\right)}{9\,c^5}\right)}{d}+\frac{a\,d^2\,\left(43\,a\,d-35\,b\,c\right)}{15\,c^4}\right)}{d}}{x\,{\left(d\,x^2+c\right)}^{3/2}}-\frac{a^2\,\sqrt{d\,x^2+c}}{5\,c^3\,x^5}-\frac{x^2\,\left(\frac{2\,d\,\left(78\,a^2\,c\,d^2-90\,a\,b\,c^2\,d+20\,b^2\,c^3\right)}{15\,c^6}-\frac{4\,a\,d^2\,\left(7\,a\,d-5\,b\,c\right)}{15\,c^5}\right)+\frac{78\,a^2\,c\,d^2-90\,a\,b\,c^2\,d+20\,b^2\,c^3}{15\,c^5}}{x\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x)
```

```
[Out] (2*a*(c + d*x^2)^(1/2)*(7*a*d - 5*b*c))/(15*c^4*x^3) - ((15*b^2*c^4 + 73*a^
2*c^2*d^2 - 80*a*b*c^3*d)/(30*c^5) - (c*((d*(15*b^2*c^4 + 73*a^2*c^2*d^2 -
80*a*b*c^3*d))/(18*c^6) + (c*((4*a*d^3*(7*a*d - 5*b*c))/(45*c^5) - (a*d^3*(
43*a*d - 35*b*c))/(9*c^5)))/d + (a*d^2*(43*a*d - 35*b*c))/(15*c^4)))/d)/(x*
(c + d*x^2)^(3/2)) - (a^2*(c + d*x^2)^(1/2))/(5*c^3*x^5) - (x^2*((2*d*(20*b
^2*c^3 + 78*a^2*c*d^2 - 90*a*b*c^2*d))/(15*c^6) - (4*a*d^2*(7*a*d - 5*b*c))
/(15*c^5)) + (20*b^2*c^3 + 78*a^2*c*d^2 - 90*a*b*c^2*d)/(15*c^5))/(x*(c + d
*x^2)^(1/2))
```

3.671 $\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx$

Optimal. Leaf size=72

$$-\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}+\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}$$

```
[Out] -a*x^2/b^2/(d*x^2)^(1/2)+1/3*x^4/b/(d*x^2)^(1/2)+a^(3/2)*x*arctan(x*b^(1/2)
/a^(1/2))/b^(5/2)/(d*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 308, 211}

$$\frac{a^{3/2}x\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}-\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -((a*x^2)/(b^2*Sqrt[d*x^2])) + x^4/(3*b*Sqrt[d*x^2]) + (a^(3/2)*x*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{x\int \frac{x^4}{a+bx^2}\,dx}{\sqrt{dx^2}}$$

$$= \frac{x\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right)dx}{\sqrt{dx^2}}$$

$$= -\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}+\frac{(a^2x)\int \frac{1}{a+bx^2}\,dx}{b^2\sqrt{dx^2}}$$

$$= -\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}+\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.78

$$\frac{x\left(\sqrt{b}\,x(-3a+bx^2)+3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b
^(5/2)*Sqrt[d*x^2])
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.75

method	result	size
default	$-\frac{x\left(-\sqrt{ab}\,b\,x^3+3\sqrt{ab}\,ax-3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)}{3\sqrt{d\,x^2}\,b^2\sqrt{ab}}$	54
risch	$\frac{x\left(\frac{1}{3}b\,x^3-ax\right)}{\sqrt{d\,x^2}\,b^2}+\frac{x\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)}{2\sqrt{d\,x^2}\,b^3}-\frac{x\sqrt{-ab}\,a\ln\left(\sqrt{-ab}\,x+a\right)}{2\sqrt{d\,x^2}\,b^3}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x*(-(a*b)^(1/2)*b*x^3+3*(a*b)^(1/2)*a*x-3*a^2*arctan(b*x/(a*b)^(1/2)))
/(d*x^2)^(1/2)/b^2/(a*b)^(1/2)
```

Maxima [A]
time = 0.49, size = 67, normalized size = 0.93

$$\frac{\frac{3\,a^2d^3\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}\,b^2}+\frac{(dx^2)^{\frac{3}{2}}bd-3\sqrt{dx^2}\,ad^2}{b^2}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(3*a^2*d^3*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b^2) + ((d*x^
2)^(3/2)*b*d - 3*sqrt(d*x^2)*a*d^2)/b^2)/d^3
```

Fricas [A]
time = 1.10, size = 147, normalized size = 2.04

$$\left[\frac{3\,ad\sqrt{-\frac{a}{bd}}\log\left(\frac{bx^2+2\sqrt{dx^2}\,b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\,(bx^2-3\,a)\sqrt{dx^2}}{6\,b^2d},\ \frac{3\,ad\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}\,b\sqrt{\frac{a}{bd}}}{a}\right)+(bx^2-3\,a)\sqrt{dx^2}}{3\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/6*(3*a*d*sqrt(-a/(b*d))*log((b*x^2 + 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a)
/(b*x^2 + a)) + 2*(b*x^2 - 3*a)*sqrt(d*x^2))/(b^2*d), 1/3*(3*a*d*sqrt(a/(b*
d))*arctan(sqrt(d*x^2)*b*sqrt(a/(b*d))/a) + (b*x^2 - 3*a)*sqrt(d*x^2))/(b^2
*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)/(d*x**2)**(1/2),x)
[Out] Integral(x**5/(sqrt(d*x**2)*(a + b*x**2)), x)
```

Giac [A]
time = 0.99, size = 56, normalized size = 0.78

$$\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2\sqrt{d}\,\mathrm{sgn}\left(x\right)}+\frac{b^2dx^3-3\,abdx}{3\,b^3d^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2*sqrt(d)*sgn(x)) + 1/3*(b^2*d*x^3 -
 3*a*b*d*x)/(b^3*d^(3/2)*sgn(x))
```

Mupad [B]
time = 0.36, size = 51, normalized size = 0.71

$$\frac{(x^2)^{3/2}}{3\,b\,\sqrt{d}}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{b^{5/2}\,\sqrt{d}}-\frac{a\,\sqrt{x^2}}{b^2\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] (x^2)^(3/2)/(3*b*d^(1/2)) + (a^(3/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(
b^(5/2)*d^(1/2)) - (a*(x^2)^(1/2))/(b^2*d^(1/2))
```

3.672 $\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx$

Optimal. Leaf size=52

$$\frac{x^2}{b\sqrt{dx^2}}-\frac{\sqrt{a}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

[Out] x^2/b/(d*x^2)^(1/2)-x*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)/(d*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 327, 211}

$$\frac{x^2}{b\sqrt{dx^2}}-\frac{\sqrt{a}\,x\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]

[Out] x^2/(b*Sqrt[d*x^2]) - (Sqrt[a]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt[d*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{x\int \frac{x^2}{a+bx^2}\,dx}{\sqrt{dx^2}}$$
$$= \frac{x^2}{b\sqrt{dx^2}} - \frac{(ax)\int \frac{1}{a+bx^2}\,dx}{b\sqrt{dx^2}}$$
$$= \frac{x^2}{b\sqrt{dx^2}} - \frac{\sqrt{a}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.85

$$\frac{x\left(\sqrt{b}\,x-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*Sqrt[d*x^2])
```

Maple [A]
time = 0.09, size = 38, normalized size = 0.73

method	result	size
default	$\frac{x\left(\sqrt{ab}\,x-a\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)}{\sqrt{d\,x^2}\,b\sqrt{ab}}$	38
risch	$\frac{x^2}{b\sqrt{d\,x^2}}+\frac{x\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{2\sqrt{d\,x^2}\,b^2}-\frac{x\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{2\sqrt{d\,x^2}\,b^2}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*((a*b)^(1/2)*x-a*arctan(b*x/(a*b)^(1/2)))/(d*x^2)^(1/2)/b/(a*b)^(1/2)
```

Maxima [A]
time = 0.52, size = 49, normalized size = 0.94

$$-\frac{\frac{ad^2\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}\,b}-\frac{\sqrt{dx^2}\,d}{b}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(a*d^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b) - sqrt(d*x^2)*d/b
)/d^2
```

Fricas [A]
time = 2.83, size = 126, normalized size = 2.42

$$\left[\frac{d\sqrt{-\frac{a}{bd}}\log\left(\frac{bx^2-2\sqrt{dx^2}\,b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\sqrt{dx^2}}{2\,bd},\,-\frac{d\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}\,b\sqrt{\frac{a}{bd}}}{a}\right)-\sqrt{dx^2}}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(d*sqrt(-a/(b*d))*log((b*x^2 - 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a)/(b*
x^2 + a)) + 2*sqrt(d*x^2))/(b*d), -(d*sqrt(a/(b*d))*arctan(sqrt(d*x^2)*b*sq
rt(a/(b*d))/a) - sqrt(d*x^2))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(d*x**2)*(a + b*x**2)), x)
```

Giac [A]
time = 0.78, size = 40, normalized size = 0.77

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b\sqrt{d}\,\mathrm{sgn}\left(x\right)}+\frac{x}{b\sqrt{d}\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b*sqrt(d)*sgn(x)) + x/(b*sqrt(d)*sgn(x)
)
```

Mupad [B]
time = 0.32, size = 37, normalized size = 0.71

$$\frac{\sqrt{x^2}}{b\,\sqrt{d}}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{b^{3/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] (x^2)^(1/2)/(b*d^(1/2)) - (a^(1/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(b^
(3/2)*d^(1/2))
```

3.673 $\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx$

Optimal. Leaf size=34

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{dx^2}}$$

```
[Out] x*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)/(d*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 211}

$$\frac{x\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx &= \frac{x\int \frac{1}{a+bx^2}\,dx}{\sqrt{dx^2}}\\ &= \frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Maple [A]
time = 0.10, size = 24, normalized size = 0.71

method	result	size
default	$\frac{x\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{d\,x^2}\,\sqrt{ab}}$	24
risch	$-\frac{x\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{d\,x^2}\,\sqrt{-ab}}+\frac{x\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{d\,x^2}\,\sqrt{-ab}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(d*x^2)^(1/2)*x/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 23, normalized size = 0.68

$$\frac{\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
[Out] arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/sqrt(a*b*d)
```

Fricas [A]
time = 1.83, size = 94, normalized size = 2.76

$$\left[-\frac{\sqrt{-abd}\,\log\left(\frac{bdx^2-ad-2\,\sqrt{-abd}\,\sqrt{dx^2}}{bx^2+a}\right)}{2\,abd},\ \frac{\sqrt{abd}\,\arctan\left(\frac{\sqrt{abd}\,\sqrt{dx^2}}{ad}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b*d)*log((b*d*x^2 - a*d - 2*sqrt(-a*b*d)*sqrt(d*x^2))/(b*x^2
+ a))/(a*b*d), sqrt(a*b*d)*arctan(sqrt(a*b*d)*sqrt(d*x^2)/(a*d))/(a*b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(d*x**2)*(a + b*x**2)), x)
```

Giac [A]
time = 0.88, size = 22, normalized size = 0.65

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,\sqrt{d}\,\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/(sqrt(a*b)*sqrt(d)*sgn(x))
```

Mupad [B]
time = 0.34, size = 23, normalized size = 0.68

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] atan((b^(1/2)*(x^2)^(1/2))/a^(1/2))/(a^(1/2)*b^(1/2)*d^(1/2))
```

3.674 $\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{a\sqrt{dx^2}}-\frac{\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}$$

```
[Out] -1/a/(d*x^2)^(1/2)-x*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(d*x^2)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 331, 211}

$$-\frac{\sqrt{b}\,x\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}-\frac{1}{a\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -(1/(a*Sqrt[d*x^2])) - (Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqr
t[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{x\int \frac{1}{x^2(a+bx^2)}\,dx}{\sqrt{dx^2}}$$
$$= -\frac{1}{a\sqrt{dx^2}} - \frac{(bx)\int \frac{1}{a+bx^2}\,dx}{a\sqrt{dx^2}}$$
$$= -\frac{1}{a\sqrt{dx^2}} - \frac{\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.92

$$-\frac{dx^2\left(\sqrt{a}+\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{a^{3/2}\left(dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
[Out] -((d*x^2*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(a^(3/2)*(d*x^2
)^(3/2)))
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.72

method	result	size
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)x+\sqrt{ab}}{\sqrt{d\,x^2}\,a\sqrt{ab}}$	36
risch	$-\frac{1}{a\sqrt{d\,x^2}}+\frac{x\left(\sum_{\_R=\mathrm{RootOf}\left(a^3\_Z^2+b\right)}\_R\ln\left(\left(3\_R^2a^3+2b\right)x+a^2\_R\right)\right)}{2\sqrt{d\,x^2}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(b*arctan(b*x/(a*b)^(1/2))*x+(a*b)^(1/2))/(d*x^2)^(1/2)/a/(a*b)^(1/2)
```

Maxima [A]
time = 0.48, size = 35, normalized size = 0.70

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a\sqrt{d}} - \frac{1}{a\sqrt{d}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")

[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*sqrt(d)) - 1/(a*sqrt(d)*x)

Fricas [**A**]
time = 1.25, size = 132, normalized size = 2.64

$$\left[\frac{dx^2\sqrt{-\frac{b}{ad}}\log\left(\frac{bx^2-2\sqrt{dx^2}\,a\sqrt{-\frac{b}{ad}}-a}{bx^2+a}\right)-2\sqrt{dx^2}}{2\,adx^2},\ -\frac{dx^2\sqrt{\frac{b}{ad}}\arctan\left(\sqrt{dx^2}\sqrt{\frac{b}{ad}}\right)+\sqrt{dx^2}}{adx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*(d*x^2*sqrt(-b/(a*d))*log((b*x^2 - 2*sqrt(d*x^2)*a*sqrt(-b/(a*d)) - a)/(b*x^2 + a)) - 2*sqrt(d*x^2))/(a*d*x^2), -(d*x^2*sqrt(b/(a*d))*arctan(sqrt(d*x^2)*sqrt(b/(a*d))) + sqrt(d*x^2))/(a*d*x^2)]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x**2+a)/(d*x**2)**(1/2),x)

[Out] Integral(1/(x*sqrt(d*x**2)*(a + b*x**2)), x)

Giac [**A**]
time = 0.82, size = 43, normalized size = 0.86

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a\sqrt{d}\,\mathrm{sgn}\left(x\right)}-\frac{1}{a\sqrt{d}\,x\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")

[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*sqrt(d)*sgn(x)) - 1/(a*sqrt(d)*x*sgn(x))

Mupad [B]
time = 0.35, size = 38, normalized size = 0.76

$$-\frac{1}{a\,\sqrt{d}\,\sqrt{x^2}}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] - 1/(a*d^(1/2)*(x^2)^(1/2)) - (b^(1/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))
/(a^(3/2)*d^(1/2))
```

3.675 $\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx$

Optimal. Leaf size=68

$$\frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}+\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}$$

```
[Out] b/a^2/(d*x^2)^(1/2)-1/3/a/x^2/(d*x^2)^(1/2)+b^(3/2)*x*arctan(x*b^(1/2)/a^(1
/2))/a^(5/2)/(d*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 331, 211}

$$\frac{b^{3/2}x\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}+\frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] b/(a^2*Sqrt[d*x^2]) - 1/(3*a*x^2*Sqrt[d*x^2]) + (b^(3/2)*x*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(a^(5/2)*Sqrt[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx &= \frac{x\int \frac{1}{x^4(a+bx^2)}\,dx}{\sqrt{dx^2}}\\ &= -\frac{1}{3ax^2\sqrt{dx^2}}-\frac{(bx)\int \frac{1}{x^2(a+bx^2)}\,dx}{a\sqrt{dx^2}}\\ &= \frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}+\frac{(b^2x)\int \frac{1}{a+bx^2}\,dx}{a^2\sqrt{dx^2}}\\ &= \frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}+\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 0.85

$$\frac{d\left(-\sqrt{a}\,(a-3bx^2)+3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{3a^{5/2}\,(dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
[Out] (d*(-(Sqrt[a]*(a - 3*b*x^2)) + 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/
(3*a^(5/2)*(d*x^2)^(3/2))
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.84

method	result	size
default	$-\frac{-3b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3-3b\sqrt{ab}\,x^2+a\sqrt{ab}}{3x^2\sqrt{d\,x^2}\,a^2\sqrt{ab}}$	57
risch	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{3a}}{\sqrt{d\,x^2}\,x^2}+\frac{x\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2+b^3\right)}\_R\ln\left(\left(3\_R^2a^5+2b^3\right)x-a^3b\_R\right)\right)}{2\sqrt{d\,x^2}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3/x^2*(-3*b^2*arctan(b*x/(a*b)^(1/2))*x^3-3*b*(a*b)^(1/2)*x^2+a*(a*b)^(1
/2))/(d*x^2)^(1/2)/a^2/(a*b)^(1/2)
```

Maxima [A]
time = 0.49, size = 52, normalized size = 0.76

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^2 \sqrt{d}} + \frac{3\, b\sqrt{d}\, x^2 - a\sqrt{d}}{3\, a^2 d x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*sqrt(d)) + 1/3*(3*b*sqrt(d)*x^2 -
a*sqrt(d))/(a^2*d*x^3)
```

Fricas [A]
time = 1.64, size = 157, normalized size = 2.31

$$\left[\frac{3\, b d x^4 \sqrt{-\frac{b}{ad}} \log\left(\frac{b x^2 + 2\sqrt{dx^2}\, a \sqrt{-\frac{b}{ad}} - a}{b x^2 + a}\right) + 2\,(3\, b x^2 - a)\sqrt{dx^2}}{6\, a^2 d x^4}, \frac{3\, b d x^4 \sqrt{\frac{b}{ad}} \arctan\left(\sqrt{dx^2}\sqrt{\frac{b}{ad}}\right) + (3\, b x^2 - a)\sqrt{dx^2}}{3\, a^2 d x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*d*x^4*sqrt(-b/(a*d))*log((b*x^2 + 2*sqrt(d*x^2)*a*sqrt(-b/(a*d))
- a)/(b*x^2 + a)) + 2*(3*b*x^2 - a)*sqrt(d*x^2))/(a^2*d*x^4), 1/3*(3*b*d*x^
4*sqrt(b/(a*d))*arctan(sqrt(d*x^2)*sqrt(b/(a*d))) + (3*b*x^2 - a)*sqrt(d*x^
2))/(a^2*d*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{dx^2}\,(a + bx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(d*x**2)*(a + b*x**2)), x)
```

Giac [A]
time = 0.93, size = 54, normalized size = 0.79

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^2 \sqrt{d}\, \mathrm{sgn}\left(x\right)} + \frac{3\, b x^2 - a}{3\, a^2 \sqrt{d}\, x^3 \mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*sqrt(d)*sgn(x)) + 1/3*(3*b*x^2 - a
)/(a^2*sqrt(d)*x^3*sgn(x))
```

Mupad [B]
time = 0.35, size = 53, normalized size = 0.78

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{a^{5/2}\,\sqrt{d}}-\frac{1}{3\,a\,\sqrt{d}\,{\left(x^2\right)}^{3/2}}+\frac{b\,x^2}{a^2\,\sqrt{d}\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] (b^(3/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(a^(5/2)*d^(1/2)) - 1/(3*a*d^
(1/2)*(x^2)^(3/2)) + (b*x^2)/(a^2*d^(1/2)*(x^2)^(3/2))
```

3.676 $\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=157

$$\frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{a^{3/2}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}-\frac{(b^2c^2+4abcd-8a^2d^2)\,\mathrm{tan}}{8b^3d^{3/2}}$$

```
[Out] -1/8*(-8*a^2*d^2+4*a*b*c*d+b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^3/
d^(3/2)+a^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*
c)^(1/2)/b^3+1/8*(-4*a*d+b*c)*x*(d*x^2+c)^(1/2)/b^2/d+1/4*x^3*(d*x^2+c)^(1/
2)/b
```

Rubi [A]

time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {489, 596, 537, 223, 212, 385, 211}

$$\frac{a^{3/2}\sqrt{bc-ad}\,\mathrm{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}-\frac{(-8a^2d^2+4abcd+b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^3d^{3/2}}+\frac{x\sqrt{c+dx^2}\,(bc-4ad)}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] ((b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2*d) + (x^3*Sqrt[c + d*x^2])/(4*b) +
 (a^(3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/b^3 - ((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(8*b^3*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx = \frac{x^3\sqrt{c+dx^2}}{4b} - \frac{\int \frac{x^2(3ac+(-bc+4ad)x^2)}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{4b}$$
$$= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d} + \frac{x^3\sqrt{c+dx^2}}{4b} + \frac{\int \frac{-ac(bc-4ad)+(-b^2c^2-4abcd+8a^2d^2)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{8b^2d}$$
$$= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d} + \frac{x^3\sqrt{c+dx^2}}{4b} + \frac{(a^2(bc-ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^3} - \frac{(b^2c^2}{}$$
$$= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d} + \frac{x^3\sqrt{c+dx^2}}{4b} + \frac{(a^2(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, -\right.}{b^3}$$
$$= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d} + \frac{x^3\sqrt{c+dx^2}}{4b} + \frac{a^{3/2}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3} - $$

Mathematica [A]
time = 0.35, size = 166, normalized size = 1.06

$$\frac{b\sqrt{d}\,x\sqrt{c+dx^2}\,(bc-4ad+2bdx^2) - 8a^{3/2}d^{3/2}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right) + (b^2c^2+4abcd-8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{8b^3d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(b*c - 4*a*d + 2*b*d*x^2) - 8*a^(3/2)*d^(3/2)*
Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqr
t[a]*Sqrt[b*c - a*d])] + (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*Log[-(Sqrt[d]*x)
 + Sqrt[c + d*x^2]])/(8*b^3*d^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(131) = 262.
time = 0.14, size = 767, normalized size = 4.89

method	result

risch	$-\frac{x(-2bd\,x^2+4ad-bc)\sqrt{d\,x^2+c}}{8d\,b^2}+\frac{\sqrt{d}\,\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)a^2}{b^3}-\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)ac}{2b^2\sqrt{d}}-\frac{\ln\left(x\sqrt{d}+\right.}{}$
default	$\frac{\frac{x\left(d\,x^2+c\right)^{\frac{3}{2}}}{4d}-\frac{c\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{4d}}{b}-\frac{a\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{b^2}+\frac{a^2\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*x*(d*x^2+c)^(3/2)/d-1/4*c/d*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*l
n(x*d^(1/2)+(d*x^2+c)^(1/2))))-a/b^2*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*l
n(x*d^(1/2)+(d*x^2+c)^(1/2)))+1/2/b^2*a^2/(-a*b)^(1/2)*((d*(x-1/b*(-a*b)^(1
/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(
-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/
b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
)-1/2/b^2*a^2/(-a*b)^(1/2)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^
(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a), x)
```

Fricas [A]
time = 1.20, size = 857, normalized size = 5.46

$$\left[\frac{4\sqrt{-abc+a^2d}\,ad^2\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-(b^2c^2+4abcd-8a^2d^2)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2(2b^2d^2x^3+(b^2cd-4abd^2)x)\sqrt{dx^2+c}}{16b^3d^2},\ \frac{2\sqrt{-abc+a^2d}\,ad^2\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)+(b^2c^2+4abcd-8a^2d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(2b^2d^2x^3+(b^2cd-4abd^2)x)\sqrt{dx^2+c}}{8b^3d^2},\ \frac{8\sqrt{abc-a^2d}\,ad^2\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-(b^2c^2+4abcd-8a^2d^2)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2(2b^2d^2x^3+(b^2cd-4abd^2)x)\sqrt{dx^2+c}}{16b^3d^2},\ \frac{4\sqrt{abc-a^2d}\,ad^2\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+(b^2c^2+4abcd-8a^2d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(2b^2d^2x^3+(b^2cd-4abd^2)x)\sqrt{dx^2+c}}{8b^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
[Out] [1/16*(4*sqrt(-a*b*c + a^2*d)*a*d^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (b^
2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqr
t(d)*x - c) + 2*(2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/
(b^3*d^2), 1/8*(2*sqrt(-a*b*c + a^2*d)*a*d^2*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x
^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^
2)) + (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x
^2 + c)) + (2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*
d^2), 1/16*(8*sqrt(a*b*c - a^2*d)*a*d^2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
 a^2*c*d)*x)) - (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x
)*sqrt(d*x^2 + c))/(b^3*d^2), 1/8*(4*sqrt(a*b*c - a^2*d)*a*d^2*arctan(1/2*s
qrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^
2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sq
rt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b^2*d^2*x^3 + (b^2*c*d - 4*a
*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{d\,x^2+c}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

```
[Out] int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2), x)
```

3.677 $\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=88

$$-\frac{a\sqrt{c+dx^2}}{b^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd}+\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

```
[Out] 1/3*(d*x^2+c)^(3/2)/b/d+a*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))
*(-a*d+b*c)^(1/2)/b^(5/2)-a*(d*x^2+c)^(1/2)/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 81, 52, 65, 214}

$$\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{a\sqrt{c+dx^2}}{b^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] -((a*Sqrt[c + d*x^2])/b^2) + (c + d*x^2)^(3/2)/(3*b*d) + (a*Sqrt[b*c - a*d]
*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)\\
&= \frac{(c+dx^2)^{3/2}}{3bd} - \frac{a\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{2b}\\
&= -\frac{a\sqrt{c+dx^2}}{b^2} + \frac{(c+dx^2)^{3/2}}{3bd} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b^2}\\
&= -\frac{a\sqrt{c+dx^2}}{b^2} + \frac{(c+dx^2)^{3/2}}{3bd} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{b^2d}\\
&= -\frac{a\sqrt{c+dx^2}}{b^2} + \frac{(c+dx^2)^{3/2}}{3bd} + \frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 85, normalized size = 0.97

$$\frac{\sqrt{c+dx^2}\,(-3ad+b(c+dx^2))}{3b^2d} + \frac{a\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]

[Out] (Sqrt[c + d*x^2]*(-3*a*d + b*(c + d*x^2)))/(3*b^2*d) + (a*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(5/2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(72) = 144$.
time = 0.11, size = 666, normalized size = 7.57

method	result
risch	$-\frac{(-bd\,x^2+3ad-bc)\sqrt{d\,x^2+c}}{3d\,b^2}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)\cdots}}{x-\frac{\sqrt{-ab}}{b}}\right)\cdots}{2b^3\sqrt{-\frac{ad-bc}{b}}\cdots}$
default	$\frac{(d\,x^2+c)^{\frac{3}{2}}}{3bd}-\frac{a\left(\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+d\left(\cdots\right.}{\sqrt{d}\cdots}\right)\cdots}{\cdots}\right.\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/3*(d*x^2+c)^(3/2)/b/d-1/2*a/b^2*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-1/2*a/b^2*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*

```
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-
a*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.56, size = 295, normalized size = 3.35

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\,(4\,b^{2}cd-3\,abd^{2})x^{2}+4\,(b^{2}dx^{2}+2\,b^{2}c-abd)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{b}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\,(bdx^{2}+bc-3\,ad)\sqrt{dx^{2}+c}}{12\,b^{2}d}, \frac{3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^{2}+2\,bc-ad)\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^{2}-acd+(bcd-ad^{2})x^{2})}\right)+2\,(bdx^{2}+bc-3\,ad)\sqrt{dx^{2}+c}}{6\,b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*
sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b*d*
x^2 + b*c - 3*a*d)*sqrt(d*x^2 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b
)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/
(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b*d*x^2 + b*c - 3*a*d)*sqrt(d*x
^2 + c))/(b^2*d)]
```

Sympy [A]
time = 3.55, size = 87, normalized size = 0.99

$$\frac{2\left(-\frac{ad^{2}\sqrt{c+dx^{2}}}{2b^{2}}+\frac{ad^{2}(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^{2}}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^{3}\sqrt{\frac{ad-bc}{b}}}+\frac{d(c+dx^{2})^{\frac{3}{2}}}{6b}\right)}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**2)/(2*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**2)/sqrt((a*d - b*c)/b))/(2*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**2)**(3
/2)/(6*b))/d**2
```

Giac [A]
time = 0.80, size = 96, normalized size = 1.09

$$-\frac{(abc - a^2d)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2} + \frac{(dx^2+c)^{\frac{3}{2}}b^2d^2 - 3\sqrt{dx^2+c}\,abd^3}{3\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b*c - a^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^2 + c)*a*b*d^
3)/(b^3*d^3)
```

Mupad [B]
time = 0.35, size = 86, normalized size = 0.98

$$\frac{(d\,x^2+c)^{3/2}}{3\,b\,d} - \frac{a\,\sqrt{d\,x^2+c}}{b^2} + \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^2+c}\,\sqrt{a\,d-b\,c}}{a^2\,d-a\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(3/2)/(3*b*d) - (a*(c + d*x^2)^(1/2))/b^2 + (a*atan((a*b^(1/2)*
(c + d*x^2)^(1/2)*(a*d - b*c)^(1/2))/(a^2*d - a*b*c))*(a*d - b*c)^(1/2))/b^
(5/2)
```

3.678 $\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=112

$$\frac{x\sqrt{c+dx^2}}{2b}-\frac{\sqrt{a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}$$

```
[Out] 1/2*(-2*a*d+b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2/d^(1/2)-arctan(x*(-
a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)*(-a*d+b*c)^(1/2)/b^2+1/2*x*
(d*x^2+c)^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {489, 537, 223, 212, 385, 211}

$$-\frac{\sqrt{a}\,\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}+\frac{x\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^2 + ((b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/S
qrt[c + d*x^2]])/(2*b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{x\sqrt{c+dx^2}}{2b} - \frac{\int \frac{ac+(-bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2b}\\
&= \frac{x\sqrt{c+dx^2}}{2b} + \frac{(bc-2ad)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2} - \frac{(a(bc-ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^2}\\
&= \frac{x\sqrt{c+dx^2}}{2b} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2b^2} - \frac{(a(bc-ad))\text{Subst}\left(\int\right.}{}\\
&= \frac{x\sqrt{c+dx^2}}{2b} - \frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+}}\right.}{2b^2\sqrt{d}}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 128, normalized size = 1.14

$$
\frac{bx\sqrt{c+dx^2}+2\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)+\frac{(-bc+2ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2] + 2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x*(S
qrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])] + ((-(b*c) + 2*a*d)
*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[d])/(2*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(90) = 180$.

time = 0.12, size = 700, normalized size = 6.25

method	result
risch	$\frac{x\sqrt{dx^2+c}}{2b} - \frac{\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)\sqrt{d}\,a}{b^2} + \frac{\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)c}{2b\sqrt{d}} - \frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\cdots\right.}{b}\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{2\sqrt{d}}}{b} - \frac{a\left(\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))-1/2
*a/(-a*b)^(1/2)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*
(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
```

```
d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+1/2*a/(-a*b)^(1/2)/b*((d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(
1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(
d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^
(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a), x)
```

Fricas [A]
time = 1.19, size = 690, normalized size = 6.16

$$\left[\frac{2\sqrt{dx^2+c}\,bdx-(bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}\,x-c\right)+\sqrt{-abc+a^2d}\,d\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)}{4b^2d}, \frac{2\sqrt{dx^2+c}\,bdx-2(bc-2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\sqrt{-abc+a^2d}\,d\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)}{4b^2d}, \frac{2\sqrt{dx^2+c}\,bdx-2\sqrt{abc-a^2d}\,d\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-(bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}\,x-c\right)}{4b^2d}, \frac{\sqrt{dx^2+c}\,bdx-\sqrt{abc-a^2d}\,d\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-(bc-2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)}{2b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*x - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt
(d*x^2 + c)*sqrt(d)*x - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8*a*b*c
*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2
*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x
^2 + a^2)))/(b^2*d), 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*(b*c - 2*a*d)*sqrt(-d
)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4
*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/(b^2*d), 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*sqrt(a*b*c
- a^2*d)*d*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*
x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - (b*c - 2*a*d)
*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d), 1/2*(sqr
t(d*x^2 + c)*b*d*x - sqrt(a*b*c - a^2*d)*d*arctan(1/2*sqrt(a*b*c - a^2*d)*(
(b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^
2 - a^2*c*d)*x)) - (b*c - 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
))/(b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{d\,x^2+c}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

```
[Out] int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2), x)
```

3.679 $\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

```
[Out] -arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(3/2)
+(d*x^2+c)^(1/2)/b
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {455, 52, 65, 214}

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)\\ &= \frac{\sqrt{c+dx^2}}{b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b}\\ &= \frac{\sqrt{c+dx^2}}{b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{bd}\\ &= \frac{\sqrt{c+dx^2}}{b} - \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 65, normalized size = 1.00

$$\frac{\sqrt{c+dx^2}}{b} - \frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sq
rt[-(b*c) + a*d]])/b^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(53) = 106$.
time = 0.10, size = 647, normalized size = 9.95

method	result
risch	$\frac{\sqrt{d x^2+c}}{b}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\right.}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right.}{2b^2\sqrt{-\frac{ad-bc}{b}}}$
default	$\frac{\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\sqrt{-ab}\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+d\left(x-\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{\right.}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b
)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-
1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2))))+1/2/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*
(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.46, size = 255, normalized size = 3.92

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\sqrt{dx^2+c}}{4\,b}, -\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\sqrt{dx^2+c}}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^
2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*
x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(d*x^2 +
 c))/b, -1/2*(sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2
*sqrt(d*x^2 + c))/b]
```

Sympy [A]
time = 2.26, size = 61, normalized size = 0.94

$$\frac{2\left(\frac{d\sqrt{c+dx^2}}{2b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**2)/(2*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**2)/sqrt((a*d
 - b*c)/b))/(2*b**2*sqrt((a*d - b*c)/b)))/d
```

Giac [A]
time = 0.63, size = 64, normalized size = 0.98

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}+\frac{\sqrt{dx^2+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b) + sqrt(d*x^2 + c)/b
```

Mupad [B]
time = 0.36, size = 53, normalized size = 0.82

$$\frac{\sqrt{d\,x^2+c}}{b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(1/2)/b - (atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2))*
(a*d - b*c)^(1/2))/b^(3/2)
```

3.680 $\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

```
[Out] arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)/b+arctan(x*(-a*d+b*c)^(1/2)/a^(1
/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/b/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {399, 223, 212, 385, 211}

$$\frac{\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sq
rt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{d\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b} - \frac{(-bc+ad)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}\\
&= \frac{d\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b} - \frac{(-bc+ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b}\\
&= \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b} + \frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 102, normalized size = 1.26

$$
-\frac{\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}} + \sqrt{d}\,\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

```
[Out] -((((Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/
(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d
*x^2]])/b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(65) = 130$.
time = 0.09, size = 653, normalized size = 8.06

method	result

default	$$\frac{\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+d\left(x-\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d \cdots}\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-a*b)^(1/2)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(
x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-1/2/(-a*b)^(1/2)*((d*(x+1/b*(-a*b)^(
1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*
(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+
1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a), x)
```

Fricas [A]
time = 1.37, size = 596, normalized size = 7.36

$$\left[\frac{2\sqrt{d}\log\left(-2dx^{2}-2\sqrt{dx^{2}+c}\sqrt{d}x-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^{2}c^{2}-8abcd+8a^{2}d^{2})x^{4}+a^{2}c^{2}-2(3abc^{2}-4a^{2}cd)x^{2}-4(a^{2}cx-(abc-2a^{2}d)x^{3})\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)}{4b},\ -\frac{4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)-\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^{2}c^{2}-8abcd+8a^{2}d^{2})x^{4}+a^{2}c^{2}-2(3abc^{2}-4a^{2}cd)x^{2}-4(a^{2}cx-(abc-2a^{2}d)x^{3})\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)}{4b},\ \frac{\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2ad)x^{2}-ac)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^{2})x^{3}+(bc^{2}-acd)x)}\right)+\sqrt{d}\log\left(-2dx^{2}-2\sqrt{dx^{2}+c}\sqrt{d}x-c\right)}{2b},\ -\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)-\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2ad)x^{2}-ac)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^{2})x^{3}+(bc^{2}-acd)x)}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + sqrt(-(b*
c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)
*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b, -1/4*(4*sqrt(-d)*ar
ctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*
c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/b, 1/2*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c
^2 - a*c*d)*x)) + sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/
b, -1/2*(2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - sqrt((b*c - a*d)/a
)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/
((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{\sqrt{-d}\,\mathrm{asin}\left(x\,\sqrt{-\frac{d}{c}}\right)}{a} & \text{if } ((c+a\,d=0\wedge b=-1)\vee a\,d=b\,c)\wedge d<0 \\ \frac{\sqrt{d}\,\ln\left(2\,\sqrt{d}\,x+2\,\sqrt{d\,x^2+c}\right)}{b}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{b\,c-a\,d}}{\sqrt{a}\,\sqrt{d\,x^2+c}}\right)\,\sqrt{b\,c-a\,d}}{\sqrt{a}\,b} & \text{if } c\neq 0\wedge(((c+a\,d\neq 0\vee b\neq -1)\wedge a\,d\neq b\,c)\vee\neg d<0) \\ \int \frac{\sqrt{d\,x^2+c}}{b\,x^2+a}\,dx & \text{if } ((((c+a\,d=0\wedge b=-1)\vee a\,d=b\,c)\wedge d<0)\vee c=0)\wedge(((c+a\,d\neq 0\vee b\neq -1)\wedge a\,d\neq b\,c)\vee\neg d<0) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2),x)
```

```
[Out] piecewise((c + a*d == 0 & b == -1 | a*d == b*c) & d < 0, ((-d)^(1/2)*asin(x
*(-d/c)^(1/2)))/a, c ~= 0 & ((c + a*d ~= 0 | b ~= -1) & a*d ~= b*c | ~d < 0
), (d^(1/2)*log(2*d^(1/2)*x + 2*(c + d*x^2)^(1/2)))/b + (atan((x*(- a*d + b
*c)^(1/2))/(a^(1/2)*(c + d*x^2)^(1/2)))*(- a*d + b*c)^(1/2))/(a^(1/2)*b), (
(c + a*d == 0 & b == -1 | a*d == b*c) & d < 0 | c == 0) & ((c + a*d ~= 0 |
b ~= -1) & a*d ~= b*c | ~d < 0), int((c + d*x^2)^(1/2)/(a + b*x^2), x))
```

3.681 $\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}$$

```
[Out] -arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)/a+arctanh(b^(1/2)*(d*x^2+c)^(1/2)
/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 85, 65, 214}

$$\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a) + (Sqrt[b*c - a*d]*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{x\,(a+bx^2)}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx,x,x^2\right)\\ &= \frac{c\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2a} - \frac{(bc-ad)\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2a}\\ &= \frac{c\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{ad} - \frac{(bc-ad)\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{ad}\\ &= -\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a} + \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 78, normalized size = 0.98

$$\frac{\frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}} - \sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

```
[Out] ((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/
Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(64) = 128$.
time = 0.11, size = 689, normalized size = 8.61

method	result

default	$-\frac{\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+d\left(x-\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*
b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x
-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2))))-1/2/a*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d
*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/a*((d*x^2+c)^(1
/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x), x)
```

Fricas [A]
time = 1.88, size = 578, normalized size = 7.22

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(b^2dx^2+2b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{4a}, \frac{4\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(b^2dx^2+2b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)}{4a}, \frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2(bc^2-acd+(bcd-ad^2)x^2)}\right)+\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{2a}, \frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2(bc^2-acd+(bcd-ad^2)x^2)}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^
2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*
x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(c)*log(
-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/a, 1/4*(4*sqrt(-c)*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) + sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*
b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 +
a^2)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqr
t(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) +
sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/a, 1/2*(sqrt(-
(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b
*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)))/a]
```

Sympy [**A**]
time = 3.88, size = 78, normalized size = 0.98

$$\frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x/(b*x**2+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**2)/sqrt(-c))/(2*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(2*a*b*sqrt((a*d - b*c)/b)))/d
```

Giac [**A**]
time = 0.84, size = 78, normalized size = 0.98

$$-\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a}+\frac{c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c +
a*b*d)*a) + c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B]
time = 0.40, size = 103, normalized size = 1.29

$$\frac{\operatorname{atanh}\left(\frac{2\,a\,b^2\,c\,d^3\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{2\,a\,b^3\,c^2\,d^3-2\,a^2\,b^2\,c\,d^4}\right)\,\sqrt{b^2\,c-a\,b\,d}}{a\,b}-\frac{\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x*(a + b*x^2)),x)
```

```
[Out] (atanh((2*a*b^2*c*d^3*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(2*a*b^3*c^2
*d^3 - 2*a^2*b^2*c*d^4))*(b^2*c - a*b*d)^(1/2))/(a*b) - (c^(1/2)*atanh((c +
 d*x^2)^(1/2)/c^(1/2)))/a
```

3.682 $\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{c+dx^2}}{ax}-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}}$$

```
[Out] -arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/a^(3/2
)-(d*x^2+c)^(1/2)/a/x
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {486, 12, 385, 211}

$$-\frac{\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*x)) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqr
t[a]*Sqrt[c + d*x^2])])/a^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
```

```
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}\,dx &= -\frac{\sqrt{c+dx^2}}{ax}+\frac{\int \frac{-bc+ad}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{a}\\ &= -\frac{\sqrt{c+dx^2}}{ax}+\frac{(-bc+ad)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{a}\\ &= -\frac{\sqrt{c+dx^2}}{ax}+\frac{(-bc+ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a}\\ &= -\frac{\sqrt{c+dx^2}}{ax}-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 89, normalized size = 1.27

$$-\frac{\sqrt{c+dx^2}}{ax}+\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*x)) + (Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d
]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/a^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(58) = 116$.
time = 0.11, size = 724, normalized size = 10.34

method	result

risch	$-\frac{\sqrt{d\,x^2+c}}{ax}-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\,\left(x-\cdots\right.}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{b\left(\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+d\left(x-\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}\cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a/(-a*b)^(1/2)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/
b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+1/2*b/a/(-a*b)^(1/2)*((d*(x+1/b
*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/
2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a
*b)^(1/2))))+1/a*(-1/c/x*(d*x^2+c)^(3/2)+2*d/c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c
/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^2), x)
```

Fricas [A]
time = 1.17, size = 273, normalized size = 3.90

$$\left[\frac{x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\sqrt{dx^2+c}}{4\,ax}, -\frac{x\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\sqrt{dx^2+c}}{2\,ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a
^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3
)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sq
rt(d*x^2 + c))/(a*x), -1/2*(x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c
^2 - a*c*d)*x)) + 2*sqrt(d*x^2 + c))/(a*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{x^2\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.
time = 1.41, size = 117, normalized size = 1.67

$$\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a}+\frac{2\,c\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b
*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) + 2*c*sqrt
(d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{x^2\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)), x)
```

3.683 $\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}$$

```
[Out] 1/2*(-a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-arctanh(b^(1/
2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/2*(d*x^
2+c)^(1/2)/a/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 101, 162, 65, 214}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}-\frac{\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/(a*x^2) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[
c]])/(2*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^2])/Sqrt[b*c - a*d]])/a^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^2}}{2ax^2} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{\sqrt{c+dx^2}}{2ax^2} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(2bc-ad)\text{Subst}\left(\int \cdots\right.}{4a\cdots}$$

$$= -\frac{\sqrt{c+dx^2}}{2ax^2} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{(2bc-ad)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{c+dx^2}}{2ax^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} - \frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\cdots}}{\sqrt{bc-a\cdots}}\right.}{a^2}$$

Mathematica [A]
time = 0.28, size = 107, normalized size = 0.95

$$\frac{-\frac{a\sqrt{c+dx^2}}{x^2} - 2\sqrt{b}\,\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right) + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^2])/x^2) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*
Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^
2]/Sqrt[c]])/Sqrt[c])/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(91) = 182$.
time = 0.12, size = 759, normalized size = 6.72

method	result
risch	$-\frac{\sqrt{d x^2+c}}{2 a x^2}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^2+c}}{x}\right) d}{2 a\sqrt{c}}+\frac{\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d x^2+c}}{x}\right) b}{a^2}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\cdots}{\cdots}}{\cdots}\right)\cdots}{\cdots}$
default	$\frac{b\left(\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+d\left(x-\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/a^2*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(
-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d
*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x-1/b*(-a*b)^(1/2))))+1/2*b/a^2*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b
```

```
*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)
/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/a*(-1/2/
c/x^2*(d*x^2+c)^(3/2)+1/2*d/c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d
*x^2+c)^(1/2))/x)))-b/a^2*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2
+c)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^3), x)
```

Fricas [A]
time = 2.39, size = 708, normalized size = 6.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="fricas")
[Out] [1/4*(sqrt(b^2*c - a*b*d)*c*x^2*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^
2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b*c - a*d)*
sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*
x^2 + c)*a*c)/(a^2*c*x^2), -1/4*(2*(2*b*c - a*d)*sqrt(-c)*x^2*arctan(sqrt(-
c)/sqrt(d*x^2 + c)) - sqrt(b^2*c - a*b*d)*c*x^2*log((b^2*d^2*x^4 + 8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*
c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 2*sqrt(d*x^2 + c)*a*c)/(a^2*c*x^2), -1/4*(2*sqrt(-b^2*c + a*b*d)*c*x^2*ar
ctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2
*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + (2*b*c - a*d)*sqrt(c)*x^2*log(
-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a
^2*c*x^2), -1/2*(sqrt(-b^2*c + a*b*d)*c*x^2*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a
*b*d^2)*x^2)) + (2*b*c - a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + sqrt(d*x^2 + c)*a*c)/(a^2*c*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{x^3 \left(a + bx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)), x)
```

Giac [A]
time = 1.38, size = 106, normalized size = 0.94

$$\frac{(b^2c-abd)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}-\frac{\sqrt{dx^2+c}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*a^2) - 1/2*(2*b*c - a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sq
rt(-c)) - 1/2*sqrt(d*x^2 + c)/(a*x^2)
```

Mupad [B]
time = 0.54, size = 268, normalized size = 2.37

$$\frac{\mathrm{atanh}\left(\frac{b^3\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{2\,\left(\frac{a\,b^3\,d^5}{2}-\frac{b^4\,c\,d^4}{2}\right)}\right)\,\sqrt{b^2\,c-a\,b\,d}}{a^2}-\frac{\sqrt{d\,x^2+c}}{2\,a\,x^2}-\frac{\mathrm{atanh}\left(\frac{b^4\,\sqrt{c}\,d^4\,\sqrt{d\,x^2+c}}{2\,\left(\frac{b^4\,c\,d^4}{2}-\frac{3\,a\,b^3\,d^5}{4}+\frac{a^2\,b^2\,d^6}{4\,c}\right)}-\frac{3\,b^3\,d^5\,\sqrt{d\,x^2+c}}{4\,\sqrt{c}\,\left(\frac{a\,b^2\,d^6}{4\,c}-\frac{3\,b^3\,d^5}{4}+\frac{b^4\,c\,d^4}{2\,a}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^2+c}}{4\,c^{3/2}\,\left(\frac{b^2\,d^6}{4\,c}-\frac{3\,b^3\,d^5}{4\,a}+\frac{b^4\,c\,d^4}{2\,a^2}\right)}\right)\,\left(a\,d-2\,b\,c\right)}{2\,a^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^3*(a + b*x^2)),x)
```

```
[Out] (atanh((b^3*d^4*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(2*((a*b^3*d^5)/2
- (b^4*c*d^4)/2)))*(b^2*c - a*b*d)^(1/2))/a^2 - (c + d*x^2)^(1/2)/(2*a*x^2)
 - (atanh((b^4*c^(1/2)*d^4*(c + d*x^2)^(1/2))/(2*((b^4*c*d^4)/2 - (3*a*b^3*
d^5)/4 + (a^2*b^2*d^6)/(4*c))) - (3*b^3*d^5*(c + d*x^2)^(1/2))/(4*c^(1/2)*(
(a*b^2*d^6)/(4*c) - (3*b^3*d^5)/4 + (b^4*c*d^4)/(2*a))) + (b^2*d^6*(c + d*x
^2)^(1/2))/(4*c^(3/2)*((b^2*d^6)/(4*c) - (3*b^3*d^5)/(4*a) + (b^4*c*d^4)/(2
*a^2))))*(a*d - 2*b*c))/(2*a^2*c^(1/2))
```

3.684 $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}}$$

```
[Out] b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/a^(5/
2)-1/3*(d*x^2+c)^(1/2)/a/x^3+1/3*(-a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/c/x
```

Rubi [A]
time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {486, 597, 12, 385, 211}

$$\frac{b\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}\,(3bc-ad)}{3a^2cx}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*Sqrt[c + d*x^2]/(a*x^3) + ((3*b*c - a*d)*Sqrt[c + d*x^2])/(3*a^2*c*x)
+ (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])
/a^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
```

```
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx &= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{\int \frac{-3bc+ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx}{3a} \\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}-\frac{\int -\frac{3bc(bc-ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3a^2c} \\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{(b(bc-ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{a^2} \\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+}}\right.}{a^2} \\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 114, normalized size = 1.09

$$\frac{\sqrt{c+dx^2}\left(3bcx^2-a(c+dx^2)\right)}{3a^2cx^3}-\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + d*x^2)))/(3*a^2*c*x^3) - (b*Sqrt[b*c -
 a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[
b*c - a*d])])/a^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(87) = 174$.

time = 0.11, size = 750, normalized size = 7.14

method	result
risch	$-\frac{\sqrt{d x^2+c}\,(a d x^2-3 c x^2 b+a c)}{3 c a^2 x^3}+\frac{b \ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2 \cdots}}{x-\frac{\sqrt{-ab}}{b}}\right.}{2a\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$
default	$\frac{b^2\left(\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\sqrt{-ab}\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+d\left(x-\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^2/(-a*b)^(1/2)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/
2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-1/2*b^2/a^2/(-a*b)^(1/2)*((d
*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2))
```

```
)/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*
b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+
1/b*(-a*b)^(1/2))))-1/3/a/c/x^3*(d*x^2+c)^(3/2)-b/a^2*(-1/c/x*(d*x^2+c)^(3/
2)+2*d/c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^4), x)
```

Fricas [A]
time = 1.43, size = 325, normalized size = 3.10

$$\left[\frac{3\,bcx^3\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,((3\,bc-ad)x^2-ac)\sqrt{dx^2+c}}{12\,a^2cx^3}, \frac{3\,bcx^3\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\,((3\,bc-ad)x^2-ac)\sqrt{dx^2+c}}{6\,a^2cx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="fricas")
[Out] [1/12*(3*b*c*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a
^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) + 4*((3*b*c - a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*c*x^3), 1/6*(3*b*c*x
^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)
*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*((3*b*c
 - a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*c*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{x^4\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a),x)
[Out] Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(87) = 174$.

time = 2.34, size = 215, normalized size = 2.05

$$-\frac{\left(b^{2} c \sqrt{d} - a b d^{\frac{3}{2}}\right) \arctan\left(\frac{\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{2} b - b c + 2 a d}{2 \sqrt{a b c d - a^{2} d^{2}}}\right)}{\sqrt{a b c d - a^{2} d^{2}} a^{2}} - \frac{2 \left(3 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{4} b c \sqrt{d} - 3 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{4} a d^{\frac{3}{2}} - 6 \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{2} b c^{2} \sqrt{d} + 3 b c^{3} \sqrt{d} - a c^{2} d^{\frac{3}{2}}\right)}{3 \left(\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{2} - c\right)^{3} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2) - 2
/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) + 3*
b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^2
)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{x^4\,\left(b\,x^2+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)), x)
```

3.685 $\int \frac{x^4 (c+dx^2)^{3/2}}{a+bx^2} \, dx$

Optimal. Leaf size=210

$$\frac{(b^2c^2-10abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3d}+\frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}+\frac{a^{3/2}(bc-ad)^{3/2}\tan^{-1}\Big(\ldots}{b^4}$$

```
[Out] a^(3/2)*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))
/b^4-1/16*(-2*a*d+b*c)*(-8*a^2*d^2+8*a*b*c*d+b^2*c^2)*arctanh(x*d^(1/2)/(d*
x^2+c)^(1/2))/b^4/d^(3/2)+1/16*(8*a^2*d^2-10*a*b*c*d+b^2*c^2)*x*(d*x^2+c)^(
1/2)/b^3/d+1/24*(-6*a*d+7*b*c)*x^3*(d*x^2+c)^(1/2)/b^2+1/6*d*x^5*(d*x^2+c)^
(1/2)/b
```

Rubi [A]

time = 0.27, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {488, 596, 537, 223, 212, 385, 211}

$$\frac{a^{3/2}(bc-ad)^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4}-\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^4d^{3/2}}+\frac{x\sqrt{c+dx^2}\,(8a^2d^2-10abcd+b^2c^2)}{16b^3d}+\frac{x^3\sqrt{c+dx^2}\,(7bc-6ad)}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b^2*c^2 - 10*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3*d) + ((7*b*c
 - 6*a*d)*x^3*Sqrt[c + d*x^2])/(24*b^2) + (d*x^5*Sqrt[c + d*x^2])/(6*b) + (
a^(3/2)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/b^4 - ((b*c - 2*a*d)*(b^2*c^2 + 8*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[
d]*x)/Sqrt[c + d*x^2]])/(16*b^4*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{dx^5\sqrt{c+dx^2}}{6b} + \frac{\int \frac{x^4(c(6bc-5ad)+d(7bc-6ad)x^2)}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{6b}$$

$$= \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} - \frac{\int \frac{x^2(3acd(7bc-6ad)-3d(b^2c^2-10abcd+8a^2d^2)x^2)}{(a+bx^2)\sqrt{c+dx^2}}}{24b^2d}$$

$$= \frac{(b^2c^2-10abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3d} + \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} +$$

$$= \frac{(b^2c^2-10abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3d} + \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} +$$

$$= \frac{(b^2c^2-10abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3d} + \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} +$$

$$= \frac{(b^2c^2-10abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3d} + \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} +$$

Mathematica [A]
time = 0.41, size = 215, normalized size = 1.02

$$\frac{b\sqrt{d}\,x\sqrt{c+dx^2}\,(24a^2d^2-6abd(5c+2dx^2)+b^2(3c^2+14cdx^2+8d^2x^4))-48a^{3/2}d^{3/2}(bc-ad)^{3/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)+3(b^3c^3+6ab^2c^2d-24a^2bcd^2+16a^3d^3)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{48b^4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(5*c + 2*d*x^2) + b^2*(3
*c^2 + 14*c*d*x^2 + 8*d^2*x^4)) - 48*a^(3/2)*d^(3/2)*(b*c - a*d)^(3/2)*ArcT
an[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d]
)] + 3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*Log[-(Sqrt[d
]*x) + Sqrt[c + d*x^2]])/(48*b^4*d^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. $2(180) = 360$.
time = 0.13, size = 1389, normalized size = 6.61

method	result

risch	$\frac{x(8b^2x^4d^2-12ab\,d^2x^2+14b^2cd\,x^2+24a^2d^2-30abcd+3b^2c^2)\sqrt{d\,x^2+c}}{48d\,b^3}-\frac{d^{\frac{3}{2}}\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)a^3}{b^4}+\frac{3\sqrt{d}\,c\ln\left(x\sqrt{\right.}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/6*x*(d*x^2+c)^(5/2)/d-1/6*c/d*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d
*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))-a/b^2*(1/4*x*(
d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x
^2+c)^(1/2))))+1/2/b^2*a^2/(-a*b)^(1/2)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/
4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-
b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1
/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^
(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*
d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2/b^2*a^2/(-a*b)^(1/2)
*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^
(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)
^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b
*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/
2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a
*b)^(1/2)))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a), x)
```

Fricas [A]
time = 5.37, size = 1119, normalized size = 5.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
[Out] [1/96*(3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(d)*lo
g(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 24*(a*b*c*d^2 - a^2*d^3)*sq
rt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c +
a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(8*b^3*d^3*x^5 + 2
*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a*b^2*c*d^2 + 8*a^2*b*
d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/48*(3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a
^2*b*c*d^2 + 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 12*(
a*b*c*d^2 - a^2*d^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2
*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3
- a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 + (8*b^3*d^3*x^5 + 2*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a
*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/96*(48*(a*b*c*d^
2 - a^2*d^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a
*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*
d)*x)) + 3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(d)*
log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^3*d^3*x^5 + 2*(7*b
^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a*b^2*c*d^2 + 8*a^2*b*d^3)*
x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/48*(24*(a*b*c*d^2 - a^2*d^3)*sqrt(a*b*c -
a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(b^3*c^3 + 6*a*
b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*
x^2 + c)) + (8*b^3*d^3*x^5 + 2*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2
*d - 10*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(c + dx^2)^{\frac{3}{2}}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d\,x^2+c\right)}^{3/2}}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

```
[Out] int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2), x)
```

3.686 $\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=115

$$-\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3}-\frac{a(c+dx^2)^{3/2}}{3b^2}+\frac{(c+dx^2)^{5/2}}{5bd}+\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

```
[Out] -1/3*a*(d*x^2+c)^(3/2)/b^2+1/5*(d*x^2+c)^(5/2)/b/d+a*(-a*d+b*c)^(3/2)*arcta
nh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)-a*(-a*d+b*c)*(d*x^2+c)
^(1/2)/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 81, 52, 65, 214}

$$\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}-\frac{a\sqrt{c+dx^2}\,(bc-ad)}{b^3}-\frac{a(c+dx^2)^{3/2}}{3b^2}+\frac{(c+dx^2)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)*Sqrt[c + d*x^2])/b^3) - (a*(c + d*x^2)^(3/2))/(3*b^2) + (c
 + d*x^2)^(5/2)/(5*b*d) + (a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)\\
&= \frac{(c+dx^2)^{5/2}}{5bd} - \frac{a\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)}{2b}\\
&= -\frac{a(c+dx^2)^{3/2}}{3b^2} + \frac{(c+dx^2)^{5/2}}{5bd} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{2b^2}\\
&= -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3} - \frac{a(c+dx^2)^{3/2}}{3b^2} + \frac{(c+dx^2)^{5/2}}{5bd} - \frac{(a(bc-ad)^2)\,\text{Subst}\left(\int \frac{}{(a+b}\right.}{2b^3}\\
&= -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3} - \frac{a(c+dx^2)^{3/2}}{3b^2} + \frac{(c+dx^2)^{5/2}}{5bd} - \frac{(a(bc-ad)^2)\,\text{Subst}\left(\int \frac{}{a-\frac{b}{c}}\right.}{b^3d}\\
&= -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3} - \frac{a(c+dx^2)^{3/2}}{3b^2} + \frac{(c+dx^2)^{5/2}}{5bd} + \frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 109, normalized size = 0.95

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2+3b^2(c+dx^2)^2-5abd(4c+dx^2)\right)}{15b^3d} - \frac{a(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 3*b^2*(c + d*x^2)^2 - 5*a*b*d*(4*c + d*x^2))
)/(15*b^3*d) - (a*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqr
t[-(b*c) + a*d]])/b^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. $2(95) = 190$.
time = 0.10, size = 1256, normalized size = 10.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(d*x^2+c)^(5/2)/b/d-1/2*a/b^2*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d
*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d
*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/
b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2*a/b^2*(1/3*(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a
*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+
1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a
*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)
/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(
1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*
c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.39, size = 397, normalized size = 3.45

$$\left[-\frac{15\,(abcd-a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(3\,b^2d^2x^4+3\,b^2c^2-20\,abcd+15\,a^2d^2+(6\,b^2cd-5\,abd^2)x^2)\sqrt{dx^2+c}}{60\,b^3d},\ \frac{15\,(abcd-a^2d^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)+2\,(3\,b^2d^2x^4+3\,b^2c^2-20\,abcd+15\,a^2d^2+(6\,b^2cd-5\,abd^2)x^2)\sqrt{dx^2+c}}{30\,b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2
*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 +
 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) - 4*(3*b^2*d^2*x^4 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*
c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^3*d), 1/30*(15*(a*b*c*d - a^2*d^2
)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*
sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(3*b^2*d^2*
x^4 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^2)*sq
rt(d*x^2 + c))/(b^3*d)]
```

Sympy [A]
time = 19.70, size = 104, normalized size = 0.90

$$-\frac{a(c+dx^2)^{\frac{3}{2}}}{3b^2}-\frac{a(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^4\sqrt{\frac{ad-bc}{b}}}+\frac{(c+dx^2)^{\frac{5}{2}}}{5bd}+\frac{\sqrt{c+dx^2}\,(a^2d-abc)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] -a*(c + d*x**2)**(3/2)/(3*b**2) - a*(a*d - b*c)**2*atan(sqrt(c + d*x**2)/sq
rt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b)) + (c + d*x**2)**(5/2)/(5*b*d)
 + sqrt(c + d*x**2)*(a**2*d - a*b*c)/b**3
```

Giac [A]
time = 0.81, size = 151, normalized size = 1.31

$$-\frac{(ab^2c^2-2\,a^2bcd+a^3d^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^3}+\frac{3\,(dx^2+c)^{\frac{5}{2}}b^4d^4-5\,(dx^2+c)^{\frac{3}{2}}ab^3d^5-15\,\sqrt{dx^2+c}\,ab^3cd^5+15\,\sqrt{dx^2+c}\,a^2b^2d^6}{15\,b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c +
 a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/15*(3*(d*x^2 + c)^(5/2)*b^4*d^4 - 5
*(d*x^2 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^2 + c)*a*b^3*c*d^5 + 15*sqrt(d*x
^2 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

Mupad [B]
time = 0.37, size = 179, normalized size = 1.56

$$\frac{(d\,x^2+c)^{5/2}}{5\,b\,d}-(d\,x^2+c)^{3/2}\left(\frac{c}{3\,b\,d}+\frac{a\,d^2-b\,c\,d}{3\,b^2\,d^2}\right)-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^2+c}\,(a\,d-b\,c)^{3/2}}{a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2}\right)\,(a\,d-b\,c)^{3/2}}{b^{7/2}}+\frac{\sqrt{d\,x^2+c}\,(a\,d^2-b\,c\,d)\,\left(\frac{c}{b\,d}+\frac{a\,d^2-b\,c\,d}{b^2\,d^2}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(5/2)/(5*b*d) - (c + d*x^2)^(3/2)*(c/(3*b*d) + (a*d^2 - b*c*d)/
(3*b^2*d^2)) - (a*atan((a*b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2))/(a^3
*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))*(a*d - b*c)^(3/2))/b^(7/2) + ((c + d*x^2)^
(1/2)*(a*d^2 - b*c*d)*(c/(b*d) + (a*d^2 - b*c*d)/(b^2*d^2)))/(b*d)
```

3.687 $\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=158

$$\frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\sqrt{c+dx^2}}{4b}-\frac{\sqrt{a}\,(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}+\frac{(3b^2c^2-12abcd+8a^2d}{8b^3}$$

```
[Out] -(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2
)/b^3+1/8*(8*a^2*d^2-12*a*b*c*d+3*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2
))/b^3/d^(1/2)+1/8*(-4*a*d+5*b*c)*x*(d*x^2+c)^(1/2)/b^2+1/4*d*x^3*(d*x^2+c)
^(1/2)/b
```

Rubi [A]
time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {488, 596, 537, 223, 212, 385, 211}

$$\frac{(8a^2d^2-12abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^3\sqrt{d}}-\frac{\sqrt{a}\,(bc-ad)^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}+\frac{x\sqrt{c+dx^2}\,(5bc-4ad)}{8b^2}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((5*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*Sqrt[c + d*x^2])/(4*b)
 - (Sqrt[a]*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/b^3 + ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/S
qrt[c + d*x^2]])/(8*b^3*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{dx^3\sqrt{c+dx^2}}{4b} + \frac{\int \frac{x^2(c(4bc-3ad)+d(5bc-4ad)x^2)}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{4b}$$

$$= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\sqrt{c+dx^2}}{4b} - \frac{\int \frac{acd(5bc-4ad)-d(3b^2c^2-12abcd+8a^2d^2)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{8b^2d}$$

$$= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\sqrt{c+dx^2}}{4b} - \frac{(a(bc-ad)^2)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^3} + \frac{(3}{}$$

$$= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\sqrt{c+dx^2}}{4b} - \frac{(a(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x,\right.}{b^3}$$

$$= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\sqrt{c+dx^2}}{4b} - \frac{\sqrt{a}\,(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}$$

Mathematica [**A**]
time = 0.33, size = 158, normalized size = 1.00

$$\frac{bx\sqrt{c+dx^2}\,(5bc-4ad+2bdx^2)+8\sqrt{a}\,(bc-ad)^{3/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)+\frac{(-3b^2c^2+12abcd-8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(5*b*c - 4*a*d + 2*b*d*x^2) + 8*Sqrt[a]*(b*c - a*d)^(3
/2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*
c - a*d])] + ((-3*b^2*c^2 + 12*a*b*c*d - 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt
[c + d*x^2]])/Sqrt[d])/(8*b^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(132) = 264$.
time = 0.12, size = 1306, normalized size = 8.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*
d^(1/2)+(d*x^2+c)^(1/2))))-1/2*a/(-a*b)^(1/2)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2)
)^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/
2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^
(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4
```

```
*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2
)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d
*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+1/2*a/(-a*b)^(1/
2)/b*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a
*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-
a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x
+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d
^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a), x)
```

Fricas [A]
time = 2.29, size = 894, normalized size = 5.66

$$\left[\frac{(3b^2c^2-12abcd+8a^2d^2)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)-4\sqrt{-abc+a^2d}(bcd-ad^2)\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)+2(2b^2d^2x^3+(5b^2cd-4abd^2)x)\sqrt{dx^2+c}}{16b^3d}, -\frac{(3b^2c^2-12abcd+8a^2d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+2\sqrt{-abc+a^2d}(bcd-ad^2)\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-(2b^2d^2x^3+(5b^2cd-4abd^2)x)\sqrt{dx^2+c}}{8b^3d}, -\frac{8\sqrt{abc-a^2d}(bcd-ad^2)\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2ad)x^3-acx)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^4+(abc^2-a^2cd)x^2)}\right)-(3b^2c^2-12abcd+8a^2d^2)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)-2(2b^2d^2x^3+(5b^2cd-4abd^2)x)\sqrt{dx^2+c}}{16b^3d}, -\frac{4\sqrt{abc-a^2d}(bcd-ad^2)\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2ad)x^3-acx)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^4+(abc^2-a^2cd)x^2)}\right)+(3b^2c^2-12abcd+8a^2d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(2b^2d^2x^3+(5b^2cd-4abd^2)x)\sqrt{dx^2+c}}{8b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(d)*x - c) - 4*sqrt(-a*b*c + a^2*d)*(b*c*d - a*d^2)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sq
```

```
rt(d*x^2 + c))/(b^3*d), -1/8*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + 2*sqrt(-a*b*c + a^2*d)*(b*c*d - a*d^2
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d
^2)*x)*sqrt(d*x^2 + c))/(b^3*d), -1/16*(8*sqrt(a*b*c - a^2*d)*(b*c*d - a*d^
2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)
/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - (3*b^2*c^2 - 12*a*b*c
*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2
*(2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d), -1/8
*(4*sqrt(a*b*c - a^2*d)*(b*c*d - a*d^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
 a^2*c*d)*x)) + (3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-
d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*
x^2 + c))/(b^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c+dx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a),x)
[Out] Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d\,x^2+c\right)}^{3/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x)
[Out] int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2), x)
```

3.688 $\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=91

$$\frac{(bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{(c+dx^2)^{3/2}}{3b}-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

```
[Out] 1/3*(d*x^2+c)^(3/2)/b-(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*
d+b*c)^(1/2))/b^(5/2)+(-a*d+b*c)*(d*x^2+c)^(1/2)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {455, 52, 65, 214}

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{b^2}+\frac{(c+dx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (c + d*x^2)^(3/2)/(3*b) - ((b*c - a*d)^
(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)\\
&= \frac{(c+dx^2)^{3/2}}{3b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{2b}\\
&= \frac{(bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{(c+dx^2)^{3/2}}{3b} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b^2}\\
&= \frac{(bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{(c+dx^2)^{3/2}}{3b} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{b^2d}\\
&= \frac{(bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{(c+dx^2)^{3/2}}{3b} - \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.14, size = 82, normalized size = 0.90

$$
\frac{\sqrt{c+dx^2}\,(4bc-3ad+bdx^2)}{3b^2} + \frac{(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((-(b*c) + a*d)^(3/2)
*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(5/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(75) = 150$.
time = 0.10, size = 1237, normalized size = 13.59 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-
a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-
a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x
-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^
(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(
1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*
(-a*b)^(1/2)))))+1/2/b*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a
*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)
/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-
(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/
b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 1.45, size = 303, normalized size = 3.33

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(bdx^2+4\,bc-3\,ad)\sqrt{dx^2+c}}{12\,b^2},\ -\frac{3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\,(bdx^2+4\,bc-3\,ad)\sqrt{dx^2+c}}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c
- a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
- 4*(b*d*x^2 + 4*b*c - 3*a*d)*sqrt(d*x^2 + c))/b^2, -1/6*(3*(b*c - a*d)*sqr
t(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(
-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(b*d*x^2 + 4*b*c
 - 3*a*d)*sqrt(d*x^2 + c))/b^2]
```

Sympy [A]
time = 11.39, size = 80, normalized size = 0.88

$$\frac{(c+dx^2)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx^2}\,(-ad+bc)}{b^2}+\frac{(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**(3/2)/(3*b) + sqrt(c + d*x**2)*(-a*d + b*c)/b**2 + (a*d - b*c
)**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b**3*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 0.58, size = 112, normalized size = 1.23

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}+\frac{(dx^2+c)^{\frac{3}{2}}b^2+3\sqrt{dx^2+c}\,b^2c-3\sqrt{dx^2+c}\,abd}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2 + 3*sqrt(d*x^2
+ c)*b^2*c - 3*sqrt(d*x^2 + c)*a*b*d)/b^3
```

Mupad [B]
time = 0.35, size = 98, normalized size = 1.08

$$\frac{(d\,x^2+c)^{3/2}}{3\,b}-\frac{\sqrt{d\,x^2+c}\,\left(a\,d-b\,c\right)}{b^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a\,d-b\,c)^{3/2}}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}\right)\,\left(a\,d-b\,c\right)^{3/2}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(3/2)/(3*b) - ((c + d*x^2)^(1/2)*(a*d - b*c))/b^2 + (atan((b^(1
/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))*(
a*d - b*c)^(3/2))/b^(5/2)
```

3.689 $\int \frac{(c+dx^2)^{3/2}}{a+bx^2} dx$

Optimal. Leaf size=113

$$\frac{dx\sqrt{c+dx^2}}{2b}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^2}+\frac{\sqrt{d}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2}$$

```
[Out] (-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b^2/a^(
1/2)+1/2*(-2*a*d+3*b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)/b^2+1/2*
d*x*(d*x^2+c)^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {427, 537, 223, 212, 385, 211}

$$\frac{(bc-ad)^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^2}+\frac{\sqrt{d}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/(2*b) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b^2) + (Sqrt[d]*(3*b*c - 2*a*d)*ArcTa
nh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx &= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{\int \frac{c(2bc-ad)+d(3bc-2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2b}\\
&= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{(d(3bc-2ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2}+\frac{(bc-ad)^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^2}\\
&= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{(d(3bc-2ad))\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{(bc-ad)^2\text{Subst}\left(\right.}{}\\
&= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^2}+\frac{\sqrt{d}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{c}}\right.}{2b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 129, normalized size = 1.14

$$
\frac{bdx\sqrt{c+dx^2}-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}}+\sqrt{d}\,(-3bc+2ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

```
[Out] (b*d*x*Sqrt[c + d*x^2] - (2*(b*c - a*d)^(3/2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt
[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*(-3
*b*c + 2*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(91) = 182$.
time = 0.12, size = 1243, normalized size = 11.00 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-a*b)^(1/2)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1
/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/
2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1
/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*
c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)
^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x-1/b*(-a*b)^(1/2)))))-1/2/(-a*b)^(1/2)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/
b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*
(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2))
)/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*
(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a), x)

Fricas [**A**]

time = 1.48, size = 721, normalized size = 6.38

$$\left[\frac{2\sqrt{dx^2+c}\,bdx-(3bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)-(bc-ad)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4(a^2cx-(abc-2a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2abx^2+a^2}\right)}{4b^2}, \frac{2\sqrt{dx^2+c}\,bdx-2(3bc-2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(bc-ad)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4(a^2cx-(abc-2a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2abx^2+a^2}\right)}{4b^2}, \frac{2\sqrt{dx^2+c}\,bdx+2(bc-ad)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^2)x^3+(bc^2-acd)x)}\right)-(3bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)}{4b^2}, \frac{\sqrt{dx^2+c}\,bdx-(3bc-2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(bc-ad)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^2)x^3+(bc^2-acd)x)}\right)}{2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*x - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c - a*d)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b^2, 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*(3*b*c - 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*c - a*d)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b^2, 1/4*(2*sqrt(d*x^2 + c)*b*d*x + 2*(b*c - a*d)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/b^2, 1/2*(sqrt(d*x^2 + c)*b*d*x - (3*b*c - 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b*c - a*d)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/b^2]

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a),x)

[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2), x)

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2), x)
```

3.690 $\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx$

Optimal. Leaf size=96

$$\frac{d\sqrt{c+dx^2}}{b}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}$$

```
[Out] -c^(3/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a+(-a*d+b*c)^(3/2)*arctanh(b^(1/2
)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(3/2)+d*(d*x^2+c)^(1/2)/b
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 86, 162, 65, 214}

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\sqrt{c+dx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*Sqrt[c + d*x^2])/b - (c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*
c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(3/
2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)}\,dx, x, x^2\right) \\
&= \frac{d\sqrt{c+dx^2}}{b} + \frac{\text{Subst}\left(\int \frac{bc^2+d(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b} \\
&= \frac{d\sqrt{c+dx^2}}{b} + \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{2ab} \\
&= \frac{d\sqrt{c+dx^2}}{b} + \frac{c^2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{ad} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,\right.}{abd} \\
&= \frac{d\sqrt{c+dx^2}}{b} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a} + \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 103, normalized size = 1.07

$$\frac{a\sqrt{b}\,d\sqrt{c+dx^2} - (-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right) - b^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ab^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

[Out] (a*Sqrt[b]*d*Sqrt[c + d*x^2] - (-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a*b^(3/2))

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(78) = 156$.

time = 0.10, size = 1293, normalized size = 13.47

method	result
default	$-\frac{\left(d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}}{3}+ \frac{d\sqrt{-ab}\left(\frac{\left(2d\left(x-\frac{\sqrt{-ab}}{b}\right)+\frac{2d\sqrt{-ab}}{b}\right)\sqrt{d\left(x-\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(3/2)/x/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/2/a*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2/a*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1

```
/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1
/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/a*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^
(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x), x)
```

Fricas [A]
time = 2.34, size = 682, normalized size = 7.10

$$\left[\frac{2\,bc^{\frac{3}{2}}\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2c}{x^2}\right)+4\,\sqrt{dx^2+c}\,ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\,(4b^2cd-3abd^2)x^2-4\,(b^2dx^2+2b^2c-abd)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)}{4ab}, \frac{4b\sqrt{-c}\,c\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+4\,\sqrt{dx^2+c}\,ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\,(4b^2cd-3abd^2)x^2-4\,(b^2dx^2+2b^2c-abd)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)}{4ab}, \frac{bc^{\frac{3}{2}}\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2c}{x^2}\right)+2\,\sqrt{dx^2+c}\,ad+(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)}{2ab}, \frac{2b\sqrt{-c}\,c\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+2\,\sqrt{dx^2+c}\,ad+(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)}{2ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c^(3/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*s
qrt(d*x^2 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b
^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2
 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x
^2 + a^2)))/(a*b), 1/4*(4*b*sqrt(-c)*c*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 4
*sqrt(d*x^2 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8
*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x
^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b
*x^2 + a^2)))/(a*b), 1/2*(b*c^(3/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
 + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arc
tan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)))/(a*b), 1/2*(2*b*sqrt(-c)*c*arctan(sqrt(-
c)/sqrt(d*x^2 + c)) + 2*sqrt(d*x^2 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)
/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b
)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)))/(a*b)]
```

Sympy [A]

time = 11.13, size = 92, normalized size = 0.96

$$\frac{d\sqrt{c+dx^2}}{b}+\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{(ad-bc)^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x/(b*x**2+a),x)
```

```
[Out] d*sqrt(c + d*x**2)/b + c**2*atan(sqrt(c + d*x**2)/sqrt(-c))/(a*sqrt(-c)) -
(a*d - b*c)**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*b**2*sqrt((a*d
 - b*c)/b))
```

Giac [A]
time = 0.58, size = 110, normalized size = 1.15

$$\frac{c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{\sqrt{dx^2+c}\,d}{b}-\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)) + sqrt(d*x^2 + c)*d/b - (
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d
))/(sqrt(-b^2*c + a*b*d)*a*b)
```

Mupad [B]
time = 0.42, size = 711, normalized size = 7.41

$$\frac{d\sqrt{dx^2+c}}{b}-\frac{\mathrm{atanh}\left(\frac{2a^3d^6\sqrt{dx^2+c}\sqrt{c^3}}{2a^3c^2d^6-8a^2bc^3d^5+12ab^2c^4d^4-6b^3c^5d^3}+\frac{8a^2cd^5\sqrt{dx^2+c}\sqrt{c^3}}{8a^2c^3d^5+6b^2c^5d^3-\frac{2a^3c^2d^6}{b}-12abc^4d^4}+\frac{6b^2c^3d^3\sqrt{dx^2+c}\sqrt{c^3}}{8a^2c^3d^5+6b^2c^5d^3-\frac{2a^3c^2d^6}{b}-12abc^4d^4}-\frac{12abc^2d^4\sqrt{dx^2+c}\sqrt{c^3}}{8a^2c^3d^5+6b^2c^5d^3-\frac{2a^3c^2d^6}{b}-12abc^4d^4}\right)\sqrt{c^3}}{a}+\frac{\mathrm{atanh}\left(\frac{6c^3d^3\sqrt{dx^2+c}\sqrt{-a^3b^3d^3+3a^2b^4cd^2-3ab^5c^2d+b^6c^3}}{6b^3c^5d^3-10a^3c^2d^6-18ab^2c^4d^4+20a^2bc^3d^5+\frac{2a^4cd^7}{b}}-\frac{6ac^2d^4\sqrt{dx^2+c}\sqrt{-a^3b^3d^3+3a^2b^4cd^2-3ab^5c^2d+b^6c^3}}{2a^4cd^7-10a^3bc^2d^6+20a^2b^2c^3d^5-18ab^3c^4d^4+6b^4c^5d^3}+\frac{2a^2cd^5\sqrt{dx^2+c}\sqrt{-a^3b^3d^3+3a^2b^4cd^2-3ab^5c^2d+b^6c^3}}{2a^4bcd^7-10a^3b^2c^2d^6+20a^2b^3c^3d^5-18ab^4c^4d^4+6b^5c^5d^3}\right)\sqrt{-b^3(ad-bc)^3}}{ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x*(a + b*x^2)),x)
```

```
[Out] (d*(c + d*x^2)^(1/2))/b - (atanh((2*a^3*d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2))/
(2*a^3*c^2*d^6 - 6*b^3*c^5*d^3 + 12*a*b^2*c^4*d^4 - 8*a^2*b*c^3*d^5) + (8*a
^2*c*d^5*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(8*a^2*c^3*d^5 + 6*b^2*c^5*d^3 - (2
*a^3*c^2*d^6)/b - 12*a*b*c^4*d^4) + (6*b^2*c^3*d^3*(c + d*x^2)^(1/2)*(c^3)^
(1/2))/(8*a^2*c^3*d^5 + 6*b^2*c^5*d^3 - (2*a^3*c^2*d^6)/b - 12*a*b*c^4*d^4)
 - (12*a*b*c^2*d^4*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(8*a^2*c^3*d^5 + 6*b^2*c^
5*d^3 - (2*a^3*c^2*d^6)/b - 12*a*b*c^4*d^4))*(c^3)^(1/2))/a + (atanh((6*c^3
```

```
*d^3*(c + d*x^2)^(1/2)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c
^2*d)^(1/2))/(6*b^3*c^5*d^3 - 10*a^3*c^2*d^6 - 18*a*b^2*c^4*d^4 + 20*a^2*b*
c^3*d^5 + (2*a^4*c*d^7)/b) - (6*a*c^2*d^4*(c + d*x^2)^(1/2)*(b^6*c^3 - a^3*
b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/(2*a^4*c*d^7 + 6*b^4*c^5*
d^3 - 18*a*b^3*c^4*d^4 - 10*a^3*b*c^2*d^6 + 20*a^2*b^2*c^3*d^5) + (2*a^2*c*
d^5*(c + d*x^2)^(1/2)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^
2*d)^(1/2))/(6*b^5*c^5*d^3 - 18*a*b^4*c^4*d^4 + 20*a^2*b^3*c^3*d^5 - 10*a^3
*b^2*c^2*d^6 + 2*a^4*b*c*d^7))*(-b^3*(a*d - b*c)^3)^(1/2))/(a*b^3)
```

3.691 $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=102

$$-\frac{c\sqrt{c+dx^2}}{ax}-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

```
[Out] -(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2
)/b+d^(3/2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b-c*(d*x^2+c)^(1/2)/a/x
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {485, 537, 223, 212, 385, 211}

$$-\frac{(bc-ad)^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/b
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx &= -\frac{c\sqrt{c+dx^2}}{ax}+\frac{\int \frac{-c(bc-2ad)+ad^2x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{a}\\
&= -\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b}-\frac{(bc-ad)^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{ab}\\
&= -\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^2\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b}-\frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{ab}\\
&= -\frac{c\sqrt{c+dx^2}}{ax}-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 129, normalized size = 1.26

$$
\frac{(bc-ad)^{3/2}x\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)-\sqrt{a}\left(bc\sqrt{c+dx^2}+ad^{3/2}x\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)\right)}{a^{3/2}bx}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] ((b*c - a*d)^(3/2)*x*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))
/(Sqrt[a]*Sqrt[b*c - a*d])] - Sqrt[a]*(b*c*Sqrt[c + d*x^2] + a*d^(3/2)*x*Lo
g[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(a^(3/2)*b*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. $2(84) = 168$.
time = 0.12, size = 1330, normalized size = 13.04

method	result
risch	$-\frac{c\sqrt{d\,x^{2}+c}}{a x}+\frac{d^{\frac{3}{2}}\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{b}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(\right.}}{x-\sqrt{}}\right.}{2\sqrt{-ab}\,b}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a/(-a*b)^(1/2)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*
b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/
d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*
b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a
*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(
-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d
*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+1/2*b/a/(-a*b)^(1/2)*(1/3*(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*
b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1
/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*
b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/
b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1
```

```
/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c
)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/a*(-1/
c/x*(d*x^2+c)^(5/2)+4*d/c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/
2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^2), x)
```

Fricas [A]
time = 1.48, size = 718, normalized size = 7.04

$$\left[\frac{2ad^{\frac{3}{2}}x\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)-(bc-ad)x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4(a^2cx-(abc-2a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2abx^2+a^2}\right)-4\sqrt{dx^2+c}\,bc}{4abx},\ -\frac{4a\sqrt{-d}\,dx\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(bc-ad)x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4(a^2cx-(abc-2a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2abx^2+a^2}\right)+4\sqrt{dx^2+c}\,bc}{4abx},\ \frac{ad^{\frac{3}{2}}x\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)-(bc-ad)x\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^2)x^3+(bc^2-acd)x)}\right)-2\sqrt{dx^2+c}\,bc}{2abx},\ -\frac{2a\sqrt{-d}\,dx\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(bc-ad)x\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\sqrt{dx^2+c}\,bc}{2abx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*d^(3/2)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c
- a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^
3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*s
qrt(d*x^2 + c)*b*c)/(a*b*x), -1/4*(4*a*sqrt(-d)*d*x*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) + (b*c - a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (
a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) + 4*sqrt(d*x^2 + c)*b*c)/(a*b*x), 1/2*(a*d^(3/2)*x*log(-2*d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c - a*d)*x*sqrt((b*c - a*d)/a)*a
rctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b
*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - 2*sqrt(d*x^2 + c)*b*c)/(a*b*x), -
1/2*(2*a*sqrt(-d)*d*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b*c - a*d)*x*sq
rt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt
((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*sqrt(d*x^2 +
 c)*b*c)/(a*b*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{2}\right)^{\frac{3}{2}}}{x^{2}\left(a + bx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}}{x^2\,\left(b\,x^2+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)), x)
```

3.692 $\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=114

$$-\frac{c\sqrt{c+dx^2}}{2ax^2}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}$$

```
[Out] -(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(
1/2)+1/2*(-3*a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)/a^2-1/2*c*
(d*x^2+c)^(1/2)/a/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 100, 162, 65, 214}

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{c\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*(c*Sqrt[c + d*x^2])/(a*x^2) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c
+ d*x^2]/Sqrt[c]])/(2*a^2) - ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^2])/Sqrt[b*c - a*d]])/(a^2*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(2bc-3ad)+\frac{1}{2}d(bc-2ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} - \frac{(c(2bc-3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^2} + \frac{(bc-ad)^2\text{Subst}\left(\int \right.}{}$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} - \frac{(c(2bc-3ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2a^2d} + \frac{(bc-ad)^2\text{Subst}}{}$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} + \frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2} - \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{}}{\sqrt{bc}}\right.}{a^2\sqrt{b}}$$

Mathematica [A]

time = 0.25, size = 108, normalized size = 0.95

$$\frac{-\frac{ac\sqrt{c+dx^2}}{x^2}+\frac{2(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}}+\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*c*Sqrt[c + d*x^2])/x^2) + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqr
t[c + d*x^2])/Sqrt[-(b*c) + a*d]])/Sqrt[b] + Sqrt[c]*(2*b*c - 3*a*d)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. 2(92) = 184.
time = 0.13, size = 1377, normalized size = 12.08

method	result
risch	$-\frac{c\sqrt{d\,x^2+c}}{2a\,x^2}-\frac{3\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)d}{2a}+\frac{c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)b}{a^2}-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-(}}{}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/a^2*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*
d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((
d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2))
)/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*
b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-
1/b*(-a*b)^(1/2)))))+1/2*b/a^2*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1
```

```
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x
+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*
d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b
+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/a*(-1/2/c/x^2*(d*x^2+c)^(5/2)
+3/2*d/c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*
(d*x^2+c)^(1/2))/x))))-b/a^2*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2
)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^3), x)
```

Fricas [A]
time = 2.24, size = 732, normalized size = 6.42

$$\left[-\frac{(bc-ad)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(b^2dx^2+2b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+(2bc-3ad)\sqrt{c}\,x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\sqrt{dx^2+c}\,ac}{4a^2x^2}, -\frac{2(2bc-3ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(bc-ad)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(b^2dx^2+2b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{dx^2+c}\,ac}{4a^2x^2}, -\frac{2(bc-ad)x^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2(bc^2-acd+(bcd-ad^2)x^2)}\right)+(2bc-3ad)\sqrt{c}\,x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\sqrt{dx^2+c}\,ac}{4a^2x^2}, -\frac{(bc-ad)x^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2(bc^2-acd+(bcd-ad^2)x^2)}\right)+(2bc-3ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\sqrt{dx^2+c}\,ac}{2a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c - a*d)*x^2*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c
 - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*
c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/4*(2*(2*b*c - 3*a*d)*sqrt(-c
)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b*c - a*d)*x^2*sqrt((b*c - a*d)/b
)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d
)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/4
*(2*(b*c - a*d)*x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d
)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2
```

```
)) + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) +
2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/2*((b*c - a*d)*x^2*sqrt(-(
b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (2*b*c - 3*a*d)*sqrt(-
c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + sqrt(d*x^2 + c)*a*c)/(a^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{x^3\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**3*(a + b*x**2)), x)
```

Giac [A]
time = 0.75, size = 120, normalized size = 1.05

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}-\frac{\sqrt{dx^2+c}\,c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/2*(2*b*c^2 - 3*a*c*d)*arctan(sqrt(d*x^2
+ c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/2*sqrt(d*x^2 + c)*c/(a*x^2)
```

Mupad [B]
time = 0.64, size = 560, normalized size = 4.91

$$-\frac{c\sqrt{d\,x^2+c}}{2\,a\,x^2}-\frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{29\,b^2\,c^{3/2}\,d^6\,\sqrt{d\,x^2+c}}{4\left(\frac{29\,b^2\,c^2\,d^6}{4}-3\,a\,b\,c\,d^7-\frac{23\,b^3\,c^3\,d^5}{4\,a}+\frac{3\,b^4\,c^4\,d^4}{2\,a^2}\right)}+\frac{23\,b^3\,c^{5/2}\,d^5\,\sqrt{d\,x^2+c}}{4\left(\frac{23\,b^3\,c^3\,d^5}{4}-\frac{29\,a\,b^2\,c^2\,d^6}{4}-\frac{3\,b^4\,c^4\,d^4}{2\,a}+3\,a^2\,b\,c\,d^7\right)}+\frac{3\,b^4\,c^{7/2}\,d^4\,\sqrt{d\,x^2+c}}{2\left(-3\,a^3\,b\,c\,d^7+\frac{29\,a^2\,b^2\,c^2\,d^6}{4}-\frac{23\,a\,b^3\,c^3\,d^5}{4}+\frac{3\,b^4\,c^4\,d^4}{2}\right)}-\frac{3\,a\,b\,\sqrt{c}\,d^7\,\sqrt{d\,x^2+c}}{\frac{29\,b^2\,c^2\,d^6}{4}-3\,a\,b\,c\,d^7-\frac{23\,b^3\,c^3\,d^5}{4\,a}+\frac{3\,b^4\,c^4\,d^4}{2\,a^2}}\right)(3\,a\,d-2\,b\,c)}{2\,a^2}-\frac{\mathrm{atanh}\left(\frac{3\,b^2\,c^2\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b\,d^3+3\,a^2\,b^2\,c\,d^2-3\,a\,b^3\,c^2\,d+b^4\,c^3}}{2\left(-2\,a^3\,b\,c\,d^7+\frac{11\,a^2\,b^2\,c^2\,d^6}{2}-5\,a\,b^3\,c^3\,d^5+\frac{3\,b^4\,c^4\,d^4}{2}\right)}+\frac{2\,b\,c\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b\,d^3+3\,a^2\,b^2\,c\,d^2-3\,a\,b^3\,c^2\,d+b^4\,c^3}}{5\,b^3\,c^3\,d^5-\frac{11\,a\,b^2\,c^2\,d^6}{2}-\frac{3\,b^4\,c^4\,d^4}{2\,a}+2\,a^2\,b\,c\,d^7}\right)\sqrt{-b\,{\left(a\,d-b\,c\right)}^3}}{a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x)
```

```
[Out] - (c*(c + d*x^2)^(1/2))/(2*a*x^2) - (c^(1/2)*atanh((29*b^2*c^(3/2)*d^6*(c +
 d*x^2)^(1/2))/(4*((29*b^2*c^2*d^6)/4 - 3*a*b*c*d^7 - (23*b^3*c^3*d^5)/(4*a
) + (3*b^4*c^4*d^4)/(2*a^2))) + (23*b^3*c^(5/2)*d^5*(c + d*x^2)^(1/2))/(4*(
(23*b^3*c^3*d^5)/4 - (29*a*b^2*c^2*d^6)/4 - (3*b^4*c^4*d^4)/(2*a) + 3*a^2*b
*c*d^7)) + (3*b^4*c^(7/2)*d^4*(c + d*x^2)^(1/2))/(2*((3*b^4*c^4*d^4)/2 - (2
3*a*b^3*c^3*d^5)/4 + (29*a^2*b^2*c^2*d^6)/4 - 3*a^3*b*c*d^7)) - (3*a*b*c^(1
/2)*d^7*(c + d*x^2)^(1/2))/((29*b^2*c^2*d^6)/4 - 3*a*b*c*d^7 - (23*b^3*c^3*
```

```
d^5)/(4*a) + (3*b^4*c^4*d^4)/(2*a^2)))*(3*a*d - 2*b*c))/(2*a^2) - (atanh((3
*b^2*c^2*d^4*(c + d*x^2)^(1/2)*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a
*b^3*c^2*d)^(1/2))/(2*((3*b^4*c^4*d^4)/2 - 5*a*b^3*c^3*d^5 + (11*a^2*b^2*c^
2*d^6)/2 - 2*a^3*b*c*d^7)) + (2*b*c*d^5*(c + d*x^2)^(1/2)*(b^4*c^3 - a^3*b*
d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)^(1/2))/(5*b^3*c^3*d^5 - (11*a*b^2*c^
2*d^6)/2 - (3*b^4*c^4*d^4)/(2*a) + 2*a^2*b*c*d^7))*(-b*(a*d - b*c)^3)^(1/2)
)/(a^2*b)
```

3.693 $\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=102

$$-\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}$$

```
[Out] (-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)
-1/3*c*(d*x^2+c)^(1/2)/a/x^3+1/3*(-4*a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/x
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {485, 597, 12, 385, 211}

$$\frac{(bc-ad)^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}\,(3bc-4ad)}{3a^2x}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*(c*Sqrt[c + d*x^2])/(a*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d*x^2])/(3*a^2
*x) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/a^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
```

```
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx &= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{\int \frac{-c(3bc-4ad)-d(2bc-3ad)x^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx}{3a}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}-\frac{\int -\frac{3c(bc-ad)^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3a^2c}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{a^2}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a^2}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 111, normalized size = 1.09

$$
\frac{\sqrt{c+dx^2}\left(3bcx^2-a(c+4dx^2)\right)}{3a^2x^3}-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + 4*d*x^2)))/(3*a^2*x^3) - ((b*c - a*d)^
(3/2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[
b*c - a*d])])/a^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1438 vs. $2(84) = 168$.

time = 0.12, size = 1439, normalized size = 14.11

method	result
risch	$-\frac{\sqrt{d x^2+c}\,(4ad x^2-3c x^2 b+ac)}{3a^2x^3}-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2}}{x-\frac{\sqrt{-ab}}{b}}\right.}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^2/(-a*b)^(1/2)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(
-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/
b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))
-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/
b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2*b^2/a^2/(-a*b)^(1/2)*(1/3*(d*(x+1/b
*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)
-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1
/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^
2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)
^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-
```

```
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(
a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1
/a*(-1/3/c/x^3*(d*x^2+c)^(5/2)+2/3*d/c*(-1/c/x*(d*x^2+c)^(5/2)+4*d/c*(1/4*x
*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d
*x^2+c)^(1/2))))))-b/a^2*(-1/c/x*(d*x^2+c)^(5/2)+4*d/c*(1/4*x*(d*x^2+c)^(3/
2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^4), x)
```

Fricas [A]
time = 1.93, size = 331, normalized size = 3.25

$$\left[-\frac{3\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,((3\,bc-4\,ad)x^2-ac)\sqrt{dx^2+c}}{12\,a^2x^3},\ \frac{3\,(bc-ad)x^3\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\,((3\,bc-4\,ad)x^2-ac)\sqrt{dx^2+c}}{6\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*
b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) - 4*((3*b*c - 4*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*x^3), 1/6
*(3*(b*c - a*d)*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d
)*x)) + 2*((3*b*c - 4*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{2}\right)^{\frac{3}{2}}}{x^{4}\left(a + bx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**4*(a + b*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(84) = 168.
time = 1.66, size = 256, normalized size = 2.51

$$-\frac{\left(b^{2}c^{2}\sqrt{d}-2\,abcd^{\frac{3}{2}}+a^{2}d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\sqrt{abcd-a^{2}d^{2}}\,a^{2}}-\frac{2\left(3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}bc^{2}\sqrt{d}-6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}acd^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc^{3}\sqrt{d}+6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ac^{2}d^{\frac{3}{2}}+3\,bc^{4}\sqrt{d}-4\,ac^{3}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*sqrt(d) - 2*a*b*c*d^(3/2) + a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x -
 sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d
 - a^2*d^2)*a^2) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c^2*sqrt(d) - 6
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*b*c^3*sqrt(d) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c^2*d^(3/2) + 3*b
*c^4*sqrt(d) - 4*a*c^3*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^
2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}}{x^4\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)), x)
```

3.694 $\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2} dx$

Optimal. Leaf size=291

$$\frac{(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3)\,x\sqrt{c+dx^2}}{128b^4d}+\frac{(59b^2c^2-104abcd+48a^2d^2)\,x^3\sqrt{c+dx^2}}{192b^3}+\frac{d(11bc}{}$$

```
[Out] 1/8*d*x^5*(d*x^2+c)^(3/2)/b+a^(3/2)*(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(1
/2)/a^(1/2)/(d*x^2+c)^(1/2))/b^5-1/128*(-128*a^4*d^4+320*a^3*b*c*d^3-240*a^
2*b^2*c^2*d^2+40*a*b^3*c^3*d+5*b^4*c^4)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/
b^5/d^(3/2)+1/128*(-64*a^3*d^3+144*a^2*b*c*d^2-88*a*b^2*c^2*d+5*b^3*c^3)*x*
(d*x^2+c)^(1/2)/b^4/d+1/192*(48*a^2*d^2-104*a*b*c*d+59*b^2*c^2)*x^3*(d*x^2+
c)^(1/2)/b^3+1/48*d*(-8*a*d+11*b*c)*x^5*(d*x^2+c)^(1/2)/b^2
```

Rubi [A]
time = 0.37, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {488, 595, 596, 537, 223, 212, 385, 211}

$$\frac{a^{3/2}(bc-ad)^{5/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^5}+\frac{x^3\sqrt{c+dx^2}\,(48a^2d^2-104abcd+59b^2c^2)}{192b^3}+\frac{x\sqrt{c+dx^2}\,(-64a^3d^3+144a^2bcd^2-88ab^2c^2d+5b^3c^3)}{128b^4d}-\frac{(-128a^4d^4+320a^3bcd^3-240a^2b^2c^2d^2+40ab^3c^3d+5b^4c^4)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128b^5d^{3/2}}+\frac{dx^5\sqrt{c+dx^2}\,(11bc-8ad)}{48b^2}+\frac{dx^5(c+dx^2)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((5*b^3*c^3 - 88*a*b^2*c^2*d + 144*a^2*b*c*d^2 - 64*a^3*d^3)*x*Sqrt[c + d*x
^2])/(128*b^4*d) + ((59*b^2*c^2 - 104*a*b*c*d + 48*a^2*d^2)*x^3*Sqrt[c + d*
x^2])/(192*b^3) + (d*(11*b*c - 8*a*d)*x^5*Sqrt[c + d*x^2])/(48*b^2) + (d*x^
5*(c + d*x^2)^(3/2))/(8*b) + (a^(3/2)*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^5 - ((5*b^4*c^4 + 40*a*b^3*c^3*d - 24
0*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(128*b^5*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{dx^5(c+dx^2)^{3/2}}{8b} + \frac{\int \frac{x^4\sqrt{c+dx^2}\,(c(8bc-5ad)+d(11bc-8ad)x^2)}{a+bx^2}\,dx}{8b}$$

$$= \frac{d(11bc-8ad)x^5\sqrt{c+dx^2}}{48b^2} + \frac{dx^5(c+dx^2)^{3/2}}{8b} + \frac{\int \frac{x^4\left(c\left(48b^2c^2-85abcd+40a^2d^2\right)+d\left(59b^2c^2-1\right.\right.}{(a+bx^2)\sqrt{c+dx^2}}}{48b^2}$$

$$= \frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x^3\sqrt{c+dx^2}}{192b^3} + \frac{d(11bc-8ad)x^5\sqrt{c+dx^2}}{48b^2} + \frac{dx^5(c+}{8\ldots}$$

$$= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d} + \frac{(59b^2c^2-104abcd+48a^2d}{192b^3}$$

$$= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d} + \frac{(59b^2c^2-104abcd+48a^2d}{192b^3}$$

$$= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d} + \frac{(59b^2c^2-104abcd+48a^2d}{192b^3}$$

$$= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d} + \frac{(59b^2c^2-104abcd+48a^2d}{192b^3}$$

Mathematica [**A**]
time = 0.64, size = 266, normalized size = 0.91

$$\frac{\frac{bx\sqrt{c+dx^2}\,\left(-192a^3d^3+48a^2bd^2(9c+2dx^2)-8ab^2d(33c^2+26cdx^2+8d^2x^4)+b^3(15c^3+118c^2dx^2+136cd^2x^4+48d^3x^6)\right)}{d}-384a^{3/2}(bc-ad)^{5/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)+\frac{3\left(5b^4c^4+40ab^3c^3d-240a^2b^2c^2d^2+320a^3bcd^3-128a^4d^4\right)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{d^{3/2}}}{384b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2]*(-192*a^3*d^3 + 48*a^2*b*d^2*(9*c + 2*d*x^2) - 8*a*b^
2*d*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) + b^3*(15*c^3 + 118*c^2*d*x^2 + 136*c
*d^2*x^4 + 48*d^3*x^6)))/d - 384*a^(3/2)*(b*c - a*d)^(5/2)*ArcTan[(a*Sqrt[d
] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])] + (3*(5*b
^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d
^4)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/d^(3/2))/(384*b^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2251 vs. $2(257) = 514$.
time = 0.14, size = 2252, normalized size = 7.74

method	result	size
risch	Expression too large to display	1560
default	Expression too large to display	2252

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*x*(d*x^2+c)^(7/2)/d-1/8*c/d*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d
*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^
2+c)^(1/2))))))-a/b^2*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3
/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+
1/2/b^2*a^2/(-a*b)^(1/2)*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d*(x-1/b*(
-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a
/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)
^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-
4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/
2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*
(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*
c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2
)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1
/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))))-1/2/b^2*a^2/(-a*b)^(1/2)*
(1/5*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(5/2)-d*(-a*b)^(1/2)/b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(
1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-
a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b
)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))
)-(a*d-b*c)/b*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2)
)-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
```

```
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*
ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)
/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^
(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/
b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x+1/b*(-a*b)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a), x)
```

Fricas [A]
time = 12.58, size = 1443, normalized size = 4.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
[Out] [-1/768*(3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*
d^3 - 128*a^4*d^4)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
- 192*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(-a*b*c + a^2*d)*log(((
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*
x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8*a*b^3
*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3 + 3*(
5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)*sqrt(
d*x^2 + c))/(b^5*d^2), 1/384*(3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c
^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x
^2 + c)) + 96*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(-a*b*c + a^2*d
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8
*a*b^3*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3
 + 3*(5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)
*sqrt(d*x^2 + c))/(b^5*d^2), 1/768*(384*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^
```

```
3*d^4)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^
2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))
 - 3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 -
128*a^4*d^4)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(4
8*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8*a*b^3*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 10
4*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3 + 3*(5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 1
44*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)*sqrt(d*x^2 + c))/(b^5*d^2), 1/384*(192*
(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(a*b*c - a^2*d)*arctan(1/2*sq
rt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2
*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a
^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*sqrt(-d)*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)) + (48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8*a*b^3*d^4)*x^5 + 2
*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3 + 3*(5*b^4*c^3*d -
 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)*sqrt(d*x^2 + c))/(
b^5*d^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(c+dx^2)^{\frac{5}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a),x)
[Out] Integral(x**4*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d\,x^2+c\right)}^{5/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x)
[Out] int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2), x)
```

3.695 $\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=144

$$-\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4}-\frac{a(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^3}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}+\frac{a(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\ldots}{\ldots}\right)}{b^{9/2}}$$

```
[Out] -1/3*a*(-a*d+b*c)*(d*x^2+c)^(3/2)/b^3-1/5*a*(d*x^2+c)^(5/2)/b^2+1/7*(d*x^2+
c)^(7/2)/b/d+a*(-a*d+b*c)^(5/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^
(1/2))/b^(9/2)-a*(-a*d+b*c)^2*(d*x^2+c)^(1/2)/b^4
```

Rubi [A]
time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 81, 52, 65, 214}

$$\frac{a(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{9/2}}-\frac{a\sqrt{c+dx^2}\,(bc-ad)^2}{b^4}-\frac{a\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^3}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^2*Sqrt[c + d*x^2])/b^4) - (a*(b*c - a*d)*(c + d*x^2)^(3/2)
)/(3*b^3) - (a*(c + d*x^2)^(5/2))/(5*b^2) + (c + d*x^2)^(7/2)/(7*b*d) + (a*
(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(9/
2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^{5/2}}{a+bx}\,dx, x, x^2\right)$$

$$= \frac{(c+dx^2)^{7/2}}{7bd} - \frac{a\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{a+bx}\,dx, x, x^2\right)}{2b}$$

$$= -\frac{a(c+dx^2)^{5/2}}{5b^2} + \frac{(c+dx^2)^{7/2}}{7bd} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)}{2b^2}$$

$$= -\frac{a(bc-ad)\,(c+dx^2)^{3/2}}{3b^3} - \frac{a(c+dx^2)^{5/2}}{5b^2} + \frac{(c+dx^2)^{7/2}}{7bd} - \frac{(a(bc-ad)^2)\,\text{Subst}\left(\int \right.}{2b^3}$$

$$= -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4} - \frac{a(bc-ad)\,(c+dx^2)^{3/2}}{3b^3} - \frac{a(c+dx^2)^{5/2}}{5b^2} + \frac{(c+dx^2)^{7/2}}{7bd} -$$

$$= -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4} - \frac{a(bc-ad)\,(c+dx^2)^{3/2}}{3b^3} - \frac{a(c+dx^2)^{5/2}}{5b^2} + \frac{(c+dx^2)^{7/2}}{7bd} -$$

$$= -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4} - \frac{a(bc-ad)\,(c+dx^2)^{3/2}}{3b^3} - \frac{a(c+dx^2)^{5/2}}{5b^2} + \frac{(c+dx^2)^{7/2}}{7bd} +$$

Mathematica [A]
time = 0.20, size = 140, normalized size = 0.97

$$\frac{\sqrt{c+dx^2}\left(-105a^3d^3+15b^3(c+dx^2)^3+35a^2bd^2(7c+dx^2)-7ab^2d(23c^2+11cdx^2+3d^2x^4)\right)}{105b^4d}+\frac{a(-bc+ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-105*a^3*d^3 + 15*b^3*(c + d*x^2)^3 + 35*a^2*b*d^2*(7*c +
 d*x^2) - 7*a*b^2*d*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)))/(105*b^4*d) + (a*(-
(b*c) + a*d)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^
(9/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2086 vs. $2(120) = 240$.
time = 0.11, size = 2087, normalized size = 14.49

method	result
risch	$-\frac{\left(-15d^3b^3x^6+21a\,b^2d^3x^4-45b^3c\,d^2x^4-35a^2b\,d^3x^2+77a\,b^2c\,d^2x^2-45b^3c^2d\,x^2+105a^3d^3-245a^2bc\,d^2+161a\,b^2c^2d-15b^3c^3\right)\sqrt{d\,x}}{105d\,b^4}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(d*x^2+c)^(7/2)/b/d-1/2*a/b^2*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d
*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/
b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(
-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))
^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2
)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(
1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*
d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)
```

```
))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*
(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))))-1/2*a/b^2*(1/5*(
d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(5/2)-d*(-a*b)^(1/2)/b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b
)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(
1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3
/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-(a*d
-b*c)/b*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*
(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d
*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d
*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2))
)/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*
b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+
1/b*(-a*b)^(1/2))))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 1.64, size = 527, normalized size = 3.66

$$\left[\frac{105 \, {\left(a b^{2} c^{2} d - 2 \, a^{2} b c d^{2} + a^{3} d^{3}\right)} \sqrt{\frac{b c - a d}{b}} \log\left(\frac{b^{2} d^{2} x^{4} + 8 \, b^{2} c^{2} - 8 \, a b c d + a^{2} d^{2} + 2 \, {\left(4 \, b^{2} c d - 3 \, a b d^{2}\right)} x^{2} + 4 \, {\left(b^{2} d x^{2} + 2 \, b^{2} c - a b d\right)} \sqrt{d x^{2} + c} \sqrt{\frac{b c - a d}{b}}}{b^{2} x^{4} + 2 \, a b x^{2} + a^{2}}\right) + 4 \, {\left(15 \, b^{3} d^{3} x^{6} + 15 \, b^{3} c^{3} - 161 \, a b^{2} c^{2} d + 245 \, a^{2} b c d^{2} - 105 \, a^{3} d^{3} + 3 \, {\left(15 \, b^{3} c d^{2} - 7 \, a b^{2} d^{3}\right)} x^{4} + {\left(45 \, b^{3} c^{2} d - 77 \, a b^{2} c d^{2} + 35 \, a^{2} b d^{3}\right)} x^{2}\right)} \sqrt{d x^{2} + c}}{420 \, b^{4} d}, \frac{105 \, {\left(a b^{2} c^{2} d - 2 \, a^{2} b c d^{2} + a^{3} d^{3}\right)} \sqrt{-\frac{b c - a d}{b}} \arctan\left(-\frac{{\left(b d x^{2} + 2 \, b c - a d\right)} \sqrt{d x^{2} + c} \sqrt{-\frac{b c - a d}{b}}}{2 \, {\left(b c^{2} - a c d + {\left(b c d - a d^{2}\right)} x^{2}\right)}}\right) + 2 \, {\left(15 \, b^{3} d^{3} x^{6} + 15 \, b^{3} c^{3} - 161 \, a b^{2} c^{2} d + 245 \, a^{2} b c d^{2} - 105 \, a^{3} d^{3} + 3 \, {\left(15 \, b^{3} c d^{2} - 7 \, a b^{2} d^{3}\right)} x^{4} + {\left(45 \, b^{3} c^{2} d - 77 \, a b^{2} c d^{2} + 35 \, a^{2} b d^{3}\right)} x^{2}\right)} \sqrt{d x^{2} + c}}{210 \, b^{4} d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/420*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt((b*c - a*d)/b)*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)
*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))
/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(15*b^3*d^3*x^6 + 15*b^3*c^3 - 161*a*b^2*
c^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3 + 3*(15*b^3*c*d^2 - 7*a*b^2*d^3)*x^4
+ (45*b^3*c^2*d - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*sqrt(d*x^2 + c))/(b^4
*d), 1/210*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-(b*c - a*d)/b
)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/
(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(15*b^3*d^3*x^6 + 15*b^3*c^3 - 1
61*a*b^2*c^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3 + 3*(15*b^3*c*d^2 - 7*a*b^2*
d^3)*x^4 + (45*b^3*c^2*d - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*sqrt(d*x^2 +
 c))/(b^4*d)]
```

Sympy [A]
time = 35.01, size = 144, normalized size = 1.00

$$-\frac{a\left(c+dx^{2}\right)^{\frac{5}{2}}}{5b^{2}}+\frac{a\left(ad-bc\right)^{3}\operatorname{atan}\left(\frac{\sqrt{c+dx^{2}}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^{5}\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^{2}\right)^{\frac{7}{2}}}{7bd}+\frac{\left(c+dx^{2}\right)^{\frac{3}{2}}\left(a^{2}d-abc\right)}{3b^{3}}+\frac{\sqrt{c+dx^{2}}\left(-a^{3}d^{2}+2a^{2}bcd-ab^{2}c^{2}\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] -a*(c + d*x**2)**(5/2)/(5*b**2) + a*(a*d - b*c)**3*atan(sqrt(c + d*x**2)/sq
rt((a*d - b*c)/b))/(b**5*sqrt((a*d - b*c)/b)) + (c + d*x**2)**(7/2)/(7*b*d)
 + (c + d*x**2)**(3/2)*(a**2*d - a*b*c)/(3*b**3) + sqrt(c + d*x**2)*(-a**3*
d**2 + 2*a**2*b*c*d - a*b**2*c**2)/b**4
```

Giac [A]
time = 0.52, size = 228, normalized size = 1.58

$$-\frac{\left(ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,b^{4}}+\frac{15\left(dx^{2}+c\right)^{\frac{7}{2}}b^{6}d^{6}-21\left(dx^{2}+c\right)^{\frac{5}{2}}ab^{5}d^{7}-35\left(dx^{2}+c\right)^{\frac{3}{2}}ab^{5}cd^{7}-105\sqrt{dx^{2}+c}\,ab^{5}c^{2}d^{7}+35\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}b^{4}d^{8}+210\sqrt{dx^{2}+c}\,a^{2}b^{4}cd^{8}-105\sqrt{dx^{2}+c}\,a^{3}b^{3}d^{9}}{105\,b^{7}d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(sqrt(d*x^2
+ c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 1/105*(15*(d*x^2
+ c)^(7/2)*b^6*d^6 - 21*(d*x^2 + c)^(5/2)*a*b^5*d^7 - 35*(d*x^2 + c)^(3/2)*
a*b^5*c*d^7 - 105*sqrt(d*x^2 + c)*a*b^5*c^2*d^7 + 35*(d*x^2 + c)^(3/2)*a^2*
```

```
b^4*d^8 + 210*sqrt(d*x^2 + c)*a^2*b^4*c*d^8 - 105*sqrt(d*x^2 + c)*a^3*b^3*d
^9)/(b^7*d^7)
```

Mupad [B]
time = 0.34, size = 251, normalized size = 1.74

$$\frac{\left(d\,x^2+c\right)^{7/2}}{7\,b\,d}-\left(d\,x^2+c\right)^{5/2}\,\left(\frac{c}{5\,b\,d}+\frac{a\,d^2-b\,c\,d}{5\,b^2\,d^2}\right)+\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^{5/2}}{a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^{5/2}}{b^{9/2}}+\frac{{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d^2-b\,c\,d\right)\,\left(\frac{c}{b\,d}+\frac{a\,d^2-b\,c\,d}{b^2\,d^2}\right)}{3\,b\,d}-\frac{\sqrt{d\,x^2+c}\,{\left(a\,d^2-b\,c\,d\right)}^2\,\left(\frac{c}{b\,d}+\frac{a\,d^2-b\,c\,d}{b^2\,d^2}\right)}{b^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(7/2)/(7*b*d) - (c + d*x^2)^(5/2)*(c/(5*b*d) + (a*d^2 - b*c*d)/
(5*b^2*d^2)) + (a*atan((a*b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(5/2))/(a^4
*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))*(a*d - b*c)^(5/2))/b^(
9/2) + ((c + d*x^2)^(3/2)*(a*d^2 - b*c*d)*(c/(b*d) + (a*d^2 - b*c*d)/(b^2*d
^2)))/(3*b*d) - ((c + d*x^2)^(1/2)*(a*d^2 - b*c*d)^2*(c/(b*d) + (a*d^2 - b*
c*d)/(b^2*d^2)))/(b^2*d^2)
```

3.696 $\int \frac{x^2 \left(c+dx^2\right)^{5/2}}{a+bx^2} \, dx$

Optimal. Leaf size=217

$$\frac{\left(11b^2c^2-18abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3}+\frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}-\frac{\sqrt{a}\,(bc-ad)^{5/2}\tan}{b}$$

```
[Out] 1/6*d*x^3*(d*x^2+c)^(3/2)/b-(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1
/2)/(d*x^2+c)^(1/2))*a^(1/2)/b^4+1/16*(-16*a^3*d^3+40*a^2*b*c*d^2-30*a*b^2*
c^2*d+5*b^3*c^3)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^4/d^(1/2)+1/16*(8*a^2
*d^2-18*a*b*c*d+11*b^2*c^2)*x*(d*x^2+c)^(1/2)/b^3+1/8*d*(-2*a*d+3*b*c)*x^3*
(d*x^2+c)^(1/2)/b^2
```

Rubi [A]
time = 0.26, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {488, 595, 596, 537, 223, 212, 385, 211}

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{16b^3}+\frac{\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^4\sqrt{d}}-\frac{\sqrt{a}\,(bc-ad)^{5/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^4}+\frac{dx^3\sqrt{c+dx^2}\,(3bc-2ad)}{8b^2}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((11*b^2*c^2 - 18*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3) + (d*(3*
b*c - 2*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*(c + d*x^2)^(3/2))/(6*b)
 - (Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/b^4 + ((5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)
*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*b^4*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{dx^3(c+dx^2)^{3/2}}{6b} + \frac{\int \frac{x^2\sqrt{c+dx^2}\,(3c(2bc-ad)+3d(3bc-2ad)x^2)}{a+bx^2}\,dx}{6b}$$

$$= \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3(c+dx^2)^{3/2}}{6b} + \frac{\int \frac{x^2(3c(8b^2c^2-13abcd+6a^2d^2)+3d(11b^2c^2-18}{(a+bx^2)\sqrt{c+dx^2}}}{24b^2}$$

$$= \frac{(11b^2c^2-18abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3} + \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3(c+dx^2)}{6b}$$

$$= \frac{(11b^2c^2-18abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3} + \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3(c+dx^2)}{6b}$$

$$= \frac{(11b^2c^2-18abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3} + \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3(c+dx^2)}{6b}$$

$$= \frac{(11b^2c^2-18abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3} + \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3(c+dx^2)}{6b}$$

Mathematica [A]
time = 0.41, size = 206, normalized size = 0.95

$$\frac{bx\sqrt{c+dx^2}\,(24a^2d^2-6abd(9c+2dx^2)+b^2(33c^2+26cdx^2+8d^2x^4))+48\sqrt{a}\,(bc-ad)^{5/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)+\frac{3(-5b^3c^3+30ab^2c^2d-40a^2bcd^2+16a^3d^3)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{48b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(9*c + 2*d*x^2) + b^2*(33*c^2 +
26*c*d*x^2 + 8*d^2*x^4)) + 48*Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(a*Sqrt[d] +
 b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])] + (3*(-5*b^3
*c^3 + 30*a*b^2*c^2*d - 40*a^2*b*c*d^2 + 16*a^3*d^3)*Log[-(Sqrt[d]*x) + Sqr
t[c + d*x^2]])/Sqrt[d])/(48*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2152 vs. $2(187) = 374$.
time = 0.13, size = 2153, normalized size = 9.92

method	result	size
risch	Expression too large to display	1469
default	Expression too large to display	2153

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/b*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))-1/2*a/(-a*b)^(1/2)/b*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))))+1/2*a/(-a*b)^(1/2)/b*(1/5*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-d*(-a*b)^(1/2)/b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))

```
+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

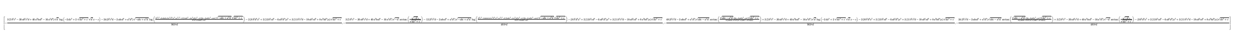
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a), x)
```

Fricas [A]
time = 4.85, size = 1161, normalized size = 5.35



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(d
)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 24*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c
*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*
(8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*
a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d), -1/48*(3*(5*b^3*c^3
 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x
/sqrt(d*x^2 + c)) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(-a*b*c + a^
2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*
x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 -
 6*a*b^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt
(d*x^2 + c))/(b^4*d), -1/96*(48*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(a*
b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(
d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*b^3*c^
3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(d)*log(-2*d*x^2 + 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b
^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2
 + c))/(b^4*d), -1/48*(24*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(a*b*c -
a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*b^3*c^3 - 30
*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
```

```
(d*x^2 + c)) - (8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(11*
b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c+dx^2)^{\frac{5}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(d\,x^2+c\right)}^{5/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

```
[Out] int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2), x)
```

3.697 $\int \frac{x(c+dx^2)^{5/2}}{a+bx^2} \, dx$

Optimal. Leaf size=119

$$\frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3}+\frac{(bc-ad)\,(c+dx^2)^{3/2}}{3b^2}+\frac{(c+dx^2)^{5/2}}{5b}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

```
[Out] 1/3*(-a*d+b*c)*(d*x^2+c)^(3/2)/b^2+1/5*(d*x^2+c)^(5/2)/b-(-a*d+b*c)^(5/2)*a
rctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)+(-a*d+b*c)^2*(d*x^
2+c)^(1/2)/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {455, 52, 65, 214}

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\sqrt{c+dx^2}\,(bc-ad)^2}{b^3}+\frac{(c+dx^2)^{3/2}\,(bc-ad)}{3b^2}+\frac{(c+dx^2)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/b^3 + ((b*c - a*d)*(c + d*x^2)^(3/2))/(3*b^
2) + (c + d*x^2)^(5/2)/(5*b) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{a+bx}\,dx, x, x^2\right)$$

$$= \frac{(c+dx^2)^{5/2}}{5b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)}{2b}$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{(c+dx^2)^{5/2}}{5b} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{2b^2}$$

$$= \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{(c+dx^2)^{5/2}}{5b} + \frac{(bc-ad)^3\text{Subst}\left(\int \ldots\right.}{2\ldots}$$

$$= \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{(c+dx^2)^{5/2}}{5b} + \frac{(bc-ad)^3\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{(c+dx^2)^{5/2}}{5b} - \frac{(bc-ad)^{5/2}\tanh^{-1}\left(\ldots\right.}{b^{7/2}}$$

Mathematica [A]
time = 0.13, size = 116, normalized size = 0.97

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2-5abd(7c+dx^2)+b^2(23c^2+11cdx^2+3d^2x^4)\right)}{15b^3} - \frac{(-bc+ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 - 5*a*b*d*(7*c + d*x^2) + b^2*(23*c^2 + 11*c*d
*x^2 + 3*d^2*x^4)))/(15*b^3) - ((-(b*c) + a*d)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2067 vs. $2(99) = 198$.

time = 0.10, size = 2068, normalized size = 17.38

method	result
risch	$\frac{(3b^2x^4d^2-5ab\,d^2x^2+11b^2cd\,x^2+15a^2d^2-35abcd+23b^2c^2)\sqrt{d\,x^2+c}}{15b^3}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-}}{}\right)}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-
a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-
1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d
^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-
1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^
(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(
3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^
(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-
b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*
b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x
-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x-1/b*(-a*b)^(1/2))))))+1/2/b*(1/5*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-d*(-a*b)^(1/2)/b*(1/8*(
2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*
c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
```

```
(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1
/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x+1/b*(-a*b)^(1
/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^
(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*
b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*
(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^
(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*l
n((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2)
)^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.36, size = 405, normalized size = 3.40

$$\left[\frac{15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(3\,b^2d^2x^4+23\,b^2c^2-35\,abcd+15\,a^2d^2+(11\,b^2cd-5\,abd^2)x^2)\sqrt{dx^2+c}}{60\,b^3},\ -\frac{15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\,(3\,b^2d^2x^4+23\,b^2c^2-35\,abcd+15\,a^2d^2+(11\,b^2cd-5\,abd^2)x^2)\sqrt{dx^2+c}}{30\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(
b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) + 4*(3*b^2*d^2*x^4 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d
^2 + (11*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/b^3, -1/30*(15*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c -
 a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)
```

```
*x^2)) - 2*(3*b^2*d^2*x^4 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*
c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/b^3]
```

Sympy [A]
time = 23.58, size = 117, normalized size = 0.98

$$\frac{(c+dx^2)^{\frac{5}{2}}}{5b}+\frac{(c+dx^2)^{\frac{3}{2}}(-ad+bc)}{3b^2}+\frac{\sqrt{c+dx^2}\,(a^2d^2-2abcd+b^2c^2)}{b^3}-\frac{(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^4\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**(5/2)/(5*b) + (c + d*x**2)**(3/2)*(-a*d + b*c)/(3*b**2) + sqr
t(c + d*x**2)*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/b**3 - (a*d - b*c)**3*ata
n(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 0.55, size = 184, normalized size = 1.55

$$\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^3}+\frac{3\,(dx^2+c)^{\frac{5}{2}}b^4+5\,(dx^2+c)^{\frac{3}{2}}b^4c+15\,\sqrt{dx^2+c}\,b^4c^2-5\,(dx^2+c)^{\frac{3}{2}}ab^3d-30\,\sqrt{dx^2+c}\,ab^3cd+15\,\sqrt{dx^2+c}\,a^2b^2d^2}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*
b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/15*(3*(d*x^2 + c)^(5
/2)*b^4 + 5*(d*x^2 + c)^(3/2)*b^4*c + 15*sqrt(d*x^2 + c)*b^4*c^2 - 5*(d*x^2
 + c)^(3/2)*a*b^3*d - 30*sqrt(d*x^2 + c)*a*b^3*c*d + 15*sqrt(d*x^2 + c)*a^2
*b^2*d^2)/b^5
```

Mupad [B]
time = 0.34, size = 137, normalized size = 1.15

$$\frac{{\left(d\,x^2+c\right)}^{5/2}}{5\,b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^{5/2}}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^{5/2}}{b^{7/2}}-\frac{{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d-b\,c\right)}{3\,b^2}+\frac{\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(5/2)/(5*b) - (atan((b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(5/2
))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(a*d - b*c)^(5/2))/
b^(7/2) - ((c + d*x^2)^(3/2)*(a*d - b*c))/(3*b^2) + ((c + d*x^2)^(1/2)*(a*d
 - b*c)^2)/b^3
```

3.698 $\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=156

$$\frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx(c+dx^2)^{3/2}}{4b}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^3}+\frac{\sqrt{d}\,(15b^2c^2-20abcd+8a}{8}$$

```
[Out] 1/4*d*x*(d*x^2+c)^(3/2)/b+(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2
)/(d*x^2+c)^(1/2))/b^3/a^(1/2)+1/8*(8*a^2*d^2-20*a*b*c*d+15*b^2*c^2)*arctan
h(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)/b^3+1/8*d*(-4*a*d+7*b*c)*x*(d*x^2+c)^(
1/2)/b^2
```

Rubi [A]
time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {427, 542, 537, 223, 212, 385, 211}

$$\frac{\sqrt{d}\,(8a^2d^2-20abcd+15b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^3}+\frac{(bc-ad)^{5/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^3}+\frac{dx\sqrt{c+dx^2}\,(7bc-4ad)}{8b^2}+\frac{dx(c+dx^2)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

```
[Out] (d*(7*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x*(c + d*x^2)^(3/2))/(4*
b) + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*b^3) + (Sqrt[d]*(15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*ArcTanh[
(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{dx(c+dx^2)^{3/2}}{4b} + \frac{\int \frac{\sqrt{c+dx^2}\,(c(4bc-ad)+d(7bc-4ad)x^2)}{a+bx^2}\,dx}{4b}$$

$$= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx(c+dx^2)^{3/2}}{4b} + \frac{\int \frac{c(8b^2c^2-9abcd+4a^2d^2)+d(15b^2c^2-20abcd+8a^2d^2)x^{}}{(a+bx^2)\sqrt{c+dx^2}}}{8b^2}$$

$$= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx(c+dx^2)^{3/2}}{4b} + \frac{(bc-ad)^3\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^3} + \frac{(d(15}{}$$

$$= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx(c+dx^2)^{3/2}}{4b} + \frac{(bc-ad)^3\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{}{\sqrt{c}}\right.}{b^3}$$

$$= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx(c+dx^2)^{3/2}}{4b} + \frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^3} + -$$

Mathematica [A]
time = 0.35, size = 160, normalized size = 1.03

$$\frac{bdx\sqrt{c+dx^2}\,(9bc-4ad+2bdx^2) - \frac{8(bc-ad)^{5/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{\sqrt{a}} - \sqrt{d}\,(15b^2c^2-20abcd+8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

```
[Out] (b*d*x*Sqrt[c + d*x^2]*(9*b*c - 4*a*d + 2*b*d*x^2) - (8*(b*c - a*d)^(5/2)*A
rcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a
*d])])/Sqrt[a] - Sqrt[d]*(15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*Log[-(Sqrt[d
]*x) + Sqrt[c + d*x^2]])/(8*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2073 vs. 2(130) = 260.
time = 0.13, size = 2074, normalized size = 13.29

method	result

risch	$-\frac{dx(-2bd\,x^2+4ad-9bc)\sqrt{d\,x^2+c}}{8b^2}+\frac{d^{\frac{5}{2}}\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)a^2}{b^3}-\frac{5d^{\frac{3}{2}}\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)ac}{2b^2}+\frac{15\sqrt{c}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-a*b)^(1/2)*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d*(x-1/b*(-a*b)^(1
/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1
/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^
2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d
-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(
1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-
1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d
^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-
1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*
(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))))-1/2/(-a*b)^(1/2)*(1/5*(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-d
*(-a*b)^(1/2)/b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+
1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3
/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*
(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d
*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1
/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
```

```
*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/
2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1
/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^
(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+
(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^
2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2))))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/(b*x^2 + a), x)
```

Fricas [**A**]
time = 2.56, size = 931, normalized size = 5.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/16*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(
d*x^2 + c)*sqrt(d)*x - c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c -
a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqr
t(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d^2*x^3 + (9*b^2
*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, -1/8*((15*b^2*c^2 - 20*a*b*c*d +
 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 2*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a
^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) - (2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, 1/16
*(8*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c -
2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3
+ (b*c^2 - a*c*d)*x)) + (15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-
2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 + (9*b^2*c*d
```

```
- 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, -1/8*((15*b^2*c^2 - 20*a*b*c*d + 8*a^
2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 4*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d
*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) -
(2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(a + b*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x^2+c)^{5/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(a + b*x^2),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(a + b*x^2), x)
```

3.699 $\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{d(c+dx^2)^{3/2}}{3b}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}$$

```
[Out] 1/3*d*(d*x^2+c)^(3/2)/b-c^(5/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a+(-a*d+b*
c)^(5/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(5/2)+d*(-a*
d+2*b*c)*(d*x^2+c)^(1/2)/b^2
```

Rubi [A]
time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 86, 159, 162, 65, 214}

$$\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}+\frac{d\sqrt{c+dx^2}\,(2bc-ad)}{b^2}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d(c+dx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (d*(c + d*x^2)^(3/2))/(3*b) - (c^(5
/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
```

```
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x(a+bx)}\,dx,x,x^2\right)\\
&= \frac{d(c+dx^2)^{3/2}}{3b} + \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\,(bc^2+d(2bc-ad)x)}{x(a+bx)}\,dx,x,x^2\right)}{2b}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{d(c+dx^2)^{3/2}}{3b} + \frac{\text{Subst}\left(\int \frac{\frac{b^2c^3}{2}+\frac{1}{2}d(3b^2c^2-3abcd+a^2d^2)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{b^2}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{d(c+dx^2)^{3/2}}{3b} + \frac{c^3\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2a} - \frac{(bc-ad)}{}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{d(c+dx^2)^{3/2}}{3b} + \frac{c^3\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{ad} - \frac{(bc-}{}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{d(c+dx^2)^{3/2}}{3b} - \frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a} + \frac{(bc-ad)^{5/2}\tan}{}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 114, normalized size = 0.92

$$
\frac{d\sqrt{c+dx^2}\,(7bc-3ad+bdx^2)}{3b^2} + \frac{(-bc+ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{ab^{5/2}} - \frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*Sqrt[c + d*x^2]*(7*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((-(b*c) + a*d)^(5/
2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(a*b^(5/2)) - (c^(
5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2137 vs. $2(102) = 204$.
time = 0.10, size = 2138, normalized size = 17.24

method	result	size
default	Expression too large to display	2138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(
-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x
-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*
d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x
-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)
^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^
(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)
^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d
-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a
*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(
x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2))))))-1/2/a*(1/5*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-d*(-a*b)^(1/2)/b*(1/8*
(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b
*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+
1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x+1/b*(-a*b)^(
1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)
^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8
*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)
^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*
ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/
b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))))+1/a*(1/5*
(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2
*c^(1/2)*(d*x^2+c)^(1/2))/x))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x), x)
```

Fricas [A]
time = 2.99, size = 837, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="fricas")
[Out] [1/12*(6*b^2*c^(5/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^
2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) + 4*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b
^2), 1/12*(12*b^2*sqrt(-c)*c^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 3*(b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2
*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) + 4*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2), 1/6*
(3*b^2*c^(5/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 3*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b
*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*
d^2)*x^2)) + 2*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^
2), 1/6*(6*b^2*sqrt(-c)*c^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 3*(b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a
*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x
^2)) + 2*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2)]
```

Sympy [A]
time = 22.29, size = 119, normalized size = 0.96

$$\frac{d\left(c+dx^2\right)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx^2}\left(-ad^2+2bcd\right)}{b^2}+\frac{c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{\left(ad-bc\right)^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x/(b*x**2+a),x)
```

```
[Out] d*(c + d*x**2)**(3/2)/(3*b) + sqrt(c + d*x**2)*(-a*d**2 + 2*b*c*d)/b**2 + c
**3*atan(sqrt(c + d*x**2)/sqrt(-c))/(a*sqrt(-c)) + (a*d - b*c)**3*atan(sqrt
(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*b**3*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 0.61, size = 163, normalized size = 1.31

$$\frac{c^3 \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,ab^2} + \frac{(dx^2+c)^{\frac{3}{2}}b^2d + 6\sqrt{dx^2+c}\,b^2cd - 3\sqrt{dx^2+c}\,abd^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)) - (b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))
/(sqrt(-b^2*c + a*b*d)*a*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d + 6*sqrt(d*x^2
 + c)*b^2*c*d - 3*sqrt(d*x^2 + c)*a*b*d^2)/b^3
```

Mupad [B]
time = 0.52, size = 2094, normalized size = 16.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x*(a + b*x^2)),x)
```

```
[Out] (atan((((c^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^
5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 -
6*a^5*b*c*d^7))/b^3 + (((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c
^2*d^4)/b^3 + ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5)^(1
/2))/(a*b^3))*(c^5)^(1/2))/(2*a))*1i)/(2*a) + ((c^5)^(1/2)*((2*(c + d*x^2)^
(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*
a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 - (((4*a^4*b^3*c
*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^3 - 8*a^
2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5)^(1/2))/(a*b^3))*(c^5)^(1/2))/(2*a))*1i
)/(2*a))/((2*(a^5*c^3*d^8 - 3*b^5*c^8*d^3 + 12*a*b^4*c^7*d^4 - 6*a^4*b*c^4*
d^7 - 19*a^2*b^3*c^6*d^5 + 15*a^3*b^2*c^5*d^6))/b^3 - ((c^5)^(1/2)*((2*(c +
 d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d
^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 + (((4*a
^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 + ((4*a^3*b^5*d^
3 - 8*a^2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5)^(1/2))/(a*b^3))*(c^5)^(1/2))/(
2*a)))/(2*a) + ((c^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2
```

- 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 - (((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5)^(1/2))/(a*b^3))*(c^5)^(1/2))/(2*a)))/(2*a)))*(c^5)^(1/2)*1i)/a + (d*(c + d*x^2)^(3/2))/(3*b) + (atan((((-b^5*(a*d - b*c)^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 + ((-b^5*(a*d - b*c)^5)^(1/2)*((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 + ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2))/(a*b^8)))/(2*a*b^5))*1i)/(2*a*b^5) + ((-b^5*(a*d - b*c)^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 - ((-b^5*(a*d - b*c)^5)^(1/2)*((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2))/(a*b^8)))/(2*a*b^5))*1i)/(2*a*b^5))/((2*(a^5*c^3*d^8 - 3*b^5*c^8*d^3 + 12*a*b^4*c^7*d^4 - 6*a^4*b*c^4*d^7 - 19*a^2*b^3*c^6*d^5 + 15*a^3*b^2*c^5*d^6))/b^3 - ((-b^5*(a*d - b*c)^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 + ((-b^5*(a*d - b*c)^5)^(1/2)*((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 + ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5) + ((-b^5*(a*d - b*c)^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 - ((-b^5*(a*d - b*c)^5)^(1/2)*((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5)))*(-b^5*(a*d - b*c)^5)^(1/2)*1i)/(a*b^5) - (d*(c + d*x^2)^(1/2)*(a*d - 2*b*c))/b^2

3.700 $\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=145

$$\frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab}-\frac{c(c+dx^2)^{3/2}}{ax}-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\tanh^{-1}\left(\cdots\right.}{2b^2}$$

```
[Out] -c*(d*x^2+c)^(3/2)/a/x-(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(
d*x^2+c)^(1/2))/a^(3/2)/b^2+1/2*d^(3/2)*(-2*a*d+5*b*c)*arctanh(x*d^(1/2)/(d
*x^2+c)^(1/2))/b^2+1/2*d*(a*d+2*b*c)*x*(d*x^2+c)^(1/2)/a/b
```

Rubi [A]
time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {485, 542, 537, 223, 212, 385, 211}

$$-\frac{(bc-ad)^{5/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}\,(ad+2bc)}{2ab}-\frac{c(c+dx^2)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] (d*(2*b*c + a*d)*x*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))/(a*x) -
 ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(a^(3/2)*b^2) + (d^(3/2)*(5*b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx = -\frac{c(c+dx^2)^{3/2}}{ax} + \frac{\int \frac{\sqrt{c+dx^2}\,(-c(bc-4ad)+d(2bc+ad)x^2)}{a+bx^2}\,dx}{a}$$

$$= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{ax} + \frac{\int \frac{-c(2b^2c^2-6abcd+a^2d^2)+ad^2(5bc-2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2ab}$$

$$= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{ax} + \frac{(d^2(5bc-2ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2} - \frac{(bc-a}{}$$

$$= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{ax} + \frac{(d^2(5bc-2ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{}{\sqrt{c+}}\right.}{2b^2}$$

$$= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{ax} - \frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b^2} + \frac{d^{3}}{}$$

Mathematica [A]
time = 0.34, size = 148, normalized size = 1.02

$$\frac{\frac{b\sqrt{c+dx^2}\,(-2bc^2+ad^2x^2)}{ax} + \frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{a^{3/2}} + d^{3/2}(-5bc+2ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] ((b*Sqrt[c + d*x^2]*(-2*b*c^2 + a*d^2*x^2))/(a*x) + (2*(b*c - a*d)^(5/2)*Ar
cTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*
d])])/a^(3/2) + d^(3/2)*(-5*b*c + 2*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]
])/(2*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2176 vs. 2(121) = 242.
time = 0.13, size = 2177, normalized size = 15.01

method	result

risch	$\frac{\sqrt{d\,x^2+c}\,(a\,d^2x^2-2b\,c^2)}{2bax}-\frac{a\,d^{\frac{5}{2}}\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b^2}+\frac{5d^{\frac{3}{2}}\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)c}{2b}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\cdots}{\cdots}\right)}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a/(-a*b)^(1/2)*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d*(x-1/b*(-a*
b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)
/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1
/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d
*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2))
)/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*
d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/
b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)
/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))))+1/2*b/a/(-a*b)^(1/2)*(1/5*(d
*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(5/2)-d*(-a*b)^(1/2)/b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)
/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(1
/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/
2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(
1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-(a*d-
b*c)/b*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
```

```
))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(
-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*
(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))
/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2))))))+1/a*(-1/c/x*(d*x^2+c)^(7/2)+6*d/c*(1/6*x*(d*x^2+c)^(5/
2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*
ln(x*d^(1/2)+(d*x^2+c)^(1/2))))))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^2), x)
```

<u>Fricas</u> [A]
time = 2.03, size = 887, normalized size = 6.12

$$\left[-\frac{(5abcd-2a^2d^2)\sqrt{d}\,x\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}\,x-c\right)-(b^2c^2-2abcd+a^2d^2)x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4(a^2cx-(abc-2a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2abx^2+a^2}\right)-2(abd^2x^2-2b^2c^2)\sqrt{dx^2+c}}{4ab^2x}, \ldots\right]$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((5*a*b*c*d - 2*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*s
qrt(d)*x - c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c - a*d)/a)*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*
d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(a*b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^
2 + c))/(a*b^2*x), -1/4*(2*(5*a*b*c*d - 2*a^2*d^2)*sqrt(-d)*x*arctan(sqrt(-
d)*x/sqrt(d*x^2 + c)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c - a*d)
/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(
b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(a*b*d^2*x^2 - 2*b^2*c^2)*s
qrt(d*x^2 + c))/(a*b^2*x), -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt((
b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*
```

```
c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + (5*a*b*c*d - 2*a^2
*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(a*b*d^
2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x), -1/2*((5*a*b*c*d - 2*a^2*d^2
)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - (a*
b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{x^2\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**2*(a + b*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^{5/2}}{x^2\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)), x)
```

3.701 $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=144

$$\frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab}-\frac{c(c+dx^2)^{3/2}}{2ax^2}+\frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}}$$

```
[Out] -1/2*c*(d*x^2+c)^(3/2)/a/x^2+1/2*c^(3/2)*(-5*a*d+2*b*c)*arctanh((d*x^2+c)^(
1/2)/c^(1/2))/a^2-(-a*d+b*c)^(5/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*
c)^(1/2))/a^2/b^(3/2)+1/2*d*(2*a*d+b*c)*(d*x^2+c)^(1/2)/a/b
```

Rubi [A]
time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 100, 159, 162, 65, 214}

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}}+\frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}+\frac{d\sqrt{c+dx^2}\,(2ad+bc)}{2ab}-\frac{c(c+dx^2)^{3/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))/(2*a*x^2)
 + (c^(3/2)*(2*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2) - ((b
*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2*b^
(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x^2(a+bx)}\,dx,x,x^2\right)$$

$$= -\frac{c(c+dx^2)^{3/2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(\frac{1}{2}c(2bc-5ad)-\frac{1}{2}d(bc+2ad)x\right)}{x(a+bx)}\,dx,x,x^2\right)}{2a}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{4}bc^2(2bc-5ad)+\frac{1}{4}d(b^2c^2-6abcd+2a^2d^2)x}{x(a+bx)\sqrt{c+dx}}\,dx\right.}{ab}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{2ax^2} - \frac{(c^2(2bc-5ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x\right.}{4a^2}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{2ax^2} - \frac{(c^2(2bc-5ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+}\right.}{2a^2d}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{2ax^2} + \frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2} - \frac{(bc}{}$$

Mathematica [A]
time = 0.24, size = 125, normalized size = 0.87

$$\frac{\frac{a\sqrt{c+dx^2}\,(-bc^2+2ad^2x^2)}{bx^2} - \frac{2(-bc+ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}} + c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(-(b*c^2) + 2*a*d^2*x^2))/(b*x^2) - (2*(-(b*c) + a*d)^(
5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(3/2) + c^(3/2
)*(2*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2235 vs. 2(118) = 236.
time = 0.14, size = 2236, normalized size = 15.53

method	result

risch	$-\frac{c^{2}\sqrt{d\,x^{2}+c}}{2a\,x^{2}}+\frac{d^{2}\sqrt{d\,x^{2}+c}}{b}-\frac{5c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)d}{2a}+\frac{c^{\frac{5}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)b}{a^{2}}+\frac{a\ln\left(\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/a^2*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*
d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d
*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d
*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a
*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)
/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a
*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(
a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*
(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2))))))+1/2*b/a^2*(1/5*(d*(x+1/b*(-a*b)^(1/2))^2
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-d*(-a*b)^(1/2)/
b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d
*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b
)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/
b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d
```

```
*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+
1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b
*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(
1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*
b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*
d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))))+1/
a*(-1/2/c/x^2*(d*x^2+c)^(7/2)+5/2*d/c*(1/5*(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)
^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))))
-b/a^2*(1/5*(d*x^2+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/
2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^3), x)
```

Fricas [A]
time = 3.24, size = 891, normalized size = 6.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b*c - a*d)/b)*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(
b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) - (2*b^2*c^2 - 5*a*b*c*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2
 + c))/(a^2*b*x^2), -1/4*(2*(2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c)*x^2*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b*c - a*
d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3
*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c -
 a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d
*x^2 + c))/(a^2*b*x^2), -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt(-(
```

```
b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (2*b^2*c^2 - 5*a*b*c*d
)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a^
2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2), -1/2*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*
sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2))
 + (2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) -
(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{x^3\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**3*(a + b*x**2)), x)
```

Giac [A]
time = 0.65, size = 158, normalized size = 1.10

$$\frac{\sqrt{dx^2+c}\,d^2}{b}-\frac{\sqrt{dx^2+c}\,c^2}{2\,ax^2}-\frac{(2\,bc^3-5\,ac^2d)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}+\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(d*x^2 + c)*d^2/b - 1/2*sqrt(d*x^2 + c)*c^2/(a*x^2) - 1/2*(2*b*c^3 - 5*
a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)) + (b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*b)
```

Mupad [B]
time = 0.74, size = 1428, normalized size = 9.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x)
```

```
[Out] (d^2*(c + d*x^2)^(1/2))/b + (atan((a^3*d^9*(c + d*x^2)^(1/2)*(c^3)^(1/2)*5i
)/(5*a^3*c^2*d^9 - (395*b^3*c^5*d^6)/4 + 87*a*b^2*c^4*d^7 - 32*a^2*b*c^3*d^
8 + (185*b^4*c^6*d^5)/(4*a) - (15*b^5*c^7*d^4)/(2*a^2)) + (a^2*c*d^8*(c + d
*x^2)^(1/2)*(c^3)^(1/2)*32i)/(32*a^2*c^3*d^8 + (395*b^2*c^5*d^6)/4 - (185*b
```

^3*c^6*d^5)/(4*a) - (5*a^3*c^2*d^9)/b + (15*b^4*c^7*d^4)/(2*a^2) - 87*a*b*c ^4*d^7) + (b^2*c^3*d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2)*395i)/(4*(32*a^2*c^3*d ^8 + (395*b^2*c^5*d^6)/4 - (185*b^3*c^6*d^5)/(4*a) - (5*a^3*c^2*d^9)/b + (1 5*b^4*c^7*d^4)/(2*a^2) - 87*a*b*c^4*d^7)) - (b^3*c^4*d^5*(c + d*x^2)^(1/2)* (c^3)^(1/2)*185i)/(4*(32*a^3*c^3*d^8 - (185*b^3*c^6*d^5)/4 + (395*a*b^2*c^5 *d^6)/4 - 87*a^2*b*c^4*d^7 + (15*b^4*c^7*d^4)/(2*a) - (5*a^4*c^2*d^9)/b)) + (b^4*c^5*d^4*(c + d*x^2)^(1/2)*(c^3)^(1/2)*15i)/(2*(32*a^4*c^3*d^8 + (15*b ^4*c^7*d^4)/2 - (185*a*b^3*c^6*d^5)/4 - 87*a^3*b*c^4*d^7 + (395*a^2*b^2*c^5 *d^6)/4 - (5*a^5*c^2*d^9)/b)) - (a*b*c^2*d^7*(c + d*x^2)^(1/2)*(c^3)^(1/2)* 87i)/(32*a^2*c^3*d^8 + (395*b^2*c^5*d^6)/4 - (185*b^3*c^6*d^5)/(4*a) - (5*a ^3*c^2*d^9)/b + (15*b^4*c^7*d^4)/(2*a^2) - 87*a*b*c^4*d^7))*(5*a*d - 2*b*c) *(c^3)^(1/2)*1i)/(2*a^2) - (atan((c^3*d^5*(c + d*x^2)^(1/2)*(b^8*c^5 - a^5* b^3*d^5 + 5*a^4*b^4*c*d^4 + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 - 5*a*b ^7*c^4*d)^(1/2)*20i)/((185*a*b^3*c^5*d^6)/2 - (85*b^4*c^6*d^5)/2 - 16*a^4*c ^2*d^9 + 56*a^3*b*c^3*d^8 + (2*a^5*c*d^10)/b - (199*a^2*b^2*c^4*d^7)/2 + (1 5*b^5*c^7*d^4)/(2*a)) - (c^2*d^6*(c + d*x^2)^(1/2)*(b^8*c^5 - a^5*b^3*d^5 + 5*a^4*b^4*c*d^4 + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 - 5*a*b^7*c^4*d) ^(1/2)*10i)/(2*a^4*c*d^10 + (185*b^4*c^5*d^6)/2 - (199*a*b^3*c^4*d^7)/2 - 1 6*a^3*b*c^2*d^9 + 56*a^2*b^2*c^3*d^8 - (85*b^5*c^6*d^5)/(2*a) + (15*b^6*c^7 *d^4)/(2*a^2)) - (c^4*d^4*(c + d*x^2)^(1/2)*(b^8*c^5 - a^5*b^3*d^5 + 5*a^4* b^4*c*d^4 + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 - 5*a*b^7*c^4*d)^(1/2)* 15i)/(2*(56*a^4*c^3*d^8 + (15*b^4*c^7*d^4)/2 - (85*a*b^3*c^6*d^5)/2 - (199* a^3*b*c^4*d^7)/2 + (2*a^6*c*d^10)/b^2 + (185*a^2*b^2*c^5*d^6)/2 - (16*a^5*c ^2*d^9)/b)) + (a*c*d^7*(c + d*x^2)^(1/2)*(b^8*c^5 - a^5*b^3*d^5 + 5*a^4*b^4 *c*d^4 + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 - 5*a*b^7*c^4*d)^(1/2)*2i) /((185*b^5*c^5*d^6)/2 - (199*a*b^4*c^4*d^7)/2 + 56*a^2*b^3*c^3*d^8 - 16*a^3 *b^2*c^2*d^9 - (85*b^6*c^6*d^5)/(2*a) + (15*b^7*c^7*d^4)/(2*a^2) + 2*a^4*b* c*d^10))*(-b^3*(a*d - b*c)^5)^(1/2)*1i)/(a^2*b^3) - (b*c^2*d*(c + d*x^2)^(1 /2))/(2*a*(b*(c + d*x^2) - b*c))

3.702 $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=130

$$\frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x}-\frac{c(c+dx^2)^{3/2}}{3ax^3}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

```
[Out] -1/3*c*(d*x^2+c)^(3/2)/a/x^3+(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(
1/2)/(d*x^2+c)^(1/2))/a^(5/2)/b+d^(5/2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/
b+c*(-2*a*d+b*c)*(d*x^2+c)^(1/2)/a^2/x
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {485, 594, 537, 223, 212, 385, 211}

$$\frac{(bc-ad)^{5/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{c\sqrt{c+dx^2}\,(bc-2ad)}{a^2x}-\frac{c(c+dx^2)^{3/2}}{3ax^3}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (c*(b*c - 2*a*d)*Sqrt[c + d*x^2])/(a^2*x) - (c*(c + d*x^2)^(3/2))/(3*a*x^3)
 + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/(a^(5/2)*b) + (d^(5/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx = -\frac{c(c+dx^2)^{3/2}}{3ax^3} + \frac{\int \frac{\sqrt{c+dx^2}\,(-3c(bc-2ad)+3ad^2x^2)}{x^2(a+bx^2)}\,dx}{3a}$$

$$= \frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x} - \frac{c(c+dx^2)^{3/2}}{3ax^3} + \frac{\int \frac{3c(b^2c^2-3abcd+3a^2d^2)+3a^2d^3x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3a^2}$$

$$= \frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x} - \frac{c(c+dx^2)^{3/2}}{3ax^3} + \frac{d^3\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b} + \frac{(bc-ad)^3\int \frac{1}{(a+bx^2)\sqrt{c+}}}{a^2b}$$

$$= \frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x} - \frac{c(c+dx^2)^{3/2}}{3ax^3} + \frac{d^3\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b} + \frac{(bc-}{}$$

$$= \frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x} - \frac{c(c+dx^2)^{3/2}}{3ax^3} + \frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b} + \frac{d^{5/2}\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.38, size = 145, normalized size = 1.12

$$-\frac{c\sqrt{c+dx^2}\,(-3bcx^2+a(c+7dx^2))}{3a^2x^3} - \frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}b} - \frac{d^{5/2}\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*(c*Sqrt[c + d*x^2]*(-3*b*c*x^2 + a*(c + 7*d*x^2)))/(a^2*x^3) - ((b*c -
 a*d)^(5/2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(a^(5/2)*b) - (d^(5/2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^
2]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2301 vs. 2(108) = 216.
time = 0.13, size = 2302, normalized size = 17.71

method	result

risch	$-\frac{c\sqrt{d\,x^2+c}\,(7ad\,x^2-3c\,x^2b+ac)}{3a^2x^3}+\frac{d^{\frac{5}{2}}\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2}{}\right)}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^2/(-a*b)^(1/2)*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d*(x-1/b*(
-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a
/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)
^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-
4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/
2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*
(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*
c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2
)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1
/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))))-1/2*b^2/a^2/(-a*b)^(1/2)*
(1/5*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(5/2)-d*(-a*b)^(1/2)/b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(
1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-
a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b
)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))
)-(a*d-b*c)/b*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
```

```
b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2)
)-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*
ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)
/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^
(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/
b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x+1/b*(-a*b)^(1/2))))))+1/a*(-1/3/c/x^3*(d*x^2+c)^(7/2)+4/3*d/c*(-1/c/x
*(d*x^2+c)^(7/2)+6*d/c*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*(d*x^2+c)^(3/2)+
3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))
))-b/a^2*(-1/c/x*(d*x^2+c)^(7/2)+6*d/c*(1/6*x*(d*x^2+c)^(5/2)+5/6*c*(1/4*x*
(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*
x^2+c)^(1/2))))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^4), x)
```

Fricas [**A**]
time = 2.38, size = 901, normalized size = 6.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a^2*d^(5/2)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a
^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*
x^2 + c))/(a^2*b*x^3), -1/12*(12*a^2*sqrt(-d)*d^2*x^3*arctan(sqrt(-d)*x/sqr
t(d*x^2 + c)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c - a*d)/a)*
log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c
- a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c
```

```
*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3), 1/6*(3*a^2*d^(5/2)*x^3*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*
sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sq
rt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - 2*(a*b*c^2 -
 (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3), -1/6*(6*a^2*sqr
t(-d)*d^2*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 3*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt
(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x))
+ 2*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{x^4(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**4*(a + b*x**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^{5/2}}{x^4\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)), x)
```

3.703 $\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/3*(d*x^2+c)^(3/2)/b/d^2-a^2*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1
/2))/b^(5/2)/(-a*d+b*c)^(1/2)-(a*d+b*c)*(d*x^2+c)^(1/2)/b^2/d^2
```

Rubi [A]
time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 90, 65, 214}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}\,(ad+bc)}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(((b*c + a*d)*Sqrt[c + d*x^2])/(b^2*d^2)) + (c + d*x^2)^(3/2)/(3*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(5/2)*Sqrt[b*c
- a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx,x,x^2\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 88, normalized size = 0.88

$$
\frac{\sqrt{c+dx^2}\,(-2bc-3ad+bdx^2)}{3b^2d^2}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{5/2}\sqrt{-bc+ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
[Out] (Sqrt[c + d*x^2]*(-2*b*c - 3*a*d + b*d*x^2))/(3*b^2*d^2) + (a^2*ArcTan[(Sqr
t[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(b^(5/2)*Sqrt[-(b*c) + a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(84) = 168.
time = 0.10, size = 361, normalized size = 3.61

method	result
risch	$-\frac{(-bd\,x^2+3ad+2bc)\sqrt{d\,x^2+c}}{3d^2b^2}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2b^3\sqrt{-\frac{ad-bc}{b}}}$
default	$\frac{\frac{x^2\sqrt{d\,x^2+c}}{3d}-\frac{2c\sqrt{d\,x^2+c}}{3d^2}}{b}-\frac{a\sqrt{d\,x^2+c}}{b^2d}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d}}{x-}\right)}{2b^3\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3*x^2/d*(d*x^2+c)^(1/2)-2/3*c/d^2*(d*x^2+c)^(1/2))-a/b^2/d*(d*x^2+c)
^(1/2)-1/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2)))-1/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(84) = 168$.

time = 1.10, size = 390, normalized size = 3.90

$$\left[\frac{3\,\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^2)\sqrt{dx^2+c}}{12\,(b^4cd^2-ab^3d^3)},-\frac{3\,\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)+2\,(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^2)\sqrt{dx^2+c}}{6\,(b^4cd^2-ab^3d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c
*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sq
rt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^3*
c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))
/(b^4*c*d^2 - a*b^3*d^3), -1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(-1/2*
(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b
*c*d + (b^2*c*d - a*b*d^2)*x^2)) + 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 -
 (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^2)\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.81, size = 105, normalized size = 1.05

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}+\frac{(dx^2+c)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^2+c}\,b^2cd^4-3\,\sqrt{dx^2+c}\,abd^5}{3\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^
2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^2 + c)*b^2*c*d^4 - 3*sqrt(
d*x^2 + c)*a*b*d^5)/(b^3*d^6)
```

Mupad [B]
time = 0.39, size = 100, normalized size = 1.00

$$\frac{{\left(d\,x^2+c\right)}^{3/2}}{3\,b\,d^2}-\left(\frac{2\,c}{b\,d^2}+\frac{a\,d^3-b\,c\,d^2}{b^2\,d^4}\right)\,\sqrt{d\,x^2+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (c + d*x^2)^(3/2)/(3*b*d^2) - ((2*c)/(b*d^2) + (a*d^3 - b*c*d^2)/(b^2*d^4))
*(c + d*x^2)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2
)))/(b^(5/2)*(a*d - b*c)^(1/2))
```

3.704 $\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{c+dx^2}}{bd}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}$$

```
[Out] a*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(1/2
)+(d*x^2+c)^(1/2)/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 81, 65, 214}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d
]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{c+dx^2}}{bd} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b}$$
$$= \frac{\sqrt{c+dx^2}}{bd} - \frac{a\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{bd}$$
$$= \frac{\sqrt{c+dx^2}}{bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.08, size = 69, normalized size = 1.01

$$\frac{\sqrt{c+dx^2}}{bd} - \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a
*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(56) = 112.
time = 0.10, size = 318, normalized size = 4.68

method	result

default	$\frac{\sqrt{d\,x^2+c}}{bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\,(x}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right.}{2b^2\sqrt{-\frac{ad-bc}{b}}}$
risch	$\frac{\sqrt{d\,x^2+c}}{bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\,(x}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right.}{2b^2\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)/b/d+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^
(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b
*(-a*b)^(1/2)))+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a
*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.
time = 1.16, size = 306, normalized size = 4.50

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(b^2c-abd)\sqrt{dx^2+c}}{4\,(b^3cd-ab^2d^2)},\ \frac{\sqrt{-b^2c+abd}\,ad\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)+2\,(b^2c-abd)\sqrt{dx^2+c}}{2\,(b^3cd-ab^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^
2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*
c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b^2*c - a*b*d
)*sqrt(d*x^2 + c))/(b^3*c*d - a*b^2*d^2), 1/2*(sqrt(-b^2*c + a*b*d)*a*d*arc
tan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*
c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + 2*(b^2*c - a*b*d)*sqrt(d*x^2 +
c))/(b^3*c*d - a*b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.68, size = 64, normalized size = 0.94

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^2+c}}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b) - sqrt(d*x^2 + c)/b)/d
```

Mupad [B]
time = 0.37, size = 57, normalized size = 0.84

$$\frac{\sqrt{d\,x^2+c}}{b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (c + d*x^2)^(1/2)/(b*d) - (a*atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(
1/2)))/(b^(3/2)*(a*d - b*c)^(1/2))
```

3.705 $\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a*
d]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]]/(Sqrt[b]*Sqrt[-(b*c) +
 a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(39) = 78.
time = 0.09, size = 300, normalized size = 6.12

method	result
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad}{}}}{x-\frac{\sqrt{-ab}}{b}}\right.}{2b\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
```

```
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2/b/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.
time = 1.25, size = 231, normalized size = 4.71

$$\left[\frac{\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,\sqrt{b^2c-abd}}, -\frac{\sqrt{-b^2c+abd}\,\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)}{2\,(b^2c-abd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*
a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c
))/(b^2*x^4 + 2*a*b*x^2 + a^2))/sqrt(b^2*c - a*b*d), -1/2*sqrt(-b^2*c + a*b
*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c
)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2))/(b^2*c - a*b*d)]
```

Sympy [A]
time = 2.57, size = 36, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
[Out] atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 0.69, size = 39, normalized size = 0.80

$$\frac{\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B]
time = 0.37, size = 39, normalized size = 0.80

$$\frac{\operatorname{atan}\left(\frac{b\,\sqrt{d\,x^2+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
[Out] atan((b*(c + d*x^2)^(1/2))/(a*b*d - b^2*c)^(1/2))/(a*b*d - b^2*c)^(1/2)
```

3.706 $\int \frac{1}{x(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}$$

```
[Out] -arctanh((d*x^2+c)^(1/2)/c^(1/2))/a/c^(1/2)+arctanh(b^(1/2)*(d*x^2+c)^(1/2)
/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 88, 65, 214}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*Sqrt[c])) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2a} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2a}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{ad} - \frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.11, size = 78, normalized size = 0.98

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*
c) + a*d] + ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/Sqrt[c])/a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(64) = 128.
time = 0.10, size = 331, normalized size = 4.14

method	result

default	$\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad}{}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2a\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2/a/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/a/c^(1/2)*ln((
2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x), x)
```

<u>Fricas</u> [A]
time = 1.80, size = 603, normalized size = 7.54

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(2b^2c^2-3abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{4ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(2b^2c^2-3abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+4\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{4ac}, -\frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2(bdx^2+bc)}\right)-\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{2ac}, -\frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2(bdx^2+bc)}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{2ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*
d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 +
(b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) + 2*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2))/(a*c), 1/4*(c*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d
```

```
+ a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/
(a*c), -1/2*(c*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - sqrt(c)*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a*c), -1/2*(c*sqrt(-b/(b*c - a*d))*a
rctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d
*x^2 + b*c)) - 2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(a*c)]
```

Sympy [A]
time = 4.89, size = 63, normalized size = 0.79

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*sqrt((a*d - b*c)/b)) + atan(
sqrt(c + d*x**2)/sqrt(-c))/(a*sqrt(-c))
```

Giac [A]
time = 1.02, size = 70, normalized size = 0.88

$$-\frac{b\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a)
+ arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B]
time = 0.50, size = 651, normalized size = 8.14

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a\,\sqrt{c}}-\frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{a^2\,d-a\,b\,c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{a^2\,d-a\,b\,c}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}}\right)\,\sqrt{b^2\,c-a\,b\,d}\,1{}\mathrm{i}}{a^2\,d-a\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^2)^(1/2)/c^(1/2))/(a*c^(1/2)) - (atan((((b^2*c - a*b*d)^(1
/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 -
((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2)
)/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c)))*1i)/(a^2*d - a*b*c) + ((b^2*c
- a*b*d)^(1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) + ((b^2*c - a*b*d)^(1/2)*(2*a^2
*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^2)^(1/2)*(b^2*c - a
*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c)))*1i)/(a^2*d - a*b*c)
)/(((b^2*c - a*b*d)^(1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) - ((b^2*c - a*b*d)^(
1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^2)^(1/2)
*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c))))/(a^2*d
- a*b*c) - ((b^2*c - a*b*d)^(1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) + ((b^2*c -
a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^
2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c))))
/(a^2*d - a*b*c)))*(b^2*c - a*b*d)^(1/2)*1i)/(a^2*d - a*b*c)
```

3.707 $\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{c+dx^2}}{2acx^2}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}$$

```
[Out] 1/2*(a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-b^(3/2)*arctan
h(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/2*(d*x^2
+c)^(1/2)/a/c/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 105, 162, 65, 214}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}}-\frac{\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/(a*c*x^2) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqr
t[c]])/(2*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*
c - a*d]])/(a^2*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(2bc+ad)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{(2bc+ad)\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+}}{\sqrt{bc-a}}\right.}{a^2\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.24, size = 109, normalized size = 0.95

$$\frac{-\frac{a\sqrt{c+dx^2}}{cx^2} + \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^2])/(c*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2
])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[c
+ d*x^2]/Sqrt[c]])/c^(3/2))/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(93) = 186$.
time = 0.12, size = 384, normalized size = 3.34

method	result
default	$-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2a^2\sqrt{-\frac{ad-bc}{b}}}$
risch	$-\frac{\sqrt{d\,x^2+c}}{2c\,x^2a}+\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)d}{2a\,c^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{a^2\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\right.}{b}}{}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2
*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/a*(-1
/2/c/x^2*(d*x^2+c)^(1/2)+1/2*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x
))+b/a^2/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^3), x)
```

Fricas [A]
time = 1.35, size = 734, normalized size = 6.38

$$\left[\frac{bc^{2}x^{2}\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^{2}d^{2}x^{4}+8b^{2}c^{2}-8abcd+a^{2}d^{2}+2(4b^{2}cd-3abd^{2})x^{2}-4(2b^{2}c^{2}-3abcd+a^{2}d^{2}+(b^{2}cd-abd^{2})x^{2})\sqrt{dx^{2}+c}\sqrt{\frac{b}{bc-ad}}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)+(2bc+ad)\sqrt{c}\,x^{2}\log\left(-\frac{dx^{2}+2\sqrt{dx^{2}+c}\sqrt{c}+2c}{x^{2}}\right)-2\sqrt{dx^{2}+c}\,ac}{4a^{2}c^{2}x^{2}}, \frac{bc^{2}x^{2}\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^{2}d^{2}x^{4}+8b^{2}c^{2}-8abcd+a^{2}d^{2}+2(4b^{2}cd-3abd^{2})x^{2}-4(2b^{2}c^{2}-3abcd+a^{2}d^{2}+(b^{2}cd-abd^{2})x^{2})\sqrt{dx^{2}+c}\sqrt{\frac{b}{bc-ad}}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)-2(2bc+ad)\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)-2\sqrt{dx^{2}+c}\,ac}{4a^{2}c^{2}x^{2}}, \frac{2bc^{2}x^{2}\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^{2}+2bc-ad)\sqrt{dx^{2}+c}\sqrt{-\frac{b}{bc-ad}}}{2(bdx^{2}+bc)}\right)+(2bc+ad)\sqrt{c}\,x^{2}\log\left(-\frac{dx^{2}+2\sqrt{dx^{2}+c}\sqrt{c}+2c}{x^{2}}\right)-2\sqrt{dx^{2}+c}\,ac}{4a^{2}c^{2}x^{2}}, \frac{bc^{2}x^{2}\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^{2}+2bc-ad)\sqrt{dx^{2}+c}\sqrt{-\frac{b}{bc-ad}}}{2(bdx^{2}+bc)}\right)-(2bc+ad)\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)-\sqrt{dx^{2}+c}\,ac}{2a^{2}c^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c^2*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^
2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) + (2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*
x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/4*(b
*c^2*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2
*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 +
 (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2
*a*b*x^2 + a^2)) - 2*(2*b*c + a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2
+ c)) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/4*(2*b*c^2*x^2*sqrt(-b/(b*c
 - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c -
a*d))/(b*d*x^2 + b*c)) + (2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*x
^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/2*(b*
c^2*x^2*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2
+ c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - (2*b*c + a*d)*sqrt(-c)*x^2*arc
tan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(a+bx^{2}\right)\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.98, size = 103, normalized size = 0.90

$$\frac{b^{2}\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,a^{2}}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{2\,a^{2}\sqrt{-c}\,c}-\frac{\sqrt{dx^{2}+c}}{2\,acx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^
2) - 1/2*(2*b*c + a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c) -
1/2*sqrt(d*x^2 + c)/(a*c*x^2)
```

Mupad [B]
time = 0.63, size = 396, normalized size = 3.44

$$\frac{\ln\left(\sqrt{d\,x^2+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{2\,a^3\,d-2\,a^2\,b\,c}-\frac{\ln\left(\sqrt{d\,x^2+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{2\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^2+c}}{2\,a\,c\,x^2}-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^2+c}\,3{}\mathrm{i}}{2\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{2\,c}+\frac{5\,a\,b^3\,d^5}{4\,c^2}+\frac{a^2\,b^2\,d^6}{4\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{4\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{4\,a}+\frac{b^2\,d^6}{4\,c}+\frac{3\,b^4\,c\,d^4}{2\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^2+c}\,5{}\mathrm{i}}{4\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{2\,a}+\frac{5\,b^3\,d^5}{4\,c}+\frac{a\,b^2\,d^6}{4\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{2\,a^2\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (log((c + d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(2*a^3*d - 2*a^2*b*c) - (log((c + d*x^2
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(2*(a^3*d - a^2*b*c)) - (c + d*x^2)^(1/2)/(2*a*c*x^2)
- (atan((b^4*d^4*(c + d*x^2)^(1/2)*3i)/(2*(c^3)^(1/2)*((3*b^4*d^4)/(2*c) +
(5*a*b^3*d^5)/(4*c^2) + (a^2*b^2*d^6)/(4*c^3))) + (b^2*d^6*(c + d*x^2)^(1/2
)*1i)/(4*(c^3)^(1/2)*((5*b^3*d^5)/(4*a) + (b^2*d^6)/(4*c) + (3*b^4*c*d^4)/(
2*a^2))) + (b^3*d^5*(c + d*x^2)^(1/2)*5i)/(4*(c^3)^(1/2)*((3*b^4*d^4)/(2*a)
 + (5*b^3*d^5)/(4*c) + (a*b^2*d^6)/(4*c^2))))*(a*d + 2*b*c)*1i)/(2*a^2*(c^3
)^(1/2))
```

3.708 $\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=114

$$\frac{x\sqrt{c+dx^2}}{2bd}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}$$

```
[Out] -1/2*(2*a*d+b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2/d^(3/2)+a^(3/2)*arc
tan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b^2/(-a*d+b*c)^(1/2)+1/2*x*
(d*x^2+c)^(1/2)/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {490, 537, 223, 212, 385, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}+\frac{x\sqrt{c+dx^2}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/(b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/(2*b^2*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \frac{x\sqrt{c+dx^2}}{2bd} - \frac{\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2bd}\\ &= \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^2} - \frac{(bc+2ad)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2d}\\ &= \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b^2} - \frac{(bc+2ad)\text{Subst}\left(\right.}{}\\ &= \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right.}{2b^2d^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 130, normalized size = 1.14

$$\frac{\frac{bx\sqrt{c+dx^2}}{d} - \frac{2a^{3/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{(bc+2ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{d^{3/2}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2])/d - (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x -
Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] + ((b*c + 2*a
*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/d^(3/2))/(2*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(92) = 184$.
time = 0.13, size = 385, normalized size = 3.38

method	result
default	$\frac{\frac{x\sqrt{d x^2+c}}{2d}-\frac{c\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{2d^{\frac{3}{2}}}}{b}-\frac{a\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{b^2\sqrt{d}}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-a}}{b}\right.}{b}}{}\right.}{}$
risch	$\frac{x\sqrt{d x^2+c}}{2bd}-\frac{a\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{b^2\sqrt{d}}-\frac{\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)c}{2b\,d^{\frac{3}{2}}}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-}}{}\right.}{b}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*x/d*(d*x^2+c)^(1/2)-1/2*c/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))-a
/b^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*
(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2/b^2*a^2/(-a*b)^(1
/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [A]
time = 1.43, size = 717, normalized size = 6.29

$$\left[\frac{ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4\left((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{dx^2+c}\,bdx+(bc+2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)}{4b^2d^2}, \frac{ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4\left((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{dx^2+c}\,bdx+2(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)}{4b^2d^2}, -\frac{2ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)-2\sqrt{dx^2+c}\,bdx-(bc+2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)}{4b^2d^2}, -\frac{ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)-\sqrt{dx^2+c}\,bdx-(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)}{2b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4
 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^
2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*b*d*x + (b*c + 2*a*d)*sqrt(
d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d^2), 1/4*(a*d^2*s
qrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 -
(a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a
*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*b*d*x + 2*(b*c + 2*a*d)*sqrt(-d)*arctan(
sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^2*d^2), -1/4*(2*a*d^2*sqrt(a/(b*c - a*d))*a
rctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a
*d*x^3 + a*c*x)) - 2*sqrt(d*x^2 + c)*b*d*x - (b*c + 2*a*d)*sqrt(d)*log(-2*d
*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d^2), -1/2*(a*d^2*sqrt(a/(b*c
 - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
- a*d))/(a*d*x^3 + a*c*x)) - sqrt(d*x^2 + c)*b*d*x - (b*c + 2*a*d)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.709 $\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}$$

```
[Out] arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b/d^(1/2)-arctan(x*(-a*d+b*c)^(1/2)/a^(1
/2)/(d*x^2+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {494, 223, 212, 385, 211}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[b
*c - a*d])) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \frac{\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b} - \frac{a\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b} - \frac{a\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 102, normalized size = 1.24

$$\frac{\frac{\sqrt{a}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*S
qrt[b*c - a*d])])/Sqrt[b*c - a*d] - Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]/Sqr
t[d])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(66) = 132.
time = 0.10, size = 337, normalized size = 4.11

method	result
default	$\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b\sqrt{d}}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d}{}}}{x-\frac{\sqrt{-ab}}{b}}\right.}{2\sqrt{-ab}\,b\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/b*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)+1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/((b*x^2 + a)*sqrt(d*x^2 + c)), x)

Fricas [A]
time = 1.24, size = 616, normalized size = 7.51

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)}{4bd},\ \frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)-4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)}{4bd},\ \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)+\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)}{2bd},\ \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)-2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)}{2bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [1/4*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b*d), 1/4*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*

```
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*
b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*
c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)))/(b*d), 1/2*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*
x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + sqrt(d)
*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b*d), 1/2*(d*sqrt(a/(b*c
 - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
- a*d))/(a*d*x^3 + a*c*x)) - 2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))
/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
[Out] int(x^2/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.710 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}$$

```
[Out] arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(1/2)/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {385, 211}

$$\frac{\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 70, normalized size = 1.43

$$-\frac{\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c
- a*d])]/(Sqrt[a]*Sqrt[b*c - a*d]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(39) = 78.
time = 0.09, size = 306, normalized size = 6.24

method	result
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/
b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2
)))+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(39) = 78$.
time = 1.84, size = 241, normalized size = 4.92

$$\left[-\frac{\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,(abc-a^2d)},\ \frac{\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)}{2\,\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-
a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b*c - a^2*d
), 1/2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))/sqrt(a*b*c - a^2*d)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.87, size = 70, normalized size = 1.43

$$-\frac{\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(
a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{b\,c-a\,d}}{\sqrt{a}\,\sqrt{d\,x^2+c}}\right)}{\sqrt{-a\,(a\,d-b\,c)}} & \text{if } 0<b\,c-a\,d \\ \frac{\ln\left(\frac{\sqrt{a\,(d\,x^2+c)}+x\sqrt{a\,d-b\,c}}{\sqrt{a\,(d\,x^2+c)}-x\sqrt{a\,d-b\,c}}\right)}{2\sqrt{a\,(a\,d-b\,c)}} & \text{if } b\,c-a\,d<0 \\ \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx & \text{if } b\,c-a\,d\notin\mathbb{R}\vee a\,d=b\,c \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - a*d + b*c, atan((x*(- a*d + b*c)^(1/2))/(a^(1/2)*(c + d*x^2
)^(1/2)))/(-a*(a*d - b*c))^(1/2), - a*d + b*c < 0, log(((a*(c + d*x^2))^(1/
2) + x*(a*d - b*c)^(1/2))/((a*(c + d*x^2))^(1/2) - x*(a*d - b*c)^(1/2)))/(2
*(a*(a*d - b*c))^(1/2)), ~in(- a*d + b*c, 'real') | a*d == b*c, int(1/((a +
 b*x^2)*(c + d*x^2)^(1/2)), x))
```

3.711 $\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{c+dx^2}}{acx}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}$$

```
[Out] -b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(1
/2)-(d*x^2+c)^(1/2)/a/c/x
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {491, 12, 385, 211}

$$-\frac{b\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*c*x)) - (b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
+ d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
```

```
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{c+dx^2}}{acx}-\frac{\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{ac}\\ &= -\frac{\sqrt{c+dx^2}}{acx}-\frac{b\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{a}\\ &= -\frac{\sqrt{c+dx^2}}{acx}-\frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a}\\ &= -\frac{\sqrt{c+dx^2}}{acx}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 93, normalized size = 1.26

$$-\frac{\sqrt{c+dx^2}}{acx}+\frac{b\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{3/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*c*x)) + (b*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c
 + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(a^(3/2)*Sqrt[b*c - a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(62) = 124$.
time = 0.10, size = 334, normalized size = 4.51

method	result

default	$\frac{b \ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad\cdots}{}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2a\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$
risch	$\frac{b \ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad\cdots}{}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2a\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(
1/2)))-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1
/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(
-a*b)^(1/2)))-(d*x^2+c)^(1/2)/a/c/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(62) = 124$.
time = 1.09, size = 324, normalized size = 4.38

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(abc-a^2d)\sqrt{dx^2+c}}{4\,(a^2bc^2-a^3cd)x}, -\frac{\sqrt{abc-a^2d}\,bcx\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+2\,(abc-a^2d)\sqrt{dx^2+c}}{2\,(a^2bc^2-a^3cd)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b*c + a^2*d)*b*c*x*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^
4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)
*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*
b*c - a^2*d)*sqrt(d*x^2 + c))/((a^2*b*c^2 - a^3*c*d)*x), -1/2*(sqrt(a*b*c -
 a^2*d)*b*c*x*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt
(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(a*b*c -
 a^2*d)*sqrt(d*x^2 + c))/((a^2*b*c^2 - a^3*c*d)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [**A**]
time = 0.89, size = 111, normalized size = 1.50

$$d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,ad}+\frac{2}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] d^(3/2)*(b*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x - sqrt
(d*x^2 + c))^2 - c)*a*d))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.712 $\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}$$

```
[Out] b^2*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(
1/2)-1/3*(d*x^2+c)^(1/2)/a/c/x^3+1/3*(2*a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/c^2/
x
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {491, 597, 12, 385, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\,(2ad+3bc)}{3a^2c^2x}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^2]/(a*c*x^3) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^2])/(3*a^2*c
^2*x) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2
)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a+bx^2) \sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{\int \frac{-3bc-2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx}{3ac}\\
&= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}-\frac{\int -\frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3a^2c^2}\\
&= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{a^2}\\
&= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{}{\sqrt{c}}\right.}{a^2}\\
&= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 117, normalized size = 1.06

$$\frac{\sqrt{c+dx^2}\,(-ac+3bcx^2+2adx^2)}{3a^2c^2x^3}-\frac{b^2\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*c) + 3*b*c*x^2 + 2*a*d*x^2))/(3*a^2*c^2*x^3) - (b^2*A
rcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a
*d])])/(a^(5/2)*Sqrt[b*c - a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(92) = 184$.
time = 0.11, size = 378, normalized size = 3.44

method	result
risch	$-\frac{\sqrt{d x^2+c}\,(-2ad x^2-3c x^2 b+ac)}{3c^2a^2x^3} - \frac{b^2 \ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right.}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2a^2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{b^2 \ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2a^2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2
))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a
*b)^(1/2)))+1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/
b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))+1/a*(-1/3/c/x^3*(d*x^2+c)^(1/2)+2/3*d/c^2/x*(d*x^2+c
)^(1/2))+b/a^2/c/x*(d*x^2+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(92) = 184$.
time = 1.99, size = 414, normalized size = 3.76

$$\left[-\frac{3\sqrt{-abc+a^2d}\,b^2c^2x^3\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(a^2bc^2-a^3cd-(3\,ab^2c^2-a^2bcd-2\,a^3d^2)x^2)\sqrt{dx^2+c}}{12\,(a^3bc^3-a^4c^2d)x^3}, \frac{3\sqrt{abc-a^2d}\,b^2c^2x^3\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-2\,(a^2bc^2-a^3cd-(3\,ab^2c^2-a^2bcd-2\,a^3d^2)x^2)\sqrt{dx^2+c}}{6\,(a^3bc^3-a^4c^2d)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3
 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)*sqr
t(d*x^2 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^3), 1/6*(3*sqrt(a*b*c - a^2*d)*b^2
*c^2*x^3*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(a^2*b*c^2 -
a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)*sqrt(d*x^2 + c))/((a^3
*b*c^3 - a^4*c^2*d)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(92) = 184$.
time = 1.86, size = 195, normalized size = 1.77

$$-\frac{1}{3}\,d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2d^2}+\frac{2\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+3\,bc^2+2\,acd\right)}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2
*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*b - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 6*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)*x - sqr
t(d*x^2 + c))^2 - c)^3*a^2*d^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.713 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{cx}{d(bc-ad)\sqrt{c+dx^2}}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}$$

```
[Out] a^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b/(-a*d+b*c)^(3/
2)+arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b/d^(3/2)-c*x/d/(-a*d+b*c)/(d*x^2+c)^
(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {481, 537, 223, 212, 385, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}-\frac{cx}{d\sqrt{c+dx^2}\,(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((c*x)/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]
*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*(b*c - a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]]/(b*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}} + \frac{\int \frac{ac+(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{d(bc-ad)}$$

$$= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}} + \frac{\int \frac{1}{\sqrt{c+dx^2}}\,dx}{bd} + \frac{a^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b(bc-ad)}$$

$$= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{bd} + \frac{a^2\mathrm{Subst}\left(\int \frac{}{a-}\right.}{}$$

$$= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right.}{bd^{3/2}}$$

Mathematica [A]

time = 0.35, size = 132, normalized size = 1.21

$$\frac{cx}{d(-bc+ad)\sqrt{c+dx^2}}-\frac{a^{3/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{b(bc-ad)^{3/2}}-\frac{\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (c*x)/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) - (a^(3/2)*ArcTan[(a*Sqrt[d] + b*x
*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*(b*c - a*d)^
(3/2)) - Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]/(b*d^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(91) = 182$.
time = 0.09, size = 804, normalized size = 7.38

method	result
default	$\frac{-\frac{x}{d\sqrt{d\,x^2+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{d^{\frac{3}{2}}}}{b}-\frac{ax}{b^2c\sqrt{d\,x^2+c}}+\frac{a^2\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-a}\cdots}{\cdots}}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))-a/b^2*x/
c/(d*x^2+c)^(1/2)+1/2/b^2*a^2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)
^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-
a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a
*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d
*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x-1/b*(-a*b)^(1/2))))-1/2/b^2*a^2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d
```

```
*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/
2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(91) = 182.
time = 1.63, size = 977, normalized size = 8.96

$$\left[-\frac{4\sqrt{dx^2+c}\,bcdx-2\left(bc^2-acd+(bcd-ad^2)x^2\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+(ad^3x^2+acd^2)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4\left((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)}{4\left(b^2c^2d^2-abcd^3+(b^2cd^3-abd^4)x^2\right)}, -\frac{4\sqrt{dx^2+c}\,bcdx+4\left(bc^2-acd+(bcd-ad^2)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(ad^3x^2+acd^2)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4\left((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)}{4\left(b^2c^2d^2-abcd^3+(b^2cd^3-abd^4)x^2\right)}, -\frac{2\sqrt{dx^2+c}\,bcdx+(ad^3x^2+acd^2)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2ad)x^2-ac\right)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)-\left(bc^2-acd+(bcd-ad^2)x^2\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)}{2\left(b^2c^2d^2-abcd^3+(b^2cd^3-abd^4)x^2\right)}, -\frac{2\sqrt{dx^2+c}\,bcdx+2\left(bc^2-acd+(bcd-ad^2)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(ad^3x^2+acd^2)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2ad)x^2-ac\right)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)}{2\left(b^2c^2d^2-abcd^3+(b^2cd^3-abd^4)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(d*x^2 + c)*b*c*d*x - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*
sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*d^3*x^2 + a*c*
d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*
x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)
, -1/4*(4*sqrt(d*x^2 + c)*b*c*d*x + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*d^3*x^2 + a*c*d^2)*sqrt(-
a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*
c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^
2 + a^2)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2), -1/2*(2*s
qrt(d*x^2 + c)*b*c*d*x + (a*d^3*x^2 + a*c*d^2)*sqrt(a/(b*c - a*d))*arctan(-
1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3
+ a*c*x)) - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b
*d^4)*x^2), -1/2*(2*sqrt(d*x^2 + c)*b*c*d*x + 2*(b*c^2 - a*c*d + (b*c*d - a
*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*d^3*x^2 + a*c*d
```

```
^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*
c*d^3 - a*b*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.714 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}} dx$

Optimal. Leaf size=77

$$-\frac{c}{d(bc-ad)\sqrt{c+dx^2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}$$

```
[Out] a*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^(1/2
)-c/d/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 79, 65, 214}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}-\frac{c}{d\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(c/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{c}{d(bc-ad)\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2(bc-ad)}\\
&= -\frac{c}{d(bc-ad)\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d(bc-ad)}\\
&= -\frac{c}{d(bc-ad)\sqrt{c+dx^2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.13, size = 76, normalized size = 0.99

$$\frac{c}{d(-bc+ad)\sqrt{c+dx^2}} + \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] c/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/
Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(65) = 130.
time = 0.09, size = 746, normalized size = 9.69

method	result
default	$-\frac{1}{bd\sqrt{d\,x^2+c}}-\frac{a\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{\cdots}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}\cdots\right.}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d/(d*x^2+c)^(1/2)-1/2*a/b^2*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)
/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+
4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-
a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x-1/b*(-a*b)^(1/2))))-1/2*a/b^2*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/
(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4
*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
+1/b*(-a*b)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(65) = 130$.
time = 1.48, size = 428, normalized size = 5.56

$$\left[-\frac{\left(ad^{2}x^{2}+acd\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\,\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}-4\,\left(bdx^{2}+2\,bc-ad\right)\sqrt{b^{2}c-abd}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\,\left(b^{2}c^{2}-abcd\right)\sqrt{dx^{2}+c}}{4\,\left(b^{3}c^{3}d-2\,ab^{2}c^{2}d^{2}+a^{2}bcd^{3}+\left(b^{3}c^{2}d^{2}-2\,ab^{2}cd^{3}+a^{2}bd^{4}\right)x^{2}\right)},\ \frac{\left(ad^{2}x^{2}+acd\right)\sqrt{-b^{2}c+abd}\,\arctan\left(-\frac{\left(bdx^{2}+2\,bc-ad\right)\sqrt{-b^{2}c+abd}\,\sqrt{dx^{2}+c}}{2\,\left(b^{2}c^{2}-abcd+\left(b^{2}cd-abd^{2}\right)x^{2}\right)}\right)-2\,\left(b^{2}c^{2}-abcd\right)\sqrt{dx^{2}+c}}{2\,\left(b^{3}c^{3}d-2\,ab^{2}c^{2}d^{2}+a^{2}bcd^{3}+\left(b^{3}c^{2}d^{2}-2\,ab^{2}cd^{3}+a^{2}bd^{4}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*d^2*x^2 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2
 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c
 - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
 4*(b^2*c^2 - a*b*c*d)*sqrt(d*x^2 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*
b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2), 1/2*((a*d^2*x^2 +
 a*c*d)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*
c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) -
 2*(b^2*c^2 - a*b*c*d)*sqrt(d*x^2 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*
b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(a+bx^{2}\right)\left(c+dx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 1.34, size = 78, normalized size = 1.01

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,(bc-ad)}+\frac{c}{\sqrt{dx^{2}+c}\,(bc-ad)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
(b*c - a*d)) + c/(sqrt(d*x^2 + c)*(b*c - a*d)))/d
```

Mupad [B]
time = 0.43, size = 64, normalized size = 0.83

$$\frac{c}{d\sqrt{d\,x^2+c}\ \left(a\,d-b\,c\right)}+\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\ \sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{\sqrt{b}\ {\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] c/(d*(c + d*x^2)^(1/2)*(a*d - b*c)) + (a*atan((b^(1/2)*(c + d*x^2)^(1/2))/(
a*d - b*c)^(1/2)))/(b^(1/2)*(a*d - b*c)^(3/2))
```

3.715 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}} \, dx$

Optimal. Leaf size=74

$$\frac{x}{(bc-ad)\sqrt{c+dx^2}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

```
[Out] -arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)/(-a*d+b*c)^(3/2
)+x/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {482, 12, 385, 211}

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)} - \frac{\sqrt{a}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] x/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx &= \frac{x}{(bc-ad)\sqrt{c+dx^2}}-\frac{\int \frac{a}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad}\\ &= \frac{x}{(bc-ad)\sqrt{c+dx^2}}-\frac{a\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad}\\ &= \frac{x}{(bc-ad)\sqrt{c+dx^2}}-\frac{a\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}\\ &= \frac{x}{(bc-ad)\sqrt{c+dx^2}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.19, size = 93, normalized size = 1.26

$$\frac{x}{(bc-ad)\sqrt{c+dx^2}}+\frac{\sqrt{a}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] x/((b*c - a*d)*Sqrt[c + d*x^2]) + (Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]
*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 757 vs. 2(62) = 124.
time = 0.10, size = 758, normalized size = 10.24

method	result

default	$\frac{x}{bc\sqrt{d\,x^2+c}} - \frac{a\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}} + \frac{}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*x/c/(d*x^2+c)^(1/2)-1/2*a/(-a*b)^(1/2)/b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*
b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*
(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*
(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+1/2*a/(-a*b)^(1/2)/b*(-1/(a*d-b*c)*b/(d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)
/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [A]
time = 1.66, size = 334, normalized size = 4.51

$$\left[-\frac{(dx^2+c)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)-4\sqrt{dx^2+c}\,x}{4(bc^2-acd+(bcd-ad^2)x^2)}, \frac{(dx^2+c)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)+2\sqrt{dx^2+c}\,x}{2(bc^2-acd+(bcd-ad^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((d*x^2 + c)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d
^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d
 + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a
*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c)*x)/(b*c^2 - a*c*d +
(b*c*d - a*d^2)*x^2), 1/2*((d*x^2 + c)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)
) + 2*sqrt(d*x^2 + c)*x)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 1.06, size = 103, normalized size = 1.39

$$-\frac{a\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{x}{\sqrt{dx^2+c}\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -a*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + x/(sqrt(d*x^
2 + c)*(b*c - a*d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
[Out] int(x^2/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.716 $\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{(bc-ad)\sqrt{c+dx^2}}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

```
[Out] -arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(3/2)
+1/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {455, 53, 65, 214}

$$\frac{1}{\sqrt{c+dx^2}\,(bc-ad)}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] 1/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{1}{(bc-ad)\sqrt{c+dx^2}} + \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2(bc-ad)}\\
&= \frac{1}{(bc-ad)\sqrt{c+dx^2}} + \frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d(bc-ad)}\\
&= \frac{1}{(bc-ad)\sqrt{c+dx^2}} - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 1.00

$$\frac{1}{(bc-ad)\sqrt{c+dx^2}} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] 1/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(60) = 120$.
time = 0.10, size = 727, normalized size = 10.10

method	result
default	$-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\left(2d\left(x-\right.\right.}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x-\frac{\sqrt{-}}{}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*
b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)
^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d
-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+1/2/b
*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1
/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)
*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.66, size = 323, normalized size = 4.49

$$\left[-\frac{(dx^2+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(2\,b^2c^2-3\,abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)-4\sqrt{dx^2+c}}{4\,(bc^2-acd+(bcd-ad^2)x^2)},\ \frac{(dx^2+c)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^2+bc)}\right)+2\sqrt{dx^2+c}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((d*x^2 + c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b
*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d +
 a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b
^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c))/(b*c^2 - a*c*d + (b*c*d - a
*d^2)*x^2), 1/2*((d*x^2 + c)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b
*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*sqrt(d*
x^2 + c))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

Sympy [A]
time = 6.69, size = 61, normalized size = 0.85

$$-\frac{1}{\sqrt{c+dx^2}\,(ad-bc)}-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] -1/(sqrt(c + d*x**2)*(a*d - b*c)) - atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/
b))/(sqrt((a*d - b*c)/b)*(a*d - b*c))
```

Giac [A]
time = 0.99, size = 71, normalized size = 0.99

$$\frac{b\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{1}{\sqrt{dx^2+c}\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
 - a*d)) + 1/(sqrt(d*x^2 + c)*(b*c - a*d))
```

Mupad [B]
time = 0.43, size = 61, normalized size = 0.85

$$-\frac{1}{\sqrt{d\,x^2+c}\,\left(a\,d-b\,c\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
[Out] - 1/((c + d*x^2)^(1/2)*(a*d - b*c)) - (b^(1/2)*atan((b^(1/2)*(c + d*x^2)^(1
/2))/(a*d - b*c)^(1/2)))/(a*d - b*c)^(3/2)
```

3.717 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}$$

```
[Out] b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(3/2)/a^(1/
2)-d*x/c/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {390, 385, 211}

$$\frac{b\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad}$$

$$= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}$$

$$= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

Mathematica [A]
time = 0.04, size = 99, normalized size = 1.25

$$\frac{dx}{c(-bc+ad)\sqrt{c+dx^2}} - \frac{b\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*x)/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) - (b*ArcTan[(a*Sqrt[d] + b*x*(Sqrt
[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d)^
(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 732 vs. 2(67) = 134.
time = 0.09, size = 733, normalized size = 9.28

method	result

default	$-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\left(2d\left(\right.\right.}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x-\sqrt{\right.}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*
(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x
-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2))))-1/2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)
*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)
/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2)
)^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(67) = 134.
time = 1.57, size = 442, normalized size = 5.59

$$\left[-\frac{4\,(abcd-a^2d^2)\sqrt{dx^2+c}\,x-(bcdx^2+bc^2)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3)x^2)},-\frac{2\,(abcd-a^2d^2)\sqrt{dx^2+c}\,x-(bcdx^2+bc^2)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)}{2\,(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(-
a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^2*c^4 - 2*a^2*b*c^3*
d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2), -1/2*(2
*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(a*b*c - a
^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a*b^2*c^4 - 2*a^2*
b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 1.11, size = 107, normalized size = 1.35

$$\frac{b\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,(bc-ad)}-\frac{dx}{(bc^2-acd)\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) - d*x/((b*c^2 -
 a*c*d)*sqrt(d*x^2 + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.718 $\int \frac{1}{x(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{d}{c(bc-ad)\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}$$

```
[Out] -arctanh((d*x^2+c)^(1/2)/c^(1/2))/a/c^(3/2)+b^(3/2)*arctanh(b^(1/2)*(d*x^2+
c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(3/2)-d/c/(-a*d+b*c)/(d*x^2+c)^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 87, 162, 65, 214}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}-\frac{d}{c\sqrt{c+dx^2}\,(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*Sqrt[c + d*x^2])) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*
c^(3/2)) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*
(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^2\right)$$

$$= -\frac{d}{c(bc-ad)\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2c(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2ac} - \frac{b^2\text{Subst}\left(\int \frac{\ldots}{(a+b\ldots}\right.}{2a(\ldots}$$

$$= -\frac{d}{c(bc-ad)\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{acd} - \frac{b^2\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{d}{c(bc-ad)\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\ldots}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}$$

Mathematica [A]
time = 0.25, size = 106, normalized size = 0.99

$$\frac{d}{c(-bc+ad)\sqrt{c+dx^2}} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{a(-bc+ad)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]

[Out] d/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[-(b*c) + a*d]])/(a*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c + d*x^
2]/Sqrt[c]]/(a*c^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(89) = 178$.
time = 0.10, size = 773, normalized size = 7.22

method	result
default	$-\frac{-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\,\left(2d\right.}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x-\right.}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/a*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a
*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*
d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-1/2/
a*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(
1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c
)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/a*(1/c/
(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(89) = 178.
time = 1.64, size = 959, normalized size = 8.96

$$\left[-\frac{4\sqrt{dx^2+c}\,acd+(bc^2dx^2+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2-4(2b^2c^2-3abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)-2(bc^2-acd+(bcd-ad^2)x^2)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{4(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^2)}, -\frac{4\sqrt{dx^2+c}\,acd-4(bc^2-acd+(bcd-ad^2)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(bc^2dx^2+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2-4(2b^2c^2-3abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)}{4(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^2)}, -\frac{2\sqrt{dx^2+c}\,acd+(bc^2dx^2+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2(bdx^2+bc)}\right)-(bc^2-acd+(bcd-ad^2)x^2)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{2(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^2)}, -\frac{2\sqrt{dx^2+c}\,acd+(bc^2dx^2+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2(bdx^2+bc)}\right)-2(bc^2-acd+(bcd-ad^2)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{2(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(d*x^2 + c)*a*c*d + (b*c^2*d*x^2 + b*c^3)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqr
t(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(b*c^2 -
 a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(
c) + 2*c)/x^2))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/4
*(4*sqrt(d*x^2 + c)*a*c*d - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b*c^2*d*x^2 + b*c^3)*sqrt(b/(b*c - a*
d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a
*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)
*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b*c^
4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/2*(2*sqrt(d*x^2 + c)*a*c
*d + (b*c^2*d*x^2 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c
 - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - (b*c^2 - a*
c*d + (b*c*d - a*d^2)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
+ 2*c)/x^2))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/2*(2
*sqrt(d*x^2 + c)*a*c*d + (b*c^2*d*x^2 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
 b*c)) - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/s
qrt(d*x^2 + c)))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2)]
```

Sympy [A]
time = 6.95, size = 94, normalized size = 0.88

$$\frac{d}{c\sqrt{c+dx^2}\,(ad-bc)}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{a\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}+\frac{\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] d/(c*sqrt(c + d*x**2)*(a*d - b*c)) + b*atan(sqrt(c + d*x**2)/sqrt((a*d - b*
c)/b))/(a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + atan(sqrt(c + d*x**2)/sqrt(-c)
)/(a*c*sqrt(-c))
```

Giac [A]
time = 0.77, size = 110, normalized size = 1.03

$$-\frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(abc-a^2d)\sqrt{-b^2c+abd}}-\frac{d}{(bc^2-acd)\sqrt{dx^2+c}}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c - a^2*d)*sqrt(-
b^2*c + a*b*d)) - d/((b*c^2 - a*c*d)*sqrt(d*x^2 + c)) + arctan(sqrt(d*x^2 +
 c)/sqrt(-c))/(a*sqrt(-c)*c)
```

Mupad [B]
time = 0.82, size = 2296, normalized size = 21.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] (atan((((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*
a*b^7*c^7*d^3 + 26*a^2*b^6*c^6*d^4 - 22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^
6 - 2*a^5*b^3*c^3*d^7))/2 - ((-b^3*(a*d - b*c)^3)^(1/2)*(18*a^3*b^6*c^8*d^4
 - 4*a^2*b^7*c^9*d^3 - 32*a^4*b^5*c^7*d^5 + 28*a^5*b^4*c^6*d^6 - 12*a^6*b^3
*c^5*d^7 + 2*a^7*b^2*c^4*d^8 + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2
)*(16*a^2*b^8*c^11*d^2 - 88*a^3*b^7*c^10*d^3 + 200*a^4*b^6*c^9*d^4 - 240*a^
5*b^5*c^8*d^5 + 160*a^6*b^4*c^7*d^6 - 56*a^7*b^3*c^6*d^7 + 8*a^8*b^2*c^5*d^
8))/(4*a*(a*d - b*c)^3)))/(2*a*(a*d - b*c)^3))*1i)/(a*(a*d - b*c)^3) + ((-b
^3*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*a*b^7*c^7*d
^3 + 26*a^2*b^6*c^6*d^4 - 22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^6 - 2*a^5*b
^3*c^3*d^7))/2 - ((-b^3*(a*d - b*c)^3)^(1/2)*(4*a^2*b^7*c^9*d^3 - 18*a^3*b^
6*c^8*d^4 + 32*a^4*b^5*c^7*d^5 - 28*a^5*b^4*c^6*d^6 + 12*a^6*b^3*c^5*d^7 -
2*a^7*b^2*c^4*d^8 + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b
^8*c^11*d^2 - 88*a^3*b^7*c^10*d^3 + 200*a^4*b^6*c^9*d^4 - 240*a^5*b^5*c^8*d
^5 + 160*a^6*b^4*c^7*d^6 - 56*a^7*b^3*c^6*d^7 + 8*a^8*b^2*c^5*d^8))/(4*a*(a
*d - b*c)^3)))/(2*a*(a*d - b*c)^3))*1i)/(a*(a*d - b*c)^3))/(2*b^7*c^6*d^3 -
 6*a*b^6*c^5*d^4 + 6*a^2*b^5*c^4*d^5 - 2*a^3*b^4*c^3*d^6 + ((-b^3*(a*d - b*
c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*a*b^7*c^7*d^3 + 26*a^2*
```

b^6*c^6*d^4 - 22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^6 - 2*a^5*b^3*c^3*d^7))
/2 - ((-b^3*(a*d - b*c)^3)^(1/2)*(18*a^3*b^6*c^8*d^4 - 4*a^2*b^7*c^9*d^3 -
32*a^4*b^5*c^7*d^5 + 28*a^5*b^4*c^6*d^6 - 12*a^6*b^3*c^5*d^7 + 2*a^7*b^2*c^
4*d^8 + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^8*c^11*d^2
- 88*a^3*b^7*c^10*d^3 + 200*a^4*b^6*c^9*d^4 - 240*a^5*b^5*c^8*d^5 + 160*a^6
*b^4*c^7*d^6 - 56*a^7*b^3*c^6*d^7 + 8*a^8*b^2*c^5*d^8))/(4*a*(a*d - b*c)^3)
))/(2*a*(a*d - b*c)^3)))/(a*(a*d - b*c)^3) - ((-b^3*(a*d - b*c)^3)^(1/2)*((
(c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*a*b^7*c^7*d^3 + 26*a^2*b^6*c^6*d^4 -
22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^6 - 2*a^5*b^3*c^3*d^7))/2 - ((-b^3*(a
*d - b*c)^3)^(1/2)*(4*a^2*b^7*c^9*d^3 - 18*a^3*b^6*c^8*d^4 + 32*a^4*b^5*c^7
*d^5 - 28*a^5*b^4*c^6*d^6 + 12*a^6*b^3*c^5*d^7 - 2*a^7*b^2*c^4*d^8 + ((-b^3
*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^8*c^11*d^2 - 88*a^3*b^7*c
^10*d^3 + 200*a^4*b^6*c^9*d^4 - 240*a^5*b^5*c^8*d^5 + 160*a^6*b^4*c^7*d^6 -
56*a^7*b^3*c^6*d^7 + 8*a^8*b^2*c^5*d^8))/(4*a*(a*d - b*c)^3)))/(2*a*(a*d -
b*c)^3)))/(a*(a*d - b*c)^3)))*(-b^3*(a*d - b*c)^3)^(1/2)*1i)/(a*(a*d - b*c
)^3) - atanh((6*b^7*c^7*d^3*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(6*b^7*c^6*d^3
- 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^5*c^4*d^5 - 30*a^3*b^4*c^3*
d^6 + 12*a^4*b^3*c^2*d^7)) - (24*a*b^6*c^6*d^4*(c + d*x^2)^(1/2))/((c^3)^(1
/2)*(6*b^7*c^6*d^3 - 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^5*c^4*d^
5 - 30*a^3*b^4*c^3*d^6 + 12*a^4*b^3*c^2*d^7)) + (38*a^2*b^5*c^5*d^5*(c + d*
x^2)^(1/2))/((c^3)^(1/2)*(6*b^7*c^6*d^3 - 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^
8 + 38*a^2*b^5*c^4*d^5 - 30*a^3*b^4*c^3*d^6 + 12*a^4*b^3*c^2*d^7)) - (30*a^
3*b^4*c^4*d^6*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(6*b^7*c^6*d^3 - 24*a*b^6*c^5
*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^5*c^4*d^5 - 30*a^3*b^4*c^3*d^6 + 12*a^4*b
^3*c^2*d^7)) + (12*a^4*b^3*c^3*d^7*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(6*b^7*c
^6*d^3 - 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^5*c^4*d^5 - 30*a^3*b
^4*c^3*d^6 + 12*a^4*b^3*c^2*d^7)) - (2*a^5*b^2*c^2*d^8*(c + d*x^2)^(1/2))/(
(c^3)^(1/2)*(6*b^7*c^6*d^3 - 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^
5*c^4*d^5 - 30*a^3*b^4*c^3*d^6 + 12*a^4*b^3*c^2*d^7)))/(a*(c^3)^(1/2)) - d/
((c + d*x^2)^(1/2)*(b*c^2 - a*c*d))

3.719 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -b^2*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c)^
(3/2)-d/c/(-a*d+b*c)/x/(d*x^2+c)^(1/2)-(-2*a*d+b*c)*(d*x^2+c)^(1/2)/a/c^2/(
-a*d+b*c)/x
```

Rubi [A]
time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {483, 597, 12, 385, 211}

$$-\frac{b^2\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}\,(bc-2ad)}{ac^2x(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*x*Sqrt[c + d*x^2])) - ((b*c - 2*a*d)*Sqrt[c + d*x^2])/(a
*c^2*(b*c - a*d)*x) - (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(a^(3/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}+\frac{\int \frac{bc-2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx}{c(bc-ad)} \\
&= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{\int \frac{b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{ac^2(bc-ad)} \\
&= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{a(bc-ad)} \\
&= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{a(bc-} \\
&= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx}}\right.}{a^{3/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.42, size = 127, normalized size = 1.02

$$
\frac{-bc(c+dx^2)+ad(c+2dx^2)}{ac^2(bc-ad)x\sqrt{c+dx^2}}+\frac{b^2\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{a^{3/2}(bc-ad)^{3/2}}
$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]

[Out] (-(b*c*(c + d*x^2)) + a*d*(c + 2*d*x^2))/(a*c^2*(b*c - a*d)*x*Sqrt[c + d*x^
2]) + (b^2*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*
Sqrt[b*c - a*d])])/(a^(3/2)*(b*c - a*d)^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(110) = 220$.
time = 0.12, size = 779, normalized size = 6.28

method	result
risch	$-\frac{\sqrt{d x^2+c}}{c^2 a x}-\frac{b d^2\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 d+2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}}{2c^2\left(b\sqrt{-cd}+\sqrt{-ab}\,d\right)\left(b\sqrt{-cd}-\sqrt{-ab}\,d\right)\left(x-\frac{\sqrt{-cd}}{d}\right)}-\frac{b^3 d\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-a\cdots}}{\cdots}}{\cdots}\right)}{\cdots}$
default	$-\frac{b\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\cdots}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x\cdots\right)}}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2*b/a/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*
(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))
^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b

```
)^(1/2))))+1/2*b/a/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/
(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4
*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
+1/b*(-a*b)^(1/2))))+1/a*(-1/c/x/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(110) = 220$.
time = 1.11, size = 560, normalized size = 4.52

$$\left[\frac{(b^2c^2dx^3+b^2c^3x)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(ab^2c^2d-3\,a^2bcd^2+2\,a^3d^3)x^2)\sqrt{dx^2+c}}{4\,((a^2b^2c^4d-2\,a^3bc^3d^2+a^4c^2d^3)x^3+(a^2b^2c^5-2\,a^3bc^4d+a^4c^3d^2)x)},-\frac{(b^2c^2dx^3+b^2c^3x)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(ab^2c^2d-3\,a^2bcd^2+2\,a^3d^3)x^2)\sqrt{dx^2+c}}{2\,((a^2b^2c^4d-2\,a^3bc^3d^2+a^4c^2d^3)x^3+(a^2b^2c^5-2\,a^3bc^4d+a^4c^3d^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*
b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c
- 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (a*b^2*c^2*d - 3
*a^2*b*c*d^2 + 2*a^3*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^4*d - 2*a^3*b*c
^3*d^2 + a^4*c^2*d^3)*x^3 + (a^2*b^2*c^5 - 2*a^3*b*c^4*d + a^4*c^3*d^2)*x),
 -1/2*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*
c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x
^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (
a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^
4*d - 2*a^3*b*c^3*d^2 + a^4*c^2*d^3)*x^3 + (a^2*b^2*c^5 - 2*a^3*b*c^4*d + a
^4*c^3*d^2)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\,(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
[Out] Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 2.48, size = 152, normalized size = 1.23

$$-\frac{b^2\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,(abc-a^2d)}+\frac{d^2x}{(bc^3-ac^2d)\sqrt{dx^2+c}}+\frac{2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] -b^2*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(a*b*c - a^2*d)) + d^2*x/
((b*c^3 - a*c^2*d)*sqrt(d*x^2 + c)) + 2*sqrt(d)/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^2 - c)*a*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
[Out] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.720 $\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}}-\frac{1}{2acx^2\sqrt{c+dx^2}}+\frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{3/2}}$$

```
[Out] 1/2*(3*a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(5/2)-b^(5/2)*arct
anh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)-1/2*d*(-
3*a*d+b*c)/a/c^2/(-a*d+b*c)/(d*x^2+c)^(1/2)-1/2/a/c/x^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 105, 157, 162, 65, 214}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}-\frac{d(bc-3ad)}{2ac^2\sqrt{c+dx^2}\,(bc-ad)}-\frac{1}{2acx^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(d*(b*c - 3*a*d))/(a*c^2*(b*c - a*d)*Sqrt[c + d*x^2]) - 1/(2*a*c*x^2*S
qrt[c + d*x^2]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2
*c^(5/2)) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a
^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{1}{2acx^2\sqrt{c+dx^2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+3ad)+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}} - \frac{1}{2acx^2\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}(bc-ad)(2bc+3ad)}{x(a+bx)\sqrt{c}}\right.}{ac^2(bc-}$$

$$= -\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}} - \frac{1}{2acx^2\sqrt{c+dx^2}} + \frac{b^3\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{2a^2(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}} - \frac{1}{2acx^2\sqrt{c+dx^2}} + \frac{b^3\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{a^2d(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}} - \frac{1}{2acx^2\sqrt{c+dx^2}} + \frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{}}\right.}{2a^2c^{5/2}}$$

Mathematica [A]
time = 0.39, size = 142, normalized size = 0.91

$$\frac{\frac{a(-bc(c+dx^2)+ad(c+3dx^2))}{c^2(bc-ad)x^2\sqrt{c+dx^2}} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((a*(-(b*c*(c + d*x^2)) + a*d*(c + 3*d*x^2)))/(c^2*(b*c - a*d)*x^2*Sqrt[c +
 d*x^2]) - (2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])
/(-(b*c) + a*d)^(3/2) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/
c^(5/2))/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(130) = 260.
time = 0.14, size = 847, normalized size = 5.43

method	result

risch	$-\frac{\sqrt{d x^2+c}}{2c^2 a x^2}+\frac{b d^3 \sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 d+2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}}{2c^2\left(b\sqrt{-cd}+\sqrt{-ab}\,d\right)\left(\sqrt{-ab}\,d-b\sqrt{-cd}\right)\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}+\frac{3\ln\left(\frac{2c+2\sqrt{c}\sqrt{d}\cdots}{x}\right)}{2c^{\frac{5}{2}}a\cdots}$
default	$b\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\left(2\cdots\right.}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x-\cdots\right.}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/a^2*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*
(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-
a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/
(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+1
/2*b/a^2*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(
-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(
a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/
a*(-1/2/c/x^2/(d*x^2+c)^(1/2)-3/2*d/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*
c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))-b/a^2*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((
2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(130) = 260$.
time = 2.27, size = 1291, normalized size = 8.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^3*d*x^4 + b^2*c^4*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 +
 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2
*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(
b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - ((2*b^2*c^2*d + a*b*c*d^2 -
3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(c)*log(-(d
*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b*c^3 - a^2*c^2*d + (a*
b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x
^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), -1/4*(2*((2*b^2*c^2*d + a*b*c*d^2 - 3*a^
2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) + (b^2*c^3*d*x^4 + b^2*c^4*x^2)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqr
t(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(a*b*c^3
 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*
d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), 1/4*(2*(b^2*c^3*d*x^4
+ b^2*c^4*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + ((2*b^2*c^2*d + a*b*c*d
^2 - 3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(c)*lo
g(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a*b*c^3 - a^2*c^2*d
+ (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3*d
^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), 1/2*((b^2*c^3*d*x^4 + b^2*c^4*x^2)*
sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqr
t(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)
*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/
sqrt(d*x^2 + c)) - (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sq
rt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x
^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^2) (c + dx^2)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 1.28, size = 172, normalized size = 1.10

$$\frac{b^3 \arctan\left(\frac{\sqrt{dx^2+c}\, b}{\sqrt{-b^2c+abd}}\right)}{(a^2bc - a^3d)\sqrt{-b^2c+abd}} - \frac{(dx^2+c)bcd - 3\,(dx^2+c)ad^2 + 2\,acd^2}{2\,(abc^3 - a^2c^2d)\left((dx^2+c)^{\frac{3}{2}} - \sqrt{dx^2+c}\; c\right)} - \frac{(2\,bc + 3\,ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}\; c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c - a^3*d)*sqrt(
-b^2*c + a*b*d)) - 1/2*((d*x^2 + c)*b*c*d - 3*(d*x^2 + c)*a*d^2 + 2*a*c*d^2
)/((a*b*c^3 - a^2*c^2*d)*((d*x^2 + c)^(3/2) - sqrt(d*x^2 + c)*c)) - 1/2*(2*
b*c + 3*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

Mupad [B]
time = 1.21, size = 2500, normalized size = 16.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] (d^2/(b*c^2 - a*c*d) + (d*(c + d*x^2)*(3*a*d - b*c))/(2*a*c^2*(a*d - b*c)))
/(c*(c + d*x^2)^(1/2) - (c + d*x^2)^(3/2)) + (atan((((-b^5*(a*d - b*c)^3)^(
1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*d^2 - 320*a^4*b^9*c^12*d^3 + 16
*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 160*a^7*b^6*c^9*d^6 - 544*a^8*b^
5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b^3*c^6*d^9))/2 - ((-b^5*(a*d -
b*c)^3)^(1/2)*(416*a^8*b^6*c^12*d^5 - 32*a^6*b^8*c^14*d^3 - 1024*a^9*b^5*c^
11*d^6 + 1056*a^10*b^4*c^10*d^7 - 512*a^11*b^3*c^9*d^8 + 96*a^12*b^2*c^8*d^
9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(512*a^7*b^8*c^16*d^2 - 2
816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14*d^4 - 7680*a^10*b^5*c^13*d^5 + 512
0*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^11*d^7 + 256*a^13*b^2*c^10*d^8))/(4*a
^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b*c)^3))*1i)/(a^2*(a*d - b*c)^3) + ((-b^5
```

*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*d^2 - 320*a^4*
b^9*c^12*d^3 + 16*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 160*a^7*b^6*c^9
*d^6 - 544*a^8*b^5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b^3*c^6*d^9))/2
 - ((-b^5*(a*d - b*c)^3)^(1/2)*(32*a^6*b^8*c^14*d^3 - 416*a^8*b^6*c^12*d^5
+ 1024*a^9*b^5*c^11*d^6 - 1056*a^10*b^4*c^10*d^7 + 512*a^11*b^3*c^9*d^8 - 9
6*a^12*b^2*c^8*d^9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(512*a^7
*b^8*c^16*d^2 - 2816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14*d^4 - 7680*a^10*b
^5*c^13*d^5 + 5120*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^11*d^7 + 256*a^13*b^
2*c^10*d^8))/(4*a^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b*c)^3))*1i)/(a^2*(a*d -
 b*c)^3))/(32*a^2*b^10*c^11*d^3 - 144*a^3*b^9*c^10*d^4 + 96*a^4*b^8*c^9*d^5
 + 256*a^5*b^7*c^8*d^6 - 384*a^6*b^6*c^7*d^7 + 144*a^7*b^5*c^6*d^8 - ((-b^5
*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*d^2 - 320*a^4*
b^9*c^12*d^3 + 16*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 160*a^7*b^6*c^9
*d^6 - 544*a^8*b^5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b^3*c^6*d^9))/2
 - ((-b^5*(a*d - b*c)^3)^(1/2)*(416*a^8*b^6*c^12*d^5 - 32*a^6*b^8*c^14*d^3
- 1024*a^9*b^5*c^11*d^6 + 1056*a^10*b^4*c^10*d^7 - 512*a^11*b^3*c^9*d^8 + 9
6*a^12*b^2*c^8*d^9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(512*a^7
*b^8*c^16*d^2 - 2816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14*d^4 - 7680*a^10*b
^5*c^13*d^5 + 5120*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^11*d^7 + 256*a^13*b^
2*c^10*d^8))/(4*a^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b*c)^3)))/(a^2*(a*d - b*
c)^3) + ((-b^5*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*
d^2 - 320*a^4*b^9*c^12*d^3 + 16*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 1
60*a^7*b^6*c^9*d^6 - 544*a^8*b^5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b
^3*c^6*d^9))/2 - ((-b^5*(a*d - b*c)^3)^(1/2)*(32*a^6*b^8*c^14*d^3 - 416*a^8
*b^6*c^12*d^5 + 1024*a^9*b^5*c^11*d^6 - 1056*a^10*b^4*c^10*d^7 + 512*a^11*b
^3*c^9*d^8 - 96*a^12*b^2*c^8*d^9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^
(1/2)*(512*a^7*b^8*c^16*d^2 - 2816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14*d^4
 - 7680*a^10*b^5*c^13*d^5 + 5120*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^11*d^7
 + 256*a^13*b^2*c^10*d^8))/(4*a^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b*c)^3)))/
(a^2*(a*d - b*c)^3)))*(-b^5*(a*d - b*c)^3)^(1/2)*1i)/(a^2*(a*d - b*c)^3) +
(atanh((440*a^4*b^8*c^11*d^5*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*c
^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7
+ 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a^
10*b^2*c^3*d^11)) - (240*a^3*b^9*c^12*d^4*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(
440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6
*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*
d^10 + 216*a^10*b^2*c^3*d^11)) + (480*a^5*b^7*c^10*d^6*(c + d*x^2)^(1/2))/(
(c^5)^(1/2)*(440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d
^6 - 1464*a^6*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864
*a^9*b^3*c^4*d^10 + 216*a^10*b^2*c^3*d^11)) - (1464*a^6*b^6*c^9*d^7*(c + d*
x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*
a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*
c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a^10*b^2*c^3*d^11)) + (496*a^7*b^5*c^8
d^8(c + d*x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^1
0*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 +

```
936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a^10*b^2*c^3*d^11)) + (936
*a^8*b^4*c^7*d^9*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*c^9*d^5 - 240
*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7 + 496*a^7*b^
5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a^10*b^2*c^3*d
^11)) - (864*a^9*b^3*c^6*d^10*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*
c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7
 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a
^10*b^2*c^3*d^11)) + (216*a^10*b^2*c^5*d^11*(c + d*x^2)^(1/2))/((c^5)^(1/2)
*(440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a
^6*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^
4*d^10 + 216*a^10*b^2*c^3*d^11)))*(3*a*d + 2*b*...
```

3.721 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}}-\frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3}+\frac{(3bc-4ad)(bc+2ad)\sqrt{c+dx^2}}{3a^2c^3(bc-ad)x}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] b^3*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(
3/2)-d/c/(-a*d+b*c)/x^3/(d*x^2+c)^(1/2)-1/3*(-4*a*d+b*c)*(d*x^2+c)^(1/2)/a/
c^2/(-a*d+b*c)/x^3+1/3*(-4*a*d+3*b*c)*(2*a*d+b*c)*(d*x^2+c)^(1/2)/a^2/c^3/(
-a*d+b*c)/x
```

Rubi [A]
time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {483, 597, 12, 385, 211}

$$\frac{b^3\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\,(3bc-4ad)(2ad+bc)}{3a^2c^3x(bc-ad)}-\frac{\sqrt{c+dx^2}\,(bc-4ad)}{3ac^2x^3(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*x^3*Sqrt[c + d*x^2])) - ((b*c - 4*a*d)*Sqrt[c + d*x^2])/
(3*a*c^2*(b*c - a*d)*x^3) + ((3*b*c - 4*a*d)*(b*c + 2*a*d)*Sqrt[c + d*x^2])
/(3*a^2*c^3*(b*c - a*d)*x) + (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} + \frac{\int \frac{bc-4ad-4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} - \frac{\int \frac{(3bc-4ad)(bc+2ad)+2bd(}{x^2(a+bx^2)\sqrt{c+}}}{3ac^2(bc-a}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} + \frac{(3bc-4ad)(bc+2ad}{3a^2c^3(bc-a}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} + \frac{(3bc-4ad)(bc+2ad}{3a^2c^3(bc-a}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} + \frac{(3bc-4ad)(bc+2ad}{3a^2c^3(bc-a}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} + \frac{(3bc-4ad)(bc+2ad}{3a^2c^3(bc-a}$$

Mathematica [A]

time = 0.51, size = 175, normalized size = 0.99

$$\frac{3b^2c^2x^2(c+dx^2)+a^2d(c^2-4cdx^2-8d^2x^4)+abc(-c^2+cdx^2+2d^2x^4)}{3a^2c^3(bc-ad)x^3\sqrt{c+dx^2}}-\frac{b^3\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (3*b^2*c^2*x^2*(c + d*x^2) + a^2*d*(c^2 - 4*c*d*x^2 - 8*d^2*x^4) + a*b*c*(-
c^2 + c*d*x^2 + 2*d^2*x^4))/(3*a^2*c^3*(b*c - a*d)*x^3*Sqrt[c + d*x^2]) - (
b^3*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*
c - a*d])])/(a^(5/2)*(b*c - a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(156) = 312.
time = 0.14, size = 847, normalized size = 4.81

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\,(-5ad\,x^2-3c\,x^2b+ac)}{3c^3a^2x^3}-\frac{b\,d^3\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 d+2\sqrt{-cd}\,\left(x-\frac{\sqrt{-cd}}{d}\right)}}{2c^3\left(b\sqrt{-cd}+\sqrt{-ab}\,d\right)\left(\sqrt{-ab}\,d-b\sqrt{-cd}\right)\left(x-\frac{\sqrt{-cd}}{d}\right)}-\frac{b^4d\ln\left(\ldots\right.}{\ldots}$
default	$b^2\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\,\left(2d\ldots\right.}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x-\ldots\right.}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*
c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/
b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/
2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-
a*b)^(1/2))))-1/2*b^2/a^2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)
^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b
*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+
1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2))))+1/a*(-1/3/c/x^3/(d*x^2+c)^(1/2)-4/3*d/c*(-1/c/x
/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2)))-b/a^2*(-1/c/x/(d*x^2+c)^(1/2)-
2*d/c^2*x/(d*x^2+c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(156) = 312.
time = 1.24, size = 706, normalized size = 4.01

$$\left[\frac{3\,(b^3c^3dx^5+b^3c^4x^3)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2-(3\,ab^3c^3d-a^2b^2c^2d^2-10\,a^3bcd^3+8\,a^4d^4)x^4-(3\,ab^3c^4-2\,a^2b^2c^3d-5\,a^3bc^2d^2+4\,a^4cd^3)x^2)\sqrt{dx^2+c}}{12\,((a^3b^2c^5d-2\,a^4bc^4d^2+a^5c^3d^3)x^5+(a^3b^2c^6-2\,a^4bc^5d+a^5c^4d^2)x^3)},\ \frac{3\,(b^3c^3dx^5+b^3c^4x^3)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-2\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2-(3\,ab^3c^3d-a^2b^2c^2d^2-10\,a^3bcd^3+8\,a^4d^4)x^4-(3\,ab^3c^4-2\,a^2b^2c^3d-5\,a^3bc^2d^2+4\,a^4cd^3)x^2)\sqrt{dx^2+c}}{6\,((a^3b^2c^5d-2\,a^4bc^4d^2+a^5c^3d^3)x^5+(a^3b^2c^6-2\,a^4bc^5d+a^5c^4d^2)x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(
(b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) - 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 - (3*a*b^
3*c^3*d - a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 8*a^4*d^4)*x^4 - (3*a*b^3*c^4
- 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((
a^3*b^2*c^5*d - 2*a^4*b*c^4*d^2 + a^5*c^3*d^3)*x^5 + (a^3*b^2*c^6 - 2*a^4*b
*c^5*d + a^5*c^4*d^2)*x^3), 1/6*(3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(a*b*c
 - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x
```

```
^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(a^2*b^2*c^4
 - 2*a^3*b*c^3*d + a^4*c^2*d^2 - (3*a*b^3*c^3*d - a^2*b^2*c^2*d^2 - 10*a^3*
b*c*d^3 + 8*a^4*d^4)*x^4 - (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2
 + 4*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c^5*d - 2*a^4*b*c^4*d^2 + a
^5*c^3*d^3)*x^5 + (a^3*b^2*c^6 - 2*a^4*b*c^5*d + a^5*c^4*d^2)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 1.60, size = 275, normalized size = 1.56

$$\frac{b^3\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bc-a^3d\right)\sqrt{abcd-a^2d^2}}-\frac{d^3x}{\left(bc^4-ac^3d\right)\sqrt{dx^2+c}}-\frac{2\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2acd^{\frac{3}{2}}+3\,bc^3\sqrt{d}+5\,ac^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b^3*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt(a*b*c*d - a^2*d^2)) - d^3*x
/((b*c^4 - a*c^3*d)*sqrt(d*x^2 + c)) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*b*c*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*
c*d^(3/2) + 3*b*c^3*sqrt(d) + 5*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 +
c))^2 - c)^3*a^2*c^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.722 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] -1/3*c*x/d/(-a*d+b*c)/(d*x^2+c)^(3/2)+a^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(
1/2)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(5/2)+1/3*(-4*a*d+b*c)*x/d/(-a*d+b*c)^2/(d
*x^2+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {481, 541, 12, 385, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}+\frac{x(bc-4ad)}{3d\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{cx}{3d\,(c+dx^2)^{3/2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(c*x)/(d*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((b*c - 4*a*d)*x)/(3*d*(b*c
- a*d)^2*Sqrt[c + d*x^2]) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sq
rt[c + d*x^2])])/(b*c - a*d)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx &= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{\int \frac{ac+(bc-3ad)x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{3d(bc-ad)}\\
&= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{\int \frac{3a^2cd}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3cd(bc-ad)^2}\\
&= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}}{(bc-ad)^2}\\
&= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^2\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)}\right.}{(bc-}\\
&= -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-a}}{\sqrt{a}\sqrt{c+}}\right.}{(bc-ad)^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.37, size = 115, normalized size = 0.98

$$
\frac{-3acx+bcx^3-4adx^3}{3(bc-ad)^2(c+dx^2)^{3/2}}-\frac{a^{3/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*a*c*x + b*c*x^3 - 4*a*d*x^3)/(3*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (a^(
3/2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b
*c - a*d])])/(b*c - a*d)^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(99) = 198$.
time = 0.10, size = 1502, normalized size = 12.84

method	result	size
default	Expression too large to display	1502

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*x/d/(d*x^2+c)^(3/2)+1/2*c/d*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d
*x^2+c)^(1/2)))-a/b^2*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))+1
/2/b^2*a^2/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c
)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^
2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(
1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/
b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/
(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^
(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2/b^2*a^2/(-a*b)^(1/2)*(-1/3/(a*
d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-
2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*
d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1
/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d
*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2
```

```
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1
/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(99) = 198.
time = 1.86, size = 524, normalized size = 4.48

$$\left[\frac{3\,(ad^2x^4 + 2\,acdx^2 + ac^2)\sqrt{-\frac{a}{bc - ad}}\log\left(\frac{(b^2c^2 - 8\,abcd + 8\,a^2d^2)x^4 + a^2c^2 - 2\,(3\,abc^2 - 4\,a^2cd)x^2 + 4\,((b^2c^2 - 3\,abcd + 2\,a^2d^2)x^3 - (abc^2 - a^2cd)x)\sqrt{dx^2 + c}\sqrt{-\frac{a}{bc - ad}}}{b^2x^4 + 2\,abx^2 + a^2}\right) + 4\,((bc - 4\,ad)x^3 - 3\,acx)\sqrt{dx^2 + c}}{12\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^4 + 2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}, -\frac{3\,(ad^2x^4 + 2\,acdx^2 + ac^2)\sqrt{\frac{a}{bc - ad}}\arctan\left(-\frac{((bc - 2\,ad)x^2 - ac)\sqrt{dx^2 + c}\sqrt{\frac{a}{bc - ad}}}{2\,(adx^3 + acx)}\right) - 2\,((bc - 4\,ad)x^3 - 3\,acx)\sqrt{dx^2 + c}}{6\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^4 + 2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 +
 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x
^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((b*c - 4*a*
d)*x^3 - 3*a*c*x)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (
b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a
^2*c*d^3)*x^2), -1/6*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(a/(b*c - a*d
))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d)
)/(a*d*x^3 + a*c*x)) - 2*((b*c - 4*a*d)*x^3 - 3*a*c*x)*sqrt(d*x^2 + c))/(b^
2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x
^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + bx^2)\,(c + dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(99) = 198.

time = 1.42, size = 304, normalized size = 2.60

$$-\frac{a^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{abcd-a^2d^2}}+\frac{\left(\frac{\left(b^3c^4d-6\,ab^2c^3d^2+9\,a^2bc^2d^3-4\,a^3cd^4\right)x^2}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}-\frac{3\left(ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3\right)}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -a^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^2
*d^2)) + 1/3*((b^3*c^4*d - 6*a*b^2*c^3*d^2 + 9*a^2*b*c^2*d^3 - 4*a^3*c*d^4)
*x^2/(b^4*c^5*d - 4*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a
^4*c*d^5) - 3*(a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)/(b^4*c^5*d - 4*
a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(d*x^2
+ c)^(3/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)\,\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.723 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{c}{3d(bc-ad)\,(c+dx^2)^{3/2}}-\frac{a}{(bc-ad)^2\sqrt{c+dx^2}}+\frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] -1/3*c/d/(-a*d+b*c)/(d*x^2+c)^(3/2)+a*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d
+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(5/2)-a/(-a*d+b*c)^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 79, 53, 65, 214}

$$-\frac{a}{\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{c}{3d\,(c+dx^2)^{3/2}\,(bc-ad)}+\frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*c/(d*(b*c - a*d)*(c + d*x^2)^(3/2)) - a/((b*c - a*d)^2*Sqrt[c + d*x^2]
) + (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a
*d)^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{c}{3d(bc-ad)(c+dx^2)^{3/2}} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2(bc-ad)}$$

$$= -\frac{c}{3d(bc-ad)(c+dx^2)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c}}\right.}{2(bc-a}$$

$$= -\frac{c}{3d(bc-ad)(c+dx^2)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{d(bc-a}$$

$$= -\frac{c}{3d(bc-ad)(c+dx^2)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} + \frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{bc-}}\right.}{(bc-ad)^{5/2}}$$

Mathematica [A]

time = 0.23, size = 100, normalized size = 0.97

$$\frac{-bc^2-ad(2c+3dx^2)}{3d(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(b*c^2) - a*d*(2*c + 3*d*x^2))/(3*d*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (a
*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1408 vs. 2(87) = 174.

time = 0.09, size = 1409, normalized size = 13.68

method	result	size
default	Expression too large to display	1409

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b/d/(d*x^2+c)^(3/2)-1/2*a/b^2*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2)
)^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/
2)/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-
b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1
/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)
*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b
)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2*a/b^2*(-1/3/(a*d-b*
c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*
(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*
c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-
1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+
```

```
1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/
b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(87) = 174.
time = 1.45, size = 535, normalized size = 5.19

$$\left[\frac{3\,(ad^3x^4 + 2\,acd^2x^2 + ac^2d)\sqrt{\frac{b}{bc - ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(2\,b^2c^2-3\,abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2 + c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right) - 4\,(3\,ad^2x^2 + bc^2 + 2\,acd)\sqrt{dx^2 + c}}{12\,(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3 + (b^2c^2d^3 - 2\,abcd^4 + a^2d^5)x^4 + 2\,(b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^2)}, -\frac{3\,(ad^3x^4 + 2\,acd^2x^2 + ac^2d)\sqrt{-\frac{b}{bc - ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2 + c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^2+bc)}\right) + 2\,(3\,ad^2x^2 + bc^2 + 2\,acd)\sqrt{dx^2 + c}}{6\,(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3 + (b^2c^2d^3 - 2\,abcd^4 + a^2d^5)x^4 + 2\,(b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt(b/(b*c - a*d))*log((b^2
*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2
+ 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2
+ c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a*d^2*x^2 + b
*c^2 + 2*a*c*d)*sqrt(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 +
 (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3
 + a^2*c*d^4)*x^2), -1/6*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt(-b/(
b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c
 - a*d))/(b*d*x^2 + b*c)) + 2*(3*a*d^2*x^2 + b*c^2 + 2*a*c*d)*sqrt(d*x^2 +
c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 +
 a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + bx^2)\,(c + dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [A]
time = 1.07, size = 127, normalized size = 1.23

$$-\frac{\frac{3\,abd\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{bc^2+3\,(dx^2+c)ad-acd}{(b^2c^2-2\,abcd+a^2d^2)(dx^2+c)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (b*c^2 + 3*(d*x^2 + c)*a*d - a*c
*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 + c)^(3/2)))/d
```

Mupad [B]
time = 0.54, size = 110, normalized size = 1.07

$$\frac{\frac{c}{3\,(a\,d-b\,c)}-\frac{a\,d\,(d\,x^2+c)}{(a\,d-b\,c)^2}}{d\,(d\,x^2+c)^{3/2}}-\frac{a\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^{5/2}}\right)}{(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] (c/(3*(a*d - b*c)) - (a*d*(c + d*x^2))/(a*d - b*c)^2)/(d*(c + d*x^2)^(3/2))
 - (a*b^(1/2)*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*
d))/(a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2)
```

3.724 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}}-\frac{\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] 1/3*x/(-a*d+b*c)/(d*x^2+c)^(3/2)-b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2
+c)^(1/2))*a^(1/2)/(-a*d+b*c)^(5/2)+1/3*(a*d+2*b*c)*x/c/(-a*d+b*c)^2/(d*x^2
+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {482, 541, 12, 385, 211}

$$-\frac{\sqrt{a}\,b\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}+\frac{x(ad+2bc)}{3c\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{x}{3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] x/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((2*b*c + a*d)*x)/(3*c*(b*c - a*d)^2*
Sqrt[c + d*x^2]) - (Sqrt[a]*b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(b*c - a*d)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)\left(c+dx^2\right)^{5/2}}\,dx = \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{a-2bx^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{3(bc-ad)}$$

$$= \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{\int \frac{3abc}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3c(bc-ad)^2}$$

$$= \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,d}{(bc-ad)^2}$$

$$= \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{(bc-ad}$$

$$= \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+d}}\right.}{(bc-ad)^{5/2}}$$

Mathematica [A]
time = 0.38, size = 122, normalized size = 1.06

$$\frac{ad^2x^3+bcx(3c+2dx^2)}{3c(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{\sqrt{a}\,b\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (a*d^2*x^3 + b*c*x*(3*c + 2*d*x^2))/(3*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) +
 (Sqrt[a]*b*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(b*c - a*d)^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1438 vs. $2(97) = 194$.
time = 0.09, size = 1439, normalized size = 12.51

method	result	size
default	Expression too large to display	1439

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))-1/2*a/(-a*b)^(1/2)/
b*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a
*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*
d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)
/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*
d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^
2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)
^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/
b*(-a*b)^(1/2)))))+1/2*a/(-a*b)^(1/2)/b*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b
)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*
(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d
*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d
-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*
(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(97) = 194$.
time = 1.38, size = 550, normalized size = 4.78

$$\left[\frac{3\,(bcd^2x^4 + 2\,bc^2dx^2 + bc^3)\sqrt{-\frac{a}{bc - ad}}\log\left(\frac{(b^2c^2 - 8\,abcd + 8\,a^2d^2)x^4 + a^2c^2 - 2\,(3\,abc^2 - 4\,a^2cd)x^2 - 4\,((b^2c^2 - 3\,abcd + 2\,a^2d^2)x^3 - (abc^2 - a^2cd)x)\sqrt{dx^2 + c}\sqrt{-\frac{a}{bc - ad}}}{b^2x^4 + 2\,abx^2 + a^2}\right) + 4\,(3\,bc^2x + (2\,bcd + ad^2)x^3)\sqrt{dx^2 + c}}{12\,(b^2c^5 - 2\,abc^4d + a^2c^3d^2 + (b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^4 + 2\,(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3)x^2)}, \frac{3\,(bcd^2x^4 + 2\,bc^2dx^2 + bc^3)\sqrt{\frac{a}{bc - ad}}\arctan\left(-\frac{((bc - 2\,ad)x^2 - ac)\sqrt{dx^2 + c}\sqrt{\frac{a}{bc - ad}}}{2\,(adx^3 + acx)}\right) + 2\,(3\,bc^2x + (2\,bcd + ad^2)x^3)\sqrt{dx^2 + c}}{6\,(b^2c^5 - 2\,abc^4d + a^2c^3d^2 + (b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^4 + 2\,(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 + b*c^3)*sqrt(-a/(b*c - a*d))*log(((b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x
^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt
(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b*c^2
*x + (2*b*c*d + a*d^2)*x^3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c
^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a
*b*c^3*d^2 + a^2*c^2*d^3)*x^2), 1/6*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 + b*c^3
)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)
*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*(3*b*c^2*x + (2*b*c*d + a*d^2)*
x^3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 -
 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^
3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + bx^2)\,(c + dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(97) = 194.
time = 0.96, size = 291, normalized size = 2.53

$$\frac{ab\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}}+\frac{\left(\frac{(2\,b^3c^3d^2-3\,ab^2c^2d^3+a^3d^5)x^2}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}+\frac{3\,(b^3c^4d-2\,ab^2c^3d^2+a^2bc^2d^3)}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] a*b*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^2*
d^2)) + 1/3*((2*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + a^3*d^5)*x^2/(b^4*c^5*d - 4
*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5) + 3*(b^3*
c^4*d - 2*a*b^2*c^3*d^2 + a^2*b*c^2*d^3)/(b^4*c^5*d - 4*a*b^3*c^4*d^2 + 6*a
^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(d*x^2 + c)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(x^2/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.725 $\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{1}{3(bc-ad)(c+dx^2)^{3/2}}+\frac{b}{(bc-ad)^2\sqrt{c+dx^2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] 1/3/(-a*d+b*c)/(d*x^2+c)^(3/2)-b^(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*
d+b*c)^(1/2))/(-a*d+b*c)^(5/2)+b/(-a*d+b*c)^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {455, 53, 65, 214}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{b}{\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{1}{3\,(c+dx^2)^{3/2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] 1/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + b/((b*c - a*d)^2*Sqrt[c + d*x^2]) - (
b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2(bc-ad)}$$

$$= \frac{1}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{b}{(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} \cdots\right.}{2(bc-ad)^2}$$

$$= \frac{1}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{b}{(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \cdots\right.}{d(bc-ad)^2}$$

$$= \frac{1}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{b}{(bc-ad)^2\sqrt{c+dx^2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx \cdots}}{\sqrt{bc-ad}}\right.}{(bc-ad)^{5/2}}$$

Mathematica [A]
time = 0.16, size = 90, normalized size = 0.92

$$\frac{4bc-ad+3bdx^2}{3(bc-ad)^2(c+dx^2)^{3/2}} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (4*b*c - a*d + 3*b*d*x^2)/(3*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (b^(3/2)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. $2(82) = 164$.
time = 0.10, size = 1390, normalized size = 14.18

method	result	size
default	Expression too large to display	1390

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x-1/b
*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1
6/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(
1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)
/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+
4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-
a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x-1/b*(-a*b)^(1/2)))))+1/2/b*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*
d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a
*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2
)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(82) = 164.
time = 1.37, size = 511, normalized size = 5.21

$$\left[\frac{3\,(bd^2x^4+2\,bcdx^2+bc^2)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(2\,b^2c^2-3\,abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(3\,bdx^2+4\,bc-ad)\sqrt{dx^2+c}}{12\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^4+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^2)},\ \frac{3\,(bd^2x^4+2\,bcdx^2+bc^2)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^2+bc)}\right)+2\,(3\,bdx^2+4\,bc-ad)\sqrt{dx^2+c}}{6\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^4+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2
*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*
(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)
*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b*d*x^2 + 4*b*c -
 a*d)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^
2), 1/6*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(-b/(b*c - a*d))*arctan(1/
2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b
*c)) + 2*(3*b*d*x^2 + 4*b*c - a*d)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d
+ a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d -
2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

Sympy [**A**]
time = 9.68, size = 85, normalized size = 0.87

$$\frac{b}{\sqrt{c+dx^2}\,(ad-bc)^2}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}-\frac{1}{3\,(c+dx^2)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] b/(sqrt(c + d*x**2)*(a*d - b*c)**2) + b*atan(sqrt(c + d*x**2)/sqrt((a*d - b
*c)/b))/(sqrt((a*d - b*c)/b)*(a*d - b*c)**2) - 1/(3*(c + d*x**2)**(3/2)*(a*
d - b*c))
```

Giac [A]
time = 0.76, size = 118, normalized size = 1.20

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{3\,(dx^2+c)b+bc-ad}{3\,(b^2c^2-2\,abcd+a^2d^2)(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*(3*(d*x^2 + c)*b + b*c - a*d)/((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 + c)^(3/2))
```

Mupad [B]
time = 0.52, size = 103, normalized size = 1.05

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)}{{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{1}{3\,\left(a\,d-b\,c\right)}-\frac{b\,\left(d\,x^2+c\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] (b^(3/2)*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(
a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2) - (1/(3*(a*d - b*c)) - (b*(c + d*x^2))
/(a*d - b*c)^2)/(c + d*x^2)^(3/2)
```

3.726 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{5/2}}$$

```
[Out] -1/3*d*x/c/(-a*d+b*c)/(d*x^2+c)^(3/2)+b^2*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)
/(d*x^2+c)^(1/2))/(-a*d+b*c)^(5/2)/a^(1/2)-1/3*d*(-2*a*d+5*b*c)*x/c^2/(-a*d
+b*c)^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {425, 541, 12, 385, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{5/2}}-\frac{dx(5bc-2ad)}{3c^2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{dx}{3c\,(c+dx^2)^{3/2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(d*x)/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(5*b*c - 2*a*d)*x)/(3*c^2
*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx &= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}+\frac{\int \frac{3bc-2ad-2bdx^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{3c(bc-ad)}\\
&= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}}+\frac{\int \frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3c^2(bc-ad)^2}\\
&= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}}+\frac{b^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}}{(bc-ad)^2}\\
&= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}}+\frac{b^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)}\right.}{(bc-}\\
&= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+}}\right.}{\sqrt{a}(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 132, normalized size = 1.08

$$\frac{dx(ad(3c+2dx^2)-bc(6c+5dx^2))}{3c^2(bc-ad)^2(c+dx^2)^{3/2}}-\frac{b^2\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}\,(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*x*(a*d*(3*c + 2*d*x^2) - b*c*(6*c + 5*d*x^2)))/(3*c^2*(b*c - a*d)^2*(c +
 d*x^2)^(3/2)) - (b^2*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2])
)/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(104) = 208$.
time = 0.10, size = 1396, normalized size = 11.44

method	result	size
default	Expression too large to display	1396

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*
(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2
*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(
-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(
a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x+
1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3
/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2
)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*
a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2)
)^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)
*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^
(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1
/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*
c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(104) = 208.
time = 1.52, size = 764, normalized size = 6.26

$$\left[-\frac{3\,(b^2c^2d^2x^4 + 2\,b^2c^3dx^2 + b^2c^4)\sqrt{-abc + a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right) + 4\,((5\,ab^2c^2d^2 - 7\,a^2bcd^3 + 2\,a^3d^4)x^3 + 3\,(2\,ab^2c^3d - 3\,a^2bc^2d^2 + a^3cd^3)x)\sqrt{dx^2+c}}{12\,(ab^3c^7 - 3\,a^2b^2c^6d + 3\,a^3bc^5d^2 - a^4c^4d^3 + (ab^3c^5d^2 - 3\,a^2b^2c^4d^3 + 3\,a^3bc^3d^4 - a^4c^2d^5)x^4 + 2\,(ab^3c^6d - 3\,a^2b^2c^5d^2 + 3\,a^3bc^4d^3 - a^4c^3d^4)x^2)}, \frac{3\,(b^2c^2d^2x^4 + 2\,b^2c^3dx^2 + b^2c^4)\sqrt{abc - a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right) - 2\,((5\,ab^2c^2d^2 - 7\,a^2bcd^3 + 2\,a^3d^4)x^3 + 3\,(2\,ab^2c^3d - 3\,a^2bc^2d^2 + a^3cd^3)x)\sqrt{dx^2+c}}{6\,(ab^3c^7 - 3\,a^2b^2c^6d + 3\,a^3bc^5d^2 - a^4c^4d^3 + (ab^3c^5d^2 - 3\,a^2b^2c^4d^3 + 3\,a^3bc^3d^4 - a^4c^2d^5)x^4 + 2\,(ab^3c^6d - 3\,a^2b^2c^5d^2 + 3\,a^3bc^4d^3 - a^4c^3d^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] [-1/12*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-a*b*c + a^2*d
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((5*a*b^2*c^2*d^2 - 7*a^2*b*c*d^3 +
 2*a^3*d^4)*x^3 + 3*(2*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + a^3*c*d^3)*x)*sqrt(d
*x^2 + c))/(a*b^3*c^7 - 3*a^2*b^2*c^6*d + 3*a^3*b*c^5*d^2 - a^4*c^4*d^3 + (
a*b^3*c^5*d^2 - 3*a^2*b^2*c^4*d^3 + 3*a^3*b*c^3*d^4 - a^4*c^2*d^5)*x^4 + 2*
(a*b^3*c^6*d - 3*a^2*b^2*c^5*d^2 + 3*a^3*b*c^4*d^3 - a^4*c^3*d^4)*x^2), 1/6
*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(a*b*c - a^2*d)*arcta
n(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c
*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((5*a*b^2*c^2*d^2 - 7*a^2*b
*c*d^3 + 2*a^3*d^4)*x^3 + 3*(2*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + a^3*c*d^3)*x
)*sqrt(d*x^2 + c))/(a*b^3*c^7 - 3*a^2*b^2*c^6*d + 3*a^3*b*c^5*d^2 - a^4*c^4
*d^3 + (a*b^3*c^5*d^2 - 3*a^2*b^2*c^4*d^3 + 3*a^3*b*c^3*d^4 - a^4*c^2*d^5)*
x^4 + 2*(a*b^3*c^6*d - 3*a^2*b^2*c^5*d^2 + 3*a^3*b*c^4*d^3 - a^4*c^3*d^4)*x
^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^2)\,(c + dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2),x)
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(104) = 208.
time = 0.76, size = 321, normalized size = 2.63

$$-\frac{b^{2} \sqrt{d} \arctan\left(\frac{\left(\sqrt{d} x-\sqrt{dx^{2}+c}\right)^{2} b-bc+2 \, ad}{2 \, \sqrt{abcd-a^{2} d^{2}}}\right)}{\left(b^{2} c^{2}-2 \, abcd+a^{2} d^{2}\right) \sqrt{abcd-a^{2} d^{2}}}-\frac{\left(\frac{\left(5 \, b^{3} c^{3} d^{3}-12 \, ab^{2} c^{2} d^{4}+9 \, a^{2} bcd^{5}-2 \, a^{3} d^{6}\right) x^{2}}{b^{4} c^{6} d-4 \, ab^{3} c^{5} d^{2}+6 \, a^{2} b^{2} c^{4} d^{3}-4 \, a^{3} bc^{3} d^{4}+a^{4} c^{2} d^{5}}+\frac{3 \, \left(2 \, b^{3} c^{4} d^{2}-5 \, ab^{2} c^{3} d^{3}+4 \, a^{2} bc^{2} d^{4}-a^{3} cd^{5}\right)}{b^{4} c^{6} d-4 \, ab^{3} c^{5} d^{2}+6 \, a^{2} b^{2} c^{4} d^{3}-4 \, a^{3} bc^{3} d^{4}+a^{4} c^{2} d^{5}}\right) x}{3 \, \left(dx^{2}+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -b^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^2
*d^2)) - 1/3*((5*b^3*c^3*d^3 - 12*a*b^2*c^2*d^4 + 9*a^2*b*c*d^5 - 2*a^3*d^6
)*x^2/(b^4*c^6*d - 4*a*b^3*c^5*d^2 + 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*d^4 +
a^4*c^2*d^5) + 3*(2*b^3*c^4*d^2 - 5*a*b^2*c^3*d^3 + 4*a^2*b*c^2*d^4 - a^3*c
*d^5)/(b^4*c^6*d - 4*a*b^3*c^5*d^2 + 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*d^4 +
a^4*c^2*d^5))*x/(d*x^2 + c)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.727 $\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{d}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{5/2}}$$

```
[Out] -1/3*d/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a/c^(5
/2)+b^(5/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^
(5/2)-d*(-a*d+2*b*c)/c^2/(-a*d+b*c)^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 87, 157, 162, 65, 214}

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{5/2}}-\frac{d(2bc-ad)}{c^2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{d}{3c\,(c+dx^2)^{3/2}\,(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*d/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d))/(c^2*(b*c - a*
d)^2*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*c^(5/2)) + (b^(
5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*(b*c - a*d)^(5/
2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2c(bc-ad)}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)^2+}{x(a+bx)\sqrt{}}\right.}{c^2(bc}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\right.}{2ac^2}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,\right.}{ac^2d}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Mathematica [A]
time = 0.42, size = 138, normalized size = 0.95

$$\frac{d\left(ad\left(4c+3dx^2\right)-bc\left(7c+6dx^2\right)\right)}{3c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{a(-bc+ad)^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(a*d*(4*c + 3*d*x^2) - b*c*(7*c + 6*d*x^2)))/(3*c^2*(b*c - a*d)^2*(c + d
*x^2)^(3/2)) - (b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]
])/(a*(-(b*c) + a*d)^(5/2)) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1454 vs. 2(123) = 246.
time = 0.10, size = 1455, normalized size = 10.03

method	result	size
default	Expression too large to display	1455

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x-1/
b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+
16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^
(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2
)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x-1/b*(-a*b)^(1/2)))))-1/2/a*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a
*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/
b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-
a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d
*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/
2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*
c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/a*(1/3/c/(d*x^2+c)^(3/2)+1
/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

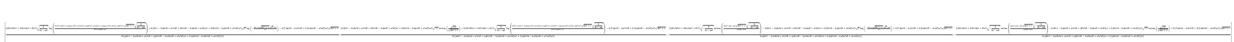
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(123) = 246$.
time = 2.39, size = 1711, normalized size = 11.80



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 6*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), 1/12*(12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), -1/6*(3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 3*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), -1/6*(3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 6*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2)]

Sympy [A]

time = 9.86, size = 133, normalized size = 0.92

$$\frac{d}{3c\left(c+dx^2\right)^{\frac{3}{2}}\left(ad-bc\right)}+\frac{d(ad-2bc)}{c^2\sqrt{c+dx^2}\,\left(ad-bc\right)^2}-\frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{a\sqrt{\frac{ad-bc}{b}}\,\left(ad-bc\right)^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{ac^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] d/(3*c*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*(a*d - 2*b*c)/(c**2*sqrt(c + d*
x**2)*(a*d - b*c)**2) - b**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*
sqrt((a*d - b*c)/b)*(a*d - b*c)**2) + atan(sqrt(c + d*x**2)/sqrt(-c))/(a*c*
*2*sqrt(-c))
```

Giac [A]
time = 0.55, size = 176, normalized size = 1.21

$$-\frac{b^3\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{-b^2c+abd}}-\frac{6\left(dx^2+c\right)bcd+bc^2d-3\left(dx^2+c\right)ad^2-acd^2}{3\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b^2*c^2 - 2*a^2*b*c
*d + a^3*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(6*(d*x^2 + c)*b*c*d + b*c^2*d -
3*(d*x^2 + c)*a*d^2 - a*c*d^2)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^
2 + c)^(3/2)) + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*c^2)
```

Mupad [B]
time = 1.44, size = 2500, normalized size = 17.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] (atan((((-b^5*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^13*c^16*d^2 - 3
2*a*b^12*c^15*d^3 + 120*a^2*b^11*c^14*d^4 - 280*a^3*b^10*c^13*d^5 + 450*a^4
*b^9*c^12*d^6 - 516*a^5*b^8*c^11*d^7 + 422*a^6*b^7*c^10*d^8 - 240*a^7*b^6*c
^9*d^9 + 90*a^8*b^5*c^8*d^10 - 20*a^9*b^4*c^7*d^11 + 2*a^10*b^3*c^6*d^12))/
2 + ((-b^5*(a*d - b*c)^5)^(1/2)*(6*a^2*b^12*c^18*d^3 - 54*a^3*b^11*c^17*d^4
 + 218*a^4*b^10*c^16*d^5 - 520*a^5*b^9*c^15*d^6 + 812*a^6*b^8*c^14*d^7 - 86
```

8*a^7*b^7*c^13*d^8 + 644*a^8*b^6*c^12*d^9 - 328*a^9*b^5*c^11*d^10 + 110*a^10*b^4*c^10*d^11 - 22*a^11*b^3*c^9*d^12 + 2*a^12*b^2*c^8*d^13 - ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^13*c^21*d^2 - 168*a^3*b^12*c^20*d^3 + 800*a^4*b^11*c^19*d^4 - 2280*a^5*b^10*c^18*d^5 + 4320*a^6*b^9*c^17*d^6 - 5712*a^7*b^8*c^16*d^7 + 5376*a^8*b^7*c^15*d^8 - 3600*a^9*b^6*c^14*d^9 + 1680*a^10*b^5*c^13*d^10 - 520*a^11*b^4*c^12*d^11 + 96*a^12*b^3*c^11*d^12 - 8*a^13*b^2*c^10*d^13))/(4*a*(a*d - b*c)^5)))/(2*a*(a*d - b*c)^5))*1i)/(a*(a*d - b*c)^5) + ((-b^5*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^13*c^16*d^2 - 32*a*b^12*c^15*d^3 + 120*a^2*b^11*c^14*d^4 - 280*a^3*b^10*c^13*d^5 + 450*a^4*b^9*c^12*d^6 - 516*a^5*b^8*c^11*d^7 + 422*a^6*b^7*c^10*d^8 - 240*a^7*b^6*c^9*d^9 + 90*a^8*b^5*c^8*d^10 - 20*a^9*b^4*c^7*d^11 + 2*a^10*b^3*c^6*d^12))/2 - ((-b^5*(a*d - b*c)^5)^(1/2)*(6*a^2*b^12*c^18*d^3 - 54*a^3*b^11*c^17*d^4 + 218*a^4*b^10*c^16*d^5 - 520*a^5*b^9*c^15*d^6 + 812*a^6*b^8*c^14*d^7 - 868*a^7*b^7*c^13*d^8 + 644*a^8*b^6*c^12*d^9 - 328*a^9*b^5*c^11*d^10 + 110*a^10*b^4*c^10*d^11 - 22*a^11*b^3*c^9*d^12 + 2*a^12*b^2*c^8*d^13 + ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^13*c^21*d^2 - 168*a^3*b^12*c^20*d^3 + 800*a^4*b^11*c^19*d^4 - 2280*a^5*b^10*c^18*d^5 + 4320*a^6*b^9*c^17*d^6 - 5712*a^7*b^8*c^16*d^7 + 5376*a^8*b^7*c^15*d^8 - 3600*a^9*b^6*c^14*d^9 + 1680*a^10*b^5*c^13*d^10 - 520*a^11*b^4*c^12*d^11 + 96*a^12*b^3*c^11*d^12 - 8*a^13*b^2*c^10*d^13))/(4*a*(a*d - b*c)^5)))/(2*a*(a*d - b*c)^5))*1i)/(a*(a*d - b*c)^5))/(4*b^12*c^13*d^3 - 26*a*b^11*c^12*d^4 + 72*a^2*b^10*c^11*d^5 - 110*a^3*b^9*c^10*d^6 + 100*a^4*b^8*c^9*d^7 - 54*a^5*b^7*c^8*d^8 + 16*a^6*b^6*c^7*d^9 - 2*a^7*b^5*c^6*d^10 + ((-b^5*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^13*c^16*d^2 - 32*a*b^12*c^15*d^3 + 120*a^2*b^11*c^14*d^4 - 280*a^3*b^10*c^13*d^5 + 450*a^4*b^9*c^12*d^6 - 516*a^5*b^8*c^11*d^7 + 422*a^6*b^7*c^10*d^8 - 240*a^7*b^6*c^9*d^9 + 90*a^8*b^5*c^8*d^10 - 20*a^9*b^4*c^7*d^11 + 2*a^10*b^3*c^6*d^12))/2 + ((-b^5*(a*d - b*c)^5)^(1/2)*(6*a^2*b^12*c^18*d^3 - 54*a^3*b^11*c^17*d^4 + 218*a^4*b^10*c^16*d^5 - 520*a^5*b^9*c^15*d^6 + 812*a^6*b^8*c^14*d^7 - 868*a^7*b^7*c^13*d^8 + 644*a^8*b^6*c^12*d^9 - 328*a^9*b^5*c^11*d^10 + 110*a^10*b^4*c^10*d^11 - 22*a^11*b^3*c^9*d^12 + 2*a^12*b^2*c^8*d^13 - ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^13*c^21*d^2 - 168*a^3*b^12*c^20*d^3 + 800*a^4*b^11*c^19*d^4 - 2280*a^5*b^10*c^18*d^5 + 4320*a^6*b^9*c^17*d^6 - 5712*a^7*b^8*c^16*d^7 + 5376*a^8*b^7*c^15*d^8 - 3600*a^9*b^6*c^14*d^9 + 1680*a^10*b^5*c^13*d^10 - 520*a^11*b^4*c^12*d^11 + 96*a^12*b^3*c^11*d^12 - 8*a^13*b^2*c^10*d^13))/(4*a*(a*d - b*c)^5)))/(2*a*(a*d - b*c)^5)))/(a*(a*d - b*c)^5) - ((-b^5*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^13*c^16*d^2 - 32*a*b^12*c^15*d^3 + 120*a^2*b^11*c^14*d^4 - 280*a^3*b^10*c^13*d^5 + 450*a^4*b^9*c^12*d^6 - 516*a^5*b^8*c^11*d^7 + 422*a^6*b^7*c^10*d^8 - 240*a^7*b^6*c^9*d^9 + 90*a^8*b^5*c^8*d^10 - 20*a^9*b^4*c^7*d^11 + 2*a^10*b^3*c^6*d^12))/2 - ((-b^5*(a*d - b*c)^5)^(1/2)*(6*a^2*b^12*c^18*d^3 - 54*a^3*b^11*c^17*d^4 + 218*a^4*b^10*c^16*d^5 - 520*a^5*b^9*c^15*d^6 + 812*a^6*b^8*c^14*d^7 - 868*a^7*b^7*c^13*d^8 + 644*a^8*b^6*c^12*d^9 - 328*a^9*b^5*c^11*d^10 + 110*a^10*b^4*c^10*d^11 - 22*a^11*b^3*c^9*d^12 + 2*a^12*b^2*c^8*d^13 + ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^13*c^21*d^2 - 168*a^3*b^12*c^20*d^3 + 800*a^4*b^11*c^19*d^4

```
 - 2280*a^5*b^10*c^18*d^5 + 4320*a^6*b^9*c^17*d^6 - 5712*a^7*b^8*c^16*d^7 +
 5376*a^8*b^7*c^15*d^8 - 3600*a^9*b^6*c^14*d^9 + 1680*a^10*b^5*c^13*d^10 -
520*a^11*b^4*c^12*d^11 + 96*a^12*b^3*c^11*d^12 - 8*a^13*b^2*c^10*d^13))/(4*
a*(a*d - b*c)^5)))/(2*a*(a*d - b*c)^5)))/(a*(a*d - b*c)^5)))*(-b^5*(a*d - b
*c)^5)^(1/2)*1i)/(a*(a*d - b*c)^5) - atanh((10*b^12*c^15*d^3*(c + d*x^2)^(1
/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^1
1*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9*
b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) + (290*a^2*b^10*c^13*d^5*(c + d*x^2)^(
1/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^
11*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
 660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9
*b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) - (630*a^3*b^9*c^12*d^6*(c + d*x^2)^(
1/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^
11*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
 660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 1...
```

3.728 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{d}{3c(bc-ad)x\,(c+dx^2)^{3/2}}-\frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}}-\frac{(bc-4ad)(3bc-2ad)\sqrt{c+dx^2}}{3ac^3(bc-ad)^2x}-\frac{b^3\tan^{-1}\left(\frac{\sqrt{bc\ldots}}{\sqrt{a}\,\sqrt{\ldots}}\right)}{a^{3/2}(bc-a\ldots}$$

```
[Out] -1/3*d/c/(-a*d+b*c)/x/(d*x^2+c)^(3/2)-b^3*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)
/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(5/2)-1/3*d*(-4*a*d+7*b*c)/c^2/(-a*d+b
*c)^2/x/(d*x^2+c)^(1/2)-1/3*(-4*a*d+b*c)*(-2*a*d+3*b*c)*(d*x^2+c)^(1/2)/a/c
^3/(-a*d+b*c)^2/x
```

Rubi [A]
time = 0.15, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {483, 593, 597, 12, 385, 211}

$$-\frac{b^3\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\,(bc-4ad)(3bc-2ad)}{3ac^3x(bc-ad)^2}-\frac{d(7bc-4ad)}{3c^2x\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{d}{3cx\,(c+dx^2)^{3/2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*d/(c*(b*c - a*d)*x*(c + d*x^2)^(3/2)) - (d*(7*b*c - 4*a*d))/(3*c^2*(b*
c - a*d)^2*x*Sqrt[c + d*x^2]) - ((b*c - 4*a*d)*(3*b*c - 2*a*d)*Sqrt[c + d*x
^2])/(3*a*c^3*(b*c - a*d)^2*x) - (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} + \frac{\int \frac{3bc-4ad-4bdx^2}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx}{3c(bc-ad)}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} + \frac{\int \frac{(bc-4ad)(3bc-2ad}{x^2(a+bx^2)\sqrt{}}}{3c^2(bc}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} - \frac{(bc-4ad)(3bc-}{3ac^3(bc}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} - \frac{(bc-4ad)(3bc-}{3ac^3(bc}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} - \frac{(bc-4ad)(3bc-}{3ac^3(bc}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} - \frac{(bc-4ad)(3bc-}{3ac^3(bc}$$

Mathematica [A]
time = 0.53, size = 180, normalized size = 1.01

$$\frac{-3b^2c^2(c+dx^2)^2 - a^2d^2(3c^2+12cdx^2+8d^2x^4) + abcd(6c^2+21cdx^2+14d^2x^4)}{3ac^3(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b^3\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{a^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*b^2*c^2*(c + d*x^2)^2 - a^2*d^2*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4) + a*b*
c*d*(6*c^2 + 21*c*d*x^2 + 14*d^2*x^4))/(3*a*c^3*(b*c - a*d)^2*x*(c + d*x^2)
^(3/2)) + (b^3*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt
[a]*Sqrt[b*c - a*d])])/(a^(3/2)*(b*c - a*d)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(156) = 312$.
time = 0.16, size = 1463, normalized size = 8.22

method	result

risch	$-\frac{\sqrt{d x^2+c}}{c^3 a x}-\frac{3 a b^2 d^4 \sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 d+2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}}{4c^3\left(b\sqrt{-cd}+\sqrt{-ab}\,d\right)^2\left(\sqrt{-ab}\,d-b\sqrt{-cd}\right)^2\left(x-\frac{\sqrt{-cd}}{d}\right)}+\frac{5b^3d^3\sqrt{\left(x-\frac{\sqrt{-c}}{d}\right.}}{4c^2\left(b\sqrt{-cd}+\sqrt{-ab}\right.}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*
(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*
a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/
2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+
2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-
4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+1/2*b/a/(-a*b)^(1/2)*(-1/3/(a*d-b*c)
*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-
a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)
/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/
(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/
b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+
1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
)+1/a*(-1/c/x/(d*x^2+c)^(3/2)-4*d/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x
^2+c)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(156) = 312$.
time = 1.91, size = 934, normalized size = 5.25

$$\left[-\frac{3\,(b^3c^3d^2x^5+2\,b^3c^4dx^3+b^3c^5x)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(3\,ab^3c^5-9\,a^2b^2c^4d+9\,a^3bc^3d^2-3\,a^4c^2d^3+(3\,ab^3c^3d^2-17\,a^2b^2c^2d^3+22\,a^3bcd^4-8\,a^4d^5)x^4+3\,(2\,ab^3c^4d-9\,a^2b^2c^3d^2+11\,a^3bc^2d^3-4\,a^4cd^4)x^2)\sqrt{dx^2+c}}{12\,((a^2b^3c^6d^2-3\,a^3b^2c^5d^3+3\,a^4bc^4d^4-a^5c^3d^5)x^5+2\,(a^2b^3c^7d-3\,a^3b^2c^6d^2+3\,a^4bc^5d^3-a^5c^4d^4)x^3+(a^2b^3c^8-3\,a^3b^2c^7d+3\,a^4bc^6d^2-a^5c^5d^3)x)},\ -\frac{3\,(b^3c^3d^2x^5+2\,b^3c^4dx^3+b^3c^5x)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+2\,(3\,ab^3c^5-9\,a^2b^2c^4d+9\,a^3bc^3d^2-3\,a^4c^2d^3+(3\,ab^3c^3d^2-17\,a^2b^2c^2d^3+22\,a^3bcd^4-8\,a^4d^5)x^4+3\,(2\,ab^3c^4d-9\,a^2b^2c^3d^2+11\,a^3bc^2d^3-4\,a^4cd^4)x^2)\sqrt{dx^2+c}}{6\,((a^2b^3c^6d^2-3\,a^3b^2c^5d^3+3\,a^4bc^4d^4-a^5c^3d^5)x^5+2\,(a^2b^3c^7d-3\,a^3b^2c^6d^2+3\,a^4bc^5d^3-a^5c^4d^4)x^3+(a^2b^3c^8-3\,a^3b^2c^7d+3\,a^4bc^6d^2-a^5c^5d^3)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-a*b*c + a^2
*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x
^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*a*b^3*c^5 - 9*a^2*b^2*c^4*d +
9*a^3*b*c^3*d^2 - 3*a^4*c^2*d^3 + (3*a*b^3*c^3*d^2 - 17*a^2*b^2*c^2*d^3 + 2
2*a^3*b*c*d^4 - 8*a^4*d^5)*x^4 + 3*(2*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 11*
a^3*b*c^2*d^3 - 4*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^2*b^3*c^6*d^2 - 3*a^
3*b^2*c^5*d^3 + 3*a^4*b*c^4*d^4 - a^5*c^3*d^5)*x^5 + 2*(a^2*b^3*c^7*d - 3*a
^3*b^2*c^6*d^2 + 3*a^4*b*c^5*d^3 - a^5*c^4*d^4)*x^3 + (a^2*b^3*c^8 - 3*a^3*
b^2*c^7*d + 3*a^4*b*c^6*d^2 - a^5*c^5*d^3)*x), -1/6*(3*(b^3*c^3*d^2*x^5 + 2
*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2
*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a
*b*c^2 - a^2*c*d)*x)) + 2*(3*a*b^3*c^5 - 9*a^2*b^2*c^4*d + 9*a^3*b*c^3*d^2
- 3*a^4*c^2*d^3 + (3*a*b^3*c^3*d^2 - 17*a^2*b^2*c^2*d^3 + 22*a^3*b*c*d^4 -
8*a^4*d^5)*x^4 + 3*(2*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 11*a^3*b*c^2*d^3 -
4*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^2*b^3*c^6*d^2 - 3*a^3*b^2*c^5*d^3 +
3*a^4*b*c^4*d^4 - a^5*c^3*d^5)*x^5 + 2*(a^2*b^3*c^7*d - 3*a^3*b^2*c^6*d^2 +
 3*a^4*b*c^5*d^3 - a^5*c^4*d^4)*x^3 + (a^2*b^3*c^8 - 3*a^3*b^2*c^7*d + 3*a^
4*b*c^6*d^2 - a^5*c^5*d^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(156) = 312.
time = 1.12, size = 366, normalized size = 2.06

$$\frac{b^3\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{abcd-a^2d^2}}+\frac{\left(\frac{\left(8\,b^3c^5d^4-21\,ab^2c^4d^5+18\,a^2bc^3d^6-5\,a^3c^2d^7\right)x^2}{b^4c^9d-4\,ab^3c^8d^2+6\,a^2b^2c^7d^3-4\,a^3bc^6d^4+a^4c^5d^5}+\frac{3\left(3\,b^3c^6d^3-8\,ab^2c^5d^4+7\,a^2bc^4d^5-2\,a^3c^3d^6\right)}{b^4c^9d-4\,ab^3c^8d^2+6\,a^2b^2c^7d^3-4\,a^3bc^6d^4+a^4c^5d^5}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] b^3*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b*c*d -
a^2*d^2)) + 1/3*((8*b^3*c^5*d^4 - 21*a*b^2*c^4*d^5 + 18*a^2*b*c^3*d^6 - 5*a
^3*c^2*d^7)*x^2/(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7*d^3 - 4*a^3*b*
c^6*d^4 + a^4*c^5*d^5) + 3*(3*b^3*c^6*d^3 - 8*a*b^2*c^5*d^4 + 7*a^2*b*c^4*d
^5 - 2*a^3*c^3*d^6)/(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7*d^3 - 4*a^
3*b*c^6*d^4 + a^4*c^5*d^5))*x/(d*x^2 + c)^(3/2) + 2*sqrt(d)/(((sqrt(d)*x -
sqrt(d*x^2 + c))^2 - c)*a*c^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.729 $\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}}-\frac{1}{2acx^2\left(c+dx^2\right)^{3/2}}-\frac{d\left(b^2c^2-8abcd+5a^2d^2\right)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}}+\frac{(2bc+5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}}$$

```
[Out] -1/6*d*(-5*a*d+3*b*c)/a/c^2/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/2/a/c/x^2/(d*x^2+c
)^(3/2)+1/2*(5*a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(7/2)-b^(7
/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(5/2)-
1/2*d*(5*a^2*d^2-8*a*b*c*d+b^2*c^2)/a/c^3/(-a*d+b*c)^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 105, 157, 162, 65, 214}

$$-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{5/2}}-\frac{d\left(5a^2d^2-8abcd+b^2c^2\right)}{2ac^3\sqrt{c+dx^2}(bc-ad)^2}+\frac{(5ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}}-\frac{d(3bc-5ad)}{6ac^2\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{1}{2acx^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(d*(3*b*c - 5*a*d))/(a*c^2*(b*c - a*d)*(c + d*x^2)^(3/2)) - 1/(2*a*c*x
^2*(c + d*x^2)^(3/2)) - (d*(b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2))/(2*a*c^3*(b*c
 - a*d)^2*Sqrt[c + d*x^2]) + ((2*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[
c]])/(2*a^2*c^(7/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(a^2*(b*c - a*d)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{1}{2acx^2(c+dx^2)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+5ad)+\frac{5bdx}{2}}{x(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)(c+dx^2)^{3/2}} - \frac{1}{2acx^2(c+dx^2)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{4}(bc-ad)(2bc+}{x(a+bx}\right.}{3ac^2}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)(c+dx^2)^{3/2}} - \frac{1}{2acx^2(c+dx^2)^{3/2}} - \frac{d(b^2c^2-8abcd+5a^2c}{2ac^3(bc-ad)^2\sqrt{c+dx}}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)(c+dx^2)^{3/2}} - \frac{1}{2acx^2(c+dx^2)^{3/2}} - \frac{d(b^2c^2-8abcd+5a^2c}{2ac^3(bc-ad)^2\sqrt{c+dx}}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)(c+dx^2)^{3/2}} - \frac{1}{2acx^2(c+dx^2)^{3/2}} - \frac{d(b^2c^2-8abcd+5a^2c}{2ac^3(bc-ad)^2\sqrt{c+dx}}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)(c+dx^2)^{3/2}} - \frac{1}{2acx^2(c+dx^2)^{3/2}} - \frac{d(b^2c^2-8abcd+5a^2c}{2ac^3(bc-ad)^2\sqrt{c+dx}}$$

Mathematica [A]
time = 0.71, size = 194, normalized size = 0.92

$$\frac{-\frac{a\left(3b^2c^2(c+dx^2)^2-2abcd(3c^2+16cdx^2+12d^2x^4)+a^2d^2(3c^2+20cdx^2+15d^2x^4)\right)}{c^3(bc-ad)^2x^2(c+dx^2)^{3/2}}+\frac{6b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}+\frac{3(2bc+5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{7/2}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-((a*(3*b^2*c^2*(c + d*x^2)^2 - 2*a*b*c*d*(3*c^2 + 16*c*d*x^2 + 12*d^2*x^4
) + a^2*d^2*(3*c^2 + 20*c*d*x^2 + 15*d^2*x^4)))/(c^3*(b*c - a*d)^2*x^2*(c +
 d*x^2)^(3/2))) + (6*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) +
 a*d]])/(-(b*c) + a*d)^(5/2) + (3*(2*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/S
qrt[c]])/c^(7/2))/(6*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1547 vs. $2(181) = 362$.
time = 0.18, size = 1548, normalized size = 7.34

method	result
risch	$-\frac{\sqrt{d\,x^2+c}}{2c^3a\,x^2}-\frac{a\,b^2d^5\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2d+2\sqrt{-cd}\,\left(x-\frac{\sqrt{-cd}}{d}\right)}}{c^3\left(b\sqrt{-cd}+\sqrt{-ab}\,d\right)^2\left(\sqrt{-ab}\,d-b\sqrt{-cd}\right)^2\sqrt{-cd}\,\left(x-\frac{\sqrt{-cd}}{d}\right)}+\frac{3b^3d^4\sqrt{\left(\right.}}{2c^2\left(b\sqrt{-cd}+\right.}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/a^2*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x
-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*
b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2)
)^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(
1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c
)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/
b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x-1/b*(-a*b)^(1/2)))))+1/2*b/a^2*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(
1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*
d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x
+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*
c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a
*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/a*(-1/2/c/x^2/(d*x^
2+c)^(3/2)-5/2*d/c*(1/3/c/(d*x^2+c)^(3/2)+1/c*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2
)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))-b/a^2*(1/3/c/(d*x^2+c)^(3/2)+1/c
```

```
*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(181) = 362$.
time = 3.06, size = 2219, normalized size = 10.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] [1/12*(3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2)*sqrt(b/(b*c - a*
d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a
*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)
*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 3*((2*
b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4
*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5 + a*b^
2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqr
t(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b^2*c^5 - 6*a^2*b*c^4*d + 3*a^3*c
^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 2*(3*a*b^2
*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2
*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*c^7*d - 2*a^3*b*
c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^
2), -1/12*(6*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x
^6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 +
(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(-c)*a
rctan(sqrt(-c)/sqrt(d*x^2 + c)) - 3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^
3*c^6*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) + 2*(3*a*b^2*c^5 - 6*a^2*b*c^4*d + 3*a^3*c^3*d^2 + 3*(a*
b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 2*(3*a*b^2*c^4*d - 16*a^
2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^6*d^2 - 2*a
^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*
c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2), 1/12*(6*(b
^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2)*sqrt(-b/(b*c - a*d))*arctan
```

```
(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2
+ b*c)) + 3*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^
6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (
2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(c)*log
(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b^2*c^5 - 6*a^2*b
*c^4*d + 3*a^3*c^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*
x^4 + 2*(3*a*b^2*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2
 + c))/((a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*
c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d +
 a^4*c^6*d^2)*x^2), 1/6*(3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2
)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*s
qrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 3*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 -
8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c
^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*
a^3*c^2*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (3*a*b^2*c^5
- 6*a^2*b*c^4*d + 3*a^3*c^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^
3*c*d^4)*x^4 + 2*(3*a*b^2*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*s
qrt(d*x^2 + c))/((a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*
(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*
b*c^7*d + a^4*c^6*d^2)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\,(a+bx^2)\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [**A**]
time = 0.50, size = 211, normalized size = 1.00

$$\frac{b^4\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{-b^2c+abd}}+\frac{9\,(dx^2+c)bcd^2+bc^2d^2-6\,(dx^2+c)ad^3-acd^3}{3\,(b^2c^5-2\,abc^4d+a^2c^3d^2)(dx^2+c)^{\frac{3}{2}}}-\frac{(2\,bc+5\,ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}\,c^3}-\frac{\sqrt{dx^2+c}}{2\,ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] b^4*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b^2*c^2 - 2*a^3*b*
c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*(9*(d*x^2 + c)*b*c*d^2 + b*c^2*d
^2 - 6*(d*x^2 + c)*a*d^3 - a*c*d^3)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*
(d*x^2 + c)^(3/2)) - 1/2*(2*b*c + 5*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(
a^2*sqrt(-c)*c^3) - 1/2*sqrt(d*x^2 + c)/(a*c^3*x^2)
```

Mupad [B]
time = 2.00, size = 2500, normalized size = 11.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] (atan(((((c + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 +
 1040*a^5*b^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 +
 2688*a^8*b^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9
+ 48480*a^11*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*c^11
*d^12 - 3680*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d^14) + ((5*a*d + 2*b*c)
*(64*a^6*b^13*c^23*d^3 + 64*a^7*b^12*c^22*d^4 - 3648*a^8*b^11*c^21*d^5 + 19
520*a^9*b^10*c^20*d^6 - 53632*a^10*b^9*c^19*d^7 + 92288*a^11*b^8*c^18*d^8 -
 106624*a^12*b^7*c^17*d^9 + 84608*a^13*b^6*c^16*d^10 - 45760*a^14*b^5*c^15*
d^11 + 16192*a^15*b^4*c^14*d^12 - 3392*a^16*b^3*c^13*d^13 + 320*a^17*b^2*c^
12*d^14 - ((c + d*x^2)^(1/2)*(5*a*d + 2*b*c)*(512*a^7*b^13*c^26*d^2 - 5376*
a^8*b^12*c^25*d^3 + 25600*a^9*b^11*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 13
8240*a^11*b^9*c^22*d^6 - 182784*a^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^
8 - 115200*a^14*b^6*c^19*d^9 + 53760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^
17*d^11 + 3072*a^17*b^3*c^16*d^12 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(c^7)^(
1/2))))/(4*a^2*(c^7)^(1/2)))*(5*a*d + 2*b*c)*1i)/(4*a^2*(c^7)^(1/2)) + (((c
 + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 + 1040*a^5*b
^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 + 2688*a^8*b
^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9 + 48480*a^1
1*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*c^11*d^12 - 368
0*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d^14) - ((5*a*d + 2*b*c)*(64*a^6*b^
13*c^23*d^3 + 64*a^7*b^12*c^22*d^4 - 3648*a^8*b^11*c^21*d^5 + 19520*a^9*b^1
0*c^20*d^6 - 53632*a^10*b^9*c^19*d^7 + 92288*a^11*b^8*c^18*d^8 - 106624*a^1
2*b^7*c^17*d^9 + 84608*a^13*b^6*c^16*d^10 - 45760*a^14*b^5*c^15*d^11 + 1619
2*a^15*b^4*c^14*d^12 - 3392*a^16*b^3*c^13*d^13 + 320*a^17*b^2*c^12*d^14 + (
(c + d*x^2)^(1/2)*(5*a*d + 2*b*c)*(512*a^7*b^13*c^26*d^2 - 5376*a^8*b^12*c^
25*d^3 + 25600*a^9*b^11*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 138240*a^11*b
^9*c^22*d^6 - 182784*a^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^8 - 115200*
a^14*b^6*c^19*d^9 + 53760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^17*d^11 + 3
072*a^17*b^3*c^16*d^12 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(c^7)^(1/2))))/(4*
a^2*(c^7)^(1/2)))*(5*a*d + 2*b*c)*1i)/(4*a^2*(c^7)^(1/2)))/(32*a^2*b^15*c^1
8*d^3 - 368*a^3*b^14*c^17*d^4 + 1056*a^4*b^13*c^16*d^5 - 5600*a^6*b^11*c^14
*d^7 + 12768*a^7*b^10*c^13*d^8 - 14112*a^8*b^9*c^12*d^9 + 8704*a^9*b^8*c^11
*d^10 - 2880*a^10*b^7*c^10*d^11 + 400*a^11*b^6*c^9*d^12 - (((c + d*x^2)^(1/
2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 + 1040*a^5*b^13*c^19*d^4
+ 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 + 2688*a^8*b^10*c^16*d^7
+ 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9 + 48480*a^11*b^7*c^13*d^
10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*c^11*d^12 - 3680*a^14*b^4*c^
```

10*d^13 + 400*a^15*b^3*c^9*d^14) + ((5*a*d + 2*b*c)*(64*a^6*b^13*c^23*d^3 + 64*a^7*b^12*c^22*d^4 - 3648*a^8*b^11*c^21*d^5 + 19520*a^9*b^10*c^20*d^6 - 53632*a^10*b^9*c^19*d^7 + 92288*a^11*b^8*c^18*d^8 - 106624*a^12*b^7*c^17*d^9 + 84608*a^13*b^6*c^16*d^10 - 45760*a^14*b^5*c^15*d^11 + 16192*a^15*b^4*c^14*d^12 - 3392*a^16*b^3*c^13*d^13 + 320*a^17*b^2*c^12*d^14 - ((c + d*x^2)^(1/2)*(5*a*d + 2*b*c)*(512*a^7*b^13*c^26*d^2 - 5376*a^8*b^12*c^25*d^3 + 25600*a^9*b^11*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 138240*a^11*b^9*c^22*d^6 - 182784*a^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^8 - 115200*a^14*b^6*c^19*d^9 + 53760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^17*d^11 + 3072*a^17*b^3*c^16*d^12 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(c^7)^(1/2))))/(4*a^2*(c^7)^(1/2)))*(5*a*d + 2*b*c))/(4*a^2*(c^7)^(1/2)) + (((c + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 + 1040*a^5*b^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 + 2688*a^8*b^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9 + 48480*a^11*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*c^11*d^12 - 3680*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d^14) - ((5*a*d + 2*b*c)*(64*a^6*b^13*c^23*d^3 + 64*a^7*b^12*c^22*d^4 - 3648*a^8*b^11*c^21*d^5 + 19520*a^9*b^10*c^20*d^6 - 53632*a^10*b^9*c^19*d^7 + 92288*a^11*b^8*c^18*d^8 - 106624*a^12*b^7*c^17*d^9 + 84608*a^13*b^6*c^16*d^10 - 45760*a^14*b^5*c^15*d^11 + 16192*a^15*b^4*c^14*d^12 - 3392*a^16*b^3*c^13*d^13 + 320*a^17*b^2*c^12*d^14 + ((c + d*x^2)^(1/2)*(5*a*d + 2*b*c)*(512*a^7*b^13*c^26*d^2 - 5376*a^8*b^12*c^25*d^3 + 25600*a^9*b^11*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 138240*a^11*b^9*c^22*d^6 - 182784*a^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^8 - 115200*a^14*b^6*c^19*d^9 + 53760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^17*d^11 + 3072*a^17*b^3*c^16*d^12 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(c^7)^(1/2))))/(4*a^2*(c^7)^(1/2)))*(5*a*d + 2*b*c))/(4*a^2*(c^7)^(1/2))))*(5*a*d + 2*b*c)*1i)/(2*a^2*(c^7)^(1/2)) - ((d^2*(c + d*x^2)*(5*a*d - 8*b*c))/(3*(b*c^2 - a*c*d)^2) - d^2/(3*(b*c^2 - a*c*d)) + (d*(c + d*x^2)^2*(5*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(2*a*c^2*(b*c^2 - a*c*d)*(a*d - b*c)))/(c*(c + d*x^2)^(3/2) - (c + d*x^2)^(5/2)) + (atan(((-b^7*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*...

3.730 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=245

$$-\frac{d}{3c(bc-ad)x^3\,(c+dx^2)^{3/2}}-\frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}}-\frac{(b^2c^2-12abcd+8a^2d^2)\,\sqrt{c+dx^2}}{3ac^3(bc-ad)^2x^3}+\frac{(bc-2ad)\,(3b}{}$$

```
[Out] -1/3*d/c/(-a*d+b*c)/x^3/(d*x^2+c)^(3/2)+b^4*arctan(x*(-a*d+b*c)^(1/2)/a^(1/
2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(5/2)-d*(-2*a*d+3*b*c)/c^2/(-a*d+b*c
)^2/x^3/(d*x^2+c)^(1/2)-1/3*(8*a^2*d^2-12*a*b*c*d+b^2*c^2)*(d*x^2+c)^(1/2)/
a/c^3/(-a*d+b*c)^2/x^3+1/3*(-2*a*d+b*c)*(-8*a^2*d^2+8*a*b*c*d+3*b^2*c^2)*(d
*x^2+c)^(1/2)/a^2/c^4/(-a*d+b*c)^2/x
```

Rubi [A]

time = 0.23, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {483, 593, 597, 12, 385, 211}

$$\frac{b^4\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}+\frac{\sqrt{c+dx^2}\,(bc-2ad)\,(-8a^2d^2+8abcd+3b^2c^2)}{3a^2c^4x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\,(8a^2d^2-12abcd+b^2c^2)}{3ac^3x^3(bc-ad)^2}-\frac{d(3bc-2ad)}{c^2x^3\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{d}{3cx^3\,(c+dx^2)^{3/2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*d/(c*(b*c - a*d)*x^3*(c + d*x^2)^(3/2)) - (d*(3*b*c - 2*a*d))/(c^2*(b*
c - a*d)^2*x^3*Sqrt[c + d*x^2]) - ((b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*Sqrt[
c + d*x^2])/(3*a*c^3*(b*c - a*d)^2*x^3) + ((b*c - 2*a*d)*(3*b^2*c^2 + 8*a*b
*c*d - 8*a^2*d^2)*Sqrt[c + d*x^2])/(3*a^2*c^4*(b*c - a*d)^2*x) + (b^4*ArcTa
n[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(5/2
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{5/2}}\,dx = -\frac{d}{3c(bc-ad)x^3 (c+dx^2)^{3/2}} + \frac{\int \frac{3(bc-2ad)-6bdx^2}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx}{3c(bc-ad)}$$

$$= -\frac{d}{3c(bc-ad)x^3 (c+dx^2)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{\int \frac{3(b^2c^2-12abcd+8 \cdots}{x^4(a+bx \cdots}}{3c^2 \cdots}$$

$$= -\frac{d}{3c(bc-ad)x^3 (c+dx^2)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a} - 12bd + \cdots\right.}{3c^2(bc - \cdots}$$

$$= -\frac{d}{3c(bc-ad)x^3 (c+dx^2)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a} - 12bd + \cdots\right.}{3c^2(bc - \cdots}$$

$$= -\frac{d}{3c(bc-ad)x^3 (c+dx^2)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a} - 12bd + \cdots\right.}{3c^2(bc - \cdots}$$

$$= -\frac{d}{3c(bc-ad)x^3 (c+dx^2)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a} - 12bd + \cdots\right.}{3c^2(bc - \cdots}$$

$$= -\frac{d}{3c(bc-ad)x^3 (c+dx^2)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a} - 12bd + \cdots\right.}{3c^2(bc - \cdots}$$

Mathematica [A]
time = 0.80, size = 233, normalized size = 0.95

$$\frac{3b^3c^3x^2(c+dx^2)^2 - ab^2c^2(c-2dx^2)(c+dx^2)^2 + a^2bcd(2c^3-9c^2dx^2-36cd^2x^4-24d^3x^6) + a^3d^2(-c^3+6c^2dx^2+24cd^2x^4+16d^3x^6)}{3a^2c^4(bc-ad)^2x^3(c+dx^2)^{3/2}} - \frac{b^4\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (3*b^3*c^3*x^2*(c + d*x^2)^2 - a*b^2*c^2*(c - 2*d*x^2)*(c + d*x^2)^2 + a^2*
b*c*d*(2*c^3 - 9*c^2*d*x^2 - 36*c*d^2*x^4 - 24*d^3*x^6) + a^3*d^2*(-c^3 + 6
*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6))/(3*a^2*c^4*(b*c - a*d)^2*x^3*(c +
d*x^2)^(3/2)) - (b^4*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))
/(Sqrt[a]*Sqrt[b*c - a*d])])/(a^(5/2)*(b*c - a*d)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. 2(221) = 442.
time = 0.18, size = 1552, normalized size = 6.33

method	result
risch	$-\frac{\sqrt{d x^2+c}\,(-8ad x^2-3c x^2 b+ac)}{3c^4a^2x^3}+\frac{5a b^2 d^5\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 d+2\sqrt{-cd}\,\left(x-\frac{\sqrt{-cd}}{d}\right)}}{4c^4\left(b\sqrt{-cd}+\sqrt{-ab}\,d\right)^2\left(b\sqrt{-cd}-\sqrt{-ab}\,d\right)^2\left(x-\frac{\sqrt{-cd}}{d}\right)}-\frac{7b^{\cdots}}{4c^3(\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^2/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*
c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d
^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^
(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)
/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2*b^2/a^2/(-a*b)^(1/2)*(-1/3/(a
*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))
-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a
*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+
1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*
d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))))+1/a*(-1/3/c/x^3/(d*x^2+c)^(3/2)-2*d/c*(-1/c/x/(d*x^2+c)^(3/2)-4*d/
c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))))-b/a^2*(-1/c/x/(d*x^
```

2+c)^(3/2)-4*d/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(221) = 442.
time = 1.90, size = 1128, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(-a*b*c + a
^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d
*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d
+ 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 - (3*a*b^4*c^4*d^2 - a^2*b^3*c^3*d^3 - 26*a
^3*b^2*c^2*d^4 + 40*a^4*b*c*d^5 - 16*a^5*d^6)*x^6 - 3*(2*a*b^4*c^5*d - a^2*
b^3*c^4*d^2 - 13*a^3*b^2*c^3*d^3 + 20*a^4*b*c^2*d^4 - 8*a^5*c*d^5)*x^4 - 3*
(a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^
2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^7*d^2 - 3*a^4*b^2*c^6*d^3 + 3*a^5*
b*c^5*d^4 - a^6*c^4*d^5)*x^7 + 2*(a^3*b^3*c^8*d - 3*a^4*b^2*c^7*d^2 + 3*a^5
*b*c^6*d^3 - a^6*c^5*d^4)*x^5 + (a^3*b^3*c^9 - 3*a^4*b^2*c^8*d + 3*a^5*b*c^
7*d^2 - a^6*c^6*d^3)*x^3), 1/6*(3*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*
c^6*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*
x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x
)) - 2*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 - (3*
a*b^4*c^4*d^2 - a^2*b^3*c^3*d^3 - 26*a^3*b^2*c^2*d^4 + 40*a^4*b*c*d^5 - 16*
a^5*d^6)*x^6 - 3*(2*a*b^4*c^5*d - a^2*b^3*c^4*d^2 - 13*a^3*b^2*c^3*d^3 + 20
*a^4*b*c^2*d^4 - 8*a^5*c*d^5)*x^4 - 3*(a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^
2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^
3*c^7*d^2 - 3*a^4*b^2*c^6*d^3 + 3*a^5*b*c^5*d^4 - a^6*c^4*d^5)*x^7 + 2*(a^3
*b^3*c^8*d - 3*a^4*b^2*c^7*d^2 + 3*a^5*b*c^6*d^3 - a^6*c^5*d^4)*x^5 + (a^3*
b^3*c^9 - 3*a^4*b^2*c^8*d + 3*a^5*b*c^7*d^2 - a^6*c^6*d^3)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(221) = 442.
time = 1.30, size = 490, normalized size = 2.00

$$-\frac{b^4\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{abcd-a^2d^2}}-\frac{\left(\frac{\left(11\,b^3c^6d^5-30\,ab^2c^5d^6+27\,a^2bc^4d^7-8\,a^3c^3d^8\right)x^2}{b^4c^{11}d-4\,ab^3c^{10}d^2+6\,a^2b^2c^9d^3-4\,a^3bc^8d^4+a^4c^7d^5}+\frac{3\left(4\,b^3c^7d^4-11\,ab^2c^6d^5+10\,a^2bc^5d^6-3\,a^3c^4d^7\right)}{b^4c^{11}d-4\,ab^3c^{10}d^2+6\,a^2b^2c^9d^3-4\,a^3bc^8d^4+a^4c^7d^5}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}-\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}+6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}-18\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2acd^{\frac{3}{2}}+3\,bc^3\sqrt{d}+8\,ac^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -b^4*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b*c*d
 - a^2*d^2)) - 1/3*((11*b^3*c^6*d^5 - 30*a*b^2*c^5*d^6 + 27*a^2*b*c^4*d^7 -
 8*a^3*c^3*d^8)*x^2/(b^4*c^11*d - 4*a*b^3*c^10*d^2 + 6*a^2*b^2*c^9*d^3 - 4*
a^3*b*c^8*d^4 + a^4*c^7*d^5) + 3*(4*b^3*c^7*d^4 - 11*a*b^2*c^6*d^5 + 10*a^2
*b*c^5*d^6 - 3*a^3*c^4*d^7)/(b^4*c^11*d - 4*a*b^3*c^10*d^2 + 6*a^2*b^2*c^9*
d^3 - 4*a^3*b*c^8*d^4 + a^4*c^7*d^5))*x/(d*x^2 + c)^(3/2) - 2/3*(3*(sqrt(d)
*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d
^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) - 18*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt(d) + 8*a*c^2*d^(3/2))/(((sqrt
(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^2*c^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.731 $\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=150

$$\frac{x\sqrt{c+dx^2}}{b^2}-\frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)}-\frac{\sqrt{a}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}$$

```
[Out] 1/2*(-4*a*d+b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^3/d^(1/2)-1/2*(-4*a*d
+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)/b^3/(-a*
d+b*c)^(1/2)+x*(d*x^2+c)^(1/2)/b^2-1/2*x^3*(d*x^2+c)^(1/2)/b/(b*x^2+a)
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {478, 596, 537, 223, 212, 385, 211}

$$-\frac{\sqrt{a}\,(3bc-4ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}-\frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)}+\frac{x\sqrt{c+dx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/b^2 - (x^3*Sqrt[c + d*x^2])/(2*b*(a + b*x^2)) - (Sqrt[a
]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2
*b^3*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]
)/(2*b^3*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = -\frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)} + \frac{\int \frac{x^2(3c+4dx^2)}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2b}$$

$$= \frac{x\sqrt{c+dx^2}}{b^2} - \frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)} - \frac{\int \frac{4acd-2d(bc-4ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{4b^2d}$$

$$= \frac{x\sqrt{c+dx^2}}{b^2} - \frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)} + \frac{(bc-4ad)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^3} - \frac{(a(3bc-4ad))\int \frac{}{(a+bx^2)\sqrt{}}}{2b^3}$$

$$= \frac{x\sqrt{c+dx^2}}{b^2} - \frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)} + \frac{(bc-4ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2b^3} - \frac{(a(3bc}{}$$

$$= \frac{x\sqrt{c+dx^2}}{b^2} - \frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)} - \frac{\sqrt{a}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}} + \frac{(bc-4ad)}{}$$

Mathematica [**A**]
time = 0.72, size = 154, normalized size = 1.03

$$\frac{\frac{bx(2a+bx^2)\sqrt{c+dx^2}}{a+bx^2} + \frac{\sqrt{a}\,(3bc-4ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{(-bc+4ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((b*x*(2*a + b*x^2)*Sqrt[c + d*x^2])/(a + b*x^2) + (Sqrt[a]*(3*b*c - 4*a*d)
*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c -
 a*d])])/Sqrt[b*c - a*d] + ((-(b*c) + 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*
x^2]])/Sqrt[d])/(2*b^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(124) = 248$.
time = 0.14, size = 2002, normalized size = 13.35

method	result	size
risch	Expression too large to display	1735
default	Expression too large to display	2002

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))-1
/4*a/b^3*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-
b*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b
)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+
1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-
2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln
((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))
^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-3/4/b^2*a/
(-a*b)^(1/2)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/
b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+3/4/b^2*a/(-a*b)^(1/2)*((d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2
)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/
2))))-1/4*a/b^3*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))
^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2
)/(a*d-b*c)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b
*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x-1/b*(-a*b)^(
1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3
/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(
1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a)^2, x)
```

Fricas [A]
time = 1.88, size = 1002, normalized size = 6.68

$$\left[-\frac{2\left(abc-4a^2d+(b^2c-4abd)x^2\right)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)+\left(3abcd-4a^2d^2+(3b^2cd-4abd^2)x^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4\left((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)-4(b^2dx^3+2abdx)\sqrt{dx^2+c}}{8(b^4dx^2+ab^3d)}, -\frac{4\left(abc-4a^2d+(b^2c-4abd)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+\left(3abcd-4a^2d^2+(3b^2cd-4abd^2)x^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4\left((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)-4(b^2dx^3+2abdx)\sqrt{dx^2+c}}{8(b^4dx^2+ab^3d)}, \frac{\left(3abcd-4a^2d^2+(3b^2cd-4abd^2)x^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2ad)x^2-ac\right)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)-\left(abc-4a^2d+(b^2c-4abd)x^2\right)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2(b^2dx^3+2abdx)\sqrt{dx^2+c}}{4(b^4dx^2+ab^3d)}, -\frac{2\left(abc-4a^2d+(b^2c-4abd)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(3abcd-4a^2d^2+(3b^2cd-4abd^2)x^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2ad)x^2-ac\right)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)-2(b^2dx^3+2abdx)\sqrt{dx^2+c}}{4(b^4dx^2+ab^3d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2
*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4*a
*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^
4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a
^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 + c))/(
b^4*d*x^2 + a*b^3*d), -1/8*(4*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqr
t(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*
c*d - 4*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*
c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c
- a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^
2 + c))/(b^4*d*x^2 + a*b^3*d), 1/4*((3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4
*a*b*d^2)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sq
rt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) - (a*b*c - 4*a^2*d + (
b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 + c))/(b^4*d*x^2 + a*b^3*d), -1/4
*(2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sq
rt(d*x^2 + c)) - (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4*a*b*d^2)*x^2)*sqrt
(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(
a/(b*c - a*d))/(a*d*x^3 + a*c*x)) - 2*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 +
c))/(b^4*d*x^2 + a*b^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(124) = 248.
time = 0.69, size = 288, normalized size = 1.92

$$\frac{\sqrt{dx^2+c}\,x}{2\,b^2}+\frac{\left(3\,abc\sqrt{d}-4\,a^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,b^3}-\frac{\left(bc\sqrt{d}-4\,ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{4\,b^3d}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*x/b^2 + 1/2*(3*a*b*c*sqrt(d) - 4*a^2*d^(3/2))*arctan(1/
2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2)
)/(sqrt(a*b*c*d - a^2*d^2)*b^3) - 1/4*(b*c*sqrt(d) - 4*a*d^(3/2))*log((sqrt
(d)*x - sqrt(d*x^2 + c))^2)/(b^3*d) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*
c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d)
)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b
*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

```
[Out] int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.732 $\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=136

$$\frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)}+\frac{a(c+dx^2)^{3/2}}{2b(bc-ad)(a+bx^2)}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/2*a*(d*x^2+c)^(3/2)/b/(-a*d+b*c)/(b*x^2+a)-1/2*(-3*a*d+2*b*c)*arctanh(b^(
1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)+1/2*(-3*a*d
+2*b*c)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)
```

Rubi [A]

time = 0.08, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 79, 52, 65, 214}

$$-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\,(2bc-3ad)}{2b^2(bc-ad)}+\frac{a(c+dx^2)^{3/2}}{2b\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(3/2
))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*c - a*d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right)}{4b^2}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x\right)}{2b^2d}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A]

time = 0.25, size = 98, normalized size = 0.72

$$\frac{\frac{\sqrt{b}\,(3a+2bx^2)\sqrt{c+dx^2}}{a+bx^2}+\frac{(2bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((Sqrt[b]*(3*a + 2*b*x^2)*Sqrt[c + d*x^2])/(a + b*x^2) + ((2*b*c - 3*a*d)*A
rcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d])/(2
*b^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1958 vs. 2(116) = 232.
time = 0.12, size = 1959, normalized size = 14.40

method	result	size
risch	Expression too large to display	1645
default	Expression too large to display	1959

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^3*(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b
)^(1/2)/(a*d-b*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d
*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)
^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b*(
-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/
b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
))+1/2/b^2*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*
(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+1/2/b^2*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a
```

```
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*
ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/
b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/4/b^3*(-a
*b)^(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d
-b*c)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b
)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-
1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+
2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln
((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^
2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.28, size = 436, normalized size = 3.21

$$\left[-\frac{(2\,abc-3\,a^2d+(2\,b^2c-3\,abd)x^2)\sqrt{b^2c-abd}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(3\,ab^2c-3\,a^2bd+2\,(b^3c-ab^2d)x^2)\sqrt{dx^2+c}}{8\,(ab^4c-a^2b^3d+(b^5c-ab^4d)x^2)},-\frac{(2\,abc-3\,a^2d+(2\,b^2c-3\,abd)x^2)\sqrt{-b^2c+abd}\,\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)-2\,(3\,ab^2c-3\,a^2bd+2\,(b^3c-ab^2d)x^2)\sqrt{dx^2+c}}{4\,(ab^4c-a^2b^3d+(b^5c-ab^4d)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*lo
g((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2
)*x^2 + 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x
```

```
^2)*sqrt(d*x^2 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^2), -1/4*((
2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1
/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 -
a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c -
 a*b^2*d)*x^2)*sqrt(d*x^2 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^
2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**3*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 0.61, size = 101, normalized size = 0.74

$$\frac{\sqrt{dx^2+c}\,ad}{2\left((dx^2+c)b-bc+ad\right)b^2}+\frac{(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b^2}+\frac{\sqrt{dx^2+c}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*a*d/(((d*x^2 + c)*b - b*c + a*d)*b^2) + 1/2*(2*b*c - 3*
a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b
^2) + sqrt(d*x^2 + c)/b^2
```

Mupad [B]
time = 0.51, size = 102, normalized size = 0.75

$$\frac{\sqrt{d\,x^2+c}}{b^2}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-2\,b\,c)}{2\,b^{5/2}\,\sqrt{a\,d-b\,c}}+\frac{a\,d\,\sqrt{d\,x^2+c}}{2\,\left(b^3\,\left(d\,x^2+c\right)-b^3\,c+a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

```
[Out] (c + d*x^2)^(1/2)/b^2 - (atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2)
)*(3*a*d - 2*b*c))/(2*b^(5/2)*(a*d - b*c)^(1/2)) + (a*d*(c + d*x^2)^(1/2))/
(2*(b^3*(c + d*x^2) - b^3*c + a*b^2*d))
```

3.733 $\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=120

$$-\frac{x\sqrt{c+dx^2}}{2b\,(a+bx^2)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\;x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^2\sqrt{bc-ad}}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\;x}{\sqrt{c+dx^2}}\right)}{b^2}$$

```
[Out] arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)/b^2+1/2*(-2*a*d+b*c)*arctan(x*(-
a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b^2/a^(1/2)/(-a*d+b*c)^(1/2)-1/2*x*
(d*x^2+c)^(1/2)/b/(b*x^2+a)
```

Rubi [A]
time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {478, 537, 223, 212, 385, 211}

$$\frac{(bc-2ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^2\sqrt{bc-ad}}-\frac{x\sqrt{c+dx^2}}{2b\,(a+bx^2)}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\;x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(x*Sqrt[c + d*x^2])/(b*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan[(Sqrt[b*c
 - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^2*Sqrt[b*c - a*d]) + (Sq
rt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx &= -\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{\int \frac{c+2dx^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2b}\\
&= -\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{d\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2}+\frac{(bc-2ad)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2b^2}\\
&= -\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{d\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x}\right.}{2b^2}\\
&= -\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^2\sqrt{bc-ad}}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.49, size = 138, normalized size = 1.15

$$
\frac{-\frac{bx\sqrt{c+dx^2}}{a+bx^2}+\frac{(-bc+2ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}\sqrt{bc-ad}}-2\sqrt{d}\,\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (-((b*x*Sqrt[c + d*x^2])/(a + b*x^2)) + ((-(b*c) + 2*a*d)*ArcTan[(a*Sqrt[d]
 + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*
Sqrt[b*c - a*d]) - 2*Sqrt[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1958 vs. $2(98) = 196$.
time = 0.10, size = 1959, normalized size = 16.32

method	result	size
default	Expression too large to display	1959

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b
*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)
^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1
/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2
*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln(
(-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^
2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))+1/4/(-a*b)^
(1/2)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a
*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(
x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2))))-1/4/(-a*b)^(1/2)/b*((d*(x+1/b*(-a*b)^(1/2))^2
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^
(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a
*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/4
/b^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)
```

```
*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/
2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x-1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(
-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(
-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(98) = 196.
time = 1.59, size = 1069, normalized size = 8.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
[Out] [-1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - 4*(a^2*b*c - a^3*d + (a*b^
2*c - a^2*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
 - (a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b^3*d)*x^
2), -1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + 8*(a^2*b*c - a^3*d + (a
*b^2*c - a^2*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b*c
 - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*
c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*
a*b*x^2 + a^2)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b^3*d)*x^2), -1/4*
(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - sqrt(a*b*c - a^2*d)*(a*b*c - 2*a
^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*
```

```
x)) - 2*(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b
^3*d)*x^2), -1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - sqrt(a*b*c - a^
2*d)*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*
d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*
b*c^2 - a^2*c*d)*x)) + 4*(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(-
d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c -
a^2*b^3*d)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
[Out] Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(98) = 196.
time = 0.66, size = 251, normalized size = 2.09

$$-\frac{\left(bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,b^2}-\frac{\sqrt{d}\,\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{2\,b^2}+\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
[Out] -1/2*(b*c*sqrt(d) - 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^
2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^2) -
 1/2*sqrt(d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 + ((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d^(3/2) - b
*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x
^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
[Out] int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.734 $\int \frac{x\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}$$

```
[Out] -1/2*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(1/2)-1/2*(d*x^2+c)^(1/2)/b/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {455, 43, 65, 214}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/(b*(a + b*x^2)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4b}\\ &= -\frac{\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2b}\\ &= -\frac{\sqrt{c+dx^2}}{2b(a+bx^2)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 80, normalized size = 1.00

$$-\frac{\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{3/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/(b*(a + b*x^2)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/
Sqrt[-(b*c) + a*d]])/(2*b^(3/2)*Sqrt[-(b*c) + a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1319 vs. $2(64) = 128$.
time = 0.09, size = 1320, normalized size = 16.50

method	result

default	$\frac{\sqrt{-ab}\left(\frac{b\left(d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\left(\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}}{}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*
b)^(1/2)/(a*d-b*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+
d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b
)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b*
(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1
/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a
/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b
*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)))-1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-
a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*
```

```
(-a*b)^(1/2)/(a*d-b*c)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)
/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x-1
/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^
2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(64) = 128.
time = 1.17, size = 356, normalized size = 4.45

$$\left[\frac{(bdx^2+ad)\sqrt{b^2c-abd}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(b^2c-abd)\sqrt{dx^2+c}}{8\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x^2)},-\frac{(bdx^2+ad)\sqrt{-b^2c+abd}\,\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)+2\,(b^2c-abd)\sqrt{dx^2+c}}{4\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((b*d*x^2 + a*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*
d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b
^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^2
), -1/4*((b*d*x^2 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c
- a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d -
 a*b*d^2)*x^2)) + 2*(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c - a^2*b^2*d +
 (b^4*c - a*b^3*d)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 0.52, size = 79, normalized size = 0.99

$$\frac{d\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^2+c}\,d}{2\left((dx^2+c)b-bc+ad\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b) - 1/2*sqrt(d*x^2 + c)*d/(((d*x^2 + c)*b - b*c + a*d)*b)
```

Mupad [B]
time = 0.44, size = 70, normalized size = 0.88

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{3/2}\,\sqrt{a\,d-b\,c}}-\frac{d\,\sqrt{d\,x^2+c}}{2\,\left(d\,b^2\,x^2+a\,d\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2)))/(2*b^(3/2)*(a*d - b
*c)^(1/2)) - (d*(c + d*x^2)^(1/2))/(2*(b^2*d*x^2 + a*b*d))
```

3.735 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=82

$$\frac{x\sqrt{c+dx^2}}{2a\,(a+bx^2)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

```
[Out] 1/2*c*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c)
^(1/2)+1/2*x*(d*x^2+c)^(1/2)/a/(b*x^2+a)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {386, 385, 211}

$$\frac{c\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{x\sqrt{c+dx^2}}{2a(a+bx^2)} + \frac{c\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a}$$
$$= \frac{x\sqrt{c+dx^2}}{2a(a+bx^2)} + \frac{c\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2a}$$
$$= \frac{x\sqrt{c+dx^2}}{2a(a+bx^2)} + \frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.41, size = 102, normalized size = 1.24

$$\frac{x\sqrt{c+dx^2}}{2a^2+2abx^2} - \frac{c\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*a^2 + 2*a*b*x^2) - (c*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[
d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*Sqrt[b*c -
a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. 2(66) = 132.
time = 0.09, size = 1965, normalized size = 23.96

method	result	size
default	Expression too large to display	1965

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/a*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-
b*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b
)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
```

```
)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+
1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-
2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln
((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))
^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))+1/4/a/(-a*
b)^(1/2)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-
a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*
(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x-1/b*(-a*b)^(1/2))))-1/4/b/a*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*
(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1
/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^
2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d
-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(
1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))))-1/4/a/(-a*b)^(1/2)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((
-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(66) = 132.

time = 1.26, size = 369, normalized size = 4.50

$$\left[\frac{4\,(abc-a^2d)\sqrt{dx^2+c}\,x-(bcx^2+ac)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{8\,(a^3bc-a^4d+(a^2b^2c-a^3bd)x^2)},\ \frac{2\,(abc-a^2d)\sqrt{dx^2+c}\,x+(bcx^2+ac)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)}{4\,(a^3bc-a^4d+(a^2b^2c-a^3bd)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(-a*b*c + a
^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d
*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3*b*c - a^4*d + (a^2*b^2*c - a^
3*b*d)*x^2), 1/4*(2*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x + (b*c*x^2 + a*c)*sqr
t(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*s
qrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^3*b*c
 - a^4*d + (a^2*b^2*c - a^3*b*d)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(66) = 132.
time = 1.14, size = 218, normalized size = 2.66

$$-\frac{c\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}\,a}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*c*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) - ((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d^(3/2) - b
*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x
^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^2,x)
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^2, x)
```

3.736 $\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\,\sqrt{bc-ad}}$$

```
[Out] -arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)/a^2+1/2*(-a*d+2*b*c)*arctanh(b^(1
/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(1/2)/(-a*d+b*c)^(1/2)+1/2*(d*x
^2+c)^(1/2)/a/(b*x^2+a)
```

Rubi [A]
time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 101, 162, 65, 214}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\,\sqrt{bc-ad}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]/(2*a*(a + b*x^2)) - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]])/a^2 + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}\\
&= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{4a^2}\\
&= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} + \frac{c\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx\right.}{2a^2d}\\
&= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} - \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\,\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A]

time = 0.50, size = 111, normalized size = 0.93

$$\frac{\frac{a\sqrt{c+dx^2}}{a+bx^2}+\frac{(-2bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,\sqrt{-bc+ad}}-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^2])/(a + b*x^2) + ((-2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) - 2*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2000 vs. $2(97) = 194$.
time = 0.10, size = 2001, normalized size = 16.82

method	result	size
default	Expression too large to display	2001

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(-a*b)^(1/2)/a*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(
1/2)/(a*d-b*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x
+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b*(-a*
b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/
d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-
1/2/a^2*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a
*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(
x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x-1/b*(-a*b)^(1/2))))-1/2/a^2*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln(
```

```
(-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^
2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-1/4/(-a*b)^(1/
2)/a*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)
*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/
2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x-1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(
-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(
-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))+1/a^2*((d*x^2+c)
^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(97) = 194$.
time = 1.25, size = 1054, normalized size = 8.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*log((b
^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^
2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(c
)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(a*b^2*c - a^2*b*
d)*sqrt(d*x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/
8*(8*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/s
```

```
qrt(d*x^2 + c)) - (2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*
b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*
a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/4*((2*a*b*c - a^2*d +
(2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a
*b*d^2)*x^2)) + 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(c)*log(-
(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b^2*c - a^2*b*d)*sqrt
(d*x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/4*((2*a
*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d
*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d
 + (b^2*c*d - a*b*d^2)*x^2)) + 4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2
)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*
x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x*(a + b*x**2)**2), x)
```

Giac [A]
time = 0.51, size = 113, normalized size = 0.95

$$\frac{\sqrt{dx^2+c}\,d}{2\left((dx^2+c)b-bc+ad\right)a}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a^2}+\frac{c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d/(((d*x^2 + c)*b - b*c + a*d)*a) - 1/2*(2*b*c - a*d)*a
rctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) +
c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c))
```

Mupad [B]
time = 0.64, size = 996, normalized size = 8.37

$$\frac{d\sqrt{d\,x^2+c}}{2\,a\,\left(b\,\left(d\,x^2+c\right)+a\,d-b\,c\right)}-\frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a^2}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}-\frac{\left(2\,a\,b^2\,c\,d^3-\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}+\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}+\frac{\left(2\,a\,b^2\,c\,d^3+\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}}{\frac{b^2\,c^2\,d^3-\frac{a\,b\,c\,d^4}{2}}{a^3}+\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}-\frac{\left(2\,a\,b^2\,c\,d^3-\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}-\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}+\frac{\left(2\,a\,b^2\,c\,d^3+\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}}\right)\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x*(a + b*x^2)^2),x)
```

```
[Out] (d*(c + d*x^2)^(1/2))/(2*a*(b*(c + d*x^2) + a*d - b*c)) - (c^(1/2)*atanh((c
 + d*x^2)^(1/2)/c^(1/2)))/a^2 - (atan((((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*
b^3*c^2*d^2 - 4*a*b^2*c*d^3))/(2*a^2) - ((2*a*b^2*c*d^3 - ((16*a^5*b^2*d^3
- 32*a^4*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))
/(8*a^2*(a^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a
^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c)*1i)/(4*(a^2*b^2*
c - a^3*b*d)) + ((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*b^3*c^2*d^2 - 4*a*b^2*c
*d^3))/(2*a^2) + ((2*a*b^2*c*d^3 + ((16*a^5*b^2*d^3 - 32*a^4*b^3*c*d^2)*(c
+ d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(8*a^2*(a^2*b^2*c - a^
3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^3*b*d)))*(
-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c)*1i)/(4*(a^2*b^2*c - a^3*b*d)))/((b^2*c^
2*d^3 - (a*b*c*d^4)/2)/a^3 + ((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*b^3*c^2*d^
2 - 4*a*b^2*c*d^3))/(2*a^2) - ((2*a*b^2*c*d^3 - ((16*a^5*b^2*d^3 - 32*a^4*b
^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(8*a^2*(a
^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c -
 a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^3*b*d))
 - ((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*b^3*c^2*d^2 - 4*a*b^2*c*d^3))/(2*a^2
) + ((2*a*b^2*c*d^3 + ((16*a^5*b^2*d^3 - 32*a^4*b^3*c*d^2)*(c + d*x^2)^(1/2
)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(8*a^2*(a^2*b^2*c - a^3*b*d)))*(-b*
(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c
))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^3*b*d))))*(-b*(a*d - b*c))^(1/2)*
(a*d - 2*b*c)*1i)/(2*(a^2*b^2*c - a^3*b*d))
```

3.737 $\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

```
[Out] -1/2*(-2*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5
/2)/(-a*d+b*c)^(1/2)-3/2*(d*x^2+c)^(1/2)/a^2/x+1/2*(d*x^2+c)^(1/2)/a/x/(b*x
^2+a)
```

Rubi [A]

time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {480, 597, 12, 385, 211}

$$-\frac{(3bc-2ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (-3*Sqrt[c + d*x^2])/(2*a^2*x) + Sqrt[c + d*x^2]/(2*a*x*(a + b*x^2)) - ((3*
b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5
/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{\int \frac{-3c-2dx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{\int \frac{c(3bc-2ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a^2c}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{(3bc-2ad)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a^2}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2a^2}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A]
time = 0.43, size = 123, normalized size = 1.09

$$
\frac{\left(-2a-3bx^2\right)\sqrt{c+dx^2}}{2a^2x\left(a+bx^2\right)}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{5/2}\sqrt{bc-ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] ((-2*a - 3*b*x^2)*Sqrt[c + d*x^2])/(2*a^2*x*(a + b*x^2)) + ((3*b*c - 2*a*d)
*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c
 - a*d])])/(2*a^(5/2)*Sqrt[b*c - a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2023 vs. $2(93) = 186$.
time = 0.13, size = 2024, normalized size = 17.91

method	result	size
risch	Expression too large to display	1673
default	Expression too large to display	2024

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b
*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)
^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1
/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2
*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln(
(-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^
2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))+1/4/a^2*(1/
(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*((d*(x-
1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(
1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1
/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1
/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1
/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))-3/4*b/a^2/(-a*b)^(1/2)*(
```

```
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)
))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a
*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x
-1/b*(-a*b)^(1/2))))+3/4*b/a^2/(-a*b)^(1/2)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)
/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(
1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*
c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/a^2*(-
1/c/x*(d*x^2+c)^(3/2)+2*d/c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/
2)+(d*x^2+c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(93) = 186$.
time = 1.40, size = 458, normalized size = 4.05

$$\left[\frac{((3\,b^2c-2\,abd)x^3+(3\,abc-2\,a^2d)x)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(2\,a^2bc-2\,a^3d+3\,(ab^2c-a^2bd)x^2)\sqrt{dx^2+c}}{8\,((a^3b^2c-a^4bd)x^3+(a^4bc-a^5d)x)}, -\frac{((3\,b^2c-2\,abd)x^3+(3\,abc-2\,a^2d)x)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+2\,(2\,a^2bc-2\,a^3d+3\,(ab^2c-a^2bd)x^2)\sqrt{dx^2+c}}{4\,((a^3b^2c-a^4bd)x^3+(a^4bc-a^5d)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
[Out] [1/8*(((3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*sqrt(-a*b*c + a^2*d
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^2*b*c - 2*a^3*d + 3*(a*b^2*c -
 a^2*b*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c - a^4*b*d)*x^3 + (a^4*b*c - a^5
*d)*x), -1/4*(((3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*sqrt(a*b*c
- a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(2*a^2*b*c -
2*a^3*d + 3*(a*b^2*c - a^2*b*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c - a^4*b*d
)*x^3 + (a^4*b*c - a^5*d)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(93) = 186.
time = 1.23, size = 329, normalized size = 2.91

$$\frac{\left(3\,bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,a^2}+\frac{3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4 bc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4 ad^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 bc^2\sqrt{d}+10\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 acd^{\frac{3}{2}}+3\,bc^3\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6 b-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4 bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4 ad+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 bc^2-4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 acd-bc^3\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*c*sqrt(d) - 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))
^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2)
+ (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^
2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) + 10*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt(d))/(((sqrt(d)*x
 - sqrt(d*x^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{x^2\,{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)^2), x)
```

3.738 $\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=159

$$-\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}$$

```
[Out] 1/2*(-a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^3/c^(1/2)-1/2*(-3*a*d+4
*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a^3/(-a*d+b
*c)^(1/2)-b*(d*x^2+c)^(1/2)/a^2/(b*x^2+a)-1/2*(d*x^2+c)^(1/2)/a/x^2/(b*x^2+
a)
```

Rubi [A]
time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 101, 156, 162, 65, 214}

$$-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}-\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((b*Sqrt[c + d*x^2])/(a^2*(a + b*x^2))) - Sqrt[c + d*x^2]/(2*a*x^2*(a + b*
x^2)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*Sqrt[c]) -
(Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*a^3*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
```

```
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)^2}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4bc+ad)-\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^2\right)}{2a}$$

$$= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(4bc-ad)-bd(bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2a^2(bc-ad)}$$

$$= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}+\frac{(b(4bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{4a^3}$$

$$= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}+\frac{(b(4bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right.}{2a^3d}$$

$$= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}-\frac{\sqrt{b}\,(4bc-3a}{}$$

Mathematica [A]
time = 0.56, size = 132, normalized size = 0.83

$$\frac{-\frac{a(a+2bx^2)\sqrt{c+dx^2}}{x^2(a+bx^2)}+\frac{\sqrt{b}\,(4bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*(a + 2*b*x^2)*Sqrt[c + d*x^2])/(x^2*(a + b*x^2))) + (Sqrt[b]*(4*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) +
 a*d] + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/(2*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2070 vs. 2(133) = 266.
time = 0.15, size = 2071, normalized size = 13.03

method	result	size
risch	Expression too large to display	1715

default	Expression too large to display	2071

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/a^2/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a
*b)^(1/2)/(a*d-b*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b
+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/
b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b
*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*
a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/
b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))))+b/a^3*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*
(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+b/a^3*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln
((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))
^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/4*b/a^2/(-a
*b)^(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d
-b*c)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b
)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-
1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x-1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+
2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln
((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^
2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))+1/a^2*(-1/2
```

```
/c/x^2*(d*x^2+c)^(3/2)+1/2*d/c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(
d*x^2+c)^(1/2))/x)))-2/a^3*b*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*
x^2+c)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^3), x)
```

Fricas [A]
time = 1.69, size = 1043, normalized size = 6.56

$$\left[-\frac{((4b^2c^2-3abcd)x^4+(4abc^2-3a^2cd)x^2)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(2b^2c^2-3abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+2((4b^2c-abd)x^4+(4abc-a^2d)x^2)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+4(2abcx^2+a^2c)\sqrt{dx^2+c}}{8(a^3bcx^4+a^4cx^2)}, -\frac{4((4b^2c-abd)x^4+(4abc-a^2d)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+((4b^2c^2-3abcd)x^4+(4abc^2-3a^2cd)x^2)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(2b^2c^2-3abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+4(2abcx^2+a^2c)\sqrt{dx^2+c}}{8(a^3bcx^4+a^4cx^2)}, \frac{((4b^2c^2-3abcd)x^4+(4abc^2-3a^2cd)x^2)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2(bdx^2+bc)}\right)-((4b^2c-abd)x^4+(4abc-a^2d)x^2)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)-2(2abcx^2+a^2c)\sqrt{dx^2+c}}{4(a^3bcx^4+a^4cx^2)}, \frac{((4b^2c^2-3abcd)x^4+(4abc^2-3a^2cd)x^2)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2(bdx^2+bc)}\right)-2((4b^2c-abd)x^4+(4abc-a^2d)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-2(2abcx^2+a^2c)\sqrt{dx^2+c}}{4(a^3bcx^4+a^4cx^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^2*c^2 - 3*a*b*c*d)*x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b/(
b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c
*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 2*((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(2*a*b*c*x^2 + a^2*c)*sqrt(d*x^2 +
 c))/(a^3*b*c*x^4 + a^4*c*x^2), -1/8*(4*((4*b^2*c - a*b*d)*x^4 + (4*a*b*c -
 a^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^2*c^2 - 3*a*
b*c*d)*x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(
2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*
sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a*b*c*x^2 + a^2*c)
*sqrt(d*x^2 + c))/(a^3*b*c*x^4 + a^4*c*x^2), 1/4*(((4*b^2*c^2 - 3*a*b*c*d)*
x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2
 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - ((4
*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a*b*c*x^2 + a^2*c)*sqrt(d*x^2 + c))/(a
^3*b*c*x^4 + a^4*c*x^2), 1/4*(((4*b^2*c^2 - 3*a*b*c*d)*x^4 + (4*a*b*c^2 - 3
*a^2*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2*((4*b^2*c - a*b*d)*x^
4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(2
*a*b*c*x^2 + a^2*c)*sqrt(d*x^2 + c))/(a^3*b*c*x^4 + a^4*c*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)**2), x)
```

Giac [A]
time = 0.50, size = 183, normalized size = 1.15

$$\frac{(4\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,a^3}-\frac{(4\,bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}}-\frac{2\,(dx^2+c)^{\frac{3}{2}}bd-2\,\sqrt{dx^2+c}\,bcd+\sqrt{dx^2+c}\,ad^2}{2\,\left((dx^2+c)^2b-2\,(dx^2+c)bc+bc^2+(dx^2+c)ad-acd\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqr
t(-b^2*c + a*b*d)*a^3) - 1/2*(4*b*c - a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))
/(a^3*sqrt(-c)) - 1/2*(2*(d*x^2 + c)^(3/2)*b*d - 2*sqrt(d*x^2 + c)*b*c*d +
sqrt(d*x^2 + c)*a*d^2)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x
^2 + c)*a*d - a*c*d)*a^2)
```

Mupad [B]
time = 0.95, size = 1193, normalized size = 7.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (atan((((-b*(a*d - b*c))^(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*
c^2*d^2 - 16*a*b^4*c*d^3))/a^4 - (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 -
((8*a^7*b^2*d^3 - 16*a^6*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2
)*(3*a*d - 4*b*c))/(4*a^4*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d
 - 4*b*c))/(4*(a^4*d - a^3*b*c)))*(3*a*d - 4*b*c)*1i)/(4*(a^4*d - a^3*b*c))
 + ((-b*(a*d - b*c))^(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*c^2*
d^2 - 16*a*b^4*c*d^3))/a^4 + (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 + ((8*
a^7*b^2*d^3 - 16*a^6*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(3
*a*d - 4*b*c))/(4*a^4*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4
*b*c))/(4*(a^4*d - a^3*b*c)))*(3*a*d - 4*b*c)*1i)/(4*(a^4*d - a^3*b*c)))/((
```

```
(3*a^2*b^3*d^5)/2 + 8*b^5*c^2*d^3 - 8*a*b^4*c*d^4)/a^6 - ((-b*(a*d - b*c))^
(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*c^2*d^2 - 16*a*b^4*c*d^3)
)/a^4 - (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 - ((8*a^7*b^2*d^3 - 16*a^6*
b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c))/(4*a^4
*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c))/(4*(a^4*d - a^
3*b*c)))*(3*a*d - 4*b*c))/(4*(a^4*d - a^3*b*c)) + ((-b*(a*d - b*c))^(1/2)*(
((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*c^2*d^2 - 16*a*b^4*c*d^3))/a^4 +
 (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 + ((8*a^7*b^2*d^3 - 16*a^6*b^3*c*d
^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c))/(4*a^4*(a^4*d
 - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c))/(4*(a^4*d - a^3*b*c))
)*(3*a*d - 4*b*c))/(4*(a^4*d - a^3*b*c))))*(-b*(a*d - b*c))^(1/2)*(3*a*d -
4*b*c)*1i)/(2*(a^4*d - a^3*b*c)) - ((b*d*(c + d*x^2)^(3/2))/a^2 + (d*(c + d
*x^2)^(1/2)*(a*d - 2*b*c))/(2*a^2))/((c + d*x^2)*(a*d - 2*b*c) + b*(c + d*x
^2)^2 + b*c^2 - a*c*d) + (atanh((b^2*d^6*(c + d*x^2)^(1/2))/(4*c^(3/2)*((b^
3*d^5)/a - (b^2*d^6)/(4*c))) - (b^3*d^5*(c + d*x^2)^(1/2))/(c^(1/2)*(b^3*d^
5 - (a*b^2*d^6)/(4*c))))*(a*d - 4*b*c))/(2*a^3*c^(1/2))
```

3.739 $\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{5\sqrt{c+dx^2}}{6a^2x^3}+\frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx}+\frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)}+\frac{b(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}$$

```
[Out] 1/2*b*(-4*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(
7/2)/(-a*d+b*c)^(1/2)-5/6*(d*x^2+c)^(1/2)/a^2/x^3+1/6*(-2*a*d+15*b*c)*(d*x^
2+c)^(1/2)/a^3/c/x+1/2*(d*x^2+c)^(1/2)/a/x^3/(b*x^2+a)
```

Rubi [A]
time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {480, 597, 12, 385, 211}

$$\frac{b(5bc-4ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\,(15bc-2ad)}{6a^3cx}-\frac{5\sqrt{c+dx^2}}{6a^2x^3}+\frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (-5*Sqrt[c + d*x^2])/(6*a^2*x^3) + ((15*b*c - 2*a*d)*Sqrt[c + d*x^2])/(6*a^
3*c*x) + Sqrt[c + d*x^2]/(2*a*x^3*(a + b*x^2)) + (b*(5*b*c - 4*a*d)*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx = \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} - \frac{\int \frac{-5c-4dx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{\int \frac{-c(15bc-2ad)-10bcdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^2c}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} - \frac{\int -\frac{3bc^2(5bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^3c^2}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{(b(5bc-4ad))\int \frac{\cdots}{\left(a+bx^2\right)\cdots}}{2a^3}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{(b(5bc-4ad))\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{b(5bc-4ad)\tan^{-1}\left(\frac{\cdots}{\sqrt{\cdots}}\right.}{2a^{7/2}\sqrt{bc-\cdots}}$$

Mathematica [A]

time = 0.68, size = 154, normalized size = 1.05

$$\frac{\sqrt{c+dx^2}\,(15b^2cx^4-2abx^2(-5c+dx^2)-2a^2(c+dx^2))}{6a^3cx^3\,(a+bx^2)}-\frac{b(5bc-4ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{7/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^2*c*x^4 - 2*a*b*x^2*(-5*c + d*x^2) - 2*a^2*(c + d*x^
2)))/(6*a^3*c*x^3*(a + b*x^2)) - (b*(5*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*x
*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(7/2)*Sqrt
[b*c - a*d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2051 vs. 2(123) = 246.
time = 0.13, size = 2052, normalized size = 13.96

method	result	size
risch	Expression too large to display	1701
default	Expression too large to display	2052

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/a^3*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*
d-b*c)*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a
*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d
-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))/(x+1/b*(-a*b)^(1/2))))-2*d/(a*d-b*c)*b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2)
)-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*
ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))+5/4*b^2/
a^3/(-a*b)^(1/2)*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(
x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
```

```
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-5/4*b^2/a^3/(-a*b)^(1/2)*((d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-
d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2
)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2)
)^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*
b)^(1/2))))-1/3/a^2/c/x^3*(d*x^2+c)^(3/2)-1/4*b/a^3*(1/(a*d-b*c)*b/(x-1/b*(
-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*((d*(x-1/b*(-a*b)^(1/2))^2+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(
1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d
-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-2*d/(
a*d-b*c)*b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1
/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b
)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))))-2/a^3*b*(-1/c/x*(d*x^2+c)^(3/2)+2*d/c*(1/2*x
*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(123) = 246$.
time = 1.57, size = 602, normalized size = 4.10

$$\left[\frac{3\left((5b^3c^2-4ab^2cd)x^5+(5ab^2c^2-4a^2bcd)x^3\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-4\left(2a^3bc^2-2a^4cd-(15ab^3c^2-17a^2b^2cd+2a^3bd^2)x^4-2(5a^2b^2c^2-6a^3bcd+a^4d^2)x^2\right)\sqrt{dx^2+c}}{24\left((a^4b^2c^2-a^5bcd)x^5+(a^5bc^2-a^6cd)x^3\right)}, \frac{3\left((5b^3c^2-4ab^2cd)x^5+(5ab^2c^2-4a^2bcd)x^3\right)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-2\left(2a^3bc^2-2a^4cd-(15ab^3c^2-17a^2b^2cd+2a^3bd^2)x^4-2(5a^2b^2c^2-6a^3bcd+a^4d^2)x^2\right)\sqrt{dx^2+c}}{12\left((a^4b^2c^2-a^5bcd)x^5+(a^5bc^2-a^6cd)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*((5*b^3*c^2 - 4*a*b^2*c*d)*x^5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c
+ a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^3*b*c^2 - 2
```

```
*a^4*c*d - (15*a*b^3*c^2 - 17*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4 - 2*(5*a^2*b^2
*c^2 - 6*a^3*b*c*d + a^4*d^2)*x^2)*sqrt(d*x^2 + c))/((a^4*b^2*c^2 - a^5*b*c
*d)*x^5 + (a^5*b*c^2 - a^6*c*d)*x^3), 1/12*(3*((5*b^3*c^2 - 4*a*b^2*c*d)*x^
5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*
b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)
*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b*c^2 - 2*a^4*c*d - (15*a*b^3*c^2
 - 17*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4 - 2*(5*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4
*d^2)*x^2)*sqrt(d*x^2 + c))/((a^4*b^2*c^2 - a^5*b*c*d)*x^5 + (a^5*b*c^2 - a
^6*c*d)*x^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(123) = 246.
time = 1.31, size = 361, normalized size = 2.46

$$-\frac{\left(5\,b^2c\sqrt{d}-4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,a^3}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abd^{\frac{3}{2}}-b^2c^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)a^3}-\frac{2\left(6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}+6\,bc^3\sqrt{d}-ac^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b^2*c*sqrt(d) - 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*
a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*sqrt(d) - 2*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c
))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*a*d + b*c^2)*a^3) - 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt
(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*b*c^2*sqrt(d) + 6*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{x^4\,{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)^2),x)
[Out] int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)^2), x)
```

3.740 $\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=197

$$\frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}-\frac{x^3(c+dx^2)^{3/2}}{2b(a+bx^2)}-\frac{3\sqrt{a}\,(bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^4}$$

```
[Out] -1/2*x^3*(d*x^2+c)^(3/2)/b/(b*x^2+a)+3/8*(8*a^2*d^2-8*a*b*c*d+b^2*c^2)*arct
anh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^4/d^(1/2)-3/2*(-2*a*d+b*c)*arctan(x*(-a*d+
b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)*(-a*d+b*c)^(1/2)/b^4+3/8*(-4*a*
d+3*b*c)*x*(d*x^2+c)^(1/2)/b^3+3/4*d*x^3*(d*x^2+c)^(1/2)/b^2
```

Rubi [A]
time = 0.22, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {478, 595, 596, 537, 223, 212, 385, 211}

$$\frac{3(8a^2d^2-8abcd+b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^4\sqrt{d}}-\frac{3\sqrt{a}\,(bc-2ad)\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^4}+\frac{3x\sqrt{c+dx^2}\,(3bc-4ad)}{8b^3}-\frac{x^3(c+dx^2)^{3/2}}{2b(a+bx^2)}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (3*(3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x^3*Sqrt[c + d*x^2])/(
4*b^2) - (x^3*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) - (3*Sqrt[a]*(b*c - 2*a*
d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
2*b^4) + (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(8*b^4*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = -\frac{x^3(c+dx^2)^{3/2}}{2b\,(a+bx^2)} + \frac{\int \frac{x^2\sqrt{c+dx^2}\,(3c+6dx^2)}{a+bx^2}\,dx}{2b}$$

$$= \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3(c+dx^2)^{3/2}}{2b\,(a+bx^2)} + \frac{\int \frac{x^2(6c(2bc-3ad)+6d(3bc-4ad)x^2)}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{8b^2}$$

$$= \frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3(c+dx^2)^{3/2}}{2b\,(a+bx^2)} - \frac{\int \frac{6acd(3bc-4ad)-6d(b^2c^2-}{(a+bx^2)\sqrt{c+}}}{16b^3c}$$

$$= \frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3(c+dx^2)^{3/2}}{2b\,(a+bx^2)} - (3a(bc-2ad)(bc-a$$

$$= \frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3(c+dx^2)^{3/2}}{2b\,(a+bx^2)} - (3a(bc-2ad)(bc-a$$

$$= \frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3(c+dx^2)^{3/2}}{2b\,(a+bx^2)} - 3\sqrt{a}\,(bc-2ad)\sqrt{bc}$$

Mathematica [A]
time = 0.61, size = 197, normalized size = 1.00

$$\frac{\frac{b\sqrt{c+dx^2}\,(-12a^2dx+b^2x^3(5c+2dx^2)+ab(9cx-6dx^3))}{a+bx^2} + 12\sqrt{a}\,(bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right) - \frac{3(b^2c^2-8abcd+8a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*Sqrt[c + d*x^2]*(-12*a^2*d*x + b^2*x^3*(5*c + 2*d*x^2) + a*b*(9*c*x - 6
*d*x^3)))/(a + b*x^2) + 12*Sqrt[a]*(b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(a*
Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])] - (
3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sq
rt[d])/(8*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3439 vs. $2(165) = 330$.
time = 0.15, size = 3440, normalized size = 17.46

method	result	size
risch	Expression too large to display	2648
default	Expression too large to display	3440

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

```
[Out] 1/b^2*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(
x*d^(1/2)+(d*x^2+c)^(1/2))))-1/4*a/b^3*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(5/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(
2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c
)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)
+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/
2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(
x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+
4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b
*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a
*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)))))-1/4*a/b^3*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1
/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(5/2)+3*d*(-a*b)^(1/2)/(a*d-b*c)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(
1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))
^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*
d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d
^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a
*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(
-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8
*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-
b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d
```

```
*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x
+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))))-3/4/b^2*a/(-a*b)^(1/2)*(1/3*(d*(x
-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(
x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2
))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a
*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))
+3/4/b^2*a/(-a*b)^(1/2)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-
a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b
)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))
-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1
/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a)^2, x)
```

Fricas [A]
time = 1.50, size = 1249, normalized size = 6.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*
a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 6
*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*
d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)
)/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^3*d^2*x^5 + (5*b^3*c*d - 6*a*b^2*d^
2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*d*x^2 + a*b
^4*d), -1/8*(3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*
d + 8*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + 3*(a*b*
c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) - (2*b^3*d^2*x^5 + (5*b^3*c*d - 6*a*b^2*d^2)*x^3 +
 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*d*x^2 + a*b^4*d), -
1/16*(12*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(a*b*c - a^2
*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c
)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 3*(a*b^2*c^2 - 8*a^2
*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d^2)*x^2)*sqrt(d)*log
(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(2*b^3*d^2*x^5 + (5*b^3*c*
d - 6*a*b^2*d^2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b
^5*d*x^2 + a*b^4*d), -1/8*(6*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x
^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) +
3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d
^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^3*d^2*x^5 + (5*
b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 +
c))/(b^5*d*x^2 + a*b^4*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(165) = 330.
time = 0.54, size = 394, normalized size = 2.00

$$\frac{1}{8}\sqrt{dx^2+c}\,x\left(\frac{2\,dx^2}{b^2}+\frac{5\,b^7cd^2-8\,ab^6d^3}{b^9d^2}\right)+\frac{3\left(ab^2c^2\sqrt{d}-3\,a^2bcd^{\frac{3}{2}}+2\,a^3d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}\,b^4}-\frac{3\left(b^2c^2\sqrt{d}-8\,abcd^{\frac{3}{2}}+8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^4d}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ab^2c^2\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2bcd^{\frac{3}{2}}+2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^3d^{\frac{5}{2}}-ab^2c^3\sqrt{d}+a^2bc^2d^{\frac{3}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*x*(2*d*x^2/b^2 + (5*b^7*c*d^2 - 8*a*b^6*d^3)/(b^9*d^2))
 + 3/2*(a*b^2*c^2*sqrt(d) - 3*a^2*b*c*d^(3/2) + 2*a^3*d^(5/2))*arctan(1/2*(
(sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(
sqrt(a*b*c*d - a^2*d^2)*b^4) - 3/16*(b^2*c^2*sqrt(d) - 8*a*b*c*d^(3/2) + 8*
a^2*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^4*d) - ((sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2
*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(5/2) - a*b^2*c^3*sq
rt(d) + a^2*b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2
)*b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(d\,x^2+c\right)}^{3/2}}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

```
[Out] int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2, x)
```

3.741 $\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=163

$$\frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3}+\frac{(2bc-5ad)\,(c+dx^2)^{3/2}}{6b^2(bc-ad)}+\frac{a(c+dx^2)^{5/2}}{2b(bc-ad)\,(a+bx^2)}-\frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\Big(}{2b^{7/2}}$$

```
[Out] 1/6*(-5*a*d+2*b*c)*(d*x^2+c)^(3/2)/b^2/(-a*d+b*c)+1/2*a*(d*x^2+c)^(5/2)/b/(
-a*d+b*c)/(b*x^2+a)-1/2*(-5*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*
d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+1/2*(-5*a*d+2*b*c)*(d*x^2+c)^(1/2)/b
^3
```

Rubi [A]
time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 79, 52, 65, 214}

$$-\frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{\sqrt{c+dx^2}\,(2bc-5ad)}{2b^3}+\frac{(c+dx^2)^{3/2}\,(2bc-5ad)}{6b^2(bc-ad)}+\frac{a(c+dx^2)^{5/2}}{2b\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^2])/(2*b^3) + ((2*b*c - 5*a*d)*(c + d*x^2)^(3
/2))/(6*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(5/2))/(2*b*(b*c - a*d)*(a + b*x^
2)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sq
rt[b*c - a*d]])/(2*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{a(c+dx^2)^{5/2}}{2b(bc-ad)(a+bx^2)} + \frac{(2bc-5ad)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(2bc-5ad)(c+dx^2)^{3/2}}{6b^2(bc-ad)} + \frac{a(c+dx^2)^{5/2}}{2b(bc-ad)(a+bx^2)} + \frac{(2bc-5ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\right.}{4b^2}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)(c+dx^2)^{3/2}}{6b^2(bc-ad)} + \frac{a(c+dx^2)^{5/2}}{2b(bc-ad)(a+bx^2)} + \frac{((2bc}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)(c+dx^2)^{3/2}}{6b^2(bc-ad)} + \frac{a(c+dx^2)^{5/2}}{2b(bc-ad)(a+bx^2)} + \frac{((2bc}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)(c+dx^2)^{3/2}}{6b^2(bc-ad)} + \frac{a(c+dx^2)^{5/2}}{2b(bc-ad)(a+bx^2)} - \frac{(2bc-}{}$$

Mathematica [A]
time = 0.30, size = 125, normalized size = 0.77

$$\frac{\sqrt{c+dx^2}\,(-15a^2d+ab(11c-10dx^2)+2b^2x^2(4c+dx^2))}{6b^3\,(a+bx^2)} - \frac{(2bc-5ad)\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*d + a*b*(11*c - 10*d*x^2) + 2*b^2*x^2*(4*c + d*x^
2)))/(6*b^3*(a + b*x^2)) - ((2*b*c - 5*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt
[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(2*b^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3380 vs. 2(139) = 278.
time = 0.13, size = 3381, normalized size = 20.74

method	result	size
risch	Expression too large to display	2519
default	Expression too large to display	3381

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

```
[Out] -1/4/b^3*(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3*d*(-a
*b)^(1/2)/(a*d-b*c)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)
^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^
(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*
d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(
-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a
*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d
*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2))/(x+1/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x+1/b*(-a*b)^(1
/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1
/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^
2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d
-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^
(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)))))+1/2/b^2*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/
b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2
*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/
b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x
-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+1/2/b^2*(1/3*(d*(x+1/b*(-a*b)^(1/2))
^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2
)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(
1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*
d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2
)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-
d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(
```

```
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/4/b^3*(-a*b)^
(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3*d*(-a*b)^(1/2)/(a*d-b
*c)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*
b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*
b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1
/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/
2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-
a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1
/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a
*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)
/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a
*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(...
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: ValueError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 1.43, size = 413, normalized size = 2.53

$$\left[-\frac{3\,(2\,abc-5\,a^2d+(2\,b^2c-5\,abd)x^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(2\,b^2dx^4+11\,abc-15\,a^2d+2\,(4\,b^2c-5\,abd)x^2)\sqrt{dx^2+c}}{24\,(b^4x^2+ab^3)},\ -\frac{3\,(2\,abc-5\,a^2d+(2\,b^2c-5\,abd)x^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\,(2\,b^2dx^4+11\,abc-15\,a^2d+2\,(4\,b^2c-5\,abd)x^2)\sqrt{dx^2+c}}{12\,(b^4x^2+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(2*a*b*c - 5*a^2*d + (2*b^2*c - 5*a*b*d)*x^2)*sqrt((b*c - a*d)/b)
*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*
d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)
/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*d +
2*(4*b^2*c - 5*a*b*d)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/12*(3*(2*
a*b*c - 5*a^2*d + (2*b^2*c - 5*a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2
*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*
d + (b*c*d - a*d^2)*x^2)) - 2*(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*d + 2*(4*b^2
*c - 5*a*b*d)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**3*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 0.54, size = 173, normalized size = 1.06

$$\frac{(2\,b^2c^2-7\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,b^3}+\frac{\sqrt{dx^2+c}\,abcd-\sqrt{dx^2+c}\,a^2d^2}{2\,((dx^2+c)b-bc+ad)b^3}+\frac{(dx^2+c)^{\frac{3}{2}}b^4+3\,\sqrt{dx^2+c}\,b^4c-6\,\sqrt{dx^2+c}\,ab^3d}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/2*(sqrt(d*x^2 + c)*a*b*c*d - sqr
t(d*x^2 + c)*a^2*d^2)/(((d*x^2 + c)*b - b*c + a*d)*b^3) + 1/3*((d*x^2 + c)^
(3/2)*b^4 + 3*sqrt(d*x^2 + c)*b^4*c - 6*sqrt(d*x^2 + c)*a*b^3*d)/b^6
```

Mupad [B]
time = 0.56, size = 183, normalized size = 1.12

$$\frac{{\left(d\,x^2+c\right)}^{3/2}}{3\,b^2}-\sqrt{d\,x^2+c}\,\left(\frac{c}{b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}\right)-\frac{\left(\frac{a^2\,d^2}{2}-\frac{a\,b\,c\,d}{2}\right)\,\sqrt{d\,x^2+c}}{b^4\,\left(d\,x^2+c\right)-b^4\,c+a\,b^3\,d}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\sqrt{a\,d-b\,c}\,\left(5\,a\,d-2\,b\,c\right)}{5\,a^2\,d^2-7\,a\,b\,c\,d+2\,b^2\,c^2}\right)\,\sqrt{a\,d-b\,c}\,\left(5\,a\,d-2\,b\,c\right)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

```
[Out] (c + d*x^2)^(3/2)/(3*b^2) - (c + d*x^2)^(1/2)*(c/b^2 - (2*b^2*c - 2*a*b*d)/
b^4) - (((a^2*d^2)/2 - (a*b*c*d)/2)*(c + d*x^2)^(1/2))/(b^4*(c + d*x^2) - b
^4*c + a*b^3*d) + (atan((b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(1/2)*(5*a*d
 - 2*b*c))/(5*a^2*d^2 + 2*b^2*c^2 - 7*a*b*c*d))*(a*d - b*c)^(1/2)*(5*a*d -
2*b*c))/(2*b^(7/2))
```

3.742 $\quad \int \frac{x^2 \left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=149

$$\frac{dx\sqrt{c+dx^2}}{b^2}-\frac{x(c+dx^2)^{3/2}}{2b\,(a+bx^2)}+\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^3}+\frac{\sqrt{d}\,(3bc-4ad)\tanh^{-1}\left(\frac{\ \ }{\sqrt{\ }}\right.}{2b^3}$$

[Out] -1/2*x*(d*x^2+c)^(3/2)/b/(b*x^2+a)+1/2*(-4*a*d+3*b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)/b^3+1/2*(-4*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/b^3/a^(1/2)+d*x*(d*x^2+c)^(1/2)/b^2

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {478, 542, 537, 223, 212, 385, 211}

$$\frac{(bc-4ad)\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^3}+\frac{\sqrt{d}\,(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{x(c+dx^2)^{3/2}}{2b\,(a+bx^2)}+\frac{dx\sqrt{c+dx^2}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]

[Out] (d*x*Sqrt[c + d*x^2])/b^2 - (x*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) + ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^3) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^3)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = -\frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)} + \frac{\int \frac{\sqrt{c+dx^2}(c+4dx^2)}{a+bx^2}\,dx}{2b}$$

$$= \frac{dx\sqrt{c+dx^2}}{b^2} - \frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)} + \frac{\int \frac{2c(bc-2ad)+2d(3bc-4ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{4b^2}$$

$$= \frac{dx\sqrt{c+dx^2}}{b^2} - \frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)} + \frac{(d(3bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^3} + \frac{((bc-4ad)(bc-}{}$$

$$= \frac{dx\sqrt{c+dx^2}}{b^2} - \frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)} + \frac{(d(3bc-4ad))\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2b^3} +$$

$$= \frac{dx\sqrt{c+dx^2}}{b^2} - \frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)} + \frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^3} + \cdots$$

Mathematica [A]
time = 0.56, size = 160, normalized size = 1.07

$$\frac{\frac{bx\sqrt{c+dx^2}(-bc+2ad+bdx^2)}{a+bx^2} - \frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}} + \sqrt{d}(-3bc+4ad)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2]*(-(b*c) + 2*a*d + b*d*x^2))/(a + b*x^2) - ((b*c - 4*a
*d)*Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/
(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*(-3*b*c + 4*a*d)*Log[-(Sqrt[d
]*x) + Sqrt[c + d*x^2]])/(2*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3380 vs. 2(123) = 246.
time = 0.14, size = 3381, normalized size = 22.69

method	result	size
risch	Expression too large to display	2585
default	Expression too large to display	3381

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3*d*(-a*b)^(1/2)/(a*d
-b*c)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-
a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(
-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/
d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/
b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b
)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b
*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*
a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/
b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)))))+1/4/(-a*b)^(1/2)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(
-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/
b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))
-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/
b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/4/(-a*b)^(1/2)/b*(1/3*(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a
*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+
1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a
*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)
/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(
1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*
c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
```

```
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/4/b^2
*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(
1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1
/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1
/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-
a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^
(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b
)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(
1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3
/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(
1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a)^2, x)
```

Fricas [A]
time = 1.59, size = 996, normalized size = 6.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2
 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*
x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*s
qrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*
```

```
d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/8*(4*(3
*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) + (a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(-(b*c - a*d)/
a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*
a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b
*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 - (b^2*c - 2*a*b*
d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), 1/4*((a*b*c - 4*a^2*d + (b^2*c -
4*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt
(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x))
- (3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sq
rt(d*x^2 + c)*sqrt(d)*x - c) + 2*(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x
^2 + c))/(b^4*x^2 + a*b^3), -1/4*(2*(3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d
)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b*c - 4*a^2*d + (b^
2*c - 4*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d
)*x)) - 2*(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b
^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (c + dx^2)^{\frac{3}{2}}}{(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(123) = 246.
time = 0.62, size = 336, normalized size = 2.26

$$\frac{\sqrt{dx^2+c}\, dx}{2\, b^2} - \frac{\left(3\, bc\sqrt{d} - 4\, ad^{\frac{3}{2}}\right) \log\left(\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2\right)}{4\, b^3} - \frac{\left(b^2c^2\sqrt{d} - 5\, abcd^{\frac{3}{2}} + 4\, a^2d^{\frac{5}{2}}\right) \arctan\left(\frac{\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 b - bc + 2\, ad}{2\, \sqrt{abcd - a^2d^2}}\right)}{2\, \sqrt{abcd - a^2d^2}\, b^3} + \frac{\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 b^2c^2\sqrt{d} - 3\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 abcd^{\frac{3}{2}} + 2\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 a^2d^{\frac{5}{2}} - b^2c^3\sqrt{d} + abc^2d^{\frac{3}{2}}}{\left(\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^4 b - 2\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 bc + 4\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 ad + bc^2\right) b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d*x/b^2 - 1/4*(3*b*c*sqrt(d) - 4*a*d^(3/2))*log((sqrt(d
)*x - sqrt(d*x^2 + c))^2)/b^3 - 1/2*(b^2*c^2*sqrt(d) - 5*a*b*c*d^(3/2) + 4*
a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^3) + ((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d^(
3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(5/2) - b^2*c^3*sqrt(d) + a*
```

```
b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d\,x^2+c\right)}^{3/2}}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

```
[Out] int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2, x)
```

3.743 $\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=99

$$\frac{3d\sqrt{c+dx^2}}{2b^2}-\frac{(c+dx^2)^{3/2}}{2b\,(a+bx^2)}-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}$$

```
[Out] -1/2*(d*x^2+c)^(3/2)/b/(b*x^2+a)-3/2*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*
d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(5/2)+3/2*d*(d*x^2+c)^(1/2)/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {455, 43, 52, 65, 214}

$$-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{(c+dx^2)^{3/2}}{2b\,(a+bx^2)}+\frac{3d\sqrt{c+dx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x^2])/(2*b^2) - (c + d*x^2)^(3/2)/(2*b*(a + b*x^2)) - (3*d*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5
/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^2\right)\\
&= -\frac{(c+dx^2)^{3/2}}{2b\,(a+bx^2)} + \frac{(3d)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{4b}\\
&= \frac{3d\sqrt{c+dx^2}}{2b^2} - \frac{(c+dx^2)^{3/2}}{2b\,(a+bx^2)} + \frac{(3d(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4b^2}\\
&= \frac{3d\sqrt{c+dx^2}}{2b^2} - \frac{(c+dx^2)^{3/2}}{2b\,(a+bx^2)} + \frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2b^2}\\
&= \frac{3d\sqrt{c+dx^2}}{2b^2} - \frac{(c+dx^2)^{3/2}}{2b\,(a+bx^2)} - \frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 96, normalized size = 0.97

$$\frac{\sqrt{c+dx^2}\,(-bc+3ad+2bdx^2)}{2b^2\,(a+bx^2)} - \frac{3d\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(b*c) + 3*a*d + 2*b*d*x^2))/(2*b^2*(a + b*x^2)) - (3*d*S
qrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(2*
b^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2151 vs. $2(79) = 158$.
time = 0.12, size = 2152, normalized size = 21.74

method	result	size
default	Expression too large to display	2152
risch	Expression too large to display	2185

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3*d*(-
a*b)^(1/2)/(a*d-b*c)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b
)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d
^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*
b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a
*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*
(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x+1/b*(-a*b)^(
1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(
1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))
^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*
d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d
^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)))))-1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1
/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(5/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(
1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))
```

```
^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*
d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^
(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b
)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(
2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*
c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(
x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/
b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.40, size = 333, normalized size = 3.36

$$\left[\frac{3\,(bdx^2+ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(2\,bdx^2-bc+3\,ad)\sqrt{dx^2+c}}{8\,(b^3x^2+ab^2)}, -\frac{3\,(bdx^2+ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\,(2\,bdx^2-bc+3\,ad)\sqrt{dx^2+c}}{4\,(b^3x^2+ab^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/8*(3*(b*d*x^2 + a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*
c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(d*x^2 + c))/(b^3*x^2 + a*b^2), -1/4*(3
*(b*d*x^2 + a*d)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*s
```

```
qrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2))
- 2*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(d*x^2 + c))/(b^3*x^2 + a*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 0.57, size = 122, normalized size = 1.23

$$\frac{\sqrt{dx^2+c}\,d}{b^2}+\frac{3\,(bcd-ad^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b^2}-\frac{\sqrt{dx^2+c}\,bcd-\sqrt{dx^2+c}\,ad^2}{2\,((dx^2+c)b-bc+ad)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] sqrt(d*x^2 + c)*d/b^2 + 3/2*(b*c*d - a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-
b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - 1/2*(sqrt(d*x^2 + c)*b*c*d - s
qrt(d*x^2 + c)*a*d^2)/(((d*x^2 + c)*b - b*c + a*d)*b^2)
```

Mupad [B]
time = 0.51, size = 117, normalized size = 1.18

$$\frac{\sqrt{d\,x^2+c}\,\left(\frac{a\,d^2}{2}-\frac{b\,c\,d}{2}\right)}{b^3\,\left(d\,x^2+c\right)-b^3\,c+a\,b^2\,d}+\frac{d\,\sqrt{d\,x^2+c}}{b^2}-\frac{3\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{d\,x^2+c}\,\sqrt{a\,d-b\,c}}{a\,d^2-b\,c\,d}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((a*d^2)/2 - (b*c*d)/2))/(b^3*(c + d*x^2) - b^3*c + a*b^
2*d) + (d*(c + d*x^2)^(1/2))/b^2 - (3*d*atan((b^(1/2)*d*(c + d*x^2)^(1/2)*(
a*d - b*c)^(1/2))/(a*d^2 - b*c*d))*(a*d - b*c)^(1/2))/(2*b^(5/2))
```

3.744 $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=131

$$\frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}+\frac{\sqrt{bc-ad}\,(bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

```
[Out] d^(3/2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2+1/2*(2*a*d+b*c)*arctan(x*(-a
*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/a^(3/2)/b^2+1/2*(-a
*d+b*c)*x*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {424, 537, 223, 212, 385, 211}

$$\frac{\sqrt{bc-ad}\,(2ad+bc)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{x\sqrt{c+dx^2}\,(bc-ad)}{2ab\left(a+bx^2\right)}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) + (Sqrt[b*c - a*d]*(b*c
 + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)
*b^2) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}+\frac{\int \frac{c(bc+ad)+2ad^2x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2ab}\\ &= \frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}+\frac{d^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2}+\frac{((bc-ad)(bc+2ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}}{2ab^2}\\ &= \frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}+\frac{d^2\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^2}+\frac{((bc-ad)(bc+2ad))\text{Su}}{}\\ &= \frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}+\frac{\sqrt{bc-ad}\,(bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{d^{3/2}\tanh^{-1}\left(\right.}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.52, size = 148, normalized size = 1.13

$$\frac{\frac{b(bc-ad)x\sqrt{c+dx^2}}{a(a+bx^2)}-\frac{\sqrt{bc-ad}\,(bc+2ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{3/2}}-2d^{3/2}\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*(b*c - a*d)*x*Sqrt[c + d*x^2])/(a*(a + b*x^2)) - (Sqrt[b*c - a*d]*(b*c
+ 2*a*d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sq
rt[b*c - a*d])])/a^(3/2) - 2*d^(3/2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(
2*b^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3386 vs. $2(109) = 218$.
time = 0.10, size = 3387, normalized size = 25.85

method	result	size
default	Expression too large to display	3387

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/a*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3*d*(-a*b)^(1/2)/(a*
d-b*c)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(
-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*
(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-(a*d-b*c)/b*((d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))
/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*
b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/
b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2
*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1
/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)))))+1/4/a/(-a*b)^(1/2)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*
(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a
/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
```

```
)-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1
/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/4/a/(-a*b)^(1/2)*(1/3*(d*(x+1/b*(-a*
b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-
a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b
*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-
a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2
)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b
*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))-1/4/b/
a*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*
(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(
1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(
1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d
^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^
2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)
^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/
b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^
(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(
3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^
(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^2, x)
```

Fricas [A]
time = 1.76, size = 903, normalized size = 6.89

$$\left[\frac{4\,(b^2c-abd)\sqrt{dx^2+c}\,x+4\,(abdx^2+a^2d)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+(abc+2\,a^2d+(b^2c+2\,abd)x^2)\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)}{8\,(ab^3x^2+a^2b^2)},\ \frac{4\,(b^2c-abd)\sqrt{dx^2+c}\,x-8\,(abdx^2+a^2d)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(abc+2\,a^2d+(b^2c+2\,abd)x^2)\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)}{8\,(ab^3x^2+a^2b^2)},\ \frac{2\,(b^2c-abd)\sqrt{dx^2+c}\,x+(abc+2\,a^2d+(b^2c+2\,abd)x^2)\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\,(abdx^2+a^2d)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)}{4\,(ab^3x^2+a^2b^2)},\ \frac{2\,(b^2c-abd)\sqrt{dx^2+c}\,x-4\,(abdx^2+a^2d)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(abc+2\,a^2d+(b^2c+2\,abd)x^2)\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)}{4\,(ab^3x^2+a^2b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x + 4*(a*b*d*x^2 + a^2*d)*sqrt(d)*l
og(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c + 2*a^2*d + (b^2*c
+ 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^
2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)
))/(a*b^3*x^2 + a^2*b^2), 1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x - 8*(a*b
*d*x^2 + a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*b*c + 2*a^
2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x -
(a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a
*b*x^2 + a^2)))/(a*b^3*x^2 + a^2*b^2), 1/4*(2*(b^2*c - a*b*d)*sqrt(d*x^2 +
c)*x + (a*b*c + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan
(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d
- a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d)*log(-2*d
*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a*b^3*x^2 + a^2*b^2), 1/4*(2*(b^2
*c - a*b*d)*sqrt(d*x^2 + c)*x - 4*(a*b*d*x^2 + a^2*d)*sqrt(-d)*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) + (a*b*c + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt((b*
c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
- a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/(a*b^3*x^2 + a^2*b^2)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{2}\right)^{\frac{3}{2}}}{\left(a + bx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(109) = 218$.
time = 0.50, size = 315, normalized size = 2.40

$$-\frac{d^{\frac{3}{2}} \log\left(\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2\right)}{2\,b^2} - \frac{\left(b^2c^2\sqrt{d} + abcd^{\frac{3}{2}} - 2\,a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}}\right)}{2\,\sqrt{abcd - a^2d^2}\,ab^2} - \frac{\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 b^2c^2\sqrt{d} - 3\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 abcd^{\frac{3}{2}} + 2\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 a^2d^{\frac{5}{2}} - b^2c^3\sqrt{d} + abc^2d^{\frac{3}{2}}}{\left(\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^4 b - 2\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 bc + 4\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 ad + bc^2\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 - 1/2*(b^2*c^2*sqrt(d
) + a*b*c*d^(3/2) - 2*a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*b^
2) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(5/2
) - b^2*c^3*sqrt(d) + a*b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b
- 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a*d + b*c^2)*a*b^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^2,x)
```

```
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^2, x)
```

3.745 $\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)^2} dx$

Optimal. Leaf size=129

$$\frac{(bc-ad)\sqrt{c+dx^2}}{2ab(a+bx^2)}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}$$

```
[Out] -c^(3/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2+1/2*(a*d+2*b*c)*arctanh(b^(1/
2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^2/b^(3/2)+1/2*(-a*d
+b*c)*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 100, 162, 65, 214}

$$\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{2ab(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) - (c^(3/2)*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]])/a^2 + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*b^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{x\,(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\,(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{bc^2+\frac{1}{2}d(bc+ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2ab}$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\,(a+bx^2)} + \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{((bc-ad)(2bc+ad))\text{Subst}\,(}{4a}$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\,(a+bx^2)} + \frac{c^2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{((bc-ad)(2bc+ad))\text{Su}}{}$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\,(a+bx^2)} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2} + \frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{2a^2b^{3/2}}$$

Mathematica [**A**]

time = 0.40, size = 122, normalized size = 0.95

$$\frac{\frac{a(bc-ad)\sqrt{c+dx^2}}{b(a+bx^2)}+\frac{\sqrt{-bc+ad}\,(2bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}-2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d)*Sqrt[c + d*x^2])/(b*(a + b*x^2)) + (Sqrt[-(b*c) + a*d]*(2*b
*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(3/2) - 2
*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3436 vs. $2(107) = 214$.
time = 0.10, size = 3437, normalized size = 26.64

method	result	size
default	Expression too large to display	3437

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3*d*(-a*b)
^(1/2)/(a*d-b*c)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1
/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/
2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(
1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b
*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a
*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*
b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x
+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2))/(x+1/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2)
)-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*
(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*
c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/
2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
```

```
*c)/b)^(1/2)))))-1/2/a^2*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(
-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/
b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))
-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/
b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)
*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2/a^2*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b
*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(
a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))
/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(
-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))-1/4/a/(-a*b)^(1/2)
*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(
1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1
/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1
/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-
a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^
(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b
)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(
1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3
/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(
```

1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x), x)

Fricas [A]
time = 1.29, size = 883, normalized size = 6.84

$$\left[\frac{\left(2abc+a^2d+\left(2b^2c+abd\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+4\left(b^2cx^2+abc\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+4\left(abc-a^2d\right)\sqrt{dx^2+c}}{8\left(a^2b^2x^2+a^3b\right)}, \frac{8\left(b^2cx^2+abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(2abc+a^2d+\left(2b^2c+abd\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+4\left(abc-a^2d\right)\sqrt{dx^2+c}}{8\left(a^2b^2x^2+a^3b\right)}, \frac{\left(2abc+a^2d+\left(2b^2c+abd\right)x^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)+2\left(b^2cx^2+abc\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(abc-a^2d\right)\sqrt{dx^2+c}}{4\left(a^2b^2x^2+a^3b\right)}, \frac{\left(2abc+a^2d+\left(2b^2c+abd\right)x^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)+4\left(b^2cx^2+abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+2\left(abc-a^2d\right)\sqrt{dx^2+c}}{4\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/8*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b^2*c*x^2 + a*b*c)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b), 1/8*(8*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b), 1/4*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b^2*c*x^2 + a*b*c)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b), 1/4*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 4*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x*(a + b*x**2)**2), x)
```

Giac [A]
time = 0.51, size = 154, normalized size = 1.19

$$\frac{c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}-\frac{(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a^2b}+\frac{\sqrt{dx^2+c}\,bcd-\sqrt{dx^2+c}\,ad^2}{2\left((dx^2+c)b-bc+ad\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/2*(2*b^2*c^2 - a*b*
c*d - a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
+ a*b*d)*a^2*b) + 1/2*(sqrt(d*x^2 + c)*b*c*d - sqrt(d*x^2 + c)*a*d^2)/(((d*
x^2 + c)*b - b*c + a*d)*a*b)
```

Mupad [B]
time = 0.63, size = 488, normalized size = 3.78

$$-\frac{\mathrm{atanh}\left(\frac{d^6\sqrt{d\,x^2+c}\,\sqrt{c^3}}{2\left(\frac{c^2\,d^6}{2}+\frac{b\,c^3\,d^5}{a}-\frac{3\,b^2\,c^4\,d^4}{2\,a^2}\right)}+\frac{c\,d^5\sqrt{d\,x^2+c}\,\sqrt{c^3}}{c^3\,d^5+\frac{a\,c^2\,d^6}{2\,b}-\frac{3\,b\,c^4\,d^4}{2\,a}}-\frac{3\,b\,c^2\,d^4\sqrt{d\,x^2+c}\,\sqrt{c^3}}{2\left(a\,c^3\,d^5-\frac{3\,b\,c^4\,d^4}{2}+\frac{a^2\,c^2\,d^6}{2\,b}\right)}\right)\sqrt{c^3}}{a^2}-\frac{\mathrm{atanh}\left(\frac{5\,c^2\,d^5\sqrt{d\,x^2+c}\,\sqrt{b^4\,c-a\,b^3\,d}}{4\left(\frac{a^2\,c\,d^7}{4}+\frac{b^2\,c^3\,d^5}{4}-\frac{3\,b^3\,c^4\,d^4}{2\,a}+a\,b\,c^2\,d^6\right)}+\frac{3\,c^3\,d^4\sqrt{d\,x^2+c}\,\sqrt{b^4\,c-a\,b^3\,d}}{2\left(a^2\,c^2\,d^6-\frac{3\,b^2\,c^4\,d^4}{2}+\frac{a^3\,c\,d^7}{4\,b}+\frac{a\,b\,c^3\,d^5}{4}\right)}+\frac{c\,d^6\sqrt{d\,x^2+c}\,\sqrt{b^4\,c-a\,b^3\,d}}{4\left(b^2\,c^2\,d^6+\frac{a\,b\,c\,d^7}{4}+\frac{b^3\,c^3\,d^5}{4\,a}-\frac{3\,b^4\,c^4\,d^4}{2\,a^2}\right)}\right)(a\,d+2\,b\,c)\sqrt{-b^3\,(a\,d-b\,c)}}{2\,a^2\,b^3}-\frac{d\sqrt{d\,x^2+c}\,(a\,d-b\,c)}{2\,a\,b\,\left(b\,(d\,x^2+c)+a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x)
```

```
[Out] - (atanh((d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(2*((c^2*d^6)/2 + (b*c^3*d^5)/
a - (3*b^2*c^4*d^4)/(2*a^2))) + (c*d^5*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(c^3*
d^5 + (a*c^2*d^6)/(2*b) - (3*b*c^4*d^4)/(2*a)) - (3*b*c^2*d^4*(c + d*x^2)^(
1/2)*(c^3)^(1/2))/(2*(a*c^3*d^5 - (3*b*c^4*d^4)/2 + (a^2*c^2*d^6)/(2*b))))*
(c^3)^(1/2))/a^2 - (atanh((5*c^2*d^5*(c + d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(1
/2))/(4*((a^2*c*d^7)/4 + (b^2*c^3*d^5)/4 - (3*b^3*c^4*d^4)/(2*a) + a*b*c^2*
d^6)) + (3*c^3*d^4*(c + d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(1/2))/(2*(a^2*c^2*d
^6 - (3*b^2*c^4*d^4)/2 + (a^3*c*d^7)/(4*b) + (a*b*c^3*d^5)/4)) + (c*d^6*(c
+ d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(1/2))/(4*(b^2*c^2*d^6 + (a*b*c*d^7)/4 + (
b^3*c^3*d^5)/(4*a) - (3*b^4*c^4*d^4)/(2*a^2))))*(a*d + 2*b*c)*(-b^3*(a*d -
b*c))^(1/2))/(2*a^2*b^3) - (d*(c + d*x^2)^(1/2)*(a*d - b*c))/(2*a*b*(b*(c +
 d*x^2) + a*d - b*c))
```

3.746 $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=128

$$-\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\sqrt{c+dx^2}}{2abx(a+bx^2)}-\frac{3c\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}}$$

```
[Out] -3/2*c*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/
a^(5/2)-1/2*(-a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/b/x+1/2*(-a*d+b*c)*(d*x^2+c)^(
1/2)/a/b/x/(b*x^2+a)
```

Rubi [A]

time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {479, 597, 12, 385, 211}

$$-\frac{3c\sqrt{bc-ad}\,\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}}-\frac{\sqrt{c+dx^2}\,(3bc-ad)}{2a^2bx}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{2abx(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -1/2*((3*b*c - a*d)*Sqrt[c + d*x^2])/(a^2*b*x) + ((b*c - a*d)*Sqrt[c + d*x^
2])/(2*a*b*x*(a + b*x^2)) - (3*c*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx = \frac{(bc-ad)\sqrt{c+dx^2}}{2abx(a+bx^2)} - \frac{\int \frac{-c(3bc-ad)-2bcdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx}{2ab}$$

$$= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx(a+bx^2)} + \frac{\int -\frac{3bc^2(bc-ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a^2bc}$$

$$= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx(a+bx^2)} - \frac{(3c(bc-ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a^2}$$

$$= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx(a+bx^2)} - \frac{(3c(bc-ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx\right.}{2a^2}$$

$$= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx(a+bx^2)} - \frac{3c\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}}$$

Mathematica [A]

time = 0.47, size = 122, normalized size = 0.95

$$\frac{\sqrt{c+dx^2}\,(-2ac-3bcx^2+adx^2)}{2a^2x\,(a+bx^2)}+\frac{3c\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a*c - 3*b*c*x^2 + a*d*x^2))/(2*a^2*x*(a + b*x^2)) + (3
*c*Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(
Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3461 vs. $2(108) = 216$.
time = 0.13, size = 3462, normalized size = 27.05

method	result	size
risch	Expression too large to display	2523
default	Expression too large to display	3462

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3*d*(-a*b)^(1/2)/(a*d
-b*c)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-
a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(
-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/
d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/
b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b
)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b
*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*
a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/
```

b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
)))))-3/4*b/a^2/(-a*b)^(1/2)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1
/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^
2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(
x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+3/4*b/a^2/(-a*b)^(1/2)*(1/3*(d*(x+1
/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d
(x+1/b(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x
+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2)
)^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*
b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))
+1/a^2*(-1/c/x*(d*x^2+c)^(5/2)+4*d/c*(1/4*x*(d*x^2+c)^(3/2)+3/4*c*(1/2*x*(d
*x^2+c)^(1/2)+1/2*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))))+1/4/a^2*(1/(a*
d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(1/3*(d*
(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/
d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d
(x-1/b(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b
(x-1/b(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1
/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(
-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/
b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*
(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2

```
*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^2), x)
```

Fricas [A]
time = 1.22, size = 351, normalized size = 2.74

$$\left[\frac{3\,(bcx^3+acx)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,((3\,bc-ad)x^2+2\,ac)\sqrt{dx^2+c}}{8\,(a^2bx^3+a^3x)}, -\frac{3\,(bcx^3+acx)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\,((3\,bc-ad)x^2+2\,ac)\sqrt{dx^2+c}}{4\,(a^2bx^3+a^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
[Out] [1/8*(3*(b*c*x^3 + a*c*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*
b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) - 4*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(d*x^2 + c))/(a^2*b*x^3 + a
^3*x), -1/4*(3*(b*c*x^3 + a*c*x)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a
*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (
b*c^2 - a*c*d)*x)) + 2*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(d*x^2 + c))/(a^2*b*
x^3 + a^3*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{x^2\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a)**2,x)
[Out] Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(108) = 216.
time = 1.27, size = 412, normalized size = 3.22

$$\frac{3\left(bc^2\sqrt{d}-acd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}\,a^2}+\frac{3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b^2c^2\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abcd^{\frac{3}{2}}+2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{5}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c^3\sqrt{d}+12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^2d^{\frac{3}{2}}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2cd^{\frac{5}{2}}+3\,b^2c^4\sqrt{d}-abc^3d^{\frac{3}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6b-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4ad+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc^2-4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2acd-bc^3\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*(b*c^2*sqrt(d) - a*c*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))
^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2)
+ (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(5/2)
 - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^3*sqrt(d) + 12*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*c^2*d^(3/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c*d^
(5/2) + 3*b^2*c^4*sqrt(d) - a*b*c^3*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c)
)^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 +
 c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*a*c*d - b*c^3)*a^2*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{x^2\,\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2), x)
```

3.747 $\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2(a+bx^2)}-\frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)}+\frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{bc-ad}\,(4bc-ad)\tanh}{2a^3\sqrt{}}$$

[Out] 1/2*(-3*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)/a^3-1/2*(-a*d+4*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^3/b^(1/2)-1/2*(-a*d+2*b*c)*(d*x^2+c)^(1/2)/a^2/(b*x^2+a)-1/2*c*(d*x^2+c)^(1/2)/a/x^2/(b*x^2+a)

Rubi [A]
time = 0.17, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 100, 156, 162, 65, 214}

$$\frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{bc-ad}\,(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{b}}-\frac{\sqrt{c+dx^2}\,(2bc-ad)}{2a^2(a+bx^2)}-\frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]

[Out] -1/2*((2*b*c - a*d)*Sqrt[c + d*x^2])/(a^2*(a + b*x^2)) - (c*Sqrt[c + d*x^2])/(2*a*x^2*(a + b*x^2)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*Sqrt[b])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)^2}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)+\frac{1}{2}d(3bc-2ad)x}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2(a+bx^2)} - \frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)(bc-ad)+\frac{1}{2}d(bc-ad)(2bc-ad}{x(a+bx)\sqrt{c+dx}}\right.}{2a^2(bc-ad)}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2(a+bx^2)} - \frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)} - \frac{(c(4bc-3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x,\right.}{4a^3}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2(a+bx^2)} - \frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)} - \frac{(c(4bc-3ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c}\right.}{2a^3d}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2(a+bx^2)} - \frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)} + \frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3} -$$

Mathematica [A]
time = 0.67, size = 154, normalized size = 0.91

$$\frac{\frac{a\sqrt{c+dx^2}(-ac-2bcx^2+adx^2)}{x^2(a+bx^2)} + \frac{(4b^2c^2-5abcd+a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}} + \sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(-(a*c) - 2*b*c*x^2 + a*d*x^2))/(x^2*(a + b*x^2)) + ((4
*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c
) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) + Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]])/(2*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3520 vs. 2(142) = 284.
time = 0.15, size = 3521, normalized size = 20.71

method	result	size
risch	Expression too large to display	2241

default	Expression too large to display	3521

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/4*b/a^2/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3*d*(-a*b)^(1/2)/(a*d-b*c)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))))+b/a^3*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+b/a^3*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))

```
-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*
(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+
2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/a^2*(-1/2/c/x^2
*(d*x^2+c)^(5/2)+3/2*d/c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln
((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))+1/4*b/a^2/(-a*b)^(1/2)*(1/(a*d-b*c)*
b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(1/3*(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+
d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x
-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b
*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(
1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d
-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-4*d/
(a*d-b*c)*b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+
3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*
b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^3), x)
```

Fricas [A]
time = 1.39, size = 1034, normalized size = 6.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt((b*c - a*d)/b)*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/
b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c -
3*a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
 4*(a^2*c + (2*a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2),
-1/8*(4*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(-c)*arctan
(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2
)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 +
2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2
+ c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*c + (2*a*b*
c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), -1/4*(((4*b^2*c - a
*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^
2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c
*d - a*d^2)*x^2)) + ((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqr
t(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*c + (2*a*
b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), -1/4*(((4*b^2*c -
 a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*
x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b
*c*d - a*d^2)*x^2)) + 2*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)
*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a^2*c + (2*a*b*c - a^2*d)*x
^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^3\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**3*(a + b*x**2)**2), x)
```

Giac [A]
time = 0.53, size = 216, normalized size = 1.27

$$\frac{\left(4\,b^2c^2-5\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,a^3}-\frac{\left(4\,bc^2-3\,acd\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}}-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}bcd-2\sqrt{dx^2+c}\,bc^2d-\left(dx^2+c\right)^{\frac{3}{2}}ad^2+2\sqrt{dx^2+c}\,acd^2}{2\left(\left(dx^2+c\right)^2b-2\left(dx^2+c\right)bc+bc^2+\left(dx^2+c\right)ad-acd\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/2*(4*b*c^2 - 3*a*c*d)*arctan(sqrt(
d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/2*(2*(d*x^2 + c)^(3/2)*b*c*d - 2*sq
```

```
rt(d*x^2 + c)*b*c^2*d - (d*x^2 + c)^(3/2)*a*d^2 + 2*sqrt(d*x^2 + c)*a*c*d^2
)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)*
a^2)
```

Mupad [B]
time = 0.96, size = 441, normalized size = 2.59

$$\frac{\operatorname{atanh}\left(\frac{b^2\,c^2\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{\frac{a^2\,b\,c\,d^7}{4}-\frac{5\,a\,b^2\,c^2\,d^6}{4}+b^3\,c^3\,d^5}-\frac{b\,c\,d^6\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(\frac{a\,b\,c\,d^7}{4}-\frac{5\,b^2\,c^2\,d^6}{4}+\frac{b^3\,c^3\,d^5}{a}\right)}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-4\,b\,c\right)}{2\,a^3\,b}-\frac{\sqrt{c}\,\operatorname{atanh}\left(\frac{3\,b\,\sqrt{c}\,d^7\,\sqrt{d\,x^2+c}}{4\,\left(\frac{3\,b\,c\,d^7}{4}-\frac{7\,b^2\,c^2\,d^6}{4\,a}+\frac{b^3\,c^3\,d^5}{a^2}\right)}-\frac{7\,b^2\,c^{3/2}\,d^6\,\sqrt{d\,x^2+c}}{4\,\left(\frac{3\,a\,b\,c\,d^7}{4}-\frac{7\,b^2\,c^2\,d^6}{4}+\frac{b^3\,c^3\,d^5}{a}\right)}+\frac{b^3\,c^{5/2}\,d^5\,\sqrt{d\,x^2+c}}{\frac{3\,a^2\,b\,c\,d^7}{4}-\frac{7\,a\,b^2\,c^2\,d^6}{4}+b^3\,c^3\,d^5}\right)\,\left(3\,a\,d-4\,b\,c\right)}{2\,a^3}-\frac{\frac{\left(a\,c\,d^2-b\,c^2\,d\right)\,\sqrt{d\,x^2+c}}{a^2}-\frac{d\,{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d-2\,b\,c\right)}{2\,a^2}}{\left(d\,x^2+c\right)\,\left(a\,d-2\,b\,c\right)+b\,{\left(d\,x^2+c\right)}^2+b\,c^2-a\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (atanh((b^2*c^2*d^5*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(b^3*c^3*d^5 -
 (5*a*b^2*c^2*d^6)/4 + (a^2*b*c*d^7)/4) - (b*c*d^6*(c + d*x^2)^(1/2)*(b^2*c
 - a*b*d)^(1/2))/(4*((a*b*c*d^7)/4 - (5*b^2*c^2*d^6)/4 + (b^3*c^3*d^5)/a)))
*(-b*(a*d - b*c))^(1/2)*(a*d - 4*b*c))/(2*a^3*b) - (c^(1/2)*atanh((3*b*c^(1
/2)*d^7*(c + d*x^2)^(1/2))/(4*((3*b*c*d^7)/4 - (7*b^2*c^2*d^6)/(4*a) + (b^3
*c^3*d^5)/a^2)) - (7*b^2*c^(3/2)*d^6*(c + d*x^2)^(1/2))/(4*((3*a*b*c*d^7)/4
 - (7*b^2*c^2*d^6)/4 + (b^3*c^3*d^5)/a)) + (b^3*c^(5/2)*d^5*(c + d*x^2)^(1/
2))/(b^3*c^3*d^5 - (7*a*b^2*c^2*d^6)/4 + (3*a^2*b*c*d^7)/4))*(3*a*d - 4*b*c
))/(2*a^3) - (((a*c*d^2 - b*c^2*d)*(c + d*x^2)^(1/2))/a^2 - (d*(c + d*x^2)^
(3/2)*(a*d - 2*b*c))/(2*a^2))/((c + d*x^2)*(a*d - 2*b*c) + b*(c + d*x^2)^2
+ b*c^2 - a*c*d)
```

3.748 $\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3}+\frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x}+\frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\,(a+bx^2)}+\frac{(5bc-2ad)\sqrt{bc-ad}\ \tan^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{2a^{7/2}}$$

```
[Out] 1/2*(-2*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d
+b*c)^(1/2)/a^(7/2)-1/6*(-3*a*d+5*b*c)*(d*x^2+c)^(1/2)/a^2/b/x^3+1/6*(-11*a
*d+15*b*c)*(d*x^2+c)^(1/2)/a^3/x+1/2*(-a*d+b*c)*(d*x^2+c)^(1/2)/a/b/x^3/(b*
x^2+a)
```

Rubi [A]

time = 0.16, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {479, 597, 12, 385, 211}

$$\frac{(5bc-2ad)\sqrt{bc-ad}\ \text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\ \sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}\ (15bc-11ad)}{6a^3x}-\frac{\sqrt{c+dx^2}\ (5bc-3ad)}{6a^2bx^3}+\frac{\sqrt{c+dx^2}\ (bc-ad)}{2abx^3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/6*((5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(a^2*b*x^3) + ((15*b*c - 11*a*d)*Sqr
t[c + d*x^2])/(6*a^3*x) + ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*x^3*(a + b*x
^2)) + ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]
*Sqrt[c + d*x^2])])/(2*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx = \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3(a+bx^2)} - \frac{\int \frac{-c(5bc-3ad)-2d(2bc-ad)x^2}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx}{2ab}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3(a+bx^2)} + \frac{\int \frac{-bc^2(15bc-11ad)-2bcd(5bc-3ad)x^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx}{6a^2bc}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3(a+bx^2)} - \frac{\int -\frac{3bc}{(a+}}{}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3(a+bx^2)} + \frac{((5bc-}{}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3(a+bx^2)} + \frac{((5bc-}{}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3(a+bx^2)} + \frac{(5bc-}{}$$

Mathematica [A]

time = 0.48, size = 151, normalized size = 0.91

$$\frac{\sqrt{c+dx^2}\,(15b^2cx^4+abx^2(10c-11dx^2)-2a^2(c+4dx^2))}{6a^3x^3\,(a+bx^2)}-\frac{(5bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]

[Out] (Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(10*c - 11*d*x^2) - 2*a^2*(c + 4*d *x^2)))/(6*a^3*x^3*(a + b*x^2)) - ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/ (2*a^(7/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3572 vs. 2(142) = 284.
time = 0.13, size = 3573, normalized size = 21.52

method	result	size
risch	Expression too large to display	2554
default	Expression too large to display	3573

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/4*b/a^3*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2* d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+3*d*(-a*b)^(1/2)/(a*d-b*c)*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1 /2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d *(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a* b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((- d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2- 2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/ b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2 *d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a *b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x +1/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(- a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^ (1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+ 1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d ^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x

+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(
1/2)))))+5/4*b^2/a^3/(-a*b)^(1/2)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d
(x-1/b(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d
(x-1/b(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b
)^(1/2)))-(a*d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/
b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-5/4*b^2/a^3/(-a*b)^(1/2)*(1/3*
(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/
b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)
/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/
2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2
d(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1
/2)))))-1/4*b/a^3*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2
))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-3*d*(-a*b)^
(1/2)/(a*d-b*c)*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b)^(1/
2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2
)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/
2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*d-b*c
)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^
(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/
b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x-1/b*(-a*b)^(1/2)))))-4*d/(a*d-b*c)*b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2
d(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*
d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/

```
b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^4), x)
```

Fricas [A]
time = 1.37, size = 443, normalized size = 2.67

$$\left[-\frac{3\,\left((5\,b^2c-2\,abd)x^5+(5\,abc-2\,a^2d)x^3\right)\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,\left((15\,b^2c-11\,abd)x^4-2\,a^2c+2\,(5\,abc-4\,a^2d)x^2\right)\sqrt{dx^2+c}}{24\,(a^3bx^5+a^4x^3)},\ \frac{3\,\left((5\,b^2c-2\,abd)x^5+(5\,abc-2\,a^2d)x^3\right)\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\,\left((15\,b^2c-11\,abd)x^4-2\,a^2c+2\,(5\,abc-4\,a^2d)x^2\right)\sqrt{dx^2+c}}{12\,(a^3bx^5+a^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
[Out] [-1/24*(3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*sqrt(-(b*c -
a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqr
t(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*b^2*c - 11*a*b*d)*
x^4 - 2*a^2*c + 2*(5*a*b*c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^
4*x^3), 1/12*(3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*sqrt((b
*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
 - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*((15*b^2*c - 11*a
*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^
5 + a^4*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{x^4\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a)**2,x)
[Out] Integral((c + d*x**2)**(3/2)/(x**4*(a + b*x**2)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(142) = 284.
time = 1.35, size = 442, normalized size = 2.66

$$-\frac{\left(5\,b^2c^2\sqrt{d}-7\,abcd^{\frac{3}{2}}+2\,a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,a^3}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c^2\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abcd^{\frac{3}{2}}+2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{5}{2}}-b^2c^3\sqrt{d}+abc^2d^{\frac{3}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)a^3}-\frac{4\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4bc^2\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4acd^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc^3\sqrt{d}+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ac^2d^{\frac{3}{2}}+3\,bc^4\sqrt{d}-2\,ac^3d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b^2*c^2*sqrt(d) - 7*a*b*c*d^(3/2) + 2*a^2*d^(5/2))*arctan(1/2*((sqr
t(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt
(a*b*c*d - a^2*d^2)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2*sqrt(d)
 - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*a^2*d^(5/2) - b^2*c^3*sqrt(d) + a*b*c^2*d^(3/2))/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a^3) - 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c*d^(3/2) - 6*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*b*c^3*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a*c^2*d^(3/2) + 3*b*c^4*sqrt(d) - 2*a*c^3*d^(3/2))/(((sqrt(d)*x - sqrt
(d*x^2 + c))^2 - c)^3*a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{x^4\,\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2), x)
```

3.749 $\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=258

$$\frac{(19b^2c^2-52abcd+32a^2d^2)\,x\sqrt{c+dx^2}}{16b^4}+\frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3}+\frac{2dx^3(c+dx^2)^{3/2}}{3b^2}-\frac{x^3(c+dx^2)^{5/2}}{2b\,(a+bx^2)}-\frac{\sqrt{a}}{}$$

```
[Out] 2/3*d*x^3*(d*x^2+c)^(3/2)/b^2-1/2*x^3*(d*x^2+c)^(5/2)/b/(b*x^2+a)-1/2*(-8*a
*d+3*b*c)*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2
))*a^(1/2)/b^5+1/16*(-64*a^3*d^3+120*a^2*b*c*d^2-60*a*b^2*c^2*d+5*b^3*c^3)*
arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^5/d^(1/2)+1/16*(32*a^2*d^2-52*a*b*c*d+
19*b^2*c^2)*x*(d*x^2+c)^(1/2)/b^4+1/8*d*(-8*a*d+7*b*c)*x^3*(d*x^2+c)^(1/2)/
b^3
```

Rubi [A]
time = 0.30, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {478, 595, 596, 537, 223, 212, 385, 211}

$$\frac{x\sqrt{c+dx^2}\,(32a^2d^2-52abcd+19b^2c^2)}{16b^4}+\frac{(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^5\sqrt{d}}-\frac{\sqrt{a}\,(3bc-8ad)(bc-ad)^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^5}+\frac{dx^3\sqrt{c+dx^2}\,(7bc-8ad)}{8b^3}-\frac{x^3(c+dx^2)^{5/2}}{2b\,(a+bx^2)}+\frac{2dx^3(c+dx^2)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] ((19*b^2*c^2 - 52*a*b*c*d + 32*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^4) + (d*(7
*b*c - 8*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^3) + (2*d*x^3*(c + d*x^2)^(3/2))/(3
*b^2) - (x^3*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 8*a*d
)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(2*b^5) + ((5*b^3*c^3 - 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*a^3*d^3)*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*b^5*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = -\frac{x^3(c+dx^2)^{5/2}}{2b(a+bx^2)} + \frac{\int \frac{x^2(c+dx^2)^{3/2}(3c+8dx^2)}{a+bx^2}\,dx}{2b}$$

$$= \frac{2dx^3(c+dx^2)^{3/2}}{3b^2} - \frac{x^3(c+dx^2)^{5/2}}{2b(a+bx^2)} + \frac{\int \frac{x^2\sqrt{c+dx^2}\,(6c(3bc-4ad)+6d(7bc-8ad)x^2)}{a+bx^2}\,dx}{12b^2}$$

$$= \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3(c+dx^2)^{3/2}}{3b^2} - \frac{x^3(c+dx^2)^{5/2}}{2b(a+bx^2)} + \frac{\int \frac{x^2(6c(12b^2c^2-37abcd}{}}{}$$

$$= \frac{(19b^2c^2-52abcd+32a^2d^2)\,x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3(c+dx^2}{3b^2}$$

$$= \frac{(19b^2c^2-52abcd+32a^2d^2)\,x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3(c+dx^2}{3b^2}$$

$$= \frac{(19b^2c^2-52abcd+32a^2d^2)\,x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3(c+dx^2}{3b^2}$$

$$= \frac{(19b^2c^2-52abcd+32a^2d^2)\,x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3(c+dx^2}{3b^2}$$

Mathematica [A]
time = 0.92, size = 269, normalized size = 1.04

$$\frac{\frac{bx\sqrt{c+dx^2}\,(96a^3d^2+12a^2bd(-13c+4dx^2)+ab^2(57c^2-82cdx^2-16d^2x^4)+b^3x^2(33c^2+26cdx^2+8d^2x^4))}{a+bx^2} + 24\sqrt{a}\,\sqrt{bc-ad}\,(3b^2c^2-11abcd+8a^2d^2)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right) + \frac{3(-5b^3c^3+60ab^2c^2d-120a^2bcd^2+64a^3d^3)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{48b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2]*(96*a^3*d^2 + 12*a^2*b*d*(-13*c + 4*d*x^2) + a*b^2*(5
7*c^2 - 82*c*d*x^2 - 16*d^2*x^4) + b^3*x^2*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4
)))/(a + b*x^2) + 24*Sqrt[a]*Sqrt[b*c - a*d]*(3*b^2*c^2 - 11*a*b*c*d + 8*a^
2*d^2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt
[b*c - a*d])] + (3*(-5*b^3*c^3 + 60*a*b^2*c^2*d - 120*a^2*b*c*d^2 + 64*a^3*
d^3)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[d])/(48*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5358 vs. $2(222) = 444$.
time = 0.14, size = 5359, normalized size = 20.77

method	result	size
risch	Expression too large to display	3583
default	Expression too large to display	5359

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a)^2, x)
```

Fricas [A]
time = 4.73, size = 1697, normalized size = 6.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
[Out] [-1/96*(3*(5*a*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 +
(5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(d
)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 12*(3*a*b^2*c^2*d - 11*
a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a^2*b*d^3)*x^2)
*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c
 + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(8*b^4*d^3*x^7
+ 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 48*
a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x)*
sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/48*(3*(5*a*b^3*c^3 - 60*a^2*b^2*
c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^
2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
) - 6*(3*a*b^2*c^2*d - 11*a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2
*c*d^2 + 8*a^2*b*d^3)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
```

```
8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) - (8*b^4*d^3*x^7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*
d - 82*a*b^3*c*d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c
*d^2 + 32*a^3*b*d^3)*x)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/96*(24*(
3*a*b^2*c^2*d - 11*a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2
+ 8*a^2*b*d^3)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
 a^2*c*d)*x)) + 3*(5*a*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^
4*d^3 + (5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2
)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^4*d^3*x^
7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 4
8*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x
)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/48*(12*(3*a*b^2*c^2*d - 11*a^2
*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a^2*b*d^3)*x^2)*sq
rt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*
sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*a
*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 6
0*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(-d)*arctan(sqrt
(-d)*x/sqrt(d*x^2 + c)) - (8*b^4*d^3*x^7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x
^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2
*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^
5*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (c+dx^2)^{\frac{5}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(222) = 444.
time = 0.60, size = 521, normalized size = 2.02

$$\frac{1}{48}\left(2\left(\frac{4\,d^2x^2}{b^2}+\frac{13\,b^{12}cd^5-12\,ab^{11}d^6}{b^{14}d^4}\right)x^2+\frac{3\,(11\,b^{12}c^2d^4-36\,ab^{11}cd^5+24\,a^2b^{10}d^6)}{b^{14}d^4}\right)\sqrt{dx^2+c}\,x+\frac{\left(3\,ab^3c^3\sqrt{d}-14\,a^2b^2c^2d^{\frac{3}{2}}+19\,a^3bcd^{\frac{5}{2}}-8\,a^4d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,b^5}-\frac{\left(5\,b^3c^3\sqrt{d}-60\,ab^2c^2d^{\frac{3}{2}}+120\,a^2bcd^{\frac{5}{2}}-64\,a^3d^{\frac{7}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{32\,b^5d}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ab^3c^3\sqrt{d}-4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2b^2c^2d^{\frac{3}{2}}+5\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^3bcd^{\frac{5}{2}}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^4d^{\frac{7}{2}}-ab^3c^4\sqrt{d}+2\,a^2b^2c^3d^{\frac{3}{2}}-a^3bc^2d^{\frac{5}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2/b^2 + (13*b^12*c*d^5 - 12*a*b^11*d^6)/(b^14*d^4))*x^2 +
3*(11*b^12*c^2*d^4 - 36*a*b^11*c*d^5 + 24*a^2*b^10*d^6)/(b^14*d^4))*sqrt(d*
```

```
x^2 + c)*x + 1/2*(3*a*b^3*c^3*sqrt(d) - 14*a^2*b^2*c^2*d^(3/2) + 19*a^3*b*c
*d^(5/2) - 8*a^4*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b
*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^5) - 1/32*(
5*b^3*c^3*sqrt(d) - 60*a*b^2*c^2*d^(3/2) + 120*a^2*b*c*d^(5/2) - 64*a^3*d^(
7/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^5*d) - ((sqrt(d)*x - sqrt(d*x
^2 + c))^2*a*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^2*c^
2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*b*c*d^(5/2) - 2*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a^4*d^(7/2) - a*b^3*c^4*sqrt(d) + 2*a^2*b^2*c^3*d^(3
/2) - a^3*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*
b^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d\,x^2+c\right)}^{5/2}}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
[Out] int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2, x)
```

3.750 $\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=198

$$\frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4}+\frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3}+\frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)}+\frac{a(c+dx^2)^{7/2}}{2b(bc-ad)\left(a+bx^2\right)}-$$

```
[Out] 1/6*(-7*a*d+2*b*c)*(d*x^2+c)^(3/2)/b^3+1/10*(-7*a*d+2*b*c)*(d*x^2+c)^(5/2)/
b^2/(-a*d+b*c)+1/2*a*(d*x^2+c)^(7/2)/b/(-a*d+b*c)/(b*x^2+a)-1/2*(-7*a*d+2*b
*c)*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(9
/2)+1/2*(-7*a*d+2*b*c)*(-a*d+b*c)*(d*x^2+c)^(1/2)/b^4
```

Rubi [A]
time = 0.13, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 79, 52, 65, 214}

$$-\frac{(2bc-7ad)(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{9/2}}+\frac{\sqrt{c+dx^2}\,(2bc-7ad)(bc-ad)}{2b^4}+\frac{\left(c+dx^2\right)^{3/2}(2bc-7ad)}{6b^3}+\frac{\left(c+dx^2\right)^{5/2}(2bc-7ad)}{10b^2(bc-ad)}+\frac{a(c+dx^2)^{7/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 7*a*d)*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^4) + ((2*b*c - 7*a*d)*(c
 + d*x^2)^(3/2))/(6*b^3) + ((2*b*c - 7*a*d)*(c + d*x^2)^(5/2))/(10*b^2*(b*c
 - a*d)) + (a*(c + d*x^2)^(7/2))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c -
7*a*d)*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*b^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^{5/2}}{(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{a(c+dx^2)^{7/2}}{2b(bc-ad)(a+bx^2)} + \frac{(2bc-7ad)\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{a+bx}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(2bc-7ad)(c+dx^2)^{5/2}}{10b^2(bc-ad)} + \frac{a(c+dx^2)^{7/2}}{2b(bc-ad)(a+bx^2)} + \frac{(2bc-7ad)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, \ldots\right.}{4b^2}$$

$$= \frac{(2bc-7ad)(c+dx^2)^{3/2}}{6b^3} + \frac{(2bc-7ad)(c+dx^2)^{5/2}}{10b^2(bc-ad)} + \frac{a(c+dx^2)^{7/2}}{2b(bc-ad)(a+bx^2)} + \frac{((2bc \ldots}{\ldots}$$

$$= \frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4} + \frac{(2bc-7ad)(c+dx^2)^{3/2}}{6b^3} + \frac{(2bc-7ad)(c+dx^2)^{5/\ldots}}{10b^2(bc-ad)}$$

$$= \frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4} + \frac{(2bc-7ad)(c+dx^2)^{3/2}}{6b^3} + \frac{(2bc-7ad)(c+dx^2)^{5/\ldots}}{10b^2(bc-ad)}$$

$$= \frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4} + \frac{(2bc-7ad)(c+dx^2)^{3/2}}{6b^3} + \frac{(2bc-7ad)(c+dx^2)^{5/\ldots}}{10b^2(bc-ad)}$$

Mathematica [A]
time = 0.37, size = 183, normalized size = 0.92

$$\frac{\sqrt{c+dx^2}\,(105a^3d^2+10a^2bd(-17c+7dx^2)+ab^2(61c^2-118cdx^2-14d^2x^4)+2b^3x^2(23c^2+11cdx^2+3d^2x^4))}{30b^4\,(a+bx^2)} - \frac{\sqrt{-bc+ad}\,(2b^2c^2-9abcd+7a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(105*a^3*d^2 + 10*a^2*b*d*(-17*c + 7*d*x^2) + a*b^2*(61*c^
2 - 118*c*d*x^2 - 14*d^2*x^4) + 2*b^3*x^2*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)
))/(30*b^4*(a + b*x^2)) - (Sqrt[-(b*c) + a*d]*(2*b^2*c^2 - 9*a*b*c*d + 7*a^
2*d^2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(2*b^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5283 vs. $2(170) = 340$.
time = 0.14, size = 5284, normalized size = 26.69

method	result	size

risch	Expression too large to display	3412
default	Expression too large to display	5284

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.34, size = 573, normalized size = 2.89

$$\left[\frac{15\,{\left(2\,ab^{2}c^{2}-9\,a^{2}bcd+7\,a^{3}d^{2}+{\left(2\,b^{3}c^{2}-9\,ab^{2}cd+7\,a^{2}bd^{2}\right)}x^{2}\right)}\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\,{\left(4\,b^{2}cd-3\,abd^{2}\right)}x^{2}-4\,{\left(b^{2}dx^{2}+2\,b^{2}c-abd\right)}\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{b}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\,{\left(6\,b^{3}d^{2}x^{6}+61\,ab^{2}c^{2}-170\,a^{2}bcd+105\,a^{3}d^{2}+2\,{\left(11\,b^{3}cd-7\,ab^{2}d^{2}\right)}x^{4}+2\,{\left(23\,b^{3}c^{2}-59\,ab^{2}cd+35\,a^{2}bd^{2}\right)}x^{2}\right)}\sqrt{dx^{2}+c}}{120\,{\left(b^{5}x^{2}+ab^{4}\right)}},\,-\frac{15\,{\left(2\,ab^{2}c^{2}-9\,a^{2}bcd+7\,a^{3}d^{2}+{\left(2\,b^{3}c^{2}-9\,ab^{2}cd+7\,a^{2}bd^{2}\right)}x^{2}\right)}\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{{\left(bdx^{2}+2\,bc-ad\right)}\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{b}}}{2\,{\left(bc^{2}-acd+{\left(bcd-ad^{2}\right)}x^{2}\right)}}\right)-2\,{\left(6\,b^{3}d^{2}x^{6}+61\,ab^{2}c^{2}-170\,a^{2}bcd+105\,a^{3}d^{2}+2\,{\left(11\,b^{3}cd-7\,ab^{2}d^{2}\right)}x^{4}+2\,{\left(23\,b^{3}c^{2}-59\,ab^{2}cd+35\,a^{2}bd^{2}\right)}x^{2}\right)}\sqrt{dx^{2}+c}}{60\,{\left(b^{5}x^{2}+ab^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/120*(15*(2*a*b^2*c^2 - 9*a^2*b*c*d + 7*a^3*d^2 + (2*b^3*c^2 - 9*a*b^2*c*
d + 7*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c
- a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 4*(6*b^3*d^2*x^6 + 61*a*b^2*c^2 - 170*a^2*b*c*d + 105*a^3*d^2 + 2*(11*b^3
*c*d - 7*a*b^2*d^2)*x^4 + 2*(23*b^3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2)*x^2)
*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/60*(15*(2*a*b^2*c^2 - 9*a^2*b*c*d +
 7*a^3*d^2 + (2*b^3*c^2 - 9*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)
/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b
)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(6*b^3*d^2*x^6 + 61*a*b^2*c^2
- 170*a^2*b*c*d + 105*a^3*d^2 + 2*(11*b^3*c*d - 7*a*b^2*d^2)*x^4 + 2*(23*b^
3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4
)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 264, normalized size = 1.33

$$\frac{\left(2 b^{3} c^{3}-11 a b^{2} c^{2} d+16 a^{2} b c d^{2}-7 a^{3} d^{3}\right) \arctan \left(\frac{\sqrt{d x^{2}+c}\, b}{\sqrt{-b^{2} c+a b d}}\right)}{2 \sqrt{-b^{2} c+a b d}\, b^{4}}+\frac{\sqrt{d x^{2}+c}\, a b^{2} c^{2} d-2 \sqrt{d x^{2}+c}\, a^{2} b c d^{2}+\sqrt{d x^{2}+c}\, a^{3} d^{3}}{2\left(\left(d x^{2}+c\right) b-b c+a d\right) b^{4}}+\frac{3\left(d x^{2}+c\right)^{\frac{5}{2}} b^{8}+5\left(d x^{2}+c\right)^{\frac{3}{2}} b^{8} c+15 \sqrt{d x^{2}+c}\, b^{8} c^{2}-10\left(d x^{2}+c\right)^{\frac{3}{2}} a b^{7} d-60 \sqrt{d x^{2}+c}\, a b^{7} c d+45 \sqrt{d x^{2}+c}\, a^{2} b^{6} d^{2}}{15 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*c^3 - 11*a*b^2*c^2*d + 16*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(sqrt(d
*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 1/2*(sqrt(d*
x^2 + c)*a*b^2*c^2*d - 2*sqrt(d*x^2 + c)*a^2*b*c*d^2 + sqrt(d*x^2 + c)*a^3*
d^3)/(((d*x^2 + c)*b - b*c + a*d)*b^4) + 1/15*(3*(d*x^2 + c)^(5/2)*b^8 + 5*
(d*x^2 + c)^(3/2)*b^8*c + 15*sqrt(d*x^2 + c)*b^8*c^2 - 10*(d*x^2 + c)^(3/2)
*a*b^7*d - 60*sqrt(d*x^2 + c)*a*b^7*c*d + 45*sqrt(d*x^2 + c)*a^2*b^6*d^2)/b
^10
```

Mupad [B]
time = 0.64, size = 276, normalized size = 1.39

$$\frac{\left(d x^{2}+c\right)^{5/2}}{5 b^{2}}-\sqrt{d x^{2}+c}\left(\frac{(a d-b c)^{2}}{b^{4}}+\frac{\left(2 b^{2} c-2 a b d\right)\left(\frac{c}{b^{2}}-\frac{2 b^{2} c-2 a b d}{b^{4}}\right)}{b^{2}}\right)-\left(d x^{2}+c\right)^{3/2}\left(\frac{c}{3 b^{2}}-\frac{2 b^{2} c-2 a b d}{3 b^{4}}\right)+\frac{\sqrt{d x^{2}+c}\left(\frac{a^{3} d^{3}}{2}-a^{2} b c d^{2}+\frac{a b^{2} c^{2} d}{2}\right)}{b^{5}\left(d x^{2}+c\right)-b^{5} c+a b^{4} d}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\, \sqrt{d x^{2}+c}\,(a d-b c)^{3/2}(7 a d-2 b c)}{7 a^{3} d^{3}-16 a^{2} b c d^{2}+11 a b^{2} c^{2} d-2 b^{3} c^{3}}\right)(a d-b c)^{3/2}(7 a d-2 b c)}{2 b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

```
[Out] (c + d*x^2)^(5/2)/(5*b^2) - (c + d*x^2)^(1/2)*((a*d - b*c)^2/b^4 + ((2*b^2*
c - 2*a*b*d)*(c/b^2 - (2*b^2*c - 2*a*b*d)/b^4))/b^2) - (c + d*x^2)^(3/2)*(c
/(3*b^2) - (2*b^2*c - 2*a*b*d)/(3*b^4)) + ((c + d*x^2)^(1/2)*((a^3*d^3)/2 +
 (a*b^2*c^2*d)/2 - a^2*b*c*d^2))/(b^5*(c + d*x^2) - b^5*c + a*b^4*d) - (ata
n((b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2)*(7*a*d - 2*b*c))/(7*a^3*d^3
- 2*b^3*c^3 + 11*a*b^2*c^2*d - 16*a^2*b*c*d^2))*(a*d - b*c)^(3/2)*(7*a*d -
2*b*c))/(2*b^(9/2))
```

3.751 $\int \frac{x^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx(c+dx^2)^{3/2}}{4b^2}-\frac{x(c+dx^2)^{5/2}}{2b\,(a+bx^2)}+\frac{(bc-6ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^4}$$

```
[Out] 3/4*d*x*(d*x^2+c)^(3/2)/b^2-1/2*x*(d*x^2+c)^(5/2)/b/(b*x^2+a)+1/2*(-6*a*d+b
*c)*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b^4
/a^(1/2)+1/8*(24*a^2*d^2-40*a*b*c*d+15*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)
^(1/2))*d^(1/2)/b^4+1/8*d*(-12*a*d+11*b*c)*x*(d*x^2+c)^(1/2)/b^3
```

Rubi [A]
time = 0.16, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {478, 542, 537, 223, 212, 385, 211}

$$\frac{\sqrt{d}\,(24a^2d^2-40abcd+15b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^4}+\frac{(bc-6ad)(bc-ad)^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^4}+\frac{dx\sqrt{c+dx^2}\,(11bc-12ad)}{8b^3}-\frac{x(c+dx^2)^{5/2}}{2b\,(a+bx^2)}+\frac{3dx(c+dx^2)^{3/2}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*(11*b*c - 12*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x*(c + d*x^2)^(3/2))
/(4*b^2) - (x*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) + ((b*c - 6*a*d)*(b*c -
a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a
]*b^4) + (Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(8*b^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = -\frac{x(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{\int \frac{(c+dx^2)^{3/2}(c+6dx^2)}{a+bx^2}\,dx}{2b}$$

$$= \frac{3dx(c+dx^2)^{3/2}}{4b^2} - \frac{x(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{\int \frac{\sqrt{c+dx^2}\,(2c(2bc-3ad)+2d(11bc-12ad)x^2)}{a+bx^2}\,dx}{8b^2}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx(c+dx^2)^{3/2}}{4b^2} - \frac{x(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{\int \frac{2c(4b^2c^2-17abcd+1}{(a}}{}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx(c+dx^2)^{3/2}}{4b^2} - \frac{x(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{((bc-6ad)(bc-}{}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx(c+dx^2)^{3/2}}{4b^2} - \frac{x(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{((bc-6ad)(bc-}{}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx(c+dx^2)^{3/2}}{4b^2} - \frac{x(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{(bc-6ad)(bc-a}{}$$

Mathematica [A]
time = 0.81, size = 221, normalized size = 1.13

$$-\frac{\frac{bx\sqrt{c+dx^2}\,(12a^2d^2+abd(-17c+6dx^2)+b^2(4c^2-9cdx^2-2d^2x^4))}{a+bx^2} + \frac{4\sqrt{bc-ad}\,(b^2c^2-7abcd+6a^2d^2)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{\sqrt{a}} + \sqrt{d}\,(15b^2c^2-40abcd+24a^2d^2)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] -1/8*((b*x*Sqrt[c + d*x^2]*(12*a^2*d^2 + a*b*d*(-17*c + 6*d*x^2) + b^2*(4*c
^2 - 9*c*d*x^2 - 2*d^2*x^4)))/(a + b*x^2) + (4*Sqrt[b*c - a*d]*(b^2*c^2 - 7
*a*b*c*d + 6*a^2*d^2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2])
)/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d +
24*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5283 vs. $2(163) = 326$.
time = 0.14, size = 5284, normalized size = 27.10

method	result	size
risch	Expression too large to display	3498
default	Expression too large to display	5284

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a)^2, x)
```

Fricas [A]
time = 3.61, size = 1379, normalized size = 7.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*c
*d + 24*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x
- c) + 2*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*
a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^
2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 2*(2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*
b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/8*((15*a*
b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b
*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b^2*c^2 - 7*a^2
*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt(-(b*c
- a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*s
qrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^3*d^2*x^5 + 3*(3*b
^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sq
rt(d*x^2 + c))/(b^5*x^2 + a*b^4), 1/16*(4*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*
d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan
(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d
- a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + (15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*
d^2 + (15*b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2
```

```
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*
b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c)
)/(b^5*x^2 + a*b^4), -1/8*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*
b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - 2*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*
c*d + 6*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a
*c*d)*x)) - (2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 -
 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(163) = 326.
time = 0.53, size = 446, normalized size = 2.29

$$\frac{1}{8}\sqrt{dx^2+c}\left(\frac{2\,d^2x^2}{b^2}+\frac{9\,b^7cd^3-8\,ab^6d^4}{b^9d^2}\right)x-\frac{\left(15\,b^2c^2\sqrt{d}-40\,abcd^{\frac{3}{2}}+24\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^4}-\frac{\left(b^3c^3\sqrt{d}-8\,ab^2c^2d^{\frac{3}{2}}+13\,a^2bcd^{\frac{5}{2}}-6\,a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}\,b^4}+\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^3c^3\sqrt{d}-4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ab^2c^2d^{\frac{3}{2}}+5\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2bcd^{\frac{5}{2}}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^3d^{\frac{7}{2}}-b^3c^4\sqrt{d}+2\,ab^2c^3d^{\frac{3}{2}}-a^2bc^2d^{\frac{5}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*(2*d^2*x^2/b^2 + (9*b^7*c*d^3 - 8*a*b^6*d^4)/(b^9*d^2))
*x - 1/16*(15*b^2*c^2*sqrt(d) - 40*a*b*c*d^(3/2) + 24*a^2*d^(5/2))*log((sqr
t(d)*x - sqrt(d*x^2 + c))^2)/b^4 - 1/2*(b^3*c^3*sqrt(d) - 8*a*b^2*c^2*d^(3/
2) + 13*a^2*b*c*d^(5/2) - 6*a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^
2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2
)*b^4) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^
2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(7/2) - b^3*c^4*sqr
t(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*d + b*c^2)*b^4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d\,x^2+c\right)}^{5/2}}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
[Out] int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2, x)
```

3.752 $\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=126

$$\frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3}+\frac{5d(c+dx^2)^{3/2}}{6b^2}-\frac{(c+dx^2)^{5/2}}{2b\,(a+bx^2)}-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}$$

```
[Out] 5/6*d*(d*x^2+c)^(3/2)/b^2-1/2*(d*x^2+c)^(5/2)/b/(b*x^2+a)-5/2*d*(-a*d+b*c)^
(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)+5/2*d*(-a*d
+b*c)*(d*x^2+c)^(1/2)/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {455, 43, 52, 65, 214}

$$-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{5d\sqrt{c+dx^2}\,(bc-ad)}{2b^3}-\frac{(c+dx^2)^{5/2}}{2b\,(a+bx^2)}+\frac{5d(c+dx^2)^{3/2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^3) + (5*d*(c + d*x^2)^(3/2))/(6*b^2)
 - (c + d*x^2)^(5/2)/(2*b*(a + b*x^2)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx,x,x^2\right)$$

$$= -\frac{(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{(5d)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^2\right)}{4b}$$

$$= \frac{5d(c+dx^2)^{3/2}}{6b^2} - \frac{(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{(5d(bc-ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^2\right)}{4b^2}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3} + \frac{5d(c+dx^2)^{3/2}}{6b^2} - \frac{(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{(5d(bc-ad)^2)\,\text{Subst}\left(\int \frac{\quad}{(a+\cdots}\right.}{4b^3}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3} + \frac{5d(c+dx^2)^{3/2}}{6b^2} - \frac{(c+dx^2)^{5/2}}{2b\,(a+bx^2)} + \frac{(5(bc-ad)^2)\,\text{Subst}\left(\int \frac{\quad}{a-\frac{bc}{d}\cdots}\right.}{2b^3}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3} + \frac{5d(c+dx^2)^{3/2}}{6b^2} - \frac{(c+dx^2)^{5/2}}{2b\,(a+bx^2)} - \frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{\cdots}}\right.}{2b^{7/2}}$$

Mathematica [A]

time = 0.30, size = 128, normalized size = 1.02

$$-\frac{\sqrt{c+dx^2}\,(15a^2d^2+10abd(-2c+dx^2)+b^2(3c^2-14cdx^2-2d^2x^4))}{6b^3\,(a+bx^2)}+\frac{5d(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] -1/6*(Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x^2) + b^2*(3*c^2 -
14*c*d*x^2 - 2*d^2*x^4)))/(b^3*(a + b*x^2)) + (5*d*(-(b*c) + a*d)^(3/2)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(2*b^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3223 vs. 2(102) = 204.
time = 0.13, size = 3224, normalized size = 25.59

method	result	size
risch	Expression too large to display	3059
default	Expression too large to display	3224

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(7/2)+5*d*(-
a*b)^(1/2)/(a*d-b*c)*(1/5*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)-d*(-a*b)^(1/2)/b*(1/8*(2*d*(x+1/b*(-a*b
)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/
d*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*
(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2))
)/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))))-(a*d-b*c)/b*(1/3*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/b*(1/4*(2*
d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/
b+4*d^2*a/b)/d^(3/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+
(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)))-(a*d-b*c)/b*((d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d^(1/2)*(-a*b)^(1/2)/b*ln((-d*(-a*b)^(1/
2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b
```

```
*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))))-6*d/(a*d-b*c)*b*(1/12*(2*d
*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+5/24*(-4*d*(a*d-b*c)/
b+4*d^2*a/b)/d*(1/8*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/d*(d*(x+1
/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(
-a*b)^(1/2)/b)/d*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((-d*
(-a*b)^(1/2)/b+d*(x+1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))))-1/4*(-a*b)^(1
/2)/a/b^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(7/2)-5*d*(-a*b)^(1/2)/(a
*d-b*c)*(1/5*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(5/2)+d*(-a*b)^(1/2)/b*(1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*
(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(
x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4
*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(
x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2))))-(a*d-b*c)/b*(1/3*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/b*(1/4*(2*d*(x-1/b*(-a*b
)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d
^(3/2)*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-(a*
d-b*c)/b*((d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2)+d^(1/2)*(-a*b)^(1/2)/b*ln((d*(-a*b)^(1/2)/b+d*(x-1/b*(-
a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*
(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x-1/b*(-a*b)^(1/2))))))-6*d/(a*d-b*c)*b*(1/12*(2*d*(x-1/b*(-a*b)^(
1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(5/2)+5/24*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d*(
1/8*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d*(d*(x-1/b*(-a*b)^(1/2))
^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+3/16*(-4*d*(a
*d-b*c)/b+4*d^2*a/b)/d*(1/4*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/d
*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)+1/8*(-4*d*(a*d-b*c)/b+4*d^2*a/b)/d^(3/2)*ln((d*(-a*b)^(1/2)/b+d*
(x-1/b*(-a*b)^(1/2)))/d^(1/2)+(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))))))
```

Maxima [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.96, size = 453, normalized size = 3.60

$$\left[-\frac{15\,(abcd - a^2d^2 + (b^2cd - abd^2)x^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right) - 4\,(2\,b^2d^2x^4 - 3\,b^2c^2 + 20\,abcd - 15\,a^2d^2 + 2\,(7\,b^2cd - 5\,abd^2)x^2)\sqrt{dx^2+c}}{24\,(b^4x^2+ab^3)}, -\frac{15\,(abcd - a^2d^2 + (b^2cd - abd^2)x^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right) - 2\,(2\,b^2d^2x^4 - 3\,b^2c^2 + 20\,abcd - 15\,a^2d^2 + 2\,(7\,b^2cd - 5\,abd^2)x^2)\sqrt{dx^2+c}}{12\,(b^4x^2+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt((b*c - a*d)/b
)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d
)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^2*d^2*x^4 - 3*b^2*c^2 + 20*a*b*
c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2
 + a*b^3), -1/12*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(-(b
*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c
 - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(2*b^2*d^2*x^4 - 3*b^
2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2
 + c))/(b^4*x^2 + a*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(c + dx^2)^{\frac{5}{2}}}{(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*(c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 0.52, size = 197, normalized size = 1.56

$$\frac{5\,(b^2c^2d - 2\,abcd^2 + a^2d^3)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,b^3} - \frac{\sqrt{dx^2+c}\,b^2c^2d - 2\,\sqrt{dx^2+c}\,abcd^2 + \sqrt{dx^2+c}\,a^2d^3}{2\,((dx^2+c)b - bc + ad)b^3} + \frac{(dx^2+c)^{\frac{3}{2}}b^4d + 6\,\sqrt{dx^2+c}\,b^4cd - 6\,\sqrt{dx^2+c}\,ab^3d^2}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 1/2*(sqrt(d*x^2 + c)*b^2*c^2*d - 2
*sqrt(d*x^2 + c)*a*b*c*d^2 + sqrt(d*x^2 + c)*a^2*d^3)/(((d*x^2 + c)*b - b*c
 + a*d)*b^3) + 1/3*((d*x^2 + c)^(3/2)*b^4*d + 6*sqrt(d*x^2 + c)*b^4*c*d - 6
*sqrt(d*x^2 + c)*a*b^3*d^2)/b^6
```

Mupad [B]
time = 0.57, size = 172, normalized size = 1.37

$$\frac{d\,{\left(d\,x^2+c\right)}^{3/2}}{3\,b^2}-\frac{\sqrt{d\,x^2+c}\,\left(\frac{a^2\,d^3}{2}-a\,b\,c\,d^2+\frac{b^2\,c^2\,d}{2}\right)}{b^4\,\left(d\,x^2+c\right)-b^4\,c+a\,b^3\,d}+\frac{5\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^{3/2}}{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{2\,b^{7/2}}+\frac{d\,\sqrt{d\,x^2+c}\,\left(2\,b^2\,c-2\,a\,b\,d\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

```
[Out] (d*(c + d*x^2)^(3/2))/(3*b^2) - ((c + d*x^2)^(1/2)*((a^2*d^3)/2 + (b^2*c^2*
d)/2 - a*b*c*d^2))/(b^4*(c + d*x^2) - b^4*c + a*b^3*d) + (5*d*atan((b^(1/2)
*d*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2))/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)
)*(a*d - b*c)^(3/2))/(2*b^(7/2)) + (d*(c + d*x^2)^(1/2)*(2*b^2*c - 2*a*b*d)
)/b^4
```

3.753 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2} dx$

Optimal. Leaf size=174

$$-\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2}+\frac{(bc-ad)x(c+dx^2)^{3/2}}{2ab\,(a+bx^2)}+\frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3}+\frac{d^{3/2}(\cdots}{\cdots}$$

```
[Out] 1/2*(-a*d+b*c)*x*(d*x^2+c)^(3/2)/a/b/(b*x^2+a)+1/2*(-a*d+b*c)^(3/2)*(4*a*d+
b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/b^3+1/2*d^(
3/2)*(-4*a*d+5*b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^3-1/2*d*(-2*a*d+b*
c)*x*(d*x^2+c)^(1/2)/a/b^2
```

Rubi [A]

time = 0.14, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {424, 542, 537, 223, 212, 385, 211}

$$\frac{(bc-ad)^{3/2}(4ad+bc)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3}+\frac{d^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{dx\sqrt{c+dx^2}\,(bc-2ad)}{2ab^2}+\frac{x(c+dx^2)^{3/2}\,(bc-ad)}{2ab\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(d*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(a*b^2) + ((b*c - a*d)*x*(c + d*x^
2)^(3/2))/(2*a*b*(a + b*x^2)) + ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sq
rt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*b^3) + (d^(3/2)*(5*
b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{\int \frac{\sqrt{c+dx^2}\,(c(bc+ad)-2d(bc-2ad)x^2)}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{\int \frac{2c(b^2c^2+2abcd-2a^2d^2)+2ad^2(5bc-4ad}{(a+bx^2)\sqrt{c+dx^2}}}{4ab^2}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{(d^2(5bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^3}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{(d^2(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\right.}{2b^3}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\right.}{2a^{3/2}b^3}$$

Mathematica [A]
time = 0.62, size = 191, normalized size = 1.10

$$\frac{\frac{bx\sqrt{c+dx^2}\,(b^2c^2+2a^2d^2+abd(-2c+dx^2))}{a(a+bx^2)} - \frac{\sqrt{bc-ad}\,(b^2c^2+3abcd-4a^2d^2)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{a^{3/2}} + d^{3/2}(-5bc+4ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2]*(b^2*c^2 + 2*a^2*d^2 + a*b*d*(-2*c + d*x^2)))/(a*(a +
 b*x^2)) - (Sqrt[b*c - a*d]*(b^2*c^2 + 3*a*b*c*d - 4*a^2*d^2)*ArcTan[(a*Sqr
t[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/a^(3/
2) + d^(3/2)*(-5*b*c + 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5289 vs. $2(146) = 292$.
time = 0.16, size = 5290, normalized size = 30.40

method	result	size
risch	Expression too large to display	3442
default	Expression too large to display	5290

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/(b*x^2 + a)^2, x)
```

Fricas [A]
time = 2.09, size = 1228, normalized size = 7.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(d
)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b^2*c^2 + 3*a^2*b*c*
d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d
)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-
(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*d^2*x^3 + (b^3*c^2
- 2*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), -1/
8*(4*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-d)*a
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 +
(b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 +
 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d +
2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), 1/4*((a*b^2*c^2 + 3
*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt((b
*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
 - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - (5*a^2*b*c*d - 4*a^
3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^
2 + c)*sqrt(d)*x - c) + 2*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d + 2*a^2*b
*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), -1/4*(2*(5*a^2*b*c*d - 4*a
^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d
*x^2 + c)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d
- 4*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d
)*x)) - 2*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*
x^2 + c))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^2)^{\frac{5}{2}}}{(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(146) = 292.
time = 0.52, size = 407, normalized size = 2.34

$$\frac{\sqrt{dx^2+c}\, d^2 x}{2\, b^2} - \frac{\left(5\, bcd^{\frac{3}{2}} - 4\, ad^{\frac{5}{2}}\right) \log\left(\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2\right)}{4\, b^3} - \frac{\left(b^3 c^3 \sqrt{d} + 2\, ab^2 c^2 d^{\frac{3}{2}} - 7\, a^2 bcd^{\frac{5}{2}} + 4\, a^3 d^{\frac{7}{2}}\right) \arctan\left(\frac{\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 b - bc + 2\, ad}{2\, \sqrt{abcd - a^2 d^2}}\right)}{2\, \sqrt{abcd - a^2 d^2}\, ab^3} - \frac{\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 b^3 c^3 \sqrt{d} - 4\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 ab^2 c^2 d^{\frac{3}{2}} + 5\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 a^2 bcd^{\frac{5}{2}} - 2\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 a^3 d^{\frac{7}{2}} - b^3 c^4 \sqrt{d} + 2\, ab^2 c^3 d^{\frac{3}{2}} - a^2 bc^2 d^{\frac{5}{2}}}{\left(\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^4 b - 2\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 bc + 4\left(\sqrt{d}\, x - \sqrt{dx^2+c}\right)^2 ad + bc^2\right) ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d^2*x/b^2 - 1/4*(5*b*c*d^(3/2) - 4*a*d^(5/2))*log((sqrt
(d)*x - sqrt(d*x^2 + c))^2)/b^3 - 1/2*(b^3*c^3*sqrt(d) + 2*a*b^2*c^2*d^(3/2
) - 7*a^2*b*c*d^(5/2) + 4*a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*
a*b^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^
2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(7/2) - b^3*c^4*sqr
t(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*d + b*c^2)*a*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\, x^2 + c)^{5/2}}{(b\, x^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(a + b*x^2)^2,x)
```

```
[Out] int((c + d*x^2)^(5/2)/(a + b*x^2)^2, x)
```

3.754 $\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=160

$$-\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{(bc-ad)^{3/2}(2bc+3ad)\tanh^{-1}}{2a^2b^{5/2}}$$

```
[Out] 1/2*(-a*d+b*c)*(d*x^2+c)^(3/2)/a/b/(b*x^2+a)-c^(5/2)*arctanh((d*x^2+c)^(1/2
)/c^(1/2))/a^2+1/2*(-a*d+b*c)^(3/2)*(3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)
^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(5/2)-1/2*d*(-3*a*d+b*c)*(d*x^2+c)^(1/2)/a/b
^2
```

Rubi [A]

time = 0.15, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 100, 159, 162, 65, 214}

$$\frac{(bc-ad)^{3/2}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}-\frac{d\sqrt{c+dx^2}\,(bc-3ad)}{2ab^2}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] -1/2*(d*(b*c - 3*a*d)*Sqrt[c + d*x^2])/(a*b^2) + ((b*c - a*d)*(c + d*x^2)^(
3/2))/(2*a*b*(a + b*x^2)) - (c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a^2
+ ((b*c - a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/(2*a^2*b^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x(a+bx)^2}\,dx,x,x^2\right)$$

$$= \frac{(bc-ad)(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\,(bc^2-\frac{1}{2}d(bc-3ad)x)}{x(a+bx)}\,dx,x,x^2\right)}{2ab}$$

$$= -\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{\frac{b^2c^3}{2}+\frac{1}{4}d(b^2c^2+4abcd-3a^2d^2)x}{x(a+bx)\sqrt{c+dx}}\,c\right.}{ab^2}$$

$$= -\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{c^3\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2a^2} -$$

$$= -\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{c^3\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right.}{a^2d}$$

$$= -\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2ab(a+bx^2)} - \frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2} + \frac{(bc-}{}$$

Mathematica [A]
time = 0.33, size = 146, normalized size = 0.91

$$\frac{\frac{a\sqrt{c+dx^2}\,(b^2c^2+3a^2d^2+2abd(-c+dx^2))}{b^2(a+bx^2)} - \frac{(-bc+ad)^{3/2}(2bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{5/2}} - 2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(b^2*c^2 + 3*a^2*d^2 + 2*a*b*d*(-c + d*x^2)))/(b^2*(a +
 b*x^2)) - ((-(b*c) + a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d
*x^2])/Sqrt[-(b*c) + a*d]])/b^(5/2) - 2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqr
t[c]])/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5353 vs. 2(134) = 268.
time = 0.10, size = 5354, normalized size = 33.46

method	result	size
default	Expression too large to display	5354

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x), x)
```

Fricas [A]
time = 3.17, size = 1132, normalized size = 7.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^
2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d
+ a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)
*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^3
*c^2*x^2 + a*b^2*c^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c
)/x^2) - 4*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x
^2 + c))/(a^2*b^3*x^2 + a^3*b^2), 1/8*(8*(b^3*c^2*x^2 + a*b^2*c^2)*sqrt(-c)
*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 +
(2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2
*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*
(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) + 4*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3
*d^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^2 + a^3*b^2), 1/4*((2*a*b^2*c^2 + a^2*b*c
*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*
d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)
/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b^3*c^2*x^2 + a*b^2*c^2)*sq
rt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*a^2*b*d^2*
x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^2 +
a^3*b^2), 1/4*((2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*
d - 3*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a
```

```
*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x
^2)) + 4*(b^3*c^2*x^2 + a*b^2*c^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + 2*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 +
c))/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^2)^{\frac{5}{2}}}{x (a + bx^2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x*(a + b*x**2)**2), x)
```

Giac [A]
time = 0.50, size = 206, normalized size = 1.29

$$\frac{c^3 \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}} + \frac{\sqrt{dx^2+c}\, d^2}{b^2} - \frac{(2\,b^3c^3 - ab^2c^2d - 4\,a^2bcd^2 + 3\,a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,a^2b^2} + \frac{\sqrt{dx^2+c}\,b^2c^2d - 2\,\sqrt{dx^2+c}\,abcd^2 + \sqrt{dx^2+c}\,a^2d^3}{2\,((dx^2+c)b - bc + ad)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)) + sqrt(d*x^2 + c)*d^2/b
^2 - 1/2*(2*b^3*c^3 - a*b^2*c^2*d - 4*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(sqrt(
d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*b^2) + 1/2*(sq
rt(d*x^2 + c)*b^2*c^2*d - 2*sqrt(d*x^2 + c)*a*b*c*d^2 + sqrt(d*x^2 + c)*a^2
*d^3)/(((d*x^2 + c)*b - b*c + a*d)*a*b^2)
```

Mupad [B]
time = 0.70, size = 1321, normalized size = 8.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x)
```

```
[Out] (d^2*(c + d*x^2)^(1/2))/b^2 + (atan((a^2*d^8*(c + d*x^2)^(1/2)*(c^5)^(1/2)*
9i)/(2*((9*a^2*c^3*d^8)/2 + 5*b^2*c^5*d^6 + (10*b^3*c^6*d^5)/a - (15*b^4*c^
7*d^4)/(2*a^2) - 12*a*b*c^4*d^7)) + (c^2*d^6*(c + d*x^2)^(1/2)*(c^5)^(1/2)*
5i)/(5*c^5*d^6 - (12*a*c^4*d^7)/b + (10*b*c^6*d^5)/a - (15*b^2*c^7*d^4)/(2*
a^2) + (9*a^2*c^3*d^8)/(2*b^2)) + (c^3*d^5*(c + d*x^2)^(1/2)*(c^5)^(1/2)*10
i)/(10*c^6*d^5 + (5*a*c^5*d^6)/b - (15*b*c^7*d^4)/(2*a) - (12*a^2*c^4*d^7)/
b^2 + (9*a^3*c^3*d^8)/(2*b^3)) - (a*c*d^7*(c + d*x^2)^(1/2)*(c^5)^(1/2)*12i
```

)/(5*b*c^5*d^6 - 12*a*c^4*d^7 + (10*b^2*c^6*d^5)/a + (9*a^2*c^3*d^8)/(2*b)
- (15*b^3*c^7*d^4)/(2*a^2)) - (b*c^4*d^4*(c + d*x^2)^(1/2)*(c^5)^(1/2)*15i)
/(2*(10*a*c^6*d^5 - (15*b*c^7*d^4)/2 + (5*a^2*c^5*d^6)/b - (12*a^3*c^4*d^7)
/b^2 + (9*a^4*c^3*d^8)/(2*b^3))))*(c^5)^(1/2)*1i)/a^2 + ((c + d*x^2)^(1/2)*
(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/(2*a*(b^3*(c + d*x^2) - b^3*c + a*b^2*
d)) - (atan((c^4*d^5*(c + d*x^2)^(1/2)*(b^8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6*c
*d^2 - 3*a*b^7*c^2*d)^(1/2)*35i)/(4*(9*a^3*b*c^3*d^8 - (25*b^4*c^6*d^5)/4 -
(85*a*b^3*c^5*d^6)/4 - (81*a^4*c^2*d^9)/4 + (27*a^5*c*d^10)/(4*b) + (49*a^
2*b^2*c^4*d^7)/2 + (15*b^5*c^7*d^4)/(2*a))) - (c^3*d^6*(c + d*x^2)^(1/2)*(b
^8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6*c*d^2 - 3*a*b^7*c^2*d)^(1/2)*45i)/(4*((27*
a^4*c*d^10)/4 - (85*b^4*c^5*d^6)/4 + (49*a*b^3*c^4*d^7)/2 - (81*a^3*b*c^2*d
^9)/4 + 9*a^2*b^2*c^3*d^8 - (25*b^5*c^6*d^5)/(4*a) + (15*b^6*c^7*d^4)/(2*a^
2))) + (c^5*d^4*(c + d*x^2)^(1/2)*(b^8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6*c*d^2
- 3*a*b^7*c^2*d)^(1/2)*15i)/(2*(9*a^4*c^3*d^8 + (15*b^4*c^7*d^4)/2 - (25*a*
b^3*c^6*d^5)/4 + (49*a^3*b*c^4*d^7)/2 + (27*a^6*c*d^10)/(4*b^2) - (85*a^2*b
^2*c^5*d^6)/4 - (81*a^5*c^2*d^9)/(4*b))) + (a^2*c*d^8*(c + d*x^2)^(1/2)*(b^
8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6*c*d^2 - 3*a*b^7*c^2*d)^(1/2)*27i)/(4*((49*a
*b^5*c^4*d^7)/2 - (85*b^6*c^5*d^6)/4 + (27*a^4*b^2*c*d^10)/4 + 9*a^2*b^4*c^
3*d^8 - (81*a^3*b^3*c^2*d^9)/4 - (25*b^7*c^6*d^5)/(4*a) + (15*b^8*c^7*d^4)/
(2*a^2))) - (a*c^2*d^7*(c + d*x^2)^(1/2)*(b^8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6
*c*d^2 - 3*a*b^7*c^2*d)^(1/2)*27i)/(4*((49*a*b^4*c^4*d^7)/2 - (85*b^5*c^5*d
^6)/4 + 9*a^2*b^3*c^3*d^8 - (81*a^3*b^2*c^2*d^9)/4 - (25*b^6*c^6*d^5)/(4*a)
+ (15*b^7*c^7*d^4)/(2*a^2) + (27*a^4*b*c*d^10)/4)))*(-b^5*(a*d - b*c)^3)^(
1/2)*(3*a*d + 2*b*c)*1i)/(2*a^2*b^5)

3.755 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=168

$$-\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)}-\frac{(bc-ad)^{3/2}(3bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}+\frac{d^{5/2}\tan}{}$$

```
[Out] 1/2*(-a*d+b*c)*(d*x^2+c)^(3/2)/a/b/x/(b*x^2+a)-1/2*(-a*d+b*c)^(3/2)*(2*a*d+
3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/b^2+d^(5/
2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2-1/2*c*(-a*d+3*b*c)*(d*x^2+c)^(1/2
)/a^2/b/x
```

Rubi [A]

time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {479, 594, 537, 223, 212, 385, 211}

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}-\frac{c\sqrt{c+dx^2}\,(3bc-ad)}{2a^2bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx\left(a+bx^2\right)}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -1/2*(c*(3*b*c - a*d)*Sqrt[c + d*x^2])/(a^2*b*x) + ((b*c - a*d)*(c + d*x^2)
^(3/2))/(2*a*b*x*(a + b*x^2)) - ((b*c - a*d)^(3/2)*(3*b*c + 2*a*d)*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*b^2) + (d^(5/2)*A
rcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}\,dx = \frac{(bc-ad)(c+dx^2)^{3/2}}{2abx(a+bx^2)} - \frac{\int \frac{\sqrt{c+dx^2}(-c(3bc-ad)-2ad^2x^2)}{x^2(a+bx^2)}\,dx}{2ab}$$

$$= -\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2abx(a+bx^2)} - \frac{\int \frac{c(3b^2c^2-4abcd-a^2d^2)-2a^2d^3x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a^2b}$$

$$= -\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2abx(a+bx^2)} + \frac{d^3\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2} - \frac{((bc-ad)^2(3$$

$$= -\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2abx(a+bx^2)} + \frac{d^3\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right.}{b^2}$$

$$= -\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2abx(a+bx^2)} - \frac{(bc-ad)^{3/2}(3bc+2ad)\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{2a^{5/2}b^2}$$

Mathematica [A]
time = 0.60, size = 192, normalized size = 1.14

$$\frac{-\frac{b\sqrt{c+dx^2}(3b^2c^2x^2+a^2d^2x^2+2abc(c-dx^2))}{a^2x(a+bx^2)} + \frac{\sqrt{bc-ad}(3b^2c^2-abcd-2a^2d^2)\tan^{-1}\left(\frac{a\sqrt{d}+bx(\sqrt{d}x-\sqrt{c+dx^2})}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}} - 2d^{5/2}\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (-((b*Sqrt[c + d*x^2]*(3*b^2*c^2*x^2 + a^2*d^2*x^2 + 2*a*b*c*(c - d*x^2)))/
(a^2*x*(a + b*x^2))) + (Sqrt[b*c - a*d]*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*A
rcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a
*d])])/a^(5/2) - 2*d^(5/2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5380 vs. $2(142) = 284$.
time = 0.13, size = 5381, normalized size = 32.03

method	result	size
risch	Expression too large to display	3403
default	Expression too large to display	5381

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

[Out] result too large to display

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

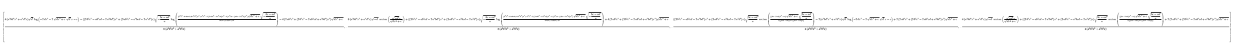
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^2), x)
```

Fricas [A]
time = 1.72, size = 1184, normalized size = 7.05



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c
)*sqrt(d)*x - c) - ((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^
2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*
d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x -
 (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) - 4*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^
2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x), -1/8*(8*(a^2*b*d^2*x^3 + a^3
*d^2*x)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((3*b^3*c^2 - a*b^2*c
*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-(b*c
 - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*
sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a*b^2*c^2 + (3*b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b
^2*x), -1/4*(((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^
2*b*c*d - 2*a^3*d^2)*x)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a
*c*d)*x)) - 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x
^2 + c)*sqrt(d)*x - c) + 2*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x), -1/4*(4*(a^2*b*d^2*x^
3 + a^3*d^2*x)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((3*b^3*c^2 -
a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqr
t((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(
(b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*(2*a*b^2*c^2
+ (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3
+ a^3*b^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{x^2(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**2*(a + b*x**2)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^{5/2}}{x^2\,{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2), x)
```

3.756 $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=180

$$-\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b\,(a+bx^2)}-\frac{c(c+dx^2)^{3/2}}{2ax^2\,(a+bx^2)}+\frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{(bc-ad)^{3/2}(4bc$$

```
[Out] -1/2*c*(d*x^2+c)^(3/2)/a/x^2/(b*x^2+a)+1/2*c^(3/2)*(-5*a*d+4*b*c)*arctanh((
d*x^2+c)^(1/2)/c^(1/2))/a^3-1/2*(-a*d+b*c)^(3/2)*(a*d+4*b*c)*arctanh(b^(1/2
)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/b^(3/2)-1/2*(-a*d+b*c)*(-a*d+2*b*c)
*(d*x^2+c)^(1/2)/a^2/b/(b*x^2+a)
```

Rubi [A]
time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 100, 154, 162, 65, 214}

$$-\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3b^{3/2}}+\frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{c+dx^2}\,(bc-ad)(2bc-ad)}{2a^2b\,(a+bx^2)}-\frac{c(c+dx^2)^{3/2}}{2ax^2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*((b*c - a*d)*(2*b*c - a*d)*Sqrt[c + d*x^2])/(a^2*b*(a + b*x^2)) - (c*(
c + d*x^2)^(3/2))/(2*a*x^2*(a + b*x^2)) + (c^(3/2)*(4*b*c - 5*a*d)*ArcTanh[
Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3) - ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*b^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x^2(a+bx)^2}\,dx,x,x^2\right) \\
&= -\frac{c(c+dx^2)^{3/2}}{2ax^2(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\,\left(\frac{1}{2}c(4bc-5ad)+\frac{1}{2}d(bc-2ad)x\right)}{x(a+bx)^2}\,dx,x,x^2\right)}{2a} \\
&= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b(a+bx^2)} - \frac{c(c+dx^2)^{3/2}}{2ax^2(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}bc^2(4bc-5ad)-\frac{1}{2}d(2b^2c^2-}{x(a+bx)\sqrt{c+d}}\right.}{2a^2b} \\
&= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b(a+bx^2)} - \frac{c(c+dx^2)^{3/2}}{2ax^2(a+bx^2)} - \frac{(c^2(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+}}\right.}{4a^3} \\
&= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b(a+bx^2)} - \frac{c(c+dx^2)^{3/2}}{2ax^2(a+bx^2)} - \frac{(c^2(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{2a^3d} \\
&= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b(a+bx^2)} - \frac{c(c+dx^2)^{3/2}}{2ax^2(a+bx^2)} + \frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+}}{\sqrt{c}}\right.}{2a^3}
\end{aligned}
$$

Mathematica [A]
time = 0.49, size = 160, normalized size = 0.89

$$
\frac{-\frac{a\sqrt{c+dx^2}\,(2b^2c^2x^2+a^2d^2x^2+abc(c-2dx^2))}{bx^2(a+bx^2)}+\frac{(-bc+ad)^{3/2}(4bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}+c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*Sqrt[c + d*x^2]*(2*b^2*c^2*x^2 + a^2*d^2*x^2 + a*b*c*(c - 2*d*x^2)))/
(b*x^2*(a + b*x^2))) + ((-(b*c) + a*d)^(3/2)*(4*b*c + a*d)*ArcTan[(Sqrt[b]*
Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(3/2) + c^(3/2)*(4*b*c - 5*a*d)*Arc
Tanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5451 vs. $2(152) = 304$.
time = 0.16, size = 5452, normalized size = 30.29

method	result	size
risch	Expression too large to display	3084

default	Expression too large to display	5452

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

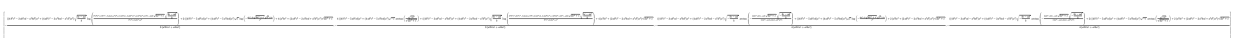
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^3), x)
```

Fricas [A]
time = 2.50, size = 1266, normalized size = 7.03



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*
c*d - a^3*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c -
a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
2*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(c)
*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*b*c^2 + (2*a*
b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b
*x^2), -1/8*(4*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)
*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^3*c^2 - 3*a*b^2*c*d
 - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt((b*c -
a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c
 - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*
a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^2), -1/4*
(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d -
a^3*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (
(4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(c)*lo
g(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*b*c^2 + (2*a*b^2
*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^
```

```
2), -1/4*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2
*b*c*d - a^3*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*
x^2)) + 2*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)
*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a^2*b*c^2 + (2*a*b^2*c^2 -
2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^2\right)^{\frac{5}{2}}}{x^3 \left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**3*(a + b*x**2)**2), x)
```

Giac [A]
time = 0.52, size = 283, normalized size = 1.57

$$-\frac{\left(4\,bc^3 - 5\,ac^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}} + \frac{\left(4\,b^3c^3 - 7\,ab^2c^2d + 2\,a^2bcd^2 + a^3d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,a^3b} - \frac{2\,(dx^2+c)^{\frac{3}{2}}b^2c^2d - 2\,\sqrt{dx^2+c}\,b^2c^3d - 2\,(dx^2+c)^{\frac{3}{2}}abcd^2 + 3\,\sqrt{dx^2+c}\,abc^2d^2 + (dx^2+c)^{\frac{3}{2}}a^2d^3 - \sqrt{dx^2+c}\,a^2cd^3}{2\,\left((dx^2+c)^2b - 2\,(dx^2+c)bc + bc^2 + (dx^2+c)ad - acd\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b*c^3 - 5*a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c))
+ 1/2*(4*b^3*c^3 - 7*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*arctan(sqrt(d*x
^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3*b) - 1/2*(2*(d*x^
2 + c)^(3/2)*b^2*c^2*d - 2*sqrt(d*x^2 + c)*b^2*c^3*d - 2*(d*x^2 + c)^(3/2)*
a*b*c*d^2 + 3*sqrt(d*x^2 + c)*a*b*c^2*d^2 + (d*x^2 + c)^(3/2)*a^2*d^3 - sqr
t(d*x^2 + c)*a^2*c*d^3)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*
x^2 + c)*a*d - a*c*d)*a^2*b)
```

Mupad [B]
time = 1.08, size = 1152, normalized size = 6.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (((c + d*x^2)^(1/2)*(a^2*c*d^3 + 2*b^2*c^3*d - 3*a*b*c^2*d^2))/(2*a^2*b) -
(d*(c + d*x^2)^(3/2)*(a^2*d^2 + 2*b^2*c^2 - 2*a*b*c*d))/(2*a^2*b))/((c + d*
x^2)*(a*d - 2*b*c) + b*(c + d*x^2)^2 + b*c^2 - a*c*d) - (atanh((5*d^9*(c +
d*x^2)^(1/2)*(c^3)^(1/2))/(4*((5*c^2*d^9)/4 + (4*b*c^3*d^8)/a - (33*b^2*c^4
```

```
*d^7)/(2*a^2) + (65*b^3*c^5*d^6)/(4*a^3) - (5*b^4*c^6*d^5)/a^4)) + (4*c*d^8
*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(4*c^3*d^8 + (5*a*c^2*d^9)/(4*b) - (33*b*c^
4*d^7)/(2*a) + (65*b^2*c^5*d^6)/(4*a^2) - (5*b^3*c^6*d^5)/a^3) + (65*b^2*c^
3*d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(4*(4*a^2*c^3*d^8 + (65*b^2*c^5*d^6)/4
 - (5*b^3*c^6*d^5)/a + (5*a^3*c^2*d^9)/(4*b) - (33*a*b*c^4*d^7)/2)) - (5*b^
3*c^4*d^5*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(4*a^3*c^3*d^8 - 5*b^3*c^6*d^5 + (
65*a*b^2*c^5*d^6)/4 - (33*a^2*b*c^4*d^7)/2 + (5*a^4*c^2*d^9)/(4*b)) - (33*b
*c^2*d^7*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(2*(4*a*c^3*d^8 - (33*b*c^4*d^7)/2
+ (65*b^2*c^5*d^6)/(4*a) + (5*a^2*c^2*d^9)/(4*b) - (5*b^3*c^6*d^5)/a^2)))*(
5*a*d - 4*b*c)*(c^3)^(1/2))/(2*a^3) - (atanh((15*c^3*d^6*(c + d*x^2)^(1/2)*
(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/(4*((7*a^3
*c^2*d^9)/4 + (55*b^3*c^5*d^6)/4 - (41*a*b^2*c^4*d^7)/4 - (a^2*b*c^3*d^8)/2
 + (a^4*c*d^10)/(4*b) - (5*b^4*c^6*d^5)/a)) + (9*c^2*d^7*(c + d*x^2)^(1/2)*
(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/(4*((a^3*c
*d^10)/4 - (41*b^3*c^4*d^7)/4 - (a*b^2*c^3*d^8)/2 + (7*a^2*b*c^2*d^9)/4 + (
55*b^4*c^5*d^6)/(4*a) - (5*b^5*c^6*d^5)/a^2)) + (5*c^4*d^5*(c + d*x^2)^(1/2
)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/((a^3*c^
3*d^8)/2 + 5*b^3*c^6*d^5 - (55*a*b^2*c^5*d^6)/4 + (41*a^2*b*c^4*d^7)/4 - (a
^5*c*d^10)/(4*b^2) - (7*a^4*c^2*d^9)/(4*b)) - (c*d^8*(c + d*x^2)^(1/2)*(b^6
*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/(4*((b^3*c^3*d
^8)/2 - (7*a*b^2*c^2*d^9)/4 + (41*b^4*c^4*d^7)/(4*a) - (55*b^5*c^5*d^6)/(4*
a^2) + (5*b^6*c^6*d^5)/a^3 - (a^2*b*c*d^10)/4)))*(-b^3*(a*d - b*c)^3)^(1/2)
*(a*d + 4*b*c))/(2*a^3*b^3)
```

3.757 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=176

$$-\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3}+\frac{(15b^2c^2-20abcd+3a^2d^2)\sqrt{c+dx^2}}{6a^3bx}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)}+\frac{5c(bc-ad)^{3/2}\tan}{}$$

```
[Out] 1/2*(-a*d+b*c)*(d*x^2+c)^(3/2)/a/b/x^3/(b*x^2+a)+5/2*c*(-a*d+b*c)^(3/2)*arc
tan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(7/2)-1/6*c*(-3*a*d+5*b*c
)*(d*x^2+c)^(1/2)/a^2/b/x^3+1/6*(3*a^2*d^2-20*a*b*c*d+15*b^2*c^2)*(d*x^2+c)
^(1/2)/a^3/b/x
```

Rubi [A]
time = 0.16, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {479, 594, 597, 12, 385, 211}

$$\frac{5c(bc-ad)^{3/2}\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}-\frac{c\sqrt{c+dx^2}\,(5bc-3ad)}{6a^2bx^3}+\frac{\sqrt{c+dx^2}\,(3a^2d^2-20abcd+15b^2c^2)}{6a^3bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/6*(c*(5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(a^2*b*x^3) + ((15*b^2*c^2 - 20*a*
b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^
(3/2))/(2*a*b*x^3*(a + b*x^2)) + (5*c*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx = \frac{(bc-ad)(c+dx^2)^{3/2}}{2abx^3(a+bx^2)} - \frac{\int \frac{\sqrt{c+dx^2}(-c(5bc-3ad)-2bcdx^2)}{x^4(a+bx^2)}dx}{2ab}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2abx^3(a+bx^2)} - \frac{\int \frac{c(15b^2c^2-20abcd+3a^2d^2)+2bcd(5bc-6c}{x^2(a+bx^2)\sqrt{c+dx^2}}}{6a^2b}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15b^2c^2-20abcd+3a^2d^2)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)(c+d}{2abx^3(a+bx}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15b^2c^2-20abcd+3a^2d^2)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)(c+d}{2abx^3(a+bx}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15b^2c^2-20abcd+3a^2d^2)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)(c+d}{2abx^3(a+bx}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15b^2c^2-20abcd+3a^2d^2)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)(c+d}{2abx^3(a+bx}$$

Mathematica [A]
time = 0.69, size = 157, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\,(15b^2c^2x^4+10abcx^2(c-2dx^2)+a^2(-2c^2-14cdx^2+3d^2x^4))}{6a^3x^3(a+bx^2)} - \frac{5c(bc-ad)^{3/2}\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c - 2*d*x^2) + a^2*(-2*c^2
 - 14*c*d*x^2 + 3*d^2*x^4)))/(6*a^3*x^3*(a + b*x^2)) - (5*c*(b*c - a*d)^(3/
2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c
 - a*d])])/(2*a^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5507 vs. $2(152) = 304$.
time = 0.14, size = 5508, normalized size = 31.30

method	result	size
risch	Expression too large to display	3404
default	Expression too large to display	5508

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^4), x)

Fricas [A]
time = 1.66, size = 483, normalized size = 2.74

$$\left[-\frac{15\,((b^2c^2-abcd)x^5+(abc^2-a^2cd)x^3)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,((15\,b^2c^2-20\,abcd+3\,a^2d^2)x^4-2\,a^2c^2+2\,(5\,abc^2-7\,a^2cd)x^2)\sqrt{dx^2+c}}{24\,(a^3bx^5+a^4x^3)}, \frac{15\,((b^2c^2-abcd)x^5+(abc^2-a^2cd)x^3)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\,((15\,b^2c^2-20\,abcd+3\,a^2d^2)x^4-2\,a^2c^2+2\,(5\,abc^2-7\,a^2cd)x^2)\sqrt{dx^2+c}}{12\,(a^3bx^5+a^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")

[Out] [-1/24*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a*b*c^2 - a^2*c*d)*x^3)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^2 - 7*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^4*x^3), 1/12*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a*b*c^2 - a^2*c*d)*x^3)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*((15*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^2 - 7*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^4*x^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^2)^{\frac{5}{2}}}{x^4\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a)**2,x)

```
[Out] Integral((c + d*x**2)**(5/2)/(x**4*(a + b*x**2)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(152) = 304$.

time = 1.42, size = 496, normalized size = 2.82

$$-\frac{5\left(b^{2}c^{3}\sqrt{d}-2\,abc^{2}d^{\frac{3}{2}}+a^{2}cd^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\sqrt{abcd-a^{2}d^{2}}\,a^{3}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b^{3}c^{3}\sqrt{d}-4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ab^{2}c^{2}d^{\frac{3}{2}}+5\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}a^{2}bcd^{\frac{5}{2}}-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}a^{3}d^{\frac{7}{2}}-b^{3}c^{4}\sqrt{d}+2\,ab^{2}c^{3}d^{\frac{3}{2}}-a^{2}bc^{2}d^{\frac{5}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)a^{3}b}-\frac{2\left(6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}bc^{3}\sqrt{d}-9\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}ac^{2}d^{\frac{3}{2}}-12\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc^{4}\sqrt{d}+12\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ac^{3}d^{\frac{3}{2}}+6\,bc^{5}\sqrt{d}-7\,ac^{4}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -5/2*(b^2*c^3*sqrt(d) - 2*a*b*c^2*d^(3/2) + a^2*c*d^(5/2))*arctan(1/2*((sqr
t(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt
(a*b*c*d - a^2*d^2)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d)
 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a^2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(
7/2) - b^3*c^4*sqrt(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d
)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a^3*b) - 2/3*(6*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*b*c^3*sqrt(d) - 9*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c^2*d^(3/
2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^4*sqrt(d) + 12*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a*c^3*d^(3/2) + 6*b*c^5*sqrt(d) - 7*a*c^4*d^(3/2))/(((sqrt(
d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^{5/2}}{x^4\,{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2), x)
```

3.758 $\int \frac{x^4}{(a+bx^2)^2 \sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=132

$$\frac{ax\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)}-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

```
[Out] -1/2*(-2*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1
/2)/b^2/(-a*d+b*c)^(3/2)+arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2/d^(1/2)+1/2
*a*x*(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {481, 537, 223, 212, 385, 211}

$$-\frac{\sqrt{a}\,(3bc-2ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*x*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*b^2*(b*c - a*
d)^(3/2)) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx &= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)(a+bx^2)} - \frac{\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2b(bc-ad)} \\
&= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)(a+bx^2)} + \frac{\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2} - \frac{(a(3bc-2ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}
\\
&= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)(a+bx^2)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b^2} - \frac{(a(3bc-2ad))
\\
&= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)(a+bx^2)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}
\end{aligned}
$$

Mathematica [A]

time = 0.75, size = 148, normalized size = 1.12

$$\frac{\frac{abx\sqrt{c+dx^2}}{(bc-ad)(a+bx^2)}+\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{2\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*b*x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + (Sqrt[a]*(3*b*c - 2*a*
d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c
 - a*d])])/(b*c - a*d)^(3/2) - (2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt
[d])/(2*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. 2(110) = 220.

time = 0.09, size = 843, normalized size = 6.39

method	result
default	$\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b^2\sqrt{d}}-\frac{a\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(-2 \cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)-1/4*a/b^3*(1/(a*d-b*c)*b/(x+1/b
*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^
```

```
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-3/4/b^2*a/(-a*b)^(1/2)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2
*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/4*a/b^3*(1/(a*d-b*
c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-
b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [**A**]
time = 1.54, size = 1053, normalized size = 7.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(d*x^2 + c)*a*b*d*x + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*s
qrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (3*a*b*c*d - 2*a^2
*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^
2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)
*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^3*c*d - a^2*b^2*d
^2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/8*(4*sqrt(d*x^2 + c)*a*b*d*x - 8*(a*b*c
- a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
+ (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d)
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d
)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a
*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/4*(2*sqrt(d*x^2 + c)
*a*b*d*x + (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(a/(b*
c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
 - a*d))/(a*d*x^3 + a*c*x)) + 2*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(
```

```
d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a*b^3*c*d - a^2*b^2*d^
2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*d*x - 4*(a*b*c -
 a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) +
 (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(a/(b*c - a*d))*
arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(
a*d*x^3 + a*c*x)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(110) = 220.
time = 0.55, size = 284, normalized size = 2.15

$$-\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(b^3c-ab^2d)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)(b^3c-ab^2d)}-\frac{\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{2\,b^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2
 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqrt(
a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt(d) - 2*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/2*log((sqrt(d)*x
 - sqrt(d*x^2 + c))^2)/(b^2*sqrt(d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.759 $\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^2}}{2b(bc-ad)\,(a+bx^2)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/2*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/2*a*(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {457, 79, 65, 214}

$$\frac{a\sqrt{c+dx^2}}{2b\,(a+bx^2)\,(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= \frac{a\sqrt{c+dx^2}}{2b(bc-ad)(a+bx^2)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4b(bc-ad)}\\ &= \frac{a\sqrt{c+dx^2}}{2b(bc-ad)(a+bx^2)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2bd(bc-ad)}\\ &= \frac{a\sqrt{c+dx^2}}{2b(bc-ad)(a+bx^2)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 100, normalized size = 1.01

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^2}}{(bc-ad)(a+bx^2)} - \frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(2
*b^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(83) = 166.
time = 0.09, size = 816, normalized size = 8.24

method	result
default	$-\frac{\sqrt{-ab}\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}}{}}{}\right)}{}\right)}{4b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^3*(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d*(-a*b
)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2
))))-1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1
/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4/b^3
*(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d*(-a*b)^(1/2)/
(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(83) = 166$.
time = 1.83, size = 450, normalized size = 4.55

$$\left[\frac{\left(2\,abc - a^{2}d + \left(2\,b^{2}c - abd\right)x^{2}\right)\sqrt{b^{2}c - abd}\,\log\left(\frac{b^{2}d^{2}x^{4} + 8\,b^{2}c^{2} - 8\,abcd + a^{2}d^{2} + 2\,\left(4\,b^{2}cd - 3\,abd^{2}\right)x^{2} - 4\,\left(bdx^{2} + 2\,bc - ad\right)\sqrt{b^{2}c - abd}\,\sqrt{dx^{2} + c}}{b^{2}x^{4} + 2\,abx^{2} + a^{2}}\right) + 4\,\left(ab^{2}c - a^{2}bd\right)\sqrt{dx^{2} + c}}{8\,\left(ab^{4}c^{2} - 2\,a^{2}b^{3}cd + a^{3}b^{2}d^{2} + \left(b^{5}c^{2} - 2\,ab^{4}cd + a^{2}b^{3}d^{2}\right)x^{2}\right)}, -\frac{\left(2\,abc - a^{2}d + \left(2\,b^{2}c - abd\right)x^{2}\right)\sqrt{-b^{2}c + abd}\,\arctan\left(-\frac{\left(bdx^{2} + 2\,bc - ad\right)\sqrt{-b^{2}c + abd}\,\sqrt{dx^{2} + c}}{2\,\left(b^{2}c^{2} - abcd + \left(b^{2}cd - abd^{2}\right)x^{2}\right)}\right) - 2\,\left(ab^{2}c - a^{2}bd\right)\sqrt{dx^{2} + c}}{4\,\left(ab^{4}c^{2} - 2\,a^{2}b^{3}cd + a^{3}b^{2}d^{2} + \left(b^{5}c^{2} - 2\,ab^{4}cd + a^{2}b^{3}d^{2}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*log((b^
2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2
 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) + 4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c))/(a*b^4*c^2 - 2
*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2), -1
/4*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 -
 a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c
))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*
b^3*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(a + bx^{2}\right)^{2}\sqrt{c + dx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.52, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^{2} + c}\,ad^{2}}{\left(b^{2}c - abd\right)\left(\left(dx^{2} + c\right)b - bc + ad\right)} + \frac{\left(2\,bcd - ad^{2}\right)\arctan\left(\frac{\sqrt{dx^{2} + c}\,b}{\sqrt{-b^{2}c + abd}}\right)}{\left(b^{2}c - abd\right)\sqrt{-b^{2}c + abd}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x^2 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^2 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B]
time = 0.50, size = 93, normalized size = 0.94

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)\,(a\,d-2\,b\,c)}{2\,b^{3/2}\,(a\,d-b\,c)^{3/2}}-\frac{a\,d\,\sqrt{d\,x^2+c}}{2\,b\,(a\,d-b\,c)\,\left(b\,\left(d\,x^2+c\right)+a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(2*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^2)^(1/2))/(2*b*(a*d - b*c)*(b*(c + d
*x^2) + a*d - b*c))
```

3.760 $\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{x\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{3/2}}$$

```
[Out] 1/2*c*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(3/2)/a
^(1/2)-1/2*x*(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {482, 12, 385, 211}

$$\frac{c\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx &= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)(a+bx^2)}+\frac{\int \frac{c}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2(bc-ad)}\\
&= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)(a+bx^2)}+\frac{c\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2(bc-ad)}\\
&= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)(a+bx^2)}+\frac{c\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2(bc-ad)}\\
&= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)(a+bx^2)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.49, size = 109, normalized size = 1.22

$$
-\frac{x\sqrt{c+dx^2}}{2(bc-ad)(a+bx^2)}-\frac{c\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2\sqrt{a}\,(bc-ad)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) - (c*ArcTan[(a*Sqrt[d] +
 b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*Sqrt[a]*
(b*c - a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(73) = 146$.
time = 0.10, size = 816, normalized size = 9.17

method	result

default	$\frac{\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}}{}\right.}{}}{4b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d*(-a*b)^(1/2)/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-1/4/(-a*b
)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/
(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
+1/4/b^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d*(-a*b)^(1/2)/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(73) = 146.
time = 1.20, size = 418, normalized size = 4.70

$$\left[-\frac{4\,(abc-a^{2}d)\sqrt{dx^{2}+c}\,x-(bcx^{2}+ac)\sqrt{-abc+a^{2}d}\,\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{4}+a^{2}c^{2}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{2}+4\,((bc-2\,ad)x^{3}-acx)\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)}{8\,(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}+(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2})x^{2})},\ -\frac{2\,(abc-a^{2}d)\sqrt{dx^{2}+c}\,x-(bcx^{2}+ac)\sqrt{abc-a^{2}d}\,\arctan\left(\frac{\sqrt{abc-a^{2}d}\,((bc-2\,ad)x^{2}-ac)\sqrt{dx^{2}+c}}{2\,((abcd-a^{2}d^{2})x^{3}+(abc^{2}-a^{2}cd)x)}\right)}{4\,(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}+(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(-a*b*c +
a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
- 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(
d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), -1/4*(2*(a*b*c - a^2*d)
*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*
b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)
*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b
^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(73) = 146.
time = 1.21, size = 231, normalized size = 2.60

$$\frac{c\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)(b^2c-abd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + ((sqrt(d)
*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d
^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(
b^2*c - a*b*d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
[Out] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.761 $\int \frac{x}{(a+bx^2)^2 \sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{c+dx^2}}{2(bc-ad)\,(a+bx^2)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,(bc-ad)^{3/2}}$$

```
[Out] 1/2*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/2*(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {455, 44, 65, 214}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}}{2\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/((b*c - a*d)*(a + b*x^2)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4(bc-ad)}\\
&= -\frac{\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2(bc-ad)}\\
&= -\frac{\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 86, normalized size = 0.99

$$\frac{1}{2}\left(-\frac{\sqrt{c+dx^2}}{(bc-ad)\left(a+bx^2\right)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(Sqrt[c + d*x^2]/((b*c - a*d)*(a + b*x^2))) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(71) = 142.
time = 0.09, size = 524, normalized size = 6.02

method	result

default	$\frac{\sqrt{-ab}\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(\right.}{}}{}\right.}{}\right.}{4ab^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d*(-a*
b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/
2))))-1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-
d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2)
)^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*
b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(71) = 142$.

time = 1.26, size = 404, normalized size = 4.64

$$\left[-\frac{(bdx^2+ad)\sqrt{b^2c-abd}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(b^2c-abd)\sqrt{dx^2+c}}{8\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)}, \frac{(bdx^2+ad)\sqrt{-b^2c+abd}\,\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)-2\,(b^2c-abd)\sqrt{dx^2+c}}{4\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*d*x^2 + a*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a
*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(
b^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), 1/4*((b*d*x^2 + a*d)*sqrt(-b^2*c
+ a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^
2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(b^2*c - a*b*d)*s
qrt(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.72, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx^2+c}\,d}{2\,((dx^2+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/2*sqrt(d*x^2 + c)*d/(((d*x^2 + c)*b - b*c + a*d)*(b*c - a
*d))
```

Mupad [B]
time = 0.43, size = 82, normalized size = 0.94

$$\frac{d\sqrt{d\,x^2+c}}{2\,\left(a\,d-b\,c\right)\,\left(b\,\left(d\,x^2+c\right)+a\,d-b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] (d*(c + d*x^2)^(1/2))/(2*(a*d - b*c)*(b*(c + d*x^2) + a*d - b*c)) + (d*atan
((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2)))/(2*b^(1/2)*(a*d - b*c)^(3/
2))
```

3.762 $\int \frac{1}{(a+bx^2)^2 \sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=100

$$\frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\,(a+bx^2)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/2*(-2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)
/(-a*d+b*c)^(3/2)+1/2*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {390, 385, 211}

$$\frac{(bc-2ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(3/
2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx &= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)}+\frac{(bc-2ad)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}\\ &= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)}+\frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2a(bc-ad)}\\ &= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 122, normalized size = 1.22

$$-\frac{bx\sqrt{c+dx^2}}{2a(-bc+ad)(a+bx^2)}+\frac{(-bc+2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(b*x*Sqrt[c + d*x^2])/(a*(-(b*c) + a*d)*(a + b*x^2)) + ((-(b*c) + 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b
*c - a*d])])/(2*a^(3/2)*(b*c - a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(84) = 168$.
time = 0.09, size = 822, normalized size = 8.22

method	result

default	$-\frac{\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\ \ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}}{}\right)}{}}{4ba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/a*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d*(-a*b)^(1/2)/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-1/4/a/(-
a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)
^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4
/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)-1/4/b/a*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d*(-a*b)^(1/2)/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(84) = 168.
time = 1.33, size = 459, normalized size = 4.59

$$\left[\frac{4\,(ab^{2}c-a^{2}bd)\sqrt{dx^{2}+c}\,x-(abc-2\,a^{2}d+(b^{2}c-2\,abd)x^{2})\sqrt{-abc+a^{2}d}\,\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{4}+a^{2}c^{2}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{2}-4\,((bc-2\,ad)x^{3}-acx)\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)}{8\,(a^{3}b^{2}c^{2}-2\,a^{4}bcd+a^{5}d^{2}+(a^{2}b^{3}c^{2}-2\,a^{3}b^{2}cd+a^{4}bd^{2})x^{2})},\ \frac{2\,(ab^{2}c-a^{2}bd)\sqrt{dx^{2}+c}\,x+\sqrt{abc-a^{2}d}\,(abc-2\,a^{2}d+(b^{2}c-2\,abd)x^{2})\arctan\left(\frac{\sqrt{abc-a^{2}d}\,((bc-2\,ad)x^{2}-ac)\sqrt{dx^{2}+c}}{2\,((abcd-a^{2}d^{2})x^{3}+(abc^{2}-a^{2}cd)x)}\right)}{4\,(a^{3}b^{2}c^{2}-2\,a^{4}bcd+a^{5}d^{2}+(a^{2}b^{3}c^{2}-2\,a^{3}b^{2}cd+a^{4}bd^{2})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - (a*b*c - 2*a^2*d + (b^2*c -
 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3
*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2
)*x^2), 1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + sqrt(a*b*c - a^2*d)*
(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((
b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2
 - a^2*c*d)*x)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^
3*b^2*c*d + a^4*b*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(84) = 168.
time = 0.52, size = 225, normalized size = 2.25

$$-\frac{1}{2}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad-bc^2\right)}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)\left(abcd-a^2d^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sqrt
(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d - b*
c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2))
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.763 $\int \frac{1}{x(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=130

$$\frac{b\sqrt{c+dx^2}}{2a(bc-ad)\,(a+bx^2)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}+\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}$$

```
[Out] 1/2*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/2*b*(
d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 105, 162, 65, 214}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}+\frac{b\sqrt{c+dx^2}}{2a\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) - ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]]/(a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(b(2bc-3ad))\text{Su}}{}$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{(b(2bc-3ad)}{}$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\right.}{2a^2(bc-ad)^3}$$

Mathematica [A]
time = 0.37, size = 124, normalized size = 0.95

$$\frac{-\frac{ab\sqrt{c+dx^2}}{(-bc+ad)(a+bx^2)} + \frac{\sqrt{b}\,(2bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a*b*Sqrt[c + d*x^2])/((-(b*c) + a*d)*(a + b*x^2))) + (Sqrt[b]*(2*b*c -
3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)
^(3/2) - (2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(108) = 216$.
time = 0.10, size = 847, normalized size = 6.52

method	result
default	$\frac{\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\ \ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right.}{b}}{\ }\right.}{\ }}{4a\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d*(-a*b)^(
1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))
)-1/4/a/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(
1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d*(-a*b)
^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b
*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)
)))+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/
2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))-1/a^2/c^
(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x), x)
```

Fricas [A]
time = 1.85, size = 1037, normalized size = 7.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(d*x^2 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b
*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(c)*log(-(
d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a^3*b*c^2 - a^4*c*d + (a^2*
b^2*c^2 - a^3*b*c*d)*x^2), 1/8*(4*sqrt(d*x^2 + c)*a*b*c + 8*(a*b*c - a^2*d
+ (b^2*c - a*b*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*a*b*c
^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*
d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 +
 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 +
 c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3*b*c^2 - a^4*c*d
 + (a^2*b^2*c^2 - a^3*b*c*d)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*c - (2*a*b*c^
2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/
2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b
*c)) + 2*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*
b*c*d)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*c - (2*a*b*c^2 - 3*a^2*c*d + (2*b^2
*c^2 - 3*a*b*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a
*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 4*(a*b*c - a^2*
d + (b^2*c - a*b*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(a^3*b*
c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^{2}\right)^{2}\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.56, size = 138, normalized size = 1.06

$$\frac{\sqrt{dx^2+c}\,bd}{2\,(abc-a^2d)((dx^2+c)b-bc+ad)}-\frac{(2\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,(a^2bc-a^3d)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*d/((a*b*c - a^2*d)*((d*x^2 + c)*b - b*c + a*d)) - 1/2
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*s
qrt(-c))
```

Mupad [B]
time = 1.07, size = 3023, normalized size = 23.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^5 - 6*a^5*b^
3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c +
d*x^2)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 - 64*
a^6*b^3*c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2 + a^2
*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*
c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^
3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(
1/2)*1i)/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((
((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(2*(a
^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 +
 2*a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^2)^(1
/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*c
*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2 + a^2*b^2*c^2
- 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*
(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3
*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/
(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((3*a*b^3*d
```

```
^4)/2 - b^4*c*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c + d*x^2)^(
1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(2*(a^4*d^2 + a^2*b
^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2
*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*x^2)^(1/2)*(3*a*d - 2
*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*c*d^4 - 32*a^4*
b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)
*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c
)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3
*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2
*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^2)^(1/2)*(13*a^2
*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)) + (((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(a^5*d
^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^2)^(1/2)*(3*a*d - 2*b*c)*(-b*(a
*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*c*d^4 - 32*a^4*b^5*c^3*d^2
+ 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 -
a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d -
 b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2
)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^3 + 3
*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)
*1i)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan(
(((((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(2*(a^5*d^2 + a^3
*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^2)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*
c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*a^2*c^(1/2)*(a^4*d^2 +
 a^2*b^2*c^2 - 2*a^3*b*c*d)))/(2*a^2*c^(1/2)) - ((c + d*x^2)^(1/2)*(13*a^2*
b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^
3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*
b^4*c^2*d^3)/(2*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^2)^(1/2)
*(16*a^7*b^2*d^5 - 64*a^6*b^3*c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d
^3))/(8*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(2*a^2*c^(1/2))
 + ((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(4
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)))/(((3*a*b^3*d^4)
/2 - b^4*c*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + (((4*a^6*b^2*d^5 -
6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(2*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*
d)) - ((c + d*x^2)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*c*d^4 - 32*a^4*b^5*c^
3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*
b*c*d)))/(2*a^2*c^(1/2)) - ((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d
^2 - 20*a*b^4*c*d^3))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/
2)) + (((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(2*(a^5*d^2 +
 a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^2)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*
b^3*c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*a^2*c^(1/2)*(a^4*d
^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(2*a^2*c^(1/2)) + ((c + d*x^2)^(1/2)*(13*
a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(4*(a^4*d^2 + a^2*b^2*c^2 -
2*a^3*b*c*d)))/(a^2*c^(1/2))))*1i)/(a^2*c^(1/2)) - (b*d*(c + d*x^2)^(1/2))/
(2*(a^2*d - a*b*c)*(b*(c + d*x^2) + a*d - b*c))
```

3.764 $\int \frac{1}{x^2(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=147

$$-\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\,(a+bx^2)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/2*b*(-4*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^
(5/2)/(-a*d+b*c)^(3/2)-1/2*(-2*a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c)/
x+1/2*b*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/x/(b*x^2+a)
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {483, 597, 12, 385, 211}

$$-\frac{b(3bc-4ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}\,(3bc-2ad)}{2a^2cx(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax\,(a+bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*((3*b*c - 2*a*d)*Sqrt[c + d*x^2])/(a^2*c*(b*c - a*d)*x) + (b*Sqrt[c +
d*x^2])/(2*a*(b*c - a*d)*x*(a + b*x^2)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b
*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{(b(3bc-4ad))\int \frac{\quad}{(a+bx^2 \ldots}}{2a^2(bc-a \ldots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{(b(3bc-4ad))\text{Subst}\left(\ldots\right.}{2 \ldots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\ldots\right.}{2a^{5/2}(bc-a \ldots}$$

Mathematica [A]
time = 0.74, size = 156, normalized size = 1.06

$$\frac{\sqrt{c+dx^2}\left(2abc-2a^2d+3b^2cx^2-2abdx^2\right)}{2a^2c(-bc+ad)x\left(a+bx^2\right)} + \frac{b(3bc-4ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^2 - 2*a*b*d*x^2))/(2*a^2*c*
(-(b*c) + a*d)*x*(a + b*x^2)) + (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*Sq
rt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2)*(b*
c - a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(127) = 254$.
time = 0.12, size = 838, normalized size = 5.70

method	result
default	$\frac{\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right.}{b}}{}\right.}{}}{4a^2}$
risch	$-\frac{\sqrt{d\,x^2+c}}{a^2cx}+\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{4a^2(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d*(-a*b)^(1/2)/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b
)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+3/4*b/a^2
/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-
3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^
```

```
2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2)))-1/a^2/c/x*(d*x^2+c)^(1/2)+1/4/a^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/
2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)-d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/
b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x-1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(127) = 254$.
time = 1.53, size = 600, normalized size = 4.08

$$\left[-\frac{((3b^3c^2-4ab^2cd)x^3+(3ab^2c^2-4a^2bcd)x)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)+4(2a^2b^2c^2-4a^3bcd+2a^4d^2+(3ab^3c^2-5a^2b^2cd+2a^3bd^2)x^2)\sqrt{dx^2+c}}{8((a^3b^3c^3-2a^4b^2c^2d+a^5bcd^2)x^3+(a^4b^2c^3-2a^5bc^2d+a^6cd^2)x)}, -\frac{((3b^3c^2-4ab^2cd)x^3+(3ab^2c^2-4a^2bcd)x)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+2(2a^2b^2c^2-4a^3bcd+2a^4d^2+(3ab^3c^2-5a^2b^2cd+2a^3bd^2)x^2)\sqrt{dx^2+c}}{4((a^3b^3c^3-2a^4b^2c^2d+a^5bcd^2)x^3+(a^4b^2c^3-2a^5bc^2d+a^6cd^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt
(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^
2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a^2*b^2*c^2 - 4*a
^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^2)*sqr
t(d*x^2 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^3 + (a^4*b^2
*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x), -1/4*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^3
+ (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c
- a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3
 + (a*b*c^2 - a^2*c*d)*x)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2 + (
3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*
c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^3 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a
^6*c*d^2)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^2\right)^2 \sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(127) = 254.
time = 1.25, size = 396, normalized size = 2.69

$$\frac{1}{2}\,d^{\frac{5}{2}}\left(\frac{\left(3\,b^{2}c-4\,abd\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{2\left(3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b^{2}c-4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}abd-6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c^{2}+14\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}abcd-8\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}a^{2}d^{2}+3\,b^{2}c^{3}-2\,abc^{2}d\right)}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{6}b-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}ad+3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}-4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}acd-bc^{3}\right)\left(a^{2}bcd^{2}-a^{3}d^{3}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^
2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt(a
*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c - 4*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*b*d - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2
+ 14*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d - 8*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^6*
b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.765 $\int \frac{1}{x^3(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=185

$$-\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)\,(a+bx^2)}-\frac{\sqrt{c+dx^2}}{2acx^2\,(a+bx^2)}+\frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{2a^3(bc-ad)^{3/2}}$$

```
[Out] 1/2*(a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/2*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/2*b*(-a*d+2*b*c)*(d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^2+a)-1/2*
(d*x^2+c)^(1/2)/a/c/x^2/(b*x^2+a)
```

Rubi [A]

time = 0.16, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 105, 156, 162, 65, 214}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}}-\frac{b\sqrt{c+dx^2}\,(2bc-ad)}{2a^2c\,(a+bx^2)\,(bc-ad)}-\frac{\sqrt{c+dx^2}}{2acx^2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(b*(2*b*c - a*d)*Sqrt[c + d*x^2])/(a^2*c*(b*c - a*d)*(a + b*x^2)) - Sq
rt[c + d*x^2]/(2*a*c*x^2*(a + b*x^2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x
^2]/Sqrt[c]])/(2*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^2)^2 \sqrt{c+dx^2}} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}} \, dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}} \, dx, x, x^2\right)}{2ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)(a+bx^2)} - \frac{\sqrt{c+dx^2}}{2acx^2(a+bx^2)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd}{x(a+bx)\sqrt{c+c}}\right.}{2a^2c(bc-a}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)(a+bx^2)} - \frac{\sqrt{c+dx^2}}{2acx^2(a+bx^2)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \frac{}{(a+}\right.}{4a^3(bc-}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)(a+bx^2)} - \frac{\sqrt{c+dx^2}}{2acx^2(a+bx^2)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \frac{}{a-}\right.}{2a^3d(bc-}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)(a+bx^2)} - \frac{\sqrt{c+dx^2}}{2acx^2(a+bx^2)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right.}{2a^3c^{3/2}}$$

Mathematica [A]
time = 0.75, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^2}(-a^2d+2b^2cx^2+ab(c-dx^2))}{c(-bc+ad)x^2(a+bx^2)} - \frac{b^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(-(a^2*d) + 2*b^2*c*x^2 + a*b*(c - d*x^2)))/(c*(-(b*c)
+ a*d)*x^2*(a + b*x^2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcTanh
[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2))/(2*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(157) = 314.
time = 0.14, size = 903, normalized size = 4.88

method	result

risch	$-\frac{\sqrt{d x^2+c}}{2 c a^2 x^2}-\frac{b^2 \sqrt{d \left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2 d \sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{4 a^2 \sqrt{-ab}\,(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}-\frac{bd \ln\left(-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-a}}{}\right.}{}$
default	$-\frac{b\left(\frac{b \sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\sqrt{}\right.}{b}\right.}{}\right.}{4a^2\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/a^2/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d*(-a
*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1
/2))))-b/a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-b
/a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/a^2*(-1
/2/c/x^2*(d*x^2+c)^(1/2)+1/2*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x
))+2/a^3*b/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)+1/4*b/a^2/(-a*b)^(
1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d*(-a*b)^(1/2)/(a*d-b*c)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
```

```
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3), x)
```

<u>Fricas</u> [A]
time = 2.23, size = 1407, normalized size = 7.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)
*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2
*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*
d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) + 2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*
a^2*b*c*d - a^3*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) +
 2*c)/x^2) - 4*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2)*sqrt(d
*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*d)*x^2),
 -1/8*(4*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*
b*c*d - a^3*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - ((4*b^3*c
^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqrt(b/(b*c -
a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3
*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^
2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a
^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2)*sqrt(d*x^2 + c))/((a^3*
b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*d)*x^2), 1/4*(((4*b^3*c^3
 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqrt(-b/(b*c - a
*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d)
)/(b*d*x^2 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*
c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*s
qrt(c) + 2*c)/x^2) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2
)*sqrt(d*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*
d)*x^2), 1/4*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2
*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2
 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(-c)*arcta
```

```
n(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b
*c*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 -
 a^5*c^2*d)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a+bx^2)^2 \sqrt{c+dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.55, size = 257, normalized size = 1.39

$$\frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,(a^3bc-a^4d)\sqrt{-b^2c+abd}}-\frac{2\,(dx^2+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^2+c}\,b^2c^2d-(dx^2+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^2+c}\,abcd^2-\sqrt{dx^2+c}\,a^2d^3}{2\,(a^2bc^2-a^3cd)\left((dx^2+c)^2b-2\,(dx^2+c)bc+bc^2+(dx^2+c)ad-acd\right)}-\frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/2*(2*(d*x^2 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^2 + c)*b^2*c^2*d - (d*x^2 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^2 + c
)*a*b*c*d^2 - sqrt(d*x^2 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^2 + c)^
2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)) - 1/2*(4*b*c +
a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B]
time = 1.58, size = 2500, normalized size = 13.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] (((c + d*x^2)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^2)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^2)*(a*d - 2*b*c) + b*(c + d*x^2)^2 + b*c^2 - a*c*d) + (atan((((-b
^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2)^(1/2)*(a^4*b^3*d^6 +
32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/
(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/
2)*(5*a*d - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^
```

4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/4 + 8*b^7*c^3*d^3 - 12*a*b^6*c^2*d^4 + (3*a^2*b^5*c*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(2*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d*x^2)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^2)^(1/2)*(a*d + 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*

```
a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c)
)/(4*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(4*a^3*(c^3)^(1/2)) + ((((c + d*x^
2)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5
 + 26*a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (
((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d
^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^2)^(1/2)*(a*d +
 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*
a^9*b^2*c^2*d^5))/(8*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c
^3*d)))*(a*d + 4*b*c))/(4*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(4*a^3*(c^3)^
(1/2)))/(((5*a^3*b^4*d^6)/4 + 8*b^7*c^3*d^3 - 12*a*b^6*c^2*d^4 + (3*a^2*b^5
*c*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((((c + d*x^2)^(1/
2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*
a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((2*a^
9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a
^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c...
```

3.766 $\int \frac{1}{x^4(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=206

$$-\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3}+\frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x}+\frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3(a+bx^2)}+\frac{b^2(5bc-6ad)\tan^{-1}}{2a^{7/2}(b}$$

```
[Out] 1/2*b^2*(-6*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a
^(7/2)/(-a*d+b*c)^(3/2)-1/6*(-2*a*d+5*b*c)*(d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c)
/x^3+1/6*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^2+c)^(1/2)/a^3/c^2/(-a*d+b*
c)/x+1/2*b*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^2+a)
```

Rubi [A]
time = 0.16, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {483, 597, 12, 385, 211}

$$\frac{b^2(5bc-6ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}(5bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{\sqrt{c+dx^2}(-4a^2d^2-8abcd+15b^2c^2)}{6a^3c^2x(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax^3(a+bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/6*((5*b*c - 2*a*d)*Sqrt[c + d*x^2])/(a^2*c*(b*c - a*d)*x^3) + ((15*b^2*c
^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*c^2*(b*c - a*d)*x) + (b
*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (b^2*(5*b*c - 6*a*d)*
ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*(b*c - a*
d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}}\, dx = \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3(a+bx^2)} - \frac{\int \frac{-5bc+2ad-4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}}\, dx}{2a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3(a+bx^2)} + \frac{\int \frac{-15b^2c^2+8abcd+4a^2d^2 \cdots}{x^2(a+bx^2)\sqrt{c \cdots}}}{6a^2c(bc \cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b \cdots}{2a(bc- \cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b \cdots}{2a(bc- \cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b \cdots}{2a(bc- \cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b \cdots}{2a(bc- \cdots}$$

Mathematica [A]

time = 0.90, size = 199, normalized size = 0.97

$$-\frac{\sqrt{c+dx^2}\,(15b^3c^2x^4+2ab^2cx^2(5c-4dx^2)+2a^3d(c-2dx^2)-2a^2b(c^2+3cdx^2+2d^2x^4))}{6a^3c^2(-bc+ad)x^3\,(a+bx^2)}-\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{7/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/6*(Sqrt[c + d*x^2]*(15*b^3*c^2*x^4 + 2*a*b^2*c*x^2*(5*c - 4*d*x^2) + 2*a
^3*d*(c - 2*d*x^2) - 2*a^2*b*(c^2 + 3*c*d*x^2 + 2*d^2*x^4)))/(a^3*c^2*(-(b*
c) + a*d)*x^3*(a + b*x^2)) - (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b*x*(
Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(7/2)*(b*c -
 a*d)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(182) = 364$.
time = 0.12, size = 885, normalized size = 4.30

method	result
risch	$-\frac{\sqrt{d\,x^2+c}\,\left(-2ad\,x^2-6c\,x^2b+ac\right)}{3c^2a^3x^3}-\frac{b^2\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{4a^3(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)}+\frac{bd\sqrt{-\cdots}}{\cdots}$
default	$-\frac{b\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x+\frac{\sqrt{-\cdots}}{b}\right)}{b}\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{4a^3\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/a^3*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))*(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+d*(-a*b)^(1/2)/(a*
d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/
2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-5/4*b^
2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*
d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2)))+5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2)))-1/4*b/a^3*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))*(d*(x-1/b*(-a
*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-d*(
-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^
(1/2))))+1/a^2*(-1/3/c/x^3*(d*x^2+c)^(1/2)+2/3*d/c^2/x*(d*x^2+c)^(1/2))+2/a
^3*b/c/x*(d*x^2+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^4), x)
```

Fricas [A]
time = 1.73, size = 758, normalized size = 3.68

$$\left[-\frac{3\,\left({\left(5\,b^{4}c^{3}-6\,ab^{3}c^{2}d\right)}x^{5}+{\left(5\,ab^{3}c^{3}-6\,a^{2}b^{2}c^{2}d\right)}x^{3}\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)}x^{4}+a^{2}c^{2}-2\,{\left(3\,abc^{2}-4\,a^{2}cd\right)}x^{2}-4\,{\left({\left(bc-2\,ad\right)}x^{3}-acx\right)}\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\,{\left(2\,a^{3}b^{2}c^{3}-4\,a^{4}bc^{2}d+2\,a^{5}cd^{2}-{\left(15\,ab^{4}c^{3}-23\,a^{2}b^{3}c^{2}d+4\,a^{3}b^{2}cd^{2}+4\,a^{4}bd^{3}\right)}x^{4}-2\,{\left(5\,a^{2}b^{3}c^{3}-8\,a^{3}b^{2}c^{2}d+a^{4}bcd^{2}+2\,a^{5}d^{3}\right)}x^{2}\right)}\sqrt{dx^{2}+c}}{24\,{\left({\left(a^{4}b^{3}c^{4}-2\,a^{5}b^{2}c^{3}d+a^{6}bc^{2}d^{2}\right)}x^{5}+{\left(a^{5}b^{2}c^{4}-2\,a^{6}bc^{3}d+a^{7}c^{2}d^{2}\right)}x^{3}\right)}},\ \frac{3\,\left({\left(5\,b^{4}c^{3}-6\,ab^{3}c^{2}d\right)}x^{5}+{\left(5\,ab^{3}c^{3}-6\,a^{2}b^{2}c^{2}d\right)}x^{3}\right)\sqrt{abc-a^{2}d}\,\arctan\left(\frac{\sqrt{abc-a^{2}d}\,{\left({\left(bc-2\,ad\right)}x^{2}-ac\right)}\sqrt{dx^{2}+c}}{2\,{\left({\left(abcd-a^{2}d^{2}\right)}x^{3}+{\left(abc^{2}-a^{2}cd\right)}x\right)}}\right)-2\,{\left(2\,a^{3}b^{2}c^{3}-4\,a^{4}bc^{2}d+2\,a^{5}cd^{2}-{\left(15\,ab^{4}c^{3}-23\,a^{2}b^{3}c^{2}d+4\,a^{3}b^{2}cd^{2}+4\,a^{4}bd^{3}\right)}x^{4}-2\,{\left(5\,a^{2}b^{3}c^{3}-8\,a^{3}b^{2}c^{2}d+a^{4}bcd^{2}+2\,a^{5}d^{3}\right)}x^{2}\right)}\sqrt{dx^{2}+c}}{12\,{\left({\left(a^{4}b^{3}c^{4}-2\,a^{5}b^{2}c^{3}d+a^{6}bc^{2}d^{2}\right)}x^{5}+{\left(a^{5}b^{2}c^{4}-2\,a^{6}bc^{3}d+a^{7}c^{2}d^{2}\right)}x^{3}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d
)*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^
2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(
-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a^3*b^
2*c^3 - 4*a^4*b*c^2*d + 2*a^5*c*d^2 - (15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*
a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^4 - 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4
*b*c*d^2 + 2*a^5*d^3)*x^2)*sqrt(d*x^2 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d
 + a^6*b*c^2*d^2)*x^5 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^3), 1
/12*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x
```

```
^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) -
2*(2*a^3*b^2*c^3 - 4*a^4*b*c^2*d + 2*a^5*c*d^2 - (15*a*b^4*c^3 - 23*a^2*b^3
*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^4 - 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*
c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2)*sqrt(d*x^2 + c))/((a^4*b^3*c^4 - 2*a^
5*b^2*c^3*d + a^6*b*c^2*d^2)*x^5 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d
^2)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + bx^2)^2 \sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(182) = 364.
time = 1.41, size = 375, normalized size = 1.82

$$\frac{1}{6}\, d^{\frac{7}{2}} \left(\frac{3\,(5\,b^3c - 6\,ab^2d)\arctan\left(-\frac{\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}}\right)}{(a^3bcd^3 - a^4d^4)\sqrt{abcd - a^2d^2}} - \frac{6\left(\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 b^3c - 2\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 ab^2d - b^3c^2\right)}{(a^3bcd^3 - a^4d^4)\left(\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^4 b - 2\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 bc + 4\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 ad + bc^2\right)} - \frac{8\left(3\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^4 b - 6\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 bc - 3\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 ad + 3\,bc^2 + acd\right)}{\left(\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 - c\right)^3 a^3d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 +
 c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4)*s
qrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c - 2*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d^4)*((s
qrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*b - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 3*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)
^3*a^3*d^3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.767 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}}+\frac{ax}{2b(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}}-\frac{3\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}$$

```
[Out] -3/2*c*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)/(-a*d+b*c
)^(5/2)+1/2*(a*d+2*b*c)*x/b/(-a*d+b*c)^2/(d*x^2+c)^(1/2)+1/2*a*x/b/(-a*d+b*
c)/(b*x^2+a)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {481, 541, 12, 385, 211}

$$-\frac{3\sqrt{a}\,c\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}+\frac{x(ad+2bc)}{2b\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{ax}{2b\,(a+bx^2)\,\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((2*b*c + a*d)*x)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (a*x)/(2*b*(b*c - a
*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - (3*Sqrt[a]*c*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{ax}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{\int \frac{ac-2bcx^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{2b(bc-ad)}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{\int \frac{3abc^2}{(a+bx^2)\sqrt{c+}}}{2bc(bc-a}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{(3ac)\int \frac{}{(a+bx^2)}}{2(bc-}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{(3ac)\text{Subst}\left(\int\right.}{}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{3\sqrt{a}\,c\tan^{-1}\left(\right.}{2(bc-}$$

Mathematica [A]

time = 0.68, size = 126, normalized size = 0.97

$$\frac{1}{2}\left(\frac{3acx+2bcx^3+adx^3}{(bc-ad)^2\,(a+bx^2)\,\sqrt{c+dx^2}}+\frac{3\sqrt{a}\,c\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((3*a*c*x + 2*b*c*x^3 + a*d*x^3)/((b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2]
) + (3*Sqrt[a]*c*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sq
rt[a]*Sqrt[b*c - a*d])])/(b*c - a*d)^(5/2))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1942 vs. $2(110) = 220$.
time = 0.10, size = 1943, normalized size = 14.95

method	result	size
default	Expression too large to display	1943

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*x/c/(d*x^2+c)^(1/2)-1/4*a/b^3*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2
)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)
^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4/b^2*a/(-a*b)^(1/2)*(-
1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2)
)+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^
2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+3/4/b^2*a/(-a
```

```
*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*
(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/
(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-1
/4*a/b^3*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3*d*(-a*b)^(1/2)/(a*
d-b*c)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a
*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*
d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+4*d/
(a*d-b*c)*b*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(110) = 220$.
time = 1.32, size = 552, normalized size = 4.25

$$\left[\frac{3\,(bcdx^4 + ac^2 + (bc^2 + acd)x^2)\sqrt{-\frac{a}{bc - ad}}\,\log\left(\frac{(b^2c^2 - 8\,abcd + 8\,a^2d^2)x^4 + a^2c^2 - 2\,(3\,abc^2 - 4\,a^2cd)x^2 - 4\,((b^2c^2 - 3\,abcd + 2\,a^2d^2)x^3 - (abc^2 - a^2cd)x)\sqrt{dx^2 + c}\sqrt{-\frac{a}{bc - ad}}}{b^2x^4 + 2\,abx^2 + a^2}\right) + 4\,((2\,bc + ad)x^3 + 3\,acx)\sqrt{dx^2 + c}}{8\,(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^4 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x^2)},\ \frac{3\,(bcdx^4 + ac^2 + (bc^2 + acd)x^2)\sqrt{\frac{a}{bc - ad}}\,\arctan\left(-\frac{((bc - 2\,ad)x^2 - ac)\sqrt{dx^2 + c}\sqrt{\frac{a}{bc - ad}}}{2\,(adx^3 + acx)}\right) + 2\,((2\,bc + ad)x^3 + 3\,acx)\sqrt{dx^2 + c}}{4\,(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^4 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*c*d*x^4 + a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(-a/(b*c - a*d))*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*s
qrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((2*b
*c + a*d)*x^3 + 3*a*c*x)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*
```

```
c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*
d - a^2*b*c*d^2 + a^3*d^3)*x^2), 1/4*(3*(b*c*d*x^4 + a*c^2 + (b*c^2 + a*c*d
)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*((2*b*c + a*d)*x^3 + 3*a*c
*x)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d -
2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3
*d^3)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^2)^2 (c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(110) = 220.
time = 1.28, size = 298, normalized size = 2.29

$$\frac{3\,ac\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}}+\frac{cx}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{dx^2+c}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 abc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4 b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 ad+bc^2\right)(b^3c^2-2\,ab^2cd+a^2bd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 3/2*a*c*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d -
a^2*d^2)) + c*x/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)) - ((sqrt(
d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sq
rt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d +
b*c^2)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.768 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}}+\frac{a}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,(bc-ad)^{5/2}}$$

```
[Out] -1/2*(a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*
c)^(5/2)/b^(1/2)+1/2*(a*d+2*b*c)/b/(-a*d+b*c)^2/(d*x^2+c)^(1/2)+1/2*a/b/(-a
*d+b*c)/(b*x^2+a)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 79, 53, 65, 214}

$$\frac{a}{2b\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{ad+2bc}{2b\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (2*b*c + a*d)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + a/(2*b*(b*c - a*d)*(a +
 b*x^2)*Sqrt[c + d*x^2]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{(2bc+ad)\text{Sub}}{}$$

$$= \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{(2bc+ad)\text{Sub}}{}$$

$$= \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{(2bc+ad)\tan}{2\sqrt{b}}$$

Mathematica [A]

time = 0.32, size = 110, normalized size = 0.82

$$\frac{1}{2}\left(\frac{3ac+2bcx^2+adx^2}{(bc-ad)^2\,(a+bx^2)\,\sqrt{c+dx^2}}+\frac{(2bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{5/2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]

[Out] ((3*a*c + 2*b*c*x^2 + a*d*x^2)/((b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2]) + ((2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(5/2)))/2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(114) = 228$.
time = 0.09, size = 1922, normalized size = 14.34

method	result	size
default	Expression too large to display	1922

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4/b^3*(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/b^3*(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x-1/b*(-a

```
*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/2/
b^2*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)
^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(
1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b
*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2
)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+1/2/b^2
*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1
/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2
))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)
*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(114) = 228.
time = 1.61, size = 732, normalized size = 5.46

$$\left[\frac{\left((2\,b^2cd+abd^2)x^4+2\,abc^2+a^2cd+(2\,b^2c^2+3\,abcd+a^2d^2)x^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(3\,ab^2c^2-3\,a^2bcd+(2\,b^3c^2-ab^2cd-a^2bd^2)x^2)\sqrt{dx^2+c}}{8\,(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4)x^4+(b^5c^4-2\,ab^4c^3d+2\,a^3b^2cd^3-a^4bd^4)x^2)},\ -\frac{\left((2\,b^2cd+abd^2)x^4+2\,abc^2+a^2cd+(2\,b^2c^2+3\,abcd+a^2d^2)x^2\right)\sqrt{-b^2c+abd}\,\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)-2\,(3\,ab^2c^2-3\,a^2bcd+(2\,b^3c^2-ab^2cd-a^2bd^2)x^2)\sqrt{dx^2+c}}{4\,(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4)x^4+(b^5c^4-2\,ab^4c^3d+2\,a^3b^2cd^3-a^4bd^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((2*b^2*c*d + a*b*d^2)*x^4 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b
*c*d + a^2*d^2)*x^2)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a
*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d
)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*
a*b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^2)*sqrt(d*x
^2 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (
b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^4 + (b^5*c^4
```

```
 - 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^2), -1/4*(((2*b^2*c*d + a
*b*d^2)*x^4 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^2)*
sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*
d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(3*a*
b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^2)*sqrt(d*x^2
 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^
5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^4 + (b^5*c^4 -
 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^2)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
[Out] Integral(x**3/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

Giac [A]

time = 0.53, size = 181, normalized size = 1.35

$$\frac{\frac{(2\,bcd+ad^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{2\,(dx^2+c)bcd-2\,bc^2d+(dx^2+c)ad^2+2\,acd^2}{(b^2c^2-2\,abcd+a^2d^2)\left((dx^2+c)^{\frac{3}{2}}b-\sqrt{dx^2+c}\,bc+\sqrt{dx^2+c}\,ad\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] 1/2*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^2 + c)*b*c*d -
2*b*c^2*d + (d*x^2 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*c + sqrt(d*x^2 + c)*a*d)))/d
```

Mupad [B]

time = 0.62, size = 142, normalized size = 1.06

$$\frac{\frac{c}{a\,d-b\,c}+\frac{(d\,x^2+c)\,(a\,d+2\,b\,c)}{2\,(a\,d-b\,c)^2}}{b\,(d\,x^2+c)^{3/2}+\sqrt{d\,x^2+c}\,\,(a\,d-b\,c)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^{5/2}}\right)\,(a\,d+2\,b\,c)}{2\,\sqrt{b}\,\,(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
[Out] (c/(a*d - b*c) + ((c + d*x^2)*(a*d + 2*b*c))/(2*(a*d - b*c)^2))/(b*(c + d*x
^2)^(3/2) + (c + d*x^2)^(1/2)*(a*d - b*c)) + (atan((b^(1/2)*(c + d*x^2)^(1/
2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2))*(a*d + 2*b*c))/(2*b^
(1/2)*(a*d - b*c)^(5/2))
```

3.769 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{x}{2(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}}+\frac{(bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{5/2}}$$

```
[Out] 1/2*(2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/(-a*d+b*
c)^(5/2)/a^(1/2)-3/2*d*x/(-a*d+b*c)^2/(d*x^2+c)^(1/2)-1/2*x/(-a*d+b*c)/(b*x
^2+a)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {482, 541, 12, 385, 211}

$$\frac{(2ad+bc)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{5/2}}-\frac{x}{2\,(a+bx^2)\,\sqrt{c+dx^2}\,(bc-ad)}-\frac{3dx}{2\sqrt{c+dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*d*x)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - x/(2*(b*c - a*d)*(a + b*x^2)*S
qrt[c + d*x^2]) + ((b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
 + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = -\frac{x}{2(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{\int \frac{c-2dx^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{x}{2(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{\int \frac{c(bc+2ad)}{(a+bx^2)\sqrt{c+d}}}{2c(bc-ad}$$

$$= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{x}{2(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{(bc+2ad)\int \frac{}{(a}}{2(bc}$$

$$= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{x}{2(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{(bc+2ad)\text{Subs}}{}$$

$$= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{x}{2(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{(bc+2ad)\tan^{-}}{2\sqrt{a}\,(}$$

Mathematica [A]
time = 0.59, size = 133, normalized size = 1.08

$$-\frac{x(bc+2ad+3bdx^2)}{2(bc-ad)^2(a+bx^2)\sqrt{c+dx^2}} + \frac{(-bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+b\sqrt{d}\,x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{2\sqrt{a}\,(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(x*(b*c + 2*a*d + 3*b*d*x^2))/((b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^
2]) + ((-(b*c) - 2*a*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*
x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*Sqrt[a]*(b*c - a*d)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(103) = 206$.
time = 0.09, size = 1922, normalized size = 15.63

method	result	size
default	Expression too large to display	1922

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(
-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3*d*(-a*b)^(1/2)/(a*d
-b*c)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*
b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)
^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d
-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+4*d/(
a*d-b*c)*b*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+
4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))+1/4/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b
*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
-3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d
-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2
*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b
*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)
/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)
/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b*(-1/(a*d-b*c
)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*
b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(
```

```
a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-1/4/(-a*b)^(1/2)/b*(-1/
(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-
2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(103) = 206.
time = 1.98, size = 744, normalized size = 6.05

$$\left[-\frac{\left(\left(b^{2} c d + 2 \, a b d^{2}\right) x^{4} + a b c^{2} + 2 \, a^{2} c d + \left(b^{2} c^{2} + 3 \, a b c d + 2 \, a^{2} d^{2}\right) x^{2}\right) \sqrt{-a b c + a^{2} d} \log\left(\frac{\left(b^{2} c^{2} - 8 \, a b c d + 8 \, a^{2} d^{2}\right) x^{4} + a^{2} c^{2} - 2 \, \left(3 \, a b c^{2} - 4 \, a^{2} c d\right) x^{2} - 4 \, \left(\left(b c - 2 \, a d\right) x^{3} - a c x\right) \sqrt{-a b c + a^{2} d} \sqrt{d x^{2} + c}}{b^{2} x^{4} + 2 \, a b x^{2} + a^{2}}\right) + 4 \, \left(3 \, \left(a b^{2} c d - a^{2} b d^{2}\right) x^{3} + \left(a b^{2} c^{2} + a^{2} b c d - 2 \, a^{3} d^{2}\right) x\right) \sqrt{d x^{2} + c}}{8 \, \left(a^{2} b^{3} c^{4} - 3 \, a^{3} b^{2} c^{3} d + 3 \, a^{4} b c^{2} d^{2} - a^{5} c d^{3} + \left(a b^{4} c^{3} d - 3 \, a^{2} b^{3} c^{2} d^{2} + 3 \, a^{3} b^{2} c d^{3} - a^{4} b d^{4}\right) x^{4} + \left(a b^{4} c^{4} - 2 \, a^{2} b^{3} c^{3} d + 2 \, a^{4} b c d^{3} - a^{5} d^{4}\right) x^{2}\right)}, \frac{\left(\left(b^{2} c d + 2 \, a b d^{2}\right) x^{4} + a b c^{2} + 2 \, a^{2} c d + \left(b^{2} c^{2} + 3 \, a b c d + 2 \, a^{2} d^{2}\right) x^{2}\right) \sqrt{a b c - a^{2} d} \arctan\left(\frac{\sqrt{a b c - a^{2} d} \left(\left(b c - 2 \, a d\right) x^{2} - a c\right) \sqrt{d x^{2} + c}}{2 \, \left(\left(a b c d - a^{2} d^{2}\right) x^{3} + \left(a b c^{2} - a^{2} c d\right) x\right)}\right) - 2 \, \left(3 \, \left(a b^{2} c d - a^{2} b d^{2}\right) x^{3} + \left(a b^{2} c^{2} + a^{2} b c d - 2 \, a^{3} d^{2}\right) x\right) \sqrt{d x^{2} + c}}{4 \, \left(a^{2} b^{3} c^{4} - 3 \, a^{3} b^{2} c^{3} d + 3 \, a^{4} b c^{2} d^{2} - a^{5} c d^{3} + \left(a b^{4} c^{3} d - 3 \, a^{2} b^{3} c^{2} d^{2} + 3 \, a^{3} b^{2} c d^{3} - a^{4} b d^{4}\right) x^{4} + \left(a b^{4} c^{4} - 2 \, a^{2} b^{3} c^{3} d + 2 \, a^{4} b c d^{3} - a^{5} d^{4}\right) x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((b^2*c*d + 2*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (b^2*c^2 + 3*a*b*
c*d + 2*a^2*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3
 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(3*(a*b^2*c*d - a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2)*
x)*sqrt(d*x^2 + c))/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*
c*d^3 + (a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 3*a^3*b^2*c*d^3 - a^4*b*d^4)*x^4
 + (a*b^4*c^4 - 2*a^2*b^3*c^3*d + 2*a^4*b*c*d^3 - a^5*d^4)*x^2), 1/4*(((b^2
*c*d + 2*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (b^2*c^2 + 3*a*b*c*d + 2*a^2*
d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*
x)) - 2*(3*(a*b^2*c*d - a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2
)*x)*sqrt(d*x^2 + c))/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^
5*c*d^3 + (a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 3*a^3*b^2*c*d^3 - a^4*b*d^4)*x
^4 + (a*b^4*c^4 - 2*a^2*b^3*c^3*d + 2*a^4*b*c*d^3 - a^5*d^4)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(103) = 206.
time = 1.25, size = 299, normalized size = 2.43

$$-\frac{dx}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{dx^2+c}}-\frac{\left(bc\sqrt{d}+2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}}+\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)(b^2c^2-2\,abcd+a^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -d*x/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)) - 1/2*(b*c*sqrt(d) +
 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^
2*d^2)) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^
4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a*d + b*c^2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.770 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{1}{2(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

```
[Out] 3/2*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^
(5/2)-3/2*d/(-a*d+b*c)^2/(d*x^2+c)^(1/2)-1/2/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)
^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {455, 44, 53, 65, 214}

$$-\frac{3d}{2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{1}{2\,(a+bx^2)\,\sqrt{c+dx^2}\,(bc-ad)}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*d)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - 1/(2*(b*c - a*d)*(a + b*x^2)*Sqr
t[c + d*x^2]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a
*d]])/(2*(b*c - a*d)^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx,x,x^2\right)$$

$$=-\frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{(3d)\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx,x,x^2\right)}{4(bc-ad)}$$

$$=-\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{(3bd)\text{Subst}\left(\int$$

$$=-\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{(3b)\text{Subst}\left(\int$$

$$=-\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{3\sqrt{b}\,d\tanh^{-1}}{2(bc}$$

Mathematica [A]

time = 0.34, size = 102, normalized size = 0.90

$$\frac{1}{2}\left(\frac{-2ad-b(c+3dx^2)}{(bc-ad)^2\,(a+bx^2)\,\sqrt{c+dx^2}}-\frac{3\sqrt{b}\,d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((-2*a*d - b*(c + 3*d*x^2))/((b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2]) - (
3*Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) +
 a*d)^(5/2))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(93) = 186$.

time = 0.10, size = 1203, normalized size = 10.65

method	result
default	$\sqrt{-ab}\left(\frac{b}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}-\frac{3d\sqrt{-ab}\left(-\ldots\right.}{(ad-\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3*d*(-
a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(
2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(
d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^
2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4
*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/
(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1
/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x-1/b*
(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-
a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(93) = 186$.
time = 1.38, size = 537, normalized size = 4.75

$$\left[\frac{3\,(bd^2x^4 + acd + (bcd + ad^2)x^2)\sqrt{\frac{b}{bc - ad}}\log\left(\frac{b^2d^2x^4 + 8\,b^2c^2 - 8\,abcd + a^2d^2 + 2\,(4\,b^2cd - 3\,abd^2)x^2 + 4\,(2\,b^2c^2 - 3\,abcd + a^2d^2 + (b^2cd - abd^2)x^2)\sqrt{dx^2 + c}\sqrt{\frac{b}{bc - ad}}}{b^2x^4 + 2\,abx^2 + a^2}\right) - 4\,(3\,bdx^2 + bc + 2\,ad)\sqrt{dx^2 + c}}{8\,(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^4 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x^2)}, -\frac{3\,(bd^2x^4 + acd + (bcd + ad^2)x^2)\sqrt{-\frac{b}{bc - ad}}\arctan\left(\frac{(bdx^2 + 2\,bc - ad)\sqrt{dx^2 + c}\sqrt{-\frac{b}{bc - ad}}}{2\,(bdx^2 + bc)}\right) + 2\,(3\,bdx^2 + bc + 2\,ad)\sqrt{dx^2 + c}}{4\,(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^4 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((
b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x
^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x
^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*b*d*x^2 +
b*c + 2*a*d)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3
*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*
d^2 + a^3*d^3)*x^2), -1/4*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt
(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b
/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(3*b*d*x^2 + b*c + 2*a*d)*sqrt(d*x^2 + c
))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^
2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 0.50, size = 153, normalized size = 1.35

$$-\frac{3\,bd\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}-\frac{3\,(dx^2+c)bd-2\,bcd+2\,ad^2}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\left((dx^2+c)^{\frac{3}{2}}b-\sqrt{dx^2+c}\,bc+\sqrt{dx^2+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -3/2*b*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) - 1/2*(3*(d*x^2 + c)*b*d - 2*b*c*d + 2*
a*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 +
 c)*b*c + sqrt(d*x^2 + c)*a*d))
```

Mupad [B]
time = 0.56, size = 130, normalized size = 1.15

$$-\frac{\frac{d}{a\,d-b\,c}+\frac{3\,b\,d\,\left(d\,x^2+c\right)}{2\,{\left(a\,d-b\,c\right)}^2}}{b\,{\left(d\,x^2+c\right)}^{3/2}+\sqrt{d\,x^2+c}\,\left(a\,d-b\,c\right)}-\frac{3\,\sqrt{b}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)}{2\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] - (d/(a*d - b*c) + (3*b*d*(c + d*x^2))/(2*(a*d - b*c)^2))/(b*(c + d*x^2)^(3
/2) + (c + d*x^2)^(1/2)*(a*d - b*c)) - (3*b^(1/2)*d*atan((b^(1/2)*(c + d*x^
2)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2)))/(2*(a*d - b*c
)^(5/2))
```

3.771 $\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{3/2}} \, dx$

Optimal. Leaf size=142

$$\frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}}+\frac{bx}{2a(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}}+\frac{b(bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}$$

```
[Out] 1/2*b*(-4*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(5/2)+1/2*d*(2*a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)^(1/2)+1/
2*b*x/a/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {425, 541, 12, 385, 211}

$$\frac{b(bc-4ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}+\frac{dx(2ad+bc)}{2ac\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{bx}{2a\,(a+bx^2)\,\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (b*x)/(2*a*(b*c
 - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + (b*(b*c - 4*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{bx}{2a(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{\int \frac{-bc+2ad-2bdx^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{\int \frac{bc(bc-4}{(a+bx^2)\sqrt{c}}}{2ac(bc-}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{(b(bc-4ad)}{2}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{(b(bc-4ad)}{}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{b(bc-4ad)}{2a}$$

Mathematica [A]
time = 0.58, size = 154, normalized size = 1.08

$$\frac{x(2a^2d^2+2abd^2x^2+b^2c(c+dx^2))}{2ac(bc-ad)^2(a+bx^2)\sqrt{c+dx^2}} - \frac{b(bc-4ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x*(2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(c + d*x^2)))/(2*a*c*(b*c - a*d)^2*(a
 + b*x^2)*Sqrt[c + d*x^2]) - (b*(b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt
[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*(b*c - a*d
)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(122) = 244$.
time = 0.09, size = 1928, normalized size = 13.58

method	result	size
default	Expression too large to display	1928

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/a*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3*d*(-a*b)^(1/2)/(a*
d-b*c)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a
*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*
d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(
1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+4*d/
(a*d-b*c)*b*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))+1/4/a/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^
(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a
*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*
d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*
(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))/(x-1/b*(-a*b)^(1/2))))-1/4/a/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x+1/b
*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(
-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))-1/4/b/a*(1/(a*d-b*c)*b/(x-1/b*(-a*b)
^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(
```

```
a*d-b*c)/b)^(1/2)+3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*
b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*
(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*
(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x-1/b*(-a*b)^(1/2))
+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(122) = 244$.
time = 1.70, size = 854, normalized size = 6.01

$$\left[\frac{(ab^2c^3-4a^2bc^2d+(b^3c^2d-4ab^2cd^2)x^4+(b^3c^3-3ab^2c^2d-4a^2bcd^2)x^2)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)+4((ab^3c^2d+a^2b^2cd^2-2a^3bd^3)x^3+(ab^3c^3-a^2b^2c^2d+2a^3bcd^2-2a^4d^3)x)\sqrt{dx^2+c}}{8(a^3b^3c^5-3a^4b^2c^4d+3a^5bc^3d^2-a^6c^2d^3+(a^2b^4c^4d-3a^3b^3c^3d^2+3a^4b^2c^2d^3-a^5bcd^4)x^4+(a^2b^4c^5-2a^3b^3c^4d+2a^5bc^2d^3-a^6cd^4)x^2)}, \frac{(ab^2c^3-4a^2bc^2d+(b^3c^2d-4ab^2cd^2)x^4+(b^3c^3-3ab^2c^2d-4a^2bcd^2)x^2)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+2((ab^3c^2d+a^2b^2cd^2-2a^3bd^3)x^3+(ab^3c^3-a^2b^2c^2d+2a^3bcd^2-2a^4d^3)x)\sqrt{dx^2+c}}{4(a^3b^3c^5-3a^4b^2c^4d+3a^5bc^3d^2-a^6c^2d^3+(a^2b^4c^4d-3a^3b^3c^3d^2+3a^4b^2c^2d^3-a^5bcd^4)x^4+(a^2b^4c^5-2a^3b^3c^4d+2a^5bc^2d^3-a^6cd^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*x^4 + (b^3*c
^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4
*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) + 4*((a*b^3*c^2*d + a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^3 +
 (a*b^3*c^3 - a^2*b^2*c^2*d + 2*a^3*b*c*d^2 - 2*a^4*d^3)*x)*sqrt(d*x^2 + c)
)/(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3 + (a^2*b^4
*c^4*d - 3*a^3*b^3*c^3*d^2 + 3*a^4*b^2*c^2*d^3 - a^5*b*c*d^4)*x^4 + (a^2*b^
4*c^5 - 2*a^3*b^3*c^4*d + 2*a^5*b*c^2*d^3 - a^6*c*d^4)*x^2), 1/4*((a*b^2*c^
3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 3*a*b^2*c^
2*d - 4*a^2*b*c*d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d
)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b
*c^2 - a^2*c*d)*x)) + 2*((a*b^3*c^2*d + a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^3 +
(a*b^3*c^3 - a^2*b^2*c^2*d + 2*a^3*b*c*d^2 - 2*a^4*d^3)*x)*sqrt(d*x^2 + c))
/(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3 + (a^2*b^4*
```

```
c^4*d - 3*a^3*b^3*c^3*d^2 + 3*a^4*b^2*c^2*d^3 - a^5*b*c*d^4)*x^4 + (a^2*b^4
*c^5 - 2*a^3*b^3*c^4*d + 2*a^5*b*c^2*d^3 - a^6*c*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^2\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(122) = 244.
time = 1.23, size = 318, normalized size = 2.24

$$\frac{d^2x}{(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{dx^2+c}}-\frac{\left(b^2c\sqrt{d}-4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abd^{\frac{3}{2}}-b^2c^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)(ab^2c^2-2\,a^2bcd+a^3d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] d^2*x/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(d*x^2 + c)) - 1/2*(b^2*c*sq
rt(d) - 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt
(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*sqrt(d) - 2*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(((sqrt(d)*x
- sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.772 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}}+\frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}+\frac{b^{3/2}(2bc-5ad)\tanh^{-1}}{2a^2(bc-}$$

[Out] -arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(3/2)+1/2*b^(3/2)*(-5*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(5/2)+1/2*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/(d*x^2+c)^(1/2)+1/2*b/a/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(1/2)

Rubi [A]

time = 0.15, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 105, 157, 162, 65, 214}

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}+\frac{b}{2a\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{d(2ad+bc)}{2ac\sqrt{c+dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]

[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + b/(2*a*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a^2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(5/2))

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\,(a+bx^2)^2\,(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b}{2a(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\right.}{a^2}$$

Mathematica [A]
time = 0.77, size = 157, normalized size = 0.92

$$\frac{\frac{a(2a^2d^2+2abd^2x^2+b^2c(c+dx^2))}{c(bc-ad)^2(a+bx^2)\sqrt{c+dx^2}} - \frac{b^{3/2}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((a*(2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(c + d*x^2)))/(c*(b*c - a*d)^2*(a +
b*x^2)*Sqrt[c + d*x^2]) - (b^(3/2)*(2*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) - (2*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]])/c^(3/2))/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1967 vs. $2(144) = 288$.
time = 0.10, size = 1968, normalized size = 11.58

method	result	size

default	Expression too large to display	1968

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/4/a/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/2/a^2*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-1/2/a^2*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/a^2*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))-1/4/a/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(144) = 288$.
time = 3.55, size = 1992, normalized size = 11.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 +
 (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^2)*sqrt(b/(b*c - a*d))*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)
*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d
*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(c)*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^
2*c^2*d + 2*a^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d
 + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 +
(a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2), 1/8*(8*(a
*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d
^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(-c)*arc
tan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d
 - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^2
)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 +
2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c
*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^
2 + a^2)) + 4*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^2)
*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4
*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d
- a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2), -1/4*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (
2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c
^2*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d
*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2*(a*b^2*c^3 - 2*a^2*b*c^
2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 -
```

```
a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^
2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*
a^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d
^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 + (a^2*b^3*c^5
 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2), -1/4*((2*a*b^2*c^4 -
5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*
c^3*d - 5*a^2*b*c^2*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*
b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 4*(a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*
x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(-c)*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) - 2*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a
^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^
2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 + (a^2*b^3*c^5
- a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 0.56, size = 225, normalized size = 1.32

$$-\frac{\left(2\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{-b^2c+abd}}+\frac{(dx^2+c)b^2cd+2\,(dx^2+c)abd^2-2\,abcd^2+2\,a^2d^3}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)\left((dx^2+c)^{\frac{3}{2}}b-\sqrt{dx^2+c}\,bc+\sqrt{dx^2+c}\,ad\right)}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/2*((d*x^2 +
 c)*b^2*c*d + 2*(d*x^2 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*c + s
qrt(d*x^2 + c)*a*d)) + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c)
```

Mupad [B]
time = 1.90, size = 2500, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)

[Out] atanh((240*a^3*b^11*c^11*d^4*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c *d^13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^ 6 + 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) - (2080*a^4*b^10*c^10*d^5*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^ 13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10 160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) + (7760*a^5*b^9*c^9*d^6*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 1638 4*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a ^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) - (16384 *a^6*b^8*c^8*d^7*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240* a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^ 6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b ^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) + (21584*a^7 *b^7*c^7*d^8*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3* b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^ 8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c ^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) - (18400*a^8*b^6 *c^6*d^9*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11 *c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^ 7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d ^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) + (10160*a^9*b^5*c^5 *d^10*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11*c^ 10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^7*d ^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) - (3520*a^10*b^4*c^4*d^ 11*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11*c^10* d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) + (704*a^11*b^3*c^3*d^12*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^7*d^7 - 21 584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d^10 + 3520 *a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) - (64*a^12*b^2*c^2*d^13*(c + d *x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11*c^10*d^4 + 208 0*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^7*d^7 - 21584*a ^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10 *b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)))/(a^2*(c^3)^(1/2)) - (d^2/(b*c^2 - a*c*d) + (d*(c + d*x^2)*(b^2*c + 2*a*b*d))/(2*a*(b*c^2 - a*c*d)*(a*d - b*c)

))/(b*(c + d*x^2)^(3/2) + (c + d*x^2)^(1/2)*(a*d - b*c)) - (atan((((-b^3*(a*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*((c + d*x^2)^(1/2)*(128*a^3*b^13*c^13*d^2 - 1344*a^4*b^12*c^12*d^3 + 6160*a^5*b^11*c^11*d^4 - 16160*a^6*b^10*c^10*d^5 + 26800*a^7*b^9*c^9*d^6 - 29312*a^8*b^8*c^8*d^7 + 21424*a^9*b^7*c^7*d^8 - 10400*a^10*b^6*c^6*d^9 + 3280*a^11*b^5*c^5*d^10 - 640*a^12*b^4*c^4*d^11 + 64*a^13*b^3*c^3*d^12) + ((-b^3*(a*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*(64*a^6*b^12*c^14*d^3 - 896*a^7*b^11*c^13*d^4 + 4992*a^8*b^10*c^12*d^5 - 15360*a^9*b^9*c^11*d^6 + 29568*a^10*b^8*c^10*d^7 - 37632*a^11*b^7*c^9*d^8 + 32256*a^12*b^6*c^8*d^9 - 18432*a^13*b^5*c^7*d^10 + 6720*a^14*b^4*c^6*d^11 - 1408*a^15*b^3*c^5*d^12 + 128*a^16*b^2*c^4*d^13 - ((-b^3*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 2*b*c)*(512*a^7*b^13*c^16*d^2 - 5376*a^8*b^12*c^15*d^3 + 25600*a^9*b^11*c^14*d^4 - 72960*a^10*b^10*c^13*d^5 + 138240*a^11*b^9*c^12*d^6 - 182784*a^12*b^8*c^11*d^7 + 172032*a^13*b^7*c^10*d^8 - 115200*a^14*b^6*c^9*d^9 + 53760*a^15*b^5*c^8*d^10 - 16640*a^16*b^4*c^7*d^11 + 3072*a^17*b^3*c^6*d^12 - 256*a^18*b^2*c^5*d^13))/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4))))/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4)))*1i)/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4)) + ((-b^3*(a*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*((c + d*x^2)^(1/2)*(128*a^3*b^13*c^13*d^2 - 1344*a^4*b^12*c^12*d^3 + 6160*a^5*b^11*c^11*d^4 - 16160*a^6*b^10*c^10*d^5 + 26800*a^7*b^9*c^9*d^6 - 29312*a^8*b^8*c^8*d^7 + 21424*a^9*b^7*c^7*d^8 - 10400*a^10*b^6*c^6*d^9 + 3280*a^11*b^5*c^5*d^10 - 640*a^12*b^4*c^4*d^11 + 64*a^13*b^3*c^3*d^12) - ((-b^3*(a*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*(64*a^6*b^12*c^14*d^3 - 896*a^7*b^11*c^13*d^4 + 4992*a^8*b^10*c^12*d^5 - 15360*a^9*b^9*c^11*d^6 + 29568*a^10*b^8*c^10*d^7 - 37632*a^...

3.773 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=205

$$\frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}}+\frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{\left(3b^2c^2-4abcd+4a^2d^2\right)\sqrt{c+dx^2}}{2a^2c^2(bc-ad)^2x}-\frac{3b^2(bc$$

```
[Out] -3/2*b^2*(-2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^
(5/2)/(-a*d+b*c)^(5/2)+1/2*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/x/(d*x^2+c)^(1/2)
+1/2*b/a/(-a*d+b*c)/x/(b*x^2+a)/(d*x^2+c)^(1/2)-1/2*(4*a^2*d^2-4*a*b*c*d+3*
b^2*c^2)*(d*x^2+c)^(1/2)/a^2/c^2/(-a*d+b*c)^2/x
```

Rubi [A]
time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {483, 593, 597, 12, 385, 211}

$$-\frac{3b^2(bc-2ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\left(4a^2d^2-4abcd+3b^2c^2\right)}{2a^2c^2x(bc-ad)^2}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2acx\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x*Sqrt[c + d*x^2]) + b/(2*a*(b*c - a
*d)*x*(a + b*x^2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 4*a*b*c*d + 4*a^2*d^2)*S
qrt[c + d*x^2])/(2*a^2*c^2*(b*c - a*d)^2*x) - (3*b^2*(b*c - 2*a*d)*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(5/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{b}{2a(bc-ad)x(a+bx^2)\sqrt{c+dx^2}} - \frac{\int \frac{-3bc+2ad-4bdx^2}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2 x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x(a+bx^2)\sqrt{c+dx^2}} - \frac{\int \frac{-3b^2c}{\ldots}}{\ldots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2 x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x(a+bx^2)\sqrt{c+dx^2}} - \frac{(3b^2c^2}{\ldots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2 x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x(a+bx^2)\sqrt{c+dx^2}} - \frac{(3b^2c^2}{\ldots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2 x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x(a+bx^2)\sqrt{c+dx^2}} - \frac{(3b^2c^2}{\ldots}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2 x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x(a+bx^2)\sqrt{c+dx^2}} - \frac{(3b^2c^2}{\ldots}$$

Mathematica [A]
time = 0.95, size = 216, normalized size = 1.05

$$\frac{-3b^3c^2x^2(c+dx^2) - 2a^3d^2(c+2dx^2) + 2a^2bd(2c^2+cdx^2-2d^2x^4) + 2ab^2c(-c^2+cdx^2+2d^2x^4)}{2a^2c^2(bc-ad)^2x(a+bx^2)\sqrt{c+dx^2}} + \frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*b^3*c^2*x^2*(c + d*x^2) - 2*a^3*d^2*(c + 2*d*x^2) + 2*a^2*b*d*(2*c^2 +
c*d*x^2 - 2*d^2*x^4) + 2*a*b^2*c*(-c^2 + c*d*x^2 + 2*d^2*x^4))/(2*a^2*c^2*(
b*c - a*d)^2*x*(a + b*x^2)*Sqrt[c + d*x^2]) + (3*b^2*(b*c - 2*a*d)*ArcTan[(
a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/
(2*a^(5/2)*(b*c - a*d)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1961 vs. $2(181) = 362$.
time = 0.15, size = 1962, normalized size = 9.57

method	result	size
risch	Expression too large to display	1712

default	Expression too large to display	1962

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/4/a^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+1/4/a^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-3/4*b/a^2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+3/4*b/a^2/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/a^2*(-1/c/x/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(181) = 362$.
time = 1.88, size = 1018, normalized size = 4.97

$$\left[\frac{3\left((b^4c^3d-2ab^3c^2d^2)x^5+(b^4c^4-ab^3c^3d-2a^2b^2c^2d^2)x^3+(ab^3c^4-2a^2b^2c^3d)x\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-4\left(2a^2b^3c^4-6a^3b^2c^3d+6a^4bc^2d^2-2a^5cd^3+(3ab^4c^3d-7a^2b^3c^2d^2+8a^3b^2cd^3-4a^4bd^4)x^4+(3ab^4c^4-5a^2b^3c^3d+6a^4bcd^3-4a^5d^4)x^2\right)\sqrt{dx^2+c}}{8\left((a^3b^4c^5d-3a^4b^3c^4d^2+3a^5b^2c^3d^3-a^6bc^2d^4)x^5+(a^3b^4c^6-2a^4b^3c^5d+2a^6bc^3d^3-a^7c^2d^4)x^3+(a^4b^3c^6-3a^5b^2c^5d+3a^6bc^4d^2-a^7c^3d^3)x\right)}, -\frac{3\left((b^4c^3d-2ab^3c^2d^2)x^5+(b^4c^4-ab^3c^3d-2a^2b^2c^2d^2)x^3+(ab^3c^4-2a^2b^2c^3d)x\right)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}\left((bc-2ad)x^2-ac\right)\sqrt{dx^2+c}}{2\left((abcd-a^2d^2)x^3+(abc^2-a^2cd)x\right)}\right)+2\left(2a^2b^3c^4-6a^3b^2c^3d+6a^4bc^2d^2-2a^5cd^3+(3ab^4c^3d-7a^2b^3c^2d^2+8a^3b^2cd^3-4a^4bd^4)x^4+(3ab^4c^4-5a^2b^3c^3d+6a^4bcd^3-4a^5d^4)x^2\right)\sqrt{dx^2+c}}{4\left((a^3b^4c^5d-3a^4b^3c^4d^2+3a^5b^2c^3d^3-a^6bc^2d^4)x^5+(a^3b^4c^6-2a^4b^3c^5d+2a^6bc^3d^3-a^7c^2d^4)x^3+(a^4b^3c^6-3a^5b^2c^5d+3a^6bc^4d^2-a^7c^3d^3)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^4*c^3*d - 2*a*b^3*c^2*d^2)*x^5 + (b^4*c^4 - a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2)*x^3 + (a*b^3*c^4 - 2*a^2*b^2*c^3*d)*x)*sqrt(-a*b*c + a^2*d)*l
og(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*
c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 +
c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^2*b^3*c^4 - 6*a^3*b^2*c^3*d + 6*a
^4*b*c^2*d^2 - 2*a^5*c*d^3 + (3*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 8*a^3*b^2
*c*d^3 - 4*a^4*b*d^4)*x^4 + (3*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 6*a^4*b*c*d^3
- 4*a^5*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^5*d - 3*a^4*b^3*c^4*d^2 + 3*
a^5*b^2*c^3*d^3 - a^6*b*c^2*d^4)*x^5 + (a^3*b^4*c^6 - 2*a^4*b^3*c^5*d + 2*a
^6*b*c^3*d^3 - a^7*c^2*d^4)*x^3 + (a^4*b^3*c^6 - 3*a^5*b^2*c^5*d + 3*a^6*b*
c^4*d^2 - a^7*c^3*d^3)*x), -1/4*(3*((b^4*c^3*d - 2*a*b^3*c^2*d^2)*x^5 + (b^
4*c^4 - a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2)*x^3 + (a*b^3*c^4 - 2*a^2*b^2*c^3*d
)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2
- a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) +
 2*(2*a^2*b^3*c^4 - 6*a^3*b^2*c^3*d + 6*a^4*b*c^2*d^2 - 2*a^5*c*d^3 + (3*a*
b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 8*a^3*b^2*c*d^3 - 4*a^4*b*d^4)*x^4 + (3*a*b
^4*c^4 - 5*a^2*b^3*c^3*d + 6*a^4*b*c*d^3 - 4*a^5*d^4)*x^2)*sqrt(d*x^2 + c))
/((a^3*b^4*c^5*d - 3*a^4*b^3*c^4*d^2 + 3*a^5*b^2*c^3*d^3 - a^6*b*c^2*d^4)*x
^5 + (a^3*b^4*c^6 - 2*a^4*b^3*c^5*d + 2*a^6*b*c^3*d^3 - a^7*c^2*d^4)*x^3 +
(a^4*b^3*c^6 - 3*a^5*b^2*c^5*d + 3*a^6*b*c^4*d^2 - a^7*c^3*d^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(181) = 362$.

time = 1.38, size = 554, normalized size = 2.70

$$-\frac{d^3x}{(b^2c^4-2\,abc^3d+a^2c^2d^2)\sqrt{dx^2+c}}+\frac{3\left(b^3c\sqrt{d}-2\,ab^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{abcd-a^2d^2}}+\frac{3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b^3c^2\sqrt{d}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4ab^2cd^{\frac{3}{2}}+2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2bd^{\frac{5}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^3c^3\sqrt{d}+18\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ab^2c^2d^{\frac{3}{2}}-20\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2bcd^{\frac{5}{2}}+8\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^3d^{\frac{7}{2}}+3\,b^3c^4\sqrt{d}-4\,ab^2c^3d^{\frac{3}{2}}+2\,a^2bc^2d^{\frac{5}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6b-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4ad+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc^2-4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2acd-bc^3\right)\left(a^2b^2c^3-2\,a^3bc^2d+a^4cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -d^3*x/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(d*x^2 + c)) + 3/2*(b^3*c
*sqrt(d) - 2*a*b^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^
2)*sqrt(a*b*c*d - a^2*d^2)) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^3*c^2*sq
rt(d) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b^2*c*d^(3/2) + 2*(sqrt(d)*x -
sqrt(d*x^2 + c))^4*a^2*b*d^(5/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^
3*sqrt(d) + 18*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) - 20*(sqrt
(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c*d^(5/2) + 8*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*a^3*d^(7/2) + 3*b^3*c^4*sqrt(d) - 4*a*b^2*c^3*d^(3/2) + 2*a^2*b*c^2*d^(
5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*(a^2*b^2*c
^3 - 2*a^3*b*c^2*d + a^4*c*d^2))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.774 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{d(2b^2c^2-2abcd+3a^2d^2)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}}-\frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{(4bc+3ad)}{}$$

```
[Out] 1/2*(3*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^3/c^(5/2)-1/2*b^(5/2)*
(-7*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(5/2)-1/2*d*(3*a^2*d^2-2*a*b*c*d+2*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(d*x^
2+c)^(1/2)-1/2*b*(-a*d+2*b*c)/a^2/c/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(1/2)-1/
2/a/c/x^2/(b*x^2+a)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 105, 156, 157, 162, 65, 214}

$$-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{5/2}}-\frac{d(3a^2d^2-2abcd+2b^2c^2)}{2a^2c^2\sqrt{c+dx^2}(bc-ad)^2}-\frac{b(2bc-ad)}{2a^2c\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}-\frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sqrt[c
+ d*x^2]) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x
^2]) - 1/(2*a*c*x^2*(a + b*x^2)*Sqrt[c + d*x^2]) + ((4*b*c + 3*a*d)*ArcTanh
[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{3/2}}\, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{3/2}}\, dx, x, x^2\right)$$

$$= -\frac{1}{2acx^2 (a+bx^2) \sqrt{c+dx^2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+3ad)+\frac{5bdx}{2}}{x(a+bx)^2(c+dx)^{3/2}}\, dx, x, x^2\right)}{2ac}$$

$$= -\frac{b(2bc-ad)}{2a^2c(bc-ad) (a+bx^2) \sqrt{c+dx^2}} - \frac{1}{2acx^2 (a+bx^2) \sqrt{c+dx^2}} - \frac{\text{Sub}}{}$$

$$= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad) (a+bx^2) \sqrt{c+dx^2}} - \frac{}{2ac}$$

$$= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad) (a+bx^2) \sqrt{c+dx^2}} - \frac{}{2ac}$$

$$= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad) (a+bx^2) \sqrt{c+dx^2}} - \frac{}{2ac}$$

$$= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad) (a+bx^2) \sqrt{c+dx^2}} - \frac{}{2ac}$$

Mathematica [A]
time = 0.96, size = 223, normalized size = 0.93

$$\frac{-\frac{a(2b^3c^2x^2(c+dx^2)+a^3d^2(c+3dx^2)+ab^2c(c^2-cdx^2-2d^2x^4)+a^2bd(-2c^2-cdx^2+3d^2x^4))}{c^2(bc-ad)^2x^2(a+bx^2)\sqrt{c+dx^2}}+\frac{b^{5/2}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}+\frac{(4bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-((a*(2*b^3*c^2*x^2*(c + d*x^2) + a^3*d^2*(c + 3*d*x^2) + a*b^2*c*(c^2 - c
*d*x^2 - 2*d^2*x^4) + a^2*b*d*(-2*c^2 - c*d*x^2 + 3*d^2*x^4)))/(c^2*(b*c -
a*d)^2*x^2*(a + b*x^2)*Sqrt[c + d*x^2])) + (b^(5/2)*(4*b*c - 7*a*d)*ArcTan[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) + ((4*b
*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2))/(2*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2041 vs. $2(209) = 418$.
time = 0.17, size = 2042, normalized size = 8.47

method	result	size
risch	Expression too large to display	1764
default	Expression too large to display	2042

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/a^2/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3*d*(
-a*b)^(1/2)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*
(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/
(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))
^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b
)^(1/2))))+4*d/(a*d-b*c)*b*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-
4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))+b/a^3*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(
-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(
a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+b/a^3*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/
2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)
^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b
*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+
1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1
/2))/(x+1/b*(-a*b)^(1/2))))+1/a^2*(-1/2/c/x^2/(d*x^2+c)^(1/2)-3/2*d/c*(1/c/
(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)))+1/4*b/a^2
/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3*d*(-a*b)^(1/2
)/(a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/
b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+
1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))
```

```
+4*d/(a*d-b*c)*b*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b
*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))-2/a^3*b*(1/c/(d*x^2+c)^(1/2)-1/c^(3/2)*ln((2*c+
2*c^(1/2)*(d*x^2+c)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 591 vs. 2(209) = 418.
time = 6.46, size = 2554, normalized size = 10.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
7*a^2*b^2*c^3*d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(b/(b*c -
 a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x
^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(
(4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*b^
4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 + (4
*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(c)*
log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*b^2*c^4 - 2*a^
3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^
3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*s
qrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^6 +
(a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^4 + (a^4*b^2*
c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), -1/8*(4*((4*b^4*c^3*d - 5*a*b^3*c^
2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a
^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 + (4*a*b^3*c^4 - 5*a^2*b^2*c^
3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x
^2 + c)) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*
d - 7*a^2*b^2*c^3*d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(b/(b
*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*
d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^
2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
```

```
 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*
c^2*d^2 + 3*a^3*b*c*d^3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2
 + 3*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 +
 a^5*b*c^3*d^3)*x^6 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^
3*d^3)*x^4 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), 1/4*(((4*b^4
*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*
d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan
(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2
+ b*c)) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*
x^6 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^
4)*x^4 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^
2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a^2*b^2
*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3
*a^3*b*c*d^3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*
d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*
d^3)*x^6 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^4
+ (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), 1/4*(((4*b^4*c^4*d - 7*
a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^4 +
(4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x
^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2
*((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*
b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 +
(4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(-
c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*
c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^4 + (2*a*b^
3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/
((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^6 + (a^3*b^3*c^6 - a
^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^4 + (a^4*b^2*c^6 - 2*a^5*b*c^
5*d + a^6*c^4*d^2)*x^2)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^2)^2 (c + dx^2)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

Giac [A]

time = 0.53, size = 367, normalized size = 1.52

$$\frac{(4\,b^4c - 7\,ab^3d)\arctan\left(\frac{\sqrt{dx^2 + c}\,b}{\sqrt{-b^2c + abd}}\right)}{2\,(a^3b^2c^2 - 2\,a^4bcd + a^5d^2)\sqrt{-b^2c + abd}} - \frac{2\,(dx^2 + c)^2b^3c^2d - 2\,(dx^2 + c)b^3c^3d - 2\,(dx^2 + c)^2ab^2cd^2 + 3\,(dx^2 + c)ab^2c^2d^2 + 3\,(dx^2 + c)^2a^2bd^3 - 7\,(dx^2 + c)a^2bcd^3 + 2\,a^2bc^2d^3 + 3\,(dx^2 + c)a^3d^4 - 2\,a^3cd^4}{2\,(a^2b^2c^4 - 2\,a^3bc^3d + a^4c^2d^2)\left((dx^2 + c)^{\frac{5}{2}}b - 2\,(dx^2 + c)^{\frac{3}{2}}bc + \sqrt{dx^2 + c}\,bc^2 + (dx^2 + c)^{\frac{3}{2}}ad - \sqrt{dx^2 + c}\,acd\right)} - \frac{(4\,bc + 3\,ad)\arctan\left(\frac{\sqrt{dx^2 + c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")

[Out] 1/2*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - 1/2*(2*(d*x^2 + c)^2*b^3*c^2*d - 2*(d*x^2 + c)*b^3*c^3*d - 2*(d*x^2 + c)^2*a*b^2*c*d^2 + 3*(d*x^2 + c)*a*b^2*c^2*d^2 + 3*(d*x^2 + c)^2*a^2*b*d^3 - 7*(d*x^2 + c)*a^2*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^2 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*((d*x^2 + c)^(5/2)*b - 2*(d*x^2 + c)^(3/2)*b*c + sqrt(d*x^2 + c)*b*c^2 + (d*x^2 + c)^(3/2)*a*d - sqrt(d*x^2 + c)*a*c*d)) - 1/2*(4*b*c + 3*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c^2)

Mupad [B]
time = 2.75, size = 2500, normalized size = 10.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)

[Out] (atan((((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*((c + d*x^2)^(1/2)*(512*a^6*b^15*c^18*d^2 - 4608*a^7*b^14*c^17*d^3 + 17824*a^8*b^13*c^16*d^4 - 38144*a^9*b^12*c^15*d^5 + 47680*a^10*b^11*c^14*d^6 - 31808*a^11*b^10*c^13*d^7 + 4624*a^12*b^9*c^12*d^8 + 8032*a^13*b^8*c^11*d^9 - 3536*a^14*b^7*c^10*d^10 - 2560*a^15*b^6*c^9*d^11 + 2896*a^16*b^5*c^8*d^12 - 1056*a^17*b^4*c^7*d^13 + 144*a^18*b^3*c^6*d^14) + ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*(128*a^10*b^13*c^19*d^3 - 1216*a^11*b^12*c^18*d^4 + 4800*a^12*b^11*c^17*d^5 - 9792*a^13*b^10*c^16*d^6 + 9216*a^14*b^9*c^15*d^7 + 2688*a^15*b^8*c^14*d^8 - 18816*a^16*b^7*c^13*d^9 + 24960*a^17*b^6*c^12*d^10 - 18048*a^18*b^5*c^11*d^11 + 7744*a^19*b^4*c^10*d^12 - 1856*a^20*b^3*c^9*d^13 + 192*a^21*b^2*c^8*d^14 - ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(7*a*d - 4*b*c)*(512*a^12*b^13*c^21*d^2 - 5376*a^13*b^12*c^20*d^3 + 25600*a^14*b^11*c^19*d^4 - 72960*a^15*b^10*c^18*d^5 + 138240*a^16*b^9*c^17*d^6 - 182784*a^17*b^8*c^16*d^7 + 172032*a^18*b^7*c^15*d^8 - 115200*a^19*b^6*c^14*d^9 + 53760*a^20*b^5*c^13*d^10 - 16640*a^21*b^4*c^12*d^11 + 3072*a^22*b^3*c^11*d^12 - 256*a^23*b^2*c^10*d^13))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4)))*1i)/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4)) + ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*((c + d*x^2)^(1/2)*(512*a^6*b^15*c^18*d^2 - 4608*a^7*b^14*c^17*d^3 + 17824*a^8*b^13*c^16*d^4 - 38144*a^9*b^12*c^15*d^5 + 47680*a^10*b^11*c^14*d^6 - 31808*a^11*b^10*c^13*d^7 + 4624*a^12*b^9*c^12*d^8 + 8032*a^13*b^8*c^11*d^9 - 3536*a^14*b^7*c^10*d^10 - 2560*a^15*b^6*c^9*d^11 + 2896*a^16*b^5*c^8*d^12

- 1056*a^17*b^4*c^7*d^13 + 144*a^18*b^3*c^6*d^14) - ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*(128*a^10*b^13*c^19*d^3 - 1216*a^11*b^12*c^18*d^4 + 4800*a^12*b^11*c^17*d^5 - 9792*a^13*b^10*c^16*d^6 + 9216*a^14*b^9*c^15*d^7 + 2688*a^15*b^8*c^14*d^8 - 18816*a^16*b^7*c^13*d^9 + 24960*a^17*b^6*c^12*d^10 - 18048*a^18*b^5*c^11*d^11 + 7744*a^19*b^4*c^10*d^12 - 1856*a^20*b^3*c^9*d^13 + 192*a^21*b^2*c^8*d^14 + ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(7*a*d - 4*b*c)*(512*a^12*b^13*c^21*d^2 - 5376*a^13*b^12*c^20*d^3 + 25600*a^14*b^11*c^19*d^4 - 72960*a^15*b^10*c^18*d^5 + 138240*a^16*b^9*c^17*d^6 - 182784*a^17*b^8*c^16*d^7 + 172032*a^18*b^7*c^15*d^8 - 115200*a^19*b^6*c^14*d^9 + 53760*a^20*b^5*c^13*d^10 - 16640*a^21*b^4*c^12*d^11 + 3072*a^22*b^3*c^11*d^12 - 256*a^23*b^2*c^10*d^13))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4)))*1i)/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4)))/(((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*((c + d*x^2)^(1/2)*(512*a^6*b^15*c^18*d^2 - 4608*a^7*b^14*c^17*d^3 + 17824*a^8*b^13*c^16*d^4 - 38144*a^9*b^12*c^15*d^5 + 47680*a^10*b^11*c^14*d^6 - 31808*a^11*b^10*c^13*d^7 + 4624*a^12*b^9*c^12*d^8 + 8032*a^13*b^8*c^11*d^9 - 3536*a^14*b^7*c^10*d^10 - 2560*a^15*b^6*c^9*d^11 + 2896*a^16*b^5*c^8*d^12 - 1056*a^17*b^4*c^7*d^13 + 144*a^18*b^3*c^6*d^14) - ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*(128*a^10*b^13*c^19*d^3 - 1216*a^11*b^12*c^18*d^4 + 4800*a^12*b^11*c^17*d^5 - 9792*a^13*b^10*c^16*d^6 + 9216*a^14*b^9*c^15*d^7 + 2688*a^15*b^8*c^14*d^8 - 18816*a^16*b^7*c^13*d^9 + 24960*a^17*b^6*c^12*d^10 - 18048*a^18*b^5*c^11*d^11 + 7744*a^19*b^4*c^10*d^12 - 1856*a^20*b^3*c^9*d^13 + 192*a^21*b^2*c^8*d^14 + ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(7*a*d - 4*b*c)*(512*a^12*b^13*c^21*d^2 - 5376*a^13*b^12*c^20*d^3 + 25600*a^14*b^11*c^19*d^4 - 72960*a^15*b^10*c^18*d^5 + 138240*a^16*b^9*c^17*d^6 - 182784*a^17*b^8*c^16*d^7 + 172032*a^18*b^7*c^15*d^8 - 115200*a^19*b^6*c^14*d^9 + 53760*a^20*b^5*c^13*d^10 - 16640*a^21*b^4*c^12*d^11 + 3072*a^22*b^3*c^11*d^12 - 256*a^23*b^2*c^10*d^13))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4)) - ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*((c + d*x^2)^(1/2)*(512*a^6*b^15*c^18*d^2 - 4608*a^7*b^14*c^17*d^3 + 17824*a^8*b^13*c^16*d^4 - 38144*a^9*b^12*c^15*d^5 + 47680*a^10*b^11*c^14*d^6 - 31808*a^11*b^10*c^13*d^7 + 4624*a^12*b^9*c^12*d^8 + 8032*a^13*b^8*c^11*d^9 - 3536*a^14*b^7*c^10*d^10 - 2560*a^15*b^6*c^9*d^11 + 2896*a^16*b^5*c^8*d^12 - 1056*a^17*b^4*c^7*d^13 + 144*a^18*b^3*c^6*d^14) + ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*(128*a^10*b^13*c^19*d^3 - 1216*a^11*b^12*c^18*d^4 + 4800*a^12*b^11*c^17*d^5 - 9792*a^13*b^10*c^16*d^6 + 9216*a^14*b^9*c^15*d^7 + 2688*a^15*b^8*c^14*d^8 - 18816*a^16*b^7*c^13*d^9 + 24960*a^17*b^6*c^12*d^10...

3.775 $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}}+\frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{\left(5b^2c^2-4abcd+8a^2d^2\right)\sqrt{c+dx^2}}{6a^2c^2(bc-ad)^2x^3}+\frac{(15b$$

```
[Out] 1/2*b^3*(-8*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a
^(7/2)/(-a*d+b*c)^(5/2)+1/2*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/x^3/(d*x^2+c)^(1
/2)+1/2*b/a/(-a*d+b*c)/x^3/(b*x^2+a)/(d*x^2+c)^(1/2)-1/6*(8*a^2*d^2-4*a*b*c
*d+5*b^2*c^2)*(d*x^2+c)^(1/2)/a^2/c^2/(-a*d+b*c)^2/x^3+1/6*(16*a^3*d^3-8*a^
2*b*c*d^2-14*a*b^2*c^2*d+15*b^3*c^3)*(d*x^2+c)^(1/2)/a^3/c^3/(-a*d+b*c)^2/x
```

Rubi [A]

time = 0.26, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {483, 593, 597, 12, 385, 211}

$$\frac{b^3(5bc-8ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-4abcd+5b^2c^2\right)}{6a^2c^2x^3(bc-ad)^2}+\frac{\sqrt{c+dx^2}\left(16a^3d^3-8a^2bcd^2-14ab^2c^2d+15b^3c^3\right)}{6a^3c^3x(bc-ad)^2}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2acx^3\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x^3*Sqrt[c + d*x^2]) + b/(2*a*(b*c -
 a*d)*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 4*a*b*c*d + 8*a^2*d^
2)*Sqrt[c + d*x^2])/(6*a^2*c^2*(b*c - a*d)^2*x^3) + ((15*b^3*c^3 - 14*a*b^2
*c^2*d - 8*a^2*b*c*d^2 + 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^3*c^3*(b*c - a*d
)^2*x) + (b^3*(5*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-5bc+2ad-6bdx^2}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-5b}{}}{}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(5b^2c}{}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(5b^2c}{}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(5b^2c}{}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(5b^2c}{}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(5b^2c}{}$$

Mathematica [A]
time = 1.45, size = 265, normalized size = 0.96

$$\frac{15b^4c^3x^4(c+dx^2) - 2a^2b^2c(c+dx^2)^2\left(c+4dx^2\right) + 2ab^3c^2x^2(5c^2-2cdx^2-7d^2x^4) + 2a^4d^2(-c^2+4cdx^2+8d^2x^4) + 2a^3bd(2c^3-3c^2dx^2+8d^3x^6)}{6a^3c^3(bc-ad)^2x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{7/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (15*b^4*c^3*x^4*(c + d*x^2) - 2*a^2*b^2*c*(c + d*x^2)^2*(c + 4*d*x^2) + 2*a
*b^3*c^2*x^2*(5*c^2 - 2*c*d*x^2 - 7*d^2*x^4) + 2*a^4*d^2*(-c^2 + 4*c*d*x^2
+ 8*d^2*x^4) + 2*a^3*b*d*(2*c^3 - 3*c^2*d*x^2 + 8*d^3*x^6))/(6*a^3*c^3*(b*c
 - a*d)^2*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) - (b^3*(5*b*c - 8*a*d)*ArcTan[(a
*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(
2*a^(7/2)*(b*c - a*d)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2031 vs. 2(249) = 498.
time = 0.20, size = 2032, normalized size = 7.34

method	result	size
risch	Expression too large to display	1730
default	Expression too large to display	2032

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/a^3*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-3*d*(-a*b)^(1/2)/(
a*d-b*c)*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(
-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(
a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+4*
d/(a*d-b*c)*b*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)
/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2))+5/4*b^2/a^3/(-a*b)^(1/2)*(-1/(a*d-b*c)*b/(d*(x-1/b
*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(
-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))-5/4*b^2/a^3/(-a*b)^(1/2)*(-1/(a*d-b*
c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a
*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-
(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))))+1/a^2*(-1/3/c/x^3/(d*x
^2+c)^(1/2)-4/3*d/c*(-1/c/x/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2)))-1/4
*b/a^3*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-
a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+3*d*(-a*b)^(1/2)/(a*d-
b*c)*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b
)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^
(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-
b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-
a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2))))+4*d/(a
```

```
*d-b*c)*b*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4
*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
-(a*d-b*c)/b)^(1/2))-2/a^3*b*(-1/c/x/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1
/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 606 vs. 2(249) = 498.
time = 2.00, size = 1252, normalized size = 4.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((5*b^5*c^4*d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*a*b^4*c^4*d
- 8*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c^4*d)*x^3)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sq
rt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^3*b^3*c^5 - 6*a^4*b^2*
c^4*d + 6*a^5*b*c^3*d^2 - 2*a^6*c^2*d^3 - (15*a*b^5*c^4*d - 29*a^2*b^4*c^3*
d^2 + 6*a^3*b^3*c^2*d^3 + 24*a^4*b^2*c*d^4 - 16*a^5*b*d^5)*x^6 - (15*a*b^5*
c^5 - 19*a^2*b^4*c^4*d - 14*a^3*b^3*c^3*d^2 + 18*a^4*b^2*c^2*d^3 + 16*a^5*b
*c*d^4 - 16*a^6*d^5)*x^4 - 2*(5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*
c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^4*b^4*c^
6*d - 3*a^5*b^3*c^5*d^2 + 3*a^6*b^2*c^4*d^3 - a^7*b*c^3*d^4)*x^7 + (a^4*b^4
*c^7 - 2*a^5*b^3*c^6*d + 2*a^7*b*c^4*d^3 - a^8*c^3*d^4)*x^5 + (a^5*b^3*c^7
- 3*a^6*b^2*c^6*d + 3*a^7*b*c^5*d^2 - a^8*c^4*d^3)*x^3), 1/12*(3*((5*b^5*c^
4*d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*a*b^4*c^4*d - 8*a^2*b^3*c^3*d^2
)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c^4*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2
*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d -
a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b^3*c^5 - 6*a^4*b^2*c^4*d
 + 6*a^5*b*c^3*d^2 - 2*a^6*c^2*d^3 - (15*a*b^5*c^4*d - 29*a^2*b^4*c^3*d^2 +
 6*a^3*b^3*c^2*d^3 + 24*a^4*b^2*c*d^4 - 16*a^5*b*d^5)*x^6 - (15*a*b^5*c^5 -
 19*a^2*b^4*c^4*d - 14*a^3*b^3*c^3*d^2 + 18*a^4*b^2*c^2*d^3 + 16*a^5*b*c*d^
4 - 16*a^6*d^5)*x^4 - 2*(5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d
^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^4*b^4*c^6*d -
```

```
 3*a^5*b^3*c^5*d^2 + 3*a^6*b^2*c^4*d^3 - a^7*b*c^3*d^4)*x^7 + (a^4*b^4*c^7
- 2*a^5*b^3*c^6*d + 2*a^7*b*c^4*d^3 - a^8*c^3*d^4)*x^5 + (a^5*b^3*c^7 - 3*a
^6*b^2*c^6*d + 3*a^7*b*c^5*d^2 - a^8*c^4*d^3)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

Giac [A]
time = 1.53, size = 486, normalized size = 1.75

$$\frac{d^4x}{\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)\sqrt{dx^2+c}}-\frac{\left(5\,b^4c\sqrt{d}-8\,ab^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^4c\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ab^3d^{\frac{3}{2}}-b^4c^2\sqrt{d}}{\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)}-\frac{2\left(6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2acd^{\frac{3}{2}}+6\,bc^3\sqrt{d}+5\,ac^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] d^4*x/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(d*x^2 + c)) - 1/2*(5*b^4*
c*sqrt(d) - 8*a*b^3*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b
- b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d
^2)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^4*c*sqrt(
d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^3*d^(3/2) - b^4*c^2*sqrt(d))/((a
^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d
 + b*c^2)) - 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 3*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c
^2*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 6*b*c^3*sqrt(
d) + 5*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3*c^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.776 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{(2bc+3ad)x}{6b(bc-ad)^2(c+dx^2)^{3/2}}+\frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}}+\frac{(4bc+11ad)x}{6(bc-ad)^3\sqrt{c+dx^2}}-\frac{\sqrt{a}\,(3bc+2ad)\tan^{-1}\cdots}{2(bc-\cdots}$$

[Out] 1/6*(3*a*d+2*b*c)*x/b/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/2*a*x/b/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(3/2)-1/2*(2*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)/(-a*d+b*c)^(7/2)+1/6*(11*a*d+4*b*c)*x/(-a*d+b*c)^3/(d*x^2+c)^(1/2)

Rubi [A]
time = 0.14, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {481, 541, 12, 385, 211}

$$-\frac{\sqrt{a}\,(2ad+3bc)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2(bc-ad)^{7/2}}+\frac{x(11ad+4bc)}{6\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{x(3ad+2bc)}{6b\,(c+dx^2)^{3/2}\,(bc-ad)^2}+\frac{ax}{2b\,(a+bx^2)\,(c+dx^2)^{3/2}\,(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]

[Out] ((2*b*c + 3*a*d)*x)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (a*x)/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((4*b*c + 11*a*d)*x)/(6*(b*c - a*d)^3*Sqrt[c + d*x^2]) - (Sqrt[a]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(7/2))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{\int \frac{ac-2(bc+ad)x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx}{2b(bc-ad)}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{\int \frac{5abc^2-2bc(2\cdots}{(a+bx^2)(c\cdots}}{6bc(bc\cdots}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{(4bc+1\cdots}{6(bc-ad)^3\cdots}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{(4bc+1\cdots}{6(bc-ad)^3\cdots}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{(4bc+1\cdots}{6(bc-ad)^3\cdots}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{(4bc+1\cdots}{6(bc-ad)^3\cdots}$$

Mathematica [A]

time = 1.14, size = 176, normalized size = 1.01

$$\frac{1}{6}\left(\frac{x(2b^2cx^2(3c+2dx^2)+2a^2d(3c+4dx^2)+ab(9c^2+16cdx^2+11d^2x^4))}{(bc-ad)^3\,(a+bx^2)\,(c+dx^2)^{3/2}}+\frac{3\sqrt{a}\,(3bc+2ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((x*(2*b^2*c*x^2*(3*c + 2*d*x^2) + 2*a^2*d*(3*c + 4*d*x^2) + a*b*(9*c^2 + 1
6*c*d*x^2 + 11*d^2*x^4)))/((b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (
3*Sqrt[a]*(3*b*c + 2*a*d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x
^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*c - a*d)^(7/2))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3521 vs. $2(150) = 300$.
time = 0.10, size = 3522, normalized size = 20.24

method	result	size
default	Expression too large to display	3522

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))-1/4*a/b^3*(1/(a*d
-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/3/(a*
d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-
2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*
d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1
/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d
*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(
x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2
*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1
/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/
(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(
x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^
2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2
```

d(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-3/4/b^2*a/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+3/4/b^2*a/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))-1/4*a/b^3*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1

```
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^
2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(150) = 300$.
time = 2.32, size = 1008, normalized size = 5.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] [-1/24*(3*((3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*d + (6*b^2
*c^2*d + 7*a*b*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*a*b*c^2*d + 4*a^2*c*
d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
 a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^4 + 2*a*b*x^2 + a^2)) - 4*((4*b^2*c*d + 11*a*b*d^2)*x^5 + 2*(3*b^2*c^2
+ 8*a*b*c*d + 4*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + 2*a^2*c*d)*x)*sqrt(d*x^2 + c)
)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d
^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*
a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 -
 a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), 1/1
2*(3*((3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*d + (6*b^2*c^2*
d + 7*a*b*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*a*b*c^2*d + 4*a^2*c*d^2)*
x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*((4*b^2*c*d + 11*a*b*d^2)*x^
5 + 2*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + 2*a^2*c*d)*x
)*sqrt(d*x^2 + c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2
*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 +
(2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)
*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4
*c*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a + bx^2\right)^2 \left(c + dx^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(150) = 300.
time = 1.34, size = 594, normalized size = 3.41

$$\frac{\left(\frac{2\,\left(b^{4} c^{5} d^{2}-a b^{3} c^{4} d^{3}-3\,a^{2} b^{2} c^{3} d^{4}+5\,a^{3} b c^{2} d^{5}-2\,a^{4} c d^{6}\right) x^{2}}{b^{6} c^{7} d-6\,a b^{5} c^{6} d^{2}+15\,a^{2} b^{4} c^{5} d^{3}-20\,a^{3} b^{3} c^{4} d^{4}+15\,a^{4} b^{2} c^{3} d^{5}-6\,a^{5} b c^{2} d^{6}+a^{6} c d^{7}}+\frac{3\,\left(b^{4} c^{6} d-2\,a b^{3} c^{5} d^{2}+2\,a^{3} b c^{3} d^{4}-a^{4} c^{2} d^{5}\right)}{b^{6} c^{7} d-6\,a b^{5} c^{6} d^{2}+15\,a^{2} b^{4} c^{5} d^{3}-20\,a^{3} b^{3} c^{4} d^{4}+15\,a^{4} b^{2} c^{3} d^{5}-6\,a^{5} b c^{2} d^{6}+a^{6} c d^{7}}\right) x}{3\,\left(d x^{2}+c\right)^{\frac{3}{2}}}+\frac{\left(3\,a b c \sqrt{d}+2\,a^{2} d^{\frac{3}{2}}\right) \arctan\left(\frac{\left(\sqrt{d} x-\sqrt{d x^{2}+c}\right)^{2} b-b c+2\,a d}{2\,\sqrt{a b c d-a^{2} d^{2}}}\right)}{2\,\left(b^{3} c^{3}-3\,a b^{2} c^{2} d+3\,a^{2} b c d^{2}-a^{3} d^{3}\right) \sqrt{a b c d-a^{2} d^{2}}}-\frac{\left(\sqrt{d} x-\sqrt{d x^{2}+c}\right)^{2} a b c \sqrt{d}-2\,\left(\sqrt{d} x-\sqrt{d x^{2}+c}\right)^{2} a^{2} d^{\frac{3}{2}}-a b c^{2} \sqrt{d}}{\left(b^{3} c^{3}-3\,a b^{2} c^{2} d+3\,a^{2} b c d^{2}-a^{3} d^{3}\right)\left(\left(\sqrt{d} x-\sqrt{d x^{2}+c}\right)^{4} b-2\,\left(\sqrt{d} x-\sqrt{d x^{2}+c}\right)^{2} b c+4\,\left(\sqrt{d} x-\sqrt{d x^{2}+c}\right)^{2} a d+b c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^4*c^5*d^2 - a*b^3*c^4*d^3 - 3*a^2*b^2*c^3*d^4 + 5*a^3*b*c^2*d^5 -
 2*a^4*c*d^6)*x^2/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^
3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7) + 3*(b^4*
c^6*d - 2*a*b^3*c^5*d^2 + 2*a^3*b*c^3*d^4 - a^4*c^2*d^5)/(b^6*c^7*d - 6*a*b
^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 -
 6*a^5*b*c^2*d^6 + a^6*c*d^7))*x/(d*x^2 + c)^(3/2) + 1/2*(3*a*b*c*sqrt(d) +
 2*a^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt
(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*d + b*c^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.777 $\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=170

$$\frac{2bc+3ad}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}}+\frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{2bc+3ad}{2(bc-ad)^3\sqrt{c+dx^2}}-\frac{\sqrt{b}\,(2bc+3ad)\tanh}{2(bc-}$$

```
[Out] 1/6*(3*a*d+2*b*c)/b/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/2*a/b/(-a*d+b*c)/(b*x^2+
a)/(d*x^2+c)^(3/2)-1/2*(3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+
b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(7/2)+1/2*(3*a*d+2*b*c)/(-a*d+b*c)^3/(d*x^2+
c)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 79, 53, 65, 214}

$$\frac{a}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{3ad+2bc}{2\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{3ad+2bc}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{b}\,(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (2*b*c + 3*a*d)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + a/(2*b*(b*c - a*d)*
(a + b*x^2)*(c + d*x^2)^(3/2)) + (2*b*c + 3*a*d)/(2*(b*c - a*d)^3*Sqrt[c +
d*x^2]) - (Sqrt[b]*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= \frac{a}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{(2bc+3ad)\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx, x, \right.}{4b(bc-ad)}$$

$$= \frac{2bc+3ad}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{a}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{(2bc+3ad)}{}$$

$$= \frac{2bc+3ad}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{a}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{2bc+}{2(bc-ad)^3}$$

$$= \frac{2bc+3ad}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{a}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{2bc+}{2(bc-ad)^3}$$

$$= \frac{2bc+3ad}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{a}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{2bc+}{2(bc-ad)^3}$$

Mathematica [A]
time = 0.47, size = 154, normalized size = 0.91

$$\frac{1}{6}\left(\frac{2a^2d(2c+3dx^2)+2b^2cx^2(4c+3dx^2)+ab(11c^2+16cdx^2+9d^2x^4)}{(bc-ad)^3\,(a+bx^2)\,(c+dx^2)^{3/2}}-\frac{3\sqrt{b}\,(2bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((2*a^2*d*(2*c + 3*d*x^2) + 2*b^2*c*x^2*(4*c + 3*d*x^2) + a*b*(11*c^2 + 16*
c*d*x^2 + 9*d^2*x^4))/((b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) - (3*Sq
rt[b]*(2*b*c + 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])
/(-(b*c) + a*d)^(7/2))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3482 vs. $2(146) = 292$.
time = 0.10, size = 3483, normalized size = 20.49

method	result	size
default	Expression too large to display	3483

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^3*(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5*d*(-a
*b)^(1/2)/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2
/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/
b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2)
)-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*
d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*
d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x+1/b*(-a*b
)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/
(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b
)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*
```

c)/b)^(1/2)))+1/4/b^3*(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(
x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(3/2)+5*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^
2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)
/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*
c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b
*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/
2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b
/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^
(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d
-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d
(x-1/b(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(
x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(
-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)))+1/2/b^2*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2)
)^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/
2)/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-
b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1
/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)
b/(d(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b
)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a
*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+1/2/b^2*(-1/3/(a*d-b*c)
b/(d(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
c)/b)^(3/2)-d(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-
a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-
a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)
/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a
*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/
(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/
b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+
1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x

```
+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(146) = 292.
time = 1.44, size = 993, normalized size = 5.84

$$\left[-\frac{3\,\left((2\,b^{2}cd^{2}+3\,abd^{3})x^{6}+2\,abc^{3}+3\,a^{2}c^{2}d+(4\,b^{2}c^{2}d+8\,abcd^{2}+3\,a^{2}d^{3})x^{4}+(2\,b^{2}c^{3}+7\,abc^{2}d+6\,a^{2}cd^{2})x^{2}\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\,(4\,b^{2}cd-3\,abd^{2})x^{2}+4\,(2\,b^{2}c^{2}-3\,abcd+a^{2}d^{2}+(b^{2}cd-abd^{2})x^{2})\sqrt{dx^{2}+c}\sqrt{\frac{b}{bc-ad}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\,\left(3\,(2\,b^{2}cd+3\,abd^{2})x^{4}+11\,abc^{2}+4\,a^{2}cd+2\,(4\,b^{2}c^{2}+8\,abcd+3\,a^{2}d^{2})x^{2}\right)\sqrt{dx^{2}+c}}{24\,\left(ab^{3}c^{5}-3\,a^{2}b^{2}c^{4}d+3\,a^{3}bc^{3}d^{2}-a^{4}c^{2}d^{3}+(b^{4}c^{3}d^{2}-3\,ab^{3}c^{2}d^{3}+3\,a^{2}b^{2}cd^{4}-a^{3}bd^{5})x^{6}+(2\,b^{4}c^{4}d-5\,ab^{3}c^{3}d^{2}+3\,a^{2}b^{2}c^{2}d^{3}+a^{3}bcd^{4}-a^{4}d^{5})x^{4}+(b^{4}c^{5}-ab^{3}c^{4}d-3\,a^{2}b^{2}c^{3}d^{2}+5\,a^{3}bc^{2}d^{3}-2\,a^{4}cd^{4})x^{2}\right)},\ \frac{3\,\left((2\,b^{2}cd^{2}+3\,abd^{3})x^{6}+2\,abc^{3}+3\,a^{2}c^{2}d+(4\,b^{2}c^{2}d+8\,abcd^{2}+3\,a^{2}d^{3})x^{4}+(2\,b^{2}c^{3}+7\,abc^{2}d+6\,a^{2}cd^{2})x^{2}\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^{2}+2\,bc-ad)\sqrt{dx^{2}+c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^{2}+bc)}\right)+2\,\left(3\,(2\,b^{2}cd+3\,abd^{2})x^{4}+11\,abc^{2}+4\,a^{2}cd+2\,(4\,b^{2}c^{2}+8\,abcd+3\,a^{2}d^{2})x^{2}\right)\sqrt{dx^{2}+c}}{12\,\left(ab^{3}c^{5}-3\,a^{2}b^{2}c^{4}d+3\,a^{3}bc^{3}d^{2}-a^{4}c^{2}d^{3}+(b^{4}c^{3}d^{2}-3\,ab^{3}c^{2}d^{3}+3\,a^{2}b^{2}cd^{4}-a^{3}bd^{5})x^{6}+(2\,b^{4}c^{4}d-5\,ab^{3}c^{3}d^{2}+3\,a^{2}b^{2}c^{2}d^{3}+a^{3}bcd^{4}-a^{4}d^{5})x^{4}+(b^{4}c^{5}-ab^{3}c^{4}d-3\,a^{2}b^{2}c^{3}d^{2}+5\,a^{3}bc^{2}d^{3}-2\,a^{4}cd^{4})x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((2*b^2*c*d^2 + 3*a*b*d^3)*x^6 + 2*a*b*c^3 + 3*a^2*c^2*d + (4*b^2
*c^2*d + 8*a*b*c*d^2 + 3*a^2*d^3)*x^4 + (2*b^2*c^3 + 7*a*b*c^2*d + 6*a^2*c*
d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^
2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
+ (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
2*a*b*x^2 + a^2)) - 4*(3*(2*b^2*c*d + 3*a*b*d^2)*x^4 + 11*a*b*c^2 + 4*a^2*c
*d + 2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^3*c^5
 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3
*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^
2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d
 - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), 1/12*(3*((2*b^2
*c*d^2 + 3*a*b*d^3)*x^6 + 2*a*b*c^3 + 3*a^2*c^2*d + (4*b^2*c^2*d + 8*a*b*c*
d^2 + 3*a^2*d^3)*x^4 + (2*b^2*c^3 + 7*a*b*c^2*d + 6*a^2*c*d^2)*x^2)*sqrt(-b
/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b
*c - a*d))/(b*d*x^2 + b*c)) + 2*(3*(2*b^2*c*d + 3*a*b*d^2)*x^4 + 11*a*b*c^2
 + 4*a^2*c*d + 2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/
(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2
 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*
b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a
*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

Giac [A]
time = 0.68, size = 260, normalized size = 1.53

$$\frac{\frac{3\sqrt{dx^2+c}\,abd^2}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)((dx^2+c)b-bc+ad)}+\frac{3\,(2\,b^2cd+3\,abd^2)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-b^2c+abd}}+\frac{2\,(3\,(dx^2+c)bcd+bc^2d+3\,(dx^2+c)ad^2-acd^2)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx^2+c)^{\frac{3}{2}}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(3*sqrt(d*x^2 + c)*a*b*d^2/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*((d*x^2 + c)*b - b*c + a*d)) + 3*(2*b^2*c*d + 3*a*b*d^2)*arctan(sq
rt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 2*(3*(d*x^2 + c)*b*c*d + b*c^2*d +
3*(d*x^2 + c)*a*d^2 - a*c*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*(d*x^2 + c)^(3/2)))/d
```

Mupad [B]
time = 0.79, size = 193, normalized size = 1.14

$$-\frac{\frac{(d\,x^2+c)\,(3\,a\,d+2\,b\,c)}{3\,(a\,d-b\,c)^2}-\frac{c}{3\,(a\,d-b\,c)}+\frac{b\,(d\,x^2+c)^2\,(3\,a\,d+2\,b\,c)}{2\,(a\,d-b\,c)^3}}{b\,(d\,x^2+c)^{5/2}+(d\,x^2+c)^{3/2}\,(a\,d-b\,c)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^{7/2}}\right)\,(3\,a\,d+2\,b\,c)}{2\,(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] - (((c + d*x^2)*(3*a*d + 2*b*c))/(3*(a*d - b*c)^2) - c/(3*(a*d - b*c)) + (b
*(c + d*x^2)^2*(3*a*d + 2*b*c))/(2*(a*d - b*c)^3))/(b*(c + d*x^2)^(5/2) + (
c + d*x^2)^(3/2)*(a*d - b*c)) - (b^(1/2)*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2))*(3*a*
d + 2*b*c))/(2*(a*d - b*c)^(7/2))
```

3.778 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{d(13bc+2ad)x}{6c(bc-ad)^3\sqrt{c+dx^2}}+\frac{b(bc+4ad)\tan^{-1}\left(\cdots\right.}{2\sqrt{a}\,(bc-\cdots}$$

```
[Out] -5/6*d*x/(-a*d+b*c)^2/(d*x^2+c)^(3/2)-1/2*x/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^
(3/2)+1/2*b*(4*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/
(-a*d+b*c)^(7/2)/a^(1/2)-1/6*d*(2*a*d+13*b*c)*x/c/(-a*d+b*c)^3/(d*x^2+c)^(1
/2)
```

Rubi [A]
time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {482, 541, 12, 385, 211}

$$\frac{b(4ad+bc)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{7/2}}-\frac{dx(2ad+13bc)}{6c\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5dx}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*d*x)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - x/(2*(b*c - a*d)*(a + b*x^2)
*(c + d*x^2)^(3/2)) - (d*(13*b*c + 2*a*d)*x)/(6*c*(b*c - a*d)^3*Sqrt[c + d*
x^2]) + (b*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/(2*Sqrt[a]*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{5/2}} \, dx = -\frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{\int \frac{c-4dx^2}{(a+bx^2)(c+dx^2)^{5/2}} \, dx}{2(bc-ad)}$$

$$= -\frac{5dx}{6(bc-ad)^2 (c+dx^2)^{3/2}} - \frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{\int \frac{c(3bc+2ad)-}{(a+bx^2)(c+}}{6c(bc-}$$

$$= -\frac{5dx}{6(bc-ad)^2 (c+dx^2)^{3/2}} - \frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{d(13bc-}{6c(bc-ad)}$$

$$= -\frac{5dx}{6(bc-ad)^2 (c+dx^2)^{3/2}} - \frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{d(13bc-}{6c(bc-ad)}$$

$$= -\frac{5dx}{6(bc-ad)^2 (c+dx^2)^{3/2}} - \frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{d(13bc-}{6c(bc-ad)}$$

$$= -\frac{5dx}{6(bc-ad)^2 (c+dx^2)^{3/2}} - \frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{d(13bc-}{6c(bc-ad)}$$

Mathematica [A]

time = 1.02, size = 183, normalized size = 1.12

$$-\frac{x(2a^2d^3x^2+2abd(6c^2+5cdx^2+d^2x^4)+b^2c(3c^2+18cdx^2+13d^2x^4))}{6c(bc-ad)^3(a+bx^2)(c+dx^2)^{3/2}}-\frac{b(bc+4ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2\sqrt{a}\,(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(x*(2*a^2*d^3*x^2 + 2*a*b*d*(6*c^2 + 5*c*d*x^2 + d^2*x^4) + b^2*c*(3*c
^2 + 18*c*d*x^2 + 13*d^2*x^4)))/(c*(b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3
/2)) - (b*(b*c + 4*a*d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2
]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*Sqrt[a]*(b*c - a*d)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3482 vs. 2(139) = 278.
time = 0.10, size = 3483, normalized size = 21.37

method	result	size
default	Expression too large to display	3483

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5*d*(-a*b)^(1/2)/(a*d
-b*c)*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b
*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1
6/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(
1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)
/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+
4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(
-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c
)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))+
1/4/(-a*b)^(1/2)/b*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
```

1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/4/(-a*b)^(1/2)/b*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/4/b^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b

```
)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(139) = 278$.
time = 2.75, size = 1292, normalized size = 7.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] [1/24*(3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c^2*d^2 + 4*a*b^2*c*d^3)*x^6 + (
2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 6*a*b^2*c^3
*d + 8*a^2*b*c^2*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) - 4*((13*a*b^3*c^2*d^2 - 11*a^2*b^2*c*d^3 - 2*a^3*b*d^4)*x^5 + 2*(9*
a*b^3*c^3*d - 4*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*x^3 + 3*(a*b^3*c
^4 + 3*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2)*x)*sqrt(d*x^2 + c))/(a^2*b^4*c^7 -
4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4*d^3 + a^6*c^3*d^4 + (a*b^
5*c^5*d^2 - 4*a^2*b^4*c^4*d^3 + 6*a^3*b^3*c^3*d^4 - 4*a^4*b^2*c^2*d^5 + a^5
*b*c*d^6)*x^6 + (2*a*b^5*c^6*d - 7*a^2*b^4*c^5*d^2 + 8*a^3*b^3*c^4*d^3 - 2*
a^4*b^2*c^3*d^4 - 2*a^5*b*c^2*d^5 + a^6*c*d^6)*x^4 + (a*b^5*c^7 - 2*a^2*b^4
*c^6*d - 2*a^3*b^3*c^5*d^2 + 8*a^4*b^2*c^4*d^3 - 7*a^5*b*c^3*d^4 + 2*a^6*c^
2*d^5)*x^2), 1/12*(3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c^2*d^2 + 4*a*b^2*c*
d^3)*x^6 + (2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3)*x^4 + (b^3*c^4 +
 6*a*b^2*c^3*d + 8*a^2*b*c^2*d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(
a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^
2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((13*a*b^3*c^2*d^2 - 11*a^2*b^2*c*d^3
- 2*a^3*b*d^4)*x^5 + 2*(9*a*b^3*c^3*d - 4*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 -
 a^4*d^4)*x^3 + 3*(a*b^3*c^4 + 3*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2)*x)*sqrt(d
*x^2 + c))/(a^2*b^4*c^7 - 4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4
*d^3 + a^6*c^3*d^4 + (a*b^5*c^5*d^2 - 4*a^2*b^4*c^4*d^3 + 6*a^3*b^3*c^3*d^4
 - 4*a^4*b^2*c^2*d^5 + a^5*b*c*d^6)*x^6 + (2*a*b^5*c^6*d - 7*a^2*b^4*c^5*d^
2 + 8*a^3*b^3*c^4*d^3 - 2*a^4*b^2*c^3*d^4 - 2*a^5*b*c^2*d^5 + a^6*c*d^6)*x^
```

```
4 + (a*b^5*c^7 - 2*a^2*b^4*c^6*d - 2*a^3*b^3*c^5*d^2 + 8*a^4*b^2*c^4*d^3 -
7*a^5*b*c^3*d^4 + 2*a^6*c^2*d^5)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(139) = 278.
time = 1.45, size = 595, normalized size = 3.65

$$-\frac{\left(\frac{(5\,b^4c^4d^3-14\,ab^3c^3d^4+12\,a^2b^2c^2d^5-2\,a^3bcd^6-a^4d^7)x^2}{b^6c^7d-6\,ab^5c^6d^2+15\,a^2b^4c^5d^3-20\,a^3b^3c^4d^4+15\,a^4b^2c^3d^5-6\,a^5bc^2d^6+a^6cd^7}+\frac{6\,(b^4c^5d^2-3\,ab^3c^4d^3+3\,a^2b^2c^3d^4-a^3bc^2d^5)}{b^6c^7d-6\,ab^5c^6d^2+15\,a^2b^4c^5d^3-20\,a^3b^3c^4d^4+15\,a^4b^2c^3d^5-6\,a^5bc^2d^6+a^6cd^7}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}}-\frac{\left(b^2c\sqrt{d}+4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{abcd-a^2d^2}}+\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abd^{\frac{3}{2}}-b^2c^2\sqrt{d}}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((5*b^4*c^4*d^3 - 14*a*b^3*c^3*d^4 + 12*a^2*b^2*c^2*d^5 - 2*a^3*b*c*d^
6 - a^4*d^7)*x^2/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3
*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7) + 6*(b^4*c
^5*d^2 - 3*a*b^3*c^4*d^3 + 3*a^2*b^2*c^3*d^4 - a^3*b*c^2*d^5)/(b^6*c^7*d -
6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*
d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7))*x/(d*x^2 + c)^(3/2) - 1/2*(b^2*c*sqrt(d
) + 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*
a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(a*b*c*d - a^2*d^2)) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*s
qrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((sqrt(d)*x - sqrt(d*x^
2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*a*d + b*c^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.779 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{5d}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{5bd}{2(bc-ad)^3\sqrt{c+dx^2}}+\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\ \cdots}{\sqrt{\cdots}}\right.}{2(bc-ad)\cdots}$$

```
[Out] -5/6*d/(-a*d+b*c)^2/(d*x^2+c)^(3/2)-1/2/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(3/2
)+5/2*b^(3/2)*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c
)^(7/2)-5/2*b*d/(-a*d+b*c)^3/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {455, 44, 53, 65, 214}

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}-\frac{5bd}{2\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{1}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5d}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*d)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x^2)*(
c + d*x^2)^(3/2)) - (5*b*d)/(2*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (5*b^(3/2)*
d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{1}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{(5d)\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)}{4(bc-ad)}$$

$$= -\frac{5d}{6(bc-ad)^2(c+dx^2)^{3/2}} - \frac{1}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{(5bd)\text{Subst}(}{}$$

$$= -\frac{5d}{6(bc-ad)^2(c+dx^2)^{3/2}} - \frac{1}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{5b}{2(bc-ad)^3}$$

$$= -\frac{5d}{6(bc-ad)^2(c+dx^2)^{3/2}} - \frac{1}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{5b}{2(bc-ad)^3}$$

$$= -\frac{5d}{6(bc-ad)^2(c+dx^2)^{3/2}} - \frac{1}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{5b}{2(bc-ad)^3}$$

Mathematica [A]

time = 0.30, size = 137, normalized size = 0.98

$$\frac{2a^2d^2 - 2abd(7c + 5dx^2) - b^2(3c^2 + 20cdx^2 + 15d^2x^4)}{6(bc - ad)^3 (a + bx^2) (c + dx^2)^{3/2}} + \frac{5b^{3/2}d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2(-bc + ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (2*a^2*d^2 - 2*a*b*d*(7*c + 5*d*x^2) - b^2*(3*c^2 + 20*c*d*x^2 + 15*d^2*x^4
))/(6*(b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (5*b^(3/2)*d*ArcTan[(S
qrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(2*(-(b*c) + a*d)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2100 vs. $2(116) = 232$.
time = 0.10, size = 2101, normalized size = 15.01

method	result	size
default	Expression too large to display	2101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5*d*(-
a*b)^(1/2)/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(
2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a
/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2
))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2
*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4
*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/
2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x+1/b*(-a*
b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)
^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d
/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/
b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(1/2)))-1/4*(-a*b)^(1/2)/a/b^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
```

```
b)^(3/2)+5*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2
))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1
/2)/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d
-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*
b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-
1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c
)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*
b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(
a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(
2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*
d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b
)^(1/2))-(a*d-b*c)/b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: ValueError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(116) = 232$.
time = 1.54, size = 895, normalized size = 6.39

$$\left[-\frac{15\,{\left(b^{2} d^{3} x^{6} + a b c^{2} d + {\left(2\,b^{2} c d^{2} + a b d^{3}\right)} x^{4} + {\left(b^{2} c^{2} d + 2\,a b c d^{2}\right)} x^{2}\right)} \sqrt{\frac{b}{b c - a d}} \log\left(\frac{b^{2} d^{2} x^{4} + 8\,b^{2} c^{2} - 8\,a b c d + a^{2} d^{2} + 2\,{\left(4\,b^{2} c d - 3\,a b d^{2}\right)} x^{2} - 4\,{\left(2\,b^{2} c^{2} - 3\,a b c d + a^{2} d^{2} + {\left(b^{2} c d - a b d^{2}\right)} x^{2}\right)} \sqrt{d x^{2} + c} \sqrt{\frac{b}{b c - a d}}}{b^{2} x^{4} + 2\,a b x^{2} + a^{2}}\right) + 4\,{\left(15\,b^{2} d^{2} x^{4} + 3\,b^{2} c^{2} + 14\,a b c d - 2\,a^{2} d^{2} + 10\,{\left(2\,b^{2} c d + a b d^{2}\right)} x^{2}\right)} \sqrt{d x^{2} + c}}{24\,{\left(a b^{3} c^{5} - 3\,a^{2} b^{2} c^{4} d + 3\,a^{3} b c^{3} d^{2} - a^{4} c^{2} d^{3} + {\left(b^{4} c^{3} d^{2} - 3\,a b^{3} c^{2} d^{3} + 3\,a^{2} b^{2} c d^{4} - a^{3} b d^{5}\right)} x^{6} + {\left(2\,b^{4} c^{4} d - 5\,a b^{3} c^{3} d^{2} + 3\,a^{2} b^{2} c^{2} d^{3} + a^{3} b c d^{4} - a^{4} d^{5}\right)} x^{4} + {\left(b^{4} c^{5} - a b^{3} c^{4} d - 3\,a^{2} b^{2} c^{3} d^{2} + 5\,a^{3} b c^{2} d^{3} - 2\,a^{4} c d^{4}\right)} x^{2}\right)}}, -\frac{15\,{\left(b^{2} d^{3} x^{6} + a b c^{2} d + {\left(2\,b^{2} c d^{2} + a b d^{3}\right)} x^{4} + {\left(b^{2} c^{2} d + 2\,a b c d^{2}\right)} x^{2}\right)} \sqrt{-\frac{b}{b c - a d}} \arctan\left(\frac{{\left(b d x^{2} + 2\,b c - a d\right)} \sqrt{d x^{2} + c} \sqrt{-\frac{b}{b c - a d}}}{2\,{\left(b d x^{2} + b c\right)}}\right) + 2\,{\left(15\,b^{2} d^{2} x^{4} + 3\,b^{2} c^{2} + 14\,a b c d - 2\,a^{2} d^{2} + 10\,{\left(2\,b^{2} c d + a b d^{2}\right)} x^{2}\right)} \sqrt{d x^{2} + c}}{12\,{\left(a b^{3} c^{5} - 3\,a^{2} b^{2} c^{4} d + 3\,a^{3} b c^{3} d^{2} - a^{4} c^{2} d^{3} + {\left(b^{4} c^{3} d^{2} - 3\,a b^{3} c^{2} d^{3} + 3\,a^{2} b^{2} c d^{4} - a^{3} b d^{5}\right)} x^{6} + {\left(2\,b^{4} c^{4} d - 5\,a b^{3} c^{3} d^{2} + 3\,a^{2} b^{2} c^{2} d^{3} + a^{3} b c d^{4} - a^{4} d^{5}\right)} x^{4} + {\left(b^{4} c^{5} - a b^{3} c^{4} d - 3\,a^{2} b^{2} c^{3} d^{2} + 5\,a^{3} b c^{2} d^{3} - 2\,a^{4} c d^{4}\right)} x^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(15*(b^2*d^3*x^6 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^4 + (b^2*c^
2*d + 2*a*b*c*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*
```

```
c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(15*b^2*d^2*x^4 + 3*b^2*c^2 + 14*a*b*c*
d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^3*c^5 -
 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c
^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2
+ 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d -
 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), -1/12*(15*(b^2*d^
3*x^6 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^4 + (b^2*c^2*d + 2*a*b*c*d^2)
*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 +
c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(15*b^2*d^2*x^4 + 3*b^2*c^2 +
14*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a*
b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 -
3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3
*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^
3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + bx^2\right)^2 \left(c + dx^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

Giac [A]
time = 0.62, size = 226, normalized size = 1.61

$$-\frac{5\, b^2 d \arctan\left(\frac{\sqrt{dx^2+c}\, b}{\sqrt{-b^2c+abd}}\right)}{2\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{-b^2c+abd}} - \frac{\sqrt{dx^2+c}\, b^2 d}{2\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)((dx^2+c)b - bc + ad)} - \frac{6\,(dx^2+c)bd + bcd - ad^2}{3\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -5/2*b^2*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - 1/2*sqrt(d*x^2
+ c)*b^2*d/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x^2 + c
)*b - b*c + a*d)) - 1/3*(6*(d*x^2 + c)*b*d + b*c*d - a*d^2)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x^2 + c)^(3/2))
```

Mupad [B]
time = 0.68, size = 171, normalized size = 1.22

$$\frac{\frac{5\,b^2\,d\,{\left(d\,x^2+c\right)}^2}{2\,{\left(a\,d-b\,c\right)}^3}-\frac{d}{3\,\left(a\,d-b\,c\right)}+\frac{5\,b\,d\,\left(d\,x^2+c\right)}{3\,{\left(a\,d-b\,c\right)}^2}}{b\,{\left(d\,x^2+c\right)}^{5/2}+{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d-b\,c\right)}+\frac{5\,b^{3/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^{7/2}}\right)}{2\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((5*b^2*d*(c + d*x^2)^2)/(2*(a*d - b*c)^3) - d/(3*(a*d - b*c)) + (5*b*d*(c
+ d*x^2))/(3*(a*d - b*c)^2))/(b*(c + d*x^2)^(5/2) + (c + d*x^2)^(3/2)*(a*d
- b*c)) + (5*b^(3/2)*d*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a^3*d^3 - b^3*c^3 +
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(2*(a*d - b*c)^(7/2))
```

3.780 $\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{5/2}} \, dx$

Optimal. Leaf size=201

$$\frac{d(3bc+2ad)x}{6ac(bc-ad)^2\,(c+dx^2)^{3/2}}+\frac{bx}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)^{3/2}}+\frac{d(3b^2c^2+16abcd-4a^2d^2)\,x}{6ac^2(bc-ad)^3\sqrt{c+dx^2}}+\frac{b^2(bc-6ad)}{2a}$$

```
[Out] 1/6*d*(2*a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/2*b*x/a/(-a*d+b*c)
/(b*x^2+a)/(d*x^2+c)^(3/2)+1/2*b^2*(-6*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a
^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(7/2)+1/6*d*(-4*a^2*d^2+16*a*b*c
*d+3*b^2*c^2)*x/a/c^2/(-a*d+b*c)^3/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {425, 541, 12, 385, 211}

$$\frac{b^2(bc-6ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{7/2}}+\frac{dx(-4a^2d^2+16abcd+3b^2c^2)}{6ac^2\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{bx}{2a\,(a+bx^2)\,(c+dx^2)^{3/2}\,(bc-ad)}+\frac{dx(2ad+3bc)}{6ac\,(c+dx^2)^{3/2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d)*x)/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (b*x)/(2*a*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^2 + 16*a*b*c*d - 4
*a^2*d^2)*x)/(6*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (b^2*(b*c - 6*a*d)*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d
)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{\int \frac{-bc+2ad-4bdx^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2(c+dx^2)^{3/2}} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{\int \frac{-3b^2c^2+1}{}}{}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2(c+dx^2)^{3/2}} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{d(3b^2c^2+}{6ac^2(bc-}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2(c+dx^2)^{3/2}} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{d(3b^2c^2+}{6ac^2(bc-}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2(c+dx^2)^{3/2}} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{d(3b^2c^2+}{6ac^2(bc-}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2(c+dx^2)^{3/2}} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{d(3b^2c^2+}{6ac^2(bc-}$$

Mathematica [A]

time = 1.16, size = 214, normalized size = 1.06

$$\frac{x\left(3b^3c^2(c+dx^2)^2-2a^3d^3(3c+2dx^2)+2ab^2cd^2x^2(9c+8dx^2)+2a^2bd^2(9c^2+5cdx^2-2d^2x^4)\right)}{6ac^2(bc-ad)^3\,(a+bx^2)\,(c+dx^2)^{3/2}}-\frac{b^2(bc-6ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*(3*b^3*c^2*(c + d*x^2)^2 - 2*a^3*d^3*(3*c + 2*d*x^2) + 2*a*b^2*c*d^2*x^2
*(9*c + 8*d*x^2) + 2*a^2*b*d^2*(9*c^2 + 5*c*d*x^2 - 2*d^2*x^4)))/(6*a*c^2*(
b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) - (b^2*(b*c - 6*a*d)*ArcTan[(a*
Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2
*a^(3/2)*(b*c - a*d)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3488 vs. $2(177) = 354$.
time = 0.10, size = 3489, normalized size = 17.36

method	result	size
default	Expression too large to display	3489

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/a*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5*d*(-a*b)^(1/2)/(a*
d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/
b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+
16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^
(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2
)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x+1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*
(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*
c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
```

-1/4/b/a*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5*d*(-a*b)^(1/2)/(a*
d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x-1/
b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+
16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^
(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2
)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/
(x-1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*
(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*
(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*
c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(
-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))
+1/4/a/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^
(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2
/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/
b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b
*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2)
)+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-
a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*
d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*
d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2
)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*
c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/4/a/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(
d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)
^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4
*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d
-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-
a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*
b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a
*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b

```
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(177) = 354$.
time = 2.83, size = 1434, normalized size = 7.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] [-1/24*(3*(a*b^3*c^5 - 6*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 6*a*b^3*c^2*d^3)*x^
6 + (2*b^4*c^4*d - 11*a*b^3*c^3*d^2 - 6*a^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 4
*a*b^3*c^4*d - 12*a^2*b^2*c^3*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*
((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) - 4*((3*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 - 20*a^3*b^2
*c*d^4 + 4*a^4*b*d^5)*x^5 + 2*(3*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^
2*c^2*d^3 - 7*a^4*b*c*d^4 + 2*a^5*d^5)*x^3 + 3*(a*b^4*c^5 - a^2*b^3*c^4*d +
 6*a^3*b^2*c^3*d^2 - 8*a^4*b*c^2*d^3 + 2*a^5*c*d^4)*x)*sqrt(d*x^2 + c))/(a^
3*b^4*c^8 - 4*a^4*b^3*c^7*d + 6*a^5*b^2*c^6*d^2 - 4*a^6*b*c^5*d^3 + a^7*c^4
*d^4 + (a^2*b^5*c^6*d^2 - 4*a^3*b^4*c^5*d^3 + 6*a^4*b^3*c^4*d^4 - 4*a^5*b^2
*c^3*d^5 + a^6*b*c^2*d^6)*x^6 + (2*a^2*b^5*c^7*d - 7*a^3*b^4*c^6*d^2 + 8*a^
4*b^3*c^5*d^3 - 2*a^5*b^2*c^4*d^4 - 2*a^6*b*c^3*d^5 + a^7*c^2*d^6)*x^4 + (a
^2*b^5*c^8 - 2*a^3*b^4*c^7*d - 2*a^4*b^3*c^6*d^2 + 8*a^5*b^2*c^5*d^3 - 7*a^
6*b*c^4*d^4 + 2*a^7*c^3*d^5)*x^2), 1/12*(3*(a*b^3*c^5 - 6*a^2*b^2*c^4*d + (
b^4*c^3*d^2 - 6*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 11*a*b^3*c^3*d^2 - 6*a^
2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 4*a*b^3*c^4*d - 12*a^2*b^2*c^3*d^2)*x^2)*sq
rt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*
sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((3*
a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 - 20*a^3*b^2*c*d^4 + 4*a^4*b*d^5)*x^5 +
2*(3*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^2*c^2*d^3 - 7*a^4*b*c*d^4 +
2*a^5*d^5)*x^3 + 3*(a*b^4*c^5 - a^2*b^3*c^4*d + 6*a^3*b^2*c^3*d^2 - 8*a^4*b
*c^2*d^3 + 2*a^5*c*d^4)*x)*sqrt(d*x^2 + c))/(a^3*b^4*c^8 - 4*a^4*b^3*c^7*d
+ 6*a^5*b^2*c^6*d^2 - 4*a^6*b*c^5*d^3 + a^7*c^4*d^4 + (a^2*b^5*c^6*d^2 - 4*
```

```
a^3*b^4*c^5*d^3 + 6*a^4*b^3*c^4*d^4 - 4*a^5*b^2*c^3*d^5 + a^6*b*c^2*d^6)*x^
6 + (2*a^2*b^5*c^7*d - 7*a^3*b^4*c^6*d^2 + 8*a^4*b^3*c^5*d^3 - 2*a^5*b^2*c^
4*d^4 - 2*a^6*b*c^3*d^5 + a^7*c^2*d^6)*x^4 + (a^2*b^5*c^8 - 2*a^3*b^4*c^7*d
 - 2*a^4*b^3*c^6*d^2 + 8*a^5*b^2*c^5*d^3 - 7*a^6*b*c^4*d^4 + 2*a^7*c^3*d^5)
*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)^2\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(177) = 354.
time = 1.30, size = 619, normalized size = 3.08

$$\frac{\left(\frac{2\,(4\,b^4c^4d^4-13\,ab^3c^3d^5+15\,a^2b^2c^2d^6-7\,a^3bcd^7+a^4d^8)x^2}{b^6c^8d-6\,ab^5c^7d^2+15\,a^2b^4c^6d^3-20\,a^3b^3c^5d^4+15\,a^4b^2c^4d^5-6\,a^5bc^3d^6+a^6c^2d^7}+\frac{3\,(3\,b^4c^5d^3-10\,ab^3c^4d^4+12\,a^2b^2c^3d^5-6\,a^3bc^2d^6+a^4cd^7)}{b^6c^8d-6\,ab^5c^7d^2+15\,a^2b^4c^6d^3-20\,a^3b^3c^5d^4+15\,a^4b^2c^4d^5-6\,a^5bc^3d^6+a^6c^2d^7}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}}+\frac{\left(b^3c\sqrt{d}-6\,ab^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^3c\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ab^2d^{\frac{3}{2}}-b^3c^2\sqrt{d}}{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(4*b^4*c^4*d^4 - 13*a*b^3*c^3*d^5 + 15*a^2*b^2*c^2*d^6 - 7*a^3*b*c*d
^7 + a^4*d^8)*x^2/(b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^
3*b^3*c^5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7) + 3*(3*
b^4*c^5*d^3 - 10*a*b^3*c^4*d^4 + 12*a^2*b^2*c^3*d^5 - 6*a^3*b*c^2*d^6 + a^4
*c*d^7)/(b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^3*b^3*c^5*
d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7))*x/(d*x^2 + c)^(3
/2) + 1/2*(b^3*c*sqrt(d) - 6*a*b^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a*b^3*c^3 - 3*a^2
*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*
x - sqrt(d*x^2 + c))^2*b^3*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*
b^2*d^(3/2) - b^3*c^2*sqrt(d))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^
2 - a^4*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2
 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.781 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{d(3bc+2ad)}{6ac(bc-ad)^2(c+dx^2)^{3/2}}+\frac{b}{2a(bc-ad)(a+bx^2)(c+dx^2)^{3/2}}+\frac{d(b^2c^2+6abcd-2a^2d^2)}{2ac^2(bc-ad)^3\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}}$$

```
[Out] 1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/2*b/a/(-a*d+b*c)/(b*
x^2+a)/(d*x^2+c)^(3/2)-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(5/2)+1/2*b^(
5/2)*(-7*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(
-a*d+b*c)^(7/2)+1/2*d*(-2*a^2*d^2+6*a*b*c*d+b^2*c^2)/a/c^2/(-a*d+b*c)^3/(d*
x^2+c)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 105, 157, 162, 65, 214}

$$\frac{b^{5/2}(2bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{7/2}}+\frac{d(-2a^2d^2+6abcd+b^2c^2)}{2ac^2\sqrt{c+dx^2}(bc-ad)^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}}+\frac{b}{2a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ac(c+dx^2)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + b/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 6*a*b*c*d - 2*a^2*d^2)
)/(2*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
]/(a^2*c^(5/2)) + (b^(5/2)*(2*b*c - 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{5bdx}{2}}{x(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(bc-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(bc-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(bc-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(bc-}$$

Mathematica [**A**]
time = 0.75, size = 215, normalized size = 0.96

$$\frac{\frac{a\left(3b^3c^2\left(c+dx^2\right)^2-2a^3d^3\left(4c+3dx^2\right)+2ab^2cd^2x^2\left(10c+9dx^2\right)+2a^2bd^2\left(10c^2+5cdx^2-3d^2x^4\right)\right)}{c^2(bc-ad)^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{3b^{5/2}(2bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((a*(3*b^3*c^2*(c + d*x^2)^2 - 2*a^3*d^3*(4*c + 3*d*x^2) + 2*a*b^2*c*d^2*x^
2*(10*c + 9*d*x^2) + 2*a^2*b*d^2*(10*c^2 + 5*c*d*x^2 - 3*d^2*x^4)))/(c^2*(b
*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (3*b^(5/2)*(2*b*c - 7*a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2) - (
6*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2))/(6*a^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3547 vs. $2(195) = 390$.
time = 0.16, size = 3548, normalized size = 15.77

method	result	size
default	Expression too large to display	3548

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1
/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5*d*(-a*b
)^(1/2)/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3
*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)
/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+
2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*
b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*
(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*
(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*
(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x-1/b*(-a*b)^
(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1
/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-
4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/
(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)))+1/4/a/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/
b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2
)-5*d*(-a*b)^(1/2)/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*
d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b
+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a
*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2
)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c
)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x+
1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/
```

```
b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2
)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b
)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)))-1/2/a^2*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(
a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)
/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(
-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(
d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1
/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1
/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b
*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-1/2/a^2*(-1/3/(a*d-b*c)*b/(
d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/
b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)
^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4
*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^
(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d
-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-
a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*
b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a
*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(195) = 390$.
time = 9.20, size = 3403, normalized size = 15.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^3)
*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*c^
6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d
^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 +
4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 +
c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 12*(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)*sqrt(c)*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(3*a*b^3*c^5 + 20*a^3*b*c^3*d^2 -
8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 2*a^3*b*c*d^4)*x^4 +
 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 3*a^4*c*d^4)*x^2
)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c
^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c
^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^
5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c
^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2), 1/24*(24*(a*b^3*c^5 - 3*a^2
*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3
 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^
2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2
*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)
/sqrt(d*x^2 + c)) + 3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a
*b^3*c^3*d^3)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^
4 + (2*b^4*c^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(b/(b*c - a*d
))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*
b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*
sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*a*
b^3*c^5 + 20*a^3*b*c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c
^2*d^3 - 2*a^3*b*c*d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3
*b*c^2*d^3 - 3*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^
7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3
+ 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6
*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8
- a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2)
, -1/12*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^
3)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*
c^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
 b*c)) - 6*(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (
b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c
```

```
^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (
b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*
x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b
^3*c^5 + 20*a^3*b*c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^
2*d^3 - 2*a^3*b*c*d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*
b*c^2*d^3 - 3*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7
*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 +
 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*
d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 -
 a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2),
 -1/12*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^3
)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*c
^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1
/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
b*c)) - 12*(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (
b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c
^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (
b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*
x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(3*a*b^3*c^5 + 20*a^3*b*
c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 2*a^3*b*c*
d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 3*a^4*
c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d
^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4
 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^
5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3
*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

Giac [A]

time = 1.12, size = 298, normalized size = 1.32

$$\frac{\sqrt{dx^2+c}\,b^3d}{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\left((dx^2+c)b-bc+ad\right)}-\frac{\left(2\,b^4c-7\,ab^3d\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\sqrt{-b^2c+abd}}+\frac{9\left(dx^2+c\right)bcd^2+bc^2d^2-3\left(dx^2+c\right)ad^3-acd^3}{3\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^3*d/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a
^4*d^3)*((d*x^2 + c)*b - b*c + a*d)) - 1/2*(2*b^4*c - 7*a*b^3*d)*arctan(sqr
t(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^
4*b*c*d^2 - a^5*d^3)*sqrt(-b^2*c + a*b*d)) + 1/3*(9*(d*x^2 + c)*b*c*d^2 + b
*c^2*d^2 - 3*(d*x^2 + c)*a*d^3 - a*c*d^3)/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2
*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^(3/2)) + arctan(sqrt(d*x^2 + c)/sqrt(
-c))/(a^2*sqrt(-c)*c^2)
```

Mupad [B]
time = 2.94, size = 2500, normalized size = 11.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((d^2*(c + d*x^2)*(3*a*d - 8*b*c))/(3*(b*c^2 - a*c*d)^2) - d^2/(3*(b*c^2 -
a*c*d)) + (d*(c + d*x^2)^2*(b^3*c^2 - 2*a^2*b*d^2 + 6*a*b^2*c*d))/(2*a*c*(b
*c^2 - a*c*d)*(a*d - b*c)^2))/(b*(c + d*x^2)^(5/2) + (c + d*x^2)^(3/2)*(a*d
 - b*c)) - atanh((560*a^3*b^16*c^19*d^4*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(56
0*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 14
9184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d^
9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^8
*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^1
4*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17*
b^2*c^3*d^18)) - (7280*a^4*b^15*c^18*d^5*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(5
60*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 1
49184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d
^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^
8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^
14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17
*b^2*c^3*d^18)) + (42560*a^5*b^14*c^17*d^6*(c + d*x^2)^(1/2))/((c^5)^(1/2)*
(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 -
 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12
*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*
b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*
a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^
17*b^2*c^3*d^18)) - (149184*a^6*b^13*c^16*d^7*(c + d*x^2)^(1/2))/((c^5)^(1/
2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^
6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c
^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^
11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 358
40*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64
*a^17*b^2*c^3*d^18)) + (351904*a^7*b^12*c^15*d^8*(c + d*x^2)^(1/2))/((c^5)^
(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15
```

*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) - (593440*a^8*b^11*c^14*d^9*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) + (741120*a^9*b^10*c^13*d^10*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) - (699840*a^10*b^9*c^12*d^11*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) + (505008*a^11*b^8*c^11*d^12*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) - (278768*a^12*b^7*c^10*d^13*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) + (116480*a^13*b^6*c^9*d^14*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b...

3.782 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=279

$$\frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}}+\frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{d(3b^2c^2+20abcd-8a^2d^2)}{6ac^2(bc-ad)^3x\sqrt{c+dx^2}}-\frac{(9b^3c^3-18$$

```
[Out] 1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/x/(d*x^2+c)^(3/2)+1/2*b/a/(-a*d+b*c)/x
/(b*x^2+a)/(d*x^2+c)^(3/2)-1/2*b^3*(-8*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)
/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(7/2)+1/6*d*(-8*a^2*d^2+20*a*b
*c*d+3*b^2*c^2)/a/c^2/(-a*d+b*c)^3/x/(d*x^2+c)^(1/2)-1/6*(-16*a^3*d^3+40*a^
2*b*c*d^2-18*a*b^2*c^2*d+9*b^3*c^3)*(d*x^2+c)^(1/2)/a^2/c^3/(-a*d+b*c)^3/x
```

Rubi [A]
time = 0.29, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {483, 593, 597, 12, 385, 211}

$$-\frac{b^3(3bc-8ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}+\frac{d(-8a^2d^2+20abcd+3b^2c^2)}{6ac^2x\sqrt{c+dx^2}(bc-ad)^3}-\frac{\sqrt{c+dx^2}(-16a^3d^3+40a^2bcd^2-18ab^2c^2d+9b^3c^3)}{6a^2c^3x(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6acx\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x*(c + d*x^2)^(3/2)) + b/(2*a*(b*c
 - a*d)*x*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^2 + 20*a*b*c*d - 8*a
^2*d^2))/(6*a*c^2*(b*c - a*d)^3*x*Sqrt[c + d*x^2]) - ((9*b^3*c^3 - 18*a*b^2
*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c - a*
d)^3*x) - (b^3*(3*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-3bc+2ad-6bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \int =$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d(3}{6ac}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d(3}{6ac}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d(3}{6ac}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d(3}{6ac}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d(3}{6ac}$$

<u>Mathematica</u> [**A**]
time = 1.26, size = 287, normalized size = 1.03

$$\frac{-9b^4c^3x^2\left(c+dx^2\right)^2-6ab^3c^2\left(c-3dx^2\right)\left(c+dx^2\right)^2+2a^4d^3\left(3c^2+12cdx^2+8d^2x^4\right)+2a^2b^2cd\left(9c^3+9c^2dx^2-21cd^2x^4-20d^3x^6\right)-2a^3bd^2\left(9c^3+27c^2dx^2+8cd^2x^4-8d^3x^6\right)}{6a^2c^3(bc-ad)^3x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{b^3(3bc-8ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{5/2}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-9*b^4*c^3*x^2*(c + d*x^2)^2 - 6*a*b^3*c^2*(c - 3*d*x^2)*(c + d*x^2)^2 + 2
*a^4*d^3*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4) + 2*a^2*b^2*c*d*(9*c^3 + 9*c^2*d*
x^2 - 21*c*d^2*x^4 - 20*d^3*x^6) - 2*a^3*b*d^2*(9*c^3 + 27*c^2*d*x^2 + 8*c*
d^2*x^4 - 8*d^3*x^6))/(6*a^2*c^3*(b*c - a*d)^3*x*(a + b*x^2)*(c + d*x^2)^(3
/2)) + (b^3*(3*b*c - 8*a*d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d
*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```

<u>Maple</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3543 vs. $2(251) = 502$.
time = 0.26, size = 3544, normalized size = 12.70

method	result	size
risch	Expression too large to display	2357
default	Expression too large to display	3544

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5*d*(-a*b)^(1/2)/(a*d
-b*c)*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b
*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(
-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+1
6/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(
1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(
a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)
/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+
4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-
2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(
x+1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(
-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c
)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)))-
3/4*b/a^2/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*
b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)
*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2
*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1
/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/
b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b
*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(
-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(
1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+3/4*b/a^2/(-a*b)^(1/2)*(-1/3/(a*d-b
*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*
d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d
```

```
*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d
*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b
*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x
+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1
/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)
))))+1/4/a^2*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5*d*(-a*b)^(1/2)
/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(
x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-
1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3
/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a
*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2
))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^
(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*
c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^
(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/
2))/(x-1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+
2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+
2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*
d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(251) = 502$.
time = 2.56, size = 1662, normalized size = 5.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((3*b^5*c^4*d^2 - 8*a*b^4*c^3*d^3)*x^7 + (6*b^5*c^5*d - 13*a*b^4*
c^4*d^2 - 8*a^2*b^3*c^3*d^3)*x^5 + (3*b^5*c^6 - 2*a*b^4*c^5*d - 16*a^2*b^3*
c^4*d^2)*x^3 + (3*a*b^4*c^6 - 8*a^2*b^3*c^5*d)*x)*sqrt(-a*b*c + a^2*d)*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))
/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(6*a^2*b^4*c^6 - 24*a^3*b^3*c^5*d + 36*a^
4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 6*a^6*c^2*d^4 + (9*a*b^5*c^4*d^2 - 27*a^
2*b^4*c^3*d^3 + 58*a^3*b^3*c^2*d^4 - 56*a^4*b^2*c*d^5 + 16*a^5*b*d^6)*x^6 +
 2*(9*a*b^5*c^5*d - 24*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 13*a^4*b^2*c^
2*d^4 - 16*a^5*b*c*d^5 + 8*a^6*d^6)*x^4 + 3*(3*a*b^5*c^6 - 5*a^2*b^4*c^5*d
- 4*a^3*b^3*c^4*d^2 + 24*a^4*b^2*c^3*d^3 - 26*a^5*b*c^2*d^4 + 8*a^6*c*d^5)*
x^2)*sqrt(d*x^2 + c))/((a^3*b^5*c^7*d^2 - 4*a^4*b^4*c^6*d^3 + 6*a^5*b^3*c^5
*d^4 - 4*a^6*b^2*c^4*d^5 + a^7*b*c^3*d^6)*x^7 + (2*a^3*b^5*c^8*d - 7*a^4*b^
4*c^7*d^2 + 8*a^5*b^3*c^6*d^3 - 2*a^6*b^2*c^5*d^4 - 2*a^7*b*c^4*d^5 + a^8*c
^3*d^6)*x^5 + (a^3*b^5*c^9 - 2*a^4*b^4*c^8*d - 2*a^5*b^3*c^7*d^2 + 8*a^6*b^
2*c^6*d^3 - 7*a^7*b*c^5*d^4 + 2*a^8*c^4*d^5)*x^3 + (a^4*b^4*c^9 - 4*a^5*b^3
*c^8*d + 6*a^6*b^2*c^7*d^2 - 4*a^7*b*c^6*d^3 + a^8*c^5*d^4)*x), -1/12*(3*((
3*b^5*c^4*d^2 - 8*a*b^4*c^3*d^3)*x^7 + (6*b^5*c^5*d - 13*a*b^4*c^4*d^2 - 8*
a^2*b^3*c^3*d^3)*x^5 + (3*b^5*c^6 - 2*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2)*x^3
 + (3*a*b^4*c^6 - 8*a^2*b^3*c^5*d)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a
*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2
)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(6*a^2*b^4*c^6 - 24*a^3*b^3*c^5*d + 36*
a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 6*a^6*c^2*d^4 + (9*a*b^5*c^4*d^2 - 27*
a^2*b^4*c^3*d^3 + 58*a^3*b^3*c^2*d^4 - 56*a^4*b^2*c*d^5 + 16*a^5*b*d^6)*x^6
 + 2*(9*a*b^5*c^5*d - 24*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 13*a^4*b^2*
c^2*d^4 - 16*a^5*b*c*d^5 + 8*a^6*d^6)*x^4 + 3*(3*a*b^5*c^6 - 5*a^2*b^4*c^5*
d - 4*a^3*b^3*c^4*d^2 + 24*a^4*b^2*c^3*d^3 - 26*a^5*b*c^2*d^4 + 8*a^6*c*d^5
)*x^2)*sqrt(d*x^2 + c))/((a^3*b^5*c^7*d^2 - 4*a^4*b^4*c^6*d^3 + 6*a^5*b^3*c
^5*d^4 - 4*a^6*b^2*c^4*d^5 + a^7*b*c^3*d^6)*x^7 + (2*a^3*b^5*c^8*d - 7*a^4*
b^4*c^7*d^2 + 8*a^5*b^3*c^6*d^3 - 2*a^6*b^2*c^5*d^4 - 2*a^7*b*c^4*d^5 + a^8
*c^3*d^6)*x^5 + (a^3*b^5*c^9 - 2*a^4*b^4*c^8*d - 2*a^5*b^3*c^7*d^2 + 8*a^6*
b^2*c^6*d^3 - 7*a^7*b*c^5*d^4 + 2*a^8*c^4*d^5)*x^3 + (a^4*b^4*c^9 - 4*a^5*b
^3*c^8*d + 6*a^6*b^2*c^7*d^2 - 4*a^7*b*c^6*d^3 + a^8*c^5*d^4)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(251) = 502.
time = 3.63, size = 938, normalized size = 3.36

$$-\frac{\left(\frac{\left(11\,b^{4}c^{6}d^{5}-38\,ab^{3}c^{5}d^{6}+48\,a^{2}b^{2}c^{4}d^{7}-26\,a^{3}bc^{3}d^{8}+5\,a^{4}c^{2}d^{9}\right)x^{2}}{b^{6}c^{11}d-6\,ab^{5}c^{10}d^{2}+15\,a^{2}b^{4}c^{9}d^{3}-20\,a^{3}b^{3}c^{8}d^{4}+15\,a^{4}b^{2}c^{7}d^{5}-6\,a^{5}bc^{6}d^{6}+a^{6}c^{5}d^{7}}+\frac{6\left(2\,b^{4}c^{7}d^{4}-7\,ab^{3}c^{6}d^{5}+9\,a^{2}b^{2}c^{5}d^{6}-5\,a^{3}bc^{4}d^{7}+a^{4}c^{3}d^{8}\right)}{b^{6}c^{11}d-6\,ab^{5}c^{10}d^{2}+15\,a^{2}b^{4}c^{9}d^{3}-20\,a^{3}b^{3}c^{8}d^{4}+15\,a^{4}b^{2}c^{7}d^{5}-6\,a^{5}bc^{6}d^{6}+a^{6}c^{5}d^{7}}\right)x}{3\,\left(dx^{2}+c\right)^{\frac{3}{2}}}+\frac{\left(3\,b^{4}c\sqrt{d}-8\,ab^{3}d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\left(a^{2}b^{3}c^{3}-3\,a^{3}b^{2}c^{2}d+3\,a^{4}bcd^{2}-a^{5}d^{3}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b^{4}c^{3}\sqrt{d}-8\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ab^{3}c^{2}d^{\frac{3}{2}}+6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{2}b^{2}cd^{\frac{5}{2}}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{3}bd^{\frac{7}{2}}-6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{4}c^{4}\sqrt{d}+22\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ab^{3}c^{3}d^{\frac{3}{2}}-36\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{2}b^{2}c^{2}d^{\frac{5}{2}}+28\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{3}bcd^{\frac{7}{2}}-8\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{4}d^{\frac{9}{2}}+3\,b^{4}c^{5}\sqrt{d}-6\,ab^{3}c^{4}d^{\frac{3}{2}}+6\,a^{2}b^{2}c^{3}d^{\frac{5}{2}}-2\,a^{3}bc^{2}d^{\frac{7}{2}}}{\left(a^{2}b^{3}c^{5}-3\,a^{3}b^{2}c^{4}d+3\,a^{4}bc^{3}d^{2}-a^{5}c^{2}d^{3}\right)\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}b-3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ad+3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}-4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}acd-bc^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((11*b^4*c^6*d^5 - 38*a*b^3*c^5*d^6 + 48*a^2*b^2*c^4*d^7 - 26*a^3*b*c^
3*d^8 + 5*a^4*c^2*d^9)*x^2/(b^6*c^11*d - 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^9*
d^3 - 20*a^3*b^3*c^8*d^4 + 15*a^4*b^2*c^7*d^5 - 6*a^5*b*c^6*d^6 + a^6*c^5*d
^7) + 6*(2*b^4*c^7*d^4 - 7*a*b^3*c^6*d^5 + 9*a^2*b^2*c^5*d^6 - 5*a^3*b*c^4*
d^7 + a^4*c^3*d^8)/(b^6*c^11*d - 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^9*d^3 - 20
*a^3*b^3*c^8*d^4 + 15*a^4*b^2*c^7*d^5 - 6*a^5*b*c^6*d^6 + a^6*c^5*d^7))*x/(
d*x^2 + c)^(3/2) + 1/2*(3*b^4*c*sqrt(d) - 8*a*b^3*d^(3/2))*arctan(1/2*((sqr
t(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2
*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b*c*d - a^2*d^
2)) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^4*c^3*sqrt(d) - 8*(sqrt(d)*x - s
qrt(d*x^2 + c))^4*a*b^3*c^2*d^(3/2) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2
*b^2*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*b*d^(7/2) - 6*(sqrt(
d)*x - sqrt(d*x^2 + c))^2*b^4*c^4*sqrt(d) + 22*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*a*b^3*c^3*d^(3/2) - 36*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^2*c^2*d^(5
/2) + 28*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*b*c*d^(7/2) - 8*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a^4*d^(9/2) + 3*b^4*c^5*sqrt(d) - 6*a*b^3*c^4*d^(3/2) + 6
*a^2*b^2*c^3*d^(5/2) - 2*a^3*b*c^2*d^(7/2))/((a^2*b^3*c^5 - 3*a^3*b^2*c^4*d
 + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^6*b - 3*(s
qrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d +
 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*a*c*d - b*c^3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.783 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{5/2}} \, dx$

Optimal. Leaf size=304

$$-\frac{d(6b^2c^2-6abcd+5a^2d^2)}{6a^2c^2(bc-ad)^2(c+dx^2)^{3/2}}-\frac{b(2bc-ad)}{2a^2c(bc-ad)(a+bx^2)(c+dx^2)^{3/2}}-\frac{1}{2acx^2(a+bx^2)(c+dx^2)^{3/2}}-\frac{d(2bc-}{2a^2c}$$

```
[Out] -1/6*d*(5*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(d*x^2+c)^(3/2)
-1/2*b*(-a*d+2*b*c)/a^2/c/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(3/2)-1/2/a/c/x^2/
(b*x^2+a)/(d*x^2+c)^(3/2)+1/2*(5*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2)
)/a^3/c^(7/2)-1/2*b^(7/2)*(-9*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-
a*d+b*c)^(1/2))/a^3/(-a*d+b*c)^(7/2)-1/2*d*(-a*d+2*b*c)*(5*a^2*d^2-a*b*c*d+
b^2*c^2)/a^2/c^3/(-a*d+b*c)^3/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 105, 156, 157, 162, 65, 214}

$$-\frac{b^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{7/2}}+\frac{(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{7/2}}-\frac{d(5a^2d^2-6abcd+6b^2c^2)}{6a^2c^2(c+dx^2)^{3/2}(bc-ad)^2}-\frac{d(2bc-ad)(5a^2d^2-abcd+b^2c^2)}{2a^2c^3\sqrt{c+dx^2}(bc-ad)^3}-\frac{b(2bc-ad)}{2a^2c(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}-\frac{1}{2acx^2(a+bx^2)(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(d*(6*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*(c + d*
x^2)^(3/2)) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b*x^2)*(c + d*x^2
)^(3/2)) - 1/(2*a*c*x^2*(a + b*x^2)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d)*(
b^2*c^2 - a*b*c*d + 5*a^2*d^2))/(2*a^2*c^3*(b*c - a*d)^3*Sqrt[c + d*x^2]) +
 ((4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(7/2)) - (b^(7
/2)*(4*b*c - 9*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*
a^3*(b*c - a*d)^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{5/2}}\,dx,x,x^2\right)$$

$$= -\frac{1}{2acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+5ad)+\frac{7bdx}{2}}{x(a+bx)^2(c+dx)^{5/2}}\,dx,x,x^2\right)}{2ac}$$

$$= -\frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \text{Su}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - 2a$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - 2a$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - 2a$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - 2a$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - 2a$$

Mathematica [A]
time = 1.36, size = 294, normalized size = 0.97

$$\frac{\frac{a\left(-6b^4c^3x^2\left(c+dx^2\right)^2-3ab^3c^2\left(c-3dx^2\right)\left(c+dx^2\right)^2+a^4d^3\left(3c^2+20cdx^2+15d^2x^4\right)+a^2b^2cd\left(9c^3+9c^2dx^2-35cd^2x^4-33d^3x^6\right)+a^3bd^2\left(-9c^3-41c^2dx^2-13cd^2x^4+15d^3x^6\right)\right)}{c^3(bc-ad)^3x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{3b^{7/2}(4bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}+\frac{3(4bc+5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{7/2}}}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((a*(-6*b^4*c^3*x^2*(c + d*x^2)^2 - 3*a*b^3*c^2*(c - 3*d*x^2)*(c + d*x^2)^2
 + a^4*d^3*(3*c^2 + 20*c*d*x^2 + 15*d^2*x^4) + a^2*b^2*c*d*(9*c^3 + 9*c^2*d
*x^2 - 35*c*d^2*x^4 - 33*d^3*x^6) + a^3*b*d^2*(-9*c^3 - 41*c^2*d*x^2 - 13*c
*d^2*x^4 + 15*d^3*x^6)))/(c^3*(b*c - a*d)^3*x^2*(a + b*x^2)*(c + d*x^2)^(3/
2)) - (3*b^(7/2)*(4*b*c - 9*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*
```

```
c) + a*d]])/(-(b*c) + a*d)^(7/2) + (3*(4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^
2]/Sqrt[c]])/c^(7/2))/(6*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3640 vs. $2(268) = 536$.
time = 0.31, size = 3641, normalized size = 11.98

method	result	size
risch	Expression too large to display	2420
default	Expression too large to display	3641

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/a^2/(-a*b)^(1/2)*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5*d*(
-a*b)^(1/2)/(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b
)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*
(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*
a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/
2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*
(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-
2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-
4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x+1/b*(-a
*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b
)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*
d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)
/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)))+b/a^3*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*
(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*
a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d
-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/
2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*
(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+
2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-
4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln(
```

```
(-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1
/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))+b/a^3*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-
a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*
(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(
-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x
+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2
*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1
/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))
-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))))+1/a^2*(-1/2/c
/x^2/(d*x^2+c)^(3/2)-5/2*d/c*(1/3/c/(d*x^2+c)^(3/2)+1/c*(1/c/(d*x^2+c)^(1/2
)-1/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))))+1/4*b/a^2/(-a*b)^(1/2)
*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5*d*(-a*b)^(1/2)/(a*d-b*c)*(
-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)
^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x-1/b*(-a*b)
^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(
1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(
-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)
/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(1/2)/(a*d-b
*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a
/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-
b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/b*(-a*b)^(1
/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(
-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(
1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(268) = 536$.

time = 18.98, size = 4115, normalized size = 13.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*c^6*d - 14*a*b^4*c
^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*d - 18*a^2*b^3*c^5
*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(b/(b*c - a*d))*log((b
^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^
2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^
2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 6*((4*b^5*c^4*d^
2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x
^8 + (8*b^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*
d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*
c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b
^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2
*d^4)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*
(3*a^2*b^3*c^6 - 9*a^3*b^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3*(2*a
*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)*x^6
+ (12*a*b^4*c^5*d - 15*a^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b*c^2*
d^4 - 15*a^5*c*d^5)*x^4 + (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c^4*d^
2 + 41*a^4*b*c^3*d^3 - 20*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^7*
d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*a^3*b
^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*
d^5)*x^6 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d
^3 - 2*a^7*c^5*d^4)*x^4 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2
- a^7*c^6*d^3)*x^2), -1/24*(12*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^
3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a*b^4*c
^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)
*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 +
17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a
^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) - 3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*
c^6*d - 14*a*b^4*c^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*
d - 18*a^2*b^3*c^5*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(b/(
b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c
*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 4*(3*a^2*b^3*c^6 - 9*a^3*b^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3*
(2*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)*
x^6 + (12*a*b^4*c^5*d - 15*a^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b*
```

```
c^2*d^4 - 15*a^5*c*d^5)*x^4 + (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c^
4*d^2 + 41*a^4*b*c^3*d^3 - 20*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*
c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*a
^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*
c^4*d^5)*x^6 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c
^6*d^3 - 2*a^7*c^5*d^4)*x^4 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*
d^2 - a^7*c^6*d^3)*x^2), 1/12*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (
8*b^5*c^6*d - 14*a*b^4*c^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^
4*c^6*d - 18*a^2*b^3*c^5*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sq
rt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(
-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 3*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*
a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a
*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^
5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*
d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d
 - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*sqrt(c)*log(-
(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a^2*b^3*c^6 - 9*a^3*b
^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3*(2*a*b^4*c^4*d^2 - 3*a^2*b^3
*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)*x^6 + (12*a*b^4*c^5*d - 15*a
^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b*c^2*d^4 - 15*a^5*c*d^5)*x^4
+ (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c^4*d^2 + 41*a^4*b*c^3*d^3 - 2
0*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3
+ 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7
*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x^6 + (a^3*b^4*c^9
- a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^4
+ (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^2), 1/1
2*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*c^6*d - 14*a*b^4*c^5*d
^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*d - 18*a^2*b^3*c^5*d^2
)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/
2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b
*c)) - 6*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

Giac [A]

time = 1.18, size = 505, normalized size = 1.66

$$\frac{(4\,b^5c-9\,ab^4d)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3)\sqrt{-b^2c+abd}}-\frac{2\,(dx^2+c)^{\frac{3}{2}}b^4c^3d-2\,\sqrt{dx^2+c}\,b^4c^4d-3\,(dx^2+c)^{\frac{3}{2}}ab^3c^2d^2+4\,\sqrt{dx^2+c}\,ab^3c^3d^2+3\,(dx^2+c)^{\frac{3}{2}}a^2b^2cd^3-6\,\sqrt{dx^2+c}\,a^2b^2c^2d^3-(dx^2+c)^{\frac{3}{2}}a^3bd^4+4\,\sqrt{dx^2+c}\,a^3bcd^4-\sqrt{dx^2+c}\,a^4d^5}{2\,(a^2b^3c^6-3\,a^3b^2c^5d+3\,a^4bc^4d^2-a^5c^3d^3)\left((dx^2+c)^2b-2\,(dx^2+c)bc+bc^2+(dx^2+c)ad-acd\right)}-\frac{12\,(dx^2+c)bcd^3+bc^2d^3-6\,(dx^2+c)ad^4-acd^4}{3\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)(dx^2+c)^{\frac{3}{2}}}-\frac{(4\,bc+5\,ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*b^5*c - 9*a*b^4*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(-b^2*c + a*b*
d)) - 1/2*(2*(d*x^2 + c)^(3/2)*b^4*c^3*d - 2*sqrt(d*x^2 + c)*b^4*c^4*d - 3*
(d*x^2 + c)^(3/2)*a*b^3*c^2*d^2 + 4*sqrt(d*x^2 + c)*a*b^3*c^3*d^2 + 3*(d*x^
2 + c)^(3/2)*a^2*b^2*c*d^3 - 6*sqrt(d*x^2 + c)*a^2*b^2*c^2*d^3 - (d*x^2 + c
)^(3/2)*a^3*b*d^4 + 4*sqrt(d*x^2 + c)*a^3*b*c*d^4 - sqrt(d*x^2 + c)*a^4*d^5
)/((a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*((d*x^2
+ c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)) - 1/3*(12*
(d*x^2 + c)*b*c*d^3 + b*c^2*d^3 - 6*(d*x^2 + c)*a*d^4 - a*c*d^4)/((b^3*c^6
- 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^(3/2)) - 1/2*(
4*b*c + 5*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c^3)
```

Mupad [B]
time = 3.76, size = 2500, normalized size = 8.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((5*d^3*(c + d*x^2)*(a*d - 2*b*c))/(3*(b*c^2 - a*c*d)^2) - d^3/(3*(b*c^2 -
a*c*d)) + (d*(c + d*x^2)^2*(15*a^4*d^4 + 6*b^4*c^4 + 64*a^2*b^2*c^2*d^2 - 1
2*a*b^3*c^3*d - 58*a^3*b*c*d^3))/(6*a^2*(b*c^2 - a*c*d)^3) + (d*(c + d*x^2)
^3*(a*d - 2*b*c)*(b^3*c^2 + 5*a^2*b*d^2 - a*b^2*c*d))/(2*a^2*(b*c^2 - a*c*d
)^3))/(b*(c + d*x^2)^(7/2) + (c + d*x^2)^(3/2)*(b*c^2 - a*c*d) + (c + d*x^2
)^(5/2)*(a*d - 2*b*c)) - (atan((a^19*c^15*d^19*(c + d*x^2)^(1/2)*125i + a^3
*b^16*c^31*d^3*(c + d*x^2)^(1/2)*420i - a^4*b^15*c^30*d^4*(c + d*x^2)^(1/2)
*4515i + a^5*b^14*c^29*d^5*(c + d*x^2)^(1/2)*20916i - a^6*b^13*c^28*d^6*(c
+ d*x^2)^(1/2)*52836i + a^7*b^12*c^27*d^7*(c + d*x^2)^(1/2)*71070i - a^8*b^
11*c^26*d^8*(c + d*x^2)^(1/2)*19530i - a^9*b^10*c^25*d^9*(c + d*x^2)^(1/2)*
107740i + a^10*b^9*c^24*d^10*(c + d*x^2)^(1/2)*212608i - a^11*b^8*c^23*d^11
*(c + d*x^2)^(1/2)*184563i + a^12*b^7*c^22*d^12*(c + d*x^2)^(1/2)*40965i +
a^13*b^6*c^21*d^13*(c + d*x^2)^(1/2)*91560i - a^14*b^5*c^20*d^14*(c + d*x^2
)^(1/2)*126720i + a^15*b^4*c^19*d^15*(c + d*x^2)^(1/2)*87276i - a^16*b^3*c^
18*d^16*(c + d*x^2)^(1/2)*37776i + a^17*b^2*c^17*d^17*(c + d*x^2)^(1/2)*104
40i - a^18*b*c^16*d^18*(c + d*x^2)^(1/2)*1700i)/(c^7*(c^7)^(1/2)*(c^7*(c^7*
(212608*a^10*b^9*d^10 - 107740*a^9*b^10*c*d^9 + 420*a^3*b^16*c^7*d^3 - 4515
*a^4*b^15*c^6*d^4 + 20916*a^5*b^14*c^5*d^5 - 52836*a^6*b^13*c^4*d^6 + 71070
*a^7*b^12*c^3*d^7 - 19530*a^8*b^11*c^2*d^8) + 10440*a^17*b^2*d^17 - 37776*a
^16*b^3*c*d^16 - 184563*a^11*b^8*c^6*d^11 + 40965*a^12*b^7*c^5*d^12 + 91560
*a^13*b^6*c^4*d^13 - 126720*a^14*b^5*c^3*d^14 + 87276*a^15*b^4*c^2*d^15) +
125*a^19*c^5*d^19 - 1700*a^18*b*c^6*d^18)))*(5*a*d + 4*b*c)*1i)/(2*a^3*(c^7
```

)^(1/2)) + (atan((((-b^7*(a*d - b*c)^7)^(1/2)*((c + d*x^2)^(1/2)*(512*a^6*b^20*c^26*d^2 - 6656*a^7*b^19*c^25*d^3 + 38560*a^8*b^18*c^24*d^4 - 129920*a^9*b^17*c^23*d^5 + 275920*a^10*b^16*c^22*d^6 - 363440*a^11*b^15*c^21*d^7 + 235312*a^12*b^14*c^20*d^8 + 85360*a^13*b^13*c^19*d^9 - 316400*a^14*b^12*c^18*d^10 + 205840*a^15*b^11*c^17*d^11 + 152384*a^16*b^10*c^16*d^12 - 430816*a^17*b^9*c^15*d^13 + 444080*a^18*b^8*c^14*d^14 - 281680*a^19*b^7*c^13*d^15 + 118640*a^20*b^6*c^12*d^16 - 32656*a^21*b^5*c^11*d^17 + 5360*a^22*b^4*c^10*d^18 - 400*a^23*b^3*c^9*d^19) + ((-b^7*(a*d - b*c)^7)^(1/2)*(9*a*d - 4*b*c)*(128*a^10*b^18*c^28*d^3 - 1792*a^11*b^17*c^27*d^4 + 10624*a^12*b^16*c^26*d^5 - 33280*a^13*b^15*c^25*d^6 + 47936*a^14*b^14*c^24*d^7 + 40448*a^15*b^13*c^23*d^8 - 368896*a^16*b^12*c^22*d^9 + 948992*a^17*b^11*c^21*d^10 - 1531200*a^18*b^10*c^20*d^11 + 1754368*a^19*b^9*c^19*d^12 - 1485440*a^20*b^8*c^18*d^13 + 939008*a^21*b^7*c^17*d^14 - 439616*a^22*b^6*c^16*d^15 + 148480*a^23*b^5*c^15*d^16 - 34304*a^24*b^4*c^14*d^17 + 4864*a^25*b^3*c^13*d^18 - 320*a^26*b^2*c^12*d^19 - ((-b^7*(a*d - b*c)^7)^(1/2)*(c + d*x^2)^(1/2)*(9*a*d - 4*b*c)*(512*a^12*b^18*c^31*d^2 - 7936*a^13*b^17*c^30*d^3 + 57600*a^14*b^16*c^29*d^4 - 259840*a^15*b^15*c^28*d^5 + 815360*a^16*b^14*c^27*d^6 - 1886976*a^17*b^13*c^26*d^7 + 3331328*a^18*b^12*c^25*d^8 - 4576000*a^19*b^11*c^24*d^9 + 4942080*a^20*b^10*c^23*d^10 - 4209920*a^21*b^9*c^22*d^11 + 2818816*a^22*b^8*c^21*d^12 - 1467648*a^23*b^7*c^20*d^13 + 582400*a^24*b^6*c^19*d^14 - 170240*a^25*b^5*c^18*d^15 + 34560*a^26*b^4*c^17*d^16 - 4352*a^27*b^3*c^16*d^17 + 256*a^28*b^2*c^15*d^18))/(4*(a^10*d^7 - a^3*b^7*c^7 + 7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6))))/(4*(a^10*d^7 - a^3*b^7*c^7 + 7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6)))*(9*a*d - 4*b*c)*1i)/(4*(a^10*d^7 - a^3*b^7*c^7 + 7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6)) + ((-b^7*(a*d - b*c)^7)^(1/2)*((c + d*x^2)^(1/2)*(512*a^6*b^20*c^26*d^2 - 6656*a^7*b^19*c^25*d^3 + 38560*a^8*b^18*c^24*d^4 - 129920*a^9*b^17*c^23*d^5 + 275920*a^10*b^16*c^22*d^6 - 363440*a^11*b^15*c^21*d^7 + 235312*a^12*b^14*c^20*d^8 + 85360*a^13*b^13*c^19*d^9 - 316400*a^14*b^12*c^18*d^10 + 205840*a^15*b^11*c^17*d^11 + 152384*a^16*b^10*c^16*d^12 - 430816*a^17*b^9*c^15*d^13 + 444080*a^18*b^8*c^14*d^14 - 281680*a^19*b^7*c^13*d^15 + 118640*a^20*b^6*c^12*d^16 - 32656*a^21*b^5*c^11*d^17 + 5360*a^22*b^4*c^10*d^18 - 400*a^23*b^3*c^9*d^19) - ((-b^7*(a*d - b*c)^7)^(1/2)*(9*a*d - 4*b*c)*(128*a^10*b^18*c^28*d^3 - 1792*a^11*b^17*c^27*d^4 + 10624*a^12*b^16*c^26*d^5 - 33280*a^13*b^15*c^25*d^6 + 47936*a^14*b^14*c^24*d^7 + 40448*a^15*b^13*c^23*d^8 - 368896*a^16*b^12*c^22*d^9 + 948992*a^17*b^11*c^21*d^10 - 1531200*a^18*b^10*c^20*d^11 + 1754368*a^19*b^9*c^19*d^12 - 1485440*a^20*b^8*c^18*d^13 + 939008*a^21*b^7*c^17*d^14 - 439616*a^22*b^6*c^16*d^15 + 148480*a^23*b^5*c^15*d^16 - 34304*a^24*b^4*c^14*d^17 + 4864*a^25*b^3*c^13*d^18 - 320*a^26*b^2*c^12*d^19 + ((-b^7*(a*d - b*c)^7)^(1/2)*(c + d*x^2)^(1/2)*(9*a*d - 4*b*c)*(512*a^12*b^18*c^31*d^2 - 7936*a^13*b^17*c^30*d^3 + 57600*a^14*b^16*c^29*d^4 - 259840*a^15*b^15*c^28*d^5 + 815360*a^16*b^14*c^27*d^6 - 1886976*a^17*b^13*c^26*d^7 + 3331...

3.784 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=362

$$\frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}}+\frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}+\frac{d(b^2c^2+8abcd-4a^2d^2)}{2ac^2(bc-ad)^3x^3\sqrt{c+dx^2}}-\frac{(5b^3c^3-$$

```
[Out] 1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/x^3/(d*x^2+c)^(3/2)+1/2*b/a/(-a*d+b*c)
/x^3/(b*x^2+a)/(d*x^2+c)^(3/2)+5/2*b^4*(-2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/
2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(7/2)/(-a*d+b*c)^(7/2)+1/2*d*(-4*a^2*d^2+8*a*
b*c*d+b^2*c^2)/a/c^2/(-a*d+b*c)^3/x^3/(d*x^2+c)^(1/2)-1/6*(-16*a^3*d^3+32*a
^2*b*c*d^2-6*a*b^2*c^2*d+5*b^3*c^3)*(d*x^2+c)^(1/2)/a^2/c^3/(-a*d+b*c)^3/x^
3+1/6*(-32*a^4*d^4+64*a^3*b*c*d^3-12*a^2*b^2*c^2*d^2-20*a*b^3*c^3*d+15*b^4*
c^4)*(d*x^2+c)^(1/2)/a^3/c^4/(-a*d+b*c)^3/x
```

Rubi [A]
time = 0.41, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {483, 593, 597, 12, 385, 211}

$$\frac{5b^4(bc-2ad)\text{ArcTan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{7/2}}+\frac{d(-4a^2d^2+8abcd+b^2c^2)}{2ac^2x^3\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{\sqrt{c+dx^2}\,(-16a^3d^3+32a^2bcd^2-6ab^2c^2d+5b^3c^3)}{6a^2c^3x^3(bc-ad)^3}+\frac{\sqrt{c+dx^2}\,(-32a^4d^4+64a^3bcd^3-12a^2b^2c^2d^2-20ab^3c^3d+15b^4c^4)}{6a^3c^4x(bc-ad)^3}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6acx^3\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^(3/2)) + b/(2*a*(b
*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 8*a*b*c*d - 4*
a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*x^3*Sqrt[c + d*x^2]) - ((5*b^3*c^3 - 6*a*b
^2*c^2*d + 32*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c -
a*d)^3*x^3) + ((15*b^4*c^4 - 20*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 64*a^3*b
*c*d^3 - 32*a^4*d^4)*Sqrt[c + d*x^2])/(6*a^3*c^4*(b*c - a*d)^3*x) + (5*b^4*
(b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
7/2)*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-5bc+2ad-8bdx^2}{x^4(a+bx^2)(c+dx^2)^{5/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \int$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{}{2a}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{}{2a}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{}{2a}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{}{2a}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{}{2a}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{}{2a}$$

Mathematica [A]
time = 1.74, size = 348, normalized size = 0.96

$$\frac{15b^5c^4x^4(c+dx^2)^2+10ab^4c^3x^2(c-2dx^2)(c+dx^2)^2-2a^2b^3c^2(c+dx^2)^3(c+6dx^2)+2a^5d^3(c^3-6c^2dx^2-24cd^2x^4-16d^3x^6)+2a^4bd^2(-3c^4+13c^3dx^2+42c^2d^2x^4+8cd^3x^6-16d^4x^8)+2a^3b^2cd(3c^4-3c^3dx^2+3c^2d^2x^4+42cd^3x^6+32d^4x^8)}{6a^3c^4(bc-ad)^3x^3(a+bx^2)(c+dx^2)^{3/2}} - \frac{5b^4(bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}\,x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (15*b^5*c^4*x^4*(c + d*x^2)^2 + 10*a*b^4*c^3*x^2*(c - 2*d*x^2)*(c + d*x^2)^
2 - 2*a^2*b^3*c^2*(c + d*x^2)^3*(c + 6*d*x^2) + 2*a^5*d^3*(c^3 - 6*c^2*d*x^
2 - 24*c*d^2*x^4 - 16*d^3*x^6) + 2*a^4*b*d^2*(-3*c^4 + 13*c^3*d*x^2 + 42*c^
2*d^2*x^4 + 8*c*d^3*x^6 - 16*d^4*x^8) + 2*a^3*b^2*c*d*(3*c^4 - 3*c^3*d*x^2
+ 3*c^2*d^2*x^4 + 42*c*d^3*x^6 + 32*d^4*x^8))/(6*a^3*c^4*(b*c - a*d)^3*x^3*
(a + b*x^2)*(c + d*x^2)^(3/2)) - (5*b^4*(b*c - 2*a*d)*ArcTan[(a*Sqrt[d] + b
*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(7/2)*(b
*c - a*d)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3634 vs. $2(330) = 660$.
time = 0.35, size = 3635, normalized size = 10.04

method	result	size
risch	Expression too large to display	2375
default	Expression too large to display	3635

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/a^3*(1/(a*d-b*c)*b/(x+1/b*(-a*b)^(1/2))/(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-5*d*(-a*b)^(1/2)/(
a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+
1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+
1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/
b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2
)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b
)^(1/2)/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))
^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1
/2)/(a*d-b*c)*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)
/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1
/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b
*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
)/(x+1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x+1/b*(-a*b)^(1/2))-2*
d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2))^2-2*
d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-
b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*(x+1/b
*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)
))+5/4*b^2/a^3/(-a*b)^(1/2)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d
*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d
-b*c)*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+
4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*
b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*
(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(
x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^
(1/2)+2*d*(-a*b)^(1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)
/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)
/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))
```

```
-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))))-5/4*b^2/a^3/(-a*b)^(1/2)*(-1/3
/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/
2))-(a*d-b*c)/b)^(3/2)-d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x+1/b*(-a*b)^(1/
2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x+1/b*(-a*b)^(1/2)
)^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+16/3*d/(-4*d
*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(d*
(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)
^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-2*d*(-a*b)^(1/2)/(a*d-b*c)*
(2*d*(x+1/b*(-a*b)^(1/2))-2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/
(d*(x+1/b*(-a*b)^(1/2))^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)
/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^
(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x+1/b*(-a*b)^(1/2))
^2-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b
)^(1/2)))))+1/a^2*(-1/3/c/x^3/(d*x^2+c)^(3/2)-2*d/c*(-1/c/x/(d*x^2+c)^(3/2)
-4*d/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))))-2/a^3*b*(-1/c/
x/(d*x^2+c)^(3/2)-4*d/c*(1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2))
)-1/4*b/a^3*(1/(a*d-b*c)*b/(x-1/b*(-a*b)^(1/2))/(d*(x-1/b*(-a*b)^(1/2))^2+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+5*d*(-a*b)^(1/2)/
(a*d-b*c)*(-1/3/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x
-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/2)+d*(-a*b)^(1/2)/(a*d-b*c)*(2/3*(2*d*(x
-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)/(d*(x-1
/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(3/
2)+16/3*d/(-4*d*(a*d-b*c)/b+4*d^2*a/b)^2*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*
b)^(1/2)/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2
))-(a*d-b*c)/b)^(1/2))-1/(a*d-b*c)*b*(-1/(a*d-b*c)*b/(d*(x-1/b*(-a*b)^(1/2)
)^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+2*d*(-a*b)^(
1/2)/(a*d-b*c)*(2*d*(x-1/b*(-a*b)^(1/2))+2*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c
)/b+4*d^2*a/b)/(d*(x-1/b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)+1/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c
)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(d*(x-1/
b*(-a*b)^(1/2))^2+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2
))/(x-1/b*(-a*b)^(1/2)))))+4*d/(a*d-b*c)*b*(2/3*(2*d*(x-1/b*(-a*b)^(1/2))+2
*d*(-a*b)^(1/2)/b)/(-4*d*(a*d-b*c)/b+4*d^2*a/b)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(330) = 660$.

time = 4.01, size = 1890, normalized size = 5.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(15*((b^6*c^5*d^2 - 2*a*b^5*c^4*d^3)*x^9 + (2*b^6*c^6*d - 3*a*b^5*c^
5*d^2 - 2*a^2*b^4*c^4*d^3)*x^7 + (b^6*c^7 - 4*a^2*b^4*c^5*d^2)*x^5 + (a*b^5
*c^7 - 2*a^2*b^4*c^6*d)*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a
*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) + 4*(2*a^3*b^4*c^7 - 8*a^4*b^3*c^6*d + 12*a^5*b^2*c^5*d^2 - 8*a^6*
b*c^4*d^3 + 2*a^7*c^3*d^4 - (15*a*b^6*c^5*d^2 - 35*a^2*b^5*c^4*d^3 + 8*a^3*
b^4*c^3*d^4 + 76*a^4*b^3*c^2*d^5 - 96*a^5*b^2*c*d^6 + 32*a^6*b*d^7)*x^8 - 2
*(15*a*b^6*c^6*d - 30*a^2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3 + 61*a^4*b^3*c^3*
d^4 - 34*a^5*b^2*c^2*d^5 - 24*a^6*b*c*d^6 + 16*a^7*d^7)*x^6 - 3*(5*a*b^6*c^
7 - 5*a^2*b^5*c^6*d - 14*a^3*b^4*c^5*d^2 + 16*a^4*b^3*c^4*d^3 + 26*a^5*b^2*
c^3*d^4 - 44*a^6*b*c^2*d^5 + 16*a^7*c*d^6)*x^4 - 2*(5*a^2*b^5*c^7 - 14*a^3*
b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a
^7*c^2*d^5)*x^2)*sqrt(d*x^2 + c))/((a^4*b^5*c^8*d^2 - 4*a^5*b^4*c^7*d^3 + 6
*a^6*b^3*c^6*d^4 - 4*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6)*x^9 + (2*a^4*b^5*c^9*
d - 7*a^5*b^4*c^8*d^2 + 8*a^6*b^3*c^7*d^3 - 2*a^7*b^2*c^6*d^4 - 2*a^8*b*c^5
*d^5 + a^9*c^4*d^6)*x^7 + (a^4*b^5*c^10 - 2*a^5*b^4*c^9*d - 2*a^6*b^3*c^8*d
^2 + 8*a^7*b^2*c^7*d^3 - 7*a^8*b*c^6*d^4 + 2*a^9*c^5*d^5)*x^5 + (a^5*b^4*c^
10 - 4*a^6*b^3*c^9*d + 6*a^7*b^2*c^8*d^2 - 4*a^8*b*c^7*d^3 + a^9*c^6*d^4)*x
^3), 1/12*(15*((b^6*c^5*d^2 - 2*a*b^5*c^4*d^3)*x^9 + (2*b^6*c^6*d - 3*a*b^5
*c^5*d^2 - 2*a^2*b^4*c^4*d^3)*x^7 + (b^6*c^7 - 4*a^2*b^4*c^5*d^2)*x^5 + (a*
b^5*c^7 - 2*a^2*b^4*c^6*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c -
 a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3
+ (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b^4*c^7 - 8*a^4*b^3*c^6*d + 12*a^5*b^2
*c^5*d^2 - 8*a^6*b*c^4*d^3 + 2*a^7*c^3*d^4 - (15*a*b^6*c^5*d^2 - 35*a^2*b^5
*c^4*d^3 + 8*a^3*b^4*c^3*d^4 + 76*a^4*b^3*c^2*d^5 - 96*a^5*b^2*c*d^6 + 32*a
^6*b*d^7)*x^8 - 2*(15*a*b^6*c^6*d - 30*a^2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3
+ 61*a^4*b^3*c^3*d^4 - 34*a^5*b^2*c^2*d^5 - 24*a^6*b*c*d^6 + 16*a^7*d^7)*x^
6 - 3*(5*a*b^6*c^7 - 5*a^2*b^5*c^6*d - 14*a^3*b^4*c^5*d^2 + 16*a^4*b^3*c^4*
d^3 + 26*a^5*b^2*c^3*d^4 - 44*a^6*b*c^2*d^5 + 16*a^7*c*d^6)*x^4 - 2*(5*a^2*
b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^
6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^2)*sqrt(d*x^2 + c))/((a^4*b^5*c^8*d^2 - 4*a^
5*b^4*c^7*d^3 + 6*a^6*b^3*c^6*d^4 - 4*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6)*x^9
+ (2*a^4*b^5*c^9*d - 7*a^5*b^4*c^8*d^2 + 8*a^6*b^3*c^7*d^3 - 2*a^7*b^2*c^6*
d^4 - 2*a^8*b*c^5*d^5 + a^9*c^4*d^6)*x^7 + (a^4*b^5*c^10 - 2*a^5*b^4*c^9*d
- 2*a^6*b^3*c^8*d^2 + 8*a^7*b^2*c^7*d^3 - 7*a^8*b*c^6*d^4 + 2*a^9*c^5*d^5)*
x^5 + (a^5*b^4*c^10 - 4*a^6*b^3*c^9*d + 6*a^7*b^2*c^8*d^2 - 4*a^8*b*c^7*d^3
 + a^9*c^6*d^4)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\left(a+bx^{2}\right)^{2}\left(c+dx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 789 vs. 2(330) = 660.
time = 4.54, size = 789, normalized size = 2.18

$$\frac{\left(\frac{2\left(7b^{4}c^{7}d^{6}-25ab^{3}c^{6}d^{7}+33a^{2}b^{2}c^{5}d^{8}-19a^{3}bc^{4}d^{9}+4a^{4}c^{3}d^{10}\right)x^{2}}{b^{6}c^{13}d-6ab^{5}c^{12}d^{2}+15a^{2}b^{4}c^{11}d^{3}-20a^{3}b^{3}c^{10}d^{4}+15a^{4}b^{2}c^{9}d^{5}-6a^{5}bc^{8}d^{6}+a^{6}c^{7}d^{7}}+\frac{3\left(5b^{4}c^{8}d^{5}-18ab^{3}c^{7}d^{6}+24a^{2}b^{2}c^{6}d^{7}-14a^{3}bc^{5}d^{8}+3a^{4}c^{4}d^{9}\right)}{b^{6}c^{13}d-6ab^{5}c^{12}d^{2}+15a^{2}b^{4}c^{11}d^{3}-20a^{3}b^{3}c^{10}d^{4}+15a^{4}b^{2}c^{9}d^{5}-6a^{5}bc^{8}d^{6}+a^{6}c^{7}d^{7}}\right)x}{3\left(dx^{2}+c\right)^{\frac{3}{2}}}-\frac{5\left(b^{5}c\sqrt{d}-2ab^{4}d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2ad}{2\sqrt{abcd-a^{2}d^{2}}}\right)}{2\left(a^{3}b^{3}c^{3}-3a^{4}b^{2}c^{2}d+3a^{5}bcd^{2}-a^{6}d^{3}\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{5}c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ab^{4}d^{\frac{3}{2}}-b^{5}c^{2}\sqrt{d}}{\left(a^{3}b^{3}c^{3}-3a^{4}b^{2}c^{2}d+3a^{5}bcd^{2}-a^{6}d^{3}\right)\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)}-\frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}bc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}\sqrt{d}-9\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}acd^{\frac{3}{2}}+3bc^{3}\sqrt{d}+4ac^{2}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(7*b^4*c^7*d^6 - 25*a*b^3*c^6*d^7 + 33*a^2*b^2*c^5*d^8 - 19*a^3*b*c^
4*d^9 + 4*a^4*c^3*d^10)*x^2/(b^6*c^13*d - 6*a*b^5*c^12*d^2 + 15*a^2*b^4*c^1
1*d^3 - 20*a^3*b^3*c^10*d^4 + 15*a^4*b^2*c^9*d^5 - 6*a^5*b*c^8*d^6 + a^6*c^
7*d^7) + 3*(5*b^4*c^8*d^5 - 18*a*b^3*c^7*d^6 + 24*a^2*b^2*c^6*d^7 - 14*a^3*
b*c^5*d^8 + 3*a^4*c^4*d^9)/(b^6*c^13*d - 6*a*b^5*c^12*d^2 + 15*a^2*b^4*c^11
*d^3 - 20*a^3*b^3*c^10*d^4 + 15*a^4*b^2*c^9*d^5 - 6*a^5*b*c^8*d^6 + a^6*c^7
*d^7))*x/(d*x^2 + c)^(3/2) - 5/2*(b^5*c*sqrt(d) - 2*a*b^4*d^(3/2))*arctan(1
/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2
))/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(a*b*c*d
- a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^5*c*sqrt(d) - 2*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*a*b^4*d^(3/2) - b^5*c^2*sqrt(d))/((a^3*b^3*c^3 - 3*a^
4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b -
 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*
a*d + b*c^2)) - 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 3*(sqr
t(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b
*c^2*sqrt(d) - 9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt
(d) + 4*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3*c^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.785 $\int (ex)^{3/2}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=212

$$\frac{4a(11Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^2}}{231b^2}+\frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be}-\frac{2a^{7/4}(11Ab-}{}$$

```
[Out] 2/11*B*(e*x)^(5/2)*(b*x^2+a)^(3/2)/b/e+2/77*(11*A*b-5*B*a)*(e*x)^(5/2)*(b*x
^2+a)^(1/2)/b/e+4/231*a*(11*A*b-5*B*a)*e*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^2-2/
231*a^(7/4)*(11*A*b-5*B*a)*e^(3/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4
)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*Ell
ipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(
1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(
1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {470, 285, 327, 335, 226}

$$-\frac{2a^{7/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(11Ab-5aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{4ae\sqrt{ex}\,\sqrt{a+bx^2}\,(11Ab-5aB)}{231b^2}+\frac{2(ex)^{5/2}\sqrt{a+bx^2}\,(11Ab-5aB)}{77be}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (4*a*(11*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (2*(11*A*b -
 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*B*(e*x)^(5/2)*(a + b*x^2
)^(3/2))/(11*b*e) - (2*a^(7/4)*(11*A*b - 5*a*B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{3/2}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be} - \frac{\left(2\left(-\frac{11Ab}{2}+\frac{5aB}{2}\right)\right)\int (ex)^{3/2}\sqrt{a+bx^2}\,dx}{11b}$$

$$= \frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be} + \frac{(2a(11A$$

$$= \frac{4a(11Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^2}}{231b^2} + \frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be}$$

$$= \frac{4a(11Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^2}}{231b^2} + \frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be}$$

$$= \frac{4a(11Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^2}}{231b^2} + \frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 110, normalized size = 0.52

$$\frac{2e\sqrt{ex}\,\sqrt{a+bx^2}\left(-\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,(-11Ab+5aB-7bBx^2)\right)+a(-11Ab+5aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (2*e*Sqrt[e*x]*Sqrt[a + b*x^2]*(-((a + b*x^2)*Sqrt[1 + (b*x^2)/a]*(-11*A*b
+ 5*a*B - 7*b*B*x^2)) + a*(-11*A*b + 5*a*B)*Hypergeometric2F1[-1/2, 1/4, 5/
4, -((b*x^2)/a)]))/(77*b^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.13, size = 276, normalized size = 1.30

method	result
risch	$\frac{2(21b^2Bx^4+33Ab^2x^2+6Babx^2+22abA-10a^2B)x\sqrt{bx^2+a}\,e^2}{231b^2\sqrt{ex}} - \frac{2a^2(11Ab-5Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2(\cdots}{\cdots}}\cdots}{23\cdots}$
default	$-\frac{2e\sqrt{ex}\left(-21Bb^4x^7+11A\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\cdots\right.\right.}{\cdots}$
elliptic	$\frac{\sqrt{ex}\sqrt{(bx^2+a)ex}\left(\frac{2Bex^4\sqrt{bex^3+aex}}{11}+\frac{2\left((Ab+Ba)e^2-\frac{9Be^2a}{11}\right)x^2\sqrt{bex^3+aex}}{7be}+\frac{2\left(Aae^2-\frac{5\left((Ab+Ba)e^2-\frac{9\cdots}{}\right)}{7b}\cdots\right)}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231*e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-21*B*b^4*x^7+11*A*(-a*b)^(1/2)*((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1
```

```
/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))*a^2*b-33*A*b^4*x^5-5*B*(-a*b)^(1/2)*((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*a^3-27*B*a*b^3*x^5-55*A*a*b^3*x^3+4*B*a^2*b^2*x^3-22*A*a^2*b^
2*x+10*B*a^3*b*x)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 94, normalized size = 0.44

$$\frac{2\left(2\,(5\,Ba^3-11\,Aa^2b)\sqrt{b}\,e^{\frac{3}{2}}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+(21\,Bb^3x^4-10\,Ba^2b+22\,Aab^2+3\,(2\,Bab^2+11\,Ab^3)x^2)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{3}{2}}\right)}{231\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(2*(5*B*a^3 - 11*A*a^2*b)*sqrt(b)*e^(3/2)*weierstrassPInverse(-4*a/b,
 0, x) + (21*B*b^3*x^4 - 10*B*a^2*b + 22*A*a*b^2 + 3*(2*B*a*b^2 + 11*A*b^3)
*x^2)*sqrt(b*x^2 + a)*sqrt(x)*e^(3/2))/b^3
```

Sympy [C] Result contains complex when optimal does not.
time = 6.48, size = 97, normalized size = 0.46

$$\frac{A\sqrt{a}\,e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{B\sqrt{a}\,e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(9/4)) + B*sqrt(a)*e**(3/2)*x**(9/2)*gamma(9/4)*hy
per((-1/2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*x^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^2+A\right)\,{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(e*x)^(3/2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int((A + B*x^2)*(e*x)^(3/2)*(a + b*x^2)^(1/2), x)
```

3.786 $\int \sqrt{ex}\,\sqrt{a+bx^2}\,(A+Bx^2)\,dx$

Optimal. Leaf size=337

$$\frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be}+\frac{4a(3Ab-aB)\sqrt{ex}\,\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}-\frac{4a^{5/4}(3Ab-aB)}{}$$

```
[Out] 2/9*B*(e*x)^(3/2)*(b*x^2+a)^(3/2)/b/e+2/15*(3*A*b-B*a)*(e*x)^(3/2)*(b*x^2+a
)^(1/2)/b/e+4/15*a*(3*A*b-B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/(a^(1/2)
+x*b^(1/2))-4/15*a^(5/4)*(3*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1
/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*E
llipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a
^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(
b*x^2+a)^(1/2)+2/15*a^(5/4)*(3*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a
^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))
)*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))
*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4
)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 285, 335, 311, 226, 1210}

$$\frac{2a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(3Ab-aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}-\frac{4a^{5/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(3Ab-aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}+\frac{4a\sqrt{ex}\,\sqrt{a+bx^2}\,(3Ab-aB)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}\,(3Ab-aB)}{15be}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (2*(3*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*b*e) + (4*a*(3*A*b - a*B)
*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + (2*B*(e*x)
^(3/2)*(a + b*x^2)^(3/2))/(9*b*e) - (4*a^(5/4)*(3*A*b - a*B)*Sqrt[e]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2]
) + (2*a^(5/4)*(3*A*b - a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)dx = \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{3aB}{2}\right)\right)\int\sqrt{ex}\sqrt{a+bx^2}\,dx}{9b}$$

$$= \frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be} + \frac{(2a(3Ab-}{}$$

$$= \frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be} + \frac{(4a(3Ab-}{}$$

$$= \frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be} + \frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be} + \frac{(4a^{3/2}(3Ab}{}$$

$$= \frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be} + \frac{4a(3Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{2B}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 93, normalized size = 0.28

$$\frac{2x\sqrt{ex}\sqrt{a+bx^2}\left(B(a+bx^2)\sqrt{1+\frac{bx^2}{a}}+(3Ab-aB)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{9b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (2*x*Sqrt[e*x]*Sqrt[a + b*x^2]*(B*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (3*A*b
- a*B)*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x
^2)/a])
```

Maple [A]
time = 0.12, size = 414, normalized size = 1.23

method	result
risch	$\frac{2x^2(5bB\,x^2+9Ab+2Ba)\sqrt{b\,x^2+a}\,e}{45b\sqrt{ex}}+\frac{2a(3Ab-Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-a}}}}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(\frac{2B\,x^3\sqrt{be\,x^3+aex}}{9}+\frac{2\left((Ab+Ba)e-\frac{7Bae}{9}\right)x\sqrt{be\,x^3+aex}}{5be}+\frac{\left(Aae-\frac{3\left((Ab+Ba)e-\frac{7Bae}{9}\right)a}{5b}\right)\sqrt{}}{}\right.}{}$
default	$\frac{2\sqrt{ex}\left(5B\,b^3x^6+18A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2b-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(e*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(5*B*b^3*x^6+18*A*((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/
(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*a^2*b-9*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-6*B*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b
/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
```

```
2^(1/2))*a^3+3*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+9*A*b^3*x^4+7*B*a*b^2*x^4+
9*A*a*b^2*x^2+2*B*a^2*b*x^2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 80, normalized size = 0.24

$$\frac{2\left(6\,(Ba^2-3\,Aab)\sqrt{b}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(5\,Bb^2x^3+(2\,Bab+9\,Ab^2)x\right)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{1}{2}}\right)}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(6*(B*a^2 - 3*A*a*b)*sqrt(b)*e^(1/2)*weierstrassZeta(-4*a/b, 0, weiers
trassPInverse(-4*a/b, 0, x)) + (5*B*b^2*x^3 + (2*B*a*b + 9*A*b^2)*x)*sqrt(b
*x^2 + a)*sqrt(x)*e^(1/2))/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.78, size = 95, normalized size = 0.28

$$\frac{A\sqrt{a}\,(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e\Gamma\left(\frac{7}{4}\right)}+\frac{B\sqrt{a}\,(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^3\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x)**(1/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_pol
ar(I*pi)/a)/(2*e*gamma(7/4)) + B*sqrt(a)*(e*x)**(7/2)*gamma(7/4)*hyper((-1/
2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**3*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(x)*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^2+A\right)\,\sqrt{e\,x}\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(e*x)^(1/2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int((A + B*x^2)*(e*x)^(1/2)*(a + b*x^2)^(1/2), x)
```

3.787 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=176

$$\frac{2(7Ab-aB)\sqrt{ex}\,\sqrt{a+bx^2}}{21be}+\frac{2B\sqrt{ex}\,(a+bx^2)^{3/2}}{7be}+\frac{2a^{3/4}(7Ab-aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}}{21b^{5/4}\sqrt{e}\,\sqrt{a+bx^2}}$$

```
[Out] 2/7*B*(b*x^2+a)^(3/2)*(e*x)^(1/2)/b/e+2/21*(7*A*b-B*a)*(e*x)^(1/2)*(b*x^2+a
)^(1/2)/b/e+2/21*a^(3/4)*(7*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1
/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*E
llipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a
^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/e^(1/2)/(
b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 285, 335, 226}

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(7Ab-aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{e}\,\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\,\sqrt{a+bx^2}\,(7Ab-aB)}{21be}+\frac{2B\sqrt{ex}\,(a+bx^2)^{3/2}}{7be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (2*(7*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b*e) + (2*B*Sqrt[e*x]*(a +
b*x^2)^(3/2))/(7*b*e) + (2*a^(3/4)*(7*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(21*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{\sqrt{ex}}\,dx = \frac{2B\sqrt{ex}\,(a+bx^2)^{3/2}}{7be} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{ex}}\,dx}{7b}$$

$$= \frac{2(7Ab-aB)\sqrt{ex}\,\sqrt{a+bx^2}}{21be} + \frac{2B\sqrt{ex}\,(a+bx^2)^{3/2}}{7be} + \frac{(2a(7Ab-aB))\int \frac{}{\sqrt{e}}}{21b}$$

$$= \frac{2(7Ab-aB)\sqrt{ex}\,\sqrt{a+bx^2}}{21be} + \frac{2B\sqrt{ex}\,(a+bx^2)^{3/2}}{7be} + \frac{(4a(7Ab-aB))\text{Subst}}{}$$

$$= \frac{2(7Ab-aB)\sqrt{ex}\,\sqrt{a+bx^2}}{21be} + \frac{2B\sqrt{ex}\,(a+bx^2)^{3/2}}{7be} + \frac{2a^{3/4}(7Ab-aB)\left(\sqrt{a}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 93, normalized size = 0.53

$$\frac{2x\sqrt{a+bx^2}\left(B(a+bx^2)\sqrt{1+\frac{bx^2}{a}}+(7Ab-aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{7b\sqrt{ex}\,\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(B*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (7*A*b - a*B)*Hyp
ergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[e*x]*Sqrt[1 + (b*x
^2)/a])
```

Maple [A]
time = 0.11, size = 246, normalized size = 1.40

method	result
risch	$\frac{2(3bBx^2+7Ab+2Ba)x\sqrt{bx^2+a}}{21b\sqrt{ex}}+\frac{2a(7Ab-Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-a}}}}{21b^2\sqrt{bex^3+aex}\sqrt{ex}}$
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(\frac{2Bx^2\sqrt{bex^3+aex}}{7e}+\frac{2\left(Ab+\frac{2Ba}{7}\right)\sqrt{bex^3+aex}}{3be}+\frac{\left(Aa-\frac{a\left(Ab+\frac{2Ba}{7}\right)}{3b}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-a}}}}{}\right.}{\sqrt{ex}\sqrt{bx^2+a}}$
default	$\frac{\frac{2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,ab}{3}-\frac{2B\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/(b*x^2+a)^(1/2)*(7*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a*b-B*((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2+3*B*b^3*x^5+7*A*b^3*x^3+5*B*a*b^
2*x^3+7*A*a*b^2*x+2*B*a^2*b*x)/(e*x)^(1/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 68, normalized size = 0.39

$$-\frac{2\left(2\,(Ba^2-7\,Aab)\sqrt{b}\,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(3\,Bb^2x^2+2\,Bab+7\,Ab^2)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{21\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21*(2*(B*a^2 - 7*A*a*b)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (3*B
*b^2*x^2 + 2*B*a*b + 7*A*b^2)*sqrt(b*x^2 + a)*sqrt(x))*e^(-1/2)/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.85, size = 97, normalized size = 0.55

$$\frac{A\sqrt{a}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{B\sqrt{a}\,x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(e)*gamma(5/4)) + B*sqrt(a)*x**(5/2)*gamma(5/4)*hyper((-1/2,
5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*e^(-1/2)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(1/2),x)
[Out] int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(1/2), x)
```

3.788 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=333

$$\frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3}+\frac{4(5Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2A(a+bx^2)^{3/2}}{ae\sqrt{ex}}-\frac{4\sqrt[4]{a}\,(5Ab+aB)\left(\sqrt{a}+\sqrt{\right.}}{}$$

```
[Out] -2*A*(b*x^2+a)^(3/2)/a/e/(e*x)^(1/2)+2/5*(5*A*b+B*a)*(e*x)^(3/2)*(b*x^2+a)^
(1/2)/a/e^3+4/5*(5*A*b+B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/e^2/b^(1/2)/(a^(1/2
)+x*b^(1/2))-4/5*a^(1/4)*(5*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1
/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*E
llipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a
^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/e^(3/2)/(
b*x^2+a)^(1/2)+2/5*a^(1/4)*(5*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^
(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))
*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*
(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/e^(3/2)
/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 285, 335, 311, 226, 1210}

$$\frac{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(aB+5Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(aB+5Ab)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}\,(aB+5Ab)}{5ae^3}+\frac{4\sqrt{ex}\,\sqrt{a+bx^2}\,(aB+5Ab)}{5\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2A(a+bx^2)^{3/2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(3/2),x]
```

```
[Out] (2*(5*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^3) + (4*(5*A*b + a*B)*
Sqrt[e*x]*Sqrt[a + b*x^2])/(5*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(a
+ b*x^2)^(3/2))/(a*e*Sqrt[e*x]) - (4*a^(1/4)*(5*A*b + a*B)*(Sqrt[a] + Sqrt[
b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)
*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) +
 (2*a^(1/4)*(5*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1
/2])/(5*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{(ex)^{3/2}}\,dx = -\frac{2A(a+bx^2)^{3/2}}{ae\sqrt{ex}} + \frac{(5Ab+aB)\int \sqrt{ex}\,\sqrt{a+bx^2}\,dx}{ae^2}$$

$$= \frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3} - \frac{2A(a+bx^2)^{3/2}}{ae\sqrt{ex}} + \frac{(2(5Ab+aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx}}}{5e^2}$$

$$= \frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3} - \frac{2A(a+bx^2)^{3/2}}{ae\sqrt{ex}} + \frac{(4(5Ab+aB))\text{Subst}\left(\int -\right.}{5e}$$

$$= \frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3} - \frac{2A(a+bx^2)^{3/2}}{ae\sqrt{ex}} + \frac{\left(4\sqrt{a}\,(5Ab+aB)\right)\text{Subst}}{5}$$

$$= \frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3} + \frac{4(5Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2A(a+bx^2)}{ae\sqrt{ex}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.54, size = 96, normalized size = 0.29

$$\frac{2x\sqrt{a+bx^2}\left(-3A(a+bx^2)\sqrt{1+\frac{bx^2}{a}} + (5Ab+aB)x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a(ex)^{3/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(-3*A*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (5*A*b + a*B)*
x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(3*a*(e*x)^(3/2)*Sqrt
[1 + (b*x^2)/a])
```

Maple [A]
time = 0.12, size = 391, normalized size = 1.17

method	result
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-B\,x^{2}+5A\right)}{5e\sqrt{ex}}+\frac{\left(2Ab+\frac{2Ba}{5}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\left(\right.}{b\sqrt{}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)ex}\,\left(-\frac{2\left(be\,x^{2}+ae\right)A}{e^{2}\sqrt{x\left(be\,x^{2}+ae\right)}}+\frac{2Bx\sqrt{be\,x^{3}+aex}}{5e^{2}}+\frac{\left(\frac{Ab+Ba}{e}+\frac{bA}{e}-\frac{3Ba}{5e}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}}{}\right.}{}$
default	$\frac{4A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab-2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(10*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a
*b+2*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))
```

```
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^
(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+b^
2*B*x^4-5*A*b^2*x^2+B*a*b*x^2-5*a*b*A)/(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 67, normalized size = 0.20

$$-\frac{2\left(2\,(Ba+5\,Ab)\sqrt{b}\,x\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-(Bbx^2-5\,Ab)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{5\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(2*(B*a + 5*A*b)*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInve
rse(-4*a/b, 0, x)) - (B*b*x^2 - 5*A*b)*sqrt(b*x^2 + a)*sqrt(x))*e^(-3/2)/(b
*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.88, size = 100, normalized size = 0.30

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{B\sqrt{a}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(3/2),x)
```

```
[Out] A*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)
/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + B*sqrt(a)*x**(3/2)*gamma(3/4)*hyper((-1/
2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*e^(-3/2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{(e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(3/2),x)
[Out] int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(3/2), x)
```

3.789 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=172

$$\frac{2(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{3ae^3}-\frac{2A(a+bx^2)^{3/2}}{3ae(ex)^{3/2}}+\frac{2(Ab+aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{3\sqrt[4]{a}\sqrt[4]{b}\,e^{5/2}\sqrt{a+bx^2}}$$

```
[Out] -2/3*A*(b*x^2+a)^(3/2)/a/e/(e*x)^(3/2)+2/3*(A*b+B*a)*(e*x)^(1/2)*(b*x^2+a)^
(1/2)/a/e^3+2/3*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(si
n(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/e^(5/2)/(b*x
^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$,
Rules used = {464, 285, 335, 226}

$$\frac{2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(aB+Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt[4]{b}\,e^{5/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}(aB+Ab)}{3ae^3}-\frac{2A(a+bx^2)^{3/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (2*(A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*a*e^3) - (2*A*(a + b*x^2)^(3/2
))/(3*a*e*(e*x)^(3/2)) + (2*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x
^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4
)*Sqrt[e])], 1/2])/(3*a^(1/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{(ex)^{5/2}}\,dx = -\frac{2A(a+bx^2)^{3/2}}{3ae(ex)^{3/2}} + \frac{(Ab+aB)\int \frac{\sqrt{a+bx^2}}{\sqrt{ex}}\,dx}{ae^2}$$

$$= \frac{2(Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{3ae^3} - \frac{2A(a+bx^2)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2(Ab+aB))\int \frac{1}{\sqrt{ex}\,\sqrt{a+}}}{3e^2}$$

$$= \frac{2(Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{3ae^3} - \frac{2A(a+bx^2)^{3/2}}{3ae(ex)^{3/2}} + \frac{(4(Ab+aB))\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{3e^3}$$

$$= \frac{2(Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{3ae^3} - \frac{2A(a+bx^2)^{3/2}}{3ae(ex)^{3/2}} + \frac{2(Ab+aB)\left(\sqrt{a}+\sqrt{b}\,x\right)}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 9.82, size = 82, normalized size = 0.48

$$\frac{2x\sqrt{a+bx^2}\left(-A(a+bx^2)+\frac{3(Ab+aB)x^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{3a(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(-(A*(a + b*x^2)) + (3*(A*b + a*B)*x^2*Hypergeometric2
F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(3*a*(e*x)^(5/2))
```

Maple [A]
time = 0.12, size = 234, normalized size = 1.36

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-B\,x^2+A)}{3x\,e^2\sqrt{ex}}+\frac{\left(\frac{2Ab}{3}+\frac{2Ba}{3}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\;\mathrm{Ellip}}{b\sqrt{be\,x^3+aex}\;e^2\sqrt{ex}\;\sqrt{b\,x^2+}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(-\frac{2A\sqrt{be\,x^3+aex}}{3e^3x^2}+\frac{2B\sqrt{be\,x^3+aex}}{3e^3}+\frac{\left(\frac{Ab+Ba}{e^2}-\frac{bA}{3e^2}-\frac{Ba}{3e^2}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}}{}\right.}{\sqrt{ex}\;\sqrt{b\,x^2+a}}$
default	$\frac{\frac{2A\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\;\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)bx}{3}+\frac{2B\sqrt{-ab}}{}}{\sqrt{b\,x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)/x*(A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/
2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*b*x+B*(-a
*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
```

```
2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*x+b^2*B*x^4-A*b^2*x^2+B*a*b*x^2-a*b
*A)/e^2/(e*x)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
[Out] e^(-5/2)*integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 59, normalized size = 0.34

$$\frac{2\left(2\left(Ba+Ab\right)\sqrt{b}\,x^2\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(Bbx^2-Ab\right)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{3\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
[Out] 2/3*(2*(B*a + A*b)*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) + (B*b*x^2
 - A*b)*sqrt(b*x^2 + a)*sqrt(x))*e^(-5/2)/(b*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.83, size = 100, normalized size = 0.58

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{B\sqrt{a}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(5/2),x)
[Out] A*sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)
/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/
2, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*e^(-5/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}}{{\left(e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(5/2),x)
[Out] int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(5/2), x)
```

3.790 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=338

$$-\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}}+\frac{4\sqrt{b}\,(Ab+5aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5ae^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2A(a+bx^2)^{3/2}}{5ae(ex)^{5/2}}-\frac{4\sqrt[4]{b}\,(Ab+5aB)\left(\sqrt{a}+\cdots\right.}{\cdots}$$

```
[Out] -2/5*A*(b*x^2+a)^(3/2)/a/e/(e*x)^(5/2)-2/5*(A*b+5*B*a)*(b*x^2+a)^(1/2)/a/e^
3/(e*x)^(1/2)+4/5*(A*b+5*B*a)*b^(1/2)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a/e^4/(a^
(1/2)+x*b^(1/2))-4/5*b^(1/4)*(A*b+5*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/
a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2)
)*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/e^(7/
2)/(b*x^2+a)^(1/2)+2/5*b^(1/4)*(A*b+5*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2
)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/
2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/
2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/e^(
7/2)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 283, 335, 311, 226, 1210}

$$\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(5aB+Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(5aB+Ab)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}+\frac{4\sqrt{b}\,\sqrt{ex}\,\sqrt{a+bx^2}\,(5aB+Ab)}{5ae^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2\sqrt{a+bx^2}\,(5aB+Ab)}{5ae^3\sqrt{ex}}-\frac{2A(a+bx^2)^{3/2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(7/2),x]
```

```
[Out] (-2*(A*b + 5*a*B)*Sqrt[a + b*x^2])/(5*a*e^3*Sqrt[e*x]) + (4*Sqrt[b]*(A*b +
5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a*e^4*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(a
 + b*x^2)^(3/2))/(5*a*e*(e*x)^(5/2)) - (4*b^(1/4)*(A*b + 5*a*B)*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^
(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^
2]) + (2*b^(1/4)*(A*b + 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(5*a^(3/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{(ex)^{7/2}}\,dx = -\frac{2A(a+bx^2)^{3/2}}{5ae(ex)^{5/2}} + \frac{(Ab+5aB)\int \frac{\sqrt{a+bx^2}}{(ex)^{3/2}}\,dx}{5ae^2}$$

$$= -\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}} - \frac{2A(a+bx^2)^{3/2}}{5ae(ex)^{5/2}} + \frac{(2b(Ab+5aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}$$

$$= -\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}} - \frac{2A(a+bx^2)^{3/2}}{5ae(ex)^{5/2}} + \frac{(4b(Ab+5aB))\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{5ae^5}$$

$$= -\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}} - \frac{2A(a+bx^2)^{3/2}}{5ae(ex)^{5/2}} + \frac{\left(4\sqrt{b}\,(Ab+5aB)\right)\text{Subst}\left(\int\right.}{5\sqrt{a}}$$

$$= -\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}} + \frac{4\sqrt{b}\,(Ab+5aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5ae^4\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2A(a+bx^2}{5ae(ex)^{5/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 95, normalized size = 0.28

$$-\frac{2x\sqrt{a+bx^2}\left(A(a+bx^2)\sqrt{1+\frac{bx^2}{a}}+(Ab+5aB)x^2\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)\right)}{5a(ex)^{7/2}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(7/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*(A*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (A*b + 5*a*B)*x^
2*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[
1 + (b*x^2)/a])
```

Maple [A]
time = 0.12, size = 417, normalized size = 1.23

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,(2Ab\,x^2+5Ba\,x^2+Aa)}{5x^2a\,e^3\sqrt{ex}}+\frac{2(Ab+5Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-a \cdots}}}\cdots}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(-\frac{2A\sqrt{be\,x^3+aex}}{5e^4x^3}-\frac{2\left(be\,x^2+ae\right)(2Ab+5Ba)}{5e^4a\sqrt{x\,(be\,x^2+ae)}}+\frac{\left(\frac{Bb}{e^3}+\frac{b(2Ab+5Ba)}{5a\,e^3}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\cdots}{\cdots}\cdots\right.}{\cdots}$
default	$\frac{\frac{4A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\;\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2}{5}-\frac{2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\cdots}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/x^2*(2*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-A*((b*x+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))*a*b*x^2+10*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
```

```
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*x^2-5*B*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*a^2*x^2-2*A*b^2*x^4-5*B*a*b*x^4-3*a*A*b*x^2-5*B*a^2*x^2-a^2*A)/(b
*x^2+a)^(1/2)/e^3/(e*x)^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
[Out] e^(-7/2)*integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 74, normalized size = 0.22

$$-\frac{2\left(2\,(5\,Ba+Ab)\sqrt{b}\,x^3\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+((5\,Ba+2\,Ab)x^2+Aa)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{5\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
[Out] -2/5*(2*(5*B*a + A*b)*sqrt(b)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPIn
verse(-4*a/b, 0, x)) + ((5*B*a + 2*A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt(x))
*e^(-7/2)/(a*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.12, size = 107, normalized size = 0.32

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(7/2),x)
[Out] A*sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a
)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + B*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1
/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(7/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)*e^(-7/2)/x^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{(e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(7/2),x)
[Out] int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(7/2), x)
```

3.791 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{9/2}}\,dx$

Optimal. Leaf size=152

$$\frac{2(Ab-7aB)\sqrt{a+bx^2}}{21ax^{3/2}}-\frac{2A(a+bx^2)^{3/2}}{7ax^{7/2}}-\frac{2b^{3/4}(Ab-7aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\right.}{21a^{5/4}\sqrt{a+bx^2}}$$

```
[Out] -2/7*A*(b*x^2+a)^(3/2)/a/x^(7/2)+2/21*(A*b-7*B*a)*(b*x^2+a)^(1/2)/a/x^(3/2)
-2/21*b^(3/4)*(A*b-7*B*a)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/a^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {464, 283, 335, 226}

$$-\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(Ab-7aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}\,(Ab-7aB)}{21ax^{3/2}}-\frac{2A(a+bx^2)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(9/2),x]
```

```
[Out] (2*(A*b - 7*a*B)*Sqrt[a + b*x^2])/(21*a*x^(3/2)) - (2*A*(a + b*x^2)^(3/2))/
(7*a*x^(7/2)) - (2*b^(3/4)*(A*b - 7*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]
, 1/2])/(21*a^(5/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{9/2}}\,dx = -\frac{2A(a+bx^2)^{3/2}}{7ax^{7/2}} - \frac{\left(2\left(\frac{Ab}{2}-\frac{7aB}{2}\right)\right)\int\frac{\sqrt{a+bx^2}}{x^{5/2}}\,dx}{7a}$$
$$= \frac{2(Ab-7aB)\sqrt{a+bx^2}}{21ax^{3/2}} - \frac{2A(a+bx^2)^{3/2}}{7ax^{7/2}} - \frac{(2b(Ab-7aB))\int\frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}$$
$$= \frac{2(Ab-7aB)\sqrt{a+bx^2}}{21ax^{3/2}} - \frac{2A(a+bx^2)^{3/2}}{7ax^{7/2}} - \frac{(4b(Ab-7aB))\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+b}}\right.}{21a}$$
$$= \frac{2(Ab-7aB)\sqrt{a+bx^2}}{21ax^{3/2}} - \frac{2A(a+bx^2)^{3/2}}{7ax^{7/2}} - \frac{2b^{3/4}(Ab-7aB)\left(\sqrt{a}+\sqrt{b}\,x\right)}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 79, normalized size = 0.52

$$\frac{2\sqrt{a+bx^2}\left(-3A(a+bx^2)+\frac{(Ab-7aB)x^2\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{21ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(9/2),x]
```

```
[Out] (2*Sqrt[a + b*x^2]*(-3*A*(a + b*x^2) + ((A*b - 7*a*B)*x^2*Hypergeometric2F1
[-3/4, -1/2, 1/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(21*a*x^(7/2))
```

Maple [A]
time = 0.12, size = 242, normalized size = 1.59

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,(2Ab\,x^2+7Ba\,x^2+3Aa)}{21x^{\frac{7}{2}}a}-\frac{2(Ab-7Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\cdots}}{21a\sqrt{b\,x^3+ax}\,\sqrt{x}\cdots}$
elliptic	$\frac{\sqrt{x\,(b\,x^2+a)}\left(-\frac{2A\sqrt{b\,x^3+ax}}{7x^4}-\frac{2(2Ab+7Ba)\sqrt{b\,x^3+ax}}{21a\,x^2}+\frac{\left(Bb-\frac{b(2Ab+7Ba)}{21a}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\cdots}{\cdots}\right.}{\sqrt{x}\,\sqrt{b\,x^2+a}\cdots}$
default	$-\frac{2\left(A\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b\,x^3-7B\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/(b*x^2+a)^(1/2)*(A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/
2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*b*x^3-7*B
*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*x^3+2*A*b^2*x^4+7*B*a*b*x^4+5*a
*A*b*x^2+7*B*a^2*x^2+3*a^2*A)/x^(7/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 67, normalized size = 0.44

$$\frac{2\left(2\,(7\,Ba-Ab)\sqrt{b}\,x^4\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-((7\,Ba+2\,Ab)x^2+3\,Aa)\sqrt{bx^2+a}\,\sqrt{x}\right)}{21\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="fricas")
[Out] 2/21*(2*(7*B*a - A*b)*sqrt(b)*x^4*weierstrassPInverse(-4*a/b, 0, x) - ((7*B
*a + 2*A*b)*x^2 + 3*A*a)*sqrt(b*x^2 + a)*sqrt(x))/(a*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.72, size = 97, normalized size = 0.64

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(9/2),x)
[Out] A*sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**2*exp_polar(I*pi)/a
)/(2*x**(7/2)*gamma(-3/4)) + B*sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4
,), b*x**2*exp_polar(I*pi)/a)/(2*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(9/2),x)
[Out] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(9/2), x)
```

3.792 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{11/2}}\,dx$

Optimal. Leaf size=331

$$\frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}}+\frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}}-\frac{4b^{3/2}(Ab-3aB)\sqrt{x}\,\sqrt{a+bx^2}}{15a^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}}+\frac{4b^{5/}}{}$$

```
[Out] -2/9*A*(b*x^2+a)^(3/2)/a/x^(9/2)+2/15*(A*b-3*B*a)*(b*x^2+a)^(1/2)/a/x^(5/2)
+4/15*b*(A*b-3*B*a)*(b*x^2+a)^(1/2)/a^2/x^(1/2)-4/15*b^(3/2)*(A*b-3*B*a)*x^
(1/2)*(b*x^2+a)^(1/2)/a^2/(a^(1/2)+x*b^(1/2))+4/15*b^(5/4)*(A*b-3*B*a)*(cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a
^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^2+a)^(
1/2)-2/15*b^(5/4)*(A*b-3*B*a)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x
^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(
1/2))^2)^(1/2)/a^(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {464, 283, 331, 335, 311, 226, 1210}

$$-\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(Ab-3aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}}+\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(Ab-3aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}}-\frac{4b^{3/2}\sqrt{x}\,\sqrt{a+bx^2}\,(Ab-3aB)}{15a^2\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{4b\sqrt{a+bx^2}\,(Ab-3aB)}{15a^2\sqrt{x}}+\frac{2\sqrt{a+bx^2}\,(Ab-3aB)}{15ax^{5/2}}-\frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(11/2),x]
```

```
[Out] (2*(A*b - 3*a*B)*Sqrt[a + b*x^2])/(15*a*x^(5/2)) + (4*b*(A*b - 3*a*B)*Sqrt[
a + b*x^2])/(15*a^2*Sqrt[x]) - (4*b^(3/2)*(A*b - 3*a*B)*Sqrt[x]*Sqrt[a + b*
x^2])/(15*a^2*(Sqrt[a] + Sqrt[b]*x)) - (2*A*(a + b*x^2)^(3/2))/(9*a*x^(9/2)
) + (4*b^(5/4)*(A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*
a^(7/4)*Sqrt[a + b*x^2]) - (2*b^(5/4)*(A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(15*a^(7/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

}, x] && PosQ[c/a]

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{11/2}}\,dx = -\frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}} - \frac{\left(2\left(\frac{3Ab}{2}-\frac{9aB}{2}\right)\right)\int \frac{\sqrt{a+bx^2}}{x^{7/2}}\,dx}{9a}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} - \frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}} - \frac{(2b(Ab-3aB))\int \frac{1}{x^{3/2}\sqrt{a+bx^2}}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} + \frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}} - \frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}} - \frac{(2b}{}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} + \frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}} - \frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}} - \frac{(4b}{}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} + \frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}} - \frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}} - \frac{(4b}{}$$

$$= \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} + \frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}} - \frac{4b^{3/2}(Ab-3aB)\sqrt{x}}{15a^2\left(\sqrt{a}+\sqrt{b}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 80, normalized size = 0.24

$$\frac{2\sqrt{a+bx^2}\left(-5A(a+bx^2)+\frac{3(Ab-3aB)x^2\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{45ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(11/2),x]
```

```
[Out] (2*Sqrt[a + b*x^2]*(-5*A*(a + b*x^2) + (3*(A*b - 3*a*B)*x^2*Hypergeometric2
F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(45*a*x^(9/2))
```

Maple [A]
time = 0.12, size = 439, normalized size = 1.33

method	result
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-6A\,b^{2}x^{4}+18Bab\,x^{4}+2aAb\,x^{2}+9B\,a^{2}x^{2}+5a^{2}A\right)}{45x^{\frac{9}{2}}a^{2}}-\frac{2b(Ab-3Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\right.}{\sqrt{}}}}{}$
elliptic	$\frac{\sqrt{x\,(b\,x^{2}+a)}\left(-\frac{2A\sqrt{b\,x^{3}+ax}}{9x^{5}}-\frac{2(2Ab+9Ba)\sqrt{b\,x^{3}+ax}}{45a\,x^{3}}+\frac{4\left(b\,x^{2}+a\right)b(Ab-3Ba)}{15a^{2}\sqrt{x\,(b\,x^{2}+a)}}-\frac{2b(Ab-3Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\right.}{}}}{}\right.}{}$
default	$-\frac{2\left(6A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^{2}x^{4}-3A\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*x^4-3*A*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^2*x^4-18*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
```

```
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*x^4+9*B*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*a^2*b*x^4-6*A*b^3*x^6+18*B*a*b^2*x^6-4*A*a*b^2*x^4+27*B*a^2
*b*x^4+7*A*a^2*b*x^2+9*B*a^3*x^2+5*A*a^3)/(b*x^2+a)^(1/2)/x^(9/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 99, normalized size = 0.30

$$-\frac{2\left(6\,(3\,Bab-Ab^2)\sqrt{b}\,x^5\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(6\,(3\,Bab-Ab^2)x^4+5\,Aa^2+(9\,Ba^2+2\,Aab)x^2\right)\sqrt{bx^2+a}\,\sqrt{x}\right)}{45\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*(6*(3*B*a*b - A*b^2)*sqrt(b)*x^5*weierstrassZeta(-4*a/b, 0, weierstra
ssPInverse(-4*a/b, 0, x)) + (6*(3*B*a*b - A*b^2)*x^4 + 5*A*a^2 + (9*B*a^2 +
 2*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(x))/(a^2*x^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 32.69, size = 100, normalized size = 0.30

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{9}{2}}\Gamma\left(-\frac{5}{4}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(11/2),x)
```

```
[Out] A*sqrt(a)*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**2*exp_polar(I*pi)/a
)/(2*x**(9/2)*gamma(-5/4)) + B*sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/
4,), b*x**2*exp_polar(I*pi)/a)/(2*x**(5/2)*gamma(-1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(11/2),x)
[Out] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(11/2), x)
```

3.793 $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{13/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}}+\frac{4b(5Ab-11aB)\sqrt{a+bx^2}}{231a^2x^{3/2}}-\frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}}+\frac{2b^{7/4}(5Ab-11aB)\left(\sqrt{a}+\sqrt{b}\,x\right.}{\quad}$$

```
[Out] -2/11*A*(b*x^2+a)^(3/2)/a/x^(11/2)+2/77*(5*A*b-11*B*a)*(b*x^2+a)^(1/2)/a/x^
(7/2)+4/231*b*(5*A*b-11*B*a)*(b*x^2+a)^(1/2)/a^2/x^(3/2)+2/231*b^(7/4)*(5*A
*b-11*B*a)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/
2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9
/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {464, 283, 331, 335, 226}

$$\frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(5Ab-11aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231a^{9/4}\sqrt{a+bx^2}}+\frac{4b\sqrt{a+bx^2}\,(5Ab-11aB)}{231a^2x^{3/2}}+\frac{2\sqrt{a+bx^2}\,(5Ab-11aB)}{77ax^{7/2}}-\frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(13/2),x]
```

```
[Out] (2*(5*A*b - 11*a*B)*Sqrt[a + b*x^2])/(77*a*x^(7/2)) + (4*b*(5*A*b - 11*a*B)
*Sqrt[a + b*x^2])/(231*a^2*x^(3/2)) - (2*A*(a + b*x^2)^(3/2))/(11*a*x^(11/2
)) + (2*b^(7/4)*(5*A*b - 11*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(231*a^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{13/2}}\,dx &= -\frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}} - \frac{\left(2\left(\frac{5Ab}{2}-\frac{11aB}{2}\right)\right)\int \frac{\sqrt{a+bx^2}}{x^{9/2}}\,dx}{11a} \\
&= \frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}} - \frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}} - \frac{(2b(5Ab-11aB))\int \frac{1}{x^{5/2}\sqrt{a+}}}{77a} \\
&= \frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}} + \frac{4b(5Ab-11aB)\sqrt{a+bx^2}}{231a^2x^{3/2}} - \frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}} + \\
&= \frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}} + \frac{4b(5Ab-11aB)\sqrt{a+bx^2}}{231a^2x^{3/2}} - \frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}} + \\
&= \frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}} + \frac{4b(5Ab-11aB)\sqrt{a+bx^2}}{231a^2x^{3/2}} - \frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}} +
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 80, normalized size = 0.43

$$\frac{2\sqrt{a+bx^2}\left(-7A(a+bx^2)+\frac{(5Ab-11aB)x^2\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{77ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(13/2),x]
```

```
[Out] (2*Sqrt[a + b*x^2]*(-7*A*(a + b*x^2) + ((5*A*b - 11*a*B)*x^2*Hypergeometric
2F1[-7/4, -1/2, -3/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(77*a*x^(11/2))
```

Maple [A]
time = 0.12, size = 270, normalized size = 1.44

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,\left(-10A\,b^2x^4+22Bab\,x^4+6aAb\,x^2+33B\,a^2x^2+21a^2A\right)}{231x^{\frac{11}{2}}\,a^2}+\frac{2b(5Ab-11Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\cdots}}{\cdots}$
elliptic	$\frac{\sqrt{x\,(b\,x^2+a)}\left(-\frac{2A\sqrt{b\,x^3+ax}}{11x^6}-\frac{2(2Ab+11Ba)\sqrt{b\,x^3+ax}}{77a\,x^4}+\frac{4b(5Ab-11Ba)\sqrt{b\,x^3+ax}}{231a^2x^2}+\frac{2b(5Ab-11Ba)\sqrt{-a\cdots}}{\cdots}\right.}{\sqrt{x}\,\sqrt{b\,x^2+\cdots}}$
default	$\frac{\frac{10A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b^2x^5}{231}-2B\sqrt{\frac{bx\cdots}{\cdots}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231/(b*x^2+a)^(1/2)*(5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*
((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elliptic
```

```
F(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b^2*x^5
-11*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a*b*x^5+10*A*b^3*x^6-22*B*a*
b^2*x^6+4*A*a*b^2*x^4-55*B*a^2*b*x^4-27*A*a^2*b*x^2-33*B*a^3*x^2-21*A*a^3)/
x^(11/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 92, normalized size = 0.49

$$-\frac{2\left(2\,(11\,Bab-5\,Ab^{2})\sqrt{b}\,x^{6}\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(2\,(11\,Bab-5\,Ab^{2})x^{4}+21\,Aa^{2}+3\,(11\,Ba^{2}+2\,Aab)x^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{x}\right)}{231\,a^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="fricas")
[Out] -2/231*(2*(11*B*a*b - 5*A*b^2)*sqrt(b)*x^6*weierstrassPInverse(-4*a/b, 0, x
) + (2*(11*B*a*b - 5*A*b^2)*x^4 + 21*A*a^2 + 3*(11*B*a^2 + 2*A*a*b)*x^2)*sq
rt(b*x^2 + a)*sqrt(x))/(a^2*x^6)
```

Sympy [C] Result contains complex when optimal does not.
time = 85.90, size = 100, normalized size = 0.53

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{11}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2x^{\frac{11}{2}}\Gamma\left(-\frac{7}{4}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{7}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(13/2),x)
[Out] A*sqrt(a)*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**2*exp_polar(I*pi)
/a)/(2*x**(11/2)*gamma(-7/4)) + B*sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (
-3/4,), b*x**2*exp_polar(I*pi)/a)/(2*x**(7/2)*gamma(-3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(13/2),x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(13/2), x)
```

3.794 $\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=252

$$\frac{8a^2(3Ab-aB)e\sqrt{ex}\,\sqrt{a+bx^2}}{231b^2}+\frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be}+\frac{2(3Ab-aB)(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{33be}+\frac{2B(e\cdots}{}$$

```
[Out] 2/33*(3*A*b-B*a)*(e*x)^(5/2)*(b*x^2+a)^(3/2)/b/e+2/15*B*(e*x)^(5/2)*(b*x^2+
a)^(5/2)/b/e+4/77*a*(3*A*b-B*a)*(e*x)^(5/2)*(b*x^2+a)^(1/2)/b/e+8/231*a^2*(
3*A*b-B*a)*e*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^2-4/231*a^(11/4)*(3*A*b-B*a)*e^(
3/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e
*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^
(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 285, 327, 335, 226}

$$-\frac{4a^{11/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(3Ab-aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{8a^2e\sqrt{ex}\,\sqrt{a+bx^2}\,(3Ab-aB)}{231b^2}+\frac{4a(ex)^{5/2}\sqrt{a+bx^2}\,(3Ab-aB)}{77be}+\frac{2(ex)^{5/2}\left(a+bx^2\right)^{3/2}(3Ab-aB)}{33be}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (8*a^2*(3*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(231*b^2) + (4*a*(3*A*b -
 a*B)*(e*x)^(5/2)*Sqrt[a + b*x^2])/(77*b*e) + (2*(3*A*b - a*B)*(e*x)^(5/2)*
(a + b*x^2)^(3/2))/(33*b*e) + (2*B*(e*x)^(5/2)*(a + b*x^2)^(5/2))/(15*b*e)
- (4*a^(11/4)*(3*A*b - a*B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/
(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sq
rt[e])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx = \frac{2B(ex)^{5/2} \left(a+bx^2\right)^{5/2}}{15be} - \frac{\left(2\left(-\frac{15Ab}{2}+\frac{5aB}{2}\right)\right) \int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \, dx}{15b}$$

$$= \frac{2(3Ab-aB)(ex)^{5/2} \left(a+bx^2\right)^{3/2}}{33be} + \frac{2B(ex)^{5/2} \left(a+bx^2\right)^{5/2}}{15be} + \frac{(2a(3A}{}$$

$$= \frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2(3Ab-aB)(ex)^{5/2} \left(a+bx^2\right)^{3/2}}{33be}$$

$$= \frac{8a^2(3Ab-aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be}$$

$$= \frac{8a^2(3Ab-aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be}$$

$$= \frac{8a^2(3Ab-aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 114, normalized size = 0.45

$$\frac{2e\sqrt{ex}\sqrt{a+bx^2}\left(-\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}\left(-15Ab+5aB-11bBx^2\right)+5a^2(-3Ab+aB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{165b^2\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (2*e*Sqrt[e*x]*Sqrt[a + b*x^2]*(-((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*(-15*A*
b + 5*a*B - 11*b*B*x^2)) + 5*a^2*(-3*A*b + a*B)*Hypergeometric2F1[-3/2, 1/4
, 5/4, -((b*x^2)/a)]))/(165*b^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A]
time = 0.10, size = 300, normalized size = 1.19

method	result

risch	$\frac{2(77B\,b^3x^6+105A\,b^3x^4+119Ba\,b^2x^4+195Aa\,b^2x^2+12B\,a^2b\,x^2+60A\,a^2b-20B\,a^3)x\sqrt{b\,x^2+a}\,e^2}{1155b^2\sqrt{ex}} - \frac{4a^3(3Ab-Ba)\sqrt{-ab}}{}$
default	$-\frac{2e\sqrt{ex}\left(-77b^5B\,x^9-105A\,b^5x^7-196Ba\,b^4x^7+30A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(\frac{2bBe\,x^6\sqrt{be\,x^3+aex}}{15}+\frac{2\left(b(Ab+2Ba)e^2-\frac{13bB\,e^2a}{15}\right)x^4\sqrt{be\,x^3+aex}}{11be}+\frac{2\left(a(2Ab+Ba)e^2-\frac{9(}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155*e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-77*b^5*B*x^9-105*A*b^5*x^7-196*B*
a*b^4*x^7+30*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^3*b-300*A*a*b^4*x
^5-10*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^4-131*B*a^2*b^3*x^5-255*
A*a^2*b^3*x^3+8*B*a^3*b^2*x^3-60*A*a^3*b^2*x+20*B*a^4*b*x)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 117, normalized size = 0.46

$$\frac{2\left(20\left(Ba^4-3\,Aa^3b\right)\sqrt{b}\,e^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(77\,Bb^4x^6-20\,Ba^3b+60\,Aa^2b^2+7\left(17\,Bab^3+15\,Ab^4\right)x^4+3\left(4\,Ba^2b^2+65\,Aab^3\right)x^2\right)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{3}{2}}\right)}{1155\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/1155*(20*(B*a^4 - 3*A*a^3*b)*sqrt(b)*e^(3/2)*weierstrassPInverse(-4*a/b,
0, x) + (77*B*b^4*x^6 - 20*B*a^3*b + 60*A*a^2*b^2 + 7*(17*B*a*b^3 + 15*A*b^
4)*x^4 + 3*(4*B*a^2*b^2 + 65*A*a*b^3)*x^2)*sqrt(b*x^2 + a)*sqrt(x)*e^(3/2))
/b^3
```

Sympy [C] Result contains complex when optimal does not.
time = 13.92, size = 199, normalized size = 0.79

$$\frac{Aa^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{A\sqrt{a}\, be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)} + \frac{Ba^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)} + \frac{B\sqrt{a}\, be^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{13}{4} \\ \frac{17}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(3/2)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(9/4)) + A*sqrt(a)*b*e**(3/2)*x**(9/2)*gamma(9/4)
*hyper((-1/2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(13/4)) + B*
a**(3/2)*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/4)) + B*sqrt(a)*b*e**(3/2)*x**(13/2)*gamma(13/
4)*hyper((-1/2, 13/4), (17/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(17/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*x^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^2+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(e*x)^(3/2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int((A + B*x^2)*(e*x)^(3/2)*(a + b*x^2)^(3/2), x)
```

3.795 $\int \sqrt{ex} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=377

$$\frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be}+\frac{8a^2(13Ab-3aB)\sqrt{ex}\,\sqrt{a+bx^2}}{195b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{117be}$$

```
[Out] 2/117*(13*A*b-3*B*a)*(e*x)^(3/2)*(b*x^2+a)^(3/2)/b/e+2/13*B*(e*x)^(3/2)*(b*
x^2+a)^(5/2)/b/e+4/195*a*(13*A*b-3*B*a)*(e*x)^(3/2)*(b*x^2+a)^(1/2)/b/e+8/1
95*a^2*(13*A*b-3*B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/(a^(1/2)+x*b^(1/2
))-8/195*a^(9/4)*(13*A*b-3*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e
^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*Ellipt
icE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2
)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2
+a)^(1/2)+4/195*a^(9/4)*(13*A*b-3*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^
(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))
*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*
(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)
/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 285, 335, 311, 226, 1210}

$$\frac{4a^{9/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(13Ab-3aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}-\frac{8a^{9/4}\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(13Ab-3aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}+\frac{8a^2\sqrt{ex}\,\sqrt{a+bx^2}\,(13Ab-3aB)}{195b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}(13Ab-3aB)}{117be}+\frac{4a(ex)^{3/2}\sqrt{a+bx^2}\,(13Ab-3aB)}{195be}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (4*a*(13*A*b - 3*a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(195*b*e) + (8*a^2*(13*A
*b - 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(195*b^(3/2)*(Sqrt[a] + Sqrt[b]*x))
+ (2*(13*A*b - 3*a*B)*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(117*b*e) + (2*B*(e*x)
^(3/2)*(a + b*x^2)^(5/2))/(13*b*e) - (8*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*A
rcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b
*x^2]) + (4*a^(9/4)*(13*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a
^(1/4)*Sqrt[e])], 1/2])/(195*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}-\frac{\left(2\left(-\frac{13Ab}{2}+\frac{3aB}{2}\right)\right)\int\sqrt{ex}\left(a+bx^2\right)^{3/2}\,dx}{13b}$$

$$=\frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{117be}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}+\frac{(2a(\cdots}{}$$

$$=\frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be}+\frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right.}{117be}$$

$$=\frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be}+\frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right.}{117be}$$

$$=\frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be}+\frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right.}{117be}$$

$$=\frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be}+\frac{8a^2(13Ab-3aB)\sqrt{ex}\sqrt{a+bx\cdots}}{195b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 97, normalized size = 0.26

$$\frac{2x\sqrt{ex}\sqrt{a+bx^2}\left(3B(a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}+a(13Ab-3aB)\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{39b\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (2*x*Sqrt[e*x]*Sqrt[a + b*x^2]*(3*B*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] + a*(
13*A*b - 3*a*B)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(39*b*Sqr
t[1 + (b*x^2)/a])
```

Maple [A]
time = 0.11, size = 438, normalized size = 1.16

method	result
risch	$\frac{2x^{2}\left(45b^{2}B\,x^{4}+65A\,b^{2}x^{2}+75Bab\,x^{2}+143abA+12a^{2}B\right)\sqrt{b\,x^{2}+a}\,e}{585b\sqrt{ex}}+\frac{4a^{2}(13Ab-3Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\cdots}}{\cdots}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^{2}+a)\,ex}\left(\frac{2bB\,x^{5}\sqrt{be\,x^{3}+aex}}{13}+\frac{2\left(b(Ab+2Ba)e-\frac{11Bbae}{13}\right)x^{3}\sqrt{be\,x^{3}+aex}}{9be}+\frac{2\left(a(2Ab+Ba)e-\frac{7\left(b(Ab+2\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{2\sqrt{ex}\left(45B\,b^{4}x^{8}+65A\,b^{4}x^{6}+120Ba\,b^{3}x^{6}+156A\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-a\cdots}}{\sqrt{-ab}}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/585/(b*x^2+a)^(1/2)*(e*x)^(1/2)/b^2*(45*B*b^4*x^8+65*A*b^4*x^6+120*B*a*b^
3*x^6+156*A*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*a^3*b-78*A*EllipticF(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*a^3*b-36*B*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2
))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a
```

```
*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*a^4+18*B*EllipticF(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(
1/2)*a^4+208*A*a*b^3*x^4+87*B*a^2*b^2*x^4+143*A*a^2*b^2*x^2+12*B*a^3*b*x^2)
/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 105, normalized size = 0.28

$$\frac{2\left(12\,(3\,Ba^{3}-13\,Aa^{2}b)\sqrt{b}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(45\,Bb^{3}x^{5}+5\,(15\,Bab^{2}+13\,Ab^{3})x^{3}+(12\,Ba^{2}b+143\,Aab^{2})x)\sqrt{bx^{2}+a}\,\sqrt{x}\,e^{\frac{1}{2}}\right)}{585\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(12*(3*B*a^3 - 13*A*a^2*b)*sqrt(b)*e^(1/2)*weierstrassZeta(-4*a/b, 0,
 weierstrassPInverse(-4*a/b, 0, x)) + (45*B*b^3*x^5 + 5*(15*B*a*b^2 + 13*A*
b^3)*x^3 + (12*B*a^2*b + 143*A*a*b^2)*x)*sqrt(b*x^2 + a)*sqrt(x)*e^(1/2))/b
^2
```

Sympy [C] Result contains complex when optimal does not.
time = 4.09, size = 197, normalized size = 0.52

$$\frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2e\Gamma\left(\frac{7}{4}\right)}+\frac{A\sqrt{a}\,b(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2e^{3}\Gamma\left(\frac{11}{4}\right)}+\frac{Ba^{\frac{3}{2}}(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2e^{3}\Gamma\left(\frac{11}{4}\right)}+\frac{B\sqrt{a}\,b(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2e^{5}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)*(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_po
lar(I*pi)/a)/(2*e*gamma(7/4)) + A*sqrt(a)*b*(e*x)**(7/2)*gamma(7/4)*hyper((
-1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**3*gamma(11/4)) + B*a**
(3/2)*(e*x)**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*e**3*gamma(11/4)) + B*sqrt(a)*b*(e*x)**(11/2)*gamma(11/4)*hyper
((-1/2, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**5*gamma(15/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(x)*e^(1/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^2+A\right)\,\sqrt{e\,x}\,\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(e*x)^(1/2)*(a + b*x^2)^(3/2),x)
[Out] int((A + B*x^2)*(e*x)^(1/2)*(a + b*x^2)^(3/2), x)
```

3.796 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=214

$$\frac{4a(11Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{77be}+\frac{2(11Ab-aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{77be}+\frac{2B\sqrt{ex}\,(a+bx^2)^{5/2}}{11be}+\frac{4a^{7/4}(11Ab-aB}{}$$

```
[Out] 2/77*(11*A*b-B*a)*(b*x^2+a)^(3/2)*(e*x)^(1/2)/b/e+2/11*B*(b*x^2+a)^(5/2)*(e
*x)^(1/2)/b/e+4/77*a*(11*A*b-B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b/e+4/77*a^(7
/4)*(11*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arcta
n(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((
b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/e^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 285, 335, 226}

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(11Ab-aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{e}\,\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\,(a+bx^2)^{3/2}(11Ab-aB)}{77be}+\frac{4a\sqrt{ex}\,\sqrt{a+bx^2}\,(11Ab-aB)}{77be}+\frac{2B\sqrt{ex}\,(a+bx^2)^{5/2}}{11be}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (4*a*(11*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(77*b*e) + (2*(11*A*b - a*B)
*Sqrt[e*x]*(a + b*x^2)^(3/2))/(77*b*e) + (2*B*Sqrt[e*x]*(a + b*x^2)^(5/2))/
(11*b*e) + (4*a^(7/4)*(11*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(77*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx = \frac{2B\sqrt{ex}\,(a+bx^2)^{5/2}}{11be} - \frac{\left(2\left(-\frac{11Ab}{2}+\frac{aB}{2}\right)\right)\int\frac{(a+bx^2)^{3/2}}{\sqrt{ex}}\,dx}{11b}$$

$$= \frac{2(11Ab-aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{77be} + \frac{2B\sqrt{ex}\,(a+bx^2)^{5/2}}{11be} + \frac{(6a(11Ab-aB))}{77}$$

$$= \frac{4a(11Ab-aB)\sqrt{ex}\,\sqrt{a+bx^2}}{77be} + \frac{2(11Ab-aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{77be} + \frac{2B\sqrt{e}}{}$$

$$= \frac{4a(11Ab-aB)\sqrt{ex}\,\sqrt{a+bx^2}}{77be} + \frac{2(11Ab-aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{77be} + \frac{2B\sqrt{e}}{}$$

$$= \frac{4a(11Ab-aB)\sqrt{ex}\,\sqrt{a+bx^2}}{77be} + \frac{2(11Ab-aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{77be} + \frac{2B\sqrt{e}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 96, normalized size = 0.45

$$\frac{2x\sqrt{a+bx^2}\left(B(a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}+a(11Ab-aB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{11b\sqrt{ex}\sqrt{1+\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/Sqrt[e*x],x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(B*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] + a*(11*A*b - a*B
)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[e*x]*Sqrt[1
+ (b*x^2)/a])
```

Maple [A]
time = 0.11, size = 272, normalized size = 1.27

method	result
risch	$\frac{2(7b^2Bx^4+11Ab^2x^2+13Babx^2+33abA+4a^2B)x\sqrt{bx^2+a}}{77b\sqrt{ex}}+\frac{4a^2(11Ab-Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\right.}{}}}{77b^2}$
default	$\frac{\frac{2Bb^4x^7}{11}+\frac{4A\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2b}{7}+\frac{2A}{}}{}$
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(\frac{2bBx^4\sqrt{bex^3+aex}}{11e}+\frac{2\left(b^2A+\frac{13}{11}abB\right)x^2\sqrt{bex^3+aex}}{7be}+\frac{2\left(2abA+a^2B-\frac{5a\left(b^2A+\frac{13}{11}abB\right)}{7b}\right)\sqrt{be}}{3be}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77/(b*x^2+a)^(1/2)*(7*B*b^4*x^7+22*A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a
*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
```

```
/2))*a^2*b+11*A*b^4*x^5-2*B*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^
(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+20
*B*a*b^3*x^5+44*A*a*b^3*x^3+17*B*a^2*b^2*x^3+33*A*a^2*b^2*x+4*B*a^3*b*x)/b^
2/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 91, normalized size = 0.43

$$-\frac{2\left(4\,(Ba^3-11\,Aa^2b)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(7\,Bb^3x^4+4\,Ba^2b+33\,Aab^2+(13\,Bab^2+11\,Ab^3)x^2)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{77\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*(B*a^3 - 11*A*a^2*b)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (
7*B*b^3*x^4 + 4*B*a^2*b + 33*A*a*b^2 + (13*B*a*b^2 + 11*A*b^3)*x^2)*sqrt(b*
x^2 + a)*sqrt(x))*e^(-1/2)/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 4.61, size = 199, normalized size = 0.93

$$\frac{Aa^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{A\sqrt{a}\,bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}+\frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}+\frac{B\sqrt{a}\,bx^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(e)*gamma(5/4)) + A*sqrt(a)*b*x**(5/2)*gamma(5/4)*hyper((-1/
2, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(9/4)) + B*a**(3
/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a
)/(2*sqrt(e)*gamma(9/4)) + B*sqrt(a)*b*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4
), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*e^(-1/2)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{e\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(1/2),x)
[Out] int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(1/2), x)
```

3.797 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=367

$$\frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3}+\frac{8a(9Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{15\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(9Ab+aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9ae^3}-\frac{2A(a+b}{ae\sqrt{e}}$$

```
[Out] 2/9*(9*A*b+B*a)*(e*x)^(3/2)*(b*x^2+a)^(3/2)/a/e^3-2*A*(b*x^2+a)^(5/2)/a/e/(
e*x)^(1/2)+4/15*(9*A*b+B*a)*(e*x)^(3/2)*(b*x^2+a)^(1/2)/e^3+8/15*a*(9*A*b+B
*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/e^2/b^(1/2)/(a^(1/2)+x*b^(1/2))-8/15*a^(5/4
)*(9*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b
^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x
^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/e^(3/2)/(b*x^2+a)^(1/2)+4/15*a^(
5/4)*(9*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arcta
n(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((
b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/e^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {464, 285, 335, 311, 226, 1210}

$$\frac{4a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(aB+9Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{8a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(aB+9Ab)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}}+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}(aB+9Ab)}{9ae^3}+\frac{4(ex)^{3/2}\sqrt{a+bx^2}\,(aB+9Ab)}{15e^3}+\frac{8a\sqrt{ex}\,\sqrt{a+bx^2}\,(aB+9Ab)}{15\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(3/2),x]
```

```
[Out] (4*(9*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(15*e^3) + (8*a*(9*A*b + a*B)
*Sqrt[e*x]*Sqrt[a + b*x^2])/(15*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) + (2*(9*
A*b + a*B)*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(9*a*e^3) - (2*A*(a + b*x^2)^(5/2
))/(a*e*Sqrt[e*x]) - (8*a^(5/4)*(9*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(
a^(1/4)*Sqrt[e])], 1/2])/(15*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + (4*a^(5/4)*
(9*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(
3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx = -\frac{2A(a+bx^2)^{5/2}}{ae\sqrt{ex}} + \frac{(9Ab+aB)\int \sqrt{ex}\,(a+bx^2)^{3/2}\,dx}{ae^2}$$

$$= \frac{2(9Ab+aB)(ex)^{3/2}(a+bx^2)^{3/2}}{9ae^3} - \frac{2A(a+bx^2)^{5/2}}{ae\sqrt{ex}} + \frac{(2(9Ab+aB))\int \sqrt{ex}$$

$$= \frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3} + \frac{2(9Ab+aB)(ex)^{3/2}(a+bx^2)^{3/2}}{9ae^3} - \frac{2A(a+}{ae}$$

$$= \frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3} + \frac{2(9Ab+aB)(ex)^{3/2}(a+bx^2)^{3/2}}{9ae^3} - \frac{2A(a+}{ae}$$

$$= \frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3} + \frac{2(9Ab+aB)(ex)^{3/2}(a+bx^2)^{3/2}}{9ae^3} - \frac{2A(a+}{ae}$$

$$= \frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3} + \frac{8a(9Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{15\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{2(9Ab+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 84, normalized size = 0.23

$$\frac{2x\sqrt{a+bx^2}\left(-\frac{3A(a+bx^2)^2}{a} + \frac{(9Ab+aB)x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{3(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*((-3*A*(a + b*x^2)^2)/a + ((9*A*b + a*B)*x^2*Hypergeom
etric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(3*(e*x)^(3/2
))
```

Maple [A]
time = 0.11, size = 421, normalized size = 1.15

method	result
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-5bB\,x^{4}-9Ab\,x^{2}-11Ba\,x^{2}+45Aa\right)}{45e\sqrt{ex}}+\frac{4a(9Ab+Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)\cdots}{\sqrt{-ab}}}\cdots}{\cdots}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)ex}\,\left(-\frac{2\left(be\,x^{2}+ae\right)aA}{e^{2}\sqrt{x\left(be\,x^{2}+ae\right)}}+\frac{2bB\,x^{3}\sqrt{be\,x^{3}+aex}}{9e^{2}}+\frac{2\left(\frac{b(Ab+2Ba)}{e}-\frac{7bBa}{9e}\right)x\sqrt{be\,x^{3}+aex}}{5be}+\cdots\left(\frac{a(\cdots}{\cdots}\cdots\right)\cdots\right)}{\cdots}$
default	$\frac{\frac{2B\,b^{3}x^{6}}{9}+\frac{24A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^{2}b}{5}-\frac{12A\sqrt{\frac{bx+\cdots}{\cdots}}\cdots}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(5*B*b^3*x^6+108*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-54*A*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
```

```
2),1/2*2^(1/2))*a^2*b+12*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*
((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elliptic
E(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-6*B*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*a^3+9*A*b^3*x^4+16*B*a*b^2*x^4-36*A*a*b^2*x^2+11*B*a^2*b*x^
2-45*A*a^2*b)/(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 90, normalized size = 0.25

$$-\frac{2\left(12\,(Ba^2+9\,Aab)\sqrt{b}\,x\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-(5\,Bb^2x^4-45\,Aab+(11\,Bab+9\,Ab^2)x^2)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{45\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45*(12*(B*a^2 + 9*A*a*b)*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstras
sPInverse(-4*a/b, 0, x)) - (5*B*b^2*x^4 - 45*A*a*b + (11*B*a*b + 9*A*b^2)*x
^2)*sqrt(b*x^2 + a)*sqrt(x))*e^(-3/2)/(b*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.78, size = 202, normalized size = 0.55

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{A\sqrt{a}\,bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{B\sqrt{a}\,bx^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a
)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + A*sqrt(a)*b*x**(3/2)*gamma(3/4)*hyper((
-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(7/4)) + B*a
**(3/2)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*p
i)/a)/(2*e**(3/2)*gamma(7/4)) + B*sqrt(a)*b*x**(7/2)*gamma(7/4)*hyper((-1/2
, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*e^(-3/2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\left(b\,x^2+a\right)^{3/2}}{\left(e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(3/2),x)
[Out] int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(3/2), x)
```

3.798 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=210

$$\frac{4(7Ab+3aB)\sqrt{ex}\,\sqrt{a+bx^2}}{21e^3}+\frac{2(7Ab+3aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{21ae^3}-\frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}}+\frac{4a^{3/4}(7Ab+3aB)\left(\sqrt{a}\right.}{}$$

```
[Out] -2/3*A*(b*x^2+a)^(5/2)/a/e/(e*x)^(3/2)+2/21*(7*A*b+3*B*a)*(b*x^2+a)^(3/2)*(
e*x)^(1/2)/a/e^3+4/21*(7*A*b+3*B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/e^3+4/21*a^
(3/4)*(7*A*b+3*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*ar
ctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/e^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {464, 285, 335, 226}

$$\frac{4a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(3aB+7Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{b}\,e^{5/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\,(a+bx^2)^{3/2}(3aB+7Ab)}{21ae^3}+\frac{4\sqrt{ex}\,\sqrt{a+bx^2}\,(3aB+7Ab)}{21e^3}-\frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (4*(7*A*b + 3*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*e^3) + (2*(7*A*b + 3*a*B)
*Sqrt[e*x]*(a + b*x^2)^(3/2))/(21*a*e^3) - (2*A*(a + b*x^2)^(5/2))/(3*a*e*(
e*x)^(3/2)) + (4*a^(3/4)*(7*A*b + 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/
4)*Sqrt[e])], 1/2])/(21*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx = -\frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}} + \frac{(7Ab+3aB)\int \frac{(a+bx^2)^{3/2}}{\sqrt{ex}}\,dx}{3ae^2}$$

$$= \frac{2(7Ab+3aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{21ae^3} - \frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}} + \frac{(2(7Ab+3aB))\int \frac{\sqrt{a}}{}}{7e^2}$$

$$= \frac{4(7Ab+3aB)\sqrt{ex}\,\sqrt{a+bx^2}}{21e^3} + \frac{2(7Ab+3aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{21ae^3} - \frac{2A(a}{3ae(}$$

$$= \frac{4(7Ab+3aB)\sqrt{ex}\,\sqrt{a+bx^2}}{21e^3} + \frac{2(7Ab+3aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{21ae^3} - \frac{2A(a}{3ae(}$$

$$= \frac{4(7Ab+3aB)\sqrt{ex}\,\sqrt{a+bx^2}}{21e^3} + \frac{2(7Ab+3aB)\sqrt{ex}\,(a+bx^2)^{3/2}}{21ae^3} - \frac{2A(a}{3ae(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 85, normalized size = 0.40

$$\frac{2x\sqrt{a+bx^2}\left(-\frac{A(a+bx^2)^2}{a}+\frac{(7Ab+3aB)x^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^2]*(-((A*(a + b*x^2)^2)/a) + ((7*A*b + 3*a*B)*x^2*Hyperge
ometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(3*(e*x)^(5
/2))
```

Maple [A]
time = 0.12, size = 255, normalized size = 1.21

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-3bB\,x^4-7Ab\,x^2-9Ba\,x^2+7Aa)}{21x\,e^2\sqrt{ex}}+\frac{4a(7Ab+3Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}}{21b\sqrt{be\,x^3+ae}}$
default	$\frac{\frac{4A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,abx}{3}+4B\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}}{\sqrt{}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(-\frac{2aA\sqrt{be\,x^3+aex}}{3e^3x^2}+\frac{2Bb\,x^2\sqrt{be\,x^3+aex}}{7e^3}+\frac{2\left(\frac{b(Ab+2Ba)}{e^2}-\frac{5Bba}{7e^2}\right)\sqrt{be\,x^3+aex}}{3be}+\frac{\left(\frac{a(2Ab+}{e^2}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/(b*x^2+a)^(1/2)/x*(14*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a*b*x
```

```
+6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2*x+3*B*b^3*x^6+7*A*b^3*x^4
+12*B*a*b^2*x^4+9*B*a^2*b*x^2-7*A*a^2*b)/b/e^2/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="maxima")
[Out] e^(-5/2)*integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 84, normalized size = 0.40

$$\frac{2\left(4\left(3\,Ba^2+7\,Aab\right)\sqrt{b}\,x^2\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(3\,Bb^2x^4-7\,Aab+\left(9\,Bab+7\,Ab^2\right)x^2\right)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{21\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="fricas")
[Out] 2/21*(4*(3*B*a^2 + 7*A*a*b)*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) +
 (3*B*b^2*x^4 - 7*A*a*b + (9*B*a*b + 7*A*b^2)*x^2)*sqrt(b*x^2 + a)*sqrt(x))
*e^(-5/2)/(b*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 6.96, size = 202, normalized size = 0.96

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{A\sqrt{a}\,b\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{Ba^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{B\sqrt{a}\,bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(5/2),x)
[Out] A*a**(3/2)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/a
)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + A*sqrt(a)*b*sqrt(x)*gamma(1/4)*hyper((
-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(5/4)) + B*a
**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi
)/a)/(2*e**(5/2)*gamma(5/4)) + B*sqrt(a)*b*x**(5/2)*gamma(5/4)*hyper((-1/2,
 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*e^(-5/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{3/2}}{{\left(e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(5/2), x)
```

3.799 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=365

$$\frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5}+\frac{24\sqrt{b}\,(Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5e^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2(Ab+aB)\left(a+bx^2\right)^{3/2}}{ae^3\sqrt{ex}}-\frac{2A(a+bx^2)}{5ae(ex)^{5/}}$$

```
[Out] -2/5*A*(b*x^2+a)^(5/2)/a/e/(e*x)^(5/2)-2*(A*b+B*a)*(b*x^2+a)^(3/2)/a/e^3/(e
*x)^(1/2)+12/5*b*(A*b+B*a)*(e*x)^(3/2)*(b*x^2+a)^(1/2)/a/e^5+24/5*(A*b+B*a)
*b^(1/2)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/e^4/(a^(1/2)+x*b^(1/2))-24/5*a^(1/4)*b
^(1/4)*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arcta
n(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((
b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/e^(7/2)/(b*x^2+a)^(1/2)+12/5*a^(1/4)*
b^(1/4)*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arct
an(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*(
(b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/e^(7/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {464, 283, 285, 335, 311, 226, 1210}

$$\frac{12\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(aB+Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5e^{7/2}\sqrt{a+bx^2}}-\frac{24\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(aB+Ab)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5e^{7/2}\sqrt{a+bx^2}}+\frac{12b(ex)^{3/2}\sqrt{a+bx^2}\,(aB+Ab)}{5ae^5}+\frac{24\sqrt{b}\,\sqrt{ex}\,\sqrt{a+bx^2}\,(aB+Ab)}{5e^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2(a+bx^2)^{3/2}(aB+Ab)}{ae^3\sqrt{ex}}-\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(7/2),x]
```

```
[Out] (12*b*(A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*a*e^5) + (24*Sqrt[b]*(A*b
 + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*e^4*(Sqrt[a] + Sqrt[b]*x)) - (2*(A*b
+ a*B)*(a + b*x^2)^(3/2))/(a*e^3*Sqrt[e*x]) - (2*A*(a + b*x^2)^(5/2))/(5*a*
e*(e*x)^(5/2)) - (24*a^(1/4)*b^(1/4)*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(5*e^(7/2)*Sqrt[a + b*x^2]) + (12*a^(1/4)*b^(1/
4)*(A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*e^(
7/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx = -\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}} + \frac{(Ab+aB)\int \frac{(a+bx^2)^{3/2}}{(ex)^{3/2}}\,dx}{ae^2}$$

$$= -\frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}} - \frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}} + \frac{(6b(Ab+aB))\int \sqrt{ex}\,\sqrt{a}}{ae^4}$$

$$= \frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5} - \frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}} - \frac{2A(a+bx^2)}{5ae(ex)^{5/2}}$$

$$= \frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5} - \frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}} - \frac{2A(a+bx^2)}{5ae(ex)^{5/2}}$$

$$= \frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5} - \frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}} - \frac{2A(a+bx^2)}{5ae(ex)^{5/2}}$$

$$= \frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5} + \frac{24\sqrt{b}\,(Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5e^4\left(\sqrt{a}+\sqrt{b}\,x\right)} - 2(A$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 84, normalized size = 0.23

$$\frac{2x\sqrt{a+bx^2}\left(-\frac{A(a+bx^2)^2}{a} - \frac{5(Ab+aB)x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{5(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(7/2),x]
```

[Out] (2*x*Sqrt[a + b*x^2]*(-((A*(a + b*x^2)^2)/a) - (5*(A*b + a*B)*x^2*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(5*(e*x)^(7/2))

Maple [A]

time = 0.11, size = 422, normalized size = 1.16

method	result
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-bB\,x^{4}+7Ab\,x^{2}+5Ba\,x^{2}+Aa\right)}{5x^{2}e^{3}\sqrt{ex}}+\frac{12(Ab+Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)\,ex}\,\left(-\frac{2aA\sqrt{be\,x^{3}+aex}}{5e^{4}x^{3}}-\frac{2\left(be\,x^{2}+ae\right)(7Ab+5Ba)}{5e^{4}\sqrt{x\,(be\,x^{2}+ae)}}+\frac{2bBx\sqrt{be\,x^{3}+aex}}{5e^{4}}+\frac{\left(\frac{b(Ab+2Ba)}{e^{3}}+\frac{b(7Ab+5Ba)}{5e^{3}}\right.}{}\right.}{}$
default	$\frac{\frac{24A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^{2}}{5}-\frac{12A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/5/x^2*(12*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b

```
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-6*A*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*a*b*x^2+12*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*x^2-6*B*((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*a^2*x^2+b^2*B*x^6-7*A*b^2*x^4-4*B*a*b*x^4-8*a*A*b*x^2-5*B*a^2*
x^2-a^2*A)/(b*x^2+a)^(1/2)/e^3/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/x^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 79, normalized size = 0.22

$$-\frac{2\left(12\,(Ba+Ab)\sqrt{b}\,x^3\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)-(Bbx^4-(5\,Ba+7\,Ab)x^2-Aa)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{5\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(12*(B*a + A*b)*sqrt(b)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInv
erse(-4*a/b, 0, x)) - (B*b*x^4 - (5*B*a + 7*A*b)*x^2 - A*a)*sqrt(b*x^2 + a)
*sqrt(x))*e^(-7/2)/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 21.93, size = 212, normalized size = 0.58

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{B\sqrt{a}\,bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(7/2),x)
```

```
[Out] A*a**(3/2)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**2*exp_polar(I*pi)/
a)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + A*sqrt(a)*b*gamma(-1/4)*hyper((-1/2,
 -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(7/2)*sqrt(x)*gamma(3/4)) +
```

```
B*a**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/
a)/(2*e**(7/2)*sqrt(x)*gamma(3/4)) + B*sqrt(a)*b*x**(3/2)*gamma(3/4)*hyper(
(-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(7/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*e^(-7/2)/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{3/2}}{{\left(e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(7/2),x)
[Out] int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(7/2), x)
```

3.800 $\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=338

$$\frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}-\frac{2a(9Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^2}}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2a^{5/4}(9Ab-7}{}$$

```
[Out] 2/45*(9*A*b-7*B*a)*e*(e*x)^(3/2)*(b*x^2+a)^(1/2)/b^2+2/9*B*(e*x)^(7/2)*(b*x
^2+a)^(1/2)/b/e-2/15*a*(9*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^(5/2
)/(a^(1/2)+x*b^(1/2))+2/15*a^(5/4)*(9*A*b-7*B*a)*e^(5/2)*(cos(2*arctan(b^(1
/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)
/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/
2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/
2)/b^(11/4)/(b*x^2+a)^(1/2)-1/15*a^(5/4)*(9*A*b-7*B*a)*e^(5/2)*(cos(2*arcta
n(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)
^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)
/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^
2)^(1/2)/b^(11/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 327, 335, 311, 226, 1210}

$$-\frac{a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(9Ab-7aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}+\frac{2a^{5/4}e^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(9Ab-7aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}-\frac{2ae^2\sqrt{ex}\sqrt{a+bx^2}(9Ab-7aB)}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2e(ex)^{3/2}\sqrt{a+bx^2}(9Ab-7aB)}{45b^2}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*(9*A*b - 7*a*B)*e*(e*x)^(3/2)*Sqrt[a + b*x^2])/(45*b^2) + (2*B*(e*x)^(7/
2)*Sqrt[a + b*x^2])/(9*b*e) - (2*a*(9*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b
*x^2])/(15*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) + (2*a^(5/4)*(9*A*b - 7*a*B)*e^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt
[a + b*x^2]) - (a^(5/4)*(9*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x]]
/(a^(1/4)*Sqrt[e])], 1/2])/(15*b^(11/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{(ex)^{5/2}}{\sqrt{a+bx^2}}\,dx}{9b}$$

$$= \frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{(a(9Ab-7aB)e^2)\int}{15b^2}$$

$$= \frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{(2a(9Ab-7aB)e)\mathrm{S}}{}$$

$$= \frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{(2a^{3/2}(9Ab-7aB)}{}$$

$$= \frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{2a(9Ab-7aB)e^2\sqrt{}}{15b^{5/2}\left(\sqrt{a}+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 96, normalized size = 0.28

$$\frac{2e(ex)^{3/2}\left(-\left(\left(a+bx^2\right)\left(-9Ab+7aB-5bBx^2\right)\right)+a(-9Ab+7aB)\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{45b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*e*(e*x)^(3/2)*(-((a + b*x^2)*(-9*A*b + 7*a*B - 5*b*B*x^2)) + a*(-9*A*b +
 7*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])
)/(45*b^2*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.11, size = 417, normalized size = 1.23

method	result

risch	$\frac{2x^2(5bBx^2+9Ab-7Ba)\sqrt{bx^2+a}\,e^3}{45b^2\sqrt{ex}} - \frac{a(9Ab-7Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{}$
elliptic	$\frac{\sqrt{ex}\sqrt{(bx^2+a)ex}\left(\frac{2Be^2x^3\sqrt{bex^3+aex}}{9b}+\frac{2\left(Ae^3-\frac{7Be^3a}{9b}\right)x\sqrt{bex^3+aex}}{5be}-\frac{3\left(Ae^3-\frac{7Be^3a}{9b}\right)a\sqrt{-ab}\sqrt{}}{}\right.}{}$
default	$-\frac{e^2\sqrt{ex}\left(-10Bb^3x^6+54A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/45*e^2/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)/b^3*(-10*B*b^3*x^6+54*A*((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2),1/2*2^(1/2))*a^2*b-27*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)
*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipti
cF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-42*B*((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2))*a^3+21*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
```

```
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-18*A*b^3*x^4+4
*B*a*b^2*x^4-18*A*a*b^2*x^2+14*B*a^2*b*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] e^(5/2)*integrate((B*x^2 + A)*x^(5/2)/sqrt(b*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 83, normalized size = 0.25

$$-\frac{2\left(3\left(7\,Ba^2-9\,Aab\right)\sqrt{b}\,e^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)-\left(5\,Bb^2x^3-\left(7\,Bab-9\,Ab^2\right)x\right)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{5}{2}}\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -2/45*(3*(7*B*a^2 - 9*A*a*b)*sqrt(b)*e^(5/2)*weierstrassZeta(-4*a/b, 0, wei
erstrassPInverse(-4*a/b, 0, x)) - (5*B*b^2*x^3 - (7*B*a*b - 9*A*b^2)*x)*sqr
t(b*x^2 + a)*sqrt(x)*e^(5/2))/b^3
```

Sympy [C] Result contains complex when optimal does not.
time = 14.89, size = 94, normalized size = 0.28

$$\frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}+\frac{Be^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
[Out] A*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*sqrt(a)*gamma(11/4)) + B*e**(5/2)*x**(11/2)*gamma(11/4)*hyper((
1/2, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(5/2)*e^(5/2)/sqrt(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(e\,x\right)}^{5/2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(1/2),x)
[Out] int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(1/2), x)
```

3.801 $\int \frac{(ex)^{3/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=174

$$\frac{2(7Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^2}}{21b^2}+\frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be}-\frac{a^{3/4}(7Ab-5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\right.}}}{21b^{9/4}\sqrt{a+bx^2}}$$

```
[Out] 2/7*B*(e*x)^(5/2)*(b*x^2+a)^(1/2)/b/e+2/21*(7*A*b-5*B*a)*e*(e*x)^(1/2)*(b*x
^2+a)^(1/2)/b^2-1/21*a^(3/4)*(7*A*b-5*B*a)*e^(3/2)*(cos(2*arctan(b^(1/4)*(e
*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1
/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(
9/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 327, 335, 226}

$$-\frac{a^{3/4}e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(7Ab-5aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}+\frac{2e\sqrt{ex}\,\sqrt{a+bx^2}\,(7Ab-5aB)}{21b^2}+\frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*(7*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b^2) + (2*B*(e*x)^(5/2)
*Sqrt[a + b*x^2])/(7*b*e) - (a^(3/4)*(7*A*b - 5*a*B)*e^(3/2)*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(21*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^2}}\,dx}{7b}$$

$$= \frac{2(7Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^2}}{21b^2} + \frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be} - \frac{\left(a(7Ab-5aB)e^2\right)\int -}{21b^2}$$

$$= \frac{2(7Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^2}}{21b^2} + \frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be} - \frac{(2a(7Ab-5aB)e)\mathrm{Subs}}{}$$

$$= \frac{2(7Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^2}}{21b^2} + \frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be} - \frac{a^{3/4}(7Ab-5aB)e^{3/2}\Big(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 96, normalized size = 0.55

$$\frac{2e\sqrt{ex}\left(-\left(\left(a+bx^2\right)\left(-7Ab+5aB-3bBx^2\right)\right)+a(-7Ab+5aB)\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{21b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*e*Sqrt[e*x]*(-((a + b*x^2)*(-7*A*b + 5*a*B - 3*b*B*x^2)) + a*(-7*A*b + 5
*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/
(21*b^2*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.10, size = 250, normalized size = 1.44

method	result
risch	$\frac{2(3bBx^2+7Ab-5Ba)x\sqrt{bx^2+a}\,e^2}{21b^2\sqrt{ex}} - \frac{a(7Ab-5Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{x}{\sqrt{-}}}}{21b^3\sqrt{bex^3+aex}\sqrt{e}}$
elliptic	$\frac{\sqrt{ex}\sqrt{(bx^2+a)ex}\left(\frac{2Bex^2\sqrt{bex^3+aex}}{7b}+\frac{2\left(Ae^2-\frac{5Be^2a}{7b}\right)\sqrt{bex^3+aex}}{3be}-\frac{\left(Ae^2-\frac{5Be^2a}{7b}\right)a\sqrt{-ab}\sqrt{\left(\right.}}{}\right.}{ex\sqrt{bx^2+a}}$
default	$-\frac{e\sqrt{ex}\left(7A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(7*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2)
)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*
b)^(1/2)*a*b-5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2-6*B*b^3*x^5-1
4*A*b^3*x^3+4*B*a*b^2*x^3-14*A*a*b^2*x+10*B*a^2*b*x)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^2 + A)*x^(3/2)/sqrt(b*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 69, normalized size = 0.40

$$\frac{2\left((5\,Ba^2-7\,Aab)\sqrt{b}\,e^{\frac{3}{2}}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(3\,Bb^2x^2-5\,Bab+7\,Ab^2)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{3}{2}}\right)}{21\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*((5*B*a^2 - 7*A*a*b)*sqrt(b)*e^(3/2)*weierstrassPInverse(-4*a/b, 0, x)
 + (3*B*b^2*x^2 - 5*B*a*b + 7*A*b^2)*sqrt(b*x^2 + a)*sqrt(x)*e^(3/2))/b^3
```

Sympy [C] Result contains complex when optimal does not.
time = 4.15, size = 94, normalized size = 0.54

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(a)*gamma(9/4)) + B*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((1/2,
 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)*e^(3/2)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(e\,x\right)}^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(1/2),x)
[Out] int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(1/2), x)
```

3.802 $\int \frac{\sqrt{ex}\,(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=299

$$\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}+\frac{2(5Ab-3aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2\sqrt[4]{a}\,(5Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\right.}}}{5b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] 2/5*B*(e*x)^(3/2)*(b*x^2+a)^(1/2)/b/e+2/5*(5*A*b-3*B*a)*(e*x)^(1/2)*(b*x^2+
a)^(1/2)/b^(3/2)/(a^(1/2)+x*b^(1/2))-2/5*a^(1/4)*(5*A*b-3*B*a)*(cos(2*arcta
n(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)
^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)
/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b
^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)+1/5*a^(1/4)*(5*A*b-3*B*a)*(cos(2*a
rctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(
1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{a}\,\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(5Ab-3aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{a}\,\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(5Ab-3aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\,\sqrt{a+bx^2}\,(5Ab-3aB)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*B*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*b*e) + (2*(5*A*b - 3*a*B)*Sqrt[e*x]*Sq
rt[a + b*x^2])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) - (2*a^(1/4)*(5*A*b - 3*a*
B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)
*Sqrt[a + b*x^2]) + (a^(1/4)*(5*A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\,(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{5b}$$

$$= \frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be} + \frac{(2(5Ab-3aB))\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5be}$$

$$= \frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be} + \frac{\left(2\sqrt{a}\,(5Ab-3aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^{3/2}} -$$

$$= \frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be} + \frac{2(5Ab-3aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2\sqrt[4]{a}\,(5Ab-3aB)\sqrt{e}\,\left(\sqrt{\right.}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 80, normalized size = 0.27

$$\frac{2x\sqrt{ex}\left(3B(a+bx^2)+(5Ab-3aB)\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{15b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (2*x*Sqrt[e*x]*(3*B*(a + b*x^2) + (5*A*b - 3*a*B)*Sqrt[1 + (b*x^2)/a]*Hyper
geometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)]))/(15*b*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.10, size = 379, normalized size = 1.27

method	result

risch	$\frac{2B x^2 \sqrt{b x^2+a}\, e}{5b\sqrt{ex}}+\frac{(5Ab-3Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}}{\ldots}\right.}{5b^2\sqrt{be\,x^3 \ldots}}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(\frac{2Bx\sqrt{be\,x^3+aex}}{5b}+\frac{\left(Ae-\frac{3Bae}{5b}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\ldots}{\ldots}\right.}{ex\sqrt{b\,x \ldots}}$
default	$\frac{\sqrt{ex}\left(10A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab-5A\sqrt{\frac{bx-\ldots}{\ldots}}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(e*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(10*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-5
*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+3*B*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)
```

```
^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))*a^2+2*b^2*B*x^4+2*B*a*b*x^2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] e^(1/2)*integrate((B*x^2 + A)*sqrt(x)/sqrt(b*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 55, normalized size = 0.18

$$\frac{2\left(\sqrt{bx^2+a}\,Bbx^{\frac{3}{2}}e^{\frac{1}{2}}+(3\,Ba-5\,Ab)\sqrt{b}\,e^{\frac{1}{2}}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 2/5*(sqrt(b*x^2 + a)*B*b*x^(3/2)*e^(1/2) + (3*B*a - 5*A*b)*sqrt(b)*e^(1/2)*
weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)))/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.73, size = 92, normalized size = 0.31

$$\frac{A(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,e\Gamma\left(\frac{7}{4}\right)}+\frac{B(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,e^3\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
[Out] A*(e*x)**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/
a)/(2*sqrt(a)*e*gamma(7/4)) + B*(e*x)**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (
11/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**3*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(x)*e^(1/2)/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{e\,x}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(1/2),x)
[Out] int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(1/2), x)
```

3.803 $\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}} dx$

Optimal. Leaf size=139

$$\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}+\frac{(3Ab-aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

```
[Out] 2/3*B*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b/e+1/3*(3*A*b-B*a)*(cos(2*arctan(b^(1/4)
*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^
(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/
a^(1/4)/b^(5/4)/e^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {470, 335, 226}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(3Ab-aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,b^{5/4}\sqrt{e}\sqrt{a+bx^2}}+\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[e*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*B*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b*e) + ((3*A*b - a*B)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*S
qrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(1/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x
^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx = \frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be} - \frac{\left(2\left(-\frac{3Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{3b}$$

$$= \frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be} + \frac{(2(3Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3be}$$

$$= \frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be} + \frac{(3Ab-aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\right.}{3\sqrt[4]{a}\,b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 79, normalized size = 0.57

$$\frac{2x\left(B(a+bx^2)+(3Ab-aB)\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[e*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*x*(B*(a + b*x^2) + (3*A*b - a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b*Sqrt[e*x]*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.11, size = 214, normalized size = 1.54

method	result

risch	$\frac{2Bx\sqrt{b\,x^2+a}}{3b\sqrt{ex}}+\frac{(3Ab-Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\right.}{}}\right.}{3b^2\sqrt{be\,x^3+aex}\,\sqrt{ex}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(\frac{2B\sqrt{be\,x^3+aex}}{3be}+\frac{\left(A-\frac{aB}{3b}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-a}}}}{b\sqrt{be\,x^3+aex}}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$
default	$\frac{3A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b-B\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-a}}}}{3\sqrt{b\,x^2+a}\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(b*x^2+a)^(1/2)*(3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b-B*((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a+2*b^2*B*x^3+2*B*a*b*x)/(e*x)^(1/2)/b^
2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 45, normalized size = 0.32

$$\frac{2\left(\sqrt{bx^2+a}\,Bb\sqrt{x}-(Ba-3\,Ab)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)e^{\left(-\frac{1}{2}\right)}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(b*x^2 + a)*B*b*sqrt(x) - (B*a - 3*A*b)*sqrt(b)*weierstrassPInvers
e(-4*a/b, 0, x))*e^(-1/2)/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.29, size = 94, normalized size = 0.68

$$\frac{A\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2
*sqrt(a)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4
,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*sqrt(e)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*e^(-1/2)/(sqrt(b*x^2 + a)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{\sqrt{e\,x}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(1/2)), x)
```

3.804 $\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=290

$$-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}+\frac{2(Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{a\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2(Ab+aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\Big(2\tan^{-1}\Big(}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

```
[Out] -2*A*(b*x^2+a)^(1/2)/a/e/(e*x)^(1/2)+2*(A*b+B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2
)/a/e^2/b^(1/2)/(a^(1/2)+x*b^(1/2))-2*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)
^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/
e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*
2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4
)/b^(3/4)/e^(3/2)/(b*x^2+a)^(1/2)+(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/
2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1
/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1
/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^
(3/4)/e^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {464, 335, 311, 226, 1210}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(aB+Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(aB+Ab)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\,\sqrt{a+bx^2}\,(aB+Ab)}{a\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^2])/(a*e*Sqrt[e*x]) + (2*(A*b + a*B)*Sqrt[e*x]*Sqrt[a +
b*x^2])/(a*Sqrt[b]*e^2*(Sqrt[a] + Sqrt[b]*x)) - (2*(A*b + a*B)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a +
 b*x^2]) + ((A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2
])/(a^(3/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx = -\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}} + \frac{(Ab+aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{ae^2}$$

$$= -\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}} + \frac{(2(Ab+aB))\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae^3}$$

$$= -\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}} + \frac{(2(Ab+aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{a}\sqrt{b}\,e^2} - \frac{(2(Ab+a}{}$$

$$= -\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}} + \frac{2(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{a\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2(Ab+aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\cdots}}{a^{3/4}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 82, normalized size = 0.28

$$\frac{x\left(-6A(a+bx^2)+2(Ab+aB)x^2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a(ex)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (x*(-6*A*(a + b*x^2) + 2*(A*b + a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric
2F1[1/2, 3/4, 7/4, -((b*x^2)/a)]))/(3*a*(e*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.10, size = 378, normalized size = 1.30

method	result

risch	$-\frac{2A\sqrt{b\,x^2+a}}{ae\sqrt{ex}}+\frac{(Ab+Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{El}}{}\right.}{ab\sqrt{be\,x^3+a}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(-\frac{2\left(be\,x^2+ae\right)A}{e^2a\sqrt{x\,(be\,x^2+ae)}}+\frac{\left(\frac{B}{e}+\frac{bA}{ae}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-}}{}\right.}{\sqrt{ex}\,\sqrt{b\,x}}$
default	$\frac{2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab-A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b+2*B*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))*a^2-B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2
^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*E
```

```
llipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-2*A*b^2*x
^2-2*a*b*A)/(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 58, normalized size = 0.20

$$-\frac{2\left((Ba+Ab)\sqrt{b}\,x\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^2+a}\,Ab\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -2*((B*a + A*b)*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4
*a/b, 0, x)) + sqrt(b*x^2 + a)*A*b*sqrt(x))*e^(-3/2)/(a*b*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.90, size = 97, normalized size = 0.33

$$\frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{Bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(1/2),x)
[Out] A*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*e**(3/2)*sqrt(x)*gamma(3/4)) + B*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (
7/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**(3/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*e^(-3/2)/(sqrt(b*x^2 + a)*x^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(1/2)),x)
[Out] int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(1/2)), x)
```

3.805 $\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}}-\frac{(Ab-3aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt[4]{b}\,e^{5/2}\sqrt{a+bx^2}}$$

```
[Out] -2/3*A*(b*x^2+a)^(1/2)/a/e/(e*x)^(3/2)-1/3*(A*b-3*B*a)*(cos(2*arctan(b^(1/4
)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a
^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
)),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)
/a^(5/4)/b^(1/4)/e^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 335, 226}

$$-\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(Ab-3aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt[4]{b}\,e^{5/2}\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^2])/(3*a*e*(e*x)^(3/2)) - ((A*b - 3*a*B)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(5/4)*b^(1/4)*e^(5/2)*Sqrt[a +
b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}} - \frac{(Ab-3aB)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{3ae^2}$$

$$= -\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}} - \frac{(2(Ab-3aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3ae^3}$$

$$= -\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}} - \frac{(Ab-3aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{3a^{5/4}\sqrt[4]{b}\,e^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 81, normalized size = 0.59

$$-\frac{2x\left(A(a+bx^2)+(Ab-3aB)x^2\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3a(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*(A*(a + b*x^2) + (A*b - 3*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*a*(e*x)^(5/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.12, size = 223, normalized size = 1.62

method	result
risch	$-\frac{2A\sqrt{b\,x^2+a}}{3ax\,e^2\sqrt{ex}}-\frac{(Ab-3Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\ldots}\right.}{3ab\sqrt{be\,x^3+aex}\,e^2\sqrt{ex}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(-\frac{2A\sqrt{be\,x^3+aex}}{3e^3a\,x^2}+\frac{\left(\frac{B}{e^2}-\frac{bA}{3a\,e^2}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\ldots}}{b\sqrt{be\,x^3+aex}}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$
default	$-\frac{A\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)bx-3B\sqrt{-ab}\ldots}{3\sqrt{b\,x^2+a}\,x\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(b*x^2+a)^(1/2)/x*(A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*b*x-3*B*
(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*x+2*A*b^2*x^2+2*a*b*A)/b/a/e^2/(
e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.24, size = 55, normalized size = 0.40

$$\frac{2\left(\left(3\,Ba-Ab\right)\sqrt{b}\,x^{2}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\sqrt{bx^{2}+a}\,Ab\sqrt{x}\,\right)e^{\left(-\frac{5}{2}\right)}}{3\,abx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*((3*B*a - A*b)*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x
^2 + a)*A*b*sqrt(x))*e^(-5/2)/(a*b*x^2)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 4.80, size = 97, normalized size = 0.70

$$\frac{A\Gamma\left(-\frac{3}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}\,e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{B\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}\,e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (
5/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**(5/2)*gamma(5/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*e^(-5/2)/(sqrt(b*x^2 + a)*x^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(1/2)), x)
```

3.806 $\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=342

$$-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}+\frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}}-\frac{2\sqrt{b}\,(3Ab-5aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5a^2e^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2\sqrt[4]{b}\,(3Ab-5aB)\left(\sqrt{a}+\cdots\right.}{\cdots}$$

```
[Out] -2/5*A*(b*x^2+a)^(1/2)/a/e/(e*x)^(5/2)+2/5*(3*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^
2/e^3/(e*x)^(1/2)-2/5*(3*A*b-5*B*a)*b^(1/2)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a^2
/e^4/(a^(1/2)+x*b^(1/2))+2/5*b^(1/4)*(3*A*b-5*B*a)*(cos(2*arctan(b^(1/4)*(e
*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1
/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(
7/4)/e^(7/2)/(b*x^2+a)^(1/2)-1/5*b^(1/4)*(3*A*b-5*B*a)*(cos(2*arctan(b^(1/4
)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a
^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
)),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)
/a^(7/4)/e^(7/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 331, 335, 311, 226, 1210}

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(3Ab-5aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}}+\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(3Ab-5aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{b}\,\sqrt{ex}\,\sqrt{a+bx^2}\,(3Ab-5aB)}{5a^2e^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2\sqrt{a+bx^2}\,(3Ab-5aB)}{5a^2e^3\sqrt{ex}}-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^2])/(5*a*e*(e*x)^(5/2)) + (2*(3*A*b - 5*a*B)*Sqrt[a + b*
x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*Sqrt[b]*(3*A*b - 5*a*B)*Sqrt[e*x]*Sqrt[a +
 b*x^2])/(5*a^2*e^4*(Sqrt[a] + Sqrt[b]*x)) + (2*b^(1/4)*(3*A*b - 5*a*B)*(Sq
rt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*Ar
cTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[
a + b*x^2]) - (b^(1/4)*(3*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^
2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)
*Sqrt[e])], 1/2])/(5*a^(7/4)*e^(7/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx = -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}} - \frac{(3Ab-5aB)\int \frac{1}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx}{5ae^2}$$

$$= -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}} + \frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}} - \frac{(b(3Ab-5aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{5a^2e^4}$$

$$= -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}} + \frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}} - \frac{(2b(3Ab-5aB))\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{b}{\ldots}}}\ldots\right)}{5a^2e^5}$$

$$= -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}} + \frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}} - \frac{\left(2\sqrt{b}\,(3Ab-5aB)\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt{a\ldots}}\ldots\right)}{5a^{3/2}e^4}$$

$$= -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}} + \frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}} - \frac{2\sqrt{b}\,(3Ab-5aB)\sqrt{ex}\,\sqrt{a+bx^2}}{5a^2e^4\left(\sqrt{a}+\sqrt{b}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 82, normalized size = 0.24

$$-\frac{2x\left(A(a+bx^2)+(-3Ab+5aB)x^2\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^2}{a}\right)\right)}{5a(ex)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*(A*(a + b*x^2) + (-3*A*b + 5*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.12, size = 417, normalized size = 1.22

method	result

risch	$-\frac{2\sqrt{b\,x^2+a}\,(-3Ab\,x^2+5Ba\,x^2+Aa)}{5a^2x^2e^3\sqrt{ex}}-\frac{(3Ab-5Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{}{\sqrt{}}}}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(-\frac{2A\sqrt{be\,x^3+aex}}{5e^4a\,x^3}+\frac{2\left(be\,x^2+ae\right)(3Ab-5Ba)}{5e^4a^2\sqrt{x\,(be\,x^2+ae)}}-\frac{(3Ab-5Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-}}{}\right.}{}$
default	$-\frac{6A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-3A\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-a}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^2*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-3*A*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*a*b*x^2-10*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*x^2+5*B*((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
```

```
-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*a^2*x^2-6*A*b^2*x^4+10*B*a*b*x^4-4*a*A*b*x^2+10*B*a^2*x^2+2*a^
2*A)/(b*x^2+a)^(1/2)/e^3/(e*x)^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 74, normalized size = 0.22

$$-\frac{2\left(\left(5\,Ba-3\,Ab\right)\sqrt{b}\,x^{3}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(\left(5\,Ba-3\,Ab\right)x^{2}+Aa\right)\sqrt{bx^{2}+a}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{5\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*((5*B*a - 3*A*b)*sqrt(b)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPIn
verse(-4*a/b, 0, x)) + ((5*B*a - 3*A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt(x))
*e^(-7/2)/(a^2*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 17.33, size = 104, normalized size = 0.30

$$\frac{A\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{B\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}\,e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt
(a)*e**(7/2)*x**(5/2)*gamma(-1/4)) + B*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,
), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**(7/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*e^(-7/2)/(sqrt(b*x^2 + a)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{{\left(e\,x\right)}^{7/2}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(1/2)), x)
```

3.807 $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}}+\frac{5(7Ab-9aB)e^3\sqrt{ex}\,\sqrt{a+bx^2}}{21b^3}-\frac{5a^{3/4}(7Ab-9aB)e^{7/2}\left(\sqrt{a}+\sqrt{\ldots}\right.}{\ldots}$$

[Out] -1/7*(7*A*b-9*B*a)*e*(e*x)^(5/2)/b^2/(b*x^2+a)^(1/2)+2/7*B*(e*x)^(9/2)/b/e/(b*x^2+a)^(1/2)+5/21*(7*A*b-9*B*a)*e^3*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^3-5/42*a^(3/4)*(7*A*b-9*B*a)*e^(7/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(13/4)/(b*x^2+a)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 294, 327, 335, 226}

$$-\frac{5a^{3/4}e^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(7Ab-9aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{42b^{13/4}\sqrt{a+bx^2}}+\frac{5e^3\sqrt{ex}\,\sqrt{a+bx^2}\,(7Ab-9aB)}{21b^3}-\frac{e(ex)^{5/2}(7Ab-9aB)}{7b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]

[Out] -1/7*((7*A*b - 9*a*B)*e*(e*x)^(5/2))/(b^2*Sqrt[a + b*x^2]) + (2*B*(e*x)^(9/2))/(7*b*e*Sqrt[a + b*x^2]) + (5*(7*A*b - 9*a*B)*e^3*Sqrt[e*x]*Sqrt[a + b*x^2])/(21*b^3) - (5*a^(3/4)*(7*A*b - 9*a*B)*e^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(42*b^(13/4)*Sqrt[a + b*x^2])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{(ex)^{7/2}}{(a+bx^2)^{3/2}}\,dx}{7b}$$

$$= -\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} + \frac{(5(7Ab-9aB)e^2)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^2}}\,dx}{14b^2}$$

$$= -\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} + \frac{5(7Ab-9aB)e^3\sqrt{ex}\sqrt{a+bx^2}}{21b^3} - \frac{(5}{}$$

$$= -\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} + \frac{5(7Ab-9aB)e^3\sqrt{ex}\sqrt{a+bx^2}}{21b^3} - \frac{(5}{}$$

$$= -\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} + \frac{5(7Ab-9aB)e^3\sqrt{ex}\sqrt{a+bx^2}}{21b^3} - \frac{5a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 111, normalized size = 0.53

$$\frac{e^3\sqrt{ex}\left(-45a^2B+ab(35A-18Bx^2)+2b^2x^2(7A+3Bx^2)+5a(-7Ab+9aB)\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{21b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(-45*a^2*B + a*b*(35*A - 18*B*x^2) + 2*b^2*x^2*(7*A + 3*B*x^
2) + 5*a*(-7*A*b + 9*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5
/4, -((b*x^2)/a)]))/(21*b^3*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.14, size = 252, normalized size = 1.19

method	result
default	$-\frac{e^3\sqrt{ex}\left(35A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a\right.}{}$

elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(\frac{e^4xa(Ab-Ba)}{b^3\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{2B\,e^3x^2\sqrt{be\,x^3+aex}}{7b^2}+\frac{2\left(\frac{(Ab-Ba)e^4}{b^2}-\frac{5B\,e^4a}{7b^2}\right)\sqrt{be\,x^3+aex}}{3be}\right.}{}$
risch	$\frac{2(3bB\,x^2+7Ab-12Ba)x\sqrt{b\,x^2+a}\;e^4}{21b^3\sqrt{ex}}-\frac{a\left(\frac{28A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}}{\sqrt{be\,x^3+aex}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/42*e^3/x*(e*x)^(1/2)*(35*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/
2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellip
ticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a*b-
45*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2-12*B*b^3*x^5-28*A*b^3*x^3
+36*B*a*b^2*x^3-70*A*a*b^2*x+90*B*a^2*b*x)/(b*x^2+a)^(1/2)/b^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 125, normalized size = 0.59

$$\frac{5\,(9\,Ba^3-7\,Aa^2b+(9\,Ba^2b-7\,Aab^2)x^2)\sqrt{b}\;e^{\frac{7}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+(6\,Bb^3x^4-45\,Ba^2b+35\,Aab^2-2\,(9\,Bab^2-7\,Ab^3)x^2)\sqrt{bx^2+a}\;\sqrt{x}\;e^{\frac{7}{2}}}{21\,(b^5x^2+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(5*(9*B*a^3 - 7*A*a^2*b + (9*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b)*e^(7/2)
*weierstrassPInverse(-4*a/b, 0, x) + (6*B*b^3*x^4 - 45*B*a^2*b + 35*A*a*b^2
 - 2*(9*B*a*b^2 - 7*A*b^3)*x^2)*sqrt(b*x^2 + a)*sqrt(x)*e^(7/2))/(b^5*x^2 +
 a*b^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 180.20, size = 94, normalized size = 0.45

$$\frac{Ae^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}+\frac{Be^{\frac{7}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{13}{4}\\\frac{17}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*e**(7/2)*x**(9/2)*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(3/2)*gamma(13/4)) + B*e**(7/2)*x**(13/2)*gamma(13/4)*hyper(
(3/2, 13/4), (17/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(17/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)*e^(7/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(e\,x\right)}^{7/2}}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(7/2))/(a + b*x^2)^(3/2),x)
```

```
[Out] int(((A + B*x^2)*(e*x)^(7/2))/(a + b*x^2)^(3/2), x)
```

3.808 $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=337

$$-\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}+\frac{3(5Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^2}}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{3\sqrt[4]{a}(5Ab-7aB)e^{5/2}\left(\sqrt{a}+\right.}{}$$

```
[Out] -1/5*(5*A*b-7*B*a)*e*(e*x)^(3/2)/b^2/(b*x^2+a)^(1/2)+2/5*B*(e*x)^(7/2)/b/e/
(b*x^2+a)^(1/2)+3/5*(5*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^(5/2)/(
a^(1/2)+x*b^(1/2))-3/5*a^(1/4)*(5*A*b-7*B*a)*e^(5/2)*(cos(2*arctan(b^(1/4)*
(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(
1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))
,1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b
^(11/4)/(b*x^2+a)^(1/2)+3/10*a^(1/4)*(5*A*b-7*B*a)*e^(5/2)*(cos(2*arctan(b^
(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/
2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(
1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(
1/2)/b^(11/4)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 294, 335, 311, 226, 1210}

$$\frac{3\sqrt[4]{a}\,e^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(5Ab-7aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{a}\,e^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(5Ab-7aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{a+bx^2}}+\frac{3e^2\sqrt{ex}\sqrt{a+bx^2}\,(5Ab-7aB)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{e(ex)^{3/2}(5Ab-7aB)}{5b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/5*((5*A*b - 7*a*B)*e*(e*x)^(3/2))/(b^2*Sqrt[a + b*x^2]) + (2*B*(e*x)^(7/
2))/(5*b*e*Sqrt[a + b*x^2]) + (3*(5*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x
^2])/(5*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) - (3*a^(1/4)*(5*A*b - 7*a*B)*e^(5/2)
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[
2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*b^(11/4)*Sqrt[a +
 b*x^2]) + (3*a^(1/4)*(5*A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(
a^(1/4)*Sqrt[e])], 1/2])/(10*b^(11/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{(ex)^{5/2}}{(a+bx^2)^{3/2}}\,dx}{5b}$$

$$= -\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} + \frac{\left(3(5Ab-7aB)e^2\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{10b^2}$$

$$= -\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} + \frac{(3(5Ab-7aB)e)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx}{e}}}\right.}{5b^2}$$

$$= -\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} + \frac{\left(3\sqrt{a}\,(5Ab-7aB)e^2\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{5b^{5/2}}$$

$$= -\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} + \frac{3(5Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^2}}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 84, normalized size = 0.25

$$\frac{2e(ex)^{3/2}\left(5Ab-7aB+bBx^2+(-5Ab+7aB)\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{5b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*e*(e*x)^(3/2)*(5*A*b - 7*a*B + b*B*x^2 + (-5*A*b + 7*a*B)*Sqrt[1 + (b*x^
2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[a + b*x^
2])
```

Maple [A]
time = 0.13, size = 391, normalized size = 1.16

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(-\frac{e^3x^2(Ab-Ba)}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{2B\,e^2x\sqrt{be\,x^3+aex}}{5b^2}+\frac{\left(\frac{3(Ab-Ba)e^3}{2b^2}-\frac{3B\,e^3a}{5b^2}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{}}{}\right.}{\sqrt{}}}}{}\right.}{}$
default	$\frac{e^2\sqrt{ex}\left(30A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab-15A\sqrt{\frac{bx}{}}\right.}{}$
risch	$\frac{2B\,x^2\sqrt{b\,x^2+a}\,e^3}{5b^2\sqrt{ex}}+\frac{\left(\frac{(5Ab-8Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}}{}\right.}{b\sqrt{be\,x^3+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*e^2/x*(e*x)^(1/2)*(30*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-15*A*((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))*a*b-42*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)
*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipti
cE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+21*B*((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))*a^2+4*b^2*B*x^4-10*A*b^2*x^2+14*B*a*b*x^2)/(b*x^2+a)^(1/2
)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 109, normalized size = 0.32

$$\frac{3\,(7\,Ba^2-5\,Aab+(7\,Bab-5\,Ab^2)x^2)\sqrt{b}\,e^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(2\,Bb^2x^3+(7\,Bab-5\,Ab^2)x)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{5}{2}}}{5\,(b^4x^2+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(3*(7*B*a^2 - 5*A*a*b + (7*B*a*b - 5*A*b^2)*x^2)*sqrt(b)*e^(5/2)*weiers
trassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (2*B*b^2*x^3 + (7
*B*a*b - 5*A*b^2)*x)*sqrt(b*x^2 + a)*sqrt(x)*e^(5/2))/(b^4*x^2 + a*b^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 52.69, size = 94, normalized size = 0.28

$$\frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}+\frac{Be^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(3/2)*gamma(11/4)) + B*e**(5/2)*x**(11/2)*gamma(11/4)*hyper(
(3/2, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(15/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(5/2)*e^(5/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(e\,x\right)}^{5/2}}{{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(3/2),x)
[Out] int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(3/2), x)
```

3.809 $\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{(3Ab-5aB)e\sqrt{ex}}{3b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}+\frac{(3Ab-5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\cdots\right.\right.}{6\sqrt[4]{a}\,b^{9/4}\sqrt{a+bx^2}}$$

```
[Out] 2/3*B*(e*x)^(5/2)/b/e/(b*x^2+a)^(1/2)-1/3*(3*A*b-5*B*a)*e*(e*x)^(1/2)/b^2/(
b*x^2+a)^(1/2)+1/6*(3*A*b-5*B*a)*e^(3/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/
a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2)
)*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(9/
4)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 294, 335, 226}

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(3Ab-5aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}\,b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}\,(3Ab-5aB)}{3b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/3*((3*A*b - 5*a*B)*e*Sqrt[e*x])/(b^2*Sqrt[a + b*x^2]) + (2*B*(e*x)^(5/2)
)/(3*b*e*Sqrt[a + b*x^2]) + ((3*A*b - 5*a*B)*e^(3/2)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(6*a^(1/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}-\frac{\left(2\left(-\frac{3Ab}{2}+\frac{5aB}{2}\right)\right)\int\frac{(ex)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{3b}$$

$$= -\frac{(3Ab-5aB)e\sqrt{ex}}{3b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}+\frac{\left((3Ab-5aB)e^2\right)\int\frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{6b^2}$$

$$= -\frac{(3Ab-5aB)e\sqrt{ex}}{3b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}+\frac{((3Ab-5aB)e)\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,d\right.}{3b^2}$$

$$= -\frac{(3Ab-5aB)e\sqrt{ex}}{3b^2\sqrt{a+bx^2}}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}+\frac{(3Ab-5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{}{\left(\sqrt{\right.}}}}{6\sqrt[4]{a}\,b^{9/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 85, normalized size = 0.49

$$\frac{e\sqrt{ex}\left(-3Ab+5aB+2bBx^2+(3Ab-5aB)\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]

[Out] (e*Sqrt[e*x]*(-3*A*b + 5*a*B + 2*b*B*x^2 + (3*A*b - 5*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b^2*Sqrt[a + b*x^2])

Maple [A]
time = 0.12, size = 225, normalized size = 1.29

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(-\frac{e^2x(Ab-Ba)}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{2Be\sqrt{be\,x^3+aex}}{3b^2}+\frac{\left(\frac{(Ab-Ba)e^2}{2b^2}-\frac{B\,e^2a}{3b^2}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}}{}\right.}{ex\sqrt{b\,x^2+a}}$
default	$\frac{e\sqrt{ex}\left(3A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b-5\right.}{6x}$
risch	$\frac{2Bx\sqrt{b\,x^2+a}\,e^2}{3b^2\sqrt{ex}}+\frac{\left(\frac{3A\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{}}{}\right.}{\sqrt{}}}\right.}{\sqrt{be\,x^3+aex}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/6*e/x*(e*x)^(1/2)*(3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b-5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a+4*b^2*B*x^3-6*A*b^2*x+10*B*a*b*x)/(b*x^2+a)^(1/2)/b^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 98, normalized size = 0.56

$$-\frac{(5\,Ba^2-3\,Aab+(5\,Bab-3\,Ab^2)x^2)\sqrt{b}\,e^{\frac{3}{2}}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(2\,Bb^2x^2+5\,Bab-3\,Ab^2)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{3}{2}}}{3\,(b^4x^2+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*((5*B*a^2 - 3*A*a*b + (5*B*a*b - 3*A*b^2)*x^2)*sqrt(b)*e^(3/2)*weierst
rassPInverse(-4*a/b, 0, x) - (2*B*b^2*x^2 + 5*B*a*b - 3*A*b^2)*sqrt(b*x^2 +
 a)*sqrt(x)*e^(3/2))/(b^4*x^2 + a*b^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 12.58, size = 94, normalized size = 0.54

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*gamma(9/4)) + B*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((3/2
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)*e^(3/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^2+A)\,\left(e\,x\right)^{3/2}}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(3/2),x)
[Out] int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(3/2), x)
```

3.810 $\int \frac{\sqrt{ex}\,(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\sqrt{ex}\,\sqrt{a+bx^2}}{ab^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{(Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\Big(2\,\mathrm{t}}{a^{3/4}b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] (A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^2+a)^(1/2)-(A*b-3*B*a)*(e*x)^(1/2)*(b*x^2+
a)^(1/2)/a/b^(3/2)/(a^(1/2)+x*b^(1/2))+(A*b-3*B*a)*(cos(2*arctan(b^(1/4)*(e
*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1
/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(
1/2)/a^(3/4)/b^(7/4)/(b*x^2+a)^(1/2)-1/2*(A*b-3*B*a)*(cos(2*arctan(b^(1/4)*
(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(
1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))
,1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)
^(1/2)/a^(3/4)/b^(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {468, 335, 311, 226, 1210}

$$-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(Ab-3aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(Ab-3aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{7/4}\sqrt{a+bx^2}}-\frac{\sqrt{ex}\,\sqrt{a+bx^2}\,(Ab-3aB)}{ab^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{(ex)^{3/2}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(a*b*e*Sqrt[a + b*x^2]) - ((A*b - 3*a*B)*Sqrt[e*x
]*Sqrt[a + b*x^2])/(a*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A*b - 3*a*B)*Sqrt[
e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a^(3/4)*b^(7/4)*S
qrt[a + b*x^2]) - ((A*b - 3*a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*
x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/
4)*Sqrt[e])], 1/2])/(2*a^(3/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\,(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}} + \frac{\left(-\frac{Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}} - \frac{(Ab-3aB)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{abe}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}} - \frac{(Ab-3aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{a}\,b^{3/2}} + \frac{(Ab-3aB}{}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}} - \frac{(Ab-3aB)\sqrt{ex}\sqrt{a+bx^2}}{ab^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{(Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 76, normalized size = 0.25

$$\frac{2x\sqrt{ex}\left(3aB+(Ab-3aB)\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[e*x]*(3*a*B + (A*b - 3*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1
[3/4, 3/2, 7/4, -((b*x^2)/a)]))/(3*a*b*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.10, size = 382, normalized size = 1.27

method	result

elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(\frac{e\,x^2(Ab-Ba)}{ba\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\left(\frac{Be}{b}-\frac{(Ab-Ba)e}{2ab}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}}{}\right)}{ex\sqrt{}}$
default	$-\frac{\sqrt{ex}\left(2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\;\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab-A\sqrt{\frac{bx}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(e*x)^(1/2)*(2*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-A*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b
/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*a*b-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+3*B*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*a^2-2*A*b^2*x^2+2*B*a*b*x^2)/(b*x^2+a)^(1/2)/b^2/x/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 98, normalized size = 0.33

$$-\frac{(Bab-Ab^2)\sqrt{bx^2+a}\,x^{\frac{3}{2}}e^{\frac{1}{2}}+(3\,Ba^2-Aab+(3\,Bab-Ab^2)x^2)\sqrt{b}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{ab^3x^2+a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -((B*a*b - A*b^2)*sqrt(b*x^2 + a)*x^(3/2)*e^(1/2) + (3*B*a^2 - A*a*b + (3*B
*a*b - A*b^2)*x^2)*sqrt(b)*e^(1/2)*weierstrassZeta(-4*a/b, 0, weierstrassPI
nverse(-4*a/b, 0, x)))/(a*b^3*x^2 + a^2*b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.61, size = 94, normalized size = 0.31

$$\frac{A\sqrt{e}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{B\sqrt{e}\,x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/2)*gamma(7/4)) + B*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((3/2,
7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*sqrt(x)*e^(1/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,\sqrt{e\,x}}{{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(3/2),x)
```

```
[Out] int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(3/2), x)
```

3.811 $\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{(Ab-aB)\sqrt{ex}}{abe\sqrt{a+bx^2}}+\frac{(Ab+aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{5/4}b^{5/4}\sqrt{e}\,\sqrt{a+bx^2}}$$

```
[Out] (A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(1/2)+1/2*(A*b+B*a)*(cos(2*arctan(b^(
1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2
)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1
/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/a^(5/4)/b^(5/4)/e^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {468, 335, 226}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(aB+Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{5/4}b^{5/4}\sqrt{e}\,\sqrt{a+bx^2}}+\frac{\sqrt{ex}\,(Ab-aB)}{abe\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] ((A*b - a*B)*Sqrt[e*x])/(a*b*e*Sqrt[a + b*x^2]) + ((A*b + a*B)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(5/4)*b^(5/4)*Sqrt[e]*Sqrt[a
 + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{3/2}}\,dx &= \frac{(Ab-aB)\sqrt{ex}}{abe\sqrt{a+bx^2}}+\frac{(Ab+aB)\int\frac{1}{\sqrt{ex}\,\sqrt{a+bx^2}}\,dx}{2ab}\\
&= \frac{(Ab-aB)\sqrt{ex}}{abe\sqrt{a+bx^2}}+\frac{(Ab+aB)\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{abe}\\
&= \frac{(Ab-aB)\sqrt{ex}}{abe\sqrt{a+bx^2}}+\frac{(Ab+aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\cdots\right.\right.}{2a^{5/4}b^{5/4}\sqrt{e}\,\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 75, normalized size = 0.52

$$
\frac{x\left(Ab-aB+(Ab+aB)\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{ab\sqrt{ex}\,\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x*(A*b - a*B + (A*b + a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((b*x^2)/a)]))/(a*b*Sqrt[e*x]*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.10, size = 213, normalized size = 1.48

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(\frac{x(Ab-Ba)}{ba\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\left(\frac{B}{b}+\frac{Ab-Ba}{2ab}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-}}{b\sqrt{be\,x^3+aex}}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$
default	$\frac{A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b+B\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}}{2\sqrt{b\,x^2+a}\,a\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b+B*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a
*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*(-a*b)^(1/2)*a+2*A*b^2*x-2*B*a*b*x)/(b*x^2+a)^(1/2)/a/(e*x)^(1/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 84, normalized size = 0.58

$$\frac{\left((Ba^2+Aab+(Bab+Ab^2)x^2)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-(Bab-Ab^2)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{ab^3x^2+a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] ((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(b)*weierstrassPInverse(-4*a/b,
0, x) - (B*a*b - A*b^2)*sqrt(b*x^2 + a)*sqrt(x))*e^(-1/2)/(a*b^3*x^2 + a^2*
b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.15, size = 94, normalized size = 0.65

$$\frac{A\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(3/2)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/
4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*sqrt(e)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*e^(-1/2)/((b*x^2 + a)^(3/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{\sqrt{e\,x}\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(3/2)), x)
```

3.812 $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}-\frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}}+\frac{(3Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{a^2\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{(3Ab-aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{\cdots}{(\cdots}}}{a^{7/4}b^{3\cdots}}$$

```
[Out] -(3*A*b-B*a)*(e*x)^(3/2)/a^2/e^3/(b*x^2+a)^(1/2)-2*A/a/e/(e*x)^(1/2)/(b*x^2
+a)^(1/2)+(3*A*b-B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/e^2/b^(1/2)/(a^(1/2)+
x*b^(1/2))-(3*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2
*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/
2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/b^(3/4)/e^(3/2)/(b*x^2+
a)^(1/2)+1/2*(3*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin
(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(
1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/b^(3/4)/e^(3/2)/(b*x^
2+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 296, 335, 311, 226, 1210}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(3Ab-aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(3Ab-aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{(ex)^{3/2}(3Ab-aB)}{a^2e^3\sqrt{a+bx^2}}+\frac{\sqrt{ex}\sqrt{a+bx^2}(3Ab-aB)}{a^2\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^2]) - ((3*A*b - a*B)*(e*x)^(3/2))/(a^2*e
^3*Sqrt[a + b*x^2]) + ((3*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(a^2*Sqrt[b
]*e^2*(Sqrt[a] + Sqrt[b]*x)) - ((3*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(
a^(1/4)*Sqrt[e])], 1/2])/(a^(7/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((3*A*
b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(7/4)*b
^(3/4)*e^(3/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{3/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}} - \frac{(3Ab-aB)\int \frac{\sqrt{ex}}{(a+bx^2)^{3/2}}\,dx}{ae^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}} - \frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}} + \frac{(3Ab-aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{2a^2e^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}} - \frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}} + \frac{(3Ab-aB)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\right.}{a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}} - \frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}} + \frac{(3Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\right.}{a^{3/2}\sqrt{b}\,e^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}} - \frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}} + \frac{(3Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{a^2\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 77, normalized size = 0.23

$$\frac{x\left(-6aA+2(-3Ab+aB)x^2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a^2(ex)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x*(-6*a*A + 2*(-3*A*b + a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4
, 3/2, 7/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(3/2)*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.13, size = 386, normalized size = 1.16

method	result

elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(-\frac{x^2(Ab-Ba)}{e\,a^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}-\frac{2\left(be\,x^2+ae\right)A}{a^2e^2\sqrt{x\,(be\,x^2+ae)}}+\frac{\left(\frac{Ab-Ba}{2a^2e}+\frac{bA}{a^2e}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}}{}\right.}{}$
default	$\frac{6A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab-3A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}}{}$
risch	$-\frac{2A\sqrt{b\,x^2+a}}{a^2e\sqrt{ex}}+\frac{\left(\frac{A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\right.}{}\right.}{\sqrt{be\,x^3+aex}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
```

```
))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b-3*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*
b-2*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(
1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-6*A
*b^2*x^2+2*B*a*b*x^2-4*a*b*A)/(b*x^2+a)^(1/2)/b/e/(e*x)^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 109, normalized size = 0.33

$$\frac{\left(\left((Bab-3\,Ab^2)x^3+(Ba^2-3\,Aab)x\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-\left(2\,Aab-(Bab-3\,Ab^2)x^2\right)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{a^2b^2x^3+a^3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (((B*a*b - 3*A*b^2)*x^3 + (B*a^2 - 3*A*a*b)*x)*sqrt(b)*weierstrassZeta(-4*a
/b, 0, weierstrassPInverse(-4*a/b, 0, x)) - (2*A*a*b - (B*a*b - 3*A*b^2)*x^
2)*sqrt(b*x^2 + a)*sqrt(x))*e^(-3/2)/(a^2*b^2*x^3 + a^3*b*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.40, size = 97, normalized size = 0.29

$$\frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{Bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3
/2)*e**(3/2)*sqrt(x)*gamma(3/4)) + B*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2),
(7/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*e**(3/2)*gamma(7/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*e^(-3/2)/((b*x^2 + a)^(3/2)*x^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x)
[Out] int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)), x)
```

3.813 $\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}-\frac{(5Ab-3aB)\sqrt{ex}}{3a^2e^3\sqrt{a+bx^2}}-\frac{(5Ab-3aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\Big(2\tan^{-1}}{6a^{9/4}\sqrt[4]{b}\,e^{5/2}\sqrt{a+bx^2}}$$

```
[Out] -2/3*A/a/e/(e*x)^(3/2)/(b*x^2+a)^(1/2)-1/3*(5*A*b-3*B*a)*(e*x)^(1/2)/a^2/e^
3/(b*x^2+a)^(1/2)-1/6*(5*A*b-3*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*El
lipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^
(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/b^(1/4)/e^
(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {464, 296, 335, 226}

$$-\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(5Ab-3aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt[4]{b}\,e^{5/2}\sqrt{a+bx^2}}-\frac{\sqrt{ex}\,(5Ab-3aB)}{3a^2e^3\sqrt{a+bx^2}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*Sqrt[e*x])/(3
*a^2*e^3*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a
^(1/4)*Sqrt[e])], 1/2])/(6*a^(9/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{3/2}}\,dx}{3ae^2}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\sqrt{ex}}{3a^2e^3\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\int \frac{1}{\sqrt{ex}\,\sqrt{a+bx^2}}}{6a^2e^2}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\sqrt{ex}}{3a^2e^3\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx}{e}}}\right.}{3a^2e^3}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\sqrt{ex}}{3a^2e^3\sqrt{a+bx^2}} - \frac{(5Ab-3aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{}}{6a^{9/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 91, normalized size = 0.52

$$\frac{x\left(-2aA-5Abx^2+3aBx^2+(-5Ab+3aB)x^2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3a^2(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x]

[Out] (x*(-2*a*A - 5*A*b*x^2 + 3*a*B*x^2 + (-5*A*b + 3*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(5/2)*Sqrt[a + b*x^2])

Maple [A]
time = 0.13, size = 232, normalized size = 1.32

method	result
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(-\frac{x(Ab-Ba)}{e^2a^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{bex^3+aex}}{3a^2e^3x^2}+\frac{\left(-\frac{Ab-Ba}{2a^2e^2}-\frac{bA}{3a^2e^2}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\cdots}{\cdots}\right.}{\sqrt{ex}\sqrt{bx^2+a}\cdots}$
default	$-\frac{5A\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)bx-3B\sqrt{-\cdots}}{6x\sqrt{bx^2\cdots}}$
risch	$-\frac{2A\sqrt{bx^2+a}}{3a^2xe^2\sqrt{ex}}-\frac{\left(\frac{A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\sqrt{-\cdots}\right.}{\sqrt{-\cdots}}}\cdots\right.}{\sqrt{bex^3+aex}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/6/x*(5*A*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*b*x-3*B*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*x+10*A*b^2*x^2-6*B*a*b*x^2+4*a*b*A)/(b*x^2+a)^(1/2)/b/a^2/e^2/(e*x)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 109, normalized size = 0.62

$$\frac{\left(\left((3\,Bab-5\,Ab^2)x^4+(3\,Ba^2-5\,Aab)x^2\right)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\left(2\,Aab-(3\,Bab-5\,Ab^2)x^2\right)\sqrt{bx^2+a}\ \sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{3\,(a^2b^2x^4+a^3bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((3*B*a*b - 5*A*b^2)*x^4 + (3*B*a^2 - 5*A*a*b)*x^2)*sqrt(b)*weierstras
sPInverse(-4*a/b, 0, x) - (2*A*a*b - (3*B*a*b - 5*A*b^2)*x^2)*sqrt(b*x^2 +
a)*sqrt(x))*e^(-5/2)/(a^2*b^2*x^4 + a^3*b*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 22.65, size = 97, normalized size = 0.55

$$\frac{A\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{B\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3
/2)*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2),
(5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*e**(5/2)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*e^(-5/2)/((b*x^2 + a)^(3/2)*x^(5/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{{\left(e\,x\right)}^{5/2}\,{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x)
[Out] int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)), x)
```

3.814 $\int \frac{A+Bx^2}{(ex)^{7/2}(a+bx^2)^{3/2}}\,dx$

<u>**Optimal**</u>. Leaf size=379

$$-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}-\frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}}+\frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}}-\frac{3\sqrt{b}\,(7Ab-5aB)\sqrt{ex}\sqrt{a+bx^2}}{5a^3e^4\left(\sqrt{a}+\sqrt{b}\,x\right)}$$

```
[Out] -2/5*A/a/e/(e*x)^(5/2)/(b*x^2+a)^(1/2)+1/5*(-7*A*b+5*B*a)/a^2/e^3/(e*x)^(1/
2)/(b*x^2+a)^(1/2)+3/5*(7*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^3/e^3/(e*x)^(1/2)-3/
5*(7*A*b-5*B*a)*b^(1/2)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a^3/e^4/(a^(1/2)+x*b^(1
/2))+3/5*b^(1/4)*(7*A*b-5*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^
(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*Ellipti
cE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)
+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/e^(7/2)/(b*x^2
+a)^(1/2)-3/10*b^(1/4)*(7*A*b-5*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1
/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*E
llipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a
^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/e^(7/2)/
(b*x^2+a)^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.21, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {464, 296, 331, 335, 311, 226, 1210}

$$-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(7Ab-5aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{10a^{11/4}e^{7/2}\sqrt{a+bx^2}}+\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(7Ab-5aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{11/4}e^{7/2}\sqrt{a+bx^2}}-\frac{3\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}(7Ab-5aB)}{5a^3e^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{3\sqrt{a+bx^2}(7Ab-5aB)}{5a^3e^3\sqrt{ex}}-\frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^2]) - (7*A*b - 5*a*B)/(5*a^2*e^3*Sqr
t[e*x]*Sqrt[a + b*x^2]) + (3*(7*A*b - 5*a*B)*Sqrt[a + b*x^2])/(5*a^3*e^3*Sq
rt[e*x]) - (3*Sqrt[b]*(7*A*b - 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(5*a^3*e^4
*(Sqrt[a] + Sqrt[b]*x)) + (3*b^(1/4)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(5*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2]) - (3*b
^(1/4)*(7*A*b - 5*a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2]
)/(10*a^(11/4)*e^(7/2)*Sqrt[a + b*x^2])
```

<u>Rule 226</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{(7Ab-5aB)\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx}{5ae^2}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} - \frac{(3(7Ab-5aB))\int \frac{1}{(ex)^{3/2}\sqrt{a+}}}{10a^2e^2}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} + \frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}} - ($$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} + \frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}} - ($$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} + \frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}} - ($$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}} - \frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}} + \frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}} - 3$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 78, normalized size = 0.21

$$\frac{x\left(-2aA+2(7Ab-5aB)x^2\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^2}{a}\right)\right)}{5a^2(ex)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x]

[Out] (x*(-2*a*A + 2*(7*A*b - 5*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)]))/(5*a^2*(e*x)^(7/2)*Sqrt[a + b*x^2])

Maple [A]

time = 0.14, size = 417, normalized size = 1.10

method	result
elliptic	$\frac{\sqrt{(b x^2+a) e x}\left(\frac{b x^2 (Ab-Ba)}{e^3 a^3 \sqrt{\left(x^2+\frac{a}{b}\right) bex}}-\frac{2A\sqrt{be x^3+aex}}{5a^2 e^4 x^3}+\frac{2\left(be x^2+ae\right)(8Ab-5Ba)}{5a^3 e^4 \sqrt{x\left(be x^2+ae\right)}}+\frac{\left(-\frac{b(Ab-Ba)}{2a^3e^3}-\frac{b(8Ab-5}{5a^3e^3}\right.}{}\right.}{}$
default	$-\frac{42A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab x^2-21A\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}}{}$

risch	$-\frac{2\sqrt{b\,x^2+a}\,(-8Ab\,x^2+5Ba\,x^2+Aa)}{5a^3x^2e^3\sqrt{ex}}-\frac{b^2\left(\frac{(8Ab-5Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{}{\sqrt{}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/x^2*(42*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-21*A*((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*
b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*a*b*x^2-30*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((
```

```
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*x^2+15*B*((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2),1/2*2^(1/2))*a^2*x^2-42*A*b^2*x^4+30*B*a*b*x^4-28*a*A*b*x^2+20*B*a^2*x^
2+4*a^2*A)/(b*x^2+a)^(1/2)/e^3/(e*x)^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 132, normalized size = 0.35

$$-\frac{\left(3\left((5\,Bab-7\,Ab^2)x^5+(5\,Ba^2-7\,Aab)x^3\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(3\,(5\,Bab-7\,Ab^2)x^4+2\,Aa^2+2\,(5\,Ba^2-7\,Aab)x^2\right)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{5\,(a^3bx^5+a^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*((5*B*a*b - 7*A*b^2)*x^5 + (5*B*a^2 - 7*A*a*b)*x^3)*sqrt(b)*weierst
rassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (3*(5*B*a*b - 7*A*
b^2)*x^4 + 2*A*a^2 + 2*(5*B*a^2 - 7*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(x))*e^
(-7/2)/(a^3*b*x^5 + a^4*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 63.73, size = 104, normalized size = 0.27

$$\frac{A\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},\frac{3}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{B\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(
3/2)*e**(7/2)*x**(5/2)*gamma(-1/4)) + B*gamma(-1/4)*hyper((-1/4, 3/2), (3/4
,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*e**(7/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*e^(-7/2)/((b*x^2 + a)^(3/2)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int((A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)), x)
```

3.815 $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=208

$$-\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2(a+bx^2)^{3/2}}+\frac{2B(ex)^{9/2}}{3be(a+bx^2)^{3/2}}-\frac{5(Ab-3aB)e^3\sqrt{ex}}{6b^3\sqrt{a+bx^2}}+\frac{5(Ab-3aB)e^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a\cdots}{\left(\sqrt{a}\cdots\right.}}}{12\sqrt[4]{a}\,b^{13/4}\sqrt{\cdots}}$$

[Out] -1/3*(A*b-3*B*a)*e*(e*x)^(5/2)/b^2/(b*x^2+a)^(3/2)+2/3*B*(e*x)^(9/2)/b/e/(b*x^2+a)^(3/2)-5/6*(A*b-3*B*a)*e^3*(e*x)^(1/2)/b^3/(b*x^2+a)^(1/2)+5/12*(A*b-3*B*a)*e^(7/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(13/4)/(b*x^2+a)^(1/2)

Rubi [A]
time = 0.09, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {470, 294, 335, 226}

$$\frac{5e^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(Ab-3aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{12\sqrt[4]{a}\,b^{13/4}\sqrt{a+bx^2}}-\frac{5e^3\sqrt{ex}\,(Ab-3aB)}{6b^3\sqrt{a+bx^2}}-\frac{e(ex)^{5/2}(Ab-3aB)}{3b^2(a+bx^2)^{3/2}}+\frac{2B(ex)^{9/2}}{3be(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]

[Out] -1/3*((A*b - 3*a*B)*e*(e*x)^(5/2))/(b^2*(a + b*x^2)^(3/2)) + (2*B*(e*x)^(9/2))/(3*b*e*(a + b*x^2)^(3/2)) - (5*(A*b - 3*a*B)*e^3*Sqrt[e*x])/(6*b^3*Sqrt[a + b*x^2]) + (5*(A*b - 3*a*B)*e^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(1/4)*b^(13/4)*Sqrt[a + b*x^2])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} - \frac{\left(2\left(-\frac{3Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{(ex)^{7/2}}{\left(a+bx^2\right)^{5/2}}\,dx}{3b}$$

$$= -\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} + \frac{\left(5(Ab-3aB)e^2\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{6b^2}$$

$$= -\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} - \frac{5(Ab-3aB)e^3\sqrt{ex}}{6b^3\sqrt{a+bx^2}} + \frac{(5(Ab-3aB}{\quad}$$

$$= -\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} - \frac{5(Ab-3aB)e^3\sqrt{ex}}{6b^3\sqrt{a+bx^2}} + \frac{(5(Ab-3aB}{\quad}$$

$$= -\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2\left(a+bx^2\right)^{3/2}} + \frac{2B(ex)^{9/2}}{3be\left(a+bx^2\right)^{3/2}} - \frac{5(Ab-3aB)e^3\sqrt{ex}}{6b^3\sqrt{a+bx^2}} + \frac{5(Ab-3aB)}{\quad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.14, size = 116, normalized size = 0.56

$$\frac{e^3\sqrt{ex}\left(15a^2B+b^2x^2(-7A+4Bx^2)+a(-5Ab+21bBx^2)+5(Ab-3aB)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{6b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(15*a^2*B + b^2*x^2*(-7*A + 4*B*x^2) + a*(-5*A*b + 21*b*B*x^
2) + 5*(A*b - 3*a*B)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4,
 1/2, 5/4, -((b*x^2)/a)]))/(6*b^3*(a + b*x^2)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(209) = 418.
time = 0.16, size = 439, normalized size = 2.11

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(\frac{a\,e^3(Ab-Ba)\sqrt{be\,x^3+aex}}{3b^5\left(x^2+\frac{a}{b}\right)^2}-\frac{e^4x(7Ab-13Ba)}{6b^3\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{2B\,e^3\sqrt{be\,x^3+aex}}{3b^3}+\frac{\left(\frac{(Ab-2Ba)}{b^3}\right.}{}\right.}{}$
default	$\frac{\left(5A\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,b^2x^2-15B\,\mathrm{E}\right.}{}$
risch	$\frac{2Bx\sqrt{b\,x^2+a}\;e^4}{3b^3\sqrt{ex}}+\frac{\left(\frac{3A\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\;\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\right.}{\sqrt{}}}\right.}{\sqrt{be\,x^3+aex}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(5*A*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-
a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*b^2*x^2-15*B*EllipticF((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*(-x*b/(-a*b)^(1/2))^(1/2)*a*b*x^2+5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))
^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b
)^(1/2)*a*b-15*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2+8*B*b^3*x^5-1
4*A*b^3*x^3+42*B*a*b^2*x^3-10*A*a*b^2*x+30*B*a^2*b*x)*e^3/x*(e*x)^(1/2)/b^4
/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((B*x^2 + A)*x^(7/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 156, normalized size = 0.75

$$-\frac{5\,((3\,Bab^2-Ab^3)x^4+3\,Ba^3-Aa^2b+2\,(3\,Ba^2b-Aab^2)x^2)\sqrt{b}\,e^{\frac{7}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(4\,Bb^3x^4+15\,Ba^2b-5\,Aab^2+7\,(3\,Bab^2-Ab^3)x^2)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{7}{2}}}{6\,(b^6x^4+2\,ab^5x^2+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(5*((3*B*a*b^2 - A*b^3)*x^4 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A*a*b
^2)*x^2)*sqrt(b)*e^(7/2)*weierstrassPInverse(-4*a/b, 0, x) - (4*B*b^3*x^4 +
 15*B*a^2*b - 5*A*a*b^2 + 7*(3*B*a*b^2 - A*b^3)*x^2)*sqrt(b*x^2 + a)*sqrt(x
)*e^(7/2))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(7/2)*e^(7/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^2+A\right)\,{\left(e\,x\right)}^{7/2}}{{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(7/2))/(a + b*x^2)^(5/2),x)
[Out] int(((A + B*x^2)*(e*x)^(7/2))/(a + b*x^2)^(5/2), x)
```

3.816 $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=349

$$\frac{(Ab-aB)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/2}}+\frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}}-\frac{(Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^2}}{2ab^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{(Ab-7aB)e^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{}}{2a^{3/}}$$

```
[Out] 1/3*(A*b-B*a)*(e*x)^(7/2)/a/b/e/(b*x^2+a)^(3/2)+1/6*(A*b-7*B*a)*e*(e*x)^(3/
2)/a/b^2/(b*x^2+a)^(1/2)-1/2*(A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a/
b^(5/2)/(a^(1/2)+x*b^(1/2))+1/2*(A*b-7*B*a)*e^(5/2)*(cos(2*arctan(b^(1/4)*(
e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1
/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),
1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^
(3/4)/b^(11/4)/(b*x^2+a)^(1/2)-1/4*(A*b-7*B*a)*e^(5/2)*(cos(2*arctan(b^(1/4
)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a
^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
)),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)
/a^(3/4)/b^(11/4)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {468, 294, 335, 311, 226, 1210}

$$-\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(Ab-7aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{11/4}\sqrt{a+bx^2}}+\frac{e^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(Ab-7aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{11/4}\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\,\sqrt{a+bx^2}\,(Ab-7aB)}{2ab^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{e(ex)^{3/2}(Ab-7aB)}{6ab^2\sqrt{a+bx^2}}+\frac{(ex)^{7/2}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b - 7*a*B)*e*(e
*x)^(3/2))/(6*a*b^2*Sqrt[a + b*x^2]) - ((A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a
+ b*x^2])/(2*a*b^(5/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A*b - 7*a*B)*e^(5/2)*(Sqr
t[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(3/4)*b^(11/4)*Sqrt[
a + b*x^2]) - ((A*b - 7*a*B)*e^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(4*a^(3/4)*b^(11/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{(Ab-aB)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{\left(-\frac{Ab}{2}+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{(a+bx^2)^{3/2}}\,dx}{3ab}$$

$$= \frac{(Ab-aB)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}} - \frac{((Ab-7aB)e^2)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{4ab^2}$$

$$= \frac{(Ab-aB)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}} - \frac{((Ab-7aB)e)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\right.}{2ab^2}$$

$$= \frac{(Ab-aB)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}} - \frac{((Ab-7aB)e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\right.}{2\sqrt{a}\,b^{5/2}}$$

$$= \frac{(Ab-aB)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}} - \frac{(Ab-7aB)e^2\sqrt{ex}\,\sqrt{a+bx^2}}{2ab^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{(Ab}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 97, normalized size = 0.28

$$-\frac{2e(ex)^{3/2}\left(a(Ab-7aB-3bBx^2)+(-Ab+7aB)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*e*(e*x)^(3/2)*(a*(A*b - 7*a*B - 3*b*B*x^2) + (-(A*b) + 7*a*B)*(a + b*x^
2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*
a*b^2*(a + b*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(363) = 726.
time = 0.12, size = 767, normalized size = 2.20

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(-\frac{e^2x(Ab-Ba)\sqrt{be\,x^3+aex}}{3b^4\left(x^2+\frac{a}{b}\right)^2}+\frac{e^3x^2(Ab-3Ba)}{2b^2a\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\left(\frac{B\,e^3}{b^2}-\frac{e^3(Ab-3Ba)}{4b^2a}\right)\sqrt{-ab}\,\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
default	$-\frac{\left(6A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^2x^2-3A\sqrt{\frac{bx+\cdots}{\sqrt{\cdots}}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*x^2-3*A*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^2*x^2-42*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*x^2+21*B*((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2),1/2*2^(1/2))*a^2*b*x^2+6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1
/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-3*A*((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))*a^2*b-42*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
```

```
lipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+21*B*((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))*a^3-6*A*b^3*x^4+18*B*a*b^2*x^4-2*A*a*b^2*x^2+14*B*a^
2*b*x^2)*e^2/x*(e*x)^(1/2)/b^3/a/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((B*x^2 + A)*x^(5/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 160, normalized size = 0.46

$$-\frac{3\left((7\,Bab^2-Ab^3)x^4+7\,Ba^3-Aa^2b+2\,(7\,Ba^2b-Aab^2)x^2\right)\sqrt{b}\,e^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(3\,(3\,Bab^2-Ab^3)x^3+(7\,Ba^2b-Aab^2)x\right)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{5}{2}}}{6\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*((7*B*a*b^2 - A*b^3)*x^4 + 7*B*a^3 - A*a^2*b + 2*(7*B*a^2*b - A*a*b
^2)*x^2)*sqrt(b)*e^(5/2)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*
a/b, 0, x)) + (3*(3*B*a*b^2 - A*b^3)*x^3 + (7*B*a^2*b - A*a*b^2)*x)*sqrt(b*
x^2 + a)*sqrt(x)*e^(5/2))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 190.73, size = 94, normalized size = 0.27

$$\frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}+\frac{Be^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 5/2), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(5/2)*gamma(11/4)) + B*e**(5/2)*x**(11/2)*gamma(11/4)*hyper(
(5/2, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(5/2)*e^(5/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,{(e\,x)}^{5/2}}{{(b\,x^2+a)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(5/2),x)
```

```
[Out] int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(5/2), x)
```

3.817 $\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{(Ab-aB)(ex)^{5/2}}{3abe\,(a+bx^2)^{3/2}}-\frac{(Ab+5aB)e\sqrt{ex}}{6ab^2\sqrt{a+bx^2}}+\frac{(Ab+5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.\right.}{12a^{5/4}b^{9/4}\sqrt{a+bx^2}}$$

```
[Out] 1/3*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^2+a)^(3/2)-1/6*(A*b+5*B*a)*e*(e*x)^(1/
2)/a/b^2/(b*x^2+a)^(1/2)+1/12*(A*b+5*B*a)*e^(3/2)*(cos(2*arctan(b^(1/4)*(e*
x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4
)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/
2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5
/4)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {468, 294, 335, 226}

$$\frac{e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(5aB+Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{9/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}\,(5aB+Ab)}{6ab^2\sqrt{a+bx^2}}+\frac{(ex)^{5/2}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/2)) - ((A*b + 5*a*B)*e*Sq
rt[e*x])/(6*a*b^2*Sqrt[a + b*x^2]) + ((A*b + 5*a*B)*e^(3/2)*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(5/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{(Ab-aB)(ex)^{5/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\int \frac{(ex)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{3ab}$$

$$= \frac{(Ab-aB)(ex)^{5/2}}{3abe\,(a+bx^2)^{3/2}} - \frac{(Ab+5aB)e\sqrt{ex}}{6ab^2\sqrt{a+bx^2}} + \frac{((Ab+5aB)e^2)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^2}}\,dx}{12ab^2}$$

$$= \frac{(Ab-aB)(ex)^{5/2}}{3abe\,(a+bx^2)^{3/2}} - \frac{(Ab+5aB)e\sqrt{ex}}{6ab^2\sqrt{a+bx^2}} + \frac{((Ab+5aB)e)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\right.}{6ab^2}$$

$$= \frac{(Ab-aB)(ex)^{5/2}}{3abe\,(a+bx^2)^{3/2}} - \frac{(Ab+5aB)e\sqrt{ex}}{6ab^2\sqrt{a+bx^2}} + \frac{(Ab+5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{\cdots}{(\cdots}}}{12a^{5/4}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.14, size = 105, normalized size = 0.57

$$\frac{e\sqrt{ex}\left(-5a^2B+Ab^2x^2-ab(A+7Bx^2)+(Ab+5aB)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{6ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]

[Out] (e*Sqrt[e*x]*(-5*a^2*B + A*b^2*x^2 - a*b*(A + 7*B*x^2) + (A*b + 5*a*B)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*a*b^2*(a + b*x^2)^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(192) = 384.
time = 0.12, size = 429, normalized size = 2.32

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\left(-\frac{e(Ab-Ba)\sqrt{be\,x^3+aex}}{3b^4\left(x^2+\frac{a}{b}\right)^2}+\frac{e^2x(Ab-7Ba)}{6b^2a\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\left(\frac{B\,e^2}{b^2}+\frac{e^2(Ab-7Ba)}{12b^2a}\right)\sqrt{-ab}\,\sqrt{\left(x-\right.}}{}\right.}{ex\sqrt{b\,x^2+a}}$
default	$\left(A\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,b^2x^2+5B\,\mathrm{Ellipti}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/12*(A*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*b^2*x^2+5*B*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*a*b*x^2+A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a*b+5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2

```
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2+2*A*b^3*x^3-14*B*a*
b^2*x^3-2*A*a*b^2*x-10*B*a^2*b*x)*e/x*(e*x)^(1/2)/a/b^3/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((B*x^2 + A)*x^(3/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 144, normalized size = 0.78

$$\frac{((5\,Bab^2+Ab^3)x^4+5\,Ba^3+Aa^2b+2\,(5\,Ba^2b+Aab^2)x^2)\sqrt{b}\,e^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(5\,Ba^2b+Aab^2+(7\,Bab^2-Ab^3)x^2)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{3}{2}}}{6\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(((5*B*a*b^2 + A*b^3)*x^4 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)
*x^2)*sqrt(b)*e^(3/2)*weierstrassPInverse(-4*a/b, 0, x) - (5*B*a^2*b + A*a*
b^2 + (7*B*a*b^2 - A*b^3)*x^2)*sqrt(b*x^2 + a)*sqrt(x)*e^(3/2))/(a*b^5*x^4
+ 2*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 60.39, size = 94, normalized size = 0.51

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\\frac{9}{4}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{9}{4},\frac{5}{2}\\\frac{13}{4}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/2), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/2)*gamma(9/4)) + B*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((9/4
, 5/2), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(3/2)*e^(3/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^2+A)\,{(e\,x)}^{3/2}}{{(b\,x^2+a)}^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(5/2),x)
[Out] int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(5/2), x)
```

3.818 $\int \frac{\sqrt{ex}\,(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=344

$$\frac{(Ab-aB)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/2}}+\frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}}-\frac{(Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{2a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{(Ab+aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a}{\left(\sqrt{a}\right.}}}{2a^{7/4}b^{7/4}}$$

```
[Out] 1/3*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^2+a)^(3/2)+1/2*(A*b+B*a)*(e*x)^(3/2)/a
^2/b/e/(b*x^2+a)^(1/2)-1/2*(A*b+B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/b^(3/2
)/(a^(1/2)+x*b^(1/2))+1/2*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*El
lipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^
(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/b^
(7/4)/(b*x^2+a)^(1/2)-1/4*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*El
lipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^
(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/b^
(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {468, 296, 335, 311, 226, 1210}

$$-\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(aB+Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{7/4}\sqrt{a+bx^2}}+\frac{\sqrt{e}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(aB+Ab)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{7/4}\sqrt{a+bx^2}}-\frac{\sqrt{ex}\,\sqrt{a+bx^2}\,(aB+Ab)}{2a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{(ex)^{3/2}(aB+Ab)}{2a^2be\sqrt{a+bx^2}}+\frac{(ex)^{3/2}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b + a*B)*(e*x)^
(3/2))/(2*a^2*b*e*Sqrt[a + b*x^2]) - ((A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^2]
)/(2*a^2*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + ((A*b + a*B)*Sqrt[e]*(Sqrt[a] + S
qrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2
]) - ((A*b + a*B)*Sqrt[e]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1
/2])/(4*a^(7/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^2)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\,(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{(Ab-aB)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{(Ab+aB)\int \frac{\sqrt{ex}}{(a+bx^2)^{3/2}}\,dx}{2ab}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}} - \frac{(Ab+aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^2}}\,dx}{4a^2b}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}} - \frac{(Ab+aB)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{\right.}{2a^2be}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}} - \frac{(Ab+aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{\right.}{2a^{3/2}b^{3/2}}$$

$$= \frac{(Ab-aB)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/2}} + \frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}} - \frac{(Ab+aB)\sqrt{ex}\,\sqrt{a+bx^2}}{2a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{(Ab+aB}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 84, normalized size = 0.24

$$\frac{2x\sqrt{ex}\left(-a^2B+(Ab+aB)\,(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a^2b\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*x*Sqrt[e*x]*(-(a^2*B) + (A*b + a*B)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hype
rgeometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a^2*b*(a + b*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 763 vs. 2(358) = 716.
time = 0.12, size = 764, normalized size = 2.22

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)\,ex}\,\left(\frac{x(Ab-Ba)\sqrt{be\,x^3+aex}}{3a\,b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{e\,x^2(Ab+Ba)}{2b\,a^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}-\frac{e(Ab+Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}}{}\right.}{}$
default	$-\frac{\left(6A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^2x^2-3A\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*x^2-3*A*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^2*x^2+6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*x^2-3*B*((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))*a^2*b*x^2+6*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellipt
icE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-3*A*((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2))*a^2*b+6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/
2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Ellip
```

```
ticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-3*B*((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))*a^3-6*A*b^3*x^4-6*B*a*b^2*x^4-10*A*a*b^2*x^2-2*B*a^2*b*x
^2)*(e*x)^(1/2)/b^2/a^2/x/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((B*x^2 + A)*sqrt(x)/(b*x^2 + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 153, normalized size = 0.44

$$\frac{3\left((Bab^2+Ab^3)x^4+Ba^3+Aa^2b+2\left(Ba^2b+Aab^2\right)x^2\right)\sqrt{b}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(3\left(Bab^2+Ab^3\right)x^3+\left(Ba^2b+5\,Aab^2\right)x\right)\sqrt{bx^2+a}\,\sqrt{x}\,e^{\frac{1}{2}}}{6\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*((B*a*b^2 + A*b^3)*x^4 + B*a^3 + A*a^2*b + 2*(B*a^2*b + A*a*b^2)*x^2
)*sqrt(b)*e^(1/2)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0,
 x)) + (3*(B*a*b^2 + A*b^3)*x^3 + (B*a^2*b + 5*A*a*b^2)*x)*sqrt(b*x^2 + a)*
sqrt(x)*e^(1/2))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 24.32, size = 94, normalized size = 0.27

$$\frac{A\sqrt{e}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{2}\\\frac{7}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{B\sqrt{e}\,x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{7}{4},\frac{5}{2}\\\frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/2)*gamma(7/4)) + B*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4,
5/2), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(x)*e^(1/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^2+A)\,\sqrt{e\,x}}{(b\,x^2+a)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(5/2),x)
[Out] int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(5/2), x)
```

3.819 $\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{(Ab-aB)\sqrt{ex}}{3abe\,(a+bx^2)^{3/2}}+\frac{(5Ab+aB)\sqrt{ex}}{6a^2be\sqrt{a+bx^2}}+\frac{(5Ab+aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{e}\dots}{\sqrt[4]{a}\,\sqrt{\dots}}\right.\right.}{12a^{9/4}b^{5/4}\sqrt{e}\,\sqrt{a+bx^2}}$$

```
[Out] 1/3*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(3/2)+1/6*(5*A*b+B*a)*(e*x)^(1/2)
/a^2/b/e/(b*x^2+a)^(1/2)+1/12*(5*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)
/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2
)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2
))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/b^(5
/4)/e^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {468, 296, 335, 226}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(aB+5Ab)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{9/4}b^{5/4}\sqrt{e}\,\sqrt{a+bx^2}}+\frac{\sqrt{ex}\,(aB+5Ab)}{6a^2be\sqrt{a+bx^2}}+\frac{\sqrt{ex}\,(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] ((A*b - a*B)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/2)) + ((5*A*b + a*B)*Sqrt[e
*x])/(6*a^2*b*e*Sqrt[a + b*x^2]) + ((5*A*b + a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x
])/(a^(1/4)*Sqrt[e])], 1/2])/(12*a^(9/4)*b^(5/4)*Sqrt[e]*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{5/2}}\,dx = \frac{(Ab-aB)\sqrt{ex}}{3abe\,(a+bx^2)^{3/2}} + \frac{\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{3/2}}\,dx}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{ex}}{3abe\,(a+bx^2)^{3/2}} + \frac{(5Ab+aB)\sqrt{ex}}{6a^2be\sqrt{a+bx^2}} + \frac{(5Ab+aB)\int \frac{1}{\sqrt{ex}\,\sqrt{a+bx^2}}\,dx}{12a^2b}$$

$$= \frac{(Ab-aB)\sqrt{ex}}{3abe\,(a+bx^2)^{3/2}} + \frac{(5Ab+aB)\sqrt{ex}}{6a^2be\sqrt{a+bx^2}} + \frac{(5Ab+aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{e^2}}}\,dx, x, \cdots\right.}{6a^2be}$$

$$= \frac{(Ab-aB)\sqrt{ex}}{3abe\,(a+bx^2)^{3/2}} + \frac{(5Ab+aB)\sqrt{ex}}{6a^2be\sqrt{a+bx^2}} + \frac{(5Ab+aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+b\cdots}{\left(\sqrt{a}+\cdots\right.}}}{12a^{9/4}b^{5/4}\sqrt{e}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 108, normalized size = 0.58

$$\frac{-a^2Bx+5Ab^2x^3+abx(7A+Bx^2)+(5Ab+aB)x(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{6a^2b\sqrt{ex}\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-(a^2*B*x) + 5*A*b^2*x^3 + a*b*x*(7*A + B*x^2) + (5*A*b + a*B)*x*(a + b*x^
2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/(6*a
^2*b*Sqrt[e*x]*(a + b*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(194) = 388$.
time = 0.12, size = 425, normalized size = 2.27

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)\,ex}\left(\frac{(Ab-Ba)\sqrt{be\,x^3+aex}}{3ae\,b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{x(5Ab+Ba)}{6b\,a^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{(5Ab+Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\cdots}}{\cdots}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$
default	$\frac{5A\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,b^2x^2+B\,\mathrm{Ellipti}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(5*A*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-
a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*b^2*x^2+B*EllipticF(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*(-x*b/(-a*b)^(1/2))^(1/2)*a*b*x^2+5*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(
1/2)*a*b+B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
```

```
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2+10*A*b^3*x^3+2*B*a*
b^2*x^3+14*A*a*b^2*x-2*B*a^2*b*x)/(e*x)^(1/2)/a^2/b^2/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 144, normalized size = 0.77

$$\frac{\left(((Bab^2+5\,Ab^3)x^4+Ba^3+5\,Aa^2b+2\,(Ba^2b+5\,Aab^2)x^2)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(Ba^2b-7\,Aab^2-(Bab^2+5\,Ab^3)x^2)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{6\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(((B*a*b^2 + 5*A*b^3)*x^4 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)
*x^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (B*a^2*b - 7*A*a*b^2 - (B
*a*b^2 + 5*A*b^3)*x^2)*sqrt(b*x^2 + a)*sqrt(x))*e^(-1/2)/(a^2*b^4*x^4 + 2*a
^3*b^3*x^2 + a^4*b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 42.55, size = 94, normalized size = 0.50

$$\frac{A\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{5}{4}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\\frac{9}{4}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(5/2)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*hyper((5/4, 5/2), (9/
4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*sqrt(e)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*e^(-1/2)/((b*x^2 + a)^(5/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{\sqrt{e\,x}\,\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/2)), x)
```

3.820 $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}} \, dx$

Optimal. Leaf size=377

$$-\frac{2A}{ae\sqrt{ex}\,(a+bx^2)^{3/2}}-\frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3(a+bx^2)^{3/2}}-\frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}}+\frac{(7Ab-aB)\sqrt{ex}\,\sqrt{a+bx^2}}{2a^3\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{(7Ab-aB)}{}$$

```
[Out] -1/3*(7*A*b-B*a)*(e*x)^(3/2)/a^2/e^3/(b*x^2+a)^(3/2)-2*A/a/e/(b*x^2+a)^(3/2
)/(e*x)^(1/2)-1/2*(7*A*b-B*a)*(e*x)^(3/2)/a^3/e^3/(b*x^2+a)^(1/2)+1/2*(7*A*
b-B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a^3/e^2/b^(1/2)/(a^(1/2)+x*b^(1/2))-1/2*
(7*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(
1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2
+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/b^(3/4)/e^(3/2)/(b*x^2+a)^(1/2)+1
/4*(7*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(
b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*
x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/b^(3/4)/e^(3/2)/(b*x^2+a)^(1/2
)
```

Rubi [A]
time = 0.21, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 296, 335, 311, 226, 1210}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(7Ab-aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{4a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,(7Ab-aB)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{a}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}-\frac{(ex)^{3/2}(7Ab-aB)}{2a^3e^3\sqrt{a+bx^2}}+\frac{\sqrt{ex}\,\sqrt{a+bx^2}\,(7Ab-aB)}{2a^3\sqrt{b}\,e^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{(ex)^{3/2}(7Ab-aB)}{3a^2e^3\left(a+bx^2\right)^{3/2}}-\frac{2A}{ae\sqrt{ex}\,\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(3/2))/(3*a
^2*e^3*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*(e*x)^(3/2))/(2*a^3*e^3*Sqrt[a +
 b*x^2]) + ((7*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^2])/(2*a^3*Sqrt[b]*e^2*(Sq
rt[a] + Sqrt[b]*x)) - ((7*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(2*a^(11/4)*b^(3/4)*e^(3/2)*Sqrt[a + b*x^2]) + ((7*A*b - a*
B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(4*a^(11/4)*b^(3/4
)*e^(3/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)\int \frac{\sqrt{ex}}{(a+bx^2)^{5/2}}\,dx}{ae^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)\int \frac{\sqrt{ex}}{(a+bx^2)^{3/2}}\,dx}{2a^2e^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}} + \frac{(7Ab-aB}{}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}} + \frac{(7Ab-aB}{}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}} + \frac{(7Ab-aB}{}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}} + \frac{(7Ab-aB}{2a^3\sqrt{b}\,e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 86, normalized size = 0.23

$$\frac{x\left(-6a^2A+2(-7Ab+aB)x^2(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a^3(ex)^{3/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (x*(-6*a^2*A + 2*(-7*A*b + a*B)*x^2*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hyperge
ometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a^3*(e*x)^(3/2)*(a + b*x^2)^(3
/2))
```

Maple [A]
time = 0.15, size = 771, normalized size = 2.05 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(42*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*x^2-21*A*((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*a*b^2*x^2-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*x^2+3*B*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*a^2*b*x^2+42*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1
/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*Elli
pticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b-21*A*((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))*a^2*b-6*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*El
lipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+3*B*((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2))*a^3-42*A*b^3*x^4+6*B*a*b^2*x^4-70*A*a*b^2*x^2+10*B*a^
2*b*x^2-24*A*a^2*b)/b/a^3/e/(e*x)^(1/2)/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 167, normalized size = 0.44

$$\frac{\left(3\left((Bab^2 - 7Ab^3)x^5 + 2(Ba^2b - 7Aab^2)x^3 + (Ba^3 - 7Aa^2b)x\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-\frac{4a}{b}, 0, \mathrm{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + \left(3(Bab^2 - 7Ab^3)x^4 - 12Aa^2b + 5(Ba^2b - 7Aab^2)x^2\right)\sqrt{bx^2 + a}\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{6\left(a^3b^3x^5 + 2a^4b^2x^3 + a^5bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*((B*a*b^2 - 7*A*b^3)*x^5 + 2*(B*a^2*b - 7*A*a*b^2)*x^3 + (B*a^3 - 7*
A*a^2*b)*x)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
0, x)) + (3*(B*a*b^2 - 7*A*b^3)*x^4 - 12*A*a^2*b + 5*(B*a^2*b - 7*A*a*b^2)*
x^2)*sqrt(b*x^2 + a)*sqrt(x))*e^(-3/2)/(a^3*b^3*x^5 + 2*a^4*b^2*x^3 + a^5*b
*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 79.60, size = 97, normalized size = 0.26

$$\frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{Bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5
/2)*e**(3/2)*sqrt(x)*gamma(3/4)) + B*x**(3/2)*gamma(3/4)*hyper((3/4, 5/2),
(7/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*e**(3/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*e^(-3/2)/((b*x^2 + a)^(5/2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)), x)
```

3.821 $\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}}-\frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/2}}-\frac{5(3Ab-aB)\sqrt{ex}}{6a^3e^3\sqrt{a+bx^2}}-\frac{5(3Ab-aB)\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a\ \cdots}{\left(\sqrt{a}\ \cdots\right.}}}{12a^{13/4}\sqrt[4]{b}\,e^{5\cdots}}$$

```
[Out] -2/3*A/a/e/(e*x)^(3/2)/(b*x^2+a)^(3/2)-1/3*(3*A*b-B*a)*(e*x)^(1/2)/a^2/e^3/
(b*x^2+a)^(3/2)-5/6*(3*A*b-B*a)*(e*x)^(1/2)/a^3/e^3/(b*x^2+a)^(1/2)-5/12*(3
*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/
4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a
)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(13/4)/b^(1/4)/e^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {464, 296, 335, 226}

$$-\frac{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}(3Ab-aB)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{12a^{13/4}\sqrt[4]{b}\,e^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{ex}\,(3Ab-aB)}{6a^3e^3\sqrt{a+bx^2}}-\frac{\sqrt{ex}\,(3Ab-aB)}{3a^2e^3\left(a+bx^2\right)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/2)) - ((3*A*b - a*B)*Sqrt[e*x])/(3
*a^2*e^3*(a + b*x^2)^(3/2)) - (5*(3*A*b - a*B)*Sqrt[e*x])/(6*a^3*e^3*Sqrt[a
 + b*x^2]) - (5*(3*A*b - a*B)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[
a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])
], 1/2])/(12*a^(13/4)*b^(1/4)*e^(5/2)*Sqrt[a + b*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx = -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{5/2}}\,dx}{ae^2}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{(5(3Ab-aB))\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}}{6a^2e^2}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{5(3Ab-aB)\sqrt{ex}}{6a^3e^3\sqrt{a+bx^2}} - \frac{(5(3Ab-}{}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{5(3Ab-aB)\sqrt{ex}}{6a^3e^3\sqrt{a+bx^2}} - \frac{(5(3Ab-}{}$$

$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}} - \frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/2}} - \frac{5(3Ab-aB)\sqrt{ex}}{6a^3e^3\sqrt{a+bx^2}} - \frac{5(3Ab-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.08, size = 120, normalized size = 0.56

$$\frac{x\left(-15Ab^2x^4+a^2(-4A+7Bx^2)+a(-21Abx^2+5bBx^4)+5(-3Ab+aB)x^2(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{6a^3(ex)^{5/2}\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (x*(-15*A*b^2*x^4 + a^2*(-4*A + 7*B*x^2) + a*(-21*A*b*x^2 + 5*b*B*x^4) + 5*
(-3*A*b + a*B)*x^2*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1
/2, 5/4, -((b*x^2)/a)]))/(6*a^3*(e*x)^(5/2)*(a + b*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(214) = 428$.
time = 0.14, size = 446, normalized size = 2.09

method	result
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(-\frac{(Ab-Ba)\sqrt{bex^3+aex}}{3a^2e^3b^2\left(x^2+\frac{a}{b}\right)^2}-\frac{x(11Ab-5Ba)}{6e^2a^3\sqrt{\left(x^2+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{bex^3+aex}}{3a^3e^3x^2}+\frac{\left(-\frac{11Ab-5Ba}{12a^3e^2}-\frac{bA}{3a^3e^2}\cdots\right.}{\cdots}\right.}{\sqrt{ex}\,\sqrt{bx\cdots}}$
default	$-\frac{15A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b^2x^3-5B\cdots}{\cdots}$
risch	$-\frac{2A\sqrt{bx^2+a}}{3a^3xe^2\sqrt{ex}}-\frac{\left(\frac{A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-\cdots}}{\cdots}\right.}{\sqrt{-\cdots}}}\cdots\right.}{\sqrt{bex^3+aex}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(15*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b^2*x^3-5*B*((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a*b*x^3+15*A*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a
*b)^(1/2)*a*b*x-5*B*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2*x+30*A*b^3
*x^4-10*B*a*b^2*x^4+42*A*a*b^2*x^2-14*B*a^2*b*x^2+8*A*a^2*b)/x/e^2/(e*x)^(1
/2)/a^3/b/(b*x^2+a)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 163, normalized size = 0.77

$$\frac{\left(5\left((Bab^2-3Ab^3)x^6+2\left(Ba^2b-3Aab^2\right)x^4+\left(Ba^3-3Aa^2b\right)x^2\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(5\left(Bab^2-3Ab^3\right)x^4-4Aa^2b+7\left(Ba^2b-3Aab^2\right)x^2\right)\sqrt{bx^2+a}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{6\left(a^3b^3x^6+2a^4b^2x^4+a^5bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(5*((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2)*x^4 + (B*a^3 - 3*
A*a^2*b)*x^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (5*(B*a*b^2 - 3*A
*b^3)*x^4 - 4*A*a^2*b + 7*(B*a^2*b - 3*A*a*b^2)*x^2)*sqrt(b*x^2 + a)*sqrt(x
))*e^(-5/2)/(a^3*b^3*x^6 + 2*a^4*b^2*x^4 + a^5*b*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 148.76, size = 97, normalized size = 0.46

$$\frac{A\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{5}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{B\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5
/2)*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(x)*gamma(1/4)*hyper((1/4, 5/2),
(5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*e**(5/2)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*e^(-5/2)/((b*x^2 + a)^(5/2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{{\left(e\,x\right)}^{5/2}\,{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)), x)
```

3.822 $\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2}\, dx$

Optimal. Leaf size=288

$$\frac{4c(11a^2d^2+bc(3bc-10ad))\,e\sqrt{ex}\,\sqrt{c+dx^2}}{231d^3}+\frac{2(11a^2d^2+bc(3bc-10ad))\,(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e}-\frac{2b(3bc-10a}{}$$

```
[Out] -2/55*b*(-10*a*d+3*b*c)*(e*x)^(5/2)*(d*x^2+c)^(3/2)/d^2/e+2/15*b^2*(e*x)^(9
/2)*(d*x^2+c)^(3/2)/d/e^3+2/77*(11*a^2*d^2+b*c*(-10*a*d+3*b*c))*(e*x)^(5/2)
*(d*x^2+c)^(1/2)/d^2/e+4/231*c*(11*a^2*d^2+b*c*(-10*a*d+3*b*c))*e*(e*x)^(1/
2)*(d*x^2+c)^(1/2)/d^3-2/231*c^(7/4)*(11*a^2*d^2+b*c*(-10*a*d+3*b*c))*e^(3/
2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arcta
n(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1
/2)+x*d^(1/2))^2)^(1/2)/d^(13/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {475, 470, 285, 327, 335, 226}

$$-\frac{2c^{7/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(11a^2d^2+bc(3bc-10ad))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2(ex)^{5/2}\sqrt{c+dx^2}\,(11a^2d^2+bc(3bc-10ad))}{77d^2e}+\frac{4ce\sqrt{ex}\,\sqrt{c+dx^2}\,(11a^2d^2+bc(3bc-10ad))}{231d^3}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{3/2}(3bc-10ad)}{55d^2e}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (4*c*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*
d^3) + (2*(11*a^2*d^2 + b*c*(3*b*c - 10*a*d))*(e*x)^(5/2)*Sqrt[c + d*x^2])/
(77*d^2*e) - (2*b*(3*b*c - 10*a*d)*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(55*d^2*e
) + (2*b^2*(e*x)^(9/2)*(c + d*x^2)^(3/2))/(15*d*e^3) - (2*c^(7/4)*(11*a^2*d
^2 + b*c*(3*b*c - 10*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(
Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(231*d^(13/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
```

```
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2}\, dx = \frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3} + \frac{2\int (ex)^{3/2}\sqrt{c+dx^2}\left(\frac{15a^2d}{2} - \frac{3}{2}b(3bc-10ad)x\right.}{15d}$$

$$= -\frac{2b(3bc-10ad)(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{55d^2e} + \frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3} + \frac{1}{11}\left(1\right.$$

$$= \frac{2\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)(ex)^{5/2}\sqrt{c+dx^2}}{77e} - \frac{2b(3bc-10ad)(ex)^{5/2}\left(c+d\right.}{55d^2e}$$

$$= \frac{4c\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d} + \frac{2\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)(ex)^5}{77e}$$

$$= \frac{4c\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d} + \frac{2\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)(ex)^5}{77e}$$

$$= \frac{4c\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d} + \frac{2\left(11a^2+\frac{bc(3bc-10ad)}{d^2}\right)(ex)^5}{77e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.25, size = 225, normalized size = 0.78

$$\frac{(ex)^{3/2}\left(\frac{2\sqrt{x}\,(c+dx^2)\left(55a^2d^2(2c+3dx^2)+10abd(-10c^2+6cdx^2+21d^2x^4)+b^2(30c^3-18c^2dx^2+14cd^2x^4+77d^3x^6)\right)}{5d^3} - \frac{4ic^2(3b^2c^2-10abcd+11a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^3}\right)}{231x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(55*a^2*d^2*(2*c + 3*d*x^2) + 10*a*b*d
*(-10*c^2 + 6*c*d*x^2 + 21*d^2*x^4) + b^2*(30*c^3 - 18*c^2*d*x^2 + 14*c*d^2
*x^4 + 77*d^3*x^6)))/(5*d^3) - ((4*I)*c^2*(3*b^2*c^2 - 10*a*b*c*d + 11*a^2*
d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sq
rt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(231*x^(3/2)*Sqrt[c + d*x^2]
)
```

Maple [A]

time = 0.14, size = 448, normalized size = 1.56

method	result
risch	$\frac{2(77b^2x^6d^3+210ab\,d^3x^4+14b^2c\,d^2x^4+165a^2d^3x^2+60abc\,d^2x^2-18b^2c^2d\,x^2+110a^2c\,d^2-100ab\,c^2d+30b^2c^3)x\sqrt{d\,x^2+c}\,e^2}{1155d^3\sqrt{ex}} -$
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2e\,x^6\sqrt{de\,x^3+cex}}{15}+\frac{2\left(b(2ad+bc)e^2-\frac{13b^2e^2c}{15}\right)x^4\sqrt{de\,x^3+cex}}{11de}+\frac{2\left(a(ad+2bc)e^2-\frac{9\left(b(2a\right.}{}\right.}{}\right.}{}$
default	$-\frac{2e\sqrt{ex}\left(-77b^2d^5x^9-210ab\,d^5x^7-91b^2c\,d^4x^7+55\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\operatorname{EllipticF}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155*e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-77*b^2*d^5*x^9-210*a*b*d^5*x^7-91
*b^2*c*d^4*x^7+55*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c^2*d^2-50*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^3*d+15*((d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*
d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(
1/2))*(-c*d)^(1/2)*b^2*c^4-165*a^2*d^5*x^5-270*a*b*c*d^4*x^5+4*b^2*c^2*d^3*
x^5-275*a^2*c*d^4*x^3+40*a*b*c^2*d^3*x^3-12*b^2*c^3*d^2*x^3-110*a^2*c^2*d^3
*x+100*a*b*c^3*d^2*x-30*b^2*c^4*d*x)/d^4
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 160, normalized size = 0.56

$$-\frac{2\left(10\left(3b^{2}c^{4}-10abc^{3}d+11a^{2}c^{2}d^{2}\right)\sqrt{d}\,e^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-\left(77b^{2}d^{4}x^{6}+30b^{2}c^{3}d-100abc^{2}d^{2}+110a^{2}cd^{3}+14\left(b^{2}cd^{3}+15abd^{4}\right)x^{4}-3\left(6b^{2}c^{2}d^{2}-20abcd^{3}-55a^{2}d^{4}\right)x^{2}\right)\sqrt{dx^{2}+c}\,\sqrt{x}\,e^{\frac{3}{2}}\right)}{1155\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(10*(3*b^2*c^4 - 10*a*b*c^3*d + 11*a^2*c^2*d^2)*sqrt(d)*e^(3/2)*wei
erstrassPInverse(-4*c/d, 0, x) - (77*b^2*d^4*x^6 + 30*b^2*c^3*d - 100*a*b*c
^2*d^2 + 110*a^2*c*d^3 + 14*(b^2*c*d^3 + 15*a*b*d^4)*x^4 - 3*(6*b^2*c^2*d^2
 - 20*a*b*c*d^3 - 55*a^2*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(x)*e^(3/2))/d^4
```

Sympy [C] Result contains complex when optimal does not.
time = 12.45, size = 150, normalized size = 0.52

$$\frac{a^{2}\sqrt{c}\,e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{ab\sqrt{c}\,e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{\Gamma\left(\frac{13}{4}\right)}+\frac{b^{2}\sqrt{c}\,e^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2
*exp_polar(I*pi)/c)/(2*gamma(9/4)) + a*b*sqrt(c)*e**(3/2)*x**(9/2)*gamma(9/
4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/gamma(13/4) + b**2
*sqrt(c)*e**(3/2)*x**(13/2)*gamma(13/4)*hyper((-1/2, 13/4), (17/4,), d*x**2
*exp_polar(I*pi)/c)/(2*gamma(17/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}\,\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
[Out] int((e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

3.823 $\int \sqrt{ex} \left(a+bx^2\right)^2 \sqrt{c+dx^2} \, dx$

Optimal. Leaf size=425

$$\frac{2\left(39a^2d^2+bc(7bc-26ad)\right)(ex)^{3/2}\sqrt{c+dx^2}}{195d^2e}+\frac{4c\left(39a^2d^2+bc(7bc-26ad)\right)\sqrt{ex}\,\sqrt{c+dx^2}}{195d^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2b(7bc-26ad}{}$$

```
[Out] -2/117*b*(-26*a*d+7*b*c)*(e*x)^(3/2)*(d*x^2+c)^(3/2)/d^2/e+2/13*b^2*(e*x)^(
7/2)*(d*x^2+c)^(3/2)/d/e^3+2/195*(39*a^2*d^2+b*c*(-26*a*d+7*b*c))*(e*x)^(3/
2)*(d*x^2+c)^(1/2)/d^2/e+4/195*c*(39*a^2*d^2+b*c*(-26*a*d+7*b*c))*(e*x)^(1/
2)*(d*x^2+c)^(1/2)/d^(5/2)/(c^(1/2)+x*d^(1/2))-4/195*c^(5/4)*(39*a^2*d^2+b*
c*(-26*a*d+7*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arc
tan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*
e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(11/4)/(d*x^2+c)^(1/2)+2/
195*c^(5/4)*(39*a^2*d^2+b*c*(-26*a*d+7*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1
/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(
1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d
^(11/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {475, 470, 285, 335, 311, 226, 1210}

$$\frac{2c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(39a^2d^2+bc(7bc-26ad)\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}}-\frac{4c^{5/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(39a^2d^2+bc(7bc-26ad)\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}\left(39a^2d^2+bc(7bc-26ad)\right)}{195d^2e}+\frac{4c\sqrt{ex}\,\sqrt{c+dx^2}\left(39a^2d^2+bc(7bc-26ad)\right)}{195d^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{3/2}(7bc-26ad)}{117d^2e}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (2*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(195*d^
2*e) + (4*c*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/
(195*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - (2*b*(7*b*c - 26*a*d)*(e*x)^(3/2)*(c
+ d*x^2)^(3/2))/(117*d^2*e) + (2*b^2*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(13*d*e
^3) - (4*c^(5/4)*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c] + Sqr
t[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(11/4)*Sqrt[c + d*x^2]) + (2
*c^(5/4)*(39*a^2*d^2 + b*c*(7*b*c - 26*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(11/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx = \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}+\frac{2\int\sqrt{ex}\,\sqrt{c+dx^2}\,\left(\frac{13a^2d}{2}-\frac{1}{2}b(7bc-26ad)x^2\right)}{13d}$$

$$= -\frac{2b(7bc-26ad)(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{117d^2e}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}+\frac{1}{39}\Big(39a$$

$$= \frac{2\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{195e}-\frac{2b(7bc-26ad)(ex)^{3/2}\left(c+dx\right.}{117d^2e}$$

$$= \frac{2\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{195e}-\frac{2b(7bc-26ad)(ex)^{3/2}\left(c+dx\right.}{117d^2e}$$

$$= \frac{2\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{195e}-\frac{2b(7bc-26ad)(ex)^{3/2}\left(c+dx\right.}{117d^2e}$$

$$= \frac{2\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{195e}+\frac{4c\left(39a^2+\frac{bc(7bc-26ad)}{d^2}\right)\sqrt{ex}}{195\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.12, size = 145, normalized size = 0.34

$$\frac{2\sqrt{ex}\left(-x\left(c+dx^2\right)\left(-117a^2d^2-26abd\left(2c+5dx^2\right)+b^2\left(14c^2-10cdx^2-45d^2x^4\right)\right)+6c\left(7b^2c^2-26abcd+39a^2d^2\right)\sqrt{1+\frac{c}{dx^2}}\,x\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{585d^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[e*x]*(-(x*(c + d*x^2)*(-117*a^2*d^2 - 26*a*b*d*(2*c + 5*d*x^2) + b^
2*(14*c^2 - 10*c*d*x^2 - 45*d^2*x^4))) + 6*c*(7*b^2*c^2 - 26*a*b*c*d + 39*a
```

```
^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2)
)]]))/(585*d^2*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.12, size = 658, normalized size = 1.55

method	result
risch	$\frac{2x^2\left(45b^2x^4d^2+130ab\,d^2x^2+10b^2cd\,x^2+117a^2d^2+52abcd-14b^2c^2\right)\sqrt{d\,x^2+c}\;e}{585d^2\sqrt{ex}}+\frac{2c\left(39a^2d^2-26abcd+7b^2c^2\right)\sqrt{-cd}\;\sqrt{\frac{\left(x\right.}{}}}{}$
elliptic	$\frac{\sqrt{ex\left(d\,x^2+c\right)}\;\sqrt{ex}\left(\frac{2b^2x^5\sqrt{de\,x^3+cex}}{13}+\frac{2\left(b(2ad+bc)e-\frac{11b^2ce}{13}\right)x^3\sqrt{de\,x^3+cex}}{9de}+\frac{2\left(a(ad+2bc)e-\frac{7\left(b(2ad+bc\right.}{}\right.}{}\right.}{}$
default	$\frac{2\sqrt{ex}\left(45b^2d^4x^8+130ab\,d^4x^6+55b^2c\,d^3x^6+234\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\;\sqrt{2}\;\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\;\sqrt{-\frac{xd}{\sqrt{-cd}}}\;\text{EllipticE}\left(\sqrt{\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/585*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^3*(45*b^2*d^4*x^8+130*a*b*d^4*x^6+55*b^
2*c*d^3*x^6+234*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
```

```
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-156*((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(
-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a*b*c^3*d+42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-117*((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*a^2*c^2*d^2+78*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-21*
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4+117*a^2*d^4*x^4+182*a*b*c*d^3*x^4-4*b^2
*c^2*d^2*x^4+117*a^2*c*d^3*x^2+52*a*b*c^2*d^2*x^2-14*b^2*c^3*d*x^2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 134, normalized size = 0.32

$$-\frac{2\left(6\left(7\,b^2c^3-26\,abc^2d+39\,a^2cd^2\right)\sqrt{d}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)-\left(45\,b^2d^3x^5+10\left(b^2cd^2+13\,abd^3\right)x^3-\left(14\,b^2c^2d-52\,abcd^2-117\,a^2d^3\right)x\right)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{1}{2}}\right)}{585\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/585*(6*(7*b^2*c^3 - 26*a*b*c^2*d + 39*a^2*c*d^2)*sqrt(d)*e^(1/2)*weierst
rassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (45*b^2*d^3*x^5 +
10*(b^2*c*d^2 + 13*a*b*d^3)*x^3 - (14*b^2*c^2*d - 52*a*b*c*d^2 - 117*a^2*d^
3)*x)*sqrt(d*x^2 + c)*sqrt(x)*e^(1/2))/d^3
```

Sympy [C] Result contains complex when optimal does not.
time = 3.09, size = 148, normalized size = 0.35

$$\frac{a^2\sqrt{c}\,(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e\Gamma\left(\frac{7}{4}\right)}+\frac{ab\sqrt{c}\,(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^3\Gamma\left(\frac{11}{4}\right)}+\frac{b^2\sqrt{c}\,(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^5\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(e*x)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_
polar(I*pi)/c)/(2*e*gamma(7/4)) + a*b*sqrt(c)*(e*x)**(7/2)*gamma(7/4)*hyper
((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(e**3*gamma(11/4)) + b**2*
sqrt(c)*(e*x)**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_p
olar(I*pi)/c)/(2*e**5*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(x)*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\,x}\,\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

```
[Out] int((e*x)^(1/2)*(a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

3.824 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx$

Optimal. Leaf size=244

$$\frac{2(5b^2c^2-22abcd+77a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{231d^2e}-\frac{2b(5bc-22ad)\sqrt{ex}\,(c+dx^2)^{3/2}}{77d^2e}+\frac{2b^2(ex)^{5/2}\,(c+dx^2)^{3/2}}{11de^3}+\frac{2c^3}{}$$

```
[Out] 2/11*b^2*(e*x)^(5/2)*(d*x^2+c)^(3/2)/d/e^3-2/77*b*(-22*a*d+5*b*c)*(d*x^2+c)
^(3/2)*(e*x)^(1/2)/d^2/e+2/231*(77*a^2*d^2-22*a*b*c*d+5*b^2*c^2)*(e*x)^(1/2
)*(d*x^2+c)^(1/2)/d^2/e+2/231*c^(3/4)*(77*a^2*d^2-22*a*b*c*d+5*b^2*c^2)*(co
s(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1
/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2
)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*
d^(1/2))^2)^(1/2)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {475, 470, 285, 335, 226}

$$\frac{2c^{3/4}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(77a^2d^2-22abcd+5b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\,\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\,\sqrt{c+dx^2}\,(77a^2d^2-22abcd+5b^2c^2)}{231d^2e}-\frac{2b\sqrt{ex}\,(c+dx^2)^{3/2}\,(5bc-22ad)}{77d^2e}+\frac{2b^2(ex)^{5/2}\,(c+dx^2)^{3/2}}{11de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/Sqrt[e*x],x]
```

```
[Out] (2*(5*b^2*c^2 - 22*a*b*c*d + 77*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d^
2*e) - (2*b*(5*b*c - 22*a*d)*Sqrt[e*x]*(c + d*x^2)^(3/2))/(77*d^2*e) + (2*b
^2*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(11*d*e^3) + (2*c^(3/4)*(5*b^2*c^2 - 22*a
*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt
[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/
(231*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
```

```
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx = \frac{2b^2(ex)^{5/2}(c+dx^2)^{3/2}}{11de^3} + \frac{2\int \frac{\sqrt{c+dx^2}\left(\frac{11a^2d}{2}-\frac{1}{2}b(5bc-22ad)x^2\right)}{\sqrt{ex}}dx}{11d}$$

$$= -\frac{2b(5bc-22ad)\sqrt{ex}(c+dx^2)^{3/2}}{77d^2e} + \frac{2b^2(ex)^{5/2}(c+dx^2)^{3/2}}{11de^3} - \frac{1}{77}\Big(-77a^2 - \frac{b}{}$$

$$= \frac{2\left(77a^2+\frac{bc(5bc-22ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{231e} - \frac{2b(5bc-22ad)\sqrt{ex}(c+dx^2)^{3/2}}{77d^2e} + \frac{2}{}$$

$$= \frac{2\left(77a^2+\frac{bc(5bc-22ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{231e} - \frac{2b(5bc-22ad)\sqrt{ex}(c+dx^2)^{3/2}}{77d^2e} + \frac{2}{}$$

$$= \frac{2\left(77a^2+\frac{bc(5bc-22ad)}{d^2}\right)\sqrt{ex}\sqrt{c+dx^2}}{231e} - \frac{2b(5bc-22ad)\sqrt{ex}(c+dx^2)^{3/2}}{77d^2e} + \frac{2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 189, normalized size = 0.77

$$\frac{\sqrt{x}\left(\frac{2\sqrt{x}(c+dx^2)(77a^2d^2+22abd(2c+3dx^2)+b^2(-10c^2+6cdx^2+21d^2x^4))}{d^2} + \frac{4ic(5b^2c^2-22abcd+77a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^2}\right)}{231\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/Sqrt[e*x],x]
```

```
[Out] (Sqrt[x]*((2*Sqrt[x]*(c + d*x^2)*(77*a^2*d^2 + 22*a*b*d*(2*c + 3*d*x^2) + b
^2*(-10*c^2 + 6*c*d*x^2 + 21*d^2*x^4)))/d^2 + ((4*I)*c*(5*b^2*c^2 - 22*a*b*
c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]
)/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^2)))/(231*Sqrt[e*x]*
Sqrt[c + d*x^2])
```

Maple [A]
time = 0.11, size = 401, normalized size = 1.64

method	result
risch	$\frac{2(21b^2x^4d^2+66ab\,d^2x^2+6b^2cd\,x^2+77a^2d^2+44abcd-10b^2c^2)x\sqrt{d\,x^2+c}}{231d^2\sqrt{ex}}+\frac{2c(77a^2d^2-22abcd+5b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{\cdot}}{\cdot}\right)}{\sqrt{-\cdot}}}}{\cdots}$
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\left(\frac{2b^2x^4\sqrt{de\,x^3+cex}}{11e}+\frac{2\left(2abd+\frac{2}{11}b^2c\right)x^2\sqrt{de\,x^3+cex}}{7de}+\frac{2\left(a^2d+2abc-\frac{5c\left(2abd+\frac{2}{11}b^2c\right)}{7d}\right)\sqrt{de\,x^3}}{3de}\cdots\right.}{\cdots}$
default	$\frac{\frac{2b^2d^4x^7}{11}+\frac{2\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2cd^2}{3}-\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231/(d*x^2+c)^(1/2)*(21*b^2*d^4*x^7+77*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^
(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)
^(1/2)*a^2*c*d^2-22*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*d+5*((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3+66*a*b*d^4*x^5+27*b^2*c*d^3*
x^5+77*a^2*d^4*x^3+110*a*b*c*d^3*x^3-4*b^2*c^2*d^2*x^3+77*a^2*c*d^3*x+44*a*
b*c^2*d^2*x-10*b^2*c^3*d*x)/(e*x)^(1/2)/d^3
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 119, normalized size = 0.49

$$\frac{2\left(2\left(5b^2c^3-22abc^2d+77a^2cd^2\right)\sqrt{d}\ \mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)+\left(21b^2d^3x^4-10b^2c^2d+44abcd^2+77a^2d^3+6\left(b^2cd^2+11abd^3\right)x^2\right)\sqrt{dx^2+c}\ \sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{231d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(2*(5*b^2*c^3 - 22*a*b*c^2*d + 77*a^2*c*d^2)*sqrt(d)*weierstrassPInve
rse(-4*c/d, 0, x) + (21*b^2*d^3*x^4 - 10*b^2*c^2*d + 44*a*b*c*d^2 + 77*a^2*
d^3 + 6*(b^2*c*d^2 + 11*a*b*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-1/2)/d^3
```

Sympy [C] Result contains complex when optimal does not.
time = 3.46, size = 150, normalized size = 0.61

$$\frac{a^2\sqrt{c}\ \sqrt{x}\ \Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\ \Gamma\left(\frac{5}{4}\right)}+\frac{ab\sqrt{c}\ x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{e}\ \Gamma\left(\frac{9}{4}\right)}+\frac{b^2\sqrt{c}\ x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\ \Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_polar
(I*pi)/c)/(2*sqrt(e)*gamma(5/4)) + a*b*sqrt(c)*x**(5/2)*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(e)*gamma(9/4)) + b**2*sq
rt(c)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi
)/c)/(2*sqrt(e)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*e^(-1/2)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(1/2),x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(1/2), x)
```

3.825 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{(ex)^{3/2}} \, dx$

Optimal. Leaf size=421

$$-\frac{2(b^2c^2-3ad(2bc+5ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3}-\frac{4(b^2c^2-3ad(2bc+5ad))\,\sqrt{ex}\,\sqrt{c+dx^2}}{15d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a^2(c+dx^2)^{3/2}}{ce\sqrt{ex}}$$

```
[Out] 2/9*b^2*(e*x)^(3/2)*(d*x^2+c)^(3/2)/d/e^3-2*a^2*(d*x^2+c)^(3/2)/c/e/(e*x)^(
1/2)-2/15*(b^2*c^2-3*a*d*(5*a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(1/2)/c/d/e^3
-4/15*(b^2*c^2-3*a*d*(5*a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/d^(3/2)/e^2
/(c^(1/2)+x*d^(1/2))+4/15*c^(1/4)*(b^2*c^2-3*a*d*(5*a*d+2*b*c))*(cos(2*arct
an(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4
)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))
^2)^(1/2)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)-2/15*c^(1/4)*(b^2*c^2-3*a*d*(5*a*
d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c
)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 470, 285, 335, 311, 226, 1210}

$$-\frac{2a^2(c+dx^2)^{3/2}}{ce\sqrt{ex}}-\frac{2\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(b^2c^2-3ad(5ad+2bc))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{4\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(b^2c^2-3ad(5ad+2bc))\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{4\sqrt{ex}\,\sqrt{c+dx^2}\,(b^2c^2-3ad(5ad+2bc))}{15d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2(ex)^{3/2}\sqrt{c+dx^2}\,(b^2c^2-3ad(5ad+2bc))}{15cde^3}+\frac{2b^2(ex)^{3/2}\,(c+dx^2)^{3/2}}{9de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(3/2),x]
```

```
[Out] (-2*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(15*c*d*
e^3) - (4*(b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*
d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(c*e*Sqrt[e*
x]) + (2*b^2*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(9*d*e^3) + (4*c^(1/4)*(b^2*c^2
 - 3*a*d*(2*b*c + 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
 Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1
/2])/(15*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - (2*c^(1/4)*(b^2*c^2 - 3*a*d*(2*
b*c + 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(
7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}{(ex)^{3/2}}\,dx = -\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}+\frac{2\int \sqrt{ex}\left(\frac{1}{2}a(2bc+5ad)+\frac{1}{2}b^2cx^2\right)\sqrt{c+dx^2}\,dx}{ce^2}$$

$$= -\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de^3}-\frac{\left(b^2c^2-3ad(2bc+5ad)\right)\int \sqrt{ex}}{3cde^2}$$

$$= -\frac{2\left(b^2c^2-3ad(2bc+5ad)\right)(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}}{9}$$

$$= -\frac{2\left(b^2c^2-3ad(2bc+5ad)\right)(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}}{9}$$

$$= -\frac{2\left(b^2c^2-3ad(2bc+5ad)\right)(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}}{9}$$

$$= -\frac{2\left(b^2c^2-3ad(2bc+5ad)\right)(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3}-\frac{4\left(b^2c^2-3ad(2bc+5ad)\right)\sqrt{e}}{15d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.10, size = 129, normalized size = 0.31

$$\frac{x\left(2\left(c+dx^2\right)\left(-45a^2d+18abdx^2+b^2x^2\left(2c+5dx^2\right)\right)+12\left(-b^2c^2+6abcd+15a^2d^2\right)\sqrt{1+\frac{c}{dx^2}}\,x^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{45d(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(3/2),x]
```

```
[Out] (x*(2*(c + d*x^2)*(-45*a^2*d + 18*a*b*d*x^2 + b^2*x^2*(2*c + 5*d*x^2)) + 12
*(-(b^2*c^2) + 6*a*b*c*d + 15*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(45*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.12, size = 624, normalized size = 1.48

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(-5b^{2}d\,x^{4}-18abd\,x^{2}-2b^{2}c\,x^{2}+45a^{2}d\right)}{45de\sqrt{ex}}+\frac{2\left(15a^{2}d^{2}+6abcd-b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\right.}{\sqrt{}}}}{}$
elliptic	$\frac{\sqrt{ex\,(d\,x^{2}+c)}\left(-\frac{2\left(de\,x^{2}+ce\right)a^{2}}{e^{2}\sqrt{x\,(de\,x^{2}+ce)}}+\frac{2b^{2}x^{3}\sqrt{de\,x^{3}+cex}}{9e^{2}}+\frac{2\left(\frac{b(2ad+bc)}{e}-\frac{7b^{2}c}{9e}\right)x\sqrt{de\,x^{3}+cex}}{5de}+\frac{\left(\frac{a(ad+2}{e}\right.}{}\right.}{}$
default	$\frac{\frac{2b^{2}x^{6}d^{3}}{9}+4\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}cd^{2}+\frac{8\sqrt{\frac{dx+}{\sqrt{}}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(5*b^2*x^6*d^3+90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
```

```
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+36*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*a*b*c^2*d-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-45*((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*a^2*c*d^2-18*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+3*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+18*a*b*d^3*x^4+7*b^2*c*d^2*x^4-45*a^2*d^
3*x^2+18*a*b*c*d^2*x^2+2*b^2*c^2*d*x^2-45*a^2*c*d^2)/(d*x^2+c)^(1/2)/d^2/e/
(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 108, normalized size = 0.26

$$\frac{2\left(6\left(b^2c^2-6\,abcd-15\,a^2d^2\right)\sqrt{d}\;x\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+\left(5\,b^2d^2x^4-45\,a^2d^2+2\left(b^2cd+9\,abd^2\right)x^2\right)\sqrt{dx^2+c}\;\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{45\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
[Out] 2/45*(6*(b^2*c^2 - 6*a*b*c*d - 15*a^2*d^2)*sqrt(d)*x*weierstrassZeta(-4*c/d
, 0, weierstrassPInverse(-4*c/d, 0, x)) + (5*b^2*d^2*x^4 - 45*a^2*d^2 + 2*(
b^2*c*d + 9*a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-3/2)/(d^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.68, size = 153, normalized size = 0.36

$$\frac{a^2\sqrt{c}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{ab\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{b^2\sqrt{c}\,x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(3/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi)
/c)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + a*b*sqrt(c)*x**(3/2)*gamma(3/4)*hyper
((-1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(e**(3/2)*gamma(7/4)) + b**
2*sqrt(c)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(
I*pi)/c)/(2*e**(3/2)*gamma(11/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*e^(-3/2)/x^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}{{\left(e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(3/2),x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(3/2), x)
```

3.826 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{2(b^2c^2-7ad(2bc+ad))\sqrt{ex}\sqrt{c+dx^2}}{21cde^3}-\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}}+\frac{2b^2\sqrt{ex}(c+dx^2)^{3/2}}{7de^3}-\frac{2(b^2c^2-7ad(2bc+ad}{}$$

```
[Out] -2/3*a^2*(d*x^2+c)^(3/2)/c/e/(e*x)^(3/2)+2/7*b^2*(d*x^2+c)^(3/2)*(e*x)^(1/2
)/d/e^3-2/21*(b^2*c^2-7*a*d*(a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/c/d/e^
3-2/21*(b^2*c^2-7*a*d*(a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4
)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*Ell
ipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(
1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(1/4)/d^(5/4)/e^(
5/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {473, 470, 285, 335, 226}

$$-\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}}-\frac{2\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(b^2c^2-7ad(ad+2bc)\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}\,d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(b^2c^2-7ad(ad+2bc)\right)}{21cde^3}+\frac{2b^2\sqrt{ex}(c+dx^2)^{3/2}}{7de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(5/2),x]
```

```
[Out] (-2*(b^2*c^2 - 7*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*c*d*e^3)
 - (2*a^2*(c + d*x^2)^(3/2))/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*(c + d*
x^2)^(3/2))/(7*d*e^3) - (2*(b^2*c^2 - 7*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[
d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(21*c^(1/4)*d^(5/4)*e^(5/2)*Sqrt[c +
d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
```

```
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx = -\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}a(2bc+ad)+\frac{3}{2}b^2cx^2\right)\sqrt{c+dx^2}}{\sqrt{ex}}\,dx}{3ce^2}$$

$$= -\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\,(c+dx^2)^{3/2}}{7de^3} - \frac{(b^2c^2-7ad(2bc+ad))\int \frac{\sqrt{c+d}}{\sqrt{ex}}}{7cde^2}$$

$$= -\frac{2(b^2c^2-7ad(2bc+ad))\sqrt{ex}\,\sqrt{c+dx^2}}{21cde^3} - \frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\,(c+}{7de^3}$$

$$= -\frac{2(b^2c^2-7ad(2bc+ad))\sqrt{ex}\,\sqrt{c+dx^2}}{21cde^3} - \frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\,(c+}{7de^3}$$

$$= -\frac{2(b^2c^2-7ad(2bc+ad))\sqrt{ex}\,\sqrt{c+dx^2}}{21cde^3} - \frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\,(c+}{7de^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 171, normalized size = 0.73

$$\frac{x^{5/2}\left(\frac{2(c+dx^2)(-7a^2d+14abdx^2+b^2x^2(2c+3dx^2))}{dx^{3/2}} + \frac{4i(-b^2c^2+14abcd+7a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d}\right)}{21(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(5/2),x]
```

```
[Out] (x^(5/2)*((2*(c + d*x^2)*(-7*a^2*d + 14*a*b*d*x^2 + b^2*x^2*(2*c + 3*d*x^2)
))/(d*x^(3/2)) + ((4*I)*(-(b^2*c^2) + 14*a*b*c*d + 7*a^2*d^2)*Sqrt[1 + c/(d
*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt
[(I*Sqrt[c])/Sqrt[d]]*d)))/(21*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.12, size = 383, normalized size = 1.64

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(-3b^{2}d\,x^{4}-14abd\,x^{2}-2b^{2}c\,x^{2}+7a^{2}d\right)}{21dx\,e^{2}\sqrt{ex}}+\frac{2\left(7a^{2}d^{2}+14abcd-b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{}}{}\right.}{\sqrt{}}}}{21d^{2}\sqrt{de\,x}}$
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\left(-\frac{2a^{2}\sqrt{de\,x^{3}+cex}}{3e^{3}x^{2}}+\frac{2b^{2}x^{2}\sqrt{de\,x^{3}+cex}}{7e^{3}}+\frac{2\left(\frac{b(2ad+bc)}{e^{2}}-\frac{5b^{2}c}{7e^{2}}\right)\sqrt{de\,x^{3}+cex}}{3de}+\frac{\left(\frac{a(ad+2bc)}{e^{2}}-\right.}{}\right.}{\sqrt{}}$
default	$\frac{\frac{2\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}d^{2}x}{3}+\frac{4\sqrt{-cd}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/(d*x^2+c)^(1/2)/x*(7*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*d^2*
x+14*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d*x-(-c*d)^(1/2)*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*b^2*c^2*x+3*b^2*x^6*d^3+14*a*b*d^3*x^4+5*b^2*c*d^2*x^
4-7*a^2*d^3*x^2+14*a*b*c*d^2*x^2+2*b^2*c^2*d*x^2-7*a^2*c*d^2)/e^2/(e*x)^(1/
2)/d^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
[Out] e^(-5/2)*integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 103, normalized size = 0.44

$$-\frac{2\left(2\left(b^2c^2-14\,abcd-7\,a^2d^2\right)\sqrt{d}\,x^2\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-\left(3\,b^2d^2x^4-7\,a^2d^2+2\left(b^2cd+7\,abd^2\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{21\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
[Out] -2/21*(2*(b^2*c^2 - 14*a*b*c*d - 7*a^2*d^2)*sqrt(d)*x^2*weierstrassPInverse
(-4*c/d, 0, x) - (3*b^2*d^2*x^4 - 7*a^2*d^2 + 2*(b^2*c*d + 7*a*b*d^2)*x^2)*
sqrt(d*x^2 + c)*sqrt(x))*e^(-5/2)/(d^2*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.91, size = 153, normalized size = 0.65

$$\frac{a^2\sqrt{c}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{ab\sqrt{c}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{b^2\sqrt{c}\,x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(5/2),x)
[Out] a**2*sqrt(c)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), d*x**2*exp_polar(I*pi)
/c)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + a*b*sqrt(c)*sqrt(x)*gamma(1/4)*hyper
((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/c)/(e**(5/2)*gamma(5/4)) + b**
2*sqrt(c)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I
*pi)/c)/(2*e**(5/2)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*e^(-5/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}{{\left(e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(5/2),x)
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(5/2), x)
```

3.827 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=421

$$\frac{2(b^2c^2+ad(10bc+ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5}+\frac{4(b^2c^2+ad(10bc+ad))\,\sqrt{ex}\,\sqrt{c+dx^2}}{5c\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}}-2a($$

```
[Out] -2/5*a^2*(d*x^2+c)^(3/2)/c/e/(e*x)^(5/2)-2/5*a*(a*d+10*b*c)*(d*x^2+c)^(3/2)
/c^2/e^3/(e*x)^(1/2)+2/5*(b^2*c^2+a*d*(a*d+10*b*c))*(e*x)^(3/2)*(d*x^2+c)^(
1/2)/c^2/e^5+4/5*(b^2*c^2+a*d*(a*d+10*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/c/e
^4/d^(1/2)/(c^(1/2)+x*d^(1/2))-4/5*(b^2*c^2+a*d*(a*d+10*b*c))*(cos(2*arctan
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/
e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2
)^(1/2)/c^(3/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)+2/5*(b^2*c^2+a*d*(a*d+10*b*
c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(
1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 464, 285, 335, 311, 226, 1210}

$$-\frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}}+\frac{2\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(ad(ad+10bc)+b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{4\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(ad(ad+10bc)+b^2c^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}\,(ad(ad+10bc)+b^2c^2)}{5c^2e^5}+\frac{4\sqrt{ex}\,\sqrt{c+dx^2}\,(ad(ad+10bc)+b^2c^2)}{5c\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a(c+dx^2)^{3/2}\,(ad+10bc)}{5c^2e^3\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(7/2),x]
```

```
[Out] (2*(b^2*c^2 + a*d*(10*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5*c^2*e^5)
+ (4*(b^2*c^2 + a*d*(10*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c*Sqrt[d]
*e^4*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(5*c*e*(e*x)^(5/2))
 - (2*a*(10*b*c + a*d)*(c + d*x^2)^(3/2))/(5*c^2*e^3*Sqrt[e*x]) - (4*(b^2*c
^2 + a*d*(10*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(5*c^(3/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (2*(b^2*c^2 + a*d*(10*b*c
 + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d
^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx = -\frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} + \frac{2\int \frac{\left(\frac{1}{2}a(10bc+ad)+\frac{5}{2}b^2cx^2\right)\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx}{5ce^2}$$

$$= -\frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} - \frac{2a(10bc+ad)\,(c+dx^2)^{3/2}}{5c^2e^3\sqrt{ex}} + \frac{(b^2c^2+ad(10bc+ad))\int \sqrt{}}{c^2e^4}$$

$$= \frac{2(b^2c^2+ad(10bc+ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5} - \frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} - \frac{2a(10bc+ad}{5c^2e^3}$$

$$= \frac{2(b^2c^2+ad(10bc+ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5} - \frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} - \frac{2a(10bc+ad}{5c^2e^3}$$

$$= \frac{2(b^2c^2+ad(10bc+ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5} - \frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} - \frac{2a(10bc+ad}{5c^2e^3}$$

$$= \frac{2(b^2c^2+ad(10bc+ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5} + \frac{4(b^2c^2+ad(10bc+ad))\sqrt{ex}\,\sqrt{}}{5c\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.10, size = 125, normalized size = 0.30

$$\frac{x\left(-2(c+dx^2)\left(10abcx^2-b^2cx^4+a^2(c+2dx^2)\right)+4(b^2c^2+10abcd+a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^4\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{5c(ex)^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(7/2),x]
```

[Out] (x*(-2*(c + d*x^2)*(10*a*b*c*x^2 - b^2*c*x^4 + a^2*(c + 2*d*x^2)) + 4*(b^2*c^2 + 10*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^4*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(5*c*(e*x)^(7/2)*Sqrt[c + d*x^2])

Maple [A]
time = 0.12, size = 648, normalized size = 1.54

method	result
risch	$-\frac{2\sqrt{d x^2+c}\,\left(-b^2 c x^4+2a^2 d x^2+10abc x^2+a^2 c\right)}{5x^2 c e^3\sqrt{ex}}+\frac{2\left(a^2d^2+10abcd+b^2c^2\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-\cdots}}{\cdots}\right.}{\sqrt{-\cdots}}}\cdots}{\cdots}$
elliptic	$\frac{\sqrt{ex\left(dx^2+c\right)}\left(-\frac{2a^2\sqrt{dex^3+cex}}{5e^4x^3}-\frac{4\left(dex^2+ce\right)a(ad+5bc)}{5e^4c\sqrt{x\left(dex^2+ce\right)}}+\frac{2b^2x\sqrt{dex^3+cex}}{5e^4}+\frac{\left(\frac{b(2ad+bc)}{e^3}+\frac{2da(ad+5bc)}{5c\,e^3}-\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{4\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2cd^2x^2}{5}+8\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/5/x^2*(2*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1

```
/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2+20*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a*b*c^2*d*x^2+2*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2-((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2),1/2*2^(1/2))*a^2*c*d^2*x^2-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*
EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d*x^
2-((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2+b^2*c*d^2*x^6-2*a^2*d^3*x^4-10*a*
b*c*d^2*x^4+b^2*c^2*d*x^4-3*a^2*c*d^2*x^2-10*a*b*c^2*d*x^2-a^2*c^2*d)/(d*x^
2+c)^(1/2)/d/e^3/(e*x)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
[Out] e^(-7/2)*integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 110, normalized size = 0.26

$$-\frac{2\left(2\,(b^2c^2+10\,abcd+a^2d^2)\sqrt{d}\,x^3\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)-(b^2cdx^4-a^2cd-2\,(5\,abcd+a^2d^2)x^2)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{5\,cdx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
[Out] -2/5*(2*(b^2*c^2 + 10*a*b*c*d + a^2*d^2)*sqrt(d)*x^3*weierstrassZeta(-4*c/d
, 0, weierstrassPInverse(-4*c/d, 0, x)) - (b^2*c*d*x^4 - a^2*c*d - 2*(5*a*b
*c*d + a^2*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-7/2)/(c*d*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.35, size = 160, normalized size = 0.38

$$\frac{a^2\sqrt{c}\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{ab\sqrt{c}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{b^2\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), d*x**2*exp_polar(I*pi
)/c)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + a*b*sqrt(c)*gamma(-1/4)*hyper((-1/
2, -1/4), (3/4,), d*x**2*exp_polar(I*pi)/c)/(e**(7/2)*sqrt(x)*gamma(3/4)) +
 b**2*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_pol
ar(I*pi)/c)/(2*e**(7/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*e^(-7/2)/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}{{\left(e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(7/2),x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(7/2), x)
```

3.828 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{9/2}} \, dx$

Optimal. Leaf size=213

$$\frac{2(7b^2c^2+ad(14bc-ad))\sqrt{x}\sqrt{c+dx^2}}{21c^2}-\frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}}-\frac{2a(14bc-ad)(c+dx^2)^{3/2}}{21c^2x^{3/2}}+\frac{2(7b^2c^2+ad(14bc}{}$$

```
[Out] -2/7*a^2*(d*x^2+c)^(3/2)/c/x^(7/2)-2/21*a*(-a*d+14*b*c)*(d*x^2+c)^(3/2)/c^2
/x^(3/2)+2/21*(7*b^2*c^2+a*d*(-a*d+14*b*c))*x^(1/2)*(d*x^2+c)^(1/2)/c^2+2/2
1*(7*b^2*c^2+a*d*(-a*d+14*b*c))*(cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))^2)^
(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d
^(1/2))^2)^(1/2)/c^(5/4)/d^(1/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 210, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {473, 464, 285, 335, 226}

$$-\frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}}+\frac{2\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(ad(14bc-ad)+7b^2c^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt[4]{d}\sqrt{c+dx^2}}+\frac{2}{21}\sqrt{x}\sqrt{c+dx^2}\left(\frac{ad(14bc-ad)}{c^2}+7b^2\right)-\frac{2a(c+dx^2)^{3/2}(14bc-ad)}{21c^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(9/2),x]
```

```
[Out] (2*(7*b^2 + (a*d*(14*b*c - a*d))/c^2)*Sqrt[x]*Sqrt[c + d*x^2])/21 - (2*a^2*
(c + d*x^2)^(3/2))/(7*c*x^(7/2)) - (2*a*(14*b*c - a*d)*(c + d*x^2)^(3/2))/(
21*c^2*x^(3/2)) + (2*(7*b^2*c^2 + a*d*(14*b*c - a*d))*(Sqrt[c] + Sqrt[d]*x)
*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt
[x])/c^(1/4)], 1/2])/(21*c^(5/4)*d^(1/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx = -\frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}} + \frac{2\int \frac{\left(\frac{1}{2}a(14bc-ad)+\frac{7}{2}b^2cx^2\right)\sqrt{c+dx^2}}{x^{5/2}}\,dx}{7c}$$

$$= -\frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}} - \frac{2a(14bc-ad)(c+dx^2)^{3/2}}{21c^2x^{3/2}} + \frac{1}{7}\left(7b^2+\frac{ad(14bc-ad)}{c^2}\right)\int$$

$$= \frac{2}{21}\left(7b^2+\frac{ad(14bc-ad)}{c^2}\right)\sqrt{x}\sqrt{c+dx^2} - \frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}} - \frac{2a(14bc-ad)}{21c^2x}$$

$$= \frac{2}{21}\left(7b^2+\frac{ad(14bc-ad)}{c^2}\right)\sqrt{x}\sqrt{c+dx^2} - \frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}} - \frac{2a(14bc-ad)}{21c^2x}$$

$$= \frac{2}{21}\left(7b^2+\frac{ad(14bc-ad)}{c^2}\right)\sqrt{x}\sqrt{c+dx^2} - \frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}} - \frac{2a(14bc-ad)}{21c^2x}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 160, normalized size = 0.75

$$\frac{2\left((c+dx^2)\left(-14abcx^2+7b^2cx^4-a^2(3c+2dx^2)\right)+\frac{2i(7b^2c^2+14abcd-a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^{9/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21cx^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(9/2),x]
```

```
[Out] (2*((c + d*x^2)*(-14*a*b*c*x^2 + 7*b^2*c*x^4 - a^2*(3*c + 2*d*x^2)) + ((2*I
)*(7*b^2*c^2 + 14*a*b*c*d - a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(9/2)*EllipticF[
I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[c])/Sqrt[d]
]))/(21*c*x^(7/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 385, normalized size = 1.81

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,(-7b^2c\,x^4+2a^2d\,x^2+14abc\,x^2+3a^2c)}{21x^{\frac{7}{2}}c}-\frac{2(a^2d^2-14abcd-7b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\right.}{}}}{21cd\sqrt{}}$
elliptic	$\frac{\sqrt{x\,(d\,x^2+c)}\left(-\frac{2a^2\sqrt{d\,x^3+cx}}{7x^4}-\frac{4a(ad+7bc)\sqrt{d\,x^3+cx}}{21c\,x^2}+\frac{2b^2\sqrt{d\,x^3+cx}}{3}+\frac{\left(2abd+\frac{2b^2c}{3}-\frac{2da(ad+7bc)}{21c}\right)\sqrt{-c}}{}\right.}{\sqrt{x}\,\sqrt{d\,x^2+}}$
default	$-\frac{2\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^2x^3-14\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/(d*x^2+c)^(1/2)*(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*d^2*x^3
-14*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c*d*x^3-7*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^2*x^3-7*b^2*c*d^2*x^6+2*a^2*d^3*x^4+14*a*b*c
*d^2*x^4-7*b^2*c^2*d*x^4+5*a^2*c*d^2*x^2+14*a*b*c^2*d*x^2+3*a^2*c^2*d)/x^(7
/2)/c/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 102, normalized size = 0.48

$$\frac{2\left(2\left(7\,b^2c^2+14\,abcd-a^2d^2\right)\sqrt{d}\,x^4\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)+\left(7\,b^2cdx^4-3\,a^2cd-2\left(7\,abcd+a^2d^2\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)}{21\,cdx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*(2*(7*b^2*c^2 + 14*a*b*c*d - a^2*d^2)*sqrt(d)*x^4*weierstrassPInverse(
-4*c/d, 0, x) + (7*b^2*c*d*x^4 - 3*a^2*c*d - 2*(7*a*b*c*d + a^2*d^2)*x^2)*s
qrt(d*x^2 + c)*sqrt(x))/(c*d*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 12.68, size = 144, normalized size = 0.68

$$\frac{a^2\sqrt{c}\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}+\frac{ab\sqrt{c}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{b^2\sqrt{c}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(9/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), d*x**2*exp_polar(I*pi
)/c)/(2*x**(7/2)*gamma(-3/4)) + a*b*sqrt(c)*gamma(-3/4)*hyper((-3/4, -1/2),
 (1/4,), d*x**2*exp_polar(I*pi)/c)/(x**(3/2)*gamma(1/4)) + b**2*sqrt(c)*sqr
t(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/c)/(2*gam
ma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}{x^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(9/2),x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(9/2), x)
```

3.829 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx$

Optimal. Leaf size=386

$$-\frac{2(15b^2c^2+ad(6bc-ad))\sqrt{c+dx^2}}{15c^2\sqrt{x}}+\frac{4\sqrt{d}\,(15b^2c^2+ad(6bc-ad))\sqrt{x}\,\sqrt{c+dx^2}}{15c^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}}-\frac{2a(6$$

```
[Out] -2/9*a^2*(d*x^2+c)^(3/2)/c/x^(9/2)-2/15*a*(-a*d+6*b*c)*(d*x^2+c)^(3/2)/c^2/
x^(5/2)-2/15*(15*b^2*c^2+a*d*(-a*d+6*b*c))*(d*x^2+c)^(1/2)/c^2/x^(1/2)+4/15
*(15*b^2*c^2+a*d*(-a*d+6*b*c))*d^(1/2)*x^(1/2)*(d*x^2+c)^(1/2)/c^2/(c^(1/2)
+x*d^(1/2))-4/15*d^(1/4)*(15*b^2*c^2+a*d*(-a*d+6*b*c))*(cos(2*arctan(d^(1/4
)*x^(1/2)/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))*Ellipti
cE(sin(2*arctan(d^(1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*
((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/(d*x^2+c)^(1/2)+2/15*d^(1/4
)*(15*b^2*c^2+a*d*(-a*d+6*b*c))*(cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))^2)^
(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d
^(1/2))^2)^(1/2)/c^(7/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 383, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {473, 464, 283, 335, 311, 226, 1210}

$$-\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}}+\frac{2\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(ad(6bc-ad)+15b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{c+dx^2}}-\frac{4\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(ad(6bc-ad)+15b^2c^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{c+dx^2}}-\frac{2\sqrt{c+dx^2}\left(\frac{ad(6bc-ad)}{c^2}+15b^2\right)}{15\sqrt{x}}+\frac{4\sqrt{d}\,\sqrt{x}\,\sqrt{c+dx^2}\,(ad(6bc-ad)+15b^2c^2)}{15c^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a(c+dx^2)^{3/2}(6bc-ad)}{15c^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(11/2),x]
```

```
[Out] (-2*(15*b^2 + (a*d*(6*b*c - a*d))/c^2)*Sqrt[c + d*x^2])/(15*Sqrt[x]) + (4*S
qrt[d]*(15*b^2*c^2 + a*d*(6*b*c - a*d))*Sqrt[x]*Sqrt[c + d*x^2])/(15*c^2*(S
qrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(9*c*x^(9/2)) - (2*a*(6*b*
c - a*d)*(c + d*x^2)^(3/2))/(15*c^2*x^(5/2)) - (4*d^(1/4)*(15*b^2*c^2 + a*d
*(6*b*c - a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x
)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[
c + d*x^2]) + (2*d^(1/4)*(15*b^2*c^2 + a*d*(6*b*c - a*d))*(Sqrt[c] + Sqrt[d
]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*
Sqrt[x])/c^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
```

```
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx = -\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} + \frac{2\int \frac{\left(\frac{3}{2}a(6bc-ad)+\frac{9}{2}b^2cx^2\right)\sqrt{c+dx^2}}{x^{7/2}}\,dx}{9c}$$

$$= -\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} - \frac{2a(6bc-ad)\,(c+dx^2)^{3/2}}{15c^2x^{5/2}} + \frac{1}{15}\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\int$$

$$= -\frac{2\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{c+dx^2}}{15\sqrt{x}} - \frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} - \frac{2a(6bc-ad)\,(c+dx^2)^{3}}{15c^2x^{5/2}}$$

$$= -\frac{2\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{c+dx^2}}{15\sqrt{x}} - \frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} - \frac{2a(6bc-ad)\,(c+dx^2)^{3}}{15c^2x^{5/2}}$$

$$= -\frac{2\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{c+dx^2}}{15\sqrt{x}} - \frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}} - \frac{2a(6bc-ad)\,(c+dx^2)^{3}}{15c^2x^{5/2}}$$

$$= -\frac{2\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{c+dx^2}}{15\sqrt{x}} + \frac{4\sqrt{d}\,\left(15b^2+\frac{ad(6bc-ad)}{c^2}\right)\sqrt{x}\,\sqrt{c+dx^2}}{15\left(\sqrt{c}+\sqrt{d}\,x\right)} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.12, size = 148, normalized size = 0.38

$$\frac{-2(c+dx^2)\left(45b^2c^2x^4+18abcx^2(c+2dx^2)+a^2(5c^2+2cdx^2-6d^2x^4)\right)+12d(15b^2c^2+6abcd-a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^6\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)}{45c^2x^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(11/2),x]
```

```
[Out] (-2*(c + d*x^2)*(45*b^2*c^2*x^4 + 18*a*b*c*x^2*(c + 2*d*x^2) + a^2*(5*c^2 +
 2*c*d*x^2 - 6*d^2*x^4)) + 12*d*(15*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*Sqrt[1 +
 c/(d*x^2)]*x^6*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))])/(45*c^2*x^
(9/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 659, normalized size = 1.71

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(-6a^{2}d^{2}x^{4}+36abcd\,x^{4}+45b^{2}c^{2}x^{4}+2a^{2}cd\,x^{2}+18ab\,c^{2}x^{2}+5a^{2}c^{2}\right)}{45x^{\frac{9}{2}}c^{2}}-\frac{2\left(a^{2}d^{2}-6abcd-15b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\right.}{\sqrt{}}}}{}$
elliptic	$\frac{\sqrt{x\left(d\,x^{2}+c\right)}\,\left(-\frac{2a^{2}\sqrt{d\,x^{3}+cx}}{9x^{5}}-\frac{4a(ad+9bc)\sqrt{d\,x^{3}+cx}}{45c\,x^{3}}+\frac{2\left(d\,x^{2}+c\right)\left(2a^{2}d^{2}-12abcd-15b^{2}c^{2}\right)}{15c^{2}\sqrt{x\left(d\,x^{2}+c\right)}}+\frac{\left(b^{2}d-\frac{d\left(2a^{2}d^{2}-12ab\right.}{15c}\right.}{}\right.}{}$
default	$-\frac{2\left(6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c\,d^{2}x^{4}-36\sqrt{\frac{dx+}{\sqrt{}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^4-36*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a*b*c^2*d*x^4-90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^4-3*((d*x+(-c
```

```
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*a^2*c*d^2*x^4+18*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d*x
^4+45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^4-6*a^2*d^3*x^6+36*a*b*c*d^2*x^
6+45*b^2*c^2*d*x^6-4*a^2*c*d^2*x^4+54*a*b*c^2*d*x^4+45*b^2*c^3*x^4+7*a^2*c^
2*d*x^2+18*a*b*c^3*x^2+5*a^2*c^3)/(d*x^2+c)^(1/2)/x^(9/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 126, normalized size = 0.33

$$-\frac{2\left(6\,(15\,b^2c^2+6\,abcd-a^2d^2)\sqrt{d}\,x^5\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+\left(3\,(15\,b^2c^2+12\,abcd-2\,a^2d^2)x^4+5\,a^2c^2+2\,(9\,abc^2+a^2cd)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)}{45\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*(6*(15*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*sqrt(d)*x^5*weierstrassZeta(-4*
c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (3*(15*b^2*c^2 + 12*a*b*c*d -
2*a^2*d^2)*x^4 + 5*a^2*c^2 + 2*(9*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c)*s
qrt(x))/(c^2*x^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 35.30, size = 151, normalized size = 0.39

$$\frac{a^2\sqrt{c}\,\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2x^{\frac{9}{2}}\Gamma\left(-\frac{5}{4}\right)}+\frac{ab\sqrt{c}\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{b^2\sqrt{c}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(11/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), d*x**2*exp_polar(I*pi
)/c)/(2*x**(9/2)*gamma(-5/4)) + a*b*sqrt(c)*gamma(-5/4)*hyper((-5/4, -1/2),
```

```
(-1/4,), d*x**2*exp_polar(I*pi)/c)/(x**(5/2)*gamma(-1/4)) + b**2*sqrt(c)*g
amma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(x)
*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(11/2),x)
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(11/2), x)
```

3.830 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{2(77b^2c^2-22abcd+5a^2d^2)\sqrt{c+dx^2}}{231c^2x^{3/2}}-\frac{2a^2(c+dx^2)^{3/2}}{11cx^{11/2}}-\frac{2a(22bc-5ad)(c+dx^2)^{3/2}}{77c^2x^{7/2}}+\frac{2d^{3/4}(77b^2c^2-22a}{}$$

```
[Out] -2/11*a^2*(d*x^2+c)^(3/2)/c/x^(11/2)-2/77*a*(-5*a*d+22*b*c)*(d*x^2+c)^(3/2)
/c^2/x^(7/2)-2/231*(5*a^2*d^2-22*a*b*c*d+77*b^2*c^2)*(d*x^2+c)^(1/2)/c^2/x^
(3/2)+2/231*d^(3/4)*(5*a^2*d^2-22*a*b*c*d+77*b^2*c^2)*(cos(2*arctan(d^(1/4)
*x^(1/2)/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))*Elliptic
F(sin(2*arctan(d^(1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*(
(d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(9/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 213, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {473, 464, 283, 335, 226}

$$\frac{2d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(5a^2d^2-22abcd+77b^2c^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{c+dx^2}}-\frac{2a^2(c+dx^2)^{3/2}}{11cx^{11/2}}-\frac{2\sqrt{c+dx^2}\left(77b^2-\frac{ad(22bc-5ad)}{c^2}\right)}{231x^{3/2}}-\frac{2a(c+dx^2)^{3/2}(22bc-5ad)}{77c^2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(13/2),x]
```

```
[Out] (-2*(77*b^2 - (a*d*(22*b*c - 5*a*d))/c^2)*Sqrt[c + d*x^2])/(231*x^(3/2)) -
(2*a^2*(c + d*x^2)^(3/2))/(11*c*x^(11/2)) - (2*a*(22*b*c - 5*a*d)*(c + d*x^
2)^(3/2))/(77*c^2*x^(7/2)) + (2*d^(3/4)*(77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^
2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx = -\frac{2a^2(c+dx^2)^{3/2}}{11cx^{11/2}} + \frac{2\int \frac{\left(\frac{1}{2}a(22bc-5ad)+\frac{11}{2}b^2cx^2\right)\sqrt{c+dx^2}}{x^{9/2}}\,dx}{11c}$$

$$= -\frac{2a^2(c+dx^2)^{3/2}}{11cx^{11/2}} - \frac{2a(22bc-5ad)(c+dx^2)^{3/2}}{77c^2x^{7/2}} - \frac{1}{77}\Bigg(-77b^2 + \frac{ad(22bc-5}{c^2}$$

$$= -\frac{2\left(77b^2-\frac{ad(22bc-5ad)}{c^2}\right)\sqrt{c+dx^2}}{231x^{3/2}} - \frac{2a^2(c+dx^2)^{3/2}}{11cx^{11/2}} - \frac{2a(22bc-5ad)(c+}{77c^2x^{7/2}}$$

$$= -\frac{2\left(77b^2-\frac{ad(22bc-5ad)}{c^2}\right)\sqrt{c+dx^2}}{231x^{3/2}} - \frac{2a^2(c+dx^2)^{3/2}}{11cx^{11/2}} - \frac{2a(22bc-5ad)(c+}{77c^2x^{7/2}}$$

$$= -\frac{2\left(77b^2-\frac{ad(22bc-5ad)}{c^2}\right)\sqrt{c+dx^2}}{231x^{3/2}} - \frac{2a^2(c+dx^2)^{3/2}}{11cx^{11/2}} - \frac{2a(22bc-5ad)(c+}{77c^2x^{7/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 187, normalized size = 0.86

$$-\frac{2\sqrt{c+dx^2}\,(77b^2c^2x^4+22abcx^2(3c+2dx^2)+a^2(21c^2+6cdx^2-10d^2x^4))}{231c^2x^{11/2}}+\frac{4id(77b^2c^2-22abcd+5a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{231c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(13/2),x]
```

```
[Out] (-2*Sqrt[c + d*x^2]*(77*b^2*c^2*x^4 + 22*a*b*c*x^2*(3*c + 2*d*x^2) + a^2*(2
1*c^2 + 6*c*d*x^2 - 10*d^2*x^4)))/(231*c^2*x^(11/2)) + (((4*I)/231)*d*(77*b
^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[
Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*Sqr
t[c + d*x^2])
```

Maple [A]
time = 0.13, size = 403, normalized size = 1.86

method	result
risch	$-\frac{2\sqrt{dx^2+c}\,(-10a^2d^2x^4+44abcdx^4+77b^2c^2x^4+6a^2cdx^2+66abc^2x^2+21a^2c^2)}{231x^{\frac{11}{2}}c^2}+\frac{2(5a^2d^2-22abcd+77b^2c^2)\sqrt{-cd}\sqrt{\frac{(x}{}}}{}$
elliptic	$\frac{\sqrt{x(dx^2+c)}\left(-\frac{2a^2\sqrt{dx^3+cx}}{11x^6}-\frac{4a(ad+11bc)\sqrt{dx^3+cx}}{77c\,x^4}+\frac{2(10a^2d^2-44abcd-77b^2c^2)\sqrt{dx^3+cx}}{231c^2x^2}+\frac{\left(b^2d+\frac{d(10a}{}\right.}{}\right.}{\sqrt{x}}$
default	$\frac{\frac{10\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^2x^5}{231}-4\sqrt{\frac{dx+}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231/(d*x^2+c)^(1/2)*(5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*d^2*x
^5-22*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c*d*x^5+77*((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^2*x^5+10*a^2*d^3*x^6-44*a*b*c*d^2*x^6-77*
b^2*c^2*d*x^6+4*a^2*c*d^2*x^4-110*a*b*c^2*d*x^4-77*b^2*c^3*x^4-27*a^2*c^2*d
*x^2-66*a*b*c^3*x^2-21*a^2*c^3)/x^(11/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 118, normalized size = 0.54

$$\frac{2\left(2\,(77\,b^2c^2-22\,abcd+5\,a^2d^2)\sqrt{d}\,x^6\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-((77\,b^2c^2+44\,abcd-10\,a^2d^2)x^4+21\,a^2c^2+6\,(11\,abc^2+a^2cd)x^2)\sqrt{dx^2+c}\,\sqrt{x}\right)}{231\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] 2/231*(2*(77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*sqrt(d)*x^6*weierstrassPInve
rse(-4*c/d, 0, x) - ((77*b^2*c^2 + 44*a*b*c*d - 10*a^2*d^2)*x^4 + 21*a^2*c^
2 + 6*(11*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c)*sqrt(x))/(c^2*x^6)
```

Sympy [C] Result contains complex when optimal does not.
time = 92.68, size = 151, normalized size = 0.70

$$\frac{a^2\sqrt{c}\,\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2x^{\frac{11}{2}}\Gamma\left(-\frac{7}{4}\right)}+\frac{ab\sqrt{c}\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}+\frac{b^2\sqrt{c}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(13/2),x)
```

```
[Out] a**2*sqrt(c)*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), d*x**2*exp_polar(I*
pi)/c)/(2*x**(11/2)*gamma(-7/4)) + a*b*sqrt(c)*gamma(-7/4)*hyper((-7/4, -1/
```

```
2), (-3/4,), d*x**2*exp_polar(I*pi)/c)/(x**(7/2)*gamma(-3/4)) + b**2*sqrt(c
)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), d*x**2*exp_polar(I*pi)/c)/(2*x**(
3/2)*gamma(1/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(13/2),x)
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(13/2), x)
```

3.831 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx$

Optimal. Leaf size=441

$$-\frac{2(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^2x^{5/2}}-\frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}}+\frac{4d^{3/2}(39b^2c^2-26abcd}{195c^3\left(\sqrt{c\right.}}$$

```
[Out] -2/13*a^2*(d*x^2+c)^(3/2)/c/x^(13/2)-2/117*a*(-7*a*d+26*b*c)*(d*x^2+c)^(3/2
)/c^2/x^(9/2)-2/195*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)*(d*x^2+c)^(1/2)/c^2/x
^(5/2)-4/195*d*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)*(d*x^2+c)^(1/2)/c^3/x^(1/2
)+4/195*d^(3/2)*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)*x^(1/2)*(d*x^2+c)^(1/2)/c
^3/(c^(1/2)+x*d^(1/2))-4/195*d^(5/4)*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)*(cos
(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c
^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(
1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/(d*x^2+c)^
(1/2)+2/195*d^(5/4)*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)*(cos(2*arctan(d^(1/4)
*x^(1/2)/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))*Elliptic
F(sin(2*arctan(d^(1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*(
(d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 437, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {473, 464, 283, 331, 335, 311, 226, 1210}

$$\frac{2d^{5/4}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(7a^2d^2-26abcd+39b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{c+dx^2}}-\frac{4d^{5/4}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(7a^2d^2-26abcd+39b^2c^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{c+dx^2}}-\frac{4d\sqrt{c+dx^2}\,(7a^2d^2-26abcd+39b^2c^2)}{195c^3\sqrt{x}}+\frac{4d^{3/2}\sqrt{x}\sqrt{c+dx^2}\,(7a^2d^2-26abcd+39b^2c^2)}{195c^3\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}}-\frac{2\sqrt{c+dx^2}\left(39b^2-\frac{ad(26bc-7ad)}{c^2}\right)}{195x^{5/2}}-\frac{2a(c+dx^2)^{3/2}(26bc-7ad)}{117c^2x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(15/2),x]
```

```
[Out] (-2*(39*b^2 - (a*d*(26*b*c - 7*a*d))/c^2)*Sqrt[c + d*x^2])/(195*x^(5/2)) -
(4*d*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[c + d*x^2])/(195*c^3*Sqrt[x
]) + (4*d^(3/2)*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*Sqrt[x]*Sqrt[c + d*x^
2])/(195*c^3*(Sqrt[c] + Sqrt[d]*x)) - (2*a^2*(c + d*x^2)^(3/2))/(13*c*x^(13
/2)) - (2*a*(26*b*c - 7*a*d)*(c + d*x^2)^(3/2))/(117*c^2*x^(9/2)) - (4*d^(5
/4)*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)
], 1/2])/(195*c^(11/4)*Sqrt[c + d*x^2]) + (2*d^(5/4)*(39*b^2*c^2 - 26*a*b*c
*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x
)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(195*c^(11/4)*Sqr
t[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx = -\frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}} + \frac{2\int \frac{\left(\frac{1}{2}a(26bc-7ad)+\frac{13}{2}b^2cx^2\right)\sqrt{c+dx^2}}{x^{11/2}}\,dx}{13c}$$

$$= -\frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}} - \frac{2a(26bc-7ad)(c+dx^2)^{3/2}}{117c^2x^{9/2}} - \frac{1}{39}\left(-39b^2 + \frac{ad(26bc-7}{c^2}\right.$$

$$= -\frac{2\left(39b^2 - \frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}} - \frac{2a(26bc-7ad)(c+}{117c^2x^{9/2}}$$

$$= -\frac{2\left(39b^2 - \frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}}$$

$$= -\frac{2\left(39b^2 - \frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}}$$

$$= -\frac{2\left(39b^2 - \frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}}$$

$$= -\frac{2\left(39b^2 - \frac{ad(26bc-7ad)}{c^2}\right)\sqrt{c+dx^2}}{195x^{5/2}} - \frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 20.16, size = 182, normalized size = 0.41

$$\frac{-2(c+dx^2)\left(117b^2c^2x^4(c+2dx^2)+26abcx^2(5c^2+2cdx^2-6d^2x^4)+a^2(45c^3+10c^2dx^2-14cd^2x^4+42d^3x^6)\right)+4d^2(39b^2c^2-26abcd+7a^2d^2)\,x^8\sqrt{1+\frac{dx^2}{c}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)}{585c^3x^{13/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(15/2),x]
```

```
[Out] (-2*(c + d*x^2)*(117*b^2*c^2*x^4*(c + 2*d*x^2) + 26*a*b*c*x^2*(5*c^2 + 2*c*
d*x^2 - 6*d^2*x^4) + a^2*(45*c^3 + 10*c^2*d*x^2 - 14*c*d^2*x^4 + 42*d^3*x^6
)) + 4*d^2*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*x^8*Sqrt[1 + (d*x^2)/c]*Hy
pergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)])/(585*c^3*x^(13/2)*Sqrt[c + d*
x^2])
```

Maple [A]
time = 0.13, size = 706, normalized size = 1.60

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(42 a^{2} d^{3} x^{6}-156 a b c\,d^{2} x^{6}+234 b^{2} c^{2} d\,x^{6}-14 a^{2} c\,d^{2} x^{4}+52 a b\,c^{2} d\,x^{4}+117 b^{2} c^{3} x^{4}+10 a^{2} c^{2} d\,x^{2}+130 a b\,c^{3} x^{2}+45 a^{2} c^{3}\right)}{585 x^{\frac{13}{2}} c^{3}}+$

elliptic	$\frac{\sqrt{x\left(d\,x^{2}+c\right)}\left(-\frac{2a^{2}\sqrt{d\,x^{3}+cx}}{13x^{7}}-\frac{4a(ad+13bc)\sqrt{d\,x^{3}+cx}}{117c\,x^{5}}+\frac{2\left(14a^{2}d^{2}-52abcd-117b^{2}c^{2}\right)\sqrt{d\,x^{3}+cx}}{585c^{2}x^{3}}-\frac{4\left(d\,x^{2}+c\right.}{195c^{3}\ldots}\right.}{\ldots}$
default	$\frac{\frac{28\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c\,d^{3}x^{6}}{195}-\frac{8\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\ldots}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/585*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^6-156*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a*b*c^2*d^2*x^6+234*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^6-21*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^6+78*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b
*c^2*d^2*x^6-117*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^6-42*a^2*d^4*x^8+1
56*a*b*c*d^3*x^8-234*b^2*c^2*d^2*x^8-28*a^2*c*d^3*x^6+104*a*b*c^2*d^2*x^6-3
51*b^2*c^3*d*x^6+4*a^2*c^2*d^2*x^4-182*a*b*c^3*d*x^4-117*b^2*c^4*x^4-55*a^2
*c^3*d*x^2-130*a*b*c^4*x^2-45*a^2*c^4)/(d*x^2+c)^(1/2)/x^(13/2)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 164, normalized size = 0.37

$$-\frac{2\left(6\,(39\,b^2c^2d-26\,abcd^2+7\,a^2d^3)\sqrt{d}\,x^7\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+\left(6\,(39\,b^2c^2d-26\,abcd^2+7\,a^2d^3)x^6+45\,a^2c^3+(117\,b^2c^3+52\,abc^2d-14\,a^2cd^2)x^4+10\,(13\,abc^3+a^2c^2d)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)}{585\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] -2/585*(6*(39*b^2*c^2*d - 26*a*b*c*d^2 + 7*a^2*d^3)*sqrt(d)*x^7*weierstrass
Zeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (6*(39*b^2*c^2*d - 26*
a*b*c*d^2 + 7*a^2*d^3)*x^6 + 45*a^2*c^3 + (117*b^2*c^3 + 52*a*b*c^2*d - 14*
a^2*c*d^2)*x^4 + 10*(13*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(x))/
(c^3*x^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(15/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}}{x^{15/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(15/2),x)
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(15/2), x)
```

3.832 $\int (ex)^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=530

$$\frac{8c^2(51a^2d^2+bc(11bc-42ad))\,e(ex)^{3/2}\sqrt{c+dx^2}}{9945d^3}+\frac{4c(51a^2d^2+bc(11bc-42ad))\,(ex)^{7/2}\sqrt{c+dx^2}}{1989d^2e}-\frac{8c^3(51a$$

```
[Out] 2/663*(51*a^2*d^2+b*c*(-42*a*d+11*b*c))*(e*x)^(7/2)*(d*x^2+c)^(3/2)/d^2/e-2
/357*b*(-42*a*d+11*b*c)*(e*x)^(7/2)*(d*x^2+c)^(5/2)/d^2/e+2/21*b^2*(e*x)^(1
1/2)*(d*x^2+c)^(5/2)/d/e^3+8/9945*c^2*(51*a^2*d^2+b*c*(-42*a*d+11*b*c))*e*(
e*x)^(3/2)*(d*x^2+c)^(1/2)/d^3+4/1989*c*(51*a^2*d^2+b*c*(-42*a*d+11*b*c))*(
e*x)^(7/2)*(d*x^2+c)^(1/2)/d^2/e-8/3315*c^3*(51*a^2*d^2+b*c*(-42*a*d+11*b*c
))*e^2*(e*x)^(1/2)*(d*x^2+c)^(1/2)/d^(7/2)/(c^(1/2)+x*d^(1/2))+8/3315*c^(13
/4)*(51*a^2*d^2+b*c*(-42*a*d+11*b*c))*e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^
(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^
(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(15/4)
/(d*x^2+c)^(1/2)-4/3315*c^(13/4)*(51*a^2*d^2+b*c*(-42*a*d+11*b*c))*e^(5/2)*
(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)
+x*d^(1/2))^2)^(1/2)/d^(15/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {475, 470, 285, 327, 335, 311, 226, 1210}

$$-\frac{4c^{13/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(51a^2d^2+bc(11bc-42ad))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{15/4}\sqrt{c+dx^2}}+\frac{8c^{13/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(51a^2d^2+bc(11bc-42ad))\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{15/4}\sqrt{c+dx^2}}-\frac{8c^3e^2\sqrt{ex}\,\sqrt{c+dx^2}\,(51a^2d^2+bc(11bc-42ad))}{3315d^{7/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{8c^2e(ex)^{3/2}\sqrt{c+dx^2}\,(51a^2d^2+bc(11bc-42ad))}{9945d^3}+\frac{2(ex)^{7/2}\left(c+dx^2\right)^{3/2}(51a^2d^2+bc(11bc-42ad))}{663d^2e}+\frac{4c(ex)^{7/2}\sqrt{c+dx^2}\,(51a^2d^2+bc(11bc-42ad))}{1989d^2e}-\frac{2b(ex)^{7/2}\left(c+dx^2\right)^{5/2}(11bc-42ad)}{357d^2e}+\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (8*c^2*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/
(9945*d^3) + (4*c*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*(e*x)^(7/2)*Sqrt[c +
 d*x^2])/(1989*d^2*e) - (8*c^3*(51*a^2*d^2 + b*c*(11*b*c - 42*a*d))*e^2*Sqr
t[e*x]*Sqrt[c + d*x^2])/(3315*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) + (2*(51*a^2*d
^2 + b*c*(11*b*c - 42*a*d))*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(663*d^2*e) - (2
*b*(11*b*c - 42*a*d)*(e*x)^(7/2)*(c + d*x^2)^(5/2))/(357*d^2*e) + (2*b^2*(e
*x)^(11/2)*(c + d*x^2)^(5/2))/(21*d*e^3) + (8*c^(13/4)*(51*a^2*d^2 + b*c*(1
1*b*c - 42*a*d))*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
```

```
2])/(3315*d^(15/4)*Sqrt[c + d*x^2]) - (4*c^(13/4)*(51*a^2*d^2 + b*c*(11*b*c
 - 42*a*d))*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[
d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
3315*d^(15/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int (ex)^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx = \frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3} + \frac{2\int (ex)^{5/2}\left(c+dx^2\right)^{3/2}\left(\frac{21a^2d}{2} - \frac{1}{2}b(11bc\right.}{21d}$$

$$= -\frac{2b(11bc-42ad)(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{357d^2e} + \frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3} +$$

$$= \frac{2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{663e} - \frac{2b(11bc-42ad)(ex)^{7}}{357d^2e}$$

$$= \frac{4c\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)(ex)^{7/2}\sqrt{c+dx^2}}{1989e} + \frac{2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)}{663}$$

$$= \frac{8c^2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{9945d} + \frac{4c\left(51a^2 + \frac{bc(11bc-42a}{d^2}\right.}{19}$$

$$= \frac{8c^2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{9945d} + \frac{4c\left(51a^2 + \frac{bc(11bc-42a}{d^2}\right.}{19}$$

$$= \frac{8c^2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{9945d} + \frac{4c\left(51a^2 + \frac{bc(11bc-42a}{d^2}\right.}{19}$$

$$= \frac{8c^2\left(51a^2 + \frac{bc(11bc-42ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{9945d} + \frac{4c\left(51a^2 + \frac{bc(11bc-42a}{d^2}\right.}{19}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.16, size = 210, normalized size = 0.40

$$\frac{2e(ex)^{3/2}\left(\left(c+dx^2\right)\left(357a^2d^2(4c^2+25cdx^2+15d^2x^4)+42abd(-28c^3+20c^2dx^2+285cd^2x^4+195d^3x^6)+b^2(308c^4-220c^3dx^2+180c^2d^2x^4+4485cd^3x^6+3315d^4x^8)\right)-84c^3(11b^2c^2-42abcd+51a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{69615d^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (2*e*(e*x)^(3/2)*((c + d*x^2)*(357*a^2*d^2*(4*c^2 + 25*c*d*x^2 + 15*d^2*x^4
) + 42*a*b*d*(-28*c^3 + 20*c^2*d*x^2 + 285*c*d^2*x^4 + 195*d^3*x^6) + b^2*(
```

```
308*c^4 - 220*c^3*d*x^2 + 180*c^2*d^2*x^4 + 4485*c*d^3*x^6 + 3315*d^4*x^8))
 - 84*c^3*(11*b^2*c^2 - 42*a*b*c*d + 51*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(69615*d^3*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.12, size = 743, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/69615*e^2/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^4*(-3315*b^2*d^6*x^12-8190*a*b
*d^6*x^10-7800*b^2*c*d^5*x^10-5355*a^2*d^6*x^8-20160*a*b*c*d^5*x^8-4665*b^2
*c^2*d^4*x^8-14280*a^2*c*d^5*x^6-12810*a*b*c^2*d^4*x^6+40*b^2*c^3*d^3*x^6+4
284*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^4*d^2-3528*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^5*d+924*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^6-2142*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*a^2*c^4*d^2+1764*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^5*d-462*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*b^2*c^6-10353*a^2*c^2*d^4*x^4+336*a*b*c^3*d^3*x^4-88*
b^2*c^4*d^2*x^4-1428*a^2*c^3*d^3*x^2+1176*a*b*c^4*d^2*x^2-308*b^2*c^5*d*x^2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 210, normalized size = 0.40

$$\frac{2\left(84\,(11\,b^2c^5-42\,abc^4d+51\,a^2c^3d^2)\sqrt{d}\,e^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)+(3315\,b^2d^5x^9+195\,(23\,b^2cd^4+42\,abd^5)x^7+45\,(4\,b^2c^2d^3+266\,abcd^4+119\,a^2d^5)x^5-5\,(44\,b^2c^3d^2-168\,abc^2d^3-1785\,a^2cd^4)x^3+28\,(11\,b^2c^4d-42\,abc^3d^2+51\,a^2c^2d^3)x)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{5}{2}}\right)}{69615\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(84*(11*b^2*c^5 - 42*a*b*c^4*d + 51*a^2*c^3*d^2)*sqrt(d)*e^(5/2)*we
ierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (3315*b^2*d^5
*x^9 + 195*(23*b^2*c*d^4 + 42*a*b*d^5)*x^7 + 45*(4*b^2*c^2*d^3 + 266*a*b*c*
d^4 + 119*a^2*d^5)*x^5 - 5*(44*b^2*c^3*d^2 - 168*a*b*c^2*d^3 - 1785*a^2*c*d
^4)*x^3 + 28*(11*b^2*c^4*d - 42*a*b*c^3*d^2 + 51*a^2*c^2*d^3)*x)*sqrt(d*x^2
 + c)*sqrt(x)*e^(5/2))/d^4
```

Sympy [C] Result contains complex when optimal does not.
time = 99.08, size = 306, normalized size = 0.58

$$\frac{a^2 c^{\frac{3}{2}} e^{\frac{5}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{d x^2 e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{11}{4}\right)} + \frac{a^2 \sqrt{c}\, d e^{\frac{5}{2}} x^{\frac{11}{2}} \Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{4} \\ \frac{15}{4} \end{matrix}\middle| \frac{d x^2 e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{15}{4}\right)} + \frac{a b c^{\frac{3}{2}} e^{\frac{5}{2}} x^{\frac{11}{2}} \Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{4} \\ \frac{15}{4} \end{matrix}\middle| \frac{d x^2 e^{i\pi}}{c}\right)}{\Gamma\left(\frac{15}{4}\right)} + \frac{a b \sqrt{c}\, d e^{\frac{5}{2}} x^{\frac{15}{2}} \Gamma\left(\frac{15}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{15}{4} \\ \frac{19}{4} \end{matrix}\middle| \frac{d x^2 e^{i\pi}}{c}\right)}{\Gamma\left(\frac{19}{4}\right)} + \frac{b^2 c^{\frac{3}{2}} e^{\frac{5}{2}} x^{\frac{15}{2}} \Gamma\left(\frac{15}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{15}{4} \\ \frac{19}{4} \end{matrix}\middle| \frac{d x^2 e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{19}{4}\right)} + \frac{b^2 \sqrt{c}\, d e^{\frac{5}{2}} x^{\frac{19}{2}} \Gamma\left(\frac{19}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{19}{4} \\ \frac{23}{4} \end{matrix}\middle| \frac{d x^2 e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{23}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x*
*2*exp_polar(I*pi)/c)/(2*gamma(11/4)) + a**2*sqrt(c)*d*e**(5/2)*x**(11/2)*g
amma(11/4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(2*gamma(
15/4)) + a*b*c**(3/2)*e**(5/2)*x**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (1
5/4,), d*x**2*exp_polar(I*pi)/c)/gamma(15/4) + a*b*sqrt(c)*d*e**(5/2)*x**(1
5/2)*gamma(15/4)*hyper((-1/2, 15/4), (19/4,), d*x**2*exp_polar(I*pi)/c)/gam
ma(19/4) + b**2*c**(3/2)*e**(5/2)*x**(15/2)*gamma(15/4)*hyper((-1/2, 15/4),
 (19/4,), d*x**2*exp_polar(I*pi)/c)/(2*gamma(19/4)) + b**2*sqrt(c)*d*e**(5/
2)*x**(19/2)*gamma(19/4)*hyper((-1/2, 19/4), (23/4,), d*x**2*exp_polar(I*pi
)/c)/(2*gamma(23/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,x)^{5/2}\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] int((e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.833 $\int (ex)^{3/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx$

Optimal. Leaf size=340

$$\frac{8c^2(57a^2d^2+bc(9bc-38ad))e\sqrt{ex}\sqrt{c+dx^2}}{4389d^3}+\frac{4c(57a^2d^2+bc(9bc-38ad))(ex)^{5/2}\sqrt{c+dx^2}}{1463d^2e}+\frac{2(57a^2d^2+$$

```
[Out] 2/627*(57*a^2*d^2+b*c*(-38*a*d+9*b*c))*(e*x)^(5/2)*(d*x^2+c)^(3/2)/d^2/e-2/
285*b*(-38*a*d+9*b*c)*(e*x)^(5/2)*(d*x^2+c)^(5/2)/d^2/e+2/19*b^2*(e*x)^(9/2
)*(d*x^2+c)^(5/2)/d/e^3+4/1463*c*(57*a^2*d^2+b*c*(-38*a*d+9*b*c))*(e*x)^(5/
2)*(d*x^2+c)^(1/2)/d^2/e+8/4389*c^2*(57*a^2*d^2+b*c*(-38*a*d+9*b*c))*e*(e*x
)^(1/2)*(d*x^2+c)^(1/2)/d^3-4/4389*c^(11/4)*(57*a^2*d^2+b*c*(-38*a*d+9*b*c)
)*e^(3/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c
)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(13/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {475, 470, 285, 327, 335, 226}

$$-\frac{4c^{11/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}(57a^2d^2+bc(9bc-38ad))F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{4389d^{13/4}\sqrt{c+dx^2}}+\frac{8c^2e\sqrt{ex}\sqrt{c+dx^2}(57a^2d^2+bc(9bc-38ad))}{4389d^3}+\frac{2(ex)^{5/2}(c+dx^2)^{3/2}(57a^2d^2+bc(9bc-38ad))}{627d^2e}+\frac{4c(ex)^{5/2}\sqrt{c+dx^2}(57a^2d^2+bc(9bc-38ad))}{1463d^2e}-\frac{2b(ex)^{5/2}(c+dx^2)^{5/2}(9bc-38ad)}{285d^2e}+\frac{2b^2(ex)^{9/2}(c+dx^2)^{5/2}}{19de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (8*c^2*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(43
89*d^3) + (4*c*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*(e*x)^(5/2)*Sqrt[c + d*x
^2])/(1463*d^2*e) + (2*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*(e*x)^(5/2)*(c +
 d*x^2)^(3/2))/(627*d^2*e) - (2*b*(9*b*c - 38*a*d)*(e*x)^(5/2)*(c + d*x^2)^
(5/2))/(285*d^2*e) + (2*b^2*(e*x)^(9/2)*(c + d*x^2)^(5/2))/(19*d*e^3) - (4*
c^(11/4)*(57*a^2*d^2 + b*c*(9*b*c - 38*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*
Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(4389*d^(13/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx = \frac{2b^2(ex)^{9/2} \left(c+dx^2\right)^{5/2}}{19de^3} + \frac{2\int (ex)^{3/2} \left(c+dx^2\right)^{3/2} \left(\frac{19a^2d}{2} - \frac{1}{2}b(9bc-38\right.}{19d}$$

$$= -\frac{2b(9bc-38ad)(ex)^{5/2} \left(c+dx^2\right)^{5/2}}{285d^2e} + \frac{2b^2(ex)^{9/2} \left(c+dx^2\right)^{5/2}}{19de^3} + \frac{1}{57}\Big($$

$$= \frac{2\left(57a^2 + \frac{bc(9bc-38ad)}{d^2}\right)(ex)^{5/2} \left(c+dx^2\right)^{3/2}}{627e} - \frac{2b(9bc-38ad)(ex)^{5/2} (c}{285d^2e}$$

$$= \frac{4c\left(57a^2 + \frac{bc(9bc-38ad)}{d^2}\right)(ex)^{5/2} \sqrt{c+dx^2}}{1463e} + \frac{2\left(57a^2 + \frac{bc(9bc-38ad)}{d^2}\right)(e}{627e}$$

$$= \frac{8c^2\left(57a^2 + \frac{bc(9bc-38ad)}{d^2}\right) e\sqrt{ex}\sqrt{c+dx^2}}{4389d} + \frac{4c\left(57a^2 + \frac{bc(9bc-38ad)}{d^2}\right)(}{1463e}$$

$$= \frac{8c^2\left(57a^2 + \frac{bc(9bc-38ad)}{d^2}\right) e\sqrt{ex}\sqrt{c+dx^2}}{4389d} + \frac{4c\left(57a^2 + \frac{bc(9bc-38ad)}{d^2}\right)(}{1463e}$$

$$= \frac{8c^2\left(57a^2 + \frac{bc(9bc-38ad)}{d^2}\right) e\sqrt{ex}\sqrt{c+dx^2}}{4389d} + \frac{4c\left(57a^2 + \frac{bc(9bc-38ad)}{d^2}\right)(}{1463e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.21, size = 259, normalized size = 0.76

$$\frac{(ex)^{3/2} \left(\frac{2\sqrt{x}\,(c+dx^2)\left(285a^2d^2(4c^2+13cdx^2+7d^2x^4)+38abd(-20c^3+12c^2dx^2+119cd^2x^4+77d^3x^6)+3b^2(60c^4-36c^3dx^2+28c^2d^2x^4+539cd^3x^6+385d^4x^8)\right)}{5d^3} - \frac{8ic^3(9b^2c^2-38abcd+57a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^3}\right)}{4389x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(285*a^2*d^2*(4*c^2 + 13*c*d*x^2 + 7*d
^2*x^4) + 38*a*b*d*(-20*c^3 + 12*c^2*d*x^2 + 119*c*d^2*x^4 + 77*d^3*x^6) +
3*b^2*(60*c^4 - 36*c^3*d*x^2 + 28*c^2*d^2*x^4 + 539*c*d^3*x^6 + 385*d^4*x^8
)))/(5*d^3) - ((8*I)*c^3*(9*b^2*c^2 - 38*a*b*c*d + 57*a^2*d^2)*Sqrt[1 + c/(
d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqr
t[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(4389*x^(3/2)*Sqrt[c + d*x^2])
```

Maple [A]

time = 0.11, size = 489, normalized size = 1.44

method	result
risch	$\frac{2(1155b^2d^4x^8+2926ab\,d^4x^6+1617b^2c\,d^3x^6+1995a^2d^4x^4+4522abc\,d^3x^4+84b^2c^2d^2x^4+3705a^2c\,d^3x^2+456ab\,c^2d^2x^2-108b^2c^3d\,x^2+1}{21945d^3\sqrt{ex}}$
default	$-\frac{2e\sqrt{ex}\left(-1155b^2d^6x^{11}-2926ab\,d^6x^9-2772b^2c\,d^5x^9-1995a^2d^6x^7-7448abc\,d^5x^7-1701b^2c^2d^4x^7+570\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\right.}{}$
elliptic	$\frac{\sqrt{ex\left(d\,x^2+c\right)}\,\sqrt{ex}\left(\frac{2b^2de\,x^8\sqrt{de\,x^3+cex}}{19}+\frac{2\left(2bd(ad+bc)e^2-\frac{17b^2d\,e^2c}{19}\right)x^6\sqrt{de\,x^3+cex}}{15de}+\frac{2\left(\left(a^2d^2+4abcd+b^2\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21945*e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-1155*b^2*d^6*x^11-2926*a*b*d^6*x
^9-2772*b^2*c*d^5*x^9-1995*a^2*d^6*x^7-7448*a*b*c*d^5*x^7-1701*b^2*c^2*d^4*
x^7+570*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c^3*d^2-380*((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^4*d+90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c
```

```
*d)^(1/2)*b^2*c^5-5700*a^2*c*d^5*x^5-4978*a*b*c^2*d^4*x^5+24*b^2*c^3*d^3*x^
5-4845*a^2*c^2*d^4*x^3+304*a*b*c^3*d^3*x^3-72*b^2*c^4*d^2*x^3-1140*a^2*c^3*
d^3*x+760*a*b*c^4*d^2*x-180*b^2*c^5*d*x)/d^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 199, normalized size = 0.59

$$-\frac{2\left(20\left(9b^2c^5-38abc^4d+57a^2c^3d^2\right)\sqrt{d}\,e^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-\left(1155b^2d^5x^8+180b^2c^4d-760abc^3d^2+1140a^2c^2d^3+77\left(21b^2cd^4+38abd^5\right)x^6+7\left(12b^2c^2d^3+646abcd^4+285a^2d^5\right)x^4-3\left(36b^2c^3d^2-152abc^2d^3-1235a^2cd^4\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{3}{2}}\right)}{21945\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/21945*(20*(9*b^2*c^5 - 38*a*b*c^4*d + 57*a^2*c^3*d^2)*sqrt(d)*e^(3/2)*we
ierstrassPInverse(-4*c/d, 0, x) - (1155*b^2*d^5*x^8 + 180*b^2*c^4*d - 760*a
*b*c^3*d^2 + 1140*a^2*c^2*d^3 + 77*(21*b^2*c*d^4 + 38*a*b*d^5)*x^6 + 7*(12*
b^2*c^2*d^3 + 646*a*b*c*d^4 + 285*a^2*d^5)*x^4 - 3*(36*b^2*c^3*d^2 - 152*a*
b*c^2*d^3 - 1235*a^2*c*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(x)*e^(3/2))/d^4
```

Sympy [C] Result contains complex when optimal does not.
time = 32.52, size = 306, normalized size = 0.90

$$\frac{a^2c^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{a^2\sqrt{c}\,de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{13}{4}\right)}+\frac{abc^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{13}{4}\right)}+\frac{ab\sqrt{c}\,de^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{4}\\\frac{17}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{17}{4}\right)}+\frac{b^2c^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{4}\\\frac{17}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{17}{4}\right)}+\frac{b^2\sqrt{c}\,de^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{17}{4}\\\frac{21}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{21}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**
2*exp_polar(I*pi)/c)/(2*gamma(9/4)) + a**2*sqrt(c)*d*e**(3/2)*x**(9/2)*gamm
a(9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(2*gamma(13/4)
) + a*b*c**(3/2)*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), d
*x**2*exp_polar(I*pi)/c)/gamma(13/4) + a*b*sqrt(c)*d*e**(3/2)*x**(13/2)*gam
ma(13/4)*hyper((-1/2, 13/4), (17/4,), d*x**2*exp_polar(I*pi)/c)/gamma(17/4)
 + b**2*c**(3/2)*e**(3/2)*x**(13/2)*gamma(13/4)*hyper((-1/2, 13/4), (17/4,)
, d*x**2*exp_polar(I*pi)/c)/(2*gamma(17/4)) + b**2*sqrt(c)*d*e**(3/2)*x**(1
7/2)*gamma(17/4)*hyper((-1/2, 17/4), (21/4,), d*x**2*exp_polar(I*pi)/c)/(2*
gamma(21/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,x)^{3/2}\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
[Out] int((e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.834 $\int \sqrt{ex} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=482

$$\frac{4c(221a^2d^2+3bc(7bc-34ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{3315d^2e}+\frac{8c^2(221a^2d^2+3bc(7bc-34ad))\,\sqrt{ex}\,\sqrt{c+dx^2}}{3315d^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{2(221a^2$$

```
[Out] 2/1989*(221*a^2*d^2+3*b*c*(-34*a*d+7*b*c))*(e*x)^(3/2)*(d*x^2+c)^(3/2)/d^2/
e-2/221*b*(-34*a*d+7*b*c)*(e*x)^(3/2)*(d*x^2+c)^(5/2)/d^2/e+2/17*b^2*(e*x)^
(7/2)*(d*x^2+c)^(5/2)/d/e^3+4/3315*c*(221*a^2*d^2+3*b*c*(-34*a*d+7*b*c))*(e
*x)^(3/2)*(d*x^2+c)^(1/2)/d^2/e+8/3315*c^2*(221*a^2*d^2+3*b*c*(-34*a*d+7*b*
c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/d^(5/2)/(c^(1/2)+x*d^(1/2))-8/3315*c^(9/4)*
(221*a^2*d^2+3*b*c*(-34*a*d+7*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*El
lipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^
(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(11/4)/(
d*x^2+c)^(1/2)+4/3315*c^(9/4)*(221*a^2*d^2+3*b*c*(-34*a*d+7*b*c))*(cos(2*ar
ctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1
/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+
x*d^(1/2))^2)^(1/2)/d^(11/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {475, 470, 285, 335, 311, 226, 1210}

$$\frac{4c^{9/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(221a^2d^2+3bc(7bc-34ad))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{11/4}\sqrt{c+dx^2}}-\frac{8c^{9/4}\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(221a^2d^2+3bc(7bc-34ad))\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{11/4}\sqrt{c+dx^2}}+\frac{8c^2\sqrt{ex}\,\sqrt{c+dx^2}\,(221a^2d^2+3bc(7bc-34ad))}{3315d^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}(221a^2d^2+3bc(7bc-34ad))}{1989d^2e}+\frac{4c(ex)^{3/2}\sqrt{c+dx^2}\,(221a^2d^2+3bc(7bc-34ad))}{3315d^2e}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{5/2}(7bc-34ad)}{221d^2e}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (4*c*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(3
315*d^2*e) + (8*c^2*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d))*Sqrt[e*x]*Sqrt[c
 + d*x^2])/(3315*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) + (2*(221*a^2*d^2 + 3*b*c*(
7*b*c - 34*a*d))*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(1989*d^2*e) - (2*b*(7*b*c
- 34*a*d)*(e*x)^(3/2)*(c + d*x^2)^(5/2))/(221*d^2*e) + (2*b^2*(e*x)^(7/2)*(
c + d*x^2)^(5/2))/(17*d*e^3) - (8*c^(9/4)*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*
a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d
^(11/4)*Sqrt[c + d*x^2]) + (4*c^(9/4)*(221*a^2*d^2 + 3*b*c*(7*b*c - 34*a*d)
)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*E
```

```
llipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3315*d^(11
/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx = \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3} + \frac{2\int \sqrt{ex}\left(c+dx^2\right)^{3/2}\left(\frac{17a^2d}{2}-\frac{1}{2}b(7bc-34ad\right.}{17d}$$

$$= -\frac{2b(7bc-34ad)(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{221d^2e} + \frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3} - \frac{1}{221}\Big($$

$$= \frac{2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{1989e} - \frac{2b(7bc-34ad)(ex)^{3/2}\,(}{221d^2e}$$

$$= \frac{4c\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{3315e} + \frac{2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)}{1989e}$$

$$= \frac{4c\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{3315e} + \frac{2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)}{1989e}$$

$$= \frac{4c\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{3315e} + \frac{2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)}{1989e}$$

$$= \frac{4c\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right)(ex)^{3/2}\sqrt{c+dx^2}}{3315e} + \frac{8c^2\left(221a^2+\frac{3bc(7bc-34ad)}{d^2}\right.}{3315\sqrt{d}\,\left(\sqrt{c}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 20.13, size = 179, normalized size = 0.37

$$\frac{2\sqrt{ex}\left(-x\left(c+dx^2\right)\left(-221a^2d^2\left(11c+5dx^2\right)-102abd\left(4c^2+25cdx^2+15d^2x^4\right)+b^2\left(84c^3-60c^2dx^2-855cd^2x^4-585d^3x^6\right)\right)+12c^2\left(21b^2c^2-102abcd+221a^2d^2\right)\sqrt{1+\frac{c}{dx^2}}\,x\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{9945d^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[e*x]*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]

[Out] (2*Sqrt[e*x]*(-(x*(c + d*x^2)*(-221*a^2*d^2*(11*c + 5*d*x^2) - 102*a*b*d*(4*c^2 + 25*c*d*x^2 + 15*d^2*x^4) + b^2*(84*c^3 - 60*c^2*d*x^2 - 855*c*d^2*x^4 - 585*d^3*x^6))) + 12*c^2*(21*b^2*c^2 - 102*a*b*c*d + 221*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(9945*d^2*Sqrt[c + d*x^2])

Maple [A]

time = 0.11, size = 699, normalized size = 1.45

method	result
risch	$\frac{2x^2(585b^2x^6d^3+1530ab\,d^3x^4+855b^2c\,d^2x^4+1105a^2d^3x^2+2550abc\,d^2x^2+60b^2c^2d\,x^2+2431a^2c\,d^2+408ab\,c^2d-84b^2c^3)\sqrt{d\,x^2+c}}{9945d^2\sqrt{ex}}$
elliptic	$\sqrt{ex\,(d\,x^2+c)}\,\sqrt{ex}\,\Bigg(\frac{2b^2d\,x^7\sqrt{de\,x^3+cex}}{17}+\frac{2\left(2bd(ad+bc)e-\frac{15b^2dce}{17}\right)x^5\sqrt{de\,x^3+cex}}{13de}+\frac{2\left(\left(a^2d^2+4abcd+b^2c^2\right)\right.}{}$
default	$2\sqrt{ex}\,\Bigg(585b^2d^5x^{10}+1530ab\,d^5x^8+1440b^2c\,d^4x^8+1105a^2d^5x^6+4080abc\,d^4x^6+915b^2c^2d^3x^6+2652\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9945/(d*x^2+c)^(1/2)*(e*x)^(1/2)/d^3*(585*b^2*d^5*x^10+1530*a*b*d^5*x^8+1
440*b^2*c*d^4*x^8+1105*a^2*d^5*x^6+4080*a*b*c*d^4*x^6+915*b^2*c^2*d^3*x^6+2
652*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^3*d^2-1224*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^4*d+252*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^5-1326*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*a^2*c^3*d^2+612*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^4*d-126*((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*b^2*c^5+3536*a^2*c*d^4*x^4+2958*a*b*c^2*d^3*x^4-24*b^2
*c^3*d^2*x^4+2431*a^2*c^2*d^3*x^2+408*a*b*c^3*d^2*x^2-84*b^2*c^4*d*x^2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 173, normalized size = 0.36

$$-\frac{2\left(12\left(21\,b^{2}c^{4}-102\,abc^{3}d+221\,a^{2}c^{2}d^{2}\right)\sqrt{d}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)-\left(585\,b^{2}d^{4}x^{7}+45\left(19\,b^{2}cd^{3}+34\,abd^{4}\right)x^{5}+5\left(12\,b^{2}c^{2}d^{2}+510\,abcd^{3}+221\,a^{2}d^{4}\right)x^{3}-\left(84\,b^{2}c^{3}d-408\,abc^{2}d^{2}-2431\,a^{2}cd^{3}\right)x\right)\sqrt{dx^{2}+c}\,\sqrt{x}\,e^{\frac{1}{2}}\right)}{9945\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9945*(12*(21*b^2*c^4 - 102*a*b*c^3*d + 221*a^2*c^2*d^2)*sqrt(d)*e^(1/2)*
weierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (585*b^2*d^
4*x^7 + 45*(19*b^2*c*d^3 + 34*a*b*d^4)*x^5 + 5*(12*b^2*c^2*d^2 + 510*a*b*c*
d^3 + 221*a^2*d^4)*x^3 - (84*b^2*c^3*d - 408*a*b*c^2*d^2 - 2431*a^2*c*d^3)*
x)*sqrt(d*x^2 + c)*sqrt(x)*e^(1/2))/d^3
```

Sympy [C] Result contains complex when optimal does not.
time = 7.09, size = 304, normalized size = 0.63

$$\frac{a^2c^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e\Gamma\left(\frac{7}{4}\right)}+\frac{a^2\sqrt{c}\,d(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^3\Gamma\left(\frac{11}{4}\right)}+\frac{abc^{\frac{3}{2}}(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^3\Gamma\left(\frac{11}{4}\right)}+\frac{ab\sqrt{c}\,d(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^5\Gamma\left(\frac{15}{4}\right)}+\frac{b^2c^{\frac{3}{2}}(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^5\Gamma\left(\frac{15}{4}\right)}+\frac{b^2\sqrt{c}\,d(ex)^{\frac{15}{2}}\,\Gamma\left(\frac{15}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{15}{4}\\\frac{19}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^7\Gamma\left(\frac{19}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)*(e*x)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*(e*x)**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp
_polar(I*pi)/c)/(2*e*gamma(7/4)) + a**2*sqrt(c)*d*(e*x)**(7/2)*gamma(7/4)*h
yper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**3*gamma(11/4)) +
 a*b*c**(3/2)*(e*x)**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*ex
p_polar(I*pi)/c)/(e**3*gamma(11/4)) + a*b*sqrt(c)*d*(e*x)**(11/2)*gamma(11/
4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(e**5*gamma(15/4)
) + b**2*c**(3/2)*(e*x)**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), d*
x**2*exp_polar(I*pi)/c)/(2*e**5*gamma(15/4)) + b**2*sqrt(c)*d*(e*x)**(15/2)
*gamma(15/4)*hyper((-1/2, 15/4), (19/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**7
*gamma(19/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(x)*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\,x}\,\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] int((e*x)^(1/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.835 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{\sqrt{ex}} \, dx$

Optimal. Leaf size=286

$$\frac{4c(33a^2d^2+bc(bc-6ad))\sqrt{ex}\sqrt{c+dx^2}}{231d^2e}+\frac{2(33a^2d^2+bc(bc-6ad))\sqrt{ex}\,(c+dx^2)^{3/2}}{231d^2e}-\frac{2b(bc-6ad)\sqrt{ex}\,(}{33d^2e}$$

```
[Out] 2/15*b^2*(e*x)^(5/2)*(d*x^2+c)^(5/2)/d/e^3+2/231*(33*a^2*d^2+b*c*(-6*a*d+b*
c))*(d*x^2+c)^(3/2)*(e*x)^(1/2)/d^2/e-2/33*b*(-6*a*d+b*c)*(d*x^2+c)^(5/2)*(
e*x)^(1/2)/d^2/e+4/231*c*(33*a^2*d^2+b*c*(-6*a*d+b*c))*(e*x)^(1/2)*(d*x^2+c
)^(1/2)/d^2/e+4/231*c^(7/4)*(33*a^2*d^2+b*c*(-6*a*d+b*c))*(cos(2*arctan(d^(
1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2
)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1
/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1
/2)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {475, 470, 285, 335, 226}

$$\frac{4c^{7/4}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(33a^2d^2+bc(bc-6ad))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\,(c+dx^2)^{3/2}(33a^2d^2+bc(bc-6ad))}{231d^2e}+\frac{4c\sqrt{ex}\sqrt{c+dx^2}\,(33a^2d^2+bc(bc-6ad))}{231d^2e}-\frac{2b\sqrt{ex}\,(c+dx^2)^{5/2}(bc-6ad)}{33d^2e}+\frac{2b^2(ex)^{5/2}(c+dx^2)^{5/2}}{15de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/Sqrt[e*x],x]
```

```
[Out] (4*c*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d^2*e
) + (2*(33*a^2*d^2 + b*c*(b*c - 6*a*d))*Sqrt[e*x]*(c + d*x^2)^(3/2))/(231*d
^2*e) - (2*b*(b*c - 6*a*d)*Sqrt[e*x]*(c + d*x^2)^(5/2))/(33*d^2*e) + (2*b^2
*(e*x)^(5/2)*(c + d*x^2)^(5/2))/(15*d*e^3) + (4*c^(7/4)*(33*a^2*d^2 + b*c*(
b*c - 6*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^
2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^
(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{\sqrt{ex}}\,dx = \frac{2b^2(ex)^{5/2}(c+dx^2)^{5/2}}{15de^3} + \frac{2\int \frac{(c+dx^2)^{3/2}\left(\frac{15a^2d}{2}-\frac{5}{2}b(bc-6ad)x^2\right)}{\sqrt{ex}}\,dx}{15d}$$

$$= -\frac{2b(bc-6ad)\sqrt{ex}\,(c+dx^2)^{5/2}}{33d^2e} + \frac{2b^2(ex)^{5/2}(c+dx^2)^{5/2}}{15de^3} + \frac{1}{33}\Bigg(33a^2 + \frac{bc(b}{}$$

$$= \frac{2\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\,(c+dx^2)^{3/2}}{231e} - \frac{2b(bc-6ad)\sqrt{ex}\,(c+dx^2)^{5/2}}{33d^2e} + \frac{2b}{}$$

$$= \frac{4c\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\,\sqrt{c+dx^2}}{231e} + \frac{2\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\,(c+dx^2)^{3}}{231e}$$

$$= \frac{4c\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\,\sqrt{c+dx^2}}{231e} + \frac{2\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\,(c+dx^2)^{3}}{231e}$$

$$= \frac{4c\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\,\sqrt{c+dx^2}}{231e} + \frac{2\left(33a^2+\frac{bc(bc-6ad)}{d^2}\right)\sqrt{ex}\,(c+dx^2)^{3}}{231e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.19, size = 223, normalized size = 0.78

$$\frac{\sqrt{x}\left(\frac{2\sqrt{x}\,(c+dx^2)\left(165a^2d^2(3c+dx^2)+30abd(4c^2+13cdx^2+7d^2x^4)+b^2(-20c^3+12c^2dx^2+119cd^2x^4+77d^3x^6)\right)}{5d^2} + \frac{8ic^2(b^2c^2-6abcd+33a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^2}\right)}{231\sqrt{ex}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/Sqrt[e*x],x]
```

```
[Out] (Sqrt[x]*((2*Sqrt[x]*(c + d*x^2)*(165*a^2*d^2*(3*c + d*x^2) + 30*a*b*d*(4*c
^2 + 13*c*d*x^2 + 7*d^2*x^4) + b^2*(-20*c^3 + 12*c^2*d*x^2 + 119*c*d^2*x^4
+ 77*d^3*x^6)))/(5*d^2) + ((8*I)*c^2*(b^2*c^2 - 6*a*b*c*d + 33*a^2*d^2)*Sqr
t[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]],
-1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^2)))/(231*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.11, size = 444, normalized size = 1.55

method	result
risch	$\frac{2(77b^2x^6d^3+210ab\,d^3x^4+119b^2c\,d^2x^4+165a^2d^3x^2+390abc\,d^2x^2+12b^2c^2d\,x^2+495a^2c\,d^2+120ab\,c^2d-20b^2c^3)x\sqrt{d\,x^2+c}}{1155d^2\sqrt{ex}} + -$ 4
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\left(\frac{2b^2d\,x^6\sqrt{de\,x^3+cex}}{15e}+\frac{2\left(2ab\,d^2+\frac{17}{15}b^2cd\right)x^4\sqrt{de\,x^3+cex}}{11de}+\frac{2\left(a^2d^2+4abcd+b^2c^2-\frac{9c\left(2ab\,d^2+\frac{17}{15}b^2\right.}{11d}\right.}{7de}\right.}{}$
default	$\frac{\frac{2b^2d^5x^9}{15}+\frac{4ab\,d^5x^7}{11}+\frac{56b^2c\,d^4x^7}{165}+\frac{4\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\operatorname{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{}}{}\right.}{7}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/(d*x^2+c)^(1/2)*(77*b^2*d^5*x^9+210*a*b*d^5*x^7+196*b^2*c*d^4*x^7+33
0*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c^2*d^2-60*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*(-c*d)^(1/2)*a*b*c^3*d+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/
2)*b^2*c^4+165*a^2*d^5*x^5+600*a*b*c*d^4*x^5+131*b^2*c^2*d^3*x^5+660*a^2*c*
d^4*x^3+510*a*b*c^2*d^3*x^3-8*b^2*c^3*d^2*x^3+495*a^2*c^2*d^3*x+120*a*b*c^3
*d^2*x-20*b^2*c^4*d*x)/d^3/(e*x)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 157, normalized size = 0.55

$$\frac{2\left(20\left(b^{2}c^{4}-6\,abc^{3}d+33\,a^{2}c^{2}d^{2}\right)\sqrt{d}\ \text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)+\left(77\,b^{2}d^{4}x^{6}-20\,b^{2}c^{3}d+120\,abc^{2}d^{2}+495\,a^{2}cd^{3}+7\left(17\,b^{2}cd^{3}+30\,abd^{4}\right)x^{4}+3\left(4\,b^{2}c^{2}d^{2}+130\,abcd^{3}+55\,a^{2}d^{4}\right)x^{2}\right)\sqrt{dx^{2}+c}\ \sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{1155\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(20*(b^2*c^4 - 6*a*b*c^3*d + 33*a^2*c^2*d^2)*sqrt(d)*weierstrassPInv
erse(-4*c/d, 0, x) + (77*b^2*d^4*x^6 - 20*b^2*c^3*d + 120*a*b*c^2*d^2 + 495
*a^2*c*d^3 + 7*(17*b^2*c*d^3 + 30*a*b*d^4)*x^4 + 3*(4*b^2*c^2*d^2 + 130*a*b
*c*d^3 + 55*a^2*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-1/2)/d^3
```

Sympy [C] Result contains complex when optimal does not.
time = 9.83, size = 306, normalized size = 1.07

$$\frac{a^{2}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{a^{2}\sqrt{c}\,dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}+\frac{abc^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}+\frac{ab\sqrt{c}\,dx^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)}+\frac{b^{2}c^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)}+\frac{b^{2}\sqrt{c}\,dx^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_pola
r(I*pi)/c)/(2*sqrt(e)*gamma(5/4)) + a**2*sqrt(c)*d*x**(5/2)*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(e)*gamma(9/4)) + a
*b*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_polar
(I*pi)/c)/(sqrt(e)*gamma(9/4)) + a*b*sqrt(c)*d*x**(9/2)*gamma(9/4)*hyper((-
1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(e)*gamma(13/4)) + b**2*
c**(3/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I
*pi)/c)/(2*sqrt(e)*gamma(13/4)) + b**2*sqrt(c)*d*x**(13/2)*gamma(13/4)*hype
r((-1/2, 13/4), (17/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(e)*gamma(17/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*e^(-1/2)/sqrt(x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(1/2),x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(1/2), x)
```

3.836 $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{3/2}} \, dx$

Optimal. Leaf size=476

$$-\frac{4(3b^2c^2-13ad(2bc+9ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{195de^3}-\frac{8c(3b^2c^2-13ad(2bc+9ad))\,\sqrt{ex}\,\sqrt{c+dx^2}}{195d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2(3b^2c^2-13$$

```
[Out] -2/117*(3*b^2*c^2-13*a*d*(9*a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(3/2)/c/d/e^3
+2/13*b^2*(e*x)^(3/2)*(d*x^2+c)^(5/2)/d/e^3-2*a^2*(d*x^2+c)^(5/2)/c/e/(e*x)
^(1/2)-4/195*(3*b^2*c^2-13*a*d*(9*a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(1/2)/d
/e^3-8/195*c*(3*b^2*c^2-13*a*d*(9*a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/d
^(3/2)/e^2/(c^(1/2)+x*d^(1/2))+8/195*c^(5/4)*(3*b^2*c^2-13*a*d*(9*a*d+2*b*c
))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arcta
n(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1
/2)+x*d^(1/2))^2)^(1/2)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)-4/195*c^(5/4)*(3*b^
2*c^2-13*a*d*(9*a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/
2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(
sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*
d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(7/4)/e^(3/2)/(d*x^2+c)^
(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 470, 285, 335, 311, 226, 1210}

$$-\frac{2a^2(c+dx^2)^{5/2}}{ce\sqrt{ex}}-\frac{4c^{5/4}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(3b^2c^2-13ad(9ad+2bc))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{8c^{5/4}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(3b^2c^2-13ad(9ad+2bc))\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{8c\sqrt{ex}\,\sqrt{c+dx^2}\,(3b^2c^2-13ad(9ad+2bc))}{195d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2(ex)^{3/2}\,(c+dx^2)^{3/2}\,(3b^2c^2-13ad(9ad+2bc))}{117cde^3}-\frac{4(ex)^{3/2}\sqrt{c+dx^2}\,(3b^2c^2-13ad(9ad+2bc))}{195de^3}+\frac{2b^2(ex)^{3/2}\,(c+dx^2)^{5/2}}{13de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x]
```

```
[Out] (-4*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(195*
d*e^3) - (8*c*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2
])/(195*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) - (2*(3*b^2*c^2 - 13*a*d*(2*b*c
+ 9*a*d))*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(117*c*d*e^3) - (2*a^2*(c + d*x^2)
^(5/2))/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*(c + d*x^2)^(5/2))/(13*d*e^3)
+ (8*c^(5/4)*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqr
t[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - (4*c^(
5/4)*(3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
```

```
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(195*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx = -\frac{2a^2(c+dx^2)^{5/2}}{ce\sqrt{ex}} + \frac{2\int \sqrt{ex}\left(\frac{1}{2}a(2bc+9ad)+\frac{1}{2}b^2cx^2\right)(c+dx^2)^{3/2}\,dx}{ce^2}$$

$$= -\frac{2a^2(c+dx^2)^{5/2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}(c+dx^2)^{5/2}}{13de^3} - \frac{\left(4\left(\frac{3b^2c^2}{4}-\frac{13}{4}ad(2bc+9ad)\right)\right)}{13cde^2}$$

$$= -\frac{2(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}(c+dx^2)^{3/2}}{117cde^3} - \frac{2a^2(c+dx^2)^{5/2}}{ce\sqrt{ex}} + \frac{2b^2(e}{}$$

$$= -\frac{4(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}\sqrt{c+dx^2}}{195de^3} - \frac{2(3b^2c^2-13ad(2bc+9ad)}{117cde}$$

$$= -\frac{4(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}\sqrt{c+dx^2}}{195de^3} - \frac{2(3b^2c^2-13ad(2bc+9ad)}{117cde}$$

$$= -\frac{4(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}\sqrt{c+dx^2}}{195de^3} - \frac{2(3b^2c^2-13ad(2bc+9ad)}{117cde}$$

$$= -\frac{4(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}\sqrt{c+dx^2}}{195de^3} - \frac{8c(3b^2c^2-13ad(2bc+9ac}{195d^{3/2}e^2\left(\sqrt{c}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.13, size = 161, normalized size = 0.34

$$\frac{x\left(2(c+dx^2)\left(117a^2d(-5c+dx^2)+26abdx^2(11c+5dx^2)+3b^2x^2(4c^2+25cdx^2+15d^2x^4)\right)+24c(-3b^2c^2+26abcd+117a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{585d(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x]
```

```
[Out] (x*(2*(c + d*x^2)*(117*a^2*d*(-5*c + d*x^2) + 26*a*b*d*x^2*(11*c + 5*d*x^2)
 + 3*b^2*x^2*(4*c^2 + 25*c*d*x^2 + 15*d^2*x^4)) + 24*c*(-3*b^2*c^2 + 26*a*b
*c*d + 117*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^2*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(c/(d*x^2))]))/(585*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.12, size = 669, normalized size = 1.41

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(-45 b^{2} d^{2} x^{6}-130 a b\,d^{2} x^{4}-75 b^{2} c d\,x^{4}-117 a^{2} d^{2} x^{2}-286 a b c d\,x^{2}-12 b^{2} c^{2} x^{2}+585 a^{2} c d\right)}{585 d e\sqrt{e x}}+\frac{4c\left(117a^{2}d^{2}+26abcd-3b\right.}{}$
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\,\left(-\frac{2\left(d e\,x^{2}+c e\right) c\,a^{2}}{e^{2}\sqrt{x\left(d e\,x^{2}+c e\right)}}+\frac{2 b^{2} d\,x^{5}\sqrt{d e\,x^{3}+c e x}}{13 e^{2}}+\frac{2\left(\frac{2 b d\left(a d+b c\right)}{e}-\frac{11 b^{2} d c}{13 e}\right) x^{3}\sqrt{d e\,x^{3}+c e x}}{9 d e}+\frac{2}{}\right.}{}$
default	$\frac{\frac{2 b^{2} d^{4} x^{8}}{13}+\frac{4 a b\,d^{4} x^{6}}{9}+\frac{16 b^{2} c\,d^{3} x^{6}}{39}+\frac{24\sqrt{\frac{d x+\sqrt{-c d}}{\sqrt{-c d}}}\,\sqrt{2}\,\sqrt{\frac{-d x+\sqrt{-c d}}{\sqrt{-c d}}}\,\sqrt{-\frac{x d}{\sqrt{-c d}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{d x+\sqrt{-c d}}{\sqrt{-c d}}},\right.}{5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/585*(45*b^2*d^4*x^8+130*a*b*d^4*x^6+120*b^2*c*d^3*x^6+1404*((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*a^2*c^2*d^2+312*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellip
ticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-36*((d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2),1/2*2^(1/2))*b^2*c^4-702*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2
-156*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d+18*((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b
^2*c^4+117*a^2*d^4*x^4+416*a*b*c*d^3*x^4+87*b^2*c^2*d^2*x^4-468*a^2*c*d^3*x
^2+286*a*b*c^2*d^2*x^2+12*b^2*c^3*d*x^2-585*a^2*c^2*d^2)/(d*x^2+c)^(1/2)/d^
2/e/(e*x)^(1/2)
```

<u>**Maxima**</u> **[F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^(3/2), x)
```

<u>**Fricas**</u> **[C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 146, normalized size = 0.31

$$\frac{2\left(12\left(3b^2c^3-26abc^2d-117a^2cd^2\right)\sqrt{d}\,x\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+\left(45b^2d^3x^6-585a^2cd^2+5\left(15b^2cd^2+26abd^3\right)x^4+\left(12b^2c^2d+286abcd^2+117a^2d^3\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{585\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/585*(12*(3*b^2*c^3 - 26*a*b*c^2*d - 117*a^2*c*d^2)*sqrt(d)*x*weierstrassZ
eta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (45*b^2*d^3*x^6 - 585*a
^2*c*d^2 + 5*(15*b^2*c*d^2 + 26*a*b*d^3)*x^4 + (12*b^2*c^2*d + 286*a*b*c*d^
2 + 117*a^2*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-3/2)/(d^2*x)
```

<u>**Sympy**</u> **[C]** Result contains complex when optimal does not.
time = 10.15, size = 309, normalized size = 0.65

$$\frac{a^2c^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{a^2\sqrt{c}\,dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{abc^{\frac{3}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{ab\sqrt{c}\,dx^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}+\frac{b^2c^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}+\frac{b^2\sqrt{c}\,dx^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi
)/c)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + a**2*sqrt(c)*d*x**(3/2)*gamma(3/4)*h
yper((-1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(3/2)*gamma(7/4))
 + a*b*c**(3/2)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_p
olar(I*pi)/c)/(e**(3/2)*gamma(7/4)) + a*b*sqrt(c)*d*x**(7/2)*gamma(7/4)*hyp
er((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(e**(3/2)*gamma(11/4)) +
 b**2*c**(3/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_p
olar(I*pi)/c)/(2*e**(3/2)*gamma(11/4)) + b**2*sqrt(c)*d*x**(11/2)*gamma(11/
4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(3/2)*gamma
(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*e^(-3/2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}}{{\left(e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2), x)
```

3.837 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=288

$$-\frac{4(3b^2c^2-11ad(6bc+7ad))\sqrt{ex}\sqrt{c+dx^2}}{231de^3}-\frac{2(3b^2c^2-11ad(6bc+7ad))\sqrt{ex}\,(c+dx^2)^{3/2}}{231cde^3}-\frac{2a^2(c+dx^2)^{5}}{3ce(ex)^{3/2}}$$

```
[Out] -2/3*a^2*(d*x^2+c)^(5/2)/c/e/(e*x)^(3/2)-2/231*(3*b^2*c^2-11*a*d*(7*a*d+6*b
*c))*(d*x^2+c)^(3/2)*(e*x)^(1/2)/c/d/e^3+2/11*b^2*(d*x^2+c)^(5/2)*(e*x)^(1/
2)/d/e^3-4/231*(3*b^2*c^2-11*a*d*(7*a*d+6*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)
/d/e^3-4/231*c^(3/4)*(3*b^2*c^2-11*a*d*(7*a*d+6*b*c))*(cos(2*arctan(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))
),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/
d^(5/4)/e^(5/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {473, 470, 285, 335, 226}

$$-\frac{2a^2(c+dx^2)^{5/2}}{3ce(ex)^{3/2}}-\frac{4c^{3/4}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(3b^2c^2-11ad(7ad+6bc))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2\sqrt{ex}\,(c+dx^2)^{3/2}(3b^2c^2-11ad(7ad+6bc))}{231cde^3}-\frac{4\sqrt{ex}\sqrt{c+dx^2}\,(3b^2c^2-11ad(7ad+6bc))}{231de^3}+\frac{2b^2\sqrt{ex}\,(c+dx^2)^{5/2}}{11de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x]
```

```
[Out] (-4*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d*
e^3) - (2*(3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*Sqrt[e*x]*(c + d*x^2)^(3/2))
/(231*c*d*e^3) - (2*a^2*(c + d*x^2)^(5/2))/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqr
t[e*x]*(c + d*x^2)^(5/2))/(11*d*e^3) - (4*c^(3/4)*(3*b^2*c^2 - 11*a*d*(6*b*
c + 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]
*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(231*d^(5
/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{5/2}} dx = -\frac{2a^2(c+dx^2)^{5/2}}{3ce(ex)^{3/2}} + \frac{2\int \frac{\left(\frac{1}{2}a(6bc+7ad)+\frac{3}{2}b^2cx^2\right)(c+dx^2)^{3/2}}{\sqrt{ex}} dx}{3ce^2}$$

$$= -\frac{2a^2(c+dx^2)^{5/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}(c+dx^2)^{5/2}}{11de^3} - \frac{(3b^2c^2 - 11ad(6bc+7ad))\int \frac{(c+}{\sqrt{}}}{33cde^2}$$

$$= -\frac{2(3b^2c^2 - 11ad(6bc+7ad))\sqrt{ex}(c+dx^2)^{3/2}}{231cde^3} - \frac{2a^2(c+dx^2)^{5/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}}{}$$

$$= -\frac{4(3b^2c^2 - 11ad(6bc+7ad))\sqrt{ex}\sqrt{c+dx^2}}{231de^3} - \frac{2(3b^2c^2 - 11ad(6bc+7ad))}{231cde^3}$$

$$= -\frac{4(3b^2c^2 - 11ad(6bc+7ad))\sqrt{ex}\sqrt{c+dx^2}}{231de^3} - \frac{2(3b^2c^2 - 11ad(6bc+7ad))}{231cde^3}$$

$$= -\frac{4(3b^2c^2 - 11ad(6bc+7ad))\sqrt{ex}\sqrt{c+dx^2}}{231de^3} - \frac{2(3b^2c^2 - 11ad(6bc+7ad))}{231cde^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 202, normalized size = 0.70

$$\frac{x^{5/2}\left(\frac{2(c+dx^2)(77a^2d(-c+dx^2)+66abdx^2(3c+dx^2)+3b^2x^2(4c^2+13cdx^2+7d^2x^4))}{dx^{3/2}} + \frac{8ic(-3b^2c^2+66abcd+77a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d}\right)}{231(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x]
```

```
[Out] (x^(5/2)*((2*(c + d*x^2)*(77*a^2*d*(-c + d*x^2) + 66*a*b*d*x^2*(3*c + d*x^2
) + 3*b^2*x^2*(4*c^2 + 13*c*d*x^2 + 7*d^2*x^4)))/(d*x^(3/2)) + ((8*I)*c*(-3
*b^2*c^2 + 66*a*b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSi
```

```
nh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d)))
/(231*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 415, normalized size = 1.44

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,\left(-21 b^2 d^2 x^6-66 a b\,d^2 x^4-39 b^2 c d\,x^4-77 a^2 d^2 x^2-198 a b c d\,x^2-12 b^2 c^2 x^2+77 a^2 c d\right)}{231 d x\,e^2\sqrt{e x}}+\frac{4c\left(77 a^2 d^2+66 a b c d-3 b^2 c^2\right)}{}$
elliptic	$\frac{\sqrt{e x\left(d\,x^2+c\right)}\left(-\frac{2 c\,a^2\sqrt{d e\,x^3+c e x}}{3 e^3 x^2}+\frac{2 b^2 d\,x^4\sqrt{d e\,x^3+c e x}}{11 e^3}+\frac{2\left(\frac{2 b d(a d+b c)}{e^2}-\frac{9 b^2 d c}{11 e^2}\right)x^2\sqrt{d e\,x^3+c e x}}{7 d e}+\frac{2(}{}\right.}{}$
default	$\frac{2 b^2 d^4 x^8}{11}+\frac{4\sqrt{\frac{d x+\sqrt{-c d}}{\sqrt{-c d}}}\,\sqrt{2}\,\sqrt{\frac{-d x+\sqrt{-c d}}{\sqrt{-c d}}}\,\sqrt{-\frac{x d}{\sqrt{-c d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{d x+\sqrt{-c d}}{\sqrt{-c d}}},\frac{\sqrt{2}}{2}\right)\sqrt{-c d}\,a^2 c\,d^2 x}{3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231/(d*x^2+c)^(1/2)/x*(21*b^2*d^4*x^8+154*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c
*d)^(1/2)*a^2*c*d^2*x+132*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*
d*x-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3*x+66*a*b*d^4*x^6+60*
b^2*c*d^3*x^6+77*a^2*d^4*x^4+264*a*b*c*d^3*x^4+51*b^2*c^2*d^2*x^4+198*a*b*c
^2*d^2*x^2+12*b^2*c^3*d*x^2-77*a^2*c^2*d^2)/d^2/e^2/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 141, normalized size = 0.49

$$-\frac{2\left(4\left(3b^2c^3-66abc^2d-77a^2cd^2\right)\sqrt{d}\,x^2\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-\left(21b^2d^3x^6-77a^2cd^2+3\left(13b^2cd^2+22abd^3\right)x^4+\left(12b^2c^2d+198abcd^2+77a^2d^3\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{231\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/231*(4*(3*b^2*c^3 - 66*a*b*c^2*d - 77*a^2*c*d^2)*sqrt(d)*x^2*weierstrass
PInverse(-4*c/d, 0, x) - (21*b^2*d^3*x^6 - 77*a^2*c*d^2 + 3*(13*b^2*c*d^2 +
 22*a*b*d^3)*x^4 + (12*b^2*c^2*d + 198*a*b*c*d^2 + 77*a^2*d^3)*x^2)*sqrt(d*
x^2 + c)*sqrt(x))*e^(-5/2)/(d^2*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.52, size = 309, normalized size = 1.07

$$\frac{a^2c^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{a^2\sqrt{c}\,d\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{abc^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ab\sqrt{c}\,dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{b^2c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{b^2\sqrt{c}\,dx^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(5/2),x)
```

```
[Out] a**2*c**(3/2)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), d*x**2*exp_polar(I*pi
)/c)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + a**2*sqrt(c)*d*sqrt(x)*gamma(1/4)*h
yper((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(5/2)*gamma(5/4))
 + a*b*c**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_po
lar(I*pi)/c)/(e**(5/2)*gamma(5/4)) + a*b*sqrt(c)*d*x**(5/2)*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(e**(5/2)*gamma(9/4)) + b*
*2*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_polar
(I*pi)/c)/(2*e**(5/2)*gamma(9/4)) + b**2*sqrt(c)*d*x**(9/2)*gamma(9/4)*hype
r((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(5/2)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*e^(-5/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}{\left(e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x)
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2), x)
```

3.838 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=468

$$\frac{4(b^2c^2+9ad(2bc+ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5}+\frac{8(b^2c^2+9ad(2bc+ad))\,\sqrt{ex}\,\sqrt{c+dx^2}}{15\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{2(b^2c^2+9ad(2bc+ad}{9c^2}$$

```
[Out] 2/9*(b^2*c^2+9*a*d*(a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(3/2)/c^2/e^5-2/5*a^2
*(d*x^2+c)^(5/2)/c/e/(e*x)^(5/2)-2*a*(a*d+2*b*c)*(d*x^2+c)^(5/2)/c^2/e^3/(e
*x)^(1/2)+4/15*(b^2*c^2+9*a*d*(a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(1/2)/c/e^
5+8/15*(b^2*c^2+9*a*d*(a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/e^4/d^(1/2)/
(c^(1/2)+x*d^(1/2))-8/15*c^(1/4)*(b^2*c^2+9*a*d*(a*d+2*b*c))*(cos(2*arctan(
d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)
^(1/2)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)+4/15*c^(1/4)*(b^2*c^2+9*a*d*(a*d+2*b
*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arc
tan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^
(1/2)+x*d^(1/2))^2)^(1/2)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 464, 285, 335, 311, 226, 1210}

$$-\frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}+\frac{4\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(9ad(ad+2bc)+b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{8\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(9ad(ad+2bc)+b^2c^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{2(ex)^{3/2}(c+dx^2)^{3/2}(9ad(ad+2bc)+b^2c^2)}{9c^2e^5}+\frac{4(ex)^{3/2}\sqrt{c+dx^2}\,(9ad(ad+2bc)+b^2c^2)}{15ce^5}+\frac{8\sqrt{ex}\,\sqrt{c+dx^2}\,(9ad(ad+2bc)+b^2c^2)}{15\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a(c+dx^2)^{5/2}(ad+2bc)}{c^2e^3\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x]
```

```
[Out] (4*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(e*x)^(3/2)*Sqrt[c + d*x^2])/(15*c*e^5)
+ (8*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*Sqrt[d]
*e^4*(Sqrt[c] + Sqrt[d]*x)) + (2*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(e*x)^(3/2
)*(c + d*x^2)^(3/2))/(9*c^2*e^5) - (2*a^2*(c + d*x^2)^(5/2))/(5*c*e*(e*x)^(
5/2)) - (2*a*(2*b*c + a*d)*(c + d*x^2)^(5/2))/(c^2*e^3*Sqrt[e*x]) - (8*c^(1
/4)*(b^2*c^2 + 9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/
(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], 1/2])/(15*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + (4*c^(1/4)*(b^2*c^2 +
 9*a*d*(2*b*c + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqr
t[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])
/(15*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}}\,dx = -\frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}} + \frac{2\int \frac{\left(\frac{5}{2}a(2bc+ad)+\frac{5}{2}b^2cx^2\right)(c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx}{5ce^2}$$

$$= -\frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}} - \frac{2a(2bc+ad)(c+dx^2)^{5/2}}{c^2e^3\sqrt{ex}} + \frac{(b^2c^2+9ad(2bc+ad))\int \sqrt{}}{c^2e^4}$$

$$= \frac{2(b^2c^2+9ad(2bc+ad))(ex)^{3/2}(c+dx^2)^{3/2}}{9c^2e^5} - \frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}} - \frac{2a(2bc+a}{c^2e}$$

$$= \frac{4(b^2c^2+9ad(2bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5} + \frac{2(b^2c^2+9ad(2bc+ad))(ex)^{3/2}}{9c^2e^5}$$

$$= \frac{4(b^2c^2+9ad(2bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5} + \frac{2(b^2c^2+9ad(2bc+ad))(ex)^{3/2}}{9c^2e^5}$$

$$= \frac{4(b^2c^2+9ad(2bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5} + \frac{2(b^2c^2+9ad(2bc+ad))(ex)^{3/2}}{9c^2e^5}$$

$$= \frac{4(b^2c^2+9ad(2bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5} + \frac{8(b^2c^2+9ad(2bc+ad))\sqrt{ex}\sqrt{}}{15\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.13, size = 141, normalized size = 0.30

$$\frac{x\left(-2(c+dx^2)\left(-18abx^2(-5c+dx^2)-b^2x^4(11c+5dx^2)+9a^2(c+7dx^2)\right)+24(b^2c^2+18abcd+9a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^4\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{45(ex)^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x]
```

```
[Out] (x*(-2*(c + d*x^2)*(-18*a*b*x^2*(-5*c + d*x^2) - b^2*x^4*(11*c + 5*d*x^2) +
 9*a^2*(c + 7*d*x^2)) + 24*(b^2*c^2 + 18*a*b*c*d + 9*a^2*d^2)*Sqrt[1 + c/(d
*x^2)]*x^4*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(45*(e*x)^(7/2
)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 668, normalized size = 1.43

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(-5b^{2}d\,x^{6}-18abd\,x^{4}-11b^{2}c\,x^{4}+63a^{2}d\,x^{2}+90abc\,x^{2}+9a^{2}c\right)}{45x^{2}e^{3}\sqrt{ex}}+\frac{\left(\frac{12}{5}a^{2}d^{2}+\frac{24}{5}abcd+\frac{4}{15}b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\sqrt{\ldots}\right.}{\sqrt{\ldots}}}}{\ldots}$
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\left(-\frac{2ca^{2}\sqrt{de\,x^{3}+cex}}{5e^{4}x^{3}}-\frac{2\left(de\,x^{2}+ce\right)a\left(7ad+10bc\right)}{5e^{4}\sqrt{x\left(de\,x^{2}+ce\right)}}+\frac{2b^{2}d\,x^{3}\sqrt{de\,x^{3}+cex}}{9e^{4}}+\frac{2\left(\frac{2bd(ad+bc)}{e^{3}}-\frac{7b^{2}dc}{9e^{3}}\ldots\right.}{5\ldots}\right.}{\ldots}$
default	$\frac{\frac{2b^{2}d^{3}x^{8}}{9}+\frac{24\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c\,d^{2}x^{2}}{5}+\frac{48\sqrt{\ldots}}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/x^2*(5*b^2*d^3*x^8+108*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2+216*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d*x^2+12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2
*c^3*x^2-54*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2-108*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*a*b*c^2*d*x^2-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2+18*a*b*d
^3*x^6+16*b^2*c*d^2*x^6-63*a^2*d^3*x^4-72*a*b*c*d^2*x^4+11*b^2*c^2*d*x^4-72
*a^2*c*d^2*x^2-90*a*b*c^2*d*x^2-9*a^2*c^2*d)/(d*x^2+c)^(1/2)/d/e^3/(e*x)^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/x^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 130, normalized size = 0.28

$$-\frac{2\left(12\,(b^2c^2+18\,abcd+9\,a^2d^2)\sqrt{d}\,x^3\mathrm{weierstrassZeta}\left(-\frac{4\,c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)-(5\,b^2d^2x^6+(11\,b^2cd+18\,abd^2)x^4-9\,a^2cd-9\,(10\,abcd+7\,a^2d^2)x^2)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{45\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/45*(12*(b^2*c^2 + 18*a*b*c*d + 9*a^2*d^2)*sqrt(d)*x^3*weierstrassZeta(-4
*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (5*b^2*d^2*x^6 + (11*b^2*c*d
+ 18*a*b*d^2)*x^4 - 9*a^2*c*d - 9*(10*a*b*c*d + 7*a^2*d^2)*x^2)*sqrt(d*x^2
+ c)*sqrt(x))*e^(-7/2)/(d*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 32.51, size = 320, normalized size = 0.68

$$\frac{a^2c^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{a^2\sqrt{c}\,d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{abc^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{ab\sqrt{c}\,dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{b^2c^{\frac{3}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{b^2\sqrt{c}\,dx^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(7/2),x)
```

```
[Out] a**2*c**(3/2)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), d*x**2*exp_polar(I*p
i)/c)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + a**2*sqrt(c)*d*gamma(-1/4)*hyper(
(-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(7/2)*sqrt(x)*gamma(3
/4)) + a*b*c**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_pola
r(I*pi)/c)/(e**(7/2)*sqrt(x)*gamma(3/4)) + a*b*sqrt(c)*d*x**(3/2)*gamma(3/4
)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(e**(7/2)*gamma(7/4)
) + b**2*c**(3/2)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp
_polar(I*pi)/c)/(2*e**(7/2)*gamma(7/4)) + b**2*sqrt(c)*d*x**(7/2)*gamma(7/4
)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(7/2)*gamma(1
1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*e^(-7/2)/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}}{{\left(e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2), x)
```

3.839 $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=430

$$\frac{2(117a^2d^2+7bc(11bc-26ad))\,e(ex)^{3/2}\sqrt{c+dx^2}}{585d^3}-\frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

```
[Out] 2/585*(117*a^2*d^2+7*b*c*(-26*a*d+11*b*c))*e*(e*x)^(3/2)*(d*x^2+c)^(1/2)/d^
3-2/117*b*(-26*a*d+11*b*c)*(e*x)^(7/2)*(d*x^2+c)^(1/2)/d^2/e+2/13*b^2*(e*x)
^(11/2)*(d*x^2+c)^(1/2)/d/e^3-2/195*c*(117*a^2*d^2+7*b*c*(-26*a*d+11*b*c))*
e^2*(e*x)^(1/2)*(d*x^2+c)^(1/2)/d^(7/2)/(c^(1/2)+x*d^(1/2))+2/195*c^(5/4)*(
117*a^2*d^2+7*b*c*(-26*a*d+11*b*c))*e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1
/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1
/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(15/4)/(
d*x^2+c)^(1/2)-1/195*c^(5/4)*(117*a^2*d^2+7*b*c*(-26*a*d+11*b*c))*e^(5/2)*(
cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1
/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+
x*d^(1/2))^2)^(1/2)/d^(15/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {475, 470, 327, 335, 311, 226, 1210}

$$-\frac{c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(117a^2d^2+7bc(11bc-26ad))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}}+\frac{2c^{5/4}e^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(117a^2d^2+7bc(11bc-26ad))\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}}-\frac{2ce^2\sqrt{ex}\sqrt{c+dx^2}\,(117a^2d^2+7bc(11bc-26ad))}{195d^{7/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{2e(ex)^{3/2}\sqrt{c+dx^2}\,(117a^2d^2+7bc(11bc-26ad))}{585d^3}-\frac{2b(ex)^{7/2}\sqrt{c+dx^2}\,(11bc-26ad)}{117d^2e}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(
585*d^3) - (2*b*(11*b*c - 26*a*d)*(e*x)^(7/2)*Sqrt[c + d*x^2])/(117*d^2*e)
+ (2*b^2*(e*x)^(11/2)*Sqrt[c + d*x^2])/(13*d*e^3) - (2*c*(117*a^2*d^2 + 7*b
*c*(11*b*c - 26*a*d))*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(195*d^(7/2)*(Sqrt[c]
+ Sqrt[d]*x)) + (2*c^(5/4)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^(5/2)*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(15/4)*Sqrt[c
+ d*x^2]) - (c^(5/4)*(117*a^2*d^2 + 7*b*c*(11*b*c - 26*a*d))*e^(5/2)*(Sqrt[
c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTa
```

```
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(195*d^(15/4)*Sqrt[c + d*x^
2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}(a+bx^2)^2}{\sqrt{c+dx^2}}dx = \frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}+\frac{2\int \frac{(ex)^{5/2}\left(\frac{13a^2d}{2}-\frac{1}{2}b(11bc-26ad)x^2\right)}{\sqrt{c+dx^2}}dx}{13d}$$

$$= -\frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}-\frac{1}{117}\left(-117a^2-7b\right.$$

$$= \frac{2\left(117a^2+\frac{7bc(11bc-26ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{585d}-\frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e}$$

$$= \frac{2\left(117a^2+\frac{7bc(11bc-26ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{585d}-\frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e}$$

$$= \frac{2\left(117a^2+\frac{7bc(11bc-26ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{585d}-\frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e}$$

$$= \frac{2\left(117a^2+\frac{7bc(11bc-26ad)}{d^2}\right)e(ex)^{3/2}\sqrt{c+dx^2}}{585d}-\frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.12, size = 143, normalized size = 0.33

$$\frac{2e(ex)^{3/2}\left((c+dx^2)\left(117a^2d^2+26abd(-7c+5dx^2)+b^2(77c^2-55cdx^2+45d^2x^4)\right)-3c(77b^2c^2-182abcd+117a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{585d^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*e*(e*x)^(3/2)*((c + d*x^2)*(117*a^2*d^2 + 26*a*b*d*(-7*c + 5*d*x^2) + b^
2*(77*c^2 - 55*c*d*x^2 + 45*d^2*x^4)) - 3*c*(77*b^2*c^2 - 182*a*b*c*d + 117
*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2)
)]))/(585*d^3*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.11, size = 661, normalized size = 1.54

method	result
risch	$\frac{2x^2(45b^2x^4d^2+130ab\,d^2x^2-55b^2cd\,x^2+117a^2d^2-182abcd+77b^2c^2)\sqrt{d\,x^2+c}\,e^3}{585d^3\sqrt{ex}} - \frac{c(117a^2d^2-182abcd+77b^2c^2)\sqrt{-cd}\,\sqrt{}}{}$
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2e^2x^5\sqrt{de\,x^3+cex}}{13d}+\frac{2\left(2ab\,e^3-\frac{11b^2e^3c}{13d}\right)x^3\sqrt{de\,x^3+cex}}{9de}+\frac{2\left(a^2e^3-\frac{7\left(2ab\,e^3-\frac{11b^2e^3c}{13d}\right)}{9d}\right.}{5d}\right.}{}$
default	$-\frac{e^2\sqrt{ex}\left(-90b^2d^4x^8-260ab\,d^4x^6+20b^2c\,d^3x^6+702\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/585*e^2/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^4*(-90*b^2*d^4*x^8-260*a*b*d^4*x
^6+20*b^2*c*d^3*x^6+702*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-1092*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a*b*c^3*d+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)
*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-351
*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2+546*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)
*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*
b*c^3*d-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-234*a^2*d^4*x^4+104*a*b*c*d
^3*x^4-44*b^2*c^2*d^2*x^4-234*a^2*c*d^3*x^2+364*a*b*c^2*d^2*x^2-154*b^2*c^3
*d*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*x^2 + a)^2*x^(5/2)/sqrt(d*x^2 + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 133, normalized size = 0.31

$$\frac{2\left(3\left(77\,b^2c^3-182\,abc^2d+117\,a^2cd^2\right)\sqrt{d}\,e^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+\left(45\,b^2d^3x^5-5\left(11\,b^2cd^2-26\,abd^3\right)x^3+\left(77\,b^2c^2d-182\,abcd^2+117\,a^2d^3\right)x\right)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{5}{2}}\right)}{585\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(3*(77*b^2*c^3 - 182*a*b*c^2*d + 117*a^2*c*d^2)*sqrt(d)*e^(5/2)*weier
strassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (45*b^2*d^3*x^5
- 5*(11*b^2*c*d^2 - 26*a*b*d^3)*x^3 + (77*b^2*c^2*d - 182*a*b*c*d^2 + 117*a
^2*d^3)*x)*sqrt(d*x^2 + c)*sqrt(x)*e^(5/2))/d^4
```

Sympy [C] Result contains complex when optimal does not.

time = 31.38, size = 144, normalized size = 0.33

$$\frac{a^{2}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{11}{4}\right)}+\frac{abe^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{\sqrt{c}\,\Gamma\left(\frac{15}{4}\right)}+\frac{b^{2}e^{\frac{5}{2}}x^{\frac{15}{2}}\Gamma\left(\frac{15}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{15}{4}\\\frac{19}{4}\end{matrix}\middle|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{19}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), d*x**2*exp_pol
ar(I*pi)/c)/(2*sqrt(c)*gamma(11/4)) + a*b*e**(5/2)*x**(11/2)*gamma(11/4)*hy
per((1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*gamma(15/4)) +
 b**2*e**(5/2)*x**(15/2)*gamma(15/4)*hyper((1/2, 15/4), (19/4,), d*x**2*exp
_polar(I*pi)/c)/(2*sqrt(c)*gamma(19/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^(5/2)*e^(5/2)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x\right)}^{5/2}\,{\left(b\,x^2+a\right)}^2}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.840 $\int \frac{(ex)^{3/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=240

$$\frac{2(77a^2d^2+5bc(9bc-22ad))\,e\sqrt{ex}\,\sqrt{c+dx^2}}{231d^3}-\frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}-\frac{c^{3/}\cdots}{\cdots}$$

```
[Out] -2/77*b*(-22*a*d+9*b*c)*(e*x)^(5/2)*(d*x^2+c)^(1/2)/d^2/e+2/11*b^2*(e*x)^(9
/2)*(d*x^2+c)^(1/2)/d/e^3+2/231*(77*a^2*d^2+5*b*c*(-22*a*d+9*b*c))*e*(e*x)^
(1/2)*(d*x^2+c)^(1/2)/d^3-1/231*c^(3/4)*(77*a^2*d^2+5*b*c*(-22*a*d+9*b*c))*
e^(3/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*
arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/
(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(13/4)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {475, 470, 327, 335, 226}

$$-\frac{c^{3/4}e^{3/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(77a^2d^2+5bc(9bc-22ad))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}+\frac{2e\sqrt{ex}\,\sqrt{c+dx^2}\,(77a^2d^2+5bc(9bc-22ad))}{231d^3}-\frac{2b(ex)^{5/2}\sqrt{c+dx^2}\,(9bc-22ad)}{77d^2e}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*(77*a^2*d^2 + 5*b*c*(9*b*c - 22*a*d))*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*
d^3) - (2*b*(9*b*c - 22*a*d)*(e*x)^(5/2)*Sqrt[c + d*x^2])/(77*d^2*e) + (2*b
^2*(e*x)^(9/2)*Sqrt[c + d*x^2])/(11*d*e^3) - (c^(3/4)*(77*a^2*d^2 + 5*b*c*(
9*b*c - 22*a*d))*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(231*d^(13/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3} + \frac{2\int \frac{(ex)^{3/2}\left(\frac{11a^2d}{2}-\frac{1}{2}b(9bc-22ad)x^2\right)}{\sqrt{c+dx^2}}dx}{11d}$$

$$= -\frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e} + \frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3} - \frac{1}{77}\left(-77a^2 - \frac{5bc(9b\right.$$

$$= \frac{2\left(77a^2+\frac{5bc(9bc-22ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d} - \frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e} + \frac{2b}{}$$

$$= \frac{2\left(77a^2+\frac{5bc(9bc-22ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d} - \frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e} + \frac{2b}{}$$

$$= \frac{2\left(77a^2+\frac{5bc(9bc-22ad)}{d^2}\right)e\sqrt{ex}\sqrt{c+dx^2}}{231d} - \frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e} + \frac{2b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.19, size = 190, normalized size = 0.79

$$\frac{(ex)^{3/2}\left(\frac{2\sqrt{x}(c+dx^2)(77a^2d^2+22abd(-5c+3dx^2)+3b^2(15c^2-9cdx^2+7d^2x^4))}{d^3} - \frac{2ic(45b^2c^2-110abcd+77a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^3}\right)}{231x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(77*a^2*d^2 + 22*a*b*d*(-5*c + 3*d*x^2
) + 3*b^2*(15*c^2 - 9*c*d*x^2 + 7*d^2*x^4)))/d^3 - ((2*I)*c*(45*b^2*c^2 - 1
10*a*b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(231*x^(
3/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.10, size = 405, normalized size = 1.69

method	result
risch	$\frac{2(21b^2x^4d^2+66ab\,d^2x^2-27b^2cd\,x^2+77a^2d^2-110abcd+45b^2c^2)x\sqrt{d\,x^2+c}\,e^2}{231d^3\sqrt{ex}} - \frac{c(77a^2d^2-110abcd+45b^2c^2)\sqrt{-cd}\,\sqrt{\frac{(x \ldots}{\ldots}}}{\ldots}$
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2e\,x^4\sqrt{de\,x^3+cex}}{11d}+\frac{2\left(2ab\,e^2-\frac{9b^2e^2c}{11d}\right)x^2\sqrt{de\,x^3+cex}}{7de}+\frac{2\left(a^2e^2-\frac{5\left(2ab\,e^2-\frac{9b^2e^2c}{11d}\right)c}{7d}\ldots\right)}{3de}\ldots\right.}{\ldots}$
default	$-\frac{e\sqrt{ex}\left(-42b^2d^4x^7+77\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/231*e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-42*b^2*d^4*x^7+77*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*(-c*d)^(1/2)*a^2*c*d^2-110*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(
1/2)*a*b*c^2*d+45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3-132*a*b*
d^4*x^5+12*b^2*c*d^3*x^5-154*a^2*d^4*x^3+88*a*b*c*d^3*x^3-36*b^2*c^2*d^2*x^
3-154*a^2*c*d^3*x+220*a*b*c^2*d^2*x-90*b^2*c^3*d*x)/d^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*x^2 + a)^2*x^(3/2)/sqrt(d*x^2 + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 122, normalized size = 0.51

$$-\frac{2\left(\left(45\,b^2c^3-110\,abc^2d+77\,a^2cd^2\right)\sqrt{d}\,e^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-\left(21\,b^2d^3x^4+45\,b^2c^2d-110\,abcd^2+77\,a^2d^3-3\left(9\,b^2cd^2-22\,abd^3\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{3}{2}}\right)}{231\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/231*((45*b^2*c^3 - 110*a*b*c^2*d + 77*a^2*c*d^2)*sqrt(d)*e^(3/2)*weierst
rassPInverse(-4*c/d, 0, x) - (21*b^2*d^3*x^4 + 45*b^2*c^2*d - 110*a*b*c*d^2
 + 77*a^2*d^3 - 3*(9*b^2*c*d^2 - 22*a*b*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(x)*e
^(3/2))/d^4
```

Sympy [C] Result contains complex when optimal does not.
time = 9.23, size = 144, normalized size = 0.60

$$\frac{a^2e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{9}{4}\right)}+\frac{abe^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\,\Gamma\left(\frac{13}{4}\right)}+\frac{b^2e^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{4}\\\frac{17}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), d*x**2*exp_pola
r(I*pi)/c)/(2*sqrt(c)*gamma(9/4)) + a*b*e**(3/2)*x**(9/2)*gamma(9/4)*hyper(
(1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*gamma(13/4)) + b**2
*e**(3/2)*x**(13/2)*gamma(13/4)*hyper((1/2, 13/4), (17/4,), d*x**2*exp_pola
r(I*pi)/c)/(2*sqrt(c)*gamma(17/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^(3/2)*e^(3/2)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^2}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
[Out] int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.841 $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=375

$$-\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}+\frac{2\left(15a^2d^2+bc(7bc-18ad)\right)\sqrt{ex}\,\sqrt{c+dx^2}}{15d^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2\sqrt[4]{}}{}$$

```
[Out] -2/45*b*(-18*a*d+7*b*c)*(e*x)^(3/2)*(d*x^2+c)^(1/2)/d^2/e+2/9*b^2*(e*x)^(7/
2)*(d*x^2+c)^(1/2)/d/e^3+2/15*(15*a^2*d^2+b*c*(-18*a*d+7*b*c))*(e*x)^(1/2)*
(d*x^2+c)^(1/2)/d^(5/2)/(c^(1/2)+x*d^(1/2))-2/15*c^(1/4)*(15*a^2*d^2+b*c*(-
18*a*d+7*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)
/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(
d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1
/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(11/4)/(d*x^2+c)^(1/2)+1/15*c
^(1/4)*(15*a^2*d^2+b*c*(-18*a*d+7*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))
)*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))
*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(11/
4)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {475, 470, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(15a^2d^2+bc(7bc-18ad)\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{11/4}\sqrt{c+dx^2}}-\frac{2\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(15a^2d^2+bc(7bc-18ad)\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{11/4}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(15a^2d^2+bc(7bc-18ad)\right)}{15d^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2b(ex)^{3/2}\sqrt{c+dx^2}(7bc-18ad)}{45d^2e}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (-2*b*(7*b*c - 18*a*d)*(e*x)^(3/2)*Sqrt[c + d*x^2])/(45*d^2*e) + (2*b^2*(e*
x)^(7/2)*Sqrt[c + d*x^2])/(9*d*e^3) + (2*(15*a^2*d^2 + b*c*(7*b*c - 18*a*d)
)*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - (2*c^(1/4
)*(15*a^2*d^2 + b*c*(7*b*c - 18*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c
 + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2]) + (c^(1/4)*(15*a^2*d
^2 + b*c*(7*b*c - 18*a*d))*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(
Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(15*d^(11/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\,(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{2\int \frac{\sqrt{ex}\left(\frac{9a^2d}{2}-\frac{1}{2}b(7bc-18ad)x^2\right)}{\sqrt{c+dx^2}}dx}{9d}$$

$$= -\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{1}{15}\left(15a^2 + \frac{bc(7bc-1}{d^2}\right.$$

$$= -\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{\left(2\left(15a^2 + \frac{bc(7bc-18ad)}{d^2}\right.\right.}{}$$

$$= -\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{\left(2\sqrt{c}\left(15a^2 + \frac{bc(7bc-1}{d^2}\right.\right.}{}$$

$$= -\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{2\left(15a^2 + \frac{bc(7bc-18ad)}{d^2}\right)}{15\sqrt{d}\left(\sqrt{c} +\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.11, size = 111, normalized size = 0.30

$$\frac{2\sqrt{ex}\left(bx(c+dx^2)\left(-7bc+18ad+5bdx^2\right)+3(7b^2c^2-18abcd+15a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{45d^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[e*x]*(b*x*(c + d*x^2)*(-7*b*c + 18*a*d + 5*b*d*x^2) + 3*(7*b^2*c^2
- 18*a*b*c*d + 15*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4, 1/
2, 3/4, -(c/(d*x^2))]))/(45*d^2*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.11, size = 604, normalized size = 1.61

method	result

risch	$\frac{2b\,x^2(5bd\,x^2+18ad-7bc)\sqrt{d\,x^2+c}\,e}{45d^2\sqrt{ex}}+\frac{(15a^2d^2-18abcd+7b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}}{}$
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2x^3\sqrt{de\,x^3+cex}}{9d}+\frac{2\left(2abe-\frac{7b^2ce}{9d}\right)x\sqrt{de\,x^3+cex}}{5de}+\frac{\left(a^2e-\frac{3\left(2abe-\frac{7b^2ce}{9d}\right)c}{5d}\right)\sqrt{-cd}}{}\right.}{}$
default	$\frac{\sqrt{ex}\left(10b^2x^6d^3+90\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45*(e*x)^(1/2)/(d*x^2+c)^(1/2)/d^3*(10*b^2*x^6*d^3+90*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a^2*c*d^2-108*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+42*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*b^2*c^3-45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
```

```
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+54*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a*b*c^2*d-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*
EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+36*a
*b*d^3*x^4-4*b^2*c*d^2*x^4+36*a*b*c*d^2*x^2-14*b^2*c^2*d*x^2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*x^2 + a)^2*sqrt(x)/sqrt(d*x^2 + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 99, normalized size = 0.26

$$-\frac{2\left(3\left(7\,b^{2}c^{2}-18\,abcd+15\,a^{2}d^{2}\right)\sqrt{d}\,e^{\frac{1}{2}}\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)-\left(5\,b^{2}d^{2}x^{3}-\left(7\,b^{2}cd-18\,abd^{2}\right)x\right)\sqrt{dx^{2}+c}\,\sqrt{x}\,e^{\frac{1}{2}}\right)}{45\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*(7*b^2*c^2 - 18*a*b*c*d + 15*a^2*d^2)*sqrt(d)*e^(1/2)*weierstrassZ
eta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (5*b^2*d^2*x^3 - (7*b^2
*c*d - 18*a*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(x)*e^(1/2))/d^3
```

Sympy [C] Result contains complex when optimal does not.
time = 2.69, size = 143, normalized size = 0.38

$$\frac{a^{2}(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\sqrt{c}\,e\Gamma\left(\frac{7}{4}\right)}+\frac{ab(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{dx^{2}e^{i\pi}}{c}\right)}{\sqrt{c}\,e^{3}\Gamma\left(\frac{11}{4}\right)}+\frac{b^{2}(ex)^{\frac{11}{2}}\,\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{dx^{2}e^{i\pi}}{c}\right)}{2\sqrt{c}\,e^{5}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*(e*x)**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), d*x**2*exp_polar(I*p
i)/c)/(2*sqrt(c)*e*gamma(7/4)) + a*b*(e*x)**(7/2)*gamma(7/4)*hyper((1/2, 7/
4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**3*gamma(11/4)) + b**2*(e
```

```
*x)**(11/2)*gamma(11/4)*hyper((1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/
c)/(2*sqrt(c)*e**5*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*sqrt(x)*e^(1/2)/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}\,\left(b\,x^2+a\right)^2}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
[Out] int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.842 $\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2b(5bc-14ad)\sqrt{ex}\sqrt{c+dx^2}}{21d^2e}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3}+\frac{(5b^2c^2-14abcd+21a^2d^2)\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c}{\left(\sqrt{c}\right.}}}{21\sqrt[4]{c}\,d^{9/4}\sqrt{e}\,\sqrt{c}}$$

```
[Out] 2/7*b^2*(e*x)^(5/2)*(d*x^2+c)^(1/2)/d/e^3-2/21*b*(-14*a*d+5*b*c)*(e*x)^(1/2
)*(d*x^2+c)^(1/2)/d^2/e+1/21*(21*a^2*d^2-14*a*b*c*d+5*b^2*c^2)*(cos(2*arcta
n(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^
2)^(1/2)/c^(1/4)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {475, 470, 335, 226}

$$\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(21a^2d^2-14abcd+5b^2c^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}\,d^{9/4}\sqrt{e}\,\sqrt{c+dx^2}}-\frac{2b\sqrt{ex}\sqrt{c+dx^2}(5bc-14ad)}{21d^2e}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[e*x]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*b*(5*b*c - 14*a*d)*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*d^2*e) + (2*b^2*(e*x)
^(5/2)*Sqrt[c + d*x^2])/(7*d*e^3) + ((5*b^2*c^2 - 14*a*b*c*d + 21*a^2*d^2)*
(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2
*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(21*c^(1/4)*d^(9/4)*S
qrt[e]*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx = \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3} + \frac{2\int \frac{\frac{7a^2d}{2}-\frac{1}{2}b(5bc-14ad)x^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx}{7d}$$

$$= -\frac{2b(5bc-14ad)\sqrt{ex}\sqrt{c+dx^2}}{21d^2e} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3} - \frac{1}{21}\left(-21a^2 - \frac{bc(5bc-}{d^{}}\right.$$

$$= -\frac{2b(5bc-14ad)\sqrt{ex}\sqrt{c+dx^2}}{21d^2e} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3} + \frac{\left(2\left(21a^2 + \frac{bc(5bc-14ad)}{d^2}\right.\right.}{}$$

$$= -\frac{2b(5bc-14ad)\sqrt{ex}\sqrt{c+dx^2}}{21d^2e} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3} + \frac{\left(21a^2 + \frac{bc(5bc-14ad)}{d^2}\right)\Big(}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.20, size = 148, normalized size = 0.77

$$\frac{2x\left(-b(c+dx^2)(5bc-14ad-3bdx^2)+\frac{i(5b^2c^2-14abcd+21a^2d^2)\sqrt{1+\frac{c}{dx^2}}\sqrt{x}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21d^2\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[e*x]*Sqrt[c + d*x^2]),x]
```

```
[Out] (2*x*(-(b*(c + d*x^2)*(5*b*c - 14*a*d - 3*b*d*x^2)) + (I*(5*b^2*c^2 - 14*a*
b*c*d + 21*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(21*d^2*Sqrt[
e*x]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.10, size = 350, normalized size = 1.81

method	result
risch	$\frac{2b(3bd\,x^2+14ad-5bc)x\sqrt{d\,x^2+c}}{21d^2\sqrt{ex}}+\frac{(21a^2d^2-14abcd+5b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{\cdots}}{21d^3\sqrt{de\,x^3+cex}\sqrt{e\cdots}}$
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\left(\frac{2b^2x^2\sqrt{de\,x^3+cex}}{7de}+\frac{2\left(2ab-\frac{5b^2c}{7d}\right)\sqrt{de\,x^3+cex}}{3de}+\frac{\left(a^2-\frac{c\left(2ab-\frac{5b^2c}{7d}\right)}{3d}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-c\cdots}}{d}\right)\cdots}{\sqrt{-c\cdots}}}\cdots}{\cdots}\right)}{\sqrt{ex}\sqrt{d\,x^2+c}}$
default	$\frac{21\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^2-14\sqrt{\frac{dx+\cdots}{\cdots}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/(d*x^2+c)^(1/2)*(21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*d^2-1
4*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c*d+5*((d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*
d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(
1/2))*(-c*d)^(1/2)*b^2*c^2+6*b^2*d^3*x^5+28*a*b*d^3*x^3-4*b^2*c*d^2*x^3+28*
a*x*b*c*d^2-10*x*b^2*c^2*d)/(e*x)^(1/2)/d^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 83, normalized size = 0.43

$$\frac{2\left(\left(5\,b^2c^2-14\,abcd+21\,a^2d^2\right)\sqrt{d}\ \mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)+\left(3\,b^2d^2x^2-5\,b^2cd+14\,abd^2\right)\sqrt{dx^2+c}\ \sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{21\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*((5*b^2*c^2 - 14*a*b*c*d + 21*a^2*d^2)*sqrt(d)*weierstrassPInverse(-4*
c/d, 0, x) + (3*b^2*d^2*x^2 - 5*b^2*c*d + 14*a*b*d^2)*sqrt(d*x^2 + c)*sqrt(
x))*e^(-1/2)/d^3
```

Sympy [C] Result contains complex when optimal does not.
time = 2.68, size = 144, normalized size = 0.75

$$\frac{a^2\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{abx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\,\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}+\frac{b^2x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), d*x**2*exp_polar(I*pi)/c)
/(2*sqrt(c)*sqrt(e)*gamma(5/4)) + a*b*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4),
 (9/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*sqrt(e)*gamma(9/4)) + b**2*x**(
9/2)*gamma(9/4)*hyper((1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqr
t(c)*sqrt(e)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*e^(-1/2)/(sqrt(d*x^2 + c)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^2}{\sqrt{e\,x}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.843 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=372

$$-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3}-\frac{2(3b^2c^2-5ad(2bc+ad))\sqrt{ex}\sqrt{c+dx^2}}{5cd^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{2(3b^2c^2-5ad(2bc+a\cdots}{\cdots}$$

```
[Out] 2/5*b^2*(e*x)^(3/2)*(d*x^2+c)^(1/2)/d/e^3-2*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(
1/2)-2/5*(3*b^2*c^2-5*a*d*(a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/c/d^(3/2
)/e^2/(c^(1/2)+x*d^(1/2))+2/5*(3*b^2*c^2-5*a*d*(a*d+2*b*c))*(cos(2*arctan(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1
/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^
(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^
(1/2)/c^(3/4)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)-1/5*(3*b^2*c^2-5*a*d*(a*d+2*b
*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arc
tan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^
(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {473, 470, 335, 311, 226, 1210}

$$-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}-\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(3b^2c^2-5ad(ad+2bc))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{2\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(3b^2c^2-5ad(ad+2bc))\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{2\sqrt{ex}\sqrt{c+dx^2}\,(3b^2c^2-5ad(ad+2bc))}{5cd^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(c*e*Sqrt[e*x]) + (2*b^2*(e*x)^(3/2)*Sqrt[c + d*x^
2])/(5*d*e^3) - (2*(3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*Sqrt[e*x]*Sqrt[c + d*x
^2])/(5*c*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x)) + (2*(3*b^2*c^2 - 5*a*d*(2*b*c
 + a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d
^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*(Sqrt[
c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(3/4)*d^(7/4)*e^(3/2)*
Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}} + \frac{2\int \frac{\sqrt{ex}\left(\frac{1}{2}a(2bc+ad)+\frac{1}{2}b^2cx^2\right)}{\sqrt{c+dx^2}}dx}{ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3} - \frac{\left(4\left(\frac{3b^2c^2}{4}-\frac{5}{4}ad(2bc+ad)\right)\right)\int \frac{\sqrt{ex}}{\sqrt{c+}}}{5cde^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3} - \frac{\left(8\left(\frac{3b^2c^2}{4}-\frac{5}{4}ad(2bc+ad)\right)\right)\text{Subst}\Bigg(}{5cde^3}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3} - \frac{(2(3b^2c^2-5ad(2bc+ad)))\,\text{Subst}\Bigg(\int}{5\sqrt{c}\,d^{3/2}e^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3} - \frac{2(3b^2c^2-5ad(2bc+ad))\sqrt{ex}\sqrt{c+}}{5cd^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.11, size = 115, normalized size = 0.31

$$\frac{x\left(2(-5a^2d+b^2cx^2)(c+dx^2)+2(-3b^2c^2+10abcd+5a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{5cd(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*(2*(-5*a^2*d + b^2*c*x^2)*(c + d*x^2) + 2*(-3*b^2*c^2 + 10*a*b*c*d + 5*a
^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^
2))]))/(5*c*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.12, size = 595, normalized size = 1.60

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,(-b^{2}c\,x^{2}+5a^{2}d)}{5cde\sqrt{ex}}+\frac{(5a^{2}d^{2}+10abcd-3b^{2}c^{2})\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\cdots}}{\cdots}$
elliptic	$\frac{\sqrt{ex\,(d\,x^{2}+c)}\left(-\frac{2\left(de\,x^{2}+ce\right)a^{2}}{e^{2}c\sqrt{x\,(de\,x^{2}+ce)}}+\frac{2b^{2}x\sqrt{de\,x^{3}+cex}}{5e^{2}d}+\frac{\left(\frac{2ab}{e}+\frac{d\,a^{2}}{ce}-\frac{3b^{2}c}{5ed}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\cdots}{\cdots}\cdots\right.}{\cdots}$
default	$\frac{10\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c\,d^{2}+20\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+20*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^2*d-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d
```

```
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-5*((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d
)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1
/2))*a^2*c*d^2-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+3*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*b^2*c^3+2*b^2*c*d^2*x^4-10*a^2*d^3*x^2+2*b^2*c^2*d*x^2-10*a^2*c*
d^2)/(d*x^2+c)^(1/2)/d^2/e/(e*x)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 90, normalized size = 0.24

$$\frac{2\left((3\,b^2c^2-10\,abcd-5\,a^2d^2)\sqrt{d}\,x\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+(b^2cdx^2-5\,a^2d^2)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{5\,cd^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*((3*b^2*c^2 - 10*a*b*c*d - 5*a^2*d^2)*sqrt(d)*x*weierstrassZeta(-4*c/d,
 0, weierstrassPInverse(-4*c/d, 0, x)) + (b^2*c*d*x^2 - 5*a^2*d^2)*sqrt(d*x
^2 + c)*sqrt(x))*e^(-3/2)/(c*d^2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.98, size = 148, normalized size = 0.40

$$\frac{a^2\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{abx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\,e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{b^2x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), d*x**2*exp_polar(I*pi)/c)/(2*sq
rt(c)*e**(3/2)*sqrt(x)*gamma(3/4)) + a*b*x**(3/2)*gamma(3/4)*hyper((1/2, 3/
```

```
4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(3/2)*gamma(7/4)) + b**2*
x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(2
*sqrt(c)*e**(3/2)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*e^(-3/2)/(sqrt(d*x^2 + c)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2}{{\left(e\,x\right)}^{3/2}\,\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(1/2)),x)
[Out] int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(1/2)), x)
```

3.844 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=184

$$-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3}-\frac{(b^2c^2-6abcd+a^2d^2)\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,F\Big(2\tan^{-}}{3c^{5/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

```
[Out] -2/3*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(3/2)+2/3*b^2*(e*x)^(1/2)*(d*x^2+c)^(1/2
)/d/e^3-1/3*(a^2*d^2-6*a*b*c*d+b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))
)*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))
*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(5/4)/d^(5/4
)/e^(5/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {473, 470, 335, 226}

$$-\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(a^2d^2-6abcd+b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3c^{5/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(3*c*e*(e*x)^(3/2)) + (2*b^2*Sqrt[e*x]*Sqrt[c + d*
x^2])/(3*d*e^3) - ((b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sq
rt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*c^(5/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}} + \frac{2\int \frac{\frac{1}{2}a(6bc-ad)+\frac{3}{2}b^2cx^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx}{3ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3} - \frac{(b^2c^2-6abcd+a^2d^2)\int \frac{1}{\sqrt{ex}\sqrt{c+dx^2}}}{3cde^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3} - \frac{(2(b^2c^2-6abcd+a^2d^2))\,\mathrm{Subst}\left(\int \frac{}{\sqrt{c+}}\right.}{3cde^3}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3} - \frac{(b^2c^2-6abcd+a^2d^2)\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{}}{3c^{5/4}d^{5/4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 165, normalized size = 0.90

$$\frac{x\left(2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(-a^2d+b^2cx^2\right)\left(c+dx^2\right)-2i(b^2c^2-6abcd+a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^{5/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2/((e*x)^(5/2)*Sqrt[c + d*x^2]),x]

[Out] (x*(2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*(-(a^2*d) + b^2*c*x^2)*(c + d*x^2) - (2*I)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(5/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/(3*c*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d*(e*x)^(5/2)*Sqrt[c + d*x^2])

Maple [A]
time = 0.12, size = 352, normalized size = 1.91

method	result
risch	$-\frac{2\sqrt{d x^2+c}\,(-b^2 c x^2+a^2 d)}{3 d c x e^2 \sqrt{e x}}-\frac{(a^2 d^2-6abcd+b^2 c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{\cdots}{\sqrt{\cdots}}}}{3c d^2\sqrt{d e x^3+c e x}\, e^2\sqrt{e x}\cdots}$
elliptic	$\frac{\sqrt{e x (d x^2+c)}\left(-\frac{2a^2\sqrt{d e x^3+c e x}}{3e^3 c x^2}+\frac{2b^2\sqrt{d e x^3+c e x}}{3e^3 d}+\frac{\left(\frac{2ab}{e^2}-\frac{d a^2}{3c e^2}-\frac{b^2 c}{3e^2 d}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\cdots}{\cdots}\right.}{\sqrt{e x}\sqrt{d x^2+c}}$
default	$-\frac{\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2d^2x-6\sqrt{-\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3/(d*x^2+c)^(1/2)/x*((-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*d^2*x-6*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d*x+(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2*x-2*b^2*c*d^2*x^4+2*a^2*d^3*x^2-2*b^2*c^2*d*x^2+2*a^2*c*d^2)/c/e^2/(e*x)^(1/2)/d^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 83, normalized size = 0.45

$$-\frac{2\left((b^2c^2-6\,abcd+a^2d^2)\sqrt{d}\,x^2\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-(b^2cdx^2-a^2d^2)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{3\,cd^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*((b^2*c^2 - 6*a*b*c*d + a^2*d^2)*sqrt(d)*x^2*weierstrassPInverse(-4*c/
d, 0, x) - (b^2*c*d*x^2 - a^2*d^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-5/2)/(c*d^2
*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 6.95, size = 148, normalized size = 0.80

$$\frac{a^2\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{ab\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\,e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{b^2x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), d*x**2*exp_polar(I*pi)/c)/(2*sq
rt(c)*e**(5/2)*x**(3/2)*gamma(1/4)) + a*b*sqrt(x)*gamma(1/4)*hyper((1/4, 1/
2), (5/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(5/2)*gamma(5/4)) + b**2*
x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(2*
sqrt(c)*e**(5/2)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*e^(-5/2)/(sqrt(d*x^2 + c)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^2}{(e\,x)^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(1/2)),x)
[Out] int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(1/2)), x)
```

3.845 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=387

$$-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}-\frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}}+\frac{2(5b^2c^2+10abcd-3a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{5c^2\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2(5b^2c^2+10abcd}{\quad}$$

[Out] -2/5*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(5/2)-2/5*a*(-3*a*d+10*b*c)*(d*x^2+c)^(1/2)/c^2/e^3/(e*x)^(1/2)+2/5*(-3*a^2*d^2+10*a*b*c*d+5*b^2*c^2)*(e*x)^(1/2)*(d*x^2+c)^(1/2)/c^2/e^4/d^(1/2)/(c^(1/2)+x*d^(1/2))-2/5*(-3*a^2*d^2+10*a*b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)+1/5*(-3*a^2*d^2+10*a*b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)

Rubi [A]
time = 0.24, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {473, 464, 335, 311, 226, 1210}

$$\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(-3a^2d^2+10abcd+5b^2c^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(-3a^2d^2+10abcd+5b^2c^2\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}\left(-3a^2d^2+10abcd+5b^2c^2\right)}{5c^2\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}-\frac{2a\sqrt{c+dx^2}\,(10bc-3ad)}{5c^2e^3\sqrt{ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^2/((e*x)^(7/2)*Sqrt[c + d*x^2]),x]

[Out] (-2*a^2*Sqrt[c + d*x^2])/(5*c*e*(e*x)^(5/2)) - (2*a*(10*b*c - 3*a*d)*Sqrt[c + d*x^2])/(5*c^2*e^3*Sqrt[e*x]) + (2*(5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c^2*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) - (2*(5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) + ((5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(5*c^(7/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}} + \frac{2\int \frac{\frac{1}{2}a(10bc-3ad)+\frac{5}{2}b^2cx^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx}{5ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}} - \frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}} + \frac{(5b^2c^2+10abcd-3a^2d^2)\int \frac{\sqrt{e}}{\sqrt{c+}}}{5c^2e^4}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}} - \frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}} + \frac{(2(5b^2c^2+10abcd-3a^2d^2))\,\mathrm{Subs}}{5c^2e}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}} - \frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}} + \frac{(2(5b^2c^2+10abcd-3a^2d^2))\,\mathrm{Subs}}{5c^{3/2}\sqrt{}}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}} - \frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}} + \frac{2(5b^2c^2+10abcd-3a^2d^2)\sqrt{ex}\,\sqrt{}}{5c^2\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.11, size = 116, normalized size = 0.30

$$\frac{x\left(-2a(c+dx^2)(10bcx^2+a(c-3dx^2))+2(5b^2c^2+10abcd-3a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^4\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{5c^2(ex)^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*(-2*a*(c + d*x^2)*(10*b*c*x^2 + a*(c - 3*d*x^2)) + 2*(5*b^2*c^2 + 10*a*b
*c*d - 3*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^4*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -(c/(d*x^2))]))/(5*c^2*(e*x)^(7/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.14, size = 626, normalized size = 1.62

method	result

risch	$-\frac{2\sqrt{d\,x^{2}+c}\,a\left(-3ad\,x^{2}+10c\,x^{2}b+ac\right)}{5c^{2}x^{2}e^{3}\sqrt{ex}}-\frac{\left(3a^{2}d^{2}-10abcd-5b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)}{\sqrt{-cd}}}}{}$
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\left(-\frac{2a^{2}\sqrt{de\,x^{3}+cex}}{5e^{4}c\,x^{3}}+\frac{2\left(de\,x^{2}+ce\right)a\left(3ad-10bc\right)}{5e^{4}c^{2}\sqrt{x\left(de\,x^{2}+ce\right)}}+\frac{\left(\frac{b^{2}}{e^{3}}-\frac{da\left(3ad-10bc\right)}{5c^{2}e^{3}}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)}{\sqrt{-cd}}}}{}\right.}{}$
default	$-\frac{6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c\,d^{2}x^{2}-20\sqrt{\frac{dx+\sqrt{}}{\sqrt{-}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^2*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2-20*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*a*b*c^2*d*x^2-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
E(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2-3*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
```

```
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*
d*x^2+5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2-6*a^2*d^3*x^4+20*a*b*c*d^2*
x^4-4*a^2*c*d^2*x^2+20*a*b*c^2*d*x^2+2*a^2*c^2*d)/(d*x^2+c)^(1/2)/d/e^3/(e*
x)^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 100, normalized size = 0.26

$$-\frac{2\left((5\,b^2c^2+10\,abcd-3\,a^2d^2)\sqrt{d}\,x^3\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+(a^2cd+(10\,abcd-3\,a^2d^2)x^2)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{5\,c^2dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*((5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*sqrt(d)*x^3*weierstrassZeta(-4*c
/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (a^2*c*d + (10*a*b*c*d - 3*a^2*
d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-7/2)/(c^2*d*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 26.83, size = 155, normalized size = 0.40

$$\frac{a^2\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{ab\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\,e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{b^2x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), d*x**2*exp_polar(I*pi)/c)/(2*s
qrt(c)*e**(7/2)*x**(5/2)*gamma(-1/4)) + a*b*gamma(-1/4)*hyper((-1/4, 1/2),
```

```
(3/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(7/2)*sqrt(x)*gamma(3/4)) + b
**2*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)
/(2*sqrt(c)*e**(7/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*e^(-7/2)/(sqrt(d*x^2 + c)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2}{{\left(e\,x\right)}^{7/2}\,\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(1/2)),x)
[Out] int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(1/2)), x)
```

3.846 $\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}-\frac{2a(14bc-5ad)\sqrt{c+dx^2}}{21c^2e^3(ex)^{3/2}}+\frac{(21b^2c^2-14abcd+5a^2d^2)\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}}{21c^{9/4}\sqrt[4]{d}\,e^{9/2}\sqrt{c+dx^2}}$$

```
[Out] -2/7*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(7/2)-2/21*a*(-5*a*d+14*b*c)*(d*x^2+c)^(
1/2)/c^2/e^3/(e*x)^(3/2)+1/21*(5*a^2*d^2-14*a*b*c*d+21*b^2*c^2)*(cos(2*arct
an(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4
)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))
^2)^(1/2)/c^(9/4)/d^(1/4)/e^(9/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {473, 464, 335, 226}

$$\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(5a^2d^2-14abcd+21b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt[4]{d}\,e^{9/2}\sqrt{c+dx^2}}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}-\frac{2a\sqrt{c+dx^2}\,(14bc-5ad)}{21c^2e^3(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(7*c*e*(e*x)^(7/2)) - (2*a*(14*b*c - 5*a*d)*Sqrt[c
 + d*x^2])/(21*c^2*e^3*(e*x)^(3/2)) + ((21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2
)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF
[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(21*c^(9/4)*d^(1/4)
*e^(9/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}} + \frac{2\int \frac{\frac{1}{2}a(14bc-5ad)+\frac{7}{2}b^2cx^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx}{7ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}} - \frac{2a(14bc-5ad)\sqrt{c+dx^2}}{21c^2e^3(ex)^{3/2}} - \frac{\left(4\left(-\frac{21}{4}b^2c^2+\frac{1}{4}ad(14bc-5ad)\right)\right.}{21c^2e^4}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}} - \frac{2a(14bc-5ad)\sqrt{c+dx^2}}{21c^2e^3(ex)^{3/2}} - \frac{\left(8\left(-\frac{21}{4}b^2c^2+\frac{1}{4}ad(14bc-5ad)\right)\right.}{2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}} - \frac{2a(14bc-5ad)\sqrt{c+dx^2}}{21c^2e^3(ex)^{3/2}} + \frac{(21b^2c^2-ad(14bc-5ad))\left(\sqrt{c}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.15, size = 159, normalized size = 0.82

$$\frac{x^{9/2}\left(\frac{2a(c+dx^2)(-3ac-14bcx^2+5adx^2)}{c^2x^{7/2}}+\frac{2i(21b^2c^2-14abcd+5a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21(ex)^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x^(9/2)*((2*a*(c + d*x^2)*(-3*a*c - 14*b*c*x^2 + 5*a*d*x^2))/(c^2*x^(7/2))
 + ((2*I)*(21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Ellip
ticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^2*Sqrt[(I*Sqrt[c
])/Sqrt[d]])))/(21*(e*x)^(9/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 370, normalized size = 1.92

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,a(-5ad\,x^2+14c\,x^2b+3ac)}{21c^2x^3e^4\sqrt{ex}}+\frac{(5a^2d^2-14abcd+21b^2c^2)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)}{\sqrt{-cd}}}}{21c^2d\sqrt{de\,x^3+cex}}$
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\left(-\frac{2a^2\sqrt{de\,x^3+cex}}{7e^5c\,x^4}+\frac{2a(5ad-14bc)\sqrt{de\,x^3+cex}}{21e^5c^2x^2}+\frac{\left(\frac{b^2}{e^4}+\frac{da(5ad-14bc)}{21c^2e^4}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right.}{\sqrt{-cd}}}}{}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$\frac{5\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^2x^3-14\sqrt{\frac{dx+}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/(d*x^2+c)^(1/2)/x^3*(5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*d^
2*x^3-14*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c*d*x^3+21*((d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^2*x^3+10*a^2*d^3*x^4-28*a*b*c*d^2*x^4+
4*a^2*c*d^2*x^2-28*a*b*c^2*d*x^2-6*a^2*c^2*d)/d/c^2/e^4/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^(9/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 94, normalized size = 0.49

$$\frac{2\left((21\,b^2c^2-14\,abcd+5\,a^2d^2)\sqrt{d}\,x^4\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-(3\,a^2cd+(14\,abcd-5\,a^2d^2)x^2)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{9}{2}\right)}}{21\,c^2dx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*((21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2)*sqrt(d)*x^4*weierstrassPInverse
(-4*c/d, 0, x) - (3*a^2*c*d + (14*a*b*c*d - 5*a^2*d^2)*x^2)*sqrt(d*x^2 + c)
*sqrt(x))*e^(-9/2)/(c^2*d*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 90.59, size = 155, normalized size = 0.80

$$\frac{a^2\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,e^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}+\frac{ab\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\,e^{\frac{9}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{b^2\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,e^{\frac{9}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(9/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), d*x**2*exp_polar(I*pi)/c)/(2*s
qrt(c)*e**(9/2)*x**(7/2)*gamma(-3/4)) + a*b*gamma(-3/4)*hyper((-3/4, 1/2),
(1/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(9/2)*x**(3/2)*gamma(1/4)) +
b**2*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), d*x**2*exp_polar(I*pi)/c)
/(2*sqrt(c)*e**(9/2)*gamma(5/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*e^(-9/2)/(sqrt(d*x^2 + c)*x^(9/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^2}{{\left(e\,x\right)}^{9/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(9/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^2/((e*x)^(9/2)*(c + d*x^2)^(1/2)), x)
```

3.847 $\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=438

$$-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}-\frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}}-\frac{2(15b^2c^2-18abcd+7a^2d^2)\sqrt{c+dx^2}}{15c^3e^5\sqrt{ex}}+\frac{2\sqrt{d}\,(15b^2c^2-18ab\cdots}{15c^3e^6\,(\cdots}$$

```
[Out] -2/9*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(9/2)-2/45*a*(-7*a*d+18*b*c)*(d*x^2+c)^(
1/2)/c^2/e^3/(e*x)^(5/2)-2/15*(7*a^2*d^2-18*a*b*c*d+15*b^2*c^2)*(d*x^2+c)^(
1/2)/c^3/e^5/(e*x)^(1/2)+2/15*(7*a^2*d^2-18*a*b*c*d+15*b^2*c^2)*d^(1/2)*(e*
x)^(1/2)*(d*x^2+c)^(1/2)/c^3/e^6/(c^(1/2)+x*d^(1/2))-2/15*d^(1/4)*(7*a^2*d^
2-18*a*b*c*d+15*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))
)^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin
(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(
1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/e^(11/2)/(d*x^2+c)^(
1/2)+1/15*d^(1/4)*(7*a^2*d^2-18*a*b*c*d+15*b^2*c^2)*(cos(2*arctan(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1
/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),
1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^
(11/4)/e^(11/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 464, 331, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(7a^2d^2-18abcd+15b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15c^{11/4}e^{11/2}\sqrt{c+dx^2}}-\frac{2\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(7a^2d^2-18abcd+15b^2c^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15c^{11/4}e^{11/2}\sqrt{c+dx^2}}+\frac{2\sqrt{d}\sqrt{ex}\sqrt{c+dx^2}\,(7a^2d^2-18abcd+15b^2c^2)}{15c^3e^6\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2\sqrt{c+dx^2}\,(7a^2d^2-18abcd+15b^2c^2)}{15c^3e^5\sqrt{ex}}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}-\frac{2a\sqrt{c+dx^2}\,(18bc-7ad)}{45c^2e^3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(11/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(9*c*e*(e*x)^(9/2)) - (2*a*(18*b*c - 7*a*d)*Sqrt[c
 + d*x^2])/(45*c^2*e^3*(e*x)^(5/2)) - (2*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d
^2)*Sqrt[c + d*x^2])/(15*c^3*e^5*Sqrt[e*x]) + (2*Sqrt[d]*(15*b^2*c^2 - 18*a
*b*c*d + 7*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*c^3*e^6*(Sqrt[c] + Sqrt[
d]*x)) - (2*d^(1/4)*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d
]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4)*e^(11/2)*Sqrt[c + d*x^2])
 + (d^(1/4)*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqr
t[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(15*c^(11/4)*e^(11/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} + \frac{2\int \frac{\frac{1}{2}a(18bc-7ad)+\frac{9}{2}b^2cx^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx}{9ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{\left(4\left(-\frac{45}{4}b^2c^2+\frac{3}{4}ad(18bc-7ad)\right)\right.}{45c^2e^4}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{2(15b^2c^2-ad(18bc-7ad))\sqrt{c}}{15c^3e^5\sqrt{ex}}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{2(15b^2c^2-ad(18bc-7ad))\sqrt{c}}{15c^3e^5\sqrt{ex}}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{2(15b^2c^2-ad(18bc-7ad))\sqrt{c}}{15c^3e^5\sqrt{ex}}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}} - \frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}} - \frac{2(15b^2c^2-ad(18bc-7ad))\sqrt{c}}{15c^3e^5\sqrt{ex}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.15, size = 155, normalized size = 0.35

$$\frac{2\sqrt{ex}\left(-\left((c+dx^2)\left(45b^2c^2x^4+18abcx^2(c-3dx^2)+a^2(5c^2-7cdx^2+21d^2x^4)\right)\right)+d(15b^2c^2-18abcd+7a^2d^2)x^6\sqrt{1+\frac{dx^2}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)\right)}{45c^3e^6x^5\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(11/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (2*Sqrt[e*x]*(-((c + d*x^2)*(45*b^2*c^2*x^4 + 18*a*b*c*x^2*(c - 3*d*x^2) +
a^2*(5*c^2 - 7*c*d*x^2 + 21*d^2*x^4))) + d*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2
*d^2)*x^6*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)
]))/(45*c^3*e^6*x^5*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.14, size = 667, normalized size = 1.52

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(21a^{2}d^{2}x^{4}-54abcd\,x^{4}+45b^{2}c^{2}x^{4}-7a^{2}cd\,x^{2}+18ab\,c^{2}x^{2}+5a^{2}c^{2}\right)}{45c^{3}x^{4}e^{5}\sqrt{ex}}+\frac{\left(7a^{2}d^{2}-18abcd+15b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{\quad}}{}\right.}{\sqrt{\quad}}}}{}$
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\left(-\frac{2a^{2}\sqrt{de\,x^{3}+cex}}{9e^{6}c\,x^{5}}+\frac{2a(7ad-18bc)\sqrt{de\,x^{3}+cex}}{45e^{6}c^{2}x^{3}}-\frac{2\left(de\,x^{2}+ce\right)\left(7a^{2}d^{2}-18abcd+15b^{2}c^{2}\right)}{15e^{6}c^{3}\sqrt{x\left(de\,x^{2}+ce\right)}}+\frac{\left(7a^{2}d^{2}-18\right.}{}\right.}{}$
default	$\frac{42\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c\,d^{2}x^{4}-108\sqrt{\frac{dx+\sqrt{-}}{\sqrt{-c}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/x^4*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^4-108*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*a*b*c^2*d*x^4+90*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^4-21*((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^4+54*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)
^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c
^2*d*x^4-45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^4-42*a^2*d^3*x^6+108*a*b*
c*d^2*x^6-90*b^2*c^2*d*x^6-28*a^2*c*d^2*x^4+72*a*b*c^2*d*x^4-90*b^2*c^3*x^4
+4*a^2*c^2*d*x^2-36*a*b*c^3*x^2-10*a^2*c^3)/(d*x^2+c)^(1/2)/e^5/(e*x)^(1/2)
/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^(11/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 128, normalized size = 0.29

$$-\frac{2\left(3\left(15\,b^2c^2-18\,abcd+7\,a^2d^2\right)\sqrt{d}\,x^5\mathrm{weierstrassZeta}\left(-\frac{4\,c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)+\left(3\left(15\,b^2c^2-18\,abcd+7\,a^2d^2\right)x^4+5\,a^2c^2+\left(18\,abc^2-7\,a^2cd\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{11}{2}\right)}}{45\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*sqrt(d)*x^5*weierstrassZeta(
-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (3*(15*b^2*c^2 - 18*a*b*c*d
 + 7*a^2*d^2)*x^4 + 5*a^2*c^2 + (18*a*b*c^2 - 7*a^2*c*d)*x^2)*sqrt(d*x^2 +
c)*sqrt(x))*e^(-11/2)/(c^3*x^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(11/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*e^(-11/2)/(sqrt(d*x^2 + c)*x^(11/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2}{{\left(e\,x\right)}^{11/2}\,\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(11/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^2/((e*x)^(11/2)*(c + d*x^2)^(1/2)), x)
```

3.848 $\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=242

$$-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}-\frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}}-\frac{2(77b^2c^2-5ad(22bc-9ad))\sqrt{c+dx^2}}{231c^3e^5(ex)^{3/2}}-\frac{d^{3/4}(77b^2c^2-5ad(2}{}$$

```
[Out] -2/11*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(11/2)-2/77*a*(-9*a*d+22*b*c)*(d*x^2+c)
^(1/2)/c^2/e^3/(e*x)^(7/2)-2/231*(77*b^2*c^2-5*a*d*(-9*a*d+22*b*c))*(d*x^2+
c)^(1/2)/c^3/e^5/(e*x)^(3/2)-1/231*d^(3/4)*(77*b^2*c^2-5*a*d*(-9*a*d+22*b*c
))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arcta
n(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1
/2)+x*d^(1/2))^2)^(1/2)/c^(13/4)/e^(13/2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {473, 464, 331, 335, 226}

$$-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(77b^2c^2-5ad(22bc-9ad))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{231c^{13/4}e^{13/2}\sqrt{c+dx^2}}-\frac{2\sqrt{c+dx^2}\,(77b^2c^2-5ad(22bc-9ad))}{231c^3e^5(ex)^{3/2}}-\frac{2a\sqrt{c+dx^2}\,(22bc-9ad)}{77c^2e^3(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(13/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^2])/(11*c*e*(e*x)^(11/2)) - (2*a*(22*b*c - 9*a*d)*Sqrt
[c + d*x^2])/(77*c^2*e^3*(e*x)^(7/2)) - (2*(77*b^2*c^2 - 5*a*d*(22*b*c - 9*
a*d))*Sqrt[c + d*x^2])/(231*c^3*e^5*(e*x)^(3/2)) - (d^(3/4)*(77*b^2*c^2 - 5
*a*d*(22*b*c - 9*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sq
rt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2]
)/(231*c^(13/4)*e^(13/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} + \frac{2\int \frac{\frac{1}{2}a(22bc-9ad)+\frac{11}{2}b^2cx^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx}{11ce^2}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} - \frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}} + \frac{(77b^2c^2-5ad(22bc-9ad))\int -}{77c^2e^4}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} - \frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}} - \frac{2(77b^2c^2-5ad(22bc-9ad))\sqrt{}}{231c^3e^5(ex)^{3/2}}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} - \frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}} - \frac{2(77b^2c^2-5ad(22bc-9ad))\sqrt{}}{231c^3e^5(ex)^{3/2}}$$

$$= -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} - \frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}} - \frac{2(77b^2c^2-5ad(22bc-9ad))\sqrt{}}{231c^3e^5(ex)^{3/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 196, normalized size = 0.81

$$\frac{x^{13/2}\left(-\frac{2(c+dx^2)(77b^2c^2x^4+22abcx^2(3c-5dx^2)+3a^2(7c^2-9cdx^2+15d^2x^4))}{c^3x^{11/2}} - \frac{2id(77b^2c^2-110abcd+45a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{c^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{231(ex)^{13/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(13/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x^(13/2)*((-2*(c + d*x^2)*(77*b^2*c^2*x^4 + 22*a*b*c*x^2*(3*c - 5*d*x^2) +
 3*a^2*(7*c^2 - 9*c*d*x^2 + 15*d^2*x^4)))/(c^3*x^(11/2)) - ((2*I)*d*(77*b^2
*c^2 - 110*a*b*c*d + 45*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[
Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^3*Sqrt[(I*Sqrt[c])/Sqrt[d]])))/
(231*(e*x)^(13/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 411, normalized size = 1.70

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(45a^{2}d^{2}x^{4}-110abcd\,x^{4}+77b^{2}c^{2}x^{4}-27a^{2}cd\,x^{2}+66ab\,c^{2}x^{2}+21a^{2}c^{2}\right)}{231c^{3}x^{5}e^{6}\sqrt{ex}}-\frac{\left(45a^{2}d^{2}-110abcd+77b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\ldots}}{\ldots}$
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\left(-\frac{2a^{2}\sqrt{de\,x^{3}+cex}}{11e^{7}c\,x^{6}}+\frac{2a(9ad-22bc)\sqrt{de\,x^{3}+cex}}{77e^{7}c^{2}x^{4}}-\frac{2\left(45a^{2}d^{2}-110abcd+77b^{2}c^{2}\right)\sqrt{de\,x^{3}+cex}}{231e^{7}c^{3}x^{2}}-\ldots\right)}{\sqrt{\ldots}}$
default	$-\frac{45\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^{2}d^{2}x^{5}-110\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/231/(d*x^2+c)^(1/2)/x^5*(45*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elli
pticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2
*d^2*x^5-110*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c*d*x^5+77*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^2*x^5+90*a^2*d^3*x^6-220*a*b*c*d^2
*x^6+154*b^2*c^2*d*x^6+36*a^2*c*d^2*x^4-88*a*b*c^2*d*x^4+154*b^2*c^3*x^4-12
*a^2*c^2*d*x^2+132*a*b*c^3*x^2+42*a^2*c^3)/c^3/e^6/(e*x)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-13/2)*integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*x^(13/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 119, normalized size = 0.49

$$-\frac{2\left((77\,b^2c^2-110\,abcd+45\,a^2d^2)\sqrt{d}\,x^6\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)+((77\,b^2c^2-110\,abcd+45\,a^2d^2)x^4+21\,a^2c^2+3\,(22\,abc^2-9\,a^2cd)x^2)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{13}{2}\right)}}{231\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/231*((77*b^2*c^2 - 110*a*b*c*d + 45*a^2*d^2)*sqrt(d)*x^6*weierstrassPInv
erse(-4*c/d, 0, x) + ((77*b^2*c^2 - 110*a*b*c*d + 45*a^2*d^2)*x^4 + 21*a^2*
c^2 + 3*(22*a*b*c^2 - 9*a^2*c*d)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-13/2)/(c
^3*x^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(13/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3279 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*e^(-13/2)/(sqrt(d*x^2 + c)*x^(13/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2}{{\left(e\,x\right)}^{13/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(13/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^2/((e*x)^(13/2)*(c + d*x^2)^(1/2)), x)
```

3.849 $\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=296

$$\frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}}+\frac{5\left(117b^2c^2-198abcd+77a^2d^2\right)e^3\sqrt{ex}\,\sqrt{c+dx^2}}{231d^4}-\frac{\left(117b^2c^2-198abcd+77a^2d^2\right)e(ex)^{5/}}{77cd^3}$$

```
[Out] (-a*d+b*c)^2*(e*x)^(9/2)/c/d^2/e/(d*x^2+c)^(1/2)-1/77*(77*a^2*d^2-198*a*b*c
*d+117*b^2*c^2)*e*(e*x)^(5/2)*(d*x^2+c)^(1/2)/c/d^3+2/11*b^2*(e*x)^(9/2)*(d
*x^2+c)^(1/2)/d^2/e+5/231*(77*a^2*d^2-198*a*b*c*d+117*b^2*c^2)*e^3*(e*x)^(1
/2)*(d*x^2+c)^(1/2)/d^4-5/462*c^(3/4)*(77*a^2*d^2-198*a*b*c*d+117*b^2*c^2)*
e^(7/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*
arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/
(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(17/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {474, 470, 327, 335, 226}

$$-\frac{5c^{3/4}e^{7/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(77a^2d^2-198abcd+117b^2c^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{462d^{17/4}\sqrt{c+dx^2}}+\frac{5e^3\sqrt{ex}\,\sqrt{c+dx^2}\left(77a^2d^2-198abcd+117b^2c^2\right)}{231d^4}-\frac{e(ex)^{5/2}\sqrt{c+dx^2}\left(77a^2d^2-198abcd+117b^2c^2\right)}{77cd^3}+\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (5*(117*b^2*c^2 - 1
98*a*b*c*d + 77*a^2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2])/(231*d^4) - ((117*b
^2*c^2 - 198*a*b*c*d + 77*a^2*d^2)*e*(e*x)^(5/2)*Sqrt[c + d*x^2])/(77*c*d^3
) + (2*b^2*(e*x)^(9/2)*Sqrt[c + d*x^2])/(11*d^2*e) - (5*c^(3/4)*(117*b^2*c^
2 - 198*a*b*c*d + 77*a^2*d^2)*e^(7/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(462*d^(17/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\int \frac{(ex)^{7/2}\left(\frac{1}{2}\left(-2a^2d^2+9(bc-ad)^2\right)-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11d^2e} - \frac{(117b^2c^2-198abcd+77a^2d^2)\int \frac{(ex}{\sqrt{c+}}}{22cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} - \frac{(117b^2c^2-198abcd+77a^2d^2)\,e(ex)^{5/2}\sqrt{c+dx^2}}{77cd^3} + \frac{2b^2(ex)^{9/}}{1}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} + \frac{5(117b^2c^2-198abcd+77a^2d^2)\,e^3\sqrt{ex}\,\sqrt{c+dx^2}}{231d^4} - \frac{(117b^2c^2}{}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} + \frac{5(117b^2c^2-198abcd+77a^2d^2)\,e^3\sqrt{ex}\,\sqrt{c+dx^2}}{231d^4} - \frac{(117b^2c^2}{}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} + \frac{5(117b^2c^2-198abcd+77a^2d^2)\,e^3\sqrt{ex}\,\sqrt{c+dx^2}}{231d^4} - \frac{(117b^2c^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.19, size = 226, normalized size = 0.76

$$\frac{e^3\sqrt{ex}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(77a^2d^2(5c+2dx^2)+66abd(-15c^2-6cdx^2+2d^2x^4)+3b^2(195c^3+78c^2dx^2-26cd^2x^4+14d^3x^6)\right)-5ic(117b^2c^2-198abcd+77a^2d^2)\sqrt{1+\frac{c}{dx^2}}\sqrt{x}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{231\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(77*a^2*d^2*(5*c + 2*d*x^2) + 66*
a*b*d*(-15*c^2 - 6*c*d*x^2 + 2*d^2*x^4) + 3*b^2*(195*c^3 + 78*c^2*d*x^2 - 2
6*c*d^2*x^4 + 14*d^3*x^6)) - (5*I)*c*(117*b^2*c^2 - 198*a*b*c*d + 77*a^2*d^
2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]
]/Sqrt[x]], -1]))/(231*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^4*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.16, size = 407, normalized size = 1.38

method	result
default	$-\frac{e^{3}\sqrt{ex}\left(-84b^{2}d^{4}x^{7}+385\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right.\right.}{}$
elliptic	$\frac{\sqrt{ex\left(dx^{2}+c\right)}\,\sqrt{ex}\left(\frac{e^{4}xc\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{d^{4}\sqrt{\left(x^{2}+\frac{c}{d}\right)dex}}+\frac{2b^{2}e^{3}x^{4}\sqrt{de\,x^{3}+cex}}{11d^{2}}+\frac{2\left(\frac{b(2ad-bc)e^{4}}{d^{2}}-\frac{9b^{2}e^{4}c}{11d^{2}}\right)x^{2}\sqrt{de\,x^{3}+ce}}{7de}\right.}{}$
risch	$\frac{2\left(21b^{2}x^{4}d^{2}+66ab\,d^{2}x^{2}-60b^{2}cd\,x^{2}+77a^{2}d^{2}-264abcd+177b^{2}c^{2}\right)x\sqrt{d\,x^{2}+c}\;e^{4}}{231d^{4}\sqrt{ex}}-\frac{c\left(\frac{308a^{2}d\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/462*e^3/x*(e*x)^(1/2)*(-84*b^2*d^4*x^7+385*((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(
-c*d)^(1/2)*a^2*c*d^2-990*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*
d+585*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3-264*a*b*d^4*x^5+156*
b^2*c*d^3*x^5-308*a^2*d^4*x^3+792*a*b*c*d^3*x^3-468*b^2*c^2*d^2*x^3-770*a^2
*c*d^3*x+1980*a*b*c^2*d^2*x-1170*b^2*c^3*d*x)/(d*x^2+c)^(1/2)/d^5
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 206, normalized size = 0.70

$$-\frac{5\,(117\,b^2c^4-198\,abc^3d+77\,a^2c^2d^2+(117\,b^2c^3d-198\,abc^2d^2+77\,a^2cd^3)x^2)\sqrt{d}\,e^{\frac{7}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-(42\,b^2d^4x^6+585\,b^2c^3d-990\,abc^2d^2+385\,a^2cd^3-6\,(13\,b^2cd^3-22\,abd^4)x^4+2\,(117\,b^2c^2d^2-198\,abcd^3+77\,a^2d^4)x^2)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{7}{2}}}{231\,(d^6x^2+cd^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/231*(5*(117*b^2*c^4 - 198*a*b*c^3*d + 77*a^2*c^2*d^2 + (117*b^2*c^3*d -
198*a*b*c^2*d^2 + 77*a^2*c*d^3)*x^2)*sqrt(d)*e^(7/2)*weierstrassPInverse(-4
*c/d, 0, x) - (42*b^2*d^4*x^6 + 585*b^2*c^3*d - 990*a*b*c^2*d^2 + 385*a^2*c
*d^3 - 6*(13*b^2*c*d^3 - 22*a*b*d^4)*x^4 + 2*(117*b^2*c^2*d^2 - 198*a*b*c*d
^3 + 77*a^2*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(x)*e^(7/2))/(d^6*x^2 + c*d^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^(7/2)*e^(7/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
[Out] int(((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.850 $\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=436

$$\frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}}-\frac{(77b^2c^2-126abcd+45a^2d^2)\,e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e}+\frac{(77b^2c^2-126abc}{15d}$$

```
[Out] (-a*d+b*c)^2*(e*x)^(7/2)/c/d^2/e/(d*x^2+c)^(1/2)-1/45*(45*a^2*d^2-126*a*b*c
*d+77*b^2*c^2)*e*(e*x)^(3/2)*(d*x^2+c)^(1/2)/c/d^3+2/9*b^2*(e*x)^(7/2)*(d*x
^2+c)^(1/2)/d^2/e+1/15*(45*a^2*d^2-126*a*b*c*d+77*b^2*c^2)*e^2*(e*x)^(1/2)*
(d*x^2+c)^(1/2)/d^(7/2)/(c^(1/2)+x*d^(1/2))-1/15*c^(1/4)*(45*a^2*d^2-126*a*
b*c*d+77*b^2*c^2)*e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(si
n(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^
(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(15/4)/(d*x^2+c)^(1/2)+1/3
0*c^(1/4)*(45*a^2*d^2-126*a*b*c*d+77*b^2*c^2)*e^(5/2)*(cos(2*arctan(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))
),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/
d^(15/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {474, 470, 327, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{c}\,e^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(45a^2d^2-126abcd+77b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{30d^{15/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\,e^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(45a^2d^2-126abcd+77b^2c^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{15/4}\sqrt{c+dx^2}}+\frac{e^2\sqrt{ex}\,\sqrt{c+dx^2}\,(45a^2d^2-126abcd+77b^2c^2)}{15d^{7/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{e(ex)^{3/2}\sqrt{c+dx^2}\,(45a^2d^2-126abcd+77b^2c^2)}{45cd^3}+\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((77*b^2*c^2 - 126*
a*b*c*d + 45*a^2*d^2)*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(45*c*d^3) + (2*b^2*(e
*x)^(7/2)*Sqrt[c + d*x^2])/(9*d^2*e) + ((77*b^2*c^2 - 126*a*b*c*d + 45*a^2*
d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(15*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) - (c
^(1/4)*(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x
)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(15*d^(15/4)*Sqrt[c + d*x^2]) + (c^(1/4)*
(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[
(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], 1/2])/(30*d^(15/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\int \frac{(ex)^{5/2}\left(\frac{1}{2}\left(-2a^2d^2+7(bc-ad)^2\right)-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)\int \frac{(ex)^{5/2}}{\sqrt{c+dx^2}}}{18cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3} + \frac{2b^2(ex)^{7/2}}{9d}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3} + \frac{2b^2(ex)^{7/2}}{9d}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3} + \frac{2b^2(ex)^{7/2}}{9d}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-126abcd+45a^2d^2\right)e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3} + \frac{2b^2(ex)^{7/2}}{9d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.13, size = 133, normalized size = 0.31

$$\frac{e(ex)^{3/2}\left(-45a^2d^2+18abd\left(7c+2dx^2\right)+b^2\left(-77c^2-22cdx^2+10d^2x^4\right)+3\left(77b^2c^2-126abcd+45a^2d^2\right)\sqrt{1+\frac{c}{dx^2}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{45d^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (e*(e*x)^(3/2)*(-45*a^2*d^2 + 18*a*b*d*(7*c + 2*d*x^2) + b^2*(-77*c^2 - 22*
c*d*x^2 + 10*d^2*x^4) + 3*(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*Sqrt[1 +
c/(d*x^2)]*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(45*d^3*Sqrt[c
 + d*x^2])
```

Maple [A]

time = 0.15, size = 618, normalized size = 1.42

method	result
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\,\sqrt{ex}\left(-\frac{e^3x^2\left(a^2d^2-2abcd+b^2c^2\right)}{d^3\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2e^2x^3\sqrt{de\,x^3+cex}}{9d^2}+\frac{2\left(\frac{b(2ad-bc)e^3}{d^2}-\frac{7b^2e^3c}{9d^2}\right)x\sqrt{de\,x^3+c}}{5de}\right.}{}$
risch	$\frac{2b\,x^2\left(5bd\,x^2+18ad-16bc\right)\sqrt{d\,x^2+c}\;e^3}{45d^3\sqrt{ex}}+\frac{\left(\left(15a^2d^2-48abcd+31b^2c^2\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right.}{\sqrt{-cd}}}\right.}{}$

default	$\frac{e^2\sqrt{ex}\left(20b^2x^6d^3+270\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\text{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/90*e^2/x*(e*x)^(1/2)*(20*b^2*x^6*d^3+270*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d
)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*
c*d^2-756*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d+462*((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d
)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1
/2))*b^2*c^3-135*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2+378*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*a*b*c^2*d-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3+72*a*b*d^3*x^
4-44*b^2*c*d^2*x^4-90*a^2*d^3*x^2+252*a*b*c*d^2*x^2-154*b^2*c^2*d*x^2)/(d*x
^2+c)^(1/2)/d^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 177, normalized size = 0.41

$$-\frac{3\,(77\,b^2c^3-126\,abc^2d+45\,a^2cd^2+(77\,b^2c^2d-126\,abcd^2+45\,a^2d^3)x^2)\sqrt{d}\,e^{\frac{5}{2}}\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)-(10\,b^2d^3x^5-2\,(11\,b^2cd^2-18\,abd^3)x^3-(77\,b^2c^2d-126\,abcd^2+45\,a^2d^3)x)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{5}{2}}}{45\,(d^5x^2+cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/45*(3*(77*b^2*c^3 - 126*a*b*c^2*d + 45*a^2*c*d^2 + (77*b^2*c^2*d - 126*a
*b*c*d^2 + 45*a^2*d^3)*x^2)*sqrt(d)*e^(5/2)*weierstrassZeta(-4*c/d, 0, weie
rstrassPInverse(-4*c/d, 0, x)) - (10*b^2*d^3*x^5 - 2*(11*b^2*c*d^2 - 18*a*b
*d^3)*x^3 - (77*b^2*c^2*d - 126*a*b*c*d^2 + 45*a^2*d^3)*x)*sqrt(d*x^2 + c)*
sqrt(x)*e^(5/2))/(d^5*x^2 + c*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{5}{2}} (a + bx^2)^2}{(c + dx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
[Out] Integral((e*x)**(5/2)*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*x^(5/2)*e^(5/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}\,(b\,x^2+a)^2}{(d\,x^2+c)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
[Out] int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.851 $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=245

$$\frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}}-\frac{(45b^2c^2-70abcd+21a^2d^2)\,e\sqrt{ex}\,\sqrt{c+dx^2}}{21cd^3}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e}+\frac{(45b^2c^2-70abcd+\cdots}{\cdots}$$

```
[Out] (-a*d+b*c)^2*(e*x)^(5/2)/c/d^2/e/(d*x^2+c)^(1/2)+2/7*b^2*(e*x)^(5/2)*(d*x^2
+c)^(1/2)/d^2/e-1/21*(21*a^2*d^2-70*a*b*c*d+45*b^2*c^2)*e*(e*x)^(1/2)*(d*x^
2+c)^(1/2)/c/d^3+1/42*(21*a^2*d^2-70*a*b*c*d+45*b^2*c^2)*e^(3/2)*(cos(2*arc
tan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2)
)^2)^(1/2)/c^(1/4)/d^(13/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {474, 470, 327, 335, 226}

$$\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(21a^2d^2-70abcd+45b^2c^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{42\sqrt[4]{c}\,d^{13/4}\sqrt{c+dx^2}}-\frac{e\sqrt{ex}\,\sqrt{c+dx^2}\left(21a^2d^2-70abcd+45b^2c^2\right)}{21cd^3}+\frac{(ex)^{5/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((45*b^2*c^2 - 70*a
*b*c*d + 21*a^2*d^2)*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(21*c*d^3) + (2*b^2*(e*x)
^(5/2)*Sqrt[c + d*x^2])/(7*d^2*e) + ((45*b^2*c^2 - 70*a*b*c*d + 21*a^2*d^2)
*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*El
lipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(42*c^(1/4)*
d^(13/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\int \frac{(ex)^{3/2}\left(\frac{1}{2}(-2a^2d^2+5(bc-ad)^2)-b^2cdx^2\right)}{\sqrt{c+dx^2}}dx}{cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e} - \frac{(45b^2c^2-70abcd+21a^2d^2)\int \frac{(ex)^{3/}}{\sqrt{c+d}}}{14cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} - \frac{(45b^2c^2-70abcd+21a^2d^2)\,e\sqrt{ex}\,\sqrt{c+dx^2}}{21cd^3} + \frac{2b^2(ex)^{5/2}\sqrt{c}}{7d^2e}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} - \frac{(45b^2c^2-70abcd+21a^2d^2)\,e\sqrt{ex}\,\sqrt{c+dx^2}}{21cd^3} + \frac{2b^2(ex)^{5/2}\sqrt{c}}{7d^2e}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}} - \frac{(45b^2c^2-70abcd+21a^2d^2)\,e\sqrt{ex}\,\sqrt{c+dx^2}}{21cd^3} + \frac{2b^2(ex)^{5/2}\sqrt{c}}{7d^2e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 191, normalized size = 0.78

$$\frac{e\sqrt{ex}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(-21a^2d^2+14abd(5c+2dx^2)-3b^2(15c^2+6cdx^2-2d^2x^4)\right)+i(45b^2c^2-70abcd+21a^2d^2)\sqrt{1+\frac{c}{dx^2}}\sqrt{x}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{21\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (e*Sqrt[e*x]*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(-21*a^2*d^2 + 14*a*b*d*(5*c + 2*d*
x^2) - 3*b^2*(15*c^2 + 6*c*d*x^2 - 2*d^2*x^4)) + I*(45*b^2*c^2 - 70*a*b*c*d
 + 21*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt
[c])/Sqrt[d]]/Sqrt[x]], -1]))/(21*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3*Sqrt[c + d*
x^2])
```

Maple [A]
time = 0.15, size = 363, normalized size = 1.48

method	result

elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\,\sqrt{ex}\left(-\frac{e^2x\left(a^2d^2-2abcd+b^2c^2\right)}{d^3\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2e\,x^2\sqrt{de\,x^3+cex}}{7d^2}+\frac{2\left(\frac{b(2ad-bc)e^2}{d^2}-\frac{5b^2e^2c}{7d^2}\right)\sqrt{de\,x^3+cex}}{3de}\right.}{}$
default	$\frac{e\sqrt{ex}\left(21\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d\right.}{}$
risch	$\frac{2b(3bd\,x^2+14ad-12bc)x\sqrt{d\,x^2+c}\,e^2}{21d^3\sqrt{ex}}+\frac{\left(\frac{21a^2d\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}}{\sqrt{de\,x^3+cex}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*e/x*(e*x)^(1/2)*(21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*d^2-7
0*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c
*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c*d+45*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*(-c*d)^(1/2)*b^2*c^2+12*b^2*d^3*x^5+56*a*b*d^3*x^3-36*b^2*c*d^2*x^3-
42*a^2*d^3*x+140*a*x*b*c*d^2-90*x*b^2*c^2*d)/(d*x^2+c)^(1/2)/d^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] e^(3/2)*integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 163, normalized size = 0.67

$$\frac{(45\,b^2c^3-70\,abc^2d+21\,a^2cd^2+(45\,b^2c^2d-70\,abcd^2+21\,a^2d^3)x^2)\sqrt{d}\,e^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)+(6\,b^2d^3x^4-45\,b^2c^2d+70\,abcd^2-21\,a^2d^3-2\,(9\,b^2cd^2-14\,abd^3)x^2)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{3}{2}}}{21\,(d^5x^2+cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] 1/21*((45*b^2*c^3 - 70*a*b*c^2*d + 21*a^2*c*d^2 + (45*b^2*c^2*d - 70*a*b*c*
d^2 + 21*a^2*d^3)*x^2)*sqrt(d)*e^(3/2)*weierstrassPInverse(-4*c/d, 0, x) +
(6*b^2*d^3*x^4 - 45*b^2*c^2*d + 70*a*b*c*d^2 - 21*a^2*d^3 - 2*(9*b^2*c*d^2
- 14*a*b*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(x)*e^(3/2))/(d^5*x^2 + c*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
[Out] Integral((e*x)**(3/2)*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*x^(3/2)*e^(3/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(e\,x)}^{3/2}\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
[Out] int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.852 $\int \frac{\sqrt{ex}\,(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=384

$$\frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e}-\frac{(21b^2c^2-30abcd+5a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{5cd^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{(21b^2c^2-30abcd+\cdots}{}$$

```
[Out] (-a*d+b*c)^2*(e*x)^(3/2)/c/d^2/e/(d*x^2+c)^(1/2)+2/5*b^2*(e*x)^(3/2)*(d*x^2
+c)^(1/2)/d^2/e-1/5*(5*a^2*d^2-30*a*b*c*d+21*b^2*c^2)*(e*x)^(1/2)*(d*x^2+c)
^(1/2)/c/d^(5/2)/(c^(1/2)+x*d^(1/2))+1/5*(5*a^2*d^2-30*a*b*c*d+21*b^2*c^2)*
(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/
(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^(11/4)/(d*x^2+c)^(1/2)-1/10*(5*a^2*d
^2-30*a*b*c*d+21*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(si
n(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^
(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^(11/4)/(d*
x^2+c)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {474, 470, 335, 311, 226, 1210}

$$-\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(5a^2d^2-30abcd+21b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{3/4}d^{11/4}\sqrt{c+dx^2}}+\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(5a^2d^2-30abcd+21b^2c^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{11/4}\sqrt{c+dx^2}}-\frac{\sqrt{ex}\,\sqrt{c+dx^2}\,(5a^2d^2-30abcd+21b^2c^2)}{5cd^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*(e*x)^(3/2)*
Sqrt[c + d*x^2])/(5*d^2*e) - ((21*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2)*Sqrt[e*
x]*Sqrt[c + d*x^2])/(5*c*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) + ((21*b^2*c^2 - 30
*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[
c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], 1/2])/(5*c^(3/4)*d^(11/4)*Sqrt[c + d*x^2]) - ((21*b^2*c^2 - 30*a*b*c*d +
 5*a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[
d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(
10*c^(3/4)*d^(11/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\,(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}} - \frac{\int \frac{\sqrt{ex}\,\left(\frac{1}{2}(-2a^2d^2+3(bc-ad)^2)-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e} - \frac{(21b^2c^2-30abcd+5a^2d^2)\int \frac{\sqrt{ex}}{\sqrt{c+dx}}}{10cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e} - \frac{(21b^2c^2-30abcd+5a^2d^2)\,\mathrm{Subst}\left(\int -\right.}{5cd^2e}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e} - \frac{(21b^2c^2-30abcd+5a^2d^2)\,\mathrm{Subst}\left(\int -\right.}{5\sqrt{c}\,d^{5/2}}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e} - \frac{(21b^2c^2-30abcd+5a^2d^2)\sqrt{ex}\,\sqrt{c+}}{5cd^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.11, size = 119, normalized size = 0.31

$$\frac{\sqrt{ex}\left(x(-10abcd+5a^2d^2+b^2c(7c+2dx^2))+(-21b^2c^2+30abcd-5a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{5cd^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[e*x]*(x*(-10*a*b*c*d + 5*a^2*d^2 + b^2*c*(7*c + 2*d*x^2)) + (-21*b^2*
c^2 + 30*a*b*c*d - 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4,
 1/2, 3/4, -(c/(d*x^2))]))/(5*c*d^2*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.14, size = 597, normalized size = 1.55

method	result

elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\,\sqrt{ex}\left(\frac{e\,x^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{d^{2}c\sqrt{\left(x^{2}+\frac{c}{d}\right)dex}}+\frac{2b^{2}x\sqrt{de\,x^{3}+cex}}{5d^{2}}+\frac{\left(\frac{b(2ad-bc)e}{d^{2}}-\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)e}{2d^{2}c}-\frac{3b^{2}ce}{5d^{2}}\right)\sqrt{-}}{}\right.}{}$
risch	$\frac{2b^{2}x^{2}\sqrt{d\,x^{2}+c}\;e}{5d^{2}\sqrt{ex}}+\frac{\left(\frac{\left(10abd-8b^{2}c\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-c}}{}\right.}{d\sqrt{de\,x^{3}}}\right.}{}$
default	$-\frac{\sqrt{ex}\left(10\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\;\mathrm{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c\,d^{2}-60\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/10*(e*x)^(1/2)*(10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
```

```
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-60*((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2),1/2*2^(1/2))*a*b*c^2*d+42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-5*((d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2),1/2*2^(1/2))*a^2*c*d^2+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*El
lipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-21*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-4*b^2*c*d^2*x^4-10*a^2*d^3*x^2+20*a*b*c*
d^2*x^2-14*b^2*c^2*d*x^2)/(d*x^2+c)^(1/2)/d^3/x/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 156, normalized size = 0.41

$$\frac{(21\,b^2c^3 - 30\,abc^2d + 5\,a^2cd^2 + (21\,b^2c^2d - 30\,abcd^2 + 5\,a^2d^3)x^2)\sqrt{d}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,c}{d}, 0, \mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d}, 0, x\right)\right) + (2\,b^2cd^2x^3 + (7\,b^2c^2d - 10\,abcd^2 + 5\,a^2d^3)x)\sqrt{dx^2 + c}\,\sqrt{x}\,e^{\frac{1}{2}}}{5\,(cd^4x^2 + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*((21*b^2*c^3 - 30*a*b*c^2*d + 5*a^2*c*d^2 + (21*b^2*c^2*d - 30*a*b*c*d^
2 + 5*a^2*d^3)*x^2)*sqrt(d)*e^(1/2)*weierstrassZeta(-4*c/d, 0, weierstrassP
Inverse(-4*c/d, 0, x)) + (2*b^2*c*d^2*x^3 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 5
*a^2*d^3)*x)*sqrt(d*x^2 + c)*sqrt(x)*e^(1/2))/(c*d^4*x^2 + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\left(a + bx^2\right)^2}{\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(e*x)*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(x)*e^(1/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}\,\left(b\,x^2+a\right)^2}{\left(d\,x^2+c\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

```
[Out] int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.853 $\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=193

$$\frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2\sqrt{ex}\,\sqrt{c+dx^2}}{3d^2e}-\frac{(5b^2c^2-6abcd-3a^2d^2)\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,F\Big(2\,\text{ta}}{6c^{5/4}d^{9/4}\sqrt{e}\,\sqrt{c+dx^2}}$$

```
[Out] (-a*d+b*c)^2*(e*x)^(1/2)/c/d^2/e/(d*x^2+c)^(1/2)+2/3*b^2*(e*x)^(1/2)*(d*x^2
+c)^(1/2)/d^2/e-1/6*(-3*a^2*d^2-6*a*b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))
,1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c
^(5/4)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {474, 470, 335, 226}

$$-\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(-3a^2d^2-6abcd+5b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{6c^{5/4}d^{9/4}\sqrt{e}\,\sqrt{c+dx^2}}+\frac{\sqrt{ex}\,(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2\sqrt{ex}\,\sqrt{c+dx^2}}{3d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(c*d^2*e*Sqrt[c + d*x^2]) + (2*b^2*Sqrt[e*x]*Sqrt
[c + d*x^2])/(3*d^2*e) - ((5*b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*(Sqrt[c] + Sq
rt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(6*c^(5/4)*d^(9/4)*Sqrt[e]*Sqrt[c
+ d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}} - \frac{\int \frac{\frac{1}{2}(-2a^2d^2+(bc-ad)^2)-b^2cdx^2}{\sqrt{ex}\,\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2\sqrt{ex}\,\sqrt{c+dx^2}}{3d^2e} - \frac{(5b^2c^2-6abcd-3a^2d^2)\int \frac{1}{\sqrt{ex}\,\sqrt{c+dx}}}{6cd^2}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2\sqrt{ex}\,\sqrt{c+dx^2}}{3d^2e} - \frac{(5b^2c^2-6abcd-3a^2d^2)\,\text{Subst}\left(\int \frac{}{\sqrt{c+}}\right.}{3cd^2e}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2\sqrt{ex}\,\sqrt{c+dx^2}}{3d^2e} - \frac{(5b^2c^2-6abcd-3a^2d^2)\left(\sqrt{c}+\sqrt{d}\,x\right)}{6c^{5/4}d^9}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.12, size = 174, normalized size = 0.90

$$\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,x(-6abcd+3a^2d^2+b^2c(5c+2dx^2))+i(-5b^2c^2+6abcd+3a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^{3/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^2\sqrt{ex}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[d]]*x*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(5*c + 2*d*x^2
)) + I*(-5*b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(3/2)*Ell
ipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(3*c*Sqrt[(I*Sqrt
[c])/Sqrt[d]]*d^2*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.15, size = 341, normalized size = 1.77

method	result
elliptic	$\frac{\sqrt{ex\left(dx^2+c\right)}\left(\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{d^2c\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2\sqrt{dex^3+cex}}{3d^2e}+\frac{\left(\frac{b(2ad-bc)}{d^2}+\frac{a^2d^2-2abcd+b^2c^2}{2d^2c}-\frac{b^2c}{3d^2}\right)\sqrt{-cd}\sqrt{\frac{\left(x\right.}{}}}{}\right.}{\sqrt{ex}\sqrt{dx^2+c}}$
default	$\frac{3\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\operatorname{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^2+6\sqrt{\frac{dx+}{\sqrt{}}}}{}$
risch	$\frac{2b^2x\sqrt{dx^2+c}}{3d^2\sqrt{ex}}+\frac{\left(\frac{6ab\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\sqrt{}\right.}{\sqrt{-}}}\right.}{\sqrt{dex^3+cex}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*d^2+6*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*(-c*d)^(1/2)*a*b*c*d-5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*
b^2*c^2+4*b^2*c*d^2*x^3+6*a^2*d^3*x-12*a*x*b*c*d^2+10*x*b^2*c^2*d)/(d*x^2+c
)^(1/2)/c/(e*x)^(1/2)/d^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 144, normalized size = 0.75

$$-\frac{\left((5\,b^2c^3-6\,abc^2d-3\,a^2cd^2+(5\,b^2c^2d-6\,abcd^2-3\,a^2d^3)x^2)\sqrt{d}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-(2\,b^2cd^2x^2+5\,b^2c^2d-6\,abcd^2+3\,a^2d^3)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{3\,(cd^4x^2+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((5*b^2*c^3 - 6*a*b*c^2*d - 3*a^2*c*d^2 + (5*b^2*c^2*d - 6*a*b*c*d^2 -
 3*a^2*d^3)*x^2)*sqrt(d)*weierstrassPInverse(-4*c/d, 0, x) - (2*b^2*c*d^2*x
^2 + 5*b^2*c^2*d - 6*a*b*c*d^2 + 3*a^2*d^3)*sqrt(d*x^2 + c)*sqrt(x))*e^(-1/
2)/(c*d^4*x^2 + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{ex}\,\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(sqrt(e*x)*(c + d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*e^(-1/2)/((d*x^2 + c)^(3/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^2}{\sqrt{e\,x}\,{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(3/2)),x)
[Out] int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.854 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=393

$$-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}-\frac{(b^2c^2-2abcd+3a^2d^2)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}}+\frac{(3b^2c^2-2abcd+3a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{c^2d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{(3b^2c^2-2a}{}$$

```
[Out] -(3*a^2*d^2-2*a*b*c*d+b^2*c^2)*(e*x)^(3/2)/c^2/d/e^3/(d*x^2+c)^(1/2)-2*a^2/
c/e/(e*x)^(1/2)/(d*x^2+c)^(1/2)+(3*a^2*d^2-2*a*b*c*d+3*b^2*c^2)*(e*x)^(1/2)
*(d*x^2+c)^(1/2)/c^2/d^(3/2)/e^2/(c^(1/2)+x*d^(1/2))-(3*a^2*d^2-2*a*b*c*d+3
*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c
)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)+1/2*
(3*a^2*d^2-2*a*b*c*d+3*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*Ellipt
icF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2
)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(7/4)/e^(3/2
)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {473, 468, 335, 311, 226, 1210}

$$\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}(3a^2d^2-2abcd+3b^2c^2)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}(3a^2d^2-2abcd+3b^2c^2)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{(ex)^{3/2}(3a^2d^2-2abcd+b^2c^2)}{c^2de^3\sqrt{c+dx^2}}+\frac{\sqrt{ex}\sqrt{c+dx^2}(3a^2d^2-2abcd+3b^2c^2)}{c^2d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*Sqrt[c + d*x^2]) - ((b^2*c^2 - 2*a*b*c*d + 3*a^2*d^
2)*(e*x)^(3/2))/(c^2*d*e^3*Sqrt[c + d*x^2]) + ((3*b^2*c^2 - 2*a*b*c*d + 3*a
^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(c^2*d^(3/2)*e^2*(Sqrt[c] + Sqrt[d]*x))
- ((3*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^
2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) + ((3*b^2*c^2 -
 2*a*b*c*d + 3*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + S
qrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2
])/(2*c^(7/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}dx = -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} + \frac{2\int \frac{\sqrt{ex}\left(\frac{1}{2}a(2bc-3ad)+\frac{1}{2}b^2cx^2\right)}{(c+dx^2)^{3/2}}dx}{ce^2}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} - \frac{(b^2c^2-2abcd+3a^2d^2)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}} - \frac{\left(2ab-\frac{3b^2c}{d}-\frac{3a^2d}{c}\right)\int}{2ce^2}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} - \frac{(b^2c^2-2abcd+3a^2d^2)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}} - \frac{\left(2ab-\frac{3b^2c}{d}-\frac{3a^2d}{c}\right)\mathrm{Su}}{}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} - \frac{(b^2c^2-2abcd+3a^2d^2)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}} + \frac{(3b^2c^2-2abcd+3a^2d^2}{}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} - \frac{(b^2c^2-2abcd+3a^2d^2)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}} + \frac{(3b^2c^2-2abcd+3a^2d^2}{c^2d^{3/2}e^2\left(\sqrt{c}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 126, normalized size = 0.32

$$\frac{x\left(-3b^2c^2x^2+6abcdx^2-3a^2d(2c+3dx^2)+(3b^2c^2-2abcd+3a^2d^2)x^2\sqrt{1+\frac{dx^2}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)\right)}{3c^2d(ex)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x*(-3*b^2*c^2*x^2 + 6*a*b*c*d*x^2 - 3*a^2*d*(2*c + 3*d*x^2) + (3*b^2*c^2 -
 2*a*b*c*d + 3*a^2*d^2)*x^2*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/4,
 7/4, -((d*x^2)/c)]))/(3*c^2*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.14, size = 594, normalized size = 1.51

method	result
elliptic	$\dfrac{\sqrt{ex\,(d\,x^2+c)}\left(-\frac{x^2\left(a^2d^2-2abcd+b^2c^2\right)}{de\,c^2\sqrt{\left(x^2+\frac{c}{d}\right)dex}}-\frac{2\left(de\,x^2+ce\right)a^2}{c^2e^2\sqrt{x\,(de\,x^2+ce)}}+\frac{\left(\frac{b^2}{ed}+\frac{a^2d^2-2abcd+b^2c^2}{2d\,c^2e}+\frac{d\,a^2}{c^2e}\right)\sqrt{-cd}\,\sqrt{\left(\right.}}{}\right.}{}$

risch	$-\frac{2a^2\sqrt{d\,x^2+c}}{c^2e\sqrt{ex}}+\frac{\left(\left(a^2d^2+b^2c^2\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}}{\ldots}\right.\right.}{d^2\sqrt{de\,x^3+\ldots}}$
default	$-\frac{3\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2-2\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2-2*((d*x+(-c*d)^(1/2))/(-c*d)^
```

```
(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))
*a*b*c^2*d+3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^
(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-6*((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)
^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/
2))*a^2*c*d^2+4*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d-6*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*b^2*c^3+6*a^2*d^3*x^2-4*a*b*c*d^2*x^2+2*b^2*c^2*d*x^2+4*a^2*c*d^2)
/(d*x^2+c)^(1/2)/d^2/e/(e*x)^(1/2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 158, normalized size = 0.40

$$-\frac{\left(\left(\left(3\,b^2c^2d-2\,abcd^2+3\,a^2d^3\right)x^3+\left(3\,b^2c^3-2\,abc^2d+3\,a^2cd^2\right)x\right)\sqrt{d}\ \mathrm{weierstrassZeta}\left(-\frac{4\,c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)+\left(2\,a^2cd^2+\left(b^2c^2d-2\,abcd^2+3\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}\ \sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{c^2d^3x^3+c^3d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] -((((3*b^2*c^2*d - 2*a*b*c*d^2 + 3*a^2*d^3)*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d +
 3*a^2*c*d^2)*x)*sqrt(d)*weierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*
c/d, 0, x)) + (2*a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + 3*a^2*d^3)*x^2)*sqr
t(d*x^2 + c)*sqrt(x))*e^(-3/2)/(c^2*d^3*x^3 + c^3*d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{\left(ex\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(3/2),x)
[Out] Integral((a + b*x**2)**2/((e*x)**(3/2)*(c + d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*e^(-3/2)/((d*x^2 + c)^(3/2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2}{{\left(e\,x\right)}^{3/2}\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x)
[Out] int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)), x)
```

3.855 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=207

$$-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}-\frac{(3b^2c^2-6abcd+5a^2d^2)\sqrt{ex}}{3c^2de^3\sqrt{c+dx^2}}+\frac{(3b^2c^2+ad(6bc-5ad))\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+}{\left(\sqrt{c}+\right.}}}{6c^{9/4}d^{5/4}e^{5/2}\sqrt{c-}}$$

```
[Out] -2/3*a^2/c/e/(e*x)^(3/2)/(d*x^2+c)^(1/2)-1/3*(5*a^2*d^2-6*a*b*c*d+3*b^2*c^2
)*(e*x)^(1/2)/c^2/d/e^3/(d*x^2+c)^(1/2)+1/6*(3*b^2*c^2+a*d*(-5*a*d+6*b*c))*
(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)
+x*d^(1/2))^2)^(1/2)/c^(9/4)/d^(5/4)/e^(5/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {473, 468, 335, 226}

$$-\frac{\sqrt{ex}\,(5a^2d^2-6abcd+3b^2c^2)}{3c^2de^3\sqrt{c+dx^2}}-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(ad(6bc-5ad)+3b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{6c^{9/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 6*a*b*c*d + 5*
a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*Sqrt[c + d*x^2]) + ((3*b^2*c^2 + a*d*(6*b*
c - 5*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]
*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(6*c^(9/4
)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}}\,dx = -\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}} + \frac{2\int \frac{\frac{1}{2}a(6bc-5ad)+\frac{3}{2}b^2cx^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx}{3ce^2}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}} - \frac{(3b^2c^2-6abcd+5a^2d^2)\sqrt{ex}}{3c^2de^3\sqrt{c+dx^2}} + \frac{(3b^2c^2+ad(6bc-5a}{6}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}} - \frac{(3b^2c^2-6abcd+5a^2d^2)\sqrt{ex}}{3c^2de^3\sqrt{c+dx^2}} + \frac{(3b^2c^2+ad(6bc-5a}{}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}} - \frac{(3b^2c^2-6abcd+5a^2d^2)\sqrt{ex}}{3c^2de^3\sqrt{c+dx^2}} + \frac{(3b^2c^2+ad(6bc-5a}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.16, size = 181, normalized size = 0.87

$$\frac{x\left(-\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,(3b^2c^2x^2-6abcdx^2+a^2d(2c+5dx^2))-i(-3b^2c^2-6abcd+5a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x^{5/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{3c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x*(-(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(3*b^2*c^2*x^2 - 6*a*b*c*d*x^2 + a^2*d*(2*c
 + 5*d*x^2))) - I*(-3*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*
x^(5/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/(3*c^
2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.15, size = 353, normalized size = 1.71

method	result
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\left(-\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{d\,e^2c^2\sqrt{\left(x^2+\frac{c}{d}\right)dex}}-\frac{2a^2\sqrt{de\,x^3+cex}}{3c^2e^3x^2}+\frac{\left(\frac{b^2}{e^2d}-\frac{a^2d^2-2abcd+b^2c^2}{2d\,c^2e^2}-\frac{d\,a^2}{3c^2e^2}\right)\sqrt{-cd}\,\sqrt{\cdots}}{\cdots}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}\cdots}$

risch	$-\frac{2a^2\sqrt{d\,x^2+c}}{3c^2x\,e^2\sqrt{ex}}-\frac{\left(\frac{\left(a^2d^2-3b^2c^2\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}{d^2\sqrt{de\,x^3+cex}}\right.}{}$
default	$-\frac{5\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2d^2x-6\sqrt{-c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x*(5*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*d^2*x-6*(-c*d)^(1/2)
*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d*x-3*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*b^2*c^2*x+10*a^2*d^3*x^2-12*a*b*c*d^2*x^2+6*b^2*c^2*d*x^2+4*a^2*c*d^
2)/(d*x^2+c)^(1/2)/c^2/e^2/(e*x)^(1/2)/d^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 156, normalized size = 0.75

$$\frac{\left(\left(\left(3\,b^2c^2d+6\,abcd^2-5\,a^2d^3\right)x^4+\left(3\,b^2c^3+6\,abc^2d-5\,a^2cd^2\right)x^2\right)\sqrt{d}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-\left(2\,a^2cd^2+\left(3\,b^2c^2d-6\,abcd^2+5\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{3\left(c^2d^3x^4+c^3d^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((3*b^2*c^2*d + 6*a*b*c*d^2 - 5*a^2*d^3)*x^4 + (3*b^2*c^3 + 6*a*b*c^2*
d - 5*a^2*c*d^2)*x^2)*sqrt(d)*weierstrassPInverse(-4*c/d, 0, x) - (2*a^2*c*
d^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(x))
*e^(-5/2)/(c^2*d^3*x^4 + c^3*d^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)^2}{\left(ex\right)^{\frac{5}{2}}\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/((e*x)**(5/2)*(c + d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*e^(-5/2)/((d*x^2 + c)^(3/2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(b\,x^2+a\right)}^2}{{\left(e\,x\right)}^{5/2}\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)), x)
```

3.856 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=434

$$-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}-\frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\,\sqrt{c+dx^2}}+\frac{(5b^2c^2-3ad(10bc-7ad))\,(ex)^{3/2}}{5c^3e^5\sqrt{c+dx^2}}-\frac{(5b^2c^2-3ad(10bc-7ad}{5c^3\sqrt{d}\,e^4\left(\sqrt{c}\right.}$$

```
[Out] -2/5*a^2/c/e/(e*x)^(5/2)/(d*x^2+c)^(1/2)+1/5*(5*b^2*c^2-3*a*d*(-7*a*d+10*b*
c))*(e*x)^(3/2)/c^3/e^5/(d*x^2+c)^(1/2)-2/5*a*(-7*a*d+10*b*c)/c^2/e^3/(e*x)
^(1/2)/(d*x^2+c)^(1/2)-1/5*(5*b^2*c^2-3*a*d*(-7*a*d+10*b*c))*(e*x)^(1/2)*(d
*x^2+c)^(1/2)/c^3/e^4/d^(1/2)/(c^(1/2)+x*d^(1/2))+1/5*(5*b^2*c^2-3*a*d*(-7*
a*d+10*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^
2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)-
1/10*(5*b^2*c^2-3*a*d*(-7*a*d+10*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))
*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*
(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/d^(3/4
)/e^(7/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 464, 296, 335, 311, 226, 1210}

$$-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}-\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(5b^2c^2-3ad(10bc-7ad))\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(5b^2c^2-3ad(10bc-7ad))\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{(ex)^{3/2}\,(5b^2c^2-3ad(10bc-7ad))}{5c^3e^5\sqrt{c+dx^2}}-\frac{\sqrt{ex}\,\sqrt{c+dx^2}\,(5b^2c^2-3ad(10bc-7ad))}{5c^3\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*Sqrt[c + d*x^2]) - (2*a*(10*b*c - 7*a*d))/(5*c^
2*e^3*Sqrt[e*x]*Sqrt[c + d*x^2]) + ((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*d))*(e
*x)^(3/2))/(5*c^3*e^5*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*
d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(5*c^3*Sqrt[d]*e^4*(Sqrt[c] + Sqrt[d]*x)) +
((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(5*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2
 - 3*a*d*(10*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c]
+ Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
1/2])/(10*c^(11/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx = -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} + \frac{2\int \frac{\frac{1}{2}a(10bc-7ad)+\frac{5}{2}b^2cx^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx}{5ce^2}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))\int}{5c^2e^4}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))(ex}{5c^3e^5\sqrt{c+dx^2}}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))(ex}{5c^3e^5\sqrt{c+dx^2}}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))(ex}{5c^3e^5\sqrt{c+dx^2}}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))(ex}{5c^3e^5\sqrt{c+dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 141, normalized size = 0.32

$$\frac{x\left(15b^2c^2x^4-30abcx^2(2c+3dx^2)+a^2(-6c^2+42cdx^2+63d^2x^4)+(-5b^2c^2+30abcd-21a^2d^2)\,x^4\sqrt{1+\frac{dx^2}{c}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)\right)}{15c^3(ex)^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x]

[Out] (x*(15*b^2*c^2*x^4 - 30*a*b*c*x^2*(2*c + 3*d*x^2) + a^2*(-6*c^2 + 42*c*d*x^2 + 63*d^2*x^4) + (-5*b^2*c^2 + 30*a*b*c*d - 21*a^2*d^2)*x^4*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)]))/(15*c^3*(e*x)^(7/2)*Sqrt[c + d*x^2])

Maple [A]
time = 0.16, size = 638, normalized size = 1.47

method	result
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\left(-\frac{2a^{2}\sqrt{de\,x^{3}+cex}}{5e^{4}c^{2}x^{3}}+\frac{4\left(de\,x^{2}+ce\right)a\left(4ad-5bc\right)}{5e^{4}c^{3}\sqrt{x\left(de\,x^{2}+ce\right)}}+\frac{x^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{e^{3}c^{3}\sqrt{\left(x^{2}+\frac{c}{d}\right)dex}}+\frac{\left(-\frac{2da\left(4ad-5bc\right)}{5c^{3}e^{3}}-\frac{a^{2}}{}\right.}{}\right.}{}$
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,a\left(-8ad\,x^{2}+10c\,x^{2}b+ac\right)}{5c^{3}x^{2}e^{3}\sqrt{ex}}-\frac{\left(8a^{2}d^{2}-10abcd\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{}}{}$

default	$-\frac{42\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c\,d^{2}x^{2}-60\sqrt{\frac{dx+\sqrt{}}{\sqrt{-}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/x^2*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2-60*((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1
/2*2^(1/2))*a*b*c^2*d*x^2+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2-21*((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(
1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)
^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c
^2*d*x^2-5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2-42*a^2*d^3*x^4+60*a*b*c*
d^2*x^4-10*b^2*c^2*d*x^4-28*a^2*c*d^2*x^2+40*a*b*c^2*d*x^2+4*a^2*c^2*d)/(d*
x^2+c)^(1/2)/d/e^3/(e*x)^(1/2)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 186, normalized size = 0.43

$$\frac{\left(((5\,b^2c^2d-30\,abcd^2+21\,a^2d^3)x^5+(5\,b^2c^3-30\,abc^2d+21\,a^2cd^2)x^3)\sqrt{d}\,\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)-(2\,a^2c^2d-(5\,b^2c^2d-30\,abcd^2+21\,a^2d^3)x^4+2\,(10\,abc^2d-7\,a^2cd^2)x^2)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{5\,(c^3d^2x^5+c^4dx^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(((5*b^2*c^2*d - 30*a*b*c*d^2 + 21*a^2*d^3)*x^5 + (5*b^2*c^3 - 30*a*b*c
^2*d + 21*a^2*c*d^2)*x^3)*sqrt(d)*weierstrassZeta(-4*c/d, 0, weierstrassPIn
verse(-4*c/d, 0, x)) - (2*a^2*c^2*d - (5*b^2*c^2*d - 30*a*b*c*d^2 + 21*a^2*
d^3)*x^4 + 2*(10*a*b*c^2*d - 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(
-7/2)/(c^3*d^2*x^5 + c^4*d*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2}{(ex)^{\frac{7}{2}} (c + dx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(3/2),x)
[Out] Integral((a + b*x**2)**2/((e*x)**(7/2)*(c + d*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*e^(-7/2)/((d*x^2 + c)^(3/2)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2}{{\left(e\,x\right)}^{7/2}\,{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x)
[Out] int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)), x)
```

3.857 $\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=302

$$\frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\,(c+dx^2)^{3/2}}+\frac{(39b^2c^2-42abcd+7a^2d^2)\,e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}}+\frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}}-\frac{5(39b^2c^2-42abcd+7a^2d^2)\,e^3\sqrt{ex}}{42cd^4}$$

```
[Out] 1/3*(-a*d+b*c)^2*(e*x)^(9/2)/c/d^2/e/(d*x^2+c)^(3/2)+1/14*(7*a^2*d^2-42*a*b
*c*d+39*b^2*c^2)*e*(e*x)^(5/2)/c/d^3/(d*x^2+c)^(1/2)+2/7*b^2*(e*x)^(9/2)/d^
2/e/(d*x^2+c)^(1/2)-5/42*(7*a^2*d^2-42*a*b*c*d+39*b^2*c^2)*e^3*(e*x)^(1/2)*
(d*x^2+c)^(1/2)/c/d^4+5/84*(7*a^2*d^2-42*a*b*c*d+39*b^2*c^2)*e^(7/2)*(cos(2
*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(
1/2))^2)^(1/2)/c^(1/4)/d^(17/4)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {474, 470, 294, 327, 335, 226}

$$\frac{5e^{7/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(7a^2d^2-42abcd+39b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{84\sqrt[4]{c}\,d^{17/4}\sqrt{c+dx^2}}-\frac{5e^3\sqrt{ex}\,\sqrt{c+dx^2}\,(7a^2d^2-42abcd+39b^2c^2)}{42cd^4}+\frac{e(ex)^{5/2}\,(7a^2d^2-42abcd+39b^2c^2)}{14cd^3\sqrt{c+dx^2}}+\frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\,(c+dx^2)^{3/2}}+\frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(9/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((39*b^2*c^2 -
42*a*b*c*d + 7*a^2*d^2)*e*(e*x)^(5/2))/(14*c*d^3*Sqrt[c + d*x^2]) + (2*b^2*
(e*x)^(9/2))/(7*d^2*e*Sqrt[c + d*x^2]) - (5*(39*b^2*c^2 - 42*a*b*c*d + 7*a^
2*d^2)*e^3*Sqrt[e*x]*Sqrt[c + d*x^2])/(42*c*d^4) + (5*(39*b^2*c^2 - 42*a*b*
c*d + 7*a^2*d^2)*e^(7/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(84*c^(1/4)*d^(17/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{(ex)^{7/2}\left(-\frac{3}{2}\left(2a^2d^2-3(bc-ad)^2\right)-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)\int \frac{(ex)^{7/2}}{\left(c+dx^2\right)^{3/2}}\,dx}{14cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{(5(3}{}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{5(39}{}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{5(39}{}$$

$$= \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(39b^2c^2-42abcd+7a^2d^2\right)e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{5(39}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.22, size = 222, normalized size = 0.74

$$\frac{(ex)^{7/2}\left(\frac{\sqrt{x}\left(-7a^2d^2\left(5c+7dx^2\right)+14abd\left(15c^2+21cdx^2+4d^2x^4\right)-b^2\left(195c^3+273c^2dx^2+52cd^2x^4-12d^3x^6\right)\right)}{d^4\left(c+dx^2\right)}+\frac{5i\left(39b^2c^2-42abcd+7a^2d^2\right)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^4}\right)}{42x^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((e*x)^(7/2)*((Sqrt[x]*(-7*a^2*d^2*(5*c + 7*d*x^2) + 14*a*b*d*(15*c^2 + 21*
c*d*x^2 + 4*d^2*x^4) - b^2*(195*c^3 + 273*c^2*d*x^2 + 52*c*d^2*x^4 - 12*d^3
*x^6)))/(d^4*(c + d*x^2)) + ((5*I)*(39*b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)*Sq
rt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]],
 -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^4)))/(42*x^(7/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(297) = 594$.
time = 0.19, size = 696, normalized size = 2.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84*(35*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*d^3*x^2-210*(-c*d)^(1
/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d^2*x^2+195*(-c*d)^(1/2)*((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2),1/2*2^(1/2))*b^2*c^2*d*x^2+24*b^2*d^4*x^7+35*((d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))
*(-c*d)^(1/2)*a^2*c*d^2-210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipti
cF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^
2*d+195*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3+112*a*b*d^4*x^5-10
4*b^2*c*d^3*x^5-98*a^2*d^4*x^3+588*a*b*c*d^3*x^3-546*b^2*c^2*d^2*x^3-70*a^2
*c*d^3*x+420*a*b*c^2*d^2*x-390*b^2*c^3*d*x)*e^3/x*(e*x)^(1/2)/d^5/(d*x^2+c)
^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((b*x^2 + a)^2*x^(7/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 249, normalized size = 0.82

$$\frac{5\,(39\,b^2c^4 - 42\,abc^3d + 7\,a^2c^2d^2 + (39\,b^2c^2d^2 - 42\,abcd^3 + 7\,a^2d^4)x^4 + 2\,(39\,b^2c^3d - 42\,abc^2d^2 + 7\,a^2cd^3)x^2)\sqrt{d}\,e^{\frac{7}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d}, 0, x\right) + (12\,b^2d^4x^6 - 195\,b^2c^3d + 210\,abc^2d^2 - 35\,a^2cd^3 - 4\,(13\,b^2cd^3 - 14\,abd^4)x^4 - 7\,(39\,b^2c^2d^2 - 42\,abcd^3 + 7\,a^2d^4)x^2)\sqrt{dx^2 + c}\,\sqrt{x}\,e^{\frac{7}{2}}}{42\,(d^7x^4 + 2\,cd^6x^2 + c^2d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/42*(5*(39*b^2*c^4 - 42*a*b*c^3*d + 7*a^2*c^2*d^2 + (39*b^2*c^2*d^2 - 42*a
*b*c*d^3 + 7*a^2*d^4)*x^4 + 2*(39*b^2*c^3*d - 42*a*b*c^2*d^2 + 7*a^2*c*d^3)
*x^2)*sqrt(d)*e^(7/2)*weierstrassPInverse(-4*c/d, 0, x) + (12*b^2*d^4*x^6 -
 195*b^2*c^3*d + 210*a*b*c^2*d^2 - 35*a^2*c*d^3 - 4*(13*b^2*c*d^3 - 14*a*b*
d^4)*x^4 - 7*(39*b^2*c^2*d^2 - 42*a*b*c*d^3 + 7*a^2*d^4)*x^2)*sqrt(d*x^2 +
c)*sqrt(x)*e^(7/2))/(d^7*x^4 + 2*c*d^6*x^2 + c^2*d^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^(7/2)*e^(7/2)/(d*x^2 + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.858 $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=442

$$\frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}}+\frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}}+\frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}}-\frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e^2\sqrt{ex}}{10cd^{7/2}\left(\sqrt{c}+\sqrt{d}\,x\right.}$$

```
[Out] 1/3*(-a*d+b*c)^2*(e*x)^(7/2)/c/d^2/e/(d*x^2+c)^(3/2)+1/30*(5*a^2*d^2-70*a*b
*c*d+77*b^2*c^2)*e*(e*x)^(3/2)/c/d^3/(d*x^2+c)^(1/2)+2/5*b^2*(e*x)^(7/2)/d^
2/e/(d*x^2+c)^(1/2)-1/10*(5*a^2*d^2-70*a*b*c*d+77*b^2*c^2)*e^2*(e*x)^(1/2)*
(d*x^2+c)^(1/2)/c/d^(7/2)/(c^(1/2)+x*d^(1/2))+1/10*(5*a^2*d^2-70*a*b*c*d+77
*b^2*c^2)*e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arct
an(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*(
(d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^(15/4)/(d*x^2+c)^(1/2)-1/2
0*(5*a^2*d^2-70*a*b*c*d+77*b^2*c^2)*e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1
/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1
/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^
(15/4)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {474, 470, 294, 335, 311, 226, 1210}

$$-\frac{e^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(5a^2d^2-70abcd+77b^2c^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{20c^{3/4}d^{15/4}\sqrt{c+dx^2}}+\frac{e^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(5a^2d^2-70abcd+77b^2c^2\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{3/4}d^{15/4}\sqrt{c+dx^2}}-\frac{e^2\sqrt{ex}\sqrt{c+dx^2}\left(5a^2d^2-70abcd+77b^2c^2\right)}{10cd^{7/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{e(ex)^{3/2}\left(5a^2d^2-70abcd+77b^2c^2\right)}{30cd^3\sqrt{c+dx^2}}+\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}+\frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(7/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((77*b^2*c^2 -
70*a*b*c*d + 5*a^2*d^2)*e*(e*x)^(3/2))/(30*c*d^3*Sqrt[c + d*x^2]) + (2*b^2*
(e*x)^(7/2))/(5*d^2*e*Sqrt[c + d*x^2]) - ((77*b^2*c^2 - 70*a*b*c*d + 5*a^2*
d^2)*e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(10*c*d^(7/2)*(Sqrt[c] + Sqrt[d]*x)) +
((77*b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(
c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(3/4)*d^(15/4)*Sqrt[c + d*x^2]) - ((77*b^2*
c^2 - 70*a*b*c*d + 5*a^2*d^2)*e^(5/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2
)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(20*c^(3/4)*d^(15/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{(ex)^{5/2}\left(\frac{1}{2}\left(-6a^2d^2+7(bc-ad)^2\right)-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)\int \frac{(ex)^{5/2}}{\left(c+dx^2\right)^{3/2}}\,dx}{30cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - \frac{((7}{}$$

$$((7$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - $$

$$((7$$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - $$

$$= \frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}} - \frac{(7}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.16, size = 153, normalized size = 0.35

$$\frac{e(ex)^{3/2}\left(5a^2d^2\left(c+3dx^2\right)-10abcd\left(7c+9dx^2\right)+b^2c\left(77c^2+99cdx^2+12d^2x^4\right)-3\left(77b^2c^2-70abcd+5a^2d^2\right)\sqrt{1+\frac{c}{dx^2}}\left(c+dx^2\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{30cd^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (e*(e*x)^(3/2)*(5*a^2*d^2*(c + 3*d*x^2) - 10*a*b*c*d*(7*c + 9*d*x^2) + b^2*
c*(77*c^2 + 99*c*d*x^2 + 12*d^2*x^4) - 3*(77*b^2*c^2 - 70*a*b*c*d + 5*a^2*d
^2)*Sqrt[1 + c/(d*x^2)]*(c + d*x^2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(
d*x^2))]))/(30*c*d^3*(c + d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(450) = 900$.

time = 0.18, size = 1191, normalized size = 2.69

method	result
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\;\sqrt{ex}\;\left(-\frac{e^2 x\left(a^2 d^2-2abcd+b^2c^2\right)\sqrt{de\,x^3+cex}}{3d^5\left(x^2+\frac{c}{d}\right)^2}+\frac{e^3x^2\left(a^2d^2-6abcd+5b^2c^2\right)}{2d^3c\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2e^2x\sqrt{de\,x^3+cex}}{5d^3}\right.}{}$

risch	$\frac{2b^2x^2\sqrt{d\,x^2+c}\,e^3}{5d^3\sqrt{ex}}+\frac{\left(b(10ad-13bc)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-}}{}\right.\right.}{d\sqrt{de\,x}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^2-420*((d*x+(-c*d)^(1/2))
/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(
-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a*b*c^2*d^2*x^2+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^2-15*(
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^
(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^2+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)
*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*
b*c^2*d^2*x^2-231*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^2-24*b^2*c*d^3*x^
6+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-420*((d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*b*c^3*d+462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*
```

```
d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-15*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-
c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a^2*c^2*d^2+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-231*((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
(1/2),1/2*2^(1/2))*b^2*c^4-30*a^2*d^4*x^4+180*a*b*c*d^3*x^4-198*b^2*c^2*d^2
*x^4-10*a^2*c*d^3*x^2+140*a*b*c^2*d^2*x^2-154*b^2*c^3*d*x^2)*e^2/x*(e*x)^(1
/2)/d^4/c/(d*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*x^2 + a)^2*x^(5/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 242, normalized size = 0.55

$$\frac{3\,(77\,b^2c^4-70\,abc^3d+5\,a^2c^2d^2+(77\,b^2c^2d^2-70\,abcd^3+5\,a^2d^4)x^4+2\,(77\,b^2c^3d-70\,abc^2d^2+5\,a^2cd^3)x^2)\sqrt{d}\,e^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)+(12\,b^2cd^3x^5+3\,(33\,b^2c^2d^2-30\,abcd^3+5\,a^2d^4)x^3+(77\,b^2c^3d-70\,abc^2d^2+5\,a^2cd^3)x)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{5}{2}}}{30\,(cd^6x^4+2\,c^2d^5x^2+c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(3*(77*b^2*c^4 - 70*a*b*c^3*d + 5*a^2*c^2*d^2 + (77*b^2*c^2*d^2 - 70*a
*b*c*d^3 + 5*a^2*d^4)*x^4 + 2*(77*b^2*c^3*d - 70*a*b*c^2*d^2 + 5*a^2*c*d^3)
*x^2)*sqrt(d)*e^(5/2)*weierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d
, 0, x)) + (12*b^2*c*d^3*x^5 + 3*(33*b^2*c^2*d^2 - 30*a*b*c*d^3 + 5*a^2*d^4
)*x^3 + (77*b^2*c^3*d - 70*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)*sqrt(d*x^2 + c)*sq
rt(x)*e^(5/2))/(c*d^6*x^4 + 2*c^2*d^5*x^2 + c^3*d^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*x^(5/2)*e^(5/2)/(d*x^2 + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}\,\left(b\,x^2+a\right)^2}{\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
[Out] int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.859 $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=248

$$\frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\,(c+dx^2)^{3/2}}+\frac{(15b^2c^2-10abcd-a^2d^2)\,e\sqrt{ex}}{6cd^3\sqrt{c+dx^2}}+\frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}}-\frac{(15b^2c^2-10abcd-a^2d^2)\,e^{3/2}\Big(\sqrt{c}+}{}$$

[Out] 1/3*(-a*d+b*c)^2*(e*x)^(5/2)/c/d^2/e/(d*x^2+c)^(3/2)+2/3*b^2*(e*x)^(5/2)/d^2/e/(d*x^2+c)^(1/2)+1/6*(-a^2*d^2-10*a*b*c*d+15*b^2*c^2)*e*(e*x)^(1/2)/c/d^3/(d*x^2+c)^(1/2)-1/12*(-a^2*d^2-10*a*b*c*d+15*b^2*c^2)*e^(3/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(5/4)/d^(13/4)/(d*x^2+c)^(1/2)

Rubi [A]
time = 0.13, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {474, 470, 294, 335, 226}

$$-\frac{e^{3/2}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(-a^2d^2-10abcd+15b^2c^2)\,F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{5/4}d^{13/4}\sqrt{c+dx^2}}+\frac{e\sqrt{ex}\,(-a^2d^2-10abcd+15b^2c^2)}{6cd^3\sqrt{c+dx^2}}+\frac{(ex)^{5/2}(bc-ad)^2}{3cd^2e\,(c+dx^2)^{3/2}}+\frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Int[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]

[Out] ((b*c - a*d)^2*(e*x)^(5/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((15*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e*Sqrt[e*x])/(6*c*d^3*Sqrt[c + d*x^2]) + (2*b^2*(e*x)^(5/2))/(3*d^2*e*Sqrt[c + d*x^2]) - ((15*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(5/4)*d^(13/4)*Sqrt[c + d*x^2])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}dx = \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{(ex)^{3/2}\left(\frac{1}{2}\left(-6a^2d^2+5(bc-ad)^2\right)-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}dx}{3cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}} - \frac{\left(15b^2c^2-10abcd-a^2d^2\right)\int \frac{(ex)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{6cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(15b^2c^2-10abcd-a^2d^2\right)e\sqrt{ex}}{6cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}} - \frac{((15b^2c}{}$$

$$((15b^2c$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(15b^2c^2-10abcd-a^2d^2\right)e\sqrt{ex}}{6cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}} - $$

$$(15b^2c$$

$$= \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e\left(c+dx^2\right)^{3/2}} + \frac{\left(15b^2c^2-10abcd-a^2d^2\right)e\sqrt{ex}}{6cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}} - $$

Mathematica [C] Result contains complex when optimal does not.
time = 10.20, size = 204, normalized size = 0.82

$$\frac{(ex)^{3/2}\left(\frac{\sqrt{x}\left(a^2d^2\left(-c+dx^2\right)-2abcd\left(5c+7dx^2\right)+b^2c\left(15c^2+21cdx^2+4d^2x^4\right)\right)}{cd^3\left(c+dx^2\right)} + \frac{i\left(-15b^2c^2+10abcd+a^2d^2\right)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{c\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^3}\right)}{6x^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((e*x)^(3/2)*((Sqrt[x]*(a^2*d^2*(-c + d*x^2) - 2*a*b*c*d*(5*c + 7*d*x^2) +
b^2*c*(15*c^2 + 21*c*d*x^2 + 4*d^2*x^4)))/(c*d^3*(c + d*x^2)) + (I*(-15*b^2
*c^2 + 10*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt
[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(6
*x^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(249) = 498.
time = 0.17, size = 674, normalized size = 2.72

method	result
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\,\sqrt{ex}\left(-\frac{e\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\sqrt{de\,x^{3}+cex}}{3d^{5}\left(x^{2}+\frac{c}{d}\right)^{2}}+\frac{e^{2}x\left(a^{2}d^{2}-14abcd+13b^{2}c^{2}\right)}{6d^{3}c\sqrt{\left(x^{2}+\frac{c}{d}\right)dex}}+\frac{2b^{2}e\sqrt{de\,x^{3}+cex}}{3d^{3}}+\right.}{}$
risch	$\frac{2b^{2}x\sqrt{d\,x^{2}+c}\,e^{2}}{3d^{3}\sqrt{ex}}+\frac{\left(\frac{6ab\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\right.}{\sqrt{}}}\right.}{\sqrt{de\,x^{3}+cex}}\right.}{}$
default	$\frac{\left(\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}d^{3}x^{2}+10\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*d^3*x^2+10*(-c*d)^(1/2)*
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d^2*x^2-15*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*b^2*c^2*d*x^2+((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c*d^2
+10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-
c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*d-15*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
```

```
2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3+8*b^2*c*d^3*x^5+2*a^2*d^4*x^3-28*a*b*c*d^3*
x^3+42*b^2*c^2*d^2*x^3-2*a^2*c*d^3*x-20*a*b*c^2*d^2*x+30*b^2*c^3*d*x)*e/x*(
e*x)^(1/2)/c/d^4/(d*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*x^2 + a)^2*x^(3/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 229, normalized size = 0.92

$$-\frac{\left(15\,b^{2}c^{4}-10\,abc^{3}d-a^{2}c^{2}d^{2}+\left(15\,b^{2}c^{2}d^{2}-10\,abcd^{3}-a^{2}d^{4}\right)x^{4}+2\left(15\,b^{2}c^{3}d-10\,abc^{2}d^{2}-a^{2}cd^{3}\right)x^{2}\right)\sqrt{d}\,e^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-\left(4\,b^{2}cd^{3}x^{4}+15\,b^{2}c^{3}d-10\,abc^{2}d^{2}-a^{2}cd^{3}+\left(21\,b^{2}c^{2}d^{2}-14\,abcd^{3}+a^{2}d^{4}\right)x^{2}\right)\sqrt{dx^{2}+c}\,\sqrt{x}\,e^{\frac{3}{2}}}{6\left(cd^{6}x^{4}+2\,c^{2}d^{5}x^{2}+c^{3}d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*((15*b^2*c^4 - 10*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c
*d^3 - a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sq
rt(d)*e^(3/2)*weierstrassPInverse(-4*c/d, 0, x) - (4*b^2*c*d^3*x^4 + 15*b^2
*c^3*d - 10*a*b*c^2*d^2 - a^2*c*d^3 + (21*b^2*c^2*d^2 - 14*a*b*c*d^3 + a^2*
d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(x)*e^(3/2))/(c*d^6*x^4 + 2*c^2*d^5*x^2 + c^3
*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex\right)^{\frac{3}{2}}\left(a+bx^{2}\right)^{2}}{\left(c+dx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((e*x)**(3/2)*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*x^(3/2)*e^(3/2)/(d*x^2 + c)^(5/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.860 $\int \frac{\sqrt{ex}\,(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=403

$$\frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\,(c+dx^2)^{3/2}}-\frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}}+\frac{(7b^2c^2-2abcd-a^2d^2)\sqrt{ex}\,\sqrt{c+dx^2}}{2c^2d^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{(7b^2c^2-2abcd-}{}$$

[Out] 1/3*(-a*d+b*c)^2*(e*x)^(3/2)/c/d^2/e/(d*x^2+c)^(3/2)-1/2*(-a*d+b*c)*(a*d+3*b*c)*(e*x)^(3/2)/c^2/d^2/e/(d*x^2+c)^(1/2)+1/2*(-a^2*d^2-2*a*b*c*d+7*b^2*c^2)*(e*x)^(1/2)*(d*x^2+c)^(1/2)/c^2/d^(5/2)/(c^(1/2)+x*d^(1/2))-1/2*(-a^2*d^2-2*a*b*c*d+7*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(11/4)/(d*x^2+c)^(1/2)+1/4*(-a^2*d^2-2*a*b*c*d+7*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(11/4)/(d*x^2+c)^(1/2)

Rubi [A]

time = 0.23, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {474, 468, 335, 311, 226, 1210}

$$\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(-a^2d^2-2abcd+7b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{4c^{7/4}d^{11/4}\sqrt{c+dx^2}}-\frac{\sqrt{e}\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(-a^2d^2-2abcd+7b^2c^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{7/4}d^{11/4}\sqrt{c+dx^2}}+\frac{\sqrt{ex}\,\sqrt{c+dx^2}\,(-a^2d^2-2abcd+7b^2c^2)}{2c^2d^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}-\frac{(ex)^{3/2}(ad+3bc)(bc-ad)}{2c^2d^2e\sqrt{c+dx^2}}+\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(3/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*c + a*d)*(e*x)^(3/2))/(2*c^2*d^2*e*Sqrt[c + d*x^2]) + ((7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(2*c^2*d^(5/2)*(Sqrt[c] + Sqrt[d]*x)) - ((7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*c^(7/4)*d^(11/4)*Sqrt[c + d*x^2]) + ((7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(4*c^(7/4)*d^(11/4)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ex}\,(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{\int \frac{\sqrt{ex}\,\left(-\frac{3}{2}\left(2a^2d^2-(bc-ad)^2\right)-3b^2cdx^2\right)}{(c+dx^2)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}} + \frac{(7b^2c^2-2abcd-a^2d^2)\int \frac{\sqrt{ex}}{\sqrt{c+d}}}{4c^2d^2}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}} + \frac{(7b^2c^2-2abcd-a^2d^2)\,\mathrm{Subst}\left(\int\right.}{2c^2d^2e}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}} + \frac{(7b^2c^2-2abcd-a^2d^2)\,\mathrm{Subst}\left(\int\right.}{2c^{3/2}d^{5/}}$$

$$= \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}} + \frac{(7b^2c^2-2abcd-a^2d^2)\sqrt{ex}\,\sqrt{c}}{2c^2d^{5/2}\left(\sqrt{c}+\sqrt{d}\,x\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 20.16, size = 147, normalized size = 0.36

$$\frac{\sqrt{ex}\left(x(2abcd(c+3dx^2)+a^2d^2(5c+3dx^2)-b^2c^2(7c+9dx^2))+3(7b^2c^2-2abcd-a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x(c+dx^2)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{c}{dx^2}\right)\right)}{6c^2d^2\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[e*x]*(x*(2*a*b*c*d*(c + 3*d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*
(7*c + 9*d*x^2)) + 3*(7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[1 + c/(d*x^2)]*
x*(c + d*x^2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(6*c^2*d^2*
(c + d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. 2(417) = 834.
time = 0.13, size = 1176, normalized size = 2.92

method	result
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\,\sqrt{ex}\,\left(\frac{x\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{de\,x^3+cex}}{3c\,d^4\left(x^2+\frac{c}{d}\right)^2}+\frac{e\,x^2\left(a^2d^2+2abcd-3b^2c^2\right)}{2d^2c^2\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{\left(\frac{b^2e}{d^2}-\frac{e\left(a^2d^2+2abcd-3b^2c^2\right.}{4d^2c^2}\right.}{}\right.}{}$
default	$-\frac{\left(6\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\ \text{EllipticE}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^3x^2+12\sqrt{\frac{dx+}{\sqrt{}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^2+12*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a*b*c^2*d^2*x^2-42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE
(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^2-3*((d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^2-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*
d^2*x^2+21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^2+6*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-
```

```
c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a^2*c^2*d^2+12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-42*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*b^2*c^4-3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2-6*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a*b*c^3*d+21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*
EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-6*a^
2*d^4*x^4-12*a*b*c*d^3*x^4+18*b^2*c^2*d^2*x^4-10*a^2*c*d^3*x^2-4*a*b*c^2*d^
2*x^2+14*b^2*c^3*d*x^2)*(e*x)^(1/2)/d^3/c^2/x/(d*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*x^2 + a)^2*sqrt(x)/(d*x^2 + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 232, normalized size = 0.58

$$-\frac{3\left(7\,b^2c^4-2\,abc^3d-a^2c^2d^2+\left(7\,b^2c^2d^2-2\,abcd^3-a^2d^4\right)x^4+2\left(7\,b^2c^3d-2\,abc^2d^2-a^2cd^3\right)x^2\right)\sqrt{d}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+\left(3\left(3\,b^2c^2d^2-2\,abcd^3-a^2d^4\right)x^3+\left(7\,b^2c^3d-2\,abc^2d^2-5\,a^2cd^3\right)x\right)\sqrt{dx^2+c}\,\sqrt{x}\,e^{\frac{1}{2}}}{6\left(c^2d^5x^4+2\,c^3d^4x^2+c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(7*b^2*c^4 - 2*a*b*c^3*d - a^2*c^2*d^2 + (7*b^2*c^2*d^2 - 2*a*b*c*d
^3 - a^2*d^4)*x^4 + 2*(7*b^2*c^3*d - 2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d
)*e^(1/2)*weierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (
3*(3*b^2*c^2*d^2 - 2*a*b*c*d^3 - a^2*d^4)*x^3 + (7*b^2*c^3*d - 2*a*b*c^2*d^
2 - 5*a^2*c*d^3)*x)*sqrt(d*x^2 + c)*sqrt(x)*e^(1/2))/(c^2*d^5*x^4 + 2*c^3*d
^4*x^2 + c^4*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(e*x)*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*sqrt(x)*e^(1/2)/(d*x^2 + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}\,\left(b\,x^2+a\right)^2}{\left(d\,x^2+c\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.861 $\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{(bc-ad)^2\sqrt{ex}}{3cd^2e\,(c+dx^2)^{3/2}}-\frac{(bc-ad)(7bc+5ad)\sqrt{ex}}{6c^2d^2e\sqrt{c+dx^2}}+\frac{(5b^2c^2+2abcd+5a^2d^2)\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}}{12c^{9/4}d^{9/4}\sqrt{e}\,\sqrt{c+dx^2}}$$

```
[Out] 1/3*(-a*d+b*c)^2*(e*x)^(1/2)/c/d^2/e/(d*x^2+c)^(3/2)-1/6*(-a*d+b*c)*(5*a*d+
7*b*c)*(e*x)^(1/2)/c^2/d^2/e/(d*x^2+c)^(1/2)+1/12*(5*a^2*d^2+2*a*b*c*d+5*b^
2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*a
rctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(
c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(9/4)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {474, 468, 335, 226}

$$\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(5a^2d^2+2abcd+5b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{9/4}d^{9/4}\sqrt{e}\,\sqrt{c+dx^2}}-\frac{\sqrt{ex}\,(5ad+7bc)(bc-ad)}{6c^2d^2e\sqrt{c+dx^2}}+\frac{\sqrt{ex}\,(bc-ad)^2}{3cd^2e\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[e*x])/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(7*b
*c + 5*a*d)*Sqrt[e*x])/(6*c^2*d^2*e*Sqrt[c + d*x^2]) + ((5*b^2*c^2 + 2*a*b*
c*d + 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*
c^(9/4)*d^(9/4)*Sqrt[e]*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2\sqrt{ex}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{\int \frac{\frac{1}{2}\left(-6a^2d^2+(bc-ad)^2\right)-3b^2cdx^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx}{3cd^2}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(7bc+5ad)\sqrt{ex}}{6c^2d^2e\sqrt{c+dx^2}} + \frac{(5b^2c^2+2abcd+5a^2d^2)\int \frac{}{\sqrt{e}}}{12c^2d^2}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(7bc+5ad)\sqrt{ex}}{6c^2d^2e\sqrt{c+dx^2}} + \frac{(5b^2c^2+2abcd+5a^2d^2)\,\mathrm{Subs}}{6c}$$

$$= \frac{(bc-ad)^2\sqrt{ex}}{3cd^2e\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(7bc+5ad)\sqrt{ex}}{6c^2d^2e\sqrt{c+dx^2}} + \frac{(5b^2c^2+2abcd+5a^2d^2)\left(\sqrt{c}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.21, size = 169, normalized size = 0.79

$$\frac{x\left(-7b^2c^2+2abcd+5a^2d^2+\frac{2c(bc-ad)^2}{c+dx^2}+\frac{i(5b^2c^2+2abcd+5a^2d^2)\sqrt{1+\frac{c}{dx^2}}\sqrt{x}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{6c^2d^2\sqrt{ex}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*(-7*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2 + (2*c*(b*c - a*d)^2)/(c + d*x^2) +
(I*(5*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[c])/Sqrt[
d]]))/(6*c^2*d^2*Sqrt[e*x]*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(220) = 440.

time = 0.13, size = 660, normalized size = 3.10

method	result
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\left(\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{de\,x^3+cex}}{3ce\,d^4\left(x^2+\frac{c}{d}\right)^2}+\frac{x(5a^2d^2+2abcd-7b^2c^2)}{6d^2c^2\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{\left(\frac{b^2}{d^2}+\frac{5a^2d^2+2abcd-7b^2c^2}{12d^2c^2}\right)\sqrt{-cd}}{}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$\frac{5\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2d^3x^2+2\sqrt{-cd}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(5*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*
```

```
x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*d^3*x^2+2*(-c*d)^(1/2)
*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d^2*x^2+5*(-c*d)^(1/2)*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*b^2*c^2*d*x^2+5*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c*d
^2+2*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*d+5*((d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x
/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/
2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3+10*a^2*d^4*x^3+4*a*b*c*d^3*x^3-14*b^2*c^2*d
^2*x^3+14*a^2*c*d^3*x-4*a*b*c^2*d^2*x-10*b^2*c^3*d*x)/(e*x)^(1/2)/c^2/d^3/(
d*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 218, normalized size = 1.02

$$\frac{\left(\left(5\,b^2c^4+2\,abc^3d+5\,a^2c^2d^2+\left(5\,b^2c^2d^2+2\,abcd^3+5\,a^2d^4\right)x^4+2\left(5\,b^2c^3d+2\,abc^2d^2+5\,a^2cd^3\right)x^2\right)\sqrt{d}\ \mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-\left(5\,b^2c^3d+2\,abc^2d^2-7\,a^2cd^3+\left(7\,b^2c^2d^2-2\,abcd^3-5\,a^2d^4\right)x^2\right)\sqrt{dx^2+c}\ \sqrt{x}\right)e^{\left(-\frac{1}{2}\right)}}{6\left(c^2d^5x^4+2\,c^3d^4x^2+c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((5*b^2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + (5*b^2*c^2*d^2 + 2*a*b*c*d^
3 + 5*a^2*d^4)*x^4 + 2*(5*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^2)*sqr
t(d)*weierstrassPInverse(-4*c/d, 0, x) - (5*b^2*c^3*d + 2*a*b*c^2*d^2 - 7*a
^2*c*d^3 + (7*b^2*c^2*d^2 - 2*a*b*c*d^3 - 5*a^2*d^4)*x^2)*sqrt(d*x^2 + c)*s
qrt(x))*e^(-1/2)/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{ex}\ \left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(5/2)/(e*x)**(1/2),x)
[Out] Integral((a + b*x**2)**2/(sqrt(e*x)*(c + d*x**2)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^2*e^(-1/2)/((d*x^2 + c)^(5/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2}{\sqrt{e\,x}\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(5/2)),x)
[Out] int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(5/2)), x)
```

3.862 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=442

$$-\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{3/2}}-\frac{(b^2c^2-2abcd+7a^2d^2)\,(ex)^{3/2}}{3c^2de^3\,(c+dx^2)^{3/2}}+\frac{(b^2c^2+ad(2bc-7ad))\,(ex)^{3/2}}{2c^3de^3\sqrt{c+dx^2}}-\frac{(b^2c^2+ad(2bc-7}{2c^3d^{3/2}e^2\,(}$$

```
[Out] -1/3*(7*a^2*d^2-2*a*b*c*d+b^2*c^2)*(e*x)^(3/2)/c^2/d/e^3/(d*x^2+c)^(3/2)-2*
a^2/c/e/(d*x^2+c)^(3/2)/(e*x)^(1/2)+1/2*(b^2*c^2+a*d*(-7*a*d+2*b*c))*(e*x)^
(3/2)/c^3/d/e^3/(d*x^2+c)^(1/2)-1/2*(b^2*c^2+a*d*(-7*a*d+2*b*c))*(e*x)^(1/2
)*(d*x^2+c)^(1/2)/c^3/d^(3/2)/e^2/(c^(1/2)+x*d^(1/2))+1/2*(b^2*c^2+a*d*(-7*
a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/co
s(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(
1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2
+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)-1
/4*(b^2*c^2+a*d*(-7*a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*Ellipt
icF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2
)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/d^(7/4)/e^(3/
2)/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 468, 296, 335, 311, 226, 1210}

$$-\frac{(ex)^{3/2}\,(7a^2d^2-2abcd+b^2c^2)}{3c^2de^3\,(c+dx^2)^{3/2}}-\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{3/2}}-\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(ad(2bc-7ad)+b^2c^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{4c^{11/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\,(ad(2bc-7ad)+b^2c^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{11/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}-\frac{\sqrt{ex}\,\sqrt{c+dx^2}\,(ad(2bc-7ad)+b^2c^2)}{2c^3d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{(ex)^{3/2}\,(ad(2bc-7ad)+b^2c^2)}{2c^3de^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-2*a^2)/(c*e*Sqrt[e*x]*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*b*c*d + 7*a^2*
d^2)*(e*x)^(3/2))/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + ((b^2*c^2 + a*d*(2*b*c
- 7*a*d))*(e*x)^(3/2))/(2*c^3*d*e^3*Sqrt[c + d*x^2]) - ((b^2*c^2 + a*d*(2*b
*c - 7*a*d))*Sqrt[e*x]*Sqrt[c + d*x^2])/(2*c^3*d^(3/2)*e^2*(Sqrt[c] + Sqrt[
d]*x)) + ((b^2*c^2 + a*d*(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d
*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], 1/2])/(2*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2]) - ((b^2*c
^2 + a*d*(2*b*c - 7*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] +
 Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1
/2])/(4*c^(11/4)*d^(7/4)*e^(3/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx = -\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{3/2}} + \frac{2\int \frac{\sqrt{ex}\,\left(\frac{1}{2}a(2bc-7ad)+\frac{1}{2}b^2cx^2\right)}{(c+dx^2)^{5/2}}\,dx}{ce^2}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)\,(ex)^{3/2}}{3c^2de^3\,(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7a}{2c^2d}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)\,(ex)^{3/2}}{3c^2de^3\,(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7a}{2c^3de^3\sqrt{c+d}}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)\,(ex)^{3/2}}{3c^2de^3\,(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7a}{2c^3de^3\sqrt{c+d}}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)\,(ex)^{3/2}}{3c^2de^3\,(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7a}{2c^3de^3\sqrt{c+d}}$$

$$= -\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+7a^2d^2)\,(ex)^{3/2}}{3c^2de^3\,(c+dx^2)^{3/2}} + \frac{(b^2c^2+ad(2bc-7a}{2c^3de^3\sqrt{c+d}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 161, normalized size = 0.36

$$\frac{x\left(b^2c^2x^2(c+3dx^2)+2abcdx^2(5c+3dx^2)-a^2d(12c^2+35cdx^2+21d^2x^4)-(b^2c^2+2abcd-7a^2d^2)\,x^2(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)\right)}{6c^3d(ex)^{3/2}\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*(b^2*c^2*x^2*(c + 3*d*x^2) + 2*a*b*c*d*x^2*(5*c + 3*d*x^2) - a^2*d*(12*c
^2 + 35*c*d*x^2 + 21*d^2*x^4) - (b^2*c^2 + 2*a*b*c*d - 7*a^2*d^2)*x^2*(c +
d*x^2)*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)]))
/(6*c^3*d*(e*x)^(3/2)*(c + d*x^2)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1186 vs. $2(452) = 904$.
time = 0.17, size = 1187, normalized size = 2.69 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(42*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2
))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^2-12*((d*x+(-c*d)^(1/2))/(
-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c
*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a*b*c^2*d^2*x^2-6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^2-21*((d*x
+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/
2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^2+6*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*
d^2*x^2+3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^2+42*((d*x+(-c*d)^(1/2))/
(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-
c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a^2*c^2*d^2-12*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d-6*((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2),1/2*2^(1/2))*b^2*c^4-21*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c^2*d^2+6*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a*b*c^3*d+3*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*E
llipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4-42*a^
```

```
2*d^4*x^4+12*a*b*c*d^3*x^4+6*b^2*c^2*d^2*x^4-70*a^2*c*d^3*x^2+20*a*b*c^2*d^
2*x^2+2*b^2*c^3*d*x^2-24*a^2*c^2*d^2)/d^2/c^3/e/(e*x)^(1/2)/(d*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 244, normalized size = 0.55

$$\frac{\left(3\left((b^2c^2d^2+2\,abcd^3-7\,a^2d^4)x^5+2\,(b^2c^3d+2\,abc^2d^2-7\,a^2cd^3)x^3+(b^2c^4+2\,abc^3d-7\,a^2c^2d^2)x\right)\sqrt{d}\ \mathrm{weierstrassZeta}\left(-\frac{4\,c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)-\left(12\,a^2c^2d^2-3\,(b^2c^2d^2+2\,abcd^3-7\,a^2d^4)x^4-(b^2c^3d+10\,abc^2d^2-35\,a^2cd^3)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{3}{2}\right)}}{6\,(c^3d^4x^5+2\,c^4d^3x^3+c^5d^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*((b^2*c^2*d^2 + 2*a*b*c*d^3 - 7*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 2*a*b*
c^2*d^2 - 7*a^2*c*d^3)*x^3 + (b^2*c^4 + 2*a*b*c^3*d - 7*a^2*c^2*d^2)*x)*sqr
t(d)*weierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (12*a^
2*c^2*d^2 - 3*(b^2*c^2*d^2 + 2*a*b*c*d^3 - 7*a^2*d^4)*x^4 - (b^2*c^3*d + 10
*a*b*c^2*d^2 - 35*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-3/2)/(c^3*d^
4*x^5 + 2*c^4*d^3*x^3 + c^5*d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{(ex)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/((e*x)**(3/2)*(c + d*x**2)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*e^(-3/2)/((d*x^2 + c)^(5/2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^2}{(e\,x)^{3/2}\,(d\,x^2+c)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x)
[Out] int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)), x)
```

3.863 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}-\frac{\left(b^2c^2-2abcd+3a^2d^2\right)\sqrt{ex}}{3c^2de^3\left(c+dx^2\right)^{3/2}}+\frac{\left(b^2c^2+5ad(2bc-3ad)\right)\sqrt{ex}}{6c^3de^3\sqrt{c+dx^2}}+\frac{\left(b^2c^2+5ad(2bc-\cdots\right.}{}$$

[Out] -2/3*a^2/c/e/(e*x)^(3/2)/(d*x^2+c)^(3/2)-1/3*(3*a^2*d^2-2*a*b*c*d+b^2*c^2)*(e*x)^(1/2)/c^2/d/e^3/(d*x^2+c)^(3/2)+1/6*(b^2*c^2+5*a*d*(-3*a*d+2*b*c))*(e*x)^(1/2)/c^3/d/e^3/(d*x^2+c)^(1/2)+1/12*(b^2*c^2+5*a*d*(-3*a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(13/4)/d^(5/4)/e^(5/2)/(d*x^2+c)^(1/2)

Rubi [A]
time = 0.16, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {473, 468, 296, 335, 226}

$$-\frac{\sqrt{ex}\left(3a^2d^2-2abcd+b^2c^2\right)}{3c^2de^3\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}+\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(5ad(2bc-3ad)+b^2c^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{12c^{13/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}+\frac{\sqrt{ex}\left(5ad(2bc-3ad)+b^2c^2\right)}{6c^3de^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x]

[Out] (-2*a^2)/(3*c*e*(e*x)^(3/2)*(c + d*x^2)^(3/2)) - ((b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*Sqrt[e*x])/(3*c^2*d*e^3*(c + d*x^2)^(3/2)) + ((b^2*c^2 + 5*a*d*(2*b*c - 3*a*d))*Sqrt[e*x])/(6*c^3*d*e^3*Sqrt[c + d*x^2]) + ((b^2*c^2 + 5*a*d*(2*b*c - 3*a*d))*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(12*c^(13/4)*d^(5/4)*e^(5/2)*Sqrt[c + d*x^2])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx = -\frac{2a^2}{3ce(ex)^{3/2}(c+dx^2)^{3/2}} + \frac{2\int \frac{\frac{3}{2}a(2bc-3ad)+\frac{3}{2}b^2cx^2}{\sqrt{ex}(c+dx^2)^{5/2}}dx}{3ce^2}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+3a^2d^2)\sqrt{ex}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+5ad(2bc-}{}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+3a^2d^2)\sqrt{ex}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+5ad(2bc-}{6c^3de^3\sqrt{c+}}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+3a^2d^2)\sqrt{ex}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+5ad(2bc-}{6c^3de^3\sqrt{c+}}$$

$$= -\frac{2a^2}{3ce(ex)^{3/2}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+3a^2d^2)\sqrt{ex}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+5ad(2bc-}{6c^3de^3\sqrt{c+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.22, size = 211, normalized size = 0.82

$$\frac{x^{5/2}\left(\frac{b^2c^2x^2(-c+dx^2)+2abcdx^2(7c+5dx^2)-a^2d(4c^2+21cdx^2+15d^2x^4)}{c^3dx^{3/2}(c+dx^2)} + \frac{i(b^2c^2+10abcd-15a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right)\middle|-1\right)}{c^3\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d}\right)}{6(ex)^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x^(5/2)*((b^2*c^2*x^2*(-c + d*x^2) + 2*a*b*c*d*x^2*(7*c + 5*d*x^2) - a^2*d
*(4*c^2 + 21*c*d*x^2 + 15*d^2*x^4))/(c^3*d*x^(3/2)*(c + d*x^2)) + (I*(b^2*c
^2 + 10*a*b*c*d - 15*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqr
t[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^3*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d)))/(
6*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(259) = 518$.
time = 0.17, size = 686, normalized size = 2.66

method	result
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\left(-\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\sqrt{de\,x^{3}+cex}}{3c^{2}e^{3}d^{3}\left(x^{2}+\frac{c}{d}\right)^{2}}-\frac{x\left(11a^{2}d^{2}-10abcd-b^{2}c^{2}\right)}{6d\,e^{2}c^{3}\sqrt{\left(x^{2}+\frac{c}{d}\right)dex}}-\frac{2a^{2}\sqrt{de\,x^{3}+cex}}{3c^{3}e^{3}x^{2}}+\frac{\left(-\frac{11a^{2}d^{2}}{\cdots}\right.}{\cdots}\right.}{\sqrt{e\cdots}}$
risch	$-\frac{2a^{2}\sqrt{d\,x^{2}+c}}{3c^{3}x\,e^{2}\sqrt{ex}}-\frac{\left(\frac{a^{2}\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-\cdots}}{\cdots}\right.}{\sqrt{-\cdots}}}\right.}{\sqrt{de\,x^{3}+cex}}\right.}{\cdots}$
default	$-\frac{15\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^{2}d^{3}x^{3}-10\sqrt{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*d^3*x^3-10*((d*x+(-c
*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^
```

```
(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c*d^2*x^3-((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(
-c*d)^(1/2)*b^2*c^2*d*x^3+15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a^2*c
*d^2*x-10*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/
2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*(-c*d)^(1/2)*a*b*c^2*d*x-((d*x+(-c*d)
^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2),1/2*2^(1/2))*(-c*d)^(1/2)*b^2*c^3*x+30*a^2*d^4*x^4-20*a*b*c*d^3*x^4-2*b^
2*c^2*d^2*x^4+42*a^2*c*d^3*x^2-28*a*b*c^2*d^2*x^2+2*b^2*c^3*d*x^2+8*a^2*c^2
*d^2)/x/e^2/(e*x)^(1/2)/c^3/d^2/(d*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 238, normalized size = 0.92

$$\frac{\left(\left((b^2c^2d^2 + 10\,abcd^3 - 15\,a^2d^4)x^6 + 2\,(b^2c^3d + 10\,abc^2d^2 - 15\,a^2cd^3)x^4 + (b^2c^4 + 10\,abc^3d - 15\,a^2c^2d^2)x^2\right)\sqrt{d}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d}, 0, x\right) - \left(4\,a^2c^2d^2 - (b^2c^2d^2 + 10\,abcd^3 - 15\,a^2d^4)x^4 + (b^2c^3d - 14\,abc^2d^2 + 21\,a^2cd^3)x^2\right)\sqrt{dx^2 + c}\,\sqrt{x}\right)e^{\left(-\frac{5}{2}\right)}}{6\,(c^3d^4x^6 + 2\,c^4d^3x^4 + c^5d^2x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(((b^2*c^2*d^2 + 10*a*b*c*d^3 - 15*a^2*d^4)*x^6 + 2*(b^2*c^3*d + 10*a*b
*c^2*d^2 - 15*a^2*c*d^3)*x^4 + (b^2*c^4 + 10*a*b*c^3*d - 15*a^2*c^2*d^2)*x^
2)*sqrt(d)*weierstrassPInverse(-4*c/d, 0, x) - (4*a^2*c^2*d^2 - (b^2*c^2*d^
2 + 10*a*b*c*d^3 - 15*a^2*d^4)*x^4 + (b^2*c^3*d - 14*a*b*c^2*d^2 + 21*a^2*c
*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-5/2)/(c^3*d^4*x^6 + 2*c^4*d^3*x^4 +
 c^5*d^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^2}{\left(ex\right)^{\frac{5}{2}}\left(c + dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/((e*x)**(5/2)*(c + d*x**2)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*e^(-5/2)/((d*x^2 + c)^(5/2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2}{{\left(e\,x\right)}^{5/2}\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)), x)
```

3.864 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=489

$$-\frac{2a^2}{5ce(ex)^{5/2}\left(c+dx^2\right)^{3/2}}-\frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\,\left(c+dx^2\right)^{3/2}}+\frac{\left(5b^2c^2-70abcd+77a^2d^2\right)(ex)^{3/2}}{15c^3e^5\left(c+dx^2\right)^{3/2}}+\frac{\left(5b^2c^2-70abcd+\right.}{10c^4e^5\sqrt{c}}$$

```
[Out] -2/5*a^2/c/e/(e*x)^(5/2)/(d*x^2+c)^(3/2)+1/15*(77*a^2*d^2-70*a*b*c*d+5*b^2*
c^2)*(e*x)^(3/2)/c^3/e^5/(d*x^2+c)^(3/2)-2/5*a*(-11*a*d+10*b*c)/c^2/e^3/(d*
x^2+c)^(3/2)/(e*x)^(1/2)+1/10*(77*a^2*d^2-70*a*b*c*d+5*b^2*c^2)*(e*x)^(3/2)
/c^4/e^5/(d*x^2+c)^(1/2)-1/10*(77*a^2*d^2-70*a*b*c*d+5*b^2*c^2)*(e*x)^(1/2)
*(d*x^2+c)^(1/2)/c^4/e^4/d^(1/2)/(c^(1/2)+x*d^(1/2))+1/10*(77*a^2*d^2-70*a*
b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/
2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arcta
n(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((
d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(15/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1
/2)-1/20*(77*a^2*d^2-70*a*b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2
)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/
2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/
2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(15/4)/d^
(3/4)/e^(7/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {473, 464, 296, 335, 311, 226, 1210}

$$-\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(77a^2d^2-70abcd+5b^2c^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{20c^{15/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{\left(\sqrt{c}+\sqrt{d}\,x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}\,x\right)^2}}\left(77a^2d^2-70abcd+5b^2c^2\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{15/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}+\frac{(ex)^{3/2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{10c^4e^5\sqrt{c+dx^2}}-\frac{\sqrt{ex}\sqrt{c+dx^2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{10c^4\sqrt{d}\,e^4\left(\sqrt{c}+\sqrt{d}\,x\right)}+\frac{(ex)^{3/2}\left(77a^2d^2-70abcd+5b^2c^2\right)}{15c^3e^5\left(c+dx^2\right)^{3/2}}-\frac{2a^2}{5ce(ex)^{5/2}\left(c+dx^2\right)^{3/2}}-\frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\,\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-2*a^2)/(5*c*e*(e*x)^(5/2)*(c + d*x^2)^(3/2)) - (2*a*(10*b*c - 11*a*d))/(5
*c^2*e^3*Sqrt[e*x]*(c + d*x^2)^(3/2)) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d
^2)*(e*x)^(3/2))/(15*c^3*e^5*(c + d*x^2)^(3/2)) + ((5*b^2*c^2 - 70*a*b*c*d
+ 77*a^2*d^2)*(e*x)^(3/2))/(10*c^4*e^5*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 70*
a*b*c*d + 77*a^2*d^2)*Sqrt[e*x]*Sqrt[c + d*x^2])/(10*c^4*Sqrt[d]*e^4*(Sqrt[
c] + Sqrt[d]*x)) + ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d
]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(10*c^(15/4)*d^(3/4)*e^(7/2)*Sqrt[c +
d*x^2]) - ((5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x)*Sqrt
```

```
[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], 1/2])/(20*c^(15/4)*d^(3/4)*e^(7/2)*Sqrt[c + d*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx = -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} + \frac{2\int \frac{\frac{1}{2}a(10bc-11ad)+\frac{5}{2}b^2cx^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx}{5ce^2}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\,(c+dx^2)^{3/2}} - \frac{\left(4\left(-\frac{5}{4}b^2c^2+\frac{7}{4}ad(10bc-\right.\right.}{5c^2}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\,(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11a}{15c^3e^5(c+dx^2)}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\,(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11a}{15c^3e^5(c+dx^2)}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\,(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11a}{15c^3e^5(c+dx^2)}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\,(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11a}{15c^3e^5(c+dx^2)}$$

$$= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} - \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\,(c+dx^2)^{3/2}} + \frac{(5b^2c^2-7ad(10bc-11a}{15c^3e^5(c+dx^2)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.16, size = 181, normalized size = 0.37

$$\frac{x\left(5b^2c^2x^4(5c+3dx^2)-10abcx^2(12c^2+35cdx^2+21d^2x^4)+a^2(-12c^3+132c^2dx^2+385cd^2x^4+231d^3x^6)-(5b^2c^2-70abcd+77a^2d^2)\,x^4(c+dx^2)\sqrt{1+\frac{dx^2}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{dx^2}{c}\right)\right)}{30c^4(ex)^{7/2}\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*(5*b^2*c^2*x^4*(5*c + 3*d*x^2) - 10*a*b*c*x^2*(12*c^2 + 35*c*d*x^2 + 21*
d^2*x^4) + a^2*(-12*c^3 + 132*c^2*d*x^2 + 385*c*d^2*x^4 + 231*d^3*x^6) - (5
*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*x^4*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)]))/(30*c^4*(e*x)^(7/2)*(c + d*x^
2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(491) = 982$.
time = 0.18, size = 1231, normalized size = 2.52

method	result
elliptic	$\frac{\sqrt{ex\,(d\,x^2+c)}\left(\frac{x\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{de\,x^3+cex}}{3c^3e^4d^2\left(x^2+\frac{c}{d}\right)^2}+\frac{x^2\left(5a^2d^2-6abcd+b^2c^2\right)}{2e^3c^4\sqrt{\left(x^2+\frac{c}{d}\right)dex}}-\frac{2a^2\sqrt{de\,x^3+cex}}{5c^3e^4x^3}+\frac{2\left(de\,x^2+c\right.}{5c^4e^4\sqrt{x\,(}}\right.}{}$

risch	$-\frac{2\sqrt{d\,x^2+c}\,a(-13ad\,x^2+10c\,x^2b+ac)}{5c^4x^2e^3\sqrt{ex}}-\frac{\left(\frac{a(13ad-10bc)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(462*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1
/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(
1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^4-420*((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2
*2^(1/2))*a*b*c^2*d^2*x^4+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Ellipt
icE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^4-231*
((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)
^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d
)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^3*x^4+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2
)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a
*b*c^2*d^2*x^4-15*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-
c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*d*x^4+462*((d*x+(-c*d
)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/
2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1
/2),1/2*2^(1/2))*a^2*c^2*d^2*x^2-420*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)*EllipticE(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d*
x^2+30*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))
```

```
/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticE(((d*x+(-c*d)^(1/2)
)/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^4*x^2-231*((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)
^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/
2))*a^2*c^2*d^2*x^2+210*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF((
(d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^3*d*x^2-15*((d*x+
(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2)
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2
))^(1/2),1/2*2^(1/2))*b^2*c^4*x^2-462*a^2*d^4*x^6+420*a*b*c*d^3*x^6-30*b^2*
c^2*d^2*x^6-770*a^2*c*d^3*x^4+700*a*b*c^2*d^2*x^4-50*b^2*c^3*d*x^4-264*a^2*
c^2*d^2*x^2+240*a*b*c^3*d*x^2+24*a^2*c^3*d)/x^2/d/c^4/e^3/(e*x)^(1/2)/(d*x^
2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] e^(-7/2)*integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 273, normalized size = 0.56

$$\frac{\left(3\left(\left(5\,b^2c^2d^2-70\,abcd^3+77\,a^2d^4\right)x^7+2\left(5\,b^2c^3d-70\,abc^2d^2+77\,a^2cd^3\right)x^5+\left(5\,b^2c^4-70\,abc^3d+77\,a^2c^2d^2\right)x^3\right)\sqrt{d}\,\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+\left(3\left(5\,b^2c^2d^2-70\,abcd^3+77\,a^2d^4\right)x^6-12\,a^2c^3d+5\left(5\,b^2c^3d-70\,abc^2d^2+77\,a^2cd^3\right)x^4-12\left(10\,abc^3d-11\,a^2c^2d^2\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{x}\right)e^{\left(-\frac{7}{2}\right)}}{30\left(c^4d^3x^7+2\,c^5d^2x^5+c^6dx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/30*(3*((5*b^2*c^2*d^2 - 70*a*b*c*d^3 + 77*a^2*d^4)*x^7 + 2*(5*b^2*c^3*d -
 70*a*b*c^2*d^2 + 77*a^2*c*d^3)*x^5 + (5*b^2*c^4 - 70*a*b*c^3*d + 77*a^2*c^
2*d^2)*x^3)*sqrt(d)*weierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d,
0, x)) + (3*(5*b^2*c^2*d^2 - 70*a*b*c*d^3 + 77*a^2*d^4)*x^6 - 12*a^2*c^3*d
+ 5*(5*b^2*c^3*d - 70*a*b*c^2*d^2 + 77*a^2*c*d^3)*x^4 - 12*(10*a*b*c^3*d -
11*a^2*c^2*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(x))*e^(-7/2)/(c^4*d^3*x^7 + 2*c^5
*d^2*x^5 + c^6*d*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^2*e^(-7/2)/((d*x^2 + c)^(5/2)*x^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^2}{{\left(e\,x\right)}^{7/2}\,{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)), x)
```

3.865 $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=372

$$\frac{2(2bc-7ad)e^3\sqrt{ex}\,\sqrt{c-dx^2}}{21b^2d}-\frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b}-\frac{2\sqrt[4]{c}\left(2b^2c^2+14abcd-21a^2d^2\right)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\Big(\sin}{21b^3d^{5/4}\sqrt{c-dx^2}}$$

```
[Out] -2/7*e*(e*x)^(5/2)*(-d*x^2+c)^(1/2)/b+2/21*(-7*a*d+2*b*c)*e^3*(e*x)^(1/2)*(
-d*x^2+c)^(1/2)/b^2/d-2/21*c^(1/4)*(-21*a^2*d^2+14*a*b*c*d+2*b^2*c^2)*e^(7/
2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/d
^(5/4)/(-d*x^2+c)^(1/2)+a*c^(1/4)*(-a*d+b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1
/2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)+a*c^(1/4)*(-a*d+b*c)*e^(7/2)*EllipticPi(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x
^2/c)^(1/2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 489, 596, 537, 230, 227, 418, 1233, 1232}

$$-\frac{2\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{21b^3d^{5/4}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2e^3\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{21b^2d}-\frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*(2*b*c - 7*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2*d) - (2*e*(e*x)^(
5/2)*Sqrt[c - d*x^2])/(7*b) - (2*c^(1/4)*(2*b^2*c^2 + 14*a*b*c*d - 21*a^2*d
^2)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(21*b^3*d^(5/4)*Sqrt[c - d*x^2]) + (a*c^(1/4)*(b*c - a*d
)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt
[c - d*x^2]) + (a*c^(1/4)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^8\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} + \frac{(2e)\text{Subst}\left(\int \frac{x^4\left(5ac+\frac{(2bc-7ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{7b}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} - \frac{(2e^5)\text{Subst}\left(\int \frac{\frac{ac(2bc-7a}{e^2}}{(a}\right.}{}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} + \frac{(2a^2(bc-ad)e^3)\text{Subst}}{}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} + \frac{(a(bc-ad)e^3)\text{Subst}\left(\right.}{}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} - \frac{2\sqrt[4]{c}\,(2b^2c^2+14abcd-}{}$$

$$= \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} - \frac{2\sqrt[4]{c}\,(2b^2c^2+14abcd-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.18, size = 187, normalized size = 0.50

$$\frac{2e^3\sqrt{ex}\left(-5a(c-dx^2)(-2bc+7ad+3bdx^2)+5ac(-2bc+7ad)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+(2b^2c^2+14abcd-21a^2d^2)\,x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{105ab^2d\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*(-5*a*(c - d*x^2)*(-2*b*c + 7*a*d + 3*b*d*x^2) + 5*a*c*(-2
*b*c + 7*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*
x^2)/a] + (2*b^2*c^2 + 14*a*b*c*d - 21*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*App
ellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(105*a*b^2*d*Sqrt[c - d*x^2
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(288) = 576$.
time = 0.22, size = 1468, normalized size = 3.95

method	result
risch	$-\frac{2(3bd\,x^2+7ad-2bc)\sqrt{-d\,x^2+c}\;x\,e^4}{21d\,b^2\sqrt{ex}}+\frac{\left(\frac{\left(21a^2d^2-14abcd-2b^2c^2\right)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-}}{bd\sqrt{-de\,x^3+cex}}\right.}{}$
elliptic	$\frac{\sqrt{ex}\;\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{2e^3x^2\sqrt{-de\,x^3+cex}}{7b}-\frac{2\left(\frac{(ad-bc)e^4}{b^2}+\frac{5e^4c}{7b}\right)\sqrt{-de\,x^3+cex}}{3de}+\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/42*e^3*(e*x)^(1/2)/b^2/d*(42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c
*d)^(1/2)-70*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2
^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+24*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^
2*d*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+4*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^3*(a*b)^(1/2)*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2
)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*
2^(1/2)*a^3*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-21*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a
*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a
^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
1/2)*a^3*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+21*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+2
1*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)*(c*d)^(1/2)+12*a*b^2*d^4*x^5*(a*b)^(1/2)-12*b^3*c*d^3*x^5*
(a*b)^(1/2)+28*a^2*b*d^4*x^3*(a*b)^(1/2)-48*a*b^2*c*d^3*x^3*(a*b)^(1/2)+20*
b^3*c^2*d^2*x^3*(a*b)^(1/2)-28*a^2*b*c*d^3*x*(a*b)^(1/2)+36*a*b^2*c^2*d^2*x
*(a*b)^(1/2)-8*b^3*c^3*d*x*(a*b)^(1/2))/x/(-d*x^2+c)^(1/2)/(a*b)^(1/2)/((c*
d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

<u>Maxima</u> [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(7/2)*integrate(sqrt(-d*x^2 + c)*x^(7/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{7}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
[Out] -Integral((e*x)**(7/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-sqrt(-d*x^2 + c)*x^(7/2)*e^(7/2)/(b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{7/2}\sqrt{c-d\,x^2}}{a-b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(7/2)*(c - d*x^2)^(1/2))/(a - b*x^2),x)
[Out] int(((e*x)^(7/2)*(c - d*x^2)^(1/2))/(a - b*x^2), x)
```

3.866 $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=414

$$-\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b}-\frac{2c^{3/4}(2bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}+\frac{2c^{3/4}(2bc-5ad)e^{5/2}}{}$$

```
[Out] -2/5*e*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b-2/5*c^(3/4)*(-5*a*d+2*b*c)*e^(5/2)*El
lipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(3/4
)/(-d*x^2+c)^(1/2)+2/5*c^(3/4)*(-5*a*d+2*b*c)*e^(5/2)*EllipticF(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(3/4)/(-d*x^2+c)^(1/2)-
c^(1/4)*(-a*d+b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-
b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(5/2)/d^(1/4
)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2
)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1
/2)/b^(5/2)/d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 489, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt{a}\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(2bc-5ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(2bc-5ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}}-\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (-2*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b) - (2*c^(3/4)*(2*b*c - 5*a*d)*e^(5/
2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(5*b^2*d^(3/4)*Sqrt[c - d*x^2]) + (2*c^(3/4)*(2*b*c - 5*a*d)*e^(5
/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(5*b^2*d^(3/4)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*(b*c - a*d)*e
^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4)*Sqr
t[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^6\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{(2e)\text{Subst}\left(\int \frac{x^2\left(3ac+\frac{(2bc-5ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{(2e)\text{Subst}\left(\int \left(-\frac{(2bc-5ad)x^2}{b\sqrt{c-\frac{dx^4}{e^2}}} - \frac{5(-abc+a^2d)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) d$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} - \frac{(2(2bc-5ad)e)\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2} + \frac{(2}{}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{\left(2\sqrt{c}\,(2bc-5ad)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2\sqrt{d}}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{\left(2\sqrt{c}\,(2bc-5ad)e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\right.}{5b^2\sqrt{d}\,\sqrt{c-dx^2}}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} + \frac{2c^{3/4}(2bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right| -}{5b^2d^{3/4}\sqrt{c-dx^2}}$$

$$= -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} - \frac{2c^{3/4}(2bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right| -}{5b^2d^{3/4}\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.13, size = 143, normalized size = 0.35

$$\frac{2e(ex)^{3/2}\left(-7a(c-dx^2)+7ac\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+(2bc-5ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{35ab\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*e*(e*x)^(3/2)*(-7*a*(c - d*x^2) + 7*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4
, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + (2*b*c - 5*a*d)*x^2*Sqrt[1 - (d*x^2)
/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(35*a*b*Sqrt[c - d*
x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. 2(308) = 616.
time = 0.17, size = 1480, normalized size = 3.57

method	result
risch	$-\frac{2x^2\sqrt{-d\,x^2+c}\,e^3}{5b\sqrt{ex}}+\frac{\left(\frac{(5ad-2bc)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\left(-\frac{2\sqrt{cd}\,\mathrm{EllipticE}\ldots}{\ldots}\right.}{bd\sqrt{-de\,x^3+c\ldots}}\right.}{\ldots}$

elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{2e^2x\sqrt{-de\,x^3+cex}}{5b}-\frac{2c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,e^3a\,\mathrm{EllipticE}}{\sqrt{-de\,x^3+cex}\,b^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*e^2*(e*x)^(1/2)*(20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2-28*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a*b^2*c^2*d+8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3-10*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^
(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a
^2*b*c*d^2+14*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d-4*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c
^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2
^(1/2))*(c*d)^(1/2)*a^2*d^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d-5*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d
^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*(c*d)^(1/2)*a*b*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi
```

```
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(
1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a
*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-4*a*b^2*d^3*x^4+4*b^3*c*d^2
*x^4+4*a*b^2*c*d^2*x^2-4*b^3*c^2*d*x^2)/x/(-d*x^2+c)^(1/2)/b^2/((c*d)^(1/2)
*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(5/2)*integrate(sqrt(-d*x^2 + c)*x^(5/2)/(b*x^2 - a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
[Out] -Integral((e*x)**(5/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-sqrt(-d*x^2 + c)*x^(5/2)*e^(5/2)/(b*x^2 - a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}\,\sqrt{c-d\,x^2}}{a-b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c - d*x^2)^(1/2))/(a - b*x^2),x)
[Out] int(((e*x)^(5/2)*(c - d*x^2)^(1/2))/(a - b*x^2), x)
```

3.867 $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=315

$$-\frac{2e\sqrt{ex}\,\sqrt{c-dx^2}}{3b}-\frac{2\sqrt[4]{c}\,(2bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(bc-ad)e^{3/2}\sqrt{1}}{}$$

```
[Out] -2/3*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b-2/3*c^(1/4)*(-3*a*d+2*b*c)*e^(3/2)*El
lipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(1/4
)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2
)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2
/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/
2)/b^2/d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 489, 537, 230, 227, 418, 1233, 1232}

$$-\frac{2\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(2bc-3ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{2e\sqrt{ex}\,\sqrt{c-dx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (-2*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b) - (2*c^(1/4)*(2*b*c - 3*a*d)*e^(3/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(3*b^2*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) +
 (c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[
c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(b^2*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2e\sqrt{ex}\,\sqrt{c-dx^2}}{3b} + \frac{(2e)\text{Subst}\left(\int \frac{ac+\frac{(2bc-3ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3b}$$

$$= -\frac{2e\sqrt{ex}\,\sqrt{c-dx^2}}{3b} - \frac{(2(2bc-3ad)e)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3b^2} + \text{—}(2$$

$$= -\frac{2e\sqrt{ex}\,\sqrt{c-dx^2}}{3b} + \frac{((bc-ad)e)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right.}{b^2}$$

$$= -\frac{2e\sqrt{ex}\,\sqrt{c-dx^2}}{3b} - \frac{2\sqrt[4]{c}\,(2bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right|}{3b^2\sqrt[4]{d}\,\sqrt{c-dx^2}}$$

$$= -\frac{2e\sqrt{ex}\,\sqrt{c-dx^2}}{3b} - \frac{2\sqrt[4]{c}\,(2bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right|}{3b^2\sqrt[4]{d}\,\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 143, normalized size = 0.45

$$\frac{2e\sqrt{ex}\left(-5a(c-dx^2)+5ac\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+(2bc-3ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{15ab\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]

[Out] (2*e*Sqrt[e*x]*(-5*a*(c - d*x^2) + 5*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (2*b*c - 3*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a*b*Sqrt[c - d*x^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1274 vs. 2(237) = 474.

time = 0.15, size = 1275, normalized size = 4.05

method	result
risch	$-\frac{2\sqrt{-d\,x^2+c}\,x\,e^2}{3b\sqrt{ex}}+\frac{\left(\frac{(3ad-2bc)\sqrt{cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{c}}{}\right.}{\sqrt{c}}}\right.}{bd\sqrt{-de\,x^3+cex}}\right.}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{2e\sqrt{-de\,x^3+cex}}{3b}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}}{\sqrt{-de\,x^3+cex}\,b^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/6*e*(e*x)^(1/2)/b*(6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^

```
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)-10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a
*b*c*d*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*(c*d)^(1/2)*(
a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-3*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d+3*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2
))*(c*d)^(1/2)*a*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2+3*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d+3*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*
a*b*c*d+4*a*b*d^3*x^3*(a*b)^(1/2)-4*b^2*c*d^2*x^3*(a*b)^(1/2)-4*a*b*c*d^2*x
*(a*b)^(1/2)+4*b^2*c^2*d*x*(a*b)^(1/2))/x/(-d*x^2+c)^(1/2)/(a*b)^(1/2)/((c*
d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(3/2)*integrate(sqrt(-d*x^2 + c)*x^(3/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
[Out] -Integral((e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-sqrt(-d*x^2 + c)*x^(3/2)*e^(3/2)/(b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{3/2}\,\sqrt{c-d\,x^2}}{a-b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c - d*x^2)^(1/2))/(a - b*x^2),x)
[Out] int(((e*x)^(3/2)*(c - d*x^2)^(1/2))/(a - b*x^2), x)
```

3.868 $\int \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{a-bx^2}\,dx$

Optimal. Leaf size=365

$$\frac{2c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-\right.}{b\sqrt{c-dx^2}}$$

```
[Out] 2*c^(3/4)*d^(1/4)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*
(1-d*x^2/c)^(1/2)/b/(-d*x^2+c)^(1/2)-2*c^(3/4)*d^(1/4)*EllipticF(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b/(-d*x^2+c)^(1/2)-c
^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c
^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/a^(1/2)
/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/
2)/d^(1/4)/a^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {477, 505, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\,b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\,b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

```
[Out] (2*c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (2*c^(3/4)*d^(1/4)*
Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*
x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 505

```
Int[((x_)^2*Sqrt[(c_) + (d_.)*(x_)^4])/((a_) + (b_.)*(x_)^4), x_Symbol] :>
Dist[d/b, Int[x^2/Sqrt[c + d*x^4], x], x] + Dist[(b*c - a*d)/b, Int[x^2/((a
 + b*x^4)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a
*d, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx,x,\sqrt{ex}\right)}{e}$$

$$= \frac{(2d)\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{be} + \frac{(2(bc-ad))\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{be}$$

$$= -\frac{\left(2\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{b} + \frac{\left(2\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}}{\sqrt{}}}{\sqrt{c-}}\right.}{b}$$

$$= -\frac{\left(2\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx,x,\sqrt{ex}\right)}{b\sqrt{c-dx^2}} + \frac{\left(2\sqrt{c}\sqrt{d}\sqrt{1-\right.}{}$$

$$= -\frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}\,(bc-ad)\sqrt{e}\sqrt{1-}}{}$$

$$= \frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}} - \frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,I}{b\sqrt{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.03, size = 69, normalized size = 0.19

$$\frac{2x\sqrt{ex}\sqrt{c-dx^2}\,F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{3a\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2),x]

[Out] (2*x*Sqrt[e*x]*Sqrt[c - d*x^2]*AppellF1[3/4, -1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a])/(3*a*Sqrt[1 - (d*x^2)/c])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(269) = 538$.
time = 0.13, size = 690, normalized size = 1.89

method	result
default	$-\frac{\sqrt{ex}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,d\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)abcd-2\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.\right.}{\cdots}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{2ec\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\cdots\right.}{b\sqrt{-de\,x^3+cex}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/2*(e*x)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*d*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d-(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*d-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2+(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b*c+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d+(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*d-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2-(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b*c-4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d+4*EllipticE(((d

```
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2)/(-d*x^2+c)^(1/2)/b
/x/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(1/2)*integrate(sqrt(-d*x^2 + c)*sqrt(x)/(b*x^2 - a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
[Out] -Integral(sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-sqrt(-d*x^2 + c)*sqrt(x)*e^(1/2)/(b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}\,\sqrt{c-d\,x^2}}{a-b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c - d*x^2)^(1/2))/(a - b*x^2),x)
[Out] int(((e*x)^(1/2)*(c - d*x^2)^(1/2))/(a - b*x^2), x)
```

3.869 $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx$

Optimal. Leaf size=283

$$\frac{2\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right.}{ab\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}}$$

```
[Out] 2*c^(1/4)*d^(3/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2
/c)^(1/2)/b/e^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)
^(1/2)/a/b/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*EllipticPi(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*
x^2/c)^(1/2)/a/b/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {477, 415, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{e}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)),x]
```

```
[Out] (2*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*b*d^(1/4)*Sqrt[e]*Sqrt
[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]
*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/(a*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 415

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \frac{2\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(2d)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{be} + \frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{be}$$

$$= \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{abe} + \frac{(bc-ad)\text{Subst}\left(\int \frac{}{\left(1+\frac{\sqrt{}}{\sqrt{}}\right.}\right.}{}$$

$$= \frac{2\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{e}\,\sqrt{c-dx^2}} + \frac{\left((bc-ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}}{a}$$

$$= \frac{2\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{e}\,\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\,(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\right.}{ab\sqrt[4]{d}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.03, size = 67, normalized size = 0.24

$$\frac{2x\sqrt{c-dx^2}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{a\sqrt{ex}\,\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)),x]
[Out] (2*x*Sqrt[c - d*x^2]*AppellF1[1/4, -1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/(a*
Sqrt[e*x]*Sqrt[1 - (d*x^2)/c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(213) = 426$.

time = 0.12, size = 640, normalized size = 2.26

method	result
default	$-\frac{\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\sqrt{cd}\,\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)a\,d^{2}\sqrt{ab}-2\,\mathrm{EllipticF}\left(\cdots\right.\right.}{\cdots}$
elliptic	$\frac{\sqrt{(-d\,x^{2}+c)\,ex}\,\left(\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{-de\,x^{3}+cex}}+\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(2*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*d^2*(a*b)^(1/2)-2*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c*d*(a*b)^(1/2)+EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(
1/2)*d),1/2*2^(1/2))*a*b*d*(c*d)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*d^2
*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*(c*d)^(1/2)+EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2
)*d),1/2*2^(1/2))*b*c*d*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*d*(c
*d)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d^2*(a*b)^(1/2)+EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*b^2*c*(c*d)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c*d*(a*b)
^(1/2))/(-d*x^2+c)^(1/2)/(e*x)^(1/2)/(a*b)^(1/2)/((c*d)^(1/2)*b+(a*b)^(1/2)
*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="maxima")
[Out] -e^(-1/2)*integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(x)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)/(e*x)**(1/2),x)
[Out] -Integral(sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate(-sqrt(-d*x^2 + c)*e^(-1/2)/((b*x^2 - a)*sqrt(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-d\,x^2}}{\sqrt{e\,x}\,\left(a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/((e*x)^(1/2)*(a - b*x^2)),x)
[Out] int((c - d*x^2)^(1/2)/((e*x)^(1/2)*(a - b*x^2)), x)
```

3.870 $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)}\,dx$

Optimal. Leaf size=392

$$-\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{e}}{\sqrt[4]{c}\sqrt{}}\right.\right.}{ae^{3/2}\sqrt{c-dx^2}}$$

```
[Out] -2*(-d*x^2+c)^(1/2)/a/e/(e*x)^(1/2)-2*c^(3/4)*d^(1/4)*EllipticE(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/e^(3/2)/(-d*x^2+c)^(1/2)+2*
c^(3/4)*d^(1/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c
)^(1/2)/a/e^(3/2)/(-d*x^2+c)^(1/2)-c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(
1/2)/a^(3/2)/d^(1/4)/e^(3/2)/b^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*El
lipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2
),I)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/e^(3/2)/b^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 486, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/
c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*e^(3/2)
*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*e^(3/2)*Sqrt[c - d*x^2]) -
 (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3
/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1
- (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt
[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)}\,dx = \frac{2\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^2\left(a-\frac{bx^4}{e^2}\right)}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2\text{Subst}\left(\int \frac{x^2\left(\frac{bc-2ad}{e^2}+\frac{bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2\text{Subst}\left(\int \left(-\frac{dx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}} + \frac{(bc-ad)x^2}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)\,dx, x, \sqrt{ex}\right.}{ae}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} - \frac{(2d)\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae^3} + \frac{(2(bc-ad))\text{Subst}\left(\int \right.}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{\left(2\sqrt{c}\,\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae^2} - \frac{\left(2\sqrt{c}\,\sqrt{d}\right)\text{S}}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{\left(2\sqrt{c}\,\sqrt{d}\,\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{ae^2\sqrt{c-dx^2}} - \frac{\left(\right.}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2c^{3/4}\sqrt[4]{d}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}\,(bc-ad}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} - \frac{2c^{3/4}\sqrt[4]{d}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}} + \frac{2c^{3/4}\sqrt[4]{d}\,\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.10, size = 143, normalized size = 0.36

$$\frac{x\left(-42a(c-dx^2)+14(bc-2ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+6bdx^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{21a^2(ex)^{3/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (x*(-42*a*(c - d*x^2) + 14*(b*c - 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[3
/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 6*b*d*x^4*Sqrt[1 - (d*x^2)/c]*Appe
llF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*(e*x)^(3/2)*Sqrt[c
- d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. 2(292) = 584.
time = 0.13, size = 1263, normalized size = 3.22

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{2(-de\,x^2+ce)}{e^2a\sqrt{x\,(-de\,x^2+ce)}}+\frac{2c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticE}\left(\sqrt{\cdots}\right.}{ea\sqrt{-de\,x^3+cex}}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a
*b*c*d-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^
(1/2)*d),1/2*2^(1/2))*a*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)
```

```
*d),1/2*2^(1/2))*b^2*c^2+(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d+(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d-((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2-(c*d)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*b*c+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*El
lipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2-2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a*b*c*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2+4*a*b*d^2*x^2-4*b^2*c
*d*x^2-4*a*b*c*d+4*b^2*c^2)/(-d*x^2+c)^(1/2)/e/(e*x)^(1/2)/a/((c*d)^(1/2)*b
+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(-3/2)*integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*x^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c-dx^2}}{-a\,(ex)^{\frac{3}{2}}+bx^2\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a),x)
[Out] -Integral(sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-sqrt(-d*x^2 + c)*e^(-3/2)/((b*x^2 - a)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-d\,x^2}}{{\left(e\,x\right)}^{3/2}\,\left(a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/((e*x)^(3/2)*(a - b*x^2)),x)
[Out] int((c - d*x^2)^(1/2)/((e*x)^(3/2)*(a - b*x^2)), x)
```

3.871 $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)}\,dx$

Optimal. Leaf size=308

$$-\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{2\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3ae^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\right.}{a^2\sqrt[4]{d}\,e^{5/2}\sqrt{c-d}}$$

```
[Out] -2/3*(-d*x^2+c)^(1/2)/a/e/(e*x)^(3/2)+2/3*c^(1/4)*d^(3/4)*EllipticF(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/e^(5/2)/(-d*x^2+c)^(1/2
)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2
)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/e^(5/2)/(-d*x^2+
c)^(1/2)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),
b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/e^(5/2)/(-
d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 486, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ae^{5/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*e
^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c -
 a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqr
t[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)}\,dx=\frac{2\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^4\left(a-\frac{bx^4}{e^2}\right)}\,dx,x,\sqrt{ex}\right)}{e}$$

$$=-\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{2\text{Subst}\left(\int \frac{\frac{3bc-2ad}{e^2}-\frac{bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{3ae}$$

$$=-\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{(2d)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{3ae^3}+\frac{(2(bc-ad))\text{Subst}\left(\int\right.}{}$$

$$=-\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{a^2e^3}+\frac{(bc-a}{}$$

$$=-\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{2\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3ae^{5/2}\sqrt{c-dx^2}}+\frac{\left((bc-ad)\right.}{}$$

$$=-\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{2\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3ae^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(bc-ad}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.12, size = 146, normalized size = 0.47

$$\frac{x\left(10(3bc-2ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-2\left(5a(c-dx^2)+bdx^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)\right)}{15a^2(ex)^{5/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (x*(10*(3*b*c - 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (
d*x^2)/c, (b*x^2)/a] - 2*(5*a*(c - d*x^2) + b*d*x^4*Sqrt[1 - (d*x^2)/c]*App
ellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])))/(15*a^2*(e*x)^(5/2)*Sqrt[c
 - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1155 vs. $2(230) = 460$.
time = 0.13, size = 1156, normalized size = 3.75 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*d*(2*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a*d*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2*2^(1/2)*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x*(a*b)^(1/2
)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*2^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^
(1/2))*a*b*c*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*2^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2
*2^(1/2))*a*d*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*2^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*2^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)-3*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d*x*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)-3*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d*x*(a*b)^(
1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*2^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*b^2*c^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*2^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*b*c*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*a*d
^2*x^2*(a*b)^(1/2)-4*b*c*d*x^2*(a*b)^(1/2)-4*a*c*d*(a*b)^(1/2)+4*b*c^2*(a*b
```

```
)^(1/2))/(-d*x^2+c)^(1/2)/x/a/e^2/(e*x)^(1/2)/(a*b)^(1/2)/((c*d)^(1/2)*b+(a
*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(-5/2)*integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*x^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c - dx^2}}{-a\left(ex\right)^{\frac{5}{2}} + bx^2\left(ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a),x)
[Out] -Integral(sqrt(c - d*x**2)/(-a*(e*x)**(5/2) + b*x**2*(e*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-sqrt(-d*x^2 + c)*e^(-5/2)/((b*x^2 - a)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-d\,x^2}}{{\left(e\,x\right)}^{5/2}\,\left(a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/((e*x)^(5/2)*(a - b*x^2)),x)
[Out] int((c - d*x^2)^(1/2)/((e*x)^(5/2)*(a - b*x^2)), x)
```

3.872 $\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}(a-bx^2)}\,dx$

Optimal. Leaf size=457

$$-\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}}-\frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}}-\frac{2\sqrt[4]{d}\,(5bc-2ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}\,e^{7/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\,(5}{}$$

```
[Out] -2/5*(-d*x^2+c)^(1/2)/a/e/(e*x)^(5/2)-2/5*(-2*a*d+5*b*c)*(-d*x^2+c)^(1/2)/a
^2/c/e^3/(e*x)^(1/2)-2/5*d^(1/4)*(-2*a*d+5*b*c)*EllipticE(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(1/4)/e^(7/2)/(-d*x^2+c)^(1/2
)+2/5*d^(1/4)*(-2*a*d+5*b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),
I)*(1-d*x^2/c)^(1/2)/a^2/c^(1/4)/e^(7/2)/(-d*x^2+c)^(1/2)-c^(1/4)*(-a*d+b*c
)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d
^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/2)/(-d*x^2+c)^(1/2
)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)
*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/
2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 486, 597, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt{b}\,\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}\,e^{7/2}\sqrt{c-dx^2}}+\frac{\sqrt{b}\,\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}\,e^{7/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\,\sqrt{1-\frac{dx^2}{c}}\,(5bc-2ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}\,e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\,\sqrt{1-\frac{dx^2}{c}}\,(5bc-2ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}\,e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}\,(5bc-2ad)}{5a^2ce^3\sqrt{ex}}-\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(7/2)*(a - b*x^2)),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 2*a*d)*Sqrt[c - d*x^
2])/(5*a^2*c*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c - 2*a*d)*Sqrt[1 - (d*x^2)/c
]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(1
/4)*e^(7/2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*(5*b*c - 2*a*d)*Sqrt[1 - (d*x^2)/
c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*c^(
1/4)*e^(7/2)*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^
2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
 + (Sqrt[b]*c^(1/4)*(b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}\left(a-bx^2\right)}\,dx = \frac{2\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^6\left(a-\frac{bx^4}{e^2}\right)}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} + \frac{2\text{Subst}\left(\int \frac{\frac{5bc-2ad}{e^2}-\frac{3bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5ae}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} - \frac{2\text{Subst}\left(\int \frac{x^2\left(-\frac{5b^2c^2-10abcd+2a^2d^2}{e^4}-\frac{bd(5bc-}{e^6}\right.}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{5a^2ce}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} - \frac{2\text{Subst}\left(\int \left(\frac{d(5bc-2ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}} - \frac{5(b^2c}{e^4\left(a-\frac{bx^4}{e^2}\right.}\right.\right.}{5a^2ce}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} - \frac{(2d(5bc-2ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\right.}{5a^2ce^5}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} + \frac{\left(2\sqrt{d}\,(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-}}\right.}{5a^2\sqrt{c}\,e^4}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} + \frac{\left(2\sqrt{d}\,(5bc-2ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subs}}{5a^2\sqrt{c}\,e^4\sqrt{c}}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} + \frac{2\sqrt[4]{d}\,(5bc-2ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\right.}{5a^2\sqrt[4]{c}\,e^{7/2}\sqrt{c-dx}}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}} - \frac{2\sqrt[4]{d}\,(5bc-2ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\right.}{5a^2\sqrt[4]{c}\,e^{7/2}\sqrt{c-dx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.17, size = 190, normalized size = 0.42

$$\frac{x\left(14(5b^2c^2-10abcd+2a^2d^2)\,x^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-6\left(7a(c-dx^2)\,(ac+5bcx^2-2adx^2)+bd(-5bc+2ad)x^6\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)\right)}{105a^3c(ex)^{7/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(7/2)*(a - b*x^2)),x]
```

```
[Out] (x*(14*(5*b^2*c^2 - 10*a*b*c*d + 2*a^2*d^2)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF
1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] - 6*(7*a*(c - d*x^2)*(a*c + 5*b*c
*x^2 - 2*a*d*x^2) + b*d*(-5*b*c + 2*a*d)*x^6*Sqrt[1 - (d*x^2)/c]*AppellF1[7
/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(105*a^3*c*(e*x)^(7/2)*Sqrt[c -
d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1541 vs. $2(345) = 690$.
time = 0.13, size = 1542, normalized size = 3.37

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{2\sqrt{-de\,x^3+cex}}{5e^4a\,x^3}+\frac{2\left(-de\,x^2+ce\right)(2ad-5bc)}{5e^4c\,a^2\sqrt{x\left(-de\,x^2+ce\right)}}-\frac{4d\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{\ }\cdots}{5a\,e^3\sqrt{-\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*b*d*(5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/
2))*a*b*c^2*d*x^2-5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*c*d*x^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
```

```
)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^3*x^2+5*(c*d)^(1/2)*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c^2*x^
2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c
^2*d*x^2+5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*a*c*d*x^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^3*x^2-5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c^2*x^2-8*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*a^2*c*d^2*x^2+28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipti
cE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d*x^2-20*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*b^2*c^3*x^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2-14*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a*b*c^2*d*x^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2-8*a^2*d^3*
x^4+28*a*b*c*d^2*x^4-20*b^2*c^2*d*x^4+12*a^2*c*d^2*x^2-32*a*b*c^2*d*x^2+20*
b^2*c^3*x^2-4*a^2*c^2*d+4*a*b*c^3)/x^2/(-d*x^2+c)^(1/2)/e^3/(e*x)^(1/2)/a^2
/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(-7/2)*integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*x^(7/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c-dx^2}}{-a\,(ex)^{\frac{7}{2}}+bx^2\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(7/2)/(-b*x**2+a),x)
[Out] -Integral(sqrt(c - d*x**2)/(-a*(e*x)**(7/2) + b*x**2*(e*x)**(7/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-sqrt(-d*x^2 + c)*e^(-7/2)/((b*x^2 - a)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-d\,x^2}}{{\left(e\,x\right)}^{7/2}\,\left(a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/((e*x)^(7/2)*(a - b*x^2)),x)
[Out] int((c - d*x^2)^(1/2)/((e*x)^(7/2)*(a - b*x^2)), x)
```

3.873 $\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx$

Optimal. Leaf size=485

$$-\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2}+\frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be}-\frac{2c^{3/4}(4b^2c^2-21abcd+15a^2d^2)\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E}{15b^3d^{3/4}\sqrt{c-dx^2}}$$

```
[Out] -2/45*(-9*a*d+11*b*c)*e*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b^2+2/9*d*(e*x)^(7/2)*
(-d*x^2+c)^(1/2)/b/e-2/15*c^(3/4)*(15*a^2*d^2-21*a*b*c*d+4*b^2*c^2)*e^(5/2)
*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/d^(
3/4)/(-d*x^2+c)^(1/2)+2/15*c^(3/4)*(15*a^2*d^2-21*a*b*c*d+4*b^2*c^2)*e^(5/2
)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/d^
(3/4)/(-d*x^2+c)^(1/2)-c^(1/4)*(-a*d+b*c)^2*e^(5/2)*EllipticPi(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^
2/c)^(1/2)/b^(7/2)/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*e^(5/2)*El
lipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2
),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(7/2)/d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 488, 596, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}}-\frac{\sqrt{a}\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{2e(ex)^{3/2}\sqrt{c-dx^2}(11bc-9ad)}{45b^2}+\frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (-2*(11*b*c - 9*a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(45*b^2) + (2*d*(e*x)^(
7/2)*Sqrt[c - d*x^2])/(9*b*e) - (2*c^(3/4)*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2
*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4)*Sqrt[c - d*x^2]) + (2*c^(3/4)*(4*b^2*c
^2 - 21*a*b*c*d + 15*a^2*d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(15*b^3*d^(3/4)*Sqrt[c - d*x^2
]) - (Sqrt[a]*c^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[
-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(b^(7/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(b*c
- a*d)^2*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*
Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(7/2)*d^(1
/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^6\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} - \frac{(2e)\text{Subst}\left(\int \frac{x^6\left(-\frac{c(9bc-7ad)}{e^2}+\frac{d(11bc-9ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{9b}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2e^5\right)\text{Subst}\left(\int \frac{x^2\left(\right.}{}\right.}{}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2e^5\right)\text{Subst}\left(\int \left(-\right.\right.}{}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2a(bc-ad)^2e\right)\text{Sul}}{}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2\sqrt{c}\left(4b^2c^2-21a\right.\right.}{}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{\left(2\sqrt{c}\left(4b^2c^2-21a\right.\right.}{}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} + \frac{2c^{3/4}(4b^2c^2-21abc}{}$$

$$= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} + \frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be} - \frac{2c^{3/4}(4b^2c^2-21abc}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.17, size = 183, normalized size = 0.38

$$-\frac{2e(ex)^{3/2}\left(-7a(c-dx^2)(-11bc+9ad+5bdx^2)+7ac(-11bc+9ad)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-3(4b^2c^2-21abcd+15a^2d^2)x^2\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{315ab^2\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (-2*e*(e*x)^(3/2)*(-7*a*(c - d*x^2)*(-11*b*c + 9*a*d + 5*b*d*x^2) + 7*a*c*(
-11*b*c + 9*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c,
(b*x^2)/a] - 3*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c
]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(315*a*b^2*Sqrt[c - d
*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2171 vs. $2(373) = 746$.
time = 0.16, size = 2172, normalized size = 4.48

method	result
risch	$\frac{2x^2(5bd\,x^2+9ad-11bc)\sqrt{-d\,x^2+c}\;e^3}{45b^2\sqrt{ex}} - \frac{\left((15a^2d^2-21abcd+4b^2c^2)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/90*e^2/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(100*a*b^3*c*d^3*x^4+36*a^2*b^2*c*
d^3*x^2+44*b^4*c^3*d*x^2-20*a*b^3*d^4*x^6+20*b^4*c*d^3*x^6-36*a^2*b^2*d^4*x
^4-64*b^4*c^2*d^2*x^4-48*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^4+24*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*b^4*c^4-80*a*b^3*c^2*d^2*x^2-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a
^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
1/2)*a^3*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+90*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4
5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c
^3*d-432*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d^2+300*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^
3*c^3*d-90*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^3+216*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^
2*c^2*d^2-150*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3*d-45*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3*d+180*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a^3*b*c*d^3-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^3*(a*b
)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+90*EllipticPi(((d*x+(c*d)^(1
```

```
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(
1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+45*(c*d)^(1/2)*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*
a*b^2*c^2*d-90*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+90*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-45*
(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*a*b^2*c^2*d)/b^3/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*
b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(5/2)*integrate((-d*x^2 + c)^(3/2)*x^(5/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx-\int\left(-\frac{dx^2(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}{-a+bx^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(c*(e*x)**(5/2)*sqrt(c - d*x**2)/(-a + b*x**2), x) - Integral(-d*x
**2*(e*x)**(5/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*x^(5/2)*e^(5/2)/(b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}\,(c-d\,x^2)^{3/2}}{a-b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x)
```

```
[Out] int(((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2), x)
```

3.874 $\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx$

Optimal. Leaf size=372

$$-\frac{2(9bc-7ad)e\sqrt{ex}\,\sqrt{c-dx^2}}{21b^2}+\frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be}-\frac{2\sqrt[4]{c}\,(12b^2c^2-35abcd+21a^2d^2)\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\Big(}{21b^3\sqrt[4]{d}\,\sqrt{c-dx^2}}$$

```
[Out] 2/7*d*(e*x)^(5/2)*(-d*x^2+c)^(1/2)/b/e-2/21*(-7*a*d+9*b*c)*e*(e*x)^(1/2)*(-
d*x^2+c)^(1/2)/b^2-2/21*c^(1/4)*(21*a^2*d^2-35*a*b*c*d+12*b^2*c^2)*e^(3/2)*
EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/d^(1
/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)
/b^3/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*e^(3/2)*EllipticPi(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/
c)^(1/2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 488, 596, 537, 230, 227, 418, 1233, 1232}

$$-\frac{2\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(21a^2d^2-35abcd+12b^2c^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{21b^3\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^3\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{2e\sqrt{ex}\,\sqrt{c-dx^2}\,(9bc-7ad)}{21b^2}+\frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (-2*(9*b*c - 7*a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(21*b^2) + (2*d*(e*x)^(5/2
)*Sqrt[c - d*x^2])/(7*b*e) - (2*c^(1/4)*(12*b^2*c^2 - 35*a*b*c*d + 21*a^2*d
^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(21*b^3*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^
2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt
[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} - \frac{(2e)\text{Subst}\left(\int \frac{x^4\left(-\frac{c(7bc-5ad)}{e^2}+\frac{d(9bc-7ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{7b}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\,\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} + \frac{(2e^5)\,\text{Subst}\left(\int \frac{\frac{acd(9bc-}{e^4}}{}\right.}{}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\,\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} + \frac{(2a(bc-ad)^2e)\,\text{Subst}}{}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\,\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} + \frac{((bc-ad)^2e)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\,\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} - \frac{2\sqrt[4]{c}\,(12b^2c^2-35abc}{}$$

$$= -\frac{2(9bc-7ad)e\sqrt{ex}\,\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} - \frac{2\sqrt[4]{c}\,(12b^2c^2-35abc}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.17, size = 182, normalized size = 0.49

$$-\frac{2e\sqrt{ex}\left(-5a(c-dx^2)(-9bc+7ad+3bdx^2)+5ac(-9bc+7ad)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+(-12b^2c^2+35abcd-21a^2d^2)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{105ab^2\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (-2*e*Sqrt[e*x]*(-5*a*(c - d*x^2)*(-9*b*c + 7*a*d + 3*b*d*x^2) + 5*a*c*(-9*
b*c + 7*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x
^2)/a] + (-12*b^2*c^2 + 35*a*b*c*d - 21*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*Ap
pellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(105*a*b^2*Sqrt[c - d*x^2]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1908 vs. 2(288) = 576.

time = 0.14, size = 1909, normalized size = 5.13

method	result
risch	$\frac{2(3bd\,x^2+7ad-9bc)\sqrt{-d\,x^2+c}\,x\,e^2}{21b^2\sqrt{ex}}-\frac{\left(\frac{\left(21a^2d^2-35abcd+12b^2c^2\right)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\cdots}}{bd\sqrt{-de\,x^3+cex}}\cdots\right.}{\cdots}$
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*e*(e*x)^(1/2)/b^2*(21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d
^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3
*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-42*EllipticPi(((d*x
```

+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d
),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+42*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*
b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)*(c*d)^(1/2)+21*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d
),1/2*2^(1/2))*a*b^3*c^3*d-21*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-21*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*(c*d)^(1/2)+42*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d^2*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)+42*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d
^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-21*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3*d-21*(c*d)^(1/2)*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*a*b^2*c^2*d+42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*2^(1/2)*a^3*d^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-112*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*
c*d^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+94*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*d*(a*b)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-24*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^3*(a*b)^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)*(c*d)^(1/2)+12*a*b^2*d^4*x^5*(a*b)^(1/2)-12*b^3*c*d^3*x^5*(a*b)^
(1/2)+28*a^2*b*d^4*x^3*(a*b)^(1/2)-76*a*b^2*c*d^3*x^3*(a*b)^(1/2)+48*b^3*c^

```
2*d^2*x^3*(a*b)^(1/2)-28*a^2*b*c*d^3*x*(a*b)^(1/2)+64*a*b^2*c^2*d^2*x*(a*b)
^(1/2)-36*b^3*c^3*d*x*(a*b)^(1/2))/x/(-d*x^2+c)^(1/2)/(a*b)^(1/2)/((c*d)^(1
/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(3/2)*integrate((-d*x^2 + c)^(3/2)*x^(3/2)/(b*x^2 - a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx-\int \left(-\frac{dx^2(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}{-a+bx^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
[Out] -Integral(c*(e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2), x) - Integral(-d*x
**2*(e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-(-d*x^2 + c)^(3/2)*x^(3/2)*e^(3/2)/(b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{3/2}\,(c-d\,x^2)^{3/2}}{a-b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x)
[Out] int(((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2), x)
```

3.875 $\int \frac{\sqrt{ex}\,(c-dx^2)^{3/2}}{a-bx^2}\,dx$

Optimal. Leaf size=421

$$\frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be}+\frac{2c^{3/4}\sqrt[4]{d}\,(7bc-5ad)\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\,(7bc-5}{}$$

```
[Out] 2/5*d*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b/e+2/5*c^(3/4)*d^(1/4)*(-5*a*d+7*b*c)*E
llipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b
^2/(-d*x^2+c)^(1/2)-2/5*c^(3/4)*d^(1/4)*(-5*a*d+7*b*c)*EllipticF(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b^2/(-d*x^2+c)^(1/2)
-c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/
2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b^(5/2)/d^(1/4)/a^(
1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)
/b^(5/2)/d^(1/4)/a^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 488, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\,b^{5/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\,b^{5/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(7bc-5ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(7bc-5ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}+\frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (2*d*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b*e) + (2*c^(3/4)*d^(1/4)*(7*b*c - 5*a
*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(5*b^2*Sqrt[c - d*x^2]) - (2*c^(3/4)*d^(1/4)*(7*b*c - 5*
a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(5*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[e
]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(1/4)*
Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\,(c-dx^2)^{3/2}}{a-bx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{(2e)\text{Subst}\left(\int \frac{x^2\left(-\frac{c(5bc-3ad)}{e^2}+\frac{d(7bc-5ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{(2e)\text{Subst}\left(\int \left(-\frac{d(7bc-5ad)x^2}{be^2\sqrt{c-\frac{dx^4}{e^2}}} - \frac{5(b^2c^2-2abcd+a^2d^2)x^2}{be^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)\right.}{5b}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} + \frac{(2d(7bc-5ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2e} + \frac{(2(b}{}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{\left(2\sqrt{c}\,\sqrt{d}\,(7bc-5ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5b^2}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{\left(2\sqrt{c}\,\sqrt{d}\,(7bc-5ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\right.}{5b^2\sqrt{c-dx^2}}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} - \frac{2c^{3/4}\sqrt[4]{d}\,(7bc-5ad)\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right.}{5b^2\sqrt{c-dx^2}}$$

$$= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be} + \frac{2c^{3/4}\sqrt[4]{d}\,(7bc-5ad)\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right.}{5b^2\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order

4 in optimal.
time = 10.18, size = 155, normalized size = 0.37

$$\frac{2x\sqrt{ex}\left(7c(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3d\left(7a(c-dx^2)+(-7bc+5ad)x^2\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)\right)}{105ab\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

```
[Out] (2*x*Sqrt[e*x]*(7*c*(5*b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2,
1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*d*(7*a*(c - d*x^2) + (-7*b*c + 5*a*d)*x^2
*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(
105*a*b*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1915 vs. 2(315) = 630.
time = 0.15, size = 1916, normalized size = 4.55

method	result
risch	$\frac{2d\sqrt{-d\,x^2+c}\,x^2e}{5b\sqrt{ex}}-\frac{\left((5ad-7bc)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\left(-\frac{2\sqrt{cd}\ \mathrm{EllipticE}\left(\right.}{}\right.\right.}{b\sqrt{-de\,x^3+cex}}$
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(e*x)^(1/2)*d*(20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+
```

```
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2-48*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^2*c^2*d+28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3-10*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^
(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a
^2*b*c*d^2+24*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d-14*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*
c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*
2^(1/2))*(c*d)^(1/2)*a^2*d^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d+5*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2
*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*(c*d)^(1/2)*a^2*d^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d-5*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^
2*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a
^2*b*c*d^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1
/2))*a*b^2*c^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2
*2^(1/2))*b^3*c^3-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
```

```
2*2^(1/2))*a^2*b*c*d^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)
*d),1/2*2^(1/2))*a*b^2*c^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*b^3*c^3-4*a*b^2*d^3*x^4+4*b^3*c*d^2*x^4+4*a*b^2*c*d^2
*x^2-4*b^3*c^2*d*x^2)/(-d*x^2+c)^(1/2)/b^2/x/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/
((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -e^(1/2)*integrate((-d*x^2 + c)^(3/2)*sqrt(x)/(b*x^2 - a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{ex}\,\sqrt{c-dx^2}}{-a+bx^2}\,dx-\int \left(-\frac{dx^2\sqrt{ex}\,\sqrt{c-dx^2}}{-a+bx^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)*(e*x)**(1/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(c*sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x) - Integral(-d*x**2
*sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-(-d*x^2 + c)^(3/2)*sqrt(x)*e^(1/2)/(b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}\,\left(c-d\,x^2\right)^{3/2}}{a-b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x)
[Out] int(((e*x)^(1/2)*(c - d*x^2)^(3/2))/(a - b*x^2), x)
```

3.876 $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx$

Optimal. Leaf size=328

$$\frac{2d\sqrt{ex}\,\sqrt{c-dx^2}}{3be}+\frac{2\sqrt[4]{c}\,d^{3/4}(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(bc-ad)^2\sqrt{1-\frac{d}{}}}{}$$

```
[Out] 2/3*d*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b/e+2/3*c^(1/4)*d^(3/4)*(-3*a*d+5*b*c)*E
llipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/e^(1/
2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b^2/d^
(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2
)/a/b^2/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 427, 537, 230, 227, 418, 1233, 1232}

$$\frac{2\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,(5bc-3ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3b^2\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{ab^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{ab^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}}+\frac{2d\sqrt{ex}\,\sqrt{c-dx^2}}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)),x]
```

```
[Out] (2*d*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*e) + (2*c^(1/4)*d^(3/4)*(5*b*c - 3*a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(3*b^2*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^
2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
a*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2d\sqrt{ex}\,\sqrt{c-dx^2}}{3be} - \frac{(2e)\text{Subst}\left(\int \frac{-\frac{c(3bc-ad)}{e^2}+\frac{d(5bc-3ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3b}$$

$$= \frac{2d\sqrt{ex}\,\sqrt{c-dx^2}}{3be} + \frac{(2d(5bc-3ad))\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3b^2e} + \frac{(2(bc-\cdots}{\cdots}$$

$$= \frac{2d\sqrt{ex}\,\sqrt{c-dx^2}}{3be} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\frac{x^2}{e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ab^2e} + \cdots$$

$$= \frac{2d\sqrt{ex}\,\sqrt{c-dx^2}}{3be} + \frac{2\sqrt[4]{c}\,d^{3/4}(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\cdots\right.}{3b^2\sqrt{e}\,\sqrt{c-dx^2}}\cdots$$

$$= \frac{2d\sqrt{ex}\,\sqrt{c-dx^2}}{3be} + \frac{2\sqrt[4]{c}\,d^{3/4}(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\cdots\right.}{3b^2\sqrt{e}\,\sqrt{c-dx^2}}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.13, size = 153, normalized size = 0.47

$$\frac{2x\left(5ad(c-dx^2)+5c(3bc-ad)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+d(-5bc+3ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{15ab\sqrt{ex}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)),x]
```

[Out] (2*x*(5*a*d*(c - d*x^2) + 5*c*(3*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*(-5*b*c + 3*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a*b*Sqrt[e*x]*Sqrt[c - d*x^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1709 vs. 2(250) = 500.
time = 0.14, size = 1710, normalized size = 5.21

method	result
risch	$\frac{2d\sqrt{-d\,x^2+c}\,x}{3b\sqrt{ex}} - \frac{\left(\frac{(3ad-5bc)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right.}{\sqrt{cd}}}\right.}{b\sqrt{-de\,x^3+cex}}\right.}{}$
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(\frac{2d\sqrt{-de\,x^3+cex}}{3be} - \frac{d\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(x}{}}\right.}{\sqrt{-de\,x^3+cex}\,b^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6/b*d*(6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-16*Elliptic

F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*d*(c*d)^
(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+10*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*(c*d)^(1/2)*(a*b)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/
2)*d),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-6*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d+6*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/
2)*a*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/
2))*b^3*c^3-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)
*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2+6*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d+6*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*(c*d)^(1/2)*a*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*b^3*c^3-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c^2+4*a*b*d^3*x^3*(a*b)^(1/2)-4
*b^2*c*d^2*x^3*(a*b)^(1/2)-4*a*b*c*d^2*x*(a*b)^(1/2)+4*b^2*c^2*d*x*(a*b)^(1

```
/2))/(-d*x^2+c)^(1/2)/(e*x)^(1/2)/(a*b)^(1/2)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)
/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -e^(-1/2)*integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\,dx-\int\left(-\frac{dx^2\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(-b*x**2+a)/(e*x)**(1/2),x)
```

```
[Out] -Integral(c*sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x) - Integr
al(-d*x**2*sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*e^(-1/2)/((b*x^2 - a)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c-d\,x^2)^{3/2}}{\sqrt{e\,x}\;(a-b\,x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(3/2)/((e*x)^(1/2)*(a - b*x^2)),x)
[Out] int((c - d*x^2)^(3/2)/((e*x)^(1/2)*(a - b*x^2)), x)
```

3.877 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx$

Optimal. Leaf size=417

$$-\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}}-\frac{2c^{3/4}\sqrt[4]{d}\,(bc+ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\,(bc+ad)\sqrt{1-\frac{dx^2}{c}}}{abe^{3/2}\sqrt{c}}$$

```
[Out] -2*c*(-d*x^2+c)^(1/2)/a/e/(e*x)^(1/2)-2*c^(3/4)*d^(1/4)*(a*d+b*c)*EllipticE
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/e^(3/2)/(-d*x
^2+c)^(1/2)+2*c^(3/4)*d^(1/4)*(a*d+b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/e^(3/2)/(-d*x^2+c)^(1/2)-c^(1/4)*(-a*d+
b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1
/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^(3/2)/d^(1/4)/e^(3/2)/(-d*x^2+c)
^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),
b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^(3/2)/d^(1/4
)/e^(3/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 485, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}}-\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (-2*c*Sqrt[c - d*x^2])/(a*e*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(b*c + a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(a*b*e^(3/2)*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*(b*c + a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
a*b*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]
) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])
/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^
(3/2)*b^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^2\left(a-\frac{bx^4}{e^2}\right)}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2\text{Subst}\left(\int \frac{x^2\left(\frac{c(bc-3ad)}{e^2}+\frac{d(bc+ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2\text{Subst}\left(\int \left(-\frac{d(bc+ad)x^2}{be^2\sqrt{c-\frac{dx^4}{e^2}}} + \frac{(b^2c^2-2abcd+a^2d^2)x^2}{be^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx, x, \sqrt{\right.}{ae}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{abe^3} - \frac{(2d}{}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}(bc+ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{abe^2} - \frac{(2}{}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}(bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \right.}{abe^2\sqrt{c-dx^2}}$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} + \frac{2c^{3/4}\sqrt[4]{d}(bc+ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}} -$$

$$= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}} - \frac{2c^{3/4}\sqrt[4]{d}(bc+ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order

4 in optimal.
time = 10.11, size = 151, normalized size = 0.36

$$\frac{x\left(-42ac(c-dx^2)+14c(bc-3ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+6d(bc+ad)x^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{21a^2(ex)^{3/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x]
```

```
[Out] (x*(-42*a*c*(c - d*x^2) + 14*c*(b*c - 3*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*Appell
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 6*d*(b*c + a*d)*x^4*Sqrt[1 - (
d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*(e*x)
^(3/2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1746 vs. $2(317) = 634$.
time = 0.14, size = 1747, normalized size = 4.19

method	result	size
elliptic	Expression too large to display	1292
default	Expression too large to display	1747

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*(4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3-2
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elli
pticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3+((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(
1/2)*a^2*d^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2
)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a
*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
```

```
c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c^2-((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)
*a^2*d^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^
(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c^2-((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2+2*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^3-((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2+2*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d-((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^3+4*a*
b^2*c*d^2*x^2-4*b^3*c^2*d*x^2-4*a*b^2*c^2*d+4*b^3*c^3)/(-d*x^2+c)^(1/2)/b/e
/(e*x)^(1/2)/a/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -e^(-3/2)*integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*x^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{c-dx^2}}{-a\left(ex\right)^{\frac{3}{2}}+bx^2\left(ex\right)^{\frac{3}{2}}}\,dx-\int \left(-\frac{dx^2\sqrt{c-dx^2}}{-a\left(ex\right)^{\frac{3}{2}}+bx^2\left(ex\right)^{\frac{3}{2}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(c*sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2)), x) -
Integral(-d*x**2*sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*e^(-3/2)/((b*x^2 - a)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-d\,x^2\right)}^{3/2}}{{\left(e\,x\right)}^{3/2}\,\left(a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x)
```

```
[Out] int((c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)), x)
```

3.878 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)} \, dx$

Optimal. Leaf size=330

$$-\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{2\sqrt[4]{c}\,d^{3/4}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3abe^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\,\Pi}{a^2b\sqrt[4]{}}$$

```
[Out] -2/3*c*(-d*x^2+c)^(1/2)/a/e/(e*x)^(3/2)+2/3*c^(1/4)*d^(3/4)*(-3*a*d+b*c)*El
lipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/e^(5/2
)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1
/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b/d^(
1/4)/e^(5/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)
/a^2/b/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 485, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{a^2b\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-ad)^2\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{a^2b\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,(bc-3ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3abe^{5/2}\sqrt{c-dx^2}}-\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (-2*c*Sqrt[c - d*x^2])/(3*a*e*(e*x)^(3/2)) + (2*c^(1/4)*d^(3/4)*(b*c - 3*a*
d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(3*a*b*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x
^2]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[
c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(a^2*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{5/2}\left(a-bx^2\right)}\,dx=\frac{2\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^4\left(a-\frac{bx^4}{e^2}\right)}\,dx,x,\sqrt{ex}\right)}{e}$$

$$=-\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{2\text{Subst}\left(\int \frac{\frac{c(3bc-5ad)}{e^2}-\frac{d(bc-3ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{3ae}$$

$$=-\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{(2d(bc-3ad))\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{3abe^3}+\frac{(2(bc-a\cdots}{\cdots}$$

$$=-\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{a^2be^3}+\frac{(bc\cdots}{\cdots}$$

$$=-\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{2\sqrt[4]{c}\,d^{3/4}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3abe^{5/2}\sqrt{c-dx^2}}+\cdots$$

$$=-\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}}+\frac{2\sqrt[4]{c}\,d^{3/4}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3abe^{5/2}\sqrt{c-dx^2}}+\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.13, size = 153, normalized size = 0.46

$$\frac{x\left(-10ac(c-dx^2)+10c(3bc-5ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-2d(bc-3ad)x^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{15a^2(ex)^{5/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x]
```

```
[Out] (x*(-10*a*c*(c - d*x^2) + 10*c*(3*b*c - 5*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*Appe
llF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - 2*d*(b*c - 3*a*d)*x^4*Sqrt[1
 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a^2*(e
*x)^(5/2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. $2(252) = 504$.
time = 0.12, size = 1729, normalized size = 5.24

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{2c\sqrt{-de\,x^3+cex}}{3e^3a\,x^2}-\frac{d\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}\left(\sqrt{\ldots}\right.}{\sqrt{-de\,x^3+cex}\,e^2b}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*d*(6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/
2)*a^2*d^2*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-8*EllipticF
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*d*x*(a*b)
^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*x*(a*b)^(1/2)*(c*d)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*
c*d^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*a^2*d^2*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-6*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1
/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+6*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b*c*d*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+
```

```
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^3*c^3*x*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^2*c^2*x*
(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*2^(1/2)*a^2*b*c*d^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*2^(1/2)*a^2*d^2*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)+6*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*d*x*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)+6*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b*c*d*x*(a*b)^(
1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*2^(1/2)*b^3*c^3*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*2^(1/2)*b^2*c^2*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4
*a*b*c*d^2*x^2*(a*b)^(1/2)+4*b^2*c^2*d*x^2*(a*b)^(1/2)+4*a*b*c^2*d*(a*b)^(1
/2)-4*b^2*c^3*(a*b)^(1/2))/(-d*x^2+c)^(1/2)/x/a/e^2/(e*x)^(1/2)/(a*b)^(1/2)
/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="maxima")
[Out] -e^(-5/2)*integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*x^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{c-dx^2}}{-a\,(ex)^{\frac{5}{2}}+bx^2\,(ex)^{\frac{5}{2}}}\,dx-\int\left(-\frac{dx^2\sqrt{c-dx^2}}{-a\,(ex)^{\frac{5}{2}}+bx^2\,(ex)^{\frac{5}{2}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a),x)
[Out] -Integral(c*sqrt(c - d*x**2)/(-a*(e*x)**(5/2) + b*x**2*(e*x)**(5/2)), x) -
Integral(-d*x**2*sqrt(c - d*x**2)/(-a*(e*x)**(5/2) + b*x**2*(e*x)**(5/2)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="giac")
[Out] integrate(-(-d*x^2 + c)^(3/2)*e^(-5/2)/((b*x^2 - a)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-d\,x^2\right)}^{3/2}}{{\left(e\,x\right)}^{5/2}\,\left(a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x)
[Out] int((c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)), x)
```

3.879 $\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{7/2}\left(a-bx^2\right)}\,dx$

Optimal. Leaf size=459

$$-\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}}-\frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}}-\frac{2c^{3/4}\sqrt[4]{d}\,(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}+\cdots$$

```
[Out] -2/5*c*(-d*x^2+c)^(1/2)/a/e/(e*x)^(5/2)-2/5*(-7*a*d+5*b*c)*(-d*x^2+c)^(1/2)
/a^2/e^3/(e*x)^(1/2)-2/5*c^(3/4)*d^(1/4)*(-7*a*d+5*b*c)*EllipticE(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/e^(7/2)/(-d*x^2+c)^(1/2
)+2/5*c^(3/4)*d^(1/4)*(-7*a*d+5*b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/
e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/e^(7/2)/(-d*x^2+c)^(1/2)-c^(1/4)*(-a*d+b*c
)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)
/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/2)/b^(1/2)/(-d*x^2+c)^(1
/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(
1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/2)/b
^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 485, 597, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt{b}\sqrt[4]{d}\,e^{7/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt{b}\sqrt[4]{d}\,e^{7/2}\sqrt{c-dx^2}}+\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-\frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(5bc-7ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x]
```

```
[Out] (-2*c*Sqrt[c - d*x^2])/(5*a*e*(e*x)^(5/2)) - (2*(5*b*c - 7*a*d)*Sqrt[c - d*
x^2])/(5*a^2*e^3*Sqrt[e*x]) - (2*c^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (
d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*
a^2*e^(7/2)*Sqrt[c - d*x^2]) + (2*c^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5
*a^2*e^(7/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2
]) + (c^(1/4)*(b*c - a*d)^2*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c]
)/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a
^(5/2)*Sqrt[b]*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^6\left(a-\frac{bx^4}{e^2}\right)}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} + \frac{2\text{Subst}\left(\int \frac{\frac{c(5bc-7ad)}{e^2}-\frac{d(3bc-5ad)x^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5ae}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} - \frac{2\text{Subst}\left(\int \frac{x^2\left(-\frac{c\left(5b^2c^2-15abcd+12a^2d^2\right)}{e^4}-\frac{bcd(}{}\right.}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{5a^2ce}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} - \frac{2\text{Subst}\left(\int \left(\frac{cd(5bc-7ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}} - \frac{5\left(b^2c^3-2a\right.}{e^4\left(a-\frac{bx^4}{e^2}\right.}\right.\right.}{5a^2ce}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} - \frac{(2d(5bc-7ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\right.}{5a^2e^5}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}(5bc-7ad)\right)\text{Subst}\left(\int \frac{}{\sqrt{c}}\right.}{5a^2e^4}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} + \frac{\left(2\sqrt{c}\sqrt{d}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{S}}{5a^2e^4\sqrt{c-}}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} + \frac{2c^{3/4}\sqrt[4]{d}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{si}\right.}{5a^2e^{7/2}\sqrt{c-dx}}$$

$$= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}} - \frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}} - \frac{2c^{3/4}\sqrt[4]{d}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{si}\right.}{5a^2e^{7/2}\sqrt{c-dx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.19, size = 187, normalized size = 0.41

$$\frac{x\left(14(5b^2c^2-15abcd+12a^2d^2)\,x^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-6\left(7a(c-dx^2)\,(ac+5bcx^2-7adx^2)+bd(-5bc+7ad)x^6\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)\right)}{105a^3(ex)^{7/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x]
```

```
[Out] (x*(14*(5*b^2*c^2 - 15*a*b*c*d + 12*a^2*d^2)*x^4*Sqrt[1 - (d*x^2)/c]*Appell
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] - 6*(7*a*(c - d*x^2)*(a*c + 5*b*
c*x^2 - 7*a*d*x^2) + b*d*(-5*b*c + 7*a*d)*x^6*Sqrt[1 - (d*x^2)/c]*AppellF1[
7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(105*a^3*(e*x)^(7/2)*Sqrt[c - d
*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2016 vs. $2(347) = 694$.
time = 0.14, size = 2017, normalized size = 4.39

method	result	size
elliptic	Expression too large to display	1328
default	Expression too large to display	2017

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*d*(20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3*x^2+28*a^2*b*d^3*x^4-4*a*b^2*c^3-5*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2
))*(c*d)^(1/2)*a^2*d^2*x^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c^2*x^2+5*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^
2*d^2*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d
),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c^2*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
```

```
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2*x^2-10*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d*x^2+5*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2*x^2-10*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d*x^
2+28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2*x^2-48*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2
*d*x^2-14*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2*x^2+24*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^
2*c^2*d*x^2-20*b^3*c^3*x^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3*x^2+5*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^3*x^2+5*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^3*x^2-48
*a*b^2*c*d^2*x^4+52*a*b^2*c^2*d*x^2+4*a^2*b*c^2*d+10*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^
(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d
*x^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*(c*d)^(1/2)*a*b*c*d*x^2-32*a^2*b*c*d^2*x^2+20*b^3*c^2*d*x^4)/x^2
/(-d*x^2+c)^(1/2)/e^3/(e*x)^(1/2)/a^2/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^
(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -e^(-7/2)*integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*x^(7/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{c\sqrt{c-dx^2}}{-a\left(ex\right)^{\frac{7}{2}}+bx^2\left(ex\right)^{\frac{7}{2}}}\,dx-\int\left(-\frac{dx^2\sqrt{c-dx^2}}{-a\left(ex\right)^{\frac{7}{2}}+bx^2\left(ex\right)^{\frac{7}{2}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(7/2)/(-b*x**2+a),x)
```

```
[Out] -Integral(c*sqrt(c - d*x**2)/(-a*(e*x)**(7/2) + b*x**2*(e*x)**(7/2)), x) -
Integral(-d*x**2*sqrt(c - d*x**2)/(-a*(e*x)**(7/2) + b*x**2*(e*x)**(7/2)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(-(-d*x^2 + c)^(3/2)*e^(-7/2)/((b*x^2 - a)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(c-d\,x^2\right)}^{3/2}}{{\left(e\,x\right)}^{7/2}\,\left(a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x)
```

```
[Out] int((c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)), x)
```

3.880 $\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=305

$$\frac{2e^3\sqrt{ex}\,\sqrt{c-dx^2}}{3bd}-\frac{2\sqrt[4]{c}\,(bc+3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3b^2d^{5/4}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\Big(}{b^2}$$

```
[Out] 2/3*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b/d-2/3*c^(1/4)*(3*a*d+b*c)*e^(7/2)*El
lipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(5/4
)/(-d*x^2+c)^(1/2)+a*c^(1/4)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(1/4)/
(-d*x^2+c)^(1/2)+a*c^(1/4)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(1/4)/(-d
*x^2+c)^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 490, 537, 230, 227, 418, 1233, 1232}

$$-\frac{2\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(3ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3b^2d^{5/4}\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^2\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{2e^3\sqrt{ex}\,\sqrt{c-dx^2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*d) - (2*c^(1/4)*(b*c + 3*a*d)*e^(7/2
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(3*b^2*d^(5/4)*Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x
^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c - d*x^2]) + (a*c^(1/
4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^2*d^(1/4)*Sqrt[c
 - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} - \frac{(2e^3)\,\text{Subst}\left(\int \frac{ac-\frac{(bc+3ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3bd}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} + \frac{(2a^2e^3)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b^2} - \text{(2(}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} + \frac{(ae^3)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b^2} + \text{(}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} - \frac{2\sqrt[4]{c}\,(bc+3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-\right.}{3b^2d^{5/4}\sqrt{c-dx^2}}$$

$$= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} - \frac{2\sqrt[4]{c}\,(bc+3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-\right.}{3b^2d^{5/4}\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 147, normalized size = 0.48

$$\frac{2e^3\sqrt{ex}\left(5a(c-dx^2)-5ac\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+(bc+3ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{15abd\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*(5*a*(c - d*x^2) - 5*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4,
 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (b*c + 3*a*d)*x^2*Sqrt[1 - (d*x^2)/c]
*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a*b*d*Sqrt[c - d*x^
2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(227) = 454$.
time = 0.14, size = 842, normalized size = 2.76 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b/d*(6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*a^2*d^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*EllipticF
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*d*(c*d)^(
1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*(c*d)^(1/2)*(a*b)^(1/2)*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)
*d),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-3*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2-3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)
*a^2*d^2+4*a*b*d^3*x^3*(a*b)^(1/2)-4*b^2*c*d^2*x^3*(a*b)^(1/2)-4*a*b*c*d^2*
x*(a*b)^(1/2)+4*b^2*c^2*d*x*(a*b)^(1/2))*e^3*(e*x)^(1/2)/(-d*x^2+c)^(1/2)/x
/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -e^(7/2)*integrate(x^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{7}{2}}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
[Out] -Integral((e*x)**(7/2)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(-x^(7/2)*e^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{7/2}}{\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(1/2)),x)
[Out] int((e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.881 $\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=349

$$-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}}+\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}}-$$

```
[Out] -2*c^(3/4)*e^(5/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^
2/c)^(1/2)/b/d^(3/4)/(-d*x^2+c)^(1/2)+2*c^(3/4)*e^(5/2)*EllipticF(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(3/4)/(-d*x^2+c)^(1/2)-
c^(1/4)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(
1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/(-d*x^2+c
)^(1/2)+c^(1/4)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1
/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/(-
d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {477, 494, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt{a}\,\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt{a}\,\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}}-\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) + (2*c^(3/4)*e^(5/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sq
rt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)
*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{(2e)\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b} + \frac{(2ae)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{b}$$

$$= \frac{\left(2\sqrt{c}\,e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b\sqrt{d}} - \frac{\left(2\sqrt{c}\,e^2\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}\,x^2}{\sqrt{c}\,e}}{\sqrt{c-\frac{dx^4}{e^2}}}\right.}{b\sqrt{d}}$$

$$= \frac{\left(2\sqrt{c}\,e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{b\sqrt{d}\sqrt{c-dx^2}} - \frac{\left(2\sqrt{c}\,e^2\sqrt{1-\frac{c}{}}\right.}{}$$

$$= \frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}} - \frac{\sqrt{a}\,\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi}{b^{3/}}$$

$$= -\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}} + \frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\right.}{bd^{3/4}\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.05, size = 70, normalized size = 0.20

$$\frac{2x(ex)^{5/2}\sqrt{\frac{c-dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{7a\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*(e*x)^(5/2)*Sqrt[(c - d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c,
 (b*x^2)/a])/(7*a*Sqrt[c - d*x^2])
```

Maple [A]
time = 0.12, size = 459, normalized size = 1.32

method	result
default	$\frac{\left(\text{EllipticPi}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)abcd-\sqrt{ab}\,\text{EllipticPi}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)\sqrt{\ }\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{\left(-d\,x^{2}+c\right)ex}\left(\frac{2e^{3}c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\cdots\right.}{bd\sqrt{-de\,x^{3}+cex}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d-(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*a*d+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d+(a*b)^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*d-4*EllipticE(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d+4*EllipticE(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2+2*EllipticF(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2)*(-d*x/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*e^2*
(e*x)^(1/2)/(-d*x^2+c)^(1/2)/x/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b
+(a*b)^(1/2)*d)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] -e^(5/2)*integrate(x^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{5}{2}}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
[Out] -Integral((e*x)**(5/2)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(-x^(5/2)*e^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}}{(a-b\,x^2)\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(1/2)),x)
[Out] int((e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.882 $\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=261

$$-\frac{2\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right.}{b\sqrt[4]{d}\,\sqrt{c-dx^2}}$$

```
[Out] -2*c^(1/4)*e^(3/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^
2/c)^(1/2)/b/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*e^(3/2)*EllipticPi(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(
1/2)/b/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1
/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b/
d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {477, 494, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{2\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*S
qrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])
+ (c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*
Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{(2e)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{b}$$

$$= \frac{e\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b} + \frac{e\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c}\right.}{b}$$

$$= -\frac{2\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}}{}$$

$$= -\frac{2\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\right.}{}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.05, size = 70, normalized size = 0.27

$$\frac{2x(ex)^{3/2}\sqrt{\frac{c-dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{5a\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*(e*x)^(3/2)*Sqrt[(c - d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c,
(b*x^2)/a])/(5*a*Sqrt[c - d*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(191) = 382.
time = 0.12, size = 406, normalized size = 1.56

method	result
default	$\frac{\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)ad\sqrt{ab}-2\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)bc\sqrt{ab}+\mathrm{EllipticPi}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{c}}{}\right.\right.}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\right.}{bd\sqrt{-de\,x^3+cex}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*d*(a*
b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c
*(a*b)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*(c*d)^(1/2)-EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*
d),1/2*2^(1/2))*a*d*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*(c*d)^(1
/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d*(a*b)^(1/2))*(c*d)^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*e*(e*x)^(1/2)/(-d*x^2+c)^(1/2)/x/((c*d)^(1/2)*b-(a*
b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -e^(3/2)*integrate(x^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{3}{2}}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral((e*x)**(3/2)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^(3/2)*e^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{3/2}}{\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

```
[Out] int((e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.883 $\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=203

$$-\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}}$$

```
[Out] -c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^
(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/a^(1/2)/b^(1/2)/(-d*x^2+
c)^(1/2)+c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(
1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/a^(1/2)/b^(1/2)/(
-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {477, 504, 1233, 1232}

$$\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] -((c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]
*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])) + (c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*E
llipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
```

```
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\text{Subst}\left(\int \frac{1}{\left(\sqrt{a}\,e-\sqrt{b}\,x^2\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{b}} - \frac{e\text{Subst}\left(\int \frac{\ldots}{\left(\sqrt{a}\,e+\sqrt{b}\ldots\right.}\right.}{\ldots}$$

$$= \frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a}\,e-\sqrt{b}\,x^2\right)\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{b}\,\sqrt{c-dx^2}} - \frac{\left(e\sqrt{1\ldots}\right.}{\ldots}$$

$$= -\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.04, size = 70, normalized size = 0.34

$$\frac{2x\sqrt{ex}\,\sqrt{\frac{c-dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{3a\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*Sqrt[e*x]*Sqrt[(c - d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b
*x^2)/a])/(3*a*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(143) = 286.
time = 0.12, size = 326, normalized size = 1.61

method	result
default	$\frac{\left(\mathrm{EllipticPi}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b-\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)\sqrt{ab}\,\sqrt{cd}+\mathrm{EllipticPi}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b-\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)b\cdots\right.}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{e\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},-\cdots\right.}{2bd\sqrt{-de\,x^3+cex}\left(-\frac{\sqrt{cd}}{d}-\frac{\sqrt{ab}}{b}\right)}\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(a*b)^(1/2)*(c*d)^(1/2)+EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*b*c-(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)+
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c)*d*(-d*x/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(e
*x)^(1/2)/(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*
b)^(1/2)*d)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] -e^(1/2)*integrate(sqrt(x)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ex}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
[Out] -Integral(sqrt(e*x)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(-sqrt(x)*e^(1/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}}{\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/((a - b*x^2)*(c - d*x^2)^(1/2)),x)
[Out] int((e*x)^(1/2)/((a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.884 $\int \frac{1}{\sqrt{ex}\,(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|\right.}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

```
[Out] c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(
1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4
)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^
(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {477, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*Sqrt[e]
*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
a*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae} + \frac{\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{\ldots}}\ldots\right)}{\ldots}$$

$$= \frac{\sqrt{1-\frac{dx^2}{c}}\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{ae\sqrt{c-dx^2}} + \frac{\sqrt{1-\frac{dx^2}{c}}\ldots}{\ldots}$$

$$= \frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}}; \sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\,\sqrt{1-\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.04, size = 68, normalized size = 0.36

$$\frac{2x\sqrt{\frac{c-dx^2}{c}}\,F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{dx^2}{c}, \frac{bx^2}{a}\right)}{a\sqrt{ex}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (2*x*Sqrt[(c - d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/
(a*Sqrt[e*x]*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(136) = 272$.
time = 0.12, size = 335, normalized size = 1.78

method	result
elliptic	$\frac{\sqrt{(-d x^2+c) e x}\left(-\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},-\frac{\sqrt{}}{d\left(-\frac{\sqrt{c}}{d}\right.}\right.}{2\sqrt{ab}\,d\sqrt{-dex^3+cex}\left(-\frac{\sqrt{cd}}{d}-\frac{\sqrt{ab}}{b}\right)}\right.}{\sqrt{}}$
default	$\frac{\left(\mathrm{EllipticPi}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)\sqrt{cd}\,b-\mathrm{EllipticPi}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)\sqrt{ab}\right.}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b-EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(
1/2))*(a*b)^(1/2)*d-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b-EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*(a*b)^(1/2)*d)*(c*d)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*b/(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*
b)^(1/2)*d)/(a*b)^(1/2)/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] -e^(-1/2)*integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(x)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a\sqrt{ex}\,\sqrt{c-dx^2}+bx^2\sqrt{ex}\,\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)/(e*x)**(1/2)/(-d*x**2+c)**(1/2),x)
[Out] -Integral(1/(-a*sqrt(e*x)*sqrt(c - d*x**2) + b*x**2*sqrt(e*x)*sqrt(c - d*x*
*2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(-e^(-1/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,x}\,\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(1/2)*(a - b*x^2)*(c - d*x^2)^(1/2)),x)
[Out] int(1/((e*x)^(1/2)*(a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.885 $\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=379

$$-\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}\,e^{3/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}\,e^{3/2}\sqrt{c-dx^2}}$$

```
[Out] -2*(-d*x^2+c)^(1/2)/a/c/e/(e*x)^(1/2)-2*d^(1/4)*EllipticE(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)+
2*d^(1/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2
)/a/c^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)-c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)
/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1
/2)/a^(3/2)/d^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)+c^(1/4)*EllipticPi(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^
2/c)^(1/2)/a^(3/2)/d^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 491, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{3/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}\,e^{3/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}\,e^{3/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(a*c*e*Sqrt[e*x]) - (2*d^(1/4)*Sqrt[1 - (d*x^2)/c]*Ell
ipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(1/4)*e^(3/
2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*c^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
 - (Sqrt[b]*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2
)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{2\text{Subst}\left(\int \frac{x^2\left(\frac{bc-ad}{e^2}+\frac{bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ace}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{2\text{Subst}\left(\int \left(-\frac{dx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}} + \frac{bcx^2}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)\right.}{ace}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{(2b)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae^3} - \frac{(2d}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{\left(2\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{a\sqrt{c}\,e^2} - \frac{\left(2\sqrt{d}\right.}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{\left(2\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right.}{a\sqrt{c}\,e^2\sqrt{c-dx^2}}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} + \frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}\,e^{3/2}\sqrt{c-dx^2}} - \frac{\sqrt{b}}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}} - \frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}\,e^{3/2}\sqrt{c-dx^2}} + \frac{2\sqrt[4]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 146, normalized size = 0.39

$$\frac{x\left(-42a(c-dx^2)+14(bc-ad)x^2\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+6bdx^4\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{21a^2c(ex)^{3/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(-42*a*(c - d*x^2) + 14*(b*c - a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4
, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 6*b*d*x^4*Sqrt[1 - (d*x^2)/c]*Appell
F1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*c*(e*x)^(3/2)*Sqrt[c
- d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(279) = 558.
time = 0.12, size = 826, normalized size = 2.18

method	result
elliptic	$\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{2\left(-de\,x^2+ce\right)}{e^2ca\sqrt{x\left(-de\,x^2+ce\right)}}+\frac{2\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\text{EllipticE}\left(\sqrt{\ldots}\right.}{ea\sqrt{-de\,x^3+cex}}\right.$
default	$-\frac{\left(4\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{2}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\text{EllipticE}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)acd-4\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{2}\sqrt{\ldots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a*c*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ell
ipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2-2*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
```

```
*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*a*c*d+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*c-((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1
/2)*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c
^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c^2+
4*a*d^2*x^2-4*b*c*d*x^2-4*a*c*d+4*b*c^2)*d*b/(-d*x^2+c)^(1/2)/c/((c*d)^(1/2
)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/a/e/(e*x)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] -e^(-3/2)*integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*x^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a\left(ex\right)^{\frac{3}{2}}\sqrt{c-dx^{2}}+bx^{2}\left(ex\right)^{\frac{3}{2}}\sqrt{c-dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

```
[Out] -Integral(1/(-a*(e*x)**(3/2)*sqrt(c - d*x**2) + b*x**2*(e*x)**(3/2)*sqrt(c
- d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-e^(-3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)*x^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(e\,x)}^{3/2}\,\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

```
[Out] int(1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.886 $\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=297

$$-\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}}+\frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}}+\frac{b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{a^2\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}}$$

```
[Out] -2/3*(-d*x^2+c)^(1/2)/a/c/e/(e*x)^(3/2)+2/3*d^(3/4)*EllipticF(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(3/4)/e^(5/2)/(-d*x^2+c)^(1
/2)+b*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/
2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2
)+b*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/
a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 491, 537, 230, 227, 418, 1233, 1232}

$$\frac{b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}}+\frac{b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}}+\frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(3*a*c*e*(e*x)^(3/2)) + (2*d^(3/4)*Sqrt[1 - (d*x^2)/c]
*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(3*a*c^(3/4)
*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{\frac{3bc+ad}{e^2}-\frac{bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{3ace}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{ae^3} + \frac{(2d}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{b\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\frac{x^2}{e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{a^2e^3} + \frac{b\text{S}}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}} + \frac{\left(b\right.}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}} + \frac{b\sqrt[4]{c}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 148, normalized size = 0.50

$$\frac{x\left(10(3bc+ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-2\left(5a(c-dx^2)+bdx^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)\right)}{15a^2c(ex)^{5/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(10*(3*b*c + a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*
x^2)/c, (b*x^2)/a] - 2*(5*a*(c - d*x^2) + b*d*x^4*Sqrt[1 - (d*x^2)/c]*Appel
lF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])))/(15*a^2*c*(e*x)^(5/2)*Sqrt[c
 - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(219) = 438$.
time = 0.13, size = 729, normalized size = 2.45

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{2\sqrt{-de\,x^3+cex}}{3e^3ca\,x^2}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x\cdots\right.}{\cdots}}\cdots\right.}{3c\,e^2a\sqrt{-de\,x^3+cex}}\cdots\right.}{\cdots}$
default	$-\frac{bd\left(2\sqrt{2}\,\text{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)adx\sqrt{ab}\,\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}-2\sqrt{2}\,\text{Elli}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*d*(2*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a*d*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2*2^(1/2)*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x*(a*b)^(1/2
)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*2^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^
(1/2))*b^2*c^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*2^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2
*2^(1/2))*b*c*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*2^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*2^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
```

```
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)+4*a*d^2*x^2*(a*b)^(1/2)-4*b*c*d*x^2*(a*b)^(1/2)-4*a*c*d*(
a*b)^(1/2)+4*b*c^2*(a*b)^(1/2))/x/(-d*x^2+c)^(1/2)/c/a/((c*d)^(1/2)*b-(a*b)
^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b)^(1/2)/e^2/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] -e^(-5/2)*integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*x^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a\left(ex\right)^{\frac{5}{2}}\sqrt{c-dx^2}+bx^2\left(ex\right)^{\frac{5}{2}}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
[Out] -Integral(1/(-a*(e*x)**(5/2)*sqrt(c - d*x**2) + b*x**2*(e*x)**(5/2)*sqrt(c
- d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-e^(-5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x)^{5/2}\,\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(1/2)),x)
[Out] int(1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.887 $\int \frac{1}{(ex)^{7/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=444

$$-\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}}-\frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}}-\frac{2\sqrt[4]{d}\,(5bc+3ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}}{}$$

```
[Out] -2/5*(-d*x^2+c)^(1/2)/a/c/e/(e*x)^(5/2)-2/5*(3*a*d+5*b*c)*(-d*x^2+c)^(1/2)/
a^2/c^2/e^3/(e*x)^(1/2)-2/5*d^(1/4)*(3*a*d+5*b*c)*EllipticE(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(5/4)/e^(7/2)/(-d*x^2+c)^(1
/2)+2/5*d^(1/4)*(3*a*d+5*b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
,I)*(1-d*x^2/c)^(1/2)/a^2/c^(5/4)/e^(7/2)/(-d*x^2+c)^(1/2)-b^(3/2)*c^(1/4)*
EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(
1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/2)/(-d*x^2+c)^(1/2)+b^(3/2)*
c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1
/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 491, 597, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}\,e^{7/2}\sqrt{c-dx^2}}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^{5/2}\sqrt[4]{d}\,e^{7/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(3ad+5bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(3ad+5bc)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}-\frac{2\sqrt{c-dx^2}(3ad+5bc)}{5a^2c^2e^3\sqrt{ex}}-\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(7/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (-2*Sqrt[c - d*x^2])/(5*a*c*e*(e*x)^(5/2)) - (2*(5*b*c + 3*a*d)*Sqrt[c - d*
x^2])/(5*a^2*c^2*e^3*Sqrt[e*x]) - (2*d^(1/4)*(5*b*c + 3*a*d)*Sqrt[1 - (d*x^
2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2*
c^(5/4)*e^(7/2)*Sqrt[c - d*x^2]) + (2*d^(1/4)*(5*b*c + 3*a*d)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(5*a^2
*c^(5/4)*e^(7/2)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e^(7/2)*Sqrt[c - d*x^2]) + (b^(3
/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(5/2)*d^(1/4)*e
^(7/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{7/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^6\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} + \frac{2\text{Subst}\left(\int \frac{\frac{5bc+3ad}{e^2}-\frac{3bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{5ace}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} - \frac{2\text{Subst}\left(\int \frac{x^2\left(-\frac{5b^2c^2-5abcd-3a^2}{e^4}\right.}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c}} \right.}{5a^2}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} - \frac{2\text{Subst}\left(\int \left(\frac{d(5bc+3ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}} - \right.\right.}{}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} + \frac{(2b^2)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-}}\right.}{a^2e^5}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} + \frac{\left(2\sqrt{d}\,(5bc+3ad)\right)\text{Subst}\left(\right.}{5a^2c}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} + \frac{\left(2\sqrt{d}\,(5bc+3ad)\sqrt{1-\frac{dx}{c}}\right.}{5a^2}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} + \frac{2\sqrt[4]{d}\,(5bc+3ad)\sqrt{1-\frac{dx^2}{c}}}{5a^2c^{5/4}e^{7/}}$$

$$= -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}} - \frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}} - \frac{2\sqrt[4]{d}\,(5bc+3ad)\sqrt{1-\frac{dx^2}{c}}}{5a^2c^{5/4}e^{7/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.17, size = 188, normalized size = 0.42

$$\frac{x\left(-42a(c-dx^2)\left(5bcx^2+a(c+3dx^2)\right)+14(5b^2c^2-5abcd-3a^2d^2)\,x^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+6bd(5bc+3ad)x^6\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{105a^3c^2(ex)^{7/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(7/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(-42*a*(c - d*x^2)*(5*b*c*x^2 + a*(c + 3*d*x^2)) + 14*(5*b^2*c^2 - 5*a*b
*c*d - 3*a^2*d^2)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2
)/c, (b*x^2)/a] + 6*b*d*(5*b*c + 3*a*d)*x^6*Sqrt[1 - (d*x^2)/c]*AppellF1[7/
4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(105*a^3*c^2*(e*x)^(7/2)*Sqrt[c -
d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. 2(332) = 664.
time = 0.13, size = 1098, normalized size = 2.47 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c^2*d
*x^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^3*x^2-5*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c^2*x^2+5*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^3*x
^2+5*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*b*c^2*x^2-12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipti
cE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*c*d^2*x^2-8*((d*x
```

```
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*a*b*c^2*d*x^2+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipti
cE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3*x^2-12*a^2*d^
3*x^4-8*a*b*c*d^2*x^4+20*b^2*c^2*d*x^4+8*a^2*c*d^2*x^2+12*a*b*c^2*d*x^2-20*
b^2*c^3*x^2+4*a^2*c^2*d-4*a*b*c^3)*d*b/x^2/(-d*x^2+c)^(1/2)/c^2/((c*d)^(1/2
)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/a^2/e^3/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] -e^(-7/2)*integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*x^(7/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a\left(ex\right)^{\frac{7}{2}}\sqrt{c-dx^{2}}+bx^{2}\left(ex\right)^{\frac{7}{2}}\sqrt{c-dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
[Out] -Integral(1/(-a*(e*x)**(7/2)*sqrt(c - d*x**2) + b*x**2*(e*x)**(7/2)*sqrt(c
- d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(-e^(-7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)*x^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x)^{7/2}\,\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(7/2)*(a - b*x^2)*(c - d*x^2)^(1/2)),x)
[Out] int(1/((e*x)^(7/2)*(a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.888 $\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}} \, dx$

Optimal. Leaf size=444

$$-\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}}+\frac{c^{3/4}(3bc-2ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}(bc-ad)\sqrt{c-dx^2}}-\frac{c^{3/4}(3bc-2ad)e^{9/2}\sqrt{}}{bd^{7}}$$

```
[Out] -c*e^3*(e*x)^(3/2)/d/(-a*d+b*c)/(-d*x^2+c)^(1/2)+c^(3/4)*(-2*a*d+3*b*c)*e^(
9/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d
^(7/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-c^(3/4)*(-2*a*d+3*b*c)*e^(9/2)*EllipticF
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(7/4)/(-a*d+b
*c)/(-d*x^2+c)^(1/2)-a^(3/2)*c^(1/4)*e^(9/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)
/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(3
/2)/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+a^(3/2)*c^(1/4)*e^(9/2)*EllipticPi(
d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d
*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.56, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 481, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{a^{3/2}\sqrt[4]{c}\,e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{a^{3/2}\sqrt[4]{c}\,e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(3bc-2ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}\sqrt{c-dx^2}\,(bc-ad)}+\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(3bc-2ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}\sqrt{c-dx^2}\,(bc-ad)}-\frac{ce^3(ex)^{3/2}}{d\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((c*e^3*(e*x)^(3/2))/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)*(3*b*c -
2*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(b*d^(7/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3/4)*(3
*b*c - 2*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(7/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (a^(
3/2)*c^(1/4)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sq
rt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/
2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (a^(3/2)*c^(1/4)*e^(9/2)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)*d^(1/4)*(b*c - a*d)*Sqrt[c -
d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^{10}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{e^3\text{Subst}\left(\int \frac{x^2\left(3ac-\frac{(3bc-2ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{d(bc-ad)}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{e^3\text{Subst}\left(\int \left(\frac{(3bc-2ad)x^2}{b\sqrt{c-\frac{dx^4}{e^2}}} + \frac{2a^2dx^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.\right.}{d(bc-ad)}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{(2a^2e^3)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b(bc-ad)}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}\,(3bc-2ad)e^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{\right.}{bd^{3/2}(bc-ad)}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}\,(3bc-2ad)e^4\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\right.}}{bd^{3/2}(bc-ad)\sqrt{c-dx^2}}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} - \frac{c^{3/4}(3bc-2ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{e}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{bd^{7/4}(bc-ad)\sqrt{c-dx^2}}$$

$$= -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{c^{3/4}(3bc-2ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{e}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{bd^{7/4}(bc-ad)\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.15, size = 148, normalized size = 0.33

$$-\frac{e^3(ex)^{3/2}\left(-7ac+7ac\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+(-3bc+2ad)x^2\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{7ad(-bc+ad)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -1/7*(e^3*(e*x)^(3/2)*(-7*a*c + 7*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2
, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + (-3*b*c + 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*
AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(a*d*(-(b*c) + a*d)*Sqr
t[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(344) = 688$.
time = 0.13, size = 1030, normalized size = 2.32 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d
+6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*b^3*c^3-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2+5*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a*b^2*c^2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3+((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^
2*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*(c*d)^(1/2)*a^2*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(
```

```
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1
/2)*d),1/2*2^(1/2))*a^2*b*c*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2-2*a*b^2*c*d^2*x^2+2*b^3*c^2*d*x^2)*e^4*(
e*x)^(1/2)/x/d/(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*
b+(a*b)^(1/2)*d)/(a*d-b*c)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] -e^(9/2)*integrate(x^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{9}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
[Out] -Integral((e*x)**(9/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2) +
 b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x^(9/2)*e^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{9/2}}{(a-b\,x^2)\,\left(c-d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
[Out] int((e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.889 $\int \frac{(ex)^{7/2}}{(a-bx^2)\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=338

$$-\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(bc-2ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{bd^{5/4}(bc-ad)\sqrt{c-dx^2}}+\frac{a\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}}{b\sqrt[4]{}}$$

```
[Out] -c*e^3*(e*x)^(1/2)/d/(-a*d+b*c)/(-d*x^2+c)^(1/2)+c^(1/4)*(-2*a*d+b*c)*e^(7/
2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(
5/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+a*c^(1/4)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2
)/b/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+a*c^(1/4)*e^(7/2)*EllipticPi(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c
)^(1/2)/b/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 481, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-2ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{bd^{5/4}\sqrt{c-dx^2}\,(bc-ad)}+\frac{a\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{a\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{b\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}-\frac{ce^3\sqrt{ex}}{d\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((c*e^3*Sqrt[e*x])/(d*(b*c - a*d)*Sqrt[c - d*x^2])) + (c^(1/4)*(b*c - 2*a*
d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(b*d^(5/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7
/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*(b*c - a*d)*
Sqrt[c - d*x^2]) + (a*c^(1/4)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[
b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{e^3\text{Subst}\left(\int \frac{ac-\frac{(bc-2ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{d(bc-ad)}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{(2a^2e^3)\,\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b(bc-ad)}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{(ae^3)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\frac{x^2}{e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b(bc-ad)}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\,(bc-2ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{bd^{5/4}(bc-ad)\sqrt{c-dx^2}}$$

$$= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\,(bc-2ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{bd^{5/4}(bc-ad)\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 148, normalized size = 0.44

$$-\frac{e^3\sqrt{ex}\left(-5ac+5ac\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+(-bc+2ad)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{5ad(-bc+ad)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -1/5*(e^3*Sqrt[e*x]*(-5*a*c + 5*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2,
1, 5/4, (d*x^2)/c, (b*x^2)/a] + (-(b*c) + 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*Ap
pellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(a*d*(-(b*c) + a*d)*Sqrt[c
 - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(264) = 528$.

time = 0.13, size = 815, normalized size = 2.41

method	result
elliptic	$\dfrac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\dfrac{e^4xc}{d(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\dfrac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\cdots}{d^2\sqrt{-de\,x^3+cex}\,\cdots}\cdots\right.}{\cdots}$
default	$-\dfrac{\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,a^2d^2\sqrt{cd}\,\sqrt{ab}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}-3\,\mathrm{EllipticF}\left(\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1
/2)*a^2*d^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*d*(c*d)^(1
/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+EllipticF(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1
/2*2^(1/2))*a^2*b*c*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(
a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2-((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))
```

```
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d^2-((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d^2+2*
a*b*c*d^2*x*(a*b)^(1/2)-2*b^2*c^2*d*x*(a*b)^(1/2))*e^3*(e*x)^(1/2)/x/(-d*x^
2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b
)^(1/2)/(a*d-b*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] -e^(7/2)*integrate(x^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{7}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
[Out] -Integral((e*x)**(7/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2) +
 b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x^(7/2)*e^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{7/2}}{\left(a-b\,x^2\right)\,{\left(c-d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
[Out] int((e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.890 $\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=414

$$-\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}}+\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}(bc-ad)\sqrt{c-dx^2}}-\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{d^{3/4}(bc-ad)\sqrt{c-}}$$

```
[Out] -e*(e*x)^(3/2)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+c^(3/4)*e^(5/2)*EllipticE(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(3/4)/(-a*d+b*c)/(-d*x
^2+c)^(1/2)-c^(3/4)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I
)*(1-d*x^2/c)^(1/2)/d^(3/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-c^(1/4)*e^(5/2)*Ell
ipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2
),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/b^(1/2)/(-d*x^2+c)^(1/2)+
c^(1/4)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1
/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/b^(1/2)
/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.49, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 482, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}\,(bc-ad)}+\frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt{a}\,\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{\sqrt{a}\,\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}-\frac{e(ex)^{3/2}}{\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((e*(e*x)^(3/2))/((b*c - a*d)*Sqrt[c - d*x^2])) + (c^(3/4)*e^(5/2)*Sqrt[1
- (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(3/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c
]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*(b
*c - a*d)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) +
(Sqrt[a]*c^(1/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt
[b]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\text{Subst}\left(\int \frac{x^2\left(3a-\frac{bx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\text{Subst}\left(\int \left(\frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}} + \frac{2ax^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx,\right.}{bc-ad}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad} + \frac{(2ae)\text{Subst}\left(\right.}{}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}\,e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{d}\,(bc-ad)} + \frac{\left(\sqrt{c}\right.}{}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} - \frac{\left(\sqrt{c}\,e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{\right.}{\sqrt{d}\,(bc-ad)\sqrt{c-dx^2}}$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} - \frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}(bc-ad)\sqrt{c-dx^2}} -$$

$$= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} + \frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}(bc-ad)\sqrt{c-dx^2}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 133, normalized size = 0.32

$$-\frac{e(ex)^{3/2}\left(7a-7a\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bx^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{7a(bc-ad)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -1/7*(e*(e*x)^(3/2)*(7*a - 7*a*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/
4, (d*x^2)/c, (b*x^2)/a] + b*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1,
11/4, (d*x^2)/c, (b*x^2)/a]))/(a*(b*c - a*d)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 827 vs. 2(314) = 628.
time = 0.13, size = 828, normalized size = 2.00

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\frac{e^3x^2}{(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\text{Ellip}\cdots}{(ad-bc)d\sqrt{-de\,x^3+cex\cdots}}\cdots\right.}{\cdots}$
default	$-\frac{\left(2\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\text{EllipticE}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)abcd-2\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{2}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*d-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*E
llipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2-((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
```

```
,1/2*2^(1/2))*a*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2-(c*d)^(1/2)*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*d+(c*d)
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*a*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1
/2))*a*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*a*b*c*d+2*a*b*d^2*x^2-2*b^2*c*d*x^2)*e^2*(e*x)^(1/2)/x/(-d*x^2+c)^(1/2)
/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*d-b*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] -e^(5/2)*integrate(x^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{5}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] -Integral((e*x)**(5/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2) +
 b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x^(5/2)*e^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}}{\left(a-b\,x^2\right)\,{\left(c-d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

```
[Out] int((e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.891 $\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=314

$$-\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\,(bc-ad)\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}}\right.}{\sqrt[4]{d}\,(bc-ad)\sqrt{}}$$

```
[Out] -e*(e*x)^(1/2)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-c^(1/4)*e^(3/2)*EllipticF(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/(-d*x
^2+c)^(1/2)+c^(1/4)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),
-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/(-
d*x^2+c)^(1/2)+c^(1/4)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/
2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/
(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 482, 537, 230, 227, 418, 1233, 1232}

$$-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}\,(bc-ad)}+\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}\,(bc-ad)}+\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}\,(bc-ad)}-\frac{e\sqrt{ex}}{\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((e*Sqrt[e*x])/((b*c - a*d)*Sqrt[c - d*x^2])) - (c^(1/4)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d
^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(1/4)*(b*c - a*d)
*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\text{Subst}\left(\int \frac{a+\frac{bx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad}$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} - \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad} + \frac{(2ae)\text{Subst}\Big(}{}$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} + \frac{e\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{a}e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{bc-ad} + \frac{e\text{S}}{}$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\,(bc-ad)\sqrt{c-dx^2}} + \cdots$$

$$= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{d}\,(bc-ad)\sqrt{c-dx^2}} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.09, size = 133, normalized size = 0.42

$$\frac{e\sqrt{ex}\left(-5a+5a\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bx^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{5a(bc-ad)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (e*Sqrt[e*x]*(-5*a + 5*a*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*
x^2)/c, (b*x^2)/a] + b*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (
d*x^2)/c, (b*x^2)/a]))/(5*a*(b*c - a*d)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(240) = 480$.

time = 0.14, size = 693, normalized size = 2.21

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\,\left(\frac{e^2 x}{(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\cdots}{2(ad-bc)d\sqrt{-de\,x^3+\cdots}}\cdots\right.}{\cdots}$
default	$-\frac{b\left(\sqrt{2}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)ad\sqrt{ab}\,\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}-\sqrt{2}\,\mathrm{Ellipti}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*(2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*a*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2^(1/2)*EllipticF(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c*(a*b)^(1/2)*(c*d)^(1
/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^
(1/2)*d),1/2*2^(1/2))*a*b*c*d-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*d-((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d-(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
```

```
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d+2*a*d^2*
x*(a*b)^(1/2)-2*b*c*d*x*(a*b)^(1/2))*e*(e*x)^(1/2)/x/(-d*x^2+c)^(1/2)/((c*d
)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b)^(1/2)/(a*d-b*c
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] -e^(3/2)*integrate(x^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ex)^{\frac{3}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
[Out] -Integral((e*x)**(3/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2) +
 b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x^(3/2)*e^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{3/2}}{\left(a-b\,x^2\right)\,{\left(c-d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
[Out] int((e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.892 $\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=420

$$-\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}(bc-ad)\sqrt{c-dx^2}}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\right.\right.}{\sqrt[4]{c}(bc-ad)\sqrt{c-}}$$

```
[Out] -d*(e*x)^(3/2)/c/(-a*d+b*c)/e/(-d*x^2+c)^(1/2)+d^(1/4)*EllipticE(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/c^(1/4)/(-a*d+b*c)/(
-d*x^2+c)^(1/2)-d^(1/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^
(1/2)*(1-d*x^2/c)^(1/2)/c^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-c^(1/4)*Ellipti
cPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)
*b^(1/2)*e^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/a^(1/2)/(-d*x^2+c)^(1
/2)+c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/
a^(1/2)/d^(1/2),I)*b^(1/2)*e^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/a^(
1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 483, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)}-\frac{d(ex)^{3/2}}{ce\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((d*(e*x)^(3/2))/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) + (d^(1/4)*Sqrt[e]*Sqr
t[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(c^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^
2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(c^(1/4
)*(b*c - a*d)*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/
c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) + (Sqrt[b]*c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
Sqrt[a]*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{e\text{Subst}\left(\int \frac{x^2\left(-\frac{2bc+ad}{e^2}+\frac{bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{c(bc-ad)}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{e\text{Subst}\left(\int \left(-\frac{dx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}} - \frac{2bcx^2}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.\right.}{c(bc-ad)}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} + \frac{(2b)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{(bc-ad)e} +$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{\sqrt{c}\,(bc-ad)} + \frac{\sqrt{d}\,\text{S}}{}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{\left(\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \right.}{\sqrt{c}\,(bc-ad)\sqrt{c-dx^2}}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\,(bc-ad)\sqrt{c-dx^2}}$$

$$= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} + \frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}\,(bc-ad)\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.16, size = 148, normalized size = 0.35

$$\frac{\sqrt{ex}\left(7(2bc+ad)x\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-3dx\left(7a+bx^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)\right)}{21ac(bc-ad)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[e*x]*(7*(2*b*c + a*d)*x*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4
, (d*x^2)/c, (b*x^2)/a] - 3*d*x*(7*a + b*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[7
/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(21*a*c*(b*c - a*d)*Sqrt[c - d*x
^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(320) = 640.
time = 0.13, size = 819, normalized size = 1.95

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\frac{de\,x^2}{c(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{e\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticE}}{(ad-bc)\sqrt{-de\,x^3+cex}}\right.}{}$
default	$-\frac{\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)b\,c^2-\sqrt{\frac{dx+\sqrt{}}{\sqrt{}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c
^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(
```

```
1/2))*(c*d)^(1/2)*c+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*b*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*c+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elli
pticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*c*d-2*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*b*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2+2*a*
d^2*x^2-2*b*c*d*x^2)*d*b*(e*x)^(1/2)/(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^
(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/c/(a*d-b*c)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] -e^(1/2)*integrate(sqrt(x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ex}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

```
[Out] -Integral(sqrt(e*x)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2) + b*
c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x)*e^(1/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}}{\left(a-b\,x^2\right)\,{\left(c-d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

```
[Out] int((e*x)^(1/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.893 $\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=328

$$-\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}}-\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{c^{3/4}(bc-ad)\sqrt{e}\,\sqrt{c-dx^2}}+\frac{b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\mathrm{si}\right.}{a\sqrt[4]{d}\,(bc-ad)\sqrt{e}}$$

```
[Out] -d*(e*x)^(1/2)/c/(-a*d+b*c)/e/(-d*x^2+c)^(1/2)-d^(3/4)*EllipticF(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/c^(3/4)/(-a*d+b*c)/e^(1/2)/(
-d*x^2+c)^(1/2)+b*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b
^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/(-a*d+b*c)/e^
(1/2)/(-d*x^2+c)^(1/2)+b*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/(-a*d+b
*c)/e^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 425, 537, 230, 227, 418, 1233, 1232}

$$-\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{c^{3/4}\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\mathrm{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)}-\frac{d\sqrt{ex}}{ce\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -((d*Sqrt[e*x])/(c*(b*c - a*d)*e*Sqrt[c - d*x^2])) - (d^(3/4)*Sqrt[1 - (d*x
^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(c^(3/
4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) +
 (b*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a*d^(1/4)*(b*c -
a*d)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{e\text{Subst}\left(\int \frac{-\frac{2bc-ad}{e^2}-\frac{bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{c(bc-ad)}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{(bc-ad)e}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} + \frac{b\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{e}\ldots\right)}{a(bc-ad)e}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\ldots\right.}{c^{3/4}(bc-ad)\sqrt{e}\,\sqrt{c-dx^2}}$$

$$= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} - \frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\ldots\right.}{c^{3/4}(bc-ad)\sqrt{e}\,\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 147, normalized size = 0.45

$$\frac{-5adx+5(2bc-ad)x\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bdx^3\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{5ac(bc-ad)\sqrt{ex}\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)),x]

[Out] (-5*a*d*x + 5*(2*b*c - a*d)*x*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*x^3*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*(b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(254) = 508$.

time = 0.13, size = 697, normalized size = 2.12

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(\frac{dx}{c(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}}{2c(ad-bc)\sqrt{-de\,x^3+cex}}\right.}{}$
default	$-\frac{bd\left(\sqrt{2}\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)ad\sqrt{ab}\,\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}-\sqrt{2}\,\mathrm{Elliptic}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*b*d*(2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2-(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)

```
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c+2*a*d^
2*x*(a*b)^(1/2)-2*b*c*d*x*(a*b)^(1/2))/(-d*x^2+c)^(1/2)/c/((c*d)^(1/2)*b-(a
*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b)^(1/2)/(a*d-b*c)/(e*x)^(1/2
)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
[Out] -e^(-1/2)*integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(x)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ac\sqrt{ex}\,\sqrt{c-dx^2}+adx^2\sqrt{ex}\,\sqrt{c-dx^2}+bcx^2\sqrt{ex}\,\sqrt{c-dx^2}-bdx^4\sqrt{ex}\,\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)/(-d*x**2+c)**(3/2)/(e*x)**(1/2),x)
[Out] -Integral(1/(-a*c*sqrt(e*x)*sqrt(c - d*x**2) + a*d*x**2*sqrt(e*x)*sqrt(c -
d*x**2) + b*c*x**2*sqrt(e*x)*sqrt(c - d*x**2) - b*d*x**4*sqrt(e*x)*sqrt(c -
 d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate(-e^(-1/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,x}\,\left(a-b\,x^2\right)\,{\left(c-d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(1/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x)
[Out] int(1/((e*x)^(1/2)*(a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.894 $\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}} dx$

Optimal. Leaf size=493

$$-\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}}-\frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}}-\frac{\sqrt[4]{d}(2bc-3ad)\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|}{ac^{5/4}(bc-ad)e^{3/2}\sqrt{c-dx^2}}$$

```
[Out] -d/c/(-a*d+b*c)/e/(e*x)^(1/2)/(-d*x^2+c)^(1/2)-(-3*a*d+2*b*c)*(-d*x^2+c)^(1
/2)/a/c^2/(-a*d+b*c)/e/(e*x)^(1/2)-d^(1/4)*(-3*a*d+2*b*c)*EllipticE(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(5/4)/(-a*d+b*c)/e^(3
/2)/(-d*x^2+c)^(1/2)+d^(1/4)*(-3*a*d+2*b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(5/4)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)
^(1/2)-b^(3/2)*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1
/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/(-a*d+b*c)
/e^(3/2)/(-d*x^2+c)^(1/2)+b^(3/2)*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(3/2)/
d^(1/4)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.70, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 483, 597, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|-1\right)}{ac^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|-1\right)}{ac^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\sqrt{c-dx^2}(2bc-3ad)}{ac^2e\sqrt{ex}(bc-ad)}-\frac{d}{ce\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*e*Sqrt[e*x]*Sqrt[c - d*x^2])) - ((2*b*c - 3*a*d)*Sqrt[c
- d*x^2])/(a*c^2*(b*c - a*d)*e*Sqrt[e*x]) - (d^(1/4)*(2*b*c - 3*a*d)*Sqrt[1
 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(a*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(2*b*c - 3*a*d)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(a*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) - (b^(3/2)*c^(1/4)*
Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*(b*c - a*d
)*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(a^(3/2)*d^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{e\text{Subst}\left(\int \frac{-\frac{2bc-3ad}{e^2}-\frac{3bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \ldots\right.}{c(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{e\text{Subst}\left(\int \frac{x\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{e\text{Subst}\left(\int \left(\ldots\right.\right.}{\ldots}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{(2b^2)\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{\left(\sqrt{d}(2bc-\ldots\right.}{\ldots}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{\left(\sqrt{d}(2bc-\ldots\right.}{\ldots}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} + \frac{\sqrt[4]{d}(2bc-3\ldots}{\ldots}$$

$$= -\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} - \frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}} - \frac{\sqrt[4]{d}(2bc-3\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.19, size = 198, normalized size = 0.40

$$\frac{x\left(21a(ad(2c-3dx^2)-2bc(c-dx^2))+7(2b^2c^2-2abcd+3a^2d^2)\,x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bd(2bc-3ad)x^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{21a^2c^2(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*(21*a*(a*d*(2*c - 3*d*x^2) - 2*b*c*(c - d*x^2)) + 7*(2*b^2*c^2 - 2*a*b*c
*d + 3*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/
c, (b*x^2)/a] + 3*b*d*(2*b*c - 3*a*d)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4,
 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*c^2*(b*c - a*d)*(e*x)^(3/2)*
Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(389) = 778$.
time = 0.13, size = 1047, normalized size = 2.12 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(6*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*a^2*c*d^2-10*EllipticE(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*a*b*c^2*d+4*
EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*b^2*c^3-3*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*a^2*c*d^2+5*EllipticF((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)*a*b*c^2*d-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*b^2*c^3-(a*b)^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/
2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*b*c^2+(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*
(c*d)^(1/2)*b*c^2+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(
```

```
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)*b^2*c^3+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)*b^2*c^3+6*a^2*d^3*x^2-10*a*b*c*d^2*x^2+4*b^2*c^2*d*x^2-4*a^2*c*d^2+
8*a*b*c^2*d-4*b^2*c^3)*b*d/(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/(
(c*d)^(1/2)*b+(a*b)^(1/2)*d)/a/(a*d-b*c)/c^2/e/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] -e^(-3/2)*integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*x^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ac\left(ex\right)^{\frac{3}{2}}\sqrt{c-dx^2}+adx^2\left(ex\right)^{\frac{3}{2}}\sqrt{c-dx^2}+bcx^2\left(ex\right)^{\frac{3}{2}}\sqrt{c-dx^2}-bdx^4\left(ex\right)^{\frac{3}{2}}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
[Out] -Integral(1/(-a*c*(e*x)**(3/2)*sqrt(c - d*x**2) + a*d*x**2*(e*x)**(3/2)*sqr
t(c - d*x**2) + b*c*x**2*(e*x)**(3/2)*sqrt(c - d*x**2) - b*d*x**4*(e*x)**(3
/2)*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(-e^(-3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*x^(3/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x)^{3/2}\,\left(a-b\,x^2\right)\,\left(c-d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x)
[Out] int(1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.895 $\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=397

$$-\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}}-\frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}}+\frac{d^{3/4}(2bc-5ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|}{3ac^{7/4}(bc-ad)e^{5/2}\sqrt{c-dx^2}}$$

```
[Out] -d/c/(-a*d+b*c)/e/(e*x)^(3/2)/(-d*x^2+c)^(1/2)-1/3*(-5*a*d+2*b*c)*(-d*x^2+c
)^(1/2)/a/c^2/(-a*d+b*c)/e/(e*x)^(3/2)+1/3*d^(3/4)*(-5*a*d+2*b*c)*EllipticF
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(7/4)/(-a*d+b
*c)/e^(5/2)/(-d*x^2+c)^(1/2)+b^2*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1
/4)/(-a*d+b*c)/e^(5/2)/(-d*x^2+c)^(1/2)+b^2*c^(1/4)*EllipticPi(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2
)/a^2/d^(1/4)/(-a*d+b*c)/e^(5/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 483, 597, 537, 230, 227, 418, 1233, 1232}

$$\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}\,(bc-ad)}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a^2\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}\,(bc-ad)}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,(2bc-5ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{3ac^{7/4}e^{5/2}\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt{c-dx^2}\,(2bc-5ad)}{3ac^2e(ex)^{3/2}(bc-ad)}-\frac{d}{ce(ex)^{3/2}\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*e*(e*x)^(3/2)*Sqrt[c - d*x^2])) - ((2*b*c - 5*a*d)*Sqrt[
c - d*x^2])/(3*a*c^2*(b*c - a*d)*e*(e*x)^(3/2)) + (d^(3/4)*(2*b*c - 5*a*d)*
Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(3*a*c^(7/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/
2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(a^2*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{e\text{Subst}\left(\int \frac{-\frac{2bc-5ad}{e^2}-\frac{5bdx^4}{e^4}}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx\right.}{c(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{e\text{Subst}\left(\right.}{}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{(2b^2)\,\text{Sub}}{}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{b^2\text{Subst}\left(\right.}{}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{d^{3/4}(2bc-}{}$$

$$= -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{d^{3/4}(2bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.18, size = 197, normalized size = 0.50

$$\frac{x\left(5a\left(ad\left(2c-5dx^2\right)-2bc\left(c-dx^2\right)\right)+5\left(6b^2c^2+2abcd-5a^2d^2\right)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bd(-2bc+5ad)x^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{15a^2c^2(bc-ad)(ex)^{5/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*(5*a*(a*d*(2*c - 5*d*x^2) - 2*b*c*(c - d*x^2)) + 5*(6*b^2*c^2 + 2*a*b*c*
d - 5*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c
, (b*x^2)/a] + b*d*(-2*b*c + 5*a*d)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1
/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a^2*c^2*(b*c - a*d)*(e*x)^(5/2)*Sqr
t[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(315) = 630$.
time = 0.14, size = 885, normalized size = 2.23

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(\frac{d^2 x}{e^2 c^2 (ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right) dex}}-\frac{2\sqrt{-de\,x^3+cex}}{3c^2 e^3 a\,x^2}+\frac{d\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}}\ldots}{2\sqrt{-d}\ldots}\ldots\right)}{\ldots}$
default	$-\frac{bd\left(5\,\mathrm{EllipticF}\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,a^2 d^2 x\sqrt{ab}\,\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}-7\,\mathrm{Elliptic}\ldots\right)}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*d*(5*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^
(1/2)*a^2*d^2*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-7*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*d*x*(a
*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+2*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c^2*x*(a*b)^(1/2)*(c*d)^(1
/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^3
*c^3*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
```

```
b^2*c^2*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/
2)*d),1/2*2^(1/2))*2^(1/2)*b^3*c^3*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^2*c^2*x*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)+10*a^2*d^3*x^2*(a*b)^(1/2)-14*a*b*c*d^2*x^2*(a*b)^(1/2)+4*b
^2*c^2*d*x^2*(a*b)^(1/2)-4*a^2*c*d^2*(a*b)^(1/2)+8*a*b*c^2*d*(a*b)^(1/2)-4*
b^2*c^3*(a*b)^(1/2))/x/(-d*x^2+c)^(1/2)/c^2/a/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b)^(1/2)/(a*d-b*c)/e^2/(e*x)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] -e^(-5/2)*integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*x^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ac\left(ex\right)^{\frac{5}{2}}\sqrt{c-dx^2}+adx^2\left(ex\right)^{\frac{5}{2}}\sqrt{c-dx^2}+bcx^2\left(ex\right)^{\frac{5}{2}}\sqrt{c-dx^2}-bdx^4\left(ex\right)^{\frac{5}{2}}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
[Out] -Integral(1/(-a*c*(e*x)**(5/2)*sqrt(c - d*x**2) + a*d*x**2*(e*x)**(5/2)*sqr
t(c - d*x**2) + b*c*x**2*(e*x)**(5/2)*sqrt(c - d*x**2) - b*d*x**4*(e*x)**(5
/2)*sqrt(c - d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(-e^(-5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*x^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x)^{5/2}\,\left(a-b\,x^2\right)\,{\left(c-d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x)
[Out] int(1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.896 $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=362

$$\frac{7e^3\sqrt{ex}\,\sqrt{c-dx^2}}{6b^2}+\frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\,(a-bx^2)}+\frac{\sqrt[4]{c}\,(8bc-21ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt[4]{d}\,\sqrt{c-dx^2}}-$$

```
[Out] 1/2*e*(e*x)^(5/2)*(-d*x^2+c)^(1/2)/b/(-b*x^2+a)+7/6*e^3*(e*x)^(1/2)*(-d*x^2
+c)^(1/2)/b^2+1/6*c^(1/4)*(-21*a*d+8*b*c)*e^(7/2)*EllipticF(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)-1/4*
c^(1/4)*(-7*a*d+5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/
2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/d^(1/4)/(-d*x^
2+c)^(1/2)-1/4*c^(1/4)*(-7*a*d+5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2
)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/
d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 478, 596, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(8bc-21ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(5bc-7ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b^3\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(5bc-7ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b^3\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\,(a-bx^2)}+\frac{7e^3\sqrt{ex}\,\sqrt{c-dx^2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (7*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*(e*x)^(5/2)*Sqrt[c - d*x^2])
/(2*b*(a - b*x^2)) + (c^(1/4)*(8*b*c - 21*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*
EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*b^3*d^(1/4
)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 7*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(4*b^3*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*
c - 7*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^3*d^(1
/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^8\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b(a-bx^2)} - \frac{e\text{Subst}\left(\int \frac{x^4\left(5c-\frac{7dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= \frac{7e^3\sqrt{ex}\,\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b(a-bx^2)} + \frac{e^5\text{Subst}\left(\int \frac{-\frac{7acd}{e^2}-\frac{d(8bc-21ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x,\right.}{6b^2d}$$

$$= \frac{7e^3\sqrt{ex}\,\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b(a-bx^2)} + \frac{((8bc-21ad)e^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\right.}{6b^3}$$

$$= \frac{7e^3\sqrt{ex}\,\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b(a-bx^2)} - \frac{((5bc-7ad)e^3)\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)}\right.}{4b^3}$$

$$= \frac{7e^3\sqrt{ex}\,\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b(a-bx^2)} + \frac{\sqrt[4]{c}\,(8bc-21ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{si}\right.}{6b^3\sqrt[4]{d}\,\sqrt{c-dx}}$$

$$= \frac{7e^3\sqrt{ex}\,\sqrt{c-dx^2}}{6b^2} + \frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b(a-bx^2)} + \frac{\sqrt[4]{c}\,(8bc-21ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{si}\right.}{6b^3\sqrt[4]{d}\,\sqrt{c-dx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.17, size = 184, normalized size = 0.51

$$\frac{e^3\sqrt{ex}\left(5a(7a-4bx^2)\left(-c+dx^2\right)+35ac(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-(-8bc+21ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{30ab^2\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e^3*Sqrt[e*x]*(5*a*(7*a - 4*b*x^2)*(-c + d*x^2) + 35*a*c*(a - b*x^2)*Sqrt[
1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - (-8*b*c +
 21*a*d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*
x^2)/c, (b*x^2)/a]))/(30*a*b^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2548 vs. $2(274) = 548$.
time = 0.19, size = 2549, normalized size = 7.04

method	result
elliptic	$\sqrt{ex}\,\sqrt{\left(-d\,x^2+c\right)ex}\left(\frac{a\,e^3\sqrt{-de\,x^3+cex}}{2b^2\left(-b\,x^2+a\right)}+\frac{2e^3\sqrt{-de\,x^3+cex}}{3b^2}-\frac{7\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}}}{4 \cdots}\right.$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*e^3*(e*x)^(1/2)/b^2*(-16*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*2^(1/2)*b^3*c^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)+16*b^3*c*d^2*x^5*(a*b)^(1/2)+28*a^2*b*d^3*x^3*(a*b)^(1/2)-16*b^
3*c^2*d*x^3*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c
*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/
2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-15*Elli
```

pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)+58*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)-16*a*b^2*d^3*x^5*(a*b)^(1/2)-12*a*b^2*c*d^2*x^3*(a*b)^(1/2)-28*
a^2*b*c*d^2*x*(a*b)^(1/2)+28*a*b^2*c^2*d*x*(a*b)^(1/2)-21*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)+42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^
(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+16*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*(a*b)^(1/2)*(c*d)^(1/2)*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)-15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2
*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+21*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)-42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/

```
2))^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(
c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+15*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-58*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a
*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(7/2)*integrate(sqrt(-d*x^2 + c)*x^(7/2)/(b*x^2 - a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*x^(7/2)*e^(7/2)/(b*x^2 - a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{7/2}\,\sqrt{c-d\,x^2}}{(a-b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(7/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2,x)
```

```
[Out] int(((e*x)^(7/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2, x)
```

3.897 $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=413

$$\frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b\,(a-bx^2)}-\frac{5c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}}+\frac{5c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\Big(\text{si}}{2b^2\sqrt{c-}}$$

```
[Out] 1/2*e*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b/(-b*x^2+a)-5/2*c^(3/4)*d^(1/4)*e^(5/2)
*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/(-d
*x^2+c)^(1/2)+5/2*c^(3/4)*d^(1/4)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-5*a*d+
3*b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1
/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(5/2)/d^(1/4)/a^(1/2)/(-d*x^2+c)
^(1/2)-1/4*c^(1/4)*(-5*a*d+3*b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(5/2)/
d^(1/4)/a^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 478, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(3bc-5ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\,b^{5/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(3bc-5ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\,b^{5/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b\,(a-bx^2)}+\frac{5c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}}-\frac{5c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (5*c^(3/4)*d^(1/4)*e^(5
/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*b^2*Sqrt[c - d*x^2]) + (5*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (d
*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b
^2*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]) - (
c^(1/4)*(3*b*c - 5*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(4*Sqrt[a]*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^6\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} - \frac{e\text{Subst}\left(\int \frac{x^2\left(3c-\frac{5dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} - \frac{e\text{Subst}\left(\int \left(\frac{5dx^2}{b\sqrt{c-\frac{dx^4}{e^2}}} + \frac{(3bc-5ad)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)dx, x, \sqrt{e\right.}{2b}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} - \frac{(5de)\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2} - \frac{((3bc-5ad)e)\text{S}}{}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} + \frac{\left(5\sqrt{c}\sqrt{d}\,e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2} - \frac{\left(5\sqrt{c}\right.}{}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} + \frac{\left(5\sqrt{c}\sqrt{d}\,e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right.}{2b^2\sqrt{c-dx^2}}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} + \frac{5c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}}{}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} - \frac{5c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} + \frac{5c}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.13, size = 163, normalized size = 0.39

$$\frac{e(ex)^{3/2}\left(-7a(c-dx^2)+7c(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+5dx^2(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{14ab\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*(e*x)^(3/2)*(-7*a*(c - d*x^2) + 7*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appe
llF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 5*d*x^2*(-a + b*x^2)*Sqrt[1
- (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(14*a*b*(-
a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2529 vs. 2(303) = 606.
time = 0.12, size = 2530, normalized size = 6.13

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\frac{e^2x\sqrt{-de\,x^3+cex}}{2b\left(-b\,x^2+a\right)}+\frac{5e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticE}\left(\cdots\right.}{2b^2\sqrt{-de\,x^3+cex}}\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*e^2*(e*x)^(1/2)*d*(-4*a*b^2*c*d*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^
2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2
*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
```

```
*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^
3*c^2*x^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*b^3*c^2*x^2+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2-20*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^
3*c^2*x^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^2*x^2+4*b^3*c^2*x^2+4*a*b^2*d^2*x^4
-4*b^3*c*d*x^4-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1
/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*
(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d+5*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+5*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-20*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*a^2*b*c*d+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d-5*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2-5*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+3
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/
2))*(c*d)^(1/2)*a*b*c-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c-10*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
```

```
)^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*
c*d*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),
1/2*2^(1/2))*(c*d)^(1/2)*b^2*c*x^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*x^2+20*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*a*b^2*c*d*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(5/2)*integrate(sqrt(-d*x^2 + c)*x^(5/2)/(b*x^2 - a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}{(-a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
[Out] Integral((e*x)**(5/2)*sqrt(c - d*x**2)/(-a + b*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)*x^(5/2)*e^(5/2)/(b*x^2 - a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}\,\sqrt{c-d\,x^2}}{{\left(a-b\,x^2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2,x)
[Out] int(((e*x)^(5/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2, x)
```

3.898 $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=328

$$\frac{e\sqrt{ex}\,\sqrt{c-dx^2}}{2b\left(a-bx^2\right)}-\frac{3\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,(bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\Pi}{4ab^2\sqrt[4]{}}$$

```
[Out] 1/2*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b/(-b*x^2+a)-3/2*c^(1/4)*d^(3/4)*e^(3/2)
*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/(-d
*x^2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/
b^2/d^(1/4)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+b*c)*e^(3/2)*EllipticPi(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x
^2/c)^(1/2)/a/b^2/d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 478, 537, 230, 227, 418, 1233, 1232}

$$-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-3ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4ab^2\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-3ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4ab^2\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{e\sqrt{ex}\,\sqrt{c-dx^2}}{2b\left(a-bx^2\right)}-\frac{3\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(a - b*x^2)) - (3*c^(1/4)*d^(3/4)*e^(3/2
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(2*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 3*a*d)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2]) -
(c^(1/4)*(b*c - 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*a*b^2*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\sqrt{ex}\,\sqrt{c-dx^2}}{2b\,(a-bx^2)} - \frac{e\text{Subst}\left(\int \frac{c-\frac{3dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= \frac{e\sqrt{ex}\,\sqrt{c-dx^2}}{2b\,(a-bx^2)} - \frac{(3de)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2} - \frac{((bc-3ad)e)\text{Su}}{}$$

$$= \frac{e\sqrt{ex}\,\sqrt{c-dx^2}}{2b\,(a-bx^2)} - \frac{((bc-3ad)e)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{4ab^2}$$

$$= \frac{e\sqrt{ex}\,\sqrt{c-dx^2}}{2b\,(a-bx^2)} - \frac{3\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} - $$

$$= \frac{e\sqrt{ex}\,\sqrt{c-dx^2}}{2b\,(a-bx^2)} - \frac{3\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.13, size = 163, normalized size = 0.50

$$\frac{e\sqrt{ex}\left(-5a(c-dx^2)+5c(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3dx^2(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{10ab\,(-a+bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (e*Sqrt[e*x]*(-5*a*(c - d*x^2) + 5*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 3*d*x^2*(-a + b*x^2)*Sqrt[1 -
(d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(10*a*b*(-a +
 b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2242 vs. $2(246) = 492$.
time = 0.13, size = 2243, normalized size = 6.84

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\,\left(\frac{e\sqrt{-de\,x^3+cex}}{2b\left(-b\,x^2+a\right)}-\frac{3e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\;\mathrm{EllipticF}}{4b^2\sqrt{-de\,x^3+cex}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*e*(e*x)^(1/2)/b*d*(-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*2^(1/2)*a*b*d*x^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)+6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2
)*b^2*c*x^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+3*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*
(c*d)^(1/2)*a*b*d*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),
1/2*2^(1/2))*b^3*c^2*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+
(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c*x^2+3*((d*x+(c*d)^(1/2))/(c*d
```

```
)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+3*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a
*b*d*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
b^3*c^2*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*x^2+6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*2^(1/2)*a*b*c*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-3*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))
*(c*d)^(1/2)*a^2*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2
*2^(1/2))*a*b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)
^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d+((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*a*b*c+4*a*b*d^2*x^3*(a*b)^(1/2)-4*b^2*c*d*x^3*(a*b)^(1/2)-4*a*b*c*d
*x*(a*b)^(1/2)+4*b^2*c^2*x*(a*b)^(1/2))/x/(-d*x^2+c)^(1/2)/(-b*x^2+a)/(a*b)
```

```
^(1/2)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(3/2)*integrate(sqrt(-d*x^2 + c)*x^(3/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}} \sqrt{c - dx^2}}{(-a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
[Out] Integral((e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)*x^(3/2)*e^(3/2)/(b*x^2 - a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{3/2}\,\sqrt{c - d\,x^2}}{(a - b\,x^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2,x)
[Out] int(((e*x)^(3/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2, x)
```

3.899 $\int \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=417

$$\frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\,(a-bx^2)}-\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\right.}{2ab\sqrt{c-dx^2}}$$

```
[Out] 1/2*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/a/e/(-b*x^2+a)-1/2*c^(3/4)*d^(1/4)*Ellipti
cE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/b/(-d
*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/b/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(a*d+b*c
)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d
^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^(3/2)/d^(1/4)/(-d*x^2+c)^(1/2
)+1/4*c^(1/4)*(a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1
/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^(3/2)/d^
(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 480, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}}+\frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] ((e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) - (c^(3/4)*d^(1/4)*Sqrt[e
]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(2*a*b*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2
)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b*S
qrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/(4*a^(3/2)*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*
(b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)
*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-c-\frac{dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ae}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{dx^2}{b\sqrt{c-\frac{dx^4}{e^2}}} - \frac{(bc+ad)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)\,dx, x, \sqrt{ex}\right.}{2ae}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} - \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2abe} + \frac{(bc+ad)\text{Subst}\left(\int \right.}{}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\,\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab} - \frac{\left(\sqrt{c}\,\sqrt{d}\right)}{}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{\left(\sqrt{c}\,\sqrt{d}\,\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x, \sqrt{ex}\right)}{2ab\sqrt{c-dx^2}} -$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} + \frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}\,(b}{}$$

$$= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)} - \frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}} + \frac{c^{3/4}\sqrt[4]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 163, normalized size = 0.39

$$\frac{\sqrt{ex}\left(21ax(-c+dx^2)+7cx(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3dx^3(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a^2\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

```
[Out] (Sqrt[e*x]*(21*a*x*(-c + d*x^2) + 7*c*x*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*Ap
pellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*d*x^3*(-a + b*x^2)*Sqrt[
1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^2*
(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2521 vs. $2(307) = 614$.
time = 0.13, size = 2522, normalized size = 6.05

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\frac{x\sqrt{-de\,x^3+cex}}{2a\left(-b\,x^2+a\right)}+\frac{ec\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticE}\left(\sqrt{\cdots}\right.}{2ab\sqrt{-de\,x^3+cex}}\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(e*x)^(1/2)*d*(-4*a*b^2*c*d*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*E
llipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
```

```
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2-(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^
2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ell
ipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^2*x^2+2*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*b^3*c^2*x^2+4*b^3*c^2*x^2+4*a*b^2*d^2*x^4-4*b^3*c*d*x^4-(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2)
)*(c*d)^(1/2)*a^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d
+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*
b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c+((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*a*b*c-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2)
```

```
)/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c*x^2-((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),
1/2*2^(1/2))*b^2*c*x^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^2+((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt(-d*x^2 + c)*sqrt(x)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{\left(-a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
[Out] Integral(sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(x)*e^(1/2)/(b*x^2 - a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}\,\sqrt{c-d\,x^2}}{{\left(a-b\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2,x)
[Out] int(((e*x)^(1/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2, x)
```

3.900 $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx$

Optimal. Leaf size=335

$$\frac{\sqrt{ex}\,\sqrt{c-dx^2}}{2ae\,(a-bx^2)}+\frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(3bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{}}{\sqrt{a}\,\sqrt{}}\right.}{4a^2b\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c}}$$

```
[Out] 1/2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/a/e/(-b*x^2+a)+1/2*c^(1/4)*d^(3/4)*Ellipti
cF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/e^(1/2)/(-d
*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d+3*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b/d^(1
/4)/e^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d+3*b*c)*EllipticPi(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1
/2)/a^2/b/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 423, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(3bc-ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2b\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(3bc-ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2b\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt{ex}\,\sqrt{c-dx^2}}{2ae\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] (Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*e*(a - b*x^2)) + (c^(1/4)*d^(3/4)*Sqrt[1 -
 (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
2*a*b*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c
^(1/4)*(3*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt
[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b*
d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \frac{2\mathrm{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{2ae\,(a-bx^2)} - \frac{\mathrm{Subst}\left(\int \frac{-3c+\frac{dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ae}$$

$$= \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{2ae\,(a-bx^2)} + \frac{d\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2abe} + \frac{(3bc-ad)\mathrm{Subst}\left(\int -\right.}{}$$

$$= \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{2ae\,(a-bx^2)} + \frac{(3bc-ad)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{4a^2be} + \frac{(3}{}$$

$$= \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{2ae\,(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{e}\,\sqrt{c-dx^2}} + \frac{\left((3bc-a\right.}{}$$

$$= \frac{\sqrt{ex}\,\sqrt{c-dx^2}}{2ae\,(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{e}\,\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\,(3bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.10, size = 161, normalized size = 0.48

$$\frac{5ax(-c+dx^2)+15cx(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+dx^3(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{10a^2\sqrt{ex}\,(-a+bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] (5*a*x*(-c + d*x^2) + 15*c*x*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4,
 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*x^3*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]
*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(10*a^2*Sqrt[e*x]*(-a +
b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2238 vs. $2(253) = 506$.

time = 0.13, size = 2239, normalized size = 6.68

method	result
elliptic	$\sqrt{(-d\,x^2+c)\,ex}\left(\frac{\sqrt{-de\,x^3+cex}}{2ea\left(-b\,x^2+a\right)}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\ldots\right.}{\ldots}}\right.}{4ab\sqrt{-de\,x^3+cex}}\right.$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d*(-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*a*b*d*x^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+2*Ellipti
cF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c*x^2*(c*
d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^
(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*d*x
^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*
c^2*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),
1/2*2^(1/2))*(c*d)^(1/2)*b^2*c*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellip
```

ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*d*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*x^2+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c-4*a*b*d^2*x^3*(a*b)^(1/2)+4*b^2*c*d*x^3*(a*b)^(1/2)+4*a*b*c*d*x*(a*b)^(1/2)-4*b^2*c^2*x*(a*b)^(1/2))/(-d*x^2+c)^(1/2)/a/(e*x)^(1/2)/(a*b)^(1/2)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/(-b*x^2+a)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(-1/2)*integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - dx^2}}{\sqrt{ex}\,(-a + bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
[Out] Integral(sqrt(c - d*x**2)/(sqrt(e*x)*(-a + b*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)*e^(-1/2)/((b*x^2 - a)^2*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c - d\,x^2}}{\sqrt{e\,x}\,(a - b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/((e*x)^(1/2)*(a - b*x^2)^2),x)
[Out] int((c - d*x^2)^(1/2)/((e*x)^(1/2)*(a - b*x^2)^2), x)
```

3.901 $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

Optimal. Leaf size=444

$$-\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}}+\frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)}-\frac{5c^{3/4}\sqrt[4]{d}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}+\frac{5c^{3/4}\sqrt[4]{d}\,\sqrt{1-\frac{dx^2}{c}}}{2a}$$

```
[Out] -5/2*(-d*x^2+c)^(1/2)/a^2/e/(e*x)^(1/2)+1/2*(-d*x^2+c)^(1/2)/a/e/(-b*x^2+a)
/(e*x)^(1/2)-5/2*c^(3/4)*d^(1/4)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1
/2),I)*(1-d*x^2/c)^(1/2)/a^2/e^(3/2)/(-d*x^2+c)^(1/2)+5/2*c^(3/4)*d^(1/4)*E
llipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/e^(3/
2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+5*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^
(5/2)/d^(1/4)/e^(3/2)/b^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-3*a*d+5*b*c)*E
llipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/
2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(3/2)/b^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 480, 597, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(5bc-3ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt{b}\,\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(5bc-3ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt{b}\,\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}}+\frac{5c^{3/4}\sqrt[4]{d}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}-\frac{5c^{3/4}\sqrt[4]{d}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}-\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}}+\frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] (-5*Sqrt[c - d*x^2])/(2*a^2*e*Sqrt[e*x]) + Sqrt[c - d*x^2]/(2*a*e*Sqrt[e*x]
*(a - b*x^2)) - (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*e^(3/2)*Sqrt[c - d*x^2]) +
 (5*c^(3/4)*d^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*
c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*Sqrt[
b]*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*Sqrt[b]*d^(1/4)*e^(3/2)*Sqrt[c
- d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^2\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)} - \frac{\text{Subst}\left(\int \frac{-5c+\frac{3dx^4}{e^2}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ae}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)} + \frac{\text{Subst}\left(\int \frac{x^2\left(\frac{c(5bc-8ad)}{e^2}+\frac{5bcdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a^2ce}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)} + \frac{\text{Subst}\left(\int \left(-\frac{5cdx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}} + \frac{(5bc^2-3ac}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c}}\right.\right.}{2a^2ce}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)} - \frac{(5d)\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a^2e^3} +$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)} + \frac{\left(5\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{\right.}{2a^2e^2}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)} + \frac{\left(5\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\right.}}{2a^2e^2\sqrt{c-dx^2}}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)} + \frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{2a^2e^{3/2}\sqrt{c-dx^2}}$$

$$= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}} + \frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)} - \frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{2a^2e^{3/2}\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.14, size = 182, normalized size = 0.41

$$\frac{x\left(21a(4a-5bx^2)\left(c-dx^2\right)+7(-5bc+8ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+15bdx^4(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a^3(ex)^{3/2}\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*(21*a*(4*a - 5*b*x^2)*(c - d*x^2) + 7*(-5*b*c + 8*a*d)*x^2*(a - b*x^2)*S
qrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 15*b*
d*x^4*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/
c, (b*x^2)/a]))/(42*a^3*(e*x)^(3/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2555 vs. 2(328) = 656.
time = 0.13, size = 2556, normalized size = 5.76

method	result
elliptic	$\sqrt{\left(-d\,x^2+c\right)ex}\left(\frac{bx\sqrt{-de\,x^3+cex}}{2a^2e^2\left(-b\,x^2+a\right)}-\frac{2\left(-de\,x^2+ce\right)}{e^2a^2\sqrt{x\left(-de\,x^2+ce\right)}}+\frac{5c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{\cdots}}{2a^2e\sqrt{-\cdots}}\cdots\right.$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d*(16*a^2*b*d^2*x^2+4*a*b^2*c*d*x^2-16*b*c*d*a^2-5*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+10*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a*b^2*c^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*
2^(1/2))*a*b^2*c^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
```

c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^2*x^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^2*x^2-20*b^3*c^2*x^2-20*a*b^2*d^2*x^4+20*b^3*c*d*x^4+16*b^2*c^2*a-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^

```
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*
(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c*x^2+5*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*b^2*c*x^2-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^2+3*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(-3/2)*integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*x^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c - dx^2}}{(ex)^{\frac{3}{2}}\,(-a + bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
[Out] Integral(sqrt(c - d*x**2)/((e*x)**(3/2)*(-a + b*x**2)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)*e^(-3/2)/((b*x^2 - a)^2*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-d\,x^2}}{{\left(e\,x\right)}^{3/2}\,{\left(a-b\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/((e*x)^(3/2)*(a - b*x^2)^2),x)
[Out] int((c - d*x^2)^(1/2)/((e*x)^(3/2)*(a - b*x^2)^2), x)
```

3.902 $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

Optimal. Leaf size=355

$$-\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}}+\frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)}+\frac{7\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{6a^2e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(7bc-5ad)}{}$$

```
[Out] -7/6*(-d*x^2+c)^(1/2)/a^2/e/(e*x)^(3/2)+1/2*(-d*x^2+c)^(1/2)/a/e/(e*x)^(3/2
)/(-b*x^2+a)+7/6*c^(1/4)*d^(3/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1
/2),I)*(1-d*x^2/c)^(1/2)/a^2/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-5*a*d+7
*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/
2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*c^
(1/4)*(-5*a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)
*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/e^(5/2)/(-d*x^2+c
)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 480, 597, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}}+\frac{7\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2e^{5/2}\sqrt{c-dx^2}}-\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}}+\frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] (-7*Sqrt[c - d*x^2])/(6*a^2*e*(e*x)^(3/2)) + Sqrt[c - d*x^2]/(2*a*e*(e*x)^(
3/2)*(a - b*x^2)) + (7*c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*e^(5/2)*Sqrt[c - d*x^2
]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*a^3*d^(1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(7*b*c - 5*a*d)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*e^(5/2)*Sqrt[c - d*x
^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{\sqrt{c-\frac{dx^4}{e^2}}}{x^4\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} - \frac{\text{Subst}\left(\int \frac{-7c+\frac{5dx^4}{e^2}}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ae}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{c(21bc-8ad)}{e^2}-\frac{7bcdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right.}{6a^2ce}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{(7d)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{6a^2e^3}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{(7bc-5ad)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\frac{x^2}{e}\right)\sqrt{c-\frac{d}{e}}}\right.}{4a^3e^3}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{7\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{6a^2e^{5/2}\sqrt{c-dx^2}}$$

$$= -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}} + \frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)} + \frac{7\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{6a^2e^{5/2}\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.15, size = 181, normalized size = 0.51

$$\frac{x\left(5a(4a-7bx^2)\left(c-dx^2\right)+5(-21bc+8ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+7bdx^4(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{30a^3(ex)^{5/2}\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*(5*a*(4*a - 7*b*x^2)*(c - d*x^2) + 5*(-21*b*c + 8*a*d)*x^2*(a - b*x^2)*S
qrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 7*b*d
*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c,
(b*x^2)/a]))/(30*a^3*(e*x)^(5/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2303 vs. $2(267) = 534$.
time = 0.13, size = 2304, normalized size = 6.49

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(\frac{b\sqrt{-de\,x^3+cex}}{2e^3a^2\left(-b\,x^2+a\right)}-\frac{2\sqrt{-de\,x^3+cex}}{3e^3a^2x^2}+\frac{7\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\cdots}}{12a^2e^2\sqrt{-de\,x\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*b*d*(21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^3*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^3*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)+12*a*b*c*d*x^2*(a*b)^(1/2)+28*b^2*c*d*x^4*(a*b)^(1/2
)+16*a*b*c^2*(a*b)^(1/2)+16*a^2*d^2*x^2*(a*b)^(1/2)-28*b^2*c^2*x^2*(a*b)^(1
/2)-16*a^2*c*d*(a*b)^(1/2)-28*a*b*d^2*x^4*(a*b)^(1/2)-21*2^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(
1/2)*d),1/2*2^(1/2))*b^2*c*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)
```

^(1/2)+15*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^3*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*x^3*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+14*2^(1/2)*EllipticF(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*
(c*d)^(1/2)*(a*b)^(1/2)+15*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d
x((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)-15*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2
*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-15*2^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-15*2^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)
*(a*b)^(1/2)+14*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*b^2*c*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-15*2
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^3*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*x*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*x*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)-14*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a*b*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+1
5*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*d*x^3*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)*(c*d)^(1/2)*(a*b)^(1/2)+15*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*d*
x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)-14*2^(1/2)*EllipticF
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c*x*((d*x+(c*d)^(1/

```
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2
))*a*b*c*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)*(a*b)^(1/2)+21*2^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)
*(a*b)^(1/2))/(-d*x^2+c)^(1/2)/x/a^2/e^2/(e*x)^(1/2)/(a*b)^(1/2)/((c*d)^(1/
2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(-5/2)*integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*x^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{\frac{5}{2}}\left(-a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)
[Out] Integral(sqrt(c - d*x**2)/((e*x)**(5/2)*(-a + b*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)*e^(-5/2)/((b*x^2 - a)^2*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-d\,x^2}}{{\left(e\,x\right)}^{5/2}\,{\left(a-b\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(1/2)/((e*x)^(5/2)*(a - b*x^2)^2),x)
[Out] int((c - d*x^2)^(1/2)/((e*x)^(5/2)*(a - b*x^2)^2), x)
```

3.903 $\int \frac{(ex)^{7/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=429

$$\frac{(57bc-77ad)e^3\sqrt{ex}\,\sqrt{c-dx^2}}{42b^3}-\frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2}+\frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)}+\frac{\sqrt[4]{c}\left(48b^2c^2-259abcd+2\right.}{}$$

```
[Out] 1/2*e*(e*x)^(5/2)*(-d*x^2+c)^(3/2)/b/(-b*x^2+a)-11/14*d*e*(e*x)^(5/2)*(-d*x
^2+c)^(1/2)/b^2+1/42*(-77*a*d+57*b*c)*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^3+
1/42*c^(1/4)*(231*a^2*d^2-259*a*b*c*d+48*b^2*c^2)*e^(7/2)*EllipticF(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^4/d^(1/4)/(-d*x^2+c)^(1
/2)-1/4*c^(1/4)*(-11*a*d+5*b*c)*(-a*d+b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2
)/b^4/d^(1/4)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-11*a*d+5*b*c)*(-a*d+b*c)*e^(7/
2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d
^(1/2),I)*(1-d*x^2/c)^(1/2)/b^4/d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 478, 595, 596, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\left(231a^2d^2-259abcd+48b^2c^2\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{42b^4\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(5bc-11ad)(bc-ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b^4\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(5bc-11ad)(bc-ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b^4\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{e^3\sqrt{ex}\,\sqrt{c-dx^2}\,(57bc-77ad)}{42b^3}+\frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)}-\frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] ((57*b*c - 77*a*d)*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(42*b^3) - (11*d*e*(e*x)^
(5/2)*Sqrt[c - d*x^2])/(14*b^2) + (e*(e*x)^(5/2)*(c - d*x^2)^(3/2))/(2*b*(a
 - b*x^2)) + (c^(1/4)*(48*b^2*c^2 - 259*a*b*c*d + 231*a^2*d^2)*e^(7/2)*Sqrt
[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(42*b^4*d^(1/4)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)
*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d^(1/4)*Sqr
t[c - d*x^2]) - (c^(1/4)*(5*b*c - 11*a*d)*(b*c - a*d)*e^(7/2)*Sqrt[1 - (d*x
^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^4*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
```

```
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^8\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{e\text{Subst}\left(\int \frac{x^4\left(5c-\frac{11dx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= -\frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{e^3\text{Subst}\left(\int \frac{x^4\left(-\frac{5c(7bc-11ad)}{e^2}+\frac{d(}{}\right.}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-}}\right.}{14b^2}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\,\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/}}{2b\left(a-bx^2\right)}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\,\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/}}{2b\left(a-bx^2\right)}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\,\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/}}{2b\left(a-bx^2\right)}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\,\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/}}{2b\left(a-bx^2\right)}$$

$$= \frac{(57bc-77ad)e^3\sqrt{ex}\,\sqrt{c-dx^2}}{42b^3} - \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}\left(c-dx^2\right)^{3/}}{2b\left(a-bx^2\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order

4 in optimal.
time = 10.21, size = 233, normalized size = 0.54

$$\frac{e^3\sqrt{ex}\left(5a(c-dx^2)\left(77a^2d-12b^2x^2(-3c+dx^2)-ab(57c+44dx^2)\right)-5ac(-57bc+77ad)\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+\left(48b^2c^2-259abcd+231a^2d^2\right)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{210ab^3\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (e^3*Sqrt[e*x]*(5*a*(c - d*x^2)*(77*a^2*d - 12*b^2*x^2*(-3*c + d*x^2) - a*b
*(57*c + 44*d*x^2)) - 5*a*c*(-57*b*c + 77*a*d)*(a - b*x^2)*Sqrt[1 - (d*x^2)
/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (48*b^2*c^2 - 259*a*
b*c*d + 231*a^2*d^2)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2,
 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(210*a*b^3*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3777 vs. 2(335) = 670.
time = 0.17, size = 3778, normalized size = 8.81

method	result	size
risch	Expression too large to display	1326
elliptic	Expression too large to display	1357
default	Expression too large to display	3778

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/168*e^3*(e*x)^(1/2)/b^3*(368*a*b^3*c*d^3*x^5*(a*b)^(1/2)-360*a^2*b^2*c*d
^3*x^3*(a*b)^(1/2)-92*a*b^3*c^2*d^2*x^3*(a*b)^(1/2)-308*a^3*b*c*d^3*x*(a*b)
^(1/2)+536*a^2*b^2*c^2*d^2*x*(a*b)^(1/2)-176*a^2*b^2*d^4*x^5*(a*b)^(1/2)-19
2*b^4*c^2*d^2*x^5*(a*b)^(1/2)-48*a*b^3*d^4*x^7*(a*b)^(1/2)+48*b^4*c*d^3*x^7
*(a*b)^(1/2)+980*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(
1/2)-614*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/
2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-336*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-336*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
```

a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+144*b^4*c^3*d*x^3*(a*b)^(1/2)+336*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+308*a^3*b*d^4*x^3*(a*b)^(1/2)-228*a*b^3*c^3*d*x*(a*b)^(1/2)+96*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^4*c^3*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b^2*c*d^3*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+336*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^3*c^2*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^3*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b^2*c*d^3*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-336*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^3*c^2*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+105*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^3*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-96*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^3*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-462*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*b*d^3*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+231*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*d^3*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+231*EllipticPi(

```
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*d^3*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^
(1/2)*(c*d)^(1/2)-980*EllipticF(((d*x+(c*d)^(1/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(7/2)*integrate((-d*x^2 + c)^(3/2)*x^(7/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((-d*x^2 + c)^(3/2)*x^(7/2)*e^(7/2)/(b*x^2 - a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{7/2}\,(c-d\,x^2)^{3/2}}{(a-b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x)
[Out] int(((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2, x)
```

3.904 $\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=485

$$-\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2}+\frac{e(ex)^{3/2}\,(c-dx^2)^{3/2}}{2b\,(a-bx^2)}-\frac{3c^{3/4}\sqrt[4]{d}\,(11bc-15ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{10b^3\sqrt{c-dx^2}}$$

```
[Out] 1/2*e*(e*x)^(3/2)*(-d*x^2+c)^(3/2)/b/(-b*x^2+a)-9/10*d*e*(e*x)^(3/2)*(-d*x^
2+c)^(1/2)/b^2-3/10*c^(3/4)*d^(1/4)*(-15*a*d+11*b*c)*e^(5/2)*EllipticE(d^(1
/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/(-d*x^2+c)^(1/2)+3
/10*c^(3/4)*d^(1/4)*(-15*a*d+11*b*c)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/(-d*x^2+c)^(1/2)+3/4*c^(1/4)*(3*a^
2*d^2-4*a*b*c*d+b^2*c^2)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(7/2)/d^(1/4)/
a^(1/2)/(-d*x^2+c)^(1/2)-3/4*c^(1/4)*(3*a^2*d^2-4*a*b*c*d+b^2*c^2)*e^(5/2)*
EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1
/2),I)*(1-d*x^2/c)^(1/2)/b^(7/2)/d^(1/4)/a^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 478, 595, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{3\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(3a^2d^2-4abcd+b^2c^2)\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\,b^{7/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{3\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(3a^2d^2-4abcd+b^2c^2)\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\,b^{7/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{3c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(11bc-15ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{10b^3\sqrt{c-dx^2}}-\frac{3c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(11bc-15ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{10b^3\sqrt{c-dx^2}}+\frac{e(ex)^{3/2}\,(c-dx^2)^{3/2}}{2b\,(a-bx^2)}-\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (-9*d*e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(10*b^2) + (e*(e*x)^(3/2)*(c - d*x^2)^
(3/2))/(2*b*(a - b*x^2)) - (3*c^(3/4)*d^(1/4)*(11*b*c - 15*a*d)*e^(5/2)*Sqr
t[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(10*b^3*Sqrt[c - d*x^2]) + (3*c^(3/4)*d^(1/4)*(11*b*c - 15*a*d)*e^(5/2)
*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(10*b^3*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*a^2*
d^2)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sq
rt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(
7/2)*d^(1/4)*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2
)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(7/2)*d
^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^6\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx,x,\sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{e\text{Subst}\left(\int \frac{x^2\left(3c-\frac{9dx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx,x,\sqrt{ex}\right)}{2b}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{e^3\text{Subst}\left(\int \frac{x^2\left(-\frac{3c(5bc-9ad)}{e^2}+\frac{3d(11}{}\right.}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-}}\right.}{10b^2}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{e^3\text{Subst}\left(\int\left(-\frac{3d(11bc-15ad)x^2}{be^2\sqrt{c-\frac{dx^4}{e^2}}}\right.\right.}{}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{(3d(11bc-15ad)e)\text{Subst}\left(\int -\right.}{10b^3}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{\left(3\sqrt{c}\sqrt{d}(11bc-15ad)e^2\right)\text{S}}{}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{\left(3\sqrt{c}\sqrt{d}(11bc-15ad)e^2\sqrt{}\right.}{}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{3c^{3/4}\sqrt[4]{d}(11bc-15ad)e^{5/2}\sqrt{1}}{10b}$$

$$= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2} + \frac{e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{3c^{3/4}\sqrt[4]{d}(11bc-15ad)e^{5/2}\sqrt{1}}{10b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.21, size = 196, normalized size = 0.40

$$\frac{e(ex)^{3/2}\left(7a(c-dx^2)\left(-5bc+9ad-4bdx^2\right)-7c(-5bc+9ad)\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3d(-11bc+15ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{70ab^2\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (e*(e*x)^(3/2)*(7*a*(c - d*x^2)*(-5*b*c + 9*a*d - 4*b*d*x^2) - 7*c*(-5*b*c
+ 9*a*d)*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)
/c, (b*x^2)/a] + 3*d*(-11*b*c + 15*a*d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]
*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(70*a*b^2*(-a + b*x^2)
*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3873 vs. $2(369) = 738$.
time = 0.20, size = 3874, normalized size = 7.99

method	result	size
elliptic	Expression too large to display	1338
risch	Expression too large to display	1408
default	Expression too large to display	3874

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*e^2*(e*x)^(1/2)*d*(15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/
2)*d),1/2*2^(1/2))*a*b^3*c^3-16*a*b^3*d^3*x^6+16*b^4*c*d^2*x^6+36*a^2*b^2*d
^3*x^4+4*b^4*c^2*d*x^4+132*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^3*x^2-66*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*b^4*c^3*x^2-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^
2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-20*b^4*c^3*x^2+66*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b
^3*c^3+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
```

```
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))
*a*b^3*c^3-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1
/2))*b^4*c^3*x^2-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*b^4*c^3*x^2-132*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3-40*a*b^3*c*d^2*x^
4-36*a^2*b^2*c*d^2*x^2-60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*
x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+60*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)+56*a*b^3*c^2*d*x^2+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^
3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1
/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-60*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^
(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+45*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)-60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b
^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/
2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+45*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1
/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)-45*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d
```

```
)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+60*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2
)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(5/2)*integrate((-d*x^2 + c)^(3/2)*x^(5/2)/(b*x^2 - a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((-d*x^2 + c)^(3/2)*x^(5/2)*e^(5/2)/(b*x^2 - a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}\,(c-d\,x^2)^{3/2}}{(a-b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x)
[Out] int(((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2, x)
```

3.905 $\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=381

$$-\frac{7de\sqrt{ex}\,\sqrt{c-dx^2}}{6b^2}+\frac{e\sqrt{ex}\,\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)}-\frac{\sqrt[4]{c}\,d^{3/4}(17bc-21ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-\right.}{6b^3\sqrt{c-dx^2}}$$

```
[Out] 1/2*e*(-d*x^2+c)^(3/2)*(e*x)^(1/2)/b/(-b*x^2+a)-7/6*d*e*(e*x)^(1/2)*(-d*x^2
+c)^(1/2)/b^2-1/6*c^(1/4)*d^(3/4)*(-21*a*d+17*b*c)*e^(3/2)*EllipticF(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/(-d*x^2+c)^(1/2)-1/4
*c^(1/4)*(-7*a*d+b*c)*(-a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b^3/d^
(1/4)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-7*a*d+b*c)*(-a*d+b*c)*e^(3/2)*Elliptic
Pi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(
1-d*x^2/c)^(1/2)/a/b^3/d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 478, 542, 537, 230, 227, 418, 1233, 1232}

$$-\frac{\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(17bc-21ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{6b^3\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-7ad)(bc-ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4ab^3\sqrt[4]{d}\,\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(bc-7ad)(bc-ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4ab^3\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{e\sqrt{ex}\,\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)}-\frac{7de\sqrt{ex}\,\sqrt{c-dx^2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (-7*d*e*Sqrt[e*x]*Sqrt[c - d*x^2])/(6*b^2) + (e*Sqrt[e*x]*(c - d*x^2)^(3/2)
)/(2*b*(a - b*x^2)) - (c^(1/4)*d^(3/4)*(17*b*c - 21*a*d)*e^(3/2)*Sqrt[1 - (
d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*
b^3*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2]) -
(c^(1/4)*(b*c - 7*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(
Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(4*a*b^3*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{e\text{Subst}\left(\int \frac{\left(c-\frac{7dx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}{a-\frac{bx^4}{e^2}}\,dx, x, \sqrt{ex}\right)}{2b}$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} + \frac{e^3\text{Subst}\left(\int \frac{-\frac{c(3bc-7ad)}{e^2}+\frac{d(17bc-21ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{6b^2}$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{(d(17bc-21ad)e)\text{Subst}\left(\int \frac{}{\sqrt{c-}}\right.}{6b^3}$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{((bc-7ad)(bc-ad)e)\text{Subst}\left(\int \right.}{4}$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{\sqrt[4]{c}\,d^{3/4}(17bc-21ad)e^{3/2}\sqrt{1-}}{6b^3\sqrt{c}}$$

$$= -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2} + \frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{\sqrt[4]{c}\,d^{3/4}(17bc-21ad)e^{3/2}\sqrt{1-}}{6b^3\sqrt{c}}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.20, size = 195, normalized size = 0.51

$$\frac{e\sqrt{ex}\left(5a\left(c-dx^2\right)\left(-3bc+7ad-4bdx^2\right)-5c(-3bc+7ad)\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+d(-17bc+21ad)x^2\left(a-bx^2\right)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{30ab^2\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (e*Sqrt[e*x]*(5*a*(c - d*x^2)*(-3*b*c + 7*a*d - 4*b*d*x^2) - 5*c*(-3*b*c +
7*a*d)*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c
, (b*x^2)/a] + d*(-17*b*c + 21*a*d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*App
ellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(30*a*b^2*(-a + b*x^2)*Sqrt
[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3453 vs. $2(293) = 586$.
time = 0.16, size = 3454, normalized size = 9.07

method	result	size
elliptic	Expression too large to display	1194
risch	Expression too large to display	1291
default	Expression too large to display	3454

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*e*(e*x)^(1/2)/b^2*d*(-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*a*b^3*c^3-34*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)+16*b^3*c*d^2*x^5*(a*b)^(1/2)+28*a^2*b*d^3*x^3*(a*b)^(1/2)-4
*b^3*c^2*d*x^3*(a*b)^(1/2)-12*b^3*c^3*x*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3-3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^2+3*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^
2-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
```

))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+76*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-16*a*b^2*d^3*x^5*(a*b)^(1/2)-24*a*b^2*c*d^2*x^3*(a*b)^(1/2)-28*a^2*b*c*d^2*x*(a*b)^(1/2)+40*a*b^2*c^2*d*x*(a*b)^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+34*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-24*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-42*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(3/2)*integrate((-d*x^2 + c)^(3/2)*x^(3/2)/(b*x^2 - a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}} (c - dx^2)^{\frac{3}{2}}}{(-a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
[Out] Integral((e*x)**(3/2)*(c - d*x**2)**(3/2)/(-a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((-d*x^2 + c)^(3/2)*x^(3/2)*e^(3/2)/(b*x^2 - a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{3/2}\,(c - d\,x^2)^{3/2}}{(a - b\,x^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x)
[Out] int(((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2, x)
```

3.906 $\int \frac{\sqrt{ex}\,(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=474

$$\frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\,(a-bx^2)}-\frac{c^{3/4}\sqrt[4]{d}\,(bc-5ad)\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\,(bc-}{}$$

```
[Out] 1/2*(-a*d+b*c)*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/a/b/e/(-b*x^2+a)-1/2*c^(3/4)*d^
(1/4)*(-5*a*d+b*c)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)
*(1-d*x^2/c)^(1/2)/a/b^2/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*(-5*a*d+b*c)*
EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/
a/b^2/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-5*a^2*d^2+4*a*b*c*d+b^2*c^2)*EllipticP
i(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e
^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^(5/2)/d^(1/4)/(-d*x^2+c)^(1/2)+1/4*c^(1/
4)*(-5*a^2*d^2+4*a*b*c*d+b^2*c^2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^
(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/
b^(5/2)/d^(1/4)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {477, 479, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(-5a^2d^2+4abcd+b^2c^2)\,\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{5/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(-5a^2d^2+4abcd+b^2c^2)\,\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}b^{5/2}\sqrt[4]{d}\,\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-5ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-5ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}}+\frac{(ex)^{3/2}\sqrt{c-dx^2}\,(bc-ad)}{2abe\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) - (c^(3/4)*
d^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[c - d*x^2]) + (c^(3/4)*d
^(1/4)*(b*c - 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b
^2*c^2 + 4*a*b*c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(4*a^(3/2)*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b^2*c^
2 + 4*a*b*c*d - 5*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(4*a^(3/2)*b^(5/2)*d^(1/4)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\,(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\,(a-bx^2)} + \frac{e\text{Subst}\left(\int \frac{x^2\left(\frac{c(bc+3ad)}{e^2}+\frac{d(bc-5ad)x^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab}$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\,(a-bx^2)} + \frac{e\text{Subst}\left(\int \left(-\frac{d(bc-5ad)x^2}{be^2\sqrt{c-\frac{dx^4}{e^2}}} + \frac{(b^2c^2+4abcd-5a^2d^2)x\cdots}{be^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\cdots}}\cdots\right.\right.}{2ab}\cdots$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\,(a-bx^2)} - \frac{(d(bc-5ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab^2e}\cdots$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\,(a-bx^2)} + \frac{\left(\sqrt{c}\,\sqrt{d}\,(bc-5ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \cdots\right.}{2ab^2}\cdots$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\,(a-bx^2)} + \frac{\left(\sqrt{c}\,\sqrt{d}\,(bc-5ad)\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt{1-\cdots}}\cdots\right.}{2ab^2\sqrt{c-dx^2}}\cdots$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\,(a-bx^2)} + \frac{c^{3/4}\sqrt[4]{d}\,(bc-5ad)\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\cdots}{\sqrt[4]{c}\cdots}\right.\right.}{2ab^2\sqrt{c-dx^2}}\cdots$$

$$= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\,(a-bx^2)} - \frac{c^{3/4}\sqrt[4]{d}\,(bc-5ad)\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\cdots}{\sqrt[4]{c}\cdots}\right.\right.}{2ab^2\sqrt{c-dx^2}}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order

4 in optimal.
time = 10.19, size = 189, normalized size = 0.40

$$\frac{\sqrt{ex}\left(21a(-bc+ad)x(c-dx^2)+7c(bc+3ad)x(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3d(bc-5ad)x^3(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a^2b\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

```
[Out] (Sqrt[e*x]*(21*a*(-(b*c) + a*d)*x*(c - d*x^2) + 7*c*(b*c + 3*a*d)*x*(-a + b
*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a]
+ 3*d*(b*c - 5*a*d)*x^3*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2,
 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^2*b*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3845 vs. $2(364) = 728$.
time = 0.14, size = 3846, normalized size = 8.11

method	result	size
elliptic	Expression too large to display	1298
default	Expression too large to display	3846

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(e*x)^(1/2)*d*(-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*a*b^3*c^3+4*a^2*b^2*d^3*x^4+4*b^4*c^2*d*x^4+4*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*b^4*c^3*x^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^3*x^2-5*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*b^4*c^3*x^2
+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*
b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a
```

*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
2^(1/2)((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3-8
*a*b^3*c*d^2*x^4-4*a^2*b^2*c*d^2*x^2-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/
2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)+8*a*b^3*c^2*d*x^2+5*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2
^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2
^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c
d)^(1/2))^(1/2)((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c
d)^(1/2)((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/
2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*

```
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(1/2)*integrate((-d*x^2 + c)^(3/2)*sqrt(x)/(b*x^2 - a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,(c-dx^2)^{\frac{3}{2}}}{(-a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
[Out] Integral(sqrt(e*x)*(c - d*x**2)**(3/2)/(-a + b*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((-d*x^2 + c)^(3/2)*sqrt(x)*e^(1/2)/(b*x^2 - a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}\,\left(c-d\,x^2\right)^{3/2}}{\left(a-b\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x)
[Out] int(((e*x)^(1/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2, x)
```

3.907 $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx$

Optimal. Leaf size=366

$$\frac{(bc-ad)\sqrt{ex}\,\sqrt{c-dx^2}}{2abe\,(a-bx^2)}+\frac{\sqrt[4]{c}\,d^{3/4}(bc+3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{e}\,\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}\,(bc-ad)(bc+$$

```
[Out] 1/2*(-a*d+b*c)*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/a/b/e/(-b*x^2+a)+1/2*c^(1/4)*d^
(3/4)*(3*a*d+b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2
/c)^(1/2)/a/b^2/e^(1/2)/(-d*x^2+c)^(1/2)+3/4*c^(1/4)*(-a*d+b*c)*(a*d+b*c)*E
llipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1
/2),I)*(1-d*x^2/c)^(1/2)/a^2/b^2/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)+3/4*c^(1/
4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1
/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b^2/d^(1/4)/e^(1/2)/(-
d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 424, 537, 230, 227, 418, 1233, 1232}

$$\frac{3\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)(bc-ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2b^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)(bc-ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2b^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,(3ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt{ex}\,\sqrt{c-dx^2}\,(bc-ad)}{2abe\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*b*e*(a - b*x^2)) + (c^(1/4)*d^
(3/4)*(b*c + 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*a*b^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*c^(1/4)*(
b*c - a*d)*(b*c + a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a
^2*b^2*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b*c - a*d)*(b*c + a*d
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*b^2*d^(1/4)*Sqrt[e]*S
qrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(bc-ad)\sqrt{ex}\,\sqrt{c-dx^2}}{2abe\,(a-bx^2)} - \frac{e\text{Subst}\left(\int \frac{-\frac{c(3bc+ad)}{e^2}+\frac{d(bc+3ad)x^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab}$$

$$= \frac{(bc-ad)\sqrt{ex}\,\sqrt{c-dx^2}}{2abe\,(a-bx^2)} + \frac{(3(bc-ad)(bc+ad))\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,\right.}{2ab^2e}$$

$$= \frac{(bc-ad)\sqrt{ex}\,\sqrt{c-dx^2}}{2abe\,(a-bx^2)} + \frac{(3(bc-ad)(bc+ad))\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{4a^2b^2e}$$

$$= \frac{(bc-ad)\sqrt{ex}\,\sqrt{c-dx^2}}{2abe\,(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}(bc+3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right|}{2ab^2\sqrt{e}\,\sqrt{c-dx^2}}$$

$$= \frac{(bc-ad)\sqrt{ex}\,\sqrt{c-dx^2}}{2abe\,(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}(bc+3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right|}{2ab^2\sqrt{e}\,\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.15, size = 187, normalized size = 0.51

$$\frac{5a(-bc+ad)x(c-dx^2)+5c(3bc+ad)x(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+d(bc+3ad)x^3(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{10a^2b\sqrt{ex}\,(-a+bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

```
[Out] (5*a*(-(b*c) + a*d)*x*(c - d*x^2) + 5*c*(3*b*c + a*d)*x*(-a + b*x^2)*Sqrt[1
 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*(b*c + 3
*a*d)*x^3*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2
)/c, (b*x^2)/a])/(10*a^2*b*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(284) = 568$.
time = 0.12, size = 2519, normalized size = 6.88

method	result
elliptic	$\frac{\sqrt{(-dx^2+c)ex}\left(-\frac{(ad-bc)\sqrt{-dex^3+cex}}{2aeb(-bx^2+a)}+\frac{3d\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\ \mathrm{Elliptic}}{4\sqrt{-dex^3+cex}\ b^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b*d*(3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*a*b^3*c^3+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*2^(1/2)*b^3*c^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*a
^2*b*d^3*x^3*(a*b)^(1/2)+4*b^3*c^2*d*x^3*(a*b)^(1/2)-4*b^3*c^3*x*(a*b)^(1/2
)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^
(1/2))^(1/2)-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(
1/2))*a*b^3*c^3+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*
2^(1/2))*b^4*c^3*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
,1/2*2^(1/2))*b^4*c^3*x^2+4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
```

,1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-8*a*b^2*c*d^2*x^3*(a*b)^(1/2)-4*a^2*b*c*d^2*x*(a*b)^(1/2)+8*a*b^2*c^2*d*x*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2

```
)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(-1/2)*integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c - dx^2)^{\frac{3}{2}}}{\sqrt{ex}\,(-a + bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
[Out] Integral((c - d*x**2)**(3/2)/(sqrt(e*x)*(-a + b*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((-d*x^2 + c)^(3/2)*e^(-1/2)/((b*x^2 - a)^2*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c - d\,x^2)^{3/2}}{\sqrt{e\,x}\,(a - b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(3/2)/((e*x)^(1/2)*(a - b*x^2)^2),x)
[Out] int((c - d*x^2)^(3/2)/((e*x)^(1/2)*(a - b*x^2)^2), x)
```

3.908 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

Optimal. Leaf size=519

$$-\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}}+\frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)}-\frac{c^{3/4}\sqrt[4]{d}\,(5bc-ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}}$$

```
[Out] -1/2*(-a*d+5*b*c)*(-d*x^2+c)^(1/2)/a^2/b/e/(e*x)^(1/2)+1/2*(-a*d+b*c)*(-d*x
^2+c)^(1/2)/a/b/e/(-b*x^2+a)/(e*x)^(1/2)-1/2*c^(3/4)*d^(1/4)*(-a*d+5*b*c)*E
llipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b/e^(
3/2)/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*(-a*d+5*b*c)*EllipticF(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b/e^(3/2)/(-d*x^2+c)^(1/
2)-1/4*c^(1/4)*(-a^2*d^2-4*a*b*c*d+5*b^2*c^2)*EllipticPi(d^(1/4)*(e*x)^(1/2
)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(
5/2)/b^(3/2)/d^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a^2*d^2-4*a*b*c
*d+5*b^2*c^2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2
)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/b^(3/2)/d^(1/4)/e^(3/2)/(-d*
x^2+c)^(1/2)
```

Rubi [A]

time = 0.76, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 479, 597, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}}-\frac{\sqrt{c-dx^2}(5bc-ad)}{2a^2be\sqrt{ex}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(-a^2d^2-4abcd+5b^2c^2)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}b^{3/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(-a^2d^2-4abcd+5b^2c^2)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}b^{3/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt{c-dx^2}(bc-ad)}{2abe\sqrt{ex}\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] -1/2*((5*b*c - a*d)*Sqrt[c - d*x^2])/(a^2*b*e*Sqrt[e*x]) + ((b*c - a*d)*Sqr
t[c - d*x^2])/(2*a*b*e*Sqrt[e*x]*(a - b*x^2)) - (c^(3/4)*d^(1/4)*(5*b*c - a
*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*a^2*b*e^(3/2)*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(5*b*c - a*
d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(2*a^2*b*e^(3/2)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b^2*c^2 - 4*a*b*c
*d - a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*b^
(3/2)*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b^2*c^2 - 4*a*b*c*d -
a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
, ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*b^(3/2)*d^
(1/4)*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
```

s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^2\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} + \frac{e\text{Subst}\left(\int \frac{\frac{c(5bc-ad)}{e^2}-\frac{d(3bc+ad)x^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} - \frac{e\text{Subst}\left(\int \frac{x^2\left(-\frac{bc^2(5bc-9ad)}{e^4}-\frac{bcd(5bc-}{e^6}\right.}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{2a^2bc}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} - \frac{e\text{Subst}\left(\int \left(\frac{cd(5bc-ad)x^2}{e^4\sqrt{c-\frac{dx^4}{e^2}}} - \frac{(5b}{e^4(}\right.\right.}{2a}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} - \frac{(d(5bc-ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx}{e}}}\right.}{2a^2be^3}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} + \frac{\left(\sqrt{c}\sqrt{d}\,(5bc-ad)\right)\text{Subst}\left(\int\right.}{2a^2be^2}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} + \frac{\left(\sqrt{c}\sqrt{d}\,(5bc-ad)\sqrt{1-\frac{dx^2}{c}}\right.}{2a^2be}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} + \frac{c^{3/4}\sqrt[4]{d}\,(5bc-ad)\sqrt{1-\frac{dx^2}{c}}\,F(}{2a^2be^{3/2}\sqrt{c}}$$

$$= -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} - \frac{c^{3/4}\sqrt[4]{d}\,(5bc-ad)\sqrt{1-\frac{dx^2}{c}}\,E}{2a^2be^{3/2}\sqrt{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.18, size = 197, normalized size = 0.38

$$\frac{x\left(21a(c-dx^2)\left(4ac-5bcx^2+adx^2\right)+7c(-5bc+9ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3d(-5bc+ad)x^4(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a^3(ex)^{3/2}\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*(21*a*(c - d*x^2)*(4*a*c - 5*b*c*x^2 + a*d*x^2) + 7*c*(-5*b*c + 9*a*d)*x
^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b
*x^2)/a] + 3*d*(-5*b*c + a*d)*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[
7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^3*(e*x)^(3/2)*(-a + b*x^2)
*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3866 vs. $2(403) = 806$.
time = 0.13, size = 3867, normalized size = 7.45

method	result	size
elliptic	Expression too large to display	1345
default	Expression too large to display	3867

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d*(-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*a*b^3*c^3+16*a*b^3*c^3-16*a^2*b^2*c^2*d+4*a^2*b^2*d^3*x^4+20*b^4*c^2*d*x^
4+20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))*b^4*c^3*x^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^3*x^2-E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)-20*b^4*c^3*x^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3-5*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3+5*((d*x+(c*d)^(
```

1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^2+5*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^2-20*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*a*b^3*c^3-24*a*b^3*c*d^2*x^4+12*a^2*b^2*c*d^2*x^2+4*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)-4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d
x^2(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+8*a*b^3*c^2*d*x^2+E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c
^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*
b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a
^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*
d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^
2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*EllipticPi(((d*x

```
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d
),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)+4*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(-3/2)*integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*x^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c-dx^2)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}(-a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
[Out] Integral((c - d*x**2)**(3/2)/((e*x)**(3/2)*(-a + b*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((-d*x^2 + c)^(3/2)*e^(-3/2)/((b*x^2 - a)^2*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-d\,x^2\right)^{3/2}}{\left(e\,x\right)^{3/2}\,\left(a-b\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x)
[Out] int((c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2), x)
```

3.909 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

Optimal. Leaf size=412

$$-\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}}+\frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}\,(a-bx^2)}+\frac{\sqrt[4]{c}\,d^{3/4}(7bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-\right.}{6a^2be^{5/2}\sqrt{c-dx^2}}$$

```
[Out] -1/6*(-3*a*d+7*b*c)*(-d*x^2+c)^(1/2)/a^2/b/e/(e*x)^(3/2)+1/2*(-a*d+b*c)*(-d
*x^2+c)^(1/2)/a/b/e/(e*x)^(3/2)/(-b*x^2+a)+1/6*c^(1/4)*d^(3/4)*(-3*a*d+7*b*
c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b
/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d+b*c)*(-a*d+7*b*c)*EllipticPi(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*
x^2/c)^(1/2)/a^3/b/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d+b*c)*
(-a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)
/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/b/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2
)
```

Rubi [A]
time = 0.57, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 479, 597, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)(7bc-ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3b\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-ad)(7bc-ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3b\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2be^{5/2}\sqrt{c-dx^2}}-\frac{\sqrt{c-dx^2}\,(7bc-3ad)}{6a^2be(ex)^{3/2}}+\frac{\sqrt{c-dx^2}\,(bc-ad)}{2abe(ex)^{3/2}\,(a-bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] -1/6*((7*b*c - 3*a*d)*Sqrt[c - d*x^2])/(a^2*b*e*(e*x)^(3/2)) + ((b*c - a*d)
*Sqrt[c - d*x^2])/(2*a*b*e*(e*x)^(3/2)*(a - b*x^2)) + (c^(1/4)*d^(3/4)*(7*b
*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(6*a^2*b*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)
*(7*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*
Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(
1/4)*e^(5/2)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*(7*b*c - a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*b*d^(1/4)*e^(5/2)*Sqrt[c - d*x^
2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(c-\frac{dx^4}{e^2}\right)^{3/2}}{x^4\left(a-\frac{bx^4}{e^2}\right)^2}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{e\text{Subst}\left(\int \frac{\frac{c(7bc-3ad)}{e^2}-\frac{d(5bc-ad)x^4}{e^4}}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2ab}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} - \frac{e\text{Subst}\left(\int \frac{-\frac{bc^2(21bc-17ad)}{e^4}+\frac{bcd(7bc}{}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{d}{e}}}\right.}{6a^2bc}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{(d(7bc-3ad))\text{Subst}\left(\int \frac{}{\sqrt{c-}}\right.}{6a^2be^3}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{((bc-ad)(7bc-ad))\text{Subst}\left(\right.}{}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}(7bc-3ad)\sqrt{1-\frac{dx^2}{c}}}{6a^2be^{5/2}}$$

$$= -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}} + \frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}(7bc-3ad)\sqrt{1-\frac{dx^2}{c}}}{6a^2be^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.18, size = 199, normalized size = 0.48

$$\frac{x\left(5a(c-dx^2)(4ac-7bcx^2+3adx^2)+5c(-21bc+17ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-d(-7bc+3ad)x^4(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{30a^3(ex)^{5/2}(-a+bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

```
[Out] (x*(5*a*(c - d*x^2)*(4*a*c - 7*b*c*x^2 + 3*a*d*x^2) + 5*c*(-21*b*c + 17*a*d
)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c,
 (b*x^2)/a] - d*(-7*b*c + 3*a*d)*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(30*a^3*(e*x)^(5/2)*(-a + b*x^
2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3471 vs. $2(324) = 648$.
time = 0.13, size = 3472, normalized size = 8.43

method	result	size
elliptic	Expression too large to display	1191
default	Expression too large to display	3472

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*d*(16*a*b^2*c^3*(a*b)^(1/2)-28*b^3*c^3*x^2*(a*b)^(1/2)+28*b^3*c^2*d*x
^4*(a*b)^(1/2)-16*a^2*b*c^2*d*(a*b)^(1/2)+3*2^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2
^(1/2))*a^3*d^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+24*2^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c^2*d*x*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^3*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)-40*a*b^2*c*d^2*x^4*(a*b)^(1/2)+4*a^2*b*c*d^2*x^2*(a*b)^(1/2
)+24*a*b^2*c^2*d*x^2*(a*b)^(1/2)-3*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a
^2*b^2*c*d^2*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+24*2^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*a*b^3*c^2*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*2^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*b^3*c^2*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
```

```
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*
d)^(1/2)+14*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*b^3*c^2*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+3*2^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c*d^2*x^3*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)-24*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^2*d*x^3*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+3*2^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*a^3*b*c*d^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*2^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*a^3*d^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1
/2)-24*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c^2*d*x*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)-6*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*a^3*d^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-3*2^
(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^3*b*c*d^2*x*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)+12*a^2*b*d^3*x^4*(a*b)^(1/2)+24*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a
*b^2*c*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-20*2^(1/2)*
EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^3*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+24*2^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2
)*d),1/2*2^(1/2))*a*b^2*c*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d
)^(1/2)-24*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d*x*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+20*2^(1/2...
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] e^(-5/2)*integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*x^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c-dx^2)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}\,(-a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)
[Out] Integral((c - d*x**2)**(3/2)/((e*x)**(5/2)*(-a + b*x**2)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((-d*x^2 + c)^(3/2)*e^(-5/2)/((b*x^2 - a)^2*x^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-d\,x^2\right)}^{3/2}}{{\left(e\,x\right)}^{5/2}\,{\left(a-b\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x)
[Out] int((c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2), x)
```

3.910 $\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=484

$$\frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)}+\frac{c^{3/4}(4bc-5ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}(bc-ad)\sqrt{c-dx^2}}-\frac{c^{3/4}(4bc-5ad)e^{9/2}\sqrt{1}}{2b^2d^{3/}}$$

```
[Out] 1/2*a*e^3*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b/(-a*d+b*c)/(-b*x^2+a)+1/2*c^(3/4)*
(-5*a*d+4*b*c)*e^(9/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-
d*x^2/c)^(1/2)/b^2/d^(3/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-1/2*c^(3/4)*(-5*a*d+
4*b*c)*e^(9/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)
^(1/2)/b^2/d^(3/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-5*a*d+7*b*c)*e
^(9/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1
/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(5/2)/d^(1/4)/(-a*d+b*c)/(-d*x^2
+c)^(1/2)-1/4*c^(1/4)*(-5*a*d+7*b*c)*e^(9/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)
/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/
2)/b^(5/2)/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.60, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 481, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{\sqrt{a}\,\sqrt[4]{c}\,e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(7bc-5ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b^{5/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt{a}\,\sqrt[4]{c}\,e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(7bc-5ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b^{5/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(4bc-5ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}\,(bc-ad)}+\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(4bc-5ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}\sqrt{c-dx^2}\,(bc-ad)}+\frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(9/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (a*e^3*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2)) + (c^(3/4
)*(4*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(3/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) - (c^(3/4)*(4*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(3/4)*(b*c - a*d)*Sqrt
[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^(5/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]
) - (Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*b^(5/2)*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^{10}}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} - \frac{e^3\text{Subst}\left(\int \frac{x^2\left(3ac+\frac{(4bc-5ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} - \frac{e^3\text{Subst}\left(\int \left(-\frac{(4bc-5ad)x^2}{b\sqrt{c-\frac{dx^4}{e^2}}} - \frac{(-7abc+5a^2d)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right)\right.}{2b(bc-ad)}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} + \frac{((4bc-5ad)e^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2(bc-ad)}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} - \frac{\left(\sqrt{c}\,(4bc-5ad)e^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{e\right.}{2b^2\sqrt{d}\,(bc-ad)}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} - \frac{\left(\sqrt{c}\,(4bc-5ad)e^4\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{d}{c}\right.}{2b^2\sqrt{d}\,(bc-ad)\sqrt{c-dx^2}}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} - \frac{c^{3/4}(4bc-5ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{2b^2d^{3/4}(bc-ad)\sqrt{c-dx^2}}$$

$$= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} + \frac{c^{3/4}(4bc-5ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{2b^2d^{3/4}(bc-ad)\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.17, size = 184, normalized size = 0.38

$$\frac{e^3(ex)^{3/2}\left(-7a^2(c-dx^2)+7ac(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-(-4bc+5ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{14ab(-bc+ad)\,(a-bx^2)\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(9/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(-7*a^2*(c - d*x^2) + 7*a*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c
]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] - (-4*b*c + 5*a*d)*x^2*(
a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^
2)/a]))/(14*a*b*(-(b*c) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2943 vs. 2(374) = 748.
time = 0.13, size = 2944, normalized size = 6.08

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{a\,e^4 x\sqrt{-de\,x^3+cex}}{2b(ad-bc)\left(-b\,x^2+a\right)}-\frac{2c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,e^5\,\mathrm{EllipticE}\left(\ldots\right.}{d\sqrt{-de\,x^3+cex}\,b^2}\right.}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(4*a^2*b^2*d^3*x^4+16*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^3*x^2-8*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*b^4*c^3*x^2-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*
x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipti
```

```
cF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3-16*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a*b^3*c^3-4*a*b^3*c*d^2*x^4-4*a^2*b^2*c*d^2*x^2-7*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/
2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)+7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*
b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*a*b^3*c^2*d*x^2+5*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*
c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a
^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-7*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*
d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+7*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*
b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+7*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/
2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^
(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)
```

```
*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^
(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+7*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*
2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+20*((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))*a^2*b^2*c*d^2*x^2-36*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] e^(9/2)*integrate(x^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^(9/2)*e^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{9/2}}{{\left(a-b\,x^2\right)}^2\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
[Out] int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.911 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=376

$$\frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)}+\frac{\sqrt[4]{c}\,(4bc-3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt[4]{d}\,(bc-ad)\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,(5bc-3ad)e^{7/2}\sqrt{\ }}{}$$

```
[Out] 1/2*a*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b/(-a*d+b*c)/(-b*x^2+a)+1/2*c^(1/4)*
(-3*a*d+4*b*c)*e^(7/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-
d*x^2/c)^(1/2)/b^2/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+
5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1
/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^
(1/2)-1/4*c^(1/4)*(-3*a*d+5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(1/
4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 481, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(4bc-3ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt[4]{d}\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(5bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^2\sqrt[4]{d}\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(5bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4b^2\sqrt[4]{d}\sqrt{c-dx^2}\,(bc-ad)}+\frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(a-bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (a*e^3*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*b*(b*c - a*d)*(a - b*x^2)) + (c^(1/4)*
(4*b*c - 3*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^2*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
 - (c^(1/4)*(5*b*c - 3*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(4*b^2*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c - 3*a
*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b^2*d^(1/4)*(b*
c - a*d)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
```

```
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} - \frac{e^3\text{Subst}\left(\int \frac{ac+\frac{(4bc-3ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)}$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} + \frac{((4bc-3ad)e^3)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2(bc-ad)} -$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} - \frac{((5bc-3ad)e^3)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\frac{x^2}{e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x$$
$$4b^2(bc-ad)$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} + \frac{\sqrt[4]{c}\,(4bc-3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{2b^2\sqrt[4]{d}\,(bc-ad)\sqrt{c-dx^2}}$$

$$= \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} + \frac{\sqrt[4]{c}\,(4bc-3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{2b^2\sqrt[4]{d}\,(bc-ad)\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.17, size = 184, normalized size = 0.49

$$\frac{e^3\sqrt{ex}\left(-5a^2(c-dx^2)+5ac(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-(-4bc+3ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{10ab(-bc+ad)(a-bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e^3*Sqrt[e*x]*(-5*a^2*(c - d*x^2) + 5*a*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*
AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - (-4*b*c + 3*a*d)*x^2*(a
- b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/
a]))/(10*a*b*(-(b*c) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. $2(294) = 588$.

time = 0.12, size = 2508, normalized size = 6.67

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{e^3 a\sqrt{-de\,x^3+cex}}{2(ad-bc)b\left(-b\,x^2+a\right)}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\text{Ellipt}}{d\sqrt{-de\,x^3+cex}\,b^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b*(-8*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*b^3*c^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*a^2*b
*d^3*x^3*(a*b)^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2
*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*
b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)+14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)-4*a*b^2*c*d^2*x^3*(a*b)^(1/2)-4*a^2*b*c*d^2*x*(a*b)^(1/2)+4*a*b^2*c^2
```

```
*d*x*(a*b)^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b
)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1
/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+6*EllipticF(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*
a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^
(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+8*El
lipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^
2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/
2*2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1
/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)+3*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a
*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-6*EllipticF(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d
)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)-14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
```

```
))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-3*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] e^(7/2)*integrate(x^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^(7/2)*e^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{7/2}}{{\left(a-b\,x^2\right)}^2\,\sqrt{c-d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
[Out] int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.912 $\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=460

$$\frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)\,(a-bx^2)}-\frac{c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin\right.}{2b(bc-ad)\sqrt{c}}$$

```
[Out] 1/2*e*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-a*d+b*c)/(-b*x^2+a)-1/2*c^(3/4)*d^(1/4
)*e^(5/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2
)/b/(-a*d+b*c)/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*e^(5/2)*EllipticF(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/(-a*d+b*c)/(-d*x^2+c)
^(1/2)+1/4*c^(1/4)*(-a*d+3*b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1
/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(3/2)/d
^(1/4)/(-a*d+b*c)/a^(1/2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-a*d+3*b*c)*e^(5/2)
*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(
1/2),I)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/(-a*d+b*c)/a^(1/2)/(-d*x^2+c)^(1/
2)
```

Rubi [A]
time = 0.54, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 482, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(3bc-ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\,b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(3bc-ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\,b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}\,(bc-ad)}-\frac{c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}\,(bc-ad)}+\frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2\,(a-bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) - (c^(3/4)*d^(1
/4)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*e^(
5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - a*d)*e^(5
/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(3/2)*d^(1
/4)*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(3*b*c - a*d)*e^(5/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*b^(3/2)*d^(1/4)*(b*c - a*d)*
Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} - \frac{e\text{Subst}\left(\int \frac{x^2\left(3c-\frac{dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} - \frac{e\text{Subst}\left(\int \left(\frac{dx^2}{b\sqrt{c-\frac{dx^4}{e^2}}} + \frac{(3bc-ad)x^2}{b\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right) dx,\right.}{2(bc-ad)}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} - \frac{(de)\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)} - \frac{((3bc-ad)}{}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} + \frac{\left(\sqrt{c}\,\sqrt{d}\,e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)} - \frac{(}{}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} + \frac{\left(\sqrt{c}\,\sqrt{d}\,e^2\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx, x,\right.}{2b(bc-ad)\sqrt{c-dx^2}}$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} + \frac{c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}} +$$

$$= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} - \frac{c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 168, normalized size = 0.37

$$\frac{e(ex)^{3/2}\left(-7a(c-dx^2)+7c(a-bx^2)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+dx^2(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{14a(-bc+ad)(a-bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e*(e*x)^(3/2)*(-7*a*(c - d*x^2) + 7*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appe
llF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + d*x^2*(-a + b*x^2)*Sqrt[1 -
(d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(14*a*(-(b*c
) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2535 vs. $2(350) = 700$.
time = 0.12, size = 2536, normalized size = 5.51

method	result
elliptic	$\frac{\sqrt{ex}\sqrt{(-dx^2+c)ex}\left(-\frac{e^2x\sqrt{-dex^3+cex}}{2(ad-bc)(-bx^2+a)}-\frac{e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\text{EllipticE}}{2(ad-bc)b\sqrt{-dex^3+cex}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-4*a*b^2*c*d*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*a*b^2*c^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2-3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
```

```
-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2+3*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2+4*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*a*b^2*c^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elliptic
E(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^2*x^2+2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*b^3*c^2*x^2+4*b^3*c^2*x^2+4*a*b^2*d^2*x^4-4*b^3*c*d*x^4-((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*
d)^(1/2)*a^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(
a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*
EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d+2*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1
/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*a*b*c-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))
```

```
/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c*x^2+3
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*b^2*c*x^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^2+((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi((...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] e^(5/2)*integrate(x^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{5}{2}}}{(-a+bx^2)^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
[Out] Integral((e*x)**(5/2)/((-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)*e^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}}{{\left(a-b\,x^2\right)}^2\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

```
[Out] int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.913 $\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=363

$$\frac{e\sqrt{ex}\,\sqrt{c-dx^2}}{2(bc-ad)\,(a-bx^2)}+\frac{\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\,(bc+ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\mathrm{I}}{4ab\sqrt[4]{d}\,(}$$

```
[Out] 1/2*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-a*d+b*c)/(-b*x^2+a)+1/2*c^(1/4)*d^(3/4
)*e^(3/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2
)/b/(-a*d+b*c)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(a*d+b*c)*e^(3/2)*EllipticPi(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*
x^2/c)^(1/2)/a/b/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(a*d+b*c)*
e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1
/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 482, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4ab\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4ab\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{e\sqrt{ex}\,\sqrt{c-dx^2}}{2\,(a-bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (e*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*(b*c - a*d)*(a - b*x^2)) + (c^(1/4)*d^(3/4
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(2*b*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*e
^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c
- a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*a*b*d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} - \frac{e\text{Subst}\left(\int \frac{c+\frac{dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} + \frac{(de)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b(bc-ad)} - \frac{((bc+ad)}{}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} - \frac{((bc+ad)e)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\frac{x^2}{e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{}\right.}{4ab(bc-ad)}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}}$$

$$= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 169, normalized size = 0.47

$$-\frac{e\sqrt{ex}\left(5a(c-dx^2)+5c(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+dx^2(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{10a(-bc+ad)(a-bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(e*x)^(3/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]

[Out] -1/10*(e*Sqrt[e*x]*(5*a*(c - d*x^2) + 5*c*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(a*(-(b*c) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2245 vs. $2(281) = 562$.

time = 0.12, size = 2246, normalized size = 6.19

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{e\sqrt{-de\,x^3+cex}}{2(ad-bc)\left(-b\,x^2+a\right)}-\frac{e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}}{4(ad-bc)b\sqrt{-de\,x^3+cex}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8*d*(-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*d*x^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c*x^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*d*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)

*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)
^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*d*x^2+((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2+((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*b^2*c*x^2+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*2^(1/2)*a^2*d*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2*Ellipti
cF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*(c*d)^(
1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)
^(1/2)*a*b*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*a^2*b*c*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b
)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c-4*a*b*d^2*x^3*(a*b
)^(1/2)+4*b^2*c*d*x^3*(a*b)^(1/2)+4*a*b*c*d*x*(a*b)^(1/2)-4*b^2*c^2*x*(a*b)
^(1/2))*e*(e*x)^(1/2)/(-d*x^2+c)^(1/2)/x/(-b*x^2+a)/((c*d)^(1/2)*b-(a*b)^(1

```
/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b)^(1/2)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] e^(3/2)*integrate(x^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}}{(-a+bx^2)^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
[Out] Integral((e*x)**(3/2)/((-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^(3/2)*e^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{3/2}}{(a-b\,x^2)^2\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
[Out] int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.914 $\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=464

$$\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)}-\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a(bc-ad)\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\right.}{2a(bc-ad)}$$

```
[Out] 1/2*b*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)-1/2*c^(3/4)*d^
(1/4)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^
(1/2)/a/(-a*d+b*c)/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*EllipticF(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/(-a*d+b*c)/(-d*x^
2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/
e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/
2)/d^(1/4)/(-a*d+b*c)/b^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-3*a*d+b*c)*Ell
ipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2)
,I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/(-a*d+b*c)/b^(1/2)/(-d*x^2+c)
^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {477, 483, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-3ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-3ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt{c-dx^2}\,(bc-ad)}-\frac{c^{3/4}\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt{c-dx^2}\,(bc-ad)}+\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2ae\left(a-bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (b*(e*x)^(3/2)*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) - (c^(3/4)*
d^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)
*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(2*a*(b*c - a*d)*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - 3*a*d)*
Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
)), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*Sqrt[b]*
d^(1/4)*(b*c - a*d)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - 3*a*d)*Sqrt[e]*Sqrt[
1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)*Sqrt[b]*d^(1/4)*(b*c - a
*d)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx=\frac{2\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{e}$$

$$=\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)}+\frac{e\text{Subst}\left(\int \frac{x^2\left(\frac{bc-4ad}{e^2}+\frac{bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{2a(bc-ad)}$$

$$=\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)}+\frac{e\text{Subst}\left(\int \left(-\frac{dx^2}{e^2\sqrt{c-\frac{dx^4}{e^2}}}+\frac{(bc-3ad)x^2}{e^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.\right.}{2a(bc-ad)}$$

$$=\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)}-\frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{2a(bc-ad)e}+\frac{(bc-3ad)\text{S}}{}$$

$$=\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)}+\frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{2a(bc-ad)}-$$

$$=\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)}+\frac{\left(\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{dx^4}{ce^2}}}\,dx,x,\right.}{2a(bc-ad)\sqrt{c-dx^2}}$$

$$=\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)}+\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a(bc-ad)\sqrt{c-dx^2}}$$

$$=\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\left(a-bx^2\right)}-\frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a(bc-ad)\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.20, size = 181, normalized size = 0.39

$$\frac{x\sqrt{ex}\left(7(bc-4ad)\left(-a+bx^2\right)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3b\left(-7a\left(c-dx^2\right)+dx^2\left(-a+bx^2\right)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)\right)}{42a^2(bc-ad)\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*Sqrt[e*x]*(7*(b*c - 4*a*d)*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4
, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*(-7*a*(c - d*x^2) + d*x^2*(-a +
b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a
])))/(42*a^2*(b*c - a*d)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2532 vs. $2(354) = 708$.
time = 0.13, size = 2533, normalized size = 5.46

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{\left(-d\,x^2+c\right)ex}\left(-\frac{bx\sqrt{-de\,x^3+cex}}{2(ad-bc)a\left(-b\,x^2+a\right)}-\frac{ec\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticE}\left(\ldots\right.}{2a(ad-bc)\sqrt{-de\,x^3+cex}}\ldots\right.}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*a*b^2*c*d*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*a*b^2*c^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
```

d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2+((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2-4*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*a*b^2*c^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^2*x^2-2*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*b^3*c^2*x^2-4*b^3*c^2*x^2-4*a*b^2*d^2*x^4+4*b^3*c*d*x^4-3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)
*a^2*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b
)^(1/2)*d),1/2*2^(1/2))*a^2*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)
^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
2^(1/2)((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*El
lipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d-2*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*a^2*b*c*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1
/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*a*b*c+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(

```
c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c*x^2+((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
*2^(1/2))*b^2*c*x^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^2+3*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt(x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}}{\left(-a+bx^2\right)^2 \sqrt{c-dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
[Out] Integral(sqrt(e*x)/((-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*e^(1/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}}{{\left(a-b\,x^2\right)}^2\,\sqrt{c-d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

```
[Out] int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.915 $\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=367

$$\frac{b\sqrt{ex}\,\sqrt{c-dx^2}}{2a(bc-ad)e\,(a-bx^2)}+\frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a(bc-ad)\sqrt{e}\,\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(3bc-5ad)\sqrt{1-\frac{dx^2}{c}}\,\Pi\Big(}{4a^2\sqrt[4]{d}\,(bc-}$$

```
[Out] 1/2*b*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)+1/2*c^(1/4)*d^
(3/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/
(-a*d+b*c)/e^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-5*a*d+3*b*c)*EllipticPi(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d
*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)/e^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(
-5*a*d+3*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2
)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)/e^(1/2)/(-d*x
^2+c)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {477, 425, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(3bc-5ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(3bc-5ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{b\sqrt{ex}\,\sqrt{c-dx^2}}{2ae\,(a-bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (b*Sqrt[e*x]*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(a - b*x^2)) + (c^(1/4)*d^
(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(2*a*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c -
5*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c
- a*d)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)*Sqrt[e]*Sqrt[c - d*
x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
```

```
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{ex}\,\sqrt{c-dx^2}}{2a(bc-ad)e\,(a-bx^2)} + \frac{e\text{Subst}\left(\int \frac{\frac{3bc-4ad}{e^2}-\frac{bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)}$$

$$= \frac{b\sqrt{ex}\,\sqrt{c-dx^2}}{2a(bc-ad)e\,(a-bx^2)} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2a(bc-ad)e} + \frac{(3bc}{}$$

$$= \frac{b\sqrt{ex}\,\sqrt{c-dx^2}}{2a(bc-ad)e\,(a-bx^2)} + \frac{(3bc-5ad)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,e}\right)\sqrt{c-\frac{dx^4}{e^2}}}\right.}{4a^2(bc-ad)e}$$

$$= \frac{b\sqrt{ex}\,\sqrt{c-dx^2}}{2a(bc-ad)e\,(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right| -}{2a(bc-ad)\sqrt{e}\,\sqrt{c-dx^2}}$$

$$= \frac{b\sqrt{ex}\,\sqrt{c-dx^2}}{2a(bc-ad)e\,(a-bx^2)} + \frac{\sqrt[4]{c}\,d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right| -}{2a(bc-ad)\sqrt{e}\,\sqrt{c-dx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.16, size = 180, normalized size = 0.49

$$\frac{5abx(-c+dx^2)+5(-3bc+4ad)x(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bdx^3(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{10a^2(bc-ad)\sqrt{ex}\,(-a+bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (5*a*b*x*(-c + d*x^2) + 5*(-3*b*c + 4*a*d)*x*(a - b*x^2)*Sqrt[1 - (d*x^2)/c
]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*x^3*(a - b*x^2)*Sq
rt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(10*a^2
*(b*c - a*d)*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2253 vs. $2(285) = 570$.
time = 0.12, size = 2254, normalized size = 6.14

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{b\sqrt{-de\,x^3+cex}}{2(ad-bc)ae\left(-b\,x^2+a\right)}-\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x \ldots\right.}{\ldots}}\ldots\right.}{4a(ad-bc)\sqrt{-de\,x^3+cex}}\ldots\right)}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b*d*(-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^
(1/2)*a*b*d*x^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+2*Ellipt
icF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^2*c*x^2*(c
*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+5*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*
b*d*x^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))
*b^3*c^2*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((
```

```
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2
)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*d*x^2-3
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*
x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(
1/2)*d),1/2*2^(1/2))*b^2*c*x^2+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*a*b*c*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-5*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)
^(1/2)*a^2*d-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(
1/2))*a*b^2*c^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(
1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d+3*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+3*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2
```

```
*2^(1/2))*a*b*c-4*a*b*d^2*x^3*(a*b)^(1/2)+4*b^2*c*d*x^3*(a*b)^(1/2)+4*a*b*c
*d*x*(a*b)^(1/2)-4*b^2*c^2*x*(a*b)^(1/2))/(-d*x^2+c)^(1/2)/a/(-b*x^2+a)/((c
*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b)^(1/2)/(a*d-b
*c)/(e*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(-1/2)*integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex}\,(-a+bx^2)^2\,\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*x)*(-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,x}\,\left(a-b\,x^2\right)^2\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
[Out] int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.916 $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=535

$$-\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}}+\frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)}-\frac{\sqrt[4]{d}\,(5bc-4ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right|}{2a^2\sqrt[4]{c}\,(bc-ad)e^{3/2}\sqrt{c-dx^2}}$$

```
[Out] -1/2*(-4*a*d+5*b*c)*(-d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c)/e/(e*x)^(1/2)+1/2*b*(
-d*x^2+c)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(e*x)^(1/2)-1/2*d^(1/4)*(-4*a*d+5
*b*c)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^
2/c^(1/4)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)+1/2*d^(1/4)*(-4*a*d+5*b*c)*El
lipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(1/4
)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-7*a*d+5*b*c)*EllipticPi
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^
(1/2)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)
+1/4*c^(1/4)*(-7*a*d+5*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),
b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4
)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.75, antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 483, 597, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}\,(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}\,(bc-ad)}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}\,e^{3/2}\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}\,e^{3/2}\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt{c-dx^2}\,(5bc-4ad)}{2a^2ce\sqrt{ex}\,(bc-ad)}+\frac{b\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] -1/2*((5*b*c - 4*a*d)*Sqrt[c - d*x^2])/(a^2*c*(b*c - a*d)*e*Sqrt[e*x]) + (b
*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)) - (d^(1/4)*(5*b
*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*a^2*c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (d
^(1/4)*(5*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d
*x^2]) - (Sqrt[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-(
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2]) + (S
qrt[b]*c^(1/4)*(5*b*c - 7*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(4*a^(5/2)*d^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)} + \frac{e\text{Subst}\left(\int \frac{\frac{5bc-4ad}{e^2}-\frac{3bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \right.}{2a(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)} - \frac{e\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)} - \frac{e\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)} + \frac{(b(5bc-7}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)} + \frac{\left(\sqrt{d}\,(5bc\right.}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)} + \frac{\left(\sqrt{d}\,(5bc\right.}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)} + \frac{\sqrt[4]{d}\,(5bc-}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)} - \frac{\sqrt[4]{d}\,(5bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.23, size = 235, normalized size = 0.44

$$\frac{x\left(-21a(c-dx^2)\left(4a^2d+5b^2cx^2-4ab(c+dx^2)\right)+7(5b^2c^2-12abcd+4a^2d^2)\,x^2(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bd(-5bc+4ad)x^4(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a^3c(bc-ad)(ex)^{3/2}\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(-21*a*(c - d*x^2)*(4*a^2*d + 5*b^2*c*x^2 - 4*a*b*(c + d*x^2)) + 7*(5*b^
2*c^2 - 12*a*b*c*d + 4*a^2*d^2)*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*(-5*b*c + 4*a*d)*x^4*(a
- b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)
/a]))/(42*a^3*c*(b*c - a*d)*(e*x)^(3/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2969 vs. 2(419) = 838.
time = 0.14, size = 2970, normalized size = 5.55

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{b^2x\sqrt{-de\,x^3+cex}}{2(ad-bc)a^2e^2\left(-b\,x^2+a\right)}-\frac{2\left(-de\,x^2+ce\right)}{e^2c\,a^2\sqrt{x\left(-de\,x^2+ce\right)}}-\frac{c\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{}}{2\sqrt{-de\,x}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3*x^2-16*a^2*b*d^3*x^4-16*a^3*c*d^2-16
*a*b^2*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*
d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c^2*x^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
```

b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2*c^2*x^2+7*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d*x
^2+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^
2*c^2*d*x^2+16*a^3*d^3*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1
/2)*d),1/2*2^(1/2))*a*b^2*c^3+16*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipti
cE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*c*d^2+20*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*a*b^2*c^3-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*c*d^2-10*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*a*b^2*c^3+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2
^(1/2))*a*b^2*c^3-16*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2*x^2+36*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a*b^2*c^2*d*x^2+8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d^2*x^2-18*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))*a*b^2*c^2*d*x^2+20*b^3*c^3*x^2+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3*x
^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*
c^3*x^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))
*b^3*c^3*x^2+36*a*b^2*c*d^2*x^4-20*a*b^2*c^2*d*x^2+32*a^2*b*c^2*d-7*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)

```
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d
)^(1/2)*a*b*c*d*x^2+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d*x^2-16*a^2*b*c*d^2*x^2-20*b^3*
c^2*d*x^4+7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d
),1/2*2^(1/2))*(c*d)^(1/2)*a^2*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c^2-7*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(-3/2)*integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*x^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex)^{\frac{3}{2}}\left(-a+bx^2\right)^2\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((e*x)**(3/2)*(-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(e^(-3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*x^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x)^{3/2}\,\left(a-b\,x^2\right)^2\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
[Out] int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.917 $\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=429

$$-\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}}+\frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\,(a-bx^2)}+\frac{d^{3/4}(7bc-4ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\right|}{6a^2c^{3/4}(bc-ad)e^{5/2}\sqrt{c-dx^2}}$$

```
[Out] -1/6*(-4*a*d+7*b*c)*(-d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c)/e/(e*x)^(3/2)+1/2*b*(
-d*x^2+c)^(1/2)/a/(-a*d+b*c)/e/(e*x)^(3/2)/(-b*x^2+a)+1/6*d^(3/4)*(-4*a*d+7
*b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^
2/c^(3/4)/(-a*d+b*c)/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*b*c^(1/4)*(-9*a*d+7*b*c)*
EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(
1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/(-a*d+b*c)/e^(5/2)/(-d*x^2+c)^(1/2)+1
/4*b*c^(1/4)*(-9*a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),
b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/(-a*d+b*c)
/e^(5/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 483, 597, 537, 230, 227, 418, 1233, 1232}

$$\frac{b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(7bc-9ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}\,(bc-ad)}+\frac{b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(7bc-9ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}\,e^{5/2}\sqrt{c-dx^2}\,(bc-ad)}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,(7bc-4ad)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{3/4}e^{5/2}\sqrt{c-dx^2}\,(bc-ad)}-\frac{\sqrt{c-dx^2}\,(7bc-4ad)}{6a^2ce(ex)^{3/2}(bc-ad)}+\frac{b\sqrt{c-dx^2}}{2ae(ex)^{3/2}\,(a-bx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] -1/6*((7*b*c - 4*a*d)*Sqrt[c - d*x^2])/(a^2*c*(b*c - a*d)*e*(e*x)^(3/2)) +
(b*Sqrt[c - d*x^2])/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)) + (d^(3/4)*
(7*b*c - 4*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(3/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2])
+ (b*c^(1/4)*(7*b*c - 9*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(4*a^3*d^(1/4)*(b*c - a*d)*e^(5/2)*Sqrt[c - d*x^2]) + (b*c^(1/4)*(7*b*c -
 9*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c -
a*d)*e^(5/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)^2\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{e\text{Subst}\left(\int \frac{\frac{7bc-4ad}{e^2}-\frac{5bdx^4}{e^4}}{x^4\left(a-\frac{bx^4}{e^2}\right)\sqrt{c-\frac{dx^4}{e^2}}}\,dx, x\right.}{2a(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} - \frac{e\text{Subst}\left(\right.}{}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{(b(7bc-}{}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{(b(7bc-}{}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{d^{3/4}(7bc-}{}$$

$$= -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}} + \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{d^{3/4}(7bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.23, size = 234, normalized size = 0.55

$$\frac{x\left(-5a(c-dx^2)\left(4a^2d+7b^2cx^2-4ab(c+dx^2)\right)+5(-21b^2c^2+20abcd+4a^2d^2)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)-bd(-7bc+4ad)x^4(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{30a^3c(bc-ad)(ex)^{5/2}\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

```
[Out] (x*(-5*a*(c - d*x^2)*(4*a^2*d + 7*b^2*c*x^2 - 4*a*b*(c + d*x^2)) + 5*(-21*b
^2*c^2 + 20*a*b*c*d + 4*a^2*d^2)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - b*d*(-7*b*c + 4*a*d)*x^4*(a -
b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]
))/(30*a^3*c*(b*c - a*d)*(e*x)^(5/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2609 vs. $2(341) = 682$.

time = 0.13, size = 2610, normalized size = 6.08

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(-\frac{b^2\sqrt{-de\,x^3+cex}}{2(ad-bc)a^2e^3\left(-b\,x^2+a\right)}-\frac{2\sqrt{-de\,x^3+cex}}{3e^3c\,a^2x^2}-\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\cdots}}{4\sqrt{-de\,x^3+c\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*b*d*(-16*a*b^2*c^3*(a*b)^(1/2)+28*b^3*c^3*x^2*(a*b)^(1/2)-28*b^3*c^2*
d*x^4*(a*b)^(1/2)+32*a^2*b*c^2*d*(a*b)^(1/2)-16*a^3*c*d^2*(a*b)^(1/2)+16*a^
3*d^3*x^2*(a*b)^(1/2)-27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c^2
*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b
^4*c^3*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1
/2))*b^4*c^3*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
```

(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+44*a*b^2*c*d^2*x^4*(a*b)^(1/2)-16*a^2*b*c*d^2*x^2*(a*b)^(1/2)-28*a*b^2*c^2*d*x^2*(a*b)^(1/2)-27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^2*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-14*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^2*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^2*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c^2*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+8*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*d^2*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-16*a^2*b*d^3*x^4*(a*b)^(1/2)-27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+22*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-22*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+27*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3*x*((

```
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)+21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^
3*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-8*2^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*d^2*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(
1/2)*(c*d)^(1/2)-21*2^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*x*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(-5/2)*integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*x^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex)^{\frac{5}{2}} (-a + bx^2)^2 \sqrt{c - dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((e*x)**(5/2)*(-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(e^(-5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x)^{5/2}\,\left(a-b\,x^2\right)^2\,\sqrt{c-d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
[Out] int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.918 $\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=529

$$\frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}}+\frac{ae^3(ex)^{3/2}}{2b(bc-ad)\,(a-bx^2)\,\sqrt{c-dx^2}}-\frac{c^{3/4}(2bc+ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{2bd^{3/4}(bc-ad)^2\sqrt{c-dx^2}}$$

```
[Out] 1/2*(a*d+2*b*c)*e^3*(e*x)^(3/2)/b/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*a*e^3*(
e*x)^(3/2)/b/(-a*d+b*c)/(-b*x^2+a)/(-d*x^2+c)^(1/2)-1/2*c^(3/4)*(a*d+2*b*c)
*e^(9/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)
/b/d^(3/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*(a*d+2*b*c)*e^(9/2)*El
lipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(3/4)/
(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d+7*b*c)*e^(9/2)*EllipticPi(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1
/2)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)-1/4*c^(
1/4)*(-a*d+7*b*c)*e^(9/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^
(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/
(-a*d+b*c)^2/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 481, 593, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{\sqrt{a}\,\sqrt[4]{c}\,e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(7bc-ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}-\frac{\sqrt{a}\,\sqrt[4]{c}\,e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(7bc-ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+2bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2bd^{3/4}\sqrt{c-dx^2}\,(bc-ad)^2}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+2bc)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2bd^{3/4}\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{e^3(ex)^{3/2}(ad+2bc)}{2b\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{ae^3(ex)^{3/2}}{2b\,(a-bx^2)\,\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((2*b*c + a*d)*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (a*e^
3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) - (c^(3/4)*(2*
b*c + a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(3/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (c^
(3/4)*(2*b*c + a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b*d^(3/4)*(b*c - a*d)^2*Sqrt[c - d*x^
2]) + (Sqrt[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(4*b^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (Sqrt
[a]*c^(1/4)*(7*b*c - a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*b^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
```

```
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^{10}}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} - \frac{e^3\text{Subst}\left(\int \frac{x^2\left(3ac+\frac{(4bc-ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right.}{2b(bc-ad)}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{e^5\text{Subst}\left(\int \frac{x^{\cdots}}{\cdots}\right.}{\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{e^5\text{Subst}\left(\int \left(\cdots\right.\right.}{\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} - \frac{(a(7bc-ad)e\cdots}{\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{\left(\sqrt{c}\,(2bc+a\cdots\right.}{\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{\left(\sqrt{c}\,(2bc+\cdots\right.}{\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{c^{3/4}(2bc+ad\cdots}{\cdots}$$

$$= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} - \frac{c^{3/4}(2bc+ad\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.19, size = 189, normalized size = 0.36

$$\frac{e^3(ex)^{3/2}\left(7a(-3ac+2bcx^2+adx^2)+21ac(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+(2bc+ad)x^2(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{14a(bc-ad)^2\,(-a+bx^2)\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(7*a*(-3*a*c + 2*b*c*x^2 + a*d*x^2) + 21*a*c*(a - b*x^2)*S
qrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + (2*b*
c + a*d)*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (
d*x^2)/c, (b*x^2)/a]))/(14*a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2951 vs. 2(413) = 826.
time = 0.13, size = 2952, normalized size = 5.58

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\,\left(\frac{a\,e^4x\sqrt{-de\,x^3+cex}}{2(ad-bc)^2\left(-b\,x^2+a\right)}+\frac{e^5x^2c}{(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}}\cdots}{2\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*a^2*b^2*d^3*x^4-8*b^4*c^2*d*x^4-8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^3*x^2+4
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),1/2*2^(1/2))*b^4*c^3*x^2-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^
2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
```

```
1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3
+8*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3+4*a*b^3*c*d^2*x^4-12*a^2*b^2*c*d^2*x^2-7*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2
)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)+7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2
*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+12*a*b^3*c^2*d*
x^2+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*
b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1
/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-7*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(
1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+7*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^
(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2
*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+7*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1
/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)-7*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2
```

```
)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+7*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2
^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a^2*b^2*c*d^2*x^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] e^(9/2)*integrate(x^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^(9/2)*e^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{9/2}}{\left(a-b\,x^2\right)^2\,\left(c-d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
[Out] int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.919 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\, dx$

Optimal. Leaf size=420

$$\frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}}+\frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\,(2bc+ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{2b\sqrt[4]{d}\,(bc-ad)^2\sqrt{c-dx^2}}$$

```
[Out] 1/2*(a*d+2*b*c)*e^3*(e*x)^(1/2)/b/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*a*e^3*(
e*x)^(1/2)/b/(-a*d+b*c)/(-b*x^2+a)/(-d*x^2+c)^(1/2)+1/2*c^(1/4)*(a*d+2*b*c)
*e^(7/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)
/b/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(a*d+5*b*c)*e^(7/2)*El
lipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/
2),I)*(1-d*x^2/c)^(1/2)/b/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)-1/4*c^(1/4)
*(a*d+5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)
*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(1/4)/(-a*d+b*c)^2/(-d*x^
2+c)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 481, 541, 537, 230, 227, 418, 1233, 1232}

$$\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+2bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2b\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}-\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+5bc)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}-\frac{\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+5bc)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4b\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{e^3\sqrt{ex}\,(ad+2bc)}{2b\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{ae^3\sqrt{ex}}{2b\left(a-bx^2\right)\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((2*b*c + a*d)*e^3*Sqrt[e*x])/(2*b*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (a*e^3*
Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) + (c^(1/4)*(2*b*c
+ a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(2*b*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4
)*(5*b*c + a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/
(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*
b*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b*c + a*d)*e^(7/2)*S
qrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*b*d^(1/4)*(b*c - a*d)^2*Sqrt
[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \frac{2\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)^2 \left(c-\frac{dx^4}{e^2}\right)^{3/2}} \, dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} - \frac{e^3\text{Subst}\left(\int \frac{ac+\frac{(4bc+ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}} \, dx, x, \sqrt{ex}\right.}{2b(bc-ad)}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{e^5\text{Subst}\left(\int -\right.}{}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{((2bc+ad)e^3}{}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} - \frac{((5bc+ad)e^3}{}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\,(2bc+ad}{}$$

$$= \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\,(2bc+ad}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.18, size = 191, normalized size = 0.45

$$-\frac{e^3\sqrt{ex}\left(5a(3ac-2bcx^2-adx^2)+15ac(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+(2bc+ad)x^2(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{10a(bc-ad)^2(-a+bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] -1/10*(e^3*Sqrt[e*x]*(5*a*(3*a*c - 2*b*c*x^2 - a*d*x^2) + 15*a*c*(-a + b*x^
2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (
2*b*c + a*d)*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4
, (d*x^2)/c, (b*x^2)/a]))/(a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2517 vs. $2(332) = 664$.
time = 0.13, size = 2518, normalized size = 6.00

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\frac{a\,e^3\sqrt{-de\,x^3+cex}}{2(ad-bc)^2\left(-b\,x^2+a\right)}+\frac{e^4xc}{(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2}{\sqrt{}}}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2
)*b^3*c^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-4*a^2*b*d^
3*x^3*(a*b)^(1/2)+8*b^3*c^2*d*x^3*(a*b)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5*EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/
2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)-4*a*b^2*c*d^2*x^3*(a*b)^(1/2)+12*a^2*b*c*d^2*x*
```

(a*b)^(1/2)-12*a*b^2*c^2*d*x*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*
b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+2
*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d
^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2
*2^(1/2))*2^(1/2)*a^3*b*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),
1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)+5*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)-4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
2^(1/2)*a*b^2*c^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+5*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(
1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*
b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2*EllipticF(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*d^2*x^2*(a*b
)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1
/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-5
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)

```
,1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*
d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^2*x^2*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] e^(7/2)*integrate(x^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^(7/2)*e^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{7/2}}{\left(a-b\,x^2\right)^2\,\left(c-d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
[Out] int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.920 $\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=485

$$\frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}}+\frac{e(ex)^{3/2}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}}-\frac{3c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-\right.}{2(bc-ad)^2\sqrt{c-dx^2}}$$

```
[Out] 3/2*d*e*(e*x)^(3/2)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*e*(e*x)^(3/2)/(-a*d+b
*c)/(-b*x^2+a)/(-d*x^2+c)^(1/2)-3/2*c^(3/4)*d^(1/4)*e^(5/2)*EllipticE(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/(-a*d+b*c)^2/(-d*x^2+c)
^(1/2)+3/2*c^(3/4)*d^(1/4)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^
(1/2),I)*(1-d*x^2/c)^(1/2)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+3/4*c^(1/4)*(a*d+b
*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)
/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)^2/a^(1/2)/b^(1/2)/
(-d*x^2+c)^(1/2)-3/4*c^(1/4)*(a*d+b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(
1/4)/(-a*d+b*c)^2/a^(1/2)/b^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 482, 593, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{3c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}\,(bc-ad)^2}-\frac{3c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{3\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\,\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}-\frac{3\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\,\sqrt{b}\,\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{3de(ex)^{3/2}}{2\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{e(ex)^{3/2}}{2\left(a-bx^2\right)\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (3*d*e*(e*x)^(3/2))/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*(e*x)^(3/2))/(2*
(b*c - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) - (3*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[
1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (3*c^(3/4)*d^(1/4)*e^(5/2)*Sqrt[1 - (
d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*
(b*c - a*d)^2*Sqrt[c - d*x^2]) + (3*c^(1/4)*(b*c + a*d)*e^(5/2)*Sqrt[1 - (d
*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*Sqrt[b]*d^(1/4)*(b*c - a*d)
^2*Sqrt[c - d*x^2]) - (3*c^(1/4)*(b*c + a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*Sqrt[b]*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*
x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}}{2(bc-ad)\,(a-bx^2)\,\sqrt{c-dx^2}} - \frac{e\text{Subst}\left(\int \frac{x^2\left(3c+\frac{3dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{2(bc-ad)}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\,(a-bx^2)\,\sqrt{c-dx^2}} + \frac{e^3\text{Subst}\left(\int \frac{x^2(\cdots}{(a-\cdots}\right.}{\cdots}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\,(a-bx^2)\,\sqrt{c-dx^2}} + \frac{e^3\text{Subst}\left(\int \left(-\cdots\right.\right.}{\cdots}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\,(a-bx^2)\,\sqrt{c-dx^2}} - \frac{(3de)\text{Subst}\left(\int \cdots\right.}{2\cdots}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\,(a-bx^2)\,\sqrt{c-dx^2}} + \frac{\left(3\sqrt{c}\,\sqrt{d}\,e^2\right)\text{S}\cdots}{\cdots}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\,(a-bx^2)\,\sqrt{c-dx^2}} + \frac{\left(3\sqrt{c}\,\sqrt{d}\,e^2\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\,(a-bx^2)\,\sqrt{c-dx^2}} + \frac{3c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{\cdots}}{\cdots}$$

$$= \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)\,(a-bx^2)\,\sqrt{c-dx^2}} - \frac{3c^{3/4}\sqrt[4]{d}\,e^{5/2}\sqrt{\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.20, size = 185, normalized size = 0.38

$$\frac{e(ex)^{3/2}\left(-7a(2ad+b(c-3dx^2))+7(bc+2ad)\,(a-bx^2)\,\sqrt{1-\frac{dx^2}{c}}\,F_1\!\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bdx^2(-a+bx^2)\,\sqrt{1-\frac{dx^2}{c}}\,F_1\!\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{14a(bc-ad)^2\,(-a+bx^2)\,\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (e*(e*x)^(3/2)*(-7*a*(2*a*d + b*(c - 3*d*x^2)) + 7*(b*c + 2*a*d)*(a - b*x^2
)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*
b*d*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2
)/c, (b*x^2)/a]))/(14*a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2548 vs. 2(369) = 738.
time = 0.13, size = 2549, normalized size = 5.26

method	result
elliptic	$\dfrac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\frac{b\,e^2x\sqrt{-de\,x^3+cex}}{2(ad-bc)^2\left(-b\,x^2+a\right)}+\frac{d\,e^3x^2}{(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{3e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}\cdots}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(8*a^2*b*d^2*x^2-4*a*b^2*c*d*x^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+3
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^
2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
```

```
^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c
^2*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*
b^3*c^2*x^2-12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2+12*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c
^2*x^2-6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^2*x^2-4*b^3*c^2*x^2-12*a*b^2*d^2*x^4+12
*b^3*c*d*x^4-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2
)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*d+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a
*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d
)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d+3*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d+12*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*a^2*b*c*d-6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b*c*d-3*((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2-3*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c*d*x^2-3*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))
*(c*d)^(1/2)*a*b*c+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(
1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^
(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c+6*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
```

```
/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c*d*
x^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*
2^(1/2))*(c*d)^(1/2)*b^2*c*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(
1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/
2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*x^2-12*((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^
(1/2))*a*b^2*c*d*x^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] e^(5/2)*integrate(x^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)*e^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}}{{\left(a-b\,x^2\right)}^2\,{\left(c-d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

```
[Out] int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.921 $\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx$

Optimal. Leaf size=391

$$\frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}}+\frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-\right.}{2(bc-ad)^2\sqrt{c-dx^2}}$$

```
[Out] 3/2*d*e*(e*x)^(1/2)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*e*(e*x)^(1/2)/(-a*d+b
*c)/(-b*x^2+a)/(-d*x^2+c)^(1/2)+3/2*c^(1/4)*d^(3/4)*e^(3/2)*EllipticF(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/(-a*d+b*c)^2/(-d*x^2+c)
^(1/2)-1/4*c^(1/4)*(5*a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/
(-a*d+b*c)^2/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(5*a*d+b*c)*e^(3/2)*EllipticPi(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x
^2/c)^(1/2)/a/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 482, 541, 537, 230, 227, 418, 1233, 1232}

$$\frac{3\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2\sqrt{c-dx^2}\,(bc-ad)^2}-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(5ad+bc)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}-\frac{\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(5ad+bc)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{3de\sqrt{ex}}{2\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{e\sqrt{ex}}{2\left(a-bx^2\right)\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (3*d*e*Sqrt[e*x])/(2*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (e*Sqrt[e*x])/(2*(b*c
 - a*d)*(a - b*x^2)*Sqrt[c - d*x^2]) + (3*c^(1/4)*d^(3/4)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2
*(b*c - a*d)^2*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (
d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d
*x^2]) - (c^(1/4)*(b*c + 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(4*a*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx=\frac{2\text{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx,x,\sqrt{ex}\right)}{e}$$

$$=\frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}}-\frac{e\text{Subst}\left(\int \frac{c+\frac{5dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx,x,\sqrt{ex}\right.}{2(bc-ad)}$$

$$=\frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}}+\frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}}+\frac{e^3\text{Subst}\left(\int \frac{-\cdots}{\left(a-\cdots\right.}\right.}{\cdots}$$

$$=\frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}}+\frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}}+\frac{(3de)\text{Subst}\left(\int\cdots\right.}{2\cdots}$$

$$=\frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}}+\frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}}-\frac{((bc+5ad)e)\text{Su}\cdots}{\cdots}$$

$$=\frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}}+\frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{\cdots}}{\cdots}$$

$$=\frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}}+\frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\sqrt{c-dx^2}}+\frac{3\sqrt[4]{c}\,d^{3/4}e^{3/2}\sqrt{\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.17, size = 186, normalized size = 0.48

$$-\frac{e\sqrt{ex}\left(5a\left(2ad+b\left(c-3dx^2\right)\right)+5(bc+2ad)\left(-a+bx^2\right)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bdx^2\left(-a+bx^2\right)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{10a(bc-ad)^2\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] -1/10*(e*Sqrt[e*x]*(5*a*(2*a*d + b*(c - 3*d*x^2)) + 5*(b*c + 2*a*d)*(-a + b
*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a]
+ 3*b*d*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*
x^2)/c, (b*x^2)/a]))/(a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2264 vs. $2(303) = 606$.
time = 0.14, size = 2265, normalized size = 5.79

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\frac{be\sqrt{-de\,x^3+cex}}{2(ad-bc)^2\left(-b\,x^2+a\right)}+\frac{d\,e^2x}{(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{3e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\cdots}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*b*d*(-4*b^2*c^2*x*(a*b)^(1/2)-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(
a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+
(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*
EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)
*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^2*x^2-12*a*b*d^2*x^3*(a*b)^(1/2)+12*b^
2*c*d*x^3*(a*b)^(1/2)+8*a^2*d^2*x*(a*b)^(1/2)-4*a*b*c*d*x*(a*b)^(1/2)-5*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*
```

```
(c*d)^(1/2)*a^2*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*d+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b*c*d-6*EllipticF(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*c*(c*d)^(1/2)*(a*b)^(1/2)*((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)+6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*2^(1/2)*b^2*c*x^2*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*
a*b^2*c*d*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^
(1/2))*a*b^2*c*d*x^2-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipti
cPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b
)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c-((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c+((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/
2)*b^2*c*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*x^2+6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*d*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(
1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*d*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)*(a*b)^(1/2)*(c*d)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*d*x^2-6*Ell
ipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b*d*x^2
```

```
*(c*d)^(1/2)*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2))*e*(e*x)^(1/2)/x/(-d*x^
2+c)^(1/2)/(-b*x^2+a)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1
/2)*d)/(a*b)^(1/2)/(a*d-b*c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] e^(3/2)*integrate(x^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^(3/2)*e^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{3/2}}{{\left(a-b\,x^2\right)}^2\,{\left(c-d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
[Out] int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.922 $\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=531

$$\frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}}+\frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\sqrt{c-dx^2}}-\frac{\sqrt[4]{d}\,(bc+2ad)\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{2a\sqrt[4]{c}\,(bc-ad)^2\sqrt{c-dx^2}}$$

```
[Out] 1/2*d*(2*a*d+b*c)*(e*x)^(3/2)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(1/2)+1/2*b*(e*
x)^(3/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(1/2)-1/2*d^(1/4)*(2*a*d+b*c)
*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)
/a/c^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*d^(1/4)*(2*a*d+b*c)*EllipticF(
d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/c^(1/4)/
(-a*d+b*c)^2/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-7*a*d+b*c)*EllipticPi(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*e^(1
/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/4*c^(
1/4)*(-7*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^
(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/
(-a*d+b*c)^2/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.76, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 483, 593, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{\sqrt{b}\,\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-7ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{\sqrt{b}\,\sqrt[4]{c}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-7ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(2ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt[4]{c}\,\sqrt{c-dx^2}\,(bc-ad)^2}-\frac{\sqrt[4]{d}\,\sqrt{e}\,\sqrt{1-\frac{dx^2}{c}}\,(2ad+bc)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt[4]{c}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{d(ex)^{3/2}(2ad+bc)}{2ace\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{b(ex)^{3/2}}{2ae\left(a-bx^2\right)\sqrt{c-dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*(e*x)^(3/2))/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x^2]) + (b*
(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*x^2]) - (d^(1/4)*(b*
c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (d^
(1/4)*(b*c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^
2]) - (Sqrt[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2]) + (Sqrt
[b]*c^(1/4)*(b*c - 7*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^2*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

d, 0]

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)\sqrt{c-dx^2}} + \frac{e\text{Subst}\left(\int \frac{x^2\left(\frac{bc-4ad}{e^2}-\frac{3bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right.}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)\sqrt{c-dx^2}} - \frac{e^3\text{Subst}\left(\right.}{}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)\sqrt{c-dx^2}} - \frac{e^3\text{Subst}\left(\right.}{}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)\sqrt{c-dx^2}} + \frac{(b(bc-7a}{}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)\sqrt{c-dx^2}} + \frac{\left(\sqrt{d}(bc-\right.}{}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)\sqrt{c-dx^2}} + \frac{\left(\sqrt{d}(bc\right.}{}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)\sqrt{c-dx^2}} + \frac{\sqrt[4]{d}(bc+}{}$$

$$= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)\sqrt{c-dx^2}} - \frac{\sqrt[4]{d}(bc+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.25, size = 230, normalized size = 0.43

$$\frac{\sqrt{ex}\left(21ax(-2a^2d^2+2abd^2x^2+b^2c(-c+dx^2))+7(-b^2c^2+8abcd+2a^2d^2)x(a-bx^2)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bd(bc+2ad)x^3(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a^2c(bc-ad)^2(-a+bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[e*x]*(21*a*x*(-2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(-c + d*x^2)) + 7*(-
(b^2*c^2) + 8*a*b*c*d + 2*a^2*d^2)*x*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*(b*c + 2*a*d)*x^3*(-a +
b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a
]))/(42*a^2*c*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2937 vs. $2(415) = 830$.
time = 0.13, size = 2938, normalized size = 5.53

method	result
elliptic	$\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\frac{b^2x\sqrt{-de\,x^3+cex}}{2(ad-bc)^2a\left(-b\,x^2+a\right)}+\frac{d^2e\,x^2}{c(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}}}{2}\right.$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(4*(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*b^3*c^3*x^2-8*a^2*b*d^3*x^4-((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^2
*c^2*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*b^2*c^2*x^2-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
```

```
*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b
+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d*x^2-7*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^2*d*x^2+8*a^3*d^3*x^2-((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^3+8*((d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))*a^3*c*d^2-4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*
((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^3-4*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))*a^3*c*d^2+2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^3-((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/(
(c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^2*c^3-8*((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a^2*b*c*d^2*x^2+4*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d*x^2+4*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*a^2*b*c*d^2*x^2-2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*c^2*d*x^2-4*b^3*c^3*x^2-2*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),1/2*2^(1/2))*b^3*c^3*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)
^(1/2)*d),1/2*2^(1/2))*b^3*c^3*x^2+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellip
ticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a
*b)^(1/2)*d),1/2*2^(1/2))*b^3*c^3*x^2+4*a*b^2*c*d^2*x^4+4*a*b^2*c^2*d*x^2+7
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/
2))*(c*d)^(1/2)*a*b*c*d*x^2-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)
```

```
*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c*d*x^2-8*a^2*b*c*d^2*x^2+
4*b^3*c^2*d*x^4-7*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi
(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(
1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*c*d+((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b*c^2+7*((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt(x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*e^(1/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}}{{\left(a-b\,x^2\right)}^2\,{\left(c-d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

```
[Out] int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.923 $\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=426

$$\frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}}+\frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,\sqrt{c-dx^2}}+\frac{d^{3/4}(bc+2ad)\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right.\right.}{2ac^{3/4}(bc-ad)^2\sqrt{e}\,\sqrt{c-dx^2}}$$

```
[Out] 1/2*d*(2*a*d+b*c)*(e*x)^(1/2)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(1/2)+1/2*b*(e*
x)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(1/2)+1/2*d^(3/4)*(2*a*d+b*c)
*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(3/
4)/(-a*d+b*c)^2/e^(1/2)/(-d*x^2+c)^(1/2)+3/4*b*c^(1/4)*(-3*a*d+b*c)*Ellipti
cPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)
*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)^2/e^(1/2)/(-d*x^2+c)^(1/2)+3/4*b*
c^(1/4)*(-3*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)
*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)^2/e^(1
/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 425, 541, 537, 230, 227, 418, 1233, 1232}

$$\frac{3b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-3ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{3b\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(bc-3ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,(2ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{2ac^{3/4}\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)^2}+\frac{b\sqrt{ex}}{2ae\,(a-bx^2)\,\sqrt{c-dx^2}\,(bc-ad)}+\frac{d\sqrt{ex}\,(2ad+bc)}{2ace\sqrt{c-dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*Sqrt[e*x])/(2*a*c*(b*c - a*d)^2*e*Sqrt[c - d*x^2]) + (b*Sq
rt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*Sqrt[c - d*x^2]) + (d^(3/4)*(b*c +
2*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(2*a*c^(3/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^
(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*
d^(1/4)*(b*c - a*d)^2*Sqrt[e]*Sqrt[c - d*x^2]) + (3*b*c^(1/4)*(b*c - 3*a*d)
*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)^2*
Sqrt[e]*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\sqrt{c-dx^2}} + \frac{e\text{Subst}\left(\int \frac{\frac{3bc-4ad}{e^2}-\frac{5bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x\right.}{2a(bc-ad)}$$

$e^3\text{Su}$

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\sqrt{c-dx^2}} - \text{—}$$

$(3b($

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\sqrt{c-dx^2}} + \text{—}$$

$(3b($

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\sqrt{c-dx^2}} + \text{—}$$

$d^{3/4}$

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\sqrt{c-dx^2}} + \text{—}$$

$d^{3/4}$

$$= \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\sqrt{c-dx^2}} + \text{—}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.20, size = 229, normalized size = 0.54

$$\frac{5ax(-2a^2d^2+2abd^2x^2+b^2c(-c+dx^2))+5(3b^2c^2-8abcd+2a^2d^2)\,x(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bd(bc+2ad)x^3(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{10a^2c(bc-ad)^2\sqrt{ex}\,(-a+bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (5*a*x*(-2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(-c + d*x^2)) + 5*(3*b^2*c^2 - 8
*a*b*c*d + 2*a^2*d^2)*x*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2,
 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*(b*c + 2*a*d)*x^3*(a - b*x^2)*Sqrt[1 -
 (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(10*a^2*c*(b*
c - a*d)^2*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2541 vs. $2(338) = 676$.
time = 0.14, size = 2542, normalized size = 5.97

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)\,ex}\left(\frac{b^2\sqrt{-de\,x^3+cex}}{2(ad-bc)^2ae\left(-b\,x^2+a\right)}+\frac{d^2x}{c(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+}}{4\sqrt{-d}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*b*d*(3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2
))*a*b^3*c^3+8*a^3*d^3*x*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)-8*a^2*b*d^3*x^3*(a*b)^(1/2)+4*b^3*c^2*d*x^3*(a*b)^(1/2
)-4*b^3*c^3*x*(a*b)^(1/2)-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/
2)*d),1/2*2^(1/2))*a*b^3*c^3+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2
)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^
(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2
^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^3*x^2+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(
```

1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+9*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-
(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*2^(1/2)*a*b^2*c*d*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)+4*a*b^2*c*d^2*x^3*(a*b)^(1/2)-8*a^2*b*c*d^2*x*(a*b)^(1/2)+4*a*b
^2*c^2*d*x*(a*b)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2
d((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)+4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/
((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*2^(1/2)*a*b^2*c^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+9
*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2
)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^2*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)-4*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1
/2)*a^2*b*d^2*x^2*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-9*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/2)*((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*
b)^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-2*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b*c*d*(a*b)^(1/2)*(c*d)^(1/
2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^
3*c^2*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2+3*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)

```
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d
)^(1/2)*a*b^2*c^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(-1/2)*integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,x}\,\left(a-b\,x^2\right)^2\,\left(c-d\,x^2\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
[Out] int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.924 $\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=628

$$\frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\,\sqrt{c-dx^2}}+\frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\sqrt{c-dx^2}}-\frac{(5b^2c^2-8abcd+6a^2d^2)\sqrt{c-dx^2}}{2a^2c^2(bc-ad)^2e\sqrt{ex}}$$

```
[Out] 1/2*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/e/(e*x)^(1/2)/(-d*x^2+c)^(1/2)+1/2*b/a/(
-a*d+b*c)/e/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)-1/2*(6*a^2*d^2-8*a*b*c*
d+5*b^2*c^2)*(-d*x^2+c)^(1/2)/a^2/c^2/(-a*d+b*c)^2/e/(e*x)^(1/2)-1/2*d^(1/4
)*(6*a^2*d^2-8*a*b*c*d+5*b^2*c^2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)^2/e^(3/2)/(-d*x^2+c)^(1/2)
+1/2*d^(1/4)*(6*a^2*d^2-8*a*b*c*d+5*b^2*c^2)*EllipticF(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)^2/e^(3/2)/(-d*x
^2+c)^(1/2)-1/4*b^(3/2)*c^(1/4)*(-11*a*d+5*b*c)*EllipticPi(d^(1/4)*(e*x)^(1
/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a
^(5/2)/d^(1/4)/(-a*d+b*c)^2/e^(3/2)/(-d*x^2+c)^(1/2)+1/4*b^(3/2)*c^(1/4)*(-
11*a*d+5*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2
)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/(-a*d+b*c)^2/e^(3/2)
/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.94, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {477, 483, 593, 597, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(5bc-11ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}\,e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(6a^2d^2-8abcd+5b^2c^2)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(6a^2d^2-8abcd+5b^2c^2)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{5/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{c-dx^2}(6a^2d^2-8abcd+5b^2c^2)}{2a^2c^2e\sqrt{ex}(bc-ad)^2}+\frac{b}{2ae\sqrt{ex}(a-bx^2)\sqrt{c-dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ace\sqrt{ex}\sqrt{c-dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*Sqrt[c - d*x^2]) + b/(2*
a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]) - ((5*b^2*c^2 - 8*a*
b*c*d + 6*a^2*d^2)*Sqrt[c - d*x^2])/(2*a^2*c^2*(b*c - a*d)^2*e*Sqrt[e*x]) -
 (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*EllipticE
[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(5/4)*(b*c -
a*d)^2*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d
^2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*a^2*c^(5/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2]) - (b^(3/2)
*c^(1/4)*(5*b*c - 11*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c]
)/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
4*a^(5/2)*d^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)
*(5*b*c - 11*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]
```

```
*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(5/2)*d
^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\sqrt{c-dx^2}} + \frac{e\text{Subst}\left(\int \frac{\frac{5bc-4ad}{e^2}-\frac{7bdx^4}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)}\right.}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\,\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\sqrt{c-d}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\,\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\sqrt{c-d}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\,\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\sqrt{c-d}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\,\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\sqrt{c-d}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\,\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\sqrt{c-d}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\,\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\sqrt{c-d}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\,\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\sqrt{c-d}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.34, size = 319, normalized size = 0.51

$$\frac{x\left(21a(2a^3d^2(2c-3dx^2)-5b^3c^2x^2(c-dx^2)+4ab^2c(c^2+cdx^2-2d^2x^4)+2a^2bd(-4c^2+2cdx^2+3d^2x^4))+7(-5b^3c^3+16ab^2c^2d-8a^2bcd^2+6a^3d^3)\,x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bd(5b^2c^2-8abcd+6a^2d^2)\,x^4(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a^3c^2(bc-ad)^2(ex)^{3/2}\,(-a+bx^2)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*(21*a*(2*a^3*d^2*(2*c - 3*d*x^2) - 5*b^3*c^2*x^2*(c - d*x^2) + 4*a*b^2*c
*(c^2 + c*d*x^2 - 2*d^2*x^4) + 2*a^2*b*d*(-4*c^2 + 2*c*d*x^2 + 3*d^2*x^4))
+ 7*(-5*b^3*c^3 + 16*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 6*a^3*d^3)*x^2*(a - b*x^
2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3
*b*d*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*x^4*(-a + b*x^2)*Sqrt[1 - (d*x^2)/
c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^3*c^2*(b*c - a
*d)^2*(e*x)^(3/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3372 vs. $2(506) = 1012$.
time = 0.14, size = 3373, normalized size = 5.37

method	result	size
elliptic	Expression too large to display	1349
default	Expression too large to display	3373

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(11*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1
/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d*x^2-11*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d*x^2+4
8*a^3*b*c^2*d^2-48*a^2*b^2*c^3*d+20*b^4*c^3*d*x^4-24*a^3*b*d^4*x^4-24*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))*a^3*b*c*d^3*x^2+56*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellip
ticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d^2*x^2
-52*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^3*d*x^2+12*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
```

^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a^3*b*c*d^
3*x^2-28*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*a^2*b^2*c^2*d^2*x^2+26*((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^
(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a
*b^3*c^3*d*x^2-5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1
/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^3*c^3*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1
/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^3*c^3*x^2-11
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^
3*d*x^2-11*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)
)*a*b^3*c^3*d*x^2+5*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Elliptic
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)
^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^3-5*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^3+24*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*a^4*c*d^3-20*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Ellipt
icE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^4-12*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))*a^4*c*d^3+10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b^3*c^4-5*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^4+20*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))*b^4*c^4*x^2-10*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticF(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^4*c^4*x^2+5*((d*x+(c*d)^(1/

```
2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^4*c^4*x^2+5*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(-3/2)*integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*x^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x)^{3/2}\,(a-b\,x^2)^2\,(c-d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
[Out] int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.925 $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=512

$$\frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}}+\frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}}-\frac{\left(7b^2c^2-8abcd+10a^2d^2\right)\sqrt{c-}}{6a^2c^2(bc-ad)^2e(ex)^{3/2}}$$

```
[Out] 1/2*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/e/(e*x)^(3/2)/(-d*x^2+c)^(1/2)+1/2*b/a/(
-a*d+b*c)/e/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2)-1/6*(10*a^2*d^2-8*a*b*c
*d+7*b^2*c^2)*(-d*x^2+c)^(1/2)/a^2/c^2/(-a*d+b*c)^2/e/(e*x)^(3/2)+1/6*d^(3/
4)*(10*a^2*d^2-8*a*b*c*d+7*b^2*c^2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(7/4)/(-a*d+b*c)^2/e^(5/2)/(-d*x^2+c)^(1/
2)+1/4*b^2*c^(1/4)*(-13*a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/(-
a*d+b*c)^2/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*b^2*c^(1/4)*(-13*a*d+7*b*c)*Ellipti
cPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*
(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/(-a*d+b*c)^2/e^(5/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 483, 593, 597, 537, 230, 227, 418, 1233, 1232}

$$\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-13ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-13ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{7/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^2}-\frac{\sqrt{c-dx^2}\left(10a^2d^2-8abcd+7b^2c^2\right)}{6a^2c^2e(ex)^{3/2}(bc-ad)^2}+\frac{b}{2ae(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2ace(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*Sqrt[c - d*x^2]) + b/(
2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]) - ((7*b^2*c^2 -
8*a*b*c*d + 10*a^2*d^2)*Sqrt[c - d*x^2])/(6*a^2*c^2*(b*c - a*d)^2*e*(e*x)^(
3/2)) + (d^(3/4)*(7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*Sqrt[1 - (d*x^2)/c]*E
llipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(7/4)
*(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13*a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^2*e
^(5/2)*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(7*b*c - 13*a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^2*e^(5/2)*Sqrt[c - d*x
^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
```

```
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{3/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}} + \frac{e\text{Subst}\left(\int \frac{\frac{7bc-4ad}{e^2}-\frac{9bdx^4}{e^4}}{x^4\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right.}\right.}{2a(bc-a}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c}}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order

4 in optimal.
time = 10.37, size = 318, normalized size = 0.62

$$\frac{x\left(5a(2a^3d^2(2c-5dx^2)-7b^3c^2x^2(c-dx^2)+4ab^2c(c^2+cdx^2-2d^2x^4)+2a^2bd(-4c^2+2cdx^2+5d^2x^4))+5(21b^3c^3-32ab^2c^2d-8a^2bcd^2+10a^3d^3)\,x^2(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bd(7b^2c^2-8abcd+10a^2d^2)\,x^4(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{30a^3c^2(bc-ad)^2(ex)^{5/2}\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

```
[Out] (x*(5*a*(2*a^3*d^2*(2*c - 5*d*x^2) - 7*b^3*c^2*x^2*(c - d*x^2) + 4*a*b^2*c*
(c^2 + c*d*x^2 - 2*d^2*x^4) + 2*a^2*b*d*(-4*c^2 + 2*c*d*x^2 + 5*d^2*x^4)) +
 5*(21*b^3*c^3 - 32*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 10*a^3*d^3)*x^2*(-a + b*x
^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] +
b*d*(7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c
]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(30*a^3*c^2*(b*c - a*d
)^2*(e*x)^(5/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2858 vs. $2(418) = 836$.
time = 0.13, size = 2859, normalized size = 5.58

method	result
elliptic	$\sqrt{\left(-d\,x^{2}+c\right)ex}\,\left(\frac{b^{3}\sqrt{-de\,x^{3}+cex}}{2(ad-bc)^{2}a^{2}e^{3}\left(-b\,x^{2}+a\right)}+\frac{d^{3}x}{e^{2}c^{2}(ad-bc)^{2}\sqrt{-\left(x^{2}-\frac{c}{d}\right)dex}}-\frac{2\sqrt{-de\,x^{3}+cex}}{3c^{2}e^{3}a^{2}x^{2}}+\frac{\sqrt{cd}\,\sqrt{\cdots}}{\cdots}\right.$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*b*d*(-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^3*d*x^3*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)+48*a^3*b*c^2*d^2*(a*b)^(1/2)+40*a^4*d^4*x^2*(a*b)^(1/
2)+30*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*
a^2*b^2*c^2*d*x*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-48*a^2*b
^2*c^3*d*(a*b)^(1/2)-40*a^3*b*d^4*x^4*(a*b)^(1/2)+28*b^4*c^3*d*x^4*(a*b)^(1
/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
```

)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^5*c^4*x^3*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^5*c^4*x^3*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)+72*a^2*b^2*c*d^3*x^4*(a*b)^(1/2)-60*a*b^3*c^2*d^2*x^4*(a*b)
^(1/2)-56*a^3*b*c*d^3*x^2*(a*b)^(1/2)+44*a*b^3*c^3*d*x^2*(a*b)^(1/2)-21*Ell
ipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+
(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^4*c^3*x^3*(c*d)^(1/2)*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)*(a*b)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)
,(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^3
*d*x^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)
*b^4*c^3*x^3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+14*Elliptic
F(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^4*c^3*x^3*(c
d)^(1/2)((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+39*EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2
^(1/2))*2^(1/2)*a^2*b^3*c^3*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-39*EllipticPi(((
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^3*c^3*d*x*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+20*Ell
ipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^4*d^3*x
*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d
)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)-28*b^4*c^4*x^2*(a*b)^(1
/2)-16*a^4*c*d^3*(a*b)^(1/2)+16*a*b^3*c^4*(a*b)^(1/2)-20*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*b*d^3*x^3*(c*d)^(1/2
)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*
2^(1/2)*a*b^3*c^3*x*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+21*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^3*x*(c*d)^(1/2)*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)*(a*b)^(1/2)-14*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),1/2*2^(1/2))*2^(1/2)*a*b^3*c^3*x*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(
a*b)^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c^2*d*x*(c*d)^
(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2)

```
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/
2))*2^(1/2)*a*b^3*c^2*d*x^3*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1
/2)-21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^4*x*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)+21*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^4*x*((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2*d*x^3*(
c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(-5/2)*integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*x^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x)^{5/2}\,(a-b\,x^2)^2\,(c-d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

```
[Out] int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.926 $\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}} \, dx$

Optimal. Leaf size=568

$$\frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}}+\frac{ae^3(ex)^{3/2}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}+\frac{(bc+4ad)e^3(ex)^{3/2}}{2(bc-ad)^3\sqrt{c-dx^2}}-\frac{c^{3/4}(bc+4ad)e^{9/2}\sqrt{}}{2d^3}$$

```
[Out] 1/6*(3*a*d+2*b*c)*e^3*(e*x)^(3/2)/b/(-a*d+b*c)^2/(-d*x^2+c)^(3/2)+1/2*a*e^3
*(e*x)^(3/2)/b/(-a*d+b*c)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/2*(4*a*d+b*c)*e^3*(
e*x)^(3/2)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)-1/2*c^(3/4)*(4*a*d+b*c)*e^(9/2)*El
lipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(3/4)/(-
a*d+b*c)^3/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*(4*a*d+b*c)*e^(9/2)*EllipticF(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(3/4)/(-a*d+b*c)^3/(-
d*x^2+c)^(1/2)+1/4*c^(1/4)*(3*a*d+7*b*c)*e^(9/2)*EllipticPi(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c
)^(1/2)/d^(1/4)/(-a*d+b*c)^3/b^(1/2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(3*a*d+7*
b*c)*e^(9/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)
/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)^3/b^(1/2)/
(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.93, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 481, 593, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3}-\frac{c^{3/4}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3ad+7bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{a}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}(3ad+7bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{e^3(ex)^{3/2}(4ad+bc)}{2\sqrt{c-dx^2}(bc-ad)^3}+\frac{e^3(ex)^{3/2}(3ad+2bc)}{6b\left(c-dx^2\right)^{3/2}(bc-ad)^2}+\frac{ae^3(ex)^{3/2}}{2b\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] ((2*b*c + 3*a*d)*e^3*(e*x)^(3/2))/(6*b*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (
a*e^3*(e*x)^(3/2))/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + ((b*c
+ 4*a*d)*e^3*(e*x)^(3/2))/(2*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (c^(3/4)*(b*c
 + 4*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (c^(3/
4)*(b*c + 4*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*d^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) +
 (Sqrt[a]*c^(1/4)*(7*b*c + 3*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-(
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*Sqrt[b]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (Sqrt[a]
*c^(1/4)*(7*b*c + 3*a*d)*e^(9/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*Sqrt[b]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
```

```
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^{10}}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} - \frac{e^3\text{Subst}\left(\int \frac{x^2\left(3ac+\frac{(4bc+3ad)x^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{e\ldots}\right.}{2b(bc-ad)}$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{e^5\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{(bc+4a\ldots}{2(bc-ad)\ldots}$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{(bc+4a\ldots}{2(bc-ad)\ldots}$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{(bc+4a\ldots}{2(bc-ad)\ldots}$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{(bc+4a\ldots}{2(bc-ad)\ldots}$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{(bc+4a\ldots}{2(bc-ad)\ldots}$$

$$= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{(bc+4a\ldots}{2(bc-ad)\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.30, size = 256, normalized size = 0.45

$$-\frac{e^3(ex)^{3/2}\left(7a(a^2d(7c-9dx^2)+b^2cx^2(-5c+3dx^2)+4ab(2c^2-4cdx^2+3d^2x^4))+7a(8bc+7ad)\left(-a+bx^2\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3b(bc+4ad)x^2(a-bx^2)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a(bc-ad)^3\left(-a+bx^2\right)\left(c-dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] -1/42*(e^3*(e*x)^(3/2)*(7*a*(a^2*d*(7*c - 9*d*x^2) + b^2*c*x^2*(-5*c + 3*d*
x^2) + 4*a*b*(2*c^2 - 4*c*d*x^2 + 3*d^2*x^4)) + 7*a*(8*b*c + 7*a*d)*(-a + b
*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c,
 (b*x^2)/a] + 3*b*(b*c + 4*a*d)*x^2*(a - b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2
)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(a*(b*c - a*d)^3*(
-a + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5113 vs. $2(446) = 892$.
time = 0.15, size = 5114, normalized size = 9.00

method	result	size
elliptic	Expression too large to display	1409
default	Expression too large to display	5114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(9/2)*integrate(x^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(x^(9/2)*e^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{9/2}}{\left(a-b\,x^2\right)^2\,\left(c-d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
[Out] int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.927 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=454

$$\frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2\left(c-dx^2\right)^{3/2}}+\frac{ae^3\sqrt{ex}}{2b(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}+\frac{5(bc+2ad)e^3\sqrt{ex}}{6(bc-ad)^3\sqrt{c-dx^2}}+\frac{5\sqrt[4]{c}\,(bc+2ad)e^{7/2}\sqrt{}}{6\sqrt[4]{d}}$$

```
[Out] 1/6*(3*a*d+2*b*c)*e^3*(e*x)^(1/2)/b/(-a*d+b*c)^2/(-d*x^2+c)^(3/2)+1/2*a*e^3
*(e*x)^(1/2)/b/(-a*d+b*c)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+5/6*(2*a*d+b*c)*e^3*(
e*x)^(1/2)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)+5/6*c^(1/4)*(2*a*d+b*c)*e^(7/2)*El
lipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-
a*d+b*c)^3/(-d*x^2+c)^(1/2)-5/4*c^(1/4)*(a*d+b*c)*e^(7/2)*EllipticPi(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/
c)^(1/2)/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)-5/4*c^(1/4)*(a*d+b*c)*e^(7/2
)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^
(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 481, 541, 537, 230, 227, 418, 1233, 1232}

$$\frac{5\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(2ad+bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{6\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^3}-\frac{5\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^3}-\frac{5\sqrt[4]{c}\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{5e^3\sqrt{ex}\,(2ad+bc)}{6\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{e^3\sqrt{ex}\,(3ad+2bc)}{6b\left(c-dx^2\right)^{3/2}(bc-ad)^2}+\frac{ae^3\sqrt{ex}}{2b\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] ((2*b*c + 3*a*d)*e^3*Sqrt[e*x])/(6*b*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (a*
e^3*Sqrt[e*x])/(2*b*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (5*(b*c +
2*a*d)*e^3*Sqrt[e*x])/(6*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (5*c^(1/4)*(b*c +
 2*a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(6*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/
4)*(b*c + a*d)*e^(7/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*d
^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (5*c^(1/4)*(b*c + a*d)*e^(7/2)*Sqrt
[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*d^(1/4)*(b*c - a*d)^3*Sqrt[c -
d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
```

```
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^8}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} - \frac{e^3\text{Subst}\left(\int \frac{ac+\frac{(4bc+5ad)x^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{e\right.}{2b(bc-ad)}$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{e^5\text{Subst}\left(\right.}{}$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{5(bc+2a}{6(bc-ad)}$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{5(bc+2a}{6(bc-ad)}$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{5(bc+2a}{6(bc-ad)}$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{5(bc+2a}{6(bc-ad)}$$

$$= \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{5(bc+2a}{6(bc-ad)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.29, size = 252, normalized size = 0.56

$$\frac{e^3\sqrt{ex}\left(a(b^2cx^2(7c-5dx^2)+a^2d(-5c+7dx^2)-2ab(5c^2-8cdx^2+5d^2x^4))+5a(2bc+ad)\left(a-bx^2\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+b(bc+2ad)x^2(a-bx^2)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{6a(bc-ad)^3\left(-a+bx^2\right)\left(c-dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (e^3*Sqrt[e*x]*(a*(b^2*c*x^2*(7*c - 5*d*x^2) + a^2*d*(-5*c + 7*d*x^2) - 2*a
*b*(5*c^2 - 8*c*d*x^2 + 5*d^2*x^4)) + 5*a*(2*b*c + a*d)*(a - b*x^2)*(c - d*
x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] +
 b*(b*c + 2*a*d)*x^2*(a - b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5
/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(6*a*(b*c - a*d)^3*(-a + b*x^2)*(c
 - d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4390 vs. $2(360) = 720$.
time = 0.14, size = 4391, normalized size = 9.67

method	result
elliptic	$\sqrt{ex}\,\sqrt{(-d\,x^2+c)\,ex}\left(\frac{d\,e^3ab\sqrt{-de\,x^3+cex}}{2(ad-bc)\left(a^2d^2-2abcd+b^2c^2\right)\left(bd\,x^2-ad\right)}+\frac{c\,e^3\sqrt{-de\,x^3+cex}}{3d^2(ad-bc)^2\left(x^2-\frac{c}{d}\right)^2}-\frac{e^4x(7ad+5\ldots}{6\left(a^2d^2-2abcd+b^2c^2\right)(ad-bc)\sqrt{\ldots}}\right.$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*e^3*(e*x)^(1/2)*b*(28*b^3*c^3*d*x^3*(a*b)^(1/2)+20*a^3*c*d^3*x*(a*b)^(
1/2)+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/
2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d*x^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*
b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d*x^2-15
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/
```

2))*(c*d)^(1/2)*a*b^2*c*d^2*x^4-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c*d^2*x^4+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*b*c*d^2*x^2+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*b*c*d^2*x^2-20*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*d^3*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^3*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+20*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*c*d^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3*d*x^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^3*d*x^2+20*a^2*b*c^2*d^2*x*(a*b)^(1/2)-40*a*b^2*c^3*d*x*(a*b)^(1/2)-20*a*b^2*c*d^3*x^5*(a*b)^(1/2)-36*a^2*b*c*d^3*x^3*(a*b)^(1/2)+36*a*b^2*c^2*d^2*x^3*(a*b)^(1/2)+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^3*d^3*x^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^3*d^3*x^2-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^3*b*c*d^3*x^2-30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c^2*d^2*x^2+15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*Elliptic

```
Pi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)
^(1/2)*d),1/2*2^(1/2))*a^3*b*c*d^3*x^2+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a^2*b^2*c^2*d^2*x^2-15*((d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^3*
c*d^2-15*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] e^(7/2)*integrate(x^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(x^(7/2)*e^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{7/2}}{(a-b\,x^2)^2\,(c-d\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
[Out] int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.928 $\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=551

$$\frac{5de(ex)^{3/2}}{6(bc-ad)^2(c-dx^2)^{3/2}}+\frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}}+\frac{d(4bc+ad)e(ex)^{3/2}}{2c(bc-ad)^3\sqrt{c-dx^2}}-\frac{\sqrt[4]{d}\,(4bc+ad)e^{5/2}\sqrt{1}}{2\sqrt[4]{c}\,(}$$

```
[Out] 5/6*d*e*(e*x)^(3/2)/(-a*d+b*c)^2/(-d*x^2+c)^(3/2)+1/2*e*(e*x)^(3/2)/(-a*d+b
*c)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/2*d*(a*d+4*b*c)*e*(e*x)^(3/2)/c/(-a*d+b*c
)^3/(-d*x^2+c)^(1/2)-1/2*d^(1/4)*(a*d+4*b*c)*e^(5/2)*EllipticE(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/c^(1/4)/(-a*d+b*c)^3/(-d*x^2+c
)^(1/2)+1/2*d^(1/4)*(a*d+4*b*c)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/c^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)+1/4*c
^(1/4)*(7*a*d+3*b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
,-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*
d+b*c)^3/a^(1/2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(7*a*d+3*b*c)*e^(5/2)*Ellipti
cPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*
b^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)^3/a^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.89, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 482, 593, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$\frac{\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+4bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}\,(bc-ad)^3}-\frac{\sqrt[4]{d}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+4bc)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{\sqrt{b}\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(7ad+3bc)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}\,(bc-ad)^3}-\frac{\sqrt{b}\sqrt[4]{c}\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,(7ad+3bc)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{de(ex)^{3/2}(ad+4bc)}{2c\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{5de(ex)^{3/2}}{6\left(c-dx^2\right)^{3/2}(bc-ad)^2}+\frac{e(ex)^{3/2}}{2\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (5*d*e*(e*x)^(3/2))/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*(e*x)^(3/2))/(
2*(b*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(4*b*c + a*d)*e*(e*x)^(3/
2))/(2*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (d^(1/4)*(4*b*c + a*d)*e^(5/2)*Sq
rt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(2*c^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(4*b*c + a*d)*e^(
5/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(2*c^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(1/4)*(3
*b*c + 7*a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(S
qrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sq
rt[a]*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (Sqrt[b]*c^(1/4)*(3*b*c + 7*
a*d)*e^(5/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt
[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*Sqrt[a]*d^(1/4
)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
```

```
s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^6}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} - \frac{e\text{Subst}\left(\int \frac{x^2\left(3c+\frac{7dx^4}{e^2}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right.}{2(bc-ad)}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2(c-dx^2)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{e^3\text{Subst}\left(\int\right.}{}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2(c-dx^2)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{d(4bc+ad}{2c(bc-ad)^3}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2(c-dx^2)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{d(4bc+ad}{2c(bc-ad)^3}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2(c-dx^2)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{d(4bc+ad}{2c(bc-ad)^3}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2(c-dx^2)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{d(4bc+ad}{2c(bc-ad)^3}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2(c-dx^2)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{d(4bc+ad}{2c(bc-ad)^3}$$

$$= \frac{5de(ex)^{3/2}}{6(bc-ad)^2(c-dx^2)^{3/2}} + \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{d(4bc+ad}{2c(bc-ad)^3}$$

$$5de(ex)^{3/2} \qquad e(ex)^{3/2} \qquad d(4bc+ad$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.34, size = 278, normalized size = 0.50

$$-\frac{e(ex)^{3/2}\left(7a(a^2d^2(c-3dx^2)+abd(11c^2-10cdx^2+3d^2x^4)+b^2c(3c^2-17cdx^2+12d^2x^4))+7(3b^2c^2+11abcd+a^2d^2)\left(-a+bx^2\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bd(4bc+ad)x^2(a-bx^2)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42ac(bc-ad)^3\left(-a+bx^2\right)\left(c-dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] -1/42*(e*(e*x)^(3/2)*(7*a*(a^2*d^2*(c - 3*d*x^2) + a*b*d*(11*c^2 - 10*c*d*x
^2 + 3*d^2*x^4) + b^2*c*(3*c^2 - 17*c*d*x^2 + 12*d^2*x^4)) + 7*(3*b^2*c^2 +
 11*a*b*c*d + a^2*d^2)*(-a + b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF
1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*(4*b*c + a*d)*x^2*(a - b*
x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c,
 (b*x^2)/a]))/(a*c*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5065 vs. $2(429) = 858$.
time = 0.14, size = 5066, normalized size = 9.19

method	result	size
elliptic	Expression too large to display	1393
default	Expression too large to display	5066

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(x^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)*e^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{5/2}}{{\left(a-b\,x^2\right)}^2\,{\left(c-d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

```
[Out] int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.929 $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=447

$$\frac{5de\sqrt{ex}}{6(bc-ad)^2\,(c-dx^2)^{3/2}}+\frac{e\sqrt{ex}}{2(bc-ad)\,(a-bx^2)\,(c-dx^2)^{3/2}}+\frac{d(14bc+ad)e\sqrt{ex}}{6c(bc-ad)^3\sqrt{c-dx^2}}+\frac{d^{3/4}(14bc+ad)e^{3/2}\sqrt{1}}{6c^{3/4}}$$

```
[Out] 5/6*d*e*(e*x)^(1/2)/(-a*d+b*c)^2/(-d*x^2+c)^(3/2)+1/2*e*(e*x)^(1/2)/(-a*d+b
*c)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/6*d*(a*d+14*b*c)*e*(e*x)^(1/2)/c/(-a*d+b*
c)^3/(-d*x^2+c)^(1/2)+1/6*d^(3/4)*(a*d+14*b*c)*e^(3/2)*EllipticF(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/c^(3/4)/(-a*d+b*c)^3/(-d*x^2
+c)^(1/2)-1/4*b*c^(1/4)*(9*a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(
1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)-1/4*b*c^(1/4)*(9*a*d+b*c)*e^(3/2)*Ellipt
icPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)
*(1-d*x^2/c)^(1/2)/a/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 482, 541, 537, 230, 227, 418, 1233, 1232}

$$\frac{d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(ad+14bc)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{6c^{3/4}\sqrt{c-dx^2}\,(bc-ad)^3}-\frac{b\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(9ad+bc)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^3}-\frac{b\sqrt[4]{c}\,e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,(9ad+bc)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a\sqrt[4]{d}\,\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{de\sqrt{ex}\,(ad+14bc)}{6c\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{5de\sqrt{ex}}{6\left(c-dx^2\right)^{3/2}(bc-ad)^2}+\frac{e\sqrt{ex}}{2\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (5*d*e*Sqrt[e*x])/(6*(b*c - a*d)^2*(c - d*x^2)^(3/2)) + (e*Sqrt[e*x])/(2*(b
*c - a*d)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(14*b*c + a*d)*e*Sqrt[e*x])/(
6*c*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(3/4)*(14*b*c + a*d)*e^(3/2)*Sqrt[1
 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(6*c^(3/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/
2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)
^3*Sqrt[c - d*x^2]) - (b*c^(1/4)*(b*c + 9*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(4*a*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} - \frac{e\text{Subst}\left(\int \frac{c+\frac{9dx^4}{e^2}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right.}{2(bc-ad)}$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e^3\text{Subst}\left(\int\right.}{}$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+a}{6c(bc-ad)^3}$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+a}{6c(bc-ad)^3}$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+a}{6c(bc-ad)^3}$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+a}{6c(bc-ad)^3}$$

$$= \frac{5de\sqrt{ex}}{6(bc-ad)^2\left(c-dx^2\right)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{d(14bc+a}{6c(bc-ad)^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.36, size = 275, normalized size = 0.62

$$\frac{e\sqrt{ex}\left(5a(a^2d^2(c+dx^2)+b^2c(-3c^2+19cdx^2-14d^2x^4)-abd(13c^2-10cdx^2+d^2x^4))-5(-3b^2c^2-13abcd+a^2d^2)\left(a-bx^2\right)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bd(14bc+ad)x^2(a-bx^2)\left(c-dx^2\right)\sqrt{1-\frac{dx^2}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{30ac(bc-ad)^3\left(-a+bx^2\right)\left(c-dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (e*Sqrt[e*x]*(5*a*(a^2*d^2*(c + d*x^2) + b^2*c*(-3*c^2 + 19*c*d*x^2 - 14*d^
2*x^4) - a*b*d*(13*c^2 - 10*c*d*x^2 + d^2*x^4)) - 5*(-3*b^2*c^2 - 13*a*b*c*
d + a^2*d^2)*(a - b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2,
 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*(14*b*c + a*d)*x^2*(a - b*x^2)*(c - d*
x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))
/(30*a*c*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4390 vs. 2(353) = 706.
time = 0.14, size = 4391, normalized size = 9.82

method	result
elliptic	$\sqrt{ex}\,\sqrt{\left(-d\,x^2+c\right)ex}\,\Bigg(\frac{b^2de\sqrt{-de\,x^3+cex}}{2\left(a^2d^2-2abcd+b^2c^2\right)(ad-bc)\left(bd\,x^2-ad\right)}+\frac{e\sqrt{-de\,x^3+cex}}{3(ad-bc)^2d\left(x^2-\frac{c}{d}\right)^2}-\frac{d\,e^2x(ad+11bc}{6c\left(a^2d^2-2abcd+b^2c^2\right)(ad-bc)\sqrt{-}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*e*(e*x)^(1/2)*b*d*(76*b^3*c^3*d*x^3*(a*b)^(1/2)-4*a^3*c*d^3*x*(a*b)^(1
/2)+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c
*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2
*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d*x^2+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^2*d*x^2-27*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2
```

))*(c*d)^(1/2)*a*b^2*c*d^2*x^4-27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(
1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^
(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c*d^2*x^4+27*((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d
)^(1/2)*a^2*b*c*d^2*x^2+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((
-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*E
llipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a^2*b*c*d^2*x^2-12*b^3*c^4*x*(a*b
)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1
/2)*a^3*d^3*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+28*Ellip
ticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b^3*c^3*x^2
*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)+2*EllipticF(((d*x+(c*d)
^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3*c*d^2*((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)*(a*b)^(1/2)*(c*d)^(1/2)-28*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^2*c^3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(
1/2)*(c*d)^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^
(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^3*c^3*x^2+3*((d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^
(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*b^3*c^3*x^2-
30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*
c^3*d*x^2+30*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2
))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/
2))*a*b^3*c^3*d*x^2-3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(a*b)^(1/2)*Ellipt
icPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*
b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^3-3*((d*x+(c*d)^(1/2))/(c*d)^(
1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*(c*d)^(1/2)*a*b^2*c^3+5
6*a^2*b*c^2*d^2*x*(a*b)^(1/2)-40*a*b^2*c^3*d*x*(a*b)^(1/2)+52*a*b^2*c*d^3*x
^5*(a*b)^(1/2)-36*a^2*b*c*d^3*x^3*(a*b)^(1/2)-36*a*b^2*c^2*d^2*x^3*(a*b)^(1
/2)-27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c

```
*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*
d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a^
2*b^2*c^2*d^2*x^2+27*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*EllipticPi(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1
/2*2^(1/2))*a^2*b^2*c^2*d^2*x^2-4*a^3*d^4*x^3*(a*b)^(1/2)+4*a^2*b*d^4*x^5*(
a*b)^(1/2)-56*b^3*c^2*d^2*x^5*(a*b)^(1/2)+3*((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(
1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b^3*c^4-3*((d*x+(c*d)^(1/2))/(c*d)^(1/
2))^(1/2)*2^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] e^(3/2)*integrate(x^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}}{(-a+bx^2)^2\,(c-dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
[Out] Integral((e*x)**(3/2)/((-a + b*x**2)**2*(c - d*x**2)**(5/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)*e^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{3/2}}{\left(a-b\,x^2\right)^2\,\left(c-d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

```
[Out] int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.930 $\int \frac{\sqrt{ex}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=625

$$\frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\left(c-dx^2\right)^{3/2}}+\frac{b(ex)^{3/2}}{2a(bc-ad)e\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}+\frac{d\left(b^2c^2+5abcd-a^2d^2\right)(ex)^{3/2}}{2ac^2(bc-ad)^3e\sqrt{c-dx^2}}-\frac{\sqrt[4]{d}\left(b^2c^2\right.}{}$$

```
[Out] 1/6*d*(2*a*d+3*b*c)*(e*x)^(3/2)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(3/2)+1/2*b*(
e*x)^(3/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/2*d*(-a^2*d^2+5*a*b
*c*d+b^2*c^2)*(e*x)^(3/2)/a/c^2/(-a*d+b*c)^3/e/(-d*x^2+c)^(1/2)-1/2*d^(1/4)
*(-a^2*d^2+5*a*b*c*d+b^2*c^2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
,I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/c^(5/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)+1/2*d
^(1/4)*(-a^2*d^2+5*a*b*c*d+b^2*c^2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/c^(5/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)
-1/4*b^(3/2)*c^(1/4)*(-11*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2
)/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)+1/4*b^(3/2)*c^(1/4)*(-11*a*d+b*c)*E
llipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/
2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/
2)
```

Rubi [A]
time = 0.99, antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {477, 483, 593, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-11ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^{3/2}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-11ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{5/4}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{5/4}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d(ex)^{3/2}\left(-a^2d^2+5abcd+b^2c^2\right)}{2ac^2e\sqrt{c-dx^2}(bc-ad)^3}+\frac{b(ex)^{3/2}}{2ae\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d(ex)^{3/2}(2ad+3bc)}{6ace\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d)*(e*x)^(3/2))/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2)^(3/2)) +
 (b*(e*x)^(3/2))/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(b^
2*c^2 + 5*a*b*c*d - a^2*d^2)*(e*x)^(3/2))/(2*a*c^2*(b*c - a*d)^3*e*Sqrt[c -
 d*x^2]) - (d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2
)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*c^(
5/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*d
^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*a*c^(5/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) - (b^(3/2)*c
^(1/4)*(b*c - 11*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*a^(3/2)*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2]) + (b^(3/2)*c^(1/4)*(b*
```

```
c - 11*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^(3/2)
*d^(1/4)*(b*c - a*d)^3*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
```

```
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b(ex)^{3/2}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{e\text{Subst}\left(\int \frac{x^2\left(\frac{bc-4ad}{e^2}-\frac{7bdx^4}{e^4}\right)}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right.}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} - \frac{e^3\text{Subst}}{}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(b}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(b}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(b}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(b}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(b}$$

$$= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b(ex)^{3/2}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(b^2c^2+}{2ac^2(b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.44, size = 327, normalized size = 0.52

$$\frac{\sqrt{ex}\left(7ax\left(ab^2cd^2x^2(17c-15dx^2)+a^3d^3(5c-3dx^2)-3b^3c^2(c-dx^2)^2+a^2bd^2(-17c^2+10cdx^2+3d^2x^4)\right)+7(b^3c^3-12ab^2c^2d-5a^2bcd^2+a^3d^3)\,x(-a+bx^2)\,(c-dx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bd(b^2c^2+5abcd-a^2d^2)\,x^3(-a+bx^2)\,(c-dx^2)\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a^2c^2(bc-ad)^3\,(-a+bx^2)\,(c-dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[e*x]*(7*a*x*(a*b^2*c*d^2*x^2*(17*c - 15*d*x^2) + a^3*d^3*(5*c - 3*d*x
^2) - 3*b^3*c^2*(c - d*x^2)^2 + a^2*b*d^2*(-17*c^2 + 10*c*d*x^2 + 3*d^2*x^4
)) + 7*(b^3*c^3 - 12*a*b^2*c^2*d - 5*a^2*b*c*d^2 + a^3*d^3)*x*(-a + b*x^2)*
(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^
2)/a] + 3*b*d*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*x^3*(-a + b*x^2)*(c - d*x^2)*
Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42
*a^2*c^2*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5676 vs. 2(503) = 1006.
time = 0.14, size = 5677, normalized size = 9.08

method	result	size
elliptic	Expression too large to display	1518
default	Expression too large to display	5677

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*e^(1/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x}}{{\left(a-b\,x^2\right)}^2\,{\left(c-d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

```
[Out] int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.931 $\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=514

$$\frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}}+\frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}}+\frac{d(3b^2c^2+17abcd-5a^2d^2)\,\sqrt{ex}}{6ac^2(bc-ad)^3e\sqrt{c-dx^2}}+\frac{d^{3/4}(3}{}$$

```
[Out] 1/6*d*(2*a*d+3*b*c)*(e*x)^(1/2)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(3/2)+1/2*b*(
e*x)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/6*d*(-5*a^2*d^2+17*
a*b*c*d+3*b^2*c^2)*(e*x)^(1/2)/a/c^2/(-a*d+b*c)^3/e/(-d*x^2+c)^(1/2)+1/6*d^
(3/4)*(-5*a^2*d^2+17*a*b*c*d+3*b^2*c^2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(7/4)/(-a*d+b*c)^3/e^(1/2)/(-d*x^2+c)^(
1/2)+1/4*b^2*c^(1/4)*(-13*a*d+3*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/
(-a*d+b*c)^3/e^(1/2)/(-d*x^2+c)^(1/2)+1/4*b^2*c^(1/4)*(-13*a*d+3*b*c)*Ellip
ticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I
)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)^3/e^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {477, 425, 541, 537, 230, 227, 418, 1233, 1232}

$$\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,(-5a^2d^2+17abcd+3b^2c^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{6ac^{7/4}\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{b^2\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(3bc-13ad)\Pi\left(-\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{b^2\sqrt[4]{c}\,\sqrt{1-\frac{dx^2}{c}}\,(3bc-13ad)\Pi\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{a}\,\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,\sqrt{ex}}{\sqrt[4]{c}\,\sqrt{e}}\right)\middle|-1\right)}{4a^2\sqrt[4]{d}\,\sqrt{e}\,\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{d\sqrt{ex}\,(-5a^2d^2+17abcd+3b^2c^2)}{6ac^2e\sqrt{c-dx^2}\,(bc-ad)^3}+\frac{b\sqrt{ex}}{2ae\,(a-bx^2)\,(c-dx^2)^{3/2}\,(bc-ad)}+\frac{d\sqrt{ex}\,(2ad+3bc)}{6ace\,(c-dx^2)^{3/2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d)*Sqrt[e*x])/(6*a*c*(b*c - a*d)^2*e*(c - d*x^2)^(3/2)) + (
b*Sqrt[e*x])/(2*a*(b*c - a*d)*e*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(3*b^2*
c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqrt[e*x])/(6*a*c^2*(b*c - a*d)^3*e*Sqrt[c -
d*x^2]) + (d^(3/4)*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqrt[1 - (d*x^2)/c]
*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a*c^(7/4)
*(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13*a*d)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)^3*S
qrt[e]*Sqrt[c - d*x^2]) + (b^2*c^(1/4)*(3*b*c - 13*a*d)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(4*a^2*d^(1/4)*(b*c - a*d)^3*Sqrt[e]*Sqrt[c - d*x
^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
```

```
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{e\text{Subst}\left(\int \frac{\frac{3bc-4ad}{e^2}-\frac{9bdx^4}{e^4}}{\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x\right.}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} - \frac{e^3}{}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(}{}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(}{}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(}{}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(}{}$$

$$= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.39, size = 328, normalized size = 0.64

$$-\frac{5ax\left(3b^3c^2(c-dx^2)^2+a^3d^3(-7c+5dx^2)+ab^2cd^2x^2(-19c+17dx^2)+a^2bd^2(19c^2-10cdx^2-5d^2x^4)\right)-5(-9b^3c^3+36ab^2c^2d-17a^2bcd^2+5a^3d^3)\,x(a-bx^2)\,(c-dx^2)\,\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+bd(3b^2c^2+17abcd-5a^2d^2)\,x^3(-a+bx^2)\,(c-dx^2)\,\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{30a^2c^2(bc-ad)^3\sqrt{ex}\,(-a+bx^2)\,(c-dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] -1/30*(5*a*x*(3*b^3*c^2*(c - d*x^2)^2 + a^3*d^3*(-7*c + 5*d*x^2) + a*b^2*c*
d^2*x^2*(-19*c + 17*d*x^2) + a^2*b*d^2*(19*c^2 - 10*c*d*x^2 - 5*d^2*x^4)) -
 5*(-9*b^3*c^3 + 36*a*b^2*c^2*d - 17*a^2*b*c*d^2 + 5*a^3*d^3)*x*(a - b*x^2)
*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x
^2)/a] + b*d*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*x^3*(-a + b*x^2)*(c - d*x
^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(
a^2*c^2*(b*c - a*d)^3*Sqrt[e*x]*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4763 vs. 2(420) = 840.
time = 0.13, size = 4764, normalized size = 9.27

method	result	size
elliptic	Expression too large to display	1311
default	Expression too large to display	4764

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*b*d*(9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b
/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^5*((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^5*c^5*x^2*((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)
^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^5*((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^5*c^5*x^2*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/
2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^4*c^4
*x^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+10*EllipticF(((d*x+
(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^4*c*d^3*(a*b)^(1/2)*
((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
```

^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^3*c^4*(a*b)^(1/2)*((d*x+(c*d)^(1/2))
/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)*(c*d)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*
d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^3*c^4*(a*
b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1
/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-9*EllipticPi(((d*x+(c*d)^(1
/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(
1/2))*2^(1/2)*a*b^3*c^4*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
(-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+
20*a^3*b*d^5*x^5*(a*b)^(1/2)+12*b^4*c^3*d^2*x^5*(a*b)^(1/2)-24*b^4*c^4*d*x^
3*(a*b)^(1/2)+28*a^4*c*d^4*x*(a*b)^(1/2)-88*a^2*b^2*c*d^4*x^5*(a*b)^(1/2)+5
6*a*b^3*c^2*d^3*x^5*(a*b)^(1/2)+60*a^3*b*c*d^4*x^3*(a*b)^(1/2)+36*a^2*b^2*c
^2*d^3*x^3*(a*b)^(1/2)-52*a*b^3*c^3*d^2*x^3*(a*b)^(1/2)-104*a^3*b*c^2*d^3*x
*(a*b)^(1/2)+76*a^2*b^2*c^3*d^2*x*(a*b)^(1/2)-12*a*b^3*c^4*d*x*(a*b)^(1/2)+
39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1
/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^3*d^2*x^4*((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)-39*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)
*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^3*d^2*x^4*((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)-10*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*2^(1/2)*a^4*d^4*x^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*
d)^(1/2)-6*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(
1/2)*b^4*c^4*x^2*(a*b)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)-39*Elli
pticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(
a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a^2*b^3*c^3*d^2*x^2*((d*x+(c*d)^(1/2))/(
c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)-30*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^4*d*x^2*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)+9*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(
1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*2^(1/2)*b^4*c^4*x^2*(a*b)
^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+39*EllipticPi(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1
/2))*2^(1/2)*a^2*b^3*c^3*d^2*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)+30*EllipticPi((
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/
2)*d),1/2*2^(1/2))*2^(1/2)*a*b^4*c^4*d*x^2*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)-20*a^
4*d^5*x^3*(a*b)^(1/2)+12*b^4*c^5*x*(a*b)^(1/2)-44*EllipticF(((d*x+(c*d)^(1/
2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2*b^2*c*d^3*x^4*(a*b)^(1/2)*(
(d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)

```
*(-d*x/(c*d)^(1/2))^(1/2)*(c*d)^(1/2)+28*EllipticF(((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a*b^3*c^2*d^2*x^4*(a*b)^(1/2)*((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d)^(1/2))...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima"
)
[Out] e^(-1/2)*integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(x)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex}\,\left(-a+bx^2\right)^2\left(c-dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
[Out] Integral(1/(sqrt(e*x)*(-a + b*x**2)**2*(c - d*x**2)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(e^(-1/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(x)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,x}\,\left(a-b\,x^2\right)^2\,\left(c-d\,x^2\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
[Out] int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.932 $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

Optimal. Leaf size=735

$$\frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\,(c-dx^2)^{3/2}}+\frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}+\frac{d(3b^2c^2+19abcd-7a^2d^2)}{6ac^2(bc-ad)^3e\sqrt{ex}\,\sqrt{c-dx}}$$

```
[Out] 1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(3/2)/(e*x)^(1/2)+1/2*b/a
/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2)+1/6*d*(-7*a^2*d^2+19*
a*b*c*d+3*b^2*c^2)/a/c^2/(-a*d+b*c)^3/e/(e*x)^(1/2)/(-d*x^2+c)^(1/2)-1/2*(-
7*a^3*d^3+19*a^2*b*c*d^2-12*a*b^2*c^2*d+5*b^3*c^3)*(-d*x^2+c)^(1/2)/a^2/c^3
/(-a*d+b*c)^3/e/(e*x)^(1/2)-1/2*d^(1/4)*(-7*a^3*d^3+19*a^2*b*c*d^2-12*a*b^2
*c^2*d+5*b^3*c^3)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2
/c)^(1/2)/a^2/c^(9/4)/(-a*d+b*c)^3/e^(3/2)/(-d*x^2+c)^(1/2)+1/2*d^(1/4)*(-7
*a^3*d^3+19*a^2*b*c*d^2-12*a*b^2*c^2*d+5*b^3*c^3)*EllipticF(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(9/4)/(-a*d+b*c)^3/e^(3/2)/
(-d*x^2+c)^(1/2)-5/4*b^(5/2)*c^(1/4)*(-3*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)
/a^(5/2)/d^(1/4)/(-a*d+b*c)^3/e^(3/2)/(-d*x^2+c)^(1/2)+5/4*b^(5/2)*c^(1/4)*
(-3*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)
/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/(-a*d+b*c)^3/e^(3/2)/
(-d*x^2+c)^(1/2)
```

Rubi [A]
time = 1.24, antiderivative size = 735, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {477, 483, 593, 597, 598, 313, 230, 227, 1214, 1213, 435, 504, 1233, 1232}

$$-\frac{5b^{5/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{5b^{5/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^{5/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d(-7a^2d^2+19abcd+3b^2c^2)}{6ac^2e\sqrt{ex}\sqrt{c-dx^2}(bc-ad)^3}+\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{9/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{9/4}e^{3/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{c-dx^2}(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3)}{2a^2c^3e\sqrt{ex}(bc-ad)^3}+\frac{b}{2ae\sqrt{ex}\,(a-bx^2)(c-dx^2)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ace\sqrt{ex}\,(c-dx^2)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*Sqrt[e*x]*(c - d*x^2)^(3/2)) + b
/(2*a*(b*c - a*d)*e*Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(3*b^2*c^
2 + 19*a*b*c*d - 7*a^2*d^2))/(6*a*c^2*(b*c - a*d)^3*e*Sqrt[e*x]*Sqrt[c - d*
x^2]) - ((5*b^3*c^3 - 12*a*b^2*c^2*d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[c -
 d*x^2])/(2*a^2*c^3*(b*c - a*d)^3*e*Sqrt[e*x]) - (d^(1/4)*(5*b^3*c^3 - 12*a
*b^2*c^2*d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/4)*(b*c - a*d)^
3*e^(3/2)*Sqrt[c - d*x^2]) + (d^(1/4)*(5*b^3*c^3 - 12*a*b^2*c^2*d + 19*a^2*
b*c*d^2 - 7*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*a^2*c^(9/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d
```

```
*x^2]) - (5*b^(5/2)*c^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-(
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2]) +
(5*b^(5/2)*c^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sq
rt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(4*a^(5/2)*d^(1/4)*(b*c - a*d)^3*e^(3/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^2\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b}{2a(bc-ad)e\sqrt{ex}\,\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e\text{Subst}\left(\int \frac{\frac{5bc-4ad}{e^2}-\frac{11bc}{e^4}}{x^2\left(a-\frac{bx^4}{e^2}\right)\left(c-\frac{d}{e}\right.}\right.}{2a(bc-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\,\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,\left(a-bx^2\right)(c-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\,\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,\left(a-bx^2\right)(c-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\,\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,\left(a-bx^2\right)(c-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\,\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,\left(a-bx^2\right)(c-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\,\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,\left(a-bx^2\right)(c-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\,\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,\left(a-bx^2\right)(c-}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\,\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e\sqrt{ex}\,\left(a-bx^2\right)(c-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.91, size = 407, normalized size = 0.55

$$\frac{x\left(-\frac{7a\left(15b^4c^3x^2\left(c-dx^2\right)^2-12ab^3c^2\left(c-dx^2\right)^2\left(c+3dx^2\right)+a^4d^3\left(12c^2-35cdx^2+21d^2x^4\right)-a^3bd^2\left(36c^3-83c^2dx^2+22cd^2x^4+21d^3x^6\right)+a^2b^2cd\left(36c^3-36c^2dx^2-59cd^2x^4+57d^3x^6\right)\right)}{(a-bx^2)(c-dx^2)}-7(5b^4c^4-20ab^3c^3d+12a^2b^2c^2d^2-19a^3bcd^3+7a^4d^4)\,x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bd(-5b^3c^3+12ab^2c^2d-19a^2bcd^2+7a^3d^3)\,x^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{42a^3c^3(-bc+ad)^3(ex)^{3/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (x*((-7*a*(15*b^4*c^3*x^2*(c - d*x^2)^2 - 12*a*b^3*c^2*(c - d*x^2)^2*(c + 3
*d*x^2) + a^4*d^3*(12*c^2 - 35*c*d*x^2 + 21*d^2*x^4) - a^3*b*d^2*(36*c^3 -
83*c^2*d*x^2 + 22*c*d^2*x^4 + 21*d^3*x^6) + a^2*b^2*c*d*(36*c^3 - 36*c^2*d*
x^2 - 59*c*d^2*x^4 + 57*d^3*x^6)))/((a - b*x^2)*(c - d*x^2)) - 7*(5*b^4*c^4
 - 20*a*b^3*c^3*d + 12*a^2*b^2*c^2*d^2 - 19*a^3*b*c*d^3 + 7*a^4*d^4)*x^2*Sq
rt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*
(-5*b^3*c^3 + 12*a*b^2*c^2*d - 19*a^2*b*c*d^2 + 7*a^3*d^3)*x^4*Sqrt[1 - (d*
x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^3*c^3*(-(
b*c) + a*d)^3*(e*x)^(3/2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6321 vs. 2(607) = 1214.
time = 0.15, size = 6322, normalized size = 8.60

method	result	size
elliptic	Expression too large to display	1888
default	Expression too large to display	6322

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] e^(-3/2)*integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*x^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex)^{\frac{3}{2}} (-a+bx^2)^2 (c-dx^2)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/((e*x)**(3/2)*(-a + b*x**2)**2*(c - d*x**2)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x)^{3/2}\,(a-b\,x^2)^2\,(c-d\,x^2)^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

```
[Out] int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.933 $\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=606

$$\frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}}+\frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}+\frac{d\left(b^2c^2+7abcd-3a^2d^2\right)}{2ac^2(bc-ad)^3e(ex)^{3/2}\sqrt{c-}}$$

```
[Out] 1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/e/(e*x)^(3/2)/(-d*x^2+c)^(3/2)+1/2*b/a
/(-a*d+b*c)/e/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/2*d*(-3*a^2*d^2+7*a
*b*c*d+b^2*c^2)/a/c^2/(-a*d+b*c)^3/e/(e*x)^(3/2)/(-d*x^2+c)^(1/2)-1/6*(-15*
a^3*d^3+35*a^2*b*c*d^2-12*a*b^2*c^2*d+7*b^3*c^3)*(-d*x^2+c)^(1/2)/a^2/c^3/(
-a*d+b*c)^3/e/(e*x)^(3/2)+1/6*d^(3/4)*(-15*a^3*d^3+35*a^2*b*c*d^2-12*a*b^2*
c^2*d+7*b^3*c^3)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/
c)^(1/2)/a^2/c^(11/4)/(-a*d+b*c)^3/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*b^3*c^(1/4)
*(-17*a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^
(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/(-a*d+b*c)^3/e^(5/2)
/(-d*x^2+c)^(1/2)+1/4*b^3*c^(1/4)*(-17*a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/
a^3/d^(1/4)/(-a*d+b*c)^3/e^(5/2)/(-d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.99, antiderivative size = 606, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {477, 483, 593, 597, 537, 230, 227, 418, 1233, 1232}

$$\frac{b^3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-17ad)\Pi\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{b^3\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}(7bc-17ad)\Pi\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d\left(-3a^2d^2+7abcd+b^2c^2\right)}{2ac^2e(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)^3}+\frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{6a^2c^{11/4}e^{5/2}\sqrt{c-dx^2}(bc-ad)^3}-\frac{\sqrt{c-dx^2}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)}{6a^2c^3e(ex)^{3/2}(bc-ad)^3}+\frac{b}{2ae(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ace(ex)^{3/2}\left(c-dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*e*(e*x)^(3/2)*(c - d*x^2)^(3/2)) +
 b/(2*a*(b*c - a*d)*e*(e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)) + (d*(b^2*
c^2 + 7*a*b*c*d - 3*a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*e*(e*x)^(3/2)*Sqrt[c -
 d*x^2]) - ((7*b^3*c^3 - 12*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt
[c - d*x^2])/(6*a^2*c^3*(b*c - a*d)^3*e*(e*x)^(3/2)) + (d^(3/4)*(7*b^3*c^3
- 12*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 - (d*x^2)/c]*Ellipti
cF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(6*a^2*c^(11/4)*(b*c
 - a*d)^3*e^(5/2)*Sqrt[c - d*x^2]) + (b^3*c^(1/4)*(7*b*c - 17*a*d)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2
)*Sqrt[c - d*x^2]) + (b^3*c^(1/4)*(7*b*c - 17*a*d)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(4*a^3*d^(1/4)*(b*c - a*d)^3*e^(5/2)*Sqrt[c - d*x^2])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{x^4\left(a-\frac{bx^4}{e^2}\right)^2\left(c-\frac{dx^4}{e^2}\right)^{5/2}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}} + \frac{e\text{Subst}\left(\int \frac{\frac{7bc-4ad}{e^2}-\frac{13}{\ldots}}{x^4\left(a-\frac{bx^4}{e^2}\right)\left(c-\ldots\right.}\right.}{2a(bc-\ldots}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(\ldots\right.}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(\ldots\right.}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(\ldots\right.}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(\ldots\right.}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(\ldots\right.}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(\ldots\right.}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}\left(c-dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\left(\ldots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 11.01, size = 427, normalized size = 0.70

$$\frac{x\left(-\frac{5a\left(7b^4c^3x^2(c-dx^2)^2-4ab^3c^2(c-dx^2)^2(c+3dx^2)+a^4d^3(4c^2-21cdx^2+15d^2x^4)-a^3bd^2(12c^3-45c^2dx^2+14cd^2x^4+15d^3x^6)+a^2b^2cd(12c^3-12c^2dx^2-37cd^2x^4+35d^3x^6)\right)}{(-bc+ad)^3(a-bx^2)(c-dx^2)}+\frac{5(21b^4c^4-44ab^3c^3d-12a^2b^2c^2d^2+35a^3bcd^3-15a^4d^4)x^2\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bc-ad)^3}-\frac{bd(7b^3c^3-12ab^2c^2d+35a^2bcd^2-15a^3d^3)x^4\sqrt{1-\frac{dx^2}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{dx^2}{c},\frac{bx^2}{a}\right)}{(bc-ad)^3}\right)}{30a^3c^3(ex)^{5/2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

```
[Out] (x*((-5*a*(7*b^4*c^3*x^2*(c - d*x^2)^2 - 4*a*b^3*c^2*(c - d*x^2)^2*(c + 3*d
*x^2) + a^4*d^3*(4*c^2 - 21*c*d*x^2 + 15*d^2*x^4) - a^3*b*d^2*(12*c^3 - 45*
c^2*d*x^2 + 14*c*d^2*x^4 + 15*d^3*x^6) + a^2*b^2*c*d*(12*c^3 - 12*c^2*d*x^2
 - 37*c*d^2*x^4 + 35*d^3*x^6)))/((-(b*c) + a*d)^3*(a - b*x^2)*(c - d*x^2))
+ (5*(21*b^4*c^4 - 44*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 35*a^3*b*c*d^3 - 1
5*a^4*d^4)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b
*x^2)/a])/(b*c - a*d)^3 - (b*d*(7*b^3*c^3 - 12*a*b^2*c^2*d + 35*a^2*b*c*d^2
 - 15*a^3*d^3)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c
, (b*x^2)/a])/(b*c - a*d)^3))/(30*a^3*c^3*(e*x)^(5/2)*Sqrt[c - d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5235 vs. 2(506) = 1012.
time = 0.15, size = 5236, normalized size = 8.64

method	result	size
elliptic	Expression too large to display	1464
default	Expression too large to display	5236

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima"
)
```

```
[Out] e^(-5/2)*integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*x^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex)^{\frac{5}{2}}\,(-a+bx^2)^2\,(c-dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/((e*x)**(5/2)*(-a + b*x**2)**2*(c - d*x**2)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(e\,x)}^{5/2}\,{(a-b\,x^2)}^2\,{(c-d\,x^2)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

```
[Out] int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.934 $\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=209

$$\frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3}-\frac{(5bc+3ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24b^2d^2}+\frac{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}{6bd}$$

```
[Out] -1/16*(-a*d+b*c)*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(b*x^2+a)^(1
/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(5/2)/d^(7/2)-1/24*(3*a*d+5*b*c)*(b*x^2+a)^(
3/2)*(d*x^2+c)^(1/2)/b^2/d^2+1/6*x^2*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b/d+1/
16*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^3
```

Rubi [A]

time = 0.18, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {457, 92, 81, 52, 65, 223, 212}

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(a^2d^2+2abcd+5b^2c^2)}{16b^2d^3}-\frac{(bc-ad)\,(a^2d^2+2abcd+5b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{5/2}d^{7/2}}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(3ad+5bc)}{24b^2d^2}+\frac{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b^2
*d^3) - ((5*b*c + 3*a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b^2*d^2) +
(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)*(5*b^2*c^2 +
 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d
*x^2])])/(16*b^(5/2)*d^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= \frac{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}{6bd} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\,\left(-ac-\frac{1}{2}(5bc+3ad)x\right)}{\sqrt{c+dx}}\,dx,x,x^2\right)}{6bd}$$

$$= -\frac{(5bc+3ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}{6bd} + \frac{(5b^2c^2+2abcd+a}{}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x}{}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x}{}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x}{}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x}{}$$

Mathematica [A]
time = 1.60, size = 187, normalized size = 0.89

$$\frac{-b\sqrt{d}\,\sqrt{a+bx^2}\,(c+dx^2)\,(3a^2d^2-2abd(-2c+dx^2)+b^2(-15c^2+10cdx^2-8d^2x^4))-3(bc-ad)^{3/2}\,(5b^2c^2+2abcd+a^2d^2)\sqrt{\frac{b\,(c+dx^2)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{48b^3d^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(3*a^2*d^2 - 2*a*b*d*(-2*c + d*x^2
) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - 3*(b*c - a*d)^(3/2)*(5*b^2*c
^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d
]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*b^3*d^(7/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(177) = 354$.
time = 0.16, size = 455, normalized size = 2.18

method	result
risch	$-\frac{\left(-8b^{2}x^{4}d^{2}-2ab\,d^{2}x^{2}+10b^{2}cd\,x^{2}+3a^{2}d^{2}+4abcd-15b^{2}c^{2}\right)\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{48b^{2}d^{3}}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^{2}}{\sqrt{bd}}+\sqrt{bd\,x^{4}+\right.}{32b^{2}\sqrt{b}}\right.}{}$
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(16b^{2}d^{2}x^{4}\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{bd}+4\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,x^{2}ab\,d^{2}\sqrt{bd}-\right.}{}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{x^{4}\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+ac}}{6d}+\frac{\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+ac}\,x^{2}a}{24bd}-\frac{5\sqrt{bd\,}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(16*b^2*d^2*x^4*((b*x^2+a)*(d*x^2+c))^
(1/2)*(b*d)^(1/2)+4*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)-20*
((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)+3*d^3*ln(1/2*(2*b*d*x^2
+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3+3*ln(1
/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2
))*a^2*c*b*d^2+9*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2
)+a*d+b*c)/(b*d)^(1/2))*a*c^2*b^2*d-15*b^3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(
d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3-6*((b*x^2+a)*(d*x^2+c
))^(1/2)*a^2*d^2*(b*d)^(1/2)-8*((b*x^2+a)*(d*x^2+c))^(1/2)*a*c*b*d*(b*d)^(1
/2)+30*((b*x^2+a)*(d*x^2+c))^(1/2)*c^2*b^2*(b*d)^(1/2))/((b*x^2+a)*(d*x^2+c
))^(1/2)/d^3/b^2/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 5.44, size = 442, normalized size = 2.11

$$\left[-\frac{3\,{\left(5\,b^{3}c^{3}-3\,ab^{2}c^{2}d-a^{2}bcd^{2}-a^{3}d^{3}\right)}\sqrt{bd}\log\left(8\,b^{2}d^{2}x^{4}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+8\,{\left(b^{2}cd+abd^{2}\right)}x^{2}+4\,{\left(2\,bdx^{2}+bc+ad\right)}\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{bd}\right)-4\,{\left(8\,b^{3}d^{3}x^{4}+15\,b^{3}c^{2}d-4\,ab^{2}cd^{2}-3\,a^{2}bd^{3}-2\,{\left(5\,b^{3}cd^{2}-ab^{2}d^{3}\right)}x^{2}\right)}\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}}{192\,b^{3}d^{4}},\ \frac{3\,{\left(5\,b^{3}c^{3}-3\,ab^{2}c^{2}d-a^{2}bcd^{2}-a^{3}d^{3}\right)}\sqrt{-bd}\arctan\left(\frac{{\left(2\,bdx^{2}+bc+ad\right)}\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{-bd}}{2\,{\left(b^{2}d^{2}x^{4}+abcd+{\left(b^{2}cd+abd^{2}\right)}x^{2}\right)}}\right)+2\,{\left(8\,b^{3}d^{3}x^{4}+15\,b^{3}c^{2}d-4\,ab^{2}cd^{2}-3\,a^{2}bd^{3}-2\,{\left(5\,b^{3}cd^{2}-ab^{2}d^{3}\right)}x^{2}\right)}\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}}{96\,b^{3}d^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2
 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4
*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3*c*d
^2 - a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^4), 1/96*(3*(5
*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*
b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^
4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3*c^2*d -
 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c))/(b^3*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}\sqrt{a+bx^{2}}}{\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [A]
time = 0.98, size = 226, normalized size = 1.08

$$\frac{\left(\sqrt{b^{2}c+(bx^{2}+a)bd-abd}\,\sqrt{bx^{2}+a}\left(2\,(bx^{2}+a)\left(\frac{4\,(bx^{2}+a)}{b^{3}d}-\frac{5\,b^{7}cd^{3}+7\,ab^{6}d^{4}}{b^{9}d^{5}}\right)+\frac{3\,(5\,b^{8}c^{2}d^{2}+2\,ab^{7}cd^{3}+a^{2}b^{6}d^{4})}{b^{9}d^{5}}\right)+\frac{3\,(5\,b^{3}c^{3}-3\,ab^{2}c^{2}d-a^{2}bcd^{2}-a^{3}d^{3})\log\left(\left|-\sqrt{bx^{2}+a}\,\sqrt{bd}+\sqrt{b^{2}c+(bx^{2}+a)bd-abd}\right|\right)}{\sqrt{bd}\,b^{2}d^{3}}\right)b}{48\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)*
(4*(b*x^2 + a)/(b^3*d) - (5*b^7*c*d^3 + 7*a*b^6*d^4)/(b^9*d^5)) + 3*(5*b^8*
c^2*d^2 + 2*a*b^7*c*d^3 + a^2*b^6*d^4)/(b^9*d^5)) + 3*(5*b^3*c^3 - 3*a*b^2*
c^2*d - a^2*b*c*d^2 - a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*b/abs(b)
```

Mupad [B]
time = 27.37, size = 993, normalized size = 4.75

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{8\,b^{5/2}\,d^{7/2}}-\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,\left(\frac{a^3\,b^3\,d^3}{8}+\frac{a^2\,b^4\,c\,d^2}{8}+\frac{3\,a\,b^5\,c^2\,d}{8}-\frac{5\,b^6\,c^3}{8}\right)}{d^9\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^5\,\left(\frac{19\,a^3\,b\,d^3}{4}+\frac{275\,a^2\,b^2\,c\,d^2}{4}+\frac{313\,a\,b^3\,c^2\,d}{4}+\frac{33\,b^4\,c^3}{4}\right)}{d^7\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^5}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^7\,\left(\frac{19\,a^3\,d^3}{4}+\frac{275\,a^2\,b\,c\,d^2}{4}+\frac{313\,a\,b^2\,c^2\,d}{4}+\frac{33\,b^3\,c^3}{4}\right)}{d^6\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^7}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^3\,\left(\frac{17\,a^3\,b^2\,d^3}{24}+\frac{91\,a^2\,b^3\,c\,d^2}{8}+\frac{17\,a\,b^4\,c^2\,d}{8}-\frac{85\,b^5\,c^3}{24}\right)}{d^8\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^3}+\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^{11}\,\left(\frac{a^3\,d^3}{8}+\frac{a^2\,b\,c\,d^2}{8}+\frac{3\,a\,b^2\,c^2\,d}{8}-\frac{5\,b^3\,c^3}{8}\right)}{b^2\,d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^{11}}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^9\,\left(\frac{17\,a^3\,d^3}{24}+\frac{91\,a^2\,b\,c\,d^2}{8}+\frac{17\,a\,b^2\,c^2\,d}{8}-\frac{85\,b^3\,c^3}{24}\right)}{b\,d^5\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^9}+\frac{\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^8\,\left(16\,a^2\,d+48\,a\,b\,c\right)}{d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^8}+\frac{\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4\,\left(16\,a^2\,b^2\,d+48\,a\,b^3\,c\right)}{d^6\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}+\frac{\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^6\,\left(32\,a^2\,b\,d^2+\frac{352\,a\,b^2\,c\,d}{3}+64\,b^3\,c^2\right)}{d^6\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^6}}{\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^{12}}{{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^{12}}+\frac{b^6}{d^6}-\frac{6\,b^5\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^2}{d^5\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^2}+\frac{15\,b^4\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4}{d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}-\frac{20\,b^3\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^6}{d^3\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^6}+\frac{15\,b^2\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^8}{d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^8}-\frac{6\,b\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^{10}}{d\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)

[Out] (atanh((d^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2)
- c^(1/2))))*(a*d - b*c)*(a^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(8*b^(5/2)*d^(7
/2)) - ((((a + b*x^2)^(1/2) - a^(1/2))*((a^3*b^3*d^3)/8 - (5*b^6*c^3)/8 + (
a^2*b^4*c*d^2)/8 + (3*a*b^5*c^2*d)/8))/(d^9*((c + d*x^2)^(1/2) - c^(1/2)))
- (((a + b*x^2)^(1/2) - a^(1/2))^5*((33*b^4*c^3)/4 + (19*a^3*b*d^3)/4 + (27
5*a^2*b^2*c*d^2)/4 + (313*a*b^3*c^2*d)/4))/(d^7*((c + d*x^2)^(1/2) - c^(1/2
))^5) - (((a + b*x^2)^(1/2) - a^(1/2))^7*((19*a^3*d^3)/4 + (33*b^3*c^3)/4 +
 (313*a*b^2*c^2*d)/4 + (275*a^2*b*c*d^2)/4))/(d^6*((c + d*x^2)^(1/2) - c^(1
/2))^7) - (((a + b*x^2)^(1/2) - a^(1/2))^3*((17*a^3*b^2*d^3)/24 - (85*b^5*c
^3)/24 + (91*a^2*b^3*c*d^2)/8 + (17*a*b^4*c^2*d)/8))/(d^8*((c + d*x^2)^(1/2
) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))^11*((a^3*d^3)/8 - (5*b^3*c
^3)/8 + (3*a*b^2*c^2*d)/8 + (a^2*b*c*d^2)/8))/(b^2*d^4*((c + d*x^2)^(1/2) -
 c^(1/2))^11) - (((a + b*x^2)^(1/2) - a^(1/2))^9*((17*a^3*d^3)/24 - (85*b^3
*c^3)/24 + (17*a*b^2*c^2*d)/8 + (91*a^2*b*c*d^2)/8))/(b*d^5*((c + d*x^2)^(1
/2) - c^(1/2))^9) + (a^(1/2)*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^8*(16*a^
2*d + 48*a*b*c))/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^8) + (a^(1/2)*c^(1/2)*(
(a + b*x^2)^(1/2) - a^(1/2))^4*(16*a^2*b^2*d + 48*a*b^3*c))/(d^6*((c + d*x^
2)^(1/2) - c^(1/2))^4) + (a^(1/2)*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^6*(
64*b^3*c^2 + 32*a^2*b*d^2 + (352*a*b^2*c*d)/3))/(d^6*((c + d*x^2)^(1/2) - c
^(1/2))^6))/(((a + b*x^2)^(1/2) - a^(1/2))^12/((c + d*x^2)^(1/2) - c^(1/2))
^12 + b^6/d^6 - (6*b^5*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d^5*((c + d*x^2)^(
1/2) - c^(1/2))^2) + (15*b^4*((a + b*x^2)^(1/2) - a^(1/2))^4)/(d^4*((c + d*
x^2)^(1/2) - c^(1/2))^4) - (20*b^3*((a + b*x^2)^(1/2) - a^(1/2))^6)/(d^3*((
c + d*x^2)^(1/2) - c^(1/2))^6) + (15*b^2*((a + b*x^2)^(1/2) - a^(1/2))^8)/(
d^2*((c + d*x^2)^(1/2) - c^(1/2))^8) - (6*b*((a + b*x^2)^(1/2) - a^(1/2))^1
0)/(d*((c + d*x^2)^(1/2) - c^(1/2))^10))
```

3.935 $\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{(3bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8bd^2}+\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4bd}+\frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}$$

```
[Out] 1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c
)^(1/2))/b^(3/2)/d^(5/2)+1/4*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b/d-1/8*(a*d+3
*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^2
```

Rubi [A]

time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {457, 81, 52, 65, 223, 212}

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(ad+3bc)}{8bd^2}+\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] -1/8*((3*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(b*d^2) + ((a + b*x^2)
^(3/2)*Sqrt[c + d*x^2])/(4*b*d) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(3/2)*d^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4bd} - \frac{(3bc+ad)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^2\right)}{8bd}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{((bc-ad)(3bc+ad))\text{Su}}{}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{((bc-ad)(3bc+ad))\text{Su}}{}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{((bc-ad)(3bc+ad))\text{Su}}{}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{(bc-ad)(3bc+ad)\tanh}{8b^{3/2}}$$

Mathematica [A]
time = 0.95, size = 119, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(-3bc+ad+2bdx^2)}{8bd^2} + \frac{(3b^2c^2-2abcd-a^2d^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + a*d + 2*b*d*x^2))/(8*b*d^2) + ((
3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]
*Sqrt[c + d*x^2])])/(8*b^(3/2)*d^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(111) = 222.
time = 0.12, size = 290, normalized size = 2.12

method	result

risch	$\frac{(2bd\,x^2+ad-3bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{8b\,d^2}+\frac{\left(-\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)a^2}{16b\sqrt{bd}}-\frac{\ln\left(\frac{\frac{1}{2}ad+}{\sqrt{}}\right.}{}\right.}{}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(-4\sqrt{bd}\,\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,bd\,x^2+d^2\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}{2\sqrt{bd}}\right.\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\left(\frac{x^2\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{4d}+\frac{\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\,a}{8bd}-\frac{3\sqrt{bd\,x^4}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-4*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))
^(1/2)*b*d*x^2+d^2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1
/2)+a*d+b*c)/(b*d)^(1/2))*a^2+2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(
1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c*b*d-3*b^2*ln(1/2*(2*b*d*x^2+2*((
b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^2-2*(b*d)^(1/
2)*((b*x^2+a)*(d*x^2+c))^(1/2)*a*d+6*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2
)*b*c)/((b*x^2+a)*(d*x^2+c))^(1/2)/d^2/b/(b*d)^(1/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 6.06, size = 334, normalized size = 2.44

$$\left[-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2-4\,(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)-4\,(2\,b^2d^2x^2-3\,b^2cd+abd^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{32\,b^2d^3},\ -\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)-2\,(2\,b^2d^2x^2-3\,b^2cd+abd^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{16\,b^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4*(2*b^2*d^2*x^2 - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3), -1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(x**3*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)

Giac [A]
time = 1.75, size = 159, normalized size = 1.16

$$\frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}\left(\frac{2\,(bx^2+a)}{b^2d}-\frac{3\,b^3cd+ab^2d^2}{b^4d^3}\right)-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)b}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(b^2*d) - (3*b^3*c*d + a*b^2*d^2)/(b^4*d^3)) - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*b/abs(b)

Mupad [B]
time = 14.08, size = 639, normalized size = 4.66

$$\frac{\frac{\left(\sqrt{bx^2+a}-\sqrt{a}\right)\left(\frac{a^2b^2d^2}{4}+\frac{ab^3cd}{2}-\frac{3b^4c^2}{4}\right)}{d^6\left(\sqrt{dx^2+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{bx^2+a}-\sqrt{a}\right)^3\left(\frac{7a^2bd^2}{4}+\frac{23ab^2cd}{2}+\frac{11b^3c^2}{4}\right)}{d^5\left(\sqrt{dx^2+c}-\sqrt{c}\right)^3}+\frac{\left(\sqrt{bx^2+a}-\sqrt{a}\right)^5\left(\frac{7a^2d^2}{4}+\frac{23abcd}{2}+\frac{11b^2c^2}{4}\right)}{d^4\left(\sqrt{dx^2+c}-\sqrt{c}\right)^5}+\frac{\left(\sqrt{bx^2+a}-\sqrt{a}\right)^7\left(\frac{a^2d^2}{4}+\frac{abcd}{2}-\frac{3b^2c^2}{4}\right)}{bd^3\left(\sqrt{dx^2+c}-\sqrt{c}\right)^7}-\frac{4a^{3/2}\sqrt{c}\left(\sqrt{bx^2+a}-\sqrt{a}\right)^6}{d^2\left(\sqrt{dx^2+c}-\sqrt{c}\right)^6}-\frac{\sqrt{a}\sqrt{c}\left(\sqrt{bx^2+a}-\sqrt{a}\right)^4(16cb^2+8adb)}{d^3\left(\sqrt{dx^2+c}-\sqrt{c}\right)^4}-\frac{4a^{3/2}b^2\sqrt{c}\left(\sqrt{bx^2+a}-\sqrt{a}\right)^2}{d^4\left(\sqrt{dx^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{bx^2+a}-\sqrt{a}\right)^8}{\left(\sqrt{dx^2+c}-\sqrt{c}\right)^8}+\frac{b^4}{d^4}-\frac{4b^3\left(\sqrt{bx^2+a}-\sqrt{a}\right)^2}{d^3\left(\sqrt{dx^2+c}-\sqrt{c}\right)^2}+\frac{6b^2\left(\sqrt{bx^2+a}-\sqrt{a}\right)^4}{d^2\left(\sqrt{dx^2+c}-\sqrt{c}\right)^4}-\frac{4b\left(\sqrt{bx^2+a}-\sqrt{a}\right)^6}{d\left(\sqrt{dx^2+c}-\sqrt{c}\right)^6}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{bx^2+a}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{dx^2+c}-\sqrt{c}\right)}\right)(ad-bc)(ad+3bc)}{4b^{3/2}d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)

[Out] ((((a + b*x^2)^(1/2) - a^(1/2))*((a^2*b^2*d^2)/4 - (3*b^4*c^2)/4 + (a*b^3*c*d)/2))/(d^6*((c + d*x^2)^(1/2) - c^(1/2))) + (((a + b*x^2)^(1/2) - a^(1/2)

)^3*((11*b^3*c^2)/4 + (7*a^2*b*d^2)/4 + (23*a*b^2*c*d)/2))/(d^5*((c + d*x^2
)^(1/2) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))^5*((7*a^2*d^2)/4 + (
11*b^2*c^2)/4 + (23*a*b*c*d)/2))/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^5) + ((
(a + b*x^2)^(1/2) - a^(1/2))^7*((a^2*d^2)/4 - (3*b^2*c^2)/4 + (a*b*c*d)/2))
/(b*d^3*((c + d*x^2)^(1/2) - c^(1/2))^7) - (4*a^(3/2)*c^(1/2)*((a + b*x^2)^
(1/2) - a^(1/2))^6)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^6) - (a^(1/2)*c^(1/2
)*((a + b*x^2)^(1/2) - a^(1/2))^4*(16*b^2*c + 8*a*b*d))/(d^4*((c + d*x^2)^(
1/2) - c^(1/2))^4) - (4*a^(3/2)*b^2*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^2
)/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^8/(
(c + d*x^2)^(1/2) - c^(1/2))^8 + b^4/d^4 - (4*b^3*((a + b*x^2)^(1/2) - a^(1
/2))^2)/(d^3*((c + d*x^2)^(1/2) - c^(1/2))^2) + (6*b^2*((a + b*x^2)^(1/2) -
 a^(1/2))^4)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^4) - (4*b*((a + b*x^2)^(1/2
) - a^(1/2))^6)/(d*((c + d*x^2)^(1/2) - c^(1/2))^6)) - (atanh((d^(1/2)*((a
+ b*x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))))*(a*d -
b*c)*(a*d + 3*b*c))/(4*b^(3/2)*d^(5/2))

3.936 $\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}\,d^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/d^
(3/2)/b^(1/2)+1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {455, 52, 65, 223, 212}

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*Sqrt[b]*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4d}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{2bd}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2bd}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}\,d^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 86, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*Sqrt[b]*d^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(66) = 132$.

time = 0.11, size = 170, normalized size = 1.98

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(a\ln\left(\frac{2bd\,x^{2}+2\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)d-b\ln\left(\frac{2bd\,x^{2}+2\sqrt{\left(b\,x^{2}+a\right)\left(d\right.}}{2\sqrt{bd}}\right.\right.}{4\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,d\sqrt{bd}}$
risch	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{2d}+\frac{\left(\frac{a\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^{2}}{\sqrt{bd}}+\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+ac}\right)}{4\sqrt{bd}}-\frac{b\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^{2}}{\sqrt{bd}}+\sqrt{bd\,x^{4}}\right.}{4d\sqrt{}}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+ac}}{2d}+\frac{a\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^{2}}{\sqrt{bd}}+\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+c}\right.}{4\sqrt{bd}}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(a*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^
2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*d-b*ln(1/2*(2*b*d*x^2+2*((b*x
^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c+2*((b*x^2+a)*(d*
x^2+c))^(1/2)*(b*d)^(1/2))/((b*x^2+a)*(d*x^2+c))^(1/2)/d/(b*d)^(1/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.60, size = 259, normalized size = 3.01

$$\left[\frac{4\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,bd-(bc-ad)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)}{8\,bd^2},\ \frac{2\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,bd+(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)}{4\,bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*
(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)))/(b*d^2)
, 1/4*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c - a*d)*sqrt(-b*d)*arcta
n(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b
^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)))/(b*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [A]
time = 1.33, size = 106, normalized size = 1.23

$$\frac{b\left(\frac{(bc-ad)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}}{bd}\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*((b*c - a*d)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2
 + a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*
sqrt(b*x^2 + a)/(b*d))/abs(b)
```

Mupad [B]
time = 2.74, size = 280, normalized size = 3.26

$$\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3\,(a\,d+b\,c)}{d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,(c\,b^2+a\,d\,b)}{d^3\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{4\,\sqrt{a}\,b\,\sqrt{c}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}+\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,(a\,d-b\,c)}{\sqrt{b}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] ((((a + b*x^2)^(1/2) - a^(1/2))^3*(a*d + b*c))/(d^2*((c + d*x^2)^(1/2) - c^
(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))*(b^2*c + a*b*d))/(d^3*((c + d*x^
2)^(1/2) - c^(1/2))) - (4*a^(1/2)*b*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^2
)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^2))/((((a + b*x^2)^(1/2) - a^(1/2))^4/(
(c + d*x^2)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x^2)^(1/2) - a^(1/2
))^2)/(d*((c + d*x^2)^(1/2) - c^(1/2))^2)) + (atanh((d^(1/2)*((a + b*x^2)^(
1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(
1/2)*d^(3/2))
```

3.937 $\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

```
[Out] -arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)/c^(1/2)+a
rctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))*b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {457, 132, 65, 223, 212, 12, 95, 214}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sq
rt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2
])])/Sqrt[d]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right) + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x,\right.$$

$$= a\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right) + \text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+b\right.$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}} + \text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Mathematica [A]
time = 0.46, size = 92, normalized size = 1.00

$$-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sq
rt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2
])])/Sqrt[d]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(68) = 136$.
time = 0.11, size = 156, normalized size = 1.70

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{b\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)}{2\sqrt{bd}}-\frac{a\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{}\right.}{2}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

default	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(a\ln\left(\frac{a d\,x^{2}+c\,x^{2} b+2\sqrt{a c}\,\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}+2 a c}{x^{2}}\right)\sqrt{b d}-b\ln\left(\frac{2 b d\,x^{2}+2\sqrt{\left(b\,x^{2}\right.}}{}\right.\right.}{2\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{b d}\,\sqrt{a c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(a*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*(
(b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*(b*d)^(1/2)-b*ln(1/2*(2*b*d*x^2+2*((
b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2))/((
b*x^2+a)*(d*x^2+c))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(68) = 136$.
time = 1.33, size = 777, normalized size = 8.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*
x^2 + c)*sqrt(b/d)) + 1/4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^
4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4), -1/2*sqrt(-b/d)*arctan(
1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2
*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + 1/4*sqrt(a/c)*log(((b^2*c^2 + 6*a*
b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 +
 (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4), 1/2*
sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2
```

```
+ c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) + 1/4*sqrt(b/d)*
log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x
^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b
/d)), 1/2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - 1/2*
sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x*sqrt(c + d*x**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**B**]
time = 10.31, size = 2500, normalized size = 27.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(x*(c + d*x^2)^(1/2)),x)
```

```
[Out] (2*atanh((20*a*b^7*(b*d)^(1/2))/(34*a^(1/2)*b^8*c^(1/2) - (33*a^(3/2)*b^7*d
)/c^(1/2) - (54*b^8*c*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c
^(1/2)) + (25*b^9*c^(3/2))/(2*a^(1/2)*d) + (4*a^(5/2)*b^6*d^2)/c^(3/2) - (1
8*b^10*c^(5/2))/(a^(3/2)*d^2) + (a^(7/2)*b^5*d^3)/(2*c^(5/2)) + (20*a*b^7*d
*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) + (10*a^2*b^6
*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2))) + (23
*b^9*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a*d*((c + d*x^2)^(1/2) - c^(1/2)))
```

- (3*a^3*b^5*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x^2)^(1/2) - c^(1/2))) + (4*b^10*c^3*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d^2*((c + d*x^2)^(1/2) - c^(1/2)))) - (54*b^8*(b*d)^(1/2))/((25*b^9*c^(1/2))/(2*a^(1/2)) + (34*a^(1/2)*b^8*d)/c^(1/2) - (54*b^8*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (33*a^(3/2)*b^7*d^2)/c^(3/2) - (18*b^10*c^(3/2))/(a^(3/2)*d) + (4*a^(5/2)*b^6*d^3)/c^(5/2) + (a^(7/2)*b^5*d^4)/(2*c^(7/2)) + (23*b^9*c*((a + b*x^2)^(1/2) - a^(1/2)))/(a*((c + d*x^2)^(1/2) - c^(1/2))) + (10*a^2*b^6*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x^2)^(1/2) - c^(1/2))) + (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d*((c + d*x^2)^(1/2) - c^(1/2))) - (3*a^3*b^5*d^4*((a + b*x^2)^(1/2) - a^(1/2)))/(c^3*((c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2)))) + (23*b^9*c*(b*d)^(1/2))/((25*a^(1/2)*b^9*c^(1/2)*d)/2 - (18*b^10*c^(3/2))/a^(1/2) + (34*a^(3/2)*b^8*d^2)/c^(1/2) - (33*a^(5/2)*b^7*d^3)/c^(3/2) + (4*a^(7/2)*b^6*d^4)/c^(5/2) + (a^(9/2)*b^5*d^5)/(2*c^(7/2)) - (54*a*b^8*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) + (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a*((c + d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) + (20*a^2*b^7*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2))) + (10*a^3*b^6*d^4*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x^2)^(1/2) - c^(1/2))) - (3*a^4*b^5*d^5*((a + b*x^2)^(1/2) - a^(1/2)))/(c^3*((c + d*x^2)^(1/2) - c^(1/2)))) + (10*a^2*b^6*(b*d)^(1/2))/((4*a^(5/2)*b^6*d)/c^(1/2) - 33*a^(3/2)*b^7*c^(1/2) + (34*a^(1/2)*b^8*c^(3/2))/d + (25*b^9*c^(5/2))/(2*a^(1/2)*d^2) + (a^(7/2)*b^5*d^2)/(2*c^(3/2)) - (18*b^10*c^(7/2))/(a^(3/2)*d^3) + (10*a^2*b^6*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (54*b^8*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(d*((c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*c*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (3*a^3*b^5*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c^3*((a + b*x^2)^(1/2) - a^(1/2)))/(a*d^2*((c + d*x^2)^(1/2) - c^(1/2))) + (4*b^10*c^4*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d^3*((c + d*x^2)^(1/2) - c^(1/2)))) - (3*a^3*b^5*(b*d)^(1/2))/(4*a^(5/2)*b^6*c^(1/2) + (a^(7/2)*b^5*d)/(2*c^(1/2)) - (33*a^(3/2)*b^7*c^(3/2))/d + (34*a^(1/2)*b^8*c^(5/2))/d^2 + (25*b^9*c^(7/2))/(2*a^(1/2)*d^3) - (18*b^10*c^(9/2))/(a^(3/2)*d^4) + (10*a^2*b^6*c*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (3*a^3*b^5*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (54*b^8*c^3*((a + b*x^2)^(1/2) - a^(1/2)))/(d^2*((c + d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c^4*((a + b*x^2)^(1/2) - a^(1/2)))/(a*d^3*((c + d*x^2)^(1/2) - c^(1/2))) + (4*b^10*c^5*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d^4*((c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(d*((c + d*x^2)^(1/2) - c^(1/2)))) + (4*b^10*c^2*(b*d)^(1/2))/((25*a^(3/2)*b^9*c^(1/2)*d^2)/2 - 18*a^(1/2)*b^10*c^(3/2)*d + (34*a^(5/2)*b^8*d^3)/c^(1/2) - (33*a^(7/2)*b^7*d^4)/c^(3/2) + (4*a^(9/2)*b^6*d^5)/c^(5/2) + (a^(11/2)*b^5*d^6)/(2*c^(7/2)) + (4*b^10*c^2*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (54*a^2*b^8*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) + (20*a^3*b^7*d^4*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2)

```
- c^(1/2))) + (10*a^4*b^6*d^5*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x
^2)^(1/2) - c^(1/2))) - (3*a^5*b^5*d^6*((a + b*x^2)^(1/2) - a^(1/2)))/(c^3*
((c + d*x^2)^(1/2) - c^(1/2))) + (23*a*b^9*c*d^2*((a + b*x^2)^(1/2) - a^(1/
2)))/((c + d*x^2)^(1/2) - c^(1/2))) + (34*a^(1/2)*b^7*(b*d)^(1/2)*((a + b*x
^2)^(1/2) - a^(1/2)))/(c^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))*((34*a^(1/2)*b
^8)/c^(1/2) - (54*b^8*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c
^(1/2)) - (33*a^(3/2)*b^7*d)/c^(3/2) + (25*b^9*c^(1/2))/(2*a^(1/2)*d) - (18
*b^10*c^(3/2))/(a^(3/2)*d^2) + (4*a^(5/2)*b^6*d^2)/c^(5/2) + (a^(7/2)*b^5*d
^3)/(2*c^(7/2)) + (10*a^2*b^6*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c +
 d*x^2)^(1/2) - c^(1/2))) - (3*a^3*b^5*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(
c^3*((c + d*x^2)^(1/2) - c^(1/2))) + (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/
2)))/(a^2*d^2*((c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*d*((a + b*x^2)^(1/
2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c*((a + b*x^2)^(
1/2) - a^(1/2)))/(a*d*((c + d*x^2)^(1/2) - c^(1/2))))) - (33*a^(3/2)*b^6*(b
*d)^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(c^(3/...
```

3.938 $\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,c^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/c^
(3/2)/a^(1/2)-1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {457, 96, 95, 214}

$$-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,c^{3/2}}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2) - ((b*c - a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*c^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)}{4c}\\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2c}\\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.64, size = 89, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} + \frac{(-bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2) + ((-(b*c) + a*d)*ArcTanh[(S
qrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*c^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(69) = 138.
time = 0.14, size = 179, normalized size = 2.01

method	result
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(\ln\left(\frac{a d\,x^2+c\,x^2 b+2\sqrt{a c}\,\sqrt{(b\,x^2+a)(d\,x^2+c)}+2 a c}{x^2}\right) a d\,x^2-\ln\left(\frac{a d\,x^2+c\,x^2 b+2\sqrt{a c}\,\sqrt{(\ldots}{}\right.\right.}{4c\sqrt{(b\,x^2+a)(d\,x^2+c)}\,x^2\sqrt{a c}}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{\left(\frac{\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{x^2}\right)ad}{4c\sqrt{ac}}-\frac{\ln\left(\frac{2ac+(ad+bc)x^2+2\ldots}{}\right.}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x\ldots}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\left(-\frac{\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{x^2}\right)b}{4\sqrt{ac}}-\frac{\sqrt{bd\,x^4+(ad+bc\ldots}}{2c\,x^2}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((
b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a*d*x^2-ln((a*d*x^2+c*x^2*b+2*(a*c)^(
1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*b*c*x^2-2*(a*c)^(1/2)*((b*x^2+
a)*(d*x^2+c))^(1/2))/((b*x^2+a)*(d*x^2+c))^(1/2)/x^2/(a*c)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 4.33, size = 280, normalized size = 3.15

$$\left[-\frac{\sqrt{ac}\,(bc-ad)x^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2+4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{ac}}{x^4}\right)+4\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,ac}{8\,ac^2x^2},\ \frac{\sqrt{-ac}\,(bc-ad)x^2\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-ac}}{2\,(abcdx^4+a^2c^2+(abc^2+a^2cd)x^2)}\right)-2\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,ac}{4\,ac^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a*c)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b
*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) + 4*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*a*c)/(a*c^2*x^2), 1/4*(sqrt(-a*c)*(b*c - a*d)*x^2*arctan(1/2*((b*c + a*d
)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 + a^
2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c)/
(a*c^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**3*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(69) = 138.
time = 2.39, size = 434, normalized size = 4.88

$$-\frac{b\left(\frac{\left(\sqrt{bd}\,b^2c-\sqrt{bd}\,abd\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,bc}+\frac{2\left(\sqrt{bd}\,b^4c^2-2\sqrt{bd}\,ab^3cd+\sqrt{bd}\,a^2b^2d^2-\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^2c-\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2abd\right)}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2abd+\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4\right)c}\right)}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*((sqrt(b*d)*b^2*c - sqrt(b*d)*a*b*d)*arctan(-1/2*(b^2*c + a*b*d - (s
qrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(
-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c) + 2*(sqrt(b*d)*b^4*c^2 - 2*sqrt(b*d)*a*b
^3*c*d + sqrt(b*d)*a^2*b^2*d^2 - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - sqrt(b*d)*(sqrt(b*x^2 + a)*sq
rt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d)/((b^4*c^2 - 2*a*b
^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*
x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^4)*c))/abs(b)
```

Mupad [B]
time = 3.39, size = 477, normalized size = 5.36

$$\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{c\,b^2}{8}+\frac{a\,d\,b}{8}\right)}{\sqrt{a}\,c^{3/2}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{b^2}{8\,c\,d}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{8}-\frac{3\,a\,b\,c\,d}{8}+\frac{b^2\,c^2}{8}\right)}{a\,c^2\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2\left(a\,d+b\,c\right)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}-\frac{d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{8\,c\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{4\,a\,c^2}+\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{4\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(x^3*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((((a + b*x^2)^(1/2) - a^(1/2))*((b^2*c)/8 + (a*b*d)/8))/(a^(1/2)*c^(3/2)*d
*((c + d*x^2)^(1/2) - c^(1/2))) - b^2/(8*c*d) + (((a + b*x^2)^(1/2) - a^(1/
2))^2*((a^2*d^2)/8 + (b^2*c^2)/8 - (3*a*b*c*d)/8))/(a*c^2*d*((c + d*x^2)^(1
/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^3/((c + d*x^2)^(1/2) - c^
(1/2))^3 + (b*((a + b*x^2)^(1/2) - a^(1/2)))/(d*((c + d*x^2)^(1/2) - c^(1/2
))) - (((a + b*x^2)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c
+ d*x^2)^(1/2) - c^(1/2))^2)) - (d*((a + b*x^2)^(1/2) - a^(1/2)))/(8*c*((c
+ d*x^2)^(1/2) - c^(1/2))) - (log(((a + b*x^2)^(1/2) - a^(1/2))/((c + d*x^2
)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(4*a*c^2) + (l
og(((c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) - (a
^(1/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/((c
 + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(4*a*c
^2)
```

3.939 $\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=143

$$\frac{(bc+3ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8ac^2x^2}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4acx^4}+\frac{(bc-ad)(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}$$

```
[Out] 1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c
)^(1/2))/a^(3/2)/c^(5/2)-1/4*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/c/x^4+1/8*(3
*a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {457, 98, 96, 95, 214}

$$\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}+\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(3ad+bc)}{8ac^2x^2}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*c + 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*a*c^2*x^2) - ((a + b*x^2
)^(3/2)*Sqrt[c + d*x^2])/(4*a*c*x^4) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTanh[(
Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(3/2)*c^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4acx^4} - \frac{\left(\frac{bc}{2}+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4acx^4} - \frac{((bc-ad)(bc+3ad))\text{Sub}}{}$$

$$= \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4acx^4} - \frac{((bc-ad)(bc+3ad))\text{Sub}}{}$$

$$= \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4acx^4} + \frac{(bc-ad)(bc+3ad)\tanh^{-}}{8a^{3/2}c^5}$$

Mathematica [A]

time = 1.20, size = 125, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(-2ac-bcx^2+3adx^2)}{8ac^2x^4}+\frac{(b^2c^2+2abcd-3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2))/(8*a*c^2*x
^4) + ((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/
(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(3/2)*c^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(117) = 234$.
time = 0.12, size = 306, normalized size = 2.14

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(-3ad\,x^2+c\,x^2b+2ac)}{8c^2x^4a}+\frac{\left(-\frac{3a\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{x^2}\right)}{16c^2\sqrt{ac}}\right.}{}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(3\ln\left(\frac{ad\,x^2+c\,x^2b+2\sqrt{ac}\,\sqrt{(b\,x^2+a)\,(d\,x^2+c)}+2ac}{x^2}\right)a^2d^2x^4-2\ln\left(\frac{ad\,x^2+c\,x^2b+2\sqrt{}}{}\right.\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\left(-\frac{b\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{8ac\,x^2}+\frac{\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)}}{x^2}\right.}{8c\sqrt{ac}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2*(3*ln((a*d*x^2+c*x^2*b+2*(a*c)^
(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a^2*d^2*x^4-2*ln((a*d*x^2+c*x
^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a*b*c*d*x^4-ln((
a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*b^2*c
^2*x^4-6*d*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a*(a*c)^(1/2)+2*((b*x^2+a)*(d*x^
2+c))^(1/2)*b*c*x^2*(a*c)^(1/2)+4*((b*x^2+a)*(d*x^2+c))^(1/2)*a*c*(a*c)^(1/
2))/((b*x^2+a)*(d*x^2+c))^(1/2)/x^4/(a*c)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 4.40, size = 358, normalized size = 2.50

$$\left[-\frac{{\left(b^{2} c^{2} + 2 \, a b c d - 3 \, a^{2} d^{2}\right)} \sqrt{a c} x^{4} \log\left(\frac{{\left(b^{2} c^{2} + 6 \, a b c d + a^{2} d^{2}\right)} x^{4} + 8 \, a^{2} c^{2} + 8 \, {\left(a b c^{2} + a^{2} c d\right)} x^{2} - 4 \, {\left({\left(b c + a d\right)} x^{2} + 2 \, a c\right)} \sqrt{b x^{2} + a} \sqrt{d x^{2} + c} \sqrt{a c}}{x^{4}}\right) + 4 \, {\left(2 \, a^{2} c^{2} + {\left(a b c^{2} - 3 \, a^{2} c d\right)} x^{2}\right)} \sqrt{b x^{2} + a} \sqrt{d x^{2} + c}}{32 \, a^{2} c^{3} x^{4}}, -\frac{{\left(b^{2} c^{2} + 2 \, a b c d - 3 \, a^{2} d^{2}\right)} \sqrt{-a c} x^{4} \arctan\left(\frac{{\left({\left(b c + a d\right)} x^{2} + 2 \, a c\right)} \sqrt{b x^{2} + a} \sqrt{d x^{2} + c} \sqrt{-a c}}{2 \, {\left(a b c d x^{4} + a^{2} c^{2} + {\left(a b c^{2} + a^{2} c d\right)} x^{2}\right)}}\right) + 2 \, {\left(2 \, a^{2} c^{2} + {\left(a b c^{2} - 3 \, a^{2} c d\right)} x^{2}\right)} \sqrt{b x^{2} + a} \sqrt{d x^{2} + c}}{16 \, a^{2} c^{3} x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a
*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) + 4*(2*a^2
*c^2 + (a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^3
*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2*(
(b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*
d*x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) + 2*(2*a^2*c^2 + (a*b*c^2 - 3*a
^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^3*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b x^{2}}}{x^{5} \sqrt{c + d x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**5*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. 2(117) = 234.
time = 2.10, size = 1107, normalized size = 7.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*((sqrt(b*d)*b^3*c^2 + 2*sqrt(b*d)*a*b^2*c*d - 3*sqrt(b*d)*a^2*b*d^2)*
arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x
^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a*b*c^2) - 2*(
sqrt(b*d)*b^9*c^5 - 7*sqrt(b*d)*a*b^8*c^4*d + 18*sqrt(b*d)*a^2*b^7*c^3*d^2
- 22*sqrt(b*d)*a^3*b^6*c^2*d^3 + 13*sqrt(b*d)*a^4*b^5*c*d^4 - 3*sqrt(b*d)*a
^5*b^4*d^5 - 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^2*b^7*c^4 + 16*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^6*c^3*d - 14*sqrt(b*d)*(sqrt(b*x^
2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^5*c^2*d^2
 - 8*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
a*b*d))^2*a^3*b^4*c*d^3 + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^3*d^4 + 3*sqrt(b*d)*(sqrt(b*x^2 + a)
*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^5*c^3 - 7*sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^
4*c^2*d - 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)
*b*d - a*b*d))^4*a^2*b^3*c*d^2 - 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - s
qrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b^2*d^3 - sqrt(b*d)*(sqrt(b*x^2
 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^3*c^2 - 2*sqrt
(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6
*a*b^2*c*d + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^6*a^2*b*d^2)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(s
qrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c -
 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*
b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4
)^2*a*c^2))/abs(b)
```

Mupad [B]
time = 11.53, size = 955, normalized size = 6.68

$$\frac{\ln\left(\frac{\sqrt{b x^2+a}-\sqrt{a}}{\sqrt{d x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b^2 c^{5/2}-3\,a^{5/2}\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^2\,c^3}-\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)\left(\frac{b d}{8 a c}-\frac{3 d (a d+b c)}{32 a c^2}\right)}{\sqrt{d x^2+c}-\sqrt{c}}-\frac{\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^5\left(\frac{a^2 d^2}{8}+\frac{5 b^2 c^2}{32}-\frac{11 a b c d}{32}\right)}{a c^3\left(\sqrt{d x^2+c}-\sqrt{c}\right)^5}-\frac{b^4}{64\sqrt{a}\,c^{3/2}\,d^2}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^2\left(\frac{11 a^2 b^2 d^2}{64}-\frac{5 b^4 c^2}{64}+\frac{a b^3 c d}{16}\right)}{a^{3/2} c^{5/2} d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^2}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^3\left(\frac{a^3 b d^3}{32}-\frac{9 a^2 b^2 c d^2}{16}+\frac{3 a b^3 c^2 d}{16}+\frac{b^4 c^3}{32}\right)}{a^2 c^3 d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^3}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)\left(\frac{b^4 c}{16}-\frac{a b^3 d}{16}\right)}{a c^2 d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^4\left(-\frac{7 a^4 d^4}{64}+\frac{a^3 b c d^3}{4}+\frac{21 a^2 b^2 c^2 d^2}{64}-\frac{a b^3 c^3 d}{4}+\frac{b^4 c^4}{64}\right)}{a^{5/2} c^{7/2} d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^4}}{\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^2}{\left(\sqrt{d x^2+c}-\sqrt{c}\right)^2}+\frac{a\left(\sqrt{b x^2+a}-\sqrt{a}\right)^6}{c\left(\sqrt{d x^2+c}-\sqrt{c}\right)^6}-\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^3\left(2 c b^2+2 a d b\right)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d x^2+c}-\sqrt{c}\right)^3}-\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^5\left(2 a d+2 b c\right)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d x^2+c}-\sqrt{c}\right)^5}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^4\left(a^2 d^2+4 a b c d+b^2 c^2\right)}{a c d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^4}}-\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b x^2+a}-\sqrt{a}\,\sqrt{d x^2+c}\right)\left(b\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b x^2+a}-\sqrt{a}\right)}{\sqrt{d x^2+c}-\sqrt{c}}\right)}{\sqrt{d x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b^2 c^{5/2}-3\,a^{5/2}\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^2\,c^3}+\frac{d^2\left(\sqrt{b x^2+a}-\sqrt{a}\right)^2}{64\sqrt{a}\,c^{3/2}\left(\sqrt{d x^2+c}-\sqrt{c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(x^5*(c + d*x^2)^(1/2)),x)
```

```
[Out] (log(((a + b*x^2)^(1/2) - a^(1/2))/((c + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*
b^2*c^(5/2) - 3*a^(5/2)*c^(1/2)*d^2 + 2*a^(3/2)*b*c^(3/2)*d))/(16*a^2*c^3)
- (((a + b*x^2)^(1/2) - a^(1/2))*((b*d)/(8*a*c) - (3*d*(a*d + b*c))/(32*a*c
^2)))/((c + d*x^2)^(1/2) - c^(1/2)) - ((((a + b*x^2)^(1/2) - a^(1/2))^5*((a
^2*d^2)/8 + (5*b^2*c^2)/32 - (11*a*b*c*d)/32))/(a*c^3*((c + d*x^2)^(1/2) -
c^(1/2))^5) - b^4/(64*a^(1/2)*c^(3/2)*d^2) + (((a + b*x^2)^(1/2) - a^(1/2))
^2*((11*a^2*b^2*d^2)/64 - (5*b^4*c^2)/64 + (a*b^3*c*d)/16))/(a^(3/2)*c^(5/2
)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^2) + (((a + b*x^2)^(1/2) - a^(1/2))^3*(
```

```
(b^4*c^3)/32 + (a^3*b*d^3)/32 - (9*a^2*b^2*c*d^2)/16 + (3*a*b^3*c^2*d)/16))
/(a^2*c^3*d^2*((c + d*x^2)^(1/2) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1
/2))*((b^4*c)/16 - (a*b^3*d)/16))/(a*c^2*d^2*((c + d*x^2)^(1/2) - c^(1/2)))
 + (((a + b*x^2)^(1/2) - a^(1/2))^4*((b^4*c^4)/64 - (7*a^4*d^4)/64 + (21*a^
2*b^2*c^2*d^2)/64 - (a*b^3*c^3*d)/4 + (a^3*b*c*d^3)/4))/(a^(5/2)*c^(7/2)*d^
2*((c + d*x^2)^(1/2) - c^(1/2))^4))/(((a + b*x^2)^(1/2) - a^(1/2))^6/((c +
d*x^2)^(1/2) - c^(1/2))^6 + (b^2*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d^2*((c
+ d*x^2)^(1/2) - c^(1/2))^2) - (((a + b*x^2)^(1/2) - a^(1/2))^3*(2*b^2*c +
2*a*b*d))/(a^(1/2)*c^(1/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^3) - (((a + b*
x^2)^(1/2) - a^(1/2))^5*(2*a*d + 2*b*c))/(a^(1/2)*c^(1/2)*d*((c + d*x^2)^(1
/2) - c^(1/2))^5) + (((a + b*x^2)^(1/2) - a^(1/2))^4*(a^2*d^2 + b^2*c^2 + 4
*a*b*c*d))/(a*c*d^2*((c + d*x^2)^(1/2) - c^(1/2))^4)) - (log(((c^(1/2)*(a +
 b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b*
x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/((c + d*x^2)^(1/2) -
 c^(1/2)))*(a^(1/2)*b^2*c^(5/2) - 3*a^(5/2)*c^(1/2)*d^2 + 2*a^(3/2)*b*c^(3/
2)*d))/(16*a^2*c^3) + (d^2*((a + b*x^2)^(1/2) - a^(1/2))^2)/(64*a^(1/2)*c^(
3/2)*((c + d*x^2)^(1/2) - c^(1/2))^2)
```

3.940 $\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=343

$$\frac{(8b^2c^2-3abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2d^2\sqrt{c+dx^2}}-\frac{(4bc-ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15bd^2}+\frac{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d}-\frac{\sqrt{c}\,(8b^2}{}$$

```
[Out] 1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/d^2/(d*x^2+c)^(
1/2)+1/15*c^(3/2)*(-a*d+4*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elli
pticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2
)/b/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(-2*a^2*d^
2-3*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*
d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2
)/b^2/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(-a*d+4*
b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^2+1/5*x^3*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {489, 596, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(-2a^2d^2-3abcd+8b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\,(-2a^2d^2-3abcd+8b^2c^2)}{15b^2d^2\sqrt{c+dx^2}}+\frac{c^{3/2}\sqrt{a+bx^2}\,(4bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(4bc-ad)}{15bd^2}+\frac{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*d^2*Sqrt[c
 + d*x^2]) - ((4*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b*d^2) +
(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c
*d - 2*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(15*b^2*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]) + (c^(3/2)*(4*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d} - \frac{\int \frac{x^2(3ac+(4bc-ad)x^2)}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{5d}$$

$$= -\frac{(4bc-ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15bd^2} + \frac{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d} + \frac{\int \frac{ac(4bc-ad)+(8b^2c^2-3ab\cdots}{\sqrt{a+bx^2}\,\sqrt{c\cdots}}}{15bd^2}$$

$$= -\frac{(4bc-ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15bd^2} + \frac{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d} + \frac{(ac(4bc-ad))\int \frac{\cdots}{\sqrt{a\cdots}}}{15\cdots}$$

$$= \frac{(8b^2c^2-3abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2d^2\sqrt{c+dx^2}} - \frac{(4bc-ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15bd^2} + \frac{x^3\sqrt{a+b\cdots}}{\cdots}$$

$$= \frac{(8b^2c^2-3abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2d^2\sqrt{c+dx^2}} - \frac{(4bc-ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15bd^2} + \frac{x^3\sqrt{a+b\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.60, size = 246, normalized size = 0.72

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2)(-4bc+ad+3bdx^2) + ic(-8b^2c^2+3abcd+2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right) - ic(-8b^2c^2+7abcd+a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)}{15b\sqrt{\frac{b}{a}}\,d^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + a*d + 3*b*d*x^2) + I*c*(-8
*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-8*b^2*c^2 + 7*a*b*c*d
 + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])
```

Maple [A]
time = 0.14, size = 526, normalized size = 1.53

method	result

elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{x^3\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{5d}+\frac{\left(a-\frac{4ad+4bc}{5d}\right)x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3bd}\right.}{}$
risch	$\frac{x(3bd\,x^2+ad-4bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15b\,d^2}-\frac{\left(-\frac{(2a^2d^2+3abcd-8b^2c^2)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+4\sqrt{-\frac{b}{a}}\,ab\,d^3x^5-\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+\sqrt{-\frac{b}{a}}\,a^2d^3x^3-4\sqrt{-\frac{b}{a}}\,b^2c^2d\,x^3+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*b^2*d^3*x^7+4*(-b/a)^(
1/2)*a*b*d^3*x^5-(-b/a)^(1/2)*b^2*c*d^2*x^5+(-b/a)^(1/2)*a^2*d^3*x^3-4*(-b/
a)^(1/2)*b^2*c^2*d*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-8*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3
-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*a^2*c*d^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+(-b/a)^(1/2)*a^2*c*
d^2*x-4*(-b/a)^(1/2)*a*b*c^2*d*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d^3/b/(-b/a
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

3.941 $\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=259

$$-\frac{(2bc-ad)x\sqrt{a+bx^2}}{3bd\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3d}+\frac{\sqrt{c}\,(2bc-ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}-\frac{c^{3/}}{}$$

```
[Out] -1/3*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/b/d/(d*x^2+c)^(1/2)-1/3*c^(3/2)*(1/(1+d
*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1
/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+1/3*(-a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*
(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1
/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {489, 545, 429, 506, 422}

$$-\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(2bc-ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3d}-\frac{x\sqrt{a+bx^2}\,(2bc-ad)}{3bd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] -1/3*((2*b*c - a*d)*x*Sqrt[a + b*x^2])/(b*d*Sqrt[c + d*x^2]) + (x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3d} - \frac{\int \frac{ac+(2bc-ad)x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{3d}$$

$$= \frac{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3d} - \frac{(ac)\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{3d} - \frac{(2bc-ad)\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{c}}}{3d}$$

$$= -\frac{(2bc-ad)x\sqrt{a+bx^2}}{3bd\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3d} - \frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right.}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+d}}$$

$$= -\frac{(2bc-ad)x\sqrt{a+bx^2}}{3bd\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3d} + \frac{\sqrt{c}\,(2bc-ad)\sqrt{a+bx^2}\,E\left(\tan^{-}\right.}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.06, size = 199, normalized size = 0.77

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2) - ic(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right) + 2ic(-bc+ad)\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3\sqrt{\frac{b}{a}}\,d^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (
2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])
```

Maple [A]
time = 0.13, size = 335, normalized size = 1.29

method	result
risch	$\frac{x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3d} - \left(\frac{(ad-2bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{EllipticE}\left(x\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}\,d}\right.$

elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3d}-\frac{ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\,\sqrt{\ldots}\right.}{3d\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b\ldots}}\right.}{\sqrt{\ldots}}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(-\sqrt{-\frac{b}{a}}\,b\,d^2x^5-\sqrt{-\frac{b}{a}}\,a\,d^2x^3-\sqrt{-\frac{b}{a}}\,bcd\,x^3+2ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*b*d^2*x^5-(-b/a)^(1/2)*
a*d^2*x^3-(-b/a)^(1/2)*b*c*d*x^3+2*a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-2*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-EllipticE(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*a*c*d
+2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*b*c^2-(-b/a)^(1/2)*a*c*d*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b
/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

3.942 $\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=232

$$\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx}-\frac{\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}+\frac{b\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\right.}{a\sqrt{d}\,\sqrt{\frac{c(a+bx}{a(c+dx}}}$$

```
[Out] d*x*(b*x^2+a)^(1/2)/c/(d*x^2+c)^(1/2)+b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(
1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/
2)*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2
)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/c^(1/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x
```

Rubi [A]
time = 0.10, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {486, 21, 433, 429, 506, 422}

$$\frac{b\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])/(c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]) + (b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx} + \frac{\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx} + \frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{c}$$

$$= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx} + b\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx + \frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

$$= \frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx} + \frac{b\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

$$= \frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx} - \frac{\sqrt{d}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

Mathematica [A]
time = 1.18, size = 111, normalized size = 0.48

$$\frac{-((a+bx^2)(c+dx^2)) + \frac{bcx\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}}}{cx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a + b*x^2)*(c + d*x^2)) + (b*c*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/Sqrt[-(b/a)])/(c*x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 168, normalized size = 0.72

method	result

default	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\left(-\sqrt{-\frac{b}{a}}\, b d x^4+b c \sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\, x\, \mathrm{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right)-\sqrt{-\frac{b}{a}}\, a d x^2-\sqrt{-\cdots}}{(b d x^4+a d x^2+c x^2 b+a c) c \sqrt{-\frac{b}{a}}\, x}$
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}{c x}+\frac{b\left(\frac{c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d x^4+a d x^2+c x^2 b+a c}}-\frac{c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{\sqrt{b d x^4+a d x^2+c x^2 b+a c}}{c x}+\frac{b\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\cdots}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d x^4+a d x^2+c x^2 b+\cdots}}\right.}{\sqrt{b x^2+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*b*d*x^4+b*c*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-(-b/a)^
(1/2)*a*d*x^2-(-b/a)^(1/2)*b*c*x^2-(-b/a)^(1/2)*a*c)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)/c/(-b/a)^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2}}{x^2 \sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**2/(d*x**2+c)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/(x**2*sqrt(c + d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}}{x^2\,\sqrt{d\,x^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(1/2)),x)
[Out] int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(1/2)), x)
```

3.943 $\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=307

$$\frac{d(bc-2ad)x\sqrt{a+bx^2}}{3ac^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}-\frac{(bc-2ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2x}-\frac{\sqrt{d}\,(bc-2ad)\sqrt{a+bx^2}\,E}{3ac^{3/2}\sqrt{\frac{c\,(a+}{a\,(c+}}}$$

```
[Out] 1/3*d*(-2*a*d+b*c)*x*(b*x^2+a)^(1/2)/a/c^2/(d*x^2+c)^(1/2)-1/3*(-2*a*d+b*c)
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a/c^(3/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d
^(1/2)*(b*x^2+a)^(1/2)/a/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^
(1/2)-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^3-1/3*(-2*a*d+b*c)*(b*x^2+a)^
(1/2)*(d*x^2+c)^(1/2)/a/c^2/x
```

Rubi [A]
time = 0.18, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {486, 597, 545, 429, 506, 422}

$$-\frac{\sqrt{d}\sqrt{a+bx^2}\,(bc-2ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}-\frac{b\sqrt{d}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(bc-2ad)}{3ac^2x}+\frac{dx\sqrt{a+bx^2}\,(bc-2ad)}{3ac^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*a*c^2*Sqrt[c + d*x^2]) - (Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} + \frac{\int \frac{bc-2ad-bdx^2}{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{3c} \\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} - \frac{(bc-2ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3ac^2x} - \frac{\int \frac{abcd-bd(bc-2ad)x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{3ac^2} \\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} - \frac{(bc-2ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3ac^2x} - \frac{(bd)\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx}}}{3c} \\
&= \frac{d(bc-2ad)x\sqrt{a+bx^2}}{3ac^2\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} - \frac{(bc-2ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3ac^2x} - \\
&= \frac{d(bc-2ad)x\sqrt{a+bx^2}}{3ac^2\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} - \frac{(bc-2ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3ac^2x} -
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.79, size = 228, normalized size = 0.74

$$
\frac{-\frac{(a+bx^2)(c+dx^2)(ac+bcx^2-2adx^2)}{a} + i\sqrt{\frac{b}{a}}\,c(-bc+2ad)x^3\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right) + i\sqrt{\frac{b}{a}}\,c(bc-ad)x^3\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3c^2x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(((a + b*x^2)*(c + d*x^2)*(a*c + b*c*x^2 - 2*a*d*x^2))/a) + I*Sqrt[b/a]*c
*(-(b*c) + 2*a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*Sqrt[b/a]*c*(b*c - a*d)*x^3*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(3*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 418, normalized size = 1.36

method	result

risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(-2ad\,x^2+c\,x^2b+ac)}{3c^2x^3a}-\frac{bd\left(-\frac{(2ad-bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,}}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(-\frac{\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3c\,x^3}+\frac{(2ad-bc)\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3c^2ax}-\frac{bd\sqrt{}}{}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(2\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-\sqrt{-\frac{b}{a}}\,b^2cd\,x^6+bd\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(2*(-b/a)^(1/2)*a*b*d^2*x^6-(-b/a)^(1/2
)*b^2*c*d*x^6+b*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*x^3*a*c-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3-2*((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3+((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*b^2*c^2*x^3+2*(-b/a)^(1/2)*a^2*d^2*x^4-(-b/a)^(1/2)*b^2*c^2*x^4+(-b/a)
^(1/2)*a^2*c*d*x^2-2*(-b/a)^(1/2)*a*b*c^2*x^2-(-b/a)^(1/2)*a^2*c^2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)/c^2/(-b/a)^(1/2)/x^3/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(1/2)/x**4/(d*x**2+c)**(1/2),x)

[Out] Integral(sqrt(a + b*x**2)/(x**4*sqrt(c + d*x**2)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(1/2)),x)

[Out] int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(1/2)), x)

3.944 $\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=276

$$-\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{128b^2d^4}+\frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192b^2d^3}$$

```
[Out] 1/128*(-a*d+b*c)^2*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*arctanh(d^(1/2)*(b*x^2
+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(5/2)/d^(9/2)+1/192*(3*a^2*d^2+10*a*b*
c*d+35*b^2*c^2)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b^2/d^3-1/48*(3*a*d+7*b*c)*
(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b^2/d^2+1/8*x^2*(b*x^2+a)^(5/2)*(d*x^2+c)^(
1/2)/b/d-1/128*(-a*d+b*c)*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*(b*x^2+a)^(1/2)
*(d*x^2+c)^(1/2)/b^2/d^4
```

Rubi [A]
time = 0.23, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {457, 92, 81, 52, 65, 223, 212}

$$-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)}{128b^2d^4}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(3a^2d^2+10abcd+35b^2c^2\right)}{192b^2d^3}+\frac{(bc-ad)^2\left(3a^2d^2+10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{128b^{5/2}d^{9/2}}-\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\,(3ad+7bc)}{48b^2d^2}+\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] -1/128*((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/(b^2*d^4) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*x
^2)^(3/2)*Sqrt[c + d*x^2])/(192*b^2*d^3) - ((7*b*c + 3*a*d)*(a + b*x^2)^(5/
2)*Sqrt[c + d*x^2])/(48*b^2*d^2) + (x^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/
(8*b*d) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqr
t[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(5/2)*d^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2(a+bx^2)^{5/2}\sqrt{c+dx^2}}{8bd} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(-ac-\frac{1}{2}(7bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^2\right)}{8bd}$$

$$= -\frac{(7bc+3ad)\,(a+bx^2)^{5/2}\sqrt{c+dx^2}}{48b^2d^2} + \frac{x^2(a+bx^2)^{5/2}\sqrt{c+dx^2}}{8bd} + \frac{(35b^2c^2+10ab\cdots}{\cdots}$$

$$= \frac{(35b^2c^2+10abcd+3a^2d^2)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{192b^2d^3} - \frac{(7bc+3ad)\,(a+bx^2)^{5/2}\sqrt{c+\cdots}}{48b^2d^2}$$

$$= -\frac{(bc-ad)\,(35b^2c^2+10abcd+3a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{(35b^2c^2+10abcd+3\cdots}{\cdots}$$

$$= -\frac{(bc-ad)\,(35b^2c^2+10abcd+3a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{(35b^2c^2+10abcd+3\cdots}{\cdots}$$

$$= -\frac{(bc-ad)\,(35b^2c^2+10abcd+3a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{(35b^2c^2+10abcd+3\cdots}{\cdots}$$

$$= -\frac{(bc-ad)\,(35b^2c^2+10abcd+3a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{(35b^2c^2+10abcd+3\cdots}{\cdots}$$

Mathematica [A]
time = 3.09, size = 231, normalized size = 0.84

$$\frac{-b\sqrt{d}\sqrt{a+bx^2}\,(c+dx^2)\,(9a^3d^3+3a^2bd^2(5c-2dx^2)+ab^2d(-145c^2+92cdx^2-72d^2x^4)+b^3(105c^3-70c^2dx^2+56cd^2x^4-48d^3x^6))+3(bc-ad)^{5/2}(35b^2c^2+10abcd+3a^2d^2)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{384b^3d^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2*
d*x^2) + a*b^2*d*(-145*c^2 + 92*c*d*x^2 - 72*d^2*x^4) + b^3*(105*c^3 - 70*c
^2*d*x^2 + 56*c*d^2*x^4 - 48*d^3*x^6))) + 3*(b*c - a*d)^(5/2)*(35*b^2*c^2 +
 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(384*b^3*d^(9/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(238) = 476$.

time = 0.13, size = 658, normalized size = 2.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(96*b^3*d^3*x^6*((b*x^2+a)*(d*x^2+c))
^(1/2)*(b*d)^(1/2)+144*a*b^2*d^3*x^4*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2
)-112*b^3*c*d^2*x^4*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+12*((b*x^2+a)*(
d*x^2+c))^(1/2)*x^2*a^2*b*d^3*(b*d)^(1/2)-184*((b*x^2+a)*(d*x^2+c))^(1/2)*x
^2*a*c*b^2*d^2*(b*d)^(1/2)+140*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*c^2*b^3*d*(b
*d)^(1/2)+9*d^4*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*a^4+12*a^3*c*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c
))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*d^3+54*ln(1/2*(2*b*d*x^2+2*((b
*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c^2*b^2*d^2-
180*a*c^3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b
*c)/(b*d)^(1/2))*b^3*d+105*b^4*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1
/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^4-18*((b*x^2+a)*(d*x^2+c))^(1/2)*a^
3*d^3*(b*d)^(1/2)-30*((b*x^2+a)*(d*x^2+c))^(1/2)*a^2*c*b*d^2*(b*d)^(1/2)+29
0*((b*x^2+a)*(d*x^2+c))^(1/2)*a*c^2*b^2*d*(b*d)^(1/2)-210*((b*x^2+a)*(d*x^2
+c))^(1/2)*c^3*b^3*(b*d)^(1/2))/b^2/d^4/((b*x^2+a)*(d*x^2+c))^(1/2)/(b*d)^(
1/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 0.80, size = 574, normalized size = 2.08

$$\left[\frac{3\,(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)+4\,(48\,b^4d^4x^6-105\,b^4c^3d+145\,ab^3c^2d^2-15\,a^2b^2cd^3-9\,a^3bd^4-8\,(7\,b^4cd^3-9\,ab^3d^4)x^4+2\,(35\,b^4c^2d^2-46\,ab^3cd^3+3\,a^2b^2d^4)x^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{1536\,b^3d^5},\ -\frac{3\,(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)-2\,(48\,b^4d^4x^6-105\,b^4c^3d+145\,ab^3c^2d^2-15\,a^2b^2cd^3-9\,a^3bd^4-8\,(7\,b^4cd^3-9\,ab^3d^4)x^4+2\,(35\,b^4c^2d^2-46\,ab^3cd^3+3\,a^2b^2d^4)x^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{768\,b^3d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
```

```
+ 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqr
t(d*x^2 + c)*sqrt(b*d)) + 4*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 145*a*b^3*c^2
*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^4 +
 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c))/(b^3*d^5), -1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*
b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 +
 b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c
*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 145*a*
b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4
)*x^4 + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2)*sqrt(b*x^2
 + a)*sqrt(d*x^2 + c))/(b^3*d^5)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(x**5*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [A]

time = 2.01, size = 305, normalized size = 1.11

$$\frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}\left(2\,(bx^2+a)\left(4\,(bx^2+a)\left(\frac{6\,(bx^2+a)}{b^3d}-\frac{7\,b^7cd^5+9\,ab^6d^6}{b^9d^7}\right)+\frac{35\,b^8c^2d^4+10\,ab^7cd^5+3\,a^2b^6d^6}{b^9d^7}\right)-\frac{3\,(35\,b^9c^3d^3-25\,ab^8c^2d^4-7\,a^2b^7cd^5-3\,a^3b^6d^6)}{b^9d^7}\right)-\frac{3\,(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,b^2d^4}\right)b}{384\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] 1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)
*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b^3*d) - (7*b^7*c*d^5 + 9*a*b^6*d^6)/(b^9*d
^7)) + (35*b^8*c^2*d^4 + 10*a*b^7*c*d^5 + 3*a^2*b^6*d^6)/(b^9*d^7)) - 3*(35
*b^9*c^3*d^3 - 25*a*b^8*c^2*d^4 - 7*a^2*b^7*c*d^5 - 3*a^3*b^6*d^6)/(b^9*d^7
)) - 3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 +
3*a^4*d^4)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d)))/(sqrt(b*d)*b^2*d^4))*b/abs(b)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
[Out] int((x^5*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.945 $\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$\frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{16bd^3}-\frac{(5bc+ad)\,(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{24bd^2}+\frac{(a+bx^2)^{5/2}\,\sqrt{c+dx^2}}{6bd}-\frac{(bc-$$

```
[Out] -1/16*(-a*d+b*c)^2*(a*d+5*b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x
^2+c)^(1/2))/b^(3/2)/d^(7/2)-1/24*(a*d+5*b*c)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/
2)/b/d^2+1/6*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b/d+1/16*(-a*d+b*c)*(a*d+5*b*c
)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^3
```

Rubi [A]
time = 0.12, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {457, 81, 52, 65, 223, 212}

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{16b^{3/2}d^{7/2}}+\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)(ad+5bc)}{16bd^3}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(ad+5bc)}{24bd^2}+\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b*d^3) - ((
5*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b*d^2) + ((a + b*x^2)^(
5/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(16*b^(3/2)*d^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{6bd} - \frac{(5bc+ad)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{12bd}$$
$$= -\frac{(5bc+ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{6bd} + \frac{((bc-ad)(5bc+ad}{}$$
$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{(a+}{}$$
$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{(a+}{}$$
$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{(a+}{}$$
$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{(a+}{}$$

Mathematica [**A**]
time = 1.71, size = 148, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3a^2d^2+2abd(-11c+7dx^2)+b^2(15c^2-10cdx^2+8d^2x^4))}{48bd^3} - \frac{(bc-ad)^2(5bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{16b^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^2) + b
^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)))/(48*b*d^3) - ((b*c - a*d)^2*(5*b*c +
 a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(16*b^(
3/2)*d^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(155) = 310.
time = 0.12, size = 455, normalized size = 2.43

method	result
risch	$\frac{(8b^2x^4d^2+14abd^2x^2-10b^2cdx^2+3a^2d^2-22abcd+15b^2c^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{48bd^3}+\frac{\left(-\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx^2}{\sqrt{bd}}+\sqrt{bdx^4+}\right.}{32b\sqrt{b}}\right.}{}$
default	$-\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}\left(-16b^2d^2x^4\sqrt{(bx^2+a)(dx^2+c)}\,\sqrt{bd}-28\sqrt{(bx^2+a)(dx^2+c)}\,x^2abd^2\sqrt{}\right.}{}$
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{bx^4\sqrt{bdx^4+(ad+bc)x^2+ac}}{6d}+\frac{7x^2\sqrt{bdx^4+(ad+bc)x^2+ac}\,a}{24d}-\frac{5bx^2\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-16*b^2*d^2*x^4*((b*x^2+a)*(d*x^2+c)
)^(1/2)*(b*d)^(1/2)-28*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)+
20*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)+3*d^3*ln(1/2*(2*b*d*
x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3+9*l
n(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(
1/2))*a^2*c*b*d^2-27*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^
(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c^2*b^2*d+15*b^3*ln(1/2*(2*b*d*x^2+2*((b*x^2+
a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3-6*((b*x^2+a)*(d*x
^2+c))^(1/2)*a^2*d^2*(b*d)^(1/2)+44*((b*x^2+a)*(d*x^2+c))^(1/2)*a*c*b*d*(b*
d)^(1/2)-30*((b*x^2+a)*(d*x^2+c))^(1/2)*c^2*b^2*(b*d)^(1/2))/b/d^3/((b*x^2+
a)*(d*x^2+c))^(1/2)/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.71, size = 440, normalized size = 2.35

$$\left[\frac{3\,(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2-4\,(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)+4\,(8\,b^3d^3x^4+15\,b^3c^2d-22\,ab^2cd^2+3\,a^2bd^3-2\,(5\,b^3cd^2-7\,ab^2d^3)x^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{192\,b^2d^4},\frac{3\,(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)+2\,(8\,b^3d^3x^4+15\,b^3c^2d-22\,ab^2cd^2+3\,a^2bd^3-2\,(5\,b^3cd^2-7\,ab^2d^3)x^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{96\,b^2d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*l
og(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^
2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) +
4*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b^3*c
*d^2 - 7*a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^4), 1/96*(
3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b*d)*arctan(1
/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*
d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3*c
^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [A]
time = 0.87, size = 225, normalized size = 1.20

$$\frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}\left(2\,(bx^2+a)\left(\frac{4\,(bx^2+a)}{b^2d}-\frac{5\,b^3cd^3+ab^2d^4}{b^4d^5}\right)+\frac{3\,(5\,b^4c^2d^2-4\,ab^3cd^3-a^2b^2d^4)}{b^4d^5}\right)+\frac{3\,(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^3}\right)b}{48\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)*
(4*(b*x^2 + a)/(b^2*d) - (5*b^3*c*d^3 + a*b^2*d^4)/(b^4*d^5)) + 3*(5*b^4*c^
2*d^2 - 4*a*b^3*c*d^3 - a^2*b^2*d^4)/(b^4*d^5)) + 3*(5*b^3*c^3 - 9*a*b^2*c^
2*d + 3*a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^3))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
[Out] int((x^3*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.946 $\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{3(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8d^2}+\frac{(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{4d}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8\sqrt{b}\,d^{5/2}}$$

```
[Out] 3/8*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/d
^(5/2)/b^(1/2)+1/4*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d-3/8*(-a*d+b*c)*(b*x^2+
a)^(1/2)*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {455, 52, 65, 223, 212}

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8\sqrt{b}\,d^{5/2}}-\frac{3\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)}{8d^2}+\frac{(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + ((a + b*x^2)^(3/
2)*Sqrt[c + d*x^2])/(4*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^
2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*Sqrt[b]*d^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{8d}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{(3(bc-ad)^2)\,\text{Subst}\Big(}{}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{(3(bc-ad)^2)\,\text{Subst}\Bigg(}{}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{(3(bc-ad)^2)\,\text{Subst}\Big(}{}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{3(bc-ad)^2\tanh^{-1}\Big(}{8\sqrt{b}\,d^5}$$

Mathematica [A]

time = 0.96, size = 104, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(-3bc+5ad+2bdx^2)}{8d^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{a+bx^2}}\right)}{8\sqrt{b}\,d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + 5*a*d + 2*b*d*x^2))/(8*d^2) + (3
*(b*c - a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])]
)/(8*Sqrt[b]*d^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(99) = 198$.
time = 0.13, size = 288, normalized size = 2.30

method	result
risch	$\frac{(2bd\,x^2+5ad-3bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{8d^2}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)a^2}{16\sqrt{bd}}-\frac{3\ln\left(\frac{\frac{1}{2}ad+}{}\right.}{}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(4\sqrt{bd}\,\sqrt{(b\,x^2+a)(d\,x^2+c)}\,bd\,x^2+3d^2\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bc}}{2\sqrt{bd}}\right.\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{b\,x^2\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{4d}+\frac{5\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\,a}{8d}-\frac{3b\sqrt{bd\,x}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(4*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(
1/2)*b*d*x^2+3*d^2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1
/2)+a*d+b*c)/(b*d)^(1/2))*a^2-6*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(
1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c*b*d+3*b^2*ln(1/2*(2*b*d*x^2+2*((
b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^2+10*(b*d)^(1
/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*a*d-6*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/
2)*b*c)/((b*x^2+a)*(d*x^2+c))^(1/2)/d^2/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 0.73, size = 334, normalized size = 2.67

$$\left[\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)+4\,(2\,b^2d^2x^2-3\,b^2cd+5\,abd^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{32\,bd^3},\,-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)-2\,(2\,b^2d^2x^2-3\,b^2cd+5\,abd^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{16\,bd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c
+ a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(2*b^2*d^2*x^2 - 3*b^
2*c*d + 5*a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^3), -1/16*(3*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d
+ a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d + 5*a*b*d^2)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c))/(b*d^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [**A**]
time = 0.79, size = 149, normalized size = 1.19

$$\frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}\,\left(\frac{2\,(bx^2+a)}{bd}-\frac{3\,(bcd-ad^2)}{bd^3}\right)-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^2}\right)b}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(
b*d) - 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(a
bs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sq
rt(b*d)*d^2))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.947 $\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=133

$$\frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{b}\,(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2d^{3/2}}$$

```
[Out] -1/2*(-3*a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))*
b^(1/2)/d^(3/2)-a^(3/2)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(
1/2))/c^(1/2)+1/2*b*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {457, 104, 163, 65, 223, 212, 95, 214}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{b}\,(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - (a^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[
a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] - (Sqrt[b]*(b*c - 3*a*d)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} + \frac{\text{Subst}\left(\int \frac{a^2d-\frac{1}{2}b(bc-3ad)x}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)}{2d}$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right) - \frac{(b(bc-3ad)}{}$$

$$(bc-3ad)\text{Subst}$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} + a^2\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right) - \frac{}{}$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} - \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}} - \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,d\right.}{2d}$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} - \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}} - \frac{\sqrt{b}\,(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{2d^{3/2}}$$

Mathematica [A]
time = 1.23, size = 132, normalized size = 0.99

$$\frac{1}{2}\left(\frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{d} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c+dx^2}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}} - \frac{\sqrt{b}\,(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{a+bx^2}}\right)}{d^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/d - (2*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c
 + d*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[c] - (Sqrt[b]*(b*c - 3*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/d^(3/2))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(101) = 202$.
time = 0.12, size = 252, normalized size = 1.89

method	result

elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{b\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{2d}+\frac{3ab\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+\cdots}\right)}{4\sqrt{bd}}\cdots\right.}{\sqrt{bx\cdots}}$
default	$\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\left(3\ln\left(\frac{2bd\,x^2+2\sqrt{(bx^2+a)(dx^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}\,abd-\ln\left(\frac{2bd\,x^2+2\sqrt{(bx^2+\cdots}}{\cdots}\right.\cdots\right.}{4\sqrt{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^
2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a*b*d-ln(1/2*(2*b
*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c
)^(1/2)*b^2*c-2*(b*d)^(1/2)*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d
*x^2+c))^(1/2)+2*a*c)/x^2)*a^2*d+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)*
(a*c)^(1/2)*b)/((b*x^2+a)*(d*x^2+c))^(1/2)/d/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.25, size = 918, normalized size = 6.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*d*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2
+ 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - (b*c - 3*a*d)*sqrt(b/d)*log(8*b^2*d^
```

```
2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*
d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, 1/4*(a*d*sqrt(a/c)*log(((b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 +
(b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + (b*c
 - 3*a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqr
t(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + 2*sqrt
(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, 1/8*(4*a*d*sqrt(-a/c)*arctan(1/2*((b*c +
a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a
^2*c + (a*b*c + a^2*d)*x^2)) - (b*c - 3*a*d)*sqrt(b/d)*log(8*b^2*d^2*x^4 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2
+ b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c)*b)/d, 1/4*(2*a*d*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c +
(a*b*c + a^2*d)*x^2)) + (b*c - 3*a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*
c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b
^2*c + a*b*d)*x^2)) + 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{x\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x*sqrt(c + d*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{x\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(x*(c + d*x^2)^(1/2)),x)
[Out] int((a + b*x^2)^(3/2)/(x*(c + d*x^2)^(1/2)), x)
```

3.948 $\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2}-\frac{\sqrt{a}\,(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

```
[Out] -1/2*(-a*d+3*b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*
a^(1/2)/c^(3/2)+b^(3/2)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(
1/2))/d^(1/2)-1/2*a*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {457, 100, 163, 65, 223, 212, 95, 214}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\,(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2) - (Sqrt[a]*(3*b*c - a*d)*A
rcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2)) +
(b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/Sqrt
[d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right) \\
&= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)-b^2cx}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)}{2c} \\
&= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} + \frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right) + \frac{(a(3bc-a\cdots}{\cdots} \\
&= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} + b\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right) + \frac{(a(3bc\cdots}{\cdots} \\
&= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} - \frac{\sqrt{a}\,(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}} + b\text{Subst}\left(\int \frac{\cdots}{1\cdots}\right. \\
&= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} - \frac{\sqrt{a}\,(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}} + \frac{b^{3/2}\tanh^{-1}\left(\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.39, size = 135, normalized size = 0.99

$$
\frac{1}{2}\left(-\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx^2} + \frac{\sqrt{a}\,(-3bc+ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c+dx^2}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{c^{3/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{a+bx^2}}\right)}{\sqrt{d}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) + (Sqrt[a]*(-3*b*c + a*d)*A
rcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])])/c^(3/2) + (2*b
^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/Sqrt[d
])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(104) = 208$.
time = 0.14, size = 263, normalized size = 1.93

method	result

risch	$-\frac{a\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{\left(\frac{b^2\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)}{2\sqrt{bd}}+\frac{a^2\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{c}\ldots}{\ldots}\right)}{\ldots}\ldots\right.}{\ldots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{b^2\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)}{2\sqrt{bd}}-\frac{3a\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{\ldots}}{\ldots}\right)}{4\ldots}\ldots\right.}{\sqrt{\ldots}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(2\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^2c\,x^2\sqrt{ac}+\ln\left(\frac{ad\,x^2+c\,x^2b+2\sqrt{ac}\ldots}{\ldots}\right)\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*
x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^2*c*x^2*(a*c)^(1/2)+ln((a
*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a^2*d*
x^2*(b*d)^(1/2)-3*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(
1/2)+2*a*c)/x^2)*a*b*c*x^2*(b*d)^(1/2)-2*a*(b*d)^(1/2)*(a*c)^(1/2)*((b*x^2+
a)*(d*x^2+c))^(1/2))/((b*x^2+a)*(d*x^2+c))^(1/2)/x^2/(b*d)^(1/2)/(a*c)^(1/2
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(104) = 208.
time = 1.83, size = 958, normalized size = 7.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b*c*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(b/d)) - (3*b*c - a*d)*x^2*sqrt(a/c)*log(((b^2*c^2
+ 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*(2*a
*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4)
 - 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), -1/8*(4*b*c*x^2*sqrt(-b/d)
*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b
/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + (3*b*c - a*d)*x^2*sqrt(a/c
)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c
*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(a/c))/x^4) + 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), 1/4*(b*c*x
^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d
 + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c)*sqrt(b/d)) + (3*b*c - a*d)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c +
(a*b*c + a^2*d)*x^2)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), -1/4*
(2*b*c*x^2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (3*b*
c - a*d)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) + 2
*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**3*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(104) = 208.
time = 1.51, size = 498, normalized size = 3.66

$$-\frac{\left(\frac{\sqrt{bd}\,b\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{d}+\frac{\left(3\sqrt{bd}\,ab^2c-\sqrt{bd}\,a^2bd\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,bc}+\frac{2\left(\sqrt{bd}\,ab^4c^2-2\sqrt{bd}\,a^2b^3cd+\sqrt{bd}\,a^3b^2d^2-\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2ab^2c-\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^2bd\right)}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2abd+\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4\right)c}\right)b}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(b*d)*b*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)
*b*d - a*b*d))^2)/d + (3*sqrt(b*d)*a*b^2*c - sqrt(b*d)*a^2*b*d)*arctan(-1/2
*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d
 - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c) + 2*(sqrt(b*d)*a*b^4
*c^2 - 2*sqrt(b*d)*a^2*b^3*c*d + sqrt(b*d)*a^3*b^2*d^2 - sqrt(b*d)*(sqrt(b*
x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^2*c - sqr
t(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^
2*a^2*b*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*
sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*c))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac {{\left (b\,x^2+a\right)}^{3/2}}{x^3\,\sqrt {d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(x^3*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(x^3*(c + d*x^2)^(1/2)), x)
```

3.949 $\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=131

$$-\frac{3(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8c^2x^2}-\frac{(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{4cx^4}-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8\sqrt{a}\,c^{5/2}}$$

```
[Out] -3/8*(-a*d+b*c)^2*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/
c^(5/2)/a^(1/2)-1/4*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/c/x^4-3/8*(-a*d+b*c)*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2/x^2
```

Rubi [A]

time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {457, 96, 95, 214}

$$-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8\sqrt{a}\,c^{5/2}}-\frac{3\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)}{8c^2x^2}-\frac{(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{4cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2) - ((a + b*x^2)
^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (3*(b*c - a*d)^2*ArcTanh[(Sqrt[c]*Sqrt[
a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*Sqrt[a]*c^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} + \frac{(3(bc-ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)}{8c}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} + \frac{(3(bc-ad)^2)\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} + \frac{(3(bc-ad)^2)\,\text{Subst}\left(\int \right.}{8}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{8\sqrt{a}\,c^{5/2}}$$

Mathematica [**A**]
time = 1.44, size = 110, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(-2ac-5bcx^2+3adx^2)}{8c^2x^4} - \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8\sqrt{a}\,c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - 5*b*c*x^2 + 3*a*d*x^2))/(8*c^2*x
^4) - (3*(b*c - a*d)^2*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a +
b*x^2])])/(8*Sqrt[a]*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(105) = 210$.

time = 0.13, size = 303, normalized size = 2.31

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(-3ad\,x^2+5c\,x^2b+2ac)}{8c^2x^4}+\frac{\left(-\frac{3\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{x^2}\right)}{16c^2\sqrt{ac}}\cdots\right.}{\cdots}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(3\ln\left(\frac{ad\,x^2+c\,x^2b+2\sqrt{ac}\,\sqrt{(b\,x^2+a)\,(d\,x^2+c)}+2ac}{x^2}\right)a^2d^2x^4-6\ln\left(\frac{ad\,x^2+c\,x^2b+2\sqrt{\cdots}}{\cdots}\right)\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\left(-\frac{3\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{x^2}\right)b^2}{16\sqrt{ac}}-\frac{5b\sqrt{bd\,x^4+(ad\cdots}}{8c\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2*(3*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1
/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a^2*d^2*x^4-6*ln((a*d*x^2+c*x^2
*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a*b*c*d*x^4+3*ln((
a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*b^2*c
^2*x^4-6*d*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a*(a*c)^(1/2)+10*((b*x^2+a)*(d*x
^2+c))^(1/2)*b*c*x^2*(a*c)^(1/2)+4*((b*x^2+a)*(d*x^2+c))^(1/2)*a*c*(a*c)^(1
/2))/((b*x^2+a)*(d*x^2+c))^(1/2)/x^4/(a*c)^(1/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more
 detail
```

Fricas [A]
time = 1.99, size = 360, normalized size = 2.75

$$\left[\frac{3\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ac}\,x^4 \log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{ac}}{x^4}\right) - 4\,(2\,a^2c^2 + (5\,abc^2 - 3\,a^2cd)x^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{32\,ac^3x^4}, \frac{3\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-ac}\,x^4 \arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-ac}}{2\,(abcdx^4+a^2c^2+(abc^2+a^2cd)x^2)}\right) - 2\,(2\,a^2c^2 + (5\,abc^2 - 3\,a^2cd)x^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{16\,ac^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*a*
b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a*
d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*(2*a^2*
c^2 + (5*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*c^3*
x^4), 1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2*((b
*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*
x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*(2*a^2*c^2 + (5*a*b*c^2 - 3*a
^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*c^3*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**5*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(105) = 210$.
time = 1.83, size = 1101, normalized size = 8.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*b*(3*(sqrt(b*d)*b^3*c^2 - 2*sqrt(b*d)*a*b^2*c*d + sqrt(b*d)*a^2*b*d^2)
*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*
x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c^2) + 2*(5
*sqrt(b*d)*b^9*c^5 - 23*sqrt(b*d)*a*b^8*c^4*d + 42*sqrt(b*d)*a^2*b^7*c^3*d^
2 - 38*sqrt(b*d)*a^3*b^6*c^2*d^3 + 17*sqrt(b*d)*a^4*b^5*c*d^4 - 3*sqrt(b*d)
*a^5*b^4*d^5 - 15*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d))^2*b^7*c^4 + 28*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^6*c^3*d - 2*sqrt(b*d)*(sqrt(b*
```

```
x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^5*c^2*d
^2 - 20*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d
 - a*b*d))^2*a^3*b^4*c*d^3 + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(
b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^3*d^4 + 15*sqrt(b*d)*(sqrt(b*x^2
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^5*c^3 + sqrt(b*
d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*
b^4*c^2*d + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^4*a^2*b^3*c*d^2 - 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b^2*d^3 - 5*sqrt(b*d)*(sqrt(b
*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^3*c^2 - 6*
sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^6*a*b^2*c*d + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x
^2 + a)*b*d - a*b*d))^6*a^2*b*d^2)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 -
2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2
*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^
2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^4)^2*c^2))/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{x^5\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(x^5*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(x^5*(c + d*x^2)^(1/2)), x)
```

3.950 $\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=429

$$-\frac{2(2bc-ad)\left(4b^2c^2-4abcd-a^2d^2\right)x\sqrt{a+bx^2}}{35b^2d^3\sqrt{c+dx^2}}+\frac{\left(8b^2c^2-11abcd+a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd^3}-\frac{2(3bc-$$

```
[Out] -2/35*(-a*d+2*b*c)*(-a^2*d^2-4*a*b*c*d+4*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/d^3
/(d*x^2+c)^(1/2)-1/35*c^(3/2)*(a^2*d^2-11*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c)
)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-
b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/b/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
d*x^2+c)^(1/2)+2/35*(-a*d+2*b*c)*(-a^2*d^2-4*a*b*c*d+4*b^2*c^2)*(1/(1+d*x^2
/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b^2/d^(7/2)/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(d*x^2+c)^(1/2)+1/35*(a^2*d^2-11*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)
^(1/2)*(d*x^2+c)^(1/2)/b/d^3-2/35*(-4*a*d+3*b*c)*x^3*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/d^2+1/7*b*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

Rubi [A]
time = 0.35, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {488, 596, 545, 429, 506, 422}

$$\frac{2\sqrt{c}\sqrt{a+bx^2}(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-11abcd+8b^2c^2\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2-11abcd+8b^2c^2\right)}{35bd^3}-\frac{2x\sqrt{a+bx^2}(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)}{35b^2d^3\sqrt{c+dx^2}}-\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{35d^2}+\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-2*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*x*Sqrt[a + b*x^2])/(35*
b^2*d^3*Sqrt[c + d*x^2]) + ((8*b^2*c^2 - 11*a*b*c*d + a^2*d^2)*x*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(35*b*d^3) - (2*(3*b*c - 4*a*d)*x^3*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/(35*d^2) + (b*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) +
 (2*Sqrt[c]*(2*b*c - a*d)*(4*b^2*c^2 - 4*a*b*c*d - a^2*d^2)*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(7/2)*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(8*b^2*c^2
 - 11*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(35*b*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx^5\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{7d} + \frac{\int \frac{x^4(-a(5bc-7ad)-2b(3bc-4ad)x^2)}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{7d}$$

$$= -\frac{2(3bc-4ad)x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{35d^2} + \frac{bx^5\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{7d} - \frac{\int \frac{x^2(-6abc(3bc-4ad}{\sqrt{a+}}}{}$$

$$= \frac{(8b^2c^2-11abcd+a^2d^2)\,x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{35bd^3} - \frac{2(3bc-4ad)x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{35d^2}$$

$$= \frac{(8b^2c^2-11abcd+a^2d^2)\,x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{35bd^3} - \frac{2(3bc-4ad)x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{35d^2}$$

$$= -\frac{2(2bc-ad)\,(4b^2c^2-4abcd-a^2d^2)\,x\sqrt{a+bx^2}}{35b^2d^3\sqrt{c+dx^2}} + \frac{(8b^2c^2-11abcd+a^2d^2)\,x\sqrt{a+b}}{35bd^3}$$

$$= -\frac{2(2bc-ad)\,(4b^2c^2-4abcd-a^2d^2)\,x\sqrt{a+bx^2}}{35b^2d^3\sqrt{c+dx^2}} + \frac{(8b^2c^2-11abcd+a^2d^2)\,x\sqrt{a+b}}{35bd^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.01, size = 305, normalized size = 0.71

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)\,(c+dx^2)\,(a^2d^2+abd(-11c+8dx^2)+b^2(8c^2-6cdx^2+5d^2x^4))+2ic(8b^3c^3-12ab^2c^2d+2a^2bcd^2+a^3d^3)\,\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic(16b^3c^3-32ab^2c^2d+15a^2bcd^2+a^3d^3)\,\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{35b\sqrt{\frac{b}{a}}\,d^4\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(-11*c + 8*d*x^2) +
 b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(8*b^3*c^3 - 12*a*b^2*c^2*d
 + 2*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(16*b^3*c^3 - 32*a*b^2*c^2*d
+ 15*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^4*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 782, normalized size = 1.82 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(5*(-b/a)^(1/2)*b^3*d^4*x^9+13*(-b/a)^
(1/2)*a*b^2*d^4*x^7-(-b/a)^(1/2)*b^3*c*d^3*x^7+9*(-b/a)^(1/2)*a^2*b*d^4*x^5
-4*(-b/a)^(1/2)*a*b^2*c*d^3*x^5+2*(-b/a)^(1/2)*b^3*c^2*d^2*x^5+(-b/a)^(1/2)
*a^3*d^4*x^3-2*(-b/a)^(1/2)*a^2*b*c*d^3*x^3-9*(-b/a)^(1/2)*a*b^2*c^2*d^2*x^
3+8*(-b/a)^(1/2)*b^3*c^3*d*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3+15*((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2*d^2-32*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*a*b^2*c^3*d+16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3-4*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2*
d^2+24*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*a*b^2*c^3*d-16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4+(-b/a)^(1/2)*a^3*c*d^3*x-11*(-b
/a)^(1/2)*a^2*b*c^2*d^2*x+8*(-b/a)^(1/2)*a*b^2*c^3*d*x)/b/d^4/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + bx^2)^{\frac{3}{2}}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.951 $\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=335

$$-\frac{\left(13ac-\frac{8bc^2}{d}-\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15d\sqrt{c+dx^2}}-\frac{2(2bc-3ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15d^2}+\frac{bx^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d}-\frac{\sqrt{c}\,(8}{}$$

```
[Out] -1/15*(13*a*c-8*b*c^2/d-3*a^2*d/b)*x*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)+2/15
*c^(3/2)*(-3*a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x
*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(5/
2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(3*a^2*d^2-13*a*b*c
*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b/d^(5/
2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/15*(-3*a*d+2*b*c)*x*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2+1/5*b*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/d
```

Rubi [A]
time = 0.23, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {488, 596, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(3a^2d^2-13abcd+8b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\left(-\frac{3a^2d}{b}+13ac-\frac{8bc^2}{d}\right)}{15d\sqrt{c+dx^2}}+\frac{2c^{3/2}\sqrt{a+bx^2}\,(2bc-3ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(2bc-3ad)}{15d^2}+\frac{bx^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] -1/15*((13*a*c - (8*b*c^2)/d - (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(d*Sqrt[c +
d*x^2]) - (2*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d^2) +
(b*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 13*a*
b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]) + (2*c^(3/2)*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d} + \frac{\int \frac{x^2\left(-a(3bc-5ad)-2b(2bc-3ad)x^2\right)}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{5d}$$

$$= -\frac{2(2bc-3ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15d^2} + \frac{bx^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d} - \frac{\int \frac{-2abc(2bc-3ad)-}{\sqrt{a+b}}}{}$$

$$= -\frac{2(2bc-3ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15d^2} + \frac{bx^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5d} + \frac{(2ac(2bc-3ad)}{}$$

$$= \frac{(8b^2c^2-13abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{15bd^2\sqrt{c+dx^2}} - \frac{2(2bc-3ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15d^2} + \frac{bx^3}{}$$

$$= \frac{(8b^2c^2-13abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{15bd^2\sqrt{c+dx^2}} - \frac{2(2bc-3ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15d^2} + \frac{bx^3}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.29, size = 245, normalized size = 0.73

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)\,(c+dx^2)\,(-4bc+6ad+3bdx^2) - ic(8b^2c^2-13abcd+3a^2d^2)\,\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right) + ic(8b^2c^2-17abcd+9a^2d^2)\,\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{15\sqrt{\frac{b}{a}}\,d^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + 6*a*d + 3*b*d*x^2) - I*c*(
8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(8*b^2*c^2 - 17*a*b*c
*d + 9*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])
```

Maple [A]
time = 0.12, size = 544, normalized size = 1.62

method	result

elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{b\,x^3\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{5d}+\frac{\left(2ab-\frac{b(4ad+4bc)}{5d}\right)x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+}}{3bd}\right.}{}$
risch	$\frac{x(3bd\,x^2+6ad-4bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15d^2}-\frac{\left(\frac{(3a^2d^2-13abcd+8b^2c^2)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2}}\right.}{}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(-3\sqrt{-\frac{b}{a}}\,b^2d^3x^7-9\sqrt{-\frac{b}{a}}\,ab\,d^3x^5+\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5-6\sqrt{-\frac{b}{a}}\,a^2d^3x^3-5\sqrt{-\frac{b}{a}}\,abc\,d^2x^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*(-b/a)^(1/2)*b^2*d^3*x^7-9*(-b/a)
^(1/2)*a*b*d^3*x^5+(-b/a)^(1/2)*b^2*c*d^2*x^5-6*(-b/a)^(1/2)*a^2*d^3*x^3-5*
(-b/a)^(1/2)*a*b*c*d^2*x^3+4*(-b/a)^(1/2)*b^2*c^2*d*x^3+9*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-
17*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*a*b*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+13*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d
-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*b^2*c^3-6*(-b/a)^(1/2)*a^2*c*d^2*x+4*(-b/a)^(1/2)*a*b*c^2*d*x)/d^
3/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.952 $\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=244

$$\frac{(bc+ad)x\sqrt{a+bx^2}}{c\sqrt{c+dx^2}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}-\frac{(bc+ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{2b\sqrt{c}\sqrt{a\,\cdots}}{\sqrt{d\,\cdots}}$$

```
[Out] (a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(d*x^2+c)^(1/2)-(a*d+b*c)*(1/(1+d*x^2/c))^(1/
2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a
/d)^(1/2))*(b*x^2+a)^(1/2)/c^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(d*x^2+c)^(1/2)+2*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(
1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d
^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-a*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)/c/x
```

Rubi [A]

time = 0.11, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {485, 545, 429, 506, 422}

$$\frac{2b\sqrt{c}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(ad+bc)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}+\frac{x\sqrt{a+bx^2}(ad+bc)}{c\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(c*Sqrt[c + d*x^2]) - (a*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])/(c*x) - ((b*c + a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx = -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx} + \frac{\int \frac{2abc+b(bc+ad)x^2}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{c}$$

$$= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx} + (2ab)\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx + \frac{(b(bc+ad))\int \frac{}{\sqrt{a+bx}}}{c}$$

$$= \frac{(bc+ad)x\sqrt{a+bx^2}}{c\sqrt{c+dx^2}} - \frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx} + \frac{2b\sqrt{c}\,\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\right.}{\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

$$= \frac{(bc+ad)x\sqrt{a+bx^2}}{c\sqrt{c+dx^2}} - \frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{cx} - \frac{(bc+ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right.\right.}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.25, size = 206, normalized size = 0.84

$$\frac{-a\sqrt{\frac{b}{a}}\,d(a+bx^2)(c+dx^2) - ibc(bc+ad)x\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right) - ibc(-bc+ad)x\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\,cdx\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(a*Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)) - I*b*c*(b*c + a*d)*x*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*b*c*(-(b*c) + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*x*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])
```

Maple [A]
time = 0.12, size = 352, normalized size = 1.44

method	result
risch	$-\frac{a\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{cx} + \frac{b\left(-\frac{(ad+bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{Elliptic}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}\,d}\right.}{}$

elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{a\sqrt{bd x^4+ad x^2+c x^2 b+ac}}{cx}+\frac{2ab\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+c x^2}}\right.}{\sqrt{}}$
default	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\left(-\sqrt{-\frac{b}{a}}\,ab d^2x^4+\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcdx-\sqrt{\frac{b x^2+a}{a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*a*b*d^2*x^4+((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x
-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*b^2*c^2*x+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x-(-b/a)^(1/2)*a^2*d^2*x^
2-(-b/a)^(1/2)*a*b*c*d*x^2-(-b/a)^(1/2)*a^2*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)/c/(-b/a)^(1/2)/d/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(d*x^4 + c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{x^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**2*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{x^2\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(x^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(x^2*(c + d*x^2)^(1/2)), x)
```

3.953 $\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=311

$$\frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}}-\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3}-\frac{2(2bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3c^2x}-\frac{2\sqrt{d}\,(2bc-ad)\sqrt{a+}}{3c^{3/2}\sqrt{\frac{c}{a}}}$$

```
[Out] 2/3*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/c^2/(d*x^2+c)^(1/2)+1/3*b*(-a*d+3*b*c)
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/a/c^(1/2)/d^(1/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/3*(-a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/
d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
d*x^2+c)^(1/2)-1/3*a*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^3-2/3*(-a*d+2*b*c)
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2/x
```

Rubi [A]

time = 0.20, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {485, 597, 545, 429, 506, 422}

$$-\frac{2\sqrt{d}\,\sqrt{a+bx^2}\,(2bc-ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{a+bx^2}\,(3bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a\sqrt{c}\,\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(2bc-ad)}{3c^2x}+\frac{2dx\sqrt{a+bx^2}\,(2bc-ad)}{3c^2\sqrt{c+dx^2}}-\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*c^2*Sqrt[c + d*x^2]) - (a*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(3*c*x^3) - (2*(2*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])/(3*c^2*x) - (2*Sqrt[d]*(2*b*c - a*d)*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(3/2)*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*(3*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx = -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} + \frac{\int \frac{2a(2bc-ad)+b(3bc-ad)x^2}{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{3c}$$

$$= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} - \frac{2(2bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3c^2x} - \frac{\int \frac{-abc(3bc-ad)-2abd(2bc-}{\sqrt{a+bx^2}\,\sqrt{c+d}}}{3ac^2}$$

$$= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} - \frac{2(2bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3c^2x} + \frac{(2bd(2bc-ad))\int \frac{}{\sqrt{a}}}{3c}$$

$$= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}} - \frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} - \frac{2(2bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3c^2x}$$

$$= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}} - \frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cx^3} - \frac{2(2bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3c^2x}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.56, size = 227, normalized size = 0.73

$$\frac{\sqrt{\frac{b}{a}}\,(a+bx^2)(c+dx^2)(-ac-4bcx^2+2adx^2)+2ibc(-2bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)-ibc(-bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)}{3\sqrt{\frac{b}{a}}\,c^2x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-(a*c) - 4*b*c*x^2 + 2*a*d*x^2) + (2*I)
*b*c*(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
/(3*Sqrt[b/a]*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 433, normalized size = 1.39

method	result

elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{a\sqrt{bd x^4+ad x^2+c x^2 b+ac}}{3c x^3}+\frac{2(ad-2bc)\sqrt{bd x^4+ad x^2+c x^2 b+ac}}{3c^2 x}+\frac{(b^2 \cdots}{\cdots}\right.}{\cdots}$
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,(-2ad x^2+4c x^2 b+ac)}{3c^2 x^3}-\frac{b\left(-\frac{(2a d^2-4bcd)c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad \cdots}}\right.}{\cdots}$
default	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\left(2\sqrt{-\frac{b}{a}}\,ab d^2 x^6-4\sqrt{-\frac{b}{a}}\,b^2 cd x^6+bd\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3 \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(2*(-b/a)^(1/2)*a*b*d^2*x^6-4*(-b/a)^(1
/2)*b^2*c*d*x^6+b*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*c-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3-2*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3+
4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*b^2*c^2*x^3+2*(-b/a)^(1/2)*a^2*d^2*x^4-3*(-b/a)^(1/2)*a*b*c*d*x^4-
4*(-b/a)^(1/2)*b^2*c^2*x^4+(-b/a)^(1/2)*a^2*c*d*x^2-5*(-b/a)^(1/2)*a*b*c^2*
x^2-(-b/a)^(1/2)*a^2*c^2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/c^2/(-b/a)^(1/2)/x^
3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{x^4\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**4*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(x^4*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(x^4*(c + d*x^2)^(1/2)), x)
```

3.954 $\int \frac{x^5 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=340

$$\frac{(bc-ad)^2 (63b^2c^2+14abcd+3a^2d^2) \sqrt{a+bx^2} \sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad) (63b^2c^2+14abcd+3a^2d^2) (a+bx^2)^{3/2}}{384b^2d^4}$$

```
[Out] -1/256*(-a*d+b*c)^3*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*arctanh(d^(1/2)*(b*x^
2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(5/2)/d^(11/2)-1/384*(-a*d+b*c)*(3*a^
2*d^2+14*a*b*c*d+63*b^2*c^2)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b^2/d^4+1/480*
(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b^2/d^3-3
/80*(a*d+3*b*c)*(b*x^2+a)^(7/2)*(d*x^2+c)^(1/2)/b^2/d^2+1/10*x^2*(b*x^2+a)^
(7/2)*(d*x^2+c)^(1/2)/b/d+1/256*(-a*d+b*c)^2*(3*a^2*d^2+14*a*b*c*d+63*b^2*c
^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^5
```

Rubi [A]
time = 0.29, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {457, 92, 81, 52, 65, 223, 212}

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2(3a^2d^2+14abcd+63b^2c^2)}{256b^2d^5} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)(3a^2d^2+14abcd+63b^2c^2)}{384b^2d^4} + \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}(3a^2d^2+14abcd+63b^2c^2)}{480b^2d^3} - \frac{(bc-ad)^3(3a^2d^2+14abcd+63b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{256b^{5/2}d^{11/2}} - \frac{3(a+bx^2)^{7/2}\sqrt{c+dx^2}(ad+3bc)}{80b^2d^2} + \frac{x^2(a+bx^2)^{7/2}\sqrt{c+dx^2}}{10bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/(256*b^2*d^5) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^
2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(384*b^2*d^4) + ((63*b^2*c^2 + 14*a*b
*c*d + 3*a^2*d^2)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(480*b^2*d^3) - (3*(3*
b*c + a*d)*(a + b*x^2)^(7/2)*Sqrt[c + d*x^2])/(80*b^2*d^2) + (x^2*(a + b*x^
2)^(7/2)*Sqrt[c + d*x^2])/(10*b*d) - ((b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*
d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]
)/(256*b^(5/2)*d^(11/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2(a+bx^2)^{7/2}\sqrt{c+dx^2}}{10bd} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(-ac-\frac{3}{2}(3bc+ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^2\right)}{10bd}$$

$$= -\frac{3(3bc+ad)\,(a+bx^2)^{7/2}\sqrt{c+dx^2}}{80b^2d^2} + \frac{x^2(a+bx^2)^{7/2}\sqrt{c+dx^2}}{10bd} + \frac{(63b^2c^2+14abcd}{}$$

$$= \frac{(63b^2c^2+14abcd+3a^2d^2)\,(a+bx^2)^{5/2}\sqrt{c+dx^2}}{480b^2d^3} - \frac{3(3bc+ad)\,(a+bx^2)^{7/2}\sqrt{c+d}}{80b^2d^2}$$

$$= -\frac{(bc-ad)\,(63b^2c^2+14abcd+3a^2d^2)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{384b^2d^4} + \frac{(63b^2c^2+14abcd+3}{}$$

$$= \frac{(bc-ad)^2\,(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\,(63b^2c^2+14a}{}$$

$$= \frac{(bc-ad)^2\,(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\,(63b^2c^2+14a}{}$$

$$= \frac{(bc-ad)^2\,(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\,(63b^2c^2+14a}{}$$

$$= \frac{(bc-ad)^2\,(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\,(63b^2c^2+14a}{}$$

Mathematica [A]
time = 3.91, size = 271, normalized size = 0.80

$$\frac{\sqrt{c+dx^2}\left(-\frac{24(3bc+ad)(a+bx^2)^4}{bd}+64x^2(a+bx^2)^4+\frac{5(bc-ad)^3(63b^2c^2+14abcd+3a^2d^2)\left(\frac{2d(a+bx^2)}{bc-ad}-\frac{4d^2(a+bx^2)^2}{3(bc-ad)^2}+\frac{16d^3(a+bx^2)^3}{15(bc-ad)^3}-\frac{2\sqrt{d}\sqrt{a+bx^2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^2)}{bc-ad}}}\right)}{4bd^5}\right)}{640bd\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*((-24*(3*b*c + a*d)*(a + b*x^2)^4)/(b*d) + 64*x^2*(a + b*x
^2)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((2*d*(a + b
*x^2))/(b*c - a*d) - (4*d^2*(a + b*x^2)^2)/(3*(b*c - a*d)^2) + (16*d^3*(a +
 b*x^2)^3)/(15*(b*c - a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(
b*c - a*d)]))))/(4*b*d^5)))/(640*b*d*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(296) = 592$.
time = 0.15, size = 900, normalized size = 2.65 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7680*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(768*b^4*d^4*x^8*(b*d)^(1/2)*((b*x^2
+a)*(d*x^2+c))^(1/2)+2016*a*b^3*d^4*x^6*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(
1/2)-864*b^4*c*d^3*x^6*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+1488*a^2*b^2
*d^4*x^4*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)-2368*a*b^3*c*d^3*x^4*(b*d)
^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+1008*b^4*c^2*d^2*x^4*(b*d)^(1/2)*((b*x^2
+a)*(d*x^2+c))^(1/2)+60*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a^3*d^4*b*(b*d)^(1/
2)-1924*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a^2*c*d^3*b^2*(b*d)^(1/2)+2996*((b*
x^2+a)*(d*x^2+c))^(1/2)*x^2*a*c^2*d^2*b^3*(b*d)^(1/2)-1260*((b*x^2+a)*(d*x^
2+c))^(1/2)*x^2*c^3*d*b^4*(b*d)^(1/2)+45*d^5*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)
*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^5+75*ln(1/2*(2*b*d*x^
2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^4*c*d^4
*b+450*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)
/(b*d)^(1/2))*a^3*c^2*d^3*b^2-2250*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c)
)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c^3*d^2*b^3+2625*ln(1/2*(2*b*
d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^4*a
*d*b^4-945*b^5*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*c^5-90*((b*x^2+a)*(d*x^2+c))^(1/2)*a^4*d^4*(b*d)^(1/2
)-180*((b*x^2+a)*(d*x^2+c))^(1/2)*a^3*c*d^3*b*(b*d)^(1/2)+3128*((b*x^2+a)*(
d*x^2+c))^(1/2)*a^2*c^2*d^2*b^2*(b*d)^(1/2)-4620*((b*x^2+a)*(d*x^2+c))^(1/2
)*a*c^3*d*b^3*(b*d)^(1/2)+1890*((b*x^2+a)*(d*x^2+c))^(1/2)*c^4*b^4*(b*d)^(1
/2))/b^2/d^5/((b*x^2+a)*(d*x^2+c))^(1/2)/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more
 detail
```

Fricas [A]
time = 1.40, size = 734, normalized size = 2.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/15360*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*
b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c
+ a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4*(384*b^5*d^5*x^8 + 94
5*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4
- 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^6 + 8*(63*b^5*c^2*d^3 -
148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^4 - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*
d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c))/(b^3*d^6), 1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*
d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(-b*d)*arctan(1/2
*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^
2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(384*b^5*d^5*x^8 + 945*b^5*
c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a
^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^6 + 8*(63*b^5*c^2*d^3 - 148*a*
b^4*c*d^4 + 93*a^2*b^3*d^5)*x^4 - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 +
481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(
b^3*d^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [A]
time = 1.06, size = 398, normalized size = 1.17

$$\frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}\left(2\,(bx^2+a)\left(4\,(bx^2+a)\left(6\,(bx^2+a)\left(\frac{8\,(bx^2+a)}{b^3d}-\frac{9\,b^7cd^7+11\,ab^6d^8}{b^9d^9}\right)+\frac{63\,b^8c^2d^6+14\,ab^7cd^7+3\,a^2b^6d^8}{b^9d^9}\right)-\frac{5\,(63\,b^9c^3d^5-49\,ab^8c^2d^6-11\,a^2b^7cd^7-3\,a^3b^6d^8)}{b^9d^9}\right)+\frac{15\,(63\,b^{10}c^4d^4-112\,ab^9c^3d^5+38\,a^2b^8c^2d^6+8\,a^3b^7cd^7+3\,a^4b^6d^8)}{b^9d^9}\right)+\frac{15\,(63\,b^5c^5-175\,ab^4c^4d+150\,a^2b^3c^3d^2-30\,a^3b^2c^2d^3-5\,a^4bcd^4-3\,a^5d^5)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,b^2d^5}\right)b}{3840\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a
)*(4*(b*x^2 + a)*(6*(b*x^2 + a)*(8*(b*x^2 + a)/(b^3*d) - (9*b^7*c*d^7 + 11*
a*b^6*d^8)/(b^9*d^9)) + (63*b^8*c^2*d^6 + 14*a*b^7*c*d^7 + 3*a^2*b^6*d^8)/(
b^9*d^9)) - 5*(63*b^9*c^3*d^5 - 49*a*b^8*c^2*d^6 - 11*a^2*b^7*c*d^7 - 3*a^3
*b^6*d^8)/(b^9*d^9)) + 15*(63*b^10*c^4*d^4 - 112*a*b^9*c^3*d^5 + 38*a^2*b^8
*c^2*d^6 + 8*a^3*b^7*c*d^7 + 3*a^4*b^6*d^8)/(b^9*d^9)) + 15*(63*b^5*c^5 - 1
75*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 -
 3*a^5*d^5)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b
*d - a*b*d)))/(sqrt(b*d)*b^2*d^5))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(b\,x^2+a\right)}^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.955 $\int \frac{x^3 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}} dx$

Optimal. Leaf size=237

$$-\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4}+\frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3}-\frac{(7bc+ad)\left(a+\right.}{4}$$

```
[Out] 5/128*(-a*d+b*c)^3*(a*d+7*b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x
^2+c)^(1/2))/b^(3/2)/d^(9/2)+5/192*(-a*d+b*c)*(a*d+7*b*c)*(b*x^2+a)^(3/2)*(
d*x^2+c)^(1/2)/b/d^3-1/48*(a*d+7*b*c)*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b/d^2
+1/8*(b*x^2+a)^(7/2)*(d*x^2+c)^(1/2)/b/d-5/128*(-a*d+b*c)^2*(a*d+7*b*c)*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^4
```

Rubi [A]
time = 0.16, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {457, 81, 52, 65, 223, 212}

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{3/2}d^{9/2}}-\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2(ad+7bc)}{128bd^4}+\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)(ad+7bc)}{192bd^3}-\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(ad+7bc)}{48bd^2}+\frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(128*b*d^4
) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(192*b*
d^3) - ((7*b*c + a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(48*b*d^2) + ((a +
 b*x^2)^(7/2)*Sqrt[c + d*x^2])/(8*b*d) + (5*(b*c - a*d)^3*(7*b*c + a*d)*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(3/2)*d^(
9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{x(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{(a+bx^2)^{7/2}\sqrt{c+dx^2}}{8bd} - \frac{(7bc+ad)\mathrm{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{16bd}$$

$$= -\frac{(7bc+ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48bd^2} + \frac{(a+bx^2)^{7/2}\sqrt{c+dx^2}}{8bd} + \frac{(5(bc-ad)(7bc+ad}{}$$

$$= \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3} - \frac{(7bc+ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48bd^2} + $$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c}}{192bd^3}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c}}{192bd^3}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c}}{192bd^3}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c}}{192bd^3}$$

Mathematica [A]
time = 3.35, size = 214, normalized size = 0.90

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\,(c+dx^2)\left(15a^3d^3+a^2bd^2(-191c+118dx^2)+ab^2d(265c^2-172cdx^2+136d^2x^4)+b^3(-105c^3+70c^2dx^2-56cd^2x^4+48d^3x^6)\right)+15(bc-ad)^{7/2}(7bc+ad)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{384b^2d^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 11
8*d*x^2) + a*b^2*d*(265*c^2 - 172*c*d*x^2 + 136*d^2*x^4) + b^3*(-105*c^3 +
70*c^2*d*x^2 - 56*c*d^2*x^4 + 48*d^3*x^6)) + 15*(b*c - a*d)^(7/2)*(7*b*c +
a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sq
rt[b*c - a*d]])/(384*b^2*d^(9/2)*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(199) = 398$.

time = 0.12, size = 658, normalized size = 2.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/768*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-96*b^3*d^3*x^6*((b*x^2+a)*(d*x^2+c
))^(1/2)*(b*d)^(1/2)-272*a*b^2*d^3*x^4*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1
/2)+112*b^3*c*d^2*x^4*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)-236*((b*x^2+a
)*(d*x^2+c))^(1/2)*x^2*a^2*b*d^3*(b*d)^(1/2)+344*((b*x^2+a)*(d*x^2+c))^(1/2
)*x^2*a*c*b^2*d^2*(b*d)^(1/2)-140*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*c^2*b^3*d
*(b*d)^(1/2)+15*d^4*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(
1/2)+a*d+b*c)/(b*d)^(1/2))*a^4+60*a^3*c*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x
^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*d^3-270*ln(1/2*(2*b*d*x^2+
2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c^2*b^2
*d^2+300*a*c^3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*b^3*d-105*b^4*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c
))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^4-30*((b*x^2+a)*(d*x^2+c))^(1/
2)*a^3*d^3*(b*d)^(1/2)+382*((b*x^2+a)*(d*x^2+c))^(1/2)*a^2*c*b*d^2*(b*d)^(1
/2)-530*((b*x^2+a)*(d*x^2+c))^(1/2)*a*c^2*b^2*d*(b*d)^(1/2)+210*((b*x^2+a)*
(d*x^2+c))^(1/2)*c^3*b^3*(b*d)^(1/2))/b/d^4/((b*x^2+a)*(d*x^2+c))^(1/2)/(b*
d)^(1/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 1.03, size = 574, normalized size = 2.42

$$\left[-\frac{15\,(7\,b^4c^4 - 20\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 4\,a^3bcd^3 - a^4d^4)\sqrt{bd}\,\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\,(b^2cd + abd^2)x^2 - 4\,(2\,bdx^2 + bc + ad)\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}\,\sqrt{bd}\right) - 4\,(48\,b^4d^4x^6 - 105\,b^4c^3d + 265\,ab^3c^2d^2 - 191\,a^2b^2cd^3 + 15\,a^3bd^4 - 8\,(7\,b^4cd^3 - 17\,ab^3d^4)x^4 + 2\,(35\,b^4c^2d^2 - 86\,ab^3cd^3 + 59\,a^2b^2d^4)x^2)\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}}{1536\,b^2d^5}, -\frac{15\,(7\,b^4c^4 - 20\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 4\,a^3bcd^3 - a^4d^4)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2 + bc + ad)\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}\,\sqrt{-bd}}{2\,(b^2d^2x^4 + abcd + (b^2cd + abd^2)x^2)}\right) - 2\,(48\,b^4d^4x^6 - 105\,b^4c^3d + 265\,ab^3c^2d^2 - 191\,a^2b^2cd^3 + 15\,a^3bd^4 - 8\,(7\,b^4cd^3 - 17\,ab^3d^4)x^4 + 2\,(35\,b^4c^2d^2 - 86\,ab^3cd^3 + 59\,a^2b^2d^4)x^2)\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}}{768\,b^2d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/1536*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d
^3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
```

```
8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c)*sqrt(b*d)) - 4*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 265*a*b^3*c^2*
d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^4
 + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^2)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c))/(b^2*d^5), -1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2
+ b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 265*a
*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3
*d^4)*x^4 + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^5)]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [**A**]

time = 1.12, size = 304, normalized size = 1.28

$$\frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}\left(2\,(bx^2+a)\left(4\,(bx^2+a)\left(\frac{6\,(bx^2+a)}{b^2d}-\frac{7\,b^3cd^5+ab^2d^6}{b^4d^7}\right)+\frac{5\,(7\,b^4c^2d^4-6\,ab^3cd^5-a^2b^2d^6)}{b^4d^7}\right)-\frac{15\,(7\,b^5c^3d^3-13\,ab^4c^2d^4+5\,a^2b^3cd^5+a^3b^2d^6)}{b^4d^7}\right)-\frac{15\,(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^4}\right)b}{384\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)
*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b^2*d) - (7*b^3*c*d^5 + a*b^2*d^6)/(b^4*d^7
)) + 5*(7*b^4*c^2*d^4 - 6*a*b^3*c*d^5 - a^2*b^2*d^6)/(b^4*d^7)) - 15*(7*b^5
*c^3*d^3 - 13*a*b^4*c^2*d^4 + 5*a^2*b^3*c*d^5 + a^3*b^2*d^6)/(b^4*d^7)) - 1
5*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^
4)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*
d)))/(sqrt(b*d)*b*d^4))*b/abs(b)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(b\,x^2+a\right)}^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.956 $\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=164

$$\frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3}-\frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d}-\frac{5(bc-ad)^3}{\quad}$$

```
[Out] -5/16*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))
/d^(7/2)/b^(1/2)-5/24*(-a*d+b*c)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d^2+1/6*(b
*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/d+5/16*(-a*d+b*c)^2*(b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/d^3
```

Rubi [A]
time = 0.10, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {455, 52, 65, 223, 212}

$$-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16\sqrt{b}\,d^{7/2}}+\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}\,(bc-ad)^2}{16d^3}-\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,(bc-ad)}{24d^2}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*d^3) - (5*(b*c - a*d)
*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*d^2) + ((a + b*x^2)^(5/2)*Sqrt[c +
d*x^2])/(6*d) - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]
*Sqrt[c + d*x^2])])/(16*Sqrt[b]*d^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{6d} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{12d}$$

$$= -\frac{5(bc-ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{6d} + \frac{(5(bc-ad)^2)\,\text{Subst}}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{(a+bx^2)^{5/2}}{6d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{(a+bx^2)^{5/2}}{6d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{(a+bx^2)^{5/2}}{6d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{(a+bx^2)^{5/2}}{6d}$$

Mathematica [**A**]
time = 1.68, size = 137, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(33a^2d^2+2abd(-20c+13dx^2)+b^2(15c^2-10cdx^2+8d^2x^4))}{48d^3} - \frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{16\sqrt{b}\,d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(33*a^2*d^2 + 2*a*b*d*(-20*c + 13*d*x^2) +
 b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)))/(48*d^3) - (5*(b*c - a*d)^3*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(16*Sqrt[b]*d^(7/2)
)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(132) = 264.
time = 0.13, size = 452, normalized size = 2.76

method	result
risch	$\frac{(8b^2x^4d^2+26ab\,d^2x^2-10b^2cd\,x^2+33a^2d^2-40abcd+15b^2c^2)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{48d^3}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(}\right.}{32\sqrt{bd}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(16b^2d^2x^4\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{bd}+52\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,x^2ab\,d^2\sqrt{bd}-\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{b^2x^4\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{6d}+\frac{13b\,x^2\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\,a}{24d}-\frac{5b^2x^2}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(16*b^2*d^2*x^4*((b*x^2+a)*(d*x^2+c))^
(1/2)*(b*d)^(1/2)+52*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)-20
*((b*x^2+a)*(d*x^2+c))^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)+15*d^3*ln(1/2*(2*b*d*x
^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3-45*l
n(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(
1/2))*a^2*c*b*d^2+45*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^
(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c^2*b^2*d-15*b^3*ln(1/2*(2*b*d*x^2+2*((b*x^2+
a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3+66*((b*x^2+a)*(d*
x^2+c))^(1/2)*a^2*d^2*(b*d)^(1/2)-80*((b*x^2+a)*(d*x^2+c))^(1/2)*a*c*b*d*(b
*d)^(1/2)+30*((b*x^2+a)*(d*x^2+c))^(1/2)*c^2*b^2*(b*d)^(1/2))/d^3/((b*x^2+a
)*(d*x^2+c))^(1/2)/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.33, size = 440, normalized size = 2.68

$$\left[-\frac{15\,{\left(b^{3} c^{3} - 3\,a b^{2} c^{2} d + 3\,a^{2} b c d^{2} - a^{3} d^{3}\right)} \sqrt{b d} \log\left(8\,b^{2} d^{2} x^{4} + b^{2} c^{2} + 6\,a b c d + a^{2} d^{2} + 8\,{\left(b^{2} c d + a b d^{2}\right)} x^{2} + 4\,{\left(2\,b d x^{2} + b c + a d\right)} \sqrt{b x^{2} + a} \sqrt{d x^{2} + c} \sqrt{b d}\right) - 4\,{\left(8\,b^{3} d^{3} x^{4} + 15\,b^{3} c^{2} d - 40\,a b^{2} c d^{2} + 33\,a^{2} b d^{3} - 2\,{\left(5\,b^{3} c d^{2} - 13\,a b^{2} d^{3}\right)} x^{2}\right)} \sqrt{b x^{2} + a} \sqrt{d x^{2} + c}}{192\,b d^{4}}, \frac{15\,{\left(b^{3} c^{3} - 3\,a b^{2} c^{2} d + 3\,a^{2} b c d^{2} - a^{3} d^{3}\right)} \sqrt{-b d} \arctan\left(\frac{{\left(2\,b d x^{2} + b c + a d\right)} \sqrt{b x^{2} + a} \sqrt{d x^{2} + c} \sqrt{-b d}}{2\,{\left(b^{2} d^{2} x^{4} + a b c d + {\left(b^{2} c d + a b d^{2}\right)} x^{2}\right)}}\right) + 2\,{\left(8\,b^{3} d^{3} x^{4} + 15\,b^{3} c^{2} d - 40\,a b^{2} c d^{2} + 33\,a^{2} b d^{3} - 2\,{\left(5\,b^{3} c d^{2} - 13\,a b^{2} d^{3}\right)} x^{2}\right)} \sqrt{b x^{2} + a} \sqrt{d x^{2} + c}}{96\,b d^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*l
og(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^
2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) -
4*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*
c*d^2 - 13*a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^4), 1/96*(
15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/
2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d
^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3*c^
2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 13*a*b^2*d^3)*x^2)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a + bx^{2}\right)^{\frac{5}{2}}}{\sqrt{c + dx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [A]
time = 1.04, size = 210, normalized size = 1.28

$$\frac{\left(\sqrt{b^{2} c + (bx^{2} + a) b d - a b d}\,\sqrt{bx^{2} + a}\,\left(2\,(bx^{2} + a)\left(\frac{4\,(bx^{2} + a)}{bd} - \frac{5\,(bcd^{3} - ad^{4})}{bd^{5}}\right) + \frac{15\,(b^{2} c^{2} d^{2} - 2\,abcd^{3} + a^{2} d^{4})}{bd^{5}}\right) + \frac{15\,(b^{3} c^{3} - 3\,ab^{2} c^{2} d + 3\,a^{2} bcd^{2} - a^{3} d^{3}) \log\left(\left|-\sqrt{bx^{2} + a}\,\sqrt{bd} + \sqrt{b^{2} c + (bx^{2} + a) b d - a b d}\right|\right)}{\sqrt{bd}\, d^{3}}\right) b}{48\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)*
(4*(b*x^2 + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 15*(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d)))/(sqrt(b*d)*d^3))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(b\,x^2+a\right)}^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
[Out] int((x*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.957 $\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{b(3bc-7ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8d^2}+\frac{b(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\,(3b^2c^2}{}$$

```
[Out] 1/8*(15*a^2*d^2-10*a*b*c*d+3*b^2*c^2)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/
2)/(d*x^2+c)^(1/2))*b^(1/2)/d^(5/2)-a^(5/2)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)
/a^(1/2)/(d*x^2+c)^(1/2))/c^(1/2)+1/4*b*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d-1
/8*b*(-7*a*d+3*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {457, 104, 159, 163, 65, 223, 212, 95, 214}

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\,(15a^2d^2-10abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(3bc-7ad)}{8d^2}+\frac{b(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/8*(b*(3*b*c - 7*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/d^2 + (b*(a + b*x^
2)^(3/2)*Sqrt[c + d*x^2])/(4*d) - (a^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2]
)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d +
15*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(
8*d^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{b(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\,\left(2a^2d-\frac{1}{2}b(3bc-7ad)x\right)}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4d}$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\text{Subst}\left(\int \frac{2a^3d^2+\frac{1}{4}b(3b}{x\sqrt{a+}}\right.}{}$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{1}{2}a^3\text{Subst}\left(\int \frac{}{x\sqrt{a+}}\right.$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} + a^3\text{Subst}\left(\int \frac{1}{-a+c}\right.$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{}}{\sqrt{a}\sqrt{}}\right.}{\sqrt{c}}$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{}}{\sqrt{a}\sqrt{}}\right.}{\sqrt{c}}$$

Mathematica [A]
time = 2.12, size = 162, normalized size = 0.87

$$\frac{1}{8}\left(\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}\,(-3bc+9ad+2bdx^2)}{d^2} - \frac{8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\,(3b^2c^2-10abcd+15a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{d^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + 9*a*d + 2*b*d*x^2))/d^2 - (8*
a^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[
c] + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/d^(5/2))/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(149) = 298$.
time = 0.12, size = 390, normalized size = 2.09

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{b^2x^2\sqrt{bdx^4+(ad+bc)x^2+ac}}{4d}+\frac{9b\sqrt{bdx^4+(ad+bc)x^2+ac}\,a}{8d}-\frac{3b^2\sqrt{bc}}{}\right.}{}$
default	$\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}\left(4\sqrt{(bx^2+a)(dx^2+c)}\,\sqrt{bd}\,\sqrt{ac}\,b^2dx^2+15\ln\left(\frac{2bdx^2+2\sqrt{(bx^2+a)(dx^2+c}}{2\sqrt{bd}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(4*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(
1/2)*(a*c)^(1/2)*b^2*d*x^2+15*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/
2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a^2*b*d^2-10*ln(1/2*(2*b*d
*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^
(1/2)*a*b^2*c*d+3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/
2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b^3*c^2-8*(b*d)^(1/2)*ln((a*d*x^2+c*x^
2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a^3*d^2+18*((b*x^
2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*a*b*d-6*((b*x^2+a)*(d*x^2+c))
^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*b^2*c)/((b*x^2+a)*(d*x^2+c))^(1/2)/d^2/(b*d)
^(1/2)/(a*c)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.63, size = 1075, normalized size = 5.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(8*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2
*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b
*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + (3*b^2*c^2 - 10*a*b*c*d + 15*a^
2*d^2)*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2
*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c)*sqrt(b/d)) + 4*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c))/d^2, 1/16*(4*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d
+ a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^
2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - (3*b^2*c^
2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*
b*d)*x^2)) + 2*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(d*x^2
 + c))/d^2, 1/32*(16*a^2*d^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c +
a^2*d)*x^2)) + (3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(b/d)*log(8*b^2*d^
2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*
d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*(2*
b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/d^2, 1/16*(
8*a^2*d^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - (3*b
^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c +
a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c
 + a*b*d)*x^2)) + 2*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c))/d^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{x\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x*sqrt(c + d*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{x\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(x*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(x*(c + d*x^2)^(1/2)), x)
```

3.958 $\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$\frac{b(bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cd}-\frac{a(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{2cx^2}-\frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{b^{3/2}(\ldots}{\ldots}$$

```
[Out] -1/2*a^(3/2)*(-a*d+5*b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)
^(1/2))/c^(3/2)-1/2*b^(3/2)*(-5*a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^
(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-1/2*a*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/c/x^2+
1/2*b*(a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/d
```

Rubi [A]
time = 0.16, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {457, 100, 159, 163, 65, 223, 212, 95, 214}

$$-\frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{b^{3/2}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2d^{3/2}}-\frac{a(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{2cx^2}+\frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(ad+bc)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*d) - (a*(a + b*x^2)^(3
/2)*Sqrt[c + d*x^2])/(2*c*x^2) - (a^(3/2)*(5*b*c - a*d)*ArcTanh[(Sqrt[c]*Sq
rt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2)) - (b^(3/2)*(b*c - 5*
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/
2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-ad)-b(bc+ad)x\right)}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2c}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a^2d(5bc-ad)+}{x\sqrt{a+bx}}\right.}{2c}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{(b^2(bc-5ad))\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{(b(bc-5ad))\text{Subst}\left(\int\right.}{}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\right.}{2c^{3/2}}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\right.}{2c^{3/2}}$$

Mathematica [A]
time = 2.23, size = 166, normalized size = 0.89

$$\frac{a^{3/2}(-5bc+ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)+\frac{\sqrt{c}\left(\frac{\sqrt{d}\sqrt{a+bx^2}\left(-a^2d+b^2cx^2\right)\sqrt{c+dx^2}}{x^2}-b^{3/2}c(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)\right)}{d^{3/2}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

[Out] (a^(3/2)*(-5*b*c + a*d)*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])] + (Sqrt[c]*((Sqrt[d]*Sqrt[a + b*x^2]*(-(a^2*d) + b^2*c*x^2)*Sqrt[c + d*x^2])/x^2 - b^(3/2)*c*(b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])]))/d^(3/2))/(2*c^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(147) = 294$.
time = 0.17, size = 374, normalized size = 2.00

method	result
risch	$-\frac{a^2\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{\left(\frac{b^2\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{2d}+\frac{5b^2\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+b}\right.}{4\sqrt{bd}}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{b^2\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{2d}+\frac{5b^2\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2}\right.}{4\sqrt{bd}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(5\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a\,b^2cd\,x^2\sqrt{ac}-\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(5*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^2*c*d*x^2*(a*c)^(1/2)-ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^3*c^2*x^2*(a*c)^(1/2)+ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a^3*d^2*x^2*(b*d)^(1/2)-5*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a^2*b*c*d*x^2*(b*d)^(1/2)+2*b^2*c*x^2*((b*x^2+a)*(d*x^2+c))^(1/2)*(a*c)^(1/2)*(b*d)^(1/2)-2*a^2*d*((b*x^2+a)*(d*x^2+c))^(1/2)*(a*c)^(1/2)*(b*d)^(1/2))/((b*x^2+a)*(d*x^2+c))^(1/2)/x^2/(b*d)^(1/2)/(a*c)^(1/2)/d

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.95, size = 1097, normalized size = 5.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*
a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*
d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (5*a*b*c*d - a^2*d^2)*x^2
*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^
2 + a^2*c*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c)*sqrt(a/c))/x^4) - 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c))/(c*d*x^2), 1/8*(2*(b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(-b/d)*arctan(1/2*(
2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^
4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (5*a*b*c*d - a^2*d^2)*x^2*sqrt(a/c)*log
(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x
^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt
(a/c))/x^4) + 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x
^2), 1/8*(2*(5*a*b*c*d - a^2*d^2)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c +
(a*b*c + a^2*d)*x^2)) - (b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*log(8*b^2*d^2*x
^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2
*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*(b^2*c
*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x^2), 1/4*((5*a*b*c*d -
 a^2*d^2)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 +
a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) +
(b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d
)*x^2)) + 2*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**3*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(147) = 294$.
time = 1.45, size = 558, normalized size = 2.98

$$\frac{b\left(\frac{2\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\,b}{d}+\frac{\left(\sqrt{bd}\,b^2c-5\sqrt{bd}\,abd\right)\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{d^2}-\frac{2\left(5\sqrt{bd}\,a^2b^2c-\sqrt{bd}\,a^3bd\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,bc}-\frac{4\left(\sqrt{bd}\,a^2b^4c^2-2\sqrt{bd}\,a^3b^3cd+\sqrt{bd}\,a^4b^2d^2-\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^2b^2c-\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^3bd\right)}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2abd+\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4\right)c}\right)}{4|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*b/d + (sqrt(
b*d)*b^2*c - 5*sqrt(b*d)*a*b*d)*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
 + (b*x^2 + a)*b*d - a*b*d))^2)/d^2 - 2*(5*sqrt(b*d)*a^2*b^2*c - sqrt(b*d)*
a^3*b*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c)
- 4*(sqrt(b*d)*a^2*b^4*c^2 - 2*sqrt(b*d)*a^3*b^3*c*d + sqrt(b*d)*a^4*b^2*d^
2 - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^2*a^2*b^2*c - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b
*x^2 + a)*b*d - a*b*d))^2*a^3*b*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 -
2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2
*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^
2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^4)*c))/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{5/2}}{x^3\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(x^3*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(x^3*(c + d*x^2)^(1/2)), x)
```

3.959 $\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=192

$$-\frac{a(7bc-3ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8c^2x^2}-\frac{a(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{4cx^4}-\frac{\sqrt{a}\,(15b^2c^2-10abcd+3a^2d^2)\tanh^{-1}\left(\sqrt{\ \ }\right.}{8c^{5/2}}$$

```
[Out] -1/8*(3*a^2*d^2-10*a*b*c*d+15*b^2*c^2)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1
/2)/(d*x^2+c)^(1/2))*a^(1/2)/c^(5/2)+b^(5/2)*arctanh(d^(1/2)*(b*x^2+a)^(1/2
)/b^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)-1/4*a*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/c/
x^4-1/8*a*(-3*a*d+7*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2/x^2
```

Rubi [A]

time = 0.14, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {457, 100, 154, 163, 65, 223, 212, 95, 214}

$$-\frac{\sqrt{a}\,(3a^2d^2-10abcd+15b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8c^{5/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(7bc-3ad)}{8c^2x^2}-\frac{a(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{4cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/8*(a*(7*b*c - 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c^2*x^2) - (a*(a
+ b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (Sqrt[a]*(15*b^2*c^2 - 10*a*b*c
*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])
])/(8*c^(5/2)) + (b^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c
 + d*x^2])])/Sqrt[d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(7bc-3ad)-2b^2cx\right)}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}a(15b^2c^2 \ldots}{x\sqrt{a \ldots}} \ldots\right.}{\ldots}$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} + \frac{1}{2}b^3\text{Subst}\left(\int \frac{\ldots}{\sqrt{a \ldots}} \ldots\right.$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} + b^2\text{Subst}\left(\int \frac{\ldots}{\sqrt{c - \ldots}} \ldots\right.$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\sqrt{a}\,(15b^2c^2-10ab \ldots}{\ldots}$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\sqrt{a}\,(15b^2c^2-10ab \ldots}{\ldots}$$

Mathematica [A]
time = 3.54, size = 206, normalized size = 1.07

$$\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}\,(-2ac-9bcx^2+3adx^2)}{8c^2x^4} + \frac{(bc-ad)^{5/2}\left(\frac{b(c+dx^2)}{bc-ad}\right)^{5/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\,(c+dx^2)^{5/2}} - \frac{\sqrt{a}\,(15b^2c^2-10abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - 9*b*c*x^2 + 3*a*d*x^2))/(8*c^2
*x^4) + ((b*c - a*d)^(5/2)*((b*(c + d*x^2))/(b*c - a*d))^(5/2)*ArcSinh[(Sqr
```

```
t[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[d]*(c + d*x^2)^(5/2)) - (Sqrt
[a]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])
/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(154) = 308$.

time = 0.14, size = 408, normalized size = 2.12

method	result
risch	$-\frac{a\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(-3ad\,x^2+9c\,x^2b+2ac)}{8c^2x^4}+\frac{\left(\frac{b^3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)}{2\sqrt{bd}}-\frac{3a^3\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{b^3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)}{2\sqrt{bd}}-\frac{15a\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(8\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^3c^2x^4\sqrt{ac}-3\ln\left(\frac{ad\,x^2+c\,x^2b+2\sqrt{\cdots}}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2*(8*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*
(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^3*c^2*x^4*(a*c)^(1/2)-
3*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)
*a^3*d^2*x^4*(b*d)^(1/2)+10*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d
*x^2+c))^(1/2)+2*a*c)/x^2)*a^2*b*c*d*x^4*(b*d)^(1/2)-15*ln((a*d*x^2+c*x^2*b
+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*a*b^2*c^2*x^4*(b*d)^
(1/2)+6*a^2*d*x^2*((b*x^2+a)*(d*x^2+c))^(1/2)*(a*c)^(1/2)*(b*d)^(1/2)-18*a*
b*c*x^2*((b*x^2+a)*(d*x^2+c))^(1/2)*(a*c)^(1/2)*(b*d)^(1/2)-4*a^2*c*((b*x^2
+a)*(d*x^2+c))^(1/2)*(a*c)^(1/2)*(b*d)^(1/2))/((b*x^2+a)*(d*x^2+c))^(1/2)/x
^4/(a*c)^(1/2)/(b*d)^(1/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.70, size = 1123, normalized size = 5.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(8*b^2*c^2*x^4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^
2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)
*x^4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(a/c))/x^4) - 4*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), -1/32*(16*b^2*c^2*x^4*sqrt(-b/d)*arc
tan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/
(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (15*b^2*c^2 - 10*a*b*c*d + 3*a
^2*d^2)*x^4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2
+ 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + 4*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), 1/16*(4*b^2*c^2*x^4*sqrt(b/d)
*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*
x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
b/d)) + (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4*sqrt(-a/c)*arctan(1/2*((b
*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^
4 + a^2*c + (a*b*c + a^2*d)*x^2)) - 2*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), -1/16*(8*b^2*c^2*x^4*sqrt(-b/d)*
arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/
d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (15*b^2*c^2 - 10*a*b*c*d +
3*a^2*d^2)*x^4*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) +
 2*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^
2*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**5*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(154) = 308$.
time = 1.70, size = 1175, normalized size = 6.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*sqrt(b*d)*b^2*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2
+ a)*b*d - a*b*d))^2)/d + (15*sqrt(b*d)*a*b^3*c^2 - 10*sqrt(b*d)*a^2*b^2*c*
d + 3*sqrt(b*d)*a^3*b*d^2)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sq
rt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sq
rt(-a*b*c*d)*b*c^2) + 2*(9*sqrt(b*d)*a*b^9*c^5 - 39*sqrt(b*d)*a^2*b^8*c^4*d
 + 66*sqrt(b*d)*a^3*b^7*c^3*d^2 - 54*sqrt(b*d)*a^4*b^6*c^2*d^3 + 21*sqrt(b*
d)*a^5*b^5*c*d^4 - 3*sqrt(b*d)*a^6*b^4*d^5 - 27*sqrt(b*d)*(sqrt(b*x^2 + a)*
sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^7*c^4 + 40*sqrt(b*
d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^
2*b^6*c^3*d + 10*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2
 + a)*b*d - a*b*d))^2*a^3*b^5*c^2*d^2 - 32*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^4*c*d^3 + 9*sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^5*
b^3*d^4 + 27*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d))^4*a*b^5*c^3 + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt
(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^4*c^2*d + 21*sqrt(b*d)*(sqrt(b*x
^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b^3*c*d^2
 - 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^4*a^4*b^2*d^3 - 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
+ (b*x^2 + a)*b*d - a*b*d))^6*a*b^3*c^2 - 10*sqrt(b*d)*(sqrt(b*x^2 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a^2*b^2*c*d + 3*sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a^3*
b*d^2)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d)
 - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*s
qrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)^2*c^2))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^{5/2}}{x^5\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(x^5*(c + d*x^2)^(1/2)),x)
[Out] int((a + b*x^2)^(5/2)/(x^5*(c + d*x^2)^(1/2)), x)
```

3.960 $\int \frac{x^4 \left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=553

$$\frac{\left(128b^4c^4-328ab^3c^3d+243a^2b^2c^2d^2-25a^3bcd^3-10a^4d^4\right)x\sqrt{a+bx^2}}{315b^2d^4\sqrt{c+dx^2}}-\frac{\left(64b^3c^3-156ab^2c^2d+105a^2bcd^2-\right.}{315bd^4}$$

```
[Out] 1/315*(-10*a^4*d^4-25*a^3*b*c*d^3+243*a^2*b^2*c^2*d^2-328*a*b^3*c^3*d+128*b
^4*c^4)*x*(b*x^2+a)^(1/2)/b^2/d^4/(d*x^2+c)^(1/2)+1/315*c^(3/2)*(-5*a^3*d^3
+105*a^2*b*c*d^2-156*a*b^2*c^2*d+64*b^3*c^3)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2
/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*
(b*x^2+a)^(1/2)/b/d^(9/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1
/315*(-10*a^4*d^4-25*a^3*b*c*d^3+243*a^2*b^2*c^2*d^2-328*a*b^3*c^3*d+128*b^
4*c^4)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b^2/d^(9/2)/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/9*b*x^5*(b*x^2+a)^(3/2)*(d*
x^2+c)^(1/2)/d-1/315*(-5*a^3*d^3+105*a^2*b*c*d^2-156*a*b^2*c^2*d+64*b^3*c^3
)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^4+1/315*(75*a^2*d^2-115*a*b*c*d+48*
b^2*c^2)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3-4/63*b*(-3*a*d+2*b*c)*x^5*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2
```

Rubi [A]
time = 0.48, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {488, 595, 596, 545, 429, 506, 422}

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{315d^3}+\frac{c^{3/2}\sqrt{a+bx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315bd^{9/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)}{315bd^4}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-10a^4d^4-25a^3bcd^3+243a^2b^2c^2d^2-328ab^3c^3d+128b^4c^4\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315b^2d^{9/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{x\sqrt{a+bx^2}\left(-10a^4d^4-25a^3bcd^3+243a^2b^2c^2d^2-328ab^3c^3d+128b^4c^4\right)}{315b^2d^4\sqrt{c+dx^2}}-\frac{4bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{63d^2}+\frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 - 25*a^3*b*c*d^3 - 10
*a^4*d^4)*x*Sqrt[a + b*x^2])/(315*b^2*d^4*Sqrt[c + d*x^2]) - ((64*b^3*c^3 -
 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/(315*b*d^4) + ((48*b^2*c^2 - 115*a*b*c*d + 75*a^2*d^2)*x^3*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(315*d^3) - (4*b*(2*b*c - 3*a*d)*x^5*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(63*d^2) + (b*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9
*d) - (Sqrt[c]*(128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 - 25*a^
3*b*c*d^3 - 10*a^4*d^4)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(315*b^2*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]) + (c^(3/2)*(64*b^3*c^3 - 156*a*b^2*c^2*d + 105*a^2*b*c*d
```

```
^2 - 5*a^3*d^3)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(315*b*d^(9/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx^5(a+bx^2)^{3/2}\sqrt{c+dx^2}}{9d} + \frac{\int \frac{x^4\sqrt{a+bx^2}\,(-a(5bc-9ad)-4b(2bc-3ad)x^2)}{\sqrt{c+dx^2}}\,dx}{9d}$$

$$= -\frac{4b(2bc-3ad)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63d^2} + \frac{bx^5(a+bx^2)^{3/2}\sqrt{c+dx^2}}{9d} + \frac{\int \frac{x^4(a(40b^2c^2-9\ldots}{\ldots}}{\ldots}$$

$$= \frac{(48b^2c^2-115abcd+75a^2d^2)\,x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315d^3} - \frac{4b(2bc-3ad)x^5\sqrt{a+bx^2}\sqrt{c\ldots}}{63d^2}$$

$$= -\frac{(64b^3c^3-156ab^2c^2d+105a^2bcd^2-5a^3d^3)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{315bd^4} + \frac{(48b^2c^2-115a\ldots}{\ldots}$$

$$= -\frac{(64b^3c^3-156ab^2c^2d+105a^2bcd^2-5a^3d^3)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{315bd^4} + \frac{(48b^2c^2-115a\ldots}{\ldots}$$

$$= \frac{(128b^4c^4-328ab^3c^3d+243a^2b^2c^2d^2-25a^3bcd^3-10a^4d^4)\,x\sqrt{a+bx^2}}{315b^2d^4\sqrt{c+dx^2}} - \frac{(64b^3c^3-1\ldots}{\ldots}$$

$$= \frac{(128b^4c^4-328ab^3c^3d+243a^2b^2c^2d^2-25a^3bcd^3-10a^4d^4)\,x\sqrt{a+bx^2}}{315b^2d^4\sqrt{c+dx^2}} - \frac{(64b^3c^3-1\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.32, size = 379, normalized size = 0.69

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)\,(c+dx^2)\,(5a^3d^3+15a^2bd^2(-7c+5dx^2)+ab^2d(156c^2-115cdx^2+95d^2x^4)+b^3(-64c^3+48c^2dx^2-40cd^2x^4+35d^3x^6))+ic(-128b^4c^4+328ab^3c^3d-243a^2b^2c^2d^2+25a^3bcd^3+10a^4d^4)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic(-128b^4c^4+392ab^3c^3d-399a^2b^2c^2d^2+130a^3bcd^3+5a^4d^4)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{315b\sqrt{\frac{b}{a}}\,d^5\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(5*a^3*d^3 + 15*a^2*b*d^2*(-7*c + 5*
d*x^2) + a*b^2*d*(156*c^2 - 115*c*d*x^2 + 95*d^2*x^4) + b^3*(-64*c^3 + 48*c
^2*d*x^2 - 40*c*d^2*x^4 + 35*d^3*x^6)) + I*c*(-128*b^4*c^4 + 328*a*b^3*c^3*
d - 243*a^2*b^2*c^2*d^2 + 25*a^3*b*c*d^3 + 10*a^4*d^4)*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-
128*b^4*c^4 + 392*a*b^3*c^3*d - 399*a^2*b^2*c^2*d^2 + 130*a^3*b*c*d^3 + 5*a
^4*d^4)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)])/(315*b*Sqrt[b/a]*d^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 1047, normalized size = 1.89 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-25*(-b/a)^(1/2)*a*b^3*c*d^4*x^7-50*
(-b/a)^(1/2)*a^2*b^2*c*d^4*x^5+49*(-b/a)^(1/2)*a*b^3*c^2*d^3*x^5-128*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b^4*c^5+5*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*a^4*c*d^4-10*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^4+128*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^5-328*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))*a*b^3*c^4*d+130*(-b/a)^(1/2)*a*b^3*d^5*x^9-5*(-b/a)^(1/2)*b^4*c*d^4*x^9+
170*(-b/a)^(1/2)*a^2*b^2*d^5*x^7+8*(-b/a)^(1/2)*b^4*c^2*d^3*x^7+80*(-b/a)^(
1/2)*a^3*b*d^5*x^5-105*(-b/a)^(1/2)*a^3*b*c^2*d^3*x+156*(-b/a)^(1/2)*a^2*b^
2*c^3*d^2*x-25*(-b/a)^(1/2)*a^3*b*c*d^4*x^3-64*(-b/a)^(1/2)*a^2*b^2*c^2*d^3
*x^3+140*(-b/a)^(1/2)*a*b^3*c^3*d^2*x^3+35*(-b/a)^(1/2)*b^4*d^5*x^11+5*(-b/
a)^(1/2)*a^4*d^5*x^3+130*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d^3-399*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^3*d^2+39
2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*a*b^3*c^4*d-25*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d^3+243*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^3*d^2-16*
(-b/a)^(1/2)*b^4*c^3*d^2*x^5-64*(-b/a)^(1/2)*b^4*c^4*d*x^3+5*(-b/a)^(1/2)*a
^4*c*d^4*x-64*(-b/a)^(1/2)*a*b^3*c^4*d*x)/b/d^5/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(x**4*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
[Out] int((x^4*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.961 $\int \frac{x^2 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\, dx$

<u>**Optimal**</u>. Leaf size=436

$$-\frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{105bd^3\sqrt{c+dx^2}}+\frac{(24b^2c^2-61abcd+45a^2d^2)\,x\sqrt{a+bx^2}\,\sqrt{c+d}}{105d^3}$$

```
[Out] -1/105*(-15*a^3*d^3+103*a^2*b*c*d^2-128*a*b^2*c^2*d+48*b^3*c^3)*x*(b*x^2+a)
^(1/2)/b/d^3/(d*x^2+c)^(1/2)-1/105*c^(3/2)*(45*a^2*d^2-61*a*b*c*d+24*b^2*c^
2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*
x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/105*(-15*a^3*d^3+103*a^2*b*c*d^2-128*a*b^2*
c^2*d+48*b^3*c^3)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/
2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b/d
^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/7*b*x^3*(b*x^2+a)^
(3/2)*(d*x^2+c)^(1/2)/d+1/105*(45*a^2*d^2-61*a*b*c*d+24*b^2*c^2)*x*(b*x^2+a
)^(1/2)*(d*x^2+c)^(1/2)/d^3-2/35*b*(-5*a*d+3*b*c)*x^3*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/d^2
```

<u>**Rubi**</u> [A]

time = 0.32, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {488, 595, 596, 545, 429, 506, 422}

$$-\frac{c^{3/2}\sqrt{a+bx^2}\,(45a^2d^2-61abcd+24b^2c^2)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(45a^2d^2-61abcd+24b^2c^2)}{105d^3}+\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105bd^{7/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}\,(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3)}{105bd^3\sqrt{c+dx^2}}-\frac{2bx^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(3bc-5ad)}{35d^2}+\frac{bx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] -1/105*((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x*Sqr
t[a + b*x^2])/(b*d^3*Sqrt[c + d*x^2]) + ((24*b^2*c^2 - 61*a*b*c*d + 45*a^2*
d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - (2*b*(3*b*c - 5*a*d)*x^
3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*d^2) + (b*x^3*(a + b*x^2)^(3/2)*Sqrt
[c + d*x^2])/(7*d) + (Sqrt[c]*(48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d
^2 - 15*a^3*d^3)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(105*b*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) - (c^(3/2)*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^(7/2)*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

<u>Rule 422</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
```

```
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{bx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7d} + \frac{\int \frac{x^2\sqrt{a+bx^2}\,(-a(3bc-7ad)-2b(3bc-5ad)x^2)}{\sqrt{c+dx^2}}\,dx}{7d}$$

$$= -\frac{2b(3bc-5ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2} + \frac{bx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7d} + \frac{\int \frac{x^2(a(18b^2c^2\cdots}{\cdots}}{\cdots}$$

$$= \frac{(24b^2c^2-61abcd+45a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} - \frac{2b(3bc-5ad)x^3\sqrt{a+bx^2}\sqrt{c\cdots}}{35d^2}$$

$$= \frac{(24b^2c^2-61abcd+45a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} - \frac{2b(3bc-5ad)x^3\sqrt{a+bx^2}\sqrt{c\cdots}}{35d^2}$$

$$= -\frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{105bd^3\sqrt{c+dx^2}} + \frac{(24b^2c^2-61abcd+45\cdots}{\cdots}$$

$$= -\frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{105bd^3\sqrt{c+dx^2}} + \frac{(24b^2c^2-61abcd+45\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.06, size = 306, normalized size = 0.70

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2)(45a^2d^2+abd(-61c+45dx^2)+3b^2(8c^2-6cdx^2+5d^2x^4))-ic(-48b^3c^3+128ab^2c^2d-103a^2bcd^2+15a^3d^3)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)+4ic(-12b^3c^3+38ab^2c^2d-41a^2bcd^2+15a^3d^3)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{105\sqrt{\frac{b}{a}}\,d^4\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(45*a^2*d^2 + a*b*d*(-61*c + 45*d*x^
2) + 3*b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) - I*c*(-48*b^3*c^3 + 128*a*b^2*
c^2*d - 103*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4*I)*c*(-12*b^3*c^3 +
38*a*b^2*c^2*d - 41*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*Sqrt[b/a]*d
^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 782, normalized size = 1.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-15*(-b/a)^(1/2)*b^3*d^4*x^9-60*(-b
/a)^(1/2)*a*b^2*d^4*x^7+3*(-b/a)^(1/2)*b^3*c*d^3*x^7-90*(-b/a)^(1/2)*a^2*b*
d^4*x^5+19*(-b/a)^(1/2)*a*b^2*c*d^3*x^5-6*(-b/a)^(1/2)*b^3*c^2*d^2*x^5-45*(
-b/a)^(1/2)*a^3*d^4*x^3-29*(-b/a)^(1/2)*a^2*b*c*d^3*x^3+55*(-b/a)^(1/2)*a*b
^2*c^2*d^2*x^3-24*(-b/a)^(1/2)*b^3*c^3*d*x^3+60*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3-164*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*a^2*b*c^2*d^2+152*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^3*d-48*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-15*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3
+103*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^2*b*c^2*d^2-128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^3*d+48*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-45*(-b/a
)^(1/2)*a^3*c*d^3*x+61*(-b/a)^(1/2)*a^2*b*c^2*d^2*x-24*(-b/a)^(1/2)*a*b^2*c
^3*d*x)/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(b\,x^2+a\right)}^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.962 $\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=330

$$\frac{\left(7ab-\frac{2b^2c}{d}+\frac{3a^2d}{c}\right)x\sqrt{a+bx^2}}{3\sqrt{c+dx^2}}+\frac{b(bc+3ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3cd}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx}+\frac{(2b^2c^2-7ab}{}$$

```
[Out] 1/3*(7*a*b-2*b^2*c/d+3*a^2*d/c)*x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*(-3*a
^2*d^2-7*a*b*c*d+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d
^(3/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(-9*a*
d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-a*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)
/c/x+1/3*b*(3*a*d+b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/d
```

Rubi [A]

time = 0.20, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {485, 542, 545, 429, 506, 422}

$$\frac{\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}\,d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(\frac{3a^2d}{c}+7ab-\frac{2b^2c}{d}\right)}{3\sqrt{c+dx^2}}-\frac{b\sqrt{c}\sqrt{a+bx^2}(bc-9ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{3cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((7*a*b - (2*b^2*c)/d + (3*a^2*d)/c)*x*Sqrt[a + b*x^2])/(3*Sqrt[c + d*x^2])
 + (b*(b*c + 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*d) - (a*(a + b*
x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x) + ((2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*S
qrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*
Sqrt[c]*(b*c - 9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx = -\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx} + \frac{\int \frac{\sqrt{a+bx^2}\,(4abc+b(bc+3ad)x^2)}{\sqrt{c+dx^2}}\,dx}{c}$$

$$= \frac{b(bc+3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx} + \frac{\int \frac{-abc(bc-9ad)-b(2b^2c^2-7\ldots}{\sqrt{a+bx^2}\sqrt{c\ldots}}}{3cd}$$

$$= \frac{b(bc+3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx} - \frac{(ab(bc-9ad))\int \frac{}{\sqrt{a\ldots}}}{3d\ldots}$$

$$= \frac{\left(7ab-\frac{2b^2c}{d}+\frac{3a^2d}{c}\right)x\sqrt{a+bx^2}}{3\sqrt{c+dx^2}} + \frac{b(bc+3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd} - \frac{a(a+bx^2)^{3/2}\ldots}{cx}$$

$$= \frac{\left(7ab-\frac{2b^2c}{d}+\frac{3a^2d}{c}\right)x\sqrt{a+bx^2}}{3\sqrt{c+dx^2}} + \frac{b(bc+3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd} - \frac{a(a+bx^2)^{3/2}\ldots}{cx}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.00, size = 254, normalized size = 0.77

$$\frac{-\sqrt{\frac{b}{a}}\,d(a+bx^2)(3a^2d-b^2cx^2)(c+dx^2) - ibc(-2b^2c^2+7abcd+3a^2d^2)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right) - 2ibc(b^2c^2-4abcd+3a^2d^2)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3\sqrt{\frac{b}{a}}\,cd^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(Sqrt[b/a]*d*(a + b*x^2)*(3*a^2*d - b^2*c*x^2)*(c + d*x^2)) - I*b*c*(-2*b
^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*b*c*(b^2*c^2 - 4*a*b*
c*d + 3*a^2*d^2)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c*d^2*x*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])
```

Maple [A]
time = 0.12, size = 568, normalized size = 1.72

method	result

elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(-\frac{a^2\sqrt{bdx^4+adx^2+cx^2b+ac}}{cx}+\frac{b^2x\sqrt{bdx^4+adx^2+cx^2b+ac}}{3d}+\frac{(3a^2b\cdots}{\cdots}\cdots\right.}{\cdots}$
risch	$-\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,(-b^2cx^2+3a^2d)}{3dcx}+\frac{b\left(-\frac{(3a^2d^2+7abcd-2b^2c^2)c\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}\cdots\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bdx^4+ad\cdots}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}\,b^3cd^2x^6-3\sqrt{-\frac{b}{a}}\,a^2bd^3x^4+\sqrt{-\frac{b}{a}}\,ab^2cd^2x^4+\sqrt{-\frac{b}{a}}\,b^3c^2dx^4+6\sqrt{\frac{bx^2+a}{a}}\,\sqrt{\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*b^3*c*d^2*x^6-3*(-b/a)^(1
/2)*a^2*b*d^3*x^4+(-b/a)^(1/2)*a*b^2*c*d^2*x^4+(-b/a)^(1/2)*b^3*c^2*d*x^4+6
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*a^2*b*c*d^2*x-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d*x+2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3*x+3*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^
2*b*c*d^2*x+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d*x-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3*x-3*(-b/a)^(1/2)*a^3*d^
3*x^2-3*(-b/a)^(1/2)*a^2*b*c*d^2*x^2+(-b/a)^(1/2)*a*b^2*c^2*d*x^2-3*(-b/a)^
(1/2)*a^3*c*d^2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b/a)^(1/2)/c/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^4
 + c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2}\right)^{\frac{5}{2}}}{x^{2}\sqrt{c + dx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**2*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{5/2}}{x^2\,\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(x^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(x^2*(c + d*x^2)^(1/2)), x)
```

3.963 $\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=336

$$\frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}}-\frac{2a(3bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{3c^2x}-\frac{a(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{3cx^3}-\frac{(3b^2c^2$$

```
[Out] 1/3*(-2*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*x*(b*x^2+a)^(1/2)/c^2/(d*x^2+c)^(1/2)-
1/3*(-2*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a
)^(1/2)/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3
*b*(-a*d+9*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)
/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/c^(1/2)/d^(1/
2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*a*(b*x^2+a)^(3/2)*(d
*x^2+c)^(1/2)/c/x^3-2/3*a*(-a*d+3*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2/
x
```

Rubi [A]
time = 0.20, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {485, 594, 545, 429, 506, 422}

$$-\frac{\sqrt{a+bx^2}\,(-2a^2d^2+7abcd+3b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\,(-2a^2d^2+7abcd+3b^2c^2)}{3c^2\sqrt{c+dx^2}}+\frac{b\sqrt{a+bx^2}\,(9bc-ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}\,\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2a\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(3bc-ad)}{3c^2x}-\frac{a(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(3*c^2*Sqrt[c + d*x
^2]) - (2*a*(3*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c^2*x) - (a*(
a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(3*c*x^3) - ((3*b^2*c^2 + 7*a*b*c*d - 2*a
^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(3*c^(3/2)*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) + (b*(9*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx = -\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3cx^3} + \frac{\int \frac{\sqrt{a+bx^2}\,(2a(3bc-ad)+b(3bc+ad)x^2)}{x^2\sqrt{c+dx^2}}\,dx}{3c}$$

$$= -\frac{2a(3bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3cx^3} + \frac{\int \frac{abc(9bc-ad)+b(3b^2c^2\cdots}{\sqrt{a+bx^2}\,\sqrt{\cdots}}}{3c^{\cdots}}$$

$$= -\frac{2a(3bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3cx^3} + \frac{(ab(9bc-ad))\int \cdots}{\cdots}$$

$$= \frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}} - \frac{2a(3bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x} - \frac{a(a+bx\cdots}{\cdots}$$

$$= \frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}} - \frac{2a(3bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x} - \frac{a(a+bx\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.17, size = 261, normalized size = 0.78

$$\frac{a\sqrt{\frac{b}{a}}\,d(a+bx^2)(c+dx^2)(-ac-7bcx^2+2adx^2)+ibc(-3b^2c^2-7abcd+2a^2d^2)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ibc(-3b^2c^2+2abcd+a^2d^2)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3\sqrt{\frac{b}{a}}\,c^2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(-(a*c) - 7*b*c*x^2 + 2*a*d*x^2) + I
*b*c*(-3*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-3*b^2*c
^2 + 2*a*b*c*d + a^2*d^2)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d*x^3*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.14, size = 583, normalized size = 1.74

method	result

elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(-\frac{a^2\sqrt{bdx^4+adx^2+cx^2b+ac}}{3cx^3}+\frac{a(2ad-7bc)\sqrt{bdx^4+adx^2+cx^2b+ac}}{3c^2x}+\cdots\right.}{\cdots}$
risch	$-\frac{a\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,(-2adx^2+7cx^2b+ac)}{3c^2x^3}-\frac{b\left(-\frac{(2a^2d^2-7abcd-3b^2c^2)c\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\left(\text{EllipticF}\left(x\sqrt{\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bdx^4+\cdots}}\right.}{\cdots}$
default	$\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\left(2\sqrt{-\frac{b}{a}}\,a^2bd^3x^6-7\sqrt{-\frac{b}{a}}\,ab^2cd^2x^6+\sqrt{\frac{bx^2+a}{a}}\,\sqrt{\frac{dx^2+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(2*(-b/a)^(1/2)*a^2*b*d^3*x^6-7*(-b/a)^
(1/2)*a*b^2*c*d^2*x^6+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c*d^2*x^3+2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d*x^3-3*((b
*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*b^3*c^3*x^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c*d^2*x^3+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d*x^3+3*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
b^3*c^3*x^3+2*(-b/a)^(1/2)*a^3*d^3*x^4-6*(-b/a)^(1/2)*a^2*b*c*d^2*x^4-7*(-b
/a)^(1/2)*a*b^2*c^2*d*x^4+(-b/a)^(1/2)*a^3*c*d^2*x^2-8*(-b/a)^(1/2)*a^2*b*c
^2*d*x^2-(-b/a)^(1/2)*a^3*c^2*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/c^2/(-b/a)^(
1/2)/d/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^6
 + c*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^{\frac{5}{2}}}{x^4\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**4*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{5/2}}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(x^4*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(x^4*(c + d*x^2)^(1/2)), x)
```

3.964 $\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=99

$$-\frac{7}{135}x\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{1}{15}x^3\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{8E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{45\sqrt{3}}-\frac{2F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{27\sqrt{3}}$$

```
[Out] -8/135*(x^2)^(1/2)/x*EllipticE(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)-2/81*(
x^2)^(1/2)/x*EllipticF(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)-7/135*x*(-3*x^
2+2)^(1/2)*(3*x^2-1)^(1/2)-1/15*x^3*(-3*x^2+2)^(1/2)*(3*x^2-1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {489, 596, 538, 436, 431}

$$-\frac{2F\left(\text{ArcCos}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{27\sqrt{3}}-\frac{8E\left(\text{ArcCos}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{45\sqrt{3}}-\frac{7}{135}\sqrt{2-3x^2}\,\sqrt{3x^2-1}\,x-\frac{1}{15}\sqrt{2-3x^2}\,\sqrt{3x^2-1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-7*x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/135 - (x^3*Sqrt[2 - 3*x^2]*Sqrt[-1
 + 3*x^2])/15 - (8*EllipticE[ArcCos[Sqrt[3/2]*x], 2])/(45*Sqrt[3]) - (2*Elli
pticF[ArcCos[Sqrt[3/2]*x], 2])/(27*Sqrt[3])
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rule 436

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{1}{15}x^3\sqrt{2-3x^2}\,\sqrt{-1+3x^2} + \frac{1}{15}\int \frac{x^2(-6+21x^2)}{\sqrt{2-3x^2}\,\sqrt{-1+3x^2}}\,dx$$

$$= -\frac{7}{135}x\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{1}{15}x^3\sqrt{2-3x^2}\,\sqrt{-1+3x^2} + \frac{1}{405}\int \frac{-42+}{\sqrt{2-3x^2}\,\sqrt{}}$$

$$= -\frac{7}{135}x\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{1}{15}x^3\sqrt{2-3x^2}\,\sqrt{-1+3x^2} + \frac{2}{27}\int \frac{1}{\sqrt{2-3x^2}\,\sqrt{}}$$

$$= -\frac{7}{135}x\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{1}{15}x^3\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{8E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}\,x\right.\right.}{45\sqrt{3}}$$

Mathematica [A]
time = 0.51, size = 92, normalized size = 0.93

$$\frac{-3x\sqrt{2-3x^2}\,(-7+12x^2+27x^4)-24\sqrt{3-9x^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)+10\sqrt{3-9x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{405\sqrt{-1+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-3*x*Sqrt[2 - 3*x^2]*(-7 + 12*x^2 + 27*x^4) - 24*Sqrt[3 - 9*x^2]*EllipticE
[ArcSin[Sqrt[3/2]*x], 2] + 10*Sqrt[3 - 9*x^2]*EllipticF[ArcSin[Sqrt[3/2]*x]
, 2])/(405*Sqrt[-1 + 3*x^2])
```

Maple [A]
time = 0.18, size = 135, normalized size = 1.36

method	result
default	$-\frac{\sqrt{3x^2-1}\,\sqrt{2}\,\sqrt{-6x^2+4}\,\left(243x^7-54x^5+5\sqrt{2}\,\sqrt{3}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2},\sqrt{2}\right.\right.}{810(9x^4-9x^2+2)}$
elliptic	$\frac{\sqrt{-(3x^2-2)(3x^2-1)}\,\left(-\frac{x^3\sqrt{-9x^4+9x^2-2}}{15}-\frac{7x\sqrt{-9x^4+9x^2-2}}{135}-\frac{7\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2}}{405\sqrt{-9x^4}}\right.}{\sqrt{-3x^2+2}}$
risch	$\frac{x(9x^2+7)(3x^2-2)\sqrt{3x^2-1}\,\sqrt{(3x^2-1)(-3x^2+2)}}{135\sqrt{-(3x^2-2)(3x^2-1)}\,\sqrt{-3x^2+2}}+\frac{\left(\frac{4\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2}\right.\right.}{135\sqrt{-9x^4+9x^2-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/810*(3*x^2-1)^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(243*x^7-54*x^5+5*2^(1/2)*3
^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticF(1/2*x*2^(1/2)*3^(1/2),2^
(1/2))-12*2^(1/2)*3^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticE(1/2*x
*2^(1/2)*3^(1/2),2^(1/2))-135*x^3+42*x)/(9*x^4-9*x^2+2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2), x)
```

Fricas [A]
time = 0.13, size = 35, normalized size = 0.35

$$-\frac{(9\,x^4+7\,x^2+8)\sqrt{3\,x^2-1}\,\sqrt{-3\,x^2+2}}{135\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/135*(9*x^4 + 7*x^2 + 8)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Integral(x**4*sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{3\,x^2-1}}{\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
[Out] int((x^4*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2), x)
```

3.965 $\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2}-\frac{7}{144}\sin^{-1}\left(3-6x^2\right)$$

```
[Out] 7/144*arcsin(6*x^2-3)-1/36*(3*x^2-1)^(3/2)*(-3*x^2+2)^(1/2)-7/72*(-3*x^2+2)
^(1/2)*(3*x^2-1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 81, 52, 55, 633, 222}

$$-\frac{7}{144}\text{ArcSin}\left(3-6x^2\right)-\frac{1}{36}\sqrt{2-3x^2}\,\left(3x^2-1\right)^{3/2}-\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{3x^2-1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-7*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/72 - (Sqrt[2 - 3*x^2]*(-1 + 3*x^2)^(3
/2))/36 - (7*ArcSin[3 - 6*x^2])/144
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{-1+3x}}{\sqrt{2-3x}}\,dx, x, x^2\right)$$
$$= -\frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2} + \frac{7}{24}\text{Subst}\left(\int \frac{\sqrt{-1+3x}}{\sqrt{2-3x}}\,dx, x, x^2\right)$$
$$= -\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2} + \frac{7}{48}\text{Subst}\left(\int \frac{}{\sqrt{2-3x}}\right.$$
$$= -\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2} + \frac{7}{48}\text{Subst}\left(\int \frac{}{\sqrt{-2+9}}\right.$$
$$= -\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2} - \frac{7}{432}\text{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.$$
$$= -\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2} - \frac{7}{144}\sin^{-1}\left(3-6x^2\right)$$

Mathematica [A]
time = 0.47, size = 66, normalized size = 1.02

$$\frac{1}{72}\left(\frac{\sqrt{2-3x^2}\,(5-9x^2-18x^4)}{\sqrt{-1+3x^2}} - 7\tan^{-1}\left(\frac{\sqrt{2-3x^2}}{\sqrt{-1+3x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] ((Sqrt[2 - 3*x^2]*(5 - 9*x^2 - 18*x^4))/Sqrt[-1 + 3*x^2] - 7*ArcTan[Sqrt[2
- 3*x^2]/Sqrt[-1 + 3*x^2]])/72
```

Maple [A]
time = 0.16, size = 81, normalized size = 1.25

method	result	size
default	$\frac{\sqrt{3x^2-1}\,\sqrt{-3x^2+2}\,\left(-12x^2\sqrt{-9x^4+9x^2-2}+7\arcsin\left(6x^2-3\right)-10\sqrt{-9x^4+9x^2-2}\right)}{144\sqrt{-9x^4+9x^2-2}}$	81
elliptic	$\frac{\sqrt{-\left(3x^2-2\right)\left(3x^2-1\right)}\,\left(-\frac{x^2\sqrt{-9x^4+9x^2-2}}{12}-\frac{5\sqrt{-9x^4+9x^2-2}}{72}+\frac{7\arcsin\left(6x^2-3\right)}{144}\right)}{\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$	84
risch	$\frac{\left(6x^2+5\right)\left(3x^2-2\right)\sqrt{3x^2-1}\,\sqrt{\left(3x^2-1\right)\left(-3x^2+2\right)}}{72\sqrt{-\left(3x^2-2\right)\left(3x^2-1\right)}\,\sqrt{-3x^2+2}}+\frac{7\arcsin\left(6x^2-3\right)\sqrt{\left(3x^2-1\right)\left(-3x^2+2\right)}}{144\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/144*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(-12*x^2*(-9*x^4+9*x^2-2)^(1/2)+7*ar
csin(6*x^2-3)-10*(-9*x^4+9*x^2-2)^(1/2))/(-9*x^4+9*x^2-2)^(1/2)
```

Maxima [A]
time = 0.48, size = 46, normalized size = 0.71

$$-\frac{1}{12}\sqrt{-9\,x^4+9\,x^2-2}\,x^2-\frac{5}{72}\sqrt{-9\,x^4+9\,x^2-2}+\frac{7}{144}\arcsin\left(6\,x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(-9*x^4 + 9*x^2 - 2)*x^2 - 5/72*sqrt(-9*x^4 + 9*x^2 - 2) + 7/144*
arcsin(6*x^2 - 3)
```

Fricas [A]
time = 0.43, size = 72, normalized size = 1.11

$$-\frac{1}{72}\left(6\,x^2+5\right)\sqrt{3\,x^2-1}\,\sqrt{-3\,x^2+2}-\frac{7}{144}\arctan\left(\frac{3\,\sqrt{3\,x^2-1}\,\left(2\,x^2-1\right)\sqrt{-3\,x^2+2}}{2\,\left(9\,x^4-9\,x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) - 7/144*arctan(3/2*sqrt(
3*x^2 - 1)*(2*x^2 - 1)*sqrt(-3*x^2 + 2)/(9*x^4 - 9*x^2 + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

Giac [A]
time = 0.96, size = 40, normalized size = 0.62

$$-\frac{1}{72}\left(6\,x^2+5\right)\sqrt{3\,x^2-1}\,\sqrt{-3\,x^2+2}+\frac{7}{72}\,\arcsin\left(\sqrt{3\,x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 7/72*arcsin(sqrt(3*x^2
 - 1))
```

Mupad [B]
time = 6.86, size = 414, normalized size = 6.37

$$-\frac{7\,\mathrm{atan}\left(\frac{\sqrt{3\,x^2-1}-\mathrm{i}}{\sqrt{2}-\sqrt{2-3\,x^2}}\right)}{36}+\frac{\frac{7\,\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}{36\,\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}+\frac{143\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^3}{36\,{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^3}-\frac{143\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^5}{36\,{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^5}-\frac{7\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^7}{36\,{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^7}+\frac{\sqrt{2}\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^2\,4\mathrm{i}}{9\,{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^2}-\frac{\sqrt{2}\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^4\,40\mathrm{i}}{9\,{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^4}+\frac{\sqrt{2}\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^6\,4\mathrm{i}}{9\,{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^6}}{\frac{4\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^2}+\frac{6\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^4}+\frac{4\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^6}+\frac{{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^8}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
```

```
[Out] ((7*((3*x^2 - 1)^(1/2) - 1i))/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))) + (143*((3
*x^2 - 1)^(1/2) - 1i)^3)/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))^3) - (143*((3*x^
2 - 1)^(1/2) - 1i)^5)/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))^5) - (7*((3*x^2 - 1
)^(1/2) - 1i)^7)/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))^7) + (2^(1/2)*((3*x^2 -
1)^(1/2) - 1i)^2*4i)/(9*(2^(1/2) - (2 - 3*x^2)^(1/2))^2) - (2^(1/2)*((3*x^2
 - 1)^(1/2) - 1i)^4*40i)/(9*(2^(1/2) - (2 - 3*x^2)^(1/2))^4) + (2^(1/2)*((3
*x^2 - 1)^(1/2) - 1i)^6*4i)/(9*(2^(1/2) - (2 - 3*x^2)^(1/2))^6))/((4*((3*x^
2 - 1)^(1/2) - 1i)^2)/(2^(1/2) - (2 - 3*x^2)^(1/2))^2 + (6*((3*x^2 - 1)^(1/
2) - 1i)^4)/(2^(1/2) - (2 - 3*x^2)^(1/2))^4 + (4*((3*x^2 - 1)^(1/2) - 1i)^6
)/(2^(1/2) - (2 - 3*x^2)^(1/2))^6 + ((3*x^2 - 1)^(1/2) - 1i)^8/(2^(1/2) - (
2 - 3*x^2)^(1/2))^8 + 1) - (7*atan(((3*x^2 - 1)^(1/2) - 1i)/(2^(1/2) - (2 -
 3*x^2)^(1/2))))/36
```

3.966 $\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{9}x\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{3\sqrt{3}}-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}$$

```
[Out] -1/9*(x^2)^(1/2)/x*EllipticE(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)-1/27*(x^
2)^(1/2)/x*EllipticF(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)-1/9*x*(-3*x^2+2)
^(1/2)*(3*x^2-1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {489, 538, 436, 431}

$$-\frac{F\left(\text{ArcCos}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}-\frac{E\left(\text{ArcCos}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{3\sqrt{3}}-\frac{1}{9}\sqrt{2-3x^2}\,\sqrt{3x^2-1}\,x$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] -1/9*(x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2]) - EllipticE[ArcCos[Sqrt[3/2]*x],
2]/(3*Sqrt[3]) - EllipticF[ArcCos[Sqrt[3/2]*x], 2]/(9*Sqrt[3])
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rule 436

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx &= -\frac{1}{9}x\sqrt{2-3x^2}\,\sqrt{-1+3x^2}+\frac{1}{9}\int \frac{-2+9x^2}{\sqrt{2-3x^2}\,\sqrt{-1+3x^2}}\,dx\\ &= -\frac{1}{9}x\sqrt{2-3x^2}\,\sqrt{-1+3x^2}+\frac{1}{9}\int \frac{1}{\sqrt{2-3x^2}\,\sqrt{-1+3x^2}}\,dx+\frac{1}{3}\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\\ &= -\frac{1}{9}x\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{3\sqrt{3}}-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 86, normalized size = 1.23

$$\frac{3x(1-3x^2)\sqrt{2-3x^2}-3\sqrt{3-9x^2}\,E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)+\sqrt{3-9x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{27\sqrt{-1+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (3*x*(1 - 3*x^2)*Sqrt[2 - 3*x^2] - 3*Sqrt[3 - 9*x^2]*EllipticE[ArcSin[Sqrt[
3/2]*x], 2] + Sqrt[3 - 9*x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2])/(27*Sqrt[-
1 + 3*x^2])
```

Maple [A]
time = 0.14, size = 129, normalized size = 1.84

method	result
default	$-\frac{\sqrt{3x^2-1}\,\sqrt{2}\,\sqrt{-6x^2+4}\,\left(54x^5+\sqrt{2}\,\sqrt{3}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2},\sqrt{2}\right)-3\sqrt{}\right.}{108(9x^4-9x^2+2)}$
elliptic	$\frac{\sqrt{-(3x^2-2)(3x^2-1)}\left(-\frac{x\sqrt{-9x^4+9x^2-2}}{9}-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)}{27\sqrt{-9x^4+9x^2-2}}\right.}{\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$
risch	$\frac{x(3x^2-2)\sqrt{3x^2-1}\,\sqrt{(3x^2-1)(-3x^2+2)}}{9\sqrt{-(3x^2-2)(3x^2-1)}\,\sqrt{-3x^2+2}}+\frac{\left(\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)-\right.}{18\sqrt{-9x^4+9x^2-2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/108*(3*x^2-1)^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(54*x^5+2^(1/2)*3^(1/2)*(-6
*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticF(1/2*x*2^(1/2)*3^(1/2),2^(1/2))-3*2
^(1/2)*3^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticE(1/2*x*2^(1/2)*3^
(1/2),2^(1/2))-54*x^3+12*x)/(9*x^4-9*x^2+2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2), x)
```

Fricas [A]
time = 0.12, size = 28, normalized size = 0.40

$$-\frac{\sqrt{3\,x^2-1}\,(x^2+1)\sqrt{-3\,x^2+2}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3*x^2 - 1)*(x^2 + 1)*sqrt(-3*x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Integral(x**2*sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{3\,x^2-1}}{\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
[Out] int((x^2*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2), x)
```

3.967 $\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{1}{12}\sin^{-1}\left(3-6x^2\right)$$

```
[Out] 1/12*arcsin(6*x^2-3)-1/6*(-3*x^2+2)^(1/2)*(3*x^2-1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {455, 52, 55, 633, 222}

$$-\frac{1}{12}\text{ArcSin}\left(3-6x^2\right)-\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{3x^2-1}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] -1/6*(Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2]) - ArcSin[3 - 6*x^2]/12
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+3x}}{\sqrt{2-3x}}\,dx, x, x^2\right)\\ &= -\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{-1+3x^2} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{2-3x}\,\sqrt{-1+3x}}\,dx, x, x^2\right)\\ &= -\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{-1+3x^2} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-2+9x-9x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{1}{36}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 9(1-2x^2)\right)\\ &= -\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{-1+3x^2} - \frac{1}{12}\sin^{-1}\left(3-6x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.12, size = 53, normalized size = 1.36

$$\frac{1}{6}\left(-\sqrt{-2+9x^2-9x^4} + 2\tan^{-1}\left(\frac{\sqrt{-1+3x^2}}{-1+\sqrt{2-3x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-Sqrt[-2 + 9*x^2 - 9*x^4] + 2*ArcTan[Sqrt[-1 + 3*x^2]/(-1 + Sqrt[2 - 3*x^2
])])/6
```

Maple [A]
time = 0.14, size = 60, normalized size = 1.54

method	result	size

default	$\frac{\sqrt{3x^2-1}\,\sqrt{-3x^2+2}\,\left(\arcsin(6x^2-3)-2\sqrt{-9x^4+9x^2-2}\right)}{12\sqrt{-9x^4+9x^2-2}}$	60
elliptic	$\frac{\sqrt{-\left(3x^2-2\right)\left(3x^2-1\right)}\,\left(\frac{\arcsin\left(6x^2-3\right)}{12}-\frac{\sqrt{-9x^4+9x^2-2}}{6}\right)}{\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$	65
risch	$\frac{(3x^2-2)\sqrt{3x^2-1}\,\sqrt{(3x^2-1)(-3x^2+2)}}{6\sqrt{-(3x^2-2)(3x^2-1)}\,\sqrt{-3x^2+2}}+\frac{\arcsin(6x^2-3)\sqrt{(3x^2-1)(-3x^2+2)}}{12\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(arcsin(6*x^2-3)-2*(-9*x^4+9*x^2-2)^(
1/2))/(-9*x^4+9*x^2-2)^(1/2)
```

Maxima [A]
time = 0.48, size = 27, normalized size = 0.69

$$-\frac{1}{6}\sqrt{-9\,x^4+9\,x^2-2}+\frac{1}{12}\arcsin\left(6\,x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-9*x^4 + 9*x^2 - 2) + 1/12*arcsin(6*x^2 - 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(31) = 62$.
time = 0.42, size = 65, normalized size = 1.67

$$-\frac{1}{6}\sqrt{3\,x^2-1}\,\sqrt{-3\,x^2+2}-\frac{1}{12}\arctan\left(\frac{3\sqrt{3\,x^2-1}\,(2\,x^2-1)\sqrt{-3\,x^2+2}}{2\,(9\,x^4-9\,x^2+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) - 1/12*arctan(3/2*sqrt(3*x^2 - 1)*(2*
x^2 - 1)*sqrt(-3*x^2 + 2)/(9*x^4 - 9*x^2 + 2))
```

Sympy [A]
time = 2.12, size = 58, normalized size = 1.49

$$\frac{\begin{cases}-\frac{\sqrt{2-3x^2}\sqrt{3x^2-1}}{2}+\frac{\operatorname{asin}\left(\sqrt{3x^2-1}\right)}{2} & \text{for}\ \sqrt{3x^2-1}>-1\wedge\sqrt{3x^2-1}<1\end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2 - 3*x**2)*sqrt(3*x**2 - 1)/2 + asin(sqrt(3*x**2 - 1))/2,
 (sqrt(3*x**2 - 1) > -1) & (sqrt(3*x**2 - 1) < 1)))/3
```

Giac [A]
time = 0.81, size = 33, normalized size = 0.85

$$-\frac{1}{6}\sqrt{3x^2-1}\sqrt{-3x^2+2}+\frac{1}{6}\arcsin\left(\sqrt{3x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 1/6*arcsin(sqrt(3*x^2 - 1))
```

Mupad [B]
time = 1.49, size = 206, normalized size = 5.28

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{3x^2-1}-\mathrm{i}}{\sqrt{2}-\sqrt{2-3x^2}}\right)}{3}-\frac{-\frac{\sqrt{3x^2-1}-\mathrm{i}}{\sqrt{2}-\sqrt{2-3x^2}}+\frac{\left(\sqrt{3x^2-1}-\mathrm{i}\right)^3}{\left(\sqrt{2}-\sqrt{2-3x^2}\right)^3}+\frac{\sqrt{2}\left(\sqrt{3x^2-1}-\mathrm{i}\right)^2 4\mathrm{i}}{3\left(\sqrt{2}-\sqrt{2-3x^2}\right)^2}}{\frac{2\left(\sqrt{3x^2-1}-\mathrm{i}\right)^2}{\left(\sqrt{2}-\sqrt{2-3x^2}\right)^2}+\frac{\left(\sqrt{3x^2-1}-\mathrm{i}\right)^4}{\left(\sqrt{2}-\sqrt{2-3x^2}\right)^4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
```

```
[Out] - atan(((3*x^2 - 1)^(1/2) - 1i)/(2^(1/2) - (2 - 3*x^2)^(1/2)))/3 - (((3*x^2
 - 1)^(1/2) - 1i)^3/(2^(1/2) - (2 - 3*x^2)^(1/2))^3 - ((3*x^2 - 1)^(1/2) -
1i)/(2^(1/2) - (2 - 3*x^2)^(1/2)) + (2^(1/2)*((3*x^2 - 1)^(1/2) - 1i)^2*4i)
/(3*(2^(1/2) - (2 - 3*x^2)^(1/2))^2))/((2*((3*x^2 - 1)^(1/2) - 1i)^2)/(2^(1
/2) - (2 - 3*x^2)^(1/2))^2 + ((3*x^2 - 1)^(1/2) - 1i)^4/(2^(1/2) - (2 - 3*x
^2)^(1/2))^4 + 1)
```

3.968 $\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=241

$$-\frac{2(3b-d)x\sqrt{2+bx^2}}{3bd\sqrt{3+dx^2}}+\frac{x\sqrt{2+bx^2}\sqrt{3+dx^2}}{3d}+\frac{2\sqrt{2}(3b-d)\sqrt{2+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3bd^{3/2}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}-\frac{\sqrt{2}}{}$$

```
[Out] -2/3*(3*b-d)*x*(b*x^2+2)^(1/2)/b/d/(d*x^2+3)^(1/2)+2/3*(3*b-d)*(1/(3*d*x^2+
9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1
/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/b/d^(3/2)/((b*x^2+2)/(d*x^2+3))
^(1/2)/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*
d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1
/2)/d^(3/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+1/3*x*(b*x^2+2)^(1/
2)*(d*x^2+3)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {489, 545, 429, 506, 422}

$$-\frac{\sqrt{2}\sqrt{bx^2+2}F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{d^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{2\sqrt{2}(3b-d)\sqrt{bx^2+2}E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3bd^{3/2}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{2x(3b-d)\sqrt{bx^2+2}}{3bd\sqrt{dx^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2],x]
```

```
[Out] (-2*(3*b - d)*x*Sqrt[2 + b*x^2])/(3*b*d*Sqrt[3 + d*x^2]) + (x*Sqrt[2 + b*x^
2]*Sqrt[3 + d*x^2])/(3*d) + (2*Sqrt[2]*(3*b - d)*Sqrt[2 + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(3*b*d^(3/2)*Sqrt[(2 + b*x^2
)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(d^(3/2)*Sqrt[(2 + b*x^2)/(3 + d*x
^2)]*Sqrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{x\sqrt{2+bx^2}\,\sqrt{3+dx^2}}{3d} - \frac{\int \frac{6+2(3b-d)x^2}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx}{3d}$$

$$= \frac{x\sqrt{2+bx^2}\,\sqrt{3+dx^2}}{3d} - \frac{2\int \frac{1}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx}{d} - \frac{(2(3b-d))\int \frac{x^2}{\sqrt{2+bx^2}\,\sqrt{3+}}}{3d}$$

$$= -\frac{2(3b-d)x\sqrt{2+bx^2}}{3bd\sqrt{3+dx^2}} + \frac{x\sqrt{2+bx^2}\,\sqrt{3+dx^2}}{3d} - \frac{\sqrt{2}\,\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|\right.}{d^{3/2}\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}}$$

$$= -\frac{2(3b-d)x\sqrt{2+bx^2}}{3bd\sqrt{3+dx^2}} + \frac{x\sqrt{2+bx^2}\,\sqrt{3+dx^2}}{3d} + \frac{2\sqrt{2}\,(3b-d)\sqrt{2+bx^2}\,E\left(\tan^{-1}\right.}{3bd^{3/2}\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.82, size = 127, normalized size = 0.53

$$\frac{\sqrt{b}\,dx\sqrt{2+bx^2}\,\sqrt{3+dx^2} + 2i\sqrt{3}\,(3b-d)E\left(i\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}}\right)|\frac{2d}{3b}\right) - 2i\sqrt{3}\,(3b-2d)F\left(i\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)}{3\sqrt{b}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2] + (2*I)*Sqrt[3]*(3*b - d)*Elli
pticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] - (2*I)*Sqrt[3]*(3*b - 2
*d)*EllipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)])/(3*Sqrt[b]*d^2)
```

Maple [A]
time = 0.15, size = 306, normalized size = 1.27

method	result
risch	$\frac{x\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}{3d} - \frac{2\left(\frac{3\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3d}}{3}, \frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{2\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}} - \frac{(3b-d)\sqrt{3d}}{}\right.}{}$

elliptic	$\frac{\sqrt{(b\,x^2+2)\,(d\,x^2+3)}\left(\frac{x\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}{3d}-\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3d}}{3}\right.}{d\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\left(b^2d\,x^5\sqrt{-d}+3b^2x^3\sqrt{-d}+2bd\,x^3\sqrt{-d}+3\sqrt{2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)*(b^2*d*x^5*(-d)^(1/2)+3*b^2*x^3*(-d)^(1
/2)+2*b*d*x^3*(-d)^(1/2)+3*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2
^(1/2)*3^(1/2)*(b/d)^(1/2))*b*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)-2*2^(1/2)*Ell
ipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*d*(b*x^2+2
)^(1/2)*(d*x^2+3)^(1/2)-6*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^
(1/2)*3^(1/2)*(b/d)^(1/2))*b*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+2*2^(1/2)*Elli
pticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*d*(b*x^2+2)
^(1/2)*(d*x^2+3)^(1/2)+6*b*x*(-d)^(1/2))/(b*d*x^4+3*b*x^2+2*d*x^2+6)/d/(-d)
^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{b\,x^2+2}}{\sqrt{d\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b*x^2 + 2)^(1/2))/(d*x^2 + 3)^(1/2),x)
```

```
[Out] int((x^2*(b*x^2 + 2)^(1/2))/(d*x^2 + 3)^(1/2), x)
```

3.969 $\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=141

$$-\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2}+\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd}-\frac{(4abcd-3(bc+ad)^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}$$

```
[Out] -1/8*(4*a*b*c*d-3*(a*d+b*c)^2)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x
^2+c)^(1/2))/b^(5/2)/d^(5/2)-3/8*(a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/
b^2/d^2+1/4*x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {457, 92, 81, 65, 223, 212}

$$-\frac{(4abcd-3(ad+bc)^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad+bc)}{8b^2d^2}+\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b^2*d^2) + (x^2*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/(4*b*d) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} + \frac{\text{Subst}\left(\int \frac{-ac-\frac{3}{2}(bc+ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4bd}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} - \frac{(4abcd-3(bc}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} - \frac{(4abcd-3(bc}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} - \frac{(4abcd-3(bc}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} - \frac{(4abcd-3(bc}{}$$

Mathematica [A]
time = 1.29, size = 120, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(-3bc-3ad+2bdx^2)}{8b^2d^2} + \frac{(3b^2c^2+2abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c - 3*a*d + 2*b*d*x^2))/(8*b^2*d^2)
+ ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(S
qrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(115) = 230$.
time = 0.11, size = 291, normalized size = 2.06

method	result

risch	$-\frac{(-2bd\,x^2+3ad+3bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{8b^2d^2}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)a^2}{16b^2\sqrt{bd}}+\frac{\ln\left(\frac{1}{2}c \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\left(4\sqrt{bd}\,\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,bd\,x^2+3d^2\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2+2\ln\left(\frac{2bd\,x^2+2 \cdots}{\cdots}\right.\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\left(\frac{x^2\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{4bd}-\frac{3\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\,a}{8b^2d}-\frac{3\sqrt{bd\,x^4 \cdots}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(4*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*b*d*x^2+3*d^2*ln(1/2*(2*b*d
*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2+2*
ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^
(1/2))*a*c*b*d+3*b^2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^
(1/2)+a*d+b*c)/(b*d)^(1/2))*c^2-6*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*a
*d-6*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/(b*d)^(1/2)/d^2/b^2/((b*x^2+a)*(d*x^2+c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.11, size = 336, normalized size = 2.38

$$\left[\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)+4\,(2\,b^2d^2x^2-3\,b^2cd-3\,abd^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{32\,b^3d^3},\ -\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)-2\,(2\,b^2d^2x^2-3\,b^2cd-3\,abd^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{16\,b^3d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^
2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*
c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(2*b^2*d^2*x^2 - 3*
b^2*c*d - 3*a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^3), -1/16*((3*
b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a
*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^
2*c*d + a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(b^3*d^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [**A**]
time = 1.15, size = 160, normalized size = 1.13

$$\frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}\left(\frac{2\,(bx^2+a)}{b^3d}-\frac{3\,b^6cd+5\,ab^5d^2}{b^8d^3}\right)-\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,b^2d^2}\right)b}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(
b^3*d) - (3*b^6*c*d + 5*a*b^5*d^2)/(b^8*d^3)) - (3*b^2*c^2 + 2*a*b*c*d + 3*
a^2*d^2)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d
- a*b*d)))/(sqrt(b*d)*b^2*d^2))*b/abs(b)
```

Mupad [**B**]
time = 12.87, size = 550, normalized size = 3.90

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{4\,b^{5/2}\,d^{5/2}}-\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,\left(\frac{3\,a^2\,b\,d^2}{4}+\frac{a\,b^2\,c\,d}{2}+\frac{3\,b^3\,c^2}{4}\right)}{d^6\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^3\,\left(\frac{11\,a^2\,d^2}{4}+\frac{25\,a\,b\,c\,d}{2}+\frac{11\,b^2\,c^2}{4}\right)}{d^5\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^3}+\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^7\,\left(\frac{3\,a^2\,d^2}{4}+\frac{a\,b\,c\,d}{2}+\frac{3\,b^2\,c^2}{4}\right)}{b^2\,d^3\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^7}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^5\,\left(\frac{11\,a^2\,d^2}{4}+\frac{25\,a\,b\,c\,d}{2}+\frac{11\,b^2\,c^2}{4}\right)}{b\,d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^5}+\frac{\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4\,\left(16\,a\,d+16\,b\,c\right)}{d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}}{\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^8}{{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^8}+\frac{b^4}{d^4}-\frac{4\,b^3\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^2}{d^3\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^2}+\frac{6\,b^2\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4}{d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}-\frac{4\,b\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^6}{d\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (atanh((d^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2)
- c^(1/2))))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(4*b^(5/2)*d^(5/2)) - (((
```

```
(a + b*x^2)^(1/2) - a^(1/2))*((3*b^3*c^2)/4 + (3*a^2*b*d^2)/4 + (a*b^2*c*d)
/2))/(d^6*((c + d*x^2)^(1/2) - c^(1/2))) - (((a + b*x^2)^(1/2) - a^(1/2))^3
*((11*a^2*d^2)/4 + (11*b^2*c^2)/4 + (25*a*b*c*d)/2))/(d^5*((c + d*x^2)^(1/2
) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))^7*((3*a^2*d^2)/4 + (3*b^2*
c^2)/4 + (a*b*c*d)/2))/(b^2*d^3*((c + d*x^2)^(1/2) - c^(1/2))^7) - (((a + b
*x^2)^(1/2) - a^(1/2))^5*((11*a^2*d^2)/4 + (11*b^2*c^2)/4 + (25*a*b*c*d)/2)
)/(b*d^4*((c + d*x^2)^(1/2) - c^(1/2))^5) + (a^(1/2)*c^(1/2)*((a + b*x^2)^(
1/2) - a^(1/2))^4*(16*a*d + 16*b*c))/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^4))
/(((a + b*x^2)^(1/2) - a^(1/2))^8/((c + d*x^2)^(1/2) - c^(1/2))^8 + b^4/d^4
 - (4*b^3*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d^3*((c + d*x^2)^(1/2) - c^(1/2
))^2) + (6*b^2*((a + b*x^2)^(1/2) - a^(1/2))^4)/(d^2*((c + d*x^2)^(1/2) - c
^(1/2))^4) - (4*b*((a + b*x^2)^(1/2) - a^(1/2))^6)/(d*((c + d*x^2)^(1/2) -
c^(1/2))^6))
```

3.970 $\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

```
[Out] -1/2*(a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(
3/2)/d^(3/2)+1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d
```

Rubi [A]

time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {457, 81, 65, 223, 212}

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(3/2)*d^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4bd}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{2b^2d}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2b^2d}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.87, size = 88, normalized size = 1.00

$$
\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(3/2)*d^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(68) = 136$.
time = 0.12, size = 172, normalized size = 1.95

method	result
default	$-\frac{\left(a\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)d+b\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right.\right.}{4d\sqrt{bd}\,b\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}$
risch	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2bd}+\frac{\left(-\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)a}{4b\sqrt{bd}}-\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\right.}{4c}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}}{2bd}-\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+\right.}{4b\sqrt{bd}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c
)/(b*d)^(1/2))*d+b*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1
/2)+a*d+b*c)/(b*d)^(1/2))*c-2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2))*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/(b*d)^(1/2)/b/((b*x^2+a)*(d*x^2+c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.11, size = 256, normalized size = 2.91

$$\left[\frac{4\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,bd+(bc+ad)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2-4\,(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)}{8\,b^2d^2}, \frac{2\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,bd+(bc+ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)}{4\,b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*
(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)))/(b^2*d^
2), 1/4*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c + a*d)*sqrt(-b*d)*arc
tan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/
(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.98, size = 104, normalized size = 1.18

$$\frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}}{bd}}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((b*c + a*d)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sq
rt(b*x^2 + a)/(b*d))/abs(b)
```

Mupad [B]
time = 2.90, size = 279, normalized size = 3.17

$$\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,\left(a\,d+b\,c\right)}{d^3\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}+\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^3\,\left(a\,d+b\,c\right)}{b\,d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^3}-\frac{4\,\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^2}{d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^2}}{\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4}{{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}+\frac{b^2}{d^2}-\frac{2\,b\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^2}{d\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,\left(a\,d+b\,c\right)}{b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((((a + b*x^2)^(1/2) - a^(1/2))*(a*d + b*c))/(d^3*((c + d*x^2)^(1/2) - c^(1
/2))) + (((a + b*x^2)^(1/2) - a^(1/2))^3*(a*d + b*c))/(b*d^2*((c + d*x^2)^(
1/2) - c^(1/2))^3) - (4*a^(1/2)*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d
^2*((c + d*x^2)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^4/((c +
 d*x^2)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x^2)^(1/2) - a^(1/2))^2
)/(d*((c + d*x^2)^(1/2) - c^(1/2))^2)) - (atanh((d^(1/2)*((a + b*x^2)^(1/2)
 - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))))*(a*d + b*c))/(b^(3/2)
*d^(3/2))
```

3.971 $\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=45

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {455, 65, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt[
d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [A]
time = 0.38, size = 45, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt[
d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(33) = 66$.
time = 0.11, size = 89, normalized size = 1.98

method	result	size
default	$\frac{\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2\sqrt{bd}\,\sqrt{(b\,x^2+a)(d\,x^2+c)}}$	89

elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^{2}}{\sqrt{bd}}+\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+ac}\right)}{2\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\sqrt{bd}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b
*d)^(1/2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/((b*x^2+a)*(d*x^2+c)
)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(33) = 66$.
time = 0.98, size = 194, normalized size = 4.31

$$\left[\frac{\sqrt{bd}\log\left(8\,b^{2}d^{2}x^{4}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+8\,(b^{2}cd+abd^{2})x^{2}+4\,(2\,bdx^{2}+bc+ad)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{bd}\right)}{4\,bd},\ -\frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx^{2}+bc+ad)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{-bd}}{2\,(b^{2}d^{2}x^{4}+abcd+(b^{2}cd+abd^{2})x^{2})}\right)}{2\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*sqrt(b*d))/(b*d), -1/2*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x^2))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^{2}}\,\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 1.17, size = 54, normalized size = 1.20

$$-\frac{b\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}\,+\sqrt{b^2c+(bx^2+a)bd-abd}\,\right|\right)}{\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -b*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*
d)))/(sqrt(b*d)*abs(b))
```

Mupad [B]
time = 0.70, size = 49, normalized size = 1.09

$$-\frac{2\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -(2*atan((b*((c + d*x^2)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^2)^(1/2)
 - a^(1/2)))))/(-b*d)^(1/2)
```

3.972 $\int \frac{1}{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{c}}$$

```
[Out] -arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {457, 95, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqr
t[c]))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

Mathematica [A]
time = 0.56, size = 46, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqr
t[c]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(34) = 68$.
time = 0.11, size = 89, normalized size = 1.93

method	result	size
default	$-\frac{\ln\left(\frac{a d\,x^2+c\,x^2 b+2\sqrt{a c}\,\sqrt{(b\,x^2+a)\,(d\,x^2+c)}+2 a c}{x^2}\right)\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{2\sqrt{a c}\,\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}$	89
elliptic	$-\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\ln\left(\frac{2 a c+(a d+b c)x^2+2\sqrt{a c}\,\sqrt{b d\,x^4+(a d+b c)\,x^2+a c}}{x^2}\right)}{2\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{a c}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x
^2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(a*c)^(1/2)/((b*x^2+a)*(d*x^2+c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(34) = 68.
time = 1.09, size = 204, normalized size = 4.43

$$\left[\frac{\sqrt{ac}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{ac}}{x^4}\right)}{4\,ac}, \frac{\sqrt{-ac}\,\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-ac}}{2\,(abcdx^4+a^2c^2+(abc^2+a^2cd)x^2)}\right)}{2\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x
^2 + c)*sqrt(a*c))/x^4)/(a*c), 1/2*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^2 +
 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 + a^2*c^2 +
 (a*b*c^2 + a^2*c*d)*x^2))/(a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(34) = 68.
time = 1.20, size = 89, normalized size = 1.93

$$-\frac{\sqrt{bd}\,b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt
(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a
bs(b))
```

Mupad [B]
time = 1.97, size = 136, normalized size = 2.96

$$-\frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)-\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{2\,\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -(log(((a + b*x^2)^(1/2) - a^(1/2))/((c + d*x^2)^(1/2) - c^(1/2))) - log(((
c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) - (a^(1/2
)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/((c + d*
x^2)^(1/2) - c^(1/2))))/(2*a^(1/2)*c^(1/2))
```

3.973 $\int \frac{1}{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2acx^2}+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}$$

```
[Out] 1/2*(a*d+b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3
/2)/c^(3/2)-1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {457, 98, 95, 214}

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^2) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*c^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}-\frac{(bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)}{4ac}\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}-\frac{(bc+ad)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2ac}\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.83, size = 91, normalized size = 1.00

$$
-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^2) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*c^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(71) = 142.
time = 0.12, size = 181, normalized size = 1.99

method	result

default	$\frac{\left(\ln\left(\frac{a d x^2+c x^2 b+2\sqrt{ac}\,\sqrt{(b x^2+a)(d x^2+c)}+2ac}{x^2}\right) a d x^2+\ln\left(\frac{a d x^2+c x^2 b+2\sqrt{ac}\,\sqrt{(b x^2+a)(d x^2+c)}+2a}{x^2}\right.\right.}{4ac\sqrt{ac}\,x^2\sqrt{(b x^2+a)(d x^2+c)}}$
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}{2c x^2 a}+\frac{\left(\frac{\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd x^4+(ad+bc)x^2+ac}}{x^2}\right)d}{4c\sqrt{ac}}+\frac{\ln\left(\frac{2ac+(ad+bc)x^2+2\right.}{}\right.}{\sqrt{b x^2+a}\,\sqrt{d x}}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{\sqrt{bd x^4+(ad+bc)x^2+ac}}{2ac x^2}+\frac{\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd x^4+(ad+bc)x}}{x^2}\right.}{4c\sqrt{ac}}\right.}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a/c*(ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*
c)/x^2)*a*d*x^2+ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/
2)+2*a*c)/x^2)*b*c*x^2-2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2))*(d*x^2+c)
^(1/2)*(b*x^2+a)^(1/2)/(a*c)^(1/2)/x^2/((b*x^2+a)*(d*x^2+c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 2.27, size = 278, normalized size = 3.05

$$\left[\frac{\sqrt{ac}\,(bc+ad)x^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2+4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{ac}}{x^4}\right)-4\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,ac}{8\,a^2c^2x^2},\ -\frac{\sqrt{-ac}\,(bc+ad)x^2\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-ac}}{2\,(abcdx^4+a^2c^2+(abc^2+a^2cd)x^2)}\right)+2\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,ac}{4\,a^2c^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c)*(b*c + a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c
)*a*c)/(a^2*c^2*x^2), -1/4*(sqrt(-a*c)*(b*c + a*d)*x^2*arctan(1/2*((b*c + a
*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 +
a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) + 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c
)/(a^2*c^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(71) = 142.
time = 1.09, size = 413, normalized size = 4.54

$$\frac{\sqrt{bd}\,b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,ab^3cd}-\frac{2\left(b^3c^2-2ab^2cd+a^2bd^2-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2bc-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2ad\right)}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2abd+\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4\right)ab^2cd}\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b
))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (sqr
t(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b*c - (sq
rt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*d)/((b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

Mupad [B]
time = 3.70, size = 481, normalized size = 5.29

$$\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{c\,b^2}{8}+\frac{a\,d\,b}{8}\right)}{a^{3/2}\,c^{3/2}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{b^2}{8\,a\,c\,d}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{8}-\frac{3\,a\,b\,c\,d}{8}+\frac{b^2\,c^2}{8}\right)}{a^2\,c^2\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}+\frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{4\,a^2\,c^2}-\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{4\,a^2\,c^2}-\frac{d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{8\,a\,c\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((((a + b*x^2)^(1/2) - a^(1/2))*((b^2*c)/8 + (a*b*d)/8))/(a^(3/2)*c^(3/2)*d
*((c + d*x^2)^(1/2) - c^(1/2))) - b^2/(8*a*c*d) + (((a + b*x^2)^(1/2) - a^(
1/2))^2*((a^2*d^2)/8 + (b^2*c^2)/8 - (3*a*b*c*d)/8))/(a^2*c^2*d*((c + d*x^2
)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^3/((c + d*x^2)^(1/2)
- c^(1/2))^3 + (b*((a + b*x^2)^(1/2) - a^(1/2)))/(d*((c + d*x^2)^(1/2) - c^
(1/2))) - (((a + b*x^2)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*
((c + d*x^2)^(1/2) - c^(1/2))^2)) + (log(((a + b*x^2)^(1/2) - a^(1/2))/((c
+ d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))/(4*a^2*
c^2) - (log(((c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(
1/2) - (a^(1/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/
2))))/((c + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d
))/(4*a^2*c^2) - (d*((a + b*x^2)^(1/2) - a^(1/2)))/(8*a*c*((c + d*x^2)^(1/2
) - c^(1/2)))
```

3.974 $\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4}+\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2}-\frac{(3b^2c^2+2abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}$$

```
[Out] -1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2
)/(d*x^2+c)^(1/2))/a^(5/2)/c^(5/2)-1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/
x^4+3/8*(a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {457, 105, 156, 12, 95, 214}

$$\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{8a^2c^2x^2}-\frac{(3a^2d^2+2abcd+3b^2c^2)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/4*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^4) + (3*(b*c + a*d)*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(8*a^2*c^2*x^2) - ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d
^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2
)*c^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(bc+ad)+bdx}{x^2\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4ac}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2} + \frac{\text{Subst}\left(\int \frac{3b}{4x\sqrt{}}\right.}{}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2} + \frac{(3b^2c^2+2abc}{}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2} + \frac{(3b^2c^2+2abc}{}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2} - \frac{(3b^2c^2+2abc}{}$$

Mathematica [A]
time = 1.39, size = 126, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(-2ac+3bcx^2+3adx^2)}{8a^2c^2x^4} - \frac{(3b^2c^2+2abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c + 3*b*c*x^2 + 3*a*d*x^2))/(8*a^2*c
^2*x^4) - ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*
x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(123) = 246$.
time = 0.12, size = 306, normalized size = 2.05

method	result

risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,(-3ad\,x^{2}-3c\,x^{2}b+2ac)}{8a^{2}c^{2}x^{4}}+\frac{\left(-\frac{3\ln\left(\frac{2ac+(ad+bc)x^{2}+2\sqrt{ac}\,\sqrt{bd\,x^{4}+(ad+bc)\,x^{2}+ac}}{x^{2}}\right.}{16c^{2}\sqrt{ac}}\right.}{}$
default	$-\frac{\left(3\ln\left(\frac{ad\,x^{2}+c\,x^{2}b+2\sqrt{ac}\,\sqrt{(b\,x^{2}+a)\,(d\,x^{2}+c)}+2ac}{x^{2}}\right)a^{2}d^{2}x^{4}+2\ln\left(\frac{ad\,x^{2}+c\,x^{2}b+2\sqrt{ac}\,\sqrt{(b\,x^{2}+a)\,(d\,x^{2}+}{x^{2}}\right.\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)\,(d\,x^{2}+c)}\left(-\frac{\sqrt{bd\,x^{4}+(ad+bc)\,x^{2}+ac}}{4ac\,x^{4}}+\frac{3\sqrt{bd\,x^{4}+(ad+bc)\,x^{2}+ac}\,d}{8a\,c^{2}x^{2}}+\frac{3\sqrt{bd\,x^{4}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/a^2/c^2*(3*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1
/2)+2*a*c)/x^2)*a^2*d^2*x^4+2*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2)*((b*x^2+a)*
(d*x^2+c))^(1/2)+2*a*c)/x^2)*a*b*c*d*x^4+3*ln((a*d*x^2+c*x^2*b+2*(a*c)^(1/2
)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/x^2)*b^2*c^2*x^4-6*d*((b*x^2+a)*(d*x^2
+c))^(1/2)*x^2*a*(a*c)^(1/2)-6*((b*x^2+a)*(d*x^2+c))^(1/2)*b*c*x^2*(a*c)^(1
/2)+4*((b*x^2+a)*(d*x^2+c))^(1/2)*a*c*(a*c)^(1/2))*(d*x^2+c)^(1/2)*(b*x^2+a
)^(1/2)/(a*c)^(1/2)/x^4/((b*x^2+a)*(d*x^2+c))^(1/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 1.57, size = 360, normalized size = 2.42

$$\left[\frac{(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2})\sqrt{ac}\,x^{4}\log\left(\frac{(b^{2}c^{2}+6\,abcd+a^{2}d^{2})x^{4}+8\,a^{2}c^{2}+8\,(abc^{2}+a^{2}cd)x^{2}-4\,((bc+ad)x^{2}+2\,ac)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{ac}}{x^{4}}\right)-4\,(2\,a^{2}c^{2}-3\,(abc^{2}+a^{2}cd)x^{2})\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{32\,a^{3}c^{3}x^{4}},\ \frac{(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2})\sqrt{-ac}\,x^{4}\arctan\left(\frac{((bc+ad)x^{2}+2\,ac)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{-ac}}{2\,(abcdx^{4}+a^{2}c^{2}+(abc^{2}+a^{2}cd)x^{2})}\right)-2\,(2\,a^{2}c^{2}-3\,(abc^{2}+a^{2}cd)x^{2})\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{16\,a^{3}c^{3}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [1/32*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*(2*a^2*c^2 - 3*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^3*x^4), 1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*(2*a^2*c^2 - 3*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^3*x^4)]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(1/(x**5*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(123) = 246$.
time = 1.06, size = 1015, normalized size = 6.81

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] -1/8*sqrt(b*d)*b^6*d^2*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a^2*b^5*c^2*d^2) - 2*(3*b^8*c^5 - 9*a*b^7*c^4*d + 6*a^2*b^6*c^3*d^2 + 6*a^3*b^5*c^2*d^3 - 9*a^4*b^4*c*d^4 + 3*a^5*b^3*d^5 - 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^6*c^4 - 4*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^5*c^3*d + 26*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^4*c^2*d^2 - 4*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^3*b^3*c*d^3 - 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^2*d^4 + 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^4*c^3 + 15*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^3*c^2*d + 15*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +

```
a)*b*d - a*b*d))^4*a^2*b^2*c*d^2 + 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b*d^3 - 3*(sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*c^2 - 2*(sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a*b*c*d - 3*(sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a^2*d^2)/((b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (
b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b
^2*c + (b*x^2 + a)*b*d - a*b*d))^4)^2*a^2*b^4*c^2*d^2))/abs(b)
```

Mupad [B]
time = 11.81, size = 962, normalized size = 6.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (log(((c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) -
(a^(1/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/(
(c + d*x^2)^(1/2) - c^(1/2)))*(3*a^(1/2)*b^2*c^(5/2) + 3*a^(5/2)*c^(1/2)*d^
2 + 2*a^(3/2)*b*c^(3/2)*d))/(16*a^3*c^3) - (log(((a + b*x^2)^(1/2) - a^(1/2
))/((c + d*x^2)^(1/2) - c^(1/2)))*(3*a^(1/2)*b^2*c^(5/2) + 3*a^(5/2)*c^(1/2
)*d^2 + 2*a^(3/2)*b*c^(3/2)*d))/(16*a^3*c^3) - ((((a + b*x^2)^(1/2) - a^(1/
2))^2*((11*b^4*c^2)/64 + (11*a^2*b^2*d^2)/64 + (5*a*b^3*c*d)/16))/(a^(5/2)*
c^(5/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^2) - b^4/(64*a^(3/2)*c^(3/2)*d^2)
 + (((a + b*x^2)^(1/2) - a^(1/2))^3*((b^4*c^3)/32 + (a^3*b*d^3)/32 - (9*a^2
*b^2*c*d^2)/16 - (9*a*b^3*c^2*d)/16))/(a^3*c^3*d^2*((c + d*x^2)^(1/2) - c^(
1/2))^3) - (((a + b*x^2)^(1/2) - a^(1/2))*((b^4*c)/16 + (a*b^3*d)/16))/(a^2
*c^2*d^2*((c + d*x^2)^(1/2) - c^(1/2))) + (((a + b*x^2)^(1/2) - a^(1/2))^5*
((a^3*d^3)/8 + (b^3*c^3)/8 - (7*a*b^2*c^2*d)/32 - (7*a^2*b*c*d^2)/32))/(a^3
*c^3*d*((c + d*x^2)^(1/2) - c^(1/2))^5) + (((a + b*x^2)^(1/2) - a^(1/2))^4*
((45*a^2*b^2*c^2*d^2)/64 - (7*b^4*c^4)/64 - (7*a^4*d^4)/64 + (a*b^3*c^3*d)/
8 + (a^3*b*c*d^3)/8))/(a^(7/2)*c^(7/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^4)
)/(((a + b*x^2)^(1/2) - a^(1/2))^6/((c + d*x^2)^(1/2) - c^(1/2))^6 + (b^2*(
(a + b*x^2)^(1/2) - a^(1/2))^2)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^2) - (((
a + b*x^2)^(1/2) - a^(1/2))^3*(2*b^2*c + 2*a*b*d))/(a^(1/2)*c^(1/2)*d^2*((c
 + d*x^2)^(1/2) - c^(1/2))^3) - (((a + b*x^2)^(1/2) - a^(1/2))^5*(2*a*d + 2
*b*c))/(a^(1/2)*c^(1/2)*d*((c + d*x^2)^(1/2) - c^(1/2))^5) + (((a + b*x^2)^
(1/2) - a^(1/2))^4*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/(a*c*d^2*((c + d*x^2)^(
1/2) - c^(1/2))^4)) + (d^2*((a + b*x^2)^(1/2) - a^(1/2))^2)/(64*a^(3/2)*c^(
3/2)*((c + d*x^2)^(1/2) - c^(1/2))^2) + (3*d*((a + b*x^2)^(1/2) - a^(1/2))*
(a*d + b*c))/(32*a^2*c^2*((c + d*x^2)^(1/2) - c^(1/2)))
```

3.975 $\int \frac{x^6}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=342

$$\frac{(8b^2c^2+7abcd+8a^2d^2)\,x\sqrt{a+bx^2}}{15b^3d^2\sqrt{c+dx^2}}-\frac{4(bc+ad)x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{15b^2d^2}+\frac{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5bd}-\frac{\sqrt{c}\,(8b^2}{}$$

```
[Out] 1/15*(8*a^2*d^2+7*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^3/d^2/(d*x^2+c)^(1
/2)+4/15*c^(3/2)*(a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
F(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/b^
2/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(8*a^2*d^2+7
*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(
1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b
^3/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-4/15*(a*d+b*c)*x
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2+1/5*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)/b/d
```

Rubi [A]
time = 0.21, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {490, 596, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,(8a^2d^2+7abcd+8b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\,(8a^2d^2+7abcd+8b^2c^2)}{15b^3d^2\sqrt{c+dx^2}}+\frac{4c^{3/2}\sqrt{a+bx^2}\,(ad+bc)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4x\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(ad+bc)}{15b^2d^2}+\frac{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^3*d^2*Sqrt[c
+ d*x^2]) - (4*(b*c + a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b^2*d^2)
+ (x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (Sqrt[c]*(8*b^2*c^2 + 7*a
*b*c*d + 8*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*b^3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[c + d*x^2]) + (4*c^(3/2)*(b*c + a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/2)*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} - \frac{\int \frac{x^2(3ac+4(bc+ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5bd}$$

$$= -\frac{4(bc+ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} + \frac{\int \frac{4ac(bc+ad)+(}{\sqrt{a+}}}{}$$

$$= -\frac{4(bc+ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} + \frac{(4ac(bc+ad}{}$$

$$= \frac{(8b^2c^2+7abcd+8a^2d^2)\,x\sqrt{a+bx^2}}{15b^3d^2\sqrt{c+dx^2}} - \frac{4(bc+ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2} + \frac{x}{}$$

$$= \frac{(8b^2c^2+7abcd+8a^2d^2)\,x\sqrt{a+bx^2}}{15b^3d^2\sqrt{c+dx^2}} - \frac{4(bc+ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2} + \frac{x}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.15, size = 249, normalized size = 0.73

$$\frac{-\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2)(4bc+4ad-3bdx^2) - ic(8b^2c^2+7abcd+8a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right) + ic(8b^2c^2+3abcd+4a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)}{15a^2\left(\frac{b}{a}\right)^{5/2}d^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*b*c + 4*a*d - 3*b*d*x^2)) - I*c
*(8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(8*b^2*c^2 + 3*a*b*c
*d + 4*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^3*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])
```

Maple [A]
time = 0.12, size = 546, normalized size = 1.60

method	result

elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{x^3\sqrt{bdx^4+adx^2+cx^2b+ac}}{5db}-\frac{(4ad+4bc)x\sqrt{bdx^4+adx^2+cx^2b+ac}}{15d^2b^2}+\frac{(4a\cdots}{\cdots}\cdots\right.}{\cdots}$
risch	$-\frac{x(-3bdx^2+4ad+4bc)\sqrt{bx^2+a}\sqrt{dx^2+c}}{15b^2d^2}+\frac{\left(-\frac{(8a^2d^2+7abcd+8b^2c^2)c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdx^4+a\cdots}}\cdots\right.}{\cdots}$
default	$-\frac{\left(-3\sqrt{-\frac{b}{a}}b^2d^3x^7+\sqrt{-\frac{b}{a}}abd^3x^5+\sqrt{-\frac{b}{a}}b^2cd^2x^5+4\sqrt{-\frac{b}{a}}a^2d^3x^3+5\sqrt{-\frac{b}{a}}abcd^2x^3+4\sqrt{-\frac{b}{a}}b^2c^2dx^3+4\sqrt{\frac{bx^2}{\cdots}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-3*(-b/a)^(1/2)*b^2*d^3*x^7+(-b/a)^(1/2)*a*b*d^3*x^5+(-b/a)^(1/2)*b^
2*c*d^2*x^5+4*(-b/a)^(1/2)*a^2*d^3*x^3+5*(-b/a)^(1/2)*a*b*c*d^2*x^3+4*(-b/a
)^(1/2)*b^2*c^2*d*x^3+4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+8*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^
3-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*a^2*c*d^2-7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+4*(-b/a)^(1/2)*a^2
*c*d^2*x+4*(-b/a)^(1/2)*a*b*c^2*d*x)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3/b^
2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**6/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.976 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=261

$$-\frac{2(bc+ad)x\sqrt{a+bx^2}}{3b^2d\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}+\frac{2\sqrt{c}\,(bc+ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{c^{3/}}{}$$

```
[Out] -2/3*(a*d+b*c)*x*(b*x^2+a)^(1/2)/b^2/d/(d*x^2+c)^(1/2)-1/3*c^(3/2)*(1/(1+d*
x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)+2/3*(a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b
*x^2+a)^(1/2)/b^2/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1
/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {490, 545, 429, 506, 422}

$$\frac{2\sqrt{c}\sqrt{a+bx^2}\,(ad+bc)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2x\sqrt{a+bx^2}\,(ad+bc)}{3b^2d\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b^2*d*Sqrt[c + d*x^2]) + (x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(3*b*d) + (2*Sqrt[c]*(b*c + a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*d^(3/2)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd} - \frac{\int \frac{ac+2(bc+ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3bd}$$

$$= \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd} - \frac{(ac)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3bd} - \frac{(2(bc+ad))\int \frac{}{\sqrt{a}}}{3}$$

$$= -\frac{2(bc+ad)x\sqrt{a+bx^2}}{3b^2d\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd} - \frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\right.\right.}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$= -\frac{2(bc+ad)x\sqrt{a+bx^2}}{3b^2d\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd} + \frac{2\sqrt{c}\,(bc+ad)\sqrt{a+bx^2}}{3b^2d^{3/2}\sqrt{\frac{c(a-}{a(c+}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.55, size = 201, normalized size = 0.77

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2)+2ic(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic(2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3b\sqrt{\frac{b}{a}}\,d^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(b*c + a*d)*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
I*c*(2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

Maple [A]
time = 0.12, size = 333, normalized size = 1.28

method	result
risch	$\frac{x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3bd} - \frac{\left(-\frac{(2ad+2bc)c\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{EllipticE}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}\;d}\right.}{}$

elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{3bd}-\frac{ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\right.}{3bd\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b}}\right.}{\sqrt{b}}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3+ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+2\sqrt{\frac{b\,x^2+a}{a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-b/a)^(1/2)*b*d^2*x^5+(-b/a)^(1/2)*a*d^2*x^3+(-b/a)^(1/2)*b*c*d*x^3+a
*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*b*c^2-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*a*c*d-2*EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*b*c^2+(-b/a)^(1/2)*a*c*d
*x)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-b/a)^(1/2)/d^2/b/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.977 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=116

$$\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

```
[Out] x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(
b*x^2+a)^(1/2)/b/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {506, 422}

$$\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{c+dx^2}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{c\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{b}$$

$$= \frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\,\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{c+dx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.76, size = 122, normalized size = 1.05

$$-\frac{ic\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}\,d\sqrt{a+bx^2}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sq
rt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.10, size = 129, normalized size = 1.11

method	result
default	$\frac{\left(-\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)+\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)\right)\sqrt{\frac{d x^2+c}{c}}\,\sqrt{\frac{b x^2+a}{a}}\,c\sqrt{b x^2+a}\,\sqrt{d x^2+c}}{d\sqrt{-\frac{b}{a}}\,(bd x^4+ad x^2+c x^2 b+ac)}$
elliptic	$-\frac{\sqrt{(b x^2+a)(d x^2+c)}\,c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}}\right.\right.}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+c x^2 b+ac}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] (-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*c*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.978 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=153

$$\frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] d*x*(b*x^2+a)^(1/2)/a/c/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(
1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/
2)*(b*x^2+a)^(1/2)/a/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2
)-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x
```

Rubi [A]
time = 0.06, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {491, 12, 506, 422}

$$-\frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(a*c*Sqrt[c + d*x^2]) - (Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])/(a*c*x) - (Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}+\frac{\int \frac{bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ac}\\ &= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}+\frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ac}\\ &= \frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{d\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{a}\\ &= \frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}-\frac{\sqrt{d}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1\right.}{a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.22, size = 146, normalized size = 0.95

$$\frac{-\frac{(a+bx^2)(c+dx^2)}{cx}-ia\sqrt{\frac{b}{a}}\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

[Out] (-(((a + b*x^2)*(c + d*x^2))/(c*x)) - I*a*Sqrt[b/a]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
)

Maple [A]
time = 0.12, size = 224, normalized size = 1.46

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{acx}-\frac{b\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{\cdots}\right.\right.}{a\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}\,\sqrt{b\,x\cdots}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\left(-\frac{\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}{acx}-\frac{b\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\cdots\right.\right.}{a\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x\cdots}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,bd\,x^4-bc\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,x\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,x\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}}\cdots\right.\right.}{xc\sqrt{-\frac{b}{a}}\,a(bd\,x^4+ad\,x^2+c\,x^2b+ac)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(-b/a)^(1/2)*b*d*x^4-b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-(-b/a)^(1/2)*a*d*x^2-(-b/a)^(1/2)*b*c*x^2-(-b/a)^(1/2)*a*c)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/x/c/(-b/a)^(1/2)/a/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.979 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=307

$$-\frac{2d(bc+ad)x\sqrt{a+bx^2}}{3a^2c^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}+\frac{2(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x}+\frac{2\sqrt{d}(bc+ad)\sqrt{a+bx^2}\ldots}{3a^2c^{3/2}\sqrt{\frac{c(\ldots}{a(\ldots}}}$$

```
[Out] -2/3*d*(a*d+b*c)*x*(b*x^2+a)^(1/2)/a^2/c^2/(d*x^2+c)^(1/2)+2/3*(a*d+b*c)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/c^(3/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c
)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^
(1/2)*(b*x^2+a)^(1/2)/a^2/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)
^(1/2)-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^3+2/3*(a*d+b*c)*(b*x^2+a)^
(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x
```

Rubi [A]

time = 0.18, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {491, 597, 545, 429, 506, 422}

$$\frac{2\sqrt{d}\sqrt{a+bx^2}(ad+bc)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2c^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{d}\sqrt{a+bx^2}F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3a^2c^2x}-\frac{2dx\sqrt{a+bx^2}(ad+bc)}{3a^2c^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-2*d*(b*c + a*d)*x*Sqrt[a + b*x^2])/(3*a^2*c^2*Sqrt[c + d*x^2]) - (Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(3*a*c*x^3) + (2*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(3*a^2*c^2*x) + (2*Sqrt[d]*(b*c + a*d)*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*c^(3/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[d]*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3} + \frac{\int \frac{-2(bc+ad)-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ac}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3} + \frac{2(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x} - \frac{\int \frac{abcd+2bd(b\cdots}{\sqrt{a+bx^2}\,\cdots}}{3a^2\cdots}$$

$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3} + \frac{2(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x} - \frac{(bd)\int \frac{\cdots}{\sqrt{a+b\cdots}}}{\cdots}$$

$$= -\frac{2d(bc+ad)x\sqrt{a+bx^2}}{3a^2c^2\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3} + \frac{2(bc+ad)\sqrt{a+bx^2}\,\cdots}{3a^2c^2x}$$

$$= -\frac{2d(bc+ad)x\sqrt{a+bx^2}}{3a^2c^2\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3} + \frac{2(bc+ad)\sqrt{a+bx^2}\,\cdots}{3a^2c^2x}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.80, size = 229, normalized size = 0.75

$$\frac{\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-ac+2bcx^2+2adx^2)+2ibc(bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ibc(2bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{3a^2\sqrt{\frac{b}{a}}\,c^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-(a*c) + 2*b*c*x^2 + 2*a*d*x^2) + (2*I)
*b*c*(b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*
a^2*Sqrt[b/a]*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.13, size = 435, normalized size = 1.42

method	result

risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,(-2ad x^2-2c x^2 b+ac)}{3a^2c^2x^3}-\frac{bd\left(-\frac{(2ad+2bc)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x}}\right.}{}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{\sqrt{bd x^4+ad x^2+c x^2b+ac}}{3ac x^3}+\frac{2(ad+bc)\sqrt{bd x^4+ad x^2+c x^2b+ac}}{3a^2c^2x}-\frac{bd\sqrt{}}{}\right.}{}$
default	$\frac{\left(2\sqrt{-\frac{b}{a}}\,ab d^2x^6+2\sqrt{-\frac{b}{a}}\,b^2cd x^6+bd\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac+2\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*(-b/a)^(1/2)*a*b*d^2*x^6+2*(-b/a)^(1/2)*b^2*c*d*x^6+b*d*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a
*c+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*b^2*c^2*x^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3+2*(-b/a)^
(1/2)*a^2*d^2*x^4+3*(-b/a)^(1/2)*a*b*c*d*x^4+2*(-b/a)^(1/2)*b^2*c^2*x^4+(-b
/a)^(1/2)*a^2*c*d*x^2+(-b/a)^(1/2)*a*b*c^2*x^2-(-b/a)^(1/2)*a^2*c^2)*(d*x^2
+c)^(1/2)*(b*x^2+a)^(1/2)/x^3/c^2/(-b/a)^(1/2)/a^2/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.980 $\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=129

$$-\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2b^2d}-\frac{(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}$$

```
[Out] -1/2*(3*a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b
^(5/2)/d^(3/2)-a^2*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^2+a)^(1/2)+1/2*(b*x^
2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 91, 81, 65, 223, 212}

$$-\frac{a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}\,(bc-ad)}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*Sqrt[a + b*x^2])) + (Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(2*b^2*d) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(5/2)*d^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(bc-ad)+\frac{1}{2}b(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)}{b^2(bc-ad)}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d} - \frac{(bc+3ad)\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.}{4b}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d} - \frac{(bc+3ad)\text{Subst}\left(\int \frac{}{\sqrt{c-}}\right.}{}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d} - \frac{(bc+3ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\right.}{2b^3d}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d} - \frac{(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{b}}\right.}{2b^{5/2}d^{3/2}}$$

Mathematica [A]
time = 1.89, size = 153, normalized size = 1.19

$$\frac{\sqrt{b}\sqrt{d}\sqrt{c+dx^2}\,(-3a^2d+b^2cx^2+ab(c-dx^2)) - (b^2c^2+2abcd-3a^2d^2)\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{2b^{5/2}d^{3/2}(bc-ad)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b]*Sqrt[d]*Sqrt[c + d*x^2]*(-3*a^2*d + b^2*c*x^2 + a*b*(c - d*x^2)) -
 (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(2*b^(5/2)*d^(3/2)*(b*c - a*d)*Sqrt[a
 + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(105) = 210$.
time = 0.16, size = 511, normalized size = 3.96

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{2b^2d}-\frac{3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+\cdots}\right.}{4b^2\sqrt{bd}}\right.}{\sqrt{b x^2+a}\cdots}$
risch	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}{2b^2d}+\frac{\left(-\frac{3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)a}{4b^2\sqrt{bd}}-\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bc\cdots}\right.}{4\cdots}\right.}{\cdots}$
default	$-\frac{\left(3\ln\left(\frac{2bd\,x^2+2\sqrt{(b x^2+a)(d x^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2bd^2x^2-2\ln\left(\frac{2bd\,x^2+2\sqrt{(b x^2+a)(d x^2+c)}\,\sqrt{bd}\cdots}{2\sqrt{bd}}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c
)/(b*d)^(1/2))*a^2*b*d^2*x^2-2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1
/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^2*c*d*x^2-ln(1/2*(2*b*d*x^2+2*((b
*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^3*c^2*x^2-2*((
b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)*a*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/
2)*(b*d)^(1/2)*b^2*c*x^2+3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*
(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*d^2-2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*
(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*b*c*d-ln(1/2*(2*b*d*
x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^2*c
^2-6*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*a^2*d+2*((b*x^2+a)*(d*x^2+c))^
(1/2)*(b*d)^(1/2)*a*b*c)*(d*x^2+c)^(1/2)/b^2/(b*x^2+a)^(1/2)/d/(b*d)^(1/2)/
(a*d-b*c)/((b*x^2+a)*(d*x^2+c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(105) = 210$.
time = 1.00, size = 498, normalized size = 3.86

$$\left[\frac{(ab^2c^2 + 2a^2bcd - 3a^3d^2 + (b^3c^2 + 2ab^2cd - 3a^2bd^2)x^2)\sqrt{bd}\log\left(8b^2d^2x^4 + b^2c^2 + 6abcd + a^2d^2 + 8(b^2cd + abd^2)x^2 - 4(2bdx^2 + bc + ad)\sqrt{bx^2 + a}\sqrt{dx^2 + c}\sqrt{bd}\right) + 4(ab^2cd - 3a^2bd^2 + (b^3cd - ab^2d^2)x^2)\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{8(ab^4cd^2 - a^2b^3d^3 + (b^5cd^2 - ab^4d^3)x^2)}, \frac{(ab^2c^2 + 2a^2bcd - 3a^3d^2 + (b^3c^2 + 2ab^2cd - 3a^2bd^2)x^2)\sqrt{-bd}\arctan\left(\frac{(2bdx^2 + bc + ad)\sqrt{bx^2 + a}\sqrt{dx^2 + c}\sqrt{-bd}}{2(b^2d^2x^4 + abcd + (b^2cd + abd^2)x^2)}\right) + 2(ab^2cd - 3a^2bd^2 + (b^3cd - ab^2d^2)x^2)\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{4(ab^4cd^2 - a^2b^3d^3 + (b^5cd^2 - ab^4d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2
*b*d^2)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c)*sqrt(b*d)) + 4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*
x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d
^2 - a*b^4*d^3)*x^2), 1/4*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2
+ 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c +
a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x^2)) + 2*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2
)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c
*d^2 - a*b^4*d^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^2)^{\frac{3}{2}}\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.90, size = 192, normalized size = 1.49

$$-\frac{2\sqrt{bd}\,a^2}{\left(b^2c - abd - \left(\sqrt{bx^2 + a}\sqrt{bd} - \sqrt{b^2c + (bx^2 + a)bd - abd}\right)^2\right)b|b|} + \frac{\sqrt{b^2c + (bx^2 + a)bd - abd}\sqrt{bx^2 + a}\,|b|}{2b^4d} + \frac{\left(\sqrt{bd}\,bc + 3\sqrt{bd}\,ad\right)\log\left(\left(\sqrt{bx^2 + a}\sqrt{bd} - \sqrt{b^2c + (bx^2 + a)bd - abd}\right)^2\right)}{4b^2d^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*d)*a^2/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
+ (b*x^2 + a)*b*d - a*b*d))^2)*b*abs(b)) + 1/2*sqrt(b^2*c + (b*x^2 + a)*b*d
 - a*b*d)*sqrt(b*x^2 + a)*abs(b)/(b^4*d) + 1/4*(sqrt(b*d)*b*c + 3*sqrt(b*d)
*a*d)*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^2)/(b^2*d^2*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^2+a\right)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
[Out] int(x^5/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

3.981 $\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}$$

```
[Out] arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(3/2)/d^(1/2)+a*
(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.192$, Rules used = {457, 79, 65, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}+\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[d]*Sqrt
[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(3/2)*Sqrt[d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{2b}\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{b^2}\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{b^2}\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}
\end{aligned}$$

Mathematica [A]
time = 1.44, size = 83, normalized size = 1.00

$$\frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{b^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*Sqrt
[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])]/(b^(3/2)*Sqrt[d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(67) = 134$.
time = 0.12, size = 292, normalized size = 3.52

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd x^2}{\sqrt{bd}}+\sqrt{bd x^4+(ad+bc)x^2+ac}\right)}{2b\sqrt{bd}}+\frac{a\sqrt{bd\left(x^2+\frac{a}{b}\right)^2+(-ad+}}{b^2(-ad+bc)\left(x^2+\frac{a}{b}\right.}\right.}{\sqrt{b x^2+a}\sqrt{d x^2+c}}$
default	$\left(\ln\left(\frac{2bd x^2+2\sqrt{(b x^2+a)(d x^2+c)}\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)abd x^2-\ln\left(\frac{2bd x^2+2\sqrt{(b x^2+a)(d x^2+c)}\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(
b*d)^(1/2))*a*b*d*x^2-ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)
^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^2*c*x^2+ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^
2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*d-ln(1/2*(2*b*d*x^2+2*((b
*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b*c-2*(b*d)^(1
/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*a)/b*(d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(b*d)^
(1/2)/(a*d-b*c)/((b*x^2+a)*(d*x^2+c))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(67) = 134$.
time = 1.24, size = 367, normalized size = 4.42

$$\left[\frac{4\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,abd+\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,\left(b^2cd+abd^2\right)x^2+4\,\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)}{4\,\left(ab^3cd-a^2b^2d^2+\left(b^4cd-ab^3d^2\right)x^2\right)},\ \frac{2\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,abd-\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\,\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)}{2\,\left(ab^3cd-a^2b^2d^2+\left(b^4cd-ab^3d^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d + (a*b*c - a^2*d + (b^2*c - a
*b*d)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*
(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*
x^2 + c)*sqrt(b*d)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2),
 1/2*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d - (a*b*c - a^2*d + (b^2*c - a
*b*d)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2))
)/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(67) = 134$.
time = 0.66, size = 135, normalized size = 1.63

$$\frac{\frac{4\,\sqrt{bd}\,ab}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)|b|}-\frac{\sqrt{bd}\,\log\left(\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{d|b|}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(b*d)*a*b/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d))^2)*abs(b)) - sqrt(b*d)*log((sqrt(b*x^2 + a)
*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(d*abs(b)))/b
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^2+a\right)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
[Out] int(x^3/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

3.982 $\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{c+dx^2}}{(bc-ad)\sqrt{a+bx^2}}$$

```
[Out] -(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {455, 37}

$$-\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{c+dx^2}}{(bc-ad)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 34, normalized size = 1.00

$$-\frac{\sqrt{c+dx^2}}{(bc-ad)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

Maple [A]
time = 0.11, size = 30, normalized size = 0.88

method	result	size
gosper	$\frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,(ad-bc)}$	30
default	$\frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,(ad-bc)}$	30
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,(ad-bc)\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(a*d-b*c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.09, size = 48, normalized size = 1.41

$$-\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{abc-a^2d+(b^2c-abd)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(30) = 60.
time = 0.61, size = 70, normalized size = 2.06

$$-\frac{2\sqrt{bd}\,b}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^2)*abs(b))
```

Mupad [B]
time = 0.69, size = 45, normalized size = 1.32

$$\frac{d\,x^2+c}{\left(a\,d\,\sqrt{d\,x^2+c}-b\,c\,\sqrt{d\,x^2+c}\right)\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (c + d*x^2)/((a*d*(c + d*x^2)^(1/2) - b*c*(c + d*x^2)^(1/2))*(a + b*x^2)^(1
/2))
```

3.983 $\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\,(a+bx^2)^{3/2}}+\frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}$$

```
[Out] arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(5/2)/d^(1/2)-1/
3*a^2*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^2+a)^(3/2)+2/3*a*(-2*a*d+3*b*c)*(
d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 91, 79, 65, 223, 212}

$$-\frac{a^2\sqrt{c+dx^2}}{3b^2\,(a+bx^2)^{3/2}\,(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}+\frac{2a\sqrt{c+dx^2}\,(3bc-2ad)}{3b^2\sqrt{a+bx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/3*(a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*a*(3*b*
c - 2*a*d)*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)^2*Sqrt[a + b*x^2]) + ArcTanh
[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(5/2)*Sqrt[d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)(a+bx^2)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)+\frac{3}{2}b(bc-ad)x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{3b^2(bc-ad)}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)(a+bx^2)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{}{\sqrt{a+bx}}\right.}{2}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)(a+bx^2)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{}{\sqrt{c-\frac{a}{b}}}\right.}{}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)(a+bx^2)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,\right.}{b^3}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)(a+bx^2)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+}}{\sqrt{b}\sqrt{c+}}\right.}{b^{5/2}\sqrt{d}}$$

Mathematica [A]
time = 2.40, size = 114, normalized size = 0.83

$$-\frac{a\sqrt{c+dx^2}\left(-6bc+3ad+\frac{ab(c+dx^2)}{a+bx^2}\right)}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{b^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/3*(a*Sqrt[c + d*x^2]*(-6*b*c + 3*a*d + (a*b*(c + d*x^2))/(a + b*x^2)))/(
b^2*(b*c - a*d)^2*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqr
t[d]*Sqrt[a + b*x^2])]/(b^(5/2)*Sqrt[d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(113) = 226.
time = 0.13, size = 609, normalized size = 4.45

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)\,x^2+ac}\right)}{2b^2\sqrt{bd}}+\frac{2a\sqrt{bd\left(x^2+\frac{a}{b}\right)^2+(-ad}}{b^3(-ad+bc)\left(x^2+\right.}\right.}{\sqrt{b\,x^2+}}$
default	$\frac{\left(-8\sqrt{bd}\,a^2b\,d^2x^4+12\sqrt{bd}\,a\,b^2cd\,x^4+3\ln\left(\frac{2bd\,x^2+2\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{(b\,x^2+a)(d\,x^2+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-8*(b*d)^(1/2)*a^2*b*d^2*x^4+12*(b*d)^(1/2)*a*b^2*c*d*x^4+3*ln(1/2*(2*
b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*((b
*x^2+a)*(d*x^2+c))^(1/2)*a^2*b*d^2*x^2-6*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*
x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)
*a*b^2*c*d*x^2+3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2
)+a*d+b*c)/(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*b^3*c^2*x^2-6*(b*d)^(1/
2)*a^3*d^2*x^2+2*(b*d)^(1/2)*a^2*b*c*d*x^2+12*(b*d)^(1/2)*a*b^2*c^2*x^2+3*l
n(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(
1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*a^3*d^2-6*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*
(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1
/2)*a^2*b*c*d+3*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*((b*x^2+a)*(d*x^2+c))^(1/2)*a*b^2*c^2-6*(b*d)^(1/2)*
a^3*c*d+10*(b*d)^(1/2)*a^2*b*c^2)*(d*x^2+c)^(1/2)/b^2/(b*x^2+a)^(1/2)/(b*d)
^(1/2)/(a*d-b*c)^2/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(113) = 226$.

time = 3.10, size = 706, normalized size = 5.15

$$\left[\frac{3\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)+4\,(5\,a^2b^2cd-3\,a^3bd^2+2\,(3\,ab^3cd-2\,a^2b^2d^2)x^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{12\,(a^2b^5c^2d-2\,a^3b^4cd^2+a^4b^3d^3+(b^7c^2d-2\,ab^6cd^2+a^2b^5d^3)x^4+2\,(ab^6c^2d-2\,a^2b^5cd^2+a^3b^4d^3)x^2)},-\frac{3\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)-2\,(5\,a^2b^2cd-3\,a^3bd^2+2\,(3\,ab^3cd-2\,a^2b^2d^2)x^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{6\,(a^2b^5c^2d-2\,a^3b^4cd^2+a^4b^3d^3+(b^7c^2d-2\,ab^6cd^2+a^2b^5d^3)x^4+2\,(ab^6c^2d-2\,a^2b^5cd^2+a^3b^4d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3 + (b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a^3*b^4*d^3)*x^2), -1/6*(3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3 + (b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a^3*b^4*d^3)*x^2)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(x**5/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(113) = 226.
time = 0.70, size = 333, normalized size = 2.43

$$-\frac{\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{2\sqrt{bd}\,b|b|}+\frac{4\left(3\sqrt{bd}\,ab^4c^2-5\sqrt{bd}\,a^2b^3cd+2\sqrt{bd}\,a^3b^2d^2-6\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2ab^2c+3\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^2bd+3\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4a\right)}{3\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)^3b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] -1/2*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(b*d)*b*abs(b)) + 4/3*(3*sqrt(b*d)*a*b^4*c^2 - 5*sqrt(b*d)*a^2*b^3*c*d + 2*sqrt(b*d)*a^3*b^2*d^2 - 6*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^2*c + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b*d + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*

```
a)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*
b*d - a*b*d))^2)^3*b*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^2+a\right)^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
[Out] int(x^5/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

3.984 $\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{a\sqrt{c+dx^2}}{3b(bc-ad)\,(a+bx^2)^{3/2}}-\frac{(3bc-ad)\sqrt{c+dx^2}}{3b(bc-ad)^2\sqrt{a+bx^2}}$$

```
[Out] 1/3*a*(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)^(3/2)-1/3*(-a*d+3*b*c)*(d*x^2+
c)^(1/2)/b/(-a*d+b*c)^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {457, 79, 37}

$$\frac{a\sqrt{c+dx^2}}{3b\,(a+bx^2)^{3/2}\,(bc-ad)}-\frac{\sqrt{c+dx^2}\,(3bc-ad)}{3b\sqrt{a+bx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)*(a + b*x^2)^(3/2)) - ((3*b*c - a*d)*Sq
rt[c + d*x^2])/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$
$$= \frac{a\sqrt{c+dx^2}}{3b(bc-ad)(a+bx^2)^{3/2}} + \frac{(3bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{6b(bc-ad)}$$
$$= \frac{a\sqrt{c+dx^2}}{3b(bc-ad)(a+bx^2)^{3/2}} - \frac{(3bc-ad)\sqrt{c+dx^2}}{3b(bc-ad)^2\sqrt{a+bx^2}}$$

Mathematica [**A**]
time = 2.03, size = 54, normalized size = 0.61

$$\frac{\sqrt{c+dx^2}\,(-2ac-3bcx^2+adx^2)}{3(bc-ad)^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
[Out] (Sqrt[c + d*x^2]*(-2*a*c - 3*b*c*x^2 + a*d*x^2))/(3*(b*c - a*d)^2*(a + b*x^
2)^(3/2))
```

Maple [**A**]
time = 0.12, size = 50, normalized size = 0.56

method	result	size
default	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-a d\,x^{2}+3 c\,x^{2} b+2 a c\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(a d-b c\right)^{2}}$	50
gosper	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-a d\,x^{2}+3 c\,x^{2} b+2 a c\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}$	63
elliptic	$-\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{d\,x^{2}+c}\,\left(-a d\,x^{2}+3 c\,x^{2} b+2 a c\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\sqrt{b d\,x^{4}+a d\,x^{2}+c\,x^{2} b+a c}\,\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(d*x^2+c)^(1/2)*(-a*d*x^2+3*b*c*x^2+2*a*c)/(b*x^2+a)^(3/2)/(a*d-b*c)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.16, size = 128, normalized size = 1.44

$$-\frac{((3\,bc-ad)x^2+2\,ac)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{3\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^2*b^2*c
^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*
(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{(a+bx^2)^{\frac{5}{2}}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(77) = 154.
time = 0.96, size = 214, normalized size = 2.40

$$-\frac{2\left(3\sqrt{bd}\,b^5c^2-4\sqrt{bd}\,ab^4cd+\sqrt{bd}\,a^2b^3d^2-6\sqrt{bd}\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^3c+3\sqrt{bd}\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4b\right)}{3\left(b^2c-abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)^3b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(b*d)*b^5*c^2 - 4*sqrt(b*d)*a*b^4*c*d + sqrt(b*d)*a^2*b^3*d^2 -
 6*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d))^2*b^3*c + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d))^4*b)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^3*b*abs(b))
```

Mupad [B]
time = 0.79, size = 139, normalized size = 1.56

$$-\frac{\sqrt{b\,x^2+a}\,\left(\frac{2\,a\,c^2}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}+\frac{x^2\,\left(3\,b\,c^2+a\,d\,c\right)}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{x^4\,\left(a\,d^2-3\,b\,c\,d\right)}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}\right)}{x^4\,\sqrt{d\,x^2+c}+\frac{a^2\,\sqrt{d\,x^2+c}}{b^2}+\frac{2\,a\,x^2\,\sqrt{d\,x^2+c}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*((2*a*c^2)/(3*b^2*(a*d - b*c)^2) + (x^2*(3*b*c^2 + a*c*
d))/(3*b^2*(a*d - b*c)^2) - (x^4*(a*d^2 - 3*b*c*d))/(3*b^2*(a*d - b*c)^2)))
/(x^4*(c + d*x^2)^(1/2) + (a^2*(c + d*x^2)^(1/2))/b^2 + (2*a*x^2*(c + d*x^2
)^(1/2))/b)
```

3.985 $\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{c+dx^2}}{3(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{2d\sqrt{c+dx^2}}{3(bc-ad)^2\sqrt{a+bx^2}}$$

```
[Out] -1/3*(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)^(3/2)+2/3*d*(d*x^2+c)^(1/2)/(-a*d
+b*c)^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {455, 47, 37}

$$\frac{2d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}\,(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{3\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^2]/((b*c - a*d)*(a + b*x^2)^(3/2)) + (2*d*Sqrt[c + d*x^2]
)/(3*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^2\right)$$
$$= -\frac{\sqrt{c+dx^2}}{3(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^2\right)}{3(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^2}}{3(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2d\sqrt{c+dx^2}}{3(bc-ad)^2\sqrt{a+bx^2}}$$

Mathematica [A]
time = 1.60, size = 52, normalized size = 0.70

$$\frac{\sqrt{c+dx^2}\left(-bc+3ad+2bdx^2\right)}{3(bc-ad)^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(b*c) + 3*a*d + 2*b*d*x^2))/(3*(b*c - a*d)^2*(a + b*x^2)
^(3/2))
```

Maple [A]
time = 0.12, size = 47, normalized size = 0.64

method	result	size
default	$\frac{\sqrt{d\,x^2+c}\,\left(2bd\,x^2+3ad-bc\right)}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\left(ad-bc\right)^2}$	47
gosper	$\frac{\sqrt{d\,x^2+c}\,\left(2bd\,x^2+3ad-bc\right)}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}$	60
elliptic	$\frac{\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{d\,x^2+c}\,\left(2bd\,x^2+3ad-bc\right)}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}\,\left(a^2d^2-2abcd+b^2c^2\right)}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(2*b*d*x^2+3*a*d-b*c)/(b*x^2+a)^(3/2)/(a*d-b*c)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(62) = 124.
time = 4.06, size = 126, normalized size = 1.70

$$\frac{(2\,bdx^2 - bc + 3\,ad)\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}}{3\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^4 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^2*b^2*c^2
- 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(a*
b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + bx^2)^{\frac{5}{2}}\,\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(62) = 124.
time = 0.66, size = 129, normalized size = 1.74

$$\frac{4\left(b^2c - abd - 3\left(\sqrt{bx^2 + a}\,\sqrt{bd} - \sqrt{b^2c + (bx^2 + a)bd - abd}\right)^2\right)\sqrt{bd}\,b^2d}{3\left(b^2c - abd - \left(\sqrt{bx^2 + a}\,\sqrt{bd} - \sqrt{b^2c + (bx^2 + a)bd - abd}\right)^2\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(b^2*c - a*b*d - 3*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^3*abs(b))
```

Mupad [B]
time = 0.76, size = 137, normalized size = 1.85

$$\frac{\sqrt{b\,x^2+a}\,\left(\frac{x^2\,\left(3\,a\,d^2+b\,c\,d\right)}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{b\,c^2-3\,a\,c\,d}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,d^2\,x^4}{3\,b\,{\left(a\,d-b\,c\right)}^2}\right)}{x^4\,\sqrt{d\,x^2+c}+\frac{a^2\,\sqrt{d\,x^2+c}}{b^2}+\frac{2\,a\,x^2\,\sqrt{d\,x^2+c}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*((x^2*(3*a*d^2 + b*c*d))/(3*b^2*(a*d - b*c)^2) - (b*c^2
- 3*a*c*d)/(3*b^2*(a*d - b*c)^2) + (2*d^2*x^4)/(3*b*(a*d - b*c)^2)))/(x^4*(
c + d*x^2)^(1/2) + (a^2*(c + d*x^2)^(1/2))/b^2 + (2*a*x^2*(c + d*x^2)^(1/2)
)/b)
```

3.986 $\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=154

$$-\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}}+\frac{2a(5bc-3ad)\sqrt{c+dx^2}}{15b^2(bc-ad)^2\left(a+bx^2\right)^{3/2}}-\frac{\left(15b^2c^2-10abcd+3a^2d^2\right)\sqrt{c+dx^2}}{15b^2(bc-ad)^3\sqrt{a+bx^2}}$$

```
[Out] -1/5*a^2*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^2+a)^(5/2)+2/15*a*(-3*a*d+5*b*
c)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^2/(b*x^2+a)^(3/2)-1/15*(3*a^2*d^2-10*a*b*
c*d+15*b^2*c^2)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^3/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {457, 91, 79, 37}

$$-\frac{\sqrt{c+dx^2}\left(3a^2d^2-10abcd+15b^2c^2\right)}{15b^2\sqrt{a+bx^2}\,(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{5b^2\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}\,(5bc-3ad)}{15b^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/5*(a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*(a + b*x^2)^(5/2)) + (2*a*(5*b*
c - 3*a*d)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - ((15
*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^3*S
qrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(5bc-ad)+\frac{5}{2}b(bc-ad)x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^2\right)}{5b^2(bc-ad)}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{2a(5bc-3ad)\sqrt{c+dx^2}}{15b^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{(15b^2c^2-10ab}{}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{2a(5bc-3ad)\sqrt{c+dx^2}}{15b^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} - \frac{(15b^2c^2-10ab}{15b^2(bc-}$$

Mathematica [A]
time = 2.60, size = 91, normalized size = 0.59

$$-\frac{\sqrt{c+dx^2}\left(15b^2c^2x^4+10abcx^2(2c-dx^2)+a^2(8c^2-4cdx^2+3d^2x^4)\right)}{15(bc-ad)^3\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/15*(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(2*c - d*x^2) + a^2*(
8*c^2 - 4*c*d*x^2 + 3*d^2*x^4)))/((b*c - a*d)^3*(a + b*x^2)^(5/2))
```

Maple [A]
time = 0.13, size = 134, normalized size = 0.87

method	result	size
gosper	$\frac{\sqrt{d\,x^{2}+c}\,\left(3a^{2}d^{2}x^{4}-10abcd\,x^{4}+15b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+20ab\,c^{2}x^{2}+8a^{2}c^{2}\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	119
default	$\frac{\left(3a^{2}d^{2}x^{4}-10abcd\,x^{4}+15b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+20ab\,c^{2}x^{2}+8a^{2}c^{2}\right)\sqrt{d\,x^{2}+c}}{15\sqrt{b\,x^{2}+a}\,\left(ad-bc\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$	134
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{d\,x^{2}+c}\,\left(3a^{2}d^{2}x^{4}-10abcd\,x^{4}+15b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+20ab\,c^{2}x^{2}+8a^{2}c^{2}\right)}{15\sqrt{b\,x^{2}+a}\,\sqrt{bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac}\,\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	180

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(3*a^2*d^2*x^4-10*a*b*c*d*x^4+15*b^2*c^2*x^4-4*a^2*c*d*x^2+20*a*b*c^2*
x^2+8*a^2*c^2)*(d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(a*d-b*c)/(b^2*x^4+2*a*b*x^2
+a^2)/(a^2*d^2-2*a*b*c*d+b^2*c^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 3.50, size = 262, normalized size = 1.70

$$-\frac{{\left({\left(15 \, b^{2} c^{2} - 10 \, a b c d + 3 \, a^{2} d^{2}\right)} x^{4} + 8 \, a^{2} c^{2} + 4 \, {\left(5 \, a b c^{2} - a^{2} c d\right)} x^{2}\right)} \sqrt{b x^{2} + a} \sqrt{d x^{2} + c}}{15 \, {\left(a^{3} b^{3} c^{3} - 3 \, a^{4} b^{2} c^{2} d + 3 \, a^{5} b c d^{2} - a^{6} d^{3} + {\left(b^{6} c^{3} - 3 \, a b^{5} c^{2} d + 3 \, a^{2} b^{4} c d^{2} - a^{3} b^{3} d^{3}\right)} x^{6} + 3 \, {\left(a b^{5} c^{3} - 3 \, a^{2} b^{4} c^{2} d + 3 \, a^{3} b^{3} c d^{2} - a^{4} b^{2} d^{3}\right)} x^{4} + 3 \, {\left(a^{2} b^{4} c^{3} - 3 \, a^{3} b^{3} c^{2} d + 3 \, a^{4} b^{2} c d^{2} - a^{5} b d^{3}\right)} x^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 8*a^2*c^2 + 4*(5*a*b*c^2
 - a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c
^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2
 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^
4*b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b
*d^3)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(136) = 272.
time = 0.66, size = 597, normalized size = 3.88

$$-\frac{2\left(15\sqrt{bd}\,b^8c^4-40\sqrt{bd}\,ab^7c^3d+38\sqrt{bd}\,a^2b^6c^2d^2-16\sqrt{bd}\,a^3b^5cd^3+3\sqrt{bd}\,a^4b^4d^4-60\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^6c^3+80\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2ab^5c^2d-20\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^2b^4cd^2+90\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4b^4c^2-40\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4ab^3cd+30\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4a^2b^2d^2-60\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^6b^2c+15\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^8\right)}{15\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)^5b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(b*d)*b^8*c^4 - 40*sqrt(b*d)*a*b^7*c^3*d + 38*sqrt(b*d)*a^2*b
^6*c^2*d^2 - 16*sqrt(b*d)*a^3*b^5*c*d^3 + 3*sqrt(b*d)*a^4*b^4*d^4 - 60*sqrt
(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2
*b^6*c^3 + 80*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^2*a*b^5*c^2*d - 20*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 90*sqrt(b*d)*(sqrt
(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^4*c^2 -
40*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d))^4*a*b^3*c*d + 30*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^4*a^2*b^2*d^2 - 60*sqrt(b*d)*(sqrt(b*x^2 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*c + 15*sqrt(b*d)*(sqr
t(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^8)/((b^2*c
- a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^2)^5*b*abs(b))
```

Mupad [B]
time = 0.88, size = 220, normalized size = 1.43

$$\frac{\sqrt{b\,x^2+a}\,\left(\frac{8\,a^2\,c^3}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x^4\,\left(-a^2\,c\,d^2+10\,a\,b\,c^2\,d+15\,b^2\,c^3\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x^6\,\left(3\,a^2\,d^3-10\,a\,b\,c\,d^2+15\,b^2\,c^2\,d\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{4\,a\,c^2\,x^2\,\left(a\,d+5\,b\,c\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}\right)}{x^6\,\sqrt{d\,x^2+c}+\frac{a^3\,\sqrt{d\,x^2+c}}{b^3}+\frac{3\,a\,x^4\,\sqrt{d\,x^2+c}}{b}+\frac{3\,a^2\,x^2\,\sqrt{d\,x^2+c}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*((8*a^2*c^3)/(15*b^3*(a*d - b*c)^3) + (x^4*(15*b^2*c^3 -
 a^2*c*d^2 + 10*a*b*c^2*d))/(15*b^3*(a*d - b*c)^3) + (x^6*(3*a^2*d^3 + 15*b
^2*c^2*d - 10*a*b*c*d^2))/(15*b^3*(a*d - b*c)^3) + (4*a*c^2*x^2*(a*d + 5*b*
c))/(15*b^3*(a*d - b*c)^3)))/(x^6*(c + d*x^2)^(1/2) + (a^3*(c + d*x^2)^(1/2
))/b^3 + (3*a*x^4*(c + d*x^2)^(1/2))/b + (3*a^2*x^2*(c + d*x^2)^(1/2))/b^2)
```

3.987 $\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=138

$$\frac{a\sqrt{c+dx^2}}{5b(bc-ad)\left(a+bx^2\right)^{5/2}}-\frac{(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^2\left(a+bx^2\right)^{3/2}}+\frac{2d(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^3\sqrt{a+bx^2}}$$

```
[Out] 1/5*a*(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)^(5/2)-1/15*(-a*d+5*b*c)*(d*x^2
+c)^(1/2)/b/(-a*d+b*c)^2/(b*x^2+a)^(3/2)+2/15*d*(-a*d+5*b*c)*(d*x^2+c)^(1/2
)/b/(-a*d+b*c)^3/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {457, 79, 47, 37}

$$\frac{a\sqrt{c+dx^2}}{5b\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2d\sqrt{c+dx^2}\,(5bc-ad)}{15b\sqrt{a+bx^2}\,(bc-ad)^3}-\frac{\sqrt{c+dx^2}\,(5bc-ad)}{15b\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(5*b*(b*c - a*d)*(a + b*x^2)^(5/2)) - ((5*b*c - a*d)*Sq
rt[c + d*x^2])/(15*b*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (2*d*(5*b*c - a*d)*
Sqrt[c + d*x^2])/(15*b*(b*c - a*d)^3*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{a\sqrt{c+dx^2}}{5b(bc-ad)\,(a+bx^2)^{5/2}} + \frac{(5bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{10b(bc-ad)}$$

$$= \frac{a\sqrt{c+dx^2}}{5b(bc-ad)\,(a+bx^2)^{5/2}} - \frac{(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^2\,(a+bx^2)^{3/2}} - \frac{(d(5bc-ad))\text{Subst}\,(}{1}$$

$$= \frac{a\sqrt{c+dx^2}}{5b(bc-ad)\,(a+bx^2)^{5/2}} - \frac{(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^2\,(a+bx^2)^{3/2}} + \frac{2d(5bc-ad)\sqrt{c+d}}{15b(bc-ad)^3\sqrt{a+}}$$

Mathematica [A]
time = 2.22, size = 91, normalized size = 0.66

$$\frac{\sqrt{c+dx^2}\,(-5b^2cx^2(c-2dx^2)-5a^2d(-2c+dx^2)-2ab(c^2-13cdx^2+d^2x^4))}{15(bc-ad)^3\,(a+bx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-5*b^2*c*x^2*(c - 2*d*x^2) - 5*a^2*d*(-2*c + d*x^2) - 2*a
*b*(c^2 - 13*c*d*x^2 + d^2*x^4)))/(15*(b*c - a*d)^3*(a + b*x^2)^(5/2))
```

Maple [A]
time = 0.12, size = 140, normalized size = 1.01

method	result	size
gosper	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-2ab\,d^{2}x^{4}+10b^{2}cd\,x^{4}-5a^{2}d^{2}x^{2}+26abcd\,x^{2}-5b^{2}c^{2}x^{2}+10a^{2}cd-2ab\,c^{2}\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	125
default	$-\frac{\left(-2ab\,d^{2}x^{4}+10b^{2}cd\,x^{4}-5a^{2}d^{2}x^{2}+26abcd\,x^{2}-5b^{2}c^{2}x^{2}+10a^{2}cd-2ab\,c^{2}\right)\sqrt{d\,x^{2}+c}}{15\sqrt{b\,x^{2}+a}\,\left(ad-bc\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$	140
elliptic	$-\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{d\,x^{2}+c}\,\left(-2ab\,d^{2}x^{4}+10b^{2}cd\,x^{4}-5a^{2}d^{2}x^{2}+26abcd\,x^{2}-5b^{2}c^{2}x^{2}+10a^{2}cd-2ab\,c^{2}\right)}{15\sqrt{b\,x^{2}+a}\,\sqrt{bd\,x^{4}+ad\,x^{2}+c\,x^{2}b+ac}\,\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	186

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-2*a*b*d^2*x^4+10*b^2*c*d*x^4-5*a^2*d^2*x^2+26*a*b*c*d*x^2-5*b^2*c^2
*x^2+10*a^2*c*d-2*a*b*c^2)*(d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(a*d-b*c)/(b^2*x
^4+2*a*b*x^2+a^2)/(a^2*d^2-2*a*b*c*d+b^2*c^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(120) = 240.
time = 2.11, size = 269, normalized size = 1.95

$$\frac{\left(2\left(5\,b^{2}cd-abd^{2}\right)x^{4}-2\,abc^{2}+10\,a^{2}cd-\left(5\,b^{2}c^{2}-26\,abcd+5\,a^{2}d^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{15\left(a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}+\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}\right)x^{6}+3\left(ab^{5}c^{3}-3\,a^{2}b^{4}c^{2}d+3\,a^{3}b^{3}cd^{2}-a^{4}b^{2}d^{3}\right)x^{4}+3\left(a^{2}b^{4}c^{3}-3\,a^{3}b^{3}c^{2}d+3\,a^{4}b^{2}cd^{2}-a^{5}bd^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*b^2*c*d - a*b*d^2)*x^4 - 2*a*b*c^2 + 10*a^2*c*d - (5*b^2*c^2 - 2
6*a*b*c*d + 5*a^2*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 -
3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^
2*b^4*c*d^2 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3
```

```
*c*d^2 - a^4*b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*
d^2 - a^5*b*d^3)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^2)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(120) = 240.
time = 0.65, size = 472, normalized size = 3.42

$$\frac{4\left(5\sqrt{bd}\,b^8c^3d-11\sqrt{bd}\,ab^7c^2d^2+7\sqrt{bd}\,a^2b^6cd^3-\sqrt{bd}\,a^3b^5d^4-25\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^6c^2d+30\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2ab^5cd^2-5\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^2b^4d^3+35\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4b^4cd+5\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4ab^3d^2-15\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^6b^2d\right)}{15\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)^5b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*sqrt(b*d)*b^8*c^3*d - 11*sqrt(b*d)*a*b^7*c^2*d^2 + 7*sqrt(b*d)*a^2*
b^6*c*d^3 - sqrt(b*d)*a^3*b^5*d^4 - 25*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d)
 - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^6*c^2*d + 30*sqrt(b*d)*(sqrt(
b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^5*c*d^2
 - 5*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
a*b*d))^2*a^2*b^4*d^3 + 35*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d))^4*b^4*c*d + 5*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^3*d^2 - 15*sqrt(b*d)*(
sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*d)
/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d))^2)^5*b*abs(b))
```

Mupad [B]
time = 0.84, size = 227, normalized size = 1.64

$$-\frac{\sqrt{b\,x^2+a}\,\left(\frac{x^2\,\left(5\,a^2\,c\,d^2+24\,a\,b\,c^2\,d-5\,b^2\,c^3\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x^4\,\left(-5\,a^2\,d^3+24\,a\,b\,c\,d^2+5\,b^2\,c^2\,d\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}-\frac{2\,d^2\,x^6\,\left(a\,d-5\,b\,c\right)}{15\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,a\,c^2\,\left(5\,a\,d-b\,c\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}\right)}{x^6\,\sqrt{d\,x^2+c}+\frac{a^3\,\sqrt{d\,x^2+c}}{b^3}+\frac{3\,a\,x^4\,\sqrt{d\,x^2+c}}{b}+\frac{3\,a^2\,x^2\,\sqrt{d\,x^2+c}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*((x^2*(5*a^2*c*d^2 - 5*b^2*c^3 + 24*a*b*c^2*d))/(15*b^3
*(a*d - b*c)^3) + (x^4*(5*b^2*c^2*d - 5*a^2*d^3 + 24*a*b*c*d^2))/(15*b^3*(a
```

```
*d - b*c)^3) - (2*d^2*x^6*(a*d - 5*b*c))/(15*b^2*(a*d - b*c)^3) + (2*a*c^2*
(5*a*d - b*c))/(15*b^3*(a*d - b*c)^3)))/(x^6*(c + d*x^2)^(1/2) + (a^3*(c +
d*x^2)^(1/2))/b^3 + (3*a*x^4*(c + d*x^2)^(1/2))/b + (3*a^2*x^2*(c + d*x^2)^
(1/2))/b^2)
```

3.988 $\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{c+dx^2}}{5(bc-ad)\,(a+bx^2)^{5/2}}+\frac{4d\sqrt{c+dx^2}}{15(bc-ad)^2\,(a+bx^2)^{3/2}}-\frac{8d^2\sqrt{c+dx^2}}{15(bc-ad)^3\sqrt{a+bx^2}}$$

```
[Out] -1/5*(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)^(5/2)+4/15*d*(d*x^2+c)^(1/2)/(-a*
d+b*c)^2/(b*x^2+a)^(3/2)-8/15*d^2*(d*x^2+c)^(1/2)/(-a*d+b*c)^3/(b*x^2+a)^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {455, 47, 37}

$$-\frac{8d^2\sqrt{c+dx^2}}{15\sqrt{a+bx^2}\,(bc-ad)^3}+\frac{4d\sqrt{c+dx^2}}{15\,(a+bx^2)^{3/2}\,(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{5\,(a+bx^2)^{5/2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/5*Sqrt[c + d*x^2]/((b*c - a*d)*(a + b*x^2)^(5/2)) + (4*d*Sqrt[c + d*x^2]
)/(15*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - (8*d^2*Sqrt[c + d*x^2])/(15*(b*c -
 a*d)^3*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx,x,x^2\right)$$
$$= -\frac{\sqrt{c+dx^2}}{5(bc-ad)(a+bx^2)^{5/2}} - \frac{(2d)\text{Subst}\left(\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^2\right)}{5(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^2}}{5(bc-ad)(a+bx^2)^{5/2}} + \frac{4d\sqrt{c+dx^2}}{15(bc-ad)^2(a+bx^2)^{3/2}} + \frac{(4d^2)\text{Subst}\left(\int \frac{}{(a+b}\right.}{15(}$$
$$= -\frac{\sqrt{c+dx^2}}{5(bc-ad)(a+bx^2)^{5/2}} + \frac{4d\sqrt{c+dx^2}}{15(bc-ad)^2(a+bx^2)^{3/2}} - \frac{8d^2\sqrt{c+dx^2}}{15(bc-ad)^3\sqrt{a+}}$$

Mathematica [A]
time = 1.81, size = 83, normalized size = 0.73

$$-\frac{\sqrt{c+dx^2}\,(15a^2d^2-10abd(c-2dx^2)+b^2(3c^2-4cdx^2+8d^2x^4))}{15(bc-ad)^3(a+bx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/15*(Sqrt[c + d*x^2]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x^2) + b^2*(3*c^2 -
4*c*d*x^2 + 8*d^2*x^4)))/((b*c - a*d)^3*(a + b*x^2)^(5/2))
```

Maple [A]
time = 0.12, size = 128, normalized size = 1.13

method	result	size
gosper	$\frac{\sqrt{d\,x^2+c}\,(8b^2x^4d^2+20ab\,d^2x^2-4b^2cd\,x^2+15a^2d^2-10abcd+3b^2c^2)}{15(b\,x^2+a)^{\frac{5}{2}}(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	113
default	$\frac{(8b^2x^4d^2+20ab\,d^2x^2-4b^2cd\,x^2+15a^2d^2-10abcd+3b^2c^2)\sqrt{d\,x^2+c}}{15\sqrt{b\,x^2+a}\,(ad-bc)(b^2x^4+2ab\,x^2+a^2)(a^2d^2-2abcd+b^2c^2)}$	128

elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{d\,x^2+c}\,(8b^2x^4d^2+20ab\,d^2x^2-4b^2cd\,x^2+15a^2d^2-10abcd+3b^2c^2)}{15\sqrt{b\,x^2+a}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}\,(b^2x^4+2ab\,x^2+a^2)(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	174

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(8*b^2*d^2*x^4+20*a*b*d^2*x^2-4*b^2*c*d*x^2+15*a^2*d^2-10*a*b*c*d+3*b^
2*c^2)*(d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(a*d-b*c)/(b^2*x^4+2*a*b*x^2+a^2)/(a
^2*d^2-2*a*b*c*d+b^2*c^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(95) = 190.
time = 5.02, size = 259, normalized size = 2.29

$$-\frac{(8\,b^2d^2x^4+3\,b^2c^2-10\,abcd+15\,a^2d^2-4\,(b^2cd-5\,abd^2)x^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{15\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^6+3\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x^4+3\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*b^2*d^2*x^4 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b^2*c*d - 5
*a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2
*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
 a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*
b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d
^3)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^2)^{\frac{7}{2}}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(95) = 190$.

time = 0.59, size = 243, normalized size = 2.15

$$-\frac{16 \left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2 - 5 \left(\sqrt{bx^2+a}\,\sqrt{bd} - \sqrt{b^2c + (bx^2+a)bd - abd}\right)^2 b^2c + 5 \left(\sqrt{bx^2+a}\,\sqrt{bd} - \sqrt{b^2c + (bx^2+a)bd - abd}\right)^2 abd + 10 \left(\sqrt{bx^2+a}\,\sqrt{bd} - \sqrt{b^2c + (bx^2+a)bd - abd}\right)^4\right)\sqrt{bd}\,b^3d^2}{15 \left(b^2c - abd - \left(\sqrt{bx^2+a}\,\sqrt{bd} - \sqrt{b^2c + (bx^2+a)bd - abd}\right)^2\right)^5 |b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -16/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*x^2 + a)*sqrt(b*d)
- sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c + 5*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + 10*(sqrt(b*x^2 + a)
*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^2/((
b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d))^2)^5*abs(b))
```

Mupad [B]

time = 0.82, size = 216, normalized size = 1.91

$$\frac{\sqrt{b\,x^2+a}\,\left(\frac{15\,a^2\,c\,d^2-10\,a\,b\,c^2\,d+3\,b^2\,c^3}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^3\,x^6}{15\,b\,{\left(a\,d-b\,c\right)}^3}+\frac{x^2\,\left(15\,a^2\,d^3+10\,a\,b\,c\,d^2-b^2\,c^2\,d\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{4\,d^2\,x^4\,\left(5\,a\,d+b\,c\right)}{15\,b^2\,{\left(a\,d-b\,c\right)}^3}\right)}{x^6\,\sqrt{d\,x^2+c}+\frac{a^3\,\sqrt{d\,x^2+c}}{b^3}+\frac{3\,a\,x^4\,\sqrt{d\,x^2+c}}{b}+\frac{3\,a^2\,x^2\,\sqrt{d\,x^2+c}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*((3*b^2*c^3 + 15*a^2*c*d^2 - 10*a*b*c^2*d)/(15*b^3*(a*d
- b*c)^3) + (8*d^3*x^6)/(15*b*(a*d - b*c)^3) + (x^2*(15*a^2*d^3 - b^2*c^2*d
 + 10*a*b*c*d^2))/(15*b^3*(a*d - b*c)^3) + (4*d^2*x^4*(5*a*d + b*c))/(15*b^
2*(a*d - b*c)^3)))/(x^6*(c + d*x^2)^(1/2) + (a^3*(c + d*x^2)^(1/2))/b^3 + (
3*a*x^4*(c + d*x^2)^(1/2))/b + (3*a^2*x^2*(c + d*x^2)^(1/2))/b^2)
```

3.989 $\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=217

$$-\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\left(a+bx^2\right)^{7/2}}+\frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\left(a+bx^2\right)^{5/2}}-\frac{\left(35b^2c^2-14abcd+3a^2d^2\right)\sqrt{c+dx^2}}{105b^2(bc-ad)^3\left(a+bx^2\right)^{3/2}}+\frac{2d(35b^2c^2 -}{105b}$$

```
[Out] -1/7*a^2*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^2+a)^(7/2)+2/35*a*(-4*a*d+7*b*
c)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^2/(b*x^2+a)^(5/2)-1/105*(3*a^2*d^2-14*a*b
*c*d+35*b^2*c^2)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^3/(b*x^2+a)^(3/2)+2/105*d*(
3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^4/(b*x^2+a)
^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {457, 91, 79, 47, 37}

$$\frac{2d\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\sqrt{a+bx^2}\,(bc-ad)^4}-\frac{\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\left(a+bx^2\right)^{3/2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{7b^2\left(a+bx^2\right)^{7/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}\,(7bc-4ad)}{35b^2\left(a+bx^2\right)^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/7*(a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*(a + b*x^2)^(7/2)) + (2*a*(7*b*
c - 4*a*d)*Sqrt[c + d*x^2])/(35*b^2*(b*c - a*d)^2*(a + b*x^2)^(5/2)) - ((35
*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(105*b^2*(b*c - a*d)^3*
(a + b*x^2)^(3/2)) + (2*d*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*
x^2])/(105*b^2*(b*c - a*d)^4*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\,(a+bx^2)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(7bc-ad)+\frac{7}{2}b(bc-ad)x}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{7b^2(bc-ad)}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\,(a+bx^2)^{7/2}} + \frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\,(a+bx^2)^{5/2}} + \frac{(35b^2c^2-14abc}{}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\,(a+bx^2)^{7/2}} + \frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\,(a+bx^2)^{5/2}} - \frac{(35b^2c^2-14abc}{105b^2(bc-}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\,(a+bx^2)^{7/2}} + \frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\,(a+bx^2)^{5/2}} - \frac{(35b^2c^2-14abc}{105b^2(bc-}$$

Mathematica [A]

time = 3.67, size = 151, normalized size = 0.70

$$\frac{\sqrt{c+dx^2}\,(-35b^3c^2x^4(c-2dx^2)+7a^3d(8c^2-4cdx^2+3d^2x^4)-7ab^2cx^2(4c^2-37cdx^2+4d^2x^4)+a^2b(-8c^3+200c^2dx^2-101cd^2x^4+6d^3x^6))}{105(bc-ad)^4\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-35*b^3*c^2*x^4*(c - 2*d*x^2) + 7*a^3*d*(8*c^2 - 4*c*d*x^
2 + 3*d^2*x^4) - 7*a*b^2*c*x^2*(4*c^2 - 37*c*d*x^2 + 4*d^2*x^4) + a^2*b*(-8
*c^3 + 200*c^2*d*x^2 - 101*c*d^2*x^4 + 6*d^3*x^6)))/(105*(b*c - a*d)^4*(a +
 b*x^2)^(7/2))
```

Maple [A]

time = 0.17, size = 241, normalized size = 1.11

method	result
gosper	$\frac{\sqrt{d\,x^2+c}\,(6a^2b\,d^3x^6-28a\,b^2c\,d^2x^6+70b^3c^2d\,x^6+21a^3d^3x^4-101a^2bc\,d^2x^4+259a\,b^2c^2d\,x^4-35c^3b^3x^4-28a^3c\,d^2x^2+200a^2b\,c^2d\,x^2}{105(b\,x^2+a)^{\frac{7}{2}}(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}$
default	$\frac{(6a^2b\,d^3x^6-28a\,b^2c\,d^2x^6+70b^3c^2d\,x^6+21a^3d^3x^4-101a^2bc\,d^2x^4+259a\,b^2c^2d\,x^4-35c^3b^3x^4-28a^3c\,d^2x^2+200a^2b\,c^2d\,x^2-28a\,b^2c^3x^2+}{105\sqrt{b\,x^2+a}\,(ad-bc)(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$
elliptic	$\frac{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}\,\sqrt{d\,x^2+c}\,(6a^2b\,d^3x^6-28a\,b^2c\,d^2x^6+70b^3c^2d\,x^6+21a^3d^3x^4-101a^2bc\,d^2x^4+259a\,b^2c^2d\,x^4-35c^3b}{105\sqrt{b\,x^2+a}\,\sqrt{bd\,x^4+ad\,x^2+c\,x^2b+ac}\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)(a^4d^4-4a^3bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(6*a^2*b*d^3*x^6-28*a*b^2*c*d^2*x^6+70*b^3*c^2*d*x^6+21*a^3*d^3*x^4-1
01*a^2*b*c*d^2*x^4+259*a*b^2*c^2*d*x^4-35*b^3*c^3*x^4-28*a^3*c*d^2*x^2+200*
a^2*b*c^2*d*x^2-28*a*b^2*c^3*x^2+56*a^3*c^2*d-8*a^2*b*c^3)*(d*x^2+c)^(1/2)/
(b*x^2+a)^(1/2)/(a*d-b*c)/(b^3*x^6+3*a*b^2*x^4+3*a^2*b*x^2+a^3)/(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(193) = 386$.
time = 3.35, size = 451, normalized size = 2.08

$$\frac{\left(2\left(35 b^{3} c^{2} d-14 a b^{2} c d^{2}+3 a^{2} b d^{3}\right) x^{6}-8 a^{2} b c^{3}+56 a^{3} c^{2} d-\left(35 b^{3} c^{3}-259 a b^{2} c^{2} d+101 a^{2} b c d^{2}-21 a^{3} d^{3}\right) x^{4}-4\left(7 a b^{2} c^{3}-50 a^{2} b c^{2} d+7 a^{3} c d^{2}\right) x^{2}\right) \sqrt{b x^{2}+a} \sqrt{d x^{2}+c}}{105\left(a^{4} b^{4} c^{4}-4 a^{5} b^{3} c^{3} d+6 a^{6} b^{2} c^{2} d^{2}-4 a^{7} b c d^{3}+a^{8} d^{4}+\left(b^{8} c^{4}-4 a b^{7} c^{3} d+6 a^{2} b^{6} c^{2} d^{2}-4 a^{3} b^{5} c d^{3}+a^{4} b^{4} d^{4}\right) x^{8}+4\left(a b^{7} c^{4}-4 a^{2} b^{6} c^{3} d+6 a^{3} b^{5} c^{2} d^{2}-4 a^{4} b^{4} c d^{3}+a^{5} b^{3} d^{4}\right) x^{6}+6\left(a^{2} b^{6} c^{4}-4 a^{3} b^{5} c^{3} d+6 a^{4} b^{4} c^{2} d^{2}-4 a^{5} b^{3} c d^{3}+a^{6} b^{2} d^{4}\right) x^{4}+4\left(a^{3} b^{5} c^{4}-4 a^{4} b^{4} c^{3} d+6 a^{5} b^{3} c^{2} d^{2}-4 a^{6} b^{2} c d^{3}+a^{7} b d^{4}\right) x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(35*b^3*c^2*d - 14*a*b^2*c*d^2 + 3*a^2*b*d^3)*x^6 - 8*a^2*b*c^3 +
56*a^3*c^2*d - (35*b^3*c^3 - 259*a*b^2*c^2*d + 101*a^2*b*c*d^2 - 21*a^3*d^3
)*x^4 - 4*(7*a*b^2*c^3 - 50*a^2*b*c^2*d + 7*a^3*c*d^2)*x^2)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7
*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b
^5*c*d^3 + a^4*b^4*d^4)*x^8 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^
2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^6 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3
*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^4 + 4*(a^3*b^5*c^
4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(a+b x^{2}\right)^{\frac{9}{2}} \sqrt{c+d x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(9/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(9/2)*sqrt(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(193) = 386$.
time = 0.65, size = 1036, normalized size = 4.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/105*(35*sqrt(b*d)*b^10*c^5*d - 119*sqrt(b*d)*a*b^9*c^4*d^2 + 150*sqrt(b*d
)*a^2*b^8*c^3*d^3 - 86*sqrt(b*d)*a^3*b^7*c^2*d^4 + 23*sqrt(b*d)*a^4*b^6*c*d
^5 - 3*sqrt(b*d)*a^5*b^5*d^6 - 245*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - s
qrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^8*c^4*d + 588*sqrt(b*d)*(sqrt(b*x
^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^7*c^3*d^2
 - 462*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d))^2*a^2*b^6*c^2*d^3 + 140*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^3*b^5*c*d^4 - 21*sqrt(b*d)*(sqrt(b*
x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^4*d^5 +
 630*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
a*b*d))^4*b^6*c^3*d - 714*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
 + (b*x^2 + a)*b*d - a*b*d))^4*a*b^5*c^2*d^2 + 42*sqrt(b*d)*(sqrt(b*x^2 + a
)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^4*c*d^3 + 42*s
qrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)
)^4*a^3*b^3*d^4 - 770*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (
b*x^2 + a)*b*d - a*b*d))^6*b^4*c^2*d + 140*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a*b^3*c*d^2 - 210*sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a^2*
b^2*d^3 + 455*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^8*b^2*c*d + 105*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^8*a*b*d^2 - 105*sqrt(b*d)*(sqrt(b*x^2 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^10*d)/((b^2*c - a*b*
d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^
7*abs(b))
```

Mupad [B]
time = 1.02, size = 336, normalized size = 1.55

$$\frac{\sqrt{b\,x^2+a}\,\left(\frac{x^6\,\left(21\,a^3\,d^4-95\,a^2\,b\,c\,d^3+231\,a\,b^2\,c^2\,d^2+35\,b^3\,c^3\,d\right)}{105\,b^4\,{\left(a\,d-b\,c\right)}^4}-\frac{x^4\,\left(7\,a^3\,c\,d^3-99\,a^2\,b\,c^2\,d^2-231\,a\,b^2\,c^3\,d+35\,b^3\,c^4\right)}{105\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{8\,a^2\,c^3\,\left(7\,a\,d-b\,c\right)}{105\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{2\,d^2\,x^8\,\left(3\,a^2\,d^2-14\,a\,b\,c\,d+35\,b^2\,c^2\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,a\,c^2\,x^2\,\left(7\,a^2\,d^2+48\,a\,b\,c\,d-7\,b^2\,c^2\right)}{105\,b^4\,{\left(a\,d-b\,c\right)}^4}\right)}{x^8\,\sqrt{d\,x^2+c}+\frac{a^4\,\sqrt{d\,x^2+c}}{b^4}+\frac{4\,a\,x^6\,\sqrt{d\,x^2+c}}{b}+\frac{6\,a^2\,x^4\,\sqrt{d\,x^2+c}}{b^2}+\frac{4\,a^3\,x^2\,\sqrt{d\,x^2+c}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(9/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*((x^6*(21*a^3*d^4 + 35*b^3*c^3*d + 231*a*b^2*c^2*d^2 - 9
5*a^2*b*c*d^3))/(105*b^4*(a*d - b*c)^4) - (x^4*(35*b^3*c^4 + 7*a^3*c*d^3 -
99*a^2*b*c^2*d^2 - 231*a*b^2*c^3*d))/(105*b^4*(a*d - b*c)^4) + (8*a^2*c^3*(
7*a*d - b*c))/(105*b^4*(a*d - b*c)^4) + (2*d^2*x^8*(3*a^2*d^2 + 35*b^2*c^2
- 14*a*b*c*d))/(105*b^3*(a*d - b*c)^4) + (4*a*c^2*x^2*(7*a^2*d^2 - 7*b^2*c^
2 + 48*a*b*c*d))/(105*b^4*(a*d - b*c)^4)))/(x^8*(c + d*x^2)^(1/2) + (a^4*(c
 + d*x^2)^(1/2))/b^4 + (4*a*x^6*(c + d*x^2)^(1/2))/b + (6*a^2*x^4*(c + d*x^
2)^(1/2))/b^2 + (4*a^3*x^2*(c + d*x^2)^(1/2))/b^3)
```

3.990 $\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -arctan(d^(1/2)*(-b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {455, 65, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt
[d]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a-bx^2}\,\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= -\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+\frac{ad}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx^2}\right)}{b}\\ &= -\frac{\mathrm{Subst}\left(\int \frac{1}{1+\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\right)}{b}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{b}\,\sqrt{d}}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 46, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{a-bx^2}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
[Out] ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a - b*x^2])]/(Sqrt[b]*Sqrt[d
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.
time = 0.11, size = 92, normalized size = 1.96

method	result	size
default	$-\frac{\arctan\left(\frac{\sqrt{bd}\,\left(-2bd\,x^2+ad-bc\right)}{2bd\sqrt{\left(-b\,x^2+a\right)\left(d\,x^2+c\right)}}\right)\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}}{2\sqrt{bd}\,\sqrt{\left(-b\,x^2+a\right)\left(d\,x^2+c\right)}}$	92

elliptic	$\frac{\sqrt{(-b\,x^2+a)\,(d\,x^2+c)}\ \arctan\left(\frac{\sqrt{bd}\ \left(x^2-\frac{ad-bc}{2bd}\right)}{\sqrt{-bd\,x^4+(ad-bc)\,x^2+ac}}\right)}{2\sqrt{-b\,x^2+a}\ \sqrt{d\,x^2+c}\ \sqrt{bd}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctan(1/2*(b*d)^(1/2)*(-2*b*d*x^2+a*d-b*c)/b/d/((-b*x^2+a)*(d*x^2+c))
^(1/2))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/((-b*x^2+a)*(d*x^2+c))
^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(35) = 70$.
time = 1.40, size = 201, normalized size = 4.28

$$\left[-\frac{\sqrt{-bd}\ \log\left(8\,b^2d^2x^4+b^2c^2-6\,abcd+a^2d^2+8\,(b^2cd-abd^2)x^2-4\,(2\,bdx^2+bc-ad)\sqrt{-bx^2+a}\ \sqrt{dx^2+c}\ \sqrt{-bd}\right)}{4\,bd},-\frac{\sqrt{bd}\ \arctan\left(\frac{(2\,bdx^2+bc-ad)\sqrt{-bx^2+a}\ \sqrt{dx^2+c}\ \sqrt{bd}}{2\,(b^2d^2x^4-abcd+(b^2cd-abd^2)x^2)}\right)}{2\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 8*(b^2
*c*d - a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c - a*d)*sqrt(-b*x^2 + a)*sqrt(d*x^2
 + c)*sqrt(-b*d))/(b*d), -1/2*sqrt(b*d)*arctan(1/2*(2*b*d*x^2 + b*c - a*d)*
sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)/(b^2*d^2*x^4 - a*b*c*d + (b^2*c*
d - a*b*d^2)*x^2))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a-bx^2}\ \sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Giac [A]
time = 0.59, size = 57, normalized size = 1.21

$$\frac{b \log \left(\left|-\sqrt{-bx^2+a}\,\sqrt{-bd}+\sqrt{b^2c+(bx^2-a)bd+abd}\right|\right)}{\sqrt{-bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-sqrt(-b*x^2 + a)*sqrt(-b*d) + sqrt(b^2*c + (b*x^2 - a)*b*d + a*b
*d)))/(sqrt(-b*d)*abs(b))
```

Mupad [B]
time = 0.74, size = 48, normalized size = 1.02

$$-\frac{2 \operatorname{atan}\left(\frac{d\left(\sqrt{a-b\,x^2}-\sqrt{a}\right)}{\sqrt{b\,d}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -(2*atan((d*((a - b*x^2)^(1/2) - a^(1/2)))/((b*d)^(1/2)*((c + d*x^2)^(1/2)
- c^(1/2)))))/(b*d)^(1/2)
```

3.991 $\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] -arctanh(d^(1/2)*(-b*x^2+a)^(1/2)/b^(1/2)/(-d*x^2+c)^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {455, 65, 223, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c - d*x^2])]/(Sqrt[b]*Sqr
t[d]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a-bx}\sqrt{c-dx}}\,dx, x, x^2\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx^2}\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\right)}{b}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 48, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c-dx^2}}{\sqrt{d}\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c - d*x^2])/(Sqrt[d]*Sqrt[a - b*x^2])]/(Sqrt[b]*Sqr
t[d]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(36) = 72$.
time = 0.12, size = 95, normalized size = 1.98

method	result	size
default	$\frac{\ln\left(-\frac{-2bd\,x^2+ad+bc-2\sqrt{(-b\,x^2+a)(-d\,x^2+c)}\,\sqrt{bd}}{2\sqrt{bd}}\right)\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2+c}}{2\sqrt{bd}\,\sqrt{(-b\,x^2+a)(-d\,x^2+c)}}$	95

elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\ln\left(\frac{-\frac{1}{2}a d-\frac{1}{2}b c+b d\,x^{2}}{\sqrt{b d}}+\sqrt{b d\,x^{4}+\left(-a d-b c\right)x^{2}+a c}\right)}{2\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}\,\sqrt{b d}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(-1/2*(-2*b*d*x^2+a*d+b*c-2*((-b*x^2+a)*(-d*x^2+c))^(1/2)*(b*d)^(1/2)
)/(b*d)^(1/2))*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(b*d)^(1/2)/((-b*x^2+a)*(-
d*x^2+c))^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(36) = 72.
time = 0.82, size = 203, normalized size = 4.23

$$\left[\frac{\sqrt{bd}\,\log\left(8\,b^{2}d^{2}x^{4}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}-8\,(b^{2}cd+abd^{2})x^{2}+4\,(2\,bdx^{2}-bc-ad)\sqrt{-bx^{2}+a}\,\sqrt{-dx^{2}+c}\,\sqrt{bd}\right)}{4\,bd},-\frac{\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^{2}-bc-ad)\sqrt{-bx^{2}+a}\,\sqrt{-dx^{2}+c}\,\sqrt{-bd}}{2\,(b^{2}d^{2}x^{4}+abcd-(b^{2}cd+abd^{2})x^{2})}\right)}{2\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 - b*c - a*d)*sqrt(-b*x^2 + a)*sqrt(-d*x^2
+ c)*sqrt(b*d))/(b*d), -1/2*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 - b*c - a*d)*s
qrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d - (b^2*c
*d + a*b*d^2)*x^2))/(b*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a-bx^{2}}\,\sqrt{c-dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

Giac [A]
time = 0.60, size = 57, normalized size = 1.19

$$\frac{b\log\left(\left|-\sqrt{-bx^2+a}\,\sqrt{bd}+\sqrt{b^2c-(bx^2-a)bd-abd}\right|\right)}{\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-sqrt(-b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c - (b*x^2 - a)*b*d - a*b*
d)))/(sqrt(b*d)*abs(b))
```

Mupad [B]
time = 0.68, size = 51, normalized size = 1.06

$$\frac{2\,\mathrm{atan}\left(\frac{b\left(\sqrt{c-d\,x^2}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{a-b\,x^2}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

```
[Out] (2*atan((b*((c - d*x^2)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a - b*x^2)^(1/2)
- a^(1/2)))))/(-b*d)^(1/2)
```

3.992 $\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{x\sqrt{2+bx^2}}{b\sqrt{3+dx^2}}-\frac{\sqrt{2}\sqrt{2+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

```
[Out] x*(b*x^2+2)^(1/2)/b/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)
*EllipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)
*(b*x^2+2)^(1/2)/b/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {506, 422}

$$\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqrt[d]*Sqrt[(2 + b*x^2
)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2+bx^2}\,\sqrt{3+dx^2}}\,dx = \frac{x\sqrt{2+bx^2}}{b\sqrt{3+dx^2}} - \frac{3\int \frac{\sqrt{2+bx^2}}{(3+dx^2)^{3/2}}\,dx}{b}$$
$$= \frac{x\sqrt{2+bx^2}}{b\sqrt{3+dx^2}} - \frac{\sqrt{2}\,\sqrt{2+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\,\sqrt{\frac{2+bx^2}{3+dx^2}}\,\sqrt{3+dx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 72, normalized size = 0.65

$$-\frac{i\sqrt{3}\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)-F\left(i\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[3]*(EllipticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] - Ell
ipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)]))/(Sqrt[b]*d)
```

Maple [A]
time = 0.11, size = 70, normalized size = 0.64

method	result
default	$\frac{\left(-\text{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)+\text{EllipticE}\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\right)\sqrt{2}}{\sqrt{-d}\,b}$
elliptic	$-\frac{\sqrt{(b\,x^2+2)\,(d\,x^2+3)}\,\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(\text{EllipticF}\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)-\text{EllipticE}\left(\frac{x\cdots}{}\right.\right.}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\,\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))+Ellip
ticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2)))*2^(1/2)/(-d
)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^2+2}\,\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
[Out] Integral(x**2/(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)),x)
[Out] int(x^2/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)), x)
```

3.993 $\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}}-\frac{c\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

```
[Out] EllipticE(1/2*x,2*(-d/c)^(1/2))*(d*x^2+c)^(1/2)/d/(1+d*x^2/c)^(1/2)-c*Ellip
ticF(1/2*x,2*(-d/c)^(1/2))*(1+d*x^2/c)^(1/2)/d/(d*x^2+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {507, 437, 435, 432, 430}

$$\frac{\sqrt{c+dx^2}\,E\left(\text{ArcSin}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}-\frac{c\sqrt{\frac{dx^2}{c}+1}\,F\left(\text{ArcSin}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d*x^2)/c])
- (c*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[c + d*x^
2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{4-x^2}\,\sqrt{c+dx^2}}\,dx &= \frac{\int \frac{\sqrt{c+dx^2}}{\sqrt{4-x^2}}\,dx}{d} - \frac{c\int \frac{1}{\sqrt{4-x^2}\,\sqrt{c+dx^2}}\,dx}{d}\\
&= \frac{\sqrt{c+dx^2}\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{4-x^2}}\,dx}{d\sqrt{1+\frac{dx^2}{c}}} - \frac{\left(c\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{4-x^2}\,\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}}\\
&= \frac{\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}} - \frac{c\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.52, size = 59, normalized size = 0.68

$$
\frac{c\sqrt{1+\frac{dx^2}{c}}\left(E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)-F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)\right)}{d\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

[Out] (c*Sqrt[1 + (d*x^2)/c]*(EllipticE[ArcSin[x/2], (-4*d)/c] - EllipticF[ArcSin[x/2], (-4*d)/c]))/(d*Sqrt[c + d*x^2])

Maple [A]
time = 0.13, size = 59, normalized size = 0.68

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)+\operatorname{EllipticE}\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\right)c\sqrt{\frac{d\,x^{2}+c}{c}}}{\sqrt{d\,x^{2}+c}\;d}$
elliptic	$-\frac{\sqrt{-\left(d\,x^{2}+c\right)\left(x^{2}-4\right)}\;c\sqrt{1+\frac{d\,x^{2}}{c}}\;\left(\operatorname{EllipticF}\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)-\operatorname{EllipticE}\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)\right)}{\sqrt{d\,x^{2}+c}\;\sqrt{-d\,x^{4}-c\,x^{2}+4d\,x^{2}+4c}\;d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-EllipticF(1/2*x,2*(-d/c)^(1/2))+EllipticE(1/2*x,2*(-d/c)^(1/2)))/(d*x^2+c)^(1/2)*c*((d*x^2+c)/c)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\sqrt{-\left(x-2\right)\left(x+2\right)}\;\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{4-x^2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(x^2/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.994 $\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}-\frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c\,(4+x^2)}}}$$

[Out] x*(d*x^2+c)^(1/2)/d/(x^2+4)^(1/2)-(1/(x^2+4))^(1/2)*EllipticE(x/(x^2+4)^(1/2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2)/d/((d*x^2+c)/c/(x^2+4))^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {506, 422}

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{\sqrt{c+dx^2}\,E\left(\text{ArcTan}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c\,(x^2+4)}}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]

[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*EllipticE[ArcTan[x/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))])

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{4+x^2}\,\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}} - \frac{4\int \frac{\sqrt{c+dx^2}}{(4+x^2)^{3/2}}\,dx}{d}$$
$$= \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}} - \frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\,\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.53, size = 70, normalized size = 0.80

$$-\frac{ic\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\sinh^{-1}\left(\frac{x}{2}\right)|\frac{4d}{c}\right)-F\left(i\sinh^{-1}\left(\frac{x}{2}\right)|\frac{4d}{c}\right)\right)}{d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*c*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[x/2], (4*d)/c] - EllipticF
[I*ArcSinh[x/2], (4*d)/c]))/(d*Sqrt[c + d*x^2])
```

Maple [A]
time = 0.12, size = 76, normalized size = 0.86

method	result
default	$-\frac{2\sqrt{\frac{d\,x^2+c}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{\frac{c}{d}}}{2}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{\frac{c}{d}}}{2}\right)\right)}{\sqrt{d\,x^2+c}\,\sqrt{-\frac{d}{c}}}$
elliptic	$-\frac{2\sqrt{(d\,x^2+c)\,(x^2+4)}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+}{c}}}{2}\right.\right.}{\sqrt{d\,x^2+c}\,\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^4+c\,x^2+4d\,x^2+4c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*(EllipticF(x*(-d/c)^(1/2),1/2*(c/d)^
(1/2))-EllipticE(x*(-d/c)^(1/2),1/2*(c/d)^(1/2)))/(-d/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{c+dx^2}\,\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(x**2/(sqrt(c + d*x**2)*sqrt(x**2 + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{x^2+4}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(x^2/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.995 $\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}(x)|-\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}\,F\left(\sin^{-1}(x)|-\frac{3}{2}\right)$$

```
[Out] 1/3*EllipticE(x,1/2*I*6^(1/2))*2^(1/2)-1/3*EllipticF(x,1/2*I*6^(1/2))*2^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {507, 435, 430}

$$\frac{1}{3}\sqrt{2}\,E\left(\text{ArcSin}(x)\left|-\frac{3}{2}\right.\right)-\frac{1}{3}\sqrt{2}\,F\left(\text{ArcSin}(x)\left|-\frac{3}{2}\right.\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 - (Sqrt[2]*EllipticF[ArcSin[x], -3/2
])/3
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-x^2}}\,dx - \frac{2}{3}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$$
$$= \frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right) - \frac{1}{3}\sqrt{2}\,F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)$$

Mathematica [A]
time = 0.27, size = 24, normalized size = 0.77

$$\frac{1}{3}\sqrt{2}\left(E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right) - F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
[Out] (Sqrt[2]*(EllipticE[ArcSin[x], -3/2] - EllipticF[ArcSin[x], -3/2]))/3
```

Maple [A]
time = 0.13, size = 25, normalized size = 0.81

method	result	size
default	$-\frac{\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{6}}{2}\right)\right)\sqrt{2}}{3}$	25
elliptic	$-\frac{\sqrt{-\left(3x^2+2\right)\left(x^2-1\right)}\,\sqrt{6x^2+4}\,\left(\operatorname{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(x,\frac{i\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^2+2}\,\sqrt{-3x^4+x^2+2}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(EllipticF(x,1/2*I*6^(1/2))-EllipticE(x,1/2*I*6^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [A]
time = 0.21, size = 23, normalized size = 0.74

$$-\frac{\sqrt{3\,x^2+2}\,\sqrt{-x^2+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(x-1)(x+1)}\,\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{1-x^2}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)),x)
[Out] int(x^2/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.996 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}(x)|\frac{3}{2}\right)+\frac{1}{3}\sqrt{2}\,F\left(\sin^{-1}(x)|\frac{3}{2}\right)$$

```
[Out] -1/3*EllipticE(x,1/2*6^(1/2))*2^(1/2)+1/3*EllipticF(x,1/2*6^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {507, 435, 430}

$$\frac{1}{3}\sqrt{2}\,F\left(\text{ArcSin}(x)\,\middle|\,\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}\,E\left(\text{ArcSin}(x)\,\middle|\,\frac{3}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] -1/3*(Sqrt[2]*EllipticE[ArcSin[x], 3/2]) + (Sqrt[2]*EllipticF[ArcSin[x], 3/
2])/3
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{1-x^2}}\,dx\right) + \frac{2}{3}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$$
$$= -\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}(x)|\frac{3}{2}\right) + \frac{1}{3}\sqrt{2}\,F\left(\sin^{-1}(x)|\frac{3}{2}\right)$$

Mathematica [A]
time = 0.27, size = 37, normalized size = 1.19

$$\frac{-E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{2}{3}\right) + F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] (-EllipticE[ArcSin[Sqrt[3/2]*x], 2/3] + EllipticF[ArcSin[Sqrt[3/2]*x], 2/3]
)/Sqrt[3]
```

Maple [A]
time = 0.13, size = 23, normalized size = 0.74

method	result	size
default	$\frac{\sqrt{2}\,\left(\mathrm{EllipticF}\left(x, \frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(x, \frac{\sqrt{6}}{2}\right)\right)}{3}$	23
elliptic	$\frac{\sqrt{(3x^2-2)\,(x^2-1)}\,\sqrt{-6x^2+4}\,\left(\mathrm{EllipticF}\left(x, \frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(x, \frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{-3x^2+2}\,\sqrt{3x^4-5x^2+2}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*2^(1/2)*(EllipticF(x,1/2*6^(1/2))-EllipticE(x,1/2*6^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(x-1)(x+1)}\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(2 - 3*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{1-x^2}\,\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

```
[Out] int(x^2/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.997 $\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

```
[Out] 1/3*EllipticE(1/2*x,I*6^(1/2))*2^(1/2)-1/3*EllipticF(1/2*x,I*6^(1/2))*2^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {507, 435, 430}

$$\frac{1}{3}\sqrt{2}\,E\left(\text{ArcSin}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}\,F\left(\text{ArcSin}\left(\frac{x}{2}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[4 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 - (Sqrt[2]*EllipticF[ArcSin[x/2], -6
])/3
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx = \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{4-x^2}}\,dx - \frac{2}{3}\int \frac{1}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx$$
$$= \frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right) - \frac{1}{3}\sqrt{2}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

Mathematica [A]
time = 0.29, size = 28, normalized size = 0.80

$$\frac{1}{3}\sqrt{2}\left(E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right) - F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[4 - x^2]*Sqrt[2 + 3*x^2]),x]
[Out] (Sqrt[2]*(EllipticE[ArcSin[x/2], -6] - EllipticF[ArcSin[x/2], -6]))/3
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.83

method	result	size
default	$-\frac{\left(\mathrm{EllipticF}\left(\frac{x}{2}, i\sqrt{6}\right)-\mathrm{EllipticE}\left(\frac{x}{2}, i\sqrt{6}\right)\right)\sqrt{2}}{3}$	29
elliptic	$-\frac{\sqrt{-\left(3x^2+2\right)\left(x^2-4\right)}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(\frac{x}{2}, i\sqrt{6}\right)-\mathrm{EllipticE}\left(\frac{x}{2}, i\sqrt{6}\right)\right)}{3\sqrt{3x^2+2}\,\sqrt{-3x^4+10x^2+8}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(EllipticF(1/2*x,I*6^(1/2))-EllipticE(1/2*x,I*6^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)), x)
```

Fricas [A]
time = 0.31, size = 23, normalized size = 0.66

$$-\frac{\sqrt{3\,x^2+2}\,\sqrt{-x^2+4}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-\left(x-2\right)\left(x+2\right)}\,\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(3*x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{4-x^2}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((4 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)),x)
[Out] int(x^2/((4 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.998 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)+\frac{1}{3}\sqrt{2}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)$$

```
[Out] -1/3*EllipticE(1/2*x,6^(1/2))*2^(1/2)+1/3*EllipticF(1/2*x,6^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {507, 435, 430}

$$\frac{1}{3}\sqrt{2}\,F\left(\text{ArcSin}\left(\frac{x}{2}\right)\middle|6\right)-\frac{1}{3}\sqrt{2}\,E\left(\text{ArcSin}\left(\frac{x}{2}\right)\middle|6\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 - x^2]),x]
```

```
[Out] -1/3*(Sqrt[2]*EllipticE[ArcSin[x/2], 6]) + (Sqrt[2]*EllipticF[ArcSin[x/2],
6])/3
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{4-x^2}}\,dx\right)+\frac{2}{3}\int \frac{1}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx$$
$$= -\frac{1}{3}\sqrt{2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)+\frac{1}{3}\sqrt{2}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|6\right)$$

Mathematica [A]
time = 0.26, size = 38, normalized size = 1.09

$$-\frac{2\left(E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{1}{6}\right)-F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{1}{6}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 - x^2]),x]
```

```
[Out] (-2*(EllipticE[ArcSin[Sqrt[3/2]*x], 1/6] - EllipticF[ArcSin[Sqrt[3/2]*x], 1
/6]))/Sqrt[3]
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.94

method	result	size
default	$\frac{2\sqrt{3}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\right)}{3}$	33
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-4)}\,\sqrt{6}\,\sqrt{-6x^2+4}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\right)}{3\sqrt{-3x^2+2}\,\sqrt{3x^4-14x^2+8}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*3^(1/2)*(EllipticF(1/2*x*6^(1/2),1/6*6^(1/2))-EllipticE(1/2*x*6^(1/2),1
/6*6^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+4)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(2 - 3*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{4-x^2}\,\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((4 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

```
[Out] int(x^2/((4 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.999 $\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}-\frac{F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}$$

```
[Out] 1/6*EllipticE(2*x,1/4*I*6^(1/2))*2^(1/2)-1/6*EllipticF(2*x,1/4*I*6^(1/2))*2
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {507, 435, 430}

$$\frac{E\left(\text{ArcSin}(2x)\,\middle|-\frac{3}{8}\right)}{3\sqrt{2}}-\frac{F\left(\text{ArcSin}(2x)\,\middle|-\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticE[ArcSin[2*x], -3/8]/(3*Sqrt[2]) - EllipticF[ArcSin[2*x], -3/8]/(3*
Sqrt[2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) && !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-4x^2}\,\sqrt{2+3x^2}}\,dx = \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-4x^2}}\,dx - \frac{2}{3}\int \frac{1}{\sqrt{1-4x^2}\,\sqrt{2+3x^2}}\,dx$$
$$= \frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}} - \frac{F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}$$

Mathematica [A]
time = 0.27, size = 28, normalized size = 0.80

$$\frac{E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right) - F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 + 3*x^2]),x]
[Out] (EllipticE[ArcSin[2*x], -3/8] - EllipticF[ArcSin[2*x], -3/8])/(3*Sqrt[2])
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.83

method	result	size
default	$-\frac{\left(\mathrm{EllipticF}\left(2x, \frac{i\sqrt{6}}{4}\right)-\mathrm{EllipticE}\left(2x, \frac{i\sqrt{6}}{4}\right)\right)\sqrt{2}}{6}$	29
elliptic	$-\frac{\sqrt{-\left(3x^2+2\right)\left(4x^2-1\right)}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(2x, \frac{i\sqrt{6}}{4}\right)-\mathrm{EllipticE}\left(2x, \frac{i\sqrt{6}}{4}\right)\right)}{6\sqrt{3x^2+2}\,\sqrt{-12x^4-5x^2+2}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/6*(EllipticF(2*x,1/4*I*6^(1/2))-EllipticE(2*x,1/4*I*6^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Fricas [A]
time = 0.27, size = 23, normalized size = 0.66

$$-\frac{\sqrt{3\,x^2+2}\,\sqrt{-4\,x^2+1}}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/12*sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-\left(2x-1\right)\left(2x+1\right)}\,\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(2*x - 1)*(2*x + 1))*sqrt(3*x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\,\sqrt{1-4\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((3*x^2 + 2)^(1/2)*(1 - 4*x^2)^(1/2)),x)
[Out] int(x^2/((3*x^2 + 2)^(1/2)*(1 - 4*x^2)^(1/2)), x)
```

3.1000 $\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{E\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}+\frac{F\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}$$

```
[Out] -1/6*EllipticE(2*x,1/4*6^(1/2))*2^(1/2)+1/6*EllipticF(2*x,1/4*6^(1/2))*2^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {507, 435, 430}

$$\frac{F\left(\text{ArcSin}(2x)\,\middle|\,\frac{3}{8}\right)}{3\sqrt{2}}-\frac{E\left(\text{ArcSin}(2x)\,\middle|\,\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 - 3*x^2]),x]
```

```
[Out] -1/3*EllipticE[ArcSin[2*x], 3/8]/Sqrt[2] + EllipticF[ArcSin[2*x], 3/8]/(3*S
qrt[2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{1-4x^2}}\,dx\right)+\frac{2}{3}\int \frac{1}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx$$
$$= -\frac{E\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}+\frac{F\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}$$

Mathematica [**A**]
time = 0.26, size = 28, normalized size = 0.80

$$\frac{-E\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)+F\left(\sin^{-1}(2x)\middle|\frac{3}{8}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 - 3*x^2]),x]
[Out] (-EllipticE[ArcSin[2*x], 3/8] + EllipticF[ArcSin[2*x], 3/8])/(3*Sqrt[2])
```

Maple [**A**]
time = 0.13, size = 27, normalized size = 0.77

method	result	size
default	$\frac{\sqrt{2}\left(\text{EllipticF}\left(2x,\frac{\sqrt{6}}{4}\right)-\text{EllipticE}\left(2x,\frac{\sqrt{6}}{4}\right)\right)}{6}$	27
elliptic	$\frac{\sqrt{(3x^2-2)(4x^2-1)}\,\sqrt{-6x^2+4}\,\left(\text{EllipticF}\left(2x,\frac{\sqrt{6}}{4}\right)-\text{EllipticE}\left(2x,\frac{\sqrt{6}}{4}\right)\right)}{6\sqrt{-3x^2+2}\,\sqrt{12x^4-11x^2+2}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*(EllipticF(2*x,1/4*6^(1/2))-EllipticE(2*x,1/4*6^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(2x-1)(2x+1)}\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(2*x - 1)*(2*x + 1))*sqrt(2 - 3*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{2-3\,x^2}\,\sqrt{1-4\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2 - 3*x^2)^(1/2)*(1 - 4*x^2)^(1/2)),x)
[Out] int(x^2/((2 - 3*x^2)^(1/2)*(1 - 4*x^2)^(1/2)), x)
```

3.1001 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=42

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)-1/3*EllipticF(1/2*x*6^(1
/2),1/3*I*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {507, 435, 430}

$$\frac{E\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}-\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3] - EllipticF[ArcSin[Sqrt[3/2]*x
], -2/3]/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = -\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx + \int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx$$
$$= \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}} - \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.28, size = 37, normalized size = 0.88

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right) - F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] (EllipticE[ArcSin[Sqrt[3/2]*x], -2/3] - EllipticF[ArcSin[Sqrt[3/2]*x], -2/3
])/Sqrt[3]
```

Maple [A]
time = 0.12, size = 35, normalized size = 0.83

method	result	size
default	$-\frac{\sqrt{3}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\right)}{3}$	35
elliptic	$-\frac{\sqrt{-\left(3x^2-2\right)\left(x^2+1\right)}\,\sqrt{6}\,\sqrt{-6x^2+4}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\right)}{6\sqrt{-3x^2+2}\,\sqrt{-3x^4-x^2+2}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*3^(1/2)*(EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))-EllipticE(1/2*x*6^(1/2
),1/3*I*6^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [A]
time = 0.15, size = 21, normalized size = 0.50

$$-\frac{\sqrt{x^2+1}\,\sqrt{-3\,x^2+2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{2-3x^2}\,\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(2 - 3*x**2)*sqrt(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{x^2+1}\,\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
[Out] int(x^2/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.1002 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*I*6^(1/2))*3^(1/2)-2/3*EllipticF(1/2*x*6^(1
/2),1/6*I*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {507, 435, 430}

$$\frac{2E\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}-\frac{2F\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 + x^2]),x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3] - (2*EllipticF[ArcSin[Sqrt
[3/2]*x], -1/6])/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) && !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx = -\left(4\int \frac{1}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx\right)+\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx$$
$$= \frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}} - \frac{2F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.26, size = 38, normalized size = 0.88

$$\frac{2\left(E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)-F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{1}{6}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 + x^2]),x]
```

```
[Out] (2*(EllipticE[ArcSin[Sqrt[3/2]*x], -1/6] - EllipticF[ArcSin[Sqrt[3/2]*x], -
1/6]))/Sqrt[3]
```

Maple [A]
time = 0.13, size = 35, normalized size = 0.81

method	result	size
default	$-\frac{2\sqrt{3}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)\right)}{3}$	35
elliptic	$-\frac{\sqrt{-\left(3x^2-2\right)\left(x^2+4\right)}\,\sqrt{6}\,\sqrt{-6x^2+4}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)\right)}{3\sqrt{-3x^2+2}\,\sqrt{-3x^4-10x^2+8}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*3^(1/2)*(EllipticF(1/2*x*6^(1/2),1/6*I*6^(1/2))-EllipticE(1/2*x*6^(1/2
),1/6*I*6^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Fricas [A]
time = 0.13, size = 21, normalized size = 0.49

$$-\frac{\sqrt{x^2+4}\,\sqrt{-3\,x^2+2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{2-3x^2}\,\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+4)**(1/2),x)
[Out] Integral(x**2/(sqrt(2 - 3*x**2)*sqrt(x**2 + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{x^2+4}\,\sqrt{2-3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 4)^(1/2)*(2 - 3*x^2)^(1/2)),x)
[Out] int(x^2/((x^2 + 4)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.1003 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}$$

```
[Out] 1/12*EllipticE(1/2*x*6^(1/2),2/3*I*6^(1/2))*3^(1/2)-1/12*EllipticF(1/2*x*6^
(1/2),2/3*I*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {507, 435, 430}

$$\frac{E\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}-\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/(4*Sqrt[3]) - EllipticF[ArcSin[Sqrt[3/
2]*x], -8/3]/(4*Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx &= -\left(\frac{1}{4}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx\right)+\frac{1}{4}\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx\\ &= \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}-\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 40, normalized size = 0.85

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{8}{3}\right)-F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{8}{3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + 4*x^2]),x]
[Out] (EllipticE[ArcSin[Sqrt[3/2]*x], -8/3] - EllipticF[ArcSin[Sqrt[3/2]*x], -8/3
])/(4*Sqrt[3])
```

Maple [A]
time = 0.13, size = 35, normalized size = 0.74

method	result	size
default	$-\frac{\sqrt{3}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)\right)}{12}$	35
elliptic	$-\frac{\sqrt{-\left(3x^2-2\right)\left(4x^2+1\right)}\,\sqrt{6}\,\sqrt{-6x^2+4}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)\right)}{24\sqrt{-3x^2+2}\,\sqrt{-12x^4+5x^2+2}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/12*3^(1/2)*(EllipticF(1/2*x*6^(1/2),2/3*I*6^(1/2))-EllipticE(1/2*x*6^(1/
2),2/3*I*6^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [A]
time = 0.23, size = 23, normalized size = 0.49

$$-\frac{\sqrt{4\,x^2+1}\,\sqrt{-3\,x^2+2}}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/12*sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{2-3x^2}\,\sqrt{4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
[Out] Integral(x**2/(sqrt(2 - 3*x**2)*sqrt(4*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{2-3\,x^2}\,\sqrt{4\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2 - 3*x^2)^(1/2)*(4*x^2 + 1)^(1/2)),x)
[Out] int(x^2/((2 - 3*x^2)^(1/2)*(4*x^2 + 1)^(1/2)), x)
```

3.1004 $\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}}-\frac{\sqrt{2}\sqrt{2+3x^2}\,E\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

```
[Out] 1/3*x*(3*x^2+2)^(1/2)/(x^2+1)^(1/2)-1/3*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+
1)^(1/2),1/2*I*2^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {506, 422}

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{3x^2+2}\,E\left(\text{ArcTan}(x)\,\middle|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}} - \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{(1+x^2)^{3/2}}\,dx$$
$$= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}\,E\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 34, normalized size = 0.42

$$-\frac{1}{3}i\sqrt{2}\left(E\left(i\sinh^{-1}(x)\middle|\frac{3}{2}\right) - F\left(i\sinh^{-1}(x)\middle|\frac{3}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
[Out] (-1/3*I)*Sqrt[2]*(EllipticE[I*ArcSinh[x], 3/2] - EllipticF[I*ArcSinh[x], 3/
2])
```

Maple [A]
time = 0.11, size = 30, normalized size = 0.38

method	result	size
default	$\frac{i\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)\sqrt{2}}{3}$	30
elliptic	$\frac{i\sqrt{(3x^2+2)(x^2+1)}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^2+2}\,\sqrt{3x^4+5x^2+2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*I*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2+1}\,\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(x**2 + 1)*sqrt(3*x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{x^2+1}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] int(x^2/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.1005 $\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=82

$$\frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}}-\frac{\sqrt{2}\sqrt{2+3x^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}$$

```
[Out] 1/3*x*(3*x^2+2)^(1/2)/(x^2+4)^(1/2)-1/3*(1/(x^2+4))^(1/2)*EllipticE(x/(x^2+
4)^(1/2),I*5^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(x^2+4))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {506, 422}

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+4}}-\frac{\sqrt{2}\sqrt{3x^2+2}\,E\left(\text{ArcTan}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[4 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}} - \frac{4}{3}\int \frac{\sqrt{2+3x^2}}{(4+x^2)^{3/2}}\,dx$$
$$= \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 38, normalized size = 0.46

$$-\frac{1}{3}i\sqrt{2}\left(E\left(i\sinh^{-1}\left(\frac{x}{2}\right)\middle|6\right)-F\left(i\sinh^{-1}\left(\frac{x}{2}\right)\middle|6\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[4 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (-1/3*I)*Sqrt[2]*(EllipticE[I*ArcSinh[x/2], 6] - EllipticF[I*ArcSinh[x/2],
6])
```

Maple [A]
time = 0.11, size = 26, normalized size = 0.32

method	result	size
default	$\frac{i\left(\text{EllipticF}\left(\frac{ix}{2},\sqrt{6}\right)-\text{EllipticE}\left(\frac{ix}{2},\sqrt{6}\right)\right)\sqrt{2}}{3}$	26
elliptic	$\frac{i\sqrt{(3x^2+2)(x^2+4)}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(\frac{ix}{2},\sqrt{6}\right)-\text{EllipticE}\left(\frac{ix}{2},\sqrt{6}\right)\right)}{3\sqrt{3x^2+2}\,\sqrt{3x^4+14x^2+8}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*(EllipticF(1/2*I*x,6^(1/2))-EllipticE(1/2*I*x,6^(1/2)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2+4}\,\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(x**2 + 4)*sqrt(3*x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{x^2+4}\,\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 4)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] int(x^2/((x^2 + 4)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.1006 $\int \frac{x^2}{\sqrt{2+3x^2}\,\sqrt{1+4x^2}}\,dx$

Optimal. Leaf size=88

$$\frac{x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}}-\frac{\sqrt{2+3x^2}\,E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{2}\,\sqrt{\frac{2+3x^2}{1+4x^2}}\,\sqrt{1+4x^2}}$$

```
[Out] 1/3*x*(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2)-1/6*(1/(4*x^2+1))^(1/2)*EllipticE(2*x
/(4*x^2+1)^(1/2),1/4*10^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(4*x^2+1)
)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {506, 422}

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{\sqrt{3x^2+2}\,E\left(\text{ArcTan}(2x)\,|\,\frac{5}{8}\right)}{3\sqrt{2}\,\sqrt{\frac{3x^2+2}{4x^2+1}}\,\sqrt{4x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[2 + 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (Sqrt[2 + 3*x^2]*EllipticE[ArcTan
[2*x], 5/8])/(3*Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} - \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{(1+4x^2)^{3/2}}\,dx$$
$$= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} - \frac{\sqrt{2+3x^2}\,E\left(\tan^{-1}(2x)\middle|\frac{5}{8}\right)}{3\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 50, normalized size = 0.57

$$-\frac{i\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\frac{8}{3}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\frac{8}{3}\right)\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[2 + 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

```
[Out] ((-1/4*I)*(EllipticE[I*ArcSinh[Sqrt[3/2]*x], 8/3] - EllipticF[I*ArcSinh[Sqr
t[3/2]*x], 8/3]))/Sqrt[3]
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 36, normalized size = 0.41

method	result	size
default	$\frac{i\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)-\operatorname{EllipticE}\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)\right)\sqrt{3}}{12}$	36
elliptic	$\frac{i\sqrt{(3x^2+2)(4x^2+1)}\,\sqrt{6}\,\sqrt{6x^2+4}\,\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)-\operatorname{EllipticE}\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)\right)}{24\sqrt{3x^2+2}\,\sqrt{12x^4+11x^2+2}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*(EllipticF(1/2*I*x*6^(1/2),2/3*6^(1/2))-EllipticE(1/2*I*x*6^(1/2),2/
3*6^(1/2)))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{3x^2+2}\,\sqrt{4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(3*x**2 + 2)*sqrt(4*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{3\,x^2+2}\,\sqrt{4\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((3*x^2 + 2)^(1/2)*(4*x^2 + 1)^(1/2)),x)
```

```
[Out] int(x^2/((3*x^2 + 2)^(1/2)*(4*x^2 + 1)^(1/2)), x)
```

3.1007 $\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2}E\left(\cos^{-1}(x)\middle|\,2\right)-\frac{1}{2}F\left(\cos^{-1}(x)\middle|\,2\right)$$

```
[Out] -1/2*(x^2)^(1/2)/x*EllipticE((-x^2+1)^(1/2),2^(1/2))-1/2*(x^2)^(1/2)/x*Elli
pticF((-x^2+1)^(1/2),2^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {507, 436, 431}

$$-\frac{F(\text{ArcCos}(x)|2)}{2}-\frac{1}{2}E(\text{ArcCos}(x)|2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] -1/2*EllipticE[ArcCos[x], 2] - EllipticF[ArcCos[x], 2]/2
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rule 436

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx + \frac{1}{2}\int \frac{\sqrt{-1+2x^2}}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{1}{2}E\left(\cos^{-1}(x)\middle|\,2\right) - \frac{1}{2}F\left(\cos^{-1}(x)\middle|\,2\right)$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.27, size = 37, normalized size = 2.18

$$\frac{\sqrt{1-2x^2}\,(-E(\sin^{-1}(x)|\,2) + F(\sin^{-1}(x)|\,2))}{2\sqrt{-1+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 - 2*x^2]*(-EllipticE[ArcSin[x], 2] + EllipticF[ArcSin[x], 2]))/(2*S
qrt[-1 + 2*x^2])
```

Maple [**A**]
time = 0.14, size = 34, normalized size = 2.00

method	result	size
default	$\frac{\left(\mathrm{EllipticF}\left(x,\sqrt{2}\right)-\mathrm{EllipticE}\left(x,\sqrt{2}\right)\right)\sqrt{-2x^2+1}}{2\sqrt{2x^2-1}}$	34
elliptic	$\frac{\sqrt{-\left(2x^2-1\right)\left(x^2-1\right)}\,\sqrt{-2x^2+1}\,\left(\mathrm{EllipticF}\left(x,\sqrt{2}\right)-\mathrm{EllipticE}\left(x,\sqrt{2}\right)\right)}{2\sqrt{2x^2-1}\,\sqrt{-2x^4+3x^2-1}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(EllipticF(x,2^(1/2))-EllipticE(x,2^(1/2)))*(-2*x^2+1)^(1/2)/(2*x^2-1)^
(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Fricas [A]
time = 0.53, size = 23, normalized size = 1.35

$$-\frac{\sqrt{2\,x^2-1}\,\sqrt{-x^2+1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(x-1)(x+1)}\,\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(2*x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^2}{\sqrt{1-x^2}\,\sqrt{2\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)),x)
[Out] int(x^2/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)), x)
```

3.1008 $\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=109

$$\frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{9\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}}-\frac{9\log\left(3+x^2\right)}{4\,2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\,2^{2/3}}$$

```
[Out] 3/2*(-x^2+1)^(2/3)+3/10*(-x^2+1)^(5/3)-9/8*ln(x^2+3)*2^(1/3)+27/8*ln(2^(2/3
)-(-x^2+1)^(1/3))*2^(1/3)+9/4*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1
/2)*2^(1/3)
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 90, 57, 631, 210, 31}

$$\frac{9\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\,2^{2/3}}+\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{3}{2}\left(1-x^2\right)^{2/3}-\frac{9\log\left(x^2+3\right)}{4\,2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*(1 - x^2)^(2/3))/2 + (3*(1 - x^2)^(5/3))/10 + (9*Sqrt[3]*ArcTan[(1 + (2
- 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - (9*Log[3 + x^2])/(4*2^(2/3)) + (27*
Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}\,(3+x)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{2}{\sqrt[3]{1-x}}-(1-x)^{2/3}+\frac{9}{\sqrt[3]{1-x}\,(3+x)}\right)dx,x,x^2\right)\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx,x,x^2\right)\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}-\frac{9\log\left(3+x^2\right)}{4\ 2^{2/3}}+\frac{27}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2\right.\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}-\frac{9\log\left(3+x^2\right)}{4\ 2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{27S}{}\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{9\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}-\frac{9\log\left(3+x^2\right)}{4\ 2^{2/3}}+
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 121, normalized size = 1.11

$$\frac{1}{40}\left(72\left(1-x^2\right)^{2/3}-12x^2\left(1-x^2\right)^{2/3}+90\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+90\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-45\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (72*(1 - x^2)^(2/3) - 12*x^2*(1 - x^2)^(2/3) + 90*2^(1/3)*Sqrt[3]*ArcTan[(1
 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 90*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 4
5*2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)])/40
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.51, size = 482, normalized size = 4.42

method	result
risch	$\frac{3(x^2-6)(x^2-1)}{10(-x^2+1)^{\frac{1}{3}}}+\frac{9\,\mathrm{RootOf}\left(\_Z^3-2\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\_Z^3-2\right)^3\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-2\right)^2+4\_Z\,\mathrm{RootOf}\left(\_Z^3-2\right)+16\_Z^2\right)}{}\right.}{}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/10*(x^2-6)*(x^2-1)/(-x^2+1)^(1/3)+9/4*RootOf(_Z^3-2)*ln(-(8*RootOf(_Z^3-2
)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+48*RootOf(Root
Of(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-84*(-x^2+1
)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)
-RootOf(_Z^3-2)*x^2-6*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*
x^2-21*(-x^2+1)^(2/3)+21*RootOf(_Z^3-2)+126*RootOf(RootOf(_Z^3-2)^2+4*_Z*Ro
otOf(_Z^3-2)+16*_Z^2))/(x^2+3))+9*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-
2)+16*_Z^2)*ln((24*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^
3-2)+16*_Z^2)*x^2+64*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2
*RootOf(_Z^3-2)^2*x^2+168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootO
f(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)+15*RootOf(_Z^3-2)*x^2+40*RootOf(RootOf(_Z
^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+42*(-x^2+1)^(2/3)-63*RootOf(_Z^3-2
)-168*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2))/(x^2+3))
```

Maxima [A]
time = 0.48, size = 108, normalized size = 0.99

$$\frac{9}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}}-\frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{9}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{3}{2}\left(-x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] 9/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) + 3/10*(-x^2 + 1)^(5/3) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1
```

```
)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
+ 3/2*(-x^2 + 1)^(2/3)
```

Fricas [A]
time = 0.89, size = 102, normalized size = 0.94

$$-\frac{3}{10}\left(x^{2}-6\right)\left(-x^{2}+1\right)^{\frac{2}{3}}+\frac{9}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right)-\frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)+\frac{9}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -3/10*(x^2 - 6)*(-x^2 + 1)^(2/3) + 9/4*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*s
qrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/3))) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)
*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(-4^(1/3) + (-x^2 +
1)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\,\left(x^{2}+3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [A]
time = 0.62, size = 108, normalized size = 0.99

$$\frac{9}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{10}\left(-x^{2}+1\right)^{\frac{5}{3}}-\frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)+\frac{9}{8}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^{2}+1\right)^{\frac{1}{3}}\right)+\frac{3}{2}\left(-x^{2}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] 9/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) + 3/10*(-x^2 + 1)^(5/3) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1
)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) +
 3/2*(-x^2 + 1)^(2/3)
```

Mupad [B]
time = 0.50, size = 128, normalized size = 1.17

$$\frac{9\,2^{1/3}\,\ln\left(\frac{729\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{729\,2^{2/3}}{4}\right)}{4}+\frac{3\,{\left(1-x^2\right)}^{2/3}}{2}+\frac{3\,{\left(1-x^2\right)}^{5/3}}{10}+\frac{9\,2^{1/3}\,\ln\left(\frac{729\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{729\,2^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}-\frac{9\,2^{1/3}\,\ln\left(\frac{729\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{729\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] (9*2^(1/3)*log((729*(1 - x^2)^(1/3))/4 - (729*2^(2/3))/4))/4 + (3*(1 - x^2)
^(2/3))/2 + (3*(1 - x^2)^(5/3))/10 + (9*2^(1/3)*log((729*(1 - x^2)^(1/3))/4
 - (729*2^(2/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/8 - (9*2^(1/3)*lo
g((729*(1 - x^2)^(1/3))/4 - (729*2^(2/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1
i + 1))/8
```

3.1009 $\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=94

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{3\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\;2^{2/3}}+\frac{3\log\left(3+x^2\right)}{4\;2^{2/3}}-\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\;2^{2/3}}$$

```
[Out] -3/4*(-x^2+1)^(2/3)+3/8*ln(x^2+3)*2^(1/3)-9/8*ln(2^(2/3)-(-x^2+1)^(1/3))*2^
(1/3)-3/4*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 81, 57, 631, 210, 31}

$$-\frac{3\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\;2^{2/3}}-\frac{3}{4}\left(1-x^2\right)^{2/3}+\frac{3\log\left(x^2+3\right)}{4\;2^{2/3}}-\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\;2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]
)/(2*2^(2/3)) + (3*Log[3 + x^2])/(4*2^(2/3)) - (9*Log[2^(2/3) - (1 - x^2)^(
1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)\\
&= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)\\
&= -\frac{3}{4}\left(1-x^2\right)^{2/3} + \frac{3\log\left(3+x^2\right)}{4\ 2^{2/3}} - \frac{9}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right.\\
&= -\frac{3}{4}\left(1-x^2\right)^{2/3} + \frac{3\log\left(3+x^2\right)}{4\ 2^{2/3}} - \frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}} + \frac{9\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{2}\\
&= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{3\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} + \frac{3\log\left(3+x^2\right)}{4\ 2^{2/3}} - \frac{9\log\left(2^{2/3}-\right.}{4\ 2^{2/}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 105, normalized size = 1.12

$$-\frac{3}{8}\left(2\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(2*(1 - x^2)^(2/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/S
qrt[3]] + 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1/3)*Log[4 + 2*(2 - 2*
x^2)^(1/3) + (2 - 2*x^2)^(2/3)]))/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.61, size = 648, normalized size = 6.89

method	result	size
risch	Expression too large to display	648
trager	Expression too large to display	749

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*(x^2-1)/(-x^2+1)^(1/3)-3/4*ln((16*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_
Z^3+2)+16*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-8*RootOf(_Z^3+2)^3*x^2*RootOf(RootOf
(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)+21*RootOf(_Z^3+2)^2*(-x^2+1)^(1/3)-
10*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)*x^2+5*RootOf(_Z^3+2
)*x^2+21*(-x^2+1)^(2/3)+42*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_
Z^2)-21*RootOf(_Z^3+2))/(x^2+3))*RootOf(_Z^3+2)-3*ln((16*RootOf(RootOf(_Z^3
+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-8*RootOf(_Z^3+2)^
3*x^2*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)+21*RootOf(_Z^3+2
)^2*(-x^2+1)^(1/3)-10*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)*
x^2+5*RootOf(_Z^3+2)*x^2+21*(-x^2+1)^(2/3)+42*RootOf(RootOf(_Z^3+2)^2+4*_Z*
RootOf(_Z^3+2)+16*_Z^2)-21*RootOf(_Z^3+2))/(x^2+3))*RootOf(RootOf(_Z^3+2)^2
+4*_Z*RootOf(_Z^3+2)+16*_Z^2)+3*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)
+16*_Z^2)*ln((32*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)^2*Roo
tOf(_Z^3+2)^2*x^2+24*RootOf(_Z^3+2)^3*x^2*RootOf(RootOf(_Z^3+2)^2+4*_Z*Root
Of(_Z^3+2)+16*_Z^2)+42*RootOf(_Z^3+2)^2*(-x^2+1)^(1/3)+4*RootOf(RootOf(_Z^3
+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)*x^2+3*RootOf(_Z^3+2)*x^2+42*(-x^2+1)^(2/
3)-84*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)-63*RootOf(_Z^3+2
))/(x^2+3))
```

Maxima [A]
time = 0.48, size = 97, normalized size = 1.03

$$-\frac{3}{8} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3} \left(4^{\frac{1}{3}} + 2\left(-x^{2} + 1\right)^{\frac{1}{3}}\right)\right) + \frac{3}{16} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}\left(-x^{2} + 1\right)^{\frac{1}{3}} + \left(-x^{2} + 1\right)^{\frac{2}{3}}\right) - \frac{3}{8} \cdot 4^{\frac{2}{3}} \log\left(-4^{\frac{1}{3}} + \left(-x^{2} + 1\right)^{\frac{1}{3}}\right) - \frac{3}{4}\left(-x^{2} + 1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] -3/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) + 3/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) - 3/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3)
```

Fricas [A]
time = 0.89, size = 122, normalized size = 1.30

$$-\frac{3}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2\,(-1)^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\right)\right)-\frac{3}{16}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)+\frac{3}{8}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^{2}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -3/4*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*(-1)^(1/3)*(-
x^2 + 1)^(1/3) - 4^(1/3))) - 3/16*4^(2/3)*(-1)^(1/3)*log(4^(1/3)*(-1)^(2/3)
*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3)) + 3/8*4^(2/3)*(-
1)^(1/3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\,\left(x^{2}+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [A]
time = 0.59, size = 97, normalized size = 1.03

$$-\frac{3}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)-\frac{3}{8}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^{2}+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^{2}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] -3/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) + 3/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) - 3/8*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3)
```

Mupad [B]
time = 0.48, size = 117, normalized size = 1.24

$$-\frac{3\,2^{1/3}\,\ln\left(\frac{81\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{81\,2^{2/3}}{4}\right)}{4}-\frac{3\,{\left(1-x^2\right)}^{2/3}}{4}-\frac{3\,2^{1/3}\,\ln\left(\frac{81\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{81\,2^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}+\frac{3\,2^{1/3}\,\ln\left(\frac{81\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{81\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] (3*2^(1/3)*log((81*(1 - x^2)^(1/3))/4 - (81*2^(2/3)*(3^(1/2)*1i + 1)^2)/16)
*(3^(1/2)*1i + 1))/8 - (3*(1 - x^2)^(2/3))/4 - (3*2^(1/3)*log((81*(1 - x^2)
^(1/3))/4 - (81*2^(2/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/8 - (3*2^
(1/3)*log((81*(1 - x^2)^(1/3))/4 - (81*2^(2/3))/4))/4
```

3.1010 $\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}-\frac{\log(3+x^2)}{4\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

```
[Out] -1/8*ln(x^2+3)*2^(1/3)+3/8*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+1/4*arctan(1/
3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {455, 57, 631, 210, 31}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}-\frac{\log(x^2+3)}{4\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - Log[3 + x^2
]/(4*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$

$$= -\frac{\log(3+x^2)}{4\ 2^{2/3}} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right) - \frac{3\text{Subst}\left(\int \frac{1}{2^{2/}\ldots}\right.}{\ldots}$$

$$= -\frac{\log(3+x^2)}{4\ 2^{2/3}} + \frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\sqrt[3]{2-2x^2}\ldots\right.}{2\ 2^{2/3}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} - \frac{\log(3+x^2)}{4\ 2^{2/3}} + \frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

Mathematica [A]
time = 0.08, size = 82, normalized size = 1.04

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+2\log\left(-2+\sqrt[3]{2-2x^2}\right)-\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)}{4\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 2*Log[-2 + (2 - 2*x^2)
^(1/3)] - Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)])/(4*2^(2/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.58, size = 736, normalized size = 9.32

method	result	size
trager	Expression too large to display	736

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*RootOf(_Z^3-2)*ln(-(96*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_
Z^2)^2*RootOf(_Z^3-2)^2*x^2+8*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z
*RootOf(_Z^3-2)+16*_Z^2)*x^2+168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_
Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)+60*RootOf(RootOf(_Z^3-2)^2+4*_Z*Ro
otOf(_Z^3-2)+16*_Z^2)*x^2+42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)+5*RootOf(_Z^3-
2)*x^2-42*(-x^2+1)^(2/3)-252*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16
*_Z^2)-21*RootOf(_Z^3-2))/(x^2+3))-1/4*ln((64*RootOf(RootOf(_Z^3-2)^2+4*_Z*
RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-8*RootOf(_Z^3-2)^3*RootOf(Ro
otOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-168*(-x^2+1)^(1/3)*RootOf(R
ootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)-8*RootOf(RootOf
(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-42*RootOf(_Z^3-2)^2*(-x^2+1)^(1
/3)+RootOf(_Z^3-2)*x^2+42*(-x^2+1)^(2/3)+168*RootOf(RootOf(_Z^3-2)^2+4*_Z*R
ootOf(_Z^3-2)+16*_Z^2)-21*RootOf(_Z^3-2))/(x^2+3))*RootOf(_Z^3-2)-ln((64*Ro
otOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-8
*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-
168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*Roo
tOf(_Z^3-2)-8*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-42*R
ootOf(_Z^3-2)^2*(-x^2+1)^(1/3)+RootOf(_Z^3-2)*x^2+42*(-x^2+1)^(2/3)+168*Roo
tOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-21*RootOf(_Z^3-2))/(x^2+3
))*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)
```

Maxima [A]
time = 0.49, size = 86, normalized size = 1.09

$$\frac{1}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] 1/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/
3)) + 1/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
```

Fricas [A]
time = 0.75, size = 86, normalized size = 1.09

$$\frac{1}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 1/4*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/3
))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3
)) + 1/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
```

Sympy [A]
time = 41.52, size = 78, normalized size = 0.99

$$\begin{cases} \sqrt[3]{2}\left(\frac{\log\left(\sqrt[3]{2-2x^2}-2\right)}{4}-\frac{\log\left((2-2x^2)^{\frac{2}{3}}+2\sqrt[3]{2-2x^2}+4\right)}{8}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[3]{2-2x^2}+1\right)}{3}\right)}{4}\right) & \text{for } x>-1\wedge x<1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Piecewise((2**(1/3)*(log((2 - 2*x**2)**(1/3) - 2)/4 - log((2 - 2*x**2)**(2/
3) + 2*(2 - 2*x**2)**(1/3) + 4)/8 + sqrt(3)*atan(sqrt(3)*((2 - 2*x**2)**(1/
3) + 1)/3)/4), (x > -1) & (x < 1)))
```

Giac [A]
time = 0.55, size = 86, normalized size = 1.09

$$\frac{1}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] 1/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/
3)) + 1/8*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3))
```

Mupad [B]
time = 0.59, size = 106, normalized size = 1.34

$$\frac{2^{1/3}\,\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9\,2^{2/3}}{4}\right)}{4}+\frac{2^{1/3}\,\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9\,2^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{8}-\frac{2^{1/3}\,\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9\,2^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] (2^(1/3)*log((9*(1 - x^2)^(1/3))/4 - (9*2^(2/3))/4))/4 + (2^(1/3)*log((9*(1
 - x^2)^(1/3))/4 - (9*2^(2/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/8 -
 (2^(1/3)*log((9*(1 - x^2)^(1/3))/4 - (9*2^(2/3)*(3^(1/2)*1i + 1)^2)/16)*(3
^(1/2)*1i + 1))/8
```

3.1011 $\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=136

$$-\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{6}+\frac{\log\left(3+x^2\right)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}\right.}{4}$$

[Out] -1/6*ln(x)+1/24*ln(x^2+3)*2^(1/3)+1/4*ln(1-(-x^2+1)^(1/3))-1/8*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)-1/12*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)+1/6*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {457, 88, 57, 632, 210, 31, 631}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(x^2+3\right)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]

[Out] -1/2*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x]/6 + Log[3 + x^2]/(12*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/4 - Log[2^(2/3) - (1 - x^2)^(1/3)]/(4*2^(2/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
```

```
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(3+x)}\,dx, x, x^2\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^2\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{6} + \frac{\log(3+x^2)}{12\ 2^{2/3}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{}{1+}\right.\\
&= -\frac{\log(x)}{6} + \frac{\log(3+x^2)}{12\ 2^{2/3}} + \frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}} - \frac{1}{2}\text{S}\\
&= -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log(x)}{6} + \frac{\log(3+x^2)}{12\ 2^{2/3}} + \frac{1}{4}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 163, normalized size = 1.20

$$\frac{1}{24}\left(-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)+\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right)+4\log\left(-1+\sqrt[3]{1-x^2}\right)-2\log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 4*Sqrt[3]*Arc
Tan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)
] + 2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)] + 4*Log[-1 + (
1 - x^2)^(1/3)] - 2*Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/24
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x), x)
```

Fricas [A]
time = 0.68, size = 177, normalized size = 1.30

$$-\frac{1}{12}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}\left(-1\right)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{48}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{24}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{12}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/12*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(2*sqrt(3)*(-1)^(1/3)*(
-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) - 1/48*4^(2/3)*(-1)^(1/3)*log(4^(1/3)*(
-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3)) + 1/24*
4^(2/3)*(-1)^(1/3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) + 1/6*sqrt(3
```

```
)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/12*log((-x^2 + 1)^
(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/6*log((-x^2 + 1)^(1/3) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [A]
time = 0.57, size = 149, normalized size = 1.10

$$-\frac{1}{24}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{48}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{12}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] -1/24*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) + 1/48*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) - 1/24*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/6*sqrt(3)*arctan(1
/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/12*log((-x^2 + 1)^(2/3) + (-x^2 +
1)^(1/3) + 1) + 1/6*log(-(-x^2 + 1)^(1/3) + 1)
```

Mupad [B]
time = 0.55, size = 256, normalized size = 1.88

$$\frac{\ln\left(\frac{405}{8}-\frac{405\,(1-x^2)^{1/3}}{8}\right)}{6}+\ln\left(\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)^3\left(393660\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)^2-\frac{37179\,(1-x^2)^{1/3}}{4}\right)-\frac{243\,(1-x^2)^{1/3}}{32}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)-\ln\left(-\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)^3\left(393660\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)^2-\frac{37179\,(1-x^2)^{1/3}}{4}\right)-\frac{243\,(1-x^2)^{1/3}}{32}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)-\frac{2^{1/3}\ln\left(\frac{405\,(1-x^2)^{1/3}}{128}-\frac{405\,2^{2/3}}{128}\right)}{12}+\frac{(-1)^{1/3}\,2^{1/3}\ln\left(\frac{405\,(1-x^2)^{1/3}}{128}-\frac{405\,(-1)^{2/3}\,2^{2/3}}{128}\right)}{12}-\frac{(-1)^{1/3}\,2^{1/3}\ln\left(-\frac{\left(1+\sqrt{3}\,1i\right)^3\left(\frac{37179\,(1-x^2)^{1/3}}{4}-\frac{10935\,(-1)^{2/3}\,2^{2/3}\left(1+\sqrt{3}\,1i\right)^2}{16}\right)}{6912}-\frac{243\,(1-x^2)^{1/3}}{32}\right)\left(1+\sqrt{3}\,1i\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] log(405/8 - (405*(1 - x^2)^(1/3))/8)/6 + log(((3^(1/2)*1i)/12 - 1/12)^3*(39
3660*((3^(1/2)*1i)/12 - 1/12)^2 - (37179*(1 - x^2)^(1/3))/4) - (243*(1 - x^
2)^(1/3))/32)*((3^(1/2)*1i)/12 - 1/12) - log(- ((3^(1/2)*1i)/12 + 1/12)^3*(
393660*((3^(1/2)*1i)/12 + 1/12)^2 - (37179*(1 - x^2)^(1/3))/4) - (243*(1 -
x^2)^(1/3))/32)*((3^(1/2)*1i)/12 + 1/12) - (2^(1/3)*log((405*(1 - x^2)^(1/3
))/128 - (405*2^(2/3))/128))/12 + ((-1)^(1/3)*2^(1/3)*log((405*(1 - x^2)^(1
/3))/128 - (405*(-1)^(2/3)*2^(2/3))/128))/12 - ((-1)^(1/3)*2^(1/3)*log(- ((
3^(1/2)*1i + 1)^3*((37179*(1 - x^2)^(1/3))/4 - (10935*(-1)^(2/3)*2^(2/3)*(3
^(1/2)*1i + 1)^2)/16))/6912 - (243*(1 - x^2)^(1/3))/32)*(3^(1/2)*1i + 1))/2
4
```

3.1012 $\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=97

$$-\frac{(1-x^2)^{2/3}}{6x^2}+\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{\log(3+x^2)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}$$

```
[Out] -1/6*(-x^2+1)^(2/3)/x^2-1/72*ln(x^2+3)*2^(1/3)+1/24*ln(2^(2/3)-(-x^2+1)^(1/
3))*2^(1/3)+1/36*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {457, 105, 12, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{(1-x^2)^{2/3}}{6x^2}-\frac{\log(x^2+3)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/6*(1 - x^2)^(2/3)/x^2 + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(6*2^(2/
3)*Sqrt[3]) - Log[3 + x^2]/(36*2^(2/3)) + Log[2^(2/3) - (1 - x^2)^(1/3)]/(1
2*2^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^2(3+x)}\,dx,x,x^2\right)$$
$$= -\frac{(1-x^2)^{2/3}}{6x^2} - \frac{1}{6}\text{Subst}\left(\int -\frac{1}{3\sqrt[3]{1-x}\,(3+x)}\,dx,x,x^2\right)$$
$$= -\frac{(1-x^2)^{2/3}}{6x^2} + \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx,x,x^2\right)$$
$$= -\frac{(1-x^2)^{2/3}}{6x^2} - \frac{\log(3+x^2)}{36\ 2^{2/3}} + \frac{1}{12}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\sqrt[3]{1-x}\right.$$
$$= -\frac{(1-x^2)^{2/3}}{6x^2} - \frac{\log(3+x^2)}{36\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x\right.}{6\ 2^{2/}}$$
$$= -\frac{(1-x^2)^{2/3}}{6x^2} + \frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}} - \frac{\log(3+x^2)}{36\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right.}{12\ 2^{2/3}}$$

Mathematica [A]
time = 0.15, size = 108, normalized size = 1.11

$$\frac{1}{72}\left(-\frac{12(1-x^2)^{2/3}}{x^2}+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ((-12*(1 - x^2)^(2/3))/x^2 + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3
))/Sqrt[3]] + 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1/3)*Log[4 + 2*(2
- 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)])/72
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 13.13, size = 755, normalized size = 7.78

method	result	size
risch	Expression too large to display	755
trager	Expression too large to display	1108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(x^2-1)/x^2/(-x^2+1)^(1/3)-1/36*ln((64*RootOf(RootOf(_Z^3-2)^2+4*_Z*Roo
tOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-8*RootOf(_Z^3-2)^3*RootOf(RootO
```

```
f(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-168*(-x^2+1)^(1/3)*RootOf(Root
Of(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)-8*RootOf(RootOf(_Z
^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)
+RootOf(_Z^3-2)*x^2+42*(-x^2+1)^(2/3)+168*RootOf(RootOf(_Z^3-2)^2+4*_Z*Root
Of(_Z^3-2)+16*_Z^2)-21*RootOf(_Z^3-2))/(x^2+3))*RootOf(_Z^3-2)-1/9*ln((64*R
ootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-
8*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2
-168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*Ro
otOf(_Z^3-2)-8*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-42*
RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)+RootOf(_Z^3-2)*x^2+42*(-x^2+1)^(2/3)+168*Ro
otOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-21*RootOf(_Z^3-2))/(x^2+
3))*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+1/36*RootOf(_Z^3-2
)*ln(-(96*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^
3-2)^2*x^2+8*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+1
6*_Z^2)*x^2+168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+
16*_Z^2)*RootOf(_Z^3-2)+60*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_
Z^2)*x^2+42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)+5*RootOf(_Z^3-2)*x^2-42*(-x^2+1
)^(2/3)-252*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-21*RootOf(
_Z^3-2))/(x^2+3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^3), x)
```

Fricas [A]
time = 0.69, size = 115, normalized size = 1.19

$$\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,x^2\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}+2\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-4^{\frac{2}{3}}x^2\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+2\cdot 4^{\frac{2}{3}}x^2\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-24\left(-x^2+1\right)^{\frac{2}{3}}}{144\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 1/144*(4*4^(1/6)*sqrt(3)*x^2*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3) + 2*sqrt(3
)*(-x^2 + 1)^(1/3))) - 4^(2/3)*x^2*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) +
 (-x^2 + 1)^(2/3)) + 2*4^(2/3)*x^2*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 24*(-
x^2 + 1)^(2/3))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [A]
time = 0.59, size = 100, normalized size = 1.03

$$\frac{1}{72}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{144}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{72}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] 1/72*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/144*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) + 1/72*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 1/6*(-x^2 + 1)^(2/3)
/x^2
```

Mupad [B]
time = 0.49, size = 120, normalized size = 1.24

$$\frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{36}-\frac{2^{2/3}}{36}\right)}{36}-\frac{(1-x^2)^{2/3}}{6x^2}+\frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{36}-\frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{144}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{72}-\frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{36}-\frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{144}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] (2^(1/3)*log((1 - x^2)^(1/3)/36 - 2^(2/3)/36))/36 - (1 - x^2)^(2/3)/(6*x^2)
 + (2^(1/3)*log((1 - x^2)^(1/3)/36 - (2^(2/3)*(3^(1/2)*1i - 1)^2)/144)*(3^(
1/2)*1i - 1))/72 - (2^(1/3)*log((1 - x^2)^(1/3)/36 - (2^(2/3)*(3^(1/2)*1i +
 1)^2)/144)*(3^(1/2)*1i + 1))/72
```

3.1013 $\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=172

$$-\frac{(1-x^2)^{2/3}}{12x^4}-\frac{(1-x^2)^{2/3}}{18x^2}-\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\log(x)}{27}+\frac{\log(3+x^2)}{108\ 2^{2/3}}+\frac{1}{18}\log\left(\right.$$

```
[Out] -1/12*(-x^2+1)^(2/3)/x^4-1/18*(-x^2+1)^(2/3)/x^2-1/27*ln(x)+1/216*ln(x^2+3)
*2^(1/3)+1/18*ln(1-(-x^2+1)^(1/3))-1/72*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)-
1/108*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)+1/27*arctan(
1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {457, 105, 156, 162, 57, 632, 210, 31, 631}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{(1-x^2)^{2/3}}{18x^2}+\frac{\log(x^2+3)}{108\ 2^{2/3}}+\frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{36\ 2^{2/3}}-\frac{(1-x^2)^{2/3}}{12x^4}-\frac{\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/12*(1 - x^2)^(2/3)/x^4 - (1 - x^2)^(2/3)/(18*x^2) - ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]]/(18*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/S
qrt[3]]/(9*Sqrt[3]) - Log[x]/27 + Log[3 + x^2]/(108*2^(2/3)) + Log[1 - (1 -
 x^2)^(1/3)]/18 - Log[2^(2/3) - (1 - x^2)^(1/3)]/(36*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^3(3+x)}\,dx,x,x^2\right)$$
$$= -\frac{(1-x^2)^{2/3}}{12x^4} - \frac{1}{12}\text{Subst}\left(\int \frac{-2-\frac{4x}{3}}{\sqrt[3]{1-x}\,x^2(3+x)}\,dx,x,x^2\right)$$
$$= -\frac{(1-x^2)^{2/3}}{12x^4} - \frac{(1-x^2)^{2/3}}{18x^2} + \frac{1}{36}\text{Subst}\left(\int \frac{4+\frac{2x}{3}}{\sqrt[3]{1-x}\,x(3+x)}\,dx,x,x^2\right)$$
$$= -\frac{(1-x^2)^{2/3}}{12x^4} - \frac{(1-x^2)^{2/3}}{18x^2} - \frac{1}{54}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx,x,x^2\right) + \frac{1}{27}\text{Sul}$$
$$= -\frac{(1-x^2)^{2/3}}{12x^4} - \frac{(1-x^2)^{2/3}}{18x^2} - \frac{\log(x)}{27} + \frac{\log(3+x^2)}{108\ 2^{2/3}} - \frac{1}{36}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2}\cdots}\right.$$
$$= -\frac{(1-x^2)^{2/3}}{12x^4} - \frac{(1-x^2)^{2/3}}{18x^2} - \frac{\log(x)}{27} + \frac{\log(3+x^2)}{108\ 2^{2/3}} + \frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right)\cdots$$
$$= -\frac{(1-x^2)^{2/3}}{12x^4} - \frac{(1-x^2)^{2/3}}{18x^2} - \frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{9\sqrt{3}}\cdots$$

Mathematica [A]
time = 0.29, size = 214, normalized size = 1.24

$$-\frac{18(1-x^2)^{2/3}+12x^2(1-x^2)^{2/3}+2\sqrt[3]{2}\sqrt{3}\,x^4\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)-8\sqrt{3}\,x^4\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)+2\sqrt[3]{2}\,x^4\log\left(-2+\sqrt[3]{2-2x^2}\right)-\sqrt[3]{2}\,x^4\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)-8x^4\log\left(-1+\sqrt[3]{1-x^2}\right)+4x^4\log\left(1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}\right)}{216x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
[Out] -1/216*(18*(1 - x^2)^(2/3) + 12*x^2*(1 - x^2)^(2/3) + 2*2^(1/3)*Sqrt[3]*x^4
*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 8*Sqrt[3]*x^4*ArcTan[(1 + 2*(1 -
 x^2)^(1/3))/Sqrt[3]] + 2*2^(1/3)*x^4*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1/3)
*x^4*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)] - 8*x^4*Log[-1 + (1 -
 x^2)^(1/3)] + 4*x^4*Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/x^4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x)
[Out] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^5), x)
```

Fricas [A]
time = 0.71, size = 217, normalized size = 1.26

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^4\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}\,(-1)^{\frac{1}{3}}\,(-x^2+1)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x^4\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(-x^2+1)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right)-2\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x^4\log\left(-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}\right)-16\sqrt{3}\,x^4\arctan\left(\frac{2}{3}\sqrt{3}\,(-x^2+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+8\,x^4\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)-16\,x^4\log\left((-x^2+1)^{\frac{1}{3}}-1\right)+12\,(2\,x^2+3)(-x^2+1)^{\frac{2}{3}}}{432\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
[Out] -1/432*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(1/6*4^(1/6)*(2*sqrt(3)*(-1)
^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 4^(2/3)*(-1)^(1/3)*x^4*log(4^
(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3))
- 2*4^(2/3)*(-1)^(1/3)*x^4*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 16
*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 8*x^4*log
((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) - 16*x^4*log((-x^2 + 1)^(1/3) - 1
) + 12*(2*x^2 + 3)*(-x^2 + 1)^(2/3))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5}\sqrt[3]{-(x-1)(x+1)}\,(x^{2}+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral(1/(x**5*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [A]
time = 0.99, size = 177, normalized size = 1.03

$$-\frac{1}{216}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\,(-x^2+1)^{\frac{1}{3}}\right)\right)+\frac{1}{432}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right)-\frac{1}{216}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-(-x^2+1)^{\frac{1}{3}}\right)+\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(-x^2+1)^{\frac{1}{3}}+1\right)\right)+\frac{2\,(-x^2+1)^{\frac{5}{3}}-5\,(-x^2+1)^{\frac{2}{3}}}{36\,x^4}-\frac{1}{54}\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)+\frac{1}{27}\log\left(-(-x^2+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] -1/216*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 1/432*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) - 1/216*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/27*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/36*(2*(-x^2 + 1)^(5/3) - 5*(-x
^2 + 1)^(2/3))/x^4 - 1/54*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/
27*log(-(-x^2 + 1)^(1/3) + 1)
```

Mupad [B]
time = 0.55, size = 397, normalized size = 2.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] log(11/486 - (11*(1 - x^2)^(1/3))/486)/27 - (2^(1/3)*log(- (2^(2/3)*((2^(1/
3)*((135*2^(2/3))/4 - (1755*(1 - x^2)^(1/3))/4))/108 + 7/2))/11664 - (1 - x
^2)^(1/3)/2916))/108 + log(((3^(1/2)*1i)/54 - 1/54)^2*(((3^(1/2)*1i)/54 - 1
/54)*(393660*((3^(1/2)*1i)/54 - 1/54)^2 - (1755*(1 - x^2)^(1/3))/4) - 7/2)
- (1 - x^2)^(1/3)/2916)*((3^(1/2)*1i)/54 - 1/54) - log(- ((3^(1/2)*1i)/54 +
 1/54)^2*(((3^(1/2)*1i)/54 + 1/54)*(393660*((3^(1/2)*1i)/54 + 1/54)^2 - (17
55*(1 - x^2)^(1/3))/4) + 7/2) - (1 - x^2)^(1/3)/2916)*((3^(1/2)*1i)/54 + 1/
54) - ((5*(1 - x^2)^(2/3))/36 - (1 - x^2)^(5/3)/18)/((x^2 - 1)^2 + 2*x^2 -
1) + ((-1)^(1/3)*2^(1/3)*log(((-1)^(2/3)*2^(2/3)*(((-1)^(1/3)*2^(1/3)*((135
*(-1)^(2/3)*2^(2/3))/4 - (1755*(1 - x^2)^(1/3))/4))/108 - 7/2))/11664 - (1
- x^2)^(1/3)/2916))/108 - ((-1)^(1/3)*2^(1/3)*log(((-1)^(2/3)*2^(2/3)*(3^(1
/2)*1i + 1)^2*(((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*((1755*(1 - x^2)^(1/3))
/4 - (135*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2)/16))/216 - 7/2))/46656 - (
1 - x^2)^(1/3)/2916)*(3^(1/2)*1i + 1))/216
```

3.1014 $\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=536

$$-\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{54x}{7\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}-$$

```
[Out] -3/7*x*(-x^2+1)^(2/3)-3/4*arctanh(x)*2^(1/3)+9/4*arctanh(x/(1+2^(1/3)*(-x^2
+1)^(1/3)))*2^(1/3)+54/7*x/(1-(-x^2+1)^(1/3)-3^(1/2))+3/4*arctan(1/x*3^(1/2
))*2^(1/3)*3^(1/2)+3/4*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(1/3)
*3^(1/2)-18/7*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^(1/3)+3^(1/2
))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1)^(1/3)+(-x
^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)^(1/3))/(1-
(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+27/7*3^(1/4)*(1-(-x^2+1)^(1/3))*EllipticE(
(1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(-
x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/
2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {490, 544, 241, 310, 225, 1893, 402}

$$-\frac{18\sqrt{2}\,3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{27\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{3\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}+\frac{3\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}-\frac{3}{7}\left(1-x^2\right)^{2/3}x+\frac{54x}{7\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{9\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{3\tanh^{-1}(x)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*x*(1 - x^2)^(2/3))/7 + (54*x)/(7*(1 - Sqrt[3] - (1 - x^2)^(1/3))) + (3*
Sqrt[3]*ArcTan[Sqrt[3]/x])/(2*2^(2/3)) + (3*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^
(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)) - (3*ArcTanh[x])/(2*2^(2/3)) + (9*A
rcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) + (27*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)
)/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x
^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqrt[-((
1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (18*Sqrt[2]*3^(
3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1
- Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(
```

```
1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*x*Sqrt[-((1 - (
1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx &= -\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{3}{7}\int \frac{3-6x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx \\
&= -\frac{3}{7}x\left(1-x^2\right)^{2/3}-\frac{18}{7}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx+9\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx \\
&= -\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right.}{2\ 2^{2/3}} \\
&= -\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right.}{2\ 2^{2/3}} \\
&= -\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{54x}{7\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}+\frac{3\sqrt{3}\tan}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 5.06, size = 156, normalized size = 0.29

$$\frac{1}{7}x\left(-2x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{3\left(-1+x^2-\frac{27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{(3+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)\right)}\right)}{\sqrt[3]{1-x^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (x*(-2*x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2] + (3*(-1 + x^2 - (27*A
ppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((3 + x^2)*(-9*AppellF1[1/2, 1/3,
 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2]
- AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))))/(1 - x^2)^(1/3)))/7
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)*x^4/(x^4 + 2*x^2 - 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral(x**4/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int(x^4/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1015 $\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=515

$$-\frac{3x}{1-\sqrt{3}-\sqrt[3]{1-x^2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}+\frac{\tanh^{-1}(x)}{2\ 2^{2/3}}-\frac{3\tanh^{-1}\left(\right.}{}$$

[Out] 1/4*arctanh(x)*2^(1/3)-3/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)-3*x/(1-(-x^2+1)^(1/3)-3^(1/2))-1/4*arctan(1/x*3^(1/2))*2^(1/3)*3^(1/2)-1/4*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)+3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)-3/2*3^(1/4)*(1-(-x^2+1)^(1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {494, 241, 310, 225, 1893, 402}

$$\frac{\sqrt{2}\,3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}-\frac{3x}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}-\frac{3\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tanh^{-1}(x)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - x^2)^(1/3)*(3 + x^2)),x]

[Out] (-3*x)/(1 - Sqrt[3] - (1 - x^2)^(1/3)) - (Sqrt[3]*ArcTan[Sqrt[3]/x])/(2*2^(2/3)) - (Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(2*2^(2/3)) + ArcTanh[x]/(2*2^(2/3)) - (3*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) + (Sqrt[2]*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7

```
+ 4*Sqrt[3]])/(x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3
))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\left(3\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx\right)+\int \frac{1}{\sqrt[3]{1-x^2}}\,dx$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}} + \frac{\tanh^{-1}(x)}{2\ 2^{2/3}} - \frac{3\,\mathrm{ta}}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}} + \frac{\tanh^{-1}(x)}{2\ 2^{2/3}} - \frac{3\,\mathrm{ta}}{}$$

$$= -\frac{3x}{1-\sqrt{3}-\sqrt[3]{1-x^2}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right.}{x}\right.}{2\ 2^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 4.44, size = 28, normalized size = 0.05

$$\frac{1}{9}x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2])/9
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)^(1/3)/(x^2+3),x)
[Out] int(x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
[Out] integral(-(-x^2 + 1)^(2/3)*x^2/(x^4 + 2*x^2 - 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral(x**2/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(1-x^2\right)^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] int(x^2/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1016 $\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+
1/12*arctan(1/x*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1
/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {402}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 0.03, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 12.88, size = 938, normalized size = 8.30

method	result	size
trager	Expression too large to display	938

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*RootOf(_Z^6+108)*ln((6*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^5+108*Root
Of(_Z^6+108)^5*(-x^2+1)^(1/3)*x^4+144*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^3
-108*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^2-36*RootOf(_Z^6+108)^2*(-x^2+1)^(
1/3)*x^5-54*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x-648*RootOf(_Z^6+108)^2*(-x^
2+1)^(1/3)*x^4-864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3+648*RootOf(_Z^6+10
8)^2*(-x^2+1)^(1/3)*x^2+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+1296*(-x^2+
1)^(2/3)*x^4+9072*(-x^2+1)^(2/3)*x^3+3888*(-x^2+1)^(2/3)*x^2-3888*(-x^2+1)^
(2/3)*x+189*RootOf(_Z^6+108)^4*x^2-3402*RootOf(_Z^6+108)*x^2+72*RootOf(_Z^6
+108)^4*x^3-1296*RootOf(_Z^6+108)*x^3-RootOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6
+108)^4*x^5-225*RootOf(_Z^6+108)^4*x^4+18*RootOf(_Z^6+108)*x^6+1296*RootOf(
_Z^6+108)*x^5+4050*RootOf(_Z^6+108)*x^4-27*RootOf(_Z^6+108)^4+486*RootOf(_Z
^6+108))/(x^2+3)^3)+1/432*ln((RootOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4*
x^5-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+225*RootOf(_Z^6+108)^4*x^4+648
*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4+72*RootOf(_Z^6+108)^4*x^3+648*(-x^2+
1)^(2/3)*x^4-864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3-189*RootOf(_Z^6+108)
^4*x^2-4536*(-x^2+1)^(2/3)*x^3-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+19
44*(-x^2+1)^(2/3)*x^2+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+27*RootOf(_Z^
6+108)^4+1944*(-x^2+1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6+108)^4+1/72*ln((Root
Of(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4*x^5-36*RootOf(_Z^6+108)^2*(-x^2+1)
^(1/3)*x^5+225*RootOf(_Z^6+108)^4*x^4+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)
*x^4+72*RootOf(_Z^6+108)^4*x^3+648*(-x^2+1)^(2/3)*x^4-864*RootOf(_Z^6+108)^
```

```
2*(-x^2+1)^(1/3)*x^3-189*RootOf(_Z^6+108)^4*x^2-4536*(-x^2+1)^(2/3)*x^3-648
*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+1944*(-x^2+1)^(2/3)*x^2+324*RootOf(_
Z^6+108)^2*(-x^2+1)^(1/3)*x+27*RootOf(_Z^6+108)^4+1944*(-x^2+1)^(2/3)*x)/(x
^2+3)^3)*RootOf(_Z^6+108)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1943 vs. 2(81) = 162.
time = 1.38, size = 1943, normalized size = 17.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
```

```
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^2\right)^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1017 $\int \frac{1}{x^2 \sqrt[3]{1-x^2} \left(3+x^2\right)} \, dx$

Optimal. Leaf size=538

$$-\frac{\left(1-x^2\right)^{2/3}}{3x}+\frac{x}{3\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}(x)}{18\ 2^{2/3}}-\text{ta}$$

```
[Out] -1/3*(-x^2+1)^(2/3)/x+1/36*arctanh(x)*2^(1/3)-1/12*arctanh(x/(1+2^(1/3)*(-x
^2+1)^(1/3)))*2^(1/3)+1/3*x/(1-(-x^2+1)^(1/3)-3^(1/2))-1/36*arctan(1/x*3^(1
/2))*2^(1/3)*3^(1/2)-1/36*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(1
/3)*3^(1/2)-1/9*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^(1/3)+3^(1
/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1)^(1/3)+(
-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)^(1/3))/(
1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+1/6*3^(1/4)*(1-(-x^2+1)^(1/3))*EllipticE
((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(
-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1
/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 538, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {491, 544, 241, 310, 225, 1893, 402}

$$-\frac{\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}}+\frac{x}{3\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{\left(1-x^2\right)^{2/3}}{3x}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{6\ 2^{2/3}}+\frac{\tanh^{-1}(x)}{18\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/3*(1 - x^2)^(2/3)/x + x/(3*(1 - Sqrt[3] - (1 - x^2)^(1/3))) - ArcTan[Sqr
t[3]/x]/(6*2^(2/3)*Sqrt[3]) - ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3))
)/x]/(6*2^(2/3)*Sqrt[3]) + ArcTanh[x]/(18*2^(2/3)) - ArcTanh[x/(1 + 2^(1/3)
*(1 - x^2)^(1/3))]/(6*2^(2/3)) + (Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*S
qrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(2*3^(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 -
 Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (Sqrt[2]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 +
 (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3)
```

```
)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3]
 - (1 - x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{(1-x^2)^{2/3}}{3x} + \frac{1}{3}\int \frac{-2-\frac{x^2}{3}}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{(1-x^2)^{2/3}}{3x} - \frac{1}{9}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx - \frac{1}{3}\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{(1-x^2)^{2/3}}{3x} - \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}(x}{18\ 2^{2/3}}$$

$$= -\frac{(1-x^2)^{2/3}}{3x} - \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}(x}{18\ 2^{2/3}}$$

$$= -\frac{(1-x^2)^{2/3}}{3x} + \frac{x}{3\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\sqrt{3}\left(1-\right.\right.}{6\ 2^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.09, size = 161, normalized size = 0.30

$$-\frac{1}{81}x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{-1+x^2+\frac{18x^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{(3+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)\right)}}{3x\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/81*(x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2]) + (-1 + x^2 + (18*x^2
*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((3 + x^2)*(-9*AppellF1[1/2, 1/
3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2
] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/(3*x*(1 - x^2)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)/(x^6 + 2*x^4 - 3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(1-x^2\right)}^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] int(1/(x^2*(1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1018 $\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=556

$$-\frac{(1-x^2)^{2/3}}{9x^3}-\frac{2(1-x^2)^{2/3}}{27x}+\frac{2x}{27\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+$$

```
[Out] -1/9*(-x^2+1)^(2/3)/x^3-2/27*(-x^2+1)^(2/3)/x-1/108*arctanh(x)*2^(1/3)+1/36
*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+2/27*x/(1-(-x^2+1)^(1/3)-3^(
1/2))+1/108*arctan(1/x*3^(1/2))*2^(1/3)*3^(1/2)+1/108*arctan((1-2^(1/3)*(-x
^2+1)^(1/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)-2/81*3^(3/4)*(1-(-x^2+1)^(1/3))*Ell
ipticF((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))
*2^(1/2)*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(
1/2)/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+1/27*3^(1/4
)*(1-(-x^2+1)^(1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)
-3^(1/2)),2*I-I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/
3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-
x^2+1)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {491, 597, 544, 241, 310, 225, 1893, 402}

$$-\frac{2\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{2x}{27\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{2(1-x^2)^{2/3}}{27x}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{18\ 2^{2/3}}-\frac{(1-x^2)^{2/3}}{9x^3}-\frac{\tanh^{-1}(x)}{54\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/9*(1 - x^2)^(2/3)/x^3 - (2*(1 - x^2)^(2/3))/(27*x) + (2*x)/(27*(1 - Sqrt
[3] - (1 - x^2)^(1/3))) + ArcTan[Sqrt[3]/x]/(18*2^(2/3)*Sqrt[3]) + ArcTan[(
Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(18*2^(2/3)*Sqrt[3]) - ArcTanh[x]
/(54*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(18*2^(2/3)) + (Sq
rt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)
^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3
/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (
2*Sqrt[2]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)
```

```
)/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x
^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*x
*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{(1-x^2)^{2/3}}{9x^3}+\frac{1}{9}\int \frac{2+\frac{5x^2}{3}}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{(1-x^2)^{2/3}}{9x^3}-\frac{2(1-x^2)^{2/3}}{27x}-\frac{1}{27}\int \frac{-1+\frac{2x^2}{3}}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{(1-x^2)^{2/3}}{9x^3}-\frac{2(1-x^2)^{2/3}}{27x}-\frac{2}{81}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx+\frac{1}{9}\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{(1-x^2)^{2/3}}{9x^3}-\frac{2(1-x^2)^{2/3}}{27x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}}$$

$$= -\frac{(1-x^2)^{2/3}}{9x^3}-\frac{2(1-x^2)^{2/3}}{27x}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}}$$

$$= -\frac{(1-x^2)^{2/3}}{9x^3}-\frac{2(1-x^2)^{2/3}}{27x}+\frac{2x}{27\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}}+$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.10, size = 166, normalized size = 0.30

$$-\frac{2}{729}x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{-3+x^2+2x^4-\frac{9x^4F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{(3+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)\right)}}{27x^3\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-2*x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2])/729 + (-3 + x^2 + 2*x^4
- (9*x^4*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((3 + x^2)*(-9*AppellF1
[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2,
-1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/(27*x^3*(1 - x^2)
^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x)
[Out] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
[Out] integral(-(-x^2 + 1)^(2/3)/(x^8 + 2*x^6 - 3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral(1/(x**4*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 (1 - x^2)^{1/3} (x^2 + 3)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int(1/(x^4*(1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1019 $\int \frac{x^7}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=133

$$-\frac{3x^4(1-x^2)^{2/3}}{10(3+x^2)}+\frac{9(1-x^2)^{2/3}(69+14x^2)}{40(3+x^2)}+\frac{99\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}}-\frac{99\log(3+x^2)}{16\ 2^{2/3}}+\frac{297\log\left(2^{2/3}\right.}{16\ 2}$$

```
[Out] -3/10*x^4*(-x^2+1)^(2/3)/(x^2+3)+9/40*(-x^2+1)^(2/3)*(14*x^2+69)/(x^2+3)-99
/32*ln(x^2+3)*2^(1/3)+297/32*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+99/16*arcta
n(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {457, 102, 151, 57, 631, 210, 31}

$$\frac{99\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}+\frac{9(1-x^2)^{2/3}(14x^2+69)}{40(x^2+3)}-\frac{99\log(x^2+3)}{16\ 2^{2/3}}+\frac{297\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}-\frac{3(1-x^2)^{2/3}x^4}{10(x^2+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*x^4*(1 - x^2)^(2/3))/(10*(3 + x^2)) + (9*(1 - x^2)^(2/3)*(69 + 14*x^2))
/(40*(3 + x^2)) + (99*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2
^(2/3)) - (99*Log[3 + x^2])/(16*2^(2/3)) + (297*Log[2^(2/3) - (1 - x^2)^(1/
3)])/(16*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^7}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1-x}\,(3+x)^2}\,dx, x, x^2\right)$$
$$= -\frac{3x^4(1-x^2)^{2/3}}{10(3+x^2)} - \frac{3}{10}\text{Subst}\left(\int \frac{x(-6+7x)}{\sqrt[3]{1-x}\,(3+x)^2}\,dx, x, x^2\right)$$
$$= -\frac{3x^4(1-x^2)^{2/3}}{10(3+x^2)} + \frac{9(1-x^2)^{2/3}(69+14x^2)}{40(3+x^2)} + \frac{99}{8}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx\right.$$
$$= -\frac{3x^4(1-x^2)^{2/3}}{10(3+x^2)} + \frac{9(1-x^2)^{2/3}(69+14x^2)}{40(3+x^2)} - \frac{99\log(3+x^2)}{16\ 2^{2/3}} + \frac{297}{16}\text{Subst}\left(\int\right.$$
$$= -\frac{3x^4(1-x^2)^{2/3}}{10(3+x^2)} + \frac{9(1-x^2)^{2/3}(69+14x^2)}{40(3+x^2)} - \frac{99\log(3+x^2)}{16\ 2^{2/3}} + \frac{297\log\left(2^{2/3}\right.}{16\ 2}$$
$$= -\frac{3x^4(1-x^2)^{2/3}}{10(3+x^2)} + \frac{9(1-x^2)^{2/3}(69+14x^2)}{40(3+x^2)} + \frac{99\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}} -$$

Mathematica [A]
time = 0.27, size = 124, normalized size = 0.93

$$\frac{3}{160}\left(-\frac{4(1-x^2)^{2/3}(-207-42x^2+4x^4)}{3+x^2} + 330\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) + 330\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right) - 165\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*((-4*(1 - x^2)^(2/3)*(-207 - 42*x^2 + 4*x^4))/(3 + x^2) + 330*2^(1/3)*Sq
rt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 330*2^(1/3)*Log[-2 + (2 - 2
*x^2)^(1/3)] - 165*2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)]
))/160
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.60, size = 489, normalized size = 3.68 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/40*(4*x^4-42*x^2-207)/(x^2+3)*(-x^2+1)^(2/3)+891/4*RootOf(RootOf(_Z^3-2)
^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)*ln((72*RootOf(RootOf(_Z^3-2)^2+36*_Z*Roo
tOf(_Z^3-2)+1296*_Z^2)*RootOf(_Z^3-2)^3*x^2+1728*RootOf(RootOf(_Z^3-2)^2+36
*_Z*RootOf(_Z^3-2)+1296*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+504*(-x^2+1)^(1/3)*Roo
tOf(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)*RootOf(_Z^3-2)+5*RootO
f(_Z^3-2)*x^2+120*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)*x
```

```
^2+14*(-x^2+1)^(2/3)-21*RootOf(_Z^3-2)-504*RootOf(RootOf(_Z^3-2)^2+36*_Z*Ro
otOf(_Z^3-2)+1296*_Z^2))/(x^2+3))+99/16*RootOf(_Z^3-2)*ln((-72*RootOf(RootO
f(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)*RootOf(_Z^3-2)^3*x^2-3888*RootO
f(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+7
56*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)*R
ootOf(_Z^3-2)+RootOf(_Z^3-2)*x^2+54*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z
^3-2)+1296*_Z^2)*x^2+21*(-x^2+1)^(2/3)-21*RootOf(_Z^3-2)-1134*RootOf(RootOf
(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2))/(x^2+3))
```

Maxima [A]
time = 0.47, size = 126, normalized size = 0.95

$$\frac{99}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}}-\frac{99}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{99}{32}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{15}{4}\left(-x^2+1\right)^{\frac{2}{3}}+\frac{27\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 99/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) + 3/10*(-x^2 + 1)^(5/3) - 99/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2
+ 1)^(1/3) + (-x^2 + 1)^(2/3)) + 99/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1
/3)) + 15/4*(-x^2 + 1)^(2/3) + 27/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

Fricas [A]
time = 0.57, size = 133, normalized size = 1.00

$$\frac{3\left(660\cdot 4^{\frac{1}{6}}\sqrt{3}\left(x^2+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-165\cdot 4^{\frac{2}{3}}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+330\cdot 4^{\frac{2}{3}}\left(x^2+3\right)\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-8\left(4x^4-42x^2-207\right)\left(-x^2+1\right)^{\frac{2}{3}}\right)}{320\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 3/320*(660*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) +
2*(-x^2 + 1)^(1/3))) - 165*4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2 +
1)^(1/3) + (-x^2 + 1)^(2/3)) + 330*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 +
 1)^(1/3)) - 8*(4*x^4 - 42*x^2 - 207)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\,\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(x**7/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [A]
time = 1.37, size = 126, normalized size = 0.95

$$\frac{99}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}}-\frac{99}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{99}{32}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{15}{4}\left(-x^2+1\right)^{\frac{2}{3}}+\frac{27\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] 99/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) + 3/10*(-x^2 + 1)^(5/3) - 99/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2
+ 1)^(1/3) + (-x^2 + 1)^(2/3)) + 99/32*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/
3)) + 15/4*(-x^2 + 1)^(2/3) + 27/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

Mupad [B]
time = 0.52, size = 148, normalized size = 1.11

$$\frac{99\,2^{1/3}\ln\left(\frac{88209\,(1-x^2)^{1/3}}{64}-\frac{88209\,2^{2/3}}{64}\right)}{16}+\frac{27\,(1-x^2)^{2/3}}{8\,(x^2+3)}+\frac{15\,(1-x^2)^{2/3}}{4}+\frac{3\,(1-x^2)^{5/3}}{10}+\frac{99\,2^{1/3}\ln\left(\frac{88209\,(1-x^2)^{1/3}}{64}-\frac{88209\,2^{2/3}\left(-1+\sqrt{3}\,1i\right)^2}{256}\right)\left(-1+\sqrt{3}\,1i\right)}{32}-\frac{99\,2^{1/3}\ln\left(\frac{88209\,(1-x^2)^{1/3}}{64}-\frac{88209\,2^{2/3}\left(1+\sqrt{3}\,1i\right)^2}{256}\right)\left(1+\sqrt{3}\,1i\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (99*2^(1/3)*log((88209*(1 - x^2)^(1/3))/64 - (88209*2^(2/3))/64))/16 + (27*
(1 - x^2)^(2/3))/(8*(x^2 + 3)) + (15*(1 - x^2)^(2/3))/4 + (3*(1 - x^2)^(5/3
))/10 + (99*2^(1/3)*log((88209*(1 - x^2)^(1/3))/64 - (88209*2^(2/3)*(3^(1/2
)*1i - 1)^2)/256)*(3^(1/2)*1i - 1))/32 - (99*2^(1/3)*log((88209*(1 - x^2)^(
1/3))/64 - (88209*2^(2/3)*(3^(1/2)*1i + 1)^2)/256)*(3^(1/2)*1i + 1))/32
```

3.1020 $\int \frac{x^5}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2} \, dx$

Optimal. Leaf size=116

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}-\frac{21\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}}+\frac{21\log\left(3+x^2\right)}{16\ 2^{2/3}}-\frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

```
[Out] -3/4*(-x^2+1)^(2/3)-9/8*(-x^2+1)^(2/3)/(x^2+3)+21/32*ln(x^2+3)*2^(1/3)-63/3
2*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)-21/16*arctan(1/3*(1+(-2*x^2+2)^(1/3))*
3^(1/2))*3^(1/2)*2^(1/3)
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {457, 91, 81, 57, 631, 210, 31}

$$-\frac{21\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{9\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{21\log\left(x^2+3\right)}{16\ 2^{2/3}}-\frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (9*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (21*Sqrt[3]*Ar
cTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) + (21*Log[3 + x^2])/(16*
2^(2/3)) - (63*Log[2^(2/3) - (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}\,(3+x)^2}\,dx, x, x^2\right)$$
$$= -\frac{9(1-x^2)^{2/3}}{8\,(3+x^2)} + \frac{1}{8}\text{Subst}\left(\int \frac{-9+4x}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$
$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9(1-x^2)^{2/3}}{8\,(3+x^2)} - \frac{21}{8}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$
$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9(1-x^2)^{2/3}}{8\,(3+x^2)} + \frac{21\log(3+x^2)}{16\;2^{2/3}} - \frac{63}{16}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+$$
$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9(1-x^2)^{2/3}}{8\,(3+x^2)} + \frac{21\log(3+x^2)}{16\;2^{2/3}} - \frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\;2^{2/3}} + $$
$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9(1-x^2)^{2/3}}{8\,(3+x^2)} - \frac{21\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\;2^{2/3}} + \frac{21\log(3+x^2)}{16\;2^{2/3}}$$

Mathematica [A]
time = 0.24, size = 119, normalized size = 1.03

$$\frac{3}{32}\left(-\frac{4(1-x^2)^{2/3}(9+2x^2)}{3+x^2} - 14\sqrt[3]{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) - 14\sqrt[3]{2}\,\log\left(-2+\sqrt[3]{2-2x^2}\right) + 7\sqrt[3]{2}\,\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*((-4*(1 - x^2)^(2/3)*(9 + 2*x^2))/(3 + x^2) - 14*2^(1/3)*Sqrt[3]*ArcTan[
(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 14*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] +
 7*2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)]))/32
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.42, size = 656, normalized size = 5.66

method	result	size
trager	Expression too large to display	656
risch	Expression too large to display	662

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*(2*x^2+9)/(x^2+3)*(-x^2+1)^(2/3)-21/16*ln((1296*RootOf(RootOf(_Z^3+2)^
2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-72*RootOf(RootOf(_
```

```
Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)*RootOf(_Z^3+2)^3*x^2+21*RootOf(_Z^
3+2)^2*(-x^2+1)^(1/3)-90*RootOf(RootOf(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*
_Z^2)*x^2+5*RootOf(_Z^3+2)*x^2+21*(-x^2+1)^(2/3)+378*RootOf(RootOf(_Z^3+2)^
2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)-21*RootOf(_Z^3+2))/(x^2+3))*RootOf(_Z^3+2
)-189/4*ln((1296*RootOf(RootOf(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)^2*
RootOf(_Z^3+2)^2*x^2-72*RootOf(RootOf(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*_
Z^2)*RootOf(_Z^3+2)^3*x^2+21*RootOf(_Z^3+2)^2*(-x^2+1)^(1/3)-90*RootOf(Root
Of(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)*x^2+5*RootOf(_Z^3+2)*x^2+21*(-
x^2+1)^(2/3)+378*RootOf(RootOf(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)-21
*RootOf(_Z^3+2))/(x^2+3))*RootOf(RootOf(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296
*_Z^2)+189/4*RootOf(RootOf(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)*ln((86
4*RootOf(RootOf(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)^2*RootOf(_Z^3+2)^
2*x^2+72*RootOf(RootOf(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)*RootOf(_Z^
3+2)^3*x^2+14*RootOf(_Z^3+2)^2*(-x^2+1)^(1/3)+12*RootOf(RootOf(_Z^3+2)^2+36
*_Z*RootOf(_Z^3+2)+1296*_Z^2)*x^2+RootOf(_Z^3+2)*x^2+14*(-x^2+1)^(2/3)-252*
RootOf(RootOf(_Z^3+2)^2+36*_Z*RootOf(_Z^3+2)+1296*_Z^2)-21*RootOf(_Z^3+2))/
(x^2+3))
```

Maxima [A]
time = 0.47, size = 115, normalized size = 0.99

$$-\frac{21}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{21}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{21}{32}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}}-\frac{9\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -21/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 21/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) - 21/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(
2/3) - 9/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

Fricas [A]
time = 0.67, size = 153, normalized size = 1.32

$$-\frac{3\left(28\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\left(x^2+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\right)\right)+7\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^2+3\right)\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-14\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^2+3\right)\log\left(-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)+8\left(2x^2+9\right)\left(-x^2+1\right)^{\frac{2}{3}}\right)}{64\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -3/64*(28*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(
2*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3))) + 7*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*
log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(
2/3)) - 14*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1
)^(1/3)) + 8*(2*x^2 + 9)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [A]
time = 1.23, size = 115, normalized size = 0.99

$$-\frac{21}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{21}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{21}{32}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}}-\frac{9\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] -21/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 21/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) - 21/32*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2
/3) - 9/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

Mupad [B]
time = 0.50, size = 137, normalized size = 1.18

$$-\frac{21\,2^{1/3}\ln\left(\frac{3969\,{\left(1-x^2\right)}^{1/3}}{64}-\frac{3969\,2^{2/3}}{64}\right)}{16}-\frac{9\,{\left(1-x^2\right)}^{2/3}}{8\,\left(x^2+3\right)}-\frac{3\,{\left(1-x^2\right)}^{2/3}}{4}-\frac{21\,2^{1/3}\ln\left(\frac{3969\,{\left(1-x^2\right)}^{1/3}}{64}-\frac{3969\,2^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{256}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{32}+\frac{21\,2^{1/3}\ln\left(\frac{3969\,{\left(1-x^2\right)}^{1/3}}{64}-\frac{3969\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{256}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (21*2^(1/3)*log((3969*(1 - x^2)^(1/3))/64 - (3969*2^(2/3)*(3^(1/2)*1i + 1)^
2)/256)*(3^(1/2)*1i + 1))/32 - (9*(1 - x^2)^(2/3))/(8*(x^2 + 3)) - (3*(1 -
x^2)^(2/3))/4 - (21*2^(1/3)*log((3969*(1 - x^2)^(1/3))/64 - (3969*2^(2/3)*(
3^(1/2)*1i - 1)^2)/256)*(3^(1/2)*1i - 1))/32 - (21*2^(1/3)*log((3969*(1 - x
^2)^(1/3))/64 - (3969*2^(2/3))/64))/16
```

3.1021 $\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=101

$$\frac{3(1-x^2)^{2/3}}{8(3+x^2)}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}}-\frac{3\log(3+x^2)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

```
[Out] 3/8*(-x^2+1)^(2/3)/(x^2+3)-3/32*ln(x^2+3)*2^(1/3)+9/32*ln(2^(2/3)-(-x^2+1)^
(1/3))*2^(1/3)+3/16*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3
)
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 57, 631, 210, 31}

$$\frac{3\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}+\frac{3(1-x^2)^{2/3}}{8(x^2+3)}-\frac{3\log(x^2+3)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*(1 - x^2)^(2/3))/(8*(3 + x^2)) + (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3
))/Sqrt[3]])/(8*2^(2/3)) - (3*Log[3 + x^2])/(16*2^(2/3)) + (9*Log[2^(2/3) -
 (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\,(3+x)^2}\,dx,x,x^2\right)\\
&= \frac{3(1-x^2)^{2/3}}{8\,(3+x^2)}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx,x,x^2\right)\\
&= \frac{3(1-x^2)^{2/3}}{8\,(3+x^2)}-\frac{3\log(3+x^2)}{16\ 2^{2/3}}+\frac{9}{16}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\sqrt[3]{1-x^2}\right.\\
&= \frac{3(1-x^2)^{2/3}}{8\,(3+x^2)}-\frac{3\log(3+x^2)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}-\frac{9\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{8\ 2^{2}}\\
&= \frac{3(1-x^2)^{2/3}}{8\,(3+x^2)}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}}-\frac{3\log(3+x^2)}{16\ 2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1}\right.}{16\ 2^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 112, normalized size = 1.11

$$\frac{3}{32}\left(\frac{4(1-x^2)^{2/3}}{3+x^2}+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*((4*(1 - x^2)^(2/3))/(3 + x^2) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^
2)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1/3)*Log[4
+ 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)]))/32
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.46, size = 477, normalized size = 4.72

method	result
trager	$\frac{3(-x^2+1)^{\frac{2}{3}}}{8(x^2+3)}+\frac{27\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-2\right)^2+36\_Z\,\mathrm{RootOf}\left(\_Z^3-2\right)+1296\_Z^2\right)\ln\left(\frac{72\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-2\right)^2+36\_\cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*(-x^2+1)^(2/3)/(x^2+3)+27/4*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2
)+1296*_Z^2)*ln((72*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)
*RootOf(_Z^3-2)^3*x^2+1728*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+129
6*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+504*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3
6*_Z*RootOf(_Z^3-2)+1296*_Z^2)*RootOf(_Z^3-2)+5*RootOf(_Z^3-2)*x^2+120*Root
Of(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)*x^2+14*(-x^2+1)^(2/3)-2
1*RootOf(_Z^3-2)-504*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2
))/(x^2+3))+3/16*RootOf(_Z^3-2)*ln((-72*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootO
f(_Z^3-2)+1296*_Z^2)*RootOf(_Z^3-2)^3*x^2-3888*RootOf(RootOf(_Z^3-2)^2+36*_
Z*RootOf(_Z^3-2)+1296*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+756*(-x^2+1)^(1/3)*RootO
f(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)*RootOf(_Z^3-2)+RootOf(_Z
^3-2)*x^2+54*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootOf(_Z^3-2)+1296*_Z^2)*x^2+21
*(-x^2+1)^(2/3)-21*RootOf(_Z^3-2)-1134*RootOf(RootOf(_Z^3-2)^2+36*_Z*RootOf
(_Z^3-2)+1296*_Z^2))/(x^2+3))
```

Maxima [A]
time = 0.48, size = 104, normalized size = 1.03

$$\frac{3}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{3}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{3}{32}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{3\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 3/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 3/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) + 3/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) + 3/8*(-x^2 + 1)^(2/3)
/(x^2 + 3)
```

Fricas [A]
time = 0.51, size = 121, normalized size = 1.20

$$\frac{3 \left(4 \cdot 4^{\frac{1}{6}} \sqrt{3}\, (x^2+3) \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} \left(4^{\frac{1}{3}} + 2\,(-x^2+1)^{\frac{1}{3}}\right)\right) - 4^{\frac{2}{3}} (x^2+3) \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}} (-x^2+1)^{\frac{1}{3}} + (-x^2+1)^{\frac{2}{3}}\right) + 2 \cdot 4^{\frac{2}{3}} (x^2+3) \log\left(-4^{\frac{1}{3}} + (-x^2+1)^{\frac{1}{3}}\right) + 8\,(-x^2+1)^{\frac{2}{3}}\right)}{64\,(x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 3/64*(4*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) + 2*(
-x^2 + 1)^(1/3))) - 4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3
) + (-x^2 + 1)^(2/3)) + 2*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)
) + 8*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [A]
time = 1.26, size = 104, normalized size = 1.03

$$\frac{3}{32} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3} \left(4^{\frac{1}{3}} + 2\,(-x^2+1)^{\frac{1}{3}}\right)\right) - \frac{3}{64} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}} (-x^2+1)^{\frac{1}{3}} + (-x^2+1)^{\frac{2}{3}}\right) + \frac{3}{32} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{1}{3}} - (-x^2+1)^{\frac{1}{3}}\right) + \frac{3\,(-x^2+1)^{\frac{2}{3}}}{8\,(x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] 3/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 3/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) + 3/32*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 3/8*(-x^2 + 1)^(2/3)/
(x^2 + 3)
```

Mupad [B]
time = 0.51, size = 126, normalized size = 1.25

$$\frac{3\,2^{1/3}\,\ln\left(\frac{81\,(1-x^2)^{1/3}}{64}-\frac{81\,2^{2/3}}{64}\right)}{16}+\frac{3\,(1-x^2)^{2/3}}{8\,(x^2+3)}+\frac{3\,2^{1/3}\,\ln\left(\frac{81\,(1-x^2)^{1/3}}{64}-\frac{81\,2^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{32}-\frac{3\,2^{1/3}\,\ln\left(\frac{81\,(1-x^2)^{1/3}}{64}-\frac{81\,2^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (3*2^(1/3)*log((81*(1 - x^2)^(1/3))/64 - (81*2^(2/3))/64))/16 + (3*(1 - x^2
)^(2/3))/(8*(x^2 + 3)) + (3*2^(1/3)*log((81*(1 - x^2)^(1/3))/64 - (81*2^(2/
3)*(3^(1/2)*1i - 1)^2)/256)*(3^(1/2)*1i - 1))/32 - (3*2^(1/3)*log((81*(1 -
x^2)^(1/3))/64 - (81*2^(2/3)*(3^(1/2)*1i + 1)^2)/256)*(3^(1/2)*1i + 1))/32
```

3.1022 $\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{(1-x^2)^{2/3}}{8(3+x^2)}+\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{\log(3+x^2)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

```
[Out] -1/8*(-x^2+1)^(2/3)/(x^2+3)-1/96*ln(x^2+3)*2^(1/3)+1/32*ln(2^(2/3)-(-x^2+1)
^(1/3))*2^(1/3)+1/48*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/
3)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {455, 44, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{(1-x^2)^{2/3}}{8(x^2+3)}-\frac{\log(x^2+3)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -1/8*(1 - x^2)^(2/3)/(3 + x^2) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8
*2^(2/3)*Sqrt[3]) - Log[3 + x^2]/(48*2^(2/3)) + Log[2^(2/3) - (1 - x^2)^(1/
3)]/(16*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)^2}\,dx, x, x^2\right)\\
&= -\frac{(1-x^2)^{2/3}}{8(3+x^2)} + \frac{1}{24}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)\\
&= -\frac{(1-x^2)^{2/3}}{8(3+x^2)} - \frac{\log(3+x^2)}{48\ 2^{2/3}} + \frac{1}{16}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right.\\
&= -\frac{(1-x^2)^{2/3}}{8(3+x^2)} - \frac{\log(3+x^2)}{48\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{8\ 2^{2/3}}\\
&= -\frac{(1-x^2)^{2/3}}{8(3+x^2)} + \frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\log(3+x^2)}{48\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right.}{16\ 2^{2/3}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 112, normalized size = 1.11

$$\frac{1}{96}\left(-\frac{12(1-x^2)^{2/3}}{3+x^2}+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((-12*(1 - x^2)^(2/3))/(3 + x^2) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2
)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1/3)*Log[4 +
 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)])/96
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.67, size = 484, normalized size = 4.79

method	result
risch	$\frac{x^2-1}{8(x^2+3)(-x^2+1)^{\frac{1}{3}}}+\frac{\mathrm{RootOf}\left(\_Z^3-2\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\_Z^3-2\right)^3\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-2\right)^2+4\_Z\,\mathrm{RootOf}\left(\_Z^3-2\right)+16\_Z^2\right)}{}\right.}{}$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(x^2-1)/(x^2+3)/(-x^2+1)^(1/3)+1/48*RootOf(_Z^3-2)*ln(-(8*RootOf(_Z^3-2
)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+48*RootOf(Root
Of(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-84*(-x^2+1
)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)
-RootOf(_Z^3-2)*x^2-6*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*
x^2-21*(-x^2+1)^(2/3)+21*RootOf(_Z^3-2)+126*RootOf(RootOf(_Z^3-2)^2+4*_Z*Ro
otOf(_Z^3-2)+16*_Z^2))/(x^2+3))+1/12*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z
^3-2)+16*_Z^2)*ln((24*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(
_Z^3-2)+16*_Z^2)*x^2+64*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2
)^2*RootOf(_Z^3-2)^2*x^2+168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*Ro
otOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)+15*RootOf(_Z^3-2)*x^2+40*RootOf(RootOf
(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+42*(-x^2+1)^(2/3)-63*RootOf(_Z^
3-2)-168*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2))/(x^2+3))
```

Maxima [A]
time = 0.47, size = 104, normalized size = 1.03

$$\frac{1}{96}\cdot 4^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{192}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{96}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
```

```
2/3)) + 1/96*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 1/8*(-x^2 + 1)^(2/3
)/(x^2 + 3)
```

Fricas [A]
time = 0.44, size = 125, normalized size = 1.24

$$\frac{4 \cdot 4^{\frac{1}{6}} \sqrt{3}\,(x^2+3) \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(4^{\frac{1}{3}} \sqrt{3} + 2\sqrt{3}\,(-x^2+1)^{\frac{1}{3}}\right)\right) - 4^{\frac{2}{3}}(x^2+3)\log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}} + (-x^2+1)^{\frac{2}{3}}\right) + 2 \cdot 4^{\frac{2}{3}}(x^2+3)\log\left(-4^{\frac{1}{3}} + (-x^2+1)^{\frac{1}{3}}\right) - 24\,(-x^2+1)^{\frac{2}{3}}}{192\,(x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(4*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3) + 2*
sqrt(3)*(-x^2 + 1)^(1/3))) - 4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2
+ 1)^(1/3) + (-x^2 + 1)^(2/3)) + 2*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 +
 1)^(1/3)) - 24*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(x/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [A]
time = 1.11, size = 104, normalized size = 1.03

$$\frac{1}{96} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3}\left(4^{\frac{1}{3}} + 2\,(-x^2+1)^{\frac{1}{3}}\right)\right) - \frac{1}{192} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}} + (-x^2+1)^{\frac{2}{3}}\right) + \frac{1}{96} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{1}{3}} - (-x^2+1)^{\frac{1}{3}}\right) - \frac{(-x^2+1)^{\frac{2}{3}}}{8\,(x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) + 1/96*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 1/8*(-x^2 + 1)^(2/3)
/(x^2 + 3)
```

Mupad [B]
time = 0.48, size = 126, normalized size = 1.25

$$\frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{64} - \frac{2^{2/3}}{64}\right)}{48} - \frac{(1-x^2)^{2/3}}{8\,(x^2+3)} + \frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{64} - \frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{96} - \frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{64} - \frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (2^(1/3)*log((1 - x^2)^(1/3)/64 - 2^(2/3)/64))/48 - (1 - x^2)^(2/3)/(8*(x^2
 + 3)) + (2^(1/3)*log((1 - x^2)^(1/3)/64 - (2^(2/3)*(3^(1/2)*1i - 1)^2)/256
)*(3^(1/2)*1i - 1))/96 - (2^(1/3)*log((1 - x^2)^(1/3)/64 - (2^(2/3)*(3^(1/2
)*1i + 1)^2)/256)*(3^(1/2)*1i + 1))/96
```

3.1023 $\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=158

$$\frac{(1-x^2)^{2/3}}{24(3+x^2)}-\frac{5\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{24\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\log(x)}{18}+\frac{5\log(3+x^2)}{144\,2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-\right.$$

[Out] 1/24*(-x^2+1)^(2/3)/(x^2+3)-1/18*ln(x)+5/288*ln(x^2+3)*2^(1/3)+1/12*ln(1-(-x^2+1)^(1/3))-5/96*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)-5/144*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)+1/18*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {457, 105, 162, 57, 632, 210, 31, 631}

$$-\frac{5\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{24\,2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{(1-x^2)^{2/3}}{24(x^2+3)}+\frac{5\log(x^2+3)}{144\,2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{5\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{48\,2^{2/3}}-\frac{\log(x)}{18}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]

[Out] (1 - x^2)^(2/3)/(24*(3 + x^2)) - (5*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(24*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(6*Sqrt[3]) - Log[x]/18 + (5*Log[3 + x^2])/(144*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/12 - (5*Log[2^(2/3) - (1 - x^2)^(1/3)])/(48*2^(2/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(3+x)^2}\,dx,x,x^2\right)\\
&= \frac{(1-x^2)^{2/3}}{24\,(3+x^2)} + \frac{1}{24}\text{Subst}\left(\int \frac{4-\frac{x}{3}}{\sqrt[3]{1-x}\,x(3+x)}\,dx,x,x^2\right)\\
&= \frac{(1-x^2)^{2/3}}{24\,(3+x^2)} + \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx,x,x^2\right) - \frac{5}{72}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+\right.\\
&= \frac{(1-x^2)^{2/3}}{24\,(3+x^2)} - \frac{\log(x)}{18} + \frac{5\log(3+x^2)}{144\ 2^{2/3}} - \frac{1}{12}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1-x^2}\right)\\
&= \frac{(1-x^2)^{2/3}}{24\,(3+x^2)} - \frac{\log(x)}{18} + \frac{5\log(3+x^2)}{144\ 2^{2/3}} + \frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{5\log\left(2^{2/3}\right.}{48}\\
&= \frac{(1-x^2)^{2/3}}{24\,(3+x^2)} - \frac{5\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{24\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\log(x)}{18} +
\end{aligned}
$$

Mathematica [A]
time = 0.30, size = 184, normalized size = 1.16

$$
\frac{1}{288}\left(\frac{12(1-x^2)^{2/3}}{3+x^2} - 10\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) + 16\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - 10\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right) + 5\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right) + 16\log\left(-1+\sqrt[3]{1-x^2}\right) - 8\log\left(1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((12*(1 - x^2)^(2/3))/(3 + x^2) - 10*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2
)^(1/3))/Sqrt[3]] + 16*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - 10
*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] + 5*2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(1/3
) + (2 - 2*x^2)^(2/3)] + 16*Log[-1 + (1 - x^2)^(1/3)] - 8*Log[1 + (1 - x^2)
^(1/3) + (1 - x^2)^(2/3)])/288
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{1}{x\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x), x)
```

Fricas [A]
time = 0.44, size = 227, normalized size = 1.44

$$-\frac{20\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\,(x^2+3)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}\,(-1)^{\frac{1}{3}}\,(-x^2+1)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)+5\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^2+3)\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(-x^2+1)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right)-10\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^2+3)\log\left(-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}\right)-32\sqrt{3}\,(x^2+3)\arctan\left(\frac{2}{3}\sqrt{3}\,(-x^2+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+16\,(x^2+3)\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)-32\,(x^2+3)\log\left((-x^2+1)^{\frac{1}{3}}-1\right)-24\,(-x^2+1)^{\frac{2}{3}}}{576\,(x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/576*(20*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*arctan(1/6*4^(1/6)*(2*sqrt(
3)*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 5*4^(2/3)*(-1)^(1/3)*(
x^2 + 3)*log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x
^2 + 1)^(2/3)) - 10*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*log(-4^(1/3)*(-1)^(2/3) +
(-x^2 + 1)^(1/3)) - 32*sqrt(3)*(x^2 + 3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3
) + 1/3*sqrt(3)) + 16*(x^2 + 3)*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1
) - 32*(x^2 + 3)*log((-x^2 + 1)^(1/3) - 1) - 24*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [A]
time = 0.92, size = 167, normalized size = 1.06

$$-\frac{5}{288}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\,(-x^2+1)^{\frac{1}{3}}\right)\right)+\frac{5}{576}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right)-\frac{5}{288}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-(-x^2+1)^{\frac{1}{3}}\right)+\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(-x^2+1)^{\frac{1}{3}}+1\right)\right)+\frac{(-x^2+1)^{\frac{2}{3}}}{24\,(x^2+3)}-\frac{1}{36}\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)+\frac{1}{18}\log\left(-(-x^2+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

[Out] -5/288*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/3))) + 5/576*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) - 5/288*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/24*(-x^2 + 1)^(2/3)/(x^2 + 3) - 1/36*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/18*log(-(-x^2 + 1)^(1/3) + 1)

Mupad [B]
time = 0.55, size = 375, normalized size = 2.37

$$\frac{\ln\left(\frac{127}{512}-\frac{127\,(1-x^2)^{1/3}}{512}\right)}{18}-\frac{5\,2^{1/3}\ln\left(-\frac{25\,2^{2/3}\left(\frac{5\,2^{1/3}\left(\frac{30375\,2^{2/3}}{64}-\frac{68283\,(1-x^2)^{1/3}}{64}\right)}{144}-\frac{1647}{128}\right)}{20736}-\frac{25\,(1-x^2)^{1/3}}{384}\right)}{144}+\ln\left(\left(-\frac{1}{36}+\frac{\sqrt{3}\,1i}{36}\right)^2\left(\left(-\frac{1}{36}+\frac{\sqrt{3}\,1i}{36}\right)\left(393660\left(-\frac{1}{36}+\frac{\sqrt{3}\,1i}{36}\right)^2-\frac{68283\,(1-x^2)^{1/3}}{64}\right)+\frac{1647}{128}\right)-\frac{25\,(1-x^2)^{1/3}}{384}\right)\left(-\frac{1}{36}+\frac{\sqrt{3}\,1i}{36}\right)-\ln\left(-\left(\frac{1}{36}+\frac{\sqrt{3}\,1i}{36}\right)^2\left(\left(\frac{1}{36}+\frac{\sqrt{3}\,1i}{36}\right)\left(393660\left(\frac{1}{36}+\frac{\sqrt{3}\,1i}{36}\right)^2-\frac{68283\,(1-x^2)^{1/3}}{64}\right)-\frac{1647}{128}\right)-\frac{25\,(1-x^2)^{1/3}}{384}\right)\left(\frac{1}{36}+\frac{\sqrt{3}\,1i}{36}\right)+\frac{(1-x^2)^{2/3}}{24\,(x^2+3)}+\frac{5\,(-1)^{1/3}\,2^{1/3}\ln\left(\frac{25\,(-1)^{2/3}\,2^{2/3}\left(\frac{5\,(-1)^{1/3}\,2^{1/3}\left(\frac{30375\,(-1)^{2/3}\,2^{2/3}}{64}-\frac{68283\,(1-x^2)^{1/3}}{64}\right)}{144}+\frac{1647}{128}\right)}{20736}-\frac{25\,(1-x^2)^{1/3}}{384}\right)}{144}-\frac{5\,(-1)^{1/3}\,2^{1/3}\ln\left(-\frac{25\,(1-x^2)^{1/3}}{384}+\frac{25\,(-1)^{2/3}\,2^{2/3}\left(1+\sqrt{3}\,1i\right)^2\left(\frac{5\,(-1)^{1/3}\,2^{1/3}\left(1+\sqrt{3}\,1i\right)\left(\frac{68283\,(1-x^2)^{1/3}}{64}-\frac{30375\,(-1)^{2/3}\,2^{2/3}\left(1+\sqrt{3}\,1i\right)^2}{256}\right)}{288}+\frac{1647}{128}\right)}{82944}\right)\left(1+\sqrt{3}\,1i\right)}{288}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)

[Out] log(127/512 - (127*(1 - x^2)^(1/3))/512)/18 - (5*2^(1/3)*log(- (25*2^(2/3)*((5*2^(1/3)*((30375*2^(2/3))/64 - (68283*(1 - x^2)^(1/3))/64))/144 - 1647/128))/20736 - (25*(1 - x^2)^(1/3))/384))/144 + log(((3^(1/2)*1i)/36 - 1/36)^2*(((3^(1/2)*1i)/36 - 1/36)*(393660*((3^(1/2)*1i)/36 - 1/36)^2 - (68283*(1 - x^2)^(1/3))/64) + 1647/128) - (25*(1 - x^2)^(1/3))/384)*((3^(1/2)*1i)/36 - 1/36) - log(- ((3^(1/2)*1i)/36 + 1/36)^2*(((3^(1/2)*1i)/36 + 1/36)*(393660*((3^(1/2)*1i)/36 + 1/36)^2 - (68283*(1 - x^2)^(1/3))/64) - 1647/128) - (25*(1 - x^2)^(1/3))/384)*((3^(1/2)*1i)/36 + 1/36) + (1 - x^2)^(2/3)/(24*(x^2 + 3)) + (5*(-1)^(1/3)*2^(1/3)*log((25*(-1)^(2/3)*2^(2/3)*((5*(-1)^(1/3)*2^(1/3)*((30375*(-1)^(2/3)*2^(2/3))/64 - (68283*(1 - x^2)^(1/3))/64))/144 + 1647/128))/20736 - (25*(1 - x^2)^(1/3))/384))/144 - (5*(-1)^(1/3)*2^(1/3)*log((25*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2*((5*(-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*((68283*(1 - x^2)^(1/3))/64 - (30375*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2)/256))/288 + 1647/128))/82944 - (25*(1 - x^2)^(1/3))/384)*(3^(1/2)*1i + 1))/288

3.1024 $\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{5(1-x^2)^{2/3}}{72\,(3+x^2)}-\frac{(1-x^2)^{2/3}}{6x^2\,(3+x^2)}+\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{\log(x)}{54}-\frac{\log(3+x^2)}{48\ 2^{2/3}}-\frac{1}{36}\log$$

```
[Out] -5/72*(-x^2+1)^(2/3)/(x^2+3)-1/6*(-x^2+1)^(2/3)/x^2/(x^2+3)+1/54*ln(x)-1/96
*ln(x^2+3)*2^(1/3)-1/36*ln(1-(-x^2+1)^(1/3))+1/32*ln(2^(2/3)-(-x^2+1)^(1/3)
)*2^(1/3)+1/48*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)-1/5
4*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {457, 105, 156, 162, 57, 632, 210, 31, 631}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{(1-x^2)^{2/3}}{6x^2\,(x^2+3)}-\frac{5(1-x^2)^{2/3}}{72\,(x^2+3)}-\frac{\log(x^2+3)}{48\ 2^{2/3}}-\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\log(x)}{54}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-5*(1 - x^2)^(2/3))/(72*(3 + x^2)) - (1 - x^2)^(2/3)/(6*x^2*(3 + x^2)) + A
rcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[3]) - ArcTan[(1 + 2*
(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[x]/54 - Log[3 + x^2]/(48*2^(2/
3)) - Log[1 - (1 - x^2)^(1/3)]/36 + Log[2^(2/3) - (1 - x^2)^(1/3)]/(16*2^(2
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^2(3+x)^2}\,dx,x,x^2\right)\\
&= -\frac{(1-x^2)^{2/3}}{6x^2(3+x^2)} - \frac{1}{6}\text{Subst}\left(\int \frac{1-\frac{4x}{3}}{\sqrt[3]{1-x}\,x(3+x)^2}\,dx,x,x^2\right)\\
&= -\frac{5(1-x^2)^{2/3}}{72(3+x^2)} - \frac{(1-x^2)^{2/3}}{6x^2(3+x^2)} - \frac{1}{72}\text{Subst}\left(\int \frac{4-\frac{5x}{3}}{\sqrt[3]{1-x}\,x(3+x)}\,dx,x,x^2\right)\\
&= -\frac{5(1-x^2)^{2/3}}{72(3+x^2)} - \frac{(1-x^2)^{2/3}}{6x^2(3+x^2)} - \frac{1}{54}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx,x,x^2\right) + \frac{1}{24}\text{Subst}\\
&= -\frac{5(1-x^2)^{2/3}}{72(3+x^2)} - \frac{(1-x^2)^{2/3}}{6x^2(3+x^2)} + \frac{\log(x)}{54} - \frac{\log(3+x^2)}{48\ 2^{2/3}} + \frac{1}{36}\text{Subst}\left(\int \frac{1}{1-x}\,d\right.\\
&= -\frac{5(1-x^2)^{2/3}}{72(3+x^2)} - \frac{(1-x^2)^{2/3}}{6x^2(3+x^2)} + \frac{\log(x)}{54} - \frac{\log(3+x^2)}{48\ 2^{2/3}} - \frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right.\\
&= -\frac{5(1-x^2)^{2/3}}{72(3+x^2)} - \frac{(1-x^2)^{2/3}}{6x^2(3+x^2)} + \frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{18\sqrt{3}}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 194, normalized size = 1.06

$$\frac{1}{864}\left(-\frac{12(1-x^2)^{2/3}(12+5x^2)}{x^2(3+x^2)}+18\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)-16\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)+18\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-9\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)-16\log\left(-1+\sqrt[3]{1-x^2}\right)+8\log\left(1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
[Out] ((-12*(1 - x^2)^(2/3)*(12 + 5*x^2))/(x^2*(3 + x^2)) + 18*2^(1/3)*Sqrt[3]*Ar
cTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 16*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^
(1/3))/Sqrt[3]] + 18*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 9*2^(1/3)*Log[4
+ 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)] - 16*Log[-1 + (1 - x^2)^(1/3)] +
 8*Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/864
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^3), x)
```

Fricas [A]
time = 0.49, size = 238, normalized size = 1.30

$$\frac{36\cdot 4^{\frac{1}{6}}\sqrt{3}\,(x^4+3\,x^2)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}+2\sqrt{3}\,(-x^2+1)^{\frac{1}{3}}\right)\right)-9\cdot 4^{\frac{2}{3}}(x^4+3\,x^2)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right)+18\cdot 4^{\frac{2}{3}}(x^4+3\,x^2)\log\left(-4^{\frac{1}{3}}+(-x^2+1)^{\frac{1}{3}}\right)-32\sqrt{3}\,(x^4+3\,x^2)\arctan\left(\frac{2}{3}\sqrt{3}\,(-x^2+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+16\,(x^4+3\,x^2)\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)-32\,(x^4+3\,x^2)\log\left((-x^2+1)^{\frac{1}{3}}-1\right)-24\,(5\,x^2+12)(-x^2+1)^{\frac{2}{3}}}{1728\,(x^4+3\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1728*(36*4^(1/6)*sqrt(3)*(x^4 + 3*x^2)*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3
) + 2*sqrt(3)*(-x^2 + 1)^(1/3))) - 9*4^(2/3)*(x^4 + 3*x^2)*log(4^(2/3) + 4^
(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 18*4^(2/3)*(x^4 + 3*x^2)*log(-
4^(1/3) + (-x^2 + 1)^(1/3)) - 32*sqrt(3)*(x^4 + 3*x^2)*arctan(2/3*sqrt(3)*(
-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 16*(x^4 + 3*x^2)*log((-x^2 + 1)^(2/3) + (-
x^2 + 1)^(1/3) + 1) - 32*(x^4 + 3*x^2)*log((-x^2 + 1)^(1/3) - 1) - 24*(5*x^
2 + 12)*(-x^2 + 1)^(2/3))/(x^4 + 3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt[3]{-(x-1)(x+1)}\,(x^{2}+3)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [A]
time = 1.08, size = 190, normalized size = 1.04

$$\frac{1}{96}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\,(-x^2+1)^{\frac{1}{3}}\right)\right)-\frac{1}{192}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right)+\frac{1}{96}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-(-x^2+1)^{\frac{1}{3}}\right)-\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(-x^2+1)^{\frac{1}{3}}+1\right)\right)+\frac{5\,(-x^2+1)^{\frac{5}{3}}-17\,(-x^2+1)^{\frac{2}{3}}}{72\left((x^2-1)^2+5\,x^2-1\right)}+\frac{1}{108}\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right)-\frac{1}{54}\log\left(-(-x^2+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) + 1/96*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 1/54*sqrt(3)*arctan(
1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/72*(5*(-x^2 + 1)^(5/3) - 17*(-x^2
 + 1)^(2/3))/((x^2 - 1)^2 + 5*x^2 - 1) + 1/108*log((-x^2 + 1)^(2/3) + (-x^2
 + 1)^(1/3) + 1) - 1/54*log(-(-x^2 + 1)^(1/3) + 1)
```

Mupad [B]

time = 0.54, size = 409, normalized size = 2.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (2^(1/3)*log((2^(2/3)*((2^(1/3)*((10935*2^(2/3))/64 - (9099*(1 - x^2)^(1/3)
)/64))/48 - 665/128))/2304 + (1 - x^2)^(1/3)/576))/48 - log((985*(1 - x^2)^
(1/3))/373248 - 985/373248)/54 + log(((3^(1/2)*1i)/108 + 1/108)^2*(((3^(1/2
)*1i)/108 + 1/108)*(393660*((3^(1/2)*1i)/108 + 1/108)^2 - (9099*(1 - x^2)^(
1/3))/64) - 665/128) + (1 - x^2)^(1/3)/576)*((3^(1/2)*1i)/108 + 1/108) - lo
g((1 - x^2)^(1/3)/576 - ((3^(1/2)*1i)/108 - 1/108)^2*(((3^(1/2)*1i)/108 - 1
/108)*(393660*((3^(1/2)*1i)/108 - 1/108)^2 - (9099*(1 - x^2)^(1/3))/64) + 6
65/128))*((3^(1/2)*1i)/108 - 1/108) - ((17*(1 - x^2)^(2/3))/72 - (5*(1 - x^
2)^(5/3))/72)/((x^2 - 1)^2 + 5*x^2 - 1) + (2^(1/3)*log((1 - x^2)^(1/3)/576
+ (2^(2/3)*(3^(1/2)*1i - 1)^2*((2^(1/3)*(3^(1/2)*1i - 1)*((10935*2^(2/3)*(3
^(1/2)*1i - 1)^2)/256 - (9099*(1 - x^2)^(1/3))/64))/96 - 665/128))/9216)*(3
^(1/2)*1i - 1))/96 - (2^(1/3)*log((1 - x^2)^(1/3)/576 - (2^(2/3)*(3^(1/2)*1
i + 1)^2*((2^(1/3)*(3^(1/2)*1i + 1)*((10935*2^(2/3)*(3^(1/2)*1i + 1)^2)/256
 - (9099*(1 - x^2)^(1/3))/64))/96 + 665/128))/9216)*(3^(1/2)*1i + 1))/96
```

3.1025 $\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=208

$$\frac{(1-x^2)^{2/3}}{216\,(3+x^2)}-\frac{(1-x^2)^{2/3}}{12x^4\,(3+x^2)}-\frac{(1-x^2)^{2/3}}{36x^2\,(3+x^2)}-\frac{13\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{216\;2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{\log(x)}{54}+$$

```
[Out] 1/216*(-x^2+1)^(2/3)/(x^2+3)-1/12*(-x^2+1)^(2/3)/x^4/(x^2+3)-1/36*(-x^2+1)^
(2/3)/x^2/(x^2+3)-1/54*ln(x)+13/2592*ln(x^2+3)*2^(1/3)+1/36*ln(1-(-x^2+1)^(
1/3))-13/864*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)-13/1296*arctan(1/3*(1+(-2*x
^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)+1/54*arctan(1/3*(1+2*(-x^2+1)^(1/3))*
3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {457, 105, 156, 162, 57, 632, 210, 31, 631}

$$-\frac{13\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{216\;2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{(1-x^2)^{2/3}}{36x^2(x^2+3)}+\frac{(1-x^2)^{2/3}}{216(x^2+3)}+\frac{13\log(x^2+3)}{1296\;2^{2/3}}+\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{13\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{432\;2^{2/3}}-\frac{(1-x^2)^{2/3}}{12x^4(x^2+3)}-\frac{\log(x)}{54}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (1 - x^2)^(2/3)/(216*(3 + x^2)) - (1 - x^2)^(2/3)/(12*x^4*(3 + x^2)) - (1 -
 x^2)^(2/3)/(36*x^2*(3 + x^2)) - (13*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]
])/(216*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt
[3]) - Log[x]/54 + (13*Log[3 + x^2])/(1296*2^(2/3)) + Log[1 - (1 - x^2)^(1/
3)]/36 - (13*Log[2^(2/3) - (1 - x^2)^(1/3)])/(432*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^3(3+x)^2}\,dx, x, x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{1}{12}\text{Subst}\left(\int \frac{-1-\frac{7x}{3}}{\sqrt[3]{1-x}\,x^2(3+x)^2}\,dx, x, x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} + \frac{1}{36}\text{Subst}\left(\int \frac{6+\frac{4x}{3}}{\sqrt[3]{1-x}\,x(3+x)^2}\,dx, x, x^2\right)\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} + \frac{1}{432}\text{Subst}\left(\int \frac{24-\frac{2x}{3}}{\sqrt[3]{1-x}\,x(3-\right.\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} + \frac{1}{54}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx,\right.\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} - \frac{\log(x)}{54} + \frac{13\log\left(3+x^2\right)}{1296\ 2^{2/3}} -\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} - \frac{\log(x)}{54} + \frac{13\log\left(3+x^2\right)}{1296\ 2^{2/3}} +\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)} - \frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)} - \frac{13\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{216\ 2^{2/3}\sqrt{3}} +
\end{aligned}
$$

Mathematica [A]
time = 0.38, size = 197, normalized size = 0.95

$$
\frac{\frac{12\left(1-x^2\right)^{2/3}\left(-18-6x^2+x^4\right)}{x^4\left(3+x^2\right)} - 26\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) + 48\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - 26\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right) + 13\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right) + 48\log\left(-1+\sqrt[3]{1-x^2}\right) - 24\log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)}{2592}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((12*(1 - x^2)^(2/3)*(-18 - 6*x^2 + x^4))/(x^4*(3 + x^2)) - 26*2^(1/3)*Sqrt
[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 48*Sqrt[3]*ArcTan[(1 + 2*(1 -
 x^2)^(1/3))/Sqrt[3]] - 26*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] + 13*2^(1/3)
*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)] + 48*Log[-1 + (1 - x^2)^(
1/3)] - 24*Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/2592
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(-x^2+1\right)^{\frac{1}{3}} \left(x^2+3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^5), x)
```

Fricas [A]
time = 0.49, size = 265, normalized size = 1.27

$$-\frac{52 \cdot 4^{\frac{1}{6}} \sqrt{3} \left(-1\right)^{\frac{1}{3}} \left(x^6+3 x^4\right) \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(2 \sqrt{3} \left(-1\right)^{\frac{1}{3}} \left(-x^2+1\right)^{\frac{1}{3}} - 4^{\frac{1}{3}} \sqrt{3}\right)\right) + 13 \cdot 4^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(x^6+3 x^4\right) \log\left(4^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} \left(-x^2+1\right)^{\frac{1}{3}} - 4^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{2}{3}}\right) - 26 \cdot 4^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(x^6+3 x^4\right) \log\left(-4^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} + \left(-x^2+1\right)^{\frac{1}{3}}\right) - 96 \sqrt{3} \left(x^6+3 x^4\right) \arctan\left(\frac{2}{3} \sqrt{3} \left(-x^2+1\right)^{\frac{1}{3}} + \frac{1}{3} \sqrt{3}\right) + 48 \left(x^6+3 x^4\right) \log\left(\left(-x^2+1\right)^{\frac{2}{3}} + \left(-x^2+1\right)^{\frac{1}{3}} + 1\right) - 96 \left(x^6+3 x^4\right) \log\left(\left(-x^2+1\right)^{\frac{1}{3}} - 1\right) - 24 \left(x^4-6 x^2-18\right) \left(-x^2+1\right)^{\frac{2}{3}}}{5184 \left(x^6+3 x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/5184*(52*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^6 + 3*x^4)*arctan(1/6*4^(1/6)*(2*
sqrt(3)*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 13*4^(2/3)*(-1)^(
1/3)*(x^6 + 3*x^4)*log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(
1/3) + (-x^2 + 1)^(2/3)) - 26*4^(2/3)*(-1)^(1/3)*(x^6 + 3*x^4)*log(-4^(1/3)
*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 96*sqrt(3)*(x^6 + 3*x^4)*arctan(2/3*sqrt(
3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 48*(x^6 + 3*x^4)*log((-x^2 + 1)^(2/3)
+ (-x^2 + 1)^(1/3) + 1) - 96*(x^6 + 3*x^4)*log((-x^2 + 1)^(1/3) - 1) - 24*(
x^4 - 6*x^2 - 18)*(-x^2 + 1)^(2/3))/(x^6 + 3*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \sqrt[3]{-\left(x-1\right)\left(x+1\right)} \left(x^2+3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [A]
time = 0.81, size = 181, normalized size = 0.87

$$-\frac{13}{2592}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{13}{5184}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{13}{2592}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{216\left(x^2+3\right)}-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{36x^4}-\frac{1}{108}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{54}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] -13/2592*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1
)^(1/3))) + 13/5184*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2
+ 1)^(2/3)) - 13/2592*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/54*sqrt(3
)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/216*(-x^2 + 1)^(2/3)/(x^
2 + 3) - 1/36*(-x^2 + 1)^(2/3)/x^4 - 1/108*log((-x^2 + 1)^(2/3) + (-x^2 + 1
)^(1/3) + 1) + 1/54*log(-(-x^2 + 1)^(1/3) + 1)
```

Mupad [B]
time = 0.56, size = 416, normalized size = 2.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] log(9109/10077696 - (9109*(1 - x^2)^(1/3))/10077696)/54 - (13*2^(1/3)*log(-
 (169*2^(2/3)*((13*2^(1/3)*((2535*2^(2/3))/64 - (7419*(1 - x^2)^(1/3))/64))
/1296 - 469/3456))/1679616 - (845*(1 - x^2)^(1/3))/5038848))/1296 + ((1 - x
^2)^(5/3)/54 - (23*(1 - x^2)^(2/3))/216 + (1 - x^2)^(8/3)/216)/(6*(x^2 - 1)
^2 + (x^2 - 1)^3 + 9*x^2 - 5) + log(((3^(1/2)*1i)/108 - 1/108)^2*(((3^(1/2)
*1i)/108 - 1/108)*(393660*((3^(1/2)*1i)/108 - 1/108)^2 - (7419*(1 - x^2)^(1
/3))/64) + 469/3456) - (845*(1 - x^2)^(1/3))/5038848*((3^(1/2)*1i)/108 - 1
/108) - log(- ((3^(1/2)*1i)/108 + 1/108)^2*(((3^(1/2)*1i)/108 + 1/108)*(393
660*((3^(1/2)*1i)/108 + 1/108)^2 - (7419*(1 - x^2)^(1/3))/64) - 469/3456) -
 (845*(1 - x^2)^(1/3))/5038848*((3^(1/2)*1i)/108 + 1/108) + (13*(-1)^(1/3)
*2^(1/3)*log((169*(-1)^(2/3)*2^(2/3)*((13*(-1)^(1/3)*2^(1/3)*((2535*(-1)^(2
/3)*2^(2/3))/64 - (7419*(1 - x^2)^(1/3))/64))/1296 + 469/3456))/1679616 - (
845*(1 - x^2)^(1/3))/5038848))/1296 - (13*(-1)^(1/3)*2^(1/3)*log((169*(-1)^
(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2*((13*(-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*(
(7419*(1 - x^2)^(1/3))/64 - (2535*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2)/25
6))/2592 + 469/3456))/6718464 - (845*(1 - x^2)^(1/3))/5038848)*(3^(1/2)*1i
+ 1))/2592
```

3.1026 $\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=543

$$\frac{3x(1-x^2)^{2/3}}{8(3+x^2)}-\frac{27x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}-\frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}}-\frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}}+\frac{5\,\mathrm{t}}{}$$

[Out] 3/8*x*(-x^2+1)^(2/3)/(x^2+3)+5/16*arctanh(x)*2^(1/3)-15/16*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)-27/8*x/(1-(-x^2+1)^(1/3)-3^(1/2))-5/16*arctan(1/x*3^(1/2))*2^(1/3)*3^(1/2)-5/16*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)+9/8*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)-27/16*3^(1/4)*(1-(-x^2+1)^(1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.16, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {481, 544, 241, 310, 225, 1893, 402}

$$\frac{9\ 3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}-\frac{27\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,E\left(\mathrm{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}-\frac{5\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}}-\frac{5\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}}+\frac{3(1-x^2)^{2/3}x}{8(x^2+3)}-\frac{27x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{15\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\ 2^{2/3}}+\frac{5\tanh^{-1}(x)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((1 - x^2)^(1/3)*(3 + x^2)^2),x]

[Out] (3*x*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (27*x)/(8*(1 - Sqrt[3] - (1 - x^2)^(1/3))) - (5*Sqrt[3]*ArcTan[Sqrt[3]/x])/(8*2^(2/3)) - (5*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(8*2^(2/3)) + (5*ArcTanh[x])/(8*2^(2/3)) - (15*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(8*2^(2/3)) - (27*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) + (9*3^(3/4)*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))

```
/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*x*S
qrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{3x(1-x^2)^{2/3}}{8(3+x^2)} - \frac{1}{8}\int \frac{3-9x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= \frac{3x(1-x^2)^{2/3}}{8(3+x^2)} + \frac{9}{8}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx - \frac{15}{4}\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= \frac{3x(1-x^2)^{2/3}}{8(3+x^2)} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}} +$$

$$= \frac{3x(1-x^2)^{2/3}}{8(3+x^2)} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}} +$$

$$= \frac{3x(1-x^2)^{2/3}}{8(3+x^2)} - \frac{27x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} - \frac{5\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}} - \frac{5\sqrt{3}\tan^{-1}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 5.42, size = 157, normalized size = 0.29

$$\frac{1}{8}x\left(x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{3\left(1-x^2+\frac{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (x*(x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2] + (3*(1 - x^2 + (9*Appell
F1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1
/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2,
4/3, 1, 5/2, x^2, -1/3*x^2]))))/((1 - x^2)^(1/3)*(3 + x^2))))/8
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)*x^4/(x^6 + 5*x^4 + 3*x^2 - 9), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
[Out] Integral(x**4/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
[Out] integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(1-x^2)^{1/3}\,(x^2+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
[Out] int(x^4/((1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1027 $\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=543

$$-\frac{x(1-x^2)^{2/3}}{8(3+x^2)}+\frac{x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{24\ 2^{2/3}}+$$

```
[Out] -1/8*x*(-x^2+1)^(2/3)/(x^2+3)-1/48*arctanh(x)*2^(1/3)+1/16*arctanh(x/(1+2^(
1/3)*(-x^2+1)^(1/3)))*2^(1/3)+1/8*x/(1-(-x^2+1)^(1/3)-3^(1/2))+1/48*arctan(
1/x*3^(1/2))*2^(1/3)*3^(1/2)+1/48*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)
/x)*2^(1/3)*3^(1/2)-1/24*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^(
1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1
)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)
^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+1/16*3^(1/4)*(1-(-x^2+1)^(1/3))
*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1
/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2)
)^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {482, 544, 241, 310, 225, 1893, 402}

$$-\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}x}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{(1-x^2)^{2/3}x}{8(x^2+3)}+\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\ 2^{2/3}}-\frac{\tanh^{-1}(x)}{24\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -1/8*(x*(1 - x^2)^(2/3))/(3 + x^2) + x/(8*(1 - Sqrt[3] - (1 - x^2)^(1/3)))
+ ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 -
 x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(24*2^(2/3)) + ArcTanh[x/
(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*2^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3
] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1
 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1 - x^2
)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - ((1 - (1 - x^2)^(1/3))*Sqrt
[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]
```

```
*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))
/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = -\frac{x(1-x^2)^{2/3}}{8(3+x^2)}+\frac{1}{8}\int \frac{1-\frac{x^2}{3}}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{x(1-x^2)^{2/3}}{8(3+x^2)}-\frac{1}{24}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx+\frac{1}{4}\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{x(1-x^2)^{2/3}}{8(3+x^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(}{24\,2^{2/}}$$

$$= -\frac{x(1-x^2)^{2/3}}{8(3+x^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(}{24\,2^{2/}}$$

$$= -\frac{x(1-x^2)^{2/3}}{8(3+x^2)}+\frac{x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\right.}{}\right)}{8\,2^{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 5.00, size = 156, normalized size = 0.29

$$-\frac{1}{216}x^3F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{x\left(-1+x^2+\frac{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)}\right)}{8\sqrt[3]{1-x^2}\,(3+x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -1/216*(x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2]) + (x*(-1 + x^2 + (9*
AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(9*AppellF1[1/2, 1/3, 1, 3/2, x^
2, -1/3*x^2] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] + AppellF1
[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/(8*(1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)*x^2/(x^6 + 5*x^4 + 3*x^2 - 9), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
[Out] Integral(x**2/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
[Out] integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(1-x^2)^{1/3}\,(x^2+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
[Out] int(x^2/((1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1028 $\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=543

$$\frac{x(1-x^2)^{2/3}}{24\,(3+x^2)}-\frac{x}{24\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{24\ 2^{2/3}}+\text{ta}$$

```
[Out] 1/24*x*(-x^2+1)^(2/3)/(x^2+3)-1/48*arctanh(x)*2^(1/3)+1/16*arctanh(x/(1+2^(
1/3)*(-x^2+1)^(1/3)))*2^(1/3)-1/24*x/(1-(-x^2+1)^(1/3)-3^(1/2))+1/48*arctan
(1/x*3^(1/2))*2^(1/3)*3^(1/2)+1/48*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2
)/x)*2^(1/3)*3^(1/2)+1/72*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^
(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+
1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1
)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)-1/48*3^(1/4)*(1-(-x^2+1)^(1/3)
)*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(
1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2
)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2
))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {425, 544, 241, 310, 225, 1893, 402}

$$\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}-\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}}+\frac{(1-x^2)^{2/3}x}{24\,(x^2+3)}-\frac{x}{24\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{8\ 2^{2/3}}-\frac{\tanh^{-1}(x)}{24\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (x*(1 - x^2)^(2/3))/(24*(3 + x^2)) - x/(24*(1 - Sqrt[3] - (1 - x^2)^(1/3)))
 + ArcTan[Sqrt[3]/x]/(8*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1
- x^2)^(1/3)))/x]/(8*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(24*2^(2/3)) + ArcTanh[x
/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(8*2^(2/3)) - (Sqrt[2 + Sqrt[3]]*(1 - (1 -
x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1
- x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt
[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4)*x*Sqrt[-((1 - (1 - x^
2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) + ((1 - (1 - x^2)^(1/3))*Sqr
t[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2
```

```
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3
))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{x(1-x^2)^{2/3}}{24\,(3+x^2)} - \frac{1}{24}\int \frac{-7-\frac{x^2}{3}}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= \frac{x(1-x^2)^{2/3}}{24\,(3+x^2)} + \frac{1}{72}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx + \frac{1}{4}\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= \frac{x(1-x^2)^{2/3}}{24\,(3+x^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{24\ 2^{2/3}}$$

$$= \frac{x(1-x^2)^{2/3}}{24\,(3+x^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}(x)}{24\ 2^{2/3}}$$

$$= \frac{x(1-x^2)^{2/3}}{24\,(3+x^2)} - \frac{x}{24\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{\ }\right.}{\ }\right.}{8\ 2^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.10, size = 157, normalized size = 0.29

$$\frac{1}{648}x\left(x^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{27\left(1-x^2+\frac{63F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (x*(x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2] + (27*(1 - x^2 + (63*Appe
llF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -
1/3*x^2] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] + AppellF1[3/2
, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/((1 - x^2)^(1/3)*(3 + x^2))))/648
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] integral(-(-x^2 + 1)^(2/3)/(x^6 + 5*x^4 + 3*x^2 - 9), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(1-x^2)^{1/3}\,(x^2+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1029 $\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=563

$$-\frac{(1-x^2)^{2/3}}{8x}+\frac{(1-x^2)^{2/3}}{24x\,(3+x^2)}+\frac{x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{72\;2^{2/3}\sqrt{3}}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{72\;2^{2/3}\sqrt{3}}$$

```
[Out] -1/8*(-x^2+1)^(2/3)/x+1/24*(-x^2+1)^(2/3)/x/(x^2+3)+7/432*arctanh(x)*2^(1/3
)-7/144*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+1/8*x/(1-(-x^2+1)^(1/
3)-3^(1/2))-7/432*arctan(1/x*3^(1/2))*2^(1/3)*3^(1/2)-7/432*arctan((1-2^(1/
3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)-1/24*3^(3/4)*(1-(-x^2+1)^(1/3
))*EllipticF((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^
(1/2))*2^(1/2)*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2)
)^2)^(1/2)/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+1/16*
3^(1/4)*(1-(-x^2+1)^(1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)
^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+
1)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))
/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {483, 597, 544, 241, 310, 225, 1893, 402}

$$-\frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}-\frac{7\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{72\;2^{2/3}\sqrt{3}}-\frac{7\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{72\;2^{2/3}\sqrt{3}}+\frac{x}{8\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}-\frac{(1-x^2)^{2/3}}{8x}+\frac{(1-x^2)^{2/3}}{24(x^2+3)x}-\frac{7\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{72\;2^{2/3}}+\frac{7\tanh^{-1}(x)}{216\;2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -1/8*(1 - x^2)^(2/3)/x + (1 - x^2)^(2/3)/(24*x*(3 + x^2)) + x/(8*(1 - Sqrt[
3] - (1 - x^2)^(1/3))) - (7*ArcTan[Sqrt[3]/x])/(72*2^(2/3)*Sqrt[3]) - (7*Ar
cTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x])/(72*2^(2/3)*Sqrt[3]) + (7*
ArcTanh[x])/(216*2^(2/3)) - (7*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(7
2*2^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1
- x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Elliptic
E[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(16*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2
)^(1/3))^2)]) - ((1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2
```

```
)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*Sqr
t[2]*3^(1/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{(1-x^2)^{2/3}}{24x\,(3+x^2)} - \frac{1}{24}\int \frac{-9+\frac{5x^2}{3}}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$
$$= -\frac{(1-x^2)^{2/3}}{8x} + \frac{(1-x^2)^{2/3}}{24x\,(3+x^2)} + \frac{1}{72}\int \frac{-23-3x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$
$$= -\frac{(1-x^2)^{2/3}}{8x} + \frac{(1-x^2)^{2/3}}{24x\,(3+x^2)} - \frac{1}{24}\int \frac{1}{\sqrt[3]{1-x^2}}\,dx - \frac{7}{36}\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}$$
$$= -\frac{(1-x^2)^{2/3}}{8x} + \frac{(1-x^2)^{2/3}}{24x\,(3+x^2)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{72\ 2^{2/3}\sqrt{3}} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-\right.}}{x}\right.}{72\ 2^{2/3}\sqrt{3}}$$
$$= -\frac{(1-x^2)^{2/3}}{8x} + \frac{(1-x^2)^{2/3}}{24x\,(3+x^2)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{72\ 2^{2/3}\sqrt{3}} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-\right.}}{x}\right.}{72\ 2^{2/3}\sqrt{3}}$$
$$= -\frac{(1-x^2)^{2/3}}{8x} + \frac{(1-x^2)^{2/3}}{24x\,(3+x^2)} + \frac{x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{72\ 2^{2/3}\sqrt{3}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.12, size = 168, normalized size = 0.30

$$\frac{-x^4F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{9\left(-8+5x^2+3x^4+\frac{69x^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)}}{216x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-(x^4*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2]) + (9*(-8 + 5*x^2 + 3*x^4
+ (69*x^2*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(-9*AppellF1[1/2, 1/3,
 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2]
- AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/((1 - x^2)^(1/3)*(3 + x^2))
)/(216*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-x^{2}+1\right)^{\frac{1}{3}} \left(x^{2}+3\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
[Out] int(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
[Out] integral(-(-x^2 + 1)^(2/3)/(x^8 + 5*x^6 + 3*x^4 - 9*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \sqrt[3]{- \left(x - 1\right) \left(x + 1\right)} \left(x^{2} + 3\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
[Out] Integral(1/(x**2*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(1-x^2\right)}^{1/3}\,{\left(x^2+3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] int(1/(x^2*(1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1030 $\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$

Optimal. Leaf size=581

$$-\frac{11(1-x^2)^{2/3}}{216x^3}+\frac{11(1-x^2)^{2/3}}{648x}+\frac{(1-x^2)^{2/3}}{24x^3(3+x^2)}-\frac{11x}{648\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{11\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{11\tan^{-1}}{}$$

```
[Out] -11/216*(-x^2+1)^(2/3)/x^3+11/648*(-x^2+1)^(2/3)/x+1/24*(-x^2+1)^(2/3)/x^3/
(x^2+3)-11/1296*arctanh(x)*2^(1/3)+11/432*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/
3)))*2^(1/3)-11/648*x/(1-(-x^2+1)^(1/3)-3^(1/2))+11/1296*arctan(1/x*3^(1/2)
)*2^(1/3)*3^(1/2)+11/1296*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(1
/3)*3^(1/2)+11/1944*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^(1/3)+
3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1)^(1/
3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)^(1/3
))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)-11/1296*3^(1/4)*(1-(-x^2+1)^(1/3))*E
llipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2
))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)*(
1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^
2)^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {483, 597, 544, 241, 310, 225, 1893, 402}

$$\frac{11\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}-\frac{11\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{432\ 3^{3/4}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,x}+\frac{11\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{11\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{216\ 2^{2/3}\sqrt{3}}-\frac{11x}{648\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}+\frac{11(1-x^2)^{2/3}}{648x}+\frac{11\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{216\ 2^{2/3}}-\frac{11(1-x^2)^{2/3}}{216x^3}+\frac{(1-x^2)^{2/3}}{24(x^2+3)x^3}-\frac{11\tanh^{-1}(x)}{648\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-11*(1 - x^2)^(2/3))/(216*x^3) + (11*(1 - x^2)^(2/3))/(648*x) + (1 - x^2)^
(2/3)/(24*x^3*(3 + x^2)) - (11*x)/(648*(1 - Sqrt[3] - (1 - x^2)^(1/3))) + (
11*ArcTan[Sqrt[3]/x])/(216*2^(2/3)*Sqrt[3]) + (11*ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x])/(216*2^(2/3)*Sqrt[3]) - (11*ArcTanh[x])/(648*2^(2
/3)) + (11*ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(216*2^(2/3)) - (11*Sq
rt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)
^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(432*3^
```

```
(3/4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) +
 (11*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1
- Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(
1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/
4)*x*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{(1-x^2)^{2/3}}{24x^3(3+x^2)} - \frac{1}{24}\int \frac{-11+\frac{11x^2}{3}}{x^4\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{11(1-x^2)^{2/3}}{216x^3} + \frac{(1-x^2)^{2/3}}{24x^3(3+x^2)} + \frac{1}{216}\int \frac{-11+\frac{55x^2}{3}}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= -\frac{11(1-x^2)^{2/3}}{216x^3} + \frac{11(1-x^2)^{2/3}}{648x} + \frac{(1-x^2)^{2/3}}{24x^3(3+x^2)} - \frac{1}{648}\int \frac{-77-\frac{11x^2}{3}}{\sqrt[3]{1-x^2}\,(3+x^2)}$$

$$= -\frac{11(1-x^2)^{2/3}}{216x^3} + \frac{11(1-x^2)^{2/3}}{648x} + \frac{(1-x^2)^{2/3}}{24x^3(3+x^2)} + \frac{11\int \frac{1}{\sqrt[3]{1-x^2}}\,dx}{1944} + \frac{11}{108}$$

$$= -\frac{11(1-x^2)^{2/3}}{216x^3} + \frac{11(1-x^2)^{2/3}}{648x} + \frac{(1-x^2)^{2/3}}{24x^3(3+x^2)} + \frac{11\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{216\ 2^{2/3}\sqrt{3}} + \frac{11\tan}{}$$

$$= -\frac{11(1-x^2)^{2/3}}{216x^3} + \frac{11(1-x^2)^{2/3}}{648x} + \frac{(1-x^2)^{2/3}}{24x^3(3+x^2)} + \frac{11\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{216\ 2^{2/3}\sqrt{3}} + \frac{11\tan}{}$$

$$= -\frac{11(1-x^2)^{2/3}}{216x^3} + \frac{11(1-x^2)^{2/3}}{648x} + \frac{(1-x^2)^{2/3}}{24x^3(3+x^2)} - \frac{11x}{648\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.11, size = 173, normalized size = 0.30

$$\frac{11x^6F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+\frac{27\left(-72+72x^2+11x^4-11x^6+\frac{693x^4F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)+F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)}}{17496x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (11*x^6*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2] + (27*(-72 + 72*x^2 + 11*
x^4 - 11*x^6 + (693*x^4*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(9*Appel
lF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, x
```

```
^2, -1/3*x^2] + AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/((1 - x^2)^(1
/3)*(3 + x^2)))/(17496*x^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(-x^{2}+1\right)^{\frac{1}{3}} \left(x^{2}+3\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
[Out] int(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
[Out] integral(-(-x^2 + 1)^(2/3)/(x^10 + 5*x^8 + 3*x^6 - 9*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \sqrt[3]{- \left(x - 1\right) \left(x + 1\right)} \left(x^{2} + 3\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
[Out] Integral(1/(x**4*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(1-x^2\right)^{1/3} \left(x^2+3\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] int(1/(x^4*(1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1031 $\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=136

$$\frac{56}{243}\left(2-3x^2\right)^{3/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{2}{891}\left(2-3x^2\right)^{11/4}+\frac{32}{81}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\,\tanh^{-1}$$

```
[Out] 56/243*(-3*x^2+2)^(3/4)-16/567*(-3*x^2+2)^(7/4)+2/891*(-3*x^2+2)^(11/4)+32/
81*2^(1/4)*arctan(1/2*(2^(1/2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))+
32/81*2^(1/4)*arctanh(1/2*(2^(1/2)+(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/
4))
```

Rubi [A]
time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {451, 267, 272, 45, 450}

$$\frac{32}{81}\sqrt[4]{2}\,\text{ArcTan}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{891}\left(2-3x^2\right)^{11/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{56}{243}\left(2-3x^2\right)^{3/4}+\frac{32}{81}\sqrt[4]{2}\,\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (56*(2 - 3*x^2)^(3/4))/243 - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 - 3*x^2)^(1
1/4))/891 + (32*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*
x^2)^(1/4))])/81 + (32*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)
*(2 - 3*x^2)^(1/4))])/81
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int \left(-\frac{16x}{27\sqrt[4]{2-3x^2}} - \frac{4x^3}{9\sqrt[4]{2-3x^2}} - \frac{x^5}{3\sqrt[4]{2-3x^2}} + \frac{64x}{27\sqrt[4]{2-3x^2}\,(4-3x^2)}\right)$$

$$= -\left(\frac{1}{3}\int \frac{x^5}{\sqrt[4]{2-3x^2}}\,dx\right) - \frac{4}{9}\int \frac{x^3}{\sqrt[4]{2-3x^2}}\,dx - \frac{16}{27}\int \frac{x}{\sqrt[4]{2-3x^2}}\,dx + \frac{64}{27}\int$$

$$= \frac{32}{243}(2-3x^2)^{3/4} + \frac{32}{81}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + \frac{32}{81}\sqrt[4]{2}\,\tanh^{-1}\left(\frac{\sqrt{2}}{2^{3/}}\right.$$

$$= \frac{32}{243}(2-3x^2)^{3/4} + \frac{32}{81}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + \frac{32}{81}\sqrt[4]{2}\,\tanh^{-1}\left(\frac{\sqrt{2}}{2^{3/}}\right.$$

$$= \frac{56}{243}(2-3x^2)^{3/4} - \frac{16}{567}(2-3x^2)^{7/4} + \frac{2}{891}(2-3x^2)^{11/4} + \frac{32}{81}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}}{2^{3/}}\right.$$

Mathematica [A]
time = 0.18, size = 110, normalized size = 0.81

$$\frac{2(2-3x^2)^{3/4}(1712+540x^2+189x^4)+7392\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+7392\sqrt[4]{2}\,\tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{18711}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4)*(1712 + 540*x^2 + 189*x^4) + 7392*2^(1/4)*ArcTan[(Sqrt
[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] + 7392*2^(1/4)*ArcTanh[
(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/18711
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.25, size = 211, normalized size = 1.55

method	result
trager	$\left(\frac{2}{99}x^4+\frac{40}{693}x^2+\frac{3424}{18711}\right)\left(-3x^2+2\right)^{\frac{3}{4}}+\frac{16\,\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^3\left(-3x^2+2\right)^{\frac{3}{4}}+2\,\mathrm{RootOf}\left(\_Z^4+8\right.}{}\right.}{81}$
risch	$-\frac{2\left(189x^4+540x^2+1712\right)\left(3x^2-2\right)}{18711\left(-3x^2+2\right)^{\frac{1}{4}}}+\frac{16\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\mathrm{RootOf}\left(\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] (2/99*x^4+40/693*x^2+3424/18711)*(-3*x^2+2)^(3/4)+16/81*RootOf(_Z^4+8)*ln(-
(RootOf(_Z^4+8)^3*(-3*x^2+2)^(3/4)+2*RootOf(_Z^4+8)^2*(-3*x^2+2)^(1/2)+4*Ro
otOf(_Z^4+8)*(-3*x^2+2)^(1/4)+6*x^2)/(3*x^2-4))-16/81*RootOf(_Z^2+RootOf(_Z
^4+8)^2)*ln(-(RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(-3*x^2+2)^(3/
4)-2*RootOf(_Z^4+8)^2*(-3*x^2+2)^(1/2)-4*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(-3*
x^2+2)^(1/4)+6*x^2)/(3*x^2-4))
```

Maxima [A]
time = 0.49, size = 151, normalized size = 1.11

$$\frac{2}{891}\left(-3x^2+2\right)^{\frac{11}{4}}-\frac{16}{567}\left(-3x^2+2\right)^{\frac{7}{4}}-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{56}{243}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] 2/891*(-3*x^2 + 2)^(11/4) - 16/567*(-3*x^2 + 2)^(7/4) - 32/81*2^(1/4)*arcta
n(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 32/81*2^(1/4)*arctan(-1/2
*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 16/81*2^(1/4)*log(2^(3/4)*(-3*
x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/81*2^(1/4)*log(-2^(3/4)*(
-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 56/243*(-3*x^2 + 2)^(3/4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(102) = 204$.
time = 1.50, size = 253, normalized size = 1.86

$$\frac{2}{18711}\left(189x^4+540x^2+1712\right)\left(-3x^2+2\right)^{\frac{3}{4}}+\frac{32}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{32}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)+\frac{8}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)-\frac{8}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/18711*(189*x^4 + 540*x^2 + 1712)*(-3*x^2 + 2)^(3/4) + 32/81*8^(1/4)*sqrt(
2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*s
qrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 1)
+ 32/81*8^(1/4)*sqrt(2)*arctan(1/8*8^(1/4)*sqrt(2)*sqrt(-4*8^(3/4)*sqrt(2)*
(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2
)*(-3*x^2 + 2)^(1/4) + 1) + 8/81*8^(1/4)*sqrt(2)*log(4*8^(3/4)*sqrt(2)*(-3*
x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 8/81*8^(1/4)*sqrt(2)*l
og(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**7/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [A]
time = 0.73, size = 160, normalized size = 1.18

$$\frac{2}{891}\left(3\,x^2-2\right)^2\left(-3\,x^2+2\right)^{\frac{3}{4}}-\frac{16}{567}\left(-3\,x^2+2\right)^{\frac{7}{4}}-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{56}{243}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] 2/891*(3*x^2 - 2)^2*(-3*x^2 + 2)^(3/4) - 16/567*(-3*x^2 + 2)^(7/4) - 32/81*
2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 32/81*2^(1/4
)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 16/81*2^(1/4)*log
(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/81*2^(1/4)*l
og(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 56/243*(-3*x
^2 + 2)^(3/4)
```

Mupad [B]
time = 0.50, size = 82, normalized size = 0.60

$$\frac{56\,{\left(2-3\,x^2\right)}^{3/4}}{243}-\frac{16\,{\left(2-3\,x^2\right)}^{7/4}}{567}+\frac{2\,{\left(2-3\,x^2\right)}^{11/4}}{891}+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{32}{81}+\frac{32}{81}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{32}{81}-\frac{32}{81}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^7/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (56*(2 - 3*x^2)^(3/4))/243 - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 +
1i/2))*(32/81 + 32i/81) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/
2))*(32/81 - 32i/81) - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 - 3*x^2)^(11/4))/
891
```

3.1032 $\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=121

$$\frac{4}{27}\left(2-3x^2\right)^{3/4}-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{8}{27}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\,\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

```
[Out] 4/27*(-3*x^2+2)^(3/4)-2/189*(-3*x^2+2)^(7/4)+8/27*2^(1/4)*arctan(1/2*(2^(1/
2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))+8/27*2^(1/4)*arctanh(1/2*(2^
(1/2)+(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))
```

Rubi [A]

time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {451, 267, 272, 45, 450}

$$\frac{8}{27}\sqrt[4]{2}\,\text{ArcTan}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{4}{27}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\,\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*(2 - 3*x^2)^(3/4))/27 - (2*(2 - 3*x^2)^(7/4))/189 + (8*2^(1/4)*ArcTan[(S
qrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27 + (8*2^(1/4)*Arc
Tanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx &= \int \left(-\frac{4x}{9\sqrt[4]{2-3x^2}} - \frac{x^3}{3\sqrt[4]{2-3x^2}} + \frac{16x}{9\sqrt[4]{2-3x^2}\,(4-3x^2)}\right) dx \\ &= -\left(\frac{1}{3}\int \frac{x^3}{\sqrt[4]{2-3x^2}}\,dx\right) - \frac{4}{9}\int \frac{x}{\sqrt[4]{2-3x^2}}\,dx + \frac{16}{9}\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,d \\ &= \frac{8}{81}(2-3x^2)^{3/4} + \frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + \frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}}{2^{3/}}\right. \\ &= \frac{8}{81}(2-3x^2)^{3/4} + \frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + \frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}}{2^{3/}}\right. \\ &= \frac{4}{27}(2-3x^2)^{3/4} - \frac{2}{189}(2-3x^2)^{7/4} + \frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + \frac{8}{2}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 103, normalized size = 0.85

$$\frac{1}{189}\left(6(2-3x^2)^{3/4}(4+x^2) + 56\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + 56\sqrt[4]{2}\tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (6*(2 - 3*x^2)^(3/4)*(4 + x^2) + 56*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^
2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] + 56*2^(1/4)*ArcTanh[(2*(4 - 6*x^2)^(1/4))
/(2 + Sqrt[4 - 6*x^2])])/189
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.73, size = 204, normalized size = 1.69

method	result
trager	$\left(\frac{2x^2}{63}+\frac{8}{63}\right)(-3x^2+2)^{\frac{3}{4}} - \frac{4\,\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^3(-3x^2+2)^{\frac{3}{4}}-2\,\mathrm{RootOf}\left(\_Z^4+8\right)^2\sqrt{-3x^2+2}+}{3x^2-4}\right.}{27}$
risch	$-\frac{2(3x^2-2)(x^2+4)}{63(-3x^2+2)^{\frac{1}{4}}} + \frac{4\,\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^3(-3x^2+2)^{\frac{3}{4}}+2\,\mathrm{RootOf}\left(\_Z^4+8\right)^2\sqrt{-3x^2+2}+4\,\mathrm{RootOf}\left(\_\right.}{3x^2-4}\right.}{27}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] (2/63*x^2+8/63)*(-3*x^2+2)^(3/4)-4/27*RootOf(_Z^4+8)*ln((RootOf(_Z^4+8)^3*(
-3*x^2+2)^(3/4)-2*RootOf(_Z^4+8)^2*(-3*x^2+2)^(1/2)+4*RootOf(_Z^4+8)*(-3*x^
2+2)^(1/4)-6*x^2)/(3*x^2-4))+4/27*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((RootOf(
_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(-3*x^2+2)^(3/4)+2*RootOf(_Z^4+8)^2
*(-3*x^2+2)^(1/2)-4*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(-3*x^2+2)^(1/4)-6*x^2)/(
3*x^2-4))
```

Maxima [A]
time = 0.48, size = 140, normalized size = 1.16

$$-\frac{2}{189}\left(-3x^{2}+2\right)^{\frac{7}{4}}-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)+\frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)-\frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)+\frac{4}{27}\left(-3x^{2}+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -2/189*(-3*x^2 + 2)^(7/4) - 8/27*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-
3*x^2 + 2)^(1/4))) - 8/27*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2
+ 2)^(1/4))) + 4/27*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 4/27*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) + 4/27*(-3*x^2 + 2)^(3/4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(91) = 182.
time = 1.39, size = 246, normalized size = 2.03

$$\frac{2}{63}\left(x^{2}+4\right)\left(-3x^{2}+2\right)^{\frac{3}{4}}+\frac{8}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^{2}+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^{2}+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^{2}+2\right)^{\frac{1}{4}}-1\right)+\frac{8}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^{2}+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^{2}+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^{2}+2\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^{2}+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^{2}+2}\right)-\frac{2}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^{2}+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^{2}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/63*(x^2 + 4)*(-3*x^2 + 2)^(3/4) + 8/27*8^(1/4)*sqrt(2)*arctan(1/4*8^(1/4)
*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^
2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 1) + 8/27*8^(1/4)*sqrt(2
)*arctan(1/8*8^(1/4)*sqrt(2)*sqrt(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1
6*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) +
 1) + 2/27*8^(1/4)*sqrt(2)*log(4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sq
rt(2) + 16*sqrt(-3*x^2 + 2)) - 2/27*8^(1/4)*sqrt(2)*log(-4*8^(3/4)*sqrt(2)*
(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^5}{3x^2\sqrt[4]{2-3x^2} - 4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**5/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [A]
time = 0.83, size = 140, normalized size = 1.16

$$-\frac{2}{189}\left(-3x^2+2\right)^{\frac{7}{4}} - \frac{2}{27}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{2}{27}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) + \frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right) - \frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right) + \frac{4}{27}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -2/189*(-3*x^2 + 2)^(7/4) - 2/27*8^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-
3*x^2 + 2)^(1/4))) - 2/27*8^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2
+ 2)^(1/4))) + 4/27*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 4/27*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) + 4/27*(-3*x^2 + 2)^(3/4)
```

Mupad [B]
time = 0.09, size = 71, normalized size = 0.59

$$\frac{4\left(2-3x^2\right)^{3/4}}{27} - \frac{2\left(2-3x^2\right)^{7/4}}{189} + 2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{8}{27}+\frac{8}{27}\mathrm{i}\right) + 2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{8}{27}-\frac{8}{27}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (4*(2 - 3*x^2)^(3/4))/27 - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i
/2))*(8/27 + 8i/27) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*
(8/27 - 8i/27) - (2*(2 - 3*x^2)^(7/4))/189
```

3.1033 $\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=106

$$\frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

```
[Out] 2/27*(-3*x^2+2)^(3/4)+2/9*2^(1/4)*arctan(1/2*(2^(1/2)-(-3*x^2+2)^(1/2))*2^(
1/4)/(-3*x^2+2)^(1/4))+2/9*2^(1/4)*arctanh(1/2*(2^(1/2)+(-3*x^2+2)^(1/2))*2
^(1/4)/(-3*x^2+2)^(1/4))
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {451, 267, 450}

$$\frac{2}{9}\sqrt[4]{2}\,\text{ArcTan}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4))/27 + (2*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2
^(3/4)*(2 - 3*x^2)^(1/4))])/9 + (2*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^
2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/9
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int \left(-\frac{x}{3\sqrt[4]{2-3x^2}}+\frac{4x}{3\sqrt[4]{2-3x^2}\,(4-3x^2)}\right)dx$$
$$= -\left(\frac{1}{3}\int \frac{x}{\sqrt[4]{2-3x^2}}\,dx\right)+\frac{4}{3}\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$
$$= \frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}+\dots}{2^{3/4}\dots}\right.$$

Mathematica [A]
time = 0.10, size = 96, normalized size = 0.91

$$\frac{2}{27}\left(\left(2-3x^2\right)^{3/4}+3\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+3\sqrt[4]{2}\tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*((2 - 3*x^2)^(3/4) + 3*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/
4)*(2 - 3*x^2)^(1/4))] + 3*2^(1/4)*ArcTanh[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[
4 - 6*x^2])]))/27
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.70, size = 199, normalized size = 1.88

method	result
trager	$\frac{2(-3x^2+2)^{\frac{3}{4}}}{27}+\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\mathrm{RootOf}\left(\_Z^4+8\right)^2(-3x^2+2)^{\frac{3}{4}}+2\dots}{\dots}\right.}{9\dots}$
risch	$-\frac{2(3x^2-2)}{27(-3x^2+2)^{\frac{1}{4}}}-\frac{\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^3(-3x^2+2)^{\frac{3}{4}}-2\,\mathrm{RootOf}\left(\_Z^4+8\right)^2\sqrt{-3x^2+2}+4\,\mathrm{RootOf}\left(\_Z^4\dots\right.}{3x^2-4}\right.}{9\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*(-3*x^2+2)^(3/4)+1/9*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((RootOf(_Z^2+Roo
tOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(-3*x^2+2)^(3/4)+2*RootOf(_Z^4+8)^2*(-3*x^2
+2)^(1/2)-4*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(-3*x^2+2)^(1/4)-6*x^2)/(3*x^2-4)
)+1/9*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(-3*x^2+2)^(3/4)+2*RootOf(_Z^4+8
)^2*(-3*x^2+2)^(1/2)+4*RootOf(_Z^4+8)*(-3*x^2+2)^(1/4)+6*x^2)/(3*x^2-4))
```

Maxima [A]
time = 0.49, size = 129, normalized size = 1.22

$$-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{27}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 2/9*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/9*2^(1/4)*lo
g(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/9*2^(1/4)*lo
g(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*x^2
+ 2)^(3/4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(80) = 160$.
time = 2.03, size = 241, normalized size = 2.27

$$\frac{2}{9}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{2}{9}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)+\frac{1}{18}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)-\frac{1}{18}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)+\frac{2}{27}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/9*8^(1/4)*sqrt(2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4) - 1) + 2/9*8^(1/4)*sqrt(2)*arctan(1/8*8^(1/4)*sqrt(2)*sqrt(-4*8
^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/2
*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) + 1/18*8^(1/4)*sqrt(2)*log(4*8^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/18*8^
(1/4)*sqrt(2)*log(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*s
qrt(-3*x^2 + 2)) + 2/27*(-3*x^2 + 2)^(3/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^3}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**3/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [A]
time = 1.02, size = 129, normalized size = 1.22

$$-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)-\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)+\frac{2}{27}\left(-3x^{2}+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 2/9*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/9*2^(1/4)*lo
g(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/9*2^(1/4)*lo
g(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*x^2
+ 2)^(3/4)
```

Mupad [B]
time = 0.08, size = 60, normalized size = 0.57

$$\frac{2\left(2-3x^{2}\right)^{3/4}}{27}+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^{2}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{2}{9}+\frac{2}{9}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^{2}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{2}{9}-\frac{2}{9}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (2*(2 - 3*x^2)^(3/4))/27 - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i
/2))*(2/9 + 2i/9) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(2
/9 - 2i/9)
```

3.1034 $\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=91

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

```
[Out] 1/6*2^(1/4)*arctan(1/2*(2^(1/2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))
+1/6*2^(1/4)*arctanh(1/2*(2^(1/2)+(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4
))
```

Rubi [A]
time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {450}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^(3/4))
 + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^(3
/4))
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

Mathematica [A]
time = 0.09, size = 76, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{3\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] + ArcTanh[
(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(3*2^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.43, size = 188, normalized size = 2.07

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^3\left(-3x^2+2\right)^{\frac{3}{4}}+2\,\mathrm{RootOf}\left(\_Z^4+8\right)^2\sqrt{-3x^2+2}+4\,\mathrm{RootOf}\left(\_Z^4+8\right)\left(-3x^2+2\right)^{\frac{1}{4}}+6x^2}{3x^2-4}\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(-3*x^2+2)^(3/4)+2*RootOf(_Z^4+8)
^2*(-3*x^2+2)^(1/2)+4*RootOf(_Z^4+8)*(-3*x^2+2)^(1/4)+6*x^2)/(3*x^2-4))+1/1
2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z
^4+8)^2*(-3*x^2+2)^(3/4)+2*RootOf(_Z^4+8)^2*(-3*x^2+2)^(1/2)-4*RootOf(_Z^2+
RootOf(_Z^4+8)^2)*(-3*x^2+2)^(1/4)-6*x^2)/(3*x^2-4))
```

Maxima [A]
time = 0.50, size = 118, normalized size = 1.30

$$-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/6*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/12*2^(1/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/12*2^(1/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(69) = 138.
time = 1.65, size = 189, normalized size = 2.08

$$\frac{1}{3}\cdot 2^{\frac{1}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{3}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-4\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(4\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(-4\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/3*2^(1/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 1/3*2^(1/4)*arctan(1/2*2^
(1/4)*sqrt(-4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2))
- 2^(1/4)*(-3*x^2 + 2)^(1/4) + 1) + 1/12*2^(1/4)*log(4*2^(3/4)*(-3*x^2 + 2)
^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/12*2^(1/4)*log(-4*2^(3/4)*(-3*
x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [A]
time = 1.15, size = 118, normalized size = 1.30

$$-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/6*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/12*2^(1/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/12*2^(1/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

Mupad [B]
time = 0.08, size = 49, normalized size = 0.54

$$2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/6 - 1i/6) - 2^(1/
4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/2))*(1/6 + 1i/6)
```

3.1035 $\int \frac{1}{x\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=145

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}$$

```
[Out] 1/8*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(3/4)+1/8*2^(1/4)*arctan(1/2*(2^
(1/2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))-1/8*arctanh(1/2*2^(3/4)*(
-3*x^2+2)^(1/4))*2^(3/4)+1/8*2^(1/4)*arctanh(1/2*(2^(1/2)+(-3*x^2+2)^(1/2))
*2^(1/4)/(-3*x^2+2)^(1/4))
```

Rubi [A]
time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {451, 272, 65, 304, 209, 212, 450}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + ArcTan[(Sqrt[2] - Sqrt[2 -
3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(4*2^(3/4)) - ArcTanh[(2 - 3*x^2)^(1/4
)/2^(1/4)]/(4*2^(1/4)) + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 -
3*x^2)^(1/4))]/(4*2^(3/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{1}{x\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int \left(\frac{1}{4x\sqrt[4]{2-3x^2}} - \frac{3x}{4\sqrt[4]{2-3x^2}\,(-4+3x^2)}\right)dx$$
$$= \frac{1}{4}\int \frac{1}{x\sqrt[4]{2-3x^2}}\,dx - \frac{3}{4}\int \frac{x}{\sqrt[4]{2-3x^2}\,(-4+3x^2)}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}} + \frac{1}{8}\text{Subst}\Bigg(\int \frac{}{\sqrt[4]{2}}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}} - \frac{1}{6}\text{Subst}\Bigg(\int \frac{x}{\frac{2}{3}-}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}} - \frac{1}{4}\text{Subst}\Bigg(\int \frac{}{\sqrt{2}}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}$$

Mathematica [A]
time = 0.16, size = 117, normalized size = 0.81

$$\frac{\sqrt{2}\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right) + \tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) - \sqrt{2}\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right) + \tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
[Out] (Sqrt[2]*ArcTan[(1 - (3*x^2)/2)^(1/4)] + ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])
/(2^(3/4)*(2 - 3*x^2)^(1/4))] - Sqrt[2]*ArcTanh[(1 - (3*x^2)/2)^(1/4)] + Ar
cTanh[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(4*2^(3/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{1}{4}}\left(-3x^2+4\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4)/x,x)
```

```
[Out] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(109) = 218.
time = 1.00, size = 270, normalized size = 1.86

$$-\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+\sqrt{-3\,x^2+2}}-\frac{1}{2}\cdot 2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}\right)-\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+(-3\,x^2+2)^{\frac{1}{4}}\right)+\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{1}{4}}+(-3\,x^2+2)^{\frac{1}{4}}\right)+\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}-2^{\frac{1}{4}}(-3\,x^2+2)^{\frac{1}{4}}-1\right)+\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-4\cdot 2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+4\,\sqrt{2}+4\,\sqrt{-3\,x^2+2}}-2^{\frac{1}{4}}(-3\,x^2+2)^{\frac{1}{4}}+1\right)+\frac{1}{16}\cdot 2^{\frac{1}{4}}\log\left(4\cdot 2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+4\,\sqrt{2}+4\,\sqrt{-3\,x^2+2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\log\left(-4\cdot 2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+4\,\sqrt{2}+4\,\sqrt{-3\,x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/4*2^(3/4)*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-3*x^2 + 2)) - 1/2*2^(3
/4)*(-3*x^2 + 2)^(1/4)) - 1/16*2^(3/4)*log(2^(1/4) + (-3*x^2 + 2)^(1/4)) +
1/16*2^(3/4)*log(-2^(1/4) + (-3*x^2 + 2)^(1/4)) + 1/4*2^(1/4)*arctan(2^(1/4
)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1/4)*(
-3*x^2 + 2)^(1/4) - 1) + 1/4*2^(1/4)*arctan(1/2*2^(1/4)*sqrt(-4*2^(3/4)*(-3
*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1
/4) + 1) + 1/16*2^(1/4)*log(4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sq
rt(-3*x^2 + 2)) - 1/16*2^(1/4)*log(-4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2
) + 4*sqrt(-3*x^2 + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^3\sqrt[4]{2-3x^2}-4x\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**3*(2 - 3*x**2)**(1/4) - 4*x*(2 - 3*x**2)**(1/4)), x)
```

Giac [A]
time = 1.09, size = 216, normalized size = 1.49

$$-\frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\,(-3\,x^2+2)^{\frac{1}{4}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\,(-3\,x^2+2)^{\frac{1}{4}}\right)\right)+\frac{1}{32}\cdot 4^{\frac{3}{8}}\sqrt{2}\log\left(4^{\frac{1}{8}}\sqrt{2}\,(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{-3\,x^2+2}+4^{\frac{1}{4}}\right)-\frac{1}{32}\cdot 4^{\frac{3}{8}}\sqrt{2}\log\left(-4^{\frac{1}{8}}\sqrt{2}\,(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{-3\,x^2+2}+4^{\frac{1}{4}}\right)+\frac{1}{8}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 4^{\frac{7}{8}}(-3\,x^2+2)^{\frac{1}{4}}\right)+\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\log\left(-(-3\,x^2+2)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)-\frac{1}{16}\cdot 4^{\frac{3}{8}}\log\left((-3\,x^2+2)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/16*4^(3/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2) + 2*(-3*x
^2 + 2)^(1/4))) - 1/16*4^(3/8)*sqrt(2)*arctan(-1/8*4^(7/8)*sqrt(2)*(4^(1/8)
*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) + 1/32*4^(3/8)*sqrt(2)*log(4^(1/8)*sqrt(2
)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) - 1/32*4^(3/8)*sqrt(2)*l
og(-4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) + 1/8*
4^(1/8)*sqrt(2)*arctan(1/4*4^(7/8)*(-3*x^2 + 2)^(1/4)) + 1/16*4^(1/8)*sqrt(
2)*log(-(-3*x^2 + 2)^(1/4) + 4^(1/8)) - 1/16*4^(3/8)*log((-3*x^2 + 2)^(1/4)
 + 4^(1/8))
```

Mupad [B]
time = 0.54, size = 91, normalized size = 0.63

$$\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}}{2}\right)}{8}+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4))/2))/8 - 2^(1/4)*atan(2^(1/4)*(2 -
 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/8 - 1i/8) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)
^(1/4)*(1/2 + 1i/2))*(1/8 + 1i/8) + (2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4
)*1i)/2)*1i)/8
```

3.1036 $\int \frac{1}{x^3 \sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=163

$$-\frac{(2-3x^2)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\sqrt{\ }\right.}{1}$$

```
[Out] -1/16*(-3*x^2+2)^(3/4)/x^2+9/64*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(3/4
)+3/32*2^(1/4)*arctan(1/2*(2^(1/2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/
4))-9/64*arctanh(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(3/4)+3/32*2^(1/4)*arctanh
(1/2*(2^(1/2)+(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {451, 272, 44, 65, 304, 209, 212, 450}

$$\frac{9\text{ArcTan}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{(2-3x^2)^{3/4}}{16x^2}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/16*(2 - 3*x^2)^(3/4)/x^2 + (9*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(
1/4)) + (3*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])
/(16*2^(3/4)) - (9*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(1/4)) + (3*Ar
cTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/(16*2^(3/4)
)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int \left(\frac{1}{4x^3\sqrt[4]{2-3x^2}} + \frac{3}{16x\sqrt[4]{2-3x^2}} - \frac{9x}{16\sqrt[4]{2-3x^2}\,(-4+3x^2)}\right)dx$$

$$= \frac{3}{16}\int \frac{1}{x\sqrt[4]{2-3x^2}}\,dx + \frac{1}{4}\int \frac{1}{x^3\sqrt[4]{2-3x^2}}\,dx - \frac{9}{16}\int \frac{x}{\sqrt[4]{2-3x^2}\,(-4+3x^2)}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}} + \frac{3}{32}\text{Subst}\left(\int\right.$$

$$= -\frac{(2-3x^2)^{3/4}}{16x^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}$$

$$= -\frac{(2-3x^2)^{3/4}}{16x^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}$$

$$= -\frac{(2-3x^2)^{3/4}}{16x^2} + \frac{3\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{16\sqrt[4]{2}} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}} - \frac{3\,\mathrm{t}}{}$$

$$= -\frac{(2-3x^2)^{3/4}}{16x^2} + \frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}} - \frac{9\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.27, size = 155, normalized size = 0.95

$$\frac{-4(2-3x^2)^{3/4} + 9\ 2^{3/4}x^2\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right) + 6\sqrt[4]{2}\,x^2\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) - 9\ 2^{3/4}x^2\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right) + 6\sqrt[4]{2}\,x^2\tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{64x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*(2 - 3*x^2)^(3/4) + 9*2^(3/4)*x^2*ArcTan[(1 - (3*x^2)/2)^(1/4)] + 6*2^(
1/4)*x^2*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] -
9*2^(3/4)*x^2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] + 6*2^(1/4)*x^2*ArcTanh[(2*(4
- 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(64*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-3x^2+2\right)^{\frac{1}{4}}\left(-3x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(123) = 246$.
time = 1.03, size = 307, normalized size = 1.88

$$-\frac{36\cdot 2^{\frac{3}{4}}x^2\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}\right)+9\cdot 2^{\frac{3}{4}}x^2\log\left(2^{\frac{1}{4}}+(-3x^2+2)^{\frac{1}{4}}\right)-9\cdot 2^{\frac{3}{4}}x^2\log\left(-2^{\frac{1}{4}}+(-3x^2+2)^{\frac{1}{4}}\right)-24\cdot 2^{\frac{1}{4}}x^2\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2^{\frac{1}{4}}(-3x^2+2)^{\frac{1}{4}}-1\right)-24\cdot 2^{\frac{1}{4}}x^2\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-4\cdot 2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-2^{\frac{1}{4}}(-3x^2+2)^{\frac{1}{4}}+1\right)-6\cdot 2^{\frac{1}{4}}x^2\log\left(4\cdot 2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}\right)+6\cdot 2^{\frac{1}{4}}x^2\log\left(-4\cdot 2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}\right)+8(-3x^2+2)^{\frac{3}{4}}}{128x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/128*(36*2^(3/4)*x^2*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-3*x^2 + 2))
- 1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) + 9*2^(3/4)*x^2*log(2^(1/4) + (-3*x^2 + 2
)^(1/4)) - 9*2^(3/4)*x^2*log(-2^(1/4) + (-3*x^2 + 2)^(1/4)) - 24*2^(1/4)*x^
2*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 +
2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) - 24*2^(1/4)*x^2*arctan(1/2*2^(1/4)*s
qrt(-4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 2^(1/
4)*(-3*x^2 + 2)^(1/4) + 1) - 6*2^(1/4)*x^2*log(4*2^(3/4)*(-3*x^2 + 2)^(1/4)
 + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) + 6*2^(1/4)*x^2*log(-4*2^(3/4)*(-3*x^2 +
 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) + 8*(-3*x^2 + 2)^(3/4))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^5\sqrt[4]{2-3x^2}-4x^3\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**5*(2 - 3*x**2)**(1/4) - 4*x**3*(2 - 3*x**2)**(1/4)), x)
```

Giac [A]
time = 0.84, size = 192, normalized size = 1.18

$$\frac{9}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}\right)-\frac{9}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+(-3x^2+2)^{\frac{1}{4}}\right)+\frac{9}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}-(-3x^2+2)^{\frac{1}{4}}\right)-\frac{3}{32}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2(-3x^2+2)^{\frac{1}{4}}\right)\right)-\frac{3}{32}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2(-3x^2+2)^{\frac{1}{4}}\right)\right)+\frac{3}{64}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{3}{64}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{(-3x^2+2)^{\frac{3}{4}}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] 9/64*2^(3/4)*arctan(1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) - 9/128*2^(3/4)*log(2^(
1/4) + (-3*x^2 + 2)^(1/4)) + 9/128*2^(3/4)*log(2^(1/4) - (-3*x^2 + 2)^(1/4)
) - 3/32*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 3/3
2*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 3/64*2^(1
/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 3/64*2^(
1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/16*(
-3*x^2 + 2)^(3/4)/x^2
```

Mupad [B]
time = 0.57, size = 109, normalized size = 0.67

$$\frac{9\,2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}}{2}\right)}{64}-\frac{{\left(2-3\,x^2\right)}^{3/4}}{16\,x^2}+\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1{}\mathrm{i}}{2}\right)\,9{}\mathrm{i}}{64}-\frac{{\left(-1\right)}^{1/4}\,2^{3/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1{}\mathrm{i}}{2}\right)\,3{}\mathrm{i}}{32}-\frac{{\left(-1\right)}^{3/4}\,2^{3/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{3/4}\,2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1{}\mathrm{i}}{2}\right)\,3{}\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (9*2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4))/2))/64 - (2 - 3*x^2)^(3/4)/(16*
x^2) + (2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*9i)/64 - ((-1)^(1/4)
*2^(3/4)*atan(((-1)^(1/4)*2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*3i)/32 - ((-1)^(
3/4)*2^(3/4)*atan(((-1)^(3/4)*2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*3i)/32
```

3.1037 $\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=164

$$\frac{2}{45}x\left(2-3x^2\right)^{3/4}+\frac{4\sqrt[4]{2}\,\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\,\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{4\sqrt[4]{2}\,\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\,\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{16\sqrt[4]{2}\,E\left(\frac{1}{2}\mathrm{s}\right.}{}$$

```
[Out] 2/45*x*(-3*x^2+2)^(3/4)+4/27*2^(1/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2)
^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+4/27*2^(1/4)*arctanh(1/3*(2^(3/
4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-16/45*2^(1
/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))
*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {451, 234, 327, 406}

$$-\frac{16\sqrt[4]{2}\,E\left(\frac{1}{2}\mathrm{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{15\sqrt{3}}+\frac{4\sqrt[4]{2}\,\mathrm{ArcTan}\left(\frac{2^{3/4}-\sqrt[4]{2}\,\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{2}{45}\left(2-3x^2\right)^{3/4}x+\frac{4\sqrt[4]{2}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2 - 3*x^2)^(3/4))/45 + (4*2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 -
3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) + (4*2^(1/4)*ArcTanh[(2
^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3
]) - (16*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int \left(-\frac{4}{9\sqrt[4]{2-3x^2}}-\frac{x^2}{3\sqrt[4]{2-3x^2}}+\frac{16}{9\sqrt[4]{2-3x^2}\,(4-3x^2)}\right)dx$$

$$= -\left(\frac{1}{3}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx\right)-\frac{4}{9}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx+\frac{16}{9}\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$

$$= \frac{2}{45}x\left(2-3x^2\right)^{3/4}+\frac{4\sqrt[4]{2}\,\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\,\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{4\sqrt[4]{2}\,\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{\ldots}}{\sqrt{3}\ldots}\right)}{9\sqrt{3}}$$

$$= \frac{2}{45}x\left(2-3x^2\right)^{3/4}+\frac{4\sqrt[4]{2}\,\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\,\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{4\sqrt[4]{2}\,\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{\ldots}}{\sqrt{3}\ldots}\right)}{9\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.45, size = 184, normalized size = 1.12

$$\frac{1}{45}x\left(3\,2^{3/4}x^2F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+\frac{2\left(2-3x^2+\frac{32F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(-4+3x^2)\left(4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)\right)}\right)}{\sqrt[4]{2-3x^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (x*(3*2^(3/4)*x^2*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + (2*(2
- 3*x^2 + (32*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^
```

```
2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/
2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + AppellF1[3/2, 5/4, 1, 5/2, (3*x^2)/
2, (3*x^2)/4])))))/(2 - 3*x^2)^(1/4)))/45
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-3x^2+2)^{\frac{1}{4}}\,(-3x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
[Out] int(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
[Out] -integrate(x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
[Out] integral((-3*x^2 + 2)^(3/4)*x^4/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{3x^2\sqrt[4]{2-3x^2} - 4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
[Out] -Integral(x**4/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
[Out] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^4}{(2-3\,x^2)^{1/4}\,(3\,x^2-4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
[Out] -int(x^4/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1038 $\int \frac{x^2}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}+\frac{\sqrt[4]{2}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}-\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{3\sqrt{3}}$$

```
[Out] 1/9*2^(1/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4
)*3^(1/2))*3^(1/2)+1/9*2^(1/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^(1/2
))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-2/9*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^
(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/
2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {451, 234, 406}

$$-\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{3\sqrt{3}}+\frac{\sqrt[4]{2}\,\text{ArcTan}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}+\frac{\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^
(1/4))])/(3*Sqrt[3]) + (2^(1/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])
/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(3*Sqrt[3]) - (2*2^(1/4)*EllipticE[ArcSin[
Sqrt[3/2]*x]/2, 2])/(3*Sqrt[3])
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int \left(-\frac{1}{3\sqrt[4]{2-3x^2}} + \frac{4}{3\sqrt[4]{2-3x^2}\,(4-3x^2)}\right)dx$$
$$= -\left(\frac{1}{3}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx\right) + \frac{4}{3}\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$
$$= \frac{\sqrt[4]{2}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}} + \frac{\sqrt[4]{2}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}} - \frac{2\sqrt[4]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.01, size = 37, normalized size = 0.25

$$\frac{x^3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
[Out] (x^3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(12*2^(1/4))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-3x^2+2)^{\frac{1}{4}}\,(-3x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
[Out] int(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
[Out] integral((-3*x^2 + 2)^(3/4)*x^2/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
[Out] -Integral(x**2/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{{\left(2-3\,x^2\right)}^{1/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
[Out] -int(x^2/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1039 $\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

```
[Out] 1/12*arctan(1/6*(2-2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)/x/(-3*x^2+2)^(1/4)*3^(
1/2))*2^(1/4)*3^(1/2)+1/12*arctanh(1/6*(2+2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)
/x/(-3*x^2+2)^(1/4)*3^(1/2))*2^(1/4)*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {406}

$$\frac{\text{ArcTan}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Mathematica [A]
time = 0.03, size = 119, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{3\sqrt{2}x^2-4\sqrt{2-3x^2}}{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)+\tanh^{-1}\left(\frac{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}{3\sqrt{2}x^2+4\sqrt{2-3x^2}}\right)}{4\ 2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (ArcTan[(3*Sqrt[2]*x^2 - 4*Sqrt[2 - 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^2
)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))/(3*Sqrt[2]*x^2
+ 4*Sqrt[2 - 3*x^2])])/(4*2^(3/4)*Sqrt[3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.48, size = 188, normalized size = 1.57

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^4+72\right)\ln\left(-\frac{6\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+72\right)-\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+72\right)^3-18\sqrt{-3x^2+2}\,x+3\,\mathrm{RootOf}\left(\_Z \right.}{3x^2-4}\right.}{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*RootOf(_Z^4+72)*ln(-(6*(-3*x^2+2)^(3/4)*RootOf(_Z^4+72)-(-3*x^2+2)^(1
/4)*RootOf(_Z^4+72)^3-18*(-3*x^2+2)^(1/2)*x+3*RootOf(_Z^4+72)^2*x)/(3*x^2-4
))-1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln(-(6*(-3*x^2+2)^(3/4)*RootOf(_Z^2+
RootOf(_Z^4+72)^2)+(-3*x^2+2)^(1/4)*RootOf(_Z^4+72)^2*RootOf(_Z^2+RootOf(_Z
^4+72)^2)-18*(-3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(89) = 178.
time = 2.66, size = 553, normalized size = 4.61

$$\frac{1}{72}\cdot 18^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x^3+54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}x+12\sqrt{2}\left(3x^2-4\right)\sqrt{-3x^2+2}-72x^2+\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3+4x\right)\sqrt{-3x^2+2}-72\left(-3x^2+2\right)^{\frac{1}{4}}x^2-6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3-4x\right)-48\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{-\frac{3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}}{3x^2-4}}}{6\left(9x^4+24x^2-16\right)}\right)-\frac{1}{72}\cdot 18^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x^3-54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}x-12\sqrt{2}\left(3x^2-4\right)\sqrt{-3x^2+2}+72x^2+\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3+4x\right)\sqrt{-3x^2+2}+72\left(-3x^2+2\right)^{\frac{1}{4}}x^2-6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3-4x\right)+48\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{-\frac{3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}}{3x^2-4}}}{6\left(9x^4+24x^2-16\right)}\right)+\frac{1}{288}\cdot 18^{\frac{1}{4}}\sqrt{2}\log\left(-\frac{36\left(3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}\right)}{3x^2-4}\right)-\frac{1}{288}\cdot 18^{\frac{1}{4}}\sqrt{2}\log\left(-\frac{36\left(3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}\right)}{3x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/72*18^(3/4)*sqrt(2)*arctan(-1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 + 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x + 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) - 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) - 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) - 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) -
 1/72*18^(3/4)*sqrt(2)*arctan(1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 - 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x - 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) + 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) + 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) + 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) +
 1/288*18^(3/4)*sqrt(2)*log(-36*(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)) - 1/288*18^(3/4)*sqrt(2)*l
og(-36*(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 4*sqrt(-3
*x^2 + 2))/(3*x^2 - 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(2-3\,x^2\right)}^{1/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
[Out] -int(1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1040 $\int \frac{1}{x^2\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=166

$$-\frac{(2-3x^2)^{3/4}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\,2^{3/4}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\,2^{3/4}}-\frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\ldots}\right.\right.}{4\,2^{3/4}}$$

```
[Out] -1/8*(-3*x^2+2)^(3/4)/x+1/16*2^(1/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2)
^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+1/16*2^(1/4)*arctanh(1/3*(2^(3/
4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-1/8*2^(1/4
)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*E
llipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {451, 331, 234, 406}

$$-\frac{\sqrt{3}E\left(\left.\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}x\right)\right|2\right)}{4\,2^{3/4}}+\frac{\sqrt{3}\text{ArcTan}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\,2^{3/4}}-\frac{(2-3x^2)^{3/4}}{8x}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/8*(2 - 3*x^2)^(3/4)/x + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^
2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4)) + (Sqrt[3]*ArcTanh[(2^(3/4)
 + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4)) - (
Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx &= \int \left(\frac{1}{4x^2\sqrt[4]{2-3x^2}} - \frac{3}{4\sqrt[4]{2-3x^2}\,(-4+3x^2)}\right)dx \\
&= \frac{1}{4}\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx - \frac{3}{4}\int \frac{1}{\sqrt[4]{2-3x^2}\,(-4+3x^2)}\,dx \\
&= -\frac{(2-3x^2)^{3/4}}{8x} + \frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}} + \frac{\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{}}{\sqrt{3}\,x}\right.}{8\ 2^{3/4}} \\
&= -\frac{(2-3x^2)^{3/4}}{8x} + \frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}} + \frac{\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{}}{\sqrt{3}\,x}\right.}{8\ 2^{3/4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.05, size = 56, normalized size = 0.34

$$
-\frac{(2-3x^2)^{3/4}}{8x} + \frac{3x^3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{64\sqrt[4]{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/8*(2 - 3*x^2)^(3/4)/x + (3*x^3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*
x^2)/4])/(64*2^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-3 x^{2}+2\right)^{\frac{1}{4}} \left(-3 x^{2}+4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
[Out] int(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
[Out] integral((-3*x^2 + 2)^(3/4)/(9*x^6 - 18*x^4 + 8*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3 x^{4} \sqrt[4]{2 - 3 x^{2}} - 4 x^{2} \sqrt[4]{2 - 3 x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
[Out] -Integral(1/(3*x**4*(2 - 3*x**2)**(1/4) - 4*x**2*(2 - 3*x**2)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{x^2\,{\left(2-3\,x^2\right)}^{1/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] -int(1/(x^2*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1041 $\int \frac{1}{x^4\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$

Optimal. Leaf size=184

$$-\frac{(2-3x^2)^{3/4}}{24x^3}-\frac{3(2-3x^2)^{3/4}}{16x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}-$$

```
[Out] -1/24*(-3*x^2+2)^(3/4)/x^3-3/16*(-3*x^2+2)^(3/4)/x+3/64*2^(1/4)*arctan(1/3*
(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+3/64
*2^(1/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*
3^(1/2))*3^(1/2)-3/16*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(
1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2)
)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {451, 331, 234, 406}

$$-\frac{3\sqrt{3}E\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\ 2^{3/4}}+\frac{3\sqrt{3}\text{ArcTan}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}-\frac{3(2-3x^2)^{3/4}}{16x}+\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}-\frac{(2-3x^2)^{3/4}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/24*(2 - 3*x^2)^(3/4)/x^3 - (3*(2 - 3*x^2)^(3/4))/(16*x) + (3*Sqrt[3]*Arc
Tan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(32
*2^(3/4)) + (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]
*x*(2 - 3*x^2)^(1/4))])/(32*2^(3/4)) - (3*Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2
]*x]/2, 2])/(8*2^(3/4))
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int \left(\frac{1}{4x^4\sqrt[4]{2-3x^2}}+\frac{3}{16x^2\sqrt[4]{2-3x^2}}-\frac{9}{16\sqrt[4]{2-3x^2}\,(-4+3x^2)}\right)dx$$

$$= \frac{3}{16}\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx+\frac{1}{4}\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx-\frac{9}{16}\int \frac{1}{\sqrt[4]{2-3x^2}\,(-4+3x^2)}$$

$$= -\frac{(2-3x^2)^{3/4}}{24x^3}-\frac{3(2-3x^2)^{3/4}}{32x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{}}{}$$

$$= -\frac{(2-3x^2)^{3/4}}{24x^3}-\frac{3(2-3x^2)^{3/4}}{16x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{}}{}$$

$$= -\frac{(2-3x^2)^{3/4}}{24x^3}-\frac{3(2-3x^2)^{3/4}}{16x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 20.13, size = 156, normalized size = 0.85

$$\frac{1}{8}(2-3x^2)^{3/4}\left(-\frac{2+9x^2}{6x^3}+\frac{9xF_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(-4+3x^2)\left(4F_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+x^2\left(2F_1\left(\frac{3}{2};-\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)-3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ((2 - 3*x^2)^(3/4)*(-1/6*(2 + 9*x^2)/x^3 + (9*x*AppellF1[1/2, -3/4, 1, 3/2,
 (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2)*(4*AppellF1[1/2, -3/4, 1, 3/2, (3*x^2
)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/2, -3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4]
- 3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])))))/8
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(-3 x^{2}+2\right)^{\frac{1}{4}} \left(-3 x^{2}+4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(3/4)/(9*x^8 - 18*x^6 + 8*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3 x^{6} \sqrt[4]{2 - 3 x^{2}} - 4 x^{4} \sqrt[4]{2 - 3 x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**6*(2 - 3*x**2)**(1/4) - 4*x**4*(2 - 3*x**2)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{x^4\,{\left(2-3\,x^2\right)}^{1/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^4*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
[Out] -int(1/(x^4*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1042 $\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=78

$$\frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4}+\frac{8}{81}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

```
[Out] 14/243*(3*x^2-1)^(3/4)+8/567*(3*x^2-1)^(7/4)+2/891*(3*x^2-1)^(11/4)+8/81*ar
ctan((3*x^2-1)^(1/4))-8/81*arctanh((3*x^2-1)^(1/4))
```

Rubi [A]

time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 90, 65, 304, 209, 212}

$$\frac{8}{81}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{2}{891}\left(3x^2-1\right)^{11/4}+\frac{8}{567}\left(3x^2-1\right)^{7/4}+\frac{14}{243}\left(3x^2-1\right)^{3/4}-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (14*(-1 + 3*x^2)^(3/4))/243 + (8*(-1 + 3*x^2)^(7/4))/567 + (2*(-1 + 3*x^2)^
(11/4))/891 + (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(
1/4)])/81
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{7}{27\sqrt[4]{-1+3x}} + \frac{8}{27(-2+3x)\sqrt[4]{-1+3x}} + \frac{4}{27}(-1+3x)^{3/4}\right.\right.\\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4} + \frac{8}{567}\left(-1+3x^2\right)^{7/4} + \frac{2}{891}\left(-1+3x^2\right)^{11/4} + \frac{4}{27}\text{Subst}\Big(\\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4} + \frac{8}{567}\left(-1+3x^2\right)^{7/4} + \frac{2}{891}\left(-1+3x^2\right)^{11/4} + \frac{16}{81}\text{Subst}\Big(\\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4} + \frac{8}{567}\left(-1+3x^2\right)^{7/4} + \frac{2}{891}\left(-1+3x^2\right)^{11/4} - \frac{8}{81}\text{Subst}\Big(\\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4} + \frac{8}{567}\left(-1+3x^2\right)^{7/4} + \frac{2}{891}\left(-1+3x^2\right)^{11/4} + \frac{8}{81}\tan^{-1}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 57, normalized size = 0.73

$$\frac{2\left(\left(-1+3x^2\right)^{3/4}\left(428+270x^2+189x^4\right)+924\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-924\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\right)}{18711}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(3/4)*(428 + 270*x^2 + 189*x^4) + 924*ArcTan[(-1 + 3*x^2)^
(1/4)] - 924*ArcTanh[(-1 + 3*x^2)^(1/4)]))/18711
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.40, size = 147, normalized size = 1.88

method	result
trager	$\left(\frac{2}{99}x^4+\frac{20}{693}x^2+\frac{856}{18711}\right)\left(3x^2-1\right)^{\frac{3}{4}}-\frac{4\ln\left(-\frac{2\left(3x^2-1\right)^{\frac{3}{4}}+2\sqrt{3x^2-1}+3x^2+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{81}-\frac{4\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln$
risch	$\frac{2\left(189x^4+270x^2+428\right)\left(3x^2-1\right)^{\frac{3}{4}}}{18711}+\frac{4\ln\left(\frac{2\left(3x^2-1\right)^{\frac{3}{4}}-2\sqrt{3x^2-1}-3x^2+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{81}-\frac{4\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}(\_}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] (2/99*x^4+20/693*x^2+856/18711)*(3*x^2-1)^(3/4)-4/81*ln(-(2*(3*x^2-1)^(3/4)
+2*(3*x^2-1)^(1/2)+3*x^2+2*(3*x^2-1)^(1/4))/(3*x^2-2))-4/81*RootOf(_Z^2+1)*
ln((2*RootOf(_Z^2+1)*(3*x^2-1)^(3/4)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)+2*(3*
x^2-1)^(1/2)-3*x^2)/(3*x^2-2))
```

Maxima [A]
time = 0.47, size = 74, normalized size = 0.95

$$\frac{2}{891}\left(3x^2-1\right)^{\frac{11}{4}}+\frac{8}{567}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{14}{243}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^2 - 1)^(3/
4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/8
1*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A]
time = 0.57, size = 64, normalized size = 0.82

$$\frac{2}{18711}\left(189x^4+270x^2+428\right)\left(3x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/18711*(189*x^4 + 270*x^2 + 428)*(3*x^2 - 1)^(3/4) + 8/81*arctan((3*x^2 -
1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4) -
1)
```

Sympy [A]
time = 12.60, size = 88, normalized size = 1.13

$$\frac{2(3x^2-1)^{\frac{11}{4}}}{891}+\frac{8(3x^2-1)^{\frac{7}{4}}}{567}+\frac{14(3x^2-1)^{\frac{3}{4}}}{243}+\frac{4\log\left(\sqrt[4]{3x^2-1}-1\right)}{81}-\frac{4\log\left(\sqrt[4]{3x^2-1}+1\right)}{81}+\frac{8\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(11/4)/891 + 8*(3*x**2 - 1)**(7/4)/567 + 14*(3*x**2 - 1)**(
3/4)/243 + 4*log((3*x**2 - 1)**(1/4) - 1)/81 - 4*log((3*x**2 - 1)**(1/4) +
1)/81 + 8*atan((3*x**2 - 1)**(1/4))/81
```

Giac [A]
time = 0.71, size = 75, normalized size = 0.96

$$\frac{2}{891}\left(3x^2-1\right)^{\frac{11}{4}}+\frac{8}{567}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{14}{243}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^2 - 1)^(3/
4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/8
1*log(abs((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.07, size = 62, normalized size = 0.79

$$\frac{8\operatorname{atan}\left(\left(3x^2-1\right)^{1/4}\right)}{81}+\frac{14\left(3x^2-1\right)^{3/4}}{243}+\frac{8\left(3x^2-1\right)^{7/4}}{567}+\frac{2\left(3x^2-1\right)^{11/4}}{891}+\frac{\operatorname{atan}\left(\left(3x^2-1\right)^{1/4}\,1\mathrm{i}\right)\,8\mathrm{i}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] (8*atan((3*x^2 - 1)^(1/4)))/81 + (atan((3*x^2 - 1)^(1/4)*1i)*8i)/81 + (14*(
3*x^2 - 1)^(3/4))/243 + (8*(3*x^2 - 1)^(7/4))/567 + (2*(3*x^2 - 1)^(11/4))/
891
```

3.1043 $\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

```
[Out] 2/27*(3*x^2-1)^(3/4)+2/189*(3*x^2-1)^(7/4)+4/27*arctan((3*x^2-1)^(1/4))-4/2
7*arctanh((3*x^2-1)^(1/4))
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 90, 65, 304, 209, 212}

$$\frac{4}{27}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{2}{189}\left(3x^2-1\right)^{7/4}+\frac{2}{27}\left(3x^2-1\right)^{3/4}-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*(-1 + 3*x^2)^(7/4))/189 + (4*ArcTan[(-1 + 3*
x^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{3\sqrt[4]{-1+3x}}+\frac{4}{9(-2+3x)\sqrt[4]{-1+3x}}+\frac{1}{9}(-1+3x)^{3/4}\right) d\right.\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{2}{9}\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\right.\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{8}{27}\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{-}\right.\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}-\frac{4}{27}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1}\right.\\ &= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\text{ta}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.84

$$\frac{2}{63}\left(2+x^2\right)\left(-1+3x^2\right)^{3/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

[Out] (2*(2 + x^2)*(-1 + 3*x^2)^(3/4))/63 + (4*ArcTan[(-1 + 3*x^2)^(1/4)])/27 - (
4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.28, size = 141, normalized size = 2.24

method	result
trager	$\left(\frac{2x^2}{63}+\frac{4}{63}\right)(3x^2-1)^{\frac{3}{4}}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)(3x^2-1)^{\frac{3}{4}}-2\,\mathrm{RootOf}\left(\_Z^2+1\right)(3x^2-1)^{\frac{1}{4}}+2\sqrt{3x^2-}}{3x^2-2}\right.}{27}$
risch	$\frac{2(x^2+2)(3x^2-1)^{\frac{3}{4}}}{63}-\frac{2\ln\left(-\frac{2(3x^2-1)^{\frac{3}{4}}+2\sqrt{3x^2-1}+3x^2+2(3x^2-1)^{\frac{1}{4}}}{3x^2-2}\right)}{27}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)(3x^2}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)

[Out] (2/63*x^2+4/63)*(3*x^2-1)^(3/4)-2/27*RootOf(_Z^2+1)*ln((2*RootOf(_Z^2+1)*(3
*x^2-1)^(3/4)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)+2*(3*x^2-1)^(1/2)-3*x^2)/(3*
x^2-2))+2/27*ln((2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)-3*x^2+2*(3*x^2-1)^(1/4
))/(3*x^2-2))

Maxima [A]
time = 0.46, size = 63, normalized size = 1.00

$$\frac{2}{189}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")

[Out] 2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3*x^2 - 1)^
(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)

Fricas [A]
time = 0.70, size = 57, normalized size = 0.90

$$\frac{2}{63}\left(3x^2-1\right)^{\frac{3}{4}}\left(x^2+2\right)+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")

[Out] 2/63*(3*x^2 - 1)^(3/4)*(x^2 + 2) + 4/27*arctan((3*x^2 - 1)^(1/4)) - 2/27*lo
g((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)

Sympy [A]
time = 9.65, size = 75, normalized size = 1.19

$$\frac{2(3x^2-1)^{\frac{7}{4}}}{189}+\frac{2(3x^2-1)^{\frac{3}{4}}}{27}+\frac{2\log\left(\sqrt[4]{3x^2-1}-1\right)}{27}-\frac{2\log\left(\sqrt[4]{3x^2-1}+1\right)}{27}+\frac{4\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(7/4)/189 + 2*(3*x**2 - 1)**(3/4)/27 + 2*log((3*x**2 - 1)**
(1/4) - 1)/27 - 2*log((3*x**2 - 1)**(1/4) + 1)/27 + 4*atan((3*x**2 - 1)**(1
/4))/27
```

Giac [A]
time = 0.63, size = 64, normalized size = 1.02

$$\frac{2}{189}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3*x^2 - 1)^
(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log(abs((3*x^2 - 1)^(1/4) -
 1))
```

Mupad [B]
time = 0.49, size = 51, normalized size = 0.81

$$\frac{4\,\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{27}+\frac{2\,{\left(3\,x^2-1\right)}^{3/4}}{27}+\frac{2\,{\left(3\,x^2-1\right)}^{7/4}}{189}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1\mathrm{i}\right)\,4\mathrm{i}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] (4*atan((3*x^2 - 1)^(1/4)))/27 + (atan((3*x^2 - 1)^(1/4)*1i)*4i)/27 + (2*(3
*x^2 - 1)^(3/4))/27 + (2*(3*x^2 - 1)^(7/4))/189
```

3.1044 $\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2}{27}(-1+3x^2)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

```
[Out] 2/27*(3*x^2-1)^(3/4)+2/9*arctan((3*x^2-1)^(1/4))-2/9*arctanh((3*x^2-1)^(1/4
))
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 81, 65, 304, 209, 212}

$$\frac{2}{9}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{2}{27}(3x^2-1)^{3/4}-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(
-1 + 3*x^2)^(1/4)])/9
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4} + \frac{4}{9}\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right) + \frac{2}{9}\text{Subst}\left(\int\right.\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4} + \frac{2}{9}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{2}{9}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.92

$$\frac{2}{27}\left(\left(-1+3x^2\right)^{3/4} + 3\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - 3\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(3/4) + 3*ArcTan[(-1 + 3*x^2)^(1/4)] - 3*ArcTanh[(-1 + 3*x
^2)^(1/4)]))/27
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.36, size = 135, normalized size = 2.81

method	result
trager	$\frac{2(3x^2-1)^{\frac{3}{4}}}{27} - \frac{\mathrm{RootOf}(\_Z^2+1)\ln\left(\frac{2\,\mathrm{RootOf}(\_Z^2+1)(3x^2-1)^{\frac{3}{4}}-2\,\mathrm{RootOf}(\_Z^2+1)(3x^2-1)^{\frac{1}{4}}+2\sqrt{3x^2-1}-3x^2}{3x^2-2}\right)}{9} + \ln($
risch	$\frac{2(3x^2-1)^{\frac{3}{4}}}{27} + \frac{\ln\left(\frac{2(3x^2-1)^{\frac{3}{4}}-2\sqrt{3x^2-1}-3x^2+2(3x^2-1)^{\frac{1}{4}}}{3x^2-2}\right)}{9} + \frac{\mathrm{RootOf}(\_Z^2+1)\ln\left(-\frac{2\,\mathrm{RootOf}(\_Z^2+1)(3x^2-1)^{\frac{3}{4}}-2\sqrt{}}{}\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*(3*x^2-1)^(3/4)-1/9*RootOf(_Z^2+1)*ln((2*RootOf(_Z^2+1)*(3*x^2-1)^(3/4
)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)+2*(3*x^2-1)^(1/2)-3*x^2)/(3*x^2-2))+1/9*
ln((2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)-3*x^2+2*(3*x^2-1)^(1/4))/(3*x^2-2))
```

Maxima [A]
time = 0.47, size = 52, normalized size = 1.08

$$\frac{2}{27}\left(3\,x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A]
time = 0.57, size = 52, normalized size = 1.08

$$\frac{2}{27}\left(3\,x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [A]
time = 7.03, size = 58, normalized size = 1.21

$$\frac{2(3x^2-1)^{\frac{3}{4}}}{27}+\frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{9}-\frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{9}+\frac{2\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(3/4)/27 + log((3*x**2 - 1)**(1/4) - 1)/9 - log((3*x**2 - 1
)**(1/4) + 1)/9 + 2*atan((3*x**2 - 1)**(1/4))/9
```

Giac [A]
time = 0.75, size = 53, normalized size = 1.10

$$\frac{2}{27}\left(3\,x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\,\log\left(\left|\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log(abs((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.47, size = 36, normalized size = 0.75

$$\frac{2\,\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{9}-\frac{2\,\mathrm{atanh}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{9}+\frac{2\,{\left(3\,x^2-1\right)}^{3/4}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] (2*atan((3*x^2 - 1)^(1/4)))/9 - (2*atanh((3*x^2 - 1)^(1/4)))/9 + (2*(3*x^2
- 1)^(3/4))/27
```

3.1045 $\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

```
[Out] 1/3*arctan((3*x^2-1)^(1/4))-1/3*arctanh((3*x^2-1)^(1/4))
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {455, 65, 304, 209, 212}

$$\frac{1}{3}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\ &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x,\right.\\ &= \frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.71, size = 126, normalized size = 3.82

method	result
trager	$-\frac{\ln\left(-\frac{2(3x^2-1)^{\frac{3}{4}}+2\sqrt{3x^2-1}+3x^2+2(3x^2-1)^{\frac{1}{4}}}{3x^2-2}\right)}{6} + \frac{\mathrm{RootOf}(\_Z^2+1)\ln\left(-\frac{2\,\mathrm{RootOf}(\_Z^2+1)(3x^2-1)^{\frac{3}{4}}-2\sqrt{3x^2-1}}{3x^2-2}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(-(2*(3*x^2-1)^(3/4)+2*(3*x^2-1)^(1/2)+3*x^2+2*(3*x^2-1)^(1/4))/(3*x
^2-2))+1/6*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(3*x^2-1)^(3/4)-2*(3*x^2-1)
^(1/2)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)+3*x^2)/(3*x^2-2))
```

Maxima [A]
time = 0.47, size = 41, normalized size = 1.24

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((3
*x^2 - 1)^(1/4) - 1)
```

Fricas [A]
time = 0.68, size = 41, normalized size = 1.24

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((3
*x^2 - 1)^(1/4) - 1)
```

Sympy [A]
time = 4.18, size = 42, normalized size = 1.27

$$\frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{6}-\frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{6}+\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] log((3*x**2 - 1)**(1/4) - 1)/6 - log((3*x**2 - 1)**(1/4) + 1)/6 + atan((3*x
**2 - 1)**(1/4))/3
```

Giac [A]
time = 0.81, size = 42, normalized size = 1.27

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(ab
s((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.09, size = 25, normalized size = 0.76

$$\frac{\operatorname{atan}\left((3\,x^2-1)^{1/4}\right)}{3}-\frac{\operatorname{atanh}\left((3\,x^2-1)^{1/4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] atan((3*x^2 - 1)^(1/4))/3 - atanh((3*x^2 - 1)^(1/4))/3
```

3.1046 $\int \frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=173

$$\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3}\right.$$

```
[Out] 1/2*arctan((3*x^2-1)^(1/4))-1/2*arctanh((3*x^2-1)^(1/4))-1/4*arctan(-1+(3*x
^2-1)^(1/4)*2^(1/2))*2^(1/2)-1/4*arctan(1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2)-
1/8*ln(1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)+1/8*ln(1+(3*x^2-1
)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,
Rules used = {457, 88, 65, 303, 1176, 631, 210, 1179, 642, 304, 209, 212}

$$\frac{1}{2}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{2\sqrt{2}}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sq
rt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-1 +
 3*x^2)^(1/4)]/2 - Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(
4*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt
[2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-2+3x)\sqrt[4]{-1+3x}}\,dx,x,x^2\right)$$
$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-1+3x}}\,dx,x,x^2\right)\right)+\frac{3}{4}\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+}}\right.$$
$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx,x,\sqrt[4]{-1+3x^2}\right)\right)+\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\right.$$
$$= \frac{1}{6}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx,x,\sqrt[4]{-1+3x^2}\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx,x,\sqrt[4]{-1}\right.$$
$$= \frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{4}\text{Subst}\left(\int \frac{}{1-\sqrt{2}}\right.$$
$$= \frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{\log\left(1-\sqrt{2}\sqrt[4]{-1}\right.}{4}$$
$$= \frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{}\right.}{2}$$

Mathematica [A]

time = 0.13, size = 110, normalized size = 0.64

$$\frac{1}{4}\left(2\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{-1+\sqrt{-1+3x^2}}{\sqrt{2}\sqrt[4]{-1+3x^2}}\right)-2\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-1+3x^2}}{1+\sqrt{-1+3x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*ArcTan[(-1 + 3*x^2)^(1/4)] - Sqrt[2]*ArcTan[(-1 + Sqrt[-1 + 3*x^2])/(Sqr
t[2]*(-1 + 3*x^2)^(1/4))] - 2*ArcTanh[(-1 + 3*x^2)^(1/4)] + Sqrt[2]*ArcTanh
[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 + Sqrt[-1 + 3*x^2])])/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.18, size = 302, normalized size = 1.75

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{2\sqrt{3x^2-1}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(3x^2-1\right)^{\frac{1}{4}}-3\,\mathrm{RootOf}\left(\_Z^4+1\right)x^2+2\left(3x^2-1\right)^{\frac{3}{4}}}{x^2}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*RootOf(_Z^4+1)*ln(-(2*(3*x^2-1)^(1/2)*RootOf(_Z^4+1)^3-2*RootOf(_Z^4+1
)^2*(3*x^2-1)^(1/4)-3*RootOf(_Z^4+1)*x^2+2*(3*x^2-1)^(3/4)+2*RootOf(_Z^4+1)
)/x^2)+1/4*RootOf(_Z^4+1)^3*ln(-(3*RootOf(_Z^4+1)^3*x^2-2*RootOf(_Z^4+1)^3+
2*RootOf(_Z^4+1)^2*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(1/2)*RootOf(_Z^4+1)+2*(3*x^
2-1)^(3/4))/x^2)-1/4*ln(-(2*(3*x^2-1)^(3/4)+2*(3*x^2-1)^(1/2)+3*x^2+2*(3*x^
2-1)^(1/4))/(3*x^2-2))+1/4*RootOf(_Z^4+1)^2*ln(-(2*RootOf(_Z^4+1)^2*(3*x^2-
1)^(1/2)-3*RootOf(_Z^4+1)^2*x^2+2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/4))/(3*x^2
-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x), x)
```

Fricas [A]
time = 0.59, size = 215, normalized size = 1.24

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}-1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{8}\sqrt{2}\log\left(4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)-\frac{1}{8}\sqrt{2}\log\left(-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)+\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1)
 + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqr
t(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(2)*(3*x^2 -
1)^(1/4) + 1) + 1/8*sqrt(2)*log(4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2
- 1) + 4) - 1/8*sqrt(2)*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1
) + 4) + 1/2*arctan((3*x^2 - 1)^(1/4)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1
/4*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(3x^2-2)\,\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [A]
time = 0.74, size = 155, normalized size = 0.90

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,(3\,x^2-1)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,(3\,x^2-1)^{\frac{1}{4}}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\,(3\,x^2-1)^{\frac{1}{4}}+\sqrt{3\,x^2-1}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\,(3\,x^2-1)^{\frac{1}{4}}+\sqrt{3\,x^2-1}+1\right)+\frac{1}{2}\arctan\left((3\,x^2-1)^{\frac{1}{4}}\right)-\frac{1}{4}\log\left((3\,x^2-1)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left|(3\,x^2-1)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 1/4*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) + 1/8*sqrt(2)*log(
sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/8*sqrt(2)*log(-sqrt(2)
*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 1/2*arctan((3*x^2 - 1)^(1/4)) -
 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log(abs((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.10, size = 77, normalized size = 0.45

$$\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{2}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(3\,x^2-1\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}+\frac{1}{4}{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(3\,x^2-1\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}-\frac{1}{4}{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] atan((3*x^2 - 1)^(1/4))/2 + (atan((3*x^2 - 1)^(1/4)*1i)*1i)/2 - 2^(1/2)*ata
n(2^(1/2)*(3*x^2 - 1)^(1/4)*(1/2 - 1i/2))*(1/4 - 1i/4) - 2^(1/2)*atan(2^(1/
2)*(3*x^2 - 1)^(1/4)*(1/2 + 1i/2))*(1/4 + 1i/4)
```

3.1047 $\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=191

$$-\frac{(-1+3x^2)^{3/4}}{4x^2}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)+\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}}$$

```
[Out] -1/4*(3*x^2-1)^(3/4)/x^2+3/4*arctan((3*x^2-1)^(1/4))-3/4*arctanh((3*x^2-1)^
(1/4))-9/16*arctan(-1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2)-9/16*arctan(1+(3*x^2
-1)^(1/4)*2^(1/2))*2^(1/2)-9/32*ln(1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2
))*2^(1/2)+9/32*ln(1+(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {457, 105, 162, 65, 303, 1176, 631, 210, 1179, 642, 304, 209, 212}

$$\frac{3}{4}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{9\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{8\sqrt{2}}-\frac{9\text{ArcTan}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{8\sqrt{2}}-\frac{(3x^2-1)^{3/4}}{4x^2}-\frac{9\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{9\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/4*(-1 + 3*x^2)^(3/4)/x^2 + (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 + (9*ArcTan[
1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (9*ArcTan[1 + Sqrt[2]*(-1 +
3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 - (9*Log[1 -
 Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2]) + (9*Log[1 +
Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)$$
$$= -\frac{(-1+3x^2)^{3/4}}{4x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{9}{2}+\frac{9x}{4}}{x(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)$$
$$= -\frac{(-1+3x^2)^{3/4}}{4x^2} - \frac{9}{16}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-1+3x}}\,dx, x, x^2\right) + \frac{9}{8}\text{Subst}\left(\int \frac{}{(}\right.$$
$$= -\frac{(-1+3x^2)^{3/4}}{4x^2} - \frac{3}{4}\text{Subst}\left(\int \frac{x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) + \frac{3}{2}\text{Subst}\left(\int\right.$$
$$= -\frac{(-1+3x^2)^{3/4}}{4x^2} + \frac{3}{8}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) - \frac{3}{8}\text{Subst}\left(\int\right.$$
$$= -\frac{(-1+3x^2)^{3/4}}{4x^2} + \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) -$$
$$= -\frac{(-1+3x^2)^{3/4}}{4x^2} + \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) -$$
$$= -\frac{(-1+3x^2)^{3/4}}{4x^2} + \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right.}{8\sqrt{2}}$$

Mathematica [A]
time = 0.19, size = 127, normalized size = 0.66

$$\frac{1}{16}\left(-\frac{4(-1+3x^2)^{3/4}}{x^2} + 12\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - 9\sqrt{2}\tan^{-1}\left(\frac{-1+\sqrt{-1+3x^2}}{\sqrt{2}\sqrt[4]{-1+3x^2}}\right) - 12\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) + 9\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-1+3x^2}}{1+\sqrt{-1+3x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ((-4*(-1 + 3*x^2)^(3/4))/x^2 + 12*ArcTan[(-1 + 3*x^2)^(1/4)] - 9*Sqrt[2]*Ar
cTan[(-1 + Sqrt[-1 + 3*x^2])/(Sqrt[2]*(-1 + 3*x^2)^(1/4))] - 12*ArcTanh[(-1
 + 3*x^2)^(1/4)] + 9*Sqrt[2]*ArcTanh[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 + Sqrt
[-1 + 3*x^2])])/16
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 8.08, size = 315, normalized size = 1.65

method	result

trager	$-\frac{(3x^2-1)^{\frac{3}{4}}}{4x^2}+\frac{9\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x^2-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2(3x^2-1)^{\frac{1}{4}}-2\sqrt{3x}}{x^2}}{16}$
risch	$-\frac{(3x^2-1)^{\frac{3}{4}}}{4x^2}+\frac{9\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(\frac{2\sqrt{3x^2-1}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2(3x^2-1)^{\frac{1}{4}}-3\,\mathrm{RootOf}\left(\_Z^4+1\right)x}{x^2}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(3*x^2-1)^(3/4)/x^2+9/16*RootOf(_Z^4+1)^3*ln(-(3*RootOf(_Z^4+1)^3*x^2-
2*RootOf(_Z^4+1)^3+2*RootOf(_Z^4+1)^2*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(1/2)*Roo
tOf(_Z^4+1)+2*(3*x^2-1)^(3/4))/x^2)+9/16*RootOf(_Z^4+1)*ln((2*(3*x^2-1)^(1/
2)*RootOf(_Z^4+1)^3+2*RootOf(_Z^4+1)^2*(3*x^2-1)^(1/4)-3*RootOf(_Z^4+1)*x^2
-2*(3*x^2-1)^(3/4)+2*RootOf(_Z^4+1))/x^2)-3/8*ln(-(2*(3*x^2-1)^(3/4)+2*(3*x
^2-1)^(1/2)+3*x^2+2*(3*x^2-1)^(1/4))/(3*x^2-2))+3/8*RootOf(_Z^4+1)^2*ln(-(2
*RootOf(_Z^4+1)^2*(3*x^2-1)^(1/2)-3*RootOf(_Z^4+1)^2*x^2+2*(3*x^2-1)^(3/4)-
2*(3*x^2-1)^(1/4))/(3*x^2-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^3), x)
```

Fricas [A]
time = 0.55, size = 252, normalized size = 1.32

$$\frac{36\sqrt{2}\,x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}-1\right)+36\sqrt{2}\,x^2\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4}-\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+1\right)+9\sqrt{2}\,x^2\log\left(4\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)-9\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)+24x^2\arctan\left((3x^2-1)^{\frac{1}{4}}\right)-12x^2\log\left((3x^2-1)^{\frac{1}{4}}+1\right)+12x^2\log\left((3x^2-1)^{\frac{1}{4}}-1\right)-8(3x^2-1)^{\frac{3}{4}}}{32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/32*(36*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3
*x^2 - 1) + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 36*sqrt(2)*x^2*arctan(1/2
*sqrt(2)*sqrt(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(
2)*(3*x^2 - 1)^(1/4) + 1) + 9*sqrt(2)*x^2*log(4*sqrt(2)*(3*x^2 - 1)^(1/4) +
 4*sqrt(3*x^2 - 1) + 4) - 9*sqrt(2)*x^2*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4) +
4*sqrt(3*x^2 - 1) + 4) + 24*x^2*arctan((3*x^2 - 1)^(1/4)) - 12*x^2*log((3*x
```

```
^2 - 1)^(1/4) + 1) + 12*x^2*log((3*x^2 - 1)^(1/4) - 1) - 8*(3*x^2 - 1)^(3/4
))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \cdot (3x^2 - 2) \sqrt[4]{3x^2 - 1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [A]
time = 0.75, size = 169, normalized size = 0.88

$$-\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{9}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{9}{32}\sqrt{2}\log\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{9}{32}\sqrt{2}\log\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{\left(3x^2-1\right)^{\frac{3}{4}}}{4x^2}+\frac{3}{4}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{3}{8}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{3}{8}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] -9/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 9/16*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) + 9/32*sqrt(2)*l
og(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 9/32*sqrt(2)*log(-sqr
t(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/4*(3*x^2 - 1)^(3/4)/x^2 +
 3/4*arctan((3*x^2 - 1)^(1/4)) - 3/8*log((3*x^2 - 1)^(1/4) + 1) + 3/8*log(a
bs((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.12, size = 82, normalized size = 0.43

$$\frac{3\,\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{4}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{4}-\frac{{\left(3\,x^2-1\right)}^{3/4}}{4\,x^2}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,9{}\mathrm{i}}{8}-\frac{{\left(-1\right)}^{3/4}\,\mathrm{atan}\left({\left(-1\right)}^{3/4}\,{\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,9{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] (3*atan((3*x^2 - 1)^(1/4)))/4 + (atan((3*x^2 - 1)^(1/4)*1i)*3i)/4 - (3*x^2
- 1)^(3/4)/(4*x^2) - ((-1)^(1/4)*atan((-1)^(1/4)*(3*x^2 - 1)^(1/4)*1i)*9i)/
8 - ((-1)^(3/4)*atan((-1)^(3/4)*(3*x^2 - 1)^(1/4)*1i)*9i)/8
```

3.1048 $\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=244

$$\frac{2}{45}x\left(-1+3x^2\right)^{3/4}+\frac{8x\sqrt[4]{-1+3x^2}}{15\left(1+\sqrt{-1+3x^2}\right)}-\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)$$

```
[Out] 2/45*x*(3*x^2-1)^(3/4)-1/27*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1
/27*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)+8/15*x*(3*x^2-1)^(1/4)/(
1+(3*x^2-1)^(1/2))-8/45*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arct
an((3*x^2-1)^(1/4)))*EllipticE(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*
(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)+4/45*(cos(2
*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF
(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3
*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {451, 236, 311, 226, 1210, 327, 407}

$$-\frac{1}{9}\sqrt{\frac{2}{3}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)+\frac{4\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{15\sqrt{3}\,x}-\frac{8\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{15\sqrt{3}\,x}+\frac{2}{45}\left(3x^2-1\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-1}\,x}{15\left(\sqrt{3x^2-1}+1\right)}-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2)^(3/4))/45 + (8*x*(-1 + 3*x^2)^(1/4))/(15*(1 + Sqrt[-1 + 3
*x^2])) - (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 - (Sqrt[2/
3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 - (8*Sqrt[x^2/(1 + Sqrt[-1
+ 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)],
 1/2])/(15*Sqrt[3]*x) + (4*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1
+ 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(15*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \left(\frac{2}{9\sqrt[4]{-1+3x^2}} + \frac{x^2}{3\sqrt[4]{-1+3x^2}} + \frac{4}{9(-2+3x^2)\sqrt[4]{-1+3x^2}}\right) dx$$

$$= \frac{2}{9}\int \frac{1}{\sqrt[4]{-1+3x^2}}\,dx + \frac{1}{3}\int \frac{x^2}{\sqrt[4]{-1+3x^2}}\,dx + \frac{4}{9}\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x}}$$

$$= \frac{2}{45}x(-1+3x^2)^{3/4} - \frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\Bigg(\frac{}{\sqrt[4]{}}$$

$$= \frac{2}{45}x(-1+3x^2)^{3/4} - \frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\Bigg(\frac{}{\sqrt[4]{}}$$

$$= \frac{2}{45}x(-1+3x^2)^{3/4} + \frac{4x\sqrt[4]{-1+3x^2}}{9\left(1+\sqrt{-1+3x^2}\right)} - \frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right.$$

$$= \frac{2}{45}x(-1+3x^2)^{3/4} + \frac{8x\sqrt[4]{-1+3x^2}}{15\left(1+\sqrt{-1+3x^2}\right)} - \frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.14, size = 177, normalized size = 0.73

$$\frac{2x\left(-1+3x^2-3x^2\sqrt[4]{1-3x^2}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)-\frac{4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(-2+3x^2)\left(2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)\right)}\right)}{45\sqrt[4]{-1+3x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2 - 3*x^2*(1 - 3*x^2)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2
, (3*x^2)/2] - (4*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^
2)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[3/2, 1
/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, 3*x^2, (3*x^2)/2
])))))/(45*(-1 + 3*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(3x^2-2)(3x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
[Out] int(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
[Out] integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 - 1)^(3/4)*x^4/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
[Out] Integral(x**4/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(3x^2-1)^{1/4}(3x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] int(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1049 $\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=224

$$\frac{2x\sqrt[4]{-1+3x^2}}{3\left(1+\sqrt{-1+3x^2}\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{2\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\,\left(1+\sqrt{}\right.}{}$$

```
[Out] -1/18*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/18*arctanh(1/2*x*6^(1
/2)/(3*x^2-1)^(1/4))*6^(1/2)+2/3*x*(3*x^2-1)^(1/4)/(1+(3*x^2-1)^(1/2))-2/9*
(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*Ell
ipticE(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2
/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)+1/9*(cos(2*arctan((3*x^2-1)^(1/4)))
^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF(sin(2*arctan((3*x^2-1)^(
1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x
*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {451, 236, 311, 226, 1210, 407}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}+\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{3\sqrt{3}\,x}-\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{3\sqrt{3}\,x}+\frac{2\sqrt[4]{3x^2-1}\,x}{3\left(\sqrt{3x^2-1}+1\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/(3*(1 + Sqrt[-1 + 3*x^2])) - ArcTan[(Sqrt[3/2]*x)/
(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]
/(3*Sqrt[6]) - (2*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])
*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(3*Sqrt[3]*x) + (Sqrt[x^2/(1
 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x
^2)^(1/4)], 1/2])/(3*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \left(\frac{1}{3\sqrt[4]{-1+3x^2}} + \frac{2}{3(-2+3x^2)\sqrt[4]{-1+3x^2}}\right) dx$$

$$= \frac{1}{3}\int \frac{1}{\sqrt[4]{-1+3x^2}}\,dx + \frac{2}{3}\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}} + \frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}} + \frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2x\sqrt[4]{-1+3x^2}}{3\left(1+\sqrt{-1+3x^2}\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 5.75, size = 52, normalized size = 0.23

$$-\frac{x^3\sqrt[4]{1-3x^2}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{6\sqrt[4]{-1+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
[Out] -1/6*(x^3*(1 - 3*x^2)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/2])/(
-1 + 3*x^2)^(1/4)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(3x^2-2)(3x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
[Out] int(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
[Out] integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 - 1)^(3/4)*x^2/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(3\,x^2-1\right)}^{1/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
[Out] int(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1050 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

```
[Out] -1/12*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^(1
/2)/(3*x^2-1)^(1/4))*6^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {407}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/Sqrt[6] - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 0.92

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{-1+3x^2}}{x}\right)-\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (ArcTan[(Sqrt[2/3]*(-1 + 3*x^2)^(1/4))/x] - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x
^2)^(1/4)])/(2*Sqrt[6])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.94, size = 138, normalized size = 2.26

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\left(3x^2-1\right)^{\frac{3}{4}}-3\sqrt{3x^2-1}\,x+\mathrm{RootOf}\left(\_Z^2-6\right)\left(3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2-2}\right)}{12}-\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*RootOf(_Z^2-6)*ln(-(RootOf(_Z^2-6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)*x
+RootOf(_Z^2-6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))-1/12*RootOf(_Z^2+6)*ln((Roo
tOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x-RootOf(_Z^2+6)*(3*x^2-1)^(1
/4)-3*x)/(3*x^2-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(43) = 86.
time = 8.40, size = 104, normalized size = 1.70

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}}{3x}\right)+\frac{1}{24}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}\,x^2-4\sqrt{6}\left(3x^2-1\right)^{\frac{3}{4}}x+12x^2-4}{9x^4-12x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/24*sqrt(6)*log(-(9
*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6)
*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(3\,x^2-1\right)}^{1/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1051 $\int \frac{1}{x^2(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=246

$$-\frac{(-1+3x^2)^{3/4}}{2x}+\frac{3x\sqrt[4]{-1+3x^2}}{2\left(1+\sqrt{-1+3x^2}\right)}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-$$

```
[Out] -1/2*(3*x^2-1)^(3/4)/x-1/8*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/
8*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)+3/2*x*(3*x^2-1)^(1/4)/(1+(
3*x^2-1)^(1/2))-1/2*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((
3*x^2-1)^(1/4)))*EllipticE(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(
3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)+1/4*(cos(2*arct
an((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF(sin(
2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-
1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {451, 331, 236, 311, 226, 1210, 407}

$$-\frac{1}{4}\sqrt{\frac{3}{2}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{4x}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}+\frac{3\sqrt[4]{3x^2-1}\,x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{(3x^2-1)^{3/4}}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*(-1 + 3*x^2)^(3/4)/x + (3*x*(-1 + 3*x^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x^
2])) - (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3/2]*
ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[
-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4
)], 1/2])/(2*x) + (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1
+ 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(4*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \left(-\frac{1}{2x^2\sqrt[4]{-1+3x^2}} + \frac{3}{2(-2+3x^2)\sqrt[4]{-1+3x^2}}\right)dx$$

$$= -\left(\frac{1}{2}\int \frac{1}{x^2\sqrt[4]{-1+3x^2}}\,dx\right) + \frac{3}{2}\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$$

$$= -\frac{(-1+3x^2)^{3/4}}{2x} - \frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{}{\sqrt[4]{-}}\right.$$

$$= -\frac{(-1+3x^2)^{3/4}}{2x} - \frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{}{\sqrt[4]{-}}\right.$$

$$= -\frac{(-1+3x^2)^{3/4}}{2x} - \frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{}{\sqrt[4]{-}}\right.$$

$$= -\frac{(-1+3x^2)^{3/4}}{2x} + \frac{3x\sqrt[4]{-1+3x^2}}{2\left(1+\sqrt{-1+3x^2}\right)} - \frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.03, size = 64, normalized size = 0.26

$$\frac{4-12x^2-3x^4\sqrt[4]{1-3x^2}\,F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{8x\sqrt[4]{-1+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
[Out] (4 - 12*x^2 - 3*x^4*(1 - 3*x^2)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*
x^2)/2])/(8*x*(-1 + 3*x^2)^(1/4))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(3x^2-2)(3x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(3/4)/(9*x^6 - 9*x^4 + 2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \cdot (3x^2 - 2)\sqrt[4]{3x^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,(3\,x^2-1)^{1/4}\,(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
[Out] int(1/(x^2*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1052 $\int \frac{1}{x^4(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=264

$$-\frac{(-1+3x^2)^{3/4}}{6x^3}-\frac{3(-1+3x^2)^{3/4}}{2x}+\frac{9x\sqrt[4]{-1+3x^2}}{2\left(1+\sqrt{-1+3x^2}\right)}-\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\right.$$

```
[Out] -1/6*(3*x^2-1)^(3/4)/x^3-3/2*(3*x^2-1)^(3/4)/x-3/16*arctan(1/2*x*6^(1/2)/(3
*x^2-1)^(1/4))*6^(1/2)-3/16*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)+
9/2*x*(3*x^2-1)^(1/4)/(1+(3*x^2-1)^(1/2))-3/2*(cos(2*arctan((3*x^2-1)^(1/4)
))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticE(sin(2*arctan((3*x^2-1)
^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)
/x*3^(1/2)+3/4*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2
-1)^(1/4)))*EllipticF(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2
-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {451, 331, 236, 311, 226, 1210, 407}

$$-\frac{3}{8}\sqrt{\frac{3}{2}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)+\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{4x}-\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}+\frac{9\sqrt[4]{3x^2-1}\,x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{3(3x^2-1)^{3/4}}{2x}-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)-\frac{(3x^2-1)^{3/4}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/6*(-1 + 3*x^2)^(3/4)/x^3 - (3*(-1 + 3*x^2)^(3/4))/(2*x) + (9*x*(-1 + 3*x
^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x^2])) - (3*Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(
-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/
4)])/8 - (3*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2
])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x) + (3*Sqrt[3]*Sqrt[x^
2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 +
 3*x^2)^(1/4)], 1/2])/(4*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 451

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^4(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \left(-\frac{1}{2x^4\sqrt[4]{-1+3x^2}} - \frac{3}{4x^2\sqrt[4]{-1+3x^2}} + \frac{9}{4(-2+3x^2)\sqrt[4]{-1+3x^2}}\right.$$

$$= -\left(\frac{1}{2}\int \frac{1}{x^4\sqrt[4]{-1+3x^2}}\,dx\right) - \frac{3}{4}\int \frac{1}{x^2\sqrt[4]{-1+3x^2}}\,dx + \frac{9}{4}\int \frac{}{(-2+3x}$$

$$= -\frac{(-1+3x^2)^{3/4}}{6x^3} - \frac{3(-1+3x^2)^{3/4}}{4x} - \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) -$$

$$= -\frac{(-1+3x^2)^{3/4}}{6x^3} - \frac{3(-1+3x^2)^{3/4}}{2x} - \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) -$$

$$= -\frac{(-1+3x^2)^{3/4}}{6x^3} - \frac{3(-1+3x^2)^{3/4}}{2x} - \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) -$$

$$= -\frac{(-1+3x^2)^{3/4}}{6x^3} - \frac{3(-1+3x^2)^{3/4}}{2x} + \frac{9x\sqrt[4]{-1+3x^2}}{4\left(1+\sqrt{-1+3x^2}\right)} - \frac{3}{8}\sqrt{\frac{3}{2}}\,\mathrm{ta}$$

$$= -\frac{(-1+3x^2)^{3/4}}{6x^3} - \frac{3(-1+3x^2)^{3/4}}{2x} + \frac{9x\sqrt[4]{-1+3x^2}}{2\left(1+\sqrt{-1+3x^2}\right)} - \frac{3}{8}\sqrt{\frac{3}{2}}\,\mathrm{ta}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 10.12, size = 148, normalized size = 0.56

$$\frac{1}{2}(-1+3x^2)^{3/4}\left(-\frac{1+9x^2}{3x^3} + \frac{9xF_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(-2+3x^2)\left(2F_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};-\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)-3F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ((-1 + 3*x^2)^(3/4)*(-1/3*(1 + 9*x^2)/x^3 + (9*x*AppellF1[1/2, -3/4, 1, 3/2
, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(2*AppellF1[1/2, -3/4, 1, 3/2, 3*x^2, (3
```

```
*x^2)/2] + x^2*(2*AppellF1[3/2, -3/4, 2, 5/2, 3*x^2, (3*x^2)/2] - 3*AppellF
1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/2])))))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(3x^2-2\right) \left(3x^2-1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
[Out] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 - 1)^(3/4)/(9*x^8 - 9*x^6 + 2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \cdot \left(3x^2-2\right) \sqrt[4]{3x^2-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
[Out] Integral(1/(x**4*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(3\,x^2-1\right)}^{1/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] int(1/(x^4*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1053 $\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\ \sqrt{2+3x^2}}{2\sqrt{3}\ x\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\ \sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\ \sqrt{2+3x^2}}{2\sqrt{3}\ x\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\ \sqrt{3}}$$

```
[Out] -1/18*arctan(1/6*(2*2^(3/4)+2*2^(1/4)*(3*x^2+2)^(1/2))/x/(3*x^2+2)^(1/4)*3^
(1/2))*2^(3/4)*3^(1/2)+1/18*arctanh(1/6*(2*2^(3/4)-2*2^(1/4)*(3*x^2+2)^(1/2
))/x/(3*x^2+2)^(1/4)*3^(1/2))*2^(3/4)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {452}

$$\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\ \sqrt{3x^2+2}}{2\sqrt{3}\ x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\ \sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2\sqrt[4]{2}\ \sqrt{3x^2+2}+2\ 2^{3/4}}{2\sqrt{3}\ x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\ \sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

```
[Out] -1/3*ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2
)^(1/4))]/(2^(1/4)*Sqrt[3]) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x^2
])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx=-\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\ \sqrt{2+3x^2}}{2\sqrt{3}\ x\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\ \sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\ \sqrt{2+3x^2}}{2\sqrt{3}\ x\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\ \sqrt{3}}$$

Mathematica [A]
time = 1.88, size = 109, normalized size = 0.84

$$-\frac{\tan^{-1}\left(\frac{-3\sqrt{2}\,x^2+4\sqrt{2+3x^2}}{2\,2^{3/4}\sqrt{3}\,x\sqrt[4]{2+3x^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt{3}\,x\sqrt[4]{4+6x^2}}{3x^2+2\sqrt{4+6x^2}}\right)}{6\sqrt[4]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

```
[Out] -1/6*(ArcTan[(-3*Sqrt[2]*x^2 + 4*Sqrt[2 + 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 +
 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*x*(4 + 6*x^2)^(1/4))/(3*x^2 + 2*Sqrt[4
 + 6*x^2])])/(2^(1/4)*Sqrt[3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.45, size = 186, normalized size = 1.44

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)\ln\left(\frac{\left(3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)+9\sqrt{3x^2+2}}{3x^2+4}\right.}{18}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*RootOf(_Z^2+RootOf(_Z^4+18)^2)*ln(((3*x^2+2)^(3/4)*RootOf(_Z^4+18)^2*
RootOf(_Z^2+RootOf(_Z^4+18)^2)+9*(3*x^2+2)^(1/2)*x+3*RootOf(_Z^4+18)^2*x-6*
(3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+18)^2))/(3*x^2+4))-1/18*RootOf(_Z^4
+18)*ln(-((3*x^2+2)^(3/4)*RootOf(_Z^4+18)^3-9*(3*x^2+2)^(1/2)*x+3*RootOf(_Z
^4+18)^2*x+6*RootOf(_Z^4+18)*(3*x^2+2)^(1/4))/(3*x^2+4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(88) = 176.
time = 1.78, size = 282, normalized size = 2.19

$$\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}\,x\sqrt{\frac{72^{\frac{3}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x+18\sqrt{2}\,x^2+24\sqrt{3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}-36x}{36x}\right)+\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}\,x\sqrt{-\frac{72^{\frac{3}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x-18\sqrt{2}\,x^2-24\sqrt{3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}+36x}{36x}\right)-\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x+18\sqrt{2}\,x^2+24\sqrt{3x^2+2}\right)}{x^2}\right)+\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\,(3x^2+2)^{\frac{1}{4}}x-18\sqrt{2}\,x^2-24\sqrt{3x^2+2}\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/216*72^(3/4)*sqrt(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt((72^(3/
4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(3*x^2 + 2))/x^2)
- 12*72^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4) - 36*x)/x) + 1/216*72^(3/4)*sqrt(2)
*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt(-(72^(3/4)*sqrt(2)*(3*x^2 + 2
)^(1/4)*x - 18*sqrt(2)*x^2 - 24*sqrt(3*x^2 + 2))/x^2) - 12*72^(1/4)*sqrt(2)
*(3*x^2 + 2)^(1/4) + 36*x)/x) - 1/864*72^(3/4)*sqrt(2)*log(96*(72^(3/4)*sqr
t(2)*(3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(3*x^2 + 2))/x^2) + 1/86
4*72^(3/4)*sqrt(2)*log(-96*(72^(3/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x - 18*sqrt(
2)*x^2 - 24*sqrt(3*x^2 + 2))/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(3x^2+2)^{\frac{3}{4}} \cdot (3x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(3/4)/(3*x**2+4),x)
```

```
[Out] Integral(x**2/((3*x**2 + 2)**(3/4)*(3*x**2 + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(3\,x^2+2\right)}^{3/4}\,\left(3\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((3*x^2 + 2)^(3/4)*(3*x^2 + 4)),x)
```

```
[Out] int(x^2/((3*x^2 + 2)^(3/4)*(3*x^2 + 4)), x)
```

3.1054 $\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

```
[Out] 1/18*arctan(1/6*(2-2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)/x/(-3*x^2+2)^(1/4)*3^(
1/2))*2^(3/4)*3^(1/2)-1/18*arctanh(1/6*(2+2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)
/x/(-3*x^2+2)^(1/4)*3^(1/2))*2^(3/4)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {452}

$$\frac{\text{ArcTan}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Mathematica [A]
time = 1.93, size = 109, normalized size = 0.91

$$-\frac{\tan^{-1}\left(\frac{-3\sqrt{2}\,x^2+4\sqrt{2-3x^2}}{2\ 2^{3/4}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt{3}\,x\sqrt[4]{4-6x^2}}{3x^2+2\sqrt{4-6x^2}}\right)}{6\sqrt[4]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/6*(ArcTan[(-3*Sqrt[2]*x^2 + 4*Sqrt[2 - 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 -
 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*x*(4 - 6*x^2)^(1/4))/(3*x^2 + 2*Sqrt[4
 - 6*x^2])])/(2^(1/4)*Sqrt[3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.44, size = 186, normalized size = 1.55

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^4+18\right)\ln\left(\frac{\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)^3+3\,\mathrm{RootOf}\left(\_Z^4+18\right)^2x+9\sqrt{-3x^2+2}\,x-6\,\mathrm{RootOf}\left(\_Z^4+18\right)\left(-3x^2+2\right)}{3x^2-4}\right.}{18}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*RootOf(_Z^4+18)*ln(((-3*x^2+2)^(3/4)*RootOf(_Z^4+18)^3+3*RootOf(_Z^4+1
8)^2*x+9*(-3*x^2+2)^(1/2)*x-6*RootOf(_Z^4+18)*(-3*x^2+2)^(1/4))/(3*x^2-4))+
1/18*RootOf(_Z^2+RootOf(_Z^4+18)^2)*ln(-(RootOf(_Z^4+18)^2*RootOf(_Z^2+Root
Of(_Z^4+18)^2)*(-3*x^2+2)^(3/4)+3*RootOf(_Z^4+18)^2*x-9*(-3*x^2+2)^(1/2)*x+
6*(-3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+18)^2))/(3*x^2-4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(89) = 178.
time = 1.15, size = 282, normalized size = 2.35

$$\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\,\sqrt{2}\,x\sqrt{\frac{72^{\frac{3}{4}}\sqrt{2}\,(-3\,x^2+2)^{\frac{1}{4}}x+18\,\sqrt{2}\,x^2+24\,\sqrt{-3\,x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\,(-3\,x^2+2)^{\frac{1}{4}}-36\,x}{36\,x}\right)+\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\,\sqrt{2}\,x\sqrt{-\frac{72^{\frac{3}{4}}\sqrt{2}\,(-3\,x^2+2)^{\frac{1}{4}}x-18\,\sqrt{2}\,x^2-24\,\sqrt{-3\,x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\,(-3\,x^2+2)^{\frac{1}{4}}+36\,x}{36\,x}\right)-\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\,(-3\,x^2+2)^{\frac{1}{4}}x+18\,\sqrt{2}\,x^2+24\,\sqrt{-3\,x^2+2}\right)}{x^2}\right)+\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\,(-3\,x^2+2)^{\frac{1}{4}}x-18\,\sqrt{2}\,x^2-24\,\sqrt{-3\,x^2+2}\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/216*72^(3/4)*sqrt(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt((72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) - 12*72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 36*x)/x) + 1/216*72^(3/4)*sqrt
(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt(-(72^(3/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x - 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2) - 12*72^(1/4)*sq
rt(2)*(-3*x^2 + 2)^(1/4) + 36*x)/x) - 1/864*72^(3/4)*sqrt(2)*log(96*(72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) + 1/864*72^(3/4)*sqrt(2)*log(-96*(72^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x -
 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(2-3\,x^2\right)^{3/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] -int(x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1055 $\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx$

Optimal. Leaf size=124

$$-\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\,\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}+\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\,\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

```
[Out] -1/2*arctan(1/2*(2*2^(3/4)+2*2^(1/4)*(b*x^2+2)^(1/2))/x/(b*x^2+2)^(1/4)/b^(
1/2))*2^(3/4)/b^(3/2)+1/2*arctanh(1/2*(2*2^(3/4)-2*2^(1/4)*(b*x^2+2)^(1/2))
/x/(b*x^2+2)^(1/4)/b^(1/2))*2^(3/4)/b^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {452}

$$\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\,\sqrt{bx^2+2}}{2\sqrt{b}\,x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}\,b^{3/2}}-\frac{\text{ArcTan}\left(\frac{2\sqrt[4]{2}\,\sqrt{bx^2+2}+2\ 2^{3/4}}{2\sqrt{b}\,x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

```
[Out] -(ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(
1/4))]/(2^(1/4)*b^(3/2))) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2])
/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx=-\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\,\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}+\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\,\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Mathematica [A]
time = 2.10, size = 112, normalized size = 0.90

$$-\frac{\tan^{-1}\left(\frac{-2^{3/4}bx^2+4\sqrt[4]{2}\sqrt{2+bx^2}}{4\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt{b}\,x\sqrt[4]{4+2bx^2}}{bx^2+2\sqrt{4+2bx^2}}\right)}{2\sqrt[4]{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

```
[Out] -1/2*(ArcTan[(-(2^(3/4)*b*x^2) + 4*2^(1/4)*Sqrt[2 + b*x^2])/(4*Sqrt[b]*x*(2
 + b*x^2)^(1/4))] + ArcTanh[(2*Sqrt[b]*x*(4 + 2*b*x^2)^(1/4))/(b*x^2 + 2*Sq
rt[4 + 2*b*x^2])])/(2^(1/4)*b^(3/2))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+2\right)^{\frac{3}{4}}\left(b\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)
```

```
[Out] int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(86) = 172.
time = 0.91, size = 393, normalized size = 3.17

$$\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^4\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{bx^2+2}}{x^2}}\frac{1}{b^6}^{\frac{3}{4}}x-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}(bx^2+2)^{\frac{1}{4}}b^4\frac{1}{b^6}^{\frac{3}{4}}-x}{x}\right)+\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^4x\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{bx^2+2}}{x^2}}\frac{1}{b^6}^{\frac{3}{4}}-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}(bx^2+2)^{\frac{1}{4}}b^4\frac{1}{b^6}^{\frac{3}{4}}+x}{x}\right)-\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{bx^2+2}}{2x^2}\right)+\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{bx^2+2}}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*
b^4*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2
```

```
)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x^2)*(b^(-6))^(3/4)*x - 8
*sqrt(2)*(1/8)^(3/4)*(b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) - x)/x) + sqrt(2)
*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*b^4*x*s
qrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2)^(1/
4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x^2)*(b^(-6))^(3/4) - 8*sqrt(2
)*(1/8)^(3/4)*(b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) + x)/x) - 1/4*sqrt(2)*(1
/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2
)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x
^2) + 1/4*sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^
(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x +
2*sqrt(b*x^2 + 2))/x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^2+2)^{\frac{3}{4}}\,(bx^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+2)**(3/4)/(b*x**2+4),x)
[Out] Integral(x**2/((b*x**2 + 2)**(3/4)*(b*x**2 + 4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="giac")
[Out] integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(b\,x^2+2\right)}^{3/4}\,\left(b\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2 + 2)^(3/4)*(b*x^2 + 4)),x)
[Out] int(x^2/((b*x^2 + 2)^(3/4)*(b*x^2 + 4)), x)
```

3.1056 $\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

```
[Out] 1/2*arctan(1/2*(2-2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)/x/(-b*x^2+2)^(1/4)/b^(1
/2))*2^(3/4)/b^(3/2)-1/2*arctanh(1/2*(2+2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)/x
/(-b*x^2+2)^(1/4)/b^(1/2))*2^(3/4)/b^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {452}

$$\frac{\text{ArcTan}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]
/(2^(1/4)*b^(3/2)) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]
*x*(2 - b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Mathematica [A]
time = 2.18, size = 114, normalized size = 0.96

$$-\frac{\tan^{-1}\left(\frac{-2^{3/4}bx^2+4\sqrt[4]{2}\sqrt{2-bx^2}}{4\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt{b}\,x\sqrt[4]{4-2bx^2}}{bx^2+2\sqrt{4-2bx^2}}\right)}{2\sqrt[4]{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

```
[Out] -1/2*(ArcTan[(-(2^(3/4)*b*x^2) + 4*2^(1/4)*Sqrt[2 - b*x^2])/(4*Sqrt[b]*x*(2
 - b*x^2)^(1/4))] + ArcTanh[(2*Sqrt[b]*x*(4 - 2*b*x^2)^(1/4))/(b*x^2 + 2*Sq
rt[4 - 2*b*x^2])])/(2^(1/4)*b^(3/2))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2+2\right)^{\frac{3}{4}}\left(-b\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)
```

```
[Out] int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(93) = 186.
time = 0.88, size = 403, normalized size = 3.39

$$\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^4\frac{1}{b^6}^{\frac{3}{4}}x\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(-bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{-bx^2+2}}{x^2}}-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}(-bx^2+2)^{\frac{1}{4}}b^4\frac{1}{b^6}^{\frac{3}{4}}+x}{x}\right)+\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^4x\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(-bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{-bx^2+2}}{x^2}}\frac{1}{b^6}^{\frac{3}{4}}-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}(-bx^2+2)^{\frac{1}{4}}b^4\frac{1}{b^6}^{\frac{3}{4}}-x}{x}\right)-\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(-bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{-bx^2+2}}{2x^2}\right)+\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(-bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{-bx^2+2}}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*
b^4*(b^(-6))^(3/4)*x*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)
```

```
^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2) -
 8*sqrt(2)*(1/8)^(3/4)*(-b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) + x)/x) + sqrt
(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*b^4*
x*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)
^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2)*(b^(-6))^(3/4) - 8*s
qrt(2)*(1/8)^(3/4)*(-b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) - x)/x) - 1/4*sqrt
(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*
sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2
 + 2))/x^2) + 1/4*sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4
*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(
1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{bx^2\left(-bx^2+2\right)^{\frac{3}{4}}-4\left(-bx^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+2)**(3/4)/(-b*x**2+4),x)
[Out] -Integral(x**2/(b*x**2*(-b*x**2 + 2)**(3/4) - 4*(-b*x**2 + 2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="giac")
[Out] integrate(-x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{{\left(2-b\,x^2\right)}^{3/4}\,\left(b\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2 - b*x^2)^(3/4)*(b*x^2 - 4)),x)
[Out] -int(x^2/((2 - b*x^2)^(3/4)*(b*x^2 - 4)), x)
```

3.1057 $\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx$

Optimal. Leaf size=120

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}$$

```
[Out] -1/9*arctan(1/3*a^(3/4)*(1+(3*x^2+a)^(1/2)/a^(1/2))/x/(3*x^2+a)^(1/4)*3^(1/
2))/a^(1/4)*3^(1/2)+1/9*arctanh(1/3*a^(3/4)*(1-(3*x^2+a)^(1/2)/a^(1/2))/x/(
3*x^2+a)^(1/4)*3^(1/2))/a^(1/4)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {452}

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}-\frac{\text{ArcTan}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

```
[Out] -1/3*ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^
(1/4))]/(Sqrt[3]*a^(1/4)) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]))
/(Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx = -\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}} + \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}$$

Mathematica [A]
time = 2.09, size = 119, normalized size = 0.99

$$-\frac{\tan^{-1}\left(\frac{-3x^2+2\sqrt{a}\,\sqrt{a+3x^2}}{2\sqrt{3}\,\sqrt[4]{a}\,x\sqrt[4]{a+3x^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt{3}\,\sqrt[4]{a}\,x\sqrt[4]{a+3x^2}}{3x^2+2\sqrt{a}\,\sqrt{a+3x^2}}\right)}{6\sqrt{3}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

```
[Out] -1/6*(ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a + 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a + 
 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a + 3*x^2)^(1/4))/(3*x^2 + 
2*Sqrt[a]*Sqrt[a + 3*x^2])])/(Sqrt[3]*a^(1/4))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(3x^2+a)^{\frac{3}{4}}(3x^2+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

```
[Out] int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

Fricas [A]
time = 4.47, size = 171, normalized size = 1.42

$$-\frac{2}{3}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}ax\left(-\frac{1}{a}\right)^{\frac{3}{4}}\sqrt{\frac{3x^{2}\sqrt{-\frac{1}{a}}+2\sqrt{3x^{2}+a}}{x^{2}}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}(3x^{2}+a)^{\frac{1}{4}}a\left(-\frac{1}{a}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+(3x^{2}+a)^{\frac{1}{4}}}{x}\right)+\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}-(3x^{2}+a)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*(1/36)^(1/4)*(-1/a)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a*x*(-1/a)
^(3/4)*sqrt((3*x^2*sqrt(-1/a) + 2*sqrt(3*x^2 + a))/x^2) - (1/36)^(3/4)*(3*x
^2 + a)^(1/4)*a*(-1/a)^(3/4))/x) - 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(1/
36)^(1/4)*x*(-1/a)^(1/4) + (3*x^2 + a)^(1/4))/x) + 1/6*(1/36)^(1/4)*(-1/a)^
(1/4)*log(-(3*(1/36)^(1/4)*x*(-1/a)^(1/4) - (3*x^2 + a)^(1/4))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a+3x^{2}\right)^{\frac{3}{4}}\cdot\left(2a+3x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+a)**(3/4)/(3*x**2+2*a),x)
```

```
[Out] Integral(x**2/((a + 3*x**2)**(3/4)*(2*a + 3*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(3\,x^2+2\,a\right)\,{\left(3\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2*a + 3*x^2)*(a + 3*x^2)^(3/4)),x)
```

```
[Out] int(x^2/((2*a + 3*x^2)*(a + 3*x^2)^(3/4)), x)
```

3.1058 $\int \frac{x^2}{(a-3x^2)^{3/4}(2a-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

```
[Out] 1/9*arctan(1/3*a^(3/4)*(1-(-3*x^2+a)^(1/2)/a^(1/2))/x/(-3*x^2+a)^(1/4)*3^(1
/2))/a^(1/4)*3^(1/2)-1/9*arctanh(1/3*a^(3/4)*(1+(-3*x^2+a)^(1/2)/a^(1/2))/x
/(-3*x^2+a)^(1/4)*3^(1/2))/a^(1/4)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {452}

$$\frac{\text{ArcTan}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4)
)]/(3*Sqrt[3]*a^(1/4)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/(S
qrt[3]*x*(a - 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{(a-3x^2)^{3/4}(2a-3x^2)}\,dx = \frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}} - \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

Mathematica [**A**]
time = 2.19, size = 119, normalized size = 0.99

$$-\frac{\tan^{-1}\left(\frac{-3x^2+2\sqrt{a}\sqrt{a-3x^2}}{2\sqrt{3}\sqrt[4]{a}\,x\sqrt[4]{a-3x^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt{3}\sqrt[4]{a}\,x\sqrt[4]{a-3x^2}}{3x^2+2\sqrt{a}\sqrt{a-3x^2}}\right)}{6\sqrt{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

```
[Out] -1/6*(ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a - 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a -
 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a - 3*x^2)^(1/4))/(3*x^2 +
2*Sqrt[a]*Sqrt[a - 3*x^2])])/(Sqrt[3]*a^(1/4))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-3x^2+a)^{\frac{3}{4}}(-3x^2+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
```

```
[Out] int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

Fricas [A]
time = 2.36, size = 171, normalized size = 1.42

$$-\frac{2}{3}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}ax\left(-\frac{1}{a}\right)^{\frac{3}{4}}\sqrt{\frac{3x^{2}\sqrt{-\frac{1}{a}}+2\sqrt{-3x^{2}+a}}{x^{2}}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}\left(-3x^{2}+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(-3x^{2}+a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}-\left(-3x^{2}+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*(1/36)^(1/4)*(-1/a)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a*x*(-1/a)
^(3/4)*sqrt((3*x^2*sqrt(-1/a) + 2*sqrt(-3*x^2 + a))/x^2) - (1/36)^(3/4)*(-3
*x^2 + a)^(1/4)*a*(-1/a)^(3/4))/x) - 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(
1/36)^(1/4)*x*(-1/a)^(1/4) + (-3*x^2 + a)^(1/4))/x) + 1/6*(1/36)^(1/4)*(-1/
a)^(1/4)*log(-(3*(1/36)^(1/4)*x*(-1/a)^(1/4) - (-3*x^2 + a)^(1/4))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}}{-2a\left(a-3x^{2}\right)^{\frac{3}{4}}+3x^{2}\left(a-3x^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+a)**(3/4)/(-3*x**2+2*a),x)
```

```
[Out] -Integral(x**2/(-2*a*(a - 3*x**2)**(3/4) + 3*x**2*(a - 3*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(2\,a-3\,x^2\right)\,{\left(a-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2*a - 3*x^2)*(a - 3*x^2)^(3/4)),x)
```

```
[Out] int(x^2/((2*a - 3*x^2)*(a - 3*x^2)^(3/4)), x)
```

3.1059 $\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx$

Optimal. Leaf size=115

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

```
[Out] -arctan(a^(3/4)*(1+(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+a)^(1/4)/b^(1/2))/a^(1
/4)/b^(3/2)+arctanh(a^(3/4)*(1-(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+a)^(1/4)/b
^(1/2))/a^(1/4)/b^(3/2)
```

Rubi [A]

time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {452}

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}-\frac{\text{ArcTan}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

```
[Out] -(ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/
4))]/(a^(1/4)*b^(3/2))) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))/(
Sqrt[b]*x*(a + b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx = -\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}} + \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Mathematica [A]
time = 2.09, size = 121, normalized size = 1.05

$$\frac{\tan^{-1}\left(\frac{bx^2-2\sqrt{a}\,\sqrt{a+bx^2}}{2\sqrt[4]{a}\,\sqrt{b}\,x\sqrt[4]{a+bx^2}}\right) - \tanh^{-1}\left(\frac{2\sqrt[4]{a}\,\sqrt{b}\,x\sqrt[4]{a+bx^2}}{bx^2+2\sqrt{a}\,\sqrt{a+bx^2}}\right)}{2\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

```
[Out] (ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a + b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a + b*x^2
)^(1/4))] - ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a + b*x^2)^(1/4))/(b*x^2 + 2*Sqrt
[a]*Sqrt[a + b*x^2])])/(2*a^(1/4)*b^(3/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{4}}\,(b\,x^2+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
```

```
[Out] int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(87) = 174$.

time = 1.34, size = 207, normalized size = 1.80

$$-2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{-\frac{1}{ab^6}}+2\sqrt{bx^2+a}}{x^2}}\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}(bx^2+a)^{\frac{1}{4}}ab^4\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+(bx^2+a)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}-(bx^2+a)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="fricas")
[Out] -2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*s
qrt((b^4*x^2*sqrt(-1/(a*b^6)) + 2*sqrt(b*x^2 + a))/x^2)*(-1/(a*b^6))^(3/4)
- (1/4)^(3/4)*(b*x^2 + a)^(1/4)*a*b^4*(-1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^(1
/4)*(-1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (b*x^2 +
 a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*x*
(-1/(a*b^6))^(1/4) - (b*x^2 + a)^(1/4))/x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^2)^{\frac{3}{4}}\cdot(2a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/4)/(b*x**2+2*a),x)
[Out] Integral(x**2/((a + b*x**2)**(3/4)*(2*a + b*x**2)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="giac")
[Out] integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(b\,x^2+a)^{3/4}\,(b\,x^2+2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x)
[Out] int(x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)), x)
```

3.1060 $\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

```
[Out] arctan(a^(3/4)*(1-(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x^2+a)^(1/4)/b^(1/2))/a^(
1/4)/b^(3/2)-arctanh(a^(3/4)*(1+(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x^2+a)^(1/4
)/b^(1/2))/a^(1/4)/b^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {452}

$$\frac{\text{ArcTan}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4)
)]/(a^(1/4)*b^(3/2)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(Sqr
t[b]*x*(a - b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx = \frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}} - \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Mathematica [A]
time = 2.11, size = 125, normalized size = 1.05

$$\frac{\tan^{-1}\left(\frac{bx^2-2\sqrt{a}\sqrt{a-bx^2}}{2\sqrt[4]{a}\sqrt{b}\,x\sqrt[4]{a-bx^2}}\right) - \tanh^{-1}\left(\frac{2\sqrt[4]{a}\sqrt{b}\,x\sqrt[4]{a-bx^2}}{bx^2+2\sqrt{a}\sqrt{a-bx^2}}\right)}{2\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

```
[Out] (ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a - b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a - b*x^2
)^(1/4))] - ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a - b*x^2)^(1/4))/(b*x^2 + 2*Sqrt
[a]*Sqrt[a - b*x^2])])/(2*a^(1/4)*b^(3/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-b\,x^2+a)^{\frac{3}{4}}\,(-b\,x^2+2a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

```
[Out] int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(91) = 182$.

time = 1.08, size = 211, normalized size = 1.77

$$-2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{-\frac{1}{ab^6}}+2\sqrt{-bx^2+a}}{x^2}}\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}(-bx^2+a)^{\frac{1}{4}}ab^4\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+(-bx^2+a)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}-(-bx^2+a)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="fricas")
[Out] -2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*s
qrt((b^4*x^2*sqrt(-1/(a*b^6)) + 2*sqrt(-b*x^2 + a))/x^2)*(-1/(a*b^6))^(3/4)
 - (1/4)^(3/4)*(-b*x^2 + a)^(1/4)*a*b^4*(-1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^
(1/4)*(-1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (-b*x^
2 + a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2
*x*(-1/(a*b^6))^(1/4) - (-b*x^2 + a)^(1/4))/x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{-2a\left(a-bx^2\right)^{\frac{3}{4}}+bx^2\left(a-bx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(3/4)/(-b*x**2+2*a),x)
[Out] -Integral(x**2/(-2*a*(a - b*x**2)**(3/4) + b*x**2*(a - b*x**2)**(3/4)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="giac")
[Out] integrate(-x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a-b\,x^2\right)}^{3/4}\,\left(2\,a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x)
[Out] int(x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)), x)
```

3.1061 $\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=188

$$\frac{56}{81}\sqrt[4]{2-3x^2}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{81}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)+\frac{16}{81}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\,\sqrt[4]{}\right.$$

```
[Out] 56/81*(-3*x^2+2)^(1/4)-16/405*(-3*x^2+2)^(5/4)+2/729*(-3*x^2+2)^(9/4)-16/81
*2^(3/4)*arctan(1+(-6*x^2+4)^(1/4))-16/81*2^(3/4)*arctan(-1+2^(1/4)*(-3*x^2
+2)^(1/4))+8/81*2^(3/4)*ln(-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/
2))-8/81*2^(3/4)*ln(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))
```

Rubi [A]
time = 0.14, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {454, 267, 272, 45, 455, 65, 217, 1179, 642, 1176, 631, 210}

$$-\frac{16}{81}2^{3/4}\text{ArcTan}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{16}{81}2^{3/4}\text{ArcTan}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{56}{81}\sqrt[4]{2-3x^2}+\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (56*(2 - 3*x^2)^(1/4))/81 - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 - 3*x^2)^(9/
4))/729 - (16*2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/81 + (16*2^(3/4)*ArcTa
n[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/81 + (8*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2
- 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81 - (8*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2
 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int \left(-\frac{16x}{27(2-3x^2)^{3/4}} - \frac{4x^3}{9(2-3x^2)^{3/4}} - \frac{x^5}{3(2-3x^2)^{3/4}} + \frac{64x}{27(2-3x^2)^{3/4}(4 \right.$$

$$= -\left(\frac{1}{3}\int \frac{x^5}{(2-3x^2)^{3/4}}\,dx\right) - \frac{4}{9}\int \frac{x^3}{(2-3x^2)^{3/4}}\,dx - \frac{16}{27}\int \frac{x}{(2-3x^2)^{3/4}}\,dx + $$

$$= \frac{32}{81}\sqrt[4]{2-3x^2} - \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{(2-3x)^{3/4}}\,dx, x, x^2\right) - \frac{2}{9}\text{Subst}\left(\int \frac{x}{(2-3x)^{3/4}}\right.$$

$$= \frac{32}{81}\sqrt[4]{2-3x^2} - \frac{1}{6}\text{Subst}\left(\int \left(\frac{4}{9(2-3x)^{3/4}} - \frac{4}{9}\sqrt[4]{2-3x} + \frac{1}{9}(2-3x)^{5/4}\right) dx\right.$$

$$= \frac{56}{81}\sqrt[4]{2-3x^2} - \frac{16}{405}\left(2-3x^2\right)^{5/4} + \frac{2}{729}\left(2-3x^2\right)^{9/4} - \frac{1}{81}\left(32\sqrt{2}\right)\text{Subst}\left(\int\right.$$

$$= \frac{56}{81}\sqrt[4]{2-3x^2} - \frac{16}{405}\left(2-3x^2\right)^{5/4} + \frac{2}{729}\left(2-3x^2\right)^{9/4} - \frac{1}{81}\left(16\sqrt{2}\right)\text{Subst}\left(\int\right.$$

$$= \frac{56}{81}\sqrt[4]{2-3x^2} - \frac{16}{405}\left(2-3x^2\right)^{5/4} + \frac{2}{729}\left(2-3x^2\right)^{9/4} + \frac{8}{81}2^{3/4}\log\left(\sqrt{2} - 2^{3/4}\right.$$

$$= \frac{56}{81}\sqrt[4]{2-3x^2} - \frac{16}{405}\left(2-3x^2\right)^{5/4} + \frac{2}{729}\left(2-3x^2\right)^{9/4} - \frac{16}{81}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4}\right.$$

Mathematica [A]
time = 0.18, size = 110, normalized size = 0.59

$$\frac{2\sqrt[4]{2-3x^2}\,(1136+156x^2+45x^4) + 720\ 2^{3/4}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) - 720\ 2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{3645}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(1/4)*(1136 + 156*x^2 + 45*x^4) + 720*2^(3/4)*ArcTan[(Sqrt[2
] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] - 720*2^(3/4)*ArcTanh[(2*
(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/3645
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.38, size = 212, normalized size = 1.13

method	result
trager	$\left(\frac{2}{81}x^4+\frac{104}{1215}x^2+\frac{2272}{3645}\right)\left(-3x^2+2\right)^{\frac{1}{4}}-\frac{16\,\mathrm{RootOf}\left(\_Z^4+2\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^4+2\right)\left(-3x^2+2\right)^{\frac{3}{4}}+2\,\mathrm{RootOf}\left(\_Z^4+2\right)^2 \cdots}{3\cdots}\right.}{81\cdots}$
risch	$-\frac{2\left(45x^4+156x^2+1136\right)\left(3x^2-2\right)}{3645\left(-3x^2+2\right)^{\frac{3}{4}}}-\frac{\left(\frac{16\,\mathrm{RootOf}\left(\_Z^4+2\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^4+2\right)^3\left(-27x^6+54x^4-36x^2+8\right)^{\frac{3}{4}}-6\,\mathrm{RootOf}\left(\_Z^4+2\right)^2\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] (2/81*x^4+104/1215*x^2+2272/3645)*(-3*x^2+2)^(1/4)-16/81*RootOf(_Z^4+2)*ln(
(2*RootOf(_Z^4+2)*(-3*x^2+2)^(3/4)+2*RootOf(_Z^4+2)^2*(-3*x^2+2)^(1/2)+2*Ro
otOf(_Z^4+2)^3*(-3*x^2+2)^(1/4)-3*x^2)/(3*x^2-4))+16/81*RootOf(_Z^2+RootOf(
_Z^4+2)^2)*ln(-(2*(-3*x^2+2)^(3/4)*RootOf(_Z^2+RootOf(_Z^4+2)^2)+2*RootOf(_
Z^4+2)^2*(-3*x^2+2)^(1/2)-2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*
(-3*x^2+2)^(1/4)+3*x^2)/(3*x^2-4))
```

Maxima [A]
time = 0.48, size = 151, normalized size = 0.80

$$\frac{2}{729}\left(-3x^2+2\right)^{\frac{9}{4}}-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{16}{405}\left(-3x^2+2\right)^{\frac{5}{4}}+\frac{56}{81}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] 2/729*(-3*x^2 + 2)^(9/4) - 16/81*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-
3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2
 + 2)^(1/4))) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqr
```

```
t(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + s
qrt(-3*x^2 + 2)) - 16/405*(-3*x^2 + 2)^(5/4) + 56/81*(-3*x^2 + 2)^(1/4)
```

Fricas [**A**]
time = 0.92, size = 198, normalized size = 1.05

$$\frac{32}{81} \cdot 2^{\frac{3}{4}} \arctan\left(2^{\frac{1}{4}} \sqrt{2^{\frac{3}{4}} (-3 x^2 + 2)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3 x^2 + 2}} - 2^{\frac{1}{4}} (-3 x^2 + 2)^{\frac{1}{4}} - 1\right) + \frac{32}{81} \cdot 2^{\frac{3}{4}} \arctan\left(2^{\frac{1}{4}} \sqrt{-2^{\frac{3}{4}} (-3 x^2 + 2)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3 x^2 + 2}} - 2^{\frac{1}{4}} (-3 x^2 + 2)^{\frac{1}{4}} + 1\right) - \frac{8}{81} \cdot 2^{\frac{3}{4}} \log\left(2^{\frac{3}{4}} (-3 x^2 + 2)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3 x^2 + 2}\right) + \frac{8}{81} \cdot 2^{\frac{3}{4}} \log\left(-2^{\frac{3}{4}} (-3 x^2 + 2)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3 x^2 + 2}\right) + \frac{2}{3645} (45 x^4 + 156 x^2 + 1136) (-3 x^2 + 2)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 32/81*2^(3/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 32/81*2^(3/4)*arctan(2^
(1/4)*sqrt(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1
/4)*(-3*x^2 + 2)^(1/4) + 1) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4)
 + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/3645*(45*x^4 + 156*x^2 + 1136)*(-3*x^2 +
 2)^(1/4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7}{3x^2 (2 - 3x^2)^{\frac{3}{4}} - 4 (2 - 3x^2)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**7/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [**A**]
time = 1.67, size = 160, normalized size = 0.85

$$\frac{2}{729} (3 x^2 - 2)^2 (-3 x^2 + 2)^{\frac{1}{4}} - \frac{16}{81} \cdot 2^{\frac{3}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{1}{4}} \left(2^{\frac{3}{4}} + 2 (-3 x^2 + 2)^{\frac{1}{4}}\right)\right) - \frac{16}{81} \cdot 2^{\frac{3}{4}} \arctan\left(-\frac{1}{2} \cdot 2^{\frac{1}{4}} \left(2^{\frac{3}{4}} - 2 (-3 x^2 + 2)^{\frac{1}{4}}\right)\right) - \frac{8}{81} \cdot 2^{\frac{3}{4}} \log\left(2^{\frac{3}{4}} (-3 x^2 + 2)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3 x^2 + 2}\right) + \frac{8}{81} \cdot 2^{\frac{3}{4}} \log\left(-2^{\frac{3}{4}} (-3 x^2 + 2)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3 x^2 + 2}\right) - \frac{16}{405} (-3 x^2 + 2)^{\frac{5}{4}} + \frac{56}{81} (-3 x^2 + 2)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] 2/729*(3*x^2 - 2)^2*(-3*x^2 + 2)^(1/4) - 16/81*2^(3/4)*arctan(1/2*2^(1/4)*(
2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/
4) - 2*(-3*x^2 + 2)^(1/4))) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4)
 + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/405*(-3*x^2 + 2)^(5/4) + 56/81*(-3*x^2
+ 2)^(1/4)
```

Mupad [B]
time = 0.49, size = 82, normalized size = 0.44

$$\frac{56\,{\left(2-3\,x^2\right)}^{1/4}}{81}-\frac{16\,{\left(2-3\,x^2\right)}^{5/4}}{405}+\frac{2\,{\left(2-3\,x^2\right)}^{9/4}}{729}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{16}{81}-\frac{16}{81}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{16}{81}+\frac{16}{81}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^7/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] (56*(2 - 3*x^2)^(1/4))/81 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1
i/2))*(16/81 - 16i/81) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2
))*(16/81 + 16i/81) - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 - 3*x^2)^(9/4))/72
9
```

3.1062 $\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=173

$$\frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}\left(2-3x^2\right)^{5/4}-\frac{4}{27}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)+\frac{4}{27}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{2}{27}2^{3/4}$$

```
[Out] 4/9*(-3*x^2+2)^(1/4)-2/135*(-3*x^2+2)^(5/4)-4/27*2^(3/4)*arctan(1+(-6*x^2+4
)^(1/4))-4/27*2^(3/4)*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4))+2/27*2^(3/4)*ln(-
2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))-2/27*2^(3/4)*ln(2^(3/4)*
(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))
```

Rubi [A]
time = 0.12, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {454, 267, 272, 45, 455, 65, 217, 1179, 642, 1176, 631, 210}

$$-\frac{4}{27}2^{3/4}\text{ArcTan}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{4}{27}2^{3/4}\text{ArcTan}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)-\frac{2}{135}\left(2-3x^2\right)^{5/4}+\frac{4}{9}\sqrt[4]{2-3x^2}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*(2 - 3*x^2)^(1/4))/9 - (2*(2 - 3*x^2)^(5/4))/135 - (4*2^(3/4)*ArcTan[1 +
 (4 - 6*x^2)^(1/4)])/27 + (4*2^(3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])
/27 + (2*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]
)/27 - (2*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]
])/27
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx &= \int \left(-\frac{4x}{9(2-3x^2)^{3/4}}-\frac{x^3}{3(2-3x^2)^{3/4}}+\frac{16x}{9(2-3x^2)^{3/4}(4-3x^2)}\right)dx\\
&= -\left(\frac{1}{3}\int \frac{x^3}{(2-3x^2)^{3/4}}\,dx\right)-\frac{4}{9}\int \frac{x}{(2-3x^2)^{3/4}}\,dx+\frac{16}{9}\int \frac{x}{(2-3x^2)^{3/4}(4-3
\end{aligned}$$

$$\begin{aligned}
&= \frac{8}{27}\sqrt[4]{2-3x^2}-\frac{1}{6}\mathrm{Subst}\left(\int \frac{x}{(2-3x)^{3/4}}\,dx,x,x^2\right)+\frac{8}{9}\mathrm{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(
\end{aligned}$$

$$\begin{aligned}
&= \frac{8}{27}\sqrt[4]{2-3x^2}-\frac{1}{6}\mathrm{Subst}\left(\int \left(\frac{2}{3(2-3x)^{3/4}}-\frac{1}{3}\sqrt[4]{2-3x}\right)dx,x,x^2\right)-\frac{32}{27}\mathrm{Su}
\end{aligned}$$

$$\begin{aligned}
&= \frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}(2-3x^2)^{5/4}-\frac{1}{27}\left(8\sqrt{2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx,x,\sqrt[4]{2-
\end{aligned}$$

$$\begin{aligned}
&= \frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}(2-3x^2)^{5/4}-\frac{1}{27}\left(4\sqrt{2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x+x^2}\,dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}(2-3x^2)^{5/4}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3
\end{aligned}$$

$$\begin{aligned}
&= \frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}(2-3x^2)^{5/4}-\frac{4}{27}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)+\frac{4}{27}2^{3/4}\mathrm{ta}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 105, normalized size = 0.61

$$\frac{1}{135}\left(2\sqrt[4]{2-3x^2}\left(28+3x^2\right)+20\ 2^{3/4}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-20\ 2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(1/4)*(28 + 3*x^2) + 20*2^(3/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3
*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] - 20*2^(3/4)*ArcTanh[(2*(4 - 6*x^2)^(1/
4))/(2 + Sqrt[4 - 6*x^2])])/135
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.92, size = 208, normalized size = 1.20

method	result
trager	$\left(\frac{2x^2}{45}+\frac{56}{135}\right)(-3x^2+2)^{\frac{1}{4}}+\frac{4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(-\frac{2\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)\right.}{}\right.}{}$
risch	$-\frac{2(3x^2+28)(3x^2-2)}{135(-3x^2+2)^{\frac{3}{4}}}-\frac{\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\mathrm{RootOf}\left(\_Z^4+2\right)^2(-}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] (2/45*x^2+56/135)*(-3*x^2+2)^(1/4)+4/27*RootOf(_Z^2+RootOf(_Z^4+2)^2)*ln(-(
2*(-3*x^2+2)^(3/4)*RootOf(_Z^2+RootOf(_Z^4+2)^2)+2*RootOf(_Z^4+2)^2*(-3*x^2
+2)^(1/2)-2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(-3*x^2+2)^(1/4)
+3*x^2)/(3*x^2-4))+4/27*RootOf(_Z^4+2)*ln(-(2*RootOf(_Z^4+2)^3*(-3*x^2+2)^(
1/4)-2*RootOf(_Z^4+2)^2*(-3*x^2+2)^(1/2)+2*RootOf(_Z^4+2)*(-3*x^2+2)^(3/4)+
3*x^2)/(3*x^2-4))
```

Maxima [A]
time = 0.47, size = 140, normalized size = 0.81

$$-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{2}{135}\left(-3x^2+2\right)^{\frac{5}{4}}+\frac{4}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 4/27*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 2/27*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(3/4
```

```
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2/135*(-3
*x^2 + 2)^(5/4) + 4/9*(-3*x^2 + 2)^(1/4)
```

Fricas [A]
time = 1.09, size = 193, normalized size = 1.12

$$\frac{8}{27}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}-2^{\frac{1}{4}}(-3\,x^2+2)^{\frac{1}{4}}-1\right)+\frac{8}{27}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}-2^{\frac{1}{4}}(-3\,x^2+2)^{\frac{1}{4}}+1\right)-\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{135}\,(3\,x^2+28)(-3\,x^2+2)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 8/27*2^(3/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqr
t(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 8/27*2^(3/4)*arctan(2^(1
/4)*sqrt(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1/4
)*(-3*x^2 + 2)^(1/4) + 1) - 2/27*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) + 2/135*(3*x^2 + 28)*(-3*x^2 + 2)^(1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^5}{3x^2\,(2-3x^2)^{\frac{3}{4}}-4\,(2-3x^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**5/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [A]
time = 1.96, size = 140, normalized size = 0.81

$$-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\,(-3\,x^2+2)^{\frac{1}{4}}\right)\right)-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\,(-3\,x^2+2)^{\frac{1}{4}}\right)\right)-\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{2}{135}\,(-3\,x^2+2)^{\frac{5}{4}}+\frac{4}{9}\,(-3\,x^2+2)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 4/27*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 2/27*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2/135*(-3
*x^2 + 2)^(5/4) + 4/9*(-3*x^2 + 2)^(1/4)
```

Mupad [B]
time = 0.09, size = 71, normalized size = 0.41

$$\frac{4\,{\left(2-3\,x^2\right)}^{1/4}}{9}-\frac{2\,{\left(2-3\,x^2\right)}^{5/4}}{135}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{4}{27}-\frac{4}{27}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{4}{27}+\frac{4}{27}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] (4*(2 - 3*x^2)^(1/4))/9 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/
2))*(4/27 - 4i/27) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(
4/27 + 4i/27) - (2*(2 - 3*x^2)^(5/4))/135
```

3.1063 $\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=158

$$\frac{2}{9}\sqrt[4]{2-3x^2}-\frac{1}{9}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)+\frac{1}{9}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}\right.}{9\sqrt[4]{2}}$$

```
[Out] 2/9*(-3*x^2+2)^(1/4)-1/9*2^(3/4)*arctan(1+(-6*x^2+4)^(1/4))-1/9*2^(3/4)*arc
tan(-1+2^(1/4)*(-3*x^2+2)^(1/4))+1/18*2^(3/4)*ln(-2^(3/4)*(-3*x^2+2)^(1/4)+
2^(1/2)+(-3*x^2+2)^(1/2))-1/18*2^(3/4)*ln(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+
(-3*x^2+2)^(1/2))
```

Rubi [A]
time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {454, 267, 455, 65, 217, 1179, 642, 1176, 631, 210}

$$-\frac{1}{9}2^{3/4}\text{ArcTan}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{1}{9}2^{3/4}\text{ArcTan}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(1/4))/9 - (2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/9 + (2^(3
/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/9 + Log[Sqrt[2] - 2^(3/4)*(2 - 3
*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x
^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^3}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx = \int \left(-\frac{x}{3\left(2-3x^2\right)^{3/4}}+\frac{4x}{3\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\right)dx$$

$$= -\left(\frac{1}{3}\int \frac{x}{\left(2-3x^2\right)^{3/4}}\,dx\right)+\frac{4}{3}\int \frac{x}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$$

$$= \frac{2}{9}\sqrt[4]{2-3x^2}+\frac{2}{3}\mathrm{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(4-3x)}\,dx, x, x^2\right)$$

$$= \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{8}{9}\mathrm{Subst}\left(\int \frac{1}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)$$

$$= \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{1}{9}\left(2\sqrt{2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)-\frac{1}{9}\left(2\sqrt{2}\right)$$

$$= \frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\mathrm{Subst}\left(\int \frac{2^{3/4}+2x}{-\sqrt{2}-2^{3/4}x-x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{9\sqrt[4]{2}}+\frac{\mathrm{Subst}\left(\int \frac{2^{3/}}{-\sqrt{2}+}\right.}{}$$

$$= \frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{}\right.}{}$$

$$= \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{1}{9}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)+\frac{1}{9}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x\right.$$

Mathematica [A]
time = 0.11, size = 97, normalized size = 0.61

$$\frac{1}{9}\left(2\sqrt[4]{2-3x^2}+2^{3/4}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(1/4) + 2^(3/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*
(2 - 3*x^2)^(1/4))] - 2^(3/4)*ArcTanh[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6
*x^2])])/9
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.92, size = 202, normalized size = 1.28

method	result
trager	$\frac{2(-3x^2+2)^{\frac{1}{4}}}{9}+\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(-\frac{2(-3x^2+2)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)+2\,\mathrm{RootOf}\left(\_Z^4+2 \right.}{}\right.}{9}$
risch	$-\frac{2(3x^2-2)}{9(-3x^2+2)^{\frac{3}{4}}}-\frac{\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\mathrm{RootOf}\left(\_Z^4+2\right)^2\left(-27x^6+54\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(-3*x^2+2)^(1/4)+1/9*RootOf(_Z^2+RootOf(_Z^4+2)^2)*ln(-(2*(-3*x^2+2)^(3
/4)*RootOf(_Z^2+RootOf(_Z^4+2)^2)+2*RootOf(_Z^4+2)^2*(-3*x^2+2)^(1/2)-2*Roo
tOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(-3*x^2+2)^(1/4)+3*x^2)/(3*x^2-
4))+1/9*RootOf(_Z^4+2)*ln(-(2*RootOf(_Z^4+2)^3*(-3*x^2+2)^(1/4)-2*RootOf(_Z
^4+2)^2*(-3*x^2+2)^(1/2)+2*RootOf(_Z^4+2)*(-3*x^2+2)^(3/4)+3*x^2)/(3*x^2-4)
)
```

Maxima [A]
time = 0.49, size = 129, normalized size = 0.82

$$-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{9}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/9*2^(
3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/18*2^(3/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2
 + 2)^(1/4)
```

Fricas [A]
time = 0.70, size = 186, normalized size = 1.18

$$\frac{2}{9}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}-2^{\frac{1}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{2}{9}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}-2^{\frac{1}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+1\right)-\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{9}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/9*2^(3/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 2/9*2^(3/4)*arctan(2^(1/4
)*sqrt(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1/4)*
(-3*x^2 + 2)^(1/4) + 1) - 1/18*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqr
t(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2 + 2)^(1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**3/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [A]
time = 1.18, size = 129, normalized size = 0.82

$$-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/9*2^(
3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/18*2^(3/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2
 + 2)^(1/4)
```

Mupad [B]
time = 0.09, size = 60, normalized size = 0.38

$$\frac{2\left(2-3x^2\right)^{1/4}}{9}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{9}-\frac{1}{9}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{9}+\frac{1}{9}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] (2*(2 - 3*x^2)^(1/4))/9 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/
2))*(1/9 - 1i/9) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/
9 + 1i/9)
```

3.1064 $\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=143

$$-\frac{\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{6\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{6\sqrt[4]{2}}+\frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{12\sqrt[4]{2}}-\frac{\log\left(\cdots\right.}{}$$

```
[Out] -1/12*2^(3/4)*arctan(1+(-6*x^2+4)^(1/4))-1/12*2^(3/4)*arctan(-1+2^(1/4)*(-3
*x^2+2)^(1/4))+1/24*2^(3/4)*ln(-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)
^(1/2))-1/24*2^(3/4)*ln(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))
```

Rubi [A]
time = 0.08, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {455, 65, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(\sqrt[4]{4-6x^2}+1\right)}{6\sqrt[4]{2}}+\frac{\text{ArcTan}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{6\sqrt[4]{2}}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/6*ArcTan[1 + (4 - 6*x^2)^(1/4)]/2^(1/4) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)
^(1/4)]/(6*2^(1/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*
x^2]]/(12*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x
^2]]/(12*2^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(4-3x)}\,dx, x, x^2\right)$$
$$= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)}{3\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right.}{3\sqrt{2}}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x+x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{6\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+2^{3/4}x+x^2}\,dx, x,\right.}{6\sqrt{2}}$$
$$= \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{12\sqrt[4]{2}} - \frac{\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{\right.}{12\sqrt[4]{2}}$$
$$= -\frac{\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{6\sqrt[4]{2}} + \frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{6\sqrt[4]{2}} + \frac{\log\left(\sqrt{2}-2^{3/4}\right.}{}$$

Mathematica [A]
time = 0.10, size = 78, normalized size = 0.55

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) - \tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] - ArcTanh[
(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(6*2^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.14, size = 190, normalized size = 1.33

method	result
trager	$-\frac{\text{RootOf}\left(\_Z^4+2\right)\ln\left(\frac{2\,\text{RootOf}\left(\_Z^4+2\right)\left(-3x^2+2\right)^{\frac{3}{4}}+2\,\text{RootOf}\left(\_Z^4+2\right)^2\sqrt{-3x^2+2}+2\,\text{RootOf}\left(\_Z^4+2\right)^3\left(-3x^2+2\right)^{\frac{1}{4}}-3}{3x^2-4}\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*RootOf(_Z^4+2)*ln((2*RootOf(_Z^4+2)*(-3*x^2+2)^(3/4)+2*RootOf(_Z^4+2)
^2*(-3*x^2+2)^(1/2)+2*RootOf(_Z^4+2)^3*(-3*x^2+2)^(1/4)-3*x^2)/(3*x^2-4))-1
/12*RootOf(_Z^2+RootOf(_Z^4+2)^2)*ln(-(2*RootOf(_Z^4+2)^2*(-3*x^2+2)^(1/2)+
2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(-3*x^2+2)^(1/4)-2*(-3*x^2
+2)^(3/4)*RootOf(_Z^2+RootOf(_Z^4+2)^2)+3*x^2)/(3*x^2-4))
```

Maxima [A]
time = 0.48, size = 118, normalized size = 0.83

$$-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/12*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/24*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/24*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(104) = 208$.
time = 0.71, size = 230, normalized size = 1.61

$$\frac{1}{24}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{24}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{16}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)-\frac{1}{96}\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)+\frac{1}{96}\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/24*8^(3/4)*sqrt(2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) - 1) + 1/24*8^(3/4)*sqrt(2)*arctan(1/16*8^(1/4)*sqrt(2)*sqrt(-
16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) -
 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) - 1/96*8^(3/4)*sqrt(2)*log(16*
8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) + 1/
96*8^(3/4)*sqrt(2)*log(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2)
+ 64*sqrt(-3*x^2 + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [A]
time = 1.22, size = 118, normalized size = 0.83

$$-\frac{1}{12} \cdot 2^{\frac{3}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{1}{4}} \left(2^{\frac{3}{4}} + 2 \left(-3 x^{2} + 2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{12} \cdot 2^{\frac{3}{4}} \arctan\left(-\frac{1}{2} \cdot 2^{\frac{1}{4}} \left(2^{\frac{3}{4}} - 2 \left(-3 x^{2} + 2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24} \cdot 2^{\frac{3}{4}} \log\left(2^{\frac{3}{4}} \left(-3 x^{2} + 2\right)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3 x^{2} + 2}\right) + \frac{1}{24} \cdot 2^{\frac{3}{4}} \log\left(-2^{\frac{3}{4}} \left(-3 x^{2} + 2\right)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3 x^{2} + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/12*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/24*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/24*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

Mupad [B]
time = 0.49, size = 49, normalized size = 0.34

$$2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/12 + 1i/12) - 2^(
3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/2))*(1/12 - 1i/12)
```

3.1065 $\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=197

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\ 2^{3/4}}-\frac{\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\,\sqrt[4]{2-3x^2}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\ 2^{3/4}}+\frac{\log}{}$$

```
[Out] -1/8*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1/4)-1/16*2^(3/4)*arctan(1+(-6
*x^2+4)^(1/4))-1/16*2^(3/4)*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4))-1/8*arctanh
(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1/4)+1/32*2^(3/4)*ln(-2^(3/4)*(-3*x^2+2)^
(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))-1/32*2^(3/4)*ln(2^(3/4)*(-3*x^2+2)^(1/4)+2^
(1/2)+(-3*x^2+2)^(1/2))
```

Rubi [A]
time = 0.13, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {454, 272, 65, 218, 212, 209, 455, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\ 2^{3/4}}-\frac{\text{ArcTan}\left(\sqrt[4]{4-6x^2}+1\right)}{8\sqrt[4]{2}}+\frac{\text{ArcTan}\left(1-\sqrt[4]{2}\,\sqrt[4]{2-3x^2}\right)}{8\sqrt[4]{2}}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/4*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/2^(3/4) - ArcTan[1 + (4 - 6*x^2)^(1/
4)]/(8*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)]/(8*2^(1/4)) - ArcTa
nh[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^
2)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2
)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int \left(\frac{1}{4x(2-3x^2)^{3/4}} - \frac{3x}{4(2-3x^2)^{3/4}(-4+3x^2)}\right)dx$$
$$= \frac{1}{4}\int \frac{1}{x(2-3x^2)^{3/4}}\,dx - \frac{3}{4}\int \frac{x}{(2-3x^2)^{3/4}(-4+3x^2)}\,dx$$
$$= \frac{1}{8}\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}x}\,dx, x, x^2\right) - \frac{3}{8}\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(-4+3x)}\,dx$$
$$= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{\frac{2}{3}-\frac{x^4}{3}}\,dx, x, \sqrt[4]{2-3x^2}\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-2-x^4}\,dx, x,$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx, x, \sqrt[4]{2-3x^2}\right.}{4\sqrt{2}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x+x^2}\right.}{8\sqrt{2}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} + \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3\right.}{16\sqrt[4]{2}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} - \frac{\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{8\sqrt[4]{2}} + \frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-\right.}{8\sqrt[4]{2}}$$

Mathematica [A]
time = 0.17, size = 121, normalized size = 0.61

$$\frac{-2\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-2\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{8\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-2*ArcTan[(1 - (3*x^2)/2)^(1/4)] + Sqrt[2]*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^
2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] - 2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] - Sqrt[
2]*ArcTanh[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(8*2^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 9.98, size = 562, normalized size = 2.85

method	result

trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{3\operatorname{RootOf}\left(\_Z^4-2\right)^2\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-2\right)^2\right)x^2-4\operatorname{RootOf}\left(\_Z^4-2\right)^2\operatorname{Root}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln(-(3*RootOf(_Z^4-2)^2*RootOf(_Z^2+Roo
tOf(_Z^4-2)^2)*x^2-4*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)+4*RootO
f(_Z^2+RootOf(_Z^4-2)^2)*(-3*x^2+2)^(1/2)-4*RootOf(_Z^4-2)^2*(-3*x^2+2)^(1/
4)+4*(-3*x^2+2)^(3/4))/x^2)-1/16*RootOf(_Z^4-2)*ln((3*RootOf(_Z^4-2)^3*x^2-
4*RootOf(_Z^4-2)^3-4*RootOf(_Z^4-2)^2*(-3*x^2+2)^(1/4)-4*(-3*x^2+2)^(1/2)*R
ootOf(_Z^4-2)-4*(-3*x^2+2)^(3/4))/x^2)+1/32*ln(-(4*(-3*x^2+2)^(1/2)*RootOf(
_Z^4-2)^3-4*RootOf(_Z^4-2)^2*(-3*x^2+2)^(1/4)-3*RootOf(_Z^4-2)*x^2-4*(-3*x^
2+2)^(3/4)+4*RootOf(_Z^4-2))/(3*x^2-4))*RootOf(_Z^4-2)^3+1/32*ln(-(4*(-3*x^
2+2)^(1/2)*RootOf(_Z^4-2)^3-4*RootOf(_Z^4-2)^2*(-3*x^2+2)^(1/4)-3*RootOf(_Z
^4-2)*x^2-4*(-3*x^2+2)^(3/4)+4*RootOf(_Z^4-2))/(3*x^2-4))*RootOf(_Z^4-2)^2*
RootOf(_Z^2+RootOf(_Z^4-2)^2)-1/16*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4
-2)^2)*ln((2*RootOf(_Z^4-2)^2*(-3*x^2+2)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2
)-2*(-3*x^2+2)^(1/2)*RootOf(_Z^4-2)^3-4*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootO
f(_Z^4-2)*(-3*x^2+2)^(1/4)+3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2+3*RootOf(_Z^
4-2)*x^2+4*(-3*x^2+2)^(3/4))/(3*x^2-4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(144) = 288.
time = 2.59, size = 315, normalized size = 1.60

$$\frac{1}{32}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{32}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{16}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)-\frac{1}{128}\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)+\frac{1}{128}\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)+\frac{1}{16}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{\sqrt{2}+\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)-\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(8^{\frac{3}{4}}+4\left(-3x^2+2\right)^{\frac{1}{4}}\right)+\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(-8^{\frac{3}{4}}+4\left(-3x^2+2\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/32*8^(3/4)*sqrt(2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^
```

```
2 + 2)^(1/4) - 1) + 1/32*8^(3/4)*sqrt(2)*arctan(1/16*8^(1/4)*sqrt(2)*sqrt(-
16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) -
 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) - 1/128*8^(3/4)*sqrt(2)*log(16
*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) + 1
/128*8^(3/4)*sqrt(2)*log(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2
) + 64*sqrt(-3*x^2 + 2)) + 1/16*8^(3/4)*arctan(1/2*8^(1/4)*sqrt(sqrt(2) + s
qrt(-3*x^2 + 2)) - 1/2*8^(1/4)*(-3*x^2 + 2)^(1/4)) - 1/64*8^(3/4)*log(8^(3/
4) + 4*(-3*x^2 + 2)^(1/4)) + 1/64*8^(3/4)*log(-8^(3/4) + 4*(-3*x^2 + 2)^(1/
4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^3\left(2-3x^2\right)^{\frac{3}{4}} - 4x\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
[Out] -Integral(1/(3*x**3*(2 - 3*x**2)**(3/4) - 4*x*(2 - 3*x**2)**(3/4)), x)
```

Giac [A]
time = 0.96, size = 210, normalized size = 1.07

$$-\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{32}\cdot 4^{\frac{1}{8}}\sqrt{2}\log\left(4^{\frac{1}{8}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right) + \frac{1}{32}\cdot 4^{\frac{1}{8}}\sqrt{2}\log\left(-4^{\frac{1}{8}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right) - \frac{1}{8}\cdot 4^{\frac{1}{8}}\arctan\left(\frac{1}{4}\cdot 4^{\frac{7}{8}}\left(-3x^2+2\right)^{\frac{1}{4}}\right) - \frac{1}{16}\cdot 4^{\frac{1}{8}}\log\left(\left(-3x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right) + \frac{1}{16}\cdot 4^{\frac{1}{8}}\log\left(-\left(-3x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
[Out] -1/16*4^(1/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2) + 2*(-3*x
^2 + 2)^(1/4))) - 1/16*4^(1/8)*sqrt(2)*arctan(-1/8*4^(7/8)*sqrt(2)*(4^(1/8)
*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) - 1/32*4^(1/8)*sqrt(2)*log(4^(1/8)*sqrt(2
)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) + 1/32*4^(1/8)*sqrt(2)*l
og(-4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) - 1/8*
4^(1/8)*arctan(1/4*4^(7/8)*(-3*x^2 + 2)^(1/4)) - 1/16*4^(1/8)*log((-3*x^2 +
 2)^(1/4) + 4^(1/8)) + 1/16*4^(1/8)*log(-(-3*x^2 + 2)^(1/4) + 4^(1/8))
```

Mupad [B]
time = 0.12, size = 91, normalized size = 0.46

$$-\frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}}{2}\right)}{8}+\frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1{}\mathrm{i}}{2}\right)\,1{}\mathrm{i}}{8}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{16}-\frac{1}{16}{}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{16}+\frac{1}{16}{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
[Out] (2^(1/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*1i)/8 - (2^(1/4)*atan((2^(3
/4)*(2 - 3*x^2)^(1/4))/2))/8 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2
- 1i/2))*(1/16 + 1i/16) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/
2))*(1/16 - 1i/16)
```

3.1066 $\int \frac{1}{x^3 (2-3x^2)^{3/4} (4-3x^2)} \, dx$

Optimal. Leaf size=215

$$-\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}-\frac{3\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{32\sqrt[4]{2}}-\frac{15\tanh^{-}}{}$$

```
[Out] -1/16*(-3*x^2+2)^(1/4)/x^2-15/64*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1/
4)-3/64*2^(3/4)*arctan(1+(-6*x^2+4)^(1/4))-3/64*2^(3/4)*arctan(-1+2^(1/4)*(
-3*x^2+2)^(1/4))-15/64*arctanh(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1/4)+3/128*
2^(3/4)*ln(-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))-3/128*2^(3/4
)*ln(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))
```

Rubi [A]

time = 0.16, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {454, 272, 44, 65, 218, 212, 209, 455, 217, 1179, 642, 1176, 631, 210}

$$-\frac{15\text{ArcTan}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}-\frac{3\text{ArcTan}\left(\sqrt[4]{4-6x^2}+1\right)}{32\sqrt[4]{2}}+\frac{3\text{ArcTan}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{32\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{16x^2}+\frac{3\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{3\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{15\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/16*(2 - 3*x^2)^(1/4)/x^2 - (15*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^
(3/4)) - (3*ArcTan[1 + (4 - 6*x^2)^(1/4)])/(32*2^(1/4)) + (3*ArcTan[1 - 2^(
1/4)*(2 - 3*x^2)^(1/4)])/(32*2^(1/4)) - (15*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/
4)])/(32*2^(3/4)) + (3*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3
*x^2]])/(64*2^(1/4)) - (3*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2
- 3*x^2]])/(64*2^(1/4))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
```

```
tegerQ[m/2])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^3 (2-3x^2)^{3/4} (4-3x^2)} \, dx = \int \left(\frac{1}{4x^3 (2-3x^2)^{3/4}} + \frac{3}{16x (2-3x^2)^{3/4}} - \frac{9x}{16 (2-3x^2)^{3/4} (-4+3x^2)} \right)$$

$$= \frac{3}{16} \int \frac{1}{x (2-3x^2)^{3/4}} \, dx + \frac{1}{4} \int \frac{1}{x^3 (2-3x^2)^{3/4}} \, dx - \frac{9}{16} \int \frac{x}{(2-3x^2)^{3/4} (-}$$

$$= \frac{3}{32} \text{Subst}\left(\int \frac{1}{(2-3x)^{3/4} x} \, dx, x, x^2 \right) + \frac{1}{8} \text{Subst}\left(\int \frac{1}{(2-3x)^{3/4} x^2} \, dx, x, x$$

$$= -\frac{\sqrt[4]{2-3x^2}}{16x^2} - \frac{1}{8} \text{Subst}\left(\int \frac{1}{\frac{2}{3} - \frac{x^4}{3}} \, dx, x, \sqrt[4]{2-3x^2} \right) + \frac{9}{64} \text{Subst}\left(\int \frac{}{(2}\right.$$

$$= -\frac{\sqrt[4]{2-3x^2}}{16x^2} - \frac{3}{16} \text{Subst}\left(\int \frac{1}{\frac{2}{3} - \frac{x^4}{3}} \, dx, x, \sqrt[4]{2-3x^2} \right) - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{2}} \right.}{}$$

$$= -\frac{\sqrt[4]{2-3x^2}}{16x^2} - \frac{3 \tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}} \right)}{16\ 2^{3/4}} - \frac{3 \tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}} \right)}{16\ 2^{3/4}} - \frac{3\text{Sub}}{}$$

$$= -\frac{\sqrt[4]{2-3x^2}}{16x^2} - \frac{15 \tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}} \right)}{32\ 2^{3/4}} - \frac{15 \tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}} \right)}{32\ 2^{3/4}} + \frac{3\,\mathrm{l}}{}$$

$$= -\frac{\sqrt[4]{2-3x^2}}{16x^2} - \frac{15 \tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}} \right)}{32\ 2^{3/4}} - \frac{3 \tan^{-1}\left(1 + \sqrt[4]{4-6x^2} \right)}{32\sqrt[4]{2}} + \frac{3}{}$$

Mathematica [A]
time = 0.25, size = 155, normalized size = 0.72

$$-\frac{4\sqrt[4]{2-3x^2} + 15\sqrt[4]{2}\, x^2 \tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right) - 3\ 2^{3/4} x^2 \tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + 15\sqrt[4]{2}\, x^2 \tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right) + 3\ 2^{3/4} x^2 \tanh^{-1}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{64x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/64*(4*(2 - 3*x^2)^(1/4) + 15*2^(1/4)*x^2*ArcTan[(1 - (3*x^2)/2)^(1/4)] -
 3*2^(3/4)*x^2*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4
))] + 15*2^(1/4)*x^2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] + 3*2^(3/4)*x^2*ArcTanh
[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 25.01, size = 577, normalized size = 2.68

method	result	size
trager	Expression too large to display	577
risch	Expression too large to display	1542

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-3*x^2+2)^(1/4)/x^2+15/128*RootOf(_Z^4-2)*ln(-(3*RootOf(_Z^4-2)^3*x^
2-4*RootOf(_Z^4-2)^3+4*RootOf(_Z^4-2)^2*(-3*x^2+2)^(1/4)-4*(-3*x^2+2)^(1/2)
*RootOf(_Z^4-2)+4*(-3*x^2+2)^(3/4))/x^2)-15/128*RootOf(_Z^2+RootOf(_Z^4-2)^
2)*ln(-(3*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2-4*RootOf(_Z^4-
2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)+4*RootOf(_Z^2+RootOf(_Z^4-2)^2)*(-3*x^2+
2)^(1/2)-4*RootOf(_Z^4-2)^2*(-3*x^2+2)^(1/4)+4*(-3*x^2+2)^(3/4))/x^2)+3/128
*ln(-(4*(-3*x^2+2)^(1/2)*RootOf(_Z^4-2)^3-4*RootOf(_Z^4-2)^2*(-3*x^2+2)^(1/
4)-3*RootOf(_Z^4-2)*x^2-4*(-3*x^2+2)^(3/4)+4*RootOf(_Z^4-2))/(3*x^2-4))*Roo
tOf(_Z^4-2)^3+3/128*ln(-(4*(-3*x^2+2)^(1/2)*RootOf(_Z^4-2)^3-4*RootOf(_Z^4-
2)^2*(-3*x^2+2)^(1/4)-3*RootOf(_Z^4-2)*x^2-4*(-3*x^2+2)^(3/4)+4*RootOf(_Z^4
-2))/(3*x^2-4))*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)-3/64*RootOf(
_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((2*RootOf(_Z^4-2)^2*(-3*x^2+2)^(
1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)-2*(-3*x^2+2)^(1/2)*RootOf(_Z^4-2)^3-4*Ro
otOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)*(-3*x^2+2)^(1/4)+3*RootOf(_Z^2+R
ootOf(_Z^4-2)^2)*x^2+3*RootOf(_Z^4-2)*x^2+4*(-3*x^2+2)^(3/4))/(3*x^2-4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(158) = 316.
time = 1.64, size = 352, normalized size = 1.64

$$\frac{12 \cdot 8^{\frac{3}{4}} \sqrt{2}\, x^2 \arctan\left(\frac{1}{4} \cdot 8^{\frac{1}{4}} \sqrt{2} \sqrt{8^{\frac{3}{4}} \sqrt{2} (-3x^2+2)^{\frac{1}{4}} + 4\sqrt{2} + 4\sqrt{-3x^2+2}} - \frac{1}{2} \cdot 8^{\frac{1}{4}} \sqrt{2} (-3x^2+2)^{\frac{1}{4}} - 1\right) + 12 \cdot 8^{\frac{3}{4}} \sqrt{2}\, x^2 \arctan\left(\frac{1}{16} \cdot 8^{\frac{1}{4}} \sqrt{2} \sqrt{-16 \cdot 8^{\frac{3}{4}} \sqrt{2} (-3x^2+2)^{\frac{1}{4}} + 64\sqrt{2} + 64\sqrt{-3x^2+2}} - \frac{1}{2} \cdot 8^{\frac{1}{4}} \sqrt{2} (-3x^2+2)^{\frac{1}{4}} + 1\right) - 3 \cdot 8^{\frac{3}{4}} \sqrt{2}\, x^2 \log\left(16 \cdot 8^{\frac{3}{4}} \sqrt{2} (-3x^2+2)^{\frac{1}{4}} + 64\sqrt{2} + 64\sqrt{-3x^2+2}\right) + 3 \cdot 8^{\frac{3}{4}} \sqrt{2}\, x^2 \log\left(-16 \cdot 8^{\frac{3}{4}} \sqrt{2} (-3x^2+2)^{\frac{1}{4}} + 64\sqrt{2} + 64\sqrt{-3x^2+2}\right) + 60 \cdot 8^{\frac{1}{4}} x^2 \arctan\left(\frac{1}{2} \cdot 8^{\frac{1}{4}} \sqrt{\sqrt{2} + \sqrt{-3x^2+2}} - \frac{1}{2} \cdot 8^{\frac{1}{4}} (-3x^2+2)^{\frac{1}{4}}\right) - 15 \cdot 8^{\frac{3}{4}} x^2 \log\left(8^{\frac{3}{4}} + 4(-3x^2+2)^{\frac{1}{4}}\right) + 15 \cdot 8^{\frac{3}{4}} x^2 \log\left(-8^{\frac{3}{4}} + 4(-3x^2+2)^{\frac{1}{4}}\right) - 32(-3x^2+2)^{\frac{1}{4}}}{512\, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/512*(12*8^(3/4)*sqrt(2)*x^2*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(
2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(
```

```
2)*(-3*x^2 + 2)^(1/4) - 1) + 12*8^(3/4)*sqrt(2)*x^2*arctan(1/16*8^(1/4)*sqr
t(2)*sqrt(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*
x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) - 3*8^(3/4)*sqrt(2)
*x^2*log(16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^
2 + 2)) + 3*8^(3/4)*sqrt(2)*x^2*log(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)
+ 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) + 60*8^(3/4)*x^2*arctan(1/2*8^(1/4)*sqr
t(sqrt(2) + sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*(-3*x^2 + 2)^(1/4)) - 15*8^(3/4
)*x^2*log(8^(3/4) + 4*(-3*x^2 + 2)^(1/4)) + 15*8^(3/4)*x^2*log(-8^(3/4) + 4
*(-3*x^2 + 2)^(1/4)) - 32*(-3*x^2 + 2)^(1/4))/x^2
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^{5}\left(2-3x^{2}\right)^{\frac{3}{4}}-4x^{3}\left(2-3x^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
[Out] -Integral(1/(3*x**5*(2 - 3*x**2)**(3/4) - 4*x**3*(2 - 3*x**2)**(3/4)), x)
```

Giac [A]

time = 0.84, size = 192, normalized size = 0.89

$$-\frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)+\frac{3}{128}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)-\frac{15}{64}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)-\frac{15}{128}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)+\frac{15}{128}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}-\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)-\frac{\left(-3x^{2}+2\right)^{\frac{1}{4}}}{16x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
[Out] -3/64*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 3/64*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 3/128*2^(3/4
)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 3/128*2^(3
/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 15/64*2
^(1/4)*arctan(1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) - 15/128*2^(1/4)*log(2^(1/4)
+ (-3*x^2 + 2)^(1/4)) + 15/128*2^(1/4)*log(2^(1/4) - (-3*x^2 + 2)^(1/4)) -
1/16*(-3*x^2 + 2)^(1/4)/x^2
```

Mupad [B]

time = 0.56, size = 107, normalized size = 0.50

$$-\frac{\left(2-3x^{2}\right)^{1/4}}{16x^{2}}-\frac{15\,2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,\left(2-3x^{2}\right)^{1/4}}{2}\right)}{64}+\frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,\left(2-3x^{2}\right)^{1/4}\,1\mathrm{i}}{2}\right)\,15\mathrm{i}}{64}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,\left(2-3x^{2}\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)+\frac{(-1)^{1/4}\,2^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,2^{3/4}\,\left(2-3x^{2}\right)^{1/4}}{2}\right)\,3\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
[Out] (2^(1/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*15i)/64 - (15*2^(1/4)*atan(
(2^(3/4)*(2 - 3*x^2)^(1/4))/2))/64 - (2 - 3*x^2)^(1/4)/(16*x^2) - 2^(3/4)*a
tan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(3/64 + 3i/64) + ((-1)^(1/4)*2^
(1/4)*atan(((-1)^(1/4)*2^(3/4)*(2 - 3*x^2)^(1/4))/2)*3i)/32
```

3.1067 $\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=182

$$\frac{80}{567}x\sqrt[4]{2-3x^2}+\frac{2}{63}x^3\sqrt[4]{2-3x^2}+\frac{8\ 2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}-\frac{8\ 2^{3/4}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}$$

```
[Out] 80/567*x*(-3*x^2+2)^(1/4)+2/63*x^3*(-3*x^2+2)^(1/4)+8/81*2^(3/4)*arctan(1/3
*(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-8/8
1*2^(3/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)
*3^(1/2))*3^(1/2)-160/1701*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)
/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^
(1/2))*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {454, 238, 327, 409, 452}

$$-\frac{160\ 2^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{567\sqrt{3}}+\frac{8\ 2^{3/4}\text{ArcTan}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}+\frac{80}{567}\sqrt[4]{2-3x^2}\,x-\frac{8\ 2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}+\frac{2}{63}\sqrt[4]{2-3x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (80*x*(2 - 3*x^2)^(1/4))/567 + (2*x^3*(2 - 3*x^2)^(1/4))/63 + (8*2^(3/4)*Ar
cTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(2
7*Sqrt[3]) - (8*2^(3/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3
]*x*(2 - 3*x^2)^(1/4))])/(27*Sqrt[3]) - (160*2^(3/4)*EllipticF[ArcSin[Sqrt[
3/2]*x]/2, 2])/(567*Sqrt[3])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int \left(-\frac{16}{27(2-3x^2)^{3/4}} - \frac{4x^2}{9(2-3x^2)^{3/4}} - \frac{x^4}{3(2-3x^2)^{3/4}} + \frac{64}{27(2-3x^2)^{3/4}(4\right.$$

$$= -\left(\frac{1}{3}\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx\right) - \frac{4}{9}\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx - \frac{16}{27}\int \frac{1}{(2-3x^2)^{3/4}}\,dx +$$

$$= \frac{8}{81}x\sqrt[4]{2-3x^2} + \frac{2}{63}x^3\sqrt[4]{2-3x^2} - \frac{16\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\,2\right)}{27\sqrt{3}} - \frac{4}{21}$$

$$= \frac{80}{567}x\sqrt[4]{2-3x^2} + \frac{2}{63}x^3\sqrt[4]{2-3x^2} + \frac{8\ 2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}} -$$

$$= \frac{80}{567}x\sqrt[4]{2-3x^2} + \frac{2}{63}x^3\sqrt[4]{2-3x^2} + \frac{8\ 2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 7.03, size = 190, normalized size = 1.04

$$\frac{2}{567}x\left(31\sqrt[4]{2}\,x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+\frac{80-102x^2-27x^4+\frac{1280F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(-4+3x^2)\left(4F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)\right)}}{(2-3x^2)^{3/4}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(31*2^(1/4)*x^2*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + (80
 - 102*x^2 - 27*x^4 + (1280*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4
])/((-4 + 3*x^2)*(4*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*
(2*AppellF1[3/2, 3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + 3*AppellF1[3/2, 7/4,
1, 5/2, (3*x^2)/2, (3*x^2)/4]))))/(2 - 3*x^2)^(3/4)))/567
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(-3x^2+2)^{\frac{3}{4}}(-3x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(1/4)*x^6/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6}{3x^2 (2-3x^2)^{\frac{3}{4}} - 4(2-3x^2)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
[Out] -Integral(x**6/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
[Out] integrate(-x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^6}{(2-3\,x^2)^{3/4}\,(3\,x^2-4)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^6/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
[Out] -int(x^6/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1068 $\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=164

$$\frac{2}{27}x\sqrt[4]{2-3x^2}+\frac{2\,2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{2\,2^{3/4}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-}\right.}{27}$$

```
[Out] 2/27*x*(-3*x^2+2)^(1/4)+2/27*2^(3/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2)
^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-2/27*2^(3/4)*arctanh(1/3*(2^(3/
4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-4/81*2^(3/
4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*
EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {454, 238, 327, 409, 452}

$$-\frac{4\,2^{3/4}F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{27\sqrt{3}}+\frac{2\,2^{3/4}\text{ArcTan}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{2}{27}\sqrt[4]{2-3x^2}\,x-\frac{2\,2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2 - 3*x^2)^(1/4))/27 + (2*2^(3/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 -
3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (2*2^(3/4)*ArcTanh[(2
^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3
]) - (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(27*Sqrt[3])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int \left(-\frac{4}{9(2-3x^2)^{3/4}} - \frac{x^2}{3(2-3x^2)^{3/4}} + \frac{16}{9(2-3x^2)^{3/4}(4-3x^2)}\right)dx$$

$$= -\left(\frac{1}{3}\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx\right) - \frac{4}{9}\int \frac{1}{(2-3x^2)^{3/4}}\,dx + \frac{16}{9}\int \frac{1}{(2-3x^2)^{3/4}(4- \cdots}$$

$$= \frac{2}{27}x\sqrt[4]{2-3x^2} - \frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}} - \frac{4}{27}\int \frac{1}{(2-3x^2)^{3/4}}\,dx + \cdots$$

$$= \frac{2}{27}x\sqrt[4]{2-3x^2} + \frac{2\,2^{3/4}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}} - \frac{2\,2^{3/4}\tanh^{-1}\left(\cdots\right)}{9\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.83, size = 184, normalized size = 1.12

$$\frac{2}{27}x\left(\sqrt[4]{2}\,x^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right) + \frac{2-3x^2+\frac{32F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(-4+3x^2)\left(4F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)\right)}}{(2-3x^2)^{3/4}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*x*(2^(1/4)*x^2*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + (2 - 3
*x^2 + (32*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2)*
(4*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/2,
3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + 3*AppellF1[3/2, 7/4, 1, 5/2, (3*x^2)/2
, (3*x^2)/4]))))/(2 - 3*x^2)^(3/4)))/27
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-3x^2+2)^{\frac{3}{4}}(-3x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] int(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 2)^(1/4)*x^4/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{3x^2(2-3x^2)^{\frac{3}{4}}-4(2-3x^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
[Out] -Integral(x**4/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
[Out] integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^4}{{\left(2-3\,x^2\right)}^{3/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
[Out] -int(x^4/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1069 $\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

```
[Out] 1/18*arctan(1/6*(2-2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)/x/(-3*x^2+2)^(1/4)*3^(
1/2))*2^(3/4)*3^(1/2)-1/18*arctanh(1/6*(2+2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)
/x/(-3*x^2+2)^(1/4)*3^(1/2))*2^(3/4)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {452}

$$\frac{\text{ArcTan}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Mathematica [A]
time = 0.04, size = 109, normalized size = 0.91

$$-\frac{\tan^{-1}\left(\frac{-3\sqrt{2}\,x^2+4\sqrt{2-3x^2}}{2\;2^{3/4}\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)+\tanh^{-1}\left(\frac{2\sqrt{3}\,x\sqrt[4]{4-6x^2}}{3x^2+2\sqrt{4-6x^2}}\right)}{6\sqrt[4]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/6*(ArcTan[(-3*Sqrt[2]*x^2 + 4*Sqrt[2 - 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 -
 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*x*(4 - 6*x^2)^(1/4))/(3*x^2 + 2*Sqrt[4
 - 6*x^2])])/(2^(1/4)*Sqrt[3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 186, normalized size = 1.55

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^4+18\right)\ln\left(\frac{\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)^3+3\,\mathrm{RootOf}\left(\_Z^4+18\right)^2x+9\sqrt{-3x^2+2}\,x-6\,\mathrm{RootOf}\left(\_Z^4+18\right)\left(-3x^2+\right.}{3x^2-4}\right.}{18}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*RootOf(_Z^4+18)*ln(((-3*x^2+2)^(3/4)*RootOf(_Z^4+18)^3+3*RootOf(_Z^4+1
8)^2*x+9*(-3*x^2+2)^(1/2)*x-6*RootOf(_Z^4+18)*(-3*x^2+2)^(1/4))/(3*x^2-4))+
1/18*RootOf(_Z^2+RootOf(_Z^4+18)^2)*ln(-(RootOf(_Z^4+18)^2*RootOf(_Z^2+Root
Of(_Z^4+18)^2)*(-3*x^2+2)^(3/4)+3*RootOf(_Z^4+18)^2*x-9*(-3*x^2+2)^(1/2)*x+
6*(-3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+18)^2))/(3*x^2-4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(89) = 178.
time = 0.96, size = 282, normalized size = 2.35

$$\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}\,x\sqrt{\frac{72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}\,x^2+24\sqrt{-3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-36x}{36x}\right)+\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}\,x\sqrt{-\frac{72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x-18\sqrt{2}\,x^2-24\sqrt{-3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+36x}{36x}\right)-\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}\,x^2+24\sqrt{-3x^2+2}\right)}{x^2}\right)+\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x-18\sqrt{2}\,x^2-24\sqrt{-3x^2+2}\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/216*72^(3/4)*sqrt(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt((72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) - 12*72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 36*x)/x) + 1/216*72^(3/4)*sqrt
(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt(-(72^(3/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x - 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2) - 12*72^(1/4)*sq
rt(2)*(-3*x^2 + 2)^(1/4) + 36*x)/x) - 1/864*72^(3/4)*sqrt(2)*log(96*(72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) + 1/864*72^(3/4)*sqrt(2)*log(-96*(72^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x -
 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{3x^2 (2-3x^2)^{\frac{3}{4}} - 4 (2-3x^2)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{(2-3\,x^2)^{3/4}\,(3\,x^2-4)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] -int(x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1070 $\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=148

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}+\frac{F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2\sqrt[4]{2}\sqrt{3}}$$

```
[Out] 1/24*2^(3/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/
4)*3^(1/2))*3^(1/2)-1/24*2^(3/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^(1
/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+1/12*2^(3/4)*(cos(1/2*arcsin(1/2*x
*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin
(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {409, 238, 452}

$$\frac{F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2\sqrt[4]{2}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(
4*2^(1/4)*Sqrt[3]) - ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x
*(2 - 3*x^2)^(1/4))]/(4*2^(1/4)*Sqrt[3]) + EllipticF[ArcSin[Sqrt[3/2]*x]/2,
 2]/(2*2^(1/4)*Sqrt[3])
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{1}{4}\int \frac{1}{(2-3x^2)^{3/4}}\,dx + \frac{3}{4}\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}} + \frac{F\left(\frac{1}{2}\sin^{-1}\right.}{2\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.43, size = 63, normalized size = 0.43

$$-\frac{\sqrt{x^2}\left(\Pi\left(-i;\sin^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)\middle|-1\right)+\Pi\left(i;\sin^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)\middle|-1\right)\right)}{2\sqrt[4]{2}\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
[Out] -1/2*(Sqrt[x^2]*(EllipticPi[-I, ArcSin[(1 - (3*x^2)/2)^(1/4)], -1] + Ellipt
icPi[I, ArcSin[(1 - (3*x^2)/2)^(1/4)], -1]))/(2^(1/4)*Sqrt[3]*x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(-3x^2+2)^{\frac{3}{4}}(-3x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
[Out] int(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
[Out] integral((-3*x^2 + 2)^(1/4)/(9*x^4 - 18*x^2 + 8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
[Out] -Integral(1/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{{\left(2-3\,x^2\right)}^{3/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
[Out] -int(1/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1071 $\int \frac{1}{x^2(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt[4]{2-3x^2}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}+\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\right.\right.}}{4\sqrt[4]{2}}$$

```
[Out] -1/8*(-3*x^2+2)^(1/4)/x+1/32*2^(3/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2)
^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-1/32*2^(3/4)*arctanh(1/3*(2^(3/
4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+1/8*2^(3/4
)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*E
llipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {454, 331, 238, 409, 452}

$$\frac{\sqrt{3}F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\sqrt[4]{2}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{8x}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/8*(2 - 3*x^2)^(1/4)/x + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^
2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)) - (Sqrt[3]*ArcTanh[(2^(3/4
) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)) +
 (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(1/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \int \left(\frac{1}{4x^2\left(2-3x^2\right)^{3/4}}-\frac{3}{4\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\right)dx\\
&= \frac{1}{4}\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}}\,dx-\frac{3}{4}\int \frac{1}{\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{8x}+2\left(\frac{3}{16}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\right)-\frac{9}{16}\int \frac{x^2}{\left(2-3x^2\right)^{3/4}\left(-4+3x\right.}\\
&= -\frac{\sqrt[4]{2-3x^2}}{8x}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{}}{\sqrt{3}}\right.}{16\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 20.06, size = 37, normalized size = 0.22

$$-\frac{F_1\left(-\frac{1}{2};\frac{3}{4},1;\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{4\ 2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
[Out] -1/4*AppellF1[-1/2, 3/4, 1, 1/2, (3*x^2)/2, (3*x^2)/4]/(2^(3/4)*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-3 x^{2}+2\right)^{\frac{3}{4}} \left(-3 x^{2}+4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
[Out] int(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
[Out] integral((-3*x^2 + 2)^(1/4)/(9*x^6 - 18*x^4 + 8*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3 x^{4} \left(2 - 3 x^{2}\right)^{\frac{3}{4}} - 4 x^{2} \left(2 - 3 x^{2}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
[Out] -Integral(1/(3*x**4*(2 - 3*x**2)**(3/4) - 4*x**2*(2 - 3*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{x^2\,{\left(2-3\,x^2\right)}^{3/4}\,\left(3\,x^2-4\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
[Out] -int(1/(x^2*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1072 $\int \frac{1}{x^4(2-3x^2)^{3/4}(4-3x^2)}\,dx$

Optimal. Leaf size=184

$$-\frac{\sqrt[4]{2-3x^2}}{24x^3}-\frac{\sqrt[4]{2-3x^2}}{4x}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}+\frac{11\sqrt{3}}{}$$

```
[Out] -1/24*(-3*x^2+2)^(1/4)/x^3-1/4*(-3*x^2+2)^(1/4)/x+3/128*2^(3/4)*arctan(1/3*
(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-3/12
8*2^(3/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)
*3^(1/2))*3^(1/2)+11/64*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/co
s(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/
2))*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {454, 331, 238, 409, 452}

$$\frac{11\sqrt{3}F\left(\frac{1}{2}\text{ArcSin}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}+\frac{3\sqrt{3}\,\text{ArcTan}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{4x}-\frac{3\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/24*(2 - 3*x^2)^(1/4)/x^3 - (2 - 3*x^2)^(1/4)/(4*x) + (3*Sqrt[3]*ArcTan[(
2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(64*2^(1
/4)) - (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2
 - 3*x^2)^(1/4))])/(64*2^(1/4)) + (11*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]
/2, 2])/(32*2^(1/4))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 452

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{1}{x^4 (2-3x^2)^{3/4} (4-3x^2)} \, dx = \int \left(\frac{1}{4x^4 (2-3x^2)^{3/4}} + \frac{3}{16x^2 (2-3x^2)^{3/4}} - \frac{9}{16 (2-3x^2)^{3/4} (-4+3x^2)} \right)$$

$$= \frac{3}{16} \int \frac{1}{x^2 (2-3x^2)^{3/4}} \, dx + \frac{1}{4} \int \frac{1}{x^4 (2-3x^2)^{3/4}} \, dx - \frac{9}{16} \int \frac{1}{(2-3x^2)^{3/4} (\cdot}$$

$$= -\frac{\sqrt[4]{2-3x^2}}{24x^3} - \frac{3\sqrt[4]{2-3x^2}}{32x} + 2\left(\frac{9}{64} \int \frac{1}{(2-3x^2)^{3/4}} \, dx \right) + \frac{5}{16} \int \frac{}{x^2 (2-}$$

$$= -\frac{\sqrt[4]{2-3x^2}}{24x^3} - \frac{\sqrt[4]{2-3x^2}}{4x} + \frac{3\sqrt{3} \tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}} \right)}{64\sqrt[4]{2}} - \frac{3\sqrt{3}}{}$$

$$= -\frac{\sqrt[4]{2-3x^2}}{24x^3} - \frac{\sqrt[4]{2-3x^2}}{4x} + \frac{3\sqrt{3} \tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}\,x\sqrt[4]{2-3x^2}} \right)}{64\sqrt[4]{2}} - \frac{3\sqrt{3}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 20.06, size = 37, normalized size = 0.20

$$-\frac{F_1\left(-\frac{3}{2};\frac{3}{4},1;-\frac{1}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\ 2^{3/4}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
[Out] -1/12*AppellF1[-3/2, 3/4, 1, -1/2, (3*x^2)/2, (3*x^2)/4]/(2^(3/4)*x^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-3x^2+2\right)^{\frac{3}{4}}\left(-3x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
[Out] int(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
[Out] integral((-3*x^2 + 2)^(1/4)/(9*x^8 - 18*x^6 + 8*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^6\left(2-3x^2\right)^{\frac{3}{4}}-4x^4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
[Out] -Integral(1/(3*x**6*(2 - 3*x**2)**(3/4) - 4*x**4*(2 - 3*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{x^4\,{\left(2-3\,x^2\right)}^{3/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^4*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
[Out] -int(1/(x^4*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1073 $\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

```
[Out] 1/18*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/18*arctanh(1/2*x*6^(1/
2)/(3*x^2-1)^(1/4))*6^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {453}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [A]
time = 1.77, size = 54, normalized size = 0.89

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)] - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x
^2)^(1/4)])/(3*Sqrt[6])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.41, size = 138, normalized size = 2.26

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\left(3x^2-1\right)^{\frac{3}{4}}-3\sqrt{3x^2-1}\,x+\mathrm{RootOf}\left(\_Z^2-6\right)\left(3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2-2}\right)}{18}+\frac{\mathrm{RootOf}\left(\_Z^2+6\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*RootOf(_Z^2-6)*ln(-(RootOf(_Z^2-6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)*x
+RootOf(_Z^2-6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))+1/18*RootOf(_Z^2+6)*ln((Roo
tOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x-RootOf(_Z^2+6)*(3*x^2-1)^(1
/4)-3*x)/(3*x^2-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(43) = 86.
time = 0.62, size = 104, normalized size = 1.70

$$-\frac{1}{18}\sqrt{6}\arctan\left(\frac{\sqrt{6}\,(3x^2-1)^{\frac{1}{4}}}{3x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\,(3x^2-1)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}\,x^2-4\sqrt{6}\,(3x^2-1)^{\frac{3}{4}}x+12x^2-4}{9x^4-12x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/36*sqrt(6)*log(-(
9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6
)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(3\,x^2-1\right)}^{3/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1074 $\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}$$

```
[Out] 1/18*arctan(1/2*x*6^(1/2)/(-3*x^2-1)^(1/4))*6^(1/2)-1/18*arctanh(1/2*x*6^(1
/2)/(-3*x^2-1)^(1/4))*6^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {453}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [A]
time = 1.80, size = 54, normalized size = 0.89

$$-\frac{-\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/3*(-ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)] + ArcTanh[(Sqrt[3/2]*x)/(-1
 - 3*x^2)^(1/4)])/Sqrt[6]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.41, size = 139, normalized size = 2.28

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\left(-3x^2-1\right)^{\frac{3}{4}}-3\sqrt{-3x^2-1}\,x+\mathrm{RootOf}\left(\_Z^2+6\right)\left(-3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2+2}\right)}{18}+\frac{\mathrm{RootOf}\left(\_Z\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*RootOf(_Z^2+6)*ln(-(RootOf(_Z^2+6)*(-3*x^2-1)^(3/4)-3*(-3*x^2-1)^(1/2
)*x+RootOf(_Z^2+6)*(-3*x^2-1)^(1/4)-3*x)/(3*x^2+2))+1/18*RootOf(_Z^2-6)*ln(
-(RootOf(_Z^2-6)*(-3*x^2-1)^(3/4)-3*(-3*x^2-1)^(1/2)*x-RootOf(_Z^2-6)*(-3*x
^2-1)^(1/4)+3*x)/(3*x^2+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.61, size = 115, normalized size = 1.89

$$-\frac{1}{36}\sqrt{6}\log\left(\frac{\sqrt{6}\,x+2\left(-3x^2-1\right)^{\frac{1}{4}}}{2x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{\sqrt{6}\,x-2\left(-3x^2-1\right)^{\frac{1}{4}}}{2x}\right)-\frac{1}{36}i\sqrt{6}\log\left(\frac{i\sqrt{6}\,x+2\left(-3x^2-1\right)^{\frac{1}{4}}}{2x}\right)+\frac{1}{36}i\sqrt{6}\log\left(\frac{-i\sqrt{6}\,x+2\left(-3x^2-1\right)^{\frac{1}{4}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*log(1/2*(sqrt(6)*x + 2*(-3*x^2 - 1)^(1/4))/x) + 1/36*sqrt(6)*
log(-1/2*(sqrt(6)*x - 2*(-3*x^2 - 1)^(1/4))/x) - 1/36*I*sqrt(6)*log(1/2*(I*
sqrt(6)*x + 2*(-3*x^2 - 1)^(1/4))/x) + 1/36*I*sqrt(6)*log(1/2*(-I*sqrt(6)*x
 + 2*(-3*x^2 - 1)^(1/4))/x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{3x^2\left(-3x^2-1\right)^{\frac{3}{4}}+2\left(-3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2)/(-3*x**2-1)**(3/4),x)
```

```
[Out] -Integral(x**2/(3*x**2*(-3*x**2 - 1)**(3/4) + 2*(-3*x**2 - 1)**(3/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2}{\left(-3\,x^2-1\right)^{3/4}\,\left(3\,x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((- 3*x^2 - 1)^(3/4)*(3*x^2 + 2)),x)
```

```
[Out] -int(x^2/((- 3*x^2 - 1)^(3/4)*(3*x^2 + 2)), x)
```

3.1075 $\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=72

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}\,b^{3/2}}$$

```
[Out] 1/2*arctan(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))/b^(3/2)*2^(1/2)-1/2*arcta
nh(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))/b^(3/2)*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{453\}$

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTan
h[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}\,b^{3/2}}$$

Mathematica [A]
time = 2.02, size = 62, normalized size = 0.86

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

```
[Out] (ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))] - ArcTanh[(Sqrt[b]*x)/(Sq
rt[2]*(-1 + b*x^2)^(1/4))])/(Sqrt[2]*b^(3/2))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(b\,x^{2}-2\right)\left(b\,x^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(55) = 110$.
time = 0.58, size = 275, normalized size = 3.82

$$\left[-\frac{2\,\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,(bx^2-1)^{\frac{1}{4}}}{\sqrt{b}\,x}\right)-\sqrt{2}\,\sqrt{b}\,\log\left(-\frac{b^2x^4-2\,\sqrt{2}\,(bx^2-1)^{\frac{1}{4}}b^{\frac{3}{2}}x^3+4\,\sqrt{bx^2-1}\,bx^2+4\,bx^2-4\,\sqrt{2}\,(bx^2-1)^{\frac{3}{4}}\sqrt{b}\,x-4}{b^2x^4-4\,bx^2+4}\right)}{4\,b^2},\ \frac{2\,\sqrt{2}\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{2}\,(bx^2-1)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\sqrt{2}\,\sqrt{-b}\,\log\left(-\frac{b^2x^4-2\,\sqrt{2}\,(bx^2-1)^{\frac{1}{4}}\sqrt{-b}\,bx^3-4\,\sqrt{bx^2-1}\,bx^2+4\,bx^2+4\,\sqrt{2}\,(bx^2-1)^{\frac{3}{4}}\sqrt{-b}\,x-4}{b^2x^4-4\,bx^2+4}\right)}{4\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) - sq
rt(2)*sqrt(b)*log(-(b^2*x^4 - 2*sqrt(2)*(b*x^2 - 1)^(1/4)*b^(3/2)*x^3 + 4*s
qrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(b)*x - 4)
/(b^2*x^4 - 4*b*x^2 + 4)))/b^2, 1/4*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(b*x
^2 - 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 2*sqrt(2)*
(b*x^2 - 1)^(1/4)*sqrt(-b)*b*x^3 - 4*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 + 4*sq
rt(2)*(b*x^2 - 1)^(3/4)*sqrt(-b)*x - 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^2-2)\,(bx^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2-2)/(b*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/((b*x**2 - 2)*(b*x**2 - 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(b\,x^2-1\right)}^{3/4}\,\left(b\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)),x)
```

```
[Out] int(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

3.1076 $\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}\,b^{3/2}}$$

```
[Out] 1/2*arctan(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))/b^(3/2)*2^(1/2)-1/2*arct
anh(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))/b^(3/2)*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {453}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTan
h[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}\,b^{3/2}}$$

Mathematica [A]
time = 1.98, size = 65, normalized size = 0.88

$$-\frac{-\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

```
[Out] -((-ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))] + ArcTanh[(Sqrt[b]*x)/
(Sqrt[2]*(-1 - b*x^2)^(1/4))])/(Sqrt[2]*b^(3/2)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2-2\right)\left(-b\,x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
```

```
[Out] int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(57) = 114$.
time = 0.71, size = 274, normalized size = 3.70

$$\left[-\frac{2\,\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,(-bx^2-1)^{\frac{1}{4}}}{\sqrt{b}\,x}\right)-\sqrt{2}\,\sqrt{b}\,\log\left(-\frac{b^2x^4+4\,\sqrt{-bx^2-1}\,bx^2-4\,bx^2-2\,\sqrt{2}\,\left((-bx^2-1)^{\frac{1}{4}}\,bx^3+2\,(-bx^2-1)^{\frac{3}{4}}\,x\right)\sqrt{b}-4}{b^2x^4+4\,bx^2+4}\right)}{4\,b^2},\ \frac{2\,\sqrt{2}\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{2}\,(-bx^2-1)^{\frac{1}{4}}\,\sqrt{-b}}{bx}\right)-\sqrt{2}\,\sqrt{-b}\,\log\left(-\frac{b^2x^4-4\,\sqrt{-bx^2-1}\,bx^2-4\,bx^2-2\,\sqrt{2}\,\left((-bx^2-1)^{\frac{1}{4}}\,bx^3-2\,(-bx^2-1)^{\frac{3}{4}}\,x\right)\sqrt{-b}-4}{b^2x^4+4\,bx^2+4}\right)}{4\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) - s
qrt(2)*sqrt(b)*log(-(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 2*sqrt(
2)*((-b*x^2 - 1)^(1/4)*b*x^3 + 2*(-b*x^2 - 1)^(3/4)*x)*sqrt(b) - 4)/(b^2*x^
4 + 4*b*x^2 + 4)))/b^2, 1/4*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(-b*x^2 - 1)
^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 4*sqrt(-b*x^2 - 1
)*b*x^2 - 4*b*x^2 - 2*sqrt(2)*((-b*x^2 - 1)^(1/4)*b*x^3 - 2*(-b*x^2 - 1)^(3
/4)*x)*sqrt(-b) - 4)/(b^2*x^4 + 4*b*x^2 + 4)))/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{bx^2\left(-bx^2-1\right)^{\frac{3}{4}}+2\left(-bx^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2-2)/(-b*x**2-1)**(3/4),x)
[Out] -Integral(x**2/(b*x**2*(-b*x**2 - 1)**(3/4) + 2*(-b*x**2 - 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="giac")
[Out] integrate(-x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(-b\,x^2-1\right)^{3/4}\,\left(b\,x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((- b*x^2 - 1)^(3/4)*(b*x^2 + 2)),x)
[Out] -int(x^2/((- b*x^2 - 1)^(3/4)*(b*x^2 + 2)), x)
```

3.1077 $\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

```
[Out] 1/18*arctan(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(1/4)*6^(1/2)-1/18*arc
tanh(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(1/4)*6^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {453}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Mathematica [A]
time = 1.98, size = 75, normalized size = 0.88

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,\sqrt[4]{a}\,\sqrt[4]{-a+3x^2}}{x}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

```
[Out] (ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))] - ArcTanh[(Sqrt[2/3]*a^
(1/4)*(-a + 3*x^2)^(1/4))/x])/(3*Sqrt[6]*a^(1/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3x^2-2a\right)\left(3x^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(59) = 118.
time = 0.59, size = 145, normalized size = 1.71

$$\frac{2\left(\frac{1}{36}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x\sqrt{\frac{\frac{3x^2}{\sqrt{a}}+2\sqrt{3x^2-a}}{x^2}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}\left(3x^2-a\right)^{\frac{1}{4}}a^{\frac{1}{4}}\right)}{x}\right)}{3a^{\frac{1}{4}}}-\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+\left(3x^2-a\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}+\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(-\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}-\left(3x^2-a\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3*(1/36)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a^(1/4)*x*sqrt((3*x^2/sq
rt(a) + 2*sqrt(3*x^2 - a))/x^2) - (1/36)^(3/4)*(3*x^2 - a)^(1/4)*a^(1/4))/x
)/a^(1/4) - 1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (3*x^2 - a)^(1
/4))/x)/a^(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1/4)*x/a^(1/4) - (3*x^2
- a)^(1/4))/x)/a^(1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-2a+3x^2)\,(-a+3x^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2*a)/(3*x**2-a)**(3/4),x)
```

```
[Out] Integral(x**2/((-2*a + 3*x**2)*(-a + 3*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{(2\,a-3\,x^2)\,(3\,x^2-a)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2*a - 3*x^2)*(3*x^2 - a)^(3/4)),x)
```

```
[Out] -int(x^2/((2*a - 3*x^2)*(3*x^2 - a)^(3/4)), x)
```

3.1078 $\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

```
[Out] 1/18*arctan(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(1/4)*6^(1/2)-1/18*ar
ctanh(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(1/4)*6^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {453}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2a-3x^2)\,(-a-3x^2)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Mathematica [A]
time = 2.06, size = 75, normalized size = 0.88

$$-\frac{-\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}{x}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

```
[Out] -1/3*(-ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))] + ArcTanh[(Sqrt[2
/3]*a^(1/4)*(-a - 3*x^2)^(1/4))/x])/(Sqrt[6]*a^(1/4))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-3x^2-2a)\,(-3x^2-a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(59) = 118.
time = 0.62, size = 145, normalized size = 1.71

$$\frac{2\left(\frac{1}{36}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x\sqrt{\frac{\frac{3x^2}{\sqrt{a}}+2\sqrt{-3x^2-a}}{x^2}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}(-3x^2-a)^{\frac{1}{4}}a^{\frac{1}{4}}\right)}{x}\right)}{3a^{\frac{1}{4}}}-\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+(-3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}+\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(-\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}-(-3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3*(1/36)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a^(1/4)*x*sqrt((3*x^2/sq
rt(a) + 2*sqrt(-3*x^2 - a))/x^2) - (1/36)^(3/4)*(-3*x^2 - a)^(1/4)*a^(1/4))
/x)/a^(1/4) - 1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (-3*x^2 - a)
^(1/4))/x)/a^(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1/4)*x/a^(1/4) - (-3*
x^2 - a)^(1/4))/x)/a^(1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{2a\left(-a-3x^2\right)^{\frac{3}{4}}+3x^2\left(-a-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2*a)/(-3*x**2-a)**(3/4),x)
```

```
[Out] -Integral(x**2/(2*a*(-a - 3*x**2)**(3/4) + 3*x**2*(-a - 3*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(3\,x^2+2\,a\right)\,{\left(-3\,x^2-a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2*a + 3*x^2)*(- a - 3*x^2)^(3/4)),x)
```

```
[Out] -int(x^2/((2*a + 3*x^2)*(- a - 3*x^2)^(3/4)), x)
```

3.1079 $\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=96

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}$$

```
[Out] 1/2*arctan(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(1/4)/b^(3/2)*2
^(1/2)-1/2*arctanh(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(1/4)/b
^(3/2)*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {453}

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}$$

Mathematica [A]
time = 2.04, size = 83, normalized size = 0.86

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+bx^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a+bx^2}}{\sqrt{b}\,x}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

```
[Out] (ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))] - ArcTanh[(Sqrt[2
]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)])/(Sqrt[2]*a^(1/4)*b^(3/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^2-2a)\,(b\,x^2-a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(71) = 142.
time = 0.64, size = 207, normalized size = 2.16

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{\frac{1}{ab^6}}+2\sqrt{bx^2-a}}{x^2}}\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}(bx^2-a)^{\frac{1}{4}}ab^4\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+(bx^2-a)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}-(bx^2-a)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*sqr
t((b^4*x^2*sqrt(1/(a*b^6)) + 2*sqrt(b*x^2 - a))/x^2)*(1/(a*b^6))^(3/4) - (1
/4)^(3/4)*(b*x^2 - a)^(1/4)*a*b^4*(1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^(1/4)*(
1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (b*x^2 - a)^(1/
4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*x*(1/(a*b^
6))^(1/4) - (b*x^2 - a)^(1/4))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-2a + bx^2)\,(-a + bx^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2-2*a)/(b*x**2-a)**(3/4),x)
```

```
[Out] Integral(x**2/((-2*a + b*x**2)*(-a + b*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{(b\,x^2-a)^{3/4}\,(2\,a-b\,x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((b*x^2 - a)^(3/4)*(2*a - b*x^2)),x)
```

```
[Out] -int(x^2/((b*x^2 - a)^(3/4)*(2*a - b*x^2)), x)
```

3.1080 $\int \frac{x^2}{(-2a-bx^2)(-a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=98

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}$$

```
[Out] 1/2*arctan(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(1/4)/b^(3/2)*
2^(1/2)-1/2*arctanh(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(1/4)
/b^(3/2)*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {453}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2a-bx^2)(-a-bx^2)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}$$

Mathematica [A]
time = 2.00, size = 86, normalized size = 0.88

$$-\frac{-\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}{\sqrt{b}\,x}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

```
[Out] -((-ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))] + ArcTanh[(Sqr
t[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)])/(Sqrt[2]*a^(1/4)*b^(3/2)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2-2a\right)\left(-b\,x^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(73) = 146.
time = 0.52, size = 211, normalized size = 2.15

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{\frac{1}{ab^6}}+2\sqrt{-bx^2-a}}{x^2}}\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(-bx^2-a\right)^{\frac{1}{4}}ab^4\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(-bx^2-a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(-bx^2-a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*sqr
t((b^4*x^2*sqrt(1/(a*b^6)) + 2*sqrt(-b*x^2 - a))/x^2)*(1/(a*b^6))^(3/4) - (
1/4)^(3/4)*(-b*x^2 - a)^(1/4)*a*b^4*(1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^(1/4)
*(1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (-b*x^2 - a)^
(1/4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*x*(1/(a
*b^6))^(1/4) - (-b*x^2 - a)^(1/4))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{2a\left(-a-bx^2\right)^{\frac{3}{4}}+bx^2\left(-a-bx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2-2*a)/(-b*x**2-a)**(3/4),x)
[Out] -Integral(x**2/(2*a*(-a - b*x**2)**(3/4) + b*x**2*(-a - b*x**2)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="giac")
[Out] integrate(-x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{{\left(-b\,x^2-a\right)}^{3/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((- a - b*x^2)^(3/4)*(2*a + b*x^2)),x)
[Out] -int(x^2/((- a - b*x^2)^(3/4)*(2*a + b*x^2)), x)
```

3.1081 $\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{14}{81}\sqrt[4]{-1+3x^2}+\frac{8}{405}\left(-1+3x^2\right)^{5/4}+\frac{2}{729}\left(-1+3x^2\right)^{9/4}-\frac{8}{81}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

```
[Out] 14/81*(3*x^2-1)^(1/4)+8/405*(3*x^2-1)^(5/4)+2/729*(3*x^2-1)^(9/4)-8/81*arct
an((3*x^2-1)^(1/4))-8/81*arctanh((3*x^2-1)^(1/4))
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 90, 65, 218, 212, 209}

$$-\frac{8}{81}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{2}{729}\left(3x^2-1\right)^{9/4}+\frac{8}{405}\left(3x^2-1\right)^{5/4}+\frac{14}{81}\sqrt[4]{3x^2-1}-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (14*(-1 + 3*x^2)^(1/4))/81 + (8*(-1 + 3*x^2)^(5/4))/405 + (2*(-1 + 3*x^2)^(
9/4))/729 - (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(1/
4)])/81
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{7}{27(-1+3x)^{3/4}} + \frac{8}{27(-2+3x)(-1+3x)^{3/4}} + \frac{4}{27}\sqrt[4]{-1+\right.\right.\\ &= \frac{14}{81}\sqrt[4]{-1+3x^2} + \frac{8}{405}\left(-1+3x^2\right)^{5/4} + \frac{2}{729}\left(-1+3x^2\right)^{9/4} + \frac{4}{27}\text{Subst}\left(\int\right.\\ &= \frac{14}{81}\sqrt[4]{-1+3x^2} + \frac{8}{405}\left(-1+3x^2\right)^{5/4} + \frac{2}{729}\left(-1+3x^2\right)^{9/4} + \frac{16}{81}\text{Subst}\left(\int\right.\\ &= \frac{14}{81}\sqrt[4]{-1+3x^2} + \frac{8}{405}\left(-1+3x^2\right)^{5/4} + \frac{2}{729}\left(-1+3x^2\right)^{9/4} - \frac{8}{81}\text{Subst}\left(\int\right.\\ &= \frac{14}{81}\sqrt[4]{-1+3x^2} + \frac{8}{405}\left(-1+3x^2\right)^{5/4} + \frac{2}{729}\left(-1+3x^2\right)^{9/4} - \frac{8}{81}\tan^{-1}\left(\sqrt{\right.\end{aligned}$$

Mathematica [A]
time = 0.06, size = 57, normalized size = 0.73

$$\frac{2\left(\sqrt[4]{-1+3x^2}\left(284+78x^2+45x^4\right)-180\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-180\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\right)}{3645}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(1/4)*(284 + 78*x^2 + 45*x^4) - 180*ArcTan[(-1 + 3*x^2)^(1
/4)] - 180*ArcTanh[(-1 + 3*x^2)^(1/4)]))/3645
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.65, size = 147, normalized size = 1.88

method	result
trager	$\left(\frac{2}{81}x^4+\frac{52}{1215}x^2+\frac{568}{3645}\right)\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4\ln\left(-\frac{2\left(3x^2-1\right)^{\frac{3}{4}}+2\sqrt{3x^2-1}+3x^2+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{81}+\frac{4\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln}{}$
risch	$\frac{2\left(45x^4+78x^2+284\right)\left(3x^2-1\right)^{\frac{1}{4}}}{3645}+\frac{\left(4\ln\left(\frac{-27x^6+18\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^4-6\sqrt{27x^6-27x^4+9x^2-1}\,x^2+18x^4+2\left(27x^6-2\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] (2/81*x^4+52/1215*x^2+568/3645)*(3*x^2-1)^(1/4)-4/81*ln(-(2*(3*x^2-1)^(3/4)
+2*(3*x^2-1)^(1/2)+3*x^2+2*(3*x^2-1)^(1/4))/(3*x^2-2))+4/81*RootOf(_Z^2+1)*
ln((2*RootOf(_Z^2+1)*(3*x^2-1)^(3/4)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)+2*(3*
x^2-1)^(1/2)-3*x^2)/(3*x^2-2))
```

Maxima [A]
time = 0.48, size = 74, normalized size = 0.95

$$\frac{2}{729}\left(3x^2-1\right)^{\frac{9}{4}}+\frac{8}{405}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{14}{81}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2 - 1)^(1/4)
 - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*
log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A]
time = 0.50, size = 64, normalized size = 0.82

$$\frac{2}{3645}\left(45x^4+78x^2+284\right)\left(3x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3645*(45*x^4 + 78*x^2 + 284)*(3*x^2 - 1)^(1/4) - 8/81*arctan((3*x^2 - 1)^
(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**7/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A]
time = 1.51, size = 75, normalized size = 0.96

$$\frac{2}{729}\left(3x^2-1\right)^{\frac{9}{4}}+\frac{8}{405}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{14}{81}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2 - 1)^(1/4)
 - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*
log(abs((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.05, size = 62, normalized size = 0.79

$$\frac{14\left(3x^2-1\right)^{1/4}}{81}-\frac{8\,\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\right)}{81}+\frac{8\left(3x^2-1\right)^{5/4}}{405}+\frac{2\left(3x^2-1\right)^{9/4}}{729}+\frac{\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\,1\mathrm{i}\right)\,8\mathrm{i}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (atan((3*x^2 - 1)^(1/4)*1i)*8i)/81 - (8*atan((3*x^2 - 1)^(1/4)))/81 + (14*(
3*x^2 - 1)^(1/4))/81 + (8*(3*x^2 - 1)^(5/4))/405 + (2*(3*x^2 - 1)^(9/4))/72
9
```

3.1082 $\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}} dx$

Optimal. Leaf size=63

$$\frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}\left(-1+3x^2\right)^{5/4}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

```
[Out] 2/9*(3*x^2-1)^(1/4)+2/135*(3*x^2-1)^(5/4)-4/27*arctan((3*x^2-1)^(1/4))-4/27
*arctanh((3*x^2-1)^(1/4))
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 90, 65, 218, 212, 209}

$$-\frac{4}{27}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{2}{135}\left(3x^2-1\right)^{5/4}+\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 + (2*(-1 + 3*x^2)^(5/4))/135 - (4*ArcTan[(-1 + 3*x
^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(-2+3x)(-1+3x)^{3/4}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{3(-1+3x)^{3/4}}+\frac{4}{9(-2+3x)(-1+3x)^{3/4}}+\frac{1}{9}\sqrt[4]{-1+3x}\right.\right.\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}\left(-1+3x^2\right)^{5/4}+\frac{2}{9}\text{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/}}\right.\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}\left(-1+3x^2\right)^{5/4}+\frac{8}{27}\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt[4]{-1}\right.\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}\left(-1+3x^2\right)^{5/4}-\frac{4}{27}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{-1+}\right.\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}\left(-1+3x^2\right)^{5/4}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\tan
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.83

$$\frac{2}{135}\left(\sqrt[4]{-1+3x^2}\left(14+3x^2\right)-10\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-10\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

[Out] (2*((-1 + 3*x^2)^(1/4)*(14 + 3*x^2) - 10*ArcTan[(-1 + 3*x^2)^(1/4)] - 10*ArcTanh[(-1 + 3*x^2)^(1/4)]))/135

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.53, size = 142, normalized size = 2.25

method	result
trager	$\left(\frac{2x^2}{45}+\frac{28}{135}\right)(3x^2-1)^{\frac{1}{4}} - \frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)(3x^2-1)^{\frac{3}{4}}-2\sqrt{3x^2-1}-2\,\mathrm{RootOf}\left(\_Z^2+1\right)(3x}{3x^2-2}\right.}{27}$
risch	$\frac{2(3x^2+14)(3x^2-1)^{\frac{1}{4}}}{135} + \frac{\left(-\frac{2\ln\left(-\frac{27x^6+18(27x^6-27x^4+9x^2-1)^{\frac{1}{4}}x^4+6\sqrt{27x^6-27x^4+9x^2-1}\,x^2-18x^4+2(27x^6-27x^4+9x}{(3x^2-}\right.}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)

[Out] (2/45*x^2+28/135)*(3*x^2-1)^(1/4)-2/27*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)+3*x^2)/(3*x^2-2))+2/27*ln((2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)-3*x^2+2*(3*x^2-1)^(1/4))/(3*x^2-2))

Maxima [A]
time = 0.47, size = 63, normalized size = 1.00

$$\frac{2}{135}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")

[Out] 2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)

Fricas [A]
time = 0.50, size = 59, normalized size = 0.94

$$\frac{2}{135}\left(3x^2+14\right)\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")

```
[Out] 2/135*(3*x^2 + 14)*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(1/4)) - 2/2
7*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**5/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A]
time = 1.12, size = 64, normalized size = 1.02

$$\frac{2}{135}\left(3\,x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3\,x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\,\log\left(\left|\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(
1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log(abs((3*x^2 - 1)^(1/4) -
1))
```

Mupad [B]
time = 0.07, size = 51, normalized size = 0.81

$$\frac{2\,{\left(3\,x^2-1\right)}^{1/4}}{9}-\frac{4\,\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{27}+\frac{2\,{\left(3\,x^2-1\right)}^{5/4}}{135}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,4{}\mathrm{i}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (atan((3*x^2 - 1)^(1/4)*1i)*4i)/27 - (4*atan((3*x^2 - 1)^(1/4)))/27 + (2*(3
*x^2 - 1)^(1/4))/9 + (2*(3*x^2 - 1)^(5/4))/135
```

3.1083 $\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=48

$$\frac{2}{9}\sqrt[4]{-1+3x^2}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

```
[Out] 2/9*(3*x^2-1)^(1/4)-2/9*arctan((3*x^2-1)^(1/4))-2/9*arctanh((3*x^2-1)^(1/4)
)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 81, 65, 218, 212, 209}

$$-\frac{2}{9}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 - (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(-
1 + 3*x^2)^(1/4)])/9
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{4}{9}\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}-\frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\text{Subst}\left(\int \right.\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.92

$$\frac{2}{9}\left(\sqrt[4]{-1+3x^2}-\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(1/4) - ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTanh[(-1 + 3*x^2)^
(1/4)]))/9
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.42, size = 136, normalized size = 2.83

method	result
trager	$\frac{2(3x^2-1)^{\frac{1}{4}}}{9}+\frac{\ln\left(\frac{2(3x^2-1)^{\frac{3}{4}}-2\sqrt{3x^2-1}-3x^2+2(3x^2-1)^{\frac{1}{4}}}{3x^2-2}\right)}{9}-\frac{\mathrm{RootOf}(\_Z^2+1)\ln\left(-\frac{2\,\mathrm{RootOf}(\_Z^2+1)(3x^2-1)^{\frac{3}{4}}-2\sqrt{\ldots}}{\ldots}\right.}{9}$
risch	$\frac{2(3x^2-1)^{\frac{1}{4}}}{9}+\frac{\left(-\frac{\ln\left(-\frac{27x^6+18(27x^6-27x^4+9x^2-1)^{\frac{1}{4}}x^4+6\sqrt{27x^6-27x^4+9x^2-1}\,x^2-18x^4+2(27x^6-27x^4+9x^2-1)^{\frac{3}{4}}-12(\ldots}{(3x^2-2)(3x^2-1)^2\ldots}\right.}{9}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(3*x^2-1)^(1/4)+1/9*ln((2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)-3*x^2+2*(3*
x^2-1)^(1/4))/(3*x^2-2))-1/9*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(3*x^2-1)
^(3/4)-2*(3*x^2-1)^(1/2)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)+3*x^2)/(3*x^2-2))
```

Maxima [A]
time = 0.49, size = 52, normalized size = 1.08

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1)
^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Fricas [A]
time = 0.58, size = 52, normalized size = 1.08

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1)
^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**3/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A]
time = 1.04, size = 53, normalized size = 1.10

$$\frac{2}{9}\,\left(3\,x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\,\log\left(\left|\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1)
^(1/4) + 1) + 1/9*log(abs((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.06, size = 36, normalized size = 0.75

$$\frac{2\,{\left(3\,x^2-1\right)}^{1/4}}{9}-\frac{2\,\mathrm{atanh}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{9}-\frac{2\,\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (2*(3*x^2 - 1)^(1/4))/9 - (2*atanh((3*x^2 - 1)^(1/4)))/9 - (2*atan((3*x^2 -
 1)^(1/4)))/9
```

3.1084 $\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

[Out] -1/3*arctan((3*x^2-1)^(1/4))-1/3*arctanh((3*x^2-1)^(1/4))

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {455, 65, 218, 212, 209}

$$-\frac{1}{3}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]

[Out] -1/3*ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTanh[(-1 + 3*x^2)^(1/4)]/3

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x
\end{aligned}$$

$$= -\frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.00

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
[Out] -1/3*ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.55, size = 125, normalized size = 3.79

method	result
trager	$\frac{\text{RootOf}(\_Z^2+1)\ln\left(\frac{2\,\text{RootOf}(\_Z^2+1)(3x^2-1)^{\frac{3}{4}}-2\,\text{RootOf}(\_Z^2+1)(3x^2-1)^{\frac{1}{4}}+2\sqrt{3x^2-1}-3x^2}{3x^2-2}\right)}{6} - \frac{\ln\left(-\frac{2(3x^2-1)^{\frac{3}{4}}+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^2+1)*ln((2*RootOf(_Z^2+1)*(3*x^2-1)^(3/4)-2*RootOf(_Z^2+1)*(3
*x^2-1)^(1/4)+2*(3*x^2-1)^(1/2)-3*x^2)/(3*x^2-2))-1/6*ln(-(2*(3*x^2-1)^(3/4
)+2*(3*x^2-1)^(1/2)+3*x^2+2*(3*x^2-1)^(1/4))/(3*x^2-2))
```

Maxima [A]
time = 0.49, size = 41, normalized size = 1.24

$$-\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((
3*x^2 - 1)^(1/4) - 1)
```

Fricas [A]
time = 0.46, size = 41, normalized size = 1.24

$$-\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((
3*x^2 - 1)^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{(3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A]
time = 1.86, size = 42, normalized size = 1.27

$$-\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(a
bs((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.46, size = 25, normalized size = 0.76

$$-\frac{\operatorname{atan}\left((3\,x^2-1)^{1/4}\right)}{3}-\frac{\operatorname{atanh}\left((3\,x^2-1)^{1/4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] - atan((3*x^2 - 1)^(1/4))/3 - atanh((3*x^2 - 1)^(1/4))/3
```

3.1085 $\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=173

$$-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+\right.$$

```
[Out] -1/2*arctan((3*x^2-1)^(1/4))-1/2*arctanh((3*x^2-1)^(1/4))-1/4*arctan(-1+(3*
x^2-1)^(1/4)*2^(1/2))*2^(1/2)-1/4*arctan(1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2)
+1/8*ln(1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)-1/8*ln(1+(3*x^2-
1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {457, 88, 65, 217, 1179, 642, 1176, 631, 210, 218, 212, 209}

$$-\frac{1}{2}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{2\sqrt{2}}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{2\sqrt{2}}+\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/2*ArcTan[(-1 + 3*x^2)^(1/4)] + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2
*Sqrt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-
1 + 3*x^2)^(1/4)]/2 + Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]
]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*S
qrt[2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-2+3x)(-1+3x)^{3/4}}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{x(-1+3x)^{3/4}}\,dx,x,x^2\right)\right)+\frac{3}{4}\text{Subst}\left(\int \frac{1}{(-2+3x)(-1\\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{x^4}{3}}\,dx,x,\sqrt[4]{-1+3x^2}\right)\right)+\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,\\
&= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx,x,\sqrt[4]{-1+3x^2}\right)\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx,\\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{4}\text{Subst}\left(\int \frac{}{1-}\\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)+\frac{\log\left(1-\sqrt{2}\sqrt[4]{-}\right.}{}\\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1+\right.}{}\end{aligned}$$

Mathematica [A]

time = 0.12, size = 111, normalized size = 0.64

$$\frac{1}{4}\left(-2\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{-1+\sqrt{-1+3x^2}}{\sqrt{2}\sqrt[4]{-1+3x^2}}\right)-2\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-1+3x^2}}{1+\sqrt{-1+3x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(-1 + 3*x^2)^(1/4)] - Sqrt[2]*ArcTan[(-1 + Sqrt[-1 + 3*x^2])/(Sq
rt[2]*(-1 + 3*x^2)^(1/4))] - 2*ArcTanh[(-1 + 3*x^2)^(1/4)] - Sqrt[2]*ArcTan
h[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 + Sqrt[-1 + 3*x^2])])/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.68, size = 302, normalized size = 1.75

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x^2-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(3x^2-1\right)^{\frac{1}{4}}-2\sqrt{3x^2-1}\ \mathrm{RootOf}\left(\_Z\right.}{x^2}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*RootOf(_Z^4+1)*ln(-(3*RootOf(_Z^4+1)^3*x^2-2*RootOf(_Z^4+1)^3+2*RootOf(
_Z^4+1)^2*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(1/2)*RootOf(_Z^4+1)+2*(3*x^2-1)^(3/4
))/x^2)-1/4*RootOf(_Z^4+1)^3*ln(-(2*(3*x^2-1)^(1/2)*RootOf(_Z^4+1)^3-2*Root
Of(_Z^4+1)^2*(3*x^2-1)^(1/4)-3*RootOf(_Z^4+1)*x^2+2*(3*x^2-1)^(3/4)+2*RootO
f(_Z^4+1))/x^2)-1/4*ln(-(2*(3*x^2-1)^(3/4)+2*(3*x^2-1)^(1/2)+3*x^2+2*(3*x^2
-1)^(1/4))/(3*x^2-2))-1/4*RootOf(_Z^4+1)^2*ln(-(2*RootOf(_Z^4+1)^2*(3*x^2-1
)^(1/2)-3*RootOf(_Z^4+1)^2*x^2+2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/4))/(3*x^2-
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x), x)
```

Fricas [A]
time = 0.46, size = 215, normalized size = 1.24

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}-1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+1\right)-\frac{1}{8}\sqrt{2}\log\left(4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)+\frac{1}{8}\sqrt{2}\log\left(-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)-\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1)
 + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqr
t(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(2)*(3*x^2 -
1)^(1/4) + 1) - 1/8*sqrt(2)*log(4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2
- 1) + 4) + 1/8*sqrt(2)*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1
) + 4) - 1/2*arctan((3*x^2 - 1)^(1/4)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1
/4*log((3*x^2 - 1)^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A]
time = 1.06, size = 155, normalized size = 0.90

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 1/4*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) - 1/8*sqrt(2)*log(
sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 1/8*sqrt(2)*log(-sqrt(2)
*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/2*arctan((3*x^2 - 1)^(1/4)) -
 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log(abs((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.51, size = 77, normalized size = 0.45

$$-\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{2}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(3\,x^2-1\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}-\frac{1}{4}{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(3\,x^2-1\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}+\frac{1}{4}{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (atan((3*x^2 - 1)^(1/4)*1i)*1i)/2 - atan((3*x^2 - 1)^(1/4))/2 - 2^(1/2)*ata
n(2^(1/2)*(3*x^2 - 1)^(1/4)*(1/2 - 1i/2))*(1/4 + 1i/4) - 2^(1/2)*atan(2^(1/
2)*(3*x^2 - 1)^(1/4)*(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.1086 $\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=191

$$-\frac{\sqrt[4]{-1+3x^2}}{4x^2}-\frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)+\frac{15\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}}-\frac{15\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right.}{8\sqrt{2}}$$

```
[Out] -1/4*(3*x^2-1)^(1/4)/x^2-3/4*arctan((3*x^2-1)^(1/4))-3/4*arctanh((3*x^2-1)^
(1/4))-15/16*arctan(-1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2)-15/16*arctan(1+(3*x
^2-1)^(1/4)*2^(1/2))*2^(1/2)+15/32*ln(1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(
1/2))*2^(1/2)-15/32*ln(1+(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {457, 105, 162, 65, 217, 1179, 642, 1176, 631, 210, 218, 212, 209}

$$-\frac{3}{4}\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)+\frac{15\text{ArcTan}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{8\sqrt{2}}-\frac{15\text{ArcTan}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{8\sqrt{2}}-\frac{\sqrt[4]{3x^2-1}}{4x^2}+\frac{15\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{15\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/4*(-1 + 3*x^2)^(1/4)/x^2 - (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 + (15*ArcTan
[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (15*ArcTan[1 + Sqrt[2]*(-1
+ 3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 + (15*Log[
1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2]) - (15*Log[
1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{15}{2}+\frac{27x}{4}}{x(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{15}{16}\text{Subst}\left(\int \frac{1}{x(-1+3x)^{3/4}}\,dx, x, x^2\right) + \frac{9}{8}\text{Subst}\left(\int\right.$$
$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{5}{4}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) + \frac{3}{2}\text{Subst}\left(\int\right.$$
$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{5}{8}\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) - \frac{5}{8}\text{Subst}\left(\int\right.$$
$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) -$$
$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) +$$
$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{15\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+\right.}{8\sqrt{2}}$$

Mathematica [A]
time = 0.19, size = 127, normalized size = 0.66

$$\frac{1}{16}\left(-\frac{4\sqrt[4]{-1+3x^2}}{x^2} - 12\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - 15\sqrt{2}\tan^{-1}\left(\frac{-1+\sqrt{-1+3x^2}}{\sqrt{2}\sqrt[4]{-1+3x^2}}\right) - 12\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - 15\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-1+3x^2}}{1+\sqrt{-1+3x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ((-4*(-1 + 3*x^2)^(1/4))/x^2 - 12*ArcTan[(-1 + 3*x^2)^(1/4)] - 15*Sqrt[2]*A
rcTan[(-1 + Sqrt[-1 + 3*x^2])/(Sqrt[2]*(-1 + 3*x^2)^(1/4))] - 12*ArcTanh[(-
1 + 3*x^2)^(1/4)] - 15*Sqrt[2]*ArcTanh[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 + Sq
rt[-1 + 3*x^2])])/16
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 10.11, size = 314, normalized size = 1.64

method	result

trager	$-\frac{(3x^2-1)^{\frac{1}{4}}}{4x^2}-\frac{15\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x^2-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2(3x^2-1)^{\frac{1}{4}}-2\sqrt{3x^2}}{x^2}\right.}{16}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(3*x^2-1)^(1/4)/x^2-15/16*RootOf(_Z^4+1)*ln((3*RootOf(_Z^4+1)^3*x^2-2*
RootOf(_Z^4+1)^3-2*RootOf(_Z^4+1)^2*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(1/2)*RootO
f(_Z^4+1)-2*(3*x^2-1)^(3/4))/x^2)-15/16*RootOf(_Z^4+1)^3*ln(-(2*(3*x^2-1)^(
1/2)*RootOf(_Z^4+1)^3-2*RootOf(_Z^4+1)^2*(3*x^2-1)^(1/4)-3*RootOf(_Z^4+1)*x
^2+2*(3*x^2-1)^(3/4)+2*RootOf(_Z^4+1))/x^2)-3/8*ln(-(2*(3*x^2-1)^(3/4)+2*(3
*x^2-1)^(1/2)+3*x^2+2*(3*x^2-1)^(1/4))/(3*x^2-2))+3/8*RootOf(_Z^4+1)^2*ln((
2*RootOf(_Z^4+1)^2*(3*x^2-1)^(1/2)-3*RootOf(_Z^4+1)^2*x^2-2*(3*x^2-1)^(3/4)
+2*(3*x^2-1)^(1/4))/(3*x^2-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^3), x)
```

Fricas [A]
time = 0.51, size = 252, normalized size = 1.32

$$\frac{60\sqrt{2}\,x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}-1\right)+60\sqrt{2}\,x^2\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4}-\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+1\right)-15\sqrt{2}\,x^2\log\left(4\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)+15\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,(3x^2-1)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)-24x^2\arctan\left((3x^2-1)^{\frac{1}{4}}\right)-12x^2\log\left((3x^2-1)^{\frac{1}{4}}+1\right)+12x^2\log\left((3x^2-1)^{\frac{1}{4}}-1\right)-8(3x^2-1)^{\frac{1}{4}}}{32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/32*(60*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3
*x^2 - 1) + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 60*sqrt(2)*x^2*arctan(1/2
*sqrt(2)*sqrt(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(
2)*(3*x^2 - 1)^(1/4) + 1) - 15*sqrt(2)*x^2*log(4*sqrt(2)*(3*x^2 - 1)^(1/4)
+ 4*sqrt(3*x^2 - 1) + 4) + 15*sqrt(2)*x^2*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4)
+ 4*sqrt(3*x^2 - 1) + 4) - 24*x^2*arctan((3*x^2 - 1)^(1/4)) - 12*x^2*log((3
*x^2 - 1)^(1/4) + 1) + 12*x^2*log((3*x^2 - 1)^(1/4) - 1) - 8*(3*x^2 - 1)^(1
/4))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \cdot (3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [A]
time = 1.32, size = 169, normalized size = 0.88

$$-\frac{15}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,(3\,x^2-1)^{\frac{1}{4}}\right)\right)-\frac{15}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,(3\,x^2-1)^{\frac{1}{4}}\right)\right)-\frac{15}{32}\sqrt{2}\log\left(\sqrt{2}\,(3\,x^2-1)^{\frac{1}{4}}+\sqrt{3\,x^2-1}+1\right)+\frac{15}{32}\sqrt{2}\log\left(-\sqrt{2}\,(3\,x^2-1)^{\frac{1}{4}}+\sqrt{3\,x^2-1}+1\right)-\frac{(3\,x^2-1)^{\frac{1}{4}}}{4\,x^2}-\frac{3}{4}\arctan\left((3\,x^2-1)^{\frac{1}{4}}\right)-\frac{3}{8}\log\left((3\,x^2-1)^{\frac{1}{4}}+1\right)+\frac{3}{8}\log\left(\left|(3\,x^2-1)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] -15/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 15/16*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) - 15/32*sqrt(2
)*log(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 15/32*sqrt(2)*log(
-sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/4*(3*x^2 - 1)^(1/4)/x
^2 - 3/4*arctan((3*x^2 - 1)^(1/4)) - 3/8*log((3*x^2 - 1)^(1/4) + 1) + 3/8*l
og(abs((3*x^2 - 1)^(1/4) - 1))
```

Mupad [B]
time = 0.11, size = 81, normalized size = 0.42

$$-\frac{3\,\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{4}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{4}-\frac{{\left(3\,x^2-1\right)}^{1/4}}{4\,x^2}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(3\,x^2-1\right)}^{1/4}\right)\,15{}\mathrm{i}}{8}-\frac{{\left(-1\right)}^{3/4}\,\mathrm{atan}\left({\left(-1\right)}^{3/4}\,{\left(3\,x^2-1\right)}^{1/4}\right)\,15{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (atan((3*x^2 - 1)^(1/4)*1i)*3i)/4 - (3*atan((3*x^2 - 1)^(1/4)))/4 - (3*x^2
- 1)^(1/4)/(4*x^2) + ((-1)^(1/4)*atan((-1)^(1/4)*(3*x^2 - 1)^(1/4))*15i)/8
- ((-1)^(3/4)*atan((-1)^(3/4)*(3*x^2 - 1)^(1/4))*15i)/8
```

3.1087 $\int \frac{x^6}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=165

$$\frac{40}{567}x\sqrt[4]{-1+3x^2}+\frac{2}{63}x^3\sqrt[4]{-1+3x^2}+\frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-\frac{2}{27}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)+$$

```
[Out] 40/567*x*(3*x^2-1)^(1/4)+2/63*x^3*(3*x^2-1)^(1/4)+2/81*arctan(1/2*x*6^(1/2)
/(3*x^2-1)^(1/4))*6^(1/2)-2/81*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/
2)+40/1701*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^
(1/4)))*EllipticF(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^
(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {454, 240, 226, 327, 409, 453}

$$\frac{2}{27}\sqrt{\frac{2}{3}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)+\frac{40\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{567\sqrt{3}\,x}+\frac{40}{567}\sqrt[4]{3x^2-1}\,x-\frac{2}{27}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)+\frac{2}{63}\sqrt[4]{3x^2-1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (40*x*(-1 + 3*x^2)^(1/4))/567 + (2*x^3*(-1 + 3*x^2)^(1/4))/63 + (2*Sqrt[2/3
]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 - (2*Sqrt[2/3]*ArcTanh[(Sqrt
[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 + (40*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(
1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(567*Sq
rt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{x^6}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \left(\frac{4}{27(-1+3x^2)^{3/4}} + \frac{2x^2}{9(-1+3x^2)^{3/4}} + \frac{x^4}{3(-1+3x^2)^{3/4}} + \frac{}{27(-2+}\right.$$

$$= \frac{4}{27}\int \frac{1}{(-1+3x^2)^{3/4}}\,dx + \frac{2}{9}\int \frac{x^2}{(-1+3x^2)^{3/4}}\,dx + \frac{8}{27}\int \frac{1}{(-2+3x^2)(-}$$

$$= \frac{4}{81}x\sqrt[4]{-1+3x^2} + \frac{2}{63}x^3\sqrt[4]{-1+3x^2} + \frac{4}{81}\int \frac{1}{(-1+3x^2)^{3/4}}\,dx + \frac{2}{21}\int$$

$$= \frac{40}{567}x\sqrt[4]{-1+3x^2} + \frac{2}{63}x^3\sqrt[4]{-1+3x^2} + \frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)$$

$$= \frac{40}{567}x\sqrt[4]{-1+3x^2} + \frac{2}{63}x^3\sqrt[4]{-1+3x^2} + \frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)$$

$$= \frac{40}{567}x\sqrt[4]{-1+3x^2} + \frac{2}{63}x^3\sqrt[4]{-1+3x^2} + \frac{2}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.81, size = 184, normalized size = 1.12

$$\frac{2x\left(-20+51x^2+27x^4-31x^2(1-3x^2)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)-\frac{80F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(-2+3x^2)\left(2F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)\right)}\right)}{567(-1+3x^2)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*(-20 + 51*x^2 + 27*x^4 - 31*x^2*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1
, 5/2, 3*x^2, (3*x^2)/2] - (80*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2]
)/((-2 + 3*x^2)*(2*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*Ap
pellF1[3/2, 3/4, 2, 5/2, 3*x^2, (3*x^2)/2] + 3*AppellF1[3/2, 7/4, 1, 5/2, 3
*x^2, (3*x^2)/2])))))/(567*(-1 + 3*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(3 x^{2}-2\right) \left(3 x^{2}-1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(1/4)*x^6/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(3 x^{2}-2\right) \left(3 x^{2}-1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**6/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(3\,x^2-1\right)^{3/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] int(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1088 $\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=147

$$\frac{2}{27}x\sqrt[4]{-1+3x^2}+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)+\frac{2\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}}{}$$

```
[Out] 2/27*x*(3*x^2-1)^(1/4)+1/27*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1
/27*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)+2/81*(cos(2*arctan((3*x^
2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF(sin(2*arctan
((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2)
)^2)^(1/2)/x*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {454, 240, 226, 327, 409, 453}

$$\frac{1}{9}\sqrt{\frac{2}{3}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)+\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)\middle|\frac{1}{2}\right)}{27\sqrt{3}\,x}+\frac{2}{27}\sqrt[4]{3x^2-1}\,x-\frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2)^(1/4))/27 + (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^
(1/4)])/9 - (Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 + (2*Sq
rt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[
(-1 + 3*x^2)^(1/4)], 1/2])/(27*Sqrt[3]*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \left(\frac{2}{9(-1+3x^2)^{3/4}} + \frac{x^2}{3(-1+3x^2)^{3/4}} + \frac{4}{9(-2+3x^2)(-1+3x^2)^{3/4}}\right) d$$

$$= \frac{2}{9}\int \frac{1}{(-1+3x^2)^{3/4}}\,dx + \frac{1}{3}\int \frac{x^2}{(-1+3x^2)^{3/4}}\,dx + \frac{4}{9}\int \frac{1}{(-2+3x^2)(-1+$$

$$= \frac{2}{27}x\sqrt[4]{-1+3x^2} + \frac{2}{27}\int \frac{1}{(-1+3x^2)^{3/4}}\,dx - \frac{2}{9}\int \frac{1}{(-1+3x^2)^{3/4}}\,dx + \frac{2}{3}$$

$$= \frac{2}{27}x\sqrt[4]{-1+3x^2} + \frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{-}}\right.$$

$$= \frac{2}{27}x\sqrt[4]{-1+3x^2} + \frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{9}\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{-}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.61, size = 179, normalized size = 1.22

$$\frac{2x\left(-1+3x^2-2x^2(1-3x^2)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)-\frac{4F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(-2+3x^2)\left(2F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+3F_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)\right)}\right)}{27(-1+3x^2)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*x*(-1 + 3*x^2 - 2*x^2*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2
, (3*x^2)/2] - (4*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^
2)*(2*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[3/2, 3
/4, 2, 5/2, 3*x^2, (3*x^2)/2] + 3*AppellF1[3/2, 7/4, 1, 5/2, 3*x^2, (3*x^2)
/2])))))/(27*(-1 + 3*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
[Out] int(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
[Out] integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
[Out] integral((3*x^2 - 1)^(1/4)*x^4/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
[Out] Integral(x**4/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
[Out] integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(3\,x^2-1\right)}^{3/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
[Out] int(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1089 $\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}} dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

```
[Out] 1/18*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/18*arctanh(1/2*x*6^(1/
2)/(3*x^2-1)^(1/4))*6^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {453}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.89

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)] - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x
^2)^(1/4)])/(3*Sqrt[6])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.00, size = 138, normalized size = 2.26

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\left(3x^2-1\right)^{\frac{3}{4}}-3\sqrt{3x^2-1}\,x+\mathrm{RootOf}\left(\_Z^2-6\right)\left(3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2-2}\right)}{18}+\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*RootOf(_Z^2-6)*ln(-(RootOf(_Z^2-6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)*x
+RootOf(_Z^2-6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))+1/18*RootOf(_Z^2+6)*ln((Roo
tOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x-RootOf(_Z^2+6)*(3*x^2-1)^(1
/4)-3*x)/(3*x^2-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(43) = 86.
time = 1.17, size = 104, normalized size = 1.70

$$-\frac{1}{18}\sqrt{6}\arctan\left(\frac{\sqrt{6}\,(3x^2-1)^{\frac{1}{4}}}{3x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\,(3x^2-1)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}\,x^2-4\sqrt{6}\,(3x^2-1)^{\frac{3}{4}}x+12x^2-4}{9x^4-12x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/36*sqrt(6)*log(-(
9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6
)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(3\,x^2-1\right)}^{3/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1090 $\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=127

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}-\frac{\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)F\left(2\tan^{-1}\left(\right.\right.}{2\sqrt{3}\,x}$$

[Out] 1/12*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/6*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {409, 240, 226, 453}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2\sqrt{3}\,x}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]

[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - (Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*Sqrt[3]*x)

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\left(\frac{1}{2}\int \frac{1}{(-1+3x^2)^{3/4}}\,dx\right)+\frac{3}{2}\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{}{\sqrt{1}} \right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3\right.}}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.12, size = 68, normalized size = 0.54

$$\frac{\sqrt[4]{-1}\,\sqrt{x^2}\left(\Pi\left(-i;\sin^{-1}\left((-1)^{3/4}\sqrt[4]{-1+3x^2}\right)\middle|-1\right)+\Pi\left(i;\sin^{-1}\left((-1)^{3/4}\sqrt[4]{-1+3x^2}\right)\middle|-1\right)\right)}{\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[x^2]*(EllipticPi[-I, ArcSin[(-1)^(3/4)*(-1 + 3*x^2)^(1/4)]
, -1] + EllipticPi[I, ArcSin[(-1)^(3/4)*(-1 + 3*x^2)^(1/4)], -1]))/(Sqrt[3]
*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(1/4)/(9*x^4 - 9*x^2 + 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(3\,x^2-1\right)^{3/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] int(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1091 $\int \frac{1}{x^2(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt[4]{-1+3x^2}}{2x}+\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}}{}$$

```
[Out] -1/2*(3*x^2-1)^(1/4)/x+1/8*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/
8*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/2*(cos(2*arctan((3*x^2-1
)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF(sin(2*arctan((3
*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2
)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {454, 331, 240, 226, 409, 453}

$$\frac{1}{4}\sqrt{\frac{3}{2}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}-\frac{\sqrt[4]{3x^2-1}}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/2*(-1 + 3*x^2)^(1/4)/x + (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1
/4)])/4 - (Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3
]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*Arc
Tan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{1}{x^2(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \left(-\frac{1}{2x^2(-1+3x^2)^{3/4}} + \frac{3}{2(-2+3x^2)(-1+3x^2)^{3/4}}\right) dx$$

$$= -\left(\frac{1}{2}\int \frac{1}{x^2(-1+3x^2)^{3/4}}\,dx\right) + \frac{3}{2}\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{2x} - 2\left(\frac{3}{4}\int \frac{1}{(-1+3x^2)^{3/4}}\,dx\right) + \frac{9}{4}\int \frac{x^2}{(-2+3x^2)(-1+$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{2x} + \frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\Bigg(\sqrt[4]{-$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{2x} + \frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\Bigg(\sqrt[4]{-$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.06, size = 52, normalized size = 0.35

$$\frac{(1-3x^2)^{3/4} F_1\left(-\frac{1}{2};\frac{3}{4},1;\frac{1}{2};3x^2,\frac{3x^2}{2}\right)}{2x(-1+3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ((1 - 3*x^2)^(3/4)*AppellF1[-1/2, 3/4, 1, 1/2, 3*x^2, (3*x^2)/2])/(2*x*(-1
+ 3*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
[Out] integral((3*x^2 - 1)^(1/4)/(9*x^6 - 9*x^4 + 2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \cdot (3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
[Out] Integral(1/(x**2*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(3\,x^2-1\right)}^{3/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
[Out] int(1/(x^2*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1092 $\int \frac{1}{x^4(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=165

$$-\frac{\sqrt[4]{-1+3x^2}}{6x^3}-\frac{2\sqrt[4]{-1+3x^2}}{x}+\frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)-\frac{11\sqrt{3}\sqrt{\cdots}}{\cdots}$$

```
[Out] -1/6*(3*x^2-1)^(1/4)/x^3-2*(3*x^2-1)^(1/4)/x+3/16*arctan(1/2*x*6^(1/2)/(3*x
^2-1)^(1/4))*6^(1/2)-3/16*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-11
/8*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*
EllipticF(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(
x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {454, 331, 240, 226, 409, 453}

$$\frac{3}{8}\sqrt{\frac{3}{2}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)-\frac{11\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\text{ArcTan}\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{8x}-\frac{2\sqrt[4]{3x^2-1}}{x}-\frac{3}{8}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)-\frac{\sqrt[4]{3x^2-1}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/6*(-1 + 3*x^2)^(1/4)/x^3 - (2*(-1 + 3*x^2)^(1/4))/x + (3*Sqrt[3/2]*ArcTa
n[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)
/(-1 + 3*x^2)^(1/4)])/8 - (11*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1
 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(8*x)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[2*(Sqrt[(-b)*(x^2/a)]/(
b*x)), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 409

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 453

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rule 454

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{1}{x^4(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \left(-\frac{1}{2x^4(-1+3x^2)^{3/4}} - \frac{3}{4x^2(-1+3x^2)^{3/4}} + \frac{9}{4(-2+3x^2)(-1+3x$$

$$= -\left(\frac{1}{2}\int \frac{1}{x^4(-1+3x^2)^{3/4}}\,dx\right) - \frac{3}{4}\int \frac{1}{x^2(-1+3x^2)^{3/4}}\,dx + \frac{9}{4}\int \frac{}{(-2}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{6x^3} - \frac{3\sqrt[4]{-1+3x^2}}{4x} - 2\left(\frac{9}{8}\int \frac{1}{(-1+3x^2)^{3/4}}\,dx\right) - \frac{5}{4}\int \frac{}{x}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{6x^3} - \frac{2\sqrt[4]{-1+3x^2}}{x} + \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{3}{8}\sqrt{}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{6x^3} - \frac{2\sqrt[4]{-1+3x^2}}{x} + \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{3}{8}\sqrt{}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{6x^3} - \frac{2\sqrt[4]{-1+3x^2}}{x} + \frac{3}{8}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right) - \frac{3}{8}\sqrt{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 10.05, size = 52, normalized size = 0.32

$$\frac{(1-3x^2)^{3/4}\,F_1\left(-\frac{3}{2};\frac{3}{4},1;-\frac{1}{2};3x^2,\frac{3x^2}{2}\right)}{6x^3(-1+3x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ((1 - 3*x^2)^(3/4)*AppellF1[-3/2, 3/4, 1, -1/2, 3*x^2, (3*x^2)/2])/(6*x^3*(
-1 + 3*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] int(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 1)^(1/4)/(9*x^8 - 9*x^6 + 2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \cdot (3x^2 - 2) (3x^2 - 1)^{\frac{3}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x**4*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(3\,x^2-1\right)^{3/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
[Out] int(1/(x^4*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1093 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=173

$$\frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be}+\frac{3a(8bc-7ad)e^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}-\frac{3a(8bc-$$

```
[Out] 1/16*(-7*a*d+8*b*c)*e*(e*x)^(3/2)*(b*x^2+a)^(1/4)/b^2+1/4*d*(e*x)^(7/2)*(b*
x^2+a)^(1/4)/b/e+3/32*a*(-7*a*d+8*b*c)*e^(5/2)*arctan(b^(1/4)*(e*x)^(1/2)/(
b*x^2+a)^(1/4)/e^(1/2))/b^(11/4)-3/32*a*(-7*a*d+8*b*c)*e^(5/2)*arctanh(b^(1
/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(11/4)
```

Rubi [A]
time = 0.08, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {470, 327, 335, 338, 304, 211, 214}

$$\frac{3ae^{5/2}(8bc-7ad)\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}-\frac{3ae^{5/2}(8bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}+\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}(8bc-7ad)}{16b^2}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((8*b*c - 7*a*d)*e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b^2) + (d*(e*x)^(7/2)
*(a + b*x^2)^(1/4))/(4*b*e) + (3*a*(8*b*c - 7*a*d)*e^(5/2)*ArcTan[(b^(1/4)*
Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^(11/4)) - (3*a*(8*b*c - 7*a*
d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^
(11/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

/b, 0]

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{\left(-4bc+\frac{7ad}{2}\right)\int \frac{(ex)^{5/2}}{(a+bx^2)^{3/4}}\,dx}{4b}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{(3a(8bc-7ad)e^2)\int \frac{\sqrt{ex}}{(a+bx^2}}{32b^2}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{(3a(8bc-7ad)e)\text{Subst}\left(\int\right.}{16}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{(3a(8bc-7ad)e)\text{Subst}\left(\int\right.}{16}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{(3a(8bc-7ad)e^3)\text{Subst}\left(\right.}{3}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} + \frac{3a(8bc-7ad)e^{5/2}\tan^{-1}\left(\right.}{32b^{11/4}}$$

Mathematica [A]
time = 0.64, size = 131, normalized size = 0.76

$$\frac{(ex)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}\,(8bc-7ad+4bdx^2)-3a(-8bc+7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+3a(-8bc+7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{32b^{11/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((e*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4)*(8*b*c - 7*a*d + 4*b*d*x^
2) - 3*a*(-8*b*c + 7*a*d)*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + 3*a
*(-8*b*c + 7*a*d)*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(32*b^(11/
4)*x^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{5}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(117) = 234$.
time = 0.49, size = 275, normalized size = 1.59

$$\frac{1}{64}\left(\left(d\left(\frac{4\left(\frac{11\,(bx^2+a)^{\frac{1}{4}}a^2b}{\sqrt{x}}-\frac{7\,(bx^2+a)^{\frac{5}{4}}a^2}{x^{\frac{5}{2}}}\right)}{b^4-\frac{2\,(bx^2+a)b^3}{x^2}+\frac{(bx^2+a)^2b^2}{x^4}}+\frac{21\left(\frac{2\,a^2\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{3}{4}}}-\frac{a^2\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{3}{4}}}\right)}{b^2}\right)-8\,c\left(\frac{3\left(\frac{2\,a\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{3}{4}}}\right)}{b}+\frac{4\,(bx^2+a)^{\frac{1}{4}}a}{\left(b^2-\frac{(bx^2+a)b}{x^2}\right)\sqrt{x}}\right)\right)e^{\frac{5}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] 1/64*(d*(4*(11*(b*x^2 + a)^(1/4)*a^2*b/sqrt(x) - 7*(b*x^2 + a)^(5/4)*a^2/x^(5/2))/(b^4 - 2*(b*x^2 + a)*b^3/x^2 + (b*x^2 + a)^2*b^2/x^4) + 21*(2*a^2*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(3/4) - a^2*log(-(b^(1/4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(3/4))/b^2) - 8*c*(3*(2*a*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(3/4) - a*log(-(b^(1/4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(3/4))/b + 4*(b*x^2 + a)^(1/4)*a/((b^2 - (b*x^2 + a)*b/x^2)*sqrt(x))))*e^(5/2)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")

[Out] Timed out

Sympy [C] Result contains complex when optimal does not.
time = 17.56, size = 94, normalized size = 0.54

$$\frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}+\frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(3/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hyper(
(3/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^(5/2)*e^(5/2)/(b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{5/2}\,\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
```

```
[Out] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

3.1094 $\int \frac{\sqrt{ex}\,(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=136

$$\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}-\frac{(4bc-3ad)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{(4bc-3ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}$$

```
[Out] 1/2*d*(e*x)^(3/2)*(b*x^2+a)^(1/4)/b/e-1/4*(-3*a*d+4*b*c)*arctan(b^(1/4)*(e*
x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))*e^(1/2)/b^(7/4)+1/4*(-3*a*d+4*b*c)*arctan
h(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))*e^(1/2)/b^(7/4)
```

Rubi [A]
time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 335, 338, 304, 211, 214}

$$-\frac{\sqrt{e}\,(4bc-3ad)\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{\sqrt{e}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (d*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b*e) - ((4*b*c - 3*a*d)*Sqrt[e]*ArcTan
[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(7/4)) + ((4*b*c -
3*a*d)*Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4
*b^(7/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ex}\,(c+dx^2)}{(a+bx^2)^{3/4}}\,dx &= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} - \frac{\left(-2bc+\frac{3ad}{2}\right)\int \frac{\sqrt{ex}}{(a+bx^2)^{3/4}}\,dx}{2b} \\
&= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} + \frac{(4bc-3ad)\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{2be} \\
&= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} + \frac{(4bc-3ad)\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{2be} \\
&= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} + \frac{((4bc-3ad)e)\text{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{4b^{3/2}} - \frac{((4bc-
\\
&= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} - \frac{(4bc-3ad)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} + \frac{(4bc-3ad)\sqrt{e}\ \text{t}
\end{aligned}$$

Mathematica [A]

time = 0.39, size = 112, normalized size = 0.82

$$\frac{\sqrt{ex}\left(2b^{3/4}dx^{3/2}\sqrt[4]{a+bx^2}+(-4bc+3ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{7/4}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]

[Out] (Sqrt[e*x]*(2*b^(3/4)*d*x^(3/2)*(a + b*x^2)^(1/4) + (-4*b*c + 3*a*d)*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + (4*b*c - 3*a*d)*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(4*b^(7/4)*Sqrt[x])

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,(d\,x^2+c)}{(b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)

[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(87) = 174.
time = 0.49, size = 189, normalized size = 1.39

$$\frac{1}{8}\left(4c\left(\frac{2\arctan\left(\frac{(bx^2+a)^{\frac14}}{b^{\frac14}\sqrt{x}}\right)}{b^{\frac34}}-\frac{\log\left(-\frac{b^{\frac14}-\frac{(bx^2+a)^{\frac14}}{\sqrt{x}}}{b^{\frac14}+\frac{(bx^2+a)^{\frac14}}{\sqrt{x}}}\right)}{b^{\frac34}}\right)-d\left(\frac{3\left(\frac{2a\arctan\left(\frac{(bx^2+a)^{\frac14}}{b^{\frac14}\sqrt{x}}\right)}{b^{\frac34}}-\frac{a\log\left(-\frac{b^{\frac14}-\frac{(bx^2+a)^{\frac14}}{\sqrt{x}}}{b^{\frac14}+\frac{(bx^2+a)^{\frac14}}{\sqrt{x}}}\right)}{b^{\frac34}}\right)}{b}+\frac{4(bx^2+a)^{\frac14}a}{\left(b^2-\frac{(bx^2+a)b}{x^2}\right)\sqrt{x}}\right)\right)e^{\frac12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] 1/8*(4*c*(2*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(3/4) - log(-(b^(1/4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(3/4)) - d*(3*(2*a*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(3/4) - a*

```
log(-(b^(1/4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqr
t(x)))/b^(3/4))/b + 4*(b*x^2 + a)^(1/4)*a/((b^2 - (b*x^2 + a)*b/x^2)*sqrt(x
))))*e^(1/2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 2.07, size = 92, normalized size = 0.68

$$\frac{c(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e\Gamma\left(\frac{7}{4}\right)}+\frac{d(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^3\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
[Out] c*(e*x)**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/
a)/(2*a**(3/4)*e*gamma(7/4)) + d*(e*x)**(7/2)*gamma(7/4)*hyper((3/4, 7/4),
(11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*e**3*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*sqrt(x)*e^(1/2)/(b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
[Out] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

3.1095 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=113

$$-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}$$

```
[Out] -d*arctan(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(3/4)/e^(3/2)+d*ar
ctanh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(3/4)/e^(3/2)-2*c*(b*x
^2+a)^(1/4)/a/e/(e*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {462, 335, 338, 304, 211, 214}

$$-\frac{d\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(a*e*Sqrt[e*x]) - (d*ArcTan[(b^(1/4)*Sqrt[e*x])/(S
qrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2)) + (d*ArcTanh[(b^(1/4)*Sqrt[e*
x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{d\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{e^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{(2d)\mathrm{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx,x,\sqrt{ex}\right)}{e^3}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{(2d)\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{e^3}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{d\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}\,e}-\frac{d\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{b}\,x^2}\right.}{\sqrt{b}}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}$$

Mathematica [A]
time = 0.31, size = 100, normalized size = 0.88

$$\frac{x\left(-2b^{3/4}c\sqrt[4]{a+bx^2}-ad\sqrt{x}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+ad\sqrt{x}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{ab^{3/4}(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (x*(-2*b^(3/4)*c*(a + b*x^2)^(1/4) - a*d*Sqrt[x]*ArcTan[(b^(1/4)*Sqrt[x])/(
a + b*x^2)^(1/4)] + a*d*Sqrt[x]*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)
]))/(a*b^(3/4)*(e*x)^(3/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d\,x^{2}+c}{(ex)^{\frac{3}{2}}\,(b\,x^{2}+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

Maxima [A]
time = 0.48, size = 93, normalized size = 0.82

$$\frac{1}{2}\left(d\left(\frac{2\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{3}{4}}}-\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{3}{4}}}\right)-\frac{4\,(bx^2+a)^{\frac{1}{4}}c}{a\sqrt{x}}\right)e^{\left(-\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/2*(d*(2*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(3/4) - log(-(b^(1/
4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(3
/4)) - 4*(b*x^2 + a)^(1/4)*c/(a*sqrt(x)))*e^(-3/2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 3.89, size = 85, normalized size = 0.75

$$\frac{\sqrt[4]{b}\,c\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{1}{4}\right)}{2ae^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] b**(1/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*e**(3/2)*gamma(3/4)) +
d*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(3/4)*e**(3/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-3/2)/((b*x^2 + a)^(3/4)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)), x)
```

3.1096 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}+\frac{2(4bc-5ad)\sqrt[4]{a+bx^2}}{5a^2e^3\sqrt{ex}}$$

```
[Out] -2/5*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(5/2)+2/5*(-5*a*d+4*b*c)*(b*x^2+a)^(1/4)/a
^2/e^3/(e*x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {464, 270}

$$\frac{2\sqrt[4]{a+bx^2}\,(4bc-5ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(5*a*e*(e*x)^(5/2)) + (2*(4*b*c - 5*a*d)*(a + b*x^
2)^(1/4))/(5*a^2*e^3*Sqrt[e*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx&=-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}-\frac{(4bc-5ad)\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx}{5ae^2}\\&=-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}+\frac{2(4bc-5ad)\sqrt[4]{a+bx^2}}{5a^2e^3\sqrt{ex}}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 44, normalized size = 0.66

$$-\frac{2x\sqrt[4]{a+bx^2}\,(ac-4bcx^2+5adx^2)}{5a^2(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
[Out] (-2*x*(a + b*x^2)^(1/4)*(a*c - 4*b*c*x^2 + 5*a*d*x^2))/(5*a^2*(e*x)^(7/2))
```

Maple [A]
time = 0.10, size = 39, normalized size = 0.58

method	result	size
gosper	$-\frac{2x\left(b\,x^2+a\right)^{\frac{1}{4}}\left(5ad\,x^2-4c\,x^2b+ac\right)}{5a^2\left(ex\right)^{\frac{7}{2}}}$	39
risch	$-\frac{2\left(b\,x^2+a\right)^{\frac{1}{4}}\left(5ad\,x^2-4c\,x^2b+ac\right)}{5e^3\sqrt{ex}\;a^2x^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
[Out] -2/5*x*(b*x^2+a)^(1/4)*(5*a*d*x^2-4*b*c*x^2+a*c)/a^2/(e*x)^(7/2)
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.87

$$\frac{2}{5}\left(\frac{c\left(\frac{5\,(bx^2+a)^{\frac{1}{4}}b}{\sqrt{x}}-\frac{(bx^2+a)^{\frac{5}{4}}}{x^{\frac{5}{2}}}\right)}{a^2}-\frac{5\,(bx^2+a)^{\frac{1}{4}}d}{a\sqrt{x}}\right)e^{\left(-\frac{7}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] 2/5*(c*(5*(b*x^2 + a)^(1/4)*b/sqrt(x) - (b*x^2 + a)^(5/4)/x^(5/2))/a^2 - 5*
(b*x^2 + a)^(1/4)*d/(a*sqrt(x)))*e^(-7/2)
```

Fricas [A]
time = 1.98, size = 37, normalized size = 0.55

$$\frac{2\left((4\,bc-5\,ad)x^2-ac\right)(bx^2+a)^{\frac{1}{4}}e^{\left(-\frac{7}{2}\right)}}{5\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*((4*b*c - 5*a*d)*x^2 - a*c)*(b*x^2 + a)^(1/4)*e^(-7/2)/(a^2*x^(5/2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(60) = 120$.
time = 38.40, size = 121, normalized size = 1.81

$$-\frac{\sqrt[4]{b}\, c \sqrt[4]{\frac{a}{bx^2}+1}\, \Gamma\left(-\frac{5}{4}\right)}{8ae^{\frac{7}{2}}x^2\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt[4]{b}\, d \sqrt[4]{\frac{a}{bx^2}+1}\, \Gamma\left(-\frac{1}{4}\right)}{2ae^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{b^{\frac{5}{4}} c \sqrt[4]{\frac{a}{bx^2}+1}\, \Gamma\left(-\frac{5}{4}\right)}{2a^2e^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -b**(1/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(8*a*e**(7/2)*x**2*gamma(3/
4)) + b**(1/4)*d*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*e**(7/2)*gamma(3/
4)) + b**(5/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a**2*e**(7/2)*gamma
(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-7/2)/((b*x^2 + a)^(3/4)*x^(7/2)), x)
```

Mupad [B]
time = 0.64, size = 49, normalized size = 0.73

$$-\frac{\left(\frac{2\,c}{5\,a\,e^3}+\frac{x^2\,\left(10\,a\,d-8\,b\,c\right)}{5\,a^2\,e^3}\right)\,{\left(b\,x^2+a\right)}^{1/4}}{x^2\,\sqrt{e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -(((2*c)/(5*a*e^3) + (x^2*(10*a*d - 8*b*c))/(5*a^2*e^3))*(a + b*x^2)^(1/4))
/(x^2*(e*x)^(1/2))
```

3.1097 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}+\frac{2(8bc-9ad)\sqrt[4]{a+bx^2}}{9a^2e^3(ex)^{5/2}}-\frac{8(8bc-9ad)\,(a+bx^2)^{5/4}}{45a^3e^3(ex)^{5/2}}$$

```
[Out] -2/9*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(9/2)+2/9*(-9*a*d+8*b*c)*(b*x^2+a)^(1/4)/a
^2/e^3/(e*x)^(5/2)-8/45*(-9*a*d+8*b*c)*(b*x^2+a)^(5/4)/a^3/e^3/(e*x)^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 279, 270}

$$-\frac{8(a+bx^2)^{5/4}\,(8bc-9ad)}{45a^3e^3(ex)^{5/2}}+\frac{2\sqrt[4]{a+bx^2}\,(8bc-9ad)}{9a^2e^3(ex)^{5/2}}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(9*a*e*(e*x)^(9/2)) + (2*(8*b*c - 9*a*d)*(a + b*x^
2)^(1/4))/(9*a^2*e^3*(e*x)^(5/2)) - (8*(8*b*c - 9*a*d)*(a + b*x^2)^(5/4))/(
45*a^3*e^3*(e*x)^(5/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}} - \frac{(8bc-9ad)\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx}{9ae^2}$$
$$= -\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}} + \frac{2(8bc-9ad)\sqrt[4]{a+bx^2}}{9a^2e^3(ex)^{5/2}} + \frac{(4(8bc-9ad))\int \frac{\sqrt[4]{a+bx^2}}{(ex)^{7/2}}\,dx}{9a^2e^2}$$
$$= -\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}} + \frac{2(8bc-9ad)\sqrt[4]{a+bx^2}}{9a^2e^3(ex)^{5/2}} - \frac{8(8bc-9ad)\left(a+bx^2\right)^{5/4}}{45a^3e^3(ex)^{5/2}}$$

Mathematica [A]
time = 0.37, size = 67, normalized size = 0.64

$$-\frac{2x\sqrt[4]{a+bx^2}\,(5a^2c-8abcx^2+9a^2dx^2+32b^2cx^4-36abdx^4)}{45a^3(ex)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
[Out] (-2*x*(a + b*x^2)^(1/4)*(5*a^2*c - 8*a*b*c*x^2 + 9*a^2*d*x^2 + 32*b^2*c*x^4
 - 36*a*b*d*x^4))/(45*a^3*(e*x)^(11/2))
```

Maple [A]
time = 0.11, size = 62, normalized size = 0.60

method	result	size
gosper	$-\frac{2x\left(b\,x^2+a\right)^{\frac{1}{4}}\left(-36abd\,x^4+32b^2c\,x^4+9a^2d\,x^2-8abc\,x^2+5a^2c\right)}{45a^3\left(ex\right)^{\frac{11}{2}}}$	62
risch	$-\frac{2\left(b\,x^2+a\right)^{\frac{1}{4}}\left(-36abd\,x^4+32b^2c\,x^4+9a^2d\,x^2-8abc\,x^2+5a^2c\right)}{45e^5\sqrt{ex}\;a^3x^4}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
[Out] -2/45*x*(b*x^2+a)^(1/4)*(-36*a*b*d*x^4+32*b^2*c*x^4+9*a^2*d*x^2-8*a*b*c*x^2
+5*a^2*c)/a^3/(e*x)^(11/2)
```

Maxima [**A**]
time = 0.27, size = 94, normalized size = 0.90

$$\frac{2}{45}\left(\frac{9\,d\left(\frac{5\,(bx^2+a)^{\frac{1}{4}}b}{\sqrt{x}}-\frac{(bx^2+a)^{\frac{5}{4}}}{x^{\frac{5}{2}}}\right)}{a^2}-\frac{\left(\frac{45\,(bx^2+a)^{\frac{1}{4}}b^2}{\sqrt{x}}-\frac{18\,(bx^2+a)^{\frac{5}{4}}b}{x^{\frac{5}{2}}}+\frac{5\,(bx^2+a)^{\frac{9}{4}}}{x^{\frac{9}{2}}}\right)c}{a^3}\right)e^{\left(-\frac{11}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/45*(9*d*(5*(b*x^2 + a)^(1/4)*b/sqrt(x) - (b*x^2 + a)^(5/4)/x^(5/2))/a^2 -
 (45*(b*x^2 + a)^(1/4)*b^2/sqrt(x) - 18*(b*x^2 + a)^(5/4)*b/x^(5/2) + 5*(b*
x^2 + a)^(9/4)/x^(9/2))*c/a^3)*e^(-11/2)
```

Fricas [**A**]
time = 1.21, size = 60, normalized size = 0.58

$$-\frac{2\,(4\,(8\,b^2c-9\,abd)x^4+5\,a^2c-(8\,abc-9\,a^2d)x^2)(bx^2+a)^{\frac{1}{4}}e^{\left(-\frac{11}{2}\right)}}{45\,a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(4*(8*b^2*c - 9*a*b*d)*x^4 + 5*a^2*c - (8*a*b*c - 9*a^2*d)*x^2)*(b*x^
2 + a)^(1/4)*e^(-11/2)/(a^3*x^(9/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-11/2)/((b*x^2 + a)^(3/4)*x^(11/2)), x)
```

Mupad [B]
time = 0.69, size = 75, normalized size = 0.72

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2\,c}{9\,a\,e^5}+\frac{x^2\,\left(18\,a^2\,d-16\,a\,b\,c\right)}{45\,a^3\,e^5}+\frac{x^4\,\left(64\,b^2\,c-72\,a\,b\,d\right)}{45\,a^3\,e^5}\right)}{x^4\,\sqrt{e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*((2*c)/(9*a*e^5) + (x^2*(18*a^2*d - 16*a*b*c))/(45*a^3*
e^5) + (x^4*(64*b^2*c - 72*a*b*d))/(45*a^3*e^5)))/(x^4*(e*x)^(1/2))
```

3.1098 $\int \frac{c+dx^2}{(ex)^{15/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}+\frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}}-\frac{16(12bc-13ad)\left(a+bx^2\right)^{5/4}}{65a^3e^3(ex)^{9/2}}+\frac{64(12bc-13ad)\left(a+bx^2\right)^{9/4}}{585a^4e^3(ex)^{9/2}}$$

```
[Out] -2/13*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(13/2)+2/13*(-13*a*d+12*b*c)*(b*x^2+a)^(1
/4)/a^2/e^3/(e*x)^(9/2)-16/65*(-13*a*d+12*b*c)*(b*x^2+a)^(5/4)/a^3/e^3/(e*x
)^(9/2)+64/585*(-13*a*d+12*b*c)*(b*x^2+a)^(9/4)/a^4/e^3/(e*x)^(9/2)
```

Rubi [A]
time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {464, 279, 270}

$$\frac{64\left(a+bx^2\right)^{9/4}(12bc-13ad)}{585a^4e^3(ex)^{9/2}}-\frac{16\left(a+bx^2\right)^{5/4}(12bc-13ad)}{65a^3e^3(ex)^{9/2}}+\frac{2\sqrt[4]{a+bx^2}(12bc-13ad)}{13a^2e^3(ex)^{9/2}}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(13*a*e*(e*x)^(13/2)) + (2*(12*b*c - 13*a*d)*(a +
b*x^2)^(1/4))/(13*a^2*e^3*(e*x)^(9/2)) - (16*(12*b*c - 13*a*d)*(a + b*x^2)^
(5/4))/(65*a^3*e^3*(e*x)^(9/2)) + (64*(12*b*c - 13*a*d)*(a + b*x^2)^(9/4))/
(585*a^4*e^3*(e*x)^(9/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}} - \frac{(12bc-13ad)\int \frac{1}{(ex)^{11/2}(a+bx^2)^{3/4}}\,dx}{13ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}} + \frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}} + \frac{(8(12bc-13ad))\int \frac{\sqrt[4]{a+bx^2}}{(ex)^{11/2}}\,dx}{13a^2e^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}} + \frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}} - \frac{16(12bc-13ad)\left(a+bx^2\right)^{5/4}}{65a^3e^3(ex)^{9/2}} -$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}} + \frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}} - \frac{16(12bc-13ad)\left(a+bx^2\right)^{5/4}}{65a^3e^3(ex)^{9/2}} +$$

Mathematica [A]
time = 0.46, size = 91, normalized size = 0.65

$$-\frac{2x\sqrt[4]{a+bx^2}\left(45a^3c-60a^2bcx^2+65a^3dx^2+96ab^2cx^4-104a^2bdx^4-384b^3cx^6+416ab^2dx^6\right)}{585a^4(ex)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*(45*a^3*c - 60*a^2*b*c*x^2 + 65*a^3*d*x^2 + 96*a*b^
2*c*x^4 - 104*a^2*b*d*x^4 - 384*b^3*c*x^6 + 416*a*b^2*d*x^6))/(585*a^4*(e*x
)^(15/2))
```

Maple [A]
time = 0.10, size = 86, normalized size = 0.61

method	result	size
gosper	$-\frac{2x\left(b\,x^2+a\right)^{\frac{1}{4}}\left(416a\,b^2d\,x^6-384b^3c\,x^6-104a^2bd\,x^4+96a\,b^2c\,x^4+65a^3d\,x^2-60a^2bc\,x^2+45c\,a^3\right)}{585a^4(ex)^{\frac{15}{2}}}$	86
risch	$-\frac{2\left(b\,x^2+a\right)^{\frac{1}{4}}\left(416a\,b^2d\,x^6-384b^3c\,x^6-104a^2bd\,x^4+96a\,b^2c\,x^4+65a^3d\,x^2-60a^2bc\,x^2+45c\,a^3\right)}{585e^7\sqrt{ex}\,a^4x^6}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/585*x*(b*x^2+a)^(1/4)*(416*a*b^2*d*x^6-384*b^3*c*x^6-104*a^2*b*d*x^4+96*
a*b^2*c*x^4+65*a^3*d*x^2-60*a^2*b*c*x^2+45*a^3*c)/a^4/(e*x)^(15/2)
```

Maxima [A]
time = 0.27, size = 128, normalized size = 0.91

$$-\frac{2}{585}\left(\frac{13\left(\frac{45\,(bx^2+a)^{\frac{1}{4}}b^2}{\sqrt{x}}-\frac{18\,(bx^2+a)^{\frac{5}{4}}b}{x^{\frac{5}{2}}}+\frac{5\,(bx^2+a)^{\frac{9}{4}}}{x^{\frac{9}{2}}}\right)d}{a^3}-\frac{3\left(\frac{195\,(bx^2+a)^{\frac{1}{4}}b^3}{\sqrt{x}}-\frac{117\,(bx^2+a)^{\frac{5}{4}}b^2}{x^{\frac{5}{2}}}+\frac{65\,(bx^2+a)^{\frac{9}{4}}b}{x^{\frac{9}{2}}}-\frac{15\,(bx^2+a)^{\frac{13}{4}}}{x^{\frac{13}{2}}}\right)c}{a^4}\right)e^{\left(-\frac{15}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -2/585*(13*(45*(b*x^2 + a)^(1/4)*b^2/sqrt(x) - 18*(b*x^2 + a)^(5/4)*b/x^(5/
2) + 5*(b*x^2 + a)^(9/4)/x^(9/2))*d/a^3 - 3*(195*(b*x^2 + a)^(1/4)*b^3/sqrt
(x) - 117*(b*x^2 + a)^(5/4)*b^2/x^(5/2) + 65*(b*x^2 + a)^(9/4)*b/x^(9/2) -
15*(b*x^2 + a)^(13/4)/x^(13/2))*c/a^4)*e^(-15/2)
```

Fricas [A]
time = 1.82, size = 84, normalized size = 0.60

$$\frac{2\,(32\,(12\,b^3c-13\,ab^2d)x^6-8\,(12\,ab^2c-13\,a^2bd)x^4-45\,a^3c+5\,(12\,a^2bc-13\,a^3d)x^2)(bx^2+a)^{\frac{1}{4}}e^{\left(-\frac{15}{2}\right)}}{585\,a^4x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/585*(32*(12*b^3*c - 13*a*b^2*d)*x^6 - 8*(12*a*b^2*c - 13*a^2*b*d)*x^4 - 4
5*a^3*c + 5*(12*a^2*b*c - 13*a^3*d)*x^2)*(b*x^2 + a)^(1/4)*e^(-15/2)/(a^4*x
^(13/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(15/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-15/2)/((b*x^2 + a)^(3/4)*x^(15/2)), x)
```

Mupad [B]
time = 0.67, size = 100, normalized size = 0.71

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2\,c}{13\,a\,e^7}+\frac{x^2\,\left(130\,a^3\,d-120\,a^2\,b\,c\right)}{585\,a^4\,e^7}-\frac{x^6\,\left(768\,b^3\,c-832\,a\,b^2\,d\right)}{585\,a^4\,e^7}-\frac{16\,b\,x^4\,\left(13\,a\,d-12\,b\,c\right)}{585\,a^3\,e^7}\right)}{x^6\,\sqrt{e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*((2*c)/(13*a*e^7) + (x^2*(130*a^3*d - 120*a^2*b*c))/(58
5*a^4*e^7) - (x^6*(768*b^3*c - 832*a*b^2*d))/(585*a^4*e^7) - (16*b*x^4*(13*
a*d - 12*b*c))/(585*a^3*e^7)))/(x^6*(e*x)^(1/2))
```

3.1099 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=180

$$-\frac{a(10bc-9ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{12b^3}+\frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}-\frac{a^{3/2}(10bc-9ad)}{}$$

```
[Out] 1/30*(-9*a*d+10*b*c)*e*(e*x)^(5/2)*(b*x^2+a)^(1/4)/b^2+1/5*d*(e*x)^(9/2)*(b
*x^2+a)^(1/4)/b/e-1/12*a^(3/2)*(-9*a*d+10*b*c)*e^2*(1+a/b/x^2)^(3/4)*(e*x)^
(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)
/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b
*x^2+a)^(3/4)-1/12*a*(-9*a*d+10*b*c)*e^3*(b*x^2+a)^(1/4)*(e*x)^(1/2)/b^3
```

Rubi [A]
time = 0.09, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {470, 327, 335, 243, 342, 281, 237}

$$-\frac{a^{3/2}e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-9ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{ae^3\sqrt{ex}\,\sqrt[4]{a+bx^2}\,(10bc-9ad)}{12b^3}+\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}\,(10bc-9ad)}{30b^2}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] -1/12*(a*(10*b*c - 9*a*d)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/b^3 + ((10*b*c -
 9*a*d)*e*(e*x)^(5/2)*(a + b*x^2)^(1/4))/(30*b^2) + (d*(e*x)^(9/2)*(a + b*x
^2)^(1/4))/(5*b*e) - (a^(3/2)*(10*b*c - 9*a*d)*e^2*(1 + a/(b*x^2))^(3/4)*(e
*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*b^(5/2)*(a + b*x
^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} - \frac{\left(-5bc+\frac{9ad}{2}\right)\int \frac{(ex)^{7/2}}{(a+bx^2)^{3/4}}\,dx}{5b}$$

$$= \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} - \frac{(a(10bc-9ad)e^2)\int \frac{(ex}{(a+b}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{}}{5b}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{}}{5b}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{}}{5b}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{}}{5b}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{}}{5b}$$

$$= -\frac{a(10bc-9ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{}}{5b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 123, normalized size = 0.68

$$\frac{e^3\sqrt{ex}\left((a+bx^2)\left(45a^2d+4b^2x^2(5c+3dx^2)-2ab(25c+9dx^2)\right)+5a^2(10bc-9ad)\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{60b^3(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (e^3*Sqrt[e*x]*((a + b*x^2)*(45*a^2*d + 4*b^2*x^2*(5*c + 3*d*x^2) - 2*a*b*(
25*c + 9*d*x^2)) + 5*a^2*(10*b*c - 9*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeomet
ric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{7}{2}} (d\,x^2+c)}{(b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((d*x^2 + c)*x^(7/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^5 + c*x^3)*sqrt(x)*e^(7/2)/(b*x^2 + a)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 57.56, size = 94, normalized size = 0.52

$$\frac{ce^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}+\frac{de^{\frac{7}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] c*e**(7/2)*x**(9/2)*gamma(9/4)*hyper((3/4, 9/4), (13/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(3/4)*gamma(13/4)) + d*e**(7/2)*x**(13/2)*gamma(13/4)*hyper(
(3/4, 13/4), (17/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(17/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*x^(7/2)*e^(7/2)/(b*x^2 + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{7/2}\,(d\,x^2+c)}{(b\,x^2+a)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
[Out] int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

3.1100 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=139

$$\frac{(6bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^2}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}+\frac{\sqrt{a}\,(6bc-5ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\right.}{6b^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] 1/3*d*(e*x)^(5/2)*(b*x^2+a)^(1/4)/b/e+1/6*(-5*a*d+6*b*c)*(1+a/b/x^2)^(3/4)*
(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b
^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1
/2)/b^(3/2)/(b*x^2+a)^(3/4)+1/6*(-5*a*d+6*b*c)*e*(b*x^2+a)^(1/4)*(e*x)^(1/2
)/b^2
```

Rubi [A]
time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$,
Rules used = {470, 327, 335, 243, 342, 281, 237}

$$\frac{\sqrt{a}\,(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}\,(6bc-5ad)}{6b^2}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((6*b*c - 5*a*d)*e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^2) + (d*(e*x)^(5/2)*(a
 + b*x^2)^(1/4))/(3*b*e) + (Sqrt[a]*(6*b*c - 5*a*d)*(1 + a/(b*x^2))^(3/4)*(
e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*b^(3/2)*(a + b*x
^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} - \frac{\left(-3bc+\frac{5ad}{2}\right)\int \frac{(ex)^{3/2}}{(a+bx^2)^{3/4}}\,dx}{3b}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} - \frac{(a(6bc-5ad)e^2)\int \frac{1}{\sqrt{ex}\,(a+b\cdots}}{12b^2}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} - \frac{(a(6bc-5ad)e)\text{Subst}\left(\int \frac{\cdots}{(a+\cdots}\right.}{6b^2}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} - \frac{\left(a(6bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(\cdots\right.}{6\cdots}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} + \frac{\left(a(6bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(\cdots\right.}{6\cdots}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} + \frac{\left(a(6bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(\cdots\right.}{12\cdots}$$

$$= \frac{(6bc-5ad)e\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} + \frac{\sqrt{a}\,(6bc-5ad)\left(1+\frac{a}{bx^2}\right)^{3/4}\cdots}{6b^{3/2}(a\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 97, normalized size = 0.70

$$\frac{e\sqrt{ex}\left(-\left(\left(a+bx^2\right)\left(5ad-2b\left(3c+dx^2\right)\right)\right)+a(-6bc+5ad)\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{6b^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (e*Sqrt[e*x]*(-((a + b*x^2)*(5*a*d - 2*b*(3*c + d*x^2))) + a*(-6*b*c + 5*a*
d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(
6*b^2*(a + b*x^2)^(3/4))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}\,(d\,x^{2}+c)}{(b\,x^{2}+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((d*x^2 + c)*x^(3/2)/(b*x^2 + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c*x)*sqrt(x)*e^(3/2)/(b*x^2 + a)^(3/4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.94, size = 94, normalized size = 0.68

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}+\frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((3/4
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(13/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^(3/2)*e^(3/2)/(b*x^2 + a)^(3/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{3/2}\,(d\,x^2+c)}{(b\,x^2+a)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
```

```
[Out] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

3.1101 $\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=102

$$\frac{d\sqrt{ex}\,\sqrt[4]{a+bx^2}}{be}-\frac{(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,e^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] -(-a*d+2*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x
*b^(1/2)/a^(1/2))),2^(1/2))/e^2/(b*x^2+a)^(3/4)/a^(1/2)/b^(1/2)+d*(b*x^2+a)
^(1/4)*(e*x)^(1/2)/b/e
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 335, 243, 342, 281, 237}

$$\frac{d\sqrt{ex}\,\sqrt[4]{a+bx^2}}{be}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,e^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (d*Sqrt[e*x]*(a + b*x^2)^(1/4))/(b*e) - ((2*b*c - a*d)*(1 + a/(b*x^2))^(3/4
)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]
*e^2*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx = \frac{d\sqrt{ex}\,\sqrt[4]{a+bx^2}}{be} - \frac{\left(-bc+\frac{ad}{2}\right)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx}{b}$$

$$= \frac{d\sqrt{ex}\,\sqrt[4]{a+bx^2}}{be} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{be}$$

$$= \frac{d\sqrt{ex}\,\sqrt[4]{a+bx^2}}{be} + \frac{\left((2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2}{bx^4}\right)^{3/4}x^3}\,dx, x\right.}{be\,(a+bx^2)^{3/4}}$$

$$= \frac{d\sqrt{ex}\,\sqrt[4]{a+bx^2}}{be} - \frac{\left((2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ae^2x^4}{b}\right)^{3/4}}\,dx, x\right.}{be\,(a+bx^2)^{3/4}}$$

$$= \frac{d\sqrt{ex}\,\sqrt[4]{a+bx^2}}{be} - \frac{\left((2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2x^2}{b}\right)^{3/4}}\,dx, x\right.}{2be\,(a+bx^2)^{3/4}}$$

$$= \frac{d\sqrt{ex}\,\sqrt[4]{a+bx^2}}{be} - \frac{(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,e^2\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 77, normalized size = 0.75

$$\frac{dx(a+bx^2)+(2bc-ad)x\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{b\sqrt{ex}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (d*x*(a + b*x^2) + (2*b*c - a*d)*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[
1/4, 3/4, 5/4, -((b*x^2)/a)])/(b*Sqrt[e*x]*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{\sqrt{ex}\,\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(x)*e^(-1/2)/(b*x^3 + a*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.20, size = 78, normalized size = 0.76

$$-\frac{c\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{3}{4}}\sqrt{e}\,x}+\frac{dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -c*hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*sqrt(e)*
x) + d*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(I*pi)
/a)/(2*a**(3/4)*sqrt(e)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-1/2)/((b*x^2 + a)^(3/4)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{\sqrt{e\,x}\,\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(3/4)),x)
[Out] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(3/4)), x)
```

3.1102 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=107

$$-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}+\frac{2\sqrt{b}\,(2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2/3*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(3/2)+2/3*(-3*a*d+2*b*c)*(1+a/b/x^2)^(3/4)
*(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*
b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(
1/2)/a^(3/2)/e^4/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {464, 335, 243, 342, 281, 237}

$$\frac{2\sqrt{b}\,(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(3*a*e*(e*x)^(3/2)) + (2*Sqrt[b]*(2*b*c - 3*a*d)*(
1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(3*a^(3/2)*e^4*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}} - \frac{(2bc-3ad)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx}{3ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}} - \frac{(2(2bc-3ad))\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{3ae^3}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}} - \frac{\left(2(2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2}{bx^4}\right)^{3/4}x^3}\,dx, x\right.}{3ae^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}} + \frac{\left(2(2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ae^2x^4}{b}\right)^{3/4}}\,dx, x\right.}{3ae^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}} + \frac{\left((2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2x^2}{b}\right)^{3/4}}\,dx, x,\right.}{3ae^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}} + \frac{2\sqrt{b}\,(2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 84, normalized size = 0.79

$$\frac{x\left(-2c(a+bx^2)+2(-2bc+3ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3a(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (x*(-2*c*(a + b*x^2) + 2*(-2*b*c + 3*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*Hyperge
ometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*a*(e*x)^(5/2)*(a + b*x^2)^(3/4
))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{(ex)^{\frac{5}{2}}\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*x^(5/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(x)*e^(-5/2)/(b*x^5 + a*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.24, size = 82, normalized size = 0.77

$$-\frac{d\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{3}{4}}e^{\frac{5}{2}}x}+\frac{c\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -d*hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*e**(5/2)
*x) + c*gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(3/4)*e**(5/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-5/2)/((b*x^2 + a)^(3/4)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{5/2}\,{\left(b\,x^2+a\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x)
[Out] int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)), x)
```

3.1103 $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=144

$$-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}+\frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}}-\frac{4b^{3/2}(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}e^6\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2/7*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(7/2)+2/21*(-7*a*d+6*b*c)*(b*x^2+a)^(1/4)/
a^2/e^3/(e*x)^(3/2)-4/21*b^(3/2)*(-7*a*d+6*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(3/
2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^
(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/e^6/(
b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {464, 331, 335, 243, 342, 281, 237}

$$-\frac{4b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(6bc-7ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{2\sqrt[4]{a+bx^2}\,(6bc-7ad)}{21a^2e^3(ex)^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(7*a*e*(e*x)^(7/2)) + (2*(6*b*c - 7*a*d)*(a + b*x^
2)^(1/4))/(21*a^2*e^3*(e*x)^(3/2)) - (4*b^(3/2)*(6*b*c - 7*a*d)*(1 + a/(b*x
^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(
5/2)*e^6*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} - \frac{(6bc-7ad)\int \frac{1}{(ex)^{5/2}(a+bx^2)^{3/4}}\,dx}{7ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} + \frac{(2b(6bc-7ad))\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,d}{21a^2e^4}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} + \frac{(4b(6bc-7ad))\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/}}\right.}{21a^2e^5}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} + \frac{\left(4b(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/}\right.}{21a^2e^5\,(}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} - \frac{\left(4b(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/}\right.}{21a^2e^5\,(}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} - \frac{\left(2b(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/}\right.}{21a^2e^5\,(a}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}} + \frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} - \frac{4b^{3/2}(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3}}{21a^{5/2}e^6\,(a+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 88, normalized size = 0.61

$$-\frac{2\sqrt{ex}\left(3c(a+bx^2)+(-6bc+7ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)\right)}{21ae^5x^4\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(3*c*(a + b*x^2) + (-6*b*c + 7*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)
*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x^2)/a)]))/(21*a*e^5*x^4*(a + b*x^2
)^(3/4))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{(ex)^{\frac{9}{2}}\,(b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*x^(9/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(x)*e^(-9/2)/(b*x^7 + a*x^5), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 127.39, size = 85, normalized size = 0.59

$$-\frac{c\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{5b^{\frac{3}{4}}e^{\frac{9}{2}}x^5}+\frac{d\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{\frac{9}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -c*hyper((3/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(3/4)*e**(9/
2)*x**5) + d*gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**2*exp_polar(I*pi)/
a)/(2*a**(3/4)*e**(9/2)*x**(3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-9/2)/((b*x^2 + a)^(3/4)*x^(9/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{9/2}\,{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x)
[Out] int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)), x)
```

3.1104 $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=182

$$-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}+\frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}}-\frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}}+\frac{8b^{5/2}(10bc-11ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex}{77a^{7/2}e^8\,(a+}$$

```
[Out] -2/11*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(11/2)+2/77*(-11*a*d+10*b*c)*(b*x^2+a)^(1
/4)/a^2/e^3/(e*x)^(7/2)-4/77*b*(-11*a*d+10*b*c)*(b*x^2+a)^(1/4)/a^3/e^5/(e*
x)^(3/2)+8/77*b^(5/2)*(-11*a*d+10*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(3/2)*(cos(1
/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*E
llipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/e^8/(b*x^2+a)^
(3/4)
```

Rubi [A]
time = 0.09, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {464, 331, 335, 243, 342, 281, 237}

$$\frac{8b^{5/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}e^8\left(a+bx^2\right)^{3/4}}-\frac{4b\sqrt[4]{a+bx^2}\,(10bc-11ad)}{77a^3e^5(ex)^{3/2}}+\frac{2\sqrt[4]{a+bx^2}\,(10bc-11ad)}{77a^2e^3(ex)^{7/2}}-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(11*a*e*(e*x)^(11/2)) + (2*(10*b*c - 11*a*d)*(a +
b*x^2)^(1/4))/(77*a^2*e^3*(e*x)^(7/2)) - (4*b*(10*b*c - 11*a*d)*(a + b*x^2)
^(1/4))/(77*a^3*e^5*(e*x)^(3/2)) + (8*b^(5/2)*(10*b*c - 11*a*d)*(1 + a/(b*x
^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*a^(
7/2)*e^8*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} - \frac{(10bc-11ad)\int \frac{1}{(ex)^{9/2}(a+bx^2)^{3/4}}\,dx}{11ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} + \frac{(6b(10bc-11ad))\int \frac{1}{(ex)^{5/2}(a+bx^2)}}{77a^2e^4}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} - \frac{(4}{}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} - \frac{(8}{}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} - \frac{(8}{}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} + \frac{(8}{}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} + \frac{(4}{}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} + \frac{8b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 88, normalized size = 0.48

$$-\frac{2\sqrt{ex}\left(7c(a+bx^2)+(-10bc+11ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};-\frac{bx^2}{a}\right)\right)}{77ae^7x^6(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(7*c*(a + b*x^2) + (-10*b*c + 11*a*d)*x^2*(1 + (b*x^2)/a)^(3/
4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x^2)/a)]))/(77*a*e^7*x^6*(a + b*
x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{13}{2}} \left(b\,x^{2}+a \right)^{\frac{3}{4}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x)
[Out] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] e^(-13/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*x^(13/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(x)*e^(-13/2)/(b*x^9 + a*x^7), x
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(3/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-13/2)/((b*x^2 + a)^(3/4)*x^(13/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{13/2}\,{\left(b\,x^2+a\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x)
[Out] int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)), x)
```

3.1105 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=171

$$-\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}+\frac{(4bc-5ad)e^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{(4bc-5ad)e^{3/2}\tanh^{-1}\left(\cdots\right.}{4b^{9/4}}$$

```
[Out] 1/2*d*(e*x)^(5/2)/b/e/(b*x^2+a)^(1/4)+1/4*(-5*a*d+4*b*c)*e^(3/2)*arctan(b^(
1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(9/4)+1/4*(-5*a*d+4*b*c)*e^(3/2
)*arctanh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(9/4)-1/2*(-5*a*d+
4*b*c)*e*(e*x)^(1/2)/b^2/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.07, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {470, 294, 335, 246, 218, 214, 211}

$$\frac{e^{3/2}(4bc-5ad)\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{e^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{e\sqrt{ex}\,(4bc-5ad)}{2b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] -1/2*((4*b*c - 5*a*d)*e*Sqrt[e*x])/(b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(5/2)
)/(2*b*e*(a + b*x^2)^(1/4)) + ((4*b*c - 5*a*d)*e^(3/2)*ArcTan[(b^(1/4)*Sqrt
[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4)) + ((4*b*c - 5*a*d)*e^(3/2)
*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} - \frac{\left(-2bc+\frac{5ad}{2}\right)\int \frac{(ex)^{3/2}}{(a+bx^2)^{5/4}}\,dx}{2b}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{\left((4bc-5ad)e^2\right)\int \frac{1}{\sqrt{ex}\,\sqrt[4]{a+bx^2}}\,dx}{4b^2}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{((4bc-5ad)e)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{e^2}}}\,dx, x \ldots\right.}{2b^2}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{((4bc-5ad)e)\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{\ldots}}{\sqrt[4]{a \ldots}}\right.}{2b^2}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{\left((4bc-5ad)e^2\right)\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx, x, \ldots\right.}{4b^2}$$

$$= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{(4bc-5ad)e^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}$$

Mathematica [A]
time = 0.77, size = 148, normalized size = 0.87

$$\frac{(ex)^{3/2}\left(2\sqrt[4]{b}\,\sqrt{x}\left(-4bc+5ad+bdx^2\right)+(4bc-5ad)\sqrt[4]{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+(4bc-5ad)\sqrt[4]{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{9/4}x^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] ((e*x)^(3/2)*(2*b^(1/4)*Sqrt[x]*(-4*b*c + 5*a*d + b*d*x^2) + (4*b*c - 5*a*d
)*(a + b*x^2)^(1/4)*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + (4*b*c -
5*a*d)*(a + b*x^2)^(1/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(4*
b^(9/4)*x^(3/2)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)

[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)

Maxima [A]
time = 0.49, size = 229, normalized size = 1.34

$$\frac{1}{8}\left(\left(d\left(\frac{4\left(4\,ab-\frac{5\,(bx^2+a)a}{x^2}\right)}{\frac{(bx^2+a)^{\frac{1}{4}}b^3}{\sqrt{x}}-\frac{(bx^2+a)^{\frac{5}{4}}b^2}{x^{\frac{5}{2}}}}+\frac{5\,a\left(\frac{2\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}\right)-4\,c\left(\frac{\frac{2\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{1}{4}}}}{b}+\frac{4\sqrt{x}}{(bx^2+a)^{\frac{1}{4}}b}\right)\right)e^{\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")

[Out] 1/8*(d*(4*(4*a*b - 5*(b*x^2 + a)*a/x^2)/((b*x^2 + a)^(1/4)*b^3/sqrt(x) - (b*x^2 + a)^(5/4)*b^2/x^(5/2)) + 5*a*(2*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(1/4) + log(-(b^(1/4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(1/4))/b^2) - 4*c*((2*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(1/4) + log(-(b^(1/4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(1/4))/b + 4*sqrt(x)/((b*x^2 + a)^(1/4)*b)))*e^(3/2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 891 vs. 2(115) = 230.
time = 1.68, size = 891, normalized size = 5.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")

[Out] 1/8*(4*(b*d*x^2 - 4*b*c + 5*a*d)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(3/2) + 4*(b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(1/4)*arctan(((4*b^8*c - 5*a*b^7*d)*(b*x^2 + a)^(3/4)*sqrt(x)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(3/4)*e^6 + (b^8*x^2 + a*b^7)*sqrt(((16*b^2*c^2 - 40*a*b*c*d + 25*a^2*d^2)*sqrt(b*x^2 + a)*x*e^3 + (b^5*x^2 + a*b^4)*sqrt((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)*e^3)/(b*x^2 + a))*((256*b^4*c^4 - 1280*a*b^

```
3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(3/4)
*e^(9/2))*e^(-6)/(256*a*b^4*c^4 - 1280*a^2*b^3*c^3*d + 2400*a^3*b^2*c^2*d^2
 - 2000*a^4*b*c*d^3 + 625*a^5*d^4 + (256*b^5*c^4 - 1280*a*b^4*c^3*d + 2400*
a^2*b^3*c^2*d^2 - 2000*a^3*b^2*c*d^3 + 625*a^4*b*d^4)*x^2))*e^(3/2) + (b^3*
x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000
*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(1/4)*e^(3/2)*log(-((b*x^2 + a)^(3/4)*(4*b
*c - 5*a*d)*sqrt(x)*e^(3/2) + (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*
c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(1/4)*e
^(3/2))/(b*x^2 + a)) - (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d +
 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(1/4)*e^(3/2)*
log(-((b*x^2 + a)^(3/4)*(4*b*c - 5*a*d)*sqrt(x)*e^(3/2) - (b^3*x^2 + a*b^2)
*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3
 + 625*a^4*d^4)/b^9)^(1/4)*e^(3/2))/(b*x^2 + a)))/(b^3*x^2 + a*b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.02, size = 94, normalized size = 0.55

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}+\frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((5/4
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*x^(3/2)*e^(3/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(e\,x)}^{3/2}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x)
[Out] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4), x)
```

3.1106 $\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=122

$$\frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}$$

```
[Out] d*arctan(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(5/4)/e^(1/2)+d*arc
tanh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(5/4)/e^(1/2)+2*(-a*d+b
*c)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {463, 335, 246, 218, 214, 211}

$$\frac{d\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{2\sqrt{ex}\,(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(a*b*e*(a + b*x^2)^(1/4)) + (d*ArcTan[(b^(1/4)*Sq
rt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e]) + (d*ArcTanh[(b^(1
/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx &= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{d\int \frac{1}{\sqrt{ex}\,\sqrt[4]{a+bx^2}}\,dx}{b}\\
&= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{(2d)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{e^2}}}\,dx,x,\sqrt{ex}\right)}{be}\\
&= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{(2d)\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{e^2}}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{be}\\
&= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{d\text{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b}+\frac{d\text{Subst}\left(\int \frac{1}{e+\sqrt{b}\,x^2}\right.}{b}\\
&= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}
\end{aligned}
$$

Mathematica [A]
time = 0.44, size = 103, normalized size = 0.84

$$\frac{\sqrt{x}\left(\frac{2\sqrt[4]{b}\,(bc-ad)\sqrt{x}}{a\sqrt[4]{a+bx^2}}+d\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{b^{5/4}\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (Sqrt[x]*((2*b^(1/4)*(b*c - a*d)*Sqrt[x])/(a*(a + b*x^2)^(1/4)) + d*ArcTan[
(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + d*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x
^2)^(1/4)]))/(b^(5/4)*Sqrt[e*x])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{\sqrt{ex}\,\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

Maxima [A]
time = 0.50, size = 114, normalized size = 0.93

$$-\frac{1}{2}\left(d\left(\frac{\frac{2\arctan\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{1}{4}}}}{b}+\frac{4\sqrt{x}}{\left(bx^2+a\right)^{\frac{1}{4}}b}\right)-\frac{4\,c\sqrt{x}}{\left(bx^2+a\right)^{\frac{1}{4}}a}\right)e^{\left(-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] -1/2*(d*((2*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(1/4) + log(-(b^(
1/4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^
(1/4))/b + 4*sqrt(x)/((b*x^2 + a)^(1/4)*b)) - 4*c*sqrt(x)/((b*x^2 + a)^(1/4
)*a))*e^(-1/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(79) = 158$.
time = 1.48, size = 332, normalized size = 2.72

$$\frac{\left(4\left(bx^{2}+a\right)^{\frac{3}{4}}(bc-ad)\sqrt{x}-4\left(ab^{2}x^{2}+a^{2}b\right)\left(\frac{d^{4}}{b^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^{2}+a\right)^{\frac{3}{4}}b^{4}d\sqrt{x}\left(\frac{d^{4}}{b^{5}}\right)^{\frac{3}{4}}-\left(b^{5}x^{2}+ab^{4}\right)\sqrt{\frac{\sqrt{bx^{2}+a}\,d^{2}x+\left(b^{3}x^{2}+ab^{2}\right)\sqrt{\frac{d^{4}}{b^{5}}}}{bx^{2}+a}}\left(\frac{d^{4}}{b^{5}}\right)^{\frac{3}{4}}}{bd^{4}x^{2}+ad^{4}}\right)+\left(ab^{2}x^{2}+a^{2}b\right)\left(\frac{d^{4}}{b^{5}}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^{2}+a\right)^{\frac{3}{4}}d\sqrt{x}+\left(b^{2}x^{2}+ab\right)\left(\frac{d^{4}}{b^{5}}\right)^{\frac{1}{4}}}{bx^{2}+a}\right)-\left(ab^{2}x^{2}+a^{2}b\right)\left(\frac{d^{4}}{b^{5}}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^{2}+a\right)^{\frac{3}{4}}d\sqrt{x}-\left(b^{2}x^{2}+ab\right)\left(\frac{d^{4}}{b^{5}}\right)^{\frac{1}{4}}}{bx^{2}+a}\right)\right)e^{\left(-\frac{1}{2}\right)}}{2\left(ab^{2}x^{2}+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(b*x^2 + a)^(3/4)*(b*c - a*d)*sqrt(x) - 4*(a*b^2*x^2 + a^2*b)*(d^4/b
^5)^(1/4)*arctan(-((b*x^2 + a)^(3/4)*b^4*d*sqrt(x)*(d^4/b^5)^(3/4) - (b^5*x
^2 + a*b^4)*sqrt((sqrt(b*x^2 + a)*d^2*x + (b^3*x^2 + a*b^2)*sqrt(d^4/b^5))/
(b*x^2 + a))*(d^4/b^5)^(3/4))/(b*d^4*x^2 + a*d^4)) + (a*b^2*x^2 + a^2*b)*(d
^4/b^5)^(1/4)*log(((b*x^2 + a)^(3/4)*d*sqrt(x) + (b^2*x^2 + a*b)*(d^4/b^5)^
(1/4))/(b*x^2 + a)) - (a*b^2*x^2 + a^2*b)*(d^4/b^5)^(1/4)*log(((b*x^2 + a)^
(3/4)*d*sqrt(x) - (b^2*x^2 + a*b)*(d^4/b^5)^(1/4))/(b*x^2 + a)))*e^(-1/2)/(
a*b^2*x^2 + a^2*b)
```

Sympy [C] Result contains complex when optimal does not.
time = 6.92, size = 83, normalized size = 0.68

$$\frac{c\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\,\sqrt{e}\,\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*gamma(1/4)/(2*a*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4)) + d*
x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(5/4)*sqrt(e)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-1/2)/((b*x^2 + a)^(5/4)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{\sqrt{e\,x}\,\left(b\,x^2+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/4)),x)
[Out] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/4)), x)
```

3.1107 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(4bc-3ad)\sqrt{ex}}{3a^2e^3\sqrt[4]{a+bx^2}}$$

```
[Out] -2/3*c/a/e/(e*x)^(3/2)/(b*x^2+a)^(1/4)-2/3*(-3*a*d+4*b*c)*(e*x)^(1/2)/a^2/e
^3/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {464, 270}

$$-\frac{2\sqrt{ex}\,(4bc-3ad)}{3a^2e^3\sqrt[4]{a+bx^2}}-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(1/4)) - (2*(4*b*c - 3*a*d)*Sqrt[e*x]
)/(3*a^2*e^3*(a + b*x^2)^(1/4))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}} - \frac{(4bc-3ad)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx}{3ae^2}$$
$$= -\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}} - \frac{2(4bc-3ad)\sqrt{ex}}{3a^2e^3\sqrt[4]{a+bx^2}}$$

Mathematica [A]
time = 0.27, size = 45, normalized size = 0.67

$$\frac{2x(-ac-4bcx^2+3adx^2)}{3a^2(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*x*(-(a*c) - 4*b*c*x^2 + 3*a*d*x^2))/(3*a^2*(e*x)^(5/2)*(a + b*x^2)^(1/4)
)
```

Maple [A]
time = 0.12, size = 39, normalized size = 0.58

method	result	size
gosper	$-\frac{2x(-3ad\,x^2+4c\,x^2b+ac)}{3(b\,x^2+a)^{\frac{1}{4}}a^2(ex)^{\frac{5}{2}}}$	39
risch	$-\frac{2c(b\,x^2+a)^{\frac{3}{4}}}{3a^2x\,e^2\sqrt{ex}}+\frac{2x(ad-bc)}{a^2e^2\sqrt{ex}\,(b\,x^2+a)^{\frac{1}{4}}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*x*(-3*a*d*x^2+4*b*c*x^2+a*c)/(b*x^2+a)^(1/4)/a^2/(e*x)^(5/2)
```

Maxima [A]
time = 0.29, size = 60, normalized size = 0.90

$$-\frac{2}{3}\left(c\left(\frac{3\,b\sqrt{x}}{(bx^2+a)^{\frac{1}{4}}a^2}+\frac{(bx^2+a)^{\frac{3}{4}}}{a^2x^{\frac{3}{2}}}\right)-\frac{3\,d\sqrt{x}}{(bx^2+a)^{\frac{1}{4}}a}\right)e^{\left(-\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] -2/3*(c*(3*b*sqrt(x)/((b*x^2 + a)^(1/4)*a^2) + (b*x^2 + a)^(3/4)/(a^2*x^(3/
2))) - 3*d*sqrt(x)/((b*x^2 + a)^(1/4)*a))*e^(-5/2)
```

Fricas [A]
time = 1.29, size = 51, normalized size = 0.76

$$-\frac{2\left(\left(4\,bc-3\,ad\right)x^{2}+ac\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{x}\,e^{\left(-\frac{5}{2}\right)}}{3\left(a^{2}bx^{4}+a^{3}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/3*((4*b*c - 3*a*d)*x^2 + a*c)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(-5/2)/(a^2*b*
x^4 + a^3*x^2)
```

Sympy [A]
time = 32.60, size = 117, normalized size = 1.75

$$c\left(\frac{\Gamma\left(-\frac{3}{4}\right)}{8a\sqrt[4]{b}\,e^{\frac{5}{2}}x^{2}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2a^{2}e^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(\frac{5}{4}\right)}\right)+\frac{d\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\,e^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*(gamma(-3/4)/(8*a*b**(1/4)*e**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(5/
4)) + b**(3/4)*gamma(-3/4)/(2*a**2*e**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(5
/4))) + d*gamma(1/4)/(2*a*b**(1/4)*e**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(5
/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-5/2)/((b*x^2 + a)^(5/4)*x^(5/2)), x)
```

Mupad [B]
time = 0.65, size = 70, normalized size = 1.04

$$-\frac{{\left(b\,x^2+a\right)}^{3/4}\,\left(\frac{2\,c}{3\,a\,b\,e^2}-\frac{x^2\,\left(6\,a\,d-8\,b\,c\right)}{3\,a^2\,b\,e^2}\right)}{x^3\,\sqrt{e\,x}+\frac{a\,x\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(3*a*b*e^2) - (x^2*(6*a*d - 8*b*c))/(3*a^2*b*e^2
)))/(x^3*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)
```

3.1108 $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}+\frac{8(8bc-7ad)\left(a+bx^2\right)^{3/4}}{21a^3e^3(ex)^{3/2}}$$

```
[Out] -2/7*c/a/e/(e*x)^(7/2)/(b*x^2+a)^(1/4)-2/7*(-7*a*d+8*b*c)/a^2/e^3/(e*x)^(3/
2)/(b*x^2+a)^(1/4)+8/21*(-7*a*d+8*b*c)*(b*x^2+a)^(3/4)/a^3/e^3/(e*x)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {464, 279, 270}

$$\frac{8\left(a+bx^2\right)^{3/4}(8bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(1/4)) - (2*(8*b*c - 7*a*d))/(7*a^2*e
^3*(e*x)^(3/2)*(a + b*x^2)^(1/4)) + (8*(8*b*c - 7*a*d)*(a + b*x^2)^(3/4))/(
21*a^3*e^3*(e*x)^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{(ex)^{9/2}\,(a+bx^2)^{5/4}}\,dx &= -\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{(8bc-7ad)\int \frac{1}{(ex)^{5/2}(a+bx^2)^{5/4}}\,dx}{7ae^2}\\ &= -\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{(4(8bc-7ad))\int \frac{1}{(ex)^{5/2}\sqrt[4]{a+}}}{7a^2e^2}\\ &= -\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}+\frac{8(8bc-7ad)\,(a+bx^2)^{3/4}}{21a^3e^3(ex)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.44, size = 67, normalized size = 0.64

$$-\frac{2x(3a^2c-8abcx^2+7a^2dx^2-32b^2cx^4+28abdx^4)}{21a^3(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
[Out] (-2*x*(3*a^2*c - 8*a*b*c*x^2 + 7*a^2*d*x^2 - 32*b^2*c*x^4 + 28*a*b*d*x^4))/
(21*a^3*(e*x)^(9/2)*(a + b*x^2)^(1/4))
```

Maple [A]
time = 0.11, size = 62, normalized size = 0.60

method	result	size
gosper	$-\frac{2x\left(28abd\,x^4-32b^2c\,x^4+7a^2d\,x^2-8abc\,x^2+3a^2c\right)}{21\left(b\,x^2+a\right)^{\frac{1}{4}}a^3\left(ex\right)^{\frac{9}{2}}}$	62
risch	$-\frac{2\left(b\,x^2+a\right)^{\frac{3}{4}}\left(7ad\,x^2-11c\,x^2b+3ac\right)}{21a^3x^3e^4\sqrt{ex}}-\frac{2bx(ad-bc)}{a^3e^4\sqrt{ex}\,\left(b\,x^2+a\right)^{\frac{1}{4}}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
[Out] -2/21*x*(28*a*b*d*x^4-32*b^2*c*x^4+7*a^2*d*x^2-8*a*b*c*x^2+3*a^2*c)/(b*x^2+
a)^(1/4)/a^3/(e*x)^(9/2)
```

Maxima [A]
time = 0.28, size = 101, normalized size = 0.97

$$-\frac{2}{21}\left(7\,d\left(\frac{3\,b\sqrt{x}}{(bx^2+a)^{\frac{1}{4}}a^2}+\frac{(bx^2+a)^{\frac{3}{4}}}{a^2x^{\frac{3}{2}}}\right)-c\left(\frac{21\,b^2\sqrt{x}}{(bx^2+a)^{\frac{1}{4}}a^3}+\frac{\frac{14\,(bx^2+a)^{\frac{3}{4}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx^2+a)^{\frac{7}{4}}}{x^{\frac{7}{2}}}}{a^3}\right)\right)e^{\left(-\frac{9}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] -2/21*(7*d*(3*b*sqrt(x)/((b*x^2 + a)^(1/4)*a^2) + (b*x^2 + a)^(3/4)/(a^2*x^
(3/2))) - c*(21*b^2*sqrt(x)/((b*x^2 + a)^(1/4)*a^3) + (14*(b*x^2 + a)^(3/4)
*b/x^(3/2) - 3*(b*x^2 + a)^(7/4)/x^(7/2))/a^3))*e^(-9/2)
```

Fricas [A]
time = 1.55, size = 74, normalized size = 0.71

$$\frac{2\,(4\,(8\,b^2c-7\,abd)x^4-3\,a^2c+(8\,abc-7\,a^2d)x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{x}\,e^{\left(-\frac{9}{2}\right)}}{21\,(a^3bx^6+a^4x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/21*(4*(8*b^2*c - 7*a*b*d)*x^4 - 3*a^2*c + (8*a*b*c - 7*a^2*d)*x^2)*(b*x^2
 + a)^(3/4)*sqrt(x)*e^(-9/2)/(a^3*b*x^6 + a^4*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-9/2)/((b*x^2 + a)^(5/4)*x^(9/2)), x)
```

Mupad [B]
time = 0.66, size = 101, normalized size = 0.97

$$-\frac{(b\,x^2+a)^{3/4}\,\left(\frac{2\,c}{7\,a\,b\,e^4}+\frac{x^2\,(14\,a^2\,d-16\,a\,b\,c)}{21\,a^3\,b\,e^4}-\frac{x^4\,(64\,b^2\,c-56\,a\,b\,d)}{21\,a^3\,b\,e^4}\right)}{x^5\,\sqrt{e\,x}+\frac{a\,x^3\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(7*a*b*e^4) + (x^2*(14*a^2*d - 16*a*b*c))/(21*a^
3*b*e^4) - (x^4*(64*b^2*c - 56*a*b*d))/(21*a^3*b*e^4)))/(x^5*(e*x)^(1/2) +
(a*x^3*(e*x)^(1/2))/b)
```

3.1109 $\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}-\frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}+\frac{16(12bc-11ad)\left(a+bx^2\right)^{3/4}}{33a^3e^3(ex)^{7/2}}-\frac{64(12bc-11ad)\,(a+b}{231a^4e^3(ex)^{7/2}}$$

```
[Out] -2/11*c/a/e/(e*x)^(11/2)/(b*x^2+a)^(1/4)-2/11*(-11*a*d+12*b*c)/a^2/e^3/(e*x
)^(7/2)/(b*x^2+a)^(1/4)+16/33*(-11*a*d+12*b*c)*(b*x^2+a)^(3/4)/a^3/e^3/(e*x
)^(7/2)-64/231*(-11*a*d+12*b*c)*(b*x^2+a)^(7/4)/a^4/e^3/(e*x)^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 279, 270}

$$-\frac{64\left(a+bx^2\right)^{7/4}(12bc-11ad)}{231a^4e^3(ex)^{7/2}}+\frac{16\left(a+bx^2\right)^{3/4}(12bc-11ad)}{33a^3e^3(ex)^{7/2}}-\frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(1/4)) - (2*(12*b*c - 11*a*d))/(11*
a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (16*(12*b*c - 11*a*d)*(a + b*x^2)^
(3/4))/(33*a^3*e^3*(e*x)^(7/2)) - (64*(12*b*c - 11*a*d)*(a + b*x^2)^(7/4))/
(231*a^4*e^3*(e*x)^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{(12bc-11ad)\int \frac{1}{(ex)^{9/2}(a+bx^2)^{5/4}}\,dx}{11ae^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} - \frac{(8(12bc-11ad))\int \frac{}{(ex)^9}}{11a^2e^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} + \frac{16(12bc-11ad)(a+bx}{33a^3e^3(ex)^{7/2}}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} + \frac{16(12bc-11ad)(a+bx}{33a^3e^3(ex)^{7/2}}$$

Mathematica [A]
time = 0.69, size = 91, normalized size = 0.65

$$-\frac{2x(21a^3c-36a^2bcx^2+33a^3dx^2+96ab^2cx^4-88a^2bdx^4+384b^3cx^6-352ab^2dx^6)}{231a^4(ex)^{13/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
[Out] (-2*x*(21*a^3*c - 36*a^2*b*c*x^2 + 33*a^3*d*x^2 + 96*a*b^2*c*x^4 - 88*a^2*b
*d*x^4 + 384*b^3*c*x^6 - 352*a*b^2*d*x^6))/(231*a^4*(e*x)^(13/2)*(a + b*x^2
)^(1/4))
```

Maple [A]
time = 0.12, size = 86, normalized size = 0.61

method	result	size
gosper	$-\frac{2x\left(-352a\,b^2d\,x^6+384b^3c\,x^6-88a^2bd\,x^4+96a\,b^2c\,x^4+33a^3d\,x^2-36a^2bc\,x^2+21c\,a^3\right)}{231\left(b\,x^2+a\right)^{\frac{1}{4}}a^4\left(ex\right)^{\frac{13}{2}}}$	86
risch	$-\frac{2\left(b\,x^2+a\right)^{\frac{3}{4}}\left(-121abd\,x^4+153b^2c\,x^4+33a^2d\,x^2-57abc\,x^2+21a^2c\right)}{231a^4x^5e^6\sqrt{ex}}+\frac{2b^2x(ad-bc)}{a^4e^6\sqrt{ex}\,\left(b\,x^2+a\right)^{\frac{1}{4}}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231*x*(-352*a*b^2*d*x^6+384*b^3*c*x^6-88*a^2*b*d*x^4+96*a*b^2*c*x^4+33*a
^3*d*x^2-36*a^2*b*c*x^2+21*a^3*c)/(b*x^2+a)^(1/4)/a^4/(e*x)^(13/2)
```

Maxima [A]
time = 0.28, size = 138, normalized size = 0.98

$$\frac{2}{231}\left(11\,d\left(\frac{21\,b^2\sqrt{x}}{(bx^2+a)^{\frac{1}{4}}a^3}+\frac{\frac{14\,(bx^2+a)^{\frac{3}{4}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx^2+a)^{\frac{7}{4}}}{x^{\frac{7}{2}}}}{a^3}\right)-3\,c\left(\frac{77\,b^3\sqrt{x}}{(bx^2+a)^{\frac{1}{4}}a^4}+\frac{\frac{77\,(bx^2+a)^{\frac{3}{4}}b^2}{x^{\frac{3}{2}}}-\frac{33\,(bx^2+a)^{\frac{7}{4}}b}{x^{\frac{7}{2}}}+\frac{7\,(bx^2+a)^{\frac{11}{4}}}{x^{\frac{11}{2}}}}{a^4}\right)\right)e^{\left(-\frac{13}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 2/231*(11*d*(21*b^2*sqrt(x)/((b*x^2 + a)^(1/4)*a^3) + (14*(b*x^2 + a)^(3/4)
*b/x^(3/2) - 3*(b*x^2 + a)^(7/4)/x^(7/2))/a^3) - 3*c*(77*b^3*sqrt(x)/((b*x^
2 + a)^(1/4)*a^4) + (77*(b*x^2 + a)^(3/4)*b^2/x^(3/2) - 33*(b*x^2 + a)^(7/4
)*b/x^(7/2) + 7*(b*x^2 + a)^(11/4)/x^(11/2))/a^4))*e^(-13/2)
```

Fricas [A]
time = 2.02, size = 99, normalized size = 0.70

$$-\frac{2\,(32\,(12\,b^3c-11\,ab^2d)x^6+8\,(12\,ab^2c-11\,a^2bd)x^4+21\,a^3c-3\,(12\,a^2bc-11\,a^3d)x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{x}\,e^{\left(-\frac{13}{2}\right)}}{231\,(a^4bx^8+a^5x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/231*(32*(12*b^3*c - 11*a*b^2*d)*x^6 + 8*(12*a*b^2*c - 11*a^2*b*d)*x^4 +
21*a^3*c - 3*(12*a^2*b*c - 11*a^3*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(-13/
2)/(a^4*b*x^8 + a^5*x^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4963 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-13/2)/((b*x^2 + a)^(5/4)*x^(13/2)), x)
```

Mupad [B]
time = 0.69, size = 125, normalized size = 0.89

$$-\frac{(b\,x^2+a)^{3/4}\,\left(\frac{2\,c}{11\,a\,b\,e^6}-\frac{16\,x^4\,(11\,a\,d-12\,b\,c)}{231\,a^3\,e^6}+\frac{x^2\,(66\,a^3\,d-72\,a^2\,b\,c)}{231\,a^4\,b\,e^6}+\frac{x^6\,(768\,b^3\,c-704\,a\,b^2\,d)}{231\,a^4\,b\,e^6}\right)}{x^7\,\sqrt{e\,x}+\frac{a\,x^5\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(11*a*b*e^6) - (16*x^4*(11*a*d - 12*b*c))/(231*a
^3*e^6) + (x^2*(66*a^3*d - 72*a^2*b*c))/(231*a^4*b*e^6) + (x^6*(768*b^3*c -
 704*a*b^2*d))/(231*a^4*b*e^6)))/(x^7*(e*x)^(1/2) + (a*x^5*(e*x)^(1/2))/b)
```

3.1110 $\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=180

$$-\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}}+\frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}-\frac{7a^{3/2}(10bc-11ad)e^4\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}}{20b^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -7/60*a*(-11*a*d+10*b*c)*e^3*(e*x)^(3/2)/b^3/(b*x^2+a)^(1/4)+1/30*(-11*a*d+
10*b*c)*e*(e*x)^(7/2)/b^2/(b*x^2+a)^(1/4)+1/5*d*(e*x)^(11/2)/b/e/(b*x^2+a)^
(1/4)-7/20*a^(3/2)*(-11*a*d+10*b*c)*e^4*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x
*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(si
n(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/b^(7/2)/(b*x^2+a)^(1/
4)
```

Rubi [A]

time = 0.07, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 291, 290, 342, 202}

$$-\frac{7a^{3/2}e^4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\,(10bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}-\frac{7ae^3(ex)^{3/2}(10bc-11ad)}{60b^3\sqrt[4]{a+bx^2}}+\frac{e(ex)^{7/2}(10bc-11ad)}{30b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (-7*a*(10*b*c - 11*a*d)*e^3*(e*x)^(3/2))/(60*b^3*(a + b*x^2)^(1/4)) + ((10*
b*c - 11*a*d)*e*(e*x)^(7/2))/(30*b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(11/2))/
(5*b*e*(a + b*x^2)^(1/4)) - (7*a^(3/2)*(10*b*c - 11*a*d)*e^4*(1 + a/(b*x^2)
)^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(7/2)*
(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{9/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} - \frac{\left(-5bc+\frac{11ad}{2}\right)\int \frac{(ex)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{5b}$$

$$= \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} - \frac{\left(7a(10bc-11ad)e^2\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{60b^2}$$

$$= -\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} + \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} + \frac{(7a^2(10bc}{}$$

$$= -\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} + \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} + \frac{\left(7a^2(10b\right.}{}$$

$$= -\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} + \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} - \frac{\left(7a^2(10b\right.}{}$$

$$= -\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} + \frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}} - \frac{7a^{3/2}(10b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.11, size = 112, normalized size = 0.62

$$\frac{e^3(ex)^{3/2}\left(77a^2d+4b^2x^2(5c+3dx^2)-2ab(35c+11dx^2)+7a(10bc-11ad)\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{60b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(77*a^2*d + 4*b^2*x^2*(5*c + 3*d*x^2) - 2*a*b*(35*c + 11*d
*x^2) + 7*a*(10*b*c - 11*a*d)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4,
5/4, 7/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{9}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] e^(9/2)*integrate((d*x^2 + c)*x^(9/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^6 + c*x^4)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(9/2)/(b^2*x^4 + 2*a*b
*x^2 + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*x^(9/2)*e^(9/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{9/2}\,(d\,x^2+c)}{(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x)
[Out] int(((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4), x)
```

3.1111 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{5/4}} dx$

Optimal. Leaf size=142

$$\frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}+\frac{\sqrt{a}\,(6bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 1/6*(-7*a*d+6*b*c)*e*(e*x)^(3/2)/b^2/(b*x^2+a)^(1/4)+1/3*d*(e*x)^(7/2)/b/e/
(b*x^2+a)^(1/4)+1/2*(-7*a*d+6*b*c)*e^2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*
b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin
(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(e*x)^(1/2)/b^(5/2)/(b*x^2
+a)^(1/4)
```

Rubi [A]
time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 291, 290, 342, 202}

$$\frac{\sqrt{a}\,e^2\sqrt{ex}\,\sqrt[4]{\frac{a}{bx^2}+1}\,(6bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}(6bc-7ad)}{6b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] ((6*b*c - 7*a*d)*e*(e*x)^(3/2))/(6*b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(7/2))
/(3*b*e*(a + b*x^2)^(1/4)) + (Sqrt[a]*(6*b*c - 7*a*d)*e^2*(1 + a/(b*x^2))^(
1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(5/2)*(a +
 b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} - \frac{\left(-3bc+\frac{7ad}{2}\right)\int \frac{(ex)^{5/2}}{(a+bx^2)^{5/4}}\,dx}{3b}$$

$$= \frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} - \frac{\left(a(6bc-7ad)e^2\right)\int \frac{\sqrt{ex}}{(a+bx^2)^{5/4}}\,dx}{4b^2}$$

$$= \frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} - \frac{\left(a(6bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\right)\int \frac{}{\left(1+\frac{a}{bx^2}\right.}}{4b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} + \frac{\left(a(6bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\right)\text{Subst}\Big(}{4b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} + \frac{\sqrt{a}\,(6bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\Big(\frac{1}{2}\text{co}}{2b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.09, size = 85, normalized size = 0.60

$$\frac{e(ex)^{3/2}\left(6bc-7ad+2bdx^2+(-6bc+7ad)\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{6b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (e*(e*x)^(3/2)*(6*b*c - 7*a*d + 2*b*d*x^2 + (-6*b*c + 7*a*d)*(1 + (b*x^2)/a
)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)
^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{5}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((d*x^2 + c)*x^(5/2)/(b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c*x^2)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(5/2)/(b^2*x^4 + 2*a*b
*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 49.07, size = 94, normalized size = 0.66

$$\frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}+\frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(5/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hyper(
(5/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^(5/2)*e^(5/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{5/2}\,(d\,x^2+c)}{(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x)
```

```
[Out] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4), x)
```

3.1112 $\int \frac{\sqrt{ex}\,(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=99

$$\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}-\frac{(2bc-3ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] d*(e*x)^(3/2)/b/e/(b*x^2+a)^(1/4)-(-3*a*d+2*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2
*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*Ell
ipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/b^(3/2)/(b*x
^2+a)^(1/4)/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 290, 342, 202}

$$\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}-\frac{\sqrt{ex}\,\sqrt[4]{\frac{a}{bx^2}+1}\,(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (d*(e*x)^(3/2))/(b*e*(a + b*x^2)^(1/4)) - ((2*b*c - 3*a*d)*(1 + a/(b*x^2))^
(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*b^(3/
2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ex}\,(c+dx^2)}{(a+bx^2)^{5/4}}\,dx &= \frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}} - \frac{\left(-bc+\frac{3ad}{2}\right)\int \frac{\sqrt{ex}}{(a+bx^2)^{5/4}}\,dx}{b}\\ &= \frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}} - \frac{\left(\left(-bc+\frac{3ad}{2}\right)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{b^2\sqrt[4]{a+bx^2}}\\ &= \frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}} + \frac{\left(\left(-bc+\frac{3ad}{2}\right)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{b^2\sqrt[4]{a+bx^2}}\\ &= \frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,b^{3/2}\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 77, normalized size = 0.78

$$\frac{x\sqrt{ex}\left(3ad+(2bc-3ad)\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3ab\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (x*Sqrt[e*x]*(3*a*d + (2*b*c - 3*a*d)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2
F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(3*a*b*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] e^(1/2)*integrate((d*x^2 + c)*sqrt(x)/(b*x^2 + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(x)*e^(1/2)/(b^2*x^4 + 2*a*b*x^2
 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.89, size = 94, normalized size = 0.95

$$\frac{c\sqrt{e}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{d\sqrt{e}\,x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
[Out] c*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/4)*gamma(7/4)) + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((5/4,
7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(11/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*sqrt(x)*e^(1/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x)
[Out] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(5/4), x)
```

3.1113 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=103

$$-\frac{2c}{ae\sqrt{ex}\,\sqrt[4]{a+bx^2}}+\frac{2(2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}\,e^2\sqrt[4]{a+bx^2}}$$

```
[Out] -2*c/a/e/(b*x^2+a)^(1/4)/(e*x)^(1/2)+2*(-a*d+2*b*c)*(1+a/b/x^2)^(1/4)*(cos(
1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*
EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/a^(3/2)/e
^2/(b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {464, 290, 342, 202}

$$\frac{2\sqrt{ex}\,\sqrt[4]{\frac{a}{bx^2}+1}\,(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}\,e^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (2*(2*b*c - a*d)*(1 + a/(b*x^2))
^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*Sqrt
[b]*e^2*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{(2bc-ad)\int \frac{\sqrt{ex}}{(a+bx^2)^{5/4}}\,dx}{ae^2}$$

$$= -\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}} - \frac{\left((2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{abe^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{\left((2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x,$$

$$= -\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{2(2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}\,e^2\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 77, normalized size = 0.75

$$\frac{x\left(-6ac+2(-2bc+ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a^2(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (x*(-6*a*c + 2*(-2*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3
/4, 5/4, 7/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(3/2)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{3}{2}} \left(b\,x^{2}+a \right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*x^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(x)*e^(-3/2)/(b^2*x^6 + 2*a*b*x^
4 + a^2*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.46, size = 82, normalized size = 0.80

$$-\frac{d\,{}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{3}{2} \end{matrix}\middle| {\frac{a e^{i\pi}}{b x^{2}}} \right)}{b^{\frac{5}{4}} e^{\frac{3}{2}} x} + \frac{c \Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{3}{4} \end{matrix}\middle| {\frac{b x^{2} e^{i\pi}}{a}} \right)}{2 a^{\frac{5}{4}} e^{\frac{3}{2}} \sqrt{x} \Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -d*hyper((1/2, 5/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(5/4)*e**(3/2)
*x) + c*gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(5/4)*e**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-3/2)/((b*x^2 + a)^(5/4)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x)
[Out] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)), x)
```

3.1114 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=144

$$-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}+\frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}-\frac{4\sqrt{b}\,(6bc-5ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}e^4\sqrt[4]{a+bx^2}}$$

```
[Out] -2/5*c/a/e/(e*x)^(5/2)/(b*x^2+a)^(1/4)+2/5*(-5*a*d+6*b*c)/a^2/e^3/(b*x^2+a)
^(1/4)/(e*x)^(1/2)-4/5*(-5*a*d+6*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(e*x)^(1/2)/a^(5/2)/e^4/(b*
x^2+a)^(1/4)
```

Rubi [A]

time = 0.05, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {464, 292, 290, 342, 202}

$$-\frac{4\sqrt{b}\,\sqrt{ex}\,\sqrt[4]{\frac{a}{bx^2}+1}\,(6bc-5ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}e^4\sqrt[4]{a+bx^2}}+\frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(1/4)) + (2*(6*b*c - 5*a*d))/(5*a^2*e
^3*Sqrt[e*x]*(a + b*x^2)^(1/4)) - (4*Sqrt[b]*(6*b*c - 5*a*d)*(1 + a/(b*x^2)
)^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*e
^4*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
 + 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{(6bc-5ad)\int \frac{1}{(ex)^{3/2}(a+bx^2)^{5/4}}\,dx}{5ae^2}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}} + \frac{(2b(6bc-5ad))\int \frac{\sqrt{ex}}{(a+bx^2)^{5/4}}\,dx}{5a^2e^4}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}} + \frac{\left(2(6bc-5ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{e\ldots}\right.}{5a^2e^4\sqrt[4]{a+b\ldots}}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}} - \frac{\left(2(6bc-5ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{e\ldots}\right.}{5a^2e^4\ldots}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}} - \frac{4\sqrt{b}\,(6bc-5ad)\sqrt[4]{1+\frac{a}{bx^2}}\ldots}{5a^{5/2}e^4\sqrt[4]{\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 78, normalized size = 0.54

$$\frac{x\left(-2ac+2(6bc-5ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};-\frac{bx^2}{a}\right)\right)}{5a^2(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (x*(-2*a*c + 2*(6*b*c - 5*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[
-1/4, 5/4, 3/4, -((b*x^2)/a)]))/(5*a^2*(e*x)^(7/2)*(a + b*x^2)^(1/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{(ex)^{\frac{7}{2}}\,(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*x^(7/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(x)*e^(-7/2)/(b^2*x^8 + 2*a*b*x^
6 + a^2*x^4), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 91.73, size = 85, normalized size = 0.59

$$-\frac{c_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{5b^{\frac{5}{4}}e^{\frac{7}{2}}x^5}+\frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -c*hyper((5/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(5/4)*e**(7/
2)*x**5) + d*gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**2*exp_polar(I*pi)/
a)/(2*a**(5/4)*e**(7/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-7/2)/((b*x^2 + a)^(5/4)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)), x)
```

3.1115 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=182

$$-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}+\frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\,\sqrt[4]{a+bx^2}}+\frac{8b^{3/2}(10bc-9ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{}}{15a^{7/2}e^6\sqrt[4]{a}}$$

```
[Out] -2/9*c/a/e/(e*x)^(9/2)/(b*x^2+a)^(1/4)+2/45*(-9*a*d+10*b*c)/a^2/e^3/(e*x)^(
5/2)/(b*x^2+a)^(1/4)-4/15*b*(-9*a*d+10*b*c)/a^3/e^5/(b*x^2+a)^(1/4)/(e*x)^(
1/2)+8/15*b^(3/2)*(-9*a*d+10*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*
arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/a^(7/2)/e^6/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.07, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {464, 292, 290, 342, 202}

$$\frac{8b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\,(10bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{7/2}e^6\sqrt[4]{a+bx^2}}-\frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\,\sqrt[4]{a+bx^2}}+\frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(1/4)) + (2*(10*b*c - 9*a*d))/(45*a^2
*e^3*(e*x)^(5/2)*(a + b*x^2)^(1/4)) - (4*b*(10*b*c - 9*a*d))/(15*a^3*e^5*Sq
rt[e*x]*(a + b*x^2)^(1/4)) + (8*b^(3/2)*(10*b*c - 9*a*d)*(1 + a/(b*x^2))^(1
/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(7/2)*e^6*
(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
 + 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} - \frac{(10bc-9ad)\int \frac{1}{(ex)^{7/2}(a+bx^2)^{5/4}}\,dx}{9ae^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} + \frac{(2b(10bc-9ad))\int \frac{}{(ex)^{3/2}(}}{15a^2e^4}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\,\sqrt[4]{a+bx^2}} -$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\,\sqrt[4]{a+bx^2}} -$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\,\sqrt[4]{a+bx^2}} +$$

$$= -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}} + \frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\,\sqrt[4]{a+bx^2}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 82, normalized size = 0.45

$$-\frac{2\sqrt{ex}\left(5ac+(-10bc+9ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)\right)}{45a^2e^6x^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(5*a*c + (-10*b*c + 9*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*Hypergeo
metric2F1[-5/4, 5/4, -1/4, -((b*x^2)/a)]))/(45*a^2*e^6*x^5*(a + b*x^2)^(1/4
))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{(ex)^{\frac{11}{2}}\,(b\,x^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*x^(11/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(x)*e^(-11/2)/(b^2*x^10 + 2*a*b*
x^8 + a^2*x^6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(5/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3279 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-11/2)/((b*x^2 + a)^(5/4)*x^(11/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{11/2}\,{\left(b\,x^2+a\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x)
[Out] int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)), x)
```

3.1116 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=184

$$\frac{2(bc-ad)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/4}}-\frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2}-\frac{(4bc-7ad)e^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}+\frac{(4bc-7ad)e$$

```
[Out] 2/3*(-a*d+b*c)*(e*x)^(7/2)/a/b/e/(b*x^2+a)^(3/4)-1/6*(-7*a*d+4*b*c)*e*(e*x)
^(3/2)*(b*x^2+a)^(1/4)/a/b^2-1/4*(-7*a*d+4*b*c)*e^(5/2)*arctan(b^(1/4)*(e*x
)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(11/4)+1/4*(-7*a*d+4*b*c)*e^(5/2)*arctan
h(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(11/4)
```

Rubi [A]
time = 0.08, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {468, 327, 335, 338, 304, 211, 214}

$$-\frac{e^{5/2}(4bc-7ad)\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}+\frac{e^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}-\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}\,(4bc-7ad)}{6ab^2}+\frac{2(ex)^{7/2}(bc-ad)}{3abe\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((4*b*c - 7*a*d)*
e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(6*a*b^2) - ((4*b*c - 7*a*d)*e^(5/2)*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(11/4)) + ((4*b*c
- 7*a*d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/
(4*b^(11/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/4}} + \frac{\left(2\left(-2bc+\frac{7ad}{2}\right)\right)\int \frac{(ex)^{5/2}}{(a+bx^2)^{3/4}}\,dx}{3ab}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{((4bc-7ad)e^2)\int \frac{\sqrt{ex}}{(a+bx^2)^{3/4}}}{4b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{((4bc-7ad)e)\text{Subst}\Big(\int \frac{}{(a}}{2b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{((4bc-7ad)e)\text{Subst}\Big(\int \frac{x}{1-}}{2b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{((4bc-7ad)e^3)\,\text{Subst}\Big(\int \frac{}{e}}{4b^5}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} - \frac{(4bc-7ad)e^{5/2}\tan^{-1}\Big(\frac{}{\sqrt{e}}}{4b^{11/4}}$$

Mathematica [A]
time = 0.78, size = 129, normalized size = 0.70

$$\frac{(ex)^{5/2}\left(\frac{2b^{3/4}x^{3/2}(-4bc+7ad+3bdx^2)}{(a+bx^2)^{3/4}} + 3(-4bc+7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) + 3(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{12b^{11/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] ((e*x)^(5/2)*((2*b^(3/4)*x^(3/2)*(-4*b*c + 7*a*d + 3*b*d*x^2))/(a + b*x^2)^
(3/4) + 3*(-4*b*c + 7*a*d)*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + 3*
(4*b*c - 7*a*d)*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(12*b^(11/4)
*x^(5/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{5}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [A]
time = 0.49, size = 233, normalized size = 1.27

$$\frac{1}{24}\left(d\left(\frac{4\left(4ab-\frac{7(bx^2+a)a}{x^2}\right)}{\frac{(bx^2+a)^{\frac{3}{4}}b^3}{x^{\frac{3}{2}}}-\frac{(bx^2+a)^{\frac{7}{4}}b^2}{x^{\frac{7}{2}}}}-\frac{21\left(\frac{2a\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{3}{4}}}\right)}{b^2}\right)+4c\left(\frac{3\left(\frac{2\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{3}{4}}}-\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{3}{4}}}\right)}{b}-\frac{4x^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}b}\right)\right)e^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] 1/24*(d*(4*(4*a*b - 7*(b*x^2 + a)*a/x^2)/((b*x^2 + a)^(3/4)*b^3/x^(3/2) - (
b*x^2 + a)^(7/4)*b^2/x^(7/2)) - 21*(2*a*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*s
qrt(x)))/b^(3/4) - a*log(-(b^(1/4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) +
(b*x^2 + a)^(1/4)/sqrt(x)))/b^(3/4))/b^2) + 4*c*(3*(2*arctan((b*x^2 + a)^(1
/4)/(b^(1/4)*sqrt(x)))/b^(3/4) - log(-(b^(1/4) - (b*x^2 + a)^(1/4)/sqrt(x))
/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(3/4))/b - 4*x^(3/2)/((b*x^2 + a)
^(3/4)*b)))*e^(5/2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] Timed out
```

Sympy [C] Result contains complex when optimal does not.
time = 54.57, size = 94, normalized size = 0.51

$$\frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}+\frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{4},\frac{11}{4}\\\frac{15}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(7/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hyper(
(7/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^(5/2)*e^(5/2)/(b*x^2 + a)^(7/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{5/2}\,(d\,x^2+c)}{(b\,x^2+a)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
```

```
[Out] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

3.1117 $\int \frac{\sqrt{ex}\,(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=125

$$\frac{2(bc-ad)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/4}}-\frac{d\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}$$

```
[Out] 2/3*(-a*d+b*c)*(e*x)^(3/2)/a/b/e/(b*x^2+a)^(3/4)-d*arctan(b^(1/4)*(e*x)^(1/
2)/(b*x^2+a)^(1/4)/e^(1/2))*e^(1/2)/b^(7/4)+d*arctanh(b^(1/4)*(e*x)^(1/2)/(
b*x^2+a)^(1/4)/e^(1/2))*e^(1/2)/b^(7/4)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {463, 335, 338, 304, 211, 214}

$$-\frac{d\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{2(ex)^{3/2}(bc-ad)}{3abe\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - (d*Sqrt[e]*ArcTan
[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7/4) + (d*Sqrt[e]*Arc
Tanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7/4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ex}\,(c+dx^2)}{(a+bx^2)^{7/4}}\,dx &= \frac{2(bc-ad)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/4}}+\frac{d\int \frac{\sqrt{ex}}{(a+bx^2)^{3/4}}\,dx}{b}\\
&= \frac{2(bc-ad)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/4}}+\frac{(2d)\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx,x,\sqrt{ex}\right)}{be}\\
&= \frac{2(bc-ad)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/4}}+\frac{(2d)\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{be}\\
&= \frac{2(bc-ad)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/4}}+\frac{(de)\text{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^{3/2}}-\frac{(de)\text{Subst}\left(\int \frac{1}{e+\sqrt{\ldots}}\right.}{}\\
&= \frac{2(bc-ad)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/4}}-\frac{d\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right.}{b^{7/4}}
\end{aligned}$$

Mathematica [A]

time = 0.50, size = 108, normalized size = 0.86

$$\frac{\sqrt{ex}\left(\frac{2b^{3/4}(bc-ad)x^{3/2}}{a(a+bx^2)^{3/4}}-3d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+3d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{3b^{7/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (Sqrt[e*x]*((2*b^(3/4)*(b*c - a*d)*x^(3/2))/(a*(a + b*x^2)^(3/4)) - 3*d*Arc
Tan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + 3*d*ArcTanh[(b^(1/4)*Sqrt[x])/(a
 + b*x^2)^(1/4)]))/(3*b^(7/4)*Sqrt[x])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [A]
time = 0.49, size = 116, normalized size = 0.93

$$\frac{1}{6}\left(d\left(\frac{3\left(\frac{2\arctan\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{3}{4}}}-\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{3}{4}}}\right)}{b}-\frac{4\,x^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}b}\right)+\frac{4\,cx^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}a}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] 1/6*(d*(3*(2*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(3/4) - log(-(b^
(1/4) - (b*x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b
^(3/4))/b - 4*x^(3/2)/((b*x^2 + a)^(3/4)*b)) + 4*c*x^(3/2)/((b*x^2 + a)^(3/
4)*a))*e^(1/2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**C**] Result contains complex when optimal does not.
time = 11.01, size = 87, normalized size = 0.70

$$\frac{c\sqrt{e}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}{2a^{\frac{7}{4}}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{d\sqrt{e}\,x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*sqrt(e)*x**(3/2)*gamma(3/4)/(2*a**(7/4)*(1 + b*x**2/a)**(3/4)*gamma(7/4))
 + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4, 7/4), (11/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*a**(7/4)*gamma(11/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(x)*e^(1/2)/(b*x^2 + a)^(7/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
```

```
[Out] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

3.1118 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=65

$$-\frac{2c}{ae\sqrt{ex}\,(a+bx^2)^{3/4}}-\frac{2(4bc-ad)(ex)^{3/2}}{3a^2e^3\,(a+bx^2)^{3/4}}$$

```
[Out] -2/3*(-a*d+4*b*c)*(e*x)^(3/2)/a^2/e^3/(b*x^2+a)^(3/4)-2*c/a/e/(b*x^2+a)^(3/
4)/(e*x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {464, 270}

$$-\frac{2(ex)^{3/2}(4bc-ad)}{3a^2e^3\,(a+bx^2)^{3/4}}-\frac{2c}{ae\sqrt{ex}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/4)) - (2*(4*b*c - a*d)*(e*x)^(3/2))/(3
*a^2*e^3*(a + b*x^2)^(3/4))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{3/2}\,(a+bx^2)^{7/4}}\,dx = -\frac{2c}{ae\sqrt{ex}\,\left(a+bx^2\right)^{3/4}} - \frac{(4bc-ad)\int \frac{\sqrt{ex}}{(a+bx^2)^{7/4}}\,dx}{ae^2}$$
$$= -\frac{2c}{ae\sqrt{ex}\,\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-ad)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/4}}$$

Mathematica [A]
time = 0.36, size = 44, normalized size = 0.68

$$\frac{2x(-3ac-4bcx^2+adx^2)}{3a^2(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
[Out] (2*x*(-3*a*c - 4*b*c*x^2 + a*d*x^2))/(3*a^2*(e*x)^(3/2)*(a + b*x^2)^(3/4))
```

Maple [A]
time = 0.12, size = 40, normalized size = 0.62

method	result	size
gosper	$-\frac{2x\left(-ad\,x^2+4c\,x^2b+3ac\right)}{3\left(b\,x^2+a\right)^{\frac{3}{4}}a^2(ex)^{\frac{3}{2}}}$	40
risch	$-\frac{2c\left(b\,x^2+a\right)^{\frac{1}{4}}}{a^2e\sqrt{ex}}+\frac{2(ad-bc)x^2}{3a^2e\sqrt{ex}\,\left(b\,x^2+a\right)^{\frac{3}{4}}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x,method=_RETURNVERBOSE)
[Out] -2/3*x*(-a*d*x^2+4*b*c*x^2+3*a*c)/(b*x^2+a)^(3/4)/a^2/(e*x)^(3/2)
```

Maxima [A]
time = 0.27, size = 60, normalized size = 0.92

$$-\frac{2}{3}\left(c\left(\frac{bx^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}a^2}+\frac{3\,(bx^2+a)^{\frac{1}{4}}}{a^2\sqrt{x}}\right)-\frac{dx^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}a}\right)e^{\left(-\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] -2/3*(c*(b*x^(3/2)/((b*x^2 + a)^(3/4)*a^2) + 3*(b*x^2 + a)^(1/4)/(a^2*sqrt(
x))) - d*x^(3/2)/((b*x^2 + a)^(3/4)*a))*e^(-3/2)
```

Fricas [A]
time = 2.07, size = 50, normalized size = 0.77

$$-\frac{2\,((4\,bc-ad)x^2+3\,ac)(bx^2+a)^{\frac{1}{4}}\sqrt{x}\,e^{\left(-\frac{3}{2}\right)}}{3\,(a^2bx^3+a^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] -2/3*((4*b*c - a*d)*x^2 + 3*a*c)*(b*x^2 + a)^(1/4)*sqrt(x)*e^(-3/2)/(a^2*b*
x^3 + a^3*x)
```

Sympy [A]
time = 33.58, size = 119, normalized size = 1.83

$$c\left(\frac{3\Gamma\left(-\frac{1}{4}\right)}{8ab^{\frac{3}{4}}e^{\frac{3}{2}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt[4]{b}\,\Gamma\left(-\frac{1}{4}\right)}{2a^2e^{\frac{3}{2}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}\right)+\frac{d\Gamma\left(\frac{3}{4}\right)}{2ab^{\frac{3}{4}}e^{\frac{3}{2}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*(3*gamma(-1/4)/(8*a*b**(3/4)*e**(3/2)*x**2*(a/(b*x**2) + 1)**(3/4)*gamma(
7/4)) + b**(1/4)*gamma(-1/4)/(2*a**2*e**(3/2)*(a/(b*x**2) + 1)**(3/4)*gamma
(7/4))) + d*gamma(3/4)/(2*a*b**(3/4)*e**(3/2)*(a/(b*x**2) + 1)**(3/4)*gamma
(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-3/2)/((b*x^2 + a)^(7/4)*x^(3/2)), x)
```

Mupad [B]
time = 0.63, size = 69, normalized size = 1.06

$$-\frac{{\left(b\,x^2+a\right)}^{1/4}\,\left(\frac{2\,c}{a\,b\,e}-\frac{x^2\,\left(2\,a\,d-8\,b\,c\right)}{3\,a^2\,b\,e}\right)}{x^2\,\sqrt{e\,x}+\frac{a\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*((2*c)/(a*b*e) - (x^2*(2*a*d - 8*b*c))/(3*a^2*b*e)))/(x
^2*(e*x)^(1/2) + (a*(e*x)^(1/2))/b)
```

3.1119 $\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}}+\frac{8(8bc-5ad)\sqrt[4]{a+bx^2}}{15a^3e^3\sqrt{ex}}$$

```
[Out] -2/5*c/a/e/(e*x)^(5/2)/(b*x^2+a)^(3/4)-2/15*(-5*a*d+8*b*c)/a^2/e^3/(b*x^2+a
)^(3/4)/(e*x)^(1/2)+8/15*(-5*a*d+8*b*c)*(b*x^2+a)^(1/4)/a^3/e^3/(e*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 279, 270}

$$\frac{8\sqrt[4]{a+bx^2}(8bc-5ad)}{15a^3e^3\sqrt{ex}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(3/4)) - (2*(8*b*c - 5*a*d))/(15*a^2*
e^3*Sqrt[e*x]*(a + b*x^2)^(3/4)) + (8*(8*b*c - 5*a*d)*(a + b*x^2)^(1/4))/(1
5*a^3*e^3*Sqrt[e*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx &= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{(8bc-5ad)\int \frac{1}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx}{5ae^2}\\ &= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}}-\frac{(4(8bc-5ad))\int \frac{1}{(ex)^{3/2}(a}}{15a^2e^2}\\ &= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}}+\frac{8(8bc-5ad)\sqrt[4]{a+bx^2}}{15a^3e^3\sqrt{ex}}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 67, normalized size = 0.64

$$-\frac{2x(3a^2c-24abcx^2+15a^2dx^2-32b^2cx^4+20abdx^4)}{15a^3(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
[Out] (-2*x*(3*a^2*c - 24*a*b*c*x^2 + 15*a^2*d*x^2 - 32*b^2*c*x^4 + 20*a*b*d*x^4)
)/(15*a^3*(e*x)^(7/2)*(a + b*x^2)^(3/4))
```

Maple [A]
time = 0.11, size = 62, normalized size = 0.60

method	result	size
gosper	$-\frac{2x(20abd\,x^4-32b^2c\,x^4+15a^2d\,x^2-24abc\,x^2+3a^2c)}{15(b\,x^2+a)^{\frac{3}{4}}a^3(ex)^{\frac{7}{2}}}$	62
risch	$-\frac{2(b\,x^2+a)^{\frac{1}{4}}(5ad\,x^2-9c\,x^2b+ac)}{5a^3x^2e^3\sqrt{ex}}-\frac{2b(ad-bc)x^2}{3a^3e^3\sqrt{ex}\,(b\,x^2+a)^{\frac{3}{4}}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x,method=_RETURNVERBOSE)
[Out] -2/15*x*(20*a*b*d*x^4-32*b^2*c*x^4+15*a^2*d*x^2-24*a*b*c*x^2+3*a^2*c)/(b*x^
2+a)^(3/4)/a^3/(e*x)^(7/2)
```

Maxima [A]
time = 0.27, size = 102, normalized size = 0.98

$$-\frac{2}{15}\left(5\,d\left(\frac{bx^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}a^2}+\frac{3\,(bx^2+a)^{\frac{1}{4}}}{a^2\sqrt{x}}\right)-c\left(\frac{5\,b^2x^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}a^3}+\frac{3\left(\frac{10\,(bx^2+a)^{\frac{1}{4}}b}{\sqrt{x}}-\frac{(bx^2+a)^{\frac{5}{4}}}{x^{\frac{5}{2}}}\right)}{a^3}\right)\right)e^{\left(-\frac{7}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] -2/15*(5*d*(b*x^(3/2)/((b*x^2 + a)^(3/4)*a^2) + 3*(b*x^2 + a)^(1/4)/(a^2*sq
rt(x))) - c*(5*b^2*x^(3/2)/((b*x^2 + a)^(3/4)*a^3) + 3*(10*(b*x^2 + a)^(1/4
)*b/sqrt(x) - (b*x^2 + a)^(5/4)/x^(5/2))/a^3))*e^(-7/2)
```

Fricas [A]
time = 0.86, size = 75, normalized size = 0.72

$$\frac{2\,(4\,(8\,b^2c-5\,abd)x^4-3\,a^2c+3\,(8\,abc-5\,a^2d)x^2)(bx^2+a)^{\frac{1}{4}}\sqrt{x}\,e^{\left(-\frac{7}{2}\right)}}{15\,(a^3bx^5+a^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] 2/15*(4*(8*b^2*c - 5*a*b*d)*x^4 - 3*a^2*c + 3*(8*a*b*c - 5*a^2*d)*x^2)*(b*x
^2 + a)^(1/4)*sqrt(x)*e^(-7/2)/(a^3*b*x^5 + a^4*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(99) = 198.
time = 166.52, size = 469, normalized size = 4.51

$$c\left(-\frac{3a^3b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{32a^5b^4e^{\frac{7}{2}}x^2\Gamma\left(\frac{7}{4}\right)+64a^4b^5e^{\frac{7}{2}}x^4\Gamma\left(\frac{7}{4}\right)+32a^3b^6e^{\frac{7}{2}}x^6\Gamma\left(\frac{7}{4}\right)}+\frac{21a^2b^{\frac{21}{4}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{32a^5b^4e^{\frac{7}{2}}x^2\Gamma\left(\frac{7}{4}\right)+64a^4b^5e^{\frac{7}{2}}x^4\Gamma\left(\frac{7}{4}\right)+32a^3b^6e^{\frac{7}{2}}x^6\Gamma\left(\frac{7}{4}\right)}+\frac{56ab^{\frac{25}{4}}x^4\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{32a^5b^4e^{\frac{7}{2}}x^2\Gamma\left(\frac{7}{4}\right)+64a^4b^5e^{\frac{7}{2}}x^4\Gamma\left(\frac{7}{4}\right)+32a^3b^6e^{\frac{7}{2}}x^6\Gamma\left(\frac{7}{4}\right)}+\frac{32b^{\frac{29}{4}}x^6\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{32a^5b^4e^{\frac{7}{2}}x^2\Gamma\left(\frac{7}{4}\right)+64a^4b^5e^{\frac{7}{2}}x^4\Gamma\left(\frac{7}{4}\right)+32a^3b^6e^{\frac{7}{2}}x^6\Gamma\left(\frac{7}{4}\right)}\right)+d\left(\frac{3\Gamma\left(-\frac{1}{4}\right)}{8ab^{\frac{3}{4}}e^{\frac{7}{2}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt[4]{b}\,\Gamma\left(-\frac{1}{4}\right)}{2a^2e^{\frac{7}{2}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*(-3*a**3*b**(17/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(32*a**5*b**4*e**(
7/2)*x**2*gamma(7/4) + 64*a**4*b**5*e**(7/2)*x**4*gamma(7/4) + 32*a**3*b**6
*e**(7/2)*x**6*gamma(7/4)) + 21*a**2*b**(21/4)*x**2*(a/(b*x**2) + 1)**(1/4)
*gamma(-5/4)/(32*a**5*b**4*e**(7/2)*x**2*gamma(7/4) + 64*a**4*b**5*e**(7/2)
*x**4*gamma(7/4) + 32*a**3*b**6*e**(7/2)*x**6*gamma(7/4)) + 56*a*b**(25/4)*
x**4*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(32*a**5*b**4*e**(7/2)*x**2*gamma(
7/4) + 64*a**4*b**5*e**(7/2)*x**4*gamma(7/4) + 32*a**3*b**6*e**(7/2)*x**6*g
amma(7/4)) + 32*b**(29/4)*x**6*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(32*a**5
*b**4*e**(7/2)*x**2*gamma(7/4) + 64*a**4*b**5*e**(7/2)*x**4*gamma(7/4) + 32
```

```
*a**3*b**6*e**(7/2)*x**6*gamma(7/4))) + d*(3*gamma(-1/4)/(8*a*b**(3/4)*e**(
7/2)*x**2*(a/(b*x**2) + 1)**(3/4)*gamma(7/4)) + b**(1/4)*gamma(-1/4)/(2*a**
2*e**(7/2)*(a/(b*x**2) + 1)**(3/4)*gamma(7/4)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-7/2)/((b*x^2 + a)^(7/4)*x^(7/2)), x)
```

Mupad [B]
time = 0.67, size = 101, normalized size = 0.97

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2\,c}{5\,a\,b\,e^3}+\frac{x^2\,\left(30\,a^2\,d-48\,a\,b\,c\right)}{15\,a^3\,b\,e^3}-\frac{x^4\,\left(64\,b^2\,c-40\,a\,b\,d\right)}{15\,a^3\,b\,e^3}\right)}{x^4\,\sqrt{e\,x}+\frac{a\,x^2\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*((2*c)/(5*a*b*e^3) + (x^2*(30*a^2*d - 48*a*b*c))/(15*a^
3*b*e^3) - (x^4*(64*b^2*c - 40*a*b*d))/(15*a^3*b*e^3)))/(x^4*(e*x)^(1/2) +
(a*x^2*(e*x)^(1/2))/b)
```

3.1120 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{2c}{9ae(ex)^{9/2}(a+bx^2)^{3/4}}-\frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}(a+bx^2)^{3/4}}+\frac{16(4bc-3ad)\sqrt[4]{a+bx^2}}{9a^3e^3(ex)^{5/2}}-\frac{64(4bc-3ad)(a+bx^2)^{5/4}}{45a^4e^3(ex)^{5/2}}$$

```
[Out] -2/9*c/a/e/(e*x)^(9/2)/(b*x^2+a)^(3/4)-2/9*(-3*a*d+4*b*c)/a^2/e^3/(e*x)^(5/
2)/(b*x^2+a)^(3/4)+16/9*(-3*a*d+4*b*c)*(b*x^2+a)^(1/4)/a^3/e^3/(e*x)^(5/2)-
64/45*(-3*a*d+4*b*c)*(b*x^2+a)^(5/4)/a^4/e^3/(e*x)^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 279, 270}

$$-\frac{64(a+bx^2)^{5/4}(4bc-3ad)}{45a^4e^3(ex)^{5/2}}+\frac{16\sqrt[4]{a+bx^2}(4bc-3ad)}{9a^3e^3(ex)^{5/2}}-\frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}(a+bx^2)^{3/4}}-\frac{2c}{9ae(ex)^{9/2}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(3/4)) - (2*(4*b*c - 3*a*d))/(9*a^2*e
^3*(e*x)^(5/2)*(a + b*x^2)^(3/4)) + (16*(4*b*c - 3*a*d)*(a + b*x^2)^(1/4))/
(9*a^3*e^3*(e*x)^(5/2)) - (64*(4*b*c - 3*a*d)*(a + b*x^2)^(5/4))/(45*a^4*e^
3*(e*x)^(5/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{(4bc-3ad)\int \frac{1}{(ex)^{7/2}(a+bx^2)^{7/4}}\,dx}{3ae^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}} - \frac{(8(4bc-3ad))\int \frac{}{(ex)^{7/2}}}{9a^2e^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}} + \frac{16(4bc-3ad)\sqrt[4]{a+bx^2}}{9a^3e^3(ex)^{5/2}}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}} + \frac{16(4bc-3ad)\sqrt[4]{a+bx^2}}{9a^3e^3(ex)^{5/2}}$$

Mathematica [A]
time = 0.78, size = 91, normalized size = 0.65

$$-\frac{2x(5a^3c-12a^2bcx^2+9a^3dx^2+96ab^2cx^4-72a^2bdx^4+128b^3cx^6-96ab^2dx^6)}{45a^4(ex)^{11/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
[Out] (-2*x*(5*a^3*c - 12*a^2*b*c*x^2 + 9*a^3*d*x^2 + 96*a*b^2*c*x^4 - 72*a^2*b*d
*x^4 + 128*b^3*c*x^6 - 96*a*b^2*d*x^6))/(45*a^4*(e*x)^(11/2)*(a + b*x^2)^(3
/4))
```

Maple [A]
time = 0.12, size = 86, normalized size = 0.61

method	result	size
gosper	$-\frac{2x\left(-96a\,b^2d\,x^6+128b^3c\,x^6-72a^2bd\,x^4+96a\,b^2c\,x^4+9a^3d\,x^2-12a^2bc\,x^2+5c\,a^3\right)}{45\left(b\,x^2+a\right)^{\frac{3}{4}}a^4(ex)^{\frac{11}{2}}}$	86
risch	$-\frac{2\left(b\,x^2+a\right)^{\frac{1}{4}}\left(-81abd\,x^4+113b^2c\,x^4+9a^2d\,x^2-17abc\,x^2+5a^2c\right)}{45a^4x^4e^5\sqrt{ex}}+\frac{2(ad-bc)b^2x^2}{3a^4e^5\sqrt{ex}\,\left(b\,x^2+a\right)^{\frac{3}{4}}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x,method=_RETURNVERBOSE)
```

[Out] -2/45*x*(-96*a*b^2*d*x^6+128*b^3*c*x^6-72*a^2*b*d*x^4+96*a*b^2*c*x^4+9*a^3*d*x^2-12*a^2*b*c*x^2+5*a^3*c)/(b*x^2+a)^(3/4)/a^4/(e*x)^(11/2)

Maxima [A]
time = 0.28, size = 139, normalized size = 0.99

$$\frac{2}{45}\left(3\,d\left(\frac{5\,b^2x^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}a^3}+\frac{3\left(\frac{10\,(bx^2+a)^{\frac{1}{4}}b}{\sqrt{x}}-\frac{(bx^2+a)^{\frac{5}{4}}}{x^{\frac{5}{2}}}\right)}{a^3}\right)-c\left(\frac{15\,b^3x^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}a^4}+\frac{\frac{135\,(bx^2+a)^{\frac{1}{4}}b^2}{\sqrt{x}}-\frac{27\,(bx^2+a)^{\frac{5}{4}}b}{x^{\frac{5}{2}}}+\frac{5\,(bx^2+a)^{\frac{9}{4}}}{x^{\frac{9}{2}}}}{a^4}\right)\right)e^{\left(-\frac{11}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")

[Out] 2/45*(3*d*(5*b^2*x^(3/2)/((b*x^2 + a)^(3/4)*a^3) + 3*(10*(b*x^2 + a)^(1/4)*b/sqrt(x) - (b*x^2 + a)^(5/4)/x^(5/2))/a^3) - c*(15*b^3*x^(3/2)/((b*x^2 + a)^(3/4)*a^4) + (135*(b*x^2 + a)^(1/4)*b^2/sqrt(x) - 27*(b*x^2 + a)^(5/4)*b/x^(5/2) + 5*(b*x^2 + a)^(9/4)/x^(9/2))/a^4))*e^(-11/2)

Fricas [A]
time = 1.77, size = 99, normalized size = 0.70

$$-\frac{2\,(32\,(4\,b^3c-3\,ab^2d)x^6+24\,(4\,ab^2c-3\,a^2bd)x^4+5\,a^3c-3\,(4\,a^2bc-3\,a^3d)x^2)(bx^2+a)^{\frac{1}{4}}\sqrt{x}\,e^{\left(-\frac{11}{2}\right)}}{45\,(a^4bx^7+a^5x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")

[Out] -2/45*(32*(4*b^3*c - 3*a*b^2*d)*x^6 + 24*(4*a*b^2*c - 3*a^2*b*d)*x^4 + 5*a^3*c - 3*(4*a^2*b*c - 3*a^3*d)*x^2)*(b*x^2 + a)^(1/4)*sqrt(x)*e^(-11/2)/(a^4*b*x^7 + a^5*x^5)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(7/4),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4063 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-11/2)/((b*x^2 + a)^(7/4)*x^(11/2)), x)
```

Mupad [B]
time = 0.69, size = 125, normalized size = 0.89

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2\,c}{9\,a\,b\,e^5}-\frac{16\,x^4\,\left(3\,a\,d-4\,b\,c\right)}{15\,a^3\,e^5}+\frac{x^2\,\left(18\,a^3\,d-24\,a^2\,b\,c\right)}{45\,a^4\,b\,e^5}+\frac{x^6\,\left(256\,b^3\,c-192\,a\,b^2\,d\right)}{45\,a^4\,b\,e^5}\right)}{x^6\,\sqrt{e\,x}+\frac{a\,x^4\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x)
[Out] -((a + b*x^2)^(1/4)*((2*c)/(9*a*b*e^5) - (16*x^4*(3*a*d - 4*b*c))/(15*a^3*e
^5) + (x^2*(18*a^3*d - 24*a^2*b*c))/(45*a^4*b*e^5) + (x^6*(256*b^3*c - 192*
a*b^2*d))/(45*a^4*b*e^5)))/(x^6*(e*x)^(1/2) + (a*x^4*(e*x)^(1/2))/b)
```

3.1121 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=192

$$\frac{2(bc-ad)(ex)^{9/2}}{3abe\left(a+bx^2\right)^{3/4}}+\frac{5(2bc-3ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^3}-\frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2}+\frac{5\sqrt{a}\,(2bc-3ad)e^2(1$$

```
[Out] 2/3*(-a*d+b*c)*(e*x)^(9/2)/a/b/e/(b*x^2+a)^(3/4)-1/3*(-3*a*d+2*b*c)*e*(e*x)
^(5/2)*(b*x^2+a)^(1/4)/a/b^2+5/6*(-3*a*d+2*b*c)*e^2*(1+a/b/x^2)^(3/4)*(e*x)
^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/b
^(5/2)/(b*x^2+a)^(3/4)+5/6*(-3*a*d+2*b*c)*e^3*(b*x^2+a)^(1/4)*(e*x)^(1/2)/b
^3
```

Rubi [A]

time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {468, 327, 335, 243, 342, 281, 237}

$$\frac{5\sqrt{a}\,e^2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6b^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{5e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}\,(2bc-3ad)}{6b^3}-\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}\,(2bc-3ad)}{3ab^2}+\frac{2(ex)^{9/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(3*a*b*e*(a + b*x^2)^(3/4)) + (5*(2*b*c - 3*a*d
)*e^3*Sqrt[e*x]*(a + b*x^2)^(1/4))/(6*b^3) - ((2*b*c - 3*a*d)*e*(e*x)^(5/2)
*(a + b*x^2)^(1/4))/(3*a*b^2) + (5*Sqrt[a]*(2*b*c - 3*a*d)*e^2*(1 + a/(b*x^
2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*b^(5/
2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{9/2}}{3abe\,(a+bx^2)^{3/4}} + \frac{\left(2\left(-3bc+\frac{9ad}{2}\right)\right)\int \frac{(ex)^{7/2}}{(a+bx^2)^{3/4}}\,dx}{3ab}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\,(a+bx^2)^{3/4}} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{(5(2bc-3ad)e^2)\int \frac{(ex)^{3/2}}{(a+bx^2)^{3/}}}{6b^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\,(a+bx^2)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+b}}{3ab^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\,(a+bx^2)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+b}}{3ab^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\,(a+bx^2)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+b}}{3ab^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\,(a+bx^2)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+b}}{3ab^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\,(a+bx^2)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+b}}{3ab^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{3abe\,(a+bx^2)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\,\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+b}}{3ab^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.12, size = 110, normalized size = 0.57

$$\frac{e^3\sqrt{ex}\left(-15a^2d+ab(10c-9dx^2)+2b^2x^2(3c+dx^2)+5a(-2bc+3ad)\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{6b^3\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (e^3*Sqrt[e*x]*(-15*a^2*d + a*b*(10*c - 9*d*x^2) + 2*b^2*x^2*(3*c + d*x^2)
+ 5*a*(-2*b*c + 3*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/
4, -((b*x^2)/a)]))/(6*b^3*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{7}{2}} \left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((d*x^2 + c)*x^(7/2)/(b*x^2 + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^5 + c*x^3)*(b*x^2 + a)^(1/4)*sqrt(x)*e^(7/2)/(b^2*x^4 + 2*a*b
*x^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 173.34, size = 94, normalized size = 0.49

$$\frac{ce^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix}\frac{7}{4}, \frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{13}{4}\right)} + \frac{de^{\frac{7}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix}\frac{7}{4}, \frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*e**(7/2)*x**(9/2)*gamma(9/4)*hyper((7/4, 9/4), (13/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(7/4)*gamma(13/4)) + d*e**(7/2)*x**(13/2)*gamma(13/4)*hyper(
(7/4, 13/4), (17/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*gamma(17/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*x^(7/2)*e^(7/2)/(b*x^2 + a)^(7/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{7/2}\,(d\,x^2+c)}{(b\,x^2+a)^{7/4}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
[Out] int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

3.1122 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=152

$$\frac{2(bc-ad)(ex)^{5/2}}{3abe\,(a+bx^2)^{3/4}}-\frac{(2bc-5ad)e\sqrt{ex}\,\sqrt[4]{a+bx^2}}{3ab^2}-\frac{(2bc-5ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,b^{3/2}\,(a+bx^2)^{3/4}}$$

```
[Out] 2/3*(-a*d+b*c)*(e*x)^(5/2)/a/b/e/(b*x^2+a)^(3/4)-1/3*(-5*a*d+2*b*c)*(1+a/b/
x^2)^(3/4)*(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2
*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^
(1/2))/b^(3/2)/(b*x^2+a)^(3/4)/a^(1/2)-1/3*(-5*a*d+2*b*c)*e*(b*x^2+a)^(1/4)
*(e*x)^(1/2)/a/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {468, 327, 335, 243, 342, 281, 237}

$$-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,b^{3/2}\,(a+bx^2)^{3/4}}-\frac{e\sqrt{ex}\,\sqrt[4]{a+bx^2}\,(2bc-5ad)}{3ab^2}+\frac{2(ex)^{5/2}(bc-ad)}{3abe\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((2*b*c - 5*a*d)*
e*Sqrt[e*x]*(a + b*x^2)^(1/4))/(3*a*b^2) - ((2*b*c - 5*a*d)*(1 + a/(b*x^2))
^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*
b^(3/2)*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{5/2}}{3abe(a+bx^2)^{3/4}} + \frac{\left(2\left(-bc+\frac{5ad}{2}\right)\right)\int \frac{(ex)^{3/2}}{(a+bx^2)^{3/4}}\,dx}{3ab}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe(a+bx^2)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{((2bc-5ad)e^2)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{3/4}}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe(a+bx^2)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{((2bc-5ad)e)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right.}}{3b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe(a+bx^2)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{\left((2bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(ex}{3b}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe(a+bx^2)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} - \frac{\left((2bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(ex}{3b}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe(a+bx^2)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} - \frac{\left((2bc-5ad)e\left(1+\frac{a}{bx^2}\right)^{3/4}(ex}{6b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{3abe(a+bx^2)^{3/4}} - \frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2} - \frac{(2bc-5ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3}}{3\sqrt{a}\,b^{3/2}(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 85, normalized size = 0.56

$$\frac{e\sqrt{ex}\left(-2bc+5ad+3bdx^2+(2bc-5ad)\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b^2(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (e*Sqrt[e*x]*(-2*b*c + 5*a*d + 3*b*d*x^2 + (2*b*c - 5*a*d)*(1 + (b*x^2)/a)^
(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*b^2*(a + b*x^2)^(
3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}} \left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((d*x^2 + c)*x^(3/2)/(b*x^2 + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c*x)*(b*x^2 + a)^(1/4)*sqrt(x)*e^(3/2)/(b^2*x^4 + 2*a*b*x
^2 + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 20.42, size = 94, normalized size = 0.62

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{4}, \frac{7}{4}\\ \frac{9}{4}\end{matrix}\right| \frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{9}{4}\right)} + \frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{7}{4}, \frac{9}{4}\\ \frac{13}{4}\end{matrix}\right| \frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 7/4), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(7/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((7/4
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*gamma(13/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*x^(3/2)*e^(3/2)/(b*x^2 + a)^(7/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{3/2}\,(d\,x^2+c)}{(b\,x^2+a)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
[Out] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

3.1123 $\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=116

$$\frac{2(bc-ad)\sqrt{ex}}{3abe\,(a+bx^2)^{3/4}}-\frac{2(2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\sqrt{b}\,e^2\,(a+bx^2)^{3/4}}$$

```
[Out] -2/3*(a*d+2*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^
(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcco
t(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/e^2/(b*x^2+a)^(3/4)/b^(1/2)+2/3*(-a*
d+b*c)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(3/4)
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {468, 335, 243, 342, 281, 237}

$$\frac{2\sqrt{ex}\,(bc-ad)}{3abe\,(a+bx^2)^{3/4}}-\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(ad+2bc)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\sqrt{b}\,e^2\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(7/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/4)) - (2*(2*b*c + a*d)*(1
 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/(3*a^(3/2)*Sqrt[b]*e^2*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{7/4}}\,dx = \frac{2(bc-ad)\sqrt{ex}}{3abe\,(a+bx^2)^{3/4}} + \frac{\left(2\left(bc+\frac{ad}{2}\right)\right)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx}{3ab}$$

$$= \frac{2(bc-ad)\sqrt{ex}}{3abe\,(a+bx^2)^{3/4}} + \frac{(2(2bc+ad))\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{3abe}$$

$$= \frac{2(bc-ad)\sqrt{ex}}{3abe\,(a+bx^2)^{3/4}} + \frac{\left(2(2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2}{bx^4}\right)^{3/4}x^3}\,dx,\right.}{3abe\,(a+bx^2)^{3/4}}$$

$$= \frac{2(bc-ad)\sqrt{ex}}{3abe\,(a+bx^2)^{3/4}} - \frac{\left(2(2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ae^2x^4}{b}\right)^{3/4}}\,dx,\right.}{3abe\,(a+bx^2)^{3/4}}$$

$$= \frac{2(bc-ad)\sqrt{ex}}{3abe\,(a+bx^2)^{3/4}} - \frac{\left((2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ae^2x^2}{b}\right)^{3/4}}\,dx, x\right.}{3abe\,(a+bx^2)^{3/4}}$$

$$= \frac{2(bc-ad)\sqrt{ex}}{3abe\,(a+bx^2)^{3/4}} - \frac{2(2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)}{3a^{3/2}\sqrt{b}\,e^2\,(a+bx^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 79, normalized size = 0.68

$$\frac{2x\left(bc-ad+(2bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3ab\sqrt{ex}\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(7/4)),x]
```

```
[Out] (2*x*(b*c - a*d + (2*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4
, 3/4, 5/4, -((b*x^2)/a)]))/(3*a*b*Sqrt[e*x]*(a + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{\sqrt{ex}\,(b\,x^2+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(x)*e^(-1/2)/(b^2*x^5 + 2*a*b*x^
3 + a^2*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 16.64, size = 78, normalized size = 0.67

$$-\frac{d\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{7}{4}}\sqrt{e}\,x}+\frac{c\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] -d*hyper((1/2, 7/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(7/4)*sqrt(e)*
x) + c*sqrt(x)*gamma(1/4)*hyper((1/4, 7/4), (5/4,), b*x**2*exp_polar(I*pi)/
a)/(2*a**(7/4)*sqrt(e)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-1/2)/((b*x^2 + a)^(7/4)*sqrt(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{\sqrt{e\,x}\,\left(b\,x^2+a\right)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(7/4)),x)
[Out] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(7/4)), x)
```

3.1124 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{7/4}}\,dx$

Optimal. Leaf size=144

$$-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}}+\frac{4\sqrt{b}\,(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{5/2}e^4\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2/3*c/a/e/(e*x)^(3/2)/(b*x^2+a)^(3/4)+4/3*(-a*d+2*b*c)*(1+a/b/x^2)^(3/4)*(
e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^
(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/
2)/a^(5/2)/e^4/(b*x^2+a)^(3/4)-2/3*(-a*d+2*b*c)*(e*x)^(1/2)/a^2/e^3/(b*x^2+
a)^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {464, 296, 335, 243, 342, 281, 237}

$$\frac{4\sqrt{b}\,(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{5/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{ex}\,(2bc-ad)}{3a^2e^3\left(a+bx^2\right)^{3/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/4)) - (2*(2*b*c - a*d)*Sqrt[e*x])/
(3*a^2*e^3*(a + b*x^2)^(3/4)) + (4*Sqrt[b]*(2*b*c - a*d)*(1 + a/(b*x^2))^(3
/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(5/2)*e^4
*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{(2bc-ad)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{7/4}}\,dx}{ae^2}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} - \frac{(2(2bc-ad))\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{3/4}}}{3a^2e^2}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} - \frac{(4(2bc-ad))\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{e^2}\right)}\right.}{3a^2e^3}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} - \frac{\left(4(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3}\right.}{3a^2e^{}}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} + \frac{\left(4(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3}\right.}{3a^2e}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} + \frac{\left(2(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3}\right.}{3a^2e^3\,(}$$

$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}} - \frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}} + \frac{4\sqrt{b}\,(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(e}{3a^{5/2}e^4\,(a+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 91, normalized size = 0.63

$$\frac{x\left(-2ac-4bcx^2+2adx^2+4(-2bc+ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3a^2(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (x*(-2*a*c - 4*b*c*x^2 + 2*a*d*x^2 + 4*(-2*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(
3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(5/2)*(a
 + b*x^2)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d\,x^{2}+c}{\left(ex\right)^{\frac{5}{2}} \left(b\,x^{2}+a\right)^{\frac{7}{4}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x)
[Out] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] e^(-5/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*x^(5/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(x)*e^(-5/2)/(b^2*x^7 + 2*a*b*x^
5 + a^2*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 59.63, size = 97, normalized size = 0.67

$$\frac{c\Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{7}{4} \\ \frac{1}{4} \end{matrix}\middle| {\frac{bx^{2} e^{i\pi}}{a}} \right)}{2a^{\frac{7}{4}} e^{\frac{5}{2}} x^{\frac{3}{2}} \Gamma\left(\frac{1}{4}\right)} + \frac{d\sqrt{x}\,\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{7}{4} \\ \frac{5}{4} \end{matrix}\middle| {\frac{bx^{2} e^{i\pi}}{a}} \right)}{2a^{\frac{7}{4}} e^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(7/4),x)
[Out] c*gamma(-3/4)*hyper((-3/4, 7/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7
/4)*e**(5/2)*x**(3/2)*gamma(1/4)) + d*sqrt(x)*gamma(1/4)*hyper((1/4, 7/4),
(5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*e**(5/2)*gamma(5/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-5/2)/((b*x^2 + a)^(7/4)*x^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{5/2}\,{\left(b\,x^2+a\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x)
[Out] int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)), x)
```

3.1125 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=181

$$-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}-\frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{4(10bc-7ad)\sqrt[4]{a+bx^2}}{21a^3e^3(ex)^{3/2}}-\frac{8b^{3/2}(10bc-7ad)\left(1+\frac{a}{bx}\right.}{21a^{7}}$$

```
[Out] -2/7*c/a/e/(e*x)^(7/2)/(b*x^2+a)^(3/4)-2/21*(-7*a*d+10*b*c)/a^2/e^3/(e*x)^(
3/2)/(b*x^2+a)^(3/4)+4/21*(-7*a*d+10*b*c)*(b*x^2+a)^(1/4)/a^3/e^3/(e*x)^(3/
2)-8/21*b^(3/2)*(-7*a*d+10*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(3/2)*(cos(1/2*arcc
ot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*Elliptic
F(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/e^6/(b*x^2+a)^(3/4)
```

Rubi [A]

time = 0.09, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {464, 296, 331, 335, 243, 342, 281, 237}

$$-\frac{8b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(10bc-7ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{7/2}e^6\left(a+bx^2\right)^{3/4}}+\frac{4\sqrt[4]{a+bx^2}\,(10bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(3/4)) - (2*(10*b*c - 7*a*d))/(21*a^2
*e^3*(e*x)^(3/2)*(a + b*x^2)^(3/4)) + (4*(10*b*c - 7*a*d)*(a + b*x^2)^(1/4)
)/(21*a^3*e^3*(e*x)^(3/2)) - (8*b^(3/2)*(10*b*c - 7*a*d)*(1 + a/(b*x^2))^(3
/4)*(e*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(7/2)*e^
6*(a + b*x^2)^(3/4))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{7/4}}\,dx = -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{3/4}} - \frac{(10bc-7ad)\int \frac{1}{(ex)^{5/2}(a+bx^2)^{7/4}}\,dx}{7ae^2}$$

$$= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}(a+bx^2)^{3/4}} - \frac{(2(10bc-7ad))\int \frac{}{(ex}}{7a^2e^2}$$

$$= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}(a+bx^2)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+b}}{21a^3e^3(ex)^{3/2}}$$

$$= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}(a+bx^2)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+b}}{21a^3e^3(ex)^{3/2}}$$

$$= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}(a+bx^2)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+b}}{21a^3e^3(ex)^{3/2}}$$

$$= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}(a+bx^2)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+b}}{21a^3e^3(ex)^{3/2}}$$

$$= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}(a+bx^2)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+b}}{21a^3e^3(ex)^{3/2}}$$

$$= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}(a+bx^2)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+b}}{21a^3e^3(ex)^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 82, normalized size = 0.45

$$-\frac{2\sqrt{ex}\left(3ac+(-10bc+7ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(-\frac{3}{4},\frac{7}{4};\frac{1}{4};-\frac{bx^2}{a}\right)\right)}{21a^2e^5x^4(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(3*a*c + (-10*b*c + 7*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*Hypergeo
metric2F1[-3/4, 7/4, 1/4, -((b*x^2)/a)]))/(21*a^2*e^5*x^4*(a + b*x^2)^(3/4)
)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d\,x^{2}+c}{\left(ex\right)^{\frac{9}{2}}\left(b\,x^{2}+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x)
[Out] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] e^(-9/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*x^(9/2)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(x)*e^(-9/2)/(b^2*x^9 + 2*a*b*x^
7 + a^2*x^5), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(7/4),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-9/2)/((b*x^2 + a)^(7/4)*x^(9/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{9/2}\,{\left(b\,x^2+a\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x)
```

```
[Out] int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)), x)
```

3.1126 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=221

$$\frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}}-\frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}}+\frac{(4bc-9ad)e^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}+\frac{(4b$$

[Out] 2/5*(-a*d+b*c)*(e*x)^(9/2)/a/b/e/(b*x^2+a)^(5/4)-1/10*(-9*a*d+4*b*c)*e*(e*x)^(5/2)/a/b^2/(b*x^2+a)^(1/4)+1/4*(-9*a*d+4*b*c)*e^(7/2)*arctan(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(13/4)+1/4*(-9*a*d+4*b*c)*e^(7/2)*arctanh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(13/4)-1/2*(-9*a*d+4*b*c)*e^3*(e*x)^(1/2)/b^3/(b*x^2+a)^(1/4)

Rubi [A]
time = 0.09, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {468, 291, 294, 335, 246, 218, 214, 211}

$$\frac{e^{7/2}(4bc-9ad)\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}+\frac{e^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}-\frac{e^3\sqrt{ex}\,(4bc-9ad)}{2b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{5/2}(4bc-9ad)}{10ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]

[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((4*b*c - 9*a*d)*e^3*Sqrt[e*x])/(2*b^3*(a + b*x^2)^(1/4)) - ((4*b*c - 9*a*d)*e*(e*x)^(5/2))/(10*a*b^2*(a + b*x^2)^(1/4)) + ((4*b*c - 9*a*d)*e^(7/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4)) + ((4*b*c - 9*a*d)*e^(7/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4))

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{\left(2\left(-2bc+\frac{9ad}{2}\right)\right)\int \frac{(ex)^{7/2}}{(a+bx^2)^{5/4}}\,dx}{5ab}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^2\right)\int \frac{(ex)^{3/2}}{(a+bx^2)^{5/4}}\,dx}{4b^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{((4bc-9ad)e^4)}{}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{((4bc-9ad)e^3)}{}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{((4bc-9ad)e^3)}{}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{((4bc-9ad)e^4)}{}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{(4bc-9ad)e^{7/2}}{}$$

Mathematica [A]
time = 1.69, size = 150, normalized size = 0.68

$$\frac{(ex)^{7/2}\left(\frac{2\sqrt[4]{b}\sqrt{x}\left(45a^2d+b^2x^2(-24c+5dx^2)+ab(-20c+54dx^2)\right)}{(a+bx^2)^{5/4}}+5(4bc-9ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+5(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{20b^{13/4}x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] ((e*x)^(7/2)*((2*b^(1/4)*Sqrt[x]*(45*a^2*d + b^2*x^2*(-24*c + 5*d*x^2) + a*
b*(-20*c + 54*d*x^2)))/(a + b*x^2)^(5/4) + 5*(4*b*c - 9*a*d)*ArcTan[(b^(1/4
)*Sqrt[x])/(a + b*x^2)^(1/4)] + 5*(4*b*c - 9*a*d)*ArcTanh[(b^(1/4)*Sqrt[x])
/(a + b*x^2)^(1/4)]))/(20*b^(13/4)*x^(7/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{7}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [A]
time = 0.49, size = 263, normalized size = 1.19

$$-\frac{1}{40}\left(4\,c\left(\frac{4\left(b+\frac{5\,(bx^2+a)}{x^2}\right)x^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}b^2}+\frac{5\left(\frac{2\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}\right)-d\left(\frac{4\left(4\,ab^2+\frac{36\,(bx^2+a)ab}{x^2}-\frac{45\,(bx^2+a)^2a}{x^4}\right)}{\frac{(bx^2+a)^{\frac{5}{4}}b^4}{x^{\frac{5}{2}}}-\frac{(bx^2+a)^{\frac{9}{4}}b^3}{x^{\frac{9}{2}}}}+\frac{45\,a\left(\frac{2\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{1}{4}}}\right)}{b^3}\right)\right)e^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] -1/40*(4*c*(4*(b + 5*(b*x^2 + a)/x^2)*x^(5/2)/((b*x^2 + a)^(5/4)*b^2) + 5*(
2*arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(1/4) + log(-(b^(1/4) - (b*
x^2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(1/4))/b^2
) - d*(4*(4*a*b^2 + 36*(b*x^2 + a)*a*b/x^2 - 45*(b*x^2 + a)^2*a/x^4)/((b*x^
2 + a)^(5/4)*b^4/x^(5/2) - (b*x^2 + a)^(9/4)*b^3/x^(9/2)) + 45*a*(2*arctan(
(b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(1/4) + log(-(b^(1/4) - (b*x^2 + a)^
(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(1/4))/b^3))*e^(7/2
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(156) = 312$.
time = 1.17, size = 959, normalized size = 4.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/40*(4*(5*b^2*d*x^4 - 20*a*b*c + 45*a^2*d - 6*(4*b^2*c - 9*a*b*d)*x^2)*(b*
x^2 + a)^(3/4)*sqrt(x)*e^(7/2) + 20*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*((256
*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 65
61*a^4*d^4)/b^13)^(1/4)*arctan(((4*b^11*c - 9*a*b^10*d)*(b*x^2 + a)^(3/4)*s
qrt(x)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*
b*c*d^3 + 6561*a^4*d^4)/b^13)^(3/4)*e^14 + (b^11*x^2 + a*b^10)*sqrt(((16*b^
2*c^2 - 72*a*b*c*d + 81*a^2*d^2)*sqrt(b*x^2 + a)*x*e^7 + (b^7*x^2 + a*b^6)*
```

```
sqrt((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c
*d^3 + 6561*a^4*d^4)/b^13)*e^7)/(b*x^2 + a))*((256*b^4*c^4 - 2304*a*b^3*c^3
*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)/b^13)^(3/4)*e
^(21/2))*e^(-14)/(256*a*b^4*c^4 - 2304*a^2*b^3*c^3*d + 7776*a^3*b^2*c^2*d^2
 - 11664*a^4*b*c*d^3 + 6561*a^5*d^4 + (256*b^5*c^4 - 2304*a*b^4*c^3*d + 777
6*a^2*b^3*c^2*d^2 - 11664*a^3*b^2*c*d^3 + 6561*a^4*b*d^4)*x^2))*e^(7/2) + 5
*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*
a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)/b^13)^(1/4)*e^(7/2)*log
(-((b*x^2 + a)^(3/4)*(4*b*c - 9*a*d)*sqrt(x)*e^(7/2) + (b^4*x^2 + a*b^3)*((
256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 +
 6561*a^4*d^4)/b^13)^(1/4)*e^(7/2))/(b*x^2 + a)) - 5*(b^5*x^4 + 2*a*b^4*x^2
 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664
*a^3*b*c*d^3 + 6561*a^4*d^4)/b^13)^(1/4)*e^(7/2)*log(-((b*x^2 + a)^(3/4)*(4
*b*c - 9*a*d)*sqrt(x)*e^(7/2) - (b^4*x^2 + a*b^3)*((256*b^4*c^4 - 2304*a*b^
3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)/b^13)^(1
/4)*e^(7/2))/(b*x^2 + a)))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*x^(7/2)*e^(7/2)/(b*x^2 + a)^(9/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x\right)}^{7/2}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
[Out] int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1127 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=149

$$\frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}$$

```
[Out] 2/5*(-a*d+b*c)*(e*x)^(5/2)/a/b/e/(b*x^2+a)^(5/4)+d*e^(3/2)*arctan(b^(1/4)*(
e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(9/4)+d*e^(3/2)*arctanh(b^(1/4)*(e*x)
^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(9/4)-2*d*e*(e*x)^(1/2)/b^2/(b*x^2+a)^(1/
4)
```

Rubi [A]
time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {463, 294, 335, 246, 218, 214, 211}

$$\frac{de^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*d*e*Sqrt[e*x])
/(b^2*(a + b*x^2)^(1/4)) + (d*e^(3/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(
a + b*x^2)^(1/4))])/b^(9/4) + (d*e^(3/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[
e]*(a + b*x^2)^(1/4))])/b^(9/4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{5/2}}{5abe\,(a+bx^2)^{5/4}} + \frac{d\int \frac{(ex)^{3/2}}{(a+bx^2)^{5/4}}\,dx}{b}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\,(a+bx^2)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{(de^2)\int \frac{1}{\sqrt{ex}\,\sqrt[4]{a+bx^2}}\,dx}{b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\,(a+bx^2)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{(2de)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\,(a+bx^2)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{(2de)\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^2}$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\,(a+bx^2)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{(de^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^2} +$$

$$= \frac{2(bc-ad)(ex)^{5/2}}{5abe\,(a+bx^2)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{9/4}} + \frac{de^{3/2}\tanh^{-}}{}$$

Mathematica [A]
time = 1.07, size = 124, normalized size = 0.83

$$\frac{e\sqrt{ex}\left(\frac{2\sqrt[4]{b}\,(-5a^2d+b^2cx^2-6abdx^2)}{a(a+bx^2)^{5/4}} + \frac{5d\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{x}} + \frac{5d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{x}}\right)}{5b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e*Sqrt[e*x]*((2*b^(1/4)*(-5*a^2*d + b^2*c*x^2 - 6*a*b*d*x^2))/(a*(a + b*x^
2)^(5/4)) + (5*d*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)])/Sqrt[x] + (5*
d*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)])/Sqrt[x]))/(5*b^(9/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}\,(d\,x^2+c)}{(b\,x^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [A]
time = 0.49, size = 129, normalized size = 0.87

$$-\frac{1}{10}\left(d\left(\frac{4\left(b+\frac{5\,(bx^2+a)}{x^2}\right)x^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}b^2}+\frac{5\left(\frac{2\arctan\left(\frac{(bx^2+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}\sqrt{x}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}{b^{\frac{1}{4}}+\frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{x}}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}\right)-\frac{4\,cx^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}a}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] -1/10*(d*(4*(b + 5*(b*x^2 + a)/x^2)*x^(5/2)/((b*x^2 + a)^(5/4)*b^2) + 5*(2*
arctan((b*x^2 + a)^(1/4)/(b^(1/4)*sqrt(x)))/b^(1/4) + log(-(b^(1/4) - (b*x^
2 + a)^(1/4)/sqrt(x))/(b^(1/4) + (b*x^2 + a)^(1/4)/sqrt(x)))/b^(1/4))/b^2)
- 4*c*x^(5/2)/((b*x^2 + a)^(5/4)*a))*e^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(99) = 198.
time = 1.98, size = 428, normalized size = 2.87

$$-\frac{4\,(5\,a^2d-(b^2c-6\,abd)x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{x}\,e^{\frac{3}{2}}+20\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)\left(\frac{d^4}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left((bx^2+a)^{\frac{3}{4}}b^7d\sqrt{x}\left(\frac{d^4}{b^9}\right)^{\frac{3}{4}}e^6-(b^8x^2+ab^7)\sqrt{\frac{\sqrt{bx^2+a}\,d^2xe^3+(b^5x^2+ab^4)\sqrt{\frac{d^4}{b^9}}\,e^3}{bx^2+a}}\left(\frac{d^4}{b^9}\right)^{\frac{3}{4}}e^{\frac{9}{2}}\right)e^{(-6)}}{bd^4x^2+ad^4}\right)e^{\frac{3}{2}}-5\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)\left(\frac{d^4}{b^9}\right)^{\frac{1}{4}}e^{\frac{3}{2}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}d\sqrt{x}\,e^{\frac{3}{2}}+(b^3x^2+ab^2)\left(\frac{d^4}{b^9}\right)^{\frac{1}{4}}e^{\frac{3}{2}}}{bx^2+a}\right)+5\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)\left(\frac{d^4}{b^9}\right)^{\frac{1}{4}}e^{\frac{3}{2}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}d\sqrt{x}\,e^{\frac{3}{2}}-(b^3x^2+ab^2)\left(\frac{d^4}{b^9}\right)^{\frac{1}{4}}e^{\frac{3}{2}}}{bx^2+a}\right)}{10\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -1/10*(4*(5*a^2*d - (b^2*c - 6*a*b*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(3/2
) + 20*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4/b^9)^(1/4)*arctan(-((b*x^
```

```
2 + a)^(3/4)*b^7*d*sqrt(x)*(d^4/b^9)^(3/4)*e^6 - (b^8*x^2 + a*b^7)*sqrt((sq
rt(b*x^2 + a)*d^2*x*e^3 + (b^5*x^2 + a*b^4)*sqrt(d^4/b^9)*e^3)/(b*x^2 + a))
*(d^4/b^9)^(3/4)*e^(9/2))*e^(-6)/(b*d^4*x^2 + a*d^4))*e^(3/2) - 5*(a*b^4*x^
4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4/b^9)^(1/4)*e^(3/2)*log(((b*x^2 + a)^(3/4)
*d*sqrt(x)*e^(3/2) + (b^3*x^2 + a*b^2)*(d^4/b^9)^(1/4)*e^(3/2))/(b*x^2 + a)
) + 5*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4/b^9)^(1/4)*e^(3/2)*log(((b
*x^2 + a)^(3/4)*d*sqrt(x)*e^(3/2) - (b^3*x^2 + a*b^2)*(d^4/b^9)^(1/4)*e^(3/
2))/(b*x^2 + a)))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 70.32, size = 116, normalized size = 0.78

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}{2a^{\frac{9}{4}}\sqrt[4]{1+\frac{bx^2}{a}}\,\Gamma\left(\frac{9}{4}\right)+2a^{\frac{5}{4}}bx^2\sqrt[4]{1+\frac{bx^2}{a}}\,\Gamma\left(\frac{9}{4}\right)}+\frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)/(2*a**(9/4)*(1 + b*x**2/a)**(1/4)*gamma(9/4)
 + 2*a**(5/4)*b*x**2*(1 + b*x**2/a)**(1/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2
)*gamma(9/4)*hyper((9/4, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(9/
4)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*x^(3/2)*e^(3/2)/(b*x^2 + a)^(9/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{3/2}\,(d\,x^2+c)}{(b\,x^2+a)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
[Out] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1128 $\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=79

$$\frac{2(bc-ad)\sqrt{ex}}{5abe\,(a+bx^2)^{5/4}}+\frac{2(4bc+ad)\sqrt{ex}}{5a^2be\sqrt[4]{a+bx^2}}$$

```
[Out] 2/5*(-a*d+b*c)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(5/4)+2/5*(a*d+4*b*c)*(e*x)^(1/2
)/a^2/b/e/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {468, 270}

$$\frac{2\sqrt{ex}\,(ad+4bc)}{5a^2be\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}\,(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(5*a*b*e*(a + b*x^2)^(5/4)) + (2*(4*b*c + a*d)*Sq
rt[e*x])/(5*a^2*b*e*(a + b*x^2)^(1/4))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{9/4}}\,dx = \frac{2(bc-ad)\sqrt{ex}}{5abe\,(a+bx^2)^{5/4}} + \frac{\left(2\left(2bc+\frac{ad}{2}\right)\right)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx}{5ab}$$
$$= \frac{2(bc-ad)\sqrt{ex}}{5abe\,(a+bx^2)^{5/4}} + \frac{2(4bc+ad)\sqrt{ex}}{5a^2be\sqrt[4]{a+bx^2}}$$

Mathematica [A]
time = 0.43, size = 44, normalized size = 0.56

$$\frac{2x(5ac+4bcx^2+adx^2)}{5a^2\sqrt{ex}\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
[Out] (2*x*(5*a*c + 4*b*c*x^2 + a*d*x^2))/(5*a^2*Sqrt[e*x]*(a + b*x^2)^(5/4))
```

Maple [A]
time = 0.10, size = 39, normalized size = 0.49

method	result	size
gosper	$\frac{2x(ad\,x^2+4c\,x^2b+5ac)}{5(b\,x^2+a)^{\frac{5}{4}}a^2\sqrt{ex}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x,method=_RETURNVERBOSE)
[Out] 2/5*x*(a*d*x^2+4*b*c*x^2+5*a*c)/(b*x^2+a)^(5/4)/a^2/(e*x)^(1/2)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.68

$$-\frac{2}{5}\left(\frac{\left(b-\frac{5\,(bx^2+a)}{x^2}\right)cx^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}a^2} - \frac{dx^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}a}\right)e^{\left(-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
[Out] -2/5*((b - 5*(b*x^2 + a)/x^2)*c*x^(5/2)/((b*x^2 + a)^(5/4)*a^2) - d*x^(5/2)
/((b*x^2 + a)^(5/4)*a))*e^(-1/2)
```

Fricas [A]
time = 1.04, size = 58, normalized size = 0.73

$$\frac{2\left((4\,bc+ad)x^2+5\,ac\right)(bx^2+a)^{\frac{3}{4}}\sqrt{x}\,e^{\left(-\frac{1}{2}\right)}}{5\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/5*((4*b*c + a*d)*x^2 + 5*a*c)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(-1/2)/(a^2*b^2
*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(71) = 142.
time = 71.81, size = 230, normalized size = 2.91

$$c\left(\frac{5a\Gamma\left(\frac{1}{4}\right)}{8a^3\sqrt[4]{b}\,\sqrt{e}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)+8a^2b^{\frac{5}{4}}\sqrt{e}\,x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}+\frac{4bx^2\Gamma\left(\frac{1}{4}\right)}{8a^3\sqrt[4]{b}\,\sqrt{e}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)+8a^2b^{\frac{5}{4}}\sqrt{e}\,x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}\right)+\frac{d\Gamma\left(\frac{5}{4}\right)}{\frac{2a^2\sqrt[4]{b}\,\sqrt{e}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}{x^2}+2ab^{\frac{5}{4}}\sqrt{e}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] c*(5*a*gamma(1/4)/(8*a**3*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/
4) + 8*a**2*b**(5/4)*sqrt(e)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(9/4)) + 4*b
*x**2*gamma(1/4)/(8*a**3*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4
) + 8*a**2*b**(5/4)*sqrt(e)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(9/4))) + d*g
amma(5/4)/(2*a**2*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4)/x**2
+ 2*a*b**(5/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-1/2)/((b*x^2 + a)^(9/4)*sqrt(x)), x)
```

Mupad [B]
time = 0.65, size = 79, normalized size = 1.00

$$\frac{{\left(b\,x^2+a\right)}^{3/4}\,\left(\frac{x^3\,\left(2\,a\,d+8\,b\,c\right)}{5\,a^2\,b^2}+\frac{2\,c\,x}{a\,b^2}\right)}{x^4\,\sqrt{e\,x}+\frac{a^2\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^2\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(9/4)),x)
```

```
[Out] ((a + b*x^2)^(3/4)*((x^3*(2*a*d + 8*b*c))/(5*a^2*b^2) + (2*c*x)/(a*b^2)))/(
x^4*(e*x)^(1/2) + (a^2*(e*x)^(1/2))/b^2 + (2*a*x^2*(e*x)^(1/2))/b)
```

3.1129 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2(8bc-3ad)\sqrt{ex}}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{8(8bc-3ad)\sqrt{ex}}{15a^3e^3\sqrt[4]{a+bx^2}}$$

```
[Out] -2/3*c/a/e/(e*x)^(3/2)/(b*x^2+a)^(5/4)-2/15*(-3*a*d+8*b*c)*(e*x)^(1/2)/a^2/
e^3/(b*x^2+a)^(5/4)-8/15*(-3*a*d+8*b*c)*(e*x)^(1/2)/a^3/e^3/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 279, 270}

$$-\frac{8\sqrt{ex}\,(8bc-3ad)}{15a^3e^3\sqrt[4]{a+bx^2}}-\frac{2\sqrt{ex}\,(8bc-3ad)}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (2*(8*b*c - 3*a*d)*Sqrt[e*x]
)/(15*a^2*e^3*(a + b*x^2)^(5/4)) - (8*(8*b*c - 3*a*d)*Sqrt[e*x])/(15*a^3*e^
3*(a + b*x^2)^(1/4))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{(8bc-3ad)\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{9/4}}\,dx}{3ae^2}$$
$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{2(8bc-3ad)\sqrt{ex}}{15a^2e^3\left(a+bx^2\right)^{5/4}} - \frac{(4(8bc-3ad))\int \frac{1}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx}{15a^2e^2}$$
$$= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{2(8bc-3ad)\sqrt{ex}}{15a^2e^3\left(a+bx^2\right)^{5/4}} - \frac{8(8bc-3ad)\sqrt{ex}}{15a^3e^3\sqrt[4]{a+bx^2}}$$

Mathematica [A]
time = 0.42, size = 67, normalized size = 0.64

$$\frac{2x\left(-5a^2c-40abcx^2+15a^2dx^2-32b^2cx^4+12abdx^4\right)}{15a^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
[Out] (2*x*(-5*a^2*c - 40*a*b*c*x^2 + 15*a^2*d*x^2 - 32*b^2*c*x^4 + 12*a*b*d*x^4)
)/(15*a^3*(e*x)^(5/2)*(a + b*x^2)^(5/4))
```

Maple [A]
time = 0.12, size = 62, normalized size = 0.60

method	result	size
gosper	$-\frac{2x\left(-12abd\,x^4+32b^2c\,x^4-15a^2d\,x^2+40abc\,x^2+5a^2c\right)}{15\left(b\,x^2+a\right)^{\frac{5}{4}}a^3(ex)^{\frac{5}{2}}}$	62
risch	$-\frac{2c\left(b\,x^2+a\right)^{\frac{3}{4}}}{3a^3x\,e^2\sqrt{ex}}+\frac{2\left(4abd\,x^2-9b^2c\,x^2+5a^2d-10abc\right)x}{5\left(b\,x^2+a\right)^{\frac{5}{4}}a^3e^2\sqrt{ex}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x,method=_RETURNVERBOSE)
[Out] -2/15*x*(-12*a*b*d*x^4+32*b^2*c*x^4-15*a^2*d*x^2+40*a*b*c*x^2+5*a^2*c)/(b*x
^2+a)^(5/4)/a^3/(e*x)^(5/2)
```

Maxima [A]
time = 0.28, size = 91, normalized size = 0.88

$$\frac{2}{15}\left(c\left(\frac{3\left(b^2-\frac{10\,(bx^2+a)b}{x^2}\right)x^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}a^3}-\frac{5\,(bx^2+a)^{\frac{3}{4}}}{a^3x^{\frac{3}{2}}}\right)-\frac{3\left(b-\frac{5\,(bx^2+a)}{x^2}\right)dx^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}a^2}\right)e^{\left(-\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] 2/15*(c*(3*(b^2 - 10*(b*x^2 + a)*b/x^2)*x^(5/2)/((b*x^2 + a)^(5/4)*a^3) - 5
*(b*x^2 + a)^(3/4)/(a^3*x^(3/2))) - 3*(b - 5*(b*x^2 + a)/x^2)*d*x^(5/2)/((b
*x^2 + a)^(5/4)*a^2))*e^(-5/2)
```

Fricas [A]
time = 3.10, size = 86, normalized size = 0.83

$$-\frac{2\,(4\,(8\,b^2c-3\,abd)x^4+5\,a^2c+5\,(8\,abc-3\,a^2d)x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{x}\,e^{\left(-\frac{5}{2}\right)}}{15\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -2/15*(4*(8*b^2*c - 3*a*b*d)*x^4 + 5*a^2*c + 5*(8*a*b*c - 3*a^2*d)*x^2)*(b*
x^2 + a)^(3/4)*sqrt(x)*e^(-5/2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(100) = 200$.
time = 222.43, size = 435, normalized size = 4.18

$$c\left(\frac{5a^2b^{\frac{19}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{32a^5b^4e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)+64a^4b^5e^{\frac{5}{2}}x^2\Gamma\left(\frac{9}{4}\right)+32a^3b^6e^{\frac{5}{2}}x^4\Gamma\left(\frac{9}{4}\right)}+\frac{40ab^{\frac{23}{4}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{32a^5b^4e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)+64a^4b^5e^{\frac{5}{2}}x^2\Gamma\left(\frac{9}{4}\right)+32a^3b^6e^{\frac{5}{2}}x^4\Gamma\left(\frac{9}{4}\right)}+\frac{32b^{\frac{27}{4}}x^4\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{32a^5b^4e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)+64a^4b^5e^{\frac{5}{2}}x^2\Gamma\left(\frac{9}{4}\right)+32a^3b^6e^{\frac{5}{2}}x^4\Gamma\left(\frac{9}{4}\right)}\right)+d\left(\frac{5a\Gamma\left(\frac{1}{4}\right)}{8a^3\sqrt[4]{b}\,e^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)+8a^2b^{\frac{5}{4}}e^{\frac{5}{2}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}+\frac{4bx^2\Gamma\left(\frac{1}{4}\right)}{8a^3\sqrt[4]{b}\,e^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)+8a^2b^{\frac{5}{4}}e^{\frac{5}{2}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] c*(5*a**2*b**(19/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-3/4)/(32*a**5*b**4*e**(5
/2)*gamma(9/4) + 64*a**4*b**5*e**(5/2)*x**2*gamma(9/4) + 32*a**3*b**6*e**(5
/2)*x**4*gamma(9/4)) + 40*a*b**(23/4)*x**2*(a/(b*x**2) + 1)**(3/4)*gamma(-3
/4)/(32*a**5*b**4*e**(5/2)*gamma(9/4) + 64*a**4*b**5*e**(5/2)*x**2*gamma(9/
4) + 32*a**3*b**6*e**(5/2)*x**4*gamma(9/4)) + 32*b**(27/4)*x**4*(a/(b*x**2)
 + 1)**(3/4)*gamma(-3/4)/(32*a**5*b**4*e**(5/2)*gamma(9/4) + 64*a**4*b**5*e
**(5/2)*x**2*gamma(9/4) + 32*a**3*b**6*e**(5/2)*x**4*gamma(9/4))) + d*(5*a*
gamma(1/4)/(8*a**3*b**(1/4)*e**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4) + 8
*a**2*b**(5/4)*e**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(9/4)) + 4*b*x**2
*gamma(1/4)/(8*a**3*b**(1/4)*e**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4) +
8*a**2*b**(5/4)*e**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(9/4)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-5/2)/((b*x^2 + a)^(9/4)*x^(5/2)), x)
```

Mupad [B]
time = 0.71, size = 115, normalized size = 1.11

$$-\frac{(b\,x^2+a)^{3/4}\,\left(\frac{2\,c}{3\,a\,b^2\,e^2}-\frac{x^2\,(30\,a^2\,d-80\,a\,b\,c)}{15\,a^3\,b^2\,e^2}+\frac{x^4\,(64\,b^2\,c-24\,a\,b\,d)}{15\,a^3\,b^2\,e^2}\right)}{x^5\,\sqrt{e\,x}+\frac{2\,a\,x^3\,\sqrt{e\,x}}{b}+\frac{a^2\,x\,\sqrt{e\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(3*a*b^2*e^2) - (x^2*(30*a^2*d - 80*a*b*c))/(15*
a^3*b^2*e^2) + (x^4*(64*b^2*c - 24*a*b*d))/(15*a^3*b^2*e^2)))/(x^5*(e*x)^(1
/2) + (2*a*x^3*(e*x)^(1/2))/b + (a^2*x*(e*x)^(1/2))/b^2)
```

3.1130 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}-\frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}+\frac{64(12bc-7ad)\left(a+bx^2\right)^{3/4}}{105a^4e^3(ex)^{3/2}}$$

```
[Out] -2/7*c/a/e/(e*x)^(7/2)/(b*x^2+a)^(5/4)-2/35*(-7*a*d+12*b*c)/a^2/e^3/(e*x)^(
3/2)/(b*x^2+a)^(5/4)-16/35*(-7*a*d+12*b*c)/a^3/e^3/(e*x)^(3/2)/(b*x^2+a)^(1
/4)+64/105*(-7*a*d+12*b*c)*(b*x^2+a)^(3/4)/a^4/e^3/(e*x)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 279, 270}

$$\frac{64\left(a+bx^2\right)^{3/4}(12bc-7ad)}{105a^4e^3(ex)^{3/2}}-\frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (2*(12*b*c - 7*a*d))/(35*a^2
*e^3*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (16*(12*b*c - 7*a*d))/(35*a^3*e^3*(e*
x)^(3/2)*(a + b*x^2)^(1/4)) + (64*(12*b*c - 7*a*d)*(a + b*x^2)^(3/4))/(105*
a^4*e^3*(e*x)^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{9/4}}\,dx &= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{5/4}} - \frac{(12bc-7ad)\int \frac{1}{(ex)^{5/2}(a+bx^2)^{9/4}}\,dx}{7ae^2} \\
&= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{5/4}} - \frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}(a+bx^2)^{5/4}} - \frac{(8(12bc-7ad))\int \frac{1}{(ex)}}{35a^2e^2} \\
&= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{5/4}} - \frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}(a+bx^2)^{5/4}} - \frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+b}} \\
&= -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{5/4}} - \frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}(a+bx^2)^{5/4}} - \frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+b}}
\end{aligned}$$

Mathematica [A]
time = 0.61, size = 88, normalized size = 0.62

$$-\frac{2x(-384b^3cx^6+32ab^2x^4(-15c+7dx^2)+5a^3(3c+7dx^2)+a^2b(-60cx^2+280dx^4))}{105a^4(ex)^{9/2}(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*x*(-384*b^3*c*x^6 + 32*a*b^2*x^4*(-15*c + 7*d*x^2) + 5*a^3*(3*c + 7*d*x
^2) + a^2*b*(-60*c*x^2 + 280*d*x^4)))/(105*a^4*(e*x)^(9/2)*(a + b*x^2)^(5/4
))
```

Maple [A]
time = 0.12, size = 86, normalized size = 0.61

method	result	size
gosper	$-\frac{2x(224a\,b^2d\,x^6-384b^3c\,x^6+280a^2bd\,x^4-480a\,b^2c\,x^4+35a^3d\,x^2-60a^2bc\,x^2+15c\,a^3)}{105(b\,x^2+a)^{\frac{5}{4}}a^4(ex)^{\frac{9}{2}}}$	86
risch	$-\frac{2(b\,x^2+a)^{\frac{3}{4}}(7ad\,x^2-18c\,x^2b+3ac)}{21a^4x^3e^4\sqrt{ex}}-\frac{2b(9abd\,x^2-14b^2c\,x^2+10a^2d-15abc)x}{5(b\,x^2+a)^{\frac{5}{4}}a^4e^4\sqrt{ex}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*x*(224*a*b^2*d*x^6-384*b^3*c*x^6+280*a^2*b*d*x^4-480*a*b^2*c*x^4+35*
a^3*d*x^2-60*a^2*b*c*x^2+15*a^3*c)/(b*x^2+a)^(5/4)/a^4/(e*x)^(9/2)
```

Maxima [A]
time = 0.31, size = 135, normalized size = 0.96

$$\frac{2}{105}\left(7\,d\left(\frac{3\left(b^2-\frac{10\,(bx^2+a)b}{x^2}\right)x^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}a^3}-\frac{5\,(bx^2+a)^{\frac{3}{4}}}{a^3x^{\frac{3}{2}}}\right)-3\,c\left(\frac{7\left(b^3-\frac{15\,(bx^2+a)b^2}{x^2}\right)x^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}a^4}-\frac{5\left(\frac{7\,(bx^2+a)^{\frac{3}{4}}b}{x^{\frac{3}{2}}}-\frac{(bx^2+a)^{\frac{7}{4}}}{x^{\frac{7}{2}}}\right)}{a^4}\right)\right)e^{\left(-\frac{9}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] 2/105*(7*d*(3*(b^2 - 10*(b*x^2 + a)*b/x^2)*x^(5/2)/((b*x^2 + a)^(5/4)*a^3)
- 5*(b*x^2 + a)^(3/4)/(a^3*x^(3/2))) - 3*c*(7*(b^3 - 15*(b*x^2 + a)*b^2/x^2
)*x^(5/2)/((b*x^2 + a)^(5/4)*a^4) - 5*(7*(b*x^2 + a)^(3/4)*b/x^(3/2) - (b*x
^2 + a)^(7/4)/x^(7/2))/a^4))*e^(-9/2)
```

Fricas [A]
time = 1.30, size = 110, normalized size = 0.78

$$\frac{2\,(32\,(12\,b^3c-7\,ab^2d)x^6+40\,(12\,ab^2c-7\,a^2bd)x^4-15\,a^3c+5\,(12\,a^2bc-7\,a^3d)x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{x}\,e^{\left(-\frac{9}{2}\right)}}{105\,(a^4b^2x^8+2\,a^5bx^6+a^6x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/105*(32*(12*b^3*c - 7*a*b^2*d)*x^6 + 40*(12*a*b^2*c - 7*a^2*b*d)*x^4 - 15
*a^3*c + 5*(12*a^2*b*c - 7*a^3*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(-9/2)/(
a^4*b^2*x^8 + 2*a^5*b*x^6 + a^6*x^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3279 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-9/2)/((b*x^2 + a)^(9/4)*x^(9/2)), x)
```

Mupad [B]
time = 0.69, size = 144, normalized size = 1.02

$$-\frac{\left(b\,x^2+a\right)}^{3/4}\,\left(\frac{2\,c}{7\,a\,b^2\,e^4}+\frac{16\,x^4\,\left(7\,a\,d-12\,b\,c\right)}{21\,a^3\,b\,e^4}+\frac{x^2\,\left(70\,a^3\,d-120\,a^2\,b\,c\right)}{105\,a^4\,b^2\,e^4}-\frac{x^6\,\left(768\,b^3\,c-448\,a\,b^2\,d\right)}{105\,a^4\,b^2\,e^4}\right)}{x^7\,\sqrt{e\,x}+\frac{a^2\,x^3\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^5\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(7*a*b^2*e^4) + (16*x^4*(7*a*d - 12*b*c))/(21*a^
3*b*e^4) + (x^2*(70*a^3*d - 120*a^2*b*c))/(105*a^4*b^2*e^4) - (x^6*(768*b^3
*c - 448*a*b^2*d))/(105*a^4*b^2*e^4)))/(x^7*(e*x)^(1/2) + (a^2*x^3*(e*x)^(1
/2))/b^2 + (2*a*x^5*(e*x)^(1/2))/b)
```

3.1131 $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=178

$$-\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}}-\frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}-\frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}+\frac{64(16bc-11ad)\left(a+bx^2\right.}{55a^4e^3(ex)^{7/2}}$$

```
[Out] -2/11*c/a/e/(e*x)^(11/2)/(b*x^2+a)^(5/4)-2/55*(-11*a*d+16*b*c)/a^2/e^3/(e*x
)^(7/2)/(b*x^2+a)^(5/4)-24/55*(-11*a*d+16*b*c)/a^3/e^3/(e*x)^(7/2)/(b*x^2+a
)^(1/4)+64/55*(-11*a*d+16*b*c)*(b*x^2+a)^(3/4)/a^4/e^3/(e*x)^(7/2)-256/385*
(-11*a*d+16*b*c)*(b*x^2+a)^(7/4)/a^5/e^3/(e*x)^(7/2)
```

Rubi [A]
time = 0.06, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {464, 279, 270}

$$-\frac{256\left(a+bx^2\right)^{7/4}(16bc-11ad)}{385a^5e^3(ex)^{7/2}}+\frac{64\left(a+bx^2\right)^{3/4}(16bc-11ad)}{55a^4e^3(ex)^{7/2}}-\frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(5/4)) - (2*(16*b*c - 11*a*d))/(55*
a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (24*(16*b*c - 11*a*d))/(55*a^3*e^3
*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (64*(16*b*c - 11*a*d)*(a + b*x^2)^(3/4))/
(55*a^4*e^3*(e*x)^(7/2)) - (256*(16*b*c - 11*a*d)*(a + b*x^2)^(7/4))/(385*a
^5*e^3*(e*x)^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{(16bc-11ad)\int \frac{1}{(ex)^{9/2}(a+bx^2)^{9/4}}\,dx}{11ae^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{(12(16bc-11ad))}{55\ldots}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{24(16bc-11a\ldots}{55a^3e^3(ex)^{7/2}\sqrt[4]{a-\ldots}}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{24(16bc-11a\ldots}{55a^3e^3(ex)^{7/2}\sqrt[4]{a-\ldots}}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{24(16bc-11a\ldots}{55a^3e^3(ex)^{7/2}\sqrt[4]{a-\ldots}}$$

Mathematica [A]
time = 1.06, size = 115, normalized size = 0.65

$$-\frac{2x(35a^4c-80a^3bcx^2+55a^4dx^2+320a^2b^2cx^4-220a^3bdx^4+2560ab^3cx^6-1760a^2b^2dx^6+2048b^4cx^8-1408ab^3dx^8)}{385a^5(ex)^{13/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*x*(35*a^4*c - 80*a^3*b*c*x^2 + 55*a^4*d*x^2 + 320*a^2*b^2*c*x^4 - 220*a
^3*b*d*x^4 + 2560*a*b^3*c*x^6 - 1760*a^2*b^2*d*x^6 + 2048*b^4*c*x^8 - 1408*
a*b^3*d*x^8))/(385*a^5*(e*x)^(13/2)*(a + b*x^2)^(5/4))
```

Maple [A]
time = 0.14, size = 110, normalized size = 0.62

method	result	size
gosper	$-\frac{2x\left(-1408a\,b^{3}d\,x^{8}+2048b^{4}c\,x^{8}-1760a^{2}b^{2}d\,x^{6}+2560a\,b^{3}c\,x^{6}-220a^{3}bd\,x^{4}+320a^{2}b^{2}c\,x^{4}+55a^{4}d\,x^{2}-80a^{3}bc\,x^{2}+35c\,a^{4}\right)}{385\left(b\,x^{2}+a\right)^{\frac{5}{4}}a^{5}\left(ex\right)^{\frac{13}{2}}}$	110

risch	$-\frac{2(b\,x^2+a)^{\frac{3}{4}}(-66abd\,x^4+117b^2c\,x^4+11a^2d\,x^2-30abc\,x^2+7a^2c)}{77a^5x^5e^6\sqrt{ex}}+\frac{2b^2(14abd\,x^2-19b^2c\,x^2+15a^2d-20abc)x}{5(b\,x^2+a)^{\frac{5}{4}}a^5e^6\sqrt{ex}}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/385*x*(-1408*a*b^3*d*x^8+2048*b^4*c*x^8-1760*a^2*b^2*d*x^6+2560*a*b^3*c*
x^6-220*a^3*b*d*x^4+320*a^2*b^2*c*x^4+55*a^4*d*x^2-80*a^3*b*c*x^2+35*a^4*c)
/(b*x^2+a)^(5/4)/a^5/(e*x)^(13/2)
```

Maxima [A]
time = 0.29, size = 172, normalized size = 0.97

$$-\frac{2}{385}\left(11\,d\left(\frac{7\left(b^3-\frac{15\,(bx^2+a)b^2}{x^2}\right)x^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}a^4}-\frac{5\left(\frac{7\,(bx^2+a)^{\frac{3}{4}}b}{x^{\frac{3}{2}}}-\frac{(bx^2+a)^{\frac{7}{4}}}{x^{\frac{7}{2}}}\right)}{a^4}\right)-c\left(\frac{77\left(b^4-\frac{20\,(bx^2+a)b^3}{x^2}\right)x^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}a^5}-\frac{5\left(\frac{154\,(bx^2+a)^{\frac{3}{4}}b^2}{x^{\frac{3}{2}}}-\frac{44\,(bx^2+a)^{\frac{7}{4}}b}{x^{\frac{7}{2}}}+\frac{7\,(bx^2+a)^{\frac{11}{4}}}{x^{\frac{11}{2}}}\right)}{a^5}\right)\right)e^{\left(-\frac{13}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] -2/385*(11*d*(7*(b^3 - 15*(b*x^2 + a)*b^2/x^2)*x^(5/2)/((b*x^2 + a)^(5/4)*a
^4) - 5*(7*(b*x^2 + a)^(3/4)*b/x^(3/2) - (b*x^2 + a)^(7/4)/x^(7/2))/a^4) -
c*(77*(b^4 - 20*(b*x^2 + a)*b^3/x^2)*x^(5/2)/((b*x^2 + a)^(5/4)*a^5) - 5*(1
54*(b*x^2 + a)^(3/4)*b^2/x^(3/2) - 44*(b*x^2 + a)^(7/4)*b/x^(7/2) + 7*(b*x^
2 + a)^(11/4)/x^(11/2))/a^5))*e^(-13/2)
```

Fricas [A]
time = 1.30, size = 134, normalized size = 0.75

$$-\frac{2\,(128\,(16\,b^4c-11\,ab^3d)x^8+160\,(16\,ab^3c-11\,a^2b^2d)x^6+35\,a^4c+20\,(16\,a^2b^2c-11\,a^3bd)x^4-5\,(16\,a^3bc-11\,a^4d)x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{x}\,e^{\left(-\frac{13}{2}\right)}}{385\,(a^5b^2x^{10}+2\,a^6bx^8+a^7x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -2/385*(128*(16*b^4*c - 11*a*b^3*d)*x^8 + 160*(16*a*b^3*c - 11*a^2*b^2*d)*x
^6 + 35*a^4*c + 20*(16*a^2*b^2*c - 11*a^3*b*d)*x^4 - 5*(16*a^3*b*c - 11*a^4
*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(-13/2)/(a^5*b^2*x^10 + 2*a^6*b*x^8 +
a^7*x^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7143 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-13/2)/((b*x^2 + a)^(9/4)*x^(13/2)), x)
```

Mupad [B]
time = 0.73, size = 156, normalized size = 0.88

$$\frac{(b\,x^2+a)^{3/4}\left(\frac{64\,x^6\,(11\,a\,d-16\,b\,c)}{77\,a^4\,e^6}-\frac{2\,c}{11\,a\,b^2\,e^6}+\frac{8\,x^4\,(11\,a\,d-16\,b\,c)}{77\,a^3\,b\,e^6}-\frac{x^2\,(110\,a^4\,d-160\,a^3\,b\,c)}{385\,a^5\,b^2\,e^6}+\frac{256\,b\,x^8\,(11\,a\,d-16\,b\,c)}{385\,a^5\,e^6}\right)}{x^9\,\sqrt{e\,x}+\frac{a^2\,x^5\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^7\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x)
```

```
[Out] ((a + b*x^2)^(3/4)*((64*x^6*(11*a*d - 16*b*c))/(77*a^4*e^6) - (2*c)/(11*a*b
^2*e^6) + (8*x^4*(11*a*d - 16*b*c))/(77*a^3*b*e^6) - (x^2*(110*a^4*d - 160*
a^3*b*c))/(385*a^5*b^2*e^6) + (256*b*x^8*(11*a*d - 16*b*c))/(385*a^5*e^6)))
/(x^9*(e*x)^(1/2) + (a^2*x^5*(e*x)^(1/2))/b^2 + (2*a*x^7*(e*x)^(1/2))/b)
```

3.1132 $\int \frac{(ex)^{13/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=230

$$\frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}}+\frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}}-\frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}}-\frac{77a^{3/2}(2bc-$$

```
[Out] 2/5*(-a*d+b*c)*(e*x)^(15/2)/a/b/e/(b*x^2+a)^(5/4)-77/60*a*(-3*a*d+2*b*c)*e^
5*(e*x)^(3/2)/b^4/(b*x^2+a)^(1/4)+11/30*(-3*a*d+2*b*c)*e^3*(e*x)^(7/2)/b^3/
(b*x^2+a)^(1/4)-1/5*(-3*a*d+2*b*c)*e*(e*x)^(11/2)/a/b^2/(b*x^2+a)^(1/4)-77/
20*a^(3/2)*(-3*a*d+2*b*c)*e^6*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcc
ot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/b^(9/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.08, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {468, 291, 290, 342, 202}

$$-\frac{77a^{3/2}e^6\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\,(2bc-3ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20b^{9/2}\sqrt[4]{a+bx^2}}-\frac{77ae^5(ex)^{3/2}(2bc-3ad)}{60b^4\sqrt[4]{a+bx^2}}+\frac{11e^3(ex)^{7/2}(2bc-3ad)}{30b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{11/2}(2bc-3ad)}{5ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{15/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(15/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (77*a*(2*b*c - 3
*a*d)*e^5*(e*x)^(3/2))/(60*b^4*(a + b*x^2)^(1/4)) + (11*(2*b*c - 3*a*d)*e^3
*(e*x)^(7/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((2*b*c - 3*a*d)*e*(e*x)^(11/2))
/(5*a*b^2*(a + b*x^2)^(1/4)) - (77*a^(3/2)*(2*b*c - 3*a*d)*e^6*(1 + a/(b*x^
2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*b^(9/2
)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^{13/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{\left(2\left(-5bc+\frac{15ad}{2}\right)\right)\int \frac{(ex)^{13/2}}{(a+bx^2)^{5/4}}\,dx}{5ab}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}} + \frac{\left(11(2bc-3ad)e^2\right)\int \frac{(ex)^{9/2}}{(a+bx^2)^{5/4}}\,dx}{10b^2}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}} - \frac{(77a(2bc-\ldots}{\ldots}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-\ldots}{5ab\ldots}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-\ldots}{5ab\ldots}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-\ldots}{5ab\ldots}$$

$$= \frac{2(bc-ad)(ex)^{15/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-\ldots}{5ab\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.13, size = 140, normalized size = 0.61

$$\frac{e^5(ex)^{3/2}\left(1155a^3d-110a^2b(7c-3dx^2)+8b^3x^4(5c+3dx^2)-20ab^2x^2(11c+3dx^2)-385a(-2bc+3ad)\left(a+bx^2\right)\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{120b^4\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^5*(e*x)^(3/2)*(1155*a^3*d - 110*a^2*b*(7*c - 3*d*x^2) + 8*b^3*x^4*(5*c +
 3*d*x^2) - 20*a*b^2*x^2*(11*c + 3*d*x^2) - 385*a*(-2*b*c + 3*a*d)*(a + b*x
^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/
(120*b^4*(a + b*x^2)^(5/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{13}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] e^(13/2)*integrate((d*x^2 + c)*x^(13/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^8 + c*x^6)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(13/2)/(b^3*x^6 + 3*a*
b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(13/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4962 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^(13/2)*e^(13/2)/(b*x^2 + a)^(9/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^{13/2}\,(d\,x^2+c)}{(b\,x^2+a)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
[Out] int(((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1133 $\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=192

$$\frac{2(bc-ad)(ex)^{11/2}}{5abe\,(a+bx^2)^{5/4}}+\frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}}-\frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}}+\frac{7\sqrt{a}\,(6bc-11ad)e^4\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E}{10b^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 2/5*(-a*d+b*c)*(e*x)^(11/2)/a/b/e/(b*x^2+a)^(5/4)+7/30*(-11*a*d+6*b*c)*e^3*
(e*x)^(3/2)/b^3/(b*x^2+a)^(1/4)-1/15*(-11*a*d+6*b*c)*e*(e*x)^(7/2)/a/b^2/(b
*x^2+a)^(1/4)+7/10*(-11*a*d+6*b*c)*e^4*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*
b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin
(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(e*x)^(1/2)/b^(7/2)/(b*x^2
+a)^(1/4)
```

Rubi [A]
time = 0.06, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {468, 291, 290, 342, 202}

$$\frac{7\sqrt{a}\,e^4\sqrt{ex}\,\sqrt[4]{\frac{a}{bx^2}+1}\,(6bc-11ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{10b^{7/2}\sqrt[4]{a+bx^2}}+\frac{7e^3(ex)^{3/2}(6bc-11ad)}{30b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{7/2}(6bc-11ad)}{15ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{11/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(11/2))/(5*a*b*e*(a + b*x^2)^(5/4)) + (7*(6*b*c - 11*a
*d)*e^3*(e*x)^(3/2))/(30*b^3*(a + b*x^2)^(1/4)) - ((6*b*c - 11*a*d)*e*(e*x)
^(7/2))/(15*a*b^2*(a + b*x^2)^(1/4)) + (7*Sqrt[a]*(6*b*c - 11*a*d)*e^4*(1 +
 a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(1
0*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{11/2}}{5abe\,(a+bx^2)^{5/4}} + \frac{\left(2\left(-3bc+\frac{11ad}{2}\right)\right)\int \frac{(ex)^{9/2}}{(a+bx^2)^{5/4}}\,dx}{5ab}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\,(a+bx^2)^{5/4}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} + \frac{(7(6bc-11ad)e^2)\int \frac{(ex)^{5/2}}{(a+bx^2)^{5/4}}\,dx}{30b^2}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\,(a+bx^2)^{5/4}} + \frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} - \frac{(7a(6bc-}{}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\,(a+bx^2)^{5/4}} + \frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} - \frac{\Big(7a(6bc}{}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\,(a+bx^2)^{5/4}} + \frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} + \frac{\Big(7a(6bc}{}$$

$$= \frac{2(bc-ad)(ex)^{11/2}}{5abe\,(a+bx^2)^{5/4}} + \frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} + \frac{7\sqrt{a}\,(6b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.12, size = 116, normalized size = 0.60

$$\frac{e^3(ex)^{3/2}\left(-77a^2d+ab(42c-22dx^2)+4b^2x^2(3c+dx^2)+7(-6bc+11ad)\,(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{12b^3\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^3*(e*x)^(3/2)*(-77*a^2*d + a*b*(42*c - 22*d*x^2) + 4*b^2*x^2*(3*c + d*x^
2) + 7*(-6*b*c + 11*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F
1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(12*b^3*(a + b*x^2)^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{9}{2}}\,(d\,x^2+c)}{(b\,x^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
[Out] int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
[Out] e^(9/2)*integrate((d*x^2 + c)*x^(9/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
[Out] integral((d*x^6 + c*x^4)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(9/2)/(b^3*x^6 + 3*a*b
^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*x^(9/2)*e^(9/2)/(b*x^2 + a)^(9/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{9/2}\,(d\,x^2+c)}{(b\,x^2+a)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
[Out] int(((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1134 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=155

$$\frac{2(bc-ad)(ex)^{7/2}}{5abe\,(a+bx^2)^{5/4}}-\frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}}-\frac{3(2bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}\,b^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 2/5*(-a*d+b*c)*(e*x)^(7/2)/a/b/e/(b*x^2+a)^(5/4)-1/5*(-7*a*d+2*b*c)*e*(e*x)
^(3/2)/a/b^2/(b*x^2+a)^(1/4)-3/5*(-7*a*d+2*b*c)*e^2*(1+a/b/x^2)^(1/4)*(cos(
1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*
EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/b^(5/2)/(
b*x^2+a)^(1/4)/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {468, 291, 290, 342, 202}

$$-\frac{3e^2\sqrt{ex}\,\sqrt[4]{\frac{a}{bx^2}+1}\,(2bc-7ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}\,b^{5/2}\sqrt[4]{a+bx^2}}-\frac{e(ex)^{3/2}(2bc-7ad)}{5ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{7/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((2*b*c - 7*a*d)*
e*(e*x)^(3/2))/(5*a*b^2*(a + b*x^2)^(1/4)) - (3*(2*b*c - 7*a*d)*e^2*(1 + a/
(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sq
rt[a]*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 291

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Dist[2*a*c^2*((m - 1)/(
b*(2*m - 3))), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}}+\frac{\left(2\left(-bc+\frac{7ad}{2}\right)\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{5ab}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}}+\frac{\left(3(2bc-7ad)e^2\right)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/4}}\,dx}{10b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}}+\frac{\left(3(2bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\int}{10b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}}-\frac{\left(3(2bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\right)\mathrm{Su}}{10b^3\sqrt[4]{a+bx}}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}}-\frac{3(2bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\,E\left(\frac{1}{2}\right.}{5\sqrt{a}\,b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.10, size = 98, normalized size = 0.63

$$\frac{e(ex)^{3/2}\left(a(-2bc+7ad+2bdx^2)+(2bc-7ad)\left(a+bx^2\right)\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{2ab^2\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e*(e*x)^(3/2)*(a*(-2*b*c + 7*a*d + 2*b*d*x^2) + (2*b*c - 7*a*d)*(a + b*x^2
)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(2
*a*b^2*(a + b*x^2)^(5/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{5}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((d*x^2 + c)*x^(5/2)/(b*x^2 + a)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c*x^2)*(b*x^2 + a)^(3/4)*sqrt(x)*e^(5/2)/(b^3*x^6 + 3*a*b
^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 169.86, size = 94, normalized size = 0.61

$$\frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{9}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{11}{4}\right)}+\frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 9/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(9/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hyper(
(9/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(9/4)*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*x^(5/2)*e^(5/2)/(b*x^2 + a)^(9/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{5/2}\,(d\,x^2+c)}{(b\,x^2+a)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
```

```
[Out] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1135 $\int \frac{\sqrt{ex}\,(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=114

$$\frac{2(bc-ad)(ex)^{3/2}}{5abe\,(a+bx^2)^{5/4}}-\frac{2(2bc+3ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 2/5*(-a*d+b*c)*(e*x)^(3/2)/a/b/e/(b*x^2+a)^(5/4)-2/5*(3*a*d+2*b*c)*(1+a/b/x
^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(
1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^
(1/2)/a^(3/2)/b^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {468, 290, 342, 202}

$$\frac{2(ex)^{3/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}-\frac{2\sqrt{ex}\,\sqrt[4]{\frac{a}{bx^2}+1}\,(3ad+2bc)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*(2*b*c + 3*a*d
)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(5*a^(3/2)*b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ex}\,(c+dx^2)}{(a+bx^2)^{9/4}}\,dx &= \frac{2(bc-ad)(ex)^{3/2}}{5abe\,(a+bx^2)^{5/4}}+\frac{\left(2\left(bc+\frac{3ad}{2}\right)\right)\int\frac{\sqrt{ex}}{(a+bx^2)^{5/4}}\,dx}{5ab}\\ &= \frac{2(bc-ad)(ex)^{3/2}}{5abe\,(a+bx^2)^{5/4}}+\frac{\left(2\left(bc+\frac{3ad}{2}\right)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\right)\int\frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{5ab^2\sqrt[4]{a+bx^2}}\\ &= \frac{2(bc-ad)(ex)^{3/2}}{5abe\,(a+bx^2)^{5/4}}-\frac{\left(2\left(bc+\frac{3ad}{2}\right)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{5ab^2\sqrt[4]{a+bx^2}}\\ &= \frac{2(bc-ad)(ex)^{3/2}}{5abe\,(a+bx^2)^{5/4}}-\frac{2(2bc+3ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.09, size = 86, normalized size = 0.75

$$\frac{x\sqrt{ex}\left(-3a^2d+(2bc+3ad)\,(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a^2b\,(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (x*Sqrt[e*x]*(-3*a^2*d + (2*b*c + 3*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*
Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(3*a^2*b*(a + b*x^2)^(5/4)
)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e x}\, \left(d \,x^{2}+c \right)}{\left(b \,x^{2}+a \right)^{\frac{9}{4}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((d*x^2 + c)*sqrt(x)/(b*x^2 + a)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(x)*e^(1/2)/(b^3*x^6 + 3*a*b^2*x
^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 44.57, size = 94, normalized size = 0.82

$$\frac{c\sqrt{e}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{d\sqrt{e}\,x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{4},\frac{9}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] c*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 9/4), (7/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(9/4)*gamma(7/4)) + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4,
9/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(9/4)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*sqrt(x)*e^(1/2)/(b*x^2 + a)^(9/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
[Out] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1136 $\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=142

$$-\frac{2c}{ae\sqrt{ex}\,\left(a+bx^2\right)^{5/4}}-\frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}}+\frac{4(6bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt{b}\,e^2\sqrt[4]{a+bx^2}}$$

```
[Out] -2/5*(-a*d+6*b*c)*(e*x)^(3/2)/a^2/e^3/(b*x^2+a)^(5/4)-2*c/a/e/(b*x^2+a)^(5/
4)/(e*x)^(1/2)+4/5*(-a*d+6*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*ar
ccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/a^(5/2)/e^2/(b*x^2+a)^(1/4)/b
^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {464, 296, 290, 342, 202}

$$\frac{4\sqrt{ex}\,\sqrt[4]{\frac{a}{bx^2}+1}\,(6bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt{b}\,e^2\sqrt[4]{a+bx^2}}-\frac{2(ex)^{3/2}(6bc-ad)}{5a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{ae\sqrt{ex}\,\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(5/4)) - (2*(6*b*c - a*d)*(e*x)^(3/2))/(5
*a^2*e^3*(a + b*x^2)^(5/4)) + (4*(6*b*c - a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e
*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*Sqrt[b]*e^2*(a
+ b*x^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{ae\sqrt{ex}\,\left(a+bx^2\right)^{5/4}} - \frac{(6bc-ad)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{9/4}}\,dx}{ae^2}$$

$$= -\frac{2c}{ae\sqrt{ex}\,\left(a+bx^2\right)^{5/4}} - \frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}} - \frac{(2(6bc-ad))\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{5/4}}\,dx}{5a^2e^2}$$

$$= -\frac{2c}{ae\sqrt{ex}\,\left(a+bx^2\right)^{5/4}} - \frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}} - \frac{\left(2(6bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\right)}{5a^2be^2\sqrt[4]{a+bx}}$$

$$= -\frac{2c}{ae\sqrt{ex}\,\left(a+bx^2\right)^{5/4}} - \frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}} + \frac{\left(2(6bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\right)}{5a^2be^2\sqrt[4]{}}$$

$$= -\frac{2c}{ae\sqrt{ex}\,\left(a+bx^2\right)^{5/4}} - \frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}} + \frac{4(6bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}\,E\left(\right.}{5a^{5/2}\sqrt{b}\,e^2\sqrt[4]{a-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 85, normalized size = 0.60

$$\frac{2x\left(-3a^2c+(-6bc+ad)x^2(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(-3*a^2*c + (-6*b*c + a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hyper
geometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(3*a^3*(e*x)^(3/2)*(a + b*x^2)^
(5/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{(ex)^{\frac{3}{2}}\left(b\,x^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*x^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(x)*e^(-3/2)/(b^3*x^8 + 3*a*b^2*
x^6 + 3*a^2*b*x^4 + a^3*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 128.49, size = 97, normalized size = 0.68

$$\frac{c\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{9}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] c*gamma(-1/4)*hyper((-1/4, 9/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(9
/4)*e**(3/2)*sqrt(x)*gamma(3/4)) + d*x**(3/2)*gamma(3/4)*hyper((3/4, 9/4),
(7/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(9/4)*e**(3/2)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-3/2)/((b*x^2 + a)^(9/4)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x)
```

```
[Out] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)), x)
```

3.1137 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=181

$$-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}}-\frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}}+\frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{24\sqrt{b}\,(2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{ex}}{5a^{7/2}e^4\sqrt[4]{a+}}$$

```
[Out] -2/5*c/a/e/(e*x)^(5/2)/(b*x^2+a)^(5/4)-2/5*(-a*d+2*b*c)/a^2/e^3/(b*x^2+a)^(
5/4)/(e*x)^(1/2)+12/5*(-a*d+2*b*c)/a^3/e^3/(b*x^2+a)^(1/4)/(e*x)^(1/2)-24/5
*(-a*d+2*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2
)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(
1/2))),2^(1/2))*b^(1/2)*(e*x)^(1/2)/a^(7/2)/e^4/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.06, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 296, 292, 290, 342, 202}

$$-\frac{24\sqrt{b}\,\sqrt{ex}\,\sqrt[4]{\frac{a}{bx^2}+1}\,(2bc-ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{7/2}e^4\sqrt[4]{a+bx^2}}+\frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(5/4)) - (2*(2*b*c - a*d))/(5*a^2*e^3
*Sqrt[e*x]*(a + b*x^2)^(5/4)) + (12*(2*b*c - a*d))/(5*a^3*e^3*Sqrt[e*x]*(a
+ b*x^2)^(1/4)) - (24*Sqrt[b]*(2*b*c - a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]
*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(7/2)*e^4*(a + b*x^2)^(1
/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
 + 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{(2bc-ad)\int \frac{1}{(ex)^{3/2}(a+bx^2)^{9/4}}\,dx}{ae^2}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} - \frac{(6(2bc-ad))\int \frac{1}{(ex)^{3/2}(a+b}}{5a^2e^2}$$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} + \frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}} + $$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} + \frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}} + $$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} + \frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}} - $$

$$= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\left(a+bx^2\right)^{5/4}} + \frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 86, normalized size = 0.48

$$\frac{2x\left(-a^2c-5(-2bc+ad)x^2(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\ {}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{3}{4};-\frac{bx^2}{a}\right)\right)}{5a^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(-(a^2*c) - 5*(-2*b*c + a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hyp
ergeometric2F1[-1/4, 9/4, 3/4, -((b*x^2)/a)]))/(5*a^3*(e*x)^(7/2)*(a + b*x^
2)^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{(ex)^{\frac{7}{2}}\left(b\,x^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x)
[Out] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
[Out] e^(-7/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*x^(7/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(x)*e^(-7/2)/(b^3*x^10 + 3*a*b^2
*x^8 + 3*a^2*b*x^6 + a^3*x^4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(9/4),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*e^(-7/2)/((b*x^2 + a)^(9/4)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x)
[Out] int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)), x)
```

3.1138 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=219

$$-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}-\frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}}+\frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{8b(14bc-9ad)}{15a^4e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\cdots$$

```
[Out] -2/9*c/a/e/(e*x)^(9/2)/(b*x^2+a)^(5/4)-2/45*(-9*a*d+14*b*c)/a^2/e^3/(e*x)^(
5/2)/(b*x^2+a)^(5/4)+4/45*(-9*a*d+14*b*c)/a^3/e^3/(e*x)^(5/2)/(b*x^2+a)^(1/
4)-8/15*b*(-9*a*d+14*b*c)/a^4/e^5/(b*x^2+a)^(1/4)/(e*x)^(1/2)+16/15*b^(3/2)
*(-9*a*d+14*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(
1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/
a^(1/2))),2^(1/2))*(e*x)^(1/2)/a^(9/2)/e^6/(b*x^2+a)^(1/4)
```

Rubi [A]
time = 0.08, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 296, 292, 290, 342, 202}

$$\frac{16b^{3/2}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\,(14bc-9ad)E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{9/2}e^6\sqrt[4]{a+bx^2}}-\frac{8b(14bc-9ad)}{15a^4e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(5/4)) - (2*(14*b*c - 9*a*d))/(45*a^2
*e^3*(e*x)^(5/2)*(a + b*x^2)^(5/4)) + (4*(14*b*c - 9*a*d))/(45*a^3*e^3*(e*x
)^(5/2)*(a + b*x^2)^(1/4)) - (8*b*(14*b*c - 9*a*d))/(15*a^4*e^5*Sqrt[e*x]*(
a + b*x^2)^(1/4)) + (16*b^(3/2)*(14*b*c - 9*a*d)*(1 + a/(b*x^2))^(1/4)*Sqrt
[e*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(9/2)*e^6*(a + b*x
^2)^(1/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 290

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4))), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 292

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[b*((2*m + 1)/(2*a*c^2*(m
 + 1))), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{(14bc-9ad)\int \frac{1}{(ex)^{7/2}(a+bx^2)^{9/4}}\,dx}{9ae^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} - \frac{(2(14bc-9ad))\int \frac{\ }{(e}}{9a^2e^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+b}}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+b}}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+b}}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+b}}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 87, normalized size = 0.40

$$\frac{2x\left(-5a^2c-(-14bc+9ad)x^2(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)\right)}{45a^3(ex)^{11/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(-5*a^2*c - (-14*b*c + 9*a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hy
pergeometric2F1[-5/4, 9/4, -1/4, -((b*x^2)/a)]))/(45*a^3*(e*x)^(11/2)*(a +
b*x^2)^(5/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2+c}{(ex)^{\frac{11}{2}}\left(b\,x^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*x^(11/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(x)*e^(-11/2)/(b^3*x^12 + 3*a*b^
2*x^10 + 3*a^2*b*x^8 + a^3*x^6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4963 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*e^(-11/2)/((b*x^2 + a)^(9/4)*x^(11/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{11/2}\,{\left(b\,x^2+a\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x)
[Out] int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)), x)
```

3.1139 $\int (ex)^m (a+bx^2)^p (c+dx^2)^q \, dx$

Optimal. Leaf size=101

$$\frac{(ex)^{1+m} (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{1+m}{2}; -p, -q; \frac{3+m}{2}; -\frac{bx^2}{a}, -\frac{dx^2}{c}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2+1/2*m,-p,-q,3/2+1/2*m,-b*x
^2/a,-d*x^2/c)/e/(1+m)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\frac{(ex)^{m+1} (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{m+1}{2}; -p, -q; \frac{m+3}{2}; -\frac{bx^2}{a}, -\frac{dx^2}{c}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((e*x)^(1 + m)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 + m)/2, -p, -q, (3 +
 m)/2, -((b*x^2)/a), -((d*x^2)/c)])/(e*(1 + m)*(1 + (b*x^2)/a)^p*(1 + (d*x^
2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (ex)^m \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int (ex)^m \left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q \, dx$$
$$= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int (ex)^m \left(1+\frac{bx^2}{a}\right.$$
$$= \frac{(ex)^{1+m} \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{1+m}{2}; -p, -\right.}{e(1+m)}$$

Mathematica [A]
time = 0.12, size = 97, normalized size = 0.96

$$\frac{x(ex)^m \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{1+m}{2}; -p, -q; \frac{3+m}{2}; -\frac{bx^2}{a}, -\frac{dx^2}{c}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x]
[Out] (x*(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 + m)/2, -p, -q, (3 + m)/
2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + m)*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^
q)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (ex)^m \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x)
[Out] int((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(x*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*(x*e)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**2+a)**p*(d*x**2+c)**q,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(x*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^m\,\left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x)
[Out] int((e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1140 $\int x^4(a+bx^2)^p\,(c+dx^2)^q\,dx$

Optimal. Leaf size=84

$$\frac{1}{5}x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] 1/5*x^5*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(5/2,-p,-q,7/2,-b*x^2/a,-d*x^2/c)/(
(1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{1}{5}x^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{5}{2};-p,-q;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a), -
((d*x^2)/c)])/(5*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int x^4 (a+bx^2)^p (c+dx^2)^q \, dx = \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^4 \left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q \, dx$$

$$= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int x^4 \left(1+\frac{bx^2}{a}\right)^p \left(1\right.$$

$$= \frac{1}{5} x^5 (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{5}{2}; -p, -q; \frac{7}{2}; -\frac{b}{}\right.$$

Mathematica [A]
time = 0.08, size = 86, normalized size = 1.02

$$\frac{1}{5} x^5 (a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(\frac{5}{2}; -p, -q; \frac{7}{2}; -\frac{bx^2}{a}, -\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^p*(c + d*x^2)^q,x]
[Out] (x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a), -
((d*x^2)/c)])/(5*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^4 (b x^2 + a)^p (d x^2 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x)
[Out] int(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

```
[Out] int(x^4*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1141 $\int x^2(a+bx^2)^p\,(c+dx^2)^q\;dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] 1/3*x^3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/(
(1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\frac{1}{3}x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx$$

$$=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\left(\right.$$

$$=\frac{1}{3}x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\right.$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 1.02

$$\frac{1}{3}x^3\left(a+bx^2\right)^p\left(\frac{a+bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^p*(c + d*x^2)^q,x]
[Out] (x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])/(3*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2\left(b\,x^2+a\right)^p\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x)
[Out] int(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

```
[Out] int(x^2*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1142 $\int (a+bx^2)^p (c+dx^2)^q \, dx$

Optimal. Leaf size=79

$$x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/((1+b*x
^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((
d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q\,dx = \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx$$
$$= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right.$$
$$= x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.
time = 0.05, size = 172, normalized size = 2.18

$$\frac{3acx(a+bx^2)^p\left(c+dx^2\right)^qF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3acF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(bcpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (3*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a
), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)
/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^p\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(c + d*x^2)^q,x)
[Out] int((a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1143 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2} dx$

Optimal. Leaf size=82

$$-\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

```
[Out] -(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(-1/2,-p,-q,1/2,-b*x^2/a,-d*x^2/c)/x/((1+b
*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a), -(
(d*x^2)/c)])/(x*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2}\,dx = \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^2}\,dx$$

$$= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right.}{x^2}$$

$$= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx}{c}\right.}{x}$$

Mathematica [A]
time = 0.08, size = 84, normalized size = 1.02

$$-\frac{(a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a), -(
(d*x^2)/c)])/(x*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x^2,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x)
[Out] int(((a + b*x^2)^p*(c + d*x^2)^q)/x^2, x)
```

3.1144 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

```
[Out] -1/3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(-3/2,-p,-q,-1/2,-b*x^2/a,-d*x^2/c)/x^
3/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x]
```

```
[Out] -1/3*((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((b*x^2)/a)
, -((d*x^2)/c)])/(x^3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^4}\,dx = \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^4}\,dx$$

$$= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)}{x^4}$$

$$= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{d}{}\right.}{3x^3}$$

Mathematica [A]
time = 0.10, size = 86, normalized size = 1.02

$$-\frac{(a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x]
[Out] -1/3*((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((b*x^2)/a)
, -((d*x^2)/c)])/(x^3*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p (d\,x^2+c)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x^4,x)
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^4,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^4,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x)
[Out] int(((a + b*x^2)^p*(c + d*x^2)^q)/x^4, x)
```

3.1145 $\int x^5(a+bx^2)^p(c+dx^2)^q\,dx$

Optimal. Leaf size=242

$$-\frac{(bc(2+p)+ad(2+q))(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)}+\frac{x^2(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2bd(3+p+q)}+\frac{(b^2c^2(2+3p+p^2)+2abc}{}$$

```
[Out] -1/2*(b*c*(2+p)+a*d*(2+q))*(b*x^2+a)^(1+p)*(d*x^2+c)^(1+q)/b^2/d^2/(2+p+q)/
(3+p+q)+1/2*x^2*(b*x^2+a)^(1+p)*(d*x^2+c)^(1+q)/b/d/(3+p+q)+1/2*(b^2*c^2*(p
^2+3*p+2)+2*a*b*c*d*(1+p)*(1+q)+a^2*d^2*(q^2+3*q+2))*(b*x^2+a)^(1+p)*(d*x^2
+c)^q*hypergeom([-q, 1+p],[2+p],-d*(b*x^2+a)/(-a*d+b*c))/b^3/d^2/(1+p)/(2+p
+q)/(3+p+q)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

Rubi [A]
time = 0.21, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 92, 81, 72, 71}

$$\frac{(a+bx^2)^{p+1}(c+dx^2)^q(a^2d^2(q^2+3q+2)+2abcd(p+1)(q+1)+b^2c^2(p^2+3p+2))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{d(bx^2+a)}{bc-ad}\right)}{2b^3d^2(p+1)(p+q+2)(p+q+3)}-\frac{(a+bx^2)^{p+1}(c+dx^2)^{q+1}(ad(q+2)+bc(p+2))}{2b^2d^2(p+q+2)(p+q+3)}+\frac{x^2(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{2bd(p+q+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] -1/2*((b*c*(2 + p) + a*d*(2 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/
(b^2*d^2*(2 + p + q)*(3 + p + q)) + (x^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1
 + q))/(2*b*d*(3 + p + q)) + ((b^2*c^2*(2 + 3*p + p^2) + 2*a*b*c*d*(1 + p)*
(1 + q) + a^2*d^2*(2 + 3*q + q^2))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hyperg
eometric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b^3*d^2*(
1 + p)*(2 + p + q)*(3 + p + q)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 (a+bx^2)^p (c+dx^2)^q \, dx = \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p(c+dx)^q \, dx, x, x^2\right)$$

$$= \frac{x^2(a+bx^2)^{1+p} (c+dx^2)^{1+q}}{2bd(3+p+q)} + \frac{\text{Subst}\left(\int (a+bx)^p(c+dx)^q(-ac-(bc(2+}{2bd(3+p+q)}$$

$$= -\frac{(bc(2+p)+ad(2+q)) (a+bx^2)^{1+p} (c+dx^2)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)} + \frac{x^2(a+bx^2)^{1+p} (c+}{2bd(3+p+q}$$

$$= -\frac{(bc(2+p)+ad(2+q)) (a+bx^2)^{1+p} (c+dx^2)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)} + \frac{x^2(a+bx^2)^{1+p} (c+}{2bd(3+p+q}$$

$$= -\frac{(bc(2+p)+ad(2+q)) (a+bx^2)^{1+p} (c+dx^2)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)} + \frac{x^2(a+bx^2)^{1+p} (c+}{2bd(3+p+q}$$

Mathematica [**A**]

time = 0.23, size = 195, normalized size = 0.81

$$\frac{(a+bx^2)^{1+p}(c+dx^2)^q\left(-\frac{(bc(2+p)+ad(2+q))(c+dx^2)}{bd(2+p+q)}+x^2(c+dx^2)+\frac{(b^2c^2(2+3p+p^2)+2abcd(1+p)(1+q)+a^2d^2(2+3q+q^2))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;\frac{d(a+bx^2)}{-bc+ad}\right)}{b^2d(1+p)(2+p+q)}\right)}{2bd(3+p+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*(-(((b*c*(2 + p) + a*d*(2 + q))*(c + d*x
^2))/(b*d*(2 + p + q))) + x^2*(c + d*x^2) + ((b^2*c^2*(2 + 3*p + p^2) + 2*a
*b*c*d*(1 + p)*(1 + q) + a^2*d^2*(2 + 3*q + q^2))*Hypergeometric2F1[1 + p,
-q, 2 + p, (d*(a + b*x^2))/(-(b*c) + a*d)])/(b^2*d*(1 + p)*(2 + p + q)*((b*
(c + d*x^2))/(b*c - a*d))^q)))/(2*b*d*(3 + p + q))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^5\left(b\,x^2+a\right)^p\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,{\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

```
[Out] int(x^5*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1146 $\int x^3(a+bx^2)^p(c+dx^2)^q\,dx$

Optimal. Leaf size=146

$$\frac{(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2bd(2+p+q)}-\frac{(bc(1+p)+ad(1+q))(a+bx^2)^{1+p}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p\right.}{2b^2d(1+p)(2+p+q)}$$

```
[Out] 1/2*(b*x^2+a)^(1+p)*(d*x^2+c)^(1+q)/b/d/(2+p+q)-1/2*(b*c*(1+p)+a*d*(1+q))*(
b*x^2+a)^(1+p)*(d*x^2+c)^q*hypergeom([-q, 1+p],[2+p],-d*(b*x^2+a)/(-a*d+b*c
))/b^2/d/(1+p)/(2+p+q)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

Rubi [A]
time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {457, 81, 72, 71}

$$\frac{(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{2bd(p+q+2)}-\frac{(a+bx^2)^{p+1}(c+dx^2)^q(ad(q+1)+bc(p+1))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{d(bx^2+a)}{bc-ad}\right)}{2b^2d(p+1)(p+q+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(2*b*d*(2 + p + q)) - ((b*c*(1 +
p) + a*d*(1 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p
, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(2*b^2*d*(1 + p)*(2 + p + q)*
((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3(a+bx^2)^p(c+dx^2)^q\,dx = \frac{1}{2}\text{Subst}\left(\int x(a+bx)^p(c+dx)^q\,dx, x, x^2\right)$$
$$= \frac{(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2bd(2+p+q)} - \frac{(bc(1+p)+ad(1+q))\text{Subst}\left(\int(a+bx)^p(c+\right.}{2bd(2+p+q)}$$
$$= \frac{(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2bd(2+p+q)} - \frac{\left((bc(1+p)+ad(1+q))(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right.\right.}{2bd(2}$$
$$= \frac{(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2bd(2+p+q)} - \frac{(bc(1+p)+ad(1+q))(a+bx^2)^{1+p}(c+dx^2}{2b^2d(1+p}$$

Mathematica [A]
time = 0.14, size = 118, normalized size = 0.81

$$\frac{(a+bx^2)^{1+p}(c+dx^2)^q\left(b(c+dx^2)-\frac{(bc(1+p)+ad(1+q))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;\frac{d(a+bx^2)}{-bc+ad}\right)}{1+p}\right)}{2b^2d(2+p+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^p*(c + d*x^2)^q,x]
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*(b*(c + d*x^2) - ((b*c*(1 + p) + a*d*(1
+ q))*Hypergeometric2F1[1 + p, -q, 2 + p, (d*(a + b*x^2))/(-(b*c) + a*d)])/
((1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q)))/(2*b^2*d*(2 + p + q))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^3(b\,x^2+a)^p\,(d\,x^2+c)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p*(c + d*x^2)^q,x)
[Out] int(x^3*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1147 $\int x(a+bx^2)^p (c+dx^2)^q \, dx$

Optimal. Leaf size=85

$$\frac{(a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} {}_2F_1\left(1+p,-q;2+p;-\frac{d(a+bx^2)}{bc-ad}\right)}{2b(1+p)}$$

```
[Out] 1/2*(b*x^2+a)^(1+p)*(d*x^2+c)^q*hypergeom([-q, 1+p],[2+p],-d*(b*x^2+a)/(-a*
d+b*c))/b/(1+p)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {455, 72, 71}

$$\frac{(a+bx^2)^{p+1} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} {}_2F_1\left(p+1,-q;p+2;-\frac{d(bx^2+a)}{bc-ad}\right)}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((d
*(a + b*x^2))/(b*c - a*d))])/(2*b*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rubi steps

$$\int x\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx=\frac{1}{2}\text{Subst}\left(\int(a+bx)^p(c+dx)^q\,dx,x,x^2\right)$$
$$=\frac{1}{2}\left(\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)\right)$$
$$=\frac{\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;-\frac{d(a+bx^2)}{bc-ad}\right)}{2b(1+p)}$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 0.99

$$\frac{\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;\frac{d(a+bx^2)}{-bc+ad}\right)}{2b(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^p*(c + d*x^2)^q,x]
[Out] ((a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, (d*(
a + b*x^2))/(-(b*c) + a*d)])/(2*b*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(b\,x^2+a\right)^p\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p*(d*x^2+c)^q,x)
[Out] int(x*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p*(d*x**2+c)**q,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p*(c + d*x^2)^q,x)
[Out] int(x*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1148 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{x} \, dx$

Optimal. Leaf size=97

$$-\frac{(a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,1;2+p;-\frac{d(a+bx^2)}{bc-ad},\frac{a+bx^2}{a}\right)}{2a(1+p)}$$

```
[Out] -1/2*(b*x^2+a)^(1+p)*(d*x^2+c)^q*AppellF1(1+p,1,-q,2+p,(b*x^2+a)/a,-d*(b*x^
2+a)/(-a*d+b*c))/a/(1+p)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {457, 142, 141}

$$-\frac{(a+bx^2)^{p+1} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(p+1;-q,1;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x,x]
```

```
[Out] -1/2*((a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*
(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(a*(1 + p)*((b*(c + d*x^2))/(b*c
 - a*d))^q)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x} dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x} dx, x, x^2\right)$$

$$= \frac{1}{2}\Bigg((c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \Bigg) \text{Subst}\Bigg(\int \frac{(a+bx)^p \left(\frac{bc}{bc-ad} + \frac{bdx}{bc-ad}\right)^q}{x} dx, x, x^2$$

$$= -\frac{(a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\Big(1+p; -q, 1; 2+p; -\frac{d(a+bx^2)}{bc-ad}, \frac{a+bx^2}{a}\Big)}{2a(1+p)}$$

Mathematica [A]
time = 0.08, size = 95, normalized size = 0.98

$$\frac{\left(1+\frac{a}{bx^2}\right)^{-p} \left(1+\frac{c}{dx^2}\right)^{-q} (a+bx^2)^p (c+dx^2)^q F_1\left(-p-q; -p, -q; 1-p-q; -\frac{a}{bx^2}, -\frac{c}{dx^2}\right)}{2(p+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x,x]
```

```
[Out] ((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -(a/(b*x^2
)), -(c/(d*x^2))])/(2*(p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x,x)
[Out] Integral((a + b*x**2)**p*(c + d*x**2)**q/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(c + d*x^2)^q)/x,x)
[Out] int(((a + b*x^2)^p*(c + d*x^2)^q)/x, x)
```

3.1149 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^3}\,dx$

Optimal. Leaf size=98

$$\frac{b(a+bx^2)^{1+p}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,2;2+p;-\frac{d(a+bx^2)}{bc-ad},\frac{a+bx^2}{a}\right)}{2a^2(1+p)}$$

```
[Out] 1/2*b*(b*x^2+a)^(1+p)*(d*x^2+c)^q*AppellF1(1+p,2,-q,2+p,(b*x^2+a)/a,-d*(b*x
^2+a)/(-a*d+b*c))/a^2/(1+p)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {457, 142, 141}

$$\frac{b(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a
+ b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^2*(1 + p)*((b*(c + d*x^2))/(b*
c - a*d))^q)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\left((c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,dx, x, x\right.$$
$$= \frac{b(a+bx^2)^{1+p}\,(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,2;2+p;-\frac{d(a+bx^2)}{bc-ad},\frac{a+bx^2}{a}\right.}{2a^2(1+p)}$$

Mathematica [**A**]
time = 0.09, size = 100, normalized size = 1.02

$$\frac{\left(1+\frac{a}{bx^2}\right)^{-p}\left(1+\frac{c}{dx^2}\right)^{-q}(a+bx^2)^p\,(c+dx^2)^q\,F_1\left(1-p-q;-p,-q;2-p-q;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{2(-1+p+q)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x]
```

```
[Out] ((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1 - p - q, -p, -q, 2 - p - q, -(a/(b*
x^2)), -(c/(d*x^2))])/(2*(-1 + p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q*x
^2)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x^3,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x)
```

```
[Out] int(((a + b*x^2)^p*(c + d*x^2)^q)/x^3, x)
```

3.1150 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^5} dx$

Optimal. Leaf size=100

$$-\frac{b^2(a+bx^2)^{1+p}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,3;2+p;-\frac{d(a+bx^2)}{bc-ad},\frac{a+bx^2}{a}\right)}{2a^3(1+p)}$$

```
[Out] -1/2*b^2*(b*x^2+a)^(1+p)*(d*x^2+c)^q*AppellF1(1+p,3,-q,2+p,(b*x^2+a)/a,-d*(
b*x^2+a)/(-a*d+b*c))/a^3/(1+p)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {457, 142, 141}

$$-\frac{b^2(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(p+1;-q,3;p+2;-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x]
```

```
[Out] -1/2*(b^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -
((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(a^3*(1 + p)*((b*(c + d*x^2)
)/(b*c - a*d))^q)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\left(\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^3}\,dx, x, x^2$$

$$= -\frac{b^2(a+bx^2)^{1+p}\,(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,3;2+p;-\frac{d(a+bx^2)}{bc-ad},\frac{a+bx^2}{a}\right.}{2a^3(1+p)}$$

Mathematica [A]
time = 0.11, size = 100, normalized size = 1.00

$$\frac{\left(1+\frac{a}{bx^2}\right)^{-p}\left(1+\frac{c}{dx^2}\right)^{-q}(a+bx^2)^p\,(c+dx^2)^q\,F_1\left(2-p-q;-p,-q;3-p-q;-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{2(-2+p+q)x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x]
[Out] ((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[2 - p - q, -p, -q, 3 - p - q, -(a/(b*
x^2)), -(c/(d*x^2))])/(2*(-2 + p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q*x
^4)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/x^5,x)
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/x**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x)
```

```
[Out] int(((a + b*x^2)^p*(c + d*x^2)^q)/x^5, x)
```

3.1151 $\int (ex)^{5/2} (a+bx^2)^p (c+dx^2)^q \, dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{7/2} (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}$$

```
[Out] 2/7*(e*x)^(7/2)*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(7/4,-p,-q,11/4,-b*x^2/a,-d
*x^2/c)/e/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{2(ex)^{7/2} (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{7}{4};-p,-q;\frac{11}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*(e*x)^(7/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -p, -q, 11/4, -((b
*x^2)/a), -((d*x^2)/c)])/(7*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int (ex)^{5/2} \left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q \, dx$$

$$= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int (ex)^{5/2} \left(1+\right.$$

$$= \frac{2(ex)^{7/2} \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{7}{4}; -p, -q;\right.}{7e}$$

Mathematica [A]
time = 0.19, size = 91, normalized size = 1.00

$$\frac{2}{7} x (ex)^{5/2} \left(a+bx^2\right)^p \left(\frac{a+bx^2}{a}\right)^{-p} \left(c+dx^2\right)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(\frac{7}{4}; -p, -q; \frac{11}{4}; -\frac{bx^2}{a}, -\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
[Out] (2*x*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -p, -q, 11/4, -(
(b*x^2)/a), -((d*x^2)/c)])/(7*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (ex)^{\frac{5}{2}} \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
[Out] int((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
[Out] e^(5/2)*integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^(5/2)*e^(5/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{5/2}\,{\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^q \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x)
[Out] int((e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1152 $\int (ex)^{3/2} (a+bx^2)^p (c+dx^2)^q \, dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{5/2} (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}$$

```
[Out] 2/5*(e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(5/4,-p,-q,9/4,-b*x^2/a,-d*
x^2/c)/e/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]

time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{2(ex)^{5/2} (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} F_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -p, -q, 9/4, -((b*
x^2)/a), -((d*x^2)/c)])/(5*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int (ex)^{3/2}\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx$$
$$=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int (ex)^{3/2}\left(1+\frac{b}{\ }\right.$$
$$=\frac{2(ex)^{5/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{5}{4};-p,-q;\frac{9}{4}\right.}{5e}$$

Mathematica [**A**]
time = 0.16, size = 91, normalized size = 1.00

$$\frac{2}{5}x(ex)^{3/2}\left(a+bx^2\right)^p\left(\frac{a+bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(\frac{5}{4};-p,-q;\frac{9}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
[Out] (2*x*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -p, -q, 9/4, -((
b*x^2)/a), -((d*x^2)/c)])/(5*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int (ex)^{\frac{3}{2}}\left(b\,x^2+a\right)^p\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
[Out] int((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
[Out] e^(3/2)*integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^(3/2)*e^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{3/2}\,{\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x)
[Out] int((e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1153 $\int \sqrt{ex}\,(a+bx^2)^p\,(c+dx^2)^q\,dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{3/2}\,(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}$$

```
[Out] 2/3*(e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/4,-p,-q,7/4,-b*x^2/a,-d*
x^2/c)/e/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{2(ex)^{3/2}\,(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p, -q, 7/4, -((b*
x^2)/a), -((d*x^2)/c)])/(3*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx = \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \sqrt{ex}\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx$$
$$= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int \sqrt{ex}\left(1+\frac{bx^2}{a}\right.$$
$$= \frac{2(ex)^{3/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4}\right.}{3e}$$

Mathematica [A]
time = 0.11, size = 91, normalized size = 1.00

$$\frac{2}{3}x\sqrt{ex}\left(a+bx^2\right)^p\left(\frac{a+bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (2*x*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p, -q, 7/4, -((b*
x^2)/a), -((d*x^2)/c)])/(3*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{ex}\,\left(b\,x^2+a\right)^p\left(d\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*x^2 + a)^p*(d*x^2 + c)^q*sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*sqrt(x)*e^(1/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*sqrt(x)*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,x}\,{\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(a + b*x^2)^p*(c + d*x^2)^q,x)
[Out] int((e*x)^(1/2)*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1154 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{\sqrt{ex}}\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{ex}\,(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}$$

```
[Out] 2*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/4,-p,-q,5/4,-b*x^2/a,-d*x^2/c)*(e*x)^(
1/2)/e/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\frac{2\sqrt{ex}\,(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (2*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^
2)/a), -((d*x^2)/c)])/(e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{\sqrt{ex}} dx = \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{\sqrt{ex}} dx$$

$$= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)}{\sqrt{ex}}$$

$$= \frac{2\sqrt{ex}\,(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^2}{a}, -\right.}{e}$$

Mathematica [A]
time = 0.11, size = 89, normalized size = 1.00

$$\frac{2x(a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^2}{a}, -\frac{dx^2}{c}\right)}{\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^2)/a), -
((d*x^2)/c)])/(Sqrt[e*x]*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*e^(-1/2)/sqrt(x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*e^(-1/2)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(1/2),x)
[Out] int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(1/2), x)
```

3.1155 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}$$

```
[Out] -2*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(-1/4,-p,-q,3/4,-b*x^2/a,-d*x^2/c)/e/((1
+b*x^2/a)^p)/((1+d*x^2/c)^q)/(e*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$-\frac{2(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4, -((b*x^2)/a), -
((d*x^2)/c)])/(e*Sqrt[e*x]*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{3/2}}\,dx = \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{(ex)^{3/2}}\,dx$$

$$= \left((a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right.}{(ex)^{3/2}}$$

$$= -\frac{2(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\right.}{e\sqrt{ex}}$$

Mathematica [A]
time = 0.17, size = 89, normalized size = 1.00

$$-\frac{2x(a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x]
[Out] (-2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4, -((b*x^2)/a),
 -((d*x^2)/c)])/((e*x)^(3/2)*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x)
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*e^(-3/2)/x^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*e^(-3/2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{(e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x)
[Out] int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2), x)
```

3.1156 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e(ex)^{3/2}}$$

```
[Out] -2/3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(-3/4,-p,-q,1/4,-b*x^2/a,-d*x^2/c)/e/(
e*x)^(3/2)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$-\frac{2(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4, -((b*x^2)/a), -
((d*x^2)/c)])/(3*e*(e*x)^(3/2)*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{(ex)^{5/2}}\,dx = \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p(c+dx^2)^q}{(ex)^{5/2}}\,dx$$

$$= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)}{(ex)^{5/2}}$$

$$= -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx}{c}\right.}{3e(ex)^{3/2}}$$

Mathematica [A]
time = 0.19, size = 91, normalized size = 1.00

$$-\frac{2x(a+bx^2)^p\left(\frac{a+bx^2}{a}\right)^{-p}(c+dx^2)^q\left(\frac{c+dx^2}{c}\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4, -((b*x^2)/a),
 -((d*x^2)/c)])/(3*(e*x)^(5/2)*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q*e^(-5/2)/x^(5/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q*e^(-5/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{(e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x)
[Out] int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)  #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)  #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)  #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```